A DISCUSSION OF SOME PROBLEMS IN SPONTANEOUS FLUCTUATIONS OF ELECTRICITY

by

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ABSTRACT

Temperature measurements below about 1^{o}K. have always been a problem to physicists and there were many proposals and attempts to remedy this situation.

One of such proposals was the use of noise due to thermal agitation of electrons in a conductor as a thermometric quantity. This was first proposed in 1946 by Lawson and Long, and there was a number of attempts to realize their idea with an increasing success. The latest achievement was that of H.J. Fink who succeeded in measuring the temperature of about 1.2^{o}K with 1% precision. This achievement brought the noise thermometer to the threshold of temperatures which it was supposed to measure.

This thesis contains a review of almost all published work in noise thermometry up to the beginning of 1961, and a review of the unpublished work of C.T.J. Alkemade. In addition to this a discussion is included of a number of aspects of noise thermometry, affecting the precision of measurements.

A research proposal is included concerning the development of a two-channel noise amplifier having theoretica low temperature limit below 0.001^{o}K.

ACKNOWLEDGEMENT

I wish to express my gratitude to Dr. D.K.C. MacDonald who, by his own example, has shown me the meaning of the word "scholarship". If this thesis in part reaches the standard required of such work, it is due to his guidance and influence.

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1. Introduction

The subject of this thesis is the consideration of the possibility of using thermal fluctuations of electrons in electrical conductors for the purpose of temperature measurements and of other related topics. Very low temperatures are of particular interest here, since there are no easy and entirely reliable methods of absolute temperature measurements below about $1^{o}K$.

The temperature of a body is regarded as a basic property, one of the fundamental parameters by which the state of that body can be described (cf. e.g. A.G. Worthing, 1940, R.C. Tolman, 1917).

The usual method of absolute temperature measurements is to use the dependence between the pressure P, volume V and the temperature T of gas, expressed by the equation of state

$$PV = RT \qquad 1.1$$

where R is the gas constant and the quantity of one mole of gas is implied.

Gases for which (1.1) holds are called ideal gases. The behavious of real gases, particularly in the region of temperatures and pressures close to their boiling point, has to be represented by a relation more complicated than 1.1, e.g.

$$PV = A(1 + Bp + Cp^2 + \ldots) \qquad 1.2$$

where A, B, C etc., are called virial coefficients and can be found experimentally. The use of the formula 1.2 for thermometry is less straightforward than that of 1.1, but it is being used in the absence of simpler means of temperature determination.

The equation of state 1.2 is the basis of the constant volume gas thermometer. Very accurate measurements with the gas thermometer can be made and are being made in order to determine certain fixed points on the temperature scale (in the low temperatures region the boiling point of oxygen, hydrogen, helium etc.). An accurate determination of temperature with a gas thermometer is, however, a very laborious process and there are few laboratories in the world which make these measurements. Apart from the difficulties of correcting for the non-ideality of the gas used and for the thermo-molecular pressure difference * there are many other corrections which have to be applied and difficulties to be overcome before a temperature can be determined to the high accuracy of about a few parts in a million (see e.g. Beattie, 1955).

An absolute measurement of the temperature with the gas thermometer is therefore impractical as a routine method. Fortunately there exist a convenient method of temperature determination by means of vapour pressure measurements. The vapour pressures of commonly used liquids (He, H, O, N, etc.) have been measured and tabulated as functions of the absolute

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* The pressure difference between the ends of the capillary tube connecting the thermometer bulb to the manometer, existin when the mean free path of the gas molecules is sufficiently large to be comparable with the width of the capillary.

thermodynamic temperature so that they form a reliable series of "primary" scales of temperature within certain limits. Helium is the only liquid having a measurable vapour pressure below about $10^{o}K$. But even the vapour pressure of helium falls down to 1 mm Hg at $1.27^{o}K$ and becomes a hundred times lower than this at about $0.8^{o}K$. The practical limit of temperature determination from the vapour pressure of He is therefore between 1^{o} and $2^{o}K$.

The absolute temperature measurements below that limit, down to about $0.01^{o}K$ are made through the fundamental relation between the heat Q, entropy S and temperature T ($T = dQ/dS$). The experimental procedure is to obtain first the dependence between Q and S on the magnetic susceptibility for a paramagnetic salt and then to deduce T from the slope of the resultant Q - S curve. Accuracies of the order of 6 - 5% can be obtained (Ambler and Hudson, 1957).

Temperatures below about $0.01^{o}K$ cannot really be measured. They can only be estimated. The lowest temperatures obtained experimentally were of the order of $20.10^{-6}\ ^{o}K$ (cf Kurti et al, 1956).

There is therefore a real need for a thermometer suitable for absolute measurements of temperatures below $2^{o}K$ and also for reliable routine measurements of temperatures in the same range.

It was thought that the thermal fluctuations of electrons in electrical conductors could be used for filling that need (Lawson and Long, 1946). The expectations were very optimisti The results of subsequent experiments by other workers, howeve fell short of the expectations. The lowest temperature that was measured with a noise thermometer up to now was about 1.3°K (Fink, 1959). This seems to be a long way from the temperatures of the order of millidegrees which were forcast as the lower limit of the sensitivity of noise thermometers.

The conduction electrons in an electrical conductor are free to move throughout the crystal lattice and, statistically, have properties similar to those of an ideal gas so that these electrons are sometimes called an "electron gas". Since however the electron gas in metals is in all cases highly degenerate * its properties are essentially different from those of an ordinary gas (Born, 1955). Nevertheless one could draw a parallel between the random motion of molecules in a gas and the random motion of free electrons in a metal.

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* Among various stationary states of electrons there may be some which correspond to the same values of energy, but differ in the values of some other physical quantities. Such eigenvalues of energy levels of the system, to which several different stationary states correspond are said to be degenerate (Landau and Lifshitz, 1958). An electron gas is said to be degenerate when its temperature is well below the Fermi temperature and its statistical properties have to be described by the Fermi distribution. The characteristic of such a state is that a large proportion of electrons completely fill the lower energy levels and are unable to take part in any physical process until excited out of these levels (Van Nostrand Sci. Enc., 1958).

The random motion of gas molecules results in what we call pressure. The random motion of electrons in a conductor results in an accumulation of charges distributed randomly throughout the volume of the conductor. These charges give rise to potential differences between any two points in a conductor. The time average of the square of these potential difference fluctuations may be compared to the pressure of an ordinary gas.

Both the gas pressure P and the mean square of the potenti fluctuations $\overline{V}^2$ are directly proportional to the absolute temperature T (other parameters being constant)

$$P = \left(\frac{R}{V}\right)T \qquad 1.2$$

$$\overline{V}^2 = (4kRB)T \qquad 1.3$$

(the bracketed symbols will be defined later in the text).

The random fluctuations in the distribution of electronic charges throughout the body of a conductor result in the flow of minute random currents between any two points. If the charg at one point of the conductor changes at a rate dq/dt with respect to another point of the body, then the time variation of the charge is equivalent to a current i = dq/dt flowing between these two points. If the electrical resistance betwee these two points is R, the voltage drop across this resistance is v = Ri = R(dq/dt). Since the charge fluctuation is random in time therefore both the current and the voltage between the two points is random in time. The mean square value of that voltage can be calculated and measured and was found to be (Johnson, Nyquist, 1928)

$$V^2 = 4kTRB \qquad 1.4$$

The mean square noise voltage given by 1.4 depends on three quantities: R - the ohmic resistance between the two points across which the voltage is measured; T - the equilibrium temperature of R with its surroundings and B - the effective bandwidth within which the noise voltage is measured. k is the Boltzmann constant.

We have here four quantities connected in one simple relation which should certainly be valid for small fluctuations and the validity of which has been checked for a wide range of parameters T, R and B. The accuracy with which the formula 1.4 was found to be true was about 10% in 1928 (Johnson) and the most recent measurements claim an accuracy of the order of 0.1% * (Garrison and Lawson, 1949; Fink, 1959).

When we have such a fundamental relation connecting four quantities then, by fixing two of these quantities, we can regard the other two as mutually dependent and use one for determining the other. A number of methematical relations can be written of which the one of immediate interest here is

$$\overline{V}^2 = f(T) \qquad 1.5$$

If we assume that R and B remain constant and independent of the temperature then

$$\overline{V}^2 = (4kRB)T \qquad 1.6$$

giving $\overline{V}^2$ linearly dependent on T.

The relation 1.6 is the basis of the noise thermometer. The equivalent noise bandwidth B is, probably without exceptio

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* For comments on this accuracy see Section 4.

independent of $\underline{v}$. The resistance R may be a function of the voltage developed across it and usually is a function of the temperature. The relation 1.6 should therefore be written strictly as

$$\bar{v}^2 = 4kR(v,T)BT \qquad 1.7$$

Such a more general nonlinear case was considered independently by MacDonald (1953, 1957), Van Kampen (1958) and Alkemade (1958) in which R was assumed to depend on the amplitude of fluctuations. All experimental work, however, on thermal noise generated within electrical conductors was carried out with the assumption that the resistance of the conductor was strictly linear, i.e. that the conductor obeys Ohm's law at least within the limited voltage range of thermal fluctuations. Since the accuracy of present day noise measurements is not very great (0.1% at the best) the assumption of linearity was never questioned. As the accuracy of measurements becomes greater it might become mandatory to allow for the nonlinearity of the noise generating resistance, particularl in systems such as diode rectifiers and p-n junctions.

The dependence of R on the temperature T presents no difficulties since experiments can, and usually are carried out at constant temperature.

In what follows it will be assumed that the resistance R is linear within the range of amplitudes of the thermal voltage fluctuations.

There is one more factor in the relation 1.6 which should be mentioned here, i.e. the quantum effect.

The simple relation $\overline{V}^2 = 4kTRB$ was derived on the basis of the equipartition law * according to which the total energy of a harmonic oscillator per degree of freedom is kT. If, however, the energy of a harmonic oscillator per degree of freedom, according to quantum mechanics, be taken as $hf/(\exp hf/kT - 1)$ ** where f is the frequency of vibrations of the harmonic oscillator, then the expression for the mean square fluctuation of the thermal voltage becomes (Nyquist, 1928)

$$\overline{V}^2 = \frac{4Rfh\,df}{e^{\frac{hf}{kT}} - 1} \qquad 1.8$$

At the lowest temperature measured with a noise thermometer up to now, i.e. about 1.3°K, and at the frequency used of about 5 kc (Fink, 1959), the magnitude of the factor hf/kT is about $0.2 \cdot 10^{-6}$ which is much less than unity and therefore for all practical purposes the formula 1.6, which is the approximate form of 1.8, holds (see e.g. Van der Ziel, 1954).

To the best of my knowledge there has been, up to now, no published experimental confirmation of the necessity of using the quantum correction.

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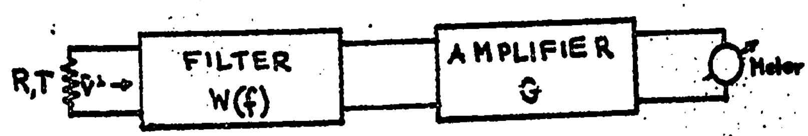
* The equipartition law may be stated as follows: "If we have any set of M classical systems in the assembly, each of s degrees of freedom, whose energy (in Hamiltonian form) consists of the sum of t square terms ($s \leq t \leq 2s$), then in equilibrium the mean energy of the set is Mt(1/2)kT or (1/2)kT for each square term in the energy" (cf. Fowler, 1955). A harmonic oscillator contains two square terms in the expression for its energy per degree of freedom (one for the kinetic and one for the static energy). The total energy therefore is 2(1/2)kT = kT

** Sometimes called "Planck's Factor".

The case of the noise thermometer may, in future, become similar to that of the gas thermometer. As the accuracy of the gas thermometer was being made greater and greater, more corrections had to be applied and more complicated the actual working formula became. By the time the noise thermometer reaches the degree of accuracy of the gas thermometer, i.e. increases its precision by about three orders of magnitude, the simple Nyquist relation 1.6 may have to be replaced by an elaborate formula allowing for the quantum effect, nonlinearity of the resistance and its temperature dependence.

A simple noise thermometer will, in general, have the components as shown in Fig. 1.9



1.9

The resistance R, in thermal equilibrium with its surroundings at temperature T, is connected to the input of an amplifier through some frequency dependent network (Filter). The output of the amplifier is measured with a square law meter. The indications of the meter are then, within the limitations indicated previously, directly proportional to the absolute temperature of R.

The physical form and the arrangement of the component parts of the noise thermometer introduce certain complications and becomes a source of different practical solutions for such a thermometer. *

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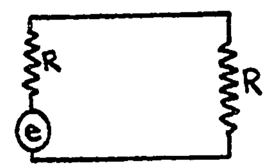
* It should be mentioned here that a radically new approach to Noise Thermometry was made by J. Fink, which approach is described in Section 2.

The range of values of physically available resistors is of the order of 1 : 10^{23}, which gives an enormous range of choice. On the other hand the maximum noise power available from the resistor is constant at a given temperature and independent from the value of the resistance. The maximum power can be obtained from a source of emf. when the load resistance is equal to the internal resistance of the generator. If we represent the thermal noise as a voltage source in series with the resistance producing it, then, for the maximum transfer of power the load resistance should have the same value.



1.10

The power transferred from left to right will be

$$P_a = \frac{e^2}{4R} \qquad 1.11$$

If the source is a noise voltage then

$$P_a = \frac{4kTRB}{4R} = kTB \text{ watts} \quad \text{or} \quad kT \text{ watts/cycle} \qquad 1.12$$

that is the maximum power which can be obtained from such a noise source depends on the temperature and bandwidth only.

For the use in temperature measurements the temperature becomes an independent variable and the only factor which can be controlled in order to produce the most desired effect is the bandwidth of the noise measuring device.

The bandwidth of an electrical circuit is intimately connected with its impedance. In general, a low impedance is associated with a wide bandwidth and a high impedance is associated with a narrow bandwidth.

If we consider a simple electrical circuit consisting of a resistance and the associated capacitance (Fig. 1.13)

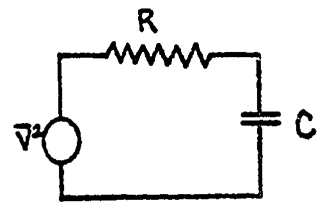


Fig. 1.13

then the noise generated across its terminals is

$$\overline{V}_c^2 = 4kTR\int_0^\infty |W(f)|^2 df = \frac{kT}{C} \qquad 1.14$$

where the immitance of the RC circuit $W(f) = 1/(j\omega RC + 1)$ (cf. e.g. Freeman, 1958).

The above result shows that in a real physical circuit it is the capacitance and not the resistance which limits the magnitude of the available noise voltage. The mean square value of the noise voltage calculated from 1.14 is about $1.4 \cdot 10^{-11}$ v^2 at one degree Kelvin across 1 $\mu\mu F$, or about 3 microvolts rms.

The order of magnitude of the capacitance associated with an amplifier input is usually assumed, for estimating purposes, to be about 30 $\mu\mu F$. The maximum rms noise voltage obtainable across a resistance connected to such an amplifier is therefore about 0.7 μV at $1^{\circ}K$, or about 12 μV at room temperature.*

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* It should be emphasized here that such magnitudes of the noise voltage could be measured with the aid of an

The noise power available from a resistor can be used either in the form of voltage fluctuations, in which case a voltage amplifier is required, or in the form of current fluctuations. Up to now all attempts of constructing noise thermometers, which are known to me, were based on the use of the noise power in the form of voltage fluctuations across certain resistance. In such case it is intuitively obvious that the highest possible resistance should be chosen in order to obtain as large voltage fluctuations as possible. Various practical solutions for that approach are given in the review of literature in section 2.

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amplifier only if the bandwidth of the amplifier were not smaller than that of the noise generating RC input circuit. Nominally an infinite amplifier bandwidth would be required. Practically a bandwidth ten times greater than the nominal bandwidth of theinput circuit (calculated from $\omega RC = 1$) would include more than 93% of the noise contributions from the noise generating resistance. In order to extract 99% of the noise power from the input circuit the amplifier bandwidth would have to be sixty times greater, or one for which $\omega RC = 60$. The disposable noise contributions from the amplifier itself could be filtered out by an RC circuit at the output. Thus for a 100 $k\Omega$ resistance and 30 $\mu\mu F$ input capacitance the nominal input bandwidth would be about 50 kc. An amplifier having a bandwidth of 500 kc could be used with another 50 kc RC network at the output to compensate for the differences in the frequency characteristics.

There is, however, no reason why the fluctuations of electrons could not be used in their natural form, i.e. as charge or current fluctuations. In such case current amplifying devices would have to be used instead of voltage amplifiers. The most widely used commercially available devices for current amplification are transistors and the work on this thesis was partly based on my proposal to use transistors in noise thermometry (see section 5). Even transistors, however, will not produce true lossless current amplification. It is my hope, however, that the investigation of the noise behavious of p-n junctions as a function of temperature will contribute to the field of noise thermometry.

2. A review of publications on Noise Thermometry.

A short review of the publications on Noise Thermometry published in 1958 (Celinski and MacDonald) surveyed the work on noise thermometry up to 1956. A photostat copy of this work is enclosed. The review is brought up-to-date by the description of publications which were found in the literature since that time.

ON THE USE OF TRANSISTORS IN LOW TEMPERATURE NOISE THERMOMETRY.

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Abstract

The use of thermal noise for measurements of very low temperatures was suggested in 1946 by Lawson and Long.

Experimental attempts to construct such a low-temperature thermometer have, so far, proved to be unsuccessful. The same proposal when applied to higher temperatures, above the liquid nitrogen point, produced satisfactory and accurate noise thermometers.

The paper gives a review of publications on noise thermometry and then describes the experimental investigation of the DC characteristics of germanium junction diodes at liquid air and liquid helium temperatures.

A noise ratio for these diodes is estimated theoretically, and its behaviour predicted for low temperatures.

Review of publications on Noise Thermometry

Summary

April 1946 - Dicke used thermal microwave radiation for determining the equivalent black-body temperature of a hot body.

May 1946 - Dicke, Beringer, Kuhl and Vane used the microwave noise spectrum in studies of the absorption of the atmosphere.

July 1946 - Lawson and Long, suggested the use of the Brownian motion for measurements of very low temperatures below 1°K. A crystal bar was suggested as a source of noise.

October 1946 - Brown and MacDonald criticised the analysis and practicability of the suggestions of Lawson and Long and pointed out the circuit approach to the problem.

January 1947 - Gerjuoy and Forrester quantitatively investigated the suggestions of Lawson and Long from the circuit point of view. A quartz crystal was assumed as the source of noise.

August 1948 - Cook, Greenspan and Weissler discounted the possibility of using a quartz crystal as a source of noise for low temperature measurements.

July 1949 - Garrison and Lawson constructed a noise thermometer for high temperature measurements in the region of 1000°K with an accuracy of 0.1%.

February 1950 - Endt points out the relation of the amplifier's bandwidth to the accuracy with which the mean square value of the noise may be determined. Estimates the lowest measurable temperature as 0.02°K.

September 1951 - Troitsky discusses the use of a radio receiver for temperature measurements of hot bodies. Estimates the minimum measurable temperature as 3°K with an accuracy of 0.3°K.

March 1954 - Aumont, Romand and Vodar constructed a wide range noise thermometer based on that described by Garrison and Lawson for the range 90° - 780°K. with an accuracy of 1°K.

February 1956 - Alkemade attempted to construct a low temperature noise thermometer with an expected range between 4°K and 90°K.

Summary of publications

The idea of a "Noise Thermometer" for the measurements of low temperatures was specifically considered in 1946 by Lawson and Long.[3]

Prior to this, Dicke[1] published an account of his experiments on the measurement of thermal radiation at microwave frequencies. Dicke's work was based on the fact that an antenna in thermal equilibrium with a black body, Fig. 1, exhibits a certain radiation resistance to which we can assign the temperature of the black body.

In general, an antenna may be represented by an equivalent resistance R_e at a temperature T_e. A thermal noise voltage is produced across the terminals of this resistance; the mean square voltage is equal to $4kT_eR_e$ per unit frequency-interval. The equivalent circuit is that of Fig. 2.

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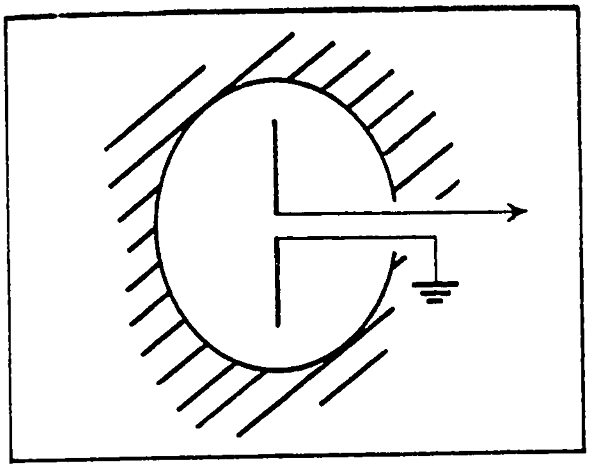

Figure 1 Antenna surrounded by a black body

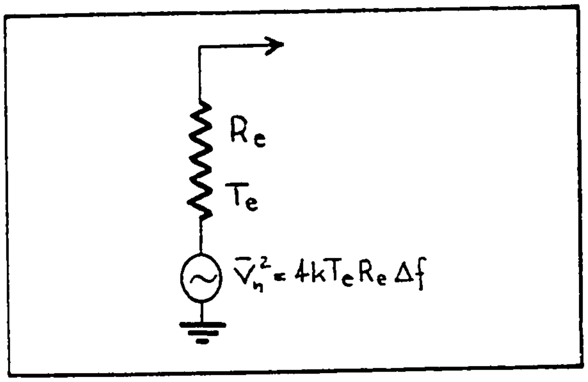

Figure 2 Equivalent circuit of an antenna

The noise across the resistance R_e is spread uniformly throughout the frequency spectrum, ignoring quantum corrections and the frequency characteristics of the antenna itself (ie. the dependence of R_e on frequency) The use of microwaves or any other particular frequency range may be determined by transmission properties of the medium between the black body and the antenna. Indeed, by selecting different frequencies, it is possible to investigate the transmission and absorption properties of the medium, e.g. the atmosphere, as was done by Dicke, Kuhl and Vane.[2]

From the concept of the temperature measurement of a black body by thermal noise we may go directly to the problem of the measurement of resistor temperature if the antenna is assumed to be replaced by a resistor R_T. If further the temperature of that resistor is determined by its surroundings then we arrive at the idea of the "Noise Thermometer". The resistor R_T is inserted in the medium, the temperature of which we wish to determine, and after the resistor comes into thermal equilibrium with the surroundings, the mean square noise voltage across the terminals of the resistor is measured. From Nyquist's formula

$$\bar{V}_n^2 = 4kTR_T\Delta f \qquad (1)$$

and

$$T = \frac{\bar{V}_n^2}{4kTR_T\Delta f} \qquad (2)$$

Lawson and Long,[3,5] appear to have been the first to suggest explicitly the use of that relation for the measurements of very low temperatures. Today the determination of absolute temperatures below 1°K is based on a combination of calorimetric and magnetic measurements which require elaborate and time consuming techniques; moreover serious difficulties still exist in the calibration and standardisation at the lowest temperatures (cf eg. D. deklerk, N. Kurte).[23,24] The use of the noise voltage developed across a resistor seemed, therefore, to offer a simple and elegant solution. If an amplifier could be constructed which would amplify that noise without contributing too much of its own, a new absolute thermometer would be obtained which would be of the greatest value.

An ideal noiseless amplifier would make possible the measurement of temperatures as low as desired. Since, however, real amplifiers inevitably contribute noise of their own there is a limit to the lowest temperature that could be measured with such a thermometer, and that limit is imposed by the amplifier's background noise. The realisation of the noise thermometer is, therefore, intimately connected with the realisation of a suitable amplifier.

To be able to override the background noise voltage of the amplifier very high resistances would have to be used as the noise thermometer elements, and this fact led to the idea of using a quartz crystal bar as a source of thermal fluctuations. Lawson and Long estimated that by using such a crystal bar, temperatures of the order of 0.001°K could be measured, and that with careful design that limit could be extended to even lower temperatures by a factor of ten.

The suggestion of Lawson and Long was criticised by Brown and MacDonald[4] who pointed out that the problem could be treated from the circuit point of view using Nyquists' theorem directly. Attention was also drawn to the existence of the flicker effect* which would make it mandatory to use amplifier frequencies high enough to eliminate the influence of that effect on the measurements. That, in turn, would point out to the use of a dynamic impedance, rather than a purely ohmic resistor. The use of a resonant circuit, possibly a superconductor, was suggested.

Further discussion came from Gerjuoy and Forrester,[6] who also independently considered the problem from the circuit point of view.

* i.e. increasing tube noise at low frequencies (typically below a few thousand cycles/sec.) due to secular variations in the cathode emission.

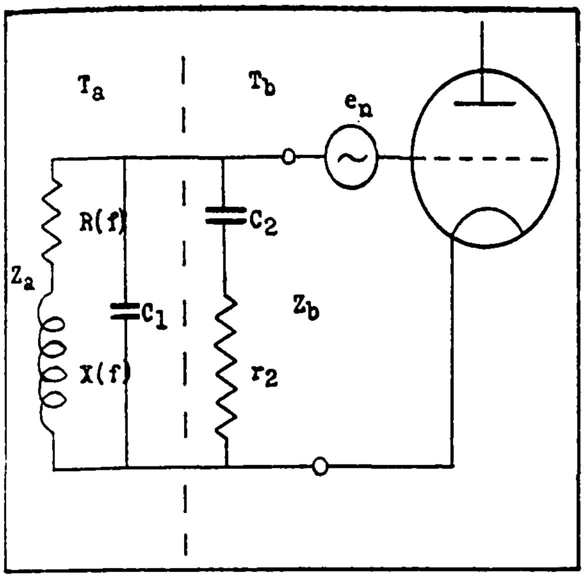

Figure 3 Equivalent circuit of an amplifier's input

A quartz crystal was considered as the thermal noise generator. The equivalent circuit assumed is given in Fig. 3. The conditions imposed on that circuit were:

(a) The low temperature noise originating within the crystal itself (temperature T_a) must be much greater than the noise originating in the other parts of the circuit (temperature T_b).

