BROADENING THE MATHEMATICAL ASPECT OF THE CONCEPT-SKILL CORRELATION AT ONE GRADE ALGEBRA LEVEL

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CURRICULUM STUDIORUM

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INTRODUCTION

1. The problem of the relation between mastery of fundamental concepts and computational skills in High School Algebra has been considered an important field of investigation for a long time.[1] It has not yet been investigated by objective methods.

The problem has been explored to some extent in the field of arithmetic,[2] but the results cannot be applied directly to Algebra, whatever arguments could be quoted to support such an application, without an investigation of the field by objective and adequate means that would correspond to the nature of Algebra, which, due to the general symbols involved, differs from Arithmetic.

Two main facts seem to make the problem difficult for the investigator: its broadness, and complexity. The broadness lies above all in a multitude of algebraic concepts and computations involved in the area of intermediate algebra, and dispersed along a number of grades. The complexity of the problem seems to lie above all in the

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[1] Walter I Monroe, Ed., Encyclopedia of Educational Research, a Project of the American Educational Research Association, New York, Macmillan, 1950, p.717-723.

[2] C.H.Butler, Mastery of Certain Mathematical Concepts by Pupils at the Junior High School Level, University of Missouri, 1931, v-56 p.

coincidence of many psychological aspects of mastering the concept or acquiring a computational skill in algebra (intelligence, memory, imagination, thinking, reasoning - are involved in both) as well as in some confusion in the field of algebraic concepts in today's school practice. The concepts suffer from the scarcity of clarity of their basic components. A substantiation of the latter is presented in Appendix 1.

2. A proper approach to undertaking the investigation of the mutual relation between the comprehension of the algebraic concepts and the ability to compute with those concepts seems to be a two-fold one:

(a) narrowing the broad area of investigation to one grade algebra, fairly representative as to the selection of the algebraic material, and fairly distant from arithmetic on the one hand, and from higher algebra on the other;

(b) limiting the investigation to a single determinate aspect of the complex set of factors involved in the problem.

Thus, only the mathematical aspect of the problem at one grade level was chosen for investigation.

The coefficient of correlation between the understanding of algebraic concepts and corresponding skills in computations at one Grade XI algebra group is to be determined first.

In turn, a three-group division method is to be applied to the investigation of the correlation: the whole group of testees to be ranked on the two tests and divided into three numerically equal and comparable subgroups (upper, middle and lower).

The findings concerning the mathematical side of the comparison of the sub-groups are to be confronted with those obtained by the coefficient of correlation.

It is expected to obtain by such a confrontation: a broadening of the mathematical aspect of the concept-skill correlation in high school algebra.

3. The whole task falls into two main divisions:

(a) to obtain adequate tools for the investigation of the problem, as there are no standardized tests for the topic available; and

(b) to investigate the problem as such.

The following procedure has been adopted:

(i) Choosing the algebraic material for the investigation of the concept-skill correlation;

(ii) Constructing two achievement tests coming up to the requirements of satisfactory measuring tools: the Algebraic Concept Test for measuring the understanding of the algebraic concepts, and the Algebraic Skill Test for measuring the ability in algebraic computations;

(iii) Trying out the tests and validating the test-items;

(iv) Administering the tests to a representative group of the English-speaking Catholic High School population in Grade XI in the City of Ottawa;

(v) Scoring the tests, determining the nature of the distributions obtained, and correlating them to obtain the coefficient of correlation of the two distributions;

(vi) Comparing the upper, middle and lower groups on both tests vertically, that is within the tests, and horizontally, that is comparing the performance of the upper, middle and lower groups on one test with their performance on the other test;

(vii) Confronting the mathematical aspect of the correlation as obtained by the coefficient of correlation with that obtained through the vertical and horizontal comparisons by the three-group division method;

(viii) Analysing the results of that confrontation and producing a synthetizing formula or graph as a summary of the results;

(ix) Drawing conclusions as to the possibility of a generalization of the findings, the method of the investigation applied, and others - according to the results of the experiment.

4. By the algebraic concept are meant those notions which appear only in algebra and do not appear in arithmetic or geometry, or which - according to Butler's formulation[3] - appear mainly in problems that are essentially algebraic and not arithmetical or geometric. The understanding of general symbols, functional dependence, and quantative relations are implied here.

A most concise and workable definition of the algebraic concept in this study is: <u>a general symbol of a quantative operation</u>.

By the term "skill in computation" is meant those abilities that are essential in solving situations that need basically an algebraic insight consisting of the manipulative and reasoning power in algebraic computations. Thus, the basic mastery of the techniques of computational operations, as well as the ability to select and to apply the proper techniques for solving algebraic problems, are included in the idea of the computational skill.

This meaning of the computational skill is consistent with the ideas represented by Edward L. Thorndike[4] in his book <u>Psychology of Algebra</u>.

The literature dealing with the algebraic concept and skill, as far as it pertains to this study, is presented in the following chapter.

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[3] C. H. Butler, <u>op.cit.</u>, pp 41-42.

[4] Edward L. Thorndike <u>et al</u>., <u>The Psychology of Algebra.</u> New York, Macmillan, 1923, pp 166-191.

CHAPTER I

SURVEY OF THE LITERATURE

Literature dealing <u>exactly</u> with the correlation between the comprehension of the algebraic concepts and the mastery of the algebraic computations, in reality does not exist.

The books and periodicals deal only with the algebraic concepts or the computational skills separately or jointly and stress the importance of the two aspects, but they hardly go further than to assume or to point at their mutual connection and they never deal exhaustively with the nature of that connection and never do they undertake any measure of the concept-skill correlation.

The available literature can be divided in two parts: one dealing mainly with the evaluation of the role of algebra in general education, and the other dealing mainly with devices and methods of improving the teaching of algebra by embracing its theoretical and social aspects.

The common feature of both seems to be in the psychological approach to the problem, and thus, the methodical aspect is reflected in both groups, though in some different ways. It concentrates in the first group of works mainly on discovering weaknesses and oversights in the teaching, whereas the search for efficient remedies prevails in the second group.

1. Algebraic Concept-Skill Correlation - Assumed

The classical and basic works of the first type are considered:

A. Edward L. Thorndike, Margaret V. Cobb, Jacob I. Orleans, Percival M. Symonds, Elva Wald, and Ella Woodyard, The Psychology of Algebra, New York, Macmillan, 1924, xi-483 p.

In this work, the correlation between the understanding of the algebraic concept and the computational skill is treated by the authors in a way that could be, perhaps quite appropriately, called a unifying or synthetic way.

There is no straight definition of the algebraic concept or of the algebraic skill to be found in Thorndike's book. That definition can only be deducted from the multifarious psychological approaches to both of them. Rather, from the partial elements of the whole concepts, an idea of the wholes can be inferred fairly exhaustively. For example, generality, as a very essential aspect of Algebra, is presented by the author rather indirectly in the statement "The Algebra as a tool is considered a tool for scientific work, for thinking about general relations".[1]

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[1] Edward L. Thorndike et al., The Psychology of Algebra, New York, Macmillan, 1924, p.47.

However, both the full meaning of "concept" and "skill", and their importance in teaching high school algebra, are approached from so many angles, and supported by numerous objective proofs, as to leave little doubt as to the conclusions arrived at.

The importance of the pupil understanding the algebraic concepts is set forth as basic to computing. The importance of general algebraic symbols (thus: π, logs, a^n, and so on) is basically acknowledged by such a statement as "as we have to read them, and perhaps twice for every once that we compute with them".[2] The importance of understanding such basic algebraic concepts as equations, function, and others, find their substantiation in the objectively-conducted proofs, ending with strong and clear conclusions of the sort "... the understanding of equations as the expression of relations, goes straight to the heart of all applied mathematics".[3] There can be no doubt either that, in the authors' view, the comprehension of algebraic concepts is basic to the computational skills.[4]

Algebraic skill is set forth as an ability to deal with symbols, quantative relations, generalizations, and with selection and organization of ideas and habits to solve algebraic situations.[5] The skill is much more "than

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[2] Ibid., p.89.
[3] Ibid., p.107.
[4] Ibid., p.p.172-194.
[5] Ibid., p.198.

to use a practical tool. It is an evidence of understanding of the algebraic principles learned and an aid in learning others"[6] (...) "The skill in algebraic computation involves very much the same abilities that problem solving does".[7]

Perhaps the most essential factor, from the authors' didactic point of view, is the understanding of the concept and the understanding of the way it can be used. That is just the understanding of both - the concept and the operation performed - that links the basic algebraic concepts and skills organically into one didactically effective and worthy whole, being a cornerstone of "the knowledge in Algebra".

In such a treatment of the problem, the existence of a very close and positive correlation between the algebraic concepts and the algebraic computational skill seems to be implicit.

However, there is no attempt made by the authors to determine exactly the closeness of that correlation and especially to measure its extent.

Thorndike's work is reflected, then, in this research as far as the definitions and importance of the algebraic concept and the algebraic skill, as well as the determination of the choice of basic concepts and skills in high school algebra, are concerned, but it has no direct bearing on the measure of the concept-skill correlation that has been undertaken in this study.

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[6] Ibid., p.328.
[7] Ibid., p.448.

B. Progressive Education Association, Mathematics in General Education, A Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum, New York, Appleton-Century-Crofts, 1938, xiv-423 p.

The Committee sets forth the criteria for selecting a set of a small number of fundamental concepts in High School Algebra, and then a set of fundamental algebraic concepts as well.

The idea of the correlation between the algebraic concept and skill is reflected here in two main aspects:

(a) there is a definite distinction between them as the two important didactic aspects in teaching algebra, but no separation;

(b) their mutual relation is considered to be close, but there are no objective proofs of the closeness quoted and no measurement of it undertaken, or reported.

As far as point (a) is concerned, the authors state in their report:

> The techniques involved (in mathematical operations) are so frequently demanded and the concepts basic to the operations they help to perform are so essential that they cannot be omitted in a consideration of mathematics as a method of problem-solving.[8]

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[8] Progressive Education Association, Mathematics in General Education, A Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum, New York, Appleton-Century-Crofts, 1938, p.168.

They state further

> ... The emphasis of this Report is designed to help correct this situation (overemphasizing techniques) by stressing concepts basic to operations no less than the techniques by which they may be performed, and by considering those concepts among a number of others fundamental in the study of mathematics ... In order to secure both understanding and skill, there must be appropriate emphasis on each.[9]

As to the importance of that distinctness of the concept from the skill their statement may well be quoted:

> The distinction between techniques for performing operations and the concepts basic to them is important at all levels. Even the student who had advanced to the study of calculus may become expert in the techniques of differentiation without a clear notion of the limit concept which is basic to the operation. Only by careful attention to the development of the underlying concepts can teachers make operations really meaningful to students.[10]

A unifying didactic link of the two aspects is expressed by the authors perhaps most clearly in their statement "... when concepts and skills are acquired in isolated contexts the pupil may be unable to put them together when faced with a complex problem".[11]

Finally, a testing of growth in understanding of the concept and of the ability to use the concept is strongly recommended by the authors.[12]

In brief, aside from the stress laid on the didactic importance of the concept and the skill in Algebra, and that of distinguishing between them without separating them, and

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[9] *Ibid.*, p.169

[10] *Ibid.*, pp 180-181.

[11] *Ibid.*, p.399.

[12] *Ibid.*, p.341.

the need for objective measurement of the two aspects, there are no close ties between the ideas of this book and the present research.

2. Concept-Skill Correlation - Suitability for Measurement

The second part of the literature, that dealing with the improvement of teaching of algebra, is represented by:

A. G. T. Buswell, Editor, Supplementary Educational Monographs, Arithmetic, 1947, Chicago University Press, 1949, 11-73 p.

This collective work may be said to represent fairly distinctly an attempt towards distinguishing more exactly the aspect of the comprehension of the mathematical concept from the computational skill. The idea of the relationship between algebraic concept and computational skill as two different, but close, didactical aspects seems to germinate here more distinctly.

In the article "Improving the Mathematics Program in Junior High School Grades", Maurice L. Hartung sets forth the didactical necessity for a "reasonable understanding" of basic concepts in algebra, that should be achieved by a procedure "guided by a few fundamental and unifying principles".[13] Those principles are the gathering of a vast

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[13] M. L. Hartung, "Improving the Mathematics Program in Junior High School Grades", in Supplementary Educational Monographs, Chicago University Press, 1949, pp 49 to 58.

number of details and more or less specific operational rules around the "elementary" algebraic concepts, which should be "taught in such a way that they stand out explicitly in the minds of the pupils".[13A] In the summary of his article the author says:

> Above all, a relatively small number of the really fundamental concepts and principles of mathematics must be made to stand out clearly like fire-observation towers above the forest of details.[14]

Thus, while the didactic role of the comprehension of basic algebraic concepts and of the understanding of the computations performed is sufficiently stressed, the problem of the correlation between the two is not developed by the author.

In the article "Place Value and the Number System", appearing in the same publication, H. Van Engen brings out the didactic aspect of the understanding of the fundamental concepts as basic to skilful operations in arithmetic and algebra, using the concept of the number system as an example for a "basic" or "key-concept" of instruction.

There can be no doubt that the author had in mind the relation of the concept and computational skill as an essential didactic problem in elementary mathematics.

L. J. Brueckner, in the article "Arithmetic in Elementary and Junior High Schools", sets forth the understanding of concepts as basic to skilful operations in

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[13A] M.L.Hartung, "Improving the Mathematics Program in Junior High School Grades", in <u>Supplementary Educational Monographs.</u> Chicago University Press, 1949, pp 49 to 58.

[14] <u>Ibid.</u>

arithmetic. Mastery of number and number operations are closely related; the meaningful number system to the pupil enables him to use it in quantative operations more skilfully. This is the way leading in consequence through school algebra experiences to the "intellectual and sound development of the individual".[15]

B. G.T.Buswell and Maurice T. Hartung, Editors, Supplementary Educational Monographs, Arithmetic, 1949, Chicago, University Press, 1950, 11-100 p.

Buswell's article "Methods of Studying Pupils' Thinking in Arithmetic",[16] sets out one fact clearly, apart from the methodical side which is the main point of the article, as far as this research is concerned: the teacher must know how the pupil (the child) thinks in order to be able to make the mathematical concepts and operations more meaningful to the learner.

Thus, the importance of both the concepts and computational skills, finds another didactical stress here.

M. L. Hartung inserts the article "Major Instructional Problems in Arithmetic in the Middle Grades"[17] in this same monograph series. From the standpoint of this research one aspect of the problem handled by the author seems to be significant, and this is the "mathematical understanding"

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[15] L.J.Brueckner "Arithmetic in Elementary and Junior High Schools", in Supplementary Educational Monographs, Arithmetic, 1949, Chicago University Press, 1950, p.9

[16] G.T.Buswell, "Methods of Studying Pupils' Thinking in Arithmetic", ibid, pp 55 to 63.

[17] M.L.Hartung, "Major Instructional Problems in Arithmetic in the Middle Grades", ibid., pp 80-86.

and the "use of number in solving the problems" which is the use of the mathematical concept in computations. The author stresses that a further "cooperative research" for improving the understanding of concepts and operational techniques for solving problems "met by persons, families, communities, and states", is seriously needed.

Vincent J. Glennon presents an article "Testing Meanings in Arithmetic".[18] The need for measuring "the mastery of certain basic mathematical understandings possessed by representative groups" is definitely acknowledged by the author. He further stresses the need for tests that "the items did not require computation" in order to discriminate "between the student who performed on a rote level and the student who performed on a rational or understanding level".

The idea of a separate measure of the concept and of the skill seems to underlie the main point of the article. Although this has not been stated by the author explicitly, the final suggestion of the use of the tests with multiple-choice items seems to indicate that such a separate measuring-device could have been the author's real intention. The idea of such measurement seems to germinate most clearly in this article.

Robert Lee Morton, in the article "The Place of Arithmetic in Various Types of Elementary-School Curriculum"[19]

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[18] V.J.Glennon, "Testing Meanings in Arithmetic", Ibid., pp 64-75.

[19] R.L.Morton, "The Place of Arithmetic in Various Types of Elementary-School Curriculums", Ibid., pp 1-20.

sets forth the didactic necessity for "building understandings gradually, one step at a time" (obviously referring to the basic concepts (numbers) and operations); and teaching "for understanding" and avoiding "mechanical tricks and devices" (obviously referring to computational skills). The author points out - as the method of teaching is concerned - the necessity for "teaching pupils to discover new truths for themselves" to ground and to deepen their understanding, and to do that by having pupils face realistic and real problems in school and out of school situations.

From the standpoint of this research, the mathematical concepts and skills are considered by the author as basic didactic aspects in elementary arithmetic and algebra.

In all the articles quoted under this second section, a more advanced distinction between the understanding of the algebraic concept and the computational skill seems to come out. It culminates in a suggestion for their measurement by objective means.

C. C.H.Butler, *Mastery of Certain Mathematical Concepts by Pupils at the Junior High School Level,* University of Missouri, 1931, 56 p.

This doctoral dissertation is a research work on the achievement in mastery of some algebraic concepts by high school pupils. The need for such a type of research is quoted by the author as follows:

> Ninth grade algebra stands out as one of the most difficult subjects of the modern curriculum (...) The fundamental concepts are not sufficiently clear to the learner (...) Hence the pupil fails to acquire a full understanding of the meaning of such important concepts as "signed number", "exponent", "equation", "relationship".[20]

The author quotes the following as essentially algebraic concepts:

> root of an equation
> formula
> a solution of an equation
> exponent
> algebraic factor
> a quadratic
> a power of a number
> a root of a number [21]

At the end of the book the author determines the concept-skill correlation as a real contribution to didactic of algebra: "A real contribution might be made by conducting an investigation of the relation of the mastery of mathematical concepts and the mastery of computational skills".[22] The book presents an objective way of measuring the understanding of some algebraic concepts on High School level by means of specially constructed tests. However, there is no indication given in what way the relation between mastery of mathematical concepts and the mastery of computational skills could be or should be investigated.

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20 C.H.Butler, Mastery of Certain Mathematical Concepts by Pupils at the Junior High School Level, University of Missouri, 1931, pp 3-6.

21 Ibid., pp 41-42.

22 Ibid., p.55.

D. C. H. Butler and F. L. Wren, The Teaching of Secondary Mathematics, New York, McGraw-Hill, 1951, xiv-550 p.

The author presents as the main methods of teaching mathematics, the lecture method, the heuristic method, the genetic and the laboratory methods. He considers these as basically devoted to the understanding of the mathematical concepts and principles as well as to their applications to concrete situations.[23]

From the point of this research, one aspect of the entire teaching of secondary mathematics seems to be of major significance: the author distinguishes between the understanding of the concept and the skill of using it in solving problems. This is perhaps most clearly evident in the statement in Chapter XVIII where the authors say "... It must not be forgotten that the use of a theorem and the proof of that same theorem are two entirely different matters and that things may often be extremely useful without being completely understood".[24]

The author sees an important didactic value in equal attention to the concepts and skills in teaching mathematics. This can be seen quite clearly from the statement: "The student who gains a real understanding of the meanings" of mathematical concepts and operations "will derive a far

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[23] Charles H. Butler and F.L.Wren, The Teaching of Secondary Mathematics, New York, McGraw-Hill, 1951, pp 158-167.

[24] Ibid., p.536.

richer experience and a far more adequate basis for further work in either applied or theoretical mathematics than the student who works by rote alone".[25]

Thus, the importance of the two aspects (concepts and skills) and distinguishing between them as generally different, though closely correlated, problems, seems to provide some basis for the problem of measuring the correlation between these two didactically important aspects of mathematical instruction.

However, that correlation is, in fact, only assumed by the author as being close, thus leaving the objective measurement of their closeness in algebra open to further investigation.

E. John Wesley Young, Lectures on Fundamental Concepts of Algebra and Geometry, New York, Macmillan, 1920, vii-247 p.

A purely mathematical, deep, work on fundamental algebraical and geometrical concepts, designed for College and University students. The algebraic concepts are not treated from the didactical point of view and not for the high school level. The main bearing of this book upon this study consists in the insight it gives into the necessity for a thorough and systematic dealing with basic mathematical concepts when determining their basic components.

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[25] Butler and Wren, op.cit., p.513.

No measurement of understanding the concept is of concern to the author.

F. Burton W. Jones, Elementary Concepts of Mathematics, New York, Macmillan, 1947, xiii-294 p.

The main idea of concept-skill correlation in algebra does not find any reflection in this book, which treats exclusively of the strictly mathematical side of selected algebraic and geometric concepts for College level.

Some of the basic algebraic concepts, like algebraic fractions, algebraic powers, and the like are presented in nearly complete make-up of their essential components. Some, however, are presented rather superficially or even wrongly. For example, the definition of the equation given "An algebraic equation is an equality between two algebraic expressions"[26] is not correct. Thus, only the mathematical side, and only of some algebraic concepts, was of some use in this study.

G. Bruce E. Meserve, Fundamental Concepts of Algebra, Cambridge 42, Mass., Addison-Wesley Publishing Company, 1953, ix-294 p.

This work is basically concerned with the concepts of higher algebra and higher analysis. The approach to the problems is purely mathematical and so no psychological nor didactical side of the algebraic concepts are developed.

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[26] Burton W. Jones, Elementary Concepts of Mathematics, New York, Macmillan, 1947, p.141.

It helps a clear distinction of the partial algebraic concept at the high school level, from the full mathematical meaning of the concept, and by so much it is reflected in this study.

H. J.P.Everett, The Fundamental Skills in Algebra, Teachers' College, Columbia University, New York, 1928, pp vii-109.

The author brings out the obscurity of the word "skill". "It is easy to recognize the person who possesses skill, but it is by no means easy to tell either what comprises the skill or how it may be imparted to another."[27]

Everett distinguishes clearly between the manipulative skill and the associative skill, and presents the objective way in which the latter has been found as something distinct from the former.

Into the solution of a simple equation in particular from the cases of the following sort:

Problem	Percent of Correct Responses (1204 pupils tested)
$4x + 5 = 17$	97.5
$8x = 5x + 12$	89.3
$\frac{Y}{3} + 2 = 5$	64.3
$\frac{x}{5} - 5 = 9$	53.7

it is evident that "there enters some ability that is outside of, and distinct from, the ability to add, subtract,

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[27] J.P.Everett, The Fundamental Skills in Algebra, New York, Teachers' College, Columbia University, 1928, p.8.

multiply, or divide. To abilities of this nature the name associative skills apply".[28]

In the case, for example, of the equation 3x = 14, "the real substance of the statement resides in the fact that the expression 3x = 14 means that the second number of the equation is 3 times x. The recognition of this fact is the associative skill of this relationship".[29]

These are some of the examples quoted by the author to illustrate the distinctness of the two kinds of skill.

But"elementary algebra requires that the pupil shall be able to add, subtract, multiply, and divide algebraic numbers, either as found in independent expressions or in equations".[30]

The obvious conclusions would seem to be that the skill required from the pupil in algebraic computing should consist of both manipulative and associative skill. Without the latter the operation is not meaningful to the learner and deprives him of the ability to deal successfully with algebraic situations.

Logically, the latter aspect should be reasonably reflected in the formulation of test-items, to measure the algebraic skill at High School level.

Finally, it could probably be said rightly that the associative skill, being of "high order" - as it undoubtedly

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[28] Ibid., Chapter III, p.16.

[29] Ibid., p.87.

[30] Ibid., p.87.

is - can be closely referred to the basic concept underlying the computation performed. The better and broader comprehension of the concept, the more likely is the associative skill for computing with it. Furthermore, the more adequate the care in developing the associative skill in algebraic computations, the better and fuller will be the comprehension of the concept connected with it. This latter view, though not included in Everett's work, has been adopted as one of the ideas basic to the construction lines of the tests for this research.

I. James G. Umstattd, <u>Secondary School Teaching</u>, Boston, Ginn & Co., 1944, xii-488 p.

The only bearing of the content of this work on the present research is the kind of method applied to it. The author sets forth convincingly the experimental method as "unquestionably the most valuable method of educational research".[31]

In particular, the procedure of determining the significance of difference between the means for two groups of testees, which has been applied by the author in the chapter entitled "Equivalent-Group Method" has been applied to several similar cases in this research.

The works of other authors on educational measurement have also been of much help in applying the methods used in this research, but they are not quoted in the <u>Bibliography</u> as they do not make for the research itself.

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[31] J.G.Umstattd, <u>Secondary School Teaching</u>, Boston Ginn and Company, 1944, p.409.

In summarizing the review of available literature, it should be noted that all the more or less strong and convincing suggestions made by different scientists and authors as to the worthiness and appropriateness of the investigation on the concept-skill correlation in mathematics, point at the fact that such investigation is necessary and though it is considered by competent writers to be anything but an easy task, nonetheless it seems to be a purposeful effort.

The objective investigation of the problem, however, requires tests as the tools of the work. The following chapter will present,the steps taken to evolve possibly adequate and accurate measuring devices.

CHAPTER II

CONSTRUCTION OF THE TESTS

1. Choice of the Algebraic Material

Grade XI Algebra material has been chosen for this investigation as a representative material of the intermediate course of algebra. This is the level at which familiarity with the primary algebraic concepts has already been achieved and at which computations with general symbols are fairly independent from arithmetical skill in computation. At this level the solution of algebraic problems rests rather with the algebraic insight marked by knowledge of algebraic concepts and abilities in using them in algebraic computations, leaving to the figures and arithmetical computing a relatively secondary role. In that area appear distinctly the concepts of basic value with reference to the general role of algebra in education which lies in development of one's general intellectual ability in "verbal, mathematical and other abstract and symbolic situations".[1]

Thus, the aspects of symbolism: $a, [, \{, (, \neq, >, <,$ as well as of generalization, in terms of formulas, and of $y = ax$, $y = ax^2$ relations, and functionality - are all

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[1] Edward L. Thorndike, *et al.*, *The Psychology of Algebra*, New York, Macmillan, 1923, p.448.

included in eleventh grade algebra, and consequently in the tests. That choice of the algebraic material is also consistent with the principles generally brought out on the role of mathematics in general education.[2] It is also consistent with the curriculum[3] of high schools issued in 1952, and now in force in schools. Thus, the exact list of the material of instruction of algebra, contained in the tests, is as follows: (1) Bracket as a symbol of algebraic wholeness; (2) concealed bracket; (3) power of a number: a^n; (4) square of a binomial $(a \pm b)^2$; (5) cube of a binomial $(a \pm b)^3$; (6) square, cube, and higher roots: $\sqrt{a} = a^{1/2}$, $\sqrt[3]{a} = a^{1/3}$, $\sqrt[n]{a} = a^{1/n}$; (7) difference of squares $(a^2 - b^2)$; (8) sum and difference of cubes $(a^3 \pm b^3)$; (9) algebraic fraction; (10) ratio; (11) inverse of fraction; (12) direct and inverse variation; (13) proportion; (14) linear equation: $ax + by = c$; (15) equation of the first degree; (16) quadratic equation $ax^2 + bx + c = 0$, as a case of $y = ax^2 + bx + c$; (17) graphic solution of the equations; (18) the highest common factor and the lowest common multiple.

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[2] Progressive Education Association, Mathematics in General Education, A Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum, New York, Appleton-Century-Crofts, 1938, xiv-423 p.

[3] Ontario Department of Education, Courses of Study, Grades XI and XII Mathematics, issued by authority of the Minister of Education for the Province of Ontario, Toronto, 1954, pp 3-10.

2. Tools for Objective Investigation

No standardized tests for measuring the comprehension of algebraic concepts exclusively, nor for measuring skill in algebraic computation, are available. The existing standardized tests[4] on algebra measure both the achievement in understanding the algebraic concept and in the computational skill simultaneously, placing only a greater emphasis on either of them, and doing that according to their particular criteria and aims of testing. They serve mainly the improvement of instruction, methods of teaching, and better organization of the material in the curriculum, or promotion and guidance purposes. Moreover, they all employ the algebraic computation in their test items for both kinds of achievement, in understanding and in computing. This fact represents a disadvantage as for the consideration of the mutual relation between the two main aspects of the problem.

Therefore, two separate tests are needed for an investigation of the relation between the understanding of the algebraic concepts and the skill in algebraic computations. The *Algebraic Concepts Test* is to serve mainly the

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[4] Educational Testing Service, *The Cooperative Algebra Tests Manual of Directions.* Princeton, New Jersey, 1951, 12 p.

Science Research Associates, *Examiner's Manual for the Basic Skills in Arithmetic Test.* 57 W. Grand Ave., Chicago, 1945, 8 p.

--------, *Examiner's Manual for Test 4 - Ability to do Quantative Thinking.* Chicago, 1951, 9 p.

measure of the comprehension of the concept, The Algebraic Skill Test is to measure mainly the ability in using the concept in algebraic computations. The first test is to measure mainly the power of understanding; the second test is to measure mainly the power of operation. In the chapters to follow, the first test will be called in short: The Concept-Test, and the second test will be called: The Skill-Test.

The problem requiring an immediate answer at this stage seems to be the exact determination of the kind of problems in which the computational skill is to be exerted: the algebraic computing or solving the verbal problem? Both kinds, with a majority of the first type, are applied in the skill-test. This is done on the basis that algebraic computing employs very much the same abilities needed for the solving of problems. Thorndike says: "In fact, algebraic computation involves very much the same abilities that problem solving does".[5]

The main construction lines of the two tests are entirely different in one respect. The Concept-Test consists basically of verbal questions and verbal distractors. The algebraic expressions used in the Concept-Test do not entail any algebraic computations. The Skill-Test consists of algebraic expressions exclusively. Thus, the algebraic

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[5] Thorndike et al., op.cit., p.448.

computation is eliminated as far as possible from the Concept-Test, and placed only in the Skill-Test. An attempt is made, therefore, in this experiment, to avoid the disadvantage of the common computational aspect in both tests, which exists in all standardized tests.

A random arithmetical computing employed in some Concept-Test items is limited to a negligible extent in comparison with the weight of the algebraic aspect involved in those items.

3. Main Structure Lines of the Tests

Both tests consist of sixty-five, four-choice items each. Their validity and reliability is to be secured by application of all corrections derived from the tryout and their subsequent statistical evaluation. The procedure applied to that purpose is shown in Appendix 2 and in the enclosed Manual and the Booklets of the tests.

The criteria of measuring a given concept in The Algebraic Concepts Test, and a corresponding ability in The Algebraic Skill Test, are quoted in Appendix 2. An attempt has been made to fix them on approximately three grades of difficulty.

Each Concept-Test item is assigned a position of one of the following three levels of difficulty:

(1) Recognition of the concept;
(2) Interpretation or evaluation of the concept;
(3) Application of the concept.

Each Skill-Test item has been fixed respectively on one of the three grades of difficulty:

(1) Computing *on* the concept; (corresponding to the Recognition level above);
(2) Computing *of* the concept; (corresponding to the Interpretation or Evaluation level);
(3) Computing *by* the concept (corresponding to the Application level, above).

This difficulty-grading of the concepts is based on the findings of Thorndike,[6] Ross,[7] and Adkins[8] in psychology and measurement in education.

It should, however, be made clear that this three-step difficulty grading is basically a tentative operation. Its justification lies in logical and practical motives of a constructional nature. They correspond, in principle, to the steps applied by the teachers of the schools concerned in actual teaching practice, and they are, according to these teachers' opinion "measured-out" to fit the possibilities of their average student. Thus, they tend to conform to the two measurement-principles postulated by Ross:[9] the average pupil in the group should "make about 50 per cent of the possible score"; and "about half of the group should 'know' the answer to each item, while the other half should not".

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[6] Thorndike *et al.*, *op.cit.*, pp 126-131, 194-198, 328.

[7] C. C. Ross, *Measurement in Today's Schools*, New York, Prentice-Hall, 1954, pp 164-167.

