CAHIER DE RECHERCHE ♯1101E
Département de science économique
Faculté des sciences sociales
Université d'Ottawa

WORKING PAPER ♯1101E
Department of Economics
Faculty of Social Sciences
University of Ottawa

A Fuzzy Approach to the Measurement of Leakages for North American Health Systems

Paul Makdissi[*], Myra Yazbeck[†] and Hugo Coldeboeuf[‡]

February 2011

[*]CIRPÉE, ERF and Department of Economics, University of Ottawa, 55 Laurier E. (10125), Ottawa, Ontario, Canada, K1N 6N5; Email: paul.makdissi@uottawa.ca.
[†] CIRPÉE and Department of Epidemiology Biostatistics and Occupational Health, Purvis Hall, McGill University, Montreal, Canada, H3A 1A2; Email: myra.yazbeck@mcgill.ca
[‡] Department of Economics, University of Ottawa, 55 Laurier E. (10125), Ottawa, Ontario, Canada, K1N 6N5; Email: hcold094@uottawa.ca.

Abstract

This paper uses a fuzzy-fuzzy stochastic dominance approach to compare patients' leakages in the Canadian and the U.S health care systems. Leakages are defined in terms of individuals who are in bad health and could not have access to health care when needed. To carry this comparison we rely on the assumption that Canada is a strong counterfactual for the U.S. We first develop a class of fuzzy leakages indices and incorporate them in a stochastic dominance framework to derive the dominance criterion. We then use the derived criterion to perform inter-country comparisons on the global level. To provide more insight, we decompose the analysis with respect to gender, ethnicity, income and education. Intra-country comparisons reveal the presence of income based leakage inequalities in both countries yet, gender, ethnic and education based disparities appear to be present in the U.S only. As for inter-country comparison, results are in general consistent with the hypothesis that leakages are less important under the Canadian health care system.

Key words: *Health care resources, Fuzzy sets, Leakage*

JEL Classification: D63, I18, I19

Résumé

Cet article propose une approche de dominance stochastique doublement floue afin de comparer les fuites dans les systèmes de santé au Canada et aux États-Unis. Les fuites sont définies en terme d'individus en mauvaise santé et ayant des problèmes d'accès aux soins lorsqu'ils en ont besoin. Afin d'effectuer cette comparaison nous faisons l'hypothèse que le Canada constitue une observation contrefactuelle pour les Etats-Unis. Dans une première étape, nous développons une classe d'indices flous de fuite et nous les incorporons dans un cadre de dominance stochastique afin de développer les conditions de dominance. Par la suite, nous utilisons ces conditions de dominance afin d'effectuer les comparaisons internationales au niveau de l'ensemble de la population. Afin d'avoir un meilleur aperçu de la situation, nous décomposons les comparaisons selon le genre, l'ethnicité, le revenu et le niveau d'éducation. Les comparaisons nationales révèlent la présence de fuites liées au revenu dans les deux pays alors que les fuites liées au genre, à l'ethnie ou au niveau d'éducation ne sont présentes qu'aux États-Unis seulement. Les comparaisons internationales indiquent que les fuites sont généralement moins importantes au Canada.

Mots clés: *Ressources pour soins de santé, ensembles flous, fuites*

Classification JEL: D63, I18, I19

1 Introduction

The debate on whether or not the United States should have a universal health care system has been open for decades and was revived with the late congressional elections. The U.S. and Canada share a lot of similarities yet, they maintained (for a long period) different health policies. These similarities and the geographical proximity have provided health economists with a quasi-natural experiment environment in which Canada appears to be a strong counterfactual for the U.S. In addition, the implementation of the Joint Canada U.S. Survey of Health (JCUSH) provides comparable data that sets the ground for the development of a precious literature on the comparison of the two health care systems.

The literature that dealt with the comparison of the Canadian and the American health care systems is still inconclusive. While a lot of studies seem to be in favour of the relative performance of the Canadian system, another body of the literature suggests that the instruments used and the methods utilized might be driving these results (Murray, 2002; Murray and Evans, 2003; Guyatt et al., 2007; O'Neill and O'Neill, 2007).[1] In addition, despite the presence of a diversified literature that deals with inter/intra-country comparisons of health outcomes and access to health services, to our best knowledge, there are a no studies that tackle these two dimensions simultaneously.[2]

The goal of this paper is to develop a "fuzzy-fuzzy" stochastic dominance approach to compare the extent to which the Canadian and the US health care systems are failing to provide access to vulnerable individuals who are in need for treatment (leakages).[3] Two reasons justify our interest in assessing leakages in these health care systems. First, ensuring adequate access to health care *per se* is an important policy objective. By focusing on individuals who leak from the health care system we study a basic question: unmet health care needs. Second, the recent implementation of the universal health care system is a raising public concerns around the possible negative repercussions for the U.S. namely: increased accessibility

[1] See also Alegria et al. (2000), Blendon et al. (2002), Schoen and Doty (2004), Sanmartin et al. (2006), Van Doorslaer et al. (2006), Eng and Feeny (2007), Katz and Hofer (1994), O'Neill and O'Neill (2007), Feeny et al. (2010), Guyatt et al. (2007), Evans and Ross (1999), Lasser et al. (2006), Ross et al. (2001),Manuel and Mao (2002) and Kunitz and Pesis-Katz (2005).
[2] See the literature review for more details
[3] Note that leakage refers to the fact that people who are not in perfect health, are in need for treatment and do not have access to it for reasons that are not related to personal choice or preference.

hurdles.[4] This controversy was reinforced by the accessibility problems often associated with the presence of a universal health care system in Canada.

To carry this comparison we rely on the assumption that Canada is a strong counterfactual for the United States. We first classify individuals according to their health status using Health Utility Index Mark3 (HUI3) in a fuzzy set framework. Thus, for all individual, we specify a gradual transition between health statuses through a membership function.[5] Second, we incorporate the specified membership function in a stochastic dominance framework and derive the stochastic dominance criterion using a "fuzzy-fuzzy" approach. To provide more insight, we decompose the stochastic dominance analysis by gender, ethnicity, income and education.

Comparing the two health care systems raises several challenges. On the methodological level, the linguistic nature of the health status (HUI3) variable implies that this variables has unclear boundaries. In fact, the HUI3 is continuous but its values can still be associated with linguistic variables whose frontiers are mistakenly defined as crisp. If the transition from one health status to another does not occur gradually, then people who are located at the margin of the cut-off point are likely to be classified into two different sets despite their strong similarities. To address this issue we use a fuzzy set framework (Zadeh, 1965) and specify a membership function that allows for gradual transitions.[6] A gradual transition imposes fuzziness on the health status variable that mimics better the reality of this non-sharply defined linguistic variable.[7] Nevertheless, the use of Zadeh's fuzzy set approach does not come without a cost: the choice of the parametric form of the membership function is quite arbitrary. Thus the results obtained may be contingent to the specific mathematical form specified. This is why we incorporate the membership function in a stochastic dominance framework and derive a stochastic dominance condition. The use of the stochastic dominance approach mutes the impact of the structure imposed on the membership function

[4]We acknowledge that only part of the U.S citizens are covered by private insurance. In this paper we don't make the distinction between insured and uninsured U.S. citizens.

[5]There is a wide variety of health measures. In this section we use a continuous preference based measure of Health Related Quality of Life: HUI3 and build a membership function using the equivalences provided in Humphries and Van Doorslaer (2000).

[6]The intuition behind this approach is that an individual can have a certain degree of non mutually exclusive belongingness to a specific state.