(b) The low temperature noise must be much greater than the overall tube noise due to the grid shot noise and plate shot noise.

From these conditions a set of three inequalities for the capacitance C_1 was drawn up. By taking a commercial tube, the Western Electric D-96475, the authors showed that the minimum measurable temperature would be about 2°K with C_1=0.02uF at a resonant frequency of 1600 c/s. The authors then speculated on the possibilities of modifying the electrometer tube and measuring circuit to obtain the best conditions. Even with these refinements Gerjuoy and Forrester conclude that "the ultimate attainable temperature will almost certainly be greater than 0.1°K."

In 1948 Cook, Greenspan and Weissler investigated the thermal voltage generated in a crystal bar. By considering an equivalent circuit of such a crystal, they found that there exists a maximum to such voltage, determined by the presence of capacities associated with the crystal and its external circuit.

It was shown that the maximum thermal voltage obtainable is given approximately by

$$\sqrt{\bar{V}_n^2} = 5.26 \times 10^{-8}\sqrt{T} \text{ volts}$$

which is equivalent to approximately 0.93µV at room temperature and to 0.0054µV at 0.01°K. Considering again the net capacity present in the circuit it may be shown that those voltages are equivalent to a charge of about 160 electrons on the equivalent quartz capacity at room temperature, and to about one electron at 0.01°K.

Now the noise voltage present at the grid of an electrometer tube (VX-41A) is approximately 0.054µV. Comparison of this voltage with that of the crystal above shows that a quartz crystal bar would not be a suitable generator of thermal voltages for low temperature measurements.

The first realisation in practice of the principles of the noise thermometer was accomplished in 1949 by Garrison and Lawson.[8] They constructed such a thermometer for the measurement of high temperatures under high pressures. The solution adopted was that of comparison of the thermal noise generated at the unknown temperature with a reference noise-level, generated in a variable resistor at a known temperature. Electromechanical vibrators were used for switching alternately the reference noise and the unknown noise on to the screen of an oscilloscope. If the noise originating in the adjustable resistance was made exactly the same magnitude as that originating in the other, the temperature of one could be found in terms of the known temperature and resistance of the other. The overall accuracy achieved was 0.1%.

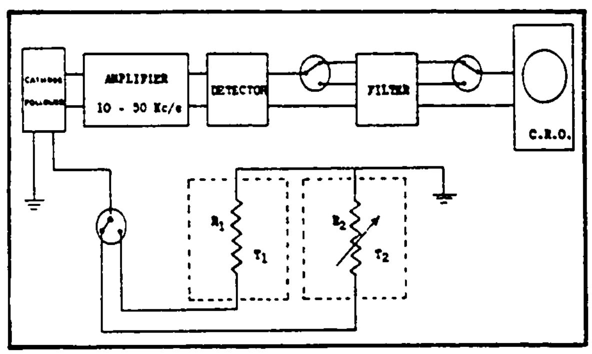

Figure 4 Noise thermometer of Garrison and Lawson

The schematic diagram (Fig.4) shows the general arrangement of the thermometer. The noise originating in R_1 and R_2 is amplified within the bandwidth of 10-50 kc/sec, detected, and passed through two low pass filters of long time constants, one for each resistor. The resulting DC output is compared on the oscilloscope. If the noise voltages from the two resistors are not equal, the DC levels at the output of the filters are different, and the oscilloscope displays a square wave-form. R_2 is now adjusted to make the DC levels identical. Since T_2 and R_2 are known, the temperature T_1 can easily be computed, or the resistance R_2 calibrated directly in degrees.

The advantages of Garrison and Lawson's thermometer were the accuracy and precision of 0.1% on an absolute scale above 100°K, the independance of the material from which the thermometer element was made and its previous history, and the independence of the absolute pressure. The disadvantageous features were the length of time (of the order of two minutes) required to obtain the precision of 0.1%, the difficulty in shielding the thermometer from extraneous pick up, and the limitations imposed by the input capacity requiring the input stage to be close to the thermometer element. Although the thermometer could not be used as a routine instrument, it could, however, provide a standard for calibrating other high temperature measuring devices.

In 1950 P.M. Endt[9] examined the problem of low temperature noise thermometry from the point of view of the amplifier's bandwidth and its relation to the statistical fluctuations of the measured noise. By considering the shot noise of the first tube of the amplifier, and minimizing the noise figure of that stage, he arrived at an optimum value of the grid leak resistance of the first tube

$$R_g = \sqrt{R'R''} \qquad (4)$$

where R' and R" are the equivalent plate and grid noise resistances respectively. By using this optimum value of the grid leak, the signal to noise ratio* at the input of the first tube is found to be:

$$\frac{\bar{V}_s^2}{\bar{V}_n^2} = \frac{T}{2T_o}\sqrt{R''/R'} \qquad (5)$$

and the bandwidth of the input network, f, due to the presence of stray capacities is given by:

$$f = \frac{1}{2\pi R_g C_g} = \frac{1}{2\pi C_g \sqrt{R'R''}} \qquad (6)$$

If now we pass the amplified noise through a square law detector and use an indicator having a time constant, t, we can measure the output with a relative error $1/\sqrt{t\Delta f}$,[25] where Δf is the bandwidth of the amplifier, in this case made identical to that of the input network.

The lowest temperature that could be measured by this method is reached when the thermal noise indication of the meter is equal to the fluctuations in the shot noise indication, that is $\bar{V}_s^2/\bar{V}_n^2 \approx 1/\Delta t \;\; f$.

Using the relations (4), (5) and (6) we obtain:

$$T_{min} \approx 2T_o\sqrt{\frac{2\pi C_g}{t}}\sqrt[4]{\frac{R'^3}{R''}} \qquad (7)$$

By taking a specific tube (Phillips AF7) P.M. Endt shows theoretically that with a grid leak $R_g = 70M\Omega$ and a bandwidth of 230 c/s the lowest measurable temperature should be about 0.02°K.

The author also shows that a quartz crystal gives essentially worse results than either a pure resistance or an L-C circuit, due to the distribution of capacities associated with the crystals, agreeing also with the conclusions of Cook et. al.[7]

In 1951 V.S. Troitsky[10] considered the capabilities of radio receivers for noise thermometry.

The reasoning adopted was the reverse of that of Dicke[1]. The author started with the analysis of a radio receiver having a resistance R_i connected to its input terminals. If an inertialess detecting device (such as an oscilloscope) is used for measuring the change in the noise due to the change of the temperature of R_i then we roughly need to double the noise level already existing at receiver's temperature T_o, to detect a change T_i. In such case

$$T_i = T_i + (F-1)\,T_o \qquad (8)$$

where F is the noise figure of the receiver. With an ideal receiver (F =1) the temperature of R_i would have to be doubled to produce a well defined indication of the detector.

If the detecting device has a finite inertia (cf. Endt[9]) we can improve the accuracy of noise level determination by a factor $Q \approx \sqrt{2t\Delta f}$. For $T_i = T_o$ we have

$$\Delta T_i = \frac{F\,T_o}{Q} \qquad (9)$$

If t, the time constant of the detector, is 5 sec; f, the receiver's bandwidth 10 Mc/s, F=10 and T_o = 300° (room temperature), then ΔT_i = 0.3°K.

If instead of R_i an antenna is connected to the receiver then it is equivalent to a resistance R_e at a temperature T_e (Fig 1,2).

If we use a directional antenna we may measure the temperature of a distant hot body by measuring the increase of the equivalent temperature T_e produced by the presence of that body within the directional pattern of the antenna.

Troitsky shows that the minimum temperature of a hot body which could be detected in such way is given by

$$T_{min} = \frac{4\pi}{\Delta\Omega}\,\frac{1}{D}\,\Delta T_e \qquad (10)$$

If the body is within 1 degree from the receiving point which gives $\Delta\Omega$ =1/3600 approx. and the antenna directivity factor D = 5000, then if ΔT_e= 0,3°K the minimum detectable temperature of a distant body T_{min}= 3°K.

* in this paper 'signal' means "wanted noise"

In 1954 Aumont, Romand and Vodar[11,12] constructed a noise thermometer based on the design of Garrison and Lawson[8]. The noise generating resistance was 2000Ω made of nickel-chrome wire. The thermometer was compared to a thermocouple standardised by the N.P.L. The temperature range covered was 90°K (the boiling point of oxygen) to 780°K. The accuracy obtained was within 1°K, that is 0.2% at 500°K. The results of their measurements are shown in table 1. The authors were constructing another thermometer which was to extand the temperature range up to 1500°K.

T°K	90	320	365	395	420	480
ΔT	-0.4	-1.0	-0.1	+1.1	+2.0	+1.2
T°K	520	560	660	690	720	780
ΔT	-0.6	+1.0	-1.1	+1.0	+0.5	+1.5

(The values in the above table were selected from a diagram showing the results of temperature measurements by Aumont, Romand, Vodar)

TABLE I

In 1956 C.T.J. Alkemade[13] made an attempt at the National Research Council, Ottawa, to construct a noise thermometer for low temperatures. The thermometer incorporated in its design suggestions of Brown and MacDonald[4]. The thermometric element was a low resistance of the order of 1 ohm in series with a superconducting coil. The preamplifier tube was an electrometer tube VX41A.

Alkemade encountered severe difficulties in realising the very high Q-factor required for the successful operation of the thermometer. The difficulties were caused by the feedback from plate to grid which, at extremely high dynamic impedances, was difficult to eliminate. That project was not completed. The present work on noise thermometry at the University of Ottawa is a continuation of Alkemade's work.

This review would not be complete without at least a short discussion of the validity of Nyquist's formula on which all the thermometric attempts depend.

The formula for the mean square noise voltage was derived by Nyquist on the assumption that the energy (potential & kinetic) per degree of freedom was kT. If the energy per degree of freedom be taken however as hf/(exp hf/kT- 1) then the expression for the mean square noise voltage becomes

$$\bar{V}_n^2 = \frac{4Rhf\Delta f}{\exp hf/kT - 1} \qquad (11)$$

Nyquist included that correction in his paper[26]. At that time, however, neither the frequencies used were sufficiently high, nor the temperatures low enough to warrant the use of the formula (11), since at normal temperatures and frequencies hf is much smaller than kT, and, when $hf \ll kT$ than equation (11) reduces to

$$\bar{V}_n^2 = 4kTR\Delta f \qquad (12)$$

The quantum correction would have to be taken into account in cases where hf becomes comparable to kT. If we assume the measured noise frequency to be 1000 Mc/sec then hf becomes equal to kT at a temperature of about 0.05°K. Both the frequency and the temperature are within the range encountered in the laboratories in the routine work. It should therefore be possible to check the quantum correction of Nyquist formula.

That quantum correction, if valid, might have to be taken into account when designing a very low temperature noise thermometer.

There is another possible source of inaccuracy in the Nyquist's formula, and that is the resistance R.

The resistance included in (12) is an idealised parameter which has been assumed to obey Ohm's law strictly. It may be asked what is the order of approximation expressed by that law. Does it hold to any degree of accuracy for any magnitude of currents and voltages? Is the resistance still constant at the level of spontaneous fluctuations, or may it depend on the voltage across the resistance terminals?

Analyses of the spontaneous fluctuations in nonlinear systems have been published by MacDonald[21] and Van Kampen[22]. Such considerations may become important when the accuracy of noise thermometers is increased, since no instrument can be more accurate than the theory on which it is based.

THE DC CHARACTERISTICS OF JUNCTION DIODES AT LOW TEMPERATURES.

(Continuation of preceding paper)

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University of Ottawa, Ottawa, Ontario.

The foregoing review of the literature on the use of vacuum tube amplifiers for low temperature noise thermometry shows that the idea is a sound one and that, theoretically at least, we should be able to produce a noise thermometer capable of temperature measurements down to at least 0.01°K.

There is a definite need for a simple, reliable and accurate thermometer for temperatures below 4.2°K. That temperature, being the boiling point of helium under normal pressure, is the lowest reliable fixed point in temperature determination. The boiling point of helium may be brought down from about 4.2°K. to approximately 1°K by the reduction of helium vapour pressure. The temperature may then be calculated from the vacuum gauge readings with the aid of tables. There is, however, a limit to pumping and beyond that limit the procedure of temperature determination becomes more and more lengthy and difficult. The lowest temperature attained up to date is of the order of 10^{-6} °K,[27]. that is six decades below 1°K. The problem of measurements of these temperatures throughout the whole range is, therefore, a formidable one.

The only attempt, known to the author, to construct a thermometer for low temperatures was that of Alkemade[13]. One of the difficulties which Alkemade encountered, was the presence of plate to grid feedback. That feedback, even with neutralization, made it impossible to achieve the high values of Q of the resonant circuit. Q of the order of 50 000 was aimed at, whereas the measured values were about 2000.

The high dynamic impedance was needed to produce a high noise voltage across it. From Nyquist relation, however, we can see that in order to obtain a high noise voltage we may increase not only the impedance across which it is developed, but also the bandwidth within which it is contained. In order to have both, high dynamic impedance and wide bandwidth, we must move up in frequency. This may be emphasized by rewriting the relation (12) in the form

$$\bar{V}_n^2 = 4kTrQ_of_o \qquad (13)$$

where r is the loss resistance of the dynamic impedance $Z_o = Q_o^2 r = (f_o/\Delta f_o)Q_o r$. A Q of one thousand at a frequency of 1000 Mc/s would produce about $(7.5\ \mu V)^2$ of thermal noise per unit loss resistance per degree K. We could obtain an even greater noise voltage at still higher frequencies. Cavity resonators might be applicable in such case. The use of very high frequencies might, however, make it necessary to take quantum correction term (11) into account.

All the theoretical considerations on noise thermometers were, up to now, limited to vacuum tubes as noise amplifying elements. There are several drawbacks to the use of thermionic tubes as preamplifiers of the thermal noise in low temperature thermometry, namely:-

a) Their large size
b) The heat dissipated in the filament
c) The necessity of using high impedances in the grid and plate circuits.

High impedances demand short leads to keep stray capacities down, and to avoid excessive pick-up. That, in turn, requires that the tube should be placed in the same low temperature bath as the thermometric element. In such case the size of the tube and the heat dissipation acquire particular significance.

The drawbacks listed above might perhaps be offset by the use of transistors as noise amplifying elements. Their use would be particularly attractive in the preamplifying stage. Fig. 5. shows a possible arrangement.

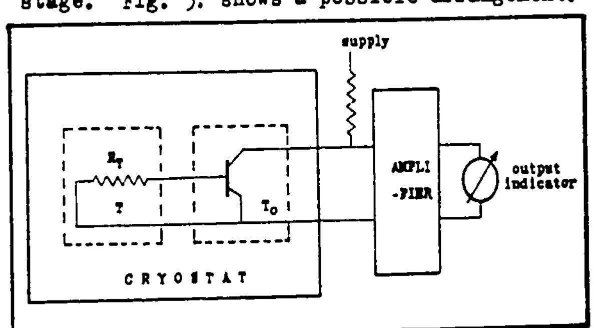

Figure 5 Noise thermometer using transistor

The thermometric element R_T is inserted in the medium at an unknown temperature T. The transistor is kept in a constant temperature bath T_1. The remainder of the circuit may then be brought outside the cryostat. Apart from offsetting the drawbacks of the thermionic tubes, we would have an additional parameter of control, the temperature T_1 of the transistor.

Were it possible to produce a very low noise transistor preamplifier stage, the noise thermometer might become a practical routine instrument. It would therefore be interesting, even from a purely practical point of view, to investigate the possibilities of using transistors for low temperature thermometry.

Apart from practical considerations, the investigation of semiconductor noise as a function of temperature should prove of theoretical interest, and it is understood that such investigations are already being carried out at the Massachusetts Institute of Technology.[27]

A noisy transistor may be represented as a four terminal network with two noise sources and a correlation impedance[28]. Since each of these noise sources is, in effect, essentially related to a noisy diode,[20] it seemed best to start the investigation from the DC diode characteristics and its dependence on the temperature.

The diodes investigated were germanium p-n-p alloy type, 2N105 RCA transistors. Each transistor was regarded as made up of two junction diodes with the base being the common point. The DC characteristics of these diodes were measured at room, liquid air and liquid helium temperatures, which correspond to about 293, 80 and 4.2 degrees Kelvin respectively.

The starting point in the analysis of experimental data was the theoretical voltage current characteristic given by:

$$I = I_0(\exp \alpha V/T - 1) \qquad (14)$$

where I is the diode current caused by the application of the voltage V across its terminals. I_0 is the saturation current, that is the current which would flow in the reverse direction for large negative values of V. The constant α is equal to e/k, the ratio of the electronic charge and Boltzmann constant. The numerical value of that ratio is about 11600 °K/volt.

The characteristics of the diodes measured showed two departures from the relation 14.

The first of these departures was the meaning which had to be given to the voltage V. Even at room temperatures the voltage across the diode had to reach a certain threshold value before the forward current started to flow and behave approximately as predicted by (14).

At lower temperatures that threshold voltage increased considerably and reached a value of about 1.1V at the liquid helium temperature. The variation of that voltage with the temperature is shown in Fig.7. It is necessary, therefore, to interpret V in the equation (14) as the excess of voltage over the threshold value V(T), that is

$$V = V_d - V(T) \qquad (15)$$

where V_d is the total voltage applied and V(T) is the threshold voltage which is a function of the temperature.

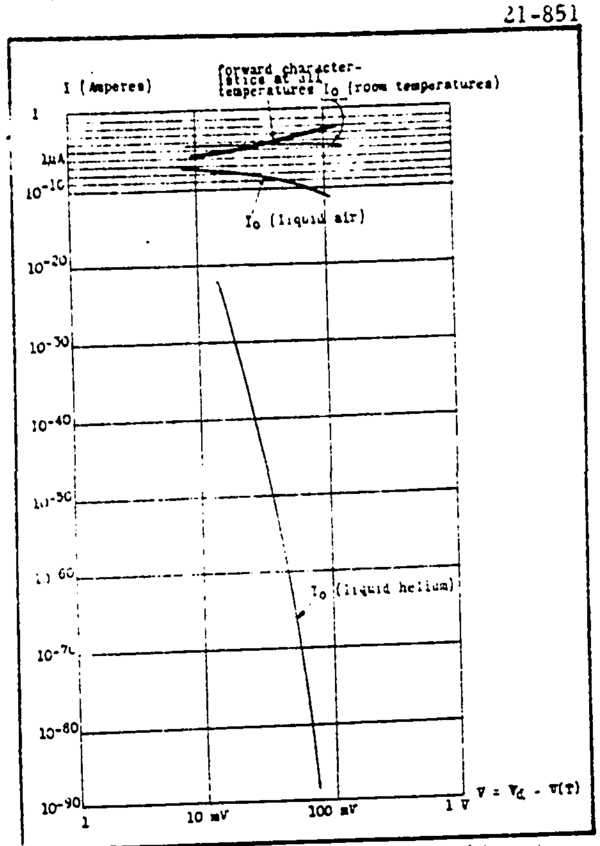

Figure 6 Forward characteristics of junction diodes at various temperatures and their saturation currents

The second departure from the equation (14) is the behaviour of the current I_0. If that current were constant it should be possible to determine it from a single measurement of I at a given V and T.

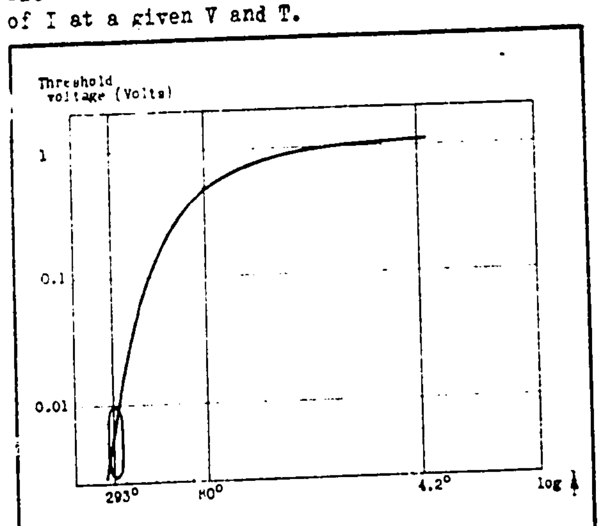

Figure 7 Variation of the threshold voltage of junction diodes with temperature

At room temperatures this is approximately true. As the temperature decreases however, I_0, as calculated from (14) exhibits very marked dependence on the applied voltage. This is shown in Fig.6. from which one could see that I_0, as defined by equation (14) decreases so much as to become meaningless at the liquid helium temperature.

It is necessary, therefore, to modify equation (14) further to take account of the variation of Io. That variation may be approximated by

$$I_o = I'T \exp(-\beta V/T) \qquad (16)$$

where I' and β are empirical constants, T is the temperature in ^oK and V the voltage as defined by (15).

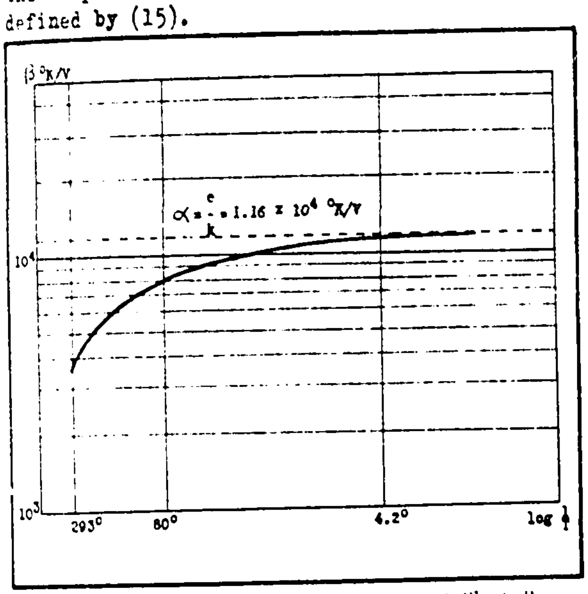

Figure 8 Variation of the exponent "beta" with temperature

Now by substituting (16) into (14) we obtain

$$I = I'T \exp(-\beta V/T)(\exp \alpha V/T - 1) \qquad (17)$$

The coefficient β in (17) is not constant. Its variation with the temperature is shown in Fig.8. The numerical value of β seems to approach that of α asymptotically as the temperature decreases.

The current I' in (14) is constant and its representative value is about $2{,}3 \times 10^{-9}$ $A/^oK$. This seems to be a much more realistic value than that of I_o calculated from (14).

The equation (17) approximates the behaviour of junction diodes only in the forward direction. It was not possible to check the reverse characteristics with the available instruments. At liquid air temperature the reverse current was of the order of 0.05 μA at -25 volts. At liquid helium temperature the reverse current was undetectable. Such behaviour might suggest that the reverse characteristic of the diode has the following form

$$I = I'T(\exp \alpha V/T - 1) \qquad (18)$$

The DC characteristic of the diode may be used for predicting its noise behaviour. Using the expression for the saturated diode noise current $I_n^2 = 2eI_{eq}\Delta f$,[19] and assuming the diode current I_{eq} to be composed of I + I_o and $-I_o$ we have for the sum of the shot noise sue to each of these currents.

$$I_n^2 = 2eI'T \exp(-\beta V/T)(\exp \alpha V/T + 1)\Delta f \qquad (19)$$

The noise ratio "n" is defined as

$$n = I_n^2 / 4kTG\Delta f \qquad (20)$$

where G is the conductance of the diode which may be calculated from (17) as

$$G = ((\alpha - \beta)I + I_o)/T \qquad (21)$$

The theoretically expected noise ratio then follows from (19) (20) and (21)

$$n = \frac{1}{2} \frac{I + 2I_o}{(1 - \beta/\alpha)I + I_o} \qquad (22)$$

If the forward current is large compared to I_o then (22) reduces to

$$n = \frac{1}{2} \frac{1}{(1 - \beta/\alpha)} \qquad (23)$$

The behaviour of the noise ratio as a function of temperature is shown in Fig. 9 for a forward current I = 100 μA, and for a large forward current when the approximation (23) may be used.

No measurements were made, as yet, to confirm the predicted noise behaviour of diodes considered. The author hopes to be able to make these measurements in the near future.

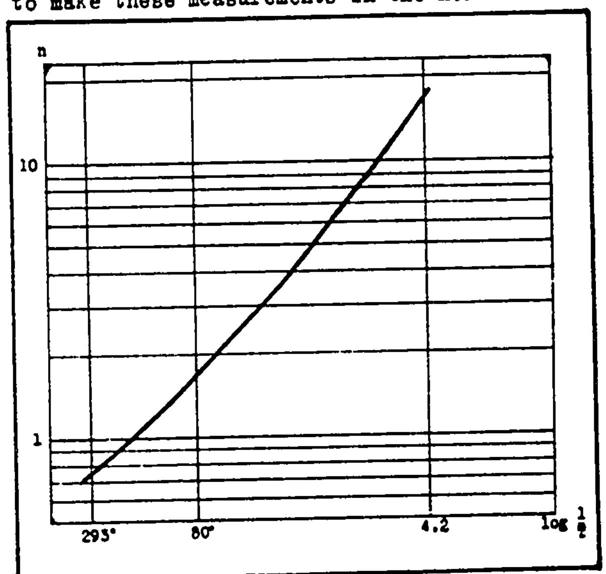

Figure 9 Variation of the noise ratio with temperature

Conclusion

The review of the publications on Noise Thermometry shows that it should be possible to make a noise thermometer for the measurements of very low temperatures, where the need for it is greatest. The use of transistors in such a thermometer might simplify certain aspects of its construction. To be able to use transistors more research is needed in order to make it possible to predict and design practical low temperatures circuits.[14,15.] A preliminary investigation into the DC behaviour of junction diodes at liquid air and liquid helium temperatures revealed interesting departures from the commonly accepted current voltage characteristics. Further research on that subject is being carried out at the University of Ottawa.

Acknowledgement

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July 1954 - Hogue analysed the factors affecting the precision and accuracy of an absolute noise thermometer and estimated the limit of precision at 0.3 - 0.5%.

December 1958 - Pursey and Pyatt described a method of measurement of the equivalent noise resistance of a noise thermometer amplifier. This was preparatory to the construction of an absolute noise thermometer with an error not greater than 0.05%.