[8] D. C. Adkins, *Construction and Analysis of Achievement Tests*, Washington, D.C., U.S.Government Printing Office, 1947, pp 50-52.

[9] Ross, *op.cit.*, p.148.

The lack of all three "steps of difficulty" with reference to some concepts in the tests, is accounted for by finding in the preliminary tryouts some items to be too easy or too hard.

The criteria underlying the formulation of each item in both tests are quoted in the test booklets on the right side of the items. (Appendix 2). They correspond to the three tentative grade difficulty levels, and consequently are mutually matched between the two tests,

The methodology of the external appearance of the test items as well as of the formulation of the problems and that of the distractors is based mainly on the principles stressed by Ross,[10] and Adkins[11]. Care has been taken to make the items simple and clear in their appearance, easily readable, containing no superfluous words and no ambiguity whatsoever, easy and clearly understandable, and containing a considerable particle of right answers in their distractors. The latter include also the most frequent mistakes made by pupils in their normal algebra course.

4. Administration and Scoring - Planned

The principles of the administration and scoring of the tests are quoted in the enclosed Algebraic Tests

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[10] Ross, op.cit., pp 179-186.

[11] Adkins, op.cit., pp 39-76.

Manual. Specific directions for each test are singled out in the Test Booklets (Appendix 2).

Conferences with teachers on curricular validity of the tests, and on all preliminary arrangements to make a thorough organization of the whole testing procedure have been considered essential to the experiment. Room, time of beginning, the seating order, supply of pencils on hand, and above all, a good atmosphere for work and the prevention of any disturbances during the testing, have been matters of care to be employed in the tryout and in the final administration of the tests.

The tests have been planned to be administered to a representative group of the English-speaking Catholic High School population of 370 pupils of Grade XI Algebra course in the City of Ottawa, consisting of 220 boys and 150 girls, ranging in age from 15 to 17 years, and representing the accomplishments of two years algebra course-work, as well as the normal range of intelligence quotients and that of educational achievement. No special selection of pupils has been planned for test purposes, as the ordinary conditions and circumstances of everyday school situations have been considered of main importance for both tests.

No time limit for the tests has been provided. About sixty minutes for each has been considered sufficient time for working out all sixty-five questions in each test.

Proctors have been used especially in those cases where the tests had to be administered in two or more rooms simultaneously.

An answer key has been prepared for hand-scoring of the tests. It is simply an unused test booklet with the correct responses filled in. The scoring of the tests is objective and thus it could be done by any trained person under supervision. One point of credit is given for each correct answer. The formula for guessing $S = C - \frac{1}{3} W$ in computing total scores is to be applied according to Ross'[12] recommendation in case "some pupils have omitted a fairly large number of items, while others have omitted few". All counting of right and wrong answers requires re-checking.

The degree of adequacy of all principles applied to the construction of both tests can be estimated in the light of the result of the tryout, and probably more clearly yet - because of the larger size of the group tested - in the final administration of the tests.

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[12] Ross, op.cit., p.158

CHAPTER III

TRYOUT AND FINAL ADMINISTRATION OF TESTS

1. Trying Out the Tests

Both tests were tried out in the City of Ottawa at St. Patrick's High School for Boys, Grades XI-XII, with 34 pupils for the Concept-Test and 31 pupils for the Skill-Test, on the 22nd of February, 1956, and at the Immaculata High School, with 25 Grade XI girl students for the Concept-Test, and 25 girl students for the Skill-Test, on 27th and 28th of February, 1956.

The group was a homogeneous sample of the entire Grade XI-XII English-speaking population in both schools. The pupils come from Ottawa and vicinity, and were not selected as to their origin, intelligence, abilities, etc. Their range of age is between 15 and 17 years. They have taken the same instruction course in algebra (1½-2 years) and have been taught basically by the same methods. The range of their IQ's is basically normal; the majority of the individuals ranging from 90 to 110, with some (a number of 7-14%) ranging higher.

A larger group, though very desirable, was unobtainable for the tryout test due to organizational reasons in the institutions concerned.

Analysis for the validity of the items, by 27% high- and 27% low-group method,[1] presented in Appendix 3, shows that there were six items in the Concept-Test, and five items in the Skill-Test, of doubtful usefulness, as the corresponding differences between the high and low responses were, in three cases, equal to zero, and in three cases equal to minus one, in the Concept-Test; and similarly one case equal to zero and five cases of small negative value in the Skill-Test.

However, as the size of the tryout group was rather small, the items have been left without change, as there seemed to be a justifiable hope that the questionable items might prove useful on a larger group of testees.

2. Final Administration of the Tests

The group for the final administration of the tests, as stated in the preceding chapter, was intended to be of 370 pupils of both sexes of Grades XI-XII of the English-speaking Catholic High School population of the City of Ottawa. The difference between Grade XI and Grade XII pupils is only external. Grade XII pupils are virtually those, who, for organizational reasons in schools, have taken only Grade XI course in Algebra. The testing included only those pupils who "ceteris paribus" had taken the same instructional course in Algebra.

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[1] C.C.Ross, Measurement in Today's Schools, New York, Prentice-Hall, 1954, p.118.

With this in mind, the tests were administered in four Ottawa High Schools:

St. Patrick's High School - to 168 boys, on 13 April 1956;

Notre Dame Convent High School - to 22 girls on 19 April 1956 (Skill-Test) and on 21 March, 1956, (Concept-Test);

Rideau Convent High School - to 53 girls on 19 April 1956 (Skill-Test), and on 28 March 1956 (Concept-Test);

Academy de LaSalle High School - to 39 boys on 13 April 1956 (Concept-Test), and on 20 April, 1956 (Skill-Test).

However, for organizational reasons in schools, it was impossible to obtain more than 320 Grade XI-XII pupils for the final administration of the tests.

For similar reasons, and because of the language factor in some schools, as well as for the advancement-differential in the algebra instruction course, a group of 109 pupils, consisting of 87 boys from St. Patrick's High School, and 22 girls from Notre Dame Convent, has been chosen as the final representative group of the English-speaking Grade XI Algebra students in the Catholic High School population of the City of Ottawa.

This group, about twice the size of that used in the tryout tests, has been selected for final analysis of the validity of the test items, and for determining the validity and reliability of the tests as a whole, as well as for the investigation of the problem of the correlation as determined in the introduction to this thesis.

As there are no external criteria scores available for computing the validity coefficients of the tests, the following procedure has been adopted:

First, analysis of the difficulty of the whole test and the test items, on the basis of the whole final group of 109 testees, has been made.

The average scores made in both tests are computed in Tables I and II. They are 35.14 in the Concept-Test, and 33.29 in the Skill-Test. As they are 54% and 51% of the maximum score possible, respectively, they seem to indicate a near-satisfactory grade of difficulty of the two tests.[2]

The difficulty of the test items has been computed in per cents in Appendix 4. There are two items of 0-15% and five items of 85-99% difficulty indices, which should have been, according to Ross,[3] removed from the Concept-Test. For the same reasons, three items of 0-15% and six items of 85-99% difficulty indices should have been removed from the Skill-Test. They have, however, been retained because the difficulty of items is relatively unimportant in this type of test.

A general picture of the dispersion of items-difficulty in both tests is presented graphically in Figure 1. Both graphs show that the extension of items difficulty, though

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[2] Ross, *op.cit.*, p.160

[3] *Ibid.*, p.119.

Concept-Test

90-99%	// //	4
80-89%	𝍸 𝍸	10
70-79%	𝍸 //	7
60-69%	𝍸 ///	8
50-59%	𝍸 ////	9
40-49%	𝍸 ////	9
30-39%	𝍸 ///	8
20-29%	𝍸 //	7
10-19%	//	2
0-9%	/	1
Total		65 items

Skill-Test

90-99%	////	4
80-89%	𝍸 //	7
70-79%	////	4
60-69%	𝍸 ///	8
50-59%	𝍸 ////	9
40-49%	𝍸 ////	9
30-39%	𝍸 ///	8
20-29%	𝍸 /	6
10-19%	𝍸 //	7
0-9%	///	3
Total		65 item

Figure 1. Illustration of the Extent of Items Difficulty Indices on the Two Tests.

not perfect, seems to approach a desirable normal extension nearly satisfactorily. The main pile-up of the items falls almost symmetrically down towards the two ends of the scale, and contains a number of nearly 45 items, out of 65, within the range of the middle 68.2% of the scale. The main distortion occurs at the sector of the easiest items, and that seem to have some partial justification in the fact that some of the easiest items have been purposely placed at the beginning of the tests.

As there could be no certainty as to whether the cause of that distortion rested only with the tests themselves, and as the difficulty of a mastery and diagnostic test is determined, according to Ross,[4] by "the importance of the subject matter than that its difficulty", no items were disregarded at the final scoring of the tests.

3. - Coefficient of Reliability of the Tests

The reliability coefficient of the Concept-Test was found by the test-retest method.

The group of thirty pupils from St.Patrick's High School was used for this purpose. Appendix 5 contains the names of the students and the scores they obtained on the test on February 22, 1956, and on April 13, 1956. The coefficient of correlation was computed in the Correlation Chart in Appendix 5, and found equal to 0.80. This falls

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[4] Ross, <u>op.cit.</u>, pp 148-149, and p.160.

within the bracket which, according to Guilford,[5] is a desirable coefficient of correlation.

As the size of the group tested was rather small, and as memory and instructional factors could also have been involved in the case, the coefficient of reliability found seems to be acceptable.

The coefficient of reliability of the Skill-Test was not computed, because a group of at least thirty students was not obtainable for organizational reasons in schools. However, as the validity of a test is much more important than its reliability and, furthermore, as the validity of the two tests has been found basically equal, it seems justifiable to assume that the reliability of the Skill-Test would not be inferior to that of the Concept-Test.

4. - Usability of the Tests

The two tests proved to be rather easy to administer. At any rate, no remarks on the impracticability of the tests have been made by teachers or pupils. In particular, there have been no complaints raised about insufficient information in the Tests Manual, or obscurity of instructions and of questions in the test booklets.

The simple scoring procedure applied is to be described next.

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[5] J.P.Guilford, Fundamental Statistics in Psychology and Education, 2nd ed., New York, McGraw-Hill, 1950, p.166.

CHAPTER IV

SCORING AND CORRELATING THE TWO SETS OF TEST SCORES

1. - Scoring Procedure

The tests have been scored by the technique described in the Algebraic Tests Manual contained in Appendix 2.

The formula for guessing has not been applied to final scoring, because, according to Ross,[1] there was no fairly large number of items "omitted by some pupils, while others have omitted few" as well as "practically all the testees have attempted every item", thus making the formula, according to D. Adkins,[2] unnecessary.

2. - Determining the Nature of the Distributions

The final group for the investigation of the problem of correlation consists of 109 pupils of St. Patrick's and Notre Dame Convent High Schools, whose names and scores, obtained on both tests, are listed in Appendix 6. The

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[1] C.C.Ross, Measurement in Today's Schools, New York, Prentice-Hall, 1954, p.158.

[2] D.C.Adkins, Construction and Analysis of Achievement Tests, Washington, D.C., U.S.Government Printing Office, 1947, p.189.

test booklets of that final group, 109 in number, of the Concept-Test, and 109 of the Skill-Test, are also enclosed.

Tables I and II present the two frequency distributions and computations of means and standard deviations. To determine the nature of the distributions, their normality or non-normality is to be determined first. If the distributions on both tests turn out to be normal, according to Guilford,[3] their linearity will be promoted.

The answer to the problem is furnished by analysis of the frequency distributions of the sets of scores obtained on both tests.

Table I furnishes the data needed for the analysis of scores obtained on the Concept-Test.

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[3] J.P.Guilford, Fundamental Statistics in Psychology and Education, 2nd ed., New York, McGraw-Hill, 1950, p.314.

Table I. - A Grouped Frequency Distribution of the Algebraic Concept-Test

Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
59	1	40	28.5	35	54.5	31	80	25	103.5
51	2	40	28.5	35	54.5	31	80	23	106
48	3	40	28.5	35	54.5	31	80	22	107
47	4	40	28.5	35	54.5	31	80	21	108
46	5	39	32	35	54.5	31	80	18	109
45	6.5	39	32	35	54.5	30	86		
45	6.5	39	32	35	54.5	30	86		
44	8.5	38	35	35	54.5	30	86		
44	8.5	38	35	35	54.5	30	86		
43	13	38	35	34	65	30	86		
43	13	37	40	34	65	29	82.5		
43	13	37	40	34	65	29	82.5		
43	13	37	40	34	65	28	93		
43	13	37	40	34	65	28	93		
43	13	37	40	34	65	28	93		
43	13	37	40	34	65	28	93		
42	18.5	37	40	33	71.5	28	93		
42	18.5	36	45.5	33	71.5	27	97.5		
42	18.5	36	45.5	33	71.5	27	97.5		
42	18.5	36	45.5	33	71.5	27	97.5		
41	23.5	36	45.5	33	71.5	27	97.5		
41	23.5	35	54.5	33	71.5	26	100.5		
41	23.5	35	54.5	32	75.5	26	100.5		
41	23.5	35	54.5	32	75.5	25	103.5		
41	23.5	35	54.5	31	80	25	103.5		
41	23.5	35	54.5	31	80	25	103.5		

Highest Score 59

Lowest Score 18

Range = Difference + 1 = 42

Class Interval $i = \frac{\text{Range}}{\text{Number of classes}} = \frac{42}{14} = 3$

Scores X	Mdpt	f.	fX	x (deviation)	f x	$f_x{}^2$
57-59	58	1	58	22.7	22.7	515.29
54-56	55	0	0	19.7	0.0	0.00
51-53	52	1	52	16.7	16.7	278.89
48-50	49	1	49	13.7	13.7	187.69
45-47	46	4	184	10.7	43.1	461.17
42-44	43	13	559	7.7	101.2	779.24
39-41	40	13	520	4.7	62.2	292.34
36-38	37	4	518	1.7	25.0	42.50
33-35	34M	27	918	- 1.2	- 32.6	39.12
30-32	31	14	434	- 4.2	- 58.9	247.38
27-29	28	11	336	- 7.2	- 79.2	570.24
24-26	25	6	125	- 10.2	- 61.2	624.24
21-23	22	3	66	- 13.2	- 39.6	520.20
18-20	19	1	19	- 16.2	- 16.2	262.44

N=109
$\Sigma X = 3,838$

$\Sigma x^2 = 4820.74$

$$\text{Mean}_c = \frac{\Sigma X}{N} = 35.211$$

$$\sigma_c = \text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{N}} = \sqrt{44.22} = \underline{\underline{6.7}}$$

When cutting off the standard deviation $\sigma_c = 6.7$ on both sides of the mean rounded to 35.2 an interval from 41.8 to 28.6 is obtained. Seventy-four scores, equal to 68.2%, of N = 109, should fall into this interval. In reality, about seventy scores are covered by that interval, and this is the first indication that the distribution approximates normality.

Moreover, when cutting off two σ_c on both sides of the mean, an interval from the midpoint, 18, to the midpoint, 53, is obtained. This interval contains 108 scores which approximates 95.4% of all scores, thus furnishing an additional indication of the near-normality of the distribution.

A grouped frequency distribution of the Algebraic Skill-Test is presented in Table II.

The analysis of data in Table II follows the pattern of that in Table I.

Table II. – A Grouped Frequency Distribution of the Algebraic Skill-Test

Score	Rank	Score	Rank	Score	Rank	Score	Rank
57	1	36	34.5	30	65.5	22	99
51	2	36	34.5	30	65.5	21	103
49	3.5	35	38	30	65.5	21	103
49	3.5	35.	38	29	72.5	21	103
47	5.5	35	38	29	72.5	21	103
47	5.5	35	38	29	72.5	21	103
46	7	35	38	29	72.5	20	106
44	8	34	44.5	29	72.5	18	107
43	9	34	44.5	29	72.5	15	108
42	11	34	44.5	28	76.5	14	109
42	11	34	44.5	28	76.5		
42	11	34	44.5	27	81		
41	14.5	34	44.5	27	81		
41	14.5	34	44.5	27	81		
41	14.5	34	44.5	27	81		
41	14.5	33	50	27	81		
40	17	33	50	27	81		
39	19.5	33	50	27	81		
39	19.5	32	55	26	86		
39	19.5	32	55	26	86		
39	19.5	32	55	26	86		
38	24.5	32	55	25	89		
38	24.5	32	55	25	89		
38	24.5	32	55	25	89		
38	24.5	32	55	24	92		
38	24.5	31	60	24	92		
38	24.5	31	60	24	92		
37	30.5	31	60	23	95.5		
37	30.5	30	65.5	23	95.5		
37	30.5	30	65.5	23	95.5		
37	30.5	30	65.5	23	95.5		
37	30.5	30	65.5	22	99		
37	30.5	30	65.5	22	99		

Highest Score 57

Lowest Score 14

Range = Difference + 1 = 44

Class Interval $i = \frac{\text{Range}}{\text{Number of classes}}$

Scores Y	Mdpt	f.	f Y	y deviation)	f_y	f_y2
57-59	58	1	58	25.7	25.7	660.49
54-56	55	0	0	22.7	0.0	0.00
51-53	52	1	52	19.7	19.7	388.09
48-50	49	2	98	16.7	33.4	557.78
45-47	46	3	138	13.7	41.2	564.44
42-44	43	5	215	10.7	53.6	573.52
39-41	40	9	360	7.7	69.6	535.92
36-38	37	14	518	4.7	66.2	311.14
33-35	34	16	544	1.7	27.7	47.09
30-32	31	18	558	- 1.2	-22.7	27.24
27-29	28	15	420	- 4.2	-63.9	268.38
24-26	25	9	225	- 7.2	-65.3	470.16
21-23	22	12	264	-10.2	-123.1	1255.62
18-20	19	2	38	-13.2	-26.5	344.80
15-17	16	1	16	-16.2	-16.2	262.44
12-14	13	1	13	-19.2	-19.2	368.64
		N=109	$\Sigma y =$ 3517			$\Sigma y^2 = 6635.75$

$\text{Mean}_s = \frac{\Sigma Y}{N} = \underline{32.265}$

$\sigma_s =$ Standard deviation $= \sqrt{\frac{\Sigma y^2}{N}} = \sqrt{60.87} = \underline{7.8}$

The standard deviation σ_s = 7.82 (the sigma of the Skill-Test), when cut off on both sides of the mean: M_s = 32.265, produces an interval ranging from the midpoint 40 to the midpoint 24, and containing 76 scores which is almost exactly 68.2% of the total number of cases. This shows that the frequency distribution Y approximates the normal. The double σ_s which equals 15.64 units, when cut off on both sides of the mean, furnishes the interval ranging from the midpoint 47 to the midpoint 16, and covers 104, or 95.4%, of the total number of cases. This in turn seems to indicate the normality of the distribution Y too.

As the numbers of cases on both sides of the mean form an almost symmetrical pattern (9,14,16 or 9,15,18), the near-normality of the frequency distribution Y seems to be acceptable.

3.- Coefficient of Correlation of the Two Sets of Scores

As the near-normality of distributions X and Y may promote[4] their linearity, and as both distributions may be regarded as continuous, the coefficient of correlation of the two sets of scores can be computed. For this purpose, the Dayhaw Correlation Chart is used, as presented in Appendix 7.

According to Guilford,[5] the coefficient of correlation being 0.70 indicates that there is a positive

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[4] *Ibid.*, p.314.

[5] *Ibid.*, p.165.

"moderate correlation" and a direct "substantial relationship" between the two distributions, and between the mastery of algebraic concepts and computational skills based on them respectively. In reality, the r_{xy} found equals 0.7044, thus being larger than 0.70; and this, according to Guilford, will indicate "high correlation; marked relationship". The latter seems to be the more acceptable, as the correlation ratio $\eta_{xy} = \eta_{yx} = 0.73$, being the maximum size of correlation index for any set of data, seems to supply a definite indication of that kind of relationship.

In the case considered the size of the group, $N = 109$, exceeds the number of classes $K_x = 14$, and $K_y = 16$ considerably. Thus, the Chi-Square test can be applied[6] to determine the linearity of the regression. Taking the numbers of freedom $(K_x - 2)$ and $(K_y - 2)$, the probability levels of about .75 and .90 respectively are found by interpolation from the Chi-Square tables. They correspond to $\chi^2 = 7.99$ and $\chi^2 = 7.79$ for variables X and Y respectively, and, as they are definitely smaller than the required Chi-Squares at the 5 (and even at the 11 and 13) per cent levels, the hypothesis of normality of the distributions seems to be quite acceptable.

The known values of the coefficient of correlation and of the means and standard deviations of the two distributions make it possible to determine now which of the tests is more difficult for the group tested, as well as whether

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[6] Ibid., p.320.

that difficulty-difference be significant or not. The answer to that problem is given by determining the significance of difference between the means of the two performances by applying the formula for computing the value of the ratio:

$$\frac{D}{\sigma_D} = \frac{D}{\sqrt{\sigma_{mc}^2 + \sigma_{ms}^2 - 2r_{cs}\,\sigma_{mc}\,\sigma_{ms}}}$$

where D is the difference of the means (35.21 and 32.26) and σ_D is the standard error of that difference.

The standard errors of the means are:

$$\sigma_{M_c} = \frac{\sigma_c}{\sqrt{N}} = 0.63$$

$$\text{and } \sigma_{M_s} = \frac{\sigma_s}{\sqrt{N}} = 0.74$$

thus, the value of $\frac{D}{\sigma_D} = \frac{2.95}{\sqrt{0.3939+0.5476-2 \times 0.7 \times 0.4662}}$

$$= \frac{2.95}{0.55}$$

$$= 5.3$$

The determined value of the ratio $\frac{D}{\sigma_D}$ being larger than 3,[7] provides the basis for stating that the difference between the means of the performances on the two tests is significant, and consequently that the Skill-Test is significantly harder than the Concept-Test.

Having established this basic fact, further steps for vertical and horizontal comparison of the performances on the two tests can be taken.

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[7] Adkins, op.cit., pp 132-133.

CHAPTER V

VERTICAL AND HORIZONTAL COMPARISON BY THE THREE-GROUP DIVISION METHOD

The correlation between the two sets of scores is to be investigated now in a two-way analysis:

(a) A Vertical Analysis - consisting of the comparison of three sub-groups in each test separately, i.e. comparing the upper with the middle, and the middle with the lower group in both the Concept-Test and the Skill-Test.

(b) A Horizontal Analysis - consisting of the comparison of the performance of the upper concept group on the Concept-Test, with its performance on the Skill-Test, and of the performance of the middle and lower concept groups with their performances on the Skill-Test.

The same comparison is to be made for three skill groups.

1. Data for Comparisons

To divide the whole test groups into three comparable groups, upper, middle, and lower, ranking of the raw scores and computation of the percentiles and standard scores are needed. This has been done in Appendix 8, and all the data obtained have been arranged in Tables III and IV. The first of these, Table III, presents the set of scores on the Concept-Test, and the second, Table IV, the set of scores on the Skill Test.

In each table, the upper group consists of 36 individuals, from No.1 to No. 36; the middle group consists of 36 individuals from No.37 to No.72, and the lower group of 37 individuals from No.73 to No.109. The numbers in Tables III and IV correspond in each test to the names of the individuals as listed in the basic list of names in Appendix 6.

Table III. - Tabulation of Raw Scores, Ranks, Percentiles, and Normalized Standard Scores of the Final 109 Pupils on the Algebraic Concept Test.

Name No.	Scores in		Rank in		Percentile in		Normalized Standard Score in	
	Concepts	Skills	Concepts	Skills	Concepts	Skills	Concepts	Skills
1	59	57	1	1	99.5	99.5	2.60	2.60
2	51	47	2	5.5	98,6	95.4	2.20	1.69
3	48	51	3	2	97.7	98.6	2.00	2.20
4	47	42	4	11	96.7	90.3	1.84	1.30
5	46	41	5	14.5	95.8	87.1	1.73	1.14
6	45	37	6.5	30.5	94.4	72.4	1.59	0.59
7	45	34	6.5	44.5	94.4	59.6	1.59	0.24
8	44	41	8.5	14.5	92.6	87.1	1.45	1.14
9	44	27	8.5	81	92.6	26.1	1.45	-0.64
10	43	49	13	3.5	88.5	97.2	1.20	1.91
11	43	49	13	3.5	88.5	97.2	1.20	1.91
12	43	38	13	24.5	88.5	77.9	1.20	0.77
13	43	39	13	19.5	88.5	82.5	1.20	0.93
14	43	29	13	72.5	88.5	33.9	1.20	-0.41
15	43	38	13	24.5	88.5	77.9	1.20	0.77
16	43	27	13	81	88.5	26.1	1.20	-0.64
17	42	44	18.5	8	83.4	93.1	0.97	1.46
18	42	37	18.5	30.5	83.4	72.4	0.97	0.59
19	42	37	18.5	30.5	83.4	72.4	0.97	0.59
20	42	34	18.5	44.5	83.4	59.6	0.97	0.24
21	41	34	23.5	44.5	78.8	59.6	0.80	0.24
22	41	40	23.5	17	78.8	84.8	0.80	1.03
23	41	42	23.5	11	78.8	90.3	0.80	1.30
24	41	38	23.5	24.5	78.8	77.9	0.80	0.77
25	41	37	23.5	30.5	78.8	72.4	0.80	0.59
26	41	32	23.5	55	78.8	50	0.80	0.00
27	40	47	28.5	5.5	74.3	95.4	0.65	1.69
28	40	46	28.5	7	74.3	94	0.65	1.55
29	40	35	28.5	38	74.3	65.5	0.65	0.26
30	40	32	28.5	55	74.3	50	0.65	0.00
31	39	38	32	24.5	71.1	77.9	0.55	0.77
32	39	30	32	65.5	71.1	40.3	0.55	-0.02
33	39	32	32	55	71.1	50	0.55	0.00
34	38	18	35	107	68.3	2.2	0.47	-2.00
35	38	39	35	19.5	68.3	82.5	0.47	0.93
36	38	33	35	50	68.3	54.5	0.47	0.11
37	37	41	40	14.5	63.7	87.1	0.35	1.13
38	37	34	40	44.5	63.7	59.6	0.35	0.24
39	37	34	40	44.5	63.7	59.6	0.35	0.24
40	37	37	40	30.5	63.7	72.4	0.35	0.59
41	37	30	40	65.5	63.7	40.3	0.35	-0.25
42	37	31	40	60	63.7	45.4	0.35	-0.12
43	37	30	40	65.5	63.7	40.3	0.35	-0.25

Table III. - Tabulation of Raw Scores, Ranks, Percentiles, and Normalized Standard Scores of the Final 109 Pupils on the Algebraic Concept-Test

Name No.	Scores on		Rank in		Percentile in		Normalized Standard Score in	
	Concepts	Skills	Concepts	Skills	Concepts	Skills	Concepts	Skills
44	36	28	45.5	76.5	58.7	30.2	0.22	-0.52
45	36	34	45.5	44.5	58.7	59.6	0.22	0.24
46	36	31	45.5	60	58.7	45.4	0.22	-0.12
47	36	33	45.5	50	58.7	54.5	0.22	0.11
48	35	29	54.5	72.5	50.4	33.9	0.03	-0.41
49	35	37	54.5	30.5	50.4	72.4	0.03	0.59
50	35	34	54.5	44.5	50.4	59.6	0.03	0.24
51	35	30	54.5	65.5	50.4	40.3	0.03	-0.25
52	35	29	54.5	72.5	50.4	33.9	0.03	-0.41
53	35	30	54.5	65.5	50.4	40.3	0.03	-0.25
54	35	41	54.5	14.5	50.4	87.1	0.03	1.13
55	35	36	54.5	34.5	50.4	68.8	0.03	0.44
56	35	32	54.5	55	50.4	50	0.03	0.00
57	35	43	54.5	9	50.4	92.2	0.03	1.42
58	35	42	54.5	11	50.4	90.3	0.03	1.30
59	35	35	54.5	38	50.4	65.5	0.03	0.40
60	35	30	54.5	65.5	50.4	40.3	0.03	-0.41
61	35	27	54.5	81	50.4	26.1	0.03	-0.64
62	34	22	65	99	40.8	9.6	-0.23	-1.30
63	34	36	65	34.5	40.8	68.8	-0.23	0.44
64	34	39	65	19.5	40.8	82.5	-0.23	0.93
65	34	29	65	72.5	40.8	33.9	-0.23	-0.41
66	34	25	65	89	40.8	18.8	-0.23	-0.88
67	34	34	65	44.5	40.8	59.6	-0.23	0.24
68	34	35	65	38	40.8	65.5	-0.23	0.40
69	33	23	71.5	95.5	34.8	12.8	-0.39	-1.13
70	33	23	71.5	95.5	34.8	12.8	-0.39	-1.13
71	33	27	71.5	81	34.8	26.1	-0.39	-0.64
72	33	25	71.5	89	34.8	18.8	-0.39	-0.88
73	33	33	71.5	50	34.8	54.5	-0.39	0.11
74	33	39	71.5	19.5	34.8	82.5	-0.39	0.93
75	32	29	75.5	72.5	31.1	33.9	-0.48	-0.41
76	32	26	75.5	86	31.1	21.5	-0.48	-0.79
77	31	30	80	65.5	27	40.3	-0.61	-0.25
78	31	25	80	89	27	18.8	-0.61	-0.88
79	31	38	80	24.5	27	77.9	-0.61	0.77
80	31	30	80	65.5	27	40.3	-0.61	-0.25
81	31	27	80	81	27	26.1	-0.61	- 0.64
82	31	27	80	81	27	26.1	-0.61	-0.64
83	31	21	80	103	27	5.9	-0.61	-1.56
84	30	26	86	86	21.5	21.5	-0.79	-0.79
85	30	32	86	55	21.5	50	-0.79	0.00

Table III. - Tabulation of Raw Scores, Ranks, Percentiles, and Normalized Standard Scores of the Final 109 Pupils on the Algebraic Concept-Test (Continued)

Name No.	Scores on		Rank in		Percentile in		Normalized Standard Score in	
	Concepts	Skills	Concepts	Skills	Concepts	Skills	Concepts	Skills
86	30	35	86	38	21.5	65.5	-0.79	0.40
87	30	28	86	76.5	21.5	30.2	-0.79	-0.83
88	30	24	86	92	21.5	16	-0.79	-1.00
89	29	20	89.5	106	18.3	3.2	-0.90	-1.85
90	29	22	89.5	99	18.3	9.6	-0.90	-1.30
91	28	26	93	86	15.1	21.5	-1.03	-0.79
92	28	32	93	55	15.1	50	-1.03	0.00
93	28	35	93	38	15.1	65.5	-1.03	0.40
94	28	23	93	95.5	15.1	12.8	-1.03	-1.13
95	28	21	93	103	15.1	5.9	-1.03	-1.56
96	27	15	97.5	108	11	1.3	-1.25	-2.20
97	27	29	97.5	72.5	11	33.9	-1.25	-0.41
98	27	24	97.5	92	11	16	-1.25	-1.00
99	27	38	97.5	24.5	11	77.9	-1.25	0.77
100	26	27	100.5	81	8.2	26.1	-1.40	-0.64
101	26	31	100.5	60	8.2	45.4	-1.40	-0.12
102	25	23	103.5	95.5	5.5	12.8	-1.60	-1.13
103	25	24	103.5	92	5.5	16	-1.60	-1.00
104	25	32	103.5	55	5.5	50	-1.60	0.00
105	25	21	103.5	103	5.5	5.9	-1.60	-1.56
106	23	21	106	103	3.3	5.9	-1.85	-1.56
107	22	21	107	103	2.2	5.9	-2.00	-1.56
108	21	14	108	109	1.3	0.4	-2.20	-2.65
109	18	22	109	99	0.4	9.6	-2.65	-1.30

The foregoing data furnish a comparable basis for examining the raw scores from the two tests.