[7]For instance, the same individual can be considered as being to some extent in good health or to some extent in bad health instead of either in good or bad health.

and produces results that are robust to any parametric membership function.[8]

In addition, from an empirical perspective, the choice of the comparison criteria is a delicate task. The accessibility measures utilized in the literature did not account for health status. Furthermore, the most frequently used health related performance measures (i.e., infant mortality and life expectancy) were often critiqued as they pick up effects that are unrelated to the health care system's quality (or accessibility) but rather the differences in health endowments, lifestyle as well as differences in environmental factors that may be endogenous to the health care system itself.[9] Consequently, it is difficult to disentangle whether the relative performance of the health care system is a cause or a consequence of the poor health outcome. While this paper does not address directly this issue, it gets around it by focusing on a specific measure (leakages) that allows us to abstract from initial health endowments and lifestyle while accounting for health status.[10]

In this study we use the HUI3 as well as information regarding unmet health care needs from JCUSH to compare the population's accessibility hurdles for those who are in need for it in Canada and the U.S. A particular advantage of this survey resides in the availability of the same variables for both countries: the variables were collected in the same way during the same period. The implementation of this survey reduced the limitations that previous studies encountered while using of variables from different surveys to conduct inter-country comparisons. The stochastic dominance results show that on a global level the Canadian health care system has fewer leakages than the American system. Given the importance of the income-health gradient and gender health disparities reported in the literature, we tested wether such a gradient and disparities exist within and between the two countries. In addition, since racial inequalities are quite prominent in the United States we checked whether there are inter/intra-country differences in leakages for the non white population. Intra-country comparisons results are consistent with the hypothesis of the presence of income based leakage disparities for both countries yet, gender,

[8]This holds for any monotonically decreasing membership form.

[9]Note that obesity rates, low birthweight (teen pregnancy), and non disease death rates are more prominent in the U.S than in Canada.

[10]It is important to note that in this paper we focus on a very specific population: individual who are to a certain extent in bad health (i.e., not in perfect health) and who have expressed that they did not have access to health care because of reasons that are not related to personal preferences or to personal choice.

ethnic, and education based inequalities appear to be present exclusively in the U.S. Further inter-country socio-demographic and socio-economic comparisons show that disparities in general are stronger in the U.S.

The remainder of the paper runs as follow. In the next section, we provide a brief review of the related literature. In section three, we build a class of fuzzy leakage indices and develop stochastic dominance conditions to test wether a ranking of two health systems will be unanimous for all indices. The fourth section applies this methodology to the comparison of the Canadian and U.S. health systems using JCUSH data. The last section of the paper concludes.

2 Review of Literature

This paper addresses the following question: which health care systems is better at providing access to individuals in poor health who are in need for treatment? It also uses an original approach to stochastic dominance namely: a *fuzzy-fuzzy* approach. While the question addressed is empirical in nature, it is important to note that it requires the development of a theoretical framework as well as specific stochastic dominance criterion on which the comparisons will be based.

In the economic literature, fuzzy set theory was extensively used in poverty measurement. It was initiated by Cerioli and Zani (1990) who were the first to point out that the transition between a state of deprivation to a state of non deprivation is gradual. In addition, Chiappero Martinetti (1994) pointed that fuzzy set approach is appropriate in the context of mutli-dimensional poverty analysis.[11] This is why most of this literature was developed mainly in the context of multi-dimentional poverty measurement.[12] Yet, the use of fuzzy set theory was not limited in the field of poverty measurement, Bérenger and Verdier-Chouchane (2007) apply it to the measures of well being such as human development indices (HDI), standards of Living (SL) and quality of Life (QL) and provide comparisons for 170 countries. While our paper is broadly inspired from this study, it remains quite distinctive from it in respect to the methodology (stochastic dominance), the measure used (HUI3) and the empirical question addressed

[11] See also, Chiappero Martinetti (1996), Martinetti (2000).

[12] Further extensions were carried to this approach by Cerioli and Zani (1990) and Cheli (1995) among others and applied by Lelli (2001), Mussard and Pi Alperin (2007) Qizilbash (2002) and Qizilbash (2003).

(leakages).

Also, there is a line of research in epidemiology applying fuzzy logic to risk estimators, decision making, probabilities (for details see Massad et al., 2003) and health status variables (Sadegh-Zadeh, 1999; Costa et al., 2004). In fact, disease diagnosis implies many levels of imprecisions and uncertainties since most diseases' description is based on linguistic terms that are often vague. The use of fuzzy set approach makes health and presence of disease complementary (to a certain extent) rather than dual (Sadegh-Zadeh, 1999). In addition, it provides gradual transition between one state and another gradual which represents better health status variables such as quality of life (Costa et al., 2004). While there is a body of literature in epidemiology that uses fuzzy logic, to our best knowledge no study incorporates it in a stochastic dominance framework.[13]

As for the literature that compares the relative performance of the Canadian and U.S. health care system most of it mainly tackled mainly two aspects: (1) health outcomes (2) access to health care.[14] The studies that focused on inter-country comparisons of population health outcomes mainly used self reported health status and health related quality of life measures but no fuzzification was performed to account for gradual transitions. Empirical evidence using these different measures reveal that on average there are no significant differences between the American and the Canadian population health. Nevertheless, there are differences in population health outcomes based on race and ethnicity and income groups. Sanmartin et al. (2006) using self reported health status pointed that health status is fairly similar in both countries yet there are higher income inequalities in the US. Similarly, Eng and Feeny (2007) using HUI3 found that on average population's health outcomes are similar in both countries but there are differences between these two populations at the low levels of education and income. Also, Lasser et al. (2006) and Armstrong et al. (2006) focused on comparing racial, education and income based health inequalities between the U.S. and Canada. They emphasize that health disparities based on race, although present in both countries, are more prominent in the U.S..[15]

[13] It is important to mention that Makdissi and Wodon (2004) is the only study in the stochastic dominance literature that incorporates fuzzy logic however they apply it to poverty measures.

[14] Some papers looked at both aspects but tackled each of them separately.

[15] Note that Lasser et al. (2006) used HUI3 measure while Armstrong et al. (2006) used a self rated health measure.

From a health care access perspective, it appears that on the global level Canadians more likely to have a regular doctor and less likely to have unmet health care needs (Lasser et al., 2006) yet the differences between the two populations are the same once we compare to the insured population in the U.S. (Sanmartin et al., 2006). One caveat in these studies is that they do focus on access without considering health measures of the individuals which may result in a overestatement of the unmet health care access needs.

While most of the literature that dealt with inter-country comparisons focus on either measure of health status or access measures this papers uses another performance measure (*leakages*) in a fuzzy fuzzy stochastic dominance approach. More specifically, we adopt a fuzzy approach to HUI3 to account for the imprecision in the health status variable. Given that the results obtained from such a transformation may be contingent to the parametric form of the fuzzification procedure, we incorporate this transformation in a stochastic dominance framework and derive robust dominance criteria.

3 Methodology

In this section, we will briefly introduce the theory of fuzzy sets, we then develop a class of fuzzy leakage indices for the comparisons of a pair of health systems. Finally we incorporate the fuzzy set approach in the stochastic dominance framework and derive a stochastic dominance criterion that identifies rankings that are unanimous for all fuzzy leakage indices. By introducing stochastic dominance framework we add an additional fuzzifying layer to the analysis such that the results will be independent from the membership function's parametric form.[16]

3.1 Fuzzy Set Theory

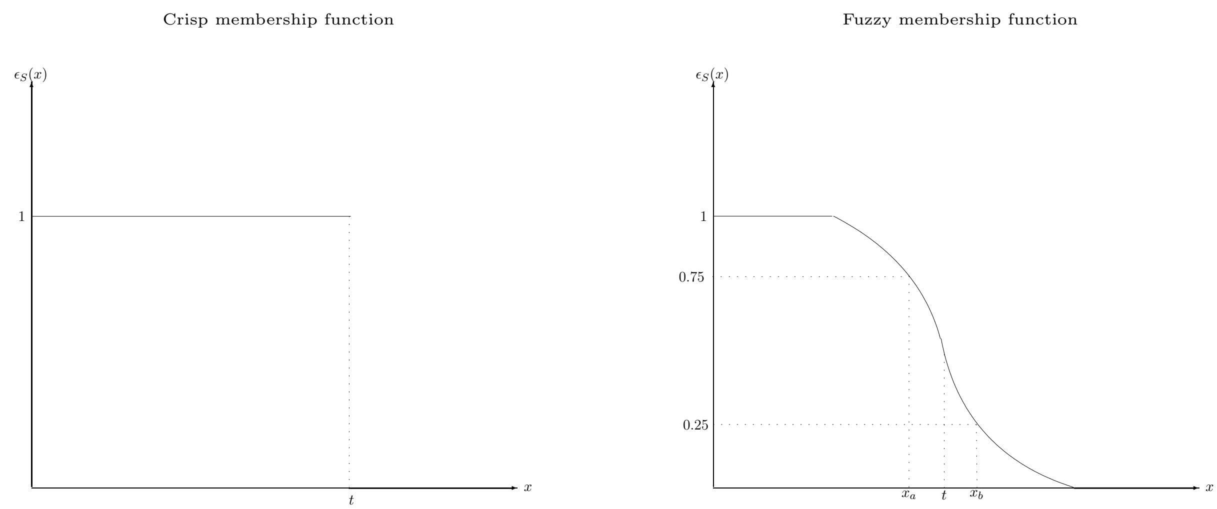
The fuzzy set theory is a way to handle unclear boundaries that arise from having deal with linguistic variables that usually refer to vague concepts and that are often context dependent. It was introduced in the seminal work by Zadeh (1965) in which he proposed a new way of dealing with the fuzziness of the information about the membership degree of a given variable x, to a particular state S.[17] Thus,

