ROTATIONAL AND QUASIROTATIONAL ENERGY LEVELS OF DEFORMED NUCLEI

by

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STATEMENT OF ORIGINALITY

The following is the list of investigations made during the course of this study which to the best of the author's knowledge have not been previously undertaken.

(1) Comparison of the two different methods of obtaining the asymmetry parameter, γ, occurring in the Davydov-Filippov model. A correlation is established between the two parameters (γ/A and A_{DF}) of this model.

(2) Krutov's model calculations for the rotational levels of the ground-state bands of even-even nuclei and a comparative study of the two asymmetric-rotor models, viz the Krutov model and the Davydov-Filippov model, in the sense as to how well they reproduce the energy levels upon evaluation of the asymmetry parameters by the two different methods.

(3) Extension of the Variable Moment-of-Inertia model by introducing anharmonicity in the potential energy and application of the proposed model , called VMI23, to the analysis of the ground-state rotational bands of 122 even-even nuclei.

(4) Application of a six degree polynomial in $(\mathcal{I}-\mathcal{I}_0)$ (where $\mathcal{I}$ denotes the moment-of-inertia of the nucleus) to derive informations on the potential energy surfaces of the "back-bending" nuclei.

(5) Proposition for an expression for the transition quadrupole moment in terms of the mass number and the moment-of-inertia of the initial and final states of the rotational levels of even-even nuclei as obtained from the variable-moment-of-inertia model.

(6) Application of the variable-moment-of-inertia model to the odd-A nuclei which have rotational levels built on the $K=\frac{1}{2}$ band.

(7) Calculations of the rotational levels of the ground-state bands of deformed even-even nuclei using the Morinigo model.

ABSTRACT

A brief outline of the present thesis is as follows. The phenomena of collective motions in nuclei along with the important aspects of its developments have been briefly described in Chapter I. This chapter also contains a short overview of the various chapters in this thesis which are based on the study of the collective motions in deformed nuclei. The various models that describe this collective behaviour among nucleons have been briefly reviewed in Chapter II. The first part of this Chapter deals with the phenomenological models and the second with the microscopic ones.

In Chapter III, the two methods of obtaining the asymmetry parameter, γ , occurring in the Davydov-Filippov model have been compared and systematics of the two parameters, γ and A_{DF}, of this model have been investigated. A relation between $(E_{2^+})A$ and γ, due to Sheline, have been re-examined and a new correlation between γ/A and A_{DF} has been found. In Chapter IV the rotational energy levels of the ground-state band of a number of deformed even-even nuclei have been calculated using the two asymmetric-rotor models, one proposed by Krutov and the other by Davydov and Filippov. The parameters occurring in these models have been evaluated by two different methods (as in Chapter III) and the relative merits of the two models have been discussed by comparing the calculated energy values with the experimental ones.

In Chapter V, the Variable Moment-of-Inertia (VMI) model for the ground-state rotational bands in even-even nuclei has been extended by adding an anharmonic term to the potential energy. A detailed application of this model, called VMI23, has been made to the ground-state bands of 122 even-even nuclei. It is shown that the inclusion of the anharmonic term improves agreement with the experimental data in many cases and provides a sensitive means of probing the potential energy surfaces of nuclei.

Chapter VI deals with those nuclei that show anomalous moment-of-inertia at high spin-states (called 'back-bending' nuclei). Informations on potential energy (P E) surfaces of such nuclei have been obtained by employing a polynomial in $(\mathcal{I}-\mathcal{I}_0)$ for the PE term in the VMI model. It is found that the PE surfaces of these nuclei show an arch type protuberance. The energy levels calculated using this modified VMI model have been compared with the experimental values for eight back-bending nuclei.

In Chapter VII an expression is proposed for the transition quadrupole moment in terms of the mass number and the moment-of-inertia of the initial and final states as obtained from the VMI model.

Chapter VIII deals with the application of the VMI model to odd- A nuclei having rotational energy levels built on the K = 1/2 band. The modified VMI energy equation, incorporating terms due to Coriolis interactions and its first order correction, for the K = 1/2 bands, gives a very good agreement of the energy values with the experimental ones.

Finally in Chapter IX, calculations have been carried out for the rotational levels of the ground-state band of some even-even nuclei, ranging from the soft ones (E_4/E_2 = 2.13) to the hard (E_4/E_2 = 3.33) ones using Morinigo's model. This model has been proposed in an attempt to give theoretical justification to the VMI model. The merits of Morinigo's model have been discussed and a critical analysis of the parameters occurring in it has been made.

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CHAPTER I

INTRODUCTION.

