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The Effect Of A Central Hub On The Onset Of
Flow Reversal In A Swirling Sudden Expansion
Flow

By

Valiallah Tavasoli

A thesis
presented to the University of Ottawa
in partial fulfillment of the
requirements for the degree of
Master of Applied Science
in
Mechanical Engineering

DEPARTMENT OF MECHANICAL ENGINEERING
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Valiallah Tavasoli, Ottawa, Canada, 1989



UNIVERSITÉ D'OTTAWA
UNIVERSITY OF OTTAWA

To My Parents And My
Wife, Elizabeth

Abstract

A series of experiments involving flow visualization and probe measurements was carried out in order to obtain the critical swirl intensity at which vortex breakdown is initiated for annular swirling jets in sudden expansions as a function of inlet blockage and expansion ratio. In the process of obtaining the critical swirl intensity, the velocity profiles at the expansion inlet were also measured. In addition, a mathematical model based on the Momentum Integral Method was developed in order to analytically predict the critical dimensionless angular momentum flux of the swirling fluid.

It was found that both sudden expansion and presence of a central hub at the throat of the expansion contribute to a lower critical swirl intensity. The critical swirl intensity was found to reach a minimum at an expansion ratio of about two, followed by a small increase in larger expansions. Increasing the central hub to throat ratio up to 0.63 resulted in a sharp drop in the critical swirl intensity. It is expected that the critical swirl intensity would decrease further as even larger hubs are applied, even though central hubs with hub to throat ratios larger than 0.7 are impractical from an engineering point of view. The mathematical model closely predicted the experimentally obtained critical swirl intensity.

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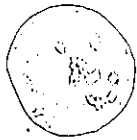
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Glossary

Roman Symbols

a_1	Inlet solid body core radius
a_2	Solid body core radius at station (2)
A_2	$\left(\frac{a_2}{r_2}\right)^2$
B	Refer to equation (3.2)
c_D	Drag coefficient
COEFV	Dynamic pressure coefficient
COFTP	Static pressure coefficient
$ERROR_u$	Refer to equation (4.75)
$ERROR_w$	Refer to equation (4.100)
g_r	Gravitational acceleration in radial direction
g_θ	Gravitational acceleration in tangential direction
g_z	Gravitational acceleration in axial direction
L	Sliding sleeve position in the swirl generator
M_a	Manometer measured axial mass flow rate
M_{ai}	Refer to equation (4.83)
M_{ao}	Refer to equation (4.80)

M_i	Refer to equation (4.84)
M_{j_0}	Refer to equation (4.81)
M_t	Manometer measured tangential mass flow rate
M_{th}	Integrated total mass flow rate at the throat
M_1	Inlet mass flow rate
M_2	Mass flow rate at station (2)
p_1	Inlet local static pressure
p_2	Local static pressure at station (2)
p_{01}	Inlet static pressure on the axis
p_{02}	Static pressure on the axis at station (2)
P_{atm}	Atmospheric pressure
P_{ax}	Manometer measured static pressure at the axial inlet
P_{ref}	Inlet average static pressure
P_t	Total pressure
P_{tan}	Manometer measured static pressure at the tangential inlet
P_w	Throat wall pressure
P_{1-5}	Pressure at probe holes 1-5
Q	Volumetric flow rate at the throat
r	Radial direction in cylindrical coordinates
r_{bi}	The inner effective boundary layer radius
r_{bo}	The outer effective boundary layer radius
r_h	Central hub radius
r_N	The last measurement point
r_1	Throat radius



r_2	Expansion radius
r_1'	First measurement point
Re	} Reynolds number
R_h	
	$-\left(\frac{r_2}{r_1}\right)^2$
Ro	Rossby number
R_2	$\left(\frac{r_2}{r_1}\right)^2$
S	Swirl number
u	Axial velocity component
u_h	Axial velocity at the edge of the wake
u_{mod}	model predicted axial velocity
u_0	Axial velocity at the centre of the wake
u_1	Inlet axial velocity near the throat wall
u_2	Axial velocity near the expansion wall
u_i'	Fitted axial velocity at each measurement point
u'	Fitted axial velocity including the boundary layer effect
U_{ref}	Average axial velocity
U_{ref}'	Fitted average axial velocity
v	Radial velocity component
V_t	Total velocity vector
w	Tangential velocity component
w_a	Tangential velocity at the inlet solid body core boundary
w'	Fitted tangential velocity including the boundary layer effect

w_h	Tangential velocity at the edge of the hub
w_{mod}	Fitted tangential velocity excluding the boundary layer effect
w_2	Tangential velocity at the solid body core boundary at station (2)
w'	Refer to equation (4.27)
w'_h	refer to equation (4.28)
x	axial direction in cylindrical coordinates

Greek Symbols

α	Fitting parameter for tangential velocity profiles
β	Refer to equation (4.43)
γ	Inclined manometer angle
Γ	Circulation
δ	Radius of viscous core
ζ	Fitting parameter for the axial velocity profiles
η	Circulation number
θ	Angular direction in cylindrical coordinates
ι	Pitch angle
κ	Shear coefficient for the axial velocity behind the wake
λ	Refer to equation (4.70)
μ	Viscosity of air
ξ	Refer to equation (4.47)

π	3.141593
ρ	Local density of air
σ	Constant dependent on mixing length and width of the wake
τ_1	Refer to equation (4.71)
τ_2	Refer to equation (4.71)
Υ	Axial flux of axial momentum
Υ_D	Dimensionless axial flux of axial momentum
Φ	$\frac{u_2}{u_1}$
χ	Yaw angle
ω	Angular velocity
Ω	axial flux of angular momentum
Ω_{Ai}	Refer to equation (4.93)
Ω_{Ao}	Refer to equation (4.88)
Ω_D	Dimensionless axial flux of angular momentum
$(\Omega_D)_{crit}$	Critical angular momentum flux
Ω_{fi}	Refer to equation (4.93)
Ω_{fo}	Refer to equation (4.89)
Ω_{func}	Angular momentum flux stream function
$(\Omega_{func})_{bi}$	Ω_{func} at $r = r_{bi}$
$(\Omega_{func})_{bo}$	Ω_{func} at $r = r_{bo}$
$(\Omega_{func})_N$	Ω_{func} at $r = r_N$
$(\Omega_{func})_{r_1}$	Ω_{func} at $r = r_1$
Ω_1	Inlet angular momentum flux
Ω_2	Angular momentum flux at station (2)

$(\Omega_{f_{unc}})_1$ $\Omega_{f_{unc}}$ at $r = r_1'$

Ω' Angular momentum flux including the boundary
layer effect

Ω'_D Dimensionless Ω'

Other Symbols

$\mathcal{R}$ $\frac{v}{w}$

$\mathcal{Z}_1$ Refer to equation (4.42)

$\mathcal{Z}_2$ Refer to equation (4.54)

$\mathcal{Z}_3$ Refer to equation (4.55)

$\mathfrak{S}_1$ Inlet axial thrust

$\mathfrak{S}_2$ Axial thrust at station (2)

Chapter 1

Introduction

A vortex with an axial velocity component is called swirling flow. It is a three dimensional phenomenon with its total velocity vector, V_t , composed of axial, radial and tangential components (Fig. (1.1) .) The flow may or may not have vorticity.

Swirling flow exists both in laminar and turbulent forms. In industrial practice, swirling flow is usually in a fully turbulent state with its fluctuations occurring in all three directions, while laminar swirling flow has mostly been studied for academic purposes.

Swirling flows are encountered in many engineering situations. In particular, swirl is used extensively as an aid to efficient, clean combustion in gasoline engines, Diesel engines, gas turbines, industrial furnaces, utility boilers and many other practical heating devices, including small domestic home furnaces. Figures (1.2) to (1.4) represent schematics of swirl flow in some of its applications.

Swirling jets have large scale effects on the flow field. Jet growth, decay , and

rate of entrainment, plus flame size and stability, are affected by the degree of swirl imparted to the flow. The degree of swirl is characterized by a dimensionless quantity called the *Swirl Number*, denoted by S and defined as:

$$S = \frac{\text{Axial Flux Of Angular Momentum}}{R.(\text{Axial Flux Of Axial Momentum})} \quad (1.1)$$

where R is the equivalent throat or nozzle radius. An alternative definition which includes the pressure force in the axial momentum term has also been used.

An idealized picture of a swirl flow is given by a Rankine vortex [1]. A swirling fluid behaves like a solid body near the axis of the flow. This region is called the forced-vortex or solid body core, in which every particle of the fluid has the same angular velocity. As a result, the tangential velocity component is directly proportional to the radial distance from the axis of rotation. Away from this region, each particle of the fluid moves freely in a circular path with its tangential velocity varying inversely with radial distance from axis of the flow. This outer zone is called a free vortex and extends from the edge of the solid body core to the tube wall.

The tangential velocity component of the swirling flow decays rapidly with downstream distance if the flow is allowed to expand. As a result, an adverse axial pressure gradient forms in regions close to the axis of the flow. In weakly swirling flows, flows with a swirl number below a critical value, this adverse pressure gradient is not large enough to cause flow reversal. Weakly swirling flows can be unstable and exhibit periodic motions, such as precession. Figure (1.5) represents a schematic of a weakly swirling flow in a sudden expansion. However, as the intensity of swirl is increased, the adverse pressure gradient becomes larger, and the

flow undergoes a transition to a complete flow reversal at the centre of the flow. The recirculation bubble formed at the axis of the swirling jet is called a *Central Toroidal Recirculation Zone*, distinguished from the *Corner Recirculation Zone* formed due to sudden expansion. Both of these recirculation zones are shown in Figure (1.6).

The central recirculation zone is only one form of a series of phenomena called vortex breakdown. A vortex breakdown is usually characterized by the development of a stagnation point on the axis, followed by a region of reversed axial flow encapsulated by a greatly expanded stream surface [2]. Six types of vortex breakdown have been reported so far [2], but the central toroidal recirculation zone is the only one which has been observed in fully turbulent flow.

The central backflow zone, when used in a fuel burner, contributes to cleaner and more efficient combustion by improving flame stabilization and by providing high rates of turbulent mixing much as a bluff body stabilizer does [1]. Hot combustion products and unburnt fuel are recirculated to the throat region where fuel and air are introduced. As a result, the fuel and air are preheated and better mixed, due to the high turbulence level in the flow, before reaching the flame. The advantages of application of swirling flow in combustion are summarized in the following three remarks:

- Reduction of flame length because of higher rates of entrainment of ambient fluid and faster mixing close to the nozzle and near recirculation zone boundaries.
- Improved flame stability because of the presence of the backflow zone which recirculates hot combustion products.

- Minimized maintenance and extended life of equipment, since no bluff body is required and flame impingement on solid surfaces (with heat and deposit problems) is avoided [1]

For these reasons, combustors employing swirling flow are almost invariably designed to produce a swirl above the critical value. For design purposes a knowledge of the minimum or critical swirl needed to produce recirculation is useful. Swirl levels much above the critical value would result in higher pressure losses without further improvement in mixing or flame stability [3]. Also, the flow field generated by many swirl combustors can be approximated by a swirling jet issuing into a sudden expansion [3]. The objective of the present research is therefore to determine the critical swirl for a swirling air jet issuing into a sudden expansion as a function of expansion ratio. The role of a central hub is also to be investigated, since the hub is representing a central fuel nozzle or vane support in a real burner. In pursuit of this goal, three different actions are undertaken:

- Use of flow visualization techniques in order to visually establish the flow conditions under which the central toroidal recirculation zone is initiated.
- Measurement of inlet velocity profiles to the expansion at the critical swirl.
- Development of a momentum integral model to predict the critical swirl.

In the following chapters, first a brief review of the literature relevant to the present investigation is given, and then both the experimental apparatus and procedure are described in chapter 3. In Chapter 4 a mathematical model based on the momentum integral method is developed together with a method for fitting simple velocity profile forms to the measured velocities. Finally, in chapter 5 the

results of both the experimental and the mathematical investigations are presented and discussed.

Typical Particle Path
In Swirling Flow

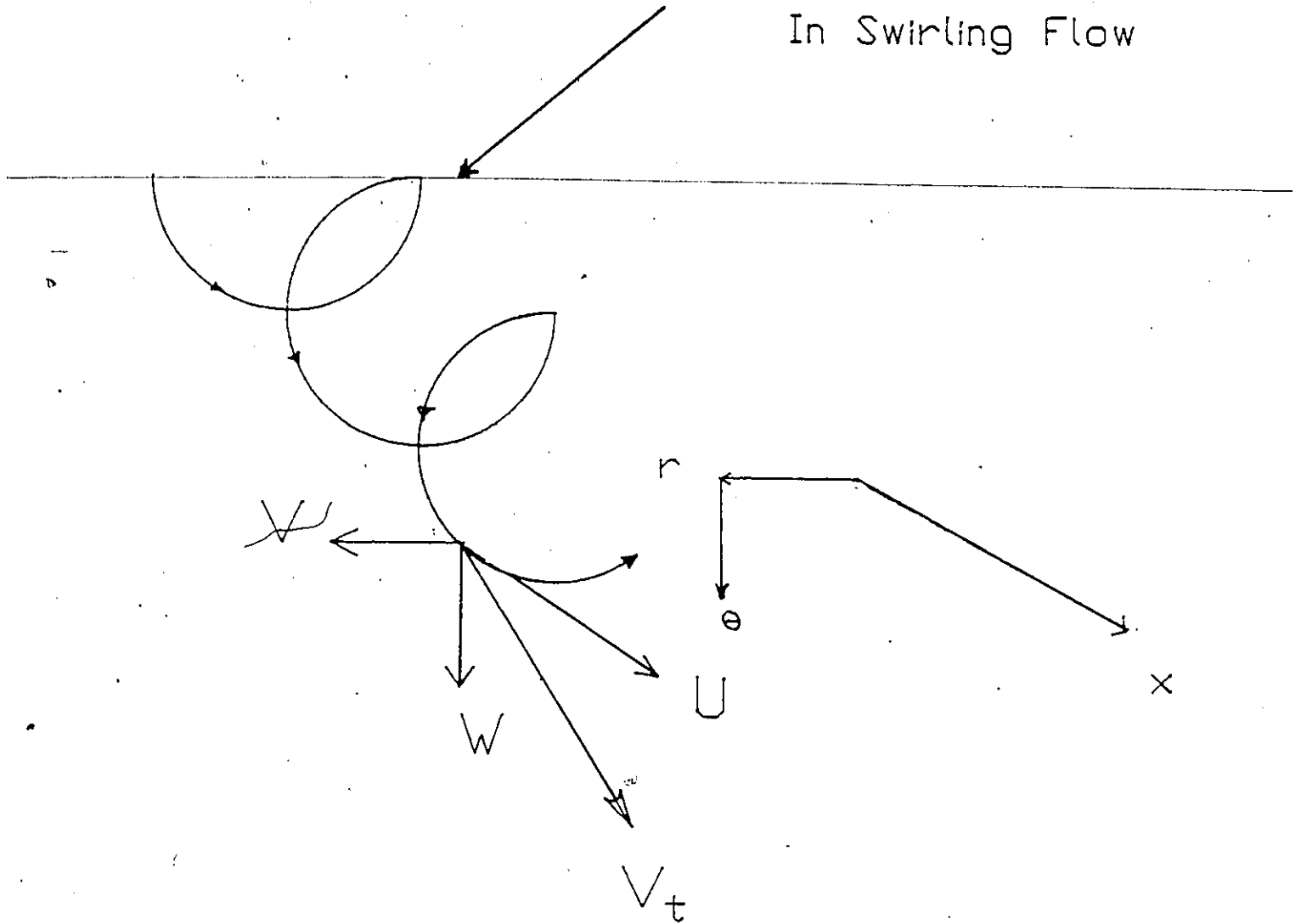


Figure 1.1: Total velocity vector and its components

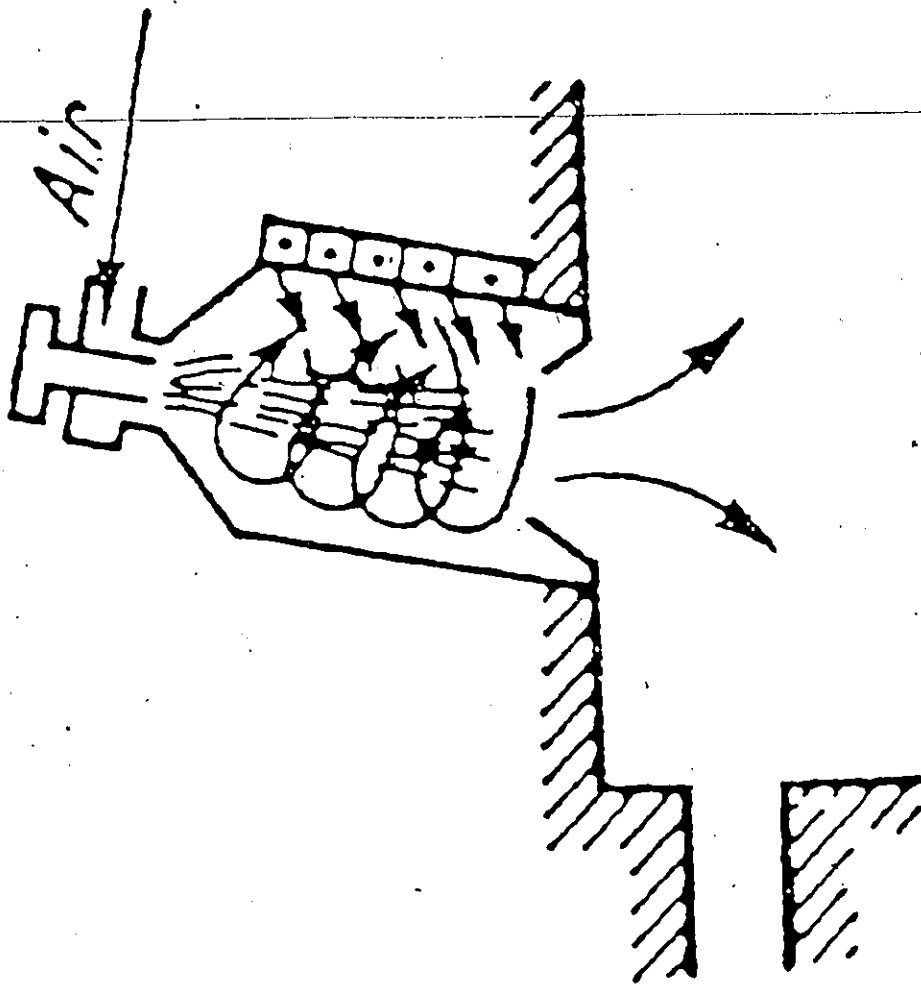


Figure 1.2: Application of swirling flow in a cyclon furnace [1].

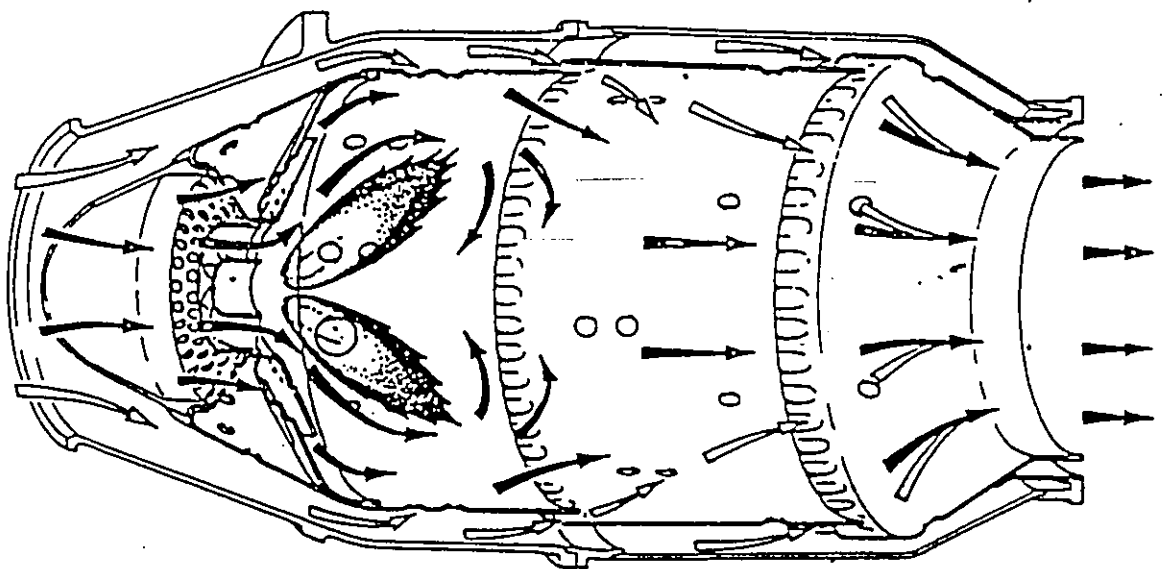


Figure 1.3: Application of swirling flow in a gas turbine engine [1].

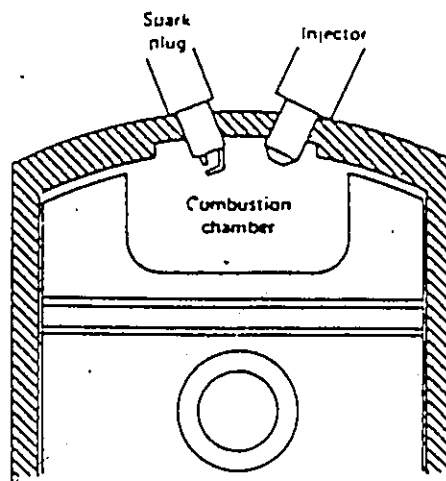
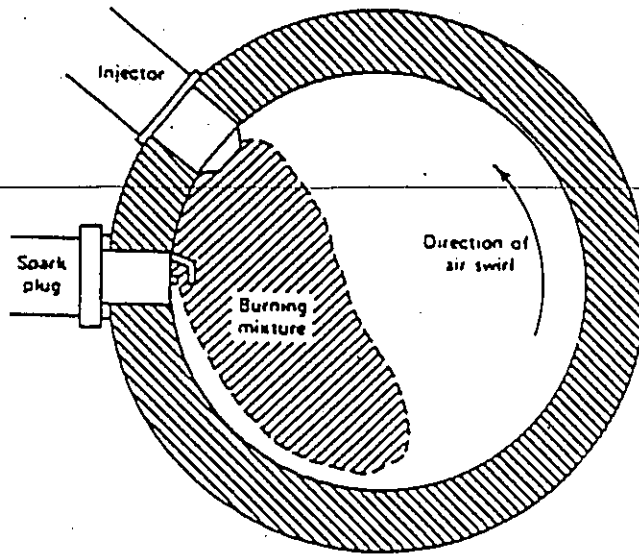


Figure 1.4: Application of swirling flow in a stratified charge engine [1].

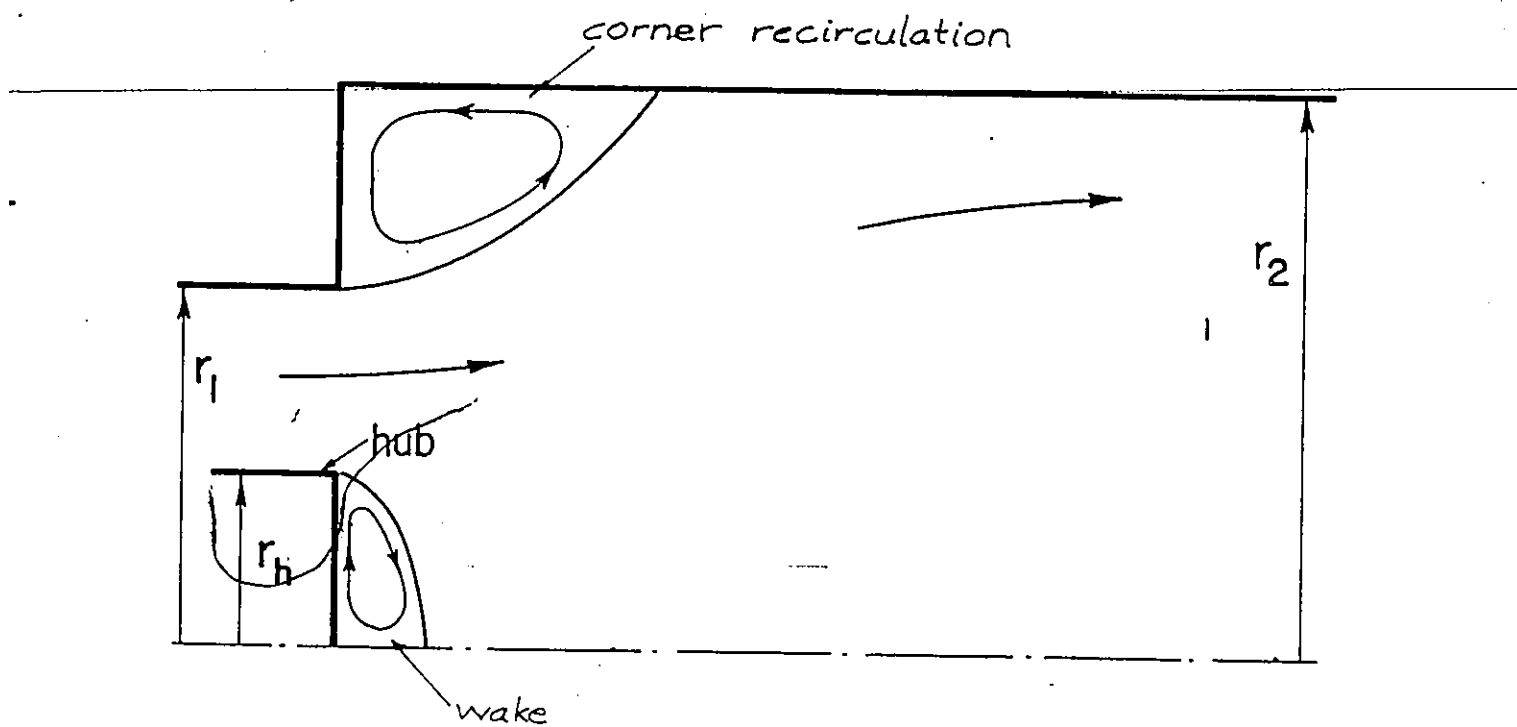


Figure 1.5: Weakly swirling flow (small expansion)

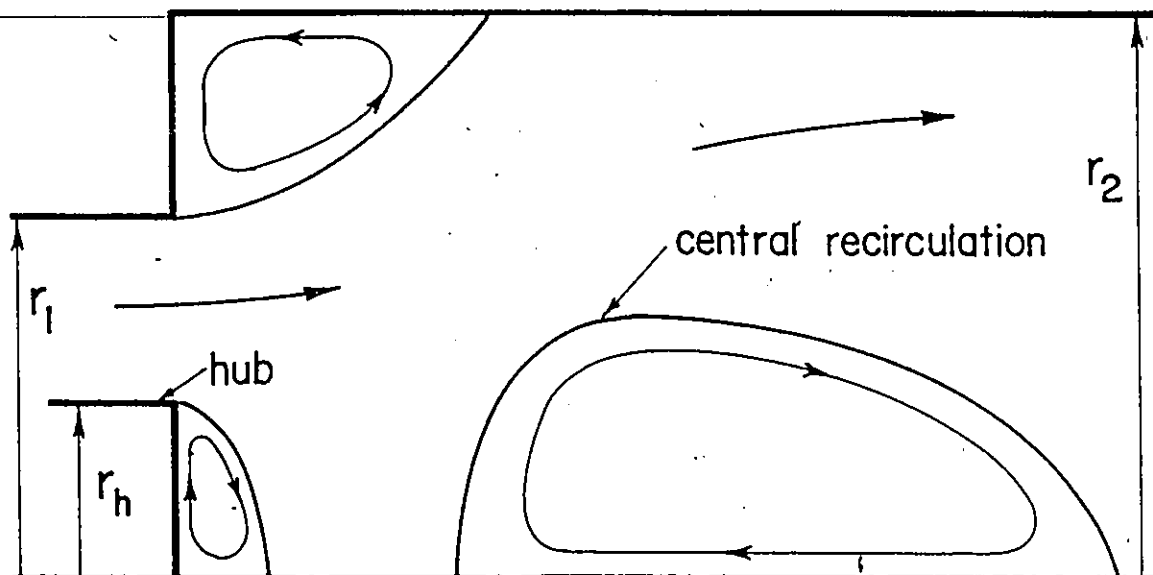


Figure 1.6: Strong swirling flow (CTRZ & CRZ)

Chapter 2

Literature Survey

Previous experimental and analytical work in axisymmetric recirculating swirling flows provides an important background for the present investigation. Information regarding turbulent swirling flows has been extensively reported. However, attention will be concentrated on papers directly related to the present work: measurements and models of the critical swirl in various geometries, the behavior of swirling flows in sudden expansions, and the effects of a central hub on expansion flows. The word 'critical' swirl as used in this thesis is defined as the minimum required to produce central recirculation; some of the authors cited here have used this word in other different senses.

2.1 Experimental Work

Gore and Ranz [4] used a hot wire anemometer to measure the velocity profiles at the onset of vortex breakdown in swirling flows moving axially through expanding cross sections. Under such conditions, regions near the axis oscillated regularly

between forward flow and back flow. Smoke introduced at the axis was observed to move first forward and then backward. Critical swirl intensities were measured in both air and water over a range of operating conditions. It was found that the critical swirl is independent of Reynolds number in the range of 20000-60000, but it is dependent on apparatus geometry.

Mathur and Maccallum [5] presented measurements of the flow fields of free turbulent swirling jets (i.e. sudden expansions of infinite ratio) issuing from hubless and annular vane swirlers for various swirl intensities, and suggested that the recirculated mass flow is dependent upon swirl number. For the hubless swirler a critical swirl number of about 0.5 can be roughly estimated by interpolating between measured axial velocity profiles, while for annular swirlers of hub to throat ratio of 0.33, the critical swirl is about 0.25. The tangential velocity profiles also suggest that the solid body core radius increases with swirl number for both types of swirlers. They also concluded that for both swirlers jet growth is enhanced by increasing swirl intensity.

In another investigation, Mathur and Maccallum [6] obtained the axial and tangential components of mean velocity in turbulent jets issuing from an annular swirler into different enclosed expansions. They related the axial location of maximum wall pressure to the impingement of the jet on the expansion section wall, while the center of the back flow zone was related to the point where wall static pressure was minimal. The axial velocity profiles indicate a critical swirl less than that observed for free swirling jets under similar conditions. Introduction of a secondary swirl issuing from the hub (co-swirl) was found to improve jet growth and reduce the critical swirl number.

Beltagui and Maccallum [7] studied the effects of sudden expansion ratios of 2.5

and 5.0 on swirling flows emerging from both hubless and annular vane swirlers. They characterized a weak swirling flow as one having an axial velocity maximum at the center of the jet, and suggested that the maximum diameter of the recirculating zone is controlled by the expansion diameter and is independent of swirl intensity once the back flow zone is established. Therefore, they suggested a swirl number based on the expansion diameter rather than the swirler throat diameter to include the effect of expansion. The presence of a central hub in the swirler was found to have little effect on the flow, although their data do not allow any conclusions to be reached about the effect on the critical swirl.

Sarpkaya [8] examined the effect of degree of divergence on the type and location of vortex breakdown for swirling flows in conical diffusers. He concluded that the adverse pressure gradient due to conical expansion affects the position of the recirculating "bubble". An increase in the angle of divergence of the tube was found to result in an increase in this adverse pressure gradient. He suggested that the occurrence of breakdown and its location depends on the flow rate and swirl, the external pressure gradient, the initial upstream conditions, and the angle of divergence of the expansion section. In another publication [9], Sarpkaya tested the stability of vortex breakdown to gradual and abrupt changes in the upstream and downstream flow conditions. These changes were introduced by means of increasing the inlet flow rate, by releasing a small air bubble from one of the guide vanes, by oscillating one of the swirl vanes, by oscillating the hypodermic tubing used for eccentric dye injection and by varying the setting of all vanes. Among the three types of breakdown observed, "double helix" and "spiral" forms were present only at low Reynold numbers and were found to be very unstable under any of the disturbances. The third type, called the "axisymmetric bubble", and equivalent

to the central recirculation being discussed in this thesis, was observed at higher Reynolds numbers and was found to be quite stable.

Faler and Leibovich [2] used flow visualization and laser Doppler anemometer measurement to study vortex breakdown in a vane-generated swirling flow issuing into a conical tube. In an apparatus modeled after Sarpkaya's, flow visualization studies revealed a total of six distinct modes of vortex breakdown. As the Reynolds number and circulation of the flow were increased, these disturbances showed a fixed order of succession and the mean breakdown position moved upstream. They described the breakdown as a "finite amplitude wave which can be held fixed in location if the upstream propagation speed of the wave vanishes. This speed depends upon both tangential and axial velocity profiles; an increase in swirl increases the propagation speed and an increase in axial velocity decreases the propagation speed. Therefore, an increase in swirl at a fixed flow rate will cause an upstream propagation of vortex breakdown." The Reynolds number range covered by their experiment went up to 8000. In all the cases in which vortex breakdown occurred, they found the flow to be laminar upstream of and in the breakdown region. It became turbulent shortly downstream of the end of the recirculation zone. Their experimental results showed that increasing flow rate at fixed vane angles produces positional changes, and increases the upstream propagation velocity of the wave.

Rhode [10] studied the existence, size and shape of both the corner recirculation zone and the central backflow zone for vane-generated turbulent swirling jets issuing into 45 and 90 degree expansions. The size and shape of the recirculation bubble for each flow field was illustrated as an artistic impression deduced from a collection of flow visualization photographs of tufts and smoke responding to the

flow. Increasing swirl intensity was found to decrease the size of the corner recirculation zone, while decreasing side wall expansion from 90 to 45 degrees practically removed the corner recirculation zone.

Analyzing a considerable amount of data obtained from flow visualization experiments performed on swirling jets issuing into a slowly diverging section, Escudier and Zehnder [11] suggested a simple criterion for the occurrence of vortex breakdown at a fixed location in the expansion section. They proposed a criterion for the breakdown based on the local Reynolds number, the circulation number, $\eta = \frac{\Gamma}{u r}$, and the ratio of radial to tangential velocities at the throat, $\mathcal{R}$:

$$Re \propto \eta^{-3} \mathcal{R}^{-1}$$

The proportionality constant was found to be independent of the divergence angle and the type of breakdown observed. The correlation was found to be well suited for Reynolds numbers ranging from 500 to 100000.

Escudier and Keller [12] measured the axial and tangential velocity components of a vane-generated annular swirling turbulent jet issuing into a sudden expansion and investigated the effects of sudden exit contractions on the velocity profiles and recirculation zone. They concluded that the shape of the recirculating bubble was affected by exit geometry. The backflow zone was found to be longer and narrower for larger contractions. In addition, by presenting streamline patterns, they showed that for a strong exit contraction and a swirl well above the critical, the recirculation zone assumed a mushroom shape, a character even more pronounced if the diameter of the annular section was reduced, while in the absence of such a center body, the recirculation zone was limited to a toroidal region without flow reversal on the axis.

Dixon et al [13] experimentally examined the aerodynamics of an unconfined turbulent swirling jet issuing from an annular divergent nozzle. The nozzle had a hub to throat ratio of 0.68 and a 30 degree angle of divergence. Measurements were taken at swirl numbers of 0.5 and 1.0 with and without the presence of a secondary swirling flow through the hub. At both swirl numbers a central recirculation zone was observed. However, the backflow zone became smaller as a greater jet flow was introduced through the central hub, and the maximum recirculated mass flow rate was found to decrease with increasing axial momentum flux ratio of hub flow to nozzle flow.

Rao et al. [14] investigated turbulent swirling flows generated by a vane swirler of 0.35 hub to throat ratio issuing into a sudden expansion to study the effect of variable vane angle and expansion variation on the occurrence and size of the backflow zone. Experimental results indicated that the solid body core radius in the expansion section is roughly the same as the radius of the backflow zone, and both are proportional to the expansion size. They also observed that an increase in swirl reduces the size of the corner recirculation zone resulting from the sudden expansion, and increases the axial velocity in the backflow zone. By interpolation between their reported flow fields, a critical swirl number of about 0.2 for an expansion ratio of 2.4 and hub to throat radius of 0.35 can be estimated.

Altgeld et al. [15] used laser Doppler anemometry to measure the three components of the velocity vector and their fluctuations in an axisymmetric swirling isothermal turbulent flow generated by a vane swirler of hub to throat ratio of about 0.5 emerging into a sudden expansion of 2.4 ratio. At a swirl number below 0.78, the effect of placing a contraction in the form of a baffle downstream of the backflow zone was tested. As a result, the backflow zone was stretched along the

axis about 3.5 times more than when the baffle was absent.

Yoon and Lilley [16], using a five-hole Pitot probe, investigated vane-generated turbulent swirling flows entering 45 and 90 degree expansions of ratios 2.0 and 4.0 equipped with downstream contractions of 45 and 90 degrees. They studied the effects of the above geometries on the shape and size of the backflow zone for different swirl intensities. They found that in a sudden expansion the size of the backflow zone increased with increasing swirl vane angle, i.e. swirl intensity, until a certain angle was reached, after which its length began decreasing while its width continued to increase. The effect of a gradual sidewall expansion on the size and shape of the backflow zone was found to be minimal. They also observed that placing a 45 degree contraction downstream of the backflow zone would result in a shorter recirculation zone and higher maximum swirl velocity, while increasing the contraction slope would cause an increase in the solid vortex core radius.

Ramos and Somer [17] used a laser Doppler anemometer to measure turbulent confined swirling flows produced by two vane swirlers capable of creating concentric co- and counter-swirling flows. Under both conditions a recirculation bubble was formed. They characterized this backflow zone by the presence of a toroidal vortex, low tangential velocity, high turbulence intensities and large dissipation rates of turbulent kinetic energy. The occurrence of a backflow zone under co-swirl conditions contradicted the results reported by Vu and Gouldin [18], who used a hot wire anemometer in the same geometry and at the same swirl. The authors attributed the difference to the disturbances introduced by the hot wire and its lack of sensitivity to flow direction.

Hallett and Günther [19] studied the effects of swirl intensity on flow and mixing patterns in a turbulent stepped sudden expansion flow. Measurements of

frequency spectra revealed the presence of a low frequency oscillation in weakly swirling flows. Increasing swirl weakened the periodicity of this oscillation, and it disappeared completely at the onset of vortex breakdown. Flow visualization revealed that this periodicity was due to precession of the flow downstream of the throat before the backflow zone settled in. They suggested that the stabilizing role of vortex breakdown was due to the fact that it caused the flow to fill the cross section immediately after the throat. They also reported a critical swirl number of about 0.38 for a 2 : 1 expansion.

Hallett and Toews [3] determined experimentally the critical swirl intensity required to produce a recirculating zone near the axis of the flow for turbulent swirling flows issuing into sudden expansions of ratio 1.25 to 3.0 as a function of expansion ratio and inlet conditions. Using an idealized Rankine vortex to describe the tangential velocity profiles, they showed that the static pressure drop in the radial direction after expansion is a function of expansion ratio and solid body vortex radius, as long as circulation Γ remains nearly constant during expansion. They also suggested that an inlet flow with a small core radius would undergo a larger decrease in radial pressure difference on expansion and hence show a lower critical swirl, while an axial velocity distribution with a maximum on the axis would require a larger pressure gradient and hence have a higher critical swirl. These predictions were confirmed by experimental results. They suggested that the axial pressure gradient increases with expansion ratio and asymptotically approaches a limit for very large expansions. Since the critical swirl intensity is inversely related to the axial pressure gradient, one would expect the critical swirl to approach an asymptote as well. Their experimental results confirmed this, but also showed that in some cases a slight minimum would occur at an expansion

ratio of about 1.5. They explained this by dividing the pressure gradient into two components , one caused by the axial velocity reduction (which would also be present in a non-swirling flow), the other due to the change in swirl velocities in expansion.

So et al. [20] investigated the mixing of a central non-swirling jet in confined turbulent swirling flows generated by a vane swirler. It was found that the existence of an external swirl had little or no effect on the general behaviour of a jet. Even though the jet decay was proportional to the inverse of axial distance travelled, as is the case in absence of an external swirl, the decay took a higher slope very close to the nozzle exit. The slope of the tangential velocity in the solid body core did not seem to be affected by the jet when the jet momentum was small compared to the axial momentum of the swirling flow. This was no longer true when the jet momentum was about one half of the axial momentum of the swirling flow. In addition to the jet studies, the presence of a center body, the jet nozzle, in the flow and its effect on the central backflow zone was investigated. It was concluded that the presence of a central hub would enhance the formation of a backflow zone in the central region of the flow due to the decrease in centerline axial velocity. The mean tangential velocity was not affected by the presence of the central hub, since the flow rotates like a solid body vortex core near the tube wall. Also, measurements of axial and tangential velocity fluctuations showed that the turbulence in the annular swirling flow is nearly isotropic.

2.2 Mathematical Modelling Of Swirling Flow

A mathematical solution of the flow field of interest has the potential of providing answers more quickly and more economically than an experimental approach to the problem. To achieve this, the model should simulate the flow in all its aspects. This may involve simultaneously solving a set of non-linear partial differential equations or assuming certain profiles for experimentally obtained velocity distributions in conjunction with more conventional methods, such as the momentum integral method, to predict the flow behaviour. Owing to the advent of the present high capacity computers, mathematical modelling and numerical solutions are now finding more favour and are being used extensively to supplement the existing experimental results.

Gore and Ranz [4] used an iterative method to solve the inviscid equation of motion for a swirling flow issuing into a conical expansion to study the effect of swirl intensity and the angle of divergence on the behaviour of the flow. The model predicted a backflow formation in the throat area, and they suggested that a realistic prediction of the critical swirl is dependent either on knowledge of the pressure distribution at the throat or of the radial distribution of axial velocity at the throat. Experimental results obtained by the authors did not agree with the mathematical predictions, especially in predicting the critical swirl.

Chow [21] analytically solved the governing differential equations to study the behaviour of a swirling fluid in tubes having axisymmetric deformation along the length. His assumptions of inviscid, incompressible and axisymmetric flow and introduction of Stoke's stream function reduced the governing differential equations to one which could be solved in conjunction with a proper equation defining the tube

radius. He characterized the intensity of swirl by the Rossby number, $Ro = \frac{u}{2R\omega}$, where u is the axial velocity and ω is the angular velocity. Ro is inversely proportional to the swirl number. Chow concluded that for a sinusoidal deformation of the tube wall, the critical Rossby number is a function of the n^{th} zero of the first order Bessel function. His results suggest that the central recirculation zone is only the first in a series of flows which exhibit an increasing number of recirculation zones as the swirl intensity increases. A similar result was obtained for flow through converging-diverging nozzles.

Bossel [22] obtained a numerical solution using the method of weighted residuals for swirling flow issuing into an expansion of variable radius of divergence. He obtained analytical solutions to the flow under the assumption of quasi-cylindrical flow (i.e negligible derivatives in the axial direction.) He predicted that stagnation on the axis, i.e. the backflow zone initiation, occurs at a critical swirl which is large for small expansions and approaches zero at large expansions. The latter prediction is physically unreasonable and results from Bossel's use of the "quasi-cylindrical" assumption by neglecting the derivatives in the axial direction.

Beltagui and Maccallum [7] proposed a swirl number based on expansion diameter rather than the throat diameter for vane generated swirling flows issuing into a sudden expansion. They also derived a relationship between the swirl number under isothermal and burning conditions. As a result, the swirl number of a swirling flow under combustion could be predicted from the experimentally or analytically obtained isothermal swirl numbers.

Morton [23] used the momentum integral method to study the effect of swirl strength, angle of divergence and secondary mass flow on the strength and size of the recirculation zone inside a diffuser. The velocity profiles were described

by simple functions, which were then used to integrate the governing differential equations. His predictions showed that increasing the swirl intensity and the angle of divergence would lead to a steeper axial pressure gradient, hence a larger and stronger recirculation zone. Also, one could notice from his plots of axial velocity distribution at various swirl intensities and divergence angles that the onset of recirculation occurred at lower swirl intensities as the divergence angle increased. Using a similar method, Domkundwar et al. [24] used wall static pressure as an indicator of the position where the onset of recirculation occurs. His conclusions in regard to critical swirl were very similar to those of Morton [23].

Nakamura and Uchida [25] employed a polynomial distribution to represent the axial and radial velocities as functions of axial and radial locations. Fitting these expressions into a simplified axial equation of motion, they predicted a polynomial pressure distribution, while applying the radial equation of motion resulted in a different representation of pressure distribution. Assuming that the circulation remained constant along a streamline, they expressed the stream function as a function of the circulation and the angular velocity of the swirling flow. Equating this representation of the stream function with the one obtained from integrating the axial velocity profile, they obtained a critical angular velocity at which the backflow zone initiated. They also found that the critical state was reached when the difference between the two pressure profiles was minimal.

Hallett [26] used a simple momentum integral model to estimate the critical swirl intensity required to produce central recirculation in a swirling sudden expansion flow. Representing the tangential velocity by a Rankine vortex and the axial velocity by a linear relationship up to the solid body core boundary and a uniform value afterwards; he derived an explicit expression for the inlet critical

swirl needed to initiate downstream flow reversal. The derivation was carried out by applying conservation of mass and momentum in conjunction with an extra assumption requiring stagnation of the flow on the axis. The critical swirl was found to be a function of expansion ratio and inlet velocity profile shape.

A number of authors have used finite difference techniques to solve the governing differential equations, generally with the aid of the $\kappa - \epsilon$ model for turbulent exchange.

Kubo and Gouldin [27] developed a numerical procedure based on stream function and vorticity with two turbulence equations to study the presence of flow reversal for annular swirling jets issuing into a sudden expansion. Effects of introducing a secondary swirling jet through the hub, of the axial velocity ratio of the two swirling flows and of Reynolds number on the formation, size, and location of the recirculation zone were investigated. The critical swirl for which the backflow zone appears was obtained as a function of axial velocity ratios. It was found that increasing the axial velocity ratio caused an initial steep drop in critical swirl followed by an approach to an asymptote as the axial velocity ratio tended to infinity.

Rhode [10] used finite difference methods to numerically solve the governing equations in order to predict the effect of side wall angle, swirl intensity, expansion ratio and the turbulence intensity of the inlet flow on swirling jets issuing into an expansion. In regions close to the throat, due to the high turbulence intensity of the flow, the predictions did not match the experimental data. However, after two throat diameters, the agreement was quite good. The deficiencies in his prediction were also attributed to an inaccurate specification of velocity and turbulence distributions at the inlet to the test section.

Dixon et al. [13] used a numerical solution of the time mean equations governing conservation of mass and momentum, supplemented by a two-equation effective viscosity turbulence model, to predict the aerodynamic behaviour of an unconfined swirling jet emerging from an annular divergent nozzle. Maps of the backflow zone and the predicted velocity profiles were presented in conjunction with experimentally obtained values to examine the effect of varying the level of swirl on the central recirculation zone. The predictions adequately represented the measured quantities.

Rao et al. [14] developed a computer code to predict the three velocity components, utilizing the κ - ϵ turbulence model for a vane-generated swirl issuing into a sudden expansion. The model accurately predicted the axial velocity profiles and the occurrence of a backflow zone for all swirl strengths, while the tangential velocity predictions indicated large deviation from experimental results at high swirl intensities and regions near the throat. They attributed this deviation to the high turbulence level of the flow under such conditions.

Ramos and Somer [17] developed computer codes using two two-equation turbulence models ($\kappa - \epsilon$ and $\kappa - l$) to predict the flow behaviour for confined concentric swirling flows under both co- and counter-swirl conditions. These models use an isotropic eddy diffusivity and solve the equations for turbulence kinetic energy and the turbulence length scale. They predicted the existence of a backflow zone under both conditions. Their predictions were verified by LDA measurements under similar conditions.

Escudier and Keller [28] proposed a uniform axial velocity and Rankine vortex model for tangential velocity distribution to derive the expression for the critical state, in a cylindrical tube. According to their findings , the flow upstream of the

back flow zone is in a state where waves can not propagate upstream.

Escudier and Zehnder [11] proposed a criterion for vortex breakdown at a fixed location in the tube. They suggested that breakdown occurs when the local Reynolds number becomes proportional to the cube of the circulation number and the radial to tangential velocity ratios.

Escudier [29] proposed a model for the tangential velocity profile given by

$$w = \frac{\Gamma(1 - e^{-r^2/\delta^2})}{2\pi r} + \frac{\omega r}{2}$$

where w is the tangential velocity, Γ is the circulation, δ is a measure of radius of viscous core, ω is the angular velocity of the vortex, and r is the radial distance from the axis of flow. The first term of this equation was derived by Burgers [28], while the second part was introduced by Escudier to account for the effect of Taylor-Görtler vortices that form on the tube wall, mix up the fluid outside the core and result in a relatively uniform and stable vorticity. The Taylor-Görtler vortices are even more pronounced for jet-driven vortices.

2.3 Summary

As both experimental and analytical work of other researchers in swirling flow shows, the focus has mostly been on the effects of various upstream and downstream conditions on size and shape of the backflow zone. Although some authors have given estimates of critical swirl intensity at which the central recirculation zone initiates, these estimates had been obtained in the process of studying the size and shape of the recirculating bubble. The only actual measurements of critical swirl in a turbulent sudden expansion flow are those of Hallett and Toews [3]. The

other results for critical swirl are for gradually diverging tubes and for laminar or nearly laminar flow. Only two mathematical models for explicit prediction of the critical swirl in a sudden expansion are available, those of Hallett [25] and Bossel [22]. Because Bossel's model assumes quasi-cylindrical flow, his prediction that the critical swirl intensity approaches zero as the expansion ratio tends to infinity is unrealistic and the validity of his model is confined to small expansion ratios. Kubo and Gouldin [27] are the only authors to use a finite difference model to predict the critical swirl but their results have not been checked against experimental results. There is therefore a need for both experimental and theoretical work on the critical swirl in a sudden expansion with a central hub. The present investigation deals particularly with measurement and prediction of the critical swirl intensity as a function of velocity distributions at the throat for an annular swirling jet issuing into sudden expansion.

Chapter 3

Experimental Procedure

The purpose of the experiments was to measure the minimum (critical) swirl required to produce central recirculation in a sudden expansion as a function of expansion ratio and hub size. Two steps were required for this : Flow visualization was first used to establish the apparatus conditions required to produce the critical swirl, after which the velocity profiles at the expansion inlet were measured and integrated to obtain the critical swirl number.

3.1 Experimental Apparatus

A schematic diagram of the experimental apparatus is shown in Fig. (3.1). Since the objective is to study the behaviour of annular swirling jets issuing into sudden expansions, three different annular sections (hubs) are used in conjunction with four different expansion tubes. Each hub is made up of Plexiglas and is equipped with a built-in tube of 0.013 m inside diameter to deliver smoke for flow visualization and to simulate fuel injection in case of mixing studies. The hubs have

diameters of 0.025, 0.050 and 0.063 m, which correspond to $\frac{r_h}{r_1}$ ratios of 0.25, 0.50 and 0.63, where r_h is the hub radius and r_1 is the throat radius. The throat diameter is kept constant throughout the investigation at 0.101 m. It is also desired to obtain a flow without a centerbody, so that a comparison can be made. To achieve this, a special hub is used which extends only to the swirl generator. A sliding smoke lance is incorporated with this hub and is placed as far possible from the throat as the visualization resolution allows. Fig. (3.2) shows the schematic of these hub sections.

Four expansion sections of 0.125, 0.163, 0.200 and 0.300 m diameter, corresponding to expansion ratios $\frac{r_2}{r_1}$ of 1.25, 1.625, 2.0 and 3.0, are also used to study the effect of expansion ratio on critical swirl. These expansion sections are also made up of Plexiglas, so that flow visualization can be carried out.

The expansion section is connected to the throat by means of a pair of circular flanges on one side and to a circular duct leading to the blower on the other side by the same arrangement (Fig. (3.3)) Different sizes of flanges are designed to accommodate each expansion section.

A blower draws air through the test rig. Air enters through a pair of ASME standard nozzles which are used for flow metering. One of the nozzles delivers air to the axial air delivery tube, while the other one delivers it to the back of the box where it enters the swirl generator. Before entering the swirl generator, tangential air has to pass through a cloth screen where both dust and larger eddies of the turbulent flow are trapped, resulting in a cleaner and more uniform air delivery to the swirl generator. The mass flow rate of air delivered through the nozzles can be varied independently by using a pair of sliding gates. Each of the flowrates is monitored by a water manometer placed on a 7.7 degree slope. The total flow rate

can also be controlled by a slider gate at the entrance to the blower. The air is exhausted to the outside atmosphere by a long circular duct.

