MULTIVARIATE STATISTICAL MODELLING OF GROUNDWATER LEVEL FLUCTUATIONS

by

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A thesis
presented to the University of Ottawa
in partial fulfillment of the
requirements for the degree of
Master of Applied Sciences
in
Civil Engineering

OTTAWA, Ontario, 1984

ABSTRACT

Starting with a mass balance equation of water fluxes, a linear groundwater level model is formulated relating the present groundwater storage to the past amount of storage, streamflow fluctuations, evapotranspiration losses, as well as to snowmelt and precipitation inputs. By modelling the streamflow influence, snowmelt, and evapotranspiration, a model relating the present amount of groundwater storage to the past amount of storage, precipitation, temperature, and hours of bright sunshine is formulated. Given time series of the dependent and independent variables, the parameters of the linear model were estimated by various techniques. These included: ordinary least squares, which is known to provide biased estimates for non-gaussian white noise; three variations of weighted least squares, two of generalized least squares, as well as the extended Kalman filter.

Since six years of data at five day intervals were available, the first five years were used for parameter estimation while the sixth year of data was used for comparison. Annual series as well as the full five years of data were used to estimate the parameters by the methods previously mentioned. Analysis of variance was performed and standard errors of the estimates were computed to quantify the re-

sults. The parameters obtained through the annual series were found to be unreliable, whereas those from five years of data appeared physically reasonable. The results indicated that the least squares methods were quite similar, but that the extended Kalman filter provided better prediction of the sixth year, especially during the large evapotranspiration periods which the least squares methods had difficulty predicting.

ACKNOWLEDGEMENTS

The author wishes to gratefully acknowledge the support, advice, and encouragement of Dr. Kazimierz Adamowski throughout the study. I am also indebted to Dr. Nicholas R. Dalezios for his advice and encouragement throughout the study.

Furthermore, I wish to thank T.E. Dabbous, graduate student at the electrical engineering department, for his comments and answers to my questions on the Kalman filter.

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SYMBOLS

B - fraction of precipitation reaching groundwater storage (ch. 3)

- parameter vector (ch. 4 & 5)

B1 to B7 - regression coefficients

BS(t) - bright sunshine at time t for evapotransp.

BSS(t) - bright sunshine at time t for snowmelt

C - proportion of groundwater storage lost to streamflow during the time increment

C(i) - terms in H matrix corresponding to Taylor series expansion

C(i,i) - matrix equal to the inverse of X' X

C.L. - confidence limits

Ct - temperature coefficient for Jensen Haise evapotranspiration model

d - constant

D - depth of water below datum

DF - degrees of freedom

DIF - sum of absolute differences

Dt - time interval

DW - Durbin-Watson statistic

e - constant

e(t) - residual at time t

ET - evapotranspiration in Jensen Haise model f - constant

F - state transition matrix

g - constant

G - daily total radiation in U.S. Army Corps of Engineers snowmelt model (ch. 3)

- control transition matrix (ch. 4 & 5)

h - constant

H - matrix relating all states to all outputs

i - constant

I - identity matrix

j - constant

J - matrix equal to the product of T and Y

K - matrix equal to the product of T and X

Kc - dimensionless crop constant for Jensen Haise evapotranspiration model

L - matrix equal to the product of T and U

Lv - latent heat of vaporization of water for Jensen Haise evapotranspiration model

m - number of parameters

M(i) - terms in F matrix corresponding to Taylor series expansion

n - number of data points

N - specific yield

p - density of water for Jensen Haise evapotransp. model (ch. 3)

- lag one autocorrelation coefficient (ch. 4 & 5)

P - estimation error covariance matrix

Pb - initial error covariance matrix of the parameters

q - constant

Q - system error covariance matrix

r - constant

R - solar radiation for Jensen Haise evapotransp. model (ch. 3)

- measurement error covaraince matrix (ch. 4 & 5)

R^2 - coefficient of determination

SDR - standard deviation of the residuals

S.E. - standard error

SM(t) - snowmelt at time t

t - time

$t(\alpha)$ - t distributed deviate with confidence

T - matrix whose product with its transpose equals the inverse of V (ch. 4)

T(t) - temperature at time t for evapotranspiration (ch. 3 & 5)

TM - daily maximum temperature in U.S. Army Corps of Engineers snowmelt model

TS(t) - temperature at time t for snowmelt

TSE - total square error

Tx - constant for Jensen Haise evapotranspiration model

u - deterministic input

ul(t) to up(t) - inputs at time t

U(t) - residuals at time t

v(t) - measurement equation noise

V - error covariance matrix

w(t) - system equation noise

W - weighting matrix

W(t) - groundwater storage above datum at time t

x - state vector

x1(t) to xn(t) - states at time t

X - measurement matrix

y - output vector

y1(t) to ym(t) - outputs at time t

Y - output vector

YA - average Y

Y(t) - observed groundwater level at time t

YP(t) - predicted groundwater level at time t

Z - instrumental variable

Z(t) - groundwater level above datum at time t

σ^2 - variance

Chapter I

INTRODUCTION

1.1 MOTIVATION

Agriculture is an activity critically dependent on groundwater levels. The large amount of water used to raise crops, even ignoring man's input in the form of irrigation, makes it possible to argue that groundwater is perhaps the most important source of water for mankind. With ninety-seven percent of the world's potable water available at any moment in time stored underground, groundwater can be an important source of water for domestic consumption, agriculture, and industry. It is estimated that thirty-nine percent of all water consumption in the United States comes from groundwater, and that ninety-six percent of rural homes are supplied by groundwater. The largest single demand on groundwater comes from irrigation, accounting for seventy-one percent of all groundwater used [60].

In Canada, surveys indicate that ten percent of municipal water systems are supplied by groundwater. In rural areas, however, its importance is considerably greater. Groundwater also provides large quantities of industrial and irrigation water. Although in most instances underground water withdrawals are small relative to surface water quantities, many communities as well as some industries are completely dependent on groundwater [14].

Aquifers, which are geologic formations that contain sufficiently saturated permeable material permitting water yields, store large quantities of water. These immmense reservoirs of underground water, however, are becoming troublesome due to their behavior. Water in the ground moves very slowly, rarely more than half a meter per day and very often merely tens of centimeters or less per year. This may on occasion lead to residence times of groundwater of a matter of centuries. The large residence time and the slow replacement of groundwater generates problems for numerous aquifers. Many are being exploited at a rate faster than nature can replenish them; that is, they are being mined. Others, because they have been contaminated by pollutants, will not yield potable water.

For water management purposes, especially as related to agriculture and drainage, accurate simulation of groundwater levels can be crucial. It would be highly advantageous to predict groundwater level fluctuations not only for agricultural areas, but also at construction and landfill sites for a multitude of reasons. This requires appropriate modelling to describe the physical interactions as well as reliable parameter estimates for simulation.

Groundwater, of course, is but one of the many stages of the hydrologic cycle, and its proper prediction depends on an accurate description of its relationships with other factors such as precipitation, evapotranspiration, snowmelt, and so on. Due to the complexity of the natural water cycle, there are limitations as to the data collection process. Evapotranspiration and snowmelt, for example, cannot be measured directly. The estimation of groundwater losses to streamflow are even more difficult to assess. For forecasting of the groundwater levels, therefore, these limitations require the formulation of a model which uses as input readily available data such as precipitation, hours of bright sunshine, and temperature.

In this study a model is developed which could be used to simulate, for water management purposes, the elevation of the water table in an aquifer. Since the variation in groundwater levels is a stochastic hydrologic process, meaning that some random element is involved, groundwater fluctuations cannot be predicted exactly. There will always exist an element of uncertainty. However, an understanding of the factors influencing the rise and fall of the water table are necessary in order to produce a model of groundwater level fluctuations.

1.2 GROUNDWATER LEVEL FLUCTUATIONS

Groundwater level fluctuations arise from a wide variety of sources, some of which are natural, while others are man-made (figure 1.1). Some of these variations are secular, meaning they extend over long time periods. Among the natural secular variations, only alternating series of wet and dry years produce long-term groundwater level fluctuations.

Groundwater levels typically increase and decrease in Canada with clear seasonal variations. There is recharge in spring from snowmelt and possibly bank storage due to flooding. This is usually followed by a gradual loss during the summer due to evapotranspiration. Frozen ground during winter can be quite a limiting factor for high groundwater levels in some areas. Diurnal variations occur due to evapotranspiration as well as atmospheric pressure changes and tides. Short-term variations can occur due to air entrapment, wind or earthquakes. Nevertheless, on a long term basis, precipitation is usually the primary recharge factor determining the groundwater table. However, in many watersheds, spring snowmelt as well as bank storage during flood periods can be equally important factors. Evapotranspiration and streamflow losses are usually the main reasons for any natural fall in the water level in an aquifer.

Natural	Man-induced
Groundwater recharge	External loading of aquifers
Air entrapment during groundwater recharge	Groundwater pumpage
Evapotranspiration	Deep-well injection
Bank storage effects near streams	Artificial recharge
Tidal effects near oceans	Agricultural irrigation and drainage
Atmospheric pressure effects	Geotechnical drainage
Earthquakes	

Modified from [22]

Figure 1.1 : MECHANISMS LEADING TO GROUNDWATER FLUCTUATIONS

Recharge of the aquifer by precipitation is dependent on various factors. Temperature and soil moisture determine how much water evaporates and runs off. Of the remaining rainfall, some is used up by plants before it reaches the groundwater table. If the groundwater level is quite low, much of the precipitation may be stored in the soil mass before it reaches the water table. The influence of man on groundwater levels is abundant and varied. The external loading of confined aquifers, groundwater pumpage, artificial recharge, irrigation and drainage, are but a few of man's activities which alter the groundwater table [22].

To forecast groundwater level fluctuations, different methods are available. These include various types of analog models, sand tank models, as well as mathematical models. This study develops a physically-based mathematical model from various time series of readily available data known to influence groundwater levels.

1.3 MATHEMATICAL MODELS IN HYDROLOGY

Mathematical models in hydrology can be either optimizing or non-optimizing. The former concern themselves with the selection of the best set of conditions to satisfy a set of objectives with a given set of constraints. The latter, which analyze particular sets of data, can be separated into statistical and physical/deterministic models. Statistical models can be further divided into stochastic, probabilistic, or regression-correlation models (figure 1.2)[20].

Non-optimizing		Optimizing
Parametric Hydrology		Systems Hydrology
Deterministic	Statistical	-Decision Theory
-Empirical	-Regression & Correlation	-Systems Analysis
-Conceptual	-Probabilistic	-Operations Research
	-Stochastic	

Modified from [20]

Figure 1.2 : MATHEMATICAL METHODS IN HYDROLOGY

The model proposed for this thesis is a statistical linear regression model, meaning that the output is a linear function of the input and that the parameters are determined through regression methods. A linear model means that if for input x1 the output is y1, and for input x2 the output is y2, then x1 plus x2 as input produces y1 plus y2 as output. It is also a physically-based model since it will incorporate within its structure some functions which explain the physical mechanisms of the hydrologic cycle.

The linear model is formulated as:

$$y = B1\ x1 + B2\ x2 + \ldots + BN\ xN + e \qquad (1.1)$$

where y is the output, B1 to BN are the regression coefficients or parameters, x1 to xN are the independent variables, and e is the error term or residual. The output is the groundwater storage, while the x's are the past groundwater storage, precipitation, bright sunshine and temperature for evapotranspiration, and bright sunshine for snowmelt; there is also a constant term.

Hydrological processes are known to be nonlinear, time-varying, and continuous. However, hydrologists have more often than not assumed time-invariance and linearity in their models in order to deal with an amenable system [12]. Therefore, a relatively simple linear regression model will be investigated in this study. Certainly, a nonlinear model could have been used, but it would have resulted in a model probably only slightly better but much more complex. What Naef [46] has reported about the small improvements provided by more complex rainfall-runoff models over simple models is very likely also applicable to groundwater level models.

Model building basically can be organized into four stages as is demonstrated by figure 1.3. The first is the selection of the type of model, which in this thesis is a statistical regression model. The second stage deals with the identification of the form of the model, which is a linear model where groundwater storages were dependent on past groundwater storage, precipitation, hours of bright sunshine, and temperature. The third step is parameter estimation, while a check of the adequacy of the model is the fourth and last step. The main objectives of this thesis are to develop a model (second stage) and to investigate various parameter estimation techniques (third stage).

1.4 PARAMETER ESTIMATION

Parameter estimation is the step in model building which results in concrete numerical results. It provides values for the regression coefficients of the model and if the parameter values are acceptable based on some criteria, then checks are performed to validate the model. Since a limited amount of data is usually available, and measurement errors

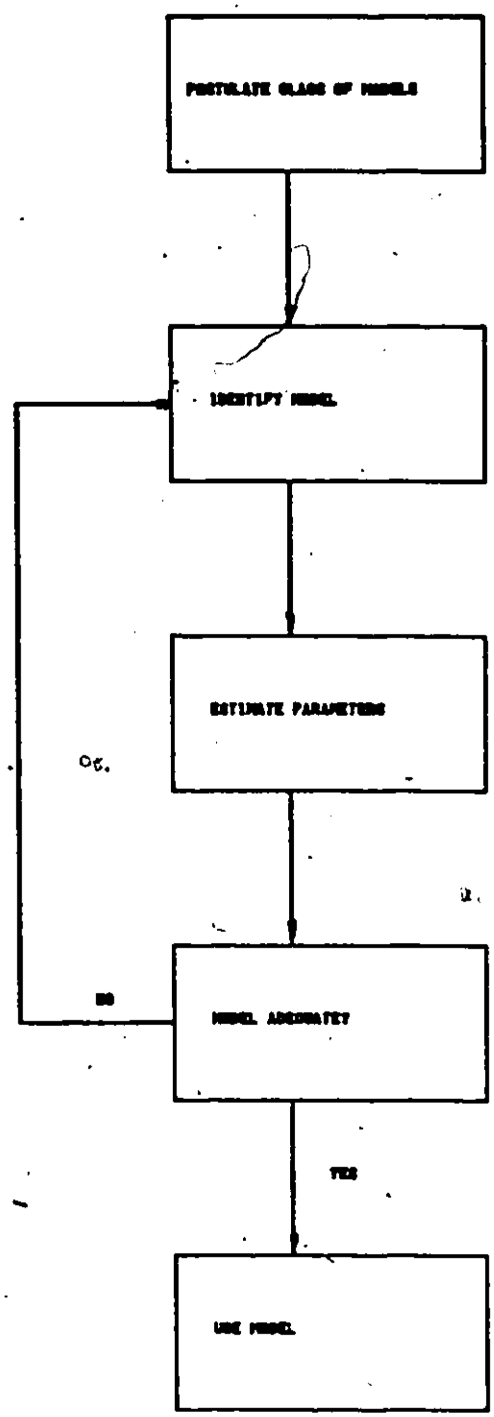


FIGURE 1.3: MODEL BUILDING STAGES

of some sort must be expected, the parameter values must be examined critically to assure that they provide realistic estimates of the quantitative behavior of the hydrologic system.

Prior to the last few decades, parameter estimation of linear regression models had mostly been through ordinary least squares. Because of its bias if the residuals are not gaussian white noise, new techniques such as generalised least squares as well as optimal estimation techniques such as the extended Kalman Filter have been investigated more recently.

The purpose of this study is to examine the various parameter estimation methods as applied to a linear groundwater level model. These parameters are used to simulate groundwater levels which are compared to the observed values. From such an analysis, a better understanding of the physical processes occuring in the natural system should be realized.

Use can be made of both on-line and off-line methods; the former generate parameter values at each time interval, while the latter merely provide a final estimate valid for the whole data. These methods result in a better understanding of the parameter variations, for clearly these depend on factors other than the input variables such as total snowfall and whether it was a wet or dry summer.

1.5 OBJECTIVES

The objectives of the research are:

(1) Starting with a mass balance equation of water fluxes, a linear groundwater level model including evapotranspiration and snowmelt will be formulated which will use as input readily available data such as hours of bright sunshine, precipitation, and temperature. The approach taken will be to begin with an already existing linear groundwater storage model and develop a seasonal evapotranspiration as well as a snowmelt portion to produce a more complex linear model. From the correlations between the groundwater levels with rainfall and snowmelt at various time lags, a final version of the model will be determined.

(2) A comparison of parameter estimation techniques for the proposed linear model shall be made in order to determine which are more appropriate in providing reliable estimates. Since ordinary least squares is known to give biased estimates for non-gaussian white noise residuals, alternative methods will be examined. Three variations of weighted least squares, two of generalised least squares, and the extended Kalman filter will be investigated and compared.

Once ordinary least squares has been applied to five years of data, the residuals shall indicate that less weights should be assigned to the measurements during certain periods of the year, during snowmelt periods, or during the evapotranspiration periods. Then, lag one correlation shall be assumed among the residuals to apply generalised least squares. Lag one correlation among the residuals at certain time periods shall also be assumed for the second variation of generalised least squares. Analysis of variance and the calculation of the standard errors will quantify the reliability of the estimates. The extended Kalman filter will also be employed and the variation of the parameters will suggest that the linear models for snowmelt and evapotranspiration were inadequate. Finally, the methods will be compared in terms of prediction of a sixth year of record and the results discussed.

Chapter II

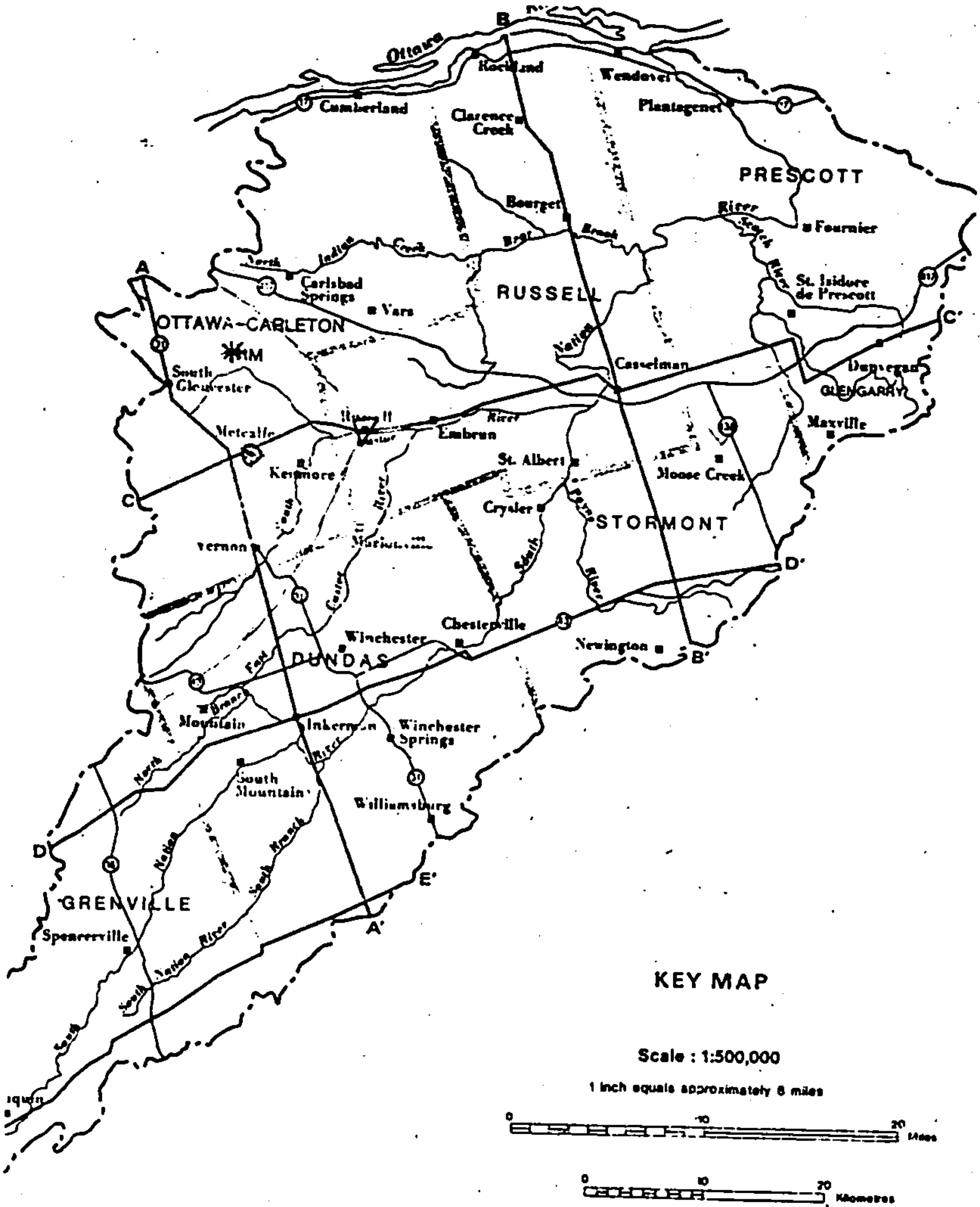
RESEARCH DATA

2.1 WATERSHED DESCRIPTION

To properly understand the model development as well as its assumptions, explanations must be provided on the research data and on the watershed itself. The prevailing conditions at the collection site of the data, where evapotranspiration and snowmelt are important factors, influenced the model formulation.

Part of the data used for this study came from the Castor River watershed, which lies east-southeast of Ottawa with approximately 782 square kilometers of total surface area drained by the Castor at the confluence of the South Nation River. The groundwater level observations were collected at Metcalf, Ontario (75 29 W - 45 13 N), which is a recharge area (figure 2.1).

The watershed is flat with a low relief comprised of numerous bogs and swamps, the nearest one to the Metcalf station being about twelve kilometers away. Limestone underlays much of the basin while the overburden consists predominantly of marine sediments considered poorly permeable [4].



▽ river gaging station

⊙ well

✱ profile site

watershed

taken from Chin (4)

FIGURE 2.1: CASTOR RIVER WATERSHED IN ONTARIO

The soil profile at a site about eleven kilometers from the Metcalf well is available. Above the bedrock of limestone and dolomite is a clayey soil of about twenty to twenty-five meters thickness with a specific yield of 0.12. Above it occurs thin layers of silty, sandy, and organic soils respectively of depth 0.50, 0.40, and 0.20 meters, with respective specific yields of 0.28, 0.35, and 0.48. These different specific yields were determined experimentally from pressure head - volumetric water content curves done for these four soils. From topographic and geologic maps, (figures 2.2 ansd 2.3), the profile, comprised of four different layers, is expected to be quite representative of the actual Metcalf site.

The region is an active farming area with barley, oats, and corn as the principal cultivated annual crops, while hay and pasture are the perennial crops. Birch, maple, and coniferous trees are generally found [26].

The only major water use within five kilometers of the Metcalf well is some industrial stream withdrawal on the Middle Castor about two kilometers from the well site. This is not expected to influence the data.

Groundwater level fluctuations show a clear annual pattern (figure 2.4). Recharge begins in the March-April period due to snowmelt floods and the highest groundwater level is usually reached in May or June. Throughout the summer, plant growth drastically reduces the groundwater

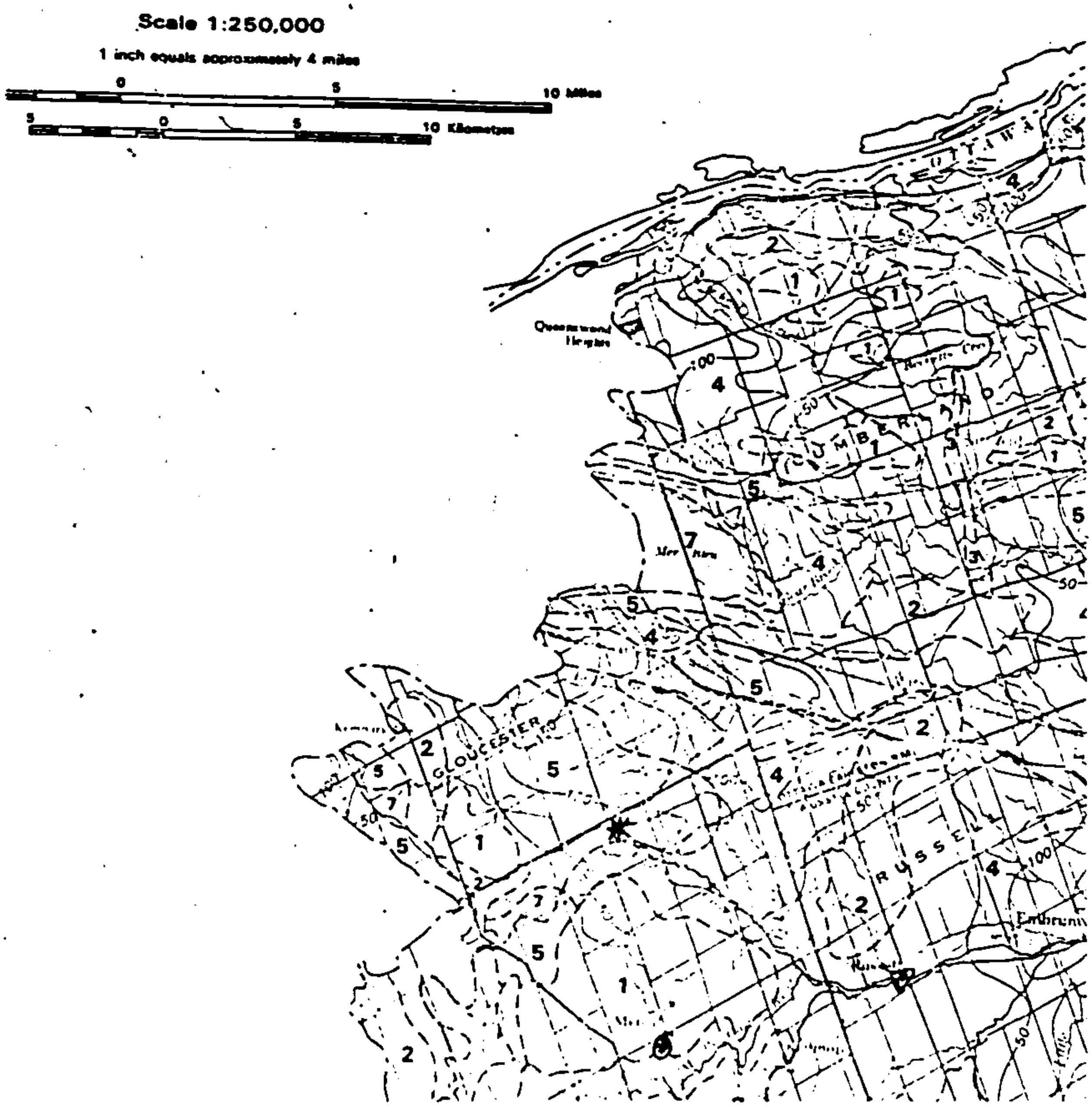
river gaging station

well

profile site

taken from Chin (4)

FIGURE 2.2: TOPOGRAPHIC MAP OF CASTOR REGION



▼ river gaging station

⊙ well

✱ profile site

taken from Chin[4]

LEGEND

CENOZOIC

QUATERNARY

7	Swamp deposits: mainly peat and muck
6	Marine beach and bar deposits: sand and gravel
5	Coarse-grained marine and deltaic deposits: sand
4	Fine-grained marine and river channel deposits: silt, clay

FIGURE 2.3: GEOLOGICAL MAP OF CASTOR REGION

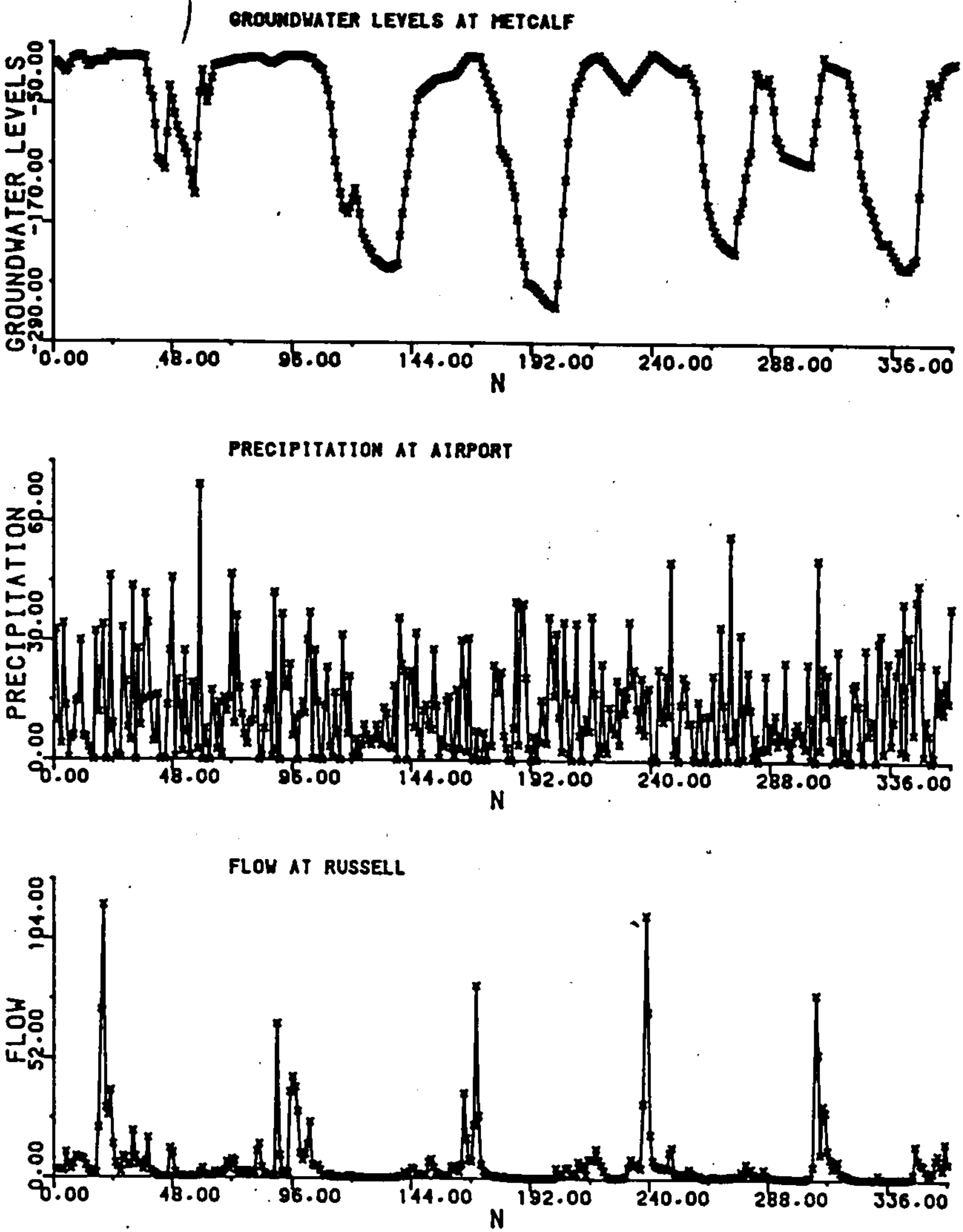


FIGURE 2.4: VARIATION OF GROUNDWATER LEVELS
PRECIPITATION, AND STREAMFLOW

level which is usually at its lowest around late September. Some recharge occurs during the fall, while the winter levels stay pretty much the same [28].

