THE UNIVERSITY OF OTTAWA

AN ANALYSIS OF COSTLESS STOCKPILING BY THE RESOURCE EXTRACTING FIRM UNDER VARIOUS MARKET STRUCTURES

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ABSTRACT

This thesis begins with an analysis of the motives conducive to stockpiling by the resource extracting firm. This is followed by an analysis of the neoclassical model of the deterministic extractive firm under perfect competition. The results derived are analyzed in light of various modifying assumptions. Then the model is amended to determine the optimal profit-maximizing behavior when there is the option of stockpiling. It is shown formally that if the market price of the resource is variable over time it may be optimal for the firm to stockpile. This conclusion does not hold when the price is constant.

The analysis of the extractive monopoly and its optimal behavior is carried out when the parameters of the model are time-autonomous. It is shown formally that a profit-maximizing monopoly may consider simultaneous sales and stockpiling as part of its optimal behavior; when this happens the extraction path increases over time while the rate of sales decreases.

Afterwards a review of the literature on imperfectly competitive resource markets concludes that it is not practical for firms to stockpile; this conclusion rests crucially on the assumption that full information prevails in the market.

Other motives for stockpiling by the firm under certainty

that are examined include stockpiling for environmental purposes, speculative purposes, or because of indivisibilities of production.

The last chapter provides an overview of the potential decrease in risk through stockpiling available to firms that operate under uncertainty. Different types of uncertainty are reviewed, and the relevant motives for stockpiling are discussed.

CHAPTER I

INTRODUCTION

Much of the analysis of nonrenewable resource markets has taken place under the assumption that there is no stockpiling. This implies a "hand-to-mouth" existence for the firm, where at every moment of time it must equate its marginal revenue from the sale of the resource with the sum of its marginal cost of extraction and marginal user cost in order to survive in the market. This leaves the firm[1] little leeway in choosing its sales strategy. While accepting that there are some cases where the assumption that no stockpiling takes place is justified (for instance, where stockpiling is prohibitively expensive or physically impossible), there are instances when a model of a resource industry might gain in realism from extending the option of stockpiling to the firms' actions. Also, the inclusion of the option of stockpiling among the range of options available to the firm to attain its profit-maximizing sales path in a theoretical model of either a firm or an industry increases the relevance of that model in analyzing and predicting the behavior of real firms in an actual resource market.

[1]Henceforth, a reference to a "firm" implies a resource extracting firm.

Yet at a more fundamental level it is relevant to examine whether, given an extractive firm's willingness to stockpile, it is possible for the firm to do so. This thesis examines the possibility for the individual resource extracting firm to stockpile its resource at zero cost once it has been taken out of the ground.

The reasons why this thesis examines mostly inventories that are held at zero stockpiling cost[1] are as follows: first the least holding cost any good incurs when it is held is the rate of interest on the value of that good: if a good is produced today for sale at some point in the future, even if it is stored at zero cost the cost of holding that good until it is sold is the difference between the good's present value today and that when it is sold at some point in the future. This difference in present value constitutes the "holding cost". Furthermore, the deletion of positive costs of maintaining an inventory from the analysis of stockpiling avoids placing the analysis in a cost-benefit context that involves the evaluation of the sum of stockpiling and holding costs compared against the price of the good at some moment in the future when the inventory is sold. Rather the object of this study is to abstract from the potential benefits of stockpiling to concentrate

[1] By "zero stockpiling cost" it is understood that the firm does not have to make any payment to any firm or entrepreneur to store its ore anywhere. This convention shall be adhered to throughout the course of this thesis, except where specifically mentioned otherwise.

instead on the implications for a firm's optimal sales path caused by offering the option of stockpiling.

Furthermore, in light of the variety of institutional structures and constraints under which a model of a mining firm or industry may be presented, the choice of the assumption of zero costs of stockpiling is further warranted.

This thesis proceeds as follows to determine the feasibility of stockpiling an exhaustible resource under varying market structures. First, the motives for any type of firm to voluntarily store some of its finished output are examined. These number eight in all. They are: (i) "strategic stockpiling" to prevent entry in the market; (ii) speculative stockpiling if the future price or output is uncertain; (iii) precautionary stockpiling, or the upkeep of a "buffer stock" to keep the consumer satisfied; (iv) stockpiling for transactions purposes (where finished inventories are used to smooth out production); (v) the price stabilization motive, where inventories may be used to decrease price fluctuations on the market; (vi) inventories maintained for environmental purposes; (vii) inventories maintained because of indivisibilities of production; (viii) intertemporal price discrimination. Hence if an individual firm departs from its "hand-to-mouth" strategy and decides to vary its sales strategy from that of equating its marginal revenue to its complete marginal costs, it must build its stockpile in function of what use it shall put it to.

The third chapter is an analysis of the competitive neoclassical extractive firm under certainty. After a review of the assumptions of the neoclassical model, the optimal extraction plans of the neoclassical firm are analyzed. The Hotelling rule is derived and limiting cases are examined. Finally, the behavior of the perfectly competitive extractive firm that has the option of stockpiling is examined. It is shown that when the price of the resource is constant over time it is never optimal for the firm to set aside any part of its extracted resource. However, if the market price of the resource is variable over time it is shown that when it is not optimal to sell the resource, it is optimal to extract it and not sell it immediately, although extraction takes place at a lesser rate than if the resource were sold immediately after extraction. This conclusion goes against the intuitive belief that in perfect competition there is no stockpiling by the individual firm.

The fourth chapter analyzes the behavior of the time-autonomous extractive monopoly. The model of the costless monopoly is derived as well as that with variable costs of extraction. Afterwards, the limiting cases to the optimal extraction path are examined. The necessary conditions to have stockpiling by a monopoly in a deterministic setting are also analyzed. It is seen that two different conditions are conducive to stockpiling for purposes of intertemporal price discrimination.

The first condition is a monopoly where the cost function is changing over time but increasing at a rate greater than that of interest. This specification may lead the producer to "bunch up" production at one point in time to avoid higher costs at a later date. If the demand function is time-dependent as well as the cost function, then the rate of increase of the cost function must be greater than that of the demand function. The second condition conducive to stockpiling is when the parameters of a time-autonomous monopoly exhibits a U-shaped average cost curve. The two conditions discussed above may hold simultaneously; i.e., it is possible that a time-dependent cost function also exhibit a U-shaped average cost curve. Afterwards, the sales and output paths of the time-autonomous monopoly that has the option to stockpile is examined. It is seen that if certain conditions hold, not only is it optimal for a monopoly to store its extracted resource in order to maximize its profits, but also exhibits an increasing extraction path.

Chapter V analyzes the structure of imperfectly competitive resource markets. As these types of markets have not been extensively analyzed in the literature, the chosen approach was to review existing models, and then examine the possibility of stockpiling within their respective frameworks. After a brief section on the relevance of dynamic consistency and the principle of optimality to the analysis of dynamic games, it is seen that in all of the models surveyed it is impossible to carry out stockpiling because of the prevalence of perfect information. Conversely, it is shown that if consumers build up a stockpile against a cartel or monopoly, that the consumer's surplus increases. Finally, the conditions necessary for stockpiling to carry out price discrimination through time are analyzed. It is shown intuitively that the conditions that make stockpiling desirable in imperfect competition are few.

Chapter VI examines stockpiling for purposes of indivisibilities of production as well as environmental motives. The analysis of stockpiles maintained for environmental motives is a corollary of the mechanism whereby a firm may decouple its sales path from its extraction path, if the curve depicting average costs of production is U-shaped. Afterward, stockpiling for reasons of indivisibilities of production is analyzed. It is seen that this motive for storage is probably one of the more pragmatic motives for stockpiling. Finally, the fact that in some cases a monopoly may set the rate of increase of the resource price greater than that of interest may represent a motive for speculators to enter the market and buy as much as possible of the resource. This particular motive reflects a relatively minor aspect of stockpiling, and will be considered only briefly.

The last chapter provides a qualitative discussion of various types of uncertainty and of how stockpiling might decrease the risk to firms of operating in markets characterized by uncertainty. After an analysis of the behavior of the extractive firm under uncertainty it is seen how, in a specific case, futures markets incite firms to base their production on a certain future price rather than on a random present price. This is followed by an analysis of motives for stockpiling under uncertainty. It is seen that only stockpiling for precautionary or speculative motives may reduce different types of uncertainties.

The main findings of this thesis bear reiterating: in a deterministic partial equilibrium model of perfect competition it is possible for a firm to store its extracted ore when the price of the resource is time-dependent. This result is contrary to the expected result concerning the actions of the individual firm in perfect competition, namely that there is no stockpiling. Furthermore, the option of stockpiling by a deterministic extractive monopoly that exhibits time-autonomous parameters is an alternative in the determination of profit-maximizing behavior. If the firm decides that stockpiling is optimal its extraction path is increasing. If on the other hand the firm decides to equate its marginal revenue from sales to the sum of its marginal user cost and the cost of extraction of the resource then the monopoly's behavior is identical in all respects to that of the neoclassical monopoly with increasing costs of extraction.

The relevance of these two findings above is significant in that it leaves the analyst considerably more freedom in charting the optimal path of the extractive firm through time. In the case of a monopoly, the finding that stockpiling may be optimal might induce a monopoly to approach a rate of sales and extraction that is more efficient than that where at each instant of time marginal revenue from sales must be equated to the sum of marginal costs of extraction and marginal user cost. The implication of this point for approaching a more efficient scale of operations than would be available without stockpiling is obvious. Finally the result that a perfectly competitive firm can contemplate stockpiling is noteworthy in the sense that the end of extraction is no longer synonymous with the end of the firm. The obvious corollary is that the firm has more scope to determine its optimal sales and extraction path. This has implications for the analysis of the deterministic extractive firm in general, under the assumption of monopoly and perfect competition.

CHAPTER II

MOTIVES FOR STOCKPILING

Introduction

The issue of stockpiling is often ignored in microeconomic analysis. However, in resource models it is particularly relevant to examine this question for the following reasons: whereas a good deal of the static microeconomic theory of the firm deals with unlimited flows ~~of~~ a good in principle, the theory of nonrenewable resources is concerned with the problem of maximizing a flow of discounted profits from a limited flow emanating from a fixed stock of a scarce resource.

A resource extracting firm's present value stream of revenue is obtained from a stock, over which it has ownership. When it contemplates stockpiling the extracted resource, the firm is essentially considering moving its stock of the resource from under to above ground. If the costs of maintaining its stockpile are nil then the only reason for the firm to dispose of its accumulated stockpile are the same that compel it to extract its reserves from the ground at a predetermined optimal rate (if abstraction is made of the holding and extraction costs.) This rate is determined by the maximization of a stream of present-value profits derived from the sale of the resource, subject to the resource availability.

There are two costs associated with maintaining an inventory: the storage cost, and the holding cost. The storage cost is the expense necessary to maintain an inventory. This expense may range from refrigeration facilities for foodstuffs, to holding tanks for crude oil[1]. The holding cost is somewhat different: it is the difference in the present value of a good between the time when it is produced (or extracted) and that time when it is taken out of the stockpile to be sold, i.e., it is the cost represented by not reaping immediately the benefits of the sale of the good when production costs have been incurred.

The concept of a "holding cost" merits some explanation. Under competitive conditions, the rate of return of a stock of capital is made up of two components: the capital gains enjoyed by that stock, and the change in extraction costs due to the increase in the present value of the extraction costs caused by the fact that this unit has been stored, not sold. This second component is equal to the change in present value between, say, a constant cost "c" at time t=0 and that at time $t=0+\theta$[2]. Hence if the rate of interest is positive,

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[1]Storage costs for nonrenewable resources are usually quite low. The U.S. Federal Energy Administration, for example, estimates oil storage costs at about \$1/bbl. capital costs (if salt domes are used) and less than \$0.01/bbl. annual charges. In an analysis of policies with regard to aluminum, chromium, platinum, and palladium, the U.S. Department of the Interior found the storage costs of those materials to be low enough to justify exclusion from their calculations. A. Nichols and R. Zeckhauser, "Stockpiling Strategies and Cartel Prices", Bell Journal of Economics 8(1) (Spring 1977): 70.

[2]This holds for a variable-cost function if it is time-autonomous.

$$ce^{-rt} > ce^{-r(t+\theta)}$$

If a unit of the stored resource is taken out of the inventory to be sold at time t+θ, the producer will receive the normal capital gains on that unit, less the present value of the cost of extraction incurred θ periods before the sale, i.e.,

$$\lambda = (P_t - c)e^{-rt} > P_t e^{-rt} - ce^{-r(t-\theta)}$$

Clearly there is no benefit in storing the resource.

Hence in an industry characterized by certainty, where the value of the industry's assets are increasing at a rate equal to that of interest less the cost of extraction (or production), the producer who stores his resource loses money on every unit he does not sell immediately after extraction. This constitutes the holding cost of the inventory. Hence the holding cost is the depreciation of a factor of production. But this is the same cost as that defined by Keynes in his appendix on user cost[1].

Therefore at first glance there is no direct benefit derived from holding an inventory in a market where certainty prevails.

But nevertheless inventories are maintained, both by firms and public organizations. The motives for the upkeep of stockpiles are diverse. One such motive is for transaction purposes, i.e., even in a deterministic context sales can fluctuate. If there are setup or adjustment costs of production,

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[1] J.M. Keynes, The General Theory of Employment, Interest, and Money. (London: Macmillan,1973): pp. 66-73. This point is examined in Ch.III below.

a firm may decide to maintain inventories to smooth out the production schedule. Nevertheless this motive is but one of the purposes for maintaining inventories.

The relevance of each motive for accumulating inventories will be examined after having examined the motives themselves.

Motives for Stockpiling

The purpose of an inventory is for a firm or an organization to sell over and above the firm's capacity to respond to the market demand. Essentially an inventory is built up when there is a decoupling of sales and output that the intertemporal substitutions of discounted marginal revenues and discounted marginal costs require[1]. It is deemed profitable to increase the stockpile when the expected yield of the stock is positive[2]. The capital value of an inventory is the sum of discounted gains to be won by substitution, and it is maximized when the value of the enterprise is maximized[3].

There are eight motives for a stockpile to be built up voluntarily in any market, given the proper conditions. These motives are:

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[1]E. Shaw, "Elements of a Theory of Inventory," Journal of Political Economy XLVIII(4) (August 1940): 469.

[2]Ibid.

[3]Ibid.

Strategic Motive

This motive for holding inventories falls within the scope of the literature on dynamic games with irreversible commitments[1]. These models of firms' behavior deal with the strategic use of product-specific capital by a monopoly to prevent entry into a market by a second firm. Essentially these models show how a firm may ward off potential entrants by committing themselves to remain in the market. This commitment takes the form of product-specific capital that is sunk in productive capacity. As this sunk capacity is unmoveable this confers credibility to the incumbent's threat to remain in the market. This threat has two components: how much sunk capacity the incumbent possesses, and how low the incumbent is willing to set the initial price in post-entry equilibrium.

The strategic use of inventory may take two forms: inventory may be used to deter entry in a market, or alternatively if entry is excluded an inventory may be used to influence future market equilibrium by building up inventories in the present. Essentially the use of inventory to deter entry illustrates how a firm will commit itself to remain in a market by "sinking" part of its production costs to influence post-entry equilibrium. Alternatively, the "pure" strategic use of inventories is much closer to the idea of dynamic games

[1]The seminal papers in this area are by A.K. Dixit, "The role of investment in entry deterrence," Economic Journal 90 (1980): 95-106; B.C. Eaton and R.G. Lipsey, "Capital, commitment, and entry equilibrium," Bell Journal of Economics 12(3) (Fall 1981): 593-604.

and how the selection of different strategy spaces by firms may lead to different types of equilibrium.

The use of inventory to deter entry in a market is a logical extension of the use of capacity to achieve the same ends. The buildup of a sales capacity (as opposed to a productive capacity) is the ideal illustration of investment of a product-specific capital; a stockpile of a manufactured good may be sold in that particular market, and represents a good whose entire costs of production have been sunk.

The advantages of holding an inventory for strategic purposes are twofold: the good is immediately available, and the marginal cost of supplying the finished good is nil[1]. The drawbacks of holding an inventory for strategic purposes become apparent in the medium to long run: every unit sold depletes the inventory by that amount. On the other hand, a productive capacity may produce the good for as long as necessary[2].

Hence, whereas an inventory is an immediate threat to potential entrants, the use of productive capacity to achieve the same ends is a much more sustainable threat.

Two articles examine the use of inventories for strategic

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[1] L. Arvan, "Some examples of dynamic Cournot duopoly with inventory," Rand Journal of Economics 16(4) (Winter 1985): 570.

[2] This remark is contingent on the product-specific capital not wearing out.

motives. First, an article by Arvan[1] examines the use of inventories by a symmetric duopoly operating in the first period to affect the outcome of the industry in the second period. The central point of the article is that in equilibrium it isn't possible for both firms to influence the market outcome through sales of their inventory at the same time: that is, if it is possible to store the output, the market equilibrium in the second period will be asymmetric[2]. The absence of symmetric equilibrium when both firms carry inventory from one period to the next is guarenteed by noting that if this were not so both firms would use their inventory to act as leader[3].

The thrust of this article is the demonstration that through the use of inventories it is possible to influence the industry outcome of duopoly. This article then is best viewed as a complement to the thesis expounded by R. Ware[4], namely that a single firm functioning as a monopoly in a market may signal to potential entrants its determination to remain in the market by building up an inventory. This constitutes the ideal expression of product-specific capital:

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[1] Ibid.

[2] Ibid, p. 572. The reason for this is that the ability to commit inventory during the first period is "tantamount to acting as a leader in the period two subgame, but only for some joint inventory vectors". As a result the firms' value functions are not concave in inventory, and hence are ill-behaved.

[3] Ibid, p. 574.

[4] R. Ware, "Inventory Holding as a Strategic Weapon to Deter Entry," Economica 52 (1985): 93-101.

an inventory held by a monopoly has no use other than on that market in particular. Whereas in Dixit's model there are no inventories in spite of its investments in capacity the firm may never set the price at a level below the variable cost of production. A further strength of the argument is as follows: the opportunity cost of a sunk cost is nil. Hence in the presence of entry in the market by a firm, the opportunity cost of the incumbent's inventory is nil. Then the incumbent may deplete as much of his inventory as he deems necessary to drive down the price, even going below variable cost so that the entrant's value function be negative.

If the holding of an inventory is too costly relative to the benefits attainable through monopoly profits then entry may be accommodated with a symmetric equilibrium holding after entry has occured. However under certain conditions it may be optimal for the incumbent to hold inventories. This is demonstrated as follows: the buildup and holding of inventories constitutes a holding cost; however after entry has occured the inventory may be sold off in such a way as to influence the payoff from the postentry game[1]. Which one of the two effects described above is greater is uncertain.

In summary, the use of inventories for strategic motives is a realistic illustration of the use of inventories for the purpose of modifying market equilibrium.

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[1]Ibid, p. 97.

Speculative Motive

This motive may take two forms. They are:
(i) In a market structure characterized by uncertainty producers will maintain stockpiles for speculative purposes if they expect the value of the product to rise[1], or if there are increasing returns to scale, or if increases in input prices are expected[2].
(ii) In a market structure characterized by certainty it is possible to have speculative stockpiling in the presence of a monopoly; for example, if the absolute value of elasticity increases with increasing output (i.e. as quantity increases the price elasticity of demand decreases) then the supplier may set a pricepath increasing faster than the rate of interest. Because of possible capital gains to be made speculators will wish to hoard the resource [3].

Precautionary Motive

When there is an uncertain demand for a product a firm may wish to maintain a buffer stock to avoid being caught short by an abrupt surge in demand. The optimal level of inventory is determined by the tradeoff between the opportunity cost of

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[1]The question of how expectations are formed is ignored here.

[2]This point is due to S. Honkapohja and T. Ito, "Inventory Dynamics in a Simple Disequilibrium Macroeconomic Model," Scandinavian Journal of Economics 82(2) (1980): 185.

[3]This example is provided by P. Dasgupta and G. Heal, Economic Theory and Exhaustible Resources (Cambridge: Cambridge University Press, 1979), pp. 329-330. The authors also add the proviso that this case is not very plausible.

being caught short and the cost of maintaining inventories. Defined in this way an inventory maintained for a precautionary motive is a "buffer stock"[1].

Transactions Motive

This motive may arise in the context of both certainty and uncertainty. If there are either anticipated or unanticipated fluctuations in demand throughout the life of the firm, adjustment costs of production may be very expensive, as well as setup costs (if another plant is needed)[2]. When these fluctuations arise inventories are used to smooth out the production schedule[3]. Even perfectly foreseen fluctuations in demand do not invalidate this motive for stockpiling.

Intertemporal Price Discrimination

This motive consists of price discrimination in time: it implies the optimum distribution of a total supply between markets that are separated in time[4]. Stockpiling for purposes

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[1]Exogeneous buffer stocks in a dynamic oligopoly model with demand inertia are analyzed by L. Phlips and J.-F. Richard, "A Dynamic Oligopoly Model with Demand Inertia and Inventories," CORE Discussion Paper No. 8603 (Université Catholique de Louvain, Belgique.) Other analyses involving buffer stocks are due to A. Blinder, "Inventories and Sticky Prices: More on the Microfoundations of Macroeconomics," American Economic Review 72 (1982); J. Green and J.-J. Laffont, "Disequilibrium Dynamics with Inventories and Anticipatory Price-Setting," European Economic Review 16 (1982).

[2]Honkapohja and Ito, "Inventory Dynamics," p. 185.

[3]A practical example of this may be startup and shutdown costs for an automobile assembly line.

[4]Shaw, "A Theory of Inventory", p. 469.

of intertemporal price discrimination constitutes a special case of present-value profit-maximization. Normally, when a firm maximizes a present value flow of revenue over time, it is straightforward to show that along the optimal sales and production paths the firm should equate its instantaneous marginal revenue with its instantaneous marginal cost. If some of the model's parameters are time-dependent then at some point it may be optimal to "bunch up" production to avoid higher costs of production or prepare for a greater demand. This is a central point in the case for stockpiling for purposes of intertemporal price discrimination[1] in a deterministic setting: the cost and demand functions may shift over time, usually upwards but sometimes downward[2]. Otherwise there is nothing to explain why an optimal sales and production path determined at the outset might not hold at some point in the firm's life[3].

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[1]The discovery of the principle of intertemporal price discrimination goes back to A. Smithies, ("The Maximization of Profits over Time with Changing Cost and Demand Functions," Econometrica 7 [1939]: 312-318) and E. Shaw ("A Theory of Inventory"). It is relevant to note that Smithies' preoccupation in discussing intertemporal price discrimination was to go beyond the standard condition for profit-maximizing in a dynamic model. Smithies states that "...the device..[...]...of assuming that an entrepreneur equates marginal cost with marginal revenue at every point in time is inadequate since it assumes that he lives a hand-to-mouth existence, producing at every point of time the amount that he sells at that time." (Smithies, "Maximization of Profits over Time", p. 312).

[2]Smithies, "Maximization of Profits over Time", p. 314.

[3]Smithies states that the demand and cost functions need not shift only upwards. However, he adds that a decrease in the cost and demand schedules over time can hardly lead a producer to stockpile. (Ibid.)

Smithies' contribution has been summarized by Phlips and proceeds as follows[1]: in a deterministic setting with time-dependent cost, demand, and price functions with a constant rate of interest the problem facing the firm is how to maximize the present value of its profit-function over the firm lifespan subject to the constraint that the total quantity sold must be equal to that produced over the life of the firm[2]. The solution to this problem determines the optimal sales and production path; it yields the <u>intertemporal discrimination rule</u>[3], namely

$$e^{-rt}\left[p + \frac{\partial p}{\partial q} q\right] = e^{-rt}\left[c + x\,\frac{\partial c}{\partial x}\right] = \lambda$$

i.e. at all points in time the discounted marginal revenue from sales must equal the discounted marginal cost from production, and is constant.

One more condition necessary to make voluntary inventories possible is that the rate of increase of the demand and the cost functions be greater than the rate of interest, with the rate of increase of the cost function greater than that of the demand function.

The derivation of a formal solution for Smithies' problem

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[1]This summary is due to L. Phlips, "Intertemporal Price Discrimination and Sticky Prices," <u>Quarterly Journal of Economics</u> XCIV(3) (1980), pp. 527-530.

[2]In the presence of a resource availability constraint this problem is solved in subsequent chapters for certain industry structures.

[3]Smithies, "Maximization of Profits over Time", p. 316.

is carried out presently: if the firm must maximize its discounted flow of profits, defined as total sales minus total revenues, subject to inventory and sales non-negativity constraints, we have

$$\max_{q,x} \int_0^T e^{-rt}\left[pq - cx\right] dt \quad \text{s.t.} \quad \begin{cases} \int_0^T (x-q)\,dt = 0 \\ S(0),\ x(0),\ q(0) = 0 \\ S(T),\ x(T),\ q(T) = 0 \end{cases}$$

where x,q,S denote, production, sales, and inventories respectively, and time subscripts have been deleted. The Hamiltonian function corresponding to this problem is

$$H = e^{-rt}(pq - cx) + \mu(x-q) + \nu(x-q)$$

where μ is the adjoint variable corresponding to inventories, and ν is a non-negativity constraint.

The necessary conditions for a maximum yield

$$\frac{\partial H}{\partial x} = -e^{-rt}\left[c + x\frac{\partial c}{\partial x}\right] + \mu + \nu \leqslant 0$$

$$\frac{\partial H}{\partial q} = e^{-rt}\left[p + q\frac{\partial p}{\partial q}\right] - \mu - \nu \leqslant 0$$

$$\mu(t) = \lambda = \nu(T)$$ i.e., $\mu(t) = \lambda$, some constant, because the adjoint equation is zero.

$$\nu(0) = 0$$

In the case where q>0, x>0 and S>0 Smithies' discrimination rule holds and we have an interior solution. If q>0, x>0, but S=0 then ν >0 and must be added to λ .

In summary, the essential aspect of intertemporal price discrimination is that the firm set a constant rate at which to equate marginal revenue and marginal cost, expressed in present-value terms.

Stockpiling for Environmental Motives

The extraction of nonrenewable resources from the ground and the preservation of the environment often constitute two activities that are at odds with one another. In an attempt to reconcile these two activities, T. Lewis examines the effect on a firm's sales and extraction path of placing a fixed cost on a firm that extracts a resource[1]. Because of the imposition of a fixed cost in an extractive firm's profit function to capture the value of the environmental damage caused extractive activities, there may emerge new optimal sales and extraction levels that are not necessarily equal. The imposition of a fixed cost results in a U-shaped average cost curve, which may be conducive to the economic stockpiling of resources as the optimal rate of extraction may exceed consumption in certain periods[2].

The object of the study is to determine whether the imposition of a fixed environmental cost affects resource consumption relative to the socially optimal rate of consumption. These results are then contrasted with those obtained when the possibility of storage is introduced through the U-shaped average cost curve.

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[1]T. Lewis, "Energy vs. the Environment," California Institute of Technology Working Paper No. 126 (1976).

[2]D. Lee and D. Orr, "The Private Discount Rate and Resource 'Conservation'", Canadian Journal of Economics VIII(3) (1975): 351-363.

Stockpiling Caused by Indivisibilities of Production

The issue of storage becomes important because of the residue left from the separation of the primary ore from the composite ore, leaving a residue[1]. It may be argued that changing technologies as well as economic growth might make the residual ore(s) potentially valuable. An additional question is whether industry or the government should undertake storage of the residual ore, since there is nothing to guarantee the industry that there may ever be a demand for the residual ore.

The Price Stabilization Motive

This area of analysis of stockpiling is concerned with attempts to stabilize commodities that are subject to systematic variability as a result of variability of supply or demand, i.e., a movement in price that is due to shifts, unforeseen or not, in either the demand or supply curves. The most prevalent example of price variability is the market for agricultural commodities, the supply of which is subject to random fluctuations, generally because of incomplete markets. So as to reduce the loss of consmer and producer surplus inherent in price variability it has been argued that it is desirable to stabilize commodity prices. This is done by the upkeep of buffer stocks[2]. Whether the upkeep of buffer stocks to reduce

[1] R.S. Pindyck, "Jointly Produced Exhaustible Resources," Journal of Environmental Economics and Management 9(4) (1982): 291-303.

[2] In May 1976 a special meeting of UNCTAD passed a

price variability is desirable or not is a matter of debate[1].

Discussion of Motives

This thesis deals exclusively with stockpiling by resource extracting firms for purposes other than price stabilization. Hence the last motive for stockpiling examined above will not be considered.

Carrying inventories for speculative purposes in a market characterized by price uncertainty has never been analyzed in a resource market.

Inventories maintained for strategic purposes constitute a subset of the literature on entry deterrence and commitments. This field lies beyond the scope of this paper, and hence will not be considered here.

The use of buffer stocks to avoid being caught short by unexpected surges in demand can only be justified in a context of uncertainty. Furthermore the range of decision rules available to the firm keeping inventories for precautionary

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resolution calling for the establishment of an Integrated Programme for Commodities in order to stabilize prices of ten "core" commodities. This resolution was endorsed by the Brandt Commission Report in 1980. (D.M.G. Newbery and J.E. Stiglitz, "Optimal Commodity Stockpiling Rules," Oxford Economic Papers 34(3) [November 1982)]: 403).

[1]Some analyses done on this subject are by S.J. Turnovsky, "The Distribution of Welfare Gains from Price Stabilization: A Survey of Some Theoretical Issues," in Stabilizing World Commodity Markets, ed. F.G. Adams and S.A. Klein (Lexington, 1978), pp. 119-148; D.M.G. Newbery and J.E. Stiglitz, The Theory of Commodity Price Stabilization (Oxford: Clarendon Press, 1981).

motives is quite large; i.e., these rules may vary in scope from the automatic stockpile of some past period's worth of sales, to estimations of consumers' expectations with respect to upcoming sales. For these reasons stockpiling for precautionary motives will not be examined here.

Throughout the course of this thesis we shall avoid dealing with the transactions motive by assuming that when there is extraction, costs of adjustment are nil.

The thrust of this thesis is placed on the analysis of stockpiling for purposes of carrying out intertemporal price discrimination by a firm under different market structures, varying from perfect competition to pure monopoly, and to some extent on ondivisibilities of production and environmental motives.

CHAPTER III

THE ANALYSIS OF STOCKPILING AND THE COMPETITIVE MINING FIRM UNDER CERTAINTY

Introduction

The theory of the single competitive mining firm operating within a competitive industry has been well-documented in the resource economics literature, but the issue of storage in the same context has received little attention[1]. The usual assumption is that no storage takes place.