Minimizing leakage in aerodynamic studies is of great importance. Any leakage into the main box would result in a higher mass entry into the system than measured by the manometers, since the entire system is under suction. In the expansion section, any leakage would result in a higher radial velocity and complex effects on the axial and tangential velocity profiles. It would also lead to a possible air flow measurement error depending on the location of the leakage. To minimize this, every joint, flange connection and any other place that could cause a leakage is heavily sealed by silicone sealant and then by a thick layer of duct tape.

3.2 Swirl Generator

The heart of the experimental rig is the swirl generator. Methods of inducing swirl in a stream of fluid can be divided into three principal categories:

1. Tangential entry of air into a cylindrical duct.
2. Axial or radial vanes in axial tube flow.
3. Rotation of mechanical devices which impart swirling motion to the fluid passing through them (eg. rotating tubes)

Tangential entry and guide vane generators are extensively used in practice while rotating bodies have only been employed in academic experiments. Figures (3.4) to (3.6) show a schematic of each type of swirl generator.

In this investigation, a tangential plus axial swirl generator, a modification of the design of Hallett [30] , is used (Fig. (3.7)). The swirl generator allows both the swirl intensity and the shape of the axial velocity profile at the swirler exit to be varied independently. Swirl is produced by admitting air through four tangential inlets of length L . The angular momentum imparted to the flow can be controlled by means of a sliding sleeve which varies L . Each tangential port is equipped with a moveable partition attached to the sleeve and a bell-mouthed entry in order to produce uniformity of the inlet velocity. By admitting a secondary non-swirling stream of air to the system (hereafter referred to as "axial air") , the swirl intensity of the resulting flow can be reduced . By regulating the proportions of air introduced tangentially and axially as well as the length L both the swirl intensity and the shape of the axial velocity profile at the swirler exit can be varied [30] . In practice the degree of variation of profile shape was found to be slight, and L was held fixed at 100 mm.

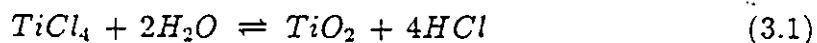
3.3 Flow Visualization

In order to ascertain the flow conditions necessary to initiate the central toroidal recirculation zone formation, flow visualization is used. The flow condition is given by the swirl intensity (i.e the proper ratio of tangential to axial air flow) for a given sliding sleeve position, L .

After unsuccessful attempts to visualize the flow with tufts and paper "flags", it was decided to use smoke introduced through the central hub to make the flow visible. Two different methods of smoke generation were tried in an attempt to maximize resolution and minimize maintenance. At first, smoke obtained from

burning wood chips was used. Figure (3.8) shows a schematic of the apparatus. The smoke was accompanied by a large amount of tar which was deposited in tubing and on the expansion section wall, causing clogging of the tubes and obstructing visualization of the flow. To minimize these problems, the incoming smoke was condensed at two different stations, leading to partial extraction of tar from the smoke. The first condenser consisted of a circular container equipped with water cooled spiral tube. The smoke entered the container from one end, passed through the water cooler unit and then exited. At the second station, tar was further condensed by allowing the smoke to impact on the surface of water in an Erlenmeyer flask. However, the tar level still remained high. Filtration using a column of charcoal was tried, but resulted in a great reduction in the visibility of the smoke. Owing to these problems, it was decided to abandon this method.

Much better success was achieved using smoke from titanium tetrachloride, $TiCl_4$ [31]. Flow visualization was carried out by means of the fumes given off by liquid titanium tetrachloride when exposed to moist air. $TiCl_4$ is a light yellow liquid with a pungent acid smell. The liquid develops dense, white TiO_2 fumes when brought in contact with moist air as a consequence of the following reaction:



The apparatus used to produce the fume is shown in Fig. (3.9). In order to delay the formation of the fume until it reaches the expansion section, the incoming air had to be dried by passing it through a column of dessicant. The fume was delivered to the expansion section by means of the smoke lance in the centerbody. This method proved to be very effective. $TiCl_4$ is non-flammable and non-explosive, but reacts strongly with water. It is very corrosive and toxic owing to the formation of hydrochloric acid. Its vapours should not be inhaled and safety goggles

should be used to avoid any possible contact with eyes.

To enhance the contrast in the expansion section, the rear interior of the plexiglass tube was painted black and the rear exterior covered by black construction paper. A box covered by black curtains was placed in front of the test section in order to minimize surface reflection due to exterior lights in the laboratory. To further enhance the contrast, two high-powered photoflood lamps were used to illuminate the test section through a narrow opening provided on the top of the expansion section. This arrangement gave a thin "sheet" of light, so that a cross section of the flow was obtained (fig. (3.10)).

To determine the required mass ratio of tangential to axial air required to initiate flow reversal at a fixed sleeve setting, the tangential gate was left fully open and the axial gate was gradually opened until the transition regime was reached. The reverse of this procedure was also followed by leaving the axial gate partially open and by gradually increasing the tangential flow until transition was reached. The reverse procedure was applied to check for hysteresis in the flow. This transition zone is characterized by an alternation of recirculating and non-recirculating flow, the latter usually being a periodically precessing flow.

3.4 Probe Measurement

3.4.1 Probe Measurement Apparatus

Once the flow condition for the onset of vortex breakdown is determined by flow visualization, a five-hole Pitot probe is inserted into the flow to measure the three velocity components and the local static pressure. The probe diameter is approx-

imately 0.005 m. Figure (3.11) shows a schematic of the probe. Two pressure transducers of 10 and 100 Torr capacities are used in conjunction with a valve board and a chart recorder in order to measure these pressures. Fig. (3.12) shows the set up for the valve board and the transducers [32].

The probe is provided with a protractor which reads the pitch angle, ι , of the flow and is fastened to a holder by means of two clamping screws. The holder is fixed onto a traversing slide by means of a support of adjustable height. The slide is used to measure the radial location of the probe head in the throat area. A schematic of the probe, slide and probe insertion in the throat area is given in Fig. (3.13).

The probe is operated in what is referred to as the null mode : the probe is rotated about its shaft until $(P_3 - P_2) = 0$, at which point the pitch angle is read off the protractor on the probe shaft. The remaining three pressures, namely $(P_1 - P_2)$, $(P_4 - P_5)$ and $(P_1 - P_w)$, where P_1 , P_2 , P_3 , P_4 , P_5 and P_w refer to the pressures at the probe holes 1, 2, 3, 4, 5 and the wall mounted pressure tap, located on the throat wall about 0.05 m from the expansion, are then read and combined to give the yaw angle, χ , of the flow which is related to the pressure coefficient, B , by means of a calibration curve. B is defined as :

$$B = \frac{P_4 - P_5}{P_1 - P_2}$$

The dynamic and total pressures are then calculated using two further coefficients

$$COEFV = \frac{P_t - P_s}{P_1 - P_2}$$

$$COFTP = \frac{P_1 - P_t}{P_t - P_s}$$

These equations are curve fit to calibration curves for the probe. The calibration curves and the method of obtaining them are discussed in the following section.

3.4.2 Probe Calibration

The probe is calibrated by a non-swirling jet of known characteristics, issuing from an ASME nozzle. The calibration procedure was developed by McGrath [32]. Fig. (3.14) shows the set up of the calibration apparatus. The probe tip is placed at the center of the potential core of the jet. To calibrate the probe, it is introduced into the jet at different yaw angles, χ , and the required pressure differences are measured. The yaw angle is defined as the angle between the normal to the nozzle centreline and the probe shaft. The measured pressures were $(P_4 - P_5)$, $(P_1 - P_2)$, $(P_1 - P_t)$ and $(P_t - P_{atm})$. The total pressure of the jet, P_t , is the nozzle back pressure in the calibration unit, and P_{atm} represents the barometric pressure which is also the static pressure in the jet. The probe was calibrated at both high and low nozzle velocities (13 and 41 m/s) to check for a Reynolds number dependence of the calibration, but none was found.

After the measurements were completed, the data were plotted. The three plots are presented in figures (3.15) to (3.17).

- $(P_4 - P_5)/(P_1 - P_2)$ vs. yaw angle, so that the yaw angle can be predicted.
- $(P_t - P_{atm})/(P_1 - P_2)$ vs yaw angle, so that the velocity pressure coefficient, $COEFV$, can be predicted from the yaw angle.
- $(P_1 - P_t)/(P_t - P_s)$ vs yaw angle, so that the total pressure coefficient, $COFTP$

, can be predicted from the yaw angle.

Each of these curves is then fitted by a ninth degree polynomial:

$$\chi = \begin{cases} \frac{(B+0.166)}{0.0326} & \text{if } B \leq 0.1779 \\ \frac{(B+0.300)}{0.0453} & \text{if } B > 0.1779 \end{cases} \quad (3.2)$$

$$\begin{aligned} COEFV = & 0.96 + (2.207 \times 10^{-3}\chi) - (8.027 \times 10^{-5}\chi^2) \\ & + (3.418 \times 10^{-6}\chi^3) + (2.407 \times 10^{-7}\chi^4) - (3.136 \times 10^{-9}\chi^5) \\ & - (1.169 \times 10^{-10}\chi^6) + (1.494 \times 10^{-14}\chi^7) + (2.875 \times 10^{-14}\chi^8) \\ & + (4.755 \times 10^{-16}\chi^9) \end{aligned} \quad (3.3)$$

$$\begin{aligned} COFTP = & -(1.000 \times 10^{-3}\chi) - (2.577 \times 10^{-4}\chi^2) - (4.026 \times 10^{-6}\chi^3) \\ & - (4.129 \times 10^{-7}\chi^4) + (8.540 \times 10^{-9}\chi^5) + (2.800 \times 10^{-10}\chi^6) \\ & - (6.683 \times 10^{-12}\chi^7) - (6.802 \times 10^{-14}\chi^8) + (1.926 \times 10^{-15}\chi^9) \end{aligned} \quad (3.4)$$

3.4.3 Probe Measurement Technique

The following steps are followed in order to proceed with flow measurements at the throat:

1. the atmospheric temperature and pressure and the readings of the two manometers connected to the flow metering nozzles are recorded.

2. After the probe is inserted into the flow, the opening in the flange is completely sealed and the whole apparatus is checked thoroughly for any possible leakage.
3. The probe is nulled (aligned with the flow) by rotating the probe until the pressure difference ($P_2 - P_3$) is zero. Once the probe is aligned , the angle of rotation (pitch angle) , is read from the attached protractor with an accuracy of ± 0.5 degrees. To set the zero angle on the protractor, the probe is nulled in a non-swirling flow (pure axial air flow.)
4. Once the probe is aligned, ($P_4 - P_5$), ($P_1 - P_2$) and ($P_1 - P_w$) are obtained by changing the valve positions on the valve board. P_w is the wall pressure just upstream of the throat and is used as a reference pressure. It is measured by a pressure tap inserted in the wall of the test section, about 2.0 cm from the throat.
5. Measurements are taken radially, starting from the hub surface and proceeding toward the throat wall in 2.0 mm increments.
6. One measurement of ($P_1 - P_{atm}$) is taken at one of the measurement stations halfway between the hub and the throat surfaces. so that the wall pressure can be calculated.
7. The traversing slide reading at each measurement point is recorded and compared with the reading at the center of the throat cross section. The slide's scale could be read with a vernier to within 0.05 mm.

3.5 Data Reduction Program

To obtain the three velocities and the static pressure from the five hole probe measurements, a computer code was developed. The code had been originally developed by McGrath [32], but was modified and converted from FORTRAN to BASIC for the present investigation.

In order to initiate the program, the following data are stored in a data file:

- Test section diameter in inches.
- Hub diameter in inches.
- Swirl generator slider opening in cm.
- Room temperature in °C.
- Barometric pressure in Torr.
- Tangential and axial manometer readings in cm. H_2O .
- Number of measurement points.
- $(P_1 - P_{atm})$ in Torr.
- Measurement of $(P_1 - P_2)$, $(P_4 - P_5)$, $(P_1 - P_w)$, pitch angle and slide reading at each measurement point.

After converting the above data into SI units, the yaw angle, static pressure and dynamic pressure were calculated using the equations in section 3.4.1

- The wall pressure, P_w , is calculated from the measured $(P_1 - P_w)$ at the location where $(P_1 - P_{atm})$ is taken, using the following equation:

$$P_w = (P_1 - P_{atm}) + (P_1 - P_w) \quad (3.5)$$

- The local density of air, ρ , is calculated from the ideal gas law.
- the total velocity, V_t , is calculated in the following manner:
 $(P_t - P_s)$ is equal to the dynamic pressure, so that

$$(P_t - P_s) = \frac{1}{2} \rho V_t^2 \quad (3.6)$$

therefore

$$V_t = \sqrt{\frac{2 COEFV (P_1 - P_2)}{\rho}} \quad (3.7)$$

- The axial velocity u , the tangential velocity w , and the radial velocity, v , are determined using the measured pitch angle and yaw angle:

$$u = V_t \cos(\chi) \cos(\iota) \quad (3.8)$$

$$v = V_t \sin(\chi) \quad (3.9)$$

$$w = V_t \cos(\chi) \sin(\iota) \quad (3.10)$$

- To have an estimate of the total mass entering the system, and also the portion of the mass that has to be swirled to initiate the vortex breakdown, the tangential and axial mass flow rates are calculated from the two manometer readings, using the following equations :

$$M_t = 7.836 \times 10^{-3} \sqrt{(P_{atm} - \frac{P_{tan}}{133.32}) \frac{P_{tan}}{T}} \quad (3.11)$$

$$M_a = 1.9056 \times 10^{-3} \sqrt{(P_{atm} - \frac{P_{ax}}{133.32}) \frac{P_{ax}}{T}} \quad (3.12)$$

where M_t and M_a correspond to the mass flow rate through the tangential and axial gates, and P_{tan} and P_{ax} are calculated in KPa from

$$P_{tan} = (98.1 \sin(\gamma) + 0.4807) LT$$

$$P_{ax} = (98.1 \sin(\gamma) + 0.4807) LA$$

where γ is the inclined manometer's angle with the horizontal and LT and LA correspond to the tangential and axial manometer readings. The total mass flow rate ($M_t + M_a$) is then calculated and is compared with that integrated from probe measurements as an estimate of probe error.

3.6 Integral Quantities

In addition to the mentioned quantities, a number of integral quantities were calculated by numerical integration of the measured velocity profiles using the trapezoid rule, so that a better understanding of the flow could be achieved:

- Mass flow rate at the throat,

$$M_{th} = 2\pi \int_{r_h}^{r_1} \rho u r dr \quad (3.13)$$

- Volumetric flow rate at the throat, Q

$$Q = 2\pi \int_{r_h}^{r_1} u r dr \quad (3.14)$$

- Reference axial velocity, U_{ref}

$$U_{ref} = \frac{Q}{\pi(r_1^2 - r_h^2)} \quad (3.15)$$

- The Reynolds Number based on the forward flow, Re

$$Re = \frac{2U_{ref}(r_1 - r_h)}{\nu} \quad (3.16)$$

- The axial flux of axial momentum, Υ

$$\Upsilon = 2\pi \int_{r_h}^{r_1} \rho u^2 r dr \quad (3.17)$$

and in non-dimensional form

$$\Upsilon_D = \frac{\Upsilon}{M_{th} U_{ref}} \quad (3.18)$$

The mass flow M_{th} integrated from the axial velocity profile is used in calculating dimensionless quantities rather than the mass flow determined from the flow meters in order to roughly cancel out any probe error in measured quantities such as Υ .

- The axial flux of tangential momentum, Ω

$$\Omega = 2\pi \int_{r_h}^{r_1} \rho u w r^2 dr \quad (3.19)$$

and in non dimensional form, Ω_D

$$\Omega_D = \frac{\Omega}{M_{th} U_{ref} r_1} \quad (3.20)$$

- *Swirl Number*, S was calculated from the definition given in equation (1).

$$S = \frac{\Omega}{r_1 \Upsilon} \quad (3.21)$$

The swirl number and Ω_D are two alternative parameters for characterizing the swirl intensity. The swirl number is the more widely used of the two. However, the dimensionless angular momentum flux has been used in this thesis, as it is easier to compare to the results of the theory to be developed

later. The two are very similar in value. Since any departure from a uniform axial velocity will cause Υ to be greater than $M_{th} U_{ref}$, the swirl number, S , is generally less than the dimensionless axial flux of angular momentum.

- The static pressure drop in the radial direction, $(P_w - P_s)$, can be theoretically estimated by a simplified form of the radial momentum equation (see chapter 4)

$$(P_s - P_w) = \int_{r_h}^r \rho \frac{w^2}{r} dr \quad (3.22)$$

- The angular momentum flux stream function, Ω_{func}

$$\Omega_{func} = \frac{2\pi \int_{r_h}^r \rho u w r^2 dr}{\Omega} \quad (3.23)$$

Each integral was calculated numerically using the Trapezoid rule. The data reduction program is presented in Appendix (A).

3.7 Probe Measurement Errors

Multi-hole probes are frequently employed for turbulent flow measurements. Such probes measure the total velocity vector's magnitude as well as its direction. Pressure and velocity gradients, flow confinement, proximity of flow boundaries, turbulence intensities and flow perturbation are among the sources of error related to the five hole probe measurement [36]. Little information is available about the effect of turbulence on the five hole probe, but the effect is known to be significant at high turbulence intensities and it is expected to result in higher measured velocities, with an error proportional to the square of turbulence intensity [37]. In

practice, although turbulence is known to influence the probe response, the effect is frequently ignored [36].

The effect of shear on a five hole probe is significant in regions of high velocity gradient and can be expected to result in an error for the direction of the total velocity vector [37]. At regions of high velocity gradient adjacent to the hub and throat surface, the probe would yield axial velocities that are high and tangential velocities that are slightly low [37].

In turbulent flow, a probe is subject to varying yaw and pitch angles and varying velocity and static pressure magnitudes. The maximum yaw and pitch angles to which the five hole probe operates are limited (± 40 degrees) and consequently experimental problems arise when the flow direction is highly variable.

Flow disturbances by the probe are another source of error in experimental measurements using five hole probes. Well-known examples of flow perturbation are probe blockage in confined flows, flow distortion when measuring in the boundary layers, generation of secondary flows, tripping of flow instabilities and local flow changes. Occurrence of each of these perturbations is subject to probe geometry, location in the flow field, the apparatus configuration and flow conditions [36]. Gouldin [36] notes for swirling flow the potential for secondary flow along the probe support toward the vortex core when a probe is introduced perpendicular to the vortex axis.

The interference of the probe in swirling flows can be avoided if the probe dimension is much less than the thickness of the vortex core [36]. Since in turbulent flow, the vortex core is found to be much larger than in laminar flow, a perturbing probe in laminar flow may be small enough to be non-perturbing in turbulent

flow [36]. In addition, the axial vortex filaments which comprise the vortex core are usually undisturbed upstream of the vortex breakdown in laminar flow. The introduction of the probe locally disturbs these filaments and the downstream propagation of such disturbances causes a significant change in the flow far away from the probe [36]. But in a fully developed turbulent flow, such filaments are already highly distorted and the insertion of the probe adds no significant new distortions [36]. The evidence indicates that turbulent flows with low or moderate swirl levels appear much less susceptible to probe perturbations than laminar swirling flows [36].

With all the probe measurements made in turbulent swirling flow, there have been very few references to significant disturbances induced by the probe. Comparisons between velocity measurements made with probes and laser Doppler velocimeters showed differences as would be expected due to the large number of possible errors associated with probe techniques, but the differences do not suggest significant probe distortions [36]. Measurements of Rhode [10] using a dummy probe of the same size and shape as the one used in the present research, proved the disturbance effect to be negligible, while Mahmud et al. [37] suggest that the perturbation effect is less than 10% .

The overall error in these measurements was estimated by comparing the mass flows measured by the manometers (equations (3.11) and (3.12)) with those calculated from the axial velocity profiles (equation (3.13)). This error reached a maximum value of about 6.5% too high. This is thought to be mainly due to the effects of turbulence on the measured pressures [36]. Hallett and Toews [3] reported a 7% error while Rhode [10] found it to be about 5%.

No experiment to determine the extent of flow disturbance by the probe was

conducted. But measurements of Rhode [10], using a dummy probe of the same size and shape, proved the disturbance effect to be negligible. Also, the pressure transducers were regularly calibrated to ensure their accuracy.

The manometers had a $\pm 1\%$ fluctuation in their readings due to turbulence, which in turn suggests a $\pm 0.5\%$ error in measuring the mass flow (the mass flow rate is proportional to the square root of the pressure read from the manometers, and therefore the error in measuring the mass flow is one half of that incurred during reading manometers.) In addition, flow visualization was most erroneous when the largest hubs or the smallest test section were being used. This was due to the large wake associated with the larger hubs and the fast dissipation of the smoke in smaller test sections. Assuming that all the velocity components are in error in the same proportions as the probe measured mass flow, one could deduce that this error would cancel out if dimensionless quantities were used to represent the flow. To non-dimensionalize, U_{ref} based on the integrated mass flow rather than on the metered mass flow was used.

Examining figure (3.16); one would expect a symmetrical behaviour, with respect to the null angle, from the probe. The manufacturer's provided calibration curves are also very similar to the present calibration curves. Therefore, one may not include such behaviour as part of the overall probe measurement error. The unsymmetrical behaviour may only be attributed to the fact that the pressure holes at the tip of the probe might have not been bored to absolute perfection.

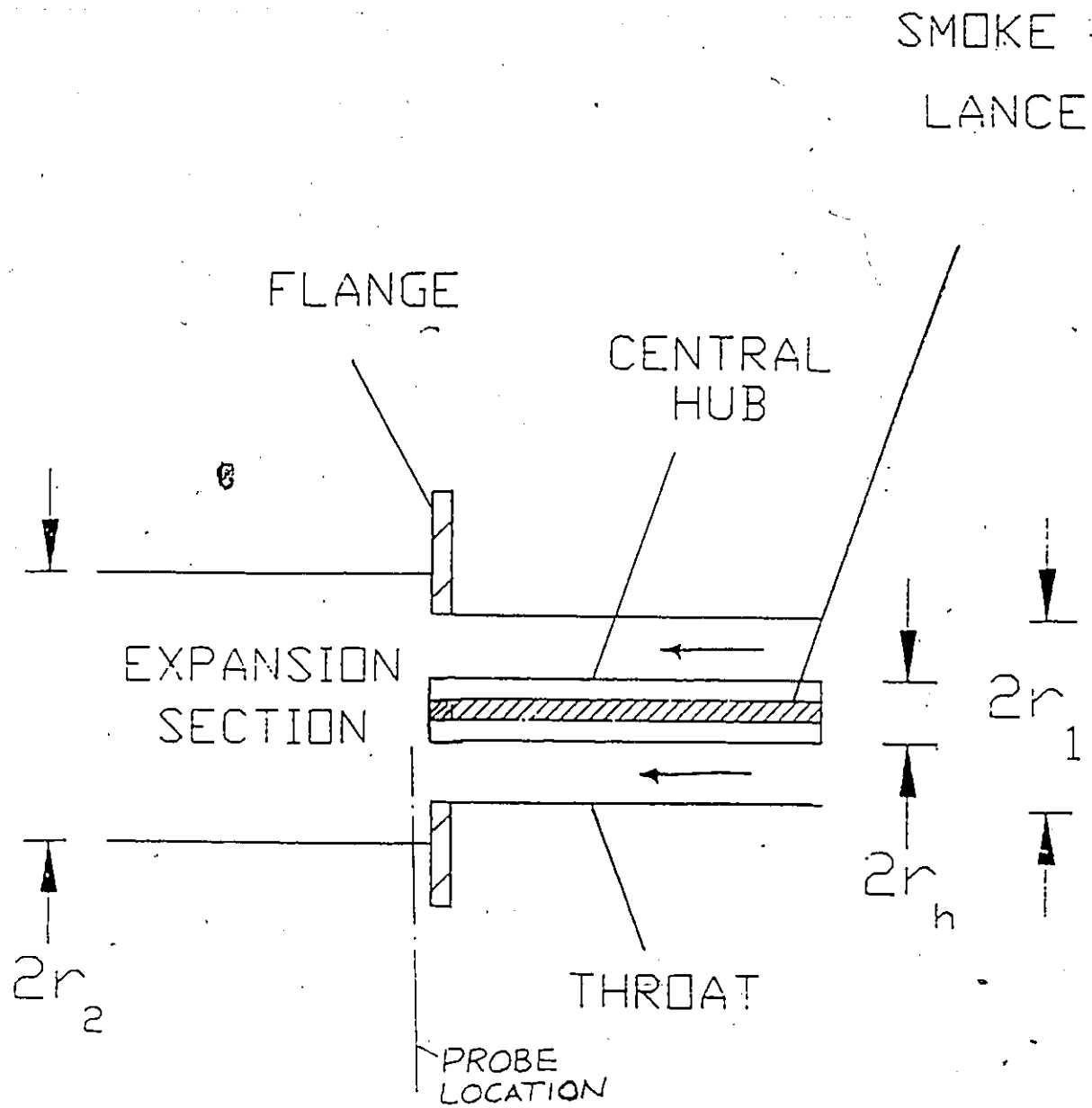
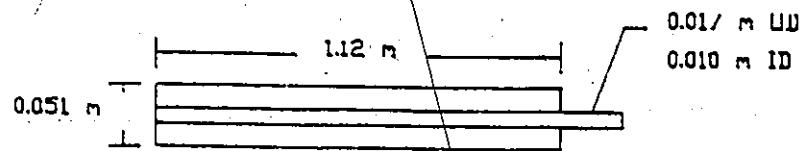
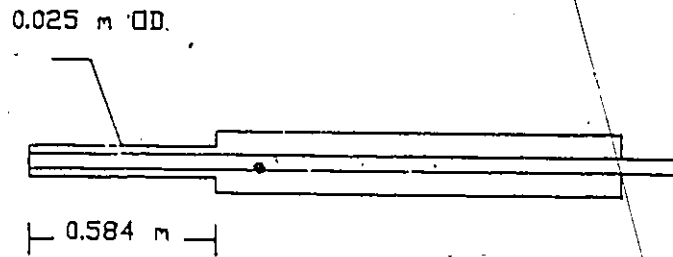


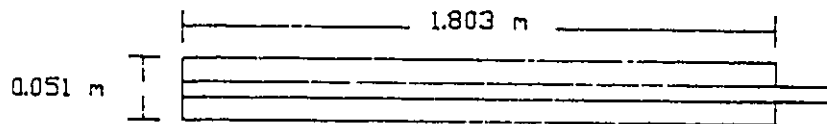
Figure 3.1: Expansion, hub and throat arrangement



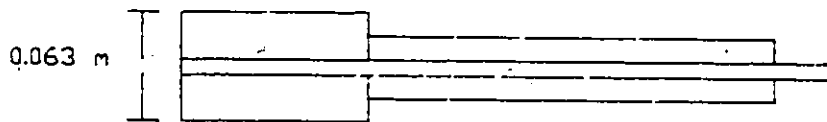
Extension For No Hub



0.025 m OD Hub



0.051 m OD Hub



0.063 m OD Hub

Figure 3.2: Annular hubs

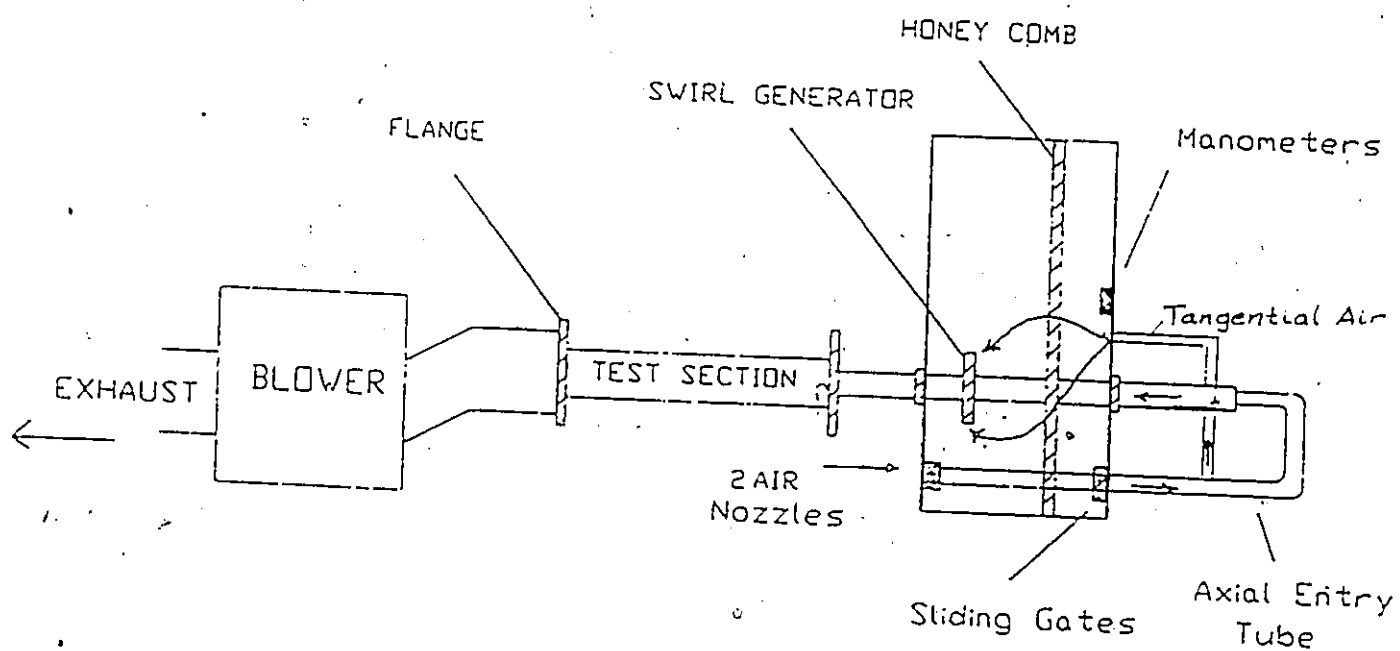


Figure 3.3: Experimental apparatus

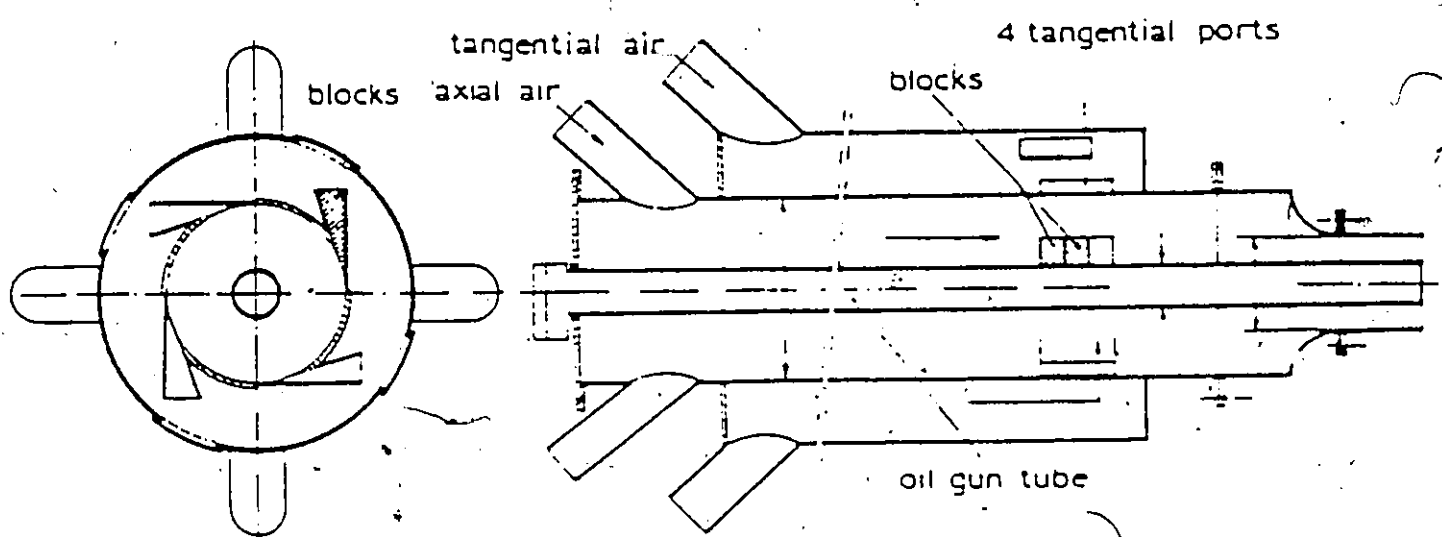


Figure 3.4: Tangential entry swirl generator [1]

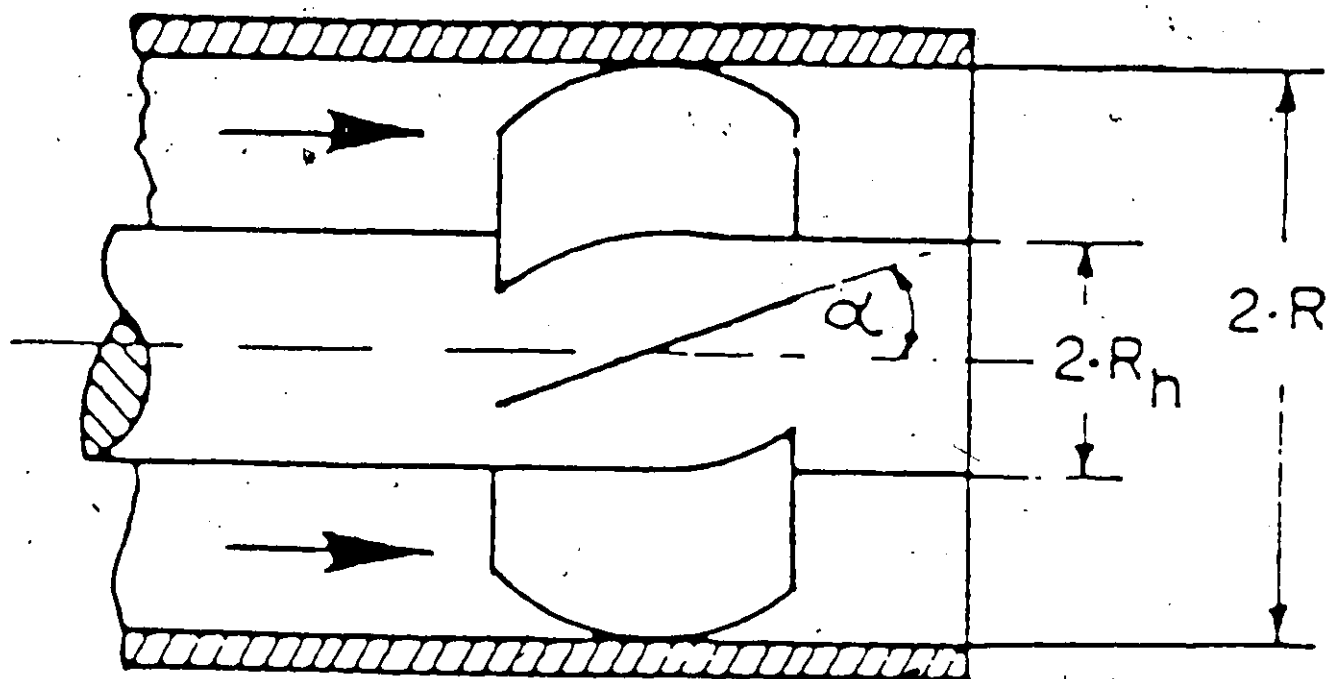


Figure 3.5: Vane guided swirl generator [1]

