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# Iterative Coded Multiuser Detection Using LDPC Codes

by

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A thesis submitted to the
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# Abstract

*Multiuser detection* (MUD) has been regarded as an effective technique for combating *cochannel interference* (CCI) in *time-division multiple access* (TDMA) systems and *multiple access interference* (MAI) in *code-division multiple access* (CDMA) systems. An optimal multiuser detector for coded multiuser systems is usually practically infeasible due to the associated complexity. An iterative receiver consisting of a soft-input soft-output (SISO) multiuser detector and a bank of SISO single user decoders can provide a system performance which approaches to that of single user system after a few iterations.

In this thesis, MUD and LDPC decoding are combined to improve the multiuser receiver performance. The soft output of the LDPC decoder is fed back to the multiuser detector to improve the detection. This leads to decision variables that have a smaller MAI component. These decision variables are then returned to the decoder and the decoding process benefits from the improvement to the decision variables. The process can be repeated many times. The resulting iterative multiuser receiver is designed based on the soft parallel interference cancellation (PIC) algorithm. For the interference reconstruction, the LDPC decoder is improved to produce the *log-likelihood ratios* (LLR) of the information bits as well as the parity bits. A sub-optimal approach is proposed to output the LLR of the parity bits with very low complexity. Thanks to the powerful error-correction ability of the LDPC decoder, the LDPC multiuser receiver can achieve a satisfactory convergence, and substantially outperforms non-iterative receivers.

Three types of SISO multiuser detectors are provided. They are: Soft Interference Cancellation (SIC) detector, SISO decorrelating detector and SISO minimum mean square error (MMSE) detector. The resulting system performance converges very quickly. The comparison of these three types of detectors is also shown in this thesis.

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# List of Acronyms

| | |
|---|---|
| AWGN | Additive White Gaussian Noise |
| BER | Bit Error Rate |
| BPA | Brief Propagation Algorithm |
| BPSK | Binary Phase Shift Keying |
| CDMA | Code Division Multiple Access |
| DS-CDMA | Direct Sequence CDMA |
| FEC | Forward Error Control |
| Hz | Hertz |
| IC | Interference Cancellation |
| LDPC | Low-Density Parity-Check |
| LLR | Log-Likelihood Ratio |
| MAI | Multiple Access Interference |
| MAP | Maximum a Posteriori |
| ML | Maximum Likelihood |
| MF | Matched Filter |
| MMSE | Minimum Mean Squared Error |
| MPA | Message Passing Algorithm |
| MUD | Multiuser Detection |
| PCCCs | Parallel Concatenated Convolutional Codes |

| | |
|---|---|
| PIC | Parallel Interference Cancellation |
| SIC | Soft Interference Cancellation |
| SUIC | SUccessive Interference Cancellation |
| SISO | Soft Input Soft Output |
| SNR | Signal to Noise Ratio |
| SPA | Sum-Produce Algorithm |
| SUD | Single User Detection |
| TDMA | Time Division Multiple Access |

# List of Symbols

| | |
|---|---|
| $(\cdot)^E$ | quantity containing extrinsic information |
| $(\cdot)^P$ | quantity obtained from the previous iteration |
| $(\cdot)_F$ | quantities passed forward from the multiuser detector forward to the LDPC decoder |
| $(\cdot)_B$ | quantities passed backward from the LDPC decoder backward to the multiuser detector. |
| $A_k$ | received amplitude of the $k$th user's signal |
| $b_k(i)$ | the $i$th modulated symbol of user $k$ |
| $\hat{b}_k$ | soft symbol estimate of $b_k$ |
| $\hat{\mathbf{b}}_k(i)$ | obtained from soft symbol estimate vector by setting the $k$th element to zero. |
| $c_i$ | $i$th variable node |
| $C_i$ | event: {check nodes connected to variable node $c_i$ } |
| $C_{i\setminus j}$ | event: {check nodes connected to variable node $c_i$, except check node $f_j$ } |
| $\hat{\mathbf{c}}$ | tentatively decoded codeword |
| $d_k(i)$ | the $i$th information bit of user $k$ |
| $\mathbf{e}_k$ | $K$-dimensional column vector of all zeros, except that the $k$th element is 1 |

| | |
|---|---|
| $F$ | length of the coded bit frame |
| $f_j$ | $j$th check node |
| $\mathbf{G}$ | generator matrix |
| $\mathbf{H}$ | parity check matrix |
| $\mathbf{I}$ | identity matrix |
| $K$ | number of users |
| $\log(\cdot)$ | natural logarithm of $(\cdot)$ |
| $L(\cdot)$ | log-likelihood ratio of $(\cdot)$ |
| $\mathbf{m}_k$ | linear minimum mean square error filter of user $k$ |
| $N_0/2$ | noise variance, equal to $\sigma^2$ |
| $n(t)$ | white Gaussian noise process |
| $\mathbf{p}_k$ | correlation between $b_k$ and $\mathbf{z}_k$ |
| $q_{ij}(b)$ | message passed from variable node $c_i$ to check node $f_j$. It is the probability that $c_i = b$ given the channel sample $y_i$ and all check node messages except $f_j$. |
| $Q$ | length of the information bit frame |
| $r_{ji}(b)$ | message passed from check node $f_j$ to variable node $c_i$. It is the probability that check equation $j$ is satisfied, given that $c_i = b$ and that the other bits are distributed according to $\{q_{ij'}\}_{j' \neq j}$. |

| | |
|---|---|
| $\mathbf{R}$ | normalized cross-correlation matrix |
| $\mathbf{R}_u$ | equivalent cross-correlation matrix of the waveform of multiple users at the output of soft interference cancellation |
| $s_k(t)$ | normalized waveform assigned to the $k$th user |
| $S_i$ | true if $\hat{\mathbf{c}}$ satisfies all parity-check equations involving $c_i$, |
| $T$ | symbol interval |
| $V_j$ | event: {variable nodes connected to check node $f_j$} |
| $V_{j\backslash i}$ | event: {variable nodes connected to check node $f_j$, except variable node $c_i$} |
| $w_c$ | column weigh in parity check matrix $\mathbf{H}$ |
| $w_r$ | row weigh in parity check matrix $\mathbf{H}$ |
| $x_k^{MMSE}$ | output value of the linear SISO MMSE detector of user $k$ |
| $\mathbf{z}_k$ | output value of the soft interference cancellation detector of user $k$ |
| $\rho$ | cross correlation of any two users in a $K$-symmetric multiuser system |
| $\sigma^2$ | white Gaussian noise variance |
| $\sigma_k^2$ | equivalent noise variance of the output of soft interference cancellation detector |
| $\sigma_{uk}^2$ | equivalent noise variance of the output of SISO decorrelator |
| $\sigma_{vk}^2$ | equivalent noise variance of the output of SISO MMSE detector |

$\Lambda_1[b_k(i)]$ — *a posteriori* LLR of $b_k(i)$ provided by the multiuser detector

$\lambda_1[b_k(i)]$ — *a priori* LLR of $b_k(i)$ provided by the multiuser detector in the previous iteration

$\lambda_1^E[b_k(i)]$ — the extrinsic LLR of $b_k(i)$ provided by the multiuser detector

$\Lambda_2[b_k(i)]$ — *a posteriori* LLR of $b_k(i)$ provided by the *k*th user's decoder

$\lambda_2^P[b_k(i)]$ — *a priori* LLR of $b_k(i)$ provided by *k*th user's decoder in the previous iteration

$\lambda_2^E[b_k(i)]$ — the extrinsic LLR of $b_k(i)$ provided by the *k*th user's decoder

$\mathbf{\Omega}_k$ — covariance of $b_k$ and $\hat{b}_k$

$\Phi(\cdot)$ — multiuser detector function

# Chapter 1
# Introduction

## 1.1 Background and Motivation

Multiuser Detection (MUD) has been studied extensively over the last decade [1]-[4]. It concerns the demodulation of one or more digital signals in the presence of multiuser interference. The need for such techniques arises notably in wireless communication channels, in which either intentional non-orthogonal signaling (e.g., Code Division Multiple Access (CDMA)) or non-ideal channel effects (e.g., multipath) lead to received signals from multiple users that are not orthogonal to one another. An optimal maximum likelihood (ML) multiuser detector is proposed by Sergio Verdu [12]. Now it is widely accepted as the optimal multiuser detector. It is optimal in the sense that the detector yields the minimum achievable probability of bit error. Unfortunately, this performance comes at the cost of high computational complexity which makes the receiver unrealizable for practical applications. Due to the complexity of the optimal receiver, a number of reduced complexity receivers that perform a linear transformation were developed. The suboptimum linear receivers such as the decorrelating and the minimum mean squared error (MMSE) receivers have been proposed to trade off the complexity and performance among the conventional and optimal receivers. A suitable remedy to alleviate the computational complexity is to apply interference cancellation (IC), which is performed in an iterative structure. An IC-based detector uses tentative information from other users to construct a new estimate by subtracting the MAI estimate. For uncoded MUD system, IC can be performed in parallel leading to a parallel interference cancellation (PIC) and successive interference cancellation (SUIC) [12].

The receiver structures discussed above generally ignore channel coding and many of these techniques provide hard decision outputs for channel decoding. In order to obtain lower bit error rate, error control coding can be applied to MUD. By viewing a synchronous CDMA channel as a block code, an iterative decoding scheme that exchanges soft information between the multiuser detector and the forward error control (FEC) decoders can be employed. For coded CDMA systems, the individual optimum multiuser detector determines the *a posteriori* probabilities (APP) for the code bits, which then passed on to the single-user decoders [19][20][25]. Each user decodes the corresponding FEC codes independently using a full complexity APP algorithm. With iterative decoding, the performance of the coded multiuser system approaches single user performance at moderate to high signal-to-noise ratio. Low-density parity-check (LDPC) codes are a class of linear block codes which provide near capacity performance on a large collection of data transmission and storage channels while simultaneously admitting implementable decoders. LDPC codes are first introduced by Gallager [5]. He also proposed an iterative decoding algorithm which results in APP decoding in cycle-free graphs. The algorithm used to decode LDPC codes works by exchanging information between variable nodes and check nodes iteratively. The detail of the decoding algorithm will be presented in the next chapter. The iterative decoding algorithm makes LDPC codes ideal for iterative coded MUD.

Over the past decade, it has been demonstrated that various multiuser detection schemes can offer significant gain in MAI suppression over conventional techniques [12]. Since error control coding is applied in most practical CDMA systems, recent research has addressed multiuser detection for coded CDMA systems [19][20][22]. With the results on iterative multiuser detection appeared, much research has been done in this area. Most work on the iterative multiuser detection considers convolutional codes [19][22][34]. Although some research work focuses on LDPC codes [26], only soft cancellation and MMSE detector are used in the iterative multiuser detector. One of the primary discussions in this thesis is regarding the design and analysis of an iterative multiuser receiver. In this thesis, the LDPC decoder and the soft-input soft-output (SISO) multiuser detector are employed to construct the iterative receiver. Following the soft interference cancellation, the decorrelating detector and linear minimum mean square error (MMSE) detector are provided to reduce the residual interference respectively. Also, the comparison of three types of receivers is provided.

## 1.2 Thesis Outline

This thesis consists of six chapters including the introduction and conclusion chapters. The focuses of the four intermediate chapters are listed as follows:

**Chapter 2** briefly introduces low-density parity-check codes fundamentals. First, linear block code are presented. Following this, LDPC codes and Tanner graph are introduced. Then, encoding and decoding algorithm are provided.

**Chapter 3** reviews the multiuser detection theory. Single user matched filter and optimal multiuser detection are first given. Then, three linear multiuser detectors and interference cancellation based multiuser detector are introduced. Furthermore, at the end of this chapter, we provide the current literature about coded multiuser detection.

**Chapter 4** proposes coded multiuser detection system model. A general iterative multisuer detection and decoding structure for LDPC-coded multiuser system is first introduced. Following this is the iterative receiver and the proposed iterative decorrelating detection algorithm is described. Finally, the Soft-Input Soft-Output (SISO) linear Minimum Mean Square Error (MMSE) detector is presented.

**Chapter 5** shows the simulation results and the corresponding discussion. Simulation results for soft interference cancellation receiver and SISO decorrelating detector are first presented. Then, results for SISO decorrelating receiver and SISO MMSE receiver are given.

**Chapter 6** concludes the thesis and discusses the probability of further research.

## 1.3 Thesis Contributions

The following are the main contributions of this thesis:

- Iterative MMSE-based joint LDPC decoding and soft interference detection is presented. The LDPC decoder is improved to yield the log-likelihood ratio (LLR) of parity bits. These LLRs are passed to the MMSE detector which users this information to reduce the residual interference.
- Performance comparison of three types of mutiuser detectors is provided. These detectors are the soft interference cancellation detector, the SISO decorelating detector and the SISO MMSE detector.
- Based on the simulation results of LDPC coded multiuser detection, we can see that the both SISO decorrelator and SISO MMSE detector have much better performance than the soft interference cancellation detector. When there are more users in the system and the cross correlation is high, SISO MMSE detector converges faster towards the single user system performance compared to the other two detectors.

# Chapter 2
# Low Density Parity Check Codes

Low Density Parity Check (LDPC) codes are a class of linear block codes. The name comes from the characteristic of their parity-check matrix which contains only a few 1's in comparison to the number of 0's. Their main advantage is that they provide a performance which is very close to the Shannon limit while maintaining a low decoding complexity compared to other iteratively decoded codes.

LDPC codes were first introduced by Gallager in his PhD thesis in 1960 and were scarcely considered in the 35 years that followed. One of the exceptions was Shu Lin's paper in 1972 [7]. Another notable exception is the important work of Tanner in 1981 [6] in which Tanner generalized LDPC codes and introduced a graphical representation of LDPC codes, now called Tanner graphs. The study of LDPC codes was resurrected in the mid-1990's with the work of MacKay [8][9], who noticed the advantages of linear block codes which possess sparse (low-density) parity-check matrices. It is shown that carefully designed irregular low density parity check codes can have near Shannon-Limit performance on memoryless channels [17][18].

## 2.1 Linear Block Codes

Since LDPC codes are a special case of linear block codes, in this section, we will provide an overview of this class of codes to provide the necessary background needed to understand LDPC encoding and decoding.

A block code consists of a set of fixed-length vectors called codewords. The length of codeword is the number of elements in the vector and is denoted by $n$. The elements of a code word are selected from an alphabet of $q$ symbols. When the alphabet consists of two symbols, 0 and 1, the code is a binary code and the elements of any code word are called bits. In this thesis, we only consider binary codes. There are $2^n$ possible codewords in a binary block code of length $n$. From these $2^n$ code words, we may select $M = 2^k$ code words ($k<n$) to form a code. Thus, a block of $k$ information bits is mapped into a code word of length $n$ selected from the set of code words. It can be characterized as an $(n,k)$ code, and the ratio $\frac{k}{n} \equiv R_c$ is defined to be the rate of the code.

### 2.1.1 Generator Matrix

Let $m_1, m_2, \ldots, m_k$ denote the $k$ information bits to be encoded into the code word **C**. Thus the vector of $k$ information bits into the encoder is denoted by

$$\mathbf{m} = [m_1, m_2, \ldots, m_k]$$

and the output of the encoder is the vector

$$\mathbf{C} = [c_1, c_2, \ldots, c_n]$$

The encoding operation performed in a linear binary block encoder can be represented by a set of $n$ equations of the form

$$c_j = m_1 g_{1j} + m_2 g_{2j} + \cdots + m_k g_{kj}, \quad j = 1,2,\cdots,n \tag{2.1}$$

where $g_{ij} = 0$ or 1. The linear equations (2.1) may also be represented in a matrix form as

$$\mathbf{C} = \mathbf{mG} \tag{2.2}$$

where **G**, called the generator matrix of the code, is

$$\mathbf{G}=\begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \\ \mathbf{g}_4 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & & & \vdots \\ g_{k1} & g_{k2} & \cdots & g_{kn} \end{bmatrix} \tag{2.3}$$

The problem of recovering the data block from a codeword can be greatly simplified through the use of systematic encoding. Consider a linear code **C** with generator matrix **G**. Using Gaussian elimination and column reordering, it is always possible to obtain a generator matrix of the form shown below.

$$\mathbf{G}=[\mathbf{P} \mid \mathbf{I_k}]=\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{,(n-k)1} & 1 & 0 & \cdots & 0 \\ p_{21} & p_{22} & \cdots & p_{2,(n-k)} & 0 & 1 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{k,(n-k)} & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{2.4}$$

where **P** is the parity array portion of the generator matrix $p_{ij}$=(0 or 1), and $\mathbf{I_k}$ is the $k \times k$ identity matrix. This can be proved by noting that the rows of a generator matrix are linearly independent and that the column rank of the matrix is equal to the row rank.

### 2.1.2 Parity Check Matrix

For a given linear block code with generator **G**, there exists another matrix **H** which is called the parity check matrix. This matrix enables us to decode the received vectors. For each $(k \times n)$ generator matrix **G**, there exists an $(n-k)\times n$ matrix **H**, such that the rows of **G** are orthogonal to the rows of **H**; that is, $\mathbf{GH}^T = \mathbf{0}$, where $\mathbf{H}^T$ is the transpose of **H**, and **0** is $k \times (n-k)$ all-zero matrix. Given a systematic generator matrix, the corresponding parity check matrix can be obtained as shown below:

$$\mathbf{H} = \left[\mathbf{I}_{n-k} \mid \mathbf{P}^T\right] = \begin{bmatrix} 1 & 0 & \cdots & 0 & p_{11} & p_{21} & \cdots & p_{k1} \\ 0 & 1 & \cdots & 0 & p_{12} & p_{22} & \cdots & p_{k2} \\ \vdots & & & & & & & \vdots \\ 0 & 0 & \cdots & 1 & p_{1,(n-k)} & p_{2,(n-k)} & \cdots & p_{k,(n-k)} \end{bmatrix} \tag{2.5}$$

Once the parity-check matrix **H** is constructed to fulfill the orthogonality requirement, we can use it to test whether a received vector is a valid member of the codeword set. **C** is a codeword generated by matrix **G** if and only if $\mathbf{CH}^T = \mathbf{0}$.

## 2.2 Low Density Parity Check Codes

Low Density Parity Check (LDPC) codes are a class of linear block codes corresponding to the parity check matrix **H**. Parity check matrix $H_{(N-K)\times N}$ consists of only zeros and ones and is very sparse which means that the density of ones in this matrix is very low.

A regular LDPC matrix is an $M \times N$ binary matrix having exactly $w_c$ ones in each column and exactly $w_r$ ones in each row, where $w_c < w_r$ and both are small compared to $N$. By this definition, every parity-check equation of a regular LDPC code involves exactly $w_r$ bits, and every bit is involved in exactly $w_c$ parity-check equations. The restriction $w_c < w_r$ is needed to ensure that more than just the all-zero codeword satisfies all of the constraints, or equivalently, to ensure a nonzero code rate. The code rate *R=1-M/N* is then $R=1-w_c/w_r$.

## 2.3 Tanner Graph

Any parity-check code (including an LDPC code) may be specified by a Tanner graph, which is essentially a visual representation of the parity check matrix **H**. Recall that an $M \times N$ parity-check matrix **H** defines a code in which the *N* bits of each codeword satisfy a set of *M* parity-check constraints. The Tanner graph contains *N* "variable" nodes, one for each bit, and *M* "check" nodes, one for each of the parity checks. The variable nodes are depicted using circles, while the check nodes are depicted using squares. The check nodes are connected to the variable nodes that they check. Specifically, a branch connects check node *m* to variable node *n* if and only if the *m*th parity check involves the *n*th bit, or in other words, if and only if $H_{m,n} = 1$. The graph is said to be bipartite because there are two distinct types of nodes, variable nodes and check nodes, and there can be no direct connect between any two nodes of the same type. Figure 2.1 shows a Tanner graph made for a simply parity check matrix **H**. The dotted path $c_2 \to f_1 \to c_5 \to f_2 \to c_2$ is an example for a short cycle. Those should usually be avoided since they are bad for decoding performance. We will cover more details later in Section 2.5.

$$H = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

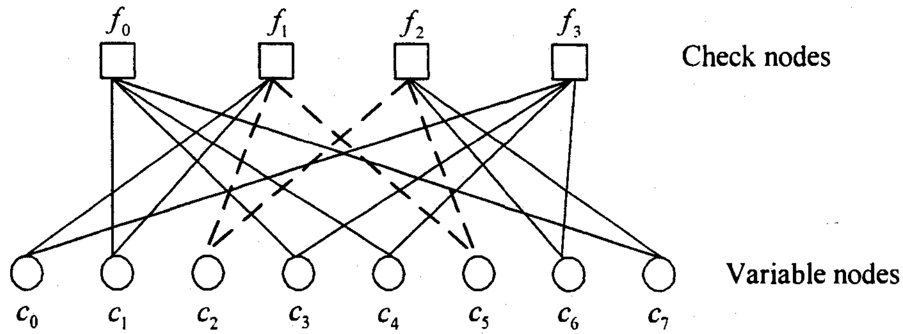


Figure 2.1 Tanner graph of a parity check matrix

## 2.4 LDPC Code Design

The parity check matrix plays a major role in the performance. Therefore, the most obvious path to the construction of an LDPC code is via the construction of a low-density parity-check matrix with prescribed properties. A large number of design techniques exist in the literature [41]. Here, we will list ways to generate a sparse matrix **H**.