This chapter examines the issue of storage in the context of the competitive extractive firm. The outline is as follows: the first section examines the assumptions underlying the classical competitive model of the mine in general equilibrium. Then the profit-maximizing behavior and the optimal extraction path of an extractive firm in a framework of partial equilibrium is analyzed. The third section discusses some of the main results of the preceding section. These include the well-known result

[1]There are two exceptions to this point. First, an article by D. Lee and D. Orr, "The private discount rate and resource 'conservation'," (Canadian Journal of Economics VIII(3) [1975]: 351-363) analyzes a monopoly that stores a processed resource above ground. Second, R. Pindyck, in "Jointly Produced Exhaustible Resources" examines an extractive firm that faces indivisibilities of production and is obliged to store one or several of its outputs. (Journal of Environmental Economics and and Management 9 (1982): 291-303.

that at the margin the user cost of the resource be the foregone value of future profits resulting from current sales. The circumstances under which the mine is exhausted at the extractive horizon are reviewed, as well as the conditions under which the mine will stop extracting. The links between the net price, the spot price and the r-percent rule is examined. This is carried over in the following section where the limiting cases to the r-percent rule are examined.

The last section examines the behavior of a competitive firm in a partial equilibrium setting that has the option of storing the resource at zero-cost above ground once it has been extracted. The result of the analysis is that the firm's decision to store the resource is contingent on present and future price. In particular, if the price of the resource is constant it is not optimal to engage in storage at any time. However, if the price of the resource is a variable it is possible that the firm stockpile its output. There is a proviso attached to the result that storage is possible, namely that the firm cannot engage in selling and stockpiling at the same time. The only time when storage is possible is when sales are nil. However when sales are positive they also occur only at the maximum rate.

The Assumptions of the Neoclassical Model

Stocks and Flows in General Equilibrium

In the context of an entire economy a deposit of ore constitutes a capital good for its owner. It is a capital good in that sense that it provides a flow of services to society over a period of time. Hence the value of a deposit of ore is the present value of future sales of that resource[1].

As the deposit provides services to society it loses its value. Thus the value of a capital good is realized in two ways: by selling the capital good or by selling its services. If the owner chooses not to exploit it, and if the market for investment goods is in equilibrium, then the value of a capital good must increase at a rate equal to that of interest[2,3].

If a resource <u>in situ</u> can be extracted costlessly then no costs of extraction need be appended on top of the intrinsic value (or the asset value) of the resource. In such cases, the price levied in the marketplace (the "spot price") is the same as

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[1]R. Solow, "The Economics of Resources or the Resources of Economics," <u>American Economic Review</u>, 64 (1974); reprinted in <u>Economics of the Environment</u>, 2nd ed., ed. R. Dorfman and N. Dorfman (New York: Norton), p. 356.

[2]If the value of one asset is increasing at a rate less than that of interest, its owner can sell it and invest the proceeds at the rate of interest.

[3]Were this chapter dealing with uncertainty the value of a deposit would increase at the same rate as any other investment in that particular risk class. However, because of the assumption of certainty, it is sufficient that the value of the deposit increase at the rate of interest.

the price of the asset (the "net price")[1].

If the value of an asset is appreciating at a rate equal to that of interest its owner will be indifferent to selling his resource today at an undiscounted price or tomorrow at the same price discounted at the prevailing rate of interest. This notion is expressed in the relation

$$p(t) = p_o e^{rt} \qquad [1\text{-}1]$$

where p_o is the endogeneously-determined net price in the current time period $t=0$. Taking the time-rate of change of (1-1) we find the "Hotelling Rule"[2]

$$\frac{\dot{p}(t)}{p(t)} = r \qquad [1\text{-}2]$$

where $p(t)=p_o e^{rt}$ is the time-pricepath of the resource in general equilibrium.

The importance of equations [1-1] and [1-2] is as follows: first these equations are a condition for stock equilibrium; that is, the value of a deposit must increase at a rate equal to that of interest. Hence net price must increase at the same rate as all other investments in the economy[3].

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[1] This convention is due to H. Hotelling, who defined net price as "...the[...]...price received after paying the cost of extracting and placing it upon the market." (H. Hotelling, "The Economics of Exhaustible Resources," Journal of Political Economy 39(2) [1931]: 141).

[2] Henceforth a dot over a variable will indicate a time-rate of change.

[3] This statement holds given a simple model of the "Hotelling" type; this means there are no costs of exploration, the grade of ore is homogeneous, and so on.

If there are positive costs of extraction then it is the price net of extraction costs that must increase at a rate equal to that of interest[1]. Secondly these two equations are a condition for flow equilibrium: if net price is increasing at the prevailing rate of interest then owners will be indifferent to extracting or holding the resource at every moment of time. This is the condition that permits the ore market to clear: at the going price demand is equal to the supply[2].

In the context of flow equilibrium a final implication of equations [1-1] and [1-2] is the following: if the rate of increase of the net price is greater than that of the rate of interest, owners will want to hold on to their resource and keep the resource in the ground; conversely, if net price is increasing slower than the rate of interest owners will wish to dispose of their resource as fast as possible[3].

The stock and flow equilibrium are thus two different markets explained through the same rule, which create a link with the theory of capital and resource deposits. Hence a resource deposit has the characteristics of an investment good, and must

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[1]This result holds only in a general equilibrium setting with a positive rate of interest. Should the rate of interest be zero then the value of a unit of the resource would be constant over time. It is admissable to assume that the time-pricepath of a resource is constant in a context of partial equilibrium.

[2]Solow, "The Economics of Resources," p. 357.

[3]Ibid., p. 361.

be treated as such.

User Cost

It has been established that a deposit of ore is a capital good. Thus to use a capital good for the purpose of providing a good or service entails a decrease in the value of that capital good. In a very broad sense this constitutes a "user cost", i.e., the cost that is incurred when capital is used to produce output[1].

It is relevant to note that a complete definition of "user cost" was presented by Keynes in the General Theory. It is:

> ...the reduction in the value of the equipment due to using it as compared with not using it, after allowing for the cost of the maintenance and improvements which it would be worth while to undertake and for purchases from other entrepreneurs.[2]

This definition was derived for the purpose of defining aggregate income of firms[3]. It is the entrepreneurs' links between the present and the future. To paraphrase Keynes, the user cost determines an entrepreneur's rate of production by

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[1]G. Heal and M. Barrow ("The Relationship between interest rates and metal price movements," Review of Economic Studies XVII [1980]: p. 161) comment that in particular, if owners of the resource regard it as a capital asset constituting part of their portfolio they will hold it only as long as the return it gives them is no less than that available elsewhere.

[2]J.M. Keynes, The General Theory, p. 70.

[3]V. Chick, Macroeconomics after Keynes (Cambridge: The M.I.T. Press, 1984), p. 47.

determining the balance to be struck between using his equipment now and preserving it for future use.

The purpose of allowing for user cost is summarized by Keynes: "...if a ton of copper is used up today it cannot be used tomorrow, and the value which the copper would have for the purposes of tomorrow must clearly be reckoned as part of the marginal cost."[2]

Hence user cost is a distinguishing feature of a resource which is exhaustible: it is an opportunity cost over time of using a resource now as opposed to using it at some date in the future: the value that a unit of ore would have had tomorrow must be counted as part of today's marginal cost of production. This is because in so doing one is sacrificing tomorrow's revenue for consumption today.

Thus the net price of a resource that has any future value tomorrow can never be zero today. This reflects the irreplaceability of the resource.

To summarize, then, the intertemporal opportunity cost of using the resource today is the user cost. Because it is a tradeoff of present sales for future revenue it is a cost associated with the exploitation of a mineral deposit because that deposit is a capital good.

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[1] J.M. Keynes, The General Theory, pp. 69-70.

[2] Ibid., pp. 72-73.

Optimal Extraction Plans of the Mine

Assumptions of the Model

This analysis is limited to partial equilibrium. It is assumed that the industry operates under perfectly competitive conditions. The assumptions underlying the behavior of the representative firm include perfect future markets, free entry and exit, a fixed amount of known reserves[1], an ore of homogeneous quality, and constant conditions over time. The last assumption includes the condition that the owner knows the costs of all inputs with certainty, and furthermore that their prices are constant. In addition the rate of interest faced by all producers is positive and uniform throughout the industry. Finally it is assumed that all capital is perfectly malleable and shiftable, i.e., the capital stock is perfectly divisible, and can be transferred to and from alternative uses at no cost[2].

The cost function of any firm is assumed to be independent

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[1]The assumption that the resource extracting industry is endowed with a fixed resource base is important; some authors have shown that when a resource base is variable the usual predictions of the theory of the extractive firm will not hold. Some authors who have analyzed this phenomenon are R. Pindyck ("The Optimal Exploration and Production of Nonrenewable Resources;" [Journal of Political Economy 86(5) (1978)]); M. Eswaran and T. Lewis, ("Ultimate Recovery of an Exhaustible Resource under Different Market Structures," [Journal of Environmental Economics and Management 11 (1984)]); R. Gilbert, ("Optimal depletion of an uncertain stock," [Review of Economic Studies 46 (1979)]); J.M. Hartwick, ("Learning about and exploiting exhaustible resource deposits of uncertain size," [Canadian Journal of Economics XVI(3)]).

[2]This assumption allows for a cost function where all

of any level of remaining reserves, time-autonomous, and either convex or ogive-shaped[1,2]. It is possible for fixed costs to prevail but they must be of the type avoidable at the end of the production run, e.g. the rent on buildings or equipment need not be paid when production has ceased[3].

The Hotelling price-rule discussed in the preceding sections deals with the industry as a whole. Because this particular analysis is limited to partial equilibrium the resource's price-path may be anything: what is important is that it be a known parameter of the model.

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capital costs are associated with variable costs. If the assumption of capital malleability is relaxed the standard results of the mine are modified. Some authors who have pursued the line of analysis are T. Puv ("On the Profitability of Exhausting Natural Resources," [Journal of Environmental Economics and Management 4 (1977)]), H. Campbell ("The Effect of Capital Intensity on the Optimal Rate of Extraction of a Mineral Deposit," [Canadian Journal of Economics XII(2) (1980)]), G. Gaudet ("Investissement optimal et coûts d'ajustement dans la théorie économique de la mine," [Canadian Journal of Economics XVI(1) (1983)]), P.J. Crabbé, ("The effect of capital intensity on the optimal rate of extraction of a mineral deposit. A comment," [Canadian Journal of Economics XV(3) (1982)]), T. Lewis ("A note on mining with investment in capital," [Canadian Journal of Economics XVIII(3) (1985)]).

[1] A. Scott, "The Theory of the Mine under Conditions of Certainty," in Extractive Resources and Taxation, ed. M. Gaffney (London: University of Wisconsin Press, 1967), p. 29.

[2] H. Burness writes that the existence of an interior minimum requires a "neoclassical" cost function, exhibiting first decreasing then increasing marginal costs and/or the presence of positive fixed costs. (H.S. Burness, "On the Taxation of Nonreplenishable Natural Resources," Journal of Environmental Economics and Management 5 [1978]: p. 291).

[3] Ibid., p. 292.

The Optimal Extraction Path of the Mine

The owner of a mine seeks to maximize the present value of his asset[1]. Accordingly he seeks to maximize the present value of his profit-function over the lifespan of the mine. Therefore the owner's problem may be represented as

$$\max \int_0^T e^{-rt} \pi(t)\,dt \qquad [2\text{-}1]$$

where T represents the lifespan of the mine. The firm's profit function is

$$\int_0^T \left[p(t)q(t) - c\big(q(t)\big) \right] e^{-rt}\,dt \qquad [2\text{-}2]$$

where p(t) is the known price-function, e^{-rt} is the continuous-time discount term, and C(q(t)) is the cost function twice differentiable and increasing in q(t). Specifically the characteristics of C(q(t)) are[2]

$$c(0) = 0 \;;\; c\big(q(t)\big) > 0 \;;\; c'\big(q(t)\big) > 0 \;;\; c''\big(q(t)\big) \geq 0. \qquad [2\text{-}3]$$

Exhaustibility implies that the firm cannot extract more than the initial deposit of ore, or

$$\int_0^T q(t)\,dt \leq R(0) \qquad [2\text{-}4]$$

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[1] Hotelling, "The Economics of Resources," p. 140.

[2] The possibility of fixed costs that are avoidable when production is nil attest to the presence of quasi-fixed costs. These costs have been mentioned in the preceding section.

where $R(0)=R_o$ denotes the fixed stock of reserves at the initial period of extraction. During the extractive phase of the mine, remaining reserves are denoted by

$$R(t) = R_o - \int_0^t q(t)\,dt. \qquad [2\text{-}4a]$$

The time-rate of change of the above is

$$\dot{R}(t) = -q(t). \qquad [2\text{-}5]$$

Thus the time-rate of change of the deposit R_o is equivalent to the amount of ore that is extracted in each period.

Given the producer's profit function and resource constraint his problem is

$$\max \int_0^T \left[p(t)q(t) - c(q(t))\right]e^{-rt} \quad s.t. \quad \begin{cases} R(0) = R_o \\ \dot{R}(t) = -q(t) \\ q(t) \geqslant 0 \\ R(T) \geqslant 0 \end{cases} \qquad [2\text{-}6]$$

where the horizon "T" is endogeneously determined. The Hamiltonian function corresponding to the profit-function above is

$$H = e^{-rt}\left[p(t)\,q(t) - c(q(t))\right] - \lambda(t)\,q(t) \qquad [2\text{-}7]$$

where λ is the adjoint variable satisfying the state equation. The necessary conditions for the problem in [2-7] are[1]

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[1] The conditions contained here are the application of Proposition 3 formulated by K.J. Arrow and M. Kurz, <u>Public Investment, the Rate of Return, and Optimal Fiscal Policy</u>, Resources for the Future (Baltimore: John Hopkins University Press, 1970): p. 38.

$$\frac{\partial H}{\partial q(t)} = \left[p(t) - c'(q(t)) \right] e^{-rt} - \lambda(t) = 0 \qquad [2\text{-}8a]$$

$$\frac{d\lambda(t)}{dt} = 0 \Rightarrow \lambda(t) = \lambda \ \forall t \qquad [2\text{-}8b]$$

$$H(T) = e^{-rT} \left[p(T) q(T) - c(q(T)) \right] - \lambda(T) q(T) = 0 \qquad [2\text{-}8c]$$

$$\lambda(T) \geqslant 0 \qquad \lambda(T) R(T) = 0 \qquad [2\text{-}8d]$$

Assuming an interior solution, [2-8a] is implied by the maximum principle, [2-8b] is the adjoint equation, and [2-8c] and [2-8d] are the transversality conditions applicable to the problem at hand.

Equation [2-8b] implies $\lambda(t) = \lambda$. Hence [2-8a] may be rewritten

$$\left[p(t) - c'(q(t)) \right] e^{-rt} = \lambda \qquad [2\text{-}9]$$

By virtue of the maximum principle the problem of dynamic optimization has been "decoupled" into a series of static maximization problems at each instant "t" within the lifespan of the mine contained in [0,T]. The function H in [2-7] is divided in two parts. The first part is the interior of the objective functional, and gives the maximum instantaneous flow of dividends, or the profit rate. Hence over time it is the flow of dividends from a capital stock. The second part is the state equation: it represents the flow of "investment" in capital, or rather, because of its negative sign, disinvestment. It is an expression of how capital depreciates as a result of choices made[1]. For this flow to be expressed in value terms it must be multiplied by the "shadow price" of the resource, λ [2]. Hence the Hamiltonian function represents the instantaneous rate of increase of the value of assets, to be maximized at each instant of time. The application of the maximum principle to [2-7] asserts that there is an optimal depletion rate of the fixed stock of the resource; this application may be interpreted as the dynamic equivalent of the static Lagrangian function[3].

The necessary conditions contained in equation [2-8a] are explained in the following way: the marginal value of a unit of the resource during any period of time is equal to the spot price p(t) minus the marginal cost of extraction C'(q(t)), both expressed in present value. The adjoint variable $\lambda(t)$ represents the value of a marginal unit of the deposit; if the amount of the deposit is reduced by one unit, the value of the deposit must decrease by $\lambda(t)$. This implies that $\lambda(t)$ represents the value of a marginal unit of the resource: the income of a firm exploiting a resource deposit would grow by that amount if one additional unit of the resource were placed on the

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[1] R. Dorfman, "An Economic Interpretation of Optimal Control Theory," American Economic Review 59 (1969): 822.

[2] C. Clark, Mathematical Bioeconomics (New York: John Wiley & Sons, 1976), p. 104.

[3] This correspondence between the Lagrangian and the Hamiltonian functions is founded in A. Takayama, Mathematical Economics (New York: Cambridge University Press, 1985), p. 601.

market. This imputes a value - a shadow price or an "imputed rent" - to the deposit: if the amount of ore in the deposit is reduced by one unit, the value of the deposit must also decrease by the same amount.

The adjoint equation [2-8b] yields information on the rate at which a unit of the resource depreciates or appreciates at a moment in time[1]. According to Dorfman's interpretation[2], the magnitude of $\dot{\lambda}$ represents the rate at which "... its potential contribution to profits becomes its past contributions."

Still from [2-8b] the adjoint equation represents the rate at which a unit of capital appreciates. Hence the value of a resource is appreciating at a constant rate. Therefore no matter what the price p(t) happens to be at any particular moment, the imputed value of the rent is constant. Then the firm produces only if the spot price is at least equal to the value of the resource <u>in situ</u> plus the costs of extraction. If the spot price is insufficient to cover marginal costs of production plus the future value of the shadow price of the resource then production is nil.

Depending on the amount of the spot price and the sum of the marginal cost of extraction plus the rent on the resource it is conceivable that firms alternate between periods

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[1]Dorfman,"Optimal Control Theory," p. 821.

[2]Ibid., p. 821.

of shutdown and production.

Expressing user cost in terms of future value we have

$$\lambda e^{rt} = p(t) + c'(q(t)). \qquad [2\text{-}10]$$

An examination of [2-10] shows how the left-hand-side (LHS) is interpreted as "user cost": it is a sacrifice of future revenues due to present sales. A unit of ore that is extracted in the present cannot be retrieved later. Thus the cost of withdrawing a unit of the stock in the present must be balanced by the gain accruing from the benefit of the sale in the future[1]. Thus exhaustion, even partial, is a cost that must be considered. Furthermore the fact that the user cost is a future-value formulation indicates that it is an intertemporal notion; if user cost is an imputed value stemming from future productivity, it is expected that the value of this cost be a function of time.

A final interpretation of [2-10] is called for: still considering the firm's mineral deposit as a capital asset, the removal of one unit of that asset for sale depletes the fixed stock by exactly the amount of the adjoint equation. In so doing the firm reduces its possibility of selling that unit at some time in the future. Thus it constitutes the "user cost".

Equation [2-8c] is the transversality condition. The

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[1]Dorfman implicitly recognized this when he wrote "...the firm should choose [extraction] at every moment so that the marginal immediate gain just equals the marginal long-run cost." ("Optimal Control Theory", p. 820).

requirement that the Hamiltonian function at the horizon be zero is explained as follows: the objective of [2-7] has been to maximize the sum of the present value flow of profits. Once the resource deposit has been physically exhausted there is no purpose in continuing production. Hence we substitute [2-9] into [2-8c], both evaluated at time T and find that

$$\frac{C(q(T))}{q(T)} = C'(q(T)). \qquad [2\text{-}11]$$

This signifies that at the terminal time of extraction the marginal cost of extraction must be equal to the average cost of extraction. This occurs for the following reasons: over time the consumption of a present value flow of payments from an exhaustible deposit implies two conflicting forces. Given a U-shaped cost curve and a positive rate of interest a firm will want to produce as much as possible in one period to maximize its one-period profit, and hence avoid receiving a discounted flow of payments. On the other hand the more a firm increases the quantity extracted the more its average costs increase (and its average profits decrease). Hence a firm seeks to produce (ideally) at the minimum of its average cost curve, thereby maximizing average profit per unit[1]. The extent to which a producer will choose to maximize total profits over minimizing costs will be dictated by the rate of interest: a high rate of

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[1]There is a limiting case where there are no costs of extraction and no upper bound on the extraction of the resource per period. This problem is analyzed later in this chapter when examining the optimal rate of depletion of a costless stockpile.

interest indicates that a remote stream of payments in the future is less attractive; therefore producers will be less concerned with the future, and production will increase towards the maximum profit rate. The opposite holds true for a low rate of interest, all other things being equal.

Much the same phenomena causes the tilt of the extraction rates of the mine towards the present: the spot price of the resource is listed in present value terms. From [2-9] it is desirable to benefit from the spot price of the resource as quickly as possible to invest it in something else. As time goes by the net price of the resource increases like compound interest and firms slide up on the "stationary" market demand curve. When this happens firms slide down on their respective average cost curves until at the horizon they are maximizing average profit. In the case of a "bell-shaped" average profit function[1,2] this result has been stated formally by N. Long for a positive rate of interest[3]: he shows that as the resource stock tends to zero, the quantity produced declines over time until, at the horizon, the quantity extracted per

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[1]The formalization of the endpoint analysis involving the average profit function is due to N. Long, "Resource Extraction under the Uncertainty about Possible Nationalization," Journal of Economic Theory 10 (1975): 42-53.

[2]The characteristics of this function are found in Long, "Resource Extraction and Nationalization," pp. 47-48.

[3]Ibid. The author also examines the same problem with zero interest rates.

period is set so as to maximize average profit[1].

The examination of the second transversality condition [2-8d] requires the determination of the value of λ, specifically whether it is zero or positive. A first explanation is as follows: from the adjoint equation [2-8b] we know that λ is a constant. Now we need to establish that it is positive. Trivially, from [2- 9] if $\lambda=0$ then price equals marginal cost at all moments of extraction. Hence there is no scarcity rent, and the resource is not "exhaustible" any more. To avoid this pitfall λ must be positive.

A second justification for a positive value of λ is provided by H. Burness[2]; he notes that if λ is nil the solving of the transversality condition in [2-8c] yields a meaningless result: to obtain equation [2-11] it is necessary that λ be positive.

It is interesting to consider a final explanation for the positive value of λ : the theory of perfect competition specifies that in short-run equilbrium the market price may be greater than marginal cost. But the optimal price charged for the resource, as witnessed in [2-9], is price equals the marginal cost plus the present value of the rent. Noticing that

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[1]This result is not new. It was demonstrated by Orris Herfindahl ("Some Fundamentals of Mineral Economics," [Land Economics XXXI(1955): 132]); R. Gordon ("A Reinterpretation of the Pure Theory of Exhaustion," [Journal of Political Economy 75 (1967): 277) states that at the horizon the firm produces nothing or produces the output that maximizes average profits.

[2]Burness, "The Taxation of Resources," p. 292.

the transversality condition in [2-11] warrants that extraction stop at the point where average cost is equal to marginal cost[1] it must be that immediately before the terminal period of extraction λ must have been positive. This seems to imply that the industry is in short-run equilibrium when extraction stops. Insofar as the levying of scarcity rents entails keeping price above marginal cost in the segment of the market considered this is not a fanciful assumption to make[2].

We now turn to the examination of the transversality conditions contained in [2-8d]. There are two cases. They are:

(i) If the constraint is not binding then $R(T)>0, \lambda(T)=0$. This is impossible in light of the preceding discussion.

(ii) If the constraint is binding then $R(T)=0, \lambda(T)\geqslant 0$. There are two possible subcases. They are:

(a) $R(T)=0; \lambda(T)=0$. This case is possible but improbable as it implies that the deposit is exhausted at the same time that the value of the resource is nil.

(b) $R(T)=0, \lambda(T)>0$. This is the only acceptable solution: the resource is still valuable but the deposits are exhausted. This is also the only solution that considers the partial equilibrium context in which this analysis is set.

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[1] Unit average costs increase when a firm produces at the left of the minimum of average cost.

[2] Burness notes that in an entire industry it is not impossible that we witness a situation of perpetual short-run equilibrium. ("The Taxation of Resources", p. 291).

In conclusion case (b) above is the only acceptable solution: the resource is exhausted while it is still valuable.

Net Price, Spot Price, and the Rate of Extraction

One of Gordon's[1] characterizations of equation [2-10] is that under the assumption of perfect competition the user cost represents marginal profit, i.e., the greater the degree of competitiveness in the market the more the price tends towards marginal revenue until, under "pure competition" the spot price is identical to marginal revenue.

Rewriting equation [2-10] in terms of marginal profit, revenue, and cost, respectively, we have

$$M\pi(t) = MR(t) - MC(t) = \lambda e^{rt}. \qquad [3\text{-}1]$$

In a non-resource-based conventional competitive market there would be no economic profit: the difference between marginal revenue (the spot price) and marginal cost is nil. Thus marginal profit is zero. In a resource industry marginal profit is equal to exactly the user cost. Hence equation [3-1] gives another dimension to the concept of user cost: it is a reminder that even under the assumption of perfect competition there is an intrinsic cost in consuming a resource. This cost is an exponential value of the shadow price of the resource.

To examine the link between [3-1] and the Hotelling rule we take the time-rate of change of [2-10] and find, after

[1]Gordon, "A Reinterpretation of Exhaustion," p. 277.

alteration to the cost function[1,2]

$$\frac{M\dot{\pi}(t)}{M\pi(t)} = \frac{\dot{p}(t)}{p(t)-c} = r. \qquad [3\text{-}2]$$

That is, the percent rate of increase of marginal profit is equal to that of price minus the constant cost of extraction, which is equal to the rate of interest. Hence in the case when the cost of extraction is constant the Hotelling rule is verified: the percent rate of increase of the spot price of the resource (minus the cost of extraction) is equal to the rate of interest.

If costs of extraction were nil, [3-2] would reduce to the simplest form of the Hotelling rule[3]

$$\frac{\dot{p}(t)}{p(t)} = r. \qquad [1\text{-}2]$$

A comparison of [1-2] and [3-2] indicates that the rate of increase of the spot price without costs of extraction is less than that with (constant) costs of extraction.

If the form of p(t) is the same in both [1-2] and [3-2], then obviously [3-2] holds for a rate of interest that is smaller than that in [1-2]. When [3-2] is reduced to the

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[1] Equation [2-10] contains two unknowns: p(t) and C(q(t)). To avoid an indeterminacy in the timepath it is necessary to set one of these variables equal to a constant. Hence C(q(t)) is equal to some constant "c".

[2] Expression [3-2] holds only if the spot price prevailing at any time in partial equilibrium coincides with that prevailing in general equilibrium.

[3] If costs of extraction were nil [3-2a] would reduce to [1-2]. The case where costs are a function of the quality of ore is taken up in the next section.

form of [1-2] we have

$$\frac{\dot{p}(t)}{p(t)} = r\left[1 - \frac{c}{p(t)}\right] \qquad [3\text{-}2a]$$

The RHS of [3-2a] is smaller than that in [1-2]. Since we cannot assume that interest rates are different, it must be that the rate of increase of [3-2] is less than that of [1-2]. Said otherwise, the Hotelling rule is not a strict equality between percent rate of change in prices and the interest rate when there are positive costs of extraction.

Exceptions to the Hotelling Rule

The Hotelling r-percent rule states that the price or the value of a unit of resource rises continuously over time, so long as the rate of interest is positive. However, empirical studies of resource pricepaths suggest that exhaustible resource pricepaths have experienced long secular declines[1].

Different theoretical models have been proposed to account for the seeming discrepancy between the theoretical predictions and the empirical results. These models invoke explanations ranging from the presence of cumulative extraction, durable exhaustible resources, exploration, and technological change.

Hotelling's article recognized that if the grade of ore

[1]This evidence is to be found in M. Slade, "Trends in Natural-Resource Commodity Prices: An Analysis of the Time Domain," Journal of Environmental Economics and Management 9(2) (1982): 122-127. A more complete study is found in H. Barnett and C. Morse, Scarcity and Growth (Baltimore: John Hopkins University Press, 1963).

is not homogeneous the cost of extraction will be a function of how much has been extracted from a mine up to a particular moment[1].

In his discussion of Hotelling, Gordon incorporates Hotelling's degradation term[2,3] into his profit function. He finds that upon maximization of his constrained profit-function that "...nothing like the r-percent rule can be derived." Nevertheless he does state that marginal profit will be reduced[4]. This implies that either the user cost will be lower, or the spot price higher, or both.

Levhari and Liviatan deal with cumulative extraction through the introduction of "full marginal cost"[5]. Through the explicit introduction of a degradation term in the cost function[6] they derive an alternative expression for marginal

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[1]Hotelling, "The Economics of Resources," p. 152.

[2]Hotelling's analysis of the problem of degradation indicates that he believed ore degradation was continuous, not by layers.

[3]From a chronological point of view it appears that Orris Herfindahl ("Depletion and Economic Theory," in <u>Extractive Resources and Taxation</u>, ed. M. Gaffney [London: University of Wisconsin Press, 1967], pp. 63-90) was the first author after Hotelling to address the problem of differing grades of ore. He found that the best ore will be used first. The same conclusion is derived in Solow ("The Resource of Economics," p. 358).

[4]Gordon, "A Reinterpretation of Exhaustion," p. 278.

[5]D. Levhari and N. Liviatan, "Notes on Hotelling's economics of exhaustible resources," <u>Canadian Journal of Economics</u> X(2) (1977): 177-192.

[6]$C[q(t), x(t)]$; $C_x \geq 0$

revenue[1]. In their case it is clear that the full marginal cost must include a term that reflects degradation costs: they interpret this last term as the present value of all future additional costs of production caused by ~~the~~ decision to extract the last unit at time t[2]. By differentiating the expression for marginal profit with respect to time the authors show that the price-path of a resource to which full marginal cost has been applied is shallower than that of resource models with no cumulative extraction.

This result coincides with the prior results obtained while discussing the r-percent rule; that is, the lower are the costs of extraction, the closer is the time-price-path to the rate of interest.

In the context of general equilibrium, where a continuum of grades of ores is available there is an analysis by Schulze worth noting: in a context of general equilibrium, where different grades of ore exist over a continuum of quality and where use is sequential, the price-path of a resource will increase at a rate equal to that of itnerest. That is, within layers the Hotelling r-percent rule will hold[3].

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[1] Levhari and Liviatan, "Notes on Hotelling," p. 181.

[2] Ibid.

[3] W. Schulze, "The Optimal Use of Non-Renewable Resources: The Theory of Extraction," Journal of Environmental Economics and Management 1(1) (1974): 53-74. The author makes several assumptions for this result to hold; in particular it is important that perfect futures markets exist to coordinate stocks and flows.

The second circumstance in which the r-percent rule will not hold is in the case of durable exhaustible resources. In an effort to explain the U-shaped characteristic of an observed resource price-path, it is suggested[1] that the demand characteristics of a nondurable resource are different from those of durable resources. Specifically, the demand for a perfectly durable resource is a stock relationship because the good provides utility as long as it is held; the demand is for a stock in circulation. Conversely, the demand for a nondurable resource is a flow, because once it is consumed it no longer provides utility; this demand is for a flow of production.

