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THERMAL CONDUCTIVITY OF TWO-PHASE AND THREE-PHASE HETEROGENEOUS SOLID MIXTURES

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ABSTRACT

A study of the thermal conductivity of two-phase and three-phase solid heterogeneous mixtures is presented. The experimental measurements of the thermal conductivity were carried out by the line-source method, which is a transient technique. A comparison is made between the experimental results and those predicted from analytical approaches. An evaluation of accuracy and reliability of the analytical approaches is also discussed.

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NOMENCLATURE

Letter Symbols

Symbol	
A	Area, ft^2
B	Variable in Eqs. (10)and (11), $\sqrt{3P_d/2}$
C	Variable in Eqs. (10) and (11), $-4\sqrt{2/3P_d}$
C_e	Euler's constant (0.5772157...)
D	Thermal diffusivity, ft^2/hr
EM	Electrical potential, MV
e	Relative error, %
I	Current, amps
$\bar{k}$	Average thermal conductivity, Btu/hr ft °F
k	Thermal conductivity, Btu/hr ft °F
L	Characteristic length, ft
Ln	Natural logarithm
log	logarithm to the base 10
n	Total number of tests, summation symbol
P	Phase volume fraction
q'	Continuous line source strength, Btu/hr ft
q	Heat flow, Btu/hr
R	Electrical resistance, ohms
r	Distance from measuring point to the line source, ft

s	Magnitude of measured error quantities
S	Sphericity
SD	Standard deviation
SM	Error criterion
t	Temperature, °F
T	Absolute temperature, °F
V	Thermocouple response, $d(EM)/dt$, MV/°F
X	Shape factor in Eq. (2)
x,y,z	Space coordinates

Greek Letters

α	Dummy variable
β	Parameter in Eq. (21), $r/2(D\theta)^{1/2}$
θ	Time, seconds
θ_c	Time correction value, seconds
ϕ	$3/S$

Subscripts

c	Continuous phase
d	Discontinuous phase
e	Equivalent
i	Number of phase or number of test
m	Mean

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I. INTRODUCTION

Problems involving heat transfer through heterogeneous systems are frequently encountered in the ceramic industry, the solid propellant industry, and other industrial and scientific endeavors. The study associated with heterogeneous systems has attracted many researchers. A large amount of research has been done recently in conduction of heat transfer in heterogeneous mixtures (1-5).*

Thermal conductivity is one of the most important parameters in the solution of conduction problems. The value of the thermal conductivity of mixtures is is not usually obtainable from handbooks because there is an infinite variety of mixtures. Therefore it is necessary to have a simple and direct method of determining the thermal conductivity of mixtures, both experimentally and theoretically, when the properties of the constituents are known.

Two-phase heterogeneous systems (continuous phase and discontinuous phase) can be classified as follows

1. Solid-solid mixtures
   (two-phase solid mixtures)

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* Number in brackets refer to reference cited

2. Suspensions: Solid particles suspended in a liquid.
3. Emulsions: Liquid droplets distributed throughout an immiscible liquid.
4. Packed beds: Solid particles resting one upon another with a flowing fluid filling the interstitial spaces.
5. Powders: Fine solid particles resting one upon another with a motionless fluid filling the interstitial spaces.
6. Porous materials

Three-phase heterogeneous systems are the extension of two-phase systems such as two-phase solid-solid mixtures with porosity, three-phase solid mixtures. Suspensions, emulsions, packed beds, and powders are also covered in the three-phase systems.

In this study, the thermal conductivity of two-phase and three-phase solid mixtures is experimentally determined by the line-source method. This is a transient technique. A comparison is made between the experimental results and those predicted from existing theoretical models. The accuracy and reliability of existing models is also discussed.

II. THEORY

GENERAL CONSIDERATION

The conduction of heat in solids, from the microscopic point of view, is a function of lattice waves (phonons) produced by atomic motion, and electron scattering. For dielectrics, the heat transfer is primarily by phonons; for metals, heat conduction is primarily by electron scattering. For alloys, semiconductors, and poorly conducting metallic solids, both the lattice waves and electron scattering are important. The details of the heat conduction mechanism are very complicated and still not completely understood. Some of the factors which govern heat conduction are boundary crystallographic imperfection of the lattice, mechanical defects of the lattice, chemical impurities, etc. (6-8).

The thermal conductivity of a solid varies with temperature and pressure, as shown in the work by Kowalczyk (7) and Klemens (9). In general, thermal conductivity increases with pressure and can increase or decrease with increasing temperature, depending on the substance.

The thermal conductivity of solid mixtures depends on particle (discontinuous phase) shape, size, orientation, proportion and distribution, and the thermal conductivity of each phase. A detailed literature survey of the thermal conductivity of two-phase and three-phase heterogeneous mixtures is presented in the next section.

TWO-PHASE HETEROGENEOUS MIXTURES

Since there exists a variety of heterogeneous mixtures, some criteria for classification must be employed. For the purpose of this discussion and review, the particles dispersed in a surrounding medium will be designated as the discontinuous phase or dispersed phase. The surrounding medium will be called the continuous phase. A method of classification, as suggested by Powers (10), will be followed hereafter.

The first criterion to be considered in the classification is the volume fractions of the phases present.

A. Dilute dispersions

If the discontinuous phase is uniformly dispersed in the continuous phase and the dispersion is of a dilute nature, the mixture is known as a dilute dispersion. The equations governing such conditions are known as dilute equations. A dilute dispersion may be defined as the volume fraction of the discontinuous phase not exceeding approximately thirty percent of the total volume.

B. "Mixtures"

If the percentage of the discontinuous phase approaches fifty percent, individual particles of this phase may be in contact. In this case, the mixture is essentially in a transitional state; that is, the continuous and discontinuous phasesare not readily distinguishable. The equations used to predict the conductivity of mixture systems are referred to as "Mixture" equations.

C. Variable dispersions

In a variable dispersion, the volume fraction of the discontinuous phase may vary widely, from zero to one hundred percent. Therefore both dilute dispersions and "mixtures" may be included in the variable dispersions category. However, in deriving the equations to predict the thermal conductivity of the variable dispersions, the continuous phase was assumed to remain continuous regardless of the concentration of the discontinuous phase. Equations of this nature are often referred to as variable dispersion equations.

A second consideration in the classification of heterogeneous mixtures is the discontinuous phase particle geometry. The particle shapes usually considered in the literature are spherical, cylindrical, and platelets.

Many models have been proposed by researchers (2,11, 13-17) in deriving theoretical equations to predict the thermal conductivity of two-phase mixtures. Some of them are based on the potential theory wherein heat flux is oriented towards or away from a particle depending on the relative conductivities of the components.

Maxwell (11) was the first to work on the problem of the conductivity of two-phase mixtures. He derived an equation

$$\frac{\frac{k_m}{k_c} - 1}{\frac{k_m}{k_c} + 2} = P_d \left[\frac{\frac{k_d}{k_c} - 1}{\frac{k_d}{k_c} + 2} \right]$$

or

$$k_m = \frac{k_c [k_d + 2\, k_c - 2\, P_d (k_c - k_d)]}{k_d + 2\, k_c + P_d (k_c - k_d)} \qquad (1)$$

to calculate the thermal conductivity of two-phase mixtures. In the derivation, Maxwell assumed that the particles were far apart and were randomly distributed small spheres so that the flux lines of heat surrounding any particle were deflected by that particle, but were not influenced by other particles. As the result of this assumption , Eq.(1) is primarily applicable for a dilute dispersion with spherical particles of discontinuous phase. However, it is found that if k_d/k_c is less than 100, this equation is reasonably accurate regardless of the particle shape (12).

Ficke (13) extended Maxwell's analysis to the case of ellipsoids suspended in a continuous phase. He obtained the equation

$$\frac{\frac{k_m}{k_c} - 1}{\frac{k_m}{k_c} + X} = P_d \left[\frac{\frac{k_d}{k_c} - 1}{\frac{k_d}{k_c} + X} \right] \qquad (2)$$

where X is a complicated function of the shape of ellipsoids and a function of k_d and k_c. Ficke concluded from his analysis that X differs from two only when the conductivities of the two components differ greatly. For the case when the conductivities of the two components are on the same order of magnitude, the value of X is equal to two. Hence Eq. (2) reduces to Eq. (1), i.e. Maxwell's equation.

Niesel (14) derived two equations to predict the thermal conductivity of two-phase mixtures with random-oriented, long thin cylindrical, dispersed particles as follows

$$P_c \left[\frac{k_c - k_m}{k_m} - \frac{4(k_c - k_m)}{k_c + k_m} \right] + P_d \left[\frac{k_d - k_m}{k_m} + \frac{4(k_d - k_m}{k_d + k_m} \right] = 0 \qquad (3)$$

$$1 - P_d = \frac{k_d - k_m}{k_d - k_c}\left[\frac{k_d + 5\,k_c}{k_d + 5\,k_m}\right]^{2/5} \qquad (4)$$

wherein Eq. (3) is applicable for a "mixture" and Eq. (4) for a variable dispersion.

Both Bruggeman (15) and Niesel (14) working independently, developed a "mixture" equation for predicting the thermal conductivity of mixtures containing randomly oriented platelets

$$P_c\left[\frac{2(k_c - k_m)}{k_m} - \frac{k_c - k_m}{k_c}\right] + P_c\left[\frac{2(k_d - k_m)}{k_m} + \frac{k_d - k_m}{k_d}\right] = 0 \qquad (5)$$

They also developed an equation for a variable dispersion aggregate composed of randomly-oriented laminae, which can be written as

$$k_m = \left[\frac{3\,k_c + 2\,P_d\,(k_d - k_c)}{3\,k_c - P_d\,(k_d - k_c)}\right] \qquad (6)$$

Powers (10) derived two equations to predict the thermal conductivity of two-phase mixtures based on the application of Ohm's law for series and parallel conductors. This application restricts the hypothesis to flux lines that are straight. For the case of a two-phase mixture consisting of laminae oriented parallel to the flux lines, the application to this system yields the equation

$$k_m = k_c P_c + k_d P_d \tag{7}$$

for the overall thermal conductivity of the mixture.

If the laminae are oriented perpendicular to the flux lines, the overall conductivity can be expressed as

$$\frac{1}{k_m} = \frac{P_c}{k_c} + \frac{P_d}{k_d} \tag{8}$$

Equations (7) and (8) place upper and lower limits on an aggregate system.

The above equations are only applicable for special discontinuous particle shape and distribution in predicting the thermal conductivity of two-phase mixtures. However, there are two general equations for predicting the thermal conductivity of two-phase mixtures that can be discussed as follows.

Hamilton (16) developed an equation for two-phase mixtures with random particle shape and orientation. His equation can be written as

$$k_m = k_c \left[\frac{k_d + (\phi - 1) k_c - (\phi - 1) P_d (k_c - k_d)}{k_d + (\phi - 1) k_c + P_d (k_c - k_d)} \right] \qquad (9)$$

In this equation, ϕ is an empirical shape factor which is expressed in the form of $\phi = 3/S$, where S is the sphericity. Sphericity is defined as the ratio of the surface area of a sphere with a volume equal to that of the particle, to the surface area of the particle. Hamilton stated that if k_d/k_c is less than 100, a value of $\phi = 3$ can be used without much error. However, ϕ should be determined for larger values of k_d/k_c. For the spherical particle, his equation reduced to Eq. (1).

Tsao (17) derived an equation to predict the thermal conductivity of two-phase mixtures by a probabilistic approach. The equation connects the thermal conductivity of two-phase mixture to the constituent conductivities and two parameters describing the spatial distribution of the phases. These two parameters must be known before the thermal conductivity can be calculated. Cheng (2) extended the analysis, by assuming a parabolic distribution of the discontinuous phase and derived two equations in predicting the thermal conductivity of two-phase mixtures. Cheng's equations are

i) When $k_c > k_d$

$$\frac{1}{k_m} = \frac{2}{\sqrt{C(k_d-k_c)[k_c+B(k_d-k_c)]}} \tan^{-1} \frac{B}{2}\sqrt{\frac{C(k_d-k_c)}{k_c+B(k_d-k_c)}} + \frac{1-B}{k_c} \qquad (10)$$

ii) When $k_d > k_c$

$$\frac{1}{k_m} = \frac{1}{\sqrt{C(k_d-k_c)[k_c+B(k_d-k_c)]}} \operatorname{Ln} \frac{\sqrt{k_c+B(k_d-k_c)} + \frac{B}{2}\sqrt{C(k_c-k_d)}}{\sqrt{k_c+B(k_d-k_c)} - \frac{B}{2}\sqrt{C(k_c-k_d)}} + \frac{1-B}{k_c} \qquad (11)$$

where $B = \sqrt{(3/2)\ P_d}$

$C = -4\sqrt{2/3\ P_d}$

Cheng's equations are appliable to a maximum discontinuous phase volume fraction of 66.6 %. He further concluded that if $k_d/k_c > 100$, the first term of Eq. (11) can be neglected and the thermal conductivity of the two-phase mixtures becomes

$$k_m \simeq \frac{k_c}{1-B} \qquad (12)$$

In Eq. (12), k_m becomes function of k_c and P_d only.

THREE- AND MULTI-PHASE HETEROGENEOUS MIXTURES

The analysis of three- and multi-phase mixtures is more complicated than that of the two-phase case. For a multi-phase system, it is very difficult to describe the phase distribution. Therefore, there has not been much work done in predicting the thermal conductivity of three- and multi-phase mixtures.

Hamilton (12) developed the following equation to predict the thermal conductivity of multi-phase mixtures.

$$k_m = \frac{\sum_{i=1}^{m} k_i P_i \frac{\left(\frac{dT}{dx}\right)_i}{\left(\frac{dT}{dX}\right)_1}}{\sum_{i=1}^{m} P_i \frac{\left(\frac{dT}{dX}\right)_i}{\left(\frac{dT}{dX}\right)_1}} \tag{13}$$

where

$$\frac{(dT/dX)_i}{(dT/dX)_1} = \frac{\phi_i k_1}{k_i + (\phi_i - 1) k_1} \tag{14}$$

By substituting Eq. (14) into Eq. (13), we get

$$k_m = \left[\frac{1 - \sum_{i=2}^{m} \frac{P_i(\phi_i - 1)(k_1 - k_i)}{k_i + (\phi_i - 1) k_1}}{1 + \sum_{i=2}^{m} \frac{P_i(k_1 - k_i)}{k_i + (\phi_i - 1) k_1}} \right] k_1 \tag{15}$$

If all the discontinuous phases are spherical particles, then $\phi_i = 3$, and Eq. (15) can be simplified to

$$k_m = k_1 \left[\frac{1 - \sum_{i=2}^{m} \frac{2 P_i(k_1 - k_i)}{k_i + 2 k_1}}{1 + \sum_{i=2}^{m} \frac{P_i(k_1 - k_i)}{k_i + 2 k_1}} \right] \tag{16}$$

Cheng (2) reduced a three-phase mixture into one or two equivalent two-phase mixtures according to the relative magnitudes of the conductivity of the constituents. Subsequently, they have been solved by the two-phase method.

From Cheng's analysis, by considering the ratio of k_{d1}/k_c and k_{d2}/k_c, four cases can be classified.

1. $k_{d1}/k_c > 100$, $k_{d2}/k_c > 100$;
2. $k_{d1}/k_c > 100$, $100 > k_{d2}/k_c > 1$;
3. $100 > k_{d1}/k_c > 1$, $100 > k_{d2}/k_c > 1$;

4. $k_{d1}/k_c < 1$ and/or $k_{d2}/k_c < 1$;

where k_{d1} refers to the discontinuous phase with higher thermal conductivity. These cases are discussed in the following.

Case 1. $k_{d1}/k_c > 100$, $k_{d2}/k_c > 100$

From Cheng's analysis of two-phase mixtures, when $k_d/k_c > 100$, the mean thermal conductivity of mixtures are function of k_c and P_d only. He extended this idea to three phase mixtures. In this case, since both k_{d1} and $k_{d2} \gg k_c$, the first discontinuous phase and the second discontinuous phase can be combined into one single discontinuous phase. Thus $P_{de} = P_{d1} + P_{d2}$, the three-phase mixture is then reduced to a two-phase mixture. By employing Eq. (12), thermal conductivity can be predicted by

$$k_m = \frac{k_c}{1 - B_e} \qquad (17)$$

where $B_e = \sqrt{(3/2)\, P_{de}}$

Case 2. $k_{d1}/k_c > 100$, $100 > k_{d2}/k_c > 1$

In this case, since $k_{d1} \gg k_c$, one can consider the three-phase mixture to be reduced to a two-phase mixture in which the continuous phase is composed of the original continuous phase and the second discontinuous phase of the three-phase mixture. The discontinuous phase of the two-phase mixture is the first discontinuous phase of the three-phase mixture. The thermal conductivity of the equivalent two-phase mixture can be predicted by

$$k_m = \frac{k_{ce}}{1 - B} \tag{18}$$

where $B = \sqrt{(3/2)\ P_{d1}}$ and k_{ce} represents the mean thermal conductivity of the continuous phase of the original three-phase mixture mixed with the second discontinuous phase of the original three-phase mixture. The value of k_{ce} must be calculated according to Eq. (11) before k_m can be determined.

An alternative method is to reduce the original three-phase mixture to a two-phase mixture in which the continuous phases consist of the continuous phase and the first discontinuous phase of the three-phase mixture. The discontinuous phase of this two-phase mixture is the second discontinuous phase of the three-phase mixture. The detailed calculation is similar to the first method. The k_m in case 2 can be taken as the arithmetic average of the values from these two methods.

Case 3. $100 > k_{d1}/k_c > 1$, $100 > k_{d2}/k_c > 1$

This case is similar to Case 2 wherein the three-phase mixture can be reduced to an equivalent two-phase mixture in which the continuous phase is composed of the original continuous phase and one of the discontinuous phases. The discontinuous phase is the remaining discontinuous phase of the original three-phase mixture. In this case, the thermal conductivity of the equivalent two-phase mixture is obtained by using Eq. (11) twice. For example, if one assumes that the continuous phase is composed of the original continuous phase and the second discontinuous phase of the three-phase mixture. The discontinuous phase of this equivalent two-phase mixture is the first discontinuous phase of the three-phase mixture. Then

$$P_d = P_{d1}$$

$$k_d = k_{d1}$$

$$P_{ce} = P_{d2} + P_c$$

and k_{ce} is obtained in the same way as in Case 2.

Case 4. $k_{d1}/k_c < 1$ and/or $k_{d2}/k_c < 1$

Case 4 can be treated exactly in the same way as Case 3. This technique of predicting the thermal conductivity of a three-phase mixture can be extended to a multi-phase system. Hamilton's equation is applicable for known shape factors of discontinuous phases, while Cheng's equation is somewhat flexible with no restriction on the particle shapes of discontinuous phases.

III. EXPERIMENTAL METHOD

INTRODUCTION

The basic law which defines heat conduction is generally attributed to the French mathematician Jean Fourier (1768-1830). The Fourier law states that the quantity of heat dQ conducted in the x-direction of a homogeneous solid in time $d\theta$ is a product of conducting area A normal to the flow path direction x, the temperature gradient dt/dx, and a property k of conducting material known as thermal conductivity. For steady heat flow, Fourier's law can be expressed as

$$q = -kA\left(\frac{\partial t}{\partial x}\right) \tag{19}$$

The negative sign is arbitrarily affixed in order that the flow of heat can be considered positive in the direction of decreasing temperature.

Thermal conductivity may be measured by either static (steady-state) or dynamic (transient) methods. In static methods, the specimen is allowed to come to steady-state condition; i.e. the temperature is a function of space coordinates only. In dynamic methods, the specimen is in an unsteady-state condition; i.e. temperature is a function of time as well as the space coodinates. For a homogeneous, isotropic material, containing no heat source or heat sink, and if the thermal conductivity is constant, the differential equation for the temperature field is

$$\frac{\partial t}{\partial \theta} = D\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right) \qquad (20)$$

where $D = k/(\rho c_p) \equiv$ thermal diffusivity

ρ = density

c_p = heat capacity at constant pressure

The temperature history is measured with an experimental arrangement which must simulate a solution of the differential equation, Eq. (20), and subsequently k can be determined.

Steady-state methods may determine thermal conductivity directly while most transient methods can determine thermal conductivity and diffusivity at the same time. However, recent literature reveals that many investigators are turning to transient methods since instruments for accurately recording rapidly changing temperature as well as computers for handling the complex solution of Eq. (20) have become available. Meanwhile, some transient methods offer the possibility of determining thermal conductivity directly from a single experiment such as thermal probe devices.

In the next section, both the steady-state methods and transient methods will be discussed and comments will be made on the different methods available.

