CAHIER DE RECHERCHE #2603E
Département de science économique
Faculté des sciences sociales
Université d’Ottawa

WORKING PAPER #2603E
Department of Economics
Faculty of Social Sciences
University of Ottawa

Optimal Stationary Contracts under One-Sided Enforcement and Persistent Adverse Selection[*]

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April 2026

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[*] This paper has been funded by the Agence Nationale de la Recherche under grant ANR-17-EURE-0010 (Investissements d'Avenir program). We thank Marco Ottaviani for helpful comments and suggestions on an earlier version of this paper. The usual disclaimer applies.

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Abstract

We characterize the optimal contract within the class of stationary mechanisms in a repeated buyer-seller relationship with persistent adverse selection and one-sided limited enforcement. A prepaid seller may breach after receiving the current transfer and terminate the relationship upon paying an enforceable penalty. In this stationary benchmark, the enforcement problem collapses to a bound on the transfer targeted to the most efficient type. This yields a three-regime characterization. With strong enforcement, the repeated static second-best contract is feasible. With weak (intermediate) enforcement, the top transfer is capped, inducing bunching among efficient types and additional downward distortions. With very weak enforcement, public penalties alone cannot sustain compliance, and the principal must leave strictly positive continuation rents, including for the least efficient type. We interpret the associated distortion as a virtual enforcement cost.

Key words: *Adverse selection; Limited enforcement; Relational contracts; Contract breach.*

JEL Classification: D82; D86; K12; C61.

1 Introduction

In many procurement, construction, and trade relationships, payment must precede at least part of performance. This timing creates a simple but important contractual friction: after receiving payment, the seller may prefer to abscond rather than deliver. We study this problem in a repeated buyer-seller relationship with persistent private information about the seller's marginal cost. Our focus is the optimal stationary mechanism when the seller's deviation takes the form of a one-sided "take-the-money-and-run" breach.

The empirical relevance of this friction is clear in environments where formal enforcement is weak. Antràs and Foley (2015) document that cash-in-advance terms are used more intensively in international transactions with weaker contractual enforcement. Macchiavello and Morjaria (2015) show, in the Kenyan rose export sector, that when formal contracts are hard to enforce, trade relies heavily on the value of future relationships and reliability improves as relationships mature. In procurement and construction, retention, performance bonds, and adjudication play related roles by trying to limit the risk that a contractor is paid and then fails to perform. These examples all point to the same issue: public enforcement and the value of future trade jointly shape which contracts are feasible in practice.

The paper characterizes the optimal contract within the class of stationary mechanisms. The key observation is that, under stationarity, the seller's enforcement problem collapses to a restriction on the transfer intended for the most efficient type. This reduction yields a sharp three-regime taxonomy. Under *strong enforcement*, the repeated static second-best contract is feasible. Under *weak enforcement*, the transfer to the most efficient type is capped by the penalty, so incentive compatibility propagates that cap into bunching among efficient types and additional downward distortions. Most distinctively, under *very weak enforcement*, public penalties alone cannot sustain compliance: continuation values must do the remaining work, and the contract must leave a strictly positive rent even for the least efficient type. That is the region in which the dynamic content of the model is sharpest.

Under full commitment and costless enforcement, stationarity is without loss in the persistent-type environment of Baron and Besanko (1984). Once enforcement itself is the friction of interest, stationarity should instead be read as a tractable benchmark that allows us to isolate clearly the consequence of considering the one-sided breach problem. Lastly, this restriction echoes real-world practices where dynamic contracts rarely keep tracks of all past history of performances; a case in order is given by franchising royalties as pointed out by Lafontaine and Shaw (1999)).

LITERATURE REVIEW. The paper is related to dynamic mechanism design and relational contracting. Under full commitment and costless enforcement, Baron and Besanko (1984) show that with persistent private information the optimal long-term contract is stationary and coincides with the repetition of the static optimum. By contrast, once enforcement is limited, stationarity is no longer innocuous.

A useful relational benchmark is Levin (2003), who studies self-enforcing contracts with hidden information and i.i.d. types. The common force in Levin's model and ours is that limited enforceability compresses the set of feasible transfers and may induce pooling. The difference is that our seller's type is persistent and the relevant one-sided deviation is post-payment breach by the seller. Under stationarity, this yields a transfer cap at the top and, in the very-weak-enforcement region, an additional enforcement rent at the bottom.

The weak-enforcement region is closely related to static screening problems with transfer caps or payment bounds, as in Thomas (2002) and Gautier and Mitra (2006). We make that connection explicit below. In that sense, the weak-enforcement region is "static-like." The most distinctive part of the present analysis is the very-weak-enforcement region, in which continuation values rather than legal penalties sustain compliance.

The paper is also closely related to Martimort et al. (2017b), which studies optimal stationary contracts under two-sided limited enforcement. Relative to that paper, the contribution here is to isolate one-sided seller breach and show that this isolation generates an additional regime in which the least efficient type receives a strictly positive enforcement rent. The paper also complements the broader literature on repeated trade under weak enforcement, including Malcomson (2016) and more recent work on "take-the-goods-and-run" frictions such as Brugues (2026).

Finally, Martimort et al. (2017a) study a binary-type model with two-sided limited enforcement in which the unrestricted optimum is non-stationary. That paper is useful here mainly as a reminder that, once enforcement is the friction of interest, non-stationary contracts may matter. Our paper therefore speaks to the optimal stationary benchmark rather than to the unrestricted dynamic optimum.

ORGANIZATION OF THE PAPER. Section 2 presents the model and derives the stationary enforcement constraint. Section 3 states the principal's stationary problem and the main theorem. Section 4 solves the model through a reduction to two auxiliary programs. Section 5 discusses interpretation, comparative statics, and the scope of the stationarity benchmark. Proofs are relegated to the Appendix.

2 Model

• PREFERENCES AND INFORMATION. We consider a long-term relationship between a buyer (the principal) and a seller (the agent). In each period, the buyer purchases a (non-durable) good q from the seller and pays a transfer t.[1] The seller and the buyer have per-period utility functions given respectively by

$$V(q,t) = S(q) - t, \qquad U(q,t,\theta) = t - \theta q,$$

where θ is the seller's marginal cost: the agent's type. Assume that the agent's type is drawn once and for all at the beginning of the relationship. The agent privately learns his cost parameter θ which is drawn from the atomless distribution $F(\cdot)$ on the interval $\Theta = [\underline{\theta}, \overline{\theta}]$ with density $f(\cdot)$. The distribution $F(\cdot)$ is common knowledge. The gross surplus function $S(\cdot)$ is increasing and strictly concave ($S'(\cdot) > 0 > S''(\cdot)$) and satisfies the Inada conditions $S'(0) = +\infty, S(0) = 0$. These assumptions ensure that the first-best surplus is always positive (i.e., $\max_{q \geq 0} S(q) - \overline{\theta} q \geq 0$).

