MATHEMATICS POSITIVITY: A NARRATIVE EXPLORATION OF ADULT MATH EXPERIENCES

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Thesis submitted to the University of Ottawa
in partial Fulfillment of the requirements for the
Master's of Art in Education

Faculty of Education
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Abstract

The following thesis uses narrative analysis and experiential turning points to explore the life stories of six adults (two women and four men, ranging in age from mid 20s to late 60s) who self-identify as mathematically positive. In semi-structured interviews, participants are asked to tell their mathematical stories, including their definition of mathematical positivity, from which a co-created narrative was written. Using a hybrid theoretical framework that originates from positive psychology's Situated Expectancy Value Theory and Hill et al.'s Framework of Mathematical Wellbeing, turning points or potential turning points in the participant's affective mathematical experience are noted. Emerging mathematical values are analyzed with Hill's Mathematical Wellbeing's seven core mathematical values.

Of the six participants, two started their narratives as mathematically negative, one was neutral and the other three were mathematically positive. All six had turning points, or potential turning points relating to mathematical values being met or not. Themes that emerged included the delineation of mathematics as being a less interpretative discipline, the importance of mathematics outside of school, the intersection of neurodiversity such as dyslexia, the importance of supportive parents and teachers and the power of perseverance. Most prevalent of Hill's core mathematical wellbeing values, with five participants, were Relationships and Perseverance. Meaning was also a significant value but what that meant depended on the participant. The participants also had mathematical values that were hard to classify within Hill's Mathematical Wellbeing Framework, such as integrity and clarity. This was presumed to be because these were adults talking about their life stories rather than children's experiences in a school setting.

Acknowledgements – Thank you to my family who put up with my absence, especially to José who cooked six straight weeks of dinner, the many researchers and theorists who informed this work, my thesis committee and especially my participants.

I'd also like thank Barbara Graves who was creative, kind and ever intellectually curious. I would have been honoured to have you examine my thesis.

Table of Contents

Chapter 1 – Introduction

1.1 Purpose of Study

There has been a growing recognition of the importance of social emotional learning in mathematics as evidenced by its inclusion as a Learning Strand – Strand A - in the Ontario Mathematics Curriculum (Government of Ontario, 2020), and the increase in focus on the affective component of learning mathematics in the literature. This is due to the recognition that negative mathematical experiences can lead to mathematical avoidance (Hunt & Maloney, 2022; Schmitz et al., 2023). Mathematical anxiety and phobia also have a negative impact on whether high school and post-secondary students choose STEM related courses and disciplines. It mirrors an ethos in the adult community where mathematical experiences are often described as aversive, such as when a presenter says 'the next slide has some math but we'll make it as painless as possible,' or the commonly held belief that there are two types of people: math and non-math people (Boaler, 2015).

Negativity in mathematics learning has a pretty robust research following (Di Martino & Zan, 2011; Schoenherr et al., 2025) but there is significantly less focus on other affective responses. And even less studied are adults who identify as mathematically positive. Two questions naturally follow. What is mathematical positivity? And how does it arise? There is an assumption that mathematical positivity has the opposite effect to what is observed by mathematical negativity, which is to draw a student toward mathematics and mathematically supported subjects in STEM (Science, Technology, Engineering, Math). There is evidence to suggest that this true in the research (Fernandez et al., 2022; Solomon, 2007; Brown et al., 2008; Dietrich & Lazarides, 2019; Wen & Dubé, 2022). Therefore, in order to both prevent

mathematical avoidance and to encourage mathematical retention, not only are mathematical anxiety and phobia a worthwhile area of research, but so is the study of mathematical positivity.

The purpose of this narrative study is to explore what mathematical positivity is in an adult population. It is an exploratory study, that if successful, could yield further directions for research.

1.2 Research Questions

It will aim to answer the following research questions about mathematical positivity:

1. What does mathematical positivity mean to adults who self-identify as being mathematically positive?
2. What narrative themes emerge that help explain the development or maintenance of mathematical positivity?
3. What narrative turning points threatened or encouraged mathematical positivity and how do they relate to the values and expectations of the participant?

1.3 Significance

To date, most research on affect in mathematics has been conducted with participants that are directly linked to the school system. They are either students in elementary or high school and, to a lesser extent undergraduate students, or they are teachers, pre-service teachers, or parents of the aforementioned students. There is significantly less research done with adults in the area of mathematical affect. Therefore, the lived mathematical experiences of adults are an important gap in this research.

Likewise, a great deal of research has been done in the area of mathematical anxiety and phobia but less on positive feelings about mathematics. Even when positive affect is included in a study, it is almost always in relation to negative affect. There seems to be very little research

that asks questions about why students are mathematically positive or what leads to mathematical positivity. One exception might be the area of Mathematical Wellbeing, which is interested in human flourishing in relation to mathematics. This is a new area of research that originates from Oceania and has almost no research done outside of this region (Campbell, 2026). It focuses on values and valuing. Given that values are socio-culturally and geographically linked, then it is important to study them in different regions and populations.

This study is focused on an understudied population with the hope of exploring a neglected part of the affective spectrum in mathematical affect. Its ultimate goal would be to open up new avenues of curricular and classroom research that can help more students grow up to identify as mathematically positive. It uses a framework that is interested not just in the attainment of mathematical achievement, but also a student's sense of wellbeing.

1.4 Overview of Methodology

By using narrative analysis, adult participants, who self-identify as mathematically positive, are asked in semi-structured interviews about their experiences with mathematics. Adults were chosen as they have a threshold of collective experiences, from which to draw their own explanations about their personal history. These transcripts were coded for affective words and phrases in the categories of emotion, attitude, beliefs, identity and value. Their definitions of mathematical positivity and mathematical values were also highlighted.

A narrative was created that was presented back to the participants for comment and correction. In this way, their mathematical stories were co-created between the researcher and the participant. Using a theoretical framework that combines Eccles & Wigfield's Situated Expectancy Value Theory (SEVT) (2020, 2024) and Hill's Mathematical Wellbeing (MWB)

Framework (Hill et al., 2021), turning point tables were made. These turning point tables chronologically list important experiences/events/episodes in the participant's mathematical story, and connect these with participants' expectations and values at the time, and whether or not there were any changes to these expectations and values. This is related to a matrix of expectations and values to determine whether or not a participant was mathematically negative, mathematically positive or if their mathematical positivity was threatened or encouraged. Any resolution or outcome is also described. From these turning point tables, a second, shorter story about the emergence of each participant's mathematical positivity was written that distills the participant's explanations of their mathematical journey.

Any mathematical values were then analyzed with the seven core mathematical values in Hill's MWB (Hill et al., 2021). Differences and major themes were described. Since this is a narrative analysis, each participant's data was analyzed separately before any comparison was made between participants.

1.5 Chapter Overview

Chapter 2 outlines the literature on mathematical affect including the often bidirectional relationship between mathematical attainment and retention (2.1). It also describes the different terms that will be used for mathematical affective coding (2.2), and how they arose. The upswing of socio-cultural research in mathematical affect is outlined in (2.3) followed by the connection between affect and experience in turning point research (2.4).

In Chapter 3, the theoretical framework is described starting with an overview of mathematical wellbeing (3.1) before focusing on Hill's Mathematical Wellbeing (MWB) Framework (Hill et al., 2021). This is followed by a description of Situated Expectancy Value

Theory (SEVT) (Eccles & Wigfield, 2020, 2024) (3.2) and how it will be used to develop a hybrid framework. SEVT's achievement performance decision making chart is simplified and modified so as to combine the two frameworks together (3.3). A chart was designed to act as a frame to order and interpret turning points in the mathematical narratives of the participants. This can be compared to a matrix that describes the different outcomes – mathematical positivity, mathematical negativity, threating or encouraging of mathematical positivity, and acts as the basis for the development of the turning point tables in Findings, (6.1). In order to analyze the participant's values with Hill's MWB core values, extended definitions of Hill's MWB core values are presented that are more congruent to an adult's lifetime versus experience at school. In order to code the affective words/phrases from the transcriptions, affective categories are defined (3.4) from the literature.

Chapter 4 outlines the Methodology and includes a summary of the methodological steps (4.1), participants and recruitment (4.3), along with data and collection (4.4) including first interview, transcription, restorying into a coherent narrative, and making participant led corrections to the narrative. Data analysis (4.5) includes analysis per participant such as the identification of affective language, description of themes and explanations and the creation of turning point tables that leads to the writing of the Emergence of Mathematical Positivity summaries. An example of this analysis can be found in Appendix 5. After individual analysis was completed, a cross comparison was done for major themes between the narratives. Lastly, a table was created that compares the participants' mathematical values with each other and with Hill's MWB core values (Table 7.1a). At the end of (4.0) can be found a description of validity and reliability referred to as Trustworthiness and Attention (4.6), ethical considerations (4.7) and limitations (4.8) of the study.

Chapter 5 presents the data in the form of participant portraits starting with a data overview (5.0) then presenting the stories in order from Ray: An Honest Player (5.1), Paul: Patterns of Planets (5.2), Claire: A Relationship with Mathematics (5.3), Martin: Guided by Curiosity (5.4), Hanna: A Drive to Understand (5.5) and James: Success through Support (5.6). Each participant portrait has data gathering contextualization and a narrative synopsis. Full narrative stories are found in Appendix 4.

Chapter 6 contains the turning point tables for each participant and the secondary stories called Emergence of Mathematical Positivity summary for each participant (6.1). This is followed by a cross thematic analysis (6.2) which describes and compares the major themes that were found in this study including What is and is Not Mathematics (6.3), Support inside and outside the classroom (6.4) Neurodiversity (6.5) and Perseverance (6.6).

Discussion of the findings is found in Chapter 7, including an analysis of the participant's values with Hill's core MWB values (7.1), in table and text format. Turning point tables and the Emergence of Mathematical Positivity summaries are compared in (7.2) followed by a focus on the participant's explanations (7.3) and the implications of this research (7.4)

Chapter 8 contains a summary of the study (8.1) and its key contributions (8.2), and a final reflection on the study (8.3)

2.0 Literature Review

2.1 Affect in Mathematics Education

> In the last decades of the twentieth century, research on emotions in achievement contexts, and in psychology more generally, has largely focused on negative emotions like anxiety. (Pekrun et al., 2024, p. 83)

Mathematics has often been treated as a rational or cognitive practice and therefore there has been a lot of focus on how students understand mathematics concepts rather than how they feel about them or how they feel about the process of learning about them (Debellis & Golden, 2006; Zan et al., 2006). Despite this, there are mentions early on about the importance of emotional responses in learning (Polya, 1957). This initial focus was weighted toward the negative in the form of mathematical anxiety and its impact (Di Martino & Zan, 2011; Schoenherr et al., 2025). Anxiety was defined as, “feelings of tension and anxiety that disrupt the manipulation of numbers and the solving of mathematical problems” (Xolocotzin Eligio, 2017, p. 5). This was relevant because mathematical anxiety was seen as negatively affecting mathematical performance. It is also important to note that mathematical affect is approached in two broad ways. Firstly, during problem solving (Debellis & Golden, 2006; Leder & Grootenboer, 2005; Mcleod & Adams, 1989; Polya, 1957), and secondly, in terms of longitudinal achievement and retention in mathematics and STEM related fields (Brown et al., 2008; Eccles & Wigfield, 2023; Pekrun et al., 2017). The latter type is often categorized as motivation theory. Of course, these two areas interact with each other such that states (momentary experiences) build to form traits that are more ingrained over time (Spieldberger et al., 1976).

Mathematical anxiety and mathematical phobia are strongly correlated with negative outcomes such as mathematical avoidance (Schmitz et al., 2023). In a meta-analysis with 8680 high school students, high mathematical anxiety strongly correlated with low performance (Zhang et al., 2019). Similarly, a meta-analysis of 1552 articles on mathematics anxiety and performance found robust and significant correlations from grade 1 to non-student adults with higher mathematical anxiety meaning lower mathematical performance (Barroso et al., 2021). Causation may be bidirectional such that lower mathematical attainment increases mathematical anxiety, just as higher mathematical anxiety lowers mathematical attainment (Carey et al., 2016)

Positive mathematical affect also impacts mathematical achievement. Data collected from 1044 Dominican high school students found the strongest positive relationship was between enjoyment, rather than difficulty, and mathematical attainment (Ma, 1997). A systematic review of 95 studies related to mathematical attitude and mathematical attainment, found that enjoyment, self-concept, confidence, perceived value and behavioural intentions were positively correlated with achievement whereas mathematical anxiety was negatively correlated to mathematical achievement (Wen & Dubé, 2022).

Poor or perceived poor performance could lead to "value appraisals" of how well (or not) a mathematical task will go the next time (Hunt & Maloney, 2022). This could lead to avoidance (Ashcroft, 2002; Hunt & Maloney, 2022; Schmitz et al, 2023). Perhaps since mathematical tasks, in school, become more challenging over time, and/or because they build on one another, there is an overall increase in negativity of mathematical affect as students progress through schooling (Dowker et al., 2016; Grootenboer & Marshman, 2015; Sumpter & Sollerman, 2023; Wen & Dubé, 2022). Put succinctly, 'I'd rather die' is the title of a study that describes reasons why a sample of 1500 sixteen-year-old UK students chose not to continue mathematics (Brown et al.,

2008). This study found that the reasons were perceived difficulty, lack of confidence (self-efficacy) as well as dislike and boredom. Importantly, enjoyment was listed as the main factor differentiating between low and high participation.

As can be seen so far, various terms have been listed under the umbrella of affect and are sorted in various ways, including negative and positive. The next section will describe some of the theoretical developments to help better define these concepts.

2.2 Categories of mathematical affect

Description and categorization of mathematical affect has been under debate for some time. According to Zan et al. (2006), a research forum at PME (Psychology of Mathematics Education) 28 Bergen found two broad themes in mathematical affect research and these were Mathematical Anxiety and Mathematical Attitude. The former drew upon theories in psychology. The latter was divided into two subthemes: thinking about Mathematical Attitude in terms of achievement and the second in terms of affective outcomes such as liking or enjoying mathematics. This latter subtheme draws from social psychology.

Mathematical anxiety is a robust field of research with slightly different underpinnings and a long history stemming from the presumption that being afraid of mathematics will lead to avoidance; a presumption born out in the literature (Ashcroft, 2002; Schmitz et al., 2023). Given that it is not uncommon to conflate mathematical anxiety with negative mathematical affect, this would mean that negative mathematical affect, in the guise of mathematical anxiety, is the more developed and studied aspect of mathematical affect. Mathematical attitude, on the other hand, is rife with definitional and operational debate. Is mathematical attitude the same as mathematical

affect or merely its most important driver? To get to this answer requires a dive into mathematical affect taxonomy.

Congruent with a general turn toward thinking about emotions in learning (Xolocotzin Eligio, 2017), there was a shift in the latter part of the 20th century toward thinking about affect in mathematical learning. Mcleod & Adams (1989)'s *Affect and Mathematical Problem Solving* represents this turning point in mathematical affect research, which had been previously (and still is) criticized for being statistically rather than theoretically driven with inconsistent use of concepts and overemphasis on the development of measurement tools leading to a difficulty comparing studies (Di Martino & Zan, 2015). Importantly, McLeod (1992) developed a taxonomy of affect where affective concepts were split into beliefs, attitudes and emotions based on temporal stability and level of emotional involvement with emotions being the most temporary and affective, and beliefs being the most stable and cognitive. Attitude is somewhere in between.

Debellis and Golden (1997) added values into the mix in the context of mathematical affect and problem solving. Their theoretical framework was developed in conjunction with an empirical study of five clinical task-based interviews, over two years, with nineteen 9-10 year olds, who were engaged in problem solving. It uses three additional terms: meta-affect, mathematical intimacy and mathematical integrity. Affect is representational in a similar way to how cognition is described. Meta affect was intended to work similarly to how meta-cognition works for cognition. In other words, there is thinking, and thinking about thinking. Similarly, there is feeling, and feeling about feeling. They consider this perhaps the most important part of persistence in mathematics. For example, it is the difference between perceiving mathematical frustration as a challenge to be met or as an indication of imminent failure. Mathematical

intimacy refers to the vulnerable relationship someone might have with mathematics and mathematical integrity refers to someone's commitment to mathematics. For the authors, emotional experiences are representational and meaningful, form pathways and affect problem solving. These networks of meanings create the affective structures of mathematical intimacy and mathematical integrity. Their tetrahedral model has emotional states, attitudes, beliefs and values as vertices. Like McLeod (1992), the first three are on a temporal scale with beliefs being the most stable. Like beliefs, values are stable but instead of being 'highly stable, cognitive structures,' they are deeply personal and cherished commitments or truths.

Beliefs and values mix together for Clarke et. al (2015) where they represent the 'mental reification' of value laden constructs gained through social interactions and are relatively stable. Despite their stability, beliefs may emerge or be enacted in a context dependent way.

The process by which emotional reactions and supportive beliefs interact, is often linked seen through attitude, which is described as a bridge between them. (Di Martino & Zan, 2011). DiMartino and Zan (2015) astutely observe that since there is a dichotomization of attitude into negative and positive and since positive attitude is associated with mathematical achievement (Wen & Dubé, 2022), then sometimes the goal changes from improving mathematical achievement to improving mathematical attitude. Though implicit in this statement is that this sleight of hand may be an issue, later research into Mathematical Wellbeing (and my own experiences as a teacher) would suggest that perhaps the goal of changing overall negative attitudes into positive ones may be a worthy goal in and of itself.

Di Martino & Zan (2010) designed a study based on students' autobiographical narratives towards mathematics called Me and Maths that uses grounded theory to develop a theoretical model of attitude. 1662 anonymous essays about 'my relationship to maths up to now' were

written by students from grades 1 to 13. They found that students refer to the following dimensions: emotional disposition, vision of mathematics, and perceived competence. Therefore, they define a negative attitude toward mathematics as disliking math, having a low perceived competence and an instrumental vision of mathematics. The last characterization is taken from Skemp's (1976) notion of instrumental versus relational understanding and compares 'rules without reason' to knowing what to do and why.

Though the tendency towards researching statistical correlations between factors and mathematical affect still exists, with its psychologically based tools and Likert scales, there has also been a new interpretative paradigm, researching attitude through narratives in the form of essays, diaries and interviews (Hannula et al., 2016). The latter is from a later edition of an influential book on mathematical affect called *Attitudes, Beliefs, Motivation and Identity in Mathematics Education* (Hannula et al., 2016), which explores research trends and issues in the field. Though perhaps not explicitly stated, of note would be the increased inclusion of socially mediated affective concepts such as identity and self-efficacy. Within this volume, Pantziara refers to Bandura's (1986) definition of self-efficacy to state that it is a "belief in one's capacity to organize and execute the courses of action required to produce given attainments," (p.7). More informally, mathematical self-efficacy could be described as a belief in self in the context of mathematics. A comparative study, using PISA 2003 data, found a reciprocal relationship between mathematical achievement and self-efficacy for 24 out of 33 countries (Williams & Williams, 2010).

Self-efficacy relates to one's identity as being someone who is or is not good at mathematics and is or is not a person who should be in a STEM related field. Identity research has focused on gender and diversity inclusion, which will be discussed in the next section.

However, it is worth noting that belief, identity and self-efficacy are highly related. Beliefs could be understood as that which someone holds as true, and self-efficacy is a belief about one's own mathematical competence. Identity are the stories we tell about ourselves (Sfard & Prusak, 2005), which can be understood as a personal myth that reconciles our traits with our goals and societal expectations (McAdams, 2008). A positive mathematical identity is linked to increased performance, participation and persistence in mathematics (Fernandez et al., 2022).

2.3 Socio-cultural Influences on Mathematics Affect (and the Child)

Whereas the latter part of the 20th century saw a shift toward recognizing the importance of the affective side of mathematics, the 21st century seems to have focused on socio-cultural influences including gender, culture and diversity. Social messages about mathematics are transmitted by society in various ways, including by socializers, such as parents and teachers, either through direct messaging or through modelling of these messages.

However, it is important to note that even in studies interested in social messaging about mathematics, affective states and influences are often also mentioned. This creates a complex causality. Feelings about mathematics can lead to perceptions of mathematics that can be explained with socio-cultural stories, and socio-cultural stories can lead to perceptions of feelings about mathematics. This complex causality will be examined further in the theoretical framework but examples, examined and otherwise, can be seen below.

There has been a particular interest in answering the question as to why there were/are not more women in STEM fields. Findings included that women have higher levels of mathematical anxiety and lower levels of mathematical self-efficacy (Beilock, et al., 2010; Huang, 2013, Fernandez et al., 2022; McMichael et al., 2026).

One study of 17, first and second year female teachers, in the US, with 107 elementary school students (52 boys and 65 girls) found that there was a drop in mathematical achievement in the students of teachers who had higher levels of mathematical anxiety, particularly in female students. Gendered beliefs, such as girls were not good at mathematics but were good at reading, appeared to be transmitted (Beilock et al., 2010). In an older cohort, 1092 grade 10, German students' professional aspirations were compared to various factors. Mother's occupation being in a more mathematical field, high mathematics grades and being male were all associated with an aspiration toward mathematics fields (Nishen et al., 2024).

Higher levels of negative mathematical affect lead to lowered persistence, participation and performance (Solomon, 2007) whereas higher levels of mathematical self-efficacy and a mathematically positive attitude can lead to overcoming challenges (Fernandez et al., 2022, Solomon, 2007). Socializers, like parents or teachers, who are mathematically anxious are correlated with poorer mathematical attainment and higher mathematical anxiety in children (Beilock et al., 2010; Maloney et al., 2015). Alternatively, having a stable, highly intrinsic motivation 'enjoyment' based style is associated with the highest level of mathematical careers and was more highly associated with male students (Dietrich & Lazarides, 2019). This is based on a study that used 751, 9th and 10th grade students, and used expectancy value-theory as their framework (described in more detail in Chapter 3.2).

One way to explain why female mathematics students might have poorer mathematical self-efficacy, higher mathematical anxiety and lowered mathematical performance is through identity. In particular, mathematics is often associated with stereotypical male attributes like being "cold, remote, hard, uncaring, rejecting, neutral, unfeeling, impersonal, empty, inhuman, absolute, objective, rule-driven, perfectly rational, male, mechanical, soulless, dead, fixed and

hierarchical' (Ernest 2004, p.18)." (de Freitas, 2008). There may be a misalignment between societal expectations and their personal experiences, creating a fragile identity in the student (Solomon et al., 2011).

Stereotype threat, where a person is afraid that they will conform to a negative stereotype about the group to which they belong or are assigned by others, is invoked to show the higher stakes for not only people of female sex but also racialized and other excluded groups such as gay men (Raymond, 2014). One of these stereotypes is the racial hierarchy in mathematics with Asians being 'too good,' white students being 'good' and Indigenous, Latin and Black students being 'not good enough' (Martin, 2009). One analysis of the 2002 Educational Longitudinal Study (ELS: 2002) of 2790 racialized students found slightly different correlations between remaining in the STEM pipeline, i.e., continuing on an educational path that requires mathematics prerequisites, and different racialized classifications, including higher self-efficacy for students identified as black/latinx, mathematical enjoyment for Asian, Hawaiian and Pacific Islander students, and a growth mindset[1] (as opposed to a fixed mindset) for indigenous students (Butler-Barnes et al., 2021).

Bringing many of these themes together is a book written about the drop in mathematical affect among middle school students. It was based on an empirical study of 1880 students ages 10-14, in school years 5-8 in New Zealand. It was found that despite girls having a high level of achievement, there was a belief that boys were better at mathematics. There were also cultural differences, with Maori and Pasifika students being more positive about mathematics overall.

[1] Mathematical Mindsets (Boaler, 2015) is a popular notion that there are two broad approaches to learning. A fixed mindset where someone considers themselves inherently either mathematical or not and a growth mindset which is emblemized in schools as *the power of yet* as in 'I am not good at fractions yet.' In other words, a growth mindset means that a student considers themselves to be constantly in the process of becoming and therefore even if they have not had success yet, they can in the future if they continue to work at it.

Other important findings had to do with the belief that mathematics was about numbers and learning multiplication was of upmost importance (Grootenboer & Marsham, 2015). They highlighted the significant role of teachers of mathematics in encouraging and facilitating positive views of mathematics and mathematical identities. It is within middle school, they argue that "we are learning to become members of communities of practice and instead of a finite end-point to learning (i.e., the acquisition of some particular knowledge), learning can be seen as a form of participation that leads to ways of knowing and becoming within particular communities of practice" (Grootenboer & Marsham, 2015, p. 117). There was a similar finding of 3500 middle schoolers, from Australia, in relation to their aspirations for mathematics, where they were particularly concerned with decimals and fractions (Wilkie, 2019) and mentioned the importance of the teacher for support. The level of belonging the student feels as a learner and user of mathematics will influence the development of their mathematical identity. Therefore, experiences within classrooms that do not foster a sense of belonging have the potential of alienating a student.

A classroom environment that a learner experiences as negative, increases mathematics anxiety. A questionnaire-based study of 216 college students found that perceived verbal aggressiveness and instructor argumentativeness negatively affect state (momentary, contextual) motivation (Lin et al., 2017). However, anxiety in the learner can also impact their perception of a situation so students with high mathematics anxiety perceive teachers as being less supportive (Luo et al., 2024; Wang et al., 2024). High enjoyment teachers, on the other hand, are found to have a positive attitude toward mathematics (Russo et al., 2023), and teachers' enjoyment of mathematics enhances students' enjoyment (Frenzel et al., 2009).

Though much of mathematics is done in school, parents and family groups are also important in the development of mathematical affect and a mathematical identity. If a family has a fixed mindset, meaning that they believe that people are inherently good or not at mathematics, rather than a growth mindset where skillsets can change (Dweck, 2006), then they are more likely to emphasize the cost (versus the value) of doing mathematics. A structural equation modelling study done with 8071 third and fourth grade students in 6 European countries, looking at mindset, as part of a longitudinal study called MATHMot, found that parents who emphasized the intrinsic value – liking or enjoying mathematics (Eccles & Wigfield, 2023) – had the most positive effect. Emphasizing the cost of mathematics, had the most negative effect on motivation. Given that motivation is correlated with achievement, this means that emphasizing the difficulty of mathematics leads to students who are less motivated and have lower mathematical achievement (Peixoto et al., 2024). In the context of family mindset, parents or siblings' fixed mindsets are also negatively correlated with motivation, engagement, and help seeking (Lin & Muenks, 2023).