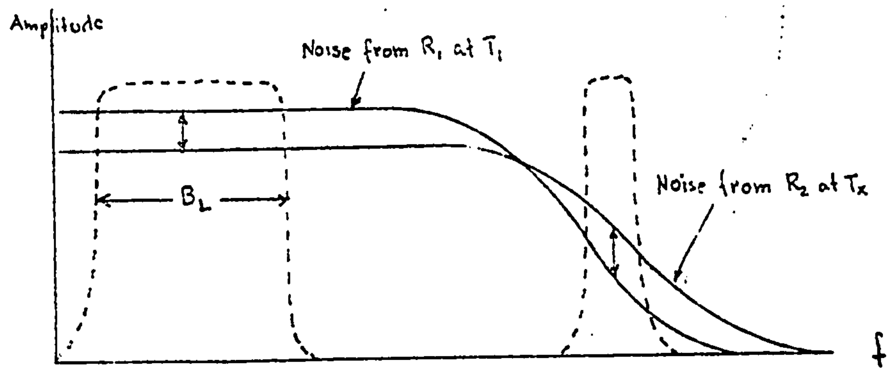
March 1959 - Patronis, Marshak, Reynolds, Sailor and Shore constructed a simple low-temperature noise thermometer and measured the lambda point of liquid helium (2.19°K) with less than 10% error.

July 1959 - Fink constructed a new absolute noise thermometer in which two amplifiers were used to amplify the noise from a simple π-network connected at the input. Under certain conditions the correlation between two noise voltages at the network terminals can be made zero. Since the amplifier noise is also noncorrelated Fink, by multiplying and averaging the two noise voltages, obtained a null condition from which the temperature could be deduced. The lowest temperature measured was 1.3°K with less than 1% error.

The work of E. W. Hogue (1954) was carried out at the National Bureau of Standards on an instrument basically the same as that of J. B. Garrison and A. W. Lawson (1949) with an addition of a synchronous rectifier and of an electronic integrator. The purpose of the work was to build a primary standard thermometer which could be used in the temperature regions beyond the range of the ordinary gas thermometer. To be useful as a standard an accuracy of at least 0.1% was required.

The temperature measurements were made by comparing the noise from a thermometric element (a known resistance at an unknown temperature) with that from a reference source (a known resistance at a known temperature). The problem considered by Hogue was to analyse the factors contributing to the inaccuracy of such a comparison.

The first condition which had to be fulfilled was the equality of the time constants (or bandwidths) of the two resistors and the associated capacities. This was accomplished by equalizing the noise from these two sources in two different channels. One (the main channel) at low frequencies, the other (the auxiliary channel) at a frequency at which the noise output from the two resistances began falling due to the capacity present. (Fig. 2.1).

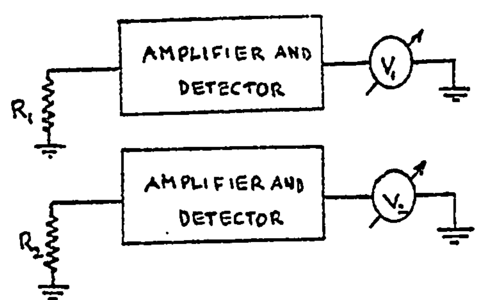


2.1

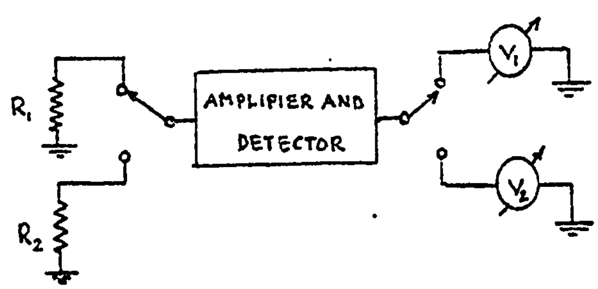
When the noise output from R_1 and R_2 was equal in both channels the time constants of both networks were the same, and knowing the value of the resistance R_2 at the unknown temperature T_x it was possible to calculate this temperature from R_1, T_1

and R_2. Hogue shows that for 0.1% precision, with such an arrangement, the amplifier bandwidth B_L can be as wide as 22% of the bandwidth of the thermometric network.

In this type of measurement the precision of the final result depends on the precision with which the unknown noise voltage can be compared to that coming from a known source. This comparison could be made by using two separate amplifier channels (one for each resistor) and by measuring the two output voltages. (Fig. 2.2). In practice it is difficult to ensure that the two channels are identical. Only one amplifie is therefore used, which is periodically switched to amplify the noise from the two resistors. (Fig. 2.3).



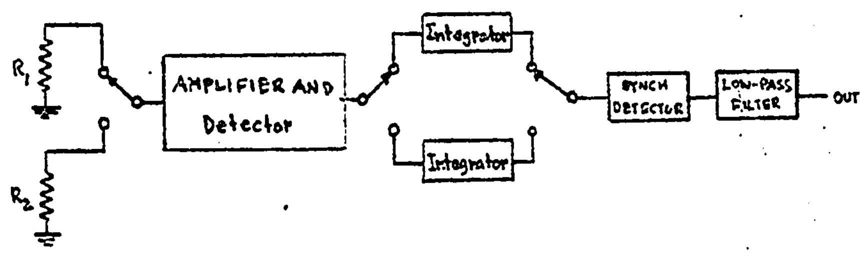
2.2



2.3

This switchin becomes a source of a number of errors which were analyzed by Hogue in detail. Another source of error arises from the fact that a common channel can be used only for amplifying frequencies high compared to the switching rate.

To accurately measure the mean square value of noise it has to be averaged after detection over a period of time which is much longer than the switching period. It is thus necessary to use additional noise processing equipment with a separate channel for each source. (Fig. 2.4).



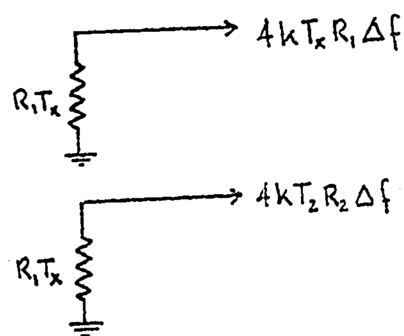
2.4

An accurate estimate of the mean-value of the fluctuating difference voltage (which should be zero) requires a very narrow band pass filter (about 10^{-4} cps in the case considered) centered at the switching frequency. Since both the construction and the use of such a filter would be impractical the center frequency of the difference voltage is shifted to DC (by means of another synchronous switch) where a low pass filter can be used, thus making the bandwidth of 10^{-4} cps practically feasible. A Miller integrator was used with a time constant of 8 min. which reduced the average amplitude of the noise fluctuations to the amplitude of 0.1% unbalance signal.

The final estimate of the time required to complete a temperature measurement with 0.1% precision was 56 minutes divided equally into two sets of measurements, one intended as a check of the other.

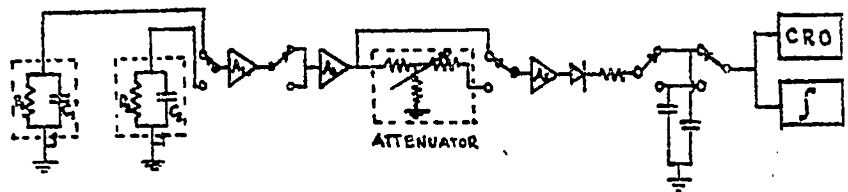
Hogue made tests to verify his calculations. In these the room temperature could be determined to within 0.5%. This means that the low temperature limit of his equipment would be in the region of 1.5°K. No mention was made about the applications of this thermometer.

The next publication connected with the Noise Thermometry (H. Pursey and E.C. Pyatt, 1959) originated at the National Physical Laboratory, England. The basic method of measurement is again similar to that of Garrison and Lawson (1949). In this case, however, the authors made a significant change. In all previous noise thermometers the unknown temperature was measured by adjusting the value of the resistance P_2 at a known temperature T_2 to give a noise output equal to that from R_1 at T_x. This led to the relation 2.4 which gave the unknown temperature in terms of three known quantities.



$$T_x = \frac{R_2}{R_1}T_2 \qquad 2.4$$

Pursey and Pyatt point out that in this type of measurement the amplifier which amplifies the noise to a measurable level is being switched between two sources of noise having different impedances. This change of input impedance may be a source of an additional error. Instead of equalising the noise at the input, the authors decided to equalise the noise from the two sources at some further point in the amplifier chain (attenuator) where such an equalization would not introduce any additional errors (Fig. 2.5)



2.5

This change, however, introduced another factor into the measurements i.e. the necessity of the knowledge of the background noise level of the amplifier.

The noise contribution of an amplifier can be represented as a resistance which, when connected to an ideal noiseless amplifier, would produce an equivalent amount of noise. It is possible to make the amplifier noise power equal to about a tenth of that generated by the low temperature source so that for 0.1% precision in temperature measurement the equivalent noise resistance of the amplifier must be determined to an accuracy better than 1%.

The authors describe in detail the measurements of the amplifier's equivalent noise resistance and the error involved in such measurements. The equipment used was quite capable of measuring its own noise with better than 1% precision; what the authors found however was the fact that the equivalent noise resistance did vary significantly over a period of a few days and that this variation was not caused by inadequate measuring techniques. As one of the possible reasons for such variations the changes in the position of the electrodes of the first tube was mentioned.

The ultimate accuracy of temperature determination which Pursey and Pyatt are hoping to achieve is within 0.01% of the true value. For this accuracy the stability of the noise amplifier would have to be improved and this would require further research into the causes of this instability.

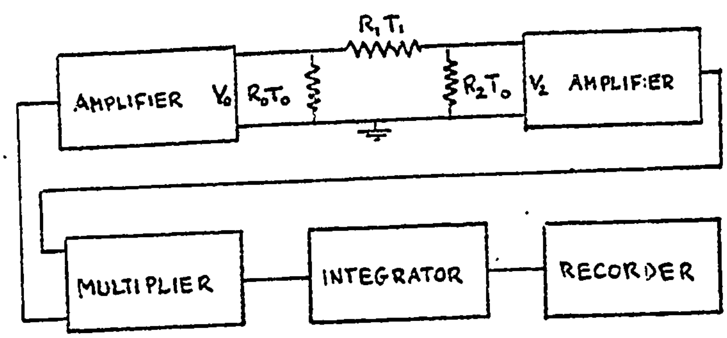
At the time of writing this review the work on the noise thermometry at the NPL was temporarily suspended.

Shortly after the publication of the preliminary results by Pursey and Pyatt an account of the construction of a complete noise thermometer was published from the Brockhaven National Laboratory by E.T. Patronis et al. in 1959. The technique used was that of straight amplification of the noise from the thermometric resistance and the comparison of its value to the noise from a known source which could be switched at the input. The amplifier input was made relatively insensitive to the variation of the input resistance by means of negative feedback in the initial stages. The mean level of the background noise of the amplifier was cancelled at the output by means of a suitable dc bias.

The results obtained by Patronis et al. were not encouraging in their accuracy. Temperature measurements within 2% were made at about 77°K (liquid nitrogen) and within 8% at about 2.2°K (lambde point of the liquid helium). The authors expressed doubt about the possibility of radical improvement within foreseeable future. No familiarity with the work of E. W. Hogue was indicated.

The last, known to me, contribution to the noise thermometry came from H. J. Fink (1959) who worked at the University of British Columbia.

The method of noise measurements used by Fink was different from those of his predecessors. In order to avoid difficulties introduced by the switches used by Garrison and Lawson and the followers, Fink amplified the noise from two thermometric elements (R_0 and R_2) arranged (together with a third resistance R_1) in the form of a π network, (Fig. 2.6) multiplied the outputs and averaged the product. The average of all noncorrelated noise products was zero, and Fink shows that under certain conditions the average of the product of the two noise voltages V_0 and V_2 appearing at the terminals of the π network can also be made zero.



2.6

In the ideal case, therefore, when the averaged product of the two amplifiers is zero, the temperature of the two resistances R_0 and R_2 is given by

$$T_0 = T_1 \frac{R_1}{R_0 + 2R_1 + R_2} \qquad 2.7$$

This method of measurement eliminates the background noise of the amplifiers and it is also a null method, both features being very desirable. Unfortunately due to a number of reasons (finite averaging time being one of them) neither the noise of the amplifier could be cancelled out completely nor could the noise voltages V_0 and V_2 be adjusted to become fully uncorrelated (grid current of the input tubes being one of the reasons). Nevertheless Fink obtained very good results in measuring liquid helium temperatures down to $1.3^{\circ}K$, with a precision of $\pm$ 1%.

This last reported work on noise thermometry seems to me as constituting the major breakthrough which was sought by the workers in this field. It seems as if this method of measurements were capable of producing results with a precision at least ten times greater than the former "conventional" methods.

Since the previous methods (Garisson and Lawson , 1949, Hogue, 1954, Pursey and Pyatt 1959) were potentially capable of accuracies within about 0.1% of the true value, the method of Fink * should, in my opinion, ultimately lead to the

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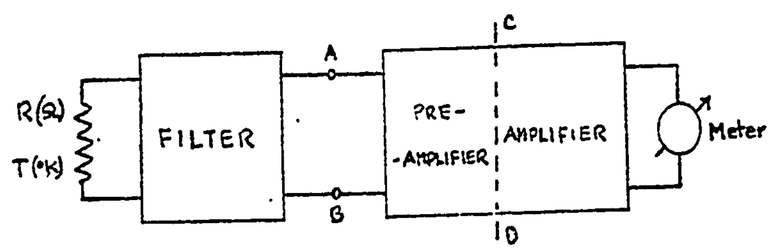
* It might perhaps be interesting to note that the idea of using the property of a π network to produce uncorrelated noise voltage, for temperature measurements, originated from J. B. Garrison, one of the creators of noise thermometry (cf. Fink).

temperature measurements with tolerances not exceeding 0.01% in the liquid helium region. Such an accuracy would put the noise thermometer in the category of instruments in which it was intended to be, i.e. an absolute thermometer capable of making accurate measurements in those temperature regions where other thermometers fail.

3. <u>Summary of the report of CTJ Alkemade "On the Construction of a Noise Thermometer for Low Temperatures"</u>.

Dr. C.T.J.Alkemade * worked on the problem of Noise Thermometer at the Low Temperature and Solid State Physics Division of the National Research Council, Ottawa, during his nine-month stay there as a post-doctorate fellow in the year 1955 under the direction of Dr. D.K.C. MacDonald. The time available to him was too short to finalize his research. The experience gained by him, however, was valuable.

The basic components of the noise thermometer constructed by Alkemade were the same as those which have been shown already in Fig. 1.4 which is repeated here.



3.1

As the noise generating element a small resistance R of the order of a few ohms was used. That resistance was inserted in series with an inductance L, thus forming a part of a resonant circuit RLC (Fig. 3.2).

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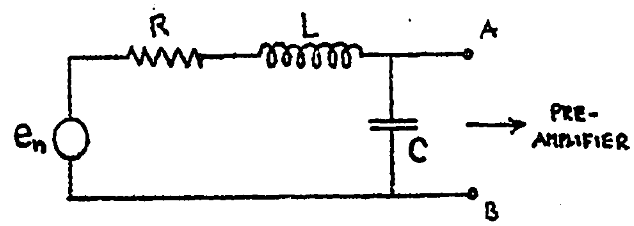
* Now in Utrecht, Holland.



3.2

Resonant circuit was used in order to "magnify" * the noise voltage generated within the resistance R and also to select the frequency band of the investigated noise in a suitable region.

The inductance L was made of copper wire coated with lead which becomes a superconductor at temperatures below about 7.2°K. At liquid helium temperatures (4.2°K and lower) the coil losses should have vanished and R should have remained as the only noise generating element. With $C = 15\ \mu\mu F$; $f_o = 1$ Mc and $R = 1\Omega$ the Q of the circuit should have been

$$Q = \frac{1}{\omega_o RC} = 10\,000 \qquad 3.3$$

The mean square noise voltage developed across the terminals AB within an infinitesimal bandwidth <u>df</u> is given by

$$d\,(V_{AB}^2) = 4kTR\,|W(f)|^2\,df \qquad 3.4$$

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* The noise power within the bandwidth of the resonant circuit remaining unchanged and equal to $4kT\Delta f$.

where W(f) is the immittance * of the input filter (see Appendix I).

The total mean square voltage over an infinite bandwidth is

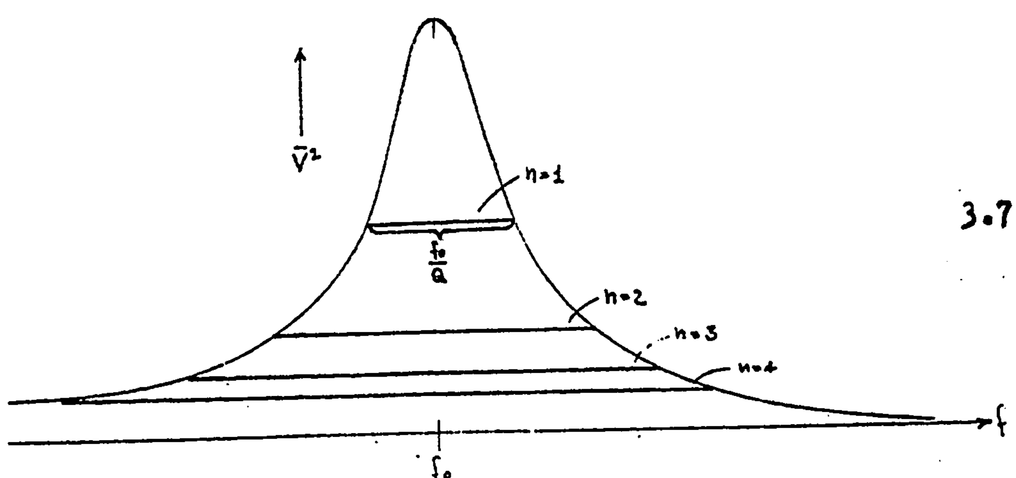
$$\bar{V}_{AB}^{2} = 4kTR\left(\frac{2\pi f_o Q}{4}\right) = kTR\,\omega_o Q \qquad 3.5$$

where $\omega_o = \frac{1}{\sqrt{LC}}$ and $Q = \frac{\omega_o L}{R}$.

If we consider only a finite bandwidth around the frequency f_o, we could write 3.5 in the form (cf. AIII.6)

$$\bar{V}_{AB}^{2} = kTR\omega_o Q\,\frac{2}{\pi}\tan^{-1} n \qquad 3.6$$

where n expresses the bandwidth involved in terms of the "nominal bandwidth" of the resonant circuit equal to f_o/Q (see Fig. 3.7).



3.7

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* The term "immittance" was first introduced by Bode and denotes either IMpedance or adMITTANCE as the case may be.

The maximum noise voltage would be obtained if we considered the bandwidth from $f = 0$ to infinity. This would, however, require an amplifier having an infinite bandwidth, or at least a very wide one. If we limit ourselves to only 99% of the total noise voltage available from the resonant circuit then we can restrict the bandwidth to about $64n$ (see Fig. AIII.10), which is practically the same as the value of $n = 20\pi \cong 62.8$ used by Alkemade. This value imposes restrictions on the Q value of the resonant circuit which should not be less than 32 *.

In such case $\frac{2}{\pi} \tan^{-1} 64 = 0.99 \cong 1$ and we can use the relation 3.6 in the approximate simple form

$$V_{AB}^2 = kTR\omega_o Q \qquad 3.8$$

where $Q \geqslant 32$ and the bandwidth implied is $\geqslant 64\, f_o/Q$.

This expression is both dimensionally and intuitively correct since the higher is the Q factor of the circuit the greater is the amplitude of the noise voltage developed across the terminals of the resonant circuit. Also, the higher is the resonant frequency the greater is the bandwidth of the circuit with a given Q and, therefore, the higher is the amplitude of the noise voltage.

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* In order to realize physically a bandwidth of $64n$ centered around the resonant frequency, we must have $f_o \pm nf_o/2Q \geqslant 0$. This gives the condition $n/2Q \leqslant 1$ i.e. $Q \geqslant 32$. This value is not an absolute minimum for Q to satisfy the realizability

The use of a resonant circuit allows the magnification of the noise voltage components in the vicinity of the resonant frequency of that circuit. The frequency discriminating property of the resonant circuit allows the choice of the band of frequencies in such a way as to exclude the flicker * effects of the amplifier and, in the case of non-metallic resistors, of the resistor R itself.

Having decided about the type of the temperature sensing element, Alkemade made a detailed investigation of the input circuit (to the left of the line CD in Fig. 3.1) in order to find the conditions for the lowest detection limit for the thermometer.

The most significant noise contributions in the apparatus was assumed to be;

a) from the thermometric element R itself,

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condition since the expression 3.6 is only an approximation for $Q > 10$ (see also Fig. AIII.10).

* The term "flicker effect" is a convenient generic term which includes all forms of low frequency noise which tend to increase with decreasing frequency and arise from irregular fluctuations of the physical parameters of the system having relatively long relaxation times (e.g. noise due to a current flowing in a thin metallic film, a carbon resistor and a semiconductor rectifier) (cf. Burgess et al., 1951).

b) from the shot noise due to the grid current

c) from the shot noise of the preamplifier tube due to the plate current.

The equivalent circuits for these components are shown in Fig. 3.9.

Two possible characteristic cases have been distinguished by Alkemade.

a) The noise bandwidth is limited by the resonant circuit at the input (amplifier bandwidth greater than that of the input circuit).

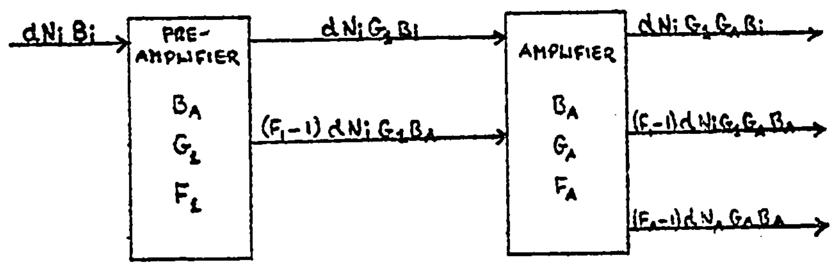
b) The noise bandwidth is limited by the amplifier (amplifier bandwidth smaller than that of the input circuit).

A short analysis of these two cases (Appendix 2) shows, that for the particular case in hand, i.e. for a given tube and circuit parameters, the second case is much more favourable than the first. The result is intuitively obvious since in the second case the amplifier accepts all the noise from the input circuit i.e. its bandwidth is filled up by the desired noise, whereas in the first case only a part of the amplifiers' bandwidth is occupied by the desired noise. Noise contributions are received from the regions of the spectrum which are thermometrically useless.

An analysis of these two cases is given here from the point of view of the noise factor of the measuring apparatus.

Figure 3.10 shows the case a), and Fig. 3.12 shows the case b). The only differences between these two cases are the relative magnitudes of B_i - the bandwidth of the input resonant circuit and B_A - the bandwidth of the amplifier.

Thus we have for the first case



3.10

where dN_i is the noise power contribution from the thermometric element, contained within an infitesimal bandwidth df.

dN_A is the thermal noise power contribution of the amplifier, contained within an infinitesimal bandwidth df.

B_i and B_A are the effective bandwidths of the input resonant circuit and of the amplifier respectively,

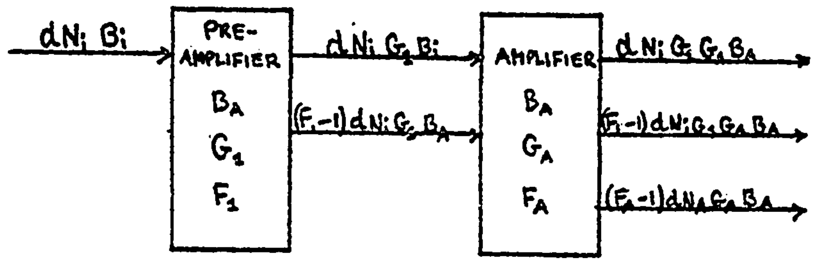
G_1, F_1, G_A, F_A are the gains and noise factors of the preamplifier and amplifier respectively.

The overall noise figure

$$F = \frac{dN_i G_1 G_A B_i + (F_1 - 1)\, dN_i G_1 G_A B_A + (F_A - 1)\, dN_A G_A B_A}{dN_i G_1 G_A B_i}$$

$$= 1 + (F_1 - 1)\frac{B_A}{B_i} + \frac{F_A - 1}{G_1}\,\frac{dN_A}{dN_i}\,\frac{B_A}{B_i} \qquad 3.11$$

In the case in which the amplifier bandwidth is smaller than the bandwidth of the input circuit we have



3.12

$$B_A < B_i$$

where from the overall noise figure

$$F = \frac{dN_i G_1 G_A B_A + (F_1 - 1)\, dN_i G_A B_A + (F_A - 1)\, dN_A G_A B_A}{dN_i G_1 G_A B_A}$$

3.13

$$= F_1 + \frac{F_A - 1}{G_1} \; \frac{dN_A}{dN_i}$$

In the first case 3.11 the amplifier contribution to the overall noise figure is increased by the factor B_A/B_1 whereas in the second case the overall noise figure is independent of that ratio (provided $B_A < B_1$).

Alkemade assumed that the limit of detection was reached when the total noise contributed by the preamplifier was equal to that originating in the thermometric element. The contribution of the main amplifier to the total noise output was not considered. Because of various side effects Alkemade never actually made any noise measurements. However, it can be seen from 3.13 that he would probably have encountered difficulties because of the noise contribution of the main amplifier itself. The amplification factor of the preamplifier tube (VX 41-A) was about unity (tube mfgr. spec.), the gain of the preamplifier G_1 could have been therefore in the order of unity at the most. Under such c nditions the noise contribution of the main amplifier would be just as important as that of the preamplifier.

Alkemade encountered several effects in his measurements which, within limited time available to him, made the temperature measurements impossible. Among these effects were:

a) The Q-factor of the input resonant circuit was realized only to 10% of the expected value (10^3 instead of 10^4). This effect was not fully explained but Alkemade, in a later communication *, suggested the electron transit time as an explanation of this.

b) An effect of cooling of the preamplifier tube causing the reduction of the plate current to about 10% of the rated value. That effect also could not be explained, but could be compensated by increasing the plate voltage of the tube or by heating its envelope. This effect was not fully reproducible.