The time horizon is infinite, discrete, and the parties have a common discount factor $\delta \in [0,1]$.

Output is observable each period. At the beginning of the relationship the buyer offers a long-term contract to the seller, who accepts or rejects after learning his cost type. We restrict attention to *stationary* mechanisms, so allocations do not depend on the history of play. Under full enforcement with persistent types this restriction is without loss (Baron and Besanko, 1984); here this restriction delineates a benchmark once enforcement itself is limited. Within that restricted class, any equilibrium allocation can be represented in truthful direct form, so we write the mechanism as $\{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta} \in \Theta}$.

- TIMING. The contracting game unfolds as follows:
  1. The seller learns his private cost parameter θ.
  2. The buyer offers a stationary mechanism $\mathcal{C} = \{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta} \in \Theta}$.
  3. The seller accepts or rejects. If he accepts, he reports $\hat{\theta}$.
  4. In each period τ, the buyer prepays $t(\hat{\theta})$ and the seller then either delivers $q(\hat{\theta})$ or breaches, pays the enforceable penalty Π, and terminates the relationship.

- INCENTIVE COMPATIBILITY AND PARTICIPATION. Because the buyer commits to a stationary mechanism, we focus on truthful stationary mechanisms $\{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta} \in \Theta}$. We denote the seller's per-period rent by

$$U(\theta) = t(\theta) - \theta q(\theta).$$

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[1] The good q can be interpreted as quantity, or quality in the case of a single unit.

His ex post participation constraint is

$$U(\theta) \geq 0 \qquad \forall \theta \in \Theta. \tag{1}$$

Lemma 1 (Incentive compatibility and monotonicity)**.** *Suppose the mechanism is truthful. Then q is non-increasing and a.e. differentiable with $\dot{q}(\theta) \leq 0$, and at any point where q and t are differentiable,*

$$\dot{t}(\theta) = \theta\, \dot{q}(\theta) \quad \textit{a.e. on } \Theta. \tag{2}$$

This lemma can equivalently be expressed as saying that U is absolutely continuous with $\dot{U}(\theta) = -q(\theta)$ a.e., and U is convex.

Definition 1 (Admissible contracts)**.** A pair (t, q) is *admissible* if: (i) q is piecewise continuous and of bounded variation with $\dot{q}(\theta) \leq 0$ a.e.; (ii) t is absolutely continuous and (2) holds a.e.; and (iii) (1) is satisfied.

Remark 1 (Continuity of the optimal schedule). Although admissibility allows for discontinuities, the optimal stationary quantity schedule is continuous and weakly decreasing. Appendix 6 provides a direct proof sketch based on concavity and the envelope condition. Thus the structural statements in the main text, such as the existence of a single bunching interval, can be read literally for continuous schedules.

• ENFORCEMENT CONSTRAINT. Accounting for the agent's option to breach after receiving the period-τ payment,

$$\frac{1}{1-\delta}\, U(\theta) \;\; \geq \;\; \max_{\hat{\theta}\in\Theta,\; \tau \geq 0} \Big\{ \sum_{s=0}^{\tau-1} \delta^s \big(t(\hat{\theta}) - \theta q(\hat{\theta})\big) \; + \; \delta^\tau \big(t(\hat{\theta}) - \Pi\big) \Big\}. \tag{3}$$

Lemma 2 (Reduction of enforcement)**.** *An admissible (t, q) is enforceable iff*

$$t(\theta) - \theta q(\theta) \;\geq\; (1-\delta)\big(t(\underline{\theta}) - \Pi\big) \qquad \forall \theta \in \Theta. \tag{4}$$

The content of Lemma 2 is that, under stationarity, the seller's enforcement problem collapses to a condition on the transfer intended for the most efficient type. A seller who deviates optimally mimics the most efficient type, collects that payment, and then breaches. This is the sense in which the one-sided enforcement problem reduces to a cap on the top transfer.

When Π is moderate, the right-hand side of (4) is positive, so enforceability constrains the set of implementable allocations. Lowering $t(\underline{\theta})$ weakens the seller's gain from a take-the-money-and-run deviation, but incentive compatibility then propagates that compression to other types. Conversely, when Π is large enough, the right-hand side of (4) is non-positive and the enforcement constraint is implied by the participation constraint. In that case, the stationary problem collapses to the usual static second-best benchmark.

3 Optimal Contract

The principal's problem is

$$(P): \qquad \max_{\{(t,q)\text{ admissible}\}} \int_{\underline{\theta}}^{\bar{\theta}} \left(S(q(\theta)) - t(\theta)\right) f(\theta) d\theta$$

subject to (1) and (4).

We extend the classical second-best screening problem by adding the stationary enforcement constraint (4). In the absence of that constraint, or whenever it is slack, the optimal stationary allocation is the repeated static contract of Baron and Myerson (1982). Let $\big(q^{os}(\theta), t^{os}(\theta)\big)$ denote that benchmark, where

$$S'\big(q^{os}(\theta)\big) = \theta + \frac{F(\theta)}{f(\theta)} \qquad \forall \theta \in \Theta,$$

and

$$t^{os}(\theta) = \theta q^{os}(\theta) + \int_{\theta}^{\bar{\theta}} q^{os}(x)\, dx.$$

The term $\theta + F(\theta)/f(\theta)$ is the usual virtual cost.

Definition 2. For any $r \geq 0$, define the generalized static output $q^{(r)}(\theta)$ by

$$S'\big(q^{(r)}(\theta)\big) = \theta + \frac{F(\theta) + r}{f(\theta)} \qquad \forall \theta \in \Theta.$$

Thus $q^{os}(\theta) = q^{(0)}(\theta)$. When $r > 0$, the output rule is more distorted than the standard second-best rule. The additional term $r/f(\theta)$ can be interpreted as a *virtual enforcement cost*: it is the shadow value of the binding top-transfer limitation, translated into the same units as the standard virtual-cost wedge.

Assumption 1. 1. The density $f(\theta)$ is differentiable.

2. The function $\dfrac{F(\theta) + r}{f(\theta)}$ is increasing in θ for every $r \in [0, r^*]$, where $r^* = \frac{1-\delta}{\delta}$.

Assumption 1 is stronger than the usual regularity condition from the unconstrained screening problem. A simple sufficient primitive condition is that the density be weakly decreasing, $f'(\theta) \leq 0$, which implies the required monotonicity for every $r \geq 0$.[2] We therefore interpret the assumption as a generalized monotonicity condition on the constrained virtual cost, rather than as an ordinary hazard-rate restriction.[3]

Next, we define the output which plays an important role in characterizing the optimal contract.