Based on this distinction the authors characterize the demand for the marginal value of services derived from a stock of a durable resource already in circulation; this enables them to determine the dynamics of demand of resources endowed with varying degress of durability[2]. Postulating a finite stock of reserves, an increasing variable cost of extraction[3], a perfectly durable resource and a static (time-autonomous)

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[1]This evidence is referred to by D. Levhari and R. Pindyck, "The Pricing of Durable Economic Resources," Quarterly Journal of Economics XCVI(3) (1981): 365-366.

[2]The authors examine the dynamics of three models. These embody, respectively, the following characteristics: perfectly durable resource, static demand; partially durable resource, static demand; partially durable resource, and growth in demand. An analysis of the three models yields a decreasing pricepath in the first case, and a U-shaped pricepath in the other two cases.

[3]If, for a perfectly durable resource the cost of extraction is constant or nil, then the stock in circulation can only increase, and eventually the price and the rent will eventually fall. (Ibid., p. 366).

demand curve, the authors find that the competitive market price falls as the stock of the resource increases; eventually the price tends asymptotically towards zero. However, while the market price decreases the rent increases at a rate equal to that of interest. This occurs because as output falls, the marginal cost of production falls faster than the price[1]. Therefore, the rate of decrease depends on the characteristics of the function depicting the marginal value of services from a stock of a durable resource already in circulation and the level of reserves of the resource. If the resource is listed as being partially durable (i.e., one tht depreciates over time but at a slower rate than a non-durable) and the demand function is time-autonomous, the price will fall initially as the stock in circulation increases and later rise as the depreciation of the stock in circulation sets in[2].

This model allows the authors to reconcile the empirically observed (U-shaped) pricepaths with the rate of increase of the price of the resource predicted by the theory.

A third instance when the r-percent rule will not necessarily hold is when there is simultaneous exploration and extraction of a resource. Instead of assuming that a producer

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[1] Ibid., p. 372.

[2] By this explanation, if the demand function increases with time it is conceivable that the use of the resource be too strong to increase the stock in circulation. In the case the resource pricepath would always increase.

is "endowed" with reserves, it is more economically meaningful to assume that new discoveries are possible during extraction. A resource model that uses this assumption is due to Pindyck[1]. Specifically, his model embodies two features that the "neoclassical" model lacks: there is a term denoting cumulative extraction (i.e., the ore is not of a uniform grade) and as exploration and discovery proceed over time, the quantity of exploratory effort is subject to decreasing returns. Hence, as opposed to costless extraction, the equation for the price dynamics must include the time-rate of change of the cost function, cumulative extraction, and the declining returns from exploration[2]. The pricepath is increasing over time. Not surprisingly, the horizon and the initial price for the producer that pursues exploration while producing are, respectively, farther and lower than a similar producer that does not carry on any exploratory activity. This is because the exploring producer can count on a larger total resource availability than the producer who does not have this option.

The determination of the optimal exploration effort presents a problem: on the one hand there is a willingness to postpone exploratory efforts (costs) as much as possible; on the other hand there is the high cost of extraction caused by nearly depleted reserves. The author presents two

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[1] R. Pindyck, "The Optimal Exploration and Production of Nonrenewable Resources," Journal of Political Economy 86(5) (1978): 841-861.

[2] Ibid., p. 845.

solutions: if initial reserves are large then exploration may be delayed in the future, with the contrary holding true for small initial reserves[1]. It is the latter case that gives rise to a U-shaped pricepath and a bell-shaped extraction path: initially output is low and price is high. As new discoveries are made extraction costs decrease and extraction increases. Eventually exploration must be subject to decreasing returns. This causes less reserves, with a correspondingly increasing pricepath and a decreasing extraction path.

A final explanation for U-shaped pricepaths has been proposed by M. Slade[2]; at the heart of the model proposed by her lies the observation that while resource prices have dropped over at least an 80-year period, the grade of the ores extracted has been either constant or has declined[3], but at the same time there has been at least one significant technological advancement in the mining of the ore - gradually introduced over a number of years - that has enabled the price of the ore to decline over a segment of the time period examined.

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[1]Ibid., p. 847.

[2]Slade, "Trends in Resource Commodity Prices".

[3]Slade cites the case of copper. In 1900 the average grade of copper ore mined was 5 percent per unit, compared to 0.7 percent today. In spite of this real copper prices fell from 1900 to 1940.

Through the use of a Hotelling-type model modified to take account of exogeneous technological change in mining technology, Slade derives an expression for a pricepath that is equivalent to the rate of change in the marginal costs of extraction due to changes in technology plus the rate of increase of the rent that accrues to the resource. If technical change is zero, the rate of increase of the price is equal to the rate of increase of the marginal value of the resource in the ground. Slade's equation depicting the rate of increase of the market price of the resource is noteworthy because a technical improvement actually appears in a negative way in the equation; if the rate of technical change is sufficiently large relative to the increase in the value of the resource, the pricepath will be negative.

In order to explain why the pricepath is U-shaped and not continuously decreasing (as befits a model exhibiting constant improvements in technology), Slade invokes once again the case of copper; she argues that most of the technological improvements in the mining of that metal were made in the first part of this century. Moreoever, a particular characteristic of the introduction of a new industrial process is its gradual introduction over a number of years. Hence the diffusion of a new process is a gradual phenomenon that occurs at a decreasing rate[1].

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[1] M. Slade, "Noninformative Trends in Natural Resource Commodity Prices: U-Shaped Price Paths Exonerated," Journal

Finally, empirical tests of Slade's theoretical model demonstrate that for the case of copper in particular, but for a mineral aggregate in general the specification of a U-shaped pricepath to explain changes in price over time is valid[1,2].

In summary, Slade's explanation for a U-shaped pricepath that is influenced by technological change is supported by empirical evidence. This indicates the importance of allowing for exogeneous technical change in the determination of resource prices over time.

Storage and the Perfectly Competitive Firm

Motives for Stockpiling

The motives for stockpiling have been examined in

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of Environmental Economics and Management 12(2) (1985): 191. This article is an answer to a critique levelled at Slade's article "Trends in Resource Commodity Prices" by M. Mueller and D. Gorin, "Informative Trends in Natural Resource Commodity Prices: A Comment on Slade,2 (Journal of Environmental Economics and Management 12(1) (1985): 89-95). Specifically, Mueller and Gorin contend that Slade's empirical model is incomplete due to the neglect of some factors that influence resource prices, such as monopoly power, political events, tariffs, etc...

[1] Slade, "U-Shaped Price Paths Exonerated", p. 191.

[2] To verify her hypotheses of resource price trends, Slade uses two types of equations. First, a linear model was used to estimate resource price paths; estijmated coefficients were significant at the 90% confidence level in half of the cases examined. Second, the use of a quadratic model to obtain fitted trends for the U-shaped price paths yielded estimated coefficients that were confident at the 90% level in all but one of the twelve cases examined.

Chapter I. Here the perfectly competitive firm is analyzed under the assumption of certainty. It is seen that the decision to engage in stockpiling hinges on the type of parameters underlying the model of the firm.

Stockpiling when there are no costs of inventory doesn't imply that the producer's costs of storage are nil: the costs of carrying a good through time is the rate of interest. If there are positive costs of extraction, depending on the type of costs, two aspects of carrying inventory through time stand out: first it is in the producer's interest to delay extraction as much as possible so that he incurs the discounted cost of extraction only. Second, if the firm were to store the resource instead of selling it then it would transcend its role as an ordinary firm and begin to act as a speculator, i.e., the firm is setting aside the resource in anticipation that its price will increase at a rate greater than or equal to the rate of interest, net of storage costs[1]. This behavior is perfectly plausible and even warranted if uncertainty prevails in the market: the firm is stockpiling for precautionary or speculative purposes. However, in the problem examined in this section there is no uncertainty, and the market is perfectly competitive. Intuitively this leaves open the question of why

[1]The basic condition for stockpiling any good is that its market value net of storage costs increase at a rate equal to or greater than the rate of interest.

such a firm might stockpile, let alone if it will exercise its option to do so. This question is solved through the analysis of the perfectly competitive firm that may stockpile at its discretion any amount of its production, at zero cost.

Through this analysis it is seen that the decision for a firm to exercise its option to stockpile depends on the autonomy or the dependence of the price with respect to time. Specifically, when the price is time-autonomous it is clearly non-optimal for a firm to stockpile its extracted resource. On the other hand a price that is a function of time leads to the conclusion that it is optimal for the firm to stockpile its extracted resource.

Assumptions of the Model

This section examines the behavior of a perfectly competitive firm identical in all respects to that in equations [2-6] and [2-7] except that it has the option of storing the resource at zero cost once it has been extracted.

The modifying assumptions that are necessary so that the perfectly competitive firm examined earlier has the option of storing the resource are the following: first it may no longer be assumed that all the ore extracted is immediately sold. Hence total revenues in the firm's profit function is the price p times the quantity sold V(t). Second, the price may be a constant or a variable. Third, it is assumed that storage is available to the firm at zero cost; hence the firm's profit function is defined over the quantity of ore <u>sold</u> minus the quantity of ore extracted. Four, bearing in mind that if the firm has stored any mineral above ground it is for the purpose of disposing of it after the mine is exhausted, the lifespan of the mine is distinguished from that of the firm. Then, contrary to the classical firm examined in the preceding sections, the terminal period (T) will denote the terminal time of the firm. The extractive horizon of the firm is referred to as $\mathcal{T} \in [0,T]$. Therefore the extractive phase of the mine takes place during $t \in [0, \mathcal{T}]$, and the pure inventory disposal problem occurs during the time period $t \in [\mathcal{T}, T]$, if applicable.

Throughout the extractive phase of the firm the inventory level, S(t), is denoted by

$$S(t) = \int_0^t q(t)\,dt - \int_0^t V(t)\,dt \qquad [4\text{-}1]$$

i.e., the stock level is determined by all the mineral extracted minus all sales. So that [4-1] never be negative we make the additional assumption[1]

$$S(0) = 0. \qquad [4\text{-}2]$$

Assumption [4-2], the inventory non-negativity rule permits us to rewrite [4-1] in the following manner:

$$S(t) = \int_0^t q(t)\,dt - \int_0^t V(t)\,dt \geq 0 \;\forall\, t \in [0, \mathcal{T}]. \qquad [4\text{-}3]$$

That is, at any time during the extractive phase of the life of

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[1]Certain models of inventory trading permit inventories to be negative. This is because the short selling of the resource is allowed. S. Sethi and G. Thompson, Optimal Control Theory (Boston: Martinus Nijhoff, 1981), p. 160.

the firm, the total amount of cumulative sales can never be greater than that of extraction.

To obtain the "inventory flow differential equation"[1] we take the time-differential of [4-3] to obtain

$$\dot{S}(t) = q(t) - V(t) \quad \forall t \in [0, \mathcal{T}]. \tag{4-4}$$

That is, throughout the extractive phase of the firm the level of the inventory may be constant, increasing or decreasing.

If an inventory has been built up during the extractive phase of the firm, before and after the mineral deposit is depleted the firm will seek to dispose of the accumulated stock. Because of the assumption that there are no costs of storage, the problem facing the firm is how to optimize the timepath of disposing of the mineral inventory. In this case the inventory time rate of change is

$$\dot{S}(t) = -V(t) \quad \forall t \in [\mathcal{T}, T]. \tag{4-5}$$

That is, after extraction has ceased the only way the stockpile can change is by decreasing. The terminal condition to [4-5] is that the level of stockpile be nonnegative, or

$$S(T) \geqslant 0. \tag{4-2a}$$

The significance of conditions [4-5] and [4-2a] is straightforward: once extraction has ceased the firm faces the problem of the "pure inventory disposal problem", i.e., over time the firm

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[1] Ibid., p. 146. An alternate expression is the "time-rate of change of the inventory".

determines at what rate to dispose of its accumulated inventory subject to the condition that it be nonnegative.

The presence of costless storage has the following effect: if the market price is below that at which the producer wishes to sell, he may keep it in storage before deciding to dispose of it on the market. Regardless of whether the costs of carrying inventory are zero or positive, the problem of the firm remains that of maximizing a present value flow of profits from a fixed stock. How this problem is solved by the producer (if an inventory is built up) is examined below.

The Possibility of Storage in Perfect Competition when the Price is a Constant

The objective function is

$$\max_{q_t, V_t} \left\{ J = \int_0^T e^{-rt} \left[pV_t - C(q_t) \right] dt + e^{-rT} W\left[S(T) \right] \right\} \qquad [4\text{-}6]$$

$$\text{s.t.} \begin{cases} R(0) = R_0 & S(0) = 0 \\ \dot{R} = -q_t & \dot{S} = q_t - V_t \\ q_t, V_t, S_t, R_t \geqslant 0 \end{cases} \qquad [4\text{-}7]$$

$$\text{where } W\left[S(T) \right] = \max_{V_t} \int_T^T p V_t \, dt \qquad [4\text{-}8]$$

$$\text{s.t.} \begin{cases} S(T) \geqslant 0 \\ V_t \geqslant 0 \\ \dot{S} = -V_t \end{cases}. \qquad [4\text{-}9]$$

Equation [4-6] is the profit-function to be maximized over the life of the firm, subject to the constraints in [4-7]. Equation [4-8] is the inventory disposal subproblem that is contained in the W-function. Specifically, the W-function (evaluated at time T) represents the sum of profits that are derived from the disposal of the stockpile, if it has been built up.

We note that if $S(t)>0$ during the extractive phase contained in $t\in[0,\mathcal{T}]$, then the state equation for inventories specified in [4-7} constrains the inventory flow to be non-negative. This ensures that there are no sales from inventory before the inventory disposal phase contained in $t\in[\mathcal{T},T]$.

The handling of the nonnegativity conditions on the inventory and its flow requires some discussion. From condition [4-7] we note that inventories may never be negative. In mathematical terms this corresponds to a pure state variable inequality constraint. This is handled as follows: at any point where $S(t)>0$ the nonnegativity constraint on inventories is not binding and can be ignored. If $S(t)=0$ at any time then the control corresponding to inventories, $\dot{S}(t)$, must be constrained so that $\dot{S}(t)\geq 0$ and $S(t)$ remain nonnegative. To handle this constraint we associate a multiplier $\theta(t)$ when $S(t)=0$, holding with complementary slackness. Therefore, we muust add the necessary conditions for complementary slackness to the application of the maximum principle:

$$\theta^*_{(t)} \geqslant 0 \qquad \theta^*_{(t)}\,\dot{S}^*_{(t)} = 0 \qquad \theta^*_{(t)}\,S^*_{(t)} = 0 .$$

We now specify the current-value augmented Hamiltonian function

$$L = pV_{(t)} - C(q_t) - \lambda(t)\,q_t + \mu(t)\left[q_t - V_t\right] + \theta(t)\left[q_t - V_t\right] \qquad [4\text{-}12\text{-}1]$$

where $\lambda_{(t)}$ is the adjoint variable corresponding to the deposit

R_o, $\mu(t)$ is the adjoint variable corresponding to the stockpile S(t), and $\theta(t)$ is an auxiliary variable assuring the nonnegativity of the stockpile S(t).

The Lagrangian function must be maximized at each moment with respect to the control variables q(t) and V(t). The first-order conditions are

$$\left.\begin{array}{l}\dfrac{\partial L}{\partial q(t)} = -\dfrac{\partial C}{\partial q(t)} - \lambda(t) + \mu(t) + \theta(t) \leq 0 \\ q(t)\left(-\dfrac{\partial C}{\partial q(t)} - \lambda(t) + \mu(t) + \theta(t)\right) = 0 .\end{array}\right] \qquad [4\text{-}12\text{-}2]$$

$$\left.\begin{array}{l}\dfrac{\partial L}{\partial V(t)} = p - \mu(t) - \theta(t) \leq 0 \\ V_t\left(p - \mu(t) - \theta(t)\right) = 0 \\ \Rightarrow V_t = \text{bang}\left[0, q(t); p - \mu(t) - \theta(t)\right]\end{array}\right] \qquad [4\text{-}12\text{-}3]$$

The adjoint equations are

$$\dot{\lambda}(t) \leqslant r\lambda(t) \qquad (\dot{\lambda}(t) - r\lambda(t))(R(t)) = 0. \qquad [4\text{-}12\text{-}4]$$

$$\dot{\mu}(t) \leqslant r\mu(t) \qquad (\dot{\mu}(t) - r\mu(t))(S(t)) = 0. \qquad [4\text{-}12\text{-}5]$$

The transversality conditions are

$$L(T) - r\left[W(S(T))\right] = 0 \qquad [4\text{-}12\text{-}6]$$

$$\mu(T) \geqslant \frac{\partial W(S(T))}{\partial S(T)} \qquad \left[\mu(T) - \frac{\partial W[S(T)]}{\partial S(T)}\right] S(T) = 0 \qquad [4\text{-}12\text{-}7]$$

$$\lambda(T) \geqslant 0 \qquad \lambda(T)R(T) = 0. \qquad [4\text{-}12\text{-}8]$$

Equation [4-12-6] determines the extractive horizon , whereas conditions [4-12-7] and [4-12-8] together determine the terminal states for the adjoint variables during the extractive phase of the mine.

Before proceeding further we must solve the boundary conditions contained in [4-12-7]. To do this the "pure inventory disposal problem" contained in equations [4-8] and [4-9] must be solved.

This is done presently. The current-value Hamiltonian function corresponding to [4-8] and [4-9] is

$$H = V(t)\left[p - \alpha(t)\right]. \qquad [4\text{-}12\text{-}8]$$

We note that there are two possible types of solution to [4-12-8]:

(A) There is an upper bound to extraction at each period of time, namely

$$0 \leqq V(t) \leqq V_{max}$$

where $V_{max} < S(t)$ and corresponds to the possibility of an impulse control. The presence of this constraint is to ensure that if it is optimal to sell the resource the entire stock will not be sold instantaneously.

The necessary conditions for a maximum are

$$V(t) = \text{bang}\left[0, V_{max}; \alpha(t)\right] \qquad [4\text{-}12\text{-}9a]$$

$$\dot{\alpha}(t) = r\,\alpha(t) \qquad [4\text{-}12\text{-}9b]$$

$$H(T) = pV(T) - \alpha(T)V(T) = 0 \qquad [4\text{-}12\text{-}9c]$$

$$\alpha(T) \geq 0 \qquad \alpha(T)\, S(T) = 0. \qquad [4\text{-}12\text{-}9d]$$

From [4-12-9a] we have the switching function

$$\sigma(t) = p - \alpha(t).$$

Hence $V(t) = \begin{cases} 0 & \text{when } \sigma < 0 \\ V_{max} & \text{when } \sigma > 0. \end{cases}$

A singular control arises when $\sigma \equiv 0$.

We now examine each solution in turn:

(i) $\sigma(t) < 0 \iff p < \alpha(t).$

This implies that the price is less than the value of the stored resource. By the adjoint equation [4-12-9b] this always remains so because $\dot{\alpha}_{(t)} = r\alpha(t)$. This solution is impossible because it implies that $W[S(\tau)]$ would be identically 0.

(ii) $\sigma(t) > 0 \iff p > \alpha(t)$

This implies that the price of the resource is greater than the value of the stored resource. The producer will sell the maximum amount until either $\alpha(t)$ reaches p or S(t) becomes nil.

(iii) $\sigma(t) \equiv 0 \iff p = \alpha(t)$

This implies $\dot{p} = \dot{\alpha} = 0$ over a time interval, and violates equation [4-12-9b]. Therefore this is only possible at time T.

In conclusion, case (ii) is the only possible scenario for the inventory disposal problem: the price of the resource is greater than its current value, and the producer sells as much as possible short of using impulse controls.

(B) There is no upper bound to the amount of ore that can be removed from the stockpile at any moment. Hence if the market price is greater than the value of the inventory everything will be sold instantaneously; we have an impulse control.

We now return to the extractive problem. From the solution of the inventory disposal problem as well as the terminal conditions[1] for the extraction problem we know that when extraction ceases, then

$$\mu(\tau) \geqslant \frac{\partial W(S(\tau))}{S(\tau)} = \alpha(\tau). \qquad [4\text{-}12\text{-}10]$$

We examine the problem of extraction shortly after time t=0. Because S(0)=0, to begin the problem we assume that q(t)>0 shortly after time t=0. Then from [4-12-2]

$$\frac{\partial C}{\partial q(t)} + \lambda(t) = \mu(t) + \theta(t).$$

We examine sales. From [4-12-3] we have the switching function

$$\sigma(t) = p - \mu(t) \quad \text{at} \quad t = 0 + \epsilon.$$

This has three possible solutions. We examine each one in turn at time $t=0+\epsilon$. They are

(i) $p < \mu(t) \implies V(t) = 0 \implies \dot{S}(t) > 0, \theta(t) = 0$; from [4-12-5] $\mu(\tau) \geqslant \alpha(\tau) > p$.

But $\dot{\mu} = r\mu$ Hence $p < \mu(t)\ \forall t$. This solution is impossible, because the resource can never be sold. This solution was ruled out by the analysis of the inventory disposal problem.

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[1] The transversality conditions conform to the current-value formulation of a problem specifying a free-end-point, a salvage value, and a one-sided constraint on the state variable at the horizon. These conditions are listed in Sethi and Thompson, Optimal Control Theory, p. 77.

(ii) $p > \mu_{(t)} \equiv V_{(t)} = q_{(t)}$, $\dot{S} = 0$ until $\mu_{(t)} = p$

(iii) $p \equiv \mu_{(t)}$ (This is a singular control). Hence, $\dot{p} = \dot{\mu} = 0$
This violates the adjoint equation in [4-12-5]. Hence this is impossible over a time interval.

Therefore the solution accepted is (ii): the price is greater than the value of a unit in storage, and sales occur at the maximum rate at all times if extraction is positive. This solution is irreversible due to the stationarity of price.

In summary the fact that the price is time-autonomous ensures that storage is nil at all times.

The Possibility of Storage in Perfect Competition When the Price is Time-Dependent

Here the analysis of the preceding section is repeated, except that the price of the extracted resource is a function of time.

The purpose of this exercise is to demonstrate how the time-dependence of the exogeneous parameters can alter the behavior of the firm with respect to storage.

It must be noted that the same inventory non-negativity constraints imposed on the preceding (time-autonomous) model contained in equation [4-12-1] apply here, as well as the complementary slackness conditions

$$\theta^*_{(t)} \geqslant 0 \qquad \theta^*_{(t)}\, \dot{S}^*_{(t)} = 0 \qquad \theta^*_{(t)}\, S^*_{(t)} = 0 .$$

The current-value augmented Hamiltonian function is

$$L = p_{(t)} V_{(t)} - c(q_t) - \lambda_{(t)} q_{(t)} + \mu_{(t)}[q_{(t)} - V_{(t)}] + \theta_{(t)}[q_{(t)} - V_{(t)}] \qquad [5\text{-}1]$$

The adjoint variables $\lambda_{(t)}$ and $\mu_{(t)}$, and the auxiliary variable $\theta(t)$ correspond to the same state equations as those in the preceding section.

The Lagrangian function must be maximized at each instant with respect to the controls $\dot{q}(t)$ and $V(t)$. The first-order conditions are

$$\left.\begin{aligned} \frac{\partial L}{\partial \dot{q}(t)} &= -\frac{\partial c}{\partial \dot{q}(t)} - \lambda(t) + \mu(t) + \theta(t) \leqslant 0 \\ q_t &\left[-\frac{\partial c}{\partial q_{(t)}} - \lambda(t) + \mu(t) + \theta(t)\right] = 0 \end{aligned}\right] \qquad [5\text{-}2\text{-}1]$$

$$\left.\begin{aligned} \frac{\partial L}{\partial V_t} &= p(t) - \mu(t) - \theta(t) \leqslant 0 \\ V_t &\left[p_{(t)} - \mu_{(t)} - \theta(t)\right] = 0 \\ V_t &= \text{bang}\left[0, q_t ; p_t - \mu(t) - \theta(t)\right] \end{aligned}\right] \qquad [5\text{-}2\text{-}2]$$

The adjoint equations are

$$\dot{\lambda}_{(t)} \leqslant r\lambda_{(t)} \quad \left[\dot{\lambda}_{(t)} - r\lambda_{(t)}\right] R_t = 0 \qquad [5\text{-}2\text{-}3]$$

$$\dot{\mu}_{(t)} \leqslant r\mu_{(t)} \quad \left[\dot{\mu}_{(t)} - r\mu_{(t)}\right] S_t = 0 \qquad [5\text{-}2\text{-}4]$$

The terminal conditions are

$$H_{(T)} - r\left[W(S_{(T)})\right] = 0 \qquad [5\text{-}2\text{-}5]$$

$$\mu_{(T)} \geqslant \frac{\partial W[S_{(T)}]}{\partial S_{(T)}} \qquad \left[\mu_{(T)} - \frac{\partial W(S_{(T)})}{\partial S_{(T)}}\right] S_{(T)} = 0 \qquad [5\text{-}2\text{-}6]$$

$$\lambda_{(T)} \geqslant 0 \qquad \lambda_{(T)} R_{(T)} = 0 \qquad [5\text{-}2\text{-}7]$$

The definitions of all the necessary conditions, adjoint equations, and terminal conditions are the same as in the preceding section and will not be repeated here.

The pure inventory disposal problem with a variable price is easily solved: the current-value Hamiltonian function is linear in sales, and is written

$$H = V(t)\left[p(t) - \alpha(t)\right] \qquad [5\text{-}3]$$

The necessary conditions for a maximum are

$$V(t) = \text{bang}\left[0, V_{max}; \alpha(t)\right] \qquad [5\text{-}4\text{-}1]$$

$$\frac{d\alpha(t)}{dt} = r\alpha(t) \qquad [5\text{-}4\text{-}2]$$

$$H(T) = V(T)\left[p(T) - \alpha(T)\right] \qquad 5\text{-}4\text{-}3]$$

$$\alpha(T) \geqslant 0 \qquad \alpha(T)S(T) = 0 \qquad [5\text{-}4\text{-}4]$$

From [5-4-1] we have

$$\sigma(t) = P(t) - \mu(t) \implies \begin{cases} V(t) = 0 \text{ when } \sigma(t) < 0 \\ V(t) = V_{max} \text{ when } \sigma(t) > 0 \end{cases}$$

There is a singular solution when

$$\sigma(t) \equiv 0$$

Similarly to the preceding section there are two types of solutions. They are:

(A) If the maximum amount of sales from the inventory are constrained so that in any period $V_{max} < S(t)$ there are three possible solutions. They are:

(i) $p(t) > \alpha(t)$ then $V(t) = V_{max}$

(ii) $p(t) < \alpha(t)$ then $V(t) = 0$

(iii) $p(t) \equiv \alpha(t)$. This is a singular solution. From the characteristics of the price function and the adjoint equation it is a coincidence if this remains so for more than a brief instant.

Given the variable price function postulated for this model, any sequence of these three phases is possible.

(B) If impulse controls are admitted then $V_{max} = S(t)$, and the inventory is sold instantaneously.

We return to the extractive problem. From the solution of the pure inventory disposal problem we know that when extraction ceases the price must be greater than the value of the stored resource. In addition, from conditions [5-2-6] we know that this is also equal to $\mu(t)$. Hence we may write

$$\mu(\tau) \geqslant \frac{\partial W(S(\tau))}{\partial S(\tau)} = \alpha(\tau) \qquad [5\text{-}2\text{-}8]$$

We now analyze the problem of the firm when there is extraction. To begin the problem we assume that at time $t = 0 + \epsilon$ $q(t) > 0$. Then from [5-2-1]

$$\frac{\partial c}{\partial q(t)} + \lambda = \mu + \theta$$

From [5-2-2] we have the switching function

$$\sigma(t) = p(t) - \mu(t) \implies \begin{cases} V(t) = 0 \text{ when } \sigma(t) < 0 \\ V(t) = q(t) \text{ when } \sigma(t) > 0 \end{cases}$$

Three cases arise. They are:

(i) If $p(t) > \mu(t)$ then $V(t)=q(t)$. Then firm sells all it extracts; hence $\dot{S}(t)=0$, $S(t)=0$, $\theta(t) \geq 0$

(ii) If $p(t) < \mu(t)$ then $V(t)=0$. But $\mu(t)>0$ since $p(t) \geq 0$. Then $\dot{S}(t)>0$, $S(t)>0$, and $\theta(t)=0$.

From [5-2-4] and [5-2-8] we know that stockpiling will take place. The rate at which stockpiling takes place is

$$\mu_t = \frac{\partial C}{\partial q_t} + \lambda_t$$

(iii) If $p(t) \equiv \mu(t)$ this is a coincidence.

In summary, when the price is higher than the complete marginal cost, sales occur at the maximum rate of extraction. If the price is below the complete marginal cost, then extraction for purposes of storage takes place. The rate at which this occurs is determined by the amount of the marginal cost of extraction.

Summary and Conclusions

The price of a resource is determined endogeneously by the entire resource market. Its time rate of change is the same as that of any other capital good, and hence depends on the rate of interest. This is the r-% rule.

A single competitive firm exploiting a resource deposit in partial equilibrium seeks to maximize the present value of a flow of profits. The firm will extract the resource only if the future value of the user cost is at least as great as the spot price plus the marginal cost of extraction. This condition holds for the duration of the life of the mine. If the resource industry is to deplete reserves at a socially optimal rate then the rate of growth of the spot price must be equal to the rate of interest. This is simultaneously a condition for stock and flow equilibrium. The r-% rule will not hold if the costs of extraction depend upon past extraction, or if the resource is "durable", or if the model allows for exploration during extraction. This leads one to question the relevance of the Hotelling rule as a criterion to use in the analysis of exhaustible resource markets.

The sales and extraction paths of the single competitive firm in partial equilibrium is analyzed when there is the option of stockpiling. When the price of the resource is a constant over time it is seen that the firm never takes up its option to set aside its extracted resource. Instead, its sales are identical to extraction, and occur at the maximum rate over the life of the firm. However, when the price of the resource is a time-dependent variable, the conclusion emerges that in some cases it is economical for the firm to stockpile. When sales are positive extraction occurs at the maximum rate. When sales are nil extraction continues at a less-than-maximum rate.

In either case - when the price is constant and when it is a variable - when sales occur it must be at the maximum rate. This ensures that when it is optimal to sell the resource it is also optimal to extract it at the maximum rate. If a stockpile has been built up it may be depleted only when extraction has ceased.