To quote Alkemade "under these circumstances it was useless to make noise measurements".

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* In a letter to D.K.C. MacDonald, April 11, 1956.

4. General comments on noise measurements and the sources of inaccuracies.

The measurements of thermal noise are, in principle, simple. All that is required is to amplify the noise to a measurable level and then measure it. In practice, however, there is a number of factors which have an adverse effect on the accuracy of such a measurement. The number and the magnitude of these effects is such as to make noise measurements not only far from simple but actually being one of the more troublesome measurements of a physical quantity. Noise measurements accuracies of 1% have been reached not very long ago in isolated cases, and a claimed 0.1% accuracy * was obtained only when the amplitude of the measured noise was relatively high (Garrison and Lawson, 1949).

The investigation of electrical noise appearing between two different points in solid bodies (conductors and semiconductors) represents one of the ways in which certain properties of these bodies can be established, confirmed or, perhaps, discovered.

At present, two properties of noise are of interest. The absolute magnitude (expressed by the mean square value) and the spectral characteristic. Little experimental work seems to have been done on other noise characteristics such as the probability distribution of instantaneous amplitudes

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* This figure was challenged by W. Hogue (1956), according to whom the accuracy of Garrison and Lawson thermometer could not have been better than 0.3 - 0.5%.

and its auto-correlation.

Most of the present-day measurements of physical quantities made in the course of scientific investigations have errors of about 1% or less. If the noise were to be used as one of the characteristic quantities specifying the properties of physical bodies or systems, then the accuracy of the routine noise measurements should be about the same, i.e. 1% or better.

It is difficult to reach this accuracy with the present-day methods, and even if it is reached, such measurements are not in the category of routine work.

It would be very desirable to have an instrument with two terminals which, when connected to the investigated sample, would give a direct reading of the absolute value of the mean square noise across the sample in a given frequency band with an accuracy of, say, 0.1%.

The uses for such an instrument would be quite numerous, temperature and resistance measurements being only two of them.

There is, at present, no such instrument available. The only commercial devices used for noise measurements are the so-called "noise figure meters", used for the determination of relative amount of noise, and these claim an accuracy not better than within $\pm$ 5% of the true value.

There are several factors which account for the absence of such a commercial noise measuring instrument. The lack of demand is, of course, the primary reason, but the difficulties inherent in the construction of such an instrument are perhaps just as important. Each of these factors contribute to both the inaccuracy of the noise measuring instrument, and to the difficulty of its construction.

A list of these factors as applied to a noise thermometer is given below. It is by no means complete, but it can be used as a starting point in investigating the problem. These factors have been discussed by some author at one time or another, and references to these discussions are given if known.

a) Noise from the thermometric element.

b) The effect of the shunt capacitance across the thermometric element.

c) The accuracy of the determination of the thermometric element's resistance and the frequency dependence of its value.

d) Temperature dependence of the resistance of the thermometric element.

e) Flicker effect in the thermometric element.

f) Grid current of the first tube.

g) The influence of Miller effect in the 1st stage.

h) Transit time effects.

i) Flicker effects in the first stage.

k) Influence of the second stage of the amplifier.

l) Background noise of the amplifier chain.

m) Bandwidth of the amplifier.

n) Effects of the filter characteristics.

o) Stability of amplification.

p) Overall feedback.

q) Extraneous signals picked up by the amplifier.

r) Non-linearity of the amplifier.

s) The detector and its law.

t) The output meter.

u) Integration time.

It seems as if it would be worthwhile to compile a fairly detailed analysis of these factors from the point of view of an experimental investigator. Particularly since the method introduced by Fink may lead to noise measurements accuracies approaching those of the measurements of other physical quantities, i.e. within about 1 - 0.1%.

The compilation given below, due to the lack of the necessary knowledge and experience, is only partial. Having, however, made a start, it should be easier to continue this compilation until a fair degree of completeness is achieved (cf. section 6).

4a. Noise from the thermometric element.

The amount of noise voltage available from the thermometric element (resistance R) is given by the Nyquist relation

$$\bar{V}^2 = 4kTRB \qquad 4a.1$$

The accuracy of the determination of this voltage depends on the precision with which the quantities k, T, R and B are known.

The value of the Boltzmann constant k is $(1.38041 \pm 0.00007) \times 10^{-16}$ erg deg^{-1}, i.e. with the relative error of $\frac{7 \times 10^{-5}}{1.38041} = 5.05 \times 10^{-4}$ or 0.00505%. (AIP Handbook, 1957).

The temperature T can be measured with an accuracy of not less than one part in a thousand, i.e. the relative error in the temperature measurement is 0.1%. This, I think, can be assumed as the upper limit of error. The accuracy of temperature measurements can be improved from this figure by at least one order of magnitude throughout the temperature range from 4.2°K up to at least 1000°K. The figure of 0.1% can therefore be safely taken as the upper limit of error in the temperature measurement.

The accuracy of the determination of the resistance R depends on the measuring instrument. If the resistance can be measured at the time of making the temperature measurement, then the error in the determination of resistance may be made as small as 0.1% or even smaller. If, however, the value of the resistance has to be determined prior to temperature measurements, then the certainty about the resistance value at the time of the measurement is much smaller. It seems to me that an error of at least 1% would have to be allowed in such case (see also 4b, c, d, g).

It follows from the above estimates that a noise thermometer should have the facility for an accurate determination of the resistance of the thermometric element at the time of temperature measurement. The magnitude of the power dissipated during such measurement may be about 10^{-6}w. (cf. Nicol and Soller, 1957).

The accuracy of knowledge of the bandwidth B is problematic. In noise measurements it is, as a rule, the bandwidth of the amplifier chain which limits the amount of noise measured. The exact determination of the equivalent bandwidth of an amplifier is difficult because of the not-so-exact measurement techniques, particularly at radio frequencies. One could probably assume the 1% figure as being quite accurate and then attempt to improve from this figure. Since the bandwidth of the amplifier is very intimately connected with its gain, an accurate bandwidth determination can only be made on an amplifier whose gain stability is better than the desired accuracy of bandwidth determination (cf. section 4m).

To sum up, the errors made in calculating the absolute mean square voltage of the noise originating in the thermometric element are given below:

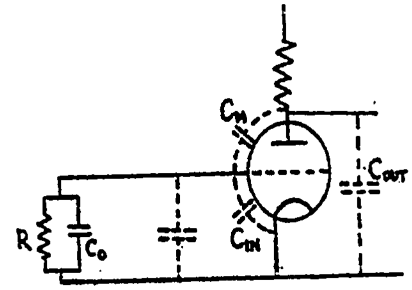
$$\Delta k = \pm 0.005$$
$$\Delta T = \pm 0.1$$
$$\Delta R = \pm 1.0$$
$$\Delta B = \pm 1.0$$
$$\text{Total error} = \pm 2.105\%$$

If the resistance R and the bandwidth B can be determined each within 0.1%, the total error made in calculating the mean, square noise from the Nyquist formula will be,

$$\Delta k = \pm 0.005$$
$$\Delta T = \pm 0.1$$
$$\Delta R = \pm 0.1$$
$$\Delta B = \pm 0.1$$
$$\text{Total error} = \pm 0.305$$

4b. <u>The effect of the shunt capacitance across the thermometric element.</u>

The thermometric element R is always shunted by a certain capacitance C. This capacitance is made up of the self capacitance of the resistance itself Co, of the stray capacities due to connecting wires C_s, and the input capacitance of the amplifier tube C_{in} and C_M. The tube input capacitance may be modified by the Miller effect (C_M).

 4b.1

As a first approximation we can assume that the total shunting capacity C_t across R is constant within the bandwidth of the measured noise. In such case the voltage across C_t is

$$\overline{V}_c^2 = \int_{f_1}^{f_2} \frac{4kTR}{(2\pi f RC_T)^2 + 1}\, df \qquad 4b.2$$

$$= \frac{2kT}{\pi C_T}\left[\tan^{-1} 2\pi RC_T f_2 - \tan^{-1} 2\pi RC_T f_1\right]$$

where it was assumed that the resistance R was independent of frequency.

If we choose the frequency band such that $\omega RC \ll 1$ then we may replace $\tan^{-1}\omega RC$ by its argument in which case

$$\overline{V}_c^2 = 4kTR\,(f_2 - f_1) \qquad 4b.3$$

If the working frequency is such that arc tangent cannot be replaced by its argument then the full formula 4b.2 has to be used.

Furthermore, the Miller capacity of the input stage is given by

$$C_M = C_{gp}\,(1 + G_V) \qquad 4b.4$$

where G_v, the voltage gain of the amplifier is given by

$$G_V = g_m R_{OUT} \frac{1}{1 + j\omega R_{OUT} C_{OUT}} \qquad 4b.5$$

where g_m and R_{out} are the mutual conductance and the effective output resistance of the input tube, and C_{out} is the total output capacitance of the stage.

Here again, if the quantity $\omega R_{out} C_{out} \ll 1$, then we may assume that G_v is essentially constant over the considered frequency range. If it is not so, however, the total capacity shunting the thermometric element R has to be considered as a function of frequency

$$C_T = C_o + C_s + C_{in} + C_M(f)$$

This case will not be considered in detail here. However, when using broadband amplifiers for noise measurements, this effect may have to be taken into consideration.

4c. The accuracy of determination of the thermometric element and the frequency dependence of its value.

The value of the thermometric element R depends, in general, on the frequency, temperature and the magnitude of the voltage across the terminals. This last relation was the subject of independent investigations by MacDonald (1953, 1957), Van Kampen (1958), Alkemade (1958) and Polder (1956) in their papers on fluctuations in nonlinear systems. Since the resistance nonlinearity is small, these effects are assumed to be of the second order of importance at the most, and I am going to disregard these effects in this thesis.

The temperature dependence of the resistance value has different character for different resistance materials. More information about this dependence is given in the next section (4d); in general, however, the thermometric element is held at a constant temperature during the temperature measurement. Therefore, even if its temperature characteristic is not known, the actual value of the resistance at the time of making the measurement can be checked and used in computations.

It is desirable to have a thermometric element whose resistance is constant throughout the temperature range in which it is to be used. This, however, can only be obtained within limited accuracy, and is not absolutely necessary.

What is really necessary, however, is a method of resistance measurement which would not upset the equilibrium between the thermometric element and the medium, the temperature of which is to be measured.

Any method of measurement which requires passing a current through the resistor will dissipate certain amount of power in it, and, therefore, upset its equilibrium with the surroundings. The only method known to me, which does not require passing an external current through the resistor, is the use of thermal noise across it. * This method, however, requires the knowledge of the resistor's

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* Verbal communication by Col. R. A. H. Galbraith, University of Ottawa.

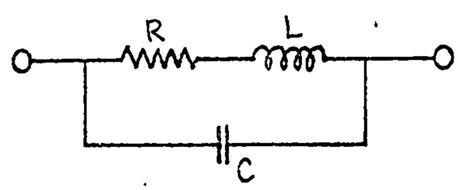
temperature and, therefore, defeats itself in this case. The only alternative seems to be the use of as little power as possible during resistance measurements and, perhaps, making use of thermal inertia of the resistance, i.e. making the measurement as short as possible. The use of short pulses at low repetition rates might be a fair solution of the problem.

There is a variety of methods which suggest themselves in connection with this. Comparison method and time constant method being two of them.

The subject of such measurements and their influence on thermal equilibrium will have to be investigated. In my estimate the inaccuracies in the resistance determination produced by these methods would be of the second order of magnitude at the most.

The factor which should acquire importance in wide band noise measurements is the frequency dependence of the resistance of the thermometric element.

The frequency dependence of wirewound resistors is quite pronounced because of the inductance of the winding and the distributed capacitance. A simple circuit which quite closely approximates the behavious of such a resistance up to a certain frequency is given in Fig. 4c.1

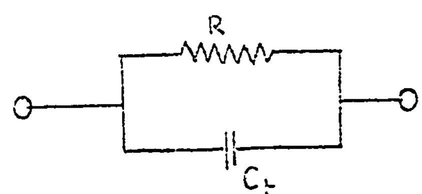


4c.1

This circuit was found to be valid up to about 17 Mc. for a 10 kohm, 10 watt, single layer power type resistor (cf. Blackburn 1949), in which case R = 10010 ohms, L = 115 μH and C = 0.64 μμF. The prediction of the high frequency behavior of wirewound resistors is, otherwise, uncertain, and for any reliable and accurate results the frequency response of the resistance sample has to be evaluated in each particular case. Only then the suitability of a wirewound resistor as a noise generator can be properly assessed.

The second type of resistors, composition type, behaves differently from the wirewound types.

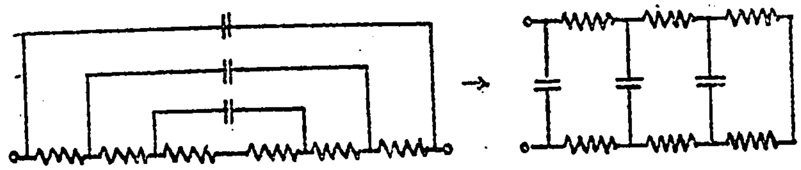
The simplest equivalent circuit for such a resistor is that of Fig. 4c.2



4c.2

This circuit is satisfactory up to certain values of resistance and frequency. At higher resistance and higher frequencies the values of both R_p and C_t decrease as compared with their low frequency values.

The next approximation in the representation of a resistor by a lumped constants circuit equivalent was made by Howe (1935, 1940).



4c.3

Fig. 4c.3 shows this approximation which can be regarded as a short circuited transmission line, half as long as the resistor. The driving point impedance may be calculated as a function of resistor's length l (cm); its distributed capacitance C (μμF per cm); the total d-c resistance R of the resistor; and the frequency f (cps). The result is given in the form of curves Fig. 4c.4 (cf. Blackburn 1945) showing how the resistance and the capacitance decrease as the product flCR increases.*

The actual measurements made on a variety of commercial composition resistors confirms the general form of the curves of Fig. 4c.4. There are, however, individual differences in the position of these curves. The only thing which can safely be said at this stage is that while the curves of the type shown here may be useful in anticipating the type of behaviour of resistors as a function of frequency, it is nevertheless necessary to find out the quantitative data in every individual case of a resistor used for noise thermometry.

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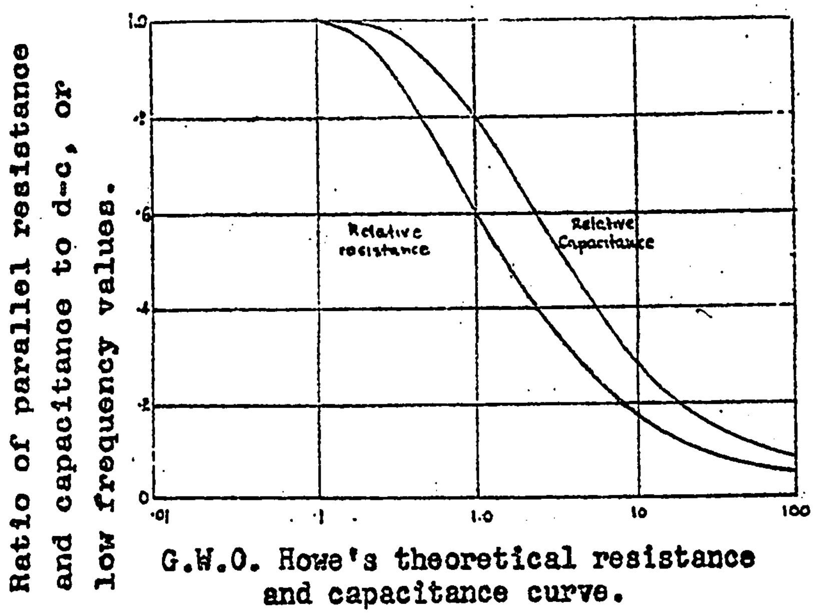
* The validity of Howe's model has been questioned (Broudy and Levinstein, 1954).



G.W.O. Howe's theoretical resistance and capacitance curve.

4c.4

The third type, metal or semiconductor film resistors, seem to have high frequency characteristics similar to those of the composition type. The transmission line model of Howe has been questioned in connection with the investi-ation of high-frequency resistance of thin films (Broudy and Levinstein, 1954). However, the general character of the resistance-frequency variation is still of the type shown in Fig. 4c.4.

Again, it seems as if the only reliable way of obtaining accurate data about the behaviour of film resistances as a function of frequency were to make these measurements on the individual sample itself.*

----

* H. J. Fink used metal film resistors manufactured by Daven Co., series 850. These resistors were found to be stable (within 1 part in 10,000) and the frequency dependence of the resistance value introduced an error not greater than 1% at 3 mc. (cf. Fink 1960).

4d. Temperature dependence of the resistance of the thermometric element.

To determine the temperature of a resistance from the measurements of thermal noise across the terminals it is necessary to know the value of the resistance at that temperature.

Ideally, the resistance of the thermometric element should be temperature insensitive. In practice, this insensitivity can be realized only to a certain approximation.

In the temperature insensitive class of resistors at low temperatures are: wire-wound, deposited carbon and boron carbon resistors (Nicol and Soller, 1957).

The resistivity of ideally pure metals (in which the resistivity is due to the effect of electron scattering caused only by thermal vibrations of the lattice) is proportional to the fifth power of temperature, at temperatures lower than about a tenth of the characteristic temperature (cf. DKC MacDonald, 1956 and also Kelly and MacDonald, 1953). This temperature is given for a few metals in the table below.

Metal	Cu	Ag	Be	Mg	Zn	Cd	Hg	Sn	Pb	Fe	Co	Ni
θ_D °K	333	223	1000	357	213	172	80	260	86	420	401	472

4d.1

(Handbook of Physics, McGraw Hill, 1958, p. 4-74)

These temperatures should be taken as a rough guide only;

it can, however, be seen that the liquid helium temperatures satisfy the condition $\Theta_D/10$ for all metals listed here.

The temperature variation of resistance of pure metals in that region can, as a good approximation, be expected to follow the fifth power law, i.e. for a one percent change in temperature we can expect about five percent change in resistance.

Extremely pure metals therefore do not seem to be suitable materials for thermometric elements in noise thermometry.

Impure metals present a very different picture. The temperature dependence of resistance of such metal may be expressed in the form (F.M. Kelly and D.K.C. MacDonald, 1953)

$$\frac{T}{R}\frac{dR}{dT} = \frac{R_i}{R_i + R_o}\;\frac{T}{R_i}\;\frac{dR_i}{dT} \qquad 4d.2$$

where $R = R_i + R_o$; R_o being the temperature independent residual resistance which, to a 1st approximation is temperature independent at low temperatures and, at these temperatures, numerically much greater than R_i, the ideal resistance of the metal.

Since $R_o \gg R_i$ the expression in 4d.2 is small. It must be pointed out that in certain cases, e.g. Cu with small concentration of Fe, (cf. Franck, Manchester and Martin 1961, and D.K.C. MacDonald 1956) the residual resistance behaves anomalously, and starts to rise again at low temperatures. This effect is by no means negligible.

but, to sum up, we can say that as a quantitative guide to the order of magnitude of the expression 4d.2 may serve the estimate that "it is improbable that the change in the resistivity of metals at temperatures below 10^{o}K will exceed about 1% per degree K, irrespective of the degree of metals' impurity, and at liquid helium temperatures all metals can be practically regarded as being impure."*

A wirewound resistor could, therefore, be used for noise thermometry at liquid He temperatures, subject to the resistance variations as above. For accurate results the temperature variation of resistance of thermometric element has to be known.

There is little information on the temperature dependence of deposited carbon and boron carbon resistors. Carbon composition resistors are being used in "resistance thermometry" where the resistance variation is a measure of the temperature. This case calls for a pronounced dependence of resistance on temperature. Because of this, the existing technical literature on carbon resistors at low temperatures deals almost exclusively with carbon composition resistors (e.g. Clement and Quinnell, 1952). I have not been able to find any references dealing with either deposited carbon or boron carbon resistors at low temperatures except for Nicol and Soller, (1957) who only state that these resistors are temperature insensitive.

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* Verbal communication from D.K.C. MacDonald (January 1961).

In order to be able to use deposited carbon or boron carbon resistors for low temperature noise thermometry it would be necessary to investigate their properties at these temperatures.

When considering the temperature dependence of resistors it might be appropriate to make some remarks on the ultimate limit of the accuracy of resistance determination which is imposed by this dependence.

The electrical resistance of an element is defined as the ratio of the voltage across the element and the current produced in the element by that voltage. This definition (Ohm's law) is the basis of all resistance measurements.

In order to measure a resistance it is necessary to apply a voltage to the element and measure the corresponding current. This procedure involves dissipating certain power in the element which alters its temperature.

The value of resistance which we measure is, therefore, made at some unknown temperature $T_x = T_o + \Delta T_o$, different from the ambient temperature T_o.

If we can measure ΔT_o caused by the dissipation of the electric power in the element, and if we know exactly the temperature dependence of the resistance of that element, then our measurement is complete.

The measurement of the temperature of the element, however, can only be made accurately by inferring its value from the measurements of some physical property which depends on the temperature of the element, such as thermal noise. Any other method of temperature measurement involves the assumption of thermal equilibrium between the element and the thermometer, which equilibrium is an idealized concept.

The circle closes when we realize that in order to measure the magnitude of thermal fluctuations of electrons in a conductor, we have to know its resistance.

In short, it seems as if it were impossible to know for certain two things at the same time, the temperature and the resistance of an element, and that there must be a theoretical limit to the accuracy with which they can both be determined.

This fundamental limit, however, seems to be very remote from the accuracies of the present-day measuring instruments. It is possible to choose such experimental conditions in which the temperature increment ΔT_o and the resistance increment ΔR_o, due to ΔT_o are both well within the tolerances of the values of T_o and R_o respectively, R_o being the resistance of the element at T_o.

4e. Flicker effect in the thermometric element.

The "flicker" effect, or what is otherwise called

1/f noise or current noise, appears only if a current flows through the resistor.

This type of noise is associated with semiconductor type resistors only. Wirewound resistors do not exhibit flicker effect. Metal film resistors, however, may exhibit current noise if the film is very thin and has a granular structure (cf. Bell 1960). This is unlikely in case of resistors having low values, of the order of a few kiloohms, which would be used as thermometric elements.

This noise has a characteristic power spectrum which is inversely proportional to frequency. For this reason, if we wish to eliminate flicker noise from noise measurements, it is necessary to cut the lower frequencies off. A safe limit seems to be a few kilocycles.

In the case of a thermometric element the current flowing through it is the grid current of the 1st tube. This current would be of the order of 10^{-8} - 10^{-15} a, or even less, depending on the tube type chosen (cf. Bell, 1960).

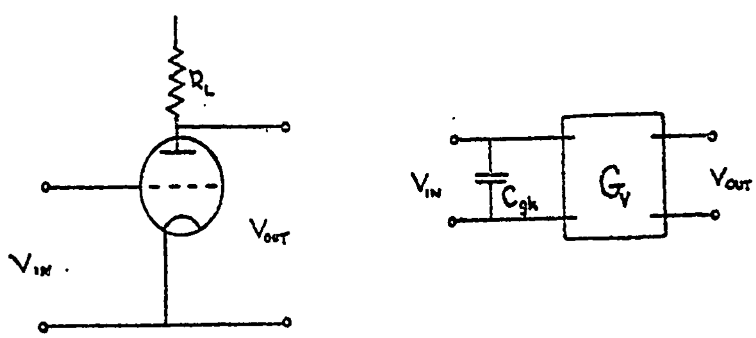
It is difficult to estimate even the order of magnitude of noise due to this current. Theoretical considerations of current noise chiefly deal with the explanations of its character. The only formulae available for numerical evaluation are empirical and, therefore, refer to particular cases.

For wide band noise measurements it is necessary to limit the low frequency response in such a way as to exclude the flicker effect. To be certain of this exclusion it is necessary to make careful measurements of the noise spectrum to find the point where it starts increasing as the frequency becomes lower. The lower frequency limit of the amplifier should be well above this point, (e.g. a decade) depending on the shape of the amplifier characteristics.

It might be remarked here that it is rather unlikely that any significant current noise would be found in metallic semiconductors at frequencies as low as 45 cps (cf. Bittel and Scheidhauer, 1956).

4f. Grid current of the first tube.

Consider a tube having a purely capacitive input impedance (Fig. 4f. 1)



4f.1

If we neglect, for the moment, the shot noise due to the cathode current, then the output voltage will be G_V times greater than that induced across the grid-to-cathode capacitance.

Assume that this capacitance is 10 $\mu\mu F$. The voltage induced across it, due to the charge of one electron, is $e/C_{gk} = V_{1e}$, where e is the electronic charge. This voltage, for the assumed case, is about 0.016μV. It is possible, in principle, to detect and even measure such a small voltage. For this purpose, however, it is necessary to use thermionic tubes with their inseparable grid current.

For a good electrometer tube the grid current is of the order of 10^{-15}A (cf. Bell, 1960). This means that in every second about 10,000 electrons, on the average, flow into the input capacitance. Disregarding the effects of the change of the working point of the tube due to this charge, the problem of measurement of potential due to a charge of one electron becomes the problem of measuring a voltage increment of 1 part in 10^4. If the measurement is to be made with an accuracy of only 10%, then the stability of the equipment used would have to be better than 1 part in 10^5.

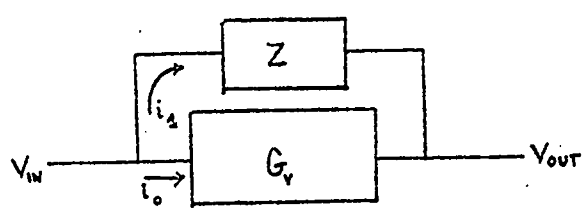
This order of stability is about the limit that can be obtained with present-day components by using amplifiers with large amounts of feedback. It seems, therefore, as if the measurement of an incremental voltage due to one electron would be feasible, if the grid current were the only factor to be considered in such a measurement. Since, however, there is a number of other factors contributing to the total fluctuations of the amplifier's output, the detection of an equivalent to the contribution of a single electron to the grid voltage would be rather problematic.

From the experimental evidence (Samaroo, 1961), however, it seems as if the measurement of an increment of one part in a thousand were feasible and there are reasons to believe that this figure could be improved somewhat by increasing the stability of the measuring equipment.