Definition 3. For any $r \geq 0$, define the output $q_r(\theta)$ by

$$q_r(\theta) = \begin{cases} q^{(r)}\left(\theta_r\right) & \text{if} \quad \theta < \theta_r, \\ q^{(r)}\left(\theta\right) & \text{if} \quad \theta_r \leq \theta \leq \overline{\theta}, \end{cases} \tag{5}$$

where $\theta_r \in \left[\underline{\theta}, \overline{\theta}\right]$ is the (minimal) solution of

$$r = \frac{F^2\left(\theta_r\right)}{\theta_r f\left(\theta_r\right) - F\left(\theta_r\right)}, \tag{6}$$

if it exists, otherwise $\theta_r = \overline{\theta}$.

When $\underline{\theta} < \theta_r < \overline{\theta}$, the output exhibits bunching for types smaller than θ_r. For types greater than θ_r, the output aligns with the generalized static output (see Figure 1). When $r = 0$, the threshold θ_r is $\underline{\theta}$ and the corresponding output $q_r(\theta)$ is the optimal static contract. If no solution to (6) exists, then the output $q_r(\theta)$ consists of complete bunching at $q^{(r)}\left(\overline{\theta}\right)$.

The transfers required to implement the output $q_r(\theta)$ are determined by

$$t_r(\theta) = \theta q_r(\theta) + \int_{\theta}^{\bar{\theta}} q_r(x)\,dx + \left(t_r(\bar{\theta}) - \bar{\theta} q_r(\bar{\theta})\right). \tag{7}$$

Let $r^* = \frac{1-\delta}{\delta}$.[4] The following lemma establishes the existence of the output $q_r(\theta)$ for all $r \in [0, r^*]$.

Lemma 3. *For any* $r \in [0, r^*]$*, either the solution* $\theta_r \leq \bar{\theta}$ *to* (6) *exists or there exists a unique* $\widehat{r} \in [0, r^*]$ *such that* $r(\bar{\theta}) = \widehat{r}$*.*

In the left panel of Figure 2, equation (6) defines the threshold θ_r as a function of r. As r increases, the bunching cutoff moves upward. In particular, the value $r^* = (1-\delta)/\delta$ determines the largest generalized distortion relevant for the stationary program.

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[2] Examples include Exponential Distribution, Beta Distributions, Gamma Distributions, Pareto Distribution and Weibull Distributions.

[3] This condition does not require the density to be globally decreasing. In particular, Gautier and Mitra (2006) provide examples of distributions with locally increasing density (such as truncated normal or logistic distributions) that satisfy their Assumption 1. These examples, with required restrictions on parameters, are also applicable in our setting.

[4] It will be shown that r^* represents the maximal value for which the optimal output is defined.

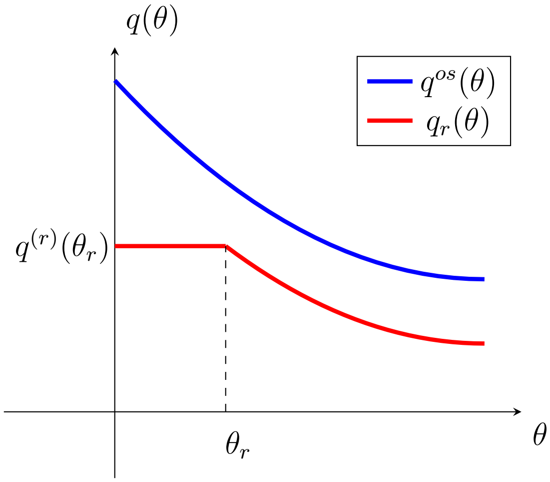

Figure 1: Optimal static contracts

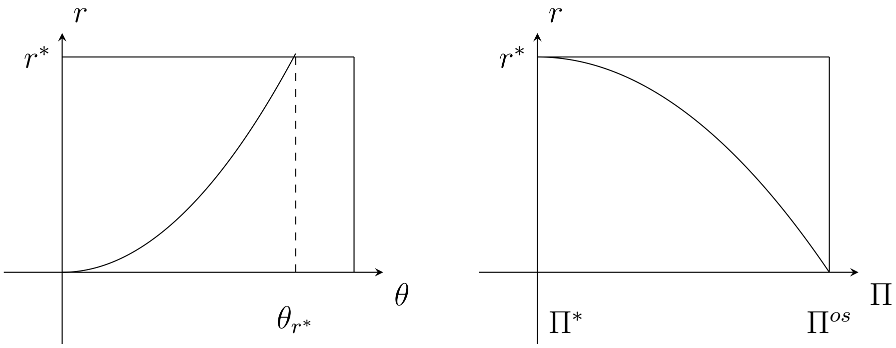

Figure 2: r as a function of θ_r and Π

In the right panel of Figure 2, each $r \in [0, r^*]$ corresponds to a penalty $\Pi \in [\Pi^*, \Pi^{os}]$ through

$$\Pi = \underline{\theta}\, q_r(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} q_r(\theta)\, d\theta. \tag{8}$$

This expression is simply $t_r(\underline{\theta})$ when the least efficient type earns zero rent. Hence the mapping from r to Π is decreasing, and we write its inverse as $r = r(\Pi)$ whenever needed.

Using (8), define Π^{os} and Π^* as the penalties associated with $r = 0$ and $r = r^*$ respectively. The threshold Π^{os} is the smallest penalty for which the static second-best contract is enforceable. The threshold Π^* is the smallest penalty for which public enforcement alone is sufficient to sustain zero rent for the least efficient type.

The optimal contract for the agent's enforcement problem can now be described.

Theorem 1. *1.* ***Strong enforcement.*** *For $\Pi \geq \Pi^{os}$, the optimal contract involves the infinite repetition of the optimal static contract.*

2. ***Weak enforcement.*** *For $\Pi^* \leq \Pi \leq \Pi^{os}$, the optimal contract is $\big(t_r(\theta), q_r(\theta)\big)$,* *where the parameter r is defined by*

$$t_r(\underline{\theta}) = \Pi. \tag{9}$$

Here, the least efficient type has zero rent, i.e., $t_r(\bar{\theta}) - \bar{\theta} q_r(\bar{\theta}) = 0$.

3. ***Very weak enforcement.*** *For* $\Pi \leq \Pi^*$, *the optimal contract is* $\big(t(\theta), q_{r^*}(\theta)\big)$. *The transfers* $t(\theta)$ *are given by*

$$t(\theta) = t_{r^*}(\theta) + \frac{1-\delta}{\delta}\left(\Pi^* - \Pi\right).$$

If $\Pi < \Pi^*$, *then the least efficient type earns a positive rent,*

$$t(\bar{\theta}) - \bar{\theta} q_{r^*}(\bar{\theta}) = \frac{1-\delta}{\delta}\left(\Pi^* - \Pi\right) > 0.$$

Theorem 1 yields a clean stationary taxonomy.