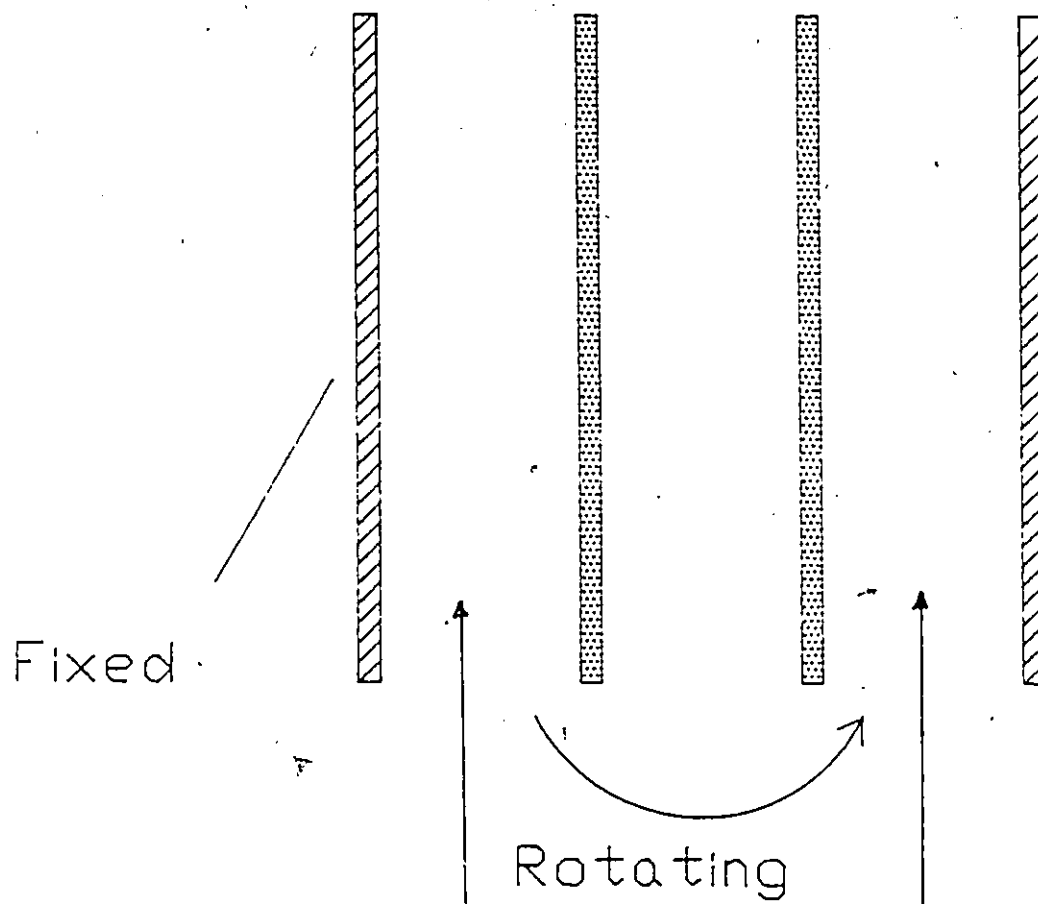


Figure 3.6: Rotating body swirl generator

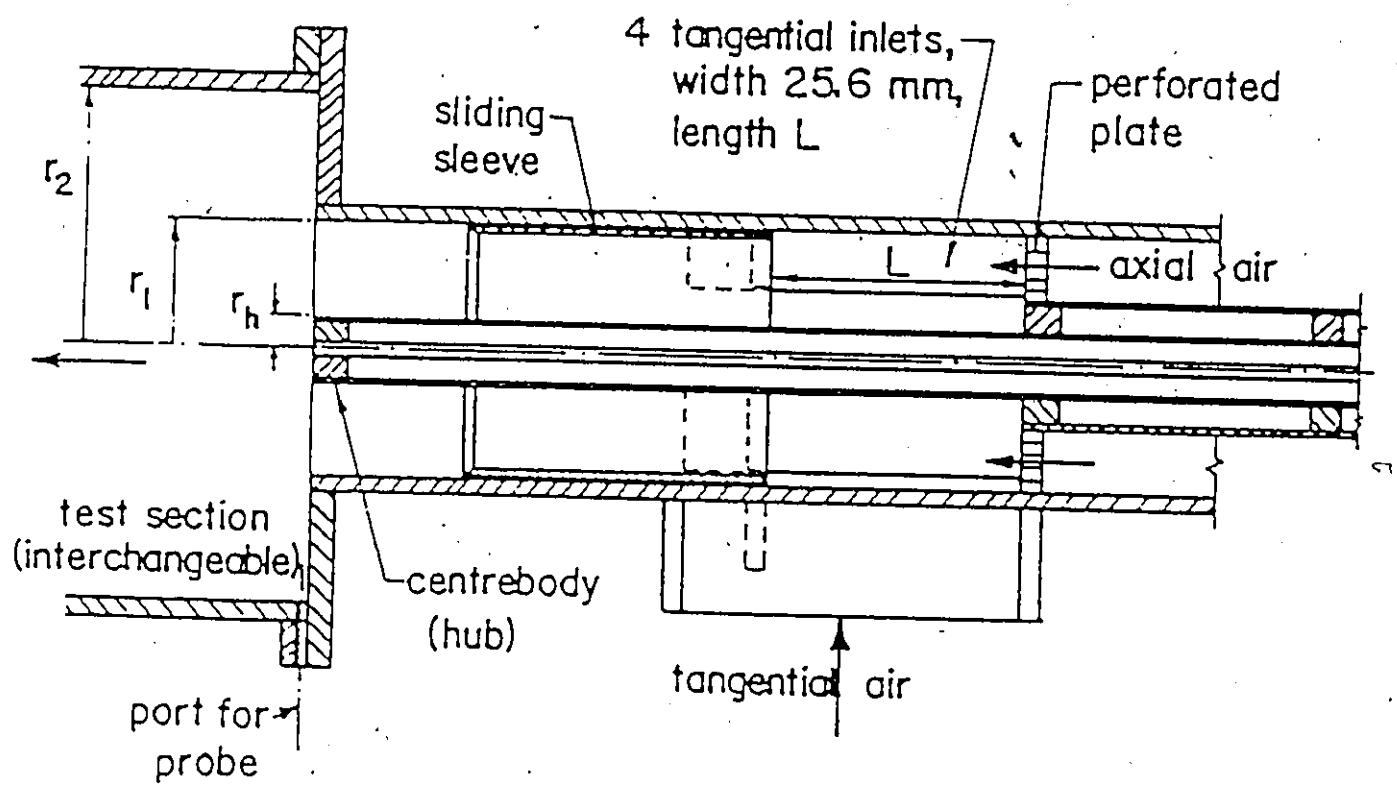


Figure 3.7: Present swirl generator

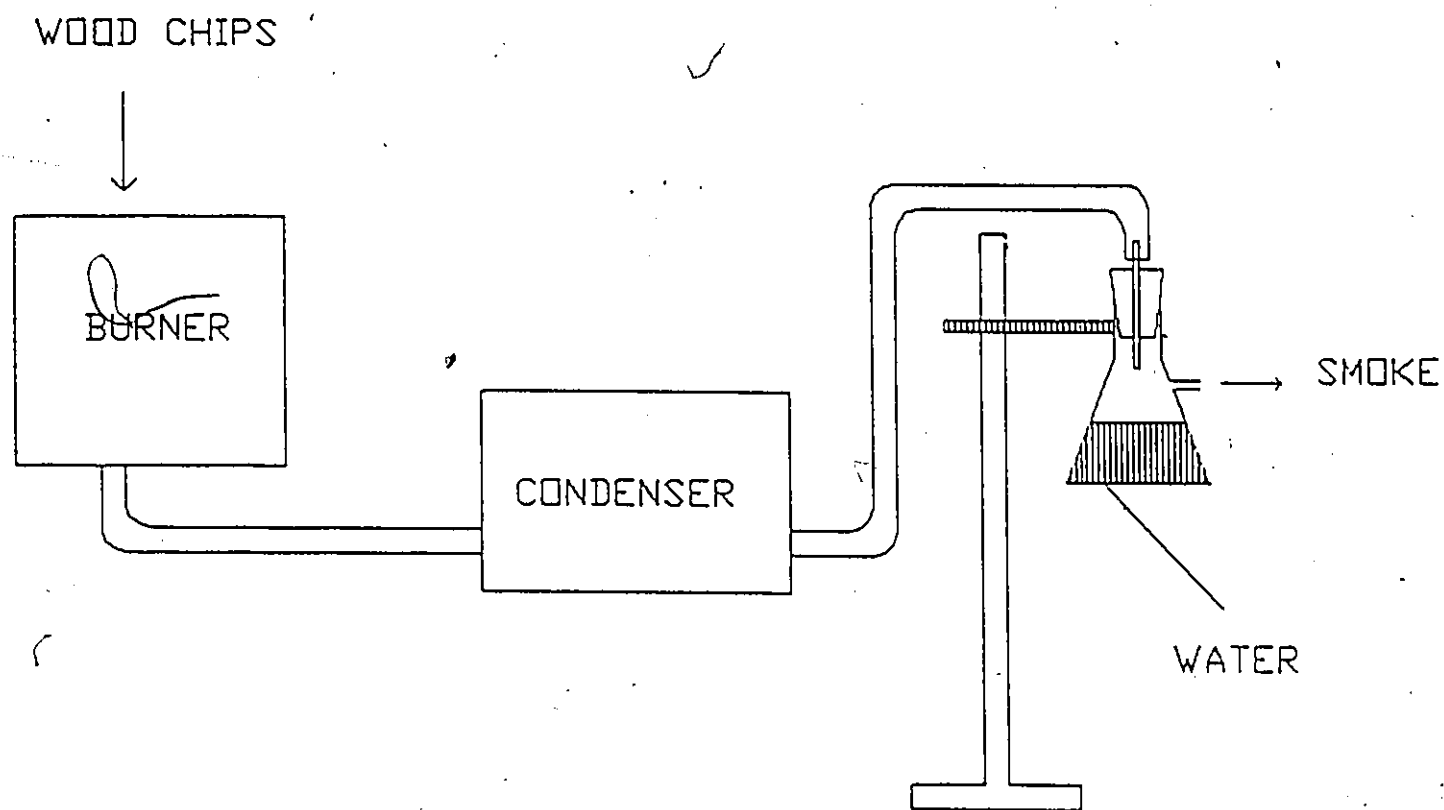


Figure 3.S: Smoke generation System

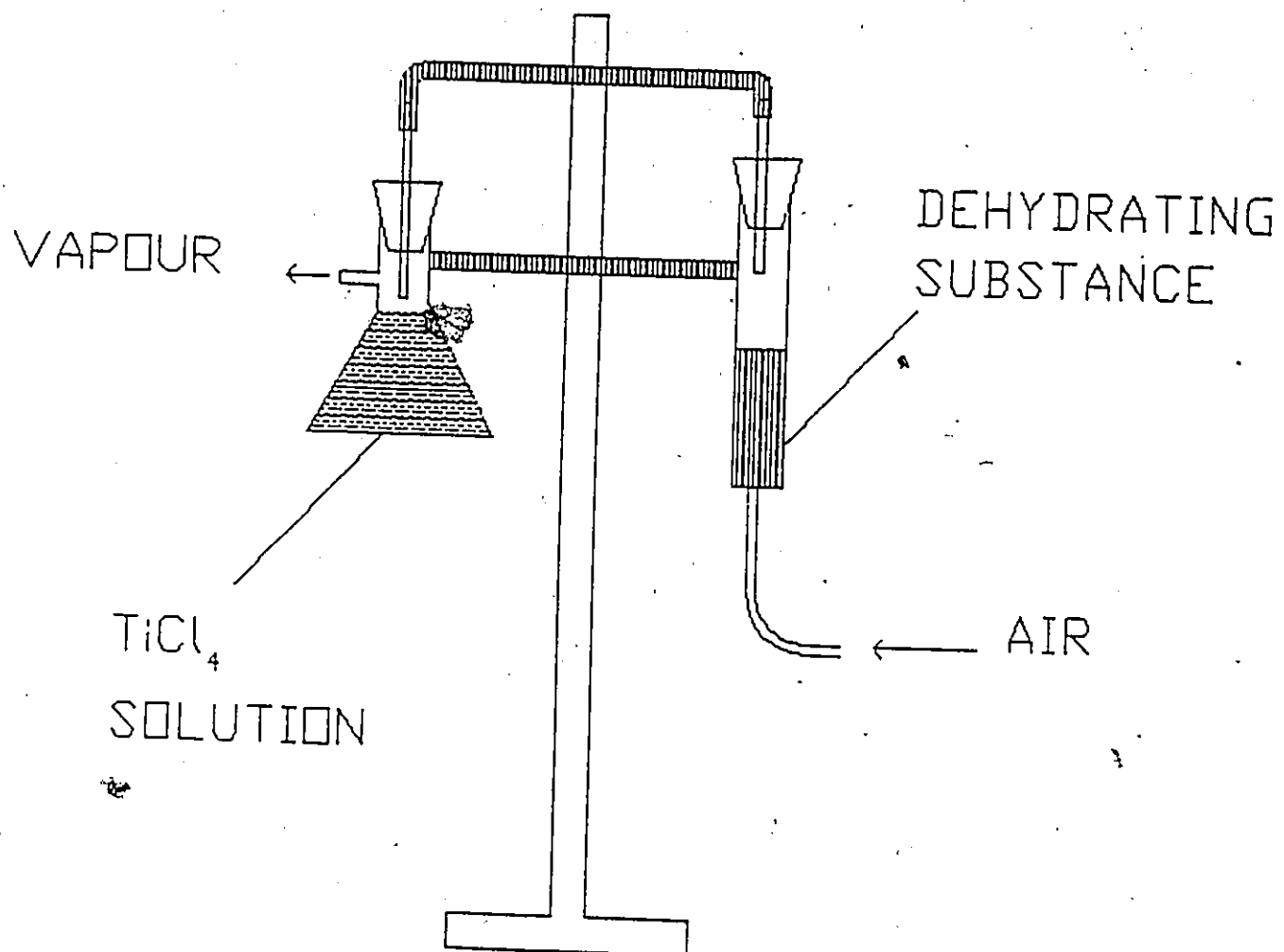


Figure 3.9: TiCl_4 fume generation unit

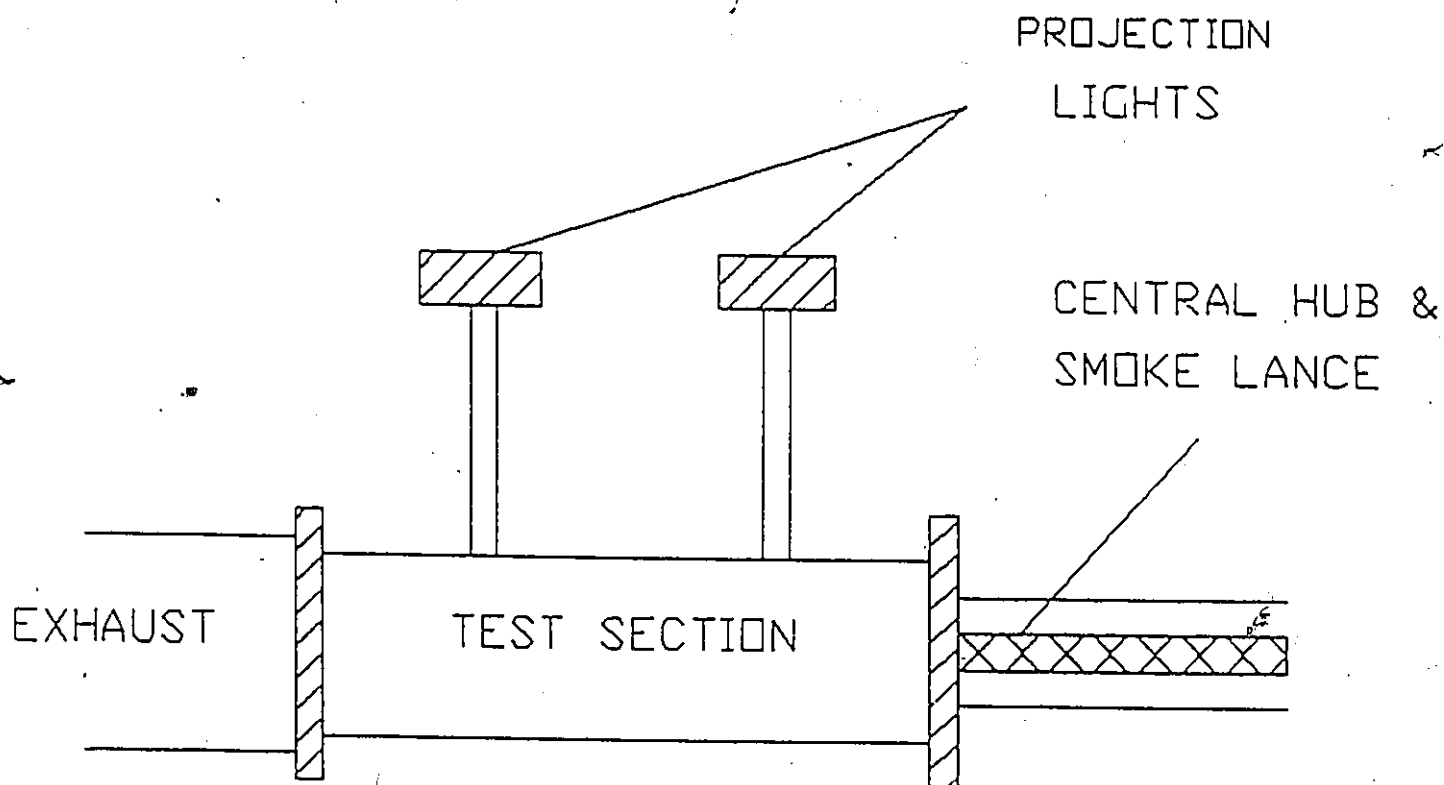


Figure 3.10: Lighting arrangement

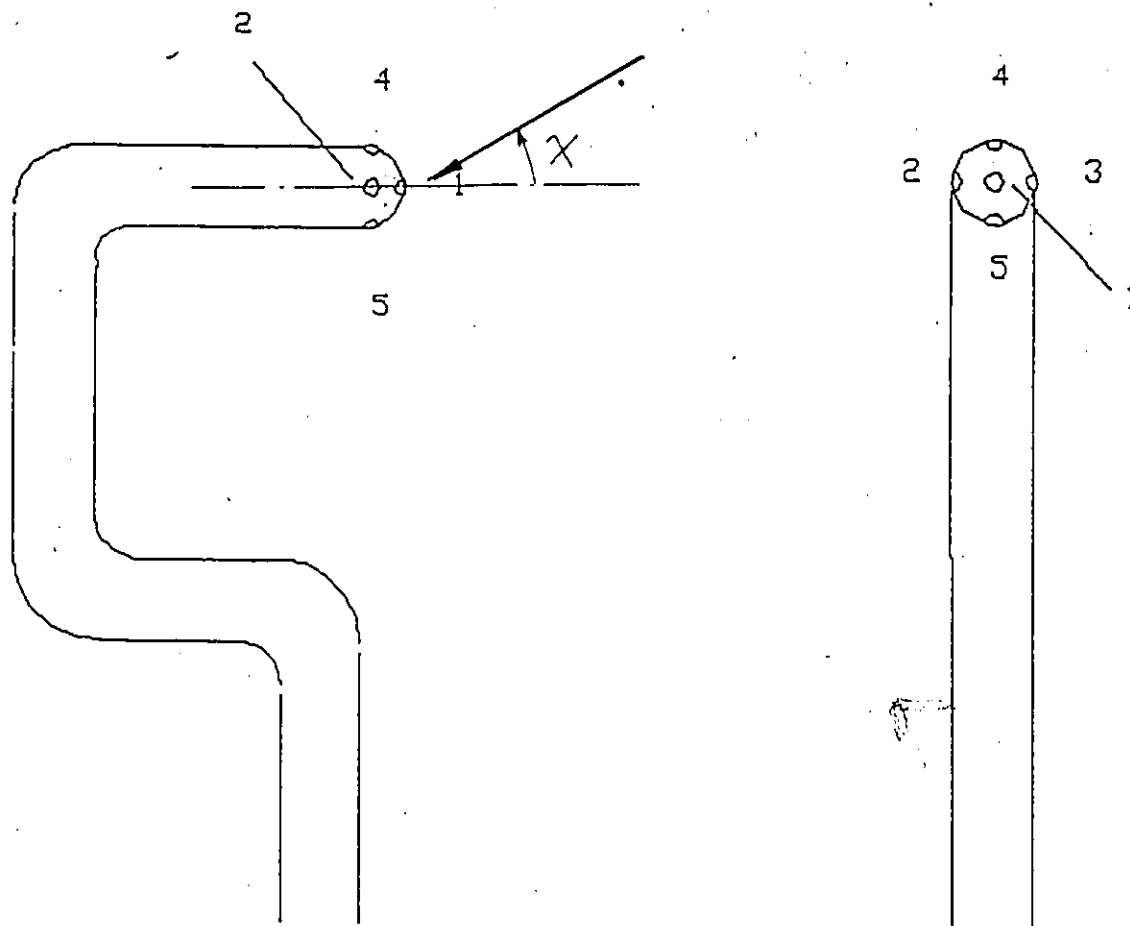


Figure 3.11: Five hole pitot probe

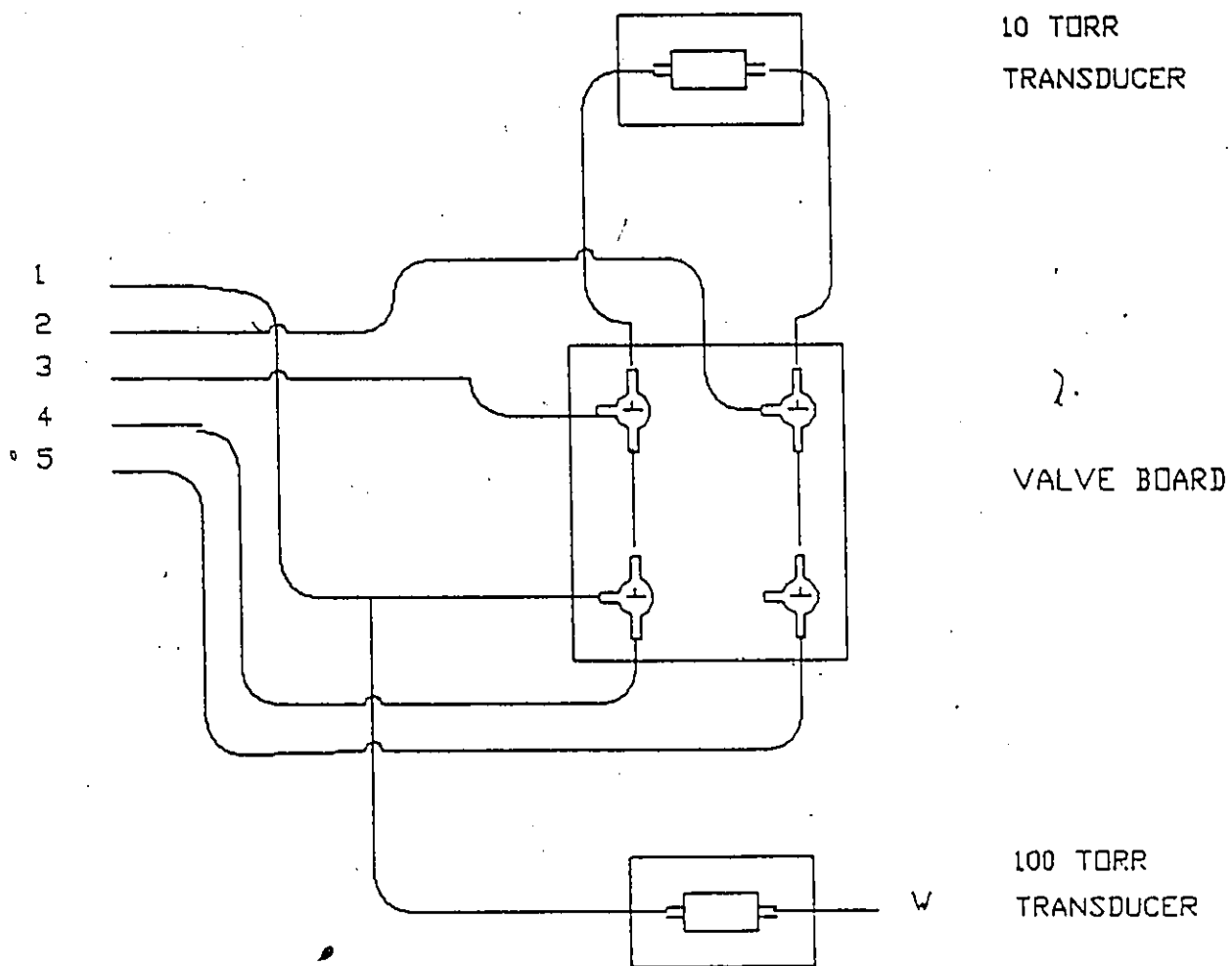


Figure 3.12: Pressure transducers and valve board

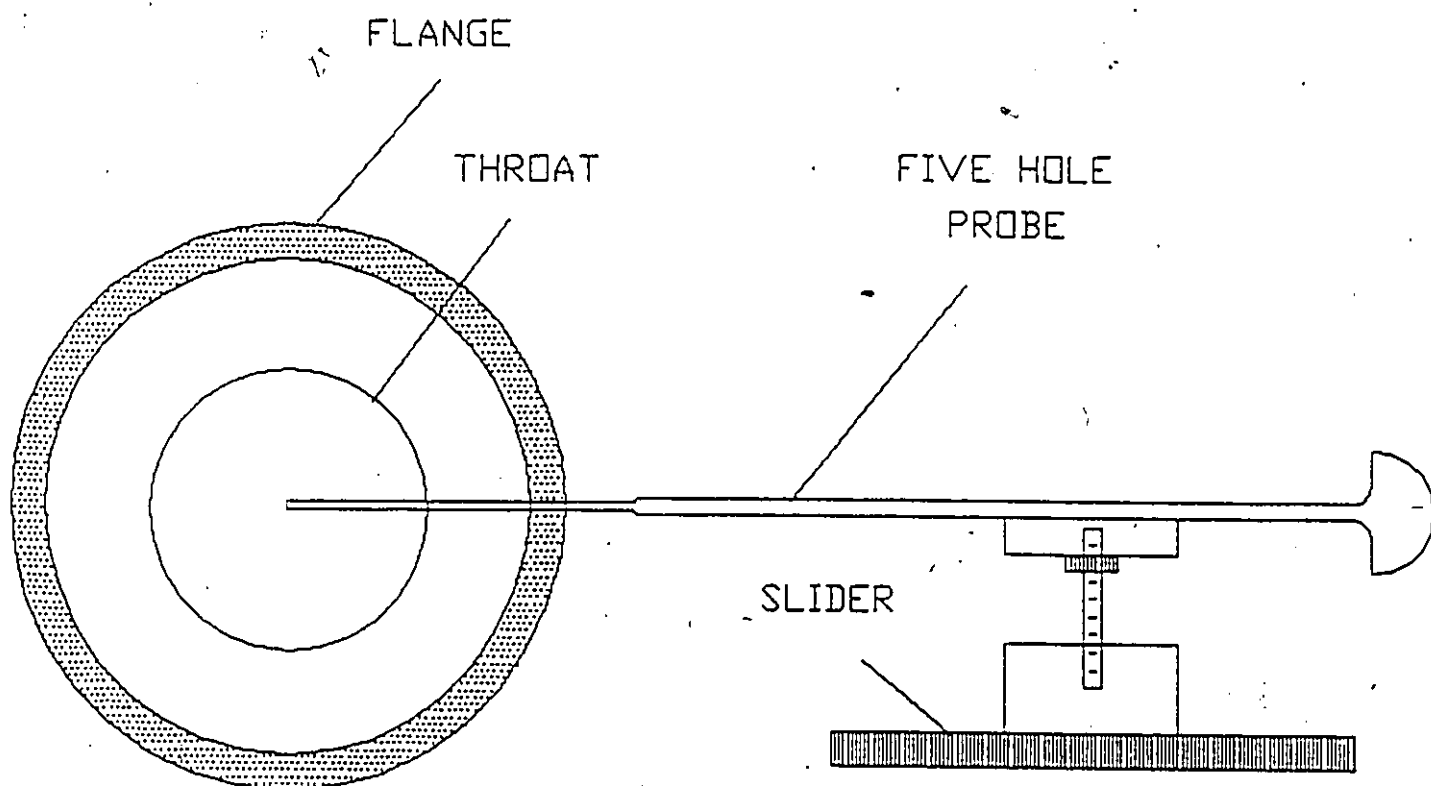


Figure 3.13: Traversing slide, probe and holder arrangement

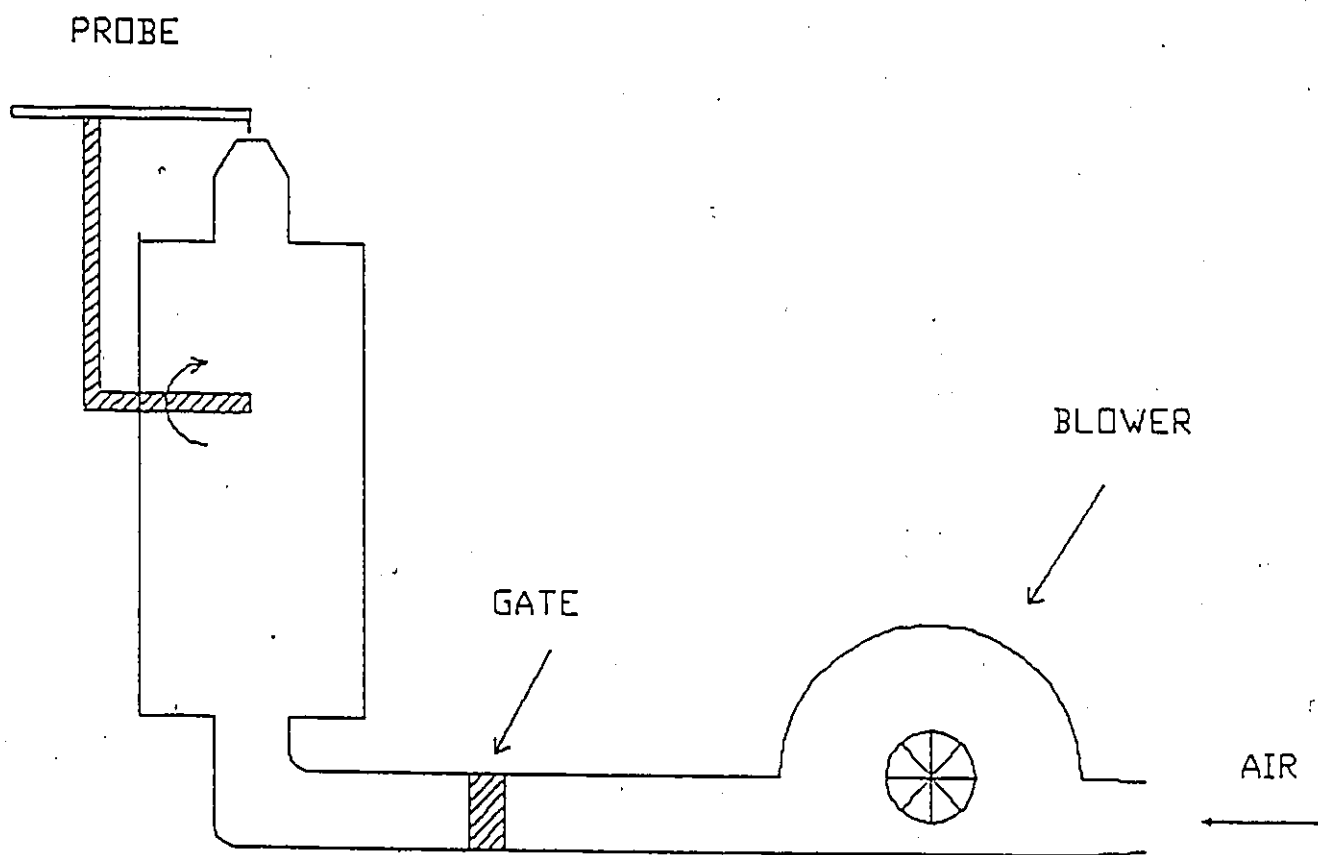


Figure 3.14: Calibration unit

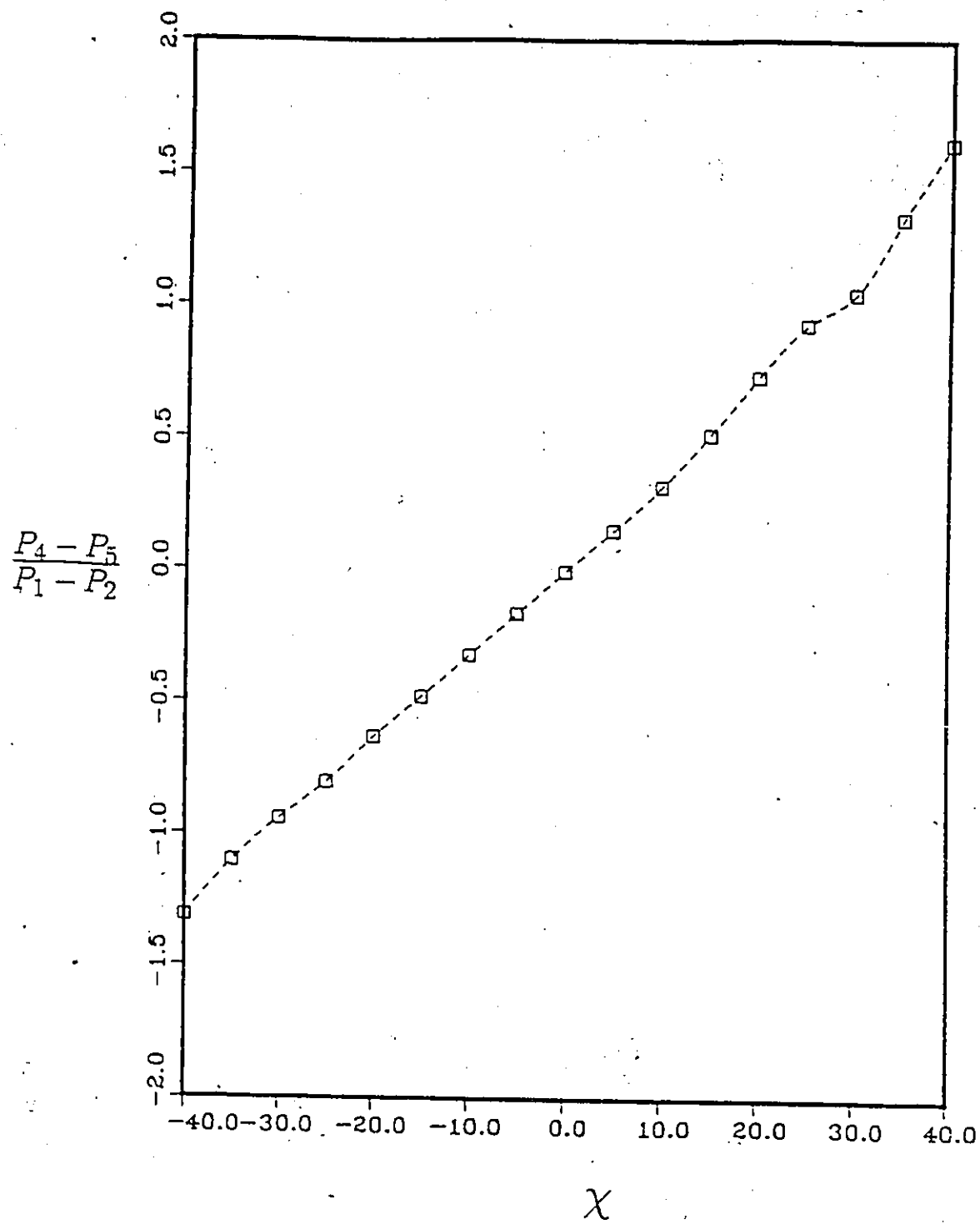


Figure 3.15: $\frac{P_4 - P_5}{P_1 - P_2}$ vs. χ

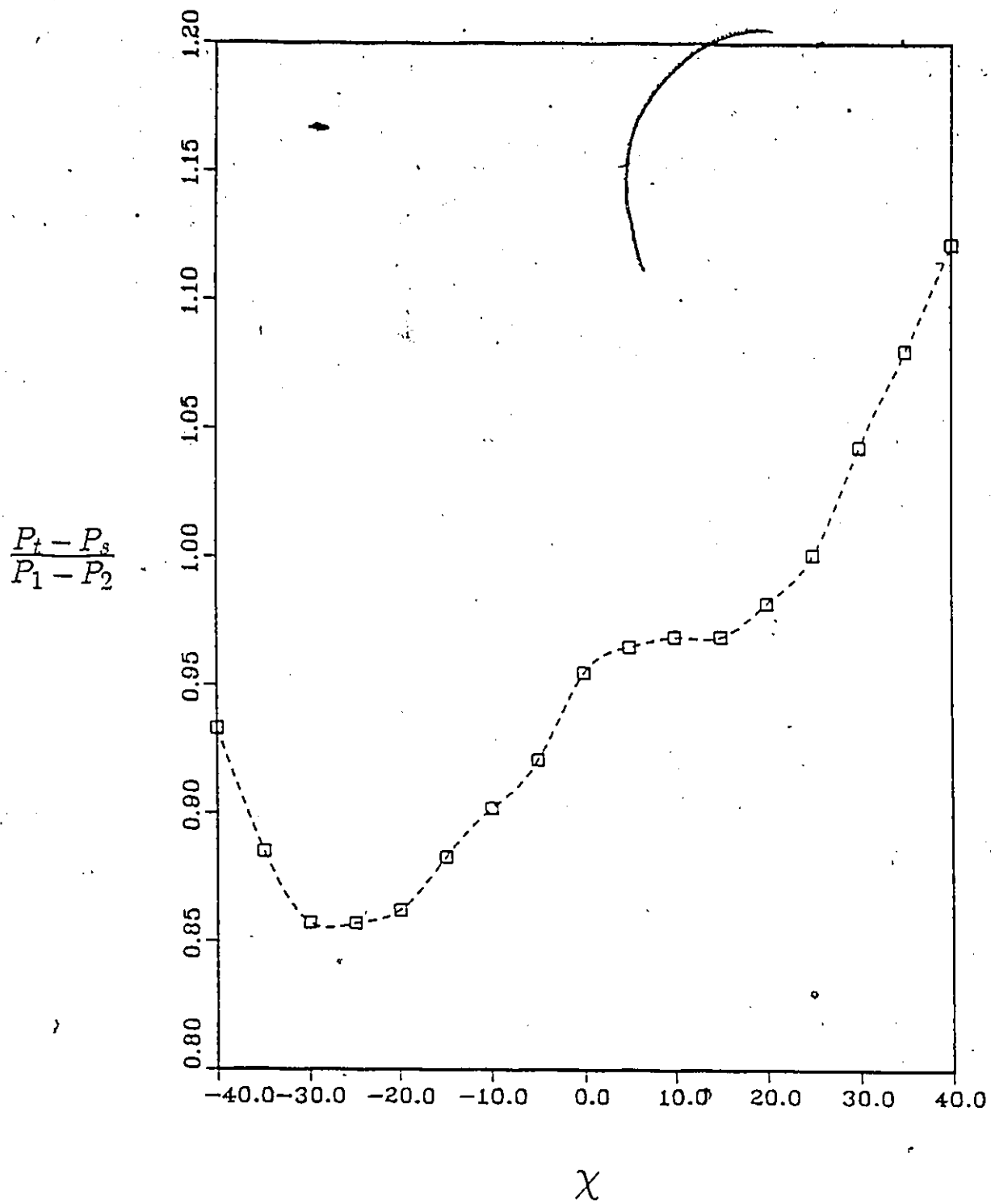


Figure 3.16: $\frac{P_t - P_s}{P_1 - P_2}$ vs. χ

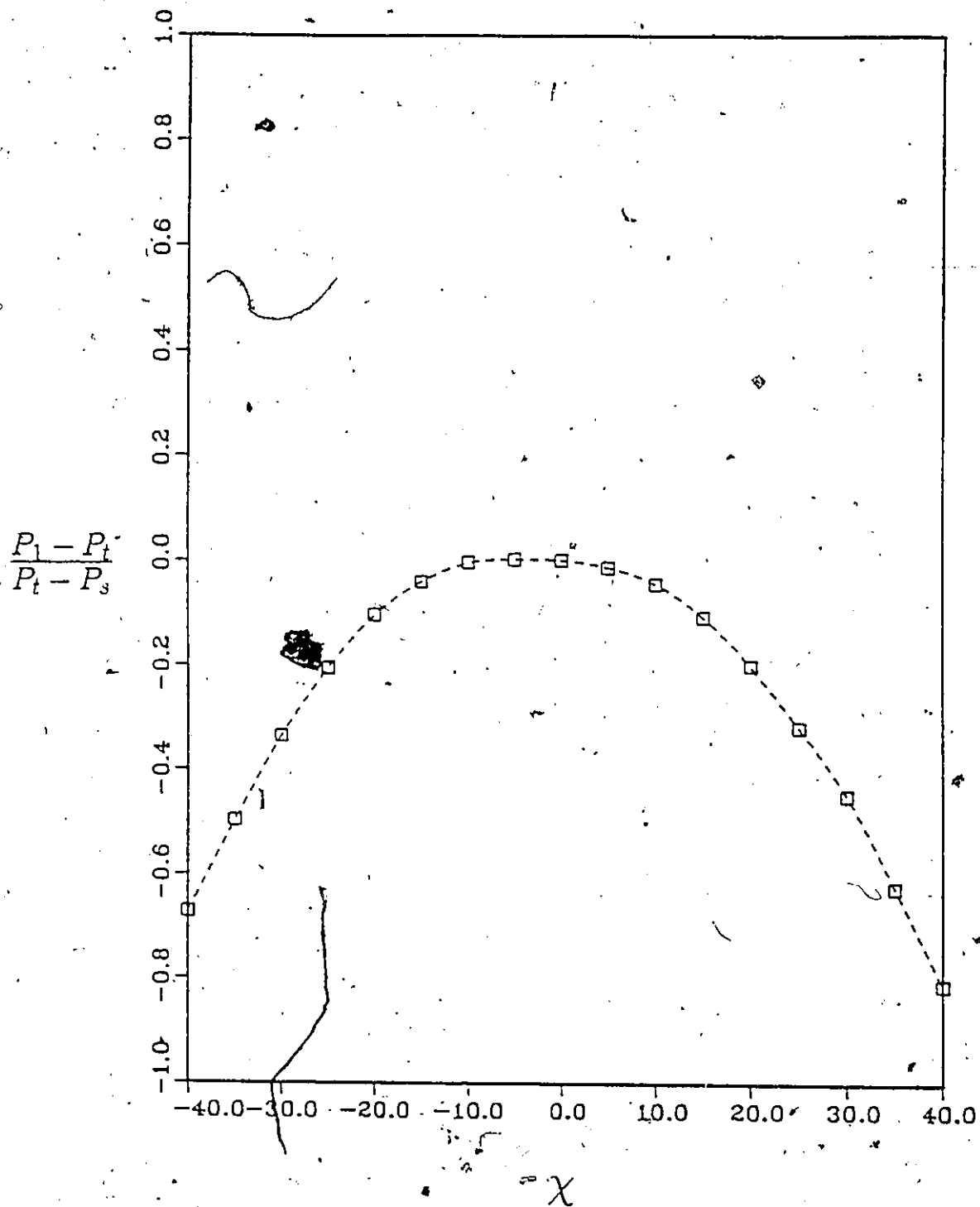


Figure.3.17: $\frac{P_1 - P_t}{P_t - P_s}$ vs. χ

Chapter 4

Development Of A Mathematical Model To Predict Critical Swirl Intensity

In this chapter a mathematical model based on the momentum integral method is developed to predict the critical dimensionless angular momentum flux of an annular swirling jet issuing into a sudden expansion. The word critical refers to the minimum swirl needed to initiate flow reversal on the axis. The model assumes simple forms for the velocity profiles at the throat and at the incipient recirculation zone, allowing the equations of motion to be integrated. The resulting algebraic equations, together with an expression for the stagnation of the flow on the axis, are then solved for the critical swirl intensity. To test the model against the experimental data, a procedure is developed for fitting the chosen velocity profile shapes to the data.

4.1 Derivation Of The Governing Equations,

The governing equations, namely the integral equations of conservation of mass and tangential and axial momenta, are directly derived from the Navier-Stokes equations in cylindrical coordinates. These equations are :

r - component

$$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} + \frac{w}{r} \frac{\partial v}{\partial \theta} - \frac{w^2}{r} + u \frac{\partial v}{\partial x} \right) = -\frac{\partial p}{\partial r} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(rv)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial w}{\partial \theta} \frac{\partial^2 v}{\partial x^2} \right] + \rho g_r \quad (4.1)$$

θ - component

$$\rho \left(\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial r} + \frac{w}{r} \frac{\partial w}{\partial \theta} + \frac{vw}{r} + u \frac{\partial w}{\partial x} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(rw)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial x^2} \right] + \rho g_\theta \quad (4.2)$$

x - component

$$\rho \left(\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial r} + \frac{w}{r} \frac{\partial u}{\partial \theta} + u \frac{\partial u}{\partial x} \right) = -\frac{\partial p}{\partial x} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial x^2} \right] + \rho g_x \quad (4.3)$$

The continuity equation is also defined as:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v) + \frac{1}{r} \frac{\partial (\rho w)}{\partial \theta} + \frac{\partial (\rho u)}{\partial x} = 0 \quad (4.4)$$

where g_x , g_θ and g_r are gravitational effects in the three directions.

The derivation is carried out under the following assumptions :

- Flow is assumed to be incompressible and inviscid.
- Flow is assumed to be axisymmetric, with changes to the properties occurring only in axial and radial directions.
- Air is assumed to behave as a perfect gas.
- Flow is assumed to be steady and isothermal.
- Gravitational effect is neglected.

Applying the above assumptions to the above equations simplifies them to :

$$v \frac{\partial v}{\partial r} - \frac{w^2}{r} + u \frac{\partial v}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \quad (4.5)$$

$$v \frac{\partial w}{\partial r} + \frac{vw}{r} + u \frac{\partial w}{\partial x} = 0 \quad (4.6)$$

$$v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (4.7)$$

$$\frac{v}{r} + \frac{\partial v}{\partial r} + \frac{\partial u}{\partial x} = 0 \quad (4.8)$$

Multiplying equation (4.8) by r and taking into account that

$$\frac{\partial r}{\partial x} = 0 \quad (4.9)$$

we have

$$r \frac{\partial u}{\partial x} = r \frac{\partial u}{\partial x} + u \frac{\partial r}{\partial x} = \frac{\partial(ur)}{\partial x} \quad (4.10)$$

$$r \frac{\partial v}{\partial r} + v = r \frac{\partial v}{\partial r} + v \frac{\partial r}{\partial r} = \frac{\partial(vr)}{\partial r} \quad (4.11)$$

Therefore, equation (4.8) simplifies to

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial r} = 0 \quad (4.12)$$

Multiplying equation (4.12) by u and equation (4.7) by r , adding the two equations and simplifying we get :

$$\frac{\partial(u^2r)}{\partial x} + \frac{\partial(ruv)}{\partial r} = -\frac{r}{\rho} \frac{\partial p}{\partial x} \quad (4.13)$$

Multiplying equation (4.8) by v , adding it to equation (4.5) and simplifying results in

$$\frac{\partial(uv)}{\partial x} + \frac{\partial v^2}{\partial r} + \frac{v^2}{r} - \frac{w^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \quad (4.14)$$

Similarly multiplying equation (4.8) by rw^2 and equation (4.6) by r^2 , adding them and simplifying results in :

$$\frac{\partial}{\partial x}(r^2uw) + \frac{\partial}{\partial r}(r^2vw) + 2rvw = 0 \quad (4.15)$$

Equations (4.12), (4.13) and (4.15) are now integrated over the cross section of a cylindrical tube of radius R . The boundary conditions are

$$u = v = w = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial w}{\partial x} = 0 \quad (4.16)$$

Substituting this into equation (4.12) results in :

$$\frac{\partial(vr)}{\partial r} = 0 \implies \frac{\partial v}{\partial r} = 0 \quad (4.17)$$

Equation (4.12) on integration becomes

$$\int_0^R \frac{\partial(ur)}{\partial x} dr + \int_0^R \frac{\partial(vr)}{\partial r} dr = 0 \quad (4.18)$$

Integrating and applying Leibniz' rule

$$\frac{d}{dx} \int_0^R ur dr = 0 \quad (4.19)$$

i.e. the volume flow rate is constant.

Equation (4.13) on integration becomes a condition for preservation of the "thrust" (axial momentum flux plus the pressure force) :

$$\frac{d}{dx} \left[\int_0^R \rho u^2 r dr + \int_0^R p r dr \right] = 0 \quad (4.20)$$

Since the flow measurements are taken at the inlet to the expansion, the swirling jet has not had a chance to expand yet and the radial velocities should be very small. Such an assumption is supported by the radial velocity profiles measured in the present experiments and those reported by Mathur and Maccallum [5] and Rao et al. [14] . Therefore the integrated equation (4.15) requires conservation of angular momentum flux.

$$\frac{d}{dx} \left(\int_0^R \rho u w r^2 dr \right) = 0 \quad (4.21)$$

Finally, if the radial velocity is small, equation (4.14) can be approximated by

$$\frac{w^2}{r} \approx \frac{1}{\rho} \frac{\partial p}{\partial r} \quad (4.22)$$

which is a balance between radial pressure gradient and centrifugal force. The following are the equations of motion of the swirling flow used in our modelling :

$$\int_0^R \rho u^2 r dr + \int_0^R p r dr = \text{const.} \quad (4.23)$$

$$\int_0^R \rho u w r^2 dr = \text{const.} \quad (4.24)$$

4.2 Prediction Of The Critical Dimensionless Axial Flux Of Angular Momentum

A momentum integral model for prediction of the critical dimensionless angular momentum flux is presented in this section and an explicit equation defining

$(\Omega_D)_{crit}$ is derived. The model is an extension of one developed for hubless flow by Hallett [26]. First, generalized equations representing the inlet tangential and axial velocity profiles and the downstream profiles at the point of commencement of vortex breakdown are proposed, and then by using conservation of mass and momenta in conjunction with the assumption of stagnation on the axis of the flow $(\Omega_D)_{crit}$ is obtained.

The inlet tangential velocity of the swirling jet is represented by a Rankine vortex having a core radius of a_1 . Measurements of Escudier and Keller [12] for $\frac{r_h}{r_1} = 0.5$ and those of Beltagui and Maccallum [7] suggest the existence of a two equation profile inside the forced vortex region for low and medium swirl. As a result, a three piece modified Rankine vortex for the tangential velocity distribution is proposed as :

$$\left\{ \begin{array}{ll} w = w_h \frac{r}{r_h} & \text{for } 0 \leq r < r_h \\ w = w_h + (w_a - w_h) \frac{r - r_h}{a_1 - r_h} & \text{for } r_h \leq r \leq a_1 \\ w = \frac{\Gamma}{r} & \text{for } a_1 \leq r \leq r_2 \end{array} \right\} \quad (4.25)$$

where w_h is the tangential velocity at the edge of the hub and w_a is the tangential velocity at the solid body core boundary defined as

$$w_a = \frac{\Gamma}{a_1} \quad (4.26)$$

Therefore, a straight line connecting the origin to the neighborhood of the first measurement point and another straight line stretching from the first measurement point to the point of maximum tangential velocity would sufficiently define the tangential velocity inside the solid body core. The geometry is shown in figure (4.1). A straight line stretching from the origin to the point of maximum tangential

velocity would have the following equation

$$w' = \frac{w_a}{a_1} r = \frac{\Gamma}{a_1^2} r \quad (4.27)$$

while such a line at the surface of the hub would give a value of

$$w'_h = \frac{\Gamma}{a_1^2} r_h \quad (4.28)$$

w_h can be defined in terms of w'_h by introducing a coefficient, α

$$w_h = \alpha w'_h = \frac{\alpha \Gamma}{a_1^2} r_h \quad (4.29)$$

Once α is calculated ($\alpha = \frac{w_h}{w'_h}$) from figure (4.1), w_h could be predicted. Therefore, equation (4.25) can be restated in terms of α :

$$\left\{ \begin{array}{ll} w = \frac{\alpha \Gamma}{a_1^2} r & \text{for } 0 \leq r < r_h \\ w = \frac{\Gamma}{a_1^2} \left[\alpha r_h + \left(\frac{a_1 - \alpha r_h}{a_1 - r_h} \right) (r - r_h) \right] & \text{for } r_h \leq r \leq a_1 \\ w = \frac{\Gamma}{r} & \text{for } a_1 \leq r \leq r_2 \end{array} \right\} \quad (4.30)$$

The axial velocity is approximated by a linear relationship suggested by inspection of the velocity profiles measured in the present experiments.

$$\left\{ \begin{array}{ll} u = 0 & \text{for } 0 \leq r \leq r_h \\ u = u_1 \left\{ \zeta - (\zeta - 1) \frac{r}{r_1} \right\} & \text{for } r_h \leq r \leq r_1 \\ u = 0 & \text{for } r_1 \leq r < r_2 \end{array} \right\} \quad (4.31)$$

where u_1 is the axial velocity at the neighbourhood of the hub surface, r_1 is the throat radius, r_2 is the expansion radius and ζ is a fitting parameter. Figure (4.1) schematically represents the velocity profiles.

It should be noted that the tangential velocity profile stretches all the way to the expansion wall, while the axial velocity profile ends at the throat boundary.

Experimental results of Beltagui and Maccallum [7], Hallett and Günther [19] and Yoon and Lilley [16] show that swirl is present for $r > r_1$ due to its upstream transport by the corner recirculation caused by the sudden expansion of the flow. Meanwhile, the same investigations reported a negligible axial velocity for $r > r_1$. For the same reason, the tangential velocity profile extends past the hub to the axis as well (see So et al. [20] .)

The following profiles are proposed for the downstream velocities :

$$\left\{ \begin{array}{ll} w = w_2(\frac{r}{a_2}) & \text{for } 0 \leq r \leq a_2 \\ w = w_2 & \text{for } a_2 \leq r \leq r_2 \\ u = u_2(\frac{r}{r_2})^2 & \text{for } 0 \leq r \leq r_2 \end{array} \right\} \quad (4.32)$$

where a_2 is the solid body core radius. These profiles are intended to represent the state of the downstream flow at the critical swirl, when stagnation of the flow on the axis has just been achieved, and are identical to those used by Hallett [26] for a hubless flow. No measurements exist of a flow at this state, but data of Beltagui and Maccallum [7], Rao et al. [14] and Hallett and Günther [19] taken at swirl not far above the critical value support the assumed profile shape. Note that the tangential velocity beyond the core radius has a flat profile, in contrast to the tangential velocity distribution at the inlet to the expansion section which had a hyperbolic distribution. The task of the model is now to solve for the inlet swirl Γ which will produce this critical flow state.

The mass flow rate at the inlet, station 1, is calculated by

$$\begin{aligned} M_1 &= 2\pi \int_{r_h}^{r_1} \rho u r dr \\ &= 2\pi \int_{r_h}^{r_1} \rho u_1 \left\{ \zeta - (\zeta - 1) \frac{r}{r_1} \right\} r dr \end{aligned} \quad (4.33)$$

$$\Rightarrow M_1 = 2\pi\rho u_1 \left\{ r_1^2 \left(\frac{\zeta}{6} + \frac{1}{3} \right) - r_h^2 \left[\frac{\zeta}{2} - \left(\frac{\zeta}{3} - \frac{1}{3} \right) \frac{r_h}{r_1} \right] \right\} \quad (4.34)$$

where M_1 is the mass flow rate at the throat. At station 2, the mass flow rate is given by :

$$\begin{aligned} M_2 &= 2\pi \int_0^{r_2} \rho u r dr \\ &= 2\pi \int_0^{r_2} \rho u_2 \left(\frac{r}{r_2} \right)^2 r dr \end{aligned} \quad (4.35)$$

$$\Rightarrow M_2 = 2\pi \rho u_2 \frac{r_2^2}{4} \quad (4.36)$$

Equating (4.34) and (4.36) and simplifying results in

$$\begin{aligned} \Phi &= \frac{u_2}{u_1} \\ &= \frac{4}{R_2} \left\{ \frac{\zeta}{6} + \frac{1}{3} - \frac{\zeta}{2} R_h + \frac{(\zeta - 1)}{3} R_h^{3/2} \right\} \end{aligned} \quad (4.37)$$

where

$$R_2 = \left(\frac{r_2}{r_1} \right)^2 \quad (4.38)$$

$$R_h = \left(\frac{r_h}{r_1} \right)^2 \quad (4.39)$$

The axial flux of angular momentum at the throat, Ω_1 , is defined by

$$\begin{aligned} \Omega_1 &= 2\pi \int_{r_h}^{r_1} \rho u w r^2 dr \\ &= 2\pi \left\{ \int_{r_h}^{a_1} \rho u_1 \left[\zeta - (\zeta - 1) \frac{r}{r_1} \right] \left[w_h + (w_a - w_h) \frac{r - r_h}{a_1 - r_h} \right] r^2 dr \right. \\ &\quad \left. + \int_{a_1}^{r_1} \rho u_1 \left[\zeta - (\zeta - 1) \frac{r}{r_1} \right] \left[\frac{\Gamma}{r} \right] r^2 dr \right\} \end{aligned} \quad (4.40)$$

and after simplifying

$$\Omega_1 = 2\pi \rho u_1 \Gamma r_1^2 Z_1 \quad (4.41)$$

where Z_1 is

$$\begin{aligned} Z_1 = & \left(\sqrt{A_1 R_h} - \frac{R_h^2}{A_1} \right) \frac{\xi}{3} (\alpha - \beta) + \\ & \frac{1}{4} \left(A_1 - \frac{R_h^2}{A_1} \right) \left[\xi \beta + (\xi - 1) (\beta - \alpha) \sqrt{R_h} \right] - \\ & \frac{\theta}{5} (\xi - 1) \left(A_1^{3/2} - \frac{R_h^{3/2}}{A_1} \right) + \frac{\xi}{2} (1 - A_1) \\ & - \frac{(\xi - 1)}{3} (1 - A_1^{3/2}) \end{aligned} \quad (4.42)$$

and

$$\beta = \frac{a_1 - \alpha r_h}{a_1 - r_h} \quad (4.43)$$

while at station 2, Ω_2 is defined as

$$\begin{aligned} \Omega_2 = 2\pi \left[\int_0^{a_2} \rho u_2 \left(\frac{r}{r_2} \right)^2 w_2 \left(\frac{r}{a_2} \right) r^2 dr + \right. \\ \left. \int_{a_2}^{r_2} \rho u_2 \left(\frac{r}{r_2} \right)^2 w_2 r^2 dr \right] \end{aligned} \quad (4.44)$$

which simplifies to

$$\Omega_2 = \Phi u_1 w_2 r_1^3 R_2^{3/2} \left[\frac{1}{5} - \frac{A_2^{5/2}}{30} \right] \quad (4.45)$$

where

$$\overline{A}_2 = \left(\frac{a_2}{r_2} \right)^2 \quad (4.46)$$

Equating (4.41) and (4.45) and simplifying results in

$$\xi = \frac{w_2 r_2}{\Gamma} = \frac{Z_1}{\Phi R_2^{3/2} \left[\frac{1}{5} - \frac{A_2^{5/2}}{30} \right]} \quad (4.47)$$

The radial momentum equation after simplification by our assumptions reduces to

$$\frac{dp}{dr} = \rho \frac{w^2}{r}$$

which after integration yields a definition for the radial distribution of static pressure at the throat

$$\left\{ \begin{array}{ll} p_1 = p_{01} + \int_0^r \rho \frac{w^2}{r} dr & \text{for } 0 \leq r \leq r_h \\ p_1 = p_{01} + \frac{\rho \Gamma^2 \alpha^2 r_h^2}{2a_1^4} + \int_{r_h}^r \rho \frac{w^2}{r} dr & \text{for } r_h \leq r \leq a_1 \\ p_1 = p_{01} + \frac{\rho \Gamma^2}{a_1^4} \left\{ r_h^2 \left[\alpha^2 + 2(\alpha - \beta)^2 \ln \left(\frac{a_1}{r_h} \right) \right] + \right. \\ \quad \left. \beta(a_1 - r_h) \left[2(\alpha - \beta)r_h + \frac{\rho}{2}(a_1 + r_h) \right] \right\} + \\ \quad \int_{a_1}^{r_2} \rho \frac{w^2}{r} dr & \text{for } r_h \leq r \leq r_2 \end{array} \right\} \quad (4.48)$$

where p_{01} is the static pressure at the axis of the flow at station (1). The pressure distribution is obtained by integrating the radial momentum equation. Equations in (4.48) are further simplified to obtain an expression for the throat static pressure profile :

$$\left\{ \begin{array}{ll} p_1 = p_{01} + \frac{\rho \Gamma^2}{a_1^4} \left(\frac{\alpha^2}{2} \right) r^2 & \text{for } 0 \leq r \leq r_h \\ p_1 = p_{01} + \frac{\rho \Gamma^2}{a_1^4} \left\{ r_h^2 \left[\frac{\alpha^2}{2} + (\alpha - \beta)^2 \ln \left(\frac{r}{r_h} \right) \right] + \right. \\ \quad \left. \beta(a_1 - r_h) \left[2(\alpha - \beta)r_h + \frac{\rho}{2}(r + r_h) \right] \right\} & \text{for } r_h \leq r \leq a_1 \\ p_1 = p_{01} + \frac{\rho \Gamma^2}{a_1^4} \left\{ r_h^2 \left[\frac{\alpha^2}{2} + (\alpha - \beta)^2 \ln \left(\frac{a_1}{r_h} \right) \right] + \right. \\ \quad \left. \beta(a_1 - r_h) \left[2(\alpha - \beta)r_h + \frac{\rho}{2}(a_1 + r_h) \right] + \right. \\ \quad \left. \frac{\alpha_1^2}{2} - \frac{a_1^4}{2r^2} \right\} & \text{for } a_1 \leq r \leq r_2 \end{array} \right\} \quad (4.49)$$

At station (2), the integration of the radial momentum equation results in :

$$\left\{ \begin{array}{ll} p_2 = p_{02} + \int_0^r \rho \frac{w^2}{r} dr & \text{for } 0 \leq r \leq a_2 \\ p_2 = p_{02} + \rho \frac{w_2^2}{2} + \int_{a_2}^r \rho \frac{w^2}{r} dr & \text{for } a_2 \leq r \leq r_2 \end{array} \right\} \quad (4.50)$$

where p_{02} is the static pressure at the axis of the flow at station (2). After simplification

$$\left\{ \begin{array}{ll} p_2 = p_{02} + \rho \frac{w_2^2}{2} \left(\frac{r}{a_2} \right)^2 & \text{for } 0 \leq r \leq a_2 \\ p_2 = p_{02} + \rho w_2^2 \left[\frac{1}{2} + \ln \left(\frac{r}{a_2} \right) \right] & \text{for } a_2 \leq r \leq r_2 \end{array} \right\} \quad (4.51)$$

The axial flux of axial momentum at the throat, $\mathfrak{S}_1$, is calculated from

$$\mathfrak{S}_1 = 2\pi \left\{ \int_{r_h}^{r_1} \rho u^2 r dr + \int_0^{r_2} p_1 r dr \right\} \quad (4.52)$$

which after substitution and simplification results in

$$\mathfrak{S}_1 = 2\pi \left\{ p_{01} \frac{r_2^2}{2} + \frac{\rho \Gamma^2}{2} \mathcal{Z}_2 + \frac{u_1^2 r_1^2 \mathcal{Z}_3}{2} \right\} \quad (4.53)$$

where

$$\begin{aligned} \mathcal{Z}_2 = & \frac{\alpha^2 R_h}{2A_1^2} \left(R_2 - \frac{R_h}{2} \right) + \frac{1}{2} \ln \left(\frac{R_2}{A_1} \right) + \\ & \frac{R_h}{A_1} (\alpha - \beta)^2 \left[\frac{R_2}{2A_1} \ln \left(\frac{A_1}{R_h} \right) - \frac{1}{2} \left(1 - \frac{R_h}{A_1} \right) \right] + \\ & \frac{2(\alpha - \beta)\beta\sqrt{R_h}}{A_1^2} \left[\frac{1}{3} (R_h^{3/2} - A_1^{3/2}) + R_2 (A_1^{1/2} - R_h^{1/2}) \right] + \\ & \frac{1}{2A_1} \left\{ \beta^2 \left(1 - \frac{R_h}{A_1} \right) \left[R_2 - \frac{1}{2} (A_1 + R_h) \right] + R_2 - A_1 \right\} \end{aligned} \quad (4.54)$$

and

$$\mathcal{Z}_3 = \frac{\zeta^2}{6} + \frac{\zeta}{3} + \frac{1}{2} - \zeta^2 R_h + \frac{4(\zeta^2 - \zeta)}{3} R_h^{3/2} - \frac{(\zeta - 1)^2}{2} R_h^2 \quad (4.55)$$

At station (2), evaluation of the axial momentum results in :

$$\begin{aligned} \mathfrak{S}_2 = & 2\pi \left\{ \int_0^{r_2} \rho u_2^2 \left(\frac{r}{r_2} \right)^4 r dr + \right. \\ & \int_0^{a_2} \left[p_{02} + \rho \frac{w_2^2}{2} \left(\frac{r}{a_2} \right)^2 \right] r dr \\ & \left. \int_{a_2}^{r_2} \left[p_{02} + \rho w_2^2 \left(\frac{1}{2} + \ln \left(\frac{r}{a_2} \right) \right) \right] r dr \right\} \end{aligned} \quad (4.56)$$

Simplification of equation (4.56) results in the definition for $\mathfrak{S}_2$:

$$\mathfrak{S}_2 = 2\pi \left\{ \rho \frac{u_1^2 r_1^2 \phi^2 R_2}{6A_1} + \rho p_{02} \frac{r_2^2}{2} + \rho \frac{\xi^2 \Gamma^2}{2} \left[\frac{A_2 R_2}{4} + \frac{R_2}{2} \ln \sqrt{\frac{R_2}{A_1}} \right] \right\} \quad (4.57)$$

Finally, a condition for stagnation of the flow along the axis is required. For a flow without a hub, this is obviously [26]

$$p_{01} + \frac{\rho u_{01}^2}{2} = p_{02} \quad (4.58)$$

With the addition of a hub, u_{01} is reduced nearly to zero, since velocities in the wake of the hub are negligibly small (So et al. [20]). However, the external flow will exert a shear stress on the wake region which must be overcome by the pressure gradient in order to stagnate the flow. The effect of this shear will be represented by $\rho \kappa \frac{u_h^2}{2}$, where u_h is the velocity at the hub surface, so that the stagnation criterion becomes

$$p_{02} = p_{01} + \rho \kappa \frac{u_h^2}{2} \quad (4.59)$$

Means for determining κ will be discussed later.

Applying conservation of axial momentum ($\mathfrak{S}_1 = \mathfrak{S}_2$), and incorporating equation (4.59) results in the following dimensionless quantity :

$$\left(\frac{\Gamma}{u_1 r_1} \right)^2 = \frac{\mathcal{Z}_3 - \frac{\Phi^2 R_2}{3} - \kappa \frac{R_2}{2} [\zeta - (\zeta - 1) \sqrt{R_h}]^2}{\xi^2 \left(\frac{A_2}{4} - \frac{1}{2} \text{Ln}(A_2) \right) - \mathcal{Z}_2} \quad (4.60)$$

Finally, the critical axial flux of angular momentum is derived by using equation (3.20)

$$(\Omega_D)_{crit} = \frac{2\pi \int_{r_h}^{r_1} u w r^2 dr}{\pi U_{ref}^2 (r_1^2 - r_h^2) r_1} \quad (4.61)$$

and substituting equation (4.41) in (4.61) results in

$$(\Omega_D)_{crit} = \frac{2u_1 \Gamma r_1^2 \mathcal{Z}_1 (1 - R_h)}{U_{ref}^2 r_1^3} \quad (4.62)$$

To find a definition for U_{ref} from the proposed profiles, we use the continuity equation :

$$2\pi \int_{r_h}^{r_1} \rho u r dr = 2\pi \int_{r_h}^{r_1} \rho U_{ref} r dr \quad (4.63)$$

from which

$$U_{ref} = \frac{\Phi u_1 R_2}{2} \quad (4.64)$$

Substituting (4.64) into (4.62) results in

$$(\Omega_D)_{crit} = \left(\frac{\Gamma}{u_1 r_1} \right) \left[\frac{8Z_1(1-R_h)}{\Phi^2 R_2^2} \right] \quad (4.65)$$

Equation (4.65) is an explicit relationship which allows for the critical swirl intensity to be predicted from the inlet conditions.