CHAPTER IV

INVENTORIES AND THE EXTRACTIVE MONOPOLY

Introduction

In the framework of a dynamic model where a firm maximizes the present value of a flow of profits, it is straightforward to show that along the optimal sales and extraction paths the marginal cost must be equated with marginal revenue from sales at every instant. If the firm is producing an exhaustible mineral, a provision must be made for the user cost to be included in the marginal cost. Nevertheless the same principle holds.

The application of this principle entails two problems. First, if some of the parameters of the model are shifting through time (i.e., are time-dependent) it might be optimal to cause a decoupling of sales and output, i.e., "bunch up" production in one period in order to sell at some point in the future. Second, the assumption that a firm always sells all it produces postulates a "hand-to-mouth" existence for a firm. This does not leave the firm much leeway in planning production over time.

If the firm admits a decoupling of sales and production at any moment, the picture is entirely different:

it may be optimal to carry out a policy of intertemporal price discrimination.

Hence this chapter proceeds as follows: the first section discusses the ordinary costless extractive monopoly. It is shown that, given certain conditions, the rate of production is less than in a similar competitive industry. Then it is seen how, if certain conditions hold, the extractive costless monopoly will produce more than a similar competitive market. Finally, a monopoly with convex (increasing) costs of extraction is analyzed.

The next section examines the conditions necessary for a monopoly to have a decoupling of sales and output: a necessary condtiion for this to occur is the presence of an increasing cost function over at least some range of output. The final section features a model of an extractive monopoly (where all the parameters are time-autonomous) but has an option of stockpiling. It is seen how stockpiling need not necessarily take place only in a time-dependent model. Indeed, this analysis of the extractive monopoly shows that the presence of a U-shaped cost curve is sufficient to ensure stockpiling in some instances.

The Classical Monopoly with No Extraction Costs

Consider the monopoly that is the sole owner of a resource that may be extracted at zero cost, with a known (inverse) stationary market demand curve P=P[Q(t)]. Furthermore the monopolist knows the time-path of the price of the asset, and owns all available assets of that resource. Finally assume that the assumptions necessary for certainty analysis hold in this case. Then the problem of the monopoly is to maximize the present value of its profit function defined over the life of the mine, i.e.,

$$\max \int_0^T P\left[Q(t)\right] Q(t) e^{-rt} dt \quad \text{s.t.} \begin{cases} \dot{R}(t) = -q(t) \\ R(0) = R_0 \\ R(T) \geqslant 0 \quad q(t) \geqslant 0. \end{cases} \qquad [6\text{-}1]$$

The current-value Hamiltonian corresponding to [5-1] is

$$H = P[Q(t)]Q(t) - \lambda(t)Q(t). \qquad [6\text{-}2]$$

The application of the maximum principle yields

$$\left.\begin{aligned} \frac{\partial H}{\partial Q(t)} &= \left[\frac{\partial P(Q(t))Q(t)}{\partial Q(t)} + P(Q(t))\right] - \lambda(t) \leqslant 0 \\ Q(t)&\left[\frac{\partial P(Q(t))}{\partial Q(t)}Q(t) + P(Q(t)) + \lambda(t)\right] = 0 \end{aligned}\right] \qquad [6\text{-}3a]$$

$$\lambda(T)R(T) = 0 \qquad \dot{\lambda} \leqslant r\lambda(t) \qquad [6\text{-}3b]$$

$$H(T) = 0. \qquad [6\text{-}3c]$$

Along the optimal sales and extraction path the marginal revenue from sales must be equal to the current value of the user cost of the resource. Alternatively, the optimal path must be such that at every instant when extraction occurs the value of an additional unit sold must be equal to the opportunity cost at this time. The adjoint equation [6-3b] implies that the user cost of the resource is monotone increasing throughout the life of the firm. The transversality condition contained in [6-3c] determines the terminal time, i.e., at time T,

$$P\left[Q(T)\right]Q(T) = Q(T)\left[\frac{dP(Q(T))}{dQ(T)} + P(Q(T))\right],$$

This is possible only if Q(T)=0. The economic significance of this result is as follows: from [6-3a] and [6-3b] the monopolist adjusts his rate of extraction so that the marginal revenue grows at a rate equal to that of the user cost. As marginal revenue increases sales decrease until at time t=T the choke point is attained along the market demand curve.

It is relevant to note the similarities between monopolistic costless extraction and the disposal of a stockpile (maintained at zero cost) at an optimal rate: if both bodies of ore are owned by monopolists then the only criterion that need to be observed while disposing of the ore is that marginal revenue from sales grows at the rate of discount. This is why, properly speaking, the problem of optimal extraction at zero cost is known as the "inventory disposal problem": in both cases a stock is placed on the market at a predetermined optimal rate, and both stocks have an element of exhaustibility. Aside from the fact that the stocked resource must be sold at a higher supply price than the ore <u>in situ</u> (to compensate the producer for the foregone value of having extracted the resource at a positive cost) there is nothing, strictly speaking, to distinguish a stored resource from an untouched deposit. Both are subject to the same criteria when placed for sale on the market.

Limiting Cases to the Optimal Extraction Path of the Neoclassical Extractive Monopoly

The results of the preceding section rest on two assumptions: that the price elasticity of demand is constant, and the costs of extraction are nil or fixed. This permits a simple extension of equation [6-3a] to obtain the monopolistic equivalent of the Hotelling rule for the costless monopoly, namely

$$\frac{\dot{MR}(t)}{MR(t)} = r.$$

In this section will be demonstrated the importance of each of the above mentionned factors so that the percent rate of increase of marginal revenue be equal to the rate of interest.

The result that a monopoly with no costs of extraction and a constant price elasticity of demand will extract as much as a competitive market is due to Stiglitz[1]. Using a simple model of an extractive monopoly he demonstrated that it is possible for a monopoly to behave in a socially optimal way[2].

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[1] J. Stiglitz, "Monopoly and the Rate of Extraction of Exhaustible Resources," American Economic Review 66 (1976): 655-661.

[2] Dasgupta, and Heal (Economic Theory and Resources, pp. 326-329) have extended Stiglitz's result when there is variable price elasticity of demand. They demonstrate that it is possible for a monopoly to produce more than is socially optimal. Whether a monopolist will choose to produce under these conditions is another matter.

For this to happen the price elasticity of demand must be constant or greater than unity[1,2].

If the price elasticity of demand is variable through time then it becomes a function of quantity extracted[3,4]. When this is taken into consideration when deriving the dynamics of price the Hotelling price-rule yields[5]

$$\frac{\dot{P}(Q(t))}{P(Q(t))} + \frac{\dot{\gamma}(Q)}{\gamma(Q)} = r .$$

The sign of all the variables are known with the exception of $\dot{\gamma}(Q)$, which may be positive or negative. It is this which permits the monopoly pricepath to increase or decrease at a rate different from that of interest, with a correspondingly slower (faster) rate of depletion than would be the case of a competitive industry. This result has been extended by

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[1] This point is made by M. Kamien and N. Schwartz, Journal of Economic Theory 15 (1977): 394-397, and Dasgupta and Heal, Economic Theory and Resources, p. 326.

[2] It is shown by Kamien and Schwartz (Ibid., p. 394) that the necessary and sufficient conditions for monopolistic and competitive extraction rates to coincide is that the production function employing the resource be of a Cobb-Douglas type. This result is achieved by demonstrating that the derived demand for an essential factor will be isoelastic if and only if the linear homogeneous production function is Cobb-Douglas.

[3] Dasgupta and Heal, Economic Theory and Resources, p. 327.

[4] It is assumed that the quantity extracted decreases over time. This assumption is made possible by assuming that so long as the resource lasts the quantity extracted must decline to ensure a rising marginal revenue.

[5] The derivation of this result may be found in Dasgupta and Heal, Economic Theory and Resources, pp. 325-331.

T. Lewis[1] by first commenting on the conditions necessary to bring about a resource depletion path steeper than that of the result, and then by incorporating into the model conditions for the monopoly extraction to be greater than the competitive rate. The main requirement for this to happen is that at some point on the industry demand curve there be a range along which the price elasticity of demand increase[2]. But Lewis cautions the reader that it has not been possible to discover general conditions that guarantee the monopolist will extract the resource faster than the competitive industry.

Another circumstance where the results predicted by the Hotelling pricepath might not hold is in the case of extraction while there is exploration. Specifically it has been suggested that the case illustrated by J. Stiglitz[3] is the dynamic analog of the static model of the limited-input monopoly of the Averch-Johnson type. An article analyzing this problem[4] introduces a modification to the Stiglitz model where exploration is permitted. With this modification the

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[1]T. Lewis, "Monopoly Exploitation of an Exhaustible Resource," Journal of Environmental Economics and Management 3 (1976): 198-204.

[2]Ibid., p. 203.

[3]Stiglitz, "Monopoly and the Rate of Extraction of Exhaustible Resources," p. 659.

[4]G. Gaudet and P. Lasserre, "On Comparing Monopoly and Competition in Exhaustible Resource Exploitation," Cahier de recherche numéro 86-02 du Groupe de Recherche en Economie de l'Energie et des Ressources Naturelles, 1986.

monopolist need not contend with the constraint imposed by the limited input. Not surprisingly, with this modification appended the Stiglitz model yields the result that a monopoly will produce less than a competitive industry.

Another way to model exploration is to postulate the effects of new discoveries of the resource on its pricepath. There are two ways to model this. One way is to postulate an uncertainty about the date of discovery of additional deposits and examine the effects on the monopoly rate of extraction[1]. Another way is to postulate that there exists unlimited potential reserves[2] but as depletion ensues further discoveries are a decreasing function of effort expended on exploration[3,4].

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[1]This is the model proposed by J. Stiglitz and P. Dasgupta, "Market Structure and Resoruce Extraction under Uncertainty," Scandinavian Journal of Economics 83(2) (1981): 318-333. In this article the authors compare the rate of extraction of a resource when there is uncertainty about the date of discovery of a new deposit of a resource under varying market structures. For our purposes we are interested only in the monopoly rate of extraction.

[2]Hence the amount of discoveries is a function of effort. This is the assumption made by R. Pindyck, "The Optimal Exploration and Production of Nonrenewable Resources," Journal of Political Economy 86(5) (1978): 841-861.

[3]Pindyck (Ibid., p. 855) then comments that viewed in this manner many "exhaustible" resources could be better thought of as inexhaustible but nonrenewable.

[4]Recently M. Eswaran and T. Lewis ("Ultimate Recovery of an Exhaustible Resource under Different Market Structures," Journal of Environmental Economics and Management 11 (1984): 55-69) have taken up the question of endogeneously determined resource deposits for an extractive industry. Their analysis deals with imperfect competition and will not be dealt with here.

Specifically Stiglitz and Dasgupta examine the effect of a monopoly discovering a new deposit of its own and a monopoly faced with a competitively owned new deposit. Under the assumptions of their model they find that in any case a monopoly will produce less than is socially optimal[1]; if the discovery is controlled by the same monopoly that is already selling the resource the rate of extraction of the resource is less than if the discovery is controlled by the same monopoly that is already controlling the resource prior to the invention. If the discovery is controlled by a competitive market the rate of extraction will be higher than in the preceding case.

The study on exploration done by Pindyck addresses the question of how discoveries are made; that is, producers are not endowed with reserves but instead must develop them. By postulating that the producers seek to maximize the sum of discounted profits the author presents the usual result that the percent rate of increase of marginal revenue will equal that of interest if there are neither costs of extraction nor costs of exploration.

The author's model states that when price for the resource is high then so is exploratory effort; the latter will tend to decrease later on when price decreases, and increase again when price starts increasing. The gist of

[1]Ibid., p. 329.

Pindyck's argument is that the price profile depends on the initial level of reserves and the magnitude and behavior of extraction costs. If reserves are initially very small the behavior of the pricepath will be U-shaped.

Given the assumptions of Pindyck's model so long as the rate of production for a monopolist is less than that for a competitive market the rate of exploration will be larger. This leads to the expectation that whether initial proved reserves are small or large it is expected that the monopolist undertake less exploratory activity than the competitive market at first, but later undertake more[1]. The introduction of exploratory activity has the effect of reducing the rate of increase of the price. This negates the notion that the percent rate of increase of price is indicative of monopoly power[2].

The model developed by Pindyck to explain the U-shaped pricepath has been criticized on methodological grounds by Swierzbinski and Mendelsohn[3] on the basis that the cost function that applies when there is no exploration is inappropriate when there is exploration[4]. Specifically the

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[1]Pindyck, "Optimal Exploration and Production of Resources," p. 848.

[2]This is measured by the inverse of the demand elasticity faced by a monopoly.

[3]J.E. Swierzbinski and R. Mendelsohn, "Cost Reducing Exploration and Exhaustible Resources: Microfoundations," Department of Economics, University of Washington Discussion Paper No. 85-1 (January 1985).

[4]Ibid., p. 3.

authors argue against the use of the cost function relevant to the traditional model of the mine - with no exploration - when there is exploration. It is this traditional specification that Pindyck used to explain the U-shaped pricepath[1]. The specifiction of a traditional cost function implicitly assumes a fixed level of reserves; the specifying of a cost function dependent on cumulative extraction along with increasing cumulative reserves ensures that in some cases the time-pricepath is decreasing. For this to happen it is crucial that the derivative of the cumulative cost function with respect to reserves be negative for some range of output. Yet such a specification ensures that the cost of production depends on the aggregate level of known reserves and not on the individual cost of extracting from new deposits[2].

As an alternative to Pindyck's formulation, the authors's model assumes that each deposit has a unit extraction and exploration cost[3]. Whereas the cost of exploration is known prior to exploration, the cost of extraction becomes known when the cost of exploration is paid[4]. This specification ensures that extraction costs are defined on the basis of costs of extraction from individual deposits, not aggregate levels of

[1] Pindyck, "Optimal Exploration and Production of Resources," pp. 844-845.

[2] Swierzbinski and Mendelsohn, "Cost Reducing Exploration and Resources," p. 20.

[3] Ibid., p.11. [4] Ibid., p. 10.

deposits. Furthermore, in Swierzbinski and Mendelsohn's model the entire costs of extraction and exploration needed to analyze the production sequence may be gathered by ranking deposits in order of their combined extraction and exploration costs[1]. This procedure ensures that the aggregate cost function includes exploration and extraction costs based on individual deposits. This way the time-pricepath ($\dot{p}$) is the difference between the price and the sum of marginal extraction cost and the marginal discovery cost. Finally, this particular pricepath is increasing at all times[2]. This result reconciles exploratory activity with the Hotelling r-% rule.

Even if costless extraction prevails along with no exploration the basic result that marginal revenue increases at a rate equal to that of interest has been disputed by some authors: it is open to debate whether the durability of the resource has any effect on its demand. Specifically the U-shaped pricepath observed for exhaustible resources has been explained by invoking the durability of some exhaustible resources. Levhari and Pindyck[3] distinguish between market demands as functions of flows of production or stocks in

[1] Ibid., p. 11.

[2] That is, $\dot{p}=rp - C(\phi)$, where ϕ is the combined marginal cost of extraction and exploration.

[3] D. Levhari and R. Pindyck, "The Pricing of Durable Exhaustible Resources," Quarterly Journal of Economics XCVI(3) (1981): 365-377.

circulation[1]. Examining a case of a durable exhaustible material--in continuous time with increasing marginal and average costs of production they find that the Hotelling rule will hold for a competitive market but not for a monopoly[2]. The reason invoked is that for a durable resource, demand is a stock relationship. Therefore it is more profitable to practice intertemporal price discrimination, with its corresponding allocation of rents away from the optimal path, expecially if the stock of the resource in circulation at the beginning of the period of extraction is small.

This result has been criticized on the grounds that the definition of marginal revenue used by Levhari and Pindyck is incomplete[3]. Chilton introduces the notion that durable resources have the characteristic that a small change in their production will affect the entire price trajectory. Bearing this in mind it is possible to redefine marginal revenue by stating thata monopolist is concerned with the time-path of marginal revenue and how a perturbation in the

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[1]The authors define a durable resource as one where their demands are for stocks in circulation rather than for flows of production, and depend on expected changes in price as well as the current price level.

[2]This result has been expanded by Moussavian and Samuelson, "On the Extraction of an Exhaustible Resource by a Monopoly," Journal of Environmental Economics and Management 11 (1984): 139-146. The authors expand on Levhari and Pindyck's conclusions that the r-% rule doesn't hold for the monopoly producers of a durable resource.

[3]J. Chilton, "The Pricing of Durable Exhaustible Resources: Comment," Quarterly Journal of Economics XCIX(3) (1984): 629-637.

production path around a particular point in time will affect the present value of revenues[1]. In this way Chilton is able to make the Hotelling rule hold for durable resources in the case of monopoly.

The Optimal Extraction Path of a Neoclassical Monopoly with Variable Costs of Extraction

Similarly to the section with costless extraction, the problem of the monopoly is to maximize the present value of a flow of profits from a body of ore. The monopoly faces a convex cost function that is identical to that specified for the competitive firm in Chatper III[2].

The monopolist's problem is

$$\max_{Q(t)} \int_0^T \left\{P[Q(t)]Q(t) - C(Q(t))\right\} e^{-rt} \quad \text{s.t.} \begin{cases} \dot{R}(t) = -Q(t) \\ R(0) = R_0 \\ Q(t) \geqslant 0 \end{cases}$$

The corresponding current-value Hamiltonian function for the above problem is

$$H = \left[P[Q(t),t]Q(t) - C(Q(t))\right] - \lambda(t)Q(t) \qquad [6\text{-}4]$$

The necessary conditions for an interior maximum are

$$\left.\begin{array}{l} \dfrac{\partial H}{\partial Q(t)} = \left[\dfrac{dP(Q(t))}{dQ(t)}Q(t) + P(Q(t)) - C'(Q(t))\right]e^{-rt} - \lambda(t) \leqslant 0 \\ Q(t)\left[\left(MR(t) - C'(Q(t))\right)e^{-rt} - \lambda(t)\right] = 0 \end{array}\right] \qquad [6\text{-}5a]$$

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[1] Ibid., p. 630.

[2] In addition, all the other assumptions made in that Chapter with respect to the quality of ore, the cost function, etc..., are assumed to hold.

$$\dot{\lambda} \leqslant r\lambda(t) \quad ; (\dot{\lambda} - r\lambda) R(t) = 0 \qquad [6\text{-}5b]$$

$$H(T) = \left[P(Q(T))\, Q(T) - C(Q(T)) \right] e^{-rt} = \lambda(T)\, Q(T) \qquad [6\text{-}5c]$$

From [6-5a] and [6-5b] we derive the classic profit-maximizing result: if extraction occurs, marginal revenue should equal the current value of the resource <u>in situ</u> net of the marginal cost of extraction.

The transversality condition for a current-value Hamiltonian function with free endpoints and an unspecified terminal time is given by [6-5c]. The solution to this equation yields

$$P[Q(T)] - \frac{C(Q(T))}{Q(T)} = \frac{d P(Q(T))Q(T)}{dQ(T)} + P(Q(T)) - C'(Q(T)).$$

That is, the terminal time of extraction is determined when average profits are equal to marginal profits.

<u>Necessary Conditions for a Decoupling of A Firm's Sales and Extraction Paths</u>

Heretofore the monopoly's sales and extraction paths have been identical, and indeed synonomous. This section examines the effect of a difference between the two paths. A decoupling of sales and extraction may be the result of two different sets of conditions that hold independently of one another but produce the same effect. The first set of conditions implies that the cost function be time-dependent at an exponential rate, and that it be non-decreasing[1].

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[1]In his 1939 article Smithies assumes that both the cost and demand schedules are shifting upward according to exponential trends, with the cost function increasing at a

Because we are operating in a context of certainty it is possible for the producer to know exactly what the price will be in each period, and correspondingly at what rate the resource will be depleted. In order to maximize the flow of discounted profits the producer will set up a pricing scheme that corresponds to the principle of the intertemporal price discrimination rule as seen in Chapter I: at every moment of time the discounted value of marginal revenue is equal to that of marginal cost[1]. The second set of conditions implies that if the parameters of the firm are time-autonomous it may be optimal to decouple sales and extraction if the cost function admits falling average production costs over some range of output[2]. Two conditions are necessary for this to happen. They are: (i) the firm's average cost must be decreasing over some range of its output; (ii) if the average cost curve is U-shaped, instantaneous marginal revenue must equal instantaneous marginal cost at an output rate less than the one at which average cost is at a minimum.

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greater rate than the demand function. A. Smithies, "The Maximization of Profits over Time with Changing Cost and Demand Functions," Econometrica 7 (1939): 312-318.

[1] Although Smithies' article does not deal with exhaustible resources it is a trivial matter to show that the scarcity rent must be added to the marginal cost.

[2] L. Arvan and L. Moses, "Inventory Investment and the Theory of the Firm," American Economic Review 72(1) (1982): 186-193.

Graphically, these conditions are the following:

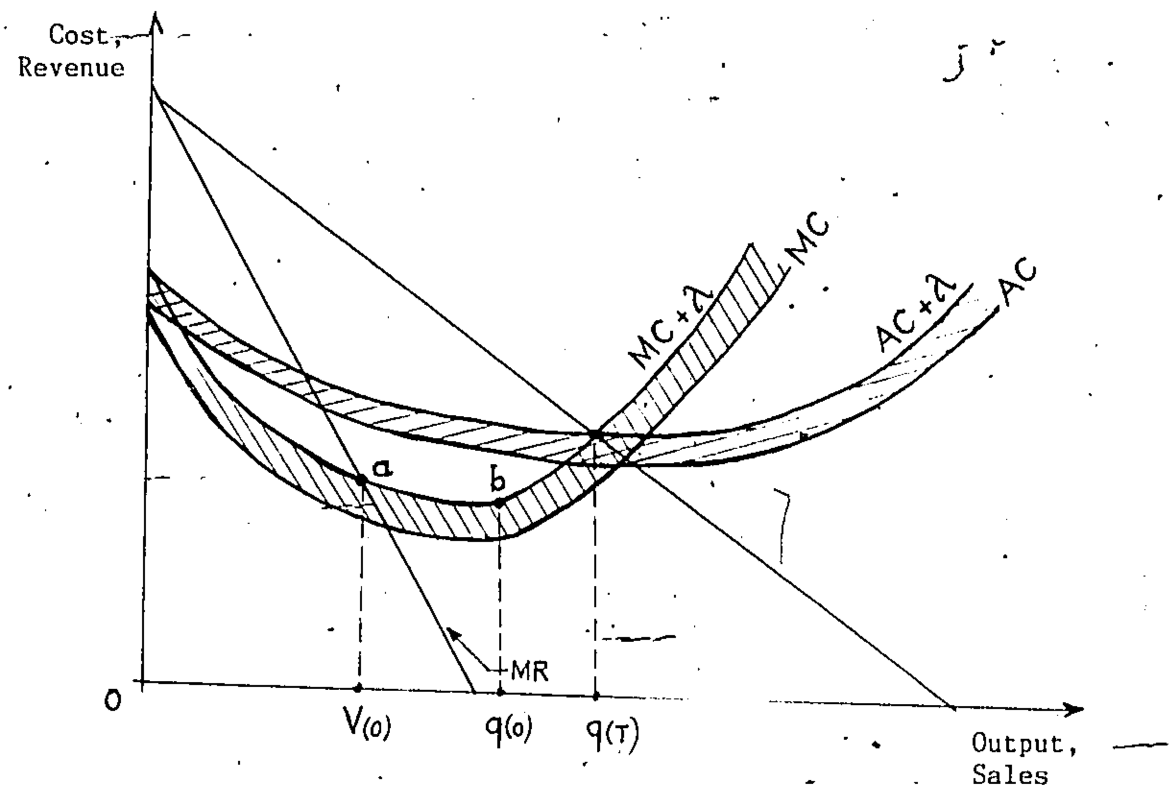

Fig. 1. The minimum of the marginal cost curve occurs at an output level less than that where the average cost curve is at a minimum. Specifically, V(0) is the sales level during the first instant of the life of the firm whereas q(0) is the production level at the same instant. Throughout the life of the firm instantaneous marginal cost is equated with instantaneous marginal revenue, but production begins at q(0) and attains the minimum of average cost at q(T), T representing the horizon of the mine.

For output and sales to be decoupled, it is necessary that the marginal cost curve intersect marginal revenue at a point where output is smaller than where average cost is at a minimum. From fig. 1 this means that the firm sells the quantity indicated at pont (a), but produces the quantity indicated by point (b), namely the minimum of the average cost curve.

If the intersection of marginal cost and marginal revenue occurs at a point greater than the minimum of the average cost curve, the profit-maximizing strategy is to synchronize sales and output. This is the topic of the preceding section of this chapter.

It is relevant to note that the synchronization of sales and output can only occur where the marginal cost and marginal revenue intersect, namely on the decreasing part of the marginal cost curve. If the marginal cost and marginal revenue intersect on the increasing portion of marginal cost then it is optimal to synchronize sales and extraction at all times.

In summary there are two different sets of conditions necessary for stockpiling to be feasible: if the model's parameters are shifting over time, or if the average cost curve is decreasing over some range of output.

A Model of the Time-Autonomous Monopoly with Storage

In this model a time-autonomous model of a monopoly with the option of storage is examined[1]. The model is analogous with that specified for the extractive monopoly with increasing costs of extraction except that the firm may exercise its option of stockpiling any amount of the extracted resource at zero cost. Hence sales and extraction horizons are not necessarily equal. Assuming the firm exists during the interval $t \in [0,T]$ it will extract during the time interval $[0,\mathcal{T}]$ and sell during the entire lifespan $[0,T]$, with $\mathcal{T} \leqslant T$ because short sales are excluded.

The type of resource deposit the firm exploits is similar to that of the competitive firm in Chapter II: likewise with that section the stock of inventory depends on cumulative extraction minus cumulative sales. Because the initial stockpile is nil, we may write with non-negativity constraint

$$S(t) = \int_0^t Q(t)\,dt - \int_0^t V(t)\,dt \geqslant 0 \quad ; \ S(0) = 0.$$

Similarly we may describe the inventory difference equation during the extractive phase by

$$\dot{S}(t) = Q(t) - V(t) \ \forall t \in [0,\mathcal{T}].$$

After extraction has ceased the above may be rewritten

$$\dot{S}(t) = -V(t) \ \forall t \in [\mathcal{T},T].$$

[1] A similar model of the time-dependent extractive monopoly has been analyzed by D. Lee and D. Orr, "The private discount rate and resource 'conservation'", Canadian Journal of Economics VIII(3) (1975): 351-363.

Costs of extraction faced by the monopoly are convex, independent of past extraction, and otherwise similar to that specified for the competitive firm in Chapter II. Costs of storage are nil.

Over the life of the firm the inverse demand curve is P[V(t)], where V(t) represents sales at each instant of time. Hence the revenue received from the sales of the resource is a function of the current sales rate at time t.

We assume that the profit function is concave in sales; i.e., net marginal revenue decreases as sales increase. Hence the market demand curve satisfies the property[1]

$$\frac{\partial^2 P[V(t)]}{\partial V(t)^2} \leqslant 0 .$$

Therefore the problem of the monopoly is as follows: during the extractive phase of the mine the firm seeks to maximize the present value of its objective profit-function subject to its resource availability constraint

$$\underset{Q(t),V(t)}{Max}\left\{ J = \int_0^T e^{-rt}\left\{P[V(t)]\,V(t) - C(Q(t))\right\}dt + e^{-rt}W[S(T)]\right\} \qquad [7\text{-}1]$$

$$s.t. \left\{\begin{array}{ll} R(o) = R_0 & \dot{R}(t) = -Q(t) \\ S(o) = 0 & \dot{S}(t) = Q(t) - V(t) \\ Q(t),\ V(t),\ S(t),\ R(t) \geqslant 0 & \end{array}\right] \qquad [7\text{-}2]$$

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[1] Dasgupta and Heal (Economic Theory and Resources, p. 324) state that with zero costs of extraction this condition holds with a strict inequality. Arvan and Moses ("Inventory Investment", p. 191) state that with positive costs of extraction the sign of the second derivative is non-positive.

where $$W(S(\tau)) = \underset{V(t)}{Max}\left\{K = \int_{\tau}^{T} e^{-r(t)} P[V(t)]V(t)\,dt\right. \quad [7\text{-}3]$$

$$\text{s.t.}\left\{\begin{array}{l}\dot{S}(t) = -V(t) \\ V, S \geqslant 0\end{array}\right] \quad [7\text{-}4]$$

The problem contained in [7-1] and [7-2] is the objective functional for the mine: when it is extracting and selling ore at the same time how will it maximize a present-value flow of profits? The subproblem contained in [7-3] and [7-4] is in principle identical to the "pure inventory disposal problem" for the monopoly. Its solution is necessary for the solving of the more general problem.

The Optimal Sales and Extraction Path of the Time-Autonomous Monopoly

To solve [7-1] and [7-2] we specify the current-value augmented Hamiltonian function

$$L = P[V(t)]V(t) - C(Q(t)) - \lambda(t)Q(t) + \mu(t)[\dot{Q}(t) - V(t)] + \theta(t)[Q(t) - V(t)] \quad [7\text{-}5]$$

The necessary conditions for the determination of an optimal sales and extraction path are

$$\left.\begin{array}{l}\dfrac{\partial L}{\partial Q(t)} = -\dfrac{\partial C}{\partial Q(t)} - \lambda(t) + \mu(t) + \theta(t) \leqslant 0 \\ Q(t)\left[-\dfrac{\partial C}{\partial Q(t)} - \lambda(t) + \mu(t) + \theta(t)\right] = 0\end{array}\right] \quad [7\text{-}6]$$

$$\left.\begin{array}{l}\dfrac{\partial L}{\partial V(t)} = \left[\dfrac{\partial P(V(t))}{\partial V(t)} V(t) + P(V(t))\right] - \mu(t) - \theta(t) \leqslant 0 \\ V(t)\left[\dfrac{\partial P}{\partial V} V + P - \mu(t) - \theta(t)\right] = 0\end{array}\right] \quad [7\text{-}7]$$

$$\dot{\lambda}_{(t)} \leqslant r\lambda_{(t)} \quad R(t)\left[\dot{\lambda}_{(t)} - r\lambda_{(t)}\right] = 0 \qquad [7\text{-}8]$$

$$\dot{\mu}_{(t)} \leqslant r\mu_{(t)} \quad S_{(t)}\left[\dot{\mu}_{(t)} - r\mu_{(t)}\right] = 0 \, . \qquad [7\text{-}9]$$

The transversality conditions are

$$H\left[R_{(T)}, S_{(T)}, V_{(T)}, P_{(T)}, \lambda_{(T)}, \mu_{(T)}, T\right] - rW\left[S_{(T)}\right] = 0 \qquad [7\text{-}10]$$

$$\mu_{(T)} \geqslant \frac{\partial W(S_{(T)})}{\partial S_{(T)}} \qquad [7\text{-}11]$$

$$\left[\mu_{(T)} - \frac{\partial W[S_{(T)}]}{\partial S_{(T)}}\right] S_{(T)} = 0 \qquad [7\text{-}12]$$

$$\lambda_{(T)} \geqslant 0 \quad \lambda_{(T)} R_{(T)} = 0 \qquad [7\text{-}13]$$

We note that the Hamiltonian in [7-5] is not linear in sales.