Therefore, there seems to be a connection between positive mathematical self-efficacy developed by positive experiences that lead to positive mathematical beliefs and a sense that mathematics is valuable to one's identity. This is fostered by a wider socio-cultural network that makes it possible for individuals to feel accepted as someone who can do mathematics, a family that supports this identity and a mathematics classroom where individuals feel included and supported.

2.4 Linking Experience to Emotion: Turning Points

Though students and teachers may not be initially aware of their mathematical affective disposition – whether it is positive, negative or mixed – they may become aware of moments which change or test it. Even the most math loving student will have difficult moments. This difficulty can be experienced at different intensities and with different results: persistence versus avoidance, anxiety versus enjoyment, building skill or not. In a student's story of mathematics, there may be experiences that shift a student from a particular affective disposition to another. Unlike most of the larger, typically statistical studies, referred to above, studies of narratives consist of detailed portraits of life stories. They can offer insight into the specificity of experiences that can change a student's perspective.

When an experience, event or episode causes a shift in something like identity, perspective or emotional tone, it could be called a turning point. This study will be looking at descriptions and shifts of general affect disposition, such as a change of emotional tone (McAdams & Bowman, 2001). This aligns a mathematical disposition, such as being mathematically positive or mathematically negative, to the emotional tone of a narrative about mathematical experiences.

In a study of 163 narratives of preservice teachers, over a 20-year period, Machalow et al. (2022) sorted narrative arc stories into "mostly positive", "mostly negative", and "mixed." Within the latter, "Taking the Reins" designated preservice teachers whose negative arcs shifted toward the positive by taking control, while "Rollercoaster" was used to account for more variable experiences. About half of the preservice teachers had positive mathematics narratives, 10% had mixed and 40% had negative narratives. Being positive was more aligned with relational teaching orientations, meaning they were interested in the whys of mathematics versus

an instrumental orientation that was more focused on rules (Skemp, 1976). However, a small number, despite being instrumental, were positive (Machalow et al., 2022). Various studies with teachers and preservice teachers show a similar trend where a turning point often involves an experience with a teacher. Either the teacher is kind and supportive or shaming and degrading (Machalow, 2020; McCulloch et al., 2013; Lo, 2021; John et al., 2020, 2022; Di Martino & Zan, 2010; Russo et al., 2023).

In another study of 210 emerging adults' mathematics education, participants were asked to write about a turning point in their mathematics education. Positive stories were linked to lower anxiety and greater mathematics motivation. Women were more likely to have higher mathematics anxiety and lower mathematical self-efficacy than men (John et al., 2020). Narratives were either consistently positive, consistently negative, involved redemption (a turn to the positive) or contamination (a turn to the negative) (McAdams & Bowman, 2001). Four themes emerged: academic performance, relevance (of the subject of mathematics), receiving help (whether it was indeed helpful or a hindrance) and study habits. The researchers noted that appraisals of experiences often led to different conclusions, which they attributed to an internal versus an external locus of control (John et al., 2020). Following on from this study, John et al. (2022) focused on 94 low points of college students and looked at the importance of self-concept in the appraisals of these low points. Those with a high mathematics self-concept, saw these low points as an opportunity for growth and betterment, whereas those with a lower mathematics self-concept (often women) used these low points as evidence of their mathematical failings and attributed the bad experience to a teacher (John et al., 2022).

A study of college faculty writing their own mathematical autobiographies described positive, negative and mixed stories. The negative experiences had to do with difficulties with

math, poor scores, unqualified or hurtful teachers, gender discrimination and general dislike. Positive experiences were about strong interest, natural aptitude, self-confidence, excellence, encouragement from parents, teachers and schools along with the importance of making mathematics meaningful by making it resonate with the particular group of students (Wilder & West, 2024). They concluded that the results demonstrated the exercise's usefulness for instructors and students in helping draw out and understand different perspectives.

Despite the usefulness of this approach, a literature review of autobiographical accounts of mathematics experiences conducted with Proquest, ERIC and Google Scholar found that most of these had teachers, preservice teachers and college/undergraduate studies as participants (Towers et al., 2017). Very few looked at younger populations or used non-interview techniques. To address this, Towers et al. (2018) used Wordle and pictures to gain insight into mathematics experiences of 124 Grade K-9 students. They reported a nuanced and changeable relationship to mathematics in terms of emotional valence. Emotion-laden pictures almost always evoked school 'as place' even when the students were explicitly asked for experiences outside of school. They also noted the dominance of head imagery, which could be related to the perspective that mathematics and the experience of mathematics are associated with the head or, by extrapolation, that it was a process associated with the head such as thinking. These concepts formed through their experiences and were influenced by those around them such as parents.

> Our findings with older students (Towers et al., 2017) also indicate that it may be a long pattern of developing difficulty or a single moment of shame (e.g. prompted by a teacher's lack of compassion towards struggle) that propels a student towards a negative relationship with mathematics, and that similar (but positive) patterns or moments can propel a student towards a more positive emotional relationship with mathematics. All of this suggests that

> students' relationships with mathematics are fragile, volatile, subject to contextual influences, and constantly in process… (Towers et al., 2018, p. 161)

Emotions are the foundational base of mathematical affect, and experience is what sparks those emotional responses. So, in order to study mathematical stories, it behooves one to pay attention not only to the influences in a participant's story but also to the moments that create turning points in their affective narrative.

3.0 Theoretical Framework

To answer the research questions, I will be drawing upon two theoretical frameworks that are linked through the concept of value and valuing. These are Situated Expectancy Value Theory (SEVT) (Eccles & Wigfield, 2020, 2024) and Hill's Framework of Mathematical Wellbeing (MWB) (Hill et al., 2021).

Much of the work done on affect in mathematics has had the goal of improving mathematical attainment, and also retention in STEM fields. As mentioned in the literature review, there has been a shift toward socio-cultural influences and student values. This shift can be seen in both the theoretical frameworks that are used for this study – Hill's MWB and SEVT. Not only do they both recognize socio-cultural influences as important, but they also highlight value as a controlling affective factor in the experiences and choices that students make about mathematics.

Firstly, Hill's MWB framework veers from attainment as a measure of success towards mathematical wellbeing as a positive outcome. Mathematical wellbeing is often used when describing the opposite of negative mathematical affect. The two terms being used together such as below, suggesting that they are very closely related if not synonyms.

> As students go through school, they increasingly suffer from mathematics anxiety (Hart & Ganley, 2019; Weir, 2023), low engagement in mathematics (National Center for Education Statistics, 2024), and negative attitudes toward mathematics (Wen & Dubé, 2022). These experiences suggest that students need support in fostering a positive sense of affect and wellbeing in mathematics. Not only can a positive sense of affect and wellbeing in mathematics support students' academic achievement (e.g., Barroso et al., 2021; Grootenboer & Hemmings, 2007; Hannula et al., 2014; Wen & Dubé, 2022), but

> it may also encourage general human flourishing (e.g., Su, 2020) and/or an increase in the number of students pursuing science and mathematics careers.
>
> (Campbell, 2025, p.2)

However, this present study takes its definition of mathematical wellbeing from the literature and allows the participants to define mathematical positivity for themselves. Getting at the participants' experiences that form and foster mathematical positivity guided the interview questions and the construction of the participants' narratives.

SEVT is then used to help anchor the complex causality of influences that may impact the participants' mathematical positivity such as societal influences, influences of socializers and personal aptitudes. Both SEVT and Hill's MWB have roots in Expectancy Value theory (Eccles, 1983; Hill & Seah, 2022) that focuses on expectancy and value as drivers of outcomes. Whereas Hill's MWB derives their values from core concepts in models of positive psychology that are associated with positive mathematical outcomes, SEVT comes from motivation theory and is interested in drawing and testing causal relationships between factors. Since this study is also interested in a longitudinal process with affective turning points and potential turning points towards or away from mathematical positivity, it reconnects these two ideas by using SEVT's achievement performance decision making chart as a basis for developing a hybrid framework with mathematical wellbeing as the outcome.

3.1 Mathematical Wellbeing

A framework that has been gaining interest of late is Mathematical Wellbeing, which comes from the Positive Psychology's field of human flourishing. It focuses on student's subjective experiences of *feeling good and functioning well in mathematics* (Hill et al., 2021) and integrates affective, social and cognitive domains. Campbell (2025) states that rather than breaking affect

into its often-overlapping components of emotion, attitude, belief, self-efficacy and so forth, mathematical wellbeing handles affect more holistically. Though there is a sense of wholeness to the concept, I would argue that it is value focused and therefore there is still a facet of the affect prism that faces forward through which you can interpret student experiences and the framework itself. For Hill & Seah (2022) valuing is the "embracing of any convictions of importance and worth" (p 386).

Initially designed as a developmental concept (Clarkson et al., 2010) based on Bloom's taxonomic approach to development, it follows five stages where learners progressively become aware and accepting of mathematics. It is a set of enabling characteristics that relates mathematical wellbeing to cognitive, affective, and emotional, educational objectives. They state that fostering positive MWB, "needs to take into account how individual values have been – and continue to be – shaped and harnessed" (Clarkson et al., 2023, p. 1132). An example, that he gives, is if a student values hard work then giving them intellectually challenging work will be positive and enabling.

MWB was then further developed into a framework by Hill et al. (2021) who used both hedonic (maximizing pleasure) and eudaimonic (one's capacity to live up to one's potential) descriptions of wellbeing (Hill et al., 2021). She conducted a scoping review to find values that were associated with mathematical wellbeing. These aligned with the Positive, Engagement, Relationships, Meaning, Accomplishment (PERMA) model (Seligman, 2011) and Engagement, Perseverance, Optimism, Connectedness and Happiness (EPOCH) model (Kern et al., 2016) of general wellbeing. Hill's MWB values are Accomplishments, Cognitions, Engagement, Meaning, Perseverance, Positive emotions and Relationships. Importantly, Hill and colleagues

stated that mathematical wellbeing occurred when the mathematical experiences of the learner were in alignment with their values (Hill et al., 2021).

Hill's MWB core values:

- Accomplishment means a sense of achievement from reaching goals or completing mathematical tasks.
- Cognitions refers to cognitive values about understanding mathematical ideas or skills.
- Engagement is about concentration, focus or deep interest.
- Meaning is seeing mathematics as being valuable or worthwhile or even having a purpose.
- Perseverance is described as grit, drive or the ability to continue.
- Positive Emotions means having fun or enjoying mathematics.
- Relationships is about being in a supportive relationship with others in regards to mathematics such as teachers, peers, parents.

Adapted from (Hill et. al, 2021, Table 1)

Hill and colleague's research of MWB is mostly centered around the geographical region of Oceania. A study of 3073 year three to year eight New Zealand students found that students mathematical wellbeing was highest for Perseverance and Relationships but lowest for Engagement and Positive Emotions (Hill & Hunter, 2024). This was despite the fact that Positive Emotions and Engagement were important ultimate values for students, suggesting an area that could be improved. They also found positive correlations between Cognitions and Perseverance, as well as Positive Emotions and Engagement meaning that understanding ideas relates to persistence and liking mathematics relates to interest. Hill & Hunter (2024) conjecture that "... as

students progress through school, many core or ‘ultimate values’ of students are increasingly not fulfilled in the mathematics classroom, potentially contributing to mathematical anxiety, disengagement, avoidance of mathematics, and mathematical illbeing” (p. 212). There were differences in MWB depending on age and ethnicity. For example, the decline in mathematical wellbeing was found to be sharpest at transition points, such as from primary school to middle school. As well, there were differences across ethnicities with Indian and Pakistani students experiencing more Positive Emotions and Engagement (Hill & Hunter, 2024). Differences in age and ethnicity, however, were not found in a self-report study of 408 US students, aged nine to fourteen. They did however find that female students had overall lower mathematical wellbeing (Campbell & Bean, 2025).

3.2 Situated Expectancy Value Theory

In Hill & Seah (2022), the authors state that,

> Our understanding of values is rooted in educational psychology, through Eccles’ (1983) expectancy-value theory [EVT]. In the popular Eccles model, subjective value (regarding what is important) are conceptualized as impacting on achievement-related choices together with expectancy beliefs (regarding the chance of success). In particular, values express importance in terms of attainment, utility and enjoyment. (p. 386)

Expectancy Value Theory (Eccles, 1983) continued to develop from the affective turn of the 1980s into the socio-cultural shift of the 2000s. Now called Situated Expectancy Value Theory (SEVT) (Eccles & Wigfield, 2020, 2024), it is situationally specific and socio-culturally informed. Part of motivation theory, it is concerned with understanding the decision-making

process underpinning mathematical engagement and the decision to continue mathematical learning into STEM fields. It relates these decisions to a student's expectation or not of success and the value they place on a task. These influences were empirically and theoretically derived and categorized from socio-cultural influences such as the conflicting identities of being female and doing mathematics, familial influences such as finding mathematics intrinsically worthwhile or not, and personal factors such as finding mathematics enjoyable. These influences form a child's perceptions and experiences, which relate to self-schemata and affective memories. They ultimately develop into expectations of success and the weighting of mathematics according to particular values and costs. This leads to decision making about mathematics activity.

> How individuals' expectancies, self-concepts of ability, and subjective task values (STVs) along with their goals and identities, depend on the specific current situation in which both conscious and non-conscious choices are being made. This also means that the range of options being considered by the individual in any given choice situation is limited by prior experience and by the cultural values, norms and characteristics that surround individuals as they mature and move through time and space. (Eccles & Wigfield, 2024, p. 4)

The initial development of SEVT was heavily influenced by trying to understand why women engage and continue in STEM subjects, however SEVT is widely used in the field of positive psychology.

Values are described as either being motivated intrinsically or extrinsically (Ryan & Deci, 2000). Intrinsic value is liking or enjoying mathematics and extrinsic utility value is how well a particular task fits into an individual's present or future plans. There is also attainment value, which is the relative personal or identity-based importance attached to engagement in an

activity. This is contrasted to the opportunity or emotional cost, which is the effort required to do the task (Eccles & Wigfield, 2024).

Since this present study explores the positive mathematical narratives of participants over a lifetime, it is useful to have a process-based framework that is interested in understanding something of the mathematical experience over time. SEVT has done "longitudinal studies designed to both describe the developmental trajectories associated with these beliefs over childhood and adolescence and then to link them to both experiences and subsequent motivated choices." (Eccles & Wigfield, 2020, p. 3). SEVT is interested in decision making such as 'I will take Advanced Functions because I want to go into Biology and that is a prerequisite' or 'I will not do my math homework because I'm more of an English kid so it won't matter in the long run anyway' whereas this study is focused on affective turning points such as 'I was really disappointed that I did not get into the advanced math class but persevered because my parents told me struggle is part of life and I love a challenge.' In other words, SEVT gives a way to organize the development and persistence of mathematical positivity despite potentially negative experiences.

3.3 Combining Hill's MWB and SEVT for this study

The concept of wellbeing allows for mathematical positivity not be aligned with the goal of good grades or progression in STEM but to be a goal in and of itself. Given that this present study did not demand any level of mathematical achievement, this fits well (though incidentally, all but one participant did have a STEM related job from accountant to environmental physicist). As this present study is a narrative analysis, stories unfold over time, with a multiplicity of influences from socio-cultural to personal preferences and aptitudes. Though running somewhat

in parallel to the categories of SEVT such as self-schemata – mathematical identity – and values surrounding mathematics, the focus of this study is on wellbeing instead of performance and achievement-based decision making. Nonetheless, the process focus of SEVT provides a useful scaffold.

In the original Chart of Achievement Performance and Choice, found in Appendix 1, by Eccles & Wigfield (2024, fig. 1, p. 52), there are many process arrows that help guide researchers interested in finding causal links between things such as child's perception of gender roles and affective memories and their relationship to different values (as categorized by SEVT). Presented below is a simplified version of the chart that does not include the causal arrows. It should be noted that on the original chart there is a high degree of interaction between various components such as influence, perceptions and interpretations. It is recommended, in deference to Eccles & Wigfield and because this model is used widely in positive psychology, that readers consult the original. This simplified version was developed to make it easier to read and also to explain the hybrid framework developed for this study.

Table 3.3a: Simplified SEVT Performance and Choice chart

<table>
<tr><td>Influences

That influence each other</td><td>Perceptions

Interpretations</td><td>Cognitive and Affective solidification</td><td>Expectations and Values</td><td>Achievement Related Choices and Performance</td></tr>
<tr><td>Situated</td><td colspan="2"></td><td>Expectancy Value</td><td>Theory</td></tr>
<tr><td colspan="5">Across the top is the development of expectations of success</td></tr>
<tr><td>Cultural Milieu</td><td rowspan="2">Child’s Perceptions</td><td rowspan="2">Child’s goals and general self-schemata</td><td rowspan="2">Expectations of Success</td><td rowspan="4">Achievement related choices and performance</td></tr>
<tr><td>Socializers Beliefs and Behaviours</td></tr>
<tr><td>Differential Aptitudes</td><td rowspan="2">Child’s Experiences</td><td rowspan="2">Child’s affective memories</td><td rowspan="2">Values (SEVT)

Interest Value, Importance Value, Utility Value, and Cost</td></tr>
<tr><td>Previous Achievement Related Experiences</td></tr>
<tr><td colspan="5">Across the bottom there is an affective value chain that can move bidirectionally</td></tr>
</table>

The hybrid framework is changed in the following ways. The outcome is not achievement and performance related decision making but mathematical positivity. The influence categories of socio-cultural, socializers (parents, teachers, for example), personal and previous experiences are understood as characters in the participant’s narratives. The outcomes arise from expectations and values associated with wellbeing rather than success or achievement. Expectations roughly correspond with the mathematical affect category of attitude with the important caveat that these

categories have fuzzy boundaries. Given that, in each case, analysis is done on a participant level, the values are those of the participant. Instead of filling the middle with connecting arrows, then testing these connections, the middle is left intentionally messy and open to the explanations of the participants, which correspond with the plotlines of the narratives and connect to the participant's Emergence of Mathematical Positivity.

Table 3.3b: Hybrid Framework: Expectation + Value = Wellbeing

<table>
<tr><td>Influences

That influence each other</td><td>The Messy Middle of Development</td><td colspan="2">Wellbeing</td></tr>
<tr><td>Characters</td><td>Plotlines</td><td colspan="2">Resolution of plotline/Outcome</td></tr>
<tr><td>Socio-cultural</td><td rowspan="4">The messy middle is defined in SEVT by being broken into a layer of perception and interpretation of experience

This leads to self-concept / identity and values, and affective response.

Participants, through a narrative, will give <u>their own stories</u> involving affective experiences and their cognitive companions.</td><td rowspan="2">Expectation of wellbeing</td><td rowspan="4">Mathematical positivity</td></tr>
<tr><td>Socializers</td></tr>
<tr><td>Personal</td><td rowspan="2">Participants' values.</td></tr>
<tr><td>Previous Experiences</td></tr>
</table>

The participants' values, will be analyzed later with Hill's MWB core values. Since these are often used in a classroom setting, there are differences between that shorter timeframe and

the longer, longitudinal timeframe of the participant's narratives; therefore, there are some alterations needed for proper interpretation as seen below:

Table 3.3c: Hill's core MWB values with Longitudinal Interpretation

Definition in Hill et al., 2021, Table 1	Longitudinal interpretation
1. Accomplishments	
Sense of achievement from reaching tasks ->	Success in mathematics
2. Cognitions	
Understanding mathematical ideas and skills ->	Intellectualism
3. Engagement	
Concentration, focus or deep interest ->	High level of interest
4. Meaning	
Seeing mathematics as worthwhile or with purpose ->	Specify purpose
5. Perseverance	
Grit, drive, ability to continue ->	Persistence with mathematics and mathematical type problems
6. Positive emotions	
Having fun, enjoying mathematics ->	Same
7. Relationships	
Supportive relationships ->	Same

The last column – Wellbeing: Resolution of Plotline /Outcome – links expectations and values to mathematical positivity through wellbeing. There is a correlation between expectations of mathematics and experience of mathematics (Eccles & Wigfield, 2020; John et al, 2022) such

as appraisals of experience in anxious students can make a mathematical experience threatening whereas they would be innocuous to a less anxious student (Hunt & Maloney, 2022).

It seems that, for the most part, having negative expectations (expecting math class to be scary, hard or otherwise unpleasant) and not having your values met (teachers being dismissive, not understanding the material, feeling distracted) may well lead to mathematical negativity. And that the opposite, having positive expectations, (expecting math class to be fun and fruitful) paired with values met (doing well on the last test and having fun working with your bestie in class) would lead to mathematical positivity.

However, there are two other potential pairings in a matrix of expectation and value. Having positive expectations "I usually do well on tests" and yet not having expectations met "I couldn't understand anything" could be threatening to mathematical positivity. Likewise, having negative expectations "I will fail like I've done before" and pairing it with values met such as "I enjoy fun lessons and this lesson was fun" could be encouraging to mathematical positivity.

Table 3.3d: Matrix of Value versus Expectation

	Negative expectations	**Positive expectations**
Values not met	Mathematical negativity	Threatening
Values met	Encouraging	Mathematical positivity

Threatening and encouraging experiences are potential turning points (McAdams & Bowman, 2001) that could change the affective tone of the narrative. Examples of this in a

mathematical story is described by McCulloch et al. (2013) who describe mathematical stories as having ‘smooth setbacks,’ ‘minor setbacks,’ ‘rollercoaster,’ ‘consistently frustrated’ or having turning points that are either negative or positive. Similar to the threatening and encouraging labels, McAdams & Bowman (2001) describe stories that involve redemption or contamination plotlines that involve a shift up or down in affective tone.

3.4 Mathematical affective categories

As can be seen in the Matrix of Value versus Expectation, there is a presumption, to be tested, that positive expectations and values met will lead to mathematical positivity through mathematical wellbeing. In order to examine turning points in terms of expectations and values and to further isolate and describe affective themes within the interview and narratives, the following affective categories will be defined:

1. Emotions: Reactions to an experience in the moment as in being scared or having fun (McLeod, 1992).

2. Attitudes: An *expectation* or anticipation of an emotional reaction such as finding mathematics interesting, loving or hating trigonometry or expecting a lesson to go well. It may also be fearing mathematics or having anxiety about mathematics as this is the anticipation of fear (DiMartino & Zan, 2010). Expectations in SEVT is an individual’s self-perception of their ability to succeed in a specific task. In fact, ‘perceived competence’ along with emotional disposition and vision of mathematics are cornerstones of DiMartino & Zan (2010)’s definition of attitude. Given that this study is

on mathematical positivity there may also be *expectations of affect* through meta-affect (Debellis & Golden, 2006) – the feelings about feelings.

3. Beliefs: These are more cognitively stable statements about things that sound like facts such as 'Math people are socially awkward.' A working definition of belief was constructed by Op't Eynde et al. who state that "students' mathematics-related beliefs are implicitly or explicitly held subjective conceptions about mathematics education, the self as a mathematician, and the social context, i.e., the class-context." (Op't Eynde et al., 2002, p. 14).

4. Identity: These are beliefs about self, such as 'I've always been good at sports or I am an arts person.' Within identity are beliefs about capacity in mathematics such as mathematical self-efficacy (Bandura, 1986; Sfard & Prusak, 2005).

5. Values: These are judgements about whether or not something is valuable. Examples include teachers being 'good' or 'bad,' something being worthwhile/important, such as 'it is very important that I get a good grade' or 'the best learning is based in curiosity' or stories about mathematics being useful versus stories about mathematics being an unnecessary abstraction (Debellis & Golden, 1997; Hill et al., 2021).

These categories are not rock solid but morph and inform one another. Feeling excitement in a mathematics class can lead to feeling excitement about a mathematics class, which in time can lead to finding mathematics exciting. Therefore, being excited in mathematics class and finding mathematics classes exciting are subtly different as they speak about feelings and

attitudes respectively. Despite an ability to categorize affect, they are highly interrelated. For example, a belief that mathematics is exciting can be subtly rearranged into ‘I am the sort of person who is excited by mathematics,’ which is an identity statement but becomes a value statement when it speaks about judgement such as ‘good teachers make mathematics exciting.’ Rather than get too caught up on the precision of these definitions, though affective statements are classified in the interviews, there is an understanding of their difficulty and fuzziness.

4.0 Methodology

4.1 Overview of Methods

This is a qualitative study about life stories that centres around the axis of mathematical positivity. It is a narrative analysis (Riessman, 2008; Polkinghorne, 1995; Andrews et al., 2008) that elicits the stories of six adult participants who self-identify as mathematically positive. It involves several layers of data gathering and analysis. Full details of which are explained in this chapter.

Data were initially collected from semi-structured interviews, which ranged from 30 to 80 minutes long, and that used an open question style (Hollway & Jefferson, 2009) intended to allow the participant to tell explanatory, experiential stories at their own pace and complexity. This first interview is coded for affective language, categorized as emotions, attitudes, beliefs, identities, and values. It is also coded for the story elements such as characters, events/episodes/experiences, explanations/plot and themes.

This first interview is restoried (Riessman, 2008; Polkinghorne, 1995; Nardi, 2016) into a cohesive narrative. This acts as another layer of analysis but also the production of more data. This narrative is presented back to the participants in a way that invites correction. This allows for cross-checking, to improve validity and reliability – referred to as trustworthiness and attention - but also inserts the researcher into the data in an intentional way.

