

Vaccine and Inclusion

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Abstract

In majoritarian democracies, popular policies may not be inclusive, and inclusive policies may not be popular. This dilemma raises the crucial question of when it is possible to design a policy that is both inclusive and popular. We address this question in the context of vaccine allocation in a polarized economy facing a pandemic. In such an economy, individuals are organized around distinct networks and groups and have in-group preferences. We provide a complete characterization of the set of inclusive and popular vaccine allocations. The findings imply that the number of vaccine doses necessary to generate an inclusive and popular vaccine allocation is greater than the one necessary to obtain an allocation that is only popular. The analysis further reveals that it is always possible to design the decision-making rule of the economy to implement an inclusive and popular vaccine allocation. Under such a rule, the composition of any group endowed with the veto power should necessarily reflect the diversity of the society.

Keywords: Inclusive vaccine policy; Leader popularity; Voting rule; Polarization; Social networks; In-group preferences; Minorities.

JEL Classification: C62, C70, D63, D71, D72.

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1 Introduction

In majoritarian democracies, the allocation of scarce resources tends to favor the interests of the majority group at the expense of those in minority groups. John Stuart Mill (1860) argued that the tyranny of the majority results in the oppression of minority factions, and he compared the power wielded by the majority to that of a despot. Painful memories of such coercion recently resulted in widespread protests in several western democratic countries around the world, forcing the leaders of these countries to consider the need for their policies to be both *just* and *inclusive*. For example, on June 25, 2021, the American President, Joseph R. Biden, issued an executive order strengthening the implementation of diversity, equity, inclusion, and accessibility in the federal workforce (The White House, 2021). In 2019, the Conservative Party in the United Kingdom (U.K.) won the most seats in parliament, offering a landslide victory to Prime Minister Boris Johnson. He responded by selecting the most diverse cabinet in U.K. political history, aiming to address some of the most pressing challenges facing the country’s polarized communities (Bokhari, 2019). Recently, in response to the COVID-19 pandemic, organizations worldwide participated in the COVAX program—a global initiative co-led by GAVI, the World Health Organization (WHO), and the Coalition for Epidemic Preparedness Innovations (CEPI)—to push for a fair and inclusive allocation of COVID-19 vaccines globally.

Motivated by these real-world examples, the natural question that arises is whether democratic political leaders who, in general, focus on short-term re-election strategies have enough incentives to implement inclusive policies. These policies are likely to be unpopular, especially if they are perceived as taking away scarce resources from the majority group and allocating them to minority groups. The unpopularity of inclusive policies may be more pronounced in societies organized around distinct networks and groups. However, if closer contacts receive priority in resource allocation, individuals may be less agitated in such communities. Hence, there exists a tradeoff between the popularity and the inclusiveness of policies in fragmented societies. This observation raises the following question: under what conditions can a political leader implement an allocation policy that is both popular and inclusive?

We address this question in the context of the allocation of vaccine doses during a pandemic. We propose a vaccine allocation model described as follows. A society consisting of a finite set of individuals or agents, $N = \{1, 2, \dots, n\}$, with $n \geq 3$, is facing the spread of a deadly virus. A safe and effective vaccine against this virus is approved for emergency use. However, the number of available doses, μ , is limited: $\mu < n$, where μ is a non-zero integer. Given N and μ , an allocation of the μ vaccine doses to individuals is any $S \subseteq N$ such that $|S| = \mu$, where $|S|$ denotes the

cardinality of S . We denote by $N(\mu)$ the set of allocations of μ vaccine doses. Each agent $i \in N$ assigns to agent $j \in N$, a real number denoted by δ_j^i . The set $\delta^i = \{\delta_j^i\}_j$ defines the order of priority desired by the agent i for access to vaccination. Given δ^i , the weighted utility provided to agent i by a given vaccine allocation X in $N(\mu)$ is defined by $u_i(X) := \sum_{j \in X} \delta_j^i$. Each agent i 's weighted utility u_i induces a *weighted preference relation* (weak ordering), which we denote by \succsim_i , over $N(\mu)$. For a given society, the *influence structure* \mathcal{I} describes the set of *influential* coalitions—groups of agents that have the power or ability to rule out any proposed allocation in $N(\mu)$ regardless of the preferences of agents outside of these coalitions. We refer to the pair $E = (N, \mathcal{I})$ as a *political economy* and the couple (E, μ) as a *vaccine allocation problem*. For any preference profile (\succsim_i) , we consider the tuple $G = \langle (E, \mu), (\succsim_i) \rangle$ as a *vaccine allocation game*. Given these concepts, the goal of a political leader is to design a vaccine allocation solution that only proposes *popular* or *stable* allocations, i.e., allocations that would not be successfully challenged by influential coalitions in society. In this general framework, we first derive some conditions under which a stable vaccine allocation exists in a homogeneous society (i.e., either a society that does not contain multiple distinct groups, or a society in which no individuals express in-group preferences). Then, to account for inclusion, we consider a polarized society organized around distinct groups where individuals have in-group preferences. We say that a vaccine allocation is *inclusive* when it contains agents from each group in the society. Then, we ask the following questions: under what conditions can a political leader consistently implement vaccine allocation policies that are both popular and inclusive? How does their ability to enforce inclusive vaccine allocation policies depend on factors such as the supply of vaccines and the distribution of power among influential groups in a polarized society?

We investigate these questions using the *core* (Shapley, 1955; Gillies, 1959) as a solution concept. In a political context, the core is a measure of both policy *stability* and *popularity*. When a policy X does not belong to the core, another policy Y is preferred to X by an influential coalition, meaning that X is unstable and unpopular. In our general vaccine allocation problem, Theorem 1, Proposition 1, and Theorem 2 show that both the number of vaccine doses and the society's influence structure are critical for a stable vaccine allocation. Using these results, we determine the existence of stable vaccine allocations in any vaccine allocation game. Theorem 2, our characterization result, shows that the core of a vaccination allocation game G is the intersection of a non-empty set $\mathcal{H}(N, \mathcal{I})$ and the set of feasible vaccine allocations, $N(\mu)$. Therefore, given the limited supply of vaccine doses, Corollary 1 derives the minimum number of doses that guarantees a stable vaccine allocation in a political economy. We extend the analysis to polarized societies organized around two distinct groups, where individuals have in-group preferences.

In-group preferences and weighted preferences all belong to the domain of other-regarding preferences.¹ Generally, factors such as ethnic, cultural, or gender proximity account for the establishment of groups in society. We can justify in-group preferences for two reasons. First, agents may display altruism towards their group members. Second, an agent's vulnerability to infection decreases with an increase in vaccination rate among closer contacts. We derive a necessary and sufficient condition for an inclusive vaccine allocation in a polarized political economy with two groups. Our first set of analyses covers one-dimensional polarized societies. Assuming that the influence structure is such that each of the two groups is influential, Theorem 3 shows that any stable vaccine allocation should provide one vaccine dose to each agent. Theorem 4 stipulates that if only one group is influential, then any stable allocation should provide a vaccine dose to each agent in that group. The latter finding implies that not being influential negatively affects the wellbeing of a group's members. Theorem 5, on the other hand, states that when no group constitutes an influential coalition, then the planner should supply at least two vaccine doses to ensure vaccine allocation stability in the society. Finally, Proposition 2 offers a characterization of the stable and inclusive (balanced or perfect inclusion, respectively) vaccine allocations for multi-dimensional polarized societies. Proposition 2 implies that the number of doses necessary to generate a stable and balanced vaccine allocation is greater than the one necessary to obtain a vaccine allocation that is both stable and inclusive, as the latter case requires a larger number of doses to ensure that the vaccine allocation is stable. Therefore, it is challenging to ensure vaccine inclusiveness in a fragmented society that does not have the means to produce or acquire a sufficiently high number of doses. When inclusiveness fails, any stable vaccine allocation favors the majority group or group with the decisive (or veto) power. This latter finding is consistent with studies showing that when voting rights are unequally distributed, individuals with greater voting power have a greater ability to induce social outcomes that maximize their preferences.²

The problem of the allocation of scarce resources during health crises is classical.³ Following the global shortage of essential medical resources as a direct consequence of the COVID-19 pandemic, governments worldwide had to enforce non-pharmaceutical control policies—lockdowns, targeted quarantines, mask wearing, and social distancing—to contain the contagion until the

¹Other-regarding preferences are widely studied; see, for example, Rabin (1993), Fehr & Schmidt (1999), Dufwenberg et al. (2011), Dimick et al. (2018), an excellent survey by Kagel & Roth (2020), and recently Bosi et al. (2021), Egorov et al. (2021), and Campos-Mercade et al. (2021). In the context of measuring social welfare from allocating perfectly divisible goods using normative principles (such as fairness and efficiency) among agents who display ordinal and other-regarding preferences, we refer to the work of Treibich (2019) and the references therein.