A twenty-five year annual water budget (1950-1974) of the whole basin indicated that thirty-nine percent of the annual precipitation occured as streamflow whereas sixty-one percent was being lost as evapotranspiration. Streamflow tends to be high in spring, but relatively even throughout the rest of the year. Precipitation is fairly constant except for the first three months of the year when it is slightly lower. Snow will typically remain on the ground during four months. Temperature follows an almost sinusoidal cycle with a peak in July and a low in January (figure 2.5). The hours of bright sunshine tend to be more frequent in summer than in winter. Precipitation exceeds potential evapotranspiration during four months of the year.

Although the Metcalf well is not at a site of concern for water resources management, numerous other areas of the watershed are either flood-prone or susceptible to contamination from waste discharges or sanitary landfills [4]. And, as mentioned earlier, agricultural activity is quite important.

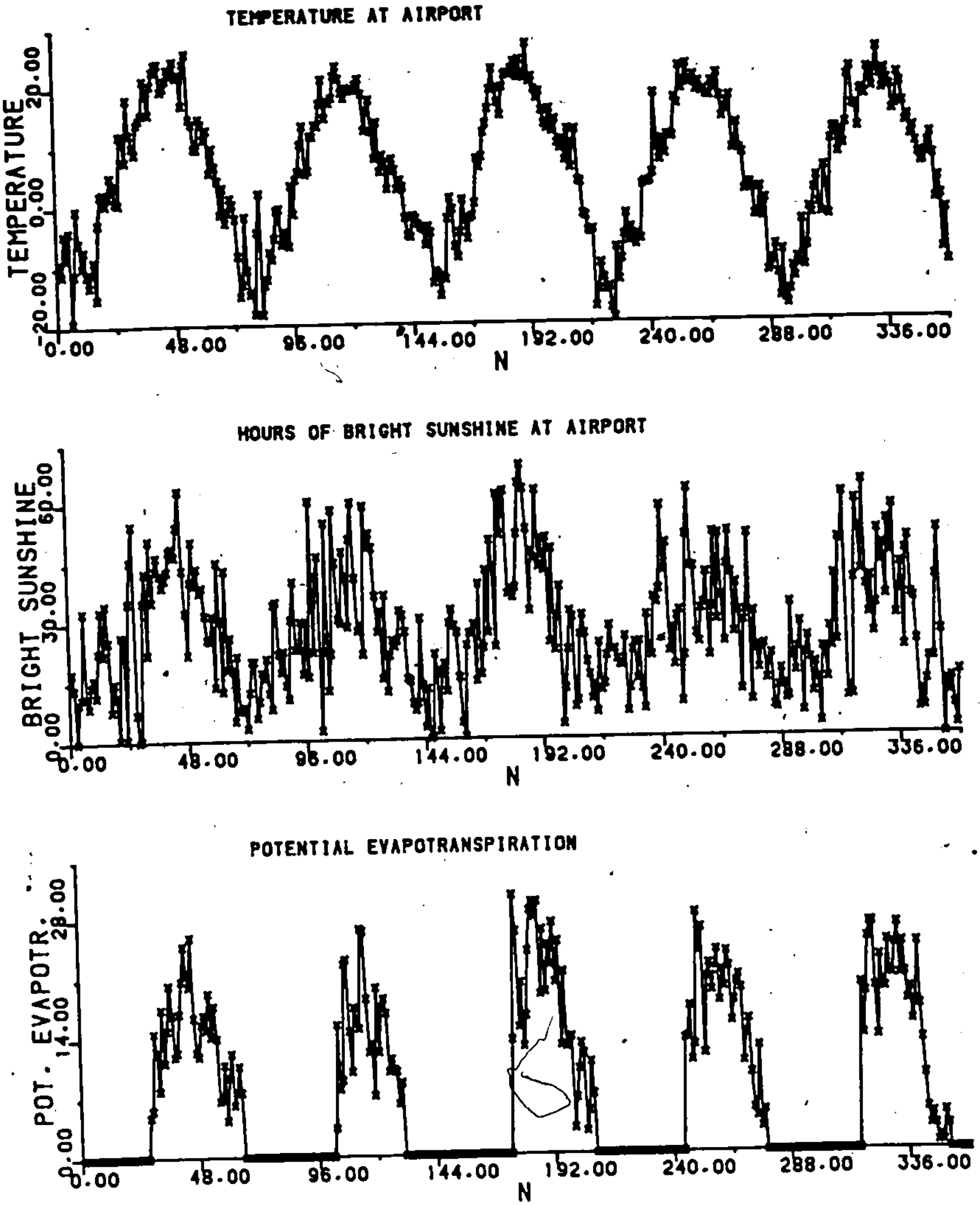


FIGURE 2.5: VARIATION OF TEMPERATURE, BRIGHT SUNSHINE, AND POT. EVAPOTR.

2.2 THE DATA BASE

The data available at the Castor watershed are five day readings of groundwater levels at the Metcalf well, as well as riverflow of the Castor River at Russell. Depth to groundwater level data was obtained six times per month, or roughly every five days. Readings were taken on the 3rd, the 8th, the 13th, the 18th, the 23rd, and at the end of the month.

Precipitation data was obtained from "Monthly Meteorological Summaries", a monthly publication of Atmospheric Environment Services, Environment Canada, for Ottawa's International Airport, hours of bright sunshine and temperature data originated from daily meteorological observations of the Atmospheric Environment Services, Environment Canada for Ottawa's International Airport. The precipitation and hours of bright sunshine are cumulative during the five day period, while the temperature readings are at every five days.

The data, which is tabulated in Appendix A, spans a period from November 1972 to November 1978 [28]. Since over six years of data were available, the first five served for calibration while the next one was used for model verification.

Since the emphasis is on modelling all factors affecting groundwater levels, including such seasonal occurences as snowmelt and evapotranspiration, the data series had to be carefully modified to delete values which physically could not be inputs to the model. These modifications are outlined below. From the temperature readings below zero, the precipitation readings which generated snow and not rainfall could easily be determined and consequently deleted to provide the rainfall sequence. This occasionally resulted in some winter rainfall.

For snowmelt, however, data selection was somewhat subjective. Based on the available riverflow measurements, the spring flood periods, consequences of snowmelt, were determined and the corresponding hours of bright sunshine and temperature were established. The recession portion of any runoff hydrograph is composed of a direct runoff section as well as a release of groundwater that had been stored in the river banks by the passing flood. The end of the snowmelt periods corresponded to the end of the direct runoff periods.

For the bright sunshine and temperature values needed for evapotranspiration, the selection was similar to rainfall. When temperature was below zero, it was ignored, unless streamflow increased due to snowmelt indicated that the five day reading of temperature was not typical of the general trend of those last five days. Due to varying specific yields, the model required that the groundwater levels be altered to the amount of groundwater storage in the soil above some selected datum. The modified data series is in Appendix B.

Chapter III

DEVELOPMENT OF GROUNDWATER LEVEL MODEL

3.1 GROUNDWATER LEVEL MODELLING

Understanding of groundwater processes has mostly come about during the last forty to fifty years, beginning with Theis who in 1935 recognized the importance of storage within an aquifer and that groundwater flow was analogous to heat flow in solids. Subsequently, more research followed using analytical solutions by Jacob, Hubbert, and Jaeger, to name but a few, who presented various quantitative methods. However, there were limitations to applying these analytical techniques to large groundwater systems. The solutions obtained were based on assumptions of homogeneous properties and simple boundary conditions, which were known to be unrealistic for large systems.

During the nineteen fifties and sixties, other methods of analysis were investigated, notably analog models using resistor- capacitor networks. Because of their great cost, they were replaced by numerical methods with the arrival of the digital computer. It was then possible to solve partial differential equations of groundwater flow, and the use of the finite difference method, as well as the finite element method became common [53]. Progress in the use of numerical methods has persisted up to today and new techniques have been developed [47].

Further developments were reported in the nineteen seventies with the use of time series analysis, which had already been used to model streamflow, precipitation, and other components of the hydrologic cycle [52]. Erikkson [15] [16] was the first to introduce time series techniques to analyze groundwater level fluctuations. He examined spectral and frequency analysis to study precipitation, river flow, and groundwater levels in wells at a Swedish esker, concluding that at the site of his analysis, groundwater level variations ran parallel to river stage variations and were lagging ten to thirty days behind.

Jackson et al. [32] used stochastic methods to study groundwater evapotranspiration, groundwater inflow rate, mean daily temperature, and daily precipitation at a groundwater discharge area in Manitoba. They concluded that in a groundwater discharge area the upper limit of the water table was controlled by evapotranspiration and baseflow to streams while the lower limit was controlled by the increase in piezometric head.

Gelhar [24] summarized recent results on stochastic analysis of flow in aquifers and emphasized that groundwater systems were highly variable both in time and space. He examined the stream aquifer system, recharge from a stream, and various flow situations with varying hydraulic conductivity. He concluded that frequency domain spectral analysis of groundwater time series was important for interpretation purposes.

Freeze [21] suggested that the variables in groundwater models ought to be stochastic; that is, there should be some probability distribution associated to them. He concluded that all soils, even those that are homogeneous, are nonuniform, and that they should be represented stochastically. He expressed doubts about the presumed accuracy of deterministic models in groundwater hydrology, explaining that it was not possible to define an equivalent uniform medium that acted in every sense like the nonuniform one. Stationarity and non-stationarity, as well as flow in various dimensions have also been studied [2] [9] [10] [44].

Recently, at the University of Ottawa, two different approaches were investigated for modelling groundwater level fluctuations at the Castor watershed, the same watershed studied in this thesis. Guerrero [26] developed a model, which used atmospheric data as well as soil-water properties, to simulate groundwater levels applying the finite-element method. He investigated the performance of the UNSAT-2 model of Neuman et al. during the growing season. Potential evapotranspiration rates were estimated by two methods. For the five simulated years, the predictions were very close but at times they lagged with the observed groundwater levels.

Hamory [28] developed a stochastic systems model of groundwater level fluctuations. However, his model was of the black box type and the relationship between groundwater levels and riverflow, precipitation, bright sunshine, temperature, and potential evapotranspiration was not explained by the actual physical processes occurring in the watershed. Because of the log transformation of the data, as well as the lack of data modification as performed for this thesis, he concluded that riverflow explained over sixty-six percent of the variance of groundwater levels while precipitation, hours of bright sunshine, temperature, and evapotranspiration were insignificant in his model.

Figure 2.4 most likely explains the reason why riverflow appeared to affect groundwater levels so much. If the logarithms of the flows are taken, then only during the spring melt periods is riverflow significant. And the high groundwater levels also occur in spring, caused by the snowmelt which produced the high riverflows.

A closer examination of the naturally occuring processes in the Castor watershed does not support the hypothesis that streamflow is the most significant variable affecting groundwater level fluctuations. The problem rests in seasonal effects which Hamory never considered. In the summer, the evapotranspiration, driven by the high temperature and the long hours of bright sunshine, is the major cause of groundwater level decrease and not the streamflow. Therefore, a physically-based approach taking in consideration seasonal effects is developed for improved forecasting of groundwater levels in the Castor watershed.

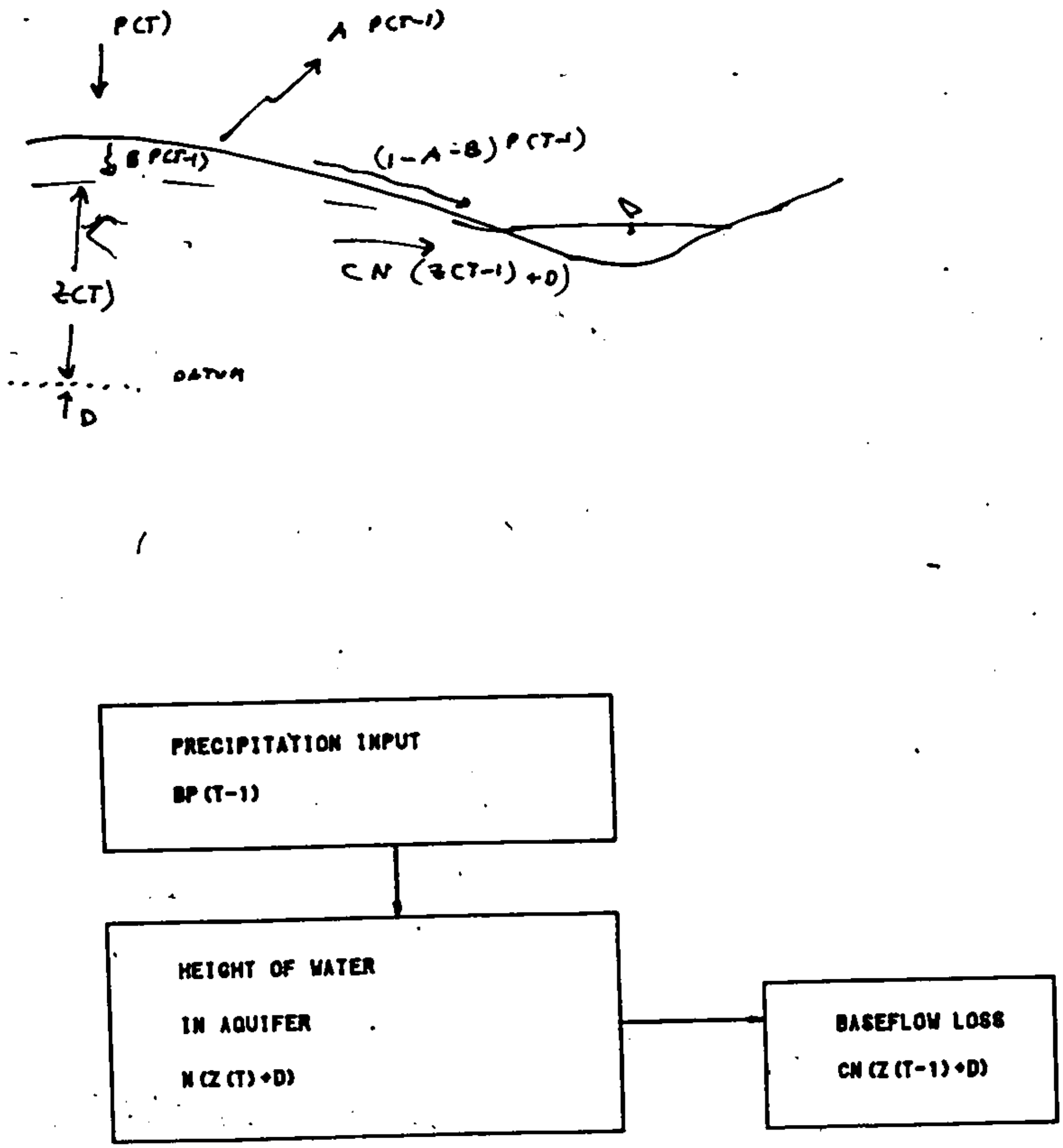
Fiering [18] presented a simple linear model for groundwater level fluctuations (figure 3.1), which was used by Kuczera [37], who examined the reliability of parameter estimation techniques. In that model, groundwater storage is dependent on past groundwater storage, a loss to streamflow proportional to the amount in groundwater storage, and some rainfall input. Rennolls et al. [50] presented a very simple linear model for drainage purposes quite similar to Fiering's. Groundwater levels were dependent on some proportion of the past groundwater levels and some rainfall input.

To the best of the author's knowledge, there is not available a linear regression model for groundwater fluctuations which incorporates evapotranspiration and snowmelt. Such a model will be formulated and developed in this thesis.

3.2 THE PHYSICAL LINEAR MODEL

The formulation of the linear groundwater level model is based on the model used by Kuczera [37], which was originally proposed by Fiering [18]. A schematic representation of this model is shown in figure 3.1.

The height of water held in the aquifer is represented by $N\ (Z(t)+D)$ where N is the specific yield, $Z(t)$ is the groundwater table above some selected datum at time t, while



THE MODEL:

$$N(Z(T)+D) = (1-C)N(Z(T-1)+D)+BP(T-1)$$

FIGURE 3.1: HYDROLOGIC MODEL OF A WATERSHED [36]

D is the depth of water below datum. At each time interval Dt, a portion of this water is lost as baseflow to an adjacent stream. This loss is proportional by C to the amount of water in the aquifer. Although the head difference between the groundwater table and the stream should determine the loss, a proportional loss can be justified considering that groundwater table and stream height variations are usually small compared to the difference in head.

Groundwater level increases are due to a rainfall input which is lagged by Dt. Kuczera [37] formulated a water balance of the previously mentioned processes as follows:

$$N\,(Z(t) + D) = (1-C)\,N\,(Z(t-1) + D) + B\,P(t-1) + U(t) \qquad (3.1)$$

where U(t) is the measurement error at time t, B is the fraction of rainfall going underground and not evaporating or running off, and C is the proportionality constant for the amount lost to streamflow.

This simple model, which uses time series of precipitation and groundwater levels, is the basis for a more complex formulation incorporating snowmelt and seasonal evapotranspiration. These last two factors are necessary in regions with snowfall and seasonal plant growth, which is the case in Canadian conditions. Since direct measurements of snowmelt and evapotranspiration were unavailable, models based on available data were formulated for them and are developed in the following subsections.

3.2.1 EVAPOTRANSPIRATION MODEL

Evapotranspiration is one of the processes of the Hydrologic cycle on which much research is still being devoted. It is known that various forms of energy, air temperature and humidity, precipitation, soil moisture, and plant processes, to name but some, influence the rate of evapotranspiration [45]. Many models are available, few of which require readily available data such as hours of bright sunshine or temperature. Jensen and Haise [40] have proposed a relatively simple model for evapotranspiration which depends on temperature and solar radiation data, and is given by:

$$ET = (Kc\ Ct\ (T-Tx)\ R)\ /\ p\ Lv \qquad (3.2)$$

where ET is the evapotranspiration in centimeters per day, Kc is a dimensionless constant dependent on the crop, Ct is a temperature coefficient per degree centigrade, T is the mean air temperature in degrees centigrade, Tx is a constant in degrees centigrade; R is the amount of solar radiation in calories per centimeter square per day, p is the density of water in grams per centimeter cubed, and Lv is the latent heat of vaporization of water (585 calories per gram).

Equation(3.2) is equivalent to the following:

$$ET = q\,T\,R - r\,R \tag{3.3}$$

where T and R are as defined previously while q and r are constants.

In this study, since the solar radiation measurements R are unavailable, the total hours of bright sunshine are used instead. Hours of bright sunshine are highly correlated with the amount of solar radiation, thus becoming an acceptable alternative, which, however, presents problems in the simple Jensen-Haise model. During a period of little or no bright sunshine, one might falsely conclude that there occurred little or no evapotranspiration, regardless of the temperature. This is entirely untrue.

Therefore, the following model relating evapotranspiration to temperature and hours of bright sunshine was used:

$$ET(T) = d\,T(T) + e\,BS(T) + f \tag{3.4}$$

where d, e, f are constants, BS is the total hours of bright sunshine during Dt, while ET and T are as previously defined.

Using this model, both temperature and hours of bright sunshine determine the evapotranspiration, and there is a constant f for the evergreens which require water during winter but whose growth most likely does not depend on temperature or hours of bright sunshine. Inclusion of evapotranspiration in the model requires modification of the data so that during winter, evapotranspiration is excluded.

3.2.2 SNOWMELT MODEL

Snowmelt is one of the most complex processes in the hydrologic cycle. The melting rate depends on a multitude of interrelated factors such as radiation, temperature, wind, vapour pressure, forest cover, and rainfall.

The U.S. Army Corps of Engineers has suggested a very simple equation relating daily snowmelt to radiation and temperature [61]:

$$SM = 0.00238\ G + 0.0245\ (TM - 77) \tag{3.5}$$

where SM is the daily snowmelt, G is the daily estimated total radiation in langleys, and TM is the daily maximum temperature in degrees Farenheit. When total radiation is replaced by hours of bright sunshine for convenience, equation (3.5) becomes of the following form:

$$SM(t) = g\ BSS(t-1) + h\ TS(t-1) + i \tag{3.6}$$

where SM is the snowmelt going to the groundwater table, BSS and TS are the hours of bright sunshine and temperature for snowmelt, while g,h, and i are constants. A lag of Dt is introduced to take into account the time necessary for the snowmelt to reach the groundwater table.

Since a general regression model will be employed all year long, the constant i has no significance in determining the input TS or BSS for snowmelt calculations, and thus can be removed, leading to:

$$SM(t) = g\ BSS(t-1) + h\ TS(t-1) \tag{3.7}$$

The simple model as given by equation (3.1) can now be modified to include both evapotranspiration and snowmelt and is expressed as:

$$\begin{aligned} N\ (Z(t) + D) = {} & (1-C)\ N\ (Z(t-1) + D) + B\ P(t-1) \\ & + d\ T(t) + e\ BSS(t-1) + g\ BSS(t-1) \\ & h\ TS(t-1) + f \end{aligned} \tag{3.8}$$

where all the terms have been previously defined.

Further modifications of the model are necessary to reflect the local geological conditions at the Castor watershed. Since the studied area of the watershed is composed of four different soil layers, each with a different specific yield, by selecting the datum at the bottom layer three meters from the top, N (Z(t)+D) becomes the summation of the N DZ + j, where DZ is a depth to which corresponds a certain constant N. Designating the sum of the N DZ at time t as W(t), then N(Z(t)+D) becomes W(t)+j, with W(t) representing the amount of water above datum in groundwater storage at time t. Therefore, equation (3.8) becomes:

$$W(t) + j = (1-C)\ (W(t-1) + j) + B\ P(t-1) + d\ T(t) + e\ BS(t) + g\ BSS(t-1) + h\ TS(t-1) + f \qquad (3.9)$$

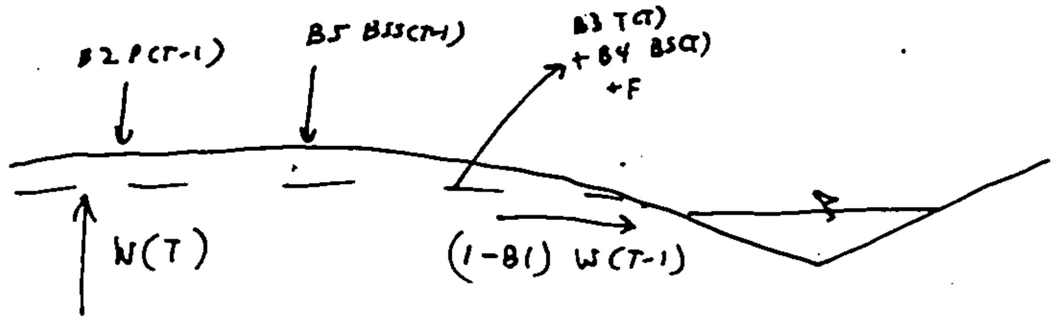
Collecting terms and grouping constants in equation (3.9) results in:

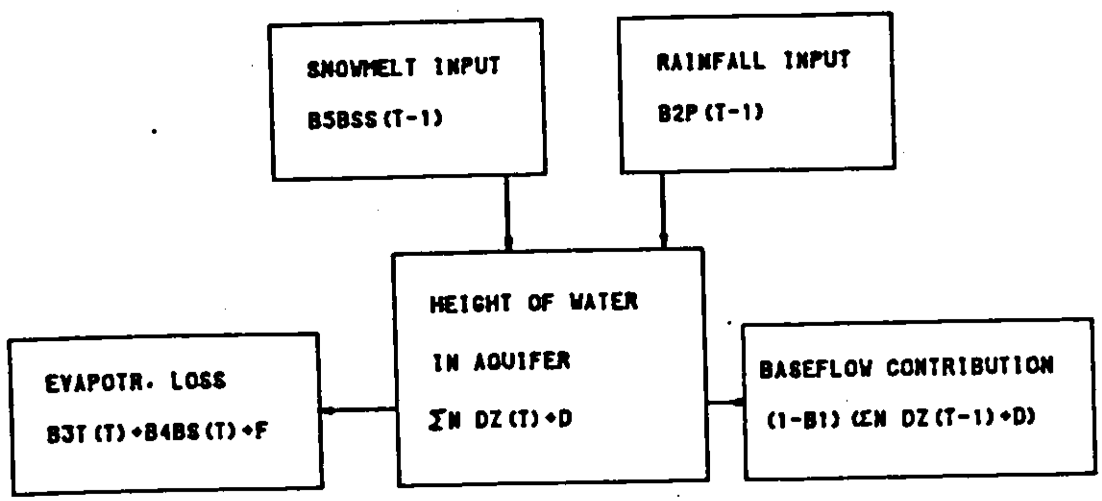
$$W(t) = B1\ W(t-1) + B2\ P(t-1) + B3\ T(t) + B4\ BS(t) + B5\ BSS(t-1) + B6\ TS(t-1) + B7 \qquad (3.10)$$

where B1 to B7 are the seven regression coefficients to be determined. This model is schematically shown in figure 3.2 with slight modifications from the one given in equation (3.10) for reasons to be explained later in this chapter.