4g. The influence of Miller effect in the 1st stage.

The interaction between input and output of a vacuum tube is often called "Miller effect" (J.M. Miller, 1919).

Assume that there is a finite impedance Z, present between the output and input terminals of an amplifier G



4g. 1

The current i_1 flowing into the amplifier in addition to the current i_o is given by

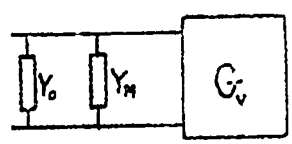
$$i_1 = \frac{V_{IN} - V_{OUT}}{Z} = V_{IN} \frac{1 - G_v}{Z} \qquad \text{4g.2}$$

where $G_v = \frac{V_{OUT}}{V_{IN}}$ is the gain of the amplifier.

The expression (2) can be written as

$$Y_M = \frac{1 - G_v}{Z} \qquad 4.\ 3$$

giving the input admittance component due to feedback. Total input admittance is therefore made up of two parts. The admittance component due to the currents flowing between input and ground, and the admittance components due to the current between input and output. This can be represented as below



4. 4

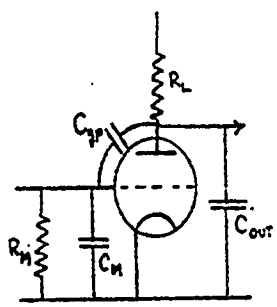
where Y_M is the equivalent admittance introduced between the input terminals, due to Miller effect.

The two admittances Y_o and Y_M are frequency dependent. Y_o usually has a capacitive character and therefore its magnitude increases with frequency. The behaviour of Y_M is more complicated since its magnitude is determined not only by passive elements but also by the gain of the amplifier, which is also frequency dependent.

In the simple case of a grounded cathode amplifier stage with capacitive load, Y_M is given by the parallel combination of R_M and C_M where

$$R_M = \frac{1 + (\omega R_{out} C_{out})^2}{\omega^2 C_{gp} C_{out} R_{out}^2 g_m} \quad \text{and} \quad C_M = \omega C_{gp} \left[\frac{2 + (\omega R_{out} C_{out})^2}{1 + (\omega R_{out} C_{out})^2} \right] g_m R_{out} \qquad 4g. 6$$

where the symbols have meaning as below.



$$R_{out} = \frac{r_p R_L}{r_p + R_L}$$

r_p - plate resistance of the tube

g_m - transconductance of the tube $\qquad$ 4g. 7

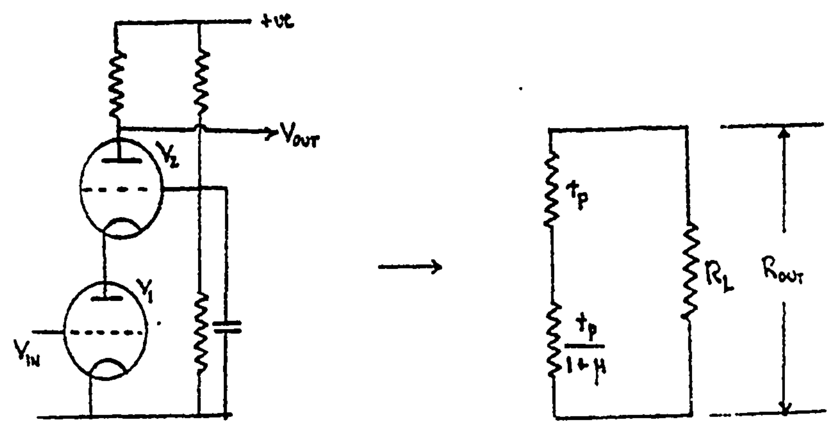
C_{gp} - grid-to-plate capacitance

C_{out} - total capacitance across R_{out}

This simple case shows that Miller effect not only magnifies the apparent tube input capacitance but also introduces a resistive component R_M. The value of R_M, calculated from 4g.6 for f = 1 mc ; R_{out} = 2 KΩ; C_{out} = 30 μμF; Cgp - 1μμF and g_m = 10ma/v is of the order of 20 KΩ; which is quite low and, moreover, varies with frequency.

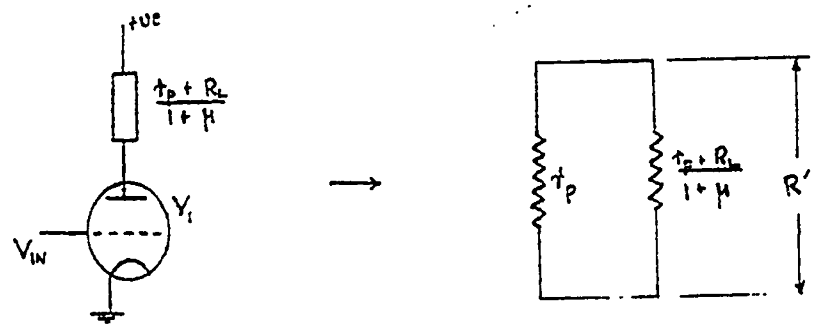
In application requiring a low noise figure of amplifiers the input stage tube configuration, almost universally used now, is the Wallman circuit, commonly called "cascode" amplifier" or "totem pole" (see Wallman et al. 1948).

This circuit is described in Fig. 4g.8 and 4g.9



4g.8

Output impedance



4g.9

Cascode circuit from the point of view of the input triode

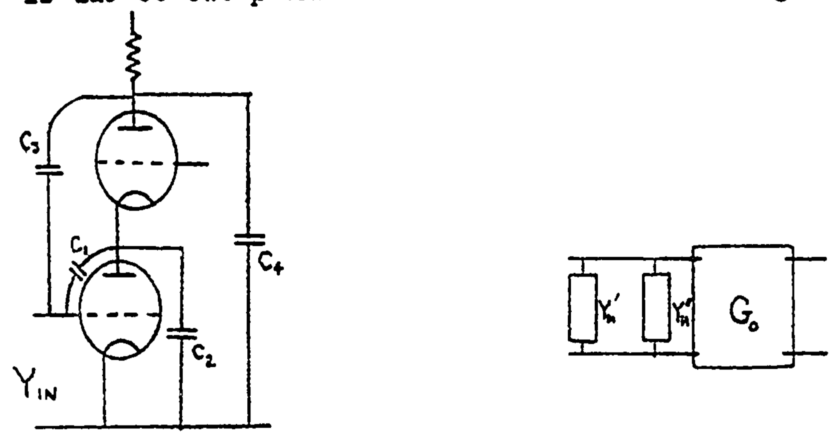
The tubes used in a cascode are usually of the same type and therefore their parameters are assumed to be identical. The gain from grid to plate of V_1, is then

$$G_1 = -g_m R' \qquad \text{4g.10}$$

where R' is as in Fig. 4g.9 and gm is the transconductance of the tube, and the overall gain from the grid of V_1, to the plate of V_2 is

$$G_o = \frac{-g_m R_L}{1 + \frac{R'}{r_p}} \qquad 4g.11$$

The Miller effect in the case of a cascode amplifier is due to two paths of feedback as shown in Fig. 4g.12



4g. 12

Here $Y_M' = j\omega C_1 (1 - G_1)$ and $Y_M'' = j\omega C_3 (1 - G_o)$

Since both G_1 and G_o are complex (because of the presence of the capacities C_2 and C_4) we can write

$$Y_M' = \frac{1}{R_{IN}'} + j\omega C_{IN}' \quad \text{and} \quad Y_M'' = \frac{1}{R_{IN}''} + j\omega C_{IN}'' \qquad 4g.13$$

where

$$R_{IN}' = \frac{1 + (\omega R' C_2)^2}{\omega^2 C_1 C_2 R'^2 g_m} \qquad C_{IN}' = \omega C_1 \left[\frac{2 + (\omega R' C_2)^2}{1 + (\omega R' C_2)^2}\right] g_m R' \qquad 4g.14$$

and

$$R'_{IN} = \frac{1 + (\omega R_{OUT} C_4)^2}{\omega^3 C_3 C_4 R_{OUT}^2 g_m} \qquad C'_{IN} = \omega C_3 \left[\frac{2 + (\omega R_{OUT} C_4)^2}{1 + (\omega R_{OUT} C_4)^2}\right] g_m R_{OUT} \qquad 4g.15$$

The meaning of symbols in these formulae is as given in 4g.8 - 4g.12.

In this case R'_{in} is very high and, because of very low value of R' (about 100 ohms) both R' and C'_{in} can be regarded as frequency independent, relative to R''_{in} and C'_{in}.

I have not checked either of these effects in a cascode circuit experimentally. This should be done, and the findings of the circuit analysis should be confronted with experimental data to confirm their usefulness in predicting the behaviour of the input impedance of a cascode circuit.

4h. Transit time effects.

In the region of audio and radio frequencies, typically below 30 mc, it is assumed that the changes in the grid potential are transferred to the plate instantaneously. Even for multistage amplifiers we usually assume that input voltage changes are reflected at the output with no delay whatever.

Actually, changes of the grid potential in a thermionic tube modulate a stream of electrons between cathode and plate. This stream has finite instantaneous and average velocities expressed usually by the "transit time" of electrons.

This transit time has undesirable effects on the behaviour of the tube because:

a) it introduces damping in the grid circuit, equivalent to the presence of a finite input resistance,

b) increases the apparent input capacitance,

c) introduces distortion of high frequency signals.

These effects are discussed in a general form by Parker (1950) and in a more detailed way by Van der Ziel (1954).

The transit time effects are of little consequence at frequencies below 10 mc where impedances associated with the grid circuit are not high ($< 10^5$ ohms). If, however, the impedance in the grid circuit is very high, such as in the Alkemade's Noise Thermometer ($> 10^8$ ohms), then the transit time effects may become noticeable even at lower frequencies (in Alkemade's case ~ 2 mc), and they may have been responsible, at least partly, for the discrepancy between the design value of the Q of the resonant circuit (50,000) and the actual value (about 1000) (see section 3).

41. Flicker effects in the first stage.

The flicker effect in the first stage will be the main factor determining the low frequency limit of the noise measurements.

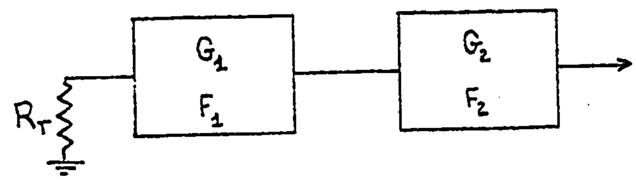
In principle this effect should be made minimum by suitable choice of components of the first and, perhaps, the second stage of the amplifier.

If noise measurements are made at low frequencies only, it may be possible to use wirewound resistors. If the frequencies of interest are in the region where the inductance of such resistors could not be tolerated, then metal film resistors would be the next choice (cf. Fink 1960).

A factor over which there is little direct control is the flicker noise originating in the cathode of the 1st tube. The magnitude of this noise will depend on the type of the cathode and will be smallest for pure tungsten cathodes and greatest for the oxide cathodes. The best advice that can be given in connection with this is to choose tubes which have low flicker noise (cf. Vand der Ziel, 1954).

4k. The influence of the second stage of the amplifier.

Consider two initial stages of an amplifier (Fig. 4k.1)



4k.1

The second stage contributes to the total background noise of the amplifier and this contribution is given by the second term in the overall noise figure F (sec e.g. Bennett 1960)

$$F = F_1 + \frac{F_2 - 1}{G_1} \qquad \frac{F_3 - 1}{G_1 G_2} \qquad \text{4k.2}$$

where F_1, F_2, F_3 are the noise figures of the 1st, 2nd and 3rd amplifier stages respectively. The effective noise contribution of the second stage is thus inversely proportional to the gain of the first stage.

In estimating the performance of a noise amplifier the second stage almost invariably has to be taken into account. To be able to neglect the noise contributions of the second stage for one percent precision in the determination of the noise contributions the gain of the first stage would have to be one hundred. This would seldom be the case.

Alkemade (see Section 3) does not seem to mention the performance of the second stage. While the failure of his equipment was due to other causes, the detection limit of his equipment was probably much worse than that calculated on the basis of the considerations of the first stage alone.

Experimentally, one can determine the performance of an amplifier by measuring its equivalent input noise resistance (Pursey and Pyatt, 1959). In such case the second stage performance is taken care of automatically.

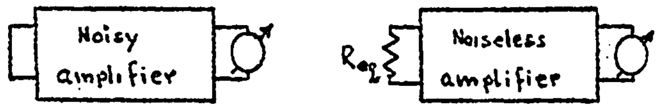
41. Background noise of the amplifier chain

The output of any amplifier exhibits fluctuations which can be traced to various causes (thermal noise, shot noise, flicker noise, etc).

Fluctuations originating in the first stage of the amplifier appear at its output magnified G_o times (G_o is the overall gain of the amplifier), those originating in the second, third etc., stage of the amplifier appear at the output magnified G_o/G_1, $G_o/(G_1G_2)$, ... $G_o/(G_1G_2 \ldots G_n)$ times respectively (G_1, $G_2 \ldots G_n$ being the respective gain of consecutive amplifier stages).

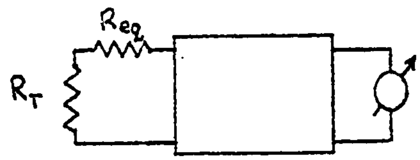
If the gain of the first stage is high ($G_1 \gg 1$) then we can often disregard noise contributions of the remaining stages, and even if not, their final effect is G_1, etc times smaller. Noise performance of an amplifier can thus be analyzed stage by stage taking account of the individual sources and of their final effect.

The overall effect of background noise of the amplifier can also be expressed in terms of an equivalent noise resistance which, if connected at the input of a noiseless amplifier would produce a noise level equivalent to that of the real amplifier (Fig. 41.1)



41.1

For the use in noise thermometry the thermometric signal is generated in another resistance R_T, which is connected in series with R_{eq} (Fig. 41.2).



41.2

The ratio of the total noise at the output of the amplifier ($\overline{V}^2_{out} \propto 4kT(R_T + R_{eq})\Delta f$) to the useful thermometric noise ($\overline{V}_T^2 \propto 4kTR_T\Delta f$) is

$$\frac{\overline{V}^2_{out}}{\overline{V}^2_T} = 1 + \frac{R_{eq}}{R_T} \qquad 41.3$$

and it seems as if any degree of measurement accuracy could be reached provided we make R_T sufficiently large compared to R_{eq}.

The measurements of R_{eq} are made with the input terminals of the amplifier short circuited. In such case the grid current of the first tube does not affect the magnitude of R_{eq} and does not have to be taken into account. As soon, however, as we open these terminals and insert the thermometric element R_T the grid current I_1 of the first tube alters the magnitude of the background noise of the amplifier by the amount $V_g^2 = 2eI_g R_T \Delta f$ *. i.e. the expression 41.3 becomes

$$\frac{\bar{V}_{out}^2}{\bar{V}_T^2} = 1 + \frac{R_{eq}}{R_T} + \frac{eI_g R_T}{2kT} \qquad 41.4$$

from which we can see that there is a value of R_T which makes the expression 41.4 a minimum, and that this value of R_T will determine the minimum temperature measurable with the equipment with a given precision. This minimum occurs when the last two terms of 41.4 become equal i.e. when

$$R_T^2 = \frac{2kTR_{eq}}{eI_g} \qquad 41.5$$

If we now impose the additional condition that the accuracy of measurement be, say, 0.1%, then the sum of the two last terms in 41.4 should be not greater than 0.001. These two conditions give us the values for R_{eq} and R_T as a function of temperature with I_g as a parameter and are shown in Fig. 41.6 for 0.1% precision of measurement.

Measurements of the equivalent noise resistance of an amplifier were made by Pursey and Pyatt (1959) in which case R_{eq} of their amplifier was found to be about 190 ohms. (dashed line in Fig 41.6). The R_{eq} of the amplifier of Verma (1958), inferred from their quoted noise figure, was about 130 ohms.

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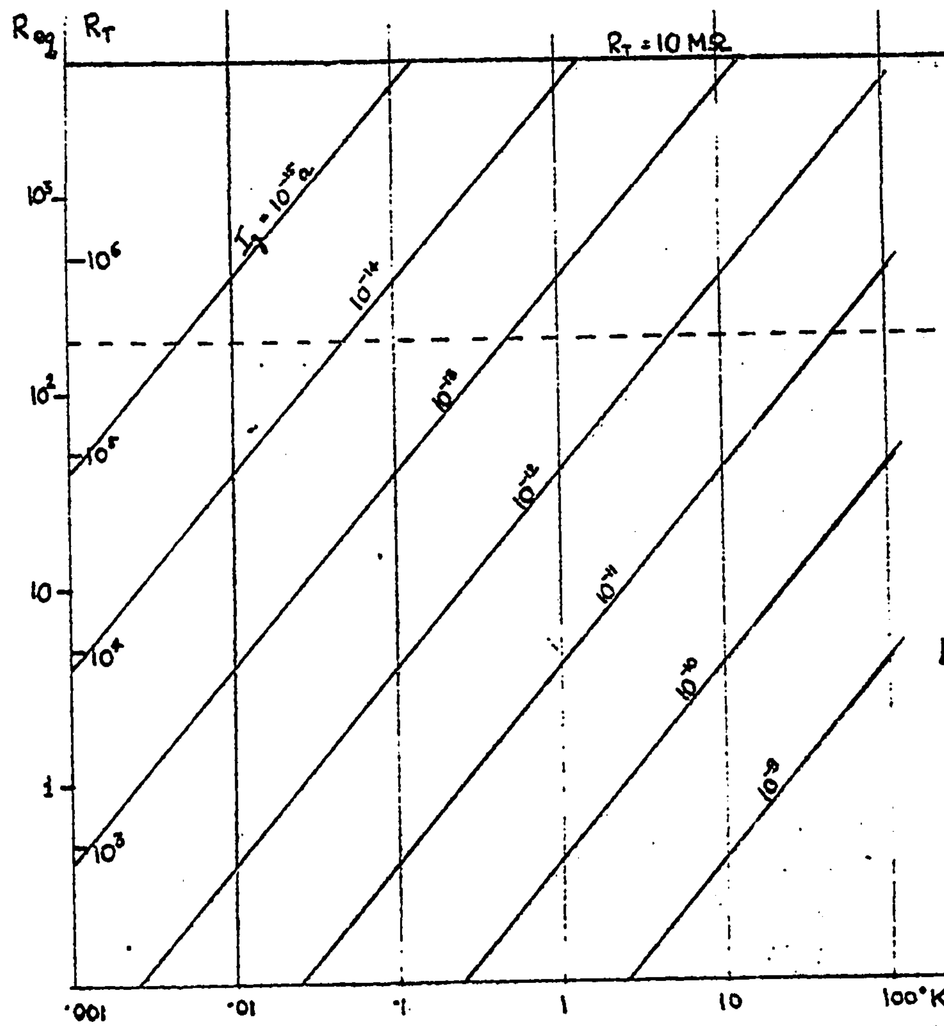
* Actually this value may be smaller by a factor Γ^2 (See D.K.C. MacDonald and R. Furth, 1947)

These two figures probably represent something close to the limit of the performance of a thermionic tube amplifier. On the other side, the value of the



41.6

Fig. 41.6 The interrelation between the thermometric element's resistance R_T, the equivalent noise resistance R_{eq}, the grid current I_g of the input tube of the amplifier and the temperature T satisfying the condition of the amplifier's background noise being not more than 0.1% of the total measured noise.

$$R_{eq} = \frac{k}{2eI_g} 10^{-6} T \quad ; \quad R_T = 2000\, R_{eq}$$

resistance of the thermometric element is limited by practical considerations of bandwidth, flicker noise and realizability of high Q resonant circuits (see section 3). The figure of 10 megohms seems to be close to a practical limit. The performance of noise measuring apparatus therefore falls somewhere between the dashed line around R_{eq} = 200 ohms and the solid line R_T = 10 megohms. We can see from there that with a good electrometer tube the measurable temperature limit would be somewhere between 0.01 and 0.1 ^{o}K. This is the ideal limit in which it is implied that the average values of the noise voltages could be determined with any desirable accuracy. Actually, since the ratio $\bar{V}^2_{OUT}/\bar{V}_T$ can only be determined with limited precision the above temperature limit will shift towards higher temperatures. These seems to be little hope, therefore, of being able to measure temperatures significantly lower than, say 0.1^{o}K with the noise measuring equipment of the type used by Garrison and Lawson (1949) and other workers up to 1959.

A different approach to the problem of background noise was made by H.J. Fink (1959) who used two amplifier chains. The output of these amplifiers was multiplied together and averaged in time. This operation eliminated all uncorrelated noise from the measurements i.e., assuming the background noise of the two amplifiers (represented above by R_{eq}) to have no correlation, eliminated R_{eq} entirely from the considerations, at least in principle.

The only major limiting factor which remains here is the grid current of the input tubes. This grid current flows through the common input network and therefore is a source of an additional correlated noise which puts a limit on the lowest detectable temperature. This limit, however, is lower than that considered above by a factor of about 10^7 (see section 6).

It seems as if the problem of the background noise were pushed aside by Fink's method and brought the grid current problem to the fore. At this stage of development, however, even this does not appear to be of immediate importance since the limit of temperature measurability imposed by the grid current here is well beyond the lowest temperatures attained by man up to this time. (cf. Kurti et al 1956).

4m. Bandwidth of the amplifier

The mean square value of the noise depends directly on the bandwidth Δf in which it is contained (4m.1)

$$\bar{V}^2 = 4kTR\Delta f \qquad 4m.1$$

Because of this the accuracy of the determination of the bandwidth has the same effect on the accuracy of noise measurements as the accuracy of the determination of R and T.

One of the methods of bandwidth measurements is to plot the frequency response of the amplifier and then to calculate the bandwidth equivalent to that response. This process is not very accurate. Frequency response has to be plotted over a wide band of frequencies in order to take into account all possible spurious responses. The precision of instruments available for this purpose is usually of $\pm$ 1 db category i.e. about $\pm$ 12%. The best precision obtainable with radio frequency equipment is $\pm$ 0.1 db i.e. about $\pm$ 1%. This is a long way from the desirable accuracy of bandwidth determination which should be better than 0.1%.

Another method of the determination of equivalent bandwidth is to use a flat spectrum noise source (e.g. noise diode) feed it into the amplifier and measure the output. If the input and the gain are known accurately, then the equivalent bandwidth can easily be calculated

$$\Delta f = \frac{\overline{V}^2_{OUT}}{G_o^2 \, \overline{V}^2_{IN}} \qquad \text{4m.2}$$

where G_o is the midband amplifier gain.

This method is simple in principle but unfortunately the gain G_o and the input voltage V_{in}^2 have to be known accurately. Accurate gain determination is almost as much troublesome as bandwidth determination, and the accuracy with which we know the input voltage V_{in}^2 is questionable. A proper approach to the determination of the equivalent bandwidth of an amplifier would be to use not less than two independent methods of measurements and infer the true value from these measurements.

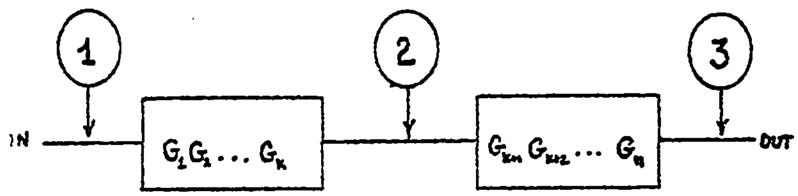
4n. Effects of filter characteristic.

One of the interesting features of noise is its spectral distribution or power spectrum.

There are two methods of measuring this power spectrum. One is to measure the value of the mean square voltage of noise within a narrow frequency band Δf centered around the frequency f_x. A set of measurements over a range of frequencies f_x will give the required power spectrum of measured noise. Another method is to measure the auto-correlation of the noise and from it compute the power spectrum (see e.g. Blackman and Tukey, 1959).

Of the two above methods it is the first which requires the use of filters.

One can distinguish three places where such a filter can be inserted in the amplifier chain (Fig. 4n.1)



4n.1

The insertion of a filter at the input of the amplifier will, in general, introduce additional noise. There may be cases in which the filter itself contains the thermometric element e.g. piezoelectric crystal (Lawson and Long, 1946) or a resonant circuit made of superconducting wire (Alkemade 1956). In these cases, however, the spectral distribution of noise was of no interest and these filters were used for different purposes. For the measurements of noise spectrum the filter would have to be introduced at some intermediate point in the amplifier chain such that the filter contribution to the output noise would be negligible, or at the output of the amplifier (points 2 and 3 in Fig. 4n.1).

The actual shape of the filter characteristics i.e. the bandwidth, the attenuation rate at the cut off points etc. should have no influence on the final result as long as the amplifier filter system is linear.

For accurate results the equivalent filter bandwidth has to be accurately known and the remarks on bandwidth measurements which have been made in section 4m also apply here.

One of the sources of difficulties in the design of filters for such an application is the fact that it is desirable to have filters having identical transfer characteristics in order to be able to compare directly the noise from different frequency regions. This is difficult to realize in a wide band of frequencies since for a given filter type it is the bandwidth relative to the center frequency $\Delta f/f_o$ which can be kept constant and not the absolute bandwidth (Δf). The constancy of bandwidth (Δf) can be achieved only at the expense of increasing the complexity of filters as $\Delta f/f_o$ becomes smaller.

4o. Stability of Amplification.

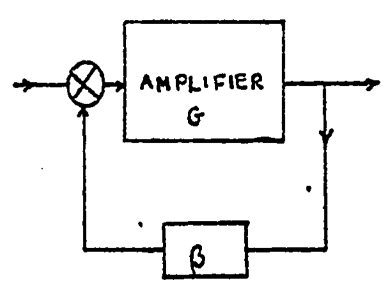
Among different quantities which can be measured in noise are:

1) statistical distribution of instantaneous amplitudes
2) mean value of the noise voltage
3) mean square value of the noise voltage
4) autocorrelation of noise
5) cross correlation of two noise sources
6) power spectrum of noise

Accurate measurements of these quantities, without exception, require relatively long time constants or repeated measurements. The measurements of mean values imply averaging in time, and for high precision measurements of noise voltages within a narrow bandwidth the averaging time may extend to several minutes or more. If a number of measurements is to be made then the measurement time will extend into hours and days. The same consideration applies to the measurements of noise correlation and power spectra.