Under *strong enforcement* ($\Pi \geq \Pi^{os}$), the enforcement constraint is slack and the repeated static second-best contract is feasible. Under *weak enforcement* ($\Pi^* \leq \Pi < \Pi^{os}$), enforceability is equivalent to a cap on the transfer intended for the most efficient type. This cap propagates through incentive compatibility and leads to bunching among efficient types and further downward distortions for the remaining types. Under *very weak enforcement* ($\Pi < \Pi^*$), public enforcement alone cannot sustain compliance: continuation values must do the remaining work, and the least efficient type earns a strictly positive rent.

STATIC BENCHMARK. The weak-enforcement region is closely related to a static screening problem with a binding upper bound on the top transfer. In that sense, bunching in this region is "static-like." The additional dynamic content appears most sharply in the very-weak-enforcement region, where continuation values generate an *enforcement rent* for the least efficient type.

WHY BUNCHING ARISES UNDER WEAK ENFORCEMENT. For $\Pi^* \leq \Pi < \Pi^{os}$, the top transfer is capped at $t(\underline{\theta}) = \Pi$. Lowering that transfer reduces the gain from a take-the-money-and-run deviation, but incentive compatibility links nearby types to the same margin. The principal therefore prefers to pool the most efficient types rather than raise the top transfer above the enforceable level.

For types above the bunching cutoff θ_r, the optimal allocation follows the generalized static rule. Relative to the standard virtual cost

$$\theta + \frac{F(\theta)}{f(\theta)},$$

enforcement adds the wedge

$$\frac{r}{f(\theta)},$$

which is naturally interpreted as a virtual enforcement cost.

RENT DECOMPOSITION. Under strong and weak enforcement, the least efficient type earns zero rent and the seller's rent is purely informational:

$$U(\theta) = \int_{\theta}^{\overline{\theta}} q_r(x)\,dx.$$

By contrast, when $\Pi < \Pi^*$, the contract must also leave a positive continuation payoff to sustain compliance. The seller's rent then decomposes as

$$U(\theta) = \underbrace{\frac{1-\delta}{\delta}(\Pi^* - \Pi)}_{\text{enforcement rent}} + \underbrace{\int_{\theta}^{\overline{\theta}} q_{r^*}(x)\,dx}_{\text{information rent}}.$$

The first term is the compliance premium required by weak public enforcement; the second term is the usual information rent.

CONNECTION TO RELATIONAL CONTRACTING. This logic connects to Levin (2003): limited enforceability compresses feasible transfers and can induce pooling. The key difference is that our environment has persistent private information and a one-sided post-payment deviation. Under stationarity, that specific deviation makes the enforcement problem collapse to a cap on the top transfer; when public enforcement is very weak, continuation values generate an additional bottom rent that is absent from a purely static transfer-cap formulation.

Example 1 (Uniform types and quadratic surplus). Suppose that $\Theta = [1, 2]$, the type distribution is uniform and the buyer's gross surplus is

$$S(q) = 5q - \frac{q^2}{2}.$$

Assume also that $\delta = \frac{4}{5}$, so that

$$r^* = \frac{1-\delta}{\delta} = \frac{1}{4}.$$

Then the optimal static output is

$$S'\big(q^{os}(\theta)\big) = 5 - q^{os}(\theta) = \theta + \frac{F(\theta)}{f(\theta)} = 2\theta - 1,$$

which yields

$$q^{os}(\theta) = 6 - 2\theta, q^{os}(1) = 4, \qquad q^{os}(2) = 2.$$

In particular, the transfer to the most efficient type under the repeated static contract is

$$t^{os}(1) = q^{os}(1) + \int_1^2 q^{os}(x)\,dx = 7.$$

Hence

$$\Pi^{os} = 7.$$

For any $r \geq 0$,

$$S'\big(q^{(r)}(\theta)\big) = 5 - q^{(r)}(\theta) = \theta + \frac{F(\theta) + r}{f(\theta)} = 2\theta - 1 + r,$$

so

$$q^{(r)}(\theta) = 6 - r - 2\theta.$$

The bunching cutoff solves (6):

$$r = \frac{F^2(\theta_r)}{\theta_r f(\theta_r) - F(\theta_r)} = (\theta_r - 1)^2,$$

that is,

$$\theta_r = 1 + \sqrt{r}.$$

Therefore,

$$q_r(\theta) = \begin{cases} 4 - r - 2\sqrt{r}, & 1 \leq \theta < 1 + \sqrt{r}, \\ 6 - r - 2\theta, & 1 + \sqrt{r} \leq \theta \leq 2. \end{cases}$$

Using (8), the penalty associated with r is

$$\Pi(r) = t_r(1) = q_r(1) + \int_1^2 q_r(x)\,dx = 7 - 2\sqrt{r} - 3r.$$

Hence

$$\Pi^* = \Pi(r^*) = \frac{21}{4}.$$

This gives three-regimes:

1. *Strong enforcement:* if $\Pi \geq 7$, the optimal contract is the repeated static contract (t^{os}, q^{os}).

2. *Weak enforcement:* if $\frac{21}{4} \leq \Pi < 7$, then $r(\Pi)$ is determined by

$$7 - 2\sqrt{r} - 3r = \Pi, \qquad r(\Pi) = \left(\frac{\sqrt{22 - 3\Pi} - 1}{3}\right)^2.$$

   For example, if $\Pi = 6$, then $r = \frac{1}{9}$ and $\theta_r = \frac{4}{3}$, so the optimal output is

$$q_r(\theta) = \begin{cases} \frac{29}{9}, & 1 \leq \theta < \frac{4}{3}, \\ \frac{53}{9} - 2\theta, & \frac{4}{3} \leq \theta \leq 2. \end{cases}$$

   Thus the most efficient types are bunched on $\left[1, \frac{4}{3}\right)$.

3. *Very weak enforcement:* if $\Pi < \frac{21}{4}$, the optimal quantity is q_{r^*}, i.e.,

$$q_{r^*}(\theta) = \begin{cases} \frac{11}{4}, & 1 \leq \theta < \frac{3}{2}, \\ \frac{23}{4} - 2\theta, & \frac{3}{2} \leq \theta \leq 2, \end{cases}$$

   and the least efficient type earns a strictly positive rent. For instance, if $\Pi = 5$, then

$$U(2) = \frac{1-\delta}{\delta}(\Pi^* - \Pi) = \frac{1}{16} > 0.$$

This example highlights the economic content of Theorem 1: stronger public enforcement relaxes the cap on the top transfer, reduces bunching, and eventually restores the repeated Baron-Myerson allocation.