### 2.4.1 Gallager Codes

Gallager [5] designed the regular LDPC codes with an **H** matrix of the form

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{w_c} \end{bmatrix} \tag{2.6}$$

where the submatrix $\mathbf{H}_1$ have the structure outline as follows.

$$\mathbf{H}_1 = \begin{bmatrix} \underbrace{111\ldots1}_{w_r} & & & \\ & \underbrace{111\ldots1}_{w_r} & & \\ & & \ddots & \\ & & & \underbrace{111\ldots1}_{w_r} \end{bmatrix}$$

The other submatrix are simply column permutations of $\mathbf{H}_1$. It is evident that $\mathbf{H}$ is regular, and has row and column weights $w_r$ and $w_c$ respectively. A proper choice of the permutations will allow the minimum distance of the code by $\mathbf{H}$ to increase beyond two. Gallager proved that a totally random choice will on average produce an excellent code [5]. He also showed that the ensemble of such codes has excellent distance properties provided $w_c>3$, and $w_r>w_c$. Further, such codes have low-complexity encoders since parity bits can be computed as a function of the user bits via the parity-check matrix[5].

Gallager codes were generalized by Tanner in 1981 [6] and were studied for application in code-division multiple-access communication systems. Gallager codes were extended by MacKay and others [8][9].

### 2.4.2 MacKay Codes

MacKay was the first to show that LDPC codes perform near capacity limits [8][9]. A large number of LDPC codes designed by MacKay can be found in [10] most of which are regular. In [9], MacKay provides algorithms which semi-randomly generate sparse **H** matrix. A few of these are listed below:

1. **H** is created by randomly generating weight-$w_c$ columns and uniform row weight.
2. **H** is created by randomly generating weight-$w_c$ columns, while ensuring weight-$w_r$ rows, while maintaining that the sum of two columns never has a weight greater than one.
3. **H** is generated as in 2, while avoiding short cycles.
4. **H** is generated as in 3, with the additional constraint that for $\mathbf{H} = [\mathbf{H}_1 \quad \mathbf{H}_2]$, $\mathbf{H}_2$ is invertible (or at least **H** is full rank).

In this thesis, the LDPC code parity check matrix **H** used is from MacKay's website [10]. Due to high complexity and latency associated with long LDPC codes, we chose the rather short (204,102) Gallager code with column weight $w_c = 3$ and row weight $w_r = 6$ to keep the simulation times reasonable. Therefore, the code rate $R$=1/2. Furthermore, we need to specify the generator matrix **G** for the encoder. The way of doing this is to reduce **H** to systematic form using Gaussian elimination and some column reordering. Radford M. Neal has created software to do the LDPC encoding using parity check matrix **H** [40]. In this thesis, we use the software from [40] to do the encoding.

## 2.5 Decoding

In addition to introducing LDPC codes, Gallager also proposed an optimal way to decode them [5]. He introduced an iterative decoding algorithm which results in *a posteriori probability* (APP) decoding on cycle-free graphs. Since that time however, many new algorithms have come to light. The names of the algorithms include: the sum-product algorithm (SPA), the belief propagation algorithm (BPA), and the message passing algorithm (MPA). The term "message passing" usually refers to all such iterative algorithms, including the SPA, BPA and its approximations. The message passing algorithm works by exchanging information between variable nodes and check nodes. More details will be presented in the next subsections.

### 2.5.1 Background and Notations

Given the received word $\mathbf{y} = [y_0\, y_1 \cdots y_{n-1}]$, we need to compute the *a posteriori* probability (APP) that a given bit in the transmitted codeword $\mathbf{c} = [c_0\, c_1 \cdots c_{n-1}]$ equals 1. The APP of *i*th bit is $\Pr(c_i = 1 \mid \mathbf{y})$. The log-APP ratio, also called the log-likelihood ratio (LLR) can be expressed as

$$L(c_i) = \log\left(\frac{\Pr(c_i = 1 \mid \mathbf{y})}{\Pr(c_i = 0 \mid \mathbf{y})}\right)$$

where here and in the rest of the thesis the natural logarithm is assumed.

The decoding algorithm for the computation of $\Pr(c_i = 1 \mid \mathbf{y})$ or $L(c_i)$ is an iterative algorithm which is based on the code's Tanner graph. It consists of two parts: check nodes update and variable nodes updates. Consider a cycle-free Tanner graph with subgraph as Figure 2.2. The variable node $c_0$ collects its input messages and passes to check node $f_2$. The information passed concerns $\Pr(c_0 = b \mid \text{input messages}), b \in \{0,1\}$ or the logarithm of the ratio of such probabilities.

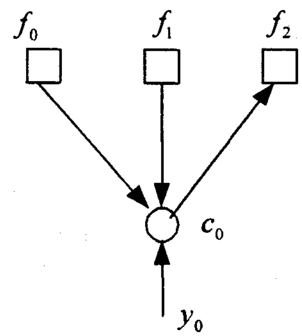


Figure 2.2 Message passing from node $c_0$ to node $f_2$

On the other hand, each check node processes its input messages and passes its resulting output messages down to its neighboring variable nodes. This is depicted in Figure 2.3 for the message from check node $f_0$ down to variable node $c_4$. The information passed concerns Pr(check equation $f_0$ is satisfied | input messages), $b \in \{0,1\}$ or the logarithm of the ratio of such probabilities.

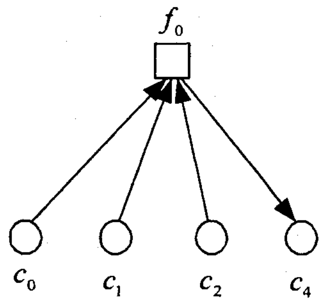


Figure 2.3 Message passing from node $f_0$ to node $c_4$

There are two ways to stop the iterative decoding. One is that the maximum number of iterations has been reached. The other is when $\hat{\mathbf{c}}\mathbf{H}^T = \mathbf{0}$, where $\hat{\mathbf{c}}$ is a tentatively decoded codeword.

Here we introduce the following notation:

- $V_j$ ={ variable nodes connected to check node $f_j$ }
- $V_{j\backslash i}$ ={ variable nodes connected to check node $f_j$, except variable node $c_i$ }
- $C_i$ ={ check nodes connected to variable node $c_i$ }
- $C_{i\backslash j}$ ={ check nodes connected to variable node $c_i$, except check node $f_j$ }
- $P_i = \Pr(c_i = 1 \mid y_i)$
- $S_i$ = true if $\hat{\mathbf{c}}$ satisfies all parity-check equations involving $c_i$,

- $q_{ij}(b)$= message passed from variable node $c_i$ to check node $f_j$. It is the probability that $c_i = b$ given the channel sample $y_i$ and all check node messages except $f_j$.

- $r_{ji}(b)$= message passed from check node $f_j$ to variable node $c_i$. It is the probability that check equation $j$ is satisfied, given that $c_i = b$ and that the other bits are distributed according to $\{q_{ij'}\}_{j' \neq j}$.

### 2.5.2 Probability-Domain Decoding

Initially, the variable node $c_i$ only has information from the channel. It passes this information to all associated check nodes $\{f_j\}_{j \in C_i}$. A check node is satisfied if the sum of all associated $c_i$ is zero, or in other words that there is an even number of "ones".

When creating the message $r_{ji}(0)$ which is to be sent back to variable node $c_i$, the check node determines the probability that there is an even number of "ones" in the other associated bits $V_{j\backslash i}$. We may express $q_{ij}(0)$ (see Figure 2.4)

$$q_{ij}(0) = \Pr(c_i = 0 \mid y_i, S_i, \text{message from all variable nodes except node } f_j)$$

$$= (1 - P_i)\Pr(S_i \mid c_i = 0, y_i, \text{message from all variable nodes except node } f_j) / \Pr(S_i)$$

$$= K_{ij}(1 - P_i)\prod_{j' \in C_{i\backslash j}} r_{j'i}(0) \quad (2.7)$$

where we used Bayes' rule twice to obtain the second line and the independence assumption to obtain the third line. Likewise when creating $r_{ji}(1)$, it determines the probability that there is an odd number of "ones".

$$q_{ij}(1) = K_{ij} P_i \prod_{j' \in C_{i \setminus j}} r_{j'i}(1) \tag{2.8}$$

The constants $K_{ij}$ are chosen to ensure $q_{ij}(0) + q_{ij}(1) = 1$.

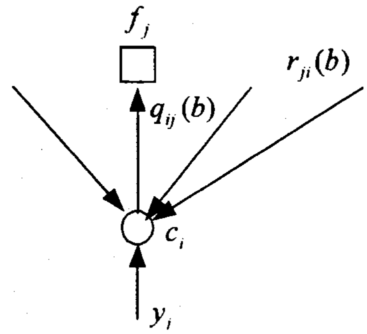


Figure 2.4 Calculation of $q_{ij}(b)$

From Gallager's paper [5], we have the following result:

(*Result*) Consider a sequence of $M$ independent binary digits $a_i$ for which $\Pr(a_i = 1) = p_i$. Then the probability that $\{a_i\}_{i=1}^{M}$ contains an *even* number of 1's is

$$\frac{1}{2} + \frac{1}{2}\prod_{i=1}^{M}(1 - 2p_i) \tag{2.9}$$

In view of this result and Figure 2.5, we have

$$r_{ji}(0) = \frac{1}{2} + \frac{1}{2}\prod_{i' \in V_{j\setminus i}} (1 - 2q_{i'j}(1)) \quad (2.10)$$

since when $c_i$=0, the bits $\{c_{i'} : i' \in V_{j\setminus i}\}$ must contain an even number of 1's in order for check equation $f_j$ to be satisfied. Clearly,

$$r_{ji}(1) = 1 - r_{ji}(0) \quad (2.11)$$

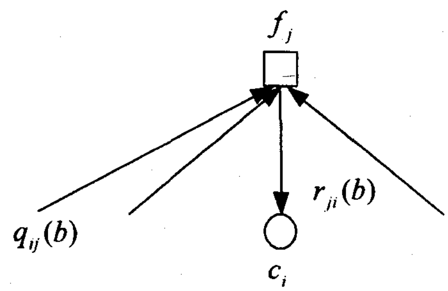


Figure 2.5 Calculation of $r_{ji}(b)$

Thus, the algorithm works as follows:

1. Initialization:

   For $i = 0, 1, \cdots, n-1$, set $P_i = \Pr(c_i = 1 \mid y_i)$, where $y_i$ is the $i$th received channel symbol. Then set $q_{ij}(0) = 1 - P_i$ and $q_{ij}(1) = P_i$ for all $i, j$ for which $h_{ij} = 1$.

2. Computation of $r_{ji}$:

$$r_{ji}(0) = \frac{1}{2} + \frac{1}{2}\prod_{i' \in V_{j\setminus i}} (1 - 2q_{i'j}(1))$$

$$r_{ji}(1) = 1 - r_{ji}(0)$$

3. Computation of $q_{ji}$:

$$q_{ij}(0) = K_{ij}(1 - P_i) \prod_{j' \in C_{i \setminus j}} r_{j'i}(0)$$

$$q_{ij}(1) = K_{ij} P_i \prod_{j' \in C_{i \setminus j}} r_{j'i}(1)$$

where $K_{ij}$ is a constant used to ensure that $q_{ij}(0) + q_{ij}(1) = 1$.

4. Computation of $Q_i$:

For all $i$:

$$Q_i(0) = K_i(1 - P_i) \prod_{j \in C_i} r_{ji}(0)$$

$$Q_i(1) = K_i P_i \prod_{j \in C_i} r_{ji}(1)$$

where $K_i$ is a constant used to ensure that $Q_i(0) + Q_i(1) = 1$.

5. Computation of the current best estimates for $c_i$, denoted as $\hat{c}_i$:

$$\hat{c}_i = \begin{cases} 0 & \text{if } Q_i(0) > 0.5 \\ 1 & \text{otherwise} \end{cases}$$

If the new $\hat{\mathbf{c}}$ satisfies $\hat{\mathbf{c}}\mathbf{H}^T = \mathbf{0}$ or the maximum number of iteration is reached, then stop. Else, return to Step 2.

### 2.5.3 Log-Domain Decoding

Use of the algorithm described in Section 2.5.2 is computationally intense, due to the large number of multiplications. Thus to improve performance and speed, a Log-Domain version is commonly used. This also reduces the number of messages that need to be sent along the edges of the graph. Log-Domain algorithm is used to do the LDPC decoding in this thesis. Instead of sending $c_i(0)$ and $c_i(1)$, the LLR of $c_i$ is sent, where the LLR of $c_i$ is

$$L(c_i) = \log\frac{\Pr(c_i = 1 \mid y_i)}{\Pr(c_i = 0 \mid y_i)} \tag{2.12}$$

Similarly, the following LLRs are used as well:

$$L(r_{ji}) = \log(\frac{r_{ji}(1)}{r_{ji}(0)})$$

$$L(q_{ij}) = \log(\frac{q_{ij}(1)}{q_{ij}(0)}) \tag{2.13}$$

$$L(Q_i) = \log(\frac{Q_i(1)}{Q_i(0)})$$

For Step 1, we first replace $r_{ji}(0)$ with $1 - r_{ji}(1)$ in (2.11) and rearrange it to obtain

$$1 - 2r_{ji}(1) = \prod_{i' \in V_{j\backslash i}} (1 - 2q_{i'j}(1))$$

Now using the fact that $\tanh(\frac{1}{2}\log(\frac{p_0}{p_1})) = p_0 - p_1 = 1-2p_1$, we may rewrite the equation above as

$$\tanh(\frac{1}{2}L(r_{ji})) = -\prod_{i' \in V_{j\backslash i}} \tanh(-\frac{1}{2}L(q_{i'j})) \tag{2.14}$$

Therefore, we have the following algorithm:

1. Initialization:

   For $i = 0,1,\cdots,n-1$, initialize $L(q_{ij})$ according to (2.13). For BI-AWGN channel, we have $L(q_{ij}) = L(c_i) = \frac{2y_i}{\sigma^2}$ [41], where $\sigma^2$ is the noise variance.

2. Compute of $L(r_{ji})$:

$$L(r_{ji}) = -2\tanh^{-1}(\prod_{i' \in V_{j\backslash i}} \tanh(-\frac{1}{2}L(q_{i'j})) \tag{2.15}$$

3. Compute of $L(q_{ij})$:

$$L(q_{ij}) = L(c_i) + \sum_{j' \in C_{i\backslash j}} L(r_{j'i}) \tag{2.16}$$

4. Compute of $L(Q_i)$:

$$L(Q_i) = L(c_i) + \sum_{j \in C_i} L(r_{ji}) \tag{2.17}$$

5. Computation of the current best estimates for $c_i$, denoted as

$$\hat{c}_i = \begin{cases} 1 & \text{if } L(Q_i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

If the new $\hat{\mathbf{c}}$ satisfies $\hat{\mathbf{c}}\mathbf{H}^T = \mathbf{0}$ or the maximum number of iteration is reached, then stop. Else, return to Step 2.

# Chapter 3
# Multiuser Detection

Multiuser detection (MUD) is used to reduce or eliminate the multiple access interference (MAI) that is present in code-division multiple access (CDMA) systems. To facilitate the reader's introduction to MUD, the examples will be based on a synchronous CDMA model in the non-fading AWGN channel with BPSK modulation. The purpose of this thesis is to combine MUD and LDPC decoding techniques. Hence this chapter focuses on different multiuser detection techniques.

## 3.1 Synchronous CDMA System

The term *multiple access communication* system describes a system that uses a common communication channel on which several transmitters simultaneously transmit information. Multiple access communication is widely used in different communication systems, especially in mobile and satellite communications. The signal sources in a multiple access channel are referred to as users. The multiple access communication scenario is depicted in Figure 3.1.

For a synchronous $K$-user multiuser system with binary phase-shift keying (BPSK) modulation, the received signal $r(t)$ can be expressed as

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), \quad t \in [0,T] \tag{3.1}$$

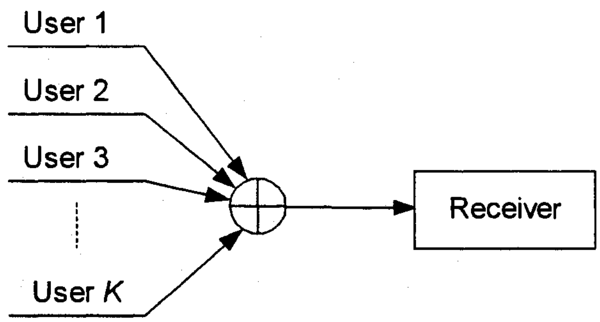


Figure 3.1 Multiple Access Communication System

where $T$ is the symbol interval, $A_k$ is the received amplitude of the $k$th user's signal, $b_k \in \{-1,+1\}$ is the BPSK modulated symbol of the $k$th user, $s_k(t)$ is the energy normalized waveform assigned to the $k$th user and is assumed to be zero outside the interval [0,T], and $n(t)$ has double-sided noise spectral density of $\frac{N_0}{2}$. Once it passes through the matched filter, it becomes a filtered sample with variance $\sigma^2$. In the receiver, we use a bank of matched filters which are matched to multiple users' waveforms, respectively. The output $y_k$ of the $k$th matched filter is

$$y_k = \int_0^T r(t)s_k(t)dt \tag{3.2}$$

Then the output vector y of a bank of matched filters can be expressed as

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{3.3}$$

where $\mathbf{R} = \{\rho_{ij}\}$ is the normalized cross-correlation matrix with diagonal elements equal to 1,

$$\rho_{ij} = \int_0^T s_i(t)s_j(t)dt,$$

$$\mathbf{y} = [y_1,...,y_K]^T,$$

$$\mathbf{b} = [b_1,...,b_K]^T,$$

$$\mathbf{A} = diag[A_1,...,A_K],$$

and $\mathbf{n}$ is a zero-mean Gaussian random vector with covariance matrix equal to [25]

$$E[\mathbf{n}\mathbf{n}^T] = \sigma^2\mathbf{R} \tag{3.4}$$

## 3.2 Matched Filter

The simplest approach to demodulate CDMA signals is with a single-user *matched filter* (MF). This is the demodulator that was first adopted in CDMA receivers. It is called the conventional detector. For the single-user CDMA channel, the MF is the optimal receiver. However, in the multiuser CDMA channel, the MF of the desired user will suffer from interference from other users. This is termed multiple access interference (MAI). The structure of CDMA MF is illustrated in Figure 3.2.

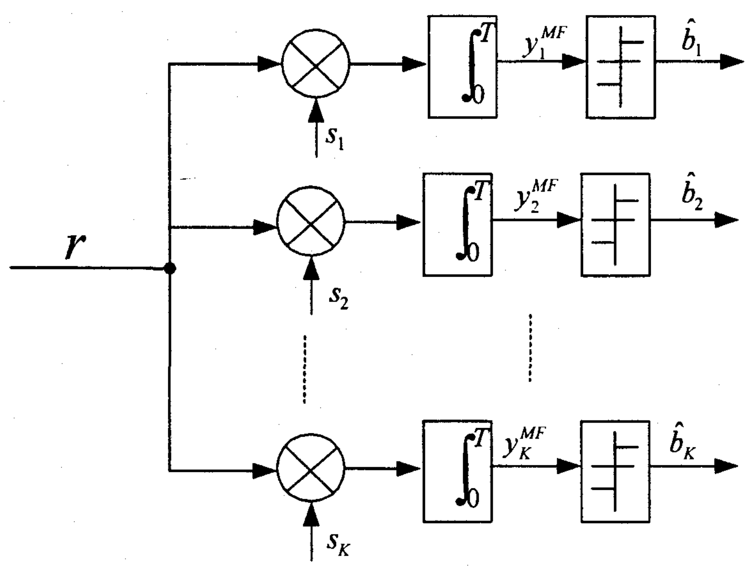


Figure 3.2: Matched Filter for CDMA Systems

The filter for a given user is matched to its signature waveform.

$$y_k^{MF} = \int_0^T r(t)s_k(t)dt = A_k b_k + \sum_{j \neq k} A_k \rho_{jk} b_j + n_k \tag{3.5}$$

where

$$n_k = \int_0^T s_k(t)n(t)dt \tag{3.6}$$

and

$$\rho_{jk} = \int_0^T s_j(t)s_k(t)dt \tag{3.7}$$

is the crosscorrelation between the *j*th and *k*th user. Assuming that hard decision are made on the received data, the output of the conventional detector is

$$\hat{b}_k = \text{sgn}(y_k^{MF}) = \text{sgn}(A_k b_k + \sum_{j \neq k} A_k \rho_{jk} b_j + n_k^{'}) \tag{3.8}$$

where $\sum_{j \neq k} A_k \rho_{jk} b_j$ is the MAI introduced in the *k*th user's signal. From (3.8), it is observed that when the absolute value of the sum of noise and MAI is bigger than that of the data, the possibility of an error arises. Hence the MF is sub-optimal for multiuser CDMA systems.

## 3.3 Optimal Multiuser Detection

The optimal multiuser detector was first proposed by Sergio Verdu[12]. Based on the criterion of maximum likelihood estimation he proposed a cost function, the maximization of which leads to jointly optimum demodulation of all users.

Consider the simple 2-user case

$$r(t) = A_1 b_1 s_1(t) + A_2 b_2 s_2(t) + n(t), \quad 0 \leq t \leq T \tag{3.9}$$

The optimum estimate of $b_1$ will minimize the probability of error and is obtained by choosing $\hat{b}_1 \in \{-1,+1\}$ such that the aposteriori probability $P(b_1 = \hat{b}_1 \mid r(t))$ is maximized. Similarly for user 2, i.e. we need to choose $\hat{b}_2$ such that $P(b_2 = \hat{b}_2 \mid r(t))$ is maximized. This detector can be term as an individually optimum multiuser detector.