2.- Comparison Within a Test - Vertical

Comparison of the standard scores contained in Table III shows that

(a) in the upper group of the Concept-Test, there are 26 cases of scores, 25 larger and 1 equal to, their scores in the Skill-Test, leaving only 10 cases of smaller scores;

(b) in the middle group of the Concept-Test, there are 22 cases of scores larger than the corresponding scores in the Skill-Test, thus leaving only 14 cases of smaller scores; and

(c) in the lower group of the Concept-Test there are 13 cases of scores, 12 larger and 1 equal to their corresponding scores in the Skill-Test, leaving 24 smaller scores.

Consequently, out of the 109 cases comprising the whole group there are 61 cases of scores, 59 greater and 2 equal, in the Concept-Test as against those in the Skill-Test, and 48 cases with lower scores in the Concept-Test than in the Skill-Test.

This seems to lend itself to a three-fold interpretation:

(a) The Concept-Test was in general easier for the upper concept-group than the Skill-Test, as more than two-thirds of all their scores in Concepts are higher than in Skills.

(b) For the middle group in the Concepts, the Concept-Test was generally easier than the Skill-Test, as a little less than two-thirds of all their scores in Concepts are higher than in Skills.

(c) For the lower group in the Concepts, the Concept-Test, generally, was more difficult than the Skill-Test, as a third of all their scores in the Concepts are larger than in Skills.

The difficulty of the Concept-Test grows when going from the strongest across the middle to the weakest group of testees. This is natural. However, some additional questions should be answered to determine this trend beyond doubt:

(1) Does this increase in difficulty occur in both the Concept and the Skill-Test, or only in one of them; and

(2) Are the differences of the difficulty of the three sub-groups significant in both tests, or not?

To answer these questions, the significance of the difference D, between the means of the upper and the middle groups in the Concept-Test is to be determined first. The formula $\frac{D}{\sigma D} = \frac{D}{\sqrt{\sigma M_1{}^2 + M_2{}^2}}$, where σ_D is the standard error of the difference between two means, and $\sigma_{M_1} = \frac{\sigma_1}{\sqrt{N}}$ and $\sigma_{M_2} = \frac{\sigma_2}{\sqrt{N}}$ are the standard errors of the means M_1 and M_2 respectively, is to be used here.

The necessary data have been computed in Appendix 9, Tables IX and X.

$M_1 = 40.41 \qquad M_2 = 35.08$

$D = 5.33$

$\sigma_1 = 4.08 \qquad \sigma_2 = 1.22$

Thus, $M_1 = \frac{4.08}{\sqrt{36}} = 0.68; \qquad M_2 = \frac{1.22}{\sqrt{36}} = 0.20$

$$\frac{D}{\sigma_D} = \frac{5.33}{\sqrt{0.68^2 + 0.20^2}} = \frac{5.33}{\sqrt{0.46 + 0.40}} = \frac{5.33}{0.92} = \underline{\underline{5.79}}$$

and this, being larger than 3, proves that the difference of the means is significant,[1] and consequently that the difficulty of *the Concept-Test is significantly greater for the middle group than for the upper group on that test*.

Similarly, the difference between the middle group and the lower in the Concept-Test is investigated subsequently. The computational details are presented in Appendix 9, Tables X and XI.

$M_2 = 35.08 \qquad M_3 = 28.08$

$D = 7.00$

$\sigma_2 = 1.22 \qquad \sigma_3 = 3.40$

Thus, $\sigma_{M_2} = 0.20; \qquad \sigma_{M_3} = \frac{3.40}{\sqrt{37}} = \frac{3.40}{6.082} = 0.55$

$$\frac{D}{\sigma_D} = \frac{7.00}{\sqrt{\sigma_{M_2}^2 + \sigma_{M_3}^2}} = \frac{7.00}{\sqrt{0.20^2 + 0.55^2}} = \frac{7}{0.58} = \underline{\underline{12}}$$

and this, being larger than the critical ratio 3, shows definitely that the difference of the means is significant,[2]

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[1] D. Adkins, *Construction and Analysis of Achievement Tests.* Washington, D.C., U.S.Govt.Printing Office, 1947, p.132

[2] *Ibid.*, p.132.

and consequently that the difficulty of the Concept-Test is significantly greater for the lower group than for the middle group on that test.

In its turn, the Skill-Test will be investigated with reference to the same difficulty grading aspect, within the three sub-groups (upper, middle and lower) of the whole group tested.

Table IV presents the rank-arrangement of scores of the Skill-Test, and the corresponding Concept-Test scores, percentiles and normalized standard scores.

Table IV. - Tabulation of Raw Scores, Ranks, Percentiles and Normalized Standard Scores on the Algebraic Skill Test.

Name No.	Scores on		Rank on		Percentile in		Normalized Standard Score on	
	Skills	Concepts	Skills	Concepts	Skills	Concepts	Skills	Concepts
1	57	59	1	1	99.5	99.5	2.60	2.60
2	51	48	2	3	98.6	97.7	2.20	2.00
3	49	43	3.5	13	97.2	88.5	1.91	1.20
4	49	43	3.5	13	97.2	88.5	1.91	1.20
5	47	51	5.5	2	95.4	98.6	1.69	2.20
6	47	40	5.5	28.5	95.4	74.3	1.69	0.65
7	46	40	7	28.5	94	74.3	1.55	0.65
8	44	42	8	18.5	93.1	83.4	1.46	0.97
9	43	35	9	54.5	92.2	50.4	1.42	0.03
10	42	41	11	23.5	90.3	78.8	1.30	0.80
11	42	35	11	54.5	90.3	50.4	1.30	0.03
12	42	47	11	4	90.3	96.7	1.30	1.84
13	41	46	14.5	5	87.1	95.8	1.14	1.73
14	41	44	14.5	8.5	87.1	92.6	1.14	1.45
15	41	37	14.5	40	87.1	63.7	1.14	0.35
16	41	35	14.5	54.5	87.1	50.4	1.14	0.03
17	40	41	17	23.5	84.8	78.8	1.03	0.80
18	39	38	19.5	35	82.5	68.3	1.03	0.47
19	39	34	19.5	65	82.5	40.8	1.03	-0.23
20	39	32	19.5	75.5	82.5	31.1	1.03	-0.48
21	39	43	19.5	13	82.5	88.5	1.03	1.20
22	38	41	24.5	23.5	77.9	78.8	0.77	0.80
23	38	31	24.5	80	77.9	27	0.77	-0.61
24	38	43	24.5	13	77.9	88.5	0.77	1.20
25	38	43	24.5	13	77.9	88.5	0.77	1.20
26	38	27	24.5	97.5	77.9	11	0.77	-1.25
27	38	39	24.5	32	77.9	71.1	0.77	0.55
28	37	45	30.5	6.5	72.4	94.4	0.59	1.59
29	37	42	30.5	18.5	72.4	83.4	0.59	0.97
30	37	42	30.5	18.5	72.4	83.4	0.59	0.97
31	37	37	30.5	40	72.4	63.7	0.59	0.35
32	37	41	30.5	23.5	72.4	78.8	0.59	0.80
33	37	35	30.5	54.5	72.4	50.4	0.59	0.03
34	36	35	34.5	54.5	68.8	50.4	0.44	0.03
35	36	34	34.5	65	68.8	40.8	0.44	-0.23
36	35	40	38	28.5	65.5	74.3	0.40	0.65
37	35	35	38	54.5	65.5	50.4	0.40	0.03
38	35	34	38	65	65.5	40.8	0.40	-0.23
39	35	30	38	86	65.5	21.5	0.40	-0.79
40	35	28	38	93	65.5	15.1	0.40	-1.03

Table IV. - Tabulation of Raw Scores, Ranks, Percentiles and Normalized Standard Scores on the Algebraic Skill Test (Continued)

Name No.	Scores on		Rank on		Percentile in		Normalized Standard Score on	
	Skills	Concepts	Skills	Concepts	Skills	Concepts	Skills	Concepts
41	34	45	44.5	6.5	59.6	94.4	0.24	1.59
42	34	41	44.5	23.5	59.6	78.8	0.24	0.80
43	34	37	44.5	40	59.6	63.7	0.24	0.35
44	34	37	44.5	40	59.6	63.7	0.24	0.35
45	34	45	44.5	6.5	59.6	94.4	0.24	1.59
46	34	35	44.5	54.5	59.6	50.4	0.24	0.03
47	34	34	44.5	65	59.6	40.8	0.24	-0.23
48	34	42	44.5	18.5	59.6	83.4	0.24	0.97
49	33	38	50	35	54.5	68.3	0.11	0.47
50	33	36	50	45.5	54.5	58.7	0.11	0.22
51	33	33	50	71.5	54.5	34.8	0.11	-0.39
52	32	40	55	18.5	50	83.4	0.00	0.97
53	32	39	55	32	50	71.1	0.00	0.55
54	32	35	55	54.5	50	50.4	0.00	0.03
55	32	30	55	86	50	21.5	0.00	-0.79
56	32	28	55	93	50	15.1	0.00	-1.03
57	32	25	55	103.5	50	5.5	0.00	-1.60
58	32	41	55	23.5	50	78.8	0.00	0.80
59	31	37	60	40	45.4	63.7	-0.12	0.35
60	31	36	60	45.5	45.4	58.7	-0.12	0.22
61	31	26	60	100.5	45.4	8.2	-0.12	-1.40
62	30	39	65.5	32	40.3	71.1	-0.25	0.55
63	30	37	65.5	40	40.3	63.7	-0.25	0.35
64	30	37	65.5	40	40.3	63.7	-0.25	0.35
65	30	35	65.5	54.5	40.3	50.4	-0.25	0.03
66	30	35	65.5	54.5	40.3	50.4	-0.25	0.03
67	30	35	65.5	54.5	40.3	50.4	-0.25	0.03
68	30	31	65.5	80	40.3	27	-0.25	-0.61
69	30	31	65.5	80	40.3	27	-0.25	-0.61
70	29	35	72.5	54.5	33.9	50.4	-0.41	0.03
71	29	34	72.5	65	33.9	40.8	-0.41	-0.23
72	29	32	72.5	75.5	33.9	31.1	-0.41	-0.48
73	29	27	72.5	97.5	33.9	11	-0.41	-1.25
74	29	43	72.5	13	33.9	88.5	-0.41	1.20
75	29	35	72.5	54.5	33.9	50.4	-0.41	0.03
76	28	36	76.5	45.5	30.2	58.7	-0.52	0.22
77	28	30	76.5	86	30.2	21.5	-0.52	-0.79
78	27	35	81	54.5	26.1	50.4	-0.64	0.03
79	27	33	81	71.5	26.1	34.8	-0.64	-0.39
80	27	31	81	80	26.1	27	-0.64	-0.61
81	27	31	81	80	26.1	27	-0.64	-0.61
82	27	26	81	100.5	26.1	8.2	-0.64	-1.40

Table III. - Tabulation of Raw Scores, Ranks, Percentiles, and Normalized Standard Scores on the Algebraic Skill Test (Continued)

Name No.	Scores on		Rank on		Percentile in		Normalized Standard Scores in	
	Skills	Concepts	Skills	Concepts	Skills	Concepts	Skills	Concepts
83	27	44	81	8.5	26.1	92.6	-0.64	1.45
84	27	43	81	13	26.1	88.5	-0.64	1.20
85	26	30	86	86	21.5	21.5	-0.79	-0.79
86	26	28	86	93	21.5	15.1	-0.79	-1.03
87	26	31	86	80	21.5	27	-0.79	-0.61
88	25	34	89	65	18.8	40.8	-0.88	-0.23
89	25	33	89	71.5	18.8	34.8	-0.88	-0.39
90	25	31	89	80	18.8	27	-0.88	-0.61
91	24	29	92	89.5	16	18.3	-1.00	-0.90
92	24	27	92	97.5	16	11	-1.00	-1.25
93	24	25	92	103.5	16	5.5	-1.00	-1.60
94	23	33	95.5	71.5	12.8	34.8	-1.13	-0.39
95	23	28	95.5	93	12.8	15.1	-1.13	-1.03
96	23	33	95.5	71.5	12.8	34.8	-1.13	-0.39
97	23	25	95.5	103.5	12.8	5.5	-1.13	-1.60
98	22	18	99	109	9.6	0.4	-1.30	-2.65
99	22	34	99	65	9.6	40.8	-1.30	-0.23
100	22	28	99	93	9.6	15.1	-1.30	-1.03
101	21	30	103	86	5.9	21.5	-1.56	-0.79
102	21	25	103	103.5	5.9	5.5	-1.56	-1.60
103	21	23	103	106	5.9	3.3	-1.56	-1.85
104	21	22	103	107	5.9	2.2	-1.56	-2.00
105	21	27	103	97.5	5.9	11	-1.56	-1.25
106	20	29	106	89.5	3.2	18.3	-1.85	-0.90
107	18	38	107	35	2.2	68.3	-2.00	0.47
108	15	27	108	97.5	1.3	11	-2.20	-1.25
109	14	21	109	108	0.4	1.3	-2.65	-2.20

From Table IV it is seen that out of 36 cases in the upper group of the Skill-Test, 22 have larger scores, and 1 has an equal score in comparison with their scores in the Concept-Test, thus leaving only 13 scores lower in the Skill-Test.

The middle group has, in 14 cases out of 36, larger scores in the Skill-Test than in the Concept-Test, thus leaving 22 scores smaller in the Skill-Test than in the Concept-Test.

The lower group in the Skill-Test has, in 13 cases out of 37, 12 higher and 1 equal score to the corresponding scores on the Concept-Test, thus leaving 24 cases of scores smaller than on the Concept-Test.

From the above it follows that out of all 109 cases, 50 scores in the Skill-Test are greater or equal to their scores in the Concept-Test, and 59 cases have lower scores in the Skill-Test than in the Concept-Test.

Using both Tables III and IV, it is easily seen that the skill scores of the upper concept group are dispersed along all three sub-groups of the Skill Test. Likewise, the dispersion of the scores of any other sub-group of either test along all sub-groups of the other test can be read directly from these tables.

Summarizing the numerical data of the above analysis, of Tables III and IV, a simple summary, Table V, is obtained.

Table V. - Verification of the Analysis of the Two Tests

GROUP	Number of Cases Having Scores In The Concept Test Larger than on ST*	Equal to ST*	Smaller than on ST*	Summary	The Skill Test Larger than on CT**	Equal to CT**	Smaller than on CT**	Summary
Upper	25	1	10	36	22	1	13	36
Middle	22	0	14	36	14	0	22	36
Lower	12	1	24	37	12	1	24	37
TOTAL	59	2	48	109	48	2	59	109

* ST = The Skill-Test

** CT = The Concept-Test

The correctness of the numbers obtained in the analysis of the two tests is thus verified.

In turn, a determination of the significance of the difference D, between the means of the upper and the middle group and that between the middle and the lower group in the Skill-Test follows. The computational details are presented in Appendix 9, Tables XII and XIII.

The formula $\frac{D}{\sigma_D} = \frac{D}{\sqrt{\sigma_{M_1}^2 + \sigma_{M_2}^2}}$; where σ_D is the standard error of the difference between two means; and $\sigma_{M_1} = \frac{\sigma_1}{\sqrt{N}}$, and $\sigma_{M_2} = \frac{\sigma_2}{\sqrt{N}}$ are the standard errors of the means M_1 and M_2 respectively, is again used.

$M_1 = 40.43$ $M_2 = 32.08$

$D = 8.35$

$\sigma_1 = 4.80$ $\sigma_2 = 1.94$

$\sigma_{M_1} = 0.80$ $\sigma_{M_2} = 0.32$

Thus, $\frac{D}{\sigma_D} = \frac{8.35}{\sqrt{0.80^2 + 0.32^2}} = \frac{8.35}{0.74} = \underline{\underline{11.2}}$, and this, being larger than the critical ratio 3, proves that the difference between the means of the upper and the middle groups in the Skill-Test is significant.

A similar exploration of the difference between the mean of the middle group, M_2, and the mean of the lower group, M_3, in the Skill-Test, produces, as computed in Appendix 9, Table XIV, the following results.

$$M_2 = 32.08 \qquad M_3 = 23.97$$

$$D = 8.11$$

$$\sigma_2 = 1.94 \qquad \sigma_3 = 3.5$$

$$\sigma_{M_2} = 0.31 \qquad \sigma_{M_3} = 0.57$$

Thus, $\frac{D}{\sigma_D} = \frac{8.11}{\sqrt{0.31^2 + 0.57^2}} = \frac{8.11}{\sqrt{0.420}} = \frac{8.110}{0.648} = \underline{\underline{12.51}},$

and this, being larger than the critical ratio 3, means that the difference in question is definitely significant.

It follows that *the difficulty of the Skill-Test grows significantly from the upper group to the middle, and from the middle to the lower group*.

It remains now to explore the significance of the differences between the means of the performances of the three groups on one test, and the means of their performances on the other test.

3. Comparison Between the Two Tests - Horizontal

The key to the horizontal comparison of corresponding groups is in the formula for the standard error of difference between two means:[3]

$$\sigma_D = \sqrt{\sigma_{M_1}^2 + \sigma_{M_2}^2 - 2r_{12}\,\sigma_{M_1}\sigma_{M_2}}$$

A new symbol in this formula is r_{12}, and it is the coefficient of the correlation of the two sets of scores; the meaning of the other symbols are the same as in the preceding analyses.

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[3] Adkins, *op.cit.*, p.131.

This formula must be used in subsequent considerations because the groups of the testees to be correlated are the same persons who have taken two different tests.

Appendix 9, Correlation Charts A, B, and C, present the computations of the three coefficients of correlation for the upper, middle, and lower groups in the Concepts-Test, and the Correlation Charts D. E, and F, present similar computations for the upper, middle, and lower groups of the Skill-Test.

Appendix 9, Table XV, presents grouped concept scores of the upper, middle and lower skill groups, whereas Table XVI provides grouped skill scores of the upper, middle and lower concept groups.

The corresponding data for the significance of the difference between the means for the upper Skill group are as follows:

Mean of the Skill scores: $M_s = 40.43$ (Appendix 9, Table XII)

Mean of the Concept Scores: $M_c = 40.25$ (Appendix 9, Table XV)

Difference of the Means $D = 0.18$

Standard deviations: $\left.\begin{array}{l}\sigma_s = 4.80 \\ \sigma_c = 5.61\end{array}\right\}$ Appendix 9 - Correlation Chart D

Standard errors of the means $\left\{\begin{array}{l}\sigma_{M_s} = \dfrac{\sigma_s}{\sqrt{N}} = \dfrac{4.80}{6} = 0.8 \\ \sigma_{M_c} = \dfrac{\sigma_c}{\sqrt{N}} = \dfrac{5.61}{6} = 0.93\end{array}\right.$

Correlation Coefficient: $r_{sc} = 0.615$ (Appendix 9, Correlation Chart A)

Standard error of difference of means:

$$\sigma_D = \sqrt{\sigma_{M_s}^2 + \sigma_{M_c}^2 - 2r_{sc}\sigma_{M_s}\sigma_{M_c}}$$

$$= \sqrt{0.80^2 + 0.93^2 - 1.488.r_{sc}}$$

$$= \sqrt{1.504 - 1.488.r_{sc}}$$

$$= 0.76$$

From this is derived the critical ratio $\frac{D}{\sigma_D} = \frac{0.18}{0.76} = 0.23$, and it is smaller than 3, which means that the difference of the means of the upper skill group on both tests is insignificant. In other words, the Skill-Test and the Concept-Test represent about the same difficulty for the upper Skill group.

In the same way the significance of the differences between the means on the Skill-Test and on the Concept-Test for the middle skill group and for the lower skill group is to be investigated. Though these differences may be expected to be of significance, as the whole Skill-Test preved to be significantly harder, nothing certain can be said without investigation of both cases.

The necessary data for the investigations are contained in Table VI, and the necessary computations from them are presented in Appendices 9 and 10.

Table VI. - Data for Determining the Significance of the Difference Between the Means of the <u>Skill Groups</u> in the Skill-Test and the Concept-Test.

STATISTICS					Computed in:
Needed		Symbol	Value in Skill Group		
			Middle	Lower	
Number of cases		N	36	37	
Mean in Skills		M_s	32.08	23.97	Appendix 9,Tables XIII & XI
Mean in Concepts		M_c	35.22	30.30	Appendix 9, Table XV
Difference of Means		$D=M_c-M_s$	3.14	6.33	
Standard Deviation	in Skill-Test	σ_s	1.94	3.50	Appendix 9 - Correlation Charts E, F.
	in Concept-Test	σ_c	4.56	5.40	
Standard Error	of M_s	σ_{M_s}	0.32	0.57	Appendix 10--points 1 and 2
	of M_c	σ_{M_c}	0.76	0.88	
Coefficient of correlation		r_{sc}	0.24	0.45	Appendix 9 - Correlation Charts E, F.
Standard Error of the Difference of Means		σ_D	0.69	0.804	Appendix 10 - points 1 and
Critical Ratio		$\frac{D}{\sigma_D}$	4.24	7.87	

From Table VI it is evident that both the middle and the lower skill groups find the Skill-Test significantly harder than the Concept-Test.

Hence, the summary of the findings on the Skill-Test follows:

For the whole group of 109 pupils, the Skill-Test is significantly harder than the Concept-Test, but for the upper group in skills there is no significant difference. It is the lower group in Skills that finds the Skill-Test most significantly harder, whereas the middle group finds it more difficult, to lesser, though significant degree.

The final problem arising out of this finding is whether the upper, middle and lower groups in the Concept-Test find a similar difficulty in the Skill-Test or not. Table VII contains the data needed to answer this question.

Table VII. - Data for Determining the Significance of the Differences between the Means of the <u>Concept-Groups</u> in the Concept-Test and the Skill-Test.

STATISTICS						
Needed		Symbol	Value in the Concept Group			Computed in:
			Upper	Middle	Lower	
Number of cases		N	36	36	37	
Mean in Concepts		M_c	40.41	35.08	28.08	Appendix 9, Tables IX,X,XI
Mean in Skills		M_s	38.08	32.11	26.78	Appendix 9, Table XVI
Difference of Means		$D=M_c-M_s$	2.33	2.97	1.30	
Standard Deviation	in Concept-Test	σ_c	4.08	1.23	3.40	Appendix 9 - Correlation Charts A,B,C.
	in Skill-Test	σ_s	6.63	5.36	5.96	
Standard Error	of M_c	$\sigma_{M_c} = \frac{\sigma_c}{\sqrt{N}}$	0.68	0.20	0.56	Appendix 10 - points 3,4,5.
	of M_s	$\sigma_{M_s} = \frac{\sigma_s}{\sqrt{N}}$	1.105	0.89	0.98	
Coefficient of Correlation		r_{cs}	0.65	0.39	0.49	Appendix 9 - Correlation Charts A,B,C.
Standard Error of the Differend of Means		σ_D	0.836	0.83	0.86	Appendix 10 - points 3,4,5.
Critical Ratio		$\frac{D}{\sigma_D}$	2.78	3.53	1.52	

Table VII makes possible the following summary of the findings on the Concept-Test. For the whole group of 109 pupils, the Concept-Test is significantly easier than the Skill-Test, and this holds true for the middle concept group as well. However, the upper concept group finds the Concept-Test only approximately easier than the Skill-Test, and the lower concept group finds it only insignificantly easier.

CHAPTER VI

SUMMARY OF THE FINDINGS

1. Relationship and Difficulty Aspects

Table VII completes the data needed for a synthesis of the vertical and horizontal comparisons and consequently of the whole investigation of the problem performed in this study.

The summary of the mathematical side of the correlation between the Algebraic Concept Test and the Algebraic Skill Test made in the preceding chapters is as follows:

(a) The correlation between the two tests is positive, and can be estimated to be "high and substantial",[1] but not very high, as the coefficient of correlation of the two distributions <u>r</u> equals 0.7044.

(b) The Skill-Test is significantly harder to the group tested than the Concept-Test.

(c) Both the Algebraic Concept Test and the Algebraic Skill Test have one feature in common. Either is significantly harder for its middle and lower groups than for the preceding group. This characteristic of both tests is apparent from the vertical comparison of the three groups (upper, middle and lower) in each test.

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[1] J.P.Guilford, <u>Fundamental Statistics in Psychology and Education</u>, 2nd ed., New York, McGraw-Hill, 1950, p.165.

(d) The closeness of the correlation between the sets of scores at the horizontal comparison is shown in Table VIII, being *r* = 0.65 and 0.61 for the two upper groups, *r* = 0.39 and 0.24 for the two middle groups, and *r* = 0.49 and 0.45 for the two lower groups. As direction of the correlation is positive in all these cases, a moderate correlation and substantial relationship[2] between the two variables can be claimed in all three divisions, upper, middle, and lower, at the horizontal comparison.

(e) The upper group in the Skill-Test finds that test only insignificantly easier than the Concept-Test. However, the middle and lower groups in the Skill-Test find the test significantly harder than the Concept-Test.

(f) The upper and the middle concept groups find the Skill-Test suggestively or significantly harder than the Concept-Test, but the lower concept group finds the Skill-Test only insignificantly harder than the Concept-Test.

As any group of either test is assigned a given grade of difficulty, according to the difference between its performance on the two tests, a simple table summarizing the data from Tables VI and VII can be constructed and appears as Table VIII.

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[2] *Ibid.*, p.165.

Table VIII. - Summary of the Performance Difficulty Grades on Both Tests.

Group	Performance of the Concept Groups		Group	Performance of the Skill Groups	
	on the Concept-Test	on the Skill-Test		on the Skill-Test	on the Concept-Test
UPPER	$M = 40.41$ $\sigma = 4.08$	$M = 38.08$ $\sigma = 6.63$	UPPER	$M = 40.43$ $\sigma = 4.80$	$M = 40.25$ $\sigma = 5.61$
	$r = 0.65$			$r = 0.61$	
	$\frac{D}{\sigma_D} = 2.78 < 3$;	suggestive of a true difference		$\frac{D}{\sigma_D} = 0.23$	insignificant difference (easier)
MIDDLE	$M = 35.08$ $\sigma = 1.22$	$M = 32.11$ $\sigma = 5.36$	MIDDLE	$M = 32.08$ $\sigma = 1.94$	$M = 35.22$ $\sigma = 4.56$
	$r = 0.39$			$r = 0.245$	
	$\frac{D}{\sigma_D} = 3.53 > 3$;	significant difference		$\frac{D}{\sigma_D} = 4.24 > 3$;	significant difference
LOWER	$M = 28.08$ $\sigma = 3.40$	$M = 26.78$ $\sigma = 5.96$	LOWER	$M = 23.97$ $\sigma = 3.5$	$M = 30.30$ $\sigma = 5.4$
	$r = 0.49$			$r = 0.45$	
	$\frac{D}{\sigma_D} = 1.52 < 2$;	insignificant difference		$\frac{D}{\sigma_D} = 7.87 > 6$;	very significant difference

M = mean

σ = standard deviation

r = coefficient of correlation

$\frac{D}{\sigma_D}$ = critical ratio

2. Regularity Trait of Difficulty Interdependency

In Table VIII the existence of the interdependency between the difficulty grades on the three corresponding group levels can be noticed. In particular, some aspects of regularity, in that interdependency, can be found.

The grades of difficulty, when placed along the straight line of equal units, furnish a comparative scale, presented in Figure 2.

By means of that scale, the difference between the lower limits of the given grades of difficulty can be expressed in units.

Furthermore, the grades of difficulties shown in Table VIII can be arranged from the upper group down to the lower, along two central cycles in the same directions, producing three pairs of grades at equal intervals. This is presented in Figure 3.

When the starting points of the two skill- and concept-cycles differ by one interval, which is 1/3 of a cycle, then the paired grades differ increasingly in the same direction by one unit, starting from the smallest difference in a pair, 3, up to 5.

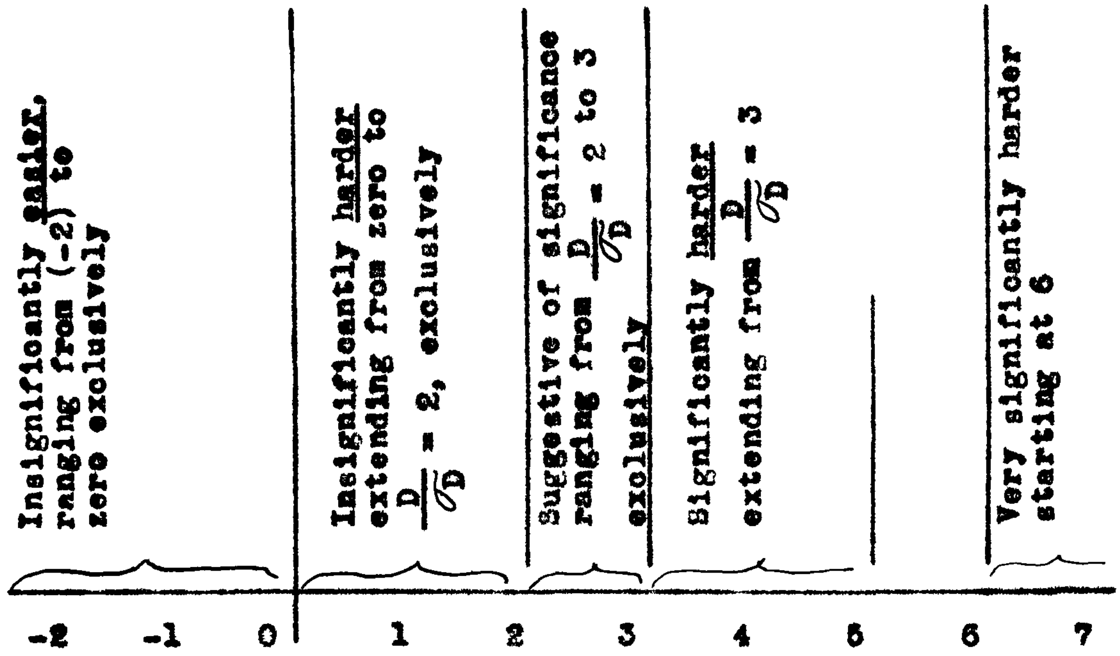

Figure 2. A Comparative Scale for Illustrating the Difficulty Grades Placement

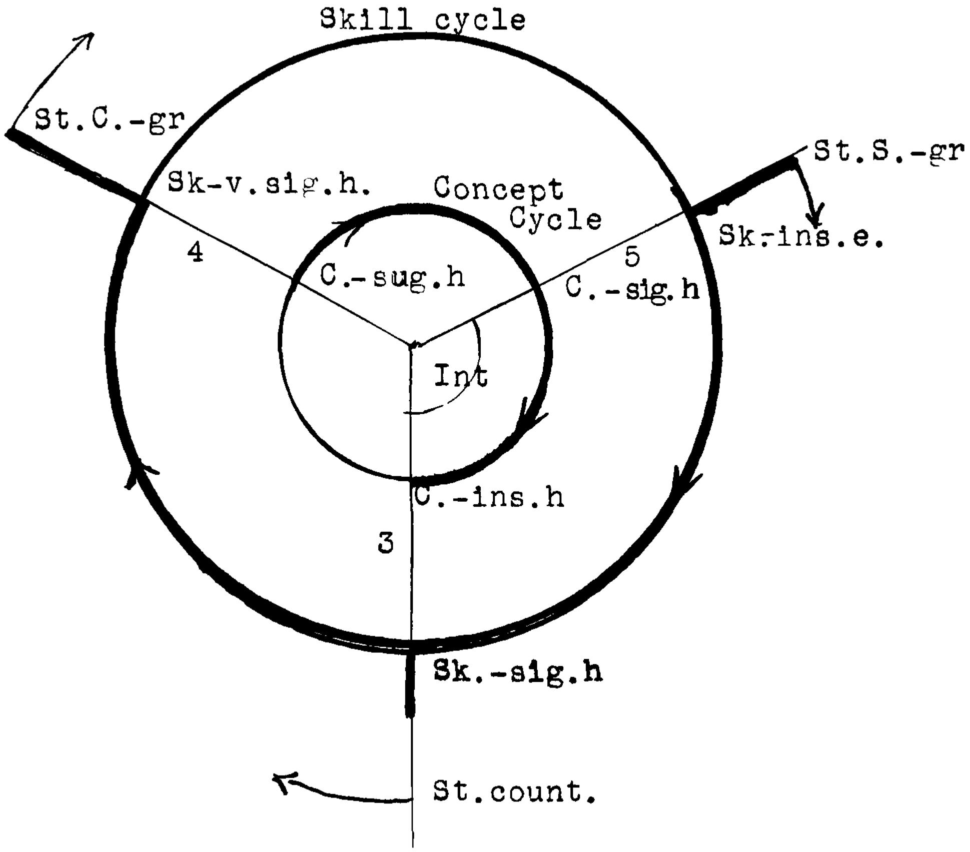

Figure 3. Graphic Illustration of the Interdependency of Difficulty Grades

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St.C.-gr = Starting Point of placing Concept Grades
St.S.-gr = Starting Point of placing Skill grades
St.count. = Starting Point of counting differences between grades
Int = One interval
Sk.-ins.e. = Skill-Test Difficulty Grade - insignificantly easier
Sk.-sig.h. = Skill-Test Difficulty Grade - significantly harder
Sk.-v.sig.h = Skill-Test Difficulty Grade - very significantly harder
C.-sug.h = Concept-Test Difficulty Grade - suggestive of significant hardness
C.-sig.h = Concept-Test Difficulty Grade - significantly harder
C.-ins.h = Concept-Test Difficulty Grade - insignificantly harder

This is so, because the differences between the lower limits of the larger and smaller difficulty grades in each pair on the cycles are as follows:

Sk.sig.h. - C.ins.h. = 3 - 0 = 3
Sk.v.sig.h. - C.sug.h. = 6 - 2 = 4
C.sig.h. - Sk.ins.e. = 3 -(-2)= 5

Thus, a set of increasing numbers, 3, 4, and 5, is obtained, and it is expressible mathematically by a finite arithmetic progression of $a_1 = 3$, and $d = 1$. It completes a trait of regularity of the final data resulting from the investigation of the mathematical aspect of the concept and skill correlation which would otherwise be scarcely discernible.