²See, e.g., Pongou & Tchantcho (2021) and Nganmeni et al. (2021).

³For the case of vaccines, see, e.g., a survey by Duijzer et al. (2018a).

approval of possible vaccines and treatments. Under such situations, scientists and policymakers suggested priority measures based on different ethical considerations. Since those allocation mechanisms involve tradeoffs, finding optimal and peaceful solutions are the primary concern for elected officials. The vaccine case is unique given its externality effect and its health dimension. Although almost all countries generally face scarcity of doses upon vaccine discovery, the problem is exacerbated in developing countries given their lack of economic, medical, and advanced technological resources (Gori et al., 2021a). For this reason, our framework could probably apply better to a developing country. However, our allocation approach can also inform political leaders in Western countries. Indeed, during a pandemic, policymakers in developed nations must calibrate vaccination policies that will not affect their popularity. Strikes due to either mandatory vaccination campaigns (Austen, 2021) or vaccine priority mechanisms perceived as unfair by some groups of the society (Lori et al., 2021), among others, can significantly reduce their chances of survival in the government. In this study, we assume that individuals have a positive attitude towards vaccines and want to receive a vaccine dose.⁴ This assumption is still very general in contexts where vaccine production is costly, with the cost being supported by taxpayers or international donors as in developing countries (Gori et al., 2021a). When governments use fiscal policy returns to support vaccine doses, we can show that all our results will remain valid if we assume that certain individuals are vaccine-hesitant. Vaccine-hesitant individuals will not receive any dose in stable vaccine allocations.

The issue of inclusion and global coordination in allocating scarce resources such as vaccines is timely. Studies show that fragmented and segregated communities have been hit hardest by the COVID-19 pandemic (Debnam Guzman et al., 2022), and restrictive policies—policies that favor population health over economic gains—benefited minority groups less in the United States (see, e.g., Pongou et al. (2022a)). As such, the issue of inclusion has received the attention of several academics and policymakers during the current COVID-19 pandemic. Our study joins the ongoing wealth of contributions pushing for inclusive actions in the fight against COVID-19. We can cite, among others, So & Woo (2020), Emanuel et al. (2020), Lancet (2020), Zard et al. (2021), Shete et al. (2021), and Norheim et al. (2021). Our findings show that implementing vaccine allocations representing a society's structure does not hurt the popularity of either a selfless or

⁴In reality, individuals may have different attitudes towards vaccination. Not everyone in the population wishes to be vaccinated. Indeed, the issue of vaccine hesitancy or the reluctance of people to receive safe and recommended available vaccines (MacDonald & The SAGE Working Group on Vaccine Hesitancy, 2015) is prevalent. Troiano & Nardi (2021) and Machingaidze & Wiysonge (2021) document several drivers of vaccine hesitancy in the era of COVID-19. For instance, the recent protests in Canada are evidence of some anti-vaccination campaigns against a compulsory mandate of the COVID-19 vaccines (Austen, 2021).

selfish political leader. Indeed, during times of crisis voters generally appreciate leaders with humanitarian principles and good leadership (Healy & Malhotra, 2009; Bechtel & Hainmueller, 2011; Bol et al., 2021). For instance, the recent landslide re-election victory of the Jacinda Ardern-led Labour government in New Zealand is evidence of the rewards political leaders can reap from good leadership during pandemics and other global crises (Goldfinch et al., 2021; Paull, 2021).

Our work contributes to the growing literature, which recommends considering economic, social, and demographic dimensions when allocating vaccines during pandemics. Along this line, some studies suggest using vaccine effectiveness (Kirwin et al., 2021; Matrajt et al., 2021), cost-benefit analyses by mixing epidemiological and economic models (Mylius et al., 2008; Medlock & Galvani, 2009; Matrajt et al., 2013; Duijzer et al., 2018b; Rao & Brandeau, 2021), mechanism design approach (Westerink-Duijzer et al., 2020; Xue & Ouellette, 2020; Akbarpour et al., 2021; Pathak et al., 2021; Castillo et al., 2021; Kremer et al., 2022), and ethics (Yi & Marathe, 2015; Wu et al., 2020; Emanuel et al., 2020; Nichol & Mermin-Bunnell, 2021; Pathak et al., 2021).⁵ In a recent study, Forman et al. (2021) highlight eleven COVID-19 vaccine challenges and potential solutions that may help decision-makers develop sound, safe, and effective vaccination campaigns against current and future disease outbreaks. Although most of these contributions cover the issue of vaccine inclusiveness, we differ in scope and analysis. We propose a framework in which political leaders could benefit from good leadership during pandemics by exploiting the society's influence structure and vaccine technology to implement popular and inclusive vaccine allocations.

Our paper also complements other studies that have proposed interacting economics and epidemiology models on designing optimal non-pharmaceutical interventions during a pandemic like COVID-19. For recent contributions, see Acemoglu et al. (2021), Gollier (2020), Gori et al. (2021a,b), Hritonenko et al. (2021), Gallic et al. (2021), and Rothert (2021), Pongou et al. (2022b); see also Boucekkine et al. (2021) on the economics of epidemics and contagious diseases.

Finally, we also contribute to the literature that uses tools from voting environments to solve allocation problems. Examples include, but are not limited to, the apportionment and proportional representation problems (Johnston, 1983; Balinski & Laraki, 2007; Florek, 2012; Brill et al., 2018; Jones et al., 2020), and the claim problems (Ju et al., 2007; Flores-Szwagrzak, 2015). Our study is closely related to the coalition structure model introduced by Aumann & Dreze (1974), and a recent modeling approach by Martin et al. (2021). In line with these studies, our conceptual

⁵See, also, Bonneuil (2021) who suggests using the total number of expected years left to live of patients eligible for ventilation as a triage protocol in the management of scarce ventilators at the peak of the COVID-19 crisis in French hospitals.

framework uses weighted preferences to consider the patterns of relationships between agents. We identify necessary and sufficient conditions under which it is possible to design a vaccine allocation policy that is popular and inclusive in a fragmented political economy.

The rest of the article is organized as follows. Section 2 describes the vaccine allocation problem. Section 3 provides general results on the existence and determination of stable vaccine allocations. Section 4 examines the issue of inclusive vaccine allocation problems in a polarized society, and Section 5 concludes.

2 Vaccine Allocation Problems

2.1 Preliminary Definitions

In our theoretical model of vaccine allocation, $N = \{1, \dots, n\}$ is a set of n **agents** indexed from 1 to n , where n is a non-zero integer, and μ is the number of vaccine doses available to these agents. We denote by 2^N the set of coalitions or non-empty subsets of N . Each agent requests a vaccine dose, but the supply of vaccines cannot satisfy total demand as $\mu < n$. A **vaccine allocation** is a possible way to choose from N , the μ agents who can receive the vaccines. If for any subset $E \in 2^N$, $|E|$ refers to the cardinality of E , then we denote the set of possible (or feasible) vaccine allocations as $N(\mu) := \{X \in 2^N : |X| = \mu\}$. Note that the available number of vaccine doses depends both on a society's economic purchasing power and its technological production capacity. For example, in less developed societies which lack the technology required to produce the vaccine, the parameter μ may be relatively low in comparison to its value in more developed countries.

2.2 Weighted Preferences and Euclidean Preferences

Each agent obtains some utility from any feasible vaccine allocation. We consider an ordinal approach in the formalization of agents' utility. We assume that agents have *ordinal preferences* over the set of feasible vaccine allocations. In our model, we distinguish two preference domains.

Weighted preferences. In this domain, agent $i \in N$ assigns to each agent $j \in N$ a real number denoted by δ_j^i , which measures how much utility accrues to i if j is vaccinated.⁶ Given $\delta^i = \{\delta_j^i\}_j$,

⁶For instance, an individual might derive more health benefits from a closer relationship being vaccinated, especially if a vaccinated person is less likely to be infected or to transmit the virus than an unvaccinated person. In a society where people are connected through a network, δ_j^i can be inversely proportional to the shortest

the weighted utility provided to agent i by a given X in $N(\mu)$ is defined by $u_i(X) := \sum_{j \in X} \delta_j^i$. Each agent i 's weighted utility u_i induces a *weighted preference* over the set $N(\mu)$, denoted \succsim_i , such that for all $X, Y \in N(\mu)$, Y is at least as preferred as X for agent i , denoted $X \succsim_i Y$, if $u_i(X) \leq u_i(Y)$. The relation \prec_i denotes the strict part of \succsim_i , and the relation \sim_i denotes the indifference part of \succsim_i .