In these measurements the influence of changes in the measuring equipment itself has to be kept below the level which would affect the accuracy of the results. The quantity which is most easily affected by the changes of parameters within the amplifier (changes of tube parameters in particular) is its gain G.

In order to keep the gain constant it is virtually mandatory to use feedback to stabilize it. To control the overall gain stability, feedback has to be applied from the output to the input. (Fig. 40.1)



40.1

Gain variations should not exceed 0.1% for a similar precision in the measurements or, if possible, should be eliminated entirely as a source of errors. Let us assume the required gain stability to be one part in 10^5 and find the feedback factor β and open loop gain G required to satisfy this condition.

Since the overall amplifier gain G_o (closed loop) is given by $G_o = G/(1-\beta G)$ then we have

$$\frac{dG_o}{G_o} = \frac{1}{1-\beta G}\frac{dG}{G} \qquad 4o.2$$

The factor dG/G is the relative change of the open loop gain. This change can be made less than 10% over long periods of time. If we assume then that $dG/G = 0.1$; $dG_o/G_o = 10^{-5}$ and that $G_o = 10^5$ then we find $G = 10^9$ and $\beta = 10^{-5}$. Thus at least nominally, we can obtain any desired stability of amplification. The limiting factor here is the stability of the feedback network. This stability cannot be made greater than about 1 part in 10^6 since the stability of good quality passive components is of that order.

4p. Overall feedback

The use of feedback in amplifiers is almost mandatory where high stability of gain is required (see section 4o). This feedback should be applied in such a way as not to distort the frequency characteristic of the amplifier. This can be achieved by making the feedback factor β constant throughout the useful bandwidth of the amplifier.

$$G_o = \frac{G}{1-\beta G} \qquad \text{4p.1}$$

where G is the open loop gain and G_o is the closed loop gain of the amplifier.

The dependence of the gain G_o on β is very pronounced since

$$\frac{dG_o}{G_o} = \frac{\beta G}{1-\beta G}\,\frac{d\beta}{\beta} \cong \frac{d\beta}{\beta} \qquad \text{4p.2}$$

i.e. the overall gain changes in the same proportion as does the feedback factor β as long as $\beta G \gg 1$.

At frequencies at which G becomes comparable to $1/\beta$ the product βG becomes comparable to unity and the expression for dG_o/G_o has to be used in its full form

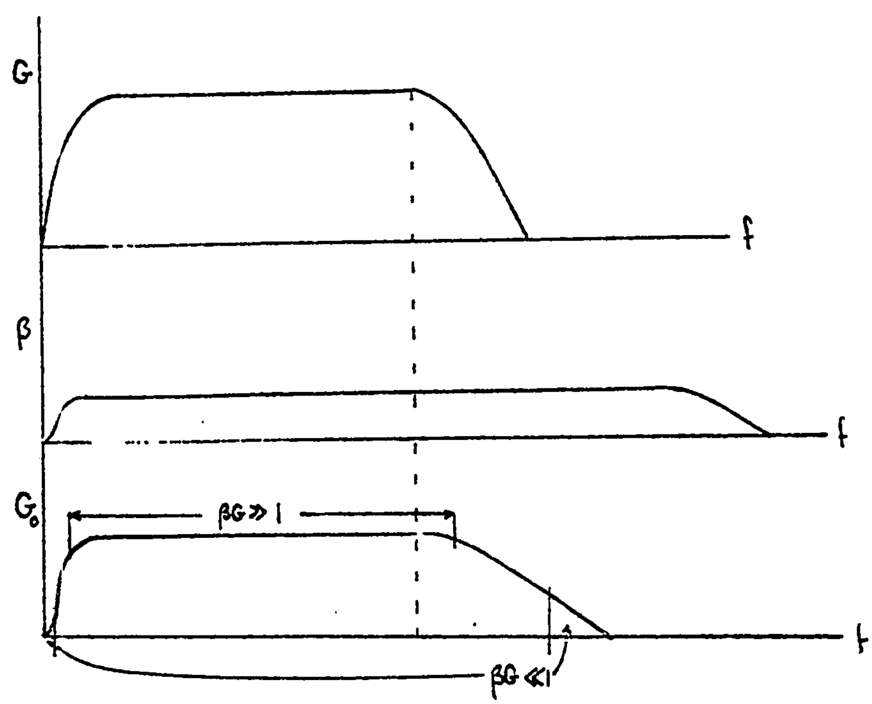
$$\frac{dG_o}{G_o} = \frac{\beta G}{1-\beta G}\,\frac{d\beta}{\beta} \qquad \text{4p.3}$$

which, for $\beta G \ll 1$ becomes

$$\frac{dG_o}{G_o} \simeq G\beta \frac{d\beta}{\beta} = G\,d\beta \qquad 4p.4$$

If, therefore, we wish to keep the gain G_o relatively independent of the changes in β factor, the latter should be kept constant as a function of frequency until the open loop gain G drops to values such that $\beta G \ll 1$. The influence of the changes in the feedback factor β is then given by 4p.4.

The above deduction is, perhaps, trivial; it does, however, provide a guide for numerical calculations for variations in G and β, both as a function of frequency and time. Graphical illustration of this is shown in Fig. 4p.5.



4p.5

It may also be remarked that the presence of feedback does not affect the signal-to-noise ratio of the amplifier (cf. Van der Ziel, 1954).

4q. <u>Extraneous signals picked up by the amplifier</u>

The extraneous signals picked up by the amplifier may considerably upset noise measurements. These signals may come from electric motors, car ignition, fluorescent lights, light switches, etc. The intensity of these disturbances will vary throughout the day, being usually lowest at night.

The only way, known to me, of making oneself independent of these disturbances is to use a good screened room which will attenuate the undresirable pick-up to a safe level in the frequency range to which the equipment is susceptible. The available commercial screened rooms are rated to attenuate electromagnetic fields from high audio frequencies (10-15kc) up to hundreds of megacycles by up to 120db. This is probably adequate for most noise measurements.

If no such room is available, or if the interference attennuation is not adequate, the only alternative seems to be to limit the time of measurements to periods in which the interference is absent or at its minimum (usually very early in the morning).

4r. Nonlinearity of amplifications

The output of an amplifier can be expressed, in general, by a relation of the form:

$$V_{OUT} = a_1 V_{IN} + a_2 V_{IN}^2 + \ldots\ldots + a_n V_{IN}^n \qquad \text{4r.1}$$

where V_{out} is the instantaneous output voltage, V_{in} is the instantaneous input voltage, and $a_1 \ldots a_n$ are coefficients, the magnitude of which, as a rule, rapidly diminishes beyond a_1 term (a_1 is the overall gain of the amplifier).

The input voltage can be represented as a sum of sinusoidal voltage e.g.

$$V_{IN} = \sum_{i=1}^{\infty} c_i \sin \omega_i t \qquad \text{4r.2}$$

The output will therefore contain these voltages magnified a_1 times and in addition to this an infinite number of components given by the second and third terms of the expression 4r.3

$$\begin{aligned} V_{OUT} = {} & a_1 \sum_{i=0}^{\infty} c_i \sin \omega_i t + \\ & + \sum_{i=0}^{\infty} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} C_{k_1} C_{k_2} \cdots C_{k_n} \sin (k_1 \omega_i \pm k_2 \omega_i \pm \cdots\cdots k_n \omega_i) + \\ & + \sum_{i=0}^{\infty} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} C'_{k_1} C'_{k_2} \cdots C_{k_n} \cos (k_1 \omega_i \pm k_2 \omega_i \pm \cdots\cdots k_n \omega_i) . \end{aligned} \qquad \text{4r.3}$$

These additional components are the harmonics of the input voltage and voltages having frequencies of the magnitude given by all the combinations of sums and differences of harmonics of the original component frequencies.

The components represented by the second and third term of the expression 4r.3 were not present at the input but are directly related to it. If, therefore, V_{in} represents random noise in which there is no correlation among individual components of the Fourier spectrum, the output voltage is no longer purely random. There are spectral components in each part of the spectrum which are directly related to spectral components in some other spectral regions.

If we square the output voltage and average it we will obtain a value which will be directly related to the mean square value of the input voltage and to the mean values of all the other even powers of the input voltage. We can represent such a relation by

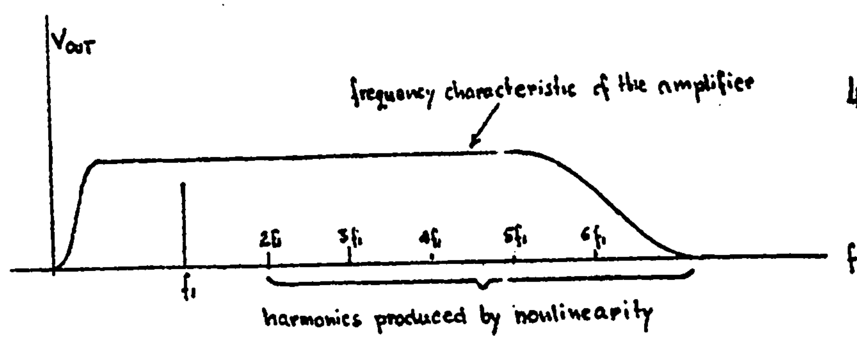
$$\bar{V}^2_{OUT} = f(\bar{V}^2_{IN}) \qquad 4r.4$$

where $f(\bar{V}^2_{in})$ is a polynomial in terms of the mean square value of the input voltage

$$f(\bar{V}^2) = b_0 + b_1(\bar{V}^2_{IN}) + b_2(\bar{V}^2_{IN})^2 + \cdot \cdot \cdot \cdot \cdot + b_n(\bar{V}^2_{IN})^n \qquad 4r.5$$

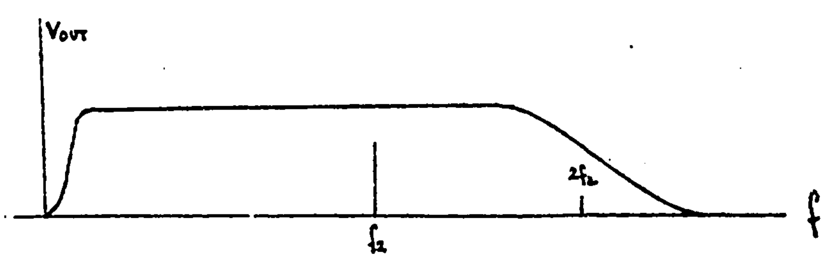
For any given amplifier this function can be experimentally found by making measurements with a known input voltage.

This voltage has to have the same spectral content as the signal for which the amplifier will be used. For white noise measurements the calibrating source should have a flat spectrum. For the measurements of noise having some other spectral distribution the spectrum of the calibrating source should be adjusted accordingly. The necessity for this can be shown in a simplifier way by considering the sinusoidal signal passed through a nonlinear amplifier (Fig. 4r.6)



4r.6

The output due to a signal having frequency f_1 at the lower end of the amplifier passband will contain 6 harmonics, the output due to a signal at a higher frequency f_2 (Fig. 4r.7) will contain only one harmonic (say). The calibration made



4r.7

with the first signal will therefore be different from the calibration made with the second one.

Similarly, precautions have to be taken when using filters for measuring the spectral distribution of noise. The amplifier has to be calibrated with each individual filter since each filter will affect the spectral components due to nonlinearities in a different way.

4s. The detector and its law

The final indication of the magnitude of the measured noise is made either by an indicating instrument (most often of the moving coil type) or by a pen recorder. Either of these devices can follow only slow fluctuations (a few cycles per second or less) and their average indication is proportional to the mean value of the current (or voltage) impressed across their terminals.

Noise is a fluctuating quantity of zero mean value. To be able to use any of the above instruments the noise voltage has to be converted into a quantity having a finite mean value bearing some definite relation to the original noise waveform.

The simplest quantity measurable in noise is its power or, what is directly proportional to it, its mean square value.

Here, the most straight forward device that can be used for measuring the noise power is a thermocouple meter. The noise power is used to heat a wire which, in turn, heats one junction of a thermocouple. Current generated in the thermocouple is passed through a moving coil meter the indications of which are directly proportional to the power dissipated in the heating element.

This is the most direct method of measuring the noise power and the only disadvantage of such meter known to me, is the fact that thermocouples are very sensitive to overload which, in noise, is difficult to predict.*

Another method of converting the noise signal into a quantity having a finite mean value is to use some nonlinear element, such as a diode.

Current-voltage characteristic of a diode does not follow any simple law. Any diode however can be used as either a "linear" or "square law" detector.

If the diode is used as a linear detector the amplitude of the applied signal has to be large enough to satisfy the relation $V_{out} = \beta V_{in}$ in the forward direction (Fig. 4s.1)

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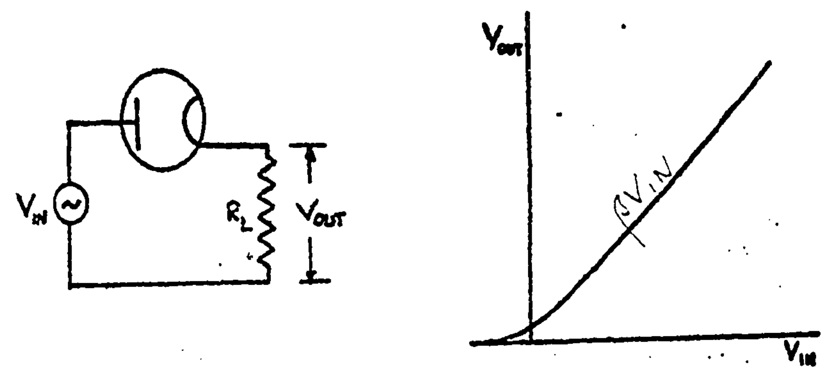
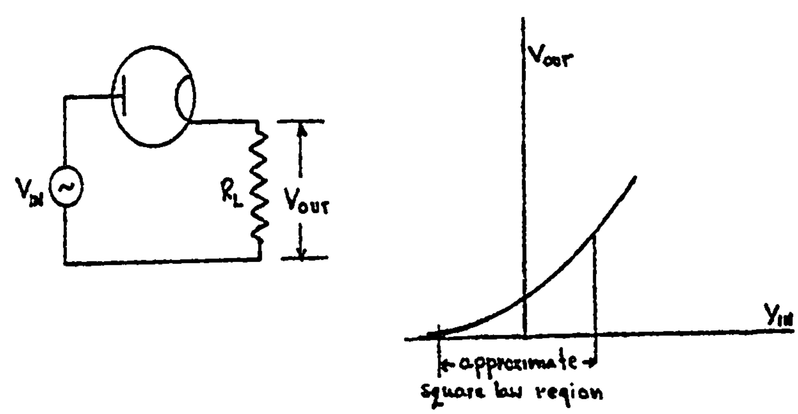
* It should be possible to construct a thermocouple meter with an automatic overload protection which would disconnect the thermocouple from the source and, at the same time, give a signal to the user that it did so.



4s.1

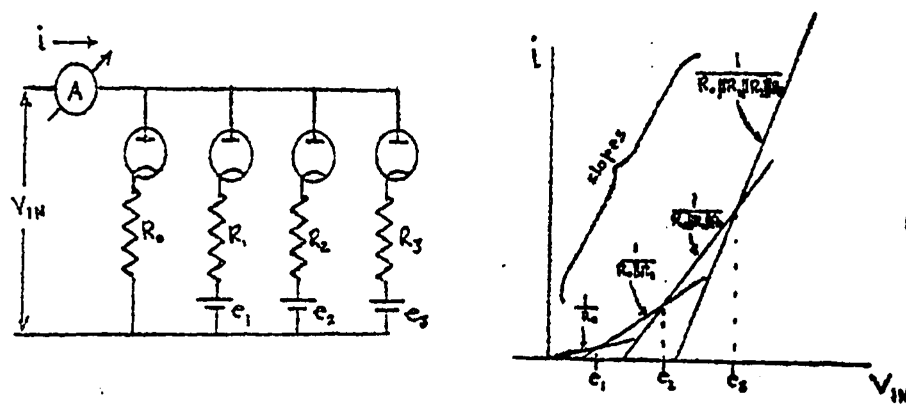
If a diode is used as a square law detector the amplitude of the signal has to be limited to the region of the current-voltage characteristic of the diode where the law $V_{out} = \beta V_{in}^2$ holds. This region is usually limited to voltages well below 1 volt (Fig. 4s.2)



4s.2

It can be shown (cf. e.g. Freeman 1958) that diodes can be used for measurements of the mean square value of noise in either case.

Since the amplitude of noise in square law diode detection is very limited it is sometimes desirable to have a detector which would maintain its square law character over a greater range of input voltages. This can be done by using a number of diodes biased in such a way that each of these diodes approximates only one segment of the "square law" parabola (Fig. 4s.3)



4s.3

This way of producing the required input-output relation was realized in a number of commercial instruments, which claim a precision of 2% in the "squareness" of their response. While the replacement of a smooth curve by straight line segments, in principle, introduces a very complicated relation between input and output, in practice this segmentation can be made as fine as desired by using sufficient number of diodes. The difference between the true parabola and its segmented approximation can thus be made as small as required.

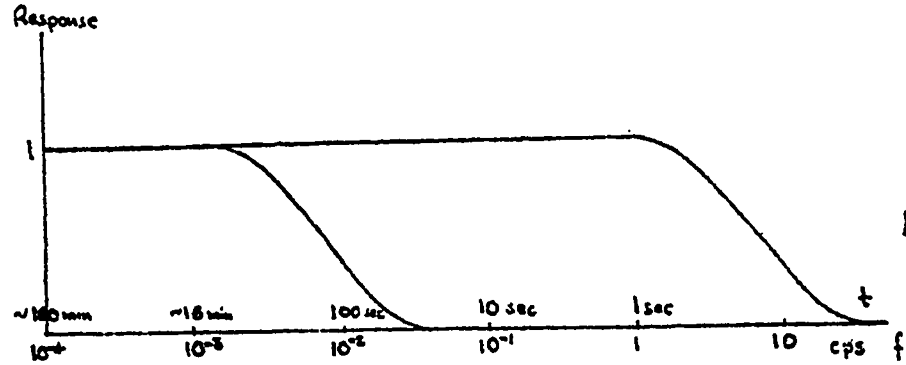
It should, perhaps, be also remarked here that the actual law which is obeyed by the detecting device is not of a paramount importance since any type of detector can be calibrated in terms of the mean square value of the input waveform. The only condition here is that the calibrating waveform has to have the same frequency spectrum as the measured quantity. If this condition can be satisfied then the actual input-output relation of the detecting device becomes of secondary importance.

4t. The output meter

The output meter may be either an indicating or a recording instrument. Because of long time constants involved the latter is used almost exclusively.

Pen recorders have, as a rule, amplifiers to increase the sensitivity of these instruments. Nominal sensitivity of 1 millivolt for full scale deflection is quite usual. It is important to check the stability of such a recorder over a prolonged time (say 24 hours or more). Long term drift can be tolerated if known, short term drifts due to amplifier instabilities, temperature and humidity changes, are inadmissible unless limited to magnitudes which do not affect the overall measurement accuracy to any considerable extent. Pen recorders are available which have a self checking and adjusting mechanism permitting long operations without the need of control and manual checking.

The frequency response of pen recorder is usually within the range of a few cycles per second. It is, therefore, a relatively broadband device compared to the passband of output averaging devices (integrators) which, for noise measurement accuracies of 0.1% and better, have to have passbands of about 10^{-3} cps or less (see Fig. 4t.1).



4t.1

4u. Integration Time

The accuracy of determination of the mean square value of the measured noise is directly connected with the length of time in which this averaging is performed. Strictly speaking it required an infinite length of time to find the average value of a random quantity

$$\overline{V}^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T V^2 \, dt \qquad 4u.1$$

whereas in fact the measurements are always limited in time i.e. we measure

$$\overline{V}_T^2 = \frac{1}{T_1}\int_0^{T_1} V^2\, dt \qquad 4u.2$$

The ratio of these two quantities is $\bar{V}_T^2/\bar{V}^2$ depends on the integration time, detector law and on the filter shape before and after rectifications (see e.g. Van der Ziel, 1954, J.J. Freeman, 1958, D.A. Bell, 1960).

5. Description of the experimental background of this thesis.

The original subject of this thesis, as suggested by Dr. D.K.C. MacDonald in 1956, was the continuation of the work of Dr. C.T.J.Alkemade on a low temperature noise thermometer.

At that time transistors were very much in vogue in the industry, and I was very interested in them. I decided therefore to investigate the possibilities of the use of transistors in such an application.

It seemed, at that time, that transistors might make possible the construction of a preamplifier which could be inserted, together with the thermometric element, into the medium the temperature of which was to be measured. It also seemed to me that, perhaps, the noise contribution of such a preamplifier itself would decrease at lower temperatures.

The problem did not seem to be too difficult at first. All that had to be done was to find out how transistors behaved at low temperatures, and then to design an amplifier which would amplify the noise from the thermometric element.

This was, in fact, what was done in the first phase of this work. A number of different transistor types from different manufacturers was used; their DC characteristics were plotted at room, liquid air and liquid helium temperatures, and then an amplifier was constructed which did work in the liquid air.

The amplifier however had a rather high level of background noise and seemed to be useless for noise thermometry. My knowledge of noise problems was much more limited at that time, and I felt that it was hopeless to try to build a complete noise thermometer from components about whose behavious at low temperatures neither I nor anybody else seemed to know much.

The next logical step was to start more basic investigations of transistor properties at low temperatures. Since transistor action is based on a suitable geometrical arrangement of two p-n junctions, the investigations were focussed on the behavious of p-n junctions as a function of temperature, the range being from room to liquid helium temperatures. For purely practical reasons (availability of liquie air and liquid helium) measurements were limited to three temperatures only, i.e. 290, 80 and 4.2 deg. K approximately.

Two questions seemed to be of immediate interest. The temperature dependence of the DC characteristics of p-n junctions, and of their noise behaviour.

The temperature dependence of the DC characteristics of germanium p-n junctions was briefly investigated in the summer of 1958. The results were published in the I.R.E. 1958 Canadian Convention Record, a photostat of which is enclosed.

These measurements were very valuable since they have shown that a p-n junction retains its character at low temperatures, but it has also shown the presence of a certain voltage barrier in a p-n junction (cf. loc. cit.). No explanation of this was found up to now.

The above measurements had a rather qualitative character and the most important single deduction which could be made from them was that far more work would have to be done in this field, the object being to find the DC current-voltage dependence in a p-n junction. It might be useful to remark here that several papers on DC current-voltage characteristics of p-n junctions have been recently published in the Soviet journals (Fizika Tverdogo Tela).

The main experimental effort has now been directed to the measurements of the noise characteristics of p-n junctions and in the spring of 1959 I obtained an NRC research grant for this purpose. In August of that year I found an assistant in the person of Mr. W. R. Samaroo, who started to work on p-n junction noise measurements.

The measurements were carried out in a conventional manner. The noise from the sample was amplified in a low noise preamplifier (cascode) and then further amplified within a 4 kc bandwidth by a radio receiver. The magnitude of noise rectified by a linear detector was measured in a way which made it possible to separate the noise originating in the p-n junction from the background noise of the amplifier. The sensitivity of the equipment was such that 50 millimicrovolts of noise could be measured with an accuracy of about 5%. This was achieved in the fall of 1960, and a series of measurements of the noise originating in p-n junctions was made.

The time required for assembling the equipment was more than twelve months. During that time we have gone through most, if not all the pitfalls through which other workers in this field have gone before. The instability of amplifiers, extraneous influences, nonrepeatibility of measurements, etc. These difficulties, their solutions and the results of measurements and their interpretation, form the basis of the M.Sc. thesis of Mr. W.R. Samaroo (1961).

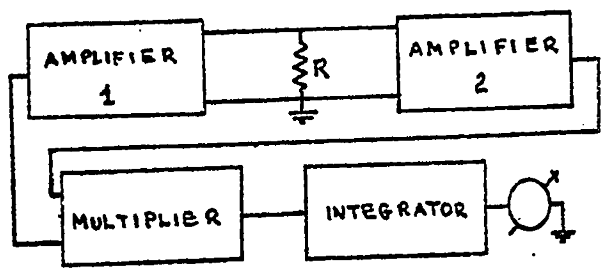
During that time the problem of the construction of low temperature noise thermometer became secondary to just plain noise measurements. Also in that time the work of H.J. Fink was published which opened a new approach to the noise measurements. Now the work is being concentrated on the construction of a two channel amplifier which could be used for measurements of the type similar to those described by Fink. This is described in more detail in section 6.

I have, for the time being, abandoned the idea of building a noise thermometer using transistors. The noise thermometer may become a by-product of the research on the subject of temperature dependence of noise in p-n junctions. The basic problem here is exactly the same as that in noise thermometry; accurate measurements of noise at different temperatures. As soon as the accuracy of noise measurements becomes better than 0.1% at liquid helium temperatures, the equipment can be used for temperature measurements. At the same time, since we intend to investigate the properties of p-n junctions, we should be able to come to a point in our research when we would know the extent to which transistors would be helpful in noise thermometry.

6. Proposal for further research.

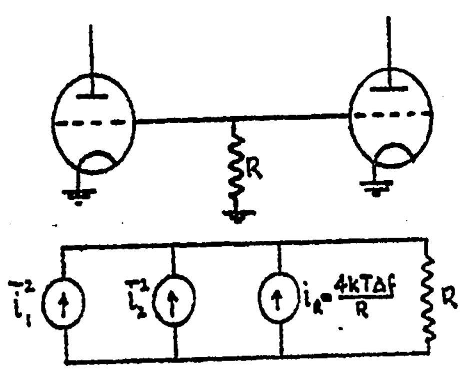
It was shown in Section 4 that the most important limitation of noise measuring apparatus is the background noise of the amplifier and the grid current of the first tube. It was also shown that it is possible to eliminate the amplifier's background noise by using two channels and by averaging the product of two uncorrelated noise backgrounds. The only remaining factor which limits the accuracy of noise measurements is the grid current of the input tube.

Let us estimate the limits of performance of the "Fink type" noise amplifier. If we assume that we can average out all noncorrelated noise in the two amplifiers in Fig. 6.1



6.1

then the only correlated noise will be the thermometric noise originating in the resistance R and the shot noise originating in the grid-to-cathode diodes of the input tubes (Fig. 6.2).