[16]Provided this membership function is monotonically decreasing.

[17]The first article considering the application of the fuzzy sets theory was also written by Zadeh in 1968

using this concept of fuzziness one can assume that there are intermediate membership values and that these values vary between 0 and 1. Zadeh's contribution to the set theory makes it possible to answer the following question: to what extent the statement "x is S" is true? One advantage of this approach over the classical set theory lies in the flexibility of the membership function. It thus makes it easier to deal with the vagueness of the linguistic variables that are usually quite difficult to classify precisely (e.g., good or bad health).[18] While in the classical set theory an individual belonging function is dichotomous, this same function becomes vague once transposed in the fuzzy set framework. For illustrative purpose, let us assume that we have two individuals ($i = a, b$). Suppose that x_a and x_b are such that $\epsilon_S(x_a) = 0.75$ and $\epsilon_S(x_b) = 0.25$ where the numerical value associated with $\epsilon_s(x_i)$ reflects how much "x is S" is true. Using the framework of fuzzy set theory, both individuals are considered to be in the same state with different degree of belonging. Yet, classifying these same individuals using the classical set theory would have yielded different results: "x is S" would have been true for individual a only.

Figure 1: Fuzzy vs crisp membership functions



Linguistic variables are frequently used and are often available in all surveys (e.g., excellent health, can hardly walk 4 km, frequently fetched water). The use of terms such as excellent, frequent, hardly are used to reflect the intensity of a particular state that can be associated with a numerical value. Let us assume that we have an adequate numerical measure of this intensity, and that we have a threshold t below which

[18]See Zadeh (1975) for more details

an individual would belong to S and above which an individual would belong to $1-S$ (see figure 1). Then using the classical set theory, individuals that are at the boundary of this threshold present a classification challenge as the boundary between these two states is not clear. The use of the fuzzy set framework is crucial to overcome such classification dilemmas as it provides a membership function to a particular state that varies in intensity (see figure1). In principle there are many membership functions that are associated with linguistic predicate, any continuous function defined between 0 and 1 is a candidate regardless of its shape.[19] In this paper, the membership function is assumed to be monotonically decreasing in health status. Apart from that, our choice of the membership is somehow arbitrary nevertheless, as we will show later, our results will be independent from the particular parametric form chosen. At this point it is important to mention that this arbitrary choice comes without a cost on the applicability of mathematical operators.[20]

3.2 Fuzzy Set Approach to Health Status

Let G be the set of individuals in good health and B be the set of individuals in bad health. Assume that we have a continuous variable h that describes objectively the health status of individuals. Then, using the traditional set theory one can define those sets as follows:

$$G := \{i : h_i > u\}, \tag{1}$$

and

$$B := \{i : h_i \leq u\}, \tag{2}$$

where h_i represents the health status of individual i and u is a threshold below which an individual is considered in bad health. Given these definitions it follows that, if $i \in G$ then $i \notin B$ (conversly if $i \notin G$ then $i \in B$). However, since the inherent linguistic distinction between being in good or bad health is less crisp than the traditional set theory, we can use fuzzy sets framework. In such a framework, we specify a continuous membership function $\epsilon_B(h) \in [0,1]$ which indicates the degree to which an individual is

[19] The most commonly used membership functions are triangular, trapezoidal, Gaussian and sigmoidal

[20] By mathematical operators we mean, fundamental operations such as union, intersections and complement and their characteristics functions commutativity, associativity, idempotency, distributivity, involution.

considered as being in bad health. Therefore, a person can be considered simultaneously in good health $(1-\epsilon_B(h))$ or in bad health $(\epsilon_B(h))$ where bad health is weighted by a membership function such that a person status can vary to a certain extent between on a continuum of degrees of bad health.

Definition 1 *Let Ξ be the set of membership functions $\epsilon_B(h)$ which are continuous, differentiable and non-increasing.*

In order to define the fuzzy set of individuals in bad health, the membership function must be drawn in Ξ. This implies that an individuals with higher h are less likely to be considered as being in bad health. The crisp definition of the set of individuals in bad health given in equation (2) is a particular case for which $\epsilon_B(h) = 1$ for $h \leq u$ and $\epsilon_B(h) = 0$ for $h > u$. There are, however, an infinity of alternative membership functions in Ξ. In this paper, we will consider the existence of two thresholds: a minimum threshold u^- under which everyone agrees that the membership function $\epsilon_B(h) = 1$, and a maximum threshold u^+ over which everyone agrees that the membership function $\epsilon_B(y) = 0$. The interval $[u^-, u^+]$ represents the area of the fuzzy frontier between the sets of individuals in bad and good health.

Definition 2 *Let $\widehat{\Xi}$ be the subset of Ξ for which*

$$\begin{array}{cc} \epsilon_B(h) = 1 & \text{for } h \leq u^- \\ \epsilon_B(h) \in [0,1] & \text{for } u^- < h < u^+ \\ \epsilon_B(h) = 0 & \text{for } h \geq u^+ \end{array}.$$

To build this membership function one has to find the correspondence between the value of the health index used and the adjectives “good” and “bad” health status. This is not an easy task since it involves normative judgement on what is to be considered a bad health status. In this context, it may be argued that differences in value judgement will result in differences in the mathematical specification of this membership function. For the time being we will abstract from this issue and assume that the membership function has the following simple functional form:

$$\epsilon_B(h) = \begin{cases} 1 & \text{for } h \leq u^- \\ \left(\frac{u^+-h}{u^+-u^-}\right)^\alpha & \text{for } u^- < h < u^+ \\ 0 & \text{for } h \geq u^+ \end{cases} \tag{3}$$

It is important to note that by setting $\alpha = 0$, we fall back in the standard definition of a crisp set with the membership function taking a value of 1 for all those whose health status is under u^+. Also, for any value of $\alpha \neq 0$, the membership function ϵ_B will take a value between 0 and 1.

Using this membership function we would like to measure the incidence of accessibility hurdles . Let $h \in [h_{min}, h_{max}]$ be the health status and c the access to health care such that $c \in \{0, 1\}$, where $c = 1$ indicates that the individual has always benefitted from health care when needed and $c = 0$ indicates that he faced, at least once, an accessibility hurdle. Also, let $f(h, c)$ represent the joint density of individuals characterized by the vector (h, c) then,

$$F(h, c) = \sum_{s=0}^{c} \int_{h_{min}}^{h} f(t, s)\, dt \tag{4}$$

indicates the joint cumulative distribution function.

In this fuzzy setting, leakage of a health system may be defined as the share of those who are in bad health who have not access to health care. Formally the leakage index for health system i takes the following form:

$$L_i(\epsilon_B) = \int_{h_{min}}^{h_{max}} \epsilon_B(h)\, f_i(h, 0)\, dh. \tag{5}$$

One can compute this statistic using the membership function defined in (3). It is important to note that the problem of comparing leakage of two health systems reduces formally to the comparison of joint distribution functions.