To predict the solid body core radius at station (2), experimental results of Rao et al. [14] and Beltagui and Maccallum [7] show that the core radius increases proportionally with the expansion ratio. Therefore, it seems reasonable to assume that

$$\frac{a_2^2}{a_1^2 - r_h^2} = \frac{r_2^2}{r_1^2 - r_h^2} \quad (4.66)$$

The above expression is used to define the downstream core radius, while two other definitions were tested as well. These were

$$\left(\frac{a_2}{a_1} \right) = \frac{r_2^2}{r_1^2 - r_h^2} \quad (4.67)$$

$$\left(\frac{a_2}{a_1} \right)^2 = \frac{r_2^2}{r_1^2 - r_h^2} \quad (4.68)$$

The model proved to be quite insensitive to the choice of a_2 in prediction of $(\Omega_D)_{crit}$. An average difference of about 8% was observed when the results of the latter two definitions were compared with those of equation (4.46).

The results of classical boundary layer theory for a circular wake [33] can be used to obtain an expression for κ . According to Schlichting [33], the axial velocity on the centreline of a circular wake is given by

$$\frac{u_h - u_0}{u_h} \propto \left[\frac{c_D \pi r_h^2}{\sigma^2 x^2} \right]^{1/3} \quad (4.69)$$

where u_h is the flow velocity at the edge of the wake, u_0 is the axial velocity at the center of the wake, c_D is the drag coefficient of the body generating the wake, σ

is a constant which depends on the mixing length and the width of the wake, and x is taken as the distance from the hub to the point of inception of breakdown. From flow visualization, it was concluded that x is independent of r_h , since the breakdown occurred at about the same location regardless of the hub size. On the other hand, findings of Rao et al. [14] and Hallett and Günther [19] suggest that x is proportional to the throat radius, r_1 , since there is no evidence that it varies significantly with the expansion ratio. Equation (4.69) is only an approximate description of the actual flow : it neglects swirl and assumes fully developed flow and a constant u_h , neither of which is fully valid. It will therefore only be used to give the form of a correlation for κ , and the power 1/3 will be transformed to an adjustable parameter λ .

Taking the constants out from equation (4.69) and solving for u_0 results in

$$\left(\frac{u_0}{u_h}\right)^2 = \kappa = 1 - 2\kappa_1 \left(\frac{r_h}{r_1}\right)^{2\lambda} + \kappa_1^2 \left(\frac{r_h}{r_1}\right)^{4\lambda} \quad (4.70)$$

or in general form

$$\kappa = 1 - \tau_1 R_h^\lambda + \tau_2 R_h^{2\lambda} \quad (4.71)$$

It can be argued that when r_h tends to r_1 , the mass flow in the annulus will become zero, and κ approaches zero. Therefore

$$1 - \tau_1 + \tau_2 = 0 \longrightarrow \tau_2 = \tau_1 - 1 \quad (4.72)$$

A trial and error procedure was followed to obtain the proper values of τ_1 and λ in order to find the most suitable value of κ . A computer code in Basic is developed to compute $(\Omega_D)_{crit}$ for each combination of hub and test section. The program is included in Appendix (B).

4.3 Fitting Model Profiles To The Experimental Data

The inlet tangential velocity of the swirling jet is represented by a Rankine vortex having a core radius of a_1 , and the axial velocity is approximated by a linear relationship as presented in the previous chapter. This section describes how these profiles were fitted to the experimental data to determine the values of $\frac{u_1}{r_1}$, ζ and α appropriate to the measured profiles.

First, a straight line must be fitted to the axial velocity profile, so that ζ and u_1 can be calculated. From the proposed equations for the axial velocity, one can notice that

$$r = r_1 \implies u = u_1$$

$$r = 0 \implies u = u_1 \zeta$$

By inspecting an axial velocity profile, one can easily give an estimate of ζ . This is achieved by passing a visual best fit straight line through the profile. Once ζ is assessed, the continuity equation can be used to calculate u_1

$$\int_{r_h}^{r_1} u_1 \left[\zeta - (\zeta - 1) \frac{r}{r_1} \right] r dr = \int_{r_h}^{r_1} U_{ref} r dr \quad (4.73)$$

After simplification

$$u_1 = \frac{U_{ref}}{\zeta - \frac{2}{3}(\zeta - 1) \left[\frac{1 - R_h^{3/2}}{1 - R_h} \right]} \quad (4.74)$$

The fitting procedure searches for the value of ζ that minimizes the error defined by

$$Error_u = \sum_{i=1}^n (u_i - u'_i)^2 \quad (4.75)$$

where u_i' is the fitted axial velocity at each measurement point, and u_i is the measured value. The fitting is carried out by the computer program listed in Appendix (C).

The fitting of tangential velocity is carried out so that the mass flow and the dimensionless axial flux of angular momentum, calculated by the data reduction program, are preserved. Using equation (3.19), the axial flux of angular momentum is

$$\Omega = 2\pi \int_{r_h}^r \rho u w r^2 dr$$

and substituting the proposed profiles for the axial and tangential velocities into equation (3.19) yields

$$\begin{aligned} \Omega = 2\pi \left\{ \int_{r_h}^{a_1} \rho u_1 \left[\zeta - (\zeta - 1) \frac{r}{r_1} \right] \left[w_h + (w_a - w_h) \frac{r - r_h}{a_1 - r_h} \right] dr + \right. \\ \left. \int_{a_1}^{r_1} \rho u_1 \left[\zeta - (\zeta - 1) \frac{r}{r_1} \right] \Gamma r dr \right\} \end{aligned} \quad (4.76)$$

Replacing w_a and w_h by their definitions, taking out the constants and evaluating the integrals, yields the final expression for the axial flux of angular momentum as

$$\Omega = 2\pi \rho u_1 \Gamma r_1^2 \mathcal{Z}_1 \quad (4.77)$$

where $\mathcal{Z}_1$ is defined as

$$\begin{aligned} \mathcal{Z}_1 = & \left(\sqrt{A_1 R_h} - \frac{R_h^2}{A_1} \right) \frac{\zeta}{3} (\alpha - \beta) + \\ & \frac{1}{4} \left(A_1 - \frac{R_h^2}{A_1} \right) \left[\zeta \beta - \frac{1}{2} (\zeta - 1)(\beta - \alpha) \sqrt{R_h} \right] - \\ & \frac{\beta}{5} (\zeta - 1) \left(A_1^{3/2} - \frac{R_h^{5/2}}{A_1} \right) + \\ & \frac{\zeta}{2} (1 - A_1) - \frac{\zeta - 1}{3} (1 - A_1^{3/2}) \end{aligned}$$

and

$$R_h = \left(\frac{r_h}{r_1}\right)^2$$

$$A_1 = \left(\frac{a_1}{r_1}\right)^2$$

Equation (4.77) is non-dimensionalized by using the average forward velocity calculated by equation (3.15). Therefore

$$\Omega_D = \frac{\Omega}{M_{th} \cdot U_{ref} \cdot r_1} \quad (4.78)$$

Substituting the expressions for Ω and M_{th} in equation (4.78) and simplifying results in

$$\Omega_D = \frac{2 u_1 \Gamma Z_1 (1 - R_h)}{U_{ref}^2 r_1} \quad (4.79)$$

The proposed model does not take into account the effect of the boundary layer at both the hub's exterior surface and the tube's interior surface, while the measured velocity profiles do possess boundary layers. To obtain the same velocities while preserving the mass flow rate, the effective boundaries of the flow are moved inwards by the boundary layer displacement thickness, as shown in figure(4.2). Subscripts (1) and (N) in figure (4.2) refer to the measurement sites adjacent to the hub and tube surface. The first measurement point is also given a superscript (') so that it will not be confused with the throat radius, r_1 . The data reduction program for the five hole probe, presented in Appendix (A), assumes a linear profile from u_N at r_N to zero at r_1 . Therefore, the mass flow between r_N and r_1 can be calculated using the trapezoid rule as

$$M_{ao} = 2\pi \int_{r_N}^{r_1} \rho u r dr = \pi \rho u_N r_N (r_1 - r_N) \quad (4.80)$$

The model assumes that at both r_N and r_{bo} , the position of the effective outer boundary layer, the axial velocity is u_N . Therefore, the mass flow rate between

r_N and r_{bo} is

$$M_{fo} = 2\pi \int_{r_N}^{r_{bo}} \rho u r dr = \pi \rho u_N (r_{bo}^2 - r_N^2) \quad (4.81)$$

Equating equations (4.80) and (4.81) results in a definition for r_{bo}

$$r_{bo} = \sqrt{r_N r_1} \quad (4.82)$$

Similarly, r_{bi} , the effective inner boundary, can be calculated by assuming u_1 at both r_1' and r_{bi} , where r_1' is the first measurement point after the hub surface, and zero axial velocity at hub's surface. Therefore, the mass flow between r_h and r_1' , and between r_{bi} and r_1' would be

$$M_{ai} = 2\pi \int_{r_h}^{r_1'} \rho u r dr = \pi \rho u_1 r_1' (r_1' - r_h) \quad (4.83)$$

$$M_{fi} = 2\pi \int_{r_{bi}}^{r_1'} \rho u r dr = \pi \rho u_1 (r_1'^2 - r_{bi}^2) \quad (4.84)$$

Equating (4.83) and (4.84) results in a definition for r_{bi}

$$r_{bi} = \sqrt{r_h r_1'} \quad (4.85)$$

Reducing the flow area, by moving the boundaries inward, affects the average axial velocity. By equating the mass flows in both geometries, the new reference velocity, U'_{ref} , can be calculated

$$\rho U'_{ref} (r_{bo}^2 - r_{bi}^2) = \rho U_{ref} (r_1^2 - r_h^2) \quad (4.86)$$

$$\rightarrow U'_{ref} = U_{ref} \frac{r_1^2 - r_h^2}{r_{bo}^2 - r_{bi}^2} \quad (4.87)$$

The axial flux of angular momentum has to be corrected for the boundary layer as well. The axial flux of angular momentum between the last measurement point, r_N and the tube wall, as calculated by the trapezoid rule, is

$$\Omega_{Ao} = 2\pi \int_{r_N}^{r_1} \rho u w r^2 dr = \pi \rho u_N w_N r_N^2 (r_1 - r_N) \quad (4.88)$$

where the subscript Ao stands for the actual momentum flux at the outer boundary. The fitted profile assumes a uniform axial velocity of u_N and a tangential velocity of $w = \frac{w_N r_N}{r}$ from r_N to r_{bo} . The modelled angular momentum flux between r_N and r_{bo} is

$$\Omega_{fo} = 2\pi \int_{r_N}^{r_{bo}} \rho u w r^2 dr = \pi \rho u_N w_N r_N (r_{bo}^2 - r_N^2) \quad (4.89)$$

The subscript fo represents the fitted angular momentum flux at the outer boundary. Using the angular momentum stream function, Ω_{func} , as defined by equation (3.23), which is an output of the data reduction program, one can conclude that

$$\frac{\Omega_{fo}}{\Omega_{Ao}} = \frac{(\Omega_{func})_{bo} - (\Omega_{func})_N}{(\Omega_{func})_{r_1} - (\Omega_{func})_N} \quad (4.90)$$

Since $(\Omega_{func})_{r_1} = 1$ by definition, and $(\Omega_{func})_N$ is already calculated by the data reduction program, therefore

$$(\Omega_{func})_{bo} = (\Omega_{func})_N + \frac{\Omega_{fo}}{\Omega_{Ao}} [1 - (\Omega_{func})_N] \quad (4.91)$$

where by virtue of equations (4.88) and (4.89) $\frac{\Omega_{fo}}{\Omega_{Ao}}$ is defined as

$$\frac{\Omega_{fo}}{\Omega_{Ao}} = \frac{r_{bo}^2 - r_N^2}{r_1 r_N - r_N^2} \quad (4.92)$$

A similar situation is faced at the hub surface. Proceeding in a similar manner as for the outer boundary layer results in

$$\frac{\Omega_{fi}}{\Omega_{Ai}} = \frac{(r_1' + r_{bi})(r_1'^2 + r_{bi}^2)}{2r_1'^3} \quad (4.93)$$

while the angular momentum stream function gives the following expression for

$$\frac{\Omega_{fi}}{\Omega_{Ai}}$$

$$\frac{\Omega_{fi}}{\Omega_{Ai}} = \frac{(\Omega_{func})_1 - (\Omega_{func})_{bi}}{(\Omega_{func})_1 - (\Omega_{func})_h} \quad (4.94)$$

where, by definition

$$(\Omega_{func})_h = 0$$

Solving equation (4.94) for $(\Omega_{func})_{bi}$ yields

$$(\Omega_{func})_{bi} = (\Omega_{func})_1 \left[1 - \frac{\Omega_{fi}}{\Omega_{Ai}} \right] \quad (4.95)$$

Due to the inward movement of the boundaries, the fitted axial flux of angular momentum of the whole flow is reduced to

$$\Omega' = [(\Omega_{func})_{bo} - (\Omega_{func})_{bi}] \Omega \quad (4.96)$$

where Ω' is the fitted angular momentum flux. Non-dimensionalizing Ω' by the fitted reference velocity yields

$$\Omega'_D = \frac{\Omega'}{r_{bo} M U'_{ref}} \quad (4.97)$$

which after substitution for U'_{ref} simplifies to

$$\Omega'_D = \{(\Omega_{func})_{bo} - (\Omega_{func})_{bi}\} \left(\frac{r_1}{r_{bo}} \right) \left\{ \frac{r_{bo}^2 - r_{bi}^2}{r_1^2 - r_h^2} \right\} \Omega_D \quad (4.98)$$

where Ω_D is the data reduction program's calculated dimensionless axial flux of angular momentum.

In order to preserve the experimentally obtained Ω_D and account for the boundary layers, it is replaced in equation (4.79) by the value of Ω'_D calculated from the experimental data. As a result, equation (4.79) can be solved to predict Γ . Equation (4.79) has two unknowns, namely a_1 and Γ , hence a trial and error procedure is followed to obtain these unknowns:

1. the first measurement location close to the axis is chosen as the site of a_1

2. Equation (4.79) is solved for Γ :

$$\Gamma = \frac{\Omega_D(1 - R_h)U_{ref}^2 r_1}{2u_1 \mathcal{Z}_1} \quad (4.99)$$

3. These values of a_1 and Γ give a trial tangential velocity profile.

4. The sum of squares of the differences between the trial fit and the experimentally measured tangential velocities is calculated using

$$Error_w = \sum_{i=1}^n (w'_i - w_i)^2 \quad (4.100)$$

where w'_i is the fitted tangential velocity at the i^{th} measurement site and

w_i is the probe measured tangential velocity at the same site.

5. The next measurement point is chosen as the site of a_1 and steps 2 to 4 are repeated until all measurement locations are covered

6. Finally, a_1 is chosen such that the model profile would yield the least squared error.

To calculate Γ from equation (4.99), the experimental parameters must be corrected for the boundary layers. Therefore the following replacements must take place before the fitted tangential velocity profile can be calculated :

- $\Omega_D \Rightarrow \Omega'_D$
- $U_{ref} \Rightarrow U'_{ref}$
- $r_1 \Rightarrow r_{bo}$
- $r_h \Rightarrow r_{bi}$

Finally, the velocity profiles based on fitted constants must be converted back to the basis of the actual reference velocity so that they may be plotted along with the experimental data

This is achieved by the following equation

$$w_{mod} = w' \left(\frac{U_{ref}}{U'_{ref}} \right) \quad (4.101)$$

$$u_{mod} = u' \left(\frac{U_{ref}}{U'_{ref}} \right) \quad (4.102)$$

where the primes refer to the velocities based on U'_{ref} which results from the fitting.

The computer code carrying out the above calculations for each hub and expansion combination is presented in Appendix (C).

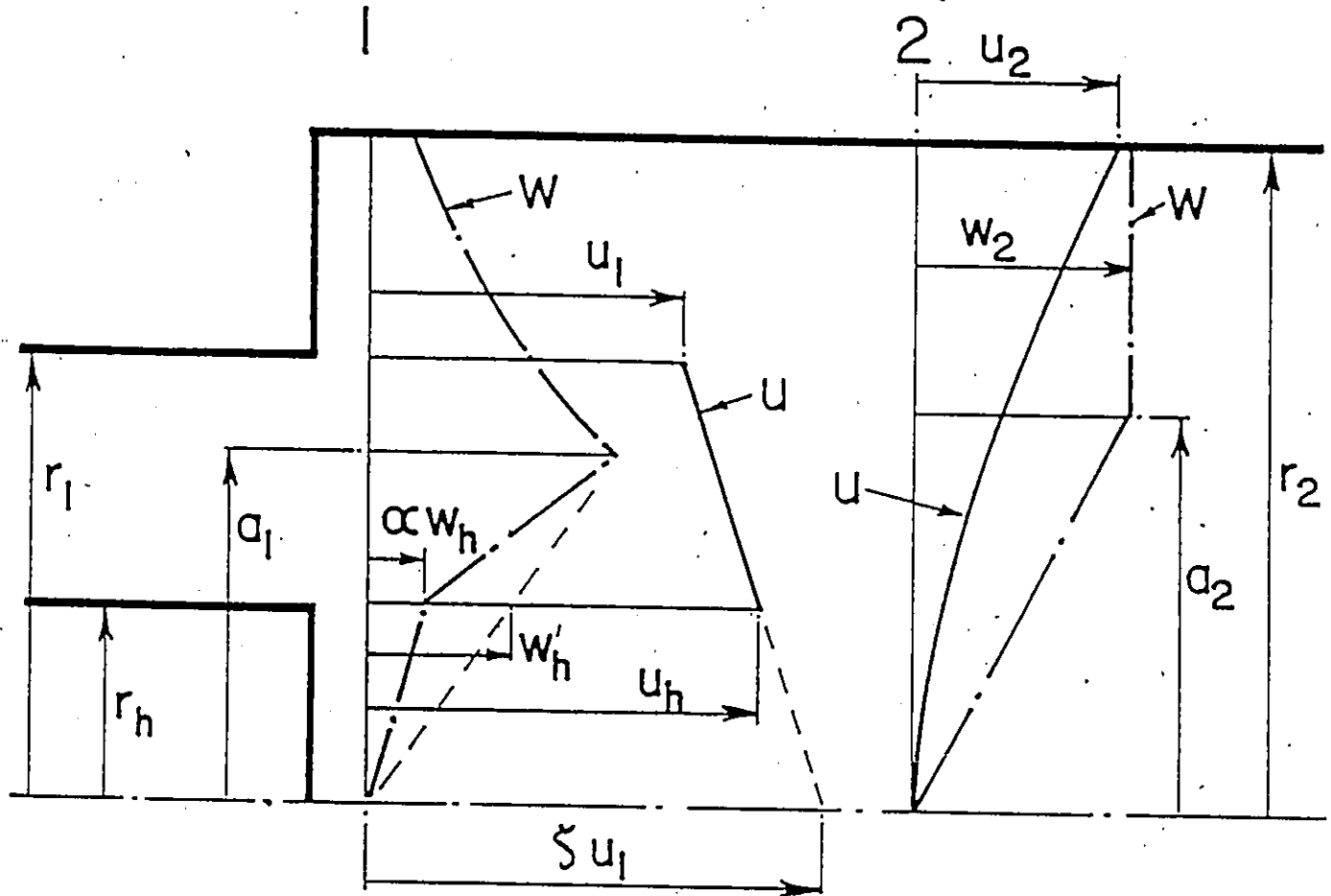


Figure 4.1: Velocity profile assumptions for the mathematical model

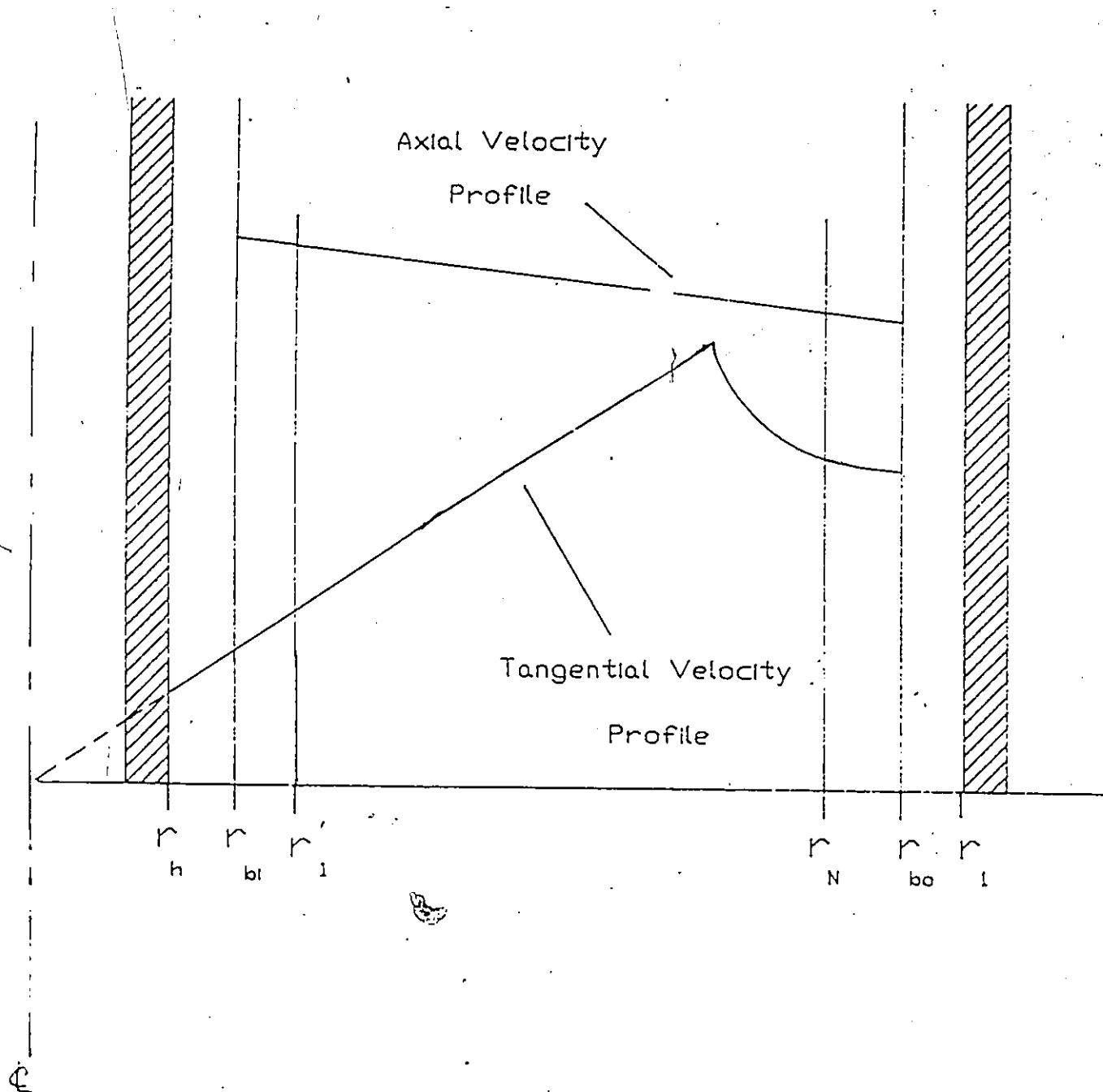


Figure 4.2: Boundary layer and measurement points geometry

Chapter 5

Results And Discussion

A series of experiments involving flow visualization and probe measurements were carried out in order to obtain the critical swirl intensity at which vortex breakdown was initiated. In the process of obtaining the critical swirl number, the three components of the total velocity vector and the static pressure at each measurement point were also calculated as explained in section 3.6. In addition, mathematical modelling, as explained in chapter 4, was carried out in order to analytically predict the critical dimensionless angular momentum flux.

5.1 Flow Visualization

The first part of each experiment involved using $TiCl_4$ as a flow visualization agent in order to obtain the proper tangential to axial mass flow ratio to initiate vortex breakdown. In the absence of swirl, the flow behaved as a simple axial jet with relatively small spread due to the high Reynolds number of the flow (82000 - 280000). Introduction of swirl caused the jet to spread quickly to fill up

the whole test section. The rapidness of the spread increased substantially as the swirl intensity was increased.

At swirl numbers below the critical value, the entire flow diverted from the axis of rotation and precessed around it. Such a phenomenon was also observed by Hallett and Günther [19]. They suggested that precession is caused by the Coanda effect, in which the jet is deflected by an asymmetrical disturbance which pushes it closer to the wall, where the swirl in the flow gives it a tangential motion. The strongest and most regular precessions were observed at low swirls, while increasing the swirl to the neighbourhood of the critical value increased the irregularity in the motion of the precessing fluid. The precession became slower and a periodic stall in its motion was observed as the swirl intensity was gradually increased. According to Hallett and Günther [19], the weakening of the precession is caused by the faster spread of the jet at higher swirl intensities, which moves mass and momentum in the flow out toward the wall, therefore limiting the possible precession amplitude. As the swirl intensity was increased a transition region was reached in which precession was stalled very frequently and a recirculation zone periodically appeared at the center of the flow. The critical swirl intensity corresponds to this transition region in which vortex breakdown is initiated. A slight increase in swirl intensity would establish a fully stable central recirculation zone downstream of the throat. The transition range, from the swirl intensity at which the first signs of disruption appeared in the precessing fluid to that at which a stable central recirculation zone was established, extended over an average 6% range of Ω_D . The emergence of the backflow zone stabilized the flow and stopped precession, since it caused the flow to fill the cross section almost immediately after leaving the throat. The fully stable recirculation zone moved

upstream as the swirl intensity was increased above the critical value and it was found to be highly turbulent and axisymmetric. Leibovich [35] observed that the fluctuation magnitudes vary considerably by being negligibly small in some parts of the recirculation zone while strong and dominant in other parts.

The advent of the recirculation zone is attributed to the establishment of an axial pressure gradient along the axis of the flow. Expansion of the flow after leaving the throat is associated with an increase in solid body core radius. Hallett and Toews[3] showed that increasing the core radius results in decrease in radial pressure drop ($p_w - p_0$). Findings of Beltagui and MacCallum [7] and Hallett and Günther [19] suggest that p_w remains nearly constant. This in turn suggests an increase in static pressure in the central region along the axis. In addition, shear due to mixing with surrounding air results in a decay in the tangential velocity. The equation for the radial pressure gradient

$$\frac{w^2}{r} \approx \frac{1}{\rho} \frac{\partial p}{\partial r}$$

shows that a reduction in w will decrease the pressure drop between wall and axis, and so increase the axial pressure gradient. Also, the general decrease in axial velocity after expansion further enhances the pressure gradient.

The effect of flow rate on the process was also tested and was found to be negligible. This insensitivity may be attributed to the fact that the flow was already in a fully turbulent mode. The Reynolds number range used varied between 82000 to 280000 depending on the hub size and the expansion ratio. The transition to recirculation showed little sign of hysteresis. An exception was found in the case of largest expansion and the largest hub size, when recirculation persisted if the swirl was decreased below the critical state.

The shape of the central recirculation zone did not change substantially as different expansions were tried. Placing a hub at the throat appeared to extend the recirculation zone all the way to the throat by combining the wake and central recirculation zone. Particularly with the smallest hubs the flow appeared to "neck-down" between the hub and the recirculation. This has also been observed by Escudier and Keller [12].

5.2 Critical Axial Flux Of Angular Momentum

$(\Omega_D)_{crit}$ was found to be dependent on both expansion ratio and central hub size. The critical angular momentum flux is plotted against both expansion ratio and hub size in figures (5.1) and (5.2).

As figure (5.1) shows, $(\Omega_D)_{crit}$ reaches a slight minimum near $\frac{r_2}{r_1} = 2.0$ for all hub sizes. Experiments of Hallett and Toews[3] for hubless flow reported such a minimum near $\frac{r_2}{r_1} = 1.63$. The experiments of Rao et al. [14] suggest a significant drop in critical swirl between $\frac{r_2}{r_1} = 1.2$ and 2.4. Meanwhile, the results of Beltagui and Maccallum [7] suggest that the critical swirl changes negligibly between $\frac{r_2}{r_1} = 2.5$ and 5.0. Hallett and Toews [3] pointed out that for a hubless flow without swirl, the pressure recovery has a maximum at $\frac{r_2}{r_1} = \sqrt{2}$; this behaviour of the pressure recovery due to axial velocity changes is responsible for the minimum in the critical swirl. $(\Omega_D)_{crit}$ was found to increase substantially at smaller expansions. A smaller pressure gradient is obtained at lower expansions, because both axial and tangential velocities undergo smaller changes, which in turn requires a higher swirl intensity to produce the necessary axial pressure gradient.

Figure (5.2) shows that $(\Omega_D)_{crit}$ is reduced significantly as $\frac{r_2}{r_1}$ is increased

for the same expansion. This is caused by the creation of a relatively low velocity region downstream of the wake. As a result, a smaller pressure gradient along the axis is needed to cause or maintain flow reversal. The larger hubs create a larger momentum deficit at the center of the flow. Therefore, a smaller pressure gradient, i.e. smaller swirl intensity, is needed to initiate the recirculation zone.

5.3 Tangential Velocity Distribution

The measured tangential velocity for each combination of central hub and expansion section is plotted against the radial distance from the axis of the test section and is presented in figures (5.3) to (5.18). The dashed line represents the fitted velocity profile.

Analyzing the figures, one can recognize a distinct solid body core, whose radius corresponds to the site of maximum tangential velocity. The tangential velocity profile inside the solid body core region increases linearly with $\frac{r}{r_1}$ from a value of zero on the axis. Outside the solid body core region, the profile deviates from the ideal Rankine vortex by being flatter. This flatness is attributed to friction.

The profiles also reveal that the core radius increased as the hub size increased. This is predictable since a larger hub directs the flow out toward the throat wall and therefore the core boundary moves further away from the axis of the flow. The expansion ratio was found to have a minimal effect on the inlet solid body core radius. The tangential velocity profiles resemble those obtained by Rao et al. [14] and Hallett and Toews [3].

5.4 Axial Velocity Distribution

Mean dimensionless axial velocity profiles are presented in figures (5.19) to (5.34) for each combination of central hub and expansion section. The dashed line represents the fitted velocity profile.

All the axial velocities present a distribution with a maximum at the edge of the hub and a gradual decrease as they approach the throat wall. This is dictated by flow processes within the swirl generator, which were not further investigated. No measurement was taken inside the wake, but Escudier and Keller [28] reported that the axial velocity in the wake and just behind a hub is negligibly small. Also, the maximum dimensionless axial velocity remained nearly constant for all cases.

5.5 Radial Velocity Distribution

The mean radial velocities are plotted against the radial distance from the axis of the flow for each hub and test section combination and are presented in figures (5.35) to (5.38).

The radial velocity is found to be very small compared to the axial velocity. This is due to the fact that measurements were taken at the throat and the flow had not had a chance to expand yet. It was found to have a maximum at the center of the flow with an asymptotic approach to zero as it moved away from the center axis. Symmetry dictates that the radial velocity be zero on the axis. The positive radial velocities in figure (5.38) are therefore indicative of probe measurement error, either an error in yaw angle χ due to the turbulence, or the presence of a secondary flow along the probe shaft due to the radial pressure gradient in the flow. The radial

velocity is known to be the least accurate of the three components [37]. Results of Rao et al. [14] support the negligibility of this component in comparison to the axial and tangential components of the total velocity vector.

5.6 Static Pressure Distribution

The static pressure distribution is presented in figures (5.39) to (5.42) by the dimensionless pressure drop in radial direction. The pressure drop $(p - p_w)$ is non-dimensionalized by a reference pressure, p_{ref} , which is calculated from

$$\begin{aligned} p_{ref} &= \frac{1}{2} \rho U_{ref}^2 \\ &= \frac{(M_{th} \cdot Q)}{2\pi^2 (r_1^2 - r_h^2)^2} \end{aligned} \quad (5.1)$$

As the figures show, the static pressure drop is the largest at the center of the flow, and approaches zero as it gets closer to the throat wall. A glance at equation (3.22), which is an integration of the simplified radial momentum equation, reveals that the low static pressure near the axis is due to the small radius in this area. The centrifugal force exerted on each particle of the flow establishes a region of low pressure near the axis of the flow. In addition, the smallest expansion requires the largest $(\Omega_D)_{crit}$ which is accompanied by larger pressure drop near the axis of the flow. In the same way, for an expansion of 2:1, the pressure drop near the axis is minimal since the least $(\Omega_D)_{crit}$ is required.

5.7 Mathematical Modelling Results

The tangential and axial velocity profiles, equations (4.25) and (4.31), were fitted to measurements as presented in figures (5.3) to (5.34) by broken lines. The tangential velocity fittings were found to be most erroneous when no hub or the smallest hub were used. This is due to the flat shape of the profile at the neighbourhood of the solid body core boundary.

The values of κ for each hub size were adjusted to obtain the best fit of the model to the experiments, resulting in:

$$\left\{ \begin{array}{ll} \kappa = 1.000 & \text{for } \frac{r_h}{r_1} = 0.00 \\ \kappa = 0.650 & \text{for } \frac{r_h}{r_1} = 0.25 \\ \kappa = 0.305 & \text{for } \frac{r_h}{r_1} = 0.50 \\ \kappa = 0.216 & \text{for } \frac{r_h}{r_1} = 0.63 \end{array} \right\} \quad (5.2)$$

These values can be closely fitted with an equation of the form of (4.71)

$$\kappa = 1 - \tau_1 R_h^\lambda + \tau_2 R_h^{2\lambda}$$

with

$$\lambda = 0.500 \quad (5.3)$$

$$\tau_1 = 1.678 \quad (5.4)$$

$$\tau_2 = 0.678 \quad (5.5)$$

A plot of κ against $\frac{r_h}{r_1}$ is presented in figure (5.43).

The predicted dimensionless angular momentum flux closely approximates the experimentally obtained ones. The largest difference was about 8% for $\frac{r_h}{r_1} =$

0.63 and $\frac{r_2}{r_1} = 3.0$, which can be attributed to the difficulty in precise recognition of the transition period for larger hubs. Also, the probe measurement error, as explained in section 3.4.5, of about 6% too high further reduces the discrepancy between the predicted and experimentally obtained $(\Omega_D)_{crit}$. Both the predicted and experimental $(\Omega_D)_{crit}$ are presented in figures (5.1) and (5.2). The mathematical model predicted a similar pattern for $(\Omega_D)_{crit}$ to that obtained experimentally in regard to expansion ratio and central hub effects. The $(\Omega_D)_{crit}$ as presented here is based on the individually fitted values of κ , rather than the value of κ obtained from equation (4.71). A maximum error of about 13% was observed when $(\Omega_D)_{crit}$ was predicted using equation (4.71).

The fitting program also gave a solid body core radius near the site of maximum tangential velocity obtained experimentally. In addition, a one-piece profile ($\alpha = 1.0$) for the forced vortex region of the tangential velocity profile was predicted for $\frac{r_h}{r_1} < 0.5$, while the value of α decreased once larger hubs were used. This was already predicted by the discussion carried out in section 4.2. A list of the parameters used in the mathematical modelling is presented in table (5.1).

Sensitivity of $(\Omega_D)_{crit}$ to the fitting parameters α , a_1 , ζ and the downstream core radius, a_2 , were also tested by plotting $(\Omega_D)_{crit}$ against each of these parameters. These plots are presented in figures (5.44) to (5.47). For each of these plots a typical test section ($\frac{r_2}{r_1} = 2.0$) was chosen and variation of $(\Omega_D)_{crit}$ with each of these parameters were plotted for the two extreme cases of $\frac{r_h}{r_1} = 0.63$ & 0.00.

The distribution of $(\Omega_D)_{crit}$ with respect to α is presented in figure (5.44). The variation is shown to be very small for $\frac{r_h}{r_1} = 0.63$, while α has no effect on $(\Omega_D)_{crit}$ in the absence of a central hub. The latter is expected, since α is a function of hub radius.

$(\Omega_D)_{crit}$ is found to increase asymptotically with inlet core radius for the hubless case and to decrease monotonically as the largest hub was used (fig. (5.45)). The result for hubless flow agrees with the results of Hallett and Toews [3], who verified that the larger core radius is accompanied by a larger critical angular momentum flux, but the trend with a hub is different. This contradiction can be explained by the momentum deficiency created by the central hub, which for large hubs appears to have a more dominant role than the core radius does in reducing the critical angular momentum flux. The critical swirl becomes infinite as a_1 approaches r_h ; a real flow will always have a_1 greater than r_h because of the hub boundary layer. In the range of our concern ($0.61 < \frac{a_1}{r_1} < 0.96$), $(\Omega_D)_{crit}$ does not show a substantial change as $\frac{a_1}{r_1}$ is varied, so that the predictions are not sensitive to $(\frac{a_1}{r_1})$.

$(\Omega_D)_{crit}$ increased monotonically as ζ was increased (fig. (5.46)). Depending on the hub size, a very low value of ζ would not result in a prediction of the critical angular momentum flux since it caused $(\frac{\Gamma}{u_1 r_1})^2$ to become negative. It can be concluded that the model is more sensitive to the choice of ζ than to the other velocity profile parameters. Meanwhile, $(\Omega_D)_{crit}$ did not change substantially with an increase in $\frac{a_2}{r_2}$ for the range of our concern ($\frac{a_2}{r_2} > 0.65$) as shown in figure (5.49). It dropped sharply at low values of the core radius at station (2) and approached a limit as the core radius was increased. This indicates the independence of the mathematical model of the choice of a_2 as explained in chapter 4.

The discrepancy between the mathematical prediction of $(\Omega_D)_{crit}$ and the experimentally obtained one can be attributed to not only the probe measurement and flow visualization errors, but also to the assumed downstream profiles of the axial and tangential velocities. Since no measurements were taken at the point of inception of vortex breakdown, certain profiles were assumed for the axial and

tangential velocity distributions as well as the radius of the solid body core .

Naturally, each of these assumptions introduces an error into the model.

$\frac{r_h}{r_1}$	$\frac{r_2}{r_1}$	$\frac{a_1}{r_1}$	U_{ref} m/sec	u_1 m/sec	w_h m/sec	w_a m/sec	Γ m^2/sec	α	ζ	κ
0.00	3.00	0.71	30.65	28.54	0.00	26.61	0.958	1.000	1.22	1.000
0.00	2.00	0.61	31.21	28.60	0.00	27.77	0.861	1.000	1.34	1.000
0.00	1.63	0.65	31.84	30.69	0.00	27.91	0.921	1.000	1.22	1.000
0.00	1.25	0.57	41.48	37.60	0.00	37.61	1.091	1.000	1.31	1.000
0.25	3.00	0.85	34.11	30.17	6.73	24.19	1.040	1.000	1.53	0.650
0.25	2.00	0.89	35.90	32.21	6.78	24.71	1.112	1.000	1.47	0.650
0.25	1.63	0.85	34.34	29.66	6.72	24.12	1.037	1.000	1.52	0.650
0.25	1.25	0.65	26.04	22.89	8.79	22.63	0.747	1.000	1.55	0.650
0.50	3.00	0.91	33.25	27.11	6.45	19.15	0.881	0.61	2.02	0.305
0.50	2.00	0.94	32.44	26.42	5.98	15.68	0.753	0.72	2.03	0.305
0.50	1.63	0.89	32.19	26.48	4.72	18.17	0.818	0.46	2.26	0.305
0.50	1.25	0.89	30.85	24.74	7.15	21.85	0.983	0.58	2.11	0.305
0.63	3.00	0.96	32.36	27.60	5.63	11.43	0.560	0.760	2.01	0.216
0.63	2.00	0.96	34.42	28.01	2.69	12.57	0.616	0.33	2.34	0.216
0.63	1.63	0.93	40.02	28.06	5.53	12.49	0.587	0.66	3.48	0.216
0.63	1.25	0.91	43.70	29.24	1.79	17.33	0.797	0.15	3.89	0.216