The roughly chronological plotline of each participants' narrative was then placed into a turning point table that records major events, changes in values and expectations and the outcome of those changes: mathematically positive, mathematically negative, or if mathematical positivity was threatened or encouraged as per the Matrix of Value Expectation (Table 3.3d). These turning point tables were linked to the explanations in the narrative, through the Hybrid

Framework (Table 3.3b) to come up with a summary of the Emergence of Mathematical Positivity. Once each participant's data was analyzed individually, the emergent themes were compared and contrasted. The values of the participants were also analyzed with Hill's seven core MWB values in order to determine which values were most commonly expressed and if any were not easily categorized by this framework.

Table 4.1a: Summary of Methodological Steps

Narrative Creation and First Interview
Step 1: First Interview
Step 2: Type up transcript then First Read
Step 3: Restorying: Determine events or episodes in the narrative then restory these into a cohesive narrative. Themes start to emerge.
Step 4: Second Interview: Present story back to participant for correction/comment. Make corrections to the story.
Narrative Analysis, Affective Language and Turning Points
Step 5: Identify affective words/phrases: attitude (expectations), value (emotion, attitude, belief, identity and value).
Step 6: Turning Point Tables and Emergence of Mathematical Positivity: Describe stories including important characters, themes and events. List narrative episodes and compare these to identified attitudes (expectations) and values.
Cross Comparison between Participants
Step 7: Describe and compare major themes across the narratives
Step 8: Compare participants' values to Hill's core values in MWB

4.2 Justification of Methodology

> "Human action is the outcome of the interaction of a person's previous learning and experiences, present-situated presses, and proposed goals and purpose." (Polkinghorne, 1995, p. 1)

The narrative turn in educational theory was a recognition of the meaning making that story serves for people. Alongside number (such as a Likert scale) and short answers (such as a survey), narrative analysis is another technique that makes stories the data (Polkinghorne, 1995). People understand their lives in a storied way, each story being unique and individual. These narratives are dynamic and change contextually – in other words they are situated and social-cultural. This present study explores the intertwining of several narratives. The first is about mathematical positivity. It is understood uniquely by each of the participants. Secondly, it is the story of how they became mathematically positive and whether or not there were any threats to that overall affective disposition. This story is temporal as it will start in the past at some point and go through a journey to the present. Experiences beget experiences and lead to further experiences (Clandinin, 2006).

These stories involve various characters such as parents, teachers, institutions and dreams. It has a protagonist which is the participant themselves. However, there are more stories. The researcher restories the participant's interview. Acting as a storyteller, they impose a layer of meaning in their choice of organization and emphasis. There is also a more academic narrative with its own elements such as theory, formality and the demands of a Master's thesis. Clandinin & Caine (2008) write that "within this space each story told and lived is stated and understood within a larger cultural, social and institutional narratives. Narrative inquiry is marked by its emphasis on relational engagement between researcher and research participants," (p. 542).

Narrative inquiry is a larger term used to describe various engagements of narrative analysis. Polkinghorne (1995) stated that narrative inquiry can be broken into two categories: analysis of narrative and narrative analysis. He drew from Bruner (1985) who distinguished research that is theoretically driven - disassembling the whole and comparing the parts and narratively driven - assembling the parts into a whole. This study mostly follows this second mode of analysis in that it constructs stories, from the participant's interviews, into data that will be understood as a whole assembled from its parts. Though narratives can be studied in a variety of ways – for instance for their dialogical, structural, visual or thematic elements (Connelly & Clandinin, 1990) – this narrative analysis is concerned with thematic elements. Within these themes are various influences including personal and socio-cultural. Storytelling allows us to hear multiple influences tied to the participant's explanation. "People are individuals that need to be understood but they cannot be understood as individuals" (Clandinin, 2006, p.46).

By intentionally choosing to honour the participants' stories and focusing on a story level of analysis, it is hoped explanations emerge. These need not be overly generalized into a recipe for success but rather they should be suggestive of the sorts of processes that may be involved in an adult who self-identifies as mathematically positive. Despite maintaining the integrity of the narrative, it is still possible to compare each story to the theoretical framework to see how value interacts with expectations and whether or not value alignment seems congruent to mathematical wellbeing, leading to mathematical positivity.

Each step in the process relates back to the original data and leads to the next step. Highlighting affective words/phrases and stories within the transcripts gives focus to the development of the narrative. Categorizing the affective words/phrases distinguishes expectations from values. Arranging the narrative into stand out experiences that relate to their

mathematical affective dispositions reveals potential turning points (Machalow et al., 2022; McCulloch et al., 2013). These potential turning points then help the researcher reflect on what the participant said about their values and expectations at the time and whether or not these were met (Hill & Hunter, 2024). The turning points also relate to the participant's own explanations about how and why they developed their mathematical positivity, what were potential the threats and how these were resolved.

4.3 Participants and Recruitment

The study invited adult participants who identified as mathematically positive, and who were no longer students, who were not grade school or high school teachers, not pre-service teachers or otherwise in the pre-postsecondary school system. They were interviewed about their own mathematical experiences and not those of their children (though references to their children did incidentally come up in the course of the conversation). The reason for focusing on this population was two-fold. There is a wealth of studies whose participants are school age or who are pre-service teachers (Xolocotzin Eligio, 2017). Secondly, this study wishes to find and define mathematical positivity that exceeds the boundaries of school. It is assumed that the ultimate goal of school and teaching is to educate citizens who will carry these experiences and skills beyond their halls. Adults have had time to reflect on the influences that went into shaping who they are. Therefore, adults have a mathematical identity and mathematical values that they could explain if asked.

The participants were selected through a snowball technique by the researcher who asked various people who she knew, if they knew anyone who considered themselves mathematically positive who was not currently working as a teacher. The original intention was to find a small number of participants (between three and five) given the rich, detailed nature of analysis,

though eight responded. Of those eight, one decided not to participate and one other was withdrawn because they were a high school teacher, leaving six participants.

4.4 Data Collection

Step 1: The First Interview

Though the first interview was audio recorded and meant to either be in person or via Zoom or Teams, all interviews took place on Zoom. They were semi-structured interviews with suggested questions that were designed to be open (Hollway & Jefferson, 2009) in order to elicit stories and were in the form of "tell me about…" and "what do you remember about…". An example of a question used in the interview was "Tell me your earliest memory of math?" Participants were encouraged to talk as long as they wanted on the subject and to flow from one topic to another naturally within the context of mathematical positivity and their mathematical experiences until there was a natural stop. The interviewer used active listening and took notes. Therefore, when there was a need for clarification, the researcher/interviewer could use the participants own words, such as "How did you get over your *terrifying experience* of *trigonometry*?" or "can you tell me about how you get people to *trust the data*?"

The interview questions were designed to answer the questions:

1) What is mathematical positivity to the participant? (definitions)
2) What was their relationship with mathematics over time? (expectations)
3) What were the important influences on that mathematical relationship? (themes)
4) Did they ever experience any mathematically related challenges? (turning points)
5) What values do they associate with mathematics? (values)

For a question script, please see Appendix 2.

Step 2: Transcription of Interviews and First Read

This first interview was transcribed by hand, by the researcher. The transcription included the words spoken, and other sounds such as laughter, pauses, up and down tones, interruptions, throat clearing, clicks. These were included in the typed-up document, though ultimately not directly used in data analysis. However, this coding did inform the researcher, when writing the narrative, as it could change the intended tone of the statement.

During the first read, different episodes and themes were highlighted and noted for later reference in order to write the narrative and construct the turning point tables. Affective language was also highlighted based on the definitions found in Table 4.5a, but were not categorized at that time. The goal was to help determine important episodes/events/experiences in the participant's narrative of mathematical positivity that would later serve to help develop the turning point tables, including the categorization of mathematical attitudes/expectations and mathematical values. For example, in the interview with Hanna, she recalled a game that she really loved before she was primary school aged, "I have no idea how old I would be, but I was pretty young. We had this, um board game that was, like, a Chip-Its/ board game, like chocolate Chip-Its."

This brackets off a time period and acts as the start of a story (Clandinin & Caine, 2008). You can also tell that a story is occurring as the participant needs very little encouragement to talk. The episode in the larger narrative unfolds naturally, ending with "Something like that. I think most of it was just addition and subtraction, though, and I remember playing it and ***absolutely loving*** *that game*"

Here the affective phrase '*absolutely loving that game*' is highlighted as affective language. This leads to an explanation of part of their mathematical journey when Hanna says it

'Helped my math skills.' Several readings may have been required to organize the transcript into the main episodes/events/experiences of the story along with potential themes. These were then listed chronologically so they could be fleshed out into a story.

Step 3: Restorying

The narratives of this study could be understood as having a master plot in which each is an Odyssey that describes the participant's mathematical journey. Within this Odyssey, there are episodes/events or experiences that have their own plots. "Plots mark off a segment of time in which events are linked together as contributors to a particular outcome" (Polkinghorne, 1995, p. 7). A plot establishes a beginning and an end as temporal boundaries, the "narrative gestalt" (Polkinghorne, 1995, p. 8). The chronological ordering of events in the restorying process presumes that making meaning means that a story has to have a beginning and work toward some end. This does not necessarily mean linearly but that there must be an antecedent that reaches an end point. The antecedents in this case are those things that lead to mathematical positivity.

According to Polkinghorne (1995), the steps to restorying are:

- Configure data into a story where the elements are arranged chronologically.
- Determine which events are contributors to the outcome, looking for "because of," "in order to" and other connectors.
- Write a story.

The function of this story is to act as a temporal gestalt and to answer how and why a

particular outcome came about (Polkinghorne, 1995, p. 19). For the purposes of creating a coherent, believable whole that aligned with what the participant was saying, sometimes different threads were introduced at different times, despite being temporally out of order.

This second order story (Scheffelaar et al., 2021) that the researcher composes from the first order story (the interview) can also be seen in terms of its broader context and specific detail. Broadening is to look for the broader context of the story, such as the participants' values, social, historical or cultural milieus (Connelly & Clandinin, 1990). It also pays attention to the details, called burrowing, in the data such as their feelings, understanding and dilemmas or the whys and hows of things happening (Connelly & Clandinin, 1990, p. 11). Broadening and burrowing are always kept in mind during the restorying process. According to Nardi (2016), restorying uses narrative elements such as problems, characters, actions and resolution. Not only is this format important for the restorying, but it influences the turning point tables, presented in the Findings (Chapter 6.1), where each story is examined for mathematical expectations, values and outcomes, namely whether they were overall mathematically negative, positive or whether their mathematical positivity was encouraged or threatened.

Whenever possible, the participants' own voices and phrasing were used (Byrne, 2017) so if there was a choice of an adjective, noun or other descriptor, the participants' own words were used rather than something that might occur to the researcher. This required constantly comparing the new story with the data. Narrative smoothing (Kim, 2016) also took place where plotlines were simplified and details that did not seem to contribute to the overall plot of mathematical positivity were not included, unless, on follow up (second interview), the participant felt they were necessary to include. For example, this happened with Claire's

narrative that initially presented a simplified version of her mother's story where Claire felt it important to expand on the detail to give overall definition to her narrative.

Step 4: Second Interview: Presenting stories back to participant then making corrections

Narrative analysis makes the relationship between the participant and the researcher more central (Lindsay & Schwind, 2016). The narrative developed from the interview is then shared with the participant. This is an interesting and vulnerable practice as the participant is hearing their own experiences, explanations and phrasing read back to them but not in the way that natural conversations take place, rather as a coherent whole. It is important at this juncture to shift the ownership of the narrative back to the participant with statements like 'this is your story' and 'please correct any details.'

The participants were aware from the first interview that there would be two interviews and the second was meant to check that the narrative was reflective of their experience. Before the second interview, any ambiguities were highlighted in the narrative so that the researcher could ask for clarification. Ambiguities could include factual uncertainties, unanswered questions or a lack of clarity about how a participant experienced something. The participants were also asked to correct any statements or story elements that they did not find reflected their experience. Each sentence was read to the participant for correction or tweaking. At regular intervals, the participant was asked if this reflected their own understanding of their experience(s) or if there was anything that could be changed or added. At the end of the reading, the participant was asked if there was anything else that could be added, emphasized, changed or just that they felt should be communicated.

4.5 Data Analysis

Each participant's analysis was done one at a time in order to maintain the coherency of each story. Therefore, categorization of affective language, explanations and themes, turning point tables, and Emergence of Mathematical Positivity summaries were done for one participant at a time before any cross analysis occurred. Afterwards, cross analysis was conducted that compared and contrasted themes between participants and analyzed each participant's values with Hill's MWB core mathematical values.

Step 5: Identifying Affective Language (words/phrases)

The highlighted affective language (words/phrases) from each interview transcript were listed in frequency tables, a word count which can be found in Appendix 3. These categorized affective language into five categories: emotion, attitude, belief, identity and value, as defined in the theoretical framework (Chapter 3.4). The reason for this was two-fold. Firstly, this helped to categorize attitude/expectation and value for the turning point tables. Secondly, as this is a story about mathematical positivity and positivity is an affective description, reflection on affective words/phrases helped identify important themes. As noted above, it is not always straightforward to categorize affective language as these categories lead from one to another over time and because the participants were talking about events that happened often in the far past therefore 'I struggled in class' might mean 'Math class felt hard' and/or 'Math classes were a struggle for me' which is the difference between emotion and identity.

Table 4.5a gives cues that were used to separate categories in the data. A ratio was also calculated for each category to overall affective word count. This analysis was not included in

the final data (it can be found in Appendix 3) but it provided further opportunity to review the data and think about how the participants used affective language.

Table: 4.5a Affect Categories Cues

Affective Definition	**Ways to identify**
Emotion: Reactions to an experience in the moment as in being scared or having fun	Phrases that suggest emotion: • "I am/was…" scared, happy, confused. • "I felt …" puzzled, overjoyed • "It was…" exciting, overwhelming
Attitude: An expectation or anticipation of an emotional reaction such as finding math interesting, loving or hating trigonometry or expecting a lesson to go well. It may also be fearing mathematics or having anxiety about mathematics as this is the anticipation of fear.	Phrases that suggest attitude: • "I expected it to be…" • "I am/was …" anxious, excited about… • "I used to be…" scared of • "I like/hate…"
Beliefs: These sound like facts such as 'Math people are socially awkward.'	Phrases that suggest identity • "I think…" • "It is / They are…"
Identity: These are beliefs about self, such as 'I've always been good at sports' or 'I am an arts person.' Within identity are beliefs about capacity in mathematics such as mathematical self-efficacy.	Phrases that suggest identity: • "I am…" someone who likes math • "They are …" nerdy
Value: These are judgements about whether or not something is valuable.	Phrases that suggest value: • "I feel… (overall)" • "It is important because…" • "I wish …" • Something being good or bad

Step 6: Turning Point Tables and Emergence of Mathematical Positivity

Before the creation of the turning point tables and the writing of the Emergence of Mathematical Positivity, each participant's narrative and transcript was examined once again to note important characters, themes and events/experiences/episodes. Important characters either appeared multiple times during the story such as Claire's mother, or the participant had emphasized their importance directly such as Paul's father, or they were associated with a potential turning point such as Ray's accounting teacher. Themes began to emerge during the first read of the story, were further developed in the narrative and then were confirmed when reading back the narrative to the participant. This was written into a connected summary, which tentatively named themes such as 'The Difficult Problem' in Hanna's sample analysis seen in Appendix 5.

Affective language, for each participant was also examined and described. The ways in which the participants described affective language categories such as emotions or attitudes was summarized, with examples. An excerpt of the affective language summary for Paul includes

> Mentions of emotional language were by and large negative and about confusion or frustration including in trigonometry (and spherical geometry), which is an area that he said he had some trouble with as he found it difficult to visualize. Most of the time, he stated that he felt familiar and comfortable with mathematics and if he was frustrated then he would just keep working on it until he figured it out. He doesn't use the words love or hate except when it came to his father who loved water and the natural world. The smallest section was beliefs and it had to do with connecting with his physics teacher 'I think we had a connection' and disconnection 'I didn't think anyone else was interested' in the eclipse that was happening outside…

Drawing from the episodes/events/experiences that were used to create the narrative, expectations about mathematics learning across the timeline and the sort of values that the participant had expressed, a turning point table was created, as adapted from the Hybrid Theoretical Framework (Table 3.3b). Turning point tables were created in order to have a strong visual representation as to how their mathematical positivity was developed, encouraged or threatened and to compare these with expectations and values at the time.

Each turning point table has three columns. The first gives a chronological tag such as 'early schooling' and important, associated events such as 'didn't get into advanced math class.' The second column describes changes in expectations and values. These may emerge around this moment and therefore are detailed or these may change and these changes are noted. If there is no change then 'no' is listed in this cell. The third column uses the Matrix of Value and Expectation Table (Table 3.3d) to determine if the participant at that time was a) mathematically negative, b) mathematically positive, c) their mathematical positivity was threatened or d) their mathematical positivity was encouraged. Threatened or encouraged mathematical positivity requires a resolution as it is a turning point so the outcome or resolution is also listed. From this turning point table, a description of how their mathematical positivity emerged is written.

Step 7: Description and comparison of major themes across narratives

After individual analysis of affective language, narratives, turning point tables, and the Emergence of Mathematical Positivity summaries, major themes emerged that were compared across the participants' stories. Examples from both the narrative and the transcript were used to support these themes.

Step 8: Comparison of participants' values to Hill's core values in MWB

Lastly, the participants' values were analyzed using the seven core values in Hill's MWB to see where they may fit in that framework, how they differed or if there is anything that does not appear in that framework.

4.6 Trustworthiness and Attention

Given the nature of qualitative data, it requires a different approach to validity and reliability. According to Andrews (2021), qualitative studies can use this list of indicators – truthfulness, trustworthiness, critical reflexivity, scholarship and accessibility, ethical sensitivity, co-construction of meaning, attention to the untold, awareness of temporal fluidity, multilayered stories and contextualization of research as indicators of quality. This is summarized as trustworthiness and attention by Parks (2023) in an article on a narrative method called Story Circles. Trustworthiness is the sense that the reader believes that the explanation is true. This was done in a number of ways as the participants helped shape and verify their stories. Additionally, interpretations of the data should feel plausible, reasonable and convincing (Riessman, 2008). Interpretations were developed step by step in a process that continually referred back to the source data. A whole story analysis was completed for each participant before any cross thematic analysis was done, consequently individual themes and explanations were already developed before they were compared. The second indicator – attention – was monitored by careful notetaking and by using the participants' own language when asking clarifying questions, writing the narrative and listing evidence for themes.

4.7 Ethical Considerations

The participants signed a consent form and were informed about the thesis, interview process and offered multiple opportunities to ask for clarification or other questions. At any point, they were able to withdraw from the study and their data would be deleted. Interviews were audiotaped via zoom and stored on a password protected computer. They will be deleted after 5 years. Given that these were stories about the participants' lives, they may have experienced uncomfortable emotions and therefore, the interviewer was cautious to allow them to lead the conversation and to talk or not talk about subjects that surrounded their story of mathematical positivity as they chose.

4.8 Limitations

As this is a qualitative study, it has its strengths and limitations. With an n=6 no broad statements can be made about generalized processes. Any conclusions should be considered only exploratory and suggestive. The demographics also self-selected and were gathered from the researcher's extended network. This meant that they were not representative of any particular population, and were not otherwise controlled.

Though specific details were not requested, according to the interviews, 5 out of 6 had professional jobs: 1 for the government, 1 employed as a professor, 1 was in accounting and 3 in STEM. 5 out of 6 had postsecondary degrees though 1 was from college, 2 had a masters' and 2 had a PhD. The master's and the PhDs were not uniformly in the same field. They ranged in age from 20s to 60s. 2 were female and 4 were male. 2 lived rurally and 4 had once lived rurally. The strongest similarity was that the majority of participants were highly educated, and this has an

impact on the results. Another striking detail was that 4 of 6 had at least one teacher as a parent though these teachers varied in the grade level, subject taught and type of teaching (Montessori, unknown). Even in this educated parent group, two of the teachers who were mothers, were partnered by fathers who had dropped out of high school so this was not a uniformly educated parent set. These fathers were, incidentally, credited by their children for passing on important mathematically applied lessons.

It is difficult to generalize even about the fact that there were more males than females in this study as each male and female is pretty unique with such small numbers. It is perhaps presumptuous to even say whether or not they were economically well off (despite having apparently high paying jobs) disabled, or considered themselves part of a particular minority as this information was not recorded. The fact that they were all Canadian would make them very broadly geographically similar though one was Francophone, one was from the East coast, and two spent time in British Columbia. It seemed that all of their mothers were employed but one was a single mother at the time. The details they included were of their own choice so though one could draw upon the literature to state that women might have higher mathematical anxiety or that cultural groups may have different mathematical values, there are too many unique variables to make one person represent a whole group. So though small numbers are a limitation, they are also a liberation as each person's story is uniquely their own.

5.0 Participant Portraits

5.0 Data Overview

Initially there were seven participants however, it was discovered, at the time of one of the initial interviews that one participant was a teacher in high school so they were informed that their data could not be used and their data was removed. This left six participants, each given a pseudonym. Their narratives are all named after an important theme in their stories.

Ray: Male, 50s, accountant - An Honest Player

Paul: Male, 50s, environmental physicist – Patterns of Planets

Claire: Female, 60s, owner of a retreat centre – A Relationship with Mathematics

Martin: Male, 40s, professor in sociology – Guided by Curiosity

Hanna: Female, 20s, works in R&D – A Drive to Understand

James: Male, 40s, data literacy lead for government– Success through Support

The six interviews ranged from 30 minutes to 80 minutes. The original co-created stories can be found in Appendix 4. Below is a summarized and contextualized version of this first layer of an analysis.

5.1 Ray: An Honest Player

<u>Data Gathering Contextualization</u>

Ray is a man in his late 50s with a friendly, jocular manner. He speaks slowly and deliberately. He has several adult children, some young grandchildren and lives with his wife in the Ottawa valley where he owns an accounting business. During the interview, it became clear that the

themes that he wanted to discuss were his identification as someone who had ADHD and the importance of "real-life" applications to mathematics in the classroom. Not only did he frame his stories in such a way in the interview, he also said that he wanted to send an extra story, by email, about an experience that his son and him had at the end of his son's schooling that directly involved Ray and an accounting teacher. His corrections to the narrative were minimal, and mostly factual. Upon hearing the story, he said "Who is this guy? I want to meet him."

Narrative Synopsis

Early on in Ray's schooling, he was more interested in reading and writing because they were more creative. He also liked history and geography and "describes himself as a big reader," saying that "reading opened up new adventures for him." On the other hand, he describes his early experiences of math as "horrible," with the exception of "his grade 4 teacher" who played a game with multiplication flashcards. "He used to imagine that he was competing against the many smart kids in his class."

He does not talk much about his family but said "his father was the first person to go from blue to white collar" and that "his parents did a lot of volunteering including starting a dart's league." This is when things began to change for him in mathematics as "sometimes someone wouldn't show up so Ray would sub in." He kept score and there were multiple types of games, which helped him practice his computational skills or as he said "Like as quickly as they're throwing darts, you're doing addition and subtraction." It got the "competitive juices flowing." The change in his identity from being "horrible" at mathematics was seen by the adults on his paper route as they were impressed that he could "bang off the numbers" on how much they owed him.

Ray feels that if he were to go to school now, “he would be classed as an ADHD student” He was “easily bored unless what was happening was going at 400m/h” and he “would challenge the teachers if they couldn’t hold his attention by disrupting the other students and being a class clown.” During much of his schooling, “he was more focused on his professional hockey career, playing until the end of the Junior A division in Ontario.”

He said that “all this changed when, in grade 11, he decided to become an accountant for ‘some odd reason’.” His accounting teacher was “amazing and patient” and “put a real-life spin on things.” He felt that this teacher had “come back to school to give back to goofballs like him.”

In college, “Ray took a double major in computers and accounting” saying “numbers are very important there.” It is here in the story that you can hear him talk about his positive relationship with mathematics. “He loved quantitative methods like statistical analysis, stating that he couldn’t get enough of it… ‘You could dig into massive amounts of data and come up with certainties you know within a small mean. If you do this, then this happens.’”

This led to a career in accounting where he identifies the value of integrity as most important in mathematics. Having owned an accounting practice for 33 years, “his name is on any number that goes out and he has a sense of pride in being able to give accurate, efficient services for customers who are looking for expertise.” He emphasizes his attention to detail and contrasts this with his early working experience where a big accounting firm said “it wasn’t worth their time if the error was within $500 dollars.”

After the interview, Ray sent a second story about helping his son with a grade 12 accounting assignment that was imminently due. Ray helped him complete it, as they would in his accounting firm, though “his son kept saying that, that’s not how his teacher would do it.” When his son received a C, he met with the teacher. “Two things happened. They slightly

reworked the entire assignment so that next semester the class was working with actual real-world situations and calculations" and the teacher offered to change his *son's* grade. Ray declined the second part but was pleased about the first.

Lastly, Ray describes math as "playful, huggable, and you can do so much with math. You can play games. You can make structures. It's all around and in everything."

5.2: Paul: Patterns of Planets

Data Gathering Contextualization

Paul is an environmental physicist somewhere in his 50s and tends to talk with a smile. He is from Nova Scotia and now lives with his wife and two young adult children in Ottawa. He was recovering from an injury so was laid-up when we had our zoom interview. His answers tended to be short and the first interview was 40 minutes long. I identified with many of his areas of interest and was equally fascinated by his life, which was quite different than mine, as he stayed in one place for his entire childhood.

Throughout the interview, he described a clear division between math and non-math people, as well as how mathematics was more clear-cut, giving more binary answers. He also emphasized the different supportive roles of each of his parents with his teacher mother encouraging his learning through things like games and his artistic father's love of patterns and the environment for inspiring his professional life. He made some basic corrections during the readback of the narrative and commented on how it reminded him of a eulogy.

Narrative Synopsis

Paul said that “he’s always found patterns in math interesting.” His earliest mathematics memories were of his teacher mother who was “a big cribbage player.” Though he was “a solid 50s student” in both English and French, he was “nominated by his class to be the student who would compete in math competitions” in grades 5 and 6. This “surprised him because he didn’t realize that other people were having challenges with it.”