3. Confrontation of the Coefficient-Method and the Three-Group Method

Finally, a comparison of the results, as to the mathematical aspect of the concept-skill correlation, obtained by the coefficient and three-group division methods can be performed.

The product moment coefficient method has revealed two basic mathematical characteristics of the data correlated:

(a) The positive direction of the correlation, which means that when one variable is growing the other does the same; and

(b) The high closeness of the correlation, indicating marked relationship between the concept and skill in the considered area of algebra.

These traits are proved by the sign and the size of the correlation coefficient $r = 0.7044$, and by the size of the correlation ratio $\eta = 0.73$, whose firmness seems to be acceptable in the light of their respective standard errors (as computed in Appendix 7).

The *three-group division method* produced:

(a) The same result as to the direction of the correlation;

(b) The closeness of the relationship at the "moderate and substantial relation" level.

Though the latter is lower than the level found by the coefficient method, they seem essentially to be not incongruous, as a group three times as large was used for computing the correlation coefficient r for the whole group of testees, and as basically a rather considerable overlap of the two adjacent grades of relationship seems to be quite justifiable.

(c) Three significantly different group-levels in either test;

(d) Disclosure of the interplay of the test-difficulty grades by determining them on three distinctly different levels, and revealing a mathematically expressible regularity trait of the actual difficulty-grades interdependency.

Finally, it should be noted that the correlation between the two variables, the comprehension of the concept and the skill in computations, is most probably higher than that shown by the two methods, as some common factors involved in both tests, especially the computational aspect, might have only reduced the size of the correlation coefficient.

CONCLUSIONS

The exploration of the problem of the concept-skill correlation and consequently the data obtained, fall in two main divisions: the first pertaining to the construction of the tools for the experiment, and the second referring to the investigation of the hypothesis of the problem presented at the beginning of this study.

Accordingly, the conclusions to be drawn from the findings of the experiment can be arranged along these two lines.

1. First, it seems justifiable to state that the problem of the concept-skill correlation in algebra can be investigated objectively only by means of tests constructed especially for that purpose. As the tests are to measure the comprehension of algebraic concepts, and the ability to apply them in computations, respectively, only these two aspects should be covered by the tests, and they kept apart as far as possible. Although concept and skill in algebra cannot be separated from each other, nonetheless they can be distinguished quite clearly. The best and most effective way to make a clear distinction between them, without cutting them off, and completely distorting, if not killing them, seems to be to remove algebraic computation from the Concept-Test as completely as possible, and to place it only in the Skill-Test.

The method of investigating the problem by two tests can be basically extended to all grades of high school algebra. It can also be used for investigation of any selected group of concepts and skills at different grades in high school algebra.

2. The mathematical aspect of the concept-skill correlation, as revealed by the product moment coefficient method, produces two basic traits of the relationship between the two variables: first, its positive direction, and second, its high degree of closeness and marked relationship.

The three-group division method proves basically the same, but it adds two main traits to the mathematical aspect already mentioned:

(a) It discloses the existence of three distinctly different, though equally-numbered, sub-groups in either test group; and

(b) it reveals the regularity trait of the interdependency between the difficulty grades on three significantly different levels, provided the tests differ significantly in difficulty for the whole group tested.

This latter trait, if produced by a stratified representative province-wide group of pupils, would set a law of the interdependency of difficulty grades in the algebraic concept-skill correlation.

Thus, a deeper insight into the mathematical side of the relationship between the two variables seems to be justifiably expected through this method.

As this is most probably the first direct investigation of the concept-skill correlation in algebra, no generalizations of the results are possible, unless special standardized tests for that purpose are available and a stratified group, representative of the entire Grade XI high school population of the province of Ontario, is obtainable.

Under the actual circumstances of the experiment, the findings of this study may be rather only indicative of what could be expected in the area investigated in a normal everyday school situation.

BIBLIOGRAPHY

BUSWELL, G.T., Editor, *Supplementary Educational Monographs, Arithmetic 1947*, Chicago, University Press, 1949, ii-73 p.

The authors, specialists and experts in the subject-matter, present, in an objective way, their findings and views on the need for:

(a) emphasis on the understanding basic to skilful operations in arithmetic and algebra (Brueckner, L.J., "Arithmetic in Elementary and Junior High Schools", p 1-9);

(b) bringing out the fundamental concepts in elementary mathematics, and understanding them by pupils, in teaching practice (Hartung, Maurice L., "Improving the Mathematics Program in Junior High School Grades", p.49-58);

(c) distinguishing the key-concepts in teaching arithmetic (Engen, H.Van, "Place Value and the Number System", p.59-73).

-------- and Maurice L. Hartung, Editors, *Supplementary Educational Monographs*, Arithmetic, 1949, Chicago, University Press, 1950, ii-100 p.

The subject-matter authorities bring out in an objective way and emphasize that mathematics must be taught meaningfully (Buswell, G.T., "Methods of Studying Pupils Thinking in Arithmetic", pp 55-63; Hartung, M.L. "Major Instructional Problems in Arithmetic in the Middle Grades", pp 80-86). The book stresses also the need for objective instruments for investigation of basic mathematical understandings (Glennon, Vincent J. "Testing Meanings in Arithmetic", p.64-74).

BUTLER, C.H., *Mastery of Certain Mathematical Concepts by Pupils at the Junior High School Level*, University of Missouri, 1931, 56 p.

This doctoral dissertation presents the extent of mastery of some algebraic concepts by pupils at the completion of the seventh, eighth and ninth grades, and its relation to mental age and to chronological age. At the end of the work the investigation of the relation between the mastery of algebraic concepts and computational skill is recommended as a field of real contribution to didactic of algebra.

-------- and F.L.Wren, The Teaching of Secondary Mathematics, New York, McGraw-Hill, 1951, xiv-550 p.

The methods of teaching mathematics are presented by the authors in a descriptive, convincingly substantiating way. The conclusion drawn is that there is no single method which fits all situations. Concepts, principles and processes are not ultimately mastered without many illustrations in varied contexts, nor without frequent applications and practice, and not without sustained intellectual effort on the part of students. The importance of algebraic concepts and principles is brought out by this work, convincingly, as a field deserving much attention and consequently further exploration.

EVERETT, J.P., The Fundamental Skills in Algebra. Teachers' College, Columbia University, New York, No.324, 1928, pp vii-109.

This objective study on the algebraic skill results in clear distinction between the manipulative and the associative skill in algebra. A definite emphasis on the latter is brought out as a basic didactical postulate in efficient teaching of algebra. It somehow loses sight of the fact that the associative skill is also conditioned to a great extent by the degree of comprehension of the algebraic concept that underlies the algebraic computation.

JONES, Burton W., Elementary Concepts of Mathematics. New York, Macmillan, 1947, xiii-294 p.

The book is designed basically for college students. An interesting work on some strictly mathematical aspects of selected algebraical and basic geometrical concepts. Didactic side of concepts is of no concern to the author, at any rate not, as high school level algebra is concerned. Some distinction between the concepts in basic algebra and those in advanced algebra had to be taken into consideration in choosing the material for this study.

MESERVE, Bruce E., Fundamental Concepts of Algebra. Cambridge 42, Mass., Addison-Wesley Press, 1953, ix-294 p.

A modern viewpoint of the algebra and the analysis. The work is designed for college level. The clearness of basic components of the concepts of intermediate algebra is indirectly brought out, but not always properly.

PROGRESSIVE EDUCATION ASSOCIATION, *Mathematics in General Education. A Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary-School Curriculum.* New York, Appleton-Century-Crofts, 1938, xiv-423 p.

The subject-matter experts and scientists give an outline of a set of fundamental concepts and principles upon which the mathematics teacher may organize the teaching in school. Those concepts and related abilities are: function, operation, proof, and symbolism. The emphasis is put on the understanding of the underlying concepts to mathematical operations, in order that the latter may be really meaningful to students. Distinction between computational techniques and the concepts basic to them, is stressed as a basic didactical postulate in teaching mathematics.

THORNDIKE, Edward L., Margaret V. Cobb, Jacob S. Orleans, Percival M. Symonds, Elva Mald, and Ella Woodyard, *The Psychology of Algebra,* New York, Macmillan, 1924, xi-483 p.

This classical work is an excellent presentation by objective methods of the role of algebra in general education, and of the predominant role of the concepts and of the algebraic computation in developing the general abilities for dealing with: symbolism, generalization, quantative dependence, functionality, selecting and organizing ideas and habits to good effect. The expressiveness of laying the algebraic concepts especially disclosing and lighting important components of some concepts is reflected in some cases in the test items of this study.

UMSTATTD, J.G., *Secondary School Teaching,* Boston, Ginn, 1944, xii-488 p.

The book contains a concise historical outline of secondary school teaching from antiquity up to date, especially with respect to the objectives and methods of teaching. As far as the educational research in modern times is concerned, the historical, the survey, and the experimental methods are outlined. The experimental method is considered to be "unquestionably the most valuable" in educational research. The method of finding the significance of the difference between two means was applied in this research.

YOUNG, John Wesley, Lectures on Fundamental Concepts of Algebra and Geometry, New York, Macmillan, 1920, vii-247 p.

A mathematical science is defined as any body of propositions arranged according to a sequence of some or all the propositions that precede it. Accordingly, the starting point of any mathematical science is always a set of one or more propositions which remain entirely unproved. That science is growing even in its smallest portions, and disclosing unexpected relations. The relation between two important algebraic aspects is included in the topic of this study.

APPENDIX 1

CONFUSION OF SOME ALGEBRAIC CONCEPTS

The existence of some confusion in the field of algebraic concepts at high school algebra level, can perhaps best be shown by some concrete examples.

1. Equation

The equation is called wrongly an "equality": "An algebraic equation is an equality between two algebraic expressions".[1] This is a definition given by Burton W. Jones in his book Elementary Concepts of Mathematics, and which is also quite commonly used in high school textbooks of algebra.[2] In actual fact, an equality is the same as an identity. An equation can be turned into equality, but this does not mean that an algebraic equation is an equality or identity.

This confusion occurs always when the basic aspect of the concept of the equation is lost to sight. The equation in its entity is a question whether the two members may become equal or not. Being a question it cannot be a statement, as are identity or equality. The answer to the question can be yes or no; and in both cases, always, sometimes, or never.

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[1] B. Jones, Elementary Concepts of Mathematics, New York, Macmillan, 1947, p.141.

[2] J.T.Crawford, A New Algebra for High Schools, Toronto Macmillan Co. of Can., 1954, xii-441 p.

Converting the equation into the "conditional" or "identical" equalities or equations, which very often occurs in text books, does not remove the confusion, as the expression "identical equality" is obviously incorrect and confusing, because it implies something else than just "equality". The term "conditional" itself obviously implies the idea that equality may occur or may not; thus the basic definition of the equation requiring that it always be equality is denied.

Joining the "conditional equation" and "identical" into one concept of equation understood as equality does not remove the confusion from the concept, whereas accepting the most essential *question aspect of the equation-concept* simplifies and clarifies the situation in all cases:

(a) $ax + b = 0$, if *a* and *b*, or only *a* are different from 0, the equation exists, and becomes identity for $x = \frac{-b}{a}$;

(b) $ax + b = 0$, $0.x + b = 0$; if $a = 0$, $b \neq 0$, the equation disappears, because *x* disappears; and an obvious inequality $b \neq 0$ arises;

(c) $ax + b = 0$, $0.x + 0 = 0$ if $a = 0$, $b = 0$ the equation disappears, because *x* disappears, and an obvious equality $0 = 0$ arises.

Thus, *the equation in an unknown x is a question for such value of x which makes the left member equal to the right.*

2. Proportion

The definition of proportion given by Crawford in his book A New Algebra for High Schools.[3] is as follows: "A proportion is an equation which expresses the equality of two ratios". This definition seems to be incorrect for two reasons:

1. The proportion is not an equation, as it is only a statement of identity, in other words, a statement of equality of two ratios, which equation can never be;
2. The equation does not express the equality of two ratios, as it only expresses the question whether the ratios are equal.

Calling $\frac{3}{4} = \frac{15}{20}$ an equation, as the author does, is obviously incorrect, and inconsistent with the concept of identity used in other places in the book.

When taking the definition of the authors as it reads, one might rightly notice:

Should an "equation which expresses the equality of two ratios" be " a proportion", then there could not be any difference between the two. They would have to be the same and interchangeable notions. Thus the two expressions

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[3] Crawford, op.cit., p.166.

$2:3 = 4:6$ (1) which are true but different proportions
$2:4 = 3:6$ (2) $(K_1 = \frac{2}{3};\ K_2 = \frac{1}{2})$

would be true but different equations.

$2:x = 4:6$ (1)
$2:4 = x:6$ (2) $(x_1 = 3;\ x_2 = 3)$

but they are true but the same equations (whose only root is $x = 3$).

So proportion is not an equation of any kind. They are different things, and after all, they differ as concepts entirely in their basic components.

In the concept of proportion, the constant K is the pivotal point; in the concept of equation, the value of x, and thus the question for unknown, is the most basic point.

Similar examples of confusion in the field of basic algebraic concepts in school practice could be extended to sets of linear equations, quadratic equations, and so on. However, this would carry the discussion beyond the necessity of establishing the existence of such confusion, which was the original intent of this appendix.

APPENDIX 2

ALGEBRAIC TESTS MANUAL AND TEST BOOKLETS

The attached Test Booklets were constructed by the writer of this report and approved by the Department of Education of the School of Psychology and Education of the University of Ottawa, for administration in schools for the purpose of this research, in March 1956.

Each booklet serves as an answer-key containing the correct choices, marked in the brackets at the right of the test items.

The criteria of particular test items are annotated in the space provided originally for the testee's rough work.

ALGEBRAIC TESTS MANUAL

I. GENERAL INFORMATION

The purpose of the two tests, (1) ALGEBRAIC CONCEPTS TEST, (2) ALGEBRAIC SKILLS TEST, is to investigate the correlation between the mastery of algebraic concepts and computational abilities at the Grade XI high school algebra level.

The tests cover the following topics:

Brackets as a symbol of algebraic wholeness;

Power of a number: a^n;

Squares and cubes of binomials: $(a \pm b)^2$; $(a \pm b)^3$;

Factors of basic binomials: (a^2-b^2); $(a^3 \pm b^3)$;

Algebraic fraction;

Variation - direct and inverse;

Proportion;

Linear equation - algebraic and graphic solution;

Quadratic equation: $ax^2 + bx + c = 0$, its algebraic and graphic solution.

The population for the final tests is to be 370 boys and girls at Grade XI-XII Ottawa High Schools.

II. CONSTRUCTION OF TESTS

The following three main principles are observed in the construction of the tests:

1. The basic concepts in the <u>Algebraic Concepts Test</u> are chosen according to the selection recommended by:

(a) The findings of research on the didactics of algebra.[1]

(b) The curriculum as set by the Minister of Education, "Courses of Study Mathematics";

(c) Textbooks such as J.T.Crawford, A New Algebra for High Schools, Toronto, Macmillan, 1954, xii-441 p.

2. The algebraic computation is eliminated from The Algebraic Concept Test, by applying the verbal formulation of questions and distracters. This is done in all cases in which an algebraic computation could interfere with the intended measure of comprehension of the algebraic concept alone.

All computation needed in The Algebraic Concept Test is limited to a random arithmetical computing, that represents a negligible per cent of the algebraic concept aspect employed in the test item.

This procedure is an attempt to avoid including the algebraic computation in both tests.

In this respect, the test differs basically from the way of measuring the algebraic concepts by the standardized tests in use.

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[1] E.L.Thorndike, The Psychology of Algebra, New York, Macmillan, 1923, xi-483 p.

Progressive Education Association, Mathematics in General Education, A Report of the Committee on the Function of Mathematics in General Education, for the Commission on Secondary School Curriculum, New York, Appleton-Century-Crofts, 1940, xiv-423 p.

G.T.Buswell, Ed., Supplementary Educational Monographs, 1947, Chicago University Press, 1949, 11-73.

C.H.Butler and F.L.Wren, The Teaching of Secondary Mathematics, New York, McGraw-Hill, 1951, xiv-550 p.

3. Three leading construction lines are observed in the making of test items:

(a) Each concept in The Algebraic Concept Test is measured on three levels of difficulty:

(1) Recognition of the concept;

(2) Interpretation or evaluation of the concept;

(3) Application of the concept.

(b) Each skill in using a concept in computation is measured on three levels of difficulty:

(1) Computation on the concept (It corresponds to the Recognition level, above);

(2) Computation of the concept; (It corresponds to the Interpretation or Evaluation level, above);

(3) Computation by the concept; (It corresponds to the Application level, above).

Thus, three levels of difficulty in both tests are "matched".

Example: The concept of proportion.

I.	II.
(1) Recognition of proportion: Item: Which of the expressions below represents an algebraic proportion?	(1) Computation on proportion: Item: In the proportion $\frac{d}{3} = \frac{2}{6}$ d equals:
1 * * *	1 * * *
2 * * *	2 * * *
3 $\frac{a}{b} = \frac{c}{d}$	3 1
4 * * * (3)	4 * * * (3)

(2) <u>Evaluation</u> or interpretation of proportion:
<u>Item</u>:
The proportion $\frac{a}{b} = \frac{c}{d}$ is true if,

1 $\frac{a}{b} = p$, and $\frac{c}{d} = p$

2 * * *

3 * * *

4 * * * (1)

(3) <u>Application</u> of the concept of proportion.
<u>Item</u>:
The proportions $\frac{3}{4} = \frac{a}{b}$ and $\frac{m}{n} = \frac{n}{2}$ are identical, if

1 * * *

2 $\frac{3}{4} = \frac{m}{n}$

3 * * *

4 * * * (2)

(2) Computation <u>of</u> a proportion:
<u>Item</u>:
By adding 1 to both sides of the proportion $\frac{a}{b} = \frac{c}{d}$, one obtains a following new proportion

1 * * *

2 * * *

3 * * *

4 $\frac{a+b}{b} = \frac{c+d}{d}$ (4)

(3) Computation <u>by</u> identical proportion.
<u>Item</u>:
The proportions $\frac{a}{b} = \frac{c}{d}$ and $\frac{a+c}{b+d} = \frac{a-c}{b-d}$ are identical, because

1 * * *

2 $\frac{a}{b} = \frac{a+c}{b+d}$

3 * * *

4 * * * (2)

That three-steps grading is reduced to one or two steps in such cases, when its application would produce obviously too easy or too hard test items.*

<u>The Algebraic Skill Test</u> is constructed under the assumption that the algebraic computation, when employing recognition, interpretation, evaluation and application of a given concept, does measure much the same abilities, which are needed for both -- the algebraic computing as well as for solving of problems.**

** The difficulty grading and the assumption as to the kind of abilities measured by the <u>Algebraic Skill Test</u> are based on the findings contained in (1) Thorndike,E.L., <u>Psychology of Algebra</u>, New York, Macmillan, 1923, xi-483 p; *(2) Ross,C.C., <u>Measurement in Today's Schools</u>, New York, Prentice-Hall, 1954, pp 164-167; (3) Adkins, D.C., <u>Construction & Analysis of Achievement Tests</u>, Washington, US Govt.Printing Office,1947,pp 51-52.

The tests consist of 65 items each. Both tests were tried out on two groups of Grade XI-XII boys and girls numbering 55-60 each in two Ottawa High Schools.

III. ADMINISTRATION

1. The pupils should be told to bring and to use ordinary pencils and good erasers.

The examiner should have some supply of pencils on hand.

The students should be seated in alternate seats so they cannot observe the answers of their neighbours.

They should understand instructions on their booklets before starting work on the test questions.

The pupils should work seriously, but without an atmosphere of nervousness.

After having obtained the test booklets, the student should:

(a) print his name and fill in the blanks on the first page;

(b) read the instructions;

(c) ask questions concerning those directions;

(d) start working on test questions, when told by the examiner.

No questions should be asked after the examination has begun.

2. One proctor is needed for every 30 pupils. He should familiarise himself in advance with what he is to do:

distributing and collecting booklets as well as keeping every student at work, and helping establish a good rapport.

3. There is no time limit for the tests. The time, however, spent by each testee on working out the whole test should be noted on his booklet.

4. All booklets should be collected after the work of pupils on the test has been finished.

IV. SCORING

To score the test a scoring key for each page is to be placed along the corresponding column of answers on the proper page of the test booklet, and corresponding answers are counted.

The sum of errors and omissions subtracted from the total number of items, 65, gives the number of right answers.

The test score equals the number of right answers minus 1/3 of wrong answers. *The test score is always a whole number*, as the fractions are rounded to the nearest unit, according to the following example: $20-5\frac{1}{3} = 15$, $30 - 5\frac{2}{3} = 24$.

The score should be put down in the space: *Total score* on the first page of the test booklet.

The maximum total score is 65; the minimum total score is zero. Negative scores are treated as zero.

TEST BOOKLET APPENDIX 2

Total Score: ________

Percentile: ________

ALGEBRAIC SKILLS

TEST

Name: ______________________________ Date ______________________

Date of birth: ______________________

City: ______________ School ________________ Grade ____________

For how many years have you studied algebra? ______________________

Read these Instructions Carefully

The purpose of this test is to find to what extent you are mastering some computations in algebra. Furthermore, the test is to help you and your fellow students in obtaining better mastery of algebraic computations. Though there is no time limit for this test, you should not spend too much time on any one problem. Space for your rough work is provided at the right of each question. No questions may be asked after your work on answers has begun.

It is important to remember how to put down the right answer to each problem of the test. Here is the explanation of it: Each problem of the test is followed by four answers, but only one of them is the correct one. You are to find that correct answer and to write the number of the line on which you have found that answer in the bracket to the right.

Example: Do your rough work in this space

The algebraic fraction is represented by the following expression:

1 $\frac{1}{2}$

2 b

3 $\frac{a}{b}$

4 $-2a$ (3)

The correct answer is $\frac{a}{b}$ and it is on the third line. Therefore, 3 has been written in the bracket to the right.

<u>Do your rough work in this space</u>

1. $-(-2a) = -[-2a]$ are both equal to:

1 a

2 $-2a$

3 $2a$

4 $-a$

<u>Criteria</u>

Recognition level:

Computation <u>on</u> synonymous brackets.

(3)

2. $-[a-(2a+b)+b]$ equals

1 a

2 $-a$

3 $-3a$

4 $3a+2b$

Interpretation level:

Computation <u>of</u> the minus bracket.

(1)

3. Where is the bracket (...) to be inserted in the binomial $[-x-1]$ to make it equal to $[-x+1]$?

1 $[(-x)-1]$

2 $[-x-(1)]$

3 $[(-x-1)]$

4 $[-(x-1)]$

Application level:

Computation <u>by</u> the bracket: inserting the bracket.

(4)

4. If $a = -\frac{b+c}{3}$, then $3a$ equals

1 $-b+c$

2 $b-c$

3 $-b-c$

4 $b+c$

Computation by invisible bracket.

(3)

- 3 -

Do your rough work in this space

Criteria:

5. $(a+b)^2$ exceeds (a^2+b^2) by

1 $(a + b)$

2 $2ab$

3 $(a-b)$

4 a^2b^2

Computation by $(a+b)^2$

(2)

6. $(a+b)^2$ exceeds $(a-b)^2$ by

1 $4ab$

2 $-2b$

3 $+b$

4 $-2ab$

Computation by $(a+b)^2$

(1)

7. What is the missing number $\sqcup$, in the product $(2x+\underline{\quad}).(2x-\underline{\quad})=(4x^2-9)$?

1 -3^2

2 $\sqrt{3}$

3 3^2

4 3

Computation by (a^2-b^2)

(4)

= = = = = = = =

8. $(a+1)^3$ equals

1 $a^3 + 3a^2b + 1^3$

2 $a^3 + 1^3$

3 $a^3 + 3a^2 + 3a +1$

4 $a^3 + a^2 + 1$

Computation of $(a+b)^3$

(3)

- 4 -

Do your rough work in this space

Criteria:

9. If x inches long edge of a cube becomes longer by 1 inch, then the cube's volume becomes larger by:

1 $(3x^2 + 3x + 1)$ inches

2 $(x + 1)^3$ inches

3 $(1 - x)$ inches

4 1^3 inches

Computation by $(a+b)^3$

(1)

10. If x inches long edge of a cube becomes shorter by 1 inch, then that cube's volume becomes smaller by:

1 1^3 inches

2 $(3x^2 - 3x + 1)$ inches

3 $(x^3 - 1^3)$ inches

4 $(-3x^2 + 3x - 1)$ inches

Computation by $(a-b)^3$

(2)

11. $(a+1)^3 + (a-1)^3$ equals

1 $2a^3$

2 $2(a^3 + 2)$

3 $2a^3 + 6a$

4 $8a^3$

Computation by $(a \pm b)^3$

(3)

= = = = = = = =

12. The sum of the cubes of (2a) and (3b) equals:

1 $2a^3 + 3b^3$

2 $(2a + 3b)^3$

3 $2^3a + 3^3b$

4 $8a^3 + 27b^3$

Computing of $(a^3 + b^3)$

(4)

Do your rough work in this space

Criteria:

13. The factor form of $(a^3 + b^3)$ tells that $(a^3 + b^3) = 0$, if

1 $a = b$ or $a^2 = b^2$

2 $(a+b) = 0$ or $(a^2-b^2) = 0$

3 $(a+b) = 0$ or $(a^2-ab+b^2) = 0$

4 $a = b$ or $-ab = 0$

Computation of the factor form of $(a^3 + b^3)$

(3)

14. The difference of volumes of two cubes, whose edges are 2 and 3 inches long, equals:

1 19 cubic inches

2 -35 cubic inches

3 17 cubic inches

4 1 cubic inch

Computation of $(a^3 - b^3)$

(1)

15. The factor form of $(a3-b3)$ tells that $(a^3-b^3) = 1$, if

1 $a^2+ab+b^2 = \frac{1}{a-b}$

2 $a-b = 1$

3 $a^2+ab+b^2 = a-b$

4 $a-b = \frac{1}{a^2+b^2}$

Computation by the factor form of $(a^3 - b^3)$

(1)

= = = = = = = =

16. The n-th part of $4n^2$, for $n = 3$, equals:

1 36

2 12

3 108

4 48

Computation on a fraction

(2)

Do your rough work in this space

Criteria:

17. $-\frac{a+b}{c} = \frac{a+b}{c}$, if a equals

1 0

2 -b

3 1

4 b

Computation of a signed fraction

(2)

18. What fraction should be put in place of f, to make the product $-\frac{a+b}{2}.f$ equal to 1?

1 $\frac{-2}{a+b}$

2 $\frac{a-b}{a+b}$

3 $\frac{-a-b}{a+b}$

4 $\frac{2}{a+b}$

Computation by a signed fraction

(1)

19. The value of $x = -\frac{\frac{m}{2}+1}{p-1}$, for $p = \frac{-m}{2}$, equals

1 -1

2 $\frac{-p+1}{p+1}$

3 +1

4 $\frac{p-1}{p+1}$

Computation by a complex fraction

(3)

20. A plane needs n hours to fly from place A to B. If it flew $\frac{n}{p}$ times faster, it would arrive at B in

1 $\frac{p}{n}$ hours

2 p hours

3 $\frac{1}{p}$ hours

4 $\frac{n}{p}$ hours

Computing by a fraction in a problem.

(2)

Do your rough work in this space

Criteria:

21. $\left(\frac{p}{q}\right) \div \left(-\frac{q}{p}\right)$ equals

1 $\frac{p^2}{q^2}$

2 -1

3 $\frac{p}{q}$

4 $-\frac{p^2}{q^2}$

Computing by the inverse of the fraction

(4)

22. By what must $\frac{b}{a}$ be multiplied to become equal to 1 ?

1 $\frac{1}{b}$

2 $\frac{a}{b}$

3 a

4 $-\frac{a}{b}$

Computing by an apparent inverse

(2)

23. The quotient $\left(a \div \frac{b}{c}\right)$ equals zero, if

1 $c = 0$

2 $b = 0$

3 $\frac{b}{c} = 0$

4 $a = \frac{-b}{c}$

Computing by "no inverse"

(1)

= = = = = = = =

24. In the proportion $\frac{d}{3} = \frac{2}{6}$, d equals

1 4

2 2

3 1

4 3

Computation on a proportion

(3)

Do your rough work in this space

Criteria:

25. By adding 1 to both sides of the proportion $\frac{a}{b} = \frac{c}{d}$, one obtains a following new proportion:

1 $\frac{a-b}{b} = \frac{c-d}{d}$

2 $\frac{a+1}{b} = \frac{c+1}{d}$

3 $\frac{b}{a+1} = \frac{d}{c+1}$

4 $\frac{a+b}{b} = \frac{c+d}{d}$

Computation of a proportion

(4)

26. What term is to be put in the vacant place $\underline{\quad}$, in the expression $\frac{3a}{4} = \frac{6a}{\underline{\quad}}$ to get a true proportion of it ?