Table 5.1 : List of parameters used in the mathematical model

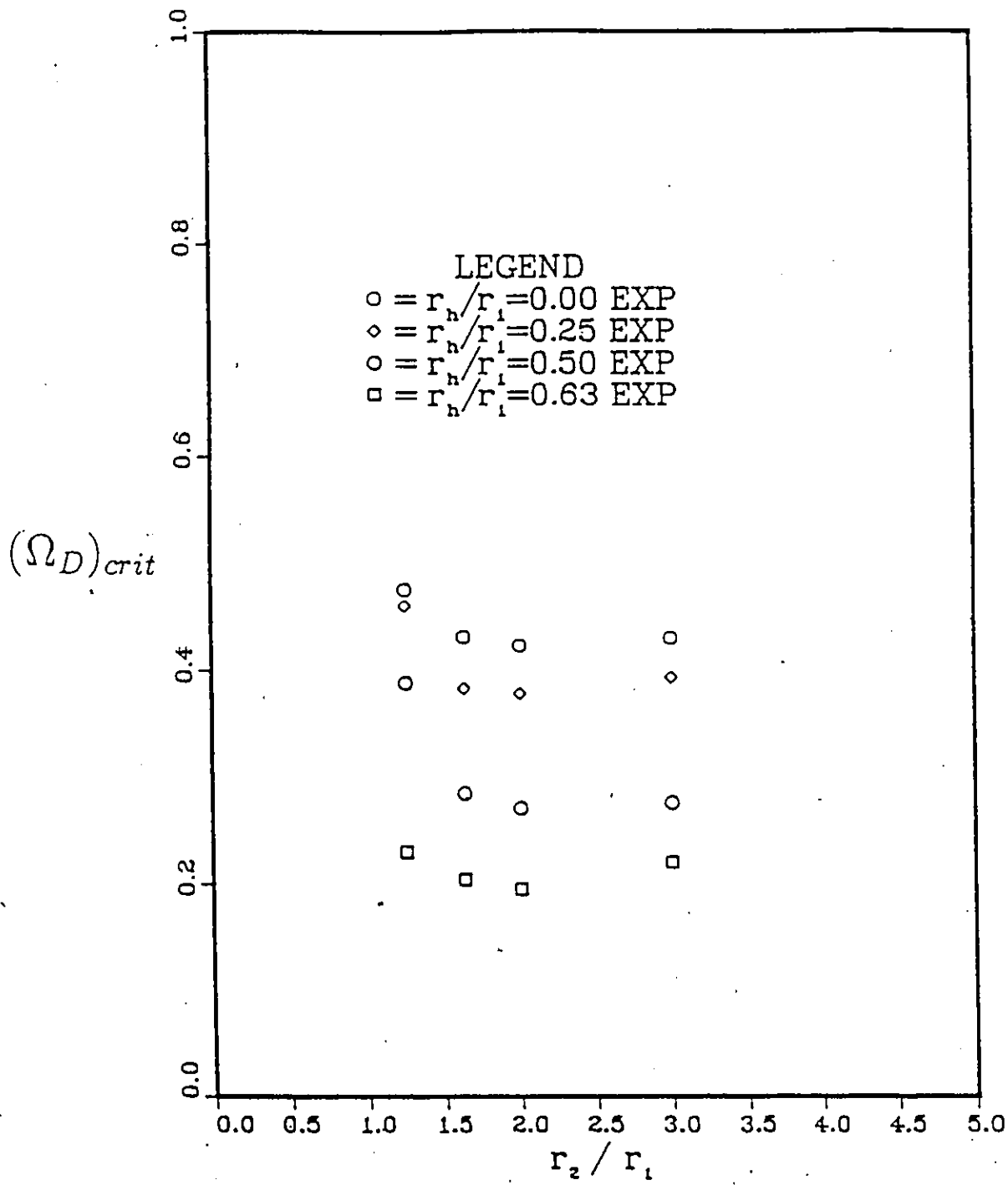


Figure 5.1: $(\Omega_D)_{crit}$ vs. $\frac{r_2}{r_1}$

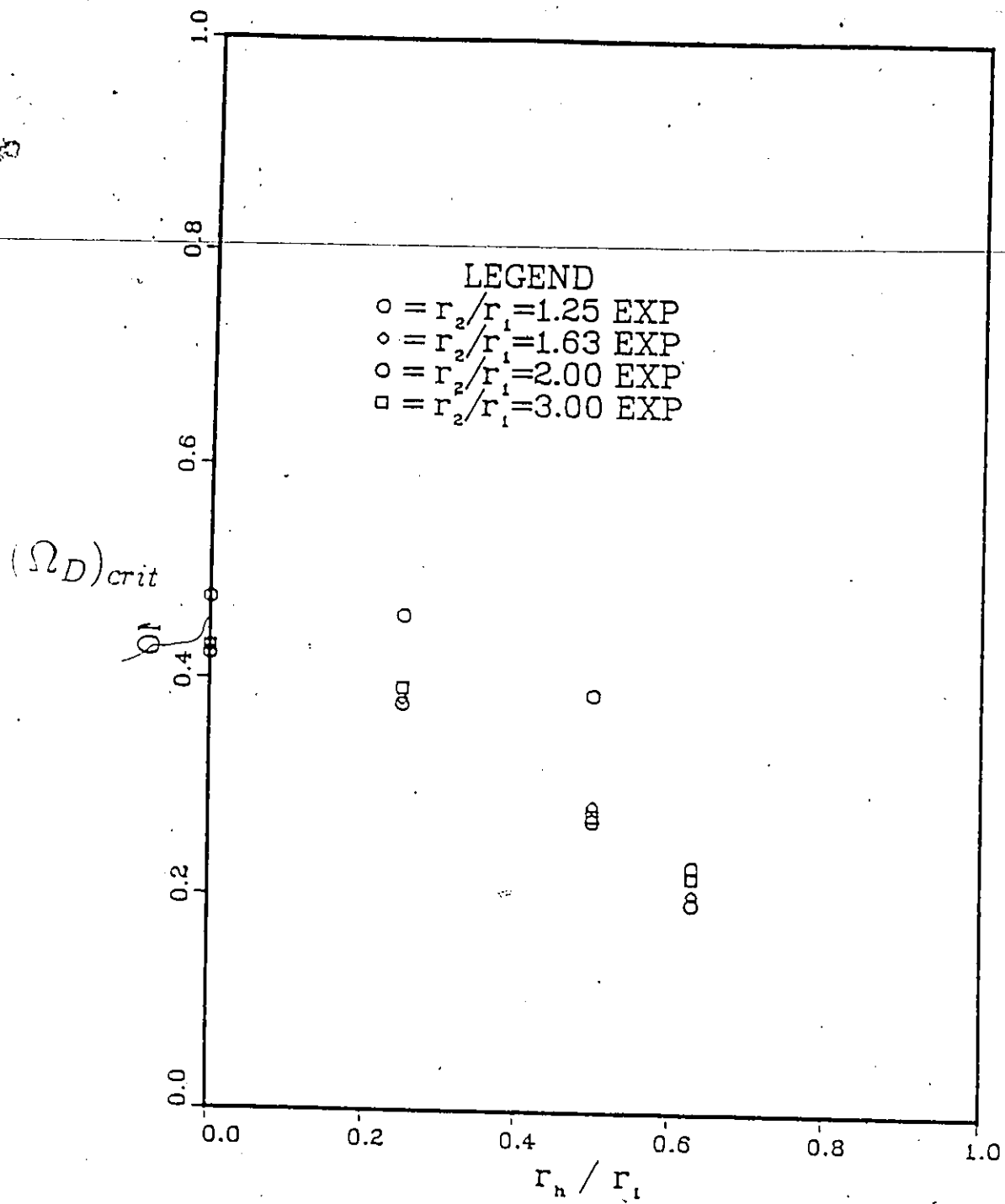


Figure 5.2: $(\Omega_D)_{crit}$ vs. $\frac{\Gamma_h}{\Gamma_l}$

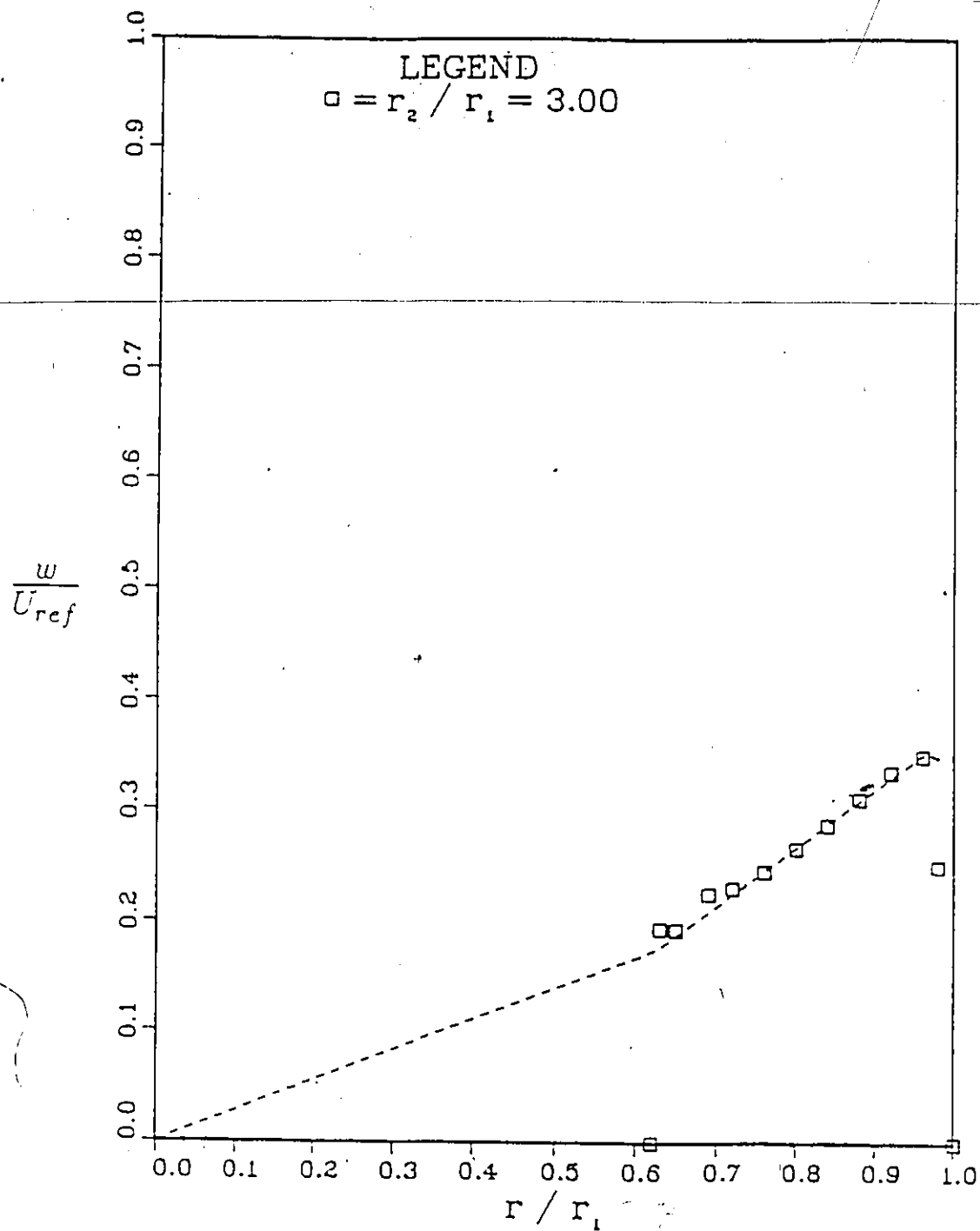


Figure 5.3: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 3.00$

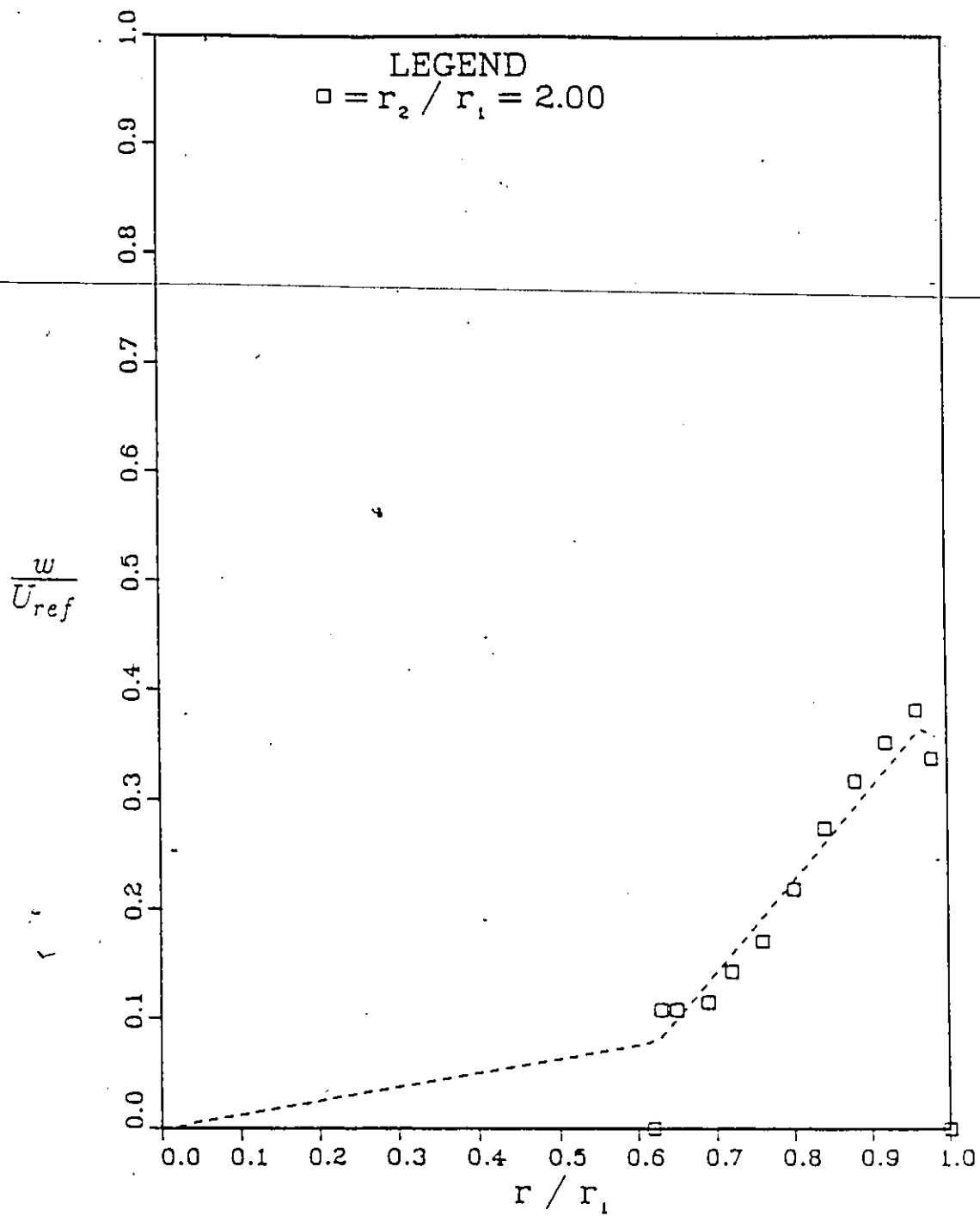


Figure 5.4: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 2.00$

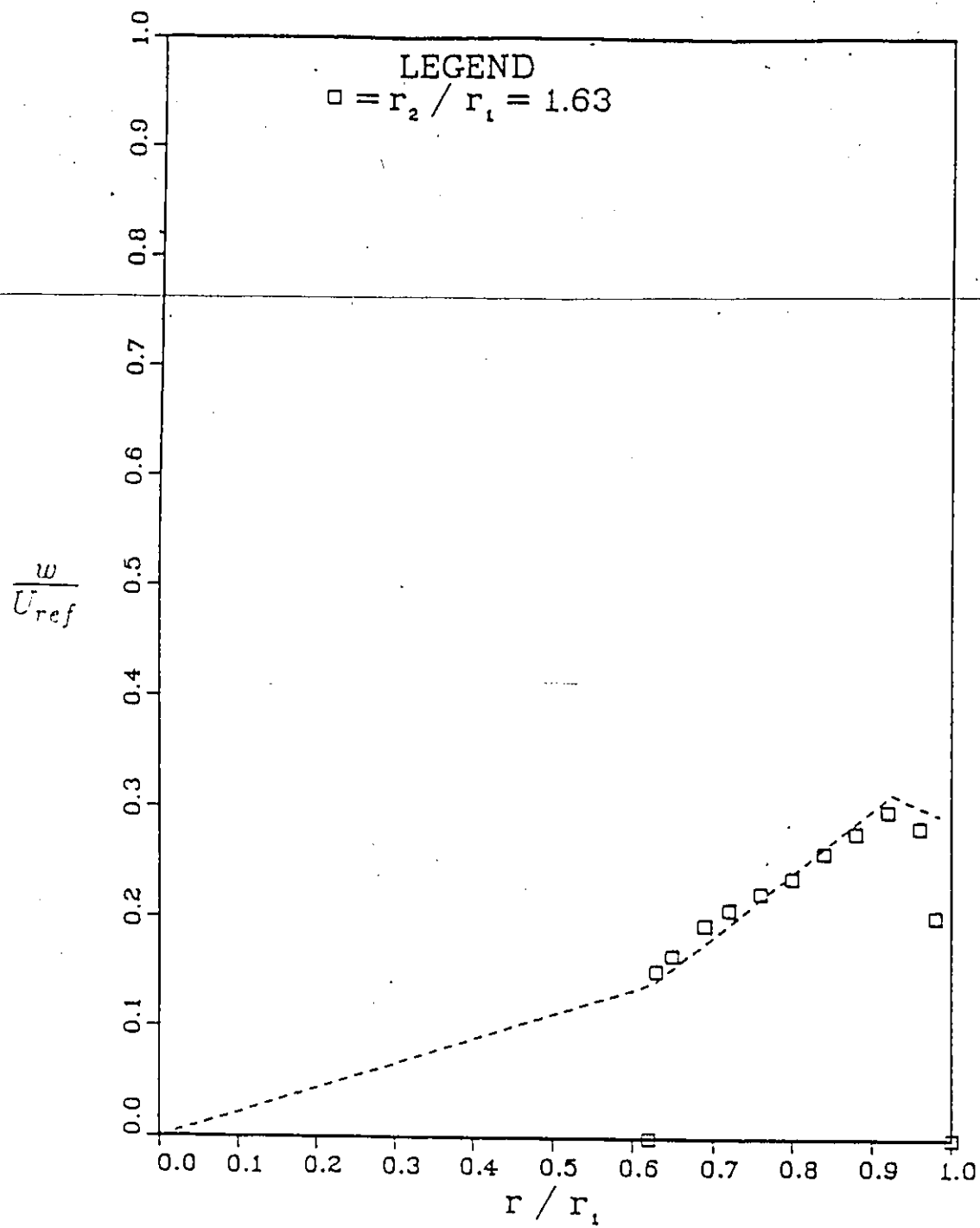


Figure 5.5: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 1.63$

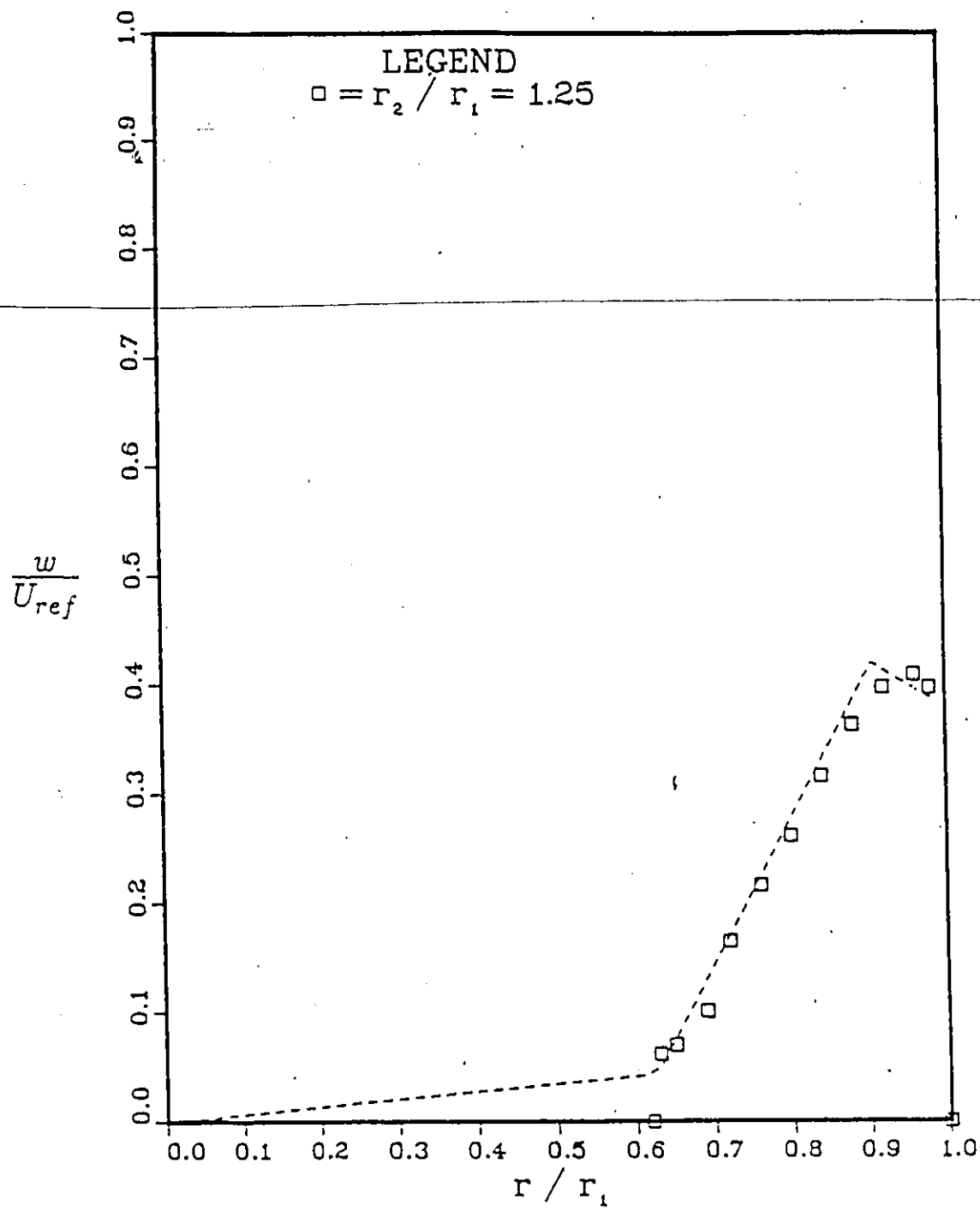


Figure 5.6: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_A}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 1.25$

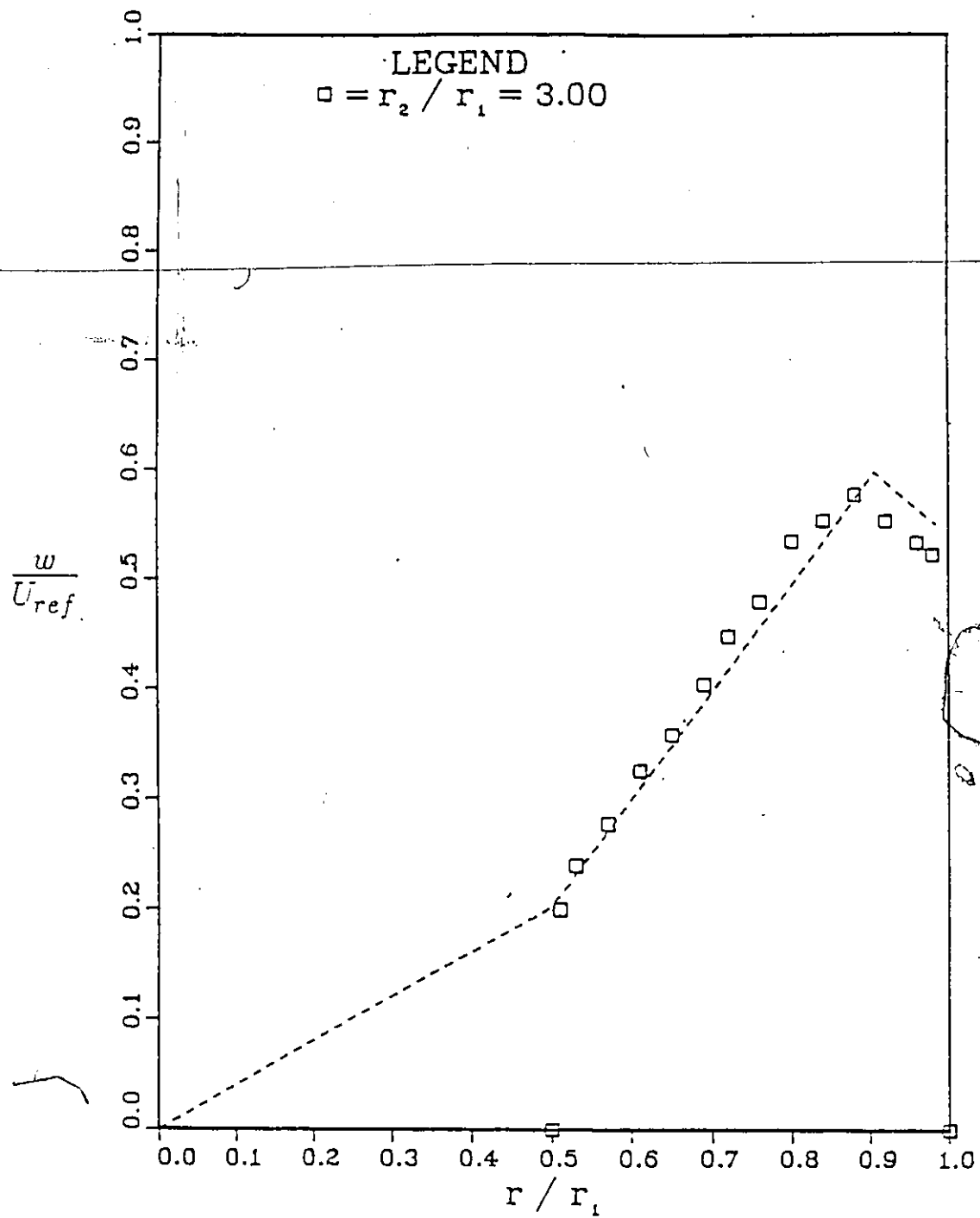


Figure 5.7: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 3.00$

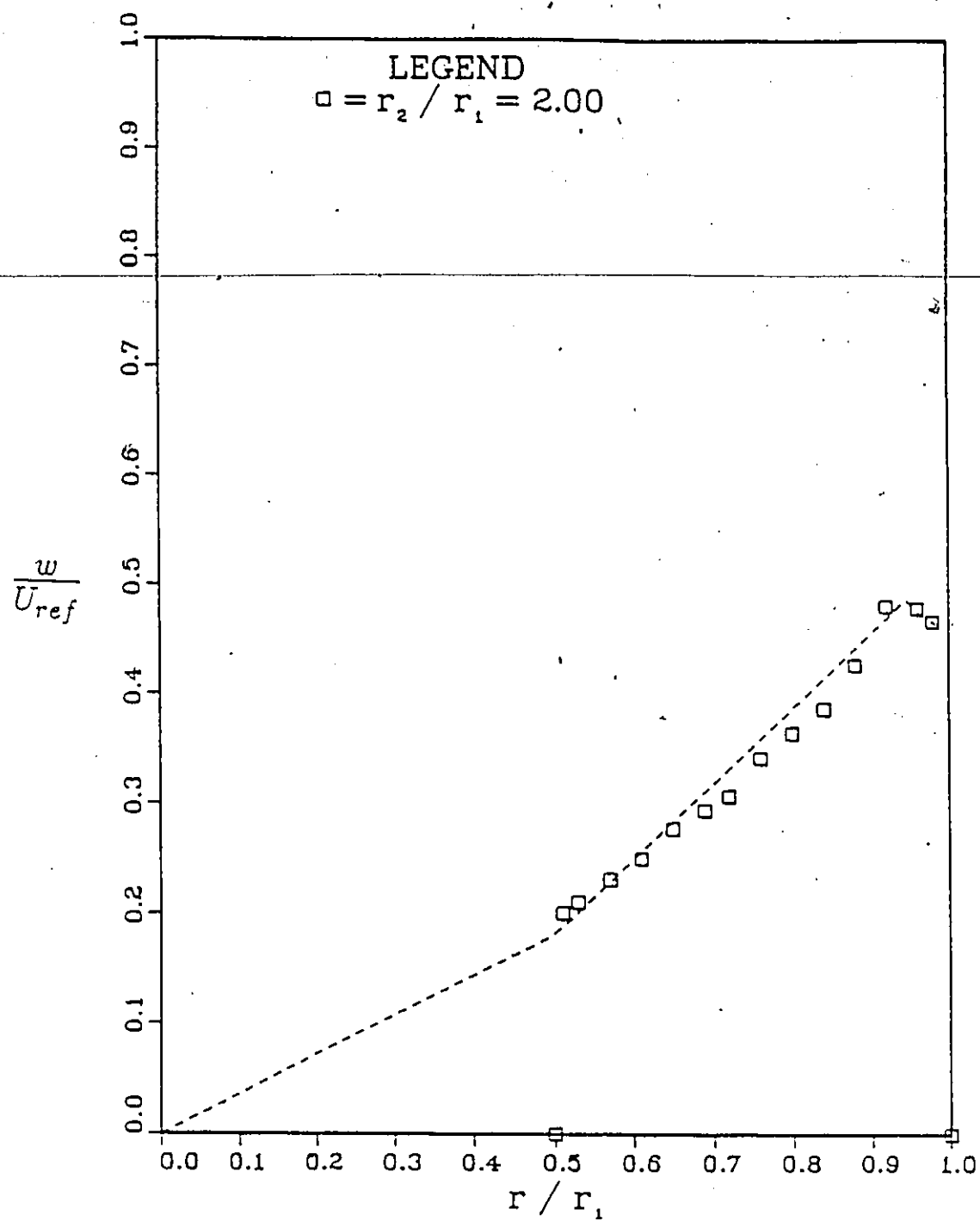


Figure 5.8: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 2.00$

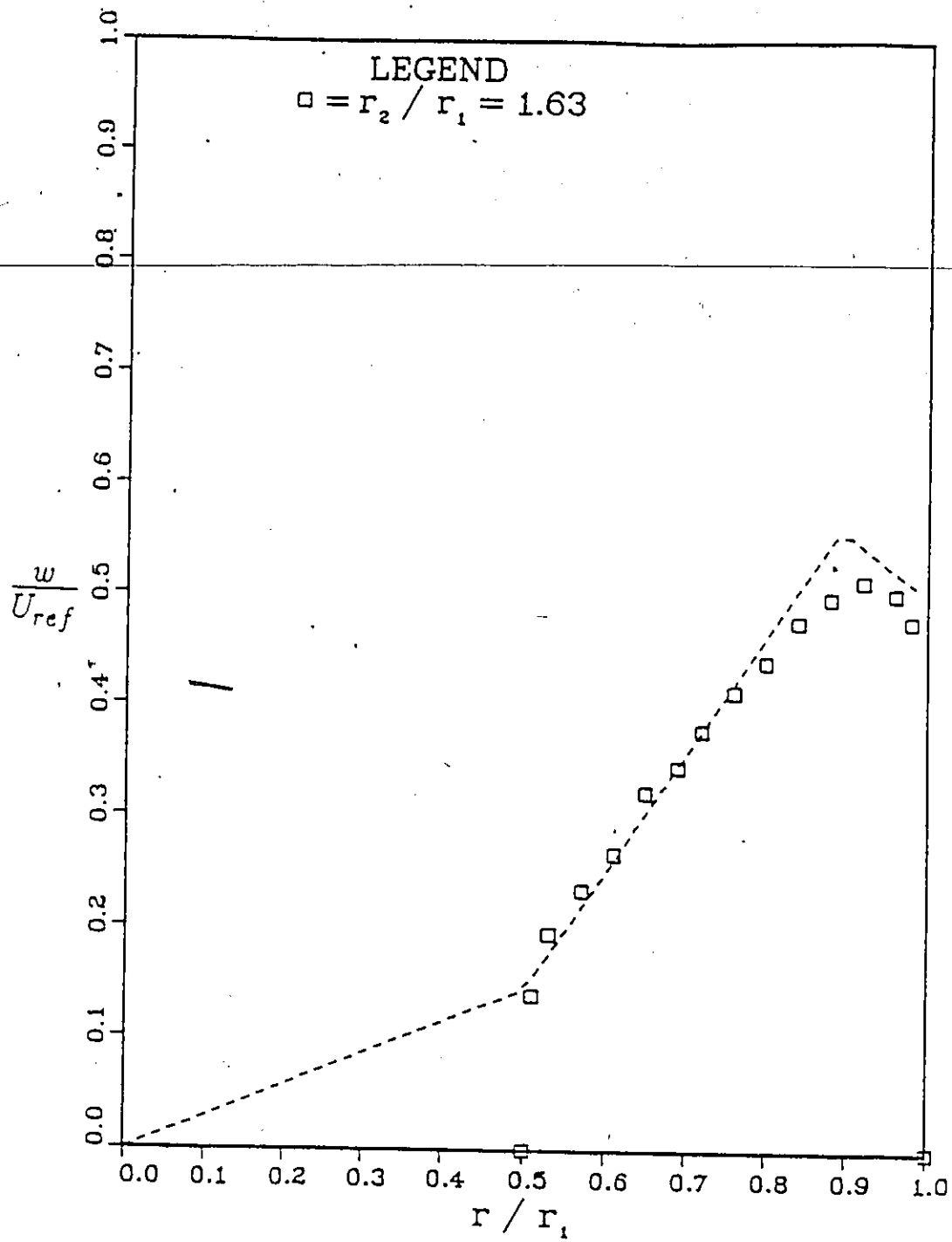


Figure 5.9: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 1.63$

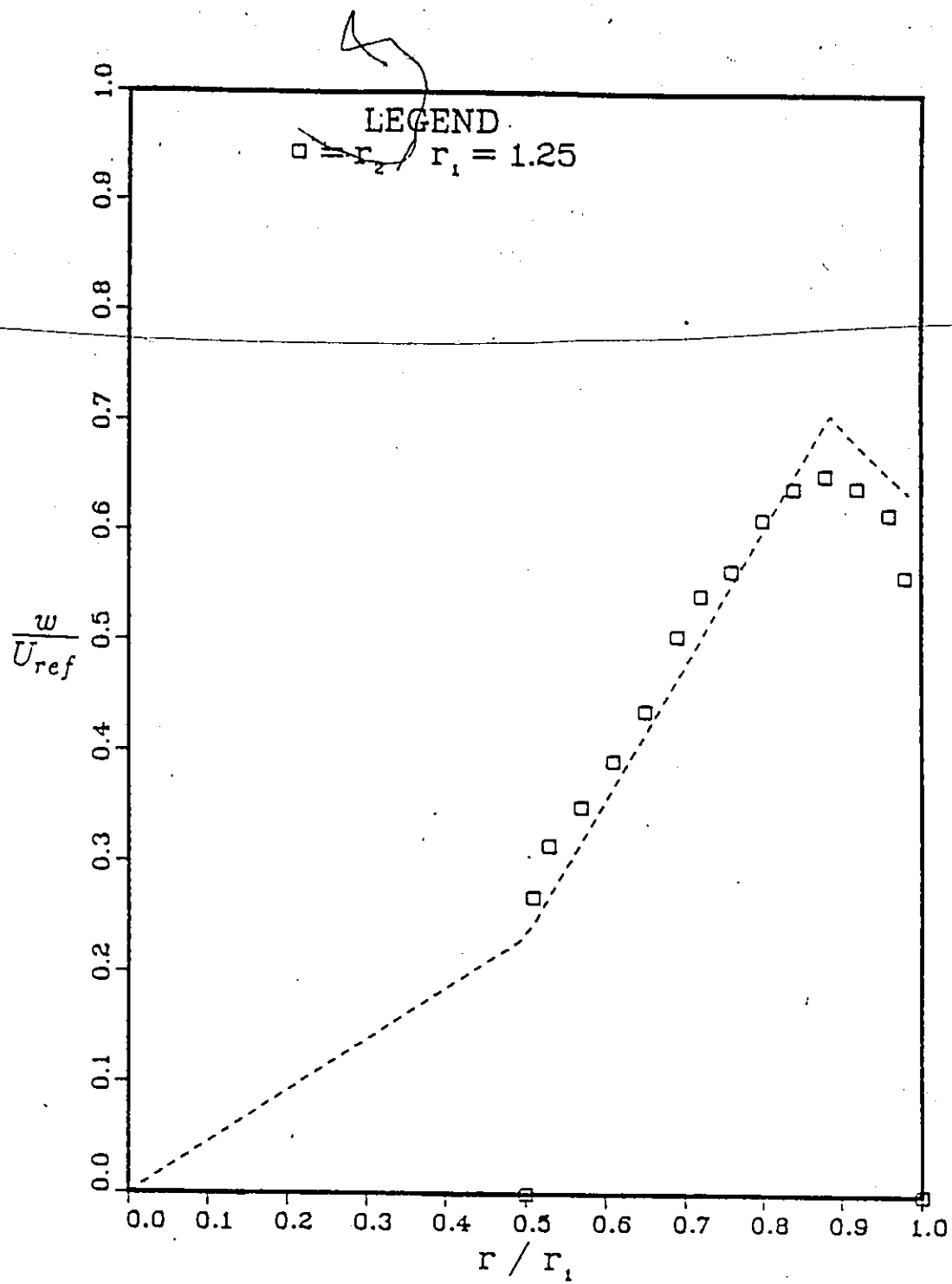


Figure 5.10: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 1.25$

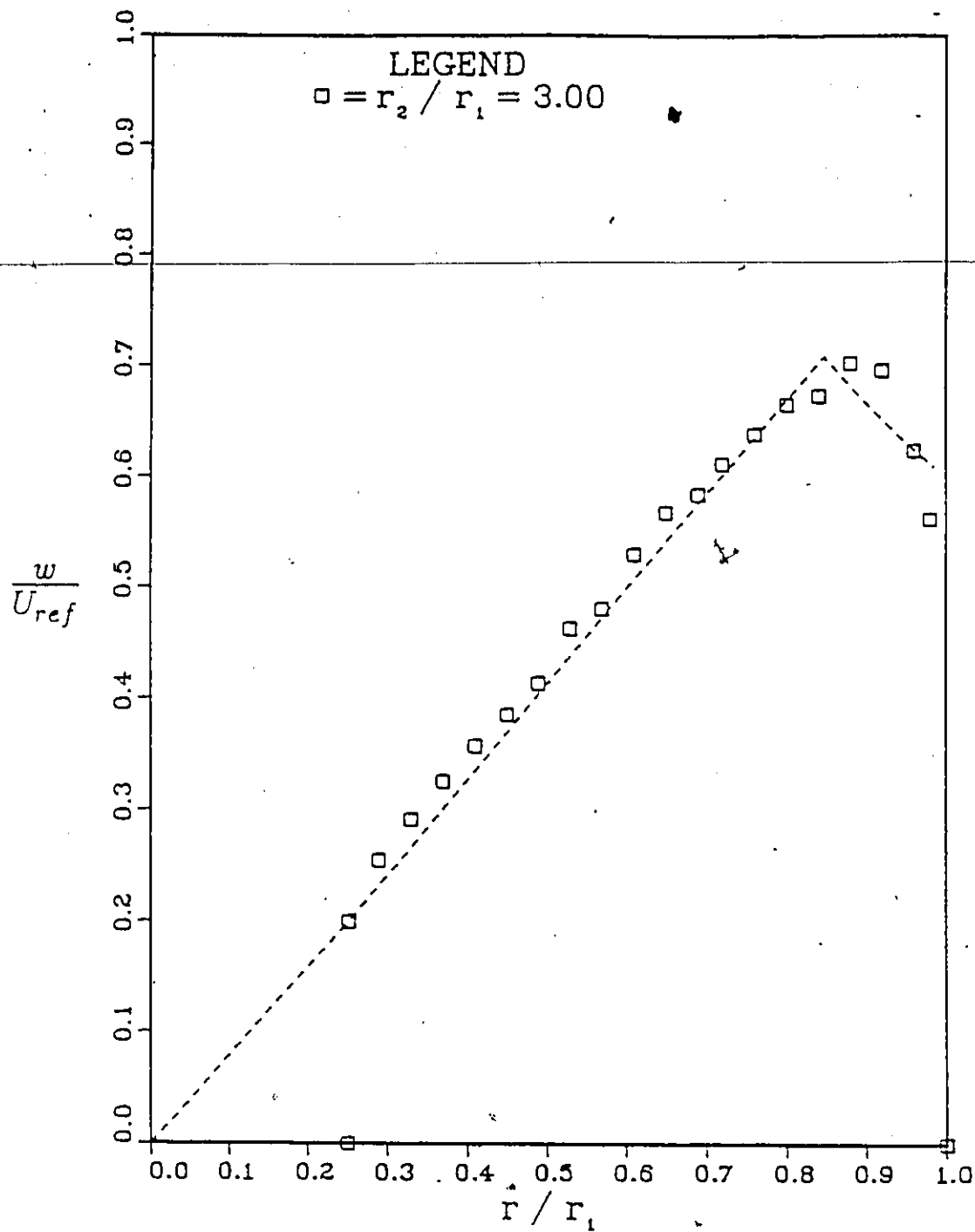


Figure 5.11: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 3.00$

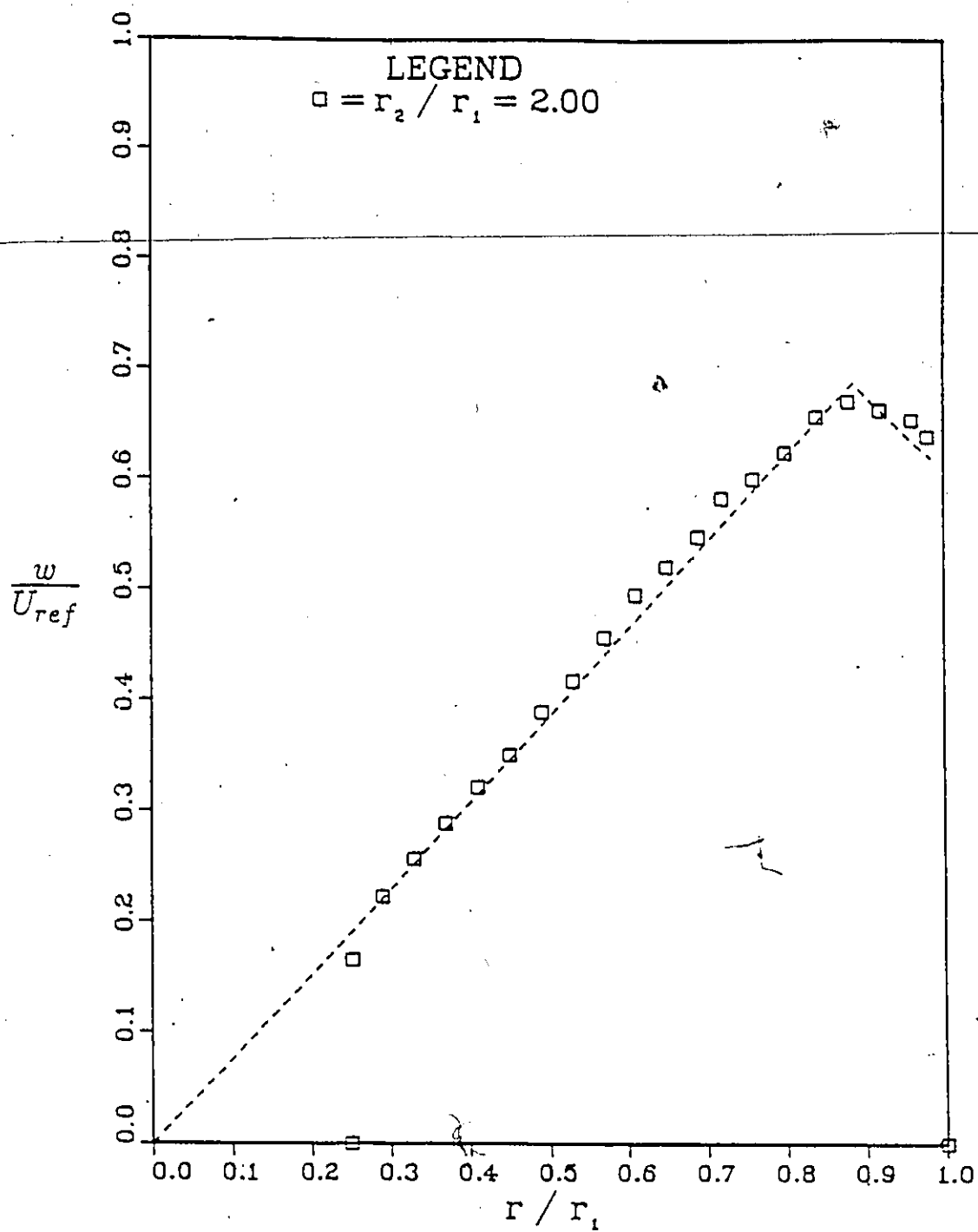


Figure 5.12: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 2.00$

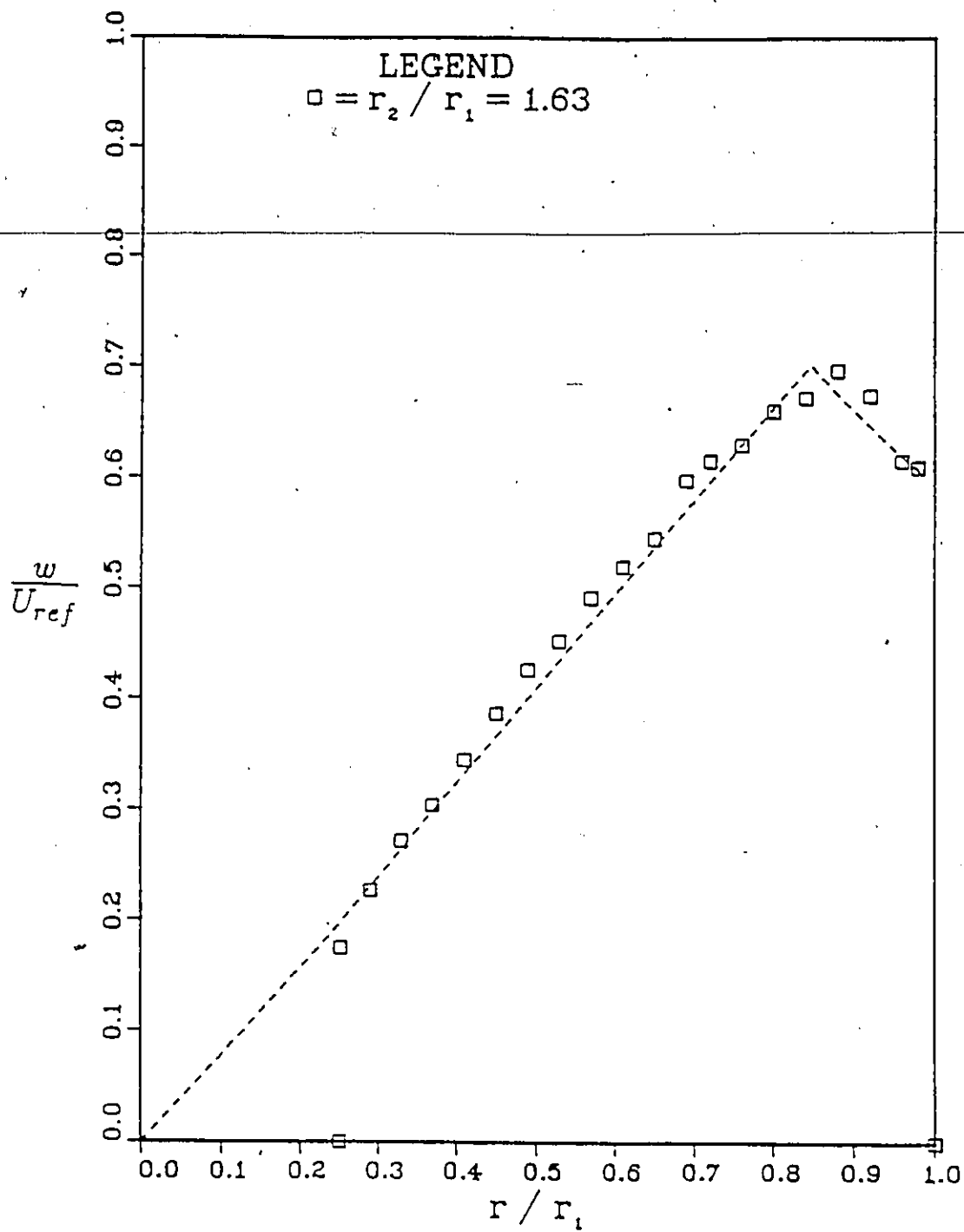


Figure 5.13: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 1.63$

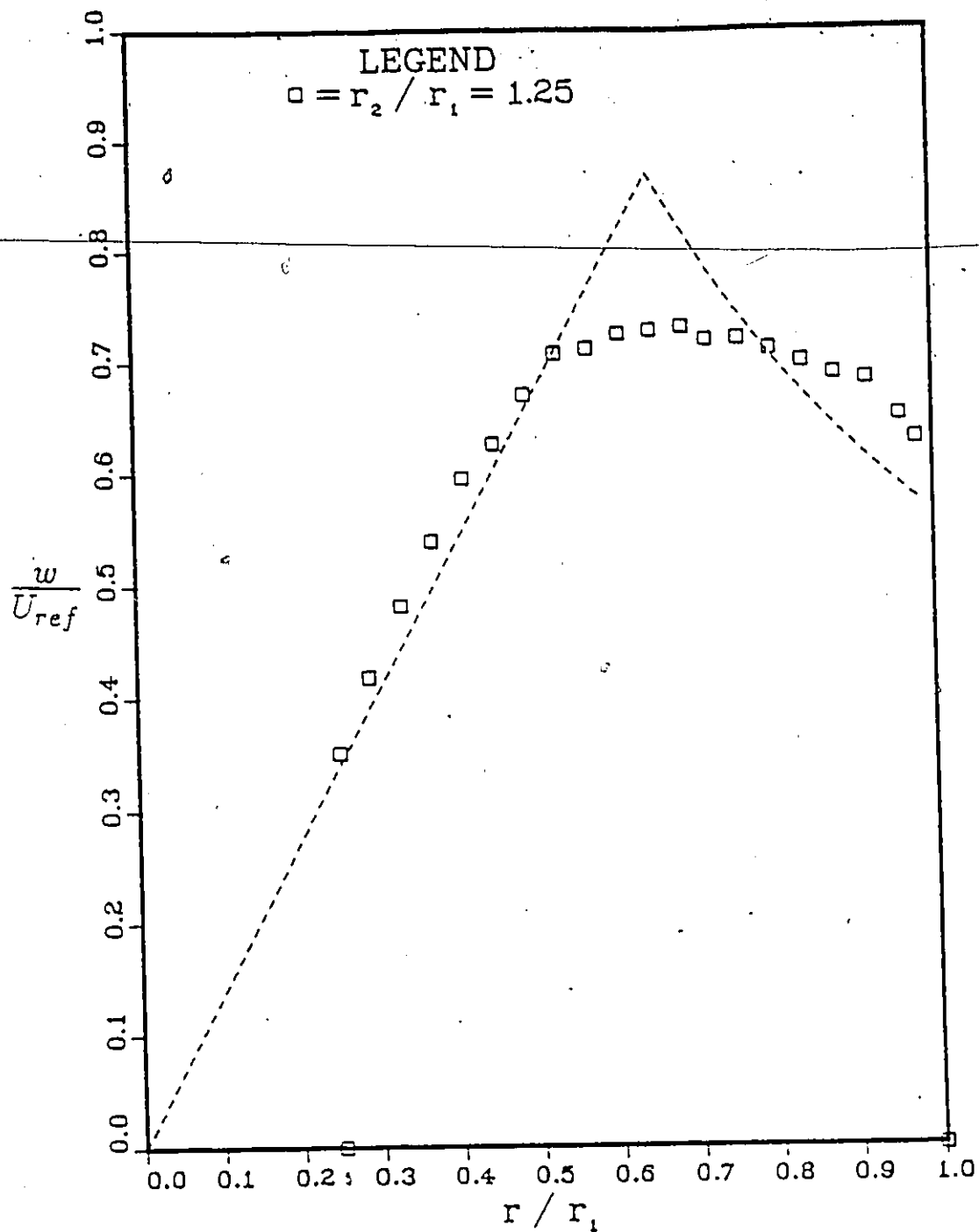


Figure 5.14: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 1.25$

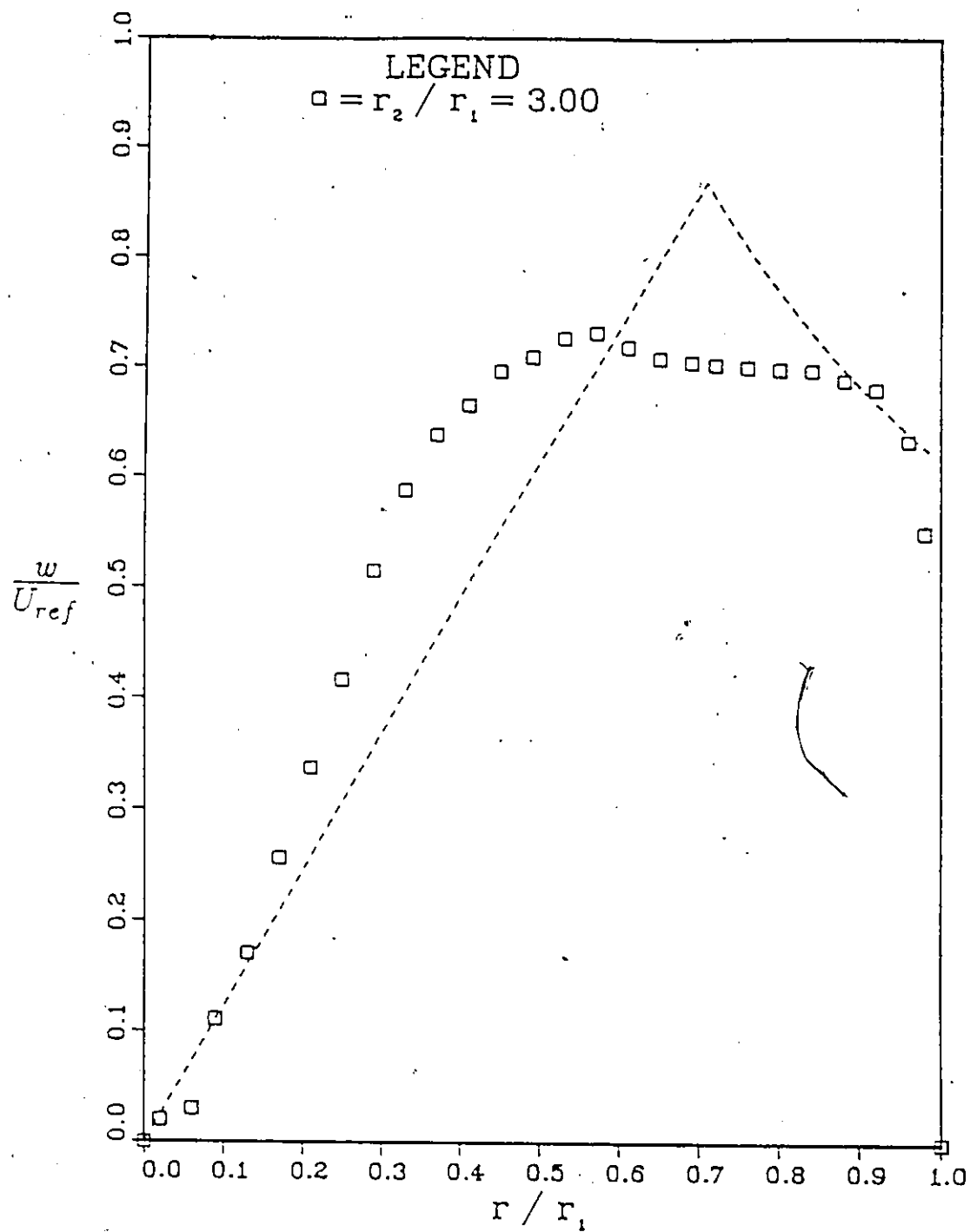


Figure 5.15: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 3.00$

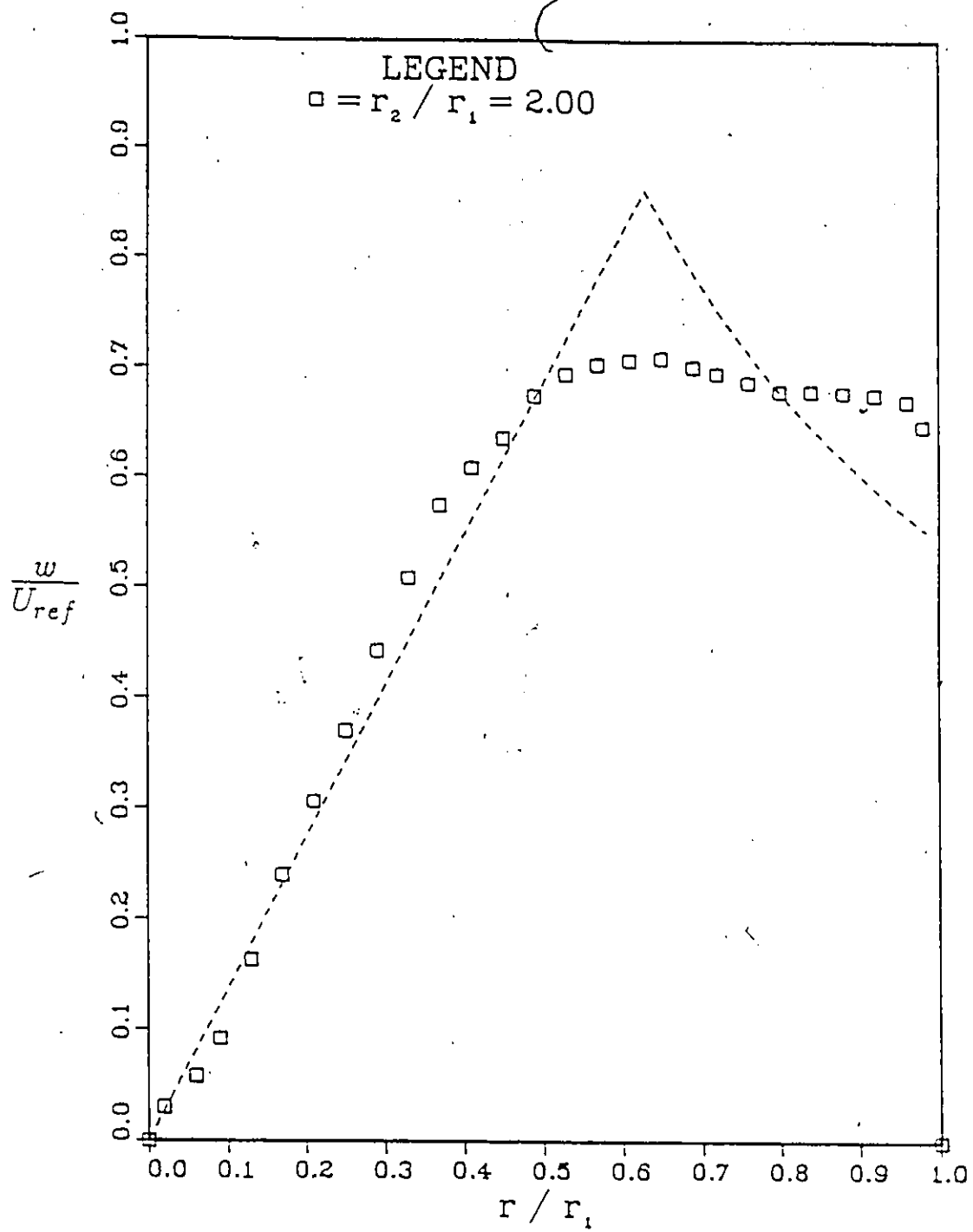


Figure 5.16: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 2.00$

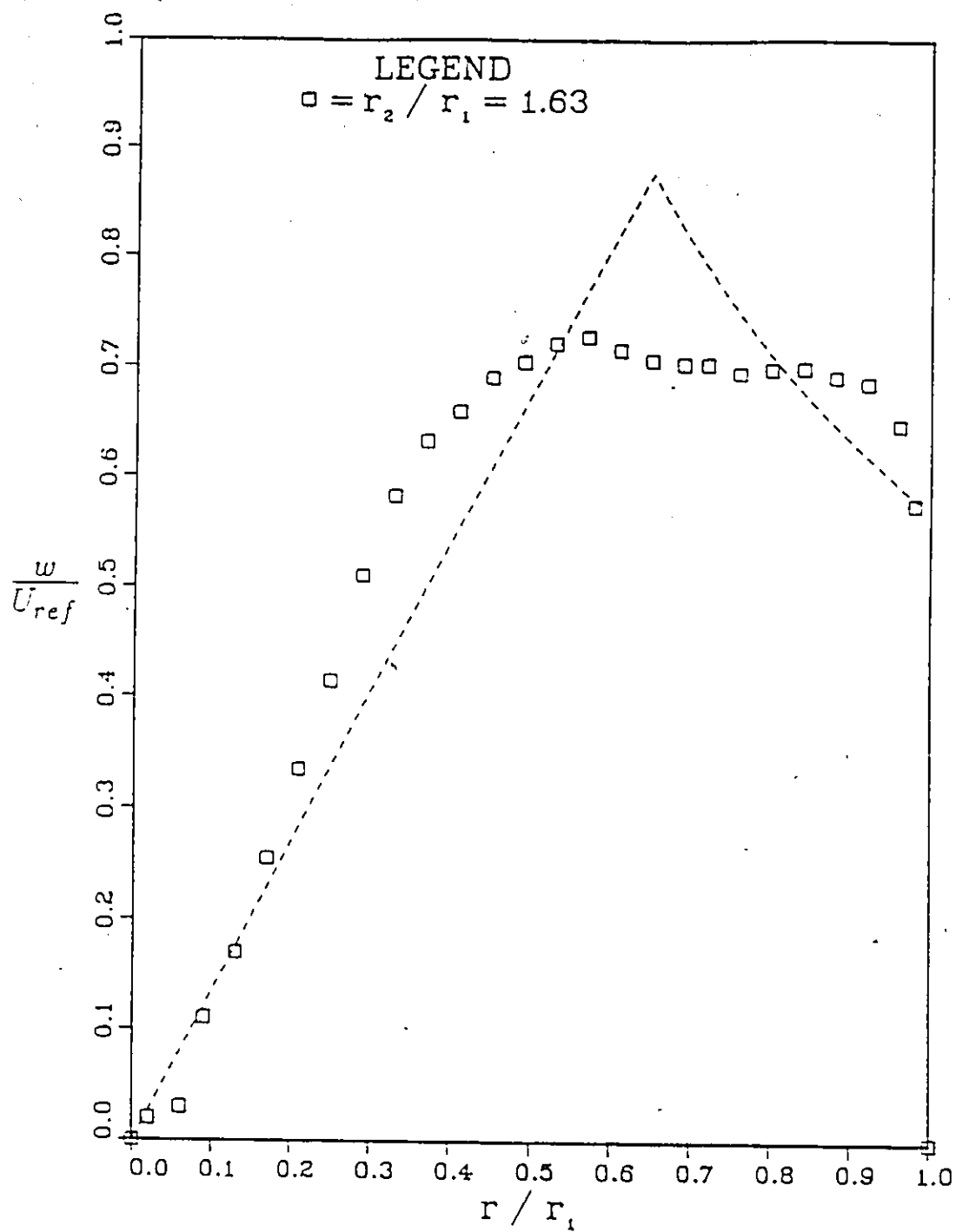


Figure 5.17: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 1.63$

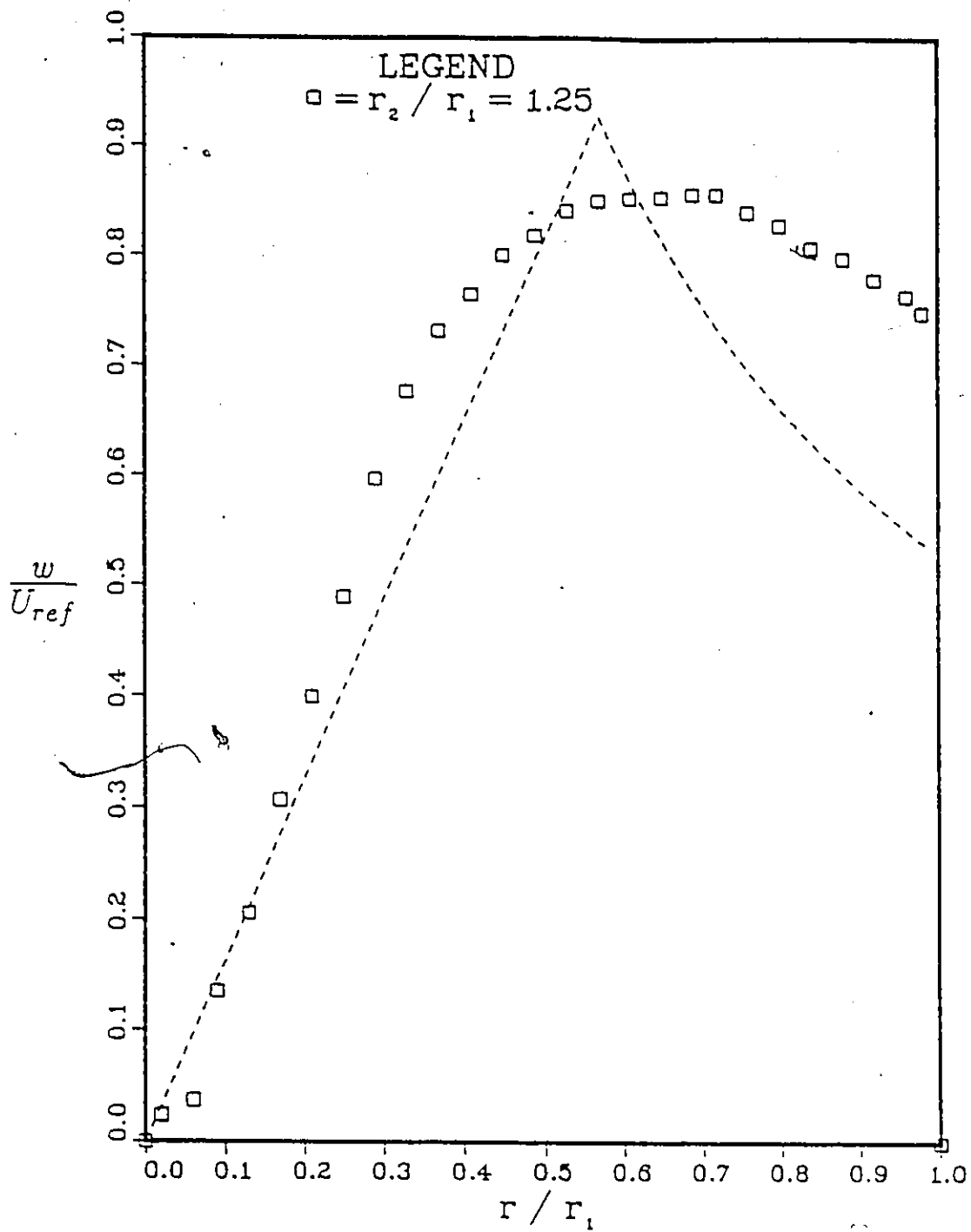


Figure 5.18: $\frac{w}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 1.25$