3.3 MODEL ASSUMPTIONS

The proposed model makes several assumptions which need to be critically examined. These assumptions are necessary in order to obtain some model, however, as explained in [36], operational models have some basic limitations, the primary one being that they are nothing more than geometric interpolations and that they work best in the range of the





THE MODEL:

W(T) = B1W(T-1)+B2P(T-1)+B3T(T)+B4BS(T)+B5BSS(T-1)+B6

FIGURE 3.2: HYDROLOGIC MODEL DEVELOPED IN THIS STUDY

empirical data.

The selection of Dt as five days was a constraint forced upon by the available data. Does rainfall and snowmelt truly take five days on average to reach the groundwater table? As will be explained later in this chapter, assuming a lag of five days was found to explain the physical processes better than no lag or a lag of ten days. A common value of lag time Dt between groundwater and rainfall was assumed throughout the whole year, regardless of the height of the water table.

An assumption which has previously been mentioned is that the regression coefficient B1 of equation (3.10), which represents unity minus the proportion of groundwater storage lost to streamflow, is a constant. However, the coefficient can be expected to vary with the difference in elevation between groundwater and river levels. Also, in spring, B1 might rise up to 1.0 and remain there during the flooding season when the river stage is extremely high. However, this is unlikely since the well site is about two kilometers from the nearest stream and is about fifteen meters higher.

The rainfall and snowmelt coefficients B2, B5,and B6 are assumed constant, and not at all dependent on temperature. However, on a warm day, evaporation will immediately reduce the amount of water available to percolate underground. Similarly, during a large storm, infiltration will soon slow down as the soil becomes saturated. Thus, the above assumptions are violated. It was also assumed that the moisture content of the soil above the water table had no effect on groundwater levels. Obviously, it influences groundwater levels as the water table rises and falls due to the effect of the capillary zone and interflow, which are ignored in the model. Moreover, it was assumed that the given soil profile is fairly homogeneous with respect to space. The variability of the specific yield N within a same layer was not considered either. It is well known that soils are never perfectly uniform.

It must also be assumed that the recorded groundwater level measurements are accurate and do reflect the actual groundwater fluctuations. The plots of average errors after applying ordinary least squares (figure 3.3) and of average groundwater levels (figure 3.4) seem to indicate the opposite. The Durbin-Watson test, applied to the seventy-two average errors by ordinary least squares, confirms the hypothesis that serial correlations exist and that the residuals are non-random. The Durbin-Watson statistic, calculated from [35]:

$$DW = \Sigma\,(e(t) - e(t-1))^2 \;/\; \Sigma\, e(t)^2 \qquad (3.11)$$

resulted in a value of 0.96, which was smaller than 1.32 which is the limit for an uncorrelated series composed of seventy-two values for a regression with five independent variables.

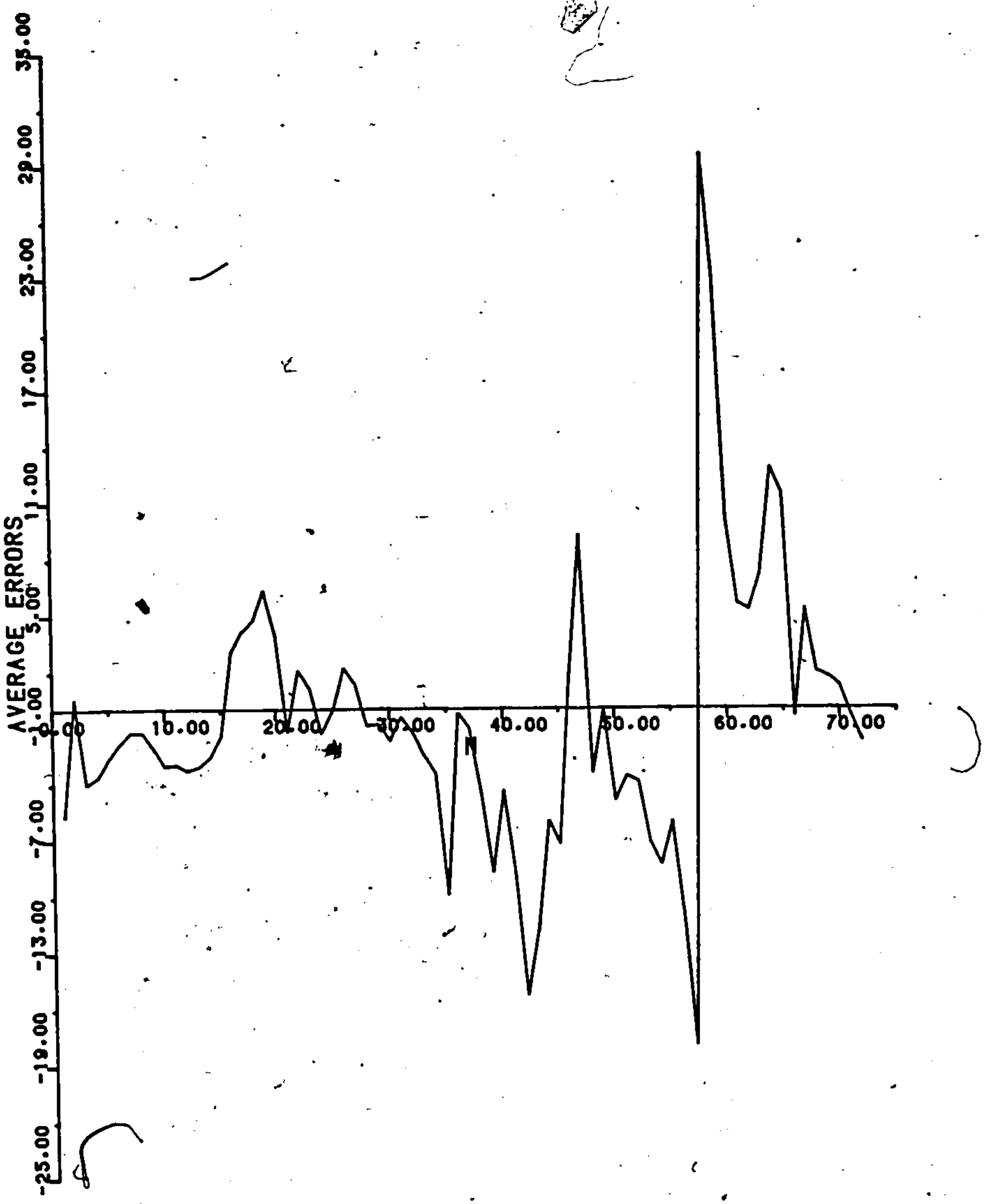


FIGURE 3.3: AVERAGE ERRORS BY LEAST SQUARES OVER FIVE YEARS

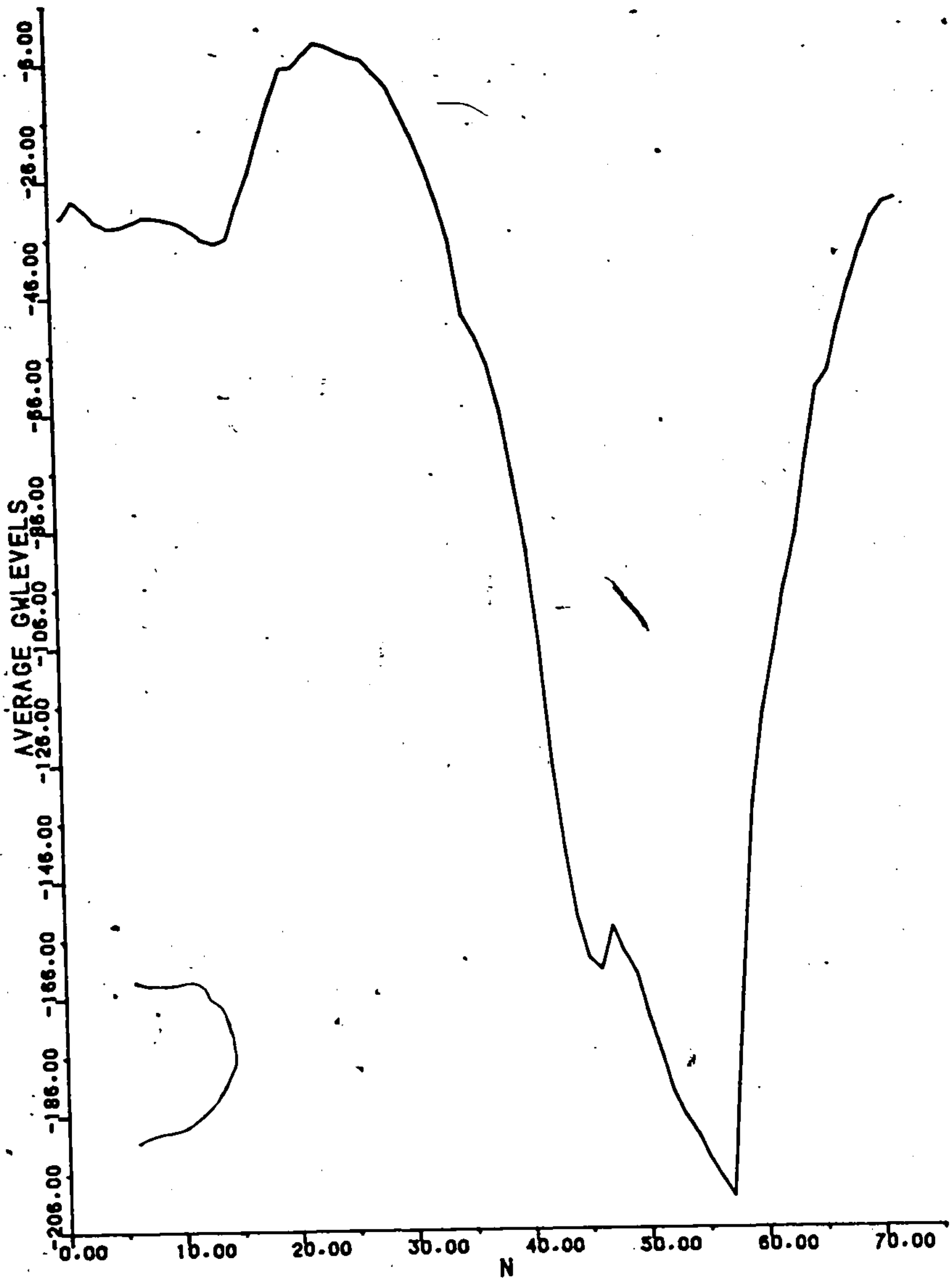


FIGURE 3.4: MEAN OBSERVED GROUNDWATER LEVELS OVER FIVE YEA

Therefore, the correlations are significant and the figures suggest that the periods of rapid change in groundwater levels seem to directly correspond to periods of large errors. The previously mentioned problem of soil moisture above the water table may be a factor contributing to those errors. Also, the simple snowmelt and evapotranspiration models may be inadequate.

Furthermore, there is the validity of using meteorological data away from the site of the groundwater level measurements. Even though the Ottawa International Airport, at a distance of sixteen kilometers, is relatively close to the Metcalf well site, hours of bright sunshine, temperature, and especially precipitation are known to vary in space.

Ideally, it can be argued that any model of the hydrologic cycle should be stochastic, nonlinear, time-varying and continuous, but practically this would result in a much too complex formulation. Therefore, it should be stressed that although the developed model is an improvement over existing models, nevertheless it has its limitations which are expressed through the simplified assumptions.

3.4 FINAL VERSION OF THE MODEL

The final version of the model in this thesis is presented in figure 3.2. It is slightly different from equation (3.10) for reasons to be explained later.

During a preliminary test run of the model by ordinary least squares on the first five years of data, one of the parameters, B6, turned out to be negative, although close to zero. All the other parameter values were physically possible except this one. The reason for B6 being negative is that the temperature reading during snowmelt periods, the variable it multiplies, is not representative of the last five days. It is possible that, for example, high temperatures can occur during four days while during the fifth, a low temperature is measured and misleads as to the actual snowmelt. Therefore, since the five day temperature reading appeared to be giving erroneous results for snowmelt in this watershed, it was decided to eliminate it.

The lag time for rainfall and snowmelt created a problem since the data was at five day intervals. In order to determine the values of lag appropriate for the analyzed data, the correlations of the first five years of groundwater levels with precipitation and hours of bright sunshine during snowmelt periods were determined for lag zero, lag Dt, and twice lag Dt. The results, presented in table 3.1, indicate that maximum correlation of groundwater levels with snowmelt occured for lag zero, while the maximum correlation between

TABLE 3.1

CORRELATIONS WITH GROUNDWATER FOR VARIOUS LAGS

LAG	GWLEVELS - RAINFALL	GWLEVELS - SNOWMELT
0	-0.0025	0.3187
1	0.0184	0.3161
2	0.0208	0.3098

groundwater levels and rainfall occured for lag two. However, both correlations for lag one were very close to their corresponding maximum correlations, and since both groundwater level increases due to rainfall and snowmelt result from infiltration, the lag time should be identical. Therefore, lag one was selected for the rainfall and snowmelt inputs in the model.

Based on the results reported in table 3.1, the following version of the model was established:

$$W(t) = B1\ W(t-1) + B2\ P(t-1) + B3\ T(t) + B4\ BS(t) + B5\ BSS(t-1) + B6 \qquad (3.12)$$

and is shown in figure 3.2. Parameter B1 corresponds to the fraction of groundwater remaining in storage and not lost to streamflow during Dt. B2 is the fraction of rainfall reaching the groundwater table. Parameters B3 and B4 correspond to the amount of evapotranspiration losses by unit input of temperature and hours of bright sunshine respectively. B5 is the amount of snowmelt reaching the groundwater table per unit input of hours of bright sunshine, while B6 is a constant corresponding to the evapotranspiration of the evergreens and the groundwater storage below datum. These parameters were subsequently determined by the methods described in the next section.

Chapter IV

PARAMETER ESTIMATION TECHNIQUES

4.1 PARAMETER ESTIMATION IN HYDROLOGY

Parameter estimation has been a major topic of interest for a long time in all fields of science and engineering, for it is these parameters which permit some output to be generated from a model. When model parameters cannot be measured directly, they must be inferred from sets of observations. These estimates of the parameters are then used for modelling purposes.

In order to compare the parameter values, various objective criteria are required and some of them need to be defined. An unbiased estimate is one whose expected value is identical to the quantity being estimated. A minimum variance estimate has its error variance less than or equal to that of any other unbiased estimate. A consistent estimate converges to the true value of the parameter as the number of measurements increase.

The various statistical estimation techniques usually are obtained on the basis of some error criterion in order to reach an optimal fit of the model to the available data. This is accomplished by minimizing some objective function, such as the sum of the squares of the errors in the ordinary least squares method. The estimation techniques also assume that the independent variables are errorless and that the dependent variable is not measured accurately. However, in reality, all the measurements can be expected to contain some degree of uncertainty. And these errors are usually expected to be normally distributed, which is not always true, especially since one often models nonlinear hydrologic processes by linear models.

Legendre in 1805 was the first to publish the theory of least squares, although Gauss had apparently used it ten years earlier to solve orbit measurements problems. The next significant contribution to estimation theory occured around 1910 when Fisher developed maximum likelihood estimation. In the last fifty years, through the work of such people as Kolmogorov and Kalman much progress has resulted leading to the present state of understanding of parameter estimation [23].

In hydrology, as reported in [31], Snyder was the first to use ordinary least squares in the development of unit hydrographs. Other parameter estimation methods followed, such as linear programming [11], and maximum likelihood. More recently, Bayesian parameter estimation techniques, the extended Kalman filter, the instrumental variable, and generalised least squares were introduced in hydrology and mentioned as worthwhile alternatives to be investigated [39][43].

Recent work on maximum likelihood has been by Sorooshian and others in [55] and [56] where they study the case of Gaussian homogeneous errors, as well as the cases for autocorrelated errors and heteroscedastic errors. Their conclusion from synthetically generated data is that maximum likelihood estimation theory provides more reliable estimates when errors are either correlated or heteroscedastic.

The classic paper by Wong and Polak [65], instigated the wide use of the instrumental variable method in the control literature. But because of the difficulty in determining an adequate instrumental variable, the recursive instrumental variable method has rather been used in hydrology with some success by Young and Whitehead [68][69][62].

The extended Kalman filter is another parameter estimation technique recently used in hydrology [8] [5]. The Kalman filter is a recursive algorithm which estimates the states of a linear system using past measurements on that system, statistical models of the system and measurement errors, as well as initial conditions in order to make more accurate statements about past, present, or future states. The extended Kalman filter, a linearized version of the Kalman filter, can solve the parameter estimation problem by providing simultaneously state and parameter estimates. First developed in the electrical engineering literature, it can be a quite powerful recursive estimation method, but may be extremely troublesome to implement, as shall be explained later in this chapter. Since it has been shown to provide on-line seasonal variations of the parameter estimates [48], it likely has an important future in hydrology.

Kuczera [37] examined how the incorporation of additional time series improved the estimation of the parameters by the use of a coupled system of equations dependent on the same variables. He also found that generalised least squares provided better estimates than ordinary least squares.

A comparison of parameter estimation techniques (some of which are inapplicable to a linear model) is presented in [30]. There have been a few papers comparing techniques in hydrology. Cooper [7] reviewed and compared recursive least squares, recursive instrumental variable, and the extended Kalman filter, concluding that recursive techniques can point out model inadequacies and parameter variations. Williams and Yeh [63] have compared linear programming, ordinary least squares, and generalised least squares, and have concluded that the latter is the superior technique.

The available parameter estimation techniques are numerous and not all have been compared together. However, not all are applicable to a certain situation. The following sections will present the parameter estimation techniques used in this thesis.

4.2 LEAST SQUARES

Parameters of a linear regression model can be determined either on-line(sequentially) or off-line(in a single step). Off-line techniques, the best known of which is ordinary least squares, are direct and provide a single estimate, while those on-line allow some insight into possible variations of the parameters.

Ordinary least squares, the most widely used parameter estimation method, results when the error covariance matrix is assumed a diagonal of constant value. When weights are given to certain observations, weighted least squares is employed. When the error covariance matrix is not a constant diagonal matrix, the method of generalised least squares results. Details are given in the following subsections.

4.2.1 ORDINARY LEAST SQUARES

Ordinary least squares can be applied to a model given by:

$$Y = X\,B + U \tag{4.1}$$

where Y are the n outputs, X is the n by m input matrix, B are the m parameters, and U are the n measurement errors, which are assumed of zero mean.

The procedure is to minimize the square of U which equals (Y-XB)'(Y- XB),where ' signifies transpose. By differentiating [27], one obtains:

$$B = inv(X'\ X)\ X'\ Y \qquad (4.2)$$

where inv signifies the inverse of that matrix.

The parameters B are unbiased, consistent, and of minimum variance as long as the error covariance matrix V is a diagonal matrix of constant value σ^2. If there is correlation among the measurement errors, the ordinary least squares solution is biased, and not necessarily consistent and of minimum variance. A recursive algorithm for determining ordinary least squares estimates on-line has been developed and is given in [17] and [7].

4.2.2 WEIGHTED LEAST SQUARES

Ordinary least squares, by minimizing the square of U, weights every measurement error equally. It is possible to modify the parameter estimates by assigning less weight to certain observations if they are considered less certain. Instead of minimizing U'U, U'WU is minimized, where W is a weighting matrix which must be positive definite.

By differentiating (Y-XB)'W(Y-XB), one obtains [29]:

$$B = inv(X'\ W\ X)\ X'\ W\ Y \qquad (4.3)$$

The special case where W is a diagonal matrix with equal values is the ordinary least squares solution. Otherwise, a different solution is obtained.

Figure 3.3 suggested the existence of measurement errors at certain of the seventy-two time periods throughout each and every year. Therefore, less weight was given to all observations which showed strong errors after the use of ordinary least squares. Consequently, for time periods 3 to 15, 28 to 45, and 50 to 65, a weight of 0.2 was arbitrarily assigned. Since the snowmelt model proposed in this study might be the most doubtful component of the groundwater storage model, it was decided to apply weighted least squares in this second manner. The regression equation of the model was run with a diagonal weighting matrix W equal to 1.0, except during snowmelt periods when it was arbitrarily chosen as 0.2. Still another possibility was to assign less weight to the observations during strong evapotranspiration periods since the model prediction during that time was less reliable.

The reasons for the selection of these special cases will become clearer once the results of ordinary least squares are presented in the next chapter.

4.2.3 GENERALISED LEAST SQUARES

The ordinary least squares method is a biased estimator when residuals are non-gaussian white noise [59] because one of its basic assumptions is that the expectation of UU' is $\sigma^2 I$, or that the errors are totally uncorrelated. In hydrology, this is rarely the case when applying a linear model, as seen from the plot of the average residuals over five years from the ordinary least squares analysis (figure 3.3).

Since, essentially, hydrologic processes are nonlinear but approximated by linear equations, this results in correlated errors at various points in time. Another problem discussed earlier is that the measurement process can introduce errors; for the developed model it seems to occur during periods of rapid change in groundwater levels.

When the expectation of UU' equals $\sigma^2 V$, where V is the positive definite covariance matrix, the method of generalised least squares must be used. This is the case here and V, the error covariance matrix, must be modelled.

In this method, as explained by Theil [59], the regression equation (4.1) is multiplied on both sides by the matrix T, where TT' equals the inverse of V, which gives:

$$T\,Y = T\,X\,B + T\,U \qquad (4.4)$$

The error expectation becomes TUU'T' which equals σ^2TVT' which equals σ^2I, the same as is required for ordinary least squares.

Introducing new transformed variables such that TY equals J, TX equals K, and TU equals L, then $J = K\,B + L$ satisfies the ordinary least squares assumptions that the errors are uncorrelated.

The ordinary least squares parameter estimation equation (4.2) then becomes:

$$B = \mathrm{inv}(K'\ K)\ K'\ J \tag{4.5a}$$ or

$$B = \mathrm{inv}(X'\ T'\ T\ X)\ X'\ T'\ T\ Y \tag{4.5b}$$

which equals:

$$B = \mathrm{inv}(X'\ \mathrm{inv}(V)\ X)\ X'\ \mathrm{inv}(V)\ Y \tag{4.6}$$

The above equation is identical to the weighted least squares solution with the inverse of V as the weighting matrix. The special case where V is a diagonal matrix of constant values equal to σ^2 results in the ordinary least squares solution. If the true formulation of the error covariance matrix V can be determined, then generalised least squares is an unbiased, minimum variance, consistent estimation technique. The practical problem, however, is in correctly assessing the error covariance matrix.

In this study, two modifications of generalised least squares were attempted. The first assumed lag one autocorrelation among the errors, while the second modelled V from the ordinary least squares residuals, resulting in an error covariance matrix with blocks corresponding to an assumed lag one autocorrelation, while the rest was zero, except for the diagonal. If one assumes lag one autocorrelation p among the errors, then V is of the shape given below [63]:

$$
V = \begin{bmatrix}
1 & p & p^2 & \cdots & & p^{N-1} \\
p & 1 & p & & & \\
\cdot & & 1 & & & \\
\cdot & & & 1 & & \\
 & & & & \ddots & \\
p^{N-1} & & & & & 1
\end{bmatrix}
\qquad (4.7)
$$

while its inverse is given as [63]:

$$
\mathrm{inv}(V) = 1/(1-\rho^2)\begin{bmatrix} 1 & -\rho & 0 \cdots & & & 0 \\ -\rho & (1+\rho^2) & -\rho & 0 \cdots & & \\ 0 & -\rho & (1+\rho^2) & -\rho & & \\ \cdot & & & & & \\ & & & & & \\ 0 & & & 0 & -\rho & 1 \end{bmatrix} \qquad (4.8)
$$

The second modification at generalised least squares assumed that the modelled errors were correlated at certain times of the year, but not at others. The shape of V was therefore as given below:

$$V = \begin{bmatrix} 1 & 0 & \cdots & & & \\ 0 & 1 & 0 & \cdots & & \\ & & B\ldots B & & & \\ & & \cdot \quad \cdot & & & \\ & & B\ldots B & & & \\ & & & 1 & & \\ & & & & & \\ & & & & 0 & 1 \end{bmatrix} \tag{4.9}$$

where the B..Bs represent a block of the same form as equation (4.7) indicating correlated errors for certain time periods only with a specific value of p. The same time periods as for weighted least squares were selected. More details will be given in the next chapter.

4.3 THE KALMAN FILTER

The Kalman filter is an optimal linear estimation technique requiring a system given in state-space representation, which is based on the Markov property; that is, the future state of a system depends on its present state. The concept of the state of a system is a mathematical entity linking inputs and outputs. Although the state, in general, has no intrinsic meaning, it is desirable for the state variables to reflect physical meaning.

If $x1(t)$ to $xn(t)$ are the states of the process at time t, and $u1(t)$ to $up(t)$ are the deterministic input or control variables to the process at time t, then the system may be described by n first order differential equations of the form given below:

$$\dot{x}1(t) = f(x1(t),\ldots,xn(t),u1(t),\ldots up(t)) \qquad (4.10)$$

meaning that the rate of change of each state may be a function of all states and all control variables.

In discrete form, it is:

$$x1(t+1) = f(x1(t),\ldots,xn(t),u1(t),\ldots,up(t)) \qquad (4.11)$$

The states, however, may not always be measurable. Therefore, the output processes $y1(t)$ to $ym(t)$ are a function of the states:

$$yl(t) = h(xl(t), \ldots, xn(t)) \tag{4.12}$$

meaning that all output processes may be a function of all states.

If the system is linear and discrete, and error terms are considered, then it takes the following form:

$$x(t+1) = F\, x(t) + G\, u(t) + w(t) \tag{4.13}$$

$$y(t) = H\, x(t) + v(t) \tag{4.14}$$

where x is the state vector, u is the external input or control variable vector, y is the measurement vector, while w and v are error vectors. These vectors must have a zero mean and be uncorrelated to one another. Matrices F and G are respectively called the state transition matrix and the control transition matrix. Matrix H relates all states to all outputs. Matrices F,G, and H actually define the system. Equations (4.13) and (4.14) are respectively called the system and measurement equations [58].

The solution of this system is given by the Kalman filter as [23]:

$$x(t+1) = F\, x(t) + G\, u(t) + P(t)\, H'\, \mathrm{inv}(H\, P(t)\, H' + R)\ .\ (y(t) - H\, x(t)) \tag{4.15}$$

where $P(t)$ is the estimation error covariance matrix of $x(t)$ defined recursively as:

$$P(t) = P(t-1) - P(t-1)\ H'\ inv(H\ P(t-1)\ H' + R)\ .\ H\ P(t-1)\ +\ Q \qquad (4.16)$$

where Q is the expectation of $w(t)w(t)'$ and R is the expectation of $v(t)v(t)'$, with both Q and R as diagonal matrices. Assuming the system (F,G, and H) is known, then initial values for the states x and the estimation error covariance matrix P must be provided. Values for Q and R must be provided as well, which are the most troublesome aspect of this technique. Adaptive methods can lessen this problem.

The Kalman filter provides unbiased, minimum variance, consistent estimates of the states as long as the system (the model, actually) and all error variances are correctly defined [23].

A comprehensive description of the Kalman filtering technique is provided in [66]. It explains that "prior to the processing of the measurement at time t, an estimate of the state at time t based on past measurements is made. The uncertainty of this estimate depends partially on the variance of the noise terms that are influencing the state equation. After the noise corrupted output variables are measured with the measurement equation, the estimate of the state based on past measurements is updated to include the current measurement. The updated estimate of the state is a weighted estimate of the uncertain prior estimate of the state and the uncertain measurements, where the weight is a function of the relative magnitudes of the variances of the state equation and measurement noise terms. If the variance of the state equation noise term is much smaller than the variance of the measurement equation noise term, then less weight will be given to the new measurement in updating the state estimator."

For the problem at hand, equation (3.12) can be transformed in state-space representation by defining groundwater storage W(t) as one of the states, and letting P(t-1), BS(t), T(t), BSS(t-1), and unity act as control variables. Matrix F is a scalar quantity whose value is B1, while G is a five by one matrix whose values are respectively B2 to B6. Since measurements of the state, groundwater storage, are available, the output y is the groundwater storage and matrix H is a scalar equal to unity.