3.3 Stochastic Dominance Condition

However, one can argue that the comparison of leakages of different health systems may depend on the choice of the membership function. To avoid making particular value judgement on the choice of a particular membership function, we develop a stochastic dominance criterion.

Proposition 1 *Let i and j be two health systems. Then $L_i \geq L_j$, for all membership functions $\epsilon_B(h) \in \widehat{\Xi}$ if and only if*

$$\int_{h_{min}}^{h} [f_i(t, 0) - f_j(t, 0)]\, dt \geq 0 \quad \forall h \in \left[u^-, u^+\right].$$

Proof. Sufficiency can be proved by integrating by parts (5), we obtain

$$L_i = -\int_{h_{min}}^{h_{max}} \epsilon_B'(h) \int_{h_{min}}^{h} f_i(t,0)dtdh. \tag{6}$$

Using this result, the difference in leakage between i and j can be written as

$$L_i - L_j = -\int_{h_{min}}^{h_{max}} \epsilon_B'(h) \int_{h_{min}}^{h} [f_i(t,0) - f_j(t,0)]\, dtdh. \tag{7}$$

Recalling definition 2 for $\widehat{\Xi}$, it is easy to see that if $\int_{h_{min}}^{h} [f_i(t,0) - f_j(t,0)]\, dt \geq 0$ for all $h \in [u^-, u^+]$, then $L_i - L_j \geq 0$. This establishes sufficiency.

In order to establish necessity, we will argue a contrario. Imagine now that $\int_{h^{min}}^{h} [f_i(t,0) - f_j(t,0)]\, dt < 0$ on an interval $\left[\overline{h}, \overline{h}+\delta\right]$, with δ arbitrarily close to 0. For the proof, we use the following membership function:

$$\epsilon_B(h) = \begin{cases} 1, & \text{if} \quad h \leq \overline{h}, \\ 1 - \frac{h-\overline{h}}{\delta}, & \text{if} \quad \overline{h} < h < \overline{h}+\delta, \\ 0, & \text{if} \quad h \geq \overline{h}+\delta, \end{cases} \tag{8}$$

where $\overline{h} \in [u^-, u^+]$. Clearly, $\epsilon_B(h) \in \widehat{\Xi}$. Differentiating this function yields:

$$\epsilon_B'(h) = \begin{cases} 0, & \text{if} \quad y \leq \overline{h}, h \neq 0, \\ -\frac{1}{\delta}, & \text{if} \quad \overline{h} < h < \overline{h}+\delta, \\ 0, & \text{if} \quad h \geq \overline{h}+\delta. \end{cases} \tag{9}$$

For $\epsilon_B(h)$ defined as in (8), $L_i - L_j$ is thus negative. Hence it cannot be that $\int_{u^-}^{h} [f_i(t,0) - f_j(t,0)]\, dt < 0$ for some $h \in \left[\overline{h}, \overline{h}+\delta\right]$ when $\overline{h} \in [u^-, u^+]$. This proves the necessity of the condition. ■

This proposition enables us to identify situations in which the leakage ordering of two health care system is robust to changes in the membership function, provided this function respects definition 2. In some cases the dominance test fail over an initial range of health status $h \in [u^-, u^+]$. In this case one may estimate critical bounds for a constrained health status range $\left[u^{min}, u^{max}\right]$ which is more restrictive than $[u^-, u^+]$. This implies that, in case of a restricted dominance result, the results do not hold for membership functions for which the fuzzy frontier is wider than the restricted domain. Consequently, one can either restrict the upper bound, the lower bound or both. When the restriction is only the upper bound, this critical bound is given by

$$u^{max} = \sup\left\{u : \int_{h_{min}}^{h} [f_i(t,0) - f_j(t,0)]\, dt \geq 0, \quad h \in \left[u^-, u\right]\right\}. \tag{10}$$

When the restriction is on the lower bound, the corresponding critical bound is given by

$$u^{min} = \inf\left\{u : \int_{h_{min}}^{h} \left[f_i\left(t,0\right) - f_j\left(t,0\right)\right] dt \geq 0, \quad h \in \left[u, u^{+}\right]\right\}. \tag{11}$$

Finally, when there is joint restriction on the two bounds, the lower and upper critical bounds are given by the joint solution of those two equations:

$$u^{max} = \sup\left\{u : \int_{h_{min}}^{h} \left[f_i\left(t,0\right) - f_j\left(t,0\right)\right] dt \geq 0, \quad h \in \left[u^{min}, u\right]\right\}, \tag{12}$$

$$u^{min} = \inf\left\{u : \int_{h_{min}}^{h} \left[f_i\left(t,0\right) - f_j\left(t,0\right)\right] dt \geq 0, \quad h \in \left[u, u^{max}\right]\right\}. \tag{13}$$

4 Empirical illustration

In this paper, the health index that we use is the Health Utility Index Mark 3 (HUI3), which covers eight attributes: vision, hearing, speech, ambulation, dexterity, emotion, cognition, and pain. Each of these attributes has five or six levels where each attribute score ranges from 0 (for instance blind for vision) to 1 (perfect vision). The HUI3 ranges from -0.36 to 1, the negative values are there to express a state that is worse than death whereas values of 0 reflect death. In order to associate each value of the HUI3 with an adjective we use the equivalences from Humphries and Van Doorslaer (2000) and set $u^{-} = 0.557$ and $u^{+} = 0.945$ (table 1).

Table 1: Distribution of HUI3 by self-assessed health

In general my health is	HUI3
Excellent	0.945
Very good	0.923
Good	0.876
Fair	0.758
Poor	0.557

In our empirical investigation we use this association to write the following membership function:

$$\epsilon_B\left(y\right) = \begin{cases} 1 & \text{for } h \leq 0.557, h \neq 0, \\ \left(\frac{0.945-h}{0.388}\right)^{\alpha} & \text{for } 0.557 < h < 0.945, \\ 0 & \text{for } h \geq 0.945. \end{cases} \tag{14}$$

Using the information from table 1, we can safely assume that everyone would agree that every individual who is above the higher bound 0.945 is in good health (thus attributed a membership value of 0). Similarly, everyone should agree that every individual below the lower bound 0.557 is in bad health (thus attributed a membership value of 1). Finally, all those who are in between the two bound will have a membership function between 0 and 1. As we vary the value of α we vary the weight that we put on the persons that are away from the lower bound. If $\alpha = 0.5$, then a individual's weight will remain important even if this later health index is far from the bound u^{-}. This means that the membership function is more sensitive to disease. Conversely, in case $\alpha = 2$ the individual's weight decays rapidly. Using such a fuzzy membership function allows us to provide a more realistic approach in health status than the crisp definitions. This is the case because individuals who are not in perfect health have different degrees of sickness that are better captured in a fuzzy context.

4.1 Data

As mentioned earlier, we use the Joint Canada/United States Surveys of Health published in 2004. This survey entails 8688 observations of which 3505 are Canadian residents and 5183 are US residents. It covers individuals between the age 18 and 85 years and information about their clinical condition as well as their demographic characteristics and their socio-economic status. This survey has the advantage of providing comparable U.S and Canadian variables provided by respectively the *United States National Center for Health Statistics* and *Statistics Canada*. Given that the survey is a (one time) random telephone survey, low income people may be under-represented.[21] There are many criteria that one can use in order to compare the U.S and Canadian health care systems. In this paper we are mainly interested in leakages of the health system therefore our key variable will be constructed using an access variable and the HUI3.[22] Since being sick is not a sufficient condition to seek health care, a person is considered as having an access problem only if she states that she had not received some type of health care when needed. More precisely, for this definition we excluded those who decided not to seek care because they don't like doctors and those who felt it would be inadequate and focus our attention on individuals who did not have access to care for reasons that are not related to preferences.

[21] Also, it is important to note that people who are institutionalized are not accounted for.
[22] We used the variable that asks individuals whether they feld they needed health care but could not have access.