4 Proof of Theorem: Reduction to Two Programs

Rewriting constraints (1) and (4) respectively yields

$$t(\theta) - \theta q(\theta) \geq 0 \quad \forall \theta \in \Theta, \tag{10}$$

$$t(\theta) - \theta q(\theta) \geq (1-\delta)(t(\underline{\theta}) - \Pi) \quad \forall \theta \in \Theta. \tag{11}$$

Constraint (11) includes, on the right-hand side, the transfer for the most efficient type $t(\underline{\theta})$, representing the seller's net gain in the event of breach. This right-hand side becomes non-positive when the net gain falls below the penalty, making the enforcement constraint ineffective. In this case, the problem becomes a second-best problem with a cap Π on transfers. If the net gain from any deviation is less than the penalty, the seller's overall benefit from breaching the contract is non-positive. Therefore, the seller has no incentive to breach as long as ex post rents are non-negative, which is assured if (10) holds.

When the seller's net gain from breach exceeds the penalty, the participation constraint follows from the enforcement constraint. The seller can breach, cover the penalty with the gain, and leave the relationship. To prevent breach, the contract must ensure that the seller obtains enough rent from continuation. This leads to two regimes of constraints:

$$(A):\begin{cases} t\left(\underline{\theta}\right)-\Pi \geq 0, \\ t(\theta)-\theta q(\theta) \geq (1-\delta)(t(\underline{\theta})-\Pi) \quad \forall \theta \in \Theta, \end{cases}$$

and

$$(B):\begin{cases} t\left(\underline{\theta}\right)-\Pi \leq 0, \\ t(\theta)-\theta q(\theta) \geq 0 \quad \forall \theta \in \Theta. \end{cases}$$

We split the problem (P) into two problems (P^A) and (P^B), obtained from (P) by replacing (10) and (11) by systems (A) and (B) respectively.

An optimal solution of (P) is necessarily optimal for either (P^A) or (P^B). Conversely, among the solutions to (P^A) and (P^B), the one that provides the highest payoff to the principal is the optimal solution of (P).

Program (P^A)**:**

Consider first (P^A). To eliminate $t\left(\underline{\theta}\right)$ from the right-hand side of (11), introduce the adjusted transfer $y(\theta)$: [5]

$$y(\theta)=t(\theta)-(1-\delta)\left(t\left(\underline{\theta}\right)-\Pi\right).$$

Then system (A) becomes

$$\begin{cases} y\left(\underline{\theta}\right)-\Pi \geq 0, \\ y(\theta)-\theta q(\theta) \geq 0 \quad \forall \theta \in \Theta. \end{cases}$$

Note that the second constraint must bind at $\overline{\theta}$:

$$y(\overline{\theta})-\overline{\theta} q(\overline{\theta})=0. \tag{12}$$

Indeed, if $y(\overline{\theta})-\overline{\theta} q(\overline{\theta})>0$, consider the contract $\big(q(\theta)+\varepsilon, y(\theta)\big)$, for ε such that $y(\overline{\theta})-\overline{\theta}(q(\overline{\theta})+\varepsilon)=0$. This change does not affect feasibility and strictly increases the principal's payoff.

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[5] Notice that $y\left(\underline{\theta}\right)=\delta t\left(\underline{\theta}\right)+(1-\delta)\Pi$ and $t\left(\underline{\theta}\right)=\frac{y(\underline{\theta})-(1-\delta)\Pi}{\delta}$. Thus $t(\underline{\theta})=\Pi$ if and only if $y(\underline{\theta})=\Pi$.

Problem (P^A) can thus be written as

$$\max_{\{y(\cdot),q(\cdot)\}\text{ admissible}} \int_{\underline{\theta}}^{\overline{\theta}} \left(S(q(\theta)) - y(\theta)\right) f(\theta) d\theta - \frac{1-\delta}{\delta}\left(y\left(\underline{\theta}\right) - \Pi\right)$$

subject to (12) and

$$y(\underline{\theta}) - \Pi \geq 0. \tag{13}$$

Problem (P^A) depends on Π, which enters both the objective and constraint (13). We treat (P^A) as an optimal control problem with boundary constraints (12)–(13) and a scrap value $-\frac{1-\delta}{\delta}\big(y\left(\underline{\theta}\right) - \Pi\big)$.

Lemma 4. 1. *When $\Pi \in [\Pi^*, \Pi^{os}]$, the optimal output for (P^A) is $q_r(\theta)$ with r determined by*

$$\theta_r\, q^{(r)}\left(\theta_r\right) + \int_{\theta_r}^{\overline{\theta}} q^{(r)}\left(\theta\right) d\theta = \Pi. \tag{14}$$

Furthermore, constraint (13) is binding: $y(\underline{\theta}) = \Pi$.

2. *For $\Pi < \Pi^*$, the optimal output is $q_{r^*}(\theta)$ and (13) is slack: $y(\underline{\theta}) > \Pi$.*

In Case 1, $t\left(\underline{\theta}\right) - \Pi \geq 0$ is binding. Enforcement is not an issue and the optimal contract does not depend on δ. Case 2, of very weak enforcement, arises when $\Pi < \Pi^*$. In this case, the output is fixed at $q_{r^*}(\theta)$. Efficiency cannot be further compromised in favor of enforcement. Given the minimal penalty, enforcement relies on the discount factor, reflected in $r^* = \frac{1-\delta}{\delta}$.

Program (P^B):

Now consider

$$(P^B): \qquad \max_{\{t(\cdot),q(\cdot)\}\text{ admissible}} \int_{\underline{\theta}}^{\bar{\theta}} \left(S(q(\theta)) - t(\theta)\right) f(\theta) d\theta$$

subject to (10) and

$$\Pi - t\left(\underline{\theta}\right) \geq 0. \tag{15}$$

Problem (P^B) is a second-best problem with a cap on transfers. It does not depend on δ. There are two differences between (P^B) and (P^A): there is no scrap value in the objective of (P^B), and constraint (15) is the reverse of (13). The optimal static contract describes the maximum payoff in the presence of the enforcement constraint. Therefore, when $\Pi \geq \Pi^{os}$ the optimal enforcement contract is simply the optimal static contract.

When $\Pi \leq \Pi^{os}$, constraint $\Pi - t\left(\underline{\theta}\right) \geq 0$ is binding. Thus, for $\Pi \in [\Pi^*, \Pi^{os}]$ the optimal outputs and transfers are identical in (P^A) and (P^B).

Lemma 5. *1. If $\Pi \leq \Pi^{os}$, the optimal output is $q_r(\theta)$, where r is defined by (14); in addition, (15) is binding: $t(\underline{\theta}) = \Pi$.*

2. If $\Pi > \Pi^{os}$, the optimal output is the static second-best contract, and (15) is strict: $t(\underline{\theta}) < \Pi$.

COMPARISON BETWEEN (P^A) AND (P^B): Intuitively, for small Π, the value derived from (P^A) exceeds that of (P^B), and the reverse holds as Π becomes large. Specifically, for $\Pi \geq \Pi^{os}$, (P^B) has the static second-best contract as its solution. For (P^A), the constraint on $t(\underline{\theta})$ distorts transfers away from the optimal static contract. The seller's gain in the event of breach, $t(\underline{\theta})$, must at least match the large penalty, leading to an upward distortion of $t^{os}(\underline{\theta})$. Consequently, for $\Pi > \Pi^{os}$, the value of (P^B) exceeds that of (P^A) and the optimal contract is the static second-best.