The form of equation (4.10) can be reached by substracting W(t-1) from both sides of equation (3.12). The rate of change of W(t) can be approximated by W(t)-W(t-1) divided by Dt, but since all the data is reported in five day intervals, the state-space representation of equation (3.12) is:

$$W(t) = (B1-1)\ W(t-1) + B2\ P(t-1) + B3\ T(t) + B4\ BS(t) + B5\ BSS(t-1) + B6 \quad (4.17)$$

For equation (4.12), y(t) simply equals the state, W(t).

4.3.1 THE EXTENDED KALMAN FILTER

Given a nonlinear system in state-space representation, the Kalman filter can be linearized, usually through Taylor series expansion, in order to solve such a problem. Since parameter estimation of a linear system given the measurements is a nonlinear problem, it therefore can be solved through the extended Kalman filter [38].

The Taylor series expansion of a function X estimated at time t+1 around the previous value estimated from X(t) keeping only the first two terms of the expansion is:

$$X(t+1) = X(t) + X'(t)\ (X(t+1) - X(t)) \quad (4.18)$$

The same principles are applied in the extended Kalman filter. If given a nonlinear system of the form:

$$x(t+1) = f(x(t),u(t),B(t)) + w(t) \quad (4.19)$$

$$y(t) = h(x(t)) + v(t) \quad (4.20)$$

then equation (4.15) remains the solution of the system under slight modification [8]:

$$x(t+1) = f(x(t),u(t),B(t)) + F\ P(t)\ H'\ \text{inv.}(H\ P(t)\ H' + R).(y(t)-h(x(t))) \qquad (4.21)$$

Matrices F and H take a different shape, as shall be explained later. Equation (4.16) changes to the following:

$$P(t) = F\ P(t-1)\ F' - F\ P(t-1)\ H'\ \text{inv}(H\ P(t-1)\ H' + R).\ H\ P(t-1)\ F' + Q \qquad (4.22)$$

For the nonlinear parameter estimation problem, the state vector X contains not only the original states of the system, but also the parameter estimates B1 to Bn, which are allowed to vary. However, under optimal conditions, these parameters will converge. The values of the elements of the F matrix correspond to the derivative of each state with respect to the other states. Similarly, H corresponds to the derivative of the measurements with respect to the states. Matrices H, P, and Q must consequently also be altered as shown below:

$$X(T) = \begin{bmatrix} X1(T) \\ \cdot \\ \cdot \\ Xn(T) \\ B1(T) \\ \cdot \\ \cdot \\ Bn(T) \end{bmatrix} \qquad (4.23)$$

$$H = \begin{bmatrix} H \\ \text{---} \\ C(i) \end{bmatrix} \qquad (4.24)$$

$$F = \left[\begin{array}{c|c} F & H(i) \\ \hline 0 & I \end{array}\right] \tag{4.25}$$

$$Pe = \left[\begin{array}{c|c} Pa & 0 \\ \hline 0 & Pb \end{array}\right] \tag{4.26}$$

$$0 = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] \tag{4.27}$$

With d as the differentiating operator:

$$M(i) = d\,(F\,x(t) + G\,u(t))/d(Bi(t)) \tag{4.28}$$

and

$$C(i) = d\,(H)/d(Bi(t)) \tag{4.29}$$

and Pb is the initial error covariance matrix of the parameters. Further details are available in [38].

The difficulty in using the extended Kalman filter is in correctly assessing R and Q in order to obtain the optimal solution. Unless an adaptive algorithm which corrects these two values is used, a trial and error procedure must be implemented. Because of the Taylor series expansion, an approximation is being dealt with. However, Ljung [38] claims that, while the method may give biased estimates, it seldom diverges if sufficiently good initial estimates are provided.

4.4 OTHER METHODS

A large number of other parameter estimation techniques exist. Some that were attempted but did not yield satisfactory results are described here.

The instrumental variable method replaces X' in the ordinary least squares solution by Z':

$$B = inv(Z' X) Z' Y \qquad (4.30)$$

where Z is a matrix highly correlated to X but uncorrelated to the noise.

Since in hydrology an attempt is made by linear models to represent nonlinear processes, one very likely gets correlated errors at some point in the analysis, which makes the instrumental variable method difficult to implement. It has been suggested [30] to use the output of the ordinary least squares model as the basis for the construction of Z. This was attempted, but since the errors were correlated, the new solution was diverging from the actual parameters.

Interestingly enough, it has been found [65] that the optimal Z equals the inverse of V times X, where V and X are respectively the error covariance matrix and the measurement matrix. This results in the generalised least squares solution and therefore reconfirms its validity as probably the best parameter estimation technique. A recursive instrumental variable method was also attempted [68], but for the same reasons as the instrumental variable method, it did not succeed.

Through linear programming, it is possible to minimize the sum of the absolute errors to obtain the parameters [11][19]. Unfortunately, because there was so much data, and the simplex algorithm was used, the system became quickly ill-conditioned. However, for models with less data, this can be a valid parameter estimation technique.

Recursive generalised least squares is a technique which fits an autoregression to the errors of the ordinary least squares fit and then, after applying this autoregression, provides a new corrected output upon which new parameters are found [17]. Since the residuals did not follow a continuous autoregressive process but were significant at certain times of the year, this technique was inappropriate.

Finally, there remains the maximum likelihood method. It was not attempted because if Gaussian errors are present, its solution is the same as ordinary least squares [56]. And clearly the cases for autocorrelated and heteroscedastic errors do not apply to the data, since there are correlations among the errors, but they only occur at certain time periods.

Chapter V

RESULTS & DISCUSSION

5.1 RESULTS

Since six years of data were available, the first five were used for parameter estimation, while the sixth was reserved for prediction and comparison with observations. The difference between the predicted and observed values were then used for model acceptance. Whenever possible, the parameters B1 to B6 were determined not only for the whole five years of data, but also for the five annual sets of seventy-two values. For reasons to be explained later, analysis of variance and the standard errors of the parameters were found only for the results obtained from an analysis of the entire five years of data by ordinary least squares, three variations of weighted least squares, and generalised least squares assuming lag one correlation among the ordinary least squares residuals.

As explained in section 3.4, the parameters B1 to B6 have physical justification. For them to make sense, they must have the correct sign and be within certain limits. Parameter B1 and B2 must be positive and less than one. B3 and B4 must be negative, while B5 and B6 must be positive. The value of B1 should be close to [redacted], while the value of B6 should be much larger than any of the other parameters.

To determine the accuracy of the prediction, two criteria were selected. The first criterion, TSE, consisted of the sum of the square errors between the measured groundwater levels and the predicted as converted from the value of W(t) obtained from equation (3.12). The second criterion consisted of the sum of the absolute differences between measured and predicted groundwater levels, and was called DIF.

In equation form, TSE and DIF become:

$$TSE = \Sigma (YP(t) - Y(t))^2 \qquad (5.1)$$

$$DIF = \Sigma \left| YP(t) - Y(t) \right| \qquad (5.2)$$

where YP(t) refers to the predicted groundwater level at time t, and Y(t) the observed groundwater level at that same time.

5.1.1 ORDINARY LEAST SQUARES

The method of ordinary least squares was used first. It is given by equation (4.2), and the results are presented in table 5.1. It was applied to the whole five years of data as well as to the data corresponding to each individual year. The obtained parameter estimates were then used to predict the groundwater levels during the sixth year and compare them with the observations.

TABLE &5.1

ORDINARY LEAST SQUARES

	PARAMETERS						YEAR 6	
YEAR	B1	B2	B3	B4	B5	B6	TSE	DIF
1	0.881	0.305	-0.515	-0.456	0.628	72.45	39619	1235
2	0.943	0.326	-1.332	0.000	0.410	32.87	3769	366
3	0.943	0.049	-1.088	-0.303	-0.054	50.44	8750	600
4	0.945	0.533	-1.550	0.661	0.180	22.40	7972	523
5	0.987	1.065	0.608	-1.039	0.443	7.87	4954	445
1-5	0.953	0.485	-0.674	-0.229	0.270	28.37	5244	430

Although the parameters from the five years of data can be physically justified, many of the parameter values for a particular year seem incongruous. A positive value of B3 for year five and of B4 for year four, a value greater than one for B2 in year five, and a negative value of B5 for year three, as well as the wide range of B6 do not make sense and shall be discussed in more detail in a later section.

Figure 5.1 is a plot of the measured and predicted groundwater levels for the sixth year using the parameters from an analysis of the whole data set. For the first half

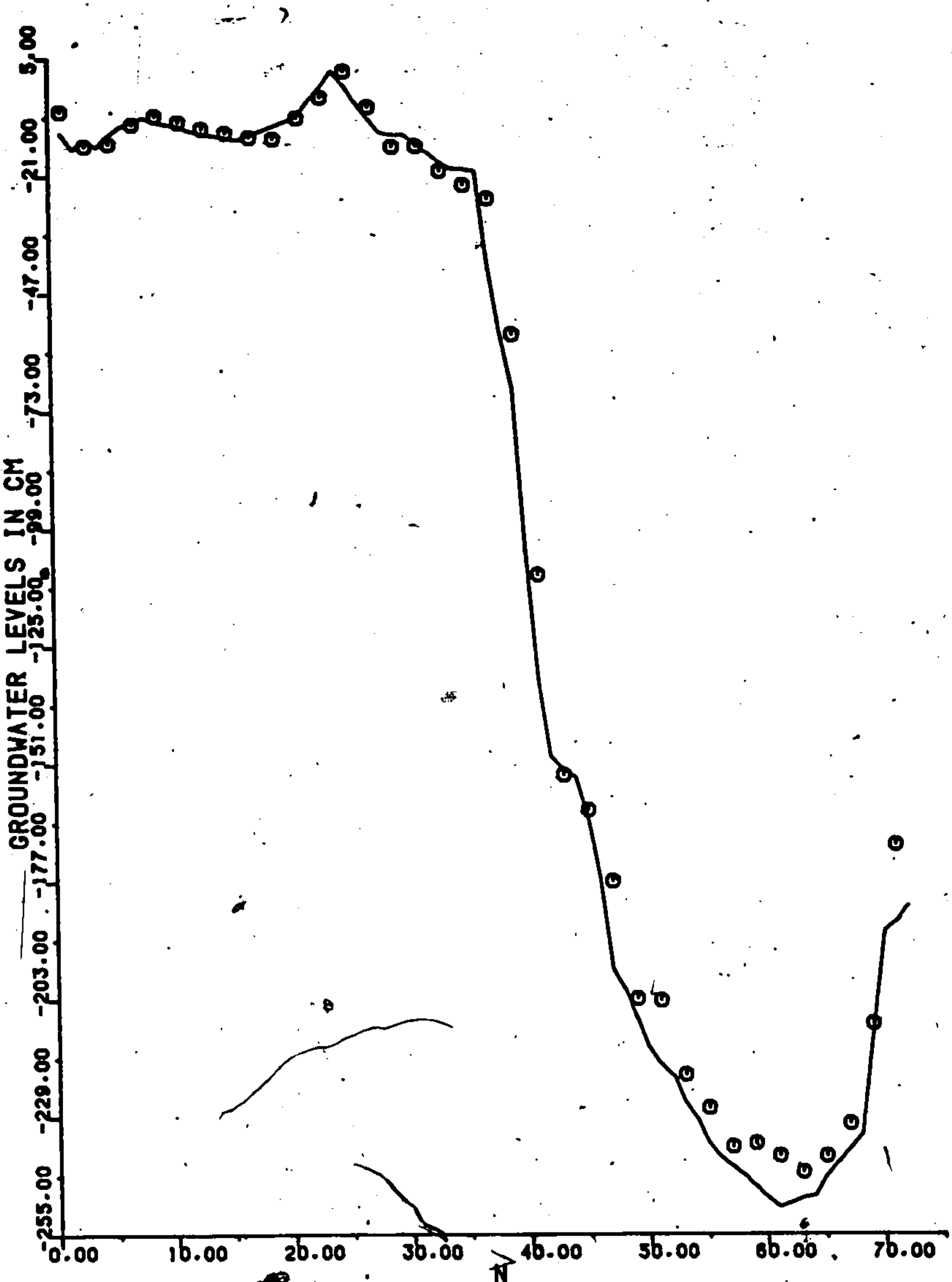


FIGURE 5.1: OBSERVED GROUNDWATER LEVELS AND PREDICTED BY ORDINARY LEAST SQUARES FOR THE SIXTH YEAR

of the year, the prediction is extremely close to the observed, but in the second, during the large evapotranspiration periods, there are noticeable differences between predicted and actual groundwater levels.

It can be argued that there is uncertainty about the evapotranspiration part of the analysis, as well as uncertainties about the snowmelt modelling, and the possible measurement errors during periods of rapid change mentioned earlier. Therefore, it was decided to assign different weights to uncertain observations which might provide better parameter estimates. This is attempted in the next section.

5.1.2 WEIGHTED LEAST SQUARES

As was done for the ordinary least squares analysis, where only the full five years of data provided sensible parameter estimates, weighted least squares was applied not only to the individual years, but also to the full set of data in the hope of determining the possible source or sources of bias. Equation (4.3) provides the solution.

Since ordinary least squares analysis revealed recurring errors at certain of the seventy-two time periods throughout the year for periods three to fifteen, twenty-eight to forty-five, and fifty to sixty-five, weighted least squares was first applied with a diagonal weighting matrix of unit value except for the previously mentioned time periods when it was selected as 0.2. The method is called WLS1 and its results are in table 5.2.

It was expected that perhaps the snowmelt portion of the model was most uncertain. This was reflected by weighted least squares where a diagonal weighting matrix equalling unity at every time step except for the snowmelt periods when the assigned weights were equal to 0.2 was applied. The method is called WLS2 and its results are in table 5.3.

Figure 5.1 revealed larger discrepancies between observed and predicted groundwater levels during large evapotranspiration periods by the ordinary least squares parameters than at other times of the year. Therefore, during the second half of the year, a weight of 0.2 was assigned. The method is called WLS3 and its results are in table 5.4.

TABLE &5.2

WLS WITH LESS WEIGHT AT CERTAIN TIME PERIODS (WLS1)

	PARAMETERS						YEAR 6	
YEAR	B1	B2	B3	B4	B5	B6	TSE	DIF
1	0.823	-0.228	-0.360	-0.275	0.534	107.32	116516	2056
2	0.932	0.124	-1.236	-0.015	0.326	43.88	7320	531
3	0.967	0.062	-0.628	-0.472	-0.041	42.47	8262	588
4	0.969	0.568	-1.188	0.622	0.134	5.31	5349	408
5	0.986	0.987	0.189	-0.601	0.027	13.21	7260	482
1-5	0.959	0.278	-0.425	-0.206	0.190	28.29	7548	528

TABLE &5.3

WLS WITH LESS WEIGHT DURING SNOWMELT (WLS2)

	PARAMETERS						YEAR 6	
YEAR	B1	B2	B3	B4	B5	B6	TSE	DIF
1	0.880	0.325	-0.536	-0.456	0.627	72.97	40065	1242
2	0.938	0.318	-1.392	-0.030	0.456	35.91	4001	386
3	0.946	-0.034	-0.783	-0.477	-0.033	49.48	8550	602
4	0.945	0.537	-1.652	0.743	0.154	22.06	8297	535
5	0.988	0.862	0.840	-1.151	0.523	8.16	4938	445
1-5	0.953	0.460	-0.602	-0.277	0.291	28.44	5147	429

TABLE &5.4

WLS WITH LESS WEIGHT DURING EVAPOTRANSPIRATION (WLS3)

	PARAMETERS						YEAR 6	
YEAR	B1	B2	B3	B4	B5	B6	TSE	DIF
1	0.937	0.121	-0.002	-0.237	0.395	36.42	18813	843
2	0.967	0.184	-0.671	0.071	0.234	18.85	3178	299
3	0.891	0.150	-2.160	0.180	0.292	67.36	14045	709
4	0.961	0.471	-0.975	0.516	0.102	12.35	7486	495
5	0.973	1.267	0.454	-1.019	0.595	9.20	5388	469
1-5	0.970	0.450	-0.486	-0.181	0.224	16.47	3611	330

As tables 5.2 and 5.3 indicate, the parameter values obtained from the full data set for the first two weighted least squares analysis are of the correct sign and within reasonable limits. However, the estimates for a particular year may not be physically reasonable, as was the case with ordinary least squares. Actually, the weighted least squares analyses appear to worsen the estimates because for the first year by the first analysis and for the third year in the second analysis a negative value of B2 was obtained, which is physically impossible. And all the other values which had the opposite sign in the ordinary least squares analysis remain.

However, the third analysis by weighted least squares did provide very interesting results. The parameter estimates from the full data set generate low TSE and DIF. While this may be only a chance occurence, it likely indicates that a linear evapotranspiration model is totally inadequate. This suggests that proper modelling of the evapotranspiration might result in a model prediction closer to the observed. Figure 5.2 shows very close prediction using the WLS3 parameters for five years during the second half of the year when evapotranspiration is important, unlike the ordinary least squares predictions of figure 5.1.

However, the estimates for the individual years appear not to have improved much, even though the prediction of the

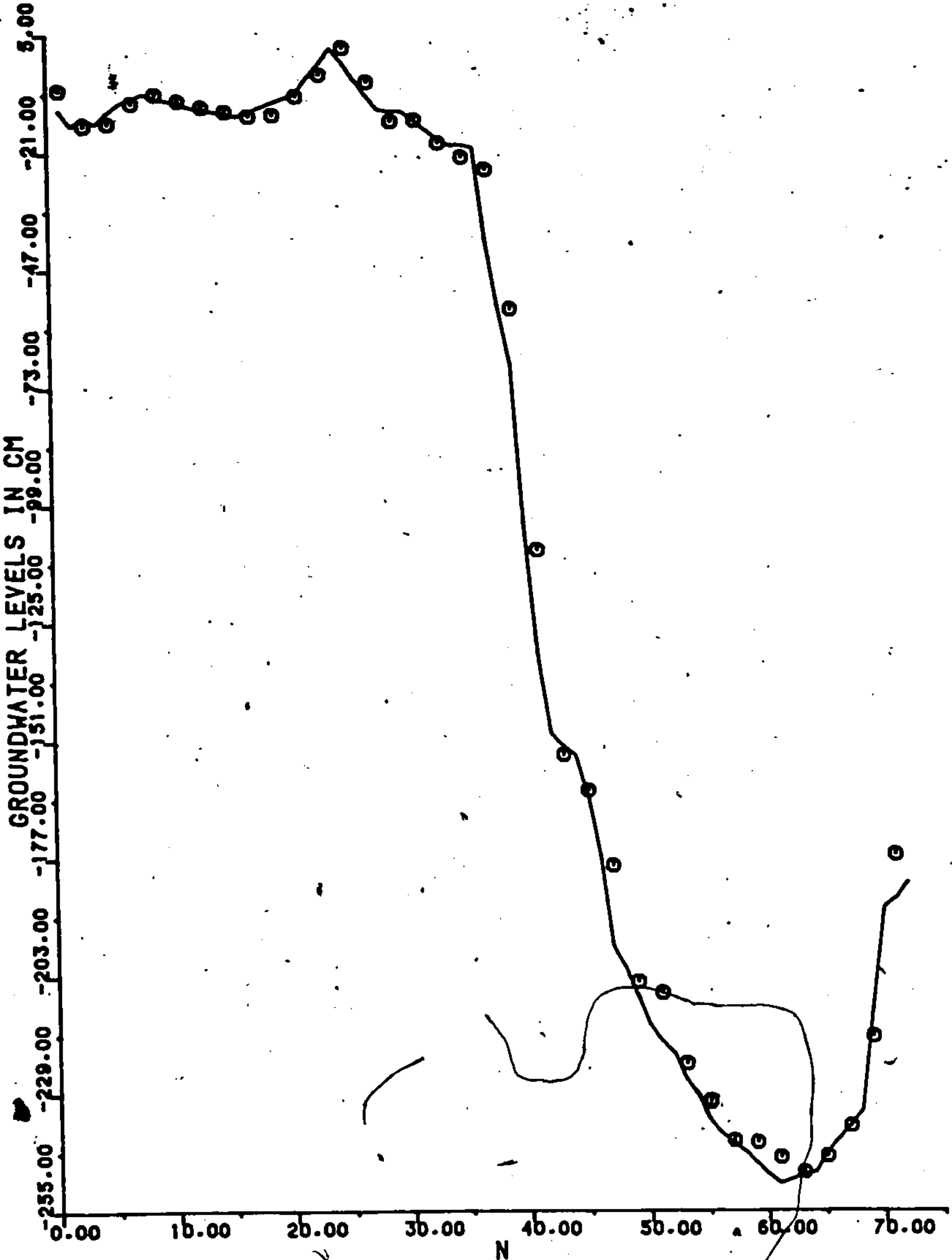


FIGURE 5.2: OBSERVED GROUNDWATER LEVELS AND PREDICTED BY WLS3 FOR THE SIXTH YEAR

sixth year with the second year parameters results in extremely low TSE and DIF as compared with the other results. Too many of the parameter values from annual analysis do not make sense, though. B1 is larger than one in year five, B3 is positive that same year, B4 is positive in years two, three, and four, while B6 still varies widely. Consequently, the improvement in parameter estimation by the third weighted least squares analysis was not very successful. Furthermore, as figures 5.3, 5.4, and 5.5 reveal, the three variations of weighted least squares have made little change on the shape of the residuals. The three plots are quite similar to the ordinary least squares residuals of figure 3.3.

Therefore, weighted least squares does not appear to improve parameter estimates. This leads to the belief that perhaps generalised least squares (GLS) is the method to use if it is desired to employ information obtained from the residuals in improving parameter estimates.

5.1.3 GENERALISED LEAST SQUARES

Generalised least squares was applied with two different assumptions about the error covariance matrix needed for the solution of equation (4.6). First, lag one autocorrelation was assumed among the residuals and applied to the total data set as well as to the five individual years. The autocorrelation coefficient of the errors p, as determined from