Euclidean preferences. Euclidean preferences constitute a specific class of weighted preferences. We use elements of the usual Euclidean affine geometry to define Euclidean preferences. For this purpose, let m be a non-zero integer. Each agent i is represented in the m -dimensional Euclidean space \mathbb{R}^m by a position q^i , also called the ideal point; see, for example, Owen (1971), Shapley (1977), and Frank & Shapley (1981). A given set of positions $\{q^i\} \equiv \{q^1, \dots, q^n\}$ corresponds to a configuration of the agents' ideal points. Let d be the usual Euclidean distance. We need to choose the weights δ such that for all $i, j \in N$, δ_j^i decreases when the distance between the ideal points of i and j increases. One can choose, for instance, $\delta_j^i = -d(q^i, q^j)$. For any coalition S and any agent i , let us define $d(i, S) = \sum_{j \in S} d(q^i, q^j)$. Remark that $d : N \times 2^N \rightarrow \mathbb{R}$ is not necessarily a metric in the mathematical sense.

2.3 Influence Structures

Generally, coalitions that decide upon which group of μ agents should receive the available vaccine doses in society have some decisive (or veto) power under a given pre-approved set of rules. An **influential coalition** (or a **winning coalition**) is any group S of agents endowed with the ability to challenge any proposed allocation $X \in N(\mu)$. \mathcal{I} denotes the set of influential coalitions, and it is fixed *a priori*. An influential structure can therefore be interpreted as a power-sharing agreement and can be designed to account for health, political, social, and economic factors. We assume that \mathcal{I} satisfies the following two conditions: (1) $\emptyset \notin \mathcal{I}$ and $\mathcal{I} \neq \emptyset$; and (2) for all coalitions S and T , if $S \in \mathcal{I}$ and $S \subseteq T$, then $T \in \mathcal{I}$. According to assumption (1), an empty coalition cannot influence the allocation of vaccines. Assumption (1) also states that at least one influential coalition exists. Assumption (2) is a classical monotonicity condition which stipulates that the addition of new agents to an influential coalition yields another influential coalition. The two assumptions together imply that the entire community, N , is an influential coalition. We consider an influence structure, \mathcal{I} , to be *weighted* if there exists a real number α called a quota, with $0 < \alpha < n$, and for each $i \in N$, there exists a non-negative weight, α_i , so that $S \in \mathcal{I}$ if

distance between i and j .

and only if $\sum_{i \in S} \alpha_i \geq \alpha$. In this case, we denote $\mathcal{I} = [\alpha; \alpha_1, \dots, \alpha_n]$, and we call \mathcal{I} a α -weighted influence structure. When $\alpha_i = \alpha_j$, for all agents i and j , \mathcal{I} is called a α -weighted symmetric influence structure.

2.4 Political Economies and Vaccine Allocation Games

A *political economy* is a pair $E = (N, \mathcal{I}, \{q^i\})$, and we refer to the couple (E, μ) as a *vaccine allocation problem*. A *vaccine allocation game*, which we denote G , is any pair $G = \langle (E, \mu), (\succsim_i) \rangle$, where $(\succsim_i) = (\succsim_1, \succsim_2, \dots, \succsim_n)$ is a preference profile over $N(\mu)$. Sometimes, we will also denote a vaccine allocation game as $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (\succsim_i) \rangle$ or $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle$, where $(u_i) = (u_1, u_2, \dots, u_n)$ represents the agents' utility profile over $N(\mu)$. We denote by \mathbb{V} the set of all vaccine allocation games.

2.5 Stable Allocations of Vaccines

Let $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle \in \mathbb{V}$. Given preference heterogeneity, implementing a vaccine allocation that satisfies all agents' societal expectations can be impossible with a limited supply of doses. Therefore, the goal of the political leader (or social planner) is to choose among $N(\mu)$, vaccine allocations that will get some degree of approval in society.

Definition 1. Let $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle \in \mathbb{V}$, and $X, Y \in N(\mu)$ be two vaccine allocations.

1. X is **challenged** by Y (or Y **challenges** X), which we denote by $X \prec Y$, if there is $S \in \mathcal{I}$ such that $u_i(Y) > u_i(X)$ for all $i \in S$.
2. A **stable** (or **popular**) vaccine allocation is not challenged in the game G .
3. The **core** of G , denoted by $C(G)$, is the set of stable vaccine allocations for G :

$$C(G) = \{X \in N(\mu) : \text{there is no } Y \in N(\mu) \text{ such that } X \prec Y\}.$$

A stable vaccine allocation will be adopted in society if it is proposed for approval. With the preference profile (u_i) or (\succsim_i) fixed in a game $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle$, we can also write the core of G as $C(E, \mu)$, where $E = (N, \mathcal{I}, \{q^i\})$ is a political economy, and μ is the number of vaccine doses. Keeping all the other elements in G fixed, we can determine the set of stable vaccine allocations, $C(E, \mu)$, for different values of μ .

2.6 Some Illustrations of the Vaccine Allocation Problem

The illustrations that we display below provide some intuition behind the characterization of stable vaccine allocations that we will derive in Section 3.

Consider two communities A and B, that consist of seven agents numbered from 1 to 7; that is, in each community, $N = \{1, \dots, 7\}$. We analyze the existence of stable vaccine allocations in some vaccine allocation games $G = \langle N, \mu, \mathcal{I}, \{q^i\}, \{u_i\} \rangle$ based on Euclidean preferences, assuming one-dimensional spatial configurations of agents' ideal points that we describe in Community A (Section 2.6.1) and Community B (Section 2.6.2). In our illustration, agents' ideal points are on the horizontal axis. Therefore, we derive agents' utilities (u_i) using the usual Euclidean distance. For simplicity, we focus on some symmetric weighted influence structures and remove (u_i) in the notation of a vaccine allocation game.

2.6.1 Vaccine Allocations in Community A

We represent Community A by a political economy $E = (N, \mathcal{I}, \{q^i\})$, where agents' ideal points $\{q^i\}$ are evenly spaced and displayed in the horizontal line in Figure 1. In this illustration, we consider allocation games that differ only with the endowed influence structure \mathcal{I} .

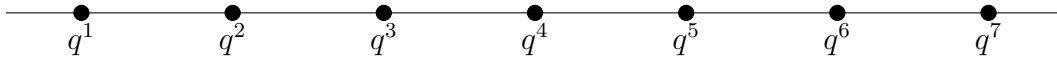


Figure 1: Configuration of agents' ideal points in Community A

Case 1A: Assume that $\mathcal{I} = [4; 1, \dots, 1]$. If $\mu = 1$, then there exists only one stable vaccine allocation that is $X = \{4\}$. Note that the influence structure \mathcal{I} is equivalent to the simple majority rule, and the stable allocation corresponds to the median agent. If $\mu = 2$, the set of stable allocations is $C(E, 2) = \{\{3, 4\}, \{4, 5\}\}$. Indeed, neither of these two allocations in $C(E, 2)$ can be challenged since any influential coalition must include agent 4. Still, agents only support peers whose positions are closer to their ideal points. Agent 4 achieves maximum satisfaction at the allocations $\{3, 4\}$ and $\{4, 5\}$. Therefore, agent 4 can't support the replacement of either $\{3, 4\}$ or $\{4, 5\}$. Alternatively, if for instance $X = \{4, 6\}$ is proposed, then $Y = \{3, 4\}$ can challenge X via the influential coalition $S = \{1, 2, 3, 4\}$. Following a similar approach, we can show that any allocation $X \notin C(E, 2)$ can be challenged by an influential coalition in \mathcal{I} . We observe that agents distant from the median agent are likely to belong to a stable allocation only if there is a sufficient number of doses. For instance, we can show that, $C(E, 3) = \{\{3, 4, 5\}\}$, $C(E, 4) = \{\{2, 3, 4, 5\}, \{3, 4, 5, 6\}\}$, $C(E, 5) = \{\{2, 3, 4, 5, 6\}\}$, etcetera.