4.2 Results

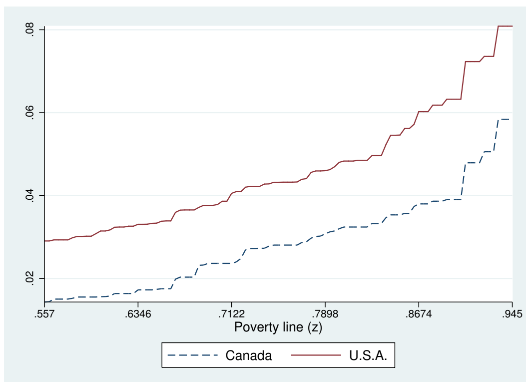
To measure to what extent the health care system is pervious we compare the membership functions for the US and Canada for different weights values ($\alpha = \{0.5, 1, 2\}$). We also perform stochastic dominance tests to check for the robustness of the results obtained.

4.2.1 The Canadian Population vs US Population

Table 2: L estimates for Canada and the U.S.A.

	Canada	U.S.A.
$\alpha = 0.5$	0.0385	0.0528
$\alpha = 1$	0.0303	0.0429
$\alpha = 2$	0.0239	0.0352

Figure 2: Dominance test, Canada vs U.S.A.



We first focus on the entire population of individuals in both countries and compare their leakages. The computed indices in table 2 have lower values for Canada for all values α. This suggests that globally, the Canadian health care system may have less leakage than the US health system. However, as argued earlier these results may not be robust to the choice of the functional form of the membership function. To assess the robustness of the results we conduct a stochastic dominance test. The absence of intersection between the two curves in figure 2 shows that for the Canadian health system there is a stochastic dominance on the entire support. This dominance result is independent of the functional form of the membership function.

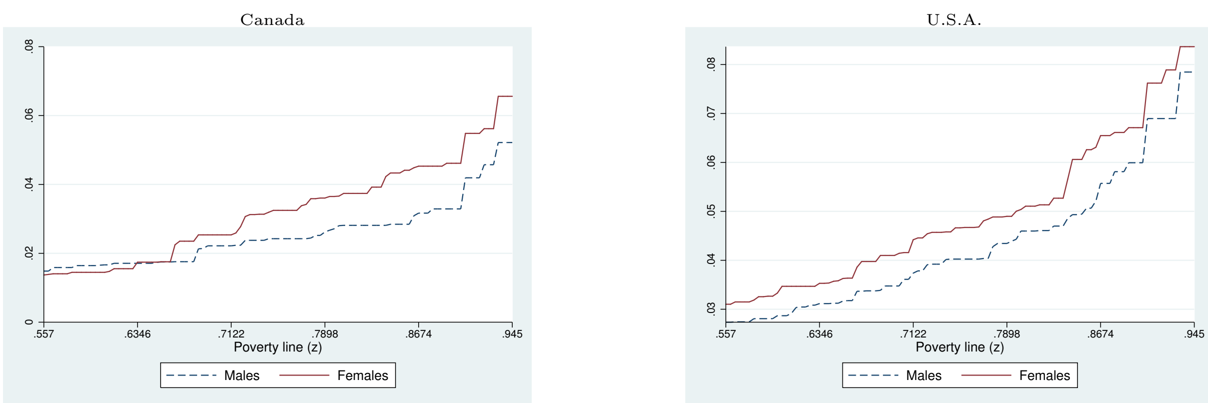
The presence of a strict stochastic dominance in leakages in health care can be shaped by many factors. To investigate the extent to which socio-demographic and socio-economic factors can explain the presence of this strict dominance we decompose our analysis by gender, ethnicity, income quintiles and education level.[23]

4.2.2 Comparison by demographic factors

Table 3: L estimates by gender for Canada and the U.S.A.

	Canada		U.S.A.	
	Females	Males	Females	Males
$\alpha = 0.5$	0.0420	0.0350	0.0577	0.0475
$\alpha = 1$	0.0329	0.0276	0.0475	0.0379
$\alpha = 2$	0.0256	0.0222	0.0394	0.0306

Figure 3: Dominance test by gender in Canada and the U.S.A.



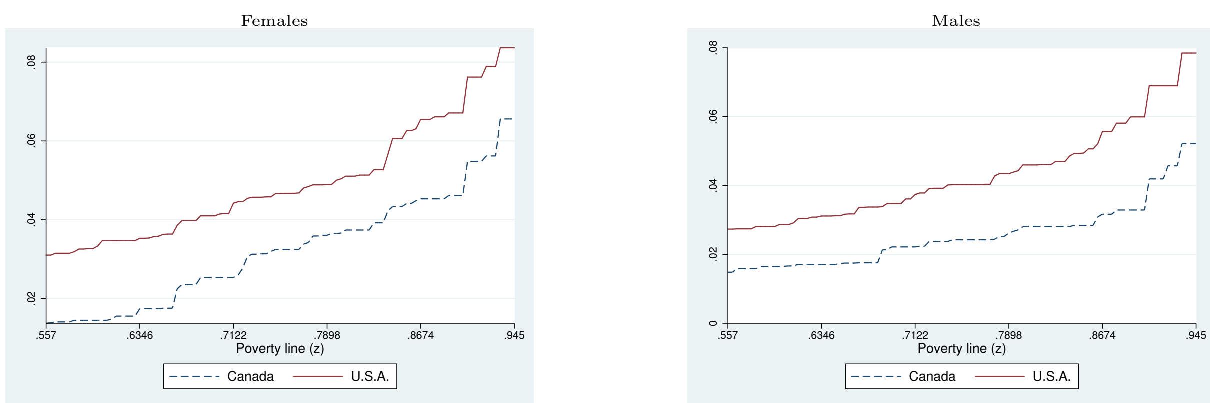
There exist a growing literature on the determinants of gender health inequalities. While the reasons behind these inequalities are still unclear, it is clear that such inequalities exist (Bryant et al., 2009). The indices reported in table 3 suggest that males have a better access in both countries. Yet, the intersection of the two curves in figure 3 shows that there is no stochastic dominance over the entire support for Canada. Therefore, for all the membership functions for which the fuzzy frontier is defined over the interval for [0.557,0.633], women are better off in Canada. On the remaining part of the support (i.e., on the interval [0.633,0.945]) there is less leakage for males in poor health. In the US the results suggested by

[23]It would be interesting to decompose in more than one dimension nevertheless the sample size at hand is too small to allow for such a decomposition.

the indices seem to be robust implying that men in poor health in the US have better access than females in poor health. The stochastic dominance test in figure 3 shows that the results are independent of the choice the membership function for the entire support. Given these gender disparities in access to health care for those who are in need for it, one is inclined to think that gender inequities in health might be partly driven by gender inequities in access to care. The literature on health care access gender inequality is quite limited, most of it is focused in income induced gender health disparities. Whilst, we acknowledge that income related gender health disparities are important, we think that further investigation need to be conducted to analyze gender disparities in access to needed health care. Women may have different medical attention needs and different accessibility constraints than men (e.g., time constraints, family burden). Thus, in order to have a better targeting of public policies it is important to understand the mechanisms that drive such disparities.

Having analyzed intra-country gender driven leakages we focus on inter-country disparities. A comparison between the first panel and the second panel of table 3 suggests that both Canadian women and men are less likely to be disadvantaged by the health care system than their US counterparts. As we can see from figure 4, these gender comparisons are robust to the choice of the membership function over the entire domain.

Figure 4: Dominance test by gender, Canada vs U.S.A.