1 8a

2 4

3 8

4 6a

Computing by the true proportion

(3)

27. The proportions $\frac{a}{b} = \frac{c}{d}$ and $\frac{a+c}{b+d} = \frac{a-c}{b-d}$ are the same, because

1 $ad = bc$

2 $\frac{a}{b} = \frac{a+c}{b+d}$

3 $(a+c)(b-d) = (a-c)(b+d)$

4 $\frac{b}{a} = \frac{d}{c}$

Computing by the identical proportion:

(2)

= = = = = = = =

28. $\frac{x}{y} = 2$ constantly. Therefore, when $y = 3p$, x equals:

1 6p

2 4p

3 $1\frac{1}{2}p$

4 5p

Computation on the formula of direct variation

( )
1

Do your rough work in this space

Criteria:

29. If s varies directly as t, then the constant c equals:

1 $c = \frac{t}{s}$

2 $c = \frac{1}{s}$

3 $c = \frac{1}{t}$

4 $c = \frac{s}{t}$

Computation of the formula of direct variation

(4)

30. A handful of h candies costs c cents. Therefore, b candies would cost:

1 $\frac{bc}{h}$ cents

2 $\frac{h}{c}$ cents

3 $\frac{h}{bc}$ cents

4 b.c cents

Computation by the concept $K = \frac{X}{y}$ (const.)

(1)

= = = = = = = =

31. x.y = 3 constantly. Thus, when x = 2a, y equals

1 $\frac{2a}{3}$

2 1

3 $\frac{3}{2a}$

4 $\frac{3}{2}$

Computation on the formula of inverse variation $x.y = K$ (const.)

(3)

32. A variable c varies inversely as t. Thus, the constant s equals:

1 $\frac{c}{t}$

2 c.t

3 $\frac{1}{ct}$

4 $\frac{t}{c}$

Computation of the formula $xy = K$ (const.)

(2)

Do your rough work in this space

Criteria:

33. A rectangular strip of land, when narrowed n-times, without changing its area, becomes n-times

1 smaller

2 longer

3 shorter

4 larger

Computation by the concept $xy = K$ (const.)

(2)

= = = = = = = =

34. The missing term ___, in the equality $\left[\frac{a}{n}x + b = \frac{ax}{n} - \_\_\_\right]$, is

1 (+b)

2 nb

3 $\frac{b}{n}$

4 (-b)

Computation by the identity

(4)

35. The root of the equation $\frac{-x}{2} = 1-x$, is equal to:

1 2

2 $\frac{1}{2}$

3 - 2

4 - 1

Computation on a simple equation of the first degree

(1)

36. The question "How many weeks does the month of February have?" is expressed by the following equation:

1 $28x = 7$

2 $7x-1 = 0$

3 $7x = 28$

4 $28 = \frac{7}{x}$

Computation of an equation of the first degree

(3)

Do your rough work in this space

Criteria:

37: $bx = b+x$, if x equals

1 0

2 $\frac{b}{b-1}$

3 1

4 $\frac{b-1}{b}$

Computation on solving an equation of the first degree

(2)

38. The value of the root of the equation $x = \sqrt{1}$, is as follows:

1 $+1^2$

2 ± 1

3 -1

4 -1^2

Evaluating the solution of an equation

(2)

39. The equations $ax+b = 0$, and $nax+nb = 0$ have the same roots equal to:

1 $-\frac{b}{a}$

2 a

3 b

4 $-\frac{a}{b}$

Computation by solving two identical equations

(1)

= = = = = = = =

40. The equation $px + q = 0$, when verified, produces the following equality:

1 $x = x$

2 $2q = 0$

3 $-q+q = 0$

4 $\frac{-q}{p} = \frac{-q}{p}$

Computation on verifying an equation of the first degree

(3)

Do your rough work in this space

Criteria:

41\. The roots x=2 and y=-3 satisfy the following set of equations:

1 $\begin{matrix} x + y = -1 \\ x - y = 0 \end{matrix}$

2 $\begin{matrix} x + 1 = 0 \\ y + x = -1 \end{matrix}$

3 $\begin{matrix} 2x + y = 1 \\ x - y = 0 \end{matrix}$

4 $\begin{matrix} 2x + y = 1 \\ 3x + 2y = 0 \end{matrix}$

Computation by verifying a set of two linear equations

(4)

42\. The set of equations $\begin{matrix} 2x + y = 1 \\ 2x + y = 2 \end{matrix}$ is satisfied by:

1 x=1; y=-1

2 x=2; y=-3

3 x=0; y=1

4 no values of x and y

Computation by verifying inconsistent system of equations

(4)

= = = = = = = =

43\. If the value of x in the equation 2x+3y = 1, equals 0 or 1, then the value of y equals:

1 $\frac{1}{3}$ or 1 respectively

2 1 or $\frac{1}{3}$ respectively

3 $\frac{1}{3}$ or $\frac{-1}{3}$ respectively

4 $\frac{-1}{3}$ or 1 respectively

Computation on plotting a linear equation

(3)

44\. The set of equations $\begin{matrix} x + y = 1 \\ 2x + y = 0 \end{matrix}$ has the roots:

1 x=-1 and y=2

2 x=+1 and y=2

3 x=-1 and y=-2

4 x=0 and y=1

Computation of the roots of a set of linear equations

(1)

Do your rough work in this space

Criteria:

45\. One straight line is presented by the graph of the following set of equations:

1 $2x + 3y = 5$, $4x + 6y = 9$

2 $x + 2y = 1$, $2x + 4y = 3$

3 $2x + y = 0$, $6x + 3y = 3$

4 $2x + 3y = 4$, $4x + 6y = 8$

Computation of indeterminate system of equations

(4)

= = = = = = = =

46\. The standard form of the quadratic equation $\frac{2}{x} = x + \frac{2}{3}$ is as follows:

1 $3x^2 + 2x + 6 = 0$

2 $3x^2 - 2x - 6 = 0$

3 $3x^2 + 2x - 6 = 0$

4 $3x^2 - 2x + 6 = 0$

Computation of the standard form of quadratic equation

(3)

47\. The product of two factors $(x-2)(x-3)$ equals zero, if

1 $x = 2$ or 4

2 $x = 3$ or 1

3 $x = 2$ or 3

4 $x = 4$ or 3

Computation on the factor form of a quadratic equation

(3)

48\. The root of the equation $3x^2+2x-6 = 0$ is as follows:

1 $\frac{-2 \pm \sqrt{4+72}}{6}$

2 $\frac{-1 \pm \sqrt{2+36}}{3}$

3 $\frac{-1 \pm \sqrt{4+72}}{3}$

4 $\frac{-2 \pm \sqrt{4-72}}{6}$

Computation on the formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

(1)

Do your rough work in this space

Criteria:

49. The discriminant $\triangle$ of the equation $ax^2+bx+c = 0$, is placed in the formula for x, right

1 before the root sign $\sqrt{}$

2 under the fraction line ——

3 under the root sign $\sqrt{}$

4 after the sign ±

Distinguishing $\triangle$ in the formula

(3)

50. The value of the discriminant $\triangle$ in the equation $x^2+x-2 = 0$, equals

1 -9

2 7

3 $\pm\sqrt{7}$

4 9

Computation of $\triangle$

(4)

51. The equation $x^2+3x-4 = 0$, has two real roots, because

1 $ac = 4$

2 $b^2 = 9$

3 $-4ac = 16$

4 $\frac{-b}{2a} = \frac{-3}{2}$

Computing by $\triangle$

(3)

= = = = = = = =

52. $(-2)^4$ is larger than (-2^4), because

1 $(-2)^4 = 16$ and $(-2^4) = -16$

2 $(-2)^4 = 16$ and $(-2^4) = -8$

3 $(-2)^4 = 8$ and $(-2^4) = -16$

4 $(-2)^4 = -8$ and $(-2^4) = -16$

Computing a^n, when $a < 0$, $n > 0$.

(1)

Do your rough work in this space

Criteria:

53. $(a-b)^{2n}$ is always positive except when

1 $n = -1$

2 $a = -b$

3 $a = b$

4 $n = \frac{1}{2}$

Evaluating an (a^n)

(3)

54. $\sqrt[3]{27a^3}$ equals

1 $3+a$

2 $3a$

3 $-3a$

4 $9a$

Computing of $\sqrt[n]{a}$

(2)

55. $\left(\frac{1}{2}\right)^{-2}$ equals

1 $\frac{1}{4}$

2 -4

3 $-\frac{1}{4}$

4 4

Application of a^{-n}

(4)

56. a^0 results from

1 $a^n + a^n$

2 $\frac{1}{a^n}$

3 $a^1 + a^{-1}$

4 $a^n - a^n$

Computing a^0 by a^n

(1)

Do your rough work in this space

Criteria:

57. The inverse of $(a^0)^n$ equals:

1 a^0

2 $\frac{1}{a}$

3 a^{-1}

4 $-a$

Computing by (a^0)

(1)

58. $\left[(x+1)^2\right]^3$, when $x = -2$, equals:

1 -1

2 1

3 -2

4 8

Computing by $(a^n)^p$

(2)

59. $\left(\frac{a}{b}\right)^n = \frac{b^n}{a^n}$, when

1 $b = a^2$

2 $a = 1$

3 $a = \pm b$

4 $b = 1$

Computing by $(\frac{a}{b})^n$

(3)

60. $(x+1)^{-2}.(x+1)^3$ equals:

1 $(x+1)^{-5}$

2 $(x+1)^1$

3 $(x+1)^{-1}$

4 $(x+1)^{-6}$

Computing by $a^n.a^p = a^{n+p}$

(2)

Do your rough work in this space

Criteria:

61. $\frac{a^n}{a^p}$ equals a, if

1 $n-p = 1$

2 $n = p^2$

3 $n-p = 0$

4 $n = -p$

(1)

Computing by $\frac{a^n}{a^p} = a^{n-p}$

= = = = = = = =

62. $a^{\frac{m}{k}}$ results from

1 $a^m + a^k$

2 $a^{\frac{-k}{m}}$

3 $\sqrt[k]{a^m}$

4 a^{k-m}

(3)

Computing $a^{\frac{m}{k}}$

63. The operation opposite to that expressed by $\sqrt[k]{a^m}$ is as follows:

1 $a^{\frac{m}{k}}$

2 $\frac{a^m}{-a^k}$

3 $\sqrt[m]{a^k}$

4 $\sqrt[-k]{a^{-m}}$

(3)

Computing the opposite to $\sqrt[k]{a^m}$

= = = = = = = =

64. $\frac{1}{x^2-1} + \frac{1}{x-1} =$

1 $x^2 + x-2$

2 $\frac{x}{x^2+1}$

3 $\frac{2}{x-1}$

4 $\frac{x+2}{x^2-1}$

(4)

Computing by L.C.M.

<u>Do your rough work in this space</u>

Criteria:

Computing by H.C.F.

65. The highest common factor of $(x+1)^3, (x+1)^2$, (x^2-1), is as follows:

1 $(x+1)$

2 (x^2-1)

3 $x-1)$

4 (x^2+1)

(1)

TEST BOOKLET

Total Score: ________

Percentile: ________

ALGEBRAIC CONCEPTS

TEST

Name: ______________________ Date ______________

Date of birth: ______________________

City: ____________ School: ______________ Grade: ________

For how many years have you studied algebra? ______________

Read these instructions carefully

The purpose of this test is to find out to what extent you are familiar with some algebraical concepts. Furthermore, the test is to help you and your fellow students in obtaining better mastery of algebraic concepts. Though there is no time-limit for this test, you should not spend too much time on any one problem. Space for your rough work is provided at the right of each question. No questions may be asked after your work on answers has begun.

Direction: Each problem of the test is followed by four choices, one of which is correct. After you have found the correct choice, put its number in the bracket to the right.

Example:

Do your rough work in this space

If $x = \pm 2$, then x^2 equals

1 ± 2

2 -4

3 4

4 -2^2 (3)

The answer "4" is correct, and it is the third choice. Therefore, "3" has been written in the bracket at the right.

Do your rough work in this space

Criteria

1. The algebraic brackets are represented by the following signs:

1 (...] , or {...)

2 (....), or [...] , or {...}

3 [(...])

4 - {...]

Recognition level:
Recognition of the algebraic bracket.

(2)

2. The correct use of the bracket (...) is shown in the following expression:

1 $y = -[-(a)]$

2 $y = (a + (b) + 1)$

3 $y = [(a+b)]$

4 $y = -(a+b)$

Evaluation level:

Appropriateness of the bracket

(4)

3. The algebraic bracket is a means of expressing

1 a change of signs

2 the importance of terms

3 the algebraic wholeness

4 the complexity of terms

Application level:

The meaning of the bracket

(3)

4. The minus fraction line implies a concealed bracket (...) on its

1 numerator only

2 numerator and denominator

3 numerator or denominator

4 denominator only

Concealed bracket

(3)

= = = = = = = =

Do your rough work in this space

Criteria:

5. The square of the sum of $\underline{a}$ and $\underline{b}$ is represented by:

1 $a^2 - b^2$

2 $a^2-2ab+b^2$

3 a^2+b^2

4 $a^2+2ab+b^2$ (4)

Recognition of $(a+b)^2$

6. The square of difference of $\underline{p}$ and $\underline{q}$ is represented by:

1 $(p-q)^2$

2 $p^2+2pq-q^2$

3 $-(p+q)^2$

4 $-p^2-q^2$ (1)

Recognition of $(a-b)^2$

7. The difference of squares of $\underline{s}$ and $\underline{z}$ is represented by:

1 $(s-z)^2$

2 $s^2+2sz-z^2$

3 s^2-z^2

4 $s^2-2sz-z^2$ (3)

Recognition of (a^2-b^2)

= = = = = = = =

8. The cube of the sum $(a+b)$ is as follows:

1 $(a^3+3ab+3ab^2+b^3)$

2 $(a+b)^3$

3 $(a^3+3a^2b+3ab+b^3)$

4 (a^3+b^3) (2)

Recognition of $(a+b)^3$

Do your rough work in this space

Criteria:

9. The cube $(a+b)^3$ exceeds (a^3+b^3) by

1 $(a+b)^2$

2 $3a^2b$

3 $3ab^2$

4 $(3a^2b + 3ab^2)$ (4)

Evaluation of $(a+b)^3$

10. The cube of difference (a-b) is represented by

1 a^3-b^3

2 $(a+b)^2(a-b)$

3 $(a-b)^2(a+b)$

4 $(a-b)^3$ (4)

Recognition of $(a-b)^3$

11. The cube $(a-b)^3$ exceeds (a^3-b^3) by

1 $(a-b)^2$

2 $3ab^2$

3 $(-3a^2b+3ab^2)$

4 $3a^2b$ (3)

Evaluation of $(a-b)^3$

= = = = = = = =

12. The sum of two algebraic cubes is represented by:

1 $(a+b)^3$

2 $(a^3+3a^2b+3ab^2+b^3)$

3 (a^3+b^3)

4 $(a^3+3ab+3a^2b+b^3)$ (3)

Recognition of (a^3+b^3)

Do your rough work in this space

Criteria:

13. (p^3+q^3) is a product of $(p+q)$ and

1 (p^2-pq+q^2)

2 $(p+q)^2$

3 (p^2+pq+q^2)

4 $(p-q)^2$ (1)

Understanding of the factor form of $(a^3 + b^3)$

14. The difference of two algebraic cubes is represented by the following expression:

1 $(x+y)^3 - (x-y)^3$

2 $(x-y)^3$

3 xy^3-x^3y

4 4^3-2^3 (1)

Recognition of $(a^3 - b^3)$

15. The factors of (p^3-q^3) are as follows:

1 $(p+q)$ and (p^2-pq+q^2)

2 $(p-q)^2$ and $(p-q)$

3 $(p-q)$ and (p^2+pq+q^2)

4 $(p+q)^2$ and $(p-q)$ (3)

Recognition of the factor form of $(a^3 - b^3)$

= = = = = = = =

16. Algebraic fractions are presented throughout the following expression:

1 $a + \frac{2b}{2} - 1$

2 $\frac{m}{2} - \frac{q}{2} + \frac{2}{m}$

3 $\frac{m}{2} + \frac{m}{p} + \frac{1}{2}$

4 $\frac{1}{2}a + \frac{b}{a} - b$ (2)

Recognition of the algebraic fraction

Do your rough work in this space

Criteria:

17. The following signed fractions are equal:

1 $+\frac{a}{b} = \frac{a}{-b}$

2 $+\frac{a}{b} = \frac{-a}{b}$

3 $+\frac{a}{b} = \frac{-a}{-b}$

4 $-\frac{a}{b} = \frac{-a}{-b}$ (3)

Understanding of the signed fraction

18. Which fraction has no meaning if <u>d</u> is any amount of dollars?

1 $\frac{-d}{-3}$

2 $\frac{d}{3}$

3 $\frac{2d}{6}$

4 $\frac{4d}{12}$ (1)

Understanding of the signed fraction in a concrete situation

19. Three equal fractions are presented in the following expression, below:

1 $n.\frac{a}{b} = \frac{na}{b} = \frac{a}{bn}$

2 $n.\frac{a}{b} = \frac{na}{b} = \frac{-a}{-\frac{b}{n}}$

3 $n.\frac{a}{b} = \frac{-a}{-bn} = \frac{na}{b}$

4 $n.\frac{a}{b} = \frac{na}{b} = \frac{-na}{-\frac{b}{n}}$ (2)

The equality of signed fractions

20. $\left(\frac{-a}{b}\right) = \left(\frac{a}{-b}\right) = \left(-\frac{a}{b}\right)$ are three equal fractions, where $b \neq 0$. Which, if any, is understandable if <u>a</u> is a number of pints of milk for sale?

1 $\left(\frac{-a}{b}\right)$

2 $\left(\frac{a}{-b}\right)$

3 $\left(-\frac{a}{b}\right)$

4 none of them (4)

Understanding of the negative fraction in a concrete situation

= = = = = = = =

Do your rough work in this space

Criteria:

21. The inverse of the fraction $\frac{a}{b}$ is as follows:

1 $\frac{-a}{b}$

2 $\frac{b}{a}$

3 $\frac{a}{-b}$

4 $-\frac{a}{b}$ (2)

Recognition of the inverse of the fraction

22. The inverse of the fraction $\frac{p}{q}$ is equal to that fraction itself, if

1 $q = 0$

2 $p = \pm 1$

3 $p = q$

4 $q = 1$ (3)

Evaluation of the inverse of algebraic fraction

23. There is no inverse of the algebraic fraction $\frac{a}{b}$, if

1 $a = b$

2 $a < b$

3 $a > b$

4 $a = 0$ (4)

Understanding of the case of no inverse of algebraic fraction

= = = = = = = =

24. The algebraic proportion is represented by the following expression:

1 $\frac{a}{b} \neq \frac{c}{d}$

2 $\frac{a}{b} = \frac{c}{d}$

3 $ad = bc$

4 $(a+b)+1 = (c+d)+1$ (2)

Recognition of proportion

Do your rough work in this space

Criteria:

25. Which, if any, of the following definitions of the proportion is the best?

1 the proportion is the equality of two ratios;

2 the proportion is an equation which states the equality of two ratios;

3 the proportion is an equation of two fractions;

4 the proportion is none of the above. (1)

Definition of proportion

26. A true proportion has always its ratios

1 positive

2 negative

3 equal

4 different (3)

The trueness of proportion

27. The following proportions are identical:

1 $\frac{2}{3} = \frac{a}{b}$ and $\frac{4}{6} = \frac{b}{a}$

2 $\frac{1}{3} = \frac{2}{6}$ and $\frac{3}{4} = \frac{6}{8}$

3 $\frac{1}{3} = \frac{a}{b}$ and $\frac{2}{6} = \frac{a}{b}$

4 $\frac{3}{5} = \frac{6}{10}$ and $\frac{1}{5} = \frac{2}{10}$ (3)

Identity and not identity of two proportions

28. The formula for direct variation of two variables x and y, is as follows:

1 $\frac{x}{y}$ = k(constant)

2 x.y = k(constant)

3 x+y = k(constant)

4 x-y = k(constant) (1)

Recognition of the formula for direct variation

Do your rough work in this space

Criteria:

29. Two variables x and y vary directly, if they simultaneously

1 change their signs

2 are equal

3 don't change

4 increase or decrease n-times (4)

The meaning of direct variation

30. A train moves uniformly, making 25 miles per hour. The distance covered by that train varies directly as

1 the width of the track

2 the load of carriages

3 the time of motion

4 the slope of the track (3)

Direct variation in the concrete

= = = = = = = =

31. The formula for inverse variation of two variables x and y is as follows:

1 $\frac{x}{y}$ = k(constant)

2 x.y = k(constant)

3 x+y = k(constant)

4 x-y = k(constant) (2)

The formula for inverse variation: ab=K (constant)

32. Two variables x and y vary inversely, if simultaneously

1 x decreases n-times, and y does the same

2 x decreases n-times, and y does not change

3 x increases n-times, and y does the same

4 x increases n-times, and y decreases n-times (4)

The meaning of inverse variation

Do your rough work in this space

Criteria:

33. The time you need to get home from your school varies inversely as

1 the speed at which you go

2 the distance of your home from the school

3 the throng you meet on your way home

4 the number of stops on your way

(**1**)

Inverse variation in the concrete

= = = = = = = =

34. The algebraic identity is represented by the following expression:

1 $ax = b$

2 $4 = 4$

3 $a = a$

4 $2+x = 1 + 2x$

(3)

Understanding of the concept of algebraic identity

35. The equation in one unknown $\underline{x}$ is represented by the following expression:

1 $x+2a = 2a+x$

2 $0 = 2x+3$

3 $\frac{b}{2}x = \frac{b}{2}x$

4 $\sqrt{x} = \sqrt{x}$

(**2**)

Recognition of the algebraic equation

36. The equation in one unknown $\underline{x}$ expresses:

1 the question about the value of $\underline{x}$ which satisfies the equation;

2 the equality of two algebraic expressions

3 the comparison of two algebraic terms

4 the operation on two algebraic expressions

(**1**)

The meaning of the equation

Do your rough work in this space

Criteria:

37. To solve an equation in unknown x, means:

1 to compare two algebraic expressions

2 to find the only true value of x

3 to transpose x to one side of the equation

4 to look for true value of x

(2)

The concept of solution of equation

38. The root of the equation in one unknown x, is:

1 the unknown x of the equation

2 the formula for solving the equation

3 the solution of the equation

4 the value of x which satisfies the equation

(4)

The concept of the root of equation

39. Two identical equations are such equations that have

1 the alike terms

2 the same way of solution

3 the same only roots

4 the same form

(3)

The meaning of identical equation

= = = = = = = =

40. To verify the equation ax + b = 0, means

1 to check if the value of x is the only correct

2 to find whether the equation is correct

3 to check on the method of solution

4 to compare both sides of the equation

(1)

Verification of the equation

Do your rough work in this space

Criteria:

41. To verify a set of two equations
$ax+by = c$
$dx+ey = f$ means

1 to check whether the equations are identical

2 to find the correct answers

3 to check whether the values of x and y are the only correct

4 to find the wrong answers (3)

Verification of two equations

42. How many solutions can a set of two equations $\begin{matrix} ax + by = c \\ dx + ey = f \end{matrix}$ possess?

1 only one

2 one, none, or unlimited number

3 only two

4 two, or more (2)

Existence of three (determinate, indeterminate, inconsistent) systems of two equations

= = = = = = = =

43. A graph of the equation $ax+by=c$ represents

1 a curve

2 a point

3 a segment

4 a straight line (4)

The concept of the graph of $ax + by = c$

44. The solution of two equations of the first degree is represented graphically by

1 a point of intersection of two straight lines

2 two parallel lines

3 one straight line

4 two short segments (1)

Graphic solution of two linear equations

Do your rough work in this space

Criteria:

45. The graph of two linear equations, which have unlimited number of the same roots, is represented by

1 two parallel lines

2 one straight line

3 unlimited number of straight lines

4 two segments (2)

The graph of identical equations

= = = = = = = =

46. The standard form of quadratic equation is represented by

1 $x^2 + x = -1$

2 $x^2 - 2 = x$

3 $ax^2 + bx + c = 0$

4 $ax^2 + bx = 3$ (3)

Standard form of quadratic equation

47. The factor form of the quadratic equation, whose roots are x = 2 and x = 3, is as follows:

1 $(x-2)(x-3) = 0$

2 $(x-2)(x+3) = 0$

3 $(x+2)(x+3) = 0$

4 $(2+x)(3-x) = 0$ (1)

The factor form of the quadratic equation

48. The formula for computing the root of the equation $ax^2+bx+c = 0$, is as follows:

1 $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2 $x = \frac{+b \pm \sqrt{b^2+4ac}}{2a}$

3 $x = \frac{-b \pm \sqrt{b^2-4ac}}{-2a}$

4 $x = \frac{+b \pm \sqrt{b^2+4ac}}{2a}$ (1)

Knowledge of the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Do your rough work in this space

Criteria:

49. The quadratic equation can have as many solutions as:

1 two

2 two or more

3 two,one, or no solution

4 two, or no solution

Number of solutions of the quadratic

(3)

50. The discriminant of the equation $ax^2+bx+c = 0$, is as follows:

1 $\pm\sqrt{b^2-4ac}$

2 (b^2-4ac)

3 $\left(-\frac{b}{2a}\right)$

4 (b^2+4ac)

Recognition of $\triangle = b^2 -4ac$

(2)

51. The discriminant of $ax^2+bx+c = 0$ tells whether that equation has

1 positive roots

2 fractional roots

3 negative roots

4 one,two,or no roots

The role of $\triangle = b^2 - 4ac$

(4)

= = = = = = = =

52. To compute a^n, where $n=1,2,3,\ldots$, means

1 to add a to itself n-times

2 to multiply a by n, a-times

3 to multiply a by itself n-times

4 to add n to itself n-times

The concept of $a^n, (n >0)$

(3)

Do your rough work in this space

Criteria:

53. $(-3)^n$ has a positive value, if

1 n is an odd integer

2 n is a positive integer

3 n is a negative integer

4 n is an even integer

(4)

Evaluation of a^n

54. The operation that is opposite to that expressed by a^n, is as follows:

1 $\sqrt[n]{a}$

2 n^a

3 $\left(\frac{1}{a}\right)^n$

4 a^{-n}

(1)

The concept of $\sqrt[n]{a}$

55. a^{-n} equals

1 $a.(-1)^n$

2 $1 \div a^n$

3 $-\frac{1}{a^n}$

4 $-a^n$

(2)

Applying a^n in the concept a^{-n}

56. a^0 equals

1 $\frac{0}{a}$

2 a

3 0

4 1

(4)

The concept of a^0

Do your rough work in this space

Criteria:

57. To raise (a^n) to the p-th power, means:

1 to compute a^{n+p}

2 to multiply a^n by a^n p-times

3 to add a^n to itself p-times

4 to multiply a^n by a^p

(2)

The concept of $(a^n)^p = a^{np}$

58. $(a.b)^n$ equals

1 $a^n.b^n$

2 $a^n.b$

3 $a.b^n$

4 a^n+b^n

(1)

The concept $(ab)^n = a^nb^n$

59. The n-th power of the fraction $\frac{a}{b}$ is represented by the following:

1 $\frac{a^n}{b^n}$

2 $\frac{a^n}{b}$

3 $\frac{a}{b^n}$

4 $\left(\frac{b}{a}\right)^n$

(1)

The concept $(\frac{a}{b})^n = \frac{a^n}{b^n}$

60. $a^n.a^q$ equals

1 a^{nq}

2 $(a^n)^q$

3 $(a^n + a^q)$

4 (a^{n+q})

(4)

The concept: $a^n.a^q = a^{n+q}$

Do your rough work in this space

Criteria:

61. $\left(\frac{a^n}{a^p}\right)$ equals

1 $(a^{\frac{n}{p}})$

2 (1^{n-p})

3 (a^n-a^p)

4 (a^{n-p})

(4)

The concept $\frac{a^n}{a^p} = a^{n-p}$

= = = = = = = =

62. $a^{\frac{m}{k}}$ equals

1 $\sqrt[k]{a^m}$

2 $a^m \div a^k$

3 $\left(a^{\frac{m}{k}}\right)^{-k}$

4 $\sqrt[m]{a^k}$

(1)

The concept $a^{\frac{m}{k}} = \sqrt[k]{a^m}$

63. $\sqrt[k]{a^m}$ has no meaning, if

1 $m = 0$

2 $k = 0$

3 $k = m$

4 $k = -m$

(2)

Evaluation of $a^{\frac{m}{k}} = \sqrt[k]{a^m}$

= = = = = = = =

64. The lowest common multiple of a^4, a^3, a^2, equals

1 a

2 $2a^2$

3 $4a^3$

4 a^4

(4)

Understanding the concept of the algebraic L.C.M.

Do your rough work in this space

65. The highest common factor of given algebraic numbers is the largest number that is

1 a product of those numbers

2 a divisor of those numbers

3 a sum of those numbers

4 a quotient of those numbers **(2)**

Criterion:

Understanding of the concept: H.C.F.

APPENDIX 3

EVALUATION OF THE TEST ITEMS

Method: 27%
High ranking 16 indiv.
Low ranking 16 indiv.

Concept-Test

Group 59 pupils = 34 boys + 25 girls

Item No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
The highest 27%	15	13	9	3	16	16	13	14	16	13	16	16	14	15	14	11	7	16	9	13	15	12
The lowest 27%	12	14	8	4	11	13	6	9	7	8	4	5	6	12	8	4	2	13	1	7	12	9
Difference	3	-1	1	-1	5	3	7	5	9	5	12	11	8	3	7	7	5	3	8	6	3	3

Item No.	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
The highest 27%	9	15	4	8	10	7	13	12	5	10	9	7	16	15	11	11	6	8	12	2	6	8
The lowest 27%	5	9	1	8	3	7	1	8	2	4	8	4	7	10	7	4	2	2	9	2	2	4
Difference	4	6	3	0	7	1	12	4	3	6	1	3	9	5	4	7	4	6	3	0	4	4

Item No.	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65
The highest 27%	6	14	13	15	3	10	4	13	10	11	2	4	7	14	11	9	9	9	4	4	15
The lowest 27%	3	10	5	8	1	1	4	10	2	2	0	2	5	13	5	6	2	4	5	1	7
Difference	3	4	8	7	2	9	0	3	8	9	2	2	2	1	6	3	7	5	-1	3	8

TOTAL: (Positive 59 + (Zero) 3 + (Minus One)3 = 65 items

EVALUATION OF THE TEST ITEMS

Method: 27%
High ranking 15 indiv.
Low ranking 15 indiv.