The individually optimum detector is not optimum since $\hat{b}_1$ and $\hat{b}_2$ are not independent conditioned on the received signal *r(t)*. Thus, we need to maximize the joint *a posteriori* probability

$$P(b_1 = \hat{b}_1, b_2 = \hat{b}_2 \mid r(t))$$

This detector can be termed as a jointly optimum detector. This is also the globally optimum multiuser detector. Now consider the general *K*-user case

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), \quad t \in [0,T]$$

For equal *a priori* probabilities of all $\mathbf{b} = [b_1,...,b_K]^T$, maximizing $P(\mathbf{b} = \hat{\mathbf{b}} \mid r(t))$ is equivalent to maximizing

$$f(r(t), 0 \le t \le T \mid \mathbf{b} = \hat{\mathbf{b}}) = \exp(-\frac{1}{2\sigma^2} \int_0^T [r(t) - \sum_{k=1}^{K} A_k \hat{b}_k s_k(t)]^2 dt) \tag{3.10}$$

That is, choosing $\hat{\mathbf{b}}$ such that $\sum_{k=1}^{K} A_k \hat{b}_k s_k(t)$ is closest to the received signal in the mean square sense. There are many combinatorial optimization problems for which there exist algorithms whose complexity grows polynomially in the size of the problem, and are thus more efficient than exhaustive search. Optimal multiuser detection however has a complexity which grows exponentially as the function of the number of users [11]. Although the optimal multi-user detector offers significant improvement in the performance of DS-CDMA systems, its high computational complexity renders it impractical for real time implementation. Research in the field of MUD thus focuses on developing algorithms which provide significantly less computational complexity, albeit at the cost of sub-optimal performance.

## 3.4 Linear Multiuser Detection

Linear multi-user detectors are an important class of suboptimal multi-user detectors [2]. These detectors apply a linear mapping to the soft output of the conventional detector to reduce the MAI seen by each user. The result of the linear transformation is known as the decision variable and is used to make a decision on the transmitted bits. Three linear multi-user detectors are popular in literature: decorrelating detector, minimum mean square error (MMSE) detector and polynomial expansion detector.

### 3.4.1 Decorrelating Detector

This detector applies the inverse of the correlation matrix to the soft output of the conventional detector. The correlation matrix is defined such that its $(i,j)$th entry is given by the correlation between the signature sequences of the $i$th and the $j$th user. Such a transformation perfectly decouples the data of different users. The detector does not need to estimate received power levels of different users and detection of each user can be implemented independent of other users. However, this detector leads to noise enhancement, which in turn leads to higher bit error rates (BER). The structure of CDMA MF is illustrated in Figure 3.3.

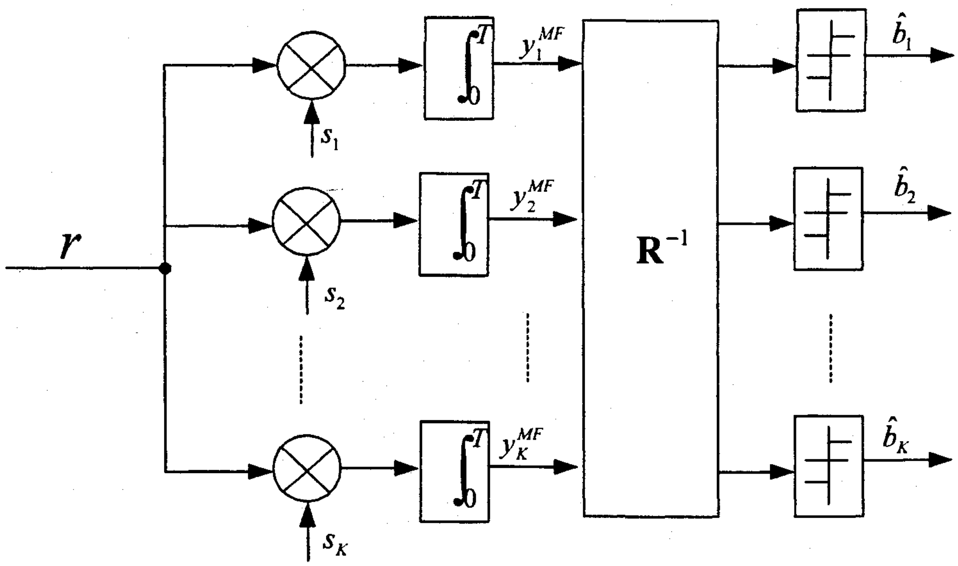


Figure 3.3 Decorrelating Detector for CDMA System

The output of the decorrelating detector is given by

$$\hat{\mathbf{b}} = \text{sgn}(\mathbf{R}^{-1}(\mathbf{RAb}) + \mathbf{n})) = \text{sgn}(\mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n}) \tag{3.11}$$

When the background noise is absent,

$$\hat{\mathbf{b}} = \text{sgn}(\mathbf{Ab})$$

i.e.,

$$\hat{\mathbf{b}} = \mathbf{b}$$

Hence, we observe that in the absence of background noise the decorrelating detector achieves perfect demodulation unlike the matched filter bank. Consider the two user case with $\mathbf{R} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, the decoded bits are

$$\begin{aligned}\hat{\mathbf{b}} &= \mathrm{sgn}(\mathbf{R}^{-1}\mathbf{y}) \\ &= \mathrm{sgn}\left(\frac{1}{1-\rho^2}\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right) \\ &= \mathrm{sgn}\left(\begin{bmatrix} \frac{1}{1-\rho^2}y_1 - \frac{\rho}{1-\rho^2}y_2 \\ -\frac{\rho}{1-\rho^2}y_1 + \frac{1}{1-\rho^2}y_2 \end{bmatrix}\right)\end{aligned}$$

We see that the decorrelating receiver performs only linear operation on the received static $\mathbf{y}$ and hence it is indeed a linear detector.

### 3.4.2 MMSE Detector

The MMSE detector, which can be regarded as an improved decorrelating detector, can solve the problem of noise enhancement in low signal-to-noise ratio (SNR). With MMSE algorithm, the mean squared error between the output and data is minimized. If the MMSE detector is expressed as

$$\mathbf{y}^{MMSE} = \mathbf{M}\mathbf{y}^{MF} \tag{3.12}$$

The problem of MMSE detection is choosing the $K \times K$ matrix $\mathbf{M}$ to achieve the minimum square-error:

$$E[\| \mathbf{Ab} - \mathbf{M}\mathbf{y}^{MF} \|^2] \tag{3.13}$$

In [12], they show that the solution of (3.13) is given by:

$$\mathbf{M} = [\mathbf{R} + \frac{\sigma^2}{\mathbf{A}^2}]^{-1} \tag{3.14}$$

where

$$\frac{\sigma^2}{\mathbf{A}^2} = diag\left\{\frac{\sigma^2}{A_1^2}, \frac{\sigma^2}{A_2^2}, \cdots, \frac{\sigma^2}{A_K^2}\right\} \tag{3.15}$$

From the equation above, we can see that the receiver needs to know the amplitudes of all users. The structure of the MMSE detector can be depicted in Figure 3.4.

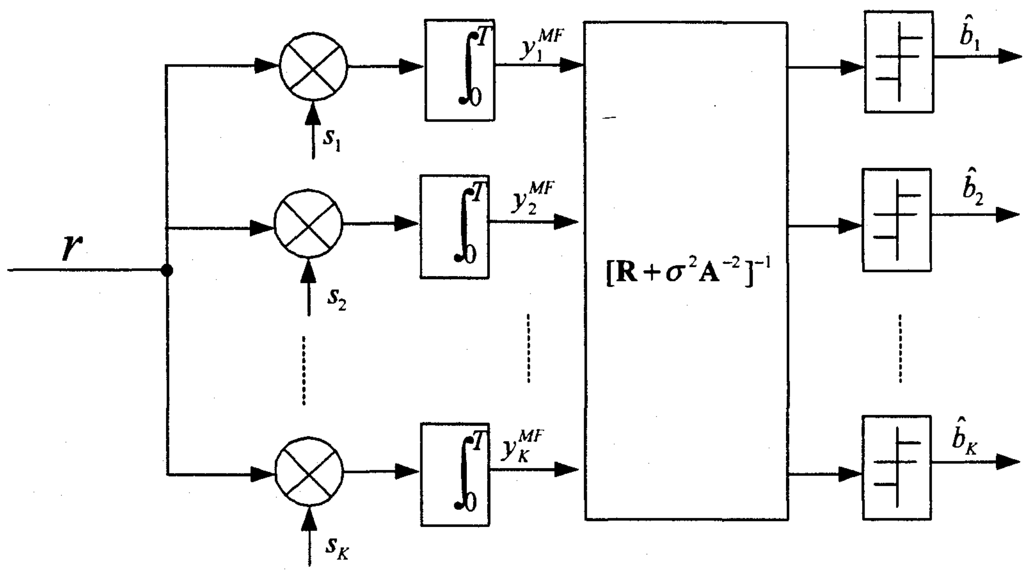


Figure 3.4 MMSE Detector for CDMA Systems

### 3.4.3 Polynomial Expansion Detector

The Polynomial Expansion (PE) Detector is based on a well known result in linear algebra, namely the Cayley-Hamilton theorem. The detector does not actually perform matrix inversion, but implements the inverse of a matrix in terms of a polynomial in the matrix. Thus, it can be used to achieve performance levels of both the decorrelating and the MMSE detector. The polynomial coefficients can be computed using some adaptive algorithm, either offline or on the fly. This detector leads to significant decrease in computational complexity and also has a simple structure which facilitates hardware implementation. The detector was first proposed by Simon Moshavi [13].

## 3.5 Interference Cancellation Based Multiuser Detection

Besides the linear detection schemes, researchers have also proposed nonlinear detectors that use the interferers' data to detect that of the desired user. *Interference cancellation* (IC) detectors employ temporary data estimates to reconstruct the interference, and then subtract it from the received signal.

Such multiuser detectors utilize feedback to reduce MAI in the received signal. The algorithms can be broken down into three classes, although the categories are not disjoint and particular realizations of sub-optimal detectors may use combination of the three classes.

### 3.5.1 Successive Interference Cancellation

The SUccessive Interference Cancellation (SUIC) detector takes a serial approach to interference cancellation [14]. At each stage of the detection process, the strongest signal is detected, regenerated and subtracted out from the received signal so that the remaining users encounter less MAI in the remaining stages. It is intuitively clear that the users must be demodulated in order of decreasing power. The SUIC detector is thus preceded by a stage which ranks users in descending order of received power. The result is that the strongest user will not benefit from any MAI reduction; the weakest user will however see a huge reduction in its MAI. The detector comprises of $K$ stages in cascade, where $K$ is the number of active users in the system. At the $k$th stage, a decision is made on the transmitted bit of the $k$th user and its signal is regenerated and subtracted from the received signal. A "cleaner" version of the signal is thus obtained, which is then used to demodulate the subsequent user. The SUIC detector offers significant performance improvements, especially when there is large disparity amongst received power levels. The detector is easily realizable in hardware. The performance is however extremely sensitive to initial bit estimates and to power level estimates. Also, the sorting operation which needs to be performed at the beginning of each bit interval leads to increase in complexity. In $K$-user successive cancellation when making a decision about the $k$th user, we assume that the decisions of users $k+1,\ldots,K$ are correct and we neglect the presence of user $1,\ldots,k-1$[12]. Therefore,

$$\hat{b}_k = \mathrm{sgn}(y_k - \sum_{j=k+1}^{K} A_j \rho_{jk} \hat{b}_j)$$

Figure 3.5 shows the structure of SUIC detector for two users.

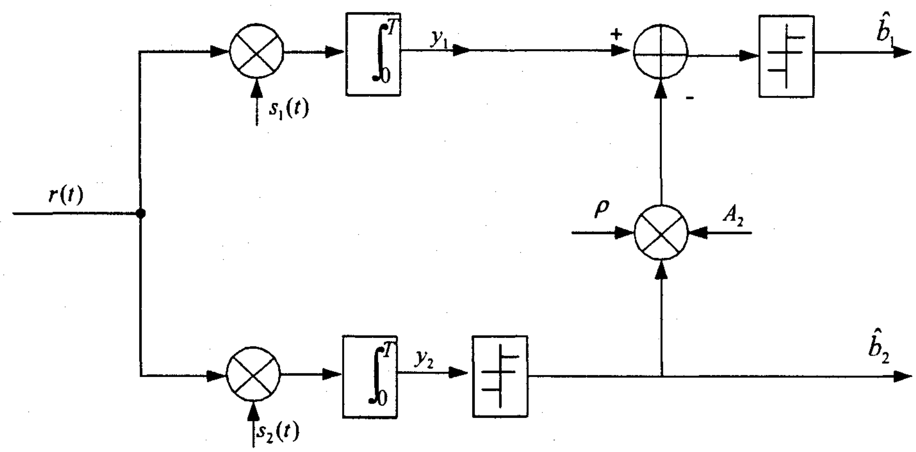


Figure 3.5 Successive Interference Cancellation for Two Synchronous Users

### 3.5.2 Parallel Interference Cancellation

In contrast to the SUIC, the PIC detector estimates and subtracts out all of the MAI for each user in parallel. The initial bit estimates are typically derived from a conventional detector. The estimated bits are scaled by received power level estimates and re-spread by signature codes. A partial summer then sums up all but one of the signals at each of the outputs, which creates the complete MAI estimate for each user. This process can be repeated for multiple stages. Each stage takes as input the bit estimates of the preceding stage and produces a new set of estimates at its output. Like the SUIC, the performance of this detector is severely affected by incorrect initial bit estimates and erroneous received power estimates. The detector performs better than SUIC in a power controlled channel. Figure 3.6 illustrates the structure of the PIC detector in a two-user scenario.

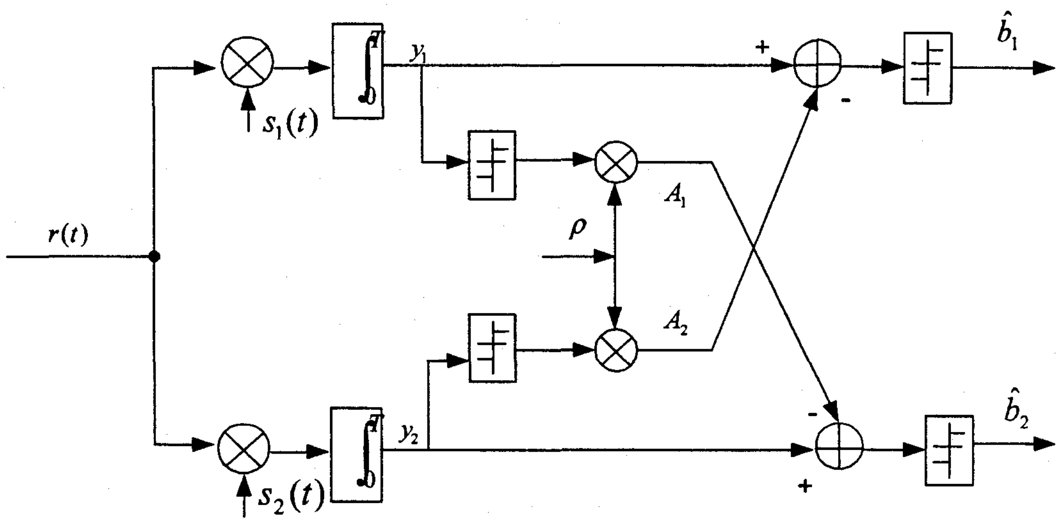


Figure 3.6 Parallel Interference Cancellation for CDMA Systems

Suppose that users are demodulated by the conventional MF in the first stage. Then the data estimate of the $k$th user is given by

$$\hat{b}_k = \text{sgn}(y_k^{MF}) \tag{3.16}$$

For the desired user $k$, remodulating the signal of user $j$ with the signature wave we get the MAI reconstruction $s_j(t)s_k(t)A_j\hat{b}_j$, which is subtracted from the received signal

$$y_k^{PIC}(t) = y_k^{MF}(t) - \sum_{j \neq k} \rho_{jk}(t) A_j \hat{b}_j^{MF} \tag{3.17}$$

$$= A_k b_k - \sum_{j \neq k} \rho_{jk}(t) A_j (b_j - \hat{b}_j) + n'(t) \tag{3.18}$$

Assuming that a hard decision is employed, the output of the two-stage PIC detector is

$$\hat{b}_k^{PIC} = \text{sgn}(y_k^{PIC}) = \text{sgn}(A_k b_k - \sum_{j \neq k} \rho_{jk}(t) A_j (b_j - \hat{b}_j) + n'(t)) \tag{3.19}$$

The PIC approach can be further iterated, by replacing the conventional MF outputs with the increasingly reliable temporary decisions of the interfering users which is made by the PIC detector in the last iteration (as shown in Figure 3.7)

$$y_{k,l}^{PIC}(t) = y_k^{MF}(t) - \sum_{j \neq k} \rho_{jk}(t) A_j \hat{b}_{j,l-1}^{PIC} \tag{3.20}$$

where $y_{k,l}^{PIC}$ is the $l$th-iteration PIC detection result and $\hat{b}_{j,l-1}^{PIC}$ is the data decision made by PIC detector in the ($l$-l)th iteration.

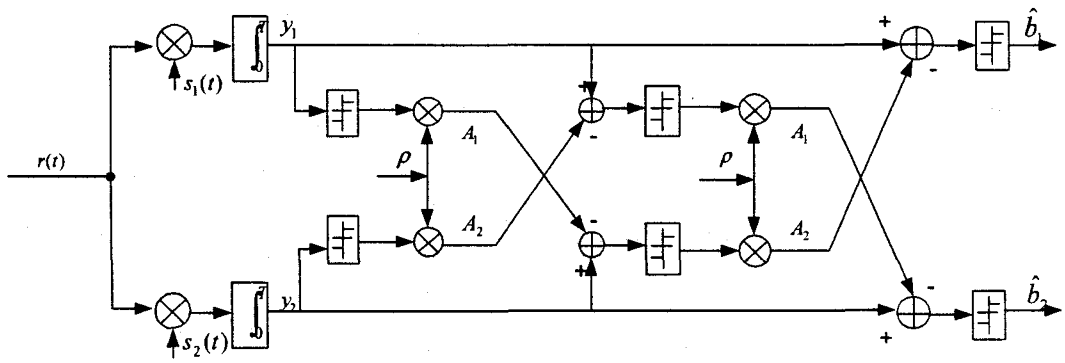


Figure 3.7 Iterative PIC Detector

The iteration can be performed as many times as needed. In general, the performance of the detector should continuously improve with increasing number of iterations.

### 3.5.3 Decision Feedback Detectors

Such detectors perform linear pre-processing followed by SUIC detection. The linear operation partially decorrelates the users without enhancing noise and the SUIC operation decisions and subtracts out the interference from one additional user at a time, in descending order of strength. This detector is based on the white noise channel model, which is widely employed in single user detection theory. The linear transformation applied in the first stage is derived from the Cholesky decomposition of the correlation matrix defined earlier. The Cholesky decomposition expresses a matrix as the product of a lower triangular and an upper triangular matrix. It can be thought of as a square root operation on the matrix. The lower triangular matrix thus obtained is used as the linear transformation, and it acts as a whitening filter for the noise present in the signal.

## 3.6 Previous Research in Coded Multiuser Detection

MAI significantly inhibits the performance and capacity of conventional DS-CDMA systems. Much research has been directed at mitigating this problem through the design of multi-user detectors. Multiuser detection provides huge gains in performance and capacity over conventional systems. The optimum multi-user detector is however not amenable to real time implementation, and several sub-optimal approaches which sacrifice performance for reduced complexity gave been proposed in literature. Some important results in this field were discussed in the preceding sections.

Most of the previous work on multiuser detection focused on uncoded CDMA system, i.e., on the demodulation of multiuser signals. Since in practice, most CDMA systems employ error control coding and interleaving, recent work in this area has addressed multiuser detection for coded CDMA system. Iterative processing techniques have received considerable attention followed by discovery of the powerful Turbo codes [15]. The so-called Turbo-principle can be successfully applied to many detection/decoding problems such as serial concatenated decoding, equalization, coded modulation, multiuser detection and joint source and channel decoding.

High interest has been raised since the results on iterative multiuser detection appeared. However, most works on the iterative multiuser detection use convolutional codes or parallel concatenated convolutional codes (PCCCs) [16]. The iterative decoding algorithm in LDPC codes make them ideal for the iterative multiuser detection. It is shown that carefully designed irregular low density parity check codes can have near Shannon-Limit performance on memoryless channels [17][18].

In order to reduce the complexity of iterative multiuser detection, soft interference cancellation, which has the lowest complexity among SISO detectors, is applied. To improve the performance of soft interference cancellation, an instantaneous linear minimum MMSE filter is applied on the output of soft interference cancellation, which further suppresses the residual multiuser interference [19].

In this thesis, besides an MMSE filter, a novel SISO decorrelator are applied on the output of soft interference cancellation. Based on updated soft inputs from a bank of single-user decoders whose reliabilities are improved in each iteration loop, the equivalent cross-correlation matrix between multiple users is also updated and has smaller cross-correlation elements. Thus noise enhancement resulting from the decorrelating process is reduced as well.

