
Economics of Base Metals

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Abstract

Doctor of Philosophy in Economics

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by Nguyen BAO ANH

In this thesis I present three papers on the Economics of the base metals industry. The thesis studies production, trading, and investment in the base metals industry, and thus explains some phenomena of the industry in an international context. Using the features of the base metals industry such as the practices in production and trading, physical properties, geology of the deposits and so on, we build theoretical models to simulate the behavior of the industry.

In Chapter One, we study the determinants and the trend of base metals prices over time by an equilibrium model of supply and demand. Because the different types of natural resources exhibit different patterns of price changes in history, we particularly simulate the long run equilibrium to study the impacts of the determinants for base metals prices. The Cobb-Douglas production function on the supply side allows substitution among production factors. The demand function for base metals from the economy is also derived. In the long run, equilibrium of aggregate supply and demand determines the systematic price trend. We show how trends of base metals prices depend on technological progress, resource scarcity, natural resource tax, and the interest rate. Assuming constant returns to scale in base metals production, the price elasticity of the supply of base metals is relatively small. Interestingly, a high natural resource tax leads

to a high price but low rate of price change over time. On the supply side, the decline of base metals relative prices can thus be explained by the inverse supply functions. On the demand side, the relative price is also declining over time as we see the implications of the inverse demand functions and our numerical illustrations. By solving the equilibrium condition, we show that the rental rate of base metals minerals in reserve may decline over time, or even not be valuable in future. The price elasticities of supply and demand are calculated and decomposed into specific effects. These are systematic components of base metal price changes in the world market.

Chapter Two deals with the fluctuations in the prices of base metals. We consider the price in the short run as an equilibrium of trade. If the long run equilibrium regulates the prices and sets them in a stabilization, then the fluctuations in price are caused by the trade and speculative activities. By simulating speculative activities and optimizing the utility of agents in international exchanges, we show that the price fluctuations are the response to risk preferences of agents and the scale of international exchanges. We find out the critical point of production investment, which depends on the market demand, profitability of the metal industry, and the distribution of base metal minerals in nature. In the specific case of the industry versus the market condition when the uncertain production is above the critical point, the price of base metal fluctuates more or less according to the number of producer offers in base metal exchanges, the speculative activities, and risk preferences of agents. In contrast, if the investment level of the base metals industry in uncertain production is below the critical point, the effects of base metal exchanges scale to the price are in the reverse direction. The comparative statics inequalities are derived to clarify the

responses of the price to the risk preferences of agents and scale of the international exchanges. Hence, the non-systematic changes of base metals prices in international exchanges are explained.

Chapter Three studies the impact of the industrial and commercial processes on investment decisions in the base metals industry. The investment decisions of investors in the primary capital market and the stock price in the secondary capital market reflect properties of the base metals industry in capital markets. We present a model of investments, which is a two stage game that incorporates Hall-Jorgenson neoclassical investment analysis and properties of the base metals industry. The paper presents a set of explanatory parameters for the properties of base metal stocks and analyzes the investment decisions. We define the industry factor and explain the empirical observations on the beta coefficient of base metal stocks. The relationships between stock prices and base metals prices are clarified using the geology of base metals deposits. The results show that there is a strong impact of the industry factor on the volatility of base metal stock prices. Economies of scale in the mining industry lead to different effects of tax policy and output prices on investment decisions. We support conclusions of the model by evidence in the base metals industry. There are policy implications that are derived from the equations of the optimal investment.

Key words : Base Metals, Price Fluctuations, Price Trends, Risk Aversion, Metals Industry, LME, International Exchange, Metal Stocks, Investment.

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List of Abbreviations

COMEX	COMmodity EXchange of New York
CRS	Constant Returns to Scale
CARA	Constant Absolute Risk Averse
CRRA	Constant Relative Risk Averse
ICMM	Interational Council of Mining and Metals
ICSG	Interational Coper Study and Group
LME	London Metal Exchange
LMEX	London Metal EXchange Index
R&D	Research and Development
SHFE	SHanghai Future Exchange
TFP	Total Factor Productivity

List of Symbols

A_t	total factor productivity
C_t	total cost of production
C_v	coefficient of variation
D_t	mineral deposits
f_m	quantities of futures that producers are selling
f_s	quantities of futures that traders are selling
g_e	expected rate of return for the base metals industry
i, i^*	the fraction of uncertain production and its optimal value
I	number of identical producers in the market
J	number of identical traders in the market
K_t, \mathbf{K}_t	capital stock used in the base metal industry and in the economy
\mathbf{L}_t	labor used in the economy
\mathbf{M}_t, m_t	quantity of base metals
p_t^i	price of investment
p_{st}	stock price
p_t^d, p_t^s	base metals price - inverse demand, inverse supply
$PV(K_t)$	present value of the firm at the capital stock K_t
R	expected value of \tilde{q} (Chapter 2)
R_t	total revenue (Chapter 1)
w_m, w_s	incomes of the representative producer and trader

β	beta coefficient (Chapter 3)
δ	depreciation rate of capital for the industry
$\varepsilon_s, \varepsilon_d$	price elasticity of supply, price elasticity of demand
ε_{pp}	price elasticity of stock price with respect to base metal price
η	the generalized risk aversion coefficient of the futures market
θ_m, θ_s	coefficients of ARA of the producers and traders
κ	quality of the minerals
μ_t	the quality of ore
π	firm's productivity (Chapter 2)
Π_t	firm's profit (Chapter 3)
\tilde{q}	extra productivity in mining extraction
ρ	discount factor
σ_m, σ_s	variance of producer and trader's incomes
σ_{sp}^2	variance of stock price
τ_t	natural resource tax
φ	the firm specific factor
Φ	the industry factor

Introduction

Base Metals in the Economy

Base metals are industrial non-ferrous metals (excluding precious metals) such as aluminum, copper, lead, nickel, tin and zinc. They are more abundant in nature and, therefore, cheaper than precious metals such as gold, silver and platinum. In the geology and mining, the term base metal refers to the inexpensive metallic elements, which appear in form of mineral deposits or ore bodies. Because of their extensive use and application in the economy, base metals are one of the major factors driving economic growth. They are used as input factors for building homes, automobiles, plants, equipment, pipes, wires, etc. Base metals' demand and changes in their prices may signal economic prosperity. As such, they are used by professional investors as a leading economic indicator.

Production of base metals uses minerals and recycled materials as inputs. Leading mining companies are listed in many countries and mainly belong to the United States, Canada, Australia, and Great Britain. Regarding the value of base metals turnover, the London Metal Exchange alone – the most important market for nonferrous metals – has a turnover value of more than US\$ 2000 billion annually. According to the International Council of Mining and Metals (ICMM), most of the mining productivity increases in the past century have been achieved through the use of more efficient processing of lower grade ores

and the use of larger scale equipment. The demand for base metals during times of economic uncertainty signals the condition of the economy. High demand during tough economic times shows that both business and consumer confidence remains high. Low demand leads to falling prices and that is a sign of fear in the market because there is still a delay period for base metals in manufacturing processes. Manufacturers' appetite for these crucial inputs decreases during economic slowdowns or unclear prosperity. However, this decline has a limit, as stockpiling for long run production drives up demand. Base metal production and demand have a symmetrical relationship. As demand for base metals increases, the producers expand their production.

Globally, the major centers for trading base metals are the Commodity Exchange of New York (COMEX), the London Metal Exchange (LME), and the Shanghai Futures Exchange (SHFE). The LME was established in 1877 in the UK, since this was the starting point of the industrial revolution. Now the LME is the world centre for base metals trading with about 80% of base metals transactions implemented over its platforms. As usual, supply and demand are the major determinants of prices. Base metals prices are also affected by speculation, monetary policy, and currency exchange rates. A reaction may be expected from the issuance of more government debt. The inverse may strengthen the demand for precious and base metals because of their perceived hedge against inflation. Policy on natural resource taxes also affects the base metals market. A weak currency from a powerful economy, such as US dollar or Japanese yen, may increase demand because investors will run for the safety of physical assets. This effect is just like the one of a fear of inflation.

In recent years, there is a tendency of investments in Latin America, Africa and parts of Asia for base metals production. These are likely to escalate in the coming decades. According to the Report of ICMM, October 2012, the global giants of the industry set criteria for the size of deposits that interest them, preferring projects with a lifespan of at least 20 years. As demand continues, it is likely that the mid-sized companies will become more important in taking on rejected but still viable projects. Because the base metals industry provides the infrastructure for the contemporary economy, the investments in the industry to supply base metal will continue to play its role in meeting society's needs. Moreover, that is the reason why this research is necessary to make the base metals industry less confusing.

Research Questions and Methodology

In this thesis, we study the price determinants of base metals in the global market, where base metals producers, manufacturers, and traders are selling, buying and speculating in well-organized commodity exchanges. The price factors determine the systematic price trend and non-systematic variation of price. The base metals industry has special properties. We find the linkages of these properties with industrial, commercial, and investment conditions. In short, we want to answer the following research questions: What drives the prices of base metals? Why do these prices fluctuate? How do the prices of base metals affect investment decisions? There will be the definitions, economic intuitions, and policy implications surrounding these issues, being presented in each of the three chapters.

Although they are a group of commodities and behave according to common economic principles, base metals have some special features which should be considered for building an economic model: (i) Base metals are recyclable, the portion of recycled materials is constant over the decades. (ii) Producers' incomes vary with the content of base metals distributed in the Earth's crust. (iii) Base metals deposits are common in nature and inexpensive to access. (iv) Prices show positive cross-elasticities of demand. (v) Base metals are homogeneous products and durable for storage. (vi) The market for these commodities is well organized with professional agents participating. Other groups of commodities do not have these characteristics, so they would not be considered in the same models. Practically, they behave differently in the markets and in the economy. For example, agricultural products are seasonal, are not homogeneous products, and are not durable for storage, so the prices vary according to many other conditions. Crude oil is an important commodity for the world economy, but there are many issues that should be taken into account including politics, collusion, and cross-impacts. In addition, crude oil is not recyclable like base metals. Using the Cobb-Douglas production function, with capital and crude oil as input factors, it looks like we can burn capital to generate outputs. We will discuss more about the other groups of natural resource commodities in the Potential Research Extension section.

The study on the trends of base metals prices in *Chapter One* uses the Cobb-Douglas production function, which is common in theoretical studies, but new for base metals production. The supply side of base metals is simulated by solving the Hotelling-style problem of the regulator. Broadly speaking, there is a movement from the industrial to the knowledge economy, where the use

of base metals is of relatively lower importance. We consider the demand side of base metals with a utilization of the model of Natural Resources and Land¹. Hence, instead of an increase of the marginal value, as per Slade (1982)², a decline of the rental rate of base metal ores is possible. We fill the gaps in the literature by imposing components on both supply and demand sides of the base metals commodities, then the equilibrium must be the right objective to look at for the trends of base metals prices in the long run. Some standard techniques are applied for calculating demand functions and price elasticities.

The inverse supply and demand functions allow us to derive the price elasticities of supply and demand. We can see the effects and mechanisms of the determinants of price changes. Under central planning and a CRS production function in minerals exploitation, the supply of base metals is inelastic. We find that the trends in base metals prices depend on technological progress, the availability of base metals in nature, natural resource taxes, and the uses of metals in the economy. We decompose the effects of the determinants of base metals prices. The mechanism and magnitude of the impacts of each individual factor allow us to illustrate the evolution of prices. By numerical illustrations, we are able to explain the downward slope of the price trend of almost every base metal.

Regarding the fluctuations in the prices of base metals, *Chapter Two* develops the theoretical model from the practice of the base metal trade described above. Hedging activities are inevitable in the international market and play a role in

¹Romer, David. "The Environment and Economic Growth." *Advanced Macroeconomics*, 3rd Edition, McGraw Hill, New York (2006): 37-44.

²Slade, Margaret E. "Trends in natural-resource commodity prices: an analysis of the time domain." *Journal of Environmental Economics and Management* 9, no. 2 (1982): 122-137.

price fluctuations. The aggregate equilibrium model is not appropriate for explaining the fluctuations in base metals prices because the plans for supply and demand are always aiming for a relatively long period of time. We depart from the framework of the model of farmers' choice, Newbery (1987)³, to develop a trade equilibrium model with the participation of professional agents in the international exchanges. These agents are the base metals producers, the manufacturers, and the traders. The producers are base metals suppliers, who smelt metals from extracted minerals and recycled materials. The manufacturers purchase base metals as the input materials for their production process, which represent the rest of the economy. The traders buy and sell base metals in two types of transactions: spot and futures contracts. As commonly known, profits for producers and manufacturers come from production, while profit for traders comes from their trading and speculative activities. We apply the Theory of Portfolio Choice for the agents to establish the equations of their optimal sales and uncertain production.

Based on this framework, we proceed to satisfy the trade equilibrium conditions of the market. We justify that there is a critical level of uncertain production in the base metals industry. We explain the price fluctuations by referring to uncertain production, the price elasticity, agents' risk preferences, and the size of international exchanges. The critical level of uncertain production is appropriately adjusted in reality, being shown with our numerical illustration. In the short-run, when more producers participate in commodity exchanges or they are less risk averse, the price of base metals fluctuates more. Conversely, with more speculative activities or if the traders are less risk averse, the price of base

³Newbery, David M. "When do futures destabilize spot prices?" *International Economic Review* (1987): 291-297.

metals fluctuates less.

The study of the base metals industry in capital markets, *Chapter Three*, addresses the impact of the industrial and commercial processes on investment decisions in the base metals industry. The investment decisions of investors in the primary capital market and the stock price in the secondary capital market reflect properties of the base metals industry accordingly. In the secondary market, the industry needs a measure to portray its characteristics. We use the fundamental analysis method to determine the possible impacts of profit on variation in stock prices and thus find the effects of the industry factor and the relevant variables. In the primary capital market, the study presents a model of investments, which is a two stage game that incorporates Hall-Jorgenson neoclassical investment analysis in Jorgenson (1963)⁴ and the properties of base metals industry.

As outcomes of the analysis, we present a set of explanatory parameters for the properties of base metal stocks and analyze the investment decisions. We define the industry factor and explain the empirical observations on the beta coefficient of base metal stocks. The relationships between stock prices and base metals prices are clarified using the geological practice of base metals deposits. The results show that there is a strong impact of the industry factor on the volatility of base metal stock prices. Economies of scale in the mining industry lead to different effects of tax policy and output prices on investment decisions. We support the arguments and the outcomes of the analysis with empirical evidence from the base metals industry.

⁴Jorgenson, Dale W. "Capital theory and investment behavior." *The American Economic Review* 53, no. 2 (1963): 247-259.

Structure of the Thesis

This Introduction section provides an overview of the base metals industry, the research questions, and the research methodologies. The rest of this thesis is organized as follows: Chapter One studies the Trends of Base Metals Prices. Chapter Two is a model of trade to explain the Fluctuations in Base Metals Prices. Chapter Three analyzes the Base Metals Industry in Capital Markets. In the Conclusion section we wrap up the research outcomes of these three chapters. Finally, the Potential Research Extensions for this subject is briefly discussed.

Chapter 1

Trends of Base Metals Prices

Abstract

The real prices of base metals exhibit a decreasing trend over time. Despite the important role of base metals in the world economy there is lack of understanding of how the prices of base metals have evolved over the last century. We model base metals prices as determined by the equilibrium of aggregate supply and demand. This allows us to study the effects of determinants of base metals prices. The study shows that the trend of base metals prices depends on technological progress, resource scarcity, natural resource taxes, the interest rate, and the demand from other sectors of the economy. In this study, the price elasticities of supply and demand are calculated and decomposed into specific effects. We show that the rental rate of base metals, which is also called the marginal value of the base metals held in reserve, may fall over time, or even not be valuable in the future. Under certain parameter restrictions we explain the decreasing trend in prices over time. This phenomenon is explained by the substitution effect and technological progress. We derive policy implications related to natural resource taxation.

1.1 Introduction

Base metals are industrial non-ferrous metals: aluminum, copper, lead, nickel, tin, and zinc. They are used for building homes, automobiles, plants, equipment, pipes, wires, and so on. Such extensive use of base metals in industry inevitably links base metals markets to aggregate economic conditions. Base metals prices play an important role in the economies of many countries, which derive the bulk of their export revenues from one or more base metals commodities. An author, Mark Riddix states that “Investors who want to know where global economies are headed should keep an eye on base metals”. Dennis Gartman, an editor and publisher in CNBC uses copper – among other base metals – as a leading economic indicator. In addition to the impacts stemming from economic growth, there may be other determinants of the prices of base metals in the long run.

Historical data shows a downward trend for almost all non-renewable resource prices within the period from 1945 until the early 1980s. Over this period, prices of natural resource commodities such as copper, iron, nickel, silver, tin, coal, natural gas, and a mineral aggregate all exhibit an upward trend. Almost all mineral prices rose from the beginning of the 1970s, especially following the 1973 oil crisis. But prices of base metals did not continue to increase, they kept going downward. This phenomenon is different from that of other non-renewable resources such as fossil fuels and forest products.

Table 1.1¹ below presents the movements of real prices of some major natural

¹Data sources: Krautkraemer (2005) and US Energy Information Administration, <http://www.eia.doe.gov>

Commodities	1960	1965	1970	1975	1980	1985	1990	1995
Base Metals								
Aluminium	100	90	91	73	83	62	67	62
Copper	100	1983	1972	84	93	63	81	75
Lead	100	153	126	96	131	59	85	55
Nickel	100	102	144	128	115	91	112	88
Tin	100	170	138	143	220	159	57	49
Zinc	100	121	99	138	89	96	127	73
Precious Metals								
Gold	100	94	83	207	491	269	223	187
Silver	100	136	160	221	648	202	109	99
Fossil Fuels								
Coal	100	88	100	222	199	157	116	88
Natural Gas	-*	-	100	187	440	532	298	232
Petroleum	-	-	100	174	320	273	187	117
* Note : "-" indicates that data is not available.								

TABLE 1.1: Prices of natural resource commodities.

resource commodities in the second half of the 20th century, from 1960 to 1995. There is a clear downward trend of prices of almost all base metals.

Because there are differences between types of commodity, from fossil fuels to precious metals in terms of applications, production, and trade, it is necessary to use different models to study the determinants of the trend in prices over time. The uses of precious metals as a tool for value storage or currency, as well as the materials in high-tech devices, require different models and analysis. On the other hand, fossil fuels have special impacts on the whole economy, with the confounding factors of geopolitics and policies of alliance. Figure 1.4 in the Appendix 1.5.1 ² shows the real historical time series of crude oil prices, revealing different and complicated movements.

²Data source: U.S. Geological Survey, http://www.usgs.gov/energy_minerals/

In this chapter we build a model for studying the trends of base metals prices and the determinants behind them. The model incorporates the following features of base metals: (i) The quality of ore, which is measured by content of base metal minerals, is the important input factor for production. (ii) The quality of base metal minerals is stable, their chemical properties and crystal structures remain unchanged in nature over millions of years. (iii) Base metals deposits are common in nature and inexpensive to access. (iv) Prices of base metals show positive cross-elasticities of demand, as they are somehow substitute goods. (v) Base metals are a homogeneous product and are durable for storage. Because of the similarity in features and applications for the economy, the prices of all the base metals always move in the same direction, not only in the long run but also in the periodical changes of price. Figure 1.1 below shows the rate of base metal price changes, annually from 1966 to 2013³. The rates of price change have almost always the same sign.

The historical price trend represents equilibrium of supply and demand. For the supply side, we consider the regulated industry with a Cobb-Douglas production function for base metals. By solving the Hotelling-style problem of the regulator, we find that the price trend in the supply of base metals depends on natural resource taxes, the interest rate, technological progress, and the degradation of ore in nature. Under constant returns to scale (CRS) production function, the supply of base metals is perfectly inelastic. The supply price is decreasing over time for realistic values of parameters. We then study the demand for base metals from the manufacturing sector. We show that, instead of increasing, the rental rate of base metal minerals underground may decrease over time.