Skill-Test

Group 56 pupils = 31 boys + 25 girls

Item No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
The highest 27%	13	9	13	5	14	12	13	13	4	2	10	14	8	11	4	4	7	9	6	5	11	12
The lowest 27%	13	10	11	1	12	5	9	11	3	0	7	9	1	5	3	1	2	4	2	2	8	7
Difference	0	-1	2	4	2	7	4	2	1	2	3	5	7	6	1	3	5	5	4	3	3	5

Item No.	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
The highest 27%	4	13	2	14	3	12	9	7	11	2	8	12	8	12	2	13	6	3	12	14	11	11
The lowest 27%	1	8	1	11	2	8	7	3	10	0	11	9	3	8	3	7	0	0	8	9	7	5
Difference	3	5	1	3	1	4	2	4	1	2	-3	3	5	4	-1	6	6	3	4	5	4	6

Item No.	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65
The highest 27%	9	5	12	6	3	4	6	9	2	11	4	2	4	7	7	6	5	8	5	7	9
The lowest 27%	1	3	2	0	0	0	0	3	1	4	0	3	1	4	1	1	1	3	0	3	7
Difference	8	2	4	6	3	4	6	6	1	7	4	-1	3	3	6	5	4	5	5	4	2

TOTAL: (Positive) 60 + (Zero) 1 + (Minus) 4 = 65 items

Ranks in Concept-Test

Score		Rank	Score		Rank
G	9	59	G	35	25.5
G	16	58	G	35	25.5
B	20	57	B	35	25.5
B	21	56	B	35	25.5
G	22	54.5	B	36	22
G	22	54.5	B	36	22
B	25	53	B	36	22
G	26	51	B	37	20
B	26	51	G	38	18.5
B	26	51	B	38	18.5
G	27	48.5	G	39	15
B	27	48.5	B	39	15
G	28	44.5	B	39	15
G	28	44.5	B	39	15
G	28	44.5	B	39	15
G	28	44.5	G	41	10.5
G	28	44.5	B	41	10.5
B	28	44.5	B	41	10.5
G	29	40	B	41	10.5
B	29	40	B	42	8
B	29	40	B	43	6.5
G	30	37	B	43	6.5
G	30	37	B	44	5
B	30	37	G	45	3.5
G	31	33.5	B	45	3.5
G	31	33.5	G	48	2
G	31	33.5	B	49	1
B	31	33.5			
B	32	30.5			
B	32	30.5			
G	34	28.5			
B	34	28.5			

the lowest 27%

The highest 27%

Total - 59 pupils
= 34 boys + 25 girls

Ranks in Skill-Test

Score		Rank	Score		Rank
G	15	55.5	G	28	23
B	15	55.5	G	28	23
G	16	54	B	28	23
G	17	53	B	28	23
B	18	52	B	28	23
G	19	50	G	29	18.5
G	19	50	B	29	18.5
G	19	50	B	29	18.5
G	20	48	B	29	18.5
G	21	45.5	G	30	15.5
G	21	45.5	B	30	15.5
G	21	45.5	B	31	12.5
B	21	45.5	B	31	12.5
B	22	42.5	B	31	12.5
B	22	42.5	B	31	12.5
G	24	39.5	G	32	9.5
G	24	39.5	B	32	9.5
B	24	39.5	B	33	7.5
B	24	39.5	B	33	7.5
B	25	36.5	B	34	6
B	25	36.5	G	37	5
G	26	33.5	G	38	4
G	26	33.5	B	39	3
B	26	33.5	G	40	2
B	26	33.5	B	43	1
G	27	28.5			
G	27	28.5			
G	27	28.5			
B	27	28.5			
B	27	28.5			
B	27	28.5			

The lowest 27%

The highest 27%

Total: 56 pupils
= 31 boys +
25 girls

APPENDIX 4

COMPUTATION OF ITEMS' DIFFICULTY INDICES

Group of 109 pupils - St.Patrick's 87 + Notre Dame 22

CONCEPT TEST

Item No.	St. Patrick's		Notre Dame				Difficulty Index in %
1	85	+	22	=	107	=	97
2	79	+	22	=	101	=	92
3	28	+	8	=	36	=	33
4	9	+	3	=	12	=	11
5	69	+	18	=	87	=	79
6	71	+	20	=	91	=	83
7	54	+	18	=	72	=	66
8	73	+	22	=	95	=	87
9	71	+	22	=	93	=	85
10	71	+	21	=	92	=	84
11	66	+	22	=	88	=	80
12	61	+	21	=	82	=	75
13	49	+	20	=	69	=	63
14	59	+	18	=	77	=	70
15	59	+	21	=	80	=	73
16	38	+	12	=	50	=	46
17	30	+	16	=	46	=	42
18	77	+	17	=	94	=	86
19	30	+	7	=	37	=	33
20	50	+	13	=	63	=	57
21	77	+	22	=	99	=	90
22	68	+	12	=	80	=	73
23	48	+	22	=	70	=	64
24	57	+	22	=	79	=	72
25	19	+	4	=	23	=	21
26	46	+	18	=	64	=	58
27	33	+	13	=	46	=	42
28	41	+	5	=	46	=	42
29	35	+	3	=	38	=	34
30	60	+	6	=	66	=	60
31	16	+	1	=	17	=	15
32	39	+	2	=	41	=	37
33	43	+	6	=	49	=	44
34	25	+	17	=	42	=	40
35	56	+	16	=	72	=	66
36	69	+	21	=	90	=	82
37	43	+	14	=	57	=	52

CONCEPT TEST (CONTINUED)

Item No.	St. Patrick's		Notre Dame				Difficulty Index in %
38	52	+	17	=	69	=	63
39	26	+	5	=	31	=	28
40	16	+	11	=	27	=	24
41	45	+	16	=	61	=	55
42	11	+	1	=	12	=	11
43	26	+	1	=	27	=	24
44	46	+	1	=	47	=	43
45	10	+	0	=	10	=	91
46	72	+	22	=	94	=	86
47	49	+	14	=	63	=	57
48	69	+	22	=	91	=	83
49	21	+	3	=	24	=	22
50	45	+	19	=	64	=	58
51	44	+	15	=	59	=	54
52	67	+	22	=	89	=	81
53	29	+	10	=	39	=	35
54	32	+	7	=	39	=	35
55	11	+	11	=	22	=	20
56	17	+	17	=	34	=	33
57	31	+	16	=	47	=	43
58	69	+	18	=	87	=	79
59	53	+	16	=	69	=	63
60	45	+	14	=	59	=	54
61	39	+	14	=	53	=	48
62	48	+	19	=	67	=	61
63	22	+	10	=	32	=	30
64	19	+	9	=	28	=	25
65	49	+	10	=	59	=	54

APPENDIX 4

COMPUTATION OF ITEMS' DIFFICULTY INDICES

Group of 109 pupils - St.Patrick's 87 + Notre Dame 22

SKILL-TEST

Item No.	St. Patrick's		Notre Dame				Difficulty Index in %
1	81	+	18	=	99	=	90
2	57	+	10	=	67	=	61
3	74	+	21	=	95	=	86
4	22	+	4	=	26	=	23
5	80	+	23	=	103	=	93
6	67	+	11	=	78	=	70
7	74	+	22	=	96	=	87
8	81	+	18	=	99	=	90
9	13	+	2	=	15	=	13
10	6	+	3	=	9	=	9
11	57	+	16	=	73	=	66
12	50	+	12	=	62	=	56
13	29	+	20	=	49	=	40
14	57	+	12	=	69	=	62
15	13	+	9	=	22	=	19
16	16	+	6	=	22	=	19
17	38	+	8	=	46	=	41
18	44	+	9	=	53	=	48
19	18	+	7	=	25	=	22
20	17	+	6	=	23	=	20
21	48	+	15	=	63	=	57
22	70	+	21	=	91	=	82
23	15	+	5	=	20	=	18
24	71	+	20	=	91	=	82
25	28	+	6	=	34	=	30
26	82	+	23	=	105	=	95
27	20	+	5	=	25	=	22
28	74	+	18	=	92	=	83
29	45	+	17	=	62	=	56
30	42	+	13	=	55	=	50
31	56	+	14	=	70	=	63
32	13	+	2	=	15	=	13
33	66	+	9	=	75	=	70
34	54	+	16	=	70	=	63
35	39	+	6	=	45	=	40
36	72	+	20	=	92	=	83
37	15	+	6	=	21	=	19

SKILL-TEST(CONTINUED)

Item No.	St. Patrick's		Notre Dame				Difficulty Index in %
38	63	+	17	=	80	=	72
39	28	+	7	=	35	=	36
40	19	+	6	=	25	=	22
41	72	+	18	=	90	=	81
42	77	+	20	=	97	=	9
43	30	+	7	=	37	=	33
44	65	+	12	=	77	=	70
45	38	+	14	=	52	=	43
46	37	+	10	=	57	=	51
47	55	+	14	=	69	=	62
48	36	+	8	=	44	=	40
49	29	+	14	=	43	=	40
50	28	+	8	=	36	=	32
51	32	+	6	=	38	=	34
52	55	+	10	=	65	=	59
53	8	+	2	=	10	=	9
54	60	+	13	=	73	=	66
55	14	+	6	=	20	=	18
56	9	+	1	=	10	=	36
57	24	+	3	=	27	=	25
58	60	+	12	=	72	=	65
59	36	+	12	=	48	=	43
60	45	+	9	=	54	=	45
61	28	+	7	=	35	=	31
62	44	+	18	=	62	=	56
63	44	+	14	=	58	=	52
64	23	+	14	=	37	=	33
65	39	+	16	=	55	=	50

APPENDIX 5

COEFFICIENT OF RELIABILITY OF THE TESTS

The enclosed list of names presents the scores obtained on two administrations of the test and the attached Dayhaw Correlation Chart shows the computation of the coefficient.

CONCEPTS - COEFFICIENT OF RELIABILITY

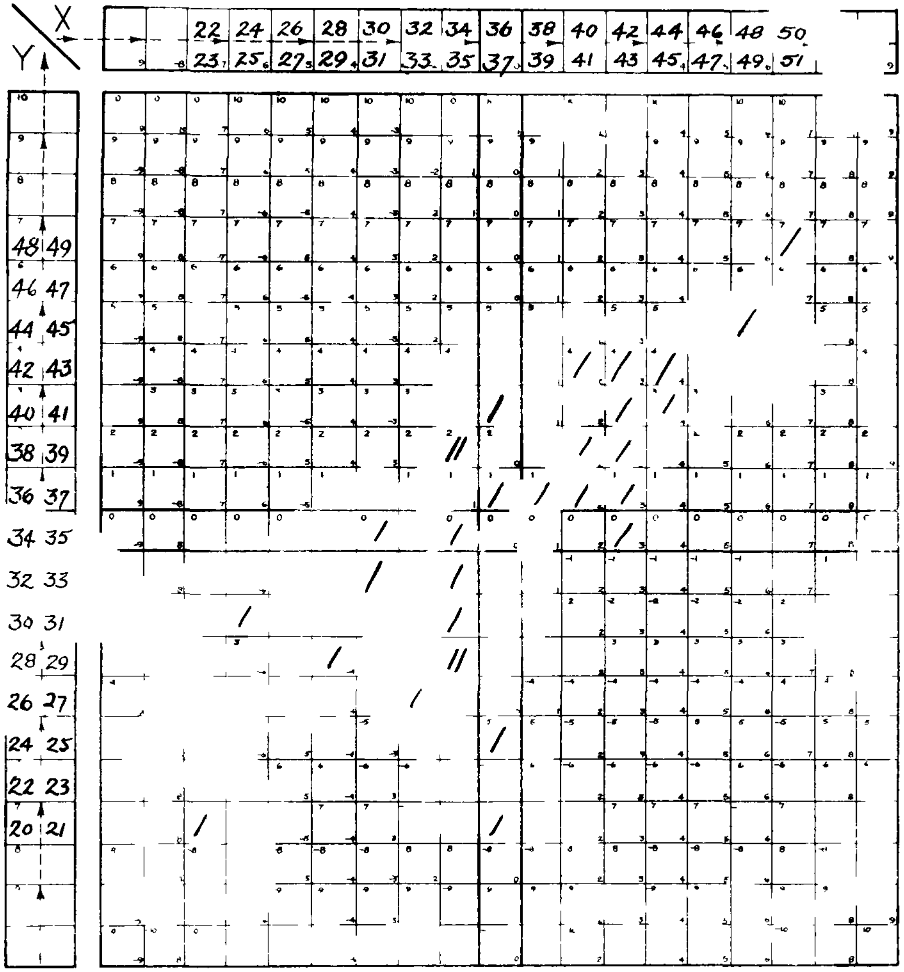
N = 30

No.	Name	First Administration	Second Administration
1	Herman Gerald Popela	49	51
2	John Clay	44	48
3	Simpson William T.	43	43
4	Ronald Le Neveu	43	45
5	Frank Cuggy	42	41
6	Peter Tobin	41	45
7	Charles Thebarge	41	42
8	Daniel Phelan	41	36
9	Gerald Fitzgerald	39	40
10	Duncan Lunan	39	35
11	Fr.John Tulatyck	39	35
12	James McGibbon	38	42
13	David McCaffrey	37	41
14	Lalonde William	36	42
15	Leo Turner	36	39
16	Richard Marks	36	37
17	Fenton Fred. Robert	35	31
18	Brown Dick	35	43
19	Peter Nolan	34	34
20	D. Glastonbury	32	34
21	Richard LeBlanc	32	30
22	Stanley Cameron	31	25
23	Peter McConnery	30	35
24	Kevin Milaney	29	29
25	Soissons Leo	29	35
26	Jony Piatek,Fr.	28	35
27	Tom Curley	26	33
28	Michael Williams	25	37
29	Phelan Pete	21	22
30	Michael Seigneur	20	36

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

Sujet (Subject) St Patrick's Tryout- Concepts Group N-30

Date June 1 1956 Per T Pozniak



f_y	y'	fy'	fy'^2	$\Sigma x'$	$\Sigma xy'$	$(\Sigma x')^2$	$\frac{(\Sigma x)^2}{f_y}$
	10						
	9						
	8						
1	7	7	49	7	49	49	49
0	6	0	0	0	0	0	0
1	5	5	25	6	30	36	36
3	4	12	48	9	36	81	27
3	3	9	27	7	21	49	16.33
4	2	8	16	3	6	9	2.25
4	1	4	4	6	6	36	9
3	0	$\Sigma(+)$ 45	0	-1	0	1	0.33
2	-1	-2	2	-4	4	16	8
2	-2	-4	8	-7	14	49	24.50
3	-3	-9	27	-6	18	36	13
1	-4	-4	16	-2	8	4	4
1	-5	-5	25	0	0	0	0
0	-6	0	0	0	0	0	0
2	-7	-14	98	-7	49	49	24.50
	-8						
	-9						
	-10						

30 N	$\Sigma(-)$ -38 7	345	11	241		21391
		Σf_y	Σf_x			$\Sigma\frac{(\Sigma x')^2}{f_y}$

43 11

305 ← $\Sigma fx'^2$

7 ← $\Sigma fy'$

241 ← $\Sigma xy'$

233.90 ← $\Sigma\frac{(\Sigma y')^2}{f_x}$

f_x			1	1	0	1	2	1	7	4	1	3	5	2	0	1	1		
x'	-9	8	-7	-6	-5	-4	3	-2	-1	0 $\Sigma(-)$	1	2	3	4	5	6	7	8	9
fx'			-7	-6	0	-4	-6	-2	-7	-32	1	6	15	8	0	6	7		
fx'^2	81	64	49 49	36 36	25 0	16	9 8	4 4	1 7	0 0	1	4 12	9 45	32	25 0	36 36	49 49	64	81
$\Sigma y'$			-7	-2	0	-3	-1	-4	-5	-8	1	7	10	7	0	5	7		
$\Sigma x'y'$			49	12	0	12	3	8	5	0	1	14	30	28	0	30	49		
$(\Sigma y')^2$			49	4	0	9	1	16	25	64	1	49	100	49	0	25	49		
$(\Sigma y')^2/f_x$			49	4	0	9	05	16	3 57	16	1	16.33	20	24.50	0	25	49		

X Y

Variable Concepts 2nd Admin, e Concepts 1st Admin.

X 36.5 i_x 15 i_x = 2 34.5 15 i_y = $\boxed{2}$

Corrections

$c'_x = \frac{\Sigma fx'}{N}$ $\frac{11}{30}$ 0 366 | c_y - $\frac{7}{30}$ - $\boxed{0\ 233}$

$c'^2_x =$ 0 133 0 054

σ

$\sigma_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$ | $\sigma'_y = \sqrt{\frac{\Sigma fy'^2}{N}\ c_y}$

$= \sqrt{\frac{305}{30} - 0133}$ 3 03 $= \sqrt{\frac{345}{30} - 0054}$ 3 38

Correlation . r_{xy}

$r_{xy} = \frac{\frac{\Sigma xy'}{N} - c_x c_y}{\sigma_x\ \sigma_y} = \frac{\frac{241}{30} - 0\ 366 \times 0\ 054}{3\ 03 \times 3\ 38}$ $\boxed{080}$

$\sigma_{r_{xy}} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0\ 64}{\sqrt{29}} = 0\ 066$

Rapport de Correlation η Correlation Ratio

$\sigma'_{\bar{x}_x} = \sqrt{\frac{\Sigma\frac{(\Sigma x)^2}{f_y}}{N} - c'^2_x}$ | $\sigma'_{\bar{y}_c} = \sqrt{\frac{\Sigma\frac{(\Sigma y)^2}{f_x}}{N} - c'^2_y}$

$= \sqrt{\frac{213\ 91}{30}}$ 0 133 2.64 $\frac{233\ 9}{30}$ 0 054 2 78

$\eta_{xy} = \frac{\sigma_{\bar{x}_x}}{\sigma'_x} = \frac{2\ 64}{3\ 03}$ 0 87 $\eta_{yx} = \frac{\sigma_{\bar{y}_c}}{\sigma_y} = \frac{2\ 78}{3\ 38}$ 0.82

$\sigma_{\eta_{xy}} = \frac{1-\eta^2_{xy}}{\sqrt{N-1}}$ $\frac{0\ 2431}{\sqrt{29}}$ 0 045 | $\sigma_{\eta_{yx}} =$ " $\frac{0\ 3276}{5\ 3852}$ 0 060

Paramètres		Statistics	
$M_x = X_0 + c'_x i_x$ =	☐	$M_y = Y_0 + c'_y i_y$ =	☐
$\sigma_x = \sigma_x \times i_x$ =	☐	$\sigma_y = \sigma_y \times i_y$ =	☐
$b_{xy} = r_{xy}\frac{\sigma_x}{\sigma_y}$ =	☐	$b_{yx} = r_{xy}\frac{\sigma_y}{\sigma_x}$ =	☐
$\chi^2 = (N - k_y)\frac{\eta^2_{xy} - r^2}{1 - \eta^2_{xy}}$		$\chi^2 = (N - k_x)\frac{\eta^2_{yx} - r^2}{1 - \eta^2_{yx}}$	
= ( ) —— =	☐	= ( ) —— =	☐

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COEFFICIENT OF RELIABILITY OF THE ALGEBRAIC CONCEPT TEST

APPENDIX 6

TWO LISTS OF NAMES AND RAW SCORES

1. Concept-Test (X)

Group of 109 pupils

List of Names and Scores

No.	Name	Concept Score	Skill Score
1	Matsumi Kondo	59	57
2	Gerald Popela	51	47
3	Tony Griffin	48	51
4	Corinne Gendron	47	42
5	Lloyd Brennan	46	41
6	Peter Tobin	45	37
7	Donald LeNeveu	45	34
8	Michael Young	44	41
9	Louis Gowers	44	27
10	Simpson William	43	49
11	Brown Dick	43	49
12	Mary El.Holt	43	38
13	Lorraine Dufour	43	39
14	Nilse Colding	43	29
15	Margaret Herbert	43	38
16	Carol Turpin	43	27
17	James McGibbon	42	44
18	Charles Thebarge	42	37
19	Donald Isbister	42	37
20	Madeleine Laplante	42	34
21	Lawrence Kolesar	41	34
22	David McCaffrey	41	40
23	Thomas Liondie	41	42
24	Frank Cuggy	41	38
25	Louise Corun	41	37
26	Virginia Mackey	41	32
27	William Jette	40	47
28	Brian Pelletier	40	46
29	McDougall Leonard	40	35
30	Gerald Fitzgerald	40	32
31	Nicole Karch	39	38
32	Patrick Gillis	39	30
33	Leo Turner	39	32
34	Mary P. Bridgman	38	18
35	William P.Diolin	38	39
36	Gerald Parent	38	33

APPENDIX 6

TWO LISTS OF NAMES AND RAW SCORES

1. Concept-Test (X)

Group of 109 pupils

List of Names and Scores

No.	Name	Concept Score	Skill Score
1	Natsumi Kondo	59	57
2	Gerald Popela	51	47
3	Tony Griffin	48	51
4	Corinne Gendron	47	42
5	Lloyd Brennan	46	41
6	Peter Tobin	45	37
7	Donald LeNeveu	45	34
8	Michael Young	44	41
9	Louis Gowers	44	27
10	Simpson William	43	49
11	Brown Dick	43	49
12	Mary El. Holt	43	38
13	Lorraine Dufour	43	39
14	Nilse Colding	43	29
15	Margaret Herbert	43	38
16	Carol Turpin	43	27
17	James McGibbon	42	44
18	Charles Theberge	42	37
19	Donald Isbister	42	37
20	Madeleine Laplante	42	34
21	Lawrence Kolesar	41	34
22	David McCaffrey	41	40
23	Thomas Liendie	41	42
24	Frank Cuggy	41	38
25	Louise Corun	41	37
26	Virginia Mackey	41	32
27	William Jette	40	47
28	Brian Pelletier	40	46
29	McDougall Leonard	40	35
30	Gerald Fitzgerald	40	32
31	Nicole Karch	39	38
32	Patrick Gillis	39	30
33	Leo Turner	39	32
34	Mary P. Bridgman	38	18
35	William P. Diolin	38	39
36	Gerald Parent	38	33

No.	Name	Concept Score	Skill Score
37	Raymond Kealy	37	41
38	Jean Closs	37	34
39	Richard Marks	37	34
40	James Gillisie	37	37
41	Richard Lord	37	30
42	Robert Campbell	37	31
43	Bob Brady	37	30
44	Martin Flood	36	28
45	Bruce Attfield	36	34
46	Douglas Goodwin	36	31
47	Joan Burgess	36	33
48	Pauline Street	35	29
49	Daisy Jarvis	35	37
50	Mangione Joseph	35	34
51	Warren L'Africain	35	30
52	Gerald Meepher	35	29
53	Fr.Jan Tulatycki	35	30
54	Patrick McTuard	35	41
55	Radburn William	35	36
56	John T. McKay	35	32
57	Duncan Treman	35	43
58	David Kelly	35	42
59	Glen Coner	35	35
60	Leo Seissons	35	30
61	Tony Piatel	35	27
62	Samora Cherry	34	22
63	Moreen Ouellette	34	36
64	John Rivet	34	39
65	Peter Nolan	34	29
66	Desmond Keon	34	25
67	Pete Lineger	34	34
68	Leo Valiquette	34	35
69	Diana Gauthier	33	23
70	Harold Kehoe	33	23
71	Martin Monagan	33	27
72	Kopil George	33	25
73	Tom Curley	33	33
74	Graeme A.McDougall	32	39
75	Engel John T.	32	29
76	Margaret Boland	31	26
77	Patrick Shea	31	30
78	John Gosselin	31	25
79	Philip O'Reilly	31	38
80	Paul Brazeau	31	30

No.	Name	Concept Score	Skill Score
81	Terry O'Connell	31	27
82	Fenton Fred	31	27
83	Paul Drapeau	30	21
84	Christopher Durham	30	26
85	William Perrault	30	32
86	Bob Flood	30	35
87	Gerald Scarcella	30	28
88	Kevin Milaney	29	24
89	Brian Ward	29	20
90	Wilma Conroy	28	22
91	Paul Geawel	28	26
92	Hendersen Jim	28	32
93	Morin Robert	28	35
94	Shaun K. Fripp	28	23
95	Sheila Gorman	27	21
96	Joe Hrobolick	27	15
97	Don Brewold	27	29
98	Tom Coulsen	27	24
99	Louis R.Saumweber	27	38
100	Peter Pyefinch	26	27
101	Robert Gauthier	26	31
102	Penny Hooper	25	23
103	Stanley Cameron	25	24
104	Lahey Terence	25	32
105	John Murphey	25	21
106	John McDermott	23	21
107	Pete Phelan	22	21
108	Trever Smith	21	14
109	Benoit John	18	22

2. Skill-Test (Y)

Group of 109 pupils

List of Names and Scores

No.	Name	Skill Score	Concept Score
1	Natsumi Kondo	57	59
2	Tony Griffin	51	48
3	Simpson William	49	43
4	Brown Dick	49	43
5	Gerald Popela	47	51
6	William Jette	47	40
7	Brian Pelletier	46	40
8	James McGibbon	44	42
9	Duncan Treman	43	35
10	Thomas Liondie	42	41
11	David Kelly	42	36
12	Corrine Gendron	42	47
13	Lloyd Brennan	41	46
14	Michael Young	41	44
15	Raymond Kealy	41	37
16	Patrick McTuard	41	35
17	David McCaffrey	40	41
18	William P. Bielan	39	38
19	John Rivet	39	34
20	Graeme A. McDougall	39	32
21	Lorraine Dufour	39	43
22	Frank Cuggy	38	41
23	Philip O'Reilly	38	31
24	Mary El.Holt	38	43
25	Margaret Herbert	38	43
26	Louis R. Saumweber	38	27
27	Nicole Karch	38	39
28	Peter Tobin	37	45
29	Charles Thebarge	37	42
30	Donald Isbister	37	42
31	James Gillisie	37	37
32	Louise Corum	37	41
33	Daisy Jarvis	37	36
34	Radburn William	36	35
35	Noreen Ouellette	36	34
36	McDougall Leonard	35	40

No.	Name	Skill Score	Concept Score
37	Glen Coner	35	35
38	Leo Valiquette	35	34
39	Bob Flood	35	30
40	Morin Robert	35	28
41	Ronald LeNeveu	34	45
42	Lawrence Kolesar	34	41
43	Jean Closs	34	37
44	Richard Marks	34	37
45	Bruce Attfield	34	45
46	Mangione Joseph	34	35
47	Pete Lineger	34	34
48	Madeleine Laplante	34	42
49	Gerald Parent	33	38
50	Joan Burgess	33	36
51	Tom Curley	33	33
52	Gerald Fitzgerald	32	40
53	Leo Turner	32	39
54	John T. McKay	32	35
55	William Perrault	32	30
56	Henderson Jim	32	28
57	Lahey Terence	32	25
58	Virginia Mackey	32	41
59	Robert Campbell	31	37
60	Douglas Goodwin	31	36
61	Robert Gauthier	31	26
62	Patrick Gillis	30	39
63	Richard Lord	30	37
64	Bob Brady	30	37
65	Warren L'Africain	30	35
66	Fr.Jan Tulatycki	30	35
67	Leo Soissons	30	35
68	Patrick Shea	30	31
69	Paul Brazeau	30	31
70	Gerald Meopher	29	35
71	Peter Nolan	29	34
72	Engel John T	29	32
73	Don Browold	29	27
74	Nilse Colding	29	43
75	Pauline Street	29	35
76	Marian Flood	28	36
77	Gerald Scarcella	28	30
78	Tony Piatel	27	35
79	Martin Monaghan	27	33
80	Terry O'Connell	27	31

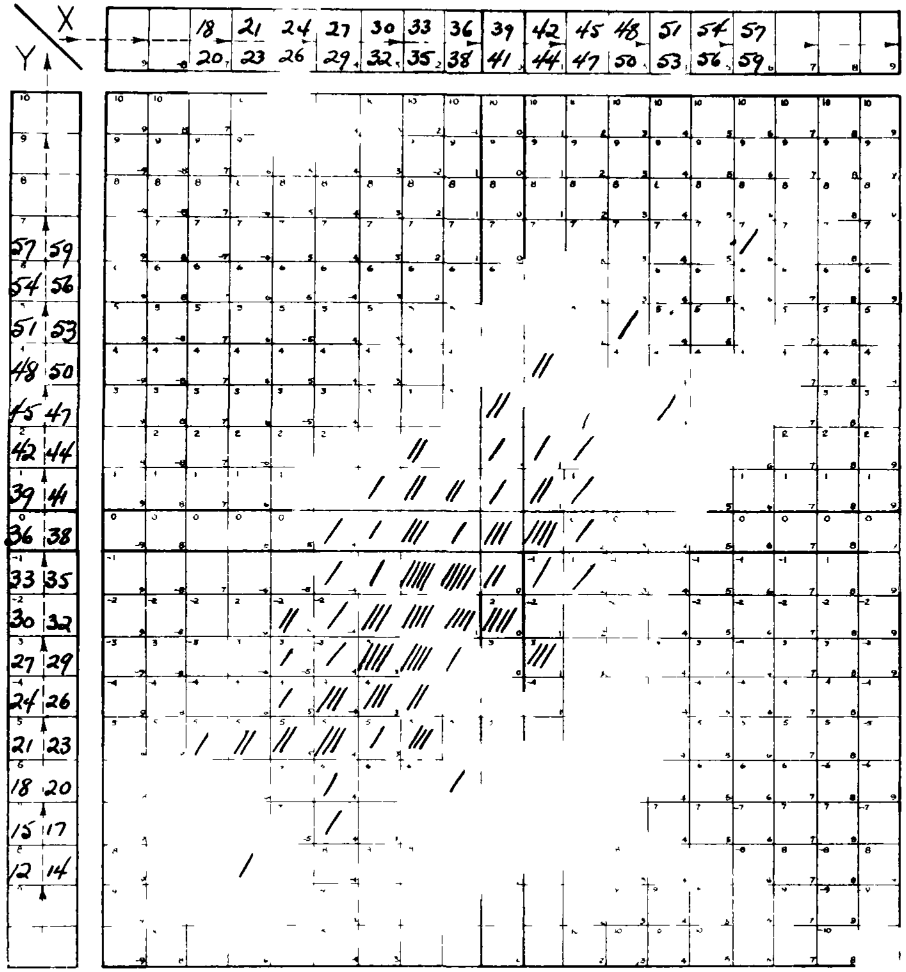
No.	Name	Skill Score	Concept Score
81	Fenton Fred	27	31
82	Peter Pyefinch	27	26
83	Louis Gowers	27	44
84	Carol Turpin	27	43
85	Christopher Durham	26	30
86	Paul Geawel	26	28
87	Margaret Boland	26	31
88	Desmond Keon	25	34
89	Kopil George	25	33
90	John Gosselin	25	31
91	Kevin Milaney	24	29
92	Tom Coulson	24	27
93	Stanley Cameron	24	25
94	Harold Kehoe	23	33
95	Shaun K. Fripp	23	28
96	Diana Gauthier	23	33
97	Penny Hooper	23	25
98	Benoit John	22	18
99	Samora Cherry	22	34
100	Wilma Conroy	22	28
101	Paul Drapeau	21	30
102	John Murphey	21	25
103	John McDermott	21	23
104	Pete Phelan	21	22
105	Sheila Gorman	21	27
106	Brian Ward	20	29
107	Mary Paula Bridgman	18	38
108	Joe Hrobonick	15	27
109	Trevor Smith	14	21

APPENDIX 7

COEFFICIENT OF CORRELATION OF THE TWO SETS OF SCORES

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

Sujets (Subjects) GRADE XI HIGH SCHOOL PUPILS N - 109
Date JUNE 7, 1956 Per: T. POŹNIAK



f_y	y'	fy'	fy'^2	$\Sigma x'$	$\Sigma x'y'$	$(\Sigma x')^2$	$\frac{(\Sigma x')^2}{f_y}$
	10						
	9						
	8						
1	7	7	49	6	42	36	36
0	6	0	0	0	0	0	~~0~~
1	5	5	25	3	15	9	9
2	4	8	32	2	8	4	2
3	3	9	27	4	12	16	5.33
5	2	10	20	-1	-2	1	0.20
9	1	9	9	-5	-5	25	2.77
14	0	$\Sigma_{(+)}$ 48	0	-8	0	64	4.57
16	-1	-16	16	-19	19	361	22.56
18	-2	-36	72	-35	70	1225	68.05
15	-3	-45	135	-29	87	841	56.06
9	-4	-36	144	-30	120	900	100
12	-5	-60	300	-50	250	2500	208.33
2	-6	-12	72	-5	30	25	12.50
1	-7	-7	49	-4	28	16	16
1	-8	-8	64	-6	48	36	36
	-9						
	-10						
109	-220 -172	1014		-177	722		579.33

f_x			1	3	6	12	14	26	14	13	13	4	1	1	0	1			
x	-9	-8	-7	-6	-5	-4	-3	-2	-1	0 $\Sigma_{(-)}$		2	3	4	5	6	7	8	9
fx'			-7	-18	-30	-48	-42	-52	-14	-211	13	8	3	4	0	6			
fx'^2	81	64	49 49	36 108	25 150	16 192	9 126	4 104	1 14	0 0	13	4 16	9 9	16 16	25 0	36 36	49	64	81
$\Sigma y'$			-5	-18	-21	-46	-35	-45	-20	-1	2	2	5	3	0	7			
$\Sigma x'y'$			35	108	105	184	105	90	20	0	2	4	15	12	0	42			
$(\Sigma y')^2$			25	324	441	2116	1225	2025	400	1	4	4	25	9	0	49			
$(\Sigma y')^2/f_x$			25	108	73.50	176.33	87.50	77.87	28.57	.07	.30	1	25	9	~~0~~	49			

Σf ↑ ↑ fx' ↑ ↑ $\Sigma\frac{(\Sigma x')^2}{f_y}$

$\Sigma_{(+)}$ 34 -177 ← fx'

833 ← f

-172 ← $\Sigma fy'$

722 ← $\Sigma x'y'$

661.14 ← $\Sigma\frac{(\Sigma y')^2}{f_x}$

	X	Y
Variable	ALGEBRAIC CONCEPT	ALGEBRAIC SKILL

X 40 k_x = 14 i_x = 3 | Y_0 37 k_y 16 i_y = $\boxed{3}$

Corrections

$c'_x = \frac{\Sigma fx'}{N} = \frac{-177}{109}$ -1.62 | $c'_y = \frac{\Sigma fy'}{N}$ $\frac{-172}{109}$ = $\boxed{-1.57}$

$c'^2_x =$ 2.62 | $c'^2_y =$ 2.46

σ'

$\sigma'_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$ | $\sigma'_y = \sqrt{\frac{\Sigma fy'^2}{N} - c'^2_y}$

$= \frac{833}{109}$ 2.62 2.24 | $\frac{1014}{109}$ 2.46 = 2.59

Correlation : r_{xy}

$r_{xy} = \frac{\frac{\Sigma x'y'}{N} - c'_x c'_y}{\sigma'_x \sigma'_y} = \frac{\frac{722}{109} - 1.62 \times 1.57}{5.80}$ = $\boxed{.704}$

$\sigma_{r_{xy}} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0.4961}{10.3923} = .048$

Rapport de Corrélation η Correlation Ratio

$\sigma'_{\bar{x}_r} = \sqrt{\frac{\Sigma\frac{(\Sigma x')^2}{f_y}}{N} - c'^2_x}$ | $\sigma'_{\bar{y}_c} = \sqrt{\frac{\Sigma\frac{(\Sigma y')^2}{f_x}}{N} - c'^2_y}$

$\frac{579.33}{109}$ 2.62 1.64 | $= \sqrt{\frac{661.14}{109}}$ 2.46 1.89

$\eta_{xy} = \frac{\sigma'_{\bar{x}_r}}{\sigma'_x} = \frac{1.64}{2.24}$ .73 | $\eta_{yx} = \frac{\sigma'_{\bar{y}_c}}{\sigma'_y} = \frac{1.89}{2.59}$ .73

$\sigma_{\eta_{xy}} = \frac{1 - \eta^2_{xy}}{\sqrt{N-1}}$ $\frac{0.46}{10.39}$ .044 | $\sigma_{\eta_{yx}} =$ $\frac{0.46}{10.39}$ .044

Paramètres Statistics

$M_x = X_0 + c'_x i_x = \boxed{35.14}$ | $M_y = Y_0 + c'_y i_y = 33.29$

$\sigma_x = \sigma'_x \times i_x = \boxed{6.72}$ | $\sigma_y = \sigma'_y \times i_y = \boxed{7.77}$

$b_{xy} = r_{xy}\frac{\sigma_x}{\sigma_y} = .61$ | $b_{yx} = r_{xy}\frac{\sigma_y}{\sigma_x}$ $\boxed{1.15}$

$\chi^2 = (N - k_y) \frac{\eta^2 - r^2}{1 - \eta^2}$ | $\chi^2 = ($

$= (93)\frac{0.04}{0.46} = \boxed{7.99}$ | $= (95\ \frac{0.04}{0.46}$ $\boxed{7.79}$

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COEFFICIENT OF CORRELATION OF THE TWO SETS OF SCORES

APPENDIX 8

COMPUTATION OF PERCENTILES AND STANDARD SCORES

1. Computation of Percentiles

The computation of percentiles for the whole group of 109 testees is presented in the attached Lists A and B for the Concept-Test and the Skill-Test, respectively.