Case 2A: Assume that $\mathcal{I} = [3; 1, \dots, 1]$. Unlike **Case 1A**, we can find two disjointed influential coalitions in \mathcal{I} . If the planner can only supply up to two vaccine doses, then there is no stable allocation; that is, $C(E, \mu) = \emptyset$ for $\mu \in \{1, 2\}$. For $\mu = 3$, the only stable vaccine allocation is $\{3, 4, 5\}$. For $\mu = 4$, there is also no stable vaccine allocation: $C(E, 4) = \emptyset$. For instance, if $X = \{2, 3, 4, 5\}$ is proposed for approval, then $Y = \{3, 4, 5, 6\}$ can challenge X via the influential coalition $S = \{5, 6, 7\}$. It follows that sometimes, an increase in the available number of vaccine doses can induce rivalries that make it challenging to obtain a stable allocation. To have a stable vaccine allocation, we must consider the same number of agents on either side of the median agent for this specific illustration. Supplying an odd number of vaccine doses (μ odd, $7 \geq \mu > 1$) in Community A can enable the planner to solve the vaccine allocation deadlock. For instance, $C(E, 5) = \{\{2, 3, 4, 5, 6\}\}$, $C(E, 6) = \emptyset$ and $C(E, 7) = \{N\}$. Notice that there is only one stable allocation in each case of available doses μ for which $C(E, \mu)$ is non-empty.

2.6.2 Vaccine Allocations in Community B

We represent Community B by a political economy $E = (N = N_1 \cup N_2, \mathcal{I}, \{q^i\})$ such that: $N_1 = \{1, 2, 3, 4\}$, $N_2 = \{5, 6, 7\}$, and agents' ideal points are represented in Figure 2. This configuration reflects a polarized society with two distinct and distant groups. In other words, the distance $d(q^4, q^5)$ is large compared to $d(q^1, q^4)$ and $d(q^5, q^7)$.

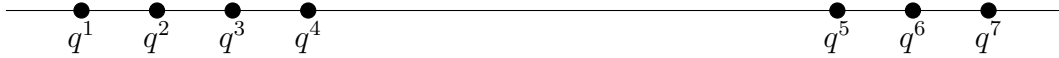


Figure 2: Configuration of agents' ideal points in Community B

Case 1B: As in **Case 1A**, we assume that $\mathcal{I} = [4; 1, \dots, 1]$. We have an influential group (N_1) and a non-influential group (N_2). Following a similar approach as in Section 2.6.1, we show that $C(E, 1) = \{\{4\}\}$, $C(E, 2) = \{\{3, 4\}\}$, $C(E, 3) = \{\{2, 3, 4\}\}$, $C(E, 4) = \{\{1, 2, 3, 4\}\}$, $C(E, 5) = \{\{1, 2, 3, 4, 5\}\}$, $C(E, 6) = \{\{1, 2, 3, 4, 5, 6\}\}$ and $C(E, 7) = \{N\}$. We note that when the supply of vaccine doses is sufficiently low in Community B, (that is, $\mu \leq 4$), agents 5, 6, and 7 are disadvantaged since they don't belong in any of the stable vaccine allocations. The latter shows that a social planner cannot implement an inclusive policy when a society is polarized, and the size of resources is low. We also note that, compared to **Case 1A**, for each supply of vaccine doses μ , the core $C(E, \mu)$ in **Case 1B** only consists of one allocation. We formalize **Case 1B** in Theorem 4.

Case 2B: As in **Case 2A**, we assume that $\mathcal{I} = [3; 1, \dots, 1]$. In this case, both groups N_1 and N_2 , are influential. This yields a chaotic situation that requires a large supply of vaccine doses to obtain a stable allocation. Precisely, for $\mu \leq 6$, $C(E, \mu) = \emptyset$ and $C(E, 7) = \{N\}$. We provide a formal result in Theorem 3.

Case 3B: We assume in this situation that $\mathcal{I} = [5; 1, \dots, 1]$. We have a polarized society but no group constitutes an influential coalition. For $1 \leq \mu \leq 3$, any subset of μ elements of $\{3, 4, 5\}$ is a stable allocation. For $4 \leq \mu \leq 7$, any subset of μ consecutive elements of N is a stable allocation. For instance, $C(E, 4)$ contains only the following allocations $\{\{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{3, 4, 5, 6\}$ and $\{4, 5, 6, 7\}\}$. Theorem 5 formalizes **Case 3B**.

3 General Characterizations and Determination of Stable Vaccine Allocations

This section provides three main results that determine the existence and characterize stable vaccine allocations in a political economy. In a vaccine allocation game $G = \langle N, \mathcal{I}, \{q_i\}, \mu, (u_i) \rangle$, we need to highlight the characteristic properties of a stable allocation. We deduce the first one directly from the definition of the concept of stable allocation according to which an allocation $X \in N(\mu)$ is stable if there is no alternative allocation $Y \in N(\mu)$ and influential coalition $S \in \mathcal{I}$ such that Y challenges X via S . In other words, if we consider any allocation Y and any influential coalition S , there is at least one agent of S who doesn't favor replacing X over Y . We can formally express these statements in the following remark.

Remark 1. An allocation X is stable if for any allocation Y and for any influential coalition S , there exists an agent $i \in S$ such that $u_i(Y) \leq u_i(X)$.

When the agents have Euclidean preferences, recall that for any coalition S and any agent $i \in N$, we set $d(i, S) = \sum_{j \in S} d(q^i, q^j)$. Given that $d(q^i, q^j) = -\delta_j^i$, we can write $d(i, S) = -\sum_{j \in S} \delta_j^i$ and then, by definition, $d(i, S) = -u_i(S)$. Hence, from Remark 1, an allocation X is stable under the Euclidean preference assumption, if for any allocation $Y \in 2^N$ and for any influential coalition $S \in \mathcal{I}$, there is an agent $i \in S$ such that $d(i, X) \leq d(i, Y)$. This property induces a well-known characterization of the elements of the usual core in a spatial voting game. Indeed, a position P is in the core of a spatial voting game if and only if P is on the convex hull of all the winning coalitions, see Saari (2014) or Martin et al. (2021). In a vaccine allocation game, if $\mu = 1$, then a set reduced to a single point located on the convex envelope of all the winning coalitions is a

stable allocation. However, it can be shown that the converse of the latter statement is not true. For an arbitrary value of $\mu \geq 1$, the use of Remark 1 in determining the stability of a vaccine allocation requires pairwise comparisons over the set $N(\mu)$. We need additional notations to provide other alternative characterizations of stable allocations. Let $u = (u_i)$ be an agents' utility profile. For any coalitions $S, T \in 2^N$, any agent $i \in N$, and any non-zero integer $k \leq \mu$, we consider the following sets:

- $B(i, T) = \{\Lambda \in 2^N : |\Lambda| = |T|, u_i(\Lambda) < u_i(T)\}$. The set $B(i, T)$ contains the subsets Λ with the same cardinality as T such that agent i prefers T to Λ . Under Euclidean preferences, we can write $B(i, T) = \{\Lambda \in 2^N : |\Lambda| = |T|, d(i, T) < d(i, \Lambda)\}$. If T contains only one element, i.e., $T = \{k\}$, then, $B(i, T)$ is the set of agents j such that $\delta_j^i < \delta_k^i$. Under Euclidean preferences, the latter implies that $d(q^i, q^k) < d(q^i, q^j)$.
- $B(S, T) = \bigcap_{i \in S} B(i, T)$. The set $B(S, T)$ contains coalitions Λ with the same cardinality as T such that any member of S prefers T to Λ .

We articulate Theorem 1.

Theorem 1. *Let $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle$ be a vaccine allocation game. A vaccine allocation $X \in N(\mu)$ is stable if and only if for any influential coalition $S \in \mathcal{I}$ and any coalition $T \in 2^N$, if there is a coalition $\Lambda \in 2^N$ such that $\Lambda \in B(S, T)$ and $\Lambda \subseteq X$, then $X \cap T \neq \emptyset$.*

Proof. We proceed by double implication.

\Rightarrow): Consider a stable allocation X , an influential coalition S , and a coalition T . Assume that there exists $\Lambda \in B(S, T)$ such that $\Lambda \subseteq X$ and let us show that $X \cap T \neq \emptyset$. If we assume, on the contrary, that $X \cap T = \emptyset$, then $Y = (X \setminus \Lambda) \cup T$ is an allocation such that $Y \setminus X = T$ and $X \setminus Y = \Lambda$. Recall that $\Lambda \in B(S, T)$ implies that $|\Lambda| = |T|$ and, then, $|Y| = |X| = \mu$. We must notice that the utility functions are additive on disjoint subsets. Implying that, for all subsets $L, L' \subseteq N$, if $L \cap L' = \emptyset$ then for any agent i , $u_i(L \cup L') = u_i(L) + u_i(L')$. Since X can be written as a disjoint union of the subsets $X \setminus Y$ and $X \cap Y$, for any $i \in S$, we have $u_i(X) = u_i(X \setminus Y) + u_i(X \cap Y) = u_i(\Lambda) + u_i(X \cap Y)$, likewise, $u_i(Y) = u_i(T) + u_i(X \cap Y)$. It follows that $u_i(Y) - u_i(X) = u_i(T) - u_i(\Lambda) > 0$ because $\Lambda \in B(S, T)$. We deduce that the allocation Y can challenge X via the influential coalition S , which contradicts the fact that X is stable. Thus, for any influential coalition S and any coalition T , if there is $\Lambda \in B(S, T)$ and $\Lambda \subseteq X$, then $X \cap T \neq \emptyset$.