For $\Pi \in [\Pi^*, \Pi^{os}]$, both (P^A) and (P^B) share the same necessary and sufficient conditions and, therefore, the same solutions. Given that $y(\underline{\theta}) - \Pi$ is binding, the scrap value in the objective of (P^A) is zero. Hence, both programs have the same value.

For any $\Pi < \Pi^*$, consider the optimal contract $\big(t(\theta), q(\theta)\big)$ for (P^B). By construction, $t(\underline{\theta}) = \Pi$, so this contract is feasible for (P^A) and attains the same objective value as in (P^B). Therefore the solution to (P^A) is also optimal for (P).

Proposition 1. *1. For $\Pi < \Pi^*$, the solution to (P^A) specified in Lemma 4 is optimal for (P).*

2. For $\Pi \in [\Pi^*, \Pi^{os}]$, both programs have the same value and yield the same optimal contract.

3. For $\Pi > \Pi^{os}$, the solution to (P^B) specified in Lemma 5 is optimal for (P).

5 Discussion

We have characterized the optimal stationary contract when the agent can commit ex ante but cannot commit not to default ex post. The optimal allocation is pinned down by the interaction between public enforcement (the breach penalty Π) and private enforcement (the continuation value governed by δ).

The model delivers regime-dependent comparative statics that map naturally into observable contract terms in procurement, construction, and trade (e.g., cash-in-advance, progress payments, retention, performance bonds, and relationship-based enforcement). Two primitives have empirical counterparts: public enforcement Π (court effectiveness, recoverability of damages, enforceable penalties) and private enforcement δ (relationship value driven by repeat trade, reputational capital, switching costs, or platform ratings).

Decomposition of rents. As discussed after Theorem 1, incentive compatibility implies the envelope condition $\dot{U}(\theta) = -q(\theta)$ a.e., hence

$$U(\theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} q(x)\, dx. \tag{16}$$

The regime-dependent term is $U(\bar{\theta})$.

- **Strong and Weak Enforcement ($\Pi \geq \Pi^*$).** In these regimes the least efficient type earns zero rent, $U(\bar{\theta}) = 0$. Thus the agent's rent is purely informational:

  $$U(\theta) = \int_{\theta}^{\bar{\theta}} q_r(x)\, dx,$$

  with $r = 0$ under strong enforcement ($\Pi \geq \Pi^{os}$) and $r = r(\Pi) \in (0, r^*(\delta)]$ under weak enforcement ($\Pi^* \leq \Pi \leq \Pi^{os}$). A higher penalty Π relaxes the transfer cap and reduces distortions (lower r), thereby increasing information rents.

- **Very Weak Enforcement ($\Pi < \Pi^*$).** Here public enforcement is insufficient to deter breach at the top; compliance must be supported by relational incentives. The optimal quantity schedule is $q_{r^*}(\theta)$ with $r^*(\delta) = \frac{1-\delta}{\delta}$, and the principal must leave a strictly positive *enforcement rent* to the least efficient type:

  $$U(\bar{\theta}) = \frac{1-\delta}{\delta}\big(\Pi^* - \Pi\big) > 0.$$

  Using (16), the agent's rent decomposes into

  $$U(\theta) = \underbrace{\frac{1-\delta}{\delta}\big(\Pi^* - \Pi\big)}_{\text{Enforcement rent}} + \underbrace{\int_{\theta}^{\bar{\theta}} q_{r^*}(x)\, dx}_{\text{Information rent}}.$$

  Holding δ fixed, raising Π reduces the enforcement rent one-for-one while leaving q_{r^*} unchanged; thus the agent is strictly worse off as Π increases within this regime, until Π reaches Π^*.

This yields the following comparative statics.[6]

Corollary 1 (Institutional improvements and surplus)**.** *Consider the optimal stationary contract at* (Π, δ)*.*

1. ***(Output and bunching.)*** *For* $\Pi \geq \Pi^{os}$*,* $q = q^{os}$ *(no bunching) and is independent of* δ*. For* $\Pi^*(\delta) \leq \Pi \leq \Pi^{os}$*, the optimal contract is* (t_r, q_r) *with* $r = r(\Pi)$ *decreasing in*

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[6] Recall that Π^* depends on δ through $r^* = \frac{1-\delta}{\delta}$. We write $\Pi^* = \Pi^*(\delta)$ emphasizing this dependence.

Π, so bunching weakly decreases with Π. For $\Pi < \Pi^*(\delta)$, the optimal quantity is $q_{r^*(\delta)}$, independent of Π, and $r^*(\delta) = \frac{1-\delta}{\delta}$ decreases in δ, implying less distortion as δ rises. Moreover, $\Pi^*(\delta)$ is increasing in δ.

2. ***(Principal payoff.)*** *The principal's expected payoff is weakly increasing in Π and weakly increasing in δ.*

3. ***(Agent payoff.)*** *Holding δ fixed, the agent's expected payoff is non-monotone in Π: it strictly decreases with Π on $\Pi < \Pi^*(\delta)$ (reduced enforcement rents), and weakly increases with Π on $\Pi^*(\delta) \leq \Pi \leq \Pi^{os}$ (higher information rents), and is constant for $\Pi \geq \Pi^{os}$. Holding Π fixed, the effect of δ on the agent's expected payoff is* locally zero *in the Strong and Weak regimes (i.e., for changes in δ that do not move the economy across the threshold $\Pi^*(\delta)$), and is generically ambiguous in the Very Weak regime.*[7]

Parts (1)–(2) follow because increasing Π or δ relaxes the enforcement constraint, expanding the feasible set and allowing the principal to do weakly better. For the agent, (16) implies that changes in $U(\bar\theta)$ shift all rents pointwise. In the very weak regime, $q = q_{r^*}$ is independent of Π while $U(\bar\theta) = \frac{1-\delta}{\delta}(\Pi^*(\delta) - \Pi)$, so increasing Π reduces $U(\bar\theta)$ and hence $U(\theta)$ for all types. In the weak regime, $U(\bar\theta) = 0$ and increasing Π lowers r, increasing q_r pointwise and therefore increasing information rents $\int_{\theta}^{\bar\theta} q_r(x)\,dx$.