# Chapter 4
# LDPC-Coded Multiuser Detection Techniques

In this chapter, we introduce iterative soft interference cancellation for a synchronous LDPC-coded CDMA system over AWGN channels. The position and task of the SISO multiuser detector in the iterative receiver for coded multiuser systems are shown in Figure 4.1. These SISO detectors are adapted from traditional multiuser detection techniques [12]. Using the soft inputs of detectors which are fedback from a bank of SISO single user decoders, these detectors further exploit the *a priori* information of coded symbols in their detection. Moreover, they calculate soft outputs, which are extrinsic log-likelihood ratios of coded symbols, and output them to SISO single user decoders in turn. Performance analysis is provided at the end of the chapter.

## 4.1 Introduction

We consider synchronous LDPC-coded multiuser systems with $K$ active users over the AWGN channel. We also assume that BPSK modulation is used by each user. This system model has been given in Section 3.1. In the following, we briefly repeat some of the relevant equations for convenience.

In the receiver, we use a bank of matched filters as the front end and sample the filter output at the symbol rate. The matched filter output vector during each symbol interval is:

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{4.1}$$

where $\mathbf{R} = \{\rho_{ij}\}$ is the $K \times K$ normalized cross-correlation matrix of multiuser waveforms. $\mathbf{y} = [y_1, \ldots, y_K]^T$ is the matched filter output vector, $\mathbf{b} = [b_1, \ldots, b_K]^T$ is the transmitted coded symbol vector, $\mathbf{A} = diag[A_1, \ldots, A_K]$ is a diagonal matrix with diagonal elements as the amplitudes of multiple users' transmitted signals, and $\mathbf{n}$ is a zero-mean Gaussian random vector with covariance matrix equal to

$$E[\mathbf{nn}^T] = \sigma^2 \mathbf{R} \tag{4.2}$$

where $\sigma^2 = \frac{N_0}{2}$ is the noise variance.

## 4.2 Iterative Multiuser Detection and Decoding Structure for LDPC coded CDMA System

We consider a $K$-user LDPC-coded CDMA system signaling through AWGN channels. The block diagram of the system is illustrated in Figure 4.1 [19]. The transmitter end is shown in the upper part of Figure 4.1. The binary information bit stream of user $k$ is denoted as $\mathbf{d}_k = [d_k(0), d_k(1), \cdots, d_k(Q-1)]$, where $Q$ is the information frame length. The coded bits of each user are modulated. The modulation can be BPSK, $M$-ary quadrature amplitude modulation ($M$-QAM) or a higher dimensional modulation. For simplicity, here we assume BPSK modulation. The vector $\mathbf{b}_k = [b_k(0), b_k(1), \cdots, b_k(F-1)]$ is one modulated symbol frame of user $k$, where $F$ is the transmitted symbol frame length. After modulation the signal is then transmitted through an AWGN channel.

The receiver consists of two parts. The soft-input soft-output multiuser detector serves as the first stage of the iterative receiver followed by a bank of $K$ single-use soft-input soft-output decoder. The SISO multiuser detector calculates the extrinsic LLR based on the received signal and the feedback from the LDPC decoders. These two parts cooperate iteratively by transferring updated extrinsic soft information of coded symbols between them. The notations will be explained in the following subsection.

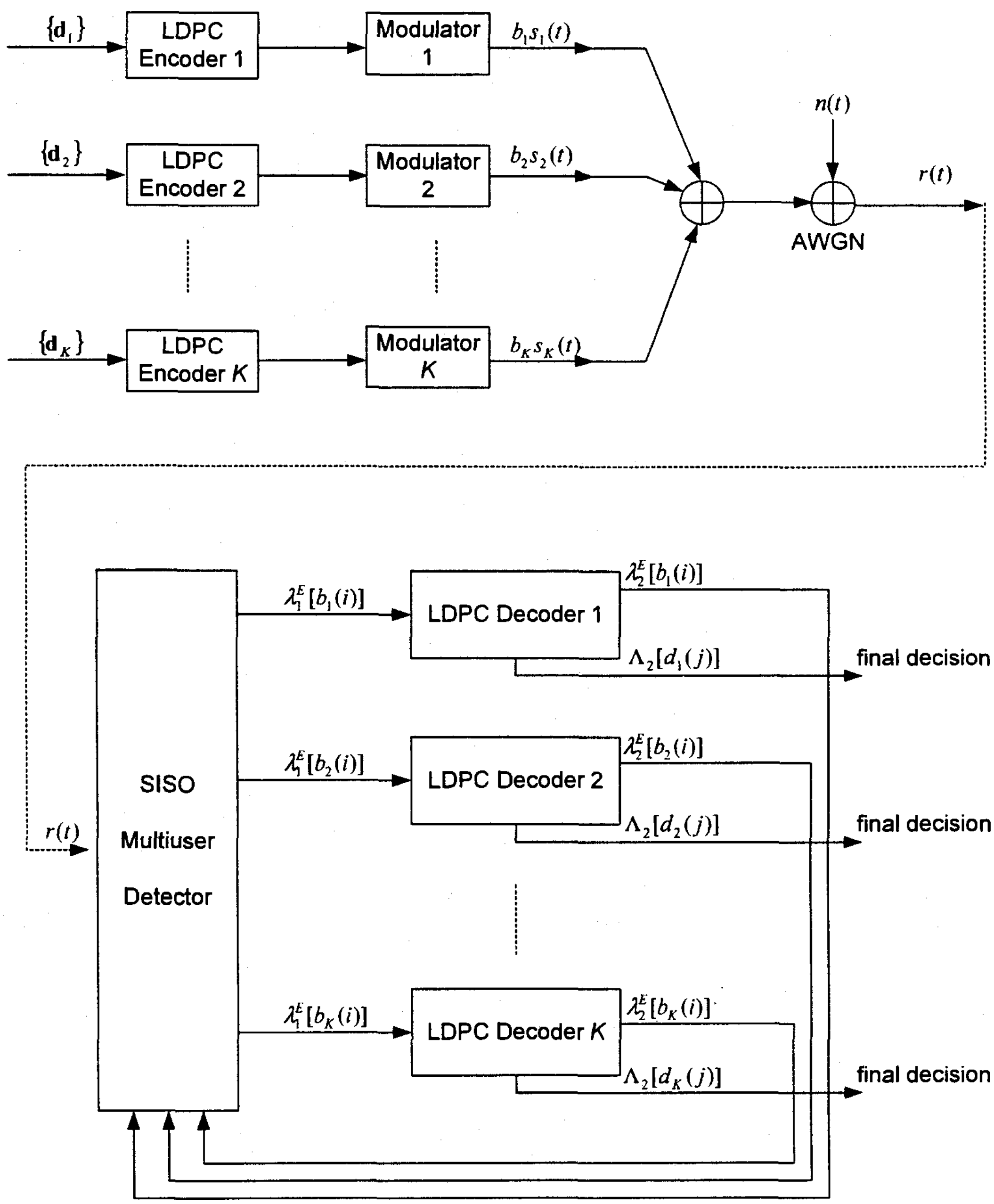


Figure 4.1 Multiuser System with Iterative Receiver

## 4.3 Iterative Receiver

The SISO multiuser detector delivers the *a posteriori* LLR of a transmitted +1 and a transmitted -1 for every coded symbol of every user, which can be expressed as:

$$\Lambda_1[b_k(i)] \equiv \log\frac{P[b_k(i)=+1\,|\,r(t)]}{P[b_k(i)=-1\,|\,r(t)]} \quad k=1,2,\cdots,K,\;\; i=0,1,\cdots,F-1 \tag{4.3}$$

Rewrite the above equation using Bayes' rule as

$$\begin{aligned}\Lambda_1[b_k(i)] &= \log\frac{P[r(t)\,|\,b_k(i)=+1]}{P[r(t)\,|\,b_k(i)=-1]}+\log\frac{P[b_k(i)=+1]}{P[b_k(i)=-1]}\\ &=\lambda_1^E[b_k(i)]+\lambda_2^P[b_k(i)]\end{aligned} \tag{4.4}$$

The first term in the above equation

$$\lambda_1^E[b_k(i)] = \log\frac{P[r(t)\,|\,b_k(i)=+1]}{P[r(t)\,|\,b_k(i)=-1]} \tag{4.5}$$

represents the extrinsic LLR of the code bit $b_k(i)$. And the second term, denoted by

$$\lambda_2^P[b_k(i)] = \log\frac{P[b_k(i)=+1]}{P[b_k(i)=-1]} \tag{4.6}$$

is the *a priori* LLR of the coded symbol $b_k(i)$, which is computed only by the LDPC decoder of the *k*th user in the previous. The superscript $^E$ denotes the extrinsic information and $^P$ denotes the quantity obtained from the previous calculation. For the first iteration, we assume equally-likely code bits, i.e., no prior information available, we have $\lambda_2^P[b_k(i)]=0$. The soft multiuser detector computed the extrinsic LLR $\lambda_1^E[b_k(i)]$

based on the received signal $r(t)$, the *a priori* information of the coded bits of all the other users, $\{\lambda_2^p[b_l(j)], l \neq k, 1 \leq j \leq F-1\}$ and the *a priori* information of the coded bits of the *k*th user other than the *i*th bit $\{\lambda_2^p[b_k(j)], j \neq i\}$. The extrinsic LLR $\lambda_1^E[b_k(i)]$ of $b_k(i)$, which is not influenced by its *a priori* LLR $\lambda_2^p[b_k(i)]$ is then fed into the *k*th user's LDPC decoder as *a priori* information in the current iteration [19][22].

The diagram of SISO multiuser detector is illustrated in Figure 4.2. In this thesis, three types of detectors are introduced. They are soft interference cancellation (SIC) detector, SISO decorrelator and SISO MMSE detector. For SIC detector, there is no filtering block which is dotted in Figure 4.2. For SISO decorrelating detector, filtering block represents the decorrelating filtering while for SISO MMSE detector it is the MMSE filtering. The key of SISO multisuer detector is to calculate the extrinsic LLR of each coded bit of each user $\{\lambda_1^E[b_k(i)], 0 \leq i \leq F-1, 1 \leq k \leq K\}$ based on the received multiuser signal $r(t)$, the *a priori* LLRs of all the other coded bits $\{\lambda_2^p[b_l(j)], l \neq k, j \neq i\}$, excluding the one of $b_k(i)$. The way of computing these extrinsic LLRs will be shown in the following subsections.

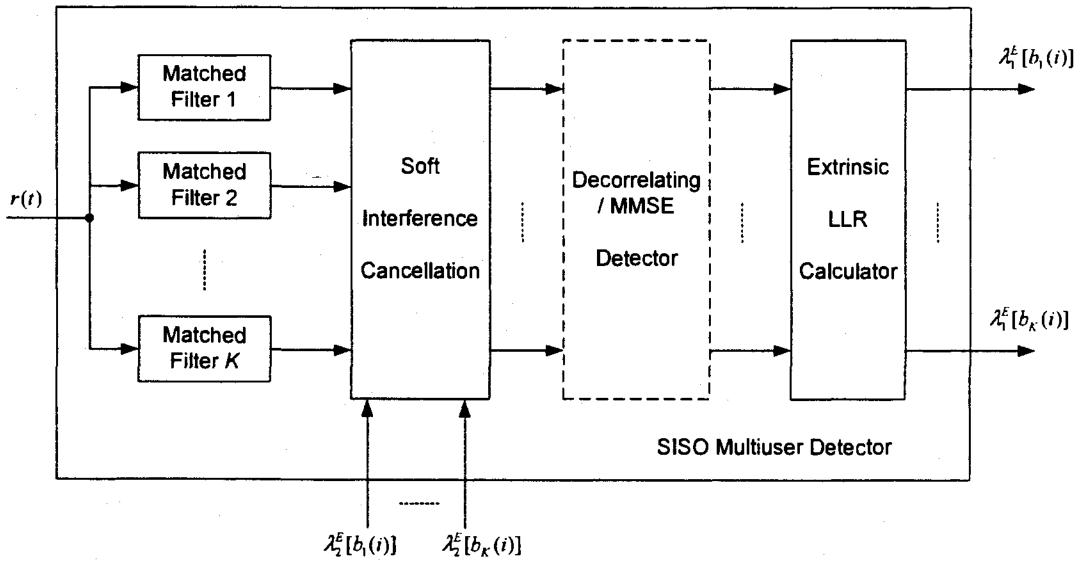


Figure 4.2 SISO Multiuser Detector

Denote $\Lambda_2[b_k(n)]$ as the *a posteriori* LLR of the LDPC decoder for coded symbol $b_k(n)$. We can calculate $\Lambda_2[b_k(n)]$ based on the *a priori* information and the structure of LDPC codes, the *k*th user's SISO decoder computes. That is,

$$\Lambda_2[b_k(n)] \equiv \log \frac{P[b_k(n)=+1 \mid \{\lambda_1^P[b_k(n)]\}_{n=0}^{F-1}]}{P[b_k(n)=-1 \mid \{\lambda_1^P[b_k(n)]\}_{n=0}^{F-1}]}$$

In [19], it has been shown that

$$\Lambda_2[b_k(n)] = \lambda_2^E[b_k(n)] + \lambda_1^P[b_k(n)] \qquad (4.7)$$

$$k = 1,2,\cdots,K, \quad n = 0,1,\cdots,F-1$$

where $\lambda_2^E[b_k(n)]$ is the extrinsic LLR provided by the decoder and $\lambda_1^P[b_k(n)]$ is the *a priori* LLR provided by the multiuser detector. The extrinsic LLRs delivered by *K* channel decoders $\{\lambda_2^E[b_k(n)]\}_{k=1}^K$ are then fed back to the SISO multiuser detector, as the *a priori* information about coded symbols of all users in the next iteration loop[19][22]. Note that at the first iteration, $\lambda_2^E[b_k(n)]$ and $\lambda_1^P[b_k(n)]$ are statistically independent. Subsequently, however, since they use the same information indirectly, they become more and more correlated and finally the improvement through the iterations diminishes.

At the end of the final iteration, single-user decoders calculate the *a posteriori* LLR of every information bit $d_k(j), j = 0,1,\cdots,F-1$,

$$\Lambda_2[d_k(j)] \equiv \log \frac{P[d_k(j)=+1 \mid \{\lambda_1^P[b_k(n)]\}_{n=0}^{F-1}]}{P[d_k(j)=-1 \mid \{\lambda_1^P[b_k(n)]\}_{n=0}^{F-1}]} \tag{4.8}$$

$$k = 1,2,\cdots,K, \quad j = 0,1,\cdots,Q-1,$$

based on which we can make the hard decision for each transmitted information bit. That is:

$$\hat{d}_k(j) = sign(\Lambda_2[d_k(j)]) \tag{4.9}$$

## 4.4 Soft Interference Cancellation

In this section we briefly review a multiuser detector which applies only soft interference cancellation [20][21].

The LLR of the decoded data are calculated by the LDPC decoder. Before they are used to reconstruct the MAI, the LLRs should be converted to the soft values of the data. In conventional methods, a hard decision is taken. Any LLR above 0 will be converted to

1 and any LLR below 0 will be converted to -1 (BPSK). Considering the uncertainty of the data, this is clearly not the optimum approach. In this case, a wrong decision will lead to an erroneous value. The LLR of a bit given by the LDPC decoder involves its reliability information as well as its value. The hard decision discards this valuable information.

Instead of hard decisions, some soft approaches have been employed to estimate the data [19]. Soft values between 1 and -1 are given. A relatively reliable data leads to a value approaching 1 or -1, while an unreliable bit will lead to a value close to 0. Obviously, giving a value close to 0 to an unreliable bit is more helpful for its potential correction, and to reduce the error propagation caused by an error. We can derive the threshold as follows. The LLR of the $k$th user's $i$th symbol is given as (4.6), and introduce

$$\lambda[b_k(i)] = \lambda_2^p[b_k(i)] = \log\frac{P[b_k(i)=+1]}{P[b_k(i)=-1]} \tag{4.10}$$

Together with

$$P[b_k(i)=+1] + P[b_k(i)=-1] = 1 \tag{4.11}$$

where we have

$$P[b_k(i)=+1] = \frac{\exp(\lambda[b_k(i)])}{1+\exp(\lambda[b_k(i)])} \tag{4.12}$$

$$P[b_k(i) = -1] = \frac{\exp(-\lambda[b_k(i)])}{1 + \exp(-\lambda[b_k(i)])} \tag{4.13}$$

If defining $\bar{b} \in \{+1,-1\}$, we can write

$$\begin{aligned}
P[b_k(i) = \bar{b}] &= \frac{\exp(\bar{b}\,\lambda[b_k(i)])}{1 + \exp(\bar{b}\,\lambda[b_k(i)])} \\
&= \frac{\exp\left(\frac{1}{2}\bar{b}\,\lambda[b_k(i)]\right)}{\exp\left(\frac{1}{2}\bar{b}\,\lambda[b_k(i)]\right) + \exp\left(-\frac{1}{2}\bar{b}\,\lambda[b_k(i)]\right)} \\
&= \frac{\cosh\left(\frac{1}{2}\lambda[b_k(i)]\right)\left[1 + \bar{b}\tanh(\frac{1}{2}\lambda[b_k(i)])\right]}{2\cosh\left(\frac{1}{2}\lambda[b_k(i)]\right)} \\
&= \frac{1}{2}\left[1 + \bar{b}\tanh(\frac{1}{2}\lambda[b_k(i)])\right]
\end{aligned} \tag{4.14}$$

Then the soft estimates of the symbol $b_k(i)$ can be given as [19]:

$$\begin{aligned}
\hat{b}_k(i) &= \sum_{\bar{b}\in\{+1,-1\}} \bar{b}P[b_k(i)] \\
&= \sum_{\bar{b}\in\{+1,-1\}} \frac{\bar{b}}{2}\left[1 + \bar{b}\tanh\left(\frac{1}{2}\lambda[b_k(i)]\right)\right] \\
&= \tanh\left(\frac{\lambda[b_k(i)]}{2}\right)
\end{aligned} \tag{4.15}$$

This threshold is depicted in Figure 4.3. We find $-1 \le \hat{b}_k(i) \le 1$, hence normalization is naturally achieved.

The soft inputs $\{\lambda_2^p(b_k), 1 \le k \le K\}$ are provided by a bank of single-user decoders. The soft estimate of each coded symbol of each user is

$$\hat{b}_k = \tanh(\frac{\lambda_2^p(b_k)}{2}), \quad k = 1,2,\cdots,K \tag{4.16}$$

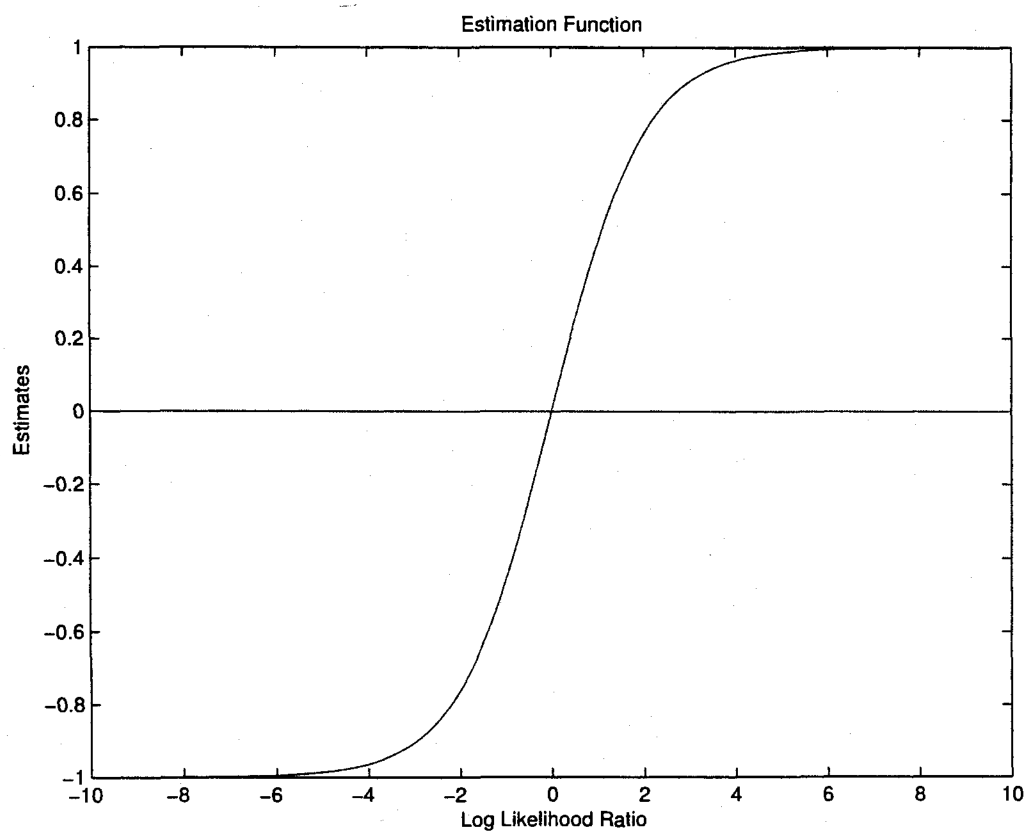


Figure 4.3 Threshold used to convert LLR to soft estimate

The interfering signal for each symbol of each user can be constructed and cancelled from its matched filter output:

$$\mathbf{z} = \mathbf{y} - (\mathbf{R} - \mathbf{I})\mathbf{A}\hat{\mathbf{b}} = \mathbf{A}\mathbf{b} + (\mathbf{R} - \mathbf{I})\mathbf{A}(\mathbf{b} - \hat{\mathbf{b}}) + \mathbf{n} \tag{4.17}$$

where $\mathbf{I}$ is the identity matrix. For the $k$th user, its output of soft interference cancellation is

$$z_k = A_k b_k + \sum_{j=1, j \neq k}^{K} \rho_{kj} A_j (b_j - \hat{b}_j) + n_k \tag{4.18}$$