\Leftarrow): Conversely, assume that for any influential coalition S and any coalition T , if there exist $\Lambda \in B(S, T)$ and $\Lambda \subseteq X$, then, $X \cap T \neq \emptyset$. We need to show that X is a stable vaccine allocation. By definition, if X is not stable, then we can find an allocation Y and an influential coalition S such that Y challenges X via S . This means that for any $i \in S$, $u_i(X) < u_i(Y)$. Let $\Lambda = X \setminus Y$ and $T = Y \setminus X$, it is clear that $\Lambda \neq \emptyset$, $T \neq \emptyset$ and $|\Lambda| = |T|$, since $|Y| = |X| = \mu$. For any $i \in S$, $u_i(X) < u_i(Y)$ is equivalent to $u_i((X \setminus Y) \cup (X \cap Y)) < u_i((Y \setminus X) \cup (X \cap Y))$ i.e., $u_i(\Lambda) < u_i(T)$. The coalitions T and Λ are such that $\Lambda \in B(S, T)$, $\Lambda \subseteq X$ and $X \cap T = \emptyset$. We get a contradiction of the initial assumption, and we conclude that X is a stable allocation. \square

Theorem 1 provides a way of checking whether a given vaccine allocation $X \in N(\mu)$ in any game $G = \langle N, \mathcal{I}, \{q_i\}, \mu, (u_i) \rangle$ is stable. We can formally use the search of coalitions $\Lambda \in B(S, T)$ given any influential coalition S and any allocation $T \in N(\mu)$ to confirm core vaccine allocations that we provided in Section 2.6. For clarity, we show in Example 1 an additional use of Theorem 1 in a simple two-dimensional vaccine allocation problem.

Example 1. Consider a political economy $E = (N, \mathcal{I}, \{q^i\})$ consisting of three agents, $N = \{1, 2, 3\}$. Agents' ideal points $\{q^1, q^2, q^3\}$ are displayed in a two-dimensional Euclidean plane (see Figure 3), where their coordinates are: $q^1 = (0, 0)$, $q^2 = (2, 0)$, and $q^3 = (1, 0)$. We consider

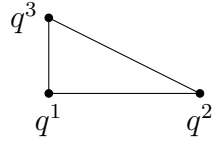


Figure 3: Configuration of agents' ideal points in community E

majority rule as the influence structure: $S \in \mathcal{I}$ if and only $|S| \geq 2$. Preferences are derived from the usual Euclidean metric given agents' ideal points. Let us assume that there are only two vaccine doses ($\mu = 2$) to be allocated in the community. In our hypothetical situation, $2^N = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$ and $N(2) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$.

- Consider $X = \{1, 2\}$. Is X a stable vaccine allocation? To answer that question, we use Theorem 1. We can note that the members of the influential coalition $\{1, 3\}$ prefer $\{3\}$ to $\{2\}$. Using Theorem 1 for $S = \{1, 3\}$, $T = \{3\}$ and $\Lambda = \{2\}$, we can conclude that $X = \{1, 2\}$ is not a stable allocation, since $X \cap T = \emptyset$.
- Consider $X = \{2, 3\}$. The members of $S = \{1, 3\}$ prefer $T = \{1\}$ to $\Lambda = \{2\}$, and $X \cap T = \emptyset$. Thanks to Theorem 1, the allocation $X = \{2, 3\}$ is not stable.

- Consider $X = \{1, 3\}$. The non-empty coalition T in 2^N that does not intersect X is $T = \{2\}$. However, there is no coalition $\Lambda \in 2^N$ such that $\Lambda \in B(S, \{2\})$ and $\Lambda \subseteq X$. Also, for either $S = \{1, 3\}$ or $\{2, 3\} \in \mathcal{I}$, there is no $\Lambda \in B(S, T)$ and $\Lambda \subseteq X$. Given that the first part of the second implication in the converse statement of Theorem 1 is false, the second implication stands. For $S = \{1, 2\} \in \mathcal{I}$, only $\Lambda = \{3\} \in B(S, \{1\})$, and $X \cap \{1\} \neq \emptyset$. It follows that $\{1, 3\}$ is the only stable vaccine allocation, and $C(G) = \{\{1, 3\}\}$.

More generally, using Theorem 1, it is straightforward to show that a vaccine allocation $X \in N(\mu)$ is not stable. We do not need to compare X with all allocations containing μ elements. Finding coalitions T and Λ containing fewer elements is enough to reach a non-stability result. However, showing the stability of an allocation requires more operations. One observation from Example 1 is that we can apply Theorem 1 by focusing on the coalitions T which do not intersect X . Such a coalition T must therefore be included in $N \setminus X$. Moreover, T cannot contain more elements than X , otherwise, we are sure that there does not exist Λ to be used with T in Theorem 1. In what follows, we formalize this alternative procedure of proving whether a vaccine allocation is stable. We need additional notations. Assume that μ is fixed in $\{1, \dots, n\}$ and consider an allocation $X \in N(\mu)$. Let n_X be the integer defined by $n_X = \min\{|X|, |N \setminus X|\} = \min\{\mu, |N \setminus X|\}$, since $X \in N(\mu)$ implies $|X| = \mu$. We also define the set $N_X = \{Y \subseteq N \setminus X : |Y| \leq n_X\}$. Proposition 1 follows.

Proposition 1. *Let $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle$ be a vaccine allocation game. A vaccine allocation $X \in N(\mu)$ is stable if and only if for any coalition $\Lambda \subseteq X$ and any coalition $T \in N_X$, if $|T| = |\Lambda|$, T does not challenge Λ .*

Proof. We proceed by double implication.

\Rightarrow): Assume that X is a stable vaccine allocation. Consider two coalitions Λ and T such that $\Lambda \subseteq X$, $T \in N_X$, and $|T| = |\Lambda|$. We want to show that T does not challenge Λ . Assume on the contrary that T challenges Λ . By Definition 1, there exists an influential coalition $S \in \mathcal{I}$ such that T challenges Λ via S . It follows that $\Lambda \in B(S, T)$. Since X is a stable vaccine allocation, Theorem 1 yields that $X \cap T \neq \emptyset$. The latter is a contradiction, because by assumption, $T \in N_X$. We conclude that T does not challenge Λ .

\Leftarrow): Assume that for any coalition $\Lambda \subseteq X$ and any coalition $T \in N_X$, if $|T| = |\Lambda|$, then T does not challenge Λ . We need to show that X is a stable vaccine allocation. Consider $S \in \mathcal{I}$, $T \in 2^N$, and assume that there exists a coalition $\Lambda \in 2^N$ such that $\Lambda \in B(S, T)$ and $\Lambda \subseteq X$.

We want to show that $X \cap T \neq \emptyset$. Assume on the contrary that $X \cap T = \emptyset$. Then $T \subseteq N \setminus X$, and $|T| \leq |N \setminus X|$. We differentiate two cases: $T \in N_X$ or $T \notin N_X$.

- First, assume that $T \in N_X$. Then, $|T| \leq \min \{\mu, |N \setminus X|\}$, and $T \in N(\mu)$. We know that there exists $\Lambda \subseteq X$ such that $\Lambda \in B(S, T)$. The latter implies that $|\Lambda| = |T|$ and $u_i(T) > u_i(\Lambda)$ for all $i \in S$, or T challenges Λ via S , which contradicts the initial assumption.

- Second, assume that $T \notin N_X$. Since $T \subseteq N \setminus X$, it follows that $|T| > n_X = \min \{\mu, |N \setminus X|\}$. Since $|T| \leq |N \setminus X|$, it holds that $|N \setminus X| \geq |T| > \mu$. Thus, T contains more elements than X , and we deduce that there is no coalition $\Lambda \subseteq X$ such that $|T| = |\Lambda|$. Therefore, there is no influential coalition $S \in \mathcal{I}$ such that $\Lambda \in B(S, T)$ and $\Lambda \subseteq X$, which contradicts our assumption. This completes the proof. \square

If there is a fixed number of μ doses of vaccine, Proposition 1 improves in some cases the procedure for checking the stability of a given allocation $X \in N(\mu)$. Indeed, X is a stable vaccine allocation if no coalition of N_X can challenge X . Formally, we must look for a subset Λ_X of X , a coalition $T_X \subseteq N \setminus X$ such that $|T_X| = |\Lambda_X|$, and an influential coalition S_X whose members unanimously prefer T_X to Λ_X . Thus, X is stable if we cannot find such coalitions Λ_X , T_X , and S_X . Next, we provide a formal characterization of the set of stable vaccine allocations in any political economy. Before stating the result, we need the following notations. Let us denote:

- for any coalition $\Lambda \in 2^N$, $\mathcal{I}(\Lambda) = \{T \in 2^N : \exists S \in \mathcal{I}, \Lambda \in B(S, T)\}$, and
- $\mathcal{H}(N, \mathcal{I}) = \{L \subseteq N : \forall \Lambda \subseteq L, \forall T \in \mathcal{I}(\Lambda), L \cap T \neq \emptyset\}$.