The formula for finding a percentile by deriving it from a rank: $P = 100 \quad \frac{n - r + 0.5}{n}$ has been used throughout,[1] by applying it to both tests.

Symbol *n* is the number of the individuals in the group; symbol *r* is the particular rank assigned to the individual's total score on the test.

2. Computation of Standard Scores

The normalized standard scores have been computed with the help of tables in the way recommended by D. Adkins[2] in her book *Construction and Analysis of Achievement Tests*. Thus, if a raw score 38 is equivalent to a percentile of 68.3 as it is in the Concept-Test at No.35, then the difference (68.3 - 50) = 18.3, divided by 109, is checked in the tables, in the column "area", and a corresponding value $\frac{x}{\sigma} = 0.47$ is computed by interpolation.

---

[1] J.P.Guilford, *Fundamental Statistics in Psychology and Education*, 2nd edition, New York, McGraw-Hill, 1950, p.553.

[2] D.C.Adkins, *Construction and Analysis of Achievement Tests,* Washington D.C., U.S.Govt.Printing Office, 1947, pp 100 and 143.

A. CONCEPT-TEST

COMPUTATION OF PERCENTILES

$$P = 100 \times \frac{N - R + 0.5}{N}$$

N = number of individuals ranked

R = particular rank assigned to the individual's total score on the test

N = 109

No.	Score	Rank		Percentile
1	59	1	$P = 100 \times \frac{109 - 1 + 0.5}{109}$	= 99.5
2	51	2	$100 \times \frac{107 + 0.5}{109}$	= 98.6
3	48	3	$\frac{10650}{109}$	= 97.7
4	47	4	$\frac{10550}{109}$	= 96.7
5	46	5	$\frac{10450}{109}$	= 95.8
6 7	45	6.5	$\frac{10300}{109}$	= 94.4
8 9	44	8.5	$\frac{10100}{109}$	= 92.6
10 11 12 13 14 15 16	43	13	$\frac{9650}{109}$	= 88.5
17 18 19 20	42	18.5	$\frac{9100}{109}$	= 83.4

No.	Score	Rank		Percentile
21 22 23 24 25 26	41	23.5	$\frac{8600}{109}$	= 78.8
27 28 29 30	40	28.5	$\frac{8100}{109}$	= 74.3
31 32 33	39	32	$\frac{7750}{109}$	= 71.1
34 35 36	38	35	$\frac{7450}{109}$	= 68.3
37 38 39 40 41 42 43	37	40	$\frac{6950}{109}$	= 63.7
44 45 46 47	36	45.5	$\frac{6400}{109}$	= 58.7
48 49 50 51 52 53 54 55 56 57 58 59 60 61	35	54.5	$\frac{5500}{109}$	= 50.4

No.	Score	Rank			Percentile
62 63 64 65 66 67 68	34	65	$\frac{4450}{109}$	=	40.8
69 70 71 72 73 74	33	71.5	$\frac{3800}{109}$	=	34.8
75 76	32	75.5	$\frac{3400}{109}$	=	31.1
77 78 79 80 81 82 83	31	80	$\frac{2950}{109}$	=	27
84 85 86 87 88	30	86	$\frac{2350}{109}$	=	21.5
89 90	29	89.5	$\frac{2000}{109}$	=	18.3
91 92 93 94 95	28	93	$\frac{1650}{109}$	=	15.1
96 97 98 99	27	97.5	$\frac{1200}{109}$	=	11
100 101	26	100.5	$\frac{900}{109}$	=	8.2

No.	Score	Rank			Percentile
102 103 104 105	25	103.5	$\frac{600}{109}$	=	5.5
106	23	106	$\frac{350}{109}$	=	3.3
107	22	107	$\frac{250}{109}$	=	2.2
108	21	108	$\frac{150}{109}$	=	1.3
109	18	109	$\frac{50}{109}$	=	0.4

B. SKILL-TEST

COMPUTATION OF PERCENTILES

$$P = 100 \times \frac{N + 0.5 - R}{N}$$

N = 109 = number of scores

R = particular rank assigned to one score

No.	Score	Rank		Percentile
1	57	1	$\frac{109.50-1}{109}\cdot 100$	= 99.5
2	51	2	$\frac{10750}{109}$	= 98.6
3 4	49	3.5	$\frac{10600}{109}$	= 97.2
5 6	47	5.5	$\frac{10400}{109}$	= 95.4
7	46	7	$\frac{10250}{109}$	= 94
8	44	8	$\frac{10150}{109}$	= 93.1
9	43	9	$\frac{10050}{109}$	= 92.2
10 11 12	42	11	$\frac{9850}{109}$	= 90.3
13 14 15 16	41	14.5	$\frac{9500}{109}$	= 87.1
17	40	17	$\frac{9250}{109}$	= 84.8
18 19 20 21	39	19.5	$\frac{9000}{109}$	= 82.5

No.	Score	Rank	Percentile
22 23 24 25 26 27	38	24.5	$\frac{8500}{109}$ = 77.9
28 29 30 31 32 33	37	30.5	$\frac{7900}{109}$ = 72.4
34 35	36	34.5	$\frac{7500}{109}$ = 68.8
36 37 38 39 40	35	38	$\frac{7150}{109}$ = 65.5
41 42 43 44 45 46 47 48	34	44.5	$\frac{6500}{109}$ = 59.6
49 50 51	33	50	$\frac{5950}{109}$ = 54.5
52 53 54 55 56 57 58	32	55	$\frac{5450}{109}$ = 50
59 60 61	31	60	$\frac{4950}{109}$ = 45.4

No.	Score	Rank		Percentile
62 63 64 65 66 67 68 69	30	65.5	$\frac{4400}{109}$	= 40.3
70 71 72 73 74 75	29	72.5	$\frac{3700}{109}$	= 33.9
76 77	28	76.5	$\frac{3300}{109}$	= 30.2
78 79 80 81 82 83 84	27	81	$\frac{2850}{109}$	= 26.1
85 86 87	26	86	$\frac{2350}{109}$	= 21.5
88 89 90	25	89	$\frac{2050}{109}$	= 18.8
91 92 93	24	92	$\frac{1750}{109}$	= 16
94 95 96 97	23	95.5	$\frac{1400}{109}$	= 12.8
98 99 100	22	99	$\frac{1050}{109}$	= 9.6

No.	Score	Rank			Percentile
101 102 103 104 105	21	103	$\frac{650}{109}$	=	5.9
106	20	106	$\frac{350}{109}$	=	3.2
107	18	107	$\frac{250}{109}$	=	2.2
108	15	108	$\frac{150}{109}$	=	1.3
109	14	109	$\frac{50}{109}$	=	0.4

APPENDIX 9

VERTICAL AND HORIZONTAL COMPARISON

/EIGHT TABLES AND SIX CORRELATION CHARTS/

Table IX. - Upper Concept Group - Computation of the Mean and Standard Deviation

Scores	Mid-point	f.	x	fx	fx^2
57-59	56	1	15.59	15.59	243.048
54-56	53	0	12.59	0.00	0.000
51-53	52	1	11.59	11.59	132.710
48-50	49	1	8.59	8.59	73.788
45-47	46	4	5.59	22.36	124.992
42-44	41	13	- 0.59	- 7.67	4.525
39-41	38	13	- 2.41	-31.33	75.505
36-38	37	3	- 3.41	-10.23	34.884

$N = 36$

$\Sigma x^2 = 689.452$

$M = \frac{\Sigma X}{N} = 40.41;$

$\sigma = \sqrt{\frac{\Sigma x^2}{N}} = \frac{26.06}{6} = 4.34$

$= \underline{4.08},$ as computed in the attached Correlation Chart A

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

Sujets (…jects) UPPER CONCEPT GROUP N 36
Dat JUNE 10, 1956 Per T POZNIAK

X Y				18 20	21 23	24 26	27 29	30 32	33 35	36 38	39 41	42 44	45 47	48 50	51 53	54 56	57 59			f_y	y'	fy'	fy'^2	$\Sigma x'$	$\Sigma x'y'$	$(\Sigma x')^2$	$\frac{(\Sigma x')^2}{f_y}$
																					10						
																					9						
																					8						
																					7						
																					6						
																					5						
57 59																	/			1	4	4	16	7	28	49	49
54 56																				0	3	0	0	0	0	0	0
51 53													/							1	2	2	4	3	6	9	9
48 50															/					1	1	1	1	5	5	25	25
45 47									/	/	/	/								4	0	Σ(+) 7		2	0	4	1
42 44							///		/	////	//	/		//						13	-1	-13	13	2	-2	4	.30
39 41								////	//	///	/	/	//							13	-2	-26	52	-1	2	1	.07
36 38				/					/		/									3	-3	-9	27	-6	18	36	12
																					-4						
																					-5						
																					-6						
																					-7						
																					-8						
																					-9						
																					-10						

f_x				1	0	0	3	4	5	8	5	3	3	2	1	0	1		
x'	-9	-8	-7	-6	-5	-4	-3	-2	-1	0 Σ(−)	1	2	3	4	5	6	7	8	9
fx'				-6	0	0	-9	-8	-5	-28	5	6	9	8	5	0	7		
fx'^2				36	0	0	-27	16	5	0	5	12	27	32	25	0	49		
$\Sigma y'$				-3	0	0	-3	-8	-8	-10	-7	-3	-2	-2	1	0	4		
$\Sigma x'y'$				18	0	0	9	16	8	0	-7	-6	-6	-8	5	0	28		
$(\Sigma y')^2$				9	0	0	9	64	64	100	49	9	4	4	1	0	16		
$(\Sigma y')^2/f_x$				9	0	0	3	16	128	12.5	98	3	1.33	2	1	0	16		

36 N
Σ(−) -48 / -41
113 ← $\Sigma fy'^2$
12 ← Σf_x
57
96.37 ← $\Sigma \frac{(\Sigma x')^2}{f_y}$
Σ(+) 40 12
180 ← $\Sigma fx'^2$
-41 ← $\Sigma fy'$
57 ← $\Sigma x'y'$
86.43 ← $\Sigma \frac{(\Sigma y')^2}{f_x}$

X Variable. SKILLS — Y Variable CONCEPTS

= 37 k_x = 14 i_x 3 | 46 k_y = 8 i_y = 3

Corrections

$c'_x = \frac{\Sigma fx}{N}$ $\frac{12}{36}$ 0.33 | c'_y N $\frac{-41}{36}$ -1.13

$c'^2_x =$ 0 1089 | ² 1 2769

σ'

$\sigma'_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$ | $\sigma'_y = \sqrt{\frac{\Sigma fy'^2}{N} - c'^2_y}$

$\frac{180}{36}$ 0 1089 2 21 | $-\sqrt{\frac{113}{36}}$ 1 2769 1 36

Correlation . r_{xy}

$$r_{xy} = \frac{\frac{\Sigma x'y'}{N} - c'_x c'_y}{\sigma_x \sigma_y} = \frac{\frac{57}{36} + 0\ 33 \times 1.13}{2.21 \times 1.36} \quad \boxed{0.65}$$

$$\sigma_{r\,xy} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0.422}{\sqrt{35}} = 0.097$$

Rapport de Correlation η Correlation Ratio

$$\sigma'_{\bar{x}c} = \sqrt{\frac{\Sigma \frac{(\Sigma x')^2}{f_y}}{N} - c'^2_x} = \frac{96\ 37}{36}\ 0\ 10\ \ 1.60$$

$$\sigma'_{\bar{y}c} = \sqrt{\frac{\Sigma \frac{(\Sigma y')^2}{f_x}}{N} - c'^2_y} \quad \frac{86.43}{36}\ 1\ 27\ \ 1.06$$

$$\eta_{xy} = \frac{\sigma'_{\bar{x}c}}{\sigma'_x} = \frac{1.60}{2\ 21} \quad 0.72$$

$$\eta_{yx} = \frac{\sigma_{\bar{y}c}}{\sigma'_y} = \frac{1.06}{1.36} \quad 0\ 77$$

$$\sigma_{\eta xy} = \frac{1-\eta^2_{xy}}{\sqrt{N-1}} \frac{1-0.5184}{5.91} \boxed{0.081}$$

$$\sigma_{\eta yx} = \frac{1-\eta^2_{yx}}{\sqrt{N-1}} \frac{1-0.5929}{5\ 91} \quad 0\ 068$$

Para TRUE 38 08 | S TRUE 40 41

$M_x = X_0 + c'_x i_x$ = 37 66 | $M_y = Y_0 + c'_y i_y$ 42.4

$\sigma_x = \sigma'_x \times i_x$ = $\boxed{6.63}$ | $\sigma_y = \sigma'_y \times i_y$ 4.08

$b_{xy} = r_{xy}\frac{\sigma_x}{\sigma_y}$ = $\boxed{1\ 05}$ | $b_{yx} = r_{xy}\frac{\sigma_y}{\sigma_x}$ .399

$\chi^2 = (N - k_y)\frac{\eta^2_{xy} - r^2}{1 - \eta^2_{xy}}$ = (28) $\frac{0\ 095}{0.481}$ 5.51 | $\chi^2 = (N - k_x)\frac{\eta^2_{yx} - r^2}{1 - \eta^2_{yx}}$ = (22) $\frac{0\ 1704}{0\ 4071}$ 9.20

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CORRELATION CHART "A"

APPENDIX 9

Table X. - Middle Concept Group - Computation of the Mean and Standard Deviation

Scores	Mid-point	f	x	x^2	fx^2
37-37	37	7	1.92	3.68	25.67
36-36	36	4	0.92	0.84	3.36
35-35	35	14	-0.08	0.006	0.08
34-34	34	7	-1.08	1.16	8.12
33-33	33	4	-2.08	4.32	17.28
		$N = 36$			$\Sigma fx^2 = 54.51$

$$M = \frac{\Sigma X}{N} = 35.08; \qquad \sigma = \sqrt{\frac{\Sigma fx^2}{N}} = \frac{7.38}{6} = \underline{1.23}$$ as

also computed in the attached Correlation Chart B.

APPENDIX 9

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

Sujets (Subject) MIDDLE CONCEPT GROUP N-36
Date JUNE 10, 1956 – Per T. POZNIAK

Y \ X	22 23	24 25	26 27	28 29	30 31	32 33	34 35	36 37	38 39	40 41	42 43	f_y	y	$f_{y'}$	$f_{y'^2}$	$\Sigma x'$	$\Sigma x'y'$	$(\Sigma x')^2$	$\frac{(\Sigma x')^2}{f_y}$
													10		100				
													9		81				
													8		64				
													7		49				
													6		36				
													5		25				
													4		16				
													3						
37 37					///		//	/		/		7	2	14	28	5	10	25	357
36 36				/	/	/	/					4	1	4	4	-2	-2	4	1.
35 35			/	//	///	/	//	//		/	//	14	0	$\Sigma(+)$ 18	0	10	0	100	7.14
34 34	/	/		/			//	/	/			7	-1	-7	7	-4	4	16	2.28
33 33	//	/	/									4	-2	-8	16	-17	34	289	7225
													-3						
													-4						
													-5		25				
													-6		36				
													-7		49				
													-8						
													-9		81				
													-10		100				

f_x					3	2	2	4	7	2	7	4	1	2	2				
x'	9	8	7	6	5	-4	-3	-2	-1	0		2	3	4	5	6	7	8	9
$f_{x'}$					-15	-8	-6	-8	-7	-44	7	8	3	8	10				
$f_{x'^2}$					75	32	18	16	7	0	7	16	9	32	50				
Σ					-5	-3	-2	0	7	1	3	1	-1	2	0				
[illegible]					25	12	6	0	-7	0	3	2	-3	8	0				
$(\Sigma\ )$					25	9	4	0	49	1	9	1	1	4	0				
$(\Sigma y')^2/f_x$					833	4.5	2	0	7	0.5	1.28	025	1	2	0				

36

$\Sigma(-)$ -15 55 -8 46

3 $\Sigma f_{y'^2}$ $\Sigma f_{x'}$

$\Sigma(+)$ 36 -8

262

3 ← $\Sigma f_{y'}$

46 ← $\Sigma x'y'$

$\boxed{26.86}$ ← $\Sigma \frac{(\Sigma y')^2}{f_x}$

8624

$\Sigma \frac{(\Sigma x')^2}{f_y}$

X	Y
Variable. SKILLS	Variable CONCEPTS
/ 30.5 $k_x = [\ 11\ ]$ i_x 2	$Y_o = \boxed{34}$ $k_y =$ 5 $i_y = [\ 1\ ]$

Corrections

$c'_x = \frac{\Sigma fx}{N}$ $-\frac{8}{36}$ -0.222

$c'^2_x =$ 0.048

$c'_y = \frac{\Sigma fy'}{N} = \frac{3}{36}$ -0.083

$c'^2_y =$ 0.006

σ'

$\sigma'_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$

$= \sqrt{\frac{262}{36}}$ 0.048 268

$\sigma'_y = \sqrt{\frac{\Sigma fy'^2}{N} - c'^2_y}$

$-\sqrt{\frac{55}{36}}$ 0 006 123

Correlation : r_{xy}

$r_{xy} = \frac{\frac{\Sigma xy'}{N} - c'_x c'_y}{\sigma_x \sigma'_y} = \frac{\frac{46}{36} + 0.0184}{3.296}$ $\boxed{039}$

$\sigma_{r_{xy}} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0.1521}{\sqrt{35}}$ — 0.143

Rapport de Correlation η Correlation Ratio

$\sigma'_{\bar{x}r} = \sqrt{\frac{\Sigma \frac{(\Sigma x')^2}{f_y}}{N} - c'^2_x}$

$\frac{8624}{36}$ 1.53

$\sigma'_{\bar{y}c} = \sqrt{\frac{\Sigma \frac{(\Sigma y')^2}{f_x}}{N} - c'^2_y}$

$\frac{26.86}{36}$ 0.006 0 86

$\eta_{xy} = \frac{\sigma_{\bar{x}r}}{\sigma'_x} = \frac{1\ 53}{2\ 68}$ 0 57

$\eta_{yx} = \frac{\sigma_{\bar{y}c}}{\sigma_y}$ $\frac{0\ 86}{1.23}$ 0.69

$\sigma_{\eta_{xy}} = \frac{1 - \eta^2_{xy}}{\sqrt{N-1}} = \frac{0\ 6751}{5.91}$ 0 114

$\sigma_{\eta_{yx}} =$ $\frac{0.5239}{5.91}$ 0 088

tres TRUE 32.11

Stati TRUE 35.08

$M_x = X_o + c'_x i_x$ = 32.06

$M_y = Y_o + c'_y i_y$ 34083

$\sigma_x = \sigma'_x \times i_x$ = 5.36

$\sigma_y = \sigma_y \times i_y$ - 1.23

$b_{xy} = r_{xy} \frac{\sigma_x}{\sigma_y}$ = 169

$b_{yx} = r_{xy} \frac{\sigma_y}{\sigma_x}$ 0.17

$\chi^2 = (N - k_y) \frac{\eta^2_{xy} - r^2}{1 - \eta^2_{xy}}$

$= (31)\ \frac{0.1728}{0\ 6781}$ $\boxed{7.93}$

$\chi^2 = (N - k_x) \frac{\eta^2_{yx} - r^2}{1}$

$= (25$ $\frac{0.324}{0.5239}$ 1.54

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CORRELATION CHART "B"

Table XI. - Lower Concept Group - Computation of the Mean and Standard Deviation

Scores	f	x	fx	fx^2
32-33	4	2	8	16
30-31	12	1	12	12
28-29	7	0	0	0
26-27	6	-1	-6	6
24-25	4	-2	-8	16
22-23	2	-3	-6	18
20-21	1	-4	-4	16
18-19	1	-5	-5	25
	N = 37		$\Sigma fx = -9$	$\Sigma fx^2 = 109$

Assumed Mean = $M^1 = 28$

$M = M^1 + i(-0.24) = 27.6$

$c = \frac{\Sigma fx}{N} = -0.24$

True Mean = $\frac{\Sigma X}{N} = 28.08$

$$\sigma = i.\sqrt{\frac{\Sigma fx^2}{N} - c^2}$$

$$= 2.\sqrt{2.94-0.05} = \underline{3.40}$$

as computed in the attached Correlation Chart C

APPENDIX 9

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

Sujets (Su jects) LOWER CONCEPT GROUP N-37

JUNE 11, 1956 Per T. POZ'NIAK

Y \ X	14 15	16 17	18 19	20 21	22 23	24 25	26 27	28 29	30 31	32 33	34 35	36 37	38 39
32 33							/	/		/			/
30 31				/		//	///	/	//	/	/		/
28 29				//	//		/			/	/		
26 27	/					/	/	/	/				/
24 25				/	/	/				/			
22 23				//									
20 21	/												
18 19					/								

f_y	y'	fy'	fy'^2	$\Sigma x'$	$\Sigma x'y'$	$(\Sigma x')^2$	$\frac{(\Sigma x')^2}{f_y}$
	10		100				
	9		81				
	8		64				
	7		49				
	6		36				
	5		25				
4	4	16	64	10	40	100	25
12	3	36	108	13	39	169	14.08
7	2	14	28	-3	-6	9	1.28
6	1	6	6	2	2	4	0 66
4	0	72		-3	0	9	2.25
2	-1	-2	2	-6	+6	36	18
1	-2	-2	4	-6	12	36	36
1	-3	-3	9	-2	6	4	4
	-4		16				
	-5		25				
	-6		36				
	-7		49				
	-8		64				
	-9		81				
	-10		100				
37 N	$\Sigma_{(-)}$ -7 65		221	5	99		101.27

$\Sigma \frac{(\Sigma x')^2}{f_y}$

f_x				2	0	0	6	4	4	6	3	3	4	2	0	3			
x'	-9	-8	-7	-6	-5	-4	-3	-2	1	0 $\Sigma_{(-)}$	1	2	3	4	5	6	7	8	9
fx'				-12	0	0	-18	-8	-4	-42	3	6	12	8	0	18			
fx'^2	81	64	49	72	0	0	54	16	4	0	3	12	36	32	0	108	49	64	81
$\Sigma y'$				-1	0	0	5	1	7	16	8	7	9	5	0	8			
$\Sigma x'y'$				6	0	0	-15	-2	-7	0	8	14	27	20	0	48			
$(\Sigma y')^2$				1	0	0	25	1	49	256	64	49	81	25	0	64			
$(\Sigma y')^2/f_x$				0.5	0	0	4.16	0.25	12.25	42.66	21.33	16.33	20.25	12 5	0	21.33			

47 5

337 ← Σf

65 ← Σfy'

99 ← Σx'y'

151.56 ← $\Sigma \frac{(\Sigma y')^2}{f_x}$

X	Y
Variable. SKILLS	Variable CONCEPTS
=26 5 k_x 13 ι_x 2	24 5 8 ι_y = 2

Corrections

$c'_x = \frac{\Sigma fx'}{N} = \frac{5}{37}$ 0 135 $c'_y = \frac{\Sigma fy'}{N} = \frac{65}{37}$ 1.75

$c'^2_x =$ 0.182 $c'^2_y =$ 3.06

σ'

$\sigma'_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$ $\sigma'_y = \sqrt{\frac{\Sigma fy'^2}{N} - c'^2_y}$

$= \frac{337}{37}$ 0 182 2 98 $= \frac{221}{37} - 3.06 - 1.70$

Correlation : r_{xy}

$$r_{xy} = \frac{\frac{\Sigma x'y'}{N} - c'_x c'_y}{\sigma_x \sigma'_y} = \frac{\frac{99}{37} - 0.135 \times 1.75}{2\ 98 \times 1.70} = \boxed{0.49}$$

$$\sigma_{r_{xy}} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0.2401}{\sqrt{37-1}} = 0\ 126$$

Rapport de Correlation η Correlation Ratio

$\sigma'_{\bar{x}r} = \sqrt{\frac{\Sigma \frac{(\Sigma x')^2}{f_y}}{N} - c'^2_x}$ $\sigma'_{\bar{y}c} = \sqrt{\frac{\Sigma \frac{(\Sigma y')^2}{f_x}}{N} - c'^2_y}$

$= \frac{101\ 27}{37}$ 0.182 1 59 $\frac{151\ 56}{37}$ 3 06 1 01

$\eta_{xy} = \frac{\sigma'_{\bar{x}r}}{\sigma'_x} = \frac{1\ 59}{2.98}$ 0 53 $\eta_{yx} = \frac{\sigma'_{\bar{y}c}}{\sigma'_y} = \frac{1\ 01}{1\ 70}$ 0.59

$\sigma_{\eta_{xy}} = \frac{1-\eta^2_{xy}}{\sqrt{N-1}}$ $\frac{0.7191}{6}$ 0.119 $= \frac{1-\eta^2_{yx}}{\sqrt{N-1}}$ $\frac{0\ 6581}{6}$ 0.109

Param	Statistics
TRUE 26.78	TRUE 28.08
$M_x = X_0 + c'_x \iota_x$ 26.67	$M_y = Y_0 + c'_y \iota_y$ 28.00
$\sigma_x = \sigma'_x \times \iota_x = \boxed{5.96}$	$\sigma_y = \sigma'_y \times \iota_y$ 3.40
$b_{xy} = r_{xy} \frac{\sigma_x}{\sigma_y} = 0\ 857$	$b_{yx} = r_{xy} \frac{\sigma_y}{\sigma_x}$ 0 279
$\chi^2 = (N-k_y) \frac{\eta^2_{xy} - r^2}{1 - \eta^2_{xy}}$	$\chi^2 = (N-k_x)$
$= (29) \frac{0.0408}{0.7191} = 1.65$	$= (24) \frac{0\ 1080}{0\ 6581} = 3.93$

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CORRELATION CHART "C"

Table XII. - Upper Skill Group - Computation of the Mean and Standard Deviation

Scores	f	y	fy	fy^2
56-57	1	8	8	64
54-55	0	7	0	0
52-53	0	6	0	0
50-51	1	5	5	25
48-49	2	4	8	32
46-47	3	3	9	27
44-45	1	2	2	4
42-43	4	1	4	4
40-41	5	0	0	0
38-39	10	-1	-10	10
36-37	2	-2	-4	8
34-35	1	-3	-3	9
	$N = 36$		$\Sigma fy = 19$	$\Sigma fy^2 = 183$

$$M = \frac{\Sigma Y}{N} = 40.43 \qquad \sigma = i.\sqrt{\frac{\Sigma fy^2}{N} - c^2}; \quad c = \frac{\Sigma fy}{N} = 0.5;$$

$$\sigma = 2.\sqrt{\frac{183}{36} - 0.25} = 2.\sqrt{4.83} = \underline{4.80}$$

as computed in the attached Correlation Chart D.

