

Aging, Dissaving, and the Role of Life Annuities

Economic Theory

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Abstract

Canada is experiencing the time of population aging currently. By purchasing life annuities, the elders can live a long life with income guarantees. At the same time, government pension system can release burdens on retirement income provision. In this paper, the world annuity markets are explored, showing that the demand for annuities remains little from past to now. The factors causing this annuity puzzle are also reviewed and the weight of these explanatory factors are gauged to illustrate that government should implement mandatory annuitization policies. Then I use an overlapping generation equilibrium model to evaluate the impact of 100% annuitization. The simulation results implicitly show that it is optimal to fully annuitize, which is consistent to the theory. Yet, 100% annuitization for people has little contribution on reducing the demographic shock on GDP per capita in Canada.

1. Introduction

Canada is one of the OECD countries that faces serious issues of population aging. Within the past two decades, the total population of Canada has grown more than 6 million from 29.9 million in 1997 to 36.7 million in 2017¹. During the same period, however, the demographic structure of the population has altered dramatically². In 1997, the proportion of the 0 to 14 age group was 20% and the corresponding percentage for the 65 years and over group was only 12.2%. Since then, the proportion of young children decreases to 16% and the senior group rose to 16.9%. Further, the population of the 65 years and over is growing at a yearly rate of 3.4%, which is around three times faster than that of children aged 0 to 14 years old or the gross growth rate of total population, which is 1.2% yearly. In other words, Canada is unavoidably entering in to an aging stage. By 2031, the proportion of seniors aged 65 and over is projected to be 23.1%³. That is, there will be one senior for every four individuals alive in Canada. It took 20 years for Canada to increase its population proportion aged 65 years and over by 4.7% from 12.2% to 16.9%. Yet, the proportion of the older group will surge by another 6.2% from 16.9% to 23.1% within the next 14 years. Thereby, aging in Canada is accelerating.

Improvements in health contributes a lot to extending life expectancy. Statistics Canada has recently released a report by Lebel and Hallman (2012) on the mortality situation for 2012/2013. The report shows that life expectancy at birth increased from 75.4 for males and 81.1 for females in 1995/1997 to 79.6 and 83.8 in 2011/2013. They found that life expectancy in Canada is among the highest in countries of the OECD, with the 13th

¹ Source: Statistics Canada. CANSIM Table 051-0001

² Data about the demographic structure in this paragraph are all from Statistics Canada, Demography Division.

³ Source: Population Projections for Canada (2013 to 2063), Statistics Canada.

rank for women and the 11th rank for men. Further they showed that the probability of surviving between ages 65 and 85 was 48% among males and 62% among females in 2011/2013. More and more people are surviving to advanced ages.

With such numbers of the elderly increasing each year, working-age people and the society will have to bear this aging burden. According to Statistics Canada, the trend of demographic dependency ratio is opposite from what was observed twenty years ago. The young dependency reduced from 29.5 per 100 workers in 1997 to 23.9 per 100 workers in 2017, and it maintained at around 23.7 for 8 years since 2010. Inversely, the old dependency ratio has steadily raised to 25.1 per 100 workers in 2017, compared with 18 per 100 workers in 1997. And this ratio is projected to be 38.5 around 2031, after all baby boomers have turned 65. This opposite trend in young and old dependency ratios will form a harmful circuit, as Hook and Weil (2012) argued that, because “increased old-age dependency reduces fertility, which further increases old-age dependency. However, as the fertility rate falls, so does the youth dependency ratio.” To prevent this from happening, one option, apart from immigration and postponing the retirement age, is to make the old sufficiently well off, so that they are no longer a burden for the working-age people. This issue is linked to retirement financial planning.

If one searches on Google for retirement planning, or goes to the website of Government of Canada⁴, he or she can obtain a fully guide on several pension revenues. First, retirees would surely receive pension incomes, at least from the Canada Pension Plan (CPP), if they have worked in Canada. The basic financial needs, however, may not be

⁴ The website lists potential sources of retirement income. See information at, <https://www.canada.ca/en/financial-consumer-agency/services/retirement-planning/sources-retirement-income.html>

satisfied by only relying on CPP (and/or other social securities), as the amount of its monthly payments is determined by the retirees' varying working years. In 2017, the maximum monthly payment amount for retirement pension (at age 65) is 1114 dollars, but the average amount is only 653 dollars⁵. Is it enough for retirees, though? A report (Shillington, 2016) on the economic circumstances of Canadian seniors shows that, if government pension benefit (including the social securities, like old age security) is the only source of income, the annual benefits for senior couples aged 65 and over was \$25746 at most in 2015. In the same year, however, the average annual expenditure per household aged 65 and over was \$55090⁶. Reliance on government pensions is not adequate, as it at most copes with only half of the retirement needs. So other sources of income will be required to fill this spread.

The straightest way to supplement the cost for the retirement is saving for it. And people are indeed accumulating lots of wealth. As data from the Survey of Financial Security (SFS) shows, the averaged net wealth (assets less debts) of households with members aged 65 and over has risen to 722k dollars⁷ in 2012 from an average of 430k dollars in 1999. Yet, an acute problem for prospective retirees is how to amortize their wealth over their retirement period in case of life length uncertainty. If they know when they will die, the solution is easy as they can split their wealth evenly by the number of months they are going to live. Until their last month of life at which time wealth will fall to zero (of course, this assumes a case without bequest motives). But people are uncertain

⁵ Contributors can choose to start receiving a reduced pension as early as age 60, or to receive their first pension at age 70 with an increased amount. These values were published by Government of Canada in 2016, based on the standard payment at age 65.

⁶ Source: Statistics Canada. Survey of household spending (SHS), household spending, by age of reference person, annual (dollars). <http://www5.statcan.gc.ca/cansim/a47>

⁷ These two values are computed based on data from Table205-0002, Statistics Canada, Survey of Financial Security (SFS). See what contents of net wealth and other information at, <http://www5.statcan.gc.ca/cansim/a47#F9>

about how long their life will last. In this paper, I will focus on the ability of people to purchase life annuities to ensure against the life length risk. Life annuity is typically an insurance product that provides the buyers with a series of payments whenever they are alive. One can treat life annuity as an investment that provides higher rate of return than the market rate (Yaari, 1965)⁸. Thereby, it is theoretically optimal for people to fully annuitize their financial wealth with insurance companies. However, few people behaved as if they believed that life annuities were of great value. Indeed, empirically it appears that people do not purchase much life annuity products. However, to show how annuity can contribute to the Canadian economy and level up people's utilities, I will adopt an OLG model (Georges and Seçkin, 2016) to simulate scenarios under two different annuitization schemes. Specifically, I will compare and analyse the results for zero-annuitization with full wealth annuitization.

The main purpose of this paper is therefore to show to what extent annuity can release the financial burden of population aging. This paper consists of seven sections. In section two, literature on life annuity is reviewed. Section three shows a general picture of annuity market in Canada and some other regions. In section four, I introduce a number of insights to explain the annuity paradox — that life annuity products are undersold. In section five, the OLG model with annuitization parameter is described. In section six, I will show the simulation results of two scenarios. At last, conclusion will be drawn and caveats as well as the directions of future research are provided.

⁸ This implication is satisfied when no bequest exists. I will give further information on Yaari's work later.

2. Literature Review

In a world of life length certainty, individuals know the date of their death so that they amortize their financial wealth by spreading/selling their assets so as to generate a constant cash stream. What should be done in the likely case of life length uncertainty? Some may decumulate their wealth too fast if they think that the chance to live to age 80 is low, while eventually they live to 100 in a great poverty for 20 years. Some, on the other side, plan to amortize their wealth for 30 years, but they die within 10 years, with large amount of unintended bequest. That is, with mortality risk, it can be hard for individuals to balance the risk of outliving total wealth against the risk of giving up a better life that they can afford while living⁹.

Life annuity is a financial product that provides retirees with stable periodic incomes when they are alive in exchange of an amount of wealth that will be transferred to the insurance company after the death of the individual. Thus, life annuity is a guarantee for retirees against the risk of outliving their wealth (Mitchell et al., 1999).

Starting from Yaari (1965), many papers discussed the optimal ratio of wealth that should be annuitized to maximize one's intertemporal utility. Yaari (1965) draws a widely cited conclusion that, if the insurance company offers a fair rate life annuity, it would be optimal for individuals to annuitize all their wealth so as to maximize the expected intertemporal utility¹⁰. He also proves that this fair actuarial rate exceeds the market rate of return by a conditional instantaneous mortality rate. Under the same assumptions of no bequest motive in a perfect fair annuity market, Yuh and Yang (2012) examined the

⁹ In the former case, people underestimate their life length; while in the latter case, they overestimate their life length.

¹⁰ This conclusion depends on the assumption that the individual does not plan to leave a bequest.

optimal annuity ratio (i.e., the optimal share of financial wealth that should be annuitized) with Korean data. They find that retirees should annuitize all the wealth to achieve maximum life utility in Korea. In addition, many papers recognised Yaari's finding by introducing life uncertainty, mortality risk and 100% wealth annuitization into OLG models (Börsch-Supan et al., 2006; Krueger and Ludwig, 2007; Georges and Seçkin, 2016).

Furthermore, a special feature of life annuities is implied that the hump-shaped consumption profile (Attanasio et al., 1999) can be smoothed by life annuities, if the rate of time preference equals the market interest rate. According to Fisher (1930) and Yaari (1965), life length uncertainty implicitly raises the degree of impatience of individual, making the individual to prefer to consume now instead of later when the risk of dying is higher. But, Yaari (1965) states that a life annuity can eliminate the life length uncertainty by providing individuals with fixed consumption, as if we were in a world of certainty. Intuitively, life annuities can smooth consumption profile. It is equivalent to the conclusion from Attanasio et al. (1999) that "neglecting uncertainty produces consumption profiles that are 'too flat'". Similarly, the result from Yuh and Yang (2012) supports this implication as well. It shows that the consumption line with 100% wealth annuitization from age 60 is nearly flat in terms of life horizon, is contrast to the declined consumption line without annuitization. Several papers converge to the same insight. For example, Hansen and İmrohoroğlu (2008) apply an OLG model under mortality uncertainty and find that the discounting of future consumption is increasing with age, which varies with an escalating mortality risk.

It is easy to blame the strict conditions that the preceding papers have assumed. In those papers, annuity is operated under completely fair market and the bequest motive is

assumed away. However, many literatures have tried to test the reliance of those findings under loose constraints. With respect to the examination of full annuitization, the work done by Davidoff et al. (2005) implies that, even if insurance company charges some higher premium rate, it is still optimal to fully annuitize, keeping other conditions the same. They also release some restrictions (not assuming expected utility axioms or intertemporal separability) on the utility function and maintain the conclusion of fully annuitization in the Arrow complete market. In presence of bequest, they state that bequest varies with time/age when wealth is not annuitized, but the existence of partial annuitization allows individuals to reduce bequest variation. Further, individuals will realize optimality by annuitizing remaining assets in addition to bequest, if actuarial annuity rate is fair. In fact, Yaari (1965) states a similar idea implicitly; certain annuitization schemes may lead to an optimal consumption path, when the marginal utilities of consumption and the bequest are equal at any moment. Bejdra and Mierau (2012) have built a large scale OLG model to analyse the effect of unfair actuarial rate. They find that this imperfection makes individuals lean on current consumption and that early decumulating of assets has a negative impact on economic growth.

On the opposite side of full annuitization, individuals sometimes choose to plan their retirement by self-annuitizing. It was first introduced by Milevsky and Robinson (2000), that individuals can manage a discretionary investment and periodically withdraw a fixed amount as retirement income¹¹. The purpose of self-annuitization is to maintain the flexibility and liquidity of accumulated wealth (Horneff et al., 2006). According to

¹¹ Individuals who choose to have such self-annuitization is actually investing their wealth in financial market. In this case, they do not consider the risk of living a longer life than their expectation.

Milevsky and Robinson (2000), probability of ruin (PoR)¹² for Canadian self-annuitants in the lifetime case is as low as a well-diversified investment portfolio. Another motivating study by Schmeiser and Post (2005) provides an interesting idea of “family strategy”¹³. An agreement among family members is made, that the retired parent bequeaths his or her wealth that remained at the end of life, while the child, as the heir, shoulders the risk of shortfall from the self-annuitization. They find that heirs are willing to sign such agreement, and that the chance for shortfall is quite low in different scenarios. They also indicate that the chance of shortfall is lower when more children are in the family. This is an argument related to Kotlikoff and Spivak (1981) who state that larger family permit to pool risk¹⁴.

Yet, no matter what Milevsky or Schmeiser find, they all admit a distinct risk of shortfall; after all, few individuals can well design a diversified portfolio. By comparing the self-annuitization portfolios with an annuity benchmark regarding German market, Albrecht and Raimond (2002) find an opposite conclusion to Milevsky’s. They set three possible rates of return for single-premium immediate life annuity as benchmarks, and then calculate the possibilities of shortfalls for three mutual funds with respect to those benchmark rates. They finally find that there is an apparent risk of outliving self-annuitants’ wealth, and that this risk is higher when individuals start to self-annuitize in older ages. In addition, the risk also raises with better performance of annuity-fund investors.

A few more literatures extend the study of self-annuitization to include additional factors like gender and risk tolerance. Agnew et al. (2008) have designed surveys for lab

¹² PoR is defined to value the probability of failure in self-annuitization. If one individual cannot live to death with adequate returns of investments, he is considered to ruin his retirement. PoR is like the probability of shortfall in following discussion.

¹³ This family strategy is largely derived from the family annuity argument of Kotlikoff and Spivak (1981).

¹⁴ Although this result seems true for many Asian countries where family bond is tight, Milevsky or Schmeiser think of that issues like family trust or cost of negotiation will influence the final outcome.

experiments and find that females are more likely to choose fair annuities, because they have higher risk aversion and are typically less financially educated than males (Dwyer et al., 2002). However, Yuh and Yang (2009) state the opposite conclusion that these two factors have no impact on the demand of annuities.

Facing the option of choosing life annuities and self-annuitization, some balanced solutions¹⁵ are also examined. Horneff et al. (2006) suggest that retirees who have relatively more risk aversion should mix self-annuitization with life annuity insurance at the beginning of retirement. Ameriks et al. (2001) also find that chance of failure in self-annuity is reduced by adding life annuity into investment portfolio. Many researchers (Ogborn and Wallas, 1955; Milevsky, 2005; Horneff and Maurer, 2008; Maurer et al., 2013) propose other solutions to complement the annuity market with other strategies. However, in my paper, I will only focus on the effects of full life annuities in comparison with self-annuitization (i.e. annuitization proportion equals to 100% and 0%).

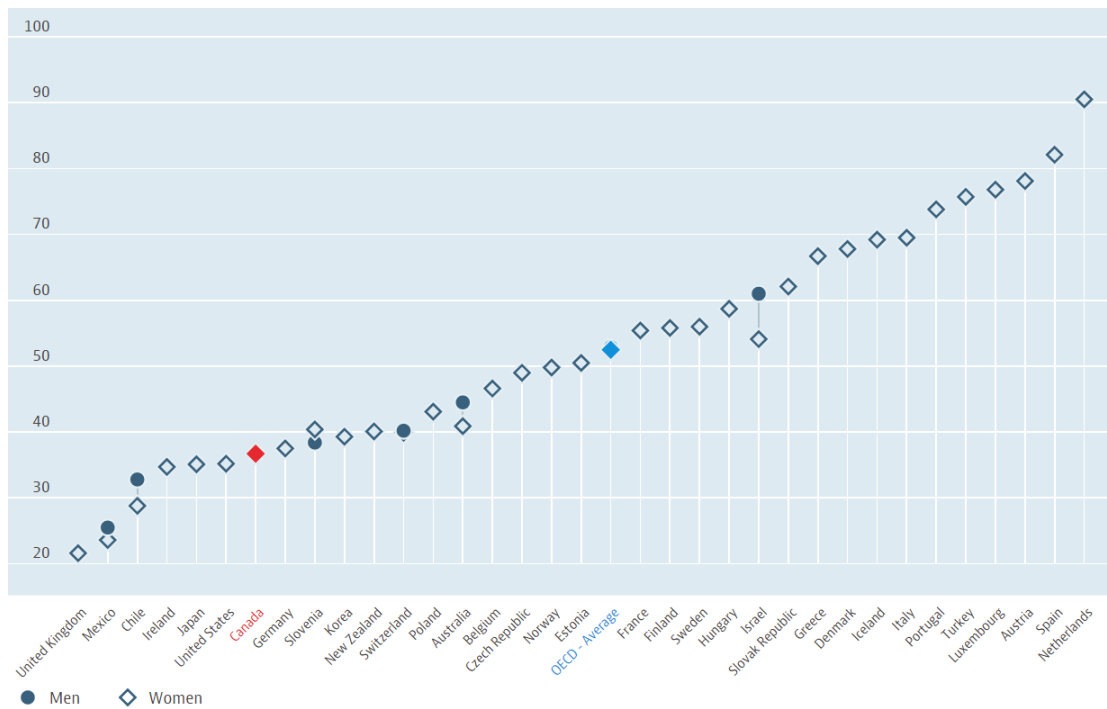
3. Life Annuity Market and Annuity Puzzle

3.1 Canadian Pension Coverage at a Glance

Shillington (2016) has pointed out that it is not enough to rely on social pensions to maintain standard of life. I have compared Shillington's estimation of total pension payments with typical retirees' expenditures and found that the pensions can only cover approximately half of the expenditure. From the income side, the gross pension replacement rate reflects the effectiveness of pension replacing the pre-retirement earnings.

¹⁵ If the only risk was longevity risk, then it would be correct to assume that people would rather prefer a full annuitization scenario. But people are facing other risks (health risk, inflation risk, etc.) and other concerns (unfair price, bequest motive, etc.), so that these offsets or reduce their demand for full annuitization. The balanced solutions here are basing on such risks and concerns. As a result, researchers suggest combining self-annuitization and life annuities.

Fig. 1 shows these rates for both men and women in OECD countries in 2014. The replacement rates for Canadian men and women are both 36.7%, meaning that the pension incomes account for only 36.7% of earnings before retirement. As shown, the benefits from Canadian pension system are small compared with many other OECD countries. The replacement rates for Canada are way lower than the average among OECD countries, which is 52.5% for both females and males. As a result, social security will not secure Canadian individuals enough pension incomes after retirement. Thereby, retirees need to seek supplements from their own savings, and annuitization of this wealth is the best strategy as discussed in section two. However, the market reaction to annuities seems inconsistent with theory.



Note: When values for men and women are the same, it shows the symbol for women only.
Source: *Pensions at a Glance, OECD Data*

Fig. 1. Gross pension replacement rates for men and women, % of pre-retirement earnings, 2014

3.2 Global Annuity Market¹⁶

As a life annuity can be a good supplement to pension plans, it is interesting to review how the annuity market has performed. Canadian Life and Health Insurance Association (CLHIA) has provided a general picture of annuity market in Canada. In their report (CLHIA, 2017), the whole annuity market includes all kinds of annuities sold directly to the public (non-registered savings) and those insured from registered plans (pension plans, RRSPs & TFSAs, RRIFs)¹⁷. The total premiums of annuity insurance in 2016 were 44.6 billion CAD, while only 17% is from non-registered savings, i.e. 7.5 billion CAD. Furtherly, Table 1 shows historical data for annuity market in Canada. The gross annuity premium from 2009 to 2016 fluctuated around 6 billion CAD. As a comparison, the total premium of life/health (L/H) insurances was way larger than that of annuities in corresponding years. In terms of percentage share, annuity market accounts for about 6% out of L/H insurance market, let alone the whole insurance market. And this share rate was decreasing in overall trend. In other words, the annuity market in Canada is quite small.

Individuals in Canada seem to have little interest in annuities. And this is a worldwide phenomenon. For example, Australia is an economy very similar to Canada in terms of economic growth and pension replacement rate, but the situation of Australian life annuity market is even worse. From 1994 to 1998, the sales of annuities (term and life) increased from 4.6 billion AUD to 9.4 billion AUD, in which life annuity accounted for

¹⁶ Since the data for life annuity is not available for most countries, then unless mentioned otherwise, the annuity data in this section involves all kinds of annuities (term & lifetime) in the market as well as pension annuities. So, the share of immediate life annuity in the whole annuity market could be much smaller. (where term annuities cover pre-determined periods/years while life annuities last until deadline; and the immediate life annuity is the one starts to be validated at the moment of purchasing.)

¹⁷ The annuity purchased with non-registered savings is the one discussed in this part. As to specific categories, the most common annuity sold in Canada is the individual immediate fixed payment annuity (OECD, 2016). There are various types of annuities introduced in this report, in case one needs more information.

roughly one thirds of the total sales (Knox, 2000). However, the lifetime annuity market size shrank dramatically from billions to millions after 2005 (Asher et al., 2013). As a result, the number of new policies for life annuity fell to 29 per year (Bateman, 2016), and there are now only two companies selling life annuity in Australia (Asher et al., 2013).

Table 1: Premiums of Selected Categories for Canada, in million CAD

Year	Annuities ^a	Life/Health Insurance ^b	Share, % ^c
2009	8,943	83,898	10.66
2010	7,815	85,298	9.16
2011	6,457	86,081	7.50
2012	5,276	88,671	5.95
2013	6,263	92,234	6.79
2014	5,562	99,371	5.60
2015	5,539	103,381	5.36
2016	7,582	105,751	7.17

^aData from 2009 to 2015 is from OECD; data for 2016 is computed with the data from CLHIA.

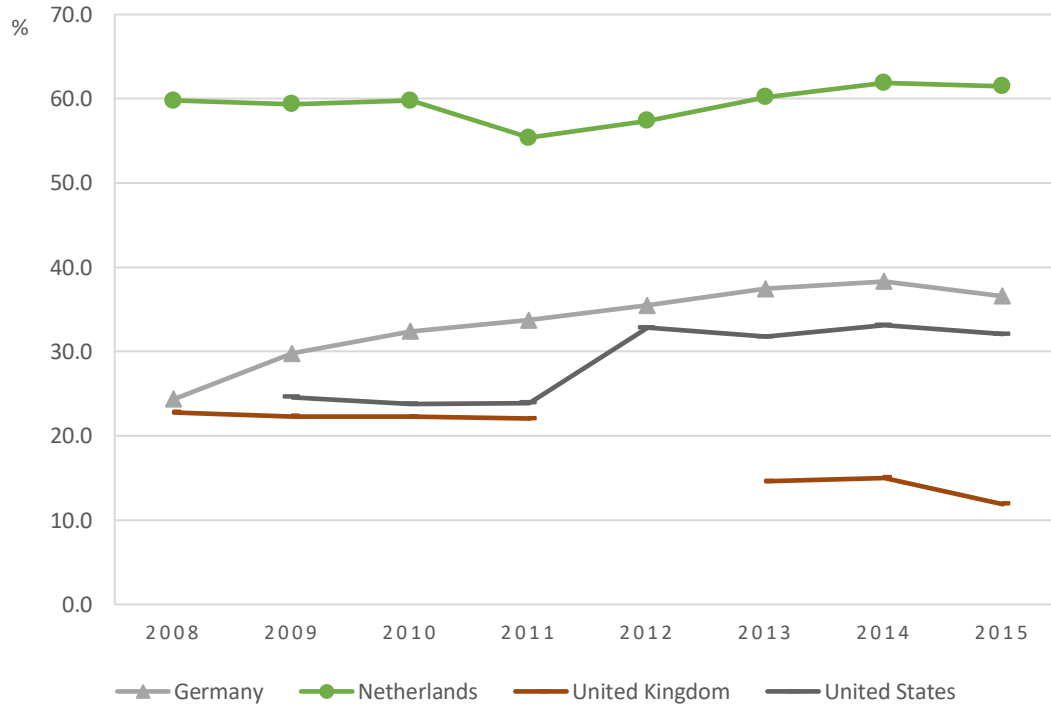
^bLife insurance includes life insurance defined by national law, annuity and pension, and health insurance. CLHIA

^cShare is the percentage ratio of annuity in life insurance.

Fig. 2 compares the shares of annuity premiums in total L/H insurance premiums for other four countries. In this chart, although annuities include term annuities and lifetime annuities, the market size of life annuities products can be reflected indirectly by the whole annuity market. For instance, according to OECD (2016) the immediate life annuity accounted for about 12% of all annuity sales in US as of 2012. Fig. 2 shows that annuities represent 32.8% of life insurance premium in the US in 2012. Then the market share of life annuity out of L/H insurance is 4%¹⁸, which is 33 billion USD. Thus, the life annuity market in the US is only a small part as well. However, the market share of annuities has

¹⁸ Annuities occupied 32.8% of L/H insurance (Appendix A), then $12\% \times 32.8\%$ is roughly 4%.

increased in recent years after years of educating and advertising to the public (Brown et al., 2007; Benartzi et al., 2011).



Note: This chart is drawn based on the data table in Appendix A. And data for UK is absent in 2012.

Fig. 2. The ratio of all annuity premiums to life insurance premiums by countries from 2008 to 2015

However, annuity markets in Netherlands and Germany are relatively huge and thriving. In fact, it is compulsory for retirees in Netherlands to annuitize all their pension wealth (Cannon et al., 2015). And we can find that Netherlands has the highest pension replacement rate among OECD countries in Fig. 1, so that there are plenty of funds flowing into annuity markets¹⁹. With respect to Germany, before 2001 the size of life annuity market was small, but it eventually reached to 2.7 billion EUR as of 2001. The boom of life annuity market started with Germany’s pension reform in 2001. Since then, pension

¹⁹However, the large annuity participation is due somewhat to the relatively fair actuarial rate in annuity market.

benefits were required to be partially annuitized as life annuities. More specifically since 2005, the annuitization ratio must not be lower than 70% (Fehr and Habermann, 2010).

The UK also has a long history of mandatory annuitization policy since 1956, at which time all the pensioners had to purchase an annuity at the latest by age 70. Then the age restriction was increased to 75 in 1976 (Hayward, 2002). Finally, this age limit was scrapped indefinitely, meaning there was no more compulsory annuitization in the UK (King, 2010). If individuals valued highly life annuities, then the market size of the whole annuity market should have remained unchanged. However, in the UK, the market share of annuity dropped substantially after 2012, with no change to the L/H premiums (Fig. 2, Appendix A). Hence, life annuity is still not that attractive to individuals whether they are required to annuitize or not. This implication is also supported by the US case (Brown et al., 2007), that retirees prefer to choose lump sum benefits given the possibility to choose between lump sums and annuities.

3.3 Annuity Puzzle

Why so little demand of annuities? In 1985, Franco Modigliani first drew the public's attention to this "annuity participation puzzle" in his Nobel Prize acceptance speech, that "*...It is a well known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest. It is still ill-understood.*" (Modigliani, 1986). Fifteen years later, James and Song (2001) examined the evidence from Canada, the UK, Switzerland, Australia, Israel, Chile, Singapore, as well as the US, and reported that "Annuities markets around the world are small". Despite the slight growth in annuity market, it still remains a puzzle. And it has been discussed widely in the literature (Benitez-

Silva, 2003; Inkmann et al., 2010; Benartzi et al., 2011; Peijnenburg, 2016; etc.). A second puzzle, according to Brown (2007), is that individuals do annuitize their wealth when offered the annuities with some guarantees. For example, individuals prefer a “10-year period certain” life annuity that guarantees annuitants to receive minimum ten years of payments whether the annuitants survive or not during that period. If the annuitants die within 10 years, insurance company will continue to pay some specified individual after the death of annuitants. This is a puzzle to Brown because in his eyes, this ten-year-payment guaranteed life annuity is equivalent to a combined product of a ten-year contingent bond and a ten-year deferred²⁰ life annuity. In other words, it is interesting that people do not like life annuity but like a guaranteed life annuity (Scott et al., 2006; Brown, 2007).

4. Argument of Explanations to Annuity Puzzle

Since Modigliani’s speech in 1985, a huge number of studies have been done attempting to explain the annuity puzzle. Brown (2007) and Inkmann et al. (2010) have provided a survey of this literature. Here, I will review some arguments starting from the perspective of consumers who are introduced to new insurance products. What factors do they face when making the decision to purchase a life annuity or not?

4.1 Pensions as substitution

The foremost and straightest possible answer for not buying annuities is that if an individual expects to receive pension incomes that function like life annuity, why would

²⁰ The deferred life annuity is a product opposite to immediate life annuity. A deferred annuity gives out its first payment in some agreed year. For example, a ten-year deferred life annuity that purchased by an individual at age 65 starts to payout when the individual is 75 years old.

they need to supplement these with a life annuity”. This concern is supported by Inkmann et al. (2010) using a simulation methodology that shows demand for voluntary life annuities is decreasing with other pension incomes. Of course, this conclusion comes from the fact that they are substitute products. In other words, pre-existing annuitization from social security crowds out private annuities (Bernheim, 1991; Heijdra et al., 2015; Hosseini, 2015). This channel is also modelled by Dushi and Webb (2004), so that the annuity demand in their model is largely affected by the pre-committed pension plans. However, they all neglect that pension is an exogenous term determined by government, so that individuals are not freely choosing between voluntary life annuity and government pensioned life annuity. As we analysed before, many theories showed that individuals are deciding about wealth annuitization at the moment of retirement. Since all available assets should be annuitized to have maximum expected utility, we can define an expression of annuitization proportion out of individuals total wealth in equation (*). In this equation, “pension” is a given (constant) exogenous variable. When the pension benefit (replacement) rate is set by the government, then, pension income is a constant.

$$(*) \quad \textit{Annuitization Proportion} = \frac{\textit{Assets at hand}}{\textit{Assets at hand} + \overline{\textit{Pension}}}$$

where, $\textit{Assets at hand} + \overline{\textit{Pension}} = \textit{Whole life disposable wealth}$.

Thus, if pension benefit rates are not varying, the annuitization proportion is stable as well. In other words, life annuity is crowded out only if the government revises the generosity of the pension benefit rate, increasing or decreasing the expected pension incomes. Alternatively, the crowding out thesis can be explained if we compare two countries, one with a high benefit rate, which implies a low annuitization proportion, with

a country with a low benefit rate, implying a high annuitization proportion. For example, the pensions in Italy replaced 70% of income before retirement, and the annuity market size in Italy was extreme small (see Fig. 1 and Appendix A). Cappelletti et al. (2013) also state that retirees in Italy are highly relying on public pay-as-you-go pensions. However, that statement fails to explain the cases where pension replacement rates are very low, like Canada, and yet annuitization rate is also very low.

Furthermore, Cappelletti et al. (2013) also conclude that wealth is a significant factor on annuitization participation, based on a large representative national survey conducted by the Bank of Italy. This influence of wealth is also noted by Inkmann et al. (2010). These two papers have the same conclusion that, the poor will annuitize less in comparison with the rich. Their findings are similar to what can be derived from equation (*). When individuals earn more life wealth, which is the denominator of equation (*), we have two cases. First, if the wealth increase is accumulated by working more hours or by the wage improvement, then the pension wealth will increase for a given pension benefit rate. Yet, according to the full annuitization theory, individual will invest the rest into life annuities, so that the demand for annuities will be raised at the same time. Second, if the wealth increase is due to better investment opportunities, leading to higher non-labour income, then pension remains constant. In this way, all the wealth increase is reallocated into life annuities. Therefore, the annuitization proportion increases in both cases. In conclusion, when there is a positive change in wealth, whether pension is increased or not, the demand for annuity will increase and life annuity is not crowding out by pension improvement. Brown (2007) also makes this point as well. He states that “it should not be surprising that these households (with low income) would wish to not annuitize what little

savings they have remaining”. On the contrary, richer households have less reasons for not annuitizing their wealth.

In conclusion, pre-committed pensions may result in zero-annuitization for the poor people, but it should not for the rich.

4.2 Bequest motives

The second reason explaining the low purchase of life annuity is that annuitants want to leave a bequest to their beloved survivors. Indeed, Yaari (1965) concludes that full annuitization is optimal only in the absence of bequest motives. The bequest motive impelled a subsequent large discussion. The prevailing opinion is that people should annuitize the amount allocated for their own future consumption and bequeath the other amount for children (Davidoff et al., 2005). Brown (2007) also commented that “while bequests clearly lead one away from the full annuitization result²¹, it does not mean that individuals will not value partial annuitization”. In fact, Yaari (1965) has provided a specific implication that certain annuitization schemes may lead to an optimal consumption path, when the marginal utility of consumption coincides with the marginal utility of bequest. Others found that the presence of children or not will not affect the likelihood of some annuitizing (Johnson et al., 2004); Nor do the number of children or existence of bequest will (Brown et al., 2007). In other words, bequest motives cannot answer why people annuitize nothing.

On the other side, researchers have never stopped their effort of evaluating the influence of bequest on annuitization. But, they cannot entirely convince me because of

²¹ Bernheim (1991) modelled the bequest into annuity function and stated the same conclusion. But, he did not say individual should not annuitize.

their methods. Heijdra et al. (2015) mixed bequest with unfair annuity rate (i.e. the adverse selection reviewed in section 4.4). They state that “with a strong bequest motive, a slight imperfection in the annuity market may inhibit the healthiest individuals from purchasing annuities”. I am not saying that their findings are wrong²², and I also agree with them that “bequest motives affect individuals’ tolerance towards imperfections in the annuity market”. But, they fail to prove that it is the existence of bequest rather than the other reasons that results in a low annuitization proportion. The same strategy of mixing factors was used by Friedman and Warshawsky (1990) and Vidal-Meliá and Lejárraga-García (2006). They all came to their conclusions that bequest mattered, under the effects of unfair annuity rates.

Another paper from Lockwood (2012) simulates the effects of various bequest levels on the annuity annuitization rate. He concludes that “modest bequest motives can eliminate purchases of available annuities”. The conclusion “bequest does explain zero-annuitization” is, however, faced with the same problem as Heijdra et al. (2015), that in his model people are facing unfair annuity rates. In his settings, every individual has half of his net wealth in a life annuity account from pension or social security (considered as compulsory annuities), and the individual is choosing at year 65 how much out of the rest of his wealth (defined as non-annuity wealth) will be voluntarily annuitized. The author succeeds in proving that the intent of bequest would prevent individuals from fully annuitizing. As other writers, he assumes unfair prices for annuities, so that it is difficult to gauge the weight of the bequest motives in explaining the annuity puzzle. To sum up

²² As what I am going to argue in next sections, the adverse selection and unfair annuity rate are the main objective explanation for a rational individual not to annuitize.

using Brown's word (2001), "there is no evidence that bequest motives are an important factor in making marginal annuity decisions". If there must be an answer for the annuity puzzle, bequest motives may lie at the bottom of the list.

Some discussion in the literature related to the bequest choice of people in different income levels is relevant. We should not expect low income people to leave much inheritance, if bequest is a luxury good (Lockwood, 2012; Auten and Joulfaian, 1996; Hurd and Smith, 2002). These individuals would better not to bequest but to fully annuitize²³. On the opposite side, for those who are so rich that they know that they can easily cover their anticipated life time consumption, then annuitization decisions are not necessarily taken into account in their utility maximization program. In this case, they may spread some wealth to purchase annuity products as a small part of their investment portfolio. But they will definitely not spend all their wealth in annuity market, since it will be a great loss for them when all the wealth is transferred to the insurance company after their death. So, the study objects should be limited to those individuals who are rational and who must live on annuity payments to cover old age consumptions. For them, they need to choose between life consumption (the life annuity need is then determined as well) and bequest. Yet, are individuals choosing bequest amount on purpose? That is, is the bequest motive introduced in the utility function? In a world of certain life length, individuals can easily decide the bequest and consumption plan in the last period of life. But, in a reality when people have no idea of when they will die, a common way is to set a stable consumption path and leave a random amount to heirs at death. Hurd (1989) has proved that "the

²³ If they behave as rational agents and face the fact that they have no extra wealth other than that planned for consumption, they will have no choice but annuitize. This is because the upper bound for life consumption is not binding yet.

marginal utility of bequests is small; therefore, desired bequests, which are estimated from model simulations, are small on average. Apparently, most bequests are accidental, the result of uncertainty about the date of death". In other words, dissaving patterns of retirees is not affected by bequest motives (Hurd, 1987).

4.3 Imperfect annuity product (liquidity shock and inflation risk)

In this part, I discuss some reasons that may explain the annuity puzzle on the basis that annuity markets are incomplete. In particular, I discuss two imperfections.

This first imperfection is related to health issues. If one individual were considering the purchase of an annuity, the first question he may think of is what will happen if he becomes ill and has no money to take care of it immediately? Since the majority who purchases life annuities are the elders, who have high risk of getting sick, it is necessary for them to save some money aside in case of an health shock. Yet, once the contract of life annuity is signed, a fixed income stream is flowing into annuitants' accounts every period, while the future annuity cannot be withdrawn ahead. So, annuity is an illiquid asset compared with other investments. For example, if someone invested his wealth into government bonds, he could sell it at any time in exchange of cash. This is the reason, as mentioned above, why self-annuitization (a portfolio of bonds and stocks, etc. that people sell progressively) can be the choice for some retirees (Heijdra, 2015). Although some solutions to health shock, which allowed annuitants to resell their annuity on the third market exist, yet it cost a lot (Brown,2007). All in all, the illiquidity of an annuity may explain why elders refuse to buy annuity (Sinclair and Smetters, 2004; Turra and Mitchell, 2005). Nowadays though, with the development of insurance products, this situation can be avoided. For example, when people need money to pay their hospital bills, the health

insurance along with life annuity can cover part of the expenses (Murtaugh et al, 2001). Another possible choice is that, people can set some money aside, and purchase life annuity with the rest of the money. Hence in this case, individuals could at least consider annuitization.

Furthermore, historically, annuity was contracted in fixed nominal value (i.e., periodic annuity is constant). Yet, the purchasing power of money in nominal term is decreasing with time because of inflation. So, with time, the real value of the annuity income decreases. However, this inflation risk problem has been solved along with the development of annuity products and market. For example, if one worries about the inflation risk, an inflation indexed life annuity would be a perfect choice (Brown et al., 2001). Because although market prices are increasing, the payment he/she will receive will be increasing proportionally. The payment of this new inflation-risk-free product will be adjusted according to the inflation rate that was set initially²⁴. However, the annuity market has remained small in the UK, even if inflation indexed annuities are widely available (Finkelstein and Poterba, 2002). Hence, inflation risk cannot truly explain the annuity paradox.

Why did the insurance companies not provide annuity products that could have responded at health and inflation risks from the start? Brown (2007) explains that it is not completely the insurance company's fault, in that insurance market was somehow growing and upgrading very slowly. For example, at first, there was no reinsurance for annuity providers to cover their own inflation risk, then it was understandable for them not to offer inflation adjusted annuities. In addition, in contrast to risks at the individual level (e.g.

²⁴ Yet this might bring an upcoming issue: the premium of the inflation indexed products will be more expensive.

longevity risks), inflation risk is an aggregate risk from the perspective of annuity providers which can hardly be diversified. In turn, that could constrain the supply of annuities.

Therefore, although the life annuity market, from its inception to now, is not perfect, there are methods that can be introduced to address these imperfections. As a rational individual, there is little reason to say “no” to those upgraded annuity products. However, since the products were introduced, little demand for them ensued. This means that these market imperfections do not explain the annuity. As a result, other explanatory factors need to be considered.

4.4 Unfair price and adverse selection

As analysed in section 4.2, when researchers (Friedman and Warshawsky, 1990; Vidal-Meliá and Lejárraga-García, 2006; Lockwood, 2012; Heijdra et al., 2015) were studying bequest motives, they blended bequest with the unfair actuarial rate for annuity. Yet, they did not make clear which factor is the dominant one. However, some inference can be obtained from Lockwood’s (2012), that unfair annuity price can profoundly eliminate annuity demand. This also applies to real life as a customer, who is planning to buy a new product, will consider if this product is worth its price. Cappelletti et al. (2013) even proved that the demand of annuity is sensitive to its price, especially for those in poverty.

How do researchers measure the value of an annuity? Theoretically, a definition is given by economists, the money’s worth (MW). MW is equal to the ratio of expected net presented value of payouts divided by the premium. If the price of annuity is fair, then expected net presented value of payouts for annuitant and the premium they are charged

should be the same. That is, MW is 1 in this case. If the price is unfair, the ratio will be less than 1. (The range of money's worth should be between 0 and 1.) The smaller the money's worth, the more unfair the product is, and therefore the more unworthy it is to purchase the product. The load is equal to one less MW and this is used to measure the fairness of annuity. So, an existing positive load will reduce the annuity demand (Brown, 2001). In real life, however, customers care more about how much payment they receive each month and how much premium in total they need to pay. Most of the customers will not notice the money's worth but notice the annuity price instead.

Heijdra et al. (2015) mention that cost (marketing cost, administration cost, etc.) for the annuity provider will increase annuity price. Some papers argue from the annuity provider's point of view, that mortality improvement will increase the cost of life annuity (Blake and Burrows, 2001; Blake et al., 2006; Blake et al., 2002; Friedberg and Webb, 2006; Brown and Orszag, 2006). When the mortality improves in the whole society and insurance companies have not yet updated mortality tables, companies will have a loss. Eventually, they will quote a higher price as they realize the changes in the mortality rates.

A common view from the literature is that the main reason why insurance company quotes such high and unfair rates is because of adverse selection. Customers have the option to choose to buy annuity products or not. If one buys an annuity product and lives a long life, he/she is making profit. If he/she lives a short life, say he/she may die the day after buying the annuity, then ex post, it was not wise for him/her to buy any annuity. That is why most of the customers will choose whether to buy an annuity or not, based on the estimation of their own health status and an expectation of their life expectancy (Finkelstein and Poterba, 2002). Empirical evidences show that those who expect to live a longer life

will voluntarily purchase life annuity (Blake et al., 2000; Finkelstein and Poterba, 2002; Brown, 2013; and Pestieau and Ponthière, 2012). While those who are worried about their own health are more likely to not purchase any life annuity, neither the long-term insurance, because they do not think they will benefit from it. Hence, for insurance companies, the individuals who buy annuities tend to live longer than the average population. Therefore, the existence of adverse selection will increase the mortality risk of insurance companies. This will lead to the increase of annuity price for insurance company to cover such risk. Heijdra et al. (2015) found out that adverse selection could have an even worse impact. “As the low health types quit the annuity market, the insurers have to set a higher price of annuities to compensate for the decrease in the average mortality rate of insurees. This, in turn, causes more individuals with low-health types to quit the annuity market”. This finally eliminates the demand of voluntary annuity.

Researchers have analysed how to eliminate the influences on price rising due to adverse selection (Blake et al., 2000; Blake et al., 2002). They suggested to put compulsory annuitization into practice to fix the problem caused by adverse selection. It is a valid method. As mentioned before, the UK had at some point applied compulsory annuitization. Finkelstein and Poterba (2002) have an analysis about the UK market, where both compulsory annuitization market and voluntary annuitization market are compared. They found out that there is a huge difference in mortality between the customers in those two markets. In addition, the effect of adverse selection in voluntary annuitization market is twice that in compulsory annuitization market. In other words, compulsory annuitization market indeed eliminates some effects of adverse selection.

As a result, unfair annuity price itself can reduce the demand for this product. Among all the reasons mentioned so far, it is the most convincing explanation for the annuity puzzle.

4.5 Education and financial literacy

Whether the price is fair or not, at first people are not willing to trust insurance companies, since they have the ability to set the prices (Heijdra et al., 2015). An evidence of this, is the fact that annuity prices vary among companies. What is more, some people believe that insurance companies can be bankrupt (Babbel and Merrill, 2006). So, even if the price is fair, will people believe? The common way the majority determine whether annuity price is fair or not is to simply compare the payment they will receive to the premium they spend on the product. That is, people will use an intuitional method to evaluate annuities rather than the scientific method, Money Worth (the expected present value of annuity divided by the premium paid). Plus, annuity calculation is indeed complicated (Brown, 2007). Lusardi, A. (2008) and Lusardi and Mitchell (2011) stated that the expectation of investment benefits is not affecting the individuals' financial decisions. Instead, their financial knowledge is the key fact on making such decisions. Researches (Inkmann et al. 2010; Cappelletti et al., 2013) found out that annuity demand will increase with financial literacy. That is, the more financial literature individual has, the more likely they will buy an annuity.

It can be concluded that people have difficulties gauging future risks, as well as difficulties to plan for retirement. In addition, one of the most important reason why people refuse to buy annuity is because they barely have any knowledge about it. A survey for Australia conducted by Bateman (2016) found that only one thirds of interviewees have

heard of life annuities; and in those, one fifth of them know that life annuities can provide a lifelong income, and just 8% of them know that the payments are guaranteed. The conclusion is that the public knows little about life annuities. In such circumstances, how can we expect annuity demand to be large? Moreover, annuity sales representatives might also lack the ability to clearly express the benefit or advantages of life annuity to potential customers (Brown, 2007). In general, both annuity sellers and buyers lack financial literacy on annuities. This explains the fact that annuity demand is inadequate.

4.6 Annuities and Behavioural economics

According to Smith and Stewart (2007) financial illiteracy is a universal phenomenon, which makes us doubt whether people are truly rational. Most of the time, people make decisions depending on their feelings. Thus, moving out of the rationality assumption will make it easier to understand the annuity puzzle. Here I will simply enumerate some interpretations based on behavioural economics.

Thaler (1985) conducted a questionnaire-based experimental study and introduced a new theory, Mental Accounting. That is, consumers intend to put gains and losses into different mental accounting when they are making decisions. Besides, consumers would narrow their considerations rather than evaluating the whole gains and losses. In the case of buying annuities, mental accounting and framing theory state that consumers tend to frame their considerations into a short period. When valuing annuities, consumers tend to think about if they can make back their investments or not in a short time (based on their expectation of life length), instead of considering the whole benefits from lifelong annuity payments in the long term (Hu and Scott, 2007). For consumers, buying an annuity is a

gamble (Brown and Warshawsky, 2001) for life length rather than an insurance against outliving.

Furthermore, Thaler's experiment reflects a phenomenon that gains and losses are considered differently. For example, if a lottery can result in a \$100 gain or a \$100 loss with half-half chance, consumers are not attracted, since the feeling of losses is twice as strong as the feeling of gains (Tversky and Kahneman, 1991). Thus, consumers are loss averse. In general, consumers will take a gamble against losses but not take it for gains (Kahneman and Tversky, 1979). Thus, for instance, individuals will not have any loss if they do not buy annuities worth of \$100000, but may have a great loss of \$100000 if they die in next day. As a result of loss aversion, individuals dislike the annuities, as they do not value as much the gains associated with living a long life and not being out of wealth.

In addition, Brown (2007) analysed that people are afraid of having to regret a decision they made. Some even thinks that it feels more secure when they hold wealth in one's hands (Brown, 2007)²⁵. Another possible concern is that many consumers are uncomfortable to accept the result that, they give out the assets accumulated from years of work in exchange for an annuity, with the possibility that they will die sooner than later, and that their wealth will be transferred to the insurance company. Behavioural Economics is very useful to understand the annuity puzzle, as it is closer to the real world. Yet, it is hard to evaluate the effect of individuals' various feelings when making decisions.

4.7 Summary

²⁵ This is also relevant to the fact that people prefer liquid wealth.

Here, I will discuss the inner connections of all motivations for not annuitizing wealth.

Among all reasons, the bequest motive fails to explain the annuity puzzle to a large extent. As I analysed before, the final amount of bequest turns out to be a random outcome. Apart from the rare case of extremely strong bequest motive (that one would rather die hungry to bequeath a targeted amount, and there is no empirical evidence or reported news showing that one could have such a strong motive), we can say, that the bequest choice is a minor factor in the decision determining the life time consumption path. Hence, the bequest motive does not affect the optimal rate of annuitization. With regards to the other reasons that explain the annuity puzzle, I have drawn a relation map (fig. 3) to illustrate the debate. In this picture, symbols of plus and minus represent a positive or negative relationship and the arrow lines show the direction of effects.

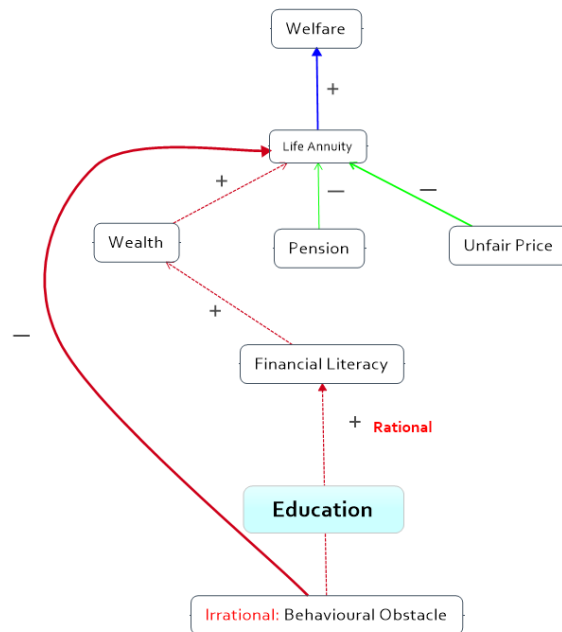


Fig. 3. Relations among Explanations

Based on the preceding discussions, the final purpose of buying annuity is to raise individuals' welfare. Fig. 3 shows the reasons that pension crowding-out, actuarial unfair rate, financial illiteracy and inadequate education will decrease the demand of life annuity. In other words, education improvement can increase financial literacy (make individuals more rational on financial decisions), and further increase wealth, then finally increase the demand of life annuity.

It is a well-documented fact that only a few people are well educated in financial issues, while a huge proportion of people are financial illiterate. For the minority of well-educated consumers, they can be assumed to be rational. The only concern for them is the existence of unfair price of life annuity, since the price is determined exogenously by insurance companies. Under these assumptions, Yaari (1965) was right to claim that rational individuals will annuitize if given a fair actuarial rate. However, most individuals in the real world are irrational and have behavioural obstacles to choosing life annuity. Since they do not receive enough education in financial knowledge, they will remain irrational in making financial decisions. Hence, the majority do not buy annuities. In one word, the annuity puzzle is to a large extent due to the fact that individuals are irrational and not receiving good educating in this area, and due small part to the unfair price of life annuities.

Although, I believe that the demand of annuities will increase if financial education is enhanced. It may take decades to achieving success by education. What else should we do then? Take pension system in Netherlands as a successful example with mandatory annuitization and annuity market regulation; the money worth of life annuity was 0.9 (Cannon et al., 2015). Thus, if the government, as rational social planner, mandates a

proportion of wealth to be annuitized for everyone, then it seems that individuals are forced back to be rational. But, this is not enough, in that there must be a fair actuarial rate. For example, in the previous analysis, when the UK stopped the compulsory annuitization policy, the annuitization rate dropped significantly, since insurance were not offering fair prices.

All in all, if the government plays a role as an annuity insurance company, and requires some rate of compulsory wealth annuitization, then all the problems of annuity puzzle will be solved. In this way, the government can level up the overall individual welfares. In the next section, I use an OLG-CGE model to compare the impact of zero versus 100% annuitization rates on the economy.

5. The Model

The OLG-CGE model used here is an out-of-the shelf model built by my supervisor, Patrick Georges. A good description of a variant of the model that I am using here is given in Georges and Seçkin (2016). This model itself is in the Auerbach and Kotlikoff (1987) tradition with endogenous labour supply. An important feature is that they introduce age-specific mortality, with a perfect annuity market, through which unintentional bequests are implicitly distributed. The theoretical description of this approach was first presented in Yaari (1965). Several OLG models have been developed and extended in many directions over the last 20 years. For a survey of these advances, see Fehr et al. (2013) and references therein. The model for this paper simplifies the Georges and Seçkin (2016) model by assuming a one-sector model with exogenous labour supply and within a closed economy framework. This simplification was analyzed by Qianzi Huang (2016) in her own major paper. Here, however, the model that I am using has the flexibility to analyze diverse

wealth annuitization schemes, unlike the framework in Qianzi Huang (2016) which assumes full wealth annuitization.

In the following, I draw heavily on the presentation of the model by Georges and Seçkin (2016), Qianzi Huang (2016), and a mimeo by Georges and Seçkin (2017) that describes how to introduce partial annuitization schemes in an OLG-CGE model.

5.1 Production sector

The production sector is characterized by a representative firm which produces a single good using a Cobb–Douglas technology. $Y_{j,\tau}$ represents the (aggregate) output (GDP) in each period τ (five-year) for country j (Canada). A is total factor productivity. In this model, it is kept constant across time. This means that the model does not attempt to capture a realistic growth process but focuses exclusively on the impact of demographic pressures. K and L represent capital stock and effective labor force, respectively. α_j , is the share of physical capital in GDP (Y). There is no intermediary good. The following is the Cobb–Douglas production function:

$$(1) \quad Y_{j,\tau} = A_{j,\tau} K_{j,\tau}^{\alpha_j} L_{j,\tau}^{1-\alpha_j}, \quad 0 < \alpha_j < 1, \quad j \in J = \{\text{Canada}\}$$

First order conditions to maximize profit provide the rental rate of capital ($re_{j,\tau}$) and the wage rate per unit of effective labor ($w_{j,\tau}$):

$$(2) \quad re_{j,\tau}/P_{j,\tau} = \alpha_j A_{j,\tau} (K_{j,\tau}/L_{j,\tau})^{\alpha_j-1}; \quad P_{j,\tau} = 1$$

$$(3) \quad w_{j,\tau}/P_{j,\tau} = (1 - \alpha_j) A_{j,\tau} (K_{j,\tau}/L_{j,\tau})^{\alpha_j}$$

5.2 Household behavior

We classify adult generations into seventeen age groups (i.e., age groups G5 = 20-24 years old, ..., G21 = 100-104). Younger generations (age groups G1 = 0-4, ..., G4 = 15-19) are fully dependent on their parents and play no active role in the model, but they

influence the age dependent components of public expenditure (health and education) and the population structure. The model introduces age-specific mortality, which is reflected by a parameter SR which represents the age-dependent survival rate of an individual.

In Georges and Seçkin (2016), individuals' utilities follow a CES type intertemporal utility function of consumption ($Con_{j,t+k,g+k}$) and leisure ($TLeis_{j,t+k,g+k}$) so that labor supply is endogenous. However, here, we assume that there is no choice between work and leisure. Therefore labor supply is exogenous. In the overlapping-generation model, each individual lives for (at most) 21 different periods and there are 21 age groups: $G1 = 0-4$ age old, $G2 = 5-9$... $G20 = 95-99$, and $G21 = 100-104$. θ_j , is the inverse of the intertemporal elasticity of substitution, which represents the decision of individuals with respect to consumption and saving. $\frac{SR_{j,t+k,g+k}}{(1+\rho_j)}$ indicates the subjective discount factor, where $SR_{j,t+k,g+k}$ represents the survival rate at any specific age. It is assumed to be 0 for the last age group (100-104), which means that if people did not die before 104, then at the end of that year they would die with probability 1. For all other age groups, the probability of survival is less than 1 and depends on age. This means that a specific share of the population of that (specific) age dies. The utility function for a representative household born at time t is:

$$(4) \quad E_t(U_j) = \frac{1}{1-\theta_j} \sum_{i=1}^{i=17} \left\{ \frac{1}{(1+\rho_j)^{i-1}} \left(\prod_{k=0}^{k=i-1} SR_{j,t+k-1,g+k-1} \right) \left(Con_{j,t+i-1,g+i-1} \right)^{1-\theta_j} \right\}$$

The household intertemporal (dynamic) budget constraint can be written as follows:

(5)

$$\begin{aligned}
& \underbrace{(1 - \lambda(1 - SR_{j,\tau,\gamma}))HA_{j,\tau+1,\gamma+1} - (HA_{j,\tau,\gamma} + UnBe_{j,\tau})}_{\text{Saving}} = \\
& \underbrace{(1 - \tau^L_{j,\tau} - Ctr_{j,\tau})Y_{j,\tau,\gamma}^L}_{\text{Labour Income}} + \underbrace{[(1 - \tau^K_{j,\tau})ri_{j,\tau}](HA_{j,\tau,\gamma} + UnBe_{j,\tau})}_{\text{Asset Income}} + Pens_{j,\tau,\gamma} - \underbrace{(1 + \tau^C_{j,\tau})Con_{j,\tau,\gamma}}_{\text{Consumption spending}} \\
& \lambda \in [0,1]; \quad \tau = t + k; \quad \gamma = g + k; \quad k = 0, \dots, 16.
\end{aligned}$$

where $HA_{j,\tau,\gamma}$ is asset holding (wealth) of an individual of age group γ *at the start* of period τ and $HA_{j,\tau+1,\gamma+1}$ is asset holding of the same individual one period later, i.e., of age group $\gamma+1$ *at the end* of period τ , or equivalently, at the start of period $\tau+1$, $Y_{j,\gamma}^L$ is gross labour income, $Pens_{j,\tau,\gamma}$ is pension benefits accruing to retired individuals, τ^L is the effective tax rate on labour income, Ctr is the worker contribution to the PAYG pension system, τ^K is the effective tax rate on asset income where ri is the rate of return on assets (defined later on in Eq. (18) with respect to the rental price of capital re and the depreciation rate of capital) and τ^C is the effective tax rate on consumption spending. Parameter λ , with $0 \leq \lambda \leq 1$, gives the wealth annuitization scheme. If $\lambda=1$, the representative individual annuitizes all of his wealth. If $\lambda=0$, the representative individual does not annuitize his wealth. All other values for λ , $0 < \lambda < 1$, represent partial wealth annuitization schemes. For example, $\lambda=0.75$ would imply that the individual annuitizes 75% of its actual wealth and keeps 25% of its wealth invested in a normal interest-bearing asset. Georges and Seçkin (2017) have proved that the term in Eq. (5), $\lambda(1 - SR_{j,\tau,\gamma})HA_{j,\tau+1,\gamma+1}$ is the (periodic) annuity payout expected by the household on its annuitized wealth.

It is assumed that people are not altruist. They do not plan to leave voluntary bequests. However, as they do not know when they will die, there could be unplanned bequests. If they fully annuitize their wealth ($\lambda=1$), they will leave all their wealth to the

insurance company. However, if they do not fully annuitize their wealth, then we need to compute the amount that will be bequeath to other agents in the model. If we assume that this amount is equally spread among all survivors of all ages, then Georges and Seçkin (2017) show that this amount can be computed as:

$$(6) \quad UnBeq_{j,\tau} = \frac{\sum_{\gamma} (1 - SR_{j,\tau-1,\gamma-1}) Pop_{j,\tau-1,\gamma-1} (1 - \lambda_j) HA_{j,\tau,\gamma}}{\sum_{\gamma} Pop_{j,\tau,\gamma}}.$$

Gross labor income per capita, $Y_{j,\gamma}^L$ (i.e., per representative household of generation γ), is defined in Eq. (7) as the standardized wage rate $w_{j,\gamma}$, (the wage rate of a new adult) multiplied by $EP_{j,\gamma}$, an age-dependent productivity (earnings) profile reflecting the individual's change in earning capacity over time, and defined as a quadratic function of age (Eq. 8). LS is the individual exogenous supply of potential non-effective units of labour. $PART$ is an age-dependent, time-invariant labour participation rate.

$$(7) \quad Y_{j,\tau,\gamma}^L = w_{j,\tau} EP_{j,\gamma} LS_{j,\gamma} PART_{j,\gamma}, \tau = t + k; \gamma = g + k; k = 0 \dots 16. \quad (7)$$

$$(8) \quad EP_{j,kw+1} = c + (\pi)(kw + 1) - (\psi)(kw + 1)^2, \quad c, \pi, \psi \geq 0 \quad kw = 0, \dots, 8$$

Based on equation (5), the household's lifetime budget constraint can be computed as:

$$(9) \quad \sum_{i=1}^{17} \left\{ \frac{\prod_{k=0}^{k=i-1} (1 - \lambda(1 - SR_{j,t+k-1,g+k-1}))}{\prod_{i=1}^{17} (1 + [(1 - \alpha_{j,t+i-1}^K) r_{j,t+i-1}])} \left[(1 - \alpha_{j,t+i-1}^L - Ctr_{j,t+i-1}) Y_{j,t+i-1,g+i-1}^L + Pens_{j,t+i-1,g+i-1} \right] \right\} + UnBe_{j,t} = 0$$

Therefore, the household solves the following Lagrangian maximisation problem where the intertemporal budget constraint is multiplied by the Lagrange parameter L :

$$\lambda = \frac{1}{1-\theta_j} \sum_{i=1}^{i=17} \left\{ \frac{1}{(1+\rho_j)^{i-1}} \left(\prod_{k=0}^{k=i-1} SR_{j,t+k-1,g+k-1} \right) (Con_{j,t+i-1,g+i-1})^{1-\theta_j} \right\} \\ + L \left[\sum_{i=1}^{17} \left\{ \frac{\prod_{k=0}^{k=i-1} (1-\lambda(1-SR_{j,t+k-1,g+k-1}))}{\prod_{i=1}^{17} (1+[(1-\tau x_{j,t+i-1}^K)ri_{j,t+i-1}])} \left[(1-\tau x_{j,t+i-1}^L - Ctr_{j,t+i-1}) Y_{j,t+i-1,g+i-1}^L + Pens_{j,t+i-1,g+i-1} \right] \right\} + UnBe_{j,t} \right]$$

The following Euler condition provides the consumption path of the household from period τ at age γ to the next period/age as:

$$(10) \quad \frac{Con_{j,\tau+1,\gamma+1}}{Con_{j,\tau,\gamma}} = \left(\frac{SR_{j,\tau,\gamma}}{(1-\lambda(1-SR_{j,\tau,\gamma}))} \frac{(1+[(1-\tau x_{j,\tau+1}^K)ri_{j,\tau+1}])}{(1+\rho_j)} \frac{(1+\tau x_{j,\tau}^C)}{(1+\tau x_{j,\tau+1}^C)} \right)^{\frac{1}{\theta_j}},$$

where, again, $\lambda \in [0,1]$ is the fraction of assets that are annuitized.

As noted In Georges and Seçkin (2017), if we set $\rho < ri$ in (10), then the individual is assumed to be more patient than the market. In this case the consumer starts out with relatively low consumption in order to save and take advantage of the high interest rates. Thus, its periodic consumption increases over the lifecycle, and this generates an upward-sloping consumption path. The introduction of life-length uncertainty ($SR_{j,\tau,\gamma} < 1$) will not change this result if $\lambda=1$ (full wealth annuitization) as the survival rates in both numerator and denominator in Eq. (10) cancel out. Indeed, it is one key result of Yaari (1965) to demonstrate that under the case of full annuitization of wealth (and no voluntary bequests) the Euler consumption equation does not depend on the survival rate or the probability of dying in a period.

Yet, partial ($\lambda < 1$) or no ($\lambda = 0$) annuitization of wealth will affect this result because the survival rate will enter the Euler equation, as obvious in Eq. (10). In this case, if $\rho < ri$ (the agent is patient), and when the individual is relatively young (so that the survival rate is high and close to 1), consumption will increase from period to period. However, with

ageing, the survival rate starts to decline progressively from period to period and this effect may eventually dominate the effect of having $\rho < ri$ on consumption dynamics. As Georges and Seçkin (2017) explain, intuitively the individual deals with longevity risk by planning to reduce future consumption in proportion to the survival probability, linked to their risk aversion. It is as if the rate of time preference (weighted by the survival rate) made the individual more impatient to consume now, instead of in the future when the survival rate will be much lower.

As we will see in the next section, this feature permits to model an age profile for consumption that is closer to what is observed in the data in the case when λ is different from 1.

5.3 Government sector and pension plans

Public expenditure on health and education (Eqs. 11-12) are age-dependent (i.e., fixed per person of a specific age). Other types of public expenditures (Eq. 13) are assumed to be age invariant, that is, they are fixed per capita, so that other total expenditure depends only on the size of the total population, not on its age structure.

$$(11) \quad GovH_{j,\tau} = \sum_{\gamma=g-4}^{g+ka} Pop_{j,\tau,\gamma} HEAC_{j,\gamma}; ka=-4, \dots 16.$$

$$(12) \quad GovE_{j,\tau} = \sum_{\gamma=g-4}^{g+ka} Pop_{j,\tau,\gamma} EDUC_{j,\gamma}; ka=-4, \dots 16.$$

$$(13) \quad Gov_{j,\tau} = TPop_{j,\tau} GEPC_j$$

In this version of the model, public debt is assumed to remain constant per capita while the government must respect its budget constraint (Eq.14). Total government spending, including interest on the public debt, must be financed each period by tax revenues from labour, capital and consumption taxes. Here, the tax rate on labour income $\tau x^L_{j,\tau}$, is the endogenous variable that fulfils the government budget constraint. In

simulation results the path for this tax rate will therefore be indicative of the fiscal pressure of ageing in Canada.

$$(14) \quad \begin{aligned} & Bond_{j,\tau+1} - Bond_{j,\tau} = Gov_{j,\tau} + GovE_{j,\tau} + GovH_{j,\tau} + ri_{j,\tau} Bond_{j,\tau} \\ & - \sum_{\gamma=g}^{g+k} Pop_{j,\tau,\gamma} \left\{ \alpha_{j,\tau}^C (Con_{j,\tau,\gamma}) + \alpha_{j,\tau}^L (w_{j,\tau} EP_{j,\gamma} LS_{j,\gamma} PART_{j,\gamma}) + \alpha_{j,\tau}^K (ri_{j,\tau} HA_{j,\tau,\gamma} + UnBeq_{j,\tau}) \right\} \end{aligned}$$

We assume a PAYG pension plan separated from the government budget constraint so as to gauge the impact of the pension pressure due to ageing independently from the rest of the budget. Retirement age is at 65 years of age. The pension benefits depend on life-time labor income and the pension replacement rate (or pension benefit rate) as follows:

$$(15) \quad Pens_{j,\tau,g+kr} = PensR_j \times \frac{1}{9} \sum_{kw=0}^8 w_{j,\tau-kr+kw,g+kw} \times EP_{j,g+kw} LS_{j,g+kw} PART_{j,g+kw} ;$$

$kr=9...16$.

The contribution rate, Ctr , applied on gross labour income, is determined endogenously so as to permit the financing of total pension benefits (Eq. 16). Hence, endogenous increases in the contribution rate will be indicative of future pension plan tensions due to population ageing (separately from other fiscal pressures)

$$(16) \quad \sum_{kr=9}^{kr=16} Pop_{j,\tau,g+kr} Pens_{j,\tau,g+kr} = Ctr_{j,\tau} \sum_{kw=0}^8 Pop_{j,\tau,g+kw} Y_{j,\tau,g+kw}^L$$

5.4 Investment and asset returns

The accumulation of Canada's capital stock ($Kstock$) is subject to depreciation (Eq 17), where Inv represents investment and δ the depreciation rate of capital. Physical capital ($Kstock$) and government bonds ($Bond$) are assumed to be perfectly substitutable. Hence the expected rate of return on bonds ri is equal to the rate of return on physical assets defined as the rental price of capital minus the depreciation rate (Eq 18).

$$(17) \quad Kstock_{j,\tau+1} = Inv_{j,\tau} + (1 - \delta_j) Kstock_{j,\tau}$$

$$(18) \quad ri_{j,\tau} = re_{j,\tau} - \delta_j$$

5.5 Market and aggregation conditions

We assume that Canada is a closed economy; therefore, there are no net exports in our model. The equilibrium in the good market and in the factor markets are given as follows:

$$(19) \quad Y_{j,\tau} = \sum_{\gamma=g}^{g+k} Pop_{j,\tau,\gamma} Con_{j,\tau,\gamma} + Inv_{j,\tau} + Gov_{j,\tau} + GovH_{j,\tau} + GovE_{j,\tau}$$

$$(20) \quad L_{j,\tau} = \sum_{\gamma=g}^{g+k} Pop_{j,\tau,\gamma} LS_{j,\gamma} EP_{j,\gamma} PART_{j,\gamma}$$

$$(21) \quad K_{j,\tau} = Kstock_{j,\tau}$$

5.6 Demographic process

In this paper, a demographic shock is simulated under different demographic variants. Following Georges and Seckin (2016), a demographic process (Eq. 22), superimposed on the OLG model, provides the exogenous shock behind the simulations results. The first law of motion simply implies that the number of children (age group $g+ka$ for $ka=-4$, i.e. age group 0-4) born at time τ is equal to the size of the preceding generation multiplied by the “birth rate”, BR , in period $\tau-1$. The second law of motion gives the size of any age group $g+ka$ beyond the first generation, as the size of this age group a period ago, multiplied by the age specific conditional “survival” rate, SR . In this model survival rates vary across time and age. For the final generation ($ka=16$), the age group 100-104, the conditional survival rate is zero. This means that for the oldest age group, everyone dies with certainty at the end of the period.

$$(23) \quad Pop_{j,\tau,g+ka} = \begin{cases} Pop_{j,\tau-1,g+ka} BR_{j,\tau-1} & \text{for } ka = -4 \\ Pop_{j,\tau-1,g+ka-1} (SR_{j,\tau-1,g+ka-1}) & \text{for } ka \in [-3, \dots, 16] \end{cases}$$

5.7 Model calibration

For model calibration purposes, we initially assume a stationary population at the start of 2015 and total population is normalised at 1. Based on the UN population data per age group for year 2015, I follow the method by Georges and Seçkin (2016, 2017) and also used in Qianzi Huang (2016) major paper and compute birth rates and age variable fertility rates so that the population structure per age group would replicate itself periods after periods as shown in Table 2a. This is the population structure that is assumed in the calibration part of the model and which is used together with the model to replicate the Social Accounting Matrix of Tables 3a/b. Then, from 2015 onwards, I apply birth rates and variable fertility rates that are consistent with the UN population data projections, as shown in Table 2b. Hence, from 2020 onwards, the population structure and population size vary.²⁶ This procedure allows precise modelling of the population size and structure within the model and replicates the UN demographic projections given in next section from 2020 onwards.

Table 2. Computing “birth” and “survival” rates from UN projections for Canada*

a) UN population structure per age group in 2015 and stationary population assumption from 2015 onwards

Years/Generations	G1 (0-4)	G2 (5-9)	...	G21 (100+)	Total Pop
2015	1942	1947		8.	35940
2020	1942	1947		8.	35940
...					
2100	1942	1947		8.	35940

b) Constructing implied birth rates and survival rates from UN population data from 2010 onwards

Years/Generations	G1 (0-4)	G2 (5-9)	...	G21 (100+)	Total Pop
2015	1942	1947		8.	35940
2020	2013	2033		10	37600
...					
2100	2402	2448		359	49668

* Numbers in cells correspond to historical (2015) and projected (UN medium variant) population sizes (in thousands) by years and generations (relabelled here G1 to G21). The “birth rates” are computed “vertically” for consecutive time periods on the basis of G1 population sizes (e.g., $BR=2013/1942= 1.036265$). “Survival rates” are computed “diagonally” across consecutive age groups and time periods (e.g., $SR=2033/1942= 1.046595$). Source: Compiled using UN Population Division data – World Population Prospects, the 2015 Revision (esa.un.org/unpd/wpp/unpp/panel_indicators.htm)

²⁶ Technically, the model is simulated from 1925 to 2260. It starts in 1925 for welfare analysis purposes and ends in 2200 to also ensure a stationary state.

Qianzi Huang (2016) has used some values for the benchmark parameters (see Table 3) and I will be using these same values for simulating the model.

Table 3. Some behavioral parameter values.

Parameters	Description	Values
α_j	Production share of capital	0.3
δ_j	Depreciation rate of capital	5.1%
$PensR_j$	Pension replacement rate	0.3
$\tau x_{j,\tau}^L (=WTxR)$	Effective tax rate on labour	24.96%
$\tau x_{j,\tau}^C$	Effective tax rate on consumption	12%
$\tau x_{j,\tau}^K$	Effective tax rate on capital	28.82%
$\theta_j = 1/Sig_j$	Inverse of the constant inter-temporal elasticity of substitution	1/0.25
$Ctr_{j,\tau}$	Contribution rate	16.8%

Table 4a/4b shows a Social Accounting Matrix (SAM) for Canada. The data used in the benchmark SAM is based on CANSIM, a socioeconomic database by Statistics Canada. This SAM was produced by Qianzi Huang (2016). I report it here for the sake of completeness of model description and the following draws heavily on her major paper. In the OLG general equilibrium model, we assume that the SAM represents the economy's initial equilibrium called the benchmark equilibrium. However, by proposing a closed economy model, we need to adjust different elements so as to maintain an initial equilibrium. Table 4b provides the same number as a proportion of GDP. The firm produces a good and its value is identical to the spending on this good: consumption spending, government spending classified by education, health, and others; and investment spending. We can also represent this spending as total demand for the commodity. As the tables show, total expenditure represents the GDP of Canada at market price. Since we

assume that the capital share of output (revenue or GDP) α is 0.3 and the labor share of output $1-\alpha$ will be 0.7, we can allocate the total output to the two production factors: capital and labor, which are also the total amount of revenues accruing to capital and labor factors, respectively. Capital revenue include capital revenues paid to the owners of capital, taxes on capital, and depreciation of capital. As mentioned above, capital costs should match the total capital revenues obtainable on the market. In the labor market, total revenue from labor depends on firm payments to wage earners, including wage taxes and pension plan contributions. Households receive their revenue from the incomes of both labor and capital, pension benefits from the government, and interest revenues on the public debt. These revenues are equal to the money they can spend on final consumption expenditure. The government sector finances its spending by collecting taxes and borrowing money. Thus, the sum of expenditures on education, health, and other spending and interest payments on debt be financed by taxes on capital revenues, labor revenues, and consumption spending. Besides, households' savings will flow into investment. In a stationary state saving is just equal to the investment needed to offset the depreciation of the capital stock.

Table 4a

	FRIM	CAPITAL	LABOUR	CON	GOV	GOVH	GOVE	INV	PEN	TOTAL
FIRM				1,070.899	263.107	126.104	102.42	143.003		1,705.533
CAPITAL	511.66									511.66
LABOUR	1,193.873									1,193.873
HOUSEHOLDS		221.194	695.192		82.279				200.741	1,199.407
TAX		147.463	297.94	128.508	32.912					606.822
SAVING		143.003								143.003
CONTR			200.741							200.741
TOTAL	1,705.533	511.66	1,193.873	1,199.407	378.298	126.104	102.42	143.003	200.741	

Table 4b

	FRIM	CAPITAL	LABOUR	CON	GOV	GOVH	GOVE	INV	PEN	TOTAL
FIRM				0.62	0.15	0.07	0.06	0.08		1.00
CAPITAL	0.30									0.30
LABOUR	0.70									0.70
HOUSEHOLDS		0.13	0.41		0.05				0.12	0.70
TAX		0.09	0.17	0.08	0.02					0.36
SAVING		0.08								0.08
CONTR			0.12							0.12
TOTAL	1.00	0.30	0.70	0.70	0.22	0.07	0.06	0.08	0.12	

6. Demographic Projections and Some Simulation Results

In this section we want to show that the assumption of full versus partial annuitization as yet another impact that was not discussed in the survey of the literature. In the particular case of Canada, it appears that assuming full annuitization tends to overestimate the negative impact of the demographic shock. Before showing some simulation results, I start by reporting some demographic projections for Canada in Fig 4 and 5.

Total population in Canada was about 36 million in 2015 and is expected to increase to about 50 million by the end of the century according to the medium variant scenario of the population division of the United Nation. Fig. 4 also reports population by age group (less than 20, 20 to 65, and 65+ age groups). Fig. 5 shows that population ageing is an important projected characteristic of the Canadian economy. For example the old age dependency ratio is projected to increase from 26% to 61 % by the end of the century. The model that was described in Section 5 replicates exactly all these projections.

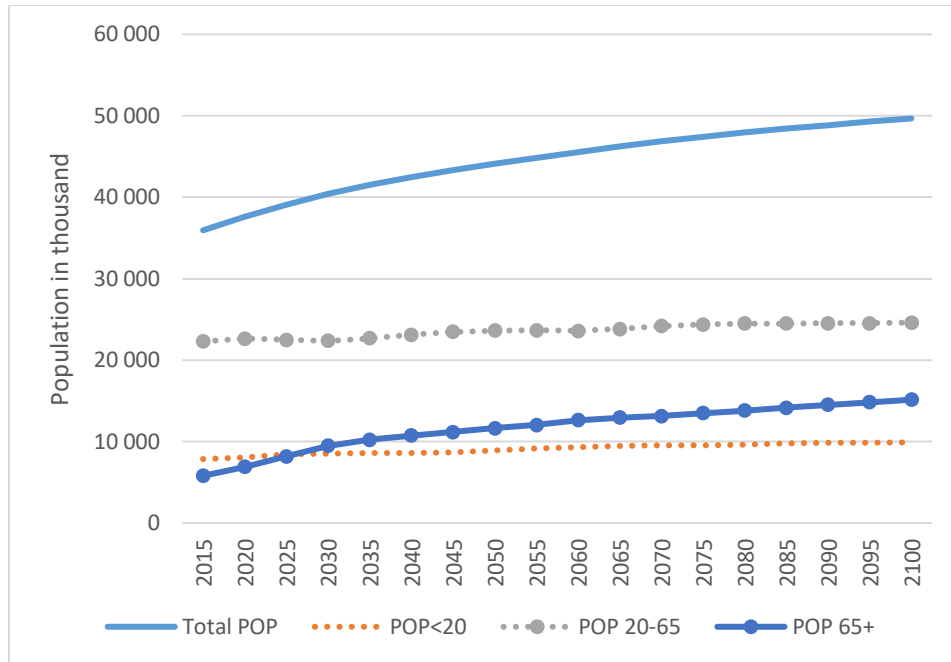


Fig. 4. CANADA Total population (TPop), 0-19, 20-65, and 65 and + (in thousand)

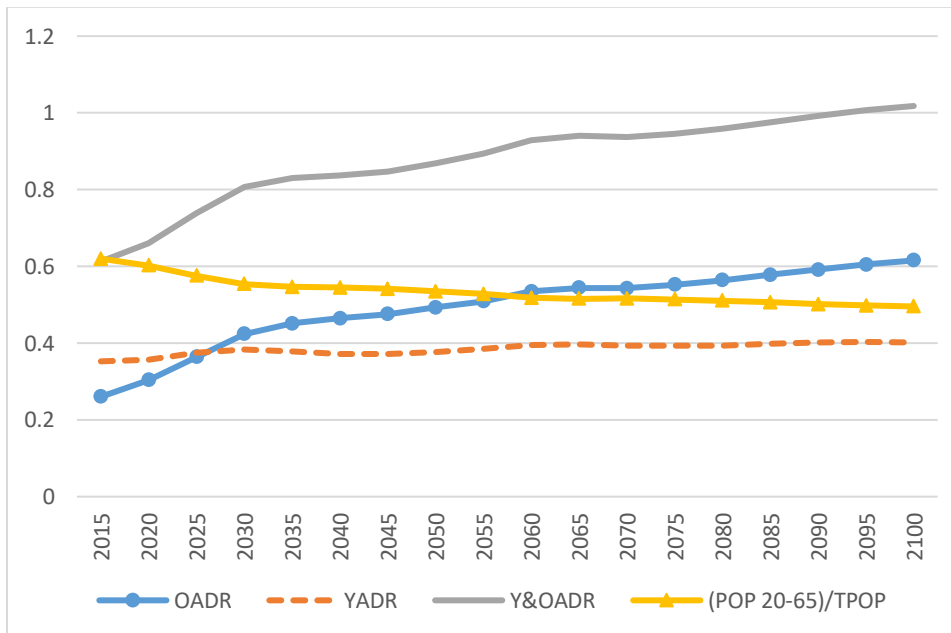


Fig. 5. CANADA. Dependency ratios and support ratio (Pop20-65/TPop)

Now, before reporting the economic impact of the demographic shock on the Canadian economy, I want to highlight a few interesting characteristics of the annuitization

schemes described in Section 3. First observe in Fig. 6 the household consumption profile per age group for different age annuitization schemes. It is clear that this graph is consistent with the discussion of the Euler equation (Eq. 10) in Section 5. In a simple OLG-CGE model with full annuitization, the consumption profile of the individual increases over time as he/she gets older. However, there is a large amount of empirical literature (see for example Lee and Mason, 2011) that shows, using household surveys data, that the consumption profile over age is normally hump-shaped. Hence introducing partial annuitization in an OLG model is one way to generate a more realist hump-shaped profile, as can be seen in Fig. 6²⁷. There are alternative ways to do that, of course. Another way to proceed in an OLG model to generate this hump-shaped profile is by introducing endogenous labor supply, as was initially proposed by Auerbach and Kotlikoff (1987).

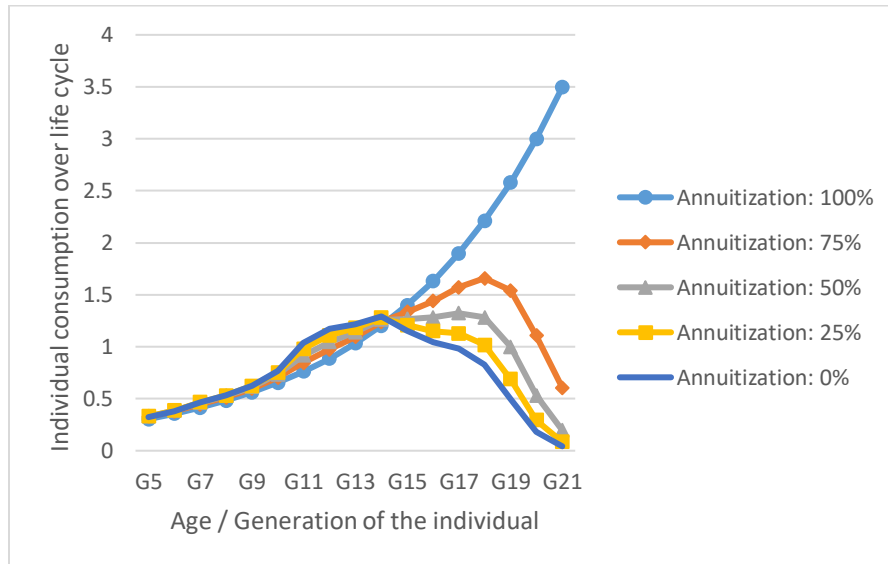


Fig. 6. CANADA. Household consumption profile over life time: diverse wealth annuitization schemes

²⁷ It is clear that this graph is consistent with the discussion of the Euler equation (Eq. 10) in Section 5. In a simple OLG-CGE model with full annuitization, the consumption profile of the individual increases over time as he/she gets older if the individual is more patient than the market ($\rho < r_i$, so that he is willing to postpone his consumption in such a way that future consumption is always larger than past consumption). Of course, if the individual was less patient than the market ($\rho > r_i$) the consumption profile would be downward sloping and if he was as patient as the market ($\rho = r_i$) it would be flat (i.e., consumption smoothing). Here we impose from the start that $\rho < r_i$ for all annuitisation scenarios so as to show that, even in this most favorable case of consumption postponement, there will be a moment when consumers start to prefer consuming now instead of the future (under less than 100% annuitization schemes).

Fig. 7 shows the age-dependent annuity payout received by the individual for two extreme scenarios (100% and 0% annuitization). Of course, in the case of zero annuitization, people receive an annuity of 0. More interestingly, this graph shows explicitly the implication of assuming full annuitization as it is routinely done in the OLG-CGE literature (e.g., Rios-Rull, 1996, 2001; Broer and Westerhout, 1997; Chauveau and Loufir, 1997; Imrohoroglu, 1998; Börsch-Supan, Ludwig and Winter, 2006; and Krueger and Ludwig, 2007) among others. Authors never report this type of results, but Fig. 7. shows an ever-increasing annuity payout as the annuitant ages. Although this result might be coherent with what Yaari (1965) meant, it is clear that this type of escalating annuity payout cannot be purchased on the annuity market. In the model of Section 5, the individual at the age of 20 already contracts out an annuity with the insurance company (although annuity payout is virtually zero until 50-55 years of age because the survival rate remains quite high in the model until that age bracket). This means that there is still room for a lot of research within the OLG-CGE literature to introduce annuitization schemes in a more realistic case, which would reflect characteristics of ‘real life’ annuity markets.

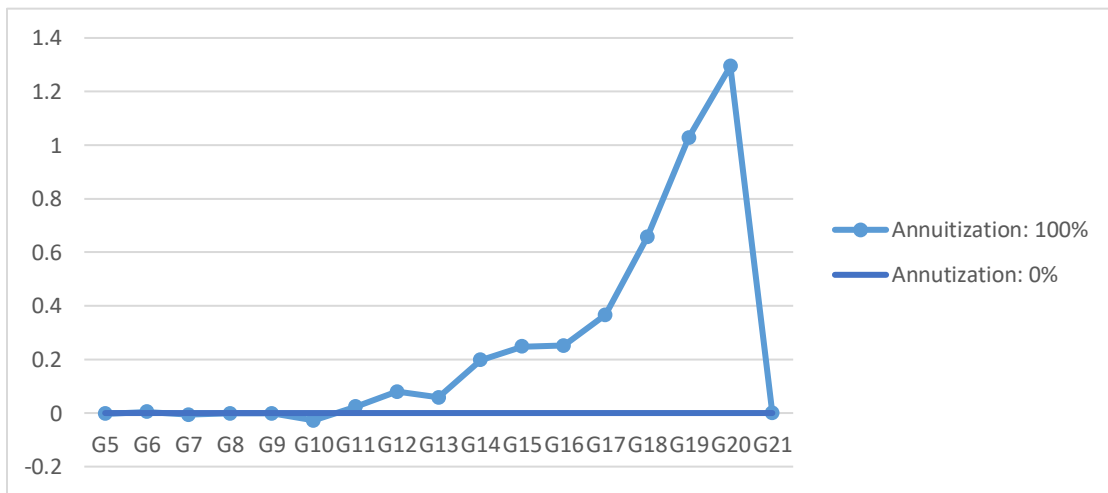


Fig. 7. CANADA. Age-dependent annuity payout accruing to the surviving annuitant (end of period)

Finally, the last result I want to report is the impact of the demographic shock on GDP per capita in Canada. The model assumes constant total factor productivity in order to focus exclusively on the demographic issue. Hence, the model does not pretend to make a forecast of the GDP per capita over the century, but illustrates instead the demographic impact on GDP per capita. This is illustrated in Fig. 8. As expected, the ageing of the population should have a negative impact on standards of life. Yet, it also shows that assuming a 100% annuitization scheme tends to accentuate the negative impact of the demographic shock. In an ageing Canada, the population density mass starts to be centered on middle age and older individuals (G11 and later i.e., 50-55 years of age and later). We observe in Fig. 6 that fully annuitized older households tend to consume more (per capita), and therefore save less (or more accurately, dis-save more)—at these ages than under a zero annuitization scheme. Consequently, we should expect a higher interest rate under the 100% annuitization scheme when compared to the 0% annuitization scheme. Concomitantly, the capital/labor ratio (K/L) is also a bit lower under the full annuitization scheme, which will decrease the output/worker ratio (Q/L) and the standard of life (Q/Pop) relative to the levels observed under the zero annuitization scenario. Although the difference between these two paths is admittedly small, it also seems to suggest that modeling routinely full annuitization in these OLG-CGE model is perhaps not the most appropriate way to deal with life-length uncertainty when we know that people, in reality, hardly purchase annuities.

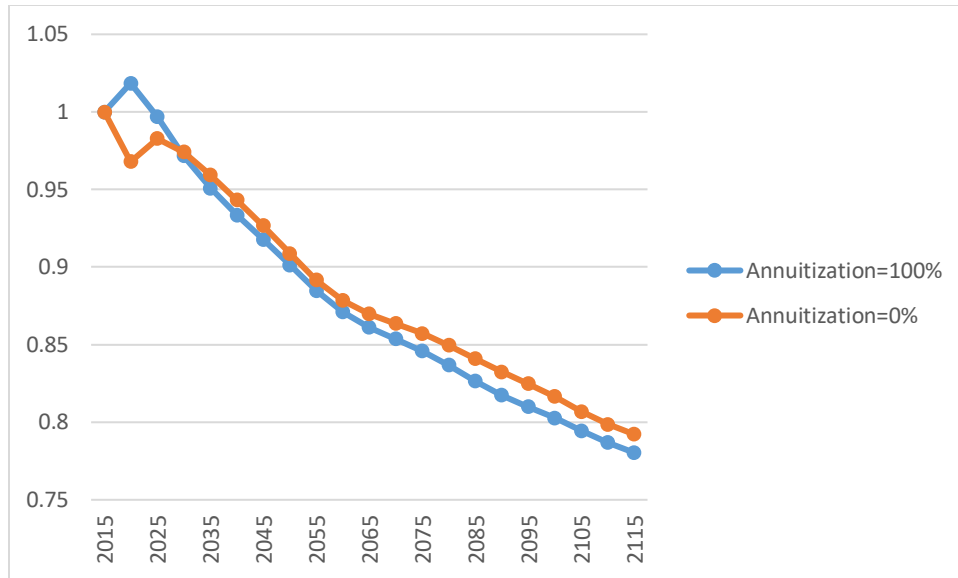


Fig. 8. CANADA. GDP per capita (Q/TPop)

7. Conclusion

This major paper provided a review of the literature on the use of annuitization as a way to insure against life-length risk. It is clear from the literature review that there is a strong annuity puzzle. Although economic theory since Yaari (1965) suggests, under specific assumptions, that the purchase of a life annuity is an optimal behavior, evidence points towards large underutilization of annuities as a way to protect against life-length risk. The simulation results in Section 6 seem to also indicate that the 100% annuitization assumption that is routinely introduced in OLG-CGE models is, perhaps, not the most appropriate technical way to deal with life-length uncertainty. It also shows for example that the negative impact of demographic changes on GDP per capita can possibly be overestimated under this assumption.

All in all, this major paper suggests two things. First, more research should be done in the OLG-CGE literature to attempt to better capture the characteristic of existing annuity products. A graph such as Fig. 7 that shows the age-profile for annuity payouts as they are

typically captured in the OLG-CGE literature, has little to do with annuities that can effectively be bought on the insurance market. Second, annuity products that offer a way to insure against life-length risk is an insurance product that merits much more consideration from individuals. More work should be done by economists and insurance companies to communicate the value of life annuities to an ageing population who has little guide on how to convert accumulated wealth into consumption flows, in the face of life-length uncertainty.

Appendix A: Annuity and L/H Premiums in Selected Countries

Year		2008	2009	2010	2011	2012	2013	2014	2015
Finland (EUR)	Annuity	1659	1127	2798	1988	1988	2458	2544	2287
	L/H	2808	3345	5215	3749	3749	5581	6092	6256
	Ratio	59.1	33.7	53.7	53.0	53.0	44.0	41.8	36.6
Germany (EUR)	Annuity	18333	23949	27898	27237	29443	32863	34019	31782
	L/H	75269	80365	86098	80705	82986	87716	88774	86859
	Ratio	24.4	29.8	32.4	33.7	35.5	37.5	38.3	36.6
Italy (EUR)	Annuity	1324	1253	1077	2309	1243	1012	956	737
	L/H	54565	81116	90114	73869	69639	85010	110424	114842
	Ratio	2.4	1.5	1.2	3.1	1.8	1.2	0.9	0.6
Netherlands (EUR)	Annuity	15911	12960	12058	12001	10892	10998	10806	9035
	L/H	26613	21832	20171	21662	18975	18272	17460	14690
	Ratio	59.8	59.4	59.8	55.4	57.4	60.2	61.9	61.5
Switzerland (CHF)	Annuity	1192	1254	873	737	677	564	441	495
	L/H	32015	29429	30164	30557	31155	32675	32649	32609
	Ratio	3.7	4.3	2.9	2.4	2.2	1.7	1.4	1.5
Korea (KRW)	Annuity		14516005	16493331	17029899	29118668	25867249	23905884	24586649
	L/H		75446826	80939219	86657362	1.07E+08	1.09E+08	1.11E+08	1.17E+08
	Ratio		19.2	20.4	19.7	27.2	23.8	21.6	21.0
United Kingdom (GBP)	Annuity	42435	36105	33344	31964		20899	22295	17278
	L/H	186336	161818	149745	144943	163481	143018	148425	145353
	Ratio	22.8	22.3	22.3	22.1		14.6	15.0	11.9
United States (USD)	Annuity		249674	237782	255059	272564	248568	261327	264944
	L/H		1015276	999954	1066620	829823	782083	788607	826618
	Ratio		24.6	23.8	23.9	32.8	31.8	33.1	32.1

Note: 1. Currencies are listed in bricks.

2. All value is in millions. All ratios are in percentage.

3. Missing values are left as blanks.

4. Ratio is equal to annuity divided by Life and Health insurance. Here, Life and Health insurance is short for L/H.

Source: OECD Insurance Yearbook, 2016

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