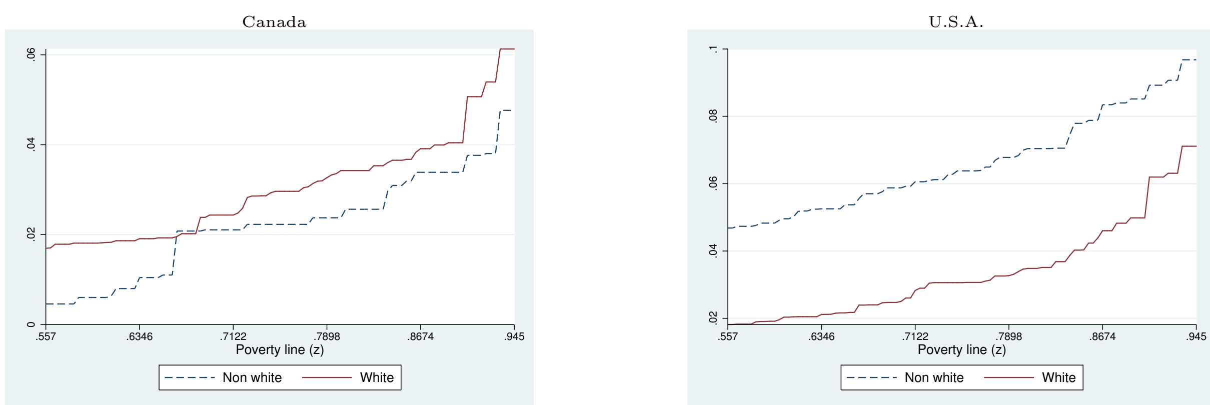


Besides gender, ethnicity is considered to be an important determinant of disparities in general and health disparities in particular. The indices in table 4 reveals that non-whites are less likely to leak from the health care system in Canada than whites. One possible explanation could be the presence of social network effect. According to Deri (2005), this effect can be produced through two mechanisms: information or norms. Thus, the concentration of a language or an ethnic group is an important factor in diffusing information or imposing a social norm. A similar story may be at the source of our results. As for the U.S the results are completely reversed: non-whites are more likely to leak from the health care system. Given the absence of universal health insurance in the U.S different mechanisms can be at the source of this ethnic based inequality in leakages (income, social networks, discrimination). The stochastic dominance test reported in figure 5 indicate that these results are robust in the case of the U.S over the entire support while they are robust for Canada on a restricted part : on the intervals defined by [0.557,0.664] and by [0.685,0.945]. This means that for Canada the results are more constrained, they are robust only if the membership function fuzzy frontier lie within the defined intervals.

Table 4: L estimates by ethnicity for Canada and the U.S.A.

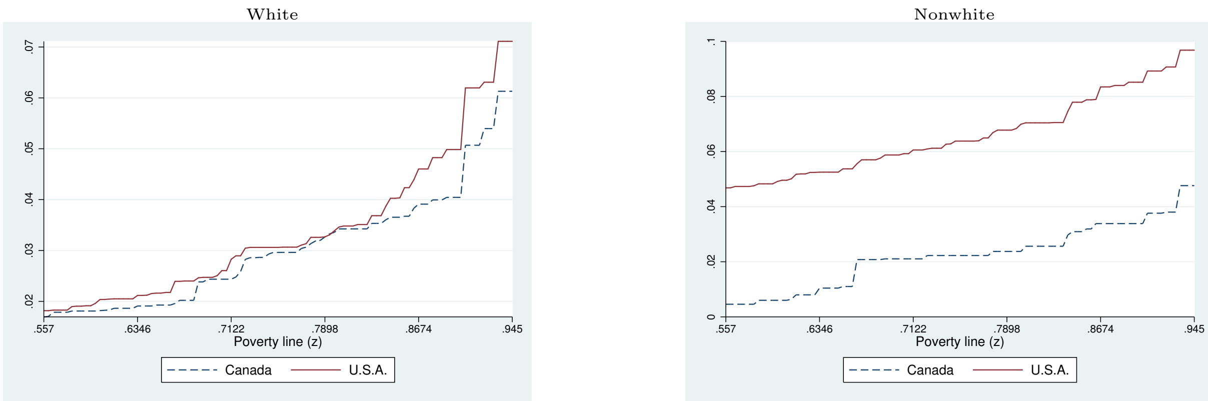
	Canada		U.S.A.	
	White	Non White	White	Non White
$\alpha = 0.5$	0.0402	0.0314	0.0448	0.0707
$\alpha = 1$	0.0318	0.0237	0.0355	0.0596
$\alpha = 2$	0.0255	0.0172	0.0285	0.0502

Figure 5: Dominance test by ethnicity in Canada and the U.S.A.



Turning to inter-country comparisons, we notice that non-white people are less likely to leak in Canada than in the U.S. As for white people, it appears that their leakages are quite comparable in both countries. This may indicate that the universal health care system in Canada provides better access to non-whites in need for health care access. It also raises an important question: can the results obtained at the global level be mainly driven by ethnic disparities?

Figure 6: Dominance test by ethnicity, Canada vs U.S.A.



4.2.3 Comparison by socio-economic status

Access to health care is often correlated to income. In the absence of a universal health care system one would expect that people from the low income groups will face higher access barriers. To provide a better understanding of the possible socio-economic forces that might be driving the strict dominance of the Canadian health system we first decompose the data by income quintiles then by education level. Given that the sample size is not very large we combine the two lowest quintiles (LQ) in a group and the three highest quintiles (HQ) in another group and compare them.[24]

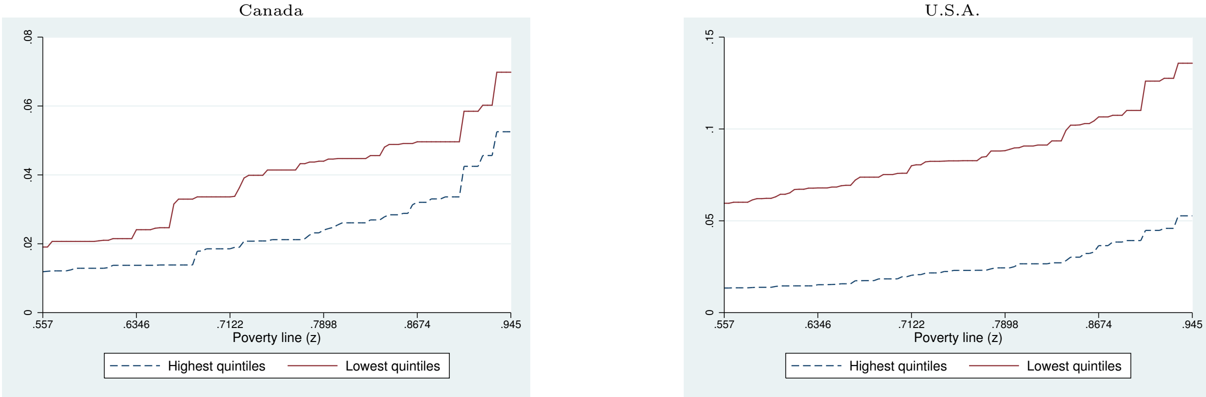
Table 5: L estimates by quintiles groups for Canada and the U.S.A.

	Canada		U.S.A.	
	Lowest quintiles	Highest quintiles	Lowest quintiles	Highest quintiles
$\alpha = 0.5$	0.0506	0.0324	0.0926	0.0337
$\alpha = 1$	0.0421	0.0242	0.0781	0.0260
$\alpha = 2$	0.0349	0.0183	0.0662	0.0203

From table 5 it appears that individuals in poor health who belong to HQ are less likely to leak from the health system regardless of their country of residence. Results in figure 7 reveal the presence of a stochastic dominance for both countries. These findings are within the expected line for the US given the absence of a universal health care system. Yet, the stochastic dominance result for Canada is a little

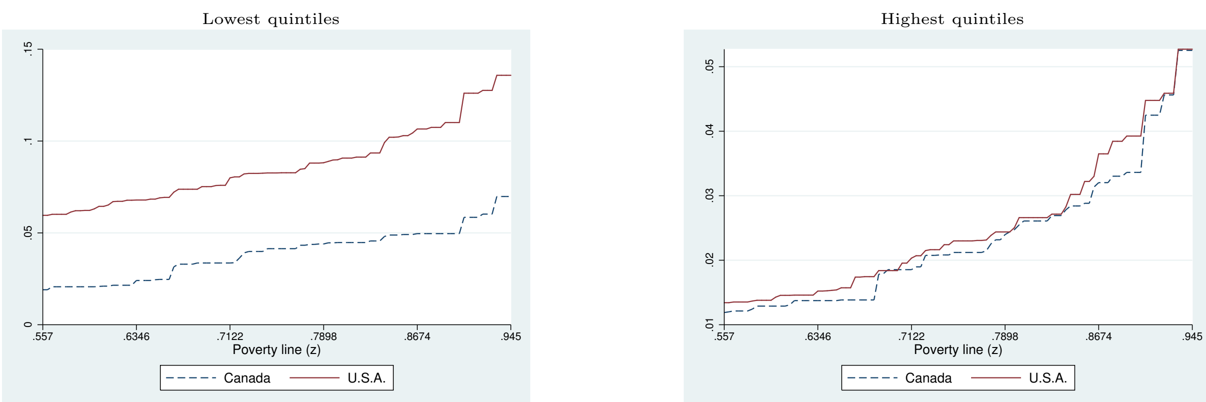
[24]A similar pattern will followed for education level.