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

Sujets (Subjects) UPPER SKILL GROUP N - 36
Date JUNE 7, 1956 Per T. POŹNIAK

Y \ X	27 29	30 32	33 35	36 38	39 41	42 44	45 47	48 50	51 53	54 56	57 59
56 57											/
54 55											
52 53											
50 51								/			
48 49						//					
46 47					//				/		
44 45						/					
42 43			//		/		/				
40 41			/	/	/	/	/				
38 39	/	//	/	/	//	///					
36 37			///	/	/	//	/				
34 35					/						

f_y	y'	fy'	fy'^2	$\Sigma x'$	$\Sigma x'y'$	$(\Sigma x')^2$	$\frac{(\Sigma x')^2}{f_y}$
	10						
	9						
	8						
	7						
	6						
1	5	5	25	4	20	16	16
0	4	0	0	0	0	0	~~0~~
0	3	0	0	0	0	0	~~0~~
1	2	2	4	1	2	1	1
2	1	2	2	-2	-2	4	2
3	0 Σ	9	0	-2	0	4	1.33
1	-1	-1	1	-1	1	1	1
4	-2	-8	16	-10	20	100	25
5	-3	-15	45	-10	30	100	20
10	-4	-40	160	-30	120	900	90
8	-5	-40	200	-19	95	361	45.12
1	-6	-6	36	-2	12	4	4
	-7						
	-8						
	-9						
	-10						
36 N	$\Sigma_{(-)}$ -110 -101		489	-71	298		205.45

$\Sigma fx'$ $\Sigma \frac{(\Sigma x')^2}{f_y}$

f_x				1	2	7	3	8	9	3	1	1	0	1					
x'	-9	-8	-7	-6	-5	-4	-3	-2	-1	0 $\Sigma_{(-)}$	1	2	3	4	5	6	7	8	9
fx'				-6	-10	-28	-9	-16	-9	-78	1	2	0	4					
fx'^2	81	64	49	36	50	112	27	32	9	0	1	4	0	16	25	36	49	64	81
$\Sigma y'$				-4	-8	-26	-12	-24	-24	-10	2	0	0	5					
$\Sigma x'y'$				24	40	104	36	48	24	0	2	0	0	20					
$(\Sigma y')^2$				16	64	676	144	576	576	100	4	0	0	25					
$(\Sigma y')^2/f_x$				16	32	96.57	48	72	64	33.33	4	0	~~0~~	25					

36 N
$\Sigma_{(+)}$ 7 -71
266 ← Σ
-101 ← $\Sigma fy'$
298 ← $\Sigma x'y'$
390.90 ← $\Sigma \frac{(\Sigma y')^2}{f_x}$

X	Y
Variable: CONCEPTS	Variable: SKILLS
X 46 k. 11 i_x = 3	46.5 12 i_y = 2

Corrections

$c'_x = \frac{\Sigma fx'}{N} = \frac{-71}{36}$ -1.97 $c'^2_x =$ 3.88 | $c'_y = \frac{\Sigma fy'}{N} = \frac{-101}{36}$ -2.80 $c'^2_y =$ 7.84

σ'

$\sigma'_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$ $= \frac{266}{36}$ 3.88 = 1.87 | $\sigma'_y = \sqrt{\frac{\Sigma fy'^2}{N} - c'^2_y}$ $= \sqrt{\frac{489}{36} - 7.87}$ 2.40

Correlation : r_{xy}

$r_{xy} = \frac{\frac{\Sigma x'y'}{N} - c'_x c'_y}{\sigma'_x \sigma'_y} = \frac{\frac{298}{36} - (-1.97)(-2.80)}{(1.87)(2.40)}$ $\boxed{0.615}$

$\sigma_{r_{xy}} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0.3782}{\sqrt{35}}$ 0.103

Rapport de Corrélation η Correlation Ratio

$\sigma'_{\bar{x}_c} = \sqrt{\frac{\Sigma \frac{(\Sigma x')^2}{f_y}}{N} - c'^2_x}$ $\frac{205.45}{36} - 3.88$ 1.34 | $\sigma'_{\bar{y}_c} = \sqrt{\frac{\Sigma \frac{(\Sigma y')^2}{f_x}}{N} - c'^2_y}$ $= \frac{390.90}{36} - 7.84$ 1.73

$\eta_{xy} = \frac{\sigma'_{\bar{x}_c}}{\sigma'_x} = \frac{1.34}{1.87}$ 0.716 | $\eta_{yx} = \frac{\sigma'_{\bar{y}_c}}{\sigma'_y} = \frac{1.73}{2.40}$ 0.720

$\sigma_{\eta_{xy}} = \frac{1 - \eta^2_{xy}}{\sqrt{N-1}}$ $\frac{0.4874}{5.91}$ 0.082 | $\sigma_{\eta_{yx}} =$ $\frac{0.4816}{5.91}$ 0.081

Paramètres St

TRUE 40.25 | TRUE 40.43

$M_x = X_0 + c'_x i_x$ = 40.09 | $M_y = Y_0 + c'_y i_y$ = 40.90

$\sigma_x = \sigma'_x \times i_x$ = 5.61 | $\sigma_y = \sigma'_y \times i_y$ = $\boxed{4.80}$

$b_{xy} = r_{xy} \frac{\sigma_x}{\sigma_y}$ = 0.67 | $b_{yx} = r_{xy} \frac{\sigma_y}{\sigma_x}$ = 0.52

$\chi^2 = (N - k_y) \frac{\eta^2 - r^2}{1 - \eta^2}$ $= (24) \frac{0.134}{0.487} = \boxed{13.57}$ | $\chi^2 = (N - k_x) \frac{\eta^2 - r^2}{1 - \eta^2}$ $= (25) \frac{0.140}{0.481} = \boxed{7.28}$

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CORRELATION CHART "D"

APPENDIX 9

Table XIII. - Middle Skill Group - Computation of the Mean and Standard Deviation

6 Scores	Mdpt.	y	y^2	f	fy^2
34-35	34.5	2.42	5.8	12	69.60
32-33	32.5	0.42	0.2	10	2.00
30-31	30.5	-1.58	2.5	11	27.50
28-29	28.5	-3.58	12.8	3	38.40
				N = 36	$\Sigma fy^2 = 137.50$

$$M = \frac{\Sigma Y}{N} = 32.08; \qquad \sigma \doteq \sqrt{\frac{\Sigma fy^2}{N}} = \frac{11.72}{6} = 1.95 \equiv 1.94$$

as computed in the attached Correlation Chart E.

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

MIDDLE SKILL GROUP N-36
te JUNE 9,1956 T. POZ'NIAK

Y \ X	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
					24 25	26 27	28 29	30 31	32 33	34 35	36 37	38 39	40 41	42 43	44 45				
34 35							/	/		////	//		/	/	//				
32 33					/		/	/	/	/	/	//	//						
30 31						/		//		///	////	/							
28 29									/	//									
f_x					1	1	2	4	2	10	7	3	3	1	2				
x'	9	-8	-7	-6	-5	-4	-3	-2	-1	0 Σ		2	3	4	5	6	7	8	9
fx'					-5	-4	-6	-8	-2	-25	7	6	9	4	10				
fx'^2	81	64	49	36	25	16	18	16	2	0	7	12	27	16	25 50	36	49	64	81
Σy					0	-1	1	-1	-2	-3	-2	-1	1	1	2				
$\Sigma x'y'$					0	4	-3	2	2	0	-2	-2	3	4	10				
$(\Sigma y')^2$					0	1	1	1	4	9	4	1	1	1	4				
$(\Sigma y')^2/f_x$					0	1	0.5	0.25	2	0.9	0.57	0.33	0.33	1	2				

f_y	y'	f_y	$f_{y'}^2$	$\Sigma x'$	$\Sigma x'y'$	$(\Sigma x')^2$	$(\Sigma x')^2/f_y$
	10						
	9						
	8		64				
	7		49				
	6		36				
	5		25				
	4		16				
	3						
	2		4				
12	1	12	12	14	14	196	16.33
10	0	Σ(+) 12	0	0	0	0	0
11	-1	-11	11	-2	2	4	0.36
3	-2	-6	12	-1	2	1	0.33
	-3		9				
	-4		16				
	-5		25				
	-6		36				
	-7		49				
	8		64				
	-9		81				
	-10						
36		-17 -5	35	11	18		17.02

$\Sigma fx'$ $\Sigma xy'$ $\Sigma \frac{(\Sigma x')^2}{f_y}$

36 | 11

189 ← $\Sigma fx'^2$

-5 ← $\Sigma fy'$

18 ← $\Sigma xy'$

888 ← $\Sigma \frac{(\Sigma y')^2}{f_x}$

	X	Y
Variable		Va ble
	X_0=34.5 k_x= 11 i_x = $\boxed{2}$	Y_0 32.5 4 =2

Corrections

$c'_x = \frac{\Sigma fx'}{N} = \frac{11}{36} = 0.305$ | $c'_y = \frac{\Sigma fy'}{N}$ $\frac{-5}{36}$ = -0.136

$c'^2_x =$ 0.093 | $c'^2_y =$ 0 018

σ'

$\sigma'_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$ | $\sigma'_y = \sqrt{\frac{\Sigma}{N}\,{}^{'2}\; c'^2_y}$

$= \sqrt{\frac{189}{36} - 0.09}$ 2.28 $\sqrt{\frac{35}{36}\; 0.0003}$ 0 97

Correlation : r_{xy}

$r_{xy} = \frac{\frac{\Sigma xy'}{N} - c'_x c'_y}{\sigma'_x \sigma'_y} = \frac{\frac{18}{36} + 0.305 \times 0.136}{2.28 \times 0\ 97} = \boxed{0\ 245}$

$\sigma_{r_{xy}} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0\ 0600}{\sqrt{35}} = 0.150$

Rapport de Correlation η Correlation Ratio

$\sigma_{x_r} = \sqrt{\frac{\Sigma \frac{(\Sigma x')^2}{f_y}}{N} - c'^2_x}$ | $\sigma_{\bar{y}_c} = \sqrt{\frac{\Sigma \frac{(\Sigma y')^2}{f_x}}{N} - c'^2_y}$

$= \frac{17.02}{36}\; 0.095$ 0.615 $\sqrt{\frac{8\,68}{36}\; 0.018}$ 0.477

$\eta_{xy} = \frac{0\ 615}{2\ 28}$ 0.26 $\eta_{yx} = \frac{\sigma_{\bar{y}_c}}{\sigma_y}\; \frac{0.477}{0.97}$ 0.48

$\sigma_{\eta_{xy}} = \frac{1 - \eta^2_{xy}}{\sqrt{N-1}} = \frac{0.932}{5.91}$ 0 517 $\frac{0.772}{5.91}$ 0 130

Paramē TRUE 35.22 s TRUE 32.08

$M_x = X_0 + c'_x i_x$ = 35 11 | $M_y = Y_0 + c'_y i_y$ = 32 22

$\sigma_x = \sigma'_x \times i_x$ = 4.56 | $\sigma_y = \sigma_y \times i_y$ - 1.94

$b_{xy} = r_{xy}\frac{\sigma_x}{\sigma_y}$ = 0.57 | $b_{yx} = r_{xy}\frac{\sigma_y}{\sigma_x}$ - 0.10

$\chi^2 = (N - k_y)\frac{\eta^2_{xy} - r^2}{1 - \eta^2_{xy}}$ | $\chi^2 = (N - k_x)\; -r^2$

$=(32\ \frac{0\ 007}{0\ 932}$ 0.22 | $=(25\ \frac{0.062}{0.772}$ 2.01

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CORRELATION CHART "E"

APPENDIX 9

Table XIV. - Lower Skill Group - Computation of the Mean and Standard Deviation

Scores	Mdpt.	y	y^2	f	fy^2
28-29	28.5	4.53	20.5	5	102.50
26-27	26.5	2.53	6.4	10	64.00
24-25	24.5	0.53	0.3	6	1.80
22-23	22.5	-1.92	3.6	7	25.20
20-21	20.5	-3.47	12.0	6	72.00
18-19	18.5	-5.47	29.9	1	29.90
16-17	16.5	-7.47	55.8	0	0.00
14-15	14.5	-9.47	89.6	2	179.20
				$N = 37$	$\Sigma fy^2 =$ 474.60

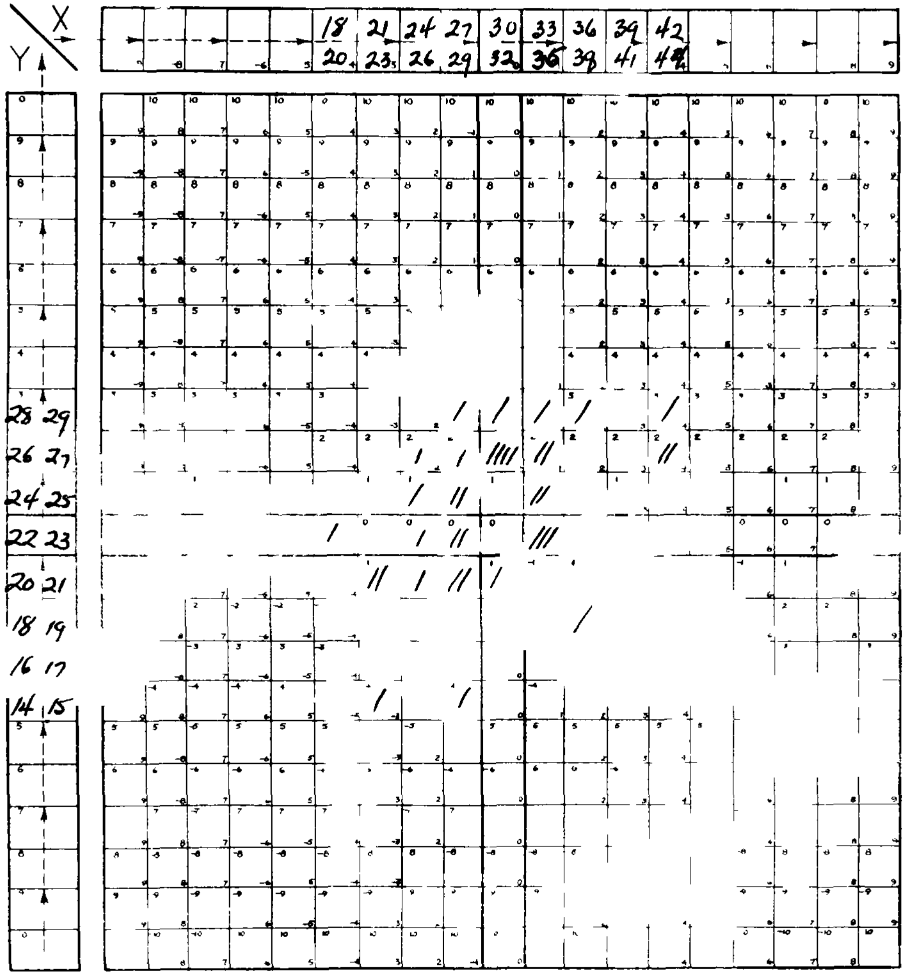
$$M = \frac{\Sigma Y}{N} = 23.97$$

$$\sigma = \sqrt{\frac{\Sigma fy^2}{N}} = \frac{21.78}{\sqrt{37}} = 3.63$$

$\equiv$ <u>3.5,</u> as computed in the attached Correlation Chart F.

LA FICHE DE CORRELATION DAYHAW — THE DAYHAW CORRELATION CHART

Sujets (Subjects) LOWER SKILL GROUP N = 37
Date JUNE 9, 1956 Per T. POŹNIAK



f_y	y	fy'	fy'^2	$\Sigma x'$	$\Sigma xy'$	$(\Sigma x')^2$	$\frac{(\Sigma x')^2}{f_y}$
	10						
	9		81				
	8		64				
	7		49				
	6		36				
	5		25				
	4		16				
5	3	15	45	6	18	36	7.20
10	2	20	40	7	14	49	4.90
6	1	6	6	-2	-2	4	0.66
7	0	Σ_+ 41		-5	0	25	3.57
6	-1	-6	6	-10	10	100	16.66
1	-2	-2	4	2	-4	4	4.00
0	-3	0	0	0	0	0	0
2	-4	-8	32	-4	16	16	8.00
	-5		25				
	-6		36				
	-7		49				
	-8		64				
	-9		81				
	-10						

f_x						1	3	4	9	7	8	2	0	3					
x'	-9	-8	-7	-6	-5	-4	-3	-2	-1	0		2	3	4	5	6	7	8	9
fx'						-4	-9	-8	-9	$\Sigma_{(-)}$ -30	8	4	0	12					
fx'^2	81	64	49	36		16	27	16	9	0	8	8	0	48	25	36	49	64	81
$\Sigma y'$						0	-6	2	1	11	9	1	0	7					
$\Sigma x'y'$						0	18	-4	-1	0	9	2	0	28					
$(\Sigma y')^2$						0	36	4	1	121	81	1	0	49					
$(\Sigma y')^2/f_x$						0	12	1	0.11	17.28	10.12	0.50	0	16.33					

37 N
$\Sigma_{(-)}$ -16
25
133 ← $\Sigma fy'^2$
-6 ← $\Sigma fx'$
52 ← $\Sigma xy'$
4499 ← $\Sigma\frac{(\Sigma x')^2}{f_y}$
Σ_+ 24 | -6
132 ← $\Sigma fx'^2$
25 ← $\Sigma fy'$
52 ← $\Sigma xy'$
5734 ← $\Sigma\frac{(\Sigma y')^2}{f_x}$

X	Y
Variable CONCEPTS	ble SKILLS

= 31 k_x = 9 i_x = 3 | 22.5 8 | i_y = 2

Corrections

$c'_x = \frac{\Sigma fx'}{N}$ $\frac{-6}{37}$ -0.162 | $c_y = \frac{\Sigma}{N}$ $\frac{25}{37}$ 0.675

$c'^2_x =$ 0.026 | 0.455

σ'

$\sigma'_x = \sqrt{\frac{\Sigma fx'^2}{N} - c'^2_x}$ | $\sigma'_y = \sqrt{\frac{\Sigma fy'^2}{N} - c'^2_y}$

$= \sqrt{\frac{132}{37}}$ 0.026 = 1.88 | $= \sqrt{\frac{133}{37}}$ 0.455 1.77

Correlation . r_{xy}

$$r_{xy} = \frac{\frac{\Sigma xy'}{N} - c_x c_y}{\sigma_x \sigma_y} = \frac{\frac{52}{37} + 0.162 \times 0.675}{1.88 \times 1.77} \quad \boxed{0.45}$$

$$\sigma_{r_{xy}} = \frac{1 - r^2_{xy}}{\sqrt{N-1}} = \frac{1 - 0.2025}{6} = 0.132$$

Rapport de Corrélation η Correlation Ratio

$\sigma'_{\bar{x}r} = \sqrt{\frac{\Sigma\frac{(\Sigma x')^2}{f_y}}{N} - c'^2_x}$ | $\sigma'_{\bar{y}c} = \sqrt{\frac{\Sigma\frac{(\Sigma y')^2}{f_x}}{N} - c'^2_y}$

$= \sqrt{\frac{4499}{37}}$ 0.026 1.09 | $\frac{57.34}{37}$ 0.455 1.04

$\eta_{xy} = \frac{\sigma'_{\bar{x}r}}{\sigma'_x} = \frac{1.09}{1.88}$ 0.57 | $= \frac{1.04}{1.77}$ 0.58

$= \frac{1-\eta^2_{xy}}{\sqrt{N}}$ $\frac{0.6751}{6}$ 0.112 | $\sigma_{\eta_{yx}} = \frac{1-\eta^2_{yx}}{\sqrt{N-1}}$ $\frac{0.6636}{6}$ 0.110

Statistics

TRUE 30.3 | TRUE 23.97

$M_x = X_0 + c'_x i_x$ = $\boxed{30.5}$ | $M_y = Y_0 + c'_y i_y$ 23.8

$\sigma_x = \sigma_x \times i_x$ = $\boxed{5.4}$ | $\sigma_y = \sigma_y \times i_y$ 3.5

$b_{xy} = r_{xy}\frac{\sigma_x}{\sigma_y}$ = $\boxed{0.69}$ | $b_{yx} = r_{xy}\frac{\sigma_y}{\sigma_x}$ 0.29

$\chi^2 = (N - k_y)$ $-r^2$ | $\chi^2 = (N - k_x)\frac{\eta^2_{yx} - r^2}{1 - \eta^2}$

$= (29)\frac{0.1224}{0.6751} = \boxed{5.25}$ | $= (28)\frac{0.1339}{0.6636}$ 5.66

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CORRELATION CHART "F"

Table XV. - Concept Scores of the Upper, Middle, and Lower Skill Groups

Concept Scores of the Upper Skill Group			Concept Scores of the Middle Skill Group			Concept Scores of the Lower Skill Group		
In order of size		Grouped	In order of size		Grouped	In order of size		Grouped
59	38	57-59	45	34	44-45	44	28	42-44
51	37	54-56	45	34	42-43	43	28	39-41
48	37	51-53	42	34	40-41	43	28	36-38
47	35	48-50	41	33	38-39	38	27	33-35
46	35	45-47	41	32	36-37	36	27	30-32
45	35	42-44	40	31	34-35	35	27	27-29
44	35	39-41	39	31	32-33	35	27	24-26
43	35	36-38	39	30	30-31	34	26	21-23
43	34	33-35	38	30	28-29	34	25	18-20
43	34	30-32	37	28	26-27	33	25	
43	32	27-29	37	28	24-25	33	25	
43	31		37	26		33	23	
42	27	i = 3	37	25	1 = 2	33	22	1 = 3
42			37			31	21	
42	N=36		36	N =36		31	18	
41			36			31		
41	$M_c = \frac{\Sigma X}{N}$ = 40.25		35	$M_c = \frac{\Sigma X}{N}$ = 35.22		31	N=37	
41			35			30	$M_c = \frac{\Sigma X}{N}$ = 30.3	
41			35			30		
40	$\sigma = \sigma_x$ = 5.61 as		35	$\sigma = \sigma_x$ = 4.56 as		30	$\sigma = \sigma_x$ = 5.4, as	
40			35			29		
40	computed in the		35	computed in the		29	computed in the	
39	attached Correlation		35	attached Correlation			attached Correla-	
	Chart D.			Chart E.			tion Chart F.	

Table XVI.- Skill Scores of the Upper, Middle, and Lower Concept Groups.

Skill Scores of the Upper Concept Group			Skill Scores of the Middle Concept Group			Skill Scores of the Lower Concept Group		
In order of size		Grouped	In order of size		Grouped	In order of size		Grouped
57	35	57-59	43	29	42-43	39	24	38-39
51	34	54-56	42	29	40-41	38	24	36-37
49	34	51-53	41	29	38-39	38	24	34-35
49	34	48-50	41	28	36-37	35	23	32-33
47	33	45-47	39	27	34-35	35	23	30-31
47	32	42-44	37	27	32-33	33	22	28-29
46	32	39-41	37	25	30-31	32	22	26-27
44	32	36-38	36	25	28-29	32	21	24-25
42	30	33-35	36	23	26-27	32	21	22-23
42	29	30-32	35	23	24-25	31	21	20-21
41	27	27-29	35	22	22-23	30	21	18-19
41	27	24-26	34			30	21	16-17
40	18	21-23	34			29	20	14-15
39		18-20	34			29	15	
39			34			28	14	
38		1 = 3	34		1 = 2	27		1 = 2
38			33			27		
38			32			27		
38			31			26		
37			31			26		
37			30			26		
37			30			25		
37			30					
			30					
			30					
	N = 36			N = 36			N = 37	
	$M_s = \frac{\Sigma X}{N} = 38.08$			$M_s = \frac{\Sigma X}{N} = 32.11$			$M_s = \frac{\Sigma X}{N} = 26.78$	
	$\sigma = \sigma_x = 6.63$ as computed in the attached Correlation Chart A			$\sigma = \sigma_x = 5.36$ as computed in the attached Correlation Chart B			$\sigma = \sigma_x = 5.96$, as computed in the attached Correlation Chart C	

APPENDIX 10

COMPUTING THE SIGNIFICANCE OF THE MEANS IN THE HORIZONTAL COMPARISON

The performance of each of the three groups, upper, middle, and lower, on one test is to be compared to its performance on the other test. In particular, the significance of difference of the means on each of such two performances is to be determined. This requires computation of the value of the ratio $\frac{D}{\sigma_D}$, where D is the difference of the means, and σ_D is the standard error of that difference equal to $\sqrt{\sigma_{M_s}^2 + \sigma_{M_c}^2 - 2r_{sc}\,\sigma_{M_s}\,\sigma_{M_c}}$; σ_{M_s} σ_{M_c} are the standard errors of the corresponding means on skills or concepts, equal to $\frac{\sigma}{\sqrt{N}}$ (sigma σ being the standard deviation of the distribution, and N its number of cases in a group.)

Thus, in each case, the same computation is performed as follows:

1. Middle Skill Group
(Its Performances on Skill-Test and on the Concept-Test)

Mean on Skills $M_s = 32.08$ (as computed in Table XVI)

Mean on Concepts $M_c = 35.22$ (as computed in Table XV)

$D = 3.14$

Standard deviation of the Skill distribution $\sigma_s = 1.94$ (as computed in Correlation Chart E

Standard deviation of the Concept distribution $\sigma_c = 4.56$ (as computed in Correlation Chart E

Standard errors of the means:

$$\sigma_{M_s} = \frac{\sigma_s}{\sqrt{N}} = \frac{1.94}{6} = 0.32$$

$$\sigma_{M_c} = \frac{\sigma_c}{\sqrt{N}} = \frac{4.56}{6} = 0.76$$

$r_{sc} = 0.245$ (as computed in Correlation Chart E)

$$\sigma_D = \sqrt{\sigma_{M_c}^2 + \sigma_{M_s}^2 - 2r_{sc}\,\sigma_{M_c}\,\sigma_{M_s}}$$

$$= \sqrt{0.76^2 + 0.32^2 - 0.49\times0.76\times0.32}$$

$$= \sqrt{0.5776+0.1024 - 0.119}$$

$$= 0.74$$

$$\frac{D}{\sigma_D} = \frac{3.14}{0.74} = 4.24 > 3$$

2. Lower Skill Group
(It's Performance on the Skill Test and on the Concept Test)

$M_s = 23.97$ (as computed in Table XIV)

$M_c = 30.30$ (as computed in Table XV)

$D = 6.33$

$\sigma_s = 3.5$ (as computed in Correlation Chart F)

$\sigma_c = 5.4$ (as computed in Correlation Chart F)

$$\sigma_{M_s} = \frac{\sigma_s}{\sqrt{N}} = \frac{3.5}{\sqrt{37}} = 0.57$$

$$\sigma_{M_c} = \frac{\sigma_c}{\sqrt{N}} = \frac{5.4}{\sqrt{37}} = 0.88$$

$r_{sc} = 0.45$ (as computed in Correlation Chart F)

$$\sigma_D = \sqrt{0.57^2+0.88^2 - 0.90\times0.57\times0.88} = 0.804$$

$$\frac{D}{\sigma_D} = \frac{6.33}{0.804} = 7.87 > 3$$

3. Upper Concept Group
(Its Performance on the Concept Test and on the Skill Test)

M_c = 40.41 (as computed in Table IX)

M_s = 38.08 (as computed in Table XVI)

$D = 2.33$

σ_c = 4.08 (as computed in Correlation Chart A)

$\sigma_s = 6.63$

$$\sigma_{M_c} = \frac{\sigma_c}{\sqrt{N}} = \frac{4.08}{6} = 0.68$$

$$\sigma_{M_s} = \frac{\sigma_s}{\sqrt{N}} = \frac{6.63}{6} = 1.105$$

r_{cs} = 0.65 (as computed in Correlation Chart A)

$$\sigma_D = \sqrt{1.105^2 + 0.68^2 - 1.30 \times 1.10 \times 0.68}$$

$$= \sqrt{1.67 - 0.97}$$

$$= \sqrt{0.70} = 0.836$$

$$\frac{D}{\sigma_D} = \frac{2.33}{0.83} = 2.78 < 3$$

4. Middle Concept Group
(Its Performance on the Concept-Test and on the Skill-Test)

M_c = 35.08 (as computed in Table X)

M_s = 32.11 (as computed in Table XVI)

$D = 2.97$

σ_c = 1.23 (as computed in Correlation Chart B)

σ_s = 5.36 (as computed in Correlation Chart B)

$$\sigma_{M_c} = \frac{\sigma_c}{\sqrt{N}} = \frac{1.23}{6} = 0.205$$

$$\sigma_{M_s} = \frac{\sigma_s}{\sqrt{N}} = \frac{5.36}{6} = 0.88$$

r_{cs} = 0.39 (as computed in Correlation Chart B)

$$\sigma_D = \sqrt{0.205^2 + 0.88^2 - 2 \times 0.39 \times 0.205 \times 0.88}$$

$$= \sqrt{0.81 - 0.14}$$

$$= 0.82$$

$$\frac{D}{\sigma_D} = \frac{2.97}{0.82} = 3.62 > 3$$

5. Lower Concept Group
(Its performance on the Concept-Test and on the Skill-Test)

M_c = 28.08 (as computed in Table XI)

M_s = 26.78 (as computed in Table XVI)

D = 1.30

σ_c = 3.40 (as computed in Correlation Chart C)

σ_s = 5.96 (as computed in Correlation Chart C)

$$\sigma_{M_c} = \frac{\sigma_c}{\sqrt{N}} = \frac{3.40}{\sqrt{37}} = 0.55$$

$$\sigma_{M_s} = \frac{\sigma_s}{\sqrt{N}} = \frac{5.96}{\sqrt{37}} = 0.98$$

r_{cs} = 0.49 (as computed in Correlation Chart C)

$$\sigma_D = \sqrt{0.55^2 + 0.98^2 - 0.98 \times 0.56 \times 0.98}$$

$$= \sqrt{0.7347}$$

$$= 0.85$$

$$\frac{D}{\sigma_D} = \frac{1.30}{0.85} = 1.52 < 2$$

APPENDIX 11

ABSTRACT OF

Broadening the Mathematical Aspect of the Concept-Skill Correlation at One Grade Algebra Level.[1]

The aim of this study was to broaden the mathematical aspect of the correlation between the mastery of basic algebraic concepts and computational skills at one grade High School Algebra level, as determined by the coefficient of correlation.

A representative algebraic material was selected at Grade XI level, and a homogenous group, representative of the Grade XI pupils of the Ottawa English-speaking Catholic High School population, was chosen for the investigation of the problem.

An Algebraic Concept Test and an Algebraic Skill Test, were constructed, tried out, and administered to a representative group of 109 pupils, to obtain the data for exploration of the problem.

A two-way procedure was applied to the investigation:

1. taking the whole tests and determining and interpreting the coefficient of correlation of data obtained, as to the closeness of the relationship and the difficulty of the tests;

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[1] Ph.D. Thesis presented by Tadeusz Poźniak, in 1956, to the School of Psychology and Education of the University of Ottawa, xii-177 pages.

2. taking three groups in the tests, upper, middle and lower, and comparing them as to the closeness of relationship, and as to tests difficulty within each test, and between the two tests.

The coefficient of correlation was found to be indicative of a positive high correlation, and a direct marked relationship.

The Algebraic Skill-Test was found to be significantly harder to the group than the Algebraic Concept-Test.

The vertical comparison, that is, within each test, of the upper, middle and lower groups, showed in both tests that these three groups differ between each other significantly in test difficulty. The Concept-Test and the Skill-Test were found significantly harder for their middle groups than for their upper groups, and also significantly harder for their lower than for their middle groups.

The horizontal comparison, that is, between the two tests, of the two respective upper groups, and likewise of the two middle and lower groups, produced exact traits of the interplay of the closeness of correlation and the tests difficulty aspects:

Direction of the correlation was found positive in all cases.

Moderate correlation and substantial relationship were found at all three, upper, middle and lower, group levels.

They are nearly equal at each level, but they are not equal between the levels.

The middle groups in both tests find the Skill-Test significantly harder than the Concept-Test. The upper and lower groups in both tests find the difficulty in other ways.

It was concluded that the mathematical aspect of the concept-skill correlation, as furnished by the coefficient of correlation method, produces two significant characteristics of the correlation:

1. The same direction;
2. A considerable closeness.

The mathematical aspect of the concept-skill correlation as furnished by the three-group division method, does the same, but, in addition, it shows the inner interplay of the closeness and difficulty aspects, and furthermore, it furnishes a mathematically expressible trait of regularity by which that difficulty interplay is marked.

It was concluded further that the exploration of the concept-skill correlation in algebra, along other grades of high school would be possible by application of two specially constructed and standardized tests: one for measuring the understanding of basic algebraic concepts, the other for measuring the algebraic computational skill. Reducing the computations in the Concept-Test to the minimum would be by all means advisable.

It was finally concluded that a full generalization of the findings could be achieved by application of both methods to a stratified truly representative sample of the Grade XI High School population on a province-wide basis.