Given a coalition Λ , the set $\mathcal{I}(\Lambda)$ contains any coalition T such that $|T| = |\Lambda|$ and there exists an influential coalition S whose members unanimously prefer T to Λ . We refer to the elements of $\mathcal{I}(\Lambda)$ as coalitions of the same size as Λ that challenge Λ . The set $\mathcal{H}(N, \mathcal{I})$ is the main component for the next characterization of stable vaccine allocations. That is the set of coalitions L such that for any coalition T , if T challenges a subset Λ of L where $|T| = |\Lambda|$, then L intersects T . Theorem 2 follows.

Theorem 2. *Let $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle$ be a vaccine allocation game. Then, the set of stable vaccine allocations of G is given by $C(G) = \mathcal{H}(N, \mathcal{I}) \cap N(\mu)$.*

Proof. We proceed by double inclusion.

\subseteq): Let X be a stable vaccine allocation. By definition, $X \in N(\mu)$. So what remains is to demonstrate that $X \in \mathcal{H}(N, \mathcal{I})$. For this purpose, consider a coalition $\Lambda \subseteq X$ and $T \in \mathcal{I}(\Lambda)$. We need to show that $X \cap T \neq \emptyset$. Since $|X| = \mu$, it is clear that $|T| \leq \mu$. Otherwise, $T \in \mathcal{I}(\Lambda)$ implies that $\Lambda \in B(S, T)$. According to Theorem 1, we have $X \cap T \neq \emptyset$, i.e., $X \in \mathcal{H}(N, \mathcal{I})$.

\supseteq): Let $Y \in \mathcal{H}(N, \mathcal{I}) \cap N(\mu)$, we must show that Y is a stable allocation. According to Theorem 1, it is sufficient to show that, for any influential coalition S and any coalition T such that $|T| \leq \mu$, if there exist $\Lambda \in B(S, T)$ and $\Lambda \subseteq Y$, then $Y \cap T \neq \emptyset$. Consider an influential coalition S , a coalition T such that $|T| \leq \mu$ and $\Lambda \in B(S, T)$ such that $\Lambda \subseteq Y$. The assumption $\Lambda \in B(S, T)$ implies that $T \in \mathcal{I}(\Lambda)$ and the assumption $Y \in \mathcal{H}(N, \mathcal{I})$ allows us to conclude that $Y \cap T \neq \emptyset$. \square

Theorem 1 and Proposition 1 allow to check the stability of a vaccine allocation. It follows that the underlying conditions of these results are equivalent. Since $\mathcal{H}(N, \mathcal{I})$ consists of all stable vaccine allocations when μ varies from 1 to $|N|$, we can also use Proposition 1 to redefine $\mathcal{H}(N, \mathcal{I})$ as the set of coalition $L \subseteq N$ such that for any $\Lambda \subseteq L$ and any $T \in N_L$, if $|\Lambda| = |T|$, then T does not challenge Λ . Thus, we can obtain another characterization of core allocations similar to Theorem 2 using Proposition 1.

Using Theorem 2, it is straightforward to show that there is no stable vaccine allocation in a game $G = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle \in \mathbb{V}$ (i.e., $C(G) = \emptyset$) when the set $\mathcal{H}(N, \mathcal{I})$ is empty, since $N(\mu) \neq \emptyset$. Given that $N \in \mathcal{H}(N, \mathcal{I})$, it is immediate that $\mathcal{H}(N, \mathcal{I})$ is non-empty. One natural question arises: what conditions on μ guarantee that the intersection $\mathcal{H}(N, \mathcal{I}) \cap N(\mu)$ is non-empty? The answer to this question depends on the parameters of the vaccine allocation game. Although we don't provide a comprehensive answer to this question, we do characterize the number of vaccine doses μ^* that guarantees the non-emptiness of $\mathcal{H}(N, \mathcal{I}) \cap N(\mu^*)$.

Let $E = (N, \mathcal{I}, \{q^i\})$ be a political economy. Let $\mu^*(E) = \min \{|L| : L \in \mathcal{H}(N, \mathcal{I})\}$ denote the minimum number of vaccine doses that guarantees the non-emptiness of \mathcal{H} . Obviously, if $\mathcal{H}(N, \mathcal{I}) = \emptyset$, then $\mu^*(E) = 0$. Additionally, $N \in \mathcal{H}(N, \mathcal{I})$ implies that $0 < \mu^*(E) \leq |N|$. Assume that the preference profile (u_i) is fixed. Corollary 1 shows that $\mu^*(E)$ is the smallest number of doses for a stable vaccine allocation.

Corollary 1. *Let $E = (N, \mathcal{I}, \{q^i\})$ be a political economy. Then, $C(E, \mu^*(E)) \neq \emptyset$, and $C(E, \mu) = \emptyset$ for any $\mu < \mu^*(E)$.*

The proof of Corollary 1 is straightforward. We illustrate the determination of $\mu^*(E)$ in the simple vaccine allocation problem in a political economy $E = (N, \mathcal{I}, \{q^i\})$ described in Example 1. However, in addition of majority rule, we consider two other influence structures: **Dictatorial rule**— $S \in \mathcal{I}$ if and only if $1 \in S$; and **unanimity rule**— $S \in \mathcal{I}$ if and only if $|S| = 3$. We check that for the dictatorial rule, as for majority rule, $\mathcal{H}(N, \mathcal{I}) = \{\{1\}, \{1, 2\}, \{1, 2, 3\}\}$, and $\mu^*(E) = 1$. For the unanimity rule, $\mathcal{H}(N, \mathcal{I}) = 2^N$, and $\mu^*(E) = 1$. In Example 1, assume that a coalition is influential if and only if it contains agent 2 or 3, i.e., $S \in \mathcal{I}$ if and only if $S \cap \{2, 3\} \neq \emptyset$ (this

type of influence structure is covered by our general model), then $\mathcal{H}(N, \mathcal{I}) = \{\{1, 2, 3\}\}$ and, $\mu^*(E) = 3$.

4 Inclusive and Balanced Vaccine Allocations in a Polarized Society

In this section, we assume that a society consisting of $|N| = n$ agents is partitioned into two non-empty and disjoint subsets (or subgroups) N_1 and N_2 . In addition, agents have in-group preferences, meaning that an agent derives more utility when a member of their subgroup is vaccinated than when a member of the other subgroup is vaccinated. Formally, $N = N_1 \cup N_2$ such that for any $t \in \{1, 2\}$, $N_t \neq \emptyset$ and $N_1 \cap N_2 = \emptyset$. For any agent i , let $t(i)$ be the unique element of $\{1, 2\}$ such that $i \in N_{t(i)}$. The example discussed in Section 2.6 shows that a non-influential group can be completely excluded from all stable vaccine allocations (see, e.g., **Case 1B** in Section 2.6.2). From a normative and ethical perspective, this situation may not be acceptable in several societies. We aim to examine the conditions under which social planners or political leaders can avoid such undesirable outcomes. In other words, we are interested in identifying conditions under which a vaccine allocation that is both stable and inclusive exists. In the scenario described above, such an inclusive allocation would need to provide opportunities for agents in both subgroups to receive vaccine doses. Below, we formally define two related concepts: inclusive vaccine allocations and balanced vaccine allocations.

Definition 2. Let $G \in \mathbb{V}$ be a vaccine allocation game involving a set N of agents partitioned into two non-empty subsets N_1 and N_2 . Consider a stable vaccine allocation $X \in C(G)$.