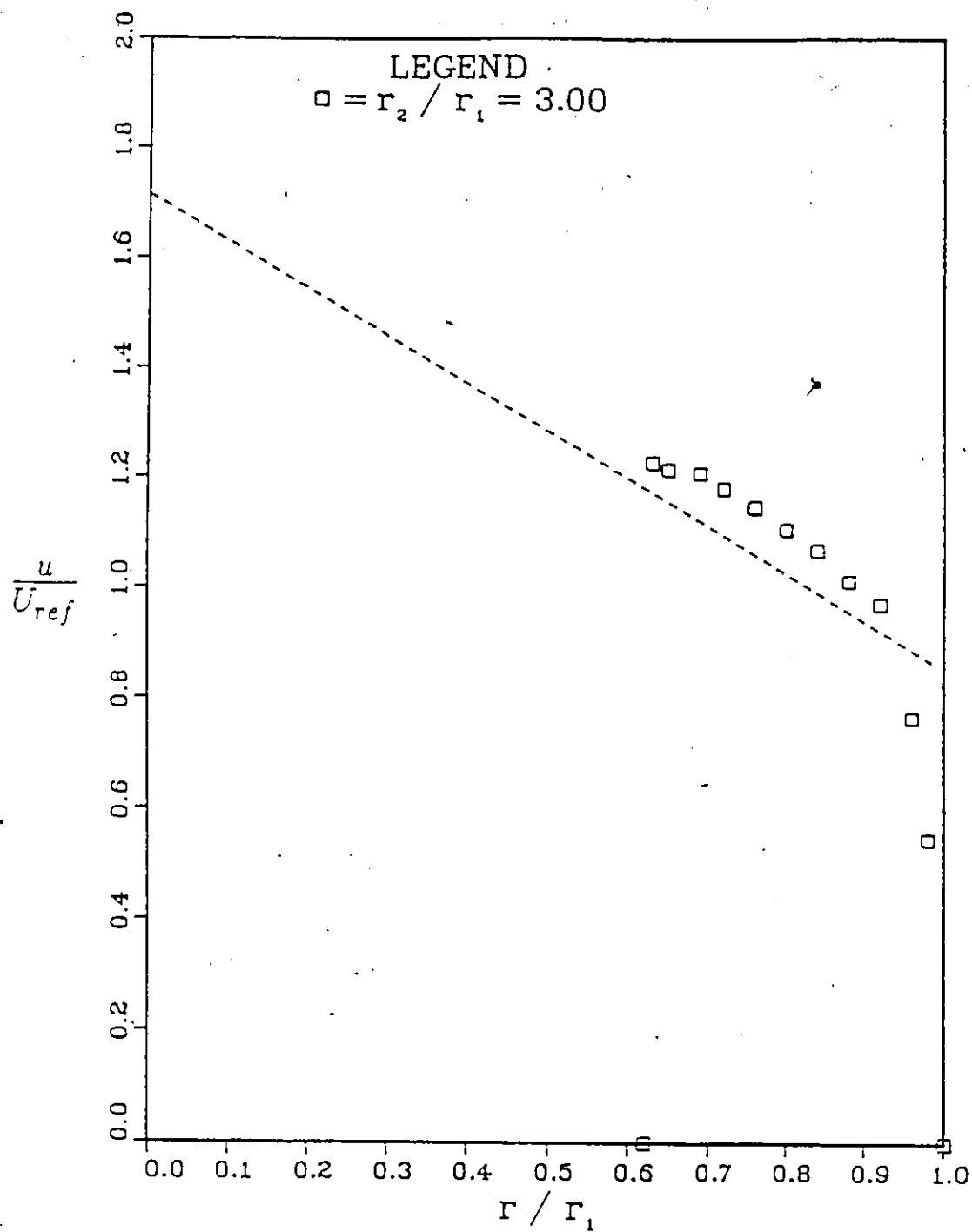


Figure 5.10: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 3.00$

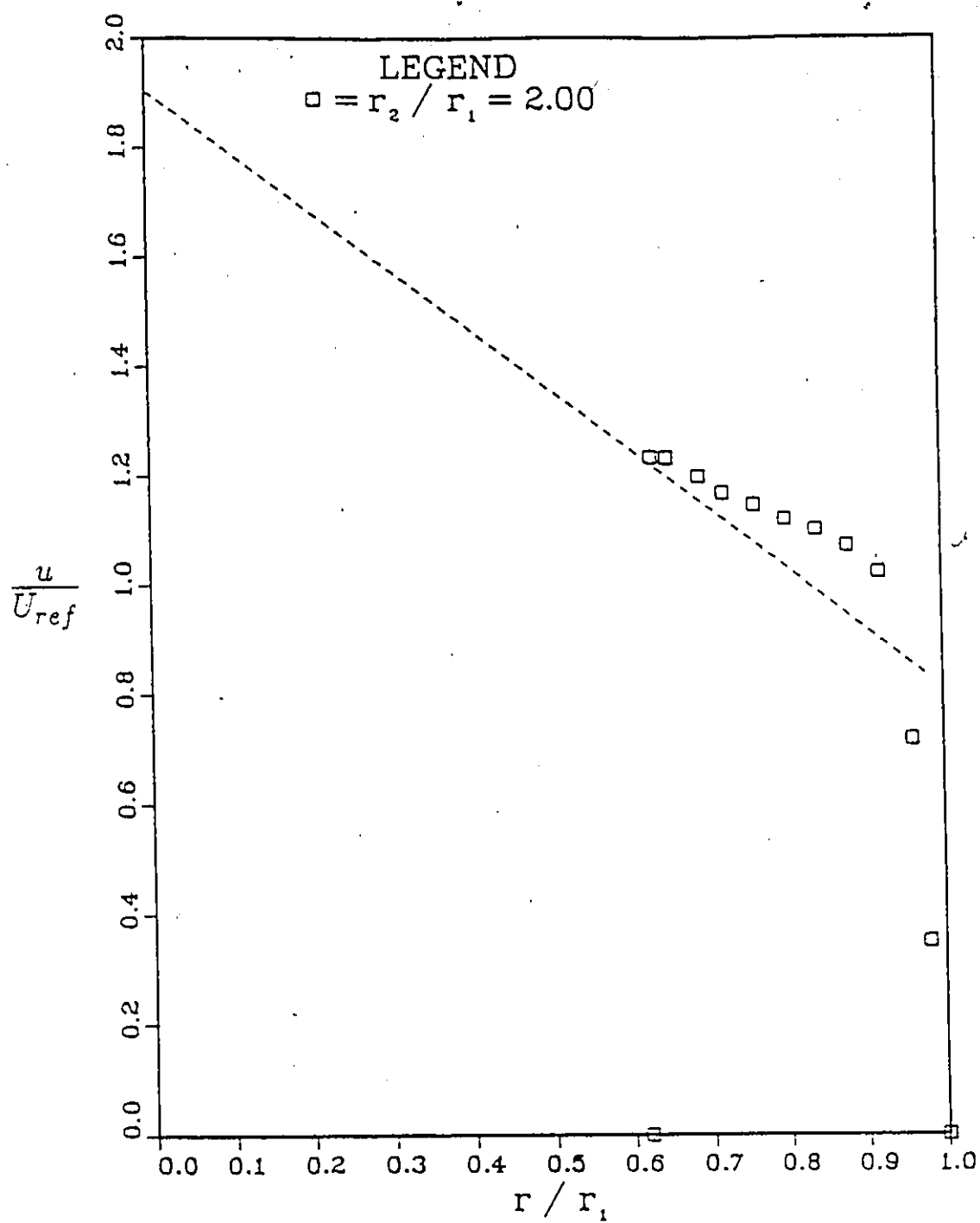


Figure 5.20: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_A}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 2.00$

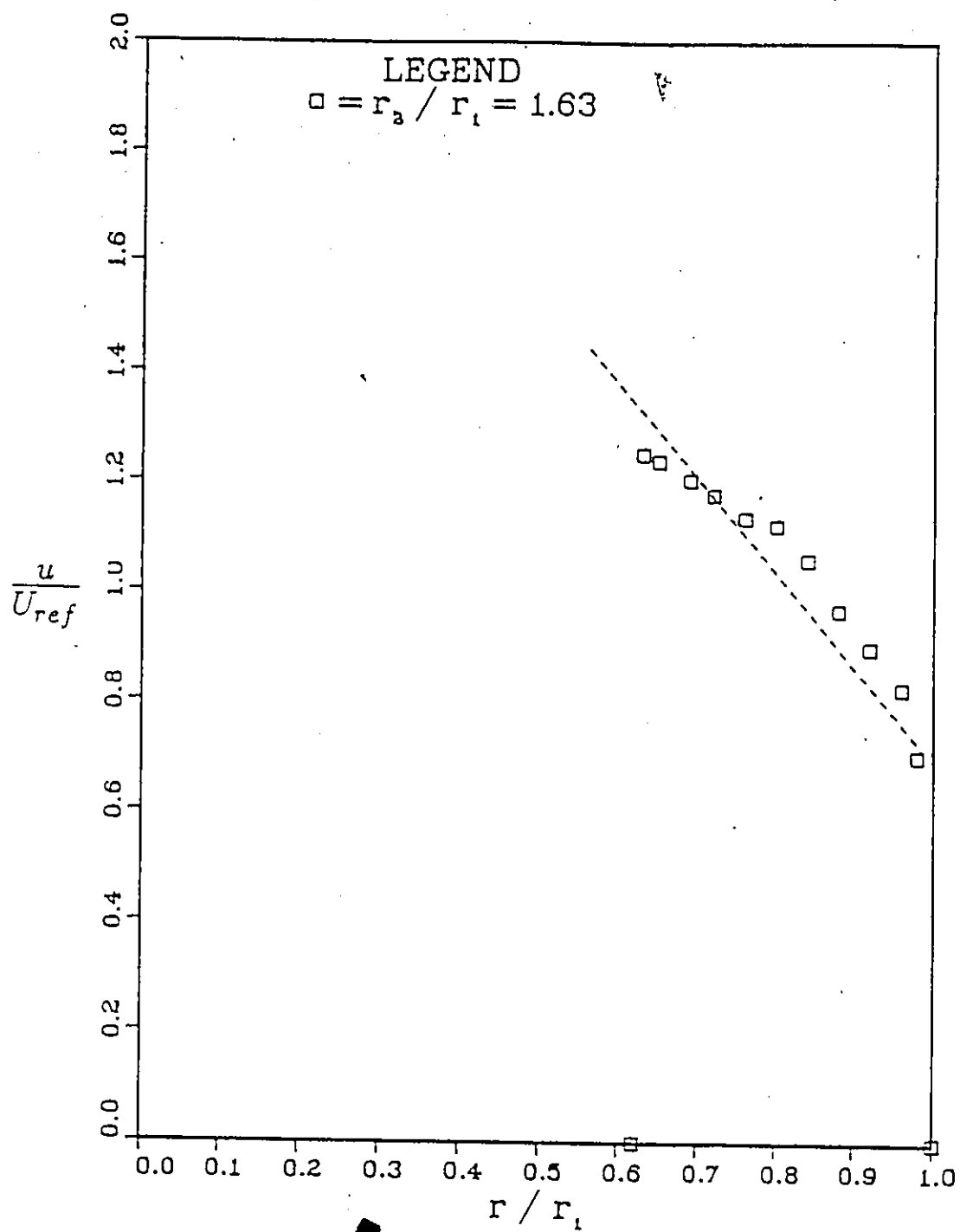


Figure 5.21: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 1.63$

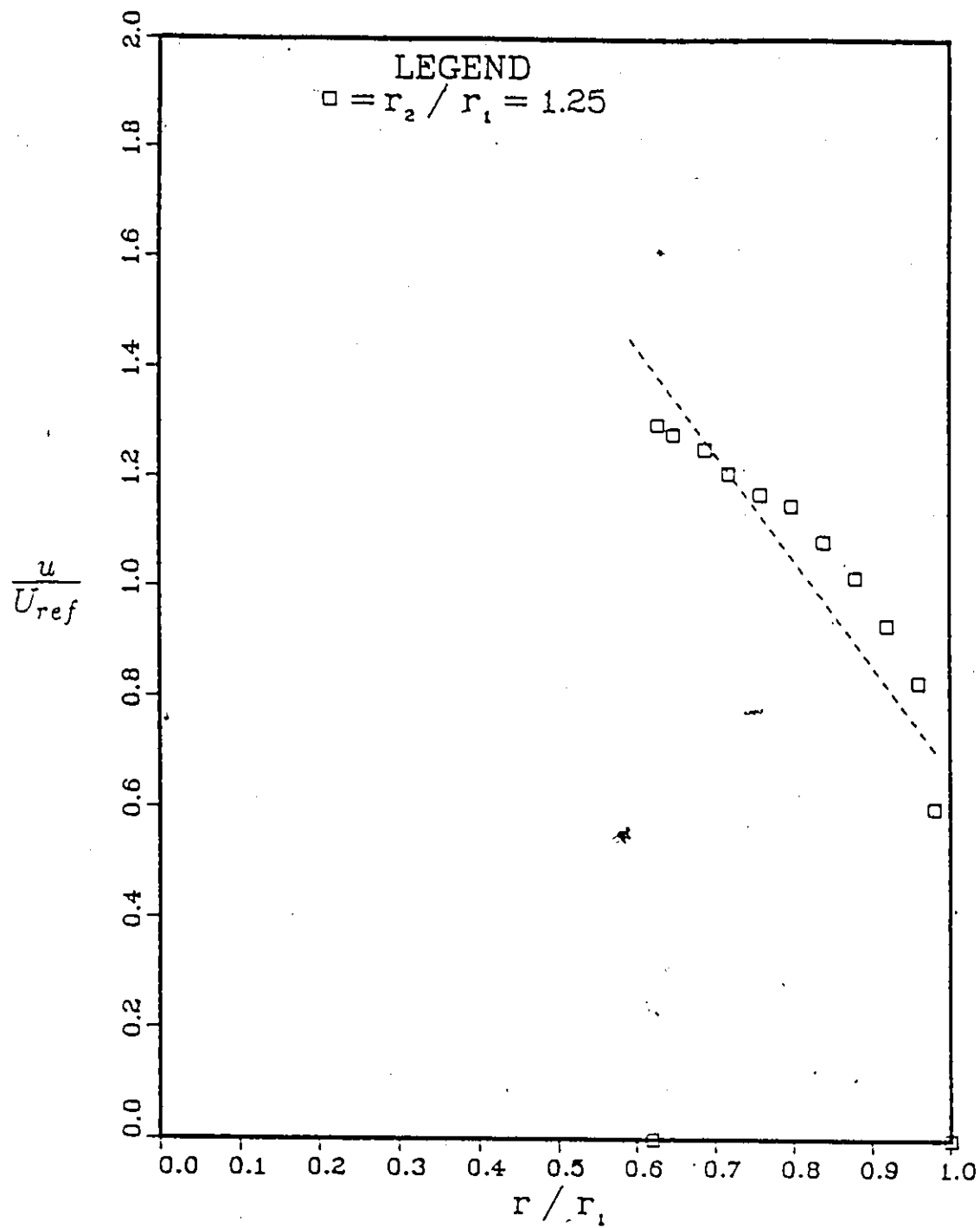


Figure 5.22: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.63$ and $\frac{r_2}{r_1} = 1.25$

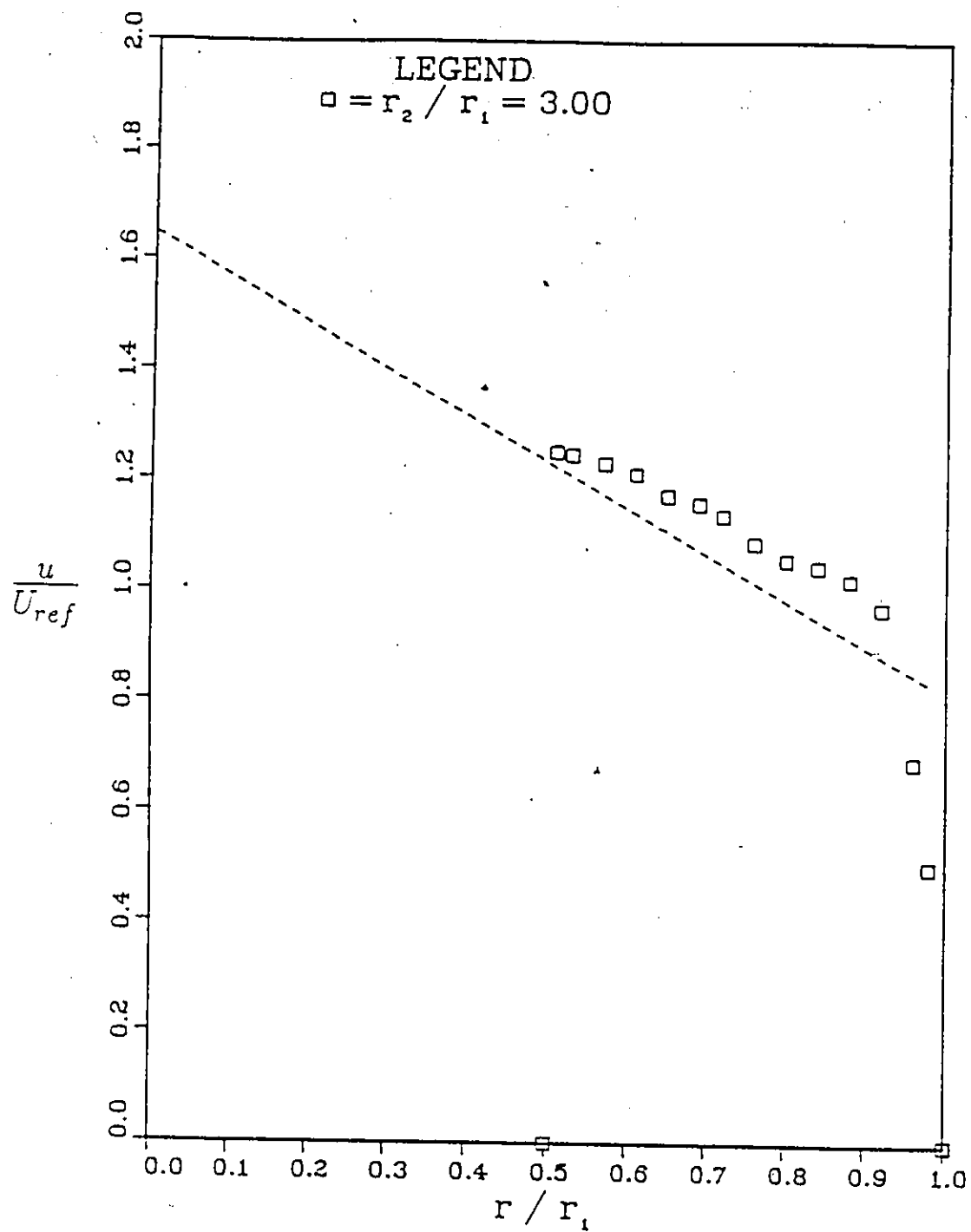


Figure 5.23: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 3.00$

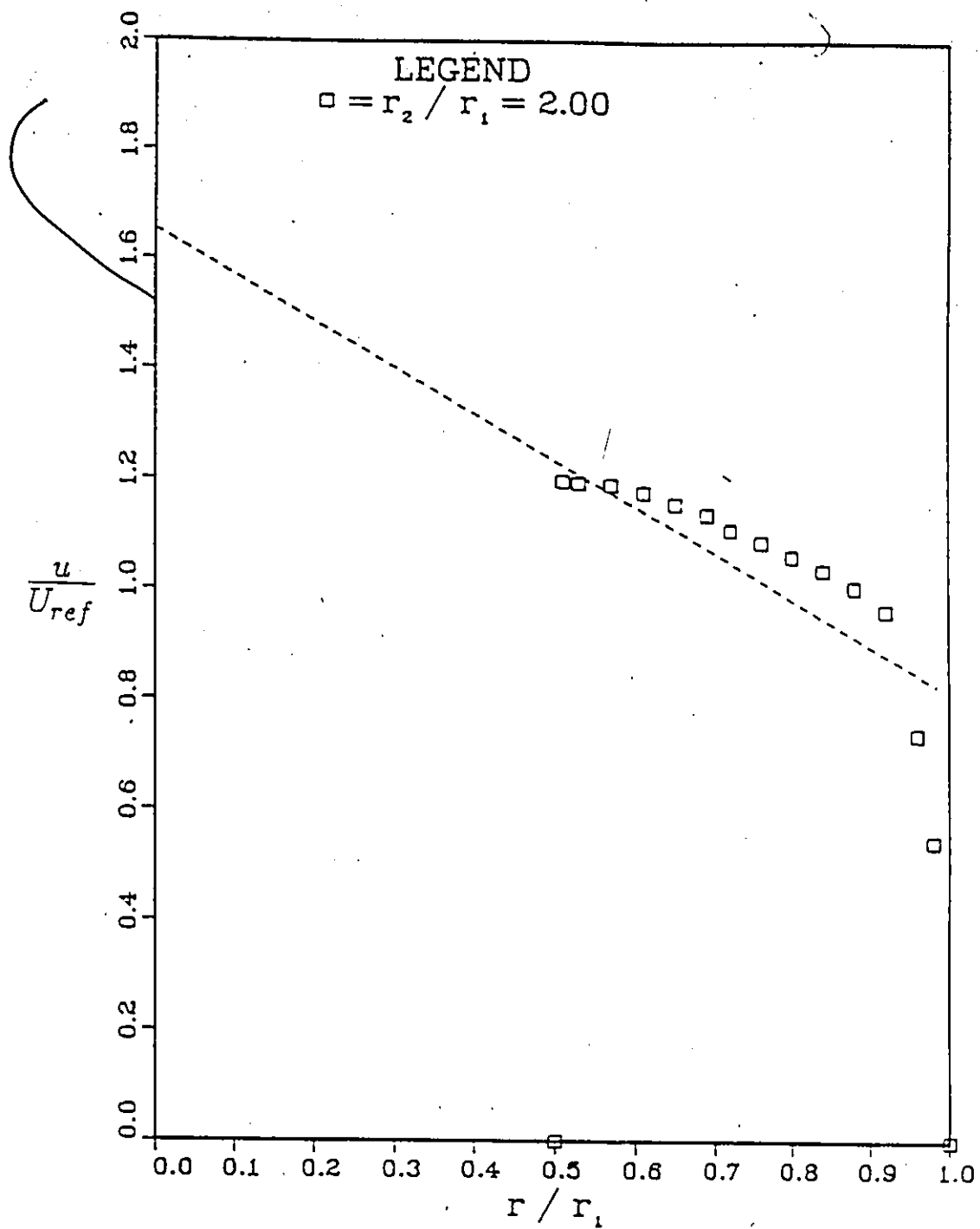


Figure 5.24: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_A}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 2.00$

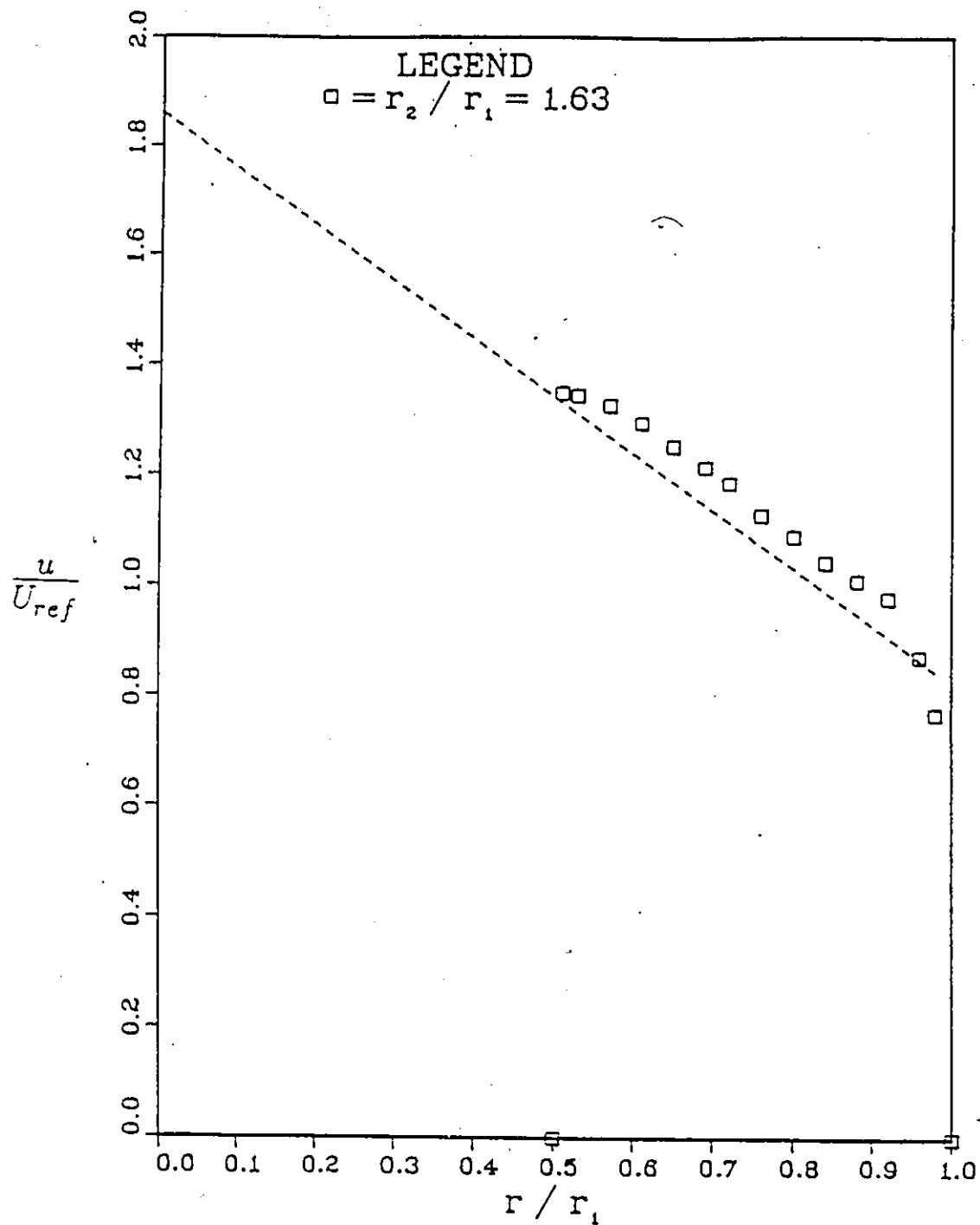


Figure 5.25: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 1.63$

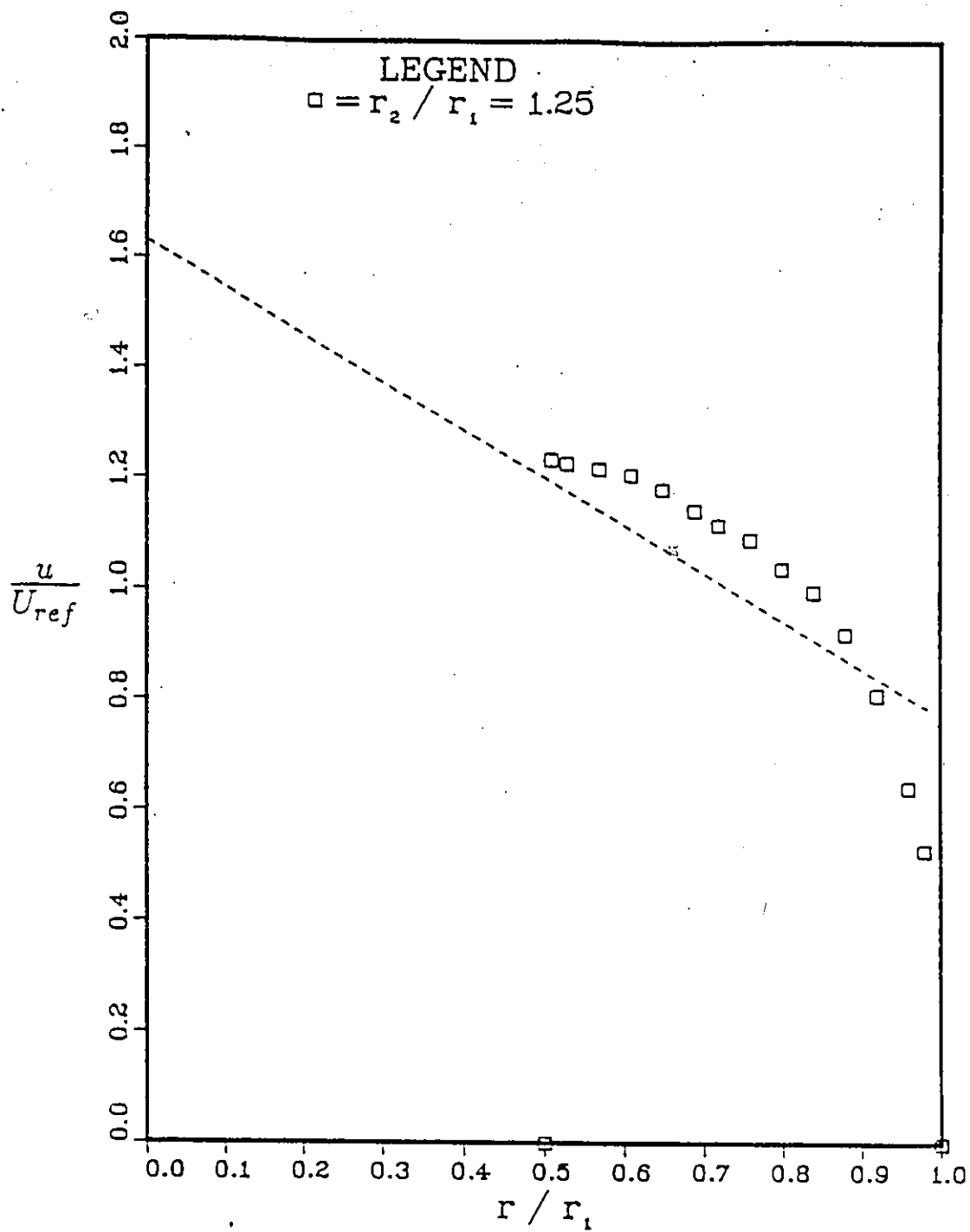


Figure 5.26: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.50$ and $\frac{r_2}{r_1} = 1.25$

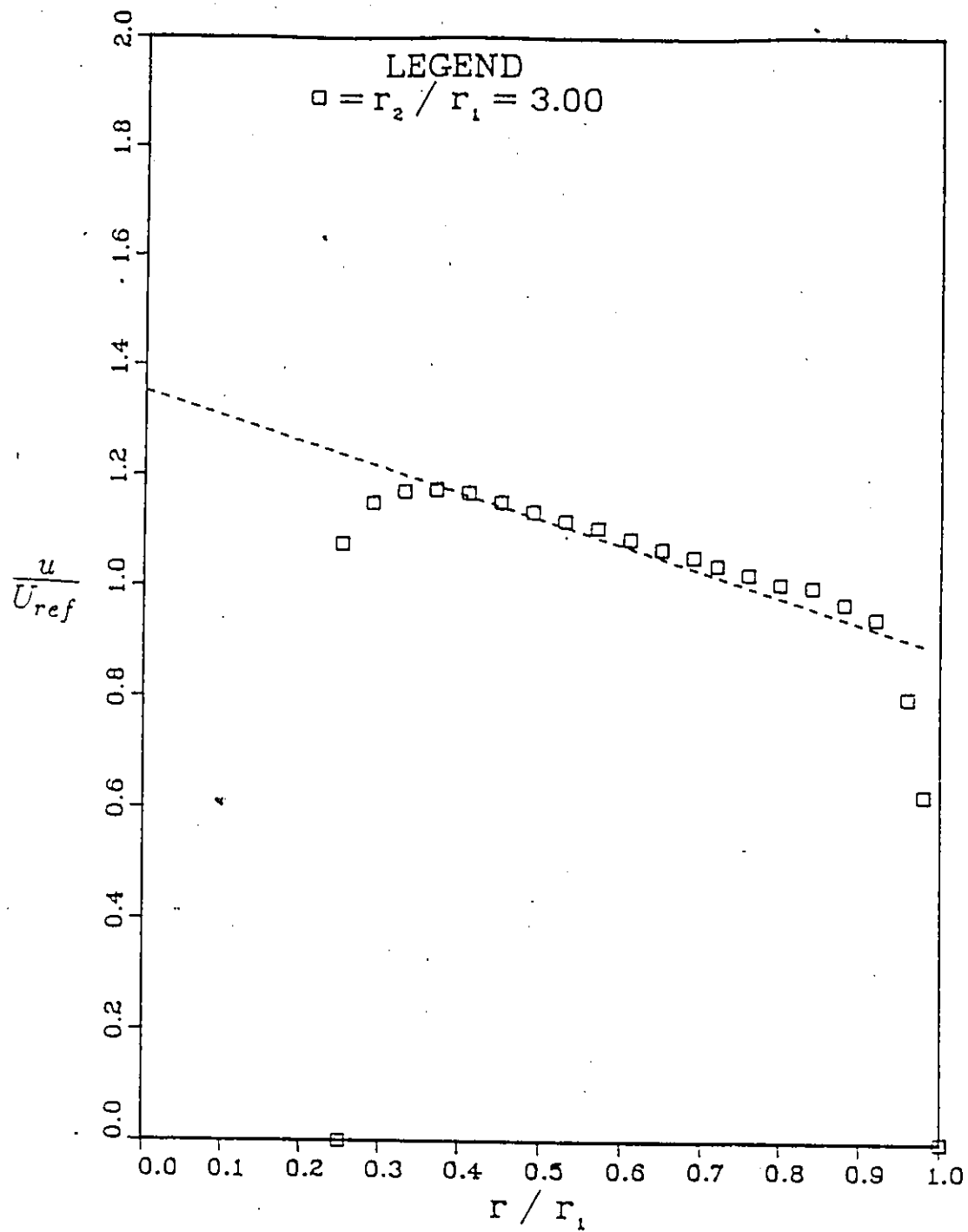


Figure 5.27: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 3.00$

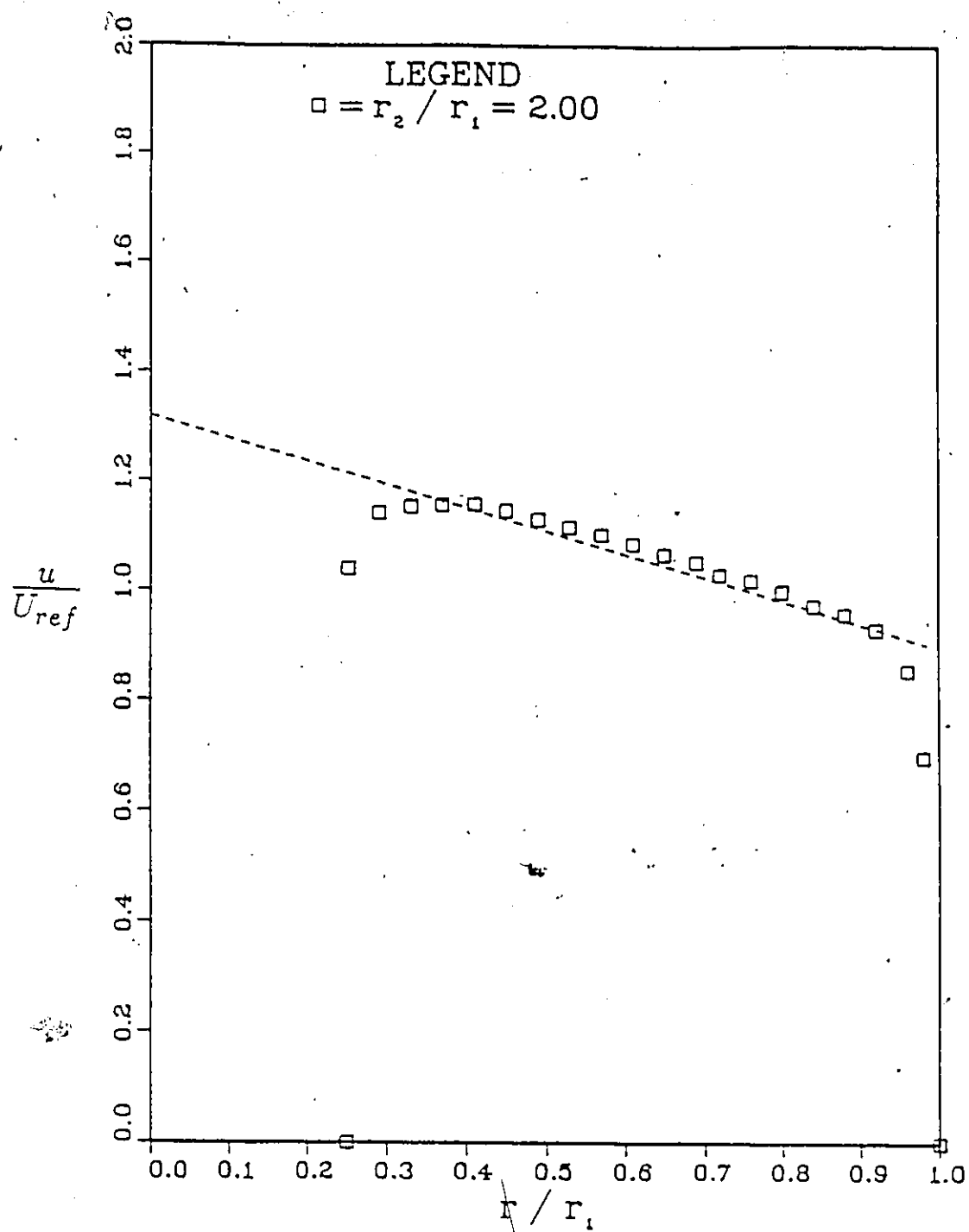


Figure 5.28: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 2.00$

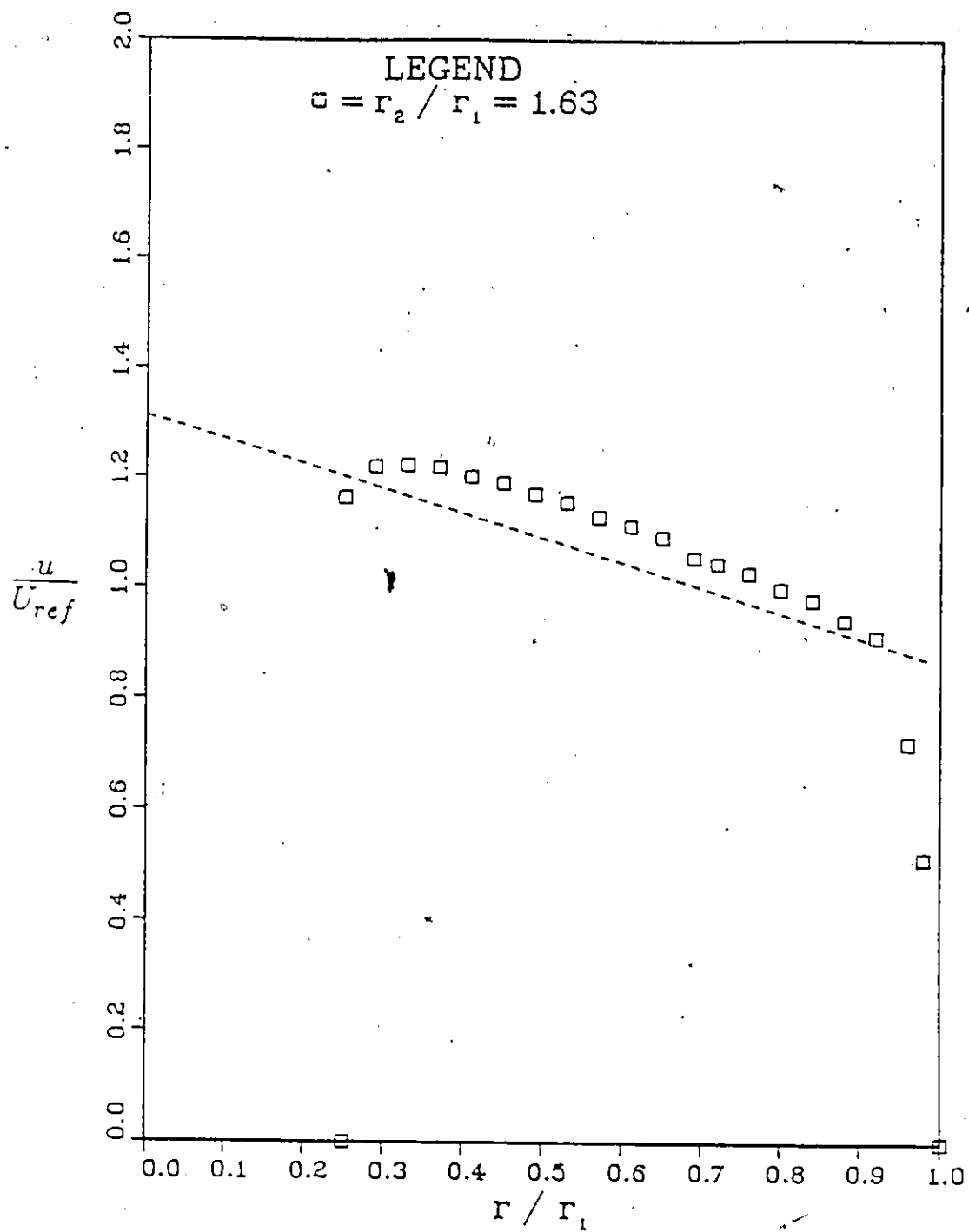


Figure 5.29: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 1.63$

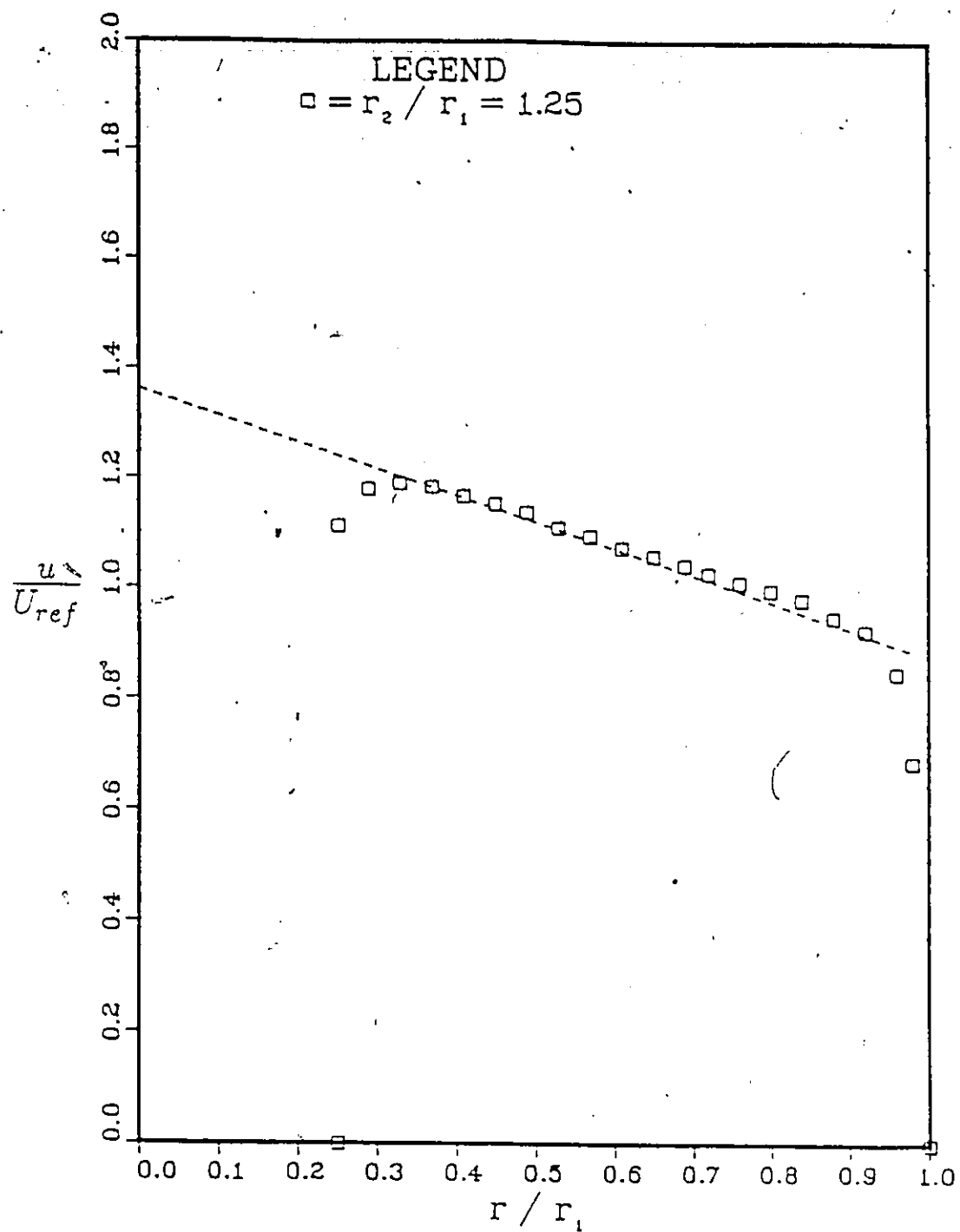


Figure 5.30: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.25$ and $\frac{r_2}{r_1} = 1.25$