GENERAL DISCUSSIONS OF MEASURING METHODS

In steady-state methods for measuring thermal conductivity, the apparatus is designed to minimize the heat transfer effects other than the conduction heat flow across the test specimen. Test specimen may take a shape for which an exact solution of Eq. (20) is possible. These shapes are generally a plane plate (slab), a sphere, and a cylinder. In flat plate arrangements, heat flows through a sample of constant cross section whose lateral area, ideally, is covered with a non-conductor of heat. Since there is no perfect thermal insulator, a technique frequently used to reduce lateral heat flow to negligible values is to provide heat guards. These heat guards are independently maintained at the same temperature as the adjacent surfaces, so that ideally no heat leakage occurs from the source, sink or specimen boundaries. To obtain adequate guarding for accurate determination of k, quite a large sample is required in this arrangement. Radial heat flow through a hollow sphere is an attractive arrangement since no heat losses occur except through the leads to the heater and thermocouples, but the forming of such a uniform heat source and fabricating of such hollow samples is often a difficult requirement. Cylindrical arrangements may lose heat at their ends. A technique to reduce the axial heat flow in cylinder to negligible values is to provide end-heat guards. Another technique to reduce axial heat flow in a cylinder is to use a long sample with small diameter and to work merely in the central section where the isothermal surface is essentially cylindrical.

In transient methods, the principal problem is to know how closely the boundary condition in an experiment match those postulated in solving Eq. (20). Advantages of transient methods are that it permits very rapid measurements, and the equipment associated with the measurement is relativly simple.

A transient method which has received considerable attention is the thermal probe device. A number of investigators (18-24) have utilized the technique to measure the conductivities of various soils (19,20), rocks and insulating materials.

Lentz (22) applied the probe to measurements on insulating materials. During the courses of his investigations, he performed tests with an enclosed heater (probe) and an unenclosed heater wire. The tests showed that the probe did not give satisfactory results in insulating materials because of the apporximate nature of correction made to allow for the finite size of the probe and the difference between the properties of the probe and the test specimen. However, good results were obtained with the unenclosed heater wire. The method which utilized an unenclosed heater wire is better known as the line-source method.

The line-source method was developed originally by Van der Held and Van Drunen (25) to measure the thermal conductivity of a liquid. The technique was used by Underwood and McTaggart (26) to establish the conductivity of polythylene and polytyrene.

From a consideration of the advantages and limitations of the various measuring methods, the line-source method can be chosen to measure the mean thermal conductivity of solid mixtures due to the following advantages:-

1. Simple and economical apparatus
2. Short experimental time

3. Easy prepartion of samples
4. Accurate results
5. Easy handling in experiment
6. Small sample size

The mathematical treatment of this method is discussed in the following section.

LINE-SOURCE METHOD

If heat is supplied to an infinite solid at a constant rate along a line, the temperature rise with respect to time of a point near the line is a function of the rate of energy input and the properties of the solid. This phenomenon is the basis for the line-source method.

Consider a line heat source of constant strength embedded in an infinite solid, the temperature is a function of spatial position, time, thermal properties of the solid and strength of the line-source. The solution of this problem was reported in Ingersoll and Zobel (22). It can be expressed as

$$t = \frac{q'}{2\pi k} \int_{\beta}^{\infty} \frac{\exp(-\alpha^2)}{\alpha} \, d\alpha \qquad (21)$$

where

$$\beta = \frac{r}{2\sqrt{D\theta}} \tag{22}$$

Eq. (21) contains a definite integral which has been previously evaluated (28) and the equation may be expressed as

$$t = \frac{q'}{2\pi k}\left[-\frac{C_e}{2} - L_n\beta + \frac{\beta^2}{2\cdot1!} - \frac{\beta^4}{4\cdot2!} + \cdots\right] \tag{23}$$

where C_e is Euler's constant = 0.5772157...

If r is small enough and θ large enough, then β becomes very small, therefore the third and subsequent terms of the series can be neglected. In this case, Eq. (23) becomes

$$t = \frac{q'}{2\pi k}\left(-\frac{C_e}{2} - L_n\,\beta\right) \tag{24}$$

Thus, for a point in the neighborhood of the line source, the temperature change between time θ_1 and θ_2 may be expressed as

$$t_2 - t_1 = \frac{q'}{2\pi k}\left[-\frac{C_e}{2} - \mathrm{Ln}\,\beta_2 + \frac{C_e}{2} + \mathrm{Ln}\,\beta_1\right]$$

or

$$t_2 - t_1 = \frac{q'}{4\pi k}\,\mathrm{Ln}\,\frac{\theta_2}{\theta_1} \tag{25}$$

It can be seen that r, the distance of the point from the line source and D, the thermal diffusivity, do not appear in Eq. (25). The remaining variables are the temperature, time and rate of heat input.

A plot of temperature versus the logarithmic of time gives a straight line with a slope of $q'/4\pi k$. Thus k can be obtained by the knowledge of this slope and the applied power per unit length of heat source. However, Eq. (25) was developed assuming constant thermal properties. It differs from the real experimental situation. A fine resistance wire which is used as a heat source wire, no matter how small, will have a finite radius and will differ in the thermal properties from the material it displaces. Therefore the heat is generated along a cylindrical surface of finite radius instead of a line. Also, there may be contact resistance between the source and the sample. Van der Held and Van Drunen (25) demonstrated that these objections cause a constant error in the observation of time. If the time error is taken into account by means of a constant value θ_c, a modified form of Eq. (25) can be written as follows

$$t_2 - t_1 = \frac{q'}{4\pi k} \operatorname{Ln}\left(\frac{\theta_2 - \theta_c}{\theta_1 - \theta_c}\right) \tag{26}$$

Eq. (26) closely describes the temperature rise in a real system if the requirement of an infinite body is satisfied. To meet this requirement, the radial dimension of the real system (test specimen) must be large enough so that the expanding temperature field does not encounter a boundary within the experimental time. Also both the system and the line-source must be of sufficient length to minimize axial heat flow at the point where the temperature rise is measured. Various effects influencing the real system will be discussed in Chapter VI.

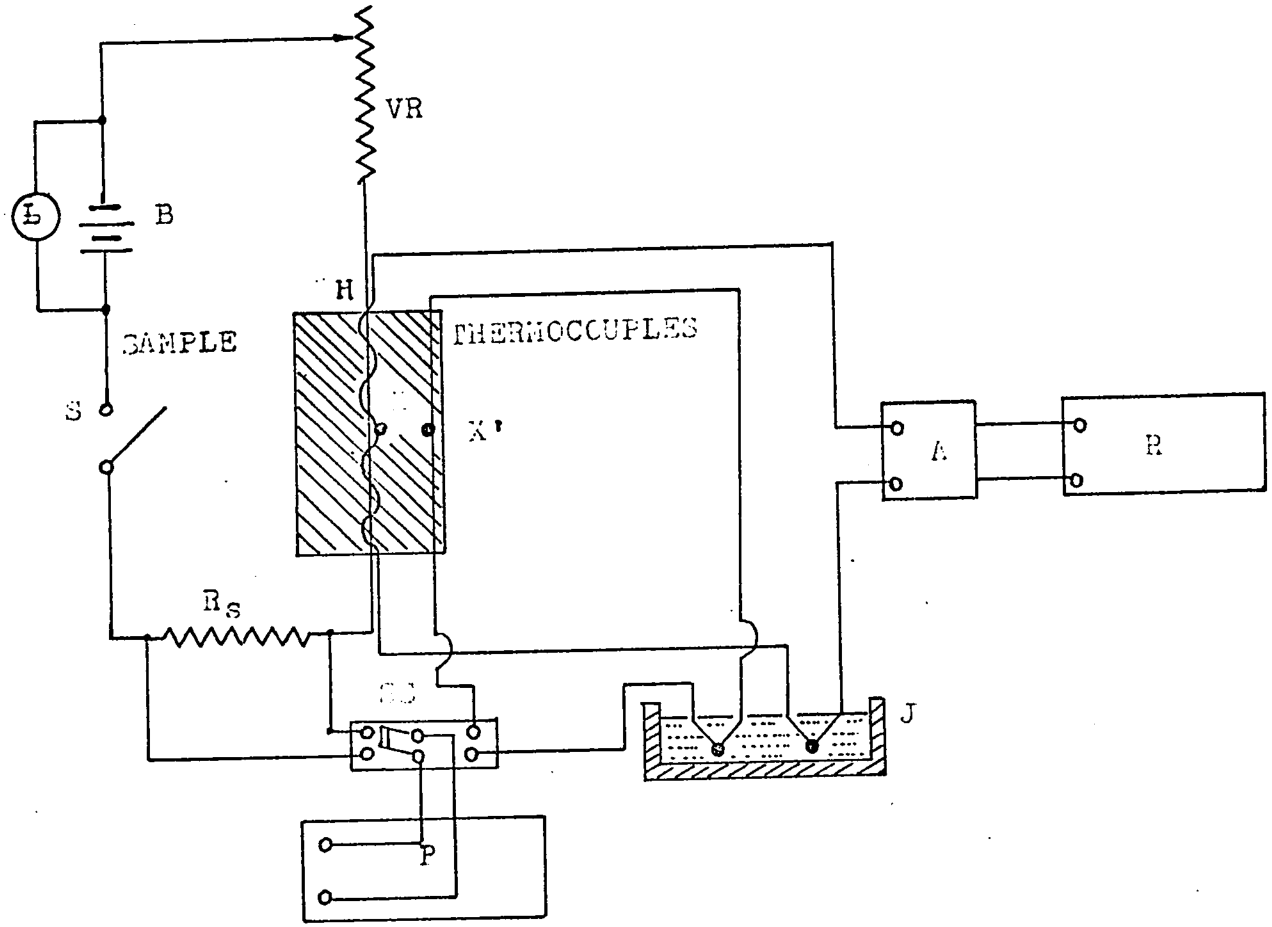
IV. MATERIALS, EQUIPMENT AND EXPERIMENTAL TECHNIQUES

EXPERIMENTAL APPARATUS

An experimental apparatus for line-source method was constructed to investigate the thermal conductivity of solid heterogeneous mixtures. The design of the apparatus follows closely a device developed by Underwood and McTaggart (26). However, some modifications have made to improve the control aspect of the experiment and to provide more complete gathering of data. These modifications can be summarised as under:-

1. Arrangement for more accurate measurement of constant power supply.
2. A second thermocouple was installed below the outer surface of the sample for measuring the ambient temperature and for indicating as to how far the sample was away from equilibrium.
3. Utilization of an automatic recorder to obtain a continuous recording of temperature versus time for the system.

A schematic diagram of the apparatus utilized for thermal conductivity measurement is shown in Fig. 1. The three basic components of the experimental apparatus are the electrical heating circuit, specimen, and the temperature



A	Amplifier
B	Battery
H	Line-Heat Source
J	Reference Junction
L	Fixed Resistor (Lamp Bulb)
P	Potentiometer
R	Recorder
S	Switch
SS	Selector Switch
R_s	Constant Resistor
VR	Variable Resistor
X	Inside Thermocouple Junction
X'	Outside Thermocouple Junction

Fig. 1. Apparatus for Line-Source Technique

measuring equipment.

ELECTRIC POWER MEASUREMENT

The value of electric power input (line-source strength) can be evaluated whenever the current which flows in the electrical heating circuit is known. The value of current can be measured by means of an ammeter connected in series with the circuit. However, due to the existence of internal resistance which occurs in any ammeter, error can not be avoided. An effort was made to increase the accuracy of the measurement by employing a constant value resistor which was connected in series in the circuit, and a potentiometer was used along with the constant resistor, for the accurate measurement of potential drop across the constant resistor, the current can be determined.

As shown in Figure 1, the electrical circuit comprises a power supply, variable resistor, heating wire, constant resistor and a switch. A twelve volt, lead plate storage battery was used as a constant voltage power supply. A variable resistor was used to facilitate the adjustment of current in the circuit. The line heat source is a piece of fine constantan wire with enamel insulation. Thirty gage constantan wire with a diameter of 0.01 inches was used to approach the theoretical line source.

The accuracy of the constant resistor R_s and the resistance of the 30 gage constantan heater wire were previously checked and calibrated. At room temperature, the constant resistor R_s, when calibrated by using a Leeds and Northrup 1 ohm standard resistor, showed 1.0573 ohms with an accuracy of ± 0.02%, while the heater wire showed a value of 3.041 ohms per foot with accuracy ± 0.06 %. Both values have been used in the final calculations.

A fixed resistor R_x was connected across the battery terminals. This resistor appeared in the form of a light bulb drawing approximately 100 milliamperes. It served to prevent an unsteady-surge of current when the switch was closed to begin the experiment.

SPECIMEN

The specimen is cylindrical in shape. The heating wire was located along the longitudinal axis of the specimen. The thermocouple for measuring temperature rise within the specimen was wrapped around the heater wire in the form of a helix with the hot junction located in the vicinity of the center of specimen. The distance of the hot junction to the heater wire was approximately 0.03 to 0.05 inches. A second thermocouple was installed just beneath the outer surface of the specimen. This second thermocouple was used in measuring the ambient temperature and in indicating whether the heat energy from the line source had reached the outer surface of the specimen or not. From the line source analysis, one knows that the experiment should stop when the heat energy reaches this location.

TEMPERATURE MEASURING EQUIPMENT

The temperature variation in the specimen was measured by means of a thermocouple with the hot junction embedded in the specimen and the reference junction in an appropriate constant temperature fluid. In this investigation, a thirty gage copper-constantan thermocouple was used. The junctions were formed by using a Fisher Scientific, Model DYNATACH 416 TC WELDER. The small junction of the thermocouple was desired to minimize the disturbance in temperature field.

The arrangement of the thermocouples is shown in Fig. 1. A Hewlett-Packard 419A dc Null Voltmeter with built-in amplifier was used as an amplifier to magnify the signal of temperature rise to be recorded. It provides 18 end scales dc voltage range in 1,3,10 sequence from 3 microvolts to 1000 volts. The rear panel output terminals provide a dc voltage (0 to ± 1 Volt) in proportion to meter deflection (input signal)

For a constant temperature reference junction of the thermocouple, an ice-and-water bath ($32^{o}F$) was initially used, the difficulty encountered in this arrangement being that the change in sample temperature at hot junction could not be obtained very accurately. For example, before heat was applied to the source line, the thermocouple gave a voltage of 0.831 MV (hot junction at 70°F), for a temperature increase of 3°F, the thermocouple response was approximately 0.900MV. By using amplifier set at a certain convenient scale, the effective amplified portion show on the strip chart recorder was (0.900-0.831)/0.900 =7.6%, Thus one could not expect to obtain a clear and accurate recording result of temperature change with respect to time in the strip chart recorder.

An effort was made to increase the sensitivity of the measurement by immersing the reference junction into a fluid bath at a temperature very close to the initial temperature of the sample before the heat was introduced. Here an oil bath in a Dewar flask (isothermal condition) was used. With this arrangement, if the difference between the hot junction in the sample and reference junction (oil bath) is 1°F, then the initial and final readings given by the thermocouple would be 0.023 MV and 0.092 MV respectively for a temperature increase of 3°F. For the amplifier set at a certain scale, the effective amplified portion showed on the strip chart recorder was (0.092-0.023)/0.092 = 75 %.

The problem involved here is the temperature stability of the oil bath. A simple experiment was carried out to check this arrangement by means of another thermocouple with junctions in the oil bath and ice-and-water bath, and from the measurement of potentiometer readings of these junctions, it was found that the temperature was very stable within fluctuation of ± 0.04°F.

With an oil bath arrangement, results of temperature change with respect to time in the strip chart recorder may less accurate than an ice-and-water bath due to the larger fluctuation of temperature change of the oil bath, but on the other hand, a more accurate or if not a comparable accuracy can be obtained in using an oil bath arrangemnt, because of the larger amplification of temperature variation.

A Hewlett-Packard Mosley 7100B Strip Chart Recorder was used to record the instantaneous changes of the input signal. The recorder has two model 17501A input modules, each input module has 16 spans from 1 MV to 100 V full scale with one megohm input resistance at null position on all fixed and variable spans. The range used in the recorder is 1V full scale since the maximum input signal from the amplifier does not exceed 1V limit.

SAMPLE PREPARATION

Samples of heterogeneous mixtures were cast for experiments. These samples were prepared from silicone rubber (as discontinuous phase). The rubber was obtained from the General Electri Company, Toronto, Ontario. This material known as RTV (Room Temperature Vulcanizing) Type-60, possesses excellent electrical and physical properties which are almost constant over a wide range of temperatures. At room temperature, it has a thermal conductivity of approximately 0.222 Btu/hr ft ^{o}F as determined in this work by the line-source method.

Four different metal powders were used as discontinuous phases in the heterogeneous mixtures. The powders were obtained from Electronic Space Products, Inc., Los Angeles, California, U.S.A. The specifications and properties of the materials used throughout this work are shown in Table I and Table II.

The samples were cast in the casting fixture made of a glass tube with $1\frac{1}{4}$ inches in diameter and 6 inches long. Before the casting of the sample, certain precautions were taken and proper placement of heating wire was accomplished by stretching the wire taut along the axis of the cylindrical mold. The thermocouple for measuring temperature within the sample was then wrapped around the heater wire in the form of a helix, with the hot junction located near the heater wire as described before. The second thermocouple was installed just inside the inner surface of the casting fixture. All the wires were pulled taut and taped to the fixture.

A certain amount of silicone rubber was poured into a cleaning mixing can. Since the rubber has a very high viscosity, 550 poises in the uncured state, ten percent of RTV-thinner(RTV-910, with density 0.096 gm/cc, General

TABLE I

SPECIFICATIONS OF METAL POWDERS USED IN EXPERIMENTS

Description	Particle Dimensions (Mean Diameter, in.)	Purity(%)
Bismuth	0.0058	99.6
Lead	0.0017	99.9
Nickel	0.0017	99.9
Aliminum	0.0037	99.5

TABLE II

PROPERTIES OF MATERIALS USED IN EXPERIMENTS

Material	Density (gm/cc)	Thermal Conductivity at 68°F (Btu/hr ft °F)
Bismuth	9.78 (29)	4.8 (30)
Lead	11.34 (29)	20.0 (30)
Nickel	8.80 (29)	52.0 (30)
Aluminum	2.7 (29)	118 (30)
Rubber (RTV-60)	1.45 (31)	0.222 *

* Experimental value from this study

Electric Company) by weightwas added to the rubber as recommended by the supplier to reduce the viscosity of rubber to an easily pourable manner. The volume of the diluted rubber in the mixing can can be calculated since the weight and density of the rubber and the thinner are known. Once the volume of the diluted rubber is known, any volume fraction of powders desired to be added to the diluted rubber can then be calculated.

Before the powder was added, it was first sifted through a nylon cloth to break any lumps. A few drops of curing agent (catalyst) were also added in the mixing can. Dibutyl Tin Dilaurate (T-12) was used as a standard catalyst which was also supplied by the General Electric Company. For three-phase mixtures, different powders were mixed well before adding to the rubber to form mixtures.

The rubber, powder and the curing agent were mixed thoroughly by using a spatula.In order to obtain uniformity, mixture was transferred twice from the mixer to another container. The mixture was deaerated immediately in a vacuum dessicator after mixing process. Thus, air bubbles and voids in the mixture can be eliminated under pressure reduction from atmosphere to vacuum, generally, thirty mintues of deaerating is adequate. It was then poured into the casting fixture.

During the process of pouring, care must be taken to prevent damage to the heater wire and the thermocouples, and the formation of air bubble should be avoided.

The curing process of the test sample is 1). twenty-four hours at room temperature, 2). five hours at 110 °F, and 3). five hours at 150 °F. When the process was complete, the sample was ready for conductivity tests.

THERMOCOUPLE CALIBRATION

There are several kinds of thermocouples which have been commonly used for the measurement of temperature, namely, Platinum-Pt 10% Rh thermocouple, Platinum-Pt 13% Rh thermocouple, Chromel-Alumel thermocouple, Iron-Constantan thermocouple, and Copper-Constantan thermocouple. Generally speaking, the first three kinds are applied for high temperature measurement. Copper-constantan thermocouple is selected in this work because not only is it economical but also, copper to copper connection to other devices such as amplifier and recorder, can eliminate unwanted thermal effects.

The calibration of thermocouples was accomplished by measuring the emf generated by thermocouples at the two known temperatures. A potentiometer was used to measure the potential difference between the hot junction and the reference junction. The melting point of ice, and the boiling point of water were selected as the thermometric fixed points for calibration purposes of the thirty gage copper-constantan thermocouples used in the experimental apparatus. The standard table which represented the temperature-emf functions of copper-constantan thermocouple given by the National Bureau Standards (32) was used in connection with the calibration. Any necessary correction to the thermocouple reading was recorded. A thermocouple calibration curve was obtained by plotting temperature corresponding to the thermocouple reading versus temperature correction.