- 1) The allocation X is **inclusive** if $X \cap N_1 \neq \emptyset$ and $X \cap N_2 \neq \emptyset$.
- 2) The allocation X is **balanced** if for any allocation Y such that $|Y| = |X|$, we have:

$$\left| \frac{|X \cap N_2|}{|N_2|} - \frac{|X \cap N_1|}{|N_1|} \right| \leq \left| \frac{|Y \cap N_2|}{|N_2|} - \frac{|Y \cap N_1|}{|N_1|} \right|.$$

An inclusive allocation is any allocation where a non-zero number of vaccines is provided to both groups of agents, while a balanced allocation is a special type of inclusive allocation, where vaccines are allocated such that the number of agents who receive a vaccine dose is almost the same in both groups. We are interested in the necessary and sufficient conditions which guarantee the existence of a stable and inclusive (resp. balanced) vaccine allocation. Our analysis covers both one-dimensional (Section 4.1) and multi-dimensional (Section 4.2) polarized societies.

4.1 One-dimensional Polarized Societies

This section considers a one-dimensional spatial framework, which generalizes the examples discussed in Section 2.6. Agents have Euclidean preferences based on ideal points located on a line or horizontal axis. Figure 2 in Section 2.6.2 represents an example of the configuration of agents' ideal points in such a polarized society. In what follows, a vaccine allocation game $G^1 \in \mathbb{V}$ is an array $G^1 = \langle N, \mathcal{I}, \{q^i\}, \mu, (u_i) \rangle$, where $N = N_1 \cup N_2$ such that for any $t \in \{1, 2\}$, $N_t \neq \emptyset$, $N_1 \cap N_2 = \emptyset$, and (u_i) is a profile of one-dimensional Euclidean preferences over $N(\mu)$. Theorem 3 below provides a necessary and sufficient condition for the existence of a stable vaccine allocation when both the two subgroups are influential. Interestingly, there exists a unique stable allocation, and this allocation is inclusive and balanced.

Theorem 3. *If $N_1 \in \mathcal{I}$ and $N_2 \in \mathcal{I}$, then the existence of a stable allocation requires $|N|$ doses of vaccines: $C(G) \neq \emptyset$ if and only if $\mu = |N|$.*

Proof. We first show that for any $\mu < n$, there is no stable allocation. Fix $\mu < n$. Any allocation X is such that $[N_1 \not\subseteq X \text{ or } N_2 \not\subseteq X]$ and $[N_1 \cap X \neq \emptyset \text{ or } N_2 \cap X \neq \emptyset]$. Without loss of generality, assume that $N_1 \not\subseteq X$ and let $i \in N_1 \setminus X$. If $N_2 \cap X \neq \emptyset$, then let $j \in N_2 \cap X$. Agent i is a member of N_1 , which is not in X , and j is a member of N_2 , which is in X . In the allocation $Y = (X \setminus \{j\}) \cup \{i\}$, a member of N_2 is replaced by a member of N_1 , and any member of N_1 prefers Y to allocation X . By assumption $N_1 \in \mathcal{I}$; therefore, X is challenged by Y via N_1 , which proves that X is not a stable allocation. If $N_2 \cap X \neq \emptyset$, then $N_1 \cap X \neq \emptyset$. In this case, choose any $k \in N_2$ and set $Z = (X \setminus \{i\}) \cup \{k\}$. Following the same reasoning, any member of N_2 prefers Z to allocation X . By assumption $N_2 \in \mathcal{I}$. So, X is challenged by Z via N_2 , which implies that X is not a stable allocation. To complete the proof, it is clear that for $\mu = |N|$, the allocation $X = \{N\}$ is stable. \square

It is straightforward to show that the stable allocation derived from Theorem 3 is both inclusive and balanced since each agent receives a vaccine dose. Theorem 4 below provides the minimum number of vaccine doses necessary to achieve an inclusive and stable allocation when only one subgroup of the two subgroups in the society is influential.

Theorem 4. *If $N_1 \in \mathcal{I}$ and $N_2 \notin \mathcal{I}$, then the existence of an inclusive and stable allocation requires $|N_1| + 1$ vaccine doses. Under this condition, there is a stable vaccine allocation in which each agent in N_1 receives a dose, and at least one agent in N_2 receives a dose.*

Proof. Assume that $N_1 \in \mathcal{I}$ and $N_2 \notin \mathcal{I}$. We prove Theorem 4 in two steps.

Step 1. Suppose that $\mu < |N_1| + 1$, i.e., $\mu \leq |N_1|$. Let $X \in N(\mu)$ be an inclusive allocation. Then we necessarily have $N_1 \not\subseteq X$ because $N_1 \cap N_2 = \emptyset$, $|N_1| \geq \mu = |X|$, and by definition of an inclusive allocation, $X \cap N_2 \neq \emptyset$. Let $i \in N_1 \setminus X$, $j \in X \cap N_2$ and $Y = (X \setminus \{j\}) \cup \{i\}$. It is straightforward that $|Y| = \mu$, and any member of N_1 prefers Y over X . Since $N_1 \in \mathcal{I}$ by assumption, it follows that Y challenges X , and X is not a stable allocation.

Step 2. Suppose that $\mu \geq |N_1| + 1$, and show that there exists a stable and inclusive vaccine allocation. In a one-dimensional space, the ideal points of the agents lie on the same line. By assumption, the society is polarized. Therefore, the ideal points of the members of N_1 are grouped on one side and those of the members of N_2 on the other side. Consider the vaccine allocation X , which contains the agents associated with the first μ ideal points which appear when we follow the axis starting from the side containing the ideal points of the members of N_1 . It is straightforward that $X \cap N_2 \neq \emptyset$, since $|N_1| < |X|$. The latter means that X is an inclusive vaccine allocation. Moreover, for any coalition $\Lambda \subseteq X$ and $T \subseteq N \setminus X$ such that $|T| = |\Lambda|$, no agent i in N_1 prefers T to Λ because, according to the polarization hypothesis and the structure of X , the ideal point of an agent $i \in N$ is closer to the ideal point of any member of X compared to that of any member of $N \setminus X$. It follows that N_2 contains the set of agents who prefer T to Λ . By assumption, N_2 is not an influential coalition. Hence, T does not challenge Λ . Thanks to Proposition 1, X is a stable vaccine allocation. Since $X \cap N_1 \neq \emptyset$ and $X \cap N_2 \neq \emptyset$, we can conclude that X is a stable and inclusive vaccine allocation. \square

Finally, Theorem 5 below deals with the existence of stable and inclusive vaccine allocations when neither of the two subgroups in the society is influential.

Theorem 5. *Assume $N_1 \notin \mathcal{I}$ and $N_2 \notin \mathcal{I}$. Then, for any $\mu \geq 1$, the set of stable vaccine allocations is non-empty. Moreover, the existence of an inclusive and stable allocation requires at least two vaccine doses.*

Proof. We note that under the assumption of a polarized society in a one-dimensional space, there exist i in N_1 and j in N_2 such that: $d(q^i, q^j) = \min \{d(q^k, q^l) : k \in N_1 \text{ and } l \in N_2\}$. We prove Theorem 5 in three steps.

Step 1. For $\mu = 1$, it is clear that we cannot have an inclusive vaccine allocation. In contrast, several vaccine allocations are stable. For instance, let the allocation $X = \{i\}$ and consider another allocation $Y = \{t\}$, where $t \in N \setminus \{i\}$. If $t \in N_1$, then the members of N_2 unanimously prefer i to t , it follows that the agents who prefer t to i is a subset of N_1 i.e. a non-influential

coalition. Likewise, if $t \in N_2$, then the members of N_1 unanimously prefer i to t , it follows that the agents who prefer t to i is a subset of N_2 i.e. a non-influential coalition. Thus, no vaccine allocation Y can challenge X , then X is a stable vaccine allocation.

Step 2. For $\mu = 2$, we can determine a stable allocation containing two elements. Let's consider the allocation $X = \{i, j\}$. We show that X is stable. For instance, consider another allocation $Y = \{k, l\}$. If $Y \subseteq N_1$, for instance, then any member of N_2 prefers X to Y ; since $N \setminus N_2 = N_1 \notin \mathcal{I}$, no influential coalition prefers Y to X , i.e., X cannot be challenged. Likewise, if $Y \subseteq N_2$, then X cannot be challenged. For the remaining case, X contains an element of N_1 and an element of N_2 . Without loss of generality, assume that $k \in N_1$ and $l \in N_2$. No member of N_1 can prefer l to j . Likewise, no member of N_2 prefers k to i , i.e., X cannot be challenged. We have proved that X is an inclusive and stable allocation for $\mu = 2$.