Figure 7: Dominance test by quintiles groups in Canada and the U.S.A.



surprising as it does not come in line with the Canadian health act that states that the “accessibility to insured medical services must not be hindered by financial considerations”. Given such results, one may be carried to think that income may be reflecting another variable such as education or presence of social networks. Social networks, might be acting as an accessibility vehicle since people from HQ are more likely to lower their accessibility hurdles through friends (or friends of friends) who are doctors or have access to doctors. In addition to social network, education can be another mediator, this possibility will be tested later in this section.

Figure 8: Dominance test by quintiles groups, Canada vs U.S.A.

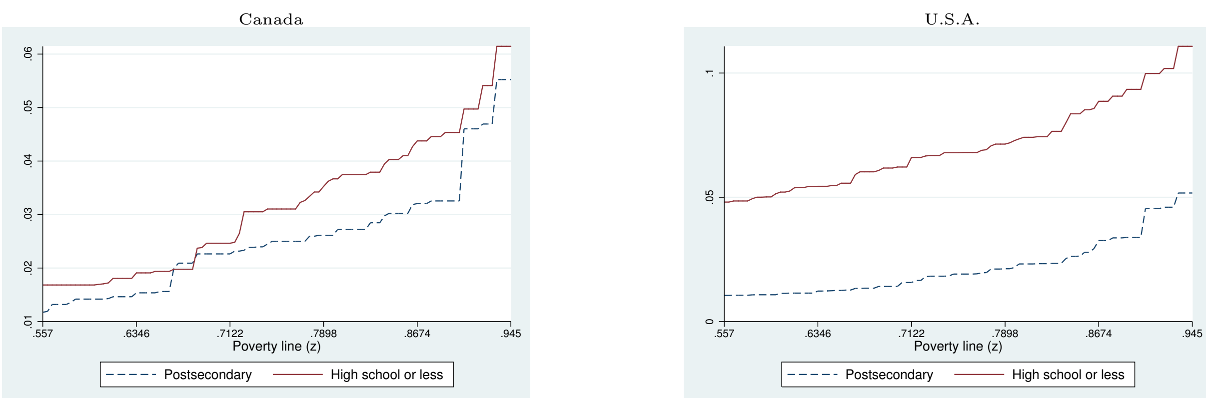


We next consider income driven differences between countries. The estimated indices for LQ in Table 6 suggests that Canada has a lower leakage than the US. This implies that when it comes to vulnerable population in poor health there is less leakage in the Canadian health system. These results seem to be robust to the choice of the membership function. Combining this information with the ones obtained on the country level one might infer that despite the fact that individuals in the HQ are less likely to leak than those in the LQ, the individuals in LQ are less likely to leak in Canada than in the US. The presence of higher income driven access inequalities comes in line with findings by O'Neill and O'Neill (2007) and Van Doorslaer et al. (2006). Despite the evidence that suggests that leakages the Canadian health care system may be income driven, it seem to perform better in capturing economically disadvantaged people in need for health care. A person in poor health with an income in the lowest two quintiles of the income distribution will be therefore less likely to leak in the Canadian health care system. As for individuals in the HQ, the indices suggest mixed conclusions with the Canadian health system performance slightly better than the US yet, figure 8 does not reveal the presence of stochastic dominance. This means that, for individuals in poor health who are in the HQ, the American and the Canadian health care systems' performance seem comparable.

Table 6: L estimates by education level for Canada and the U.S.A.

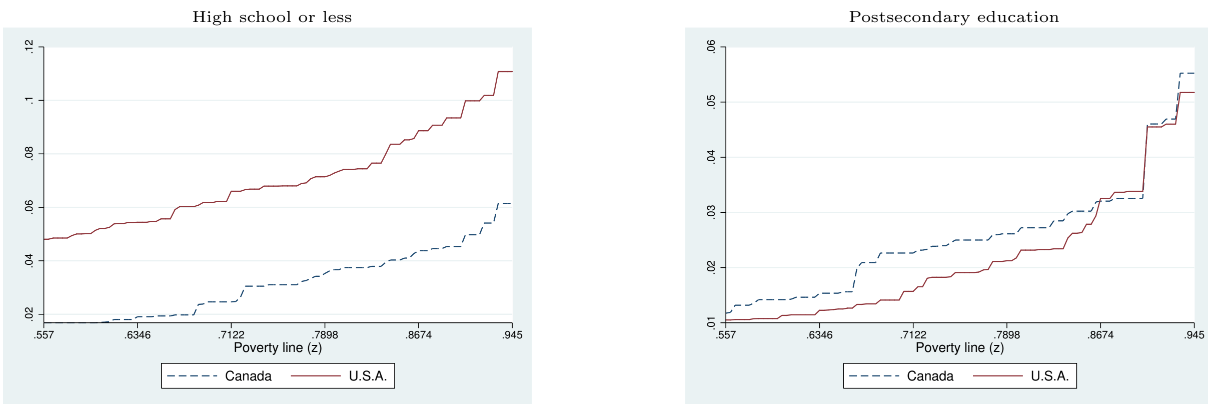
	Canada		U.S.A.	
	High school or less	Postsecondary education	High school or less	Postsecondary education
$\alpha = 0.5$	0.0434	0.0337	0.0732	0.0346
$\alpha = 1$	0.0346	0.0260	0.0620	0.0258
$\alpha = 2$	0.0274	0.0205	0.0528	0.0195

Figure 9: Dominance test by education level in Canada and the U.S.A.



In addition to income, education level is another indicator of the socio-economic status that is often used in the literature. To explore this dimension, we partition the population in two groups, individuals with high school or less (HS) and individuals with postsecondary education (PS). The presence of leakages associated to education level seem to ambiguous in Canada. It appears that the individual with PS education level have better access than those with HS education level over certain part of the support namely, over the intervals defined by $[0.557, 0.664]$ and $[0.685, 0.945]$. This excludes education as a possible mechanism driving the apparent income driven disparities leakages. For the US, the estimated indices in table 6 as well as the presence of stochastic dominance in figure 9 show that leakages are clearly associated with low education level. Having a postsecondary degree in the US provides better access to health care. This comes in line with the results obtained from the decomposition by income quintiles and are robust to any specified membership function.

Figure 10: Dominance test by education level, Canada vs U.S.A.



Focusing on the group of individuals with HS education level, we notice that the indices indicate that Canada has a lower leakage than the US. These results are reinforced by the presence of stochastic dominance depicted in figure 10 . This, once again seems to indicate that the Canadian performs better in capturing vulnerable individuals. A person with an education level of high school or less will be therefore less likely to leak from the net of the Canadian health care system. These results are completely reversed for individuals with PS education level. The estimated indices suggest that the US performs better in capturing individual with postsecondary education. Yet, the crossings in 10 show that these results valid only if we restrict the fuzzy frontier over the interval $[0.557, 0.866]$. Beyond the upper bound the presence of multiple crossings does not allow us to draw any conclusions.

5 Conclusion

This paper tackle an important empirical question regarding the comparison of leakages in the American and Canadian health care system. To conduct this study we assume that Canada is a strong counterfactual for the United States and use the HUI3 as well as information regarding unmet health care needs from JCUSH survey. Given the linguistic nature of the health status variable used, we classify individuals according to their health status using a fuzzy set framework. More precisely, we specify a gradual transition between health statuses through a membership function. We then incorporate the specified membership function in a stochastic dominance framework and derive the stochastic dominance criterion using a *fuzzy-fuzzy* approach.