The melting point of ice was observed by placing the thermocouple into an ice bath consisting of a Dewar thermo flask filled with shaved ice saturated with water. The boiling point of water was recorded by immersing the the thermocouple into the boiling liquid contained in a beaker.

The true temperatures and the corresponding emf (reference junction at 32°F) given by NBS for the calibration points, with the consideration of the atmospheric conditions during the calibration are:

Melting point of ice	32.00°F	(0.000MV)
Boiling point of water	211.93°F	(4.271MV)

Observed temperatures corresponding to measured emf genarated by 30 gage copper-constantan thermocouples are:

	Thermo-couple 1	Thermo-couple 2	Thermo-couple 3
Melting point of ice	0.000MV (32.00°F)	0.000MV (32.00°F)	0.000MV (32.00°F)
Boiling point of water	4.249MV (211.13°F)	4.251MV (211.21°F)	4.250MV (211.17°F)

Fig. 2 is a typical graph of temperature correction versus measured temperature of thermocouple 1. The graph was constructed assuming that the corrections between the thermometric points used varied linearly with temperature. Temperatures corresponding to thermocouple reading during the experiment was corrected corresponding to the correction chart when used in the thermal conductivity calculations.

CALIBRATION OF AMPLIFIER AND RECORDER

As mentioned before, in order to obtain the best recording of temperature changes with respect to time with the strip chart recorder, the range of the null voltmeter with built-in amplifier used in the measurement was selected as ± 100 microvolts full scale. The output of the amplifier will provide ± 1 volt signal full range proportion to the input signal. The input resistance is 100 kΩ in the range of ± 100 microvolts of the amplifier. A known voltage signal from a potentiometer was used to calibrate the output signal of the amplifier. The output signals were recorded on a

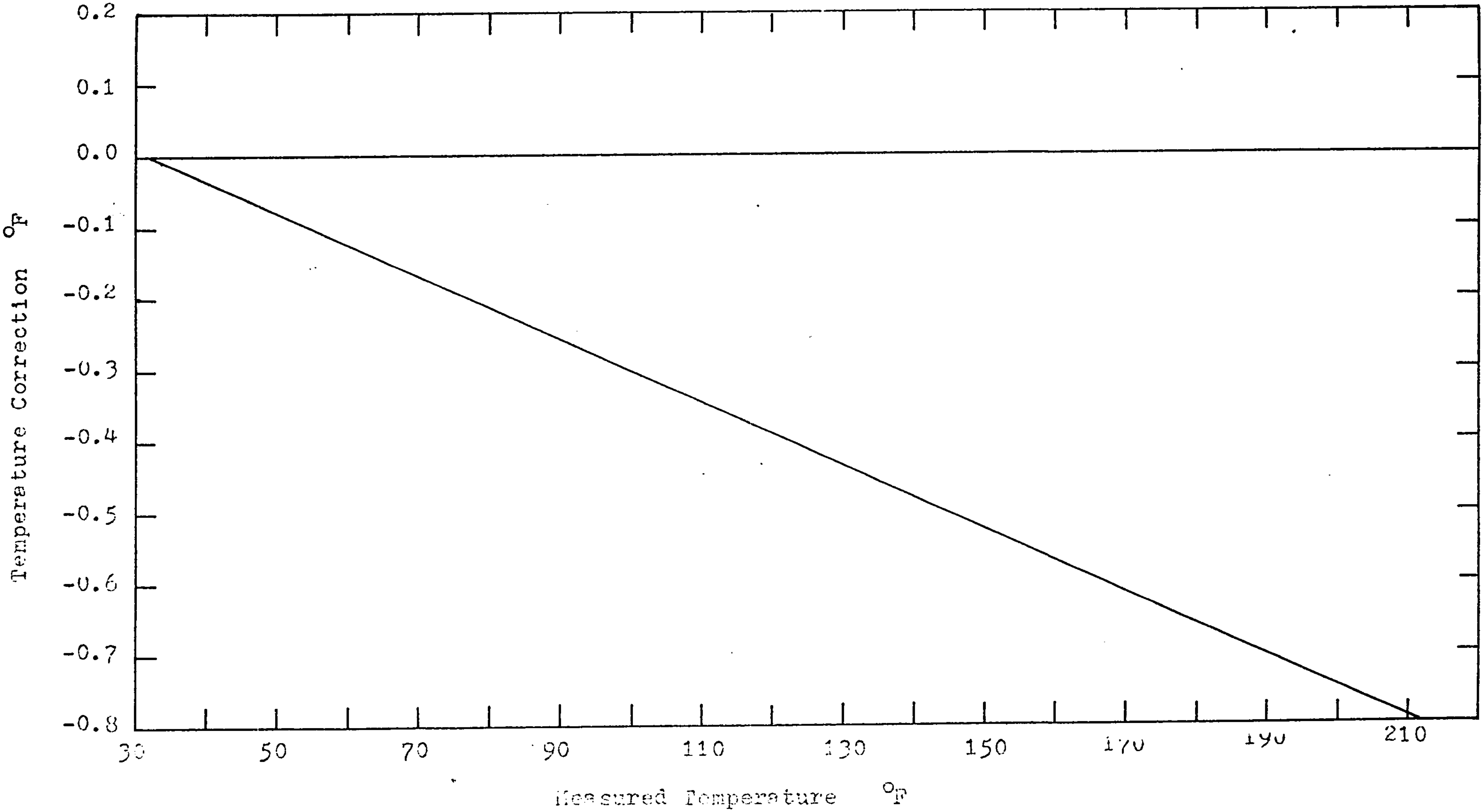


Fig. 2. Calibration Curve for 30 Gage Copper-Constantan Thermocouples

calibrated digital voltmeter (Hewlett-Packard Model 2401C Intergrating Voltmeter) and an accuracy within ± 3.5 % was obtained.

Calibration of the Hewlett-Packard Mosley 7100B strip chart recorder followed in the same manner. The range used in the recorder was 1 volt full scale. A standard known signal from a potentiometer was used to calibrate the strip chart recording of the recorder. The standard chart rolls with 10 inches of writing width where 1 inch represents 100 millivolts are used.

EXPERIMENTAL PROCEDURE

The sample must be at some uniform temperature in order to conform to the initial conditions specified in the development of Eq. (26). Considering the general arrangement of the experimental apparatus, only the test sample had to be placed in the special environment while the remainder of the equipment could be located outside. The sample was insulated with fiber-glass insulation to minimize the effects of environmental temperature. The glass fixture of the sample was not removed since the glass itself can serve as another layer of insulation.

The procedure utilized to obtain the temperature and time data for calculation purposes of thermal conductivity is discussed in the following.

The sample was checked first for equilibrium. The condition of equilibrium in the sample was determined by comparing the temperature reading obtained from the two thermocouples installed in the sample. When the two thermocouples gave the same temperature reading for a period of five minutes, equilibrium was considered to exist. When thermal equilibrium was reached, the automatic recorder was switched on and the initial traces were obtained which served as references.

The variable resistor was adjusted to give a current which would produce a temperature increase of about 2 to 3°F in one minute. The switch connecting the battery to the line source was then closed, applying current to begin the experiment. Current flow was allowed to continue until the outside thermocouple responded, and then the experiment was terminated.

During the experiment, the current was measured by means of constant resistor and the potentiometer. It was checked at regular time intervals to ensure that constant values of current were maintained throughout the experiment. Likewise, the temperature near the outer surface of the sample was regularly checked to ensure that the dynamic temperature field did not reach the outer surface within the experimental time. With the data obtained from the automatic strip chart recorder, the thermal conductivity was calculated.

V. DATA ANALYSIS AND RESULTS

COMPUTATION OF THERMAL CONDUCTIVITY FROM EXPERIMENTAL DATA

In calculating the thermal conductivity, only a portion of the experimental curve (temperature versus time) can be used. The initial point and final point of this portion are designated as lower and upper time limits respectively. The lower time limit can be estimated from Eq. (22) by knowing the value of β ,r, and D. For RTV-60 silicone rubber,r and D are known values and equal to 0.05 inches and 8.8×10^{-3} ft^2/hr (33) respectively. The value of β is determined by the method of trial and error, using the truncated series of equation (23),i.e. the 3rd and subsequent terms neglected. Assuming an acceptable upper limit of 2% for the error between two temperature ranges and using the modified Eq. (26), the value of β is found to be 0.24. The lower time limit thus calculated is approximately equaled to 30 seconds. Since the value of thermal diffusivity of the mixtures is higher than that of silicone rubber, the lower time limit of mixtures will be less than 30 seconds. The upper time limit was obtained when the thermocouple embedded near the surface of the sample responded during the experiment.

Before a value of thermal conductivity can be computed from the experimental data, the time correction factor θ_c must be determined first. Several techniques have been proposed to evaluate θ_c. One of the methods recommended and developed originally by Underwood and McTaggart (26) is present in the following.

First the experimental data obtained from the strip chart recorder are plotted on semi-logarithm graph paper. Because it is not yet corrected for θ_c, the result which represents the continuous temperature versus time is slightly curved (dotted line) as shown in Fig. 3. The data points were selected from the continuous dotted line for the purpose of calculating convenience. Following the procedure used by Underwood and McTaggart, a straightedge is laid on the graph paper and shifted about until it lies a constant number of seconds (not distance) away from each of the data points. Then, a line may be drawn along the straightedge, representing temperature rise with corrected time.

The rate of heat input to the sample from the line source is determined next. This is computed from the observed level of current and the known value for the resistance of the heater wire. The amount of heat generated per unit length of heating wire is calculated by

$$q' = 3.413\ I^2\ (R/L) \quad \text{Btu/hr ft} \tag{27}$$

where R (ohms) is the resistance of the heater wire. From Eq. (26) and Eq. (27), it follows that

$$k = \frac{3.413\ I^2 R}{4\pi (t_2 - t_1)}\ 2.303\ \log\left(\frac{\theta_{2corr}}{\theta_{1corr}}\right) \tag{28}$$

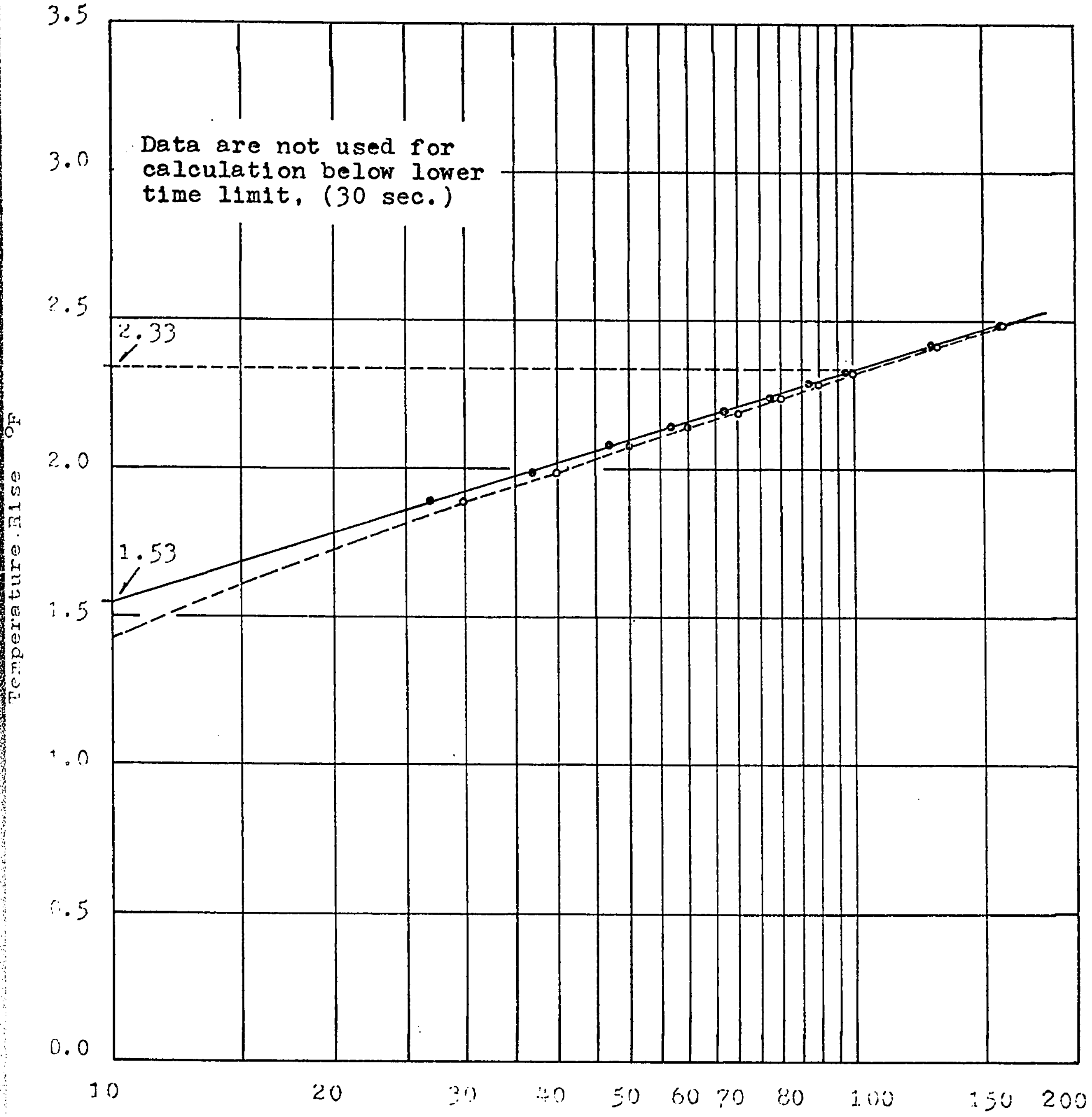


Fig. 3. Typical Variation of Temperature with Time for Rubber-Aluminum Mixture ($F_d = 0.05$)
o Original Data Points, ● Corrected Data Points, $I = 0.339$ amps, $\Delta T_{100-10} = 0.80°F$, $R_L = 3.041 \Omega/ft$, $\theta_c = 3$ sec.

The straight line which represents temperature rise with corrected time in Fig. 3 has constant slope, therefore any two points of this line can be used for determining the value of k. For convenience, if the value of $\theta_{2corr}/\theta_{1corr} = 100/10 = 10$, the logarthmic ratio is unity. Eq. (28) reduces to

$$k = \frac{0.6254\, I^2 R}{(t_2 - t_1)\, L} \tag{29}$$

In the example chosen in Fig. 4

$$k = \frac{(0.339)^2 \times 3.041 \times 0.6254}{(2.33 - 1.53)} = 0.274 \quad \text{Btu/hr ft } {}^\circ\text{F}$$

Thus the thermal conductivity of a rubber-aluminum mixture with volume fraction of aluminum 0.05 is 0.274 Btu/hr ft °F.

The method given above to achieve the results of thermal conductivity is not the most favourable since the graphical correction of the initial data points by hand may cause inaccuracies in the end results.

A computation procedure was developed to calculate the values of k by computer. This program enables the entire procedure of line-source method to be standardized. It has great significance not only in reducing the inaccuracies which were present in the graphical method, but also in reducing the amount of time required. The computer program is presented in the Appendix 1.

RESULTS

SINGLE-PHASE MATERIAL

Two samples of single-phase silicone rubber were prepared and thirty-two experiments were conducted by the line -source method. The individual results are presented in Appredix 2 as tabulated form. It should be noticed, however, that all the experiments were conducted over a small temperature range, approximately that of the surroundings. The variation of conductivity with temperature will not be large and was assumed to be negligible over the small range that was investigated.

A comparison was made between the results from this study and those from National Bureau Standard (34) as shown in Fig. 4. A good agreement is observed between the two values. For clarity, the dotted line through the NBS values is also shown. Hence the experimental values of

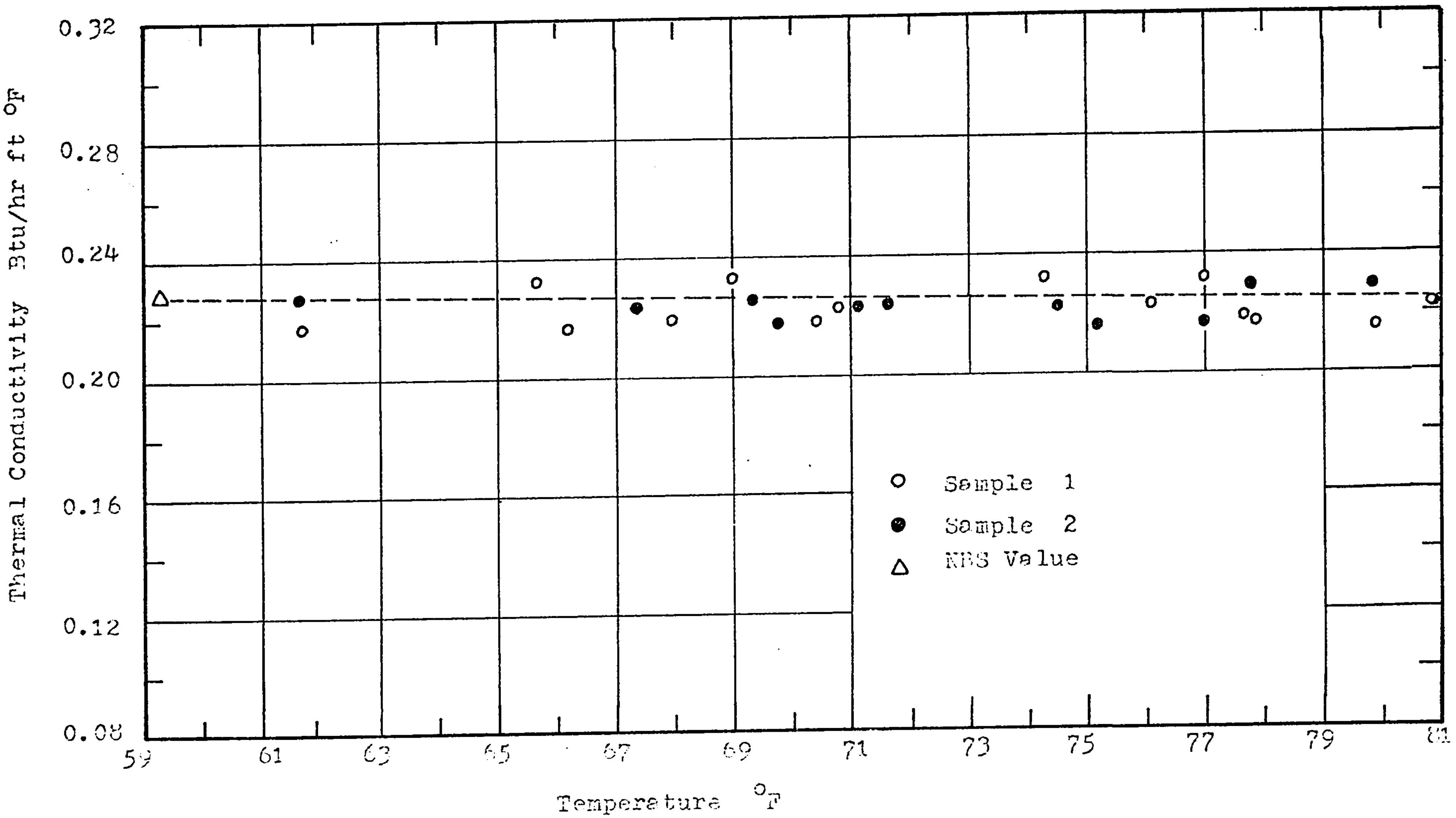


Fig. 4. Thermal Conductivity versus Temperature for RTV-60 Silicone Rubber (Measured with Current in the Range 0.18-0.34 amps)