6 Appendix

Proof of Lemma 2. Multiply both sides of (3) by $(1-\delta)$ and write $U(\hat\theta;\theta) := t(\hat\theta) - \theta q(\hat\theta)$. For any fixed report $\hat\theta$ and any integer $\tau \geq 0$,

$$(1-\delta)\sum_{s=0}^{\tau-1}\delta^s\, U(\hat\theta;\theta) + (1-\delta)\delta^\tau\big(t(\hat\theta) - \Pi\big) = (1-x)\,U(\hat\theta;\theta) \,+\, (1-\delta)x\big(t(\hat\theta) - \Pi\big),$$

where $x := \delta^\tau \in [0,1]$. For fixed $\hat\theta$ this expression is affine in x, hence its maximum over $x \in [0,1]$ is attained at an endpoint $x \in \{0,1\}$ and is worth:

$$\max\Big\{U(\hat\theta;\theta),\ (1-\delta)\big(t(\hat\theta) - \Pi\big)\Big\}.$$

Taking now the maximum over $\hat\theta \in \Theta$ yields

$$(1-\delta)\cdot\text{RHS of (3)} = \max\Big\{\underbrace{\max_{\hat\theta} U(\hat\theta;\theta)}_{=U(\theta)\ \text{by IC}},\ \underbrace{(1-\delta)\max_{\hat\theta} t(\hat\theta)}_{=(1-\delta)\,t(\underline{\theta})} - (1-\delta)\Pi\Big\}.$$

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[7] Higher δ improves efficiency but reduces enforcement rents.

The equality $\max_{\hat{\theta}} U(\hat{\theta};\theta) = U(\theta)$ uses truthfulness (Lemma 1). Since q is non-increasing and $\dot{t} = \theta\dot{q} \leq 0$ a.e., t is weakly decreasing in θ, so $\max_{\hat{\theta}} t(\hat{\theta}) = t(\underline{\theta})$. Therefore, (3) is equivalent to

$$U(\theta) \ \geq \ \max\Big\{U(\theta),\ (1-\delta)\big(t(\underline{\theta})-\Pi\big)\Big\} \quad \forall\theta,$$

which is in turn equivalent to (4). □

Proof of Lemma 3. Note first that (6) yields $r(\underline{\theta}) = 0$. Second, the denominator $\theta f(\theta) - F(\theta)$ is positive at $\theta = \underline{\theta}$. Consider an interval $[\underline{\theta}, \varepsilon]$ such that this denominator is positive for all $\theta \in [\underline{\theta}, \varepsilon]$. Differentiating (6) with respect to θ gives $r'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \varepsilon]$. Indeed,

$$r'(\theta_r)\left(\theta_r - \frac{F(\theta_r)}{f(\theta_r)}\right) = 2F(\theta_r)f(\theta_r) - r\theta_r f'(\theta_r) = 2F(\theta_r)f(\theta_r) - \frac{F^2(\theta_r) + rF(\theta_r)}{f(\theta_r)}f'(\theta_r).$$

The numerator on the right-hand side is

$$2F(\theta_r)f^2(\theta_r) - F^2(\theta_r)f'(\theta_r) - rF(\theta_r)f'(\theta_r) = F(\theta_r)\Big(2f^2(\theta_r) - F(\theta_r)f'(\theta_r) - rf'(\theta_r)\Big) > 0,$$

where the last inequality follows from Assumption 1. Proceeding by extending the interval step by step, either we reach some θ' such that $\theta' f(\theta') - F(\theta') = 0$, or for all $\theta \in \big[\underline{\theta}, \overline{\theta}\big]$ we have $\theta f(\theta) - F(\theta) > 0$. In the first case, for all $r \in [0, r^*]$ there exists a unique $\theta_r \in \big[\underline{\theta}, \overline{\theta}\big]$ such that $r = \frac{F^2(\theta_r)}{\theta_r f(\theta_r) - F(\theta_r)}$. In the second case, we can assume that $\theta_r = \overline{\theta}$ for all $r \geq \widehat{r} = r(\overline{\theta})$. □

Proof of Lemma 4. *Optimality conditions for problem* (P^A). We explicitly incorporate monotonicity by introducing $z(\theta) = \dot{q}(\theta)$. Problem (P^A) is then formulated as an optimal control problem with state variables $y(\theta)$ and $q(\theta)$ and control $z(\theta)$. The co-state variables corresponding to (18) and (19) are denoted by $\lambda_1(\theta)$ and $\lambda_2(\theta)$ respectively.

The Hamiltonian is

$$H(y, q, z, \lambda_1, \lambda_2, \theta) = (S(q) - y)f(\theta) + \lambda_1\theta z + \lambda_2 z,$$

which is concave in (y, q, z) for all θ. Let $\big(y(\theta), q(\theta), z(\theta)\big)$ be an admissible triplet with continuous, a.e. differentiable $y(\theta), q(\theta)$ and piecewise continuous $z(\theta)$. Then $\big(y(\theta), q(\theta), z(\theta)\big)$ is optimal if and only if there exist continuous, piecewise differentiable co-state variables $(\lambda_1(\theta), \lambda_2(\theta))$ such that the following conditions (see Seierstad and Sydsaeter (1986), pp. 85, 396) are satisfied:

$$z(\theta) \in \arg\max_{z} H\big(y(\theta), q(\theta), z, \lambda_1(\theta), \lambda_2(\theta), \theta\big) \quad \forall\theta, \tag{17}$$

$$\dot{y}(\theta) = \theta z(\theta), \tag{18}$$

$$\dot{q}\,(\theta) = z\,(\theta) \leq 0, \tag{19}$$

$$\dot{\lambda}_1(\theta) = f(\theta) \quad \text{a.e.}, \tag{20}$$

$$\dot{\lambda}_2(\theta) = -S'\big(q(\theta)\big)\,f(\theta) \quad \text{a.e.}, \tag{21}$$

$$\lambda_1(\underline{\theta}) = \frac{1-\delta}{\delta} - \beta\,(1-\delta)\,, \quad \lambda_2(\underline{\theta}) = 0, \quad \beta \geq 0 \;(= 0 \text{ if } y(\underline{\theta}) - \Pi > 0), \tag{22}$$

$$\lambda_1(\overline{\theta}) = \gamma, \quad \lambda_2(\overline{\theta}) = -\gamma\overline{\theta}, \quad \gamma \geq 0. \tag{23}$$

Denote

$$r = \lambda_1(\underline{\theta}) = \frac{1-\delta}{\delta} - \beta\,(1-\delta) = \frac{(1-\delta)\,(1-\beta\delta)}{\delta}. \tag{24}$$

Conditions (18), (19) and (20)–(22) imply

$$\lambda_1(\theta) = r + F(\theta), \tag{25}$$

and

$$\lambda_2(\theta) = -\int_{\underline{\theta}}^{\theta} S'\big(q(u)\big)\,f(u)\,du. \tag{26}$$

Define $\psi\,(\theta) = \lambda_1(\theta)\theta + \lambda_2(\theta)$. From (25)–(26) we get

$$\psi\,(\theta) = \big(r + F(\theta)\big)\theta - \int_{\underline{\theta}}^{\theta} S'\big(q(u)\big)\,f(u)\,du.$$

Optimality condition (17) yields

$$\psi\,(\theta)\,z\,(\theta) = 0, \qquad \psi\,(\theta) \geq 0 \quad \forall\theta. \tag{27}$$

Derivation of r^*. Note that

$$\psi\,(\underline{\theta}) = r\,\underline{\theta} \geq 0. \tag{28}$$

Thus $r \geq 0$. From (24), the minimal value $r = 0$ corresponds to $\beta = 1/\delta > 0$, and the maximal value $r^* = \frac{1-\delta}{\delta}$ corresponds to $\beta = 0$.