The adjoint equations may be interpreted as the time-rate of change of, respectively, the value of each unit of ore in storage, and each unit of ore in situ. The terminal condition [7-10] determines the terminal time T ; condition [7-11] states that at the extractive horizon a marginal unit of stored ore must be at least equal to the value to the stockpile of adding that additional unit.

Before proceeding further we solve the boundary problem contained in [7-3] and [7-4]. This is the solution of the "pure inventory disposal problem". The current-value Hamiltonian function which must be maximized with respect to V(t) at each moment of time is

$$H = P\left[V(t)\right]V(t) - \alpha(t)V(t) \qquad [7-14]$$

As [7-14] is not linear in sales we find the necessary conditions

$$\frac{\partial H}{\partial V(t)} = \frac{\partial P}{\partial V}V + P - \alpha(t) = 0 \qquad [7-15]$$

$$\frac{\dot{\alpha}(t)}{\alpha(t)} = r \qquad [7-16]$$

$$H(T) = 0 \qquad [7-17]$$

$$\alpha(T) = 0 \qquad \alpha(T)\,S(T) = 0 \qquad [7-18]$$

Condition [7-15] states that the current value of the user cost must be equated to the marginal revenue from the sale of ore in each period. Conditions [7-17] and [7-18] together imply that at time T

$$P\left[V(T)\right]V(T) - \alpha(T)V(T) = 0.$$

This holds only if total revenue is zero or if average revenue is equal to marginal revenue. But from condition [7-16] we know that at the terminal time $\alpha(T)>0$. Hence at time T the quantity sold is nil (implying the choke price is reached) or the inventory is depleted.

From condition [7-15] we know that at the initial time of sale from the inventory the marginal revenue from the sale of one unit must be equal to the value that the sale of that unit takes out from the stockpile. This means that $\alpha(\tau)$ must be at least equal to the value of the addition of a marginal unit to the stockpile,

$$\alpha(\tau) = \mu(\tau) \geqslant \frac{\partial W[S(\tau)]}{\partial S(\tau)} .$$

This is the boundary condition for the end of the extraction period.

Hence there are two possible optimal sales and extraction paths that constitute a solution to equations to [7-1] to [7-4]. They are:

(i) Production and sales are equal at each instant. Over the life of the firm stockpiling is nil. Hence $S(t)=0$, $\dot{S}(t)=0$, and $\theta(t) \geq 0$. At each point in time profits are maximized by maximizing instantaneous profits. The extraction and sales paths are identical with those of the optimal extraction path of a neoclassical monopoly with variable costs of extraction. The firm shuts down when extraction ceases. The firm's actions are identical with those of the monopoly with increasing costs of extraction.

(ii) It is optimal to stockpile. We examine the firm at a moment of time, $t=0+\epsilon$ and assume that $Q(t)>0$. Then from [7-6]

$$\frac{\partial C}{\partial Q(t)} + \lambda(t) = \mu(t) \quad \text{and} \quad \theta(t)=0,\ S(t)>0,\ \dot{S}(t)>0$$

Differentiating the above with respect to time and substituting for λ and μ from [7-8] and [7-9] we find the time-rate of change of the extraction path

$$\dot{Q} = \frac{r(\mu+\lambda)}{\frac{\partial^2 C}{\partial Q^2}} \geqslant 0$$

i.e., extraction increases over time.

We now examine marginal revenue; from [7-7] if sales are positive then marginal revenue is equal to the value of a unit in storage. That is,

$$MR(t) = \mu(t) \; ; \; \dot{S}(t), S(t) > 0 \; ; \; \theta(t) = 0 .$$

Differentiating this with respect to time and substituting for $\dot{\mu}$ from [7-9] we have

$$\dot{MR}(t) = r\mu(t) > 0 .$$

The monopolist will set marginal revenue from sales increasing at a rate equal to that of compound interest.

The combination of increasing marginal revenue from sales, an increasing extraction path, and a downward-sloping market demand curve imply that sales are <u>decreasing</u>.

Similarly to the competitive case, inventory decumulation may begin only when extraction has ceased. This ensures that there is only one sequence of extraction, followed by pure inventory decumulation. Formally, the extractive horizon τ is determined by condition [7-10]. The continuity of the profit-function ensures that the condition outlined in [7-10] may be attained only once over the life of the firm.

It is relevant to note that the treating of [7-6] and [7-7] as strict equalities, with $\theta(t)=0$, yields the condition

$$\frac{\partial C}{\partial Q(t)} + \lambda(t) = \left[\frac{\partial P}{\partial V} V + P\right] = \mu(t) .$$

At all times the marginal revenue from sales equals the marginal cost of extraction plus the user cost, and is equal to the current value of a unit in storage. This is exactly Smithies' discrimination rule for intertemporal discrimination discussed in Chapter I.

In summary the scenario of possible stockpiling indicates that while the cost of extraction, the marginal revenue, and the time-paths of extraction are increasing, sales are decreasing and therefore the firm is moving up along the (downward-sloping) industry demand curve.

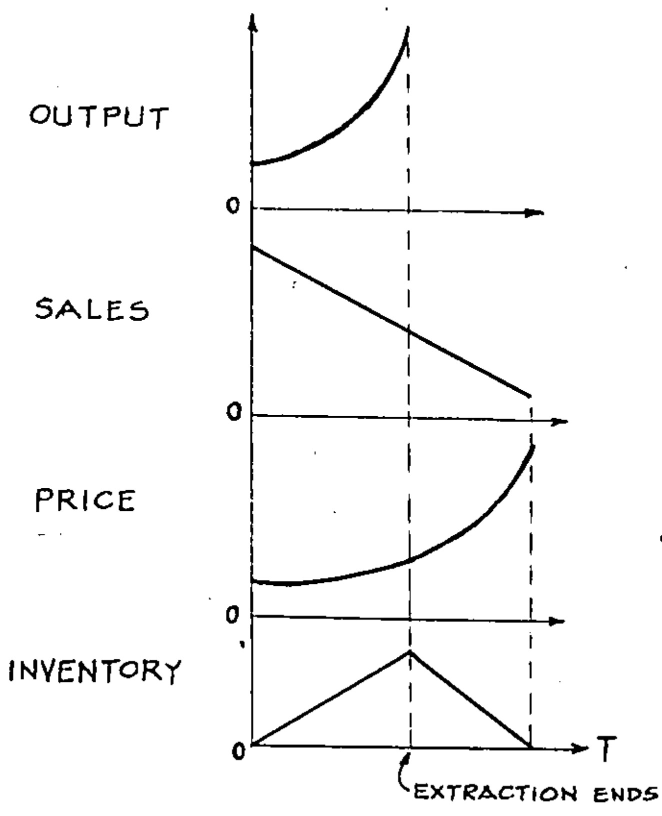

Fig. 2. Timepaths of output, sales, price, and inventory if there is stockpiling by the time-autonomous inventory.

Summary and Conclusions

It has been shown that the costless monopoly sets its pricepath increasing such that its marginal revenue increases at the rate of interest. When costs of extraction are nil the problem of maximizing a present-value flow of profits from a stock of ore in the ground is identical to the one of disposing from a stockpile of the same resource maintained at zero cost.

Some limiting cases to the monopoly that sets its pricepath increasing at the rate of interest are then examined. Some of the cases discussed involve varying specifications for the market demand curve, or the cost functions. However, when the resource-extracting firm is allowed to explore there is a problem with the proper specification of the cost function to explain a U-shaped pricepath.

The analysis of the profit-maximizing monopoly with no exploration but increasing costs of extraction yields the expected result that the rate of increase of marginal revenue is monotone increasing, and extraction ceases when marginal profit is equal to average profit.

Finally, the conditions necessary to have decoupling sales and output are examined. There are two sets of conditions that can produce this result. In a time-autonomous deterministic model it is possible and sometimes optimal to "bunch up" production during some periods of production. The two conditions necessary to have a decoupling of sales and output in a time-autonomous model are analyzed. When these conditions are present, stockpiling occurs. To what extent stockpiling does, in fact, take place depends in large mesure on the parameters of the model.

CHAPTER V

IMPERFECT COMPETITION

Introduction

It has been seen that under certain circumstances it is profitable for a firm to engage in stockpiling for purposes of price discrimination through time. Under the assumption that there are few but less than many firms in the market this chapter examines how equilibrium is reached in the market, and how stockpiling is feasible.

Following Friedman's definition we shall define "oligopoly theory" as the partial equilibrium study of markets in which the demand side is competitive, while the supply side is neither monopolized nor competitive."[1] It must be noted at the outset that the analysis in this chapter is limited to noncooperative equilibrium.

Unlike perfect competition, "imperfect competition" leaves firms in the market with a considerable amount of discretion in deciding how to formulate their output and pricing policies. Hence it is necessary to examine some notions of game theory to understand some of the firms' limitations

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[1] J. Friedman, "Oligopoly Theory," in Handbook of Mathematical Economics, ed., K. Arrow and M. Intriligator, vol. 2 (New York: North-Holland), p. 491.

when operating in dynamic imperfectly competitive markets.

A good deal of the resource economics literature dealing with imperfect competition considers a fringe/cartel type of market structure. The common link between all of these market structures is the fact that all players have at all times perfect information. This is the key to understanding why stockpiling is little used in these markets[1]. This is not to rule out that building inventories for purposes of price discrimination is ruled out here; but instead the wealth of different models and operating assumptions available to explain the behavior of firms in imperfect competition precludes any systematic analysis of how firms might react to one another. Instead, what this chapter intends to do is briefly discuss the decision rule for a firm engaging in intertemporal price discrimination. Afterward there will be a discussion on the modelling of dynamic imperfectly competitive markets. The purpose of this section is to give the reader an insight into the difficulties inherent in modelling dynamic imperfectly competitive markets. This will be followed by a review of the existing literature on resource-extracting cartels as well as of imperfectly competitive resource firms. One of the purposes of this section is to give the reader a perception of why, given the behavioral assumptions of the models just examined, it is impossible to

[1]The exception is made by the analysis of Ware, "Inventory Holding as a Strategic Weapon to Deter Entry," Economica 52 (1985): 93-101.

have any stockpiling.

One last model reviewed deals with a reversal of roles: instead of the producer stockpiling it is the consumers who build up a stockpile to (ostensibly) reduce the producers' surplus.

This chapter concludes with a section on how stockpiling might be feasible under assumptions of imperfect competition. It appears that the most logical reason for building up inventories is for strategic motives.

Problems Inherent in Modelling Imperfect Competition

Introduction

The study of noncooperative equilibria of imperfect competition poses specific problems. Abstracting from all the problems inherent in modelling classic (that is, "static") imperfect competition, one problem inherent to dynamic models is deciding how a firm will react over time; to use classical game terminology this implies the question of how are players' strategy spaces to be formulated, i.e., the assumptions that are made concerning the players' strategic behavior often influence the nature of the conclusion[1]. Specifically, the choice of strategy spaces decides if the equilibrium is dynamically consistent, this being the decisive factor in

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[1] J. Reinganum and N. Stokey ("Oligopoly Extraction of a Common Property Natural Resource," International Economic Review 26(1) [1985]: 162) write that "... the choice ...[of strategy spaces]... should not be made casually since it can dictate to a very large degree the nature of the conclusions."

deciding whether or not the equilibrium respects the principle of optimality.

Therefore this discussion on modelling dynamic imperfect competition shall be followed by an analysis of open-and-closed-loop dynamic games, leading in turn to an examination of dynamic consistency and the principle of optimality. The relevance of all the above factors will become apparent when making the link with Nash-Cournot and Stackleberg equilibria, respectively.

Open and Closed-Loop Dynamic Games

In a dynamic setting, an "open-loop" solution assumes that an agent must decide on all his future actions at the outset; he is not allowed to react to previous decisions of other agents. A "closed-loop solution" (or a "feedback strategy model") allows an agent to decide during each instant of time (or in every period, if in a discrete-time setting) what his profit-maximizing (or welfare-maximizingm, as the case may be) behavior will be[1,2]. If his decision variable

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[1] D. Gottwald and W. Guth, "Allocation of exhaustible resources in oligopolistic markets," Energy Economics, October 1980, p. 208.

[2] J. Reinganum and N. Stokey state that in dynamic games there are two strategy spaces commonly used, corresponding to two assumptions about the period of commitment. There are "path strategies", where the period of commitment is the same as the planning horizon, and "decision rule strategies", where firms observe the values of relevant state variables and respond to them immediately in choosing their current actions. In this text "path strategies" correspond to open-loop games, and "decision rule strategies" to closed-loop games. How-

is price, then that agent will decide at each instant what price to set, and likewise if the agent's decision variable is output.

The advantage of using an open-loop strategy when analyzing imperfect competition is the greater analytic tractability[1]. That is, at the beginning of the game horizon each player announces a price-and-extraction path to which he commits himself. In this case the resulting initial equilibrium is identical to that of a static model, and the players commit themselves to maintaining their initial positions throughout the course of the game.

In contrast to an open-loop strategy it is obvious that a model that allows each player to adjust his actions at each moment of time on the basis of the best response to the observed state of the world is much more realistic[2]. However relatively few models of dynamic games make use of feedback strategies because of the difficulties inherent in modelling

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these two types of strategy spaces affect equilibria will become apparent during the discussion on dynamic consistency (Reinganum and Stokey, "Common Property Resource," p. 460).

[1] M. Eswaran and T. Lewis, "Exhaustible resources and alternative equilibrium concepts," Canadian Journal of Economics XVIII(3) (1985): 460.

[2] M. Eswaran and T. Lewis state that an additional requirement for a game that is allowed feedback strategies is that future actions be incorporated in present decision-making; the decision rule for the current period embodies the equilibrium responses for future periods. This implies perfect foresight on the part of the players. (Ibid., p. 460).

players' reactions.

Therefore the inherent strength of the open-loop formulation is somewhat diminished by the implicit assumption of the seeming lack of rationality on the part of the players[1].

Finally the apparent drawback of the open-loop form has been partly corrected by the demonstration that in certain specific cases the open-and-closed-loop equilibria are identical[2]. The relevance of this conclusion will become apparent later in this chapter.

Dynamic Consistency and the Principle of Optimality

Regardless of the market structure assumed, the problem of the firm remains to maximize its present discounted profits given a stock of reserves and varying constraints. The technique of dynamic programming used to solve the problem through the application of the maximum principle finds an optimal extraction path to be followed throughout the life of the

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[1]That is, the following of an announced extraction-and-price-path throughout the length of the firm's extraction horizon, regardless of the evolution of the observed state of the world, might not appear to be entirely rational at first sight. This can be explained by the fact that in following its announced extraction path a firm is binding itself to its delivery contracts.

[2]This conclusion is due to M. Eswaran and T. Lewis ("Alternative Equilibrium Concepts," p. 461). The particular conditions necessary for this conclusion to be valid is that the resource be privately owned, that the demand curve facing the oligopolistic extractive industry be isoelastic, and that extraction costs be zero. The authors continue on to say that even when the two equilibria do not coincide the qualitative differences between the two equilibria are small.

mine. This yields an optimal policy. This optimal policy is guaranteed by the principle of optimality, which states

> "An optimal policy has the property that, whatever the initial state and decision, the remaining decisions must constitute an optimal policy starting from the state resulting from the first decision."[1]

That is, dynamic programming assumes that future values don't affect the current decision. To do otherwise is dynamically inconsistent[2]. Hence we may define a policy to be consistent if that policy satisfies the principle of optimality[3] as well as the maximum principle of control theory.

Formally speaking[4], let a dynamic system be defined by a state $x_1(t)$, $x_2(t)$, ..., $x_s(t)$, and the corresponding controls chosen through the application of the maximum principle be denoted by some variables $v_1(t)$, $v_2(t)$,..., $v_n(t)$ following the solving of some maximization problem, subject to an initial condition $x_i(0)=\bar{x}_i(0)$, a constant term at time t=0. Then the general form of the dynamic programming problem is

$$\max \int_0^\infty U\left[x_s(t), v_n(t), t\right] dt$$

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[1]R. Bellman, Dynamic Programming (Princeton: Princeton University Press, 1957), p. 28, cited in Sethi and Thompson, Optimal Control Theory, p. 16.

[2]D.M.G. Newbery, "Oil Prices, Cartels, and the Problem of Dynamic Inconsistency," Economic Journal 91 (1981): 634.

[3]L. Karp, "Optimality and Consistency in a Differential Game with Non-Renewable Resources," Journal of Economic Dynamics and Control 8(1) (1984): 75.

[4]This exposition is due to Arrow and Kurz, Public Investment, pp. 27-29.

subject to the initial state condition along with a state time-differene operator. If $v_i^*(t)$ is the control that maximizes the above integral, then those functions (the v_i^*'s) are the optimal path strategy (or open-loop control). An optimal feedback control solving the above would be of the form $v_i^*(t)=v_i^*[x^*(t),t]$ where $x^*(t)$ corresponds to the optimal rate of change of the state variable.

Intuitively speaking, the maximum principle is obvious, for if it did not define an optimal path from the first period onward there would have been a better path to choose in the first place.

For an optimal policy to be dynamically consistent throughout the lifespan of the mine it is necessary that at every moment during the extractive phase the firm ignore its self-interest and instead remain committed to its original contract; that is, after the initial period of extraction a firm will ignore its short-run self-interest as it arises, given the firm's market power and its expectations of future opportunities[1].

Hence a solution path is <u>intertemporally consistent</u> if a player whose expectations about the other players' (or country's) actions are realized while carrying out the initial

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[1]Hence the open-loop equilibrium described is also the "binding-contracts equilibrium". If firms renege on their contracts and instead seek to pursue their expectations of greater future opportunities, then the resulting equilibrium is the rational expectations equilibrium. (D.M.G. Newbery, "Oil Prices, Cartels and the Problem of Dynamic Inconsistency," <u>Economic Journal</u> 91 (1981): 618).

part of the equilibrium path will not wish to depart from the remainder of the path[1].

In summary a binding contract equilibrium is dynamically consistent; a rational expectations equilibrium need not be. Dynamic consistency works on the assumption that suppliers respect the contracts they have signed[2].

Cournot-Nash and Stackleberg Equilibria

A non-cooperative equilibrium is a Nash equilibrium[3,4]. In a dynamic setting a set of path strategies constitute a Nash equilibrium if for each player the following condition is satisfied: the path to which a player commits himself must be an optimal response to the paths to which the other players have committed themselves[5]. From the above definition, and defining a Cournot equilibrium as one where players take each other's actions as given, and a Stackleberg equilibrium as one

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[1] Vincent Crawford, Joel Sobel, and Ichiro Takahashi, "Bargaining, Strategic Reserves, and International Trade in Exhaustible Resources," American Journal of Agricultural Economics 66(4) (1984): 479.

[2] Newbery, "Oil and Dynamic Consistency," p. 617.

[3] E. Malinvaud, Lectures on Microeconomic Theory, trans. by A. Silvey, 4th ed., (New York: North-Holland, 1985): p. 150.

[4] Alternatively, "if a feasible vector of acts cannot be upset by any single-membered coalition it is said to be a Nash equilibrium." (P. Dasgupta and G. Heal, Economic Theory and Exhaustible Resources [Oxford: Cambridge University Press, 1979], p. 13).

[5] Reinganum and Stokey, "Common Property Resource," p. 163.

where each player's reactions are taken as given, it is easily established that a Cournot or a Stackleberg equilibrium is also a Nash equilibrium[1].

As has been established in the preceding section, whether or not players are operating in a closed-loop game context does not rule out the fact that all players have decision rule strategies; in an open-loop game the strategy will be decided once and for all whereas in a closed-loop game the strategy will be continuously modified.

An open-loop Nash equilibrium exists when the path to which a player commits himself is an optimal (initial) response to the paths to which other players are committed[2]. A Nash equilibrium states only that a player's decision rule must be an optimal response when viewed from the initial (date, state) pair; the continuation of his decision rule need not be an optimal response when viewed from any (date, state) pair. Furthermore, a Nash equilibrium is subgame perfect if the continuation of the given decision rules constitutes a Nash equilibrium when viewed from any intermediate pair (date, state)[3]. Therefore subgame perfect Nash equilibria satisfy the principle of optimality throughout the game; imperfect Nash equilibrium strategies don't.

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[1]H. Varian, <u>Microeconomic Theory</u>, 2nd ed. (New York: Norton), p. 101.

[2]Reinganum and Stokey, "Common Property Resource," p. 163.

[3]Ibid.

A further distinction is warranted here: a game is dynamically consistent only if the equilibrium represents the continuation of Nash equilibrium strategies. A game admits subgame perfection only if the above property holds for every equilibrium point[1].

In summary, a non-cooperative open-loop solution to a deterministic dynamic game yields a Cournot-Nash solution; such a solution to this type of game is obtained by considering the problem at hand as one of a classical optimal control problem for one player with respect to the others while considering the control variables fixed over the horizon[2]. This constitutes a binding contract equilibrium, and hence is dynamically consistent[3].

Conversely, in a closed-loop equilibrium players are continuously reacting to their opponents' chosen optimal output paths; this implies that the agent can react to previous decisions of other agents[4]. Hence the most notable feature

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[1]Ibid, p. 164.

[2]J. Lévine et J. Thépot, "Open Loop and Closed Loop Equilibria in a Dynamical Duopoly," in Optimal Control Theory and Economic Analysis, ed. Gustav Feichtinger, (New York: North-Holland, 1982), p. 145.

[3]Dynamic inconsistency arises as follows: the maximum principle finds an optimal price and extraction path which, when viewed from the present, maximizes the present discounted profit, on the assumption that that price-path will be followed. (Newbery, "Oil and Dynamic Consistency," p. 617).

[4]Gottwald and Guth ("Allocation of resources," p. 208) write that this is analyzing the "subgame perfect equilibrium behavior of dynamic market games."

of closed-loop solutions is that a producer may observe his opponent's output and price strategies, and react to them at any point to maximize his objective profit-function[1].

Given the above, a rational expectations equilibrium must be a closed-loop solution.

A closed-loop equilibrium need not be dynamically consistent. Finally, if one is to allow for rationality in a Stackleberg game, it must be remembered that the rationality of the follower implies that the closed-loop policy does not, in general, satisfy the principle of optimality.

Cartels in Exhaustible Resource Markets

Introduction

The first article dealing with a cartel-fringe type of market structure for an exhaustible resource is due to Salant[2]. Using a simplified market structure he derives a noncooperative Cournot-Nash equilibrium for a cartel and a competitive fringe. In particular the equilibrium price-path derived by Salant for the entire industry is between that of a competitive and a a monopolistic price-paths, respectively, predicted by Hotelling[3] for these industry structures. To

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[1] Karp, "Optimality with Non-Renewable Resources," p. 75.

[2] S. Salant, "Exhaustible Resources and Industrial Structure: A Nash-Cournot Approach to the World Oil Market," Journal of Political Economy 84(5) (1976): 1078-1093.

[3] This point is due to T. Lewis and R. Schmalensee, "Cartel deception in nonrenewable resource markets," Bell Journal of Economics 13(1) (1982): 263.

what extent this aspect of cartel behavior as well as others modified when some of the assumptions defining the model are relaxed will be examined in the subsequent sections.

The Basic Model of a Cartel in an Imperfectly Competitive Resource Market

Salant[1] postulates that the structure of the world oil market is neither perfectly competitive nor perfectly monopolistic. Hence he analyzes the structure of a modified model of the oil market that comprises a dominant cartel and a competitive fringe.

Before a cartel is formed there is a specified number of firms in the market that extract a nonrenewable resource from deposits of the resource of identical size at the same constant marginal cost. A cartel is formed when two or more deposits of the resource come under the control of the same management[2]. If the costs of extraction are constant across the industry, then the feature that distinguishes the cartel from the fringe is that the cartel owns more reserves[3]. In addition, Salant's model assumes fixed reserves, complete freedom of entry and exit[4], a well-defined stationary consumer demand curve, and

[1] Salant, "Resources and Industrial Structure".

[2] Ibid., p. 1080.

[3] Another advantage accrues to the cartel when costs of extraction are constant; this point is examined below.

[4] The author also adds that if the amount of available reserves is fixed, entry into the industry is difficult.

perfect information[1] on all participants' prices, costs, reserves, and outputs.

If, in addition to the "cartel" made up of at least two firms, it is assumed that the rest of the world is divided equally among a sufficiently large number of small firms, then a dominant firm model results[2]. In such a model each small firm takes the cartel's price path as given, and sets its price and extraction path so as to maximize the sum of discounted profits. The cartel also seeks to maximize its sum of discounted profits with respect to its own reserves and the fringe's output. This situation - where each sector accepts as given the other's optimal strategy - is a Nash-Cournot equilibrium[3].

One last simplifying assumption that is made for the sake of clearly demonstrating the advantage of the cartel owning more oil than the component members of the fringe is as follows: all participants in the market may extract the resource at zero marginal cost[4].

The cartel's problem is to pick a price and output path, given the fringe's sales path and the cartel's own stock of resources. To derive the demand curve on the basis of which

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[1] Lewis and Schmalensee ("Cartel Deception", p. 134) note that under the assumption of perfect information all expectations must be realized.

[2] Salant, "Resources and Industrial Structure", p. 1080.

[3] Ibid., p. 1080.

[4] Ibid., p. 1081.

it forms its sales decision, the cartel deducts the fringe's sales from the stationary demand curve to form a sequence of excess demand curves. Each one of these curves indicates the amount of the resource demand ata given price, in excess of what the fringe sells at that instant[1]. It is on the basis of these excess demand curves, subject to its resource availability, that the cartel picks its sales path over time.

The cartel's optimal strategy is to equate the discounted value of the marginal revenue from the sale of the resource at each instant of time. Furthermore, when the cartel ends its sales, the discounted marginal revenue from sales must never exceed that obtained during a period of positive sales. These two conditions ensure that profits are maximized.

The competitive sector chooses its sales path so as to maximize the sum of discounted profits, given the price path that prevails at that moment. Hence it acts like an ordinary competitive sector, where each firm takes the price path as given. Finally, speculators are allowed to operate in the market: the only limit imposed on their behavior is that they enter the market with no holdings of the resource beforehand.

The characterization of the equilibrium between the cartel and the fringe is as follows: as long as the fringe has positive reserves, both the marginal revenue and the price increase at the rate of interest; eventually both sectors exhaust their

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[1] Ibid.

reserves. This characterization of the equilibrium suggests that the cartel will always adjust its sales path to have positive reserves when the fringe's reserves are exhausted. This is demonstrated in the following way: if the cartel's reserves were exhausted before those of the fringe, after the cartel's exhaustion the fringe would continue selling the resource like an ordinary perfectly competitive industry. Compare this scenario with two moments, namely when the cartel is selling, and when the cartel's sales are nil. If the cartel has positive sales its marginal revenue is less than price; when the cartel's sales are nil its marginal revenue is equal to price. Since both the price and the marginal revenue are growing at the rate of interest, this suggests that marginal revenue has grown by more than the rate of interest to reach the level of the price[1]. For the rate of increase of marginal revenue to take a leap of this sort is indicative of the fact that the cartel's strategy is not optimal.

Hence in equilibrium the fringe must exhaust its reserves before the cartel exhausts its own[2].

The sufficiency condition for the existence of an equilibrium between the cartel and the fringe is that the consumer demand curve have a single point of unit elasticity,

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[1] Ibid., p. 1083.

[2] This result is consistent with the principle of optimality, namely that the optimal policy determined by the application of the maximum remain valid for the duration of the life of the system.

and that the elasticity along the consumer demand curve increase strictly with price[1].

In order to maintain the distribution of market shares chosen in the initial period of extraction, the price and marginal revenue must remain in the same proportions as that set initially. This occurs if the elasticity of excess demand is the same at every point chosen by the cartel along its excess demand curve. If the consumer demand curve satisfies the sufficiency condition noted above, then the point elasticity of a given excess demand curve increases as the price increases. Therefore the sales of the competitive fringe must fall as the price rises to shift the excess demand curve to restore the elasticity of excess demand to its previous ~~domand~~[2].

The effect of a cartel on a competitive fringe are well-known: the initial price of a cartel-fringe industry is set higher than that of a competitive industry, and the price expansion paths of the cartel and the fringe must cross only once. Furthermore, the existence of a cartel makes all suppliers of the resource in the market better off, in the aggregate[3].

[1]Ibid., pp. 1084-1085, and pp. 1090-1091. Salant adds that all linear or concave demand curves and many convex curves satisfy this condition.

[2]Ibid., p. 1085.

[3]In addition, the profits of every firm in the fringe jump by an even larger percentage than the cartel's. This is because fringe firms get the benefit of higher prices without having to contort their sales to obtain these higher prices. (Ibid., p. 1086). This point is stated also by T. Lewis and R. Schmalense, "Cartels and Oligopoly Pricing of Nonreplenishable Natural

It is possible for a single firm to break ranks with cartel discipline and dispose of its entire resource stock at the same rate as the rest of the fringe; by virtue of the joint assumptions of perfect information and noncooperative Nash-Cournot equilibrium, the breaking away from the ranks of other firms in the cartel causes an adjustment an adjustment downward of the entire pricepath. If all members imitate the "chiseller" to act on their own, then the entire industry structure is identical to that of perfect competition.

All of the preceding discussion has been based on the assumption of zero costs of extraction. The inclusion of constant marginal costs of extraction changes the results trivially. However, the inclusion of variable costs of extraction changes the structure of the model, because the cartel's advantage over the fringe is enhanced by the fact that it is able to extract as much as any other firm at a lower cost[1]. This increases the cartel's market power. In the absence of the formation of a cartel the industry behaves as an ordinary profit-maximizer. Although Salant doesn't prove the existence of equilibrium[2] he assumes it exists to show that

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Resources," in Dynamic Optimization and Mathematical Economics, ed. Pan-Tai Liu (New York: Plenum Press, 1980), p. 141.

[1]Salant, "Resources and Industrial Structure," p. 1088.

[2]To the knowledge of the author, no proof of existence exists for a cartel-fringe market structure endowed with increasing marginal costs of extraction.

the equilibrium has the same basic properties as the structure of the model with constant marginal costs. Specifically, the cartel will still remain in the market after the fringe is exhausted, the real price of the resource is monotone increasing, and finally the formation of a cartel raises the fringe's discounted profits by an even greater percentage than those of the cartel[1].