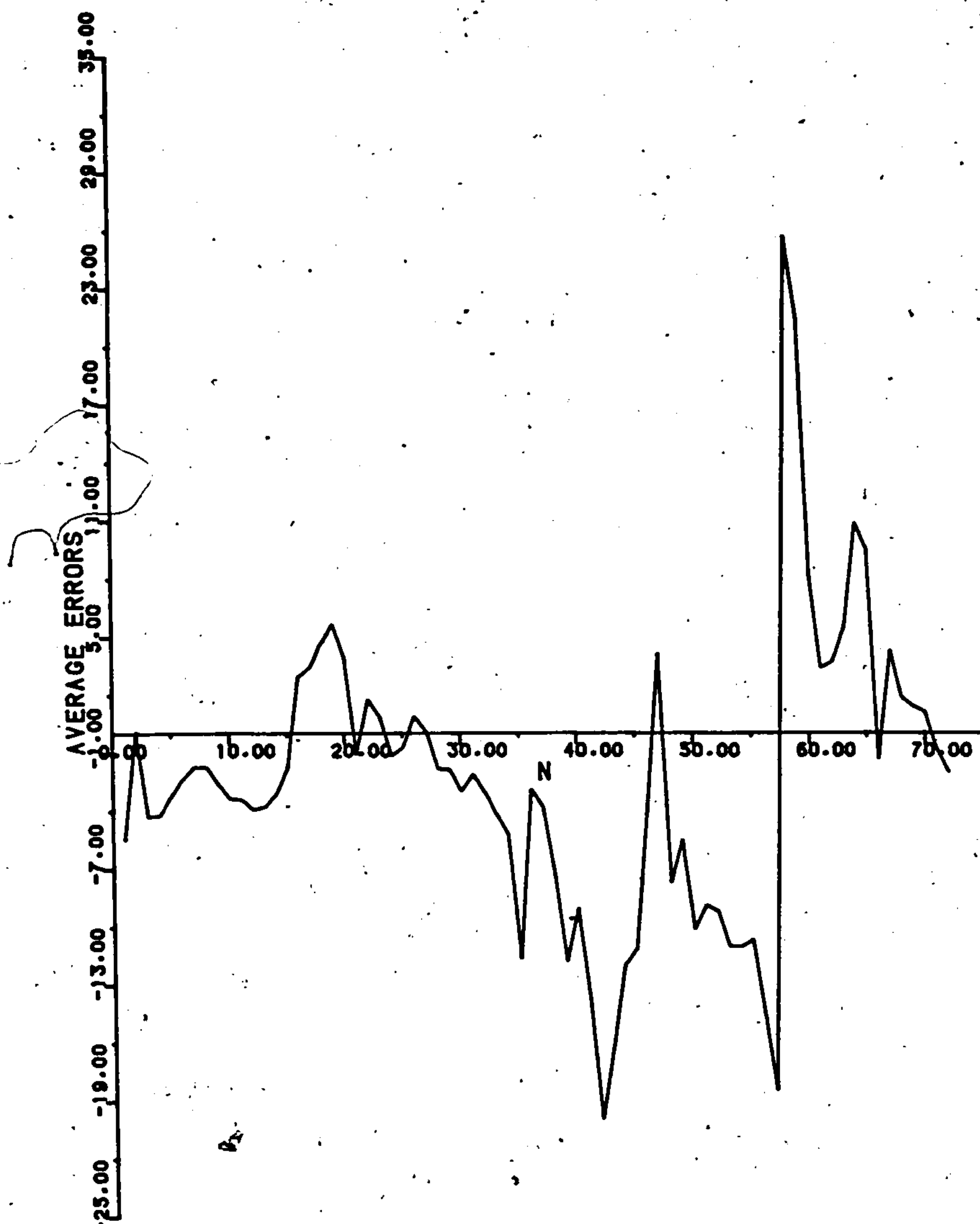


FIGURE 5.3: AVERAGE ERRORS BY WLS1 OVER FIVE YEARS

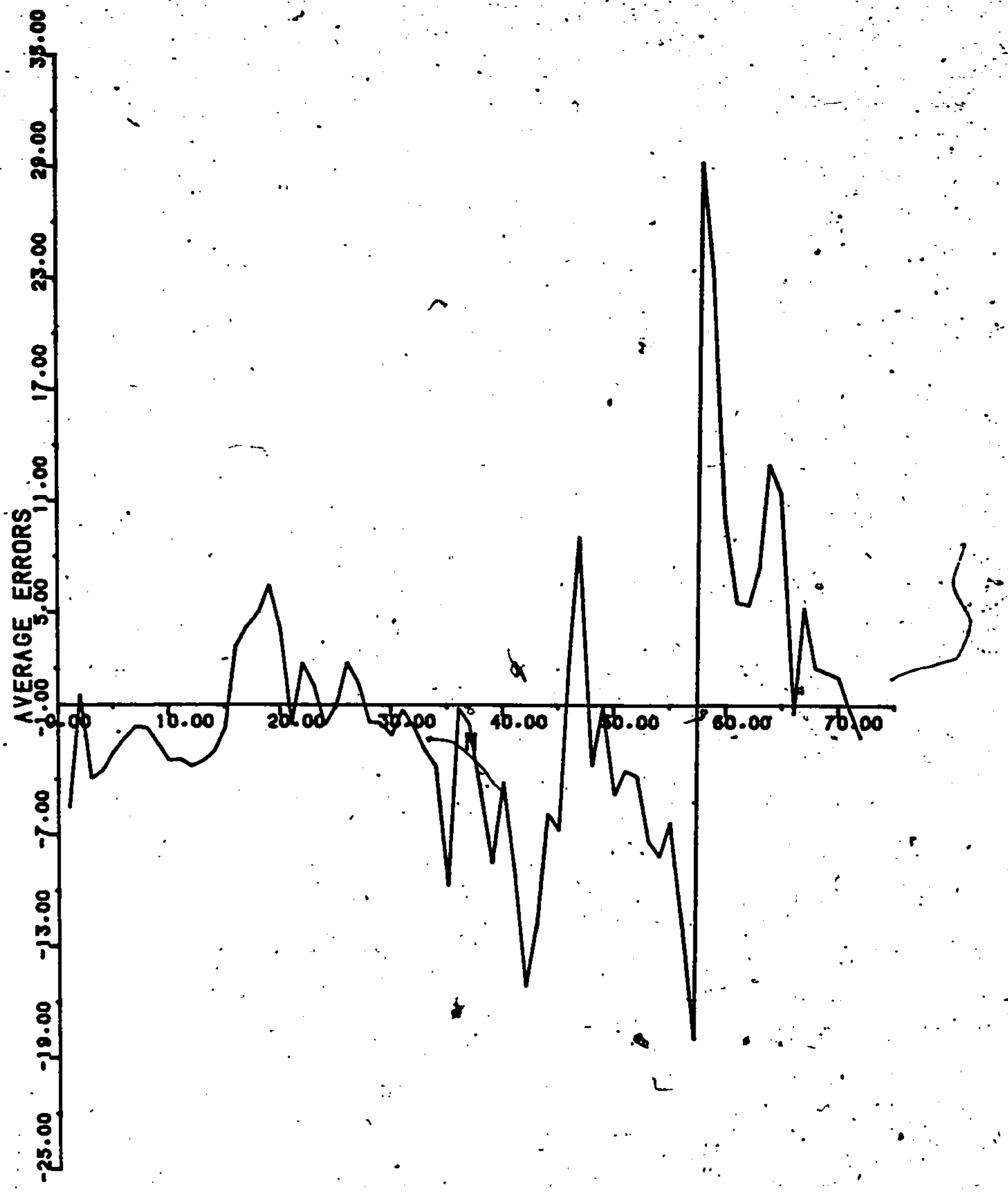


FIGURE 5.4: AVERAGE ERRORS BY WLS2 OVER FIVE YEARS

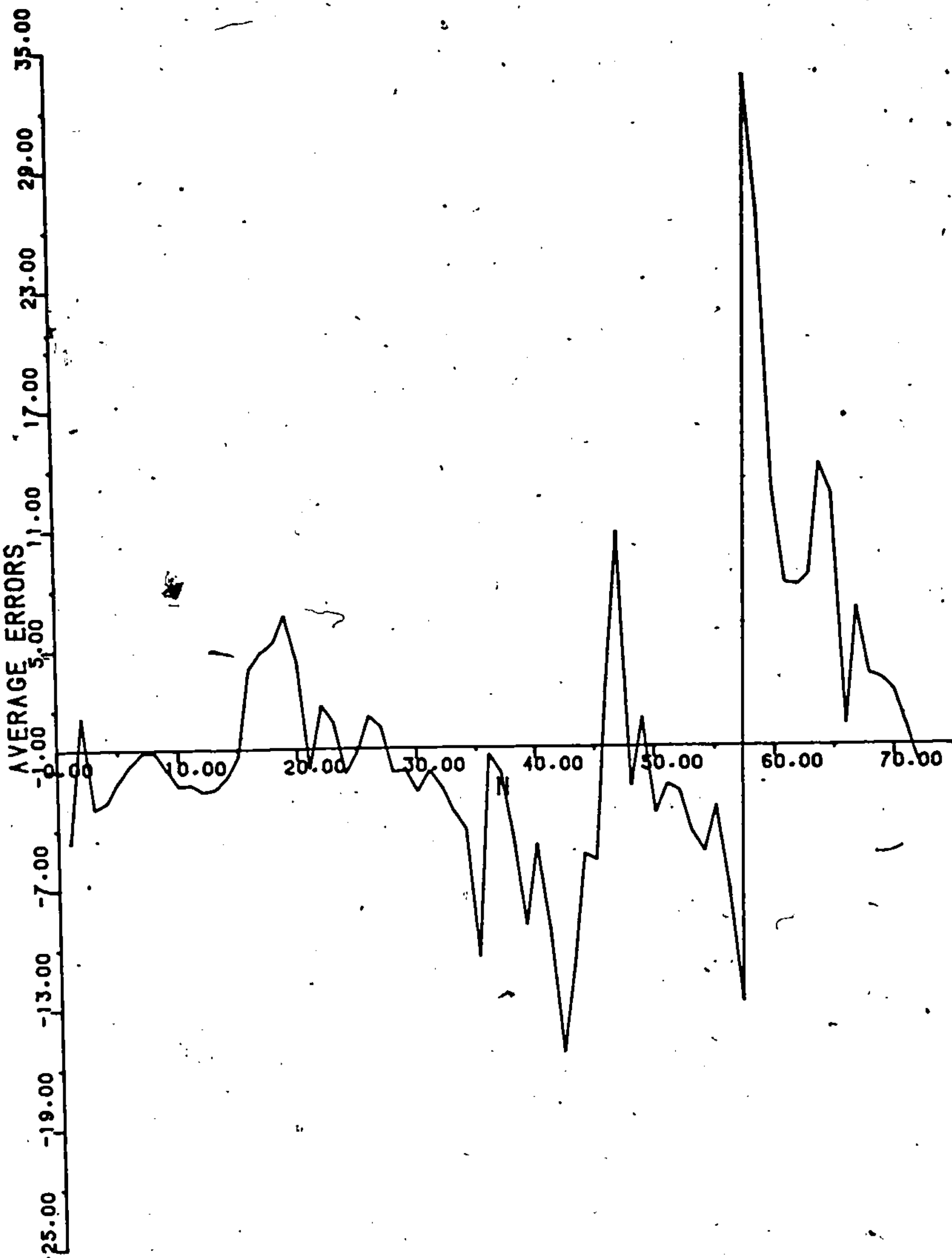


FIGURE 5.5: AVERAGE ERRORS BY WLS3 OVER FIVE YEARS

the average ordinary least squares residuals, was found to be 0.5145 using the following equation:

$$p = \Sigma(e(t)\ e(t-1)) \ / \ \Sigma(e(t-1))^2 \qquad (5.3)$$

where e(t) are the residuals or error terms at time t after applying ordinary least squares. Only lag one autocorrelation was assumed because the error covariance matrix is known [63], as given by equations (4.7) and (4.8). The method is called GLS and its results are presented in table 5.5.

TABLE &5.5

GLS ASSUMING LAG ONE AUTOCORRELATION

	PARAMETERS						YEAR 6	
YEAR	B1	B2	B3	B4	B5	B6	TSE	DIF
1	0.827	0.204	-0.878	-0.237	0.385	103.50	91595	1829
2	0.956	0.146	-0.986	-0.010	0.152	26.89	3383	337
3	0.923	-0.150	-1.104	-0.200	-0.211	58.92	14178	757
4	0.944	0.307	-1.210	0.485	0.207	23.70	7417	516
5	0.980	0.826	0.522	-0.880	0.357	10.46	4536	418
1-5	0.937	0.162	-0.595	-0.106	0.108	34.66	8749	580

Then, lag one autocorrelation was assumed for different time periods. For the time periods from three to fifteen, the autocorrelation coefficient p, as determined from equation (5.3), was found to be 1.0545, for time periods twenty-eight to forty-five it was 0.7741, for fifty to fifty-eight it was 0.6923, and finally for fifty-nine to sixty-five it was found to be 1.2260. The analysis was performed on a yearly basis because such an analysis for the full five years of data resulted in an error covariance matrix of order three hundred and sixty with a large number of small values which when inverted leads to an ill-conditioned system. The results are presented in table 5.6.

TABLE &5.6

GLS ASSUMING AUTOCORRELATIONS AT CERTAIN TIMES

	PARAMETERS						YEAR 6	
YEAR	B1	B2	B3	B4	B5	B6	TSE	DIF
1	0.706	0.240	-1.087	-0.133	0.803	170.00	285416	3132
2	0.947	0.211	-0.830	0.049	0.126	33.59	9199	588
3	0.960	0.077	-0.990	-0.237	-0.267	47.50	11055	684
4	0.970	0.193	-0.643	0.238	0.221	8.66	3995	327
5	0.936	0.460	0.160	-0.713	0.305	42.25	16769	809

Although the parameters for an individual year were at times seemingly as erroneous as for the ordinary and weighted least squares analysis, the predicted error was small for the second year using the first method of generalised least squares and also small for the fourth year using the second method. However, that second very low prediction error comes with a positive value of B4, which is physically impossible. Also, the plot of the residuals of the first method of generalised least squares in figure 5.6 shows very little change from figure 3.3. Therefore, it cannot be concluded that generalised least squares is a better estimation method, especially since a number of parameters remain with the opposite sign.

5.1.4 CONCLUSION ON LEAST SQUARES

To conclude the least squares methods, it can be observed that while the results for individual years appear too often physically impossible, at least the results for the full five years make sense. The sixth year predictions as quantified by TSE and DIF are quite similar using the parameters from five years of data. These parameters also do not show too much change, unlike the parameters from annual values which show a large range and at times reach physically impossible values.

Weighted least squares applying a lesser weight during

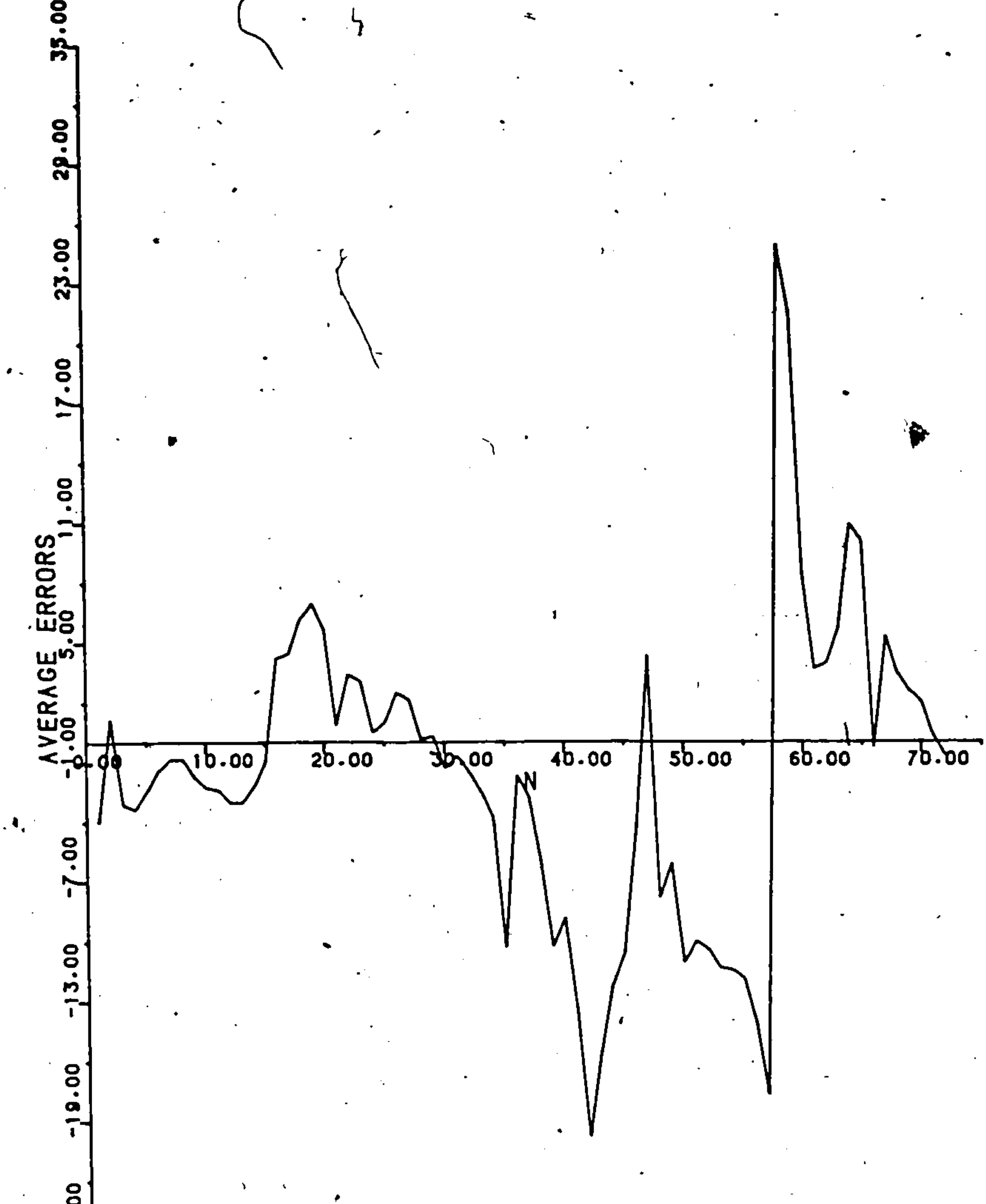


FIGURE 5.6: AVERAGE ERRORS BY GLS OVER FIVE YEARS

the second half of the year when evapotranspiration was most significant provided the lowest TSE and DIF with its five year parameters. It also predicted very closely the groundwater levels during that evapotranspiration period of the sixth year.

It appears that the parameters obtained from analyzing five years of data from all least squares methods can correctly predict with the model since they generate low error criteria (TSE and DIF) for the sixth year, while those from an annual analysis do not necessarily do so. All the methods provide parameters from analyzing the whole data set which are quite similar, but those from WLS3 predicted the sixth year much better. However, from the plots of the residuals, it cannot be concluded that weighted least squares or generalized least squares improve the parameter estimates.

Since off-line least squares methods show no improvement in the parameter estimates, an on-line method such as the extended Kalman filter may indicate source or sources of bias in the estimation process. This is attempted in the next section.

5.1.5 THE EXTENDED KALMAN FILTER

The extended Kalman filter is the next technique that was applied. Equations (4.19) and (4.20) represent the system where the combined state and parameter estimate vector x is composed of the rate of change of W(t) as defined in equation (4.17), and then the six parameters B1 to B6. The non-linear function f(x(t),u(t),B(t)) is represented by the right-hand side of equation (4.17) for the first state, and the latest estimate of the parameters themselves for the six other states as shown below:

$$f(x(T),u(T),B(T)) = \begin{bmatrix} B1(T).W(T-1) + B2(T).P(T-1) + \\ B3(T).T(T) + B4(T).BS(T) + \\ B5(T).BSS(T-1) + B6 \\ B1(T) \\ B2(T) \\ B3(T) \\ B4(T) \\ B5(T) \\ B6(T) \end{bmatrix} \qquad (5.4)$$

The F matrix, as defined by equation (4.25), is a seven by seven of the shape shown below, where F(1,2) to F(1,7) are the change terms due to the extended version of the Kalman filter obtained from equation (4.28):

$$F = \begin{array}{c} \begin{matrix} \text{B1-1} & W(T-1) & P(T-1) & T(T) & BS(T) & BSS(T-1) & 1 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad (5.5)$$

For example, for the first and only original state, groundwater storage $W(t)$, applying equation (4.28), $F(1,2)$ can be found by differentiating equation (4.17) with respect to B1, which results in $W(t-1)$. Differentiating equation (4.17) with respect to all the other parameters results in $F(1,3)$ to $F(1,7)$.

The vector H, which multiplies the state vector x in order to give the measurements y, has its first value equal to unity while the next six values are zero. The output y consists only of the amount of water above datum in groundwater storage.

Since the Kalman filtering technique requires an initial estimate for the states x, initial parameter values obtained by ordinary least squares as applied to the five years of data were selected for the second to the seventh states. For the first state, the amount of water above datum in groundwater storage, the calculated value of $W(t)$ for the starting time, was used.

An initial error covariance matrix Po of the shape of equation (4.26), with some zero terms, resulted in lack of parameter variation because this assumed that the states were independent of one another. However, any variation in one parameter will influence the groundwater storage state which will influence the other parameters. Once non-diagonal matrices were introduced for P and Q, the parameters varied and converged as expected. The initial values of the

P matrix are always chosen large because they will decrease and converge to the actual. P(1,1) was selected as 1000000, for P(1,2) to P(1,7) and P(2,1) to P(7,1) it was 100000, while for the rest of the matrix it was 1000. Q(1,2) to Q(1,7) and Q(2,1) to Q(7,1) were equal to 0.7, while the rest of the matrix was equal to 0.01. Since y was equal to the first state, the measurement error variance R(1,1) was equal to Q(1,1) the system error variance of the first state, which was varied in order to assess the optimal fit for the first five years of data.

The determination of the best results from the extended Kalman filter is rather straightforward. The use of the minimum sum of square errors between the predicted and measured groundwater storages W(t) was selected as the criteria of optimality. In other words, the minimum TSE over a realistic range of Q(1,1) and R(1,1) values provided the optimal parameters. This resulted in the optimal estimates of table 5.7 after three hundred and sixty time steps or five years, which required a value of Q(1,1) and of R(1,1) equal to one hundred and eighty-five. This value of Q(1,1) was used to run the extended Kalman filter in order to predict the sixth year.

Figure 5.7 shows the measured and the predicted groundwater levels as given by the extended Kalman filter. The predictions, besides generating a lower TSE and DIF than by

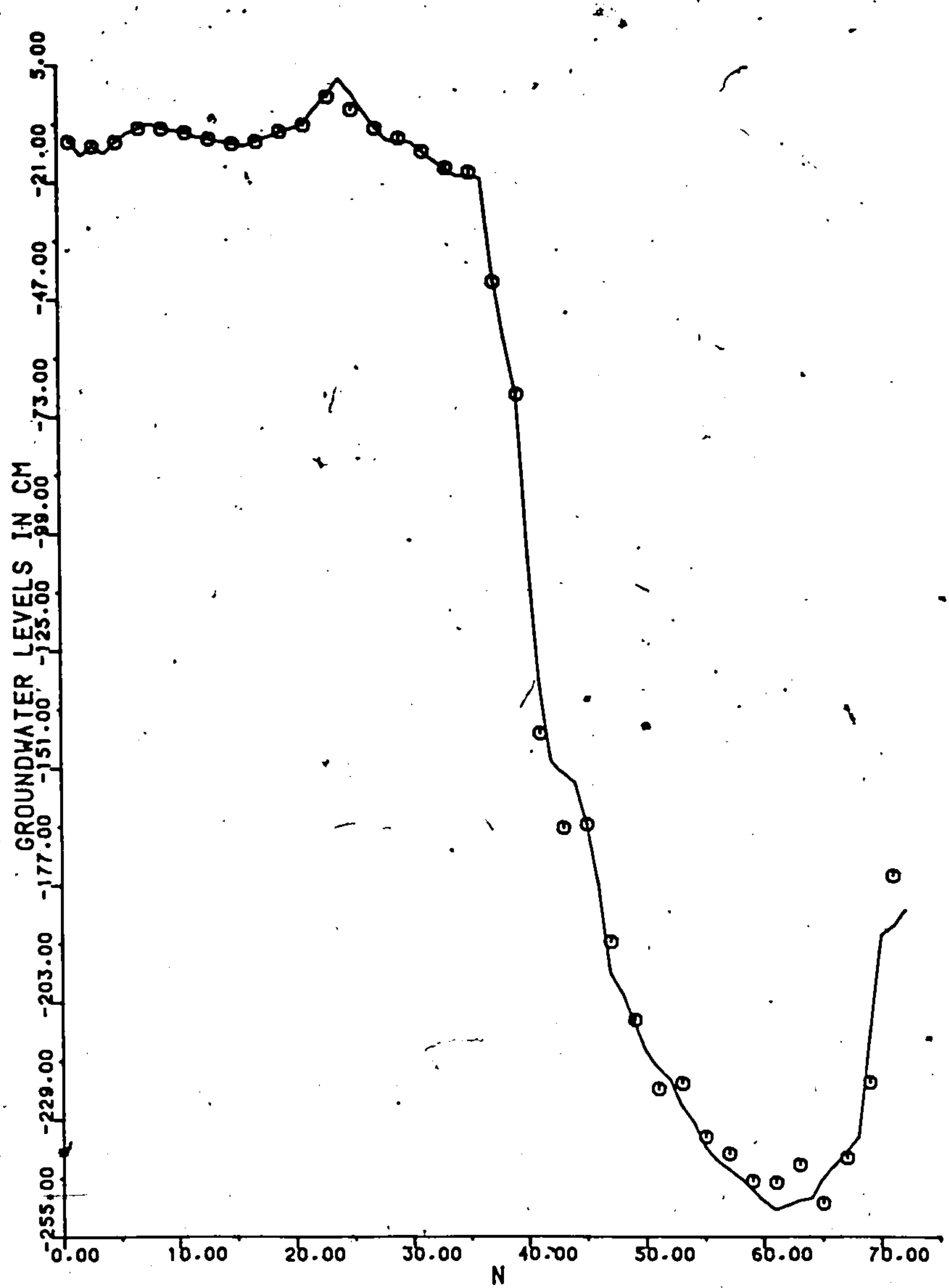


FIGURE 5.7: OBSERVED GROUNDWATER LEVELS AND PREDICTED BY KALMAN FILTER FOR THE SIXTH YEAR

TABLE 5.7

EXTENDED KALMAN FILTER ANALYSIS FOR TOTAL DATA SET

PARAMETERS						YEAR 6	
B1	B2	B3	B4	B5	B6	TSE	DIF
0.952	0.484	-0.675	-0.230	0.269	28.36	1091	165

Note: These parameter estimates were obtained after 360 time steps or five years. The parameters actually varied as shown by figures 5.8 to 5.13.
The values of TSE and DIF were not obtained from predicted groundwater levels with constant parameters, but with groundwater levels predicted with the varying parameters and the updated states given by the filter.

ordinary least squares, followed the groundwater level fluctuations very closely, especially during the large evapotranspiration periods. The obtained parameter estimates after three hundred and sixty time steps are very close to the ordinary least squares estimates with five years of data, but the parameters actually varied.

Plots of the variation of the six parameters (figures 5.8 to 5.13) indicate more or less random variations throughout the year except during the periods of rapid change in groundwater levels, most noticeably during spring. This may be that Q(1,1) and R(1,1) were inadequately assessed for these time periods, where they should be larger. Or, with nonlinear processes at work, the model formulation during that short span of time was inadequate.