The form of optimal $q(\theta)$. If $\psi\,(\theta) > 0$ over a non-degenerate interval, then $z\,(\theta) = 0$ on this interval, rendering both state variables $q\,(\theta)$ and $y\,(\theta)$ constant due to (18) and (19). Conversely, if $\psi\,(\theta) = 0$ over a non-degenerate interval Θ' it follows that $\dot{\psi}\,(\theta) = 0$, leading to

$$S'\big(q(\theta)\big) = \theta + \frac{F(\theta) + r}{f(\theta)} \quad \text{for all } \theta \in \Theta'.$$

This condition implies that $q(\theta) = q^{(r)}(\theta)$ for all $\theta \in \Theta'$.

We now show that there can be at most one bunching interval. Suppose, to the contrary, that there are two disjoint bunching intervals, denoted $\Theta_1 = [\underline{\theta}, \theta_1)$ and $\Theta_3 = \big(\theta_2, \overline{\theta}\big]$.[8]

Within the intermediate interval $\Theta_2 = (\theta_1, \theta_2)$, we have $\psi(\theta) = 0$, so $q(\theta) = q^{(r)}(\theta)$ for all $\theta \in \Theta_2$. For all $\theta \in \Theta_1$, $q(\theta) = q^{(r)}(\theta_1) = q_1$, and for all $\theta \in \Theta_3$, $q(\theta) = q^{(r)}(\theta_2) = q_3$. If Θ_3 were nonempty, then $q_3 > q^{(r)}(\theta)$ for all $\theta \in \Theta_3$, and hence $S'\big(q_3\big) < S'\big(q^{(r)}(\theta)\big)$. Thus,

$$\dot{\psi}(\theta) = r + F(\theta) + f(\theta)\theta - S'(q_3)f(\theta) > r + F(\theta) + f(\theta)\theta - S'\big(q^{(r)}(\theta)\big)f(\theta) = 0$$

for all $\theta \in \Theta_3$. Since $\psi(\theta_2) = 0$ and, by the boundary condition, $\psi(\overline{\theta}) = 0$, this is a contradiction. Thus there may be only one bunching interval $[\underline{\theta}, \theta_1]$.

Derivation of (6). The first equation to determine θ_1 and r is $\psi(\theta_1) = 0$:

$$0 = \big(r + F(\theta_1)\big)\theta_1 - \int_{\underline{\theta}}^{\theta_1} S'\big(q(u)\big)f(u)\,du. \tag{29}$$

Since $q(u) = q_1 = q^{(r)}(\theta_1)$ for $u \in [\underline{\theta}, \theta_1]$, and $S'(q_1) = \theta_1 + \frac{r+F(\theta_1)}{f(\theta_1)}$, we obtain

$$r\theta_1 = \big(r + F(\theta_1)\big)\frac{F(\theta_1)}{f(\theta_1)}, \tag{30}$$

which is equivalent to (6).

Construction of the optimal contract. We have established that the optimal output has the form in Figure 1 with $\theta_1 = \theta_r$.

For all $\Pi \geq \Pi^{os}$ we have $r = 0$. Hence (14) has only the trivial solution $\theta_r = \underline{\theta}$ and the optimal output is the static second-best contract. In this case, $\beta = \frac{1}{\delta} > 0$, so (13) binds and $y(\underline{\theta}) = \Pi^{os}$.

For all $\Pi \in [\Pi^*, \Pi^{os}]$, constraint (13) binds so $y(\underline{\theta}) = \Pi$ and $t(\underline{\theta}) = \Pi$. Define r via

$$\theta_r q^{(r)}(\theta_r) + \int_{\theta_r}^{\overline{\theta}} q^{(r)}(\theta)d\theta = \Pi.$$

Differentiating with respect to r gives $\Pi'(r) < 0$, so r is uniquely determined for $\Pi \in [\Pi^*, \Pi^{os}]$.

For $\Pi < \Pi^*$, constraint (13) is slack and the solution is the same as at Π^*. ☐

Proof of Lemma 5. Modulo the change of variables, the optimality conditions (17)–(21)

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[8] Note that θ_1 may coincide with $\underline{\theta}$.

are the same as for (P^A). The transversality condition (22) is replaced by

$$\lambda_1(\underline{\theta}) = \beta' (1-\delta), \quad \lambda_2(\underline{\theta}) = 0, \quad \beta' \geq 0 \ (= 0 \text{ if } \Pi - t(\underline{\theta}) > 0).$$

In this case the multiplier in (P^B) is formally unrestricted, but the theorem's characterization only needs the economically relevant range $r \in [0, r^*]$. If $r = 0$, then the optimal output is the static second-best contract and the corresponding transfers are $t_0(\theta)$ defined by (7) with zero rent for $\overline{\theta}$. If $r > 0$, then the optimal output is $q_r(\theta)$ and transfers are given by (7) with zero rent for $\overline{\theta}$. $\square$

Proposition 2 (Continuity)**.** *Under Assumption 1, every optimal stationary solution* (t, q) *has a quantity schedule* $q : \Theta \to \mathbb{R}_+$ *that is continuous and weakly decreasing.*

Proof sketch. Weak decrease follows from Lemma 1. For continuity, consider Program (P^A), the only case in which the pointwise constraint can bind on a nontrivial interval. Define the adjusted transfer

$$y(\theta) = t(\theta) - (1-\delta)\big(t(\underline{\theta}) - \Pi\big),$$

and the adjusted rent

$$\widetilde{U}(\theta) = y(\theta) - \theta q(\theta).$$

Then $\widetilde{U}$ is absolutely continuous and satisfies $\dot{\widetilde{U}}(\theta) = -q(\theta)$ almost everywhere. The pointwise feasibility constraint becomes

$$\widetilde{U}(\theta) \geq 0 \qquad \forall \theta \in \Theta,$$

which is linear in $(\widetilde{U}, q)$, while the objective integrand is strictly concave in q and linear in $\widetilde{U}$. If q had a jump at some interior type, replacing that jump by a local continuous interpolation over a small neighborhood would preserve implementability and feasibility and, by strict concavity of S, would strictly increase surplus. Hence an optimal schedule cannot jump. Therefore q is continuous. $\square$

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