In summary, then, Salant's model embodies five main features. They are: (i) all firms are initially of the same size and possess reserves of identical size; (ii) all firms have the same constant or variable costs of extraction; (iii) all firms in the market have perfect information at all times on all possible parameters affecting the industry; (iv) all agents accept the announced price and sales paths of the other participants in the industry; furthermore they consider their announced own sales and extraction paths as binding; (v) the inverse demand curve must be increasing with price, comprising one point of unit elasticity.

Why Stockpiling is Unfeasible in the Basic Model of an Imperfectly Competitive Resource Market

It is relevant to discuss the possibility for either

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[1]Salant, "Resources and Industrial Structure," pp. 1088-1089.

the cartel or the fringe of storing the resource above ground once it is extracted: all firms in the industry have perfect information on all parameters, and possess identical reserves. It is not likely that the fringe will store the resource above ground because of the assumption that it acts as a perfectly competitive industry[1]. On the other hand, if the cartel sets aside a portion of its production for a stockpile, the fringe will interpret the move as a decrease in the cartel's sales rate, with a change in its cost parameters. If sales rates decline then so will the time-price-path. The fringe will react by increasing its sales rate, with a corresponding increase in its time-price-path. In essence, the fringe will take up the slack caused by a decrease in the cartel's sales rate. But the fringe acts as a profit-maximizer, and hence chooses the best pricepath available given the circumstances. However we are postulating a noncooperative Nash-Cournot open-loop equilibrium: the fringe takes as given the cartel's pricepath, which in turn maximizes discounted profits based on the fringe's pricepath. It is doubtful that this type of equilibrium is feasible for two reasons: first, the cartel would be deliberately setting a time-price-path

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[1]If costs of extraction are constant it is optimal for the firm to delay extraction as much as possible.

lower than it might have in ordinary circumstances. This indicates a suboptimal behavior. Second, and more importantly such a behavior by the cartel violates the dynamic consistency of the sales path, which has defined an optimal extraction and sales policy for the cartel. A deviation from this path is not consistent with the principle of the maximization of a stream of discounted profits over the life of the firm (or the cartel).

In conclusion, any deviation from the cartel's predetermined sales and extraction path implies a suboptimal behavior.

Variants of the Basic Model of a Cartel and Results Obtained

Unequal costs and reserves among the firms in the industry

A variant of Salant's model is presented by Ulph and Folie[1]. The model is roughly the same as Salant's[2] but firms' costs and sizes of reserves vary across the industry. The authors find that in such a case the formation of a cartel decreases the value of fringe reserves. This result holds for the case of linear demand and constant marginal costs. The purpose of the last two assumptions is to ensure that any change of profits before and after the formation of the cartel

[1] A. Ulph and G. Folie, "Exhaustible resources and cartels: an intertemporal Nash-Cournot model," Canadian Journal of Economics XIII(4) (1980): 645-658.

[2] Salant, "Resources and Industrial Structure".

is due exclusively to a change on the resource rents. If convex costs of extraction are assumed it is still possible to draw conclusions about the outcome of equilibrium, but no precise statements can be made regarding the magnitude of the change of the profits (rents) of the fringe[1].

The cartel's excess average revenue curve[2] is derived from subtracting at each instant of time the given level of fringe output. While respecting the rules of a Nash-Cournot market structure, this also allows the cartel to act like a monopolist in maximizing its time profile of discounted profits.

In a market structure under analysis there are five different possible patterns of production between the cartel and the fringe[3]. The type of cost advantage given to the cartel[4] is such that only three of these five possible equilibria can occur. But the robustness of the result that cartelization decreases the value of the fringe is such that the authors find that they need not give the cartel such a cost

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[1]Ulph and Folie note that an immediate consequence of this result is that contrary to Salant, who concludes that cartel profits decrease only consumer prices, if fringe rents fall then the formation of a cartel hinders both consumers and producers.

[2]Ulph and Folie, "Resources and Cartels," p. 649.

[3]Ibid., p. 656.

[4]An implication of the assumption that the cartel has a cost advantage is that at the time the cartel is formed the fringe is not producing. (Ulph and Folie, Ibid., pp. 646-647).

advantage. Indeed they find that even if the cartel has a small cost advantage (i.e., the cartel's costs are only "slightly lower") relative to the fringe the latter will lose out because of cartelization.

Introducing well-behaved convex cost functions in the analysis leads to slightly different results: as expected, there can be only one segment of time in the life of the industry when cartel and fringe are producing at the same time. If the cartel has in principle a significant cost advantage of the type suggested above then the cartel will begin and finish production with the fringe producing at some time in between.

Within the industry just considered the difficulty with engaging in stockpiling is the same as when costs and reserves are constant: perfect information prevails, on the basis of which firms compute their optimal extraction and sales paths. The presence of a stationary (inverse) demand curve ensures that the principle of dynamic consistency on which is based the optimal sales and extraction paths need be followed in each period. Hence the model's parameters remain unchanging, and the open-loop Nash-Cournot (noncooperative) equilibrium remains valid for the duration of the industry. Then, any deviation from this path is likely to be suboptimal.

The Generalization of the Basic Model of a Cartel in an Imperfectly Competitive Resource Industry

Salant's specification for a demand curve insists on the fact that it be decreasing in elasticity. Lewis and Schmalensee[1] extend the analysis so that both constant and increasing-elasticity demand curves[2] be included in the analysis of and in the proof of existence of equilibrium, in addition to the decreasing elasticity case with which Salant carries out his analysis. In addition, Lewis and Schmalensee also prove the existence of equilibrium for the two additional types of demand curves that they consider, and characterize these equilibria.

Probably their most important contribution in their extension of the analysis of Salant's analysis[3] is that they carry out the comparative statics of cartel equilibrium.

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[1] Lewis and Schmalensee, "Cartels and Oligopoly Pricing of Resources,": 133-156.

[2] The constant elasticity demand curve specification is fairly common and merits no comment here. The increasing elasticity specification refers to a demand schedule where elasticity increases with consumption. In their article, Lewis and Schmalensee justify this specification in referring to the demand for a product for which demand might be quite inelastic for small quantities, but as the price decreases the type of market for this good has close substitutes with other goods. This particular specification has been used in an earlier paper by T. Lewis, S. Matthews, and H. Burness, "Monopoly and the Rate of Extraction of Exhaustible Resources: Note," American Economic Review 69(1) (1979): 227-230.

[3] Salant, "Resources and Industrial Structure."

They confirm the fact that total discounted cartel profits increase the greater are initial reserves of the cartel and the fringe.

The Basic Model If There is Imperfect Observability of Reserves

A crucial feature of the interaction between the cartel and the fringe has been perfect information, available at no cost to all members of the industry; this has given rise to a non-cooperative Cournot-Nash equilibrium. Lewis and Schmalensee[1] examine the consequences of the fringe having imperfect information on the cartel; specifically they assume that the reserves of the fringe are known to all participants in the market but those of the cartel only to itself, with the proviso that the cartel's sales path be consistent with its announced inventory levels. This model depends on a static result derived by Lewis and Schmalensee, namely that, all things constant, if the reserves of the cartel increase, then the initial price charged by the fringe decreases[2]. Because the fringe is allowed to observe the cartel's sales (but not

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[1] Lewis and Schmalensee, "Cartels and Oligopoly Pricing of Resources."

[2] This result is obtained in Lewis and Schmalensee (Ibid.). Letting I^m represent the level of the cartel's reserves and P_o the initial price charged by the fringe, then $\partial p_o/\partial I^m > 0$. The effect of having a lower initial price is to accelerate the initial depletion of the resource, hence decreasing the terminal time of exhaustion. In this way the cartel can enjoy monopoly profits sooner in time than if it had truthfully represented its reserves.

reserves) it has two choices. They are:

(i) Follow the classical profit-maximizing strategy, in which case the fringe immediately perceives the cartel's misrepresentation of inventories and adjusts its pricepath accordingly.

(ii) Continue cheating; to make the fraud credible the cartel's sales must be consistent with its announced reserves; hence it cannot maximize profits on all of the units it places for sale on the market before the fringe is exhausted. This implies that for the cheating strategy to be profitable the loss is less than the gains from a monopoly position for the cartel. This is due to the fact that if demand is linear the fringe's output is independent of the cartel's actual or announced reserve holdings[1]. Lewis and Schmalensee state that this is not so[2]; furthermore they prove formally that telling the truth dominates any strategy involving stock misrepresentations to the fringe during the initial stages of the game.

In summary, if cheating is carried out by the cartel it must be such that its sales path is consistent with its announced inventory levels. Hence when the cartel announces that its reserves actually are much larger than previously thought, the selling price must be readjusted downward. This behavior is hardly consistent with profit-maximization.

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[1]This point is due to Salant, "Resources and Industrial Structure," p. 1085.

[2]Lewis and Schmalensee, "Cartels and Oligopoly Pricing of Resources," p. 146.

The Basic Model If There is Covert Hoarding and Dumping of the Resource

Heretofore the discussion of cartel equilibria has assumed the existence of perfect futures markets. As mentioned in the section dealing with dynamic games, complete futures markets imply perfect foresight; this allows participants in an industry to set a price-and-output path for the duration of the extraction period, fully expecting not to have to revise it. This is a binding contract equilibrium.

The role of complete information when a producer is fixing his future plans is crucial to the determination of a Nash-Cournot-type of equilibrium.

In keeping with the reality of resource markets a more realistic assumption to make regarding total reserves is that they are unknown, and hence it is impossible to fix output plans for the duration of the extraction period. The only complete information available to the producers is the price charged in the present period[1]. Hence the assumption of an open-loop strategy on which a Nash-Cournot equilibrium is arrived at is negated; instead the model becomes one of a Stackleberg price-leadership type[2].

Given this possibility of behavior, the cartel has

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[1]Any indication of future price is unreliable because future reserves are indeterminate.

[2]Lewis and Schmalensee, "Cartel Deception," p. 265.

two choices. These are:

(i) In the presence of a profit-maximizing competitive fringe it can announce any pricepath it can afford to support given its reserves. Then for the case of zero costs of extraction in the fringe and a common rate of interest the Stackelberg equilibrium is a Cournot equilibrium: the price must increase at the rate of interest until the fringe is depleted, independently of the actions of the cartel[1]. Furthermore, given the assumptions made, the authors assert the following Lemma: "suppose the cartel can announce any pricepath p(t) that it can support given a profit-maximizing competitive fringe behavior. This pricepath must be followed for all time. Then cartel wealth is maximized by announcing the Salant equilibrium pricepath."[2]

(ii) If the cartel's output cannot be observed it is possible for the cartel to increase its wealth by announcing a false pricepath at the beginning of extraction, and revising it at some time during the production phase.

It is through this second option that suppliers can engage in covert dumping and hoarding of the resource; the

[1] This point is due to R.J. Gilbert, "Dominant firm pricing policy in a market for an exhaustible resource," Bell Journal of Economics 9(2) (1978): 388. In that article Gilbert mentions that in the more general cases a Cournot equilibrium is not a Stackleberg equilibrium.

[2] Lewis and Schmalensee, "Cartel Deception," p. 266.

essential requirement is that they falsify competitors' expectations, and voluntarily enforce dynamically inconsistent production plans[1].

The second option listed above includes two possibilities. These are:

(a) Covert dumping on the part of the cartel. This is an attempt by the cartel to benefit from the higher price available if reserves were smaller than they actually are. If the cartel manages to dispose of its entire inventory before it has to adjust its pricepath, it is able to increase its wealth relative to what it would have been if the cartel had sold its production on the basis of the Salant pricepath[2]. After the cartel's reserves are exhausted the price takes a discrete jump downward to adjust to the reality of the remaining reserves.

It is possible that when readjustments occur in the industry's pricepath the cartel has not yet disposed of its entire reserves. If this happens the authors demonstrate that it only changes the magnitude, not the direction, of the results[3].

(b) Covert hoarding on the part of the cartel. The cartel announces a pricepath based on smaller reserves than its

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[1] T. Lewis and R. Schmalensee (Ibid., p. 267) state that this option is viable only if cartel output cannot be monitored. They base their analysis on this assumption.

[2] Ibid., p. 266.

[3] Ibid., p. 268.

actual reserves. It must initially sell less of the resource than is consistent with its profit-maximizing output. However when the cartel's deception is revealed the cartel finds itself with reserves larger than would be the case had it been truthful. In accordance with the authors' finding that an initial Salant-type equilibrium must be supported for the duration of the industry, the Nash-Cournot-type pricepath is revealed after the deception. In this case the industry will revert to the original equilibrium predicted by Salant[1]. But according to the Lemma reproduced earlier, it is not in the cartel's interest to support a different equilibrium pricepath after the deception is discovered. Therefore cartel wealth must be increased through the hoarding of the resource.

Whereas the purpose of stockpiling is immediately negated by the activity of covert dumping, covert hoarding is a form of stockpiling, albeit a secret stockpile. But the cartel has consistently hidden its hoarded resource; hence when it does make its stockpile public it finds that the marginal benefit on each unit of ore is reduced due to the sudden flooding of the market by the new reserves. Therefore it is decidedly suboptimal to covertly build a stockpile, for the future price on each unit will be lower than that presently obtainable.

Cartel-Fringe Model of an Extractive Industry with Stackleberg Equilibria

The difference between Cournot and Stackleberg equilibria

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[1]Salant, "Resources and Industrial Structure".

has already been examined in the preceding section. In such models firms' price or output can be strategic variables for a number of reasons.

In an analysis[1] of a cartel-fringe market structure of the oil industry where the cartel acts as a Stackleberg leader, the cartel sets a profit-maximizing price-and-extraction path which the fringe responds to. The fringe then sets its own profit-maximizing output based on its expectations of future prices (these future prices are part of expected past, present, and future, that the fringe holds with respect to the cartel).

The mechanism by which the industry arrives at an equilibrium is as follows[2]: assuming both sectors of the industry have perfect information on the stationary inverse demand curve, the cartel announces a production path based on the supply response of the fringe. The response of the fringe, in turn, depends on its component members' individual costs of extraction[3]. The fringe, taking price as given, selects a profit-maximizing output. This output consists of the industry output less the cartel's announced output; this makes up the

[1] R.J. Gilbert, "Dominant Firm pricing in a market for an exhaustible resource," Bell Journal of Economics 9(2) (1978): 385-395.

[2] Gilbert notes that the existence of an intertemporal equilibrium doesn't imply that this equilibrium would be attained in a dynamic economy. (Ibid., p. 386).

[3] Ibid., p. 389.

residual demand function[1].

The conditions which determine whether or not the fringe will extract are identical to those of other models of resource extraction: the current rent for reserves in the competitive sector is the difference between the price and the marginal cost of extraction[2]. This sequence of events rests on two factors. These are:

(i) The supply response that is chosen by the fringe once the cartel has announcerd its pricepath[3]; this depends on the way expectations are formed[4].

(ii) Firms in the fringe possess perfect information about the past, present, and future actions of the cartel[5].

To simplify this model Gilbert makes two assumptions:

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[1] Ibid., p. 387.

[2] Ibid.

[3] A particular assumption of a Stacklebeeg equilibrium is that firms in the fringe are each individually capable of influencing the price at any date, and hence take the price as given when determining their supply plans. (Newbery, "Oil and Dynamic Consistency," pp. 620-621).

[4] Gilbert states that he doesn't know how expectations are formed; if the firms knew this they could use this information in their pricing policy. ("Dominant firm pricing," p. 386).

[5] Ibid., p. 386. In addition, T. Lewis and R. Schmalensee, state that to assume knowledge of each other's output paths in advance is a strong assumption, which is close to assuming complete futures markets. ("On Oligopolistic Markets for Nonrenewable Natural Resources," <u>Quarterly Journal of Economics</u> 95 (378-81) (1980): 477).

that at a high enough price (the "limit price") the available substitutes of the resource are greater than the present recoverable reserves, and at ranges below the limit price the elasticity of demand for the resource is below unity[1].

Given the stated parameters of the model Gilbert examines two scenarios for strategic interaction between the cartel and the fringe, namely that the fringe's marginal cost of extraction is constant or increasing. The former case corresponds to a situation where firms are operating at less than maximum capacity, while the latter case corresponds to a situation where a capacity constraint is imposed on the fringe; as it seeks to increase production, marginal costs of extraction increase.

In the first case, when the marginal cost of production is constant, the equilibrium pricepath has two phases. These are:

(i) When both sectors are extracting the industry's time-pricepath must be equal to the rate of interest until the limit price is attained. Until that happens, the market demand is determined by the residual demand[2]. During that period the cartel can vary the price-level by varying its announced output level. But the competitive fringe's pricepath is still governed by the arbitrage equation, namely that the percent

[1] Gilbert, "Dominant firm pricing," p. 386.

[2] The residual demand has been defined previously. Specifically it has two functions. They are: (i) it is the difference of total market demand less the cartel's announced demand; (ii) it constitutes the competitive fringe's demand.

time-rate of change of the pricepath is equal to the fringe's rate of time-preference[1]. That is,

$$\frac{dP[Q^f(t)+\bar{Q}^\ell(t)]\,dt}{P[Q^f(t)+\bar{Q}^\ell(t)]-m^fg^f[S^f(t)]}=r^f$$

where $Q^f(t)+\bar{Q}^\ell(t)$ constitutes the sum of the fringe and the cartel's announced output, respectively, and $m^f g^f[S^f(t)]$ is the fringe's constant marginal cost. (The analogy with the Hotelling rule for the constant-cost case is obvious).

(ii) When the limit price is reached all fringe reserves earning a positive rent with a marginal cost of extraction less than the market price will have been recovered. Then the cartel may maximize profits[2,3]. When the fringe's marginal cost of extraction is constant up to a point, and then increases, this specification is equivalent to imposing a constraint on productive capacity. This can have several effects on the behavior of the fringe. These are: (a) the production of the fringe cannot be more than its constraint; (b) the time-

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[1]Gilbert, "Dominant firm pricing," p. 390. The author notes that this is the same characteristic that governs the pricepath of a competitive industry in the presence of a backstop technology. This discussion is contained in P. Dasgupta and G. Heal, Economic Theory and Resources, pp. 175-379.

[2]Gilbert, "Dominant firm pricing," p. 390.

[3]There seems to be a problem with this solution, namely the determination of the conditions under which producers of a substitute enter the market once the limit price is reached, and in so doing how they can coexist alongside the cartel that extracts a "maximum rate of profit" even though the market price is marginally below the price of the substitute. (Ibid.)

rate of change of the fringe production is a decreasing function of time.

Because the time-path of extraction is a decreasing function of time, capacity production must always occur before less-than-capacity production[1]. During the time that the price is less than the limit price the cartel maximizes its profit by equating its marginal revenue to its marginal cost (inclusive of the rent). Bearing in mind that rents earned on the cartel's reserves are inversely related to the size of the reserves, as the size of the reserves tends to infinity the shadow price of the rent decreases so that the cartel's marginal revenue is equated to the marginal cost exclusive of the rent on the resource.

If the fringe capacity is small the cartel might ignore the effects of its decision by the fringe[2]. In this case the response of the fringe is to produce at maximum capacity; this is not its optimal response to the cartel's chosen price and extraction path. Hence there is an incentive for the fringe to increase capacity. If that happens the industry pricepath declines for a short period of time, then increases when reserves start declining. Eventually the price tends to the limit price.

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[1] Ibid., p. 393.

[2] The author states that the cartel chooses a production path, or pricing policy, that maximizes net revenues, and those revenues depend on the rate of production of the fringe. (Ibid., p. 394).

In summary two cases arise in this particular model: (i) if there is no capacity constraint in the industry then the cartel's optimal production rate is contingent on the characteristics of the fringe, or

(ii) if the fringe has a capacity constraint then the cartel may act as a classical limit-pricing firm[1], i.e., the fringe produces at capacity and the price remains constant or initially decreases.

<u>On the Unfeasibility of Stockpiling In a Cartel-Fringe Model of an Extractive Industry with Stackleberg Equilibria</u>

The key features in the Gilbert model are that the fringe has perfect information about the market demand curve, as well as on all the cartel's actions. What is of concern if there is to be stockpiling is the type of cost function as well as how the industry arrives at an equilibrium. In this model the pricepath is set by the cartel, based on the supply response of the fringe.

There are two considerations that argue against the cartel stockpiling, should it wish to do so. First, present-value considerations demand that the cost of extraction be delayed as far as possible in the future. Second, price is increasing at a rate equal to that of interest, and there is no possible future uncertainty. This invalidates the only

[1]Ibid., p. 386.

possible motive - future price uncertainty - that the cartel might have for stockpiling.

Oligopoly in Exhaustible Resource Markets

Introduction

Heretofore all markets considered are of the cartel-fringe type. If cartels are made up of a small number of firms then any cartel is potentially oligopolistic if it breaks down.

Two types of oligopoly are examined here: open- and closed-loop. The open-loop oligopoly corresponds to a Nash-Cournot market structure. The closed-loop equilibrium permits agents to decide in every period (or each moment of time) what they will extact.

Open-Loop or Cournot-Nash Oligopoly

By oligopoly is understood here a market structure where a single firm cannot dominate but any firm's actions can have an effect on equilibrium.

Following Salant's model of imperfect competition[1] Lewis and Schmalensee [2] use the same basic structure to analyze oligopoly equilibrium. Once more a crucial feature

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[1] Salant, "Resources and Industrial Structure."

[2] T. Lewis and R. Schmalensee, "Cartel and Oligopoly Pricing of Nonreplenishable Natural Resources," in Dynamic Optimization and Mathematical Economics, ed. Pan-Tai Liu (New York: Plenum Press, 1980): 133-156.

of this model is that perfect information prevails in the market[1], that the market demand curve exhibit decreasing elasticity of demand, that there are constant marginal costs of extraction, and that consumers are all passive price-takers who follow the continuous and well-known market demand curve. Each firm acts as a Cournot-Nash profit-maximizer, and takes all other firms' actions as given. Then, if reserves are equally distributed among firms, costs are constant and equal among firms, and there is a point of unit elasticity of demand along the demand curve it is shown that there exists a noncooperative oligopoly equilibrium for such a market[2]. Furthermore the rate of increase of the price is less than the rate of interest; ceteris paribus, if the number of firms increases, the initial price for the resource increases, the horizon for the industry decreases, thereby decreasing total profits[3].

These results are taken up again by T. Lewis and R. Schmalensee as a preamble to more general results[4]. By the same token the authors also give a formal extension of the Hotelling price rule involving firms between the extremes of

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[1]Ibid., p. 134.

[2]Lewis and Schmalensee,"Oligopolistic Markets for Resources."

[3]Lewis and Schmalensee, "Cartels and Oligopoly Pricing of Resources," p. 134.

[4]These results are reproduced in Lewis and Schmalensee, "Oligopolistic Markets for Resources," pp. 477-479.

perfect competition and monopoly.

These results are contingent on the symmetry of costs and reserves in the industry. If the assumption of equality of reserves is relaxed then the firms with the greater reserves will begin production earlier and cease production after the other firms[1,2]. In addition the extraction horizon is lengthened.

The presence of dissimilar constant costs in firms among the industry changes the nature of the analysis. Still assuming constant costs and a homogeneous grade of ore, if one firm in the industry has costs that are greater than other firms' individual costs then there will ensue inefficiency in equilibrium. It is intuitively plausible that a low-cost deposit must be exhausted before a higher-cost reserve[3]. But an efficient oligopoly equilibrium is

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[1] Ibid., p. 482.

[2] An examination of a not entirely dissimilar market structure proposed by Crawford, Sobel, and Takahashi, ("Bargaining and Strategic Reserves," pp. 472-480). In that model the authors conclude that the resource-rich entity will produce first until the reserves are equal for all participants (Ibid., p. 477). The difference in the conclusions between the two models rests crucially on the fact that Lewis and Schmalensee ("Oligopolistic Markets for Resources") analyze a partial equilibrium model, whereas Crawford et al analyze a general equilibrium model of a simplified world economy. Although this seemingly unfair comparison between two dissimilar models stretches the realm of credulity it is necessary because of the resemblance of the initial problem, i.e., two producers possess unequal reserves of the same resource.

[3] This point is formally demonstrated for a constant-cost industry in J.M. Hartwick, "Exploitation of Many Deposits of an Exhaustible Resource," Econometrica 46(1) (1978): 201-217.

incompatible with the exploitation of deposits containing different grades of ore for the following reasons[1,2]: assume there are two types of reserves, low grade and high grade, with N_1 firms exploiting the high-grade reserves and N_2 firms exploiting the low-grade reserves. Then all N_1 firms must cease extraction before N_2 firms take over and begin extracting at a higher cost of extraction. Because of the assumed continuity of the firms' rates of extraction, the industry output must tend to zero as the exhaustion of the first grade of deposits is nearly completed. As that happens, the price must increase, and decreases shortly after the exploitation of the new deposits begins. But this is impossible. Therefore in order for a shift of deposits to happen without a discontinuity in the industry supply, it must be that two types of firms are operating at the same time.

Two aspects of this inefficiency result are worth noting: first, this is the intertemporal analogy of market inefficiency in a Cournot oligopoly when there is asymmetry in firms in the industry. Second, there are a number of possible industry outcomes when costs differ: these include the possibility of a high-cost firm exhausting its reserves before

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[1] In this context, eficiency implies that no two deposits containing ores of different grades are exploited at the same time.

[2] This is a paraphrase of a proof offered by Lewis and Schmalensee, "Oligopolistic Markets for Resources," p. 484.

the low-cost firm[1].

If it occurs that, in spite of the full information which each firm had access to when choosing its profit-maximizing rate of output, the high-cost firm exhausts its reserves before the low-cost firm, then this leaves open the question of the relevance of assuming an open-loop equilibrium in firms' output paths, and therefore have productive inefficiency. This is not respectful of the dynamic consistency that should characterize such a model, and leaves one to doubt whether a producer would ever abide by such a dynamic inconsistency.

In summary, the open-loop oligopoly equilibrium is based on the following features: there is perfect information, each firm is a Nash-Cournot profit maximizer, and there is constant quality and equal reserves in the firms across the industry. In this case efficiency conditions argue against stockpiling for the usual reasons: the marginal cost of extraction is constant; therefore it is in the producer's best interests to postpone extraction. Furthermore the presence of perfect information ensures that there is no reason to carry out covert hoarding of the resource, because the time-price-path is based on a sum of reserves that is well-known throughout the industry. The same reasoning applies to the case of

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[1]Ibid., p. 485.

unequal reserves, due to the fact that costs of extraction are still constant.

Closed-Loop Oligopoly Equilibrium

In a closed-loop equilibrium producers are free to choose at any instant in time the output they will place on the market. An examination of such a market is useful for the insights it provides in the functioning of closed-loop models.

An analysis of how producers will react in a simple two-period market is offered by D. Gottwald and W. Guth[1]; in their analysis of a closed-loop world oil market they compute the equilibrium solutions for different situations within the oil market.

The assumptions and the parameters underlying the model are as follows: there is a fixed homogeneous stock of the resource, a small number of producers of the same size, a well-defined market demand that grows at a fixed rate if the real price of oil is constant through time, and linear marginal extraction costs that are identical throughout the industry. Finally, each firm is assumed to be a profit maximizer.

The process of market decision is as follows: at the beginning of each period each player is informed about the stocks of the other players as well as his own. Based on this information each player makes a sales decision with respect to

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[1]Gottwald and Guth, "Allocation of resources,": 208-222.

his reserve constraint. Because the "market" (the game) is assumed to be symmetric only symmetric subgames are reached[1].

The authors are particularly concerned to show that within the parameters of their model it may be proven that there is a subgame perfect equilibrium point prescribing the same strategy for each producer[2]. But this decision must still be made independently by each producer.

The nature of the solution arrived at depends crucially on a definition of scarcity, i.e., oil is not scarce if at least one producer attempts to sell more than he has[3]. If that happens, the sale in any period can be isolated from those in other period. Hence different time periods can be treated as independent periods, where trading only happens in one of the periods. (If oil is defined as being "scarce" then the optimal sales paths chosen by producers in each period are still identical from one producer to the next but the industry supply will be determined differently if oil is not scarce).

The authors' analysis derives two main results. They are:

(i) as the rate of interest inreases producers consider the rate of sales in each period to be equivalent;

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[1] Ibid., p. 209.

[2] Ibid., p. 210.

[3] Ibid.

(ii) despite the parameters of the game which demands that producers decide on their level of output in each period , no producer is myopic, i.e., no producer can ignore the rate of interest when making his output decision in each period.

The issue of stockpiling in this model is a debatable point: the purpose of this section has been to give the reader an example of closed-loop oligopoly equilibrium. The type of model presented is such that at each moment of time each firm may choose its level of extraction. All firms within the industry know the extent of reserves, and the market demand curve is well-defined. At each moment in time each player decides on his output, based on his reserve constraint. The presence of positive costs of extraction as well as perfect information about reserves combine to make it suboptimal for a producer to consider storing the extracted resource; instead it is in the producer's best interests to delay extraction as much as possible. This negates any motive for stockpiling for purposes of future price discrimination.

Cartel and Stockpiling by the Consumer

Heretofore stockpiling has been examined with a view to allow a firm to practice intertemporal price discrimination. It is taken for granted that the consumers alone are responsible for their welfare, and do not respond in an organized manner to the actions of the cartel.

However, in a multiperiod context, if in addition to facing the consumers of the good the cartel also has to contend with an autonomous agency that purchases the good for future use for the benefit of the consumer, that would change the results and the motives for stockpiling[1].

Although the authors also present a multiperiod analysis of stockpiling we will limit ourselves to their analysis of the two-period case. This is for three reasons:
(i) first we are concerned with exhaustible natural resources: the authors address the problem in a two-period context,
(ii) the two-period model has greater analytic tractability than multiperiod models analyzed in the same paper[2],
(iii) in spite of its abstraction, a two-period horizon yields insights into the process of optimal stockpiling.

The goal of the cartel is to maximize its discounted profits net of its production costs, whereas that of the consuming nation is to maximize the present value of the net consumers surplus[3]. The sequence in each period by which the market arrives at an equilibrium is as follows: the cartel

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[1]A.L. Nichols and R.J. Zeckhauser, "Stockpiling strategies and cartel prices," Bell Journal of Economics 8(1) (1977): 66-96. The participants in the market are the cartel and the consuming nation.

[2]This is stated by the authors in Ibid., pp. 69, 74.

[3]Ibid., pp. 68-69.

sets the price, the consumers purchase what they need for consumption in that period, and the government agency, acting for the consumers, selects a stockpile level, S.

Before postulating such a sequence the authors note that the government agency that chooses the level of stockpiling is farsighted, and that in each period it carries out its actions in light of future consequences[1].