An important advance in our understanding of the nuclear structure was made when Mayer (1948) and Haxel, Jensen and Suess (1949) successfully explained the "magic numbers" on the basis of a Shell model. In this model the correlation among individual nucleons is assumed to be small and the central potential needed for a shell configuration is generated by the individual fields of the nucleons. The spin-orbit interaction term and other residual interactions are added to the Hamiltonian to make the description complete. The Shell model has been very successful in correlating spins and parities of the ground-state bands of nuclei, in explaining the magic number discontinuities in binding energies and the observed islands of isomerism (Mayer and Jensen 1955). It has also very well explained the spins and parities of odd-A nuclei, the magnetic moments of odd nuclei, alpha and beta decay systematics and several other nuclear phenomena.

In spite of the several accomplishments of the Shell model, it has still exhibited many limitations. The phenomena of nuclear fission, whose main features can be successfully described in terms of the free vibrations of a 'liquid-drop' in which the particles interact strongly with each other, could not be accounted for by the Shell model. The experimentally observed nuclear quadrupole moments in most cases are much larger than the Shell model predictions, especially

in the regions between two closed shells (Gordy 1949 and Townes et al 1949). The transition probabilities of low lying states exceed the single-particle estimates very often by as much as two orders of magnitude. The existence of typical rotational and vibrational band spectra in deformed nuclei (similar to that in molecules) are not easily accounted for by the Shell model. Finally the discovery of the phenomenon called the photonuclear "giant resonances" (Baldwin and Klaiber 1947, 1948) which were observed as strong broad peaks in the photofission cross-section of the nuclei, gave impetus to the use of models other than the Shell-model in the interpretation of the various nuclear properties.

All of the above facts indicate the need for a new kind of mechanism in order to explain these nuclear properties. They can be well understood if it is assumed that they arise due to the cooperative effect of the motions of a large number of particles in the nucleus. This has lead to the new phenomenon - the collective motion - and the models which incorporate this mechanism are known as the "collective models".

The collective model had its early developments in the 1950's with the pioneering work of Bohr (1952) and Bohr and Mottelson (1953). Since those initial advances up to the present time, enormous amounts of experimental and theoretical work have been accomplished. During the last twenty years various models have been proposed to promote understanding in the mechanisms of the collective motions. The present thesis deals with a number of investigations on the collective models for the nucleus. The contents of each chapter are discussed below.

In Chapter II of this thesis we describe and review some of the important collective models and their distinguishing features; various phenomenological models of the symmetric and asymmetric rotator types have been surveyed followed by a brief discussion of some of the microscopic models.

A tremendous amount of development in the heavy ion excitation methods along with the refinements in the detection techniques has made available very precise experimental data on level energies with high angular momenta. The collective electric quadrupole transition rates have been very accurately measured using the techniques of Coulomb excitations (Breit et al 1957, Greiner and Arenhovel 1968).

The collective excitations in nuclei are, in broad sense, classified into three groups: (1) those resulting from the rotation of the permanently deformed nuclei, which are intrinsically not excited, (2) those resulting from the quadrupole vibrations of the nuclear surface, and (3) those resulting from the octupole vibrations. The first excited state that occurs in the deformed even-even nuclei is the ground-state rotational band. Rotational spectra are usually observed in three regions of the periodic table: near A = 24, the rare earth region, i.e. $150 \leq A \leq 190$, and the actinide region ($A \geq 220$). For the doubly even nuclei the spin and parity sequence of the ground-state rotational levels are characterized as 0+, 2+, 4+, 6+ ... etc. Quadrupole vibrations of the surface usually occur at higher excitation. These states are classified as β and γ-vibrational states and correspond

to the vibrations which either preserve or destroy the assumed axial symmetry of the nucleus. The β-vibrational band is characterized with spin and parity sequence 0+, 2+, 4+ etc, whereas the γ-band has levels with spins and parities 2+, 3+, 4+, 5+.. etc. The groups of energy levels which arise due to octupole vibrations of the surface occur at higher excitation and are characterized as negative parity, odd-spin states.