In general, “he did not find math frustrating though it could be confusing at times such as trigonometry with its cosines, tangents and the mysterious importance of right angles,” saying “it wasn’t just simple linear patterns anymore. Suddenly it was 2 and 3D patterns. It became more challenging at that point.” This is later echoed when he mentions the challenges of spherical geometry in his bachelor’s honour thesis in astronomy as it was “difficult to visualize the orbits and get the math to work.” Importantly, he said that he just kept working the problem until “eventually he figured these things out.”

Both his parents were important in his schooling but he especially wants to credit his father who was a high school drop-out and worked as his school’s janitor. His father was “environmentally positive and got him to want to understand things around him.” He was interested in “oceanography or lake bathymetry” and they were “always going on hikes or fishing.” “His father would point out how things related to one another, such as patterns in the forest or how wind blew across the lake making waves.” However, “he would shut right down if you brought math into it.” His father expressed his interest through art as he was “always drawing with charcoal or whittling something.”

Teachers also feature as important. “He had a conflict with his 10-12 English teacher, ‘we never interpreted anything the same way.’” However, his English teacher during his bachelor’s

"brought him right back into it with 'just his willingness to say oh that's interesting.'" He also connected with his high school physics teacher who, instead of getting him in trouble for skipping class and smuggling out a large, concave mirror to project the solar eclipse on the side of a school, got him to teach the class about the astronomical event.

After his Bachelor's, he travelled to Australia for a while as "it was as far away as I could get and 'I wasn't going to the moon' so it would have to do." After a Master's in Astronomy, he decided "there's more opportunities looking at this planet from space then looking at other planets" so took his PhD in Physics and Remote Sensing leading to his current career.

For Paul, "'life is a bunch of patterns" to extrapolate. He describes mathematical positivity as feeling familiar and comfortable with the subject.

5.3 Claire: A Relationship with Mathematics

Data Gathering Contextualization

Claire is a woman in her late 60s with a big smile and an easy laugh. She talks at length and candor about her story and uses strong language when moved. She has lived in the Ottawa valley all her life, has one daughter and owns a retreat centre.

The interview process with Claire was long and complicated. She had a lot of story to tell and very specific needs about how it was told. This led to four interviews including a follow up where she submitted a suggested rewrite of her narrative that was mostly concerned with the specific details of her chronology but also contained a re-emphasis toward the struggles of being a woman in STEM. Her full narrative is quite long (Appendix 4.3) so the Synopsis below only gives a snapshot. Though her full narrative doesn't just focus on her mathematical experiences, this was the detail that she felt necessary to capture the context of her story.

For Claire, relationships and belonging are very important including that of her mother, her teachers and her partner, who has now passed. Claire is both drawn to mathematics but feels not particularly talented at it. She values achievement, which for her, is symbolized by the missed opportunity of getting a PhD.

Narrative Synopsis

"Claire finds math fascinating, particularly number theory as well as physics and mathematical ideas more generally." She talks about Cantor, Fermat, Gödel and Erdős. She emphasizes repeatedly that though she is interested, she does not feel "naturally gifted at the subject" and that "she needs more time with it then some people and would have done better in a supportive environment with good teachers who were interested in their subject."

She tells about her brother who she sees as "much more intelligent" than her but who likely had a "undiagnosed learning disability" and her mother who preferred her brother and with whom she had a complicated relationship. She feels that her mother was impacted by social expectations about womanhood and sacrifice saying that "her mother was 'always giving others the biggest piece of cake' and felt that "her mom expected her to be like her." Claire felt that "she was told that she was very intelligent and could do anything she wanted in life but covertly the message was that men were superior, were supposed to be given more opportunities and were automatically going to be more accepted in the world in ways that women were not."

Claire went to a Hebrew school when she was very young and remembers that "she loved to read" but that "she associated math with arithmetic and fast problem solving" which "she did not like, and felt she was not good at…" Though she had a brief connection with a grade 6 teacher who "…she loved," she "avoided school" and "describes herself as being depressed." At

the end of grade 8, she applied and was accepted into an all girl's private school. This school "saved her – intellectually and emotionally" and it was here that she "describes learning the rudiments of critical thinking and analysis." She also began to understand mathematics as more than just arithmetic. Importantly, she felt that the "teachers were supportive," and "many of the teachers were women." In grade 11, she had to re-enter the public school for financial reasons and in grade 13, she dropped out to go to art school. This is one of the many decisions that she regrets.

In the narrative, Claire describes in a lot of detail the many twisting paths that her life takes including a number of jobs and educational moments such as when she returned to Ottawa to take courses that would lead to her Master's in Experimental Psychology. It was here that she finally found belonging. During her Master's, she became pregnant with her daughter, and "had to leave the lab for the duration as she was having trouble walking." She also met her partner who was a theoretical physicist and a "natural mathematician." She wishes that she would have had more mathematical understanding so they could have had deeper conversations.

Though "she applied for and won a second NSERC for her PhD" various obstacles stood in her way including the need to "'grow up' and get a real job." She tried various paths afterwards including a teacher's diploma and "was not impressed by the practicums, saying she was appalled by the way that math was taught," feeling that there were far more 'inspiring' and 'engaging' ways to teach such as 'number puzzles' and 'applying real world problems.'

"Claire regrets not doing her PhD as she felt that would have made her a 'worthier' human being." However, "she says that it was harder for women, during her time: it seemed that, although you could participate in STEM careers, you had to be at the top of your class if you were a woman."

For Claire, mathematics can be “magical” and she thinks that “it must be amazing to be a really good mathematician,” and that “these truths are beautiful.”

5.4 Martin: Guided by Curiosity

Data Contextualization

Martin is a quantitative sociology professor in his 40s with a soft-spoken and friendly manner. He lives in Ottawa with his wife and child. The initial interview was 60 minutes long and took place over zoom. It was the only one conducted from my place of work with poor reception in an echo prone office. This led to some comical responses on both sides but did not seem to affect the overall quality of the interview. Despite the fact that he does teach quantitative methods and statistics at the University level, it was determined that this was not a problem for the selection process as he is not part of the common participant demographics of elementary and high school teachers. I found myself resonating with many of the stories and themes, such as his desire to go deep into subjects and the fact that he moved frequently as a child.

On the read back of the narrative, he expressed it was accurate to the way he felt and was curious about where the project was going.

Narrative Synopsis

Martin was born in Quebec and moved to British Columbia when he was in grade 3 and attended a Francophone school. “His earliest memory in math may have been in grade one and it was converting between Fahrenheit and Celsius.” He said that he “was never particularly math oriented but was also not scared of it. Any difficulties that he faced… came from learning math in a new language when he switched from Francophone to English school later on.” Both his

parents were teachers and his family valued "words and rhetoric." Every year, his father got him to do the "Dictée des Amériques," for example.

He does not have many stories from his early schooling but he does remember a teacher who taught him functions around grade 8. This teacher said that "the students didn't need to be afraid. He made it fun and he would say things like 'you know don't feel dumb, I had difficulties too.'" Martin attended a high school that he said was "ranked quite high" and had teachers who were "very inspiring." He got along with his grade 12 math teacher but he said that they would be having "conversations where the teacher would be talking about square roots and Martin would be talking about poetry."

Curiosity was a very important thru-line in his story. It is the thing that he most values in math, wants his child/ren to possess and thinks makes for good math teachers. During his Bachelor's of Sociology, he took a number of courses out of curiosity including "calculus and linear algebra." This worried his father who "was afraid that Martin would never find a profession."

He "fell in love with math in his PhD, through programming, as he was using a method called Gini decompression, which at the time had no existing program to calculate it. 'The formula was in sigma notation and I had to program it myself and it made me feel good to be able to just take a formula and translate it into a program.'" He finds "math and programming self-reinforcing."

Martin tries to bring his approach of deep learning and curiosity to his students, who he says "have often self-selected away from math, telling them that, given that they all see themselves as philosophers then they should accept that at the entrance to Plato's academy there's a sign that says 'Only geometers may enter.'"

He remembers a quantitative sociology professor during his undergraduate that could not answer his questions such as when he asked about “cross tables and chi square.” He was surprised that the professor seemed to have no interest. Therefore, he makes a point of trying to pique his student’s curiosity by connecting math ideas to other concepts in the social domain such as how Euler’s constant is about compound interest...”

Now “he wishes that he could take some courses in the math department” but recognizes that his schedule is very full. “It would really help me to be around people who were passionate about math.” He “finds math therapeutic” and says “that as a sociologist a situation can be read in a number of ways and there’s no way to deny someone’s experience. It is very subjective.” He has a “highly intellectual orientation” and “feels our society is very focused on fun and lifestyle but would benefit from more of a knowledge focus” that perhaps would “… allow them to think something that they have never thought before.”

5.5 Hanna: A drive to understand

Data Contextualization

Hanna is in her mid 20s and is employed by a hardware R&D team. She lives with her roommate in Ottawa. Hers was the last interview that was conducted. Like the others, it was on Zoom and it was 40 minutes long. She was easy to talk with if a bit quiet at the beginning. I identified with her interest in problem solving and general can-do attitude. Her revisions of the narrative from the interview were minor. Her narrative was dominated by themes of support, speculations on what makes for a good mathematics education and her own interest areas like space. As the youngest of the participants, she also had something to say about online education.

Narrative Synopsis

When it comes to mathematics, educationally and professionally, Hanna identifies it with problem solving. She describes herself as being "a pretty good student who needed to really understand the problem and was a perfectionist." "She values perseverance and curiosity in mathematics and sees a willingness to try new things as important." She credits this, in part, to feeling supported by her parents who were both able to help her with her school work. "Her father was a high school math teacher who could help her with STEM related subjects and her mother had a degree in history…"

She stated that "she 'never had a bad experience that would discourage me from learning…'" and contrasts this with "her friends [who] did not have this sort of support," speculating that perhaps their parents "… either did not know the material or did not have time to sit down and work through it."

Hanna did not have a time in the "early years where she found math challenging. In fact, she didn't really have to put in much effort as it came pretty naturally." Her earliest memory was playing a game called "Chip it" where chocolate chips were placed on a board after solving a simple math problem. "She figured that this game probably put her ahead of the curve in grades 1 and 2" and "It wasn't until grade 10 where some concepts became more difficult such as trigonometry," which she "often found hard to visualize" as "instead of being able to jump ahead to the answer… she really had to work through trigonometry problems step by step."

Online classes were also an area that she found challenging. She experienced them twice. Once when she chose to take a class online and the second time during the COVID pandemic when she was in her Bachelor's of Astrophysics. To this day, she does not feel solid on the topics that she was meant to learn online.

Generally, she had positive things to say about schooling and teachers and commented that both "her senior physics and math teacher were quite good. In particular, she really liked how her math teacher handled homework. All assignments and tests would be set at the beginning of the unit. Students could choose if and when to do them. However, at the time of testing, everyone had to hand in a particular homework assignment of the teacher's choosing." But if they got above a particular grade on the test then that homework assignment was not marked or was marked as 100%. "This gave Hanna a lot of flexibility and freedom on how and when she chose to do her work."

Hanna also "really liked her grade 12 physics teacher describing him as fantastic." In large part this is because of the subject matter such as learning "how things interact at a very small and a very big level." Hanna was fascinated by space from a young age. "She would watch documentaries with her parents on rovers, probes, telescopes, nebulas and even a whole miniseries on quantum mechanics. The latter she found absolutely fascinating."

Though Hanna said "she has a wide range of interests that include music, drawing and painting," she ultimately went into astrophysics with a minor in music. "This led to her job on an R&D team, which, in practice she describes as quite close to engineering, saying, "'I do a lot of just physical work with our systems and their circuit boards… If something goes wrong, I have to figure out where it's wrong and if there is an issue with the design."

"She finds parallels between how she approached challenges in the past and how she approaches challenges in her job now" such as "a couple really weird failure modes that I've had to just kind of go through and slowly just eliminate piece by piece by piece until you kind of just see something, and then you're like, wait a second could this be, for whatever reason, the problem?'" She describes her job as frustrating but that frustration is "part of the process,"

attributing her "comfort with frustration" as "'if I do work hard and I put in effort, eventually I will figure something out."

5.6 James: Overcoming challenges through support

Data Contextualization

James is a man in his 50s with a throaty laugh who talks in long, articulate sentences. He lives with his wife and two children in Ottawa, and works for the government as a data and statistics communicator. He has strong ethical views on why mathematics is important that intersect with childhood influences on equity but also his own learning in history and experiences with the pandemic.

I found myself agreeing with him and identifying with many of his references from the Youtube Channel 3blue1brown to Star Trek. Underlying the entire story was the importance of his very supportive family in his own success and how he extrapolates that into mathematics communication and education.

It was very important to James that I include information on his siblings' learning disabilities as well as include the details of his families struggles such as how they lost their jobs in an economic downturn in the 80s. This was the third interview where the participant wished to put in more detail than was originally included in the narrative.

Narrative Synopsis

James is from lower, mainland BC and grew up in an agricultural area. "His mother went back to school when he was still young" to become a St. Nicholas Montessori specialist teacher. He felt that being around Montessori materials was important for his early mathematics learning and

interest. “His earliest memory of math was getting lost in the moment of counting Montessori glass beads. He just wanted to know how high you could count on them.”

His mother, recognizing his “struggles with social interaction, placed him in the 4H[2] club – an agriculturally based club where young people engage in activities like raising animals for show – at a young age, in which he participated from the ages of 9 to 17. He felt that working with animals helped him bridge that social gap and as a bonus ‘tracking progress was just numbers and data.’” His two older siblings had “severe learning disabilities” so the “scaffolding” his parents set in place “helped him excel through elementary and high school, giving him skills and tools that helped him learn things at the right time.”

“His father also helped influence his mathematical identity.” He was a high school drop-out and artist but James remembers that he restored cars for a living and ‘you measure twice then a third time before you cut. Precision and accuracy are important.” His dad also insisted that James “‘never rest on his laurels. He didn’t want me to waste my talent.’”

His first big upset in his mathematics education came when he did not make the top 24 to get into an advanced mathematics class. His family supported him and reminded him of a quote from Star Trek when captain Picard says to Data, “’You can do everything right and still fail.’ Failure, James learned from his family, or doing something wrong and asking for help is not a sign of weakness. In fact, it is a sign of strength.”

“This attitude served him well” when he was “attaining his International Baccalaureate.” His chemistry teacher was unprepared and his physics and calculus teacher was a “‘a bit kooky and he would spend half the class talking about conspiracy theories.’ He also wasn’t very good at teaching the higher math.” The IB math students started working together, often relying on one

[2] 4H stands for Head, Heart, Hands, and Health. It is a STEM and agriculturally based activity.

student who had the economic resources to afford a tutor. James ended up being the only one in this group who managed to get his IB. In contrast, his history teacher made him feel "encouraged and seen." Given that "James's family came from a leftist perspective…" and his classmates did not, "he could have easily felt like the class was ganging up on him."

James ended up taking his Bachelor's and Master's in history. However, "pursuing history was a challenge as he had undiagnosed dyslexia" and "chose not to get it formally recognized because he said that he was told that if he did, he would find himself restricted from certain kinds of courses in University." His master's had a statistical component and eventually led to his job, which he describes as "math communication" otherwise titled as "Data Literacy Lead for a governmental organization."

In this job, he says he helps people "break down ideas." It is about "helping people to use data. To not just think in terms of numbers but about connections between those things and the strength of those connections can be defined numerically." He also uses the word trust as in "building trust between people and numbers so they can make strategic decisions and accept critique." In fact, mathematical positivity to James is about "being comfortable with numbers and data." This he relates to social risk as in how people were communicated to during the pandemic saying "it was hard to see history repeat itself."

6.0 Findings

6.1 Turning Point Tables and Emergence of Mathematical Positivity

Below are the participants' turning point tables and summaries of the Emergence of Mathematical Positivity, which includes their definition of mathematical positivity. For an example of the process of how these were developed, see Hanna's data example (Appendix 5). Three participants had relatively early and stable development of mathematical positivity (James, Paul and Hanna), two started out as mathematically negative (Claire and Ray) and one was ambiguous or even neutral (Martin). All experienced some level of threat to their mathematical positivity and those who were initially mathematically negative experienced turning points that encouraged mathematical positivity.

Table 6.1a: Ray's Turning Point Table and Emergence of Mathematical Positivity

Event Chronology	**Change in expectation or value**	**Turning points Outcome**
Young child 'Horrible' at math Liked reading	No	Mathematically negative
Paper route Darts Improvement of skill	Math skill improved Values of competition met	Encouraging
Early teens Easily bored Competitive hockey	Unclear but not described	Unclear
Late teens Accounting classes The 'right' teacher	Values met: the 'right' teacher, useful math	Encouraging
College Hardworking Interested (coding, statistics)	Values met Expectations: good at math	Mathematically positive
Work as an accountant First job	Values of integrity not met	Threatening Switched jobs
Work as an accountant Has own company	Values of integrity and usefulness met.	Mathematically positive

Helping son with his accounting assignment Changing curriculum	Expectation: good at math Values met: teacher saw value in 'real-life' math, took Ray's suggestions	Mathematically positive

Ray's adult mathematical values include integrity, hard work, competition and utility. Educationally valuable teachers are relatable, patient, and use real-world applications. He defined mathematical positivity as being 'confident in my mathematical abilities.' As can be seen, he started out as mathematically negative but became encouraged through improved skill and support from adults. By college, he appears to have become mathematically positive, experiencing a steady increase in the usefulness of mathematics.

Table 6.1b: Paul's Turning Point Table and Emergence of Mathematical Positivity

Event – Time	**Change in expectation or value**	**Turning points Outcome**
Younger years Card games with math	No	Mathematically positive
Elementary school Math competitions	No	Mathematically positive
High school English teacher critical Physics teacher positive Trigonometry is frustrating	Possibly Expectation: Math is easy but trigonometry is more difficult	Threatening Outcome: Worked through it until he got it (perseverance)
Undergraduate (astronomy)	Possibly Expectation: Math is easy but Spherical geometry is difficult	Threatening Outcome: Worked until he figured it out (perseverance)
Travelled to Australia	No	Mathematically positive
PhD in remote sensing	No	Mathematically positive
Environmental Physics	No	Mathematically positive

Paul's mathematical values were not spelled out but when asked he said, "life is a bunch of patterns." He also seems to value family based on the focus and themes in his narrative. Paul defines mathematical positivity as 'being familiar and comfortable with math in everyday life.'

Mathematical positivity is an emerging default from childhood, perhaps encouraged by his mother. Threatening instances were minimal and both involved geometrical visualization, which he worked through until he figured it out. This would suggest perseverance to be added to Paul's mathematical values. Importantly, as he emphasized this, his mathematical positivity is entwined with environmental positivity as his father was the one who first showed him the importance of patterns in nature, which Paul later applied to mathematics.

Table 6.1c: Claire's Turning Point Table and Emergence of Mathematical Positivity

Event – Time	**Change in expectation or value**	**Turning points Outcome**
Early years Liked reading Didn't like arithmetic Lost her father	Expectation: math is arithmetic and she is not good at it	Mathematically negative
Public school Nice grade 6 teacher	Value of belonging met Expectation of math unclear	Unclear
Public School Grades 7/8	Value of connection not met	Unclear
Private school Academic sophistication Not so much science Math: logic and ideas	Value of belonging/connection, intellect and thinking met Change of expectation to math is logic and ideas	Mathematically positive
Grade 11/12 Liked algebra Not so much geometry but worked on it	No	Mathematically positive
Dropped out of grade 13	Value in accomplishments not met because exhausted	Threatening Outcome: Changed path
Art school Very young Northern Telecom	Not valued because didn't have a higher degree	Threatening Outcome: Changed path
Undergrad Enjoyed classes Belonging	Values of belonging and intellectual curiosity met Higher degree/education met Expectation about math: hard but interesting	Mathematically positive

Teacher's college	Value of intellectual curiosity and success not met Expectations about math: fun, interesting, worthwhile	Threatening Outcome: Changed path
Upper Ed NSERC Had a child Met a partner	Values met: belonging, interest and intellectual connection Expectations met	Mathematically positive
Did not do PhD Partner is a theoretical physicist	Values not met: success Values met: connection Expectations that math is hard but interesting unchanged	Threatening Resolution unclear Disquiet in wellbeing
Later life Interested more and more in mathematics Friends who like math Mathematical experiences	No	Mathematically positive

Claire values intelligence, abstraction, and accomplishment, such as in the form of higher degrees, as well as connection and belonging. Educational values include support, mathematics as wonder, and to a lesser extent application. She starts by describing mathematics negatively with mathematical positivity emerging in high school in a supportive school environment where she was introduced to new ideas. This is potentially threatened several times when the value of success is not met. Her definition of mathematical positivity was fragmented in her interview but it included thinking mathematics was fun and useful along with mathematics allowing for special insight.

An important theme overlaying her narrative was of missed opportunity because she was a woman both in the sense of being made to feel that she had to be better than average to succeed but also because of the responsibilities of motherhood conflicted with her interests in higher education.

Table 6.1d: Martin's Turning Point Table and Emergence of Mathematical Positivity

Event – Time	**Change in expectation or value**	**Turning points Outcome**
Early years Culture and Rhetoric School in Quebec	No	Unclear
Move to BC Math is easy Inspiring teacher	Expectation: math is easy Value: fun, supportive met	Mathematically positive
English as a second language Highschool with good teachers Learning programming	Expectation: Math is easy Value: knowledgeable, accomplished teachers Challenges (programming), intellect met	Mathematically positive
University Frustration with stats teacher	Programming is useful. Value: challenge, curiosity, accomplishment, intellect Values not met: curiosity	Threatening Outcome: Solidified importance of curiosity
PhD Programming is useful Calculation	Expectation: Math is interesting. Programming and math are useful. Values met	Mathematically positive
Teaching statistics Parenthood	No	Mathematically positive

Above all, Martin values curiosity and deep thinking but also challenge and accomplishment. He defined mathematical positivity in reverse by saying that it was someone who was not afraid. He also finds mathematics (and programming) useful. Good teachers are fun, successful, inspiring and have deep knowledge about their subject. A good society is more knowledge oriented. Mathematical positivity seems to have strengthened as he grew older as mathematics became more useful and interesting to him.

Table 6.1e: Hanna's Turning Point Table and Emergence of Mathematical Positivity

Event – Time	**Change in expectation or value**	**Turning points Outcome**
Early life Chip it math game Mathematical skill	Expectations: Math is fun Value: Support of parents	Mathematically positive
Grade school Fascinated by physics How things worked	Expectations: Math is easy Problem solving is interesting Value: curiosity, understanding the universe	Mathematically positive
High school More complex problems Working the problem	Expectations: Math is easy however this was not met Value: persistence, curiosity, autonomy, understanding the universe	Threatening as math was not as easy Outcome: problem solved through persistence
Online classes Too simple, not sufficient to explore problems Subjects never solidified	Expectations: I understand math No change in value	Threatening Online education was not sufficient Outcome: online education is the problem
University Astrophysics and music Very interesting	Expectations: I understand math and find it interesting No change in value	Mathematically positive
Job in R&D Weird failure modes Work the problem	No	Mathematically positive

Hanna values support, persistence, curiosity, autonomy, and understanding the universe. She defined mathematical positivity as not being afraid. Though this narrative seems initially to be entirely positive, upon closer examination there are several threats to her mathematical positivity that were overcome through the further solidification of the values of persistence, and by the recognition that it was the mode of delivery that was the issue. Despite encountering issues, she never describes her experiences as negative.

Table 6.1f: James' Turning Point Table and Emergence of Mathematical Positivity

Event – Time	**Change in expectation or value**	**Turning points Outcome**
Early Years Montessori influenced Counting the beads Siblings with learning disabilities Parents supported him	Expectation: Numbers are captivating Values: unclear	Mathematically positive
Elementary Montessori school Dad taught him accuracy and precision	Values: independent learning, accuracy and precision, support	Mathematically positive
4H	Values gained and met: usefulness of data	Mathematically positive
Did not get into advanced math class	Expectation: He is a math guy not met Value: independent learning, precision and accuracy, hard work, equity, support	Threatened Outcome: overcame with familial support
High school International Baccalaureate Bad teacher, poorly prepared No help. Unequal resources	Not clear what his expectations were regarding IB Value of equity and teacher support not met but value of independent learning and hard work compensated	Threatened Outcome: overcame because he was a Montessori trained kid who recognized the injustice of inequity and could work hard
Only kid to graduate with IB in his group	Value of reaching your potential met: graduated with IB	Mathematically positive
History Degree With statistics Hid his dyslexia	Values met: usefulness of data, living up to your potential	Mathematically positive
Government Job Bringing math alive Helping people trust and be comfortable with data	Expectation: He is a math and data guy Values: comfort with mathematics and sharing it with others not only was formed but met	Mathematically positive
Pandemic Poor data communication	Values not met: equity, communication	Threatened on societal level Outcome: solidified his belief in the importance of good data communication.

James' values include support from family, independent learning, precision and accuracy, hard work, reaching your potential and not giving up, equity, utility of data and the importance of clear communication. He defines mathematical positivity as finding data useful and thinks that comfort with numbers is a social right that should be shared to mitigate social risk. He starts his narrative as mathematically positive and describes a few threatening, potential turning points including when he does not get into an advanced mathematics class but overcomes this with the support of his family. Also, when he has a "bad" teacher during his International Baccalaureate, he overcomes this with his capacity to teach himself – a skill that he learned from Montessori.

6.2 Cross Thematic Analysis

From the individual data analysis, various themes emerged including a need to distinguish mathematics from other disciplines, which are collectively called What is and is not Mathematics and is found in section 6.3. In particular, mathematics was compared to subjects that required a lot of reading. This distinction was made very early in schooling. For Ray and Claire, it related to a preference for English but for Paul and James, it was a struggle to acquire literacy skills because of undiagnosed dyslexia. Furthermore, mathematics is defined as 'less interpretative' and more logical with several participants expressing that this was comforting or even therapeutic. Lastly, engagement with mathematics and data is described outside of the classroom such as using computation in card games, data analysis in 4H and score keeping in darts. Significantly, these out-of-classroom mathematical engagements often occurred early in the participant's life and helped to solidify skill or exemplify their importance. The exception to that was coding or computer programming, which was encountered later on in the participants' lives.