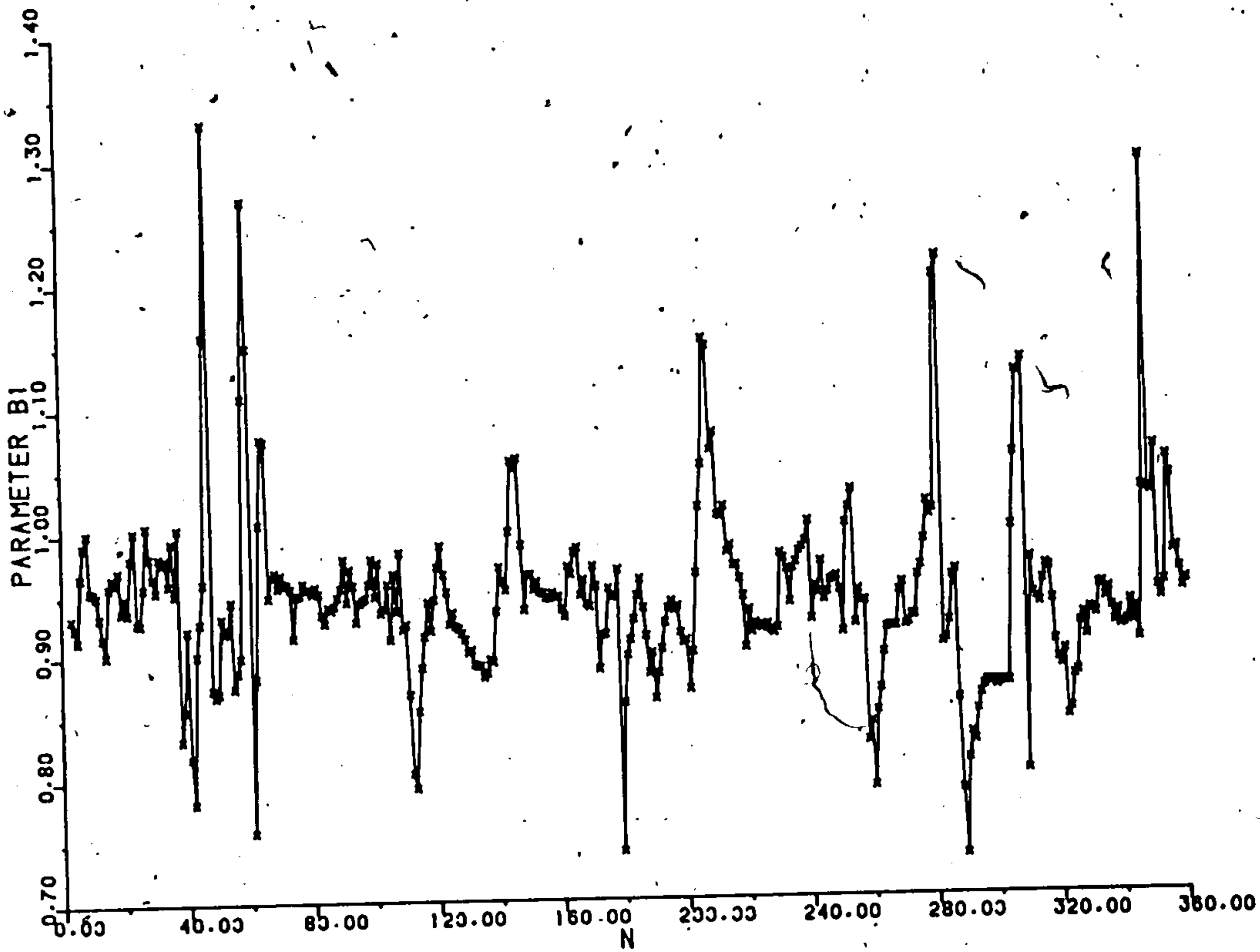


FIGURE 5.8: VARIATION OF PARAMETER B1 IN TIME

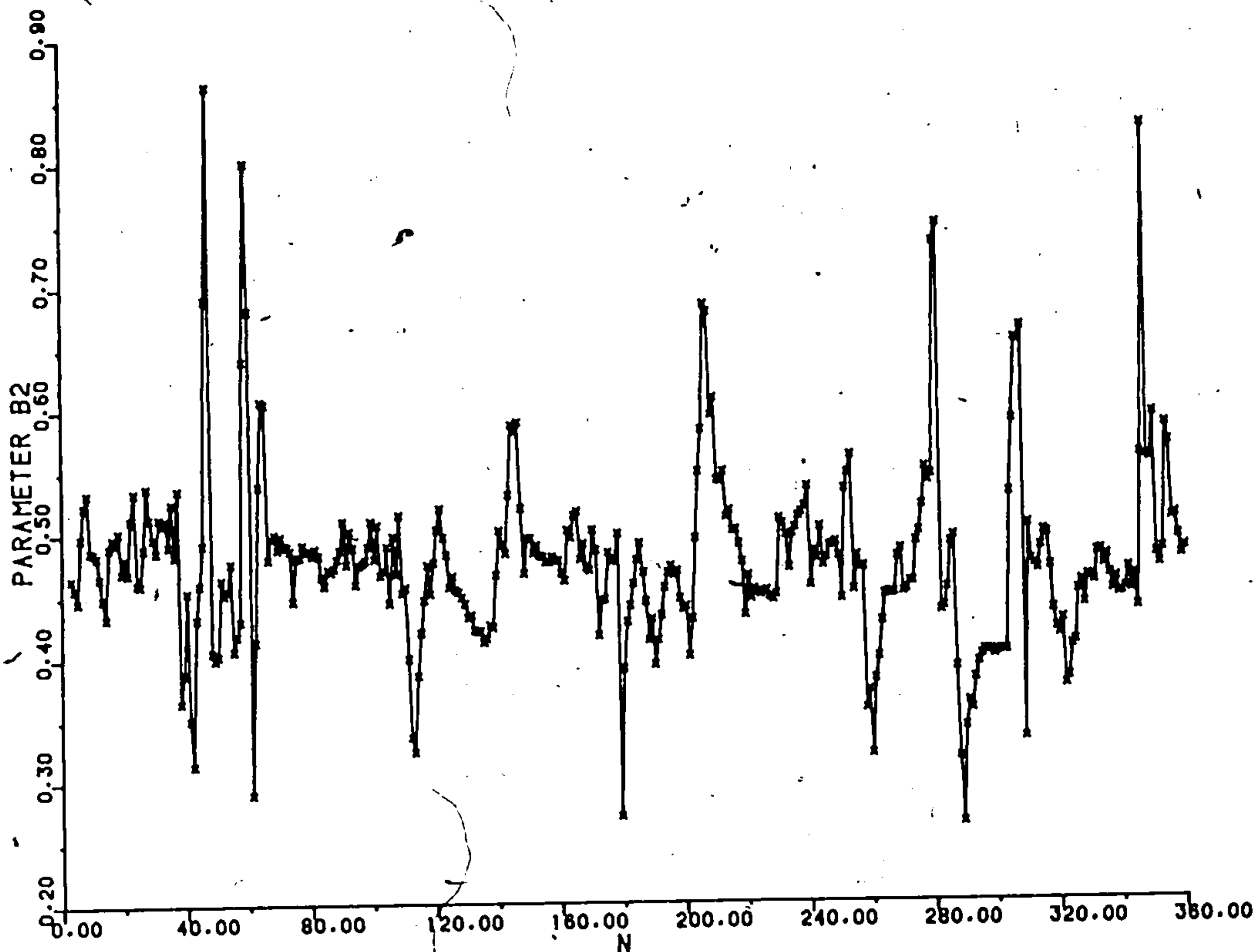


FIGURE 5.9: VARIATION OF PARAMETER B2 IN TIME

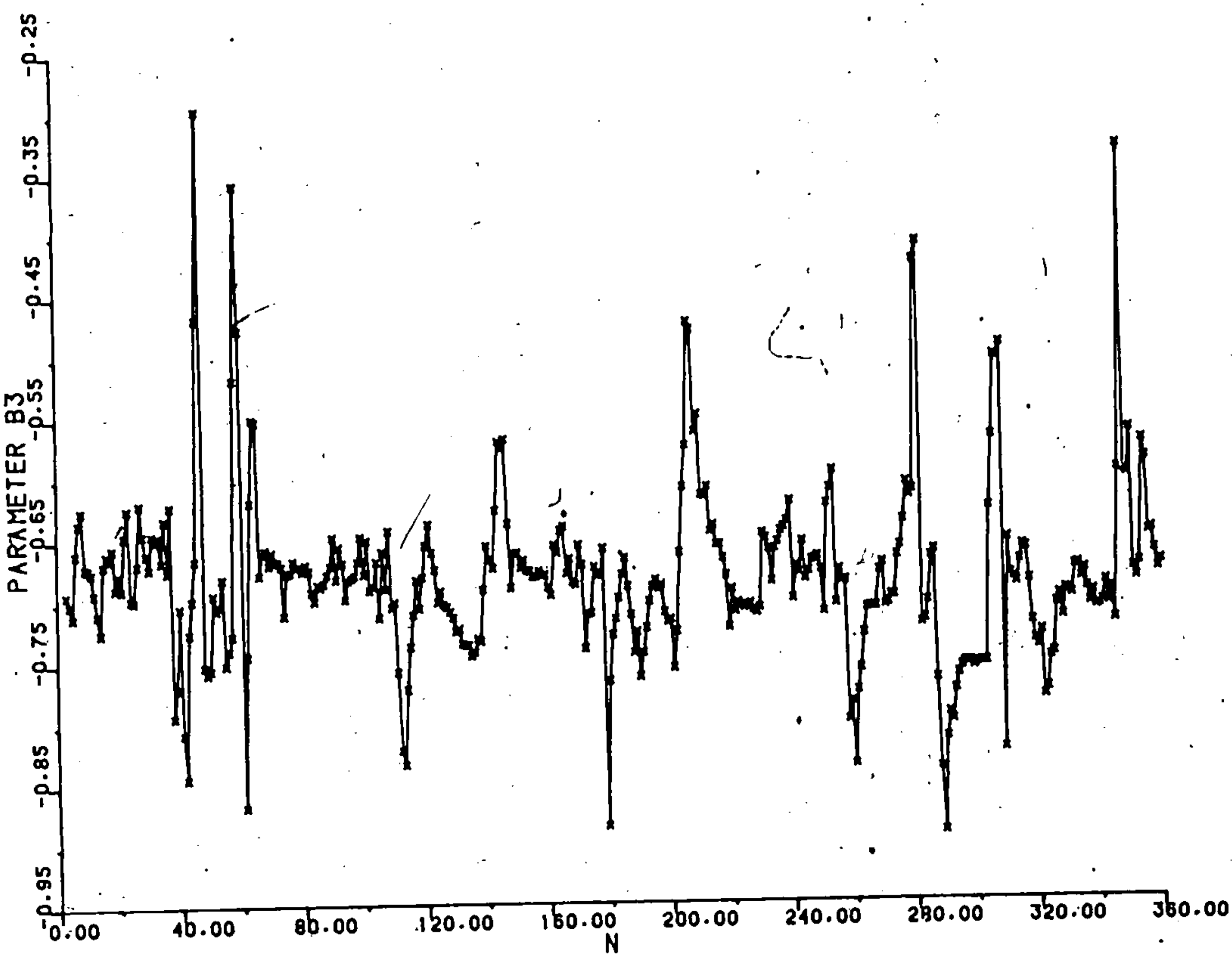


FIGURE 5.10: VARIATION OF PARAMETER B3 IN TIME

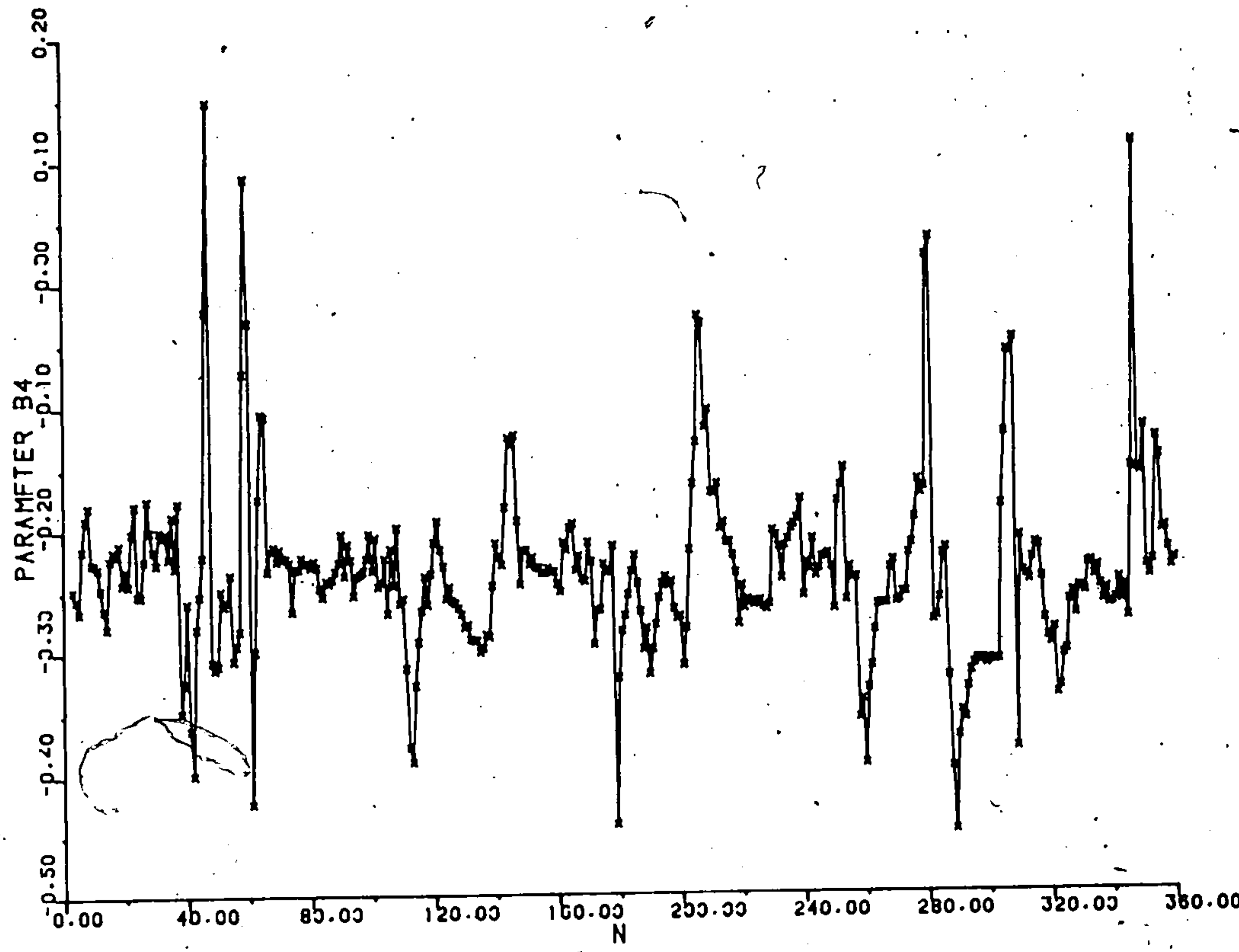


FIGURE 5.11. VARIATION OF PARAMETER B4 IN TIME

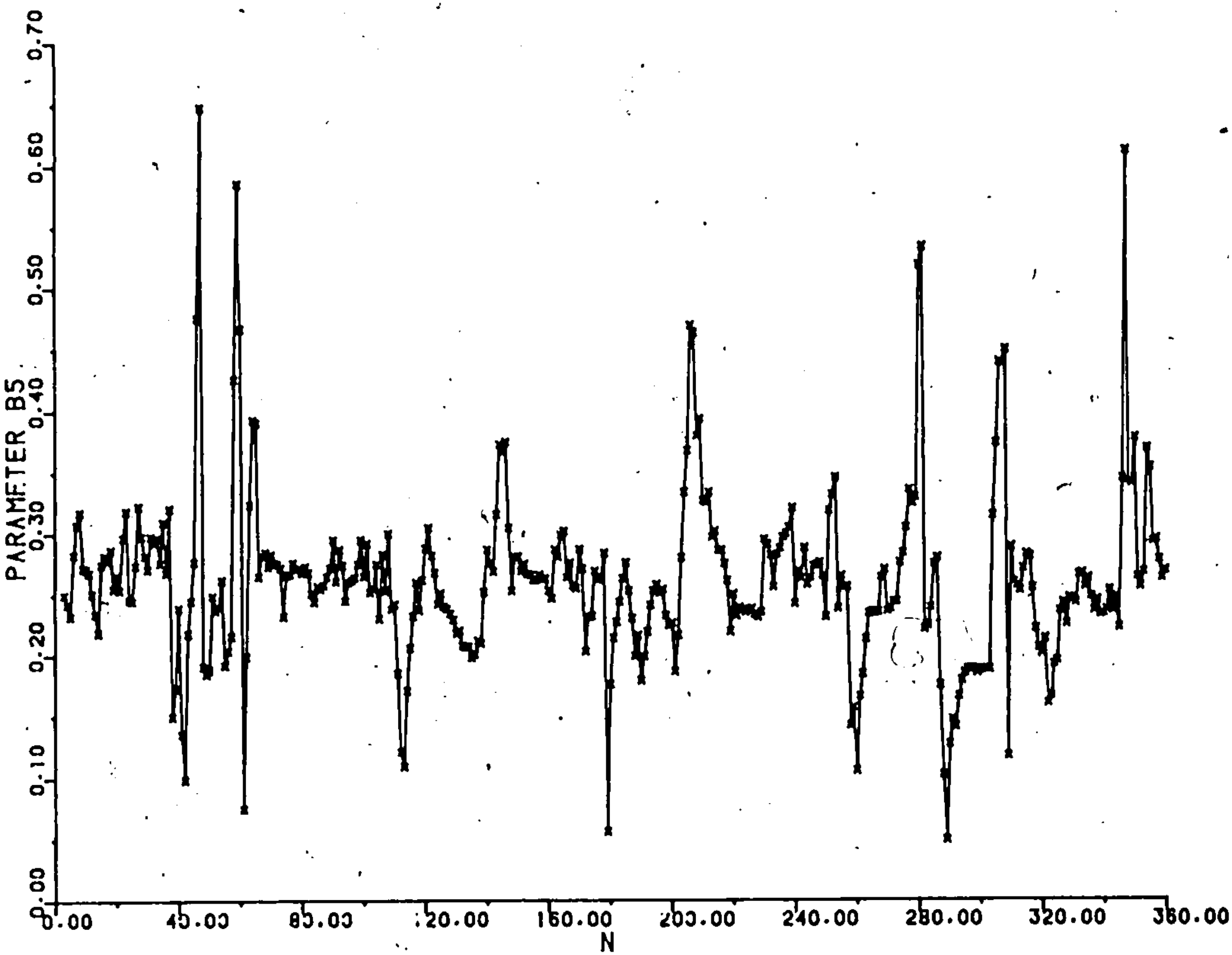


FIGURE 5.12: VARIATION OF PARAMETER B5 IN TIME

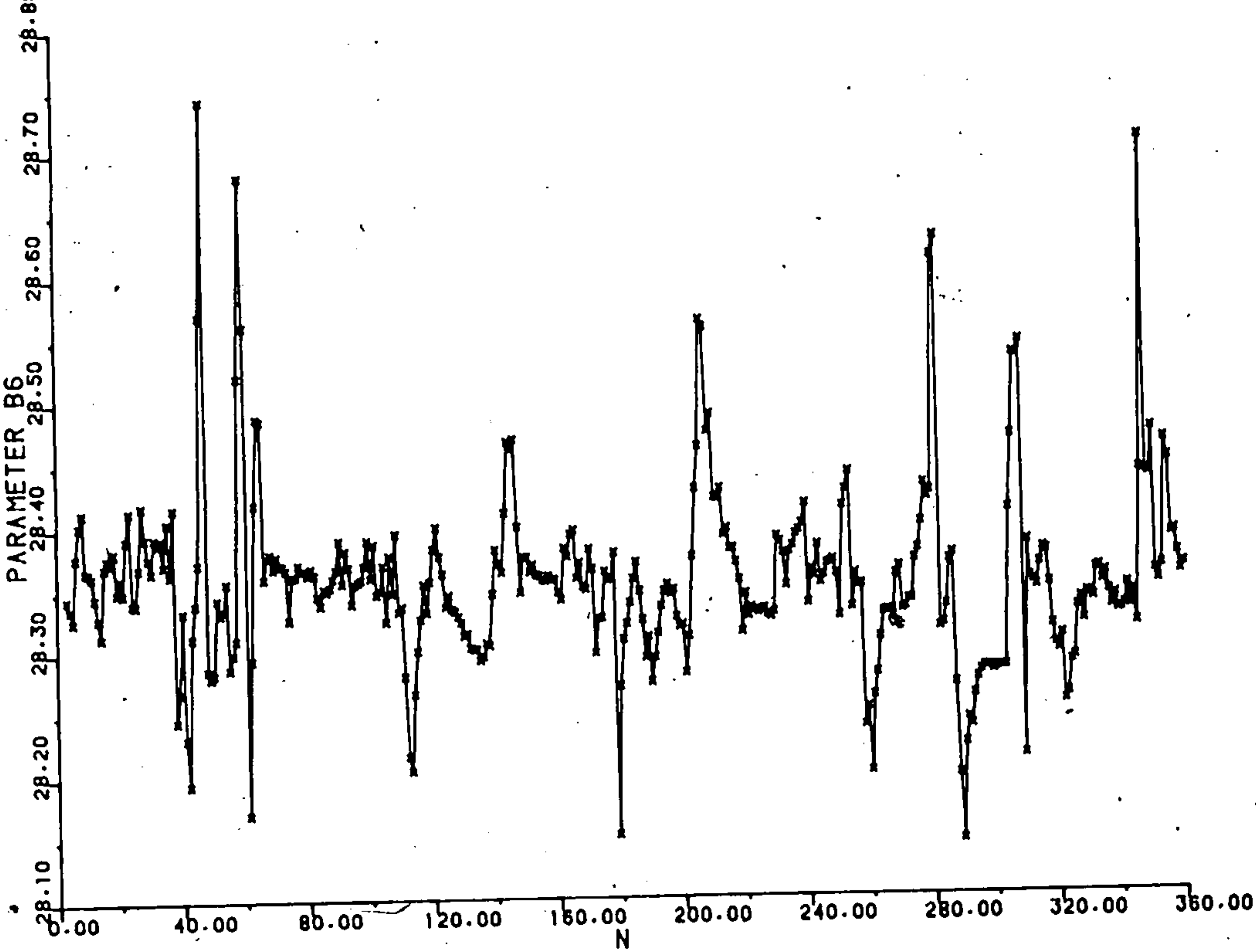


FIGURE 5.13: VARIATION OF PARAMETER B6 IN TIME

The large value of one hundred and eighty-five for the measurement error variance is presumed due to incomplete and highly approximate knowledge about the geology of the site. A slight change in either specific yields or layer thicknesses of the four soils at the site would result in much different values of the amount of water above datum in the groundwater storage. This is most likely also the reason for lack of seasonal variation of some of the parameters. It was expected that for the parameter B2 especially, which is the proportion of rainfall contributing to groundwater levels, some variation could be seen throughout the year. Instead, any possible variation probably damped by the errors, the parameters varied randomly except during periods of rapid change in groundwater levels.

In order to conclude a critical look at the results, however, standard errors must be determined for the parameter estimates and analysis of variance must be performed, which will be done in the next subsection.

5.1.6 STANDARD ERRORS OF ESTIMATES AND ANALYSIS OF VARIANCE

By definition, sixty-eight percent of the parameter estimation values can be expected to be found within plus or minus one standard error of the estimate. Given the standard error(S.E.), confidence limits(C.L.) on parameters can easily be found by multiplying the standard error by the t distributed deviate corresponding to a given percentage of confidence $t(\alpha)$ [27]:

$$C.L. = Bi \pm t(\alpha)\ (S.E.) \qquad (5.6)$$

where Bi represents the estimate of parameter i.

The procedure for determining the standard error of parameters estimated by ordinary least squares is straightforward. First, the standard deviation of the residuals(SDR) must be calculated by summing the squares of the residuals and dividing by the appropriate degrees of freedom(DF) [27]:

$$SDR = \Sigma\,(YP(t) - Y(t))^2 \;/\; DF \qquad (5.7)$$

Since part of four of the five input series, P(t-1), BS(t), T(t), and BSS(t-1), contain zero values (see Appendix B), the standard errors of parameters B2 to B5 were calculated summing only the squares of those residuals corresponding to some non-zero input and dividing by the appropriate degrees of freedom, the latter being given in table 5.8.

Once the standard deviation of the residuals corresponding to each of the six parameters have been determined, the inverse of the square of the measurement matrix, inv.(X'X), must be calculated. The diagonal elements of the six by six matrix, multiplied by the appropriate standard deviation of the residuals, result in the variances of the parameter estimates. Calling C the inverse of X' X, then the standard

TABLE &5.8

STANDARD ERROR AND CONFIDENCE LIMITS

PARAMETER	DEGREES OF FREEDOM	STANDARD ERROR	CONFIDENCE LIMIT 95%
ORDINARY LEAST SQUARES			
B1	354	0.0109	0.931 , 0.975
B2	254	0.1376	0.211 , 0.755
B3	279	0.2638	-1.191 , -0.157
B4	279	0.1512	-0.525 , 0.066
B5	42	0.0934	0.088 , 0.452
B6	354	6.0869	16.43 , 40.29
GENERALISED LEAST SQUARES			
B1	354	0.0112	0.91[illegible] , 0.959
B2	254	0.1416	-0.116 , 0.494
B3	279	0.2708	-1.126 , -0.064
B4	279	0.1552	-0.312 , 0.096
B5	42	0.1064	-0.112 , 0.314
B6	354	6.2171	22.47 , 46.85
WLS1			
B1	354	0.0111	0.937 , 0.981
B2	254	0.1397	0.004 , 0.552
B3	279	0.2669	-0.948 , 0.098
B4	279	0.1530	-0.506 , 0.094
B5	42	0.0956	0.002 , 0.377
B6	354	6.1829	16.28 , 40.30
WLS2			
B1	354	0.0109	0.931 , 0.975
B2	254	0.1376	0.190 , 0.730
B3	279	0.2640	-1.119 , -0.085
B4	279	0.1513	-0.573 , 0.019
B5	42	0.0947	0.105 , 0.477
B6	354	6.0885	16.51 , 40.37
WLS3			
B1	354	0.0110	0.948 , 0.992
B2	254	0.1398	0.176 , 0.724
B3	279	0.2690	-1.013 , 0.041
B4	279	0.1542	-0.483 , 0.121
B5	42	0.0941	0.040 , 0.408
B6	354	6.1257	4.46 , 28.48

error of the parameters (S.E.) of parameter i equals [27]:

$$S.E.(i) = C(i,i)\ SDR(i) \qquad (5.8)$$

Confidence limits can then be found by adding or substracting from the parameter estimates the product of the standard deviation of the parameter estimates and the corresponding t distributed percentile as shown in equation (5.8) [27]. This method was applied not only to the ordinary least squares estimates, but also to the weighted least squares and generalised least squares estimates. The results are in tables 5.8.

When analyzing the full five years of data, the degrees of freedom are sufficiently large. However, individual years do not contain sufficient data of certain time series to provide a large number of degrees of freedom. For BSS(t-1), during the fourth year (see Appendix B) there are only six non-zero values, and one attempts to calculate six parameters. This is the likely reason that physically impossible parameter values were found for some of the yearly regressions while those for the whole data set always made sense. The number of degrees of freedom appears to be too small for reliable annual estimates. Therefore, only the results from the analysis of the full five years of data can be considered valid, which is why only for these values was analysis of variance performed.

Analysis of variance is a standard statistical tool to assess the many contributions to variation in regression models. The total sum of squares of the predicted variable Y can be divided into the sum of squares due to the mean, the sum of squares due to regression, and the sum of squares due to residuals [27]. Calling YA the average value of Y, the previous statement is expressed in mathematical form as follows:

$$Y'\ Y = n\ YA^2 + (B'\ X'\ Y - n\ YA^2) + (Y'\ Y - B'\ X'\ Y) \quad (5.9)$$

where all the terms have been previously defined. If n is the total sample length, and m is the number of parameters, then the corresponding degrees of freedom are, respectively, one, m minus one, and n minus m.

The coefficient of determination, R^2, which ranges from zero to one, equals to the ratio of the sum of squares due to regression and the sum of squares about the mean. The latter equals either to the total sum of squares minus the sum of squares due to the mean, or to the total of the sum of squares due to regression and the sum of squares due to residuals.

The coefficient of determination can be expressed in another way. Calling YP(t) the value of Y predicted by the model at time t, then the coefficient of determination equals [27]:

TABLE &5.9

ANALYSIS OF VARIANCE FOR OLS AND GLS

SOURCE	DEGREES OF FREEDOM	OLS SUM OF SQUARES	GLS SUM OF SQUARES
MEAN	1	5.56 E07	5.56 E07
REGRESSION	354	1.16 E07	1.10 E07
RESIDUAL	5	3.39 E05	3.53 E05
TOTAL	360	6.76 E07	6.70 E07
COEFFICIENT OF DETERMINATION		0.972	0.968

TABLE &5.10

ANALYSIS OF VARIANCE FOR WLS

SOURCE	DEGREES OF FREEDOM	WLS1 SUM OF SQUARES	WLS2 SUM OF SQUARES	WLS3 SUM OF SQUARES
MEAN	1	5.56 E07	5.56 E07	5.56 E07
REGRESSION	354	1.15 E07	1.16 E07	1,18 E07
RESIDUAL	5	3.49 E05	3.39 E05	3.43 E05
TOTAL	360	6.75 E07	6.76 E07	6.78 E07
COEFFICIENT OF DETERMINATION		0.971	0.972	0.972

$$R^2 = \Sigma(YP(t)-YA)^2 / \Sigma(Y(t)-YA)^2 \quad (5.10)$$

The positive square root of the coefficient of determination is called the correlation coefficient, which is a measure of the linear relationship between the dependent and the independent variables. The results for ordinary least squares and generalised least squares assuming lag one correlation among the residuals are in table 5.9, while those for the three variations of weighted least squares are in table 5.10.

The results from the analysis of variance as well as the standard errors and the parameter estimates themselves shall be discussed in more details in the next section.

5.2 DISCUSSION

The plots of figure 5.1 and 5.2, as well as the results from tables 5.1 to 5.5 reveal that the parameters obtained from the least squares methods by analysing five years of data result in not very large deviations between measured and predicted groundwater levels in the sixth year. Figure 5.7 and table 5.7 indicate, however, that the extended Kalman filter provides more accurate groundwater level predictions than all the least squares methods. But after five years of analysis, the parameter estimates were quite similar to the ones obtained by the least squares methods. However, it is believed these estimates could be further improved.

The assumptions made in section 3.3 must be reexamined in light of the results. It was mentioned that the time step Dt had to be chosen as five days because of the data available. Clearly, if daily data were available, then the lag times for rainfall and snowmelt would be a more appropriate number of days and the model prediction should be much closer to the observed. Furthermore, the number of degrees of freedom in the regression would be much higher leading to smaller standard errors for the parameter estimates.

Soil moisture obviously has an impact on groundwater level fluctuations. If it could be measured, then a more complex model could be formulated. There would be not only storage below the groundwater table, but also between the soil surface and the groundwater table. Some of the parameters could appear in both the above groundwater table and below groundwater table storage equations, and the solution of the coupled system would result in more reliable parameters, as in Kuczera's [37] coupled system of groundwater storage and streamflow equations.

The most limiting assumption was most likely that of spatial homogeneity. Correctly defining a groundwater system is a well-known problem in hydrology. Unlike surface water systems, which are usually well defined on the basis of topographic divides, groundwater systems are much less well defined, their boundaries not necessarily coinciding with local topographic features. Furthermore, any hydrogeological system consists of a range of geologic materials with a corresponding range of hydrogeologic parameters. This range may be quite limited in homogeneous systems, or it may be varied in a very heterogeneous system [25]. The large value of R in the Kalman filter analysis suggests large errors in the groundwater storage values. If the groundwater level measurements can be assumed with little measurement error, then only better knowledge of the site's geology would improve model prediction.

Figure 3.3, as well as figures 5.3 to 5.6, plot the average residuals obtained by the least squares methods. Since they are quite similar, it appears that weighted least squares and generalised least squares do little to improve parameter estimates. This also suggests that a linear formulation is too much of a simplification. Since the time periods of errors correspond to periods of rapid change in groundwater levels, it is quite likely that the discrepancy is due to nonlinear processes. Also, it may be, as mentioned earlier, that the groundwater level measurement mechanism does not respond quickly enough to large rapid changes in groundwater storage. The variations of the parameters by the extended Kalman filter as shown in figures 5.8 to 5.13 appear to confirm this.

As explained in the last section, the use of few non-zero data for some of the annual time series resulted in irrational parameter estimates for individual years. However, figure 5.14, which plots parameter B5 obtained by ordinary least squares against total snowfall, reveals that the yearly parameter estimates may be biased, but they hold some valid information. This figure shows that the greater the winter snowfall, the greater the value of parameter B5, which intuitively makes sense, since a greater snowmelt production can be expected from greater snow depth.

Other dependences between parameters estimated from annual data and either total summer rain and/or total hours of bright sunshine were examined, but none of them were conclusive. Given more years of data, a better picture of other dependences might be created. These findings could be used to further improve the model, and perhaps even suggest some nonlinear formulations for snowmelt and evapotranspiration.

While the ninety-five percent confidence intervals of table 5.8 suggest that some of the parameters, especially B4, may be zero, this is unlikely to be physically realistic. Since the model is physically-based, it is known that hours of bright sunshine are inputs and would exert significant influences on groundwater level fluctuations. What is more likely, is that some of the parameter values may be close to zero and analysis with more years of data, or with daily readings as mentioned earlier, would shorten the confidence intervals and thereafter leave the value of zero outside these limits.