The theory of collective rotations in deformed nuclei yields a simple expression for the energy as proposed by Bohr and Mottelson (1953)

$$E(I) = \frac{\hbar^2}{2\mathcal{J}} I(I+1) \qquad (1.1)$$

where I is the spin of the level and $\mathcal{J}$ is called the "moment-of-inertia" of the nucleus. Eq. (1.1) is in accord with the description of the nucleus as a symmetric top (Bohr and Mottelson 1953, Kerman 1959). Since the real nucleus does have internal degrees of freedom, the levels with internal modes of excitation are also present along with the pure rotational ones. As mentioned earlier these are described as either deformation vibrations (β-vibration) or asymmetry -vibrations (γ-vibration). It is possible to have rotational states built on each of these vibrational states. In reality the interaction between various modes of motion takes place and experimentally no nucleus has been found to follow the I(I+1) law (Eq. (1.1)) strictly. These deviations have been attributed by various authors, to be due to the rotation-vibration interaction, the centrifugal stretching of the nucleus, the breakdown of the pairing

due to Coriolis effect, the coupling of ground-state band with quasi-particle levels, and possibly other effects.

Applying the techniques of perturbation theory Bohr and Mottelson (1953) have arrived at an energy equation incorporating an additional term to Eq. (1.1), proportional to $I^2(I+1)^2$. This correction term is small in the strongly deformed regions of nuclei but becomes rather large as one approaches the regions of transition to spherical nuclei. Subsequent analysis of the energy spectrum (Scharff-Goldhaber et al 1958, Kane et al 1960, Bjerregard et al 1963) showed that only a first order correction is inadequate to describe the energy spectrum at higher angular momentum. Various empirical studies (e.g. Stephens et al 1959, Hansen et al 1963, Gupta and Sood 1966, Nathan and Nilsson 1965) stressed the importance of higher order terms. Later it was shown (Stephens 1965) that, the comparison of the observed energies with "such a power series energy equation in I(I+1)" would be meaningless since a fitting would require almost as many terms as there are points to be fitted. It was further shown by Sood (1964), that the convergence of the series is very poor or even breaks down for high spin states of transitional and moderately deformed nuclei.

Other empirical equations for energy as a function of angular momentum were proposed by several authors with various degrees of success in explaining the experimental data. Among these: Ejiri's

(1967) equation is written as

$$E(I) = aI(I+1) + pI \qquad (1.2)$$

where p is a parameter. Sood (1968) has suggested the following equation for the ground band levels:

$$E(I) = A \left[\frac{1+(N-1)(B/A)I(I+1)}{1+N(B/A)I(I+1)}\right] I(I+1) \qquad (1.3)$$

with N = 2.85 - 0.05I. A and B are the two parameters here. Varshni's (1968) equation is of the following form,

$$E_I = aI(I+1) + pI + qI^2(I+1) \qquad (1.4)$$

where the first term is a rotational term, the second a vibrational term and the third one is a cross term. a, p and q are the three parameters.

In the year 1958 Davydov and Filippov took another approach towards explaining the collective motions in deformed even-even nuclei. They assumed the nucleus to be an axially asymmetrical body. To describe the shape of the nucleus two parameters were introduced: (1) the deformation parameter β_0 and (2) the nonaxiality parameter γ_0. In order to arrive at an energy expression for the nucleus undergoing collective motions, they made two simplifying assumptions: (1) that β_0 and γ_0 remain fixed during rotation i.e. the nucleus rotated without altering its shape, and (2) they adopted a model dependent (Hydrodynamical model) expression for the moment-of-inertia of the nucleus, which incorporated the same two parameters, β_0 and γ_0. Davydov and Filippov have also derived

expressions for the transition probability for transitions between various excited levels.

In Chapter III we discuss some features of the Davydov-Filippov (DF) model. The two methods for calculating the parameters of this model are examined in detail and a comparative study of these is made. It has been found that the values of γ obtained by using the first excited 2+ and 4+ level energies are more appropriate in reproducing the ground-state bands in this model as compared to the more common method of using the first and second excited 2+ levels. We have, in this Chapter, studied the systematics of the two parameters β_0 and γ_0 in detail. A relationship between parameters first pointed out by Sheline (1962) has been re-examined and a new correlation between γ/A and A_{DF} (the other parameter in the DF model) has been established.

Krutov (1968a,b) has adopted a different approach in describing the nuclear collective motions as a change in the density distribution of the nuclear matter with time. Assuming the nucleus to be deformed and axially asymmetric, he has set up a Hamiltonian for rotational motion as well as expressions for the moments-of-inertia . Krutov's description also incorporates two parameters: the deformations in mass and charge distributions are separately denoted by parameters β and β_e and similarly the nonaxiality parameters, by γ and γ_e respectively.

Krutov and Zackrevsky (1969 a, b) have, derived expressions for energy levels and quadrupole transition branching ratios using the above approach.