6.2

Where $\overline{i_1^2}$ and $\overline{i_2^2}$ is the grid current noise.

The total mean square noise voltage developed across the resistance R is

$$\overline{V_t^2} = \left(\overline{i_1^2} + \overline{i_2^2} + \frac{4kT\Delta f}{R}\right) R^2 \qquad 6.3$$

Of this voltage the only component which is useful thermometrically, or otherwise is $\overline{V^2} = 4kTR\Delta f$. The ratio between these two voltages

$$\frac{\overline{V_t^2}}{\overline{V^2}} = 1 + \frac{(\overline{i_1^2} + \overline{i_2^2})R}{4kT\Delta f} \qquad 6.4$$

The currents $\overline{i_1^2}$ and $\overline{i_2^2}$ are made up of a number of components (e.g. electron current, ionic current, ohmic leakage between the electrodes). For the purpose of an estimate, however, we can treat the grid-to-cathode diode as if it were working under the retarded field condition. The mean square of the fluctuations of such current is given by (cf. MacDonald and Furth, 1947)

$$\overline{i^2} = 2eI\Gamma^2\Delta f \qquad 0 < \Gamma^2 \leqslant 1 \qquad 6.5$$

where e is the electron charge, I is the average diode current and Γ is the, so called, "space charge reduction factor" which depends on the operating conditions of the diode. It is shown (loc. cit.) that true retarding-field conditions in a diode will prevail when the current is below a certain critical value, and if this is satisfied the magnitude of the space charge reduction factor Γ is unity.

For the purpose of an approximate calculation let us assume that the grid current is less than its critical value and that $\Gamma^2 = 1$ (both these assumptions seem to be well justified) and calculate the excess correlated noise in equation 6.4

$$\frac{(\overline{i_1^2} + \overline{i_2^2})R}{4kT\Delta f} = \frac{2e(I_1 + I_2)\Delta f R}{4kT\Delta f} = \frac{e}{2k}\frac{R}{T}(I_1 + I_2) \qquad 6.6$$

If we moreover assume that the grid currents of both diodes are equal then we obtain

$$\frac{\overline{V_t^2}}{\overline{V^2}} \cong 1 + \frac{e}{k}\frac{R}{T} I_g \qquad 6.7$$

where I_g is the average grid current of the input tubes.

Let us assume that the input tubes are electrometer tubes having grid current not greater than 10^{-15}a, and let us calculate the lowest temperature which could be measured with the precision of 1%,

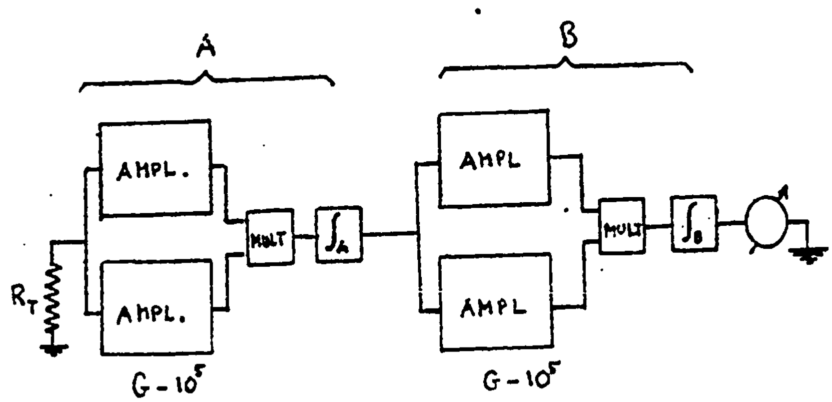
$$T = 100\frac{e}{k}RI_g = 10^2 \frac{1.6 \times 10^{-19}}{1.4 \times 10^{-23}} \cdot 100 \cdot 10^{-15} \simeq 10^{-7}\,{}^\circ K$$

in which the value of R was assumed to be 100 ohms, which seems to be a reasonable value.

The magnitude of the uncorrelated noise induced at the grids of the amplifier tubes would then be $\overline{V}^2 = kT/C =$ $= 1.4 \times 10^{-23} . 10^{-7}/100 \times 10^{-12} \simeq 1.10^{-20}$ volts ^{2} or about 0.0001 μv rms. To obtain such an amplitude of noise voltage a bandwidth of more than 10 mc would be required which is readily realizable. Total capacitance across the thermometric resistance of 100 ohm was assumed to be 100μμF. This is again reasonable but could probably be reduced to about one half of this value.

The above calculation shows the limitations of the sensitivity of the described apparatus. The measurable low temperature limit is beyond the lowest obtainable. Similarly the total amplification required of such apparatus would be about 10^5 times greater than could be realized.

The problem of amplification might perhaps be solved at least partly by using two complete systems in tandem as in Fig. 6.8

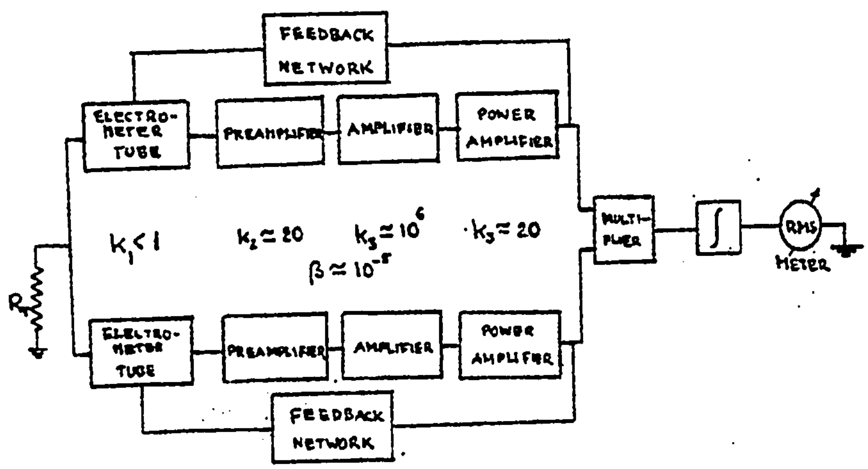


6.8

With such an arrangement it might be possible to increase the total amplification considerably. Since the system B would have to work at frequencies within the passband of the integrator $\int_A$ it would be necessary to use the system A at frequencies high enough to make the passband of $\int_A$ reasonably wide. This condition might not be difficult to realize by working in the UHF or microwave region with the system A.

A start has already been made towards realizing a practical system of the type shown in Fig. 6.1.

A more detailed block diagram of this is shown in Fig. 6.9,



6.9

Electrometer tubes are used at the input to reduce the grid current to a minimum. The preamplifier (Fig. 6.10) is a cascode stage using the tube 7308 (Phillips), which is a low noise, high gm double triode, well suited for this purpose.

The amplifier is made up of two commercial amplifiers *

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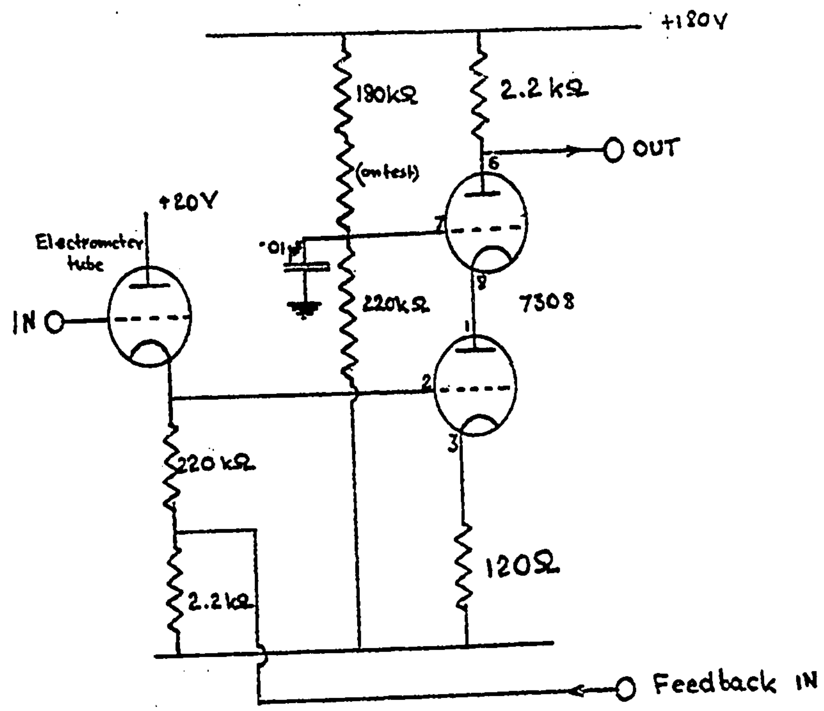
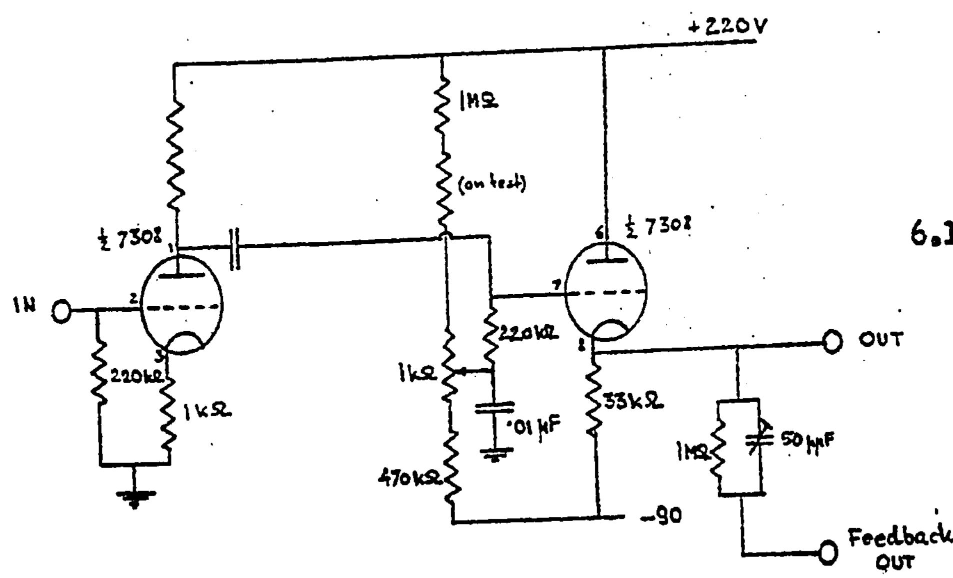
* Wide band decade amplifier, Type 500-AR, Technology Instrument Corpn.



6.10

Electrometer Tube and Preamplifier



6.11

Output Stage

each of which has a gain of 10^3 at a bandwidth of about one megacycle.

Power amplifier (Fig. 6.11) includes one voltage amplifier stage and a cathode follower output.

Open loop gain of the above chain is about 4×10^8. This gain can be reduced by means of negative feedback to a level such that the noise output does not exceed a few volts. As a startin; figure the value of the closed loop gain is proposed to be 2×10^4. The rather large amount of feedback ($\beta K = 2 \times 10^4$) should insure good stability w·ich is essential here (cf. section 4o).

The amplifier of Fig. 6.9, when completed, will be used for the investigations of noise in semiconductors. These investigations were already started and some preliminary measurements made with conventional equipment (cf. section 5). It should be possible to improve the accuracy and range of these measurements with the new apparatus.

In addition to the measurements of the mean square value of noise appearing across the terminals of a semiconductor p-n junction this equipment is also suitable for the measurements of the autocorrelation of noise. To adapt the apparatus of Fig. 6.9 to this purpose it is only necessary to insert a variable delay line into one of the two channels. By making a series of measurements with different values of delay the value of the function $\frac{1}{T}\int_0^T f(\tau) f(t-\tau)\, d\tau$ can be plotted, where $f(\tau)$ and $f(t-\tau)$ are the noise signals in the two channels delayed by τ second with respect to each other.

This function is only an approximation to the correlation function which is the limit of the above integral for an infinitely long averaging time. The degree of approximation however is limited by practical considerations only. The measurements of the above function could be used for calculating power spectra.

To sum up this research proposal, one can say that there is enough material for many years of development of accurate noise measuring amplifiers and for research on noise which will depend directly on the availability and quality of these amplifiers. A low temperature noise thermometer will, I hope, become the subject of another M.Sc. thesis in the not too distant future.

CONCLUSION

This study of problems connected with the low-temperature noise thermometry indicates that there is a very real promise of the realization of an absolute noise thermometer capable of measuring temperatures below 0.01°K with a precision of not less than 1%.

Such an instrument may not be suited to routine measurements, but may at least provide the basis for a temperature scale in the very low temperature region.

AI. Appendix 1

The power spectrum P(f) of a signal passed through a linear network is given by

$$P(f) = P_{in}(f) \, |W(f)|^2 \qquad \text{AI.1}$$

where $P_{in}(f)$ is the power spectrum of the input signal and W(f) is the transfer immitence of the network.

The term "power spectrum" or "power spectral density" describes the way in which the power of a signal is distributed in frequency. The use of the word "power" is based on the electrical concept of power dissipation in a one ohm resistance being equal to the square of the applied voltage, or to the square of the current passing through that resistance. Similarly the power spectrum is proportional to the square of the relevant signal amplitude.

It is necessary to realize that power spectrum is a quantity which is proportional to the square of current or voltage or to the power of the signal, but that its dimensions are not necessarily watts/cycle. These dimensions may also be volts or amperes squared/cycle and the term "power spectrum" will still be applied to these expressions.

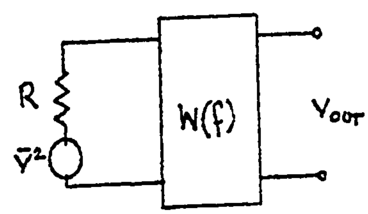
The case considered here is typical in the slight confusion introduced by the use of the word "power" where only volts squared/cycle are involved.

Consider a resistance R generating thermal noise voltage $\bar{v}_n^2 = 4kTRdf$. We assume that voltage to be independent of the frequency. The noise power developed across the resistance is

$$dP_n = \frac{\bar{v}_n^2}{R} = 4kTdf \qquad \text{AI.2}$$

The power spectrum dP_n/df is uniform in frequency and independent from it.

Assume now that we apply the noise voltage generated by R to a frequency selective network i.e. to a filter having an immitance W(f)



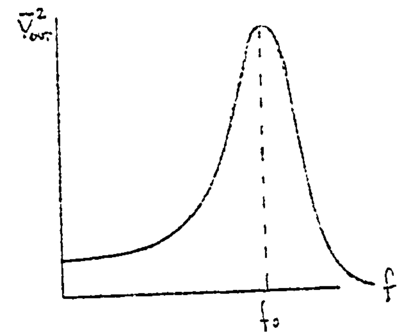
AI.3

The filter output voltage will be proportional to its frequency characteristic, and the square of the output voltage will be proportional to the square of the frequency characteristic of the filter, i.e. to the square of its immitance

$$\bar{V}_{OUT}^2 = \bar{V}_{IN}^2 \left|W(f)\right|^2 \qquad \text{AI.4}$$

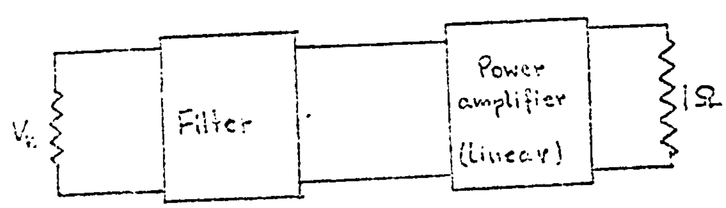
The power developed by the noise source across the output terminals of the filter is still the same as before (assuming a lossless network) and has a flat frequency spectrum.

The mean square voltage, however, may have a very pronounced frequency characteristic, e.g. as shown in Fig. AI.5



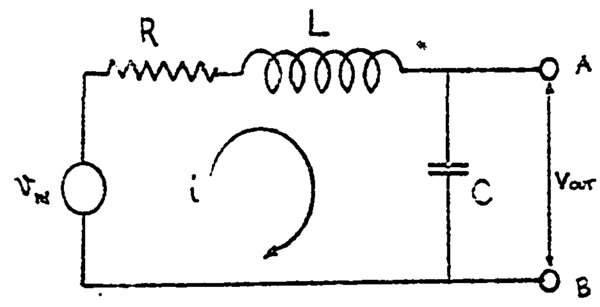
AI.5

We apply, in this case, the term "power spectrum" to the output voltage squared rather than to the output power. The dimensions of the power spectrum are, in this case volts squared. The only way in which we could come back to the original dimensions of watts/ cycle would be to put at the output of the filter a distortionless amplifier. If the output impedance of the amplifier were 1Ω then by feeding its output into a 1Ω resistance we would dissipate in that resistance power proportional to the square of the input voltage, which could be measured in watts per cycle (Fig. AI.6).



AI.6

Consider now the case of a series resonant circuit in which broad band noise is generated by the loss resistance of the circuit (Fig. AI.7)



AI.7

The power spectrum of the input signal is

$$dP_{in}(f) = 4kTRdf \qquad \text{AI.8}$$

and is assumed to be flat throughout the frequency range under the consideration.

The transfer immitance W(f), in this case, can be written in the form

$$W(f) = \frac{V_{OUT}}{V_{IN}} = \frac{1}{j\omega RC + (1-\omega^2 LC)} \qquad \text{AI.9}$$

$$|W(f)|^2 = \frac{1}{(\omega RC)^2 + (1-\omega^2 LC)^2}$$

$$= \frac{1}{1 + x^2\left(\frac{1}{Q^2} - 2\right) + x^4} \qquad \text{AI.10}$$

where $x = \omega/\omega_o$; $\omega_o^2 = 1/LC$ and $Q = \omega_o L/R = 1/\omega_o RC$.

The function AI.10 is proportional to the power spectrum of the noise across the capacitance C.

The total noise power contained within a certain bandwidth is given by the integral of the power spectrum

$$P_n' = \int_{f_1}^{f_2} P_{IN}(f)\;|W(f)|^2\,df$$

$$= P_{IN}\,f_0 \int \frac{1}{1 + x^2\left(\frac{1}{Q^2} - 2\right) + x^4}\,dx \qquad \text{AI.11}$$

The integral in AI.11 can be written in the form *

$$\int \frac{dx}{1 + x^2\left(\frac{1}{Q^2} - 2\right) + x^4} = \frac{1}{2w}\int \frac{dx}{x^2 + wx + 1} - \frac{1}{2w}\int \frac{dx}{x^2 - wx + 1} +$$

$$\frac{1}{2}\int \frac{dx}{x^2 + wx \quad 1} + \frac{1}{2}\int \frac{dx}{x^2 \quad wx \quad 1} \qquad \text{AI.12}$$

where $w = \sqrt{4 - \frac{1}{Q}2}$.

All integrals in AI.12 can be expressed in terms of elementary functions. The power P_n therefore

$$P_n' = P_{IN}\,f_0\,Q\left[\frac{1}{4w}\ln\frac{x^2+wx+1}{x^2-wx+1} + \tan^{-1} Q(2x + w) + \tan^{-1} Q(2x - w)\right] \qquad \text{AI.13}$$

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* The integral in AI.11 was solved in this form by Dr. H. Helfenstein whose help I would like to acknowledge here and thank for it.

For numerical results it is convenient to express the power P'_n as a fraction of the total power contained within the semi-infinite frequency band from 0 to $+\infty$.

$$P_n = \frac{P'_n}{P_{in}} \qquad \text{AI.14}$$

Since $P'_n = P_{in} f_o \frac{Q\pi}{2}$ then P_n can be written as

$$P_n = \frac{1}{2Q\pi W} \ln \frac{x^2 + Wx + 1}{x^2 - Wx + 1} + \frac{1}{\pi}\left[\tan^{-1} Q(2x + W) + \right. \qquad \text{AI.15}$$
$$\left. + \tan^{-1} Q(2x - W)\right]$$

The values of P_n as a function of x have been computed for a range of values of Q (Q = 1, 3, 10, 30, 100, 300, 1000) and plotted in the diagrams AI. 17 - 20.

From these diagrams it is possible to find the percentage of the total energy of the resonant circuit which is contained within any given bandwidth.

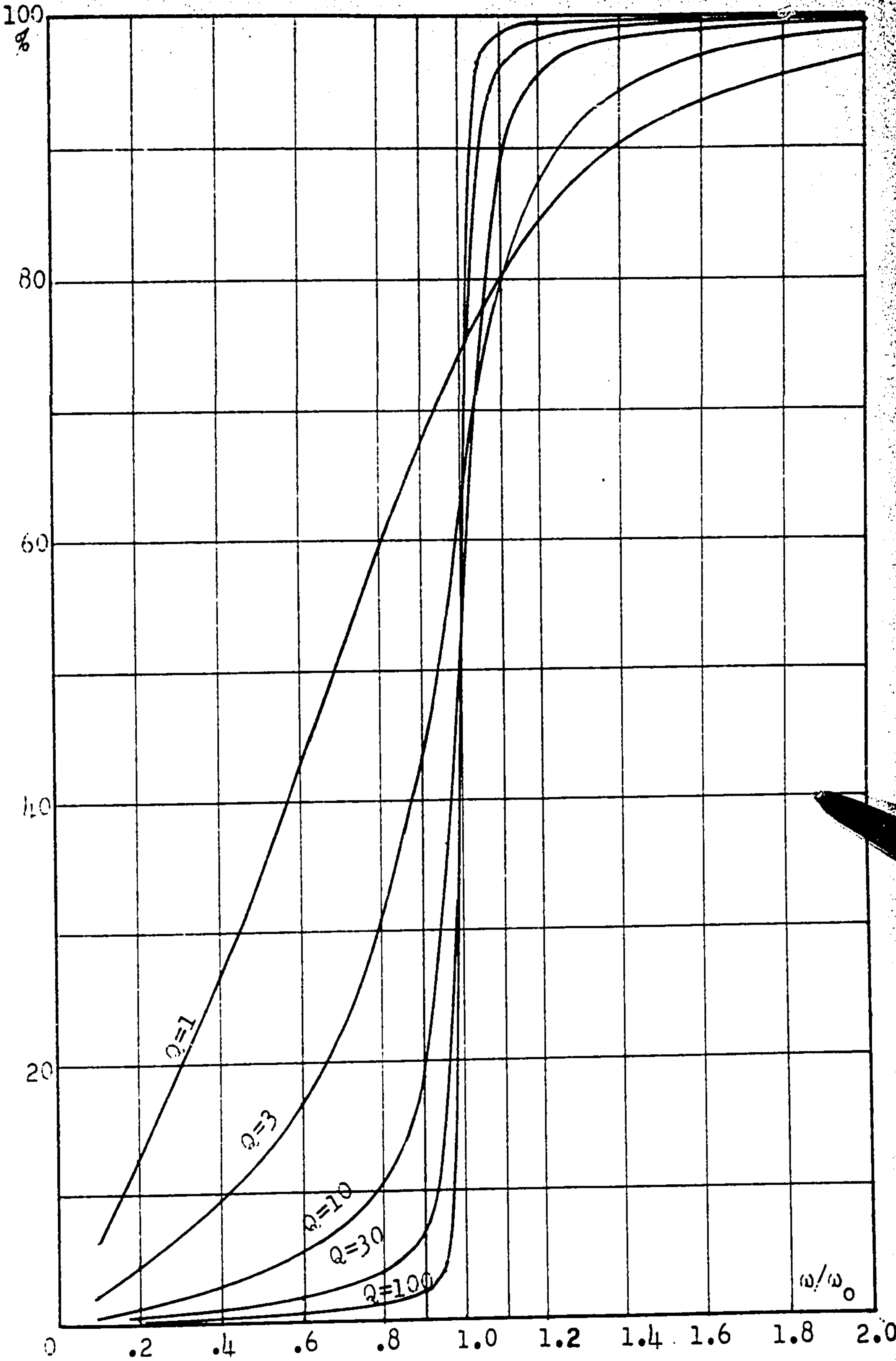


Fig. AI.17. Percentage of total energy of an RLC resonant circuit in the vicinity of resonance.

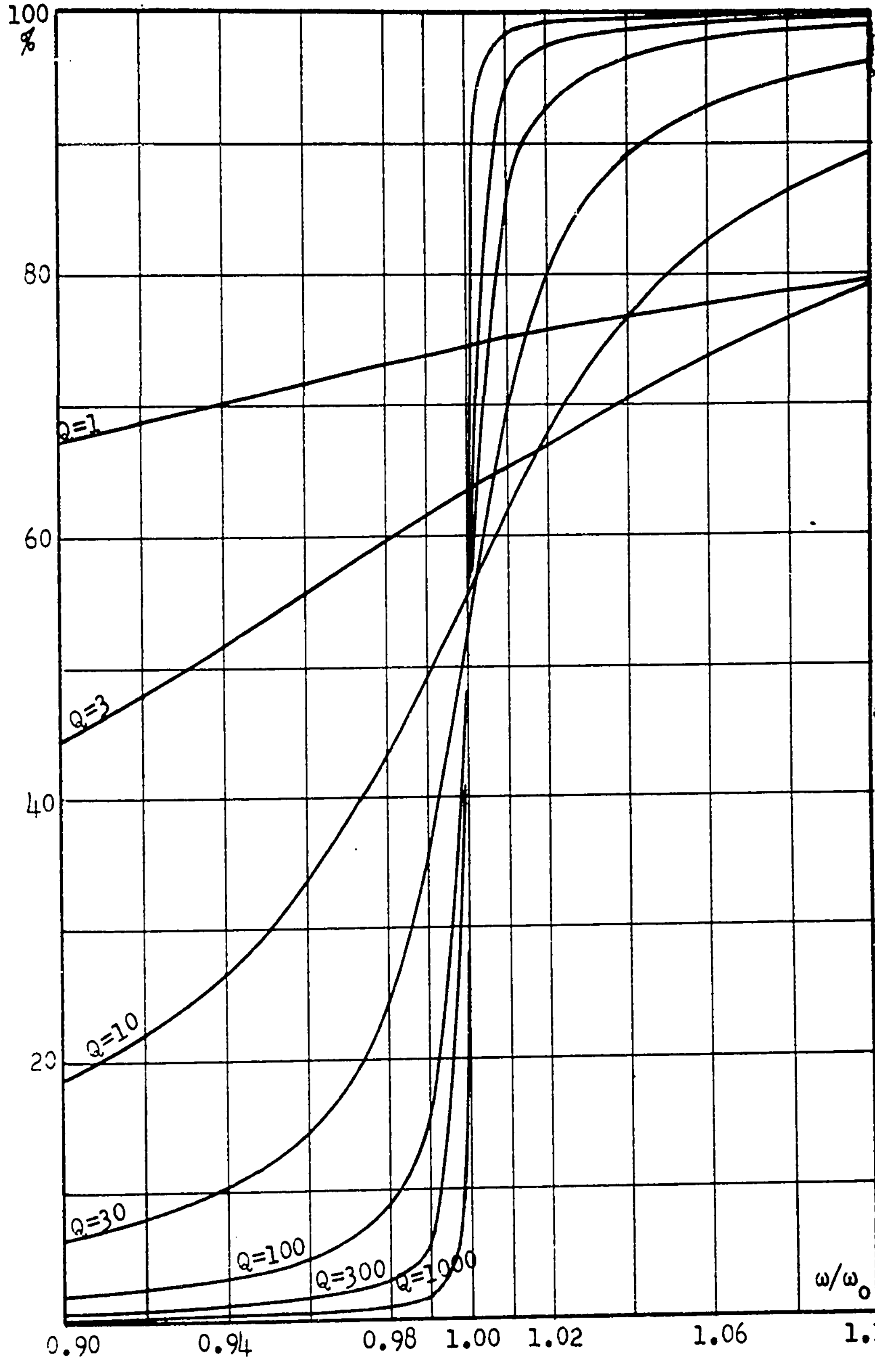


Fig. AI.18 Percentage of total energy of an RLC resonant circuit in the vicinity of resonance.