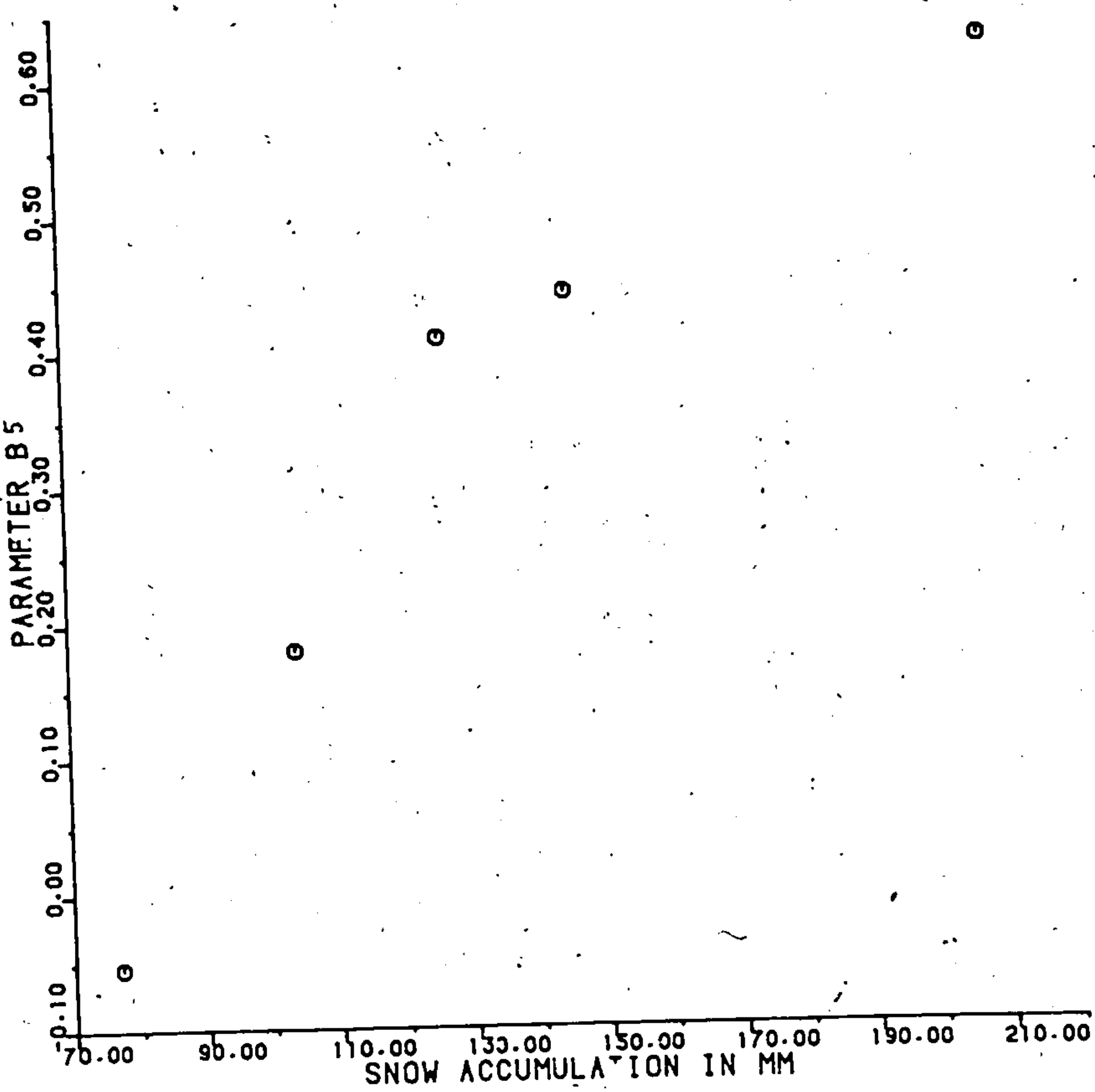


FIGURE 5.14: VARIATION OF PARAMETER B5 FROM ORDINARY LEAST SQUARES AGAINST SNOW ACCUMULATION

Since evapotranspiration is known to be a nonlinear process [45], perhaps it could be modelled separately and then incorporated in the regression analysis. If the evapotranspiration parameter is close to unity, then the calculated evapotranspiration values are likely quite good. The same may be applied to snowmelt.

The various least squares techniques appear equally effective. Their coefficient of determination for the full data set are quite similar and very high at around ninety-seven percent. But because present groundwater storages are highly dependent on past groundwater storages, these high values, while clearly indicative of a realistic and sound model formulation, may be misleading as to the accuracy of the results. Similarly, prediction of the sixth year by all the least squares methods is almost the same in terms of TSE and DIF.

The extended Kalman filter results, however, predicted the sixth year much better than the least squares methods because the parameters were allowed to vary. The predictions followed observed groundwater levels very closely as seen in figure 5.7, especially during periods of large evapotranspiration which the least squares methods had difficulty dealing with. Because there are actually nonlinear processes at work for evapotranspiration, and the parameters do not change in the least squares methods, prediction could not be as successful as for the extended Kalman filter.

However, the prediction with the least squares methods may be appropriate for most applications, especially since the use of the extended Kalman filter requires a greater amount of knowledge, effort, and computer time to obtain results.

Chapter VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

A groundwater level fluctuations model was developed based on a mass balance equation of water fluxes. A linear model was formulated relating present groundwater storage to past groundwater storage, to streamflow losses, to evapotranspiration losses, as well as to snowmelt and rainfall inputs. By modelling streamflow losses, snowmelt, and evapotranspiration, a model relating present groundwater storage to past groundwater storage, rainfall, temperature, and hours of bright sunshine resulted.

With available time series of the dependent and independent variables from the Castor watershed at a five day interval, the parameters of the linear model were determined. The following parameter estimation techniques were investigated: ordinary least squares, three variations of weighted least squares, two variations of generalised least squares, as well as the extended Kalman filter.

The following conclusions were drawn:

(1) Parameters for annual series as well as for five years of data were found, but only the latter could be considered valid. Because some of the time series consisted of

too many zero elements, the resulting degrees of freedom were too low for the annual series. Since the obtained parameters from five years of data have the correct sign and are within the expected limits, the model formulation can also be termed valid. However, because of the sharper variation of the parameters B1 to B6 with the extended Kalman filter during periods of rapid change in groundwater level, it appears that the chosen models for evapotranspiration and snowmelt were too simple.

(2) A comparison of the results from the various techniques suggests that the performance of all of the least squares methods was quite similar since their coefficient of determination as well as their prediction of the sixth year of data were about equal. However, the extended Kalman filter , with its varying parameters, provided better prediction of the sixth year, especially during the large evapotranspiration period where the least squares methods were facing difficulties in prediction. The small discrepancies between observed and predicted groundwater levels as shown in figure 5.7 result in a model able to forecast groundwater levels to within a few, generally one or two, centimeters of the actual.

(3) Conclusions can be drawn on the behavior of the hydrologic system using the extended Kalman filter parameters after three hundred and sixty time steps of table 5.7. From the value of parameter B2, which multiplies the rainfall in-

put, it appears that about forty-eight percent of the rainfall at the Castor site on the average reaches groundwater storage. From the value of parameter B1, which multiplies the past groundwater storage, it can be assumed that on average about five percent of the groundwater storage at any given time is lost to streamflow within five days. Unfortunately, no real conclusions about the physical behavior of the system can be inferred from parameters B3 to B6.

6.2 RECOMMENDATIONS FOR FUTURE RESEARCH

During this investigation, it has become evident that there are a number of areas in which continued research would be worthwhile.

(1) Daily data would be more appropriate for the application of this model at the Castor watershed or at any other site. More than five years of data is required so as to narrow as much as possible the confidence intervals on the parameters. How much more would depend on the accuracy provided by the model and the accuracy desired by the user.

(2) Also, it would serve to provide more B5 parameter values so that some curve could be prepared to determine the value of parameter B5 of a given year given a certain total snowfall if the extended Kalman filter is not to be used for prediction. Furthermore, more annual values of the parameters could be used to investigate dependence between some of the parameters and other factors.

(3) The use of daily or even hourly temperature data instead of the fifth day reading can be expected to better the results since the sum of such data would likely be a more accurate indicator of temperature during the previous five days.

(4) A further improvement in the model can be expected if the geology of the watershed can be described in more detail. Especially, accurate values of the specific yields as well as a correct value of the average depth of each soil layer over the region which influences the model are necessary. Since groundwater storage is the dependent variable and past groundwater storage is one of the independent variables in the model, accurate assessment of the geology of the area is likely to be one of the most important factors for accurate model performance.

(5) The groundwater level fluctuation model developed in this thesis could be expanded even further. Given more data as well as nonlinear models for snowmelt and evapotranspiration, it would be possible to develop a much more complex formulation. Since accepted methods exist to assess evapotranspiration, its calculation and then its inclusion as one of the independent variables could well result in much better predictions.

(6) Furthermore, if soil moisture data were available, a model consisting of water storages above and below the groundwater table could be prepared. This two-equation mo-

del would have some similar parameters, and its solution would consequently result in more reliable estimates of the parameters.

(7) And finally, seasonal considerations in the behavior of the residuals, as proposed for the second variation of generalised least squares, can be expected in other hydrologic models. Therefore, such considerations might well improve the parameter estimates of these models.

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Appendix A

ORIGINAL DATA FROM CASTOR SITE

Listed below are six years of the original data from the Castor site whose origin was explained in chapter two. GW represents groundwater levels in centimeters as measured from the ground surface at five day intervals, SF represents the streamflow in cubic meters per second as measured at the Russell station (Station No. 02LB006) every five days, PR represents the cumulative precipitation in millimeters over a five day interval, TE represents the temperature in degrees Celsius at every five days, while BS represents the five day total of hours of bright sunshine.

GW	SF	PR	TE	BS
5.5	4.72	23.60	-6.60	11.3
8.0	3.71	32.80	-9.10	17.7
10.0	3.46	9.91	-11.20	13.6
12.5	3.22	4.06	-4.40	0.0
15.0	3.08	34.30	-8.50	10.1
18.5	11.18	13.70	-3.90	11.2
16.0	5.80	0.51	-19.20	33.0
11.0	3.88	5.33	-11.20	11.4

5.0	6.01	6.10	-0.30	8.9
4.0	9.47	14.70	-5.70	14.2
3.0	8.50	15.20	-9.90	16.0
1.5	8.55	30.00	-7.20	11.6
3.5	7.98	5.84	-11.30	22.8
7.5	5.22	5.59	-13.20	33.2
13.0	3.34	3.05	-10.90	22.0
12.0	2.58	1.78	-8.90	34.7
10.0	1.88	0.00	-15.30	25.5
8.5	1.98	32.30	-2.70	7.9
7.5	21.79	11.90	2.10	15.0
7.5	73.13	22.40	2.40	11.4
8.0	118.75	34.00	0.50	0.8
8.0	30.43	0.00	1.90	26.8
4.5	27.19	7.37	5.20	26.3
0.0	37.87	46.20	3.70	0.7
1.0	14.52	9.10	0.90	35.2
3.0	6.30	0.51	0.60	45.6
4.0	4.70	1.52	12.10	54.7
3.5	2.65	0.51	12.20	7.1
2.5	9.31	33.30	7.80	0.0
3.0	8.50	19.60	18.30	0.4
3.0	5.07	8.13	12.40	35.3
2.5	6.14	4.83	10.20	42.7
3.0	20.27	43.70	9.10	22.0
2.5	9.18	0.00	14.10	50.8
2.0	5.46	27.90	14.80	35.3

2.5	6.19	8.64	15.60	43.3
4.0	3.60	15.00	21.40	46.6
4.0	7.16	41.70	19.00	42.0
20.0	17.38	34.50	15.70	39.0
38.0	3.03	15.70	20.60	41.0
45.0	1.81	4.57	22.90	43.0
72.0	1.31	5.08	24.00	48.1
106.0	0.55	16.30	24.10	48.8
108.0	0.57	0.00	19.50	46.7
111.5	0.40	0.51	20.10	54.1
116.0	0.35	0.00	21.40	63.3
80.0	0.33	13.70	23.20	43.3
33.0	12.74	27.40	22.10	32.5
47.0	10.77	45.70	24.50	21.8
61.0	2.25	0.76	21.80	40.3
74.0	0.77	20.10	22.30	50.4
81.5	0.70	13.70	17.10	39.4
88.0	0.53	2.03	24.10	43.6
96.0	0.46	27.40	25.80	37.5
101.0	0.70	7.11	14.80	38.8
119.0	0.46	1.02	14.00	31.8
133.0	0.53	19.30	9.80	25.7
141.0	0.78	18.00	9.70	25.2
84.0	0.66	2.54	14.80	31.6
39.0	2.32	69.10	13.90	13.6
17.0	4.26	0.00	11.30	45.1
35.0	1.33	7.87	12.90	30.5

49.0	1.03	4.06	5.70	12.5
39.0	0.90	0.00	9.80	43.0
24.5	0.96	17.50	6.70	18.0
12.0	2.74	12.20	4.30	26.1
11.5	1.56	2.54	-1.40	17.7
10.5	1.48	14.50	3.10	5.1
10.0	1.99	1.52	-2.80	21.4
9.5	3.17	15.70	-1.00	7.7
9.0	7.00	12.20	1.30	7.6
8.0	3.85	15.50	0.30	8.5
7.5	8.23	46.70	-2.10	3.1
6.5	7.31	9.14	-8.30	12.2
5.5	3.04	36.30	-15.10	20.0
6.0	2.38	18.00	-13.20	20.0
6.0	2.87	11.40	-1.80	5.8
5.0	2.86	6.35	-10.70	10.0
4.5	2.73	4.06	-14.80	17.2
4.0	2.61	9.14	-14.30	15.7
4.0	2.61	9.91	-18.10	20.7
3.5	2.48	18.80	-4.40	12.0
3.5	11.65	19.10	2.10	8.0
3.5	15.03	0.00	-18.20	34.2
4.5	4.72	1.02	-13.20	34.9
6.0	1.99	7.87	-12.30	21.8
7.5	1.12	12.70	-7.70	17.2
8.5	1.40	21.10	-8.90	22.4
10.0	1.04	1.02	-5.40	13.9

8.5	1.12	42.20	-0.80	10.4
7.0	67.24	2.54	-0.50	30.5
5.5	9.91	0.00	-6.50	40.0
4.0	2.62	36.80	-4.40	22.9
2.5	2.03	18.30	-6.50	23.0
1.5	2.00	19.80	-6.80	29.9
2.0	37.49	24.10	3.40	16.8
2.0	44.06	6.10	-1.40	29.9
2.0	39.62	10.70	4.40	16.0
2.0	28.91	0.00	5.70	60.3
2.0	10.91	11.40	10.30	22.2
2.0	7.69	14.70	13.40	41.4
2.5	10.44	11.70	5.30	46.4
4.0	14.91	30.20	5.90	2.4
4.5	24.50	37.10	11.70	22.2
5.0	5.88	0.25	11.60	54.8
9.5	5.07	27.70	13.60	12.5
12.0	5.54	14.70	13.40	21.7
14.5	3.43	0.25	17.20	58.1
16.0	1.24	0.00	21.20	44.8
26.5	0.77	13.70	14.70	30.4
36.0	1.56	23.40	17.60	29.4
52.0	1.66	3.30	17.20	47.5
80.0	0.59	0.76	18.10	41.2
107.5	0.66	17.00	21.80	28.5
124.0	0.62	6.60	23.40	50.7
140.0	0.42	0.00	21.30	59.9

154.0	0.57	31.50	18.90	40.8
156.0	0.54	16.00	18.00	27.5
160.0	0.57	6.60	19.70	29.0
154.0	1.38	21.10	19.30	21.6
145.0	0.92	0.76	19.30	59.0
135.5	0.42	1.27	19.40	48.5
147.0	0.34	5.33	20.30	52.0
160.5	0.30	0.51	21.20	48.5
180.0	0.30	3.81	18.70	36.3
185.5	0.31	8.89	12.50	27.3
191.5	0.27	6.35	15.80	32.4
196.0	0.27	4.83	17.70	15.4
199.5	0.25	3.81	12.30	36.5
206.0	0.24	5.33	7.80	12.1
207.5	0.25	8.89	13.90	21.2
209.5	0.24	6.10	5.60	25.1
212.0	0.26	3.81	8.40	23.5
213.5	0.28	13.20	7.00	25.0
214.5	0.35	11.70	2.70	32.6
215.5	0.30	3.30	5.80	31.4
214.0	0.31	2.89	7.70	27.2
212.0	0.43	18.80	5.90	15.3
211.0	0.44	0.00	2.60	14.7
182.0	1.08	35.80	4.10	9.1
160.0	0.83	24.10	3.00	7.3
139.0	2.82	22.40	-2.20	9.2
121.0	1.76	0.00	-5.10	30.5

99.0	1.75	21.30	-5.40	13.6
80.0	4.73	22.40	-1.90	3.0
62.0	3.97	8.13	-2.40	12.8
45.0	2.07	32.00	-3.30	0.1
40.0	1.82	6.86	-3.90	0.5
37.5	1.72	1.52	-4.20	21.5
35.0	1.64	12.40	-6.80	17.2
33.0	1.65	14.00	-3.70	1.9
31.0	7.15	8.64	-4.70	16.5
28.5	8.42	7.62	-13.20	19.0
26.0	4.86	27.90	-11.20	12.0
25.0	2.50	13.70	-11.70	29.3
24.5	1.70	0.51	-15.30	32.6
24.0	1.38	5.33	-12.10	28.5
23.0	1.13	3.30	-12.60	26.9
22.5	1.08	15.00	-2.40	15.5
22.0	1.61	16.00	1.00	4.1
21.0	5.26	3.05	-0.80	0.5
20.5	5.54	1.02	-7.10	23.6
17.0	3.92	17.80	-8.90	26.6
13.5	2.83	2.79	-4.70	26.8
10.0	6.20	30.20	0.70	28.9
6.5	37.13	19.30	-2.60	14.6
3.0	17.40	2.03	-5.80	39.0
2.0	8.27	30.70	-2.40	16.2
2.5	7.63	0.51	-2.00	22.8
2.5	23.36	0.00	0.10	42.4

3.0	83.94	7.37	7.60	26.7
3.5	27.24	1.78	6.20	50.0
13.0	6.02	0.00	7.90	43.2
20.0	3.09	6.86	12.00	23.1
27.5	2.21	0.00	13.50	61.6
34.0	1.25	3.81	16.00	51.3
41.5	1.30	3.56	19.20	59.1
49.0	1.15	23.90	22.70	62.5
54.0	0.57	21.10	19.40	36.4
95.5	0.54	17.00	17.00	37.8
99.5	1.13	22.10	14.90	35.6
103.5	0.68	6.10	19.60	38.0
108.0	0.41	3.05	21.60	51.5
120.0	0.31	0.00	22.00	64.4
132.0	0.26	0.00	21.70	69.0
143.0	0.24	8.13	22.70	63.0
166.0	0.23	39.90	23.90	52.7
188.0	0.23	3.81	21.20	32.1
199.0	0.23	38.60	24.20	47.3
211.5	0.25	39.40	21.40	45.3
229.5	0.34	20.80	21.20	62.6
230.0	0.19	3.05	26.90	43.2
232.0	0.13	0.00	20.50	50.3
234.0	0.12	6.10	21.30	42.3
237.5	0.13	0.00	19.20	50.5
240.0	0.15	4.83	17.70	24.1
244.0	0.19	15.20	18.80	47.8

247.0	0.18	10.20	15.60	22.1
250.0	0.20	4.32	13.30	30.7
251.0	0.35	36.10	12.60	38.2
252.0	0.27	28.20	15.90	3.4
254.0	0.45	16.30	12.10	12.4
230.0	4.77	32.00	14.10	22.2
198.0	2.43	10.90	10.10	32.0
158.0	1.06	2.29	9.30	30.4
126.0	2.37	34.80	10.80	7.8
88.0	4.59	17.00	7.20	9.3
58.0	4.65	1.78	12.50	26.0
46.5	1.77	0.00	6.00	30.9
32.0	2.20	15.70	10.00	26.3
27.5	2.62	34.50	12.40	19.1
20.5	7.63	6.35	3.70	15.8
11.0	6.18	0.00	3.50	9.8
8.5	4.82	8.13	-1.90	13.4
6.0	1.91	10.70	-2.20	6.4
4.0	8.55	7.37	-4.70	23.9
2.5	8.29	36.30	-4.90	11.9
2.0	8.75	17.00	-4.40	13.4
0.5	13.22	1.02	-17.40	20.5
4.5	8.26	2.03	-14.50	28.1
7.0	6.24	24.40	-10.10	22.0
11.0	3.47	3.81	-12.60	22.1
14.0	1.54	2.03	-14.70	19.6
17.0	0.70	13.50	-14.10	18.2

20.0	0.32	8.38	-17.70	18.2
23.5	0.20	6.86	-19.30	25.3
26.5	0.54	20.30	-7.50	6.5
30.0	0.65	4.06	-13.00	12.3
33.5	0.52	17.80	-9.50	21.9
36.0	0.56	11.90	-5.40	23.9
32.0	1.30	16.00	-1.90	14.5
28.0	5.25	35.00	-6.50	24.0
24.5	8.77	19.10	-4.20	12.0
22.0	5.54	22.90	-5.90	7.0
18.0	6.98	13.00	-7.10	30.7
14.5	6.04	8.00	-5.30	21.5
11.0	4.47	20.80	-6.20	20.4
7.5	32.60	6.60	3.00	35.2
4.0	114.62	17.30	3.40	33.9
-1.0	72.67	18.30	3.20	37.2
-0.5	19.23	0.25	3.60	58.2
1.0	7.95	1.27	4.90	41.9
2.5	6.18	0.00	18.30	48.8
4.5	5.30	23.10	9.20	21.4
6.0	5.16	11.40	7.10	27.4
8.0	4.40	9.91	10.60	18.0
10.0	5.49	21.60	8.10	30.3
12.0	4.26	11.20	11.30	31.8
13.0	11.63	50.00	11.00	8.7
15.0	13.83	1.78	11.00	20.2
17.5	2.97	5.59	17.50	50.9

18.0	2.06	0.00	16.50	62.6
15.5	1.24	14.20	22.80	42.9
12.0	2.09	21.10	19.00	42.7
18.0	2.45	19.30	20.30	25.8
22.0	2.69	9.40	23.10	23.7
28.0	4.16	9.65	19.90	32.1
35.0	2.82	0.51	20.40	33.6
59.0	1.79	0.00	21.20	41.3
83.0	1.28	15.00	18.70	20.3
112.0	1.69	11.20	20.10	32.6
128.0	0.84	0.00	18.40	51.4
151.0	0.45	1.52	18.80	29.4
160.0	0.37	11.40	17.90	51.0
170.0	0.40	11.40	17.80	35.4
176.5	0.97	21.60	20.20	23.8
182.0	1.16	0.00	19.00	43.3
186.5	0.61	0.00	21.50	51.7
189.0	0.62	33.80	17.60	41.1
192.0	0.88	14.20	14.20	26.1
193.5	0.61	8.64	15.50	38.6
195.5	0.62	0.25	16.70	31.5
197.0	0.79	56.60	17.70	11.6
199.0	1.56	5.33	8.90	29.0
164.0	1.42	10.40	9.70	31.7
157.0	1.06	0.00	13.10	51.0
146.0	1.55	31.80	8.80	8.9
122.0	3.63	0.51	8.10	31.2

105.0	1.58	13.70	1.90	21.8
97.0	6.96	22.40	2.90	16.8
53.0	2.77	13.00	2.90	22.4
17.5	4.74	1.27	2.10	23.0
21.5	2.53	6.10	-2.40	14.2
26.5	1.67	0.00	-2.40	18.3
29.0	2.03	2.79	2.20	20.6
26.0	1.38	3.30	-2.70	7.5
22.5	4.32	21.60	0.30	6.9
35.0	2.11	3.05	-11.80	12.4
54.0	1.15	9.14	-11.30	16.6
82.5	0.88	5.33	-9.80	12.7
86.0	0.76	11.70	-7.40	9.1
92.0	0.74	4.06	-12.30	9.7
100.0	0.64	7.11	-16.10	33.4
101.0	0.58	5.40	-8.30	19.8
102.0	0.53	24.90	-16.50	16.0
103.0	0.52	4.50	-17.30	22.0
104.0	0.41	1.20	-15.00	28.5
105.0	0.37	5.20	-11.10	6.0
106.0	0.33	8.00	-12.90	20.5
107.5	0.30	9.60	-9.60	24.9
108.0	0.28	7.20	-10.80	19.2
108.5	0.30	5.10	-3.30	10.7
109.0	0.25	3.30	-11.00	18.1
109.5	0.39	24.60	-7.70	14.1
85.0	1.06	0.60	-1.30	3.2

66.0	1.18	11.30	-2.30	22.0
43.0	5.72	0.00	1.90	21.6
20.0	80.60	50.60	3.90	14.2
1.5	54.81	3.00	-0.60	27.2
9.0	11.11	23.70	-2.20	22.1
10.0	32.25	11.60	5.60	40.4
10.5	29.46	22.10	2.50	23.6
11.0	12.89	5.80	-2.60	49.5
12.5	9.02	5.50	10.10	48.8
14.0	4.81	0.20	12.00	61.0
14.5	6.52	27.90	12.10	9.5
16.0	8.00	0.60	7.90	12.1
17.0	3.34	11.20	11.60	9.9
27.0	2.40	2.60	9.30	39.3
40.0	2.09	0.00	12.80	59.2
56.0	1.72	0.00	20.80	51.9
72.0	0.92	19.00	22.20	41.4
97.0	0.66	19.80	15.90	64.0
117.0	0.57	14.20	15.70	37.5
130.0	0.43	4.40	12.10	30.4
143.0	0.39	0.00	18.30	39.7
146.0	0.39	28.30	18.50	25.6
154.0	0.42	6.80	17.50	32.8
161.5	0.38	9.30	22.00	51.6
169.0	0.35	10.60	20.30	43.5
178.0	0.25	0.00	18.80	47.9
187.0	0.23	30.00	21.70	35.0

187.5	2.43	32.00	25.40	54.0
187.5	0.62	2.20	20.80	45.8
187.5	0.25	18.50	18.40	57.7
194.5	0.27	25.10	21.70	30.1
200.0	0.30	4.40	20.00	36.2
204.0	0.23	10.20	19.00	43.1
209.0	0.35	22.40	14.90	22.0
212.5	0.64	28.20	15.80	46.5
213.0	0.40	2.80	21.00	49.7
214.0	0.45	40.00	19.50	35.1
214.0	0.28	1.90	16.20	34.2
210.0	0.34	31.70	12.90	23.6
206.0	0.56	21.30	13.90	6.4
203.5	0.77	7.10	10.60	10.9
140.0	2.71	40.60	12.00	6.7
64.5	15.08	44.70	9.40	13.4
56.0	9.08	24.80	7.50	19.1
42.0	7.25	1.60	6.10	19.2
25.0	6.48	10.70	6.30	19.2
28.5	4.32	7.00	7.80	39.7
33.5	2.69	0.00	9.40	51.6
38.0	2.11	1.40	10.90	25.9
26.5	3.67	24.10	7.30	0.0
17.0	9.33	12.80	-0.10	13.4
12.5	11.35	18.60	3.50	10.4
10.0	7.05	12.30	0.50	14.1
8.5	4.72	20.40	-8.40	7.2

8.0	16.37	15.10	-1.80	2.2
7.0	7.46	38.90	-10.50	15.7
11.5	4.26	28.30	-13.90	7.9
15.0	4.20	0.90	-4.30	12.1
13.0	3.99	36.80	-1.30	2.8
14.5	5.17	1.20	-14.80	15.0
12.0	3.52	4.10	-14.30	10.4
10.0	5.55	56.40	-12.80	4.0
9.0	12.29	3.30	-15.00	9.7
8.0	9.58	13.20	-14.60	16.2
9.0	7.45	4.20	-6.50	13.0
9.5	10.43	30.10	-10.50	24.1
10.0	8.11	0.00	-17.50	32.9
11.0	3.90	0.60	-9.80	27.2
11.5	2.41	0.20	-11.50	36.5
12.0	1.79	0.40	-11.00	26.4
12.5	1.38	0.40	-8.10	28.5
13.0	1.19	0.00	-10.30	24.2
12.0	1.14	2.80	-12.60	39.5
11.0	1.08	0.00	-6.80	46.8
10.0	2.55	17.60	-0.80	19.7
9.0	9.62	1.60	-5.60	46.1
8.0	10.13	25.40	-3.50	30.0
4.5	10.04	18.30	0.00	27.6
1.5	29.77	21.20	-0.70	30.4
-2.0	48.90	12.00	-1.00	33.0
1.0	112.01	28.80	3.50	18.5

5.0	60.62	23.70	5.40	29.2
8.5	24.29	2.20	6.80	57.0
11.5	5.89	0.00	7.50	60.6
12.0	3.10	0.60	5.00	31.6
12.0	2.53	13.20	13.00	37.3
14.0	2.59	13.40	14.50	18.6
16.0	7.17	23.80	16.90	38.8
18.0	2.86	5.20	17.30	69.0
19.5	1.25	16.50	23.10	68.5
19.5	3.06	11.70	15.80	34.4
20.0	1.07	6.20	15.80	47.5
41.0	0.80	24.50	16.00	49.4
56.0	1.82	41.70	19.20	30.1
68.0	1.76	0.40	18.70	50.7
104.0	0.66	6.60	21.50	54.3
132.5	0.47	0.00	18.60	66.5
149.5	0.43	20.20	23.20	37.2
152.0	0.39	6.70	20.10	61.4
154.0	0.30	4.20	22.70	48.9
163.0	0.29	4.80	23.20	36.7
176.5	0.27	19.20	19.00	42.5
196.5	0.22	21.40	20.60	46.3
201.0	0.27	20.60	22.10	43.0
207.5	0.23	1.00	23.20	55.0
214.0	0.27	36.20	22.00	37.1
217.5	0.43	24.00	17.20	43.0
220.0	0.45	13.00	17.20	34.8

226.0	0.37	6.00	17.30	49.8
229.5	0.31	5.40	13.40	33.2
235.0	0.34	12.50	13.90	34.6
238.0	0.28	4.00	11.70	27.4
240.0	0.23	5.40	12.50	38.0
242.0	0.19	3.80	10.80	29.7
244.5	0.23	15.50	11.90	15.8
247.0	0.22	3.70	8.60	23.5
249.0	0.29	23.00	9.40	12.3
248.0	0.38	2.20	3.40	19.5
247.0	0.34	7.00	7.70	28.1
246.5	0.40	9.50	6.50	33.4
242.0	0.35	0.00	7.60	32.8
239.0	0.36	0.60	6.60	16.1
236.0	0.38	16.40	3.50	25.8
233.0	0.89	25.90	1.60	15.5
210.0	0.61	18.40	-5.10	19.1
188.0	0.61	10.70	-7.70	16.0
186.0	0.68	24.40	-5.20	5.1
182.5	0.78	5.10	-5.30	12.7

Appendix B

MODIFIED DATA FROM CASTOR SITE

Listed below is the modified data. From left to right, the columns represent W(t), P(t-1), W(t-1), T(t),BS(t), and BSS(t-1). W(t), P(t-1), ~~and~~ W(t-1) are in millimeters, BS(t) and BSS(t-1) are in hours, and T(t) is in degrees Celsius plus ten. A value of ten was added to differentiate between zero degrees Celsius and a lack of data.