From the equation, we can see that $z_k$ is sum of the original signal, the residual interference and the noise. The second term can be assumed to be a Gaussian random variable which is independent of $n_k$ [22]. Thus the soft output of the multiuser detector can be calculated as follows:

$$\lambda_1^E[b_k] = \log \frac{P[z_k \mid b_k = +1]}{P[z_k \mid b_k = -1]} = \frac{2A_k z_k}{\sigma_k^2}, \quad 1 \le k \le K \tag{4.19}$$

where $\sigma_k^2$ is the variance of the sum of residual interference and $n_k$ can be statistically estimated by multiuser detector. Rewrite (4.17), we have

$$\mathbf{z} = \mathbf{Ab} + \mathbf{n}_z \tag{4.20}$$

where $\mathbf{n}_z$ is the equivalent noise vector which is still a Gaussian random variable vector. The covariance matrix

$$\begin{aligned} E[\mathbf{n}_z \mathbf{n}_z^T] &= E[((\mathbf{R}-\mathbf{I})\mathbf{A}(\mathbf{b}-\hat{\mathbf{b}})+\mathbf{n})((\mathbf{R}-\mathbf{I})\mathbf{A}(\mathbf{b}-\hat{\mathbf{b}})+\mathbf{n})^T] \\ &= (\mathbf{R}-\mathbf{I})\mathbf{A}\operatorname{cov}\{\mathbf{b}-\hat{\mathbf{b}}\}\mathbf{A}(\mathbf{R}-\mathbf{I}) + \sigma^2\mathbf{R} \end{aligned} \tag{4.21}$$

and in (4.21)

$$\begin{aligned} \operatorname{cov}\{\mathbf{b}-\hat{\mathbf{b}}\} &= diag[\operatorname{var}\{b_1(i)\}, \operatorname{var}\{b_2(i)\}, \cdots, \operatorname{var}\{b_K(i)\}] \\ &= diag[1-\hat{b}_1(i)^2, 1-\hat{b}_2(i)^2, \cdots, 1-\hat{b}_K(i)^2] \end{aligned} \tag{4.22}$$

because

$$\mathrm{var}\{b_j(i)\} = E\{b_j(i)^2\} - [E\{b_j(i)\}]^2 \tag{4.23}$$

$$= 1 - \hat{b}_1(i)^2 \tag{4.24}$$

Therefore, the variance $\sigma_k^2$

$$\sigma_k^2 = [(\mathbf{R}-\mathbf{I})\mathbf{A}\,\mathrm{cov}\{\mathbf{b}-\hat{\mathbf{b}}\}\mathbf{A}(\mathbf{R}-\mathbf{I}) + \sigma^2\mathbf{R}]_{k,k}$$

$$= [\sigma^2\mathbf{R}]_{k,k} \tag{4.25}$$

Combining (4.17) to (4.25), we can have the

$$\lambda_1[b_k] = \frac{2\cdot\left(y_k - \sum_{j\neq k}\mathbf{A}_j\mathbf{R}_{j,k}\hat{b}_j\right)}{[\sigma^2\mathbf{R}]_{k,k}} \tag{4.26}$$

## 4.5 Proposed Iterative Decorrelating Detection

After the soft interference cancellation (4.17), the output $z_k$ still has residual interference.

In order to apply the decorrelating detector, rewrite **z** in an alternative way:

$$\begin{aligned}\mathbf{z} &= \mathbf{y} - (\mathbf{R}-\mathbf{I})\mathbf{A}\hat{\mathbf{b}} \\ &= \mathbf{RAb} + \mathbf{n} - (\mathbf{R}-\mathbf{I})\mathbf{A}\hat{\mathbf{b}} \\ &= \mathbf{RA}(\mathbf{b}-\hat{\mathbf{b}}) + \mathbf{A}\hat{\mathbf{b}} + \mathbf{n} \\ &= \mathbf{R_u Ab} + \mathbf{n}\end{aligned} \tag{4.27}$$

where $\mathbf{R}_u$ is a new equivalent cross-correlating matrix between waveforms of multiple users. We can express $\mathbf{R}_u$ equal to

$$\mathbf{R_u} = \mathbf{R}\cdot\mathbf{C} + \mathbf{D} \tag{4.28}$$

Both **C** and **D** are diagonal matrices and expressed as

$$\mathbf{C}=\begin{bmatrix} \frac{b_1-\hat{b}_1}{b_1} & & & 0 \\ & \frac{b_2-\hat{b}_2}{b_2} & & \\ & & \ddots & \\ 0 & & & \frac{b_K-\hat{b}_K}{b_K} \end{bmatrix}_{K\times K} \tag{4.29}$$

$$\mathbf{D}=\begin{bmatrix} \frac{\hat{b}_1}{b_1} & & & 0 \\ & \frac{\hat{b}_2}{b_2} & & \\ & & \ddots & \\ 0 & & & \frac{\hat{b}_K}{b_K} \end{bmatrix}_{K\times K} \tag{4.30}$$

$$\mathbf{C=I\text{-}D} \tag{4.31}$$

Suppose a multiuser system with three users has a symmetric cross-correlation matrix **R**:

$$\mathbf{R}=\begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{bmatrix} \tag{4.32}$$

The corresponding cross-correlation matrix $\mathbf{R_u}$ for the output of soft interference cancellation **z** is equal to

$$\mathbf{R}_\mathbf{u} = \begin{bmatrix} 1 & \rho_{12}\cdot(1-\frac{\hat{b}_2}{b_2}) & \rho_{13}\cdot(1-\frac{\hat{b}_3}{b_3}) \\ \rho_{21}\cdot(1-\frac{\hat{b}_1}{b_1}) & 1 & \rho_{23}\cdot(1-\frac{\hat{b}_3}{b_3}) \\ \rho_{31}\cdot(1-\frac{\hat{b}_1}{b_1}) & \rho_{32}\cdot(1-\frac{\hat{b}_2}{b_2}) & 1 \end{bmatrix} \tag{4.33}$$

From (4.33), it is easy to see that $\mathbf{R}_\mathbf{u}$ is the function of $b_k$, $\hat{b}_k$ and $\rho_{ij}$. However, $b_k$ is unknown. Therefore, some reasonable approximation needs to be applied. In order to get the $\mathbf{R}_\mathbf{u}$ matrix, we need to estimate

$$\frac{\hat{b}_k}{b_k}, \quad k = 1,2,\cdots,K \tag{4.34}$$

For BPSK-modulated symbols, $b_k \in \{-1,+1\}$, the following equation holds:

$$\frac{\hat{b}_k}{b_k} = \hat{b}_k \cdot b_k, \quad k = 1,2,\cdots,K \tag{4.35}$$

When the hard decision based on $\hat{b}_k$ is correct, that is, when $b_k = \mathrm{sgn}(\hat{b}_k)$, we will have

$$\frac{\hat{b}_k}{b_k} = \hat{b}_k \cdot b_k = \left|\hat{b}_k\right|, \quad k = 1,2,\cdots,K \tag{4.36}$$

If the above equation holds, the estimated cross-correlation matrix $\mathbf{R}_\mathbf{u}$ becomes

$$\mathbf{R}_\mathbf{u} = \begin{bmatrix} 1 & \rho_{12}\cdot(1-\left|\hat{b}_2\right|) & \rho_{13}\cdot(1-\left|\hat{b}_3\right|) \\ \rho_{21}\cdot(1-\left|\hat{b}_1\right|) & 1 & \rho_{23}\cdot(1-\left|\hat{b}_3\right|) \\ \rho_{31}\cdot(1-\left|\hat{b}_1\right|) & \rho_{32}\cdot(1-\left|\hat{b}_2\right|) & 1 \end{bmatrix} \tag{4.37}$$

With the generation of matrix $\mathbf{R}_u$, we can apply a decorrelator so as to further reduce the residual interference in **z**. Assume that the approximate $\mathbf{R}_u$ is perfect and its inverse $\mathbf{R}_u^{-1}$ exists, then the output of decorrelating **z** can be written as

$$\mathbf{x} = \mathbf{R}_u^{-1} \cdot \mathbf{z} = \mathbf{Ab} + \mathbf{R}_u^{-1} \cdot \mathbf{n} = \mathbf{Ab} + \mathbf{n}_u \tag{4.38}$$

The second term in the above equation is the equivalent noise vector $\mathbf{n}_u$. In [22], it is shown that the distribution of the equivalent noise is still a Gaussian random variable vector and has the covariance matrix

$$\begin{aligned} E[\mathbf{n}_u \mathbf{n}_u^T] &= E[(\mathbf{R}_u^{-1}\mathbf{n})(\mathbf{R}_u^{-1}\mathbf{n})^T] \\ &= \sigma^2 \mathbf{R}_u^{-1} \mathbf{R} (\mathbf{R}_u^{-1})^T \end{aligned} \tag{4.39}$$

Denote $n_{uk}$ as the corresponding noise variable for the $k$th user's output $x_k$. It is exactly a Gaussian random variable with distribution $N(0, \sigma_{uk}^2)$, where $\sigma_{uk}^2$ is the $k$th diagonal element of the covariance matrix (4.39). Therefore, the variance

$$\sigma_{uk}^2 = \left[\sigma^2 \mathbf{R}_u^{-1} \mathbf{R} (\mathbf{R}_u^{-1})^T\right]_{k,k} \tag{4.40}$$

The soft output of the multiuser detector during each symbol interval is

$$\lambda_1^E[b_k] = \log \frac{P[x_k \mid b_k = +1]}{P[x_k \mid b_k = -1]} = \frac{2A_k x_k}{\sigma_{uk}^2}, \quad 1 \le k \le K \tag{4.41}$$

Therefore,

$$\lambda_1^E[b_k] = \frac{2A_k \mathbf{R}_u^{-1} \cdot \mathbf{z}_k}{\left[\sigma^2 \mathbf{R}_u^{-1} \mathbf{R} (\mathbf{R}_u^{-1})^T\right]_{k,k}} \tag{4.42}$$

## 4.6 Linear SISO MMSE Multiuser Detector

In traditional linear MMSE multiuser detection [12], we assume that there is no *a priori* information of coded symbol. However, in the SISO multiuser detector, we do have the *a priori* information of coded symbols, which consists of *a priori* LLR of coded symbols and is fed back from a bank of SISO single user decoders. Different users' coded symbols are assumed independent of one another.

Rewriting (4.17), we have

$$\mathbf{z}_k(i) = \mathbf{y}(i) - \mathbf{RA}\hat{\mathbf{b}}_k(i) = \mathbf{RA}[\mathbf{b}(i) - \hat{\mathbf{b}}_k(i)] + \mathbf{n}(i) \tag{4.43}$$

where $\mathbf{b}(i) = [b_1(i), \cdots, b_K(i)]^T$ and $\hat{\mathbf{b}}_k(i) = [\hat{b}_1(i), \cdots, \hat{b}_{k-1}(i), 0, \hat{b}_{k+1}(i), \cdots, \hat{b}_K(i)]^T$. $\hat{\mathbf{b}}_k(i)$ is obtained from $\hat{\mathbf{b}}(i)$ by setting the *k*th element to zero.

In order to further suppress the residual interference in $\mathbf{z}_k(i)$, a linear MMSE filter is used to obtain

$$x_k^{MMSE}(i) = \mathbf{m}_k(i)^T \mathbf{z}_k(i) \tag{4.44}$$

$$\mathbf{m}_k = \arg\left\{\min_{m^*} E\left[b_k(i) - \mathbf{m}^{*T}\mathbf{z}_k(i)\right]^2\right\} \tag{4.45}$$

$$= \arg\left\{\min_{\mathbf{m}^*} \mathbf{m}^{*T} E\left[\mathbf{z}_k(i)\mathbf{z}_k(i)^T\right]\mathbf{m}^* - 2\mathbf{m}^{*T} E\left[b_k(i)\mathbf{z}_k(i)\right]\right\} \tag{4.46}$$

The covariance matrix $\mathbf{R}_z$ of $\mathbf{z}_k$ is

$$\mathbf{R}_z = E\left[\mathbf{z}_k(i)\mathbf{z}_k(i)^T\right]$$

$$= \mathbf{RA}\,\mathrm{cov}\{\mathbf{b}(i) - \hat{\mathbf{b}}_k(i)\}\mathbf{AR} + \sigma^2\mathbf{R} \tag{4.47}$$

and the correlation between $b_k$ and $\mathbf{z}_k$ is

$$\begin{aligned} \mathbf{p}_k &= E\{b_k(i)\mathbf{z}_k(i)\} \\ &= \mathbf{RA}E\{b_k(i)[\mathbf{b}(i) - \hat{\mathbf{b}}_k(i)]\} \\ &= \mathbf{RAe}_k \end{aligned} \tag{4.48}$$

where $\mathbf{e}_k$ denotes a $K$-dimensional column vector of all zeros, except for the $k$th element, which is 1. Denote $\mathbf{\Omega}_k \equiv \mathrm{cov}\{\mathbf{b}(i) - \hat{\mathbf{b}}_k(i)\}$, similar to (4.22), in (4.47) we have

$$\begin{aligned} \mathbf{\Omega}_k &\equiv \mathrm{cov}\{\mathbf{b}(i) - \hat{\mathbf{b}}_k(i)\} \\ &= diag[\mathrm{var}\{b_1(i)\}, \cdots, \mathrm{var}\{b_{k-1}(i)\}, 1, \mathrm{var}\{b_{k+1}(i)\}, \cdots, \mathrm{var}\{b_K(i)\}] \\ &= diag[1 - \hat{b}_1(i)^2, \cdots, 1 - \hat{b}_{k-1}(i)^2, 1, 1 - \hat{b}_{k+1}(i)^2, \cdots, 1 - \hat{b}_K(i)^2] \end{aligned} \tag{4.49}$$

Based on the optimum linear solution for the Wiener-Hopf equation in (4.45) [23], the optimum MMSE filter $\mathbf{m}_k$ for the $k$th user is

$$\begin{aligned} \mathbf{m}_k &= \mathbf{R}_z^{-1} \cdot \mathbf{p}_k \\ &= [\mathbf{RA\Omega}_k\mathbf{AR} + \sigma^2\mathbf{R}]^{-1} \cdot \mathbf{RAe}_k \\ &= [\mathbf{A\Omega}_k\mathbf{AR} + \sigma^2\mathbf{I}]^{-1} \cdot \mathbf{Ae}_k \\ &= A_{kk}\mathbf{R}^{-1}[\mathbf{A\Omega}_k\mathbf{A} + \sigma^2\mathbf{R}^{-1}]^{-1} \cdot \mathbf{e}_k \end{aligned} \tag{4.50}$$

The output of the MMSE filter is

$$\begin{aligned} x_k^{MMSE} &= \mathbf{m}_k^T\mathbf{z}_k = \mathbf{m}_k^T \cdot (\mathbf{y}(i) - \mathbf{RA}\hat{\mathbf{b}}_k(i)) \\ &= \{A_{kk}\mathbf{R}^{-1}[\mathbf{A\Omega}_k\mathbf{A} + \sigma^2\mathbf{R}^{-1}]^{-1} \cdot \mathbf{e}_k\}^T \cdot (\mathbf{y}(i) - \mathbf{RA}\hat{\mathbf{b}}_k(i)) \\ &= A_{kk}\mathbf{e}_k^T[\mathbf{A\Omega}_k\mathbf{A} + \sigma^2\mathbf{R}^{-1}]^{-1} \cdot (\mathbf{R}^{-1}\mathbf{y}(i) - \mathbf{A}\hat{\mathbf{b}}_k(i)) \end{aligned} \tag{4.51}$$

We can rewrite

$$x_k^{MMSE} = \mathbf{m}_k^T \mathbf{z}_k = \mathbf{m}_k^T \cdot \mathbf{RA} \cdot \begin{pmatrix} b_1 - \hat{b}_1 \\ \vdots \\ b_k \\ \vdots \\ b_K - \hat{b}_K \end{pmatrix} + \mathbf{m}_k^T \cdot \mathbf{n}$$

$$= \left(\mathbf{m}_k^T \mathbf{RA}\right)_k \cdot b_k + \sum_{i=1, i \neq k}^{K} \left(\mathbf{m}_k^T \mathbf{RA}\right)_i \cdot (b_i - \hat{b}_i) + \mathbf{m}_k^T \cdot \mathbf{n} \quad (4.52)$$

We can see that in (4.52) the first term is the desired signal, the second term is the residual interference after soft interference cancellation and the last one is the background noise. After assuming that the second term is also a Gaussian random variable which is independent of the background noise, we can express

$$x_k^{MMSE} = \mu_k b_k + \eta_k \quad (4.53)$$

where

$$\mu_k = \left(\mathbf{m}_k^T \mathbf{RA}\right)_k$$
$$= A_{kk}^2 \left[\left(\mathbf{A}\mathbf{\Omega}_k \mathbf{A} + \sigma^2 \mathbf{R}^{-1}\right)^{-1}\right]_{kk} \quad (4.54)$$

and $\eta_k$ is a Gaussian random variable with the distribution $N(0, \sigma_{vk}^2)$, where $\sigma_{vk}^2$ is given by

$$\sigma_{vk}^2 = \text{var}\left\{x_k^{MMSE}\right\} = E\left\{(x_k^{MMSE})^2\right\} - \mu_k^2$$
$$= \mathbf{m}_k^T E\left\{\mathbf{z}_k \mathbf{z}_k^T\right\} \mathbf{m}_k - \mu_k^2$$
$$= \mathbf{m}_k^T [\mathbf{RA}\mathbf{\Omega}_k \mathbf{AR} + \sigma^2 \mathbf{R}] \mathbf{m}_k - \mu_k^2$$
$$= \mathbf{m}_k^T [\mathbf{RA}\mathbf{\Omega}_k \mathbf{AR} + \sigma^2 \mathbf{R}][\mathbf{RA}\mathbf{\Omega}_k \mathbf{AR} + \sigma^2 \mathbf{R}]^{-1} \mathbf{RA}\mathbf{e}_k - \mu_k^2$$
$$= \mathbf{m}_k^T \mathbf{RA}\mathbf{e}_k - \mu_k^2$$

$$= \mu_k - \mu_k^2 \tag{4.55}$$

Thus, the extrinsic information delivered by the MMSE detector is

$$\begin{aligned}\lambda_1^E[b_k(i)] &= \log \frac{P[x_k^{MMSE}(i) \mid b_k(i) = +1]}{P[x_k^{MMSE}(i) \mid b_k(i) = -1]} \\ &= -\frac{\left[x_k^{MMSE}(i) - \mu_k(i)\right]^2}{2\sigma_{vk}^2} + \frac{\left[x_k^{MMSE}(i) + \mu_k(i)\right]^2}{2\sigma_{vk}^2} \\ &= \frac{2\mu_k(i) x_k^{MMSE}(i)}{\sigma_{vk}^2} = \frac{2x_k^{MMSE}(i)}{1 - \mu_k(i)}\end{aligned} \tag{4.56}$$

## 4.7 Performance Analysis

Three iterative multiuser detectors have been described in this chapter: the soft interference cancellation detector, the SISO decorrelator and the SISO MMSE detector. Since the latter two detectors are based on the soft interference cancellation detector, we expect to see that the SISO decorrelator and SISO MMSE detector have better performance than the soft interference cancellation detector. The simulation results in the next chapter show that our expectation is correct.

In SISO decorrelator, from (4.33), we can see that $\mathbf{R}_u$ is dependent on $b_k$, which is unknown. If one hard decision based on $\hat{b}_k$ is not correct, some new residual interference will be introduced by multiplying $\mathbf{R}_u^{-1}$ with $\mathbf{z}$. However, we can estimate $b_k$ without mistake unless when the symbol estimate's reliability is low, i.e.,. $|\hat{b}_k|$ is small. Therefore, the introduced residual interference will be low. The simulation results in the next chapter confirm this approximation. The reason for this is that the SISO decorrelator makes good use of soft inputs provided by the SISO single user decoders and in turn, provides soft outputs to them. As the number of iterations increases, the symbol estimate reliabilities will increase as well. At the same time, the approximate cross-correlation matrix $\mathbf{R}_u$ tends towards the true one. Therefore, after a certain number of iterations, the residual interference which may be introduced by the approximate $\mathbf{R}_u$ can be ignored.

In (4.33), $\mathbf{R}_u$ is dependent on $\frac{\hat{b}_k}{b_k}$. However, $\rho_{ij} \cdot (1-\frac{\hat{b}_j}{b_j}) \leq \rho_{ij}$. Thus, the absolute values of elements in $\mathbf{R}_u$ are less than or equal to the corresponding values in $\mathbf{R}$ except for diagonal ones. This means that cross-correlations between multiple users are reduced. Moreover, with more iteration loops, symbol estimates have greater reliability (i.e., larger $|\hat{b}_k|$), and therefore multiple users have less cross-correlations.