Stochastic dominance results from inter-country comparisons suggests that on the global level individuals who are in poor health are less likely to have unmet health care needs in Canada. In addition, it appears that there are gender, ethnic and education based disparities in leakages in the U.S. From an intra-country perspective, stochastic dominance results reveal the presence of income driven disparities in both countries yet, these disparities seem to be more accentuated in the U.S. Though, income driven disparities in leakages are less prominent in Canada, the presence universal health care system does not seem to eliminate the income gradient completely.

There are many possible extension for this paper. The stochastic dominance criterion that we derived can be applied to any other continuous health index or health outcome measure such as BMI, birthweight or z-scores. On a methodological level, one can apply a multidimensional fuzzy framework to capture the fuzziness of the substitution between each of the attributes of the HUI3. The stochastic dominance framework for such fuzzy indices are not yet available in the literature. Also, from an empirical perspective it would be important to investigate in details gender disparities in leakages as it is an important dimension of gender based health disparities that is still under-investigated. Also, it would be interesting to assess in depth the extent to which the differences in inter-country leakages are due to ethnicity. Last but not least, further research is needed as far as the mechanisms driving the presence of income based leakages in the Canadian universal health care system. In addition, the implementation of another wave of this survey with a larger sample would enable us to perform detailed analysis and provide a better picture of the differences between the two health care systems.

References

Alegria, M., Bijl, R., Lin, E., Walters, E. and Kessler, R. (2000), 'Income differences in persons seeking outpatient treatment for mental disorders: a comparison of the United States with Ontario and The Netherlands', *Archives of general psychiatry* **57**(4), 383.

Armstrong, H., Clement, W., Lin, Z. and Prus, S. (2006), 'Contrasting Inequalities: Comparing correlates of health in Canada and the United States', *Social and Economic Dimensions of an Aging Population Research Papers* .

Blendon, R., Schoen, C., DesRoches, C., Osborn, R., Scoles, K. and Zapert, K. (2002), 'Inequities in health care: a five-country survey', *Health Affairs* **21**(3), 182.

Bryant, T., Leaver, C. and Dunn, J. (2009), 'Unmet healthcare need, gender, and health inequalities in Canada', *Health Policy* **91**(1), 24–32.

Bérenger, V. and Verdier-Chouchane, A. (2007), 'Multidimensional measures of well-being: standard of living and quality of life across countries', *World Development* **35**(7), 1259–1276.

Cerioli, A. and Zani, S. (1990), A fuzzy approach to the measurement of poverty, *in* 'Income and wealth distribution, inequality and poverty: proceedings of the second International Conference on Income Distribution by Size, Generation, Distribution, Measurement, and Applications, held at the University of Pavia, Italy, September 28-30, 1989', p. 272.

Cheli, B. (1995), 'Totally fuzzy and relative measures in dynamics context', *Metron* **53**(3/4), 83–205.

Chiappero Martinetti, E. (1994), 'A new approach to evaluation of well-being and poverty by fuzzy set theory', *Giornale degli economisti e annali di economia* **53**, 367–88.

Chiappero Martinetti, E. (1996), 'Standard of living evaluation based on Sen's approach: some methodological suggestions', *Notizie di Politeia* **12**(43/44), 37–53.

Costa, A., Massad, E., Ortega, N. and Araújo, A. (2004), 'Perception of disability in a public health perspective: a model based on fuzzy logic', *International Journal of Medical Informatics* **73**(7-8), 647–656.

Deri, C. (2005), 'Social networks and health service utilization', *Journal of Health Economics* **24**(6), 1076–1107.

Eng, K. and Feeny, D. (2007), 'Comparing the health of low income and less well educated groups in the United States and Canada', *Population Health Metrics* **5**(1), 10.

Evans, R. and Ross, N. (1999), 'What is right about the Canadian health care system?', *Milbank Quarterly* **77**(3), 393–399.

Feeny, D., Kaplan, M., Huguet, N. and McFarland, B. (2010), 'Comparing population health in the United States and Canada', *Population Health Metrics* **8**(1), 8.

Guyatt, G., Devereaux, P., Lexchin, J., Stone, S., Yalnizyan, A., Himmelstein, D., Woolhandler, S., Zhou, Q., Goldsmith, L., Cook, D. et al. (2007), 'A systematic review of studies comparing health outcomes in Canada and the United States', *Open Medicine* **1**(1), e27.

Humphries, K. and Van Doorslaer, E. (2000), 'Income-related health inequality in Canada', *Social Science & Medicine* **50**(5), 663–671.

Katz, S. and Hofer, T. (1994), 'Socioeconomic disparities in preventive care persist despite universal coverage: breast and cervical cancer screening in Ontario and the United States', *Jama* **272**(7), 530.

Kunitz, S. and Pesis-Katz, I. (2005), 'Mortality of white Americans, African Americans, and Canadians: the causes and consequences for health of welfare state institutions and policies', *Milbank Quarterly* **83**(1), 5–39.

Lasser, K., Himmelstein, D. and Woolhandler, S. (2006), 'Access to care, health status, and health disparities in the United States and Canada: results of a cross-national population-based survey', *American Journal of Public Health* **96**(7), 1300.

Lelli, S. (2001), *Factor Analysis vs. Fuzzy Sets theory: Assessing the influence of different techniques on sen's functioning approach*, Citeseer.

Makdissi, P. and Wodon, Q. (2004), 'Fuzzy targeting indices and orderings', *Bulletin of economic Research* **56**(1), 41–51.

Manuel, D. and Mao, Y. (2002), 'Avoidable mortality in the United States and Canada, 1980-1996', *American journal of public health* **92**(9), 1481.

Martinetti, E. (2000), 'A multidimensional assessment of well-being based on Sen's functioning approach', *Rivista Internazionale di Scienze Sociali* **2**, 207–239.

Massad, E., Ortega, N., Struchiner, C. and Burattini, M. (2003), 'Fuzzy epidemics', *Artificial Intelligence in Medicine* **29**(3), 241–259.

Murray, C. (2002), *Summary measures of population health: concepts, ethics, measurement and applications*, World Health Organization.

Murray, C. and Evans, D. (2003), *Health systems performance assessment*, World Health Organization.

Mussard, S. and Pi Alperin, M. (2007), 'Multidimensional Poverty Decomposition: A Fuzzy Set Approach, Satistica & Applicazioni, Vol', **5**(1), 29–52.

O'Neill, J. and O'Neill, D. (2007), 'Health status, health care and inequality: Canada vs. the US', *Frontiers in Health Policy Research* **10**(1), 3.

Qizilbash, M. (2002), 'A note on the measurement of poverty and vulnerability in the South African context', *Journal of International Development* **14**(6), 757–772.

Qizilbash, M. (2003), 'Vague language and precise measurement: the case of poverty', *Journal of Economic Methodology* **10**(1), 41–58.

Ross, N., Nobrega, K. and Dunn, J. (2001), 'Income segregation, income inequality and mortality in North American metropolitan areas', *GeoJournal* **53**(2), 117–124.

Sadegh-Zadeh, K. (1999), 'Fundamentals of clinical methodology: 3. Nosology.', *Artificial intelligence in medicine* **17**(1), 87.

Sanmartin, C., Berthelot, J., Ng, E., Murphy, K., Blackwell, D., Gentleman, J., Martinez, M. and Simile, C. (2006), 'Comparing health and health care use in Canada and the United States', *Health Affairs* **25**(4), 1133.

Schoen, C. and Doty, M. (2004), 'Inequities in access to medical care in five countries: findings from the 2001 Commonwealth Fund International Health Policy Survey.', *Health Policy* **67**(3), 309.

Van Doorslaer, E., Masseria, C., Koolman, X. et al. (2006), 'Inequalities in access to medical care by income in developed countries', *Canadian Medical Association Journal* **174**(2), 177.

Zadeh, L. (1965), 'Fuzzy sets*', *Information and control* **8**(3), 338–353.

Zadeh, L. (1975), 'The concept of a linguistic variable and its application to approximate reasoning–I* 1', *Information sciences* **8**(3), 199–249.