In view of the fact that the cartel sets the price in each period, the model considered here has a resemblance with the formulation of a Stackleberg model. But there is a difference with other models of Stackleberg behavior in that the optimal price charged by the cartel in the first period is a function of its own cost and output parameters. Similarly, the purchasing of the good by the consumers is a collection of individual acts. Finally, the level of stockpiling that is selected at the end of each period is a function of demand and price parameters relevant to each particular period. Thus the decision of who will exercise leadership rests more on the sequence of actors' market behavior than by any conscious desire to exercise any strategic advantage.

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[1]Nichols and Zeckhauser, "Stockpiling Strategies," p. 71. The problem with this assumption is similar to that noted by Gilbert ("Dominant firm pricing,") who avoids explaining how expectations are formed. However, he circumvents this problem by postulatintg an intertemporal equilibrium. (Ibid., p. 394). Furthermore, in a many-period context Nichols and Zeckhauser's model of market behavior with stockpiling is intractible analytically and can only be solved for specific parameter values through numerical analysis. (Nichols and Zeckhauser, "Stockpiling strategies," p. 74).

Nichols and Zeckhauser seek to establish that if certain conditions hold, i.e., if there is no quantity constraint on the production of the resource, and no communication between the producers and consumers other than through the marketplace, then it may be established that in general the setting up of a stockpile will benefit the consumer and the producer.[1] Given the parameters of the authors' model, the derivation of this result is straightforward: for each period there is a linear demand curve

$$C_t = K - \alpha p_t$$

corresponding to consumption: "K" is a constant fixed parameter, as is "α", and p_t indicates the price in each period. The aggregate demand curves for each period are linear[2] and of the type

$$D_t = C_t \pm S \quad , \quad t = 1,2$$

That is, the aggregate demand in each period is a function of the amount purchased by consumers C_t (>0) and the amount of stockpiling, S, that takes place. Because this is a two-period model, stockpiles are built up in the first period to be released in the second period. Therefore

$$D_1 = C_1 + S$$
$$D_2 = C_2 - S$$

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[1] Nichols and Zeckhauser, "Stockpiling strategies," p. 68.

[2] The reason for such a specification is that given the multiperiod game context in which the model is set, little insight could be gained from working with general functions.

The cartel's goal is to maximize its discounted total revenue

$$Y = p_1 D_1 + \beta p_2 D_2$$

where β is the discount factor (remember that production is assumed to be costless). Based on these factors (and disposing of perfect information) the cartel will set a profit-maximizing price contingent on whether or not there is stockpiling, which determines the maximization of total revenue[1]. This price is

$$p_t^* = K/2\alpha$$

if there is no stockpiling and

$$p_t^* = (K+S)/\alpha 2$$

if stockpiling takes place.

The value function for the consuming nation, F, is made up of two components: the consumer's surplus (the triangular area under the demand curve) and the government agency's revenues (net of purchase costs) from holding the resource, i.e.,

$$F_t = \int_{P_0}^{P_1} P(Q)\,dp \pm p_t S$$

The first term above measures the area under the demand curve. Over two periods the value function is

$$F = F_1 + \beta F_2$$

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[1]This is attained as follows: in each period the cartel maximizes the expression $TR=p_t[K-\alpha p_t]$ with the obvious extension if there is negative or positive stockpiling. Maximizing this expression with respect to the price yields the optimal price.

In the absence of precise parameters defining the demand curve it is impossible to make absolute comparisons between the stockpiling and no-stockpiling case. However, relative results based on arbitrary values of the demand curve indicate that with stockpiling the first-period price is marginally higher but the second-period price is much lower[1]. Furthermore the present value of consumers' surplus increases, as does the cartel's revenues.

The benefits of stockpiling outlined above depend on the fact that there is no quantity constraint on the resource. If a quantity constraint is imposed then total consumption takes on the form

$$C_1 + C_2 \leqslant R \qquad [8\text{-}1]$$

where R represents total reserves. The quantity constraint may not be binding for two reasons: if total reserves are extremely large relative to consumption, or if total reserves are so small that no stockpiling takes place.

The analysis of stockpiling when the constraint is binding proceeds as follows[2]: considering the range of resource availability where the constraint is binding we substitute the values for consumption for each period in equation [8-1], and after rearranging we have

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[1] Nichols and Zeckhauser, "Stockpiling strategies," pp. 73-74.

[2] Ibid., pp. 94-96.

$$p_2 \geqslant \frac{2K - R - \alpha p_1}{\alpha} \quad [8\text{-}2]$$

This constitutes the price-floor in the second period if there is stockpiling. Hence during the first period it serves no purpose to stockpile beyond what can drive the price below what is indicated in [8-2]. The equation for the price in [8-2] varies inversely with the price in the first period: as p_1 decreases the price-floor for p_2 increases and vice-versa. Based on [8-2] the authors develop optimal levels for stockpiles and price. The results are: the optimal level of stockpiling varies proportionately with the resource constraint; the former increases with the latter, until the maximum resource deposit that is necessary for the constraint to be not binding is reached. When that occurs the optimal level of stockpile is constant, as is the price. The availability of the stockpiling option implies that the first-period price is greater than the price without stockpiling over all the ranges of quantity of reserves considered. Whether or not this benefits the consumer is determined as before by the level of consumer's and producer's surplus. The results for the producer's surplus are unambiguous: over most of the range where the resource is binding the producer's surplus with storage is greater than if there is no storage. Still over the relevant range of resource availability, the amount of consumer's surplus declines when there is storage; but as the level of reserves increases this changes: the amount of consumer's surplus with storage is greater than if there is no storage. This remains true as the quantity of reserves tends to infinity. Thus over a large segment of the range where the resource availability constraint is binding there are losses to the consumer if there is stockpiling.

The authors note that this result is disturbing, because most of the commodities being considered for stockpiling are constrained[1].

In summary the results of stockpiling in the presence of a resource availability constraint are ambiguous.

On the Possibility of Inventory Accumulation In Imperfect Competition

It has been seen that in some circumstances it is optimal to stockpile in both competitive and monopolistic markets. It should stand to reason that in a deterministic market structure where firms have some amount of market power, i.e., in imperfectly competitive markets, that firms have some discretion in setting output, sales, and price.

A simple framework in which inventories are maintained in an imperfectly competitive market is given by Phlips[2]. Assuming a particularly simple model of imperfect competition, Phlips assumes that there is a fixed number of firms (no entry

[1]Nichols and Zeckhauser, "Stockpiling Strategies," p. 96.

[2]Phlips, "Intertemporal Price Discrimination," pp. 530-531.

is permitted) that sell a homogeneous product at an identical price. Using a modification of the optimal control problem that was solved to verify Smithies' intertemporal discrimination rule (Chapter I) Phlips finds that the optimal pricing policy for a firm is

$$p\left(1 + \frac{q_i}{q} \cdot \frac{1}{\eta}\right) = k_i \qquad [9\text{-}1]$$

where k_i represents the i^{th} firm's "normalized" marginal cost, that is, the constant and discounted marginal cost at which discounted marginal revenues are equated, at each moment, q_i is the individual firm's output, q is the industry output, and η is the elasticity of demand for the industry.

It is relevant to note two features of the result in [9-1]. These are: (i) equation [9-1] may be a noncooperative Nash equilibrium[1]; (ii) the market share of each firm need not be equal.

Inside an imperfectly competitive industry operating under certainty, the ideal motive for stockpiling is the strategic motive[2]; if there are positive costs of extraction, the firm may wish to completely extract the resource as a signal to other firms of its determination to remain in the market. Further, the use of a stockpile may be the perfect instrument to attempt to undercut a competitor - real or imagined - in the market. Due to the range of possibilities open to the analyst this type of motive for holding inventories

[1]Ibid., p. 531.

[2]This motive has been discussed in Chapter I.

in imperfect competition will not be analyzed here.

As in all other types of markets, stockpiling for speculative purposes is well suited to imperfect competition. When the producer builds up a stockpile for a speculative purpose then in principle this activity resembles that of hedging: in both cases an actor in the marketplace seeks to reduce the risk that price variability in the market represents for him, and at most seeks to profit from anticipated price variability due to incomplete futures markets.

Still within an imperfectly competitive market structure, an inventory maintained for precautionary motives is made redundant by the presence of a known and well-defined market demand curve, along with perfect knowledge of competitors' actions and possibilities, and complete futures markets. Indeed, only if future demand (and hence price) is uncertain is there any purpose of maintaining inventories for precautionary motives.

The presence of well-defined and continuous cost functions with no startup costs invalidate the transaction motive in markets characterized by both certainty and uncertainty.

Heretofore we have seen that stockpiling for purposes of intertemporal price discrimination hinges on U-shaped cost curves as well as sufficient market power to the firm (in the form of a monopoly over the resource) to ensure that it isn't undercut by a potential entrant. It is not impossible to conceive of a duopoly, both with identical reserves and cost functions, carrying out stockpiling for the purpose of intertemporal price discrimination later on. Whether the resulting equilibrium will be cooperative or not is a matter to be determined by the model's specifications. If a cooperative equilibrium prevails, then this market is analogous to a monopoly, in which case the preceding chapter's analysis becomes relevant.

Summary and Conclusions

The models examined above rest explicitly on the assumption of full information for all participants in the market. The only divergence from this rule is the scenario that allows for imperfect observability of reserves while output remains perfectly observable. But an explicit assumption is that all firms follow a Nash-Cournot equilibrium path. Hence, knowing the rate of interest, the number of firms in the market, etc..., it is easy to deduce what the size of the firms' reserves are, and for the firm to adjust its output correspondingly.

Also, the structures of oligopoly that were examined above explicitly imply the presence of perfect information. An attempt by any firm in the market to undertake stockpiling for strategic purposes is useless because other firms would immediately notice this and adjust their price and output paths accordingly.

The discussion of the feasibility of inventories under different market structures has demonstrated that the framework of the models examined precludes any inventory buildup for purposes of intertemporal price discrimination; instead, probably the most plausible motives for holding inventories in an imperfectly competitive market are twofold: the speculative motive if the price is uncertain, and the strategic motive otherwise. Whereas the analysis of the latter is a field unto itself, the study of the speculative motive stems from price uncertainty. There is also the limiting case where consumers stockpile the resource to undercut the producers' surplus. However this analysis somewhat detracts from the main thrust of this thesis, which is to examine the actions of the individual firm.

CHAPTER VI

OTHER MOTIVES FOR STOCKPILING

Introduction

This chapter examines stockpiling by a firm for purposes other than voluntary intertemporal price discrimination[1]. The first of these purposes is the "environmental motive" which is a mechanical extension of stockpiling for intertemporal price discrimination. The second motive is that of indivisibilities of production: what are the effects on the firm of extracting a byproduct and storing it for future use? The third motive examined is that of speculative stockpiling; this section discusses the role of the elasticity of demand in determining the monopolist's pricepath over time, and the role of speculators.

Stockpiling for Environmental Purposes[2]

The extraction of minerals or energy generally causes some form of environmental damage. In an effort to examine

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[1]The question of how a firm should use its inventory will not be examined here.

[2]This discussion is based on T. Lewis, "Energy vs. the Environment," California Institute of Technology Working Paper no. 126 (May 1976): 22 pages.

the effects on a firm's sales and extraction path of paying compensation to society for damage caused to the environment by the extraction of a resource, Lewis examines the effect of imposing a fixed cost, independent of the rate of extraction, while the resource is being extracted. The effects of the imposition of this tax are examined in light of two sets of circumstances; respectively, these are when the optimal extraction and sales paths may be rearranged through the use of above-ground storage, and when any rearrangement of the optimal sales and extraction paths is impossible due to prohibitively expensive storage costs.

The model developed by Lewis is of the neoclassical type; in this case this implies the maximization of a present value stream of profits from a fixed body of ore, along with a convex variable cost function[1] dependent only on the quantity of ore extracted. The imposition of a fixed tax on the firm while it extracts the resource results in an average-cost curve that is U-shaped[2].

In the case where marginal storage costs are constant and independent of the rate of extraction, Lewis shows that the rate of consumption of the resource is not affected by the imposition of a fixed tax on the extractive firm's operations

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[1]Both the first and second derivatives are positive.

[2]Lewis, "Energy and the Environment," p. 3. In addition, the author invokes D. Lee and D. Orr ("The Private Discount Rate and Resource 'Conservation'" [Canadian Journal of Economics VIII(3) (1975): 351-363]) to claim that if the unit extraction

if storage is available at a reasonable cost, as is the horizon for the consumption of the resource. Moreoever, the rate of extraction is speeded up due to the imposition of the fixed tax[1]. However, the horizon for the consumption of the resource does not change due to the inclusion of environmental costs, if storage is available at constant costs[2].

The intuitive explanation for the result that a resource on which a fixed cost is imposed on its extraction will face a shorter extraction period but a consumption horizon of equal length than if the fixed cost was not imposed, is because whereas consumption decisions are made on the basis of marginal quantities, the marginal cost of stockpiling is independent of the level of stockpiles. This result is further developed to demonstrate that the consumption of the resource across time is allocated so as to equate the present value of marginal returns from the sale of the ore[3]. This is an enunciation of the intertemporal discrimination rule as

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cost is U-shaped it may be economical to store extracted resources if sales and extraction paths differ. The argument invoked is along the same lines as that seen in Chapter IV for the decoupling of the sales and extraction path.

[1]Furthermore, the greater are the constant marginal costs of storage the shorter are the extraction horizons.

[2]Formally, the equations describing the sales path and horizon do not depend exlicitly on the fixed environmental costs levied, and thus need not affect these paths.

[3]Lewis, "Energy and the Environment," p. 10.

examined in Chapter II: that the discounted marginal revenue from sales must equal the discounted entire marginal cost from production in time.

The purpose of the analysis of stockpiling for environmental purposes is twofold: in addition to demonstrating that there may be a social purpose in the utilization of the intertemporal discrimination rule, it also addresses the debate pitting "environmentalists" against those who argue that the conservation of the environment is incompatible with other social values, such as full employment. Specifically it shows that if storage is available at constant marginal costs, then consumption paths need not differ because of the imposition of a fixed cost on the extractive firm's operations.

Indivisibilities of Production

Introduction

It is not uncommon for a deposit of a resource to contain large amounts of a second type of ore. For example, deposits of natural gas and aluminm bauxite may contain, respectively, considerable amounts of helium and titanium. This raises a number of issues. These are:

(i) The effect on the pricepath of one resource of extracting a second resource at the same time.

(ii) What measure of scarcity is to be used when evaluating the abundance of the resource.

(iii) Whether or not to incur the storage cost of maintaining these resources for possible sale in the future. This issue is relevant when considering that the present demand for the residual ore is small and inelastic, with considerable uncertainty over future demand.

Two papers examine the market behavior of resources that possess such characteristics. The difference of their models of the industry is apparent through the different assumptions that are made regarding the dependence of the marginal cost of extraction on the level of reserves or the rate of extraction. These models are each examined in turn.

It must be recalled that the survey of the models is undertaken with a view to analyzing stockpiling, to the detriment of the associated issues when examining jointly produced exhaustible resources[1].

A Model of the Industry When The Marginal Cost of Production is Independent of the Rate of Extraction

An industry with these characteristics is analyzed by Pindyck[2]. A competitive industry extracts a composite

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[1] Some of these issues include the relevance of differing measures of scarcity when evaluating resource deposits, as well as how to evaluate future demand for the secondary resource.

[2] R.S. Pindyck, "Jointly Produced Exhaustible Resources," Journal of Environmental Economics and Management 9(3) (1982): 291-303.

homogeneous ore from a number of deposits. The composite ore is extracted at an average and marginal cost c(R), a decreasing function of the known (and fixed) reserve level.

Although the grade of ore of the resource in each deposit is homogeneous, the cost function must conform to the peculiarity that the extraction of the composite ores is distinct from the production of a particular ore.

The analysis of the necessary conditions for a competitive equilibrium yields the notion of a "transfer price"[1], i.e., the price at which the extracted raw ore should be sold to the processing firms to separate out the individual resources. To the producer of the resource, the transfer price represents the cost that it does not have to incur to obtain a marginal unit of the unprocessed ore, if it were not placed on the market by the extractor. If the cost of extraction is constant, the transfer pricepath of the resource is equal to the rate of interest less the (constant) costs of extraction. Similarly, the stored resouce's pricepath must satisfy the condition that the percent rate of increase net

[1]Ibid., p. 294.

of the cost of storage be equal to the rate of interest[1]: if there are positive costs of storage then the amount of capital gains to be had in holding the resource should just equal the total cost of holding that unit of ore.

The advantage of stockpiling is apparent when examining the extent to which the price behavior of one resource affects the availability of the other resource[2]. This assertion is illustrated by the example of helium and natural gas: at the present time the demand for the former is small but inelastic, and its storage is costly. Then, compared to what would prevail if they were produced independently, producing both resources jointly reduces the current price of the secondary resource (helium) and increases the current price of the primary resource (natural gas). Therefore the exhaustion of natural gas is delayed and that of helium is speeded up. This effect depends on the availability and costliness of storage. If much cheaper storage becomes available, it is conceivable that the price of natural gas will not be affected, and correspondingly there will be little difference in extraction paths. The advantage of storing the residual resource becomes apparent when one notices that because of depletion the marginal-cost function of each resource changes:

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[1]Ibid., p. 295.

[2]Ibid.

this affects the point of intersection of each marginal cost curve with the time-autonomous demand function. If the characteristics of the demand function for the residual resource are changed through greater accessibility of storage facilities then the initial price of the residual resource might be higher, but its initial production would also be much higher, and the rise of the initial price of the resource would be more gradual, as demonstrated in Figure 3.

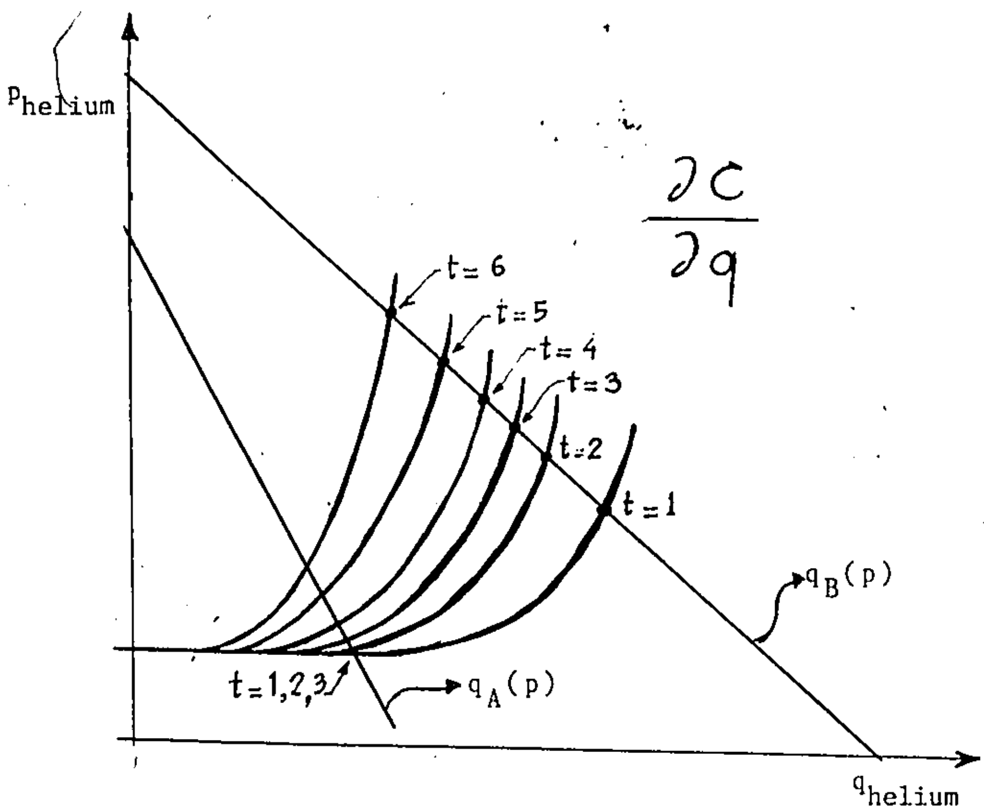

Fig. 3. $q_A(p)$ is the demand curve for helium if storage is very costly or unavailable, and $q_B(p)$ is the demand curve for helium if storage is cheap.

Figure 3 is explained as follows: as depletion ensues the marginal-cost curves move to the left (for purposes of graphical illustrations we are assuming discrete time). If there is an incentive to store helium its demand is much higher, with the contrary holding true is there is no storage. It is to be noted that in both cases helium is extracted, as it is a byproduct of natural gas.

The conclusion to draw from the study of the extraction of joint resources is that the pricepath of the composite ore must follow the Hotelling rule, as must that of the stored resource. Of course if storage is unavailable the pricepath of the residual resource may not conform to the Hotelling price rule.

Furthermore it is relevant to note that the result that the net worth of the stored resource increases at a rate equal to that of interest is an appropriate illustration of the discussion contained in the introduction to Chapter I: that is, even if an inventory is maintained at zero-cost, it doesn't explain why a stockpile has been accumulated, for the positive cost of extraction that has been incurred while extracting the resource will not be recouped until its sale. Under these conditions, the presence of indivisibilities of production is ample grounds for the justification of a stockpile.

A Model of the Industry When Marginal Costs of Extraction Depend On The Rate of Extraction

In a model of the industry that examines the effects of joint resource extraction[1], the firms' cost functions

[1] S. Thivierge et G. Gaudet, "Produits liés et exploitation optimale des ressources non renouvelables," Department of Economics, Laval University, April 1985 (Mimeographed).

resemble those of the traditional model of the mine, i.e., the marginal cost of extraction is a function of the extraction rate. Two structures of a competitive industry that extract joint resources are contrasted: one industry does not have the option to stockpile its extracted resource, whereas the other industry has that option, albeit at a constant marginal cost. Otherwise, the parameters defining the two industries are identical.

The determination of a competitive equilibrium for the industry yields the conditions necessary for each extractive firm to maximize the present value of a stream of discounted profits: in particular, the value of the rent on a marginal unit of raw ore should grow at a rate equal to that of interest.

The transfer price of the ore is the price that the extractive firm will charge the firm that separates the ores; the percent rate of increase of the transfer price inclusive of its costs of extraction is less than the rate of interest[1].

The evolution of the extraction path is determined by the anticipated evolution of the transfer price and the evolution of the rent on the mineral. Specifically, that the anticipated time-rate of change of the transfer price path be (greater than, equal to, or less than) the rate of growth of the rent on the resource implies that the extraction path is (increasing, constant, or decreasing)[2]. This relation is explained in

[1]Ibid., pp. 11-12.

[2]Ibid., p. 16.

the following ways: if the producer anticipates the increase in the transfer price to be greater than that of the rent on the ore, this indicates that the ore in situ is undervalued, and the producer will delay placing it on the market. If, on the contrary, the producer anticipates the transfer price to rise by less than the rent, then the ore is overvalued, and the producer will tilt the extraction path toward the present so as to avoid the devaluation of the rent. The obvious analogy holds if the producer anticipates the rate of increase of the transfer price to be equal to the net price of the ore in situ.

The analysis of the industry that disposes of the possibility of stockpiling the residual resource yields the standard condition for a storable good in a competitive market: the percent rate of increase of the stored good is equal to the rate of interest plus the marginal cost of stockpiling[1]. This positive cost entails, ceteris paribus, an increase in the marginal cost of extraction relative to what it would be without stockpiling. Combined with the extraction cost function, this last result entails that extraction at the horizon by the representative firm with the option of stock-piling will be higher than the representative firm of the industry that does not have this option[3]. Taken in

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[1]Ibid., p. 24.

[2]Ibid., p. 28.

[3]Ibid.

conjunction with the condition examined earlier (namely that the extraction path depends on the anticipated rate of growth of the transfer price with the actual increase in the rent) this last result allows the authors to compare the differences in the extraction paths between the industry that has the option to stockpile and that which does not[1]. Three cases arise. They are:

(i) If the extraction path is constant (and hence the anticipated increase in the transfer price of the mineral is equal to that of the ore <u>in situ</u>) then the amount extracted during every period will be greater for the stockpiling firm than for the firm that does not have this option. Furthermore, the stockpiling firm's extraction horizon is shorter than that which does not have the option to stockpile.

(ii) If the extraction path is increasing, then the producer anticipates the increase in the mineral rent to be inferior to that of the mineral's transfer price. The extraction rate will be greater at all times and the extractive horizon of the representative firm that has the option of stockpiling will be shorter than the firm that does not have this option.

(iii) If the extraction path is decreasing (and hence the anticipated increase in the value of the mineral is greater than that of the mineral's transfer price) then the amount of ore

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[1]It must be recalled that the condition that the extraction path is decreasing, constant, or increasing in relation to the anticipated increase in the time path of the transfer price and rent on the ore <u>in situ</u> holds regardless of whether or not the firm stockpiles.

extracted at each moment by the firm that has the option to stockpile is greater than if the firm does not have this option to stockpile.

In summary, in every case examined, the stockpiling firm extracts more during every period; but has a shorter extraction horizon than the firm that is not offered this option. In light of the fact that the only difference between the models is that in one case all competitive firms are offered the option of stockpiling whereas in the second case this option is not offered, two things are intuitively apparent; first, if it is optimal for one firm to stockpile, all firms will do so; second, the use of stockpiling reduces the extraction horizon compared to a similar firm that does not stockpile. Furthermore, when offered the option of stockpiling, the producers' strategy is to take advantage of the economics of scale available through stockpiling and increase their level of extraction. It is this condition that results in shorter extraction horizons for the stockpiling firms.

Monopoly Pricepaths and Speculative Stockpiling under Certainty

The Hotelling rule states that in equilibrium the pricepath of the resource must be equal to the rate of interest[1].

[1]A historical note is appropriate here: S.W. Salant and D.W. Henderson, in "Market Anticipations of Government Policies and the Price of Gold," (Journal of Political Economy 86(4) (1978): 627-648 [p. 631]) note that the type of constraint on monopolistic extractors examined here (i.e., that a

This is a condition for stock and flow equilibrium. Nevertheless, there are conditions when a monopolist might wish to set his pricepath increasing faster than the rate of interest. A possible scenario where this might occur is as follows: if the demand curve facing a monopolist is such that the elasticity of demand decreases as quantity increases (i.e., the absolute value of the demand elasticity increases as output increases)[1] then the rate of growth of the equilibrium pricepath will be greater than the rate of interest. But the price of the resource is increasing at a greater rate than other investments. Hence if speculators are allowed to operate in the market they will step in and buy as much as possible of the resource. Although this scenario is theoretically plausible, two aspects need to be pointed out: (i) it is by no means clear that a pricepath increasing faster than the rate of interest can sustain a dynamic equilibrium; (ii) even if a dynamic equilibrium is sustained it would generate large

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monopolist's ability to set a time-path is limited by the presence of speculators) is analogous to that examined by A. Smithies in 1941 ("Monopolistic Price Policy in a Spatial Market [Econometrica 9(1) (1941): 63-73]), where he considered the problem of a monopolist who wishes to discriminate in the price of one commodity sold in two markets separated by space. The monopolist knows that he can't let the price of the good diverge by more than the costs of transport because of arbitrageurs. In the context examined in this thesis, arbitrageurs are speculators and physical distance is replaced by time.

[1]This point is due to Dasgupta and Heal, Economic Theory and Resources, pp. 330-331.

investments in the resource[1].

Therefore, like all theoretical models its validity depends on the behavioral assumptions. If such a situation prevailed in real life then perhaps for a short time the percent rate of increase of the pricepath is greater than the rate of interest (i.e., $\dot{p}_t/p_t > r$) until speculators enter the market and reestablish the pricepath to its proper level. In a theoretical model the best way to avoid this sort of situation is to impose a constraint whereby the percent rate of increase of the pricepath is less than or equal to the rate of interest (i.e., $\dot{p}_t/p_t \leq r$)[2].

Summary and Conclusions

The purpose of this chapter has been to examine stockpiling in a broader context than that seen in preceding chapters. It has been seen that stockpiling for environmental purposes is a mechanical (but nontrivial) extension of the analysis of the monopoly with the option of stockpiling as in Chapter IV. The analysis of the firm that extracts joint ores reduces to the conditions which determine the amount of residual ore that will be stored at a positive cost. The section dealing with speculation in a context of certainty is

[1] Ibid., p. 330. The authors claim there is empirical evidence to indicate that purchases of large speculative stocks have been curtailed because of lack of investment capital.

[2] Ibid., pp. 330-331.

more an examination of a theoretical possibility than the analysis of a practical motive for stockpiling.

CHAPTER VII

INVENTORY AND UNCERTAINTY

Introduction

The analysis of the firm under assumptions of certainty abstracts from many factors that prevail in the marketplace. Models that assume that certainty prevails must make a number of assumptions, implicit or otherwise. Some of the more obvious assumptions made are that all reserves are discovered and accounted for, that the price function in the market is not random and is well-known to all interested parties, that information is costless and complete, and so on.

While the use of the assumption of certainty may not result in extremely precise results, the main advantage of postulating a context of certainty is that these models are more easily analyzed and yield greater analytic tractability compared to other models that assume that uncertainty prevails.

For the purposes of the analysis considered here there are seven types of uncertainty. These uncertainties are with respect to: (i) total reserves of the resource; (ii) the cost function; (iii) the demand function; (iv) the price of the resource; (v) the date of introduction of a substitute for the resource; (vi) changes in the rate of interest; (vii) uncertainty as to the date of nationalization of the resource, or as to whether there may be new uses or substitutes for the resource, or as to whether there might be disruptions in the trade of the resource.

These uncertainties may be divided into two broad categories: exogenous, and institutionally induced uncertainties[1]. These types of uncertainty and their relevance will be examined later.

Once the type of uncertainty facing the producer is agreed upon, there is the supplementary question of why a particular motive for stockpiling rather than another will be invoked. This question has particular significance in view of the fact that a firm's stockpiling decision will vary according to the market structure.

There is also the question of information to be addressed: does one agent in the marketplace know more than another, and if so did he buy that information; is his expected gain from using the information greater than the price he paid for the information? This question is disposed of in assuming that all agents' state of ignorance (or knowledge) about the market is equal: not one agent in the marketplace may dispose of any information which may procure an advantage with respect to the other agents in the marketplace[2].

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[1]This distinction is due to Dasgupta and Heal, Economic Theory and Resources, p. 395.

[2]Insofar as one may observe that in the real world large amounts of resources are spent on gathering information with

When it is possible to model the different states of the world in a number of finite outcomes, the firms' perception of expected profit determines how they will act when faced with different types of uncertainty. This may also bear on how a firm perceives the issue of stockpiling.