One main disadvantage of decorrelating detection is noise enhancement introduced by multiplying the inverse of the cross-correlation matrix. In (4.39), the equivalent noise variance $E[\mathbf{n}_u\mathbf{n}_u^T]=\sigma^2\mathbf{R}_u^{-1}\mathbf{R}(\mathbf{R}_u^{-1})^T$. If we multiplying $\mathbf{R}^{-1}$ with $\mathbf{y}$, the noise variance $E[\mathbf{n}\ \mathbf{n}^T]=\sigma^2\mathbf{R}^{-1}$. In [22], it is shown that

$$\left(\sigma^2\mathbf{R}_u^{-1}\mathbf{R}(\mathbf{R}_u^{-1})^T\right)_{kk} \leq \left(\sigma^2\mathbf{R}^{-1}\right)_{kk} \tag{4.57}$$

From (4.57), we know that the noise enhancement introduced by decorrelating $\mathbf{z}$ with $\mathbf{R}_u^{-1}$ is less than or equal to that introduced by decorrelation $\mathbf{y}$ with $\mathbf{R}^{-1}$. Therefore, as the reliabilities of coded symbols are improved noise enhancement is small, the SISO decorrelator performance will converge to the single user system quickly.

In the SISO MMSE detector, the MMSE receiver is employed after the soft interference cancellation. The LDPC decoder provides the LLR of both the information bits and the parity bits. Then the LLR values are converted to the normalized soft estimates given by (4.15). The soft information is fedback to the detector and the MAI is subtracted. At the first iteration, there is no *a priori* information. The SISO MMSE detector behaves like the traditional MMSE receiver. With the refreshed extrinsic information generated at the end of each iteration, the decoding result is expected to improve. Then the improved decoder outputs will be converted to more accurate data estimates that can be used to yield a more accurate interference cancellation. Thus, it is reasonable to believe that the SISO MMSE detector will improve continuously with increasing number of iterations. The signal to residual interference-plus-noise is denoted as $\mathrm{SNR}_k = \frac{\mu_k{}^2}{\sigma_k^2} = \frac{\mu_k}{1-\mu_k}$. It has been shown in [19] that $\mathrm{SNR}_k$ will increase as the number of iterations increases. Hence, each user's performance will be improved and converge to the single user performance quickly as the number of iterations increases. This is confirmed by Monte-Carlo simulation results shown in the next chapter.

# Chapter 5
# Simulation Results

## 5.1 Models and Conditions for System Simulation

In this chapter, we evaluate the performance of various multiuser receivers. A synchronous $K$-symmetric multiuser system is assumed for simplicity. Its cross-correlation matrix **R** has only one parameter $\rho$, indicating the cross correlation between any pair of user.

$$\mathbf{R}=\begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & & \ddots & \\ \rho & \cdots & \rho & 1 \end{bmatrix} \tag{5.1}$$

We use the Monte-Carlo technique to obtain the bit error rate performance because it is difficult to get the system performance by a closed expression of bit error probability for iterative receivers. The cross correlation $\rho$ between any two users is equal.

In the simulation, the parity check matrix **H** of LDPC code used by each user is from MacKay's website [10]. It is (204,102) Gallager codes with column weight $w_c = 3$ and row weight $w_r = 6$. Therefore, the code rate $R$=1/2. Iterative SISO single user decoders are based on the log-domain LDPC decoding algorithm which is described in Section 2.5.3. For each cycle, the LDPC decoders are limited to a maximum of 15 decoding iterations. If a decoder has not decoded the codeword by this point, the LLRs on the last decoding iteration are fedback to the multiuser detector. If the LLRs in a particular decoder converge to a valid codeword before or on the $15^{th}$ iteration, we assume that the codeword is successfully decoded and hard decisions are fedback to the multiuser detector. No additional decoding attempts will be made for successfully decoded codewords on later cycles of the multiuser/decoding algorithm. The frame size of information bits is 102 and the total number of simulated information is at least 5000 codewords. As $E_b/N_o$ increases and the BER decreases, we increase the number of codewords per simulation to maintain a reasonable confidence interval. It is assumed the received amplitudes of all users' signals are equal and normalized to be unity. In this case, since all the users have the same performance, without loss of generality, we only show bit error rates of the first user.

A subscript $_F$ denotes quantities passed from multiuser detector forward to the LDPC decoder and $_B$ denotes quantities passed from LDPC decoder backward to the multiuser detector.

The iterative multiuser detection algorithm for LDPC-coded CDMA system is as follows:

[1] Inialization: $L(r) = 0$, and $L_B[b_k(i)] = 0$.

[2] Iterative multiuser detection iteration:

[2-1] SISO multiuser detection. Use the following function

$$L_F[(b_k(i)] = \Phi(\{y(t)\}, \{L_B[(b_{k'}(i)]\}_{k' \neq k}) \tag{5.2}$$

where $\Phi(\cdot)$ denotes the multiuser detector function.

[2-2] LDPC decoding iteration.

[2-2-1] Variable nodes update:

$$L(q_{ij}) = L_F[b_k(i)] + \sum_{j' \in C_{i \setminus j}} L(r_{j'i})$$

[2-2-2] Check nodes update:

$$L(r_{ji}) = -2\tanh^{-1}(\prod_{i' \in V_{j \setminus i}} \tanh(-\frac{1}{2} L(q_{i'j}))$$

[2-3] Extrinsic messages are computed and passed back to the multiuser detector:

$$L_B[b_k(i)] = \sum_{j \in C_i} L(r_{ji})$$

[3] Final hard decisions on information and parity bits:

$$\hat{b}_k(i) = sign\{L_F[b_k(i)] + L_B[b_k(i)]\}$$

Table 5.1 Algorithm for Iterative LDPC-coded CDMA System

1) For SISO soft interference cancellation detector, (5.2) becomes

$$L_F[b_k(i)] = \frac{2 \cdot \left( y_k(i) - \sum_{j \neq k} \mathbf{A}_j \mathbf{R}_{j,k} \hat{b}_j(i) \right)}{[\sigma^2 \mathbf{R}]_{k,k}} \tag{5.3}$$

where $\hat{b}_j(i) = \tanh(\frac{L_B(b_j(i))}{2})$.

2) For SISO decorrelator, (5.2) becomes

$$L_F[b_k(i)] = \frac{2 \cdot \mathbf{R}_u^{-1} \cdot \left( y_k(i) - \sum_{j \neq k} \mathbf{A}_j \mathbf{R}_{j,k} \hat{b}_j(i) \right)}{\left[\sigma^2 \mathbf{R}_u^{-1} \mathbf{R} (\mathbf{R}_u^{-1})^T \right]_{k,k}} \tag{5.4}$$

3) For SISO MMSE detector, (5.2) becomes

$$L_F[b_k(i)] = \frac{2 x_k^{MMSE}(i)}{1 - \mu_k(i)} \tag{5.5}$$

where $$x_k^{MMSE}(i) = A_{kk} \mathbf{e}_k^T [\mathbf{A} \mathbf{\Omega}_k \mathbf{A} + \sigma^2 \mathbf{R}^{-1}]^{-1} \cdot (\mathbf{R}^{-1} y_k(i) - \mathbf{A} \hat{b}_k(i)) \tag{5.6}$$

and $$\mu_k = A_{kk}^2 \left[ (\mathbf{A} \mathbf{\Omega}_k \mathbf{A} + \sigma^2 \mathbf{R}^{-1})^{-1} \right]_{kk} \tag{5.7}$$

# 5.2 Simulation Results

## 5.2.1 Simulation Results for SIC and SISO Decorrelating Detector

In this subsection, we examine the performances of the SISO decorrelator and soft interference cancellation detector. In the first iteration, there is no soft information fedback from the single user decoders. Therefore, the SISO decorrelating detector follows the traditional decorrelating detector, while the soft interference cancellation detector follows conventional multiuser detection.

Figure 5.1 to Figure 5.8 show comparison of bit error rate performances between the soft interference cancellation detector and the SISO decorrelator for two $K$-symmetric multiuser systems with $K = 3, 5, 7, 10$ and $\rho = 0.3, 0.5$ respectively. The performance of the soft interference cancellation detector and the SISO decorrelator are represented by solid curves and dotted curves, respectively. For reference, the performance of the single user system is also shown in these figures. Compared with the soft interference cancellation detector, the SISO decorrelating detector has much better performance. The SISO decorrelator performance converges to the single user system performance much faster and has better bit error rates in low SNRs. The performance in the second iteration shows significant improvement. As we increase the number of iterations, the additional improvement from one iteration to the next decreases. At the first iteration, there is no estimation of the coded symbols. The soft interference cancellation detector behaves the traditional successive interference cancellation detector while the SISO decorrelator is the traditional decorrelator. After the first iteration, the LDPC decoders feed back the extrinsic LLRs to the SISO multiuser detector as the *a priori* information in the next iteration.

If the systems have the same cross correlation (i.e., $\rho$ is the same), as the number of the users in the system increases, the system converges to the single user system performance slowly. The soft interference cancellation detector is more sensitive to the system parameters compared to the SISO decorrelating detector. That is, as the number of users increases, the soft interference cancellation detector does not get much improvement while the SISO decorrelating detector is more stable in the convergence speed.

If the systems have the same number of users (i.e., $K$ is the same), as the cross correlation increases, the performance of soft interference cancellation detector degrades significantly while the SISO decorrelating detector is more steady and still converges to the single user detector as the SNR gets large.

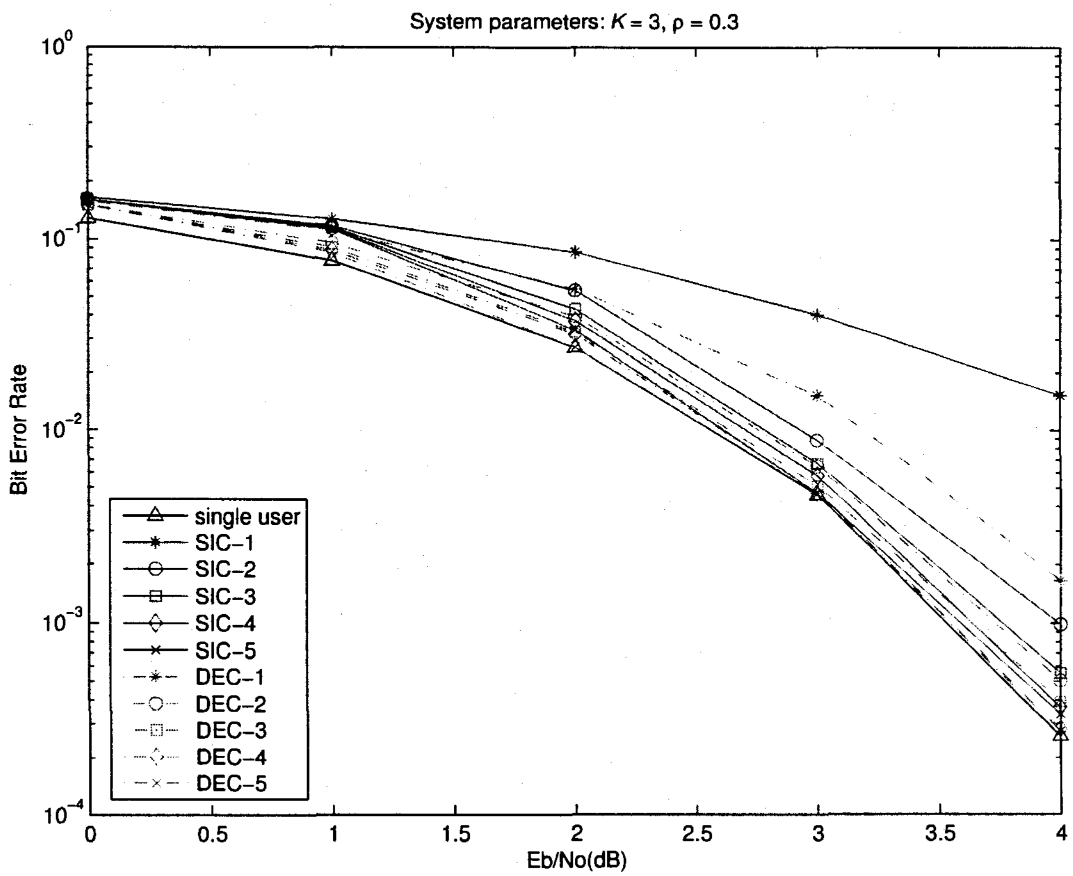


Figure 5.1 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 3$, $\rho = 0.3$

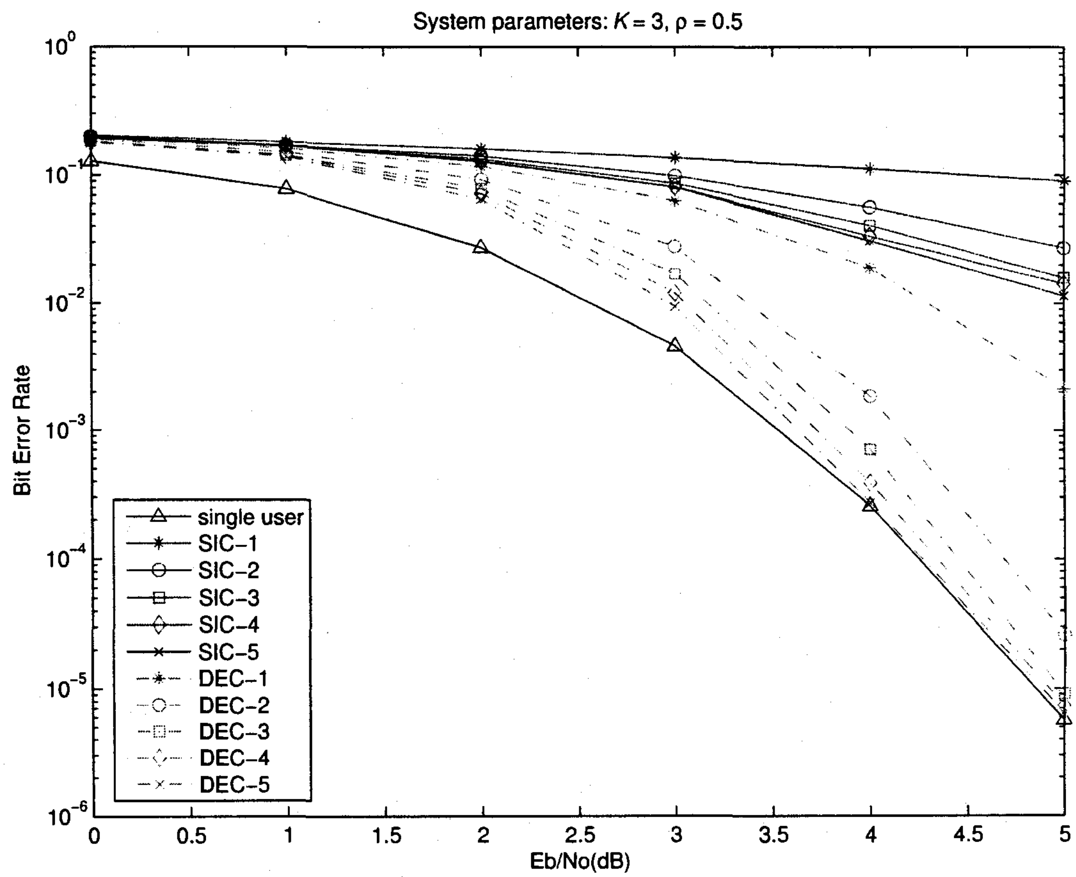


Figure 5.2 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 3$, $\rho = 0.5$

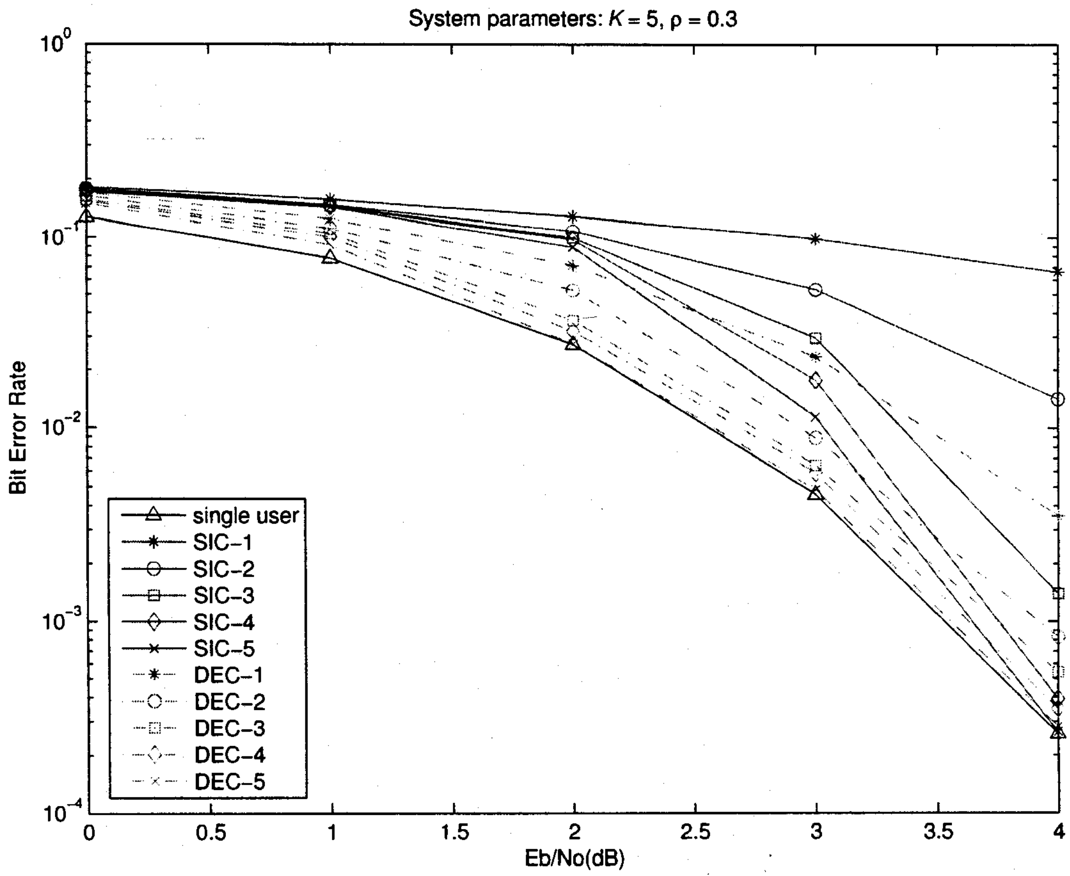


Figure 5.3 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 5$, $\rho = 0.3$

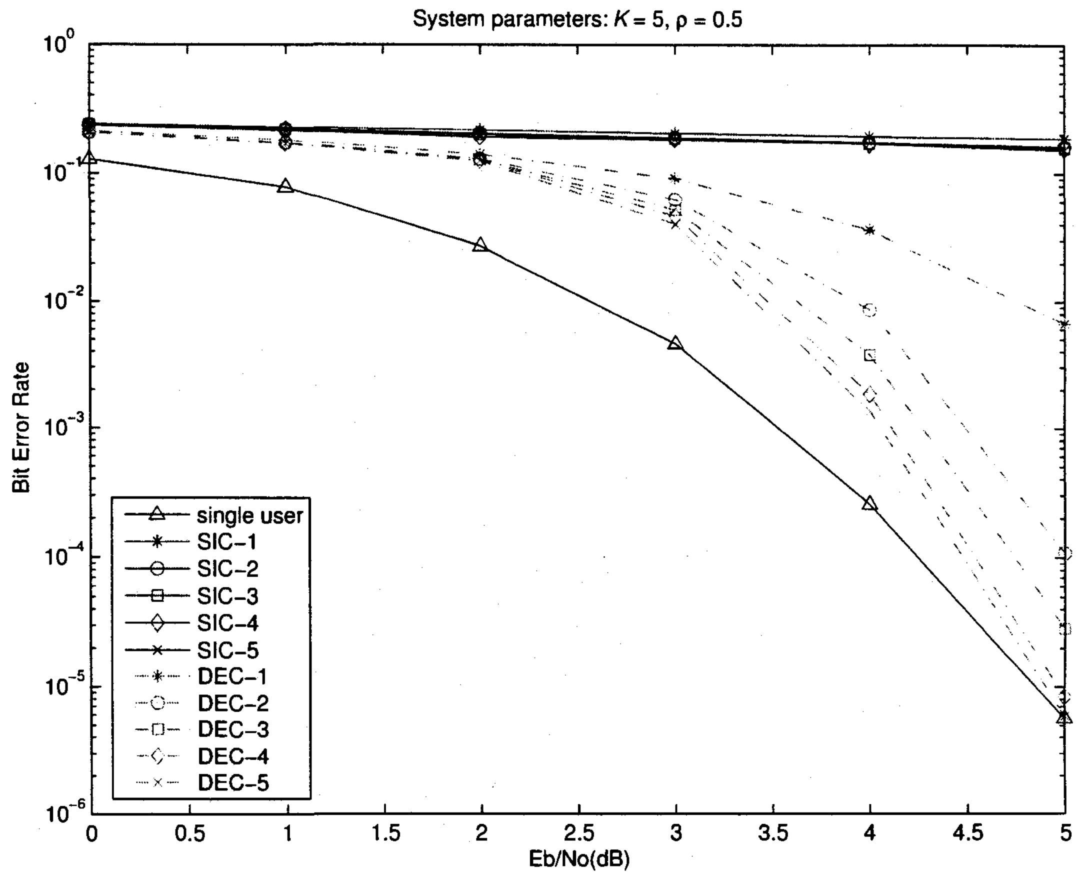


Figure 5.4 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 5$, $\rho = 0.5$