Parental and teacher support also emerges as an important theme and will be discussed in section 6.4. For several participants – James, Paul and Hanna – parental support is an important, highlighted theme that helps explain the development of their mathematical positivity. Since we were discussing their mathematical lives, and many people associate mathematics with school, there were many stories of teachers. These were not always STEM or mathematics teachers but were nevertheless included as they provide further support for Hill's core value of Relationships, and because these stories often involve turning points or potential turning points in the participants' narratives. Two smaller themes also emerge in sections 6.5 and 6.6 and these are Neurodiversity, which strongly intersects with the previous two themes, and Perseverance.

6.3 What is and is not mathematics

6.3a Mathematics versus Literacy (Paul, James, Ray, Martin, Claire)

5 out of the 6 participants mentioned ideas of literacy versus mathematics in some form or another. They all shared the fact that there was a distinction but differed on the importance of that difference. For Paul and James reading and writing were difficult with Paul saying, "I was a solid 50s" in English and French, or how James said,

> James: It was- it was a very like I focused on the math materials and using the test tube division materials a lot, and trying to figure out teaching myself how to do square roots off of the Montessori materials, the science materials were the- another area I focused in. So I was doing brain science. This is all in grade four. Yep, and yeah, I loved it. I could have focused more on English. I probably should have.

There was a sense that mathematics was a refuge where Paul and James could be successful. This was not the case for Claire and Ray who read a lot when they were young but had yet to

develop any interest in mathematics. Ray felt he was "horrible" at mathematics and Claire said "grade school, you know, it was like the sort of addition, subtraction. I hated math." However,

> She loved to read. She took out multiple books [per week] – science books, autobiographies and stories – from the library and this is where she credits much of her early learning. … In those early days, Claire associated math with arithmetic and fast problem solving. She did not like it, and felt she was no good at it.

For Martin, it would seem that all school was easy but there was still a distinction between mathematics versus literature and culture.

> Martin: …My parents were not interested in math ah.. I was more I would say culture oriented and literature to be honest um.. My dad was one of um like one of the few like one of the only- he was from six children and he was the only one who did 'l'école classique' you know at a time when [intake] Quebec was very catholic. [intake] and there was these great schools where they would learn Latin and Greek though that always impressed me so I was never particularly math oriented but I was never scared of it or particularly bad at it.

He also describes talking with his Calculus teacher by saying 'we would have discussions but I would I would be talking to him about poetry and he would [laugh] be talking to me about square roots or something.' He also describes his sociology students as inhabiting this distinction

> Martin: … I was talking to students the other day and they told me that you know a lot of people choose sociology because they don't want to **do** math so
>
> Telsing: M-hm
>
> Martin: [intake breath] so there's some self-selection. Obviously, this isn't going to the be the place where they're actually- this the kind of place where- where [intake breath] where people seek refuge from math [laugh]

But then he tries to break down his own boundary

> Martin: And um at the entrance of Plato's academy, you know this, ah it was like basically ah a sign that said only geometers may enter. You know.
>
> Telsing: Mm
>
> Martin: So and I- I tell them you know that sociologists learn to value being very philosophical and abstract but I tell them that's a bit of a joke because philosophy also involves math and you know like when I meet philosophers they'll talk to me about non Euclidian geometry and stuff and ah um see that's philosophy. So I find sociology only values one part of it. And like that.

6.3b. Mathematics is less interpretative (Ray, Paul, Martin, James)

Mathematics, and by extension STEM subjects, being in opposition with art or literature subjects, is a theme pointing to how some of the participants saw themselves as inhabiting two realms. For Hanna it is "I have a very wide range of interests, because I was also in music… I really enjoyed art, like drawing, painting and all that kind of stuff, um but definitely like math and physics an- science as well" and Claire talks about when she says her mother supported her in the arts but not in STEM and how Paul referred to some things as being "English questions" meaning interpretable in different ways.

Ray, Paul, Martin and James all referred to something about the exactness of mathematics that they found comforting, in opposition to the openness of more literacy-based subjects, whether they were called 'English' (Paul) or 'Culture' (Martin). For Paul, it is a way to live.

> Paul: [sucks in] [click] um.. probably just the way I look at life in general is just a bunch of patterns.

Telsing: mm-hmm

Paul: and to me that's mathematical

Telsing: [overlap] Yeah

Paul: because you figure out what direction things are going in or what's the likelihood that somethings going to happen so that's how I attack life/

and

Paul: [laugh] I think that's one thing I really like about math more than- than thee ah more social sciences I guess. You go from more yes no-s to maybes.

Telsing: In social sciences right

Paul: Yeah in social sciences you get more interpretation of what feels right and then that always bugged me more when I was younger than it does now but the idea that there is no right or wrong answer

Similarly, for James where 'it gives you a concrete way of expressing reality.'

James: Um [pause] and [intake] [outtake] even in probability- even when count probability and- and- and a range of possibilities- a scope of possibilities [intake] as being a correct answer- a possible answer [intake] um you can also clearly see when things are not an appropriate direction to go.

And for Martin it is 'therapeutic'

Martin: So ah the subjectivity of it is ah sometimes um.. [click] like bad for the mental health in a way because [intake] like wh- who am I to say like how people should feel in the situation or something but you don't get that with math. It's just like here's the answer/.

6.3c. Mathematics as Useful vs Abstraction. (Ray, Claire)

Ray valued mathematical integrity because it connected to "…accurate, efficient service to customers…" This connects with the above idea of mathematics giving an exact answer but it also has to do with the theme of utility and usefulness. Not only does mathematics have utility in being explanatory but *good mathematics*, and *good mathematics teaching*, is useful in real life.

> Ray: No but he was very well liked. He his ah manner of teaching – he put a real-life spin on [pause] why we were sitting down and why we had that piece of paper in front of us and on the piece of paper was a real-life situation that we were going to do for the rest of our lives.
>
> Telsing: Right
>
> Ray: And that would have to do with a lot of personal finance, personal taxes and sure enough it's something you know on a daily basis. If it's budgeting. If it's ah figuring out loans, you know mortgage.

And later in his Story 2 when he talked about helping his son with his accounting homework and then later helping the teacher rewrite his curriculum.

> Two things happened. They slightly reworked the entire assignment so that next semester the class was working with actual real-world situations and calculations and two the teacher offered to change his *son's* grade on the project, heavily discounted because though the result was correct, it was not shown in the correct format. Ray objected to the change in grade because his son had left the assignment until the last minute.

And it can be seen in Claire's description of how one should teach mathematics "in context, in situ"

> … um you know, and I came along when they started new math, whatever it was, and they thought maybe this was a better way to teach but, you know s- look- looking [pause] all these years like all these years later, looking back, I think they did a piss poor job of teaching math. It's a wonder anybody ever learned anything. [intake] I think there's wonderful ways to learn [pause] math, like in context, in situ…

So-called *good mathematics*, that is applied to real world problems, is in contrast with lofty abstractions, which is interesting because Claire also really values these headier interpretations of mathematics

> Claire: it's like you have- the human mind does have the ability to see beyond the mundane into something like these relationships that are [pause] like in a way really profound. And and I don't know it's like I hate to think that lousy math teaching for example we deny people the ability to [pause] um experience some of this because it's far more [pause] [click] to me it's far more grounding in the sense that is m- that the world does have some meaning that we can apprehend [intake]

6.3d. Outside the Math classroom: ways into math (Ray, Paul, Martin, Hanna, James)

An important theme that emerged from the data was that of influences outside of the mathematics classroom and how these influences built mathematical skill, and interest in subjects that required support from mathematics. For instance, in the days of very early schooling or even preschool, Hanna described the game Chip It where she practiced basic computation, which she felt probably "put her ahead of the curve." Likewise, Paul credits his mother with helping his math skills, intentionally or not, as "All the games we played were very

math oriented so a lot of card games. A lot of ah games where math was or patterns were significant."

For Ray and James, it occurred later, in grade school. Ray, "Ah… my math was **horrible** um.. ah.. until I started playing darts/" and James notes that

> James: My mother was a teacher retired relatively recently, I guess, and she fed my sort of passion for understanding the world through science and math. And she got me involved in activities that gave me opportunities to use math, even if it wasn't something at face value we thought might be the case. I was put in 4H at a young age, which is an agricultural youth group. And the reason for that is I struggled profoundly with social interaction, right? Okay, go figure, someone who likes numbers is- struggles the social side of things. So that was the intent. But what helped me bridge that gap was a big part of working with the animals and- and tracking progress was just recording data and numbers.

Ray, Paul and Martin mention computer programming as significant with Martin drawing an arrow toward his interest in mathematics.

> Martin: [pause] Well I- I hmm… I suppose um when I started my PhD I chose a subject that would involve a lot of stats. And I do remember falling in love a little bit with math in some ways because I had to do a lot of programming with stats and I- I enjoy that.

and

> Martin: So I think in a way programming made me ah was a stepping stone to make me love math again because I- I like programming a lot and ah what do you program well you mostly program kind of like math equations so the two were self-reinforcing I would say.

and

Martin: My dad was not math oriented but he did teach me programming and that was the great thing about parents who are teachers. We always had access to computers.

Lastly both Paul and Hanna have degrees in space science – physics & astronomy and astrophysics respectively – with Paul saying that he was interested in "…Physics, chemistry. Anything related to planets. I was interested in planets" and Hanna saying "Yeah, I really liked space from a young age." Claire also shared an interest in science, "I always assumed that I was going to grow up and go and study science." Later on, she would use mathematics – "Fourier transforms" - in her experimental psychology Master's.

6.4 Support inside and outside the classroom (James, Paul, Hanna, Claire)

6.4a Parental Support

One of the most important of Hill's core values and themes that emerged from the narratives, with all the participants in major and minor ways, was that of Relationships in the form of parental or teacher support. This is not a surprise given how important these people would be in the lives of the participants as they interacted with mathematics during their younger years. In particular James, Paul and Hanna mention parental support as being important to their success as students. For James, it is a point of continued emphasis such as "…of the resources that were available to me most important was the reinforcement from my parents that just because you don't succeed, it doesn't mean you can't come back later and try again." It was also about creating a prepared environment such that "I was young enough that I was absorbing all of these tools that they were given" in reference to his older siblings but also, he mentions the influence of having Montessori materials at hand as in,

James: I remember before she became a teacher, when I was in preschool, as she was studying to become a teacher, she would take each of us to the - to UBC with her. Sure, okay, and involve us in that. And I remember learning how to do multiplication three dimensional like doing volume calculations on volume using Montessori materials.

Similarly, for Paul, who struggled to read, he said that he "had usually read the books before we got assigned in class just by pure coincidence. My mother might have set that up. I don't know. [intake]" and it was his "father who got me wanting to understand things around me." Despite his father not being interested in math, "it was more ah my appreciation of math in a more environmental context was more driven by him."

Hanna also highlighted the importance of her parents support saying,

Hanna: Both my parents, have… college or university degrees so um… so when I was, like, learning stuff in class, if I **did** have trouble, I could go home and they would sort of help me through it. So I never really had a… **bad** experience that would… discourage me from… learning about it and that 'I just grew up… with a lot of math ah in my household.' She contrasts this with her friends whose parents perhaps could not be as helpful so "it was more frustrating for them to go in and have someone… and I think it was just the person who was trying to help them was… also getting frustrated because they weren't understanding." Parental support is also an important theme with Claire but in the negative as she did not feel that her mother really supported her ambitions in STEM.

6.4b. Good versus bad teachers, Good versus bad teaching (Claire, Ray, Martin, Paul, James)

Many value statements were about teachers and teaching, signposted by words like *good* and *bad* in the interviews and followed by descriptions of how they were good or bad. Though the teachers mentioned, were not always mathematics teachers, they were always vital to explaining the participants' draw or repulsion toward mathematics and often featured in their turning points. This is similar to the parental support as it was not always directly about mathematics but both directly and indirectly impacted it.

For Claire, Ray and Martin, it was their teachers rather than their parents who were central to the value of Relationships. For Ray, most teachers could not reach him "And so if they couldn't hold my attention in the way my brain was wired, I would disrupt the class and become a problem student because I was ah distracting the students [intake]." He went into accounting he said "or some odd reason" but upon asking for clarification he said,

> Ray: It was the right teacher. I had this ah-ah one guy. He w- he was a younger teacher [intake]
>
> Telsing: [overlap] oh
>
> Ray: in our school and ah and um I didn't get much [pause] um.. as a student I would challenge the teachers if they couldn't hold my attention and this one particular gentleman Mr. X was amazing.
>
> Telsing: Right
>
> Ray: He was patient and um [sniff in] I went okay there's something to look at here. This is cool. This is cool. [sniffs in]

And later elaborated by saying that they had an affinity with each other.

> Ray: He ah he was just very ah patient I- I'm going to go out on a limb, I believe he saw a lot of himself in me. And he had finished his schooling ah attained his accounting ah degree and came back to school and teach to kind of give back to ah goofballs like myself.

For Claire, the value of Relationships meant being seen and belonging, or even being lost such as "I don't know. I just felt so out of place. [pause] I just always felt like I didn't belong there. I couldn't figure out" versus

> Claire: But I think, you know, this was like going to an academic high school, and they used to, you know, we got tests, like, every week, and they would read our marks out and- and it wasn't like we were being punished. It was like they paid all this attention to us. I just disappeared in the crowd before that, like I didn't exist. [intake] And it was the first time somebody paid attention to me and really, like, they really, they were wonderful. Like, we got so much support, like the ex- like we had a lot expected of us,
>
> Telsing: Mm-hmm
>
> Claire: and we were pushed, and we learned how to do stuff. And [pause] I mean, as I said, I think those guys saved my life. I don't think, I don't know what the hell would have happened to me if I hadn't gone there, but I would never survived a regular high school if I'd gone straight in.

For Paul, good teachers heard him – "I don't know, just his willingness to say oh that's interesting. I don't understand it but that's interesting. You know that kind of response," and saw him, "And he caught me so he made me teach that physics class… on eclipses [laugh]." A similar sentiment is expressed by James when he says his rural history teacher "back[ed] him up." He remembers being "like the one person in the class of- you know it was an IB class of you

know 15… advocating from a leftist perspective and he would encourage me to pursue the ideas further."

For Martin, it is passion and interest in their subject that made a good teacher, which is why he was so frustrated with his statistics professor, "like how do you get that probability. Like what is the law of chi square. And here he couldn't answer me, didn't care about the reasons why." However, he did like his middle school math teacher who was,

> Martin:…like smart. It was his first year teaching and he was quite passionate. He did teach. I think he did teach math really well.'

Also,

> Martin: 'Mr. X played in a way a way of making math ah ah like a positive thing because he made it fun and you know he would say you know don't feel dumb, like I had difficulties too you know.'

James also felt let down by teachers who were underprepared especially for the rigors of IB. "It was challenging, but I found the most challenging aspects of it were the in the courses where the teachers weren't prepared to teach it," which included his advanced mathematics teacher.

Therefore, the stories of good versus bad teachers directly spoke about mathematics or STEM teachers as in the case of Paul, Ray, James and Martin or spoke about general belonging in school – such as with Claire – and this was subject independent of the mathematical domain but specific to her sense of comfort at school, which ultimately impacted her learning in all subjects, including mathematics. Similarly, Paul struggled with English and this was made worse by having a teacher who he felt didn't listen to him, potentially pushing him further towards STEM subjects.

6.5 Neurodiversity: Disability and Genius

Another theme is that of neurodiversity in many forms. Firstly, in the form of struggle. Struggle and neurodivergence is a pivotal theme in James' story as he emphasizes the fact that both his older siblings have "severe learning disabilities" and he credits his early success in schooling with the prepared environment that his parents had provided for these siblings. He connects his own struggles with dyslexia to issues of justice saying if he "went forward with a formal diagnosis, I would find myself restricted from taking certain courses that I needed to do to go on to do a Master's degree." He also identifies as someone who needed help with socializing and credits his mother for helping him with this struggle by putting him in 4H.

Social awkwardness is something that Claire describes as well though she explains it as a result of being accelerated, making her younger than her classmates. This relates to her sense of not belonging.

> Claire: I just was like- I was a really, really, really shy kid. I think I was kind of awkward. [sniff in] I was almost always like one of the [pause] two or three youngest kids in the class. [click] And I think that might not have been such a good thing/. You know, I might have been better off if I hadn't been pushed ahead/

Paul also describes dyslexia and, to a minor degree, a sense of separation between him and his classmates. For example, "I didn't think anyone else was really interested" he responded to the fact that he had not asked anyone else to help him project the eclipse on the side of the school wall.

On the other hand, Ray's struggle was not with reading. In fact, he said, "I'm a big reader" but "I would bore easily um ah at the time they just said ah I have a lot of energy. Now I would have been classed as an ADHD um student."

In the case of Ray, James, Claire and Paul, much of their explanations involve these neurodiversities such as Ray's need for speed. Only competition, like darts or the risks of the real world, could hold his attention. James makes a link between his struggles with communication and reading, and justice. Then, he overcomes the odds to become a communicator who leverages his mathematical strength. Paul accepts that he has more of an affinity for STEM than "English" though he says that he must have learnt in the meantime as he writes reports now. This early difference steered his path. And for Claire, feeling like she did not belong was an echo throughout her story.

Discussions of neurodiversity extend beyond difficulties into talents. Claire, Martin and James all mention talent, genius and special intuition when it comes to mathematics. Both Claire and Martin value intellectual knowledge and wish they had such insight. As for James, he laments at the wasted talent that a lack of support in society could produce. Martin distinguishes skill from special talent which he calls an intuitive sense.

> Martin: like I wish I would have more of an intuitive sense of like when I do factor analysis I …it's a struggle to get there. … I wish I had more- I feel like if I have any strength in math like it's doing the actual computation but it's not like… but it's not like I look at a formula and I go Eureka.

And Claire makes frequent mention of genius such as how she describes both her brother and her late partner as quite talented. She feels that "it must be amazing to be a really good mathematician [intake]." She enjoys reading about mathematical geniuses and despite being told that she was "quite smart," she wishes her "brain was better for math."

> Claire: Like when I was reading about Paul Erdős or you know like I- I loved reading about, like, his- like you know his- it's not about ah yeah, his biography, some of the other

> mathematicians, like [intake] you know, this kind of combination of being fascinated and wishing that my **brain** was better for math

This connects with the social idea that mathematics is something special people with special brains can do.

So outside of the important influences of parents and teachers, society has had an impact on who the participants thought belonged in mathematics. Martin recognized this social messaging when he said, "I think math is associated with engineers and nerds," even though he thought it should be seen as "cool." He lamented that society was focused on "lifestyle" and wished it had more of a 'knowledge focus.'

6.6 Perseverance – Work the Problem (Martin, Hanna, James, Ray)

Lastly, an important minor theme is the value of Perseverance. It is mentioned in numerous stories in a number of ways. It is embedded in Ray's story, which involves a lot of activities that require dedication such as Junior A hockey, but also in his drive to be accurate and precise. "Like ah I would spend a lot of time looking for 5 cents" when he was completing a client's accounting file. For Paul, it is more of a learning skill and explanation of how he got over challenges such as figuring out spherical geometry. In his understated style, he said that he responded to the challenge by, "Yeah more- approach it more. Try to figure it out" or "…just recognized it as a challenge that I'd be able to work through."

For James, it was described more as a conviction to "never rest on your laurels" and to work hard even when things were difficult for him. "I worked hard at it. And um [click] still struggle to this day with grammatic- with grammar and spelling. But [intake] [outtake] [click] I- I- I refuse to accept defeat [laughs]."

However, the theme of Perseverance comes most into focus in Hanna's story as this is central to her values of perseverance, autonomy, and curiosity about understanding the universe. It is used as the reason why she became "good" at mathematics -

> Hanna: Say they don't like math, and generally it's just because they… are not… good at it. Um [click] and the only reason that I find that I'm good at it is because I sat down a lot, and I just worked on it for a long amount of time. Um so I enjoyed, like, working through stepwise, like, logical…sort of problem solving. Um and yeah, it's just… it's… I have fun doing it. Um so I think that's kind of the positive aspect, is I just, like- I enjoy that logical process/, and that if it's… if it is tough. That's not a reason for me to dislike it.

-but, also, why she is able to remain in her job. She combines the idea of perseverance with coping with frustration.

> Hanna: Oh, yeah, it's great. It's really frustrating, like, during the process, but I'm also well aware that, like, that's… that's pretty normal, so, I'm… yeah, I'm kind of okay being frustrated

and

> Hanna: I think with my colleagues, it's very much the same. Like, we're all kind of in the jobs that we're in because of the outlook that we have. Like, I don't… I just don't think you would last very long if you had a different outlook, or if you did get really frustrated.

7.0 Discussion

7.1 Comparison of Participant's Values and Hill's Core Mathematical Wellbeing Values.

Below is a chart that compares the values of all six participants. By far the most commonly expressed of Hill's MWB values, across the different narratives, was that of Relationships in the form of support (or lack thereof) of parents (Paul, Claire, Hanna and James) and teachers (Ray, Paul, Claire, Martin and James). Other common values include Perseverance (Ray, Paul, Hanna and James) and Meaning (Ray, Paul, Claire, and Jay) but what that meaning meant was different depending on the participant with the utility of real-life scenarios being meaningful to Ray, environmental stewardship and understanding through patterns being meaningful for Paul, wonder and awe in mathematics being meaningful to Claire and justice with data communication being meaningful to James. These also happen to correspond with their jobs such as accounting for Ray, environmental physicist for Paul, data communication for James. It was even the case for Claire who runs a retreat centre and is often surrounded by people who seek awe and wonder in their life.

Though differently designed as a study, Hill & Hunter (2024) found highest levels of MWB for their New Zealand year 3 to 8 students in the categories of Relationships and Perseverance; however, they also stated that the students' overall MWB "was most strongly associated with [the value of] positive emotions" (Hill & Hunter, 2024, p.227). Though that study and this present study are not directly comparable, Meaning comes more to the forefront (as described above) for these adult participants rather than Positive Emotions. This is likely due to the differences in timeframes with students associating MWB with Positive Emotions such as enjoying a class whereas adults understand that even worthy goals have unpleasant moments therefore being meaningful makes it worthwhile and does not necessarily lower overall MWB.

Table 7.1a: Comparison of Values between participants

Name	Ray	Paul	Claire	Martin	Hanna	James
Math Values	Hard work Accounting Statistics Integrity Competition	Patterns Clarity Environment Persistence	Connection Belonging Success Intellectual - Curiosity Wonder-awe-magic Skill-talent	Curiosity Intellectual -depth Passion Less-ambiguity	Understand -the-universe Curiosity Work-the-question Autonomy Persistence Support	Independent-learning Precision & accuracy Hard work Don't-give-up Equity & justice Comfort-with-Data Data-Communication Concrete-reality
Math Education Values	Patience Useful Relatable	Being heard	Useful Fun	Curiosity Connection to other disciplines-things		Prepared
Hill's Values	Accompli. Cognitions Engage.* Meaning Persevere. Fun* Relation.	Accompli. Cognitions Engage. Meaning Persevere. Fun Relation.	Accompli.* Cognitions Engage. Meaning Persevere. Fun Relation.	Accompli. Cognitions Engage. Meaning Persevere. Fun Relation.	Accompli. Cognitions Engage. Meaning Persevere. Fun Relation.	Accompli. Cognitions Engage. Meaning Persevere. Fun Relation.
Other	Integrity Competition could fall under fun or engagement	Clarity	Skill-talent might fall under Accompli.	Passion Less-ambiguity		Precision & accuracy Equity & Justice Concrete-reality

Matches are highlighted blue and non-matches are coloured grey. Potential matches are given a *
To be able to compare all six participants across a single row, Hill's Mathematical Core Values are given the following short forms Accomplishments = Accompli., Cognitions = Cognitions, Engagement = Engage., Meaning = Meaning, Perseverance = Persevere., Positive Emotions = Fun. Relationships = Relation.

There were also values that were not easily categorized in Hill's MWB. These were integrity (Ray), clarity (Paul), less ambiguity (Martin), precision and accuracy (James), concrete way of expression reality (James), innate mathematical skill or talent (Claire), and justice/equity (James). Four of these seem similar and these are clarity, less ambiguity, precision and accuracy and a concrete way of expressing reality. In all cases, they refer to wanting something clearer cut and less ambiguous/interpretative than what they perceive is delivered by non-STEM practices such as humanities, social sciences or literature. This connects to the major theme of What is and is not Mathematics in section 6.3.

In Hill et al. (2021, p. 354), value and valuing in mathematics education can be broken into three subtypes: general education values (moral or ethical values to do with the objectives of education), mathematical values (math as a discipline ex. valuing mystery of mathematics) and mathematics education values (ex. learning and teaching mathematics). It was found in studies with students that mathematics educational values were the most significant of these three values. However, given that this is a non-student population, it would seem that the other two value sub-types come to the forefront in values such as justice, integrity and less interpretative conceptions of reality.

Also, not as immediately apparent is the value of Positive Emotions though it was mentioned in affective language such as when Hanna says "Um so I enjoyed, like, working through stepwise, like, logical…sort of problem solving. Um and yeah, it's just… it's… I have fun doing it" and Martin describes a grade school teacher as "fun." Claire also describes doing math as fun though this is more of a justification as there is no reason she would do it now except for fun. Therefore, it could be concluded that Claire finds mathematics fun and values Positive

Emotions. Ray found a fast multiplication game fun in school but described valuing it because it was competitive.