To examine all of the issues raised above this chapter will proceed in the following manner: first the different types of uncertainties that a firm might encounter will be reviewed. Then the behavioral assumptions of the firm under the assumption of price uncertainty are analyzed. Afterwards the behavior of the same type of firm when given the possibility of engaging in hedging is analyzed. It is seen that the main consequence of offering the option of hedging to a firm is to increase its output with respect to future price. Finally motives for the individual firm to stockpile under different types of uncertainty are reviewed. It is seen that in response to almost every type of uncertainty it is beneficial, intuitively, for the firm to stockpile.

It may be noted at the outset that this chapter is not intended to provide a systematic review of all types of uncertainties and of firms' behavior toward stockpiling. Instead, this chapter provides a brief overview of what

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the result that large firms are better informed on the state of the world than small firms, then clearly the assumption made in the text, i.e., that all agents in the marketplace have the same amount of information, is inadequate. However, insofar as one realizes that no one firm ever has sufficient information to forecast all the possible outcomes then that assumption is not inadequate.

study of uncertainty has been made in the resource market, and if and how it is possible to accommodate stockpiling in the analysis.

The Nature of Uncertainties

Including exogeneous and institutionally induced uncertainties there are seven types of uncertainties that a firm in the marketplace may be confronted with. These are: (i) Uncertainty as to the total reserves of the resource. This type of uncertainty has two components: (a) there may exist a fixed amount of undiscovered reserves, the discovery of which is a function of exploratory effort. This type of uncertainty has been examined as part of the conditions which may cause a divergence from the Hotelling pricepath, and consequently will not be examined here; (b) reserve uncertainty may be modelled by assuming that available reserves increase or decrease according to a stochastic process. This form of reserve uncertainty, modelled by Pindyck[1], is of a type such that reserves in the present are known with certainty, but the level of future reserves is subject to a stochastic process with independent increments[2]; it is of a form such that future

[1] R. S. Pindyck, "Uncertainty and Exhaustible Resource Markets," Journal of Political Economy 88(6) (1980): 1203-1225.

[2] This is the "Ito process," which requires, when used in optimization, Ito's differentiation rule for functions of stochastic processes as well as stochastic dynamic programming. (Ibid., p. 1206).

uncertainty increases with the time horizon. Hence this form of uncertainty pertains to future amounts of the reserves. Furthermore, this form of uncertainty entails that reserve levels are continuously being revised upward or downward. Formulated in this way the production process takes on the form of a dynamic stochastic optimization process.

(ii) Uncertainty as to the cost function: the cost parameter may change because of the discovery of a new technology for the extraction of the resource. If the deposits of ore exhibit decreasing returns up to a cutoff grade at which point it becomes economical to produce a substitute, the discovery of a new technology of extraction may change the parameters of the industry. In the course of this thesis the assumption has been made consistently that the cost function is known with certainty, is invariant through time, and technology is constant through time[1].

(iii) Demand uncertainties: two types of uncertainty may affect demand. These are: (a) discrete changes in demand may occur at some point in the future. These changes may in turn be broken down into two categories: permanent and transitory changes. In both cases a random discrete disturbance (either multiplicative or additive) may affect the known

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[1]M. Hoel has analyzed resource extraction when the true reserves and the costs of extraction are uncertain in "Resource extraction, uncertainty, and learning," Bell Journal of Economics 9(2) (1978): 642-645.

demand for a resource[1]. Whereas a transitory demand shock may take on the form of a labor disruption in the industry consuming the resource, a permanent disruption may take on the form of the invention of a substitute for the resource[2]. All of these disruptions are of a discrete nature, and all occur at once whether they are of a permanent or temporary nature; (b) continuous changes may occur gradually in the parameters defining the resource industry, with the result that demand may be permanently affected. These changes are usually due to events such as increases in factor prices, changes in technology, environmental considerations halting the extraction of a resource, a sudden decrease in the cost of substitutes for the resource, and other such matters.

Both types of disturbances described in (a) and (b) above are random[3].

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[1] Random fluctuations of this type commonly affect markets for agricultural commodities. A review and critique of different types of random fluctuations is provided by S. Turnovsky, "The Distribution of Welfare Gains from Price Stabilization: A Survey of Some Theoretical Issues,"in Stabilizing World Commodity Markets, ed. F.G. Adams and S.A. Klein, (Mass.: Lexington Books, 1978): 119-148.

[2] This point is examined by Dasgupta and Heal, Economic Theory and Resources.

[3] R. Pindyck ("Uncertainty and Resource Markets," pp. 1205-1206) maintains that continuous changes in demand produce different changes in resource consumption than does a discrete change in demand.

(iv) Uncertainty as to the date of invention of a substitute[1]. This type of uncertainty raises two questions: will resource extraction in a market with no uncertainty over the discovery date of a substitute resource be faster or slower than in an identical market where there is no chance of a resource substitute being discovered, and secondly, does the type of market structure have any bearing on the price (and hence the consumption) of the resource.

(v) Uncertainty as to the future price: price uncertainty occurs when the prevailing price is known to be subject to random fluctuations due to stochastic components in the determinants of supply and demand[2]. Two types of price uncertainty may occur, depending on whether the pricepath is set endogeneously (i.e., by the competitive industry) or exogeneously (either by a cartel or a monopoly)[3]. These are: (a) for an endogeneously set pricepath, random fluctuations may be modelled as either of the additive or the multiplicative type, with the disturbance term identically and independently (i.i.d.) over time, with a specified

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[1] This type of uncertainty is similar to that of uncertainty as to the date of discovery of a new resouce.

[2] J. McCall, "Probabilistic Microeconomics," Bell Journal of Economics 2(2) (1971): 404.

[3] Whereas a cartel or a monopoly has the freedom to set the pricepaths of their resource increasing at the rate of their choice, the same may not be said for markets in perfect competition, where individual firms must accept the resource industry's rate of increase of its pricepath, as well as its initial level.

density function[1]. Individual firms' beliefs about the sales price are summarized in a subjective probability distribution[2]; (b) there may be uncertainty concerning a pricepath that is exogeneously set (e.g., by a cartel or a monopoly). This uncertainty may affect the present rate of production in two ways: first, random price fluctuations lead to variations in demand, with a corresponding movement along firms' respective cost functions because of a change in output. This may influence the expected-profit-maximizing output in the present. Second, due to the fact that proven reserves may be represented as an option on future production of the resource, and that the willingness to exercise the option to extract the resource at a future date varies proportionately with its price at that time, it follows that under future price uncertainty the greater the uncertainty the greater is the incentive to hold back production and keep the option, due to the fact that under price uncertainty the current value of a unit of reserve is larger than the current price net of the cost of extraction[3].

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[1]T. Lewis, "Attitudes towards Risk and the Optimal Exploitation of an Exhaustible Resource," Journal of Environmental Economics and Management 4(2) (1977): 113.

[2]A. Sandmo, "On the Theory of the Competitive Firm under Price Uncertainty," American Economic Review 61 (1971): 65.

[3]These points are due to R. Pindyck, "The Optimal Production of an Exhaustible Resource when Price is Exogeneous and Stochastic," Scandinavian Journal of Economics 83(2) (1981): 277-288.

(vi) Uncertainty as to the future rate of interest: the expressing of a flow of revenue in present value terms discounts the future; this is why extraction of a mineral is biased towards the present. An increase in the rate of interest will shorten the horizon, and conversely for a decrease in the rate of interest. In all of the models of the firm discussed heretofore an often overlooked assumption is that the rate of interest facing the industry is assumed to be constant over the course of the life of the firm: this condition is necessary so that the optimal extraction path defined through the application of the maximum principle holds over the life of the firm. Furthermore, the same path defined through the application of the maximum principle is based on the assumption of an open-loop control with no feedback: at all moments of time during its extractive phase the firm follows the price and extraction path defined through the maximum principle. To take account of an eventual change in the rate of interest is analogous to diverging from the optimal path. However, whereas it is theoretically possible to have discontinuities in the rate of interest, they will not be examined here as that possibility diverges too far from the theoretical preoccupations at hand.

(vii) Uncertainty as to the date of nationalization of the resource, or as to whether there may be new substitutes for the resource, or whether there might be disruptions in the trade of the resource; all of these uncertainties are exogeneous (institutionally induced) and therefore may be eliminated through the use of a "risk-adjusted discount rate". The effect of possible nationalization on the extraction of a resource has been examined by N. Long[1]. In that article the uncertainty about nationalization is subdivided in three parts: uncertainty about whether the firm will be nationalized, the date of nationalization, and the compensation for the nationalization. Uncertainty about the date of nationalization is handled through the firm's subjective (joint) probability about its date of nationalization. The solving of the firm's optimal extraction path, given its subjective probability of nationalization, are that the effect of uncertainty about nationalization with no compensation are identical to a firm that faces no uncertainty but must contend with an (increasing) profit tax over time. This case is a particular example of a general proposition, namely when uncertainty is purely exogeneous stochastic probabilities may be replaced by a deterministic problem.

All of the types of uncertainties discussed above overlap to some degree: for example in a probabilistic setting there is virtually nothing to distinguish between the effects of price and demand uncertainty.

When discussing uncertainty two topics must be addressed: first, noting that as the risk-bearing attitudes

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[1]N. Long, "Resource Extraction under the Uncertainty about Possible Nationalization," Journal of Economic Theory 10(1) (1975): 42-53.

of firms have an influence on the behavior of the firm under certainty, discussions of uncertainty must include some comment on whether the firm is risk-neutral, risk-averse, or risk-loving. Second, any discussion of uncertainty must include parallel topics such as insurance markets in general and hedging in particular.

Due to the size of the body of literature on uncertainty, as well as the complexity of the modelling of the topic of uncertainty, all that will be done in subsequent sections is to present a comment on the behavior of price uncertainty and how the problem of stockpiling for different motives might be handled when dealing under such assumptions.

The Competitive Dynamic Firm under Price Uncertainty

The Competitive Dynamic Firm With No Resource Constraint

The static maximization problem of the firm's expected profit under price uncertainty is well known: depending on the firm's attitude towards risk it seeks the optimal output to maximize its expected utility-of-profit function[1].

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[1]The static theory of the firm under uncertainty is well known. Prominent articles in the field include S. Lippman and J. McCall, ("The Economics of Uncertainty: Selected Topics and Probabilistic Methods," in Handbook of Mathematical Economics, ed. K.J. Arrow and M.D. Intriligator, vol. 1 [New York: North-Holland, 1981]: 211-289); J. McCall, ("Probabilistic Microeconomics," (Bell Journal of Economics 2(2) [1971]: 403-438); A. Sandmo, ("On the Theory of the Competitive Firm under Uncertainty," (American Economic Review 61(1) [1971]: 65-73)).

It has been shown that the static problem of the firm under price uncertainty is easily dynamized under the assumption of stationary period-utility, where the behavior of the firm is assumed to be identical in all time periods[1,2]. Assuming the firm must maximize its present-value sum of discounted utilities of profit over its (infinite) lifespan, its problem may be written as

$$\int_0^{\infty} e^{-rt}\, U[\pi(t)]\, dt \qquad [10\text{-}1]$$

and the solution must satisfy for each period[3]

$$EU'[p - C'(q)] = 0, \qquad [10\text{-}2]$$

Equation [10-2] holds for a risk-neutral firm: this strict equality is replaced by an inequality depending on whether the firm is risk-averse or risk-preferring.

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[1]This is done by H.S. Burness, "Price Uncertainty and the Exhaustible Firm," Journal of Environmental Economics and Management 5(2) (1978); 132.

[2]This problem is analogous to an open-loop dynamic optimization problem, where the solution for the initial period must hold for all periods.

[3]Similarly to the static problem of the mine, the utility-of-profit function is concave, and the marginal utility of income is positive; if the firm is risk-neutral this implies that the optimal (profit-maximizing) output must equal the utility-maximizing output in all time periods. The obvious analogy holds for a risk-averse or risk-loving firm.

The Competitive Extractive Firm Under Price Uncertainty with an Exhaustive Resource Constraint

Risk Neutrality

This analysis is similar to that of the firm with no resource constraint, except that the firm can only derive expected utility from a finite stock of the resource in a fixed deposit. The firm's utility of profit function is the same as that of the preceding section, as well as the conditions dictating the conditions dictating the conduct of the firm. Nevertheless, there is a distinction to be made with the preceding section: although the owner is maximizing over an infinite period of time a sum of utilities stemming from the sale of an extracted resource, it must be recalled that the firm itself will not last that long. Therefore, adopting a solution proposed by Burness[1], we state the firm's problem as maximizing the sum of discounted utilities over the life of the mine, i.e.,

$$\max\left[\int_0^T e^{-rt}U(\pi)\,dt + \int_T^\infty e^{-rt}U_0\,dt\right] \qquad [10\text{-}3]$$

$$\text{s.t. } \dot{R}(t) = -q(t)\ ;\ R(0) = R_0\ ;\ R(T) = 0\ . \qquad [10\text{-}4]$$

Although the firm is risk-neutral there are two solutions to [10-3]. These are:

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[1]Burness, "Price Uncertainty and the Exhaustive Firm,", p. 133.

$$e^{-rt}U'(\pi)\left[p - c'(q)\right] = \lambda \qquad [10\text{-}5]$$

$$0 = U'(\pi)\left[p - c'(q)\right] - \left[U(\pi) - U_o\right]/q. \qquad [10\text{-}6]$$

Condition [10-5] is the profit-maximizing condition: for each period of production the discounted marginal utility of profit must be equal to the user cost. Condition [10-6] is the transversality condition: at the terminal time marginal utility of profit from each unit of output must be equal to average utility[1].

Risk Aversion

Risk aversion implies concave utility. Furthermore, instead of maximizing a sum of present flows of utility as in [10-3] a producer will maximize the expected utility of that equation, that is,

$$\max_{q} \int^{T} e^{-rt} E\left[U(\pi)\right] dt \qquad [10\text{-}7]$$

subject to [10-4]. Necessary conditions for an optimum require that[2]

$$\lambda = e^{-rt} E[U'](p - c'(q)) \qquad [10\text{-}8]$$

$$e^{-rt}\left\{E(U(\pi)) - E(U'(\pi))(p - c'(q))q\right\} = 0 \qquad [10\text{-}9]$$

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[1] Ibid.

[2] Ibid., p. 138.

From [10-8] the shadow price of the resource is a constant proportion of the discounted value of the resource times the expected utility. Condition [10-9] is the transversality condition indicating the end of the extraction period.

Because of the particular form of the necessary conditions it is difficult to infer anything on the relationship between the expected price and the marginal cost. However, based on the risk-bearing attitudes of the owner, it is possible to obtain some results. In particular, it is possible to establish that if the goal of the producer is to maximize the stream of expected utility, then the risk-preferring owners will deplete the stock of the resource more rapidly than risk-neutral owners[1]. On the other hand, if the criterion of maximazation of expected profit is used, the rate of resource depletion is more rapid for risk-averse than risk-neutral firms, whose rate of extraction is more rapid for risk-preferring firms[2].

The Competitive Firm Under Price Uncertainty with Hedging

Under price uncertainty a firm can counter price uncertainty in two ways: it can hedge through trade in futures markets or it can hold inventories. When trading on futures markets a firm sells at a certain future price rather

[1] T. Lewis, "Attitudes Towards Risk," pp. 114-115.

[2] Ibid., p. 117.

than maximize its profit on the basis of an expected present price. Similarly the purpose of inventories when the price is uncertain is either for speculative or precautionary motives. Whether engaging in hedging or buildups of inventories, both actions are indicative of the producer's actions to insure himself from price fluctuations.

In all of the cases examined heretofore it is seen that under uncertainty a risk-averse firm produces less than its risk-neutral counterpart[1]. These results are derived by making the implicit assumption that the firm has no access to futures markets. Here this assumption is relaxed: there is an uncertain present price and a forward price known with certainty. This section reviews the results of how a firm facing present uncertainty with access to futures markets reacts, given its attitude to risk.

At the outset it need be stated that hedging is an essentially dynamic problem: faced with a random present price and a certain future price a firm must maximize during each period the expected utility of profits. If the price and

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[1] This same result in a static context is found in Sandmo ("The Competitive Firm Under Uncertainty"), Lippman and McCall ("The Economics of Uncertainty"), and J. Hey (Uncertainty in Microeconomics [Oxford: M. Robertson, 1979]). Hey adds that if uncertainty is removed and the random price is replaced by its expected value, the optimal output would be given by $C'(q)=Ep$, i.e., marginal cost is equal to expected price. The immediate corollary is that the risk-neutral firm's optimal output is the same under uncertainty as it is for certainty. This conclusion is repeated again by Lippman and McCall, Idem., p. 249.

cost functions are time-autonomous and during each period of time during the firm's lifespan provision is made for expressing the utility-of-profit function in present value terms, then the dynamic problem is identical to the static one: find the optimal amount to hedge at each moment of time given the future price that is known with certainty.

An analysis of a von Neumann-Morgenstern utility-maximizing firm that seeks to reduce price uncertainty through hedging has been carried out by Holthausen[1]. Contrary to the usual results of the type obtained in equations [10-8] and [10-9] Holthausen finds that the utility-maximizing firm will produce a level of output which depends only on the certain forward price and is independent of the firm's degree of risk aversion and the probability density function of price[2].

The model developed by Holthausen is defined by a utility function defined over profit that the firm seeks to maximize:

$$\max_{x,h} E\,U(\pi) = \int_0^\infty U\big(p(x-h) + bh - c(x)\big) f(p)\,dp \qquad [10\text{-}10]$$

where x is the quantity of output
h is the quantity of output hedged on the forward market
b is the certain (future) price
f(p) is the known probability density function.

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[1] D.M. Holthausen, "Hedging and the Competitive Firm Under Price Uncertainty," American Economic Review 69(5) (1979): 989-995.

[2] Ibid., p. 989.

The necessary conditions for the maximization of [10-10] yield

$$\frac{\partial EU}{\partial x} = \int_0^\infty U'(\pi)(p - c'(x)) f(p)\, dp = 0 \qquad [10\text{-}11]$$

$$\frac{\partial EU}{\partial h} = \int_c^\infty U'(\pi)(b - p) f(p)\, dp = 0 . \qquad [10\text{-}12]$$

Equation [10-10] is the same as equation [10-5] except for the exhaustible resource constraint; if a futures market did not exist, the conditions for a maximum would be the same as those examined in [10-5] and which, for a static context would yield the solution that a risk-neutral firm operating in price uncertainty sets its expected price equal to its (certain) marginal cost. The presence of [10-12] changes the expected result: Holthausen finds that the firm's output level is independent of its degree of risk aversion[1]. This result is obtained by adding [10-2] and [10-3] with the result that

$$(b - c'(x)) \int_0^\infty U'(\pi) f(p)\, dp = 0 . \qquad [10\text{-}13]$$

It can be seen that [10-13] holds only if C'(x)=b. Otherwise, the assumption underlying $U'(\pi)$ would be violated, implying that marginal utility is negative. Equation [10-13] implies that the output decision is a function only of the future prices of the output. This result is interesting for several reasons. First, a firm exhibits risk-averse behavior because

[1] Ibid., p. 990.

of price uncertainty. The futures price being known with certainty, uncertainty is removed from the marketplace, (and be extension any reason to be risk-averse). Thus a producer will seek to maximize the utility of his expected profits, and produce at the point where price is equal to marginal cost, as in equation [10-2][1]. Second, if futures prices are competitive (as they are assumed to be), then they are expected to be close to the expected price Ep. In this case risk-averse firms will produce an output close to what they would produce if the expected price Ep were replaced by a certain price. This result is close to the efficient-markets hypothesis, for which one of the main requirements is that the price of a good be equal to its marginal cost.

Motives for Stockpiling Under Uncertainty

Most of the different types of uncertainty examined in the preceding section justify stockpiling as a means to reduce uncertainty. We now discuss these uncertainties: (i) Reserve Uncertainty: contingent on what degree of uncertainty prevails with respect to potential reserves, uncertainty may or may not justify stockpiling of the resource. A resource model where the only uncertain parameter is the amount of reserves undiscovered at the present time, but nevertheless are known to constitute a fixed but as yet

[1]Mathematically, this implies that the producer's utility function has a positive continuous first derivative $U'(\pi)>0$ but that $U''(\pi)=0$. Were the producer risk-averse $U'(\pi)>0$, $U''(\pi)<0$.

undiscovered amount of reserves that is contingent only on exploratory effort for their discovery, does not justify stockpiling for either speculative or precautionary motives. To be more precise, stockpiling for precautionary motives is not justified, because the amount of undiscovered reserves as well as all the other parameters are known with certainty. Nevertheless, this statement depends crucially on the notion of undiscovered reserves constituting a stock, not a flow. If the model is amended to assume that there is a limitless amount of reserves, the discovery of which is contingent only on exploratory effort, then whether or not that constitutes an incentive to stockpile depends on the structure of the industry being examined. However, this form of stockpiling is not carried out for purposes of reducing uncertainty <u>per se</u>. Furthermore, uncertainty may easily be added to the model by relaxing the assumption of either price or demand certainty. Immediately this makes stockpiling for precautionary and speculative purposes desirable. This leaves two questions unanswered first, if there is price uncertainty by what process does a firm go about determining its subjective price expectation? Second, assuming that stockpiling takes place, how is the amount of the optimal stockpile determined?

(ii) Cost Uncertainty: if certainty is assumed to hold in the industry, with the the exception of the cost-of-extraction function which may be either time-dependent and increase at a rate greater than the rate of interest, or subject to a random process, then the firm has an advantage to stockpile for precautionary motives. The purpose of this is to stockpile when the cost of extraction is very high, drawing from accumulated stockpiles to satisfy current demand.

(iii) Demand and Price Uncertainty: in either case stockpiling may be justified for precautionary and speculative purposes; building up inventories for precautionary motives is relevant if there is a sudden price decrease translating in a sudden surge of demand, at which point the buffer stocks will be depleted, whereas stockpiling for speculative purposes is warranted if the time rate of the pricepath is greater than the rate of interest.

(iv) Uncertainty as to the Future Rate of Interest: it has been discussed in the preceding section how a variable interest rate is incompatible with an open-loop maximization process that admits no feedback process. To admit the construction of a stockpile is tantamount to supposing a closed-loop process with feedback controls.

(v) Uncertainty as to the Date of Nationalization of the Resource: this type of uncertainty may be reduced to a certainty-equivalent model, and hence need not be discussed here.

A summary of these motives for stockpiling is presented below in Table 1.

TABLE 1

MOTIVES FOR STOCKPILING INVOKED BY THE INDIVIDUAL FIRM IN RESPONSE TO DIFFERENT TYPES OF UNCERTAINTY

	Motive for Stockpiling	
Type of uncertainty	Speculative	Precautionary
Reserve	x	x
Cost		x
Demand	x	x
Introduction of a substitute resource[1]		
Price	x	x
Future rate of interest		
Date of nationalization of the resource		

[1]This type of uncertainty depends on the assumption of a known date of introduction of a substitute.

Summary and Conclusions

The nature of uncertainties has been reviewed. Exogeneous and institutionally induced uncertainties were included. This was followed by a review of the behavior of the competitive extractive firm under price uncertainty. It is seen that for the risk-neutral firm the extraction path is set so that the present value of the marginal utility of profit is equal to the rent of the resource. The analogy for the risk-averse firm is examined. This is followed by an analysis of the utility-maximizing firm that has access to futures markets. It is seen that the firm's level of output is a function of the certain forward price. This is a way of reducing uncertainty.

Finally, the different motives for stockpiling to reduce uncertainty were analyzed. It is determined intuitively that in all cases where reserve, cost, demand, and price uncertainty prevail it may be beneficial to keep on hand a stockpile of the resource.

As witnessed in the preceding discussion, the scope and breadth of the study of uncertainty is quite vast. A more exhaustive discussion on varying uncertainties and their relevance to stockpiling would require that specific assumptions be made on the parameters underlying the models under analysis in order to attain definitive conclusions on the question of the pertinence of inventories in models of uncertainty.

CHAPTER VIII

CONCLUDING REMARKS

This thesis has been concerned with the availability of stockpiling at zero storage cost to an ~~ex~~tractive firm under varying market structures. Of the eight motives for stockpiling examined only the following are relevant in the context of certainty: intertemporal price discrimination, speculative, environmental, transaction motives, indivisibilities of production, and strategic motives. Only the following motives were retained for analysis: stockpiling for environmental motives, and indivisibilities of production. The strategic motive has been deleted because of its concern with non-strategic entry deterrence; its analysis requires first that more fundamental motives for holding inventories be examined. The transactions motive has been ignored as well. The motives for stockpiling for precautionary and speculative purposes are relevant mostly to firms operating under uncertainty. Due to the difficulty inherent in modelling uncertainty, a relevant analysis of these motives may be conducted only inside a framework embodying specific parameters. Hence the relevance of these motives for stockpiling within a context of uncertainty are examined qualitatively only in the last chapter. Finally, as this thesis examines stockpiling at the level of the individual firm, stockpiling for reasons of price stabilization is not considered.

The second chapter analyzes the behavior of the firm in a competitive deterministic framework. The assumptions of the neoclassical model of the mine are reviewed, and the optimal extraction path of the neoclassical deterministic firm in partial equilibrium is derived. The results of that analysis are explained, and exceptions to the general results of the firm are considered. In particular some hypotheses suggested to explain the discrepancy between empirically-observed U-shaped price paths, and the Hotelling rule that states that the price of the mineral net of extraction costs increases at a rate equal to that of interest, are examined. These hypotheses include the presence of cumulative extraction, durable exhaustible resources, the presence of technological change, and the presence of exploration while there is extraction. Then the traditional model of the competitive firm is amended to permit stockpiling at zero cost. Two situations are considered: one in which the price of the resource is constant, and one in which it is varying. In both cases it is seen that if it is profitable to sell the resource it is optimal to do so at the maximum rate. When the price of the resource is constant the analysis demonstrates that it is never optimal to stockpile the resource, because the inventory would never be sold. On the other hand, if the price of the resource is a time-dependent variable, in some circumstances it is optimal to stockpile it. Furthermore the analysis yields the conclusion that if it is profitable to sell the resource it is optimal to do so at the maximum rate, short of depleting the stockpile (this is permitted only in one continuous sequence once extraction has ceased). However, when sales are nil it is optimal to continue extraction at less than full capacity. This rate is determined by the difference between the market price of the ore and the value of a unit of ore in storage; in turn, the value of a unit of ore in storage is determined by the sum of the marginal cost of extraction and the marginal user cost of the resource.

In conclusion, storage of a resource by a deterministic firm occurs under certainty only when its market price is a time-dependent variable. In the particular model considered, sales occur at the maximum rate or not at all, whereas inventories can only be built up at a rate less than the maximum rate of extraction, concurrently with zero sales. The pertinence of these conclusions to a more general model of the extractive firm embodying features such as cumulative extraction and exploration represents an interesting area for future research.

After a derivation of the extraction and price paths of the costless monopoly, some limiting cases to the neoclassical monopoly are examined. Limiting cases to the ordinary monopoly pricepaths include the examination of the cases where the monopoly extracts more (or less) than the social optimum, and differences in the expected rate of increase of the pricepath due to durable exhaustible resources; the debate over the correct specification of the cost function when the firm carries out exploration is mentioned as well. Then the model of the neoclassical extractive firm with variable costs of extraction is presented. This is followed by an analysis of the conditions that make it possible for firms to store their extracted output for the purpose of intertemporal price discrimination. Two different sets of conditions enable a firm to decouple its sales and output. The first set of conditions is that a firm subject to time-autonomous parameters has a U-shaped average-cost curve; the second condition is that at least the cost function is time-dependent and increasing over time at a rate greater than that of interest. The analysis of the time-autonomous extractive monopoly with the option to stockpile is carried out. Two solutions are possible: if stockpiling is not optimal, the price-and-extraction-paths are identical to those of the extractive monopoly with variable costs of extraction; over the life of the firm sales and the quantity of ore extracted decline, and the value of the resource increases at an exponential rate. If the second solution holds, i.e., if stockpiling is optimal, then it is seen that while marginal revenue is monotone increasing, the extraction of the resource is constant or increasing. This striking result contradicts the accepted wisdom of maximizing a present-value flow of revenues, where there is usually a bias towards the present due to a positive rate of discount. Finally the nonnegativity constraint on the inventory, in addition to the continuity of the profit-function, ensures that extraction may cease only once over the life of the firm. The extension of this analysis to conditions including more complex cost functions and time-dependent parameters represent an interesting area for further research.

The analysis of cartels and oligopoly in exhaustible resource markets shows that the framework in which models of firms' behavior are set leave little scope for stockpiling for reasons of carrying out price discrimination. This is due to the time-autonomy of the parameters defining firms' behavior, and the presence of perfect information. Hence in the models examined, and accomodation of stockpiling for the purpose of realizing abnormal economit profits would necessitate the modification of some of the underlying parameters. In addition, the strategic motive for holding inventories represents an ideal motive for the setting up of a stockpile. Finally, the analysis of the strategic motive after suitable modifications to the models of imperfect competition should be done in the context of dynamic games and entry deterrence literature. This is especially true in light of the discussion outlining the possibility of inventory accumulation in imperfect competition. Furthermore, the discussion of consumers' stockpiling demonstrates that tangible results can be attained with simple market structures.

The analysis of other motives for stockpiling is contained in Chapter VI. The speculative motive for stockpiling under certainty is discussed, as is the environmental motive, which is a direct corollary of stockpiling when the firm is faced with a U-shaped average-cost curve. The reasons for building up inventories in the presence of indivisibilities of production are reviewed. These motives represent pragmatic reasons for maintaining inventories.

The discussion of uncertainty distinguishes between institutional and exogeneously induced uncertainties. This is followed by a discussion of the nature of uncertainties, and how stockpiling for motives specifically applied to uncertainty can reduce the risk inherent to firms operating in such markets. Additional conclusions from this particular topic would have to be preceded by a more exhaustive analysis of the behavior of the firm under certainty.

The most significant result of this thesis has been the establishing of conditions that demonstrate the possibility for a firm in a perfectly competitive resource market to stockpile the resource if the price is time-dependent, while also demonstrating that it is impossible to stockpile the extracted resource if its price is a constant over time. Similarly, in the case of a time-autonomous monopoly the stockpiling of the extracted resource above ground (and the corresponding increasing extraction path) is definitely an option to be considered on the same basis as the more usual "classical" profit-maximizing behavior, namely that of equating marginal revenue from sales to the sum of the firm's marginal costs of placing the resource on the market at every instant. It would be worthwhile to undertake further research in both of these areas - perfect competition and monopoly - to determine whether the conclusion reached that stockpiling is part of the firm's profit-maximizing behavior may be derived when more general conditions are specified, such as the presence of cumulative extraction, varying grades of ore, ill-behaved cost functions, and other like matters.

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