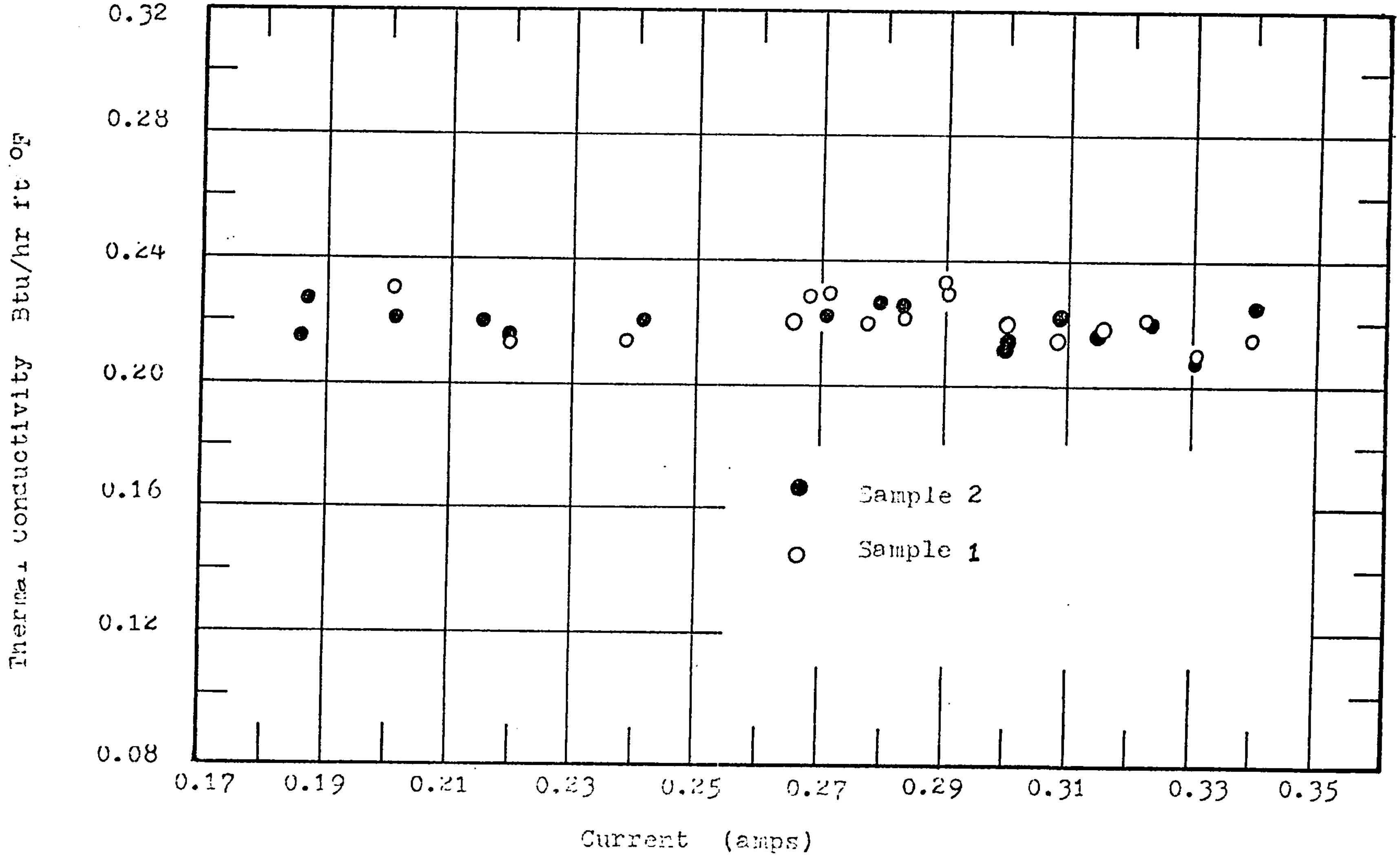


Fig. 5. Thermal Conductivity versus Current for RTV-60 Silicone Rubber (Measured in the temperature range 59-86 ^{o}F)

TABLE III

NATIONAL BUREAU OF STANDARD TEST REPORT ON THERMAL CONDUCTIVITY OF SILICONE RUBBER

Test Number	HC 31563	HC 31463	HC 31363
Mean Temperature of Specimen, ^{o}F	14.4	59.3	126.9
Thermal Conductivity, Btu/hr ft ^{o}F	0.232	0.228	0.220
Thickness, as Tested, inches	0.503	0.506	0.513
Density, as Tested, lb/ft^3	95.3	94.9	95.6
Temp. Gradient in Specimen, $^{o}F/in.$	22.5	23.1	23.8

silicone rubber obtained are to be used in connection with the following theoretical prediction of the thermal conductivity for two-phase and three-phase mixtures. A listing of NBS values is also provideed in Table III.

Lentz (22) stated that the current level may effect the value of indicated conductivity. The experimental data were examined but no evidence was found as seen from Fig. 5 to support this statement (see also Fig. 7).

TWO-PHASE MIXTURES

Twelve two-phase samples with various compositions and various fractions of discontinuous phase were cast. These samples were prepared from silicone rubber as the continuous phase and metal powders as the discontinuous phase. Fig. 6 and Figs. 8-10 show the individual experimental results of the thermal conductivities versus temperature of various two-phase mixtures.

Table IV shows the results of comparison between the experimental data from this study and the results obtained from the theoretical equations for predicting the thermal conductivity of two-phase mixtures. The results of Table IV are also plotted in Figs. 11-14. Maxwell's equation and Cheng's equation were chosen for the purpose of comparison. Since the shape factor of the discontinuous phase was not known, the particles of discontinuous phase were assumed

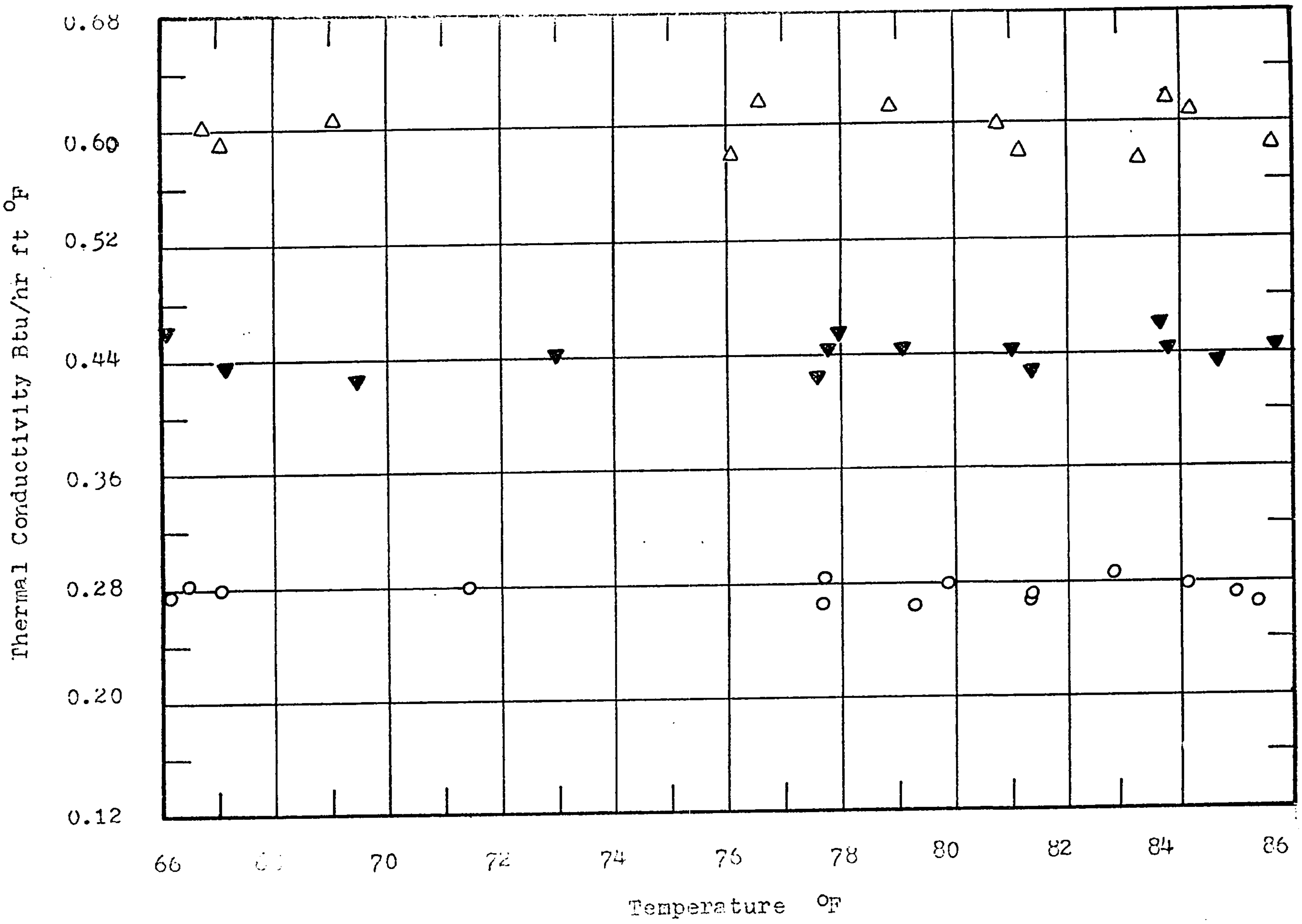


Fig. 6. Thermal Conductivity versus Temperature for Rubber-Aluminum Mixtures (Measured with Current in the Range 0.28-0.39 amps)
△ P_d = 0.28; ▼ P_d = 0.16; ○ P_d = 0.05

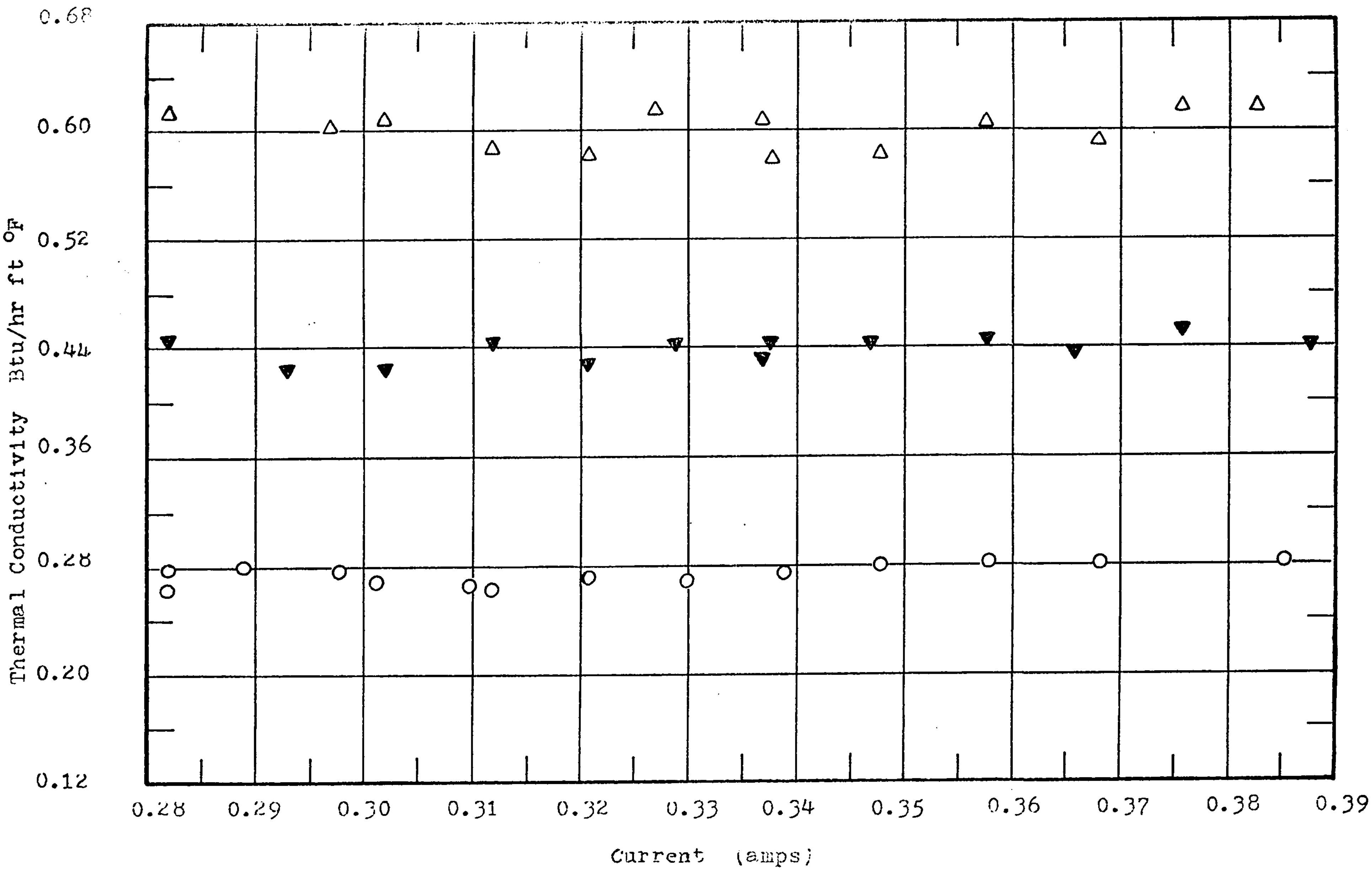


Fig. 7. Thermal Conductivity versus Current for Rubber-Aluminum Mixtures (Measured in the temperature range 66-86°F)
△ $P_d = 0.28$; ▼ $P_d = 0.16$ ○ $P_d = 0.05$

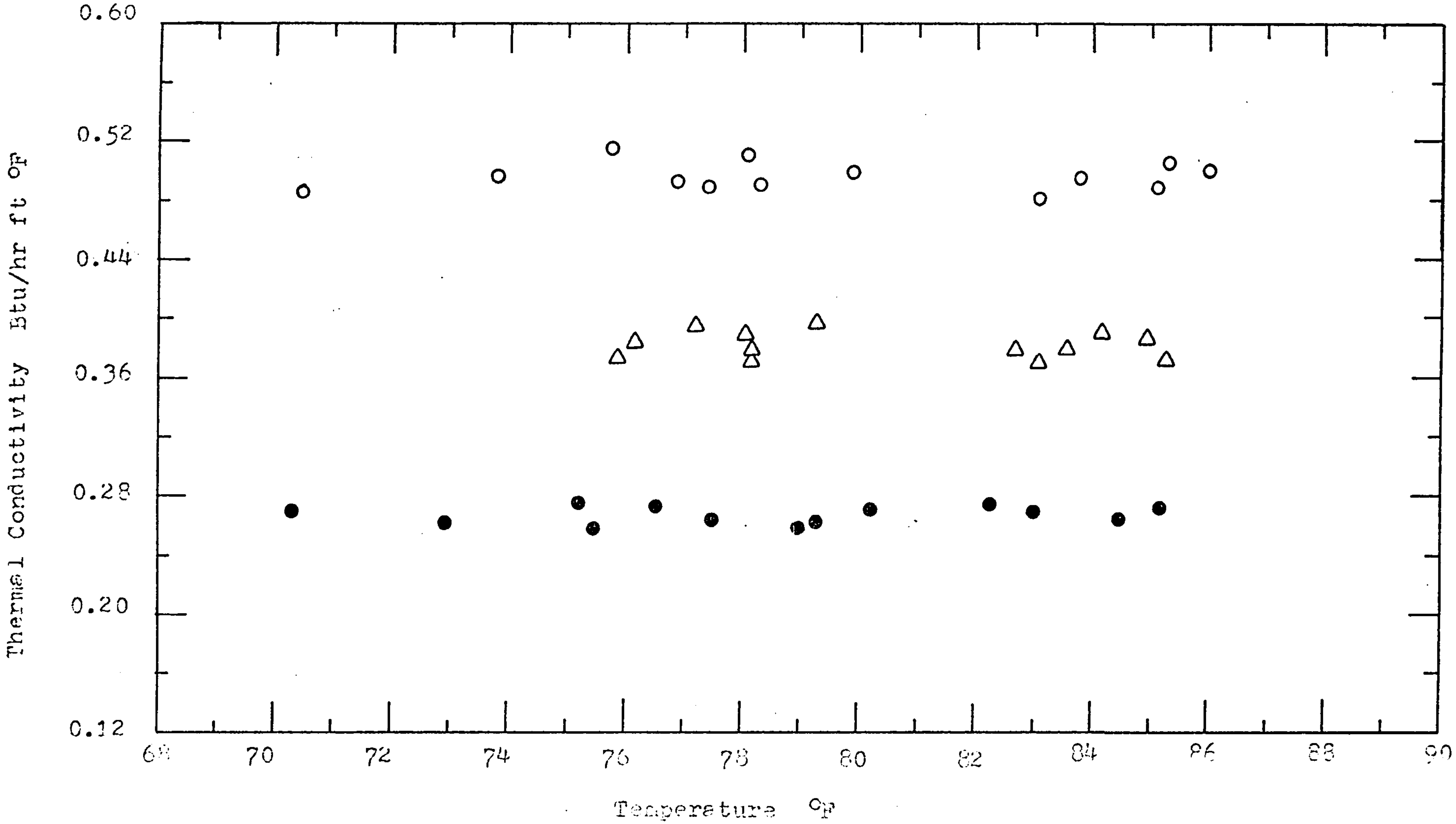


Fig. 8. Thermal Conductivity versus Temperature for Rubber-Lead Mixtures, ● $P_d = 0.24$, △ $P_d = 0.16$, ○ $P_d = 0.05$

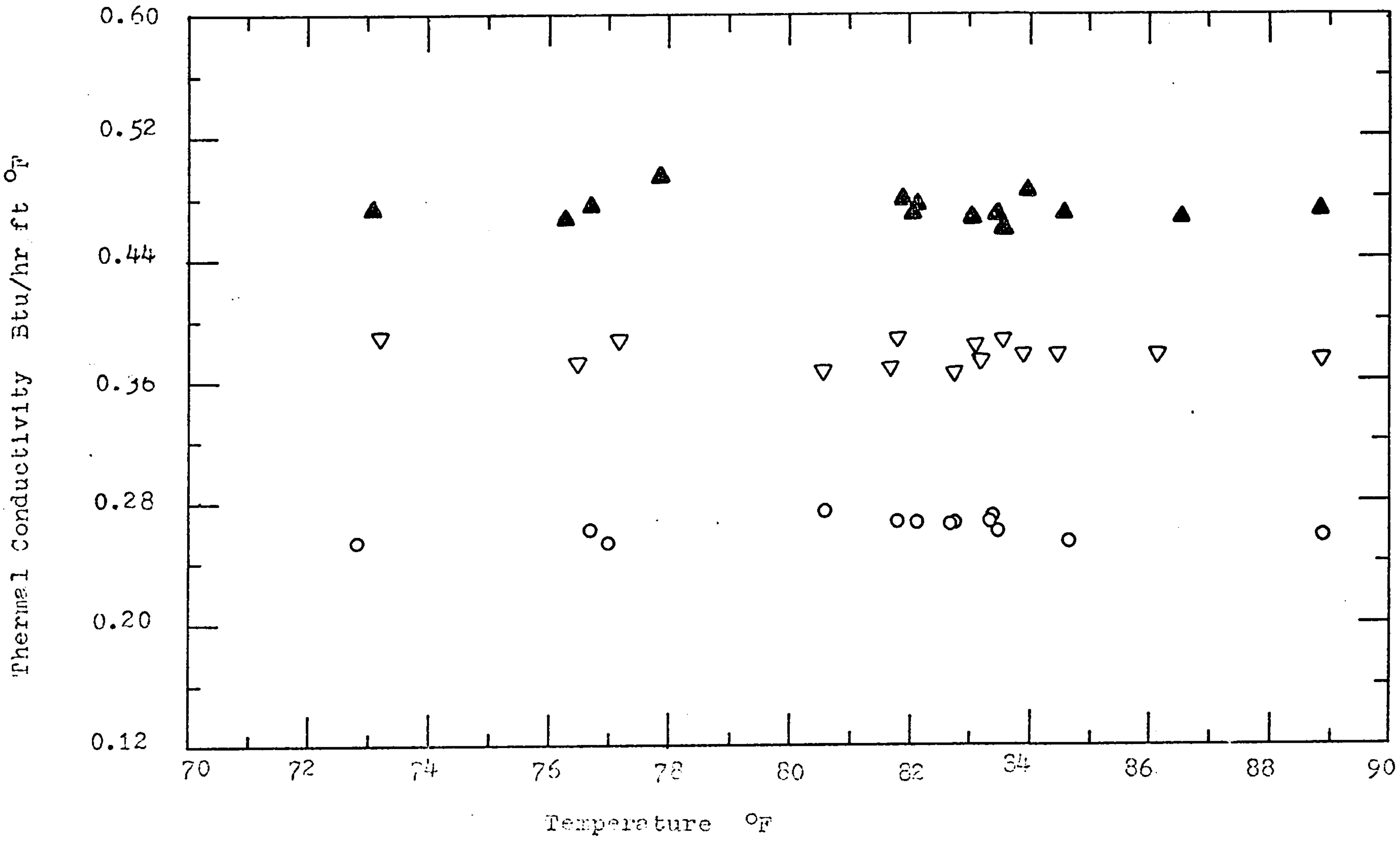


Fig. 9. Thermal Conductivity versus Temperature for Rubber-Nickel Mixture, ▲ $P_d = 0.24$, ▽ $P_d = 0.16$, ○ $P_d = 0.05$