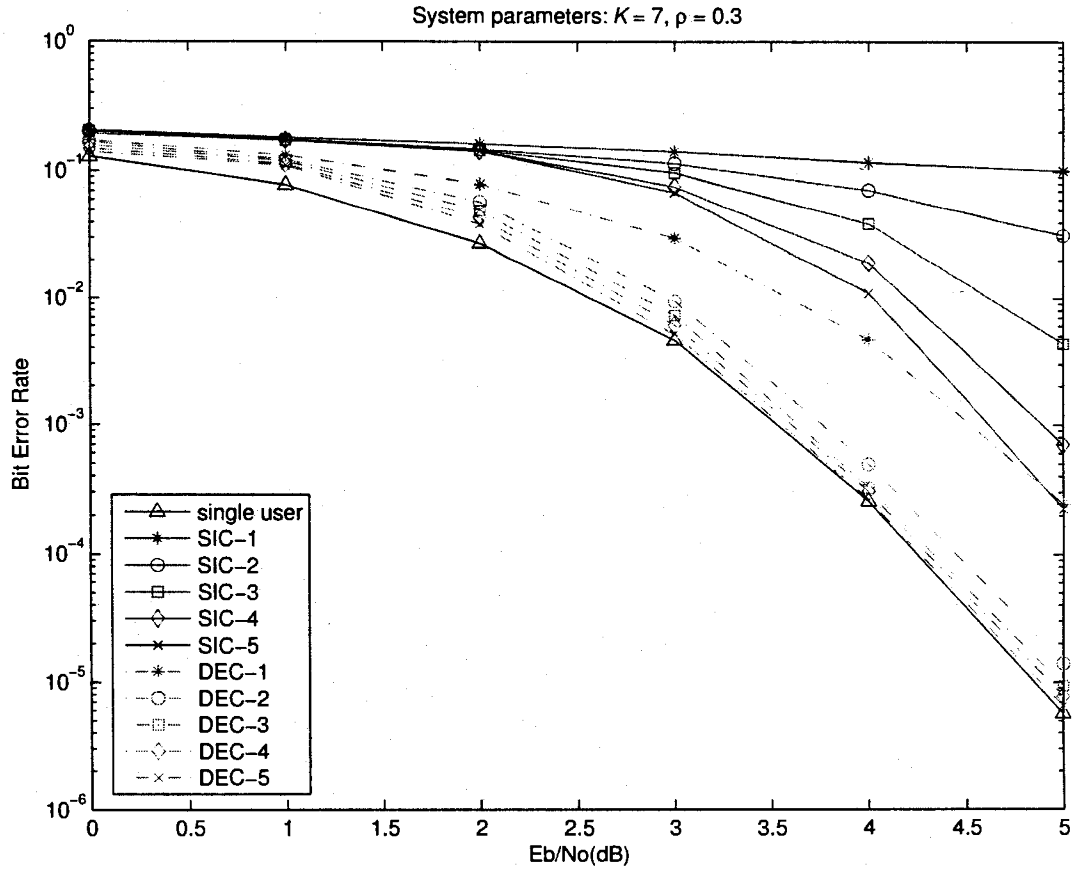


Figure 5.5 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 7$, $\rho = 0.3$

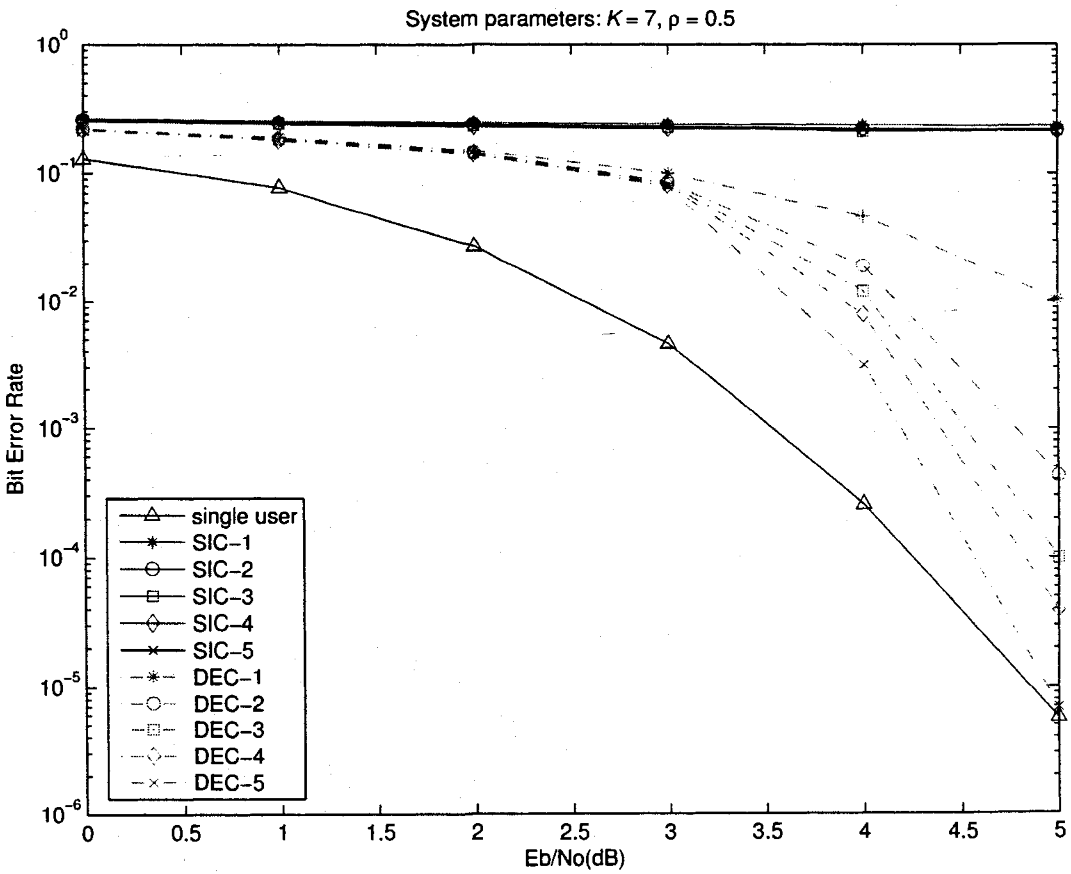


Figure 5.6 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 7$, $\rho = 0.5$

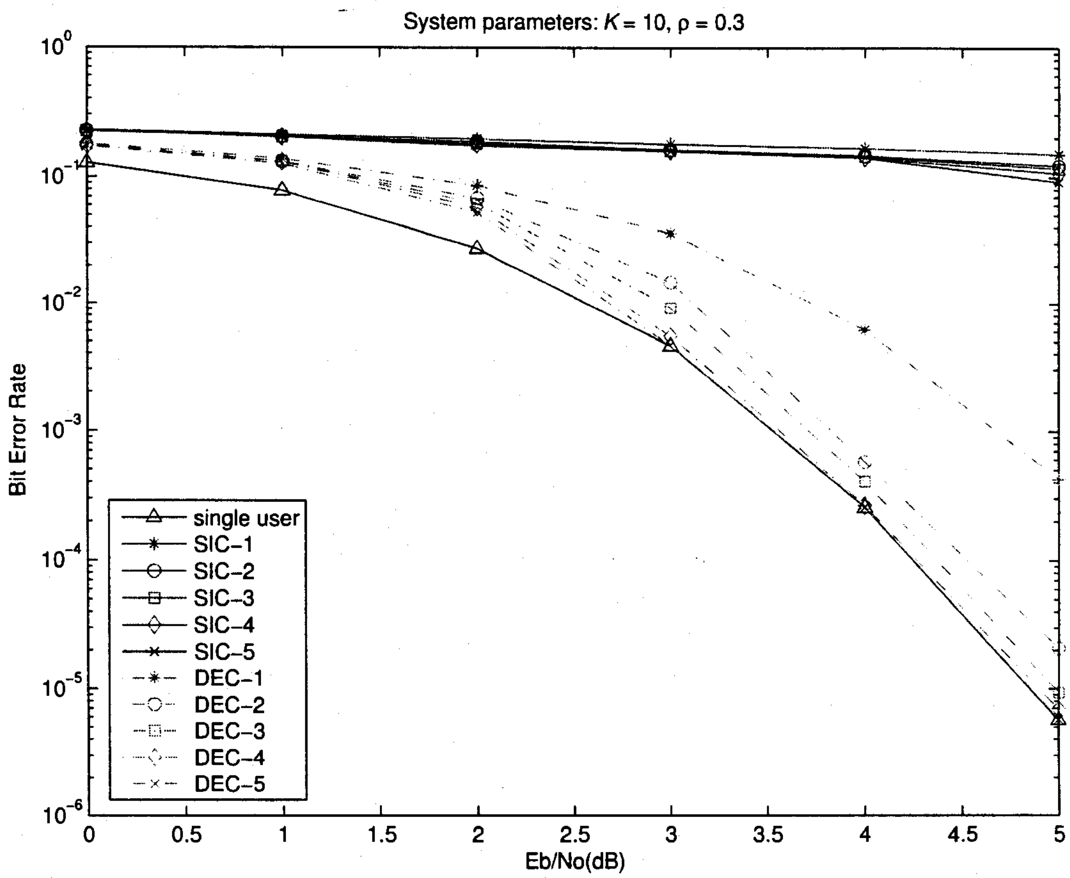


Figure 5.7 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 10$, $\rho = 0.3$

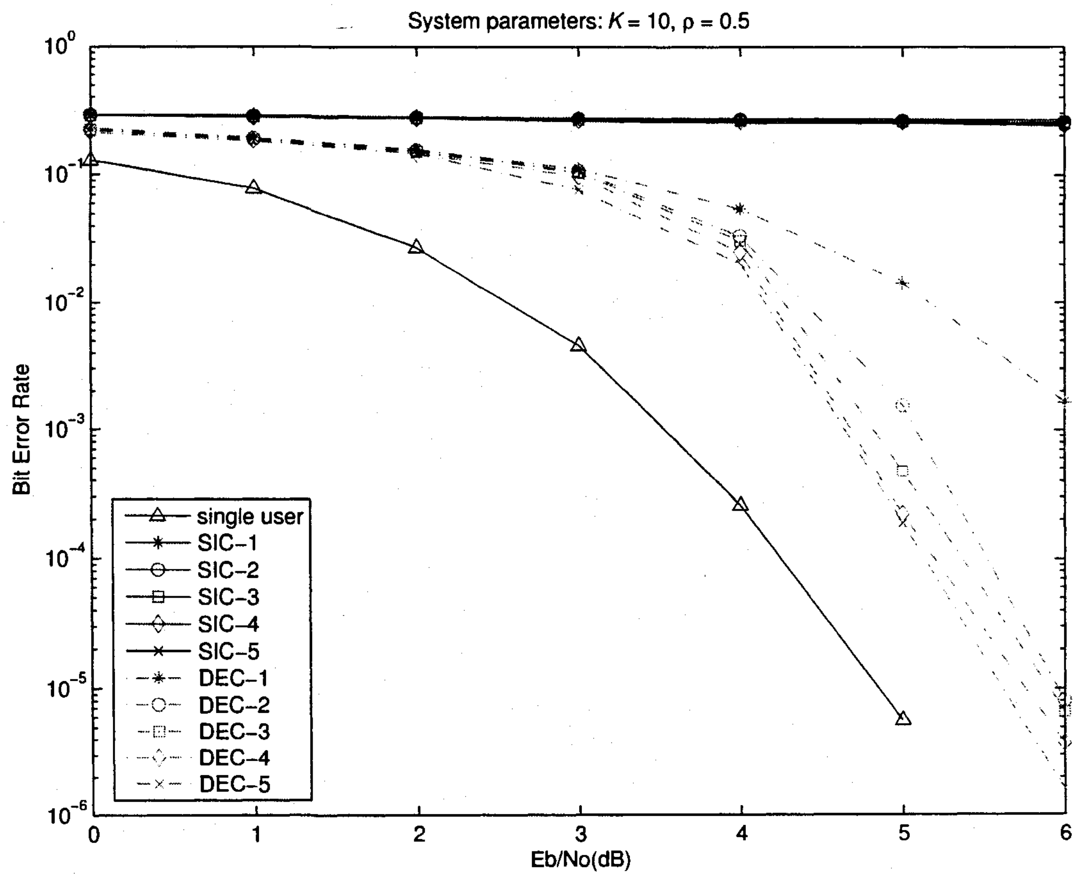


Figure 5.8 Comparison of bit error rate performances between the soft interference canceller in the first five iterations (SIC-1 – SIC-5), the SISO decorrelator (DEC-1 – DEC-5) and the single user system for $K = 10,\ \rho = 0.5$

### 5.2.2 Simulation Results for SISO Decorrelator and SISO MMSE Detector

In this subsection, we use the same system parameters and Monte-Carlo simulation scenarios as those described in subsection 5.2.1. We will compare the system performances provided by the SISO decorrelator and those provided by the linear SISO MMSE detector.

Figure 5.9 -- Figure 5.16 show comparison of bit error rate performances between the SISO decorrelator and the SISO MMSE detector for two $K$-symmetric multiuser systems with $K$ = 3, 5, 7, 10 and $\rho = 0.3, 0.5$ respectively. Solid curves denote bit error rates provided by the SISO decorrelator and dotted curves are provided by the SISO MMSE detector. The performance of the single user system is also shown in these figures. Compared with the SISO decorrelating detector, the SISO MMSE detector has better performance. The SISO MMSE detector performance converges to the single user system performance faster and has better bit error rates in low SNRs. The performance in the second iteration improves a lot compared to the latter iterations. At the first iteration, there is no estimation of the coded symbols. The SISO decorrelator is the traditional decorrelato while the SISO MMSE detector follows the traditional linear MMSE detector. After the first iteration, the LDPC decoders feed back the extrinsic LLRs to the SISO multiuser detector as the *a priori* information in the next iteration.

It is well known that the traditional decorrelator has a worse performance compared with the traditional linear MMSE detector [12]. Therefore, the conclusion we obtained in Section 5.2.2, which is that the SISO decorrelator has a slightly worse performance than the linear SISO MMSE detector, is intuitive. Another possible reason is explained as follows. Based on (4.54) and (4.55), the signal to residual interference-plus-noise ration in $x_k^{MMSE}$ is $\text{SNR}_k = \frac{\mu_k^2}{\sigma_k^2} = \frac{\mu_k}{1-\mu_k}$ of user $k$ at the output of the linear SISO MMSE detector depends only on reliabilities $\{|\hat{b}_i|, i \neq k\}$ of soft symbol estimates, excluding their hard decisions $\{\text{sgn}(\hat{b}_i), i \neq k\}$. While for SISO decorrelator, its approximate cross-correlation matrix $\mathbf{R}_\mathbf{u}$ in (4.33) is based on both reliabilities and hard decisions of soft symbol estimates. Therefore, the linear SISO MMSE detector is more robust against soft inputs with low reliabilities than the SISO decorrelator.

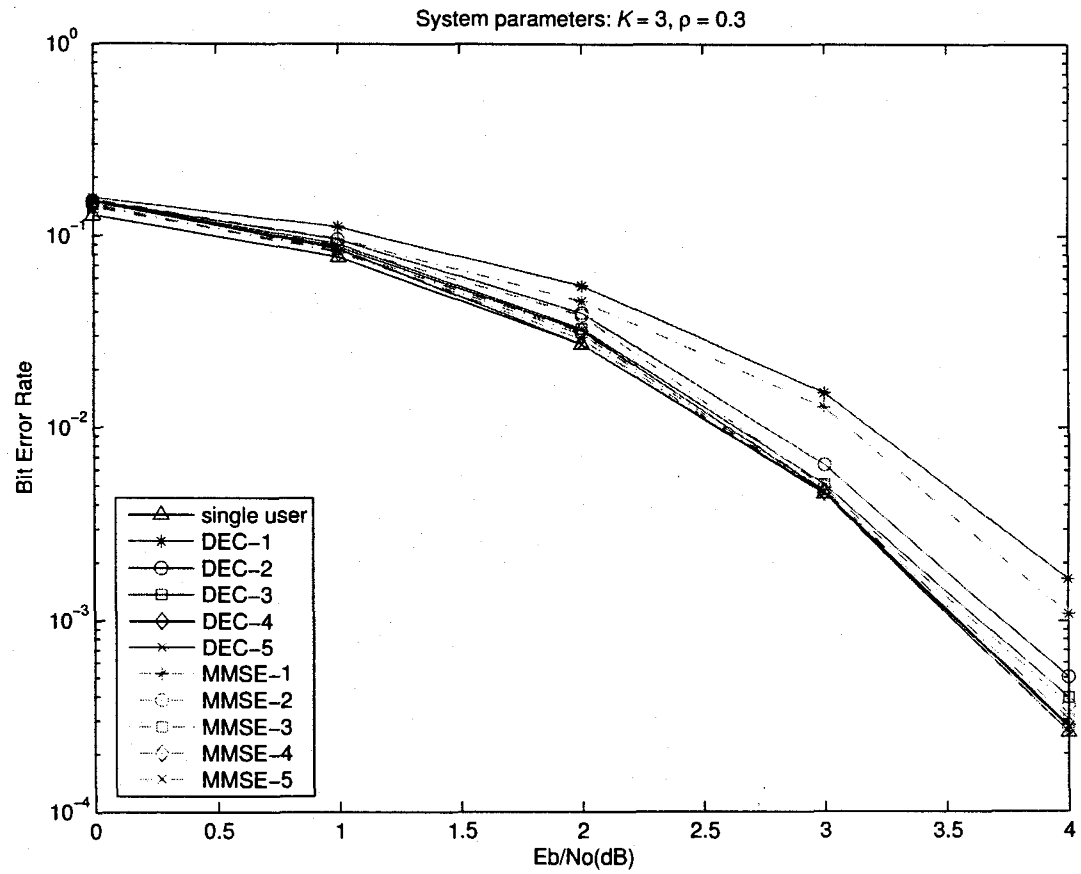


Figure 5.9 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 3$, $\rho = 0.3$

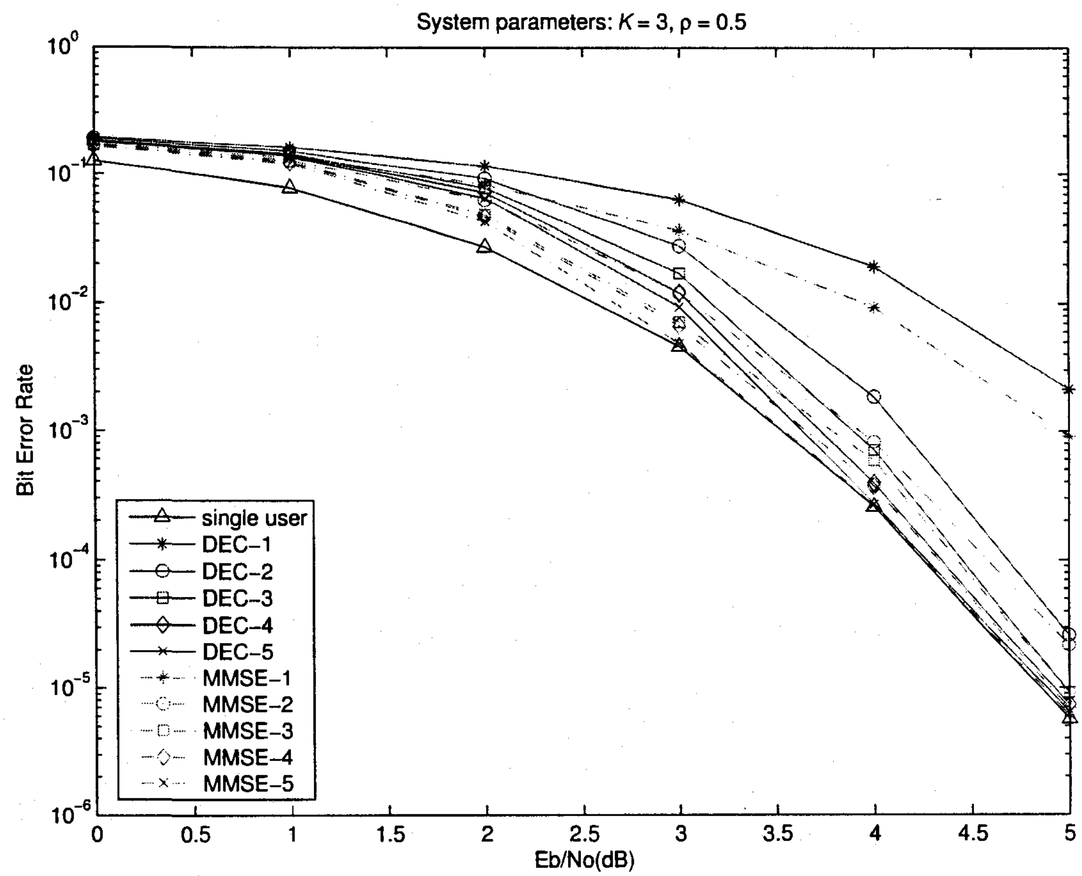


Figure 5.10 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 3$, $\rho = 0.5$