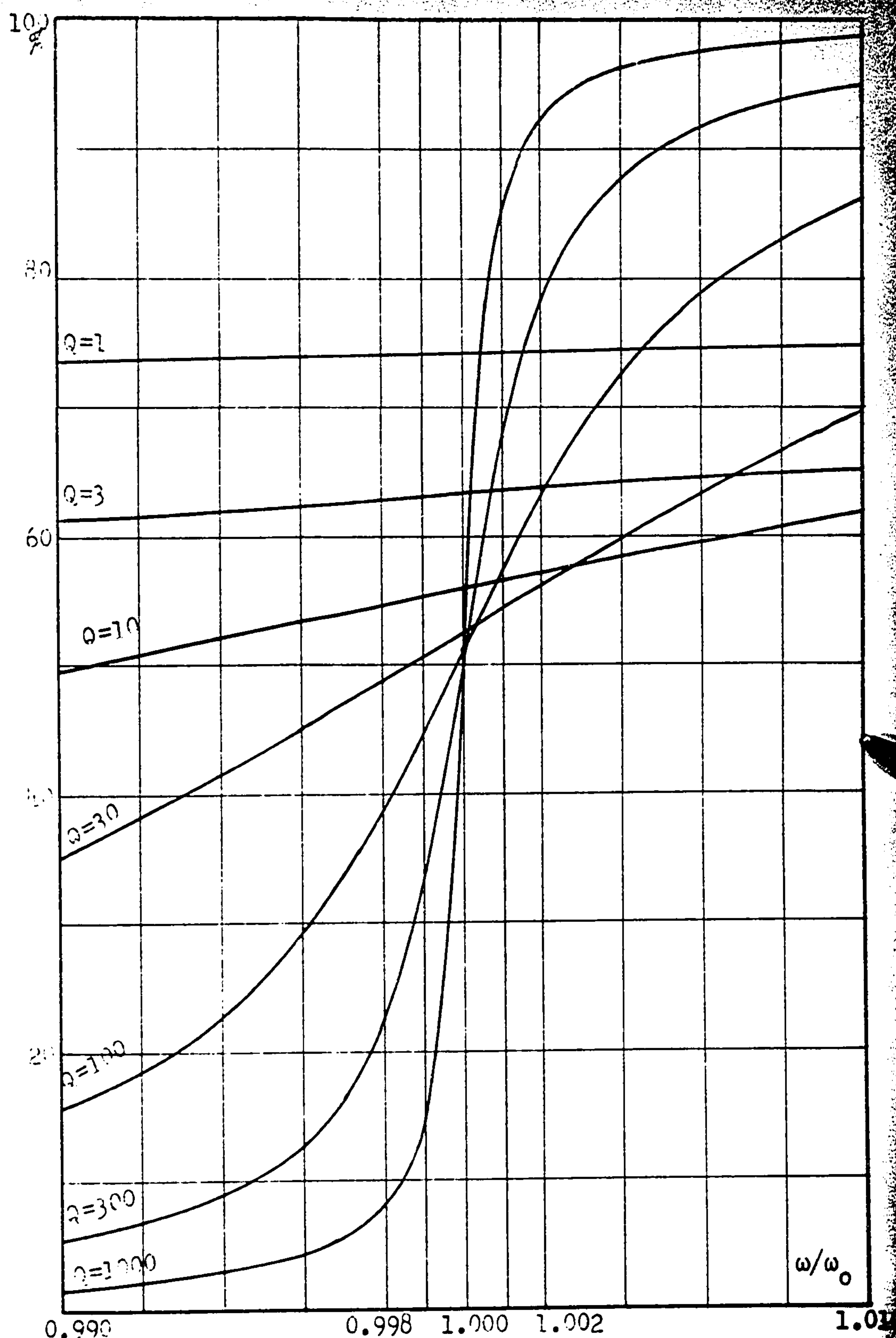


Fig. AI.19. Percentage of total energy of an RLC resonant circuit in the vicinity of resonance.

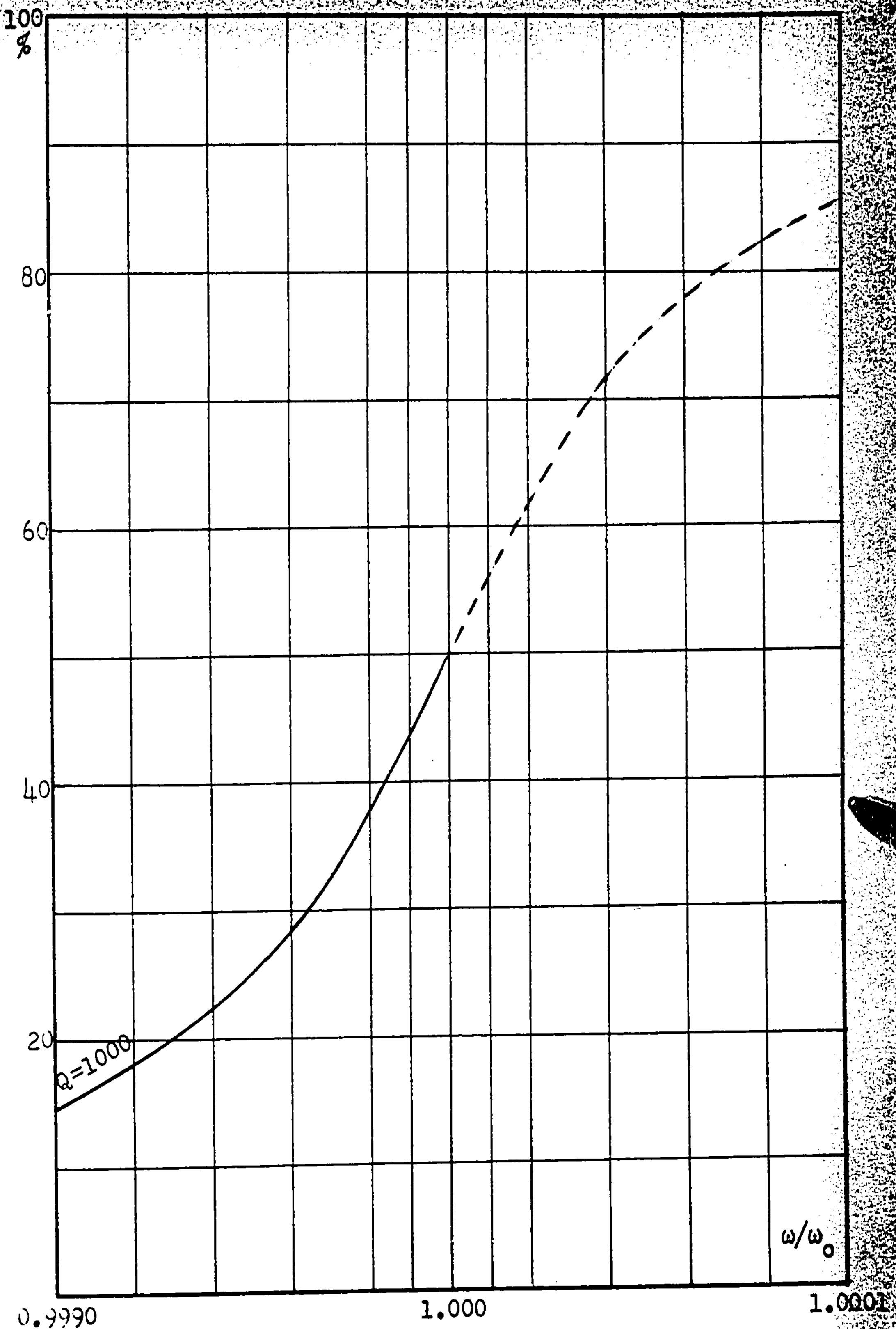


Fig. AI.20. Percentage of total energy of an RLC resonant circuit in the vicinity of resonance.

AII. Appendix 2.

There are two clearly distinguishable cases when dealing with the amplification of noise by a noisy amplifier.

Denote the input noise, i.e. the noise which we wish to amplify, by N_i and the amplifier noise, i.e. the noise which is not wanted but unavoidable, by N_A.

Both N_i and N_A are assumed to be contained within their respective bandwidths B_i and B_A. The two cases mentioned above are

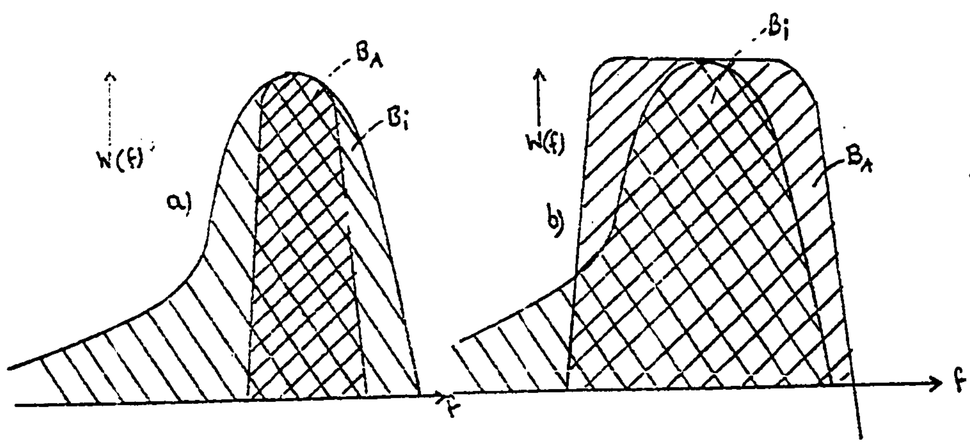
a) $B_i > B_A$

AII.1

b) $B_i < B_A$

The case $B_i = B_A$ is of little interest since it is difficult to realize in practice and if it were realizable it would be included in the case $B_i > B_A$.

The two above cases are shown diagramatically in Fig.



AII.

The cross hatched area represents the unavoidable mixture of wanted and unwanted noise. Singly hatched area, within the amplifier bandwidth B_A in case b), represents the straight amplifier noise which can be separated and rejected by a suitable filter, thus reducing the case b) to that of a).

The case b) is inadmissible when dealing with noise N_i the level of which is of the same order, or lower, than that of the amplifier noise N_A.

Three distinct sources of noise w re taken into consideration by Alkemade:

a) The noise from the thermometric element R, i.e. the information bearing noise, equal in this case to (3.6)

$$\bar{v}_i^2 = kTR\,\omega_o \eta_1 = \frac{kT}{C}\eta \qquad \text{AII.3}$$

b) The noise due to the shot effect of the (equivalent) grid current i_g

$$\bar{v}_g^2 = 2ei_g \int_{B_A} |Z(f)|^2\,df \qquad \text{AII.4}$$

where Z(f) is the impedance of the grid circuit (Fig. 3. 8b)

$$|Z(f)|^2 = R^2 \frac{1 + Q^2 x^2}{1 + (\frac{1}{Q^2} - 2)x^2 + x^4} \qquad \text{AII.5}$$

and

$$\int |Z(f)|^2 df = f_0 \frac{Q^2-1}{4w} \ln \frac{x^2-wx+1}{x^2+wx+1} + \qquad \text{AII.6}$$

$$+ \frac{Q^2+1}{2} Q\left[\tan^{-1} Q(2x+w) + \tan^{-1} Q(2x-w)\right]$$

where $x = \frac{\omega}{\omega_c}$; $\omega_o^2 = \frac{1}{LC}$; $w^2 = 4 - \frac{1}{Q^2}$ and $Q = \frac{\omega L}{R} = \frac{1}{\omega RC}$

The above integral is not very suitable for an explicit expression of its value over an arbitrary bandwidth B. Since, however, Z(f) is a resonant circuit and the Q value of that circuit is usually high, we can assume that almost all energy of the circuit is contained within the considered bandwidth B i.e.

$$\int_B |Z(f)|^2 df \simeq \int_0^\infty |Z(f)|^2 df = (Q^2+1)\frac{Q\omega_o R^2}{4} \qquad \text{AII.7}$$

If we assume that $Q > 10$, then we can neglect unity in the expression $(Q^2 + 1)$ and write

$$\int_0^\infty |Z(f)|^2 df \simeq \frac{Q^3 \omega_o R^2}{4} = \frac{L}{4RC^2} \qquad \text{AII.8}$$

If we wish to be more exact we can multiply the last expression by the factor η_2 which, similarly as in the Appendix I, expresses the fraction of energy contained within the considered bandwidth with respect to the total energy available in the semi-infinite bandwidth from 0 to $+\infty$.

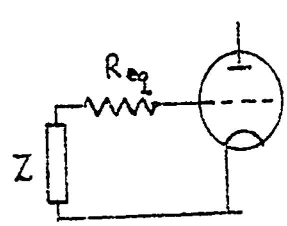
$$\int_0^{\infty} |Z(f)|^2 \, df = \frac{L}{4RC^2} \eta_2 \qquad \text{AII.9}$$

The values of η_2 can be computed from the formula AII.6 with the aid of graphs included in the Appendix I, (e.g. when $Q = 30$; $f_o = 1$ mc and $B = 100$ kc then $\eta_2 = (0.96 - 0.07) = 0.89$).

In the case considered by Alkemade the Q factor of the circuit was of the order of 1000 and, therefore η_2 was practically equal to unity. In that case

$$\overline{V_g^2} = 2 e i_g \frac{L}{4RC^2} \qquad \text{AII.10}$$

c) The second source of unwanted noise is the shot noise of the plate current. That noise source can be represented as a resistance in series with the grid circuit, the value of which is chosen in such a way as to give, at the temperature $T = T_o = 290^o K$, a noise effect equivalent to that due to plate current shot noise (Fig. AII.11).



AII.11

Since the noise voltage due to R_{eq} is $4kT_oR_{eq}B_A$ and the usual approximate value for R_{eq} is $2.5/g_m$, then we have, for the noise voltage due to the plate current shot effect, the expression

$$\bar{V}_s^2 \cong 10kT_o B_A / g_m \qquad \text{AII.12}$$

AIII. Appendix 3

Noise power contained in a given bandwidth of a resonant circuit.

The noise contained within a finite bandwidth centered around the resonant frequency of a resonant circuit can be expressed as (cf. 3.6)

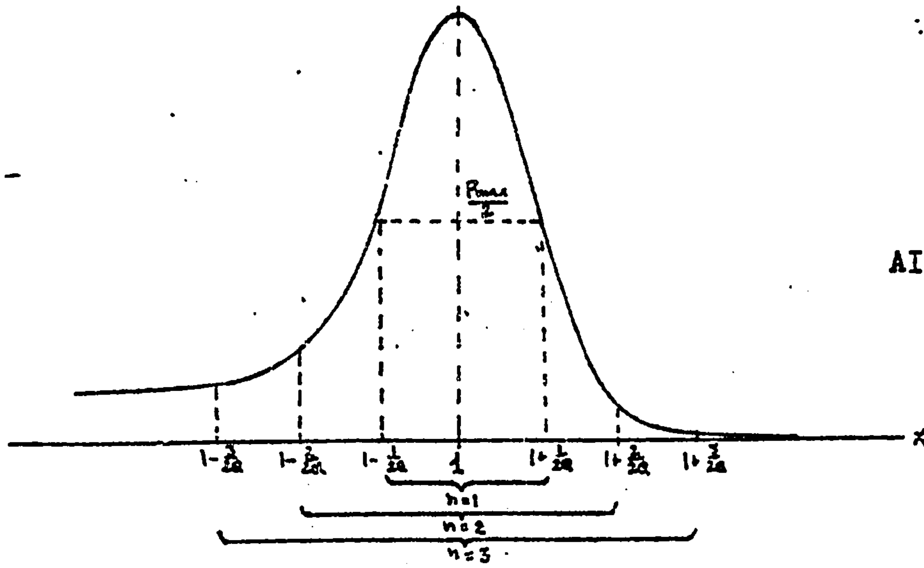
$$\overline{V}_{AB}^2 = kTR\omega_0 Q\eta_1 \qquad \text{AIII.1}$$

where $\eta_1(Q, \Delta f)$ is the percentage of the noise power contained within the considered bandwidth and is, in general, a function of the Q of the circuit and of the bandwidth.

If P_{n1} and P_{n2} are the relative noise powers contained within the frequency limits $0 - f_1$ and $0 - f_2$, then the noise power contained within the bandwidth $f_2 - f_1$ will be given by the expression

$$P_{n12} = \eta_1 = P_{n2} - P_{n1} \qquad \text{AIII.2}$$

The frequency limits f_1 and f_2 can be referred to the resonant frequency and written as $f_1/f_0 = x_1$ and $f_2/f_0 = x_2$. Further, since we shall ordinarily be concerned with the frequencies centered around the resonant frequencies we can write $x_{1,2} = 1 \pm \Delta f/f_0$. Furthermore there is a quantity, which is widely used in communications engineering, called "nominal bandwidth" of a resonant circuit and which is given by the ratio f_o/Q.



AIII.3

We can express our variable x in terms of that quantity by making $\Delta f = n(f_o/2Q)$. Then $x_{1,2} = 1 \pm n/2Q$, where n is always positive and expresses the number of nominal bandwidths (see Fig. AIII.e). The power P_{n12} then becomes a function of the quantity $n/2Q = z$. Written in full

$$P_{n12} = \eta_1 = \frac{1}{2Q\pi w} \ln \frac{z^2(w+z)^2 - (w^2-4)}{z^2(w-z)^2 - (w^2-4)}$$

$$+ \frac{1}{\pi}\left[\tan^{-1} Q(2+w+2z) - \tan^{-1} Q(2+w-2z)\right] \qquad \text{AIII.4}$$

$$+ \frac{1}{\pi}\left[\tan^{-1} Q(2-w+2z - \tan^{-1} Q(2-w-2z)\right]$$

where $w = \sqrt{(4 - 1/Q)}$.

The above expression is exact. In that form, however, it is not very suitable for analysis. An approximate expression can be derived from (AIII.4) by observing that:

a) the quantity $z = n/2Q$ has to be considered for values less than unity. If z were greater than unity then our original variable $x_1 = (1 - z)$ would become negative. We are concerned only with the positive frequencies.

b) We are usually concerned with resonant circuits having Q values definitely greater than 10, and in the particular case of Alkemade the measured Q was of the order of 10^3.

If we take the above restrictions into account ($z < 1$ and $Q > 10$) we find that the first term in (AIII.4) is less than $1/10Q$ and that the difference between the first two terms in the square brackets is of the order of $1/4Q^2$. This means that for $Q = 10$ the first term contributes only about 1%, and the second contributes only about 1/2% to P_{n12}.

The only significant terms left from the expression AIII.4 are

$$P_{n12} = \eta_1 \cong \frac{1}{\pi}\left[\tan^{-1} Q(2-w+2z) - \tan^{-1} Q(2-w-2z)\right] \qquad \text{AIII.5}$$

here again, since $2 - w = 2 - \sqrt{(4 - 1/Q^2)} = 1/2Q^2$ we may, for the sake of simplicity, assume that $2 - w = 0$ and thus obtain a very simple expression

$$\eta_1 \cong \frac{2}{\pi} \tan^{-1} n \qquad \text{AIII.6}$$

which gives us the power contained within n nominal bandwidths of a resonant circuit for $Q>10$. This expression shows that η_1 is relatively independent of Q. It should also be remarked here that the last simplification was not fully justifiable. It is valid only for $Q\gg10$. For rough approximations (AIII.6) can be further simplified by expanding $\tan^{-1}$ in series

$$\eta_1 \cong 1 - \frac{2}{\pi}\left(\frac{1}{n} - \frac{1}{3n^3}\right) \qquad \text{AIII.7}$$

or for large n

$$\eta_1 \cong 1 - \frac{2}{\pi n} \qquad \text{AIII.8}$$

The consecutive approximations given by the expressions AIII.6, AIII.7 and AIII.8 are given in the form of a table listing the quantities $\eta_1 - 1$. This was chosen rather than the straight η_1 since it is easier to see the magnitude of the departure of that quantity from unity, and also it is easier to plot $\eta_1 - 1$ in graphical form (see Fig. AIII.10).

Table AIII.9

n	.1	.4	1	4	10	40	80
computed for Q=10	.9365	.7577	.4993	.1514	.0515	.0037	
computed for Q=1000				.1559	.0635	.0159	.0080
$1\frac{2}{\pi}\tan^{-1}n$	19365	.7577	.5000	.1559	.0635	.0159	.0080
$\frac{2}{\pi}\left(\frac{1}{n} - \frac{1}{3n^3}\right)$			.4245	.1559	.0634	.0159	.0080
$-\frac{2}{\pi n}$			.6366	.1592	.0636	.0159	.0080

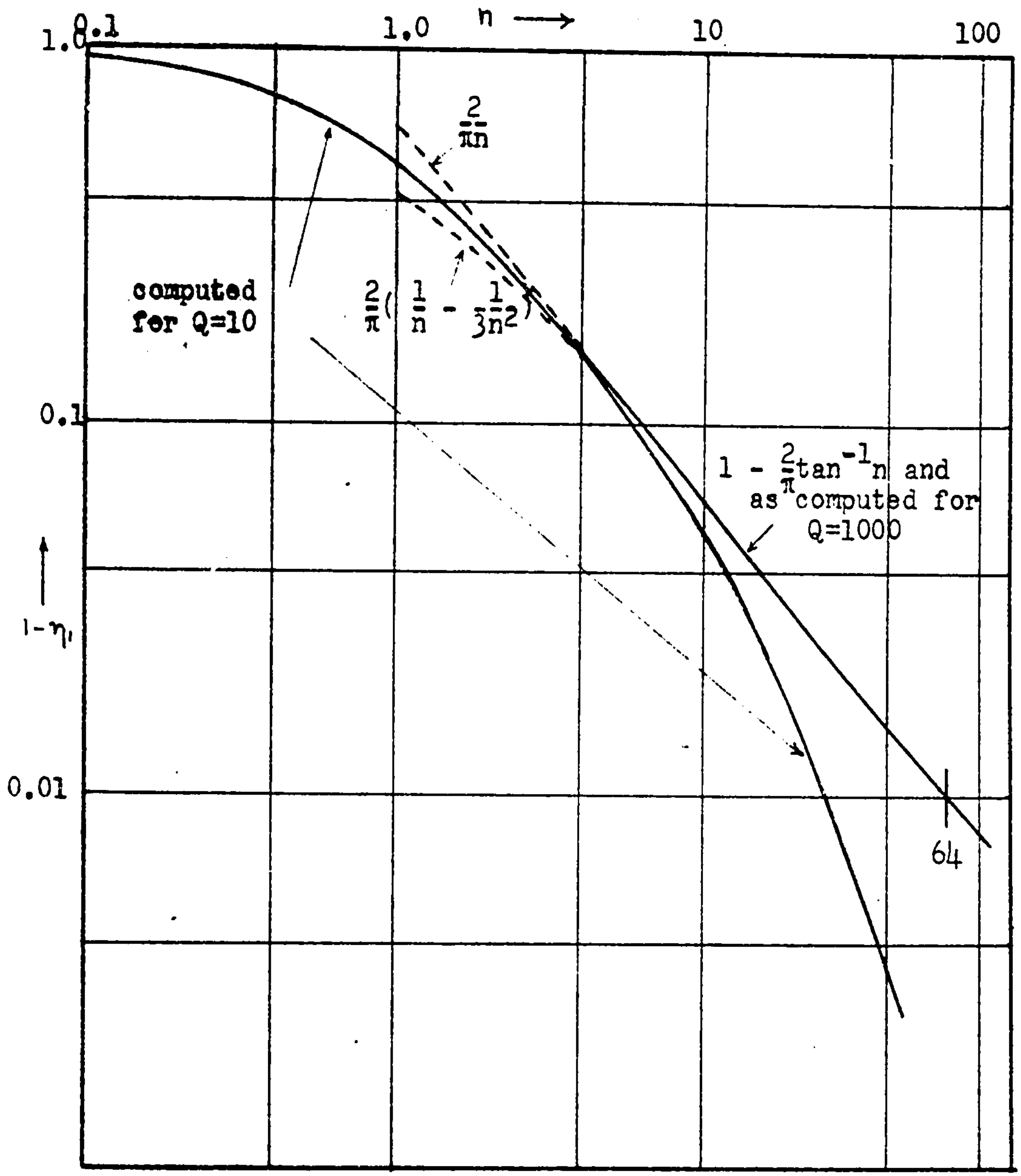


Fig. A3.10

Different approximations for expressing the power **excluded** from the resonant circuit as a function of **the** "**nominal** bandwidth" $n = f_o/Q$.

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