³Data source: U.S. Geological Survey, http://www.usgs.gov/energy_minerals/

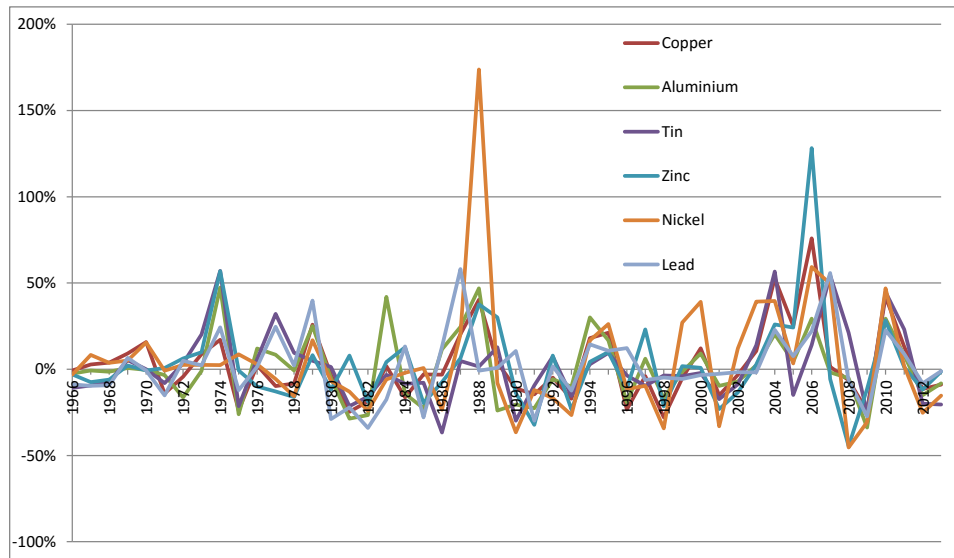


FIGURE 1.1: Rate of change in base metals prices.

It is possible that the rental rate approaches zero when the parameters of the economy and the metals industry change, or remains constant with some specific values of these parameters. The resulting demand price is also decreasing. Thus we can justify the possibility of a downward sloping price trend.

Our results are driven by the use of the Cobb-Douglas function for base metals production. The use of this production function reflects substitutability between input factors, particularly between capital and mineral deposits, which are degrading over time – thus making the use of capital more efficient.

Literature Review

Despite the important role of base metals in the world economy there is little research on evolution of base metals prices. While some empirical studies

contribute to the understanding of base metals prices, there are no theoretical studies.

Tilton (1989), Tilton, & Landsberg (1999), and Buñuel (2001) state that the decline in the mineral resource intensity of mining production provides evidence of the effect wherein the costly input (mineral deposits) is substituted for by the less costly (capital), given technological progress. For example, the solvent extraction-electrowinning (SX-EW) method for refining copper ore succeeded in reducing costs. This method allows the more efficient use of lower grade copper ore. Theoretically, Hotelling-style models predict an increasing real price for non-renewable natural resource commodities. However, empirical observations establish falling prices for these commodities. For instance, see Buñuel (2001) and Krautkraemer (2005). To explain the inconsistency in the price trends of natural resource commodities, Slade (1982) suggests a U-shaped time path for the relative prices of natural resource commodities. This paper only considers the supply side of commodities, and still leaves the question of where the prices are heading in the next period of time. However, Krautkraemer points out that there is no evidence of increasing base metals prices. He concludes that technological progress has ameliorated the scarcity of natural resource commodities, but resource amenities have become more scarce, and it is unlikely that technology alone can remedy that. We depart further from the literature by introducing the Cobb-Douglas production function, which combines the effect of technological progress with the notion of substitutability of input factors. Then we consider the demand side for base metals from the manufacturing sectors of the economy. This allows us to establish the possibility of a constant declining trend in prices.

Our study fills the gap in this literature by offering a theoretical model of base metals prices, which includes equilibrium of supply and demand. Our contributions include the clarification of the impacts of technological progress, ore reserve availability, and policy-relevant parameters on base metals prices. We also address the responsiveness of the price with respect to the determinants, by the decomposition of the price elasticities into the specific effects. We show that, instead of increasing, there is a possibility that the rental rate of base metal minerals underground decreases over time.

The rest of this chapter is organized as follows: Section 2 introduces assumptions, notation, and the theoretical model. Section 3 solves and analyzes the equilibrium condition. Section 4 summarizes the outcomes of the model. Section 5 presents mathematical calculation, graphic illustrations, and proofs of lemmas, corollaries, propositions.

1.2 The Model

The supply of base metals comes from the regulated mining industry. The regulator maximizes the discounted profit, which is the difference between revenue $R = R_t(m_t)$ and the total cost of production of the industry $C = C_t(A_t, \mu_t, m_t)$. The extraction path of the mining industry reflects the degradation of metal minerals in nature. The quality of ore at time t is μ_t (the content of metal minerals in extracted ore). Degradation of the quality over time implies that $\frac{\partial \mu}{\partial t} < 0$. The regulator chooses the time path for the extraction rate $\frac{\partial \mu}{\partial t}$. Hereinafter we

denote $\frac{\partial \mu}{\partial t}$ by μ_t .

The Cobb-Douglas production function for base metals is

$$m_t = A_t K_t^a (\mu_t D_t)^b, \quad (1.1)$$

where K_t is capital, D_t is mineral deposits, and A_t represents the total factor productivity of the mining industry. We denote by r the cost of capital⁴ and by τ the natural resource tax.

Demand for base metals comes from the manufacturing sector. The manufacturing sector has a Cobb-Douglas production function

$$Y_t = A_t \mathbf{K}_t^\alpha \mathbf{L}_t^\beta \mathbf{M}_t^\gamma, \quad (1.2)$$

where $\mathbf{K}_t, \mathbf{L}_t, \mathbf{M}_t$ are capital, labour, and base metals demand at time t in the whole economy, respectively. We assume the CRS production function for the manufacturing sector $\alpha + \beta + \gamma = 1$.

For simplification, assuming that there is a spill-over effect in the total factor productivity so that the TFP of the mining industry and that of the manufacturing sector (A_t) equal each other. The cost of capital r in this model is the same for the whole economy.

⁴Including tax on capital, if any.

1.3 Equilibrium

1.3.1 Supply

The industry's cost minimization function

$$\min_{K,D} r_t K_t + \tau_t D_t \quad (1.3)$$

$$s.t.1.1. \quad (1.4)$$

Using the standard technique we obtain the following

Lemma 1 : *The cost function of the mining sector is*

$$C_t(A_t, \mu_t, m_t) = \psi(r_t^a \tau_t^b)^{\frac{1}{a+b}} A_t^{\frac{-1}{a+b}} \mu_t^{\frac{-b}{a+b}} m_t^{\frac{1}{a+b}}, \quad (1.5)$$

where $\psi = \left(\frac{a}{b}\right)^{\frac{b}{a+b}} + \left(\frac{a}{b}\right)^{\frac{-a}{a+b}}$.

Proof: See Appendix 1.5.2.

After extracting ore from the deposit, the minerals are separated from ore by the beneficiation process. Smelters produce base metals from refined minerals. Production output m_t equals the content of metal in minerals κ , which is a function of μ_t , multiplied by the rate of degradation of ore over time $\dot{\mu}_t$

$$m_t = \dot{\mu}_t \kappa(\mu_t). \quad (1.6)$$

The regulator maximizes industry profit by choosing the extraction path. By equation 1.6, this is equivalent to choosing the rate of ore degradation $\dot{\mu}_t$. Hence,

the regulator's problem is

$$\max_{\dot{\mu}_t} W = \int_0^{\infty} e^{-rt} [R_t(m_t) - C_t(A_t, \mu_t, m_t)] dt \quad (1.7)$$

$$s.t. 1.6. \quad (1.8)$$

Solving this maximization problem yields the following (we denote by p^s the inverse supply function and we omit the subscript t henceforth)

Proposition 1 :

a) The (inverse) supply function for base metals from the mining sector is

$$p^s = \frac{\psi}{a+b} (r^a \tau^b)^{\frac{1}{a+b}} A^{\frac{-1}{a+b}} \mu^{\frac{-b}{a+b}} m^{\frac{1-a-b}{a+b}} + \frac{\lambda e^{rt}}{\kappa}, \quad (1.9)$$

$$\dot{p}^s = \frac{\partial^2 C}{\partial m \partial t} + \frac{1}{\kappa} (\lambda r e^{rt} - \frac{\partial C}{\partial \mu}). \quad (1.10)$$

b) If $a + b = 1$ then

$$p^s = \frac{\psi r^a \tau^b}{A \mu^b} + \frac{\lambda e^{rt}}{\kappa}, \quad (1.11)$$

$$\dot{p}^s = -\frac{\psi r^a \tau^b}{A \mu^b} \left[\frac{\dot{A}}{A} + \frac{\dot{\mu}}{\mu} - \frac{b \dot{m}}{\mu \kappa} \right] + \frac{\lambda r e^{rt}}{\kappa}. \quad (1.12)$$

Proof: See Appendix 1.5.3.

The term $\frac{\lambda e^{rt}}{\kappa}$ stands for the marginal value of the base metals in reserve, or the rental rate of this natural resource. In practice, base metal minerals have been formed deep in the Earth's mantle at extremely high temperatures, so they have stable crystal structures. For example, Pentlandite $(Fe, Ni)_9S_8$ is an important ore of nickel, found in a large deposit at Sudbury, Ontario, Canada. These

mineral ore bodies are assumed to be produced through magmatic segregation. When the hot liquid magma is cooling, crystals of high-density minerals such as metal sulfides fall to the bottom of the magma chamber and collect into one large metal-rich bonanza convenient for mining. Under that high temperature and pressure, base metal minerals are very hard, durably crystallized, and exist in nature for millions of years – thus the content of the useful element κ in the second term of equation 1.9 is not declining. The rental rate does not increase over time by the reduction of κ .

Essentially, we don't know how much the rental rate of the ore's marginal value is in reserve until we analyze the demand side of the economy. Intuitively, there is a doubt that the value of one ton of unextracted nickel ore remains unchanged in the next hundred years. In the future, the marginal product of input factors changes, so the rental rate will also change. By the law of diminishing returns, the marginal product of base metals is going down as time passes. Additionally, unlike almost all other natural resources, base metals are recyclable. The total base metals circulation in the economy increases as extraction continues. The use of recycled materials increases and thus the rental rate $\frac{\lambda e^{rt}}{\kappa}$ becomes relatively lower over time.

The following Corollary can be inferred from Proposition 1.

Corollary 1 :

a) The price elasticity of supply is

$$\varepsilon_s = 1 + \frac{\varepsilon_{Ap}}{1 - a - b} + \frac{\varepsilon_{\mu p}}{1 - a - b}. \quad (1.13)$$

b) If $a + b = 1$ then the supply of base metals is perfectly inelastic.

Proof: See Appendix 1.5.4.

In equation 1.13, ε_{Ap} reflects the responsiveness of total factor productivity in the mining industry to a change in p^s and $\varepsilon_{\mu p}$ reflects the percentage change in the extraction rate in response to a change in p^s .

Evaluation of the returns to scale in metals production is a difficult task. We follow a somewhat implicit method to justify the constant returns to scale hypothesis. Empirically, Stuermer (2014) finds that the price elasticity of base metals is between 0.2 and 0.8 (Aluminum 0.7 - 0.8; Copper 0.4; Lead 0.2; Tin and Zinc 0.2 - 0.4). These estimations support our assumption that the base metal production exhibits constant returns to scale. Thus we can focus on equations 1.11 and 1.12 to explain the following effects: (i) High total factor productivity and the availability of minerals in nature keep the supply of base metals relatively low. (ii) The growth rate of technology $\frac{\dot{A}}{A}$ has a negative impact on the supply increase. (iii) The degradation progress of ore in nature $|\frac{\dot{\mu}}{\mu}|$ has a positive effect on supply change⁵.

According to the International Council of Mining and Metals, most of the mining productivity increases in the past century have been achieved through the use of more efficient processing of lower grade ores and the use of larger scale equipment⁶. Figure 1.5 shows that technological progress in the mining industry has been improved significantly, while there was only a slight degradation

⁵ The degradation progress is the reduction of base metals content μ in deposits (in other words, in nature) over time, i.e. the absolute value of $\frac{\dot{\mu}}{\mu}$.

⁶Source: Raw Materials Group, Stockholm, Sweden. The trends over time for ore grades in Australia, Canada, and the USA are shown in Mudd (2009).

of ore over the last decade, illustrated in Figure 1.6. To simulate the time path of supply, consider the values $\frac{\dot{A}}{A} \approx 1\%$ and $\frac{\dot{\mu}}{\mu} \ll 1\%$ ⁷. Substituting these values into equation 1.12 we obtain a decreasing supply price over time.

Considering the policy variables of the economy: interest rate r and natural resource tax τ .

Proposition 2 : *Comparative statics in supply price*

$$\frac{\partial \dot{p}^s}{\partial \tau} < 0; \frac{\partial p^s}{\partial \tau} > 0; \frac{\partial p^s}{\partial r} > 0; \frac{\partial \dot{p}^s}{\partial r} < > 0. \quad (1.14)$$

Proof: Immediate from equations 1.11 and 1.12.

There are several policy implications from Proposition 2: (i) If the natural resource tax τ is initially high, then the rate of increase in supply price is low, causing the supply price to be relatively low in the future. (ii) High tax and a high discount rate imply a high price of base metals. (iii) The effect of the discount rate on the rate of change in supply price is ambiguous.

1.3.2 Demand

The 19th and 20th centuries witnessed a transition from the agricultural and industrial economy to the knowledge economy. This progress is happening even faster in the first decades of the 21st century. The key input factor of a knowledge economy is intellectual capability, while base metals are essential

⁷Sources: The World Bank, <http://www.worldbank.org/>. The Asian Development Bank, <http://www.adb.org/>. ICMM Report, October 2012.

to the industrial economy. As the productivity of labour and capital increase, knowledge-intensive production, R&D, and technological progress relatively reduce the use of base metals in the manufacturing sector. Hence, the adjustment of metal use in the economy is possible. While there is a convergence of metal use as a share of GDP, if technological progress is dominant then the price is declining over time.

In the manufacturing sector, the Cobb-Douglas production function is $Y_t = A_t K_t^\alpha L_t^\beta M_t^\gamma$. Assuming that the manufacturing sector's production function exhibits constant return to scale, i.e. $\alpha + \beta + \gamma = 1$. The demand for base metals comes from the manufacturing sector. Rewriting this production function in per-capita form we have

$$y_t = A_t k_t^\alpha m_t^\gamma. \quad (1.15)$$

Lemma 2 :

a) The (inverse) demand function for base metals from the manufacturing sector is

$$p^d = A^{-\frac{1}{\alpha}} \left(\frac{\gamma r}{\alpha} \right) y^{\frac{1}{\alpha}} m^{-\frac{\alpha+\gamma}{\alpha}}. \quad (1.16)$$

b) The rate of change of p^d is

$$\frac{\dot{p}^d}{p^d} = \frac{\dot{A}}{A} + (\gamma - 1) \frac{\dot{m}}{m} + \alpha \frac{\dot{k}}{k}. \quad (1.17)$$

Proof: See Appendix 1.5.5.

The following Corollary can be inferred from Lemma 2.

Corollary 2 : *The price elasticity of demand is*

$$\varepsilon_d = \frac{\varepsilon_{yp} - \varepsilon_{Ap} - \alpha}{\alpha + \gamma}. \quad (1.18)$$

Proof: See Appendix 1.5.6.

Equation 1.16 implies that, on the demand side *ceteris paribus*: (i) Technological progress reduces the demand price for base metals by shifting down the demand curve. (ii) Economic growth and the interest rate shift the demand curve up.

Equation 1.17 implies that the rate of change of p^d equals the rate of technological progress minus the growth rate of base metals demand. The growth rate of metals use negatively impacts on the rate of change p^d because $\gamma - 1 < 0$ ($\alpha, \beta, \gamma < 1$).

Expression 1.18 decomposes ε_d into three terms. ε_{Ap} reflects the responsiveness of total factor productivity in the mining industry to a change in p^d , ε_{yp} reflects the responsiveness of economic output to a change in the base metal price, and a constant value $\frac{\alpha}{\alpha + \gamma}$ reflects the share of base metals used in the economy.

In the long run, the capital growth rate equals zero in the steady state: $\frac{\dot{k}}{k} = 0$; Suppose the TFP growth rate $\frac{\dot{A}}{A} \approx 1\%$ ⁸; $\frac{\dot{m}}{m}$ equals economic growth (per capita) $\approx 2 - 4\%$ ⁹; $\gamma \approx 0.1$ ¹⁰. With these values we have negative value of $\frac{\dot{p}^d}{p^d}$. Thus the demand of base metals in the economy is decreasing over time.

⁸<http://www.worldbank.org/>, <http://www.adb.org/>.

⁹<http://www.economist.com/blogs/graphicdetail/2012/10/>.

¹⁰Empirical works in literature estimated $\alpha \approx 0.3 - 0.4$, $\beta \approx 0.5 - 0.6$, thus we assume $\gamma \approx 0.1$.

1.3.3 Equilibrium

The solution of the system of equations 1.9 and 1.16 is the long run equilibrium price. For simplification, we solve the system of equations for the case where base metals production is constant returns to scale. Accordingly, the condition for equilibrium is then leading to the following

Proposition 3 : *Over time, the declining of the economic rent of base metals resources in nature is possible*

$$\frac{\lambda e^{rt}}{\kappa} = \left(\frac{\gamma r}{\alpha}\right) \left(\frac{y}{A}\right)^{\frac{1}{\alpha}} m^{-\frac{\alpha+\gamma}{\alpha}} - \frac{\psi r^a \tau^b}{A \mu^b}. \quad (1.19)$$

The left hand side of equation 1.19 is the economic rent, or another words, the marginal value of the base metal ore underground. Instead of increasing, it is quite possible that this rental rate of the resource falls when the parameters of the economy and the metals industry change. In some cases, the rental rate remains constant with the specific values of the economy's parameters a, b, α, γ and the variables A, m, y . This rental rate equals zero if and only if the demand of base metals is

$$m = \left[\frac{\gamma y^{\frac{1}{\alpha}} \mu^b A^{\frac{\alpha-1}{\alpha}}}{\alpha \psi r^{a-1} \tau^b} \right]^{\frac{\alpha}{\alpha+\gamma}}. \quad (1.20)$$

To see that the rental rate can decrease from time to time, we analyze the saturation of base metals use in the economy. As base metals are produced from extracted minerals and recycled materials, there will be a circumstance when the fraction of recycled materials in base metals production increases to one. Because the base metals have been extracted from the Earth's crust over the

course of history, the accumulation would reach a definite amount. The demand for base metals in the economy approaches a constant level – there will eventually be no more base metals extraction necessary for the economy. If that is the case, the rent of base metal mineral deposits is zero.

Intuitively, the law of diminishing marginal productivity implies that the marginal product of base metals in the economy should decline over time. In addition, the circulation of recycled materials helps lowering demand of extracted minerals.

Turning back to the realistic progress of base metals prices, we present the supply and demand curves in the three-dimensional coordinate system (p, m, t) , Figure 1.2 below. This Figure depicts the trend of the equilibrium price .

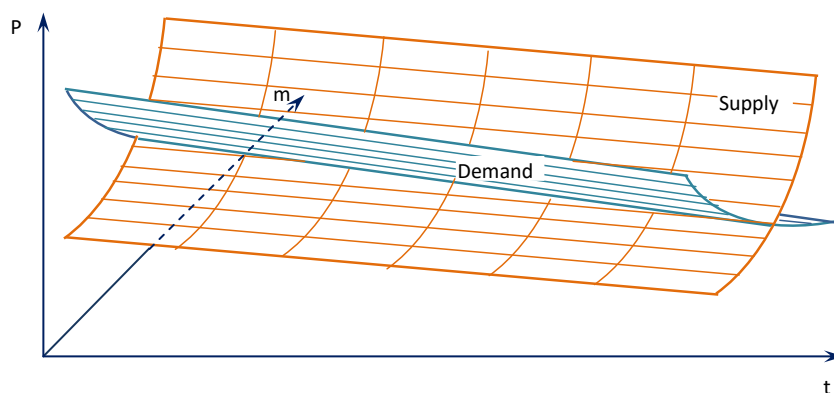


FIGURE 1.2: Aggregate supply-demand equilibrium over time.

The trend line in Figure 1.3 is the projection of the Figure 1.2 onto (p, t) coordinate system. The prices are declining over time for the empirical values of parameters $\frac{A}{A'}$, $\frac{\mu}{\mu'}$, τ , and $\frac{m}{m'}$.

To support our theoretical prediction, we use data from United States Geology Service to discover what has really happened in the past. Figure 1.3¹¹ below shows real prices and trends of aluminum, copper, and nickel in from 1900 to 2012. We use 1998 US\$ to indicate the relative prices. For each metal we build the trend line of the time series data. These trends illustrate our findings with the empirical parameters of the industry.

Notice that the prices of aluminum and copper spike during World War I (1914 - 1918). Nickel price did not increase because it was not really a necessity for war purposes (while there is no change in supply, the increase of m in equation 1.16 shifts up the demand curve). The price of base metals in periods of economic crises during the 1930s and in 1996 was below the trend line. During the financial crisis of 2007 - 2009 the prices of almost all base metals were highly volatile, which is explained in the next chapter or in Nguyen and Semenov (2015). After this period, the prices climbed up again as the economies in the world recovered, the effect of y on the demand side is enabled (see equation 1.16). Additionally, one of the reasons for the price of nickel increase is that the Indonesian government increased export taxes for nickel in 2009 (represented by τ in equation 1.9). This seriously affected a part of nickel and bauxite supply with US\$2 billion annual revenue.

¹¹Data : http://www.usgs.gov/energy_minerals/.

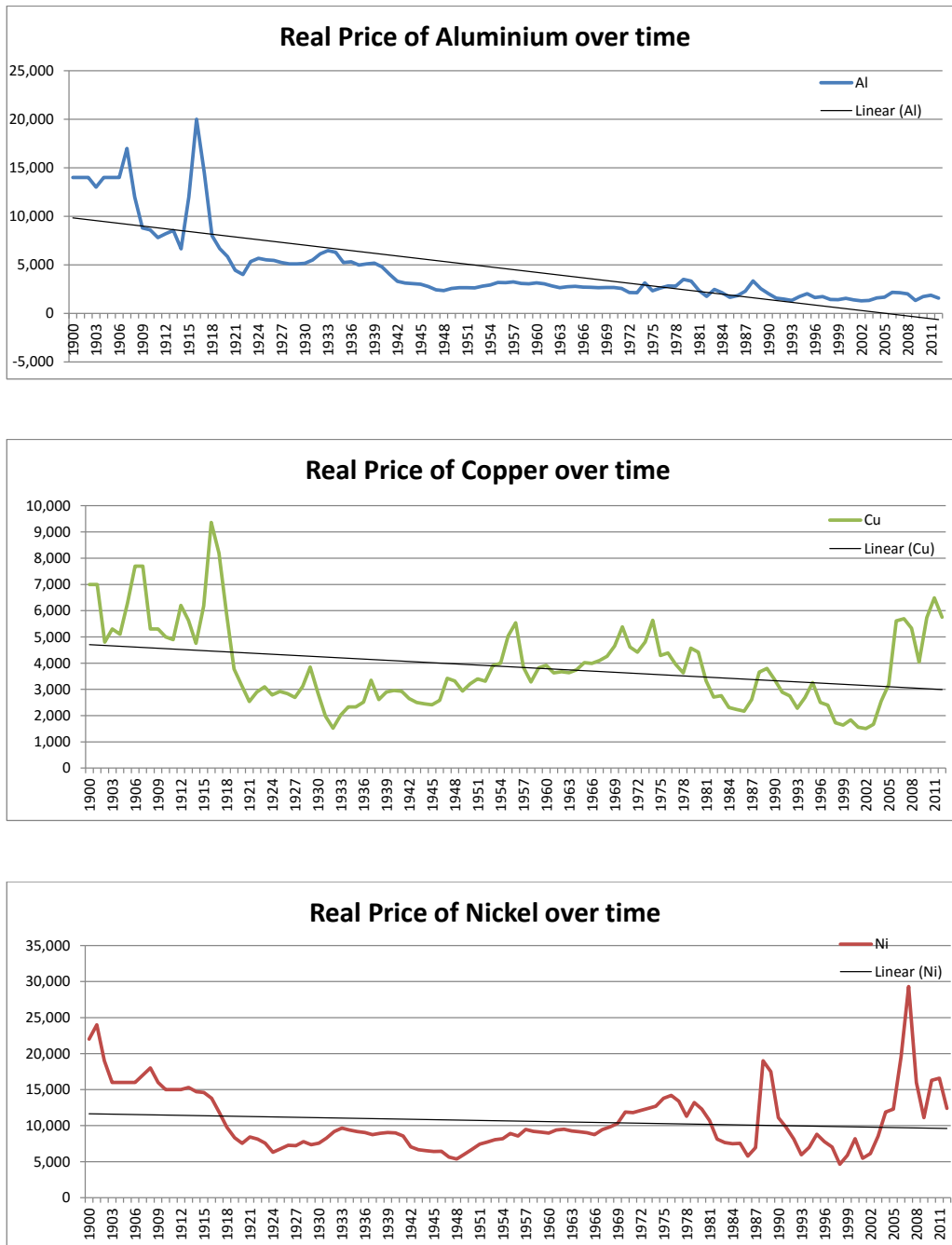


FIGURE 1.3: Real price of some base metals over time.

1.4 Conclusion

The theoretical literature has no comprehensive consideration of all relevant factors that impact the price of base metals. Normally the price is defined only by the mining industry, with a simple production function, and applicable for all types of natural resources commodities. Market demand was neither considered nor discussed.

This chapter answers the question of what drives the trends of base metals prices. By modeling the industry regulator's problem for extraction of base metal minerals in combination with the demand from the economy, we show that in the long run the price of base metals is a function of total factor productivity of the mining industry, the availability of metals in nature, natural resource policy, and demand from economy. We decompose the price elasticity of supply and demand into the responsiveness of price to changes in different price determinants. Assuming constant returns to scale, the price elasticity of the supply of base metals is relatively small ($0 < \varepsilon < 1$, inelastic). Interestingly, a high natural resource tax leads to a high price but low rate of price change over time. By solving the system of equations for equilibrium conditions, we show that the rental rate of base metals minerals in reserve can decrease over time. The mechanism and magnitude of the impacts by each individual factor allow us to illustrate the evolution of prices. Based on numerical simulations we show that prices of almost all base metals may decline over time.

1.5 Appendix

1.5.1 Crude Oil Price

While base metals prices have declining trend lines, historical crude oil prices have a different time trend. It is neither declining nor U-shaped. We may consider the geopolitical and alliance-based aspects of the real world for modeling oil prices.

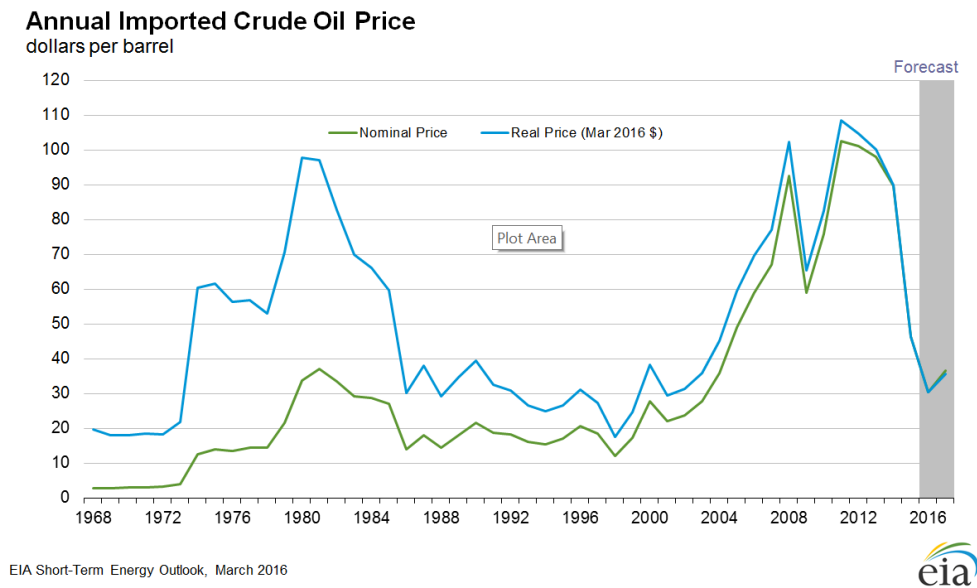


FIGURE 1.4: Real price of crude oil over time.

For precious metals or rare earth elements, different groups of natural resource commodities require different typical considerations in simulations.

1.5.2 Proof of Lemma 1

The cost minimization problem is

$$C_t(A, \mu, m) = \min_{K, D} r_t K_t + \tau_t D_t \quad (1.21)$$

$$s.t. m_t = A_t K_t^a (\mu_t D_t)^b. \quad (1.22)$$

Ignoring subscript t and solving 1.22 for D we have

$$D = A^{-\frac{1}{b}} \mu^{-1} m^{\frac{1}{b}} K^{-\frac{a}{b}}. \quad (1.23)$$

Substituting 1.23 into 1.21 yields

$$C(A, \mu, m) = \min_{K, D} rK + \tau \mu A^{-\frac{1}{b}} \mu^{-1} m^{\frac{1}{b}} K^{-\frac{a}{b}}. \quad (1.24)$$

The first order condition is

$$r - \frac{a}{b} \tau A^{-\frac{1}{b}} \mu^{-1} m^{\frac{1}{b}} K^{-\frac{a+b}{b}} = 0. \quad (1.25)$$

Thus we obtain the conditional demand function for K

$$K = A^{\frac{-1}{a+b}} \mu^{\frac{-b}{a+b}} \left(\frac{\tau a}{rb} \right)^{\frac{b}{a+b}} m^{\frac{1}{a+b}}. \quad (1.26)$$

The conditional demand function for D is

$$D = A^{\frac{-1}{a+b}} \mu^{\frac{-b}{a+b}} \left(\frac{\tau a}{rb} \right)^{\frac{-a}{a+b}} m^{\frac{1}{a+b}}. \quad (1.27)$$

Substituting 1.26 and 1.27 into 1.21 yields the cost function of the industry

$$C_t(A, \mu, m) = A_t^{\frac{-1}{a+b}} \mu_t^{\frac{-b}{a+b}} \left[\left(\frac{a}{b} \right)^{\frac{b}{a+b}} + \left(\frac{a}{b} \right)^{\frac{-a}{a+b}} \right] [r^a \tau^b m]^{\frac{1}{a+b}}. \quad (1.28)$$

Finally denote $\psi = \left(\frac{a}{b} \right)^{\frac{b}{a+b}} + \left(\frac{a}{b} \right)^{\frac{-a}{a+b}}$ to obtain 1.5.

1.5.3 Proof of Proposition 1

The Hamiltonian is

$$\mathcal{H} = e^{-rt} \{R(\dot{\mu}\kappa) - C(A, \dot{\mu}\kappa, \mu, t)\} - \lambda\dot{\mu}, \quad (1.29)$$

where λ_t is the co-state variable. The first order condition with respect to $\dot{\mu}_t$ is

$$\frac{\partial \mathcal{H}}{\partial \dot{\mu}_t} = e^{-rt} \left\{ R' \kappa - \frac{\partial C}{\partial m} \kappa \right\} - \lambda = 0. \quad (1.30)$$

The derivative of revenue equals the price of base metals $p(m)$, thus we have

$$R' = p(m) = \frac{\partial C}{\partial m} + \frac{\lambda e^{rt}}{\kappa}. \quad (1.31)$$

The price change over time is the derivative of p with respect to time (we ignore superscript s)

$$\dot{p} = \frac{\partial^2 C}{\partial m \partial t} + \frac{\dot{\lambda} e^{rt} + \lambda r e^{rt}}{\kappa} - \frac{\lambda e^{rt} \kappa' \dot{\mu}}{\kappa^2}. \quad (1.32)$$

The derivative of λ can be obtained from the first order condition of the Hamiltonian with respect to μ_t

$$\frac{\partial \mathcal{H}}{\partial \mu_t} = \dot{\lambda} = e^{-rt} \left\{ R' \dot{\mu}_t \kappa' - \frac{\partial C}{\partial m} \dot{\mu}_t \kappa' - \frac{\partial C}{\partial \mu} \right\}. \quad (1.33)$$

Substituting 1.33 into 1.32 and rearranging the RHS yields

$$\dot{p} = \frac{\partial^2 C}{\partial m \partial t} + \frac{1}{\kappa} \left(\lambda r e^{rt} - \frac{\partial C}{\partial \mu} \right). \quad (1.34)$$

Consider the cost function in equation 1.5, in case of non-CRS, or $a + b \neq 1$. Taking the derivative of $C(m)$ with respect to m and substituting into 1.31 yields

$$p = \frac{\psi}{a+b} (r^a \tau^b)^{\frac{1}{a+b}} A^{\frac{-1}{a+b}} \mu^{\frac{-b}{a+b}} m^{\frac{1-a-b}{a+b}} + \frac{\lambda e^{rt}}{\kappa}. \quad (1.35)$$

Assuming a constant returns to scale production function ($a + b = 1$), we have

$$C(A, \mu, m) = \psi r^a \tau^b A^{-1} \mu^{-b} m. \quad (1.36)$$

Thus we obtain

$$\frac{\partial C}{\partial m} = \frac{\psi r^a \tau^b}{A \mu^b}, \quad (1.37)$$

$$\frac{\partial^2 C}{\partial m \partial t} = -\psi r^a \tau^b A^{-1} \mu^{-b} \left(\frac{\dot{A}}{A} + \frac{\dot{\mu}}{\mu} \right), \quad (1.38)$$

$$\frac{\partial C}{\partial \mu} = -b \psi r^a \tau^b A^{-1} \mu^{-1-b} m. \quad (1.39)$$

Substituting 1.38 and 1.39 into 1.34 yields

$$\dot{p} = -\frac{\psi r^a \tau^b}{A \mu^b} \left\{ \frac{\dot{A}}{A} + \frac{\dot{\mu}}{\mu} - \frac{bm}{\mu \kappa} \right\} + \frac{\lambda r e^{rt}}{\kappa}. \quad (1.40)$$

Substituting 1.37 into 1.31 yields

$$p = \frac{\psi r^a \tau^b}{A \mu^b} + \frac{\lambda e^{rt}}{\kappa}. \quad (1.41)$$

1.5.4 Proof of Corollary 1

Taking the logarithm of both sides of 1.35 yields

$$\ln(p) = \ln\left(\frac{\psi}{a+b}(r^a \tau^b)^{\frac{1}{a+b}}\right) - \frac{1}{a+b} \ln(A) - \frac{b}{a+b} \ln(\mu) + \frac{1-a-b}{a+b} \ln(q) + \ln\left(\frac{\lambda e^{rt}}{\kappa}\right). \quad (1.42)$$

Differentiating both sides of 1.42 leads to

$$\frac{p'}{p} = -\frac{1}{a+b} \frac{A'}{A} - \frac{b}{a+b} \frac{\mu'}{\mu} + \frac{1-a-b}{a+b} \frac{q'}{q}. \quad (1.43)$$

Dividing both sides of 1.43 by $\frac{p'}{p}$ we obtain

$$1 = -\frac{1}{a+b} \frac{A'}{A} \frac{p}{p'} - \frac{b}{a+b} \frac{\mu'}{\mu} \frac{p}{p'} + \frac{1-a-b}{a+b} \frac{q'}{q} \frac{p}{p'}. \quad (1.44)$$

Note that $\frac{A'}{A} \frac{p}{p'} = \varepsilon_{Ap}$, $\frac{\mu'}{\mu} = \varepsilon_{\mu p}$. Rearranging 1.44 yields 1.13

$$\varepsilon_s = 1 + \frac{\varepsilon_{Ap}}{1-a-b} + \frac{\varepsilon_{\mu p}}{1-a-b}.$$

1.5.5 Proof of Lemma 2

The price of additional capital unit equals the discount rate

$$r = \frac{\partial y}{\partial k} = \alpha A k^{\alpha-1} m^\gamma. \quad (1.45)$$

The price of additional base metal unit equals the marginal product of base metals

$$p = \frac{\partial y}{\partial m} = \gamma A k^\alpha m^{\gamma-1}. \quad (1.46)$$

Taking the derivative of p with respect to time t yields

$$\dot{p} = \gamma \dot{A} k^\alpha m^{\gamma-1} + \alpha \gamma A k^{\alpha-1} \dot{k} m^{\gamma-1} + (\gamma - 1) \gamma A k^\alpha m^{\gamma-2} \dot{m}. \quad (1.47)$$

Dividing both sides of 1.47 by 1.46 we obtain 1.17.

$$\frac{\dot{p}^d}{p^d} = \frac{\dot{A}}{A} + (\gamma - 1) \frac{\dot{m}}{m} + \alpha \frac{\dot{k}}{k}.$$

Solving 1.15 for k yields

$$k = A^{\frac{1}{\alpha}} m^{-\frac{\gamma}{\alpha}} y^{\frac{1}{\alpha}}. \quad (1.48)$$

The representative firm of the manufacturing sector minimizes production cost

$$\min_{k,m} r k + p m. \quad (1.49)$$

s.t 1.48.

Substituting 1.48 into 1.49

$$\min_{k,m} r A^{\frac{1}{\alpha}} m^{-\frac{\gamma}{\alpha}} y^{\frac{1}{\alpha}} + p m.$$

The first order condition with respect to m is

$$-\frac{\gamma}{\alpha}rA^{-\frac{1}{\alpha}}m^{-\frac{\alpha+\gamma}{\alpha}}y^{\frac{1}{\alpha}} + p = 0.$$

Thus, the conditional demand function for base metals is

$$m = A^{-\frac{1}{\alpha+\gamma}}\left(\frac{\alpha p}{\gamma r}\right)^{-\frac{\alpha}{\alpha+\gamma}}y^{\frac{1}{\alpha+\gamma}}.$$

Rearranging the above equation to obtain the inverse demand function [1.16](#)

$$p = A^{-\frac{1}{\alpha}}\left(\frac{\gamma r}{\alpha}\right)^{\frac{1}{\alpha}}y^{\frac{1}{\alpha}}m^{-\frac{\alpha+\gamma}{\alpha}}.$$

1.5.6 Proof of Corollary 2

To derive the price elasticity of demand, taking the logarithm of both sides of 1.16 yields

$$\ln p = -\frac{1}{\alpha} \ln A + \ln \left(\frac{\gamma r}{\alpha} \right) + \frac{1}{\alpha} \ln y - \frac{\alpha + \gamma}{\alpha} \ln m.$$

Differentiating both sides leads to

$$\frac{p'}{p} = -\frac{1}{\alpha} \frac{A'}{A} + \frac{1}{\alpha} \frac{y'}{y} - \frac{\alpha + \gamma}{\alpha} \frac{m'}{m}.$$

Dividing by $\frac{p'}{p}$ we get

$$1 = -\frac{1}{\alpha} \frac{A'}{A} \frac{p}{p'} + \frac{1}{\alpha} \frac{y'}{y} \frac{p}{p'} - \frac{\alpha + \gamma}{\alpha} \frac{m'}{m} \frac{p}{p'}.$$

Note that $\frac{A'}{A} \frac{p}{p'} = \varepsilon_{Ap}$, $\frac{y'}{y} \frac{p}{p'} = \varepsilon_{yp}$, and $\frac{m'}{m} \frac{p}{p'} = \varepsilon_d$. Rearranging the above equation we have 1.18

$$\varepsilon_d = \frac{\varepsilon_{yp} - \varepsilon_{Ap} - \alpha}{\alpha + \gamma}.$$

1.5.7 Ore Degradation and Technological Progress

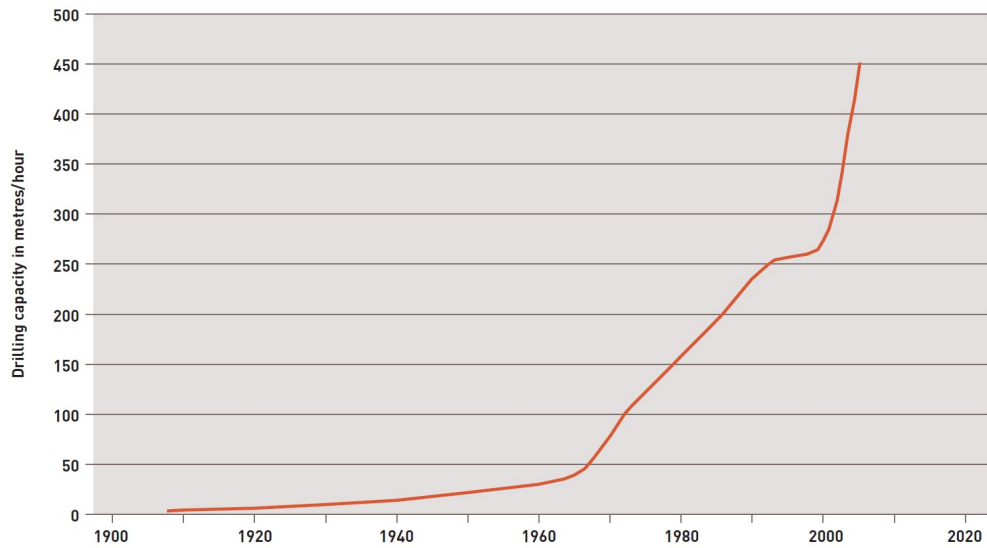


FIGURE 1.5: Mining technological progress over the last century.

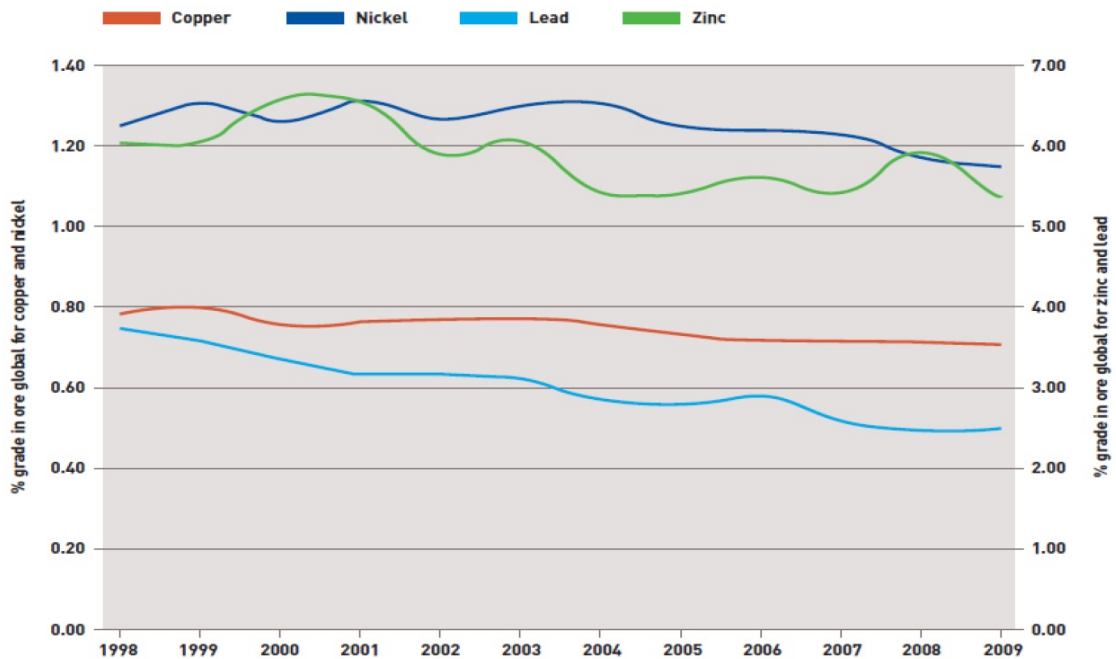


FIGURE 1.6: Ore degradation over the last decade.

Acknowledgments

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Chapter 2

Fluctuations in Base Metals Prices

Abstract

This chapter studies the fluctuations of base metals prices, in which we develop a theoretical model of short run equilibrium based on trading activities of agents in the base metals international exchanges. The results show that higher quality and more plentiful base metals in natural reserves keep the price more stable. The chapter addresses a critical value involved in choosing the fraction of material inputs from risky mining extraction and from risk-free recycling. The critical value of this fraction depends on the conditions of the market, as well as the availability and distribution of base metals in nature. The chapter shows that the scale and the structure of international exchanges play a crucial role in base metals prices stabilization. If more producers participate in the international exchange, or producers are less risk averse, the prices of base metals fluctuate more. If there are more speculative activities or traders are less risk averse, the prices of base metals fluctuate less. While there is a big question about where the prices are heading the next period, the findings in this study may shed light on price stability and movement during the recent financial crisis and possible future conditions of the economy.

2.1 Introduction

In the long run, the price of base metals depends on the total factor productivity of the mining industry, availability of metals in nature, natural resource tax, the interest rate, and demand from the manufacturing sector of the economy. These variables remain unchanged over a relatively long period of time. However, we observe the prices deviate from the trend line. Fluctuations in the base metals prices are unexpected changes that can affect the producer's profit margin and make it difficult to budget costs in manufacturing sectors. The equilibrium of aggregate supply and demand from the previous chapter cannot explain these fluctuations because the production and consumption of base metals interact for a long period of time.

Dennis Gartman of CNBC says that, prior to the 2007-2009 financial crisis, "many base metals prices moved downwards long before the data signaled weakness in the global economy". At the same time, we observe that during the crisis, the London Metal Exchange index (LMEX) exhibited strong fluctuations. While trends in base metals prices signal prosperity and economic growth, the fluctuations of base metals prices should also be of significant economic importance.

The concentrations of base metal elements in mineral deposits are determined by geological processes over millions of years. If these concentrations are deemed to be economically viable then they are known as ore bodies. According to the Geological Survey of Canada,¹ as base metal deposits have been formed by magmatic and hydrothermal processes, the contents of base metals in nature

¹Sangster D.F. (2012), *Geology of Base Metals Deposits - Volume IV*

vary widely and differentiate in parts of the Earth's crust as per Table 2.1 below.

Region	Continental Crust		Oceanic Crust	
Elements	Range	Average	Range	Average
Copper	10 - 90	24	50 - 100	90
Lead	0.5 - 24	17	1 - 10	3
Zinc	50 - 100	57	80 - 120	100
<i>Unit : parts per million (ppm).</i>				

TABLE 2.1: Base metals abundances in nature.

Hence, in a project of base metals extraction, there is always some uncertainty in the ore reserves estimation phase. Ore reserves are crucial for mining company assets and their reliable estimation is an issue during both feasibility studies and daily operations. The reserve estimate is determined by the application of certain technical and economic parameters. According to Dominy (2002), engineering aspects of reserve estimation can be accurately determined to 10%. However the majority of project risk will revolve around the natural resource. In practice, the available information and sample density are not always sufficient to allow a reliable estimate of the size, tonnage and grade of the mineralization. Even though technical standards are required in the sampling procedure, the outcomes are respectively equivalent to confidence intervals. Such geology and engineering aspects generate uncertainty in every stages of a mining and metallurgy project (see Figure 2.1 below² for the phases of a mining project).

Empirically, the portion of recycled materials used as input for base metal production has been constant over the last decade. For example, according to the

²Source : BC Ministry of Energy and Mines

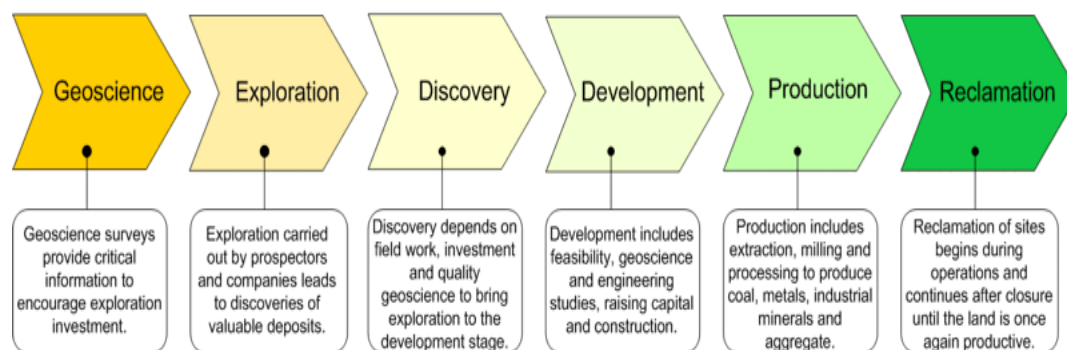


FIGURE 2.1: Phases of a mining project.

International Copper Study Group (ICSG), recycled content in copper production has remained steadily in the 33.7% - 36.8% range over the last few years. The International Zinc Association (IZA) states that 60% of zinc production comes from mined ores and the rest from secondary materials. The Yale University Center of Ecology reports that the recycling input rate of nickel production is 33%. The International Lead Association (ILA) says that the recycled content of production is 52.6%. In general, those figures have remained stable over the last ten years.³ In contrast to the stable productivity of the recycling input, a mining project with the phases showed in Figure 2.1 may generate uncertain productivity in base metals production. The state of nature, the random distribution of the useful elements in the Earth's crust, and the riskiness in exploration and exploitation processes are the root causes of this uncertainty.

The industrial non-ferrous metals such as aluminum, copper, lead, nickel, tin,

³Sources: Metal Bulletin 30 January 2012, Report of the United Nations Environment Program 2011.

and zinc are commonly traded in the international market via standard transactions. A spot contract in a base metal exchange is a contract of buying or selling base metals for settlement payment and delivery on the spot date, which is normally two business days after the trade date. Futures and options are two financial instruments commonly used to hedge against commodity price risk. There are three major centers for trading base metals: the Commodity Exchange of New York (COMEX), the London Metal Exchange (LME), and the Shanghai Futures Exchange (SHFE). More than 80% of global non-ferrous futures business is transacted on LME platforms. In 2012, shareholders of the London Metal Exchange voted overwhelmingly to sell this historic exchange to Hong Kong Exchanges Clearing for £1.4 billion.

In this chapter, we study fluctuations of base metals prices by the hedging activities in the market, where producers and traders are selling and buying in well-organized commodity exchanges. The previous chapter, “Trends in base metals prices” discovered the systematic price determinants. This chapter addresses the determinants of price fluctuations as non-systematic price components, which make the long-run trend line difficult to predict. We answer the question: What drives fluctuations in base metals prices? We link price fluctuations to agents’ risk preferences in exchanges and to trading activity. Prior to trade, producers have to decide on the structure of input materials supplying to metallurgy process. Recycling constitutes a less risky supply source for producers than mining. Producers have to choose the fraction of risky – but potentially more profitable – mining. We introduce the critical fraction of uncertain production, above which the market properly responds to an increase in risk aversion. We provide an explanation of higher fluctuations in the prices

of base metals during the crisis of 2007-2009. There are two effects which contribute to this increase in fluctuations. Firstly, during a crisis the traders become more risk averse. Secondly, there are more producers who want to get rid of stocks during a crisis. Both effects contribute positively to the increase in price fluctuations.

Literature Review

There is an empirical literature on the fluctuations of metals prices. Chen (2010) presents an empirical study of the fluctuations of 21 metals prices. He found that if fluctuation is commodity-specific rather than “global” then metals-exporting countries can smooth income via diversification. Batten et al. (2010) study the monthly price fluctuations of precious metals and the macroeconomic determinants such as the business cycle, the monetary environment, and the financial market sentiment of this volatility. Our study fills the gap in this literature by offering a theoretical treatment of pricing, which includes trade in metal exchanges.

Studies of crude oil and precious metals pricing are closely related to base metals pricing. Soytaş et al. (2009) examines co-movements and information transmission among the spot prices of precious metals, oil, and the US dollar/euro exchange rate. They find evidence of a weak relationship in the long run equilibrium but strong feedback in the short run. The spot precious metal markets respond significantly (but temporarily) to a shock in any of the prices of the other metals prices and the exchange rate. Investors may diversify away at least a portion of the risk by investing in precious metals, oil, and the euro.

Slade (1991) tests the hypothesis that prices of metals are more stable in concentrated markets and investigates whether markets in which buyers are consumers have more stable prices than those with suppliers and speculators. She offers an explanation of the increase in metal price instability through changes in market structure and organizational variables. The main reason for this is the increased reliance on commodity exchanges, the declines in concentration are of less importance.

Newbery (1987) considers a theoretical model of risky choice by farmers. Risky production increases price risk. Thus speculators will tend to increase price instability. Using the same analysis on the market as in Newbery (1987), Fung et al. (1991) study impacts of forward markets on international trade, using the optimal control approach to expose the behavior of producers. We develop the model of trade based on the framework of Newbery (1987) and Newbery and Stiglitz (1982) in the farmer's choice problem, to simulate the trade equilibrium. The model helps us to explain the fluctuations in the prices of base metals. Furthermore, we introduce the critical feature of choosing the share of risky mining in smelting, with the complement taken up by risk-free recycling. The empirical numerical examples illustrate our findings.

The rest of this chapter is organized as follows: Section 2 introduces assumptions, notation, and the theoretical model. Section 3 analyzes the trade equilibrium. Section 4 summarizes the results from this chapter. In section 5 we present mathematical calculation and proofs of propositions.

2.2 The Model

In Figure 2.2 below, we describe the processes of base metals production - trading - recycling in the economy, where the inputs for base metals production come from ore extraction and recycled materials.

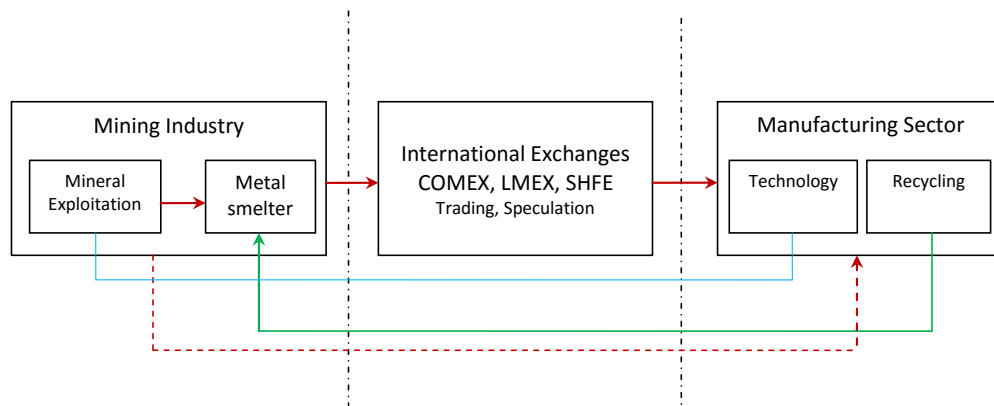


FIGURE 2.2: Base metals production and trading in the economy.

There are three sectors in the economy. The mining industry produces base metals from extracted minerals and recycled materials. The manufacturing sector consumes base metals as an input factor. The trading sector represents the transaction platforms where base metals producers, buyers, and traders meet and trade base metals commodities. Producers are the smelters who supply base metals. Buyers are enterprises, coming from the manufacturing sector who consume base metals for their inputs. Traders are the commercial agents who gain profit from speculation – buy and sell transactions. In the international exchanges, producers and traders sell and buy futures at price p^f while buyers

purchase from both other types of agents under spot contracts at price p . Note that prices of spot contracts in the future are unknown.

Base metals are produced from the smelting of ore extracted from mines and recycled materials from the economy. The producers in the mining industry choose a fraction i of extracted ore – the rest $(1 - i)$ comes from recycled materials. Recycled materials are assumed to generate a certain productivity, which is normalized to 1. Mining projects of scale i^* generate uncertain productivity $1 + \tilde{q}$ per unit of extracted ore. The expected value of \tilde{q} is $R > 0$. Producers face variation in the content of valuable elements found in nature. As illustrated in Table 2.1, the content of base metals in ore varies across mineral deposits, and even from time to time within one specific ore body.⁴ Suppose that the drilling exploration, sampling, and sample analysis reveal that \tilde{q} is normally distributed with expected value $E[1 + \tilde{q}] = 1 + R$ and variance σ^2 . We assume the price p of spot contracts, is a realization of a random variable $\tilde{p} = p(\tilde{q})$.

Suppose that in the international exchanges there are I identical producers and J identical traders. The quantities of futures that producers and traders are selling are f_m and f_s respectively. We use the subscripts m, s , which stand for mining and speculating, to indicate the *producer* and *trader*, respectively. We avoid using subscripts p, t , which can be confused with *price* and *time*. If the

⁴In the feasibility study phase of a mining and metallurgy project, drilling exploration is executed in the mineral deposit to evaluate the metal content and reserve size. The coefficient of variation C_v and standard deviation σ have been used (we observed this fifty years ago in the former Soviet Union) to evaluate the degree of risk, which directly affects the profit of the base metals producer. $\sigma = \sqrt{\frac{\sum (c_i - \bar{c})^2}{n-1}}$ and $C_v = \frac{\sigma}{\bar{c}}$, where c_i is the content of the metal in sample i , n is the number of samples, and \bar{c} is the mean value of c_i .

traders buy futures then f_s takes a negative value. The utility function of producers and traders is given by $U(w) = -e^{-\theta w}$, where $w \in \{w_m, w_s\}$ is income and $\theta \in \{\theta_m, \theta_s\}$ is the coefficient of absolute risk aversion of the producer and trader respectively.

2.3 Trade Equilibrium

Producers choose the share of extracted ore i before realization of the random variable \tilde{q} . The total productivity of the base metal industry for $i = i^*$ is

$$\tilde{\pi} = (1 - i^*) + i^*(1 + \tilde{q}) = 1 + i^*\tilde{q}. \quad (2.1)$$

Consider price elasticity $\varepsilon = |\varepsilon_d|$ at the point $(\pi = 1, p = 1)$. If the price increases by ∂p then revenue increases by $\frac{1}{\varepsilon}$ and decreases by $\frac{\pi}{\varepsilon}$, so that the price in the short run is⁵

$$\tilde{p} \approx 1 + \frac{1}{\varepsilon} - \frac{\tilde{\pi}}{\varepsilon}. \quad (2.2)$$

Therefore the price \tilde{p} is normally distributed. Substituting 2.1 into 2.2 yields the price as a function of the optimal investment level i^*

$$\tilde{p} = 1 - \frac{i^*\tilde{q}}{\varepsilon}. \quad (2.3)$$

The variance of the price is

$$\sigma_p^2 = \left(\frac{i^*}{\varepsilon}\right)^2 \sigma^2. \quad (2.4)$$

⁵This expression is similar to (A13) in Newbery (1987). However, because we are interested in short run price fluctuations, we do not postulate a linear demand function. Alternatively, if we are interested in a longer period when p is realized we can assume linear demand.

Importantly, the fluctuation in prices depends on the optimal fraction of uncertain production. This is the instrument, which will affect the prices. We obtain the following proposition.

Proposition 1 : *The price of base metals varies with the scale of uncertain production i^* , the elasticity of demand ε , and the distribution of ore content in nature σ^2*

$$\frac{\partial \sigma_p^2}{\partial i^*} > 0, \frac{\partial \sigma_p^2}{\partial \varepsilon} < 0, \frac{\partial \sigma_p^2}{\partial \sigma^2} > 0. \quad (2.5)$$

Proof: Follows directly from 2.4.

We define the generalized risk aversion coefficient η that combines the size and risk preferences of the agents in the market.

Definition 1 : *The generalized risk aversion coefficient of the futures market is a combination of the number of agents in the market and their risk aversion coefficients.*

$$\eta = \frac{J\theta_m}{J\theta_m + I\theta_s}.$$

Notice that $\eta \in [0, 1]$. If traders are infinitely risk averse, $\theta_s = \infty$, then the coefficient $\eta = 0$. If traders are risk neutral, $\theta_s = 0$, then the coefficient $\eta = 1$.

Other properties of η are

$$\frac{\partial \eta}{\partial J} > 0, \frac{\partial \eta}{\partial I} < 0, \frac{\partial \eta}{\partial \theta_s} < 0, \frac{\partial \eta}{\partial \theta_m} > 0. \quad (2.6)$$

If producers sell f_m tonnes of base metal in the futures market at price p^f to

traders and the remaining $\frac{1}{I}(m - f_m)$ tonnes will be sold at price p , then a representative producer's expected profit is

$$w_m = \frac{1}{I} \left(p^f f_m + \tilde{p}(1 + i\tilde{q} - f_m) \right). \quad (2.7)$$

A representative trader earns

$$w_s = \frac{f_s}{J} (p^f - \tilde{p}). \quad (2.8)$$

The fraction of extraction minerals i^* for input materials is chosen by producers. This generates a fraction of output which is uncertain and normally distributed, so that the expected income of the metal producers is also normally distributed. With a normal distribution of income and CARA utility function, a producer's expected utility maximization is equivalent to solving the maximization problem

$$\max_{i, f_m} \left(E(w_m) - \frac{\theta_m}{2} \sigma_m^2 \right).$$

Similarly, the representative trader's expected utility maximization problem is

$$\max_{f_s} \left(E(w_s) - \frac{\theta_s}{2} \sigma_m^2 \right).$$

Proof: See Appendix 2.5.1.

Solving these agents' problems yields the optimal choices of the representative producer and trader on the uncertain production and futures quantities

Lemma 1: *Under the conditions of the market on expected price, sales revenue, and the quantity offered by agents, the optimal choices of producer and trader on i^* , f_m , f_s are*

the following :

$$i = \frac{I}{\theta_m \sigma_{pq}^2} E(pq) - \frac{(1 - f_m) \text{Cov}(p, pq)}{\sigma_{pq}^2}, \quad (2.9)$$

$$f_m = 1 + \frac{\text{Cov}(p, pq)}{\sigma_p^2} i + \frac{I}{\theta_m \sigma_p^2} (p^f - E(p)), \quad (2.10)$$

$$f_s = \frac{J p^f - E(p)}{\theta_s \sigma_p^2}. \quad (2.11)$$

Proof : See Appendix 2.5.1.

Equilibrium of the futures market in international exchanges requires that the total selling and buying quantity of futures equals zero $f_m + f_s = 0$. Therefore, from equations 2.10 and 2.11 we have

$$1 + \frac{\text{Cov}(p, pq)}{\sigma_p^2} i + \frac{I}{\theta_m \sigma_p^2} (p^f - E(p)) + \frac{J p^f - E(p)}{\theta_s \sigma_p^2} = 0. \quad (2.12)$$

Using 2.12, the condition for the optimal choice of uncertain production i (see Appendix 2.5.3) is

$$i(1 - \eta) = \frac{R\varepsilon^2 - \varepsilon i^*(R^2 + \sigma^2)}{\frac{\theta_m}{I}(\varepsilon - 2Ri^*)^2 \sigma^2} + \frac{(1 - \eta)i^*}{\varepsilon - 2Ri^*}. \quad (2.13)$$

We can see that there are constant terms on the right hand side, denote them as

$$F_1 = \frac{R\varepsilon^2 - \varepsilon i^*(R^2 + \sigma^2)}{\frac{\theta_m}{I}(\varepsilon - 2Ri^*)^2 \sigma^2}, \quad F_2 = \frac{(1 - \eta)i^*}{\varepsilon - 2Ri^*}.$$

Rewriting equation 2.13 we have

$$i = i(\eta) = \frac{F_1}{1 - \eta} + F_2. \quad (2.14)$$

Now we are going to define the critical level of input shares. Equation 2.14 may give us a direct intuition about the relationship between the uncertain production and the market risk aversion. When the market risk aversion increases, one can expect that the equilibrium choice of i should move towards safe technology. Accordingly, we should have $\frac{\partial i}{\partial \eta} < 0$.

Equilibrium implies $i = i^*$. Taking the derivative of i with respect to η yields

$$\frac{\partial i}{\partial \eta} = \frac{F_1}{(1 - \eta)^2}. \quad (2.15)$$

Because $(1 - \eta)^2 > 0$, thus $\frac{\partial i}{\partial \eta} < 0$ only if $F_1 < 0$, which is equivalent to $i^* > \frac{R\varepsilon}{R^2 + \sigma^2}$. We define this special value as the critical fraction of the uncertain production $i^\#$.

Definition 2 : *The critical level of the uncertain production is a combination of the demand elasticity, the availability of base metals and their distribution variance in nature*

$$i^\# = \frac{R\varepsilon}{R^2 + \sigma^2}.$$

In fact, the fraction of uncertain production is large enough in reality. A numerical example for nickel production can illustrate this. For nickel mines in Canada we have $\sigma \approx 0.3^6$. In the global market $\varepsilon \approx 0.05^7$, $R \approx 0.2^8$. Thus $i^\# \approx 0.011$, and then the inequality $i^* > i^\#$ is valid because in fact $i^* \approx 0.67$ (the data is from Center for Ecology, Yale University).

⁶Canada is the world's second largest nickel producer, after Russia. More than 30% of nickel in the world is produced here.

⁷Economic Research Center, the University of Western Australia.

⁸According to Victory Nickel Inc., one of Canada's largest undeveloped sulphide nickel inventories. <http://www.victorynickel.ca/>

Considering the speculative activities with comparative statics $\frac{\partial \eta}{\partial J} > 0$, $\frac{\partial \eta}{\partial \theta_s} < 0$ and using 2.5 we have

$$\frac{\partial \sigma_p^2}{\partial J} = \frac{\partial \sigma_p^2}{\partial i^*} \frac{\partial i^*}{\partial \eta} \frac{\partial \eta}{\partial J} < 0, \quad (2.16)$$

and

$$\frac{\partial \sigma_p^2}{\partial \theta_s} = \frac{\partial \sigma_p^2}{\partial i^*} \frac{\partial i^*}{\partial \eta} \frac{\partial \eta}{\partial \theta_s} > 0. \quad (2.17)$$

We derive the following proposition about the impacts of traders on the price:

Proposition 2 : *Suppose that $i^* > i^\#$, if there are more speculative activities (J increases) or they are less risk averse (θ_s decreases), then the price of base metals fluctuates less.*

Similarly, considering the supply side with comparative statics $\frac{\partial \eta}{\partial I} < 0$, $\frac{\partial \eta}{\partial \theta_m} > 0$, using 2.5 we have

$$\frac{\partial \sigma_p^2}{\partial I} = \frac{\partial \sigma_p^2}{\partial i^*} \frac{\partial i^*}{\partial \eta} \frac{\partial \eta}{\partial I} > 0 \quad (2.18)$$

and

$$\frac{\partial \sigma_p^2}{\partial \theta_m} = \frac{\partial \sigma_p^2}{\partial i^*} \frac{\partial i^*}{\partial \eta} \frac{\partial \eta}{\partial \theta_m} < 0. \quad (2.19)$$

This leads to the proposition about the impacts from supply side on the price:

Proposition 3 : *Suppose that $i^* > i^\#$, if there are more producers (I increases) or they are less risk averse (θ_m decreases), then the price of base metals fluctuates more.*

Figure 2.3⁹ presents the dynamics of base metals prices from 2000 to 2015. The main observation is that fluctuation in price increased dramatically before and

⁹Source: Data and graph were built at Bloomberg's terminal in Jan. 2015.

during the 2006-20011 crisis. In normal conditions of the economy, before 2006 and after 2011, prices fluctuate less.



FIGURE 2.3: Fluctuations of LME index.

These observations can be explained by two results of our model. Traders tend to be more risk averse (θ_s is high) during liquidity shocks, and more producers are interested in selling futures contracts (I is high). Thus, the price fluctuates more during crises. According to Vayanos (2004), liquidity premiums increase in volatile times. Traders become more risk averse because higher fundamental fluctuation increases the likelihood that their performance falls short of the threshold. This will lead to costly withdrawal of funds.

2.4 Conclusion

By simulating speculative activities under trade equilibrium and optimizing the utility of agents in international exchanges, we show that in the short run price fluctuations respond to risk preferences of agents and the scale of international exchanges. We introduce the critical point of production investment, which depends on market demand, profitability of the metal industry and the distribution of base metal minerals in nature. In the specific case of the industry versus the market condition $i > i^\#$, the price of base metals fluctuates more or less according to the number of producer offers in base metals exchanges, the speculative activities, and risk preferences of agents. If more producers participate in international exchange, or producers are less risk averse, the price of base metals fluctuates more. If there are more speculative activities or traders are less risk averse, the price of base metals fluctuates less. In contrast, if the investment level of the metal industry is below that critical point, the effects of base metal exchanges scaled to the price are in the reverse direction. The comparative statics inequalities are derived to clarify the responses of the price to the risk preferences of agents in the international exchanges. Hence, the non-systematic changes of base metals prices in international exchanges are explained.

We consider the availability of base metal minerals in nature both in the sense of the type of distribution and the size of the reserve in nature. Higher quality and more plentiful natural reserves keep the price more stable. Variations in base metals content in mineral deposits affect the price via the expected utilities of agents.

2.5 Appendix

2.5.1 Proof of Lemma 1

Equivalence of the utility maximization problems

Suppose rational expectation on future income. The expected utility of an agent in the next period is

$$E[U(w_1)] = E[U(w_0(1 + R))].$$

The utility maximization problem of an agent is

$$\max_{i, f_m} E[U(w_m)] = E[-e^{-\theta_m w_m}]. \quad (2.20)$$

Taylor approximation implies $f(x + a) \approx \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$. Hence, the second order approximation ($n = 2$) is

$$E[U(w_1)] \approx E[U(w_0) + U'(w_0)Rw_0 + \frac{1}{2}U''(w_0)R^2w_0^2]. \quad (2.21)$$

Because $\sigma_R^2 = E[R^2] - (E[R])^2$, we have $E[R^2] = (E[R])^2 + \sigma_R^2$. Substituting this into 2.21 yields

$$E[U(w_1)] = U(w_0) + U'(w_0)E[R]w_0 + \frac{1}{2}U''(w_0)w_0^2[(E[R])^2 + \sigma_R^2]. \quad (2.22)$$

Because $U(w_0), U'(w_0)w_0$ are known, we subtract $U(w_0)$ and divide the right hand side of 2.22 by $U'(w_0)w_0$. Then maximizing 2.22 is equivalent to maximizing

$$E[R] + \frac{1}{2} \frac{U''(w_0)w_0^2}{U'(w_0)} [(E[R])^2 + \sigma_R^2].$$

For a CARA utility function, $\theta = -\frac{U''(w_0)w_0}{U'(w_0)}$, and the term $(E[R])^2$ is relatively small. Hence, the maximization problem 2.20 becomes

$$\max_{i, f_m} E w_m - \frac{\theta_m}{2} \sigma_m^2.$$

This result is applied for both agents, producer and trader.

Solving the maximization problems of agents

The metal producer maximizes his expected utility $\max_{i, f_m} E(U(w_m)) = E(-e^{-\theta_m w_m})$.

Let income be normally distributed $w_m \sim N(E(w_m), \sigma_m^2)$ and assume a CARA utility function $U(w) = -e^{-\theta w}$. Applying the second order approximation to the utility function this is equivalent to

$$\max_{i, f_m} E(w_m) - \frac{\theta_m}{2} \sigma_m^2. \quad (2.23)$$

Similarly the maximization problem of the trader is

$$\max_{f_s} E(w_s) - \frac{\theta_s}{2} \sigma_t^2. \quad (2.24)$$

From 2.7 the variance of producer's income σ_m^2 is

$$\sigma_m^2 = Var(w_m) = \frac{1}{I^2} \left((1 - f_m)^2 \sigma_p^2 + i^2 \sigma_{pq}^2 + 2i(1 - f_m) Cov(p, pq) \right). \quad (2.25)$$

Substituting 2.25 and 2.7 into 2.23 the problem of a metal producer is

$$\max_{i, f_m} p^f f_m + (1 - f_m) E(p) + i E(pq) - \frac{\theta_m}{2I} \left((1 - f_m)^2 \sigma_p^2 + i^2 \sigma_{pq}^2 + 2i(1 - f_m) Cov(p, pq) \right). \quad (2.26)$$

The first order condition with respect to i is

$$E(pq) - \frac{\theta_m}{I} \sigma_{pq}^2 i - \frac{\theta_m}{I} (1 - f_m) \text{Cov}(p, pq) = 0.$$

The optimal choice for mining extraction is

$$i^* = \frac{I}{\theta_m \sigma_{pq}^2} E(pq) - \frac{(1 - f_m) \text{Cov}(p, pq)}{\sigma_{pq}^2}. \quad (2.27)$$

The first order condition with respect to f_m is

$$p^f - E(p) + \frac{\theta_m}{I} (1 - f_m) \sigma_p^2 + \frac{i \theta_m}{I} \text{Cov}(p, pq) = 0.$$

The optimal choice of futures to be sold by producer f_m is

$$f_m = 1 + \frac{i \text{Cov}(p, pq)}{\sigma_p^2} + \frac{I}{\theta_m \sigma_p^2} (p^f - E(p)). \quad (2.28)$$

From 2.8 the variance of a trader's income is σ_s^2 is

$$\sigma_s^2 = \left(\frac{f_s}{J} \right)^2 \sigma_p^2. \quad (2.29)$$

Substituting 2.8 and 2.29 into 2.24, the problem of a trader is

$$\max_{f_s} p^f f_s - f_s E(p) - \frac{\theta_s}{2J} f_s \sigma_p^2. \quad (2.30)$$

The first order condition with respect to f_s is

$$p^f - E(p) - \frac{\theta_s}{J} f_s \sigma_p^2 = 0.$$

The optimal choice of futures to be sold by the trader f_s is

$$f_s = \frac{J}{\theta_s} \frac{p^f - E(p)}{\sigma_p^2}. \quad (2.31)$$

2.5.2 Applying Taylor approximation for the variance of pq

By definition,

$$\text{var} f(p, q) = E \left((f(p, q) - E(f(p, q)))^2 \right). \quad (2.32)$$

Using the first order Taylor approximation expanded around R we have

$$E(f(p, q)) = E(pq) \approx R(1 - i^*R), \quad (2.33)$$

$$\begin{aligned} \text{Var}(f(p, q)) &\approx E \left(\left(f(R) + f'_m(R)(p - p_R) + f'_q(R)(q - q_R) - f(R) \right)^2 \right) \\ &= E \left(f_m'^2(R)(p - p_R)^2 + f_q'^2(R)(q - q_R)^2 + 2f'_m(R)(p - p_R)f'_q(R)(q - q_R) \right) \\ &= f_m'^2 \text{Var}(p) + f_q'^2 \text{Var}(q) + 2f'_m(R)f'_q(R)\text{Cov}(p, q). \end{aligned} \quad (2.34)$$

By the definition of covariance

$$\text{Cov}(p, q) = E(pq) - E(p)E(q) = R - \frac{i^*(R^2 + \sigma^2)}{\varepsilon} - \left(1 - \frac{i^*R}{\varepsilon}\right)R = \frac{-i^*\sigma^2}{\varepsilon}. \quad (2.35)$$

Substituting $f(p, q) = pq$, $\text{Var}(p) = \left(\frac{i^*}{\varepsilon}\right)^2\sigma^2$, $\text{Var}(q) = \sigma^2$, and 2.35 into 2.34 yields

$$\begin{aligned} \text{Var}(pq) &\approx \left(\frac{i^*}{\varepsilon}\right)^2 R^2\sigma^2 + \left(1 - \frac{i^*R}{\varepsilon}\right)^2 \sigma^2 - 2R \left(1 - \frac{i^*R}{\varepsilon}\right) \frac{i^*\sigma^2}{\varepsilon} \\ &= \left(\frac{i^*}{\varepsilon}\right)^2 R^2\sigma^2 + \sigma^2 - 2\frac{i^*R}{\varepsilon}\sigma^2 + \left(\frac{i^*}{\varepsilon}\right)^2 R^2\sigma^2 - 2\frac{i^*R}{\varepsilon}\sigma^2 + 2\left(\frac{i^*}{\varepsilon}\right)^2 R^2\sigma^2 \\ &= \sigma^2 - 4\frac{i^*R}{\varepsilon}\sigma^2 + 4\left(\frac{i^*}{\varepsilon}\right)^2 R^2\sigma^2 \\ &= \left(1 - 2\frac{i^*R}{\varepsilon}\right)^2 \sigma^2. \end{aligned} \quad (2.37)$$

Thus we have

$$\text{Var}(pq) \approx \left(1 - 2\frac{i^*R}{\varepsilon}\right)^2 \sigma^2 + 2\left(\frac{i^*}{\varepsilon}\right)^2 \sigma^4. \quad (2.38)$$

2.5.3 The optimal value of i

The equilibrium condition is

$$f_m + f_s = 1 + \frac{iCov(p, pq)}{\sigma_p^2} + \frac{I}{\theta_m \sigma_p^2} (p^f - E(p)) + \frac{J}{\theta_s} \frac{p^f - E(p)}{\sigma_p^2} = 0. \quad (2.39)$$

Rearranging this yields

$$\frac{p^f - E(p)}{\theta_m \sigma_p^2} = - \left(1 + \frac{iCov(p, pq)}{\sigma_p^2} \right) \frac{\theta_s}{J\theta_m + I\theta_s}. \quad (2.40)$$

Substituting 2.40 into 2.10 for f_m and collecting the variables I, J, θ_m, θ_s leads to

$$f_m = \left(1 + \frac{iCov(p, pq)}{\sigma_p^2} \right) \left(1 - \frac{I\theta_s}{J\theta_m + I\theta_s} \right). \quad (2.41)$$

Using the definition of the coefficient η we have

$$f_m = \eta \left(1 + \frac{iCov(p, pq)}{\sigma_p^2} \right). \quad (2.42)$$

Substituting 2.42 into 2.9 gives

$$i \left(1 - \eta \left(\frac{Cov(p, pq)}{\sigma_p \sigma_{pq}} \right)^2 \right) = \frac{\frac{I}{\theta_m} E(pq) - (1 - \eta) Cov(p, pq)}{\sigma_{pq}^2}. \quad (2.43)$$

The term $\frac{cov(p, pq)}{\sigma_p \sigma_{pq}} = \rho$ is the correlation coefficient of the price and the sales of base metal extraction (p, pq) . As p and pq are price and revenue, suppose $\rho \approx -1$, i.e. perfect negative correlation. We have

$$p = 1 - \frac{i^* q}{\varepsilon}. \quad (2.44)$$

The variance of price is

$$\sigma_p^2 = \left(\frac{i}{\varepsilon}\right)^2 \sigma^2. \quad (2.45)$$

The expected value of revenue by mined extraction $E(pq)$ is

$$E(pq) = E\left(q - \frac{iq^2}{\varepsilon}\right) = R - \frac{i}{\varepsilon}(R^2 + \sigma^2). \quad (2.46)$$

The variance of revenue by mined extraction is σ_{pq}^2 . Note that p and q are not independent, so to calculate $Var(pq)$ we apply the Taylor approximation (see 2.37 above) $\sigma_{pq}^2 = \left(1 - 2\frac{iR}{\varepsilon}\right)^2 \sigma^2$. The covariance of the price and mined extraction revenue is

$$Cov(p, pq) = -\frac{i^*}{\varepsilon} \left(1 - 2\frac{i^*R}{\varepsilon}\right) \sigma^2. \quad (2.47)$$

Substituting 2.37, 2.45, 2.46, 2.47 into 2.43 yields

$$i(1 - \eta) = \frac{R\varepsilon^2 - \varepsilon i^*(R^2 + \sigma^2)}{\frac{\theta_m}{I}(\varepsilon - 2Ri^*)^2 \sigma^2} + \frac{(1 - \eta)i^*}{\varepsilon - 2Ri^*}. \quad (2.48)$$

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Chapter 3

Base Metals Industry in Capital Markets

Abstract

This chapter studies the impact of industrial and commercial processes on investment decisions in the base metals industry. The investment decisions of investors in the primary capital market, as well as the stock price in the secondary capital market, reflect features of the base metals industry. We present a model of investment in the form of a two stage game that incorporates Hall-Jorgenson neoclassical investment analysis and properties of the base metals industry. This chapter presents a set of explanatory parameters for the properties of base metal stocks and analyzes investment decisions. We define the industry factor and explain the empirical observations on the beta coefficient of base metal stocks. The relationships between stock prices and base metal prices are clarified using the geological properties of base metal deposits. The results show that there is a strong impact of the industry factor on the volatility of base metal stock prices. Economies of scale in the mining industry lead to different effects of tax policy and output prices on investment decisions. We support conclusions of the model with evidence from the base metals industry.

3.1 Introduction

We begin by observing the S&P/TSX Global Base Metals Index, which indexes global securities involved in the production and extraction of base metals. The index is an investable portfolio that provides investors with a broadly representative benchmark for global mining portfolios and a basis for innovative, index-linked investment vehicles. Figure 3.1¹ shows that there are more impulsive waves in the Base Metals Index history than in the Composite Index. The Base Metals Index is more volatile, with a downward trend in the last 5 years, this reminds us that there may be a strong correlation with base metal prices. We also observe that the Base Metals Index responds more sensitively to the status of the economy. In addition, in every stock market the beta coefficients of base metal stocks are usually greater than 1 ($\beta > 1$), as shown in Table 3.1, section 3.4.2 later².

These differences between S&P/TSX Global Base Metals Index and S&P/TSX Composite Index patterns raise questions about specific features of the base metals industry in capital markets. We believe that as the prices of natural resources are changing, investments in base metal mining companies are adjusted, and therefore optimized accordingly. Other than the prices, the production and trading of base metal commodities as well as the expected return of investors in capital markets play roles in the variation of base metal stock prices.

¹Source: S&P Dow Jones Indices LLC, <http://ca.spindices.com/indices/>

²In finance, the beta of an investment indicates whether the investment is more or less volatile than the market. A beta greater than 1 indicates that the investment is more volatile than the market.

Using the features of the base metals industry, we present the explanatory variables of the properties of base metal stocks. Specifically, we are looking for an answer to the following questions: How are base metal stocks different from other stocks, and how are investment decisions made under the specific industrial and commercial conditions of the industry?

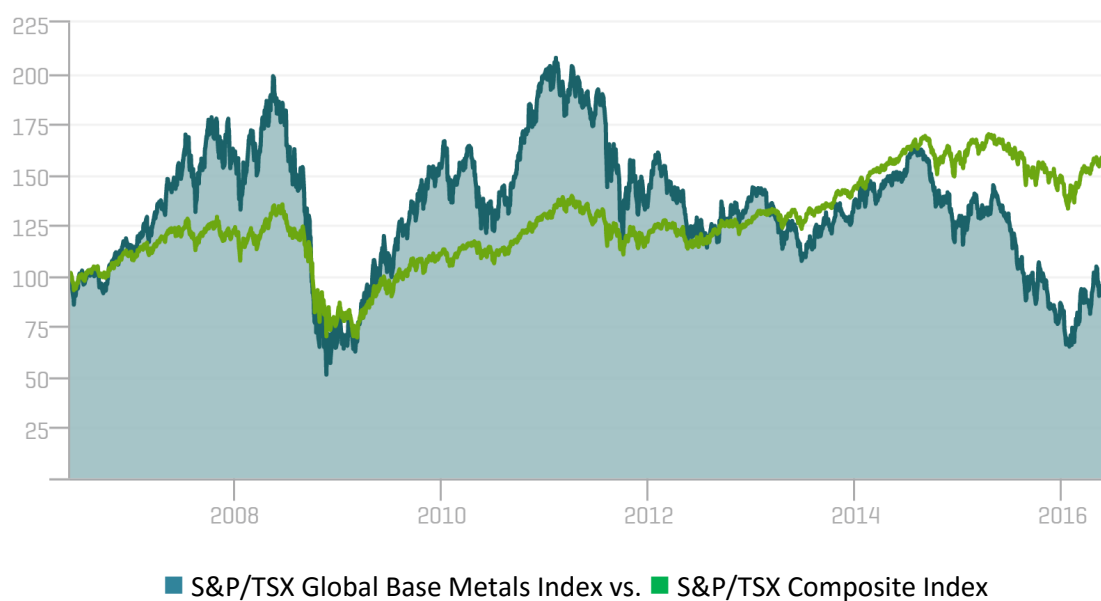


FIGURE 3.1: The Global Base Metals Index TXBM.

The theoretical and empirical literature present mixed results on how prices of commodities, such as oil and gold, affect stock prices of producers and the stock market in general. Instead of focusing on the direct relationship between base metals and stock prices, this study investigates the relations between different phases of the base metals firms' operations, including production, trading, and investment activities. This chapter identifies the contributions of production

uncertainty, commodity prices, the elasticity of demand, the expected return of investors, and the rate of capital depreciation on stock prices of firms whose primary business involves mining and trading the relevant commodities. Thus this study investigates whether the observed patterns of base metal stocks are explained by such fundamental factors. The aggregate indices of the sectors related to base metals and the capital market overall are also considered. Due to the importance that mining stocks have in its composition, the empirical evidence is taken from the Toronto Stock Exchange, which lists more mining companies than any other exchange. Our analysis confirms the impact of individual parameters on variations in base metal stocks.

In the primary market, investors set up new companies, expand production, and in the secondary market investors buy stocks. The properties of the base metals industry show up in both markets. Our results show that the proportion of uncertain production, the elasticity of base metals demand, the availability of base metals in nature, and the expected rate of return of investors in the industry all affect the pattern of stock prices and investment decisions. We introduce the Industry Factor, which represents the volatile character of the base metals industry's profitability. Using this factor we explain the empirical observation on the beta coefficient of base metal stocks. We analyze the investment decisions of firms and discover that economies of scale in the mining industry lead to different effects of tax policy and output prices on investment decisions. Finally, we study co-movement of the base metal prices and the respective stock prices using the elasticity of stock price with respect to the base metal price. The geology of base metal deposits in nature plays an important role in this relationship.

Literature Review

The literature on the commodity - stock market nexus dates back to the beginning of the 1980s and mostly focuses on gold and oil. The previous works concentrate in three groups: Empirical studies on gold mining firms, empirical studies on crude oil, and theoretical studies in general. A lack of theoretical and empirical studies on base metal stocks still persists in the literature.

Gold and silver mines are precious resources that draw attentions of investors and scholars. Tufano (1998) studies the exposure of North American gold mining firms to changes in the price of gold. He shows that the average mining stock has gold price elasticity equal to 2. However, the exposure varies considerably over time and across firms. Episcopos (1996) studies the sub-index portfolio, which includes stocks of gold and silver mines in the Toronto Stock Exchange. He found evidence that the beta coefficients of metals and mining groups are time varying. Time varying beta coefficients are computed as functions of market volatility. This indicates that the spread between beta coefficients of safe and risky sub-index portfolios may increase during periods of high aggregate volatility. Khoury (1984) argues that unstable dividends, political risks, currency exchange risks, and business risks disassociate the price of gold from the returns of the firm's securities. Rock (1988) suggests that investing in mining stocks is the worst way to expect for profits in gold due to the non-gold price related risk.

Regarding crude oil price changes, though they are considered an important factor for understanding fluctuations in stock prices, the direction of the relationship is unclear. Sadorsky (2001) finds a positive relationship between changes in oil prices and the Toronto Stock Exchange oil and gas index. He develops a multi-factor market model to estimate the expected returns to stock prices in the Canadian oil and gas industry. Results show that crude oil prices have a large and significant impact on stock price returns. An increase in the market or oil factor price increases the return to Canadian oil and gas stock prices. Boyer and Filion (2007) find a similar relationship using a sample of oil and gas companies. However, some recent studies indicate that the debate on the oil and stock market linkage is far from settled. Gorton and Rouwenhorst (2006) show that equities of commodity based companies cannot serve as substitutes for commodity futures as they have a much higher correlation with the stock market than do commodity futures. Kilian and Park (2009) conduct a study on oil price shocks and their impacts of on the US stock market. They find that the effect depends on the forces driving the change in the price of oil, which may be demand or supply shocks in the oil market. Narayan and Sharma (2011) study empirically the relationship between oil prices and stock returns for hundreds of firms in the US. They show that oil price appreciation has a positive effect on the returns of firms operating in the energy and transportation sectors.

There are few existing studies that investigate bull and bear markets in natural resource commodity prices. Lee et al (2006) find that natural resources prices are stationary around deterministic trends with structural breaks in the intercepts and trend slopes. This result is compatible with the notion of bull and bear markets. Roberts (2009) examines fluctuations in real metals prices – to

determine if they exhibit cyclical behavior – and the corresponding durations of such cycles. Ntantamis and Zhou (2014) present an empirical analysis of the Canadian stock market. The results suggest that there is little evidence that the market phases identified for the individual stocks are related to phases in the commodity prices.

In terms of theoretical work, Geman (2009) stresses the importance of natural resources availability to the market price of a company's stock. The example of Royal Dutch Shell's stock price in 2004 – after the announcement that it had overestimated its reserves – shows that the revelation of fewer reserves than expected caused higher stock prices with more fluctuations (see section 2.3 - Scarcity, Reserves and Price Volatility). Baker (2002) uses a simple model to outline the conditions under which corporate investments are sensitive to non-fundamental movements in stock prices. The key prediction is that stock prices have a stronger impact on the investment of "equity-dependent" firms – firms that need external equity to finance marginal investments. Blose and Shieh (1995) present a model that describes the influence of gold prices on the value of gold mining stocks, and test the model empirically. They find that for companies whose primary business is gold mining, the gold price elasticity of the company's stock is greater than one.

The rest of this chapter is organized as follows: Section 2 introduces assumptions, notation, and the theoretical model. Section 3 analyzes the parameters to determine their impacts on base metal stock prices and the investment decisions. Section 4 summarizes the results from this study. In section 5 we present mathematical proofs.

3.2 The Model

Assumption and Notation

To describe the production decision we assume that there is a threshold within the range of technologies being used, so that for capacity investment below that threshold the productivity is standard, and above that threshold the productivity is more risky and possibly more profitable. Recycled materials are assumed to generate a certain productivity, which is normalized to 1. Mining projects of scale i^* generate uncertain productivity $1 + \tilde{q}$ per unit of extracted ore. The expected value of \tilde{q} is $R > 0$. The price elasticity of base metals demand in the market is ε .

Compared to precious metals and rare earth elements, base metals are distributed in nature widely and have relatively low finding costs. Input factors for base metals production are capital K_t and ore deposits D_t . Technologies and metal content in ore deposits are imperfect substitutes. The regulator levies natural resource taxes on deposits D , while the real quality of ore μ is unknown to the regulator. The production function of the industry has the Cobb-Douglas form $m_t = A_t K_t^a (\mu_t D_t)^b$. The firm's short-run profit is $\Pi_t = \Pi_t(p_t, \tau_t, A_t, K_t)$. The interest rate is r , the discount factor of investors is $\rho = 1/r$. This discount factor reflects the local economic conditions for the firm.

In the base metals industry, there are professional investors with an expected rate of return g_e and risk preference, which are different from those of the common market. Thus the expected rate of return on the investment to the firm is $g_e \geq r$. The firm is currently using capital K_t , thus K_t is given at the time of the investment decision and K_{t+1}^* is the capital stock in the next period. The

marginal profit p_t^i with respect to additional investment I_t is the profit of the shareholders and the price of additional investment. Assuming *ceteris paribus*, the variation of the stock price is perfectly correlated with the profit of the firm. In other words, using the formula for pricing a stock price by fundamental analysis, the stock price³ is $p_{st} = \frac{p_{t+1}^i}{g_e}$.

The depreciation rate of capital for the industry is δ . The capital dynamics equation of the firm is

$$K_{t+1} = (1 - \delta)K_t + I_t. \quad (3.1)$$

The model of investment in base metals producers uses the framework of Hall-Jorgenson neoclassical investment analysis applied to the typical properties of the base metals industry. This allows us to go further than the classical analysis and develop the variation aspects of stock market prices.

The Model

The mining firm maximizes the present value of future cash flows of the mining and metallurgy projects.

$$PV(K_0) = E_0 \int_{t=0}^{\infty} \rho^t [\Pi_t - p_t^i I_t] dt \quad (3.2)$$

s.t. 3.1.

The investment decision can be considered as a static, two-stage game. Suppose that there are some opportunities for the expansion of production: the firm may have a new project, or a merger & acquisition deal. In the first period, the firm

³Suppose that an asset can be sold at the price p^i in period $t + 1$, the expected rate of return is g_e , then price of the asset at time t is the present value $p_{st} = \frac{p_{t+1}^i}{g_e}$.

optimizes profit Π_t with respect to ore deposits D_t .

$$\Pi_t = p_t A_t K_t^a (\mu_t D_t^*)^b - \tau_t D_t^* \quad (3.3)$$

where D^* is the optimal use of the deposit.

Lemma 1

a) The optimal use of deposit D^* is a function of metal prices, the resource tax, total factor productivity (TFP), and capital

$$D_t^*(p_t, A_t, K_t) = \left[\frac{b p_t A_t K_t^a \mu_t^b}{\tau_t} \right]^{\frac{1}{1-b}}. \quad (3.4)$$

b) Profit of the firm is

$$\Pi_t = \frac{1-b}{b} \left[\frac{b p_t A_t K_t^a \mu_t^b}{\tau_t} \right]^{\frac{1}{1-b}}. \quad (3.5)$$

Proof : Equation 3.4 is obtained directly from solving the first order condition of the profit maximization problem. Substituting 3.4 into 3.3 yields 3.5.

The objective of a listed firm is to maximize the value of the firm. Note that p_t^i is the price of additional investment. In the second period, the firm is choosing the sequence of investments $\{I_t\}_0^\infty$. Problem 3.2 can be written recursively as

$$\max_I PV(K_t) = \Pi(p_t, \tau_t, A_t, K_t) - p_t^i I_t + \rho E_t PV(K_{t+1}). \quad (3.6)$$

s.t. 3.1.

Lemma 2

In equilibrium the profit on investment and the desired stock of capital K_{t+1}^* satisfy

$$p_t^i = a\rho E_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*(a+b-1)} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} + \rho(1-\delta) E_t p_{t+1}^i. \quad (3.7)$$

Proof : See Appendix 3.5.1

Assuming that investors have full and transparent information about the firm (the listed company has to reveal all information about TFP A_{t+1} , assessment of the quality of the ore deposit μ_{t+1} , government policy τ_{t+1} , and the scale of the investment K_{t+1}), equation 3.7 implies: (i) the condition for optimal level of capital in the next period. (ii) Given information of the firm, the profit for investment varies with the price of base metals. (iii) Investors value the expected profit and price appreciation of the stock in the future.

In the next section we are going to anchor the value K_{t+1}^* to calculate the variations of investors' profit. This means that the desired level of capital – the value K_{t+1}^* – is known

3.3 Analysis

3.3.1 The Firm Specific Factor and the Industry Factor

Assuming the stationary data series of stock price p_{st} , we calculate the variance of stock price p_{st} using 3.7 (see Appendix 3.5.2)

$$\sigma_{sp}^2 \approx \frac{1}{g_e^2 [\gamma^2 - (1-\delta)^2]} \left\{ \frac{a}{1-b} \left[\frac{b^b A_{t+1} K_{t+1}^{*(a+b-1)} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \right\}^2 \left[E_t (p_{t+1})^{\frac{b}{1-b}} \right]^2 \sigma_p^2. \quad (3.8)$$

Note that $A_{t+1}, K_{t+1}, \mu_{t+1}, \tau_{t+1}$ in the term $\left\{ \frac{a}{1-b} \left[\frac{b^b A_{t+1} K_{t+1}^{*a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \right\}^2$ are the variables of information on expected productive conditions of the individual firm in the next period. We define this non-negative term as a productive information coefficient of the individual firm - the firm specific factor.

Definition 1 : *The Firm Specific Factor φ_t is a combination of parameters of an individual firm that reflects its expected productive conditions*

$$\varphi_t = \left\{ \frac{a}{1-b} \left[\frac{b^b A_{t+1} K_{t+1}^{*a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \right\}^2. \quad (3.9)$$

Clearly, the natural resource tax τ reduces the coefficient φ . The quality of mineral ore μ increases φ . In reality we have all the variables of the firm stable for a period of time, so the firm's profitability remains unchanged for a relatively long time. For that reason we ignore the subscript t in the coefficient φ_t to calculate the variance of stock price σ_{sp}^2 .

Recall that the price scheme in the short-run⁴ is

$$E(p) = 1 - \frac{i^* R}{\varepsilon}. \quad (3.10)$$

Substituting 3.9 and 3.10 into 3.8 yields

$$\sigma_{sp}^2 = \frac{\varphi}{g_e^2 [r^2 - (1 - \delta)^2]} \left(1 - \frac{i^* R}{\varepsilon} \right)^{\frac{2b}{1-b}} \sigma_p^2. \quad (3.11)$$

The set of parameters $\delta, \varepsilon, i^*, R$ in the term $\frac{1}{[r^2 - (1 - \delta)^2]} \left(1 - \frac{i^* R}{\varepsilon} \right)^{\frac{2b}{1-b}}$ represents the properties of the base metals industry sector. We define this term as a common

⁴This expression is developed in Nguyen and Semenov (2015) and in Chapter 2, page 50.

coefficient for the firms operating in the industry – the industry factor.

Definition 2 : *The Industry Factor Φ is the combination of parameters of the industry that reflects the common properties and the operating conditions of the industry*

$$\Phi = \frac{1}{[r^2 - (1 - \delta)^2]} \left(1 - \frac{i^* R}{\varepsilon}\right)^{\frac{2b}{1-b}}. \quad (3.12)$$

We derive the following properties of the industry factor

Proposition 1 : *The scale of uncertain production i^* and the depreciation of capital δ negatively affect the industry factor Φ . The price elasticity ε positively affects Φ*

$$\frac{\partial \Phi}{\partial i^*} < 0; \frac{\partial \Phi}{\partial \delta} < 0; \frac{\partial \Phi}{\partial \varepsilon} > 0.$$

Suppose that $b = 0.381$ ⁵; $R = 0.2$. Figure 3.2 describes the variations of the industry factor Φ with respect to i^* , ε and δ ⁶. We present the graph of Φ as functions of those variables. Furthermore in Table 3.2, Appendix 3.5.3 we show some values of Φ with respect to changes in those variables.

Intuitively, more uncertain production (i^* is high) leads to a lower price of base metals ($E[p]$ is low). The low price implies a stable profit (Panel A of Figure 3.2). The high price elasticity (ε is high) generates the high expected price ($E[p]$

⁵MacAvoy (2012) estimates that, for the period from 1990 to 2005, the capital share in mining industry was $a \approx 0.618$. Using this result we can assume that $b \approx 1 - 0.618 = 0.381$.

⁶In practice, the capital depreciation of many economic sectors δ is equal to 0.10 as per the accounting regulations. In the mining sector δ for equipment is higher. According to Natural Resources Canada, "Most capital assets acquired by mining and oil and gas companies are included in Class 41, which qualifies for a depreciation rate of 25% on a declining balance basis". But the lifetime of a mine is long enough to have δ smaller than normal. Source : Natural Resources Canada <https://www.nrcan.gc.ca/mining-materials/>.

is high), which increases the variance of the firm's profit (Panel C of Figure 3.2). In a different impact mechanism, if an investment depreciates quickly, another investment is required to generate profit, so the initial capital has less impact in the business process, thus the variance of profit created is small (Panel B of Figure 3.2).

Now substituting 3.12 into 3.11 yields the following proposition

Proposition 2 : *The volatility of a base metal stock varies with information relating to the profitability of the firm φ , the changes in operating and business conditions of the industry Φ , the expected rate of return for base metal stocks in the stock market g_e , and the variance of the output price σ_p^2*

$$\sigma_{sp}^2 = \frac{\varphi\Phi}{g_e^2} \sigma_p^2. \quad (3.13)$$

Equation 3.13 expresses the variation of stock price as a combination of major factors : (i) If the productive coefficient of the firm φ and the industrial factor Φ are high then the stock price p_{st} is more volatile. (ii) If the base metal price fluctuates more, the stock price becomes riskier. Suppose that the model is applicable for the oil industry, we can somehow explain the case of Shell's stock price in 2004, described in Geman (2009). The announcement of μ_t affects the firm-specific factor φ_t according to equation 3.9. This in turn leads to higher σ_{sp} as in equation 3.13, implying more fluctuation in Shell's stock price. Depending on the share of capital in production (the value of b), a change of μ can increase or decrease σ_{sp} .

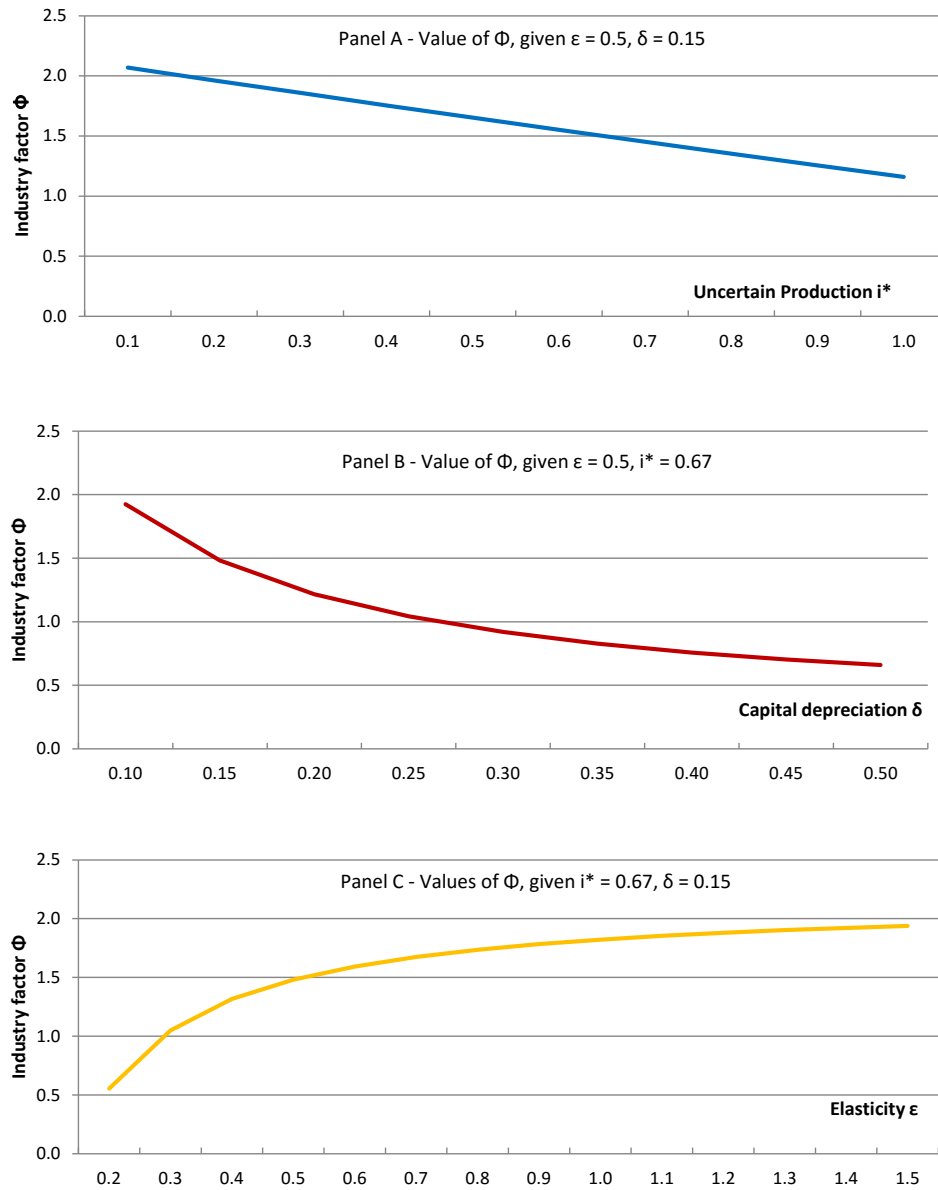


FIGURE 3.2: Variations of Φ with respect to changes of Uncertain production i^* , Capital depreciation δ , and Elasticity ε .

In chapter two we had the variance of the price $\sigma_p^2 = \left(\frac{i^*}{\varepsilon}\right)^2 \sigma^2$. The price p_t varies with the size of the risk i^* , the elasticity ε , and the state of nature σ in the base metals industry. In general, i^* and ε have two side effects on the variation of the stock price through Φ and σ_p^2 .

3.3.2 Beta coefficients of the industry

In practice, the beta coefficients of the metal stocks are greater than 1, see MacAvoy (2012) p. 116 for example. A beta coefficient of 1 suggests that the stock carries the same risk as the overall market and will earn only the market return. If a beta coefficient is smaller than 1, that suggests a below average risk (where the average means the overall market). A coefficient higher than 1 suggests an above average risk.

The beta coefficient is calculated as the covariance of a stock's return with market returns divided by the variance of the market return. A slight modification helps us to build another key relationship, which tells that the beta coefficient is equal to the correlation coefficient multiplied by the standard deviation of stock returns, divided by the standard deviation of market returns.

$$\beta = \varrho(sp, mk) \frac{\sigma_{sp}}{\sigma_{mk}}, \quad (3.14)$$

where $\varrho(sp, mk)$ is the correlation coefficient between the stock return and the market return, and σ_{mk} is the standard deviation of the market return. We show the calculation of σ_{mk} in Appendix 3.5.4.

Substituting 3.13 and 3.32 into 3.14 yields

$$\beta = \varrho(sp, mk) \frac{\sqrt{\varphi\Phi}\sigma_p}{\sqrt{\frac{\varphi_m}{\delta_m(2-\delta_m)}}} \quad (3.15)$$

In normal conditions, *ceteris paribus* suppose that the correlation coefficient $\varrho(sp, mk)$ equals 1. In a perfectly competitive capital market, the investments always go to a more profitable portfolio, leading to an equilibrium in profitability among industrial sectors so that there should be an equality between φ and φ_m . Thus we can simplify 3.15

$$\beta \approx \sqrt{\delta_m(2-\delta_m)}\Phi\sigma_p. \quad (3.16)$$

Equation 3.16 illustrates the property of base metal stock beta coefficients : (i) The fluctuations of base metal prices in the international market. (ii) The industrial factor Φ , which combines uncertain production, the elasticity, and the distribution of base metals in nature. (iii) The rate of capital depreciation in the base metals industry in comparison with that of the whole economy.

Beta coefficient (β , dated 05 July 2016)			
BHP Billiton Limited.	NYSE	BHP	1.46
Canadian Zinc Corporation	TSX	CZN	1.55
Freeport-McMoRan Inc.	NYSE	FCX	2.64
Noranda Aluminium Holding Corp.	NYSE	NOR	2.33
Rio Tinto plc.	NYSE	RIO	1.68
Vale SA	NYSE	VALE	1.65

TABLE 3.1: Beta coefficients of some base metal stocks.

In every stock market, the beta coefficients of base metal stocks are usually

greater than 1 ($\beta > 1$). Table 3.1 above⁷ presents beta coefficients of some base metal stocks, dated 25 Sep. 2015.

3.3.3 Investment decision of the firm

Multiplying both sides of 3.7 by r we obtain

$$rp_t^i = aE_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*(a+b-1)} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} + (1 - \delta) E_t p_{t+1}^i. \quad (3.17)$$

Rearranging 3.17 we have

$$aE_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*(a+b-1)} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} = p_t^i [r - 1 + \delta] - p_t^i (1 - \delta) \frac{E_t p_{t+1}^i - p_t^k}{p_t^k}. \quad (3.18)$$

The right hand side of equation 3.18 consists of the expected increase, minus the depreciation of capital after one period, that is Jorgenson's cost of capital C_t^J , see Jorgenson (1963). The transparent information of the project means that the technology A_{t+1} , quality of the mining project μ_{t+1} , and government policy τ_{t+1} are foreseen, therefore we have

$$a \left[\frac{b^b E_t [p_{t+1}] A_{t+1} K_{t+1}^{a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} = C_t^J. \quad (3.19)$$

The Hall-Jorgenson analysis concludes that the firm will choose to purchase more capital goods today in order to take advantage of the relatively lower current price p_t^i . We will go further with typical parameters of a base metal firm.

The firm is choosing the investment level I_t to obtain K_{t+1}^* such that the expected

⁷Sources : Investing News <http://investingnews.com/daily/resource-investing/>, Google Finance <https://www.google.com/finance/>, and information posted in Stock Exchanges NYSE, TSX, NASDAQ.

marginal profit is equal to C_t^J . The optimal capital K_{t+1}^* is

$$K_{t+1}^* = \left[\frac{\tau_{t+1}}{b^b E_t[p_{t+1}] A_{t+1} \mu_{t+1}^b} \left(\frac{C_t^J}{a} \right)^{1-b} \right]^{\frac{1}{a+b-1}}. \quad (3.20)$$

The feasible studies suppose that $E_t(p_{t+1}) = p_t$. Rearranging 3.20 leads to

$$K_{t+1}^* = \left[\frac{\tau_{t+1}}{b^b A_{t+1} \mu_{t+1}^b} \left(\frac{C_t^J}{a} \right)^{1-b} \right]^{\frac{1}{a+b-1}} p_t^{\frac{1}{1-a-b}}. \quad (3.21)$$

Substituting 3.21 into 3.1 yields the following proposition

Proposition 3 : *The optimal investment in the firm at time t is*

$$I_t^* = \left[\frac{\tau_{t+1}}{b^b A_{t+1} \mu_{t+1}^b} \left(\frac{C_t^J}{a} \right)^{1-b} \right]^{\frac{1}{a+b-1}} p_t^{\frac{1}{1-a-b}} - (1 - \delta) K_t. \quad (3.22)$$

Equation 3.22 expresses the relationship between the operating conditions of the firm and the investment decision. In other words, it shows how the parameters τ, A, μ, p, δ affect the optimal investment. (i) The optimal investment of the firm must cover the depreciation of the capital stock and maximize the corporate value – including the depreciation of the capital stock and the profit of the business. (ii) The optimal investment varies with changes of the base metal price p_t . Economies of scale in production determine the sign of this effect. (iii) When the firm sees a chance of increases in the future price of stock, the term $\frac{E_t p_{t+1}^i - p_t^k}{p_t^k}$ is large, meaning that the cost of capital C_t^J is low, if $a + b < 1$ then this should be time for the firm to increase investment by issuing additional stocks.

There is a controversy regarding economies of scale in mining industry. As we

discussed in chapter one, there is empirical evidence that the industry exhibits constant returns to scale. However, the World Development Report 2009 (by the World Bank, Chapter 4 ⁸) suggests that the base metals industry exhibits increasing return to scale. The international auditing firms (PriceWaterhouse-Cooper, Ernst & Young ⁹) provide reports of professionals and discussions of senior managers where they do not conclude a definite economy of scale for the mining industry but analyze the productivity of labour and capital. The above result realizes the impacts of conditions under the specific contexts.

Suppose that the industry has decreasing returns to scale, meaning $a + b < 1$, then low taxes τ , the availability of minerals in nature μ , or advanced technology A will encourage the investment decision of the firm. High prices of base metals will encourage investment in the industry. If the industry has increasing return to scale, $a + b > 1$, then low taxes, the availability of minerals in nature, and advanced technology do not always encourage the more investment. Interestingly, low prices of output may encourage more investments to obtain the optimal profit from expanding production.

3.3.4 Extension : Elasticity of the stock price w.r.t. output price

In this section we clarify the fact that the price of some base metal stocks reacts promptly to changes of the respective base metal price while others are reluctant. We do not consider the investment behavior for short-run base metal price changes, so the firm value maximization problem is not applied. Instead, we

⁸Sources: The World Bank, <http://siteresources.worldbank.org/INTWDRS/Resources>

⁹Sources: The firms PWC, <https://www.pwc.com/ee/et/publications/>, and Ernst & Young, <http://www.ey.com/Publication/>.

calculate the book value of the stock and its elasticity with respect to the respective base metal price.

The initial investment in the mining project is I_0 . The variable costs of production are allocated to the range of products and denoted by C_{vt} where subscript $v \in \{1, 2 \dots V\}$ indicates the product and $t \in \{0, 1, 2 \dots T\}$ indicates the time period. Because of the competitive market in international exchanges, the base metal producers are price takers. In a typical mining and metallurgy project, the large initial investment I_0 does not allow temporary suspension of production, so for the whole life of the mining firm – T years – we always have output $m_t > 0$. The production plan follows the mining project schedule from the beginning, with very little interference by the trading sector, thus $\frac{dm_t}{dp_t} = 0$, $\frac{dC_t}{dp_t} = 0$.

Base metals are usually found in nature in association with many other elements. It is common that a mining company has more than one product. For example copper ores frequently contain gold, silver, molybdenum, lead, zinc, and cobalt. By-product metals of nickel are cobalt, palladium, rhodium, and scandium. Although most tin is obtained from mining tin ores, it is also found in association with ores of tungsten, tantalum, and lead, and minor quantities of tin are recovered as by-products of mining these metals¹⁰.

The book value of the stock price p^k is the present value of future cash flows of the mining and metallurgy projects including total value of by-products, divided by the quantity of shares N

¹⁰Long, Keith R., John H. DeYoung Jr, and Stephen D. Ludington. Database of significant deposits of gold, silver, copper, lead, and zinc in the United States. US Geological Survey Open File Report (1998).

$$p^k = \frac{1}{N} \left[\int_{v=1}^V \int_{t=0}^T \rho^t (p_{vt} m_{vt} - C_{vt}) dt dv - I_0 \right]. \quad (3.23)$$

Consider the base metal product, $v = 1$. In the relatively short period of time for which we are considering the impact of the output price on the stock price, $\frac{\dot{A}}{A}, \frac{\dot{\mu}}{\mu}, \tau, r$ is relatively small so that $p_{vt} = p_{v0} r^t$. Substituting this into 3.23 yields

$$p^k = \frac{p_{1,0}}{N} \int_{t=0}^T m_{1t} dt - \frac{1}{N} \int_{t=0}^T \rho^t C_{1t} dt + \frac{1}{N} \int_{v=2}^V \int_{t=0}^T \rho^t (p_{vt} m_{vt} - C_{vt}) dt dv - \frac{I_0}{N}. \quad (3.24)$$

We derive the condition for ε_{pp} in Proposition 4.

Proposition 4 : *The more profitable are the by-products, the less is the elasticity of the stock price with respect to the base metal price ε_{pp} . We have $\varepsilon_{pp} = 1$ if and only if the profit from by-products is high enough to cover total production cost.*

$$\int_{t=0}^T \rho^t C_{1t} dt + I_0 = \int_{v=2}^V \int_{t=0}^T \rho^t (p_{vt} m_{vt} - C_{vt}) dt dv. \quad (3.25)$$

Proof : See Appendix 3.5.5.

This result reflects the practice that base metal stocks respond with some delays to changes in base metal prices. In some rare cases where the producer has only one base metal product as output, or the by-products are not profitable enough, then the inequality holds and $\varepsilon_{pp} \geq 1$. Figure 3.3 depicts the correlation of some base metal prices and the stock prices of large producers, respectively. FCX (Freeport-McMoRan) is the world's largest copper and molybdenum producer. VALE (Vale) is the world's largest producer of nickel and iron ore. CZN

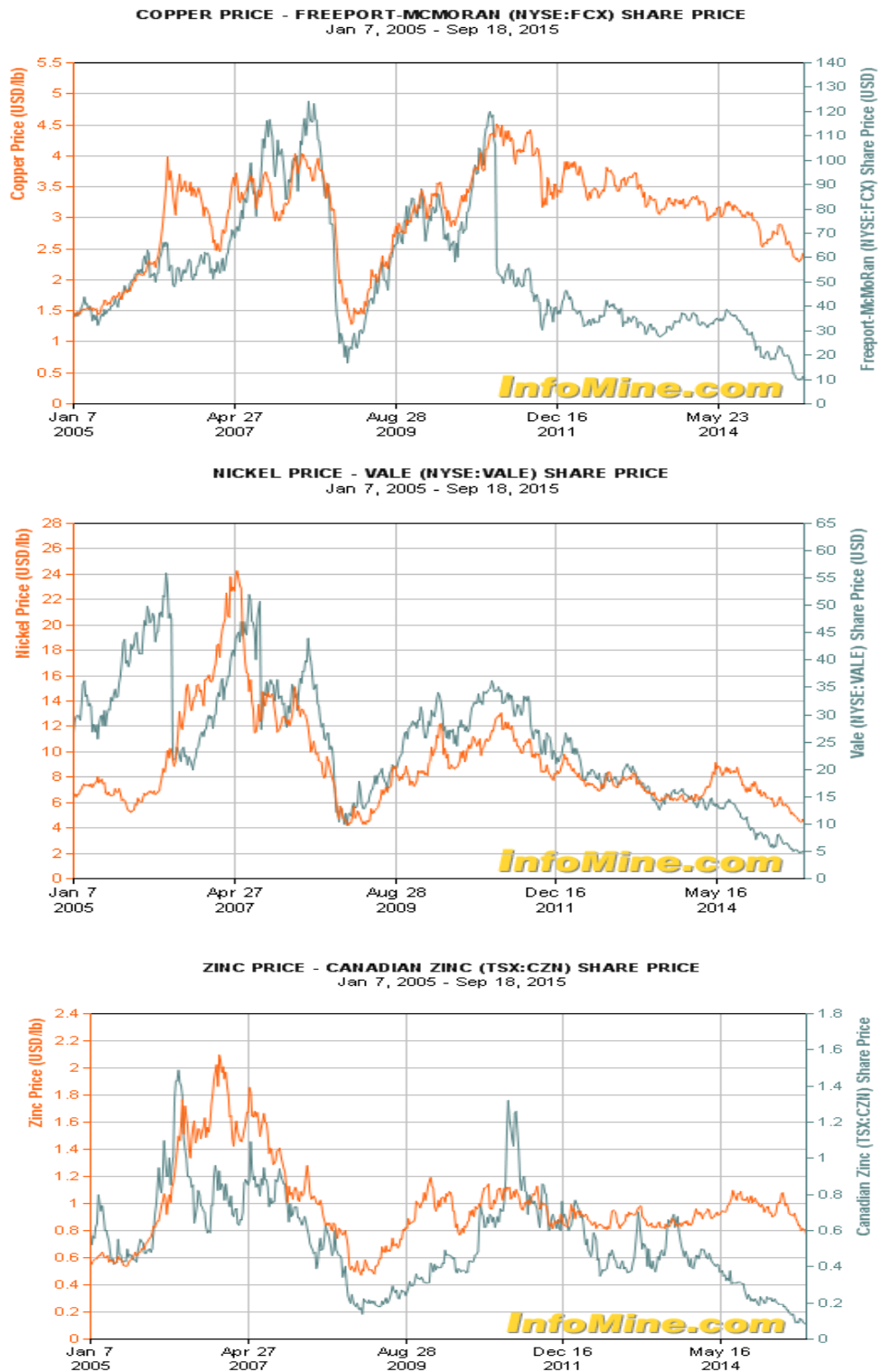


FIGURE 3.3: Correlation of the base metal price and the producer's stock price.

(Canada Zinc) - headquartered in Vancouver, BC - possess the mines, which host substantial resources of silver, zinc, and lead.

3.4 Conclusion

In this chapter we built a model of investments using a two-stage static game model, the framework of Hall-Jorgenson neoclassical investment analysis, and the typical properties of the base metals industry. Our fundamental analysis sharpens the understanding of the base metals industry, specifically the impacts of production, trading, and investment on a base metal producer's stock price and its relationship with the base metal price itself.

We define the industry factor to describe the features of the base metals industry, so that we are able to explain their empirical observation on beta coefficients of base metal stocks. We present the set of explanatory parameters of the properties of the base metals industry in capital markets, including the investment decision of the firm in the primary market and the behaviors of the stock price in the secondary market. We clarify the impacts of base metals production, trading, and investment processes on the producer's stock price and thus discover what makes the pattern of mining listed firms stocks on the capital market differ from firm to firm. The results show that there is a strong impact of the industry factor on the volatility of base metal stock prices. We also analyze the investment decision of producers given circumstances of production and markets. For different economies of scale in the industry, the availability of minerals in nature, the tax policy, and the output prices all have different effects on investment decisions of firm.

3.5 Appendix

3.5.1 Proof of Lemma 2

The firm chooses the investment level to maximize the its value

$$\max_I PV(K_t) = \Pi(p_t, \tau_t, A_t, K_t) - p_t^i I_t + \rho E_t PV(K_{t+1}).$$

$$s.t. \quad K_{t+1} = (1 - \delta)K_t + I_t.$$

The first order condition with respect to I_t is

$$\frac{\partial PV(K_t)}{\partial I_t} = -p_t^i + \rho E_t \frac{\partial PV(K_{t+1})}{\partial K} = 0. \quad (3.26)$$

Applying the Envelope Theorem, the marginal value of the firm with respect to additional investment is equal to marginal profit plus the appreciation of capital, minus the depreciation of the additional capital

$$\frac{\partial PV(K_{t+1})}{\partial K} = \frac{\partial \Pi(p_{t+1}, \tau_{t+1}, A_{t+1}, K_{t+1})}{\partial K} + (1 - \delta)p_{t+1}^i. \quad (3.27)$$

Substituting 3.27 into 3.26 yields

$$p_t^i = \rho E_t \frac{\partial \Pi(p_{t+1}, \tau_{t+1}, A_{t+1}, K_{t+1})}{\partial K} + \rho(1 - \delta)E_t p_{t+1}^i. \quad (3.28)$$

Taking the derivative of 3.5 with respect to K yields

$$\frac{\partial \Pi(p_{t+1}, \tau_{t+1}, A_{t+1}, K_{t+1})}{\partial K} = \frac{1}{1-b} \frac{1-b}{b} \left[\frac{b p_{t+1} A_{t+1} K_{t+1}^a \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{b}{1-b}} a \frac{b p_{t+1} A_{t+1} \mu_{t+1}^b}{\tau} K_{t+1}^{a-1},$$

$$\frac{\partial \Pi(p_{t+1}, \tau_{t+1}, A_{t+1}, K_{t+1})}{\partial K} = a \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}}. \quad (3.29)$$

Substituting 3.29 into 3.28 we obtain equation 3.7.

$$p_t^i = a\rho E_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*(a+b-1)} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} + \rho(1 - \delta) E_t p_{t+1}^i.$$

3.5.2 Calculation of σ_{sp}^2

By Lemma 2 we have

$$p_t^i = a\rho E_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*(a+b-1)} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} + \rho(1-\delta) E_t p_{t+1}^i.$$

Multiplying both sides by r

$$r p_t^i = a E_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*(a+b-1)} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} + (1-\delta) E_t p_{t+1}^i,$$

$$r^2 \text{Var}(p_t^i) = \text{Var} \left\{ a E_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \right\} + (1-\delta)^2 \text{Var}[E_t p_{t+1}^i].$$

The stationary process of p_t^i implies that there is no economic shock for a while and we have $\text{Var}(p_t^i) = \text{Var}(p_{t+1}^i)$

$$\text{Var}(p_t^i)[r^2 - (1-\delta)^2] = \text{Var} \left\{ a E_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{*a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \right\},$$

$$\text{Var}(p_t^i)[r^2 - (1-\delta)^2] = \left\{ a \left[\frac{b^b A_{t+1} K_{t+1}^{*a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \right\}^2 \text{Var}(p_{t+1}^{\frac{1}{1-b}}).$$

Applying Taylor expansion to approximate the left hand side¹¹ we obtain

$$\text{Var}(p_t^i) \approx \frac{1}{r^2 - (1-\delta)^2} \left\{ a \left[\frac{b^b A_{t+1} K_{t+1}^{*a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \frac{1}{1-b} \right\}^2 \left[E_t (p_{t+1})^{\frac{b}{1-b}} \right]^2 \sigma_p^2. \quad (3.30)$$

Using the formula for pricing a stock by fundamental analysis, the stock price $p_{st} = \frac{p_{t+1}^i}{g_e}$, where g_e is the expected rate of return of investors, thus the variance of stock price is

$$\sigma_{sp}^2 = \left(\frac{1}{g_e} \right)^2 \text{Var}(p_t^i). \quad (3.31)$$

¹¹Using the formula $\text{Var}[f(X)] \approx (f'(E[X]))^2 \text{Var}[X]$.

Substituting 3.30 into 3.31 yields 3.8

$$\sigma_{sp}^2 \approx \frac{1}{g_e^2[r^2 - (1 - \delta)^2]} \left\{ a \left[\frac{b^b A_{t+1} K_{t+1}^{*a+b-1} \mu_{t+1}^b}{\tau_{t+1}} \right]^{\frac{1}{1-b}} \frac{1}{1-b} \right\}^2 \left[E_t(p_{t+1})^{\frac{b}{1-b}} \right]^2 \sigma_p^2.$$

3.5.3 The industry factor (Φ) w.r.t. changes of i^* , ε , δ

Given that $b = 0.381$, $R = 0.2$ we have the value of Φ varies as the following :

Variation of the industry factor (Φ)			
Uncertain production i^*	Elasticity ε	Depreciation δ	Value of Φ
0.67	0.6	0.10	3.86
0.67	0.7	0.10	4.05
0.67	0.8	0.10	4.20
0.67	0.9	0.10	4.32
0.7	0.5	0.10	3.51
0.8	0.5	0.10	3.21
0.9	0.5	0.10	3.03
1.0	0.5	0.10	2.81
0.67	0.5	0.05	6.99
0.67	0.5	0.15	2.45
0.67	0.5	0.20	1.89
0.67	0.5	0.25	1.56

TABLE 3.2: Variation of the Industrial Factor.

3.5.4 Calculation of σ_{mk}

We calculate σ_{mk} by assuming the general production function $Y_t = A_t K_t^a L_t^b$ of the whole economy, which is represented in stock markets by listed companies. L stands for another input factor of the economy with price w . Recalculating 3.7 yields the equivalent equation

$$p_t^i = a\rho E_t \left[\frac{b^b p_{t+1} A_{t+1} K_{t+1}^{a+b-1}}{w_{t+1}} \right]^{\frac{1}{1-b}} + (1 - \delta) E_t p_{t+1}^i,$$

and repeating the calculation from 3.4 to 3.9 we obtain the equivalent coefficient φ_m for the listed companies in the market.

$$\varphi_m = \left\{ \frac{a\rho}{1-b} \left[\frac{b^b A_{t+1} K_{t+1}^{a+b-1}}{w_{t+1}} \right]^{\frac{1}{1-b}} \right\}^2.$$

Because $E_t(p_{t+1}) = p_t = 1$, then 3.8 becomes

$$\sigma_{mk}^2 \approx \frac{\varphi_m}{g_e^2} \frac{1}{\delta_m(2 - \delta_m)}. \quad (3.32)$$

3.5.5 Proof of Proposition 4

The elasticity of stock price with respect to p_{vt} is

$$\varepsilon_{pp} = \frac{dp^k}{dp_{vt}} \frac{p_{vt}}{p^k}. \quad (3.33)$$

Using 3.24 we are calculating $\frac{dp^k}{dp_{1,0}}$.

$$\frac{dp^k}{dp_{1,0}} = \frac{1}{N} \int_{t=0}^T m_{1t} dt + \frac{p_{1,0}}{N} \int_{t=0}^T \rho^t \frac{dm_{1t}}{dp_{1,0}} dt - \frac{1}{N} \int_{t=0}^T \rho^t \frac{dC_{1t}}{dp_{1,0}} dt. \quad (3.34)$$

Note that $\frac{dm_{1t}}{dp_{1t}} = 0$, $\frac{dC_{1t}}{dp_{1t}} = 0$ because in a mining project with more than one product, the operation of the project is continuous and the output of one base metal depends on the content of the base metal mineral in the crude ore.

$$\frac{dp^k}{dp_{1,0}} = \frac{1}{N} \int_{t=0}^T m_{1t} dt. \quad (3.35)$$

Substituting 3.35 into 3.33 obtains

$$\varepsilon_{pp} = \frac{p_0}{p^k N} \int_{t=0}^T m_{1t} dt. \quad (3.36)$$

Substituting 3.24 into 3.36 obtains

$$\varepsilon_{pp} = \frac{p_{1,0} \int_{t=0}^T m_{1t} dt}{p_{1,0} \int_{t=0}^T m_{1t} dt - \int_{t=0}^T \rho^t C_{1t} dt + \int_{v=2}^V \int_{t=0}^T \rho^t (p_{vt} m_{vt} - C_{vt}) dt dv - I_0}. \quad (3.37)$$

It can be seen that $\varepsilon_{pp} \geq 1$ iff

$$p_{1,0} \int_{t=0}^T m_{1t} dt \geq p_{1,0} \int_{t=0}^T m_{1t} dt - \int_{t=0}^T \rho^t C_{1t} dt + \int_{v=2}^V \int_{t=0}^T \rho^t (p_{vt} m_{vt} - C_{vt}) dt dv - I_0. \quad (3.38)$$

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Conclusion

Base Metals are essential commodities for the world economy, as they are important input factors for production of all sorts. There are typical features of this industry which allow us to build theoretical models for simulating behaviours of the agents and markets. In nature, base metal ore deposits are common and inexpensive to access. Base metals are recyclable, and the portion of recycled materials for base metals metallurgy is constant over the decades. The prices of base metals show positive cross-elasticities of demand. They are homogeneous products and durable for storage. In the international market, base metals are traded by professional agents in well-organized, concentrated exchanges. Moreover, base metals can be considered as a channel of investment for commodity speculation and corporate shares.

In this Research

The studies in this thesis employ the methodology of positive economics and the typical features of base metals. We describe and explain the main observed phenomena of this industry. We also present normative judgments on the practical observations to clarify their status and provide evidence. We emphasize noticeable policy implications of the findings. We utilize the frameworks and theories from Slade (1982), Newbery (1987), and Jorgenson (1963) to develop our models and answer the specific research questions.

By modeling the regulator's problem for the extraction of base metal minerals and the phases of metal production projects, investments, and production decisions, we show that in the long run base metals prices depend on the total factor productivity of the industry and the availability of the metals in nature. Assuming constant returns to scale, the price elasticity of the supply of base metals is relatively small. Interestingly, a high natural resource tax leads to a high price but low rate of price change over time. On the supply side, the decline of base metals relative prices thus can be explained by the increase in total factor productivity of the industry and stable quality of the base metal ore deposits in nature. On the demand side, the relative price is also declining over time because of the impacts of the growth rate of the technological progress and demand from the economy. Our numerical illustration clarifies the declining trend of base metals prices. By solving the system of equations for equilibrium conditions, we show that the rental rate of base metals minerals in reserve can decrease over time, or even not be valuable in the future. Never in the literature has an equilibrium of supply and demand been derived to show the actual prices of these special commodities. In this study, the price elasticities of supply and demand are also calculated and decomposed into specific effects. These are systematic components of base metal price changes in the international market. Using the results from this study, we are able to explain some dramatic drops and rises of base metals prices in the past.

Even though the equilibrium price is realized in the long run, the non-systematic changes in price have another complicated story because of the existence of speculative activities. We define this kind of change as fluctuations. We use

the variance of price to measure fluctuations. Since the fluctuation is the change of price in the short term, it might not be a result of the long run aggregate supply and demand equilibrium. With the presence of the traders involved in commodity exchanges, we have a full picture of the industrial and commercial processes for taking base metals into applications in the economy. By simulating speculative activities and optimizing the utility of the producers and traders in the international exchanges, we show how price fluctuations respond to risk preferences and participation of agents and the scale of international exchanges. We find out the critical point of production investment, which depends on market demand, profitability of the base metals industry and the distribution of base metal minerals in nature. We define the critical level of uncertain production as a specific case of the industry versus the market conditions. For which, if the fraction of uncertain production is higher than the critical level, then the prices of base metals fluctuate more or less according to the number of producer offers in base metal exchanges, the speculative activities, and risk preferences of agents. In contrast, if the investment level of the metal industry is below that critical point, the directions of the responses of base metal exchanges to the price are reversed. Hence, the non-systematic changes of base metals prices in international exchanges are explained. The unusual fluctuations of base metals prices in some specific periods of time in history can thus be nicely explained.

We have considered the availability of base metal minerals in nature both in the sense of the type of distribution and the size of the reserves in nature. Higher quality and more plentiful natural reserves keep the price more stable. Variation in the content of base metal elements in mineral deposits affects the price via the expected utilities of agents.

Turning to the base metals industry as an objective of investment, we study its characteristics in relationship with capital markets. We clarify the impacts of the industrial and commercial processes on investment decisions in the base metals industry. The investment decisions of investors in the primary investment market and the stock price in the securities market reflect properties of the base metals industry. A set of explanatory parameters for the properties of base metal stocks have been presented and we also analyze the investment decisions of the base metals producers. We define the industry factor to explain the empirical observations on the beta coefficients of base metal stocks. The relationships between stock prices and base metals prices are clarified using the geological features of base metal deposits. The results show that there is a strong impact of the industry factor on the volatility of base metal stock prices. Economies of scale in the mining industry lead to different effects of tax policy and output prices on investment decisions. We support conclusions of the model with empirical evidence. We present policy implications which are derived from the equations of optimal investment.

Potential Research Extensions

In this thesis we assume that a portion of production has an uncertain productivity, which is realistic and novel in the literature. The study can be further developed using a different definition of uncertainty. For instance, the form $E_t p = \lambda p_1 + \bar{\lambda} p_2$, where $\lambda + \bar{\lambda} = 1$, can be used to express the expected value of p responding to the status of nature λ .

Another potential extension is allowing different production functions for different types of natural resources. The distributions of valuable elements in the Earth's crust suggest different production functions. For example, to study the rare earth elements, the Leontief production function should be used. The Leontief production function applies to this situation because the inputs must be used in fixed proportions. Starting from those proportions, if usage of one unit of capital input is increased without ore deposit being increased, output will not change as the assessments of rare earth elements deposits are very limited. In this case, the production function is given by $Q = \text{Min}[\alpha K, \beta(\mu D)]$, where α, β reflect the proportional ratio of capital stock and the limited ore deposits. Alternatively, if we want to study precious metals such as gold or silver, their usage as a tool for value storage and their monetary characteristics as a hedge against inflation must be considered in the model. We can consider the dynamic equation of the gold stock $m_t = (\frac{1+\theta_t}{1+\pi_t})m_{t-1}$, where m is the gold stock used for value storage in the economy, θ is the growth rate of the gold stock, including the extraction amount within the period, and π is the inflation rate of the economy.

There is a disagreement regarding the form of the distribution of base metal elements in nature. Some technical materials show that the probability density functions of useful elements – and thus the income of the miners – have a log-normal form. If base metals in nature are log-normally distributed, where the shape of metal content distribution is skewed to the right, then the results could be different. Note that the utility function in that case must be defined so that the calculations are tractable and the optimization problems for the agents can be solved. For example, the combination of a CRRA utility function $U_t =$

$\frac{1}{1-\theta} E_t w_{t+1}^{1-\theta}$ and the log-normally distributed income w , lead the utility maximization problem to the same solution as the problem $\max_w E_t w_{t+1} + \frac{1}{2}(1-\theta)\sigma_{wt}^2$. There are many possible distributed forms of income, so the calculation using these assumptions may lead to some interesting results.

Empirically, in a working paper, we study the possibility of forecasting the prosperity of industries based on changes in base metals prices. Using the data set on the prices of base metals from 1960 to 2013 and a Vector Auto Regression model, we estimate the GDP growth of the world economy. The Table below shows our preliminary results

Year	Forecast	Actual	Error
2013	2.24	2.36	0.12
2014	2.83	2.49	-0.34
2015	2.61	2.40	-0.21
2016	2.85	-	-

The Granger causality Wald Tests indicate that the evidence favors the alternative that GDP growth Granger causes base metals price increases. Additionally, we can use an algorithm to isolate the fluctuation in prices of base metals from the price trend to predict the uncertainty of the economy.

This research represents an initial step in the study of the economics of base metals. The base metals industry still contains many undiscovered mysteries of great significance because it is relevant to every aspect of human life. Beyond the initial research presented in this thesis, there is a desire for understanding cross-border M&A deals in the industry, which have become common in recent years. Furthermore, the economic studies on exploration and exploitation of

offshore, substitute materials, metals technology, and so on, require interdisciplinary collaboration. More and more puzzle pieces are needed to assemble the full picture surrounding the Economics of Base Metals.