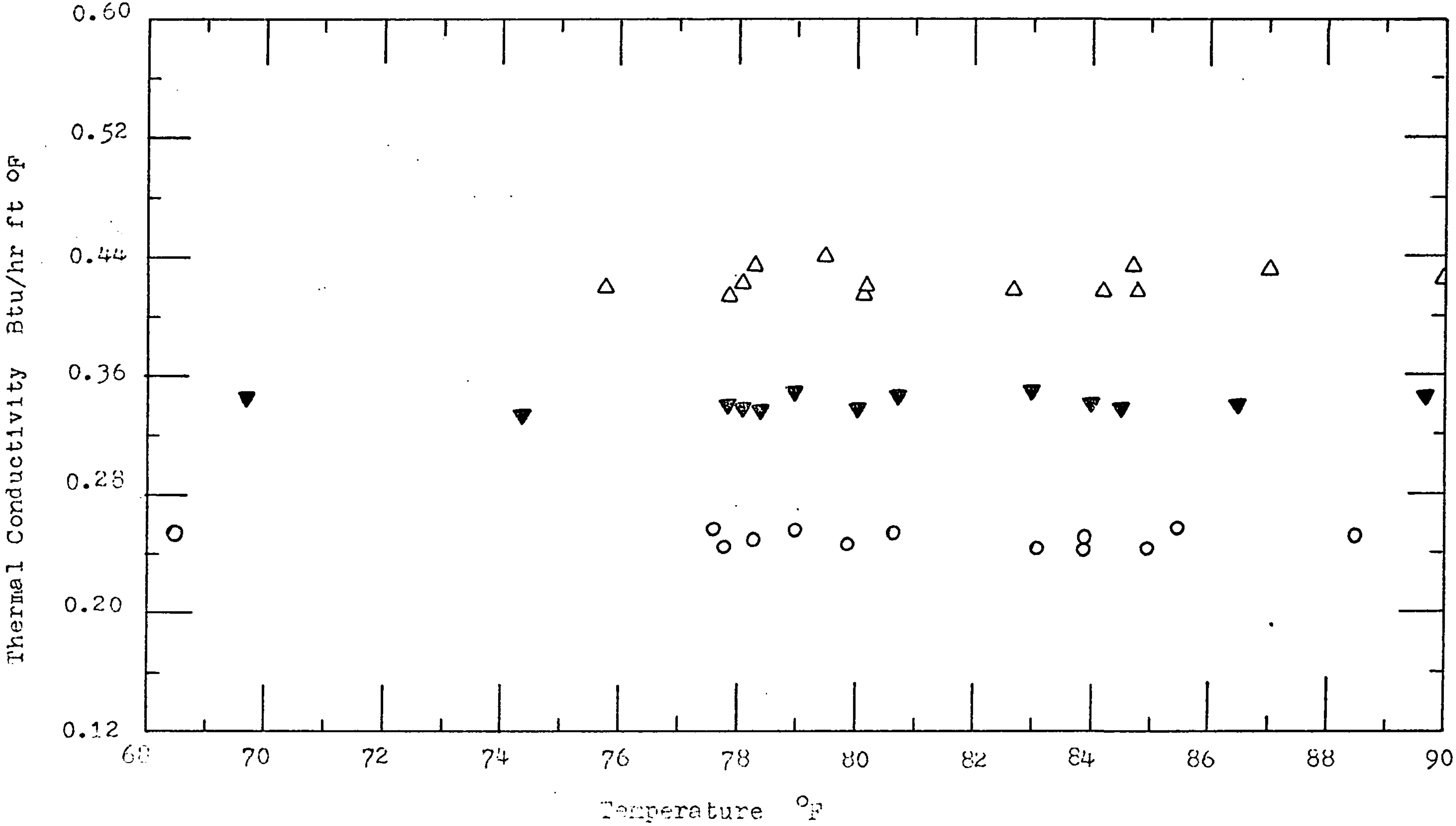


Fig. 10. Thermal Conductivity versus Temperature for Rubber-Bismuth Mixtures, △ F_d = 0.24, ▼ F_d = 0.16, ○ F_d = 0.05

TABLE IV

COMPARISON OF THE PREDICTED THERMAL CONDUCTIVITY WITH EXPERIMENTAL DATA OF TWO-PHASE SOLID MIXTURES

A. Aluminum Powder in Silicone Rubber

P_d	Experiment	Maxwell, Eq. (1)		Cheng, Eq. (11)	
	k_m (Btu/hr ft °F)	k_m	Deviation from Exp. Data (%)	k_m	Deviation from Exp. Data (%)
0.05	0.274	0.257	-6.2	0.303	+10.5
0.16	0.442	0.348	-21.3	0.430	-2.7
0.28	0.600	0.479	-20.2	0.619	+3.1

B. Lead Powder in Silicone Rubber

P_d	Experiment	Maxwell, Eq. (1)		Cheng, Eq. (11)	
	k_m (Btu/hr ft °F)	k_m	Deviation from Exp. Data (%)	k_m	Deviation from Exp. Data (%)
0.05	0.267	0.256	-4.1	0.295	+10.5
0.16	0.383	0.344	-10.2	0.412	+7.6
0.24	0.497	0.423	-14.9	0.516	+3.8

TABLE IV--Continued

C. Nickel Powder in Silicone Rubber

P_d	Experiment	Maxwell, Eq. (1)		Cheng, Eq. (11)	
	k_m (Btu/hr ft °F)	k_m	Deviation from Exp. Data (%)	k_m	Deviation from Exp. Data (%)
0.05	0.265	0.257	-3.0	0.301	+13.5
0.16	0.379	0.347	-8.5	0.424	+11.8
0.24	0.471	0.429	-8.9	0.537	+14.0

D. Bismuth Powder in Silicone Rubber

P_d	Experiment	Maxwell, Eq. (1)		Cheng, Eq. (11)	
	k_m (Btu/hr ft °F)	k_m	Deviation from Exp. Data (%)	k_m	Deviation from Exp. Data (%)
0.05	0.250	0.252	+0.8	0.278	+11.2
0.16	0.341	0.330	-3.2	0.372	+9.1
0.24	0.423	0.398	-5.9	0.451	+6.6

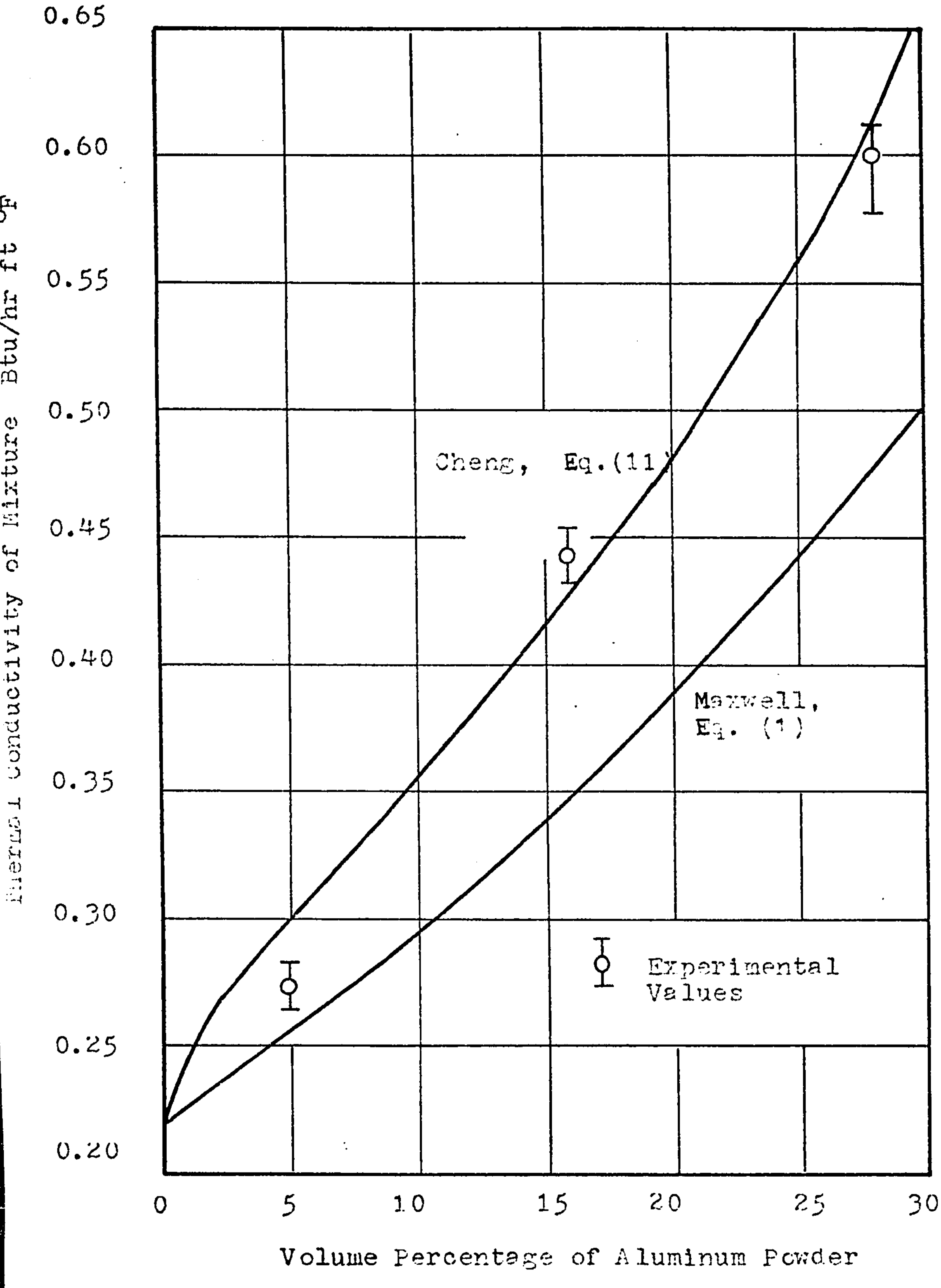


Fig. 11. Comparison of Calculated and Measured Conductivities of Aluminum-Rubber Two-Phase Mixtures

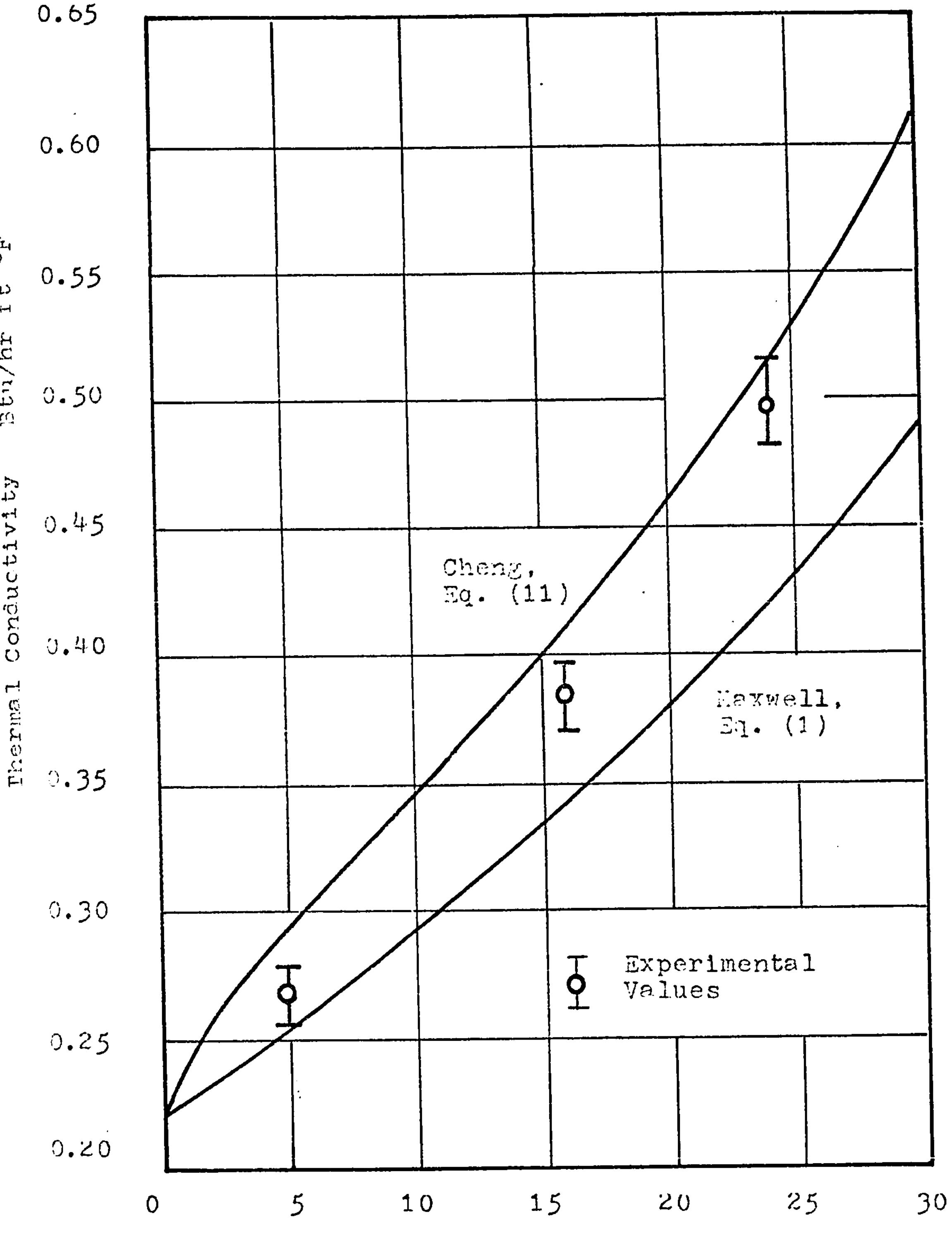
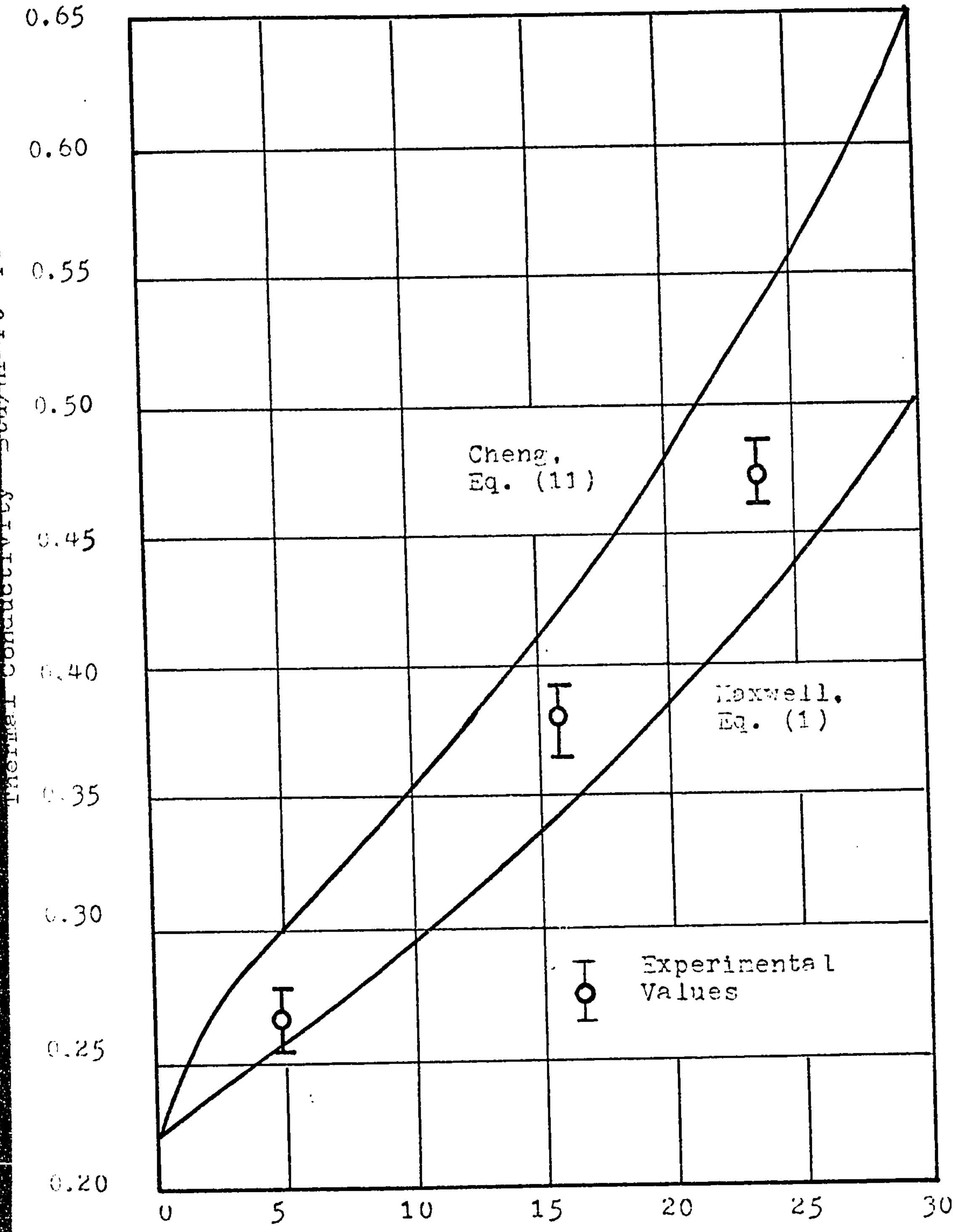
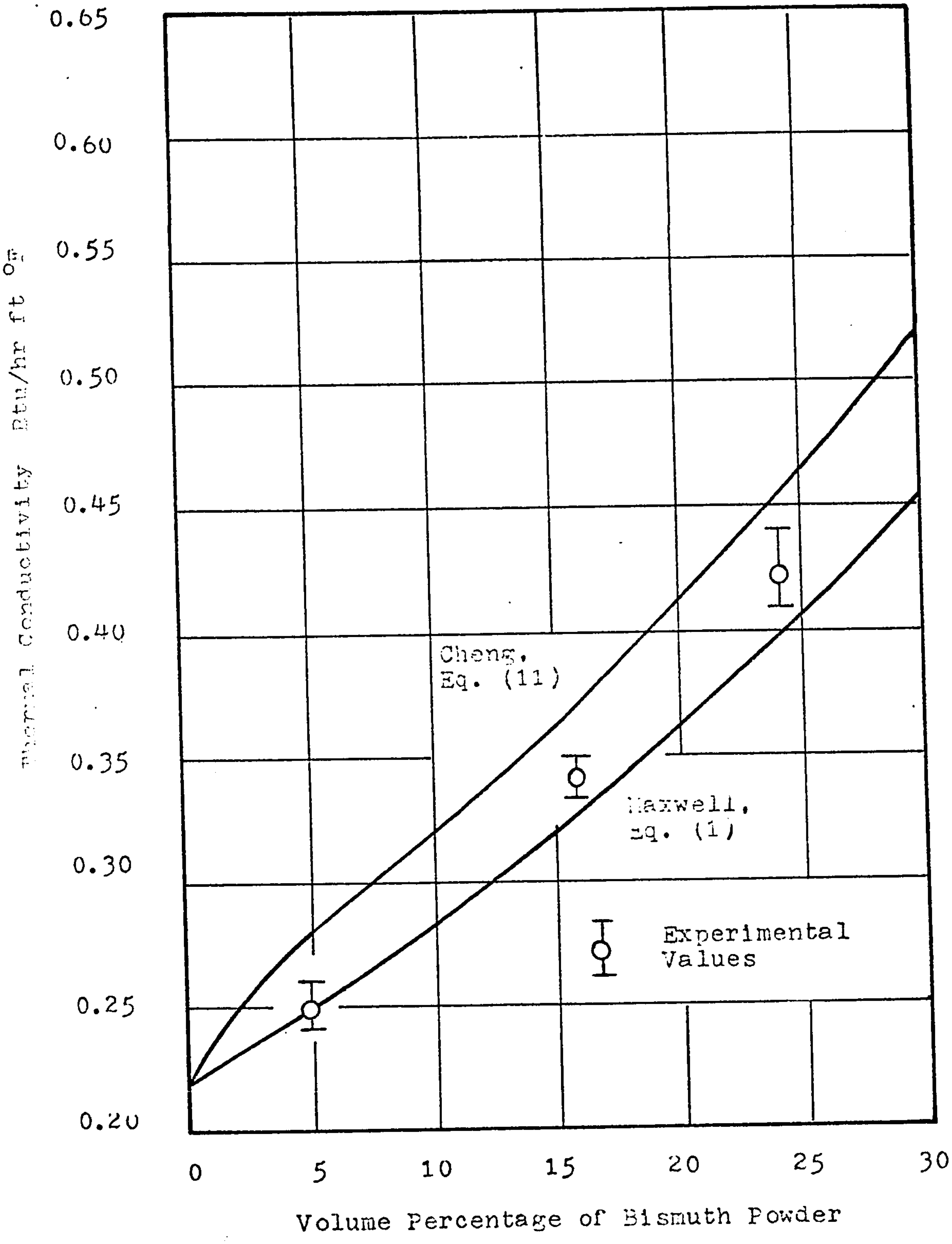


Fig. 12. Comparison of Calculated and Measured Conductivities for Rubber-Lead Two-Phase Mixtures



Volume Percentage of Nickel Powder

Fig. 13. Comparison of Calculated and Measured Conductivities for Rubber-Nickel Two-Phase Mixtures



Gig. 14. Comparison of Calculated and Measured Conductivities for Rubber-Bismuth Two-Phase Mixtures

to be randomly distribed small spheres. However there are no restrictions on the particle shape of the discontinuous phase in Cheng's equation. As pointed out by Hamilton, Maxwell's equation, Eq. (1), can be used with reasonable accuracy regardless of the particle shape if the ratio of conductivity k_d/k_c is less than 100, otherwise the discontinuous phase shape factor must be considered. This statement is quite evident from the present experimental results. On the other hand, Cheng's equation may become more favourable when the ratio of k_d/k_c is large enough.