Reasons why teachers might be *bad* seemed less about not being fun and more about being ill prepared (James), argumentative (Paul), insufficiently curious about their own subject (Martin), or lacking interest in teaching (Hanna). Here we can see echoes of how the "light bulb moments" of high enjoyment teachers and their sense of mathematical positivity model enjoyment for students (Russo et al., 2023) and how perceived teacher argumentativeness can alienate students (Lin et al., 2017)

However, school is described as boring, in that it is potentially not fun or at least not captivating, by both Ray and Claire. Their turning points did not seem so much about whether mathematics elicited more Positive Emotions but rather that teachers were relatable (Ray) or supportive (Claire). Compare this to their own descriptions of what makes a good teacher, which is for Ray someone who uses real-life scenarios and for Claire someone who presents the magic and mystery of mathematics. Finally, Martin thinks that if you are a teacher then you should be both passionate and deeply understand your subject (such as statistics). Their descriptions of what makes for good mathematics teaching overlap with what they value in mathematics itself.

Beliefs of self also often correspond with their mathematical values, corresponding somewhat with Clarke's (2015) interpretation of beliefs overlapping with value to become reifications of self. For example, Ray sees himself as having integrity and identifies integrity as an important mathematical value. Martin sees himself as being deeply, intellectually curious and sees this as the most important mathematical value. Hanna is a little harder to decipher but she describes herself as hard working and in pursuit of understanding. She also values curiosity and perseverance. James describes himself not only as being shaped by inequity but seeking equity

and ascribing equity as an important value in mathematics. It was a little harder to describe Paul's values but he identifies himself as thinking in patterns and disliking a lack of clarity such as found in more perspective-based knowledges such as literature. Claire, on the other hand, is the most interesting as she strongly values innate talent and success as evidenced by her regret about not having a PhD and her multiple mentions of talent. Innate talent is a value she also attributes to mathematics like "genius" that allows you to think things others do not. She does not, however, identify herself as especially talented since her identity is both as a smart person and somehow also as not smart (though this is also how she describes humans, as simultaneously "fucked up" and "foolish" but also capable of amazing feats of intellect). This she attributes to a lack of confidence and support by her mother and the influence of patriarchal society. She is the person who seems the least satisfied with her accomplishments.

Claire's story, in particular, relates to the extensive body of work on STEM and the patriarchy where women encounter social barriers to this male dominated academic landscape (Fernandez et al 2022, McMichael et al 2026). In all the narratives, there seems to be a strong link to the details of individual identity – stories we tell about ourselves (Sfard & Prusak, 2005) - and the larger story about mathematics, which should be no surprise given the narrative structure of this research.

7.2 Hybrid Framework: Turning points and Emergence of Mathematical Positivity

The turning point tables of each of the participants had evidence that it was values met or not met that seemed to correspond to whether or not they were mathematically positive or if this was threatened or encouraged. Of the three participants who were either negative (Claire, Ray) or ambivalent (Martin) about mathematics, it was values being met that seems to shift their expectations about mathematics, as when Ray's educational value of relatability through real-

world examples was met, shifting his expectations about mathematics from not useful to useful and even as a viable career choice. Or when Claire's value of belonging and connection, as well as intellectual curiosity, were met enabling her to be more open to learning more about logic and ideas in her all girls private school. This changed her expectations about mathematics from boring and repetitive to interesting. Claire's experience recalls Grootenboer & Marsham's (2015) findings that elementary students associated mathematics with computation. It also speaks to a successful transition into more abstract, analytical thinking and the significant role of teachers in encouraging and facilitating a positive mathematical identity.

When it came to those who were more strongly mathematically positive from near the beginning of their story (James, Hanna, Paul), values seem to act as a buffer against threats to their mathematical positivity as when Hanna's value of perseverance compensated for the challenges that she encountered during trigonometry and online schooling. "Values provide the individual with the will or reason to maintain their course of action despite challenges that might occur." (Hill et al., 2021, p. 354).

7.3 Participants' Explanations and themes

When comparing the influences – societal, socializers, personal, previous experiences (Hybrid Framework found in Table 3.3b), each participant has their own story. However, there were some important comparisons. In all instances, the participants saw themselves in the mathematics, harkening back to Debellis & Golden's (1997) description of mathematical values as being 'deeply personal' and 'cherished commitments or truths' or how SEVT's Self Schemata is pivotal in their achievement related choice (Eccles & Wigfield, 2024).

For example, Martin was not interested in mathematics until he connected deeply with the explanatory power of mathematical modelling. He became a quantitative guy who taught his students in a way that he wished he had been taught. For Paul and James, they could not see themselves (initially) as 'English class' people, given that they both identified as undiagnosed dyslexics, but leaned into their budding mathematical identities. The dichotomy of 'English' versus 'Math' is seen as early as elementary school given that students can have self-efficacy in either area broadly (Beilock et al., 2010). Hanna, just like Paul, James and even Claire, were all captivated by concepts in astronomy and physics, which is famously mathematical in nature, making it such that by pursuing and further identifying with these concepts, they also grew closer to mathematics. This could be seen as a personal interest or somehow intrinsic to the person or, more properly, as more evidence for those complex causal arrows of process maps as found in SEVT and the importance of understanding individual stories.

Each one had their own sense of what was valuable in mathematics and this heavily overlapped with their own mathematical identity. Rather than interpreting the logical and so-called objective domain of mathematics as "cold, remote, hard, uncaring, rejecting, neutral, unfeeling, impersonal, empty, inhuman, absolute, objective, rule-drive, perfectly rational, male, mechanical, soulless, dead, fixed and hierarchical (Ernest 2004, p.18)," (de Freitas, 2008), they perceived it as comforting, therapeutic, useful, and insightful. Though some characteristics remain the same - logical, rule-driven, rational – this list does not have the same sense of alienation. Their engagement with the mythos of mathematics was one where they belonged within it.

Socializers are indeed an important causal theme (Eccles & Wigfield, 2024), and Hill's core mathematical value of Relationships was the one that was most evident in the participants'

stories. Parents were supportive in a number of ways including being mathematically competent as many were teachers (Paul, James, Hanna, Martin) who engaged the young participants in games and activities that had a mathematical aspect – cards, board games (Paul, James, Hanna) or computer programming (Martin). Related is the study by Nishen et al. (2024) who found a correlation between students' mathematical career aspirations and their mother's STEM related occupation.

Just like in previous research (Hill & Hunter, 2024; Machalow et al., 2022; Towers et al., 2018; Wilder & West, 2024) teachers also featured prominently in turning points for the participants either by fostering a sense of belonging, or by being ill prepared and/or lacking interest in their topic. This also recalls findings that teachers' enjoyment and positive attitude enhances that of the students (Frenzel et al., 2009; Russo et al., 2023), whereas perceived verbal aggressiveness and instructor argumentativeness negatively impacted students (Lin et al., 2017).

Perhaps one of the most interesting findings was the participants' own emphasis on mathematics outside of school such as darts, 4H, statistical debate inside the house, or hiking and fishing with a father who loved patterns in nature. This is particularly interesting in light of Towers et al. (2017) study where the students principally thought of mathematics as being situated in school. These out-of-school activities seem to build several things including mathematical self-efficacy, noted for its importance in helping overcome mathematical challenges (Fernandez et al., 2022; Solomon, 2007), but also meaning. Findings were similar to those in the Wilder & West's (2024) study on autobiographies of college faculty, that looked at mathematics positivity, negativity and trauma, in that positive stories often involved supportive parents or teachers, good grades, seeing mathematics as a tool for understanding and the importance of making mathematics relevant (p. 41). However, it is important not to jump to a

conclusion of what is meant by making mathematics relevant because merely connecting it to the real world as in “ecologists use mathematical programs that model population,” or “understanding statistics helps you understand risk” does not have the same weight as making mathematics meaningful. This latter concept can be very personal as it connects with individual mathematical values and mathematical identities. Crucially, what is valuable and ultimately meaningful about mathematics, and how each participant became mathematically positive, was as unique as each participant.

7.4 Implications of Research

Assuming that it is a worthy goal to encourage mathematical wellbeing in the hopes of increasing the number of people who identify as mathematically positive, then what insight, if any could be gained by this study? To answer this, let us split it into two questions. As a teacher, what does this say about the practice of teaching mathematics? More generally what does this study say about the divide between those who identify as mathematically positive and those who do not.

Pedagogically, mathematics (like most other areas of study such as literature and music) is not merely contained within the school walls. It is a frame of interaction with the world. However, within the school, some effort could be made to draw these connections for students by including games, cross disciplinary studies and studying mathematics in a way that it is artistic, musical, problem solving or otherwise outside of what students normally think of as computational practice on a lined page.

Likewise, it is important to recognize the power of socializers in a student’s lives (Eccles & Wigfield, 2024). Whenever possible, a teacher should work to help a student feel heard and

seen.[3] A frustrated mathematics student can feel encouraged not only their struggle is recognized, but when a teacher takes the time to find out what is valuable to them and whether or not there is a way to incorporate those values into their lessons (Hill et al., 2021). In this way a student may make a connection first through the teacher and then to the mathematics. Conversely, a student with a proclivity towards mathematics or a related STEM subject may be supported by the teacher who shows interest or helps a student to extend their knowledge (West & Wilder, 2024).

More generally, it draws attention to the fact that those who have found meaning in mathematics do so beyond the time when they were in school. They may continue to pursue it as a hobby such as with Claire or professionally such as with Martin. It also highlights the fact that there appears to be very little work that has been done on mathematical positivity outside of the classroom and beyond schooling. Understandably since most studies are interested in school mathematics learning, values emerged from the adult participants that were not so easily contained within Hill's MWB such as Integrity, Equity and Clarity. This is a potentially interesting and underexplored area of research since presumably the ultimate goal of school mathematics is not only competence and appreciation of mathematics but also to produce adults who are in the words of Paul "… familiar and comfortable with math in everyday life." It may be

[3] Recently I put this insight into practice with a grade 8 mathematics student who was feeling frustrated and overwhelmed. He said, "why do I even need to do this?" in reference to geometry work. I gave an answer to do with applications of geometry. He said, "why can't we just do marketing?" Our school actually does have an entire class on entrepreneurship where the students run their own business. I was about to reply but stopped and said, "What's valuable to you?" He said, "I want to learn about the real world and how to make money." I replied, "Okay in your IRP, let's make sure that your project is about that." IRPs are something our school does which are independent research projects and are intended to give the student choice and autonomy in learning. He seemed relieved and smiled. "Okay." I could see the transformation in his affective response as he felt heard. I could have continued to explain how the mathematics was actually valuable despite his insistence that everything not overtly and directly to do with money was not useful but was glad that instead, I tuned in. He will still get the curriculum but I think he felt validated and like he could learn something that was important to him.

here, in the regions off the school map, where strategies and insights are uncovered that can help achieve these goals.

8.0 – Conclusion

8.1 Summary of Study

This study aimed to answer the following questions:

1. What does mathematical positivity mean to adults who self-identify as being mathematically positive?
2. What narrative themes emerge that help explain the development or maintenance of mathematical positivity?
3. What narrative turning points threatened or encouraged mathematical positivity and how do they relate to the values and expectations of the participant?

As an exploratory study, it has gathered rich, qualitative data from a sample of 6 adult participants, ranging from their mid-20s to late 60s, 2 women and 4 men, all of whom identified as being mathematically positive. It has used narrative analysis to elicit their mathematical life stories. This was done via 2 interviews. In the first, they were asked open ended questions and the researcher used active listening and the participant's own language to ask clarification questions. The first interview was coded for affective language, descriptions of mathematical positivity and mathematical values. It was also segmented into narrative experiences/episodes/events, which were restoried into a coherent narrative. This was presented back to the participants for correction. Participants spoke for 30 minutes to about 80 minutes in the first interview depending on how much detail they wanted to include.

The second interview varied from being short with some minor factual corrections to becoming a back and forth where the researcher and the participant further crafted the story until the participant felt it more adequately reflected their understanding, such as with Claire who needed to emphasize many autobiographical facts and Ray who wanted to include a second story relating to his son's experience at school. The co-created narratives were then used as secondary

data, respecting the participant's own explanations of events, and other aspects of their relationship with mathematics.

Definitions of mathematical positivity (Research question 1) had to do with being either confident (Ray) or comfortable/familiar (Paul) with mathematics or not being afraid of mathematics (Martin, Hanna). This relates to having high mathematical self-efficacy that in turn speaks to an identity as being mathematically aligned. Both Claire and James also included versions of finding mathematics useful that related to their own identity.

Their mathematical identities, values, and professions/hobbies were often in alignment: as how Paul valued the environment, saw the world in patterns and works as an environmental physicist who uses patterns in satellite data that give clues to what is happening in the environment. Similarly, Hanna, values autonomy and hard work, feels her parents supported her in school and in her hobbies such as space and physics, and graduated with an astrophysics degree. She now works for an R&D team solving hardware problems, which requires in no small measure autonomy and hard work.

In order to determine turning points or potential turning points in participants' mathematical positivity (Research Question 3), co-created narratives, affective language coding, descriptions of mathematical positivity and values were used to create turning point tables. These were the basis for each participants' Emergence of Mathematical Positivity summaries. Two of the participants started off as mathematically negative and associated early mathematics with computation. Both were encouraged toward mathematical positivity as their values were met: the value of belonging and connection for Carol, and utility for Ray. One participant, Martin, was mathematically ambivalent, but grew distinctly mathematically positive through computer programming in later high school and university when he began to associate mathematics with

challenge and deep understanding through curiosity. The last three were mathematically positive from near the beginning of their story. Two gravitated toward it, in part, because they struggled with English but found success in mathematics. Hanna was both mathematically positive but also did not express any negativity toward literature-based subjects. Nonetheless these last three had their own potential turning points where their mathematical positivity was threatened, but overcame these potential obstacles through their own value system. For Paul and Hanna, this was perseverance and for James it was both support from his family and perseverance.

Themes (Research Question 2) that emerged from the data included how the participants delineated mathematics and STEM subjects from other disciplines such as literature and cultural subjects. There was also an implicit or explicit distinction between mathematics outside and inside the school structure, and before and after formal schooling. This was seen in the theme of Outside the Math Classroom: ways into math, which described such activities as darts, board games, cards and 4H as entry points into mathematics skill and confidence. Relationships with teachers and support from parents also emerged as an important theme, with supportive parents being seen as important when it came to mathematics either because the parents had skill in the subject or because they were encouraging. Good teachers were those who supported the participant and/or related to them such as how Ray thought of his accounting teacher as being somewhat like him. An unexpected theme that emerged was that of how neurodiversity intersected with other themes in their mathematics stories such as identifying as mathematical due to dyslexia (Paul and James) or having trouble concentrating on mathematics because of ADHD (Ray). Additionally, Perseverance was mentioned in a variety of ways by Martin, Hanna, James and Ray. In all cases, this illustrated a turning toward instead of away from mathematics,

or the opposite of avoidance – one of the primary concerning outcomes of mathematical anxiety (Hunt & Maloney, 2022; Schmitz et al., 2023)

Mathematical values were then compared across the participants with Hill's MWB core values (Hill et al., 2021). Appearing in at least four participants were Cognitions, Meaning, Perseverance, and Relationships, the latter being the most prevalent in five participants. However, there were also values that did not fall into any category such as clarity, equity and justice, integrity, and passion. This could be due to the fact that these were adult participants speaking about their lives rather than children in school.

These values intersect with the emergent themes such as the value of Relationships with teachers and parents supporting their mathematical pursuits. This is well illustrated by the theme of Outside the Mathematics Classroom when parents exposed the young participants to mathematical experiences. This helped them become comfortable and familiar with mathematical concepts improving their mathematical self-efficacy and ultimately their mathematical wellbeing.

Also important are the themes of What is Math, and to a lesser extent Neurodiversity, as it crosses over into the identity question of "Who am I"? Most participants seemed to be mathematical insiders in that they felt familiar and comfortable with it and liked what they defined mathematics to be. They even identified themselves as having the same characteristics: being deeply intellectual (Martin) or involving patterns (Paul). This relates to their core values of Cognitions and Meaning.

8.2 Key Contributions

Many of the participants related experiences of mathematics outside of school as important to their mathematical success and mathematical positivity. This suggests the need to draw attention to the mathematics in children's everyday life. Just as reading storybooks to young children helps them become readers (Dahl-Leonard et al., 2025), involving children in mathematical practice in the form of games, hobbies or other activities allows them to be comfortable and familiar with mathematics in their everday lives. It also allows them to see the meaningfulness of mathematical activity and how it may relate to them. Hobbies and interests like agriculture, music, space and computer programming were all mentioned as being gateways to mathematical interest. Speaking as a mathematics teacher, there are opportunities for this sort of practice in all disciplines and it does not have to be didactic but rather can naturally occur. Related to this is the importance of relationships in the lives of students. Parents and teachers, or other important socializers, who are mathematically anxious or negative can transmit these messages to students but conversely, as this study suggests, those that are supportive of the children's efforts and interests can encourage mathematical positivity. It need not be required for the parent to be mathematical but rather to show an interest in the child. Though all stories illustrate the importance of relationships, I often think of how Paul's father presented an avenue for him to direct his mathematics toward environmental stewardship despite not being mathematically confident himself.

Though each participants' definition of mathematical positivity varied, a comfort with mathematics was a common theme. This was seen again in values where mathematics was described by several as therapeutic, calming, less interpretative and comforting. Related, perhaps, to the idea that mathematics presents the *right* answer, it could also be understood that

for these mathematically positive adults, mathematics was an important framing device that they used to navigate the world.

Adults' mathematical values differ somewhat from what has been discovered thus far about children's values, which should probably not come as a surprise, but it nonetheless demonstrates a need to broaden the concept of mathematical wellbeing to more populations. Longevity in mathematics may take seed early so seeing what it blossoms into may help encourage its growth. However, Relationships, Perseverance and Meaning were all consistent with Hill's MWB. Yet, though Meaning was important, it took on unique form for each individual.

Of particular value, for this researcher, was the development of a methodological system that allowed for the data to coalesce into themes that stayed close to the participant's own explanations.

8.3 Final Reflection

As a teacher of mathematics and science, I have long been fascinated by the affective spectrum of mathematics students and have tried my best to support them both in their cognitive understanding but also in their affective approach to the subject. Within my classroom, we try to hold space for the idea that mathematics might be fun and exciting. We also recognize that practice and challenge will be part of learning, just like in any other subject. I have listened over the years to many mathematical complaints such as "this doesn't make any sense," "when will I ever use this" "why can't you just tell me the rules," "I'm just bad at math like my dad," or "this hurts my brain." I have also heard exclamations of joy and triumph such as "I get it now!" and "I love doing these problems," and "why can't all math be like that." I've even heard "this is

boring, can't we do something more challenging" and "I can't stop now, I've almost figured it" as the bell chimes for a new class to start.

However, when I started looking into ways to make mathematics more fun, manipulatable, more intuitive and more embodied, I kept encountering the overwhelming mathematical anxiety that characterizes a large part, at least my experience, of people around me, including in the research. It did seem that there were two types of people, even in my staff room, the math and the non-math people. This was certainly not my own experience as my own mathematical narrative reads like a transformation story from someone who was not terribly good at mathematics, identified as creative and went to art school, to someone who later learned mathematics out of interest. However, never did I associate being not particularly good at mathematics as mathematically negative. I always liked it, I just couldn't always do it so I refused to define myself as not a math person and once I could do mathematics, I did not think of myself as just a math person.

Therefore, when I was thinking of what research would most benefit my own students, I thought of those seemingly mysterious group of people – the mathematically positive. Who were they and what messages did they have for us teachers? Would there be any lessons that could improve the affective experience of mathematics students?

This led me to interview a fascinating group of participants, each with their own unique story. I often think of these stories as I go about my days both teaching and as someone who is constantly investigating the world. I am grateful for their participation and forthrightness. I am also grateful for their help in crafting their own stories.

Outside of the valuable insights of each of the participants, one of the most useful parts of this thesis has been the development of the methodology. By creating a rather rigorous step by

step process, I was able to turn an unwieldly, large amount of data into something which distilled, though not effortlessly, sensibly into claims that were easy to support as they were compared and verified multiple times over multiple steps back to the data in both its raw and storied form. I am also grateful to the many, many researchers and theorists, from whom I drew inspiration, for their guidance. The development of the turning point tables was illuminating as on initial inspection some stories did not seem to have any threat to their mathematical positivity but bumps emerged when ordered carefully with values and expectations. In fact, it was so interesting and revealing that if time were lenient, I would have wished to have interviewed more participants as no doubt each story would have given further insight both on the usefulness of the methodology but also on the phenomena of mathematical positivity. It is therefore with grateful thanks to my participants that I end this thesis.

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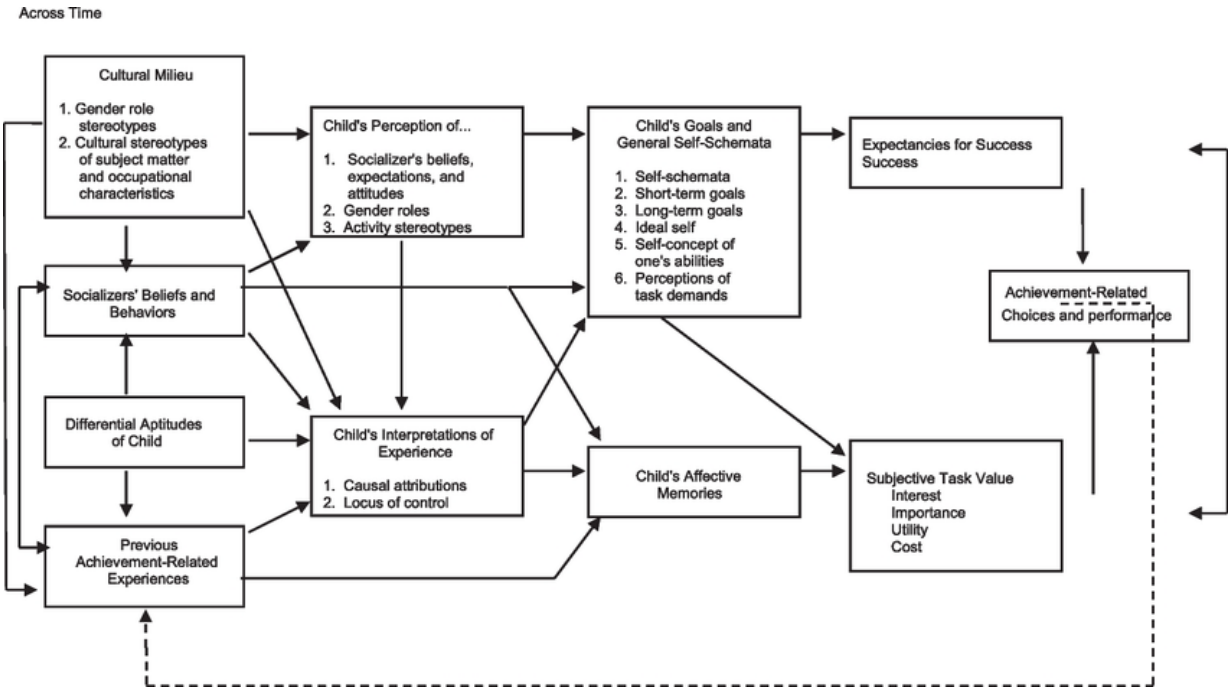
Appendices

Appendix 1: Achievement Performance and Choice

Eccles and colleague's expectancy value theory SEVT of achievement performance and choice



(Eccles & Wigfield, 2024, p. 52)

Appendix 2: List of Possible Questions

1. What does mathematics positivity mean to you?
2. Can you describe someone else you know who is mathematics positive?
3. Tell me about your… earliest mathematics memory
4. Did you always like mathematics?
5. Can you tell me what mathematics was like for you in school?
6. What do you remember about your teachers?
7. Did you go to the same school in grade school/high school?
8. Can you tell me the story about… something challenging in mathematics?
9. Tell me about values you associate with mathematics.

Appendix 3 Affect word/phrase frequency count

Index comparing frequency of affective words/phrases with word count of interview.

Name	Emotion	Attitude	Belief	Value	Identity	Word Count
Ray	11 Fe/WC 0.00254	9 Fa/WC 0.00208	7 Fb/WC 0.00162	16 Fv/WC 0.00370	14 Fi/WC 0.00323	WC: 4331 T: 40 min 57/WC 0.0132
Paul	16 Fe/WC 0.00322	11 Fa/WC 0.00221	3 Fb/WC 0.000603	5 Fv/WC 0.00101	13 Fi/WC 0.00261	WC: 4972 T: 30 min 48/WC 0.00965
Claire	73 Fe/WC 0.00505	70 Fa/WC 0.00484	25 Fb/WC 0.00173	51 Fv/WC 0.00353	31 Fi/WC 0.00214	WC: 14457 T: 80 min 250/WC 0.0173
Martin	17 Fe/WC 0.00205	34 Fa/WC 0.00410	12 Fb/WC 0.00145	24 Fv/WC 0.00289	18 Fi/WC 0.00217	WC: 8293 T: 60 min 105/WC 0.0127
Hanna	25 Fe/WC 0.00325	25 Fa/WC 0.00325	7 Fb/WC 0.00091	16 Fv/WC 0.00208	17 Fi/WC 0.00221	WC: 7683 T: 40 min 90/WC 0.0117
James	31 Fe/WC 0.00346	13 Fa/WC 0.00145	13 Fb/WC 0.00145	24 Fv/WC 0.00268	12 Fi/WC 0.00134	WC: 8970 T: 60 min 93/WC 0.0104

The above table uses word count as a proxy for the amount expressed though instances of affective word use may occur in phrases therefore using multiple words. It is meant as a rough way of comparing whether or not there was comparatively more or less affective word/phrase use and comparing the types of affective domain frequency between participants whose interviews varied in length. Gold, silver and bronze correspond to the 'winners' for frequency as in most frequent, second most frequent and third most frequent.

Appendix 4: Full Narrative Stories

A4.1 Ray: An Honest Player

Ray didn't say too much about his family other than his dad was the first person to go from blue to white collar when he trained in management. Early on in his schooling, Ray was more interested in reading and writing because it was creative. He describes himself as both easily bored and filled with child-like curiosity. He liked history and geography, saying he was a big reader and that reading opened up new adventures for him. He would look up places in the atlas, sometimes picking three spots to memorize because it was kind of fun. He also noted that cultures keep making reoccurring mistakes so reading history was like watching the present unfold in the past.