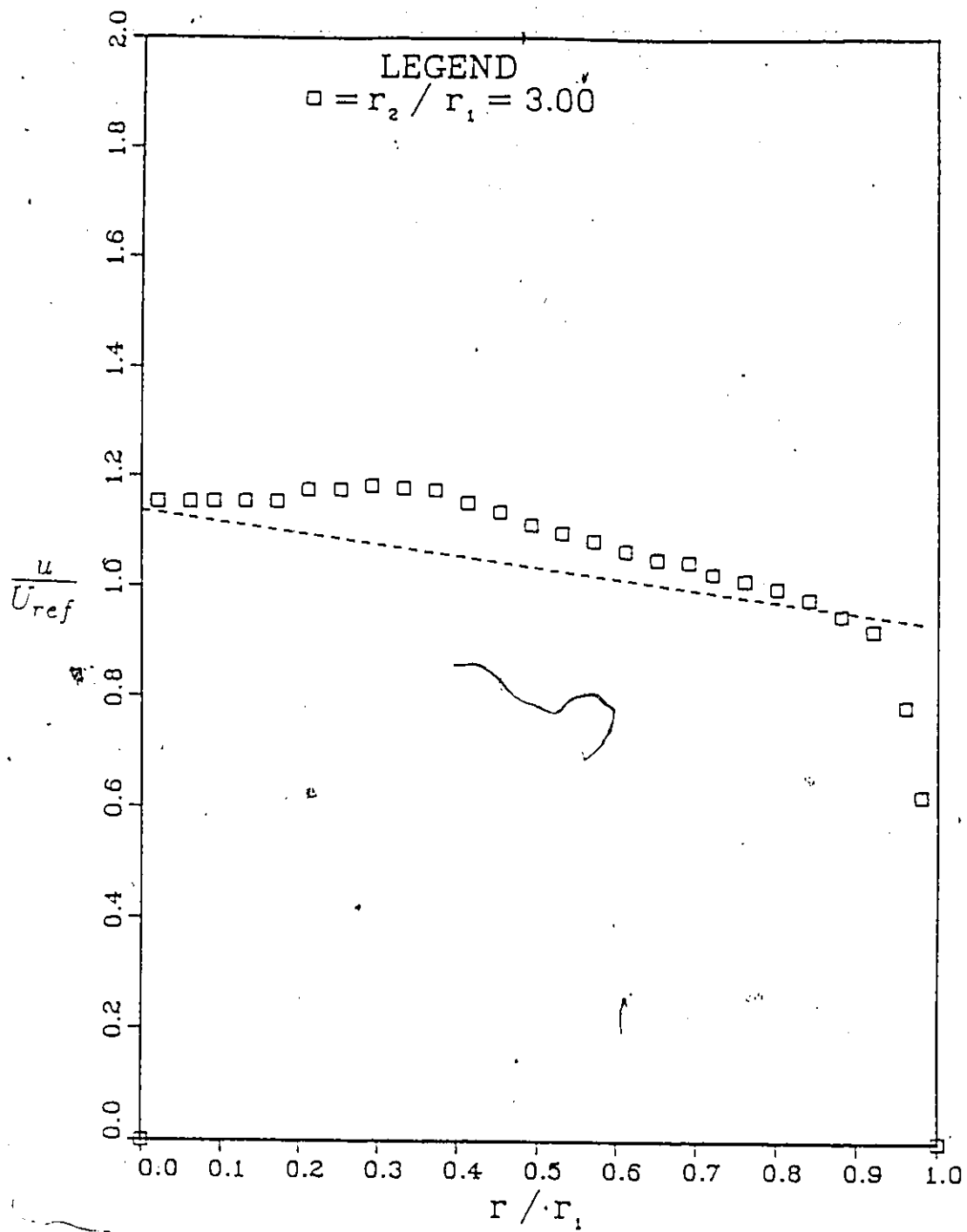


Figure 5.31: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 3.00$

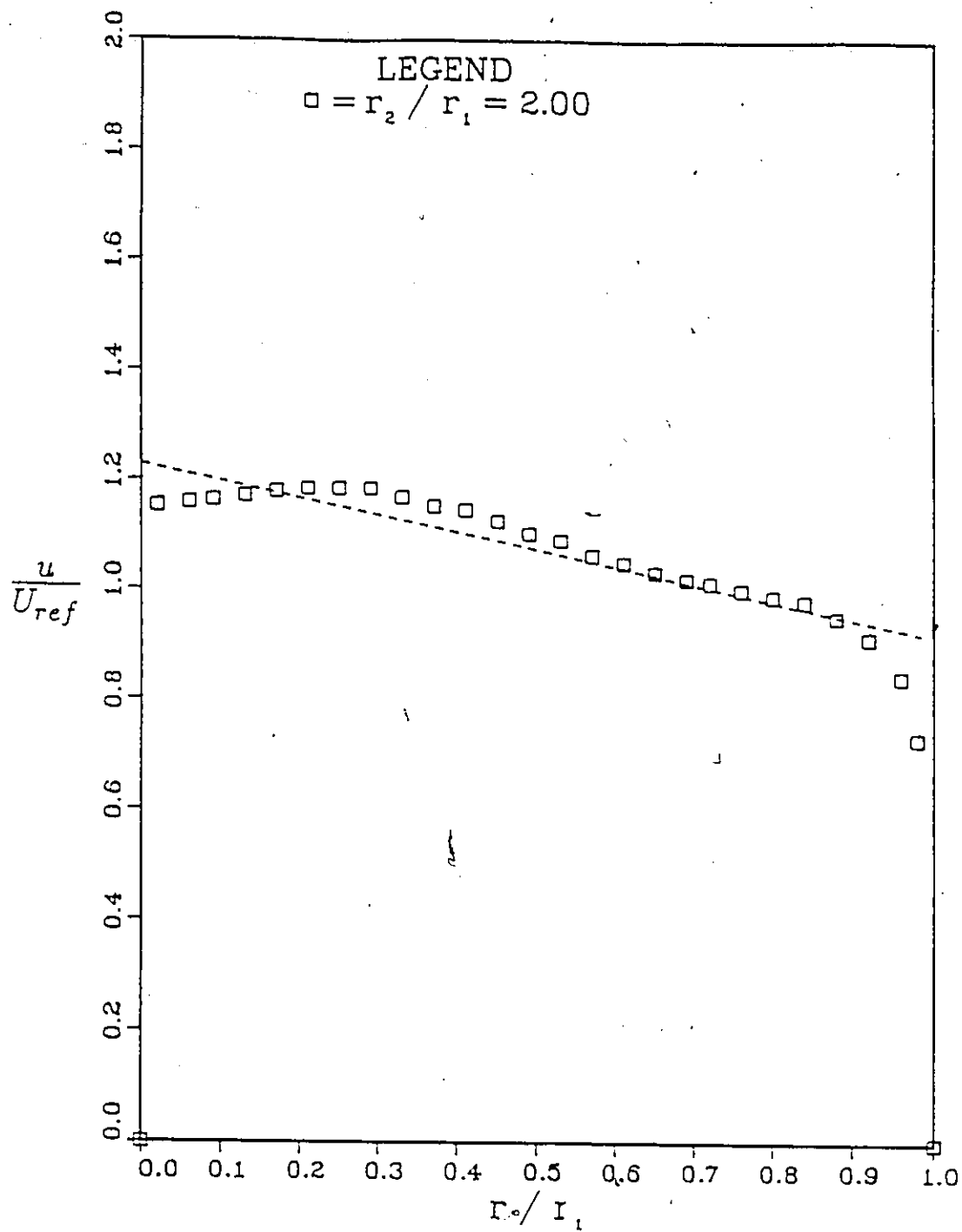


Figure 5.32: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 2.00$

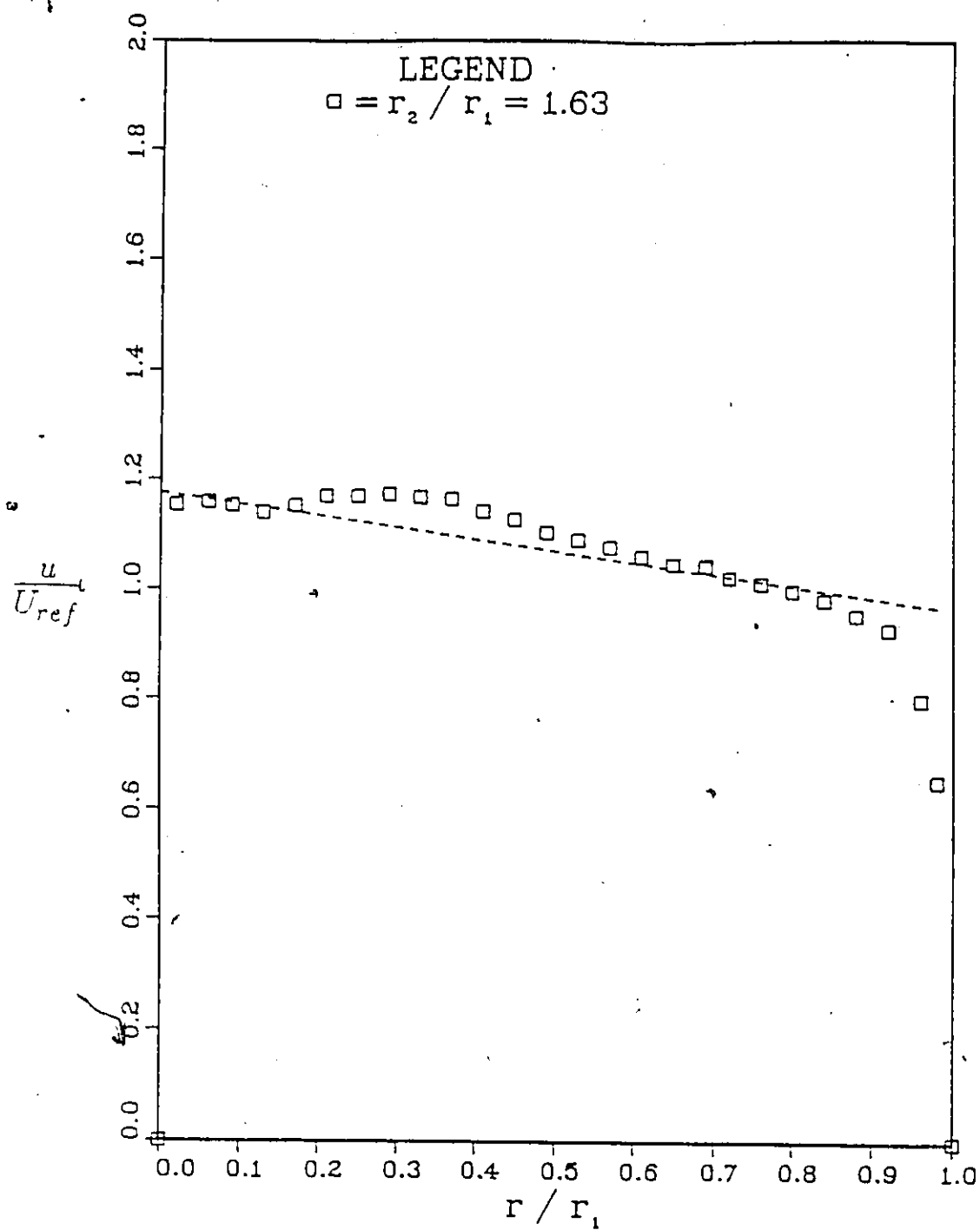


Figure 5.33: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 1.63$

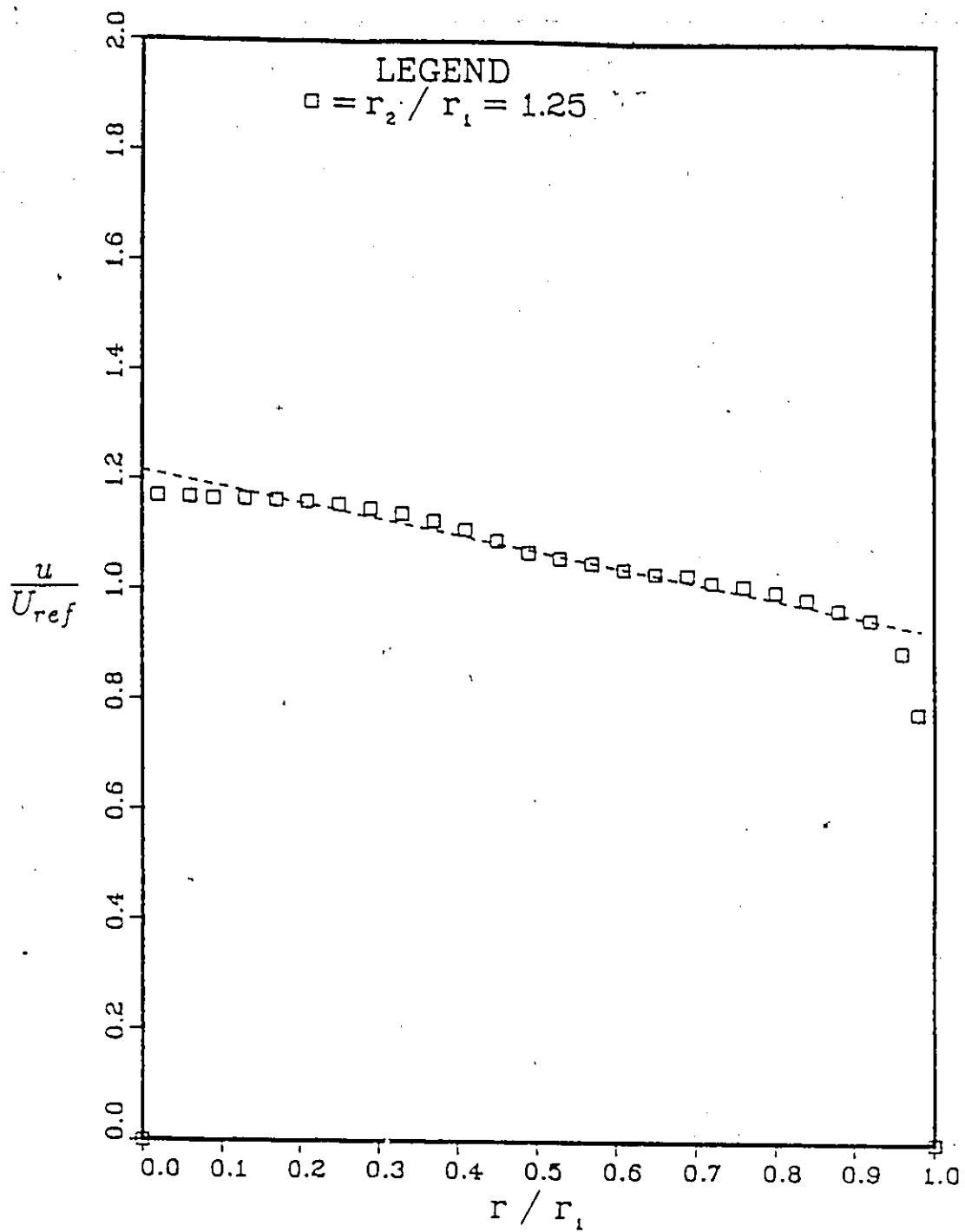


Figure 5.34: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.00$ and $\frac{r_2}{r_1} = 1.25$

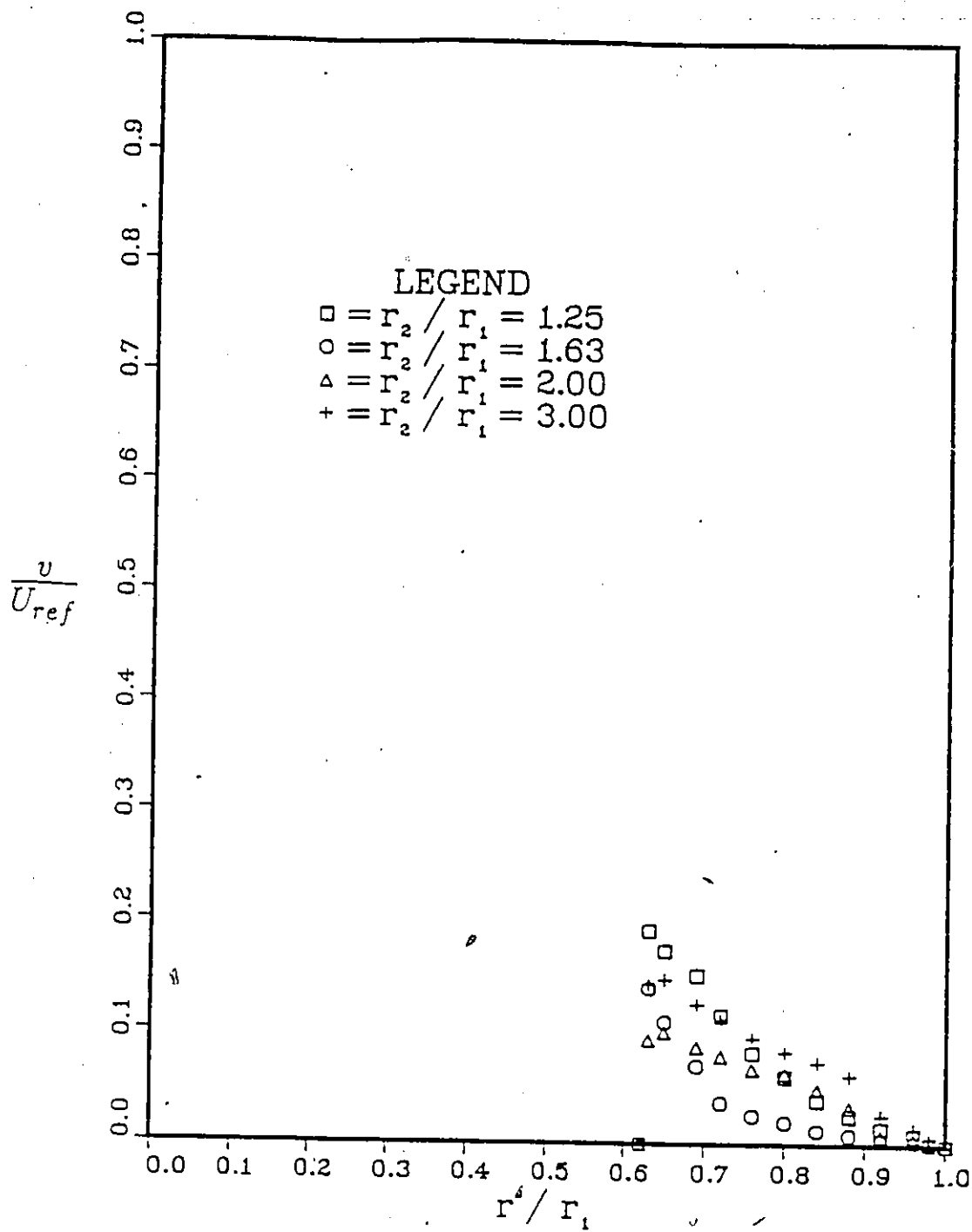


Figure 5.35: $\frac{v}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.63$

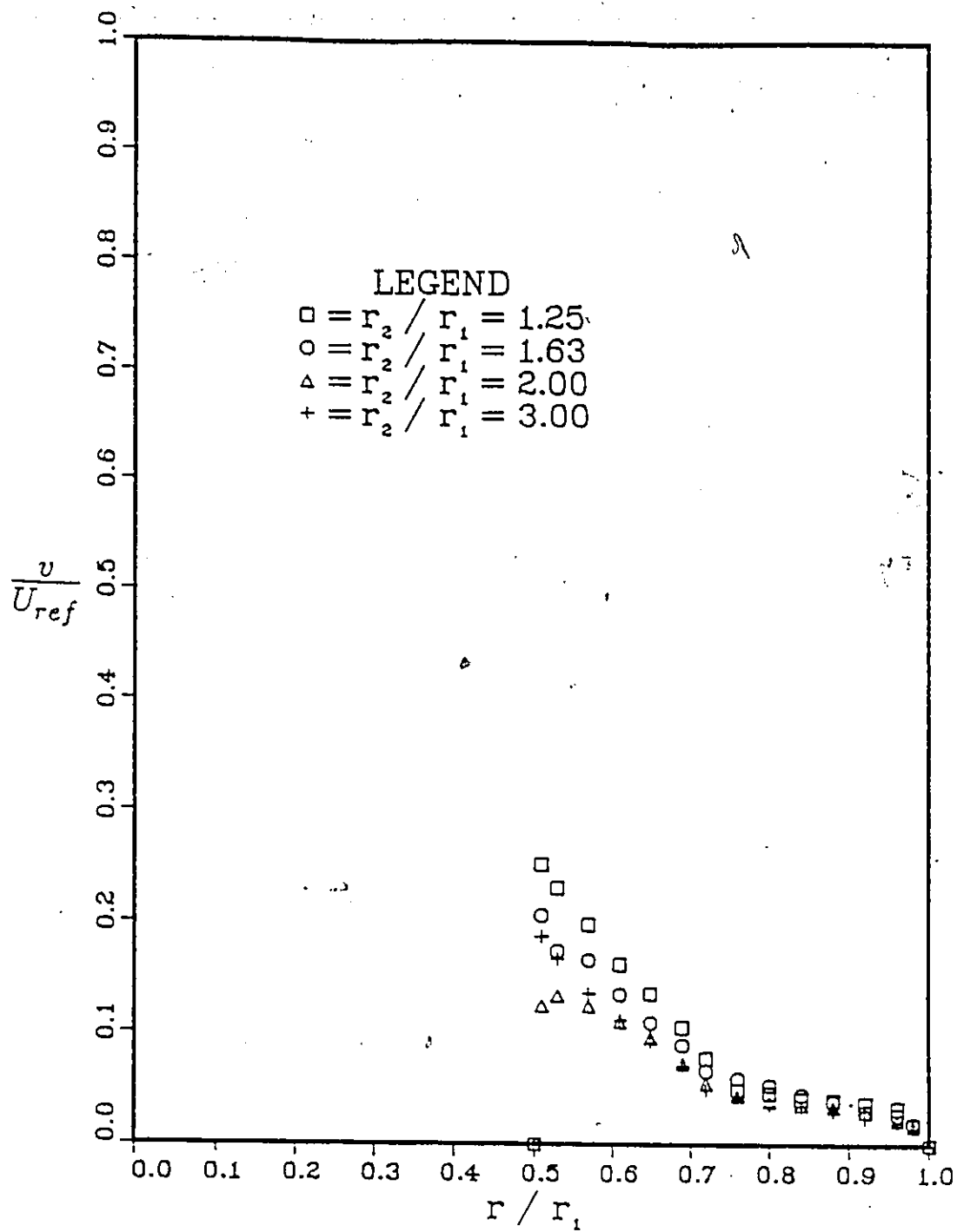


Figure 5.36: $\frac{v}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.50$

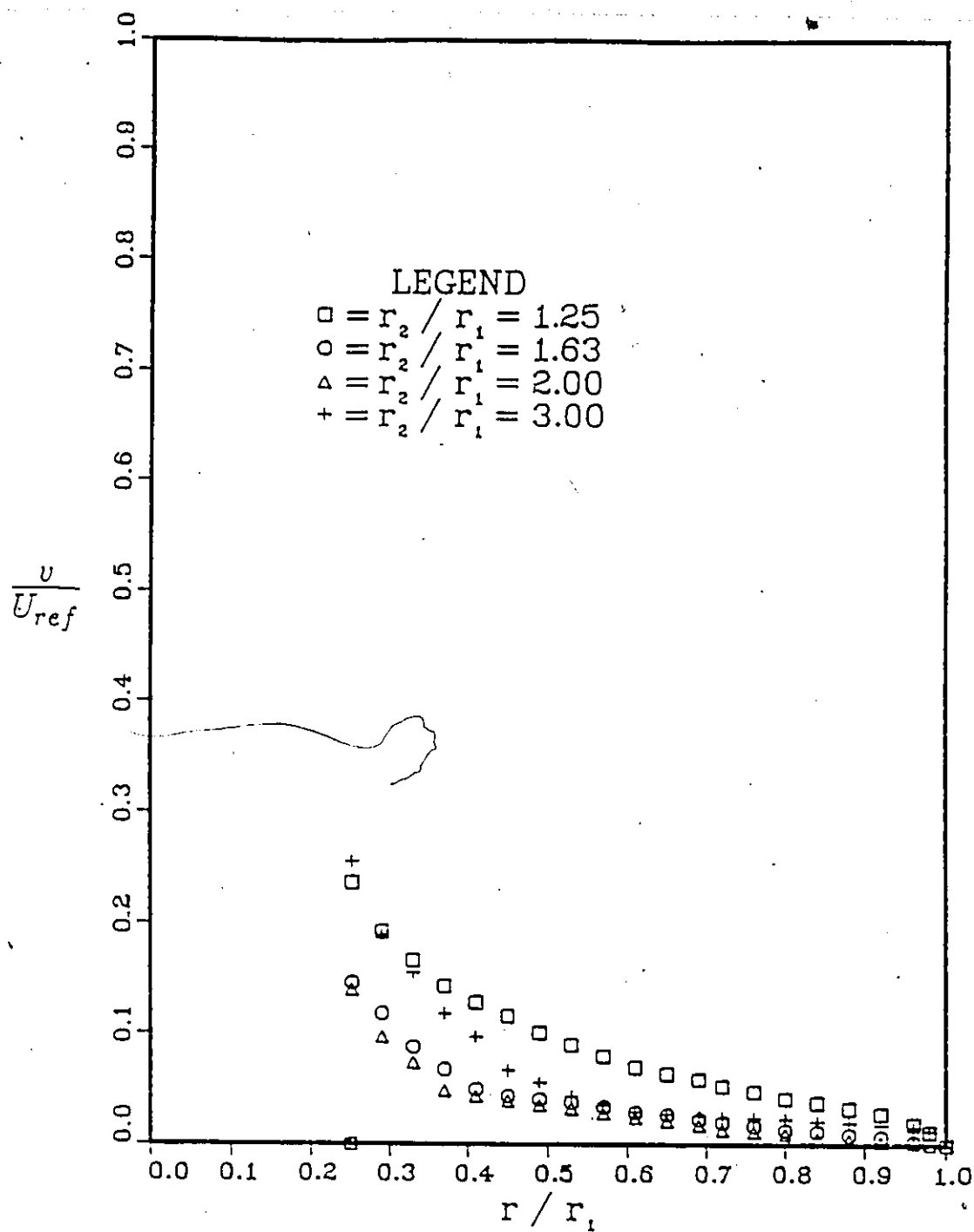


Figure 5.37: $\frac{u}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.25$

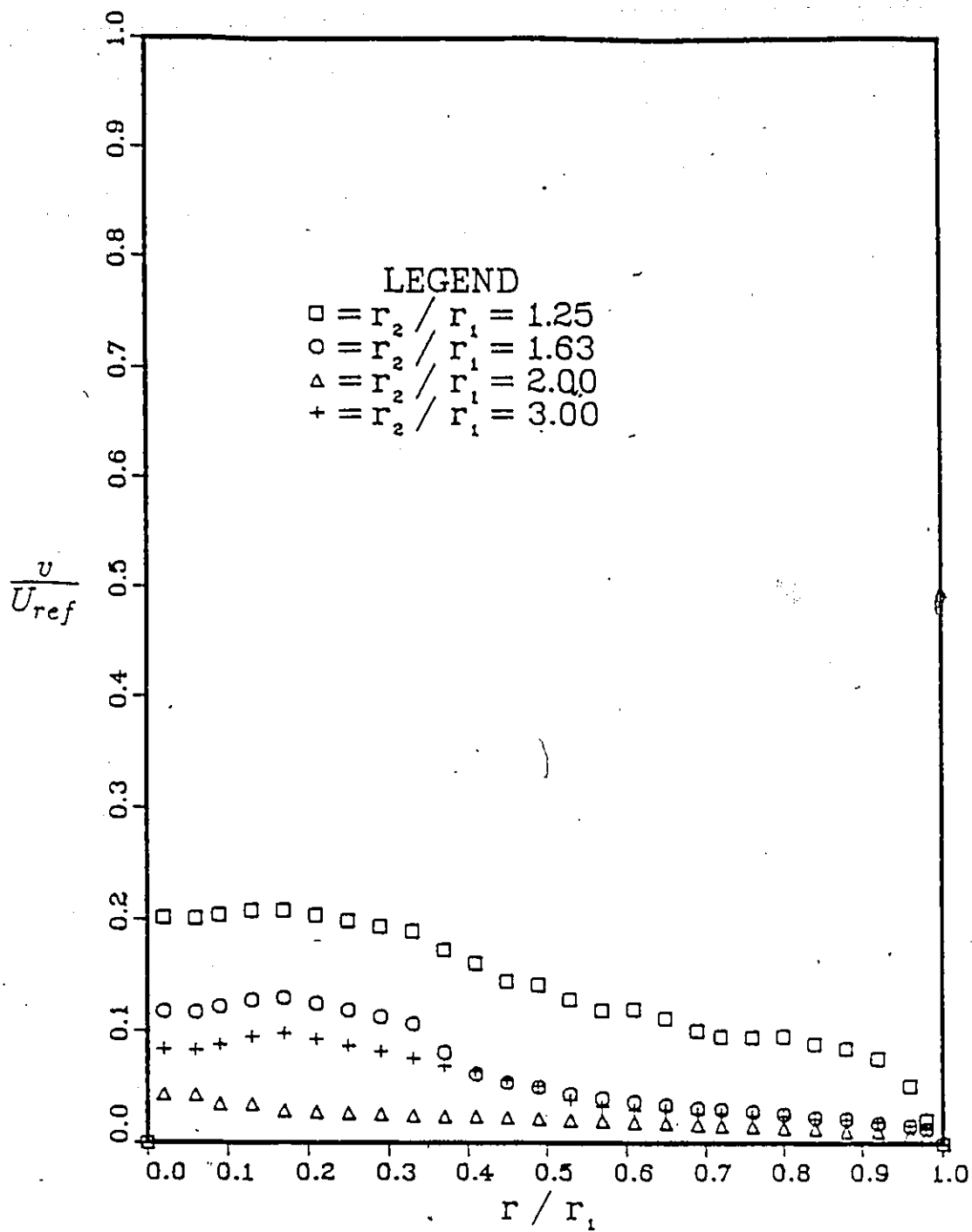


Figure 5.38: $\frac{v}{U_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.00$

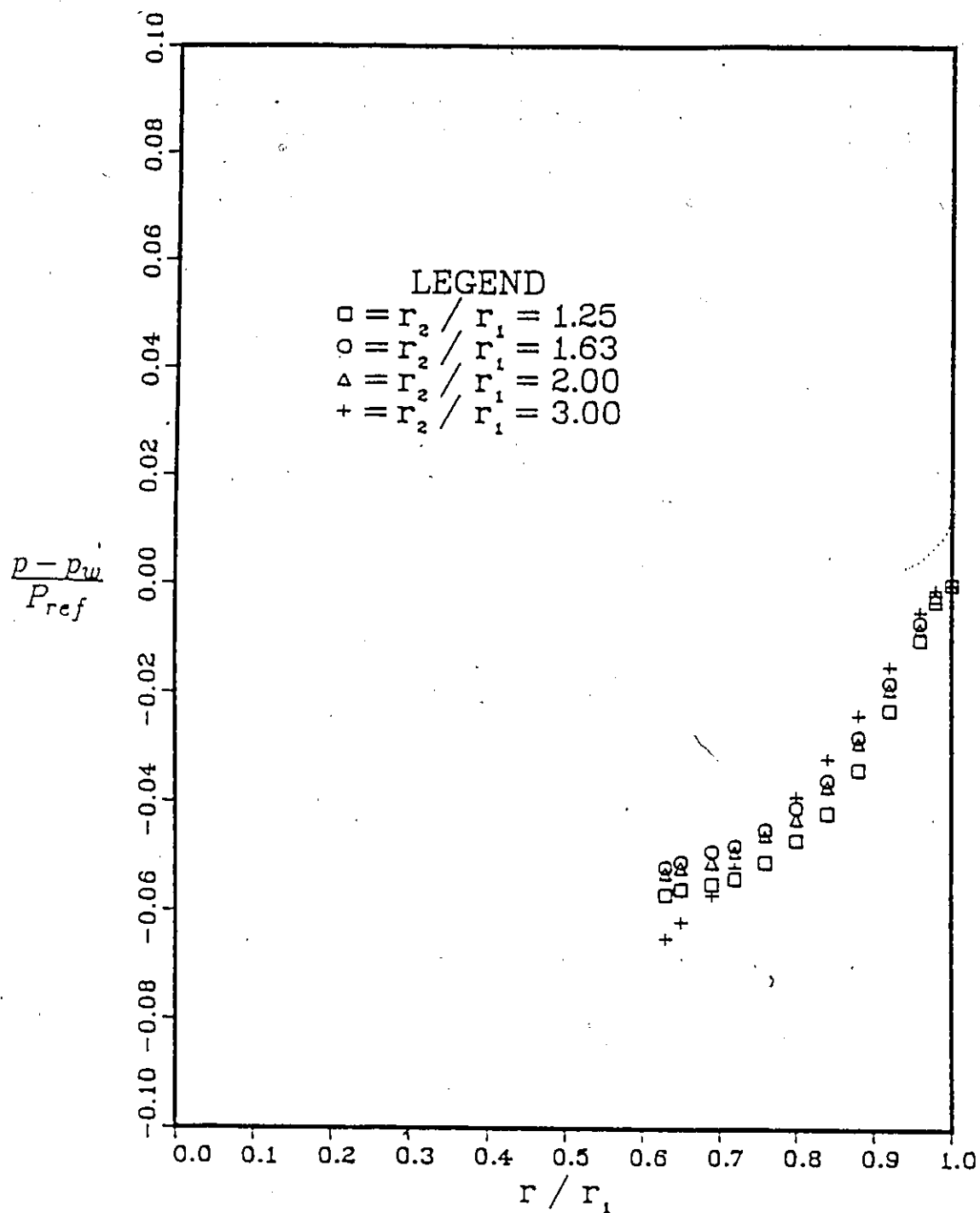


Figure 5.39: $\frac{p - p_w}{P_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_2}{r_1} = 0.63$

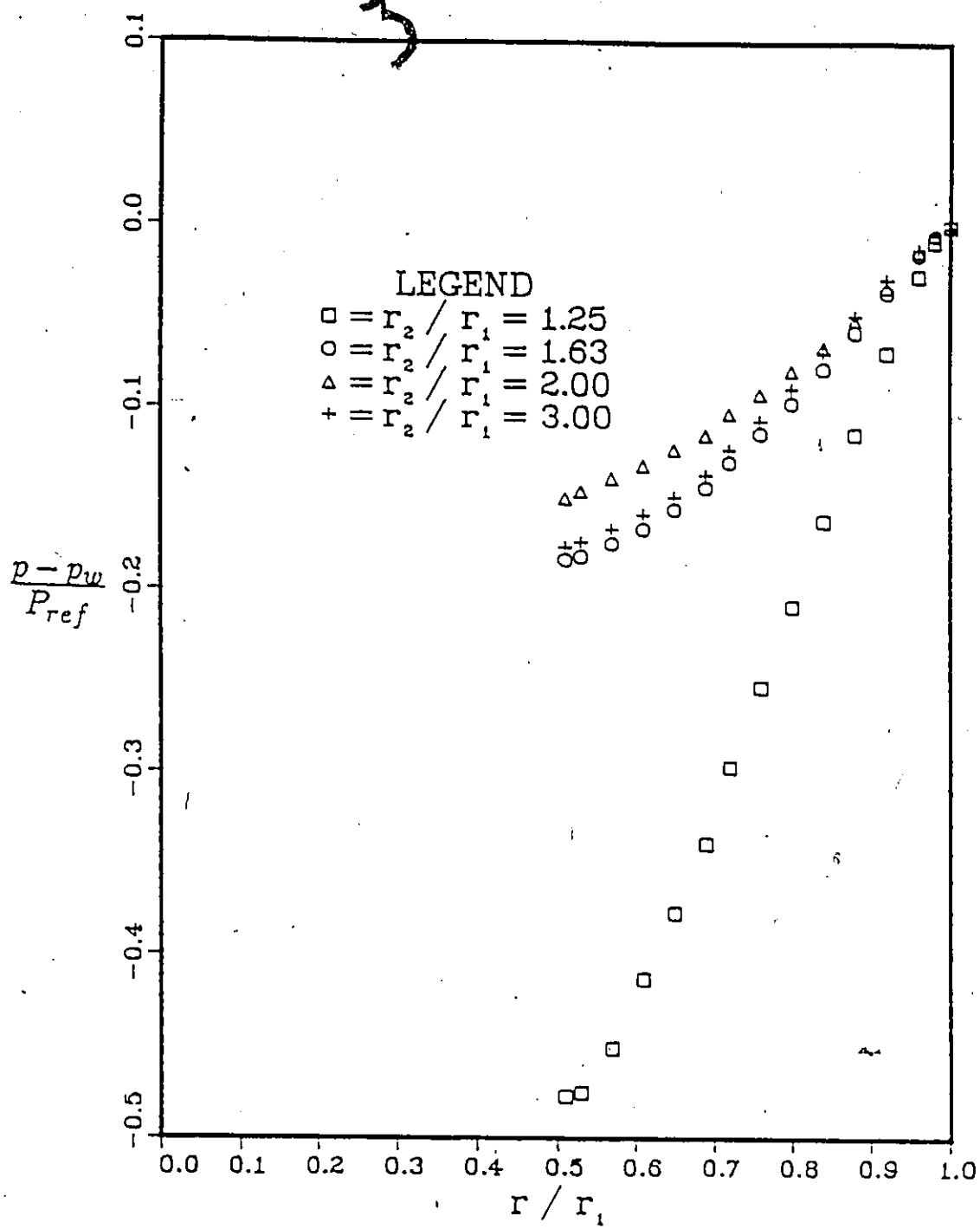


Figure 5.40: $\frac{p - p_w}{P_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.50$

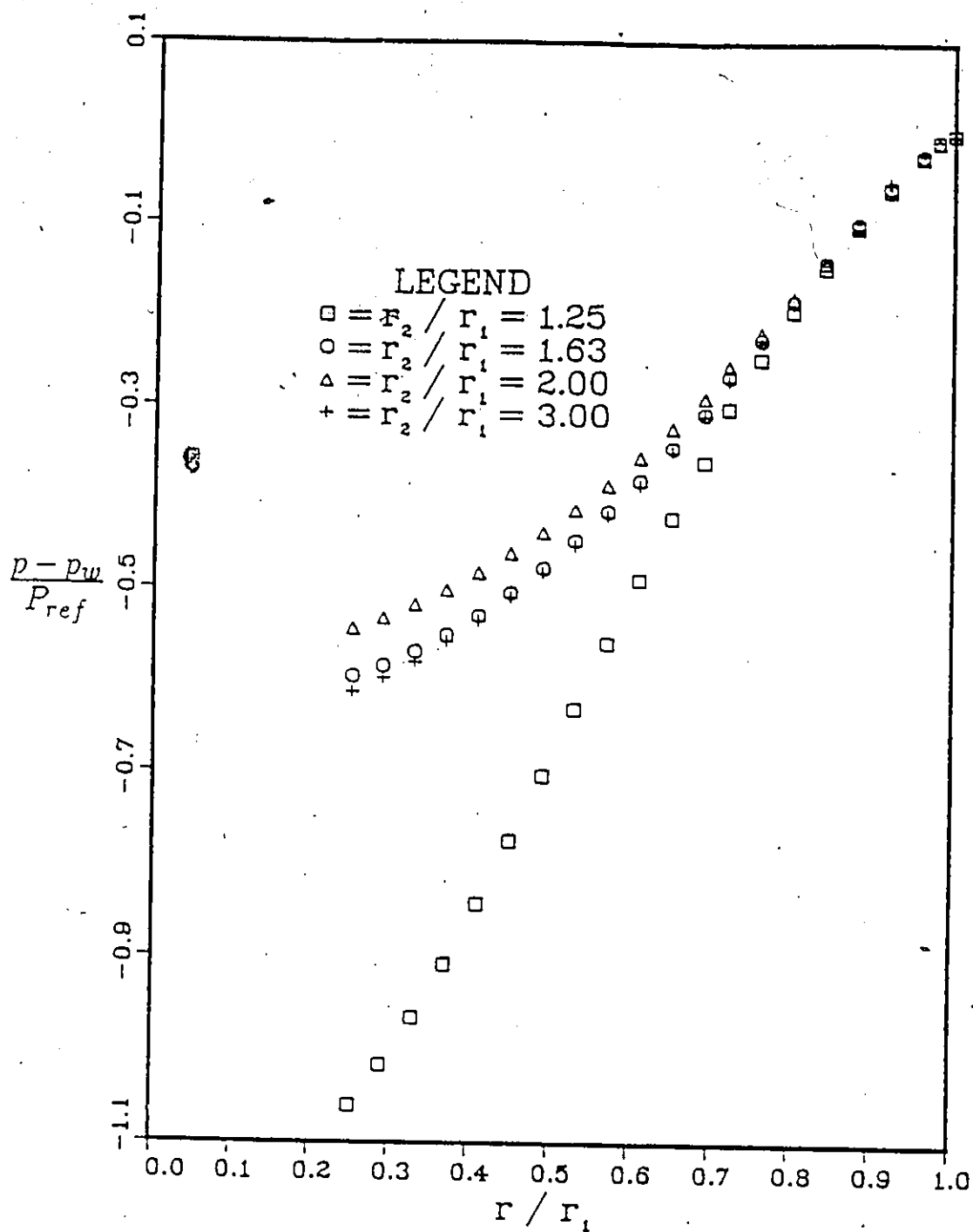


Figure 5.41: $\frac{p - p_w}{P_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.25$

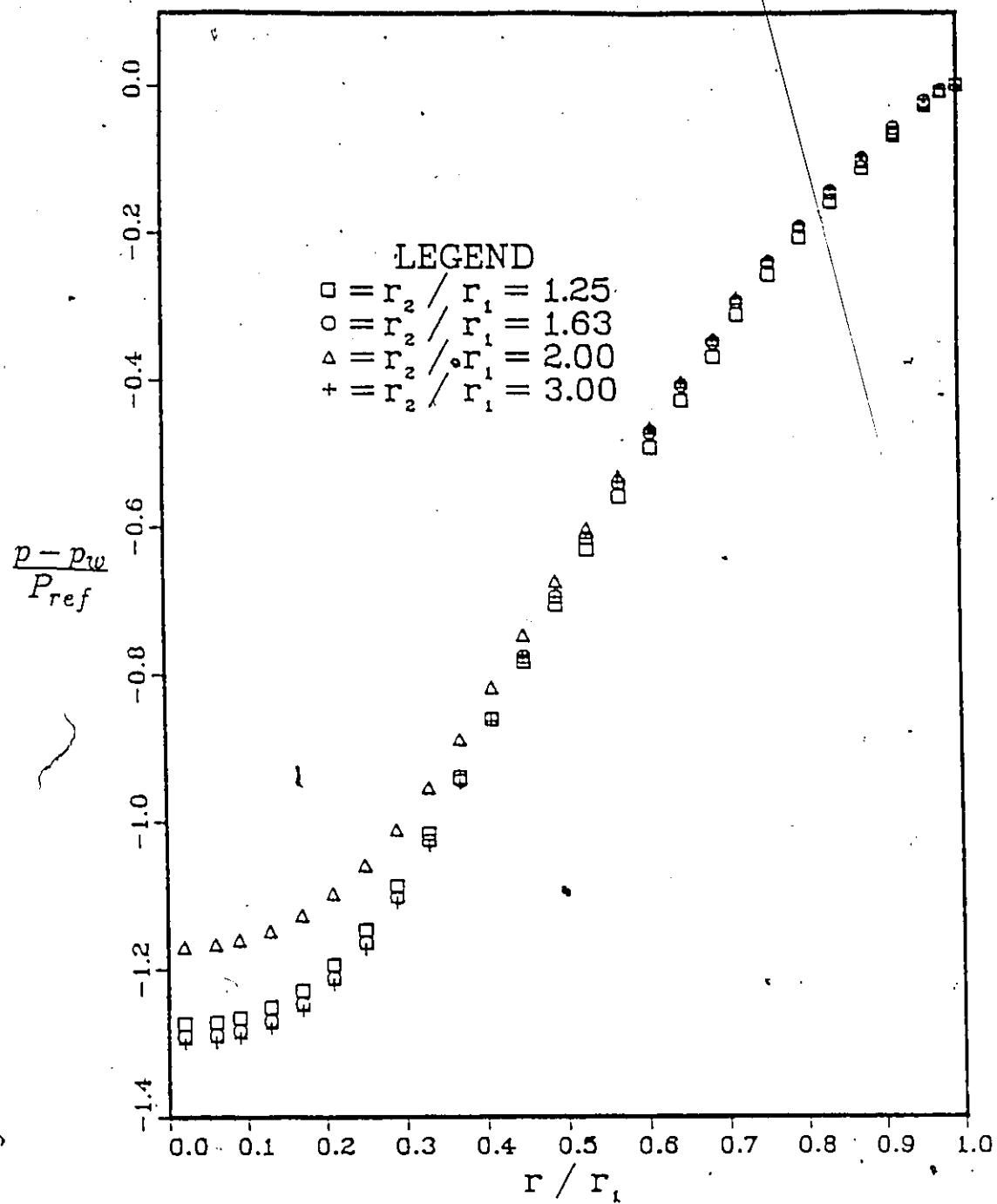


Figure 5.42: $\frac{p - p_w}{P_{ref}}$ vs. $\frac{r}{r_1}$ for $\frac{r_h}{r_1} = 0.00$

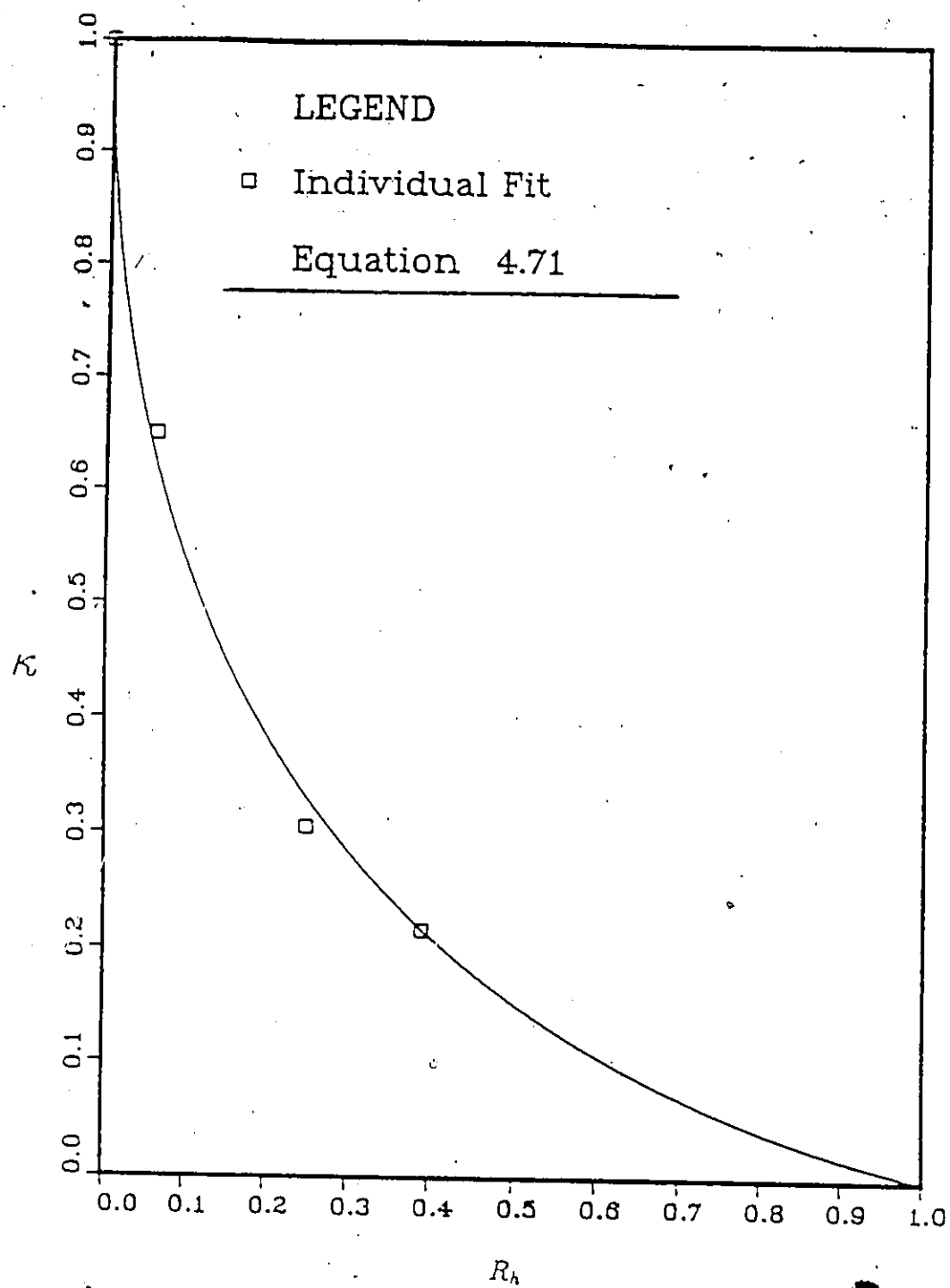


Figure 5.43: κ vs. R_h

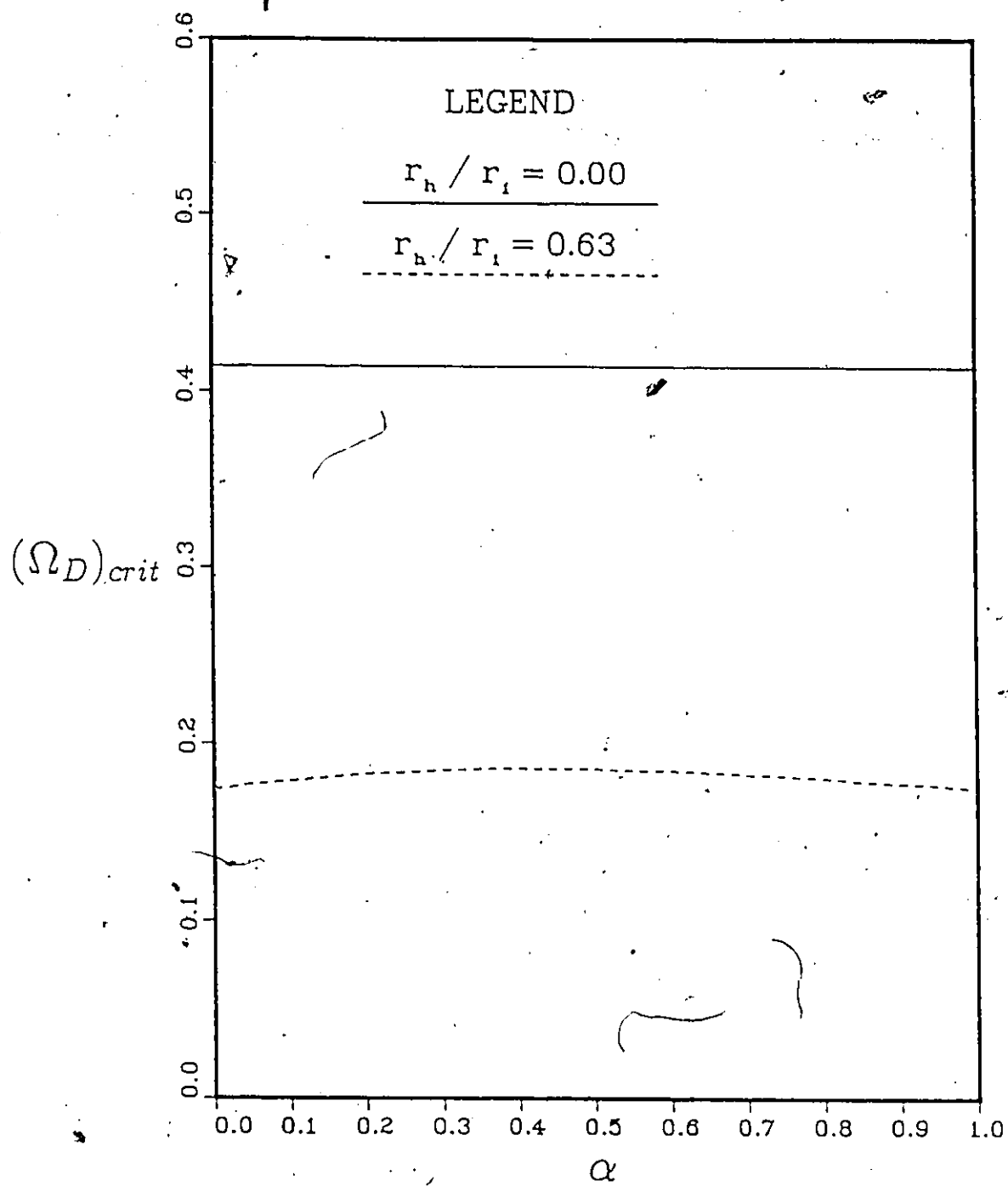


Figure 5.44: $(\Omega_D)_{crit}$ vs. α

$\zeta = 1.34 \quad \kappa = 1.000 \quad \alpha = 1.000 \quad \frac{r_h}{r_l} = 0.00$

$\zeta = 2.34 \quad \kappa = 0.216 \quad \alpha = 0.33 \quad \frac{r_h}{r_l} = 0.63$

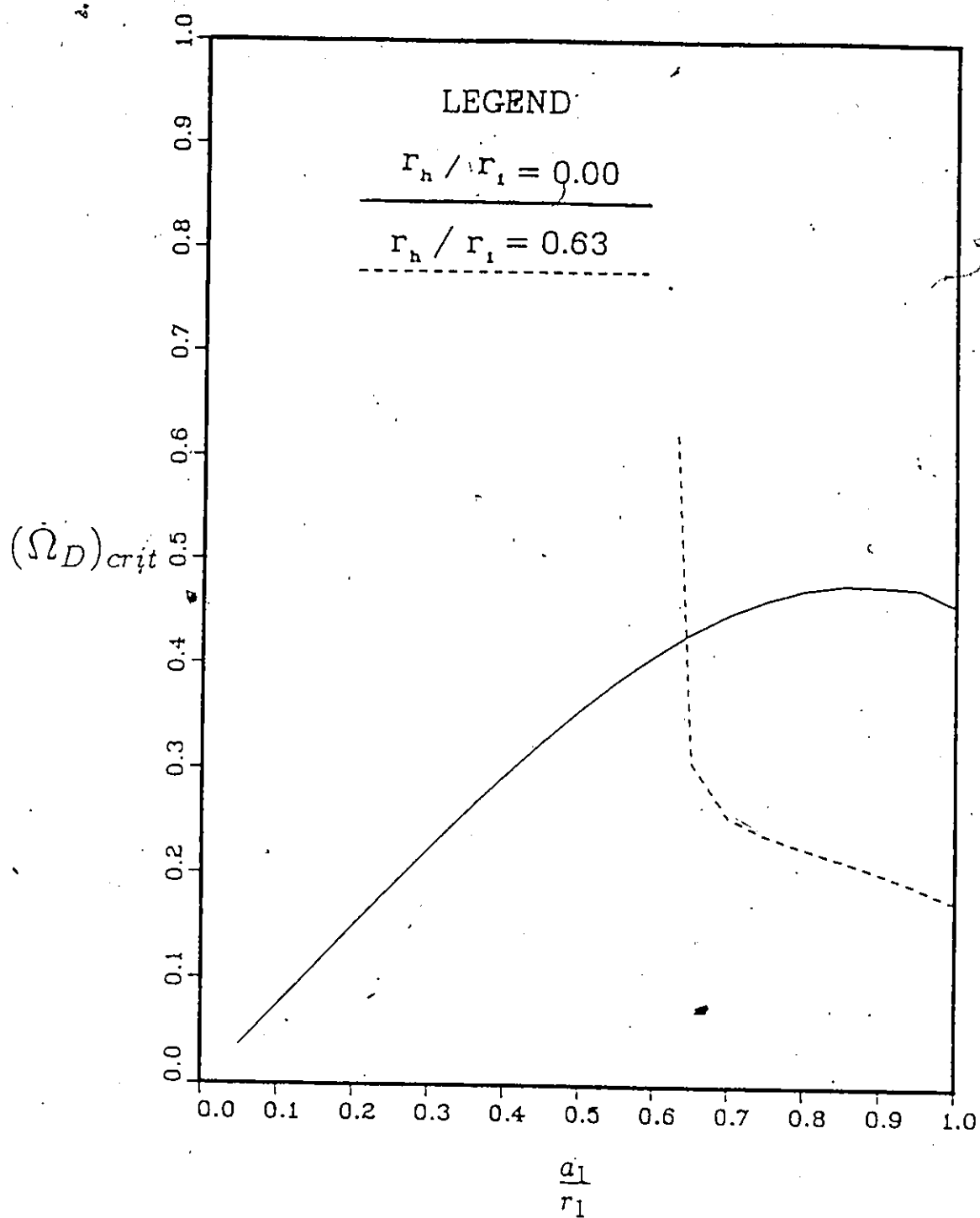


Figure 5.45: $(\Omega_D)_{crit}$ vs. $\frac{a_1}{r_1}$

$\alpha = 1.000 \quad \kappa = 1.000 \quad \frac{a_1}{r_1} = 0.61 \quad \frac{r_h}{r_1} = 0.00$ ———

$\alpha = 0.330 \quad \kappa = 0.216 \quad \frac{a_1}{r_1} = 0.96 \quad \frac{r_h}{r_1} = 0.63$ - - -

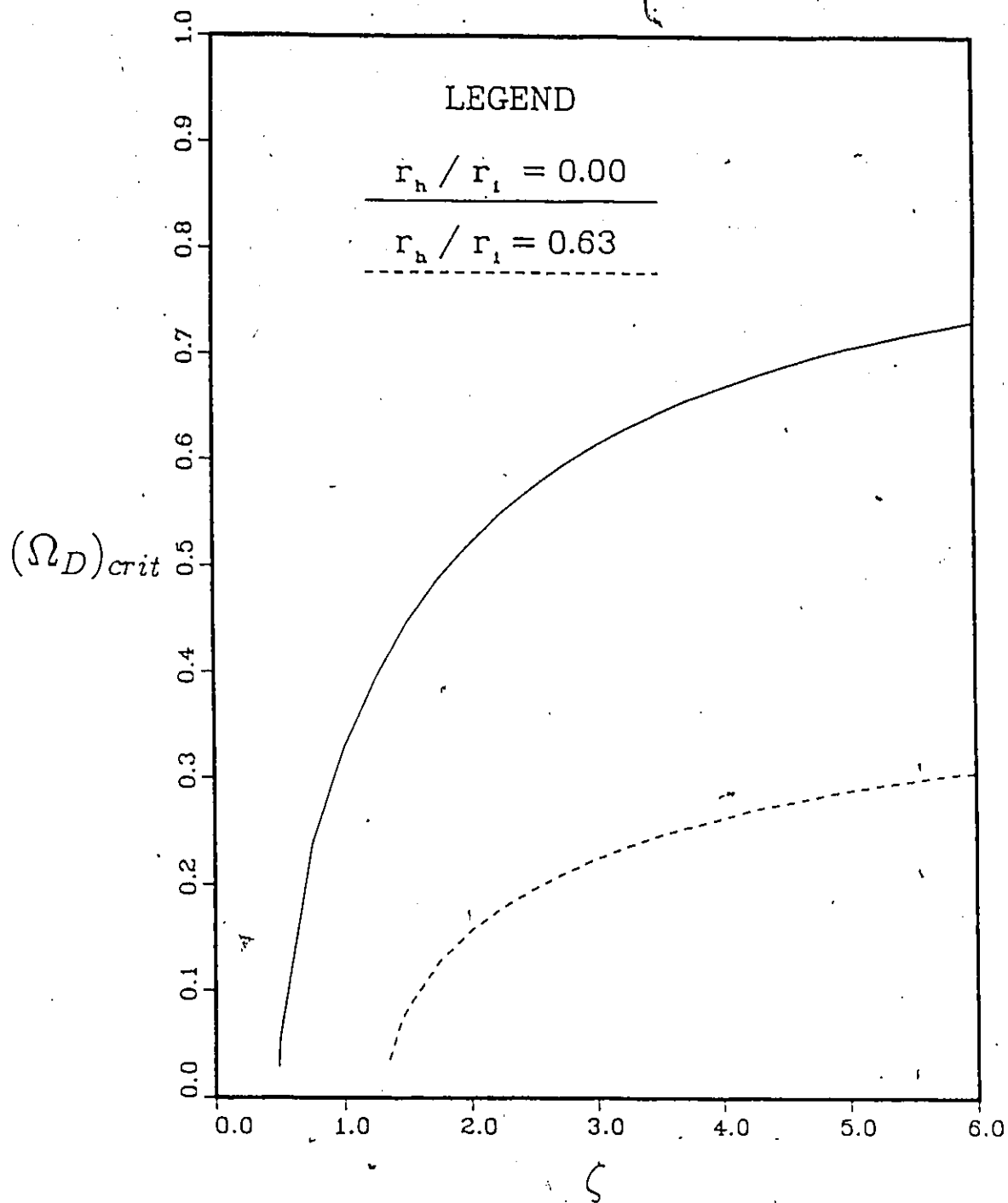


Figure 5.46: $(\Omega_D)_{crit}$ vs. ζ

$\alpha = 1.000 \quad \kappa = 1.000 \quad \frac{\alpha_1}{r_1} = 0.61 \quad \zeta = 1.34 \quad \frac{\alpha_2}{r_2} = 0.00$

$\alpha = 0.330 \quad \kappa = 0.216 \quad \frac{\alpha_1}{r_1} = 0.96 \quad \zeta = 2.34 \quad \frac{\alpha_2}{r_2} = 0.63$

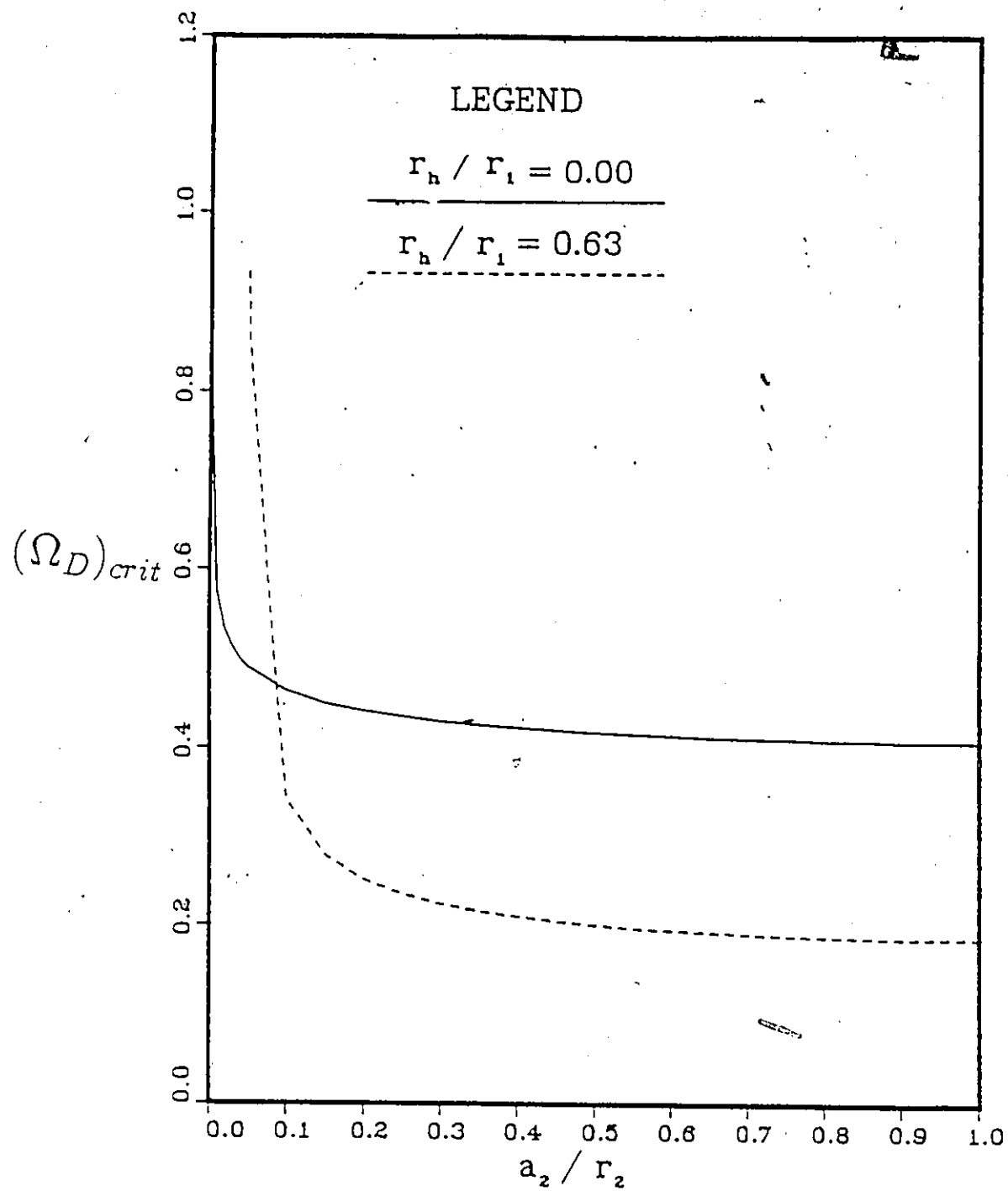


Figure 5.47: $(\Omega_D)_{crit}$ vs. $\frac{a_2}{r_2}$

Chapter 6 ✓

Conclusion And Recommendations

The present research is concerned with determining the effect of a central hub in a sudden expansion on the critical angular momentum flux. In the process of accomplishing the above task, the inlet velocity components as well as the static pressure were measured using a five hole Pitot probe. In addition, a mathematical method was developed in order to predict the critical dimensionless angular momentum flux, $(\Omega_D)_{crit}$.

6.1 Conclusion

As both the predicted and experimentally obtained $(\Omega_D)_{crit}$ shows, the presence of a central hub in a sudden expansion has a direct effect on the critical angular momentum flux. Placing a hub at the centre of the throat reduced $(\Omega_D)_{crit}$ substantially. A drop, ranging from 50% to 60% in $(\Omega_D)_{crit}$ was observed as larger

hubs were applied. For the same hub, the critical swirl decreased as higher expansions were used. This critical swirl reaches a minimum at an expansion ratio of about 2 followed by a slight increase at higher expansions. Increasing the expansion ratio from 1.25 to 2.00 resulted in a drop in $(\Omega_D)_{crit}$ ranging from 18% to 40%, depending on the hub size. The lowest value of $(\Omega_D)_{crit}$ was observed to be 0.195 at $\frac{r_2}{r_1} = 2.00$ and $\frac{r_3}{r_1} = 0.63$, while the highest $(\Omega_D)_{crit}$ occurred at $\frac{r_2}{r_1} = 1.25$ and $\frac{r_3}{r_1} = 0.00$. The mathematical model closely approximated the experimentally obtained $(\Omega_D)_{crit}$ a similar pattern was predicted.

The measured tangential velocity profiles resemble a Rankine vortex by having a forced vortex shape inside the solid body core and a decaying appearance outside of this region. Meanwhile, the axial velocity profiles suggest a distribution with a maximum at the edge of the hub, while the radial component proved to be negligibly small. In addition, the static pressure measurement indicates the existence of a low pressure area near the axis of the flow. The static pressure increased as it approached the throat wall.

6.2 Recommendations For Further Work

The present research could be continued in the following areas :

1. LDA measurement of the flow velocities would result in a more accurate measurement of $(\Omega_D)_{crit}$.
2. Taking movies of the flow visualization experiment would help in obtaining an accurate $(\Omega_D)_{crit}$ as well as improving the ability to analyze and characterize the flow instabilities and temporal behaviour of the central recirculation

zone.

3. A finite difference model could be developed to predict the $(\Omega_D)_{crit}$ as well as the inlet and downstream velocity components.
4. Experimental measurement of the velocity profiles at the point of inception of vortex breakdown will definitely improve the accuracy of the developed mathematical model.
5. Effect of combustion on $(\Omega_D)_{crit}$, the shape and size of the central recirculation zone and the location of the point of inception of vortex breakdown could be studied.
6. The effects of geometry and inlet conditions on recirculation size and mass flow could be investigated.

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Appendix A

Data Reduction Program

```
5      CLS
10     GOSUB 200 : REM SUBROUTINE 1 INITIATES THE ANALYSIS
20     GOSUB 500 : REM SUBROUTINE 2 COMPUTES THE VELOCITIES
30     GOSUB 1000 : REM SUBROUTINE 3 COMPUTES THE MASS FLOW
                     BASED ON THE INLET CONDITIONS
40     GOSUB 1100 : REM SUBROUTINE 4 COMPUTES THE MASS AND
                     MOMENTUM FLUXES
50     GOSUB 1500 : REM SUBROUTINE 5 COMPUTES THE ENERGY FLUXES
60     GOSUB 2000 : REM SUBROUTINE 6 COMPUTES THE FORCES
70     GOSUB 2500 : REM SUBROUTINE 7 COMPUTES THE STREAM FUNCTION
80     GOSUB 3200 : REM SUBROUTINE 8 PRINTS THE OUTPUT
90     REM SUBROUTINE 9 RECORDS THE OUTPUT ON A DATA FILE
100    REM SUBROUTINE 10 PERFORMS THE TRAPAZOIDAL RULE FOR INTEGRATION
110    REM SUBROUTINE 11 IS USED FOR FORMATTING PURPOSES
120    END
200    REM *****
```

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210  REM SUBROUTINE 1 INPUTS THE DATA FROM FIVE
      HOLE PRESSURE PROBE
220  REM *****
240  DIM UNI(35),AYAW(35),AP1P2(35),AP1PW(35),R(35),YAW(35),P1P2(35),
      P1PW(35),DUMMY(35),VT(35),WVREF(35),VTVREF(35),PSTAT(35)
250  DIM P(35),PSTPW(35),U(35),W(35),V(35),PR(35),UVREF(35),UBACK1(35)
260  DIM UBACK2(35),GXFUNC(35),AP4P5(35),P4P5(35),PITCH(35),VVREF(35)
      GTFUNC(35)
265  DIM A(35),B(35),C(35),D(35),RT(35),R1(35),TPSTAT(35),Y(35)
270  OPEN "B:IN83.DAT" FOR INPUT AS #1
280  INPUT#1 ,EXPANS,HUB,SLIDER,T,PBAR,LTI,LAI,LT,LA,N,P1PBAR
300  RHUB = HUB * .0254 / 2
310  TKELV = T + 273.15
320  K1 = PBAR * .9964 : REM ASME NOZZLE COEFFICIENT
380  AXIS = 213.2
390  THROAT = .0508 : REM THROAT RADIUS IN METERS
400  FOR I = 1 TO N
410      INPUT#1 ,UNI(I),AYAW(I),AP1P2(I),AP1PW(I),AP4P5(I)
460  NEXT I
465  CLOSE#1
470  MANO = 7.83
480  RETURN
500  REM *****
505  REM SUBROUTINE 2 COMPUTES THE VELOCITIES
506  REM *****
507  R(0) = RHUB : DUMMY(0) = 1
510  FOR I = 1 TO N

```

```

520      R(I) = ( UNI(I) - AXIS ) / 1000
530      YAW(I) = AYAW(I) * .017453
540      P1P2(I) = AP1P2(I) * 133.32
550      P1PW(I) = AP1PW(I) * 133.32
560      P4P5(I) = AP4P5(I) * 133.32
565      DUMMY(I) = 1
570      NEXT I
580      PW = P1PBAR * 133.32 - P1PW(5)
590      FOR J = 1 TO N
600          IF ( P1P2(J) = 0 ) THEN 610 ELSE 630
610              X = 0
620              GOTO 680
630              B = P4P5(J) / P1P2(J)
640              IF ( B <= .1779 ) THEN 650 ELSE 670
650                  X = ( B + .166 ) / .0326
660                  GOTO 680
670                  X = ( B + .3 ) / .0453
680          IF ( X < -40 ) AND ( X > 40 ) THEN 690 ELSE 730.
690              R1(J) = R(J) * 1000
700              PRINT "WARNING : A PITCH ANGLE OF ";X;" DEGREES IS
              CALCULATED AT R = ";R1(J);" MM. THIS POINT IS"
710              PRINT "ELIMINATED."
720              GOTO 860
730              COEFV1 = .96 + (.002207 * X) - (8.027E-05 * X^2) +
              (3.418E-06 * X^3) + (2.047E-07 * X^4)
731              COEFV2 = ( -.03136 * X^5) - (.0001169 * X^6) -
              (1.494E-07 * X^7) + (2.875E-07 * X^8 ) )

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      * 1.0E-07
732      COEFV3 = .0000001 * .0000001 * (.04755 * X^9)
733      COEFV = COEFV1 + COEFV2 + COEFV3
740      IF ( X >= 0 ) AND ( X <= 10 ) THEN 750 ELSE 770
750      COFTP = 0
760      GOTO 780
770      COFTP1 = - (.001 * X) - (.0002577 * X^2) - (4.062E-06
      * X^3) - (4.129E-07 * X^4) + (.0854 * X^5)
      * 1.0E-07,
771      COFTP2 = ( (.0028 * X^6) - (6.683E-05 * X^7) -
      ( 6.802E-07 * X^8 ) ) * 1.0E-07
772      COFTP3 = .0000001 * .0000001 * (.1926 * X^9)
773      COFTP = COFTP1 + COFTP2 + COFTP3
780      PSTPW(J) = P1PW(J) - P1P2(J) * COEFV * ( 1 + COFTP )
790      P(J) = .46446 * ( ( PSTPW(J) + PW ) / 133.32 + PBAR ) / TKELV
800      VT(J) = SQR ( 2 * COEFV * P1P2(J) / P(J) )
810      X = X * .017453
820      U(J) = VT(J) * COS (X) * COS(YAW(J))
830      W(J) = ABS ( VT(J) * COS(X) * SIN(YAW(J)) )
840      V(J) = - VT(J) * SIN(X)
850      PR(J) = P(J) * R(J)
860      NEXT J
861      P(0) = P(1) : PR(0) = P(0) * R(0)
862      P(N+1) = P(N) : PR(N+1) = P(N+1) * THROAT : R(N+1) = THROAT
863      U(0) = 0 : V(0) = 0 : W(0) = 0
      U(N+1) = 0 : V(N+1) = 0 : W(N+1) = 0
871      RETURN

```

```

1000 REM *****
1001 REM SUBROUTINE 3 COMPUTES THE MASS FLOWS BASED
      ON THE INLET CONDITIONS
1002 REM *****
1020 PT = ( 98.1 * SIN ( MANO * .017453 ) + .4807 ) * ( LT - LTI )
1030 PA = ( 98.1 * SIN ( MANO * .017453 ) + .4807 ) * ( LA - LAI )
1036 PRINT "PT = ";PT : PRINT " PA = ";PA : PRINT "TKELVIN=";TKELV
1037 PRINT "PBAR1 = ";PBAR1
1040 MT1 = .007836 * SQR ( (PBAR1 - PT/133.32) * PT/TKELV )
1050 MA1 = .0019056 * SQR ( (PBAR1 - PA/133.32) * PA/TKELV )
1060 MTOT1 = MT1 + MA1
1070 RETURN

1100 REM *****
1101 REM SUBROUTINE 4 COMPUTES THE MASS AND MOMENTUM
      FLUXES INSIDE THE TEST SECTION
1102 REM *****
1110 FOR J = 0 TO (N+1)
1120   A(J) = PR(J) : B(J) = U(J) : C(J) = U(J)
1130   D(J) = DUMMY(J) : RT(J) = R(J)
1140 NEXT J
1150 GOSUB 5000
1160 GX = 6.2832 * TRAP
1170 FOR J = 0 TO (N+1)
1180   C(J) = W(J) : D(J) = R(J)
1190 NEXT J
1200 GOSUB 5000

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```

1210  GT = 6.2832 * TRAP
1220  S = GT / ( THROAT * GX )
1230  FOR J = 0 TO (N+1)
1240      B(J) = U(J) : C(J) = DUMMY(J) : D(J) = DUMMY(J)
1250  NEXT J
1260  GOSUB 5000
1270  MTOT2 = 6.2832 * TRAP
1280  RATIO = MTOT2 / MTOT1
1285  PRINT "RATIO = ";RATIO
1290  ZETA = MT1 / MTOT1
1300  FOR J = 0 TO (N+1)
1310      A(J) = U(J) : B(J) = R(J) : C(J) = DUMMY(J)
1320      D(J) = DUMMY(J)
1330  NEXT J
1340  GOSUB 5000
1350  Q = 6.2832 * TRAP
1355  A = ( THROAT ^ 2 ) - ( RHUB ^ 2 )
1357  H = ( ( THROAT ^ 2 ) - ( RHUB ^ 2 ) ) ^ 2
1360  VREF = Q / ( 3.1416 * A )
1370  PREF = ( MTOT2 * Q ) / ( 19.7392 * H )
1380  GTD = GT / ( THROAT * MTOT2 * VREF )
1385  PRINT "GTD =",GTD
1390  GXD = GX / ( MTOT2 * VREF )
1400  RETURN
1500  REM *****
1501  REM SUBROUTINE 5 COMPUTES THE ENERGY FLUXES
1502  REM *****

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```

1510   FOR J = 0 TO (N+1)
1520       A(J) = PR(J) : B(J) = U(J) : C(J) = U(J)
1530       D(J) = U(J)
1540   NEXT J
1541   GOSUB 5000
1550   EKINX = 3.1416 * TRAP
1560   FOR J = 0 TO (N+1)
1570       B(J) = W(J) : D(J) = W(J)
1580   NEXT J
1590   GOSUB 5000
1600   EKINT = 3.1416 * TRAP
1610   FOR J = 0 TO (N+1)
1620       B(J) = V(J) : D(J) = V(J)
1630   NEXT J
1640   GOSUB 5000
1650   EKINR = 3.1416 * TRAP
1660   KETOT = EKINX + EKINT + EKINR
1670   FOR J = 0 TO (N+1)
1680       A(J) = U(J) : B(J) = R(J) : C(J) = PSTPW(J)
1690       D(J) = DUMMY(J)
1700   NEXT J
1710   GOSUB 5000
1720   EST = Q * PW + 6.2832 * TRAP
1730   ETOT = KETOT + EST
1740   EAV = MTOT2 * ( VREF^2 ) / 2
1750   EKINXD = EKINX / EAV
1760   EKINTD = EKINT / EAV

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1770  EGINRD = EGINR / EAV
1775  KETOTD = KETOT / EAV
1780  ESTD = EST / EAV
1790  ETOTD = ETOT / EAV
1800  RETURN
2000  REM *****
2001  REM SUBROUTINE 6 COMPUTES THE PRESSURE FORCE
      AND THRUST
2002  REM *****
2010  FOR J = 0 TO (N+1)
2020      A(J) = DUMMY(J) : B(J) = R(J) : C(J) = PSTPW(J)
2030      D(J) = DUMMY(J)
2040  NEXT J
2050  GOSUB 5000
2060  PRESSF = 6.2832 * TRAP
2070  PRESFD = PRESSF / ( MTOT2 * VREF )
2080  RE = 2 * VREF * ( THROAT - RHUB ) / .0000148
2090  THRUST = PRESSF + GX
2100  THRUSD = THRUST / ( MTOT2 * VREF )
2110  ST = GT / ( THROAT * THRUST )
2120  FOR J = 1 TO N
2130      UVREF(J) = U(J) / VREF : WVREF(J) = W(J) / VREF
2140      VVREF(J) = V(J) / VREF : VTVREF(J) = VT(J) / VREF
2150      PSTAT(J) = PSTPW(J) / PREF
2160  NEXT J
2170  TPSTAT(N+1) = 0
2190  J = N + 1

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2195 REM INTEGRATION IS PERFORMED FROM THE THROAT WALL TOWARD
      THE HUB SURFACE
2197 IF ( J < 1 ) THEN 2240
2200 M = P(J) * ( W(J) ^ 2 ) / R(J)
2210 TRAP2 = ABS ( P(J-1) * ( W(J-1)^2 ) / R(J-1) + M ) *
2210 TRAP2 = ABS ( P(J-1) ( R(J) - R(J-1) ) / 2
2220 TPSTAT(J-1) = TPSTAT(J) - TRAP2 / PREF
2230 J = J - 1
2235 GOTO 2197
2240 RETURN
2500 REM *****
2501 REM SUBROUTINE 7 COMPUTES THE STREAM FUNCTION
      AND RECIRCULATED MASS FLOW
2502 REM *****
2510 Y(0) = 0
2520 FOR J = 1 TO (N+1)
2530 TRAP2 = ( PR(J) * U(J) + PR(J-1) * U(J-1) ) * ( R(J) - R(J-1)
      ) / 2
2540 Y(J) = Y(J-1) + 6.2832 * TRAP2 / MTOT2
2550 NEXT J : PT = 10
2560 FOR J = 1 TO (N-1)
2570 IF ( Y(J) > 1 ) AND ( Y(J) < Y(J-1) ) AND ( PT = 10 ) THEN
      2580 ELSE 2590 2590
2580 PT = R(J)
2590 NEXT J
2600 FOR J = 1 TO N
2610 IF ( R(J) > = PT ) THEN 2620 ELSE 2650

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2620          UBACK1(J) = 0
2630          UBACK2(J) = ( ABS ( U(J) ) - U(J) ) / 2
2640          GOTO 2670
2650          UBACK2(J) = 0
2660          UBACK1(J) = ( ABS ( U(J) ) - U(J) ) / 2
2670      NEXT J
2680      FOR J = 0 TO (N+1)
2690          A(J) = PR(J) : B(J) = UBACK1(J) : C(J) = DUMMY(J)
2700          D(J) = DUMMY(J)
2710      NEXT J
2720      GOSUB 5000
2730      MBACK1 = 6.2832 * TRAP
2740      FOR J = 0 TO (N+1)
2750          B(J) = UBACK2(J)
2760      NEXT J
2770      GOSUB 5000
2780      MBACK2 = 6.2832 * TRAP
2790      MBACKT = MBACK1 + MBACK2
2800      RATIO1 = MBACK1 / MTOT2
2810      RATIO2 = MBACK2 / MTOT2
2820      RATIO = MBACKT / MTOT2
2830      GXFUNC(0) = 0 : GTFUNC(0) = 0
2840      FOR J = 1 TO (N+1)
2850          TRAP2 = ( PR(J) * U(J) * U(J) + PR(J-1) * U(J-1) * U(J-1) )
                * ( R(J) - R(J-1) ) / 2
2860          GXFUNC(J) = GXFUNC(J-1) + 6.2832 * TRAP2 / GX
2870      NEXT J

```

```

2880   FOR J = 1 TO (N+1)
2890       TRAP2 = ( PR(J) * U(J) * W(J) * R(J) + PR(J-1) * U(J-1) *
                W(J-1) * R(J-1) ) * ( R(J) - R(J-1) ) / 2
2900       GTFUNC(J) = GTFUNC(J-1) + 6.2832 * TRAP2 / GT
2910   NEXT J
2920   RETURN
3200   REM *****
3201   REM SUBROUTINE 8 PRINTS THE OUTPUT ON A DOT
                MATRIX PRINTER
3202   REM *****
3210   LPRINT " DIAMETER OF TEST SECTION      = ";EXPANS,"INCH" : LPRINT
3220   LPRINT " HUB DIAMETER      = ";HUB,"INCH" : LPRINT
3230   LPRINT " SLIDER SETTING      = ";SLIDER,"Cm" : LPRINT
3240   LPRINT " TEMPERATURE      = ";T,"C" : LPRINT
3250   LPRINT " BAROMETRIC PRESSURE      = ";PBAR,"mm Hg" : GOSUB 6030
3260   LPRINT TAB(5);"R";TAB(20);"U";TAB(35);"V";TAB(50);"W";TAB(65);
                "TOT-VEL"
3270   LPRINT TAB(5);"MM";TAB(19);"M/S";TAB(34);"M/S";TAB(49);"M/S";
                TAB(67);"M/S" : GOSUB 6000
3280   FOR I = 0 TO (N+1)
3290       R1(I) = R(I) * 1000
3300       LPRINT USING " *****";R1(I); : LPRINT TAB(16);"";
3301       LPRINT USING "*****";U(I); : LPRINT TAB(31);"";
3302       LPRINT USING "*****";V(I); : LPRINT TAB(46);"";
3303       LPRINT USING "*****";W(I); : LPRINT TAB(64);"";
3304       LPRINT USING "*****";VT(I)
3305       GOSUB 6000

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3310  NEXT I : GOSUB 6030
3320  LPRINT TAB(5);"R";TAB(20);"U/VREF";TAB(35);"V/VREF";TAB(50);
      "W/VREF";TAB(65);"VT/VREF"
3330  LPRINT TAB(5);"mm" : GOSUB 6000
3350  FOR I = 0 TO (N+1)
3360      LPRINT USING " *****";R1(I); : LPRINT TAB(16);"";
3361      LPRINT USING " *****";UVREF(I); : LPRINT TAB(31);"";
3362      LPRINT USING " *****";VVREF(I); : LPRINT TAB(46);"";
3363      LPRINT USING " *****";WVREF(I); : LPRINT TAB(61);"";
3364      LPRINT USING " *****";VTVREF(I)
3365      GOSUB 6000
3370  NEXT I
3375  GOSUB 6030
3380  LPRINT TAB(5);"R";TAB(20);"PST - PW";TAB(40);"( PST - PW ) / PREF"
3381  LPRINT TAB(64);"DENSITY"
3390  LPRINT TAB(5);"MM";TAB(23);"PA";TAB(38);"EXPT";TAB(54);"THEORY"
      : GOSUB 6000
3400  FOR I = 1 TO (N+1)
3410      LPRINT USING " *****";R1(I); : LPRINT TAB(19);"";
3411      LPRINT USING " *****";PSTPW(I); : LPRINT TAB(34);"";
3412      LPRINT USING " *****";PSTAT(I); : LPRINT TAB(49);"";
3413      LPRINT USING " *****";TPSTAT(I) : LPRINT TAB(64);""
3414      LPRINT USING " *****";P(I) : GOSUB 6000
3420  NEXT I
3425  GOSUB 6030
3430  LPRINT TAB(5);"R";TAB(20);"STREAM";TAB(35);"GX-FUNC";TAB(50);
      "GT-FUNC"

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3440 LPRINT TAB(5);"mm";TAB(20);"FUNCTION" : GOSUB 6000
3450 FOR I = 0 TO (N+1)
3460     LPRINT USING "    ****.***";R1(I); : LPRINT TAB(19);"";
3461     LPRINT USING "****.***";Y(I); : LPRINT TAB(34);"";
3462     LPRINT USING "****.***";GXFUNC(I); : LPRINT TAB(49);"";
3463     LPRINT USING "****.***";GTFUNC(I) : GOSUB 6000
3470 NEXT I : GOSUB 6030
3480 LPRINT " PRESSURE AT THE WALL    = ";PW,"Pa" : LPRINT : LPRINT
3490 LPRINT CHR$(27) CHR$(45) CHR$(1) " MASS FLOW RATES IN Kg/S"
3500 LPRINT CHR$(27) CHR$(45) CHR$(0) "" : LPRINT
3510 LPRINT " AT INLET    M-TAN    = ";MT1,"Kg/S" : LPRINT
3520 LPRINT " AT INLET    M-AXIAL   = ";MA1,"Kg/S" : LPRINT
3530 LPRINT " AT INLET    M-TOTAL   = ";MTOT1,"Kg/S" : LPRINT
3540 LPRINT " AT TEST SECTION M-TOTAL = ";MTOT2,"Kg/S" : LPRINT
3550 LPRINT " M-TOTAL ( TEST SECTION ) / M-TOTAL ( INLET )    = ";RATIO
3555 LPRINT " M-TAN / M-TOTAL ( INLET )    = ";ZETA : LPRINT
3560 GOSUB 6030
3570 LPRINT CHR$(27) CHR$(45) CHR$(1) " RECIRCULATION"
3580 LPRINT CHR$(27) CHR$(45) CHR$(0) "" : LPRINT ""
3590 LPRINT TAB(30);"Kg/S";TAB(50);"DIMENSIONLESS"
3600 LPRINT " M-BACK-ON AXIS    = ";TAB(27);MBACK1;TAB(50);RATIO1

3610 LPRINT " M-BACK AT WALL    = ";TAB(27);MBACK2;TAB(50);RATIO2
3620 LPRINT " TOTAL M-BACK    = ";TAB(27);MBACKT;TAB(50);RATIO : LPRINT
3630 LPRINT CHR$(27) CHR$(45) CHR$(1) " MOMENTUM FLUXES"
3640 LPRINT CHR$(27) CHR$(45) CHR$(0) TAB(60);"DIMENSIONLESS"
3650 LPRINT " ANGULAR MOMENTUM FLUX    = ";TAB(30);GT;"    N-M";TAB(60)

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;GTD : LPRINT
3660 LPRINT " AXIAL MOMENTUM FLUX      = ";TAB(30);GX;"      N";TAB(60);GXD
3670 LPRINT " PRESSURE FORCE          = ";TAB(30);PRESSF;"    N";TAB(60)
      ;PRESFD : LPRINT
3680 LPRINT " THRUST                  = ";TAB(30);THRUST;"    N";TAB(60);THRUSD
3690 LPRINT " SWIRL NUMBER            = ";S : LPRINT
3700 LPRINT " SWIRL NUMBER W.R.T THRUST = ";ST : LPRINT
3705 LPRINT " VOLUMETRIC FLOW RATE     = ";Q,"CUBIC METER / SEC"
3710 LPRINT " REFERENCE VELOCITY      = ";VREF,"M/S" : LPRINT
3715 LPRINT " REFERENCE PRESSURE      = ";PREF,"Pa " : LPRINT
3720 LPRINT " RAYNOLD'S NUMBER        = ";RE : LPRINT : LPRINT
3725 GOSUB 6030
3730 LPRINT CHR$(27) CHR$(45) CHR$(1) " ENERGY FLUXES " : LPRINT
3740 LPRINT CHR$(27) CHR$(45) CHR$(0) : LPRINT TAB(40);"J/SEC";TAB(60);
3750 LPRINT " AXIAL KINETIC ENERGY FLUX = ";TAB(38);EKINX;TAB(60)
      ;EKINXD
3760 LPRINT " TANGENTIAL KINETIC ENERGY FLUX = ";TAB(38);EKINT;
      ;TAB(60);EKINTD
3765 LPRINT " RADIAL KINETIC ENERGY FLUX = ";TAB(38);EKINR;TAB(60)
      ;EKINRD
3770 LPRINT " TGTAL KINETIC ENERGY FLUX = ";TAB(38);KETOT;TAB(60)
      ;KETOTD
3780 LPRINT " PRESSURE ENERGY FLUX     = ";TAB(38);EST;TAB(60);ESTD
3790 LPRINT " TOTAL ENERGY FLUX      = ";TAB(38);ETOT;TAB(60);ETOTD
3800 RETURN
4000 REM *****
4001 REM SUBROUTINE 9 CREATES A DATA FILE AND RECORDS

```

THE RESULTS IN IT

```

4002 REM *****
4010 OPEN "A:OUT515.DAT" FOR OUTPUT AS #1
4020 WRITE#1 ,EXPANS,HUB,SLIDER,T,PBAR
4030 FOR I = 0 TO (N+1)
4040     WRITE#1 ,R1(I),U(I),W(I),V(I),UVREF(I),VVREF(I),WVREF(I),
        VTVREF(I)
4050     WRITE#1 ,PSTPW(I),PSTAT(I),TPSTAT(I),Y(I),GXFUNC(I),GTFUNC(I)
4060 NEXT I
4070 WRITE#1 ,PW,MT1,MA1,MTOT1,MTOT2,RATIO,ZETA,MBACK1,RATIO1,MBACK2,
        RATIO2
4080 WRITE#1 ,MBACKT,RATIO2,VREF,RE,GT,GTD,GX,GXD,PRESSF,PRESFD,THRUST
        ,THRUSD
4090 WRITE#1 ,S,ST,EKINX,EKINXD,EKINT,EKINTD,KETOT,KETOTD,EST,ESTD,ETOT
        ,ETOTD,ZETA,Q
4100 CLOSE#1
4110 RETURN
5000 REM *****
5005 REM SUBROUTINE 10 PERFORMS THE TRAPAZOIDAL
        RULE FOR INTEGRATION
5006 REM *****
5010 TRAP = 0
5020 FOR J = 1 TO (N+1)
5030     TRAP = TRAP + ( A(J) * B(J) * C(J) * D(J) + A(J-1) * B(J-1) *
        C(J-1) * D(J-1) ) * ( RT(J) - RT(J-1) ) / 2
5040 NEXT J
5050 RETURN

```

```

6000 REM *****
6005 REM SUBROUTINE 11 IS USED FOR FORMATING PURPOSES
6006 REM *****
6010 FOR K = 1 TO 70 : LPRINT "-"; : NEXT K
6020 LPRINT "" : RETURN
6030 CLS : PRINT " FORWARD THE PRINTER TO THE TOP OF THE" : PRINT
6040 PRINT " FORM. PRESS 1 WHEN READY" : PRINT : PRINT
6050 INPUT Z
6060 IF ( Z <> 1 ) THEN 6070 ELSE 6090
6070 PRINT " PRESS 1 TO CONTINUE"
6080 GOTO 6050
6090 RETURN

```

Appendix B

Computer Code Used To Predict

$$(\Omega_D)_{crit}$$

```
10  DIM RHUB(20),REXP(20),U1(20),ZETA(20),A(20),GAMFIT(20)
    ,RH(20),A1(20)
20  DIM R2(20),PHI(20),Z1(20),Z2(20),B(20),A2(20),TESLA(20)
    ,RH01(20),RH02(20)
30  DIM RHO(20),OMEGA(20),GAMMA(20),Z11(20),Z12(20),Z21(20)
    ,Z22(20),REXP1(20)
32  DIM RHUB1(20),RH011(20),RH012(20),RH021(20),RH022(20)
    OMEGEX(20),ALPHA(20)
35  DIM BETA(20),Z13(20),Z14(20),Z51(20),Z52(20),Z53(20),
    ,Z54(20),Z5(20),K(20)
40  INPUT " N , A = ";N,A
70  LPRINT CHR$(14) " N = ";N : LPRINT
75  LPRINT CHR$(14) " A = ";A : LPRINT
80  FOR I = 1 TO 16
```

```

90      READ A(I),ZETA(I),U1(I),REXP1(I),RHUB1(I),ALPHA(I)
      ,OMEGEX(I)
100     RHUB(I) = RHUB1(I) * .0254 : REXP(I) = REXP1(I)
      * 0.0254
110     RH(I) = ( RHUB(I) / .0508 ) ^ 2
115     K(I) = 1 - A * ( RH(I)^N ) + ( A - 1 ) * RH(I)^( 2*N )
120     A1(I) = ( A(I) / .0508 ) ^ 2
130     R2(I) = ( REXP(I) / .0508 ) ^ 2
140     PHI(I) = (4/R2(I)) * ( ZETA(I)/6 + 1/3 - ZETA(I)*RH(I)/2
      * (ZETA(I)-1)* (RH(I)^1.5)/3 )
150     BETA(I) = ( A(I) - ALPHA(I) * RH(I) ) / ( A(I) - RH(I) )
160     Z11(I) = ( (A1(I)*RH(I))^1.5 - ( RH(I)^2 ) / A1(I) )
      * ZETA(I) * ( ALPHA(I) - BETA(I) ) / 3
170     Z12(I) = ( A1(I) - ( RH(I)^2 ) / A1(I) ) * ( ZETA(I)
      * BETA(I) + ( ZETA(I) - 1 ) * ( BETA(I) -
      - ALPHA(I) ) * ( RH(I)^0.5 ) * 0.25
180     Z13(I) = - BETA(I) * ( ZETA(I) - 1 ) * ( A1(I)^1.5 -
      ( RH(I)^2.5 ) / A1(I) ) / 5
      + ( ZETA(I) / 2 ) * ( 1 - A1(I) )
190     Z14(I) = - ( ZETA(I) - 1 ) * ( 1 - A1(I)^1.5 ) / 3
200     Z1(I) = Z11(I) + Z12(I) + Z13(I) + Z14(I)
210     REM  RHO = ( GAMMA / ( U1*r1 ) ) ^ 2
220     Z51(I) = ( ALPHA(I)^2 ) * ( RH(I) / (A1(I)^2) ) * ( R2(I)
      - .5 * RH(I) ) / 2 + .5 * LOG ( R2(I)/
230     Z52(I) = ( ( ALPHA(I) - BETA(I) )^2 ) * RH(I) / A1(I)
      * ( 0.5 * R2(I) / A1(I) * LOG ( A1(I)/RH(I) ) -
      0.5 * ( 1 - RH(I) / A1(I) ) )

```

```

240      Z53(I) = 2 * ( ALPHA(I) - BETA(I) ) * BETA(I) * SQR(RH(I))
           / ( A1(I)^2 ) * ( RH(I)^1.5 - A1(I)^1.5 ) / 3 +
           ( A1(I)^0.5 - RH(I)^0.5 ) * R2(I) )
250      Z54(I) = ( BETA(I)^2 * ( 1 - RH(I)/A1(I) ) * ( R2(I) -
           - 0.5 * ( A1(I) + RH(I) ) ) + R2(I) - A1(I) ) /
           ( 0.5 * A1(I) )
260      Z5(I) = Z51(I) + Z52(I) + Z53(I) + Z54(I)
265      Z2(I) = ( ZETA(I)^2 )/6 + ZETA(I)/3 + 1/2 - ( ZETA(I)^2 )
           * RH(I) + 4/3 * ( ZETA(I)^2 - ZETA(I) ) * ( RH(I)
           ^1.5 ) - ( ( ZETA(I) - 1 ) * RH(I) )^2 ) / 2
270      REM   B(I) IS THE CORE RADIUS AT STATION 2
280      B(I) = SQR( ( A(I)^2 - RHUB(I)^2 ) * (REXP(I)^2) /
           ( .0508^2 - RHUB(I)^2 ) )
290      A2(I) = ( B(I) / REXP(I) ) ^ 2
300      TESLA(I) = Z1(I) / ( PHI(I) * ( R2(I)^1.5 ) * ( 1/5 -
           ( A2(I)^2.5 ) / 30 ) )
310      RHO1(I) = Z2(I) - ( PHI(I)^2 ) * R2(I) / 3 - K(I) * .5 *
           R2(I) * ( ( ZETA(I) - ( ZETA(I) - 1 ) * ( RH(I)
           ^0.5 ) )^2 )
320      RHO2(I) = ( TESLA(I)^2 ) * ( A2(I) / 4 + .5
           * LOG ( A2(I) ) ) - Z5(I)
330      RHO(I) = RHO1(I) / RHO2(I)
340      IF ( RHO(I) < 0 ) THEN 370
350      OMEGA(I) = ( RHO(I)^.5 ) * 8 * Z1(I) * ( 1 - RH(I) )
           / ( ( PHI(I)^2 ) * R2(I)^2 )
360      GAMMA(I) = ( U1(I) * .0508 ) * ( RHO(I)^.5 )
370 NEXT I

```

```

380 LPRINT TAB(2);"RHUB";TAB(10);"REXP";TAB(25);"OMEGA";TAB(40)
      ;"OMEGA";TAB(50);"K"
390 LPRINT TAB(24);"EXPER";TAB(40);"MODEL"
400 FOR I = 1 TO 12
410     LPRINT "-----"
420     LPRINT TAB(24); : LPRINT USING "##." ;RHUB1(I); : LPRINT
      TAB(9) " ";
430     LPRINT USING "##." ;REXP1(I); : LPRINT TAB(24)"";
440     LPRINT USING "###" ;OMEGEX(I); : LPRINT TAB(39)"";
450     LPRINT USING "###" ;OMEGA(I); : LPRINT TAB(49)"";
455     LPRINT USING "###";K(I)
460 NEXT I
470 RESTORE
480 LPRINT : LPRINT
490 K = K - INC
510 END

```

Appendix C

Computer Code Used To Fit The Assumed Velocity Profiles Into The Experimentally Obtained Data

```
10  DIM WU(26),RR1(26),UU(26),W(26),R(26),U(26),ZETA(26)
    ,U1URFPR(26),U1UREF(26),U1(26),ERR1(26),UMOD(26,26)
    ,ALPHA(26),ERR2(26,26),A1(26,26),S(26)
20  DIM BETA(26,26),APR(26,26),Z11(26,26),Z12(26,26),Z13(26,26)
    ,Z14(26,26),Z1(26,26),GAMMA(26,26),WH(26,26),WA(26,26)
    ,WMODPR(26,26),WMOD(26,26)
30  DIM U1ZETA(26)
40  OPEN "B:W120.DAT" FOR INPUT AS #1
50  INPUT #1,EXPAN,RH,R1,N,OMEGA,UREF,GT1,GTN,ZETA,ALPHA
```

```

60  FOR I = 1 TO N
70      INPUT #1,WU(I),RR1(I),UU(I)
80      R(I) = RR1(I) * R1
90      W(I) = WU(I) * UREF
100     U(I) = UU(I) * UREF
110 -NEXT I
120  RBO = SQR ( R(N) * R1 )
130  RBI = SQR ( R(1) * RH )
140  UREFPR = UREF * ( R1^2 - RH^2 ) / ( RBO^2 - RBI^2 )
150  DFODAO = ( RBO^2 - R(N)^2 ) / ( R(N) * ( R1 - R(N) ) )
160  DFIDAI = ( R(1) + RBI ) * ( R(1)^2 + RBI^2 ) / ( 2
          * R(I)^3 )
170  GTBO = GTN * DFODAO * ( 1 - GTN )
180  GTBI = GT1 * ( 1 - DFIDAI )
190  OMEGPR = ( GTBO - GTBI ) * ( R1 / RBO ) * OMEGA *
          ( UREF / UREFPR )
200  REM
210  REM *****
220  REM *      LINES 250 - 500 FIT A STRAIGHT LINE THROUGH THE *
230  REM *      AXIAL VELOCITY PROFILE *
240  REM *****
250  REM
260  J = 1
270  RHPR = ( RBI / RBO ) ^ 2
280  PRINT TAB(3);"J";TAB(11);"ZETA";TAB(24);"U1";TAB(36);"ERROR";
          TAB(52);"K" ;TAB(56);"U1UREF"
290  FOR K = -40 TO 40 STEP 5

```

```

300     ZETA(J) = ZETA + K * ZETA / 100
310     REM    U1URFPR = U1 / UREFPR
320     U1URFPR(J) = 1 / ( ZETA(J) - 2 * ( ZETA(J) - 1 ) * ( 1 -
                        RHPR1.5 ) / ( 3 * ( 1 - RHPR ) ) )
330     U1UREF(J) = U1URFPR(J) * ( UREFPR / UREF )
340     U1(J) = U1UREF(J) * UREF
350     ERR1(1) = 0
360     FOR I = 1 TO N
370         UMOD(I,J) = U1(J) * ( ZETA(J) - ( ZETA(J) - 1 ) *
                        * R(I) / R1 )
380         ERR1(J) = ERR1(J) + ( U(I) - UMOD(I,J) ) ^ 2
390     NEXT I
400     PRINT TAB(2);J;TAB(10);ZETA(J);TAB(20);U1(J);TAB(35);
        ;ERR1(J);TAB(50);K;TAB(55);U1UREF(J)
410     J = J + 1
420 NEXT K
430 PRINT " CHOOSE THE LSE FITTING FOR  U PROFILE BY ENTERING "
440 PRINT " THE PROPER VALUE OF J FOR MINIMUM ERROR      ";
450 INPUT F : U1ZETA(F) = U1UREF(F) * ZETA(F) : PRINT U1ZETA(F)
460 LPRINT CHR$(27) CHR$(69) ""
470 LPRINT CHR$(14) "THE FITTING PARAMETERS" : LPRINT
480 LPRINT "HUB RADIUS = ";RH : LPRINT
490 LPRINT "EXPANSION DIAMETER = ";EXPAN : LPRINT
500 LPRINT "ZETA = ";ZETA(F) : LPRINT
510 LPRINT "U1 = ";U1(F) : LPRINT
520 REM
530 REM *****

```

```

540 REM *      LINES 570 - 1040 FIT A RANKIN VORTEX THROUGH *
550 REM *      THE TANGENTIAL VELOCITY PROFILE *
560 REM *****
570 REM
580 J = 1 : CLS
590 FOR K = -30 TO 30 STEP 10
600 PRINT TAB(2);"I";TAB(10);"a1";TAB(23);"GAMMA";TAB(35);
      "ERROR";TAB(51);"K"
610 ALPHA(J) = ALPHA + K * ALPHA / 100
620 FOR I = 1 TO N
630 A1(I,J) = R(I)
640 SUM = 0
650 FOR L = 1 TO N
660 BETA(L,J) = ( A1(I,J) - ALPHA(J) * RBI ) /
      ( A1(I,J) - RBI )
670 APR(L,J) = ( A1(I,J) / RBO ) ^ 2
680 Z11(L,J) = ( ( APR(L,J) * RHPR ) ^ .5 - ( RHPR ^
      ^2 ) / APR(L,J) ) * ZETA(F) * ( ALPHA(J) -
      BETA(L,J) ) / 3
690 Z12(L,J) = ( APR(L,J) - ( RHPR^2 ) / APR(L,J) )
      * ( ZETA(F) * BETA(L,J) + ( ZETA(F) - 1 )
      * ( BETA(L,J) - ALPHA(J) ) * ( RHPR^1.5 ) )
      / 4
700 Z13(L,J) = - BETA(L,J) * ( ZETA(F) - 1 ) * ( (
      APR(L,J) ^ 1.5 ) - ( RHPR^2.5 ) / APR(L,J)
      ) / 5
710 Z14(L,J) = ZETA(F) * ( 1 - APR(L,J) ) / 2 - ( ZETA(F)

```

```

-1 ) * ( 1 - ( APR(L,J) ^ 1.5 ) ) / 3
720      Z1(L,J) = Z11(L,J) + Z12(L,J) + Z13(L,J) + Z14(L,J)
730      GAMMA(L,J) = OMEGPR * ( UREFPR / U1(F) ) * UREFPR
          * RBO * ( 1 - RHPR ) / ( (Z1(L,J) * 2 )
740      WH(L,J) = ALPHA(J) * GAMMA(L,J) * RH / ( A1(I,J)
          ^ 2 )
750      WA(L,J) = GAMMA(L,J) / A1(I,J)
760      IF ( R(L) <= RBI ) THEN 770 ELSE 790
770          WMODPR(L,J) = WH(L,J) * R(L) / RBI
780          GOTO 830
790      IF ( R(L) > RBI ) AND ( R(L) <= A1(I,J) ) THEN 800
          ELSE 820
800          WMODPR(L,J) = WH(L,J) + ( WA(L,J) - WH(L,J)
          ) * ( R(L) - RBI ) / ( A1(I,J)
          - RBI )
810          GOTO 830
820          WMODPR(L,J) = GAMMA(L,J) / R(L)
830          WMOD(L,J) = WMODPR(L,J) * UREFPR / UREF
840          SUM = SUM + ( W(L) - WMOD(L,J) ) ^ 2
850      NEXT L
860      ERR2(I,J) = SUM
870      PRINT TAB(2);I;TAB(8);A1(I,J);TAB(23);GAMMA(I,J);
          TAB(35);ERR2(I,J);TAB(51);K
880      NEXT I
890      PRINT " CHOOSE THE LSE FITTING FOR W PROFILE BY ENTERING "
900      PRINT " THE PROPER VALUE OF I FOR MINIMUM ERROR ";
910      INPUT S(J) : CLS

```

```

920     J = J + 1
930 NEXT K
940 PRINT TAB(2);"J";TAB(10);"A1";TAB(23);"GAMMA";TAB(35);"ERROR";
      TAB(51);"ALPHA"
950 FOR J = 1 TO 7
960     PRINT TAB(2);J;TAB(10);A1(S(J),J);TAB(23);GAMMA(S(J),J);
      TAB(35);ERR2(S(J),J);TAB(51);ALPHA(J)
970 NEXT J
980 PRINT " CHOOSE THE LSE FITTING FOR W PROFILE BY ENTERING "
990 PRINT " THE PROPER VALUE OF J FOR MINIMUM ERROR ";
1000 INPUT Q
1010 LPRINT "A1 = ";A1(S(Q),Q) : LPRINT
1020 LPRINT "GAMMA = ";GAMMA(S(Q),Q) : LPRINT
1030 LPRINT "ALPHA = ";ALPHA(Q) : LPRINT
1040 LPRINT "WH = ";WH(S(Q),Q) : LPRINT
1050 LPRINT "WA = ";WA(S(Q),Q) : LPRINT
1060 CLOSE #1
1070 END

```