Step 3. More generally, for $\mu \geq 2$, consider any allocation $X \in N(\mu)$ such as $\{i, j\} \subseteq X$. Then, X is an inclusive vaccine allocation since $X \cap N_1 \neq \emptyset$ and $X \cap N_2 \neq \emptyset$. We can use the same reasoning as above to show that X is a stable allocation. \square

Remark that Theorems 3, 4, and 5 imply that the design of the influence structure \mathcal{I} significantly affects the existence of a stable and inclusive vaccine allocation for a fixed number of doses μ . Theorem 3 proves that, in the case of perfect inclusion (where the vaccine supply is large enough to ensure all members of a society are able to receive a dose), it is always possible to implement a vaccine allocation which is stable, inclusive, and balanced. Theorem 4 demonstrates that the group which possesses the veto power has vaccine priority over the other group. Theorem 4 also implies that if the number of doses is not large enough to satisfy influential group members' demand for doses, then the vulnerable group receives no vaccine dose. According to Theorem 5, when neither group in society constitutes an influential coalition, the leader can always implement a stable and inclusive vaccine allocation as long as they can supply at least two doses. As we might expect, Theorem 5 doesn't imply that all stable vaccine allocations are inclusive. Indeed, as we illustrate in **Case 3B** in Section 2.6.2 when $\mu = 4$, the core consists of four allocations among which only one stable vaccine allocation, $N_1 = \{1, 2, 3, 4\}$, is not inclusive since it doesn't include any agent in $N_2 = \{5, 6, 7\}$.

An implication of Theorem 4 is that under majority rule, the majority group members have vaccine priority, and members of the minority group either receive nothing or receive the residual supply of doses. We should note that majority rule is just one of many constitutional arrangements covered by Theorem 4. In other constitutional arrangements where the minority group is influential

(i.e., is able to veto any proposed vaccine allocation) then the minority group will have priority access to vaccine doses, while the majority group will receive the residual supply. We can also apply Theorems 3 and 5 to constitutional arrangements other than majority rule, as Theorem 3 includes influence structures similar to unanimity rules, and Theorem 5 covers constitutional arrangements in which neither group has decisive power. When constitutional arrangements are such that neither group has decisive power, it requires that any coalition must include members of both groups in order to be influential. Such constitutional arrangements can be viewed as inclusive and generally have a multi-cameral representation (Taylor & Zwicker, 1993; Guemmegne & Pongou, 2014). Under such arrangements, it takes two vaccine doses to achieve an inclusive and stable vaccine allocation. Therefore, our analysis reveals that it is always possible to design the decision-making rule of the political economy to implement a vaccine allocation that is both stable and inclusive. Under such a rule, the composition of any group endowed with the veto power should necessarily reflect the diversity of the society.

4.2 Multi-dimensional Polarized Societies

In a multi-dimensional polarized society, searching for an inclusive or balanced stable vaccine allocation is cumbersome. Indeed, in a high-dimensional space, the ideal points can have a disparate configuration. A simple approach is to choose among the stable allocations, those that are inclusive. To prove the existence of stable vaccine allocations in multi-dimensional polarized societies, we define the sets $\mathcal{H}^{in}(N, \mathcal{I})$ and $\mathcal{H}^{ba}(N, \mathcal{I})$ as follows: $\mathcal{H}^{in}(N, \mathcal{I}) = \{L \in \mathcal{H}(N, \mathcal{I}) : L \cap N_1 \neq \emptyset \text{ and } L \cap N_2 \neq \emptyset\}$, and $L \in \mathcal{H}^{ba}(N, \mathcal{I})$ if and only if $L \in \mathcal{H}(N, \mathcal{I})$ and for any $S \subseteq N$, $|S| = |L|$ implies $\left| \frac{|L \cap N_2|}{|N_2|} - \frac{|L \cap N_1|}{|N_1|} \right| \leq \left| \frac{|S \cap N_2|}{|N_2|} - \frac{|S \cap N_1|}{|N_1|} \right|$. Let $E = (N, \mathcal{I}, \{q^i\})$ be a political economy. For any $\mu > 0$, denote by $C^{in}(E, \mu)$ (respectively, $C^{ba}(E, \mu)$) the set of inclusive (respectively, balanced) and stable vaccine allocations. Also, let $\mu^{in*}(E) = \min \{|L| : L \in \mathcal{H}^{in}(N, \mathcal{I})\}$ and $\mu^{ba*}(E) = \min \{|L| : L \in \mathcal{H}^{ba}(N, \mathcal{I})\}$. Proposition 2 follows Theorem 2 and Corollary 1.

Proposition 2. *Let $E = (N, \mathcal{I}, \{q^i\})$ be a political economy. Then, the following assertions hold.*

- 1) $C^{in}(E, \mu^{in*}(E)) \neq \emptyset$, and $C^{in}(E, \mu) = \emptyset$ for any $\mu < \mu^{in*}(E)$.
- 2) $C^{ba}(E, \mu^{ba*}(E)) \neq \emptyset$, and $C^{ba}(E, \mu) = \emptyset$ for any $\mu < \mu^{ba*}(E)$.
- 3) $C^{in}(E, \mu) = \mathcal{H}^{in}(N, \mathcal{I}) \cap N(\mu)$ and $C^{ba}(E, \mu) = \mathcal{H}^{ba}(N, \mathcal{I}) \cap N(\mu)$.

Assertion 1) of Proposition 2 states that in a political economy, E , an inclusive and stable vaccine allocation exists if and only if the number of vaccine doses is at least equal to $\mu^{in*}(E)$.

Assertion 2) of Proposition 2 indicates that a balanced and stable allocation exists if and only if the number of vaccine doses exceeds $\mu^{ba*}(E)$. Remark that we can compute the thresholds $\mu^{in*}(E)$ and $\mu^{ba*}(E)$ for any political economy, E . Finally, assertion 3) of Proposition 2 offers a characterization of all stable and inclusive allocations and the set of stable and balanced vaccine allocations in a political economy.

One can show that $\mu^{ba*}(E)$ is greater than $\mu^{in*}(E)$, implying that implementing a stable and balanced allocation is more demanding than enacting a vaccine allocation that is both stable and inclusive. This is not surprising since any balanced vaccine allocation is inclusive, whereas the converse is generally not true. Also, note that the number of vaccine doses necessary to implement a stable vaccine allocation is smaller than the number of doses needed to reach a stable and inclusive allocation or a stable and balanced allocation. These observations imply that fragmented societies which cannot acquire a sufficiently large number of vaccine doses will find it challenging to implement vaccine allocation policies that are both stable and inclusive. In such societies, majority group members will have vaccine priority at the disadvantage of minority factions. One can test the predictions of the model using data on vaccine coverage in fragmented societies which differ in terms of their economic or technological ability to produce or acquire doses.

5 Concluding remarks

Our primary motivation is to examine stable and inclusive vaccine allocation policies in a political economy. Stable vaccine allocations represent policies that a political leader could implement without losing popularity. Our model assumes that individuals are organized around distinct groups or networks and have weighted other-regarding preferences. These preferences imply that utility derived from vaccinated agents depends on the agent's identity. Given the protective nature of vaccines, we assume that an agent's satisfaction increases as their group's members get vaccinated even if this individual does not receive a dose themselves. A straightforward rationale behind this assumption is that an agent's likelihood of getting infected by the virus decreases as the vaccinated population in their group increases, as agents interact with those within their group more frequently than with those outside their group.

First, we characterize stable vaccine allocations in any political economy. Our analysis shows that the existence of a stable vaccine allocation depends on the vaccine supply. In particular, a stable vaccine allocation may not exist if vaccine supply is very low relative to the number of eligible individuals. Our results imply that there exists a minimum number of doses that

guarantees the existence of a stable allocation. This number depends on the society's influence structure and the configuration of individual preferences.

Second, to study the existence of allocations that are both stable and inclusive, we extend our analysis to polarized societies partitioned into two distinct and disjoint groups, where individuals have in-group preferences. We characterize the set of stable and inclusive vaccine allocations, as well as the set of stable and balanced allocations, where a balanced allocation is an allocation that achieves perfect inclusion. These characterizations imply that the number of vaccine doses necessary to generate a stable and balanced allocation is greater than the number of doses necessary to obtain a stable and inclusive allocation. Similarly, the number of doses needed to achieve a stable and inclusive allocation is greater than the number of doses needed to achieve a stable allocation. These findings mean that solving the problem of vaccine inclusiveness in a fragmented society which does not have the means to acquire a sufficiently high number of vaccine doses is a challenging task. When inclusiveness fails, any stable vaccine allocation favors the majority group. Our findings also demonstrate the critical role of the economy's influence structure in implementing vaccine inclusiveness. Importantly, these results imply that one can design an influence structure to implement a stable allocation that is inclusive. In this case, the composition of any influential coalition must include members of all groups, thus reflecting the society's diversity.

Our findings imply that polarized societies which are unable to obtain a large number of vaccine doses will be less likely to implement vaccine allocation policies that include under-represented groups or communities without influence.

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Conflict of interest

The authors declare that they have no conflict of interests.

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