In Chapter IV the rotational energy levels with high angular momentum for the ground-state bands of a number of deformed even-even nuclei have been calculated using Krutov's model, and also the Davydov-Filippov model. The parameters occurring in these two models are evaluated for each case utilising the methods described in Chapter III and a comparative study of these has been made. The merits of the Krutov and the Davydov-Filippov models are compared and discussed in detail in their ability to reproduce the experimental energy levels. Since both are pure asymmetric rotator models the pattern of results obtained is, relatively speaking, similar.

In recent years there has been considerable interest in calculating the moment-of-inertia of high spin rotational states in deformed even-even nuclei. These works were stimulated by the measurements of Stephens, Lark and Diamond (1964), who succeeded in measuring energy values up to spin 18+ for some nuclei. Diamond et al (1964) have discussed in detail the relationship between moment-of-inertia and the deformation in nuclei. Since deformation, β changes with angular momentum, the moment-of-inertia would also vary accordingly.

More recently, Mariscotti, Scharff-Goldhaber and Buck (1969) have carried out an extensive analysis of the ground-state bands of many even-even nuclei and have proposed an empirical model known as

the "Variable Moment-of-Inertia" (VMI) model. Their simple two parameter energy equation has been very successful in reproducing the ground-state rotational and quasi-rotational energy levels for a large number of even-even nuclei in various mass-regions. The energy equation is written as

$$E_I(\mathcal{I}) = \frac{1}{2}\frac{I(I+1)}{\mathcal{I}} + \frac{c}{2}(\mathcal{I}-\mathcal{I}_0)^2 \qquad (1.5)$$

where c and $\mathcal{I}_0$ are the two parameters.

The details of this model appear in Chapters II and V of this thesis.

A closer scrutiny of the VMI model energies revealed that for a good number of nuclei which are termed as "soft nuclei" (e.g. ^{120}Xe, ^{122}Xe, ^{124}Xe, ^{126}Xe, ^{190}Os etc.) the agreement with the experimental energies is not satisfactory. Such deviations could be accounted for if one uses an improved form of the potential energy term in the VMI energy equation. In Chapter V of this thesis we have extended the VMI model by adding an anharmonic term to the potential energy. A detailed application of this model, called VMI23, has been made to the ground-state bands of 122 even-even nuclei. It is shown that the inclusion of the anharmonic term improves agreement with the experiment in many cases, and provides a sensitive means of probing the potential energy surfaces of nuclei. In this chapter we also discuss a new graphical method of comparing the energies calculated by various models with the experimental one, which proved to be very sensitive to even the slightest disagreement between the two.

Recently some interesting experimental studies (Johnson et al 1971, 1972, Thieberger et al 1972, and Lieder et al 1972) on the ground-state rotational band of even-even nuclei in the mass-range A = 158-168 have revealed an anomalous behaviour of the moment-of-inertia at high spin states. If the experimental data are plotted in the plane of the moment-of-inertia $2\mathcal{J}/\hbar^2$ versus the square of the nuclear rotational frequency $(\hbar\omega)^2$ this singular feature shows up with a typical "back-bending" of the curve, caused by a sharp sudden increase in $\mathcal{J}$ at a high angular momentum. The $(\mathcal{J},\omega)^2$ plot looks like an S shaped curve. Such anomalous behaviour in the rotational motion has been interpreted by two main theoretical models. The first type (Mottelson and Valatin 1960, Krumlinde and Szymanski 1971) assumes a phase transition from a superfluid to a normal state induced by the Coriolis force, taking place at some critical value of the angular momentum. The second type (Stephens and Simon 1972) is based on the mixing of collective bands with different moments-of-inertia associated with different quasi-particle excitations. These two types of models differ mainly in the effect of the Coriolis force that breaks or perturbs the pairing interaction among the nucleons.

In Chapter VI of this thesis we investigate the potential energy surfaces of the nuclei which show anomalous moment-of-inertia at high spin states. A sixth degree polynomial in powers of $(\mathcal{J}-\mathcal{J}_0)$ has been tested to derive informations about the potential energy surfaces of such nuclei. Analysis of the experimental data has been carried out

using the new model, called VMI6P ('6P' for the sixth degree polynomial representing potential energy term). The calculated energy values are also compared with the previously calculated values of Wahlborn and Gupta (1972) wherever available, and of Molinari and Regge (1972) for the nucleus ^{162}Er. A detailed discussion on the forms of the potential energy surfaces that could shed some light on the anomalous nature of the rotational motion has also been included in this chapter.