On the other hand, he mostly describes his early experience with math as horrible. It was only in context of a friendly competition that he found it fun. For example, his grade 4 teacher played a game with multiplication flashcards where the students had to hold up the correct answer and the teacher would tell everyone else, with the wrong answer, to sit down. He used to imagine that he was competing against the many smart kids who were in his class.

Things started to turn when he was about 10 or 11. His parents did a lot of volunteering including starting a dart's league. Sometimes someone wouldn't show up so Ray would sub in. He would also keep score. He laughed and said "Like as quickly as they're throwing darts, you're doing addition and subtraction. It's very stressful to have adults looking at you but it got the, "competitive juices flowing." Around this time, he began competitive darts. Darts are complex, he explained, and there are many different ways of playing, all with their own calculations. All this practice meant that he became very proficient at mental math. Around this time, he also had a paper route and the adults on that route were very impressed when he could

just ‘bang off the numbers” on how much they owed him. Though he liked the affirmation, he thought it odd that they were surprised to meet a kid who could do sums.

Math learning in school did not hold his attention. Ray was easily bored unless whatever was happening was going at 400m/h. He would challenge teachers if they couldn’t hold his attention by disrupting the other students and being a class clown. Though he got along with the teachers outside of class, he just couldn’t ‘zip it.’ He feels now he would be classed as an ADHD student. Mostly he would just ask the teacher what he needed to pass and would do that plus one since at the time he was more focused on his professional hockey career, playing until the end of the Junior A division in Ontario.

All this changed when, in grade 11, he decided to be an accountant for ‘some odd reason.’ He had an accounting teacher, who inspired him who he really liked. He was amazing and patient. Mr. A put a real-life spin on things. Ray said that “we had that piece of paper in front of us and on the piece of paper was a real-life situation and we were going to do for the rest of our lives.” It was a lot of personal finance like figuring out loans or doing personal income tax. He wondered if Mr. A saw something of himself in Ray as after the teacher had obtained his accounting degree, he had come back to school to give back to goofballs like him. Ray took accounting in grade 12 and 13 as well and when his senior accounting teacher found out that Ray had become an accountant, he was shocked and pleased. Apparently, this teacher had jokingly responded that he wasn’t sure if Ray would pass his class let alone complete his accounting degree.

In college, Ray took a double major in computers and accounting, mentioning that there is a lot of logic in coding, “numbers are very important there.” This is where his math positivity really started to shine as he loved quantitative methods like statistical analysis, stating he

"couldn't get enough of it… You could dig into massive amounts of data and you can come up with certainties you know within a small mean. If you do this, then this happens. This just fascinated me." He went from a student who just squeaked by in high school to a 90s student in college.

Ultimately, he said math is about integrity. He has owned his accounting practice for 33 years. His name is on any number that goes out and he has a sense of pride in being able to give accurate, efficient service for customers who are looking for expertise. He has an attention to detail almost to a fault and is the kind of accountant who will go looking for a missing five cents. After he had graduated accounting, he initially worked for a big firm and that they tried to beat this integrity out of him saying it wasn't worth their time if the error was within $500 dollars. This did not sit well with Ray. He is a competitive man who enjoys a game but is also an honest player.

When asked what kind of person math would be, he said that an image immediately came to mind and that was the Staypuff Marshmallow man from the Ghostbusters because it is playful, huggable and you can do so much with math. You can play games. You can make structures. It's all around and in everything."

Story 2

After the interview, Ray said that he had something that he really needed to tell me and this was a second story about his relationship with his son and mathematics. His son was taking a grade 12 math course and the teacher had given a 10-part assignment at the end of the term worth 20% of his grade. This assignment was about real-life scenarios like getting a mortgage, investing money and financing a vehicle. His son kept saying that, that's not how his teacher would do it to

Ray's sage advice. They consulted the textbook but it was not enlightening. This all happened at the 11th hour as the assignment was due the next day. Yup, just like dad, he said.

Ray presented his son with an option. They could work the problems the way he approaches them in his accounting office, showing their work and present the conclusions and file on time or take a substantial hit for being late. The papers were filed on time and the next week Ray's son was presented a passing grade in the mid-60s. That evening, Ray called the teacher into his office for a meeting. As an aside, he had been preparing the teacher's taxes for over a dozen years and helped him with tax planning for his wife's business.

Two things happened. They slightly reworked the entire assignment so that next semester the class was working with actual real-world situations and calculations and two the teacher offered to change 'Son's' grade on the project, heavily discounted because though the result was correct, it was not shown in the correct format. Ray objected to the change in grade because his son had left the assignment until the last minute.

A4.2 Paul: Pattern of Planets

Paul says that he's always found patterns in math interesting. His earliest math moments were because his mother was a big cribbage card player and adding to 15 is important in that game. He described her as math positive and said that you could see it in the games she played such as cribbage, euchre, 45 and crokinole. This latter game is pool but with chips on a board that you flick in. There's a lot of vector thinking, he says.

One example of his interest in early patterns, was in fact a misinterpretation. He was trying to figure out a math problem with primes. Since he was taught that an even number plus an even number and an odd number plus an odd number equalled an even number, and that an odd number plus an even number gave an odd number, he guessed that there must be twice as many even numbers as odd numbers. It was later that he realized that an odd+even and even+odd could be understood as two different categories.

By grades 5 and 6 he was nominated by his class to be the student who would compete in math competitions. This surprised him because he didn't realize that other people were having challenges with it. Since computers were new and exciting at the time, these contests morphed into programming contests shortly afterwards that he also participated in. His comfort with math led in a number of directions such as astronomy, programming and the industrial arts. The latter of which he said he did a lot of in school. He enjoyed lathing most of all because of the geometry.

Generally, he did not find math frustrating though it could be confusing at times such as trigonometry with its cosines, tangents, and the mysterious importance of right angles. Perhaps it was "a little bit the nomenclature and… [that] it wasn't just simple linear patterns anymore. Suddenly it was 2 and 3D patterns. It became more challenging at that point." This would later

be echoed by his mention of spherical geometry in his bachelor's honour thesis in astronomy. He said it was at times difficult to visualize the orbits and get the math to work. However, he would just keep at it until eventually he figured these things out.

This is in contrast with his frustration with learning languages where he just couldn't find a path through. In both English and French, he was a solid 50s student. Given his mother was a math and English teacher, she had trouble understanding why he was so good at one subject and not the other.

She worked at his small school in South Central Nova Scotia that had 250 kids from grades K to 12. His father, who also worked there as the janitor, had dropped out of school in grade 6. Paul says it was common for people to drop out before high school then go to work in the fishing or forestry industries. Having both parents working at his school meant that "I couldn't get away with anything." For example, one day he injured his back while playing hooky and the closest place to go for help was his school. He arrived just as his mother was leaving and his father was showing up for work!

Paul said that though his mother was math positive, more important, for Paul's future career, was that his father was environmentally positive and got him to want to understand things around him. His father was into topics like oceanography or lake bathymetry and was always exploring them like watching the news if there was a relevant topic on. They were always going on hikes or fishing and his father would point out how things related to one another, such as patterns in the forest or how wind blew across the lake making waves, but would shut right down if you brought math into it. Instead, he would express this attention to relationship and pattern through art as he was always drawing with charcoal or whittling something.

When it came to teachers, Paul really connected to his high school physics teacher. They just really got along. His class was small with just six students. It also happened that his physics class was at the same time as a total solar eclipse. Since his dad was the school janitor, Paul knew where everything was including a large concave mirror. All the students were supposed to stay inside to avoid eye injury but since Paul was so interested, he skipped physics class and smuggled the mirror outside to project the eclipse on the school wall. His physics teacher caught him and made him give an astronomy lesson to the class.

On the other hand, he had a conflict with his 10-12 English teacher, "we never interpreted anything the same way," he says. This teacher had a rigid interpretation of the text and Paul felt unheard. He just stopped reading by the end of grade 12. However, he had to take an English class during his astronomy bachelor's in Halifax. This teacher was much better and brought him right back into it with "just his willingness to say oh that's interesting. I don't understand it but that's interesting." This time Paul felt heard.

After his bachelor's in astronomy he travelled Australia for a year because "it was sort of as far away as I could get," and "I wasn't going to the moon" so it would have to do. He was deciding where to go for graduate school but everywhere he visited just didn't feel right so he ended up in Toronto where he completed a Master's in astronomy and a PhD in physics and remote sensing. Though he said that he's always loved planetary astronomy, "There's more opportunities looking at this planet from space then looking at other planets."

And though his mother was an important early influence for him, he credits his father more for his interest in math within an environmental context. His father has a love and appreciation for the outdoors that he shared. Paul now works as an environmental physicist evaluating satellite imagery with respect to environmental variables. This involves looking for

changes in spectral indices to find shifts in something like vegetative cover that may indicate insect damage or unauthorized forestry etc...

Talking with Paul, you get a sense that math is a way to interpret the world or to put it in his own words, “life is a bunch of patterns and you figure out what direction things are going … or what’s the likelihood that somethings going to happen so that’s how I attack life.” He has expressed that math positivity is about feeling familiar and comfortable but his story also suggests that it isn’t a lens with just any purpose but for Paul one of care and stewardship.

A4.3 Claire : A Relationship with Mathematics

Claire finds math fascinating, particularly number theory as well as physics and mathematical ideas more generally. She's read about various mathematicians such as Cantor, Fermat, Godel and Erdös. However, she is in two minds about her ability to really understand math. In spite of her interest she feels like she is a slow student, not naturally gifted at the subject. She needs more time with it then some people and would have done better in a supportive environment with good teachers who were interested in their subject. At the same time, she recognizes that there were obstacles in her life that may have prevented her from aligning her interest in math with her potential skill. There are frequent mentions of feeling like not a good enough student. If she had, had an aptitude, she says would have continued to study math. She said that her brother and her were very different. He always seemed "much more intelligent" to her, able to understand things easily that she struggled with. She thinks he likely had an undiagnosed learning disability that impeded his academic performance in spite of his high intelligence. Nonetheless he overcame it finally and became an engineer and successful entrepreneur.

She describes herself as occupying two intellectual spaces or rather a space between failure and success. There is a real tension between describing herself as unsuccessful, not fitting in, becoming invisible, working to exhaustion while simultaneously describing characteristics that we often associate with potential such as curiosity and prodigious reading. She was accelerated in primary school and, later, completed art college followed by an undergraduate degree and master's degree in psychology and, later, a second undergraduate B.A. in education.

Claire's mother had a fiancée who died in WWII. She then met and married Claire's father and supported him through twelve years of medical school, working in the fashion industry in various roles. She moved with him to Ottawa for his work as a medical specialist and researcher, where Claire and her brother were born, leaving behind her own career.

Unfortunately, Clare's father died of cancer when she was seven years old, leaving her mother to raise Claire and her brother on her own. Claire describes her mother as being angry and depressed, which she thinks was a consequence of losing her husband so early. Claire's mother went back to school and earned a degree in psychology. She built a second career as a social worker while she raised her kids as a single mom. Claire said that her mom didn't share much about her life before Claire was born. This may have been because she was far too busy, but it may also have been because she had a difficult childhood. Her parents were immigrants and her father struggled with manic-depression. Claire describes her mother as constantly working and trying to do everything she felt the world expected of her. She had a strong sense of the need to do things for others, to the point of sometimes neglecting her own needs. As Claire said her mother was 'always giving others the biggest piece of cake.' She felt that her mom expected her to be like her. Claire says that she understood the need to be kind and supportive to others but pushed back feeling that 'I wanted the biggest piece of cake sometimes too!' Claire felt more of an affinity with her father than with her mother who she states appeared to favour her brother. She feels that overtly she was told that she was very intelligent and could do anything she wanted in life but covertly she felt the message was that men were superior, were supposed to be given more opportunities and were automatically going to be more accepted in the world in ways that women were not.

A lot of this parallels later in this story with Claire's own sacrifice to take care of her daughter instead of pursing her own PhD. She was drawn to both STEM related fields and STEM trained men, but says that she never had the confidence that she could succeed and so chose psychology as field because it was less daunting to her. Claire says it is important to note that her mother was supportive of her art career if not her science interests. In fact, Claire was

delighted when her daughter took calculus in university and hoped she would be drawn to a STEM subject as a career but she chose commerce and social sciences in the end.

When Claire was very young, she was sent to a Hebrew school that was Hebrew half the day and English academic studies the other half. Claire did not like school very much saying several times that she hated it. Instead she said she was much more interested in the natural world and in reading. She loved to read. She took out multiple books – science books, autobiographies and stories – from the library and this is where she credits much of her early learning. Early on, based on some tests that Claire remembers being given, she was placed in an accelerated program which made the shy, awkward feeling and sensitive Claire feel more out of place, often being in classes with children who were older than her. In those early days, Claire associated math with arithmetic and fast problem solving. She did not like it, and felt she was no good at it. She remembers a time when she was about 8 when the teacher told her mother that she was behind because she didn't know her multiplication tables. Her mom spent weeks grilling her with multiplication problems to help her learn. She didn't enjoy the process and found it stressful. She recalls lying under the dining room table trying to answer questions. She especially disliked the 9 times tables and wonders why no one taught her tricks to calculate them. Later she came to distinguish this from what she calls math, which is more about abstract relationships and ideas than about rote learning.

Throughout Claire's story, math is described in positive ways. It is fun, insightful, a game or puzzle, logical, solid and reassuring as well as magical, amazing or even beautiful. She says her interest in math has increased over the years and she would still like to pursue it.

Her mother moved her to grade 6 in the public school when she was nine, almost ten, at the start of the year, again making her younger than her classmates. She loved grade 6. She had

a male teacher who she became quite attached to, likely she says because he was a father figure for her and she still badly missed her own father. In contrast, grades 7 and 8 were not good experiences. She avoided school and found excuses to stay home, often saying she didn't feel well. She describes herself as being depressed. At the end of grade 8 she heard about an all girl's private school and begged her mom to let her write the scholarship entrance exam.

She didn't win the scholarship although the school gave her a smaller scholarship for their extra-curricular fine art program. Several times, she mentioned that this school saved her—intellectually and emotionally – she learned how to study, how to write exams and essays. She describes learning the rudiments of critical thinking and analysis. She also began to understand mathematics as something more than just addition and subtraction, multiplication and division. She comments that she feels the school, with its focus on encouraging students and pushing them to do their best, may well have saved her life. Teachers were supportive and she began to learn. Many of the teachers were woman so she had positive female role models: an influential history teacher who told the class about stories about England during the war, an English teacher who made literature fascinating. The private school was a demanding school though, and the commute was over an hour, taking three different buses, a lot of waiting in the cold in winter, and a long walk at the end. In grades 9 and 10, she was often exhausted to the point of falling asleep in the bus on the way home from school every day.

At the end of grade 10, she had to leave the private school because her mother said it was too expensive. She entered he public system in grade 11. She had a heavy course load to start and found that she was a bit behind in science so taking grade 12 Chemistry was a baptism by fire. However, she said she was well ahead in English. She also stated that she found algebra easier

than geometry but eventually got the hang of it through the help of a tutor and through perseverance.

In grade 13, she was taking a full course load, including algebra and logic, which she really liked. At that point, she was living away from home, after many struggles with her mother, and was far away from her local high school, which meant another long bus ride. She had chosen a very heavy course load, three grade 13 math courses, plus physics and four other courses. This caused her to became overwhelmed with the work, which was more than she could manage. Looking back, she realizes that there was no need to take such a heavy load as she already had two grade 13 level courses and could have graduated with four more. Instead, she just quit. She thought of going back to school in January to do a half year and to finish up at another high school with a semester system, but it was the year of a big teacher's strike and the schools closed down for thirteen weeks making any course work pretty much impossible.

In the end she graduated with a grade 12 diploma at 17. Her mother had always encouraged her art, and she had taken drawing and painting and sculpture courses at the local art school during several of her high school summers. So, she applied to OCA in Toronto and was accepted, though the school was a bit reluctant to take her because of her age. She was excited but also somewhat overwhelmed by the new experience often feeling awkward in social situations. She quickly found a boyfriend, but describes him as entirely unsuitable for her.

Afterwards begins an educational odyssey that ends up at the brink of a PhD in psychology. She took a year off from art school and worked in camera stores, going back to finish her final year, doing some computer programming at a very basic level and taking electronics and holography. She still felt undirected. She worked in Toronto doing graphics for a system that was a forerunner to today's internet. She then moved to Ottawa to do the same work

but the job only lasted about a year. She then found work as a database administrator with Bell Northern Research. After two years there, unable to advance because she didn't have a university degree, she applied and was accepted provisionally into an Art Therapy Master's degree in the US, needing to take some basic university psychology courses. She had already done a computer science introductory credit course and an introduction to psychology courses at Carleton University and done well so Carleton was willing to let her take the 2nd and 3rd year courses she needed for the graduate program. She did a full year of courses including statistics in which she did very well. During that year she spoke with some local art therapists who assured her that there was no work available locally and that art therapy was really not recognized as a legitimate profession the way it was in the UK and other places.

In the end she applied and was accepted into an honor's degree program at Carleton and after finishing won an NSCRC scholarship for graduate work. She describes interviewing professors to see if she found them interesting and, as much as possible, choosing her courses that way. She found performing in school stressful, but loved her classes. She felt as though she belonged there, a new experience for her in a classroom setting. At one point she told one of her professors that she wanted to take a math class and he strongly dissuaded her, saying it would likely bring her average down.

She had to change supervisors a couple of times. The first supervisor and her just couldn't figure out how to work together. The second supervisor, who was a pilot was killed in a plane crash. She finally settled on an adjunct supervisor who worked in communications research. He had a study that he wanted her to do in an area called masking theory. The study was designed to determine compression algorithm standards for video coding compression. She recalls studying Fourier transforms at the time and understanding them. But says she can no

longer recall much about the mathematics. Sometime during her masters, she became pregnant with her daughter. The father was not interested in raising a child and she decided to have the child on her own. It was a difficult pregnancy and she was asked to leave the lab for the duration as she was having trouble walking. After her daughter's was born, she stayed home with her for some months but was finally able to find childcare and come back for her final year to finish her thesis research. She applied for and won a second NSERC for her PhD but ended up not using it. At the time she was exhausted and unsure that a PhD would give her much opportunity for work. She felt that she needed to "grow up" and get a real job. Looking back, she understands her decision, but also regrets it.

She had also applied for a management trainee program with the federal government, and was also accepted but deferred to do a one-year B.Ed. thinking that perhaps she could get teaching work. At teacher's college, she completed the program for teaching JK to grade 6. She was not impressed by the practicums, saying she was appalled by the way that math was taught and said she felt that children could be taught these subjects in ways that were significantly more inspiring. She could now see so many options for teaching math that would make it, she thought, far more engaging for children. She mentioned teaching children about alternative number bases for example, as well as number puzzles, and some actual practice applying math to real world problems. Supply teaching was an option but, even there, she could not depend on the minimum of 20-30 hours work a week necessary to maintain her daycare subsidy. Therefore, she took the management trainee program with the Federal government, which led to a position with Corrections Canada. She no longer had the option of the PhD funding as she had already had one deferment while pregnant.

It was during this time too that she met her partner, who was a theoretical physicist and natural mathematician according to her. They had long discussions about physics and mathematics and philosophy. She was always aware that her limited mathematics background limited the depth they could go into in some of these discussions. Her partner helped her understand Fourier transforms, which she needed for her Master's degree. She speaks fluently about her masters even while saying she's forgotten a lot of the technical details: she also says she felt like a mouse in a mouse hole and didn't feel like she was studying anything really profound. She felt she had less of an aptitude for some of the mathematically based engineering side, feeling out of her depth with respect to the various machines that were used for running the tests on subjects. Generally, she felt more at home studying theory rather than actually being in the lab and designing and running machinery.

Claire. regrets not doing her PhD as she felt that would have made her feel like a worthier human being. She often mentions people's accomplishments and stated somewhat jokingly that she had PhD envy of her partner. She says that it was harder for women, during her time: it seemed that, although you could participate in STEM careers, you had to be at the top of your class if you were a woman. Her partner, who had taught physics early in his career said he loved having women in his classes but admitted it was rare. He always encouraged her daughter in the sciences and helped tutor her through her university calculus course. She was happy to report that this was less of an issue now as more women seemed to be in the field.

Currently, she says she is still interested in mathematics and had been talking about relearning it. She had a good friend in her painting class, who sadly passed recently, who had been a math professor and he was excited about going back and doing his research again for fun.

There are many contrasts in Claire's story: slow and fast learning, good and bad teaching, close and distant relationships. Throughout, Claire has expressed an appreciation for the magical in math. She said that it must be amazing to be a really good mathematician. She describes having experience with the Fibonacci sequence during a meditation retreat. She wrote down her intuitions about how the sequences were related and found that most of them were correct. She describes this as feeling almost magical and/or beautiful. She says she envies mathematicians who can experience such things throughout their careers. "These truths are beautiful."

Claire's story seems to this interviewer to be a complex and deeply interwoven tale about relationships and striving to find place. Her relationship with math was a relationship with people. A father gone too soon, good teachers, thrilling ideas, and a very good friend for a partner who she wished to engage with more deeply. It is also about lost opportunities: the friend lost suddenly, a regretted choice not to go further academically and a sense of being misplaced educationally. She describes math as something of a gateway to not only fun and exciting ideas but also deeper meaning.

A4.4 Martin: Guided by Curiosity

Martin went to school in Quebec until around grade 2 and then a francophone school in grade 3 in British Columbia. It was not in a good area of town, he says. His earliest memory in math may have been in grade one and it was converting between Fahrenheit and Celsius. Martin was never particularly math oriented but was also not scared of it. Any difficulties that he faced, he said came from learning math in a new language when he switched from Francophone to English school later on. His parents were both teachers and his father, who came from a big Catholic family, had a traditional education where he learned Latin and Greek. Every year, his father got him to do the Dictée des Amériques, to spell properly. His family valued words and rhetoric. Martin was likewise more interested in culture and language.

Though he did not have many stories about his earlier schooling, he did remember a Mr. X who taught him grade 7 or 8 though it may have been grade 9. He remembers learning functions. Mr. X taught really well and he thinks may have increased his confidence in math. Mr. X said that the students didn't need to be afraid. He made it fun and he would say things like "you know don't feel dumb, I had difficulties too." The Highschool Martin attended in BC was ranked quite high and the teachers were very inspiring. He remembers a grade 12 math teacher, who had almost had a PhD in chemistry, that he got along with quite well. They would have conversations where the teacher would be talking about square roots and Martin would be talking about poetry.

Martin was very curious and interested in a great many things, taking various courses outside of his degree at University such as linguistics and Latin. During this period, he also decided to take Calculus and linear algebra. He thought that perhaps he took them because of the mystique of math being hard. His father grew exasperated since as soon as Martin got a little bit

good at something new, he would lose interest in it. His father was afraid that Martin would never find a profession.

Martin said he fell in love with math in his PhD, through programming, as he was using a method called Gini decompression, which at the time had no existing program to calculate it so he stated, “The formula was in sigma notation and I had to program it myself and it made me feel good to be able to just take a formula and translate it into a program” He says that he likes programming a lot finding math and programming self-reinforcing. In fact, this reminds him of a time when Martin was in his young twenties and his father and him learned PHP to create a program to help his father tally student’s grades. He feels that programming fueled his interest in math. Since his parents were teachers, he had access to a computer and would “play with it basically all summer, staying up to 4am programming.”

Martin sounds like a curious and tenacious student, learning widely and deeply. He spoke about being annoyed by his quantitative sociology professor, during his Bachelors, who could not answer his questions such as a time when the professor was talking about cross tables and chi square. Martin wanted to understand the relationship but the professor didn’t know. This surprised him as he expected the professor to have an interest. To Martin one of the most important intellectual traits is curiosity. In fact, he thinks it might be the most valuable trait you could have in math. He tries to teach his own students in quantitative sociology in a way that he wasn’t taught. When a student asked Martin how you got p value from Chi, saying “Is it magic” Martin went out of his way to program a simulation so that the students could see it for themselves.

Deep learning, curiosity and real understanding are entangled in Martin’s story. He relates the quote “if you can’t explain something to a child, you don’t understand it.” He

compares this to intellectuals that have a vanity in needing to sound smart so they will name drop Hegel or someone of similar heft but when questioned cannot go into much depth on the topic. “It’s all tied up with words” He says “I criticize people in my profession who don’t really love the discipline. They just love to sound smart, I feel.”

As a teacher, he likes to tease his sociology students, who have often self-selected away from math, by telling them that, given that they all see themselves as philosophers then they should accept that at the entrance to Plato’s academy there’s a sign that says “Only geometers may enter” He says that he likes to pique his student’s curiosity by connecting math ideas to other concepts more in the social domain such as how Euler’s constant is about compound interest and how the metric system, part of the origin of regression, was motivated by the French revolution.

Martin strikes one as someone who is comfortable with the struggle that surrounds gaining knowledge. Even though he has encountered concepts, in mathematics, that were not immediately obvious to him, such as complex numbers, he works through them and he has the internal motivation to gain ever more knowledge. He wishes that he could take some courses in the math department though he recognizes that this may not fit into his already busy schedule. He says that he feels that “it would really help me to be around people who were passionate about math.” Martin feels that he is quite good at computation but wishes he had more of an intuitive grasp on things like factor analysis.

Through the conversation, you could feel a sophisticated tension between the subjective and the objective. Martin says he finds math therapeutic. He says that as a sociologist a situation can be read in a number of ways and there is no way to deny someone’s experience. It is very subjective. This along with his commentary about intellectual vanity and students’ fear driving

them away from an analytical tool which could help them with their thinking and sociology and his own drive to understand deeply paint a picture of someone with a highly intellectual orientation. In fact, he says that he feels our society is very focused on fun and lifestyle but would benefit from more of knowledge focus, would benefit from a challenge. Math could indeed provide that challenge. Or in the words of Martin, “it might allow them to think something that they have never thought before.”