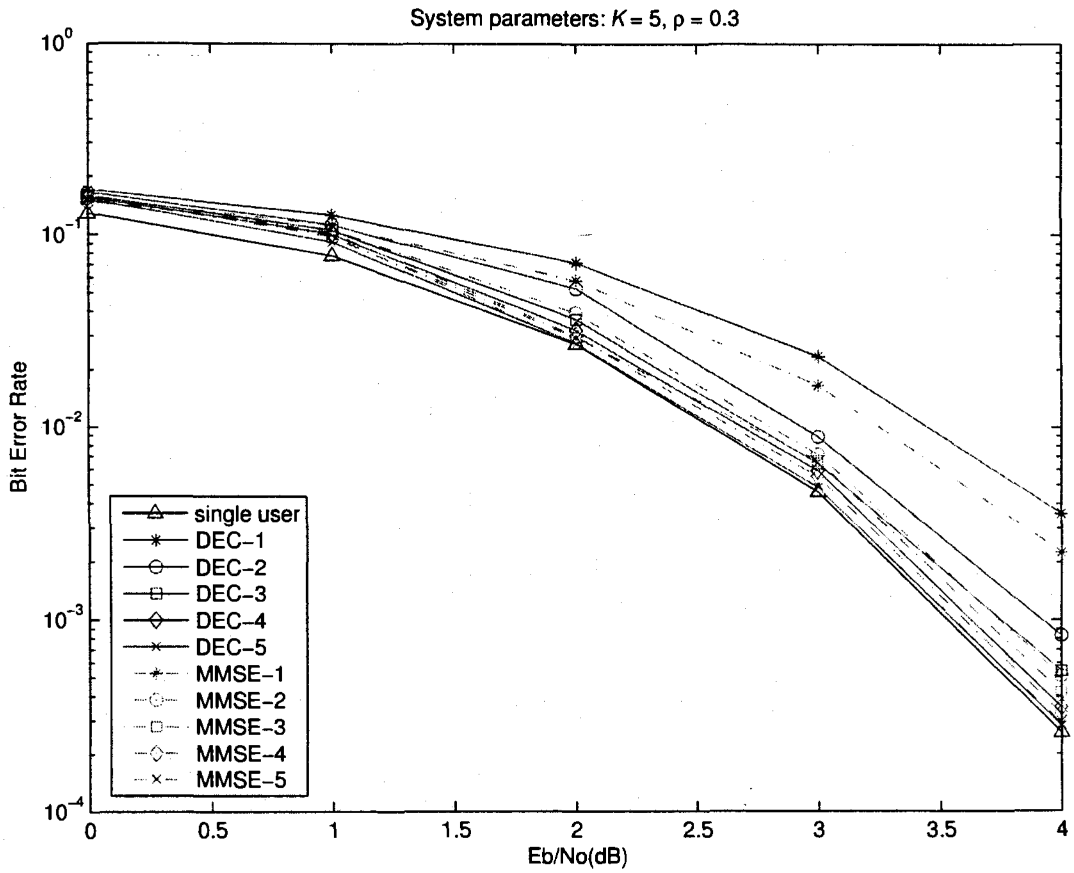


Figure 5.11 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 5$, $\rho = 0.3$

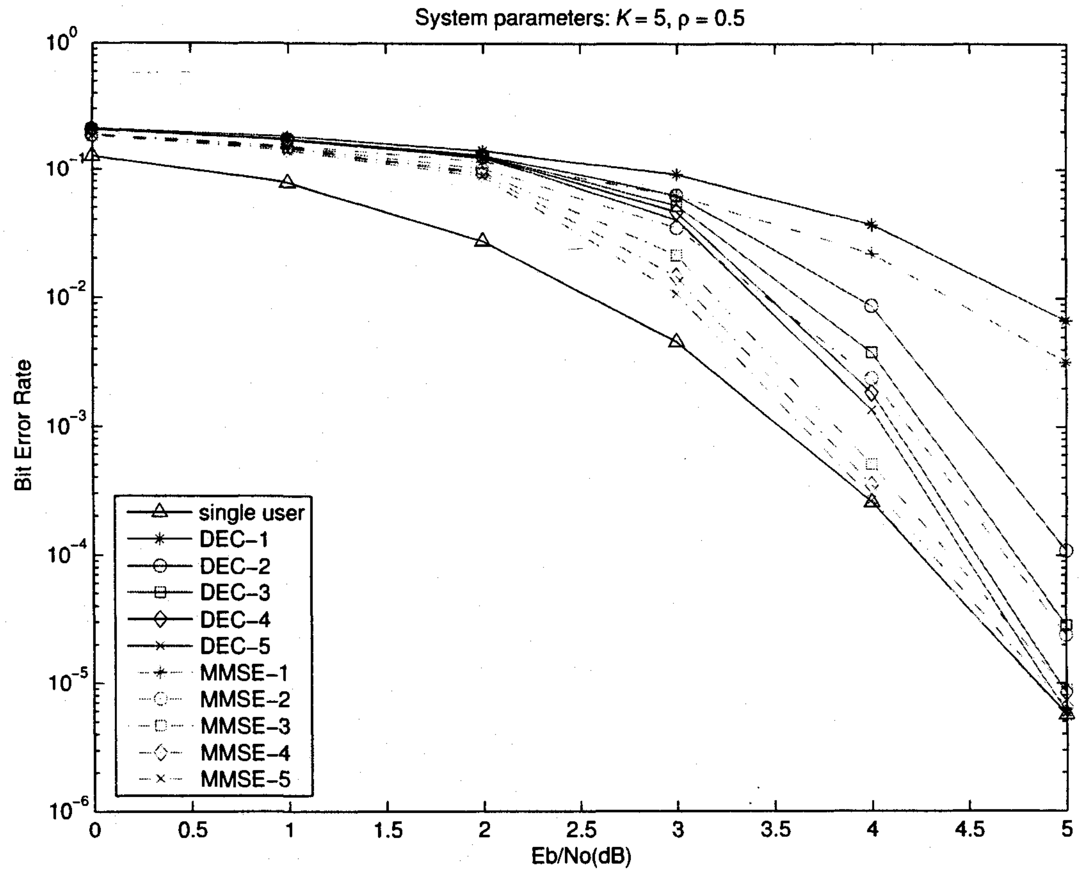


Figure 5.12 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 5$, $\rho = 0.5$

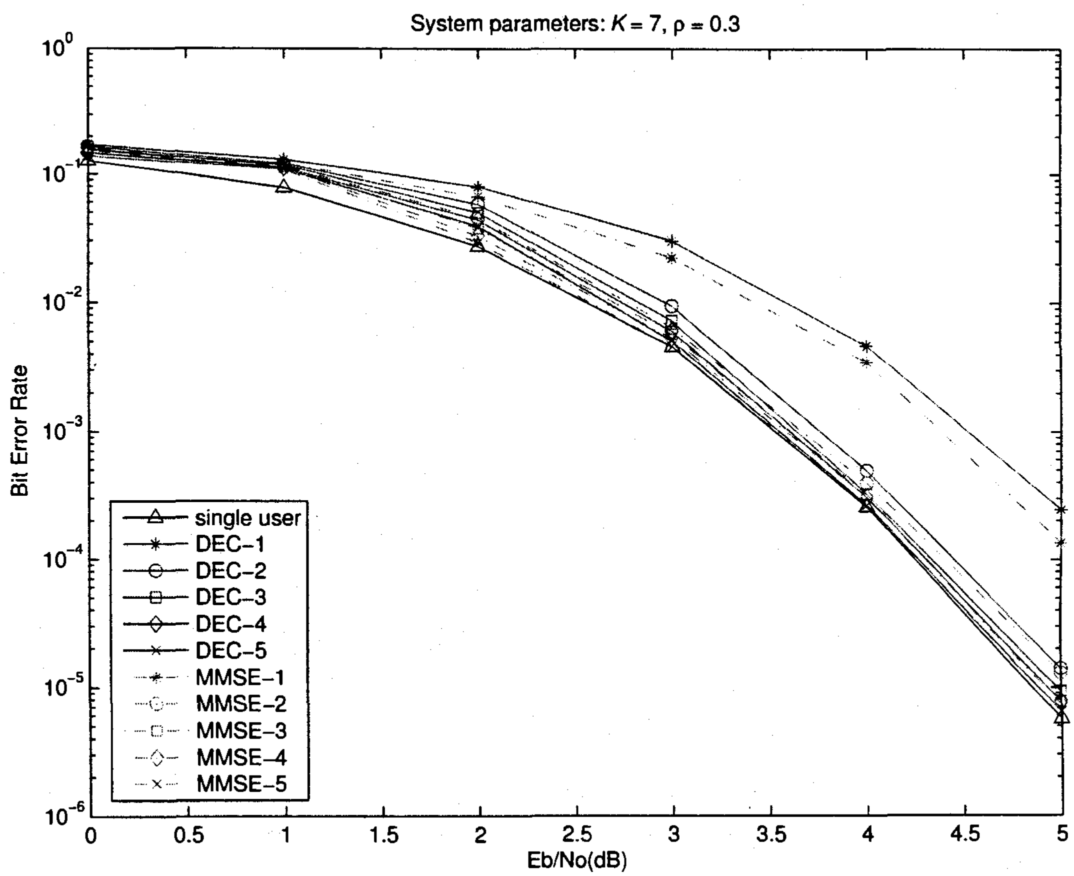


Figure 5.13 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 7$, $\rho = 0.3$

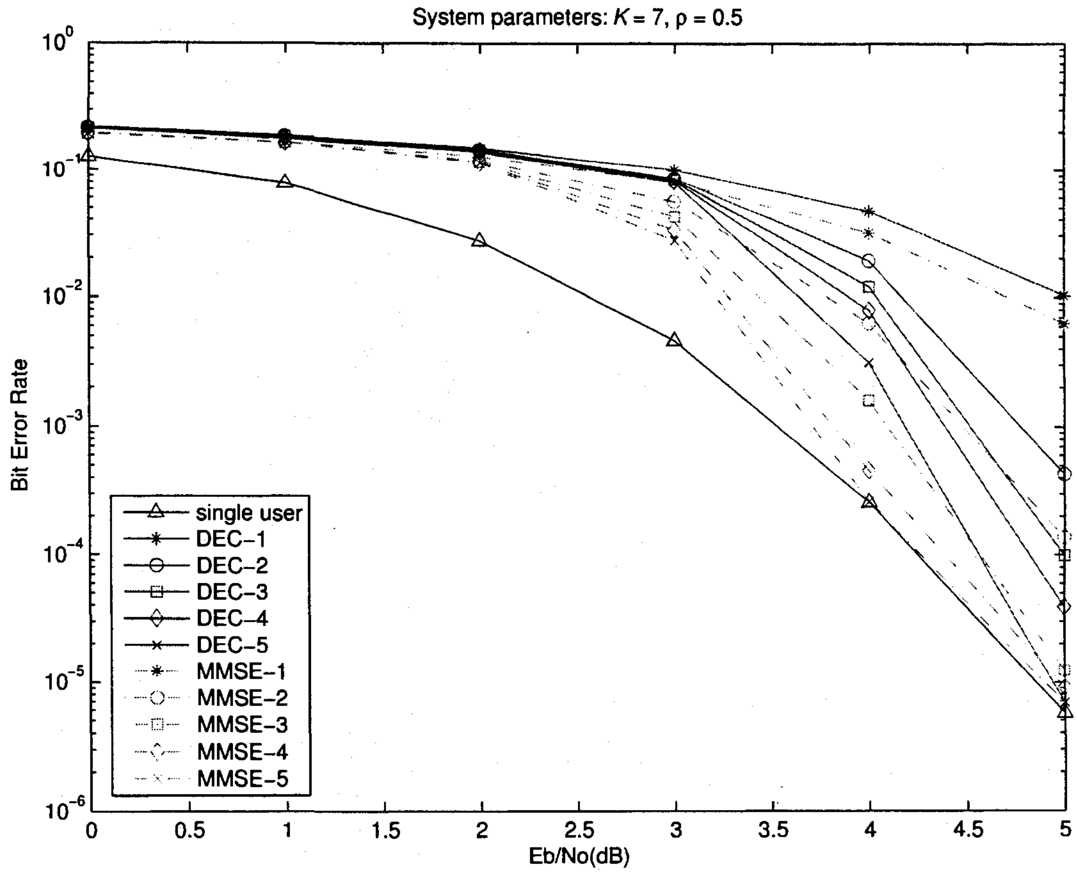


Figure 5.14 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 7,\ \rho = 0.5$

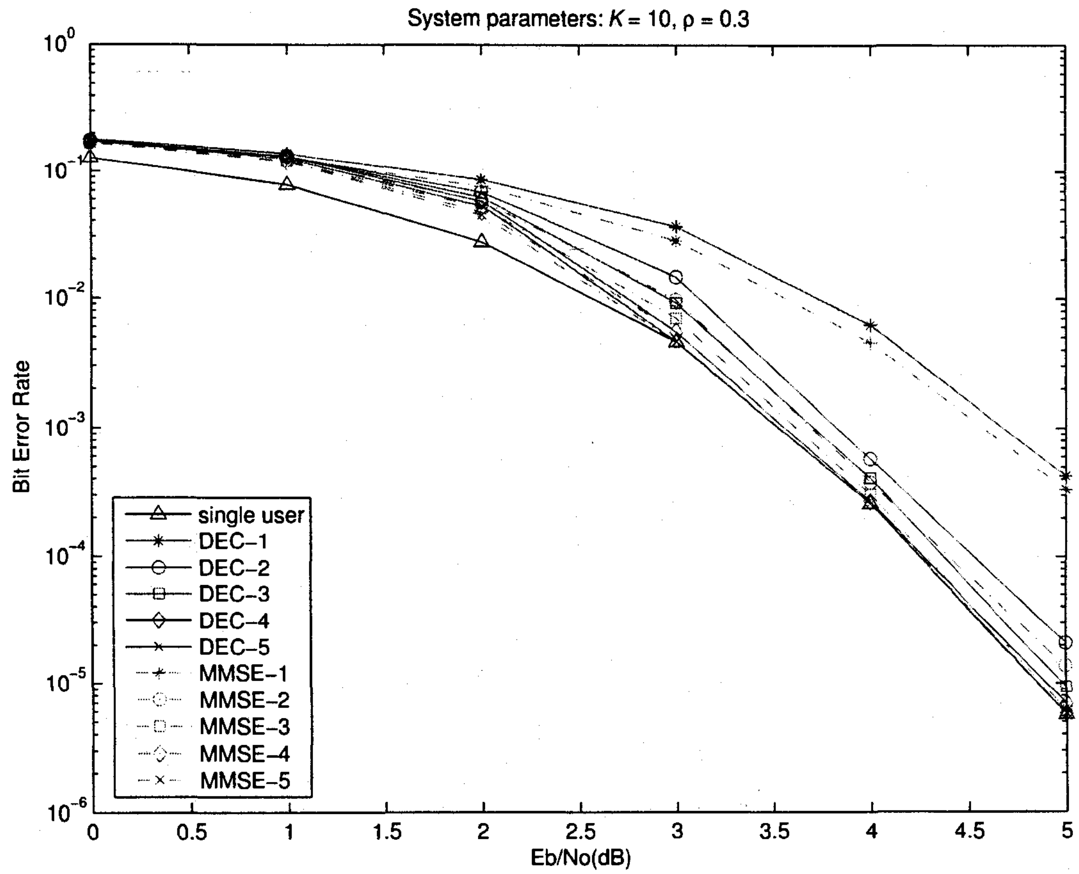


Figure 5.15 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 10$, $\rho = 0.3$

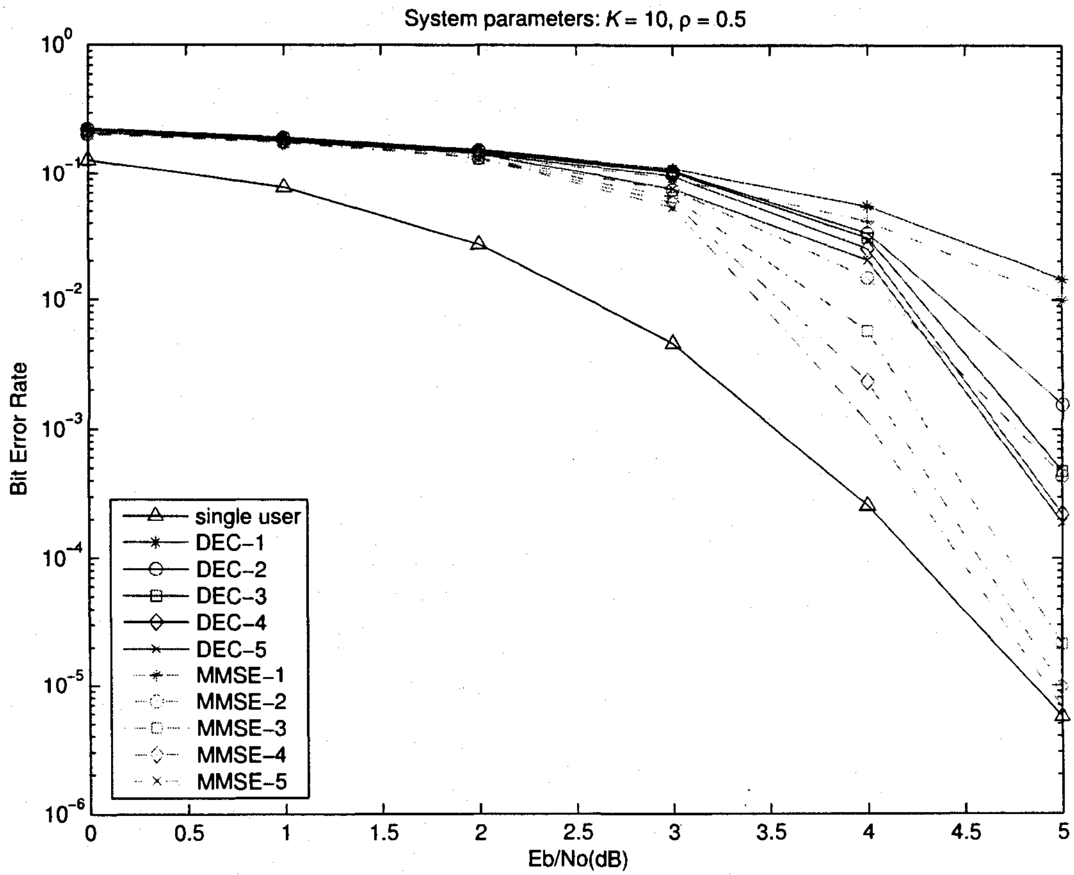


Figure 5.16 Comparison of bit error rate performances between the SISO decorrelator at the first five iterations (DEC-1 – DEC-5), the linear SISO MMSE detector (MMSE1 – MMSE-5) and the single user system for $K = 10$, $\rho = 0.5$

## 5.3 Summary of Simulation Results

In this section, we compare the three types of SISO multiuser detectors presented in the Chpater 4. All of these detectors consist of soft interference cancellation, which makes use of soft inputs of detectors provided by a bank of SISO single user decoders. In addition, they further successfully exploit these soft inputs in their own ways. This is confirmed by their performance analysis and simulation results, which show that the resulting system performance approaches to that of the single user system quickly with the increased number of iteration.

The decorrelating detectors and linear MMSE detectors are successfully adapted after the soft interference cancellation. These detectors cooperate iteratively with a bank of SISO single user decoders. The resulting system performance approaches to that of the single user system after only a few iterations for moderate and high signal-to-noise ratios. The simulation results show that the SISO MMSE detector has the best performance while the soft interference cancellation detector has the worst.

The performance of SIC is very sensitive to the number of users and the value of the cross-correlation, while the performance of the decorrelator and MMSE is more resistant to increases in these parameters. We assume that the SISO decorrelator and SISO MMSE detector will have good near-far resistant for lower power users. This has been shown in the system coded with convolutional codes [49].

# Chapter 6

# Conclusions and Future Research

## 6.1 Conclusions

In this thesis, we propose iterative coded multiuser detection employing LDPC codes. In this receiver, the MUD and the LDPC decoding are combined. In each iteration, the soft information is exchanged between these two components. Thanks to the exchange, both the multiuser detector and the LDPC decoder can be continuously improved. Therefore, the iterative joint MUD and decoding can achieve a substantial performance gain over separate detection and decoding. Performance comparison of these three SISO detectors has been provided in the thesis. The simulation results show that the system performance converges to that of single user system after a few iterations for moderate and high signal-to-noise ratios. The performance in the second iteration shows significant improvement. With the increased number of iterations, the additional improvement from one iteration to the next decreases. The SISO MMSE detector has the better performance than the SISO decorrelating detector while the SISO decorrelating detector has better performance than the soft interference cancellation detector.

## 6.2 Future Research

In this thesis, we simulated and discussed performance of LDPC-coded systems on an AWGN channel. We can further examine the performance of the different multiuser detectors in fading multipath channels which are more appropriate for wireless communication [24].

In our simulations, the LDPC codes we used is (204, 102) Gallager code. We used this code due to low complexity for simulation purposes. The performance of the different detectors with other LPDC codes should be examined.

In the described system, the users have the same received energy. The effect of strong users and weak users in a near-far situation should be studied. Near-far resistance of three different detectors can be investigated.

To further improve the performance, the use of interleavers should be investigated. Since the noise encountered by different users on the same signaling interval is correlated, all users may encounter dependent error patterns. The performance of coded MUD depends on different users being able to decode their codewodes so that they may be able to feed reliable information back to the multiuser detector. When the error patterns encountered by different users are dependent, the decoding abilities of the different users are also correlated. By assigning different interleavers to different users, we randomize the error patterns as seen by different users, making it less likely that one user's inability to decode his or her codeword affects all other users' decoding abilities.

The number of users employed in out simulation is at most 10 due to excessive running time at higher number of user. Also, the simulation uses Monte-Carlo simulation technique which has a high complexity to get the bit error rate performances. If we can recognize some underlying factors which dominantly influence performances of iterative

SISO detectors, that will be much helpful. By analyzing these factors, we can better understand and predict their performance limits and characteristics such as convergence speed.

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