In their original work on the VMI model Mariscotti, Scharff-Goldhaber and Buck (1969) have introduced another empirical law showing a relationship between intrinsic transition quadrupole moments Q_{02} and the transition moment-of-inertia $\mathcal{J}_{02}$ for a transition between $0+ \rightarrow 2+$ states. The relationship is given by

$$Q_{02} = k\, \mathcal{J}_{02}^{1/2} \qquad (1.6)$$

where k is a constant and Q_{02} are obtained from the experimental B(E2) values. Such a relationship is found useful in predicting the quadrupole moments of the deformed even-even nuclei. In Chapter VII we suggest an alternative relationship between transition quadrupole moments and the transition moments-of-inertia which provides a better description than eq. (1.6) at high mass numbers and for transitions involving higher angular momenta.

Chapter VIII deals with the odd-A nuclei where the occurrence of low lying rotational bands built on a particular intrinsic state has been a commonly observed phenomenon. Extension of the collective

models to odd-A nuclei can be made by assuming the odd proton (or neutron) coupled to the deformed even-even core. We extend the variable moment-of-inertia model to the odd-A nuclei which have rotational levels built on the K = 1/2 band. The energy equation also incorporates the Coriolis antipairing term and its first order correction. It has already been established (Davidson and Feenberg 1953) that the Coriolis term has strong effect on the level energies of the K = 1/2 band. The energy equation, containing four parameters, is fitted using non-linear least-squares procedure to the 34 K = 1/2 bands of odd-A nuclei. The satisfactory agreement between the calculated and observed energy values prove the validity for the application of the VMI model to such nuclear systems. A comparison of the parameters with the corresponding parameters of the neighbouring even-even nuclei obtained from the VMI model has also been made.

The simplicity of the empirical VMI model and the fact that it describes the rotational levels of nuclei over a wide range of the periodic table with a remarkable success have led theorists to attempt to provide a more theoretical basis to this model. One such attempt is due to Morinigo (1970). He has reformulated the basic shell-model Hamiltonian for a group of n identical particles in an oscillator well, which can interact with each other by way of two body quadrupole interactions. This Hamiltonian is then separated into two parts, one

describing the rotation and the other, the intrinsic motions. A semi-classical argument shows that the energy equation from this Hamiltonian can approximate to an equation identical to the one given in the VMI model. No energy calculations have been reported in Morinigo's paper. In Chapter IX we have applied the Morinigo model to 12 representative even-even nuclei which range from the "soft" ($E_4/E_2 \simeq 2.13$) to the "hard" ($E_4/E_2 \simeq 3.33$) region. A detailed analysis of the parameters occurring in this model has been made and the calculated energy values are compared with the three parameter VMI23 model.

CHAPTER II

NUCLEAR COLLECTIVE MODELS

The collective phenomenon in nuclei including the various modes of its excitations have been described with the help of nuclear collective models. In this chapter we shall review some of the important models developed in this field of study over the past twenty years. We have divided this chapter into two parts. The first part deals with the phenomenological models and in the second, we briefly describe some of the microscopic models. The phenomenological models have been arranged into two sections, A and B. Section A deals with the "symmetric rotor" type models and B, with those models which are termed as "asymmetric rotators".

PART I. Phenomenological models

(A) Symmetric Rotor Type:

1) The Bohr-Mottelson model

A significant shortcoming of the independent particle model was its inability to explain the large electric quadrupole moment observed in various nuclei. In order to explain this phenomenon it was earlier postulated (Rainwater 1950) that the extra core nucleons might cause a deformation in the nucleus which is considered to behave as an incompressible "liquid-drop". Bohr (1952) and Bohr and Mottelson (1953) first developed these fundamental ideas into a definite model. They considered

the collective motions in nuclei as vibrations and rotations of the "irrotational" liquid drop. In the following, we shall describe this model in detail, since this is the basis of most of the other collective models in this category.

The shape of the nucleus in a space-fixed coordinate system can be described with the help of a set of shape-coordinates (Preston 1963)

$$R(\theta',\phi') = R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} y_{\lambda\mu}(\theta',\phi')\right] \qquad (2.1)$$

where R_0 is the radius of the spherical nucleus, $y_{\lambda\mu}$'s are the spherical harmonics and $\alpha_{\lambda\mu}$'s are the parameters that describe the shape of the nucleus at a particular instant. Since the radius $R(\theta,\phi)$ is real

$$\alpha^*_{\lambda\mu} = (-1)^{\mu} \alpha_{\lambda,-\mu} \qquad (2.2)$$

Then for small surface oscillations the classical kinetic energy can be expressed as

$$T = \frac{1}{2} \sum_{\lambda,\mu} B_\lambda \left|\dot{\alpha}_{\lambda\mu}\right|^2 \qquad (2.3)$$

where B_λ is a mass parameter, associated with a deