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Stochastic and Fuzzy Analyses in Reliability Design

Thesis presented to the University of Ottawa in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering

By

©Kouroush Jenab

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Abstract

The risk analysis process involving information acquisition, modeling, analysis, and decision steps results in the product design improvement. To perform product risk assessment, this study addresses stochastic and fuzzy analyses in reliability design. Using decision-making techniques and the flow-graph concept, the main objective of this study is to develop analytical models with time varying input data, and/or fuzzy input data for reliability techniques. The models (i.e., Graph-based failure effects analysis, Group-based failure effects analysis, Imprecise-chance Markov chains, Fuzzy and stochastic fault tree analysis, Binary k-out-of-n system with self-loop units, Reversible multi-state k-out-of-n:G/F/Load sharing system, and Imprecise-chance reliability estimation) incorporate the stochastic self-healing mechanisms represented by self-loop graph, and/or conflict resolution approach. Stochastic models developed in this study compute Time-To-Event/State data made up of probability of the system failure, and mean and standard deviation of time to an event/state. To identify, prioritize and eliminate potential failures in the system, the fuzzy models presented in this study introduce aggregated/compensated approaches for mitigating conflict of input data.

The applications of the stochastic and fuzzy models are demonstrated through practical examples. Using typical, practical, and extreme values of the basic parameters of the models and performing sensitivity analysis, the end results demonstrate the robustness of and conflict resolution capability of the models.

TABLE OF CONTENTS

Chapter Nomenclature

Chapter 1:

AHP	Analytic Hierarchy Process
ATM	Asynchronous Transfer Mode
CM	Criticality Matrix
FMEA	Failure Mode and Effect Analysis
FBD	Functional Block Diagram
FEA	Failure Effects Analysis
FRA	Failure Risk Analysis
FTA	Fault Tree Analysis
GFEA	Group-based Failure Effect Analysis
PC	Pareto Chart
RAMS	Reliability, Availability, Maintainability, and Safety
RBD	Reliability Block Diagram
RPC	Rick Priority Category
RPN	Risk Priority Number
SAE	Society of Automotive Engineering
STD	State Space Diagram

Chapter 2:

A(x)	A subset of set Z(x) includes starting nodes in Gw or gw
f_j	'j'th risk criterion
G_s	Strong relational graph
g_s	Reverse of strong relational graph
G_w	Weak relational graph
g_w	Reverse of the weak relational graph
i	Index for failure, i=1,2,…., I
$I_k(f_j)$	The relative importance of 'j'th criterion according to 'k'th expert
Int[n]	Integer value of a number,'n'
j	Index for risk criterion/ root cause, j=1,2,…,J

Chapter Nomenclature

k Index for expert in group/ symptoms group $k=1,2,\ldots,K$

$O(F_i)$ Rank of 'i'th failure node in backward steps

$O(R_i)$ Rank of 'i'th root cause node in backward steps

$O(S_i)$ Rank of 'i'th symptom node in backward steps

$Ord_k(i)$ A ranked set composed of RPC's of 'i'th failure obtained from 'K' experts

$P_{ik}(f_j)$ the relative level of impact of 'i'th failure to 'j'th criterion obtained from 'k'th expert

$R(F_i)$ Rank of 'i'th failure node in forward steps

$R(R_i)$ Rank of 'i'th root cause node in forward steps

$R(S_i)$ Rank of 'i'th symptom node in forward steps

$Rnd[n]$ Nearest integer value to a number,'n'

RPC_{ik} Compensated Risk Priority Category (RPC) of the relative impact of the 'i'th failure to the system according to 'k'th expert's opinion

$RPC_{[ik]}$ 'k'th maximum compensated Risk Priority Category of the relative impact of the failure 'i' to the system

S A set of fuzzy linguistic variables (relative importance of decision factor or relative failure impact)

S_ξ Fuzzy linguistic variable for defining the relative failure impact to risk criteria or the relative importance of the functions/subsystems

$Set_k(i)$ A set composed of RPC's of 'i'th failure obtained from 'K' experts

$T(i)$ Total rank of 'i'th node

V_{ijk} Index of RPC factor for 'i'th failure, 'j'th risk criterion, and 'k'th expert

$Y(x)$ A set of all nodes that are not ranked until step x in Gs or gs

$Z(x)$ A subset of set Y(x) includes starting nodes (symptoms, root causes, failures) in step x. Starting node is a node with no entry

Ψ Maximum number of categories

ξ Index for fuzzy variable $\xi=1,2,\ldots,\Psi$

Chapter Nomenclature

ς Index for fuzzy variable $\varsigma=1,2,\ldots,\Psi$

α_{jk} Index of importance category of 'j'th risk criterion which is assigned by 'k'th expert

β_{ijk} Index of 'impact category of i'th failure to 'j'th risk criterion, which is assigned by 'k'th expert

γ Degree of aggregation

Chapter 3:

$\tilde{\leq}$ Fuzzy less than or equal operand that means left hand side is less than or equal right hand side plus certain tolerance

$\tilde{\geq}$ Fuzzy greater than or equal operand that means left hand side is greater than or equal right hand side minus certain tolerance

$E[R_i(t)]$ Expected value of the reliability of the unit 'i'

i Unit index, i=1,…,n

n Total number of units

$\tilde{Pr}ob(\,)$ Fuzzy probability function

r_i Estimated reliability for unit i

R_P Reliability of parallel network

R_S Reliability of series network

$\tilde{R}_i(t)$ Fuzzy reliability function of the unit 'i'

λ_i Failure rate of the unit i

μ_{λ_i} Mean of the failure rate of the unit i

σ_{λ_i} Standard deviation of the failure rate of the unit i

Chapter Nomenclature

$\mu_{R_i(t)}$ Membership function of fuzzy function $\tilde{R}_i(t)$

μ_{r_i} Mean of estimated reliability for unit i

σ_{r_i} Standard deviation of estimated reliability for unit i

Δ_{r_i} Tolerance for estimation of reliability of unit i

α_i Probability that fuzzy probability function of unit 'i' to meet its goal

Δ_{α_i} Tolerance of probability for fuzzy probability function of unit 'i' to meet its goal

$\Phi(\ )$ Cumulative distribution function of normal distribution

Chapter 4:

ASIC: Specific Integrated Circuits

e_{ij} Event ij in FTA

E_{ij} Self-loop basic event j under event i

fg_{ij} Time distribution function of event ij to be generated

fr_{ij} Time distribution function of event ij to be recovered

fu_{ij} Time distribution function that event ij effects on upper event i

FPGA: Field Programmable Gate Arrays

FTA: Fault Tree Analysis

i Upper event index, i=1,…,I

ij Event index

j Lower event index, j=1,…,J

Mg_{ij} MGF of time distribution function that event ij is generated

Mr_{ij} MGF of time distribution function that event ij is recovered

Mu_{ij} MGF of time distribution function that event ij effects on upper event i

MGF: Moment Generating Function

Chapter Nomenclature

PE_i	Probability of occurrence of self-loop basic event i
Pg_{ij}	Probability that event ij happens
Pr_{ij}	Probability that event ij happens and to be detected by detection means
Pu_{ij}	Probability that event ij happens and not to be detected by detection means
t	Time of transmittance of the event that is a random variable
TP:	Topological Equation
W_i	Equivalent transmittance of top Event i
W_{ij}	Equivalent transmittance of self-loop basic event ij
Wg_{ij}	Transmittance of generation link of event ij
Wr_{ij}	Transmittance of recovery link of event ij
Wu_{ij}	Transmittance of non-detected link of event ij
$\mu(e_{ij})$	Membership function of probability occurrence of Event ij in Fuzzy sets , $1 \geq \mu(e_{ij}) \geq 0$. Also it can be denoted by μ.
$\tilde{\gamma}$	Fuzzy triangular number for degree of nearness to strict logical 'AND' and 'OR'
γ_l	Lower Fuzzy degree of nearness to strict logical 'AND' and 'OR'
γ_m	Most likely Fuzzy degree of nearness to strict logical 'AND' and 'OR'
γ_u	Upper Fuzzy degree of nearness to strict logical 'AND' and 'OR'
μ_i	Mean time to occurrence of the event i that can also be denoted by $E_i(t)$ or μ_{Ei}
σ_i	Standard deviation time to occurrence of the event i
μg_{ij}	Mean time of generation of the failure in event ij
μr_{ij}	Mean time of occurrence of detectable failure in event ij
μu_{ij}	Mean time of occurrence of un-detectable failure in event ij
σg_{ij}	Standard deviation of time of generation occurrence of the event ij
σr_{ij}	Standard deviation of time of detection occurrence of the event ij
σu_{ij}	Standard deviation of time of un-detection occurrence of the event ij

Chapter Nomenclature

Chapter 5:

$E(t^2)$ Second moment generate function of unit 'i'

$E^{k/n}(t^2)$ Second moment generating of k-out-of-n:G system

fu_i Time distribution function that a failure in unit 'i' effects the system

$fu_{ij}(t)$ Transition time distribution function of unit 'u' from state 'i' to state 'j'

fr_i Time distribution function that a failure will be recovered in unit i

fg_i Time distribution function that a failure will be generated in unit i

i Unit index, i=1,…,I

i,j State indices, i/j=1,…,r

j_{sys} System state indices, j_{sys}=1,…,r'

MGF Moment generating function

$M_{fi(t)}(S)$ MGF of distribution function of unit 'i' (e.g., $f_i(t)$)

$M_{ij}^{u}(S)$ MGF of distribution function of transition time from state 'i' to state 'j' of unit 'u'

M_{gj} MGF of f_{gi}

M_{ri} MGF of f_{ri}

M_{ui} MGF of f_{ui}

$MTTS_j$ Mean Time To unit/system being in State 'j'

P_i Probability that unit 'i' fails, which can also be denoted by 'P' for identical units

$P^{k/n}$ Probability of a k-out-of-n:G system failure

$P^{k/n}_j$ Probability of Multi-State k-out-of-n system being in stare 'j'

P_{ui} Probability that unit i fails and will not be detected

P_{uij} Transition probability from state 'i' to state 'j' of unit

P_{ri} Probability that unti 'i' fails and will be detected

P_{gi} Probability of the failure generation in unit 'i'

STTF Standard deviation time to system failure (STTF)

Chapter Nomenclature

$STTS_j$	Standard deviation Time To unit/system being in State 'j'
S_{uj}	State of unit 'u' in 'j'
t	Time of occurrence of failure that is a random variable
t_{ij}	Time of transition from state'i' to state'j' that is a random variable
TP	Topological Equation
u	index of unit u=1,2, …,n
U_i	Self-Loop unit 'i'
W_{gi}	Transmittance of generation failure in unit i
W_{ri}	Transmittance of recovery failure in unit i
W_{ui}	Transmittance of non-detected failure in unit i
W_{uij}	Transmittance of transition from state 'i' to state 'j' in unit 'u'
W_{uj}	Transmittance of state 'j' in unit 'u'
W_i	Equivalent transmittance of unit i
W_p	Transmittance of the parallel system
W_{pj}	Transmittance of the parallel system being in stare 'j'
$W^{k/n}$	Transmittance of k-out-of-n:G system failure
$W^{k/n}{}_j$	Transmittance of state 'j' in Multi-state k-out-of-n systems /Transmittance of k-out-of-n systems failure being in stare 'j'
W_s	Transmittance of the series system
W_{sj}	Transmittance of the series system being in stare 'j'
μ_s	Mean time to failure of the series system that can also be denoted by Es(t)
σ_s	Standard deviation time to failure of the series system
μ_p	Mean time to failure of the parallel system that can also be denoted by Ep(t)
σ_p	Standard deviation time to failure of the parallel system
$\mu^{k/n}$	Mean time to failure of k-out-of-n:G system that can also be denoted by Ek/n(t)

Chapter Nomenclature

$\sigma^{k/n}$	Standard deviation time to failure of k-out-of-n:G system
μ_i	Mean time to failure in unit 'i' that can also be denoted by Ei(t)
σ_i	Standard deviation time to failure in the unit 'i' (STTF)
μ_{gi}	Mean time to generation of the failure occurrence of the unit i
μ_{ri}	Mean time to detectable failure occurrence of the unit i
μ_{ui}	Mean time to non-detectable failure occurrence of the unit i
σ_{gi}	Standard deviation of time of generated failure occurrence of the unit i
σ_{ri}	Standard deviation of time of detected failure occurrence in unit i
σ_{ui}	Standard deviation of time of un-detected failure occurrence in unit i

Chapter 6:

$\tilde{\leq}$	Fuzzy less than or equal operand that means left hand side is less than or equal right hand side plus certain tolerance
$\tilde{\geq}$	Fuzzy greater than or equal operand that means left hand side is greater than or equal right hand side minus certain tolerance
$E[R_i(t)]$	Expected value of the reliability of the unit 'i'
i	Unit index, i=1,…,n
n	Total number of units
$\tilde{Pr}ob()$	Fuzzy probability function
r_i	Estimated reliability for unit i
R_P	Reliability of parallel network
R_S	Reliability of series network
$\tilde{R}_i(t)$	Fuzzy reliability function of the unit 'i'
λ_i	Failure rate of the unit i

Chapter Nomenclature

μ_{λ_i} Mean of the failure rate of the unit i

σ_{λ_i} Standard deviation of the failure rate of the unit i

$\mu_{R_i(t)}$ Membership function of fuzzy function $\widetilde{R}_i(t)$

μ_{r_i} Mean of estimated reliability for unit i

σ_{r_i} Standard deviation of estimated reliability for unit i

Δ_{r_i} Tolerance for estimation of reliability of unit i

α_i Probability that fuzzy probability function of unit 'i' to meet its goal

Δ_{α_i} Tolerance of probability for fuzzy probability function of unit 'i' to meet its goal

$\Phi(\,)$ Cumulative distribution function of normal distribution

$N(\mu,\sigma)$ Normal distribution with mean μ and standard deviation σ

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CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION

The reliability of physical devices and downloaded software plays a vital role in the down time and availability of a system. Down time and availability of the systems are known as serviceability performance. This term refers to the ability of a service to be obtained when requested, for a predetermined duration, under the required quality of the service.

To obtain high serviceability performance, the design team and reliability professionals employ a variety of tools and techniques to control, manage, and to improve the probability of failure in the devices, the equipment, and the systems vis-à-vis time and cost constraints during the design phases as shown in Figure 1.1

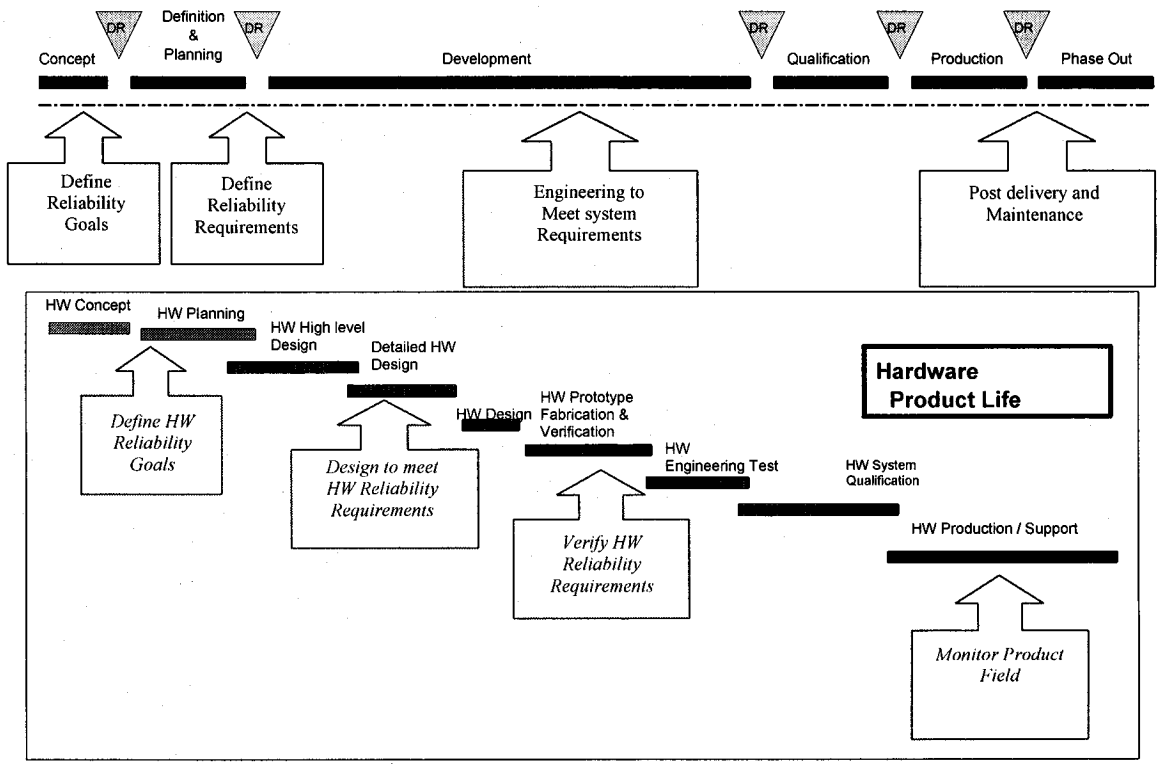

Figure 1.1. Product Development Life cycle

The high level design phase defines the structure of the system in terms of its components, their interfaces, and component functionality via functional block diagram. The detailed design phase, however, describes the internal structure of the system identified in the high level design by logic flow, schematic, etc. The reliability techniques assess quantitative measures of Reliability, Availability, Maintainability and Safety (RAMS) of the systems. The assessment of either the mechanical or electrical systems is continuously performed during the high level design and the detailed design phases. Notwithstanding, the techniques would appear formidable to RAMS because they possess certain weaknesses. These weaknesses return to either a mathematical assumption underlying them, or a fuzziness of the system and of the input data. Despite the demanding mathematical techniques used in RAMS, much of the efforts in performing RAMS on the system are extended in understanding and demonstrating the architecture of the system. RAMS analyses on the system are performed by utilizing the graphical methods, Failure Mode and Effects Analysis (FMEA), Failure Risk Assessment, Fault Tree Analysis (FTA), and redundancy modeling (i.e., k-out-of-n systems). These methods are used by reliability professional to present the system architecture and operation.

Graphical methods, such as the functional block diagram (FBD), the reliability block diagram (RBD), and the state space diagram (STD) are very useful for describing the system, and then transforming the system information into a more useful form to analyze and to make a decision. The RBD shows the flow of data, of control or of transmission throughout the system, and describes how the various components are interrelated with one another. The RBD is commonly used to represent the reliability architecture of the system. However, for most systems, there are modes of the operation that extend beyond whether the system is working or has failed. Each possible mode of the operation for a system is called a state space. The STD is a picture of all different possible states of the operations of the system. To compare RBD and STD, the RBD is useful in representing the effects of individual component on the system reliability. On the other hand, the STD is useful in representing the interdependence between the behaviors of the components. Both diagrams suffer from system complexity and uncertainty of either a transition state or a state space.

FMEA at the functional block level is the key to successfully achieving the desired objectives of the high level design. Generally, The FMEA is employed prior to the hardware specification being released and would be a base for Failure Mode and Effects, Criticality Analysis (FMECA). Given a functional block diagram, FMEA may be performed via the total or partial failures associated with the functional block of interest. Due to the presence of complexity associated with the combination of the composite failure modes in the block diagram, their severity should be categorized qualitatively. To compare FMEA and FTA, FMEA takes less time and cost to develop, and has less comprehensive analysis capabilities.

The FTA is one of the most widely used techniques by reliability professional that is applicable for risk assessments, the one shot mission finite, and clearly identifiable top events. The new advances in FTA are the ability to handle the sequential failures, common cause failures, and time dependent failure rates. Use of the binary decision diagram or Petri Nets give significant analysis power to FTA. Although these techniques lead the design process toward meeting the requirements, sometimes there is no justifiable way for improving the design reliability because of either lack of knowledge, or the shortage of time or cost. Consequently, the redundancy techniques, including k-out-of-(n+m) systems, are posed as the alternatives to meet the requirements.

The redundancy model means to configure the system with more than two devices, interfaces, services, or systems for one task. The redundant devices, and systems must be the same in functionality and might be different in the reliability measurements i.e. failure rate. Redundancy would be implemented in two ways- warm redundancy and hot redundancy with the general structure of k-out-of-(n+m):G/(H/W). In this structure, n indicates the number of the parallel units, and the 'm' is the number of standby units. The units mandate to operate the same function but they may not have the same failure rate. If the 'k' is equal to 'n', it means the structure operates like a serial structure that needs all units to work well. Otherwise, any assigned value for k less than n+1 would take the advantages of the parallel network with high reliability. The G/(H/W) shows all units are good and sane, and how the back up system works. The hot backup means all redundant units operate simultaneously, and the supervision system decides which ones should be online or offline shown by 'H'. Contrary to the hot backup, the warm backup keeps the redundant unit inactive until one of

the active units fails. A lot of studies that are classified into binary and multi-state k-out-of-n systems have been conducted on redundancy.

Because of the stochastic or fuzzy reliability input data, this thesis focuses on the reliability design techniques used by reliability professionals to improve the serviceability performance of the systems. These include fuzzy group failure effects analysis, fuzzy failure risk analysis, fuzzy Markov chains, stochastic and fuzzy FTA with self-loop basic events, Binary and Multi-state k-out-of-n systems and reversible multi-state k-out-of-n systems.

1.2 LITERATURE ANALYSIS

In recent years, because the avionics, transit, and telecommunication systems have grown larger and some have become too complex and sophisticated, careful attention has been given to RAMS analysis of the fields. Thus, design reliability techniques have been extensively studied in the literature over the years. The large number of publications on this topic precludes a comprehensive review. Only influential studies are reviewed in this thesis. The collected publications listed include conference proceedings, journals, workshops, and books. Table 1.1 illustrates the classification of references and Table 1.2 presents sources of journal and conference proceedings papers.

Table 1.1. Classification of publications on reliability design

- ***Failure analysis:***
  - Analytical:1-2, 1-29, 1-37, 1-91, 1-107, 1-114, 1-121, 1-132, 1-134
  - Descriptive:1-1, 1-3, 1-5, 1-6, 1-7, 1-8, 1-9, 1-1 1-11, 1-12, 1-13, 1-14, 1-15, 1-16, 1-17, 1-18, 1-19, 1-2 1-21, 1-23, 1-24, 1-25, 1-26, 1-27, 1-28, 1-3 1-31, 1-32, 1-33, 1-34, 1-35, 1-36, 1-38, 1-39, 1-4 1-42, 1-43, 1-44, 1-45, 1-46, 1-48, 1-49, 1-51, 1-52, 1-53, 1-54, 1-56, 1-57, 1-58, 1-59, 1-6 1-61, 1-62, 1-63, 1-64, 1-65, 1-66, 1-67, 1-68, 1-69, 1-72, 1-74, 1-75, 1-76, 1-77, 1-78, 1-79, 1-81, 1-82, 1-83, 1-84, 1-86, 1-87, 1-88, 1-89, 1-9 1-92, 1-93, 1-94, 1-95, 1-99, 1-103, 1-105, 1-106, 1-108, 1-109, 1-11 1-111, 1-115, 1-116, 1-117, 1-118, 1-119, 1-12 1-124, 1-127, 1-128, 1-133
  - Metaheuristic:1-71, 1-101, 1-102
  - Simulation:1-22, 1-41, 1-47, 1-104
  - Knowledge based system:1-5 1-55, 1-7 1-73, 1-8 1-85, 1-96, 1-97, 1-98, 1-10 1-112, 1-125

- Decision Making:1-4, 1-113, 1-122, 1-123, 1-126, 1-129, 1-13 1-131

- ***Markov model:***
  - Analytical:2-1, 2-2, 2-3, 2-4, 2-6, 2-7, 2-8, 2-9, 2-1 2-11, 2-13, 2-14, 2-15, 2-16, 2-17, 2-18, 2-19, 2-2 2-21, 2-22, 2-23, 2-25, 2-26, 2-28, 2-3 2-31, 2-32, 2-33, 2-34, 2-35, 2-37, 2-38, 2-39, 2-4 2-41, 2-43, 2-44, 2-46, 2-47, 2-50, 2-52, 2-54
  - Descriptive:2-12, 2-24
  - Metaheuristic:2-27, 2-29, 2-48, 2-53
  - Simulation:2-5
  - Knowledge based system:-
  - Decision Making:2-36, 2-42, 2-45
- ***Fault tree analysis*:**
  - Analytical:3-1, 3-2, 3-6, 3-15, 3-31, 3-32, 3-33, 3-36, 3-37, 2-1 3-42, 3-43, 3-45, 3-46, 3-47, 3-48, 3-52, 3-63, 3-67, 3-69, 3-78, 3-83, 3-85, 3-88, 3-89, 3-91, 3-94, 3-105, 3-106, 3-118, 1-106, 3-123, 2-44, 3-125, 3-129, 2-50, 3-133, 3-138
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  - Decision Making:3-14, 3-41, 3-64, 3-65, 3-79, 3-119
- ***k-out-of-n systems:***
  - Analytical:4-2, 4-3, 4-4, 4-7, 4-8, 4-9, 4-1 4-11, 4-12, 4-14, 4-15, 4-16, 4-21, 4-22, 4-23, 4-24, 4-25, 4-26, 4-27, 4-29, 4-3 4-33, 4-38, 4-4 4-41, 4-42, 4-43, 4-44, 4-45, 4-46, 4-49, 4-52, 3-32, 4-53, 4-54, 4-55, 4-56, 4-57, 4-58, 4-59, 4-6 4-63, 4-68, 4-7 4-71, 4-74, 4-78, 4-84, 4-87, 4-9 4-91, 4-92, 4-94, 4-97, 4-99, 4-10 4-109, 4-112, 4-115, 4-118, 4-119, 4-12 4-126, 4-13 4-132, 4-133, 4-135, 4-137, 4-139
  - Descriptive:4-1, 4-6, 4-34, 4-39, 4-48, 4-51, 4-65, 4-69, 4-77, 4-8 4-81, 4-83, 4-95, 4-103, 4-106, 4-107, 4-114, 4-121, 4-125, 4-129, 4-134, 4-138, 4-140
  - Metaheuristic:4-5, 4-17, 4-18, 4-19, 4-2 2-6, 4-32, 4-35, 4-36, 4-37, 4-5 4-61, 4-62, 4-64, 4-66, 4-67, 4-72, 4-73, 4-75, 4-76, 4-79, 4-82, 4-85, 4-88, 4-

89, 4-93, 4-96, 4-102, 4-104, 4-105, 4-117, 4-122, 4-127, 4-128, 4-141, 4-142
    - Simulation:4-28, 4-31, 4-98, 4-111, 4-136
    - Knowledge based system:4-13, 4-123
    - Decision Making:4-47, 4-86, 4-101, 4-108, 4-11 4-113, 4-116, 4-124, 4-131
- ***Reliability estimation:***
    - Analytical:5-1, 5-2, 5-3, 5-9, 5-1 5-11, 5-13, 5-17, 5-23, 5-27, 5-33, 5-36, 5-37, 5-38, 5-41, 5-49, 5-51, 5-52, 5-54, 5-56, 5-58, 5-59, 5-61
    - Descriptive:5-19, 5-26, 5-48
    - Metaheuristic:5-4, 5-5, 5-6, 5-8, 5-16, 5-18, 5-2 5-21, 5-25, 5-28, 5-29, 5-3 5-31, 5-39, 5-4 5-42, 5-43, 5-44, 5-45, 5-46, 5-53, 5-55, 5-60
    - Simulation:5-7, 5-15, 5-32, 5-34, 5-50
    - Knowledge based system:5-12, 5-14, 5-22, 5-24, 5-47, 5-57
    - Decision Making:5-35

Table 1.2. Sources of the most journal and conference proceeding papers listed in the references

Journals

- Asia-Pasific Journal of Operational Research.
- Bioinformatic.
- Biometrika.
- Chemical Engineering
- Computer Magazines
- Computer Assurance (COMPAS)
- Computer Integrated Manufacturing Systems
- Computers and Operations Research
- Comptes Rendus Mathematique.
- Electronics Letters
- Engineering Structures
- European Journal of Operational Research
- Fuzzy Sets and Systems
- Global Telecommunications Conference (GLOBECOM '94)
- IEE Colloquium on Applications of Expert Systems in Road Transportation
- IEE Colloquium on Applications of Model-Based Reasoning
- IEE Colloquium on Artificial Intelligence in Simulation
- IEE Colloquium on Computer Aided Engineering of Automotive Electronics
- IEE Colloquium on Radiation Protection - the Role of Safety-Related Control Systems
- IEE Colloquium on Systems Engineering of Aerospace Projects
- IEE Colloquium on the Electrical System of the Jaguar XK8 (Digest No.: 1996/281)
- IEEE Aerospace Conference
- IEEE Journal on Selected Areas in Communications
- IEEE Journals on Selected areas in communication
- IEEE Software
- IEEE Telecommunication Magazine
- IEEE Transactions on Acoustics Speech and Signal Processing
- IEEE Transactions on Aerospace and Electronic Systems
- IEEE Transactions on Applied Superconductivity
- IEEE Transactions on Communications

- IEEE Transactions on Computers
- IEEE Transactions on Engineering Management
- IEEE Transactions on Industry Applications
- IEEE Transactions on Nuclear Science
- IEEE Transactions on Parallel and Distributed Systems
- IEEE Transactions on Power Delivery
- IEEE Transactions on Reliability
- IEEE Transactions on Software Engineering
- IEEE Transactions on System MAN and Cybernetic- part B
- IEEE Transactions on System Man and Cybernetics
- IEEE Transactions on Vehicular Technology
- IIE Transactions
- Information and Computation.
- Information Science 96
- Intelligent Systems Engineering
- International Conference on Acoustics Speech and Signal Processing (ICASSP-95)
- International Journal of Quality and Reliability Management
- Journal of Applied Probability
- Journal of Applied Reliability
- Journal of Computer and System Sciences
- Journal of Loss Prevention in the Process Industries.
- Journal of Industrial Engineering
- Journal of Orofacial Orthopedics / Fortschritte der Kieferorthopädie
- Microelectronics and Reliability
- Power Engineering Journal
- Quality and Reliability Engineering International
- Reliability Engineering and System Safety

Conference Proceedings and workshops

- AFRICON '92 Proceedings 3rd AFRICON Conference
- Annual Proceedings of the 11th IEEE VLSI Test
- Annals of Combinatorics
- Symposium-Digest of Papers
- Annual Proceedings of the 21st International Computer Software and Applications Conference (COMPSAC '97)
- Annual Proceedings of the 6th IEEE International Conference on Wafer Scale Integration
- Annual Proceedings of the 8th IEEE International Conference on Innovative Systems in Silicon
- Artificial Intelligence in Medicine
- Fuzzy Information and Decision Processes- North-Holland- Amsterdam
- Fuzzy Information Process Society- Annual Meeting of the North America
- Fuzzy Information Processing Society NAFIPS '97 Annual Meeting of the North American
- IEEE First International Conference on Algorithms and Architectures for Parallel Processing (ICAPP 95)
- IEEE International Conference on Communications
- IEEE International Conference on Communications (ICC '93) Geneva
- IEEE International Conference on Computer-Aided Design (ICCAD-89)-Digest of Technical Papers
- IEEE International Conference on Intelligent Processing Systems (ICIPS '97)
- IEEE International Conference on Neural Networks

- IEEE International Conference on Systems Man and Cybernetics
- IEEE International Conference on Wafer Scale Integration
- IEEE International Joint Conference on Neural Networks
- IEEE Proceedings of Southeast Con '91
- IEEE Systems Readiness Technology Conference. 'Advancing Mission Accomplishment' Conference Record (AUTOTESTCON '90)
- International Conference on Electric Power Engineering- PowerTech Budapest 99.
- International Symposium on Electronic Materials and Packaging (EMAP 2000)
- Joint 9th IFSA World Congress and 20th NAFIPS International Conference
- Oceans '02 MTS/IEEE
- Portland International Conference on Management and Technology (PICMET)- Innovation in Technology Management - The Key to Global Leadership
- Pre Conference Proceedings of IEEE International Engineering Management Conference on Managing Projects in a Borderless World
- Proceedings American Control Conference
- Proceedings International Symposium on Product Quality and Integrity Annual
- Proceedings of Aerospace Applications Conference Part: 1
- Proceedings of Design Automation and Test in Europe Conference and Exhibition
- Proceedings of Electrotechnical Conference 'Integrating Research Industry and Education in Energy and Communication Engineering' (MELECON '89) Mediterranean
- Proceedings of IEE Computers and Digital Techniques
- Proceedings of IEEE Aerospace Conference
- Proceedings of IEEE High-Assurance Systems Engineering Workshop
- Proceedings of IEEE International Computer Performance and Dependability Symposium
- Proceedings of IEEE International Conference on Fuzzy Systems (International Joint Conference of the Fourth IEEE International Conference on Fuzzy Systems and The Second International Fuzzy Engineering Symposium)
- Proceedings of IEEE International Conference on Systems Man and Cybernetics (IEEE SMC '99)
- Proceedings of IEEE International Symposium on Defect and Fault Tolerance in VLSI Systems
- Proceedings of IEEE International Symposium on Information Theory
- Proceedings of IEEE International Symposium on Semiconductor Manufacturing
- Proceedings of IEEE International Workshop on Defect and Fault Tolerance in VLSI Systems
- Proceedings of Institute of Electrical and Electronics Engineers International Conference on Security Technology Crime Countermeasures
- Proceedings of International Canadian Reliability and Maintainability Symposium
- Proceedings of International Conference on Dependable Systems and Networks
- Proceedings of International Conference on Image Processing

- Proceedings of International Conference on Network Protocols
- Proceedings of International Conference on Systems Man and Cybernetics- 'Systems Engineering in the Service of Humans'
- Proceedings of International Symposium on Principles of Software Evolution
- Proceedings of International Test Conference
- Proceedings of TENCON 2000
- Proceedings of the 10th IEEE Pacific Rim International Symposium on Dependable Computing
- Proceedings of the 10th International Conference on Software Engineering
- Proceedings of the 10th International symposium on Software Reliability Engineering
- Proceedings of the 11th IEEE/AIAA Digital Avionics Systems Conference
- Proceedings of the 11th International Symposium on Software Reliability Engineering (ISSRE)
- Proceedings of the 12th IEEE Computer Security Foundations Workshop
- Proceedings of the 12th International Computer Software and Applications Conference (COMPSAC 88)
- Proceedings of the 13th IEEE Symposium on Fusion Engineering
- Proceedings of the 13th International Conference on Distributed Computing Systems
- Proceedings of the 14th International Telecommunication, Energy Conference
- Proceedings of the 17th AIAA/IEEE/SAE Digital Avionics Systems Conference (DASC)
- Proceedings of the 17th AIAA/IEEE/SAE Digital Avionics Systems Conference (DASC.)
- Proceedings of the 17th International Conference on VLSI Design
- Proceedings of the 18th Annual International Phoenix Conference on Computers and Communications Conference
- Proceedings of the 18th Digital Avionics Systems Conference
- Proceedings of the 18th International Conference on Software Engineering
- Proceedings of the 18th International Symposium on Software Reliability Engineering
- Proceedings of the 19th Annual International Computer Software and Applications Conference (COMPSAC)
- Proceedings of the 19th International Electronic Manufacturing Technology Symposium Competitive Manufacturing for the Next Decade (IEEE/CHMT)
- Proceedings of the 1st International Conference on Software Testing, Reliability and Quality Assurance
- Proceedings of the 20th International Computer Software and Applications Conference (COMPSAC '96).
- Proceedings of the 21st IEEE Conference on Local Computer Networks
- Proceedings of the 27th IEEE Conference on Decision and Control
- Proceedings of the 2nd Annual IEEE Symposium on Computer-Based Medical Systems
- Proceedings of the 31st Annual Symposium on Foundations of Computer Science
- Proceedings of the 35th Conference on Decision and Control
- Proceedings of the 3rd Asian Test Symposium

- Proceedings of the 3rd Electronics Packaging Technology Conference (EPTC)
- Proceedings of the 3rd IEEE Conference on Fuzzy Systems on Computational Intelligence
- Proceedings of the 3rd IEEE International High-Assurance Systems Engineering Symposium
- Proceedings of the 3rd International Conference on Wafer Scale Integration
- Proceedings of the 3rd International Symposium on Software Reliability Engineering
- Proceedings of the 3rd World Congress on Intelligent Control and Automation
- Proceedings of the 49th Electronic Components and Technology Conference
- Proceedings of the 4th International Software Metrics Symposium
- Proceedings of the 4th International Workshop on Object-Oriented Real-Time Dependable Systems
- Proceedings of the 5th International Symposium on Software Reliability Engineering
- Proceedings of the 5th International Workshop on Computer-Aided Software Engineering
- Proceedings of the 8th International Symposium On Software Reliability Engineering
- Proceedings of the Annual Reliability and Maintainability Symposium
- Proceedings of the Annual Reliability and Maintainability Symposium on Product Quality and Integrity
- Proceedings of the Eighth International Symposium on Software Reliability Engineering
- Proceedings of the First International Symposium On Environmentally Conscious Design and Inverse Manufacturing (EcoDesign)
- Proceedings of the First Regional Conference on Engineering in Medicine and Biology Society and 14th Conference of the Biomedical Engineering Society of India
- Proceedings of the Fourth Annual Conference on Computer Assurance (COMPASS)- 'Systems Integrity Software Safety and Process Security'
- Proceedings of the IEEE Aerospace and Electronics Conference (NAECON 1996)
- Proceedings of the IEEE International Conference on Management of Innovation and Technology (ICMIT)
- Proceedings of the IEEE International Conference on Neural Networks
- Proceedings of the IEEE International Symposium on Electronics and the Environment (ISEE)
- Proceedings of the IEEE National Aerospace and Electronics Conference (NAECON)
- Proceedings of the institution of radio engineers 50
- Proceedings of the International Conference on Dependable Systems and Networks
- Proceedings of the International Symposium and Workshop on Systems Engineering of Computer Based Systems
- Proceedings of the IPMU 2000
- Proceedings of the Second International Conference on Requirements Engineering
- Proceedings of the Sixth Annual Conference on Computer Assurance (COMPASS) Systems Integrity Software Safety and Process Security

- Proceedings of the Software Engineering Standards Symposium
- Proceedings of the TENCON
- Proceedings of the Third IEEE Conference on Fuzzy Systems on Computational Intelligence
- Proceedings of the Thirty Fifth Meeting of the IEEE Holm Conference on Electrical Contacts
- Proceedings of the Workshop on Application of Interval Analysis to Systems and Control Girona Spania
- Proceedings Workshop on Industrial-Strength Formal Specification Techniques
- Proceedings Workshop on the Management of Replicated Data
- The 14th IEEE International Conference on Automated Software Engineering
- The 15th AIAA/IEEE Digital Avionics Systems Conference
- The 16th AIAA/IEEE Digital Avionics Systems Conference (DASC)
- The 19th International Symposium on Fault-Tolerant Computing
- The 23rd IEEE/CPMT Electronics Manufacturing Technology Symposium
- The 23rd International Symposium on Fault-Tolerant Computing (FTCS-23) Digest of Papers
- The 25th International Symposium on Fault-Tolerant Computing (FTCS-25)
- The 25th International Symposium on Fault-Tolerant Computing- Highlights from Twenty-Five Years
- The 27th Asilomar Conference on Signals Systems and Computers Conference
- The 28th Annual International Symposium on Fault-Tolerant Computing-Digest of Papers.
- The 29th Annual International Symposium on Fault-Tolerant Computing- Digest of Papers.
- The 2nd ed. Boston Kluwer Academic publisher
- The 3rd International Workshop on Integrating Error Models with Fault Injection
- The 44th Annual conference of the Canadian Operation Research Symposium-Toronto
- The 44th IEEE Vehicular Technology Conference
- The 8th Mediterranean Electrotechnical Conference (MELECON '96)
- The Second International Conference on Intelligent Systems Engineering
- Advanced Semiconductor Manufacturing Conference and Workshop (IEEE/SEMI)
- ATM in Europe: the user handbook
- International Integrated Reliability Workshop Final Report
- Kybernetes
- Research & Engineering Center of Ford motor com
- The 35th Intersociety Energy Conversion Engineering Conference and Exhibit (IECEC)

As shown in Table 1.1, the reviewed publications are grouped into five main classes based upon the subject (Reliability technique) as follows:

- Failure analysis,
- Markov model,

- Fault tree analysis,
- k-out-of-n system, and
- Reliability assessment.

Subsequently, in a class, a publication uses a certain method and types of data for a specific application. The breakdowns of method, types of input data, and application are described in Table 1.3.

Table 1.3. A breakdown of publications in a group

Publication attribute	Breakdown
Application	Design improvement, Process improvement, General
Method	Analytical, Descriptive, Meta-heuristic, Simulation, Knowledge-based system, Decision making
Type of input data	Stochastic, Deterministic, Fuzzy number and linguistic variables, General

Figure 1.2 presents several stack charts that show number of publications for the following combinations.

(a) Reliability technique and application,

(b) Reliability technique and method,

(c) Reliability technique and type of input data,

(d) Application and method,

(e) Application and type of input data

(f) Method and type of input data

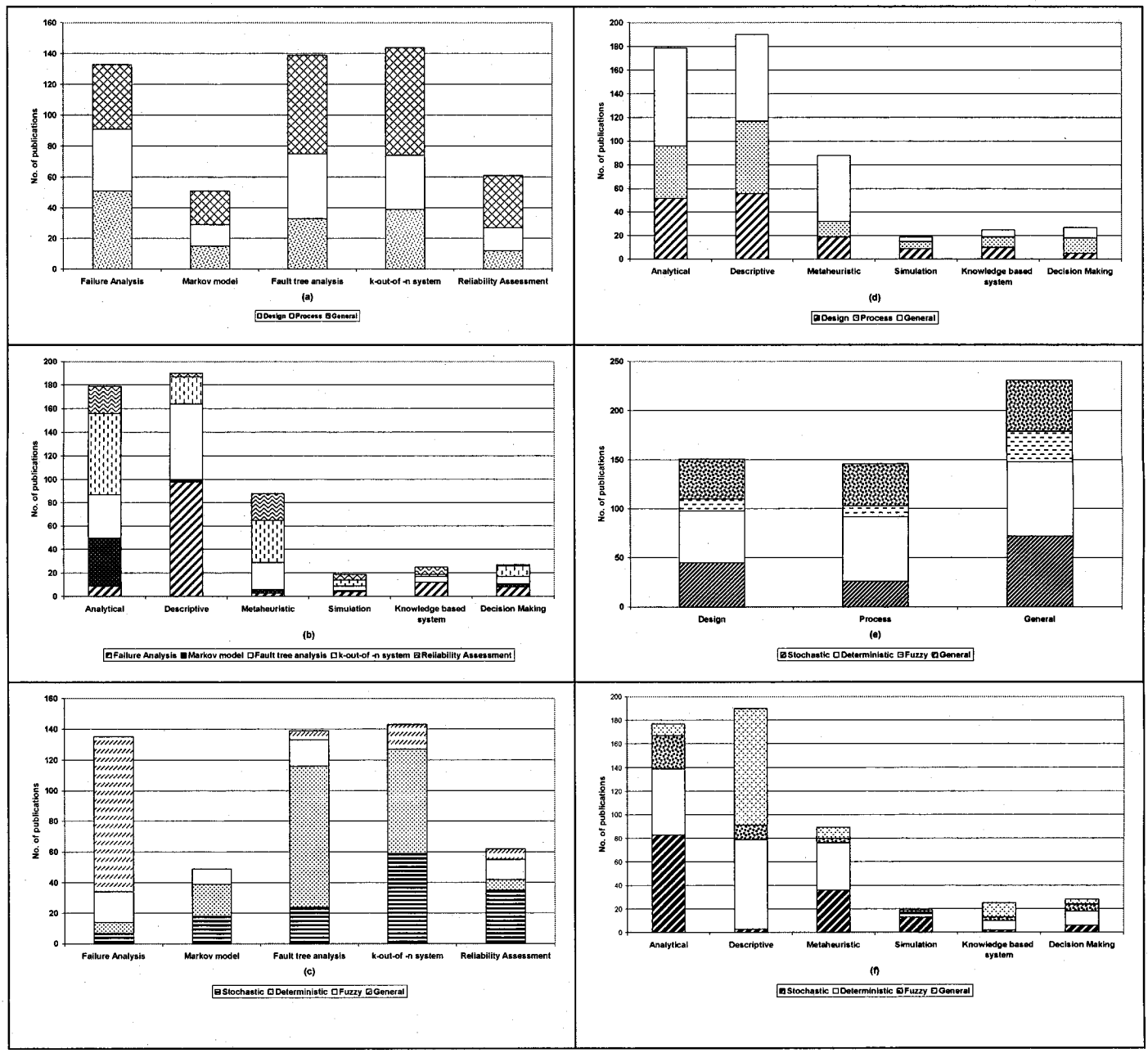


Figure 1.2. Profile of publications: (a) Reliability technique and application, (b) Reliability technique and method, (c) Reliability technique and type of input data, (d) Application and method, (e) Application and type of input data (f) Method and type of input data

This thesis focuses on the reliability design techniques i.e., Failure Analysis (FMEA, Failure Risk Assessment), Markov Models, FTA, k-out-of-n systems, and reliability estimation that can improve the serviceability performance of the system during design phases under

stochastic and fuzzy situations. In the next chapter, a snapshot of and a literature analysis of reliability design techniques have been provided. The snapshots are summary of the literature reviews that depict shortcomings of publications and required attentions on the reliability design techniques.

1.2.1 FAILURE EFFECTS ANALYSIS SNAPSHOT

Failure Effects Analysis (FEA), which results in the physical design improvement, is a fundamental risk analysis process involving information acquisition, modeling, analysis, and decision. In recent years, the published literature pertaining to FEA has been concerned with developing either deterministic or fuzzy models in order to identify, prioritize and eliminate the potential failures in the system. Moreover, some approaches focus on application of decision-making techniques leading to improve the reliability, quality, and safety.

In deterministic models, the Risk Priority Number (RPN) and Pareto Chart (PC) are used as the principal knowledge acquisition to represent and score the system failure effects. The RPN is a product of failure factors (Occurrence probability, Non-detection probability, Severity of impact, Expected cost). These factor values range between '1' and '10'. In [1-50]*, to bring design of Failure Mode and Effects Analysis (FMEA) in line with accepted probabilistic risk management theory, the Society of Automotive Engineers (SAE) has replaced the RPN metric with the Critically Matrix (CM). The CM shows severity of a failure against its probability of occurrences. However, the use of CM alone cannot compensate for the major weaknesses of the SAE approaches and others. Thus, to meet the SAE requirements and provide a robust FMEA model, the Bayesian Network was proposed and called BN-FMEA [1-121].

However, due to the shortcomings of deterministic RPN-based approaches (e.g., BN-FMEA) as pointed out in [1-40, 69, 99], the RPN has been replaced with Risk Priority Category (RPC) for performing fuzzy FEA in the system. Using decision support system, the earliest fuzzy FEA methods are classified as shown in Table 1.4.

In [1-99,123], the fuzzy method based on linguistic variables, Grey theory, and Maximin method was used to determine an RPC to evaluate the risk level of the system. The approaches allow considerable weighting of severity factor associated to a cause of failure. Due to using both linguistic variables for evaluating failure factors and Grey theory to

* [1-50] means Section 1 Reference No.50

prioritize the risk of failure without utility function, the proposed methods can be considered as a breakthrough in FEA. One drawback of these approaches is that the weight of criteria and the scores of the failure causes must be assigned subjectively.

Table 1.4. Fuzzy decision-making methods

Method Category	Method name	Reference
Maximin Methods	Chang et al approach	[1-99]
	Puente et al approach	[1-113]
	Xu et al approach	[1-124]
Linguistic Methods	Kwok et al approach	[1-126]
	Pillary et al approach	[1-128]
	Wang et al approach	[1-131]
TOPSIS Methods	Braglia et al approach	[1-129]
AHP Methods	Bozdag et al approach	[1-130]
Weighting Methods	Umano et al approach	[1-68]
	Wang et al approach	[1-133]
Outranking Methods	Takeda's approach	[1-4]
Conjunction and disjunction Methods	Turksen's approach	[1-127]

Later, in [1-112, 130], the Analytic Hierarchy Process (AHP) technique was proposed to find the preferential weight of the failures in order to determine the failure risk priority number. This approach bases upon three principles: decomposition, comparative judgments, and the synthesis of priorities. It has several shortcomings for failure effects analysis, such as man-made inconsistency in pair-wise comparisons, rank reversal when new failures are introduced. Thus, the knowledge base rule as a new approach for FEA was introduced. In [1-128], using linguistic variables, authors developed the fuzzy single-based expert rules to determine the RPC of the failure. The approach utilizes Grey theory to avoid the use of a utility function. However, FEA is a system technical risk analysis that extracts knowledge about the potential failure mode from a group of experts called an expert knowledge base system with possible conflict in experts' opinions. The knowledge based system uses the knowledge encoded in some form such as rule-based systems, decision tree. Generally, the

construction of a failure effect knowledge base has been carried out by interviewing experts in failure effect and painstakingly translating the experts' opinions into an appropriately structured set of rules (e.g., *if-then*)[1-70]. In [1-124], using min-max function and linguistic variables, authors proposed a system that comprises the expert knowledge base rule and the failure factor interfaces to perform FEA with uncertain and imprecise information. The knowledge maps into one or more *if-then* rule(s). However, the following sources of potential inconsistency may result in conflicting conclusions in the knowledge base system:

- Conflict of rules (i.e., 'if' parts of the rules are similar and 'then' parts are different)
- Subsumption (i.e., two or more rules have the same result, but one contains additional restriction on the situations in which it will succeed.)

In addition, due to the time consuming, complexity of consistency check, and difficulty of maintenance of knowledge base approach, the fuzzy TOPSIS approach for FEA was proposed to avoid the definition of a knowledge base supported by several qualitative rules [1-129]. Though TOPSIS method has some advantages, it suffers from sensitivity analysis because the criterion with the highest score has disproportionate influence in the failure ranking process. On the other hand, in [1-133], to dilute conflict in decision group, authors have presented a conflict resolution model to integrate multiple possibility distribution that can be used in Group-based Failure Effects Analysis (GFEA). The drawback of this model is the use of MINMAX function in aggregation technique, which is not adequate to study the failure effects and failure risk priority when tradeoffs exist among them [1-25]. (see basic concept of group decision making in Appendix-B).

1.2.2 MARKOV MODEL SNAPSHOT

The Markov model is useful for systems with dependent failure and repair modes when components within the system are independent, and when multi-state systems with common failures require analysis. To better describe the principle of Markov chains, consider the system including 'K' states and suppose the transition rate from any state 'i' to any state 'j' is t_{ij}. Thus, by assuming the state of the system at time 't' is denoted by X(t),

probability of moving from any state 'i' to any state 'j' (i.e., p_{ij}) is expressed by $p_{ij}(t) = P(X(s+t) = j \mid X(s) = i)$, where $t \geq 0, s \geq 0$. the matrix P which decribes the state transition is $P = \lfloor p_{ij} \rfloor_{k \times k}$, where each row maximum is one. Thus, $P^n = \lfloor p_{ij}^n \rfloor_{k \times k}$, which means the 'n' fold product of P where p_{ij}^n is the probability of being in state 'j' at step 'n' given that we started in state 'i'. Let $p^{(0)}$ is the initial probability distribution and $p^{(n)}$ is the probability distribution of being in state 'j' after 'n' steps. Thus, we have $p^{(n)} = p^{(0)} P^n$.

Due to uncertain and time-varying input reliability data, the use of fuzzy sets in Markov chains has been considered in some publications. Today the practice of fuzzy Markov chains can be classified to the use of fuzzy probability distribution or of fuzzy possibility distribution in transition matrix. In [2-1]*, the fuzzy probability transition matrix was developed based on fuzzy subsets of [0,1]. In [2-12], authors developed linguistic transition matrix along with developing the basic properties of fuzzy Markov chains. This paper gives a justification of max-min composition of fuzzy matrices for computing P^n. In [2-24], using Markov model and fuzzy transition, authors have developed a multi stage decision process. The fuzzy Markov modeling was used as a technique for analyzing fault tolerant designs under considerable uncertainty in [2-23]. In [2-44], using expert's linguistically evaluation, a fuzzy Markov model was used to demonstrate the state transition of server resources utilization because web server workload forecasting was one of the essential considerations in web server management and network upgrading. In [2-50], a finite fuzzy Markov chains was developed based on possibility theory in accordance with developing a finite horizon Markovian decision process. However, it is widely accepted that one of the major problems of fuzzy possibility distribution is associated with the benchmarking of fuzzy variables in term of membership function.

1.2.3 FAULT TREE ANALYSIS SNAPSHOT

FTA is one of the most widely used reliability design techniques in the system development life cycle. It is applicable for risk assessment in system with a clearly

* [2-1] means Section 2 Reference 1

identified single top event. Since FTA is event-oriented, differentiation between the levels of the events plays a vital role in creating the tree. By defining dependency relationship among the basic events, intermediate, and top events, the reliability measures can be estimated. The literature pertaining to FTA is concerned with sequential failure and common cause failures [3-18]**, using binary decision diagrams [3-119], Meta-heuristic i.e., knowledge-based system, expert system, neural networks, and Petri Nets [3-9], developing symbols [3-125], extending efficient algorithms to perform the FTA [3-37], and application of Fuzzy Sets [3-52]. The knowledge-based system uses the knowledge encoded in some form such as rule-based systems, decision tree. The construction of a fault tree knowledge base has been carried out by interviewing experts and been painstakingly translated the experts' opinions into an appropriately structured set of rules (e.g., if-then). However, there are some sources of potential inconsistency may result in conflicting conclusions in the knowledge base system. Alternatively, Petri Net approach is used for fault tree analysis. This approach like other Meta-heuristic approaches suffers from low accuracy in comparison with analytical approaches, painstakingly modeling of large-scale system, analysis difficulty of large-scale model, and desperately need to computer-aided tools for practical applications. On the other hand, the use of fuzzy sets is criticized because of the benchmarking of fuzzy variables in term of membership function.

1.2.4 K-OUT-OF-N SYSTEM SNAPSHOT

Reliability networks are commonly used to represent the architecture of a system. They are useful to illustrate simple and straightforward approach that illustrates the effects of all possible configurations (e.g., series, parallel, k-out-of-n, and standby) functioning and failed units on the total system. It is a well-known fact that the reliability of a series system is low and, of the parallel system is high with very high cost. Thus, k-out-of-n solution becomes more attractive to use by engineering professionals. They have developed k-out-of-n:G and k-out-of-n:F systems made up of binary or Multi-State units. In the binary k-out-of-n:G system , the system works if and only if at least 'k' units are good. On the other hand,

** [3-18] means Section 3 Reference No. 18

the binary consecutive k-out-of-n:F system fails if and only if at least k consecutive units fail. Some researchers have extended the concepts of the binary k-out-of-n system to the multi-state k-out-of-n system with some limitations. In a Multi-State k-out-of-n:G system, the state of the system remains at certain level if the required number of units stay at that state level or higher. Table 1.5 presents a classification of the methods and configurations pertaining to k-out-of-n systems in the published literature.

Today, the most challenging problem faced by researchers is to develop an efficient method for calculating the reliability of k-out-of-n:G/F systems. In [4-141,122]*, authors developed an efficient algorithm for computing reliability of a linear or circular consecutive k-out-of-n:F system. The efficiency of the algorithm is measured by time complexity function. In [4-117], the author used neural network technique to calculate the system reliability. However, the quality of the solutions depends upon the degree of network parameters required to be tuned. In [4-142], the author developed a model for finding k-out-of-n minimum cut sets. The model is simpler and more efficient than other models because it is based on graph theory. In [4-140], the author has developed a Markov model for a k-out-of-n system with 'M' standby units. Similarly, in [4-132], an analytical model was developed to evaluate the reliability of different type of k-out-of-n:F systems. In [4-120], an analytical model was developed for a load-sharing k-out-of-n:G system with non-identical units. However, this model suffers from the size of system i.e., for n>5. In [4-133], using k-out-of-n:G/F, a decision making model for the furnace maintenance was developed to estimate the remaining life time. In [4-78], the author developed a model for a k-out-of-n:G system with imperfect fault-coverage. The fault-coverage is the probability ratio of the detectable failures to the sum of all potential failures within the system. In [4-83], an analytical approach was developed to estimate the lower bound for the Binary-State k-out-of-n:G system reliability. Although the concept of Binary-State k-out-of-n has been extended to the Multi-State k-out-of-n, all rules may not be applicable to Multi-State k-out-of-n systems. Thus, in [4-127], two types of Multi-State k-out-of-n:G systems were investigated (i.e., increasing and decreasing systems). The increasing Multi-State k-out-of-n:G system is equivalent to that at least k_j units must be at least at state 'j' for the system to be in state 'j' or less where state 'i' is

* [4-141, 122] means Section 4 References 141 and 122

higher than state 'i+1'. The authors developed an analytical model for such case based on the properties of the Binary-State k-out-of-n system. The decreasing Multi-State k-out-of-n system is equivalent to that higher system state level 'j' and lower requirement on the number of units that must be at state 'j' or less. The authors developed an algorithm for evaluating such a system. In [4-111], using the Markov method, the author developed a closed form availability solution for two k-out-of-n systems with 'M' failure modes.

Table 1.5. Classification of published literature on k-out-of-n Systems

Unit Mode(s)	Method	Configuration k-out-of-n:	Achievement	Reference
Binary	Lin's method	F	Computational algorithm	[4-141]
	Chang's et al method	F	Computational algorithm	[4-122]
	Narayan's method	G	Neural Network	[4-117]
	Lee's method	G	Computational algorithm	[4-104]
	Hwang's method	F	Computational algorithm	[4-105]
	Wu's et al method	G	Computational algorithm	[4-85]
Binary	Yeh's method	G	Analytic Model	[4-142]
	Wang's method	G/Standby	Markov Model	[4-140]
	Zuo's method	F	Analytic Model	[4-132]
	Liu's method	G/Load-Sharing	Analytic Model	[4-120]
	Behr's method	G	Analytic Model	[4-114]
	Zuo's method	G/F	Analytic Model	[4-108]
	Newton's method	G	Analytic Model	[4-78]
	Chao's method	F	Analytic Model/ Survey	[4-94]
	Lipow's method	G	Analytic Model	[4-83]
Multi	Huang's et al method	G	Analytic Model	[4-127]
	Moustafa's method	G	Markov Model	[4-111]

1.2.5 RELIABILITY ESTIMATION SNAPSHOT

The reliability estimation of the system composed of subsystems/units/devices has been extensively studied in the literature over the years. In [2-10], the comparison between

reliability networks and basic operations on fuzzy sets is presented for a power generator. In [5-9]*, using fuzzy sets to define '*possibilites*' for the reliability estimation was proposed to reflect the uncertainty in the reliability data by producing a range of possibilities for reliability. In [5-1], the formal interval computation has been proposed since it has several advantages. For example, it allows simulating the whole interval-input space in reasonable time in [5-29]. The interval-input space is the histogram that is described in terms of intervals with an associated probability mass for reliability. The formal interval method used for reliability estimation is based on the interval-input space to be simulated. Although, the use of the interval-input space reduces the size of input space, but it results in the exhaustive simulation time and provides guaranteed bounds for reliability. The concept of propagating "failure probability intervals" rather than fixed values for the probability of unit failure has been shown to be consistent with fuzzy notation and fuzzy trapezoid number representing reliability data in [5-2] (see Definition A-2 in Appendix-A). In [5-29, 25] due to interval-based, estimation provides the guaranteed bounds for reliability, it shows the higher accuracy of the reliability estimation in robot fault tree analysis than that of local estimation approach

1.2.6 FAILURE EFFECTS ANALYSIS LITERATURE REVIEW

In 1981, Herrin [1-1] developed a matrix FMEA technique to provide an organized and traceable analysis from the piece-part failure mode through all indenture levels to system level failure effects.

The following year, in1982, Herrin [1-2] proposed the Matrix FMEA technique to perform a system interface analysis applicable in a telecommunication system. In the same year, Lannes [1-3] performed cost analysis versus reliability of the design that in customer point of view is configuration of the system. Takeda [1-4] used fuzzy outranking relation in multi-criteria decision for failure mode and effects analysis.

In the following year, Kreuze et al [1-5] presented the Built In Test System and its functions along with FMEA derived built in analysis.

* [5-9] means Section 5 Reference No. 9

In 1984, Collett et al [1-6] addressed a logically extension of FMEA in the Built In Test System, which tested on a number of programs and showed a satisfied result and Dussault [1-7] provided a survey on FMEA and performed feasibility study on standardized automated FMEA technique.

Two years later, in 1986, Tyron [1-8] introduced functional circuit analysis as a manual analysis technique that discussed design validation task and FMEA.

In 1988, Bednarz et al [1-9] discussed the systems reliability assessment and efficient analysis for FMEA to improve the FMEA. Strandberg et al [1-10] proposed fault simulation approach for FMEA or FTA as a complementary analysis technique.

In the following year, Raheja [1-11] performed the software analysis to minimize the downtime as well as the vulnerable hardware of circuit system via FMEA and Craig [1-12] presented quality planning issues and its relation with reliability tools, such as FMEA.

Seven publications on the analysis in reliability design appeared in 1990. Lehtela M. [1-13] developed a computer-based method of FMEA to analyze the electronic equipment and circuits with respect to the time consuming manual method. Freeman [1-14] introduced the structured qualification techniques such as FMEA to identify possible hazards in a chemical process. Prasad [1-15] developed FMEA method for improving manufacturing reliability in IC's package assembly since IC's are used in most circuit systems. Hugo et al [1-16] discussed circuit designing from the viewpoint of decision-making. Klein et al [1-17] performed FMEA on the wind turbine circuit and as a result, many changes were made to the hardware. Cambi et al [1-18] extended the reliability approach by defining main function of each subsystem based on FMEA to find the possible failures and related consequence on safety. Poon et al [1-19] addressed the symptom model based approach that correlates the failure symptom with ambiguity group using historical data.

In 1991, six papers appeared on FMEA. Mckinney et al [1-20] urged using FMEA as a remarkable way to identify all catastrophic, critical and safety related failure possibilities. As well, Kara-Zaitri et al [1-21] employed FMEA in design phase based on British standard 5760. Lee et al [1-22] as a part of their project performed FMEA and diagnosis. However, as the traditional FMEA is slow and time inefficient, they designed a qualitative electrical circuit simulator that can model the structure and behavior of a system under analysis.

Citherell [1-23] demonstrated the application of FMEA in chemical engineering. Sexton [1-24] addressed some of the practical problems associated with the application of FMEA and proposed a cost saving alternative method applicable to commercial software. The following year in 1992, eight articles presented various aspects of FMEA in the telecommunication area. McCrea [1-26] as a contractor in Kennedy Space Center performed FMEA to meet the shuttle "fail safe" programs requirements during design phase. Russomanno et al [1-27] developed an expert system for FMEA. Steinke [1-28] implemented the safety standard related to solid state control for household electric ranges by using FMEA. Bell et al [1-29] developed a tool that automates the reasoning portion of FMEA. It is built around a flexible casual reasoning module that has been adapted to the FMEA procedure. Kara-Zaitri et al [1-30] presented a methodology combing the benefits of matrix FMEA and risk priority number technique. Lishou et al [1-31] determined reliability and maintainability of a power station including electrical and electronic circuits using FMEA. Caputo et al [1-32] presented a model to assess the availability of a controlled system by employing both FTA and FMEA. Palumbo [1-33] extended FMEA simulation as an effective way to perform both FMEA and reliability analysis.

The year 1993 witnessed seven publications on FMEA. To assess the safety of embedded real time control system designed for using in automotive application, Goddard et al [1-34] adapted the traditional FMEA technique. Savakoor et al [1-35] brought the feasibility of integrating FMEA and circuit analysis into a comprehensive reliability analysis technique. This study addressed the problem of how to best combine circuit analysis and FMEA to achieve benefit in the design phase. Kukkal et al [1-36] designed a database system for storing FMEA data to improve the quality of analysis and make it more meaningful and visible. It also gave rise to the possibility of developing an integrated tool without database for related reliability analysis. Russomanno et al [1-37] highlighted the benefits of the expert system technique to develop functional reasoning in a FMEA. Hunt et al [1-38] created an automated FMEA tool to make the analyzing process more time efficient. Later, a technique for evaluating the expected effectiveness of a design team configuration was presented by Safoutin et al [1-39]. Gilchrist [1-40] presented a method for ranking failure modes based on the risk priority number.

In 1994, eight papers related to FMEA were published. Brall et al [1-41] implemented a formal reliability and maintainability program. The use of FMEA in design review and a closed loop failure reporting system had proven particularly effective in resolving reliability and maintainability issues. Palumbo [1-42] presented the current results of multi-year efforts to automate FMEA and its associated reliability analysis. The FMEA is produced through a simulation of the failure propagation process. Pizzo et al [1-43] presented probabilistic FMEA that combines concepts of load and capability analysis with other variables, such as temperature, and electrical stress. Redgate et al [1-44] implemented functional hazard analysis, FMEA, FTA, and Zonal analysis in the McDonald Douglas Aircraft Company to satisfy design requirements. Becker [1-45] discussed most approaches of FMEA in the classic military guidance, which is straightforward for electro-mechanical devices. Yet, it is not adequate to address the dynamics or interplay between hardware and software. Noaks [1-46] mentioned that there is a need for an overall system view of safety taken in design procedure along with FMEA that may lead to safety issues. Tripathi et al [1-47] proposed to perform FMEA that may be applied to a wide range of everyday problems that occurs on all process modules in order to control defect densities. Whitcomb et al [1-48] developed the systematic FMEA for Ford Motor Company that would be applied in the semiconductor manufacturing industry.

In 1995, FMEA studies yielded another eleven publications. Price et al [1-49] explained that FMEA analysis on the design of a system is often only completed after a first prototype has been developed. He extended an automated electrical FMEA for the flame system that combined hardware and software. Bowels et al [1-50] applied the fuzzy concept in FMEA since during a failure analysis process, values are often not available and qualitative thresholds and linguistic terms are usually more relevant to the design than numerical expressions. Hollick et al [1-51] presented an approach to make more efficient and effective use of the collected in service data for identifying items for which the preventive maintenance requirement is ineffective or non-applicable. Muraleedharan et al [1-52] focused on artificial heart valve failure analysis because failure of such a device can lead to serious threatening the life of the recipient. Jackson et al [1-53] standardized the FMEA and developed a guideline for air force contractor to perform FMEA efficiently. Nicol et al [1-

54] extended the new graphical reliability estimation tool and FMEA simulator that extracts details of FMEA. Bowels et al [1-55] proposed an approach based on fuzzy set theory and fuzzy cognitive maps to provide a basis for automating much of the reasoning required for performing FMEA. Gagnon et al [1-56] described a new approach in the sample make up for qualifying a new process, a process change, and a device change via FMEA. Ingleby et al [1-57] developed the calculus method that was sufficiently expressive for the articulation hazard defense rules obtained from FMEA technique. Stamatis [1-58] discussed the failure mode and effects analysis method and its applications.

The following year, in 1996, twelve publications again involved FMEA. Montgomery et al [1-59] developed automated FMEA that spans the entire design cycle for electrical/ electronic circuit systems during the design phase. Pugh et al [1-60] based on safety, reliability, and cost issues provided automated FMEA for qualitative circuit analysis. Price [1-61] described automating electrical design failure mode and effect analysis and how it would be able to save engineering efforts. Milton et al [1-62] developed a plan that focused on the FMEA and critical item list to utilize the system. Yang et al [1-63] extended the binary state reliability analysis to continuous state reliability analysis for analyzing both catastrophic failure and performance degradation simultaneously. Wegner et al [1-64] developed a practical approach to verticality, fault accountability, and diagnostic maturation of hierarchical systems applicable for aircraft system. Herman et al [1-65] used FMEA for preventing the deviation on decision levels of control indicators. The aim of control indicators in the production organization is to increase employee motivation. Demko [1-66] offered some comments for managing reliability. By combining Petri Net and FMEA, Goddard [1-67] extended an approach to assess the requirements for the safety of the real time control systems. Considering as lateral improvement in FMEA, Umano et al [1-68] introduced pair-wise group-decision making based on fuzzy preference relation. Ben-Daya et al [1-69] proposed an improvement of the risk priority number (RPN) approach for ranking failure modes by combining the RPN with the expected cost model.

In 1997, eleven publications examined FMEA. Flive [1-70] extended a method for diagnosing failures by using the descriptive knowledge base and fuzzy model. That obtained the parameters of the model from FMEA. Ammer et al [1-71] not only presented a

methodology for risk assessment in design stage but also used FMEA for severity analysis on the software. Chunping et al [1-72] demonstrated an automated analysis method and software package tool for critical software utilized in digital fly control systems. The package is developed on the basis of combination of tabular and matrix methods. Montgomery et al [1-73] employed the FMEA for reliable and maintainable product by providing a structured approach for considering failures and their effects on system. Onodera et al [1-74] evaluated over 100 studies using FMEA and categorized them in either critically method applicable in primarily at the design stage or risk priority number applicable in manufacturing stage. Fronczak [1-75] demonstrated top-down methodology, based on FTA that has been used to identify potential high consequence fault in a microprocessor-based system. Price [1-76] gave an approach to reduce design workload in electrical circuit fields by employing FMEA. Hindson [1-77] considered an application to the generic principles of concurrent engineering and team working. However, he urged some tools such as FMEA should be adapted for team working. White et al [1-78] gave some points on the product reliability, FMEA, and process controls in circuit design to improve reliability measures. Kai et al [1-79] presented new measures of reliability based on disutility and continuous degradation of state of the product by using FMEA in order to respond to customer requests. Filev [1-80] introduced a descriptive knowledge base method that used multiple-input multiple-output (MIMO) fuzzy model for diagnosing a system.

The result of a survey on FMEA presented seven publications on FMEA in 1998. Price et al [1-81] extended a method to use the approximation failure rates for components to select the most likely combinations of failures for simulation. Bowels [1-82] developed a new FMEA for design practices using Pareto ranking matrix to prioritize failures and critically matrix. Sincell et al [1-83] used FMEA for electronic assemblies that had redundancy as well as cross strap. Carson [1-84] proposed an approach for performing line maintenance that flow out of FMEA by identifying and correcting the effects of the failure. Surkan [1-85] presented an efficient, defect detective technique for power supply charge. Bowles [1-86] compared the SAE FMECA with Mil-Std-1629, risk priority number models and explained the major improvement of the SAE FMECA over others. Umano et al [1-87] proposed the pair-wise

group decision making that based upon the fuzzy preference relation, fuzzy sets computational procedure and fuzzy criteria.

Eleven publications appeared in 1999 that directly or indirectly dealt with FMEA. Childs et al [1-88] modified the FMEA to consider common cause failure risks and amalgamated this version with traditional FMEA to prioritize the single failure and common cause risks. Bot et al [1-89] introduced new approach of FMEA to support concurrent engineering environment. The new version of FMEA provided more accurate, simpler, computerized analysis tool. Price et al [1-90] developed another version of automated FMEA tools for circuit analysis in design phase. This tool is compatible with concurrent engineering and uses the qualitative simulation instead of numerical simulation. Moyer et al [1-91] developed FMEA and design of experiment to improve chip technology in both design and process. The virtual engineering included mechanical, electrical and a process simulation was used for initial design and FMEA facilitated the concurrent design development phase. Spangler et al [1-92] worked on the mathematical model for equivalent relations and partitions within the FMEA to fault detection and fault isolation. Lindahl et al [1-92] discussed demand on an efficient method, which are life cycle assessment and checklist. E-FMEA was developed as a tool for efficient design based on advantageous of the methods. In the same year, Kenyon et al [1-94] proposed a methodology that would be helpful for FMEA in microcomputer assemblies. Osterman et al [1-95] proposed up-front approach for analyzing the circuit card assembly design by using FMEA in assessment software. Trahan et al [1-96] proposed a change management system that the integral element of this system was the formal use of an inverted FMEA for the appropriate assessment methodology. Knight [1-97] presented design decision support tools, such as FMEA used to generate comparation data for benchmark purpose. Renault et al [1-98] presented a knowledge-based workbench used to assist in building reliability models by FMEA. Chang [1-99] developed a fuzzy method for FMEA based on Gray theory to derive the risk priority number from the relative weighting coefficient.

In 2000, fourteen publications pertaining to FMEA appeared. Yacoub et al [1-100] presented a methodology for risk assessment at the early stage of the development life cycle. Severity analysis was performed by using FMEA as applied to the development life cycle. Since risk

was not assessed in detail prior to commencing the project, Walker [1-101] developed FMEA to evaluate the risk of failure during the detailed design stage. Kuo et al [1-102] developed an FMEA model to monitor manufacturing systems and process monitor based on colored Petri Nets. Bellomo et al [1-103] extended FMEA to support growth of reliability and availability of a very large particle with a lot of electromagnetic subsystems. Rosing et al [1-104] extended a fault simulation and modeling of microelectromechanical systems to achieve high reliability and safety. DeWinter et al [1-105] presented design tools that could be used to improve both the reliability and repair time of a drive during the design phase. Krasich [1-106] developed more symbols for FTA and proposed that FTA be used as a failure mode analysis tool. Ruiz et al [1-107] presented an alternative method for FMEA , FTA and Markov modeling to find system level failure modes by means of building a full functional model. Goddard [1-108] recommended using FMEA for a system during the design phase. He provided SFMEA to assess the safety critical real time control systems embedded in military products. Thieme et al [1-109] implemented FMEA and FEA for linear alternator and radiation for NASA Glenn center in housing project. Mena [1-110] developed the failure mechanism methodology for integrated circuit assembly and testing. The two dimensional matrix relating to the failure mechanism was extended. Radu et al [1-111] proposed fault tolerant design using hardware redundancy with N+ 1 redundant circuit. Braglia [1-112] developed a new tool for failure mode analysis utilizing economic aspects in FMEA. Meidert et al [1-113] discussed the quality methods i.e., statistical process control, FMEA and their limits.

The following year, in 2001, a total of nine publications appeared on FMEA. Lee et al [1-114] extended a method that established Bayesian belief network theory to construct probabilistic directed acyclic graph. This graph is used to represent dependencies between internal and external states in the physical system, such as an electronic component. Bowles et al [1-115] provided a comprehensive illustration to show how the software FMEA would be effectively applied to a microprocessor based control system having little or no hardware protection. Zagray et al [1-116] presented application of FMEA for assessing impact of design changes on reliability in the early design phase. Rooney [1-117] developed Failure Mode and Effects, diagnostic Analysis for analyzing the

safety of process control plants for nuclear reactors. Nguyen [1-118] performed systematic FMEA for designed software employed in a multimedia digital distribution system. The systematic FMEA provided the list of potential failure to establish the corrective action priorities. Kraniak [1-119] reported reliability actions in the past, such as FMEA that identify the design verification activities, manufacturing quality control etc. To reduce the cost of performing FMEA, Throop et al [1-120] presented the engineering product and operations cross cutting hybrid simulation for software failure analysis. Lee [1-121] introduced the use of Bayes probabilistic networks for FMEA. He et al [1-122] discussed the setback of the conventional FMEA method for rotating machines and presented the back propagation neural networks to perform FMEA.

The year 2002 witnessed five publications on FMEA. Puente [1-123] proposed the fuzzy method based on linguistic variables, Grey theory, and Maximin method to determine an RPC for evaluating the risk level of the system. The approaches allow considerable weighing of severity factor associated with a cause of failure. Using min-max function and linguistic variables, Xu et al [1-124] proposed that a system comprises the expert knowledge base rule and the failure factor interfaces to perform FMEA with uncertain and imprecise information. The knowledge maps into one or more *if-then* rule(s). However, conflict of rules and subsumption are the sources of potential inconsistency, which may result in conflicting conclusions in the knowledge base system. In [1-125,126,127] authors presented the group decision-making models based on upper and lower exponential possibility distribution, Grey theory, etc used in the group failure analysis.

The following year, in 2002, three publications appeared on FMEA. Using linguistic variables, Pillary et al [1-128] developed the fuzzy single-based expert rules to determine the RPC of the failure. The approach utilizes Grey theory to avoid the use of utility function. Because of time consuming, complexity of consistency check, and difficulty of maintenance of knowledge base approach, Braglia et al [1-129] proposed the fuzzy TOPSIS approach for FEA to avoid the definition of a knowledge base supported by several qualitative rules. Though TOPSIS method has some advantages, it suffers from sensitivity analysis because the criterion with the highest score has disproportionate

influence in the failure ranking process. Bozdag et al [1-130] and Wang et al [1-131] developed fuzzy decision-making models with potential application in FMEA.

In 2004, to dilute conflict in a decision group, Wang et al [1-132] presented a conflict resolution model to integrate multiple possibility distributions that can be used in Group-based Failure Effects Analysis (GFEA). Wu [1-133] introduced the use of fuzzy random variables in the fuzzy Bayesian system reliability.

1.2.7 MARKOV MODEL LITERATURE REVIEW

Generally, the practice of fuzzy Markov chains can be classified by the use of a fuzzy probability distribution or of fuzzy possibility distribution in the transition matrix. In 1987, Kurse et al [2-1] developed the fuzzy probability transition matrix based on fuzzy subsets of [0,1].

In 1988, Wereley et al [2-2] extended semi Markov model reliability for faults tolerant control system that combined physical components with redundancy management. Olsson et al [2-3] used the Markov model to analyze and to find relationships between consecutive interval on 38 patient. Wei et al [2-4] presented a procedure for computing the cycle time as a performance factor of concurrent system modeled by Petri Nets. The result of the new model showed consistency of the obtained result from the Markov Model.

In the following year, in 1989, Greeing et al [2-5] developed an algorithm to optimize sequential partitioning for data flow graph and showed validity of an algorithm by using both a Markov model and a simulation model. Dhillon [2-6] developed a reliability model for repairable and non-repairable redundant systems with human error to show relationships between decreasing reliability and MTTF with increasing value of human error by applying Markov Model with five transition diagrams. Stassinopoulos et al [2-7] was the first person using Markov Model to map the requirement of zero delay on Asynchronous Transfer Mode (ATM) network.

White et al [2-8] in 1990 demonstrated a semi-Markov process that would be employed as an effective tool to construct the model of the system. Long [2-9] proposed several analyzing methods for replicable control protocol such as the state transition diagram and Markov chains.

In 1992, Ramsachandran et al [2-10] applied Fuzzy logic in reliability including network reliability and Markov Modeling with three linguistic parameters, Fuzzy union, and Fuzzy intersection operations. Sankaranarayanan et al [2-12] developed the linguistic transition matrix along with developing the basic properties of fuzzy Markov chains. This paper gives a justification of max-min composition of fuzzy matrices for computing the transition matrix. Fitz et al [2-11,13] used a Markov model to analyze the mean time to slip between attract points of digital phase lock loop base.

Four publications developed Markov Model analysis in 1993. Choi et al [2-14] discussed the notion of conditional MTTF and developed an efficient computational method of computing the absorption probability and conditional MTTF in a finite state space Markov model. Balakrishnan et al [2-15] improved the Markov model for big size of state space by reduction via lumping heuristic used to investigate the characteristics of the heuristically constructed Markov chain. Wang et al [2-16] developed a Markov model for chasing the reliability measures in channel fading systems and the Markov model result shows its consistency with the simulator. Chang et al [2-17] presented an alternate traffic model for an ATM multiplexer and showed the successful performance measures calculated by Markov Model.

In the following year, six publications focused on Markov Models. Wu et al [2-18] applied Markov analysis and transition fluid approximation methods for dynamic analysis in wireless communication networks with reserved idle single and multiple access. Chen et al [2-19] employed the Markov Model for measuring performance of 16QAM digital PLL based demodulators since the first-order digital PLL with Gaussian noise channel produces a discrete time first order Markov process. Wu et al [2-20] presented a Markov model analysis to evaluate the throughput and delay performance of the PRMA integrated voice and data system. Qiao et al [2-21] used a Markov model for analyzing the time division multiplexed interconnection network. Qi et al [2-22, 23] showed the effect of packet capture on PRMA stability and capacity obtained by using Markov Model. Yoshida [2-24] developed Markov Chains with a transition possibility measure and fuzzy dynamic programming.

In 1995, eight Markov Model related studies were published. Somani et al [2-25] proposed an efficient decomposition technique for Markov chain analysis since Markov chain tended

to grow exponentially with the number of components and beyond a certain size it become intractable. Stanshine [2-26] extended a Markov model applied to a silent failure of a telecommunication system that remained undetected. Cheng et al [2-27] presented a Markov Model for reliability assessment based on neural network techniques. Fitz et al [2-28] developed a nonlinear Markov analysis technique used to assess the statistical performance of digital phase locked loop demodulators. Malhotra et al [2-29] explained a methodology to build up dependability models using stochastic Petri Nets and Markov Models. Salzenstein et al [2-30] presented a new unsupervised Bayesian image segmentation method using Fuzzy Markov model and defining the Markovian distribution of Fuzzy picture without noise. Balakrishnan et al [2-31] compared fault tree analysis with Markov chain analysis and presented a decomposition approach to avoid a full system Markov reliability model for repairable systems with independent failure and repair processes.

The outcome of Markov Model studies yielded three publications in 1996. Fook et al [2-32] for analyzing the performance of a fully connected full duplex network with channel sensing executed a discrete time Markov chain. Kurano et al [2-33] developed a multi stage decision process with Markov model and Fuzzy transition. In completing the Aggarwal's algorithm for terminal pair reliability in network reliability, Netes et al [2-34] presented an algorithm with imperfect nodes.

In 1997, Khademi [2-35] used probabilistic methods and the Markov chain to describe reliability requirements of teleprotection system by considering that the telecommunication system is needed when there is a fault on the transmission line. Brooks et al [2-36] extended redundancy model over distributed multi sensor to minimize the cost while insuring system dependability using tabu search , simulated annealin, and Markov chains. Platis et al [2-37] developed a non-homogenous Markov chain applicable for asymptotic availability of systems. Susova et al [2-38] presented the usage of a Markov model in a redundant aircraft system. Using Markov chains showed the Aircraft state could be presented by the components failure rate and flight duration.

Four publications appeared in 1998. Kiang [2-39] addressed module testing in telecommunication as a requirement of reliability and quality assurance for functional performance verification. Leuschen et al [2-40] employed Fuzzy Markov modeling as a

technique for analyzing fault tolerant designs under considerable uncertainty. Rao [2-41] in extending Tsen's proposal presented a solution based on determining the adjoin of the reduced incident matrix of the graph that leads to an interconnection network for optimizing reliability. Rosti et al [2-42] developed saving scheduling policies for multi processors systems and analysed those polices using Markov chains.

In 1999, Lyu et al [2-43] developed reliability modeling and analysis of clustered systems by defining the hardware, operating system, and application software reliability techniques using the Markov model.

In the year 2000, Cheong et al [2-44] developed a Fuzzy Markov model to demonstrate the state transition of server resources utilization based on expert's linguistically evaluation since Web server workload forecasting was one of the essential consideration in Web server management and network upgrading. Avrachenkov et al [2-45] presented fuzzy Markov Chains properties in reliability engineering.

In the following year, five publications dealing with Markov Model appeared. He et al [2-46] developed the convergence of matrix sequences that arise in birth and death Markov chains and explained their limitation. Kipouridis et al [2-47] evaluated the size order problem of probability state vector elements of a homogenous Markov system. Ruan et al [2-48] presented a Fuzzy Markovian method for brain tissue segmentation form magnetic resonance images. Zhu et al [2-49] developed FAT, a Markov model for analyzing different kind of redundancy and system configuration to chase the reliability measures on a power network, however the model is applicable for other networks as well. Buckley et al [2-50] worked on finite Fuzzy Markov chains based on possibility theory that was a base for developing a finite horizon Markovian decision process.

In 2002, Pieczynski [2-52] introduced a pairwise Markov chain model, which was compared with hidden the Markov chain, and the triple Markov chain. The advantage of this model is better modeling.

In 2003, Desharnais et al [2-53] developed an approximation technique for the continuous-state labeled Markov process.

In 2004, Bejerano [2-54] developed an algorithm for the variable length Markov chain un-restricted to a predefined uniform depth.

1.2.8 FAULT TREE ANALYSIS LITERATUER REVIEW

In 1988, nine FTA publications related to FTA appeared. Shebalin [3-3] presented a new software safety analysis approach for distributed systems based on fault tree analysis. This technique checks the safety critical logic using the categories of the component fault events. Cha et al [3-4] developed a technique for verifying the safety of complex, and real time software using software fault tree analysis. Ramache et al [3-5] presented some initial thoughts on the development of an expert system that is a simple fault tree intended to assist a traffic or highway engineer when making decisions about the location of objects. Stanek et al [3-6] worked on the application of reliability theory to mine electrical power systems using FTA. Kuzawinski et al [3-7] demonstrated the application of FTA in the user interface tool, artificial intelligent, and expert system. Chou et al [3-8] addressed the application of the probabilistic risk assessment technique for surveillance test interval evaluation. FTA was developed and performed to find the most proper subsystem for test interval relaxation. Hura et al [3-9] presented application of Petri Nets into fault tree analysis. Sloane [3-10] extended the FMEA as a valuable tools for circuit board under design. Mulvihill [3-11] presented two qualitative and qualitative analyses for design safety enhancement.

In the following year, in 1989, eight publications were concerned with FTA. Connolly [3-12] from the Hewlett Packard company developed the systematic form of safety goal verification by using FTA for software control system to be exchanged with electronic or manual controls. Heger et al [3-13] developed an approach for calculating top-event exact probability. Carrasco [3-14] developed an algorithm to find minimal cuts of coherent fault trees. The approach used a decision tree to perform minimal cuts. Zielinkski [3-15] presented practical techniques for application in probabilistic structural mechanics to critical aircraft components. Tanaka et al [3-16] implemented a Monte Carlo method for evaluating a dynamic reliability problem where system components were operating in time. Cherniavsky [3-17] developed an approach for software design to emphasize the construction of test set for checking the conditions leading to the software safety. Page et al [3-18] proposed a model for analysis of system subject to common cause failures. To analyze the reliability of

INEL-Site power system, Galyean et al [3-19] presented a model based on FTA along with computer code. Helman [3-20] developed an algorithm to decompose the scheme for analysis of fault trees.

In 1990, Finnie [3-21] worked on the process that ensures network management of integrity of the overall system by using FTA. Lui [3-22] proposed the design consideration for fault tree analysis using spreadsheet software. In the same year, Gough et al [3-23] developed a technique for the large scale reliability block diagram instead of using the decomposition method and Hessian et al [3-24] developed a method using FTA to assess weaknesses of a new process design at any time during system development.

In 1991, eight publications appeared to analyze the system using FTA. Nakajo et al [3-25] analyzed about 700 errors in commercial measurement and control software products. He developed a model based on the cause effect relationships of errors by using FTA. Bulow [3-26] from General Electric company developed the new approach of Fault Locator and Weighting system using FTA , that is usually applicable to large software systems. Raheja [3-27] developed some reliability techniques, such as process FMEA, FTA, process analysis mapping, process design reviews, along with case study in chemical process and steel manufacturing. Leveson et al [3-28] extended FTA for verifying the application coded in ADA language. Agarwala [3-29] presented a technique for proper design and impact analysis of event sequencing for the system safety and availability. Shimeall et al [3-30] proposed the analysis method of the safety of the software in a digital control system. Guth [3-31] extended a theoretical approach based on Fuzzy concept for FTA. Schneeweiss [3-32] developed fault tree with two types of gates and repeated events.

In the following year, in 1992, seven publications appeared that proposed the FTA technique to analyze the system. Caputo et al [1-32] presented a model to assess the availability of controlled systems by employing both FTA and FMEA. Bilbao [3-33] demonstrated a risk analysis model using combination of FTA and Fuzzy set operations. Law [3-34] presented the integration of power equipment with switching equipment and how a system approach to power would reduce maintenance while providing increased system reliability. Using Boolean functions, Schneeweiss [3-35] investigated fault trees with repeated events. Patterson-hine [3-36] developed a modular technique for FTA. Iverson [3-37] proposed a

fault tree translation system with complexity using digraph. Ramsachandran et al [2-10] applied Fuzzy logic in reliability including network reliability and FTA with three linguistic parameters, Fuzzy union, and Fuzzy intersection operations.

In 1993, a total of four publications appeared. Friedman [3-38] developed automated software coded in Pascal to construct the FTA with two types of gates. Mohan [3-39] categorized the technological failures in three groups and developed FTA for technological failures. Ulieru [3-40] developed Fuzzy FTA with explanatory linguistic expert information. Coudert et al [3-41] presented an analysis method of coherent and non coherent Fault Trees.

In 1994, fourteen published papers appeared on various aspects of FTA. Min et al [3-43] presented a methodology for designing a fail-safe system. The fail-safe system rules in the system specification allows FTA to verify the fail-safeness. Mojahedbakhsh et al [3-44] extended a detailed process for software requirements safety analysis by using FTA. Weber et al, [3-45] developed a FTA based on Fuzzy logic and the Weibull distribution to represent the membership function. Dugan et al [3-46] presented a quantitative reliability analysis of a system designed to tolerate hardware and software faults. Fujino [3-47] developed a new Fuzzy gate for Fuzzy FTA to make it more efficient. Ulieru [3-48] extended an approximate reasoning model for diagnosis of continuos system based on Fuzzy extension of the fault tree analysis. Fukaya et al [3-49] proposed the assessment method for software specification based on FTA. Nolan et al [3-50] extended FTA reflecting signal varieties over a wide range for soft faults, which are usually too tough to detect. Unbehend [3-51] applied FTA in a software system for monitoring and diagnosis. Weber [3-52] presented a new type of artificial intelligence software for risk analysis by using fuzzy fault tree with Weibull membership functions. Yinghua et al [3-53] presented a fail-safe microprocessor based system for interlocking on railways. Redgate et al [3-54] performed a few analysis methods include functional hazard analysis, FMEA, and FTA for McDonald Douglas aircraft. Coudert et al [3-55] proposed an original approach to create efficiently prime covers, non-redundant prime covers, and minimal prime covers of non coherent fault trees.

In 1995, nine articles were published. Geymayr et al [3-56] presented the application of knowledge engineering and methodology for the assessment of reliability and safety of an industrial system using FTA. Henning et al [3-57] presented the principal difficulties with

cut set evaluation in large FTA and developed a program for fast analysis on large FTA with replicated and negated gates. Collins et al [3-58] explained the development of the software for high consequence application creating an urgent need for increased surety techniques such as FTA. Subramanian et al [3-59] showed that design process has four steps including the fault tree verification step. Yang et al [3-60] developed a method for processing fault diagnosis by using information from FTA and uncertainty of data. Melhart [3-61] proposed the use of FTA with external interaction model as a part of the software requirements specification, and analysis phase for development of the system. Salehfar et al [3-62] proposed FTA methodology to evalutate system design and employed the method for electro-hydraulic steering system to identify subsystem that had most potential for improving overall system reliability. Krichsteiger et al [3-63] presented the classical method for FTA to gain the high RAMS performace of the combined cycle power plants.

In the following year, in 1996, seventeen articles appeared. Pullum et al [3-64] developed several techniques to solve dynamic and static FTA applicable to the analysis of hardware, software, and humanware in computer based system. Sinnamon et al [3-65] demonstrated the use of binary decision diagram for FTA and some ways in which it can be efficiently performed on a computer. Burkett et al [3-66] extended a simple analysis and documentation procedure that would help ensure the accuracy of FTA and consequently assure the safety of the system. Walker et al [3-67] addressed the application of FTA to the robot manipulator reliability and the fault tolerant. Collet [3-68] developed two kinds of FTA's to perform the minimal cut sets union probability. Roberts et al [3-69] developed the system reliability prediction using Fuzzy arithmetic. Dutuit et al [3-70] presented a linear tie algorithm to detect the modules of fault trees that were derived from Tarjan algorithm. Ye [3-71] presented how various forms of performing fault trees and system state information affected human performance and preference during fault diagnosis. Stoddard [3-72] extended the Basili's metric for software reliability that would be employed to measure the identification. The use of reliability fault tree logic was considered for analyzing this extended method. Mohammed [3-73] improved process reliability to ensure the process would continue to deliver long term sustained performance. The reliability analysis was performed to enhance the performance of products. Luts et al [3-74] employed both FMEA and FTA softwares for

identifying requirements. Sullivan et al [3-75] developed a novel fault tree analysis tool for evaluating Microsoft's object linking and embedding technology and associated applications. Hsia et al [3-76] developed an approach to detect the class fault because the test set may fail to cover the class. Wyss et al [3-77] developed FTA based probabilistic logic modeling technique for reliability and failure modes of current generation network technologies. Yiping et al [3-78] presented dynamic FTA and provided a Markov model for performing dynamic FTA. Bouissou [3-79] developed a pre-processing of FTA that ensured the results given by different methods were consistence. Hamilton et al [3-80] demonstrated a novel method for analyzing risk associated with robotics and manufacturing applications. Kendall [3-81] performed a literature survey on methods that proposed FTA for the development phases and system safety analysis .

In the following year, in 1997, twelve publications addressed FTA. Lee et al [3-82] developed a computation method for evaluating important measues of gates in a fault tree. Vishnuvajjala et al [3-83] demonstrated the effects of system characteristics on flow analysis technique and how flow analysis supports FTA. Gerogiannis et al [3-84] presented a general framework for analyzing the safety aspect of a complex safety critical real time application. Cong et al [3-85] presented the application of fault diagnosis system for an automated assembly line. Tsuchiya et al [3-86] discussed the safety analysis of design and proposed FTA based technique to drive the safety requirements from the requirement specification, the component library, and the design documents. Fronczak [1-75] demonstrated a top-down methodology based on FTA that had been used to identify potential high consequence fault in microprocessor based system. Hasson et al [3-87] discussed the Boeing processes, which were used to assure the safety of new airplane designs. Gulati et al [3-88] addressed a new modular approach for efficient analysis of both static and dynamic fault trees and Markov model. Dugan et al [3-89] developed a software application that performs static and dynamic FTA for hardware, software, and humanware in complex systems. Kocza et al [3-90] addressed a computer aided reliability system to model the production systems along with FTA. Knight et al [3-91] evaluated all reliability techniques to create a new technique. Wilds [3-92] presented a short review of the probabilistic risk assessment and addressed the main problem areas for its application.

In 1998, eight papers were published that directly or indirectly dealt with FTA. Anand et al [3-93] developed a decomposition scheme where independent subtree of the fault tree was detected and solved hierarchically. Lyu [3-94] developed the testing and evaluating schemes for software reliability by using FTA. Liggesmeyer et al [3-95] improved system reliability with an automated FTA tool by enhancing the quality of FTA and reducing costs associated with manual FTA. Coppit et al [3-96] developed the design techniques in the collaborative development of the dynamic FTA tool. Dugan and Sullivan [3-97] extended FTA capability to the model with variety of the distribution time to failure functions. The approach includes both the binary decision diagram and Markov analysis method. Portwood [3-98] proposed the solution and the guideline for the system safety assessment on Civil Airborne system that supervised FTA, Markov modeling, etc. Mauri et al [3-99] presented a way to address the problem by relating Functional Failure Analysis, FMEA, and FTA to each other. The approach integrated them into a new technique and applied it to hierarchically systems. Hansen et al [3-100] presented the results of one safety analysis technique, i.e., FTA, and interpreted them as software safety requirements.

In 1999, seventeen articles examined FTA. Wang el al [3-101] employed FTA for analyzing the reliability of excitation controller of a synchronous generator. Sullivan et al [3-102] extended a dynamic FTA tool to combine the innovative DIFTree analysis method with built user interface. DIFTree integrated the binary decision diagram and Markov method under the notation of dynamic FTA. Amari et al [3-103] extended a new approach for incorporating imperfect fault coverage into the combinatorial model. By using the approach, the reliability engineers can use the software package for computing reliability. Dugan and Sullivan [3-104] presented a graphical design language called "reliability information embedded design language" for modeling the digital system. Designers can employ the FTA to obtain reliability analysis at an early design stage. Szabo et al [3-105] proposed a multi state based procedure that was adapted to the reconfiguration. This procedure was based on the fault tree technique that provided the treatment of the system components with adaptive properties. Manian et al [3-106] presented an accurate procedure for dynamic fault tree analysis because the semantic of dynamic FTA were complex. Kaufman et al [3-107] developed system reliability for embedded hardware and software by using FTA. Renault et

al [3-108] developed a computer program for creating a fault tree automatically. The program used a classification procedure to generate FTA for different purposes. Jianfeng et al [3-109] developed a procedure for combined FMEA and FTA to perform reliability analysis of redundant systems. Yiping et al [3-110] presented a dissimilar fault tolerant control computer system with software and hardware using dynamic fault tree analysis. Yamada et al [3-111] extended several issues on securing human safety in a human/robot system using FTA. Dugan et al [3-112] presented an approach to demonstrate issues in developing a dynamic FTA in industry. Lutz et al [3-113] amalgamated two safety analysis techniques used separately on software and hardware during the system engineering process. Copenhafer et al [3-114] discussed that the provision of tool support for automated and repeatable experiments could provide significant value to designers. Kumar [3-115] discussed how Fault Tree analysis would be used to perform reliability analysis of hardware and software systems. Verbitsky et al [3-116] extended theoretically based FTA methods to evaluate reliability of dailing system. Ponta et al [3-117] presented a model for performing reliability analysis according to cost and benefits.

In the following year, 2000, nine articles appeared. Ou et al [3-118] presented the sensitivity measures for both the binary decision DIAGram and Markov technique. Andrews et al [3-119] developed an event tree analysis approach using decision diagrams. This study showed that in some situations the analysis methods based on traditional FTA were inaccurate and insufficient. Contini et al [3-120] suggested the necessary step for identifying design modification in complex systems by using FTA and Markovian process. Zaho et al [3-121] developed a Fault Tree Analysis on the operation of manipulating a storehouse of solidified nuclear waste. Dugan and Sullivan [3-122] extended a reliability-engineering model for use by academic and industrial research labs. This tool mainly focused on dynamic fault tree analysis. Krasich [1-106] developed more gates and multiple causes to a dual failure mode and presented the application of Fault Tree analysis for evaluation of the system reliability improvement in the design phase. Meshkat et al [3-123] proposed a technique for the reliability analysis of systems with on demand and dynamic failure modes. Shaoying [3-124] extended four techniques that combine FTA with the static analysis and the testing approach

for reliability analysis. Due to variability and fuzzy workload of the web server, We et al [2-44] proposed a fuzzy Markov model to illustrate the state transitions.

In 2001, ten papers appeared. Carreras et al [3-125] presented an approach based interval methods for estimating reliability using FTA. Papadopoulos et al [3-126] extended a new technique for performing safety analysis based on computer and software FTA. Basilio et al [3-127] developed an approach for probabilistic risk assessment using FTA. A system level fault tree was generated to recognize fault scenario and failure modes. Kingman et al [3-128] discussed how to handle the problems associated with constructing and managing a reliable telecommunication network. Bogomil [3-129] employed FTA to obtain parameters, which were used in the reliability functions, and the safety functions. Kohda et al [3-130] presented the latency of basic event in FTA. Verbitsky [3-131] in order to address the dialing problem provided individual symbol of one or more random keys, and grouped the symbol failures for the complex consumer communication system. Zampino [3-132] presented the five-step FTA for system troubleshooting. Zhu et al [2-50] developed FTA for analyzing different kind of redundancy and system configuration to chase the reliability measures in the power network.

In 2002, Cepin et al [3-133], developed a new method based on fault trees and a range of risk for assessing the reliability and safety of nuclear power plant. In the same year, Wang et al [3-134] proposed a computer-aided fault tree to reduce human error in manual construction of fault trees. In the following year, 2003, Cha et al [3-135] presented a model checking technique to validate the accuracy of fault tree because the conventional techniques such as cut set, probabilistic analysis are unable to pin point flaws when failure modes are inaccurately defined. In the same year, to overcome drawbacks of computer-aided fault tree synthesis to handle the control loops, Wang et al [3-136] proposed a systematic approach to FTA. In 2004, Huang et al [3-138] proposed the use of the Posbit reliability theory in fault tree analysis for a system with small probability or limited statistical data.

1.2.9 K-OUT-OF-N SYSTEM LITERATURE REVIEW

In 1986, Rushdi [4-3] developed an approach for k-out-of-n system of non-identical switches in a symmetric switching system.

In 1988, a total of nine publications dealt with various aspects of k-out-of-n. Chan et al [4-4] presented a survey on consecutive k-out-of-n:F and developed an algorithm to obtain the system reliability and to generate minimal path and cut sets. Heijden et al [4-5] developed a k-out-of-n model for a cold stand by redundant system. The model considered a two-state component in the multi state system using Monte-Carlo simulation to validate the performance of the system. Dhillon et al [4-6] surveyed chemical publication on system reliability and categorized them into categories such as chemical system, reliability topics and the methodology used. Dhillon et al [4-7] presented some models to perform reliability analysis of repairable and non-repairable k-out-of-n systems. Papastavridis et al [4-8] analyzed a consecutive k-out-of-n:F system involving identical and independent components. Angus [4-9] presented an approach for obtaining MTBF of a k-out-of-n:G system with repairable components. Sah et al [4-10] proposed an approach to obtain the optimal value for K in the k-out-of-n system. Hwang [4-11] modified the consecutive k-out-of-n:F to relayed consecutive k-out-of-n:F line. Pham et al [4-12] presented an application of Barlow and Heidtmann algorithm for computing reliability of k-out-of-n systems.

In 1989, fourteen publications appeared on k-out-of-n systems. Chang et al [4-13] proposed a diagnosis approach for k-out-of-n system with repairable electrical components. Barkat et al [4-14] developed the detection approach using the k-out-of-n fusion rule. Viswanathan et al [4-15] presented a network of sensors that received signals from the environment and the fusion center to make the decision concerning k-out-of-n received signals. Hwang [4-16] implemented consecutive k-out-of-n units the computer network topology. Sarji et al [4-17] developed an approach to calculate the reliability of k-out-of-n systems. The approach is non-recursive and coded in FORTAN. Kossow et al [4-18] presented a topological formula for obtaining system reliability of linear and circular consecutive k-out-of-n:F networks. Papastavridis [4-19] extended an recursive method for calculating the life time distribution of a circular consecutive k-out-of-n:F system. Hwang et al [4-20] worked on an advanced k-out-of-n system called consecutive connected system. The algorithm for computing the reliability of the multi state system was proposed. Papastavridia [4-21] proposed a distribution function for a number of cold spares that were failed in a system. Chrysaphinou et al [4-22] developed a new model for relay communication station based on a k-out-of-n

system. Wood [4-23] presented several equations for availability of a system with exhaustive spares. Those equations are conservative, however they show more justifiable results than the result of simulation or Markov modeling. Chung et al [4-24] developed an algorithm for optimal replacement in k-out-of-n systems. Hwang et al [4-25] presented replacement of a consecutive k-out-of-n system with 1-out-of-n line. Dhillon [2-6] presented reliability analysis of redundant systems with repairable and nonrepairable subsystems and human error and common cause failures.

In 1990, ten publications appeared on k-out-of-n systems. Rushdi [4-26] presented some strict consecutive k-out-of-n:F systems for further development. Pham [4-27] extended an approach for finding the optimal system size for a k-out-of-n system with the components having binary states. Chang et al [4-28] developed a method for repairable components in k-out-of-n systems. Iyer [4-29] proved that the distribution of time to failure of any consecutive k-out-of-n:F system could be obtained if the life times of the components were independent exponential random variables. Kuo et al [4-30] extended a relation between two k-out-of-n:G and k-out-of-n:F systems and developed theorems for such systems. Bowerman et al [4-31] proposed an approach for calculating the binomial survivor function applicable for a large 'n' in a k-out-of-n system. Guangping et al [4-32] proposed a direct, exact approach to calculate the reliability of the consecutive k-out-of-n:F system with homogenous Markov dependence instead of recursive approach. Papastavridis [4-33] developed 'm' consecutive k-out-of-n:F systems and a recursive formula to calculate the failure probability. Salvia et al [4-34] presented two dimensional consecutive k-out-of-n:F model . Mass et al [4-35] developed an FMEA technique for analyzing the complexity of learning from counterexamples and memebership queries system.

The year 1991 witnessed sixteen publications on k-out-of-n systems. [4-36] presented an architecture for high speed and low cost processors. Murakami et al [4-37] proposed an algorithm with strictly digital neutral network to solve four coloring map problems of the combinatorial optimization problems using the set selection problem and the k-out-of-n design rule. Chen [4-38] extended a software based k-out-of-n control system involved in 'N' modules under the control of the software. In such system, it functions correctly if the software operates correctly and at least 'K' modules work well. Tsuda [4-39] suggested the

rotary replacement redundancy for the defect tolerance in tree architecture using switching scheme based on k-out-of-n redundancy. Bai et al [4-40] developed a redundancy optimization model for k-out-of-n system with common cause failures to find the optimal number of the redundant units. Haim [4-41] extended an approach based on Bayesian prediction for the mean values and confidence bounds of the system failure state probabilities. Rushdi [4-42] proposed the iterative structure of the Sarjie's algorithm for calculating the k-out-of-n system reliability. Suich et al [4-43] developed a tool for choosing redundancy level k-out-of-n:G system in order to minimize the cost. Singh et al [4-44] compared the life time of the similar structures of k-out-of-n:G for two different lifetimes. Hwang [4-45] presented two approaches to obtain the failure time distribution for a k-out-of-n:F system. Shao et al [4-46] presented a Markov model for analyzing the reliability on k-out-of-n system where subsystems share the load on the system. Kossow [4-47] extended two enhancments in the integer optimization method for spare allocation in order to minimize spare cost of k-out-of-n of system configuration. Srivastava et al [4-48] introduced a spare cost model . Hwang [4-49] presented some comments on a strict consecutive k-out-of-n:f system applicable in the networks such as telecommunication neworks. Satam [4-50] discussed the maximum number of failures that can occur when a k-out-of-n system fails. Rushdi [4-51] extended an approach for consecutive k-out-of-n:F system by using the benefit of nonrecursive method in computing the reliability.

In 1992, thirteen publications appeared on k-out-of-n systems. Akhtar [4-52] proposed a modeling technique for a shared load k-out-of-n: G system and compared it with Markov chain result. Schneeweiss [3-32] used Markov k-out-of-n systems to check whether statistical dependence, and the system MTTF were nearly that for statistically independent components. Wu et al [4-53] extended an O(k:n) algorithm for assessing the reliability of a circular consecutive k-out-of-n:F system. Hwang [4-54] argued that component reliability depend on system size. Pham [4-55] developed a mathematical approach to determine the most economical value for 'K' or 'N' in the k-out-of-n:G system when 'K' or 'N' is fixed. She et al [4-56] assessed the system reliability of a k-out-of-n warm standby system. Boehme et al [4-57] presented a practical approach and system reliability formulas by using the result of a consecutive k-out-of-n:F system. Elisa-Fuste et al [4-58] presented an

application of k-out-of-n to maximize detection probability in the sensor and the data fusion system. Shi et al [4-59] developed optimal spare allocation for defect tolerant VLSI component in k-out-of-n systems. Canfield et al [4-60] presented the newly obtained results on reliability of consecutive k-out-of-n system when both 'K' and 'N' were varying. Tesuda [4-62] suggested hierarchical redundancy using the defect tolerant replacement circuit for increasing the yield of large area LSI's. The defect tolerant replacement circuits could be constructed by using distributed switches in basic k-out-of-n redundancy schemes. Zakari et al [4-62] worked on a component importance in a linear consecutive k-out-of-n system. Ksir [4-63] proposed the upper and lower bound calculated for the k-out-of-n system reliability.

In 1993, fifteen publications appeared on k-out-of-n systems. Wu et al [4-64] proposed a method to improve the original $O(n.k^2)$ algorithm for the reliability of a circular consecutive k-out-of-n:F system. Such et al [4-65] proposed an approach to minimize the system cost by choosing optimal subsystem reliability and k-out-of-n:G subsystem redundancy. Aria et al [4-66] presented a technique using k-out-of-n design rule for neural network to find the sets of diagnostic patterns for testing VLSI circuits. Nakagawa et al [4-67] employed a k-out-of-n design rule in developing an approach for digital neurocomputer based on the paradigm of the constraint set programming in order to solving combinatorial optimization problems. Blum et al [4-68] employed k-out-of-n fusion rule for developing a method for the asymptotic relative efficiency of two centralized detection schemes. Brzezinski et al [4-69] used k-out-of-n request type for extending the general model of distributed computing. Zuo [4-70] developed a rectangular and cylindrical 2-dimensioanl consecutive k-out-of-n system.In another paper, Zuo [4-71] proposed a more efficient algorithm with O(kn) for linear k-out-of-n system regarding to the others. Hwang [4-72] developed an algorithm for computing the reliability of a circular consecutive k-out-of-n:F system with order $O(n.k^2)$. Chen [4-73] proposed a reliability model for a consecutive 2-out-of-n:F system with node and link failure. Jaisingh et al [4-74] presented the influence of the network environment on the system of non-renewable component using k-out-of-n components to meet reliability requirements. Koutras et al [4-75] presented a 2-dimensional consecutive k-out-of-n:F system with independent components. Sfakianakis et al [4-76] presented the reliability of a linear consecutive k-out-of-n:F system that includes two state components. Papastavridis and

Newton [4-77, 78] provided some comments on the concept of consecutive k-out-of-n system.

Eleven publications pertaining to telecommunication appeared in 1994. Murakami et al [4-79] presented a neural network algorithm generating a near maximal planar subgraph. Tsuda [4-80] specified the relation between WSI array and the maximum WSI integration scales with the k-out-of-n redundancy configuration. Karpovsky et al [4-81] developed a new approach for RAMS testing of a k-out-of-n pattern sensitive fault model. Wu et al [4-82] developed a new reliability model for a consecutive weighted k-out-of-n:F system with order O(n). Lipow [4-83] improved the lower bound for reliability of k-out-of-n:G systems. Kuo et al [4-84] showed some factors that would not be modeled, but they affect system reliability. Wu et al [4-85] extended an approach for calculating the reliability of a weighted k-out-of-n system with order O(n.k). Srivastava et al [4-86] presented an application of a classical method of steep-descent optimization coupled with a boundary tracking technique to solve the integer spare allocation problem. Akhtar [4-87] presented a recursive method to assess reliability measures of a k-out-of-n:G repairable systems. Cai [4-88] calculated upper and lower reliability bounds and developed a limit formula for reliability of large k-out-of-n:G systems. Wu et al [4-89] developed an algorithm to evaluate reliability of weighted k-out-of-n systems.

In 1995, seventeen publications relating to k-out-of-n configuration appeared. Hwang [4-90] presented an algorithm to solve the combinatorial problem of the counting number of the states with order $O(k^3.\log(n/k))$. Behr et al [4-91] extended a formula for the signed domination of k-out-of-n systems. Malinowski et al [4-92] developed an approach to determine the reliability of a circular consecutive connected k-out-of-n system with multi state components. Belfore [4-93] presented an efficient algorithm for calculating the reliability of a k-out-of-n system with order $O(n.(\log2(n)^2)$. Chao et al [4-94] conducted a survey on understanding, computation method, and structure of the consecutive k-out-of-n:F systems. Brzezinski et al [4-95] presented a model for deadlocks in asynchronous message communication systems that include k-out-of-n requests. Murakami et al [4-96] proposed a neural network algorithm for solving combinatorial optimization problems such as reliability of k-out-of-n. Bolan et al [4-97] investigated component redundancy versus system

redundancy which would be propagated to telecommunication environment as physical component, line, port, card, and service redundancy. Moosa et al [4-98] developed a simulation approach for integrated circuits reliability that considered process flaws and material properties. Barbour et al [4-99] developed a compound Poisson approximation for reliability of a k-out-of-n system. Kossow et al [4-100] presented approaches for determining the linear consecutive connected k-out-of-n system with multi state components. Tsuda et al [4-101] presented a spare connection approach for a k-out-of-n system applicable to totally defect tolerant arrays capable of quick broadcasting system. Tohma et al [4-102] proposed an approach for realization of fail-safe sequential machine. Newton [4-103] presented some issues on reliability of k-out-of-n:G system with imperfect fault coverage. Lee et al [4-104] presented an algorithm for calculating the reliability of k-out-of-n:G system. Hwang et al [4-105] developed an algorithm with order $O(k^3(\log(n/k)))$ time to compute the reliability of the consecutive k-out-of-n:F system. Chao et al [4-106] conducted a broad survey on consecutive k-out-of-n:F systems since 1980.

Seven publications related to k-out-of-n appeared in 1996. Tsuda [4-107] used spare connection scheme for k-out-of-n redundancies. The proposed scheme can be used for constructing various k-out-of-n configurations capable of quick broadcasting, by using spare circuits. Zuo et al [4-108] presented an application of k-out-of-n and the consecutive k-out-of-n system reliability models in chemical industry. Hsieh [4-109] extended a predication approach for failure time of k-out-of-n:F system with 'n' similar components. Coit el al [4-110] presented a genetic algorithm to optimize the reliability of a series-parallel system. Moustafa et al [4-111] developed a Markov model for analyzing the availability of a multi failure modes k-out-of-n system.

In 1997, six publications appeared. Ma [4-112] presented stochastic ordering of order statistics and its application in the stochastic comparisons of life time of k-out-of-n systems having independent components. Singh et al [4-113] presented a k-out-of-n system with indential and independent life distributions. Behr et al [4-114] offered two formulas for calculating the reliability of a k-out-of-n:G system. The formulas were developed for k-out-of-n:G coherent binary system. Salloun et al [4-115] improved their first model for finding an optimal procedure by implementing an algorithm. Posser et al [4-116] worked on a

decentralized supervisory control coined problem. The main result of the study was a set of local supervisor that can solve problems for any k-out-of-n type of event fusion rule. Narayan [4-117] proposed the use of the Hopfield neural networks for evaluating k-out-of-n systems.

In 1998, six articles on k-out-of-n configuration were published. Dao et al [4-118] proposed an approach amalgamated graph theoretic and algebraic techniques to obtain reliability of ring topology network. Boland et al [4-119] proved that number of physical minimal repairs were stochastically larger than number of statistical minimal repairs for a k-out-of-n system with similar components. Liu [4-120] extended a technique for computing the reliability of k-out-of-n :G system composed of non-identical components with an arbitrary distributed failure time. Tang et al [4-121] presented a tool that includes four software modules such as k-out-of-n and Markov model. Chang et al [4-122] developed a reliability algorithm for circular consecutive weighted k-out-of-n:F systems with order O(min[n,k].n).

In 1999, Li et al [4-123] introduced a delegation logic approach for large scale, open, and distributed system along with k-out-of-n thresholds. Mi [4-124] developed a model for active redundancy allocation in a k-out-of-n system. The objective was to assign which group of components should be active redundancy and which group of component should be used for active redundancy to achieve maximum reliability of the k-out-of-n system. The size of both groups mandated to be same. Amri et al [4-125] were the first people that extended maximizing the reliability for system with imperfect fault coverage using a k-out-of-n system. Hsun-Wen [4-126] demonstrated that optimum performance of k-out-of-n:G in line occurs when value of 'n' is between k and 2k.

The year 2000 witnessed nine articles that addressed k-out-of-n redundancy. Huanget al [4-127] presented a new definition of multi state k-out-of-n system and proposed several formulas for increasing and decreasing k-out-of-n structure. Muselli [4-128] presented improved bounds for reliability of a consecutive k-out-of-n:F system. Tsuda [4-129] suggested a spare connection scheme for k-out-of-n redundancy for constructing fault tolerant ring or mesh connected array of the processing node. Mosoko and et al [4-130] presented an analysis approach with explicit state equations by using recursive formulas. Tsuda [4-131] used a spare connection scheme for k-out-of-n redundancy in constructing

fault tolerant parallel computers with series connected, mesh connected or tree connected processing elements. Zuo et al [4-132] presented several algorithms for k-out-of-n:F consecutive k-out-of-n:F, and linearly connected(r,s)-out-of-(m,n):F systems to assess the system reliability. Huang [4-133, 136] presented a multi state k-out-of-n:G system with M+1 state components and defined algorithms for reliability assessment of a system. By using a genetic algorithm known as numerical method, Marseguerra [4-134] determined the optimal system configuration for a k-out-of-n system.

Petakos [4-135] developed a function that characterizes the failure coefficients of a general coherent k-out-of-n system. In 2001, Nguyen [4-137] extended reliability techniques for real-time distributed multimedia systems with different type of structures. In the same year, Martin et al [4-138] developed a k-out-of-n threshold scheme to protect secrets among a group of n shareholders. Higahiyama [4-139] also proved that the weighted k-out-of-n system would work, if the total weight of some operational components was at least K. He developed a method to calculate the reliability in O(n.k) computing time and in O(n.k) memory space.

In 2002 ,Wang et al [4-140] presented a k-out-of-(n+m) system and studied the reliability and avaiability of the system with 'm' cold stand by units.

In 2004, Lin [4-141] developed a heuristic algorithm for computing the reliability of both linear and circular consecutive k-out-of-n:F systems. Also, Yeh [4-142].presented an algorithm based on minimum cust sets to assess the k-out-of-n system reliability.

1.2.10 RELIABILITY ESTIMATION LITERATURE REVIEW

In 1983, Geist et al [5-3] proposed an analytic technique for estimating the reliability of electronics system.

In 1990, considering design limitations and other factors, Geist et al [5-4] evaluated the reliability estimation techniques used for fault-tolerant system.

In 1992, Lyu et al [5-5] presented the use of case tool for developing software reliability models. In the same year, Wohlin et al [5-6] presented a method for estimating software reliability in the design phases.

In 1993, Baca [5-7] developed a Monte Carlo model for solving the static reliability problems based on less prior information and yield estimators with standard deviation. In the same year, Schneidewind [5-8] presented the application of the software reliability estimation method approved by American Institute of Aeronautics and Astronutics.

In 1994, six articles addressed reliability estimation. Singer [5-9] developed a fuzzy fault tree that could picture the tolerances of the probability values of hazards. Jalote et al [5-10] developed an estimation method for software reliability based on history of the program and graph theory. Mathur et al [5-11] addressed the effect of the operational error in reliability estimation by reviewing four models i.e., Musa-Okumoto, Goal-Okumoto, Coverage enhanced, and Musa-Okumoto. To mitigate the drawbacks of estimation methods in software reliability, Muralidhara et al [5-12] proposed a model for testers to estimate the reliability of a given software. Savchul et al [5-13] introduced a Bayes reliability estimator for the binomial probability based on multiple information sources. Considering operational profiles and test case result, Woit [5-14] developed a reliability estimation model for software reliability.

Two publications related to reliability estimation appeared in 1995. Mathur et al [5-15] investigated the testing methods and software reliability estimation approaches and proposed a reliability estimation method. Nicol et al [5-16] presented a graph-based reliability estimation approach.

In the following year, 1996, four articles dealing with reliability estimation appeared. Adams [5-17] presented a confidence interval based model to estimate the reliability of the software. Using prolog programming, Azem et al [5-18] presented a reliability estimation model.

Der Kiurreghian [5-19] reviewed methods for computing the reliability in earthquake engineering. Xie et al [5-20] developed a graphical plotting of the failure data for estimating the reliability.

Four publications related to reliability estimation appeared in 1997. Using approximate confidence intervals, Coit [5-21] developed a model for estimating the system reliability. Krishnamurthy et al [5-22] developed component based reliability estimation (CBRE) for software reliability. Willits et al [5-23] proposed the use of small binomial sample for the

reliability of the series system. Kazunori et al [5-24] introduced a systematic design of the current stress level to estimate reliability.

Three publications related to reliability estimation appeared in 1999. Using interval methods to address uncertainty in input data, Carreras et al [5-25] presented a new model for robot reliability estimation. Yusen et al [5-26] presented an approach based on combination of the reliability engineering and software development life cycle. Jyh-jen et al [5-27] addressed the local estimation problems and adopted the Bayesian approach to obtain interval estimation for reliability.

In 2000, six articles appeared on reliability estimation. Carreras et al [5-29] proposed the use of fault trees and the time-varying input data for robot reliability estimation. They used interval method representing time-varying input data in the model. Considering time and budget constraints to determine unit reliability, Coit [5-30] introduced a system reliability estimation model based on the prioritizing index. The prioritizing index allows a comparison of units and a relative ranking of them. Using the Petri-net technique, Yin et al [5-31] developed a reliability estimation model for the early stage of a software development. Tzong et al [5-32] used the Markov chain technique for estimating reliability of a real time system. Considering activation energy and room ambiant temperature, Lacey et al [5-34] developed an estimation model for light emitting diodes (LED). Tal et al [5-35] addressed the drawbacks of the fixed duration testing methods and of the non-fixed duration testing methods for reliability estimation.

In 2001, Coit et al [5-36] developed a reliability prediction using the prioritization index to estimate and improve the reliability of the system. In the same year, Grandell et al [5-37] proposed an algorithm for evaluating mobile telecommunication networks. Huitian et al [5-38] introduced a real time reliability estimation model based upon time-series analysis technique. Jin et al [5-39] developed an estimation model for system reliability using confidence intervals that conceded the variance of system reliability. Rallis et al [5-40] developed a failure preconditioning and post conditioning approach to estimate software reliability during the testing period. Using life time data, Sarhan Ammar [5-41] introuduced a reliability estimation technique based on likelihood and Bayes concepts. Zhang et al [5-42] presented a methology for use in product developement life cycle.

In the following year, 2002, three articles dealing with reliability estimation appeared. Cayula et al [5-43] proposed the root mean square (RMS) error estimator for reliability estimation of environmental measurements. Trivedi [5-44] addressed the shortcomings of the conventional approaches for estimation of software reliability. They developed a tool to mitigate the drawbacks of conventional approaches and to assess the reliability of the heterogeneous systems. [5-45].

In 2003, Krummenauer [5-46] proposed an image processing system and compared with the estimated reliability to x-ray's. In the same year, Kukar [5-47] discussed the reliability of diagnosis in medical decision making and proposed a reliability estimation method for machine learning used in medical diagnostic.

The year 2004 witnessed seven articles on reliability estimation. Gokhale [5-48] investigated the drawbacks the reliability estimation techniques and used variance in reliability estimates. Quigley et al [5-54] developed conditional data analysis for reliability evaluation based on the limited expected values function. Considering a flowshop problem, Sorensen et al [5-55] proposed Petri-net and simulation methods to evaluate the quality and reliability of flowshop. Phillips [5-56] developed the Bayesian estimation method based on inverted gamma model for the conditional distribution of the failure time. Using the correlation coefficient, Evandt et al [5-57] presented a graphical test for reliability estimation. Oliveira et al [5-60] investigated various reliability estimation methods. Pearn et al [5-61] addressed an accuracy analysis of estimators using the simulation technique.

1.3 MOTIVATION AND OBJECTIVE OF THESIS

Reliability design techniques have been studied extensively over the years. Although the earlier publications made improvement in RAMS techniques, evolution in engineering fields requires improvements upon the shortcomings of FMEA, the Markov Model, FTA, k-out-of-n systems, and reliability estimation because of stochastic or fuzzy environments. These shortcomings are discussed below.

1.3.1 FAILURE EFFECTS ANALYSIS (FEA) SHORTCOMINGS

These shortcomings are as follows:

a. In standard FEA, either RPC or RPN is used to construct not only the system failure effect model but also to implement risk analysis. Most of the earlier studies used failure factors (Failure probability, Non-detection probability, Severity of effect, and Expected cost) to assess either RPC or RPN of the potential failure. However, due to the fuzzy set modeling and knowledge base system problems (i.e., benchmarking of fuzzy variables, conflict of the rules, respectively), the use of RPC is not very effective. On the other hand, the use of traditional RPN confronts failure risk analysis with some problems. RPN is a multiplication of the failure factors and their values ranging between '1' and '10'. The multiplication of factors in RPN expression is not always an appropriate way for risk priority analysis due to the following reasons:

- ❖ Evolving an identical RPN value for different sets of factors.
- ❖ Ignoring the level of relations (i.e., strong and weak relationships) between symptoms, root causes, and failures.

b. FEA is a system technical risk analysis that extracts knowledge about the potential failure mode from a group of experts with possible conflict in experts' opinions. The experts' opinions differ substantially because the experts do not often agree on the level of the failure factors and the functions/subsystems attributes (e.g., importance). Therefore, conflict always occurs in Group-based Failure Effects Analysis (GFEA). GFEA has been studied and several approaches have been proposed. The following are the drawbacks of the proposed approaches:

- Combination of the fuzzy method based on linguistic variables, Grey theory, and Maximin method to determine an RPC for evaluating the risk level of the system. The approach allows considerable weighting of severity factor associated with a cause of failure. Due to using both linguistic variables for evaluating failure factors and Grey theory to prioritize the risk of failure without utility function, the proposed method can be considered as a breakthrough in FEA. However, one drawback of these approaches is that the

weight of criteria and the scores of the failure causes must be assigned subjectively.

- The Analytic Hierarchy Process (AHP) technique was proposed to find the preferential weight of the failures in order to determine the failure risk priority number. This approach is based on three principles: decomposition, comparative judgments, and the synthesis of priorities. It has several shortcomings for failure effects analysis including man-made inconsistency in pair-wise comparisons and rank reversal when new failures are introduced.
- The knowledge based rule as a new approach for FEA was introduced. The knowledge based system uses the knowledge encoded in some form such as rule-based systems and decision tree. Generally, the construction of a failure effect knowledge base has been carried out by interviewing experts in failure effect and painstakingly translating the experts' opinions into an appropriately structured set of rules (e.g., *if-then*). The knowledge maps into one or more if-then rule(s). However, the following sources of potential inconsistency may result in conflicting conclusions in the knowledge base system:
    - Conflict of rules (i.e., '*if*' parts of the rules are similar and '*then*' parts are different)
    - Subsumption (i.e., two or more rules have the same result, but one contains additional restriction on the situations in which it will succeed.)
- The fuzzy TOPSIS approach for FEA was proposed to avoid the definition of a knowledge base supported by several qualitative rules. This method suffers from sensitivity analysis because the criterion with the highest score has disproportionate influence in the failure ranking process.

1.3.2 MARKOV CHAIN ANALYSIS SHORTCOMINGS

These shortcomings are as follows:

a. A Markov chain is useful for modeling of systems with dependent failure and repair modes when components within the system are independent, and when multi-state systems with common failures require analysis. However, due to uncertain and time-varying input reliability data, the use of fuzzy sets in Markov chains has been considered in some publications. Today the practice of fuzzy Markov chains can be classified by the use of fuzzy probability distributions or of fuzzy possibility distributions in the transition matrix. However, until now no attention has been paid to fuzzy flow-rate equation systems associated with interval chance to deal with time-varying input for the states of Markov chains and uncertain input data. For example, in electronic design, part parameters change with time due to ageing effects and stress. Furthermore, failure rates calculated in accordance with MIL-HDBK-217 do not address circuit failures caused by part changes due to ageing effect

1.3.3 FAULT TREE ANALYSIS SHORTCOMINGS

These shortcomings are as follows:

a. FTA is applicable for risk assessments in systems with a clearly identified single top event. Since FTA is event-oriented, differentiation between the levels of the events plays a vital role in creating the tree. By defining dependency relations between the basic events and the intermediate/top events, the reliability measures can be accurately estimated. The literature pertaining to FTA is concerned with sequential failures and common cause failures, using binary decision diagrams, developing a Meta heuristic algorithm, developing symbols, and application of fuzzy sets. As a result, the approach has not been explored in FTA to deal with stochastic self-loop basic events. The self-loop basic event is associated with the probabilities of the event being generated, detected, or non-detected along with their probability distribution functions. The self-loop basic event rises in FTA based on embedded detection and recovery mechanisms in the system made up of both hardware and software subsystems.

b. The application of fuzzy sets has been growing at a significant rate because fuzzy sets offer a unique approach for dealing with certain types of essential imprecision. However, for fuzzy failure impact analysis, little attention has been paid to the

compensated/aggregated logic gates in FTA. The compensated/aggregated gates are composed of 'OR' and 'AND' operators with a compensatory operator.

1.3.4 K-OUT-OF-N SYSTEM SHORTCOMINGS

These shortcomings are as follows:

a. Due to low cost and high reliability, k-out-of-n solution becomes more popular among professionals. They have developed k-out-of-n:G and k-out-of-n:F systems made up of binary or multi-state units. Some publications have extended the concepts of the binary k-out-of-n to the multi-state k-out-of-n system with some limitations. These publications can be classified based on state (i.e., binary-state, multi-state), method (Computational algorithm, Meta heuristics, Analytical), and type of system (G, F, Load sharing). Although various k-out-of-n systems have been studied corresponding to the architecture of systems, no attention has been paid to k-out-of-n systems comprised of units with failure detection, isolation, and self-healing (recovery) mechanisms. Under such scenario the unit is called a self-loop unit, the continuous checks must be performed while the unit is in service. When failures are 'Detected', hardware devices notify the software application, which takes appropriate actions and tries to pin-point the problem to a replaceable or self-healing subunit. This step is called 'Isolation' and is a key for success in service availability. For 'Self-healing' (recovery) actions, the Software (SW) application may use Hardware (HW) resources to remove the failure and return the unit to the operational mode. Within such a system, it is important to note not only the reliability and the availability of the system, but also the mean and standard deviation of the time to k-out-of-n:G system failure.

b. Although the concept of Binary-State k-out-of-n has been extended to the Multi-State k-out-of-n, all rules may not be applicable to Multi-State k-out-of-n systems. Two types of Multi-State k-out-of-n:G systems have been studied (i.e., increasing and decreasing systems). The increasing Multi-State k-out-of-n:G system is equivalent to that at least kj units must be at least at state 'j' for the system to be in state 'j' or less where state 'i' is higher than state 'i+1'. An analytical model has been developed for this structure based on the properties of the Binary-State k-out-of-n system. The decreasing Multi-State k-out-of-n system is equivalent to that higher system state level 'j' and lower requirement on the

number of units that must be at state 'j' or less. Due to complexity of this structure, Meta heuristic algorithms have been developed. Although all researchers have studied various types of k-out-of-n systems corresponding to the architecture of systems, they have overlooked k-out-of–n systems comprised of units that have 'r' states (i.e., state 1st is the best operational condition and state rth is the worst operational condition) with state monitoring and recovery functions. Under such scenario the system is called a Reversible Multi-State k-out-of-n:G system with continuous monitoring being performed while the units are in service. Within such a system, it is important to note not only the reliability and the availability of the system, but also Time-To-State data (i.e., probability, the mean and standard deviation of the time to reach a certain state).

1.3.5 RELIABILITY ESTIMATION SHORTCOMINGS

These shortcomings are as follows:

a. With the known reliability of the units, the reliability of the system can be calculated. To estimate the reliability of the subsystem/unit/device, various prediction approaches for certain type of system are available depending upon the depth of knowledge of the design and the availability of historical data on subsystem/unit/device reliability. During production development cycle, data describing the system design evolves from a qualitative description of system functions to detailed specification and drawings suitable for production. However, the lack of precise design definition, lack of knowledge of use environment and lack of field operational history are the major concerns of all prediction methods. Alternatively, the use of fuzzy sets has been proposed since it offers a unique approach for dealing with certain types of essential imprecision knowledge about design, and subsystem/unit/device reliability factors. However, to estimate the reliability of a system composed of subsystems/units/devices with uncertain reliability data, little attention has been paid to develop analytical approach to deal with imprecise-chance reliability estimation when the chance factor is not specified precisely.

1.3.6 OBJECTIVES

The major objective of this thesis is enhancing the techniques employed in reliability design. The thesis focuses on FEA, the Markov chains, FTA, k-out-of-n systems, and reliability estimation models with stochastic and fuzzy input data. Each one has specific cooperation with design phases. The objectives are as follows:

i. To present a practical Failure Risk Analysis (FRA) model using Graph-based Risk Priority Number (GRPN). GRPN depicts the relationships among symptoms, root causes, and failures and takes into account these relationships for failure risk analysis to mitigate problems associated with traditional RPN (Corresponding to Section 1.3.1, Subsec.a).

ii. Using fuzzy aggregation and fuzzy compensation techniques in the MINMAX method, to present the GFEA approach to mitigate the problem (e.g., sensitivity and consistency analysis) associated with the proposed FEA approaches (Corresponding to Section 1.3.1, Subsec.b).

iii. To present fuzzy flow-rate equation to deal with time-varying input for the states of Markov chains associated with interval chance to represent the uncertain input data (Corresponding to Section 1.3.2, Subsec.a).

iv. To develop an analytical approach based on flow-graph concept for computing Time-To-Event data in a stochastic FTA (i.e., probability, the mean and standard deviation time to Top Event) for system with self-loop basic events (Corresponding to Section 1.3.3, Subsec.a).

v. To introduce the compensated/aggregated logic gates in FTA due to the extensively growth of application of fuzzy sets in reliability design. The compensated/aggregated

gates are composed of 'OR' and 'AND' operators with a compensatory operator (Corresponding to Section 1.3.3, Subsec.b).

vi. To develop an analytical approach based on the flow-graph concept for evaluating k-out-of-n system comprised of units with failure detection, isolation, and self-healing (recovery) mechanisms, to calculate Time-To-Failure data (Corresponding to Section 1.3.4, Subsec.a).

vii. To develop an analytical approach based on the flow-graph concept for evaluating Reversible Multi-state k-out-of-n system comprised of units that have 'r' states (i.e., state 1^{st} is the best operational condition and state r^{th} is the worst operational condition presented by the Semi-Markov model) with state monitoring and recovery functions to evaluate Time-To-State data (Corresponding to Section 1.3.4, Subsec.b).

viii. To present the imprecise-chance reliability estimation method that uses imprecise-chance to express the reliability of the system (Corresponding to Section 1.3.5, Subsec.a).

1.4 THESIS STRUCTURE

This dissertation is divided into seven chapters, five reference sections, and three appendices.

Chapter 1: This chapter presents an overall and detailed view of reliability design techniques and a literature survey.

Chapter 2: This chapter presents a method for Failure Risk Analysis (FRA) by using Graph-based Risk Priority Number (GRPN). It is based on the concept of using relational graphs that depict the level of the relationship between symptoms, root causes, and failures. Furthermore, this chapter introduces the Group-based Failure Effects Analysis (GFEA). The approach uses fuzzy Risk Priority Category (RPC) and group decision-making techniques to study both

the failure effects on the functions/subsystems and the failure risk category with uncertain information. In addition, the approach uses the compensated operators to allow the tradeoffs either among failure factors or among functions/subsystems attributes.

Chapter 3: This chapter presents an imprecise-chance Markov chain with stochastic transition rates for part parameter change, by using fuzzy flow-rate equation system associated with interval chance.

Chapter 4: This chapter presents stochastic FTA with self-loop basic events and Fuzzy FTA with compensated/aggregated gate. An analytical approach is developed for performing fault tree analysis (FTA) with stochastic self-loop basic events. The proposed approach uses the flow-graph concept and moment generating function (MGF) to compute Time-To-Event data. Also, in this chapter, the two newly developed Fuzzy Fault Tree (FTA) models are introduced to study the failure trade-off among basic events when compensation is allowed. Model I compensates fuzzy 'AND', and 'OR' gates with arithmetic mean. Model II aggregates fuzzy 'OR' and 'AND' gates with degree of nearness to the logic gates. A solved example is presented to illustrate the model's application to a system with imprecise failure impacts.

Chapter 5: This chapter presents the analytical approaches to evaluate both k-out-of-n systems with self-loop units and Reversible Multi-State k-out-of-n systems. The chapter, also, presents an analytical approach for a k-out-of-n system with units having the failure detection, isolation, and recovery mechanisms. The approach uses the flow-graph concept and moment generating function (MGF) to analyze the reliability of the k-out-of-n system with self-loop units. This newly developed analytical approach provides the probability of the system failure, system mean and standard deviation of time to failure. Later, a flow-graph-based approach is introduced to analyze a Multi-State k-out-of-

n:G/F/Load-Sharing system. The Multi-State k-out-of-n:G/F/Load-Sharing system comprises 'n' identical units, which are under state monitoring and recovery function. The units have 'r' states and their state transit models (i.e., Semi-Markov) are presented with a flow-graph model. This newly developed analytical approach provides Multi-State Time-To-State information such as the probability, and mean and standard deviation time for the unit/system being in certain state.

Chapter 6: This chapter presents a new imprecise-chance method for estimating network reliability when the chance factor is not specified precisely. The method uses fuzzy operand and fuzzy probability for uncertain or fuzzy reliability data during product development life cycle.

Chapter 7: This chapter presents conclusions and future directions

CHAPTER 2

FAILURE EFFECTS ANALYSIS

2.1 BACKGROUND

Failure Effects Analysis (FEA) is a fundamental risk analysis process involving information acquisition, modeling, analysis, and decision, which result in the physical design improvement. FEA methods have been concerned with developing either deterministic or fuzzy models in order to identify, prioritize and eliminate the potential failures in the system. Moreover, some approaches focus on application of decision-making techniques leading to improve the reliability, quality, and safety.

In deterministic methods, the Risk Priority Number (RPN) and Pareto Chart (PC) were used as the principal knowledge acquisition to represent and score the system failure effects. The RPN is a product of failure factors (Occurrence probability, Non-detection probability, Severity of impact, etc.). Table 2.1 shows that the traditional FMEA uses five scales and the scores of one to ten to measure the failure factors.

Table 2.1. Scores of failure factors

Score	Likelihood of Occurrence probability	Likelihood of Non-detection probability	Severity of impact
1	0	0-5	Not detected by customer
2	1/20000	6-15	
3	1/10000	16-25	Slight annoyance
4	1/2000	26-35	
5	1/1000	36-45	Customer dissatisfaction
6	1/200	46-55	
7	1/100	56-65	High degree
8	1/20	66-75	of dissatisfaction
9	1/10	76-85	Safety consequences
10	1/2	86-100	

For example, consider the protocol engine of the general data path section in the box comprised of several electrical circuit boards. As shown in Figure 2.1, the protocol engine is made up of Field Programmable Gate Arrays (FPGA), RAM, and Interfaces that generate the cell data including the header and data sections.

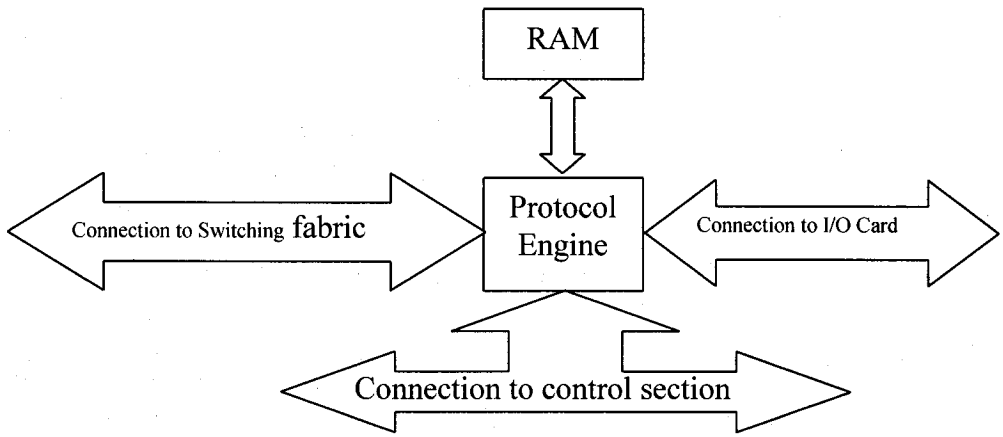


Figure 2.1. The protocol engine of data path

The potential failures that may occur in this subsection are data corruption, header corruption, power shutdown, and control error whose ranks are calculated as shown in Table 2.2:

Table 2.2. Risk Priority Number (RPN) of the failure in data path

Failure	Score			Risk Priority Number (RPN)
	Occurrence Probability	Non-detection Probability	Severity	
Data corruption	7	1	1	**7**
Header corruption	6	2	4	**48**
Power shutdown	2	1	9	**18**
Control error	3	3	9	**81**

To bring design of Failure Mode and Effects Analysis (FMEA) in line with accepted probabilistic risk management theory, the Society of Automotive Engineers (SAE) has

replaced the RPN metric with the Critically Matrix (CM). As shown in Figure 2.2, the CM shows the severity of a failure against its probability of occurrence. The failure that lies in the upper right quadrant is targeted for priority remedial action.

Pareto chart is a histogram chart that depicts the percentage of failure occurrence as shown in Figure 2.3. The failure with high percentage of occurrence is targeted for priority remedial action.

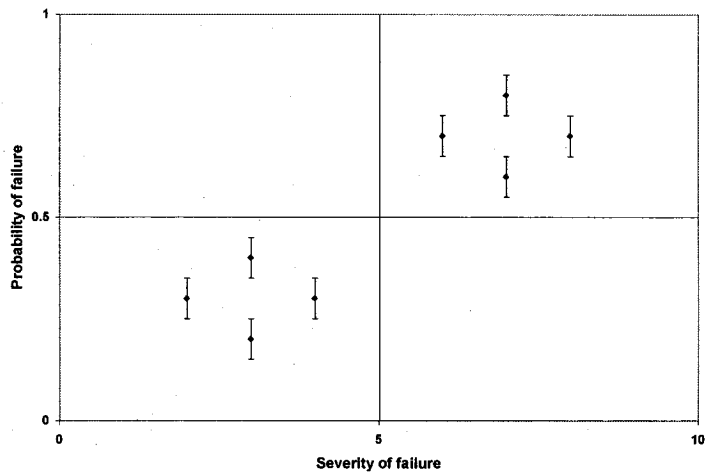


Figure 2.2.Criticality matrix

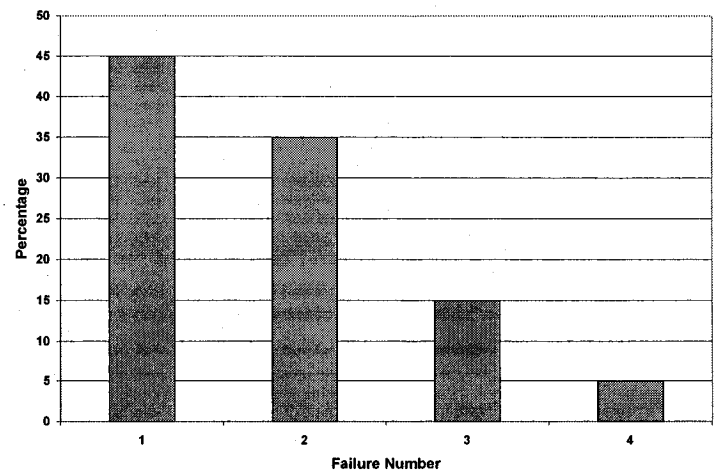


Figure 2.3. Pareto chart

However, the use of either CM or Pareto chart alone cannot compensate the major weakness of the SAE approaches.

However, RPN, CM, and Pareto chart are criticized due to the following shortcomings:

- They do not take into account the quality of the product.
- They overlook the relative importance among failure factors.
- Multiplication of factors in RPN expression is not always appropriate for risk priority analysis.
- Using RPN does not satisfy the requirements of measurement.
- In-consistency of relation for different factors in RPN (e.g., linear relation between failure probability and probability scale and non-linear relation between non-detection probability and probability scale).
- Evolvement of identical RPN value for different sets of factors.

Due to the drawbacks of deterministic RPN-based approaches, the RPN has been replaced with Risk Priority Category (RPC) for performing fuzzy FEA in the system. RPC bases upon the traditional categories i.e., very low 'VL' to very high 'VH' corresponding to the scoring for the failure factors as presented in Table 2.3.

Table 2.3. Categories for the failure factors

Occurrence probability	Non-detection probability	Severity of impact	Categories
1	1	1	VL
2,3	2,3	2,3	L
4,5,6	4,5,6	4,5,6	M
7,8	7,8	7,8	H
9,10	9,10	9,10	VH

As an RPN is multiplication of the failure factors, Table 2.4 provides a division of the traditional domain of the risk priority number (RPN) from 1 to 1,000 into nine class intervals to discriminate risks.

Table 2.4. Risk Priority Categories

RPN class interval	Class Mean	RPC category
1-50	25	VL
50-100	75	VL-L
100-150	125	L
150-250	200	L-M
250-350	300	M
350-450	400	M-H
450-600	525	H
600-800	700	H-VH
800-1000	900	VH

However, developing and benchmarking a fuzzy rule base system that maps the failure categories into RPC categories is a vital process. Moreover, FEA is a system technical risk analysis that extracts knowledge about the potential failure mode from a group of experts with possible conflict in experts' opinions. Therefore, RPC-based methods require adapting the rule base with group-based FEA system.

In the chapter, two new methods are presented for Failure Risk Analysis (FRA). The first, Graph-based Risk Priority Number (GRPN), is based on the concept of using relational graphs that depict the level of the relationship between symptoms, root causes, and failures. The second, Group-based Failure Effects Analysis (GFEA), uses fuzzy Risk Priority Category (RPC) and group decision-making techniques to study both the failure effects on the functions/subsystems and the failure risk category with uncertain information.

2.2 GRAPH-BASED FAILURE EFFECT ANALYSIS

In standard FEA, either RPC or RPN is used to construct not only the system failure effect model but also to implement risk analysis. However, due to the fuzzy set modeling and knowledge-based system problems (i.e., benchmarking of fuzzy variables, conflict of the

rules, respectively), the use of RPC is a painstaking approach. On the other hand, the use of the traditional RPN confronts failure risk analysis with problems as follows:

- Evolving an identical RPN value for different sets of factors.
- Ignoring the level of relationship between symptoms, root causes, and failures.

To improve RPN, newly developed Graph-based FEA bases its process on extracting knowledge about the potential failure in four steps. (i) Identify a set of potential failures, (ii) Determine a set of root causes for a strong relation to the failure, and a set of root causes for a weak relation to the failure. (iii) Determine a set of symptoms for strong relation to the root cause and a set of symptoms for weak relation to the root cause, (iv) Generate both strong and weak relational graphs that present the level of relationship (strong, weak) between the symptoms, root causes, and the failures as shown in Figure 2.4. The new method improves the traditional RPN by including both relational graphs into the RPN expression.

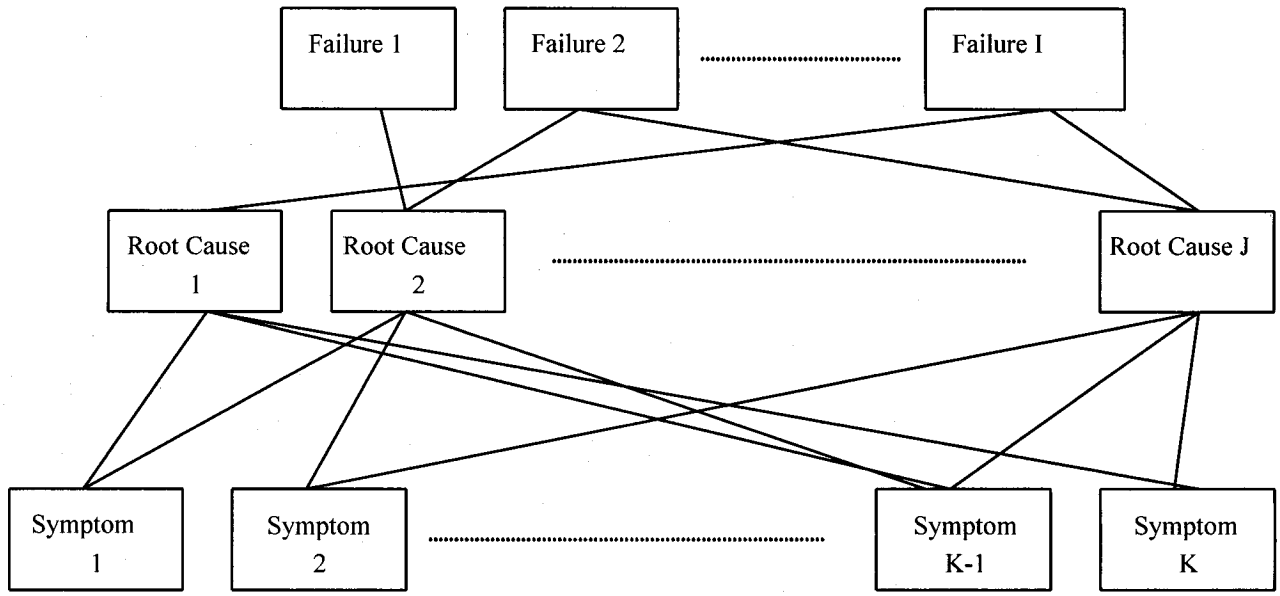


Figure 2.4. Failure relational graph

For example, consider the protocol engine of the data path section in Figure 2.1. Figure 2.5 presents the relationships among the potential failures, root causes, and symptoms that can be classified to the weak and strong graphs. Having such graphs, the objective is to present a

practical FEA model that uses Graph-based Risk Priority Number (GRPN). GRPN depicts the relationships among symptoms, root causes, and failures and takes into account these relationships for failure risk analysis to mitigate the problems associated with traditional RPN.

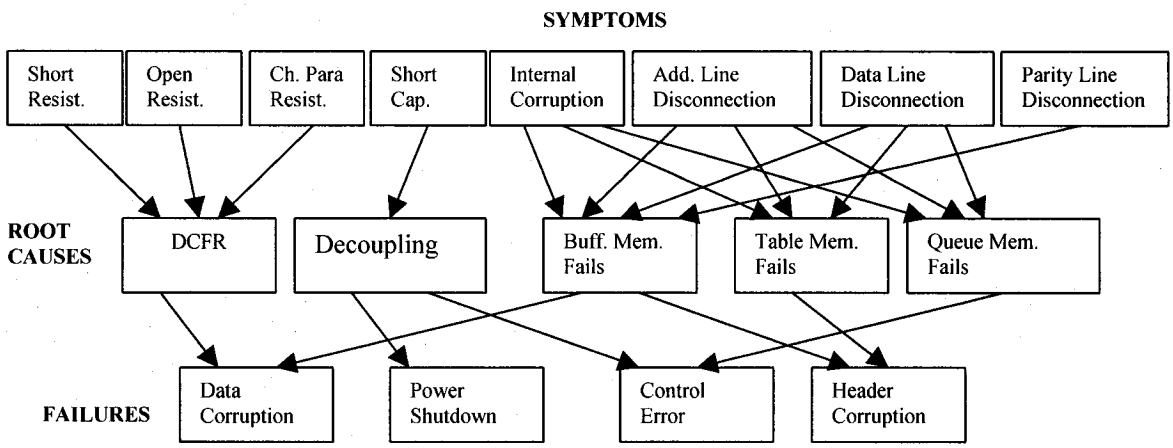


Figure 2.5. The relational graph of the protocol Engine

2.2.1 GRAPH-BASED FAILURE EFFECT ANALYSIS MODEL [1-134]

Figure 2.4 introduces the rational graphs related to a system. These graphs present a set of 'I' potential failures, a set of 'J' root causes, and a set of 'K' symptoms. Moreover, the graphs depict the strong and weak relationship between symptoms, root causes and failures in strong and weak relational graphs denoted by G_s and G_w. G_s and G_w are directed graphs that constitute nodes (i.e., failure, root cause, symptom) and arcs (i.e., relation between two nodes). Each graph (i.e., G_s, G_w) has a reverse graph (i.e., g_s, g_w) that the direction of arcs is opposite of that in the graph. On the other hand, each failure is associated with scored risk factors (i.e., Occurrence probability, Non-detection probability, Severity of impact, Expected cost) in the range of '1' and '10'. Since different sets of factors may result in identical RPN, the FEA model considers the effect of relational directed graphs in RPN expression in accordance with the following assumptions:

- The causes of failure events are equally weighted.
- No loop is allowed in relational graphs.

The model is comprised of the forward and backward steps in order to find the rank of the potential failure based on level of relation to root causes and symptoms. The GRPN is a graph-based risk priority number that takes into account the interrelationships of the symptoms, root causes, and failures into RPN.

Initial steps:

Step 1: identify the sets of failures, root causes, and symptoms

Step 2: generate G_s, G_w, g_s, and g_w

Step 3: Compute RPN(i) of all potential failures (Occurrence probability score × Non-detection probability score × Severity of impact score × Expected cost score) where i=1,2,…,I

Forward steps:

Step 4: Consider G_s, G_w

Step 5: Set x equal to one

Step 6: Generate Y(x) (A set of all nodes that are not ranked until step x in G_s), if Y(x) is empty go to backward steps

Step 7: Generate Z(x) (A subset of set Y(x) includes starting nodes (symptoms, root causes, and failures) in step x. The starting node is a node with no entry)

Step 8: Generate A(x), which is a subset of Z(x) including starting node in G_w

Step 9: Assign value of x as forward rank to all elements of set A(x) (i.e., R(i)=x $\forall i \in A(x)$)

Step 10: Remove all nodes of A(x) and associated arcs from G_s, G_w , x = x + 1

Step 11: Go to Step 6

Backward steps:

Step 12: Consider g_s, g_w

Step 13: Set x equal to one

Step 14: Generate Y(x) (A set of all nodes that are not ranked until step x in g_s), if Y(x) is empty then go to step 20.

Step 15: Generate Z(x) (A subset of set Y(x) includes starting nodes (symptoms, root causes, and failures) in step x. The starting node is a node with no entry.)

Step 16: Generate A(x), which is a subset of Z(x) including starting node in g_w

Step 17: Assign value of x as backward rank to all element of set A(x) (i.e., O(i)=x $\forall i \in A(x)$)

Step 18: Remove all nodes of A(x) and associated arcs from g_s, g_w , x = x + 1

Step 19: Go to Step 14

Ranking steps:

Step 20: For all failure nodes, calculate total the rank that constitutes the forward and backward ranks as follows:

$$T(i) = \frac{R(i) + (1 + Max - O(i))}{2} \qquad \text{for i=1,2,...,I} \tag{2.2.1}$$

where 'Max' is the maximum rank of the failures in backward steps

Step 21: For all failure nodes calculate GRPN as follows:

$$GRPN(i) = \frac{RPN(i)}{T(i)} \qquad \text{for i=1,2,...,I} \tag{2.2.2}$$

In steps 20 and 21, the FRA model encompasses all failure factors and cause factors into GRPN to improve the traditional RPN. In order to demonstrate the use of FEA Model, an example composed of four symptoms, three root causes, and five failures is presented in the next section.

2.2.2 ILLUSTRATION OF GRAPH-BASED FAILURE EFFECT ANALYSIS MODEL

Consider a system with five failures whose occurrence is based on three root causes. Figure 2.6 shows the relational graphs (G_s, G_w) that depict level of relationship between four potential symptoms and root causes and failures. S, R, and F denote the symptom, root cause, and failure.

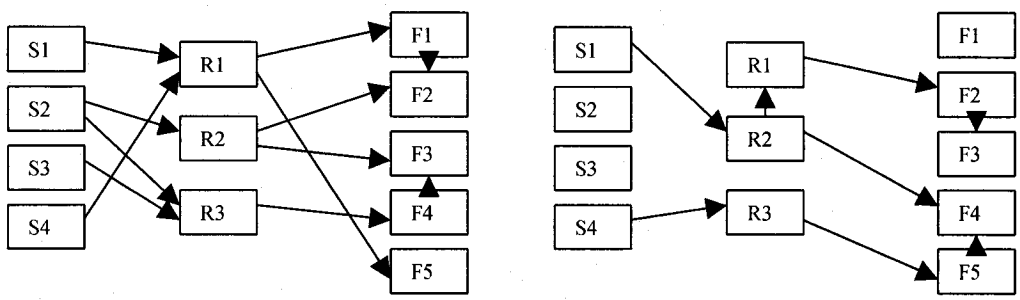


Figure 2.6. Relational graphs

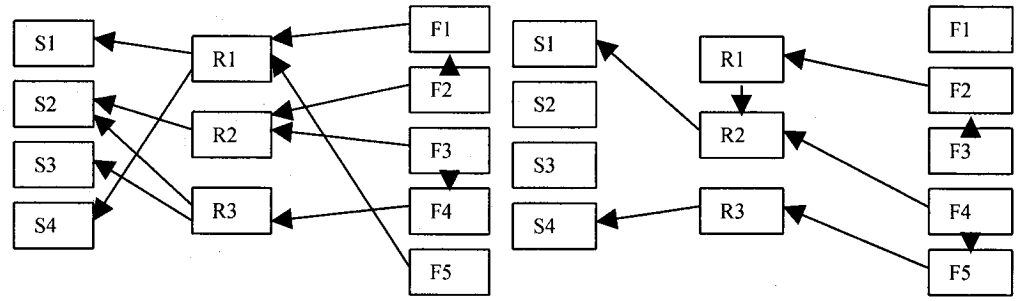
Risk Priority Number of the failures (F1, F2, F3, F4, and F5) are calculated based on Occurrence probability score × Non-detection probability score × Severity of impact score × Expected cost score and converted to a value between 1 to 10, which results in 7, 5, 7, 4, 4, respectively. Therefore, ranking of the failures is "F1-F3-F2-F4-F5" or "F1-F3-F2-F5-F4" since RPN's of both F4 and F5 are equal. Thus, by implementing the forward and backward steps listed in Tables 2.5 and 2.6, the FEA will improve the rank of failure according to their level of relationships to the root causes and the symptoms.

Table 2.5. Forward step results

Status	Iteration	Y	Z	A	R/O
Forward	1	{S1,S2,S3,S4,R1,R2,R3,F1, F2,F3,F4,F5}	{S1,S2,S3,S4}	{S1,S2,S3,S4}	R(S1)=R(S2)= R(S3)= R(S4)=1
	2	{R1,R2,R3,F1,F2,F3,F4,F5}	{R1,R2,R3}	{R1,R2,R3}	R(R1)=R(R2)= R(R3)=2
	3	{F1,F2,F3,F4,F5}	{F1,F4,F5}	{F1,F5}	R(F1)= R(F5)=3
	4	{F2,F3,F4}	{F2,F4}	{F2,F4}	R(F2)= R(F4)=4
	5	{F3}	{F3}	{F3}	R(F3)=5
	6	∅			

Since Y(6) is empty, Go to Backward steps:

Step 12: By changing the direction of arcs in G_s, and G_w, we get g_s, g_w as shown in Figure 2.7.



Reversed strong relational graph (g_s) Reversed weak relational graph (g_w)

Figure 2.7. Reversed relational graphs

Table 2.6. Backward step results

Status	Iteration	Y	Z	A	R/O
Backward	1	***{S1,S2,S3,S4,R1,R2,R3,F1, F2,F3,F4,F5}***	{F2,F3,F5}	{F3}	O(F3)=1
	2	{S1,S2,S3,S4,R1,R2,R3,F1, F2,F4,F5}	{F2,F4,F5}	{F2,F4}	O(F2)=O(F4)=2
	3	{S1,S2,S3,S4,R1,R2,R3,F1, F5}	{R2,R3,F1,F5}	{F1,F5}	O(F1)=O(F5)=3
	4	{S1,S2,S3,S4,R1,R2,R3}	{R1,R2,R3}	{R1,R3}	O(R1)=O(R3)=4
	5	{S1,S2,S3,S4,R2}	{S1,S3,S4,R2}	{S3,S4,R2}	O(S3)=O(S4)=O(R 2)=5
	6	{S1,S2}	{S1,S2}	{S1,S2}	O(S1)=O(S2)=6
	7	∅			

Since Y(7) is empty, Go to Ranking steps:

By executing ranking steps (i.e., steps 20 and 21), Table 2.7 presents the result of GRPN. Therefore, the new rank of the failures that encompasses the failure, root cause, and symptom relations is "F1-F5-F3-F2-F4". Using RPN or GRPN, the rank of F1 is the same. However, failures 4 and 5 with equal and low risk rank in traditional RPN have different

GRPN, which causes F5 to move to upper rank because of F4 effect on F5 in g_w. Moreover, the order of F2 and F3 in both RPN and GRPN is similar.

Table 2.7. Ranking step result

Failure	RPN	T	GRPN
F1	7	3.5	2
F2	5	4.5	1.11
F3	7	5.5	1.27
F4	4	4.5	0.89
F5	4	3.5	1.42

2.3 GROUP-BASED FAILURE EFFECT ANALYSIS

FEA is a system technical risk analysis process. This process comprised information acquisition, modeling, analysis, and decision, extracts knowledge about the potential failure mode from a group of experts is called Group-based Failure Effects Analysis (GFEA). In the information acquisition phase, the experts' opinions differ substantially because the experts do not often agree on the level of the failure factors and the functions/subsystems attributes (e.g., importance). Therefore, conflict always occurs in GFEA. The proposed FEA approaches can be classified into deterministic and fuzzy models. Since deterministic models are based upon the use of RPN, they suffer from drawbacks of the traditional RPN. On the other hand, due to fuzziness, and the complexity of both the failure effect and the importance of the functions/subsystems, the use of fuzzy linguistic values is the most realistic approach. Thus, fuzzy models introduce the rule base and RPC to address the GFEA requirements as follows:

- Difficulty of assigning an exact numerical value to importance of the risk criteria.
- Difficulty of expression of opinion via numerical value for the relative relation of the potential failures to the certain risk criterion.

Though, fuzzy models have been studied, the followings appear to be inefficient in addressing the requirements of the GFEA:

- Conflict of experts' opinion.
- Existence of the tradeoffs among the risk criteria.
- Sensitivity analysis.
- Conflict of rules (i.e., 'if' parts of the rules are similar and 'then' parts are different).
- Subsumption (i.e., two or more rules have the same result, but one contains additional restrictions on the situations in which it will succeed).

The objective of this thesis on GFEA model is not only to address the GFEA requirements but also to mitigate the drawbacks of fuzzy models.

2.3.1 GROUP-BASED FAILURE EFFECT ANALYSIS MODEL [1-135]

Consider a system with 'J' risk criteria (i.e., failure factors and functions/subsystems). They can be expressed as score, percentage, and probability. The risk criterion has its importance with respect to the perceptions of the member of a group (made up of 'K' experts). Also, 'I' potential failures are listed. Based on 'k'th expert's opinion, each one has a relative impact on the system relative to a risk criterion. As the experts cannot assign the exact number to either the importance of risk criteria or the failure impact on a system, the use of linguistic variables becomes the best alternative. Table 2.8 presents nine linguistic variables denoted by 'S' with subscripted index rank to express the importance of the risk criteria and the system failure impacts relative to a risk criterion. To minimize the classification error and have a consistent GFEA model, the linguistic variables were based upon the fuzzy method and the Gray relational analysis [1-113]. Gray theory provides a measure to analyze relationships between discrete quantitative and qualitative series.

Table 2.8. Fuzzy linguistic variables

Index	Linguistic Variable	Probability	Percentage	Score
S_7	Perfect - *(P)*	0.5	0-5	10
S_6	VeryHigh -*(VH)*	0.1	6-15	9
S_5	High - *(H)*	0.05	16-25	8
*	Medium-High	0.01	26-35	7
S_4	Medium - *(M)*	0.005	36-45	6
*		0.001	46-55	5
*		0.0005	56-65	4
S_3	Low - *(L)*	0.0001	66-75	3
*	VeryLow-Low	0.00005	76-85	2
S_2	VeryLow - *(VL)*	<0.00005	86-99	1
S_1	Non - *(N)*	0	100	0

*: Use next lower level

Using predefined categories, the Decision Matrices are defined according to the experts' opinion. For example, decision matrix of 'k'th expert is

$$MAT(k) = \begin{matrix} I_k(f_1) & I_k(f_2) & \ldots\ldots\ldots & I_k(f_J) \end{matrix} \begin{bmatrix} P_{1k}(f_1), & P_{1k}(f_2), & \ldots\ldots\ldots, & P_{1k}(f_J) \\ P_{2k}(f_1), & P_{2k}(f_2), & \ldots\ldots\ldots, & P_{2k}(f_J) \\ & & & \\ & & & \\ P_{Ik}(f_1), & P_{Ik}(f_2), & \ldots\ldots\ldots, & P_{Ik}(f_J) \end{bmatrix}_{I\times J}$$

Where $I_k(f_j)$ states the importance of 'j'th risk criterion and $P_{ik}(f_j)$ is the relative impact of 'i'th failure on the system relative to 'j'th risk criterion ('f_j'). In the case of divergence of decision matrices, we have developed the GFEA model not only to dilute the divergence in the failure risk analysis and but also to prioritize the system failures. The method uses the principal concept of ranking, maximum, and minimum as defined in Appendix-A.

To dilute the divergence of experts' opinion and to aggregate the opinions, the Group-based Failure Effects Analysis method has constituted the steps as follows:

Step 0. Determine a set of risk criteria and a set of potential failures

Step 1. Define importance of risk criteria

Step 2. Define the relative impact of the potential failure

Step 3. Determine risk priority category of failure 'i' according to perception of 'k'th expert. There are always tradeoffs among decision factors. The tradeoffs cause the identical RPC for different scenarios. Thus, to compute the category of the impact of failure according to certain experts' opinion, Equation (2.3.1) can be used. It encompasses the divergence of experts' opinion in decision matrices. Also, to deal with tradeoffs among the decision factors, it is equipped to compensate operations.

$$RPC_{ik} = \text{Min}\{Cmp(I_k(f_1), P_{ik}(f_1)), Cmp(I_k(f_2), P_{ik}(f_2)),\ldots, Cmp(I_k(f_J), P_{ik}(f_J))\} \quad (2.3.1)$$

where $Cmp(I_k(f_j), P_{ik}(f_j))$ is the compensated maximum function for aggregating the importance of 'j'th risk criterion (i.e., $I_k(f_j)$) with 'i'th failure effect on 'j'th risk criterion (i.e., $P_{ik}(f_j)$) according to 'k'th expert's opinion. By substituting $I_k(f_j)$ and $P_{ik}(f_j)$ with corresponding indices according to Table 2.8 (i.e., $S\alpha_{jk}$, and $S_{\beta ikj}$) in Equation(3), we get

$$RPC_{ik} = \text{Min}\{Cmp(S\alpha_{1k}, S_{\beta ik1}), Cmp(S\alpha_{2k}, S_{\beta ik2}),\ldots, Cmp(S\alpha_{Jk}, S_{\beta ikJ})\} \quad (2.3.2)$$

where

$$Cmp(S\alpha_{jk}, S_{\beta ikj}) = S_{V_{ijk}} \quad (2.3.3)$$

and

$$V_{ijk} = Rnd\left(\gamma.Max(\alpha_{jk}, \beta_{ijk}) + (1-\gamma).Int[\frac{\alpha_{jk} + \beta_{ijk}}{2}]\right) \quad (2.3.4)$$

where γ is the degree of aggregation ($0 \leq \gamma \leq 1$). For γ equal to one, Equation (2.3.3) is same as Maximum function (see Definition A-7 in Appendix-A) and setting γ equal to zero yields the arithmetic mean for Equation (2.3.3). Thus, by substituting Equations (2.3.3) and (2.3.4) into Equation (2.3.2), we have RPC_{ik} that indicates risk priority category of 'i'th failure in the system based on 'k'th expert's perception.

Step 4. Rank the RPC's of the failure

Using Definition A-7, the ranked risk priority category can be generated for each failure. Thus, the set of RPC's for 'i'th failure is expressed by

$$Set_K(i) = \{RPC_{i1}, RPC_{i2}, ...RPC_{iK}\} \quad (2.3.5)$$

with respect to Definition A-6, the ranked set is

$$Ord_K(i) = \{RPC_{[i1]}, RPC_{[i2]}, ...RPC_{[iK]}\} \quad (2.3.6)$$

where, $RPC_{[ik]}$ is the 'k'th maximum in the ranked list of the set element.

Step 5. Determine aggregation categories

Depending upon the number of experts agreed on certain level on failure impact, the aggregation category must be generated. Assume 'ε' is the total number of experts have agreed on a certain failure and criterion. Using the ratio $\frac{\varepsilon}{K}$, the aggregation category maps 'ε' to the relative linguistic set of Table 2.8. For example, the aggregation category is 'Perfect' when all experts agree on certain level of the failure effect on the system (i.e., ε = K) . To map 'ε', the Equation (2.3.7) presents the aggregation function used in the model.

$$Agg(\varepsilon) = S_{Int[1+(\varepsilon.\frac{\Psi-1}{K})]} \quad (2.3.7)$$

where 'ε' is the number of experts that satisfies aggregation function ($0 \leq \varepsilon \leq K$).

Step 6. Compute the comprehensive risk priority category of the potential failures

In the Group-based FEA, the failure is associated with a set that includes ordered elements (i.e., RPC's) corresponding to the members of a group. The comprehensive risk priority category for each failure is the aggregation of the RPC's of the certain failure. Thus, by substituting Agg(ε) function (Equation (2.3.7)) into Equation (2.3.8), the comprehensive aggregated failure impact of 'i'th failure in the system is expressed by

$$RPC_i = Max\{Min(Agg(\varepsilon), RPC_{[i\varepsilon]}), Min(Agg(\varepsilon-1), RPC_{[i(\varepsilon-1)]}), ..., Min(Agg(1), RPC_{[i1]})\} \quad (2.3.8)$$

where $RPC_{[ik]}$ is the 'k'th maximum in the ranked list of RPC of 'i'th failure and i=1,2,.....,I. By performing step 6 for all failures (i=1,2,.....,I) and using Definition A.6, a set made up of the ranked comprehensive risk priority category can be generated .

Step 7. Perform sensitivity analysis

In order to ensure the consistency of GFEA result, a slight change in either the level of risk criteria importance $I_k(f_j)$ or the level of failure impact on risk criteria $P_{ik}(f_j)$ for I=1,2,….,I , j=1,2,….,J and k=1,2,….,K (if a slight change of them produces a completely different result, tune the aggregation degree to dilute the divergence of the certain expert on the result). Moreover, by changing γ from 0 to 1 with incremental step equal to 0.1 and performing steps 3 to 6, generate RPC's corresponding to the certain value of γ. Based on Pareto Chart, if the 15% of the top ranked failures are always similar for the different values of γ, the GFEA result is consistent and independent from γ. Otherwise, conflict level of experts' opinion needs to be diluted by removing the predefined items of decision matrix ($I_k(f_j)$, $P_{ik}(f_j)$) that slight change of them causes inconsistency of the result. Perform steps 3 to 6 to find the result.

2.3.2 ILLUSTRATION OF GROUP-BASED FAILURE EFFECT ANALYSIS MODEL

Military vehicles are periodically required to have their suspension tested. Thus, a new model of suspension tester is developed to perform the test in the field without returning the vehicles to shop. This device comprises digital and analog circuitry and hydraulic subsystems as shown in Table 2.9. The device performs the test in several steps (i.e., engage, lift, release and disengage steps). In order to study the RPC of the failures and rank them for improving physical design, a group including three experts (e.g., design engineer, reliability engineer, and sales/marketing engineer) will carry out the FEA. The system has 12 risk criteria (9 subsystems and 3 failure factors) as shown in the fourth row of the decision matrix in Table 2.9. Rows 1 to 3 of the decision matrix are the relative importance (category) of those risk criteria corresponding to experts 1, 2, and 3, respectively. Moreover, the experts identified 59 potential failures that were listed in row 5 to the end. In each row, a failure is associated with category of its impact to defined criteria in format of "X_1-X_2-X_3" corresponding to experts 1, 2, and 3. For example, the relative impact of the R232(com) interface disconnection (Failure) on the Main Console (Risk criterion) is defined "H-VH-VH" by experts (i.e., expert1 assigns High(H) and experts 2, and 3 assign VeryHigh(VH) to

the relative impact of the R232(com) interface disconnection on the Main Console). Table 2.9 depicts divergence/conflict of experts' opinions that come from human brain process and the fuzziness of the failure impact.

Using GFEA model with a degree of aggregation (e.g., equal to 0.8), Table 2.10 shows subsystems such as Winsys, power supply, and circuit breaker have high ranked failures and they need to be reengineered. To ensure the robustness of the model, the sensitivity analysis is performed. Table 2.11 presents that the 15% of the top ranked failures are the same for different values of the aggregation degree and ranking differentiation for $\gamma>0.8$ is justifiable than ranking differentiation for $\gamma<0.8$. RPC of the failures is different and it corresponds to the value of γ because γ compensates the aggregation function with arithmetic mean. Furthermore, a slight change of parameters in decision matrices does not result in completely different RPCs that lead to the consistency of GFEA method.

Table 2.9. Decision Matrices

	Decision Criteria												
Expert 1	VH	P	VH	VH	H	M	VH	VH	VH	VH	VH	H	VH
Expert 2	H	VH	H	H	H	L	L	M	M	M	M	H	M
Expert 3	P	H	M	M	M	M	H	H	H	H	L	H	H
Failures	Main Console	Motor Control Console	Hull Accelerometer	Wheel Accelerpmeter	Load Pad	Accuator	Digital Breakout input	Digital Breakout Box output	Ruptured Hoses	Manif. Ass.	Failure probability	Non-detection probability	Expected cost
RS232(Com1) interface disconnected	H-VH-VH	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-M-L	H-M-L	H-M-M
Circuit Breaker input Cable	N-H-H	N-H-H	N-H-H	N-H-H	N-H-H	N-H-H	VH-H-H	N-N-N	N-N-N	N-N-N	L-VH-L	L-M-L	H-M-M
Circuit Breaker input SW 1	N-M-H	N-M-H	N-M-H	N-M-H	N-M-H	N-M-H	H-M-H	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
Circuit Breaker input SW 2	N-M-H	N-M-H	N-M-H	N-M-H	N-M-H	N-M-H	H-M-H	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
Circuit Breaker input SW 3	N-M-H	N-M-H	N-M-H	N-M-H	N-M-H	N-M-H	H-M-H	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
Acc. Cable	H-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-M-H	N-N-N	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
Acc.1A fuse	VH-N-N	N-N-N	N-N-N	N-N-N	N-N-N	VH-H-H	N-N-N	N-N-N	N-N-N	N-N-N	M-VL-L	L-M-L	H-M-M
Acc. Dual power supply	VH-N-N	N-N-N	N-N-N	N-N-N	N-N-N	VH-H-H	N-N-N	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
Acc. 1A (-15)V	N-N-N	N-N-N	VH-H-H	VH-H-H	N-N-N	VH-H-H	N-N-H	N-N-H	N-N-H	N-N-H	L-VL-H	L-M-H	H-M-H
Acc. 1A (+15)V	N-N-N	N-N-N	VH-H-H	VH-H-H	N-N-N	H-H-H	N-N-H	N-N-H	N-N-H	N-N-H	L-VL-H	L-M-H	H-M-H
APM (Analog Personality Madule)	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	VH-H-H	N-N-H	N-N-H	N-N-H	N-N-H	L-VL-H	L-M-H	H-M-H
AFF Borad	N-N-N	N-N-N	H-H-H	H-H-H	H-H-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
3A Fuse-24 VDC	M-L-M	N-N-N	N-N-N	N-N-N	M-M-M	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
3A Fuse- power supply	L-L-L	N-N-N	N-N-N	N-N-N	L-L-L	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
24VDC power supply	H-H-H	N-N-N	N-N-N	N-N-N	H-H-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
LoadPad/Wheel Acc./Hull Acc./ Cable	N-N-N	N-N-N	L-L-M	L-L-M	L-L-M	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	L-VL-L	L-M-L	H-M-M
MV10-Signal Conditioning Board	N-N-N	N-N-N	N-N-N	N-N-N	L-L-M	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
Power conditioning	N-N-N	N-N-N	N-N-N	N-N-N	M-M-M	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
1A Fuse-dual power supply-15 VDC	VH-N-N	VH-N-N	VH-M-M	VH-M-M	VH-M-M	VH-N-N	VH-N-N	VH-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
1A Fuse- input power supply	M-N-N	M-N-N	M-H-H	M-H-H	M-N-N	M-N-N	M-N-N	M-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
ICS 3150-020	M-N-N	M-N-N	M-M-M	M-M-M	M-N-N	M-N-N	M-N-N	M-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
Encoder	N-N-M	M-M-M	N-N-M	N-N-M	N-N-M	N-N-M	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
120VAC-DMC Fuse	N-L-N	L-L-L	N-L-N	N-L-N	N-L-N	N-L-N	N-L-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
120VAC-DPST Relay	H-M-H	H-M-H	H-M-H	H-M-H	H-M-H	H-M-H	H-M-H	H-N-H	H-H-H	H-N-H	H-VH-H	L-M-H	H-M-H
120VAC-Circuit Breaker	H-M-H	H-M-H	H-M-H	H-M-H	H-M-H	H-M-H	H-M-H	H-N-H	H-H-H	H-N-H	H-VH-H	L-M-H	H-M-H
120VAC-Receptable Connection	N-L-H	L-L-H	N-L-H	N-L-H	N-L-H	N-L-H	N-L-H	N-N-H	N-N-H	N-N-H	M-VH-H	L-M-H	H-M-H
Magnetic position switch on Rig	N-N-N	H-VH-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
DMC 1412	N-N-N	H-VH-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
Servo Amp	N-N-N	H-VH-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
ICM 1460	N-N-N	H-VH-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
Com1	N-N-N	H-VH-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
RTD SS8	L-M-L	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
Keypad	L-M-L	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	M-VH-L	L-M-L	H-M-M
Winsys-RTD DM6856	H-H-H	N-H-N	N-H-N	N-H-N	N-H-N	N-H-N	N-H-N	N-H-N	N-H-N	N-H-N	M-H-L	L-H-L	H-H-M
Winsys-RTD DM6430	H-H-H	L-H-N	L-H-N	L-H-N	L-H-N	L-H-N	L-H-N	L-H-N	L-H-N	N-H-N	M-H-L	L-H-L	H-H-M
Winsys-DIO	H-VH-H	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	N-N-N	M-L-L	L-M-L	H-M-M
Winsys Fuse	M-VH-M	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	N-N-N	M-L-L	L-M-L	H-M-M
Winsys-120 VCA power	M-VH-M	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	L-N-N	N-N-N	M-L-L	L-M-L	H-M-M
Digital Circuit Breaker- Cable	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	N-N-N	N-N-N	N-N-N	M-L-L	L-M-L	H-M-M
Digital Circuit Breaker- Switch 1	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	N-N-N	N-N-N	N-N-N	M-L-L	L-M-L	H-M-M
Digital Circuit Breaker- Switch 2	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	N-N-N	N-N-N	N-N-N	M-L-L	L-M-L	H-M-M
Digital Circuit Breaker- Switch 3	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	L-M-L	N-N-N	N-N-N	N-N-N	M-L-L	L-M-L	H-M-M
Electromotor	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-H-H	N-N-N	M-L-L	L-M-L	H-M-M
Pump	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-H-H	N-N-N	M-L-L	L-M-L	H-M-M
Resevior	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-H-H	N-N-N	M-L-L	L-M-L	H-M-M
Cylender Leakage	N-N-H	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-M	N-N-M	N-N-M	VH-VH-VH	VH-VH-VH	H-M-M
RAM Excessive play	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-M	N-N-M	N-N-M	M-L-M	M-L-M	H-M-M
Siezed Cylender	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-M	N-N-M	N-N-M	M-L-M	M-L-M	H-M-M
Pressure Relife Valve leakage	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-N-H	H-N-H	H-H-H	H-L-H	H-M-H	H-M-H
Pressure Relife Valve open	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-N-H	H-N-H	H-H-H	H-L-H	H-M-H	H-M-H
Solenoid	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-N-H	H-N-H	H-H-H	H-L-H	H-M-H	H-M-H
Valves	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	N-N-N	H-N-H	H-N-H	H-N-H	H-H-H	H-L-H	H-M-H	H-M-H
Circuit Breaker Output- 3 A Fuse	L-L-L	L-L-L	L-L-L	L-L-L	L-L-L	L-L-L	N-L-L	H-M-L	H-M-L	H-M-L	H-M-L	H-M-L	H-M-L
Circuit Breaker Output- 3A fuse(2)	L-L-L	L-L-L	L-L-L	L-L-L	L-L-L	L-L-L	N-L-L	H-M-L	H-M-L	H-M-L	H-M-L	H-M-L	H-M-L

Table 2.10. GFEA result for γ=0.8

Rank	Failures	Failure effect/Severity	Rank	Failures	Failure effect/Severity
1	120VAC-DPST Relay	High	3	3A Fuse-24 VDC	Low
1	120VAC-Circuit Breaker	High	3	3A Fuse- power supply	Low
2	Circuit Breaker input Cable	Medium	3	24VDC power supply	Low
2	Circuit Breaker input SW 1	Medium	3	LoadPad/Wheel Acc./Hull Acc./ Cable	Low
2	Circuit Breaker input SW 2	Medium	3	MV10-Signal Conditioning Board	Low
2	Circuit Breaker input SW 3	Medium	3	Power conditioning	Low
2	Acc. 1A (-15)V	Medium	3	1A Fuse-dual power supply-15 VDC	Low
2	Acc. 1A (+15)V	Medium	3	1A Fuse- input power supply	Low
2	APM (Analog Personality Madule)	Medium	3	ICS 3150-020	Low
2	120VAC-Receptable Connection	Medium	3	Encoder	Low
2	Winsys-RTD DM6856	Medium	3	120VAC-DMC Fuse	Low
2	Winsys-RTD DM6430	Medium	3	Magnetic position switch on Rig	Low
2	Digital Circuit Breaker- Cable	Medium	3	DMC 1412	Low
2	Digital Circuit Breaker- Switch 1	Medium	3	Servo Amp	Low
2	Digital Circuit Breaker- Switch 2	Medium	3	ICM 1460	Low
2	Digital Circuit Breaker- Switch 3	Medium	3	Com1	Low
2	Cylender Leakage	Medium	3	RTD SS8	Low
2	RAM Excessive play	Medium	3	Keypad	Low
2	Siezed Cylender	Medium	3	Winsys-DIO	Low
2	Pressure Relife Valve leakage	Medium	3	Winsys Fuse	Low
2	Pressure Relife Valve open	Medium	3	Winsys-120 VCA power	Low
2	Solenoid	Medium	3	Electromotor	Low
2	Valves	Medium	3	Pump	Low
3	RS232(Com1) interface disconnected	Low	3	Resevior	Low
3	Acc. Cable	Low	3	Circuit Breaker Output- 3 A Fuse	Low
3	Acc.1A fuse	Low	3	Circuit Breaker Output- 3A fuse(2)	Low
3	Acc. Dual power supply	Low	3	Circuit Breaker Output- 24VDC	Low
3	AFF Borad	Low	3	Circuit Breaker Output- Cable	Low

Table 2.11. Degree of aggregation sensitivity analysis

	Degree of Aggregation				
15% top Ranked Failures	0.8-1	0.7-0.8	0.5-0.6	0.3-0.4	0.0-0.2
120VAC-DPST Relay	High	High	High	Medium	Medium
120VAC-Circuit Breaker	High	High	High	Medium	Medium
Circuit Breaker input Cable	Medium	Low	Low	Low	Low
Circuit Breaker input SW 1	Medium	Low	Low	Low	Low
Circuit Breaker input SW 2	Medium	Low	Low	Low	Low
Circuit Breaker input SW 3	Medium	Low	Low	Low	Low
Acc. 1A (-15)V	Medium	Low	Low	Low	Low

To better describe the steps of the GFEA model, an example of each step is provided below.

Step 0. Determine a set of risk criteria and a set of potential failures.

Table 2.9 shows the Main Console, Main Console Motor, as effect criteria and fifty four potential failures in column one.

Step 1. Define importance of risk criteria.

Using category of Table 2.8, the 'k'th expert determines the importance of all 'J' risk criterion (i.e., $I_k(f_j)$, where k=1,2,…,K and j=1,2,…,J). For example in Table 2.8, expert 1 expressed the importance of all failure factors (Main Console, Motor Control Console, …) VH, P, VH, …., respectively.

Step 2. Define the relative impact of the potential failure.

Obtain the level of impact of the failure 'i' to the function 'j' from the 'k'th expert (e.g., $P_{ik}(f_j)$). For example, the relative impact of Circuit Breaker input cable (Failure) on the Main Console (Risk criterion) is categorized "N-H-H" by experts (i.e., expert1 assigns None(N) and experts 2, and 3 assign High(H) to the relative impact of the Circuit Breaker input cable on the Main Console).

Step 3. Determine the risk priority category of failure 'i' according to the perception of the 'k'th expert.

For example, the RPC of failure1 in Table 2.8 (i.e., RS232(Com1)) based on expert 1 is expressed by

RPC_{11}=Min{Cmp(H,VH), Cmp(N,P),Cmp(N,VH), Cmp(N,VH), Cmp(N,H), Cmp(N,M), Cmp(N,VH), Cmp(N,VH), Cmp(N,VH), Cmp(N,VH), Cmp(H,VH), Cmp(H,VH), Cmp(H,VH)}

Using indices of Table 2.8, RPC_{11} is defined as

RPC_{11}=Min{Cmp(S_5,S_6),Cmp(S_1,S_7),Cmp(S_1,S_6),Cmp(S_1,S_6),Cmp(S_1,S_5),Cmp(S_1,S_4), Cmp(S_1,S_6), Cmp(S_1,S_6), Cmp(S_1,S_6), Cmp(S_1,S_6), Cmp(S_5,S_6), Cmp(S_5,S_5), Cmp(S_5,S_6)}

With respect to Equations (2.3.5), and (2.3.6) and Definition A.6, and γ=0.8, we get

RPC_{11}=Min { S_6 ,S_5 ,S_4}= S_4

The above results mean that the risk priority category of the failure 1 is "Medium" according to expert 1. Similarly, RPC of Failure 1 is defined "Low" for experts 2 and 3.

Perform steps 1 to 3 for all experts to determine RPC_{ik} where i=1,2,….,I, and k=1,2,…,K

Step 4. Rank the RPCs of the failure.

For example, the set of RPCs for failure 1 in Table 2.9 based on three experts is

$Set_3(1) = \{M, L, L\}$

Thus, the ranked set is

$Ord_3(1) = \{M, L, L\}$

Step 5. Determine the aggregation categories.

Using Table 2.8, Table 2.12 presents the aggregation categories of the example for a group composed of three experts (K=3) and seven levels of linguistic variables (Ψ =7). As a result, the level of agreement will be increased as shown in Table 2.12 by increasing 'ε'.

Table 2.12. Aggregation Categories for group of three experts (K=3)

ε	Ψ	$Int[1+(\varepsilon.\frac{\Psi-1}{K})]$	Index	Category
0	7	1	S_1	Non (N)
1	7	3	S_3	Low (L)
2	7	5	S_5	High (H)
3	7	7	S_7	Perfect (P)

Step 6. Compute the comprehensive risk priority category of the potential failures.

For example, using Table 3 and $Ord_3(1)$ in step 4, the $RPC_{[13]}$ is defined.

RPC_1 = Max{Min(L,M), Min(M,L), Min(P,L)}=L

By performing the step 6 for all failures (i=1,2,…..,I) and using Definition A.6, a set made up of the ranked comprehensive risk priority category can be generated .

Step 7. Perform sensitivity analysis.

Table 2.11 presents that the 15% of the top ranked failures are the same for different values of the aggregation degree and the ranking differentiation for $\gamma>0.8$ is justifiable than ranking differentiation for $\gamma<0.8$. RPC. Moreover, a slight change in aggregate relative impact has no effect in the outcome of the model.

CHAPTER 3

FUZZY MARKOV CHAINS

3.1 BACKGROUND

Markov chains are useful for analyzing systems with dependent failure and repair modes when components within the system are independent, and when multi-state systems with common failures require analysis. To better describe the principle of Markov chains, consider the system with 'K' states and suppose that the transition rate from any state 'i' to any state 'j' is t_{ij}. Thus, by assuming the state of the system at time 't' is denoted by X(t), probability of moving from any state 'i' to any state 'j' (i.e., p_{ij}) is expressed by

$$p_{ij}(t) = P(X(s+t) = j \mid X(s) = i) \tag{3.1.1}$$

where $t \geq 0, s \geq 0$. the matrix P which decribes the state transition is

$$P = \lfloor p_{ij} \rfloor_{k \times k} \tag{3.1.2}$$

where each row maximum is one. Thus,

$$P^n = \lfloor p_{ij}^n \rfloor_{k \times k} \tag{3.1.3}$$

which means the 'n' fold product of P where p_{ij}^n is the probability of being in state 'j' at step 'n' given that we started in state 'i'. Let $p^{(0)}$ denote the initial probability distribution and $p^{(n)}$ denote the probability distribution of being in state 'j' after 'n' steps. Thus, we have

$$p^{(n)} = p^{(0)} P^n \tag{3.1.4}$$

Due to uncertain and time-varying input reliability data, the use of fuzzy sets in Markov chains has been studied over years. For example, in electronic design, part parameters

change with time due to aging effects and stress. Furthermore, failure rates calculated in accordance with MIL-HDBK-217 do not address circuit failures caused by part changes due to aging effect.

Today, the practice of fuzzy Markov chains can be classified by the use of a fuzzy probability distribution or of a fuzzy possibility distribution in a transition matrix (An example along with Marco is given in Appendix-E). However, it is widely accepted that one of the major problems of fuzzy possibility distribution is associated with the benchmarking of fuzzy variables in term of membership function.

Thus, to calculate the steady state distribution of Markov chains, Equation (3.1.4) can be reduced to a simple closed form that is so-called flow-rate equation system. The objective is to develop a fuzzy flow-rate equation system associated with interval chance. The imprecise-chance fuzzy flow-rate equations deal with time-varying input for the states of Markov chains. The equations are associated with interval chance to represent the uncertain input data. Thus, because of association of fuzzy flow-rate equation with interval chance, the method is called imprecise-chance Markov chains.

3.2 IMPRECISE-CHANCE MARKOV CHAINS

For the most systems, there are more modes of operation than just working or failed. Each possible mode of operation for a system is called state of the system and the set of states of the system is called state space. By transfer rate, state of system can be changed to all possible states in the state space. This process is described by a state transition diagram that uses arrows to show possible transfers as shown in Figure 3.1.

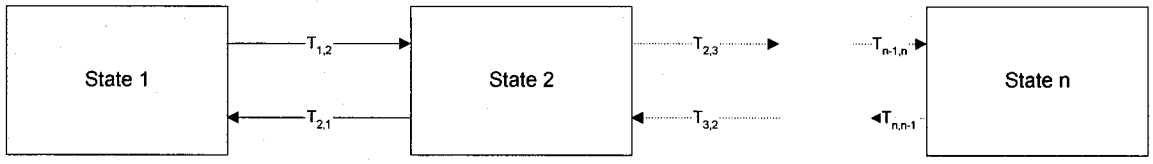


Figure 3.1. State transition diagram

There are several approaches to analyze such state transition diagram such as Markov chains. A Markov chain is specific types of Markov process that system states and times are discrete variables as described in the previous section. In steady state, the simple form of Equation (3.1.4) is expressed by,

$$[(p(1)\ p(2)\ldots.p(n)]A = 0 \qquad (3.2.1)$$

where $\lim_{t\longrightarrow\infty} p_{ij}(t) = p(j)$, and 'A' is the transition matrix. Matrix multiplication shows that this implies for all 'i', and p(i) must satisfy flow-rate equation system that constitute 'n' equations corresponding the states of the system.

$$(Rate\ of\ entry\ from\ all\ states\ to\ state\ k) - (Rate\ of\ exit\ from\ state\ k\ to\ all\ states) = 0$$

for k=1,2,…, n. (3.2.2)

Due to the uncertain and time-varying transition rate (i.e., component failure rate), the element of the transition matrix can be a fuzzy probability distribution or a fuzzy possibility distribution. Because of aging effect and stress (e.g., dynamic, electrical, thermal, etc), the failure rate of the component changes with time in electronic design. Moreover, failure rates

calculated in accordance with MIL-HDBK-217 do not address circuit failures caused by stress and aging effect, which result in fuzzy flow-rate equation system. In addition, due to uncertain input reliability data, each flow-rate equation is associated with a specified chance. Thus, to deal with such situation, this section introduces the fuzzy flow-rate equation system based on certain assumptions. The specified chance represents the possibility that flow-rate equation lies within specified interval (e.g., $[-\Delta b_k, +\Delta b_k]$). The interval may vary for the states because of aging effect and stress. Moreover, due to uncertain and time varying part failure rate, the chance is expressed by the interval method. This new approach presents Equation (3.2.5) with fuzzy operand associated with specified interval chance that is called imprecise-chance equation system.

Assumptions

- Transition rates (i.e., failure rates) are independently and normally distributed random variables.
- The state is discrete and the time is a continuous variable.

This newly developed method is described below.

3.2.1 IMPRECISE-CHANCE MARKOV CHAINS MODEL [2-51, 2-55, 2-56]

Given a Markov chain with 'n' states and random transition rates between states, $t_{i,j}$, where i represents the source and j the destination states, the goal is to find the probability of being in each state (e.g., P(1), P(2), …, P(n)). For non-fuzzy Markov chains, we can obtain an equation for each state based on the fact that, for steady state conditions, the rate of entry into the state 'k' must be equal to the rate of exit from it. Thus, we write

$$\sum_{i=1}^{n} t_{i,k}.P(i) - P(k)\sum_{j=1}^{n} t_{k,j} = 0 \qquad \text{for k=1,2,..., n} \tag{3.2.3}$$

Since the system equation is composed of 'n' equations and is indeterminate, one of the equations can be replaced by

$$\sum_{i=1}^{n} P(i) = 1 \tag{3.2.4}$$

In contrary to non-fuzzy Markov chains, Equation (3.2.3) with fuzzy operand associated with specified chance that is so-called imprecise-chance flow-rate equation is expressed by

$$\Pr \tilde{o}b(\sum_{i=1}^{n} t_{i,k}.P(i) - P(k)\sum_{j=1}^{n} t_{k,j} \cong 0) \tilde{\geq} \alpha_k \qquad \text{for} \quad \text{k=1,2,...,n} \tag{3.2.5}$$

The imprecise-chance flow-rate equation constitutes of two fuzzy parts. In the steady state situation, the first part is the flow-rate equation, which is the rate of entry into the state 'k' minus the rate of exit from state 'k'. It lies within specified interval (e.g., $[-\Delta b_k, +\Delta b_k]$) where Δb_k is specified tolerance with respect to aging and stress effects on component failure rate. The second part is the interval chance (i.e.,$(\alpha - \Delta\alpha,\ \alpha_k)$ that represents probability that flow-rate equation lies within specified interval, where $\Delta\alpha$ is specified tolerance. Similar to non-fuzzy Markov chains, one of the equations of imprecise-chance flow-rate equation can be replaced by Equation (3.2.4).

Thus, we have a fuzzy equation system composed of 'n' imprecise-chance flow-rate equations that cannot be solved by matrix technique. In order to solve such an equation system (i.e., Equations (3.2.4) and (3.2.5), it needs to be converted to non-fuzzy form in steps as follows:

Step 1: conversion of the imprecise-chance flow-rate equations (Equation (3.2.5))

Since $t_{i,k}$ are independently and normally distributed random variables with mean $\mu_{i,k}$ and standard deviation $\sigma_{i,k}$, with respect to Definition A-4 in Appendix-A, Equation (3.2.6) yields

$$\Pr\tilde{o}b\left(\frac{y_k-\mu_k}{\sigma_k}\cong\frac{b_k-\mu_k}{\sigma_k}\right)\tilde{\geq}\alpha_k \qquad \text{for k=1,2,..., n} \tag{3.2.6}$$

Where $b_k=0$, and

$$y_k=\sum_{i=1}^{n}P(i).t_{i,k}-\sum_{j=1}^{n}P(k).t_{k,j} \tag{3.2.7}$$

$$\mu_k=\sum_{i=1}^{n}P(i).\mu_{i,k}-\sum_{j=1}^{n}P(k).\mu_{k,j} \tag{3.2.8}$$

$$\sigma_k^2=\sum_{i=1}^{n}P(i)^2.\sigma_{i,k}^2+\sum_{j=1}^{n}P(k)^2.\sigma_{k,j}^2 \tag{3.2.9}$$

Using Equation (3.2.4) in Appendix-A, Equation (3.2.6) is converted to

$$\Phi(\frac{b_k + \Delta_{b_k} - \mu_k}{\sigma_k}) - \Phi(\frac{b_k - \Delta_{b_k} - \mu_k}{\sigma_k}) \geq \alpha_k - \Delta_{\alpha_k} \quad \text{for} \quad k=1,2,\ldots,n \tag{3.2.10}$$

In steady state, b_k is equal to zero, thus, we have

$$\Phi(\frac{\Delta_{b_k} - \mu_k}{\sigma_k}) - \Phi(\frac{-\Delta_{b_k} - \mu_k}{\sigma_k}) \geq \alpha_k - \Delta_{\alpha_k} \quad \text{for} \quad k=1,2,\ldots,n \tag{3.2.11}$$

Since $\Phi(-x) = 1 - \Phi(x)$, we get

$$\Phi(\frac{\Delta_{b_k} - \mu_k}{\sigma_k}) - 1 + \Phi(\frac{\Delta_{b_k} + \mu_k}{\sigma_k}) \geq \alpha_k - \Delta_{\alpha_k} \quad \text{for} \quad k=1,2,\ldots,n \tag{3.2.12}$$

The other form of Equation (3.2.12) is

$$2\Delta_{b_k} - \Phi^{-1}(1 - \alpha_k - \Delta_{\alpha_k}).\sigma_k \geq 0 \qquad \text{for } k=1,2,\ldots,n \tag{3.2.13}$$

Step 2: Determine the possibility of probability of interval for Equation (3.2.4)

With the aid of Definition A-4 in Appendix-A, the possibility of Equation (3.2.4) is defined below.

$$\mu_{\sum_{i=1}^{n} P(i)} = \begin{cases} A: 0 & for \sum_{i=1}^{n} P(i) \le 1-\Delta_p \ .or. \sum_{i=1}^{n} P(i) \ge 1+\Delta_p \\ B: \dfrac{\sum_{i=1}^{n} P(i) - (1-\Delta_p)}{\Delta_p} & for\ 1-\Delta_p < \sum_{i=1}^{n} P(i) \le 1 \\ C: \dfrac{1+\Delta_p - \sum_{i=1}^{n} P(i)}{\Delta_p} & for\ 1 \le \sum_{i=1}^{n} P(i) < 1+\Delta_p \end{cases} \quad (3.2.14)$$

where Δ_p is the specified interval for the sum of the probabilities of being in the states of the system. When the possibility is equal to one, it implies that sum of probabilities is equal to one. Therefore, Equation (3.2.4) can be replaced with Equation (3.2.14) in the equation system. To solve the converted flow-rate equation system, the possibility is considered as an objective that needs to be maximized subject to the converted fuzzy flow-rate equation system to the non-fuzzy form made up of Equation (3.2.13).

Step 3: Development of the Mathematical model

To solve the non-fuzzy equation system composed of Equations (3.2.13) and (3.2.14), a mathematical model is proposed. The model has one objective function (i.e., Equation (3.2.13) denoted by G in Equation (3.2.15)), corresponding with the possibility of sum of the probabilities to satisfy Equation (3.2.4). To have $\sum_{i=1}^{n} P(i)$ to equal one, the possibility (i.e., Equation (3.2.4)) must be maximized subject to both 'n' non-fuzzy flow-rate equations (i.e., Equation (3.2.13) demonstrated by Equation (3.2.16)) and lower/upper bounds for G and

$\sum_{i=1}^{n} P(i)$ represented by Equations (3.2.17)-(3.2.21) known as constraints. The mathematical model is expressed as follows:

Objective: Max : G (3.2.15)

Subject to:

$$2\Delta_{b_k} - \Phi^{-1}(1-\alpha_k - \Delta_{\alpha_k}).\sigma_k \geq 0 \quad \text{for} \quad k=1,2,\ldots,n \tag{2.2.16}$$

$$G \leq \frac{1+\Delta_p - \sum_{i=1}^{n} P(i)}{\Delta_p} \tag{3.2.17}$$

$$G \leq \frac{\sum_{i=1}^{n} P(i) - 1 + \Delta_p}{\Delta_p} \tag{3.2.18}$$

$$1-\Delta_p \leq \sum_{i=1}^{n} P(i) \leq 1+\Delta_p \tag{3.2.19}$$

$$P(i) \geq 0 \quad \text{for } i=1,2,\ldots,n \tag{3.2.20}$$

$$G \geq 0 \tag{3.2.21}$$

Where $\sigma_k = \sqrt{\sum_{i=1}^{n} P(i)^2.\sigma_{i,k}^2 + \sum_{j=1}^{n} P(k)^2.\sigma_{k,j}^2}$ and $G = \mu_{\sum_{i=1}^{n} P(i)}$

By solving the model the maximum possibility is obtained, which results in having the state probabilities. There are several software packages for solving the model. One example of these packages is LINGO used in this paper.

3.2.2 ILLUSTRATION OF IMPRECISE-CHANCE MARKOV CHAINS MODEL

Due to thermal effects, the physical properties of electrical components are rapidly modified and the rate of component failures doubles for every 10°C rise in temperature. Therefore, to meet the design requirements of a box made up of electrical circuit boards in accordance with standards, the use of a cooling unit is required to keep junction temperature within operational limits. Figure 3.2 shows Markov model of the box that requires airflow equal to $Constant\ Coefficient \times \frac{Power\ Dissipation}{Air\ Temprature\ Rise}$. The cooling unit composed of two fans configured in dynamic redundancy is used to maintain the airflow. Each fan constitutes mechanical parts, connectors, wire, and power circuit.

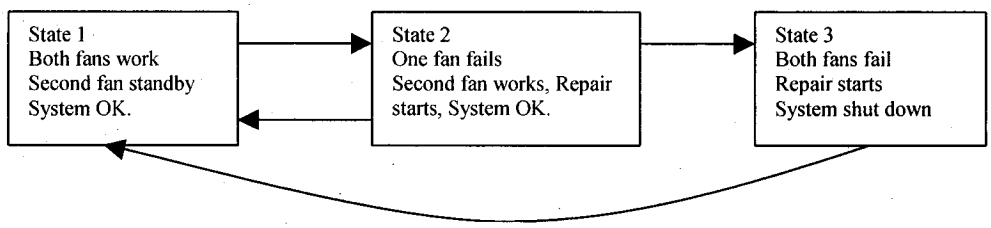


Figure 3.2. Cooling unit Markov model

Table 3.1 presents normally distributed transition rates, interval for flow-rate equation (tolerance), flow-rate equation chance, and interval for the chance.

Table 3.1. Input parameters of cooling unit Markov model

State K	Transition rate (entry) $t_{i,k}$~N($\mu_{i,k}$, $\sigma_{i,k}$)	Transition rate(exit) $t_{k,j}$~N($\mu_{k,j}$, $\sigma_{k,j}$)	Interval for equation ($b_k \pm \Delta b_k$)	Chance-interval of equation ($\alpha - \Delta\alpha, \alpha_k$)	Specified interval for sum of probabilities (Δ_p)
1	$t_{2,k}$~N(5,0) $t_{3,k}$~N(5,0)	$t_{k,2}$~N(10,0)	0 ± 5	(0.95,0.97)	0.05
2	$t_{1,k}$~N(10,0)	$t_{k,3}$~N(10,0)	0 ± 10	(0.95,0.97)	0.05
3	$t_{2,k}$~N(10,0)	$t_{k,1}$~N(5,0)	0 ± 7	(0.95,0.97)	0.05

By implementing steps 1 to 3, mathematical model is developed. We propose the use of LINGO Modeling to develop the mathematical programming as follows:

```
MODEL:! THE PRECISE-CHANCE MARKOV CHAINS;
 SETS:
        STATE/1..3/: BK, DBK, ALF, DALF, P, Y;
        SIG( STATE, STATE): S;
 ENDSETS
 DATA:
        BK=0, 0, 0;
        DBK=5, 10, 7;
        ALF=0.97, 0.97, 0.97;
        DALF=0.02, 0.02, 0.02;
        DP=0.05;
        S=0, 10, 0,
          5, 0, 10,
          7, 0, 0;
 ENDDATA

! DEFINING MODEL;
        ! THE OBJECTIVE/ EQUATION (22);
        !_____________________________
        MAX = G;

        ! CONSTRAINTS;
```

```
!_____________________________

! EQUATION (23);
@FOR(STATE(I) : P2(I)=P(I)*P(I)) ! Used for sigma equation;
@FOR(STATE(K) : (@SUM(SIG(I,J) | J #EQ# K: Y(I)*S(I,J)^2)+
        @SUM(SIG(M,N) | M #EQ# K: Y(M)*S(M,N)^2))
            <=(2/1.645)^2*DBK(K)^2);

! EQUATION (24);
@SUM(STATE(I) : P(I))<=1+DP-DP*G;
! EQUATION (25);
@SUM(STATE(I) : P(I))>=1-DP+DP*G;
! EQUATION (26);
@SUM(STATE(I) : P(I))<=1+DP;
@SUM(STATE(I) : P(I))>=1-DP;
! EQUATION 27;
@FOR(STATE(I) : P(I)>=0);
! EQUATION 28;
G>=0;
END
```

Using LINGO, Table 3.2 presents the result of the mathematical model. This result indicates that the maximum possibility is 0.9999995, which results to derive the probability of being in the states of the cooling unit.

Since cooling is functioning adequately in the states 1 and 2, the probability of the system being up is equal to 0.999. More specifically it means that the cooling unit down time in a year is 8.8 hours.

Table 3.2. Result of the mathematical model for Markov Model

Possibility value:	**0.9999995**
State No. and Description	**Probability of being in state**
1-One fan works; the other is standby: system ok	0.148
2-One fan works; the other fails: system ok	0.851
3-Both fans fail: system down	0.001

CHAPTER 4

FUZZY AND STOCHASTIC FAULT TREE ANALYSIS

4.1 BACKGROUND

Generally, tree analysis techniques are concerned with the ways either a system fails or a system succeeds. Tree analysis techniques model a failure or a success of a system via logically and deductively modeling approaches i.e., Fault Tree Analysis (FTA), and Success Tree Analysis (STA). Due to similarity of those approaches in construction and analysis method, this chapter uses the term of FTA in general. Moreover, FTA is one of the most widely used reliability techniques in the system development life cycle. It is applicable for risk assessment in systems with a clearly identified single top event. Because FTA is event-oriented, differentiation between the levels of the events plays a vital role in creating the tree. By defining dependency relations between the basic events and the intermediate/top events, the reliability measures can be accurately estimated. To perform the system risk assessment, FTA utilizes three types of symbols as follows:

- Primary event symbols
  -  Basic event: a basic initiating fault requiring no further development.
  -  Conditioning event: specific conditions that apply to any logic gate.
  -  Undeveloped event: an event that is not further developed due to insufficient consequences or lack of information.
  -  External event: an event that is normally expected to occur
  -  Intermediate event: a fault that occurs due to one or more antecedent causes acting through logic gates.

- Gate symbols
  -  AND: output fault occurs if all of the input faults occur.
  -  OR: Output fault occurs if at least one of the input faults occurs.
  -  Exclusive OR: output faults occurs if exactly one of the input faults occurs.
  -  Priority AND: output fault occurs if all input faults occur in specific sequence.
  -  Inhibit: output fault occurs if input fault occurs in presence of an enabling condition.
- Transfer symbols
  -  Transfer in: indicates that the tree is developed further at the occurrence of the corresponding transfer out.
  -  Transfer out: indicates that this portion of the tree must be attached at the corresponding transfer in.

To bring the design of FTA in line with accepted probabilistic risk management theory, the use of stochastic or fuzzy sets is proposed. First, this chapter introduces fuzzy FTA and then stochastic FTA. In fuzzy FTA, the new gates are presented to aggregate or compensate AND and OR gates. The stochastic FTA introduces a new primary symbol called Self-loop basic

event representing the failure that can be detected, isolated and removed from tree, with stochastic parameters.

4.2 FUZZY FAULT TREE ANALYSIS

Suppose we have a system with several subsystems, each having ambiguous impacts upon failure of the basic events. All subsystems of the system are made up of electronic, electrical, and mechanical devices. Since the devices have a high level of complexity, a failure of a device has an uncertain effect on the upper events through the logic gate. The strict logic gates (e.g., 'AND' and 'OR') are commonly used to indicate how those fuzzy failures effect the upper level event of fault tree. The fuzzy 'AND' and 'OR' gates use minimum and maximum functions, respectively as expressed by Equations (A.5), and (A.6) in Appendix-A. Those Equations are not adequate approach for aggregation and compensation when tradeoffs exist between the failures. Therefore, using fuzzy aggregation and compensation methods, in this chapter we have developed the FTA models.

4.2.1 COMPARISON BETWEEN FTA AND BASIC OPERATION ON FUZZY SETS

The parallel and series networks can be represented by 'AND', and 'OR' gates, respectively, in FTA. Assume that we are given two fuzzy sets of e_{11} and e_{12} with membership functions $\mu(e_{11})$ and $\mu(e_{12})$, respectively. With respect to Equation (A.6) in Appendix-A, the contribution of these events to the top level event-e_1 with a fuzzy 'AND' gate as shown in Figure 4.1 and denoted by '$(e_{11} \cap e_{12})$', is defined as

$$\mu(e_{11} \cap e_{12}) = \min\{\mu(e_{11}), \mu(e_{12})\} \tag{4.2.1}$$

Similarly, expansion of Equation (4.2.1) for J's basic events contributing to the top event through a fuzzy 'AND' gate is

$$\mu(\bigcap_{j=1}^{J} e_{1j}) = \min\{\mu(e_{11}), \mu(e_{12}), \ldots, \mu(e_{1J})\} \tag{4.2.2}$$

Table 4.1 presents the values of the membership function of the top event (e.g., e_1) with a fuzzy 'AND' gate shown in Figure 4.1. Figure 4.2 depicts the membership function of the top event, i.e., decreasing for the fuzzy 'AND' gate. In other words, because the membership function of the top event is less than the membership function of all precedence events, the chance of occurrence of the top event is less than the chance of occurrence of all precedence events.

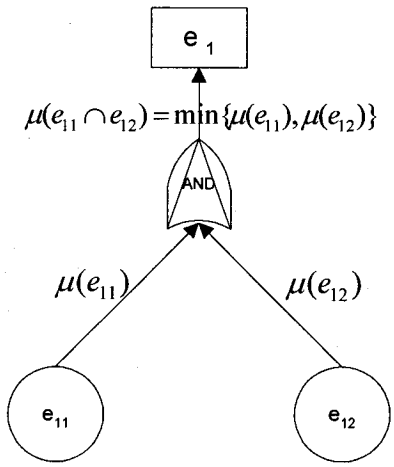


Figure 4.1. Fuzzy AND gate

Table 4.1. Membership function of Fuzzy 'AND' gate

P	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mu(e_{11})$	0.7	0.5	0.4	0.4	0.4	0.3	0.8	0	0
$\mu(e_{12})$	0.2	0.3	0.3	0.5	0.5	0.8	0.8	0.9	0.9
$\mu(e_{11} \cap e_{12})$	0.2	0.3	0.3	0.4	0.4	0.3	0.8	0	0

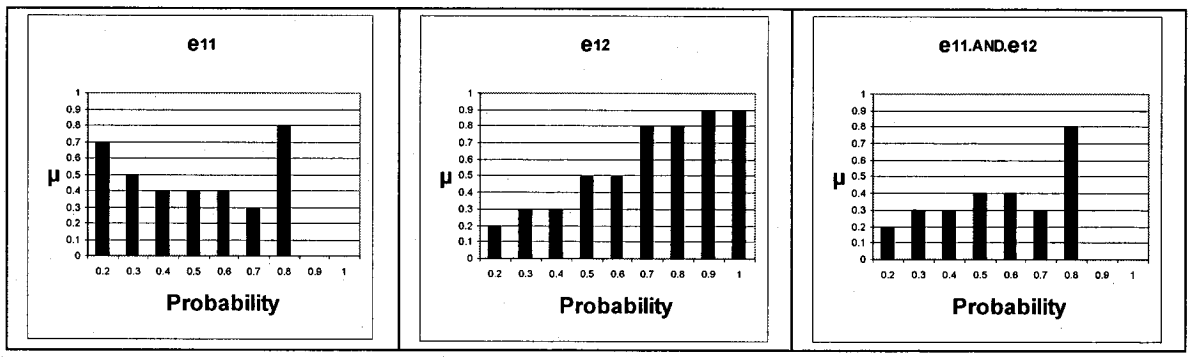


Figure 4.2. Membership function for Figure 4.1

In the same manner, using Equation (A.5) in Appendix-A, the contribution of these events to the top event e_1 with a fuzzy 'OR' gate shown in Figure 4.3, denoted by '$(e_{11} \cup e_{12})$', is expressed by

$$\mu(e_{11} \cup e_{12}) = \max\{\mu(e_{11}), \mu(e_{12})\} \tag{4.2.3}$$

Similarly, expansion of Equation (4.2.3) for J's basic event contributing to the top event through a fuzzy 'OR' gate is

$$\mu(\bigcup_{j=1}^{J} e_{1j}) = \max\{\mu(e_{11}), \mu(e_{12}), ..., \mu(e_{1J})\} \tag{4.2.4}$$

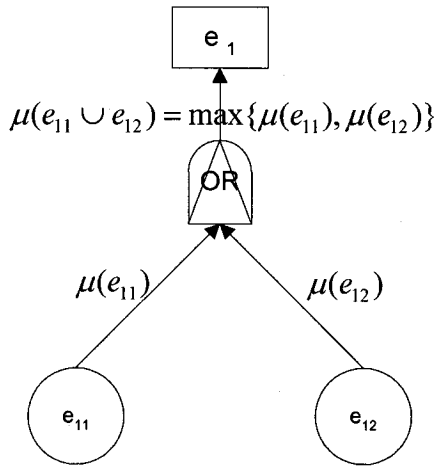


Figure 4.3. Fuzzy OR gate

Table 4.2 presents the values of the membership function of the top event (e.g., e_1) with a fuzzy 'OR' shown in Figure 4.3. Figure 4.4 shows the membership function of the top event, i.e., increasing for the fuzzy 'OR' gate. As a result, the chance of occurrence of the top event is greater than the chance of occurrence of all precedence events since membership function of the top event is greater than the membership function of all precedence events.

Table 4.2. Membership of Fuzzy 'OR'

P	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\mu(e_{11})$	0.7	0.5	0.4	0.4	0.4	0.3	0.8	0	0
$\mu(e_{12})$	0.2	0.3	0.3	0.5	0.5	0.8	0.8	0.9	0.9
$\mu(e_{11} \cup e_{12})$	0.7	0.5	0.4	0.5	0.5	0.8	0.8	0.9	0.9

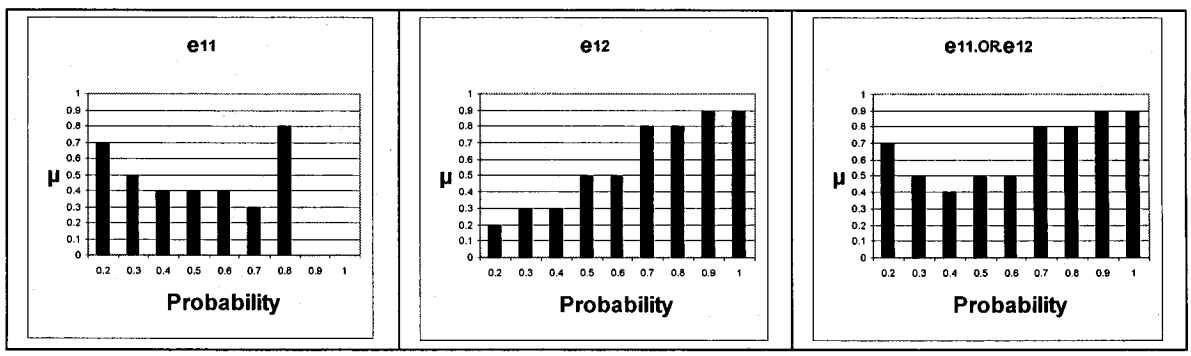


Figure 4.4. Membership function for Figure 4.3

4.2.2 FUZZY FAULT TREE ANALYSIS MODEL [3-137]

Consider a system composed of several subsystems for which the failure impact and its contribution to the system failure are not precisely known. The use of strict fuzzy 'AND' and 'OR' gates is not an adequate approach for performing fault tree analysis in this system. This newly developed fuzzy model realizes the idea of failure trade-offs among the contributing basic events when compensation is allowed. For examples, consider FTA in Figures 4.1, and 4.2, the resulting tradeoffs lie between the most optimistic lower bound and the most pessimistic upper bound as shown in Figure 4.5. The model combines the minimum and the maximum operators with other operators in order to develop the bounds. However, the non-parametric averaging operators, such as arithmetic and geometric means are not adequate for a system with imprecise failure impacts. The model amalgamates the minimum and the maximum operators with the compensatory operators. The combination of these operators in fault tree analysis leads to very good results.

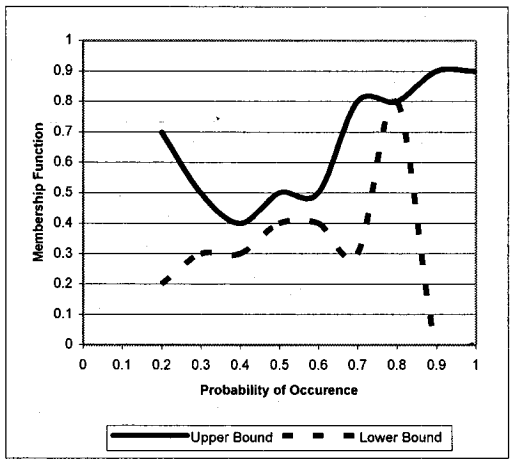


Figure 4.5. Lower and upper bounds for probability

4.2.2.1 MODEL I: COMPENSATED GATES

Suppose we have a system composed of subsystems that have fuzzy impacts on the top level of the tree through the either compensated 'AND' or compensated 'OR' gate. This makes trade-offs among the failure impacts of all subsystems. Because the minimum and maximum operators are not adequate approaches for aggregation and compensation in such system. The model rectifies the minimum and maximum operators with arithmetic means as expressed by Equations (4.2.5)-(4.2.6) for fuzzy 'AND' and 'OR' gates, respectively.

$$\mu(e_{11} \cap e_{12}) = \gamma.\min\{\mu(e_{11}), \mu(e_{12})\} + (1-\gamma)\frac{(\mu(e_{11}) + \mu(e_{12}))}{2} \tag{4.2.5}$$

$$\mu(e_{11} \cup e_{12}) = \gamma.\max\{\mu(e_{11}), \mu(e_{12})\} + (1-\gamma)\frac{(\mu(e_{11}) + \mu(e_{12}))}{2} \tag{4.2.6}$$

Where γ is the degree of nearness to the strict logical meaning of 'AND' and 'OR', respectively. By substituting γ =1 into Equations (4.2.5) and (4.2.6), the fuzzy 'AND' becomes the minimum operator; the fuzzy 'OR' reduces to the maximum operator. By setting γ = 0 yields the arithmetic mean for both Equations. Figure 4.6 shows the nearness of the compensated 'AND' and 'OR' gates to the strict logic gates for different values of γ.

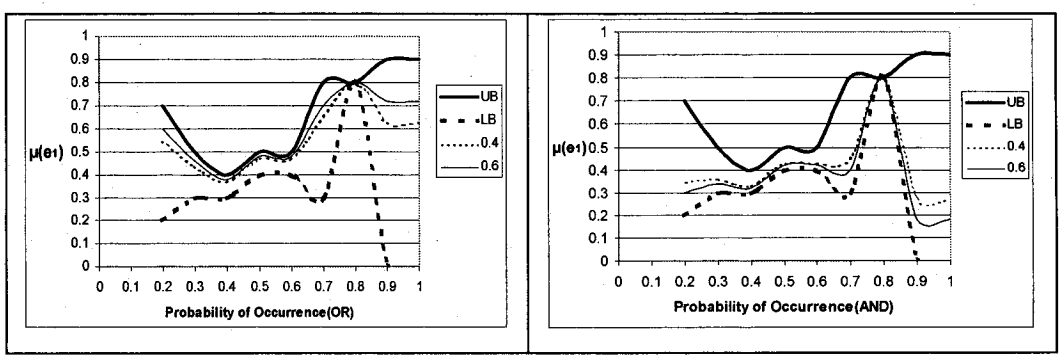


Figure 4.6. Nearness Curve for Model I

4.2.2.2 MODEL II: AGGREGATED AND/OR GATES

Model 'II' follows the idea of parameterized compensation. The compensation is defined by taking linear convex combinations of non-compensatory operators. The aggregate of two fuzzy sets e_{11} and e_{12} is generated by the convex combination of minimum and maximum operators as follows:

$$\mu(e_{11}, e_{12}) = \gamma.\min\{\mu(e_{11}), \mu(e_{12})\} + (1-\gamma)\max\{\mu(e_{11}), \mu(e_{12})\} \quad (4.2.7)$$

Where γ is the degree of nearness to the strict logical meaning of 'AND' or 'OR'. By setting $\gamma = 1$ in Equation (4.2.7), the fuzzy 'AND/OR' becomes the 'AND' operator, and for $\gamma=0$, it yields the 'OR' operator. Figure 4.7 shows the nearness of aggregated 'AND/OR' gate to the strict logic gate for different values of γ.

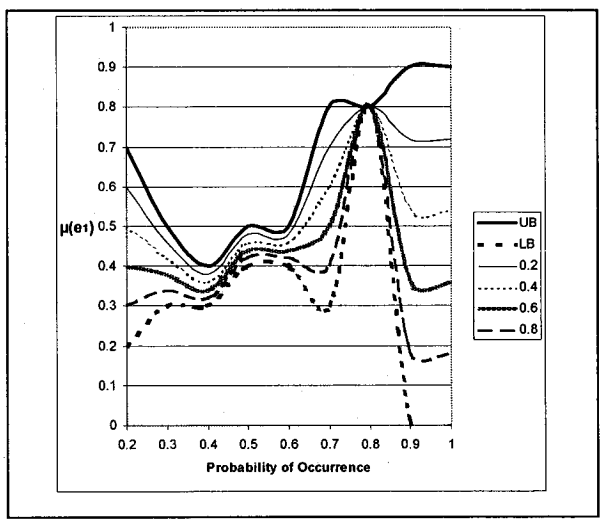


Figure 4.7. Nearness Curve for Model II

4.2.2.3 DEGREE OF NEARNESS TO THE STRICT LOGICAL MEANING OF 'AND' AND 'OR'

Models I and II use γ as a degree of compensation/aggregation of failures. The most crucial step is to assign the triangular fuzzy number for the nearness logical parameter $\tilde{\gamma}$.

Generally, the triangular fuzzy number is derived by three numbers, which are the lower limit, the most likely value, and upper limit of $\tilde{\gamma}$. In order to simplify the calculation of the compensated/aggregated gate, the average of the fuzzy number (γ) is proposed for substitution into Equations (4.2.5)-(4.2.7). The fuzzy average number (γ) is expressed by

$$\gamma = \frac{(\gamma_\ell + 4.\gamma_m + \gamma_u)}{6} \tag{4.2.8}$$

4.2.2.4 FUZZY LINGUISTIC FAILURE IMPACT PROBABILITY OF THE BASIC EVENT

Due to the system complexity and different levels of failure severity, estimating the failure impact probability of the basic event on the upper level event is difficult. In such cases, using either past experience or benchmarking, the probability may be estimated. The fuzzy model studies this issue by using fuzzy linguistic variables such as Low, Medium, High, etc. The membership functions of those linguistic variables are defined in Ref.[2-10]. Furthermore, The set of linguistic variables can be extended to VeryHigh, VeryLow, More-or-Less, Plus, and Slightly via fuzzy operators as expressed by Equations (A.7)-(A.13) in Appendix-A. Table 4.3 presents the probability of failure impact of the basic event for those linguistic variables.

Table 4.3. Membership grades of the probability of failure impact

	Probability of Failure Impact										
Linguistic Variable	**0.000%**	**0.001%**	**0.01%**	**0.25%**	**0.50%**	**0.75%**	**1%**	**8%**	**10%**	**15%**	**20%**
LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
MEDIUM	0	0	0	0	0.5	0.7	1	0.7	0.5	0	0
HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
VERYLOW	0	0	0.25	0.49	1	0.49	0.25	0	0	0	0
VERYHIGH	0	0	0	0	0	0	0	0.25	0.49	0.81	1
MORE-OR-LESS-LOW	0	0	0.71	0.84	1	0.84	0.71	0	0	0	0
MORE-OR-LESS-MEDIUM	0	0	0	0	0.71	0.84	1	0.84	0.71	0	0
MORE-OR-LESS-HIGH	0	0	0	0	0	0	0	0.71	0.84	0.95	1
PLUS-LOW	0	0	0.42	0.64	1	0.64	0.42	0	0	0	0
PLUS-HIGH	0	0	0	0	0	0	0	0.42	0.64	0.88	1
SLIGHTLY-LOW	0	0	0.35	0.52	0	0.52	0.35	0	0	0	0
SLIGHTLY-HIGH	0	0	0	0	0	0	0	0.35	0.52	0.07	0

4.2.2.5 ILLUSTRATION OF FUZZY FAULT TREE ANALYSIS MODELS

Assume that one of the critical sections of a vehicle suspension tester is a main console that controls the hydraulic system. This console is composed of complex hardware devices such as winsys-PPC55810T, Tectoral, Crydom. Figure 4.8 presents the fuzzy fault tree with aggregated gate and linguistic failure impact for the main console based on the suspension tester FTA given in Appendix-E. The gate is a fuzzy compensated 'OR' with a fuzzy degree of nearness to strict logical value of γ˜.

By using the above models, the membership grades of the occurrence probability of the main console being down can be determined. Figure 4.9 depicts the membership grades for the top event with both models. Since occurrence probability at 0.5% and availability at 99.5% have maximum grade of membership, they can be representative of reliability measures for the main console. Furthermore, membership grade for availability>99.999% is equal to zero. It means if the Main Console reliability goal is downtime less than 5 minutes per year, it does not meet the goal. Also, in general view of Figure 4.9, the pattern of Model 'II' and Model 'I' AND gate are similar. Moreover, Model 'I-OR', compared to previous models, shows the greater possibilities of occurrence. The detailed calculation is given in Appendix-E, Section 11.2.

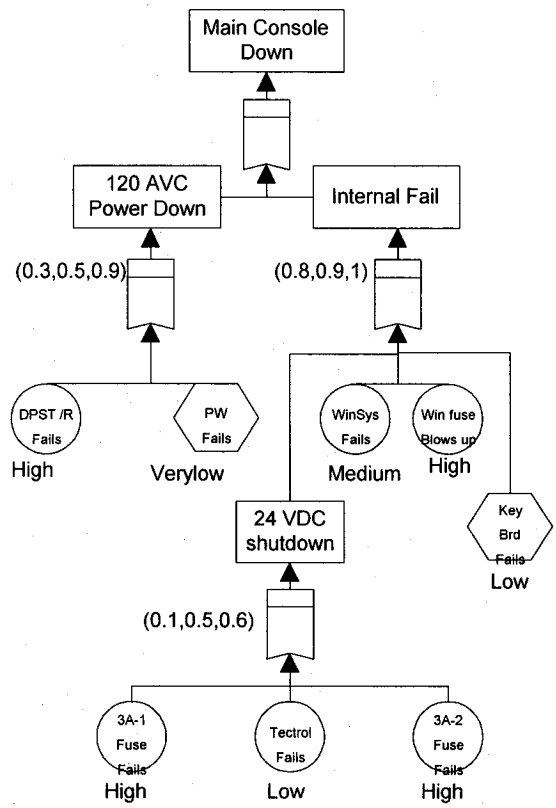


Figure 4.8. FTA of Main Console of suspension tester

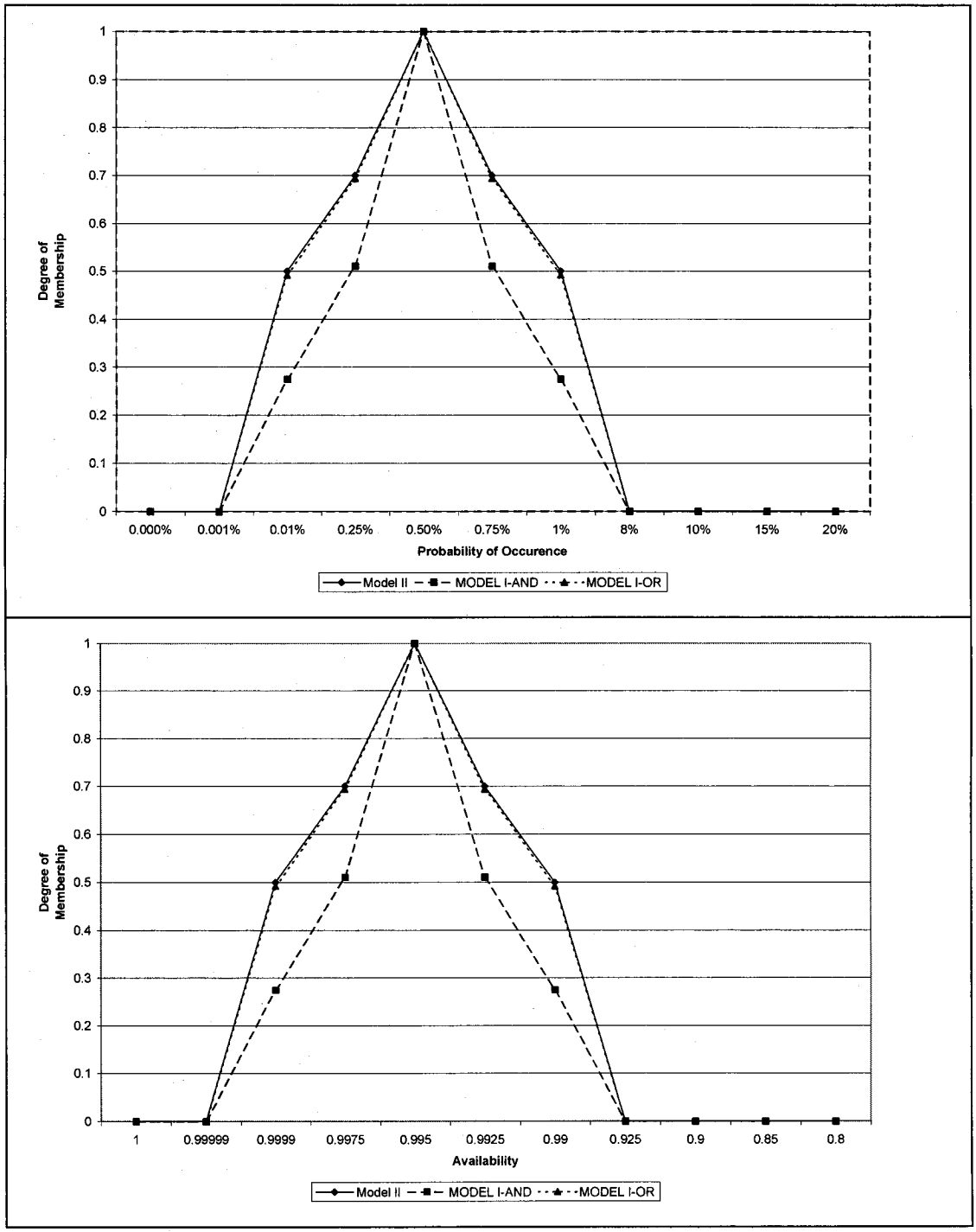


Figure 4.9. Membership grades of the probability of failure impact

4.3 STOCHASTIC FAULT TREE ANSLYSIS WITH SELF-LOOP BASIC EVENT [3-139]

Technical hardware and software systems with advanced technology are often equipped with failure detection, isolation and self-healing mechanisms. In such systems, continuous checks must be performed while the system is in service. When the failures are *'Detected'*, hardware devices notify the software application, which takes appropriate actions and tries to pin-point the problem down to a replaceable or self-healing subsystem. This step is called *'Isolation'* that is a key to success to service availability. For *'Self-healing'* actions, the software application may use the hardware resources to remove the failure and return the system to the operational mode.

Figure 4.10 introduces a stochastic FTA for such a system. If a failure occurs in the system, it may be detected, isolated and recovered with chance denoted by Pr.

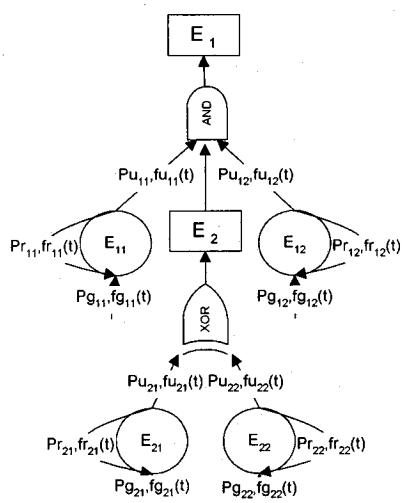


Figure 4.10. Stochastic FTA with self-loop basic event

These mechanisms often takes time, which is a random variable with certain probability distribution function (pdf) denoted by fr(t). However, the detection mechanisms are not able to detect, isolate or recover all potential failures. The non-detected/isolated/recovered failures cause the system to fail with the probability denoted by Pu associated with the effect time, which is a random variable with pdf denoted by fu(t). The effect time is required time

for a non-detected/isolated/recovered failure in order to fully participate in the system failure. Moreover, the failure may occur with chance denoted by Pg and the time pdf denoted by fg(t).

Due to the complexity of the stochastic FTA with a self-loop basic event, the traditional FTA is unable to present those embedded mechanisms. Consequently, in order to compute the probability and Time-To-Event data i.e., mean and standard deviation of time to occurrence to occurrence of the top event, an analytical approach has been developed. It provides more reliable result compared to simulation or Meta-heuristic approaches for FTA with the self-loop basic event, and the deterministic gates shown in Figure 4.10. The self-loop basic event represents embedded detection and recovery mechanisms in the system.

Assumptions:

- The occurrence times of basic events are statistically independent.
- Output of the gates are deterministic in the stochastic FTA.

4.3.1 FLOW-GRAPH MODEL OF STOCHASTIC FAULT TREE ANSLYSIS WITH SELF-LOOP BASIC EVENT

4.3.1.1 FLOW-GRAPH OF SELF-LOOP BASIC EVENT

The lower self-loop basic event is connected to the upper one through the logic node. The self-loop basic event is associated with generation, detection/recovery, and non-detection transmittances as shown in Figure 4.11. Each transmittance parameter is composed of a probability and a time distribution function. The self-loop basic event may occur by generating transmittance (e.g., 'a' in Figure 4.11). If detection mechanisms can detect the event, it will be removed from the system with detecting and recovering transmittance (e.g., 'c' in Figure 4.11) represented by a self-loop over the basic event. Nevertheless, the self-loop basic event will affect the intermediate/top event by not detecting transmittance 'b' in Figure 4.11.

This FTA is an open flow-graph. In order to make it close flow-graph and to use properties of close flow-graph, which is TP equation is equal to zero [4-2], a dummy link from the very

top event to the very bottom events is added as shown in Figure 4.11. Using the TP equation, the stochastic FTA will reduce the tree to fewer events and links. For example, consider a self-loop basic event with a, b, and c transmittance parameters in Figure 4.11. To find the equivalent transmittance of the self-loop basic event (e.g., W_{ij}) , the dummy link with transmittance (i.e., 1/ W_{ij}) can be added in order to have a close flow-graph. Since the topology function of close flow-graph is equal to zero, the equivalent transmittance of Figure 4.11 is defined by

$$W_{ij} = \frac{ab}{1-ac} \tag{4.3.1}$$

The detailed derivation of Equation (4.3.1) is given in Appendix C. It is to be noted that it is possible to reduce the FTA flow-graph by stepwise reducing it from multi-basic events to single-basic events.

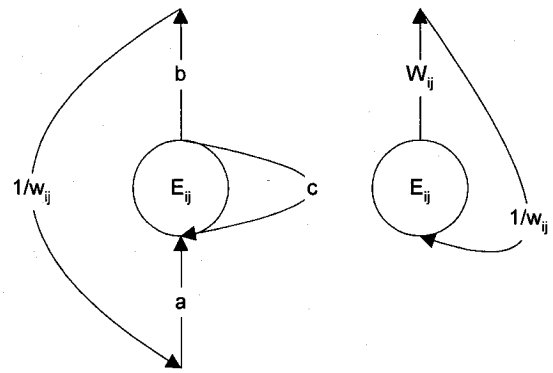


Figure 4.11. Equivalent self-loop basic event

4.3.1.2 FLOW-GRAPH OF FTA WITH AND GATE

In the theory of FTA flow-graph, three types of logic events (e.g., gate) are being discussed. The logic events, such as AND, XOR, and OR are considered deterministic or stochastic. The output of a deterministic logic event occurs if the event rises. On the other hand, there is always a chance for output in the stochastic logic event. The sum of chances

on all outputs of the stochastic logic event must equal to one. Otherwise, the logic event has absorption behavior, which is not applicable for FTA flow-graph. The deterministic AND gate provides an output only if all the input events occur. Consider an AND gate with self-loop basic events as shown in Figure 4.12. By the stepwise reducing approach, FTA flow-graph will be reduced to the new one without self-loop basic event as shown in Figure 4.13. The probability of occurrence of two independent events associated with the AND gate is the intersection of their individual probabilities. Thus,

$$P_{Ei1 \cap Ei2} = P_{Ei1}.P_{Ei2} \tag{4.3.2}$$

However, to find the probability, mean and variance of occurrence time of two independent self-loop basic events associated with 'AND' gate, the equation (4.3.1) and the basic properties of MGF are used (i.e, Equations (C.6)-(C.10)). With respect to Equations (C.6) and (C.10), the equivalent transmittance for the self-loop basic events 'E_{i1}' and 'E_{i2}' (i.e., W_{i1} and W_{i2}, receptively) can be defined as follows:

$$W_{i1} = \frac{Pg_{i1}.Pu_{i1}.Mg_{i1}(s).Mu_{i1}(s)}{1 - Pg_{i1}.\mathrm{Pr}_{i1}.Mg_{i1}(s).Mr_{i1}(s)} \tag{4.3.3}$$

where $a=Pg_{i1}.Mg_{i1}(s)$, $b=Pu_{i1}.Mu_{i1}(s)$, and $c=Pr_{i1}.Mr_{i1}(s)$

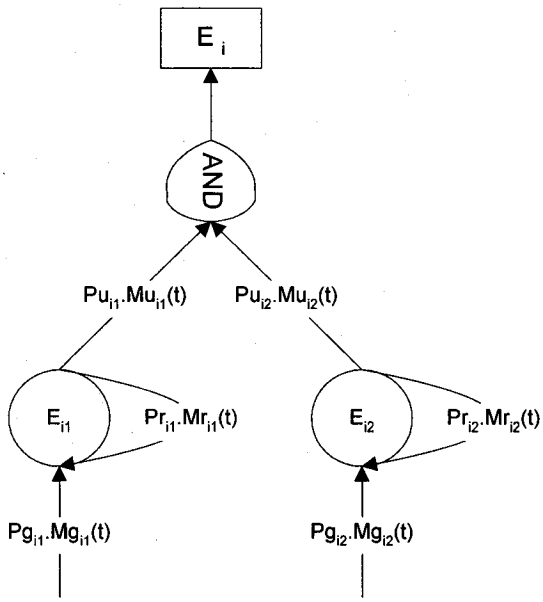


Figure 4.12. FTA flow-graph with AND gate and self-loop basic event

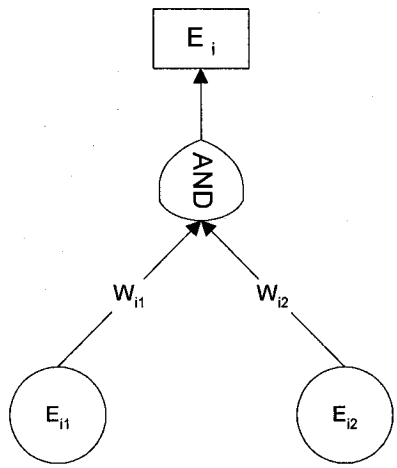


Figure 4.13.Reduced FTA flow-graph with AND gate and self-loop basic event

Similarly, the equivalent transmittance of the self-loop basic event 'E_{i2}' is defined

$$W_{i2} = \frac{Pg_{i2}.Pu_{i2}.Mg_{i2}(s).Mu_{i2}(s)}{1 - Pg_{i2}.\mathrm{Pr}_{i2}.Mg_{i2}(s).Mr_{i2}(s)} \tag{4.3.4}$$

Where $a=Pg_{i2}.Mg_{i2}(s)$, $b=Pu_{i2}.Mu_{i2}(s)$, and $c=Pr_{i2}.Mr_{i2}(s)$

With respect to Equation (4.3.2), the equivalent transmittance of the top event (e.g., Ei) is

$$Wi = \prod_{j=1}^{2} W_{ij} \tag{4.3.5}$$

In the same manner, Equation (4.3.5) may be extended for 'J' self-loop basic events as follows:

$$W_i = \prod_{j=1}^{J} W_{ij} \tag{4.3.6}$$

Thus, using Equations (C.6) to (C.10), the mean of random variable 't' and the occurrence probability of the top event are

$$P_{E_i} = \prod_j W_{ij} \Big|_{s=0} = \prod_j \frac{Pg_{ij}.Pu_{ij}}{1 - Pg_{ij}.\mathrm{Pr}_{ij}} \tag{4.3.7}$$

$$\mu_{E_i} = \sum_j \frac{(\mu_{g_{ij}} + \mu_{u_{ij}})(1 - Pg_{ij}.\mathrm{Pr}_{ij}) + (\mu_{g_{ij}} + \mu_{r_{ij}}).Pg_{ij}.\mathrm{Pr}_{ij}}{(1 - Pg_{ij}.\mathrm{Pr}_{ij})} \tag{4.3.8}$$

Where 'j' is the index for self-loop basic events.

The detailed developments of Equations (4.3.7) and (4.3.8) are presented in Appendix D.

4.3.1.3 FLOW-GRAPH OF FTA WITH XOR GATE

The XOR gate provides an output event if only one of the input events is present. Assume a XOR gate with self-loop basic events illustrated in Figure 4.14. Using stepwise reducing approach, FTA flow-graph is simplified to the FTA without self-loop basic event as shown in Figure 4.15. The probability of occurrence of two independent events tied with XOR gate is the union of their individual probabilities. Thus,

$$P_{Ei1 \cup Ei2} = P_{Ei1} + P_{Ei2} - P_{Ei1 \cap Ei2} \tag{4.3.9}$$

However, in contrast to OR gate, in XOR gate, the events are mutual exclusive; the events cannot occur simultaneously, thus, we write,

$$P_{Ei1 \cap Ei2} = 0 \tag{4.3.10}$$

When the event E_{i1} occurs, the MGF will be $M_{Ei1}(s)$ and when the event E_{i2} occurs, the MGF will be $M_{Ei2}(s)$.

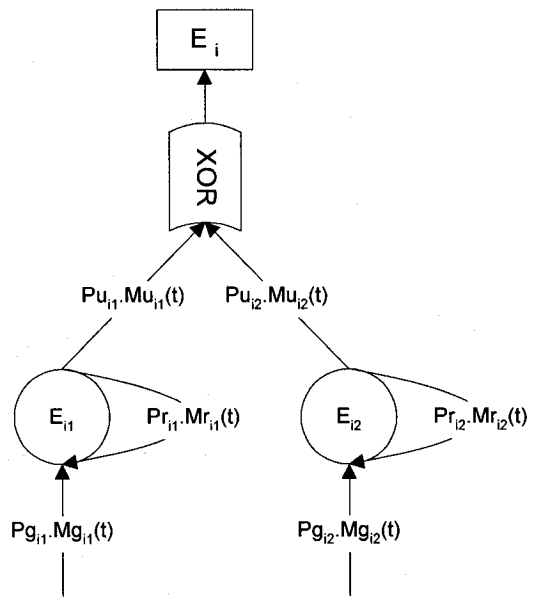


Figure 4.14. FTA flow-graph with XOR gate and self-loop basic event

To find the probability, mean, and variance time of occurrence of two independent self-loop basic events associated with 'XOR' gate, the same approach as for 'AND' gate is followed. Thus, with respect to Equations (C.6) and (C.10), the equivalent transmittance for self-loop basic events 'E_{i1}' and 'E_{i2}' (i.e., W_{i1} and W_{i2}, receptively) are

$$W_{i1} = \frac{Pg_{i1}.Pu_{i1}.Mg_{i1}(s).Mu_{i1}(s)}{1 - Pg_{i1}.\Pr_{i1}.Mg_{i1}(s).Mr_{i1}(s)} \tag{4.3.11}$$

$$W_{i2} = \frac{Pg_{i2}.Pu_{i2}.Mg_{i2}(s).Mu_{i2}(s)}{1 - Pg_{i2}.\Pr_{i2}.Mg_{i2}(s).Mr_{i2}(s)} \tag{4.3.12}$$

Using Equations (4.3.9) and (4.3.10), the equivalence transmittance of top event (e.g., Ei) is

$$Wi = \sum_{j}^{2} W_{ij} \tag{4.2.13}$$

Where W_{i1} and W_{i2} represent the transmittance of events 'E_{i1}' and 'E_{i2}' respectively, and W_i is the transmittance for event 'i' with 'XOR' gate.
Similarly, extended Equation (4.3.13) for 'J' self-loop basic events is

$$W_i = \sum_{j=1}^{J} W_{ij} \tag{4.3.14}$$

Using Equations (C.6) to (C.9), the mean of random variable 't' and the occurrence probability of the top event are expressed as

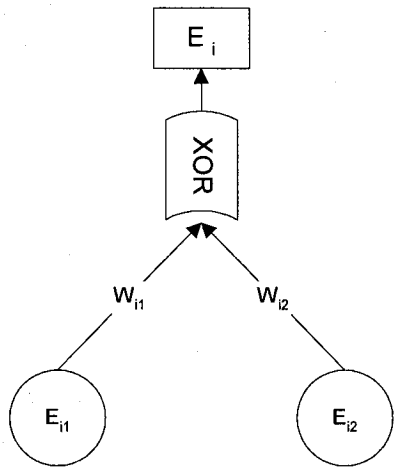


Figure 4.15. Reduced FTA flow-graph with XOR gate and self-loop basic event

$$P_{E_i} = \sum_j W_{ij}\big|_{s=0} = \sum_j \frac{Pg_{ij}.Pu_{ij}}{1 - Pg_{ij}.\Pr_{ij}} \tag{4.3.15}$$

$$\mu_{E_i} = \frac{1}{P_{E_i}} \sum_j \frac{(\mu_{g_{ij}} + \mu_{u_{ij}})(1 - Pg_{ij}.\Pr_{ij})Pg_{ij}.Pu_{ij} + (\mu_{g_{ij}} + \mu_{r_{ij}}).Pg_{ij}^{\,2}.Pu_{ij}.\Pr_{ij}}{(1 - Pg_{ij}.\Pr_{ij})^2} \tag{4.3.16}$$

Where 'j' is the index for self-loop basic events

Appendix D presents detailed derivations of Equations (4.3.15) and (4.3.16).

4.3.2 ILLUSTRATION OF STOCHASTIC FAULT TREE ANALYSIS WITH SELF-LOOP BASIC EVENT

Figure 4.16 illustrates the block diagram representing the general data path of an electrical circuit board along with Ingress and Egress paths used in a data communication networks. The board failures can be categorized as link or device failures, either of which may be detected directly or indirectly. In Appendix D, input tables list all self-loop basic events and their transmittance parameters of the failures along with their detection and

recovery mechanisms. Since the structures of both the ingress and egress paths are similar, the data path goes down if both paths fail as depicted in Figure 4.17. To study the probability of the board down as well as the mean and variance of time to occurrence of the top event, stochastic FTA flow-graph is implemented for only the ingress path because of similarity of both paths (Figure 4.18). As presented in Table 4.4 derived from Appendix-E, the probability of occurrence of ingress path down is equal to 0.0021 and the mean and standard deviation of time of Ingress path down are 243.566 and 71.38 hours, respectively. Furthermore, by using Equation (4.3.3), the probability of data path down is equal to 0.000004 and reliability of circuit meets the target (e.g., 99.999%).

Table 4.4. Result of stochastic FTA flow-graph implementation for the electrical circuit board

Event	Wij	μ (Hr)	σ^2 (Hr)	Probability	Availability	Downtime (Min/Year)
Data Process Down	0.000004EXP(487.132s)	487.132	100.94	0.000004	0.999996	2.104
Ingress path down	0.0021EXP(243.566s+2547.3719s^2)	243.566	71.38	0.0021	0.997918	1094.96
egress path down	0.0021EXP(243.566s+2547.3719s^2)	243.566	71.38	0.0021	0.997918	1094.96

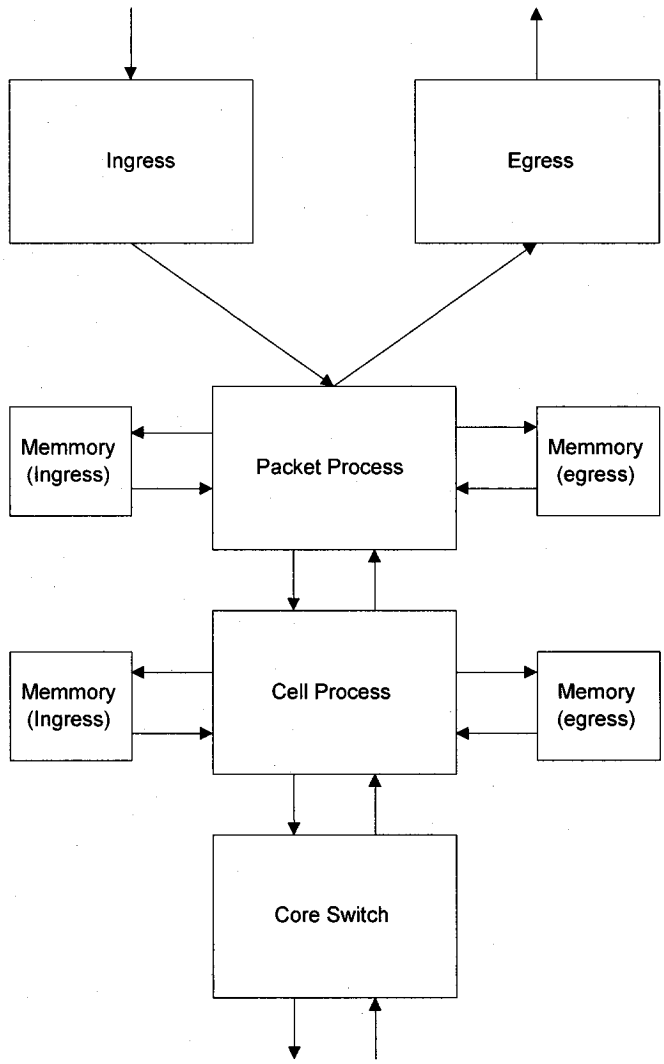


Figure 4.16. Block diagram of the electrical circuit board

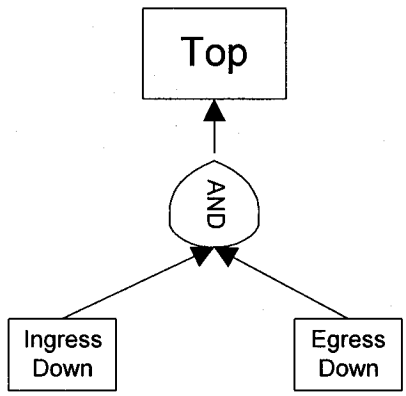


Figure 4.17. Level zero FTA

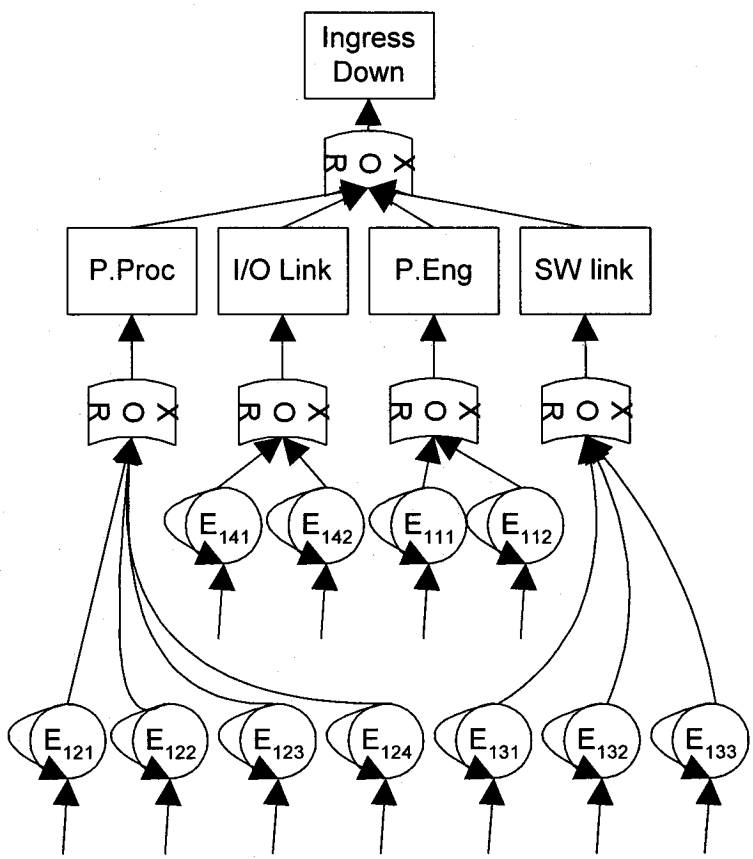


Figure 4.18. FTA of ingress and egress data path

CHAPTER 5

BINARY AND MULTI-STATE K-OUT-OF-N SYSTEMS

5.1 BACKGROUND

Reliability is useful to illustrate the effects of all possible configurations (e.g., series, parallel, k-out-of-n, and standby) functioning and failed units on the total system. It is a well-known fact that the reliability of a series system is usually low and, of the parallel system is usually high with very high cost. Thus, k-out-of-n solution becomes more attractive to use by engineering professionals. They have developed k-out-of-n:G and k-out-of-n:F systems made up of binary or Multi-State units. In the binary k-out-of-n:G system , the system works if and only if at least 'k' units are good. On the other hand, the binary consecutive k-out-of-n:F system fails if and only if at least k consecutive units fail.

Some researchers have extended the concept of the binary k-out-of-n system to the multi-state k-out-of-n system with some limitations. In a Multi-State k-out-of-n:G system, the state of the system remains at a certain level if the required number of units stay at that state level or higher. The studies pertaining to k-out-of-n systems can be classified, based upon unit mode, method, configuration, and achievement, as follows:

- Unit mode: binary state, and multi-state
- Configuration: G, F, Stand by, Load sharing
- Achievement: computational algorithm, analytical model

k-out-of-n systems have been studied the over years. These studies were concentrated on various k-out-of-n systems corresponding to the architecture of systems. However, they have overlooked two types of system:

- k-out-of-n system comprised of units with failure detection, isolation, and self-healing (recovery) mechanisms. Under such scenarios, the unit is called a self-loop unit and continuous checks must be performed while the unit is in service. When failures are 'Detected', hardware devices notify the software application, which takes appropriate actions and tries to pin-point the problem to a replaceable or self-healing

subunit. This step is called 'Isolation' and is a key for success in service availability. For 'Self-healing' (recovery) actions, the Software (SW) application may use Hardware (HW) resources to remove the failure and return the unit to the operational mode.

- k-out-of–n system comprised of units that have 'r' states (i.e., state 1^{st} is the best operational condition and state r^{th} is the worst operational condition) with state monitoring and recovery functions. The state transit process can be represented by a Semi-Markov model with the transition factors. Under such scenario the system is called a Reversible Multi-State k-out-of-n:G system with continuous monitoring being performed while the units are in service.

In this chapter, the analytical approaches based on flow-graph concept are introduced for such systems that determine not only the reliability and the availability of the system, but also time-to-state/event data (i.e., the probability, mean and standard deviation of time to event).

5.2 K-OUT-OF-N SYSTEM WITH SELF-HEALING UNITS

Consider a k-out-of-n system comprise of several independent self-loop units monitored continuously. The system performs failure detection, isolation and self-healing (Recovery) mechanisms to meet high service availability. If a failure occurs in the system, it may be detected, isolated and recovered with a chance denoted by Pr. These mechanisms often take time, which is a random variable with a probability distribution function (pdf) denoted by $fr_i(t)$. However, the detection mechanisms are not able to detect, isolate or recover all potential failures. The non-detected/isolated/recovered failures cause the safety or reliability event with the probability denoted by Pu, associated with the effect time, which is a random variable with 'pdf' denoted by fu(t). The effect time is the time required for a non-detected/isolated/recovered failure to fully participate in the system failure. Moreover, the failure may occur with a chance denoted by Pg and the time 'pdf' denoted by fg(t). Thus, the self-loop unit has three transmittances that indicate generation, detection, and participation

of the unit failures, as shown in Figure 5.1. In such a system with a series, parallel, or a mix configuration of several self-loop units, the basic reliability network methods are unable to determine reliability and time-to-event data.

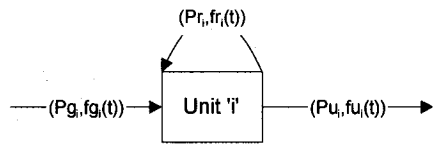


Figure 5.1. Self-loop Unit

Assumption:

- Self-loop units are statistically independent

5.2.1 ANALYTICAL APPROACH FOR K-OUT-OF-N SYSTEM WITH SELF-HEALING UNITS [4-143, 4-145]

The analytical approach bases the system reliability analysis upon the flow-graph concept and its adaptation for the self-loop unit (i.e., Self-healing unit) and k-out-of-n system. Thus, using the concept of the flow-graph and MGF properties (see Appendix-C), an analytical approach is developed to derive k-out-of-n system reliability measures.

5.2.1.1 FLOW-GRAPH TRANSMITTANCE OF A SELF-LOOP UNIT

Each link in the flow-graph is associated with two parameters (P_{ij},f_{ij}), each denoting the probability and the time distribution of the link, respectively. The probability is multiplicative while the time distribution is additive in nature. Thus, the probability components could be handled by basic properties of probability.

Nevertheless, difficulties may be anticipated with the time distribution element. Several transformers exist that will turn an additive operation into a multiplicative operation. The

one chosen for the flow-graph was MGF because of two reasons. Firstly, the units are independent. Secondly, the existence of a well-known theorem, which states that the MGF of the sum of independent random variables is equal to the product of the MGF of the individual events.

Thus, based on MGF properties, the transmittance of the self-loop unit denoted by W_i (s) is defined as follows:

$$W_i(s) = P_i.M_{fi(t)}(s) \tag{5.2.1}$$

where P_i is the probability element in unit i, and $M_{fi(t)}(S)$ is MGF of the time distribution function of the event (i.e., failure generation, failure detection, failure participation). Thus, by using Equation (5.2.1) for each transmittance of Figure 5.1 and substituting them into Equation (C.5) (in Appendix-C), the equivalent of self-loop unit is a simple unit with transmittance $W_i(s)$ as follows:

$$W_i(s) = \frac{Pg_i.Pu_i.Mg_i(s).Mu_i(s)}{1 - Pg_i.\Pr_i.Mg_i(s).Mr_i(s)} \tag{5.2.2}$$

Moreover, by stepwise reduction of the flow-graph, the reliability network will be converted to a single unit without a self-loop. This simplified method for the k-out-of-n system is presented below.

5.2.1.2 FLOW-GRAPH TRANSMITTANCE OF K-OUT-OF-N SYSTEM WITH SELF-LOOP UNITS

Figure 5.2 presents a k-out-of-n system comprised of 'n' self-loop units. In general, the system fails if more than 'k's self-loop units fail. By having the equivalent failure of k-out-of-n system denoted by $W_{k/n}$, we are able to compute the reliability, availability, mean time to failure (MTTF), and standard deviation of the time to failure (STTF) of the k-out-of-n system. Furthermore, since the k-out-of-n system can be reduced to series and parallel

networks by setting k equal to 'n' and 1, respectively. All these measures can also be computed for series and parallel networks as well. The equivalent of the k-out-of-n system and its relationship with the series and parallel equivalents is expressed by

$$W_p \leq W_{k/n} \leq W_s \tag{5.2.3}$$

where W_s, W_p are the equivalents of series and parallel network failures, respectively, and

$$W_s = \sum_{i=1}^{I} W_i \tag{5.2.4}$$

and

$$W_p = \prod_{i=1}^{I} Wi \tag{5.2.5}$$

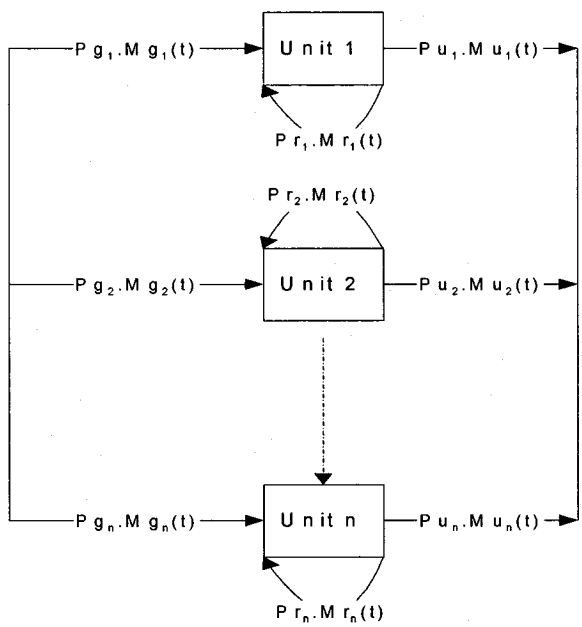


Figure 5.2. k-out-of-n system with self-loop units

Therefore, the transmittance of k-out-of-n system with identical units can be expressed by

$$W_{k/n} = \sum_{i=k}^{n} \binom{n}{i} \begin{pmatrix} Transmittance \\ of\ 'i'\ Parallel\ units \end{pmatrix} \tag{5.2.6}$$

By assuming that the units are identical and using Equations (5.2.2), (5.2.4)-(5.2.6), we have

$$W_{k/n} = \sum_{i=k}^{n} \binom{n}{i} W^{i} \tag{5.2.5}$$

Thus,

$$W_{k/n} = \sum_{i=k}^{n} \binom{n}{i} \left(\frac{Pg\ .Pu\ .Mg\ (s).Mu\ (s)}{1 - Pg\ .\mathrm{Pr}\ .Mg\ (s).Mr\ (s)} \right)^{i} \tag{5.2.8}$$

By applying MGF properties for Equation (5.2.8), the failure probability of a k-out-of-n system is given by

$$P_{k/n} = \sum_{i=k}^{n} \binom{n}{i} \left(\frac{Pg\ .Pu}{1 - Pg\ .\mathrm{Pr}} \right)^{i} \tag{5.2.9}$$

For k=1, Equation (5.2.8) leads to

$$P_{1/n} = n. \left(\frac{Pg\ .Pu}{1 - Pg\ .\mathrm{Pr}} \right) \tag{5.2.10}$$

Similarly, for k=n, Equation (5.2.8) results in

$$P_{n/n} = \left(\frac{Pg \ .Pu}{1 - Pg \ .\Pr} \right)^n \tag{5.2.11}$$

Using Equations (5.2.8) and (5.2.9), we get

$$\mu_{k/n} = \frac{1}{P_{k/n}} \sum_{i=k}^{n} A_i \tag{5.2.12}$$

$$\text{where } A_i = \binom{n}{i} i. \left(\frac{Pg.Pu}{1 - Pg.\Pr} \right)^{i-1} . \left(\frac{(\mu_g + \mu_u)(1 - Pg \ .\Pr) + (\mu_g + \mu_r).Pg \ .\Pr}{1 - Pg.\Pr} \right) \tag{5.2.13}$$

Similarly, by applying MGF properties to Equations (5.2.8) and (5.2.12), STTF of the k-out-of-n system can be

$$\sigma^2_{k/n} = E_{k/n}(t^2) - (\mu_{k/n})^2 \tag{5.2.14}$$

where $E(t^2)$ is

$$E_{k/n}(t^2) = \frac{1}{P_{k/n}} \sum_{i=k}^{n} i. \binom{n}{i} .P^i . \left(E(t^2) + (i-1).\mu^2 \right) \tag{5.2.15}$$

where $P = P_i \quad \forall i$.

In addition, using Equations (5.2.1) and (5.2.2), and the basic properties of MGF, the MGF of the self-loop unit is

$$M_i(s) = \frac{1}{P_i} . \left(\frac{Pg \ .Pu \ .Mg \ (s).Mu \ (s)}{1 - Pg \ .\Pr \ .Mg \ (s).Mr \ (s)} \right) \quad \text{for} \quad i=1,2,\ldots,n \tag{5.2.16}$$

5.2.1.3 ILLUSTRATION OF FLOW-GRAPH TRANSMITTANCE OF K-OUT-OF-N SYSTEM WITH SELF-LOOP UNITS

Consider an electrical distribution control box that includes five parallel cards configured as k-out-of-n units as shown in Figure 5.2. With the use of HW/SW detection, isolation, and recovery mechanisms, the card acts like a self-loop unit in the box. Moreover, in the card, the time of an event (i.e., failure generation, failure detection/ isolation/ recovery, and failure participation in the system failure) is a random variable. This variable is assumed normally distributed with mean μ and standard deviation σ that is denoted by N(μ , σ) in Table 5.1.

Table 5.1. Input Data: probability and distribution functions of failure generation, detection, and participation in System Failure

Unit	P_g	P_r	P_u	$f_g(t)$	$f_r(t)$	$f_u(t)$
1-5	0.1-0.65	0.9	0.1	N(100,200)	N(0.01,0)	N(0.5,0)

The distribution parameters (i.e., μ and σ) are the mean time and the standard deviation of the time to event in hours, respectively. For the purpose of computing the probability, mean time to failure (MTTF) and standard deviation of the time to failure (STTF) of the box, the proposed model is used and corresponding resulting plots are shown in Figure 5.3 when the failure generation probability varies between 0.1 and 0.65. The detailed calculation is provided in Appendix-E. Figure 5.3 shows that the box failure probability increases dramatically with increasing card failure probability, for k>3. Furthermore, the slope of the probability curve decreases with decreasing number of the required cards denoted by k.
Moreover, Figure 5.4 shows that MTTF of the box decreases with increasing card failure probability. It also demonstrates that the MTTF of the box increases with decreasing number of the required cards. On the other hand, Figure 5.5 demonstrates that the increase in the probability of the card failure can result in the increase of STTF in the box. In other words, higher STTF of the k-out-of-n box can be achieved by increasing the number of the required cards.

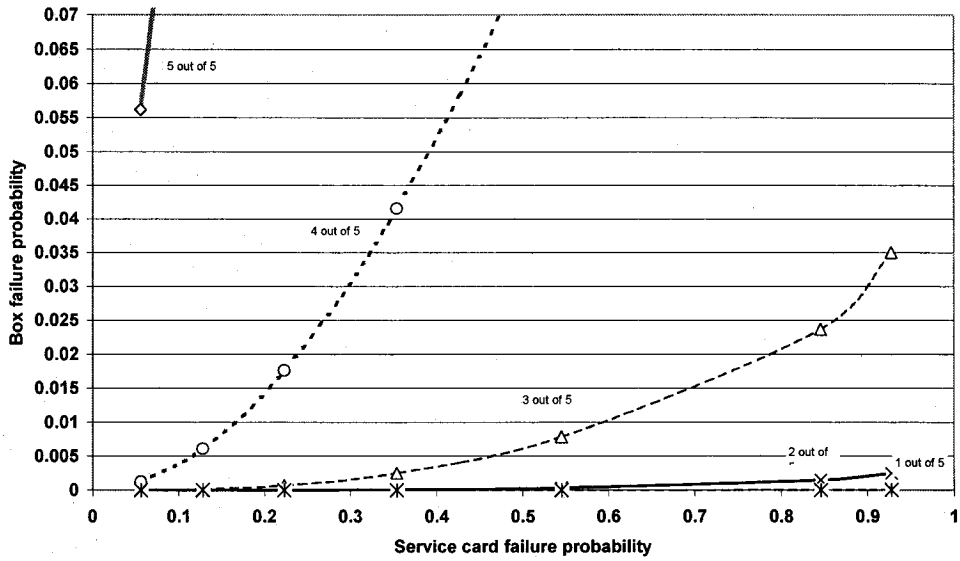


Figure 5.3. Failure probability of k-out-of-n electrical circuit box

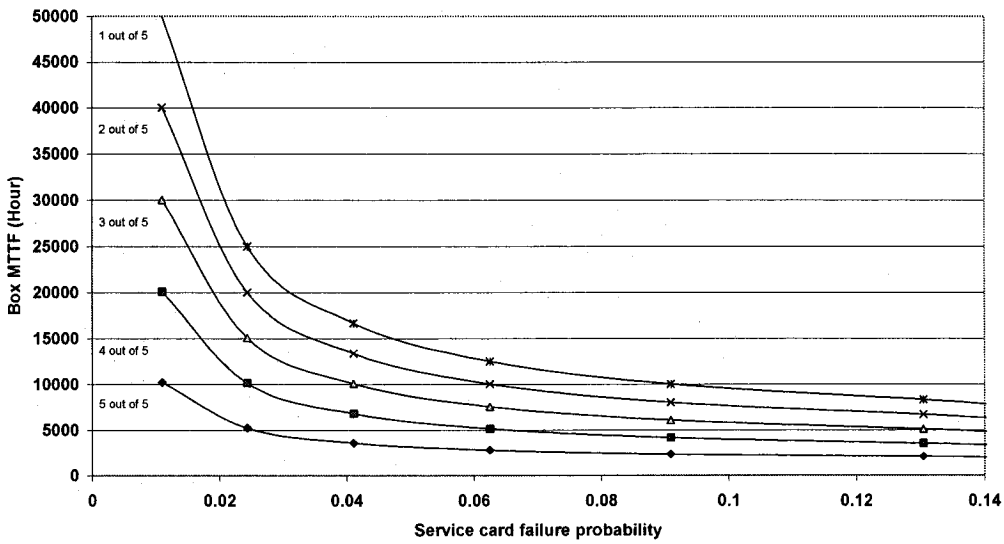


Figure 5.4. Mean time to failure (MTTF) of k-out-of-n electrical circuit box

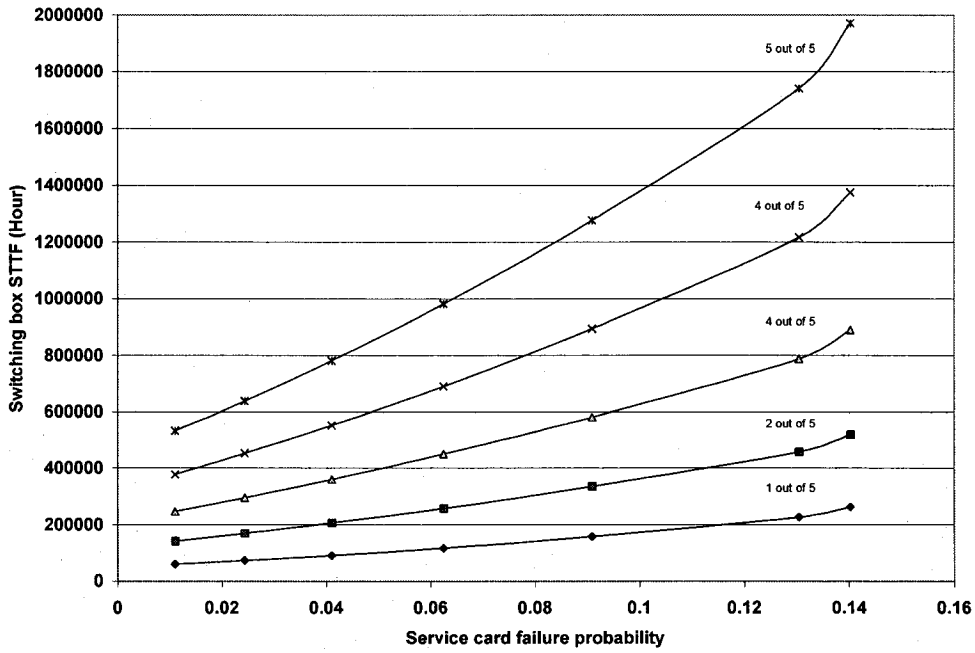


Figure 5.5. Standard deviation time to failure (STTF) of k-out-of-n electrical circuit box

5.3 REVERSIBLE MULTI-STATE K-OUT-OF-N SYSTEM

Figure 5.6 presents a Reversible Multi-State k-out-of-n:G system comprising 'n' identical, and independent units monitored continuously.

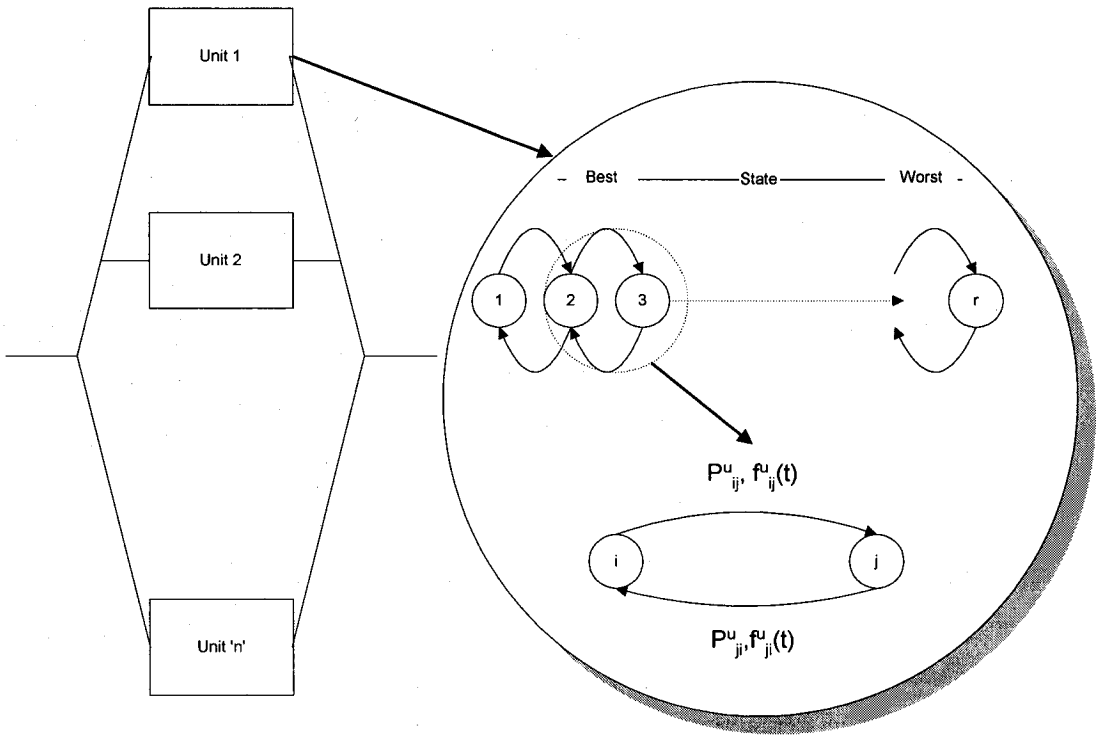


Figure 5.6. Reversible Multi-state k-out-of-n:G system

The system performs the state monitoring and recovery functions to meet high service availability. All identical and independent units are in good state as demonstrated by state '1'. Due to aging affect and stress, the state of the unit may change from state 'i' to state 'j=i+1' when level of state 'i' is better than state 'j'. Using continuous monitoring and recovery functions, the state of the unit may be returned from state 'j' to state 'i'. This state transit process can be represented by a Semi-Markov model with the transition factors (e.g., probability 'P^u_{ij}', transition time from state 'i' to 'j', which is a random variable with a probability distribution function ('pdf') denoted by $f^u_{ij}(t)$).

Assessment of the system requires an analytic approach that provides not only the reliability and the availability of the systems, but also multi-state time-to-state data (probability, the mean and standard deviation time to reach certain state in the reversible multi-state k-out-of-n:G/F/Load-Sharing systems. Therefore, in this chapter, a theoretical and practical flow-graph-based approach is introduced to assess the multi-state k-out-of-n:G/F/Load-Sharing systems with respect to the following assumptions:

Assumption:

- All units are identical and statistically independent.
- All units are good and at state '1'.
- Within a unit, transition factors are identical (Semi-Markov Transition State).

In the rest of this chapter, by using the stepwise reduction approach, the transmittance of unit state being in 'j' is defined by

$$W_j^u = \prod_{\ell=2}^{j} W_{\ell-1,\ell} \tag{5.3.1}$$

In developing multi-state k-out-of-n:G/F/Load-Sharing systems, one must first develop a flow-graph model for k-out-of-n systems and then substitute Equation (5.3.1) into the model.

5.3.1 FLOW-GRAPH MODEL FOR MULTI-STATE K-OUT-OF-N SYSTEM [4-144]

Consider a multi-state k-out-of-n:G system comprised of 'n' identical and independent units as shown in Figure 5.6. By having the equivalent transmittance of multi-state k-out-of-n system being in state 'j' denoted by $W_{j_{sys}}^{k/n}$, we were able to compute the reliability, availability, time-to-state data (e.g., MTTS, STTS) of the multi-state k-out-of-n:G/F/Load-Sharing systems. Furthermore, as multi-state k-out-of-n:G system can be reduced to the multi-state series and parallel systems by setting k to 'n' and 1, respectively,

all these measures can also be computed for multi-state series and parallel systems. The equivalent of the multi-state k-out-of-n:G system and its relationship with the multi-state series and parallel equivalents is expressed by

$$W_{j_{sys}}^{s} \leq W_{j_{sys}}^{k/n} \leq W_{j_{sys}}^{p} \tag{5.3.2}$$

where $W_{j_{sys}}^{s}, W_{j_{sys}}^{p}$ are the equivalents of series and parallel systems at state 'j'.

More specifically,

$$W_{j_{sys}}^{s} = \prod_{u=1}^{n} W_{j}^{u} \tag{5.3.3}$$

and

$$W_{j_{sys}}^{p} = \sum_{u=1}^{n} W_{j}^{u} \tag{5.3.4}$$

where W_j^u is the transmittance of unit 'u' in state 'j'

All in all, there are several multi-state k-out-of-n systems that require a new flow-graph model based on their structures.

5.3.1.1 FLOW-GRAPH MODEL FOR MULTI-STATE K-OUT-OF-N:G

The Multi-state k-out-of-n:G system is composed of two types of structures. The increasing structure is equivalent to at least k_j units must be at least at state 'i'<'j' for the system to be in state 'j' or less. On the other hand, the decreasing Multi-State k-out-of-n system structure is equivalent to that the lower system state level 'j' and lower requirement on the number of units that must be at state 'j' or less. Therefore, for both structures, transmittance of the system being in state 'j' is

$$W_{j_{sys}}^{k/n} = \sum_{i=k}^{n} \binom{n}{i} \begin{pmatrix} Transmittance \\ of\ 'i'\ Parallel\ units \\ in\ state < J_{sys} \end{pmatrix} . \begin{pmatrix} Transmittance \\ of\ 'n-i'\ Parallel\ units \\ in\ state > J_{sys} \end{pmatrix} \quad (5.3.5)$$

By assuming that the units are identical and using Equations (5.2.2), (5.3.3)-(5.3.5), we get

$$W_{j_{sys}}^{k/n} = \sum_{i=k}^{n} \binom{n}{i} . (W_{j<j_{sys}}^{u})^{i} . (W_{j>j_{sys}}^{u})^{n-i} \quad (5.3.6)$$

where $W_{j<j_{sys}}^{u} = \prod_{\ell=1}^{j_{sys}} W_{\ell}^{u}$, $W_{j>j_{sys}}^{u} = W_{j<j_{sys}}^{u} . \sum_{\ell=j_{sys}}^{r} W_{\ell}^{u}$

5.3.1.2 FLOW-GRAPH MODEL FOR CONSECUTIVE STRUCTURE OF MULTI-STATE K-OUT-OF-N:F

The linear or circular consecutive structure of multi-state k-out-of-n:F comprises 'n' identical and independent units with similar transition factors. In such a system, transmittance of system being in state 'j' based on at least 'k' consecutive units being in state 'j' or above is expressed by

$$W_{j_{sys}}^{k/n} = \binom{n}{k} . (W_{j<j_{sys}}^{u})^{n-k} . (W_{j>j_{sys}}^{u})^{k} \quad (5.3.7)$$

5.3.1.3 LOAD-SHARING STRUCTURE OF MULTI-STATE K-OUT-OF-N:G

A load-sharing system refers to a parallel system whose units equally share the system load. For example, in Binary-State 1-out-of-2 system, if a set of two parallel fans deliver 'x' air flow in a room ambient temperature, each fan delivers 'x/2' of air flow. If a minimum of 'x' air flow is required at all times, and one of the fans fails at a given time t_1, then the other fan's speed should be increased to provide 'x' air flow alone. This scenario

can be extended for a multi-state k-out-of-n:Load-Sharing system. If state of each unit 'u' (e.g., S^u_j) denotes the level of participation of that unit in the system function, the system state can be defined by an algebraic expression denoted by $L_{sys} = L(S^1_j, S^2_j, \ldots, S^n_j)$. Thus, the transmittance of the system being in state 'j' is expressed by

$$W^{k/n}_{L_{sys}} = \sum_{i=k}^{n} \binom{n}{i} \cdot \prod_{\substack{\ell=1 \\ L_{sys}<j}}^{i} W^{u}_{u^{\ell}_j} \cdot \prod_{\substack{\ell=i+1 \\ L_{sys}>j}}^{n} W^{u}_{u^{\ell}_j} \tag{5.3.8}$$

5.3.1.4 DERIVE TIME-TO-STATE DATA FROM EQUATION (5.3.8)- $W^{k/n}_{j.or.j_{sys}.or.L_{sys}}$

By applying MGF properties to Equations (5.3.6-5.3.8), the Time-To-State data can be derived from the transmittance formula as follows:

$$P^{k/n}_j = W^{k/n}_j \Big|_{s=0} \tag{5.3.9}$$

Using Equations (5.3.6-5.3.8) of k-out-of-n systems, we can derive Time-To-State 'j' data, such as $MTTS_j$, and $STTS_j$ as follows:

$$MTTS^{k/n}_j = \frac{1}{P^{k/n}_j} \cdot \frac{\partial W^{k/n}_j}{\partial s} \Big|_{s=0} \tag{5.3.10}$$

Similarly, by applying MGF properties to Equations (5.3.6-5.3.8), $STTS_j$ of the k-out-of-n system is given by

$$[STTS^{k/n}_j]^2 = \frac{1}{P^{k/n}_j} \cdot \frac{\partial^2 W^{k/n}_j}{\partial s^2} \Big|_{s=0} - (MTTS^{k/n}_j)^2 \tag{5.3.11}$$

5.3.1.5 ILLUSTRATION OF REVERSIBLE MULTI-STATE K-OUT-OF-N:G/F/LOAD SHARING MODELS

Assume that due to thermal effects, the physical properties of electrical components are rapidly altered and the rate of component failure doubles for every 10°C rise in temperature. Therefore, to meet the design requirements of an electrical power distribution control box used in aircraft in accordance with standards, the use of a cooling system must keep junction temperature within operational temperature. The box requires airflow equal to $X = Constant\ Coefficient \times \frac{Power\ Dissipation}{Air\ Temprature\ Rise}$. The cooling system comprises several fans, which are identical and independent with three states. Figure 5.7 shows a Semi-Markov model represented by a state transition diagram of the fan including probability and distribution of the transition time. Using Equations (5.2.2, 5.3.1), (5.3.9) and basic probability properties, the preliminary fan assessment including probability, Time-To-State data (Mean, Standard deviation) is performed. The result derived from Appendix-E, indicates that the probabilities of being in states 1, 2, and 3 are 0.24, 0.56, and 0.2, respectively.

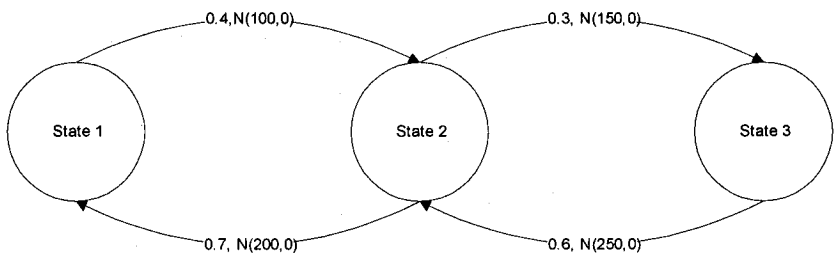


Figure 5.7. Semi-Markov model of the fan

Furthermore, mean and standard deviation of time-to-states 2 and 3 are $MTTS_2$= 215, $STTS_2$=300, $MTTS_3$=255, and $STTS_3$=329 hours, respectively. However, the cooling system composed of several fans may be represented as a Reversible multi-state k-out-of-n:G/F/Load Sharing system. For example, a multi-state 3-out-of-5:G cooling system. Thus, in this case the system works if and only if at least three fans work in state 2 or less. Using

Equations (5.3.6) and (5.3.9-5.3.0), the probability, mean, and standard deviation Time-To-State 2 for the system are

$P^{3/5}_2$ =0.443
$MTTS_2$ = 411.24 hour, and
$STTS_2$ =1302.99 hour

Another form of cooling system can be a multi-state 3-out-of-5:F system. In this case, the system fails if at least 3 fans fail. Using Equations (5.3.7) and (5.3.9-5.3.11), the probability, mean, and derivation time-to-state 2 for the system are

$P^{3/5}_2$ = 0.174
$MTTS_2$= 39.93 hour, and
$STTS_2$ =1394.2 hour

In similar manner, reliability assessment is performed for a cooling system formed as multi-state 1-out-of-2:Load sharing system If a box requires air flow denoted by 'X", which is defined by X= $Constant\ Coefficient \times \frac{Power\ Dissipation}{Air\ Temprature\ Rise}$, a set of two parallel fans deliver 'X' air flow in a room ambient , each fan delivers 'X', 'X/2', or 'O' air flow in states 1, 2, and 3 ,respectively. Thus, using Equations (5.3.8) and (5.3.9-5.3.10), the probability, mean, and standard deviation Time-To-$L_{sys=x}$ are

$P^{1/2}_{Lsys=x}$ = 0.429
$MTTS_{Lsys=x}$ = 429.1 hour, and
$STTS_{Lsys=x}$ = 457.38 hour

CHAPTER 6

RELIABILITY ESTIMATION

6.1 BACKGROUND

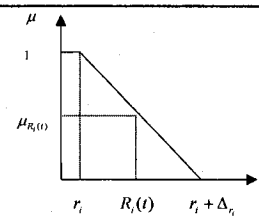
Reliability networks are commonly used to represent the reliability architecture of the systems. It is a simple approach to represent the effects of all possible configurations (e.g., series, parallel, k-out-of-n, standby) of functioning and failed units on the system operation. With the known reliability of the units, the reliability of the system can be calculated. To estimate the reliability of the subsystem/unit/device, various prediction approaches for certain type of the system are available depending upon the depth of knowledge of the design and the availability of historical data on subsystem/unit/device reliability. During the production development cycle, data describing the system design evolves from a qualitative description of system functions to detailed specification and drawings suitable for production. However, the lack of the precise design definition, lack of knowledge of use environment and lack of field operational history are the major concerns of all prediction methods. Alternatively, the use of fuzzy sets has been proposed since it offers a unique approach for dealing with certain types of essential imprecision knowledge about design, and subsystem/unit/device reliability factors. Nonetheless, in order to describe the application of the fuzzy concept, consider an inkjet printer composed of series Mechatronic components as presented in Table 6.1. The application of all components is new but the designs are based on similar types of components. In this series network each item is required for successful operation of the printer.

Due to stress or design uncertainty, the reliability of and reliability tolerance of 'i'th component are indicated with r_i and Δ_{r_i}, respectively. By assigning the possibility a chance 'α_i' with certain tolerance 'Δ_{α_i}' for the components, the estimation model is required to extract the expected value of component reliability. Thus, by reviewing the earliest studies for solving the problem described in section 6.2, the new model is developed in section 6.3 which is able to handle the imprecise probability (i.e., $\Pr\tilde{o}b(R_i(t) \ \tilde{<} r_i) \tilde{\geq} \alpha_i$) associated

with possibility of the reliability data. However, to estimate the reliability of the system composed of subsystems/units/devices with uncertain reliability data for an item like an inkjet printer, little attention has been paid to develop an analytical approach to deal with imprecise-chance reliability estimation when the chance factor is not specified precisely.

In this chapter, we introduce a network reliability problem involved in imprecise-chance reliability estimation. The reliability is associated with imprecise-chance that is expressed by a fuzzy operand and a fuzzy probability function. The fuzzy probability function expresses the fuzzy reliability of the subsystem/unit/device by using fuzzy operands.

Table 6.1. Inkjet printer components' reliability data

Component	r_i, Δ_{r_i}	α_i, Δ_{α_i}	$Prob(R_i(t) < r_i + \Delta_{r_i}) \geq \alpha_i - \Delta_{\alpha_i}$
1-Print head	0.95,0.04	0.975,0.0025	
2-Pivot assembly	0.91,0.03	0.975,0.0025	
3-Paper feed	0.92,0.01	0.975,0.0025	
4-Control electronics	0.96,0.02	0.975,0.0025	
5-Power supply	0.95,0.05	0.975,0.0025	Due to uncertainity of reliability data, the possibility of reliability is defined by $\mu_{R_i(t)} = \begin{cases} 1 \text{ for } R_i(t) \leq r_i \\ \dfrac{r_i - \Delta_{r_i} - R_i(t)}{\Delta_{r_i}} \text{ for } r_i \leq R_i(t) \leq r_i + \Delta_{r_i} \\ 0 \text{ for } R_i(t) \geq r_i + \Delta_{r_i} \end{cases}$
6-Fan assembly	0.94,0.08	0.975,0.0025	
7-Service station	0.98,0.09	0.975,0.0025	
8-Paper tray	0.97,0.01	0.975,0.0025	

6.2 IMPRECISE-CHANCE RELIABILITY ESTIMATION [5-62]

For most systems, there are usually many subsystems/devices that we have less technological knowledge or reliability data on them. The sources of lack of knowledge or reliability data usually are:

- designer's intellectual property,
- unknown areas of leading-edge of science and technology, and

- shortcomings of prediction methods in uncertainty situation.

In this case, we prefer to express the reliability of the subsystem/unit/device by fuzzy probability function (i.e., $\Pr\tilde{o}b(R_i(t) \ \tilde{<} r_i) \tilde{\geq} \alpha_i$). The expression '$R(t)_i \tilde{<} r_i$' means that the reliability of the unit 'i' is less than $r_i + \Delta_i$, where Δ_i is a tolerance for the estimation of unit reliability due to uncertainty. Similarly, the expression '$\Pr\tilde{o}b(...) \tilde{\geq} \alpha_i$' states the imprecise-chance of the reliability expression. The imprecise-chance expression implies that the probability is greater than $\alpha_i - \Delta_{\alpha_i}$,where $\Delta_{\alpha i}$ is the tolerance for estimation of probability function of unit 'i'. Consider the series and parallel networks composed of 'n' units with the imprecise-chance reliability as shown in Figure 6.1. In order to calculate the reliability of the networks, the conversion of imprecise-chance reliability of units into crisp (deterministic) reliability of units has to be accomplished.

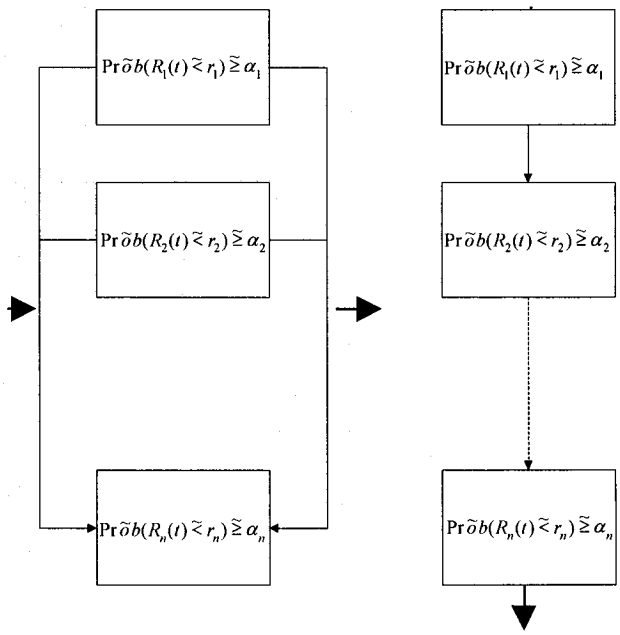


Figure 6.1. Parallel and series networks

6.2.1 RELIABILITY NETWORK WITH STOCHASTIC RELIABILITY UNITS

Consider a subsystem/unit/device 'i' with a reliability that is expressed by

$$\Pr ob(R_i(t) \ < r_i) \geq \alpha_i \tag{6.2.1}$$

where r_i is a continuous variable and α_i crisp (nonrandom).

In order to compute the reliability of the system composed of several such units in series or parallel, we propose the conversion of stochastic reliability function into a crisp. Assuming the expected reliability of unit 'i' (i.e., r_i) is a normally distributed random variable with mean μ_{r_i} and standard deviation σ_{r_i}, we get

$$\Pr ob(\frac{R_i(t) \ - r_i + \mu_{r_i} - E[R_i(t) \ ]}{\sigma_{r_i}} < \frac{\mu_{r_i} - E[R_i(t)]}{\sigma_{r_i}}) \geq \alpha_i \tag{6.2.2}$$

Thus, upper limit for the expected value of reliability of unit 'i' is expressed by

$$E[R_i(t)] \leq \mu_{r_i} - \Phi^{-1}(\alpha_i).\sigma_{r_i} \tag{6.2.3}$$

6.2.2 RELIABILITY NETWORK WITH UNITS HAVING IMPRECISE-CHANCE RELIABILITY

Consider a subsystem/unit/device 'i' with reliability that is expressed by

$$\Pr \tilde{o} b(R_i(t) \tilde{<} r_i) \tilde{\geq} \alpha_i \tag{6.2.4}$$

To calculate the reliability of the system composed of several such units in series or parallel, we propose the conversion of imprecise-chance reliability function into a crisp. With respect to the defuzzifing approach presented in Appendix-A, and using tolerances for both reliability and probability functions (i.e., $\Delta_{r_i}, \Delta_{\alpha_i}$), we get non-fuzzy of Equation (6.2.4) as follows:

$$\Pr ob(R_i(t) < r_i + \Delta_{r_i}) \geq \alpha_i - \Delta_{\alpha_i} \tag{6.2.5}$$

As in the case of reliability network with stochastic reliability units', the conversion of Equation (6.2.4) into a crisp format is

$$\Pr ob(\frac{R_i(t) - r_i - \Delta_{r_i} + \mu_{r_i} - E[R_i(t) - \Delta_{r_i}]}{\sigma_{r_i}} < \frac{\mu_{r_i} - E[R_i(t) - \Delta_{r_i}]}{\sigma_{r_i}}) \geq \alpha_i - \Delta_{\alpha_i} \tag{6.2.6}$$

Thus, Reliability of unit 'i' is

$$E[R_i(t)] \leq \mu_{r_i} - \Delta_{r_i} - \Phi^{-1}(\alpha_i - \Delta_{\alpha_i}).\sigma_{r_i} \tag{6.2.7}$$

6.2.3 ILLUSTRATION OF IMPRECISE-CHANCE RELIABILITY ESTIMATION

The application of this approach is demonstrated by considering a box made up of the electrical circuit boards shown in Figure 6.2. The box has several newly developed boards that all may be configured for only one path from source to sink. For the box to perform its function, at least one board must work normally. Based on past experiences and experts' opinion, the reliability of the new board is less than 0.9792+0.0001 with the chance that is equal to or greater than 0.975-0.0025, where 0.0001 and 0.0025 are the tolerances for reliability and probability functions, respectively, as shown in Figure A-3 in Appendix-A. With respect to Equation (6.2.7), the reliability of the box is expressed by

$$R_P(t) = 1 - \prod_{i=1}^{n} (1 - E[R_i(t)]) \tag{6.2.8}$$

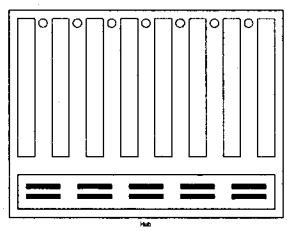


Figure 6.2. Electrical circuit box

By assuming all boards are similar, and substituting the upper limit of expected reliability for the board into Equation (6.2.8), we get

$$R_P(t) = 1 - (1 - \mu_r + \Delta_r + \Phi^{-1}(\alpha - \Delta_\alpha).\sigma_r)^n \tag{6.2.9}$$

where $n = 2, \alpha_i = \alpha = 0.975, \Delta_{\alpha_i} = \Delta_\alpha = 0.0025, \mu_{r_i} = \mu_r = 0.9792, \sigma_{r_i} = \sigma_r = 0.0001$ and $\Delta_{r_i} = \Delta_r = 0.0005 \quad \forall i$. Based on taxonomy of design approaches in the industry, we consider only two boards as static or dynamic redundancy (i.e., $n = 2$). With respect to Equation (6.2.2), the reliability of the box is equal to 0.99956. As Figure 6.3 shows, the reliability of the path from source to sink includes several boxes and is reduced when total number of the boxes is increased. The reliability is always less than 0.9995.

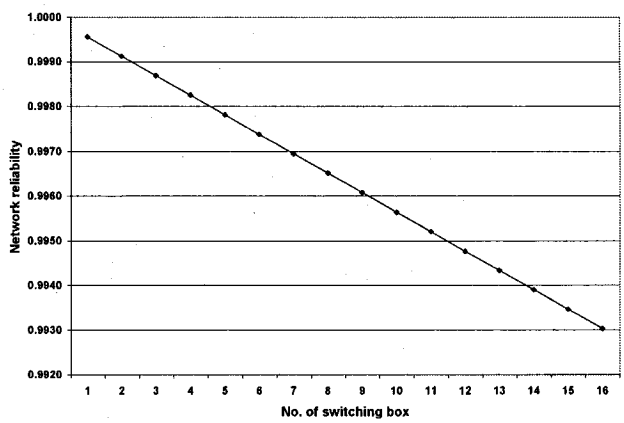


Figure 6.3. The reliability of parallel network

Another scenario is to establish a connection from source to sink through several designated boards (Series network). With respect to Equation (6.2.7), the reliability of series network is expressed by

$$R_S(t) = \prod_{i=1}^{n} E[R_i(t)] \tag{6.2.10}$$

As shown in Figure 6.4, the reliability of the path from source to sink is reduced if the total number of the boxes is increased. The reliability is less than 0.9786.

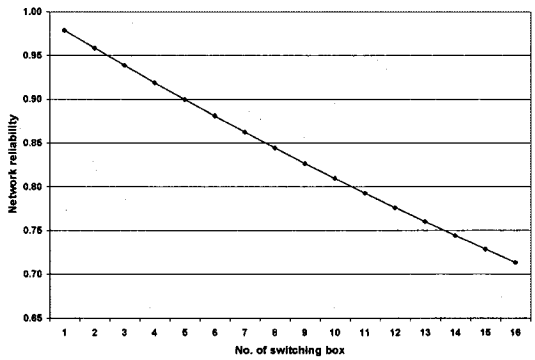


Figure 6.4. The reliability of series network

Figure 6.5 shows the result of sensitivity analysis of series and parallel networks. For both the networks the results are not sensitive to the change of $\alpha, \Delta_\alpha, \sigma_r$ and σ_r . However, Figure 6.5 shows by increasing tolerance (Δ_r), the reliability of the series and parallel networks is decreased drastically.

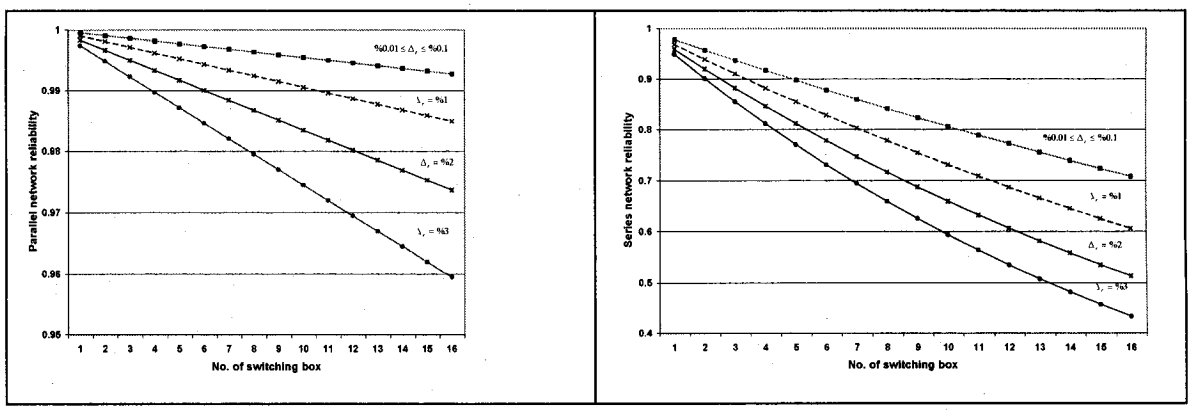


Figure 6.5. The sensitivity analysis of parallel and series networks

CHAPTER 7

CONCLUSIONS AND FUTURE DIRECTIONS

The proposed studies on stochastic and fuzzy analyses in reliability design have been carried out in chapters 2, 3, 4, 5, and 6. In chapter 2, new methods of failure analysis have been designed to identify and prioritize the potential failures in a system. The failure risk analysis (FRA) method uses Graph-based risk priority number (GRPN). Using relational graphs to mitigate problems associated with traditional RPN, GRPN depicts the relationships among symptoms, root causes, and failures and takes into account these relationships for failure risk analysis. This method can be extended by considering weights for relational graphs in future work. However, FRA is a system technical risk analysis that extracts knowledge about the potential failure mode from a group of experts called expert knowledge base system with possible conflict in experts' opinions. Therefore, Group-based failure effect analysis (GFEA) method has been developed to prioritize the potential failures in a system. Using fuzzy aggregation and fuzzy compensation techniques in the MINMAX method, GFEA approach mitigates the conflicts and the problems (e.g., sensitivity and consistency analysis) associated with the proposed FEA approaches. In future work, an algorithm can be developed to identify and remove noncompliance experts.

Chapter 3 presents a new fuzzy Markov model to deal with the aging effect and stress (e.g., dynamic, electrical, thermal, etc) in the failure rate of the component. The failure rates calculated in accordance with MIL-HDBK-217 do not address circuit failures caused by stress and aging effect, which result in the fuzzy flow-rate equation system. The flow-rate equation is associated with specified chance representing possibility that flow-rate equation

lies within specified interval. The interval may vary for the states because of aging effect and stress. To compute the probability of being in states and rate of entry into states in a system, a new fuzzy Markov Model called imprecise-chance Markov has been developed to convert the imprecise-chance equation to a nonfuzzy form. This model can be extended to the common distribution functions in future work.

Chapter 4 introduces fuzzy and stochastic fault tree analysis. Due to the extensively growth of application of fuzzy sets in reliability design, fuzzy fault tree analysis with the compensated/aggregated logic gates has been developed. The compensated/aggregated gates are composed of fuzzy 'OR' and 'AND' operators with a compensatory operator. The compensatory operator is used to mitigate the effect of the maximum or minimum functions in the fuzzy gates. This method can be extended to other types of gates in FTA for future work. On the other hand, in stochastic fault tree analysis, an analytical approach based upon flow-graph concept was developed to compute Time-To-Event data (i.e., probability, mean and standard deviation of time) for a system with self-loop events. The self-loop event represents failure detection and recovery mechanisms of a system. Because of method mathematical foundation, the proposed analytical approach is a robust method in comparison to the simulation or Meta-heuristic approaches. This method uses MGF as a transformer, which will turn an additive operation into a multiplicative operation. For future work, transformers' efficiency study in term of less calculation for common distribution functions should be studied.

Chapter 5 has developed an analytical approach based on the flow-graph concept for evaluating a binary k-out-of-n system comprised of units with failure detection, isolation,

and self-healing (recovery) mechanisms, to calculate Time-To-Failure data. This chapter has extended the developed analytical approach to the Reversible Multi-state k-out-of-n systems comprised of units that have 'r' states (i.e., state 1st is the best operational condition and state rth is the worst operational condition presented by the Semi-Markov model) with state monitoring and recovery functions to evaluate Time-To-State data. These analytical approaches are robust methods in comparison to the other methods for the increasing and decreasing structures of k-out-of-n systems. For future work, they can be extended for deferred maintenance models.

To estimate the reliability of the subsystem/unit/device, various prediction approaches for certain type of the system are available depending upon the depth of knowledge of the design and the availability of historical data on subsystem/unit/device reliability. However, the lack of precise design definition, lack of knowledge of use environment and lack of field operational history are the major concerns of all prediction methods. Chapter 6 presents a new fuzzy method to calculate the reliability of the system composed of several units in series or parallel. By converting the imprecise-chance reliability function into a nonfuzzy form, this method computes the system reliability. For future work, this method can be extended to common distribution functions.

Several practical examples have been demonstrated to present the applications of the newly developed models. The validity and robustness of the models have been verified by the obtained results based on the typical, practical and extreme values of the basic parameters of the models and performing model sensitivity analysis.

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APPENDIX A: FUZZY SETS

Most of approaches used for modeling and computing are crisp, deterministic, and precise in character. Crisp means dichotomous, which implies a statement can be true or false and nothing in between. Precision indicates the parameters of model represent exactly the characteristics of the real system in the model. Certainty means the parameters of the model are known without doubt about their values. Most of times, the parameters of the model, however, are not crisp, known, and precise. Therefore, fuzzy sets were first developed in 1962 to deal with ambiguous information and decision-making based on imprecise information. Moreover, in fuzzy sets, it can express the concepts like "critical" or "minor" for failure effect may not be precisely expressed via mathematical expression. Not only do the concepts constitute a wide range of human perception of the world, but also they play a vital role in decision-making and knowledge communication. Consider the general notation used in fuzzy sets as follows:

$\widetilde{A}$	Fuzzy set
$\mu_{\widetilde{A}}(x)$	Possibility function of fuzzy set $\widetilde{A}$
X	Universal set
x_i	Element of fuzzy set
N_A	Set of element index in fuzzy set $\widetilde{A}$
(ℓ, m, u)	Fuzzy triangular number
$\widetilde{f}(x)$	Fuzzy function
$\mu_{f(x)}$	Possibility function of fuzzy function $\widetilde{f}(x)$
G	Goal of function
Δ_G	An interval for function G

In contrary to fuzzy sets, there are crisp sets. The following crisp set terminology may also be used in reference to fuzzy sets as follows:

By defining universal set, 'X', as a finite number of elements (e.g., probability of being in the state), an individual 'x ' is a member of a set 'A' and we write

$$x \in A \tag{A.1}$$

Otherwise 'x' is not a member of a set 'A', thus,

$$x \notin A \tag{A.2}$$

Generally, a set can be described as follows:

$$A = \{x_i \mid i \in N_A\} \tag{A.3}$$

Where N_A is dimension of set 'A' and 'i' is index of element belonging to set 'A'

In other words, by setting value 1 or 0 to possibility function (e.g., $\mu_A(x)$) this indicates whether x belongs to set 'A' or not, respectively.

As defined in crisp sets, possibility function values could be 0 or 1. However, the possibility function can be generalized such that the values assigned to the elements of the universal set fall within a specified range (e.g., $0 \leq \mu_A(x) \leq 1$). Thus, for the set in question, $\mu_A(x)$ indicates the possibility grade of these elements because larger values denote higher degrees of set possibility. A set defined by such possibility is called a fuzzy set and it is expressed by

$$\widetilde{A} = \{(x, \mu(x)) \mid x \in X\} \tag{A.4}$$

Definition A-1: Fuzzy Union and Intersection of set '$\widetilde{A}$' and '$\widetilde{B}$'are denoting by $(\widetilde{A} \vee \widetilde{B})$, and $(\widetilde{A} \wedge \widetilde{B})$

The fuzzy union and intersection operators like crisp set are expressed by

$$(\widetilde{A} \vee \widetilde{B}) = \{(x, \max(\mu_A(x), \mu_B(x) \mid x \in X\} \tag{A.5}$$

$$(\widetilde{A} \wedge \widetilde{B}) = \{(x, \min(\mu_A(x), \mu_B(x) \mid x \in X\} \tag{A.6}$$

Definition A-2: Fuzzy linguistic variables

If the values of the attributes/criteria are not known precisely, using linguistic variables is most realistic approach. In [2.10], the fuzzy membership of linguistic variables 'Low', 'Medium', and 'High' are defined as follows:

Low ={(0.01%,0.5),(0.25%,0.7),(0.50%,1.0),(0.75%,0.7),(1%,0.5)}

Medium ={(0.50%,0.5),(0.75%,0.7),(1%,1.0),(8%,0.7),(10%,0.5)}

High ={8%,0.5),(10%,0.7),(15%,0.9),(20%,1.0)}

The fuzzy functions that can be used for extension of linguistic variables (i.e., VeryHigh, VeryLow, etc) is presented. To extend a fuzzy set '$\widetilde{A}$' by fuzzy terms like 'VERY', 'PLUS', 'MORE-LESS', 'SLIGHTLY', several fuzzy generator functions are developed in [1.25] as follows:

-Concentration $\mu_{con}(x) = [\mu_A(x)]^2$ (A.7)

-Dilation $\mu_{dil}(x) = [\mu_A(x)]^{1/2}$ (A.8)

-Intensification $\mu_{\text{int}}(x) = \begin{cases} 2[\mu(x)]^2 \ldots\ldots 0 \le \mu(x) \le 1/2 \\ 1-2[1-\mu(x)]^2 \ldots Otherwise \end{cases}$ (A.9)

By substituting degree of membership of set '$\widetilde{A}$' (e.g., $\mu_A(x)$) in equations (A.7)-(A.9), the new fuzzy sets can be generated. Theses sets can be bases for generating 'Very($\widetilde{A}$)', 'More or Less($\widetilde{A}$)', 'Plus($\widetilde{A}$)', 'Slightly($\widetilde{A}$)' as follows:

Very($\widetilde{A}$)=Concentrate($\widetilde{A}$) (A.10)

More or Less($\widetilde{A}$)= Dilation($\widetilde{A}$) (A.11)

Plus($\widetilde{A}$)=($[\mu(x)]^{1.25}$ (A.12)

Slightly($\widetilde{A}$)= Intensification{ Plus($\widetilde{A}$) and NotVery($\widetilde{A}$)} (A.13)

Definition A-3: Fuzzy triangular number

A fuzzy triangular number, 'x', are defined by three numbers (ℓ, m, u), pessimistic, most likely, and optimistic bounds, respectively as shown in Figure A.1.

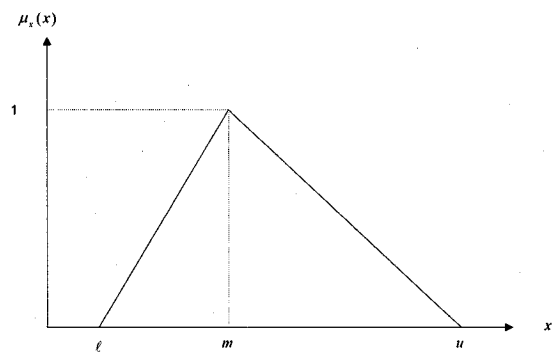

Figure A. 1. Fuzzy triangular number possibility

For $x \le \ell$, and $x \ge u$ the possibility function of x, $\mu_x(x)$, is equal to zero, which means no chance for those x to be in fuzzy set $\widetilde{A}$. Furthermore, the possibility function of m, $\mu_x(m)$, is equal to one.

Definition A-4: Fuzzy function

Consider $\tilde{f}(x)$ as a fuzzy function. Thus, by setting goal for $\tilde{f}(x)$, we need to define as an interval or tolerance to achieve the goal (e.g., Δ_G). Therefore, possibility function of $\tilde{f}(x) \geq G$ is defined in Figure A.2. By using the aspiration for fuzzy function $\tilde{f}(x)$, $\tilde{f}(x) \geq G$ can be converted to non-fuzzy form by expression

$$f(x) \geq G - \Delta_G \tag{A.14}$$

Where ΔG is the interval or tolerance for non-fuzzy function, $f(x)$.

Definition A-5: Fuzzy Probability of non-fuzzy function

Consider $f(x)$ as a non-fuzzy function. Setting the probability to achieve the goal for $f(x)$ by defining an interval or tolerance (e.g., Δ_α), we write

$$\Pr\tilde{o}b(f(x) \leq G) \tilde{\geq} \alpha \tag{A.15}$$

Thus, the possibility function of probability of non-fuzzy function can be expressed as follows:

Therefore, The Equation (A.15) can be converted to non-fuzzy form by expression

$$\Pr ob(f(x) \leq G) \geq \alpha - \Delta_\alpha \tag{A.16}$$

In case that the Prob function is normally distributed function, we write

$$\Phi(f(x) \leq G) \geq \alpha - \Delta_\alpha \tag{A.17}$$

$$\mu_{f(x)}(x) = \begin{cases} 0 & for\ f(x) \leq G - \Delta_G \\ \dfrac{f(x) - (G - \Delta_G)}{\Delta_G} & for\ G - \Delta_G \leq f(x) \leq G \\ 1 & for\ f(x) \geq G \end{cases}$$

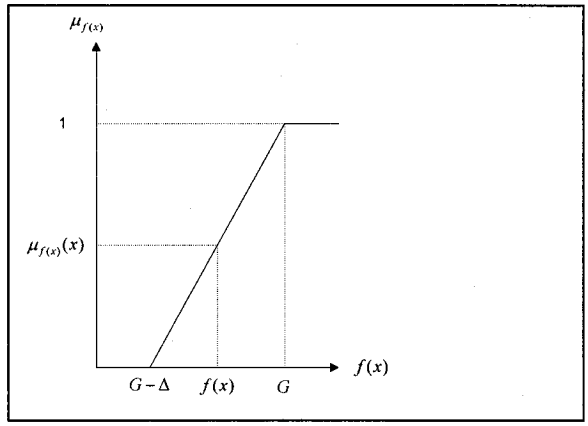

Figure A. 2. Possibility function of fuzzy function

$$\mu_{Prob(f(x)\le G)}(\alpha) = \begin{cases} 0 & for\ Prob(f(x) \le G) \le \alpha - \Delta_\alpha \\ \dfrac{Prob(f(x) \le G) - (\alpha - \Delta_\alpha)}{\Delta_\alpha} & for\ (\alpha - \Delta_\alpha) \le Prob(f(x) \le G) \le \alpha \\ 1 & for\ Prob(f(x) \le G) \ge \alpha \end{cases}$$

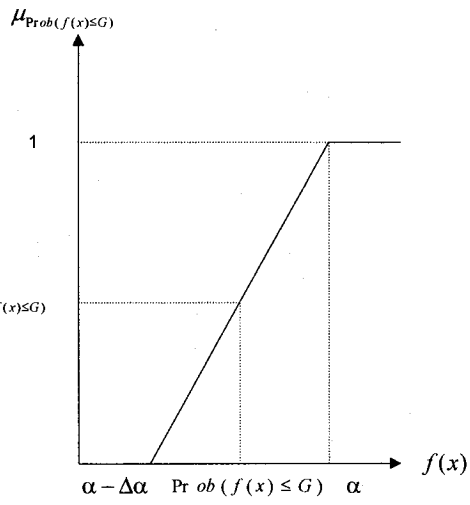

Figure A. 3. Possibility function of fuzzy probability function

Definition A-6: Ranking

Using the linguistic variables listed in Table 2.8 in Chapter 2, experts determine the importance of the risk criteria (i.e., failure factors including failure probability, non-detection probability, severity of effect, expected cost, and functions/subsystems

importance) and the system failure impacts relative to a risk criterion. Using 'S' to denote the term linguistic variable, and subscripted with an associated index rank, the rank for the failure factors and the function attributes can be expressed. For example, for impact of a failure on a system in relation to severity factor considered to be 'VeryHigh (VH)', it is represented in the index by S_6.

To assign RPC to failures based on a failure factor, we simply compare their indices. If a failure factor of failure 'A', denoted by 'S_ξ', is greater than the failure factor of failure 'B' in a system, denoted by 'S_ς', the rank of failure 'A' is higher than the rank of failure 'B'. In the other words, we write 'S_ξ' > 'S_ς' as $\xi > \varsigma$,.

For example, in Table 2.9, Chapter 2, the rank of the failure probability (failure factor) of three failures (Analog Personality Module, RS232 Com1, Power Conditioning) are assigned Low(L), High(H), and Medium(M) by expert 1. As presented in Table 2.8, the corresponding indices to 'S' for 'L', 'M', 'H' are 3, 4, and 5, respectively. Thus, in the opinion of expert 1, and within the definition that $S_5 > S_4 > S_3$, the rank of RS232 Com1 failure is higher than the rank of Analog Personality Module failure and of Power Conditioning failure relative to only failure probability factor.

Definition A-7: Maximum and minimum of two fuzzy linguistic variables

Consider two system failures with assigned system impact ranks (linguistic variables) relative to a specified failure factor (e.g., Non-detection probability). To identify the failure with higher system impact relative to the failure factor, a maximum function is required. With respect to Definition A-6, the failure with maximum impact relative to the specified failure factor in the system is the one with the highest rank (i.e. greater index of 'S'), thus,

$$\text{Max}(S_\xi , S_\varsigma)=S_\xi \qquad \text{(A.18)}$$

where index ξ is greater than index ς.

For example, using Table 2.8 to map the indices of two linguistic variables 'Perfect (P)' and 'VeryHigh (VH)', we assign indices 7 and 6 to 'S' for 'Perfect (P)' and 'VeryHigh (VH)', respectively. Thus, we write

$\text{Max}(\text{'P'},\text{'VH'})=\text{Max}(S_7 , S_6)$

With respect to Definition A-6, the rank of the linguistic variable 'Perfect (P)' is higher than that of 'VeryHigh (VH)' because $S_7 > S_6$.Thus,

Max('P','VH')='P'

Also, minimum of two fuzzy linguistic variables is equal to the lowest ranked one (i.e., smaller index of 'S'), thus,

$$\text{Min}(S_\xi , S_\varsigma)=S_\varsigma \qquad \text{(A.19)}$$

where index ξ is greater than index ς.

B. APPENDIX B: DECISION MAKING CONCEPT

Within group decision-making processes that are based on fuzzy linguistic preference, there exists a set of alternatives and a set of decision-makers that are denoted by failure and expert sets, respectively. Decision-maker (DM) bases his/her decision upon certain criteria (failure risk factors, operation, safety, and their weight). These criteria can be independent or dependent. When criteria are independent, the importance weights of criteria are treated as coefficients of an additive aggregation rule. By defining X_{ijk} as fuzzy preference relation between 'i'th alternative (the potential failures) and 'j'th criterion taken from 'k'th DM, the decision matrix for 'k'th DM can be made. What makes the decision process difficult because it involves both the competency of the alternatives and the divergence of decision makers' perception of those alternatives [1.40]. In such situations, not only is the matching technique used to find a match between alternatives and preferences for each individual decision-maker, but also aggregating techniques are implemented, as a second step to combine perceptions of all decision-makers [1.68].

Table B. 1. Decision Matrix for 'k'th Decision Maker (expert)

	DM	CRITERIA Subsystems, Safety Factors, Failure Risk Factors C_1 C_2 C_3 .....
A L T E R N A T I V E S	A_1 A_2 . . . A_I	X_{11k} X_{12k} X_{13k} ... X_{1Jk} X_{21k} X_{22k} X_{23k} ... X_{2Jk} . . . . X_{I1k} X_{I2k} X_{I3k} ... X_{IJk}

APPENDIX C: FLOW-GRAPH AND TRANSMITTANCE CONCEPTS

C.1. FLOW-GRAPH

Figure C-1 presents a flow-graph that is made up of the nodes and links associated with transmittance (i.e., a, b, etc). This closed flow-graph has the following basic properties:

- Only one start node
- Only one end node
- At least one path from the start to the end nodes
- Topological equation of closed flow-graph is equal to zero [4.2]

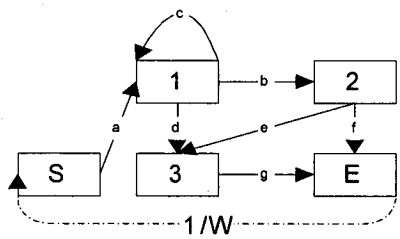

Figure C. 1. Flow-graph

In Figure C.1, the flow-graph has 5 nodes and 7 links. If one considers a dummy link from end node 'E' to start node 'S' with transmittance 1/W, where W is the equivalent of the flow-graph, the flow-graph becomes a closed flow-graph. The flow-graph has the following four loops:

- Transmittance of the loop I (L_1) = $\frac{a.b.f}{w}$
- Transmittance of the loop II (L_2) = $\frac{a.b.e.g}{w}$
- Transmittance of the loop III (L_3) = $\frac{a.d.g}{w}$
- Transmittance of the loop IV (L_4) = c

To find the equivalent of the flow-graph, the topological equation must be equal to zero. Therefore,

$$TP = 1 - \sum_{i=1}^{\ell} L_i + \sum_{i}^{\ell}\sum_{j}^{\ell} L_i.L_j - \sum_{i}^{\ell}\sum_{j}^{\ell}\sum_{k}^{\ell} L_i.L_j.L_k + ... \quad \text{(C.1)}$$

where the first order loop (i.e., L_i) is composed of only one loop and the second order loop (i.e., $L_i.L_j$) is composed of the product of two disjointed loop 'i' and 'j' . These disjointed loop 'i' and 'j' have no intersection in their nodes and links. Similarly, the 'n'th order loop can be defined as a product of 'n' disjoint loops. For example, the transmittance of the first order loops in Figure 2 is given by

$$\sum_{i=1}^{4} L_i = \frac{a.b.f}{w} + \frac{a.b.e.g}{w} + \frac{a.d.g}{w} + c \quad \text{(C.2)}$$

and there is no second order loop, since the loops are not disjointed.

Thus, by substituting the transmittance of the first order loop into the TP Equation (C.1), and setting the remaining terms equal to zero, we get

$$TP = 1 - \left(\frac{a.b.f}{w} + \frac{a.b.e.g}{w} + \frac{a.d.g}{w} + c \right) \quad \text{(C.3)}$$

By equating TP equation equal to zero, we get

$$W = \frac{a.b.f + a.b.e.g + a.d.g}{1 - c} \quad \text{(C.4)}$$

Therefore, the equivalent of the flow-graph in Figure C.1 is a single node flow-graph with the transmittance of Equation (C.4).

In a similar manner, the equivalent flow-graph of the self-loop unit 'i' (e.g., W_i) shown in Figure 1 can be expressed by

$$W = \frac{ab}{1 - ac} \quad \text{(C.5)}$$

where a=(P_{gi},$f_{gi}(t)$), b=(P_{ui}, $f_{ui}(t)$), and c=(P_{ri}, $f_{ri}(t)$).

As such, both probability and distribution functions constitute the transmittance of the self-loop unit that requires a transformer to combine them.

C.2. TRANSMITTANCE

Each link in the FTA flow-graph is associated with two parameters (P_{ij},f_{ij}) denoting the probability and the time distribution of the link, respectively. The probability is

multiplicative while the time distribution is additive in nature. The FTA flow-graph is a multiplicative system; so it would appear that the probability parts could be handled by normal FTA approach. However, the trouble could be anticipated with the time distribution element in normal FTA approach. Several transformers exist that will turn an additive operation into a multiplicative operation. The one chosen for FTA flow-graph is MGF because FTA events are independent and there is well-known theorem which states the MGF of sum of independent events is equal to the product of the MGF of the individual events. Moreover, MGF has basic properties that are used to turn additive operation into a multiplicative operation. Three basic properties of MGF are as follows:

by substituting S=0 in MGF formula (e.g., $M_{f_{ij}(t)}(s) = \int_t e^{s.t} f_{ij}(t)\,dt$), we get

$$M_{fij(t)}(s)\,|_{s=0} = 1 \tag{C.6}$$

and substituting S=0 in the first derivative of MGF yields expected value for a random variable 't'

$$E(t) = \frac{\partial M_{fij(t)}(s)}{\partial s}\,|_{s=0} \tag{C.7}$$

Similarly, by substituting S=0 in the second derivative of MGF, we get expected value for second order or a random variable 't^2'

$$E(t^2) = \frac{\partial^2 M_{fij(t)}(s)}{\partial s^s}\,|_{s=0} \tag{C.8}$$

The variance of a random variable 't' is expressed by

$$\sigma^2(t) = E(t^2) - (E(t))^2 \tag{C.9}$$

Thus, substituting expected values of 't' and 't^2' obtained from Equations (C.7) and (C.8) into Equation (C.9), we get variance of time of occurrence.

Consequently, with respect to independent events, and MGF properties, the transmittance of the FTA flow-graph denoted by W_{ij} (s) can be expressed by

$$W_{ij}(s) = P_{ij}.M_{fji(t)}(s) \tag{C.10}$$

Where P_{ij} is probability of occurrence and $M_{fji(t)}(s)$ is MGF of time of occurrence of the independent event.

APPENDIX-D: TRANSMITTANCE OF FTA GATES

SECTION 1: CALCULATION OF FLOW-GRAPH TRANSMITTANCE FOR FTA WITH AND GATE

Consider Figures 4.11, and 4.12, by substituting a=$Pg_{i1}.Mg_{i1}(s)$, b=$Pu_{i1}.Mu_{i1}(s)$, c=$Pr_{i1}.Mr_{i1}(s)$, and S=0 into Equation (C.5) and applying Equation (C.6), we get

$$W_{i1} = \frac{Pg_{i1}.Pu_{i1}.Mg_{i1}(s).Mu_{i1}(s)}{1-Pg_{i1}.\Pr_{i1}.Mg_{i1}(s).Mr_{i1}(s)} \Big|_{s=0} = \frac{Pg_{i1}.Pu_{i1}}{1-Pg_{i1}.\Pr_{i1}} \tag{D.1}$$

Where W_{i1} is denoting the equivalent transmittance for E_{i1}

Similarly, by substituting a=$Pg_{i2}.Mg_{i2}(s)$, b=$Pu_{i2}.Mu_{i2}(s)$, c=$Pr_{i2}.Mr_{i2}(s)$, and S=0 into Equation (C.5) and using Equation (C.6), we get

$$W_{i2} = \frac{Pg_{i2}.Pu_{i2}.Mg_{i2}(s).Mu_{i2}(s)}{1-Pg_{i2}.\Pr_{i2}.Mg_{i2}(s).Mr_{i2}(s)} \Big|_{s=0} = \frac{Pg_{i2}.Pu_{i2}}{1-Pg_{i2}.\Pr_{i2}} \tag{D.2}$$

Using Equation (4.3.6) and basic properties of MGF defined by Equation (C.7), the occurrence probability of event E i is

$$P_{E_i} = \prod_j W_{ij} \Big|_{s=0} = \prod_j \frac{Pg_{ij}.Pu_{ij}}{1-Pg_{ij}.\Pr_{ij}} \tag{D.3}$$

By substituting MGF of the event Ei1 obtained from Equation (C6) and Equation (C.7) into Equation (4.3.6), the mean or expected value of time that the top event occurs (i.e., Ei1) is expressed by

$$\mu_{E\,i} = E_i(t) = \frac{1}{P_{E_i}}.\frac{\partial Wi}{\partial s} \Big|_{s=0} = \frac{1}{P_{E_i}}.\sum_{j=1}\frac{\partial W_{ij}}{\partial s}.\prod_{\substack{\ell=1 \\ \ell\neq j}} W_{i\ell} \Big|_{s=0} \tag{D.4}$$

Thus, by substituting the value of P_{Ei} from Equation (D.3) into Equation (D.4), we get

$$\mu_{Ei} = \sum_j \left(\frac{1 - Pg_{ij}.\Pr_{ij}}{Pg_{ij}.Pu_{ij}}\right) \frac{\partial W_{ij}}{\partial s} \Big|_{s=0} \tag{D.5}$$

Therefore, by simplifying the Equation (D.5), we have

$$\mu_{E_i} = \sum_j \frac{(\mu_{g_{ij}} + \mu_{u_{ij}})(1 - Pg_{ij}.\Pr_{ij}) + (\mu_{g_{ij}} + \mu_{r_{ij}}).Pg_{ij}.\Pr_{ij}}{(1 - Pg_{ij}.\Pr_{ij})} \tag{D.6}$$

with respect to Equation (C.9), we need to calculate $E(t^2)$ in order to have variance of the occurrence time of the top event. Thus, by using Equation (C.8), we have

$$E_i(t^2) = \frac{1}{P_{E_i}} \left(\sum_{j=1} \left(\frac{\partial^2 W_{ij}}{\partial s^2} . \prod_{\substack{\ell=1 \\ \ell \neq j}} W_{i\ell}\right) + \frac{\partial W_{ij}}{\partial s} \left(\sum_{\substack{\ell=1 \\ \ell \neq j}} \frac{\partial W_{i\ell}}{\partial s} \prod_{\substack{m=1 \\ m \neq \ell}} W_{im}\right)\right) \tag{D.7}$$

SECTION 2: CALCULATION OF FLOW-GRAPH TRANSMITTANCE FOR FTA WIH XOR GATE

Consider Figures 4.14 and 4.15, by substituting $a=Pg_{i1}.Mg_{i1}(s)$, $b=Pu_{i1}.Mu_{i1}(s)$, $c=Pr_{i1}.Mr_{i1}(s)$, and S=0 in Equation (C.5) and using Equation (C.6), we get

$$W_{i1} = \frac{Pg_{i1}.Pu_{i1}.Mg_{i1}(s).Mu_{i1}(s)}{1 - Pg_{i1}.\Pr_{i1}.Mg_{i1}(s).Mr_{i1}(s)} \Big|_{s=0} = \frac{Pg_{i1}.Pu_{i1}}{1 - Pg_{i1}.\Pr_{i1}} \tag{D.8}$$

Where W_{i1} is denoting the equivalent transmittance for E_{i1}

Similarly, by substituting $a=Pg_{i2}.Mg_{i2}(s)$, $b=Pu_{i2}.Mu_{i2}(s)$, $c=Pr_{i2}.Mr_{i2}(s)$, and S=0 into Equation (C.5) and applying Equation (C.6), we get

$$W_{i2} = \frac{Pg_{i2}.Pu_{i2}.Mg_{i2}(s).Mu_{i2}(s)}{1 - Pg_{i2}.\Pr_{i2}.Mg_{i2}(s).Mr_{i2}(s)} \Big|_{s=0} = \frac{Pg_{i2}.Pu_{i2}}{1 - Pg_{i2}.\Pr_{i2}} \tag{D.9}$$

Using Equation (4.3.5.) and basic properties of MGF defined by Equation (C.6), the occurrence probability of event E i is

$$P_{E_i} = \sum_j W_{ij}\big|_{s=0} = \sum_j \frac{Pg_{ij}.Pu_{ij}}{1 - Pg_{ij}.\mathrm{Pr}_{ij}} \tag{D.10}$$

By substituting MGF of the event E i1 obtained from Equation (C.10) and Equation (C.7) into Equation (4.3.6), the mean or expected value of time of the top event occurrence (i.e., E i1) is expressed by

$$\mu_{Ei} = E_i(t) = \frac{1}{P_{E_i}}.\frac{\partial Wi}{\partial s}\ \Big|_{s=0} = \frac{1}{P_{E_i}}.\sum_{j=1}\frac{\partial W_{ij}}{\partial s}.\ \Big|_{s=0} \tag{D.11}$$

Thus, by substituting the value of P_{Ei} from Equation (D.10) into Equation (D.11), we get

$$\mu_{Ei} = \frac{\sum_j \frac{\partial W_{ij}}{\partial s}\ \Big|_{s=0}}{\sum_j \frac{Pg_{ij}.Pu_{ij}}{1 - Pgj.\mathrm{Pr}_{ij}}} \tag{D.12}$$

Therefore, by simplifying Equation (D.12), we have

$$\mu_{E_i} = \frac{\sum_j \frac{(\mu_{g_{ij}} + \mu_{u_{ij}})(1 - Pg_{ij}.\mathrm{Pr}_{ij})Pg_{ij}.Pu_{ij} + (\mu_{g_{ij}} + \mu_{r_{ij}}).Pg_{ij}{}^2.Pu_{ij}.\mathrm{Pr}_{ij}}{(1 - Pg_{ij}.\mathrm{Pr}_{ij})^2}}{\sum_j \frac{Pg_{ij}.Pu_{ij}}{1 - Pg_{ij}.\mathrm{Pr}_{ij}}} \tag{D.13}$$

With respect to Equation (C.9), we need to calculate $E(t^2)$ in order to have variance of 't' a random variable of the occurrence time of the top event. Thus, by using Equation (C.8), we have

$$E_i(t^2) = \frac{1}{P_{E_i}}.\frac{\partial^2 Wi}{\partial s^2}\ \Big|_{s=0} = \frac{1}{P_{E_i}}.\sum_{j=1}\frac{\partial^2 W_{ij}}{\partial s^2}.\ \Big|_{s=0} \tag{D.14}$$

SECTION 3: TABLE OF INPUT DATA

Table D. 1. Event of Data path

Event Code	Description and Recovery Mechanisms	Failure in 109(Hr)
E1	Ingress path down	-
E11	I/O link corruption	-
E111	I/O Link fails,Polling – Redundancy	7500
E112	Transceiver fails,Polling – Redundancy	20000
E12	PacketProcess(P.P) down	-
E121	P.P link fails,Polling- Redundancy	6700
E122	P.P internally fails,Polling, checksum – Redundancy	45000
E123	P.P Memory fails, Checksum- Redundancy	25000
E124	P.P Memory link fails	2000
E13	CellProcess(P.Eng) down	-
E131	P.Eng Link fails, Bounced test, Redundancy	2300
E132	P.Eng process fails, Bounced test, Redundancy	32000
P133	P.Eng memory link fails	23000
E14	S/W Core link down	-
E141	S/W Link fails, Bounced test, Redundancy	3200
E142	Cell Header, corruption, Bounced test, Redundancy	15000

Table D. 2. Event occurrence probability Distribution parameters

Event Code	Pg	μ (min)	σ(min)
E111	0.25	15.2103	2.2815
E112	0.30	5.7039	0.8556
E121	0.30	17.0264	2.5540
E122	0.40	2.5350	0.3803
E123	0.35	4.5631	0.6845
E124	0.20	57.0386	8.5558
E131	0.15	49.5987	7.4398
E132	0.25	3.5649	0.5347
P133	0.20	4.9599	0.7440
E141	0.20	35.6491	5.3474
E142	0.40	7.6051	1.1408

Table D. 3. Event detection (self-loop) probability Distribution parameters

Event Code	Pu	μ (min)	σ(min)
E111	0.25	1.90129E-09	6.6545E-10
E112	0.25	1.90129E-09	6.6545E-10
E121	0.35	1.90129E-08	6.6545E-09
E122	0.20	1.90129E-08	6.6545E-09
E123	0.20	1.90129E-08	6.6545E-09
E124	0.30	1.90129E-08	6.6545E-09
E131	0.15	1.90129E-08	6.6545E-09
E132	0.15	1.90129E-08	6.6545E-09
P133	0.30	1.90129E-08	6.6545E-09
E141	0.25	1.90129E-09	6.6545E-10
E142	0.25	1.90129E-09	6.6545E-10

Table D. 4. Event un-detection probability Distribution parameters

Event Code	Pr	μ (min)	σ(min)
E111	0.75	3.80257E-06	3.80257E-07
E112	0.75	3.80257E-06	3.80257E-07
E121	0.65	3.80257E-06	3.80257E-07
E122	0.80	3.80257E-06	3.80257E-07
E123	0.80	3.80257E-06	3.80257E-07
E124	0.70	3.80257E-06	3.80257E-07
E131	0.85	3.80257E-06	3.80257E-07
E132	0.85	3.80257E-06	3.80257E-07
P133	0.70	3.80257E-06	3.80257E-07
E141	0.75	3.80257E-06	3.80257E-07
E142	0.75	3.80257E-06	3.80257E-07

Table D. 5. Mean time and standard deviation of Ingress/Egress path events

EVENT CODE	μ (Hr)	σ(Hr)
E1	243.57	71.38
E11	26.08	16.23
E111	18.72	14.90
E112	7.36	6.45
E12	97.54	49.12
E121	21.15	17.18
E122	3.73	3.99
E123	6.34	6.27
E124	66.32	45.41
E13	67.14	37.54
E131	56.85	37.13
E132	4.53	3.85
P133	5.77	3.95
E14	52.80	31.77
E141	41.94	29.74
E142	10.86	11.19

APPENDIX-E: MACROS AND DETAILED CALCULATION OF THE EXAMPLES

E.1. FUZZY MARKOV MODEL EXAMPLE

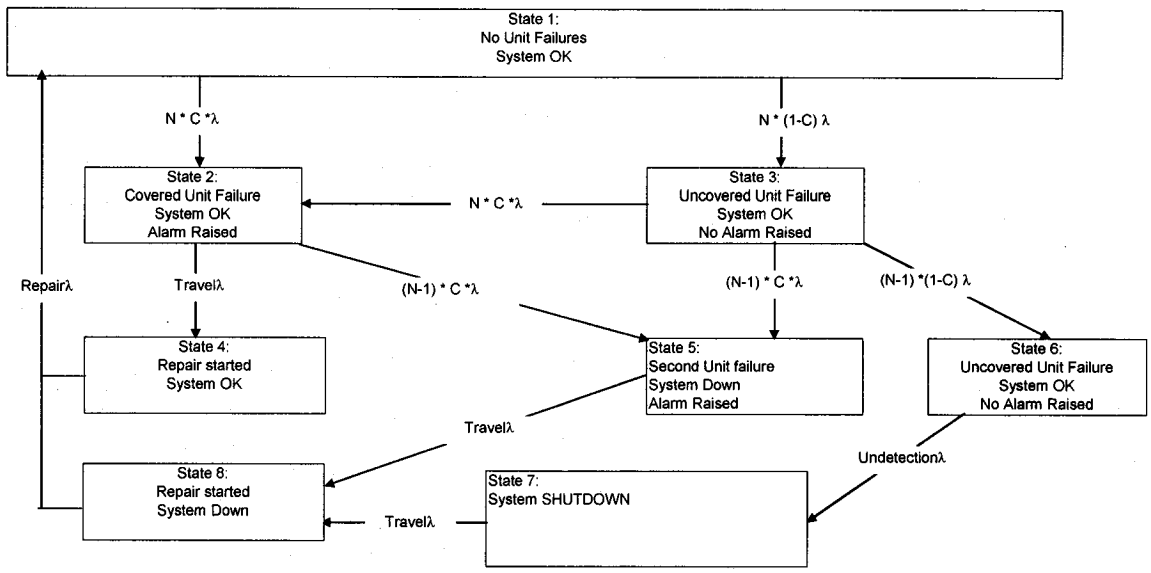

Figure E. 1. General fuzzy Markov model for a system with 'N' units and 2 failures

Table E. 1. Transition Data and Description for fuzzy Markov model of a system with 2 units and 2 failures

Transitions

From, To	Rate (FITs)	Equation	Description
1,2	1136.943889	N * C *λ	N covered unit Failure rates
1,3	83.38944444	N * (1-C) *λ	N Uncovered unit failure rates
3,2	1136.943889	N * C *λ	N covered unit Failure rates
2,4	521739130.4	travelλ	Travel rate
4,1	1090909091	repairλ	Repair rate
2,5	568.4719444	(N-1) * C *λ	N-1 covered unit Failure rates
3,5	568.4719444	(N-1) * C*λ	N-1 covered unit Failure rates
3,6	41.69472222	(N-1) * (1-C) *λ	N-1 Uncovered unit failure rates
5,8	521739130.4	travelλ	Travel rate
6,7	132187706.5	Undect	N-1 Uncovered unit failure detection time
7,8	521739130.4	travelλ	Travel rate
8,1	1090909091	repairλ	Repair rate

Transition Matrix

	From 1	From 3	From 2	From 4	From 5	From 6	From 7	From 8
To 1	0	0	0	1.09E+09	0	0	0	1.09E+09
To 3	83.38944	0	0	0	0	0	0	0
To 2	1136.944	1136.944	0	0	0	0	0	0
To 4	0	0	521739130	0	0	0	0	0
To 5	0	568.4719	568.47194	0	0	0	0	0
To 6	0	41.69472	0	0	0	0	0	0
To 7	0	0	0	0	0	132187707	0	0
To 8	0	0	0	0	521739130	0	521739130	0

| T |

Figure E. 2. Transition Matrix for Markov model of a system with 2 units and 2 failures

Calc Matrix

1	1	1	1	1	1	1	1	P1		1
83.3894	-1747.11	0	0	0	0	0	0	P3		0
1136.94	1136.944	-5.22E+08	0	0	0	0	0	P2		0
0	0	521739130	-1.1E+09	0	0	0	0	P4	=	0
0	568.4719	568.47194	0	-5.22E+08	0	0	0	P5		0
0	41.69472	0	0	0	-1.32E+08	0	0	P6		0
0	0	0	0	0	132187707	-5.22E+08	0	P7		0
0	0	0	0	521739130	0	521739130	-1.1E+09	P8		0

| A | | B | = | C |

Inverse Calc. Matrix (i.e. Inverse of A)

0.95444	0.000546	2.704E-09	8.75E-10	2.704E-09	9.925E-09	2.704E-09	8.75E-10
0.04556	-0.00055	1.291E-10	4.18E-11	1.291E-10	4.737E-10	1.291E-10	4.18E-11
2.2E-06	2.43E-15	-1.92E-09	2E-15	6.174E-15	2.266E-14	6.174E-15	2E-15
1E-06	1.16E-15	-9.17E-10	-9.2E-10	2.953E-15	1.084E-14	2.953E-15	9.55E-16
5E-08	-6E-10	-1.95E-15	4.55E-17	-1.92E-09	5.162E-16	1.406E-16	4.55E-17
1.4E-08	-1.7E-10	4.071E-17	1.32E-17	4.071E-17	-7.56E-09	4.071E-17	1.32E-17
3.6E-09	-4.4E-11	1.031E-17	3.34E-18	1.031E-17	-1.92E-09	-1.92E-09	3.34E-18
2.5E-08	-3.1E-10	-9.27E-16	2.34E-17	-9.17E-10	-9.17E-10	-9.17E-10	-9.2E-10

Figure E. 3. Matrix equation of Markov model of a system with 2 units and 2 failures

Actual Transition Rate Matrix (per year)

	From 1	From 3	From 2	From 4	From 5	From 6	From 7	From 8
To 1	0	0	0	0.009966	0	0	0	0.000244
To 3	0.0007	0	0	0	0	0	0	0
To 2	0.00951	0.000454	0	0	0	0	0	0
To 4	0	0	0.0099664	0	0	0	0	0
To 5	0	0.000227	1.086E-08	0	0	0	0	0
To 6	0	1.67E-05	0	0	0	0	0	0
To 7	0	0	0	0	0	1.665E-05	0	0
To 8	0	0	0	0	0.000227	0	1.665E-05	0

Figure E. 4. Transition Matrix of Markov model of a system with 2 units and 2 failures

Table E. 2. Results of Markov model of a system with 2 units and 2 failures

Results...			
State	Probability of Being in State	Time in State (mpy)	Rate of Entry□(per year)
1	0.954441281	501997.9361	0.0102
3	0.045555405	23960.3206	0.0007
2	2.17913E-06	1.1461	0.0100
4	1.04219E-06	0.5482	0.0100
5	4.96382E-08	0.0261	0.0002
6	1.43691E-08	0.0076	0.0000
7	3.64055E-09	0.0019	0.0000
8	2.54812E-08	0.0134	0.0002

Table E. 3. Fuzzy input data, System Availability, and States status

Number of Units	2	
Fits	610.1666667	Based on FMECA(L=508,M=628,M=641)
Repair time	0.916666667	hours(L=0.5, M=1, U=1)
Travel Time	1.916666667	hours(L=0.5, M=2, U=3)
Fuzzy Fault Coverage	93.17%	Based on FMECA & Failure severrity level
Uncovered Fault Detection Time	7.565	hours
Detection and Switch time	0.6	seconds
Down time per year	0.0490	minutes per year
detection and switch time (Cut off Call rate)	27.80	
Availability	99.9999906871%	

STATE	STATUS	TIME IN STATE		Cutoff Call Status
1	OK	501997.9361	Minutes per year	
2	OK	1.1461	Minutes per year	
3	OK	23960.3206	Minutes per year	
4	OK	0.5482	Minutes per year	
5	DOWN	0.0261	Minutes per year	Hit
6	DOWN	0.0076	Minutes per year	Hit
7	DOWN	0.0019	Minutes per year	
8	DOWN	0.0134	Minutes per year	

E.2. FUZZY FTA OF SUSPENSION TESTER

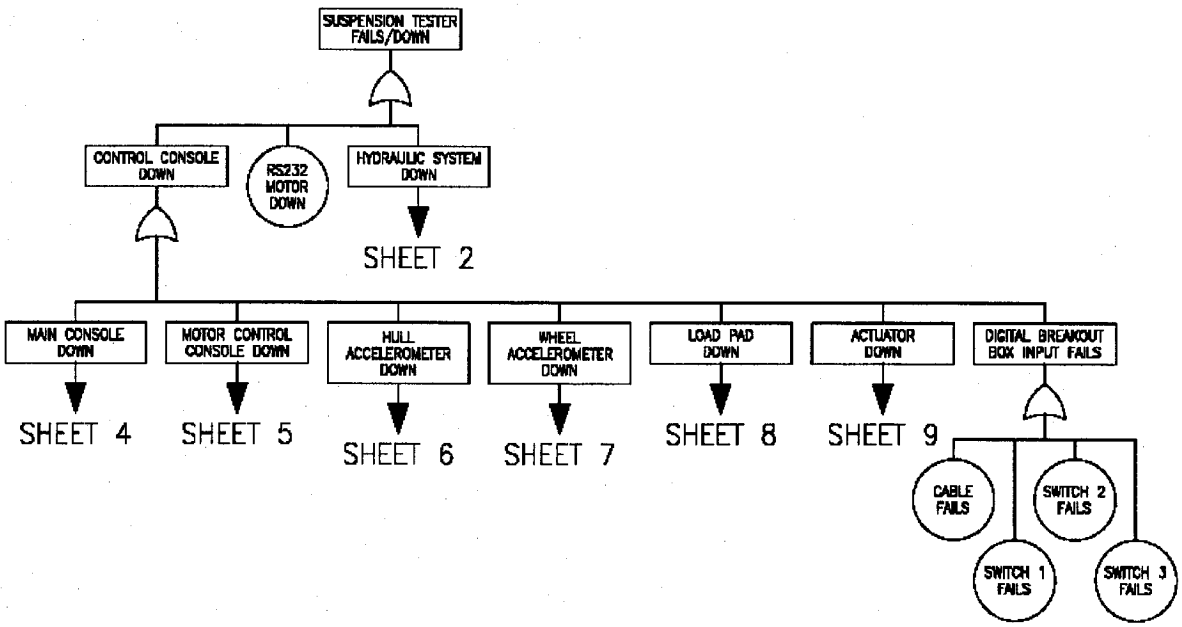

Figure E. 5. Fuzzy FTA of Suspension tester – Main page(Sheet 1)

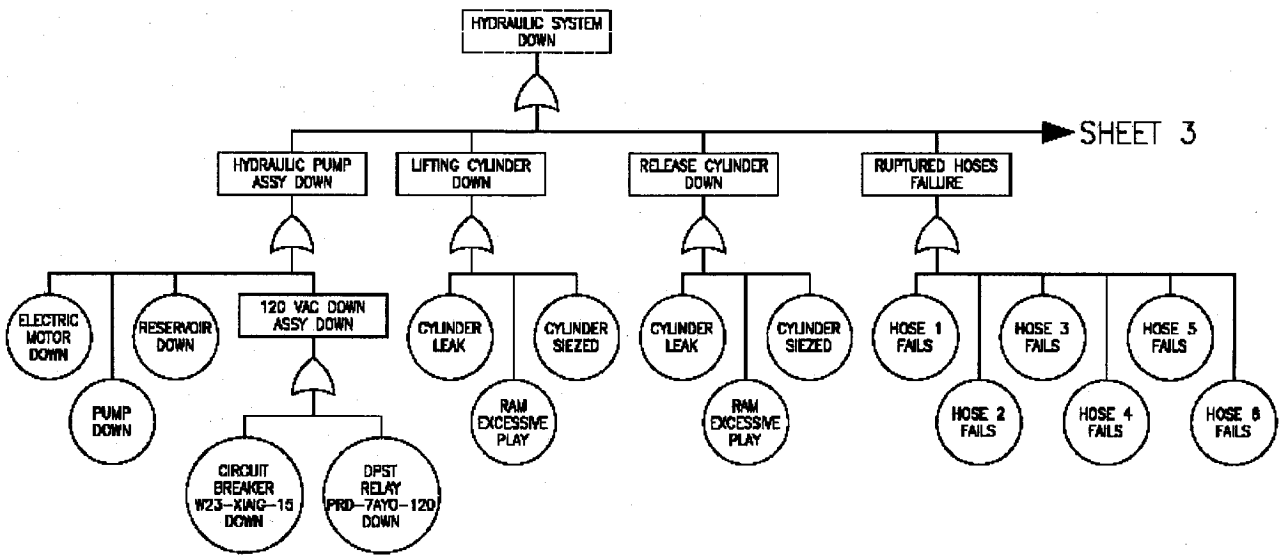

Figure E. 6. FTA of Hydraulic system down- Sheet 2

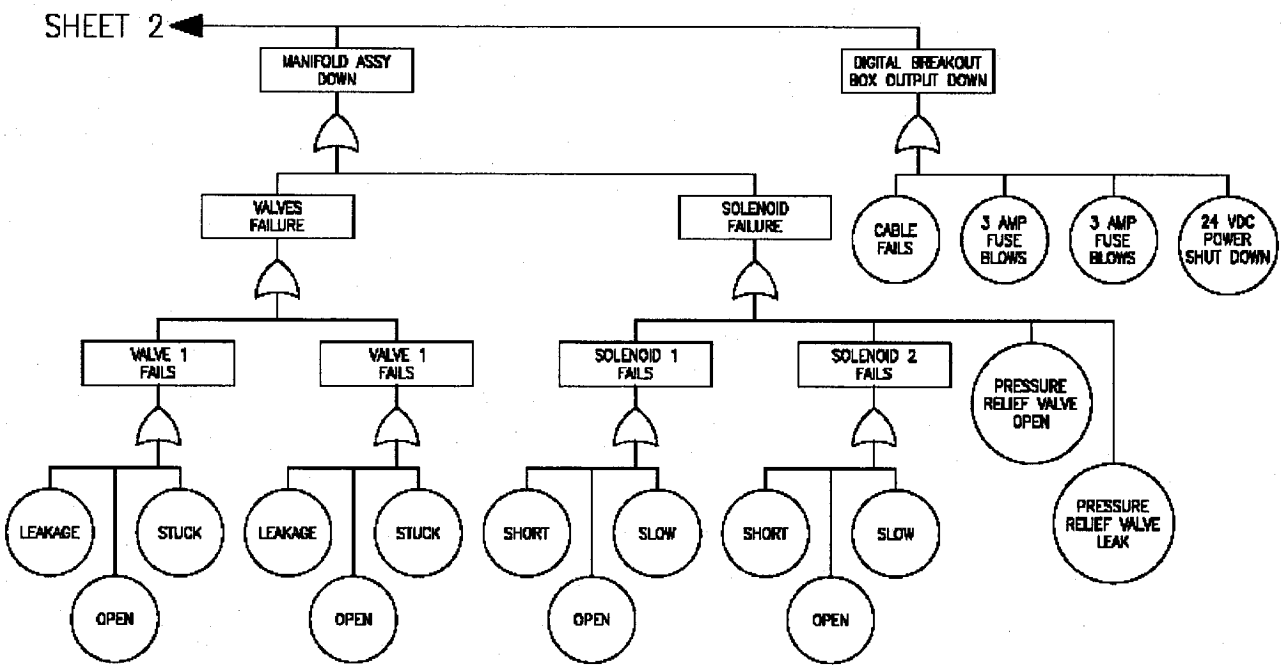

Figure E. 7. FTA of Hydraulic system down- Sheet 3

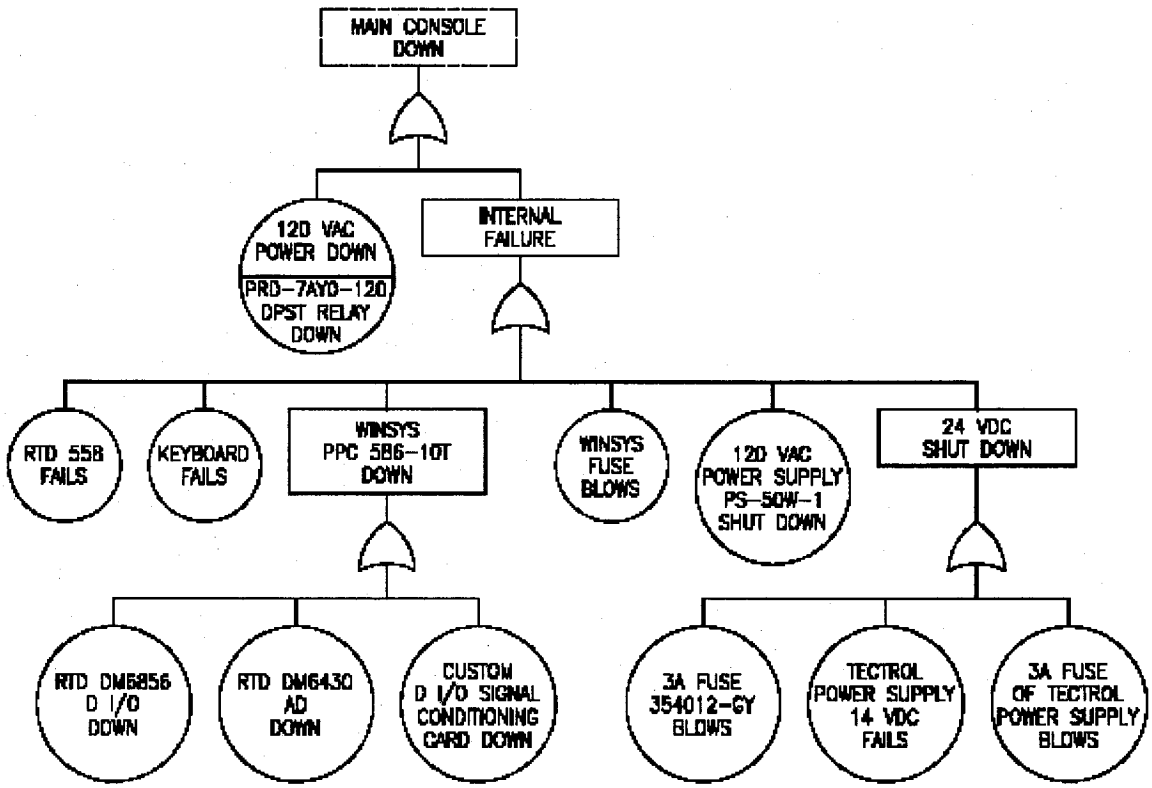

Figure E. 8. FTA of Main console down- Sheet 4

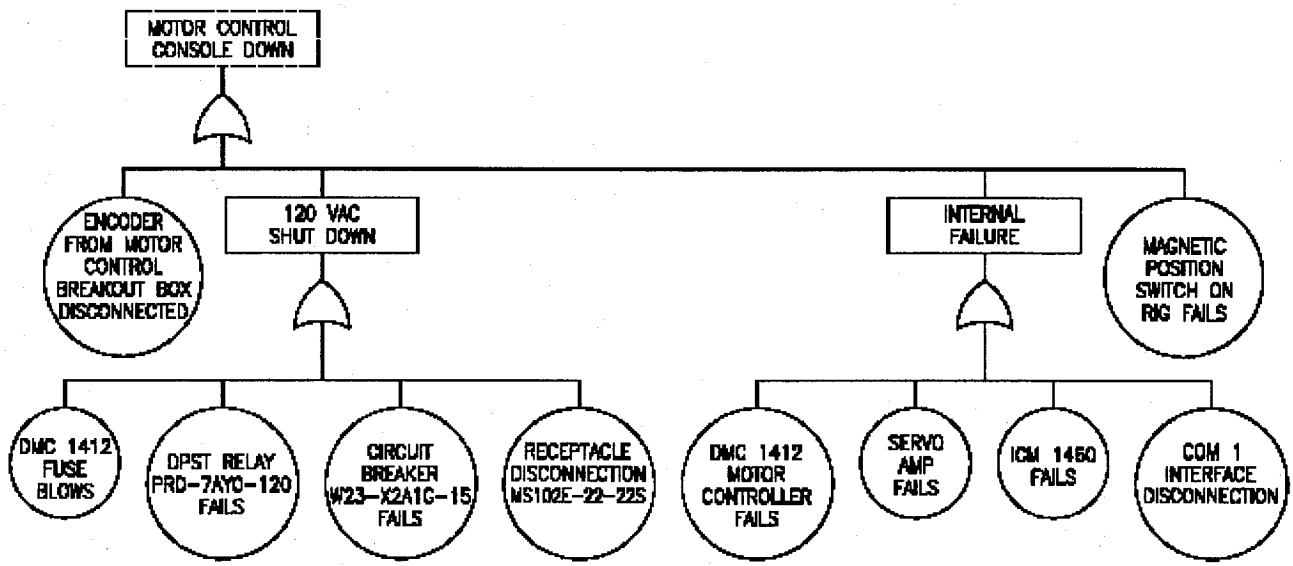

Figure E. 9. FTA of Motor control console down- Sheet 5

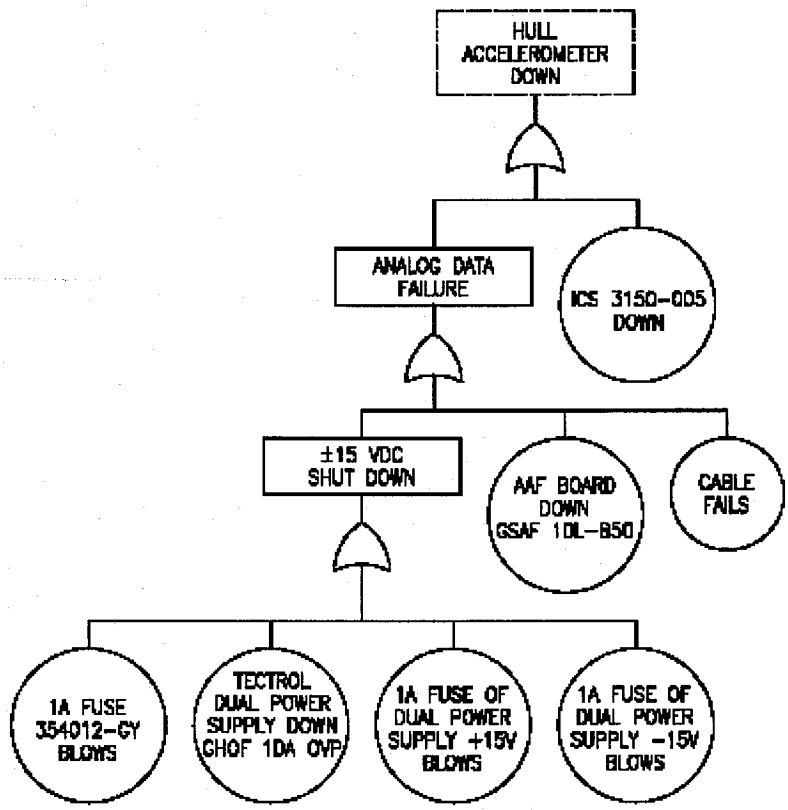

Figure E. 10. FTA of Hull accelerometer down- Sheet 6

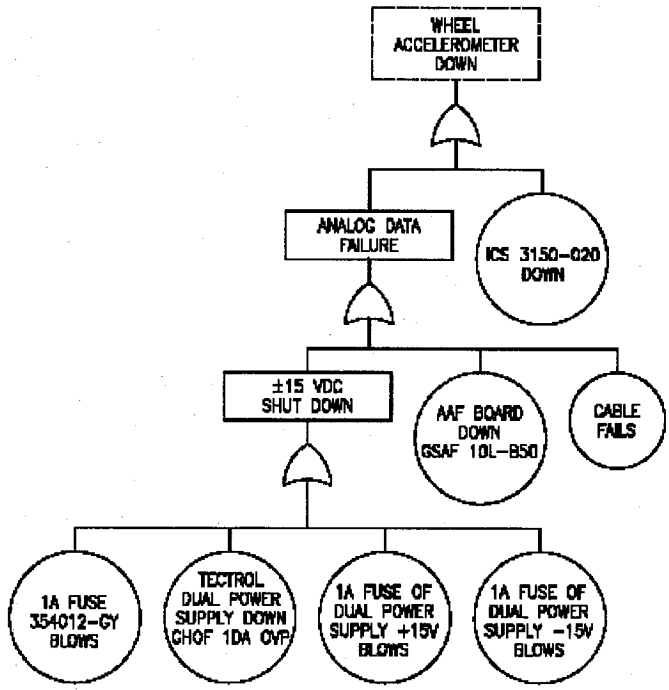

Figure E. 11. FTA of Hydraulic system down- Sheet 7

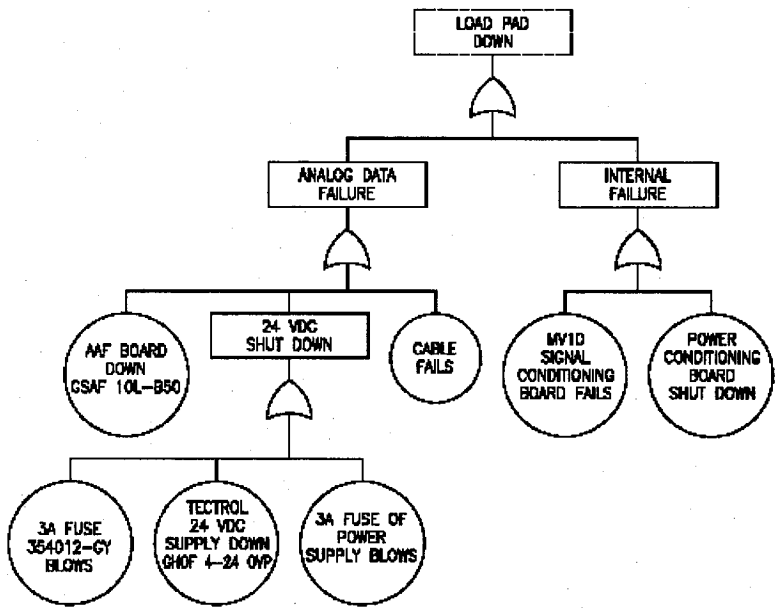

Figure E. 12. FTA of Hydraulic system down- Sheet 8

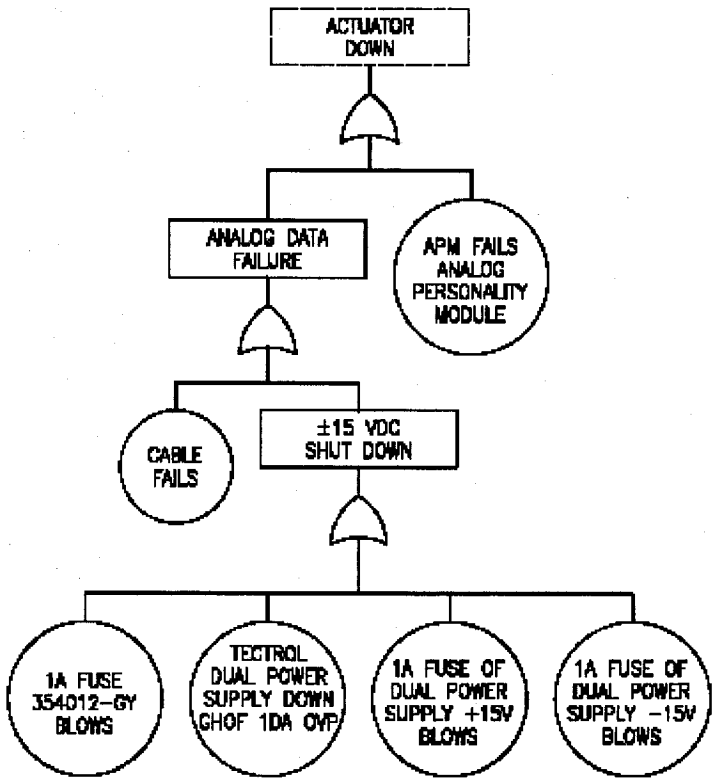

Figure E. 13. FTA of Hydraulic system down- Sheet 9

Table E. 4 . Marco results for Suspension tester down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Suspension Tester Down												
AND/OR		0	0	0.39	0.6	0.89	0.6	0.41	0.18	0.18	0.18	0.2
AND		0	0	0.32	0.53	0.85	0.53	0.37	0.07	0.07	0.07	0.08
OR		0	0	0.43	0.63	0.95	0.63	0.46	0.22	0.22	0.22	0.24
Gama	0.1	0.5	0.6	0.45								
Control Cosole down	AND/OR	0	0	0.33	0.49	0.76	0.49	0.49	0.33	0.34	0.33	0.37
Hydraulic System Down	AND/OR	0	0	0.27	0.5	0.99	0.5	0.3	0.04	0.04	0.06	0.06
RS232(Com1) interface disconnected	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0

Table E. 5 . Marco results for Control console down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Control Console Down												
AND/OR		0	0	0.33	0.49	0.76	0.49	0.49	0.33	0.34	0.33	0.37
AND		0	0	0.38	0.58	1.05	0.76	0.73	0.5	0.56	0.43	0.48
OR		0	0	0.55	0.79	1.29	0.97	0.95	0.61	0.61	0.6	0.67
Gama	0.1	0.5	0.6	0.45								
Main Console Down	AND/OR	0	0	0.5	0.7	1	0.7	0.5	0.22	0.31	0.4	0.45
Motor Control Console Down	AND/OR	0	0	0.12	0.23	0.47	0.23	0.22	0.23	0.33	0.42	0.47
Hull Accelerometer Down	AND/OR	0	0	0.21	0.29	0.64	0.62	0.72	0.44	0.36	0.13	0.14
Wheel Accelerpmeter Down	AND/OR	0	0	0.21	0.29	0.64	0.62	0.72	0.44	0.36	0.13	0.14
Load Pad Down	AND/OR	0	0	0.27	0.41	0.65	0.41	0.27	0.2	0.27	0.35	0.39
Accuator Down	AND/OR	0	0	0.21	0.29	0.64	0.62	0.72	0.44	0.36	0.13	0.14
Digital Brreakout input fails	AND/OR	0	0	0.28	0.39	0.55	0.39	0.28	0.28	0.39	0.5	0.55

Table E. 6 . Marco results for Digital breakout box output down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Digital BreakOut Box output Down												
AND/OR		0	0	0.28	0.39	0.55	0.39	0.28	0.28	0.39	0.5	0.55
AND		0	0	0.18	0.26	0.37	0.26	0.18	0.18	0.26	0.33	0.37
OR		0	0	0.41	0.57	0.82	0.57	0.41	0.41	0.57	0.74	0.82
Gama	0.1	0.5	0.6	0.45								
Cable fails	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
SW 1 fails	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
SW 2 fails	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
SW 3 fails	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0

Table E. 7 . Marco results for Load pad down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Load Pad Down												
AND/OR		0	0	0.27	0.41	0.65	0.41	0.27	0.2	0.27	0.35	0.39
AND		0	0	0.15	0.25	0.44	0.25	0.15	0.14	0.19	0.25	0.28
OR		0	0	0.27	0.4	0.59	0.4	0.27	0.18	0.25	0.32	0.35
Gama	0.1	0.5	0.6	0.45								
Data Analog Down	AND/OR	0	0	0.4	0.56	0.8	0.56	0.4	0.15	0.21	0.27	0.3
Load Pad internally fails	AND/OR	0	0	0.12	0.23	0.47	0.23	0.12	0.23	0.33	0.42	0.47

Table E. 8 . Marco results for Load pad internally fails FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Load Pad internally fails												
AND/OR		0	0	0.12	0.23	0.47	0.23	0.12	0.23	0.33	0.42	0.47
AND		0	0	0.06	0.11	0.23	0.11	0.06	0.12	0.16	0.21	0.23
OR		0	0	0.19	0.38	0.77	0.38	0.19	0.38	0.54	0.69	0.77
Gama	0.3	0.5	0.9	0.53								
MV10 / Signal conditioning board fails	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
Power conditioning board shutdown	VERYLOW	0	0	0.25	0.49	1	0.49	0.25	0	0	0	0

Table E. 9 . Marco results for Analog data failure of load pad FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Analog data failure of Load Pad												
AND/OR		0	0	0.4	0.56	0.8	0.56	0.4	0.15	0.21	0.27	0.3
AND		0	0	0.36	0.5	0.72	0.5	0.36	0.05	0.07	0.09	0.1
OR		0	0	0.46	0.64	0.92	0.64	0.46	0.17	0.24	0.31	0.35
Gama	0.1	0.5	0.6	0.45								
AFF Board Down (GSAF 10L B50)	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
Cable fails	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
24 VDC shutdown	AND/OR	0	0	0.28	0.39	0.55	0.39	0.28	0.28	0.39	0.5	0.55

Table E. 10 . Marco results for Actuator down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Actuator Down												
AND/OR		0	0	0.21	0.29	0.64	0.62	0.72	0.44	0.36	0.13	0.14
AND		0	0	0.07	0.1	0.46	0.47	0.42	0.21	0.21	0.04	0.05
OR		0	0	0.24	0.34	0.57	0.54	0.7	0.47	0.35	0.15	0.16
Gama	0.1	0.5	0.6	0.45								
APM Fails	MEDIUM	0	0	0	0	0.5	0.7	1	0.7	0.5	0	0
Data Analog Down	AND/OR	0	0	0.38	0.53	0.76	0.53	0.38	0.13	0.18	0.23	0.26

Table E. 11 . Marco results for Wheel accelerometer down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Wheel Accelerometer Down												
AND/OR		0	0	0.21	0.29	0.64	0.62	0.72	0.44	0.36	0.13	0.14
AND		0	0	0.07	0.1	0.46	0.47	0.42	0.21	0.21	0.04	0.05
OR		0	0	0.24	0.34	0.57	0.54	0.7	0.47	0.35	0.15	0.16
Gama	0.1	0.5	0.6	0.45								
ICS 3150-020 Down	MEDIUM	0	0	0	0	0.5	0.7	1	0.7	0.5	0	0
Data Analog Down	AND/OR	0	0	0.38	0.53	0.76	0.53	0.38	0.13	0.18	0.23	0.26

Table E. 12 . Marco results for Hull accelerometer down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Hull Accelerometer Down												
AND/OR		0	0	0.21	0.29	0.64	0.62	0.72	0.44	0.36	0.13	0.14
AND		0	0	0.07	0.1	0.46	0.47	0.42	0.21	0.21	0.04	0.05
OR		0	0	0.24	0.34	0.57	0.54	0.7	0.47	0.35	0.15	0.16
Gama	0.1	0.5	0.6	0.45								
ICS 3150-005 Down	MEDIUM	0	0	0	0	0.5	0.7	1	0.7	0.5	0	0
Data Analog Down	AND/OR	0	0	0.38	0.53	0.76	0.53	0.38	0.13	0.18	0.23	0.26

Table E. 13 . Marco results for Analog data failures FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Analog data failures(Hull/Wheel/Accuator)												
AND/OR		0	0	0.38	0.53	0.76	0.53	0.38	0.13	0.18	0.23	0.26
AND		0	0	0.33	0.46	0.66	0.46	0.33	0.04	0.06	0.08	0.09
OR		0	0	0.45	0.63	0.9	0.63	0.45	0.15	0.21	0.27	0.3
Gama	0.1	0.5	0.6	0.45								
AFF Board Down (GSAF 10L B50)	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
Cable fails	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
15+/- VDC shutdown	AND/OR	0	0	0.23	0.33	0.47	0.33	0.23	0.23	0.33	0.42	0.47

Table E. 14 . Marco results for +/-15 VDC shutdown FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
+/- 15 VDC Shutdown'												
AND/OR		0	0	0.23	0.33	0.47	0.33	0.23	0.23	0.33	0.42	0.47
AND		0	0	0.29	0.44	0.7	0.44	0.29	0.12	0.16	0.21	0.23
OR		0	0	0.56	0.81	1.23	0.81	0.56	0.38	0.54	0.69	0.77
Gama	0.3	0.5	0.9	0.53								
1A fuse 354012-GY blowes up	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
Tectrol Dual power supply down	VERYLOW	0	0	0.25	0.49	1	0.49	0.25	0	0	0	0
1A fuse of dual power supply -15 blowes up	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
1A fuse of dual power supply +15 blowes up	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0

Table E. 15 . Marco results for Motor control console down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Motorl Control Console down												
AND/OR		0	0	0.12	0.23	0.47	0.23	0.22	0.23	0.33	0.42	0.47
AND		0	0	0.14	0.24	0.45	0.24	0.19	0.25	0.32	0.41	0.45
OR		0	0	0.27	0.51	0.98	0.51	0.44	0.51	0.69	0.89	0.98
Gama	0.3	0.5	0.9	0.53								
Encoder from Motor .... Down	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
Magnetic position switch on rig fails	VERYLOW	0	0	0.25	0.49	1	0.49	0.25	0	0	0	0
Motor Control Internally fails	AND/OR	0	0	0.23	0.33	0.47	0.33	0.47	0.33	0.33	0.42	0.47
120 VAC Shutdown	AND/OR	0	0	0.12	0.23	0.47	0.23	0.12	0.23	0.33	0.42	0.47

Table E. 16 . Marco results for Motor control internally fails FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Motorl Control Internally Fails												
AND/OR		0	0	0.23	0.33	0.47	0.33	0.47	0.33	0.33	0.42	0.47
AND		0	0	0.18	0.28	0.58	0.44	0.41	0.28	0.28	0.21	0.23
OR		0	0	0.44	0.65	1.12	0.81	0.94	0.65	0.65	0.69	0.77
Gama	0.3	0.5	0.9	0.53								
DMC 1412 Motorl Control Fails	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
Servo Amp fails	VERYLOW	0	0	0.25	0.49	1	0.49	0.25	0	0	0	0
ICM 1460 fails	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
COM1 interface disconnection	MEDIUM	0	0	0	0	0.5	0.7	1	0.7	0.5	0	0

Table E. 17 . Marco results for 120 AVC power shutdown FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
120 AVC power Shutdown												
AND/OR		0	0	0.12	0.23	0.47	0.23	0.12	0.23	0.33	0.42	0.47
AND		0	0	0.06	0.11	0.23	0.11	0.06	0.12	0.16	0.21	0.23
OR		0	0	0.19	0.38	0.77	0.38	0.19	0.38	0.54	0.69	0.77
Gama	0.3	0.5	0.9	0.53								
Circuit Breaker W23-X1A1G-15 Down	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
DPST relay down	VERYLOW	0	0	0.25	0.49	1	0.49	0.25	0	0	0	0
DMC 1412 Fuse blowes up	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
receptable disconnection MS102E 22 22S	MEDIUM	0	0	0	0	0.5	0.7	1	0.7	0.5	0	0

Table E. 18 . Marco results for Main console fails FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Main Console Fails												
AND/OR		0	0	0.5	0.7	1	0.7	0.5	0.22	0.31	0.4	0.45
AND		0	0	0.29	0.41	0.58	0.41	0.29	0.07	0.1	0.13	0.15
OR		0	0	0.37	0.52	0.75	0.52	0.37	0.26	0.36	0.47	0.52
Gama	0.1	0.5	0.6	0.45								
120 VCA Power Down / DPST Relay Down	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
24VDC Shutdown	OR	0	0	0.32	0.44	0.63	0.44	0.32	0.41	0.57	0.74	0.82

Table E. 19 . Marco results for Main console internally fails FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Main Console Internally Fails												
AND/OR		0	0	0.28	0.39	0.55	0.39	0.55	0.39	0.39	0.5	0.55
AND		0	0	0.25	0.37	0.68	0.5	0.43	0.35	0.4	0.39	0.43
OR		0	0	0.47	0.69	1.13	0.81	0.88	0.66	0.71	0.8	0.88
Gama	0.1	0.5	0.6	0.45								
RTD SS8 fails	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
Keyboard fails	High	0	0	0	0	0	0	0	0.5	0.7	0.9	1
Winsys Fuse fails	VERYLOW	0	0	0.25	0.49	1	0.49	0.25	0	0	0	0
120 VAC powe supply PS-50W-1 Down	MEDIUM	0	0	0	0	0.5	0.7	1	0.7	0.5	0	0
Winsys PPC-586-10T Down	AND/OR	0	0	0.28	0.39	0.55	0.39	0.28	0.28	0.39	0.5	0.55
24VDC Shutdown	OR	0	0	0.32	0.44	0.63	0.44	0.32	0.41	0.57	0.74	0.82

Table E. 20 . Marco results for Winsys PPC-586-10T down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
WinSys PPC-586-10T Down												
AND/OR		0	0	0.28	0.39	0.55	0.39	0.28	0.28	0.39	0.5	0.55
AND		0	0	0.09	0.13	0.18	0.13	0.09	0.18	0.26	0.33	0.37
OR		0	0	0.32	0.44	0.63	0.44	0.32	0.41	0.57	0.74	0.82
Gama	0.1	0.5	0.6	0.45								
RTD DM6856 D I/O Down	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1
RTD DM6430 (AD) Down	LOW	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
Custom D I/O Signal conditioning card Down	HIGH	0	0	0	0	0	0	0	0.5	0.7	0.9	1

Table E. 21 . Marco results for Hydraulic system down FTA

		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Hydraulic System Down												
AND/OR		0	0	0.27	0.5	0.99	0.5	0.3	0.04	0.04	0.06	0.06
AND		0	0	0.26	0.5	0.99	0.5	0.28	0.03	0.03	0.01	0.01
OR		0	0	0.5	0.7	1	0.7	0.96	0.66	0.63	0.78	0.87
Gama	0.8	0.95	1	0.93								
Digital Breakout Box output down	OR	0	0	0.49	0.69	0.99	0.7	0.98	0.67	0.48	0	0
Ruptured Hoses Failures	OR	0	0	0.5	0.7	1	0.7	0.5	0	0	0	0
Manif. Ass. Down	OR	0	0	0.5	0.7	1	0.7	0.95	0.64	0.45	0	0
Release Cyld Down	OR	0	0	0.48	0.68	0.99	0.7	0.97	0.67	0.48	0	0
Lifting Cyld. Down	OR	0	0	0.48	0.68	0.99	0.7	0.97	0.67	0.48	0	0
Hyd.Pump Ass. Down	OR	0	0	0.25	0.49	1	0.49	0.25	0.46	0.64	0.83	0.92

E.3. STOCHASTIC FTA EXAMPLE

Table E. 22. Calculation of Time-Event- Data for Ingress path down FTA

						IN			Out			SELF-LOOP		
EVENT CODE	DESCRIPTION	Transmittance - Wij	MU (hour)	STD Deviation	P of trans.	P	MU - Min	SIGMA	P of trans.	MU- Min	SIGMA	P of trans.	MU- min	SIGMA
E1	**Ingress path down**	**0.0021EXP(243.566s+2547.3719s^2)**	**243.57**	**71.38**	0.0021		-	-	0.2082%					
E11	**I/O link corruption**	0.1737EXP(26.0802s+131.7789s^2)	26.08	16.23	0.1737				0.17					
E111	I/O Link failsPolling – Redun	0.0625EXP(15.2103s+5.2054s^2))/ 1-0.1875EXP(15.2103s+5.2054S^2)	18.72	14.90	0.0769	0.25	8000000	1E+06	0.25	0.001	0.00035	0.75	2	0.2
E112	Transceiver failsPolling – Re	0.075EXP(5.7039s+0.732s^2))/ 1-0.225EXP(5.7039s+0.732S^2)	7.36	6.45	0.0968	0.3	3000000	450000	0.25	0.001	0.00035	0.75	2	0.2
E12	**PacketProcess(P.P) down**	0.4151EXP(97.5404s+1206.1934s^2)	97.54	49.12	0.4151				0.42					
E121	P.P link failsPolling- Redund	0.105EXP(17.0264s+6.5227s^2))/ 1-0.195EXP(17.0264s+6.5227S^2)	21.15	17.18	0.1304	0.3	8955224	1E+06	0.35	0.01	0.0035	0.65	2	0.2
E122	P.P internally fails Scratch p	0.08EXP(2.535s+0.1446s^2))/ 1-0.32EXP(2.5351s+0.1446S^2)	3.73	3.99	0.1176	0.4	1333333	200000	0.20	0.01	0.0035	0.8	2	0.2
E123	P.P Memory fails Checksum	0.07EXP(4.5631s+0.4685s^2))/ 1-0.28EXP(4.5631s+0.4685S^2)	6.34	6.27	0.0972	0.35	2400000	360000	0.20	0.01	0.0035	0.8	2	0.2
E124	P.P Memory link fails	0.06EXP(57.0386s+73.2014s^2))/ 1-0.14EXP(57.0386s+73.2014S^2)	66.32	45.41	0.0698	0.2	3E+07	5E+06	0.30	0.01	0.0035	0.7	2	0.2
E13	**CellProcess(P.Eng) down**	0.1432EXP(67.1409s+704.6928s^2)	67.14	37.54	0.1432				0.14					
E131	P.Eng Link fails Bounced tes	0.0225EXP(49.5987s+55.3508s^2))/ 1-0.1275EXP(49.5987s+55.3508S^2)	56.85	37.13	0.0258	0.15	2.6E+07	4E+06	0.15	0.01	0.0035	0.85	2	0.2
E132	P.Eng process fails Bounce	0.0375EXP(3.5649s+0.2859s^2))/ 1-0.2125EXP(3.5649s+0.2859S^2)	4.53	3.85	0.0476	0.25	1875000	281250	0.15	0.01	0.0035	0.85	2	0.2
P133	P.Eng memory link fails	0.06EXP(4.9599s+0.5535s^2))/ 1-0.14EXP(4.9599s+0.5535S^2)	5.77	3.95	0.0698	0.2	2608696	391304	0.30	0.01	0.0035	0.7	2	0.2
E14	**S/W Core link down**	0.2017EXP(52.8046s+504.7068s^2)	52.80	31.77	0.2017				0.20					
E141	S/W Link fails Bounced test	0.05EXP(35.6491s+28.5943s^2))/ 1-0.15EXP(35.6491s+28.5943S^2)	41.94	29.74	0.0588	0.2	1.9E+07	3E+06	0.25	0.001	0.00035	0.75	2	0.2
E142	Cell Header corruption Boun	0.1EXP(7.6051s+1.3014s^2))/ 1-0.3EXP(7.6051s+1.3014S^2)	10.86	11.19	0.1429	0.4	4000000	600000	0.25	0.001	0.00035	0.75	2	0.2

This table shows the results of the stochastic Ingress path FTA based on the input data in Appendix-D, Section3.

E.4. RESULT OF K-OUT-OF-N SYSTEM MACRO

Table E. 23. Results of Macro for k-out-of-n system

	Punit	Punit	Punit	Punit	Punit	Punit	Punit
	0.010989	0.02439	0.041096	0.0625	0.090909	0.130435	0.140271
	Pk/n	Pk/n	Pk/n	Pk/n	Pk/n	Pk/n	Pk/n
p	0.056166	0.128047	0.223077	0.354081	0.545051	0.845982	0.927708
W5/5	0.056166	0.128047	0.223077	0.354081	0.545051	0.845982	0.99559
W4/5	0.001221	0.006096	0.017597	0.041581	0.090505	0.193809	0.254633
W3/5	1.33E-05	0.000147	0.000708	0.002519	0.007861	0.023676	0.035027
W2/5	7.31E-08	1.78E-06	1.44E-05	7.72E-05	0.000348	0.001485	0.002483
W1/5	1.6E-10	8.63E-09	1.17E-07	9.54E-07	6.21E-06	3.78E-05	7.15E-05
	Mu k/n v2	Mu k/n v2	Mu k/n v2	Mu k/n v2	Mu k/n v2	Mu k/n v2	Mu k/n v2
p	0.010989	0.02439	0.041096	0.0625	0.090909	0.130435	0.140271
W5/5	10225.3	5248.981	3612.215	2817.162	2367.687	2103.894	2042.93
W4/5	20120.77	10131.88	6813.289	5165.743	4191.17	3559.781	3391.838
W3/5	30071.14	15075.61	10082.53	7591.747	6104.085	5121.218	4853.286
W2/5	40043.58	20043.94	13379.52	10049.56	8054.237	6727.463	6362.945
W1/5	50027.05	25024.55	16690.38	12523.3	10023.05	8356.217	7896.853
	Sigma k/n	Sigma k/n	Sigma k/n	Sigma k/n	Sigma k/n	Sigma k/n	Sigma k/n
p	0.010989	0.02439	0.041096	0.0625	0.090909	0.130435	0.140271
W5/5	59850.57	72914.88	90939.36	117095.3	157674	226677.6	262949.9
W4/5	141424.3	169136.5	206224.3	258047.4	334704.8	457339.2	518863.3
W3/5	247364.2	295641.6	360086.8	449688.8	581073.8	788166.3	890639.9
W2/5	377558	452146.2	551978.1	691057.3	895155.9	1216437	1375016
W1/5	531967.8	638552.7	781705.9	981804.5	1276334	1741034	1970600

This table presents Data-To-Event for k-out-of-5 system including probability of system failure, Mean time to system failure, and standard deviation of time to system failure for k=1,2,3,4,5.

E.5. DETAILED CALCULATION OF REVERSIBEL K-OUT-OF-N SYSTEM

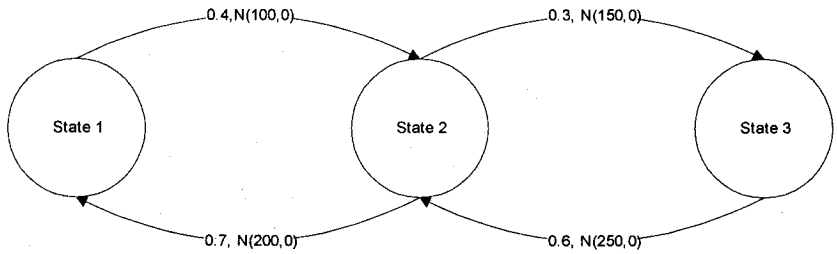

Figure E. 14. Semi-Markov model of a Reversible Multi-state k-out-of-n system

Table E. 24. Calculation of a Semi-Markov model with 3 states in Multi-state k-out-of-n system

$$W_2 = \frac{0.4e^{100s}}{1-0.28e^{300s}} \rightarrow P_2 = W_2\big|_{s=0} = 0.56,$$

$$MTTS_2 = \frac{1}{P_2}.\frac{\partial W_2}{\partial s}\Big|_{s=0} = \frac{1}{0.56}.120.37 = 215,$$

$$STTS_2^2 = \frac{1}{P_2}\left(\frac{\partial W^2{}_2}{\partial s^2}\Big|_{s=0}\right) - [MTTS_2]^2 = \frac{76311.73}{0.56} - (215)^2 = 90045.9 \rightarrow STTS_2 = 300$$

Similarly

$$W_3 = \frac{0.12e^{250s}}{(1-0.28e^{300s}) * (1-0.18e^{400s})} \rightarrow P_3 = 0.2$$

$$MTTS_3 = 255$$

$$STTS_3 = 329$$

$$P1 = 1 - P_2 - P_3 = 0.24$$

Table E. 25. Calculation of Multi-state 3-out-of-5:G system with 3 states

$$W_{j=2}^{3/5} = \sum_{i=3}^{5} W_{j\leq 2}^{i} . W_{j>2}^{5-i}$$

$$P_{j=2}^{3/5} = W_{j=2}^{3/5}\big|_{s=0} = 10*(0.56)^3(0.2)^2 + 5(0.56)^4(0.2) + 5(0.56)^5 = 0.443$$

$$MTTS_{j=2}^{3/5} = \frac{1}{P_{j=2}^{3/5}} . \frac{\partial W_{j=2}^{3/5}}{\partial s}\big|_{s=0} = \frac{1}{P_{j=2}^{3/5}} . \begin{pmatrix} 10.[3.W_{j\leq 2}^{'} . W_{j\leq 2}^{2} . W_{j>2} + 2.W_{j\leq 2}^{3} . W_{j>2}^{'} . W_{j>2}] \\ + 5.[4.W_{j\leq 2}^{'} . W_{j\leq 2}^{3} . W_{j>2} + W_{j\leq 2}^{4} . W_{j>2}^{'}] \\ + 5.W_{j\leq 2}^{'} . W_{j>2} \end{pmatrix}$$

$$MTTS_{j=2}^{3/5} = \frac{1}{0.443} \begin{pmatrix} 10*[3*120.37*0.56^2*0.2^2] + \\ 5*[4*120.37*0.56^3*0.2 + 0.56^4*32.19] + \\ 5*120.37*0.56^4 \end{pmatrix} = 411.24$$

$$\frac{\partial^2 W_{j\leq 2}^{3/5}}{\partial s^2} = 10.[3.W_{j\leq 2}^{''} . W_{j\leq 2}^{2} . W_{j>2}^{2} + 6W_{j\leq 2}^{'} . W_{j\leq 2} . W_{j>2}^{2} + 6.W_{j\leq 2}^{'} W_{j\leq 2}^{2} . W_{j>2} + 2.W_{j\leq 2}^{3} . W_{j>2}^{''} . W_{j>2}^{''}$$
$$+ 2W_{j\leq 2}^{3} . W_{j>2}^{'2}] + 5.[4.W_{j\leq 2}^{''} . W_{j\leq 2}^{3} . W_{j>2} + 12.W_{j\leq 2}^{'2} . W_{j\leq 2}^{2} . W_{j>2} + 4.W_{j\leq 2}^{'} W_{j\leq 2}^{3} . W_{j>2}^{'} + 4.W_{j\leq 2}^{'} . W_{j>2}^{3} . W_{j>2}^{'}$$
$$+ W_{j\leq 2}^{4} . W_{j>2}^{''}] + 5.W_{j\leq 2}^{''} . W_{j>2}^{4} + 20.W_{j\leq 2}^{2} . W_{j>2}^{3}$$

$$\frac{\partial^2 W_{j\leq 2}^{3/5}}{\partial s^2}\big|_{s=0} = 827032.13$$

$$STTS_{j\leq 2}^{3/5} = \left(\frac{827032.13}{0.443} - [411.24]^2 \right)^{0.5} = 1302.99$$

Table E. 26. Calculation of Multi-state 3-out-of-5:F system with 3 states

$$W_{j_{sys}=2}^{3/5} = \binom{5}{3}(W_{j\le 2}^{u})^2.(W_{j>2}^{u})^3 = 10.\left(\frac{0.4e^{100s}}{1-0.28e^{300s}}\right)^2.\left(\frac{0.12e^{250s}}{(1-0.28e^{300s}).(1-0.18e^{400s})}\right)^3$$

$$W_{j=2}^{3/5} = \frac{0.0027e^{950s}}{(1-0.28e^{300s})^5.(1-0.18e^{400s})^2} \quad \rightarrow P_{j_{sys}=2}^{3/5:F} = W_{j_{sys}=2}^{3/5}\Big|_{s=0} = 0.174$$

where:

$a = e^{950s}, b = (1-0.28e^{300s}), and\ c = (1-0.18e^{400s})$

$$\frac{\partial W_{j_{sys}=2}^{3/5:F}}{\partial s} = W'^{3/5}_{j_{sys}=2} = \frac{0.0027}{(bc)^2}.\left(a'b^5c^2 - 5ab'b^4c^2 - 2a'b^5c'c\right) = \frac{0.0027*250}{[0.72^5*0.82^2]^2} = 39.93$$

$$MTTS_{j_{sys}=2}^{3/5:F} = \frac{39.93}{0.174} = 229.18$$

where $K = \left(a'b^5c^2 - 5ab'b^4c^2 - 2a'b^5c'c\right)$

$$\frac{\partial^2 W_{j_{sys}=2}^{3/5:F}}{\partial s^2} = W''^{3/5}_{j_{sys}=2} = \frac{0.0027}{(bc)^4}\left(\begin{array}{l}(bc)^4[a''bc + 5a'b'bc + 2a'bc'c - 5a'b'bc - 5a'b''bc - \\ 20ab'bc - 2a'bc'c - 10ab'bc'c - 2abc''c - 2abc'] - \\ 2(b'c + c'b)bc.K\end{array}\right)$$

$$W''^{3/5}_{j_{sys}=2}\Big|_{s=0} = 347370.52$$

$$STTS_{j_{sys}=2}^{3/5:F} = \left(\frac{347370.52}{0.174} - [229.18]^2\right)^{0.5} = 1394.2$$

Table E. 27. Calculation of Multi-state 3-out-of-5:Load sharing system with 3 states

$$W_{L_{sys}=x}^{1/2} = \sum_{i=1}^{2}\binom{2}{i}.\prod_{j} W_{u_j^i}^{u} = \binom{2}{1} W_1 .W_3 + \binom{2}{2}(W_2)^2$$

$$W_{L_{sys}=x}^{1/2} = 2.\left(\frac{0.4e^{100s}}{1-0.28e^{300s}}.\frac{0.12e^{250s}}{(1-0.28e^{300s}).(1-0.18e^{400s})}\right) + \left(\frac{0.12e^{250s}}{(1-0.28e^{300s}).(1-0.18e^{400s})}\right)^2$$

$$P_{L_{sys}=x}^{1/2} = W_{L_{sys}=x}^{1/2}\Big|_{s=0} = 0.429$$

$$W'_{L_{sys}=x} = 2.[W'_1.W_2 + W_1.W'_2] + 2.W_2.W'_2$$

$$MTSS_{L_{sys}=x}^{1/2} = \left(2.[W'_1.W_2 + W_1.W'_2] + 2.W_2.W'_2\right)\Big|_{s=0} = \frac{184.06}{0.429} = 429.1$$

$$W''_{L_{sys}=x} = 2.[W''_1.W_2 + 2.W'_1.W'_2 + 2.W_1.W''_2 + W'^2_2 + W_2.W''_2]$$

$$W''_{L_{sys}=x}\Big|_{s=0} = 168735.55$$

$$STTS_{L_{sys}=x}^{1/2} = \left(\frac{168735.55}{0.429} - [429.1]^2\right)^{0.5} = 457.38$$