A4.5 Hanna: A Drive to Understand

Hanna describes her interest and what she finds valuable in math in similar ways. She says she enjoys the challenge of math, of working through something stepwise, problem solving and sitting with a problem. She values perseverance and curiosity in mathematics and sees a willingness to try new things as important. This carries over to her professional life as problem-solving hardware issues in an R&D unit.

She says that she was a pretty good student who needed to really understand the problem and was a perfectionist. Given that she was a good student, she was often given a lot of space in class which acted to her benefit as it gave her time to tackle the problem from different strategies such as starting backwards from the answer.

This may stem, in part, from feeling supported both at school and at home. Both of her parents had post graduate degrees. Her father was a high school math teacher who could help her with STEM related subjects and her mother had a degree in history and ended up in business so could help with anything else. In terms of math, she said, consequentially, she "never had a bad experience that would discourage me from learning about it" In contrast, many of her friends did not have this sort of support perhaps because her friends' parents either did not know the material or did not have time to sit down and work through it. She also said that in elementary school, there really isn't a lot of math supports and that by high school many students have already given up. If they chose to go on, she felt that seeking help at school would be helpful and afterwards they would have been less frustrated.

Hanna could not think of a time, especially in the early years, where she found math challenging. In fact, she didn't really have to put in much effort as it came pretty naturally. Her earliest memory in math, was a game called Chip it where you used plastic chocolate chips (and

sometimes real one) to fill up spaces and had to solve simple math problems. She figured that this game probably put her ahead of the curve in grades 1 and 2. It wasn't until grade 10 where some concepts became more difficult such as trigonometry. Given what people told her, she was expecting it to be extremely difficult but didn't find it quite so bad though it was often hard to visualize. She said that the unit circle really helped. Instead of being able to jump ahead to the answer, as she had done before, she really had to work through trigonometry problems step by step.

She also found online classes challenging and still does not feel solid in all topics presented in an online high school precalculus 12 (precal 11, calc 11 then precal 12 and then calc 12) class such as limits. Not only was it an online class, which she didn't really, like but it was also different than her peers were learning because there were curricular changes ongoing at the time. Though she understood the concepts she could get lost on what to do next procedurally however with practice the ideas that were shaky became solid but it was with practice in an in-person environment. It is important to note that the topics of her grade 12 calculus class were a bit different than what students encounter in Ontario as her British Columbia high school class covered both derivatives and integrals but in Ontario instead of integrals, they do vectors and some matrix algebra. Her grade 12 calculus was prepped to compete with a UBC class. And if you presented to UBC with transcripts that you took that class, you could challenge the UBC calculus exam and potentially be exempted from the class.

COVID happened when Hanna was in University and she remembers that she had some issue again with an online class. There are certain topics, that she still encounters, such as sequences and series that she never really learned in depth.

Nevertheless, she says that both her senior physics and math teacher were quite good. In particular, she really liked how her math teacher handled homework. All assignments and tests would be set at the beginning of the unit. Students could choose if and when to do them. However, at the time of testing, everyone had to hand in a particular homework assignment of the teacher's choosing. If they got above a particular percent then she thinks that the homework portion was just automatically given a 100% or something similar. This gave Hanna a lot of flexibility and freedom on how and when she chose to do her work but at the same time it encouraged doing homework in the class.

She also really liked her grade 12 physics teacher, describing him as fantastic. She really enjoyed what she was learning such as basic forces, and how things interact at a very small and a very big level. "Investigation is really interesting to me." This fed a pre-existing interest in space that she has had from a young age. She would watch documentaries with her parents on rovers, probes, telescopes, nebulas and even a whole miniseries on quantum mechanics. The latter she found absolutely fascinating. She even worked it into her grade 6 science project on extreme environments as her extreme environment was space. Her grade 6 teacher asked her parents how she had learned this stuff but Hanna had done it on her own through watching about it and going on to learn more.

In contrast, she didn't really enjoy her early language courses although she really likes learning them now. She said her elementary school language teachers didn't seem to like teaching. Her school in BC introduced French very late and it wasn't taught either very often or in much depth. In high school, she switched to Spanish classes which she liked more.

Besides physics, that Hanna ultimately majored in, in University, she has a wide range of interests that include music, drawing and painting. She says that math and physics are very

similar though physics is more situational and interpretative whereas with math you see the problem and know what strategy to apply. In physics, there are more assumptions so you have to read the whole scenario. When she was choosing post-secondary education, she applied for a number of programs and initially chose a bachelor's of arts undeclared at Carleton. However, after realizing that there was also an astrophysics program, she reapplied on a whim and was accepted. She graduated with a major in astrophysics and a minor in music.

This led to her job on an R&D team, which, in practice, she describes as quite close to engineering, saying, "I do a lot of just physical work with our systems and their circuit boards… If something goes wrong, I have to figure out where it's wrong and if there is an issue with the design." She finds parallels between how she approached challenges in the past and how she approaches challenges in her job now. She said that she's had "a couple really weird failure modes that I've had to just kind of go through and slowly just eliminate piece by piece by piece until you kind of just see something, and then you're like, wait a second could this be, for whatever reason, the problem?" She even describes it as often frustrating but she knows that frustration is part of the process and says that people who can't handle being frustrated wouldn't choice or stay in the field very long. She attributes her comfort with frustration from learning from a young age that "if I do work hard and I put in effort, eventually I will figure something out."

A4.6: James: Success through Support

James grew up in lower mainland BC in an agricultural area. His parents both lost their jobs shortly after he was born due to a recession. His mother went back to school when he was quite young. She worked in the court system as a stenographer. As she had the economic condition and ability to actually get into school, she become a St. Nicholas Montessori specialist teacher. There were various Montessori materials around so when he was about 4 or 5, his very earliest memory of math was getting lost in the moment of counting Montessori glass beads. He just wanted to know how high you could count on them. He also remembers doing volume calculations with the materials at the UBC while his mom was training and later on using the test tube division and square root calculating materials. James attended public school in grades 1 and 2 but then he went to Montessori private school for grades 3 and 4. During most of that time, he was taught by his mother though there was a period where another, less well-trained teacher was working in the class, which lead to some gaps in the students learning (including his).

One theme in James's story is the idea of support and availability of resources. This comes out in a number of ways including systemic lack of support such as economic availability or lack thereof of tutors for advanced mathematics, potential discrimination against learning differences or the idea of the prepared classroom such as in a Montessori setting and how the experience of independence could lead to later resilience.

James frequently mentions the importance of his parents support in overcoming obstacles and in influencing his future decision. His mother, recognizing his struggles with social interaction, placed him in 4H at a young age, in which he participated in from the ages of 9 to 17. He felt that working with animals helped him bridge that social gap and as a bonus "tracking progress was just numbers and data." His older siblings had severe learning disabilities and the

scaffolding that was in place helped him excel through elementary and high school, giving him skills and tools that helped him learn things at the right time.

His father also helped influence his mathematical identity. He was a high school drop-out who had trained as an artist but James remembers that he restored cars for a living and as James said you measure twice then a third time before you cut. Precision and accuracy are important. His dad was the one to always push him reminding him "never to rest on my laurels. He didn't want me to waste my talent."

His family was very political. They would discuss trends and statistics. If someone made a misrepresentation, they would discuss it. The emphasis on both data and its interpretation as well as misinterpretation turns out to be quite important in his future career as a math communicator. James had two goals in life. The first was to be an astrophysicist because he was fascinated by space and science and wanted to understand how it all worked. The second was to serve in government. He accomplished the latter.

After he attended Montessori school, he was put back into the public school for the rest of his education. He says that the only time he felt truly devastated was when he applied to be in an advanced math class but he didn't end up scoring within the top 24 students and therefore did not get the opportunity. However, James had an understanding that failure was part of life because of the strong support he had from his family. Quoting Captain Picard talking with Data, from Star Trek, he says "You can do everything right and still fail." Failure, James learned from his family, or doing something wrong and asking for help is not a sign of weakness. In fact, it is a sign of strength.

This attitude served him well in attaining his International Baccalaureate during high school. This was the second moment of math challenge that he could think of. He said that his

chemistry teacher didn't have all the IB materials. More problematically, his physics and calculus teacher was "a bit kooky and he would spend half the class talking about conspiracy theories." He also wasn't very good at teaching the higher math. The students who were getting their IB started working together as a group. It was a very diverse group of people with remarkably little in common and that afterwards they didn't remain friends rather it was just circumstance that had brought them together. One of the students had a private tutor and this student would pass on the teaching to the others. In the end, James was the only one to graduate with an IB in that year. He feels that his resilience came, in part, from his early experience at Montessori. In fact, he says, teachers commented often that he must be a Montessori student.

On the other hand, his favourite teacher in high school was a history teacher from England who would bring history alive as he was a child during WW2. James felt encouraged and seen by this teacher. James's family came from a leftist perspective or as James put it "anti-Mulroney, anti-Regan, ant-Thatcherite" that was quite different from the kids in the area. The teacher made space in the class for James to give his side. He said it could have easily felt like the class was ganging up on him. History was another subject that James really liked. It ended up being the degree he took in his undergraduate. Pursuing history was a challenge as he had undiagnosed dyslexia. Despite getting help in grade 5 from a special program, grammar and spelling were always difficult for him. He chose not to get it formally recognized because he said, in University, he was told that if he did, he would find himself restricted from certain kinds of courses, which he would need to do graduate school.

For his masters in History, he used a statistical approach to understanding social dynamics with a strong focus on textual analysis (statistical history, and socio-economic history). This led to his first job, which was to do textual analysis of source material. He was figuring out

a way to automate that function, to use textual elements to understand semantic channels. He loved this work but the job ended when the government did a workplace adjustment. Instead of allowing for anyone to lose their jobs, he moved his team to a new position and ended up working in oversight and developing statistical analysis and the development of program calculating tools for CPP and OAS. In his next job, he got himself a reputation as a bullshit detector by calling out errors in statistical analysis. All of this led to his current role as the Data Literacy Lead for a governmental organization

He describes his job as math communication. He says that part of his training is to confront complexity and find the simple building blocks to that complexity. “I work to break down ideas,” he says, “My focus outside of my direct work responsibilities has been building the data complexity framework for the Government of Canada.” He said it’s not just about the existing data framework, which he helped build, but about processes. It is about helping people to use the data. To not just think in terms of numbers but about connections between those things and the strength of those connections can be defined numerically.

He also sees his job as building trust between people and numbers so they can make strategic decisions and accept critique. He emphasizes the social connection in data and math communication, saying “you have to bring the numbers alive so they mean something.” Bringing us to James’ definition of math positivity, he says that he sees his job as not merely communicating about the data but helping people become comfortable with it. Math positivity to James is about being comfortable with numbers and data. And to some degree ambiguity.

But, he says, it is also important that the leadership of any organization walk the walk when it comes to clear communication about math and data. If they don’t take part in the training

themselves, to provide support to their employees then no one else is going to do it. Modelling this behaviour is really important. It is a key to growing math advocacy and communication.

Math itself, according to James "is a concrete way of expressing reality. It gives you a common chalkboard for us to have a discussion." At the end of our conversation, James spoke about how this came up in the Pandemic. Given his previous background in history, his strong political stance, his sensitivity to inequality and his belief that everyone can be good at math and numbers given the right support, he felt that there was some bad numerical communication during the pandemic, and that it was hard to see history repeat itself.

James comes across as a staunch proponent of numeracy. He feels that it is not necessary for everyone to understand all the math but to be comfortable with it so that they can ask the right questions. He believes that given equitable support, everyone can have access to the data and that this data can be the basis for productive conversation. He feels we need more and better math communicators.

Appendix 5: Individual Data Analysis Example: Hanna

Step 1-2: Excerpt of coded interview

Story: 'math and me'

Hanna: I think… For me, it's… I… like, I have an interest in…just learning how things work, and math is a good tool for understanding how things work, because it's just used in so many different applications. Um… I think it's really just not being… **scared** of it, like… I know a lot of people who- Say they don't like math, and generally it's just because they… are not… good at it. Um [click] and the only reason that I find that I'm good at it is because I sat down a lot, and I just worked on it for a long amount of time. Um so I enjoyed, like, working through stepwise, like, logical…sort of problem solving. Um and yeah, it's just… it's… I have fun doing it. Um so I think that's kind of the positive aspect, is I just, like- I enjoy that logical process/, and that if it's… if it is tough. That's not a reason for me to dislike it.

T: Okay. And why do you think that, for you, it wasn't a reason to dislike it?

Story – 'parental support'

Hanna: I think I just grew up… with a lot of math ah in my household. My dad's a math teacher. So…And… Both my parents, have… college or university degrees so um… so when I was, like, learning stuff in class, if I **did** have trouble, I could go home and they would sort of help me through it. So I never really had a… **bad** experience that would… discourage me from… learning about it.

Affect words/phrases
Parental support
Working the problem-The Difficult Problem
Application

Episodes/events/experiences

1. My system was supportive – parents and teachers. Friends frustrated
2. Curiosity about how things work – parents, teachers, physics teacher
3. Chip-it board game
4. Math gets complex – the importance of perseverance – grade 10
5. Different classes – calculus in different provinces and online work
6. Kind of a perfectionist
7. Good system – physic versus math. Good math teacher
8. Music or Astrophysics
9. Really weird failure modes – peers in grad school, colleagues at work

10. Math contests
11. Didn't like language courses because of teachers
12. Math Values

Step 3-4: Full narrative and correction

Step 5[4]: Affective words/phrases, initial identification: Hanna's data

Emotions (25) Momentary state	If I did have trouble * Never really had a bad experience… discourage * frustrating (friends) * not being scared * frustrating * that was fun * I find it absolutely fascinating * it was fascinating * I did not find it difficult * conceptually more difficult * I had no idea what to do next * limits were tough * derivatives easier * really get into * having trouble * it encourages you to practice* kind of a whim * it had a good time * it's really frustrating * increasingly frustrating * I'm glad * and just do it for fun * they were fun to do * already not really enjoyed French * I did enjoy (Spanish)	Trouble - 1 Never had a bad – 1 Frustrating – 4 Not scared - 1 Fun – 3 Fascinating – 2 Not difficult – 1 More difficult - 1 No idea what to do – 1 Easier – 1 Really get into – 1 Trouble – 1 Encourages - 1 Whim – 1 Good time – 1 Glad – 1 Not enjoy - 1 Enjoy - 1
Negative words were often used in conjunction with other people: discourage, trouble, frustrating, difficult, trouble, no idea what to do Positive words: never had a bad, not scared, not difficult, fun, fascinating, easier, really get into, encourages, whim, good time, glad, enjoy Contrasts include easy vs very difficult, frustrating vs fascinating		
Attitudes (25) expectations	Enjoyed working through stepwise, logical, problem solving * I have fun doing it * enjoy logical process * every time they went for help, it was such a frustrating experience (friends) so they didn't bother * chore to go get help * just associating bad things (with math) * people were already making the choice not to bother * I picked it because enjoyed it *I really like space * I like learning how things work * I really enjoy art * it	Enjoy – 4 Fun – 1 Frustrating – 1 A chore – 1 Associating bad - 1 Not to bother – 1 Like – 2 Didn't like - 5 Interesting – 3

[4] The order of the steps is out of sequence in this example so as to better show connections between steps. For example, though affective language was highlighted in the transcripts before the narratives were written and tentatively categorized. This was done in earnest afterwards. However, to see the connections between the transcripts and the affective word categories, they are placed, in this example, one after another.

	was interesting * absolutely loved that game * expecting it to be extremely difficult * normal people complaining (trig) *Jump to the answer… and trust I got it right * fantastic way to do stuff * really don't like sums and series * didn't like it – didn't understand it * didn't like getting things wrong * it was really interesting * I loved that system for homework* really interesting topics in music * didn't like language courses * teachers didn't like it (language teachers)	Loved – 2 Difficult – 1 Complaining – 1 Trust – 1 Fantastic - 1
When I was compiling the affect words, I noticed the tendency to say things like 'it was interesting' and this was easy or difficult as in there was an expectation that this will be easy or difficult. However, the most common words are (didn't) like and enjoy.		
Beliefs (7)	Math is a good tool for understanding how things work * math is used in so many applications * it's really just not being scared of it * People who don't like math are generally not good at it * If you don't want to learn the content, you won't go for help * If they chose to go, it was helpful * Physics is very much math	Math is a tool – 2 Math +, not scared – 1 Math and skill – 1 Don't like, help – 1 Help is helpful – 1 Physics is math - 1
Math is a useful tool for understanding how things work Math positivity is about not being afraid People who don't like math are generally not good at it and if you don't want to learn the content, you probably won't go for help. Teachers are helpful Physics is very much mathematics.		
Values (16)	Chip it was helpful * It was good (physics) * want to learn * willingness to try new things * really spend time and don't get discouraged * that was both good and probably not good: working alone is good and bad as you can sit and try new things * I think they were all quite good (teachers) * physics teacher fantastic and calculus teacher good * She had a very good system. I found this worked really well for math * It was up to each person to gauge (autonomy) * system of homework was fantastic (autonomy and practice) * questions were relevant * good practice questions * curiosity and perseverance * really good physics teacher	Games helpful - 1 Physics good – 1 Want to learn/try – 3 Perseverance – 3 Teachers good – 4 Autonomy, good – 2 Working alone bad - 1 Usefulness of Q – 2 Curiosity – 1
This section, in particular was difficult, until I recognized that statements of good and bad were value judgements not always defined. A teacher is good because they developed a system of homework that was good as it gave autonomy. Autonomy is good.		

<table>
<tr><td colspan="3">Physics is good as it allows you to investigate why and how things work therefore it is investigation and curiosity which are values.

Perseverance and willingness to try and not getting discourage could be synonyms.*

*In the initial coding, her statement about “being good at it because I sat down…” and worked through it, was placed in identity but it is also a value statement.</td></tr>
<tr><td>Identity (17)</td><td>Interest in how things work * I find that I'm good at it is because I sat down a lot, and I just worked on it for a long amount of time * if it’s tough, that’s not a reason to dislike it * I never had a bad experience that would discourage me * if they went for help, it did not help (friends) * investigation was really interesting to me * I have a wide range of interests * I have an easy time understanding STEM concepts * early on, it came naturally * I put a lot of effort into understanding stuff * I’m a perfectionist * I don’t like getting things wrong *I also was a pretty good student * I’m okay with being frustrated * if I do work hard, put in the effort, eventually I will figure something out * if I get to learn something in the course, I’m pretty engaged * math is curiosity and perseverance</td><td>Hanna:
Interest, investigation– 2
Work for a long time – 1
Not discouraged – 3
Wide ranging interests – 1
Easy understanding – 1
Came naturally – 1
Put in effort – 2
Perfectionist – 2
Good student - 1
Engaged – 1

Friends:
Family didn’t help – 1

Math:
Curiosity – 1
Perseverance - 1</td></tr>
<tr><td colspan="3">Many phrases collapse into the idea of working hard, not getting discouraged and a pursuit of understanding. She sees herself as a student who, at least in the early years, found school easy. Though mentioned less in affect statements, she had support of her family which she did not see as much in her friends’ relationships. What math is corresponds with her own strengths: curiosity and perseverance.</td></tr>
</table>

Step 6 - Themes, character, plot

The Difficult Problem is a character in this story as it appears and needs to be related to in a positive way for success. Given that Hanna values perseverance and being curious, leading to trying new things and new strategies, this means that she has a good relationship with the Difficult Problem.

Her parents are also important characters as she found them helpful and supportive throughout her schooling as her father was a high school math teacher and her mother had a

history major. They also supported her interests in space and physics. The theme of support is important as she said that many of her friends did not have parents who were able to support them when they had challenges in mathematics. By the time, these friends, went to high school, they no longer saw themselves as mathematically capable and had chosen other paths.

Mathematics itself, for the most part wasn't difficult, even if it was occasionally challenging such as trigonometry, but online schooling made learning difficult. She had to do online learning twice. First in high school because she took a course out of sequence and second during the pandemic in university. She says that there are still some ideas – limits, series and sequences – that she still feels shaky on as they were presented to her online.

Teachers were an important character in her story as well. She said that her math teacher was quite good and that she really liked how this teacher handled homework as it gave her a sense of autonomy while still supporting students' skill building. Autonomy is an interesting theme in Hanna's story as initially she is left on her own because she is fairly advanced so the teacher just let her do her own thing but this also gave Hanna a chance to develop problem solving skills. This would become extremely important later on in her life from her degree in astrophysics to her job in an R&D department troubleshooting hardware issues. As she said, someone that wasn't comfortable with the Difficult Problems would get very frustrated and leave in her job.

Her physics teacher was also "fantastic," according to her, though it was somewhat unclear what he did that was so great other than making available information about the very big and the very small – the inner workings of the universe. It is also possible that the good teachers were those that liked teaching if they are to be contrasted with her language teachers in BC who she felt didn't like teaching and therefore made her not like languages initially.

Step 6 - Affect and Value words in the Interview

Negative emotion words/phrases often were her describing other people's experiences such as someone having a "bad experience," "having trouble" or "being frustrated." Whereas she more commonly used positive emotion words/phrases to talk about her own experiences of mathematics including "not scared," "fascinating, easy" and "enjoy." This sets up a contrast to describing the experiences of others who are not mathematically positive, in her estimation, with herself who is.

Expressions of attitudes, for Hanna, most commonly involved the words (don't) like and enjoy or that she finds something interesting such as quantum mechanics or astronomy. Her beliefs were positive to neutral in nature as well such as "Math is a useful tool for understanding how things work," "Teachers are helpful," and "Physics is very much mathematics." One belief sticks out as it is "People who don't like math are generally not good at it and don't want to learn the content" so they probably won't go for help in it. Therefore, there is an equating of skill level with how much they like mathematics.

When it comes to identity, many phrases collapse into the idea of working hard, not getting discouraged and a pursuit of understanding. She sees herself as a student, who at least in the early years, found school easy.

The easiest way to identify value statements in this interview was to look for the adjective good and this would relate to what she finds valuable such as a system of homework being good as it allows for autonomy. Working by yourself is good as it allows you to try new things or strategies for problem solving. Many statements are about problem solving and imply a persistence through problem solving such as a "willingness to try new things," "spend time and

don't get discouraged," "working alone is good and bad as you can sit and try new things." One minor value mentioned is the relevance of the questions but it is unclear if this meant they were relevant to tests or relevant to life in general though I suspect the former as she mentions, almost in passing, that she was a perfectionist and doing well was important. Though she does not make it a value statement, the theme of support making her mathematical journey possible, suggests that support is valuable.

Support versus autonomy could be seen as a contradiction even within her own statement that being alone in grade school to problem solve was both good and bad though ultimately it seems to have tipped over into good.

Values: support, persistence, curiosity, autonomy, understanding the universe.

Step 6 - Hanna. Turning Point Timeline (as found in Section 6.1e)

Event – Time	**Change in expectation or value**	**Turning points Outcome**
Early life Chip it math game Mathematical skill	Expectations: Math is fun Unclear of expectations Value: Support of parents	Mathematically positive
Grade school Fascinated by physics How things worked	Expectations: Math is easy Problem solving is interesting Value: curiosity, understanding the universe	Mathematically positive
High school More complex problems Working the problem	Expectations: Math is easy Value: persistence, curiosity, autonomy, understanding the universe	Threatening as math was not as easy Outcome: problem solved through persistence
Online classes Too simple, not sufficient to explore problems Subjects never solidified	Expectations: I understand math No change in value	Threatening Online education was not sufficient

		Outcome: online education is the problem
University Astrophysics and music Very interesting	Expectations: I understand math and find it interesting No change in value	Mathematically positive
Job in R&D Weird failure modes Work the problem	No	Mathematically positive

Emergence and threat to mathematical positivity

Her definition is that mathematical positivity is about not being afraid. At first glance, it might seem that this story is overwhelmingly and simply about mathematical positivity but upon closer examination of the timeline, it becomes clear that something interesting is happening with value as her mathematical positivity encounters threats at least twice. Once in the form of more difficult subject matter that she couldn't "Jump to the answer… and trust I got it right" And a second time when she has difficulty understanding and retaining information presented in online classes. Despite these challenges, she never describes her experience as negative. Instead they seem to solidify different values that she already had such as persistence – working the problem – and the right balance of autonomy and support. Her own values of curiosity, and persistence seem to help overcome the threat to mathematical positivity.

Step 6 - Hanna: Connection to the Theoretical Framework

Her family are important socializers as her father, a high school mathematics teacher was able to help her when needed and her parents supported her interest in space and physics

> 'I'd watch…like, documentaries with my parents um and a lot of them were…on space, and so either just… um rovers and probes, some of the telescopes, also certain nebulas. There was… a whole set of, like, 4 or 5 in a row is, like, a mini-series on…quantum

> mechanics for one of the… one of the ones that we watched, and I found it absolutely fascinating'

An early experience with mathematics games may also have set her up for success. She describes herself as a "pretty, good student" who did not find mathematics in the early years challenging and if she did encounter a problem, she would work the problem and "make sure I understood every single… piece of it, to get to an answer." This suggests personal characteristics of drive, persistence and curiosity combined with previous experiences of success in mathematics. Indeed, being female and having a positive mathematical identity, as she clearly does, is correlated with increased performance, participation and persistence (Fernandez et al., 2022) Hanna demonstrates this by saying, "if I do work hard and I put in effort, eventually I will figure something out."

Her biggest obstacles in mathematics came during online classes when the lack of in-person support negatively influenced her understanding.

> COVID happened when Hanna was in University and she remembers that she had some issue again with an online class. There are certain topics, that she still encounters, such as sequences and series that she never really learned in depth.

Given that her values are persistence, autonomy, support, intellectual curiosity and understanding the university, this corresponds best with Hill's core values Cognitions in the sense of intellectual curiosity, Engagement such as deep interest, and Perseverance. Relationships are also important as she frequently references support and found online courses difficult to learn and retain information from.

Step 7: See Section 6.2 Cross Thematic Analysis

Initially this was done for each participant individually before they were compared.

Step 8: See Section 7.1: Comparison of Participants' Values and Hill's Core MWB values