THREE-PHASE MIXTURES

Six three-phase samples were tested. Figs. 15-17 show the individual experimental results of thermal conductivity versus temperature of the three-phase mixtures. Comparison between the experimental results and results obtained from theoretical analysis for predicting the thermal conductivity of three-phase mixtures were also shown in Table V. Hamilton's equation (Eq. 16) and Cheng's technique were used for theoretical predictions.

All the results of the comparisons of the predicted thermal conductivities with experimental results for two-phase and three-phase solid mixtures have been summarized as shown in Table VI.

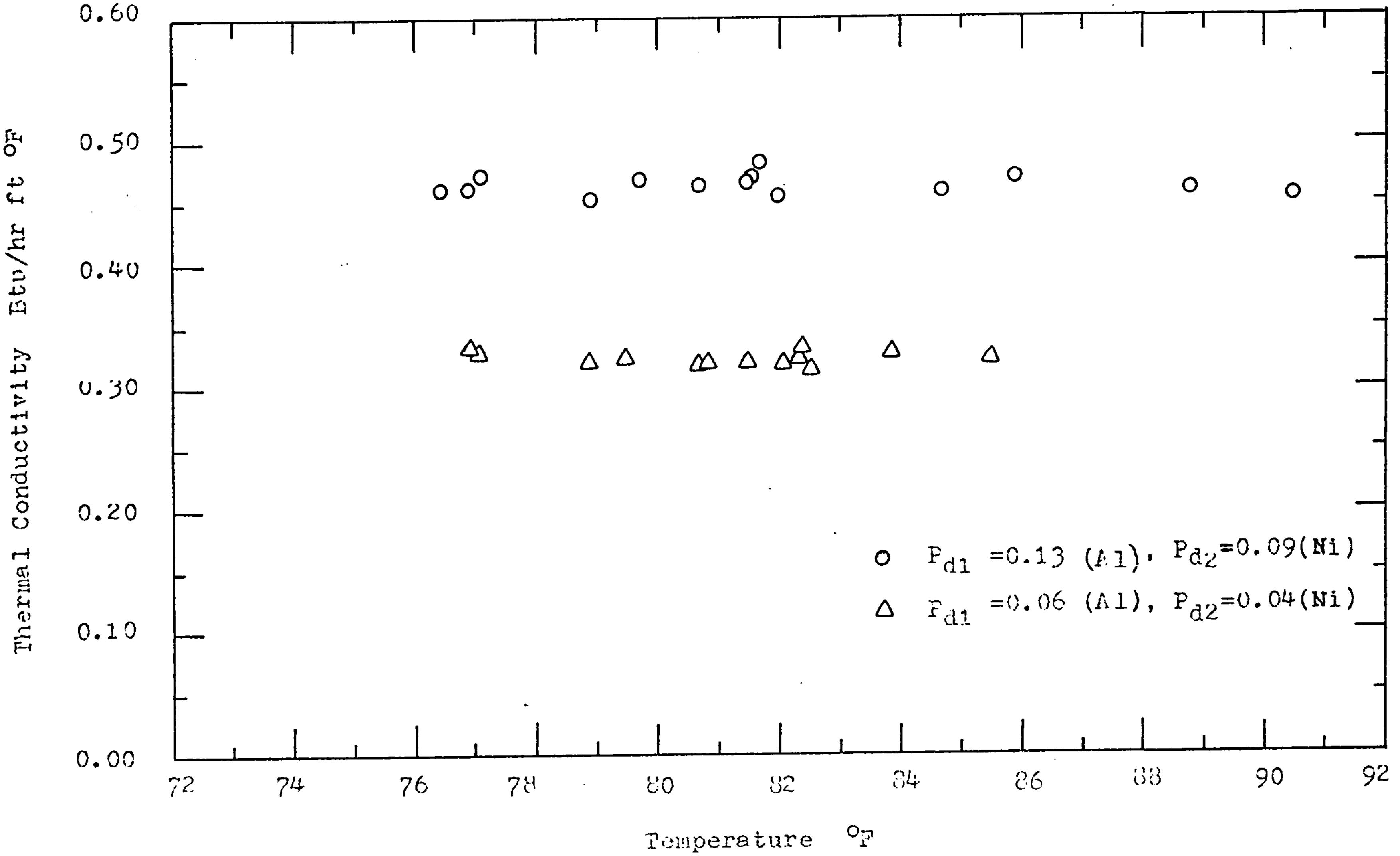


Fig. 15. Thermal Conductivity versus Temperature for Rubber-Aluminum-Nickel Mixtures.

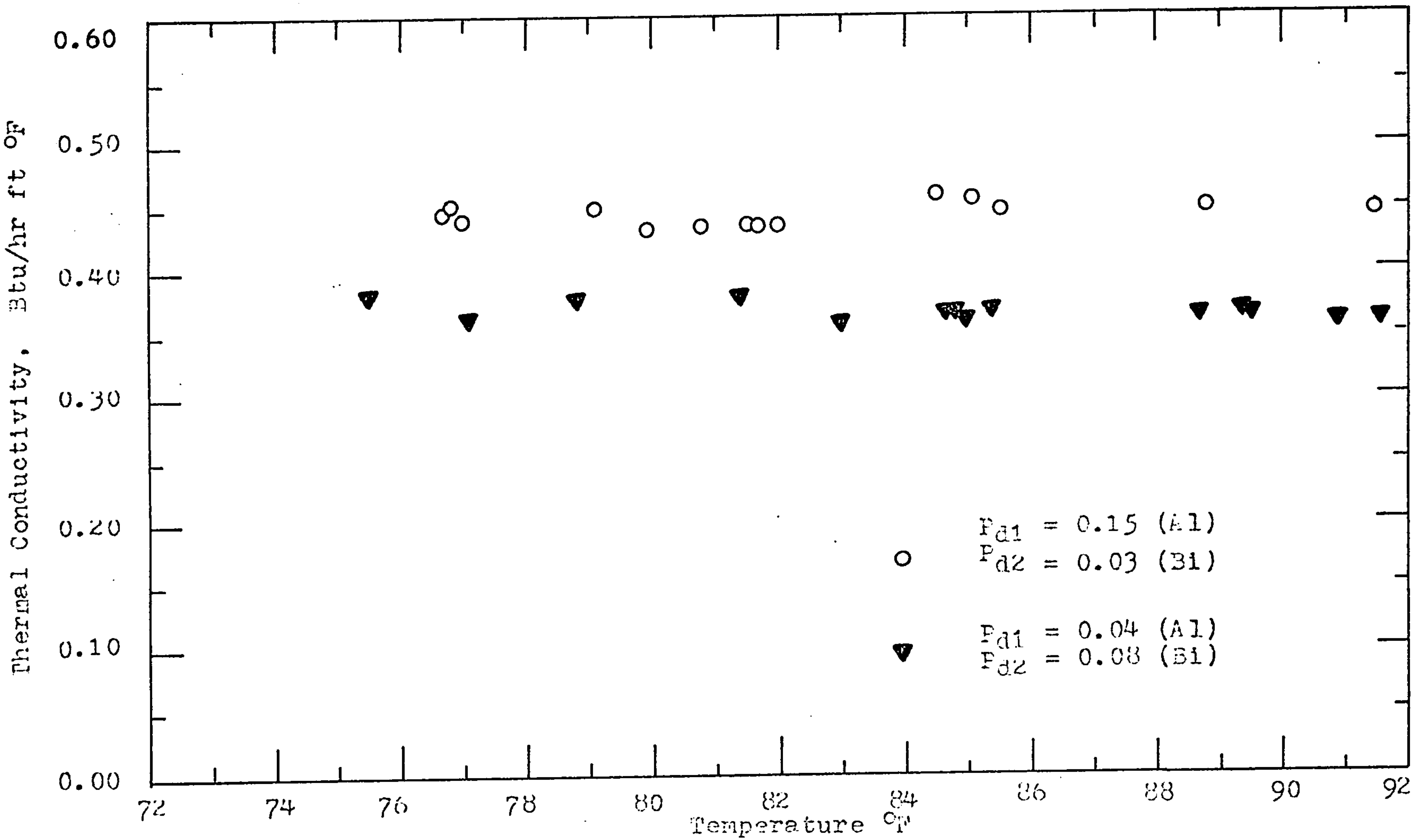


Fig. 16. Thermal Conductivity versus Temperature for Rubber-Aluminum-Bismuth Mixtures.

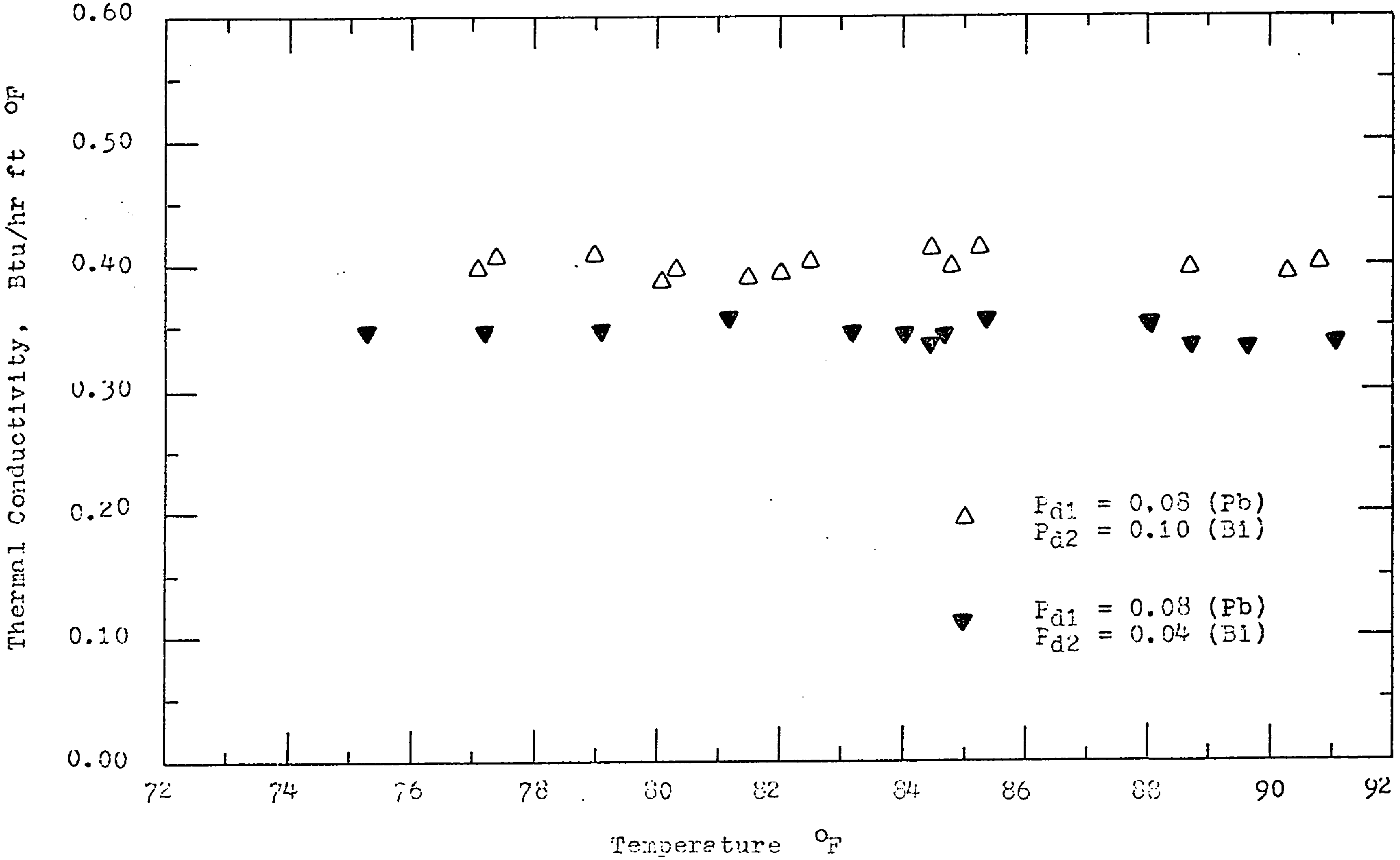


Fig. 17. Thermal Conductivity versus Temperature for Rubber-Lead-Bismuth Mixtures.

TABLE V

COMPARISON OF THE PREDICTED THERMAL CONDUCTIVITIES WITH EXPERIMENTAL DATA OF THREE-PHASE SOLID MIXTURES

A. Aluminum and Nickel Powder in Silicone Rubber

		Experiment	Hamilton,* Eq. (16)		Cheng, Eq.(17)	
P_{d1}	P_{d2}	k_m Btu/hr ft °F	k_m	Deviation from Exp. Data (%)	k_m	Deviation from Exp. Data (%)
0.06	0.04	0.323	0.295	-8.70	0.362	+12.1
0.13	0.09	0.465	0.407	-12.5	0.521	+12.0

B. Aluminum and Bismuth Powder in Silicone Rubber

		Experiment	Hamilton,* Eq. (16)		Cheng's Analysis of Case 2	
P_{d1}	P_{d2}	k_m Btu/hr ft °F	k_m	Deviation from Exp. Data (%)	k_m	Deviation from Exp. Data (%)
0.15	0.03	0.445	0.363	-18.4	0.494	+11.0
0.04	0.08	0.368	0.301	-18.2	0.401	+8.9

TABLE V--Continued

C. Lead and Bismuth Powder in Silicone Rubber

		Experiment	Hamilton,* Eq. (16)		Cheng's Analysis of Case 3	
P_{d1}	P_{d2}	k_m Btu/hr ft °F	k_m	Deviation from Exp. Data (%)	k_m	Deviation from Exp. Data (%)
0.08	0.10	0.400	0.353	-11.8	0.470	+17.5
0.08	0.04	0.342	0.307	-10.2	0.379	+10.8

* Assume Discontinuous Phases are Spherical

TABLE VI

SUMMARY OF THE COMPARISONS OF THE PREDICTED THERMAL CONDUCTIVITIES WITH EXPERIMENTAL DATA FOR TWO-PHASE AND THREE-PHASE SOLID MIXTURES

Type of Mixtures	Average Percentage of Deviation Predicted by Existing Methods
Two-Phase Solid Mixtures	8.9% by Maxwell, Eq. (1) 8.7% by Cheng, Eq. (11)
Three-Phase Solid Mixtures	13.3% by Hamilton, Eq. (16) (Assume Discontinuous Phases are Spherical) 12.05% by Cheng's Analysis

VI. DISCUSSION AND CONCLUSIONS

DISCUSSION OF VARIOUS EFFECTS

In the derivation of Eq. (26), an ideal line source of infinite length in an infinite medium was assumed. Also, all terms except the first two were neglected in Eq. (23) in order to obtain an equation in which the values of r and D do not appear. In an actual experiment, the sample is finite and the heat source has a finite length, radius and mass. Although most of these sources of error have been mentioned before, they are presented here for discussions. The possible errors other than instrumentation errors are:

1. Error due to terms dropped in the infinite series of Eq. (23). With a proper choice of the parameter β, this error may be considered to be negligible.

2. Error due to finite size of the sample. Errors of this type may be avoided with the use of a second thermocouple just beneath the outer surface of the experimental specimen. When the temperature of this junction begins to increase, the experiment should be terminated. Samples with length to diameter of 4 have been shown to approximate the condition in an infinite cylinder (35).

3. Error due to the finite length of heat source.

Blackwell (36) has shown that, for wires of small diameter and a length of two inches or more, the error due to finite length of heat source will not be noticeable.

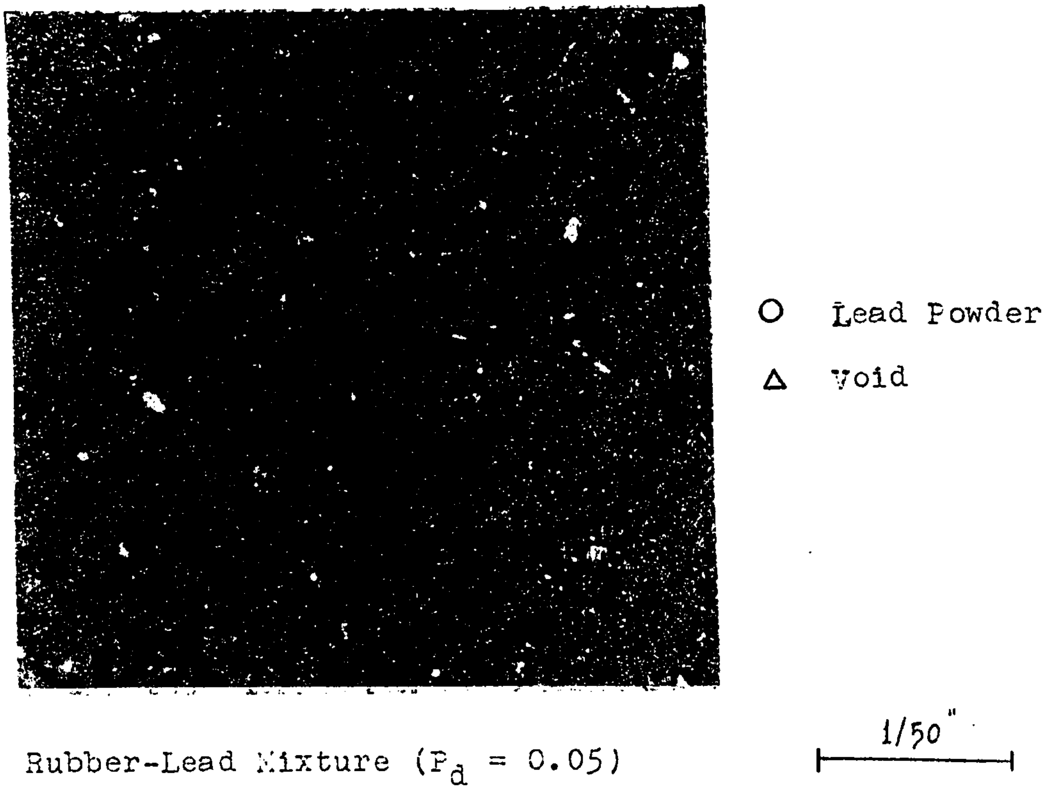
4. Error due to contact resistance between heater wire and the test material. Van der Held and Van Drunen (25) derived a time correction factor θ_c, to compensate for this effect.

Some of the samples were cut longitudinally and transversely after finishing the experiments to check the phase distribution and porosity (due to air bubbles during the casting process). It was found that there were a few tiny air pores in the sample but the distcontinuous phase in general was uniformly distributed in the continuous phase.

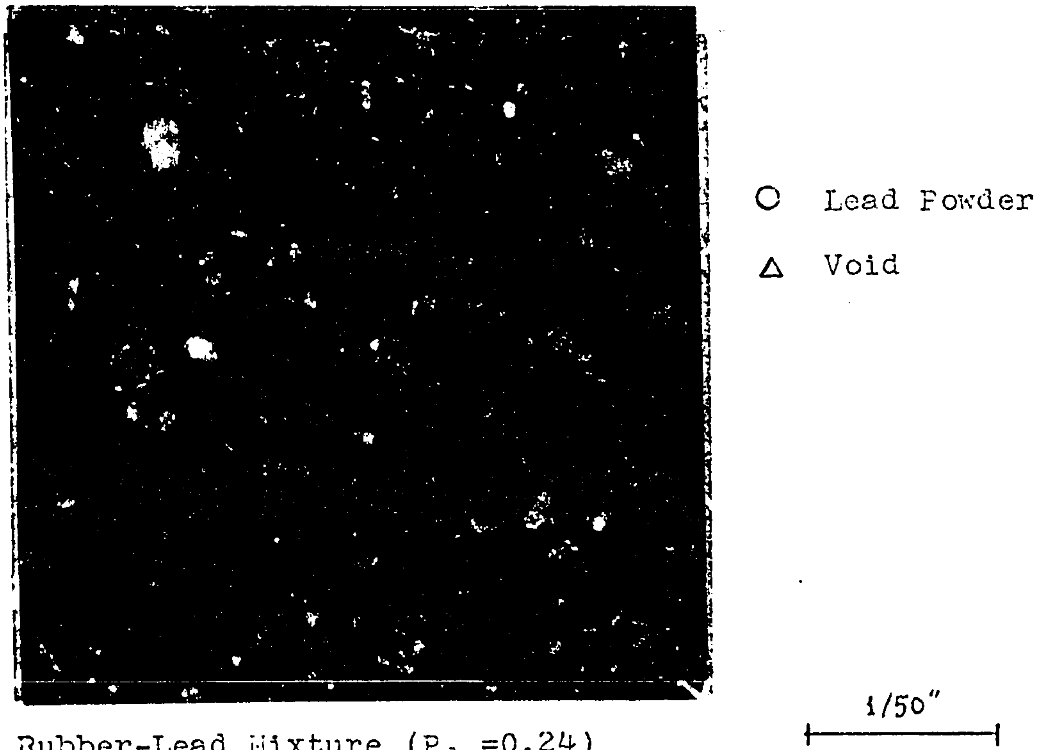
Figure.18 shows the photomicrographs of the experimental samples with magnifying 50 times of original size. It is estimated that the entrapped voids in the sample does not exceed 1% of the total volume, therefore the effect due to the entrapped air or voids in the experimental samples may be neglected.

ERROR ANALYSIS OF THE EXPERIMENTAL DATA

A general classification of the errors may be made in the error analysis, namely, systematic errors, random errors, and personal errors. Through proper design and using

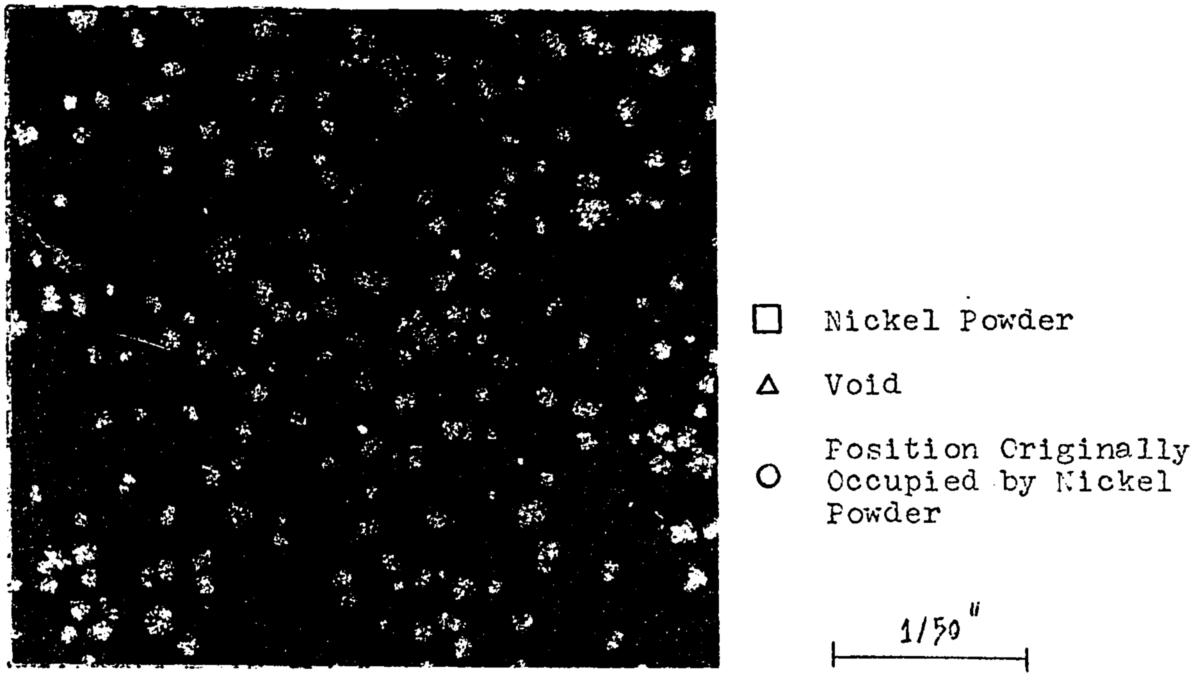


Rubber-Lead Mixture (P_d = 0.05)

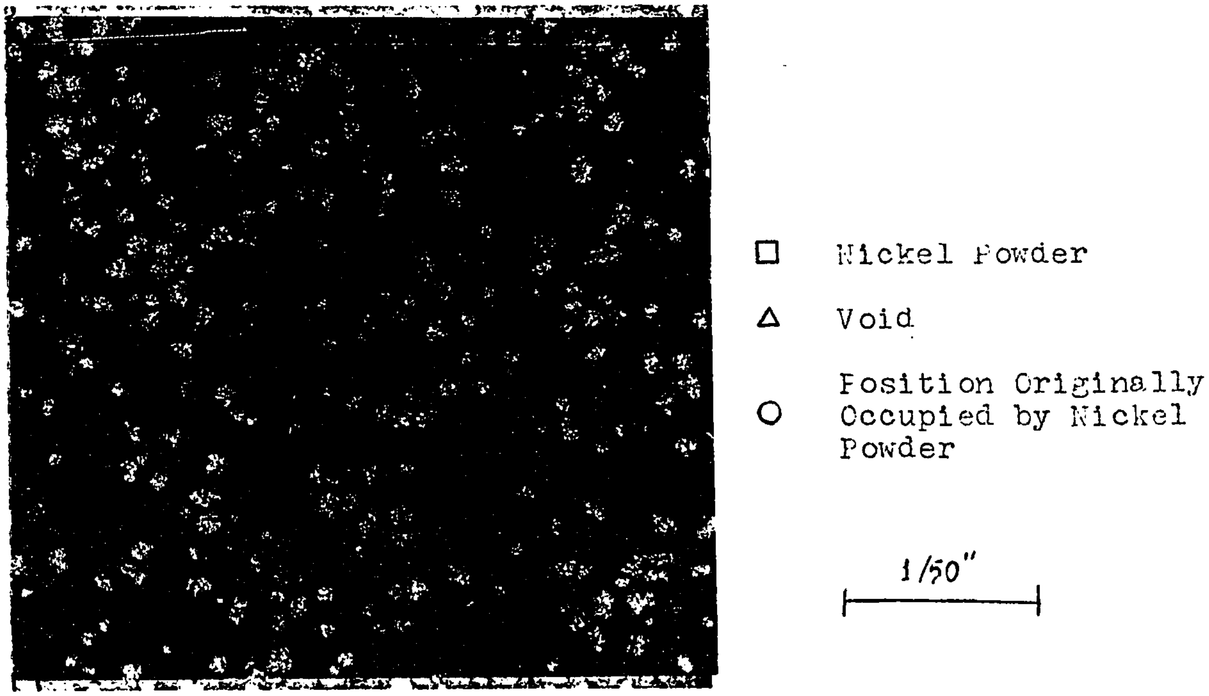


Rubber-Lead Mixture (P_d =0.24)

Fig. 18. Photomicrographs of Mixtures

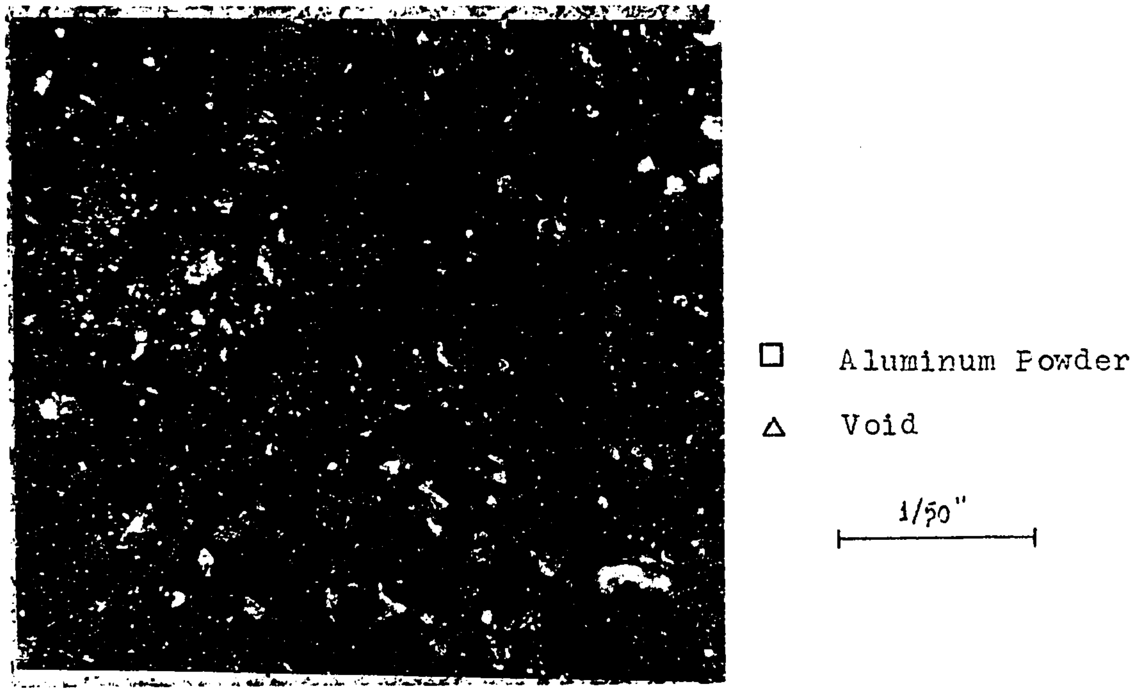


Rubber-Nickel Mixture (P_d = 0.16)

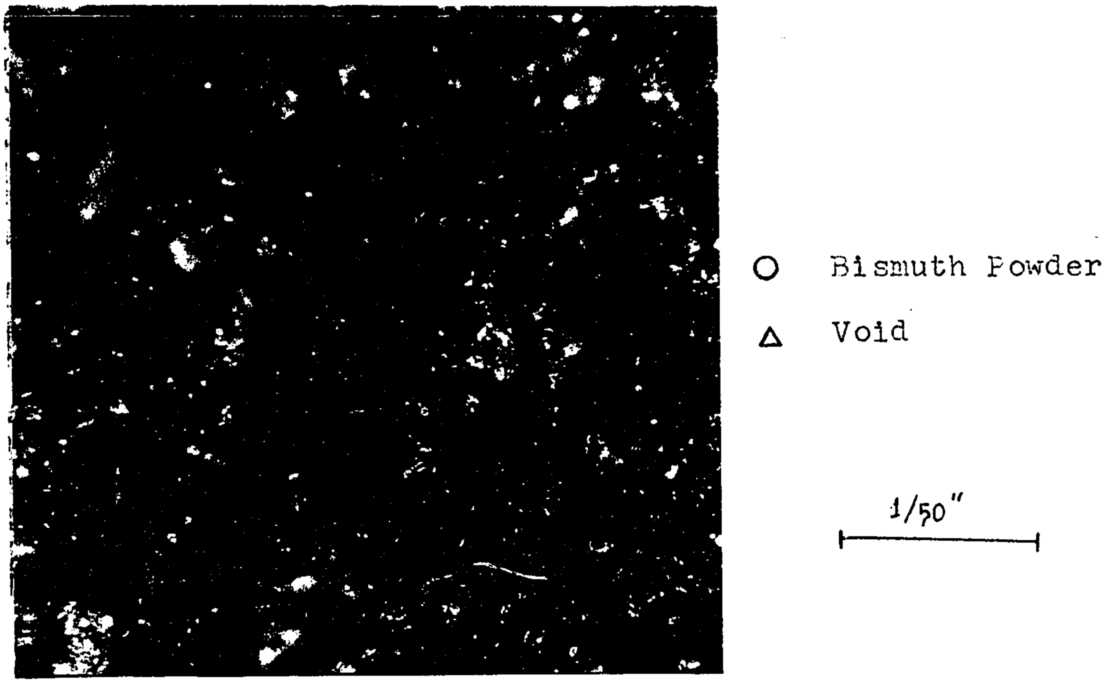


Rubber-Nickel Mixture (P_d = 0.24)

Fig. 18. (Continued) Photomicrographs of Mixtures

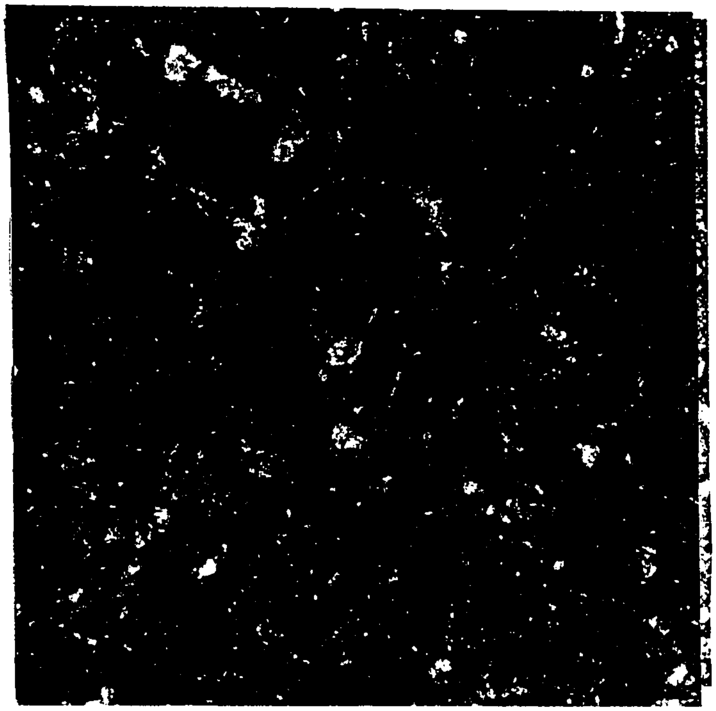


Rubber-Aluminum Mixture (P_d = 0.16)



Rubber-Bismuth Mixture (P_d = 0.24)

Fig. 18 (Continued). Photomicrographs of Mixtures



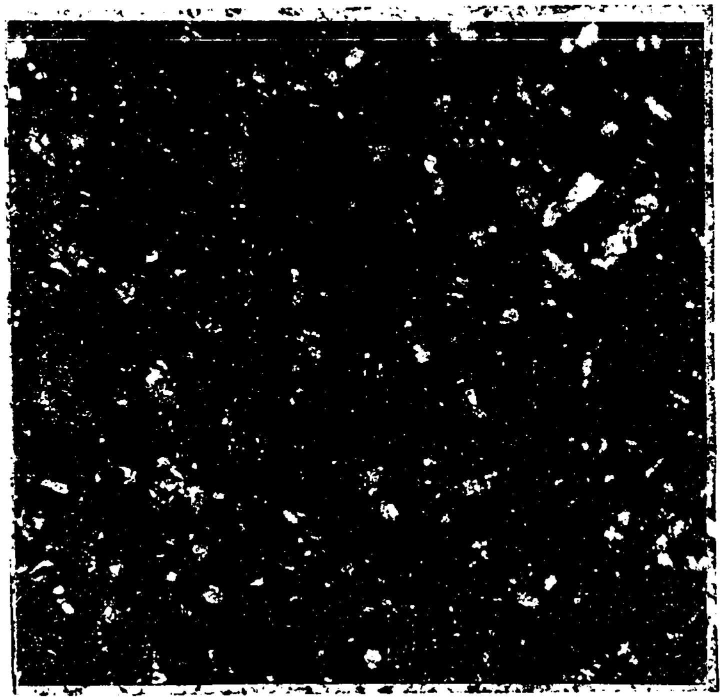
○ Aluminum Powder

□ Bismuth Powder

△ Void

Rubber-Aluminum-Bismuth Mixture with Volume Composition 82 : 15 : 3

1/50"



○ Lead Powder

□ Bismuth Powder

△ Void

Rubber-Lead-Bismuth Mixture with Volume Composition 82 : 8 : 10

1/50"

Fig. 18 (Continued). Photomicrographs of Mixtures automatic equipment for measurements in the experiment, personal errors, may be reduced to a minimum. However, systematic errors affect the experiment in one direction and random errors may occur in either direction (plus or minus). These errors are always present in any experiment and will be accounted for in the following analysis.

A procedure often used in error analysis is to give both an internal and external estimate of the errors. The internal estimate is a measure of the actual consistency of the experimental results. Here the basis for the internal estimate has been selected as the sample standard deviation, SD. This standard deviation is given by (37)

$$SD = \sqrt{\frac{\sum_{i=1}^{n} (k_i - \bar{k})^2}{n - 1}} \qquad (30)$$

The sample standard deviation is based on a finite set of n observations and will change from sample to sample.

The external estimate, however, is completely removed from any consideration of the actual experimental data and should evolve from general conditions associated with the experimental apparatus and computational procedure. Therefore, the external estimate will not change.

We consider the functional relationship

$$k = f\ (a,b,c,d)$$

where k is the indicated conductivity and a,b,c,and d are the separate measured quantities which are likely to be in error. Then we have

$$dk = \sum_{n=a}^{d} \left(\frac{\partial f}{\partial n}\right) dn \tag{31}$$

The variation in k is considered to be produced by errors in quantities a to d, Whose magnitudes are s_a to s_d. Thus,

$$S_k = \sum_{n=a}^{d} \left(\frac{\partial f}{\partial n}\right) S_n \tag{32}$$

where s_k represents the total magnitude of error involved in the function and s_n represent the magnitude of separate errors. Eq. (32) cannot be solved directly since it is not known how the error terms add up. However, s_k^2 can be evaluated:

$$S_k^2 = \sum_{n=a}^{d} \left(\frac{\partial f}{\partial n}\right)^2 S_n^2 + \sum_{n=a,\, m=a}^{d} \left(\frac{\partial f}{\partial m}\right)\left(\frac{\partial f}{\partial n}\right) S_m S_n \tag{33}$$

where m ≠ n. Cook and Rabinowicz (38) point out that when the errors are independent and symmetrical in regard to positive and negative values, the cross products in the second term of Eq. (33) tend to disappear, giving

$$S_k^2 = \sum_{n=a}^{d} \left(\frac{\partial f}{\partial n}\right)^2 S_n^2 \tag{34}$$

Eq. (34) is the general form for the resultant error in terms of its components.

The functional form of the equation currently used for calculating conductivity in the present case corresponding to Eq. (28) may be represented by the formula

$$k = f\ (a,b,c,d) = C_n(\ I^2RV/\Delta EM\ ) \tag{35}$$

where

$C_n = (3.413)(2.303)/4\pi$

$V = d(EM)/dt$, thermocouple response, MV/°F

$\Delta EM =$ Corrected millivolt change betweem θ_{2corr} and θ_{1corr} , MV

Thus Eq. (34) for the error becomes

$$s_k^2/k^2 = 4s_I^2/I^2 + s_R^2/R^2 + s_V^2/V^2 + s_{\Delta EM}^2/\Delta EM^2 \tag{36}$$

rearrange Eq. (36), we have

$$e_k = (4e_I^2 + e_R^2 + e_V^2 + e_{\Delta EM}^2)^{1/2} \qquad (37)$$

where e_k is the total percentage error and e_n, $n = I, R...$ is the percentage error associated with the quantities I, R, V, and ΔEM.

The percentage errors affect the result of conductivity calculated from measured quantities are 0.02% for current, 0.06% for resistance, 0.75% for thermocouple response(39), and 3.5% for millivolt change between θ_{2corr} and θ_{1corr} due to amplification error. Application of Eq. (37) yields

$$e_k = \pm 3.6\%$$

as the external estimate error for any sample under ideal conditions. It is quite obvious that the external estimate error is mainly due to amplification error, therefore, the accuracy may be improved if more accurate amplifier is used in the experimental arrangement. However, a comparison between external estimate and internal estimate of accuracy agrees well with the experimental results in the present work as shown in the Appendix 2.

CONCLUSIONS

From this study of the thermal conductivity of two-phase and three-phase heterogeneous mixtures, several general conclusions may be drawn as under:

1. The line-source method, which has been described and demonstrated, provides a simple and efficient method

for measuring thermal conductivity of solid mixtures. The experimental results given by the line-source method are consistent and reproducible.

2. Comparing the experimental results of the single-phase material (silicone rubber) with those obtained from the National Bureau Standard , one can conclude that the line-source method under the condition employed in this study is dependable.

3. The lin-source method allows the sample to be isolated from the measuring equipment during a test; thus the sample can be placed in any temperature environment. This makes it extremely convenient for measuring thermal conductivity at different temperatures. In addition to this, the small temperature gradients and short experimental time required by the method makes it especially suited for measurement of heat-sensitive materials, where the imposition of large temperature gradients for an extended period will alter or destroy the sample.

4. In the two-phase mixtures, if the volume fraction of discontinuous phase is small, i.e. less than ten percent and the ratio of k_d and k_c is less than 100, such as bismuth-rubber mixtures, Maxwell's equation, Eq. (1) can be applied to predict the thermal conductivity with a reasonable accuracy regardless of the particle shape as pointed out by Hamilton (12).

5. In the two-phase mixtures, if the ratio of k_d and k_c is much great than 100, such as aluminum-rubber mixtures, the shape factor of the discontinuous phase will

effect the mean thermal conductivity of the mixtures. However, since the shape factor of aluminum particles is not known, Cheng's equation, Eq. (11) gives a better prediction than Maxwell's equation since Eq. (11) was derived without considering the shape factor of discontinuous phase and taking an average value for a mixture containing all kinds of different particle shapes.

6. Comparison of the predicted thermal conductivity with experimental results of nickel-rubber mixtures shows that Maxwell's equation gives better results. This can be explained by the presence of almost spherical nickel powder particles in the photomicrographs of the samples of nickel-rubber mixtures as assumed by Maxwell in deriving his equation.

7. Comparison of the theoretical predicted thermal conductivities with experimental results of the three-phase solid mixtures shows that Cheng's analysis always over-estimates the experimental results but Hamilton's equation, Eq. (16) always under-estimates the experimental results. The prediction from these two techniques can be used as the upper and lower bounds for the theoretical prediction of the thermal conductivity of three-phase mixtures.

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APPENDIX 1

Computer Programming

The governing equation used to calculate the value of thermal conductivity from experimental data is Eq. (26). It may be written as

$$t_i - t_j = C_K \, Ln\left(\frac{\theta_i - \theta_c}{\theta_j - \theta_c}\right) \qquad (i > j) \qquad \text{(A-1)}$$

where $C_k = 3.413 \; I^2R/4\pi k$

and the subscripts i, j refer to the individual experimental data points obtained from the continuous recording of experimental curve (temperature versus time)

The least-squares technique has been applied to solve Eq. (A-1) by the computer. The summation of the sum of the squares of the error terms E_{ij} corresponding to the Eq. (A-1) is

$$SM = \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} E_{ij}^2 = \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} \left[(t_i - t_j) - C_K \, Ln\left(\frac{\theta_i - \theta_c}{\theta_j - \theta_c}\right) \right]^2 \qquad \text{(A-2)}$$

The values of C_k and θ_c are determined when the value of error criterion SM is minimum, i.e. $\partial(SM)/\partial C_k = 0$, $\partial(SM)/\partial \theta_c = 0$. Since

$$\frac{\partial(SM)}{\partial \theta_c} = (2\,C_K) \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} \left[(t_i - t_j) - C_K \, Ln\left(\frac{\theta_i - \theta_c}{\theta_j - \theta_c}\right) \right] \left(\frac{1}{\theta_i - \theta_c} - \frac{1}{\theta_j - \theta_c} \right) \qquad \text{(A-3)}$$

$$\frac{\partial(SM)}{\partial C_k} = (-2)\sum_{j=1}^{n-1}\sum_{i=j+1}^{n}\left[(t_i-t_j) - C_k \mathrm{Ln}\left(\frac{\theta_i-\theta_c}{\theta_j-\theta_c}\right)\right]\mathrm{Ln}\left(\frac{\theta_i-\theta_c}{\theta_j-\theta_c}\right) \quad (A-4)$$

Setting the two equations to zero and rearranging the summations, we obtain the following equations

$$C_k = \frac{\sum_{j=1}^{n-1}\sum_{i=j+1}^{n}(t_i-t_j)\left[\frac{1}{\theta_i-\theta_c} - \frac{1}{\theta_j-\theta_c}\right]}{\sum_{j=1}^{n-1}\sum_{i=j+1}^{n}\mathrm{Ln}\left(\frac{\theta_i-\theta_c}{\theta_j-\theta_c}\right)\left[\frac{1}{\theta_i-\theta_c} - \frac{1}{\theta_j-\theta_c}\right]} \quad (A-5)$$

$$\sum_{j=1}^{n-1}\sum_{i=j+1}^{n}(t_i-t_j)\mathrm{Ln}\left(\frac{\theta_i-\theta_c}{\theta_j-\theta_c}\right) - C_k\sum_{j=1}^{n-1}\sum_{i=j+1}^{n}\mathrm{Ln}\left(\frac{\theta_i-\theta_c}{\theta_j-\theta_c}\right)^2 = 0 \quad (A-6)$$

Thus, theoretically we have two equations from which the two unknowns C_k and θ_c can be determined. However, it is very difficult to solve C_k and θ_c directly in this case. The following approximate method (trial and error) can be used. First, substituting Eq. (A-5) into Eq. (A-6), one obtains an equation involving θ_c only. For different values of θ_c (starting from zero), different values of the left-hand side expression of Eq. (A-6) can be obtained. However, there is only one θ_c such that the left-hand side expression of Eq. (A-6) is a minimum and this is the desirable θ_c. Once θ_c is obtained, C_k can be calculated from Eq. (A-5) and therefore k is determined.

```
      C
      C       THIS PROGRAMMING APPLIES THE LEAST-SQUARES TECHNIQUE
      C       TO DETERMINE THE VALUE OF THERMAL CONDUCTIVITY FROM
      C       EQUATION A-1
      C
      C       DEFINE THE FOLLOWING ITEMS
      C       E=ERROR CRITERION
      C       AKM=THC=THERMAL CONDUCTIVITY
      C       TC=TIME CORRECTION FACTOR
      C       TEMP=TEMPERATURE
      C       AMP=CURRENT
      C       RES=RESISTANCE
      C       ERR=SMALLEST ERROR
      C
              IMPLICIT REAL*8 (A-H,O-Z)
              DIMENSION TIME(30),TEMP(30),AAA(350),AA(15,15), C(350),
             @BBB(350),BB(15,15),DDD(350),DD(15,15),E(350),AKM(350),
             @CCC(350),CC(15,15)
              READ(1,100) (TIME(K),K=1,14)
         100  FORMAT(7F6.0)
              READ(1,99) AMP
          99  FORMAT(F8.5)
              READ(1,101) (TEMP(K),K=1,14)
         101  FORMAT(7F6.3)
              TC=0.0
              DO 3 I=1,301
      C
      C       CALCULATION OF C(K) OF EQUATION A-5
      C
              AAA(I)=0.
              DO 4 K=1,13
              DO 4 L=1,13
              IF (TEMP(L+1) .LE. TEMP(K)) GOTO 4
              AA(K,L)=((TEMP(L+1)-TEMP(K))*(1./(TIME(L+1)-TC)-1./(TIME(K)-TC)))
              AAA(I)=AAA(I)+AA(K,L)
           4  CONTINUE
              BBB(I)=0.
              DO 5 K=1,13
              DO 5 L=1,13
              IF(TIME(L+1).LE.TIME(K)) GOTO 5
              BB(K,L)=DLOG((TIME(L+1)-TC)/(TIME(K)-TC))*
             @(1./(TIME(L+1)-TC)-1./(TIME(K)-TC))
              BBB(I)=BBB(I)+BB(K,L)
```

```
      5   CONTINUE
          C(I)=AAA(I)/BBB(I)
    C
    C     CALCULATION OF EQUATION A-6
    C
          DDD(I)=0.
          DO 6 K=1,13
          DO 6 L=1,13
          IF(TEMP(L+1).LE.TEMP(K)) GOTO 6
          DD(K,L)=(TEMP(L+1)-TEMP(K))*DLOG((TIME(L+1)-TC)/(TIME(K)-TC))
          DDD(I)=DDD(I)+DD(K,L)
      6   CONTINUE
          CCC(I)=0.
          DO 7 K=1,13
          DO 7 L=1,13
          IF (TIME(L+1).LE.TIME(K)) GOTO 7
          CC(K,L)=((DLOG((TIME(L+1)-TC)/(TIME(K)-TC)))**2)*C(I)
          CCC(I)=CCC(I)+CC(K,L)
      7   CONTINUE
          F(I)=DDD(I)-CCC(I)
          RES=3.041
          AKM(I)=(3.413*AMP*AMP*RES)/(4*3.1416*C(I))
          TC=TC+0.1
      3   CONTINUE
    C
    C     DETERMINATION OF THE VALUE OF THERMAL CONDUCTIVITY
    C
          ERR=DABS (F(1))
          THC=AKM(1)
          I=2
    300   IF (ERR-DABS(F(I))) 301,301,302
    302   ERR=DABS(F(I))
          THC=AKM(I)
    301   IF(I-301)303,304,304
    303   I=I+1
          GO TO 300
    304   WRITE(3,305)ERR,THC
    305   FORMAT(//,34X,'SMALLEST ERROR=',D16.8,5X,'KM=',D16.8)
          RETURN
          END
```

APPENDIX 2

Tablated Data of Experimental Results

1. Results of Conductivity for RTV-60 Silicone Rubber Sample No. 1.

Sample Standard Deviation ± 0.0051 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F	Current Amps
1	61.7	0.228	0.222	0.188
2	67.4	0.224		0.308
3	69.4	0.224		0.271
4	69.8	0.217		0.220
5	70.8	0.222		0.241
6	71.1	0.222		0.215
7	71.6	0.222		0.202
8	74.5	0.222		0.202
9	75.2	0.216		0.300
10	77.0	0.217		0.301
11	77.8	0.228		0.340
12	79.8	0.229		0.277
13	80.8	0.222		0.322
14	81.6	0.219		0.314
15	83.9	0.212		0.330
16	85.2	0.227		0.282

2. Results of Conductivity for RTV-60 Silicone Rubber Sample No. 2

Sample Standard Deviation ± 0.0067 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F	Current Amps
1	61.7	0.217	0.222	0.187
2	65.7	0.231		0.290
3	66.2	0.215		0.220
4	68.0	0.220		0.315
5	69.0	0.232		0.271
6	70.4	0.217		0.308
7	74.3	0.231		0.202
8	76.1	0.222		0.300
9	77.0	0.231		0.267
10	77.7	0.218		0.339

11	77.9	0.216	0.222	0.239
12	79.9	0.221		0.277
13	80.9	0.223		0.322
14	81.9	0.232		0.290
15	83.2	0.215		0.330
16	85.6	0.224		0.283

3. Results of Conductivity for Rubber-Aluminum Mixture with $P_d = 0.05$

Sample Standard Deviation ± 0.0068 Btu/hr ft °F

Test No.	Temperature °F	k_1 Btu/hr^{1}ft °F	$\bar{k}$ Btu/hr ft °F
1	66.2	0.275	0.274
2	66.5	0.283	
3	67.1	0.280	
4	71.5	0.278	
5	71.7	0.265	
6	77.7	0.280	
7	79.3	0.263	
8	79.9	0.279	
9	81.4	0.270	
10	81.4	0.271	
11	82.8	0.283	
12	84.1	0.274	
13	85.0	0.269	
14	85.2	0.263	

4. Results of Conductivity for Rubber-Aluminum Mixture with $P_d = 0.16$

Sample Standard Deviation ± 0.0117 Btu/hr ft °F

Test No.	Temperature °F	k_1 Btu/hr^{1}ft °F	$\bar{k}$ Btu/hr ft °F
1	66.1	0.460	0.442
2	67.1	0.435	
3	69.7	0.423	
4	73.0	0.442	
5	77.6	0.424	
6	77.8	0.444	

7	78.0	0.454	0.442
8	79.1	0.445	
9	81.1	0.441	
10	81.4	0.425	
11	83.7	0.460	
12	83.8	0.442	
13	84.7	0.432	
14	85.7	0.444	

5. Results of Conductivity for Rubber-Aluminum Mixture with $P_d = 0.28$

Sample Standard Deviation ± 0.0141 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	65.2	0.603	0.600
2	66.8	0.601	
3	67.1	0.590	
4	69.1	0.607	
5	76.2	0.580	
6	76.6	0.616	
7	78.9	0.613	
8	80.8	0.597	
9	81.2	0.579	
10	83.4	0.573	
11	83.8	0.619	
12	84.2	0.607	
13	85.6	0.584	
14	90.0	0.616	

6. Results of Conductivity for Rubber-Lead Mixture with $P_d = 0.05$

Sample Standard Deviation ± 0.0059 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	70.3	0.269	0.267
2	72.9	0.261	
3	75.2	0.275	
4	75.5	0.258	
5	76.7	0.273	
6	77.5	0.261	
7	79.0	0.259	
8	79.3	0.262	
9	82.1	0.269	
10	82.3	0.275	
11	82.5	0.268	
12	84.5	0.264	
13	85.3	0.272	
14	92.2	0.269	

7. Results of Conductivity for Rubber-Lead Mixture with $P_d = 0.16$

Sample Standard Deviation ± 0.0089 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	75.9	0.375	0.383
2	76.2	0.384	
3	76.7	0.395	
4	78.1	0.391	
5	78.2	0.372	
6	78.2	0.377	
7	79.3	0.397	
8	82.7	0.379	
9	83.1	0.370	
10	83.6	0.380	
11	84.2	0.394	
12	85.0	0.386	
13	85.3	0.373	
14	92.4	0.384	

8. Results of Conductivity for Rubber-Lead Mixture with $P_d = 0.24$

Sample Standard Deviation ± 0.0109 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	70.5	0.485	0.497
2	73.8	0.498	
3	75.8	0.514	
4	76.9	0.494	
5	77.5	0.490	
6	78.1	0.511	
7	78.3	0.489	
8	79.9	0.500	
9	83.1	0.481	
10	83.8	0.495	
11	85.1	0.490	
12	85.3	0.516	
13	86.0	0.503	
14	93.1	0.489	

9. Results of Conductivity for Rubber-Nickel Mixture with $P_d = 0.05$

Sample Standard Deviation ± 0.0069 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	72.8	0.253	0.265
2	76.7	0.262	
3	77.0	0.254	
4	80.6	0.275	
5	81.8	0.269	
6	82.1	0.269	
7	82.7	0.267	
8	82.8	0.266	
9	83.4	0.268	
10	83.4	0.269	
11	83.5	0.262	
12	84.6	0.255	
13	88.9	0.256	
14	91.9	0.262	

10. Results of Conductivity for Rubber-Nickel Mixture with $P_d = 0.16$

Sample Standard Deviation ± 0.0085 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	73.2	0.385	0.379
2	76.5	0.374	
3	76.7	0.389	
4	80.6	0.367	
5	81.7	0.370	
6	81.8	0.390	
7	82.8	0.365	
8	83.1	0.384	
9	83.1	0.372	
10	83.6	0.391	
11	83.9	0.381	
12	84.5	0.380	
13	86.2	0.376	
14	88.9	0.377	

11. Results of Conductivity for Rubber-Nickel Mixture with $P_d = 0.24$

Sample Standard Deviation ± 0.0061 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	73.1	0.473	0.471
2	76.3	0.467	
3	76.7	0.476	
4	81.9	0.479	
5	82.1	0.469	
6	82.1	0.474	
7	83.1	0.466	
8	83.5	0.474	
9	83.6	0.461	
10	84.0	0.484	
11	84.6	0.469	
12	86.6	0.468	
13	88.9	0.473	
14	93.1	0.465	

12. Results of Conductivity for Rubber-Bismuth Mixture with $P_d = 0.05$

Sample Standard Deviation ± 0.0058 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\overline{k}$ Btu/hr ft °F
1	68.5	0.254	0.250
2	77.6	0.258	
3	77.8	0.246	
4	78.3	0.250	
5	78.5	0.259	
6	79.9	0.247	
7	80.9	0.253	
8	83.1	0.243	
9	83.9	0.243	
10	83.9	0.248	
11	84.5	0.244	
12	85.5	0.259	
13	88.5	0.252	
14	92.0	0.248	

13. Results of Conductivity for Rubber-Bismuth Mixture with $P_d = 0.16$

Sample Standard Deviation ± 0.0051 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\overline{k}$ Btu/hr ft °F
1	69.7	0.335	0.341
2	74.4	0.334	
3	77.9	0.341	
4	78.1	0.337	
5	78.4	0.336	
6	79.0	0.350	
7	80.5	0.341	
8	80.7	0.346	
9	83.0	0.349	
10	84.0	0.341	
11	84.5	0.338	
12	86.5	0.341	
13	89.7	0.344	
14	92.0	0.346	

14. Results of Conductivity for Rubber-Bismuth Mixture with $P_d = 0.24$

Sample Standard Deviation ± 0.0090 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	75.8	0.420	0.423
2	78.1	0.421	
3	78.3	0.434	
4	77.9	0.414	
5	79.5	0.440	
6	80.2	0.420	
7	80.2	0.410	
8	82.7	0.416	
9	84.2	0.417	
10	84.7	0.435	
11	84.8	0.416	
12	87.1	0.432	
13	90.0	0.424	
14	91.8	0.427	

15. Results of Conductivity for Rubber-Aluminum-Nickel Mixture with Volume Composition 78 : 13 : 9

Sample Standard Deviation ± 0.0066 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	76.4	0.458	0.465
2	76.9	0.462	
3	77.1	0.472	
4	78.9	0.453	
5	79.7	0.469	
6	80.7	0.462	
7	81.5	0.463	
8	81.6	0.469	
9	81.7	0.473	
10	82.0	0.455	
11	84.7	0.468	
12	85.9	0.471	
13	88.8	0.461	
14	90.5	0.466	

16. Results of Conductivity for Rubber-Aluminum-Nickel Mixture with Volume Composition 90 : 6 : 4

Sample Standard Deviation ± 0.0054 Btu/hr ft ^oF

Test No.	Temperature ^oF	k_i Btu/hr ft ^oF	$\bar{k}$ Btu/hr ft ^oF
1	70.4	0.318	0.323
2	76.9	0.331	
3	77.1	0.330	
4	78.9	0.319	
5	79.5	0.323	
6	80.8	0.320	
7	80.8	0.321	
8	81.5	0.320	
9	82.1	0.318	
10	82.3	0.322	
11	82.4	0.334	
12	82.5	0.317	
13	83.9	0.327	
14	85.5	0.323	

17. Results of Conductivity for Rubber-Aluminum-Bismuth Mixture with Volume Composition 82 : 15 : 3

Sample Standard Deviation ± 0.0112 Btu/hr ft ^oF

Test No.	Temperature ^oF	k_i Btu/hr ft ^oF	$\bar{k}$ Btu/hr ft ^oF
1	76.7	0.446	0.445
2	76.8	0.453	
3	77.0	0.441	
4	79.1	0.451	
5	79.8	0.431	
6	80.8	0.436	
7	81.5	0.430	
8	81.7	0.430	
9	82.0	0.430	
10	84.5	0.463	
11	85.1	0.459	
12	85.4	0.450	
13	88.8	0.453	
14	90.5	0.450	

18. Results of Conductivity for Rubber-Aluminum-Bismuth Mixture with Volume Composition 88 : 4 : 8

Sample Standard Deviation ± 0.0084 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	75.5	0.382	0.368
2	77.1	0.361	
3	78.8	0.378	
4	81.4	0.384	
5	83.0	0.359	
6	84.7	0.369	
7	84.8	0.367	
8	85.0	0.358	
9	85.4	0.370	
10	88.7	0.364	
11	89.4	0.370	
12	89.5	0.367	
13	90.9	0.360	
14	91.6	0.362	

19. Results of Conductivity for Rubber-Lead-Bismuth Mixture with Volume Composition 82 : 8 :10

Sample Standard Deviation ± 0.0093 Btu/hr ft °F

Test No.	Temperature °F	k_i Btu/hr ft °F	$\bar{k}$ Btu/hr ft °F
1	77.1	0.397	0.400
2	77.4	0.409	
3	79.0	0.412	
4	80.1	0.386	
5	80.3	0.396	
6	81.5	0.389	
7	82.0	0.393	
8	82.5	0.403	
9	84.5	0.415	
10	84.8	0.398	
11	85.3	0.415	
12	88.7	0.397	
13	90.3	0.394	
14	90.8	0.403	

20. Results of Conductivity for Rubber-Lead-Bismuth Mixture with Volume Composition 88 :8 ; 4

Sample Standard Deviation ± 0.0068

Test No.	Temperature °F	k_i Btu/hr ft °F	$\overline{k}$ Btu/hr ft °F
1	75.3	0.345	0.342
2	77.2	0.343	
3	79.1	0.346	
4	81.2	0.357	
5	83.2	0.341	
6	84.1	0.339	
7	84.5	0.330	
8	84.7	0.341	
9	85.4	0.349	
10	88.1	0.347	
11	88.7	0.335	
12	89.7	0.337	
13	91.1	0.337	
14	94.1	0.347	