W(t)	P(t-1)	W(t-1)	T(t)	BS(t)	BSS(t-1)
565.599	0.000	577.599	0.000	0.000	0.000
555.999	0.000	565.599	0.000	0.000	0.000
543.999	0.000	555.999	0.000	0.000	0.000
531.999	0.000	543.999	0.000	0.000	0.000
515.199	0.000	531.999	0.000	0.000	0.000
527.199	0.000	515.199	0.000	0.000	0.000
551.199	0.000	527.199	0.000	0.000	0.000
579.999	0.000	551.199	0.000	0.000	0.000
584.799	6.100	579.999	0.000	0.000	0.000
589.599	14.699	584.799	0.000	0.000	0.000
596.799	0.000	589.599	0.000	0.000	0.000
587.199	0.000	596.799	0.000	0.000	0.000

567.999	0.000	587.199	0.000	0.000	0.000
541.599	0.000	567.999	0.000	0.000	0.000
546.399	0.000	541.599	0.000	0.000	0.000
555.999	0.000	546.399	0.000	0.000	0.000
563.199	0.000	555.999	7.300	7.900	0.000
567.999	32.300	563.199	12.100	15.000	0.000
567.999	11.899	567.999	12.399	11.399	15.000
565.599	22.399	567.999	10.500	0.800	11.399
565.599	34.000	565.599	11.899	26.800	0.800
582.399	0.000	565.599	15.199	26.300	26.800
603.999	7.370	582.399	13.699	0.700	26.300
599.199	46.199	603.999	10.899	35.199	0.700
589.599	9.100	599.199	10.599	45.600	35.199
584.799	0.510	589.599	22.099	54.699	0.000
587.199	1.520	584.799	22.199	7.100	0.000
591.999	0.510	587.199	17.799	0.000	0.000
589.599	33.300	591.999	28.300	0.400	0.000
589.599	19.600	589.599	22.399	35.300	0.000
591.999	8.130	589.599	20.199	42.699	0.000
589.599	4.830	591.999	19.099	22.000	0.000
591.999	43.699	589.599	24.099	50.800	0.000
594.399	0.000	591.999	24.799	35.300	0.000
591.999	27.899	594.399	25.599	43.300	0.000
584.799	8.640	591.999	31.399	46.600	0.000
584.799	15.000	584.799	29.000	42.000	0.000
507.999	41.699	584.799	25.699	39.000	0.000
444.999	34.500	507.999	30.600	41.000	0.000

420.499	15.699	444.999	32.899	43.000	0.000
334.399	4.570	420.499	34.000	48.100	0.000
239.199	5.080	334.399	34.100	48.800	0.000
233.599	16.300	239.199	29.500	46.699	0.000
226.199	0.000	233.599	30.100	54.100	0.000
220.800	0.510	226.199	31.399	63.300	0.000
311.999	0.000	220.800	33.199	43.300	0.000
462.499	13.699	311.999	32.100	32.500	0.000
413.499	27.399	462.499	34.500	21.800	0.000
365.199	45.699	413.499	31.800	40.300	0.000
328.799	0.760	365.199	32.300	50.399	0.000
307.799	20.100	328.799	27.100	39.399	0.000
289.599	13.699	307.799	34.100	43.600	0.000
267.199	2.030	289.599	35.800	37.500	0.000
253.199	27.399	267.199	24.799	38.800	0.000
217.199	7.110	253.199	24.000	31.800	0.000
200.399	1.020	217.199	19.799	25.699	0.000
190.800	19.300	200.399	19.699	25.199	0.000
300.799	18.000	190.800	24.799	31.600	0.000
441.499	2.540	300.799	23.899	13.600	0.000
522.399	69.100	441.499	21.299	45.100	0.000
455.499	0.000	522.399	22.899	30.500	0.000
406.499	7.870	455.499	15.699	12.500	0.000
441.499	4.060	406.499	19.799	43.000	0.000
492.249	0.000	441.499	16.699	18.000	0.000
546.399	17.500	492.249	14.300	26.100	0.000
548.799	12.199	546.399	8.600	17.699	0.000

553.599	2.540	548.799	13.100	5.100	0.000
555.999	14.500	553.599	7.200	21.399	0.000
558.399	1.520	555.999	9.000	7.700	0.000
560.799	15.699	558.399	11.300	7.600	0.000
565.599	12.199	560.799	10.299	8.500	0.000
567.999	15.500	565.599	7.900	3.100	0.000
572.799	46.699	567.999	0.000	0.000	0.000
577.599	0.000	572.799	0.000	0.000	0.000
575.199	0.000	577.599	0.000	0.000	0.000
575.199	0.000	575.199	0.000	0.000	0.000
579.999	0.000	575.199	0.000	0.000	0.000
582.399	0.000	579.999	0.000	0.000	0.000
584.799	0.000	582.399	0.000	0.000	0.000
584.799	0.000	584.799	0.000	0.000	0.000
587.199	0.000	584.799	0.000	0.000	0.000
587.199	18.800	587.199	0.000	0.000	0.000
587.199	19.100	587.199	0.000	0.000	0.000
582.399	0.000	587.199	0.000	0.000	0.000
575.199	0.000	582.399	0.000	0.000	0.000
567.999	0.000	575.199	0.000	0.000	0.000
563.199	0.000	567.999	1.100	22.399	0.000
555.999	21.100	563.199	4.600	13.899	0.000
563.199	1.020	555.999	9.200	10.399	0.000
570.399	42.199	563.199	9.500	30.500	0.000
577.599	2.540	570.399	3.500	40.000	30.500
584.799	0.000	577.599	5.600	22.899	40.000
591.999	36.800	584.799	3.500	23.000	22.899

596.799	18.300	591.999	3.200	29.899	23.000
594.399	19.800	596.799	13.399	16.800	29.899
594.399	24.100	594.399	8.600	29.899	16.800
594.399	6.100	594.399	14.399	16.000	29.899
594.399	10.699	594.399	15.699	60.300	16.000
594.399	0.000	594.399	20.299	22.199	60.300
594.399	11.399	594.399	23.399	41.399	22.199
591.999	14.699	594.399	15.300	46.399	41.399
584.799	11.699	591.999	15.899	2.400	46.399
582.399	30.199	584.799	21.699	22.199	2.400
579.999	37.100	582.399	21.599	54.800	22.199
558.399	0.250	579.999	23.599	12.500	0.000
546.399	27.699	558.399	23.399	21.699	0.000
534.399	14.699	546.399	27.199	58.100	0.000
527.199	0.250	534.399	31.199	44.800	0.000
485.249	0.000	527.199	24.699	30.399	0.000
451.999	13.699	485.249	27.600	29.399	0.000
395.999	23.399	451.999	27.199	47.500	0.000
311.999	3.300	395.999	28.100	41.199	0.000
234.999	0.760	311.999	31.800	28.500	0.000
211.199	17.000	234.999	33.399	50.699	0.000
192.000	6.600	211.199	31.300	59.899	0.000
175.199	0.000	192.000	28.899	40.800	0.000
172.800	31.500	175.199	28.000	27.500	0.000
168.000	16.000	172.800	29.699	29.000	0.000
175.199	6.600	168.000	29.300	21.600	0.000
186.000	21.100	175.199	29.300	59.000	0.000

197.399	0.760	186.000	29.399	48.500	0.000
183.600	1.270	197.399	30.300	52.000	0.000
167.399	5.330	183.600	31.199	48.500	0.000
144.000	10.510	167.399	28.699	36.300	0.000
137.399	3.810	144.000	22.500	27.300	0.000
130.199	8.890	137.399	25.799	32.399	0.000
124.800	6.350	130.199	27.699	15.399	0.000
120.599	4.830	124.800	22.299	36.500	0.000
112.800	3.810	120.599	17.799	12.100	0.000
111.000	5.330	112.800	23.899	21.199	0.000
108.599	8.890	111.000	15.600	25.100	0.000
105.599	6.100	108.599	18.399	23.500	0.000
103.800	3.810	105.599	17.000	25.000	0.000
102.599	13.199	103.800	12.699	32.600	0.000
101.399	11.699	102,599	15.800	31.399	0.000
103.199	3.300	101.399	17.699	27.199	0.000
105.599	2.890	103.199	15.899	15.300	0.000
106.800	18.800	105.599	12.600	14.699	0.000
141.599	0.000	106.800	14.100	9.100	0.000
168.000	35.800	141.599	13.000	7.300	0.000
193.199	24.100	168.000	7.800	9.200	0.000
214.800	22.399	193.199	4.900	30.500	0.000
258.799	0.000	214.800	4.600	13.600	0.000
311.999	21.300	258.799	8.100	3.000	0.000
362.399	22.399	311.999	7.600	12.800	0.000
420.499	8.130	362.399	6.700	0.100	0.000
437.999	32.000	420.499	6.100	0.500	0.000

446.749	6.860	437.999	5.800	21.500	0.000
455.499	1.520	446.749	3.200	17.199	0.000
462.499	12.399	455.499	6.300	1.900	0.000
469.499	14.000	462.499	5.300	16.500	0.000
478.249	8.640	469.499	0.000	0.000	0.000
486.999	0.000	478.249	0.000	0.000	0.000
490.499	0.000	486.999	0.000	0.000	0.000
492.249	0.000	490.499	0.000	0.000	0.000
493.999	0.000	492.249	0.000	0.000	0.000
497.499	0.000	493.999	0.000	0.000	0.000
499.249	0.000	497.499	0.000	0.000	0.000
500.999	15.000	499.249	0.000	0.000	0.000
504.499	16.000	500.999	0.000	0.000	4.100
506.249	3.050	504.499	0.000	0.000	0.500
522.399	0.000	506.249	0.000	0.000	23.600
539.199	0.000	522.399	5.300	26.800	26.600
555.999	2.790	539.199	10.699	28.899	26.800
572.799	30.199	555.999	7.400	14.600	28.899
589.599	19.300	572.799	4.200	39.000	14.600
594.399	2.030	589.599	7.600	16.199	39.000
591.999	30.699	594.399	8.000	22.800	16.199
591.999	0.510	591.999	10.099	42.399	22.800
589.599	0.000	591.999	17.599	26.699	42.399
587.199	7.370	589.599	16.199	50.000	26.699
541.599	1.780	587.199	17.899	43.199	50.000
507.999	0.000	541.599	22.000	23.100	43.199
481.749	6.860	507.999	23.500	61.600	0.000

458.999	0.000	481.749	26.000	51.300	0.000
432.749	3.810	458.999	29.199	59.100	0.000
406.499	3.560	432.749	32.699	62.500	0.000
388.999	23.899	406.499	29.399	36.399	0.000
268.599	21.100	388.999	27.000	37.800	0.000
257.399	17.000	268.599	24.899	35.600	0.000
246.199	22.100	257.399	29.600	38.000	0.000
233.599	6.100	246.199	31.600	51.500	0.000
216.000	3.050	233.599	32.000	64.399	0.000
201.600	0.000	216.000	31.699	69.000	0.000
188.399	0.000	201.600	32.699	63.000	0.000
160.800	8.130	188.399	33.899	52.699	0.000
134.399	39.899	160.800	31.199	32.100	0.000
121.199	3.810	134.399	34.199	47.300	0.000
106.199	38.600	121.199	31.399	45.300	0.000
84.599	39.399	106.199	31.199	62.600	0.000
84.000	20.800	84.599	36.899	43.199	0.000
81.599	3.050	84.000	30.500	50.300	0.000
79.199	0.000	81.599	31.300	42.300	0.000
75.000	6.100	79.199	29.199	50.500	0.000
72.000	0.000	75.000	27.699	24.100	0.000
67.199	4.830	72.000	28.800	47.800	0.000
63.599	15.199	67.199	25.599	22.100	0.000
60.000	10.199	63.599	23.299	30.699	0.000
58.799	4.320	60.000	22.599	38.199	0.000
57.599	36.100	58.799	25.899	3.400	0.000
55.199	28.199	57.599	22.099	12.399	0.000

84.000	16.300	55.199	24.099	22.199	0.000
122.399	32.000	84.000	20.099	32.000	0.000
170.399	10.899	122.399	19.299	30.399	0.000
208.800	2.290	170.399	20.799	7.800	0.000
289.599	34.800	208.800	17.199	9.300	0.000
374.999	17.000	289.599	22.500	26.000	0.000
415.249	1.780	374.999	16.000	30.899	0.000
465.999	0.000	415.249	20.000	26.300	0.000
481.749	15.699	465.999	22.399	19.100	0.000
506.249	34.500	481.749	13.699	15.800	0.000
551.199	6.350	506.249	13.500	9.800	0.000
563.199	0.000	551.199	8.100	13.399	0.000
575.199	8.130	563.199	7.800	6.400	0.000
584.799	10.699	575.199	5.300	23.899	0.000
591.999	7.370	584.799	5.100	11.899	0.000
594.399	36.300	591.999	5.600	13.399	0.000
601.599	17.000	594.399	0.000	0.000	0.000
582.399	0.000	601.599	0.000	0.000	0.000
570.399	0.000	582.399	0.000	0.000	0.000
551.199	0.000	570.399	0.000	0.000	0.000
536.799	0.000	551.199	0.000	0.000	0.000
522.399	0.000	536.799	0.000	0.000	0.000
507.999	0.000	522.399	0.000	0.000	0.000
495.749	0.000	507.999	0.000	0.000	0.000
485.249	0.000	495.749	0.000	0.000	0.000
472.999	0.000	485.249	0.000	0.000	0.000
460.749	0.000	472.999	0.000	0.000	0.000

451.999	0.000	460.749	4.600	23.899	0.000
465.999	11.899	451.999	8.100	14.500	0.000
479.999	16.000	465.999	3.500	24.000	0.000
492.249	35.000	479.999	5.800	12.000	0.000
500.999	19.100	492.249	4.100	7.000	0.000
517.599	22.899	500.999	2.900	30.699	0.000
534.399	13.000	517.599	4.700	21.500	0.000
551.199	8.000	534.399	3.800	20.399	0.000
567.999	20.800	551.199	13.000	35.199	0.000
584.799	6.600	567.999	13.399	33.899	35.199
613.999	17.300	584.799	13.199	37.199	33.899
608.999	18.300	613.999	13.600	58.199	37.199
599.199	0.250	608.999	14.899	41.899	58.199
591.999	1.270	599.199	28.300	48.800	41.899
582.399	0.000	591.999	19.199	21.399	48.800
575.199	23.100	582.399	17.099	27.399	0.000
565.599	11.399	575.199	20.599	18.000	0.000
555.999	9.910	565.599	18.099	30.300	0.000
546.399	21.600	555.999	21.299	31.800	0.000
541.599	11.199	546.399	21.000	8.700	0.000
531.999	50.000	541.599	21.000	20.199	0.000
519.999	1.780	531.999	27.500	50.899	0.000
517.599	5.590	519.999	26.500	62.600	0.000
529.599	0.000	517.599	32.800	42.899	0.000
546.399	14.199	529.599	29.000	42.699	0.000
517.599	21.100	546.399	30.300	25.800	0.000
500.999	19.300	517.599	33.100	23.699	0.000

479.999	9.400	500.999	29.899	32.100	0.000
455.499	9.650	479.999	30.399	33.600	0.000
371.499	0.510	455.499	31.199	41.300	0.000
303.599	0.000	371.499	28.699	20.300	0.000
225.600	15.000	303.599	30.100	32.600	0.000
206.399	11.199	225.600	28.399	51.399	0.000
178.800	0.000	206.399	28.800	29.399	0.000
168.000	1.520	178.800	27.899	51.000	0.000
156.000	11.399	168.000	27.800	35.399	0.000
148.199	11.399	156.000	30.199	23.800	0.000
141.599	21.600	148.199	29.000	43.300	0.000
136.199	0.000	141.599	31.500	51.699	0.000
133.199	0.000	136.199	27.600	41.100	0.000
129.599	33.800	133.199	24.199	26.100	0.000
127.800	14.199	129.599	25.500	38.600	0.000
125.399	8.640	127.800	26.699	31.500	0.000
123.599	0.250	125.399	27.699	11.600	0.000
121.199	56.600	123.599	18.899	29.000	0.000
163.199	5.330	121.199	19.699	31.699	0.000
171.600	10.399	163.199	23.099	51.000	0.000
184.800	0.000	171.600	18.799	8.900	0.000
213.600	31.800	184.800	18.099	31.199	0.000
241.999	0.510	213.600	11.899	21.800	0.000
264.399	13.699	241.999	12.899	16.800	0.000
392.499	22.399	264.399	12.899	22.399	0.000
519.999	13.000	392.499	12.100	23.000	0.000
502.749	1.270	519.999	0.000	0.000	0.000

485.249	0.000	502.749	0.000	0.000	0.000
476.499	0.000	485.249	0.000	0.000	0.000
486.999	2.790	476.499	0.000	0.000	0.000
499.249	3.300	486.999	0.000	0.000	0.000
455.499	21.600	499.249	0.000	0.000	0.000
388.999	0.000	455.499	0.000	0.000	0.000
304.999	0.000	388.999	0.000	0.000	0.000
295.199	0.000	304.999	0.000	0.000	0.000
278.399	0.000	295.199	0.000	0.000	0.000
255.999	0.000	278.399	0.000	0.000	0.000
253.199	0.000	255.999	0.000	0.000	0.000
250.399	0.000	253.199	0.000	0.000	0.000
247.599	0.000	250.399	0.000	0.000	0.000
244.799	0.000	247.599	0.000	0.000	0.000
241.999	0.000	244.799	0.000	0.000	0.000
239.199	0.000	241.999	0.000	0.000	0.000
234.999	0.000	239.199	0.000	0.000	0.000
233.599	0.000	234.999	0.000	0.000	0.000
232.199	0.000	233.599	0.000	0.000	0.000
230.799	0.000	232.199	0.000	0.000	0.000
229.399	0.000	230.799	0.000	0.000	0.000
297.999	0.000	229.399	8.700	3.200	0.000
351.199	0.600	297.999	7.700	22.000	0.000
427.499	11.300	351.199	11.899	21.600	0.000
507.999	0.000	427.499	13.899	14.199	0.000
596.799	50.600	507.999	9.400	27.199	14.199
560.799	3.000	596.799	7.800	22.100	27.199

555.999	23.[illegible]99	560.799	15.600	40.399	22.100
553.599	11.600	555.999	12.500	23.600	40.399
551.199	22.100	553.599	7.400	49.500	23.600
543.999	5.800	551.199	20.099	48.800	49.500
536.799	5.500	543.999	22.000	61.000	48.800
534.399	0.200	536.799	22.099	9.500	0.000
527.199	27.899	534.399	17.899	12.100	0.000
522.399	0.600	527.199	21.599	9.900	0.000
483.499	11.199	522.399	19.299	39.300	0.000
437.999	2.600	483.499	22.799	59.199	0.000
381.999	0.000	437.999	30.800	51.899	0.000
334.399	0.000	381.999	32.199	41.399	0.000
264.399	19.000	334.399	25.899	64.000	0.000
219.600	19.800	264.399	25.699	37.500	0.000
204.000	14.199	219.600	22.099	30.399	0.000
188.399	4.400	204.000	28.300	39.699	0.000
184.800	0.000	188.399	28.500	25.600	0.000
175.199	28.300	184.800	27.500	32.800	0.000
166.199	6.800	175.199	32.000	51.600	0.000
157.199	9.300	166.199	30.300	43.500	0.000
146.399	10.600	157.199	28.800	47.899	0.000
135.599	0.000	146.399	31.699	35.000	0.000
135.000	30.000	135.599	35.399	54.000	0.000
135.000	32.000	135.000	30.800	45.800	0.000
135.000	2.200	135.000	28.399	57.699	0.000
126.599	18.500	135.000	31.699	30.100	0.000
120.000	25.100	126.599	30.000	36.199	0.000

115.199	4.400	120.000	29.000	43.100	0.000
109.199	10.199	115.199	24.899	22.000	0.000
105.000	22.399	109.199	25.799	46.500	0.000
104.399	28.199	105.000	31.000	49.699	0.000
103.199	2.800	104.399	29.500	35.100	0.000
103.199	40.000	103.199	26.199	34.199	0.000
108.000	1.900	103.199	22.899	23.600	0.000
112.800	31.699	108.000	23.899	6.400	0.000
115.800	21.300	112.800	20.599	10.899	0.000
192.000	7.100	115.800	22.000	6.700	0.000
355.399	40.600	192.000	19.399	13.399	0.000
381.999	44.699	355.399	17.500	19.100	0.000
430.999	24.800	381.999	16.099	19.199	0.000
490.499	1.600	430.999	16.299	19.199	0.000
478.249	10.699	490.499	17.799	39.699	0.000
460.749	7.000	478.249	19.399	51.600	0.000
444.999	0.000	460.749	20.899	25.899	0.000
485.249	1.400	444.999	17.299	0.000	0.000
522.399	24.100	485.249	9.900	13.399	0.000
543.999	12.800	522.399	13.500	10.399	0.000
555.999	18.600	543.999	10.500	14.100	0.000
563.199	12.300	555.999	1.600	7.200	0.000
565.599	20.399	563.199	8.200	2.200	0.000
570.399	15.100	565.599	0.000	0.000	0.000
548.799	0.000	570.399	0.000	0.000	0.000
531.999	0.000	548.799	0.000	0.000	0.000
541.599	0.000	531.999	0.000	0.000	0.000

534.399	0.000	541.599	0.000	0.000	0.000
546.399	0.000	534.399	0.000	0.000	0.000
555.999	0.000	546.399	0.000	0.000	0.000
560.799	0.000	555.999	0.000	0.000	0.000
565.599	0.000	560.799	0.000	0.000	0.000
560.799	0.000	565.599	0.000	0.000	0.000
558.399	0.000	560.799	0.000	0.000	0.000
555.999	0.000	558.399	0.000	0.000	0.000
551.199	0.000	555.999	0.000	0.000	0.000
548.799	0.000	551.199	0.000	0.000	0.000
546.399	0.000	548.799	0.000	0.000	0.000
543.999	0.000	546.399	0.000	0.000	0.000
541.599	0.000	543.999	0.000	0.000	0.000
546.399	0.000	541.599	0.000	0.000	0.000
551.199	0.000	546.399	3.200	46.800	0.000
555.999	0.000	551.199	9.200	19.699	0.000
560.799	17.600	555.999	4.400	46.100	0.000
565.599	1.600	560.799	6.500	30.000	46.100
582.399	25.399	565.599	10.000	27.600	30.000
596.799	18.300	582.399	9.299	30.399	27.600
623.999	21.199	596.799	9.000	33.000	30.399
599.199	12.000	623.999	13.500	18.500	33.000
579.999	28.800	599.199	15.399	29.199	18.500
563.199	23.699	579.999	16.799	57.000	29.199
548.799	2.200	563.199	17.500	60.600	57.000
546.399	0.000	548.799	15.000	31.600	0.000
546.399	0.600	546.399	23.000	37.300	0.000

536.799	13.199	546.399	24.500	18.600	0.000
527.199	13.399	536.799	26.899	38.800	0.000
517.599	23.800	527.199	27.300	69.000	0.000
510.399	5.200	517.599	33.100	68.500	0.000
510.399	16.500	510.399	25.799	34.399	0.000
507.999	11.699	510.399	25.799	47.500	0.000
434.499	6.200	507.999	26.000	49.399	0.000
381.999	24.500	434.499	29.199	30.100	0.000
345.599	41.699	381.999	28.699	50.699	0.000
244.799	0.400	345.599	31.500	54.300	0.000
201.000	6.600	244.799	28.600	66.500	0.000
180.600	0.000	201.000	33.199	37.199	0.000
177.600	20.199	180.600	30.100	61.399	0.000
175.199	6.700	177.600	32.699	48.899	0.000
164.399	4.200	175.199	33.199	36.699	0.000
148.199	4.800	164.399	29.000	42.500	0.000
124.199	19.199	148.199	30.600	46.300	0.000
118.800	21.399	124.199	32.100	43.000	0.000
111.000	20.600	118.800	33.199	55.000	0.000
103.199	1.000	111.000	32.000	37.100	0.000
99.000	36.199	103.199	27.199	43.000	0.000
96.000	24.000	99.000	27.199	34.800	0.000
88.800	13.000	96.000	27.300	49.800	0.000
84.599	6.000	88.800	23.399	33.199	0.000
78.000	5.400	84.599	23.899	34.600	0.000
74.399	12.500	78.000	21.699	27.399	0.000
72.000	4.000	74.399	22.500	38.000	0.000

69.599	5.400	72.000	20.799	29.699	0.000
66.599	3.800	69.599	21.899	15.800	0.000
63.599	15.500	66.599	18.599	23.500	0.000
61.199	3.700	63.599	19.399	12.300	0.000
62.399	23.000	61.199	13.399	19.500	0.000
63.599	2.200	62.399	17.699	28.100	0.000
64.199	7.000	63.599	16.500	33.399	0.000
69.599	9.500	64.199	17.599	32.800	0.000
73.199	0.000	69.599	16.599	16.100	0.000
76.800	0.600	73.199	13.500	25.800	0.000
80.399	16.399	76.800	11.600	15.500	0.000
108.000	25.899	80.399	4.900	19.100	0.000
134.399	18.399	108.000	2.300	16.000	0.000
136.800	10.699	134.399	4.800	5.100	0.000
141.000	24.399	136.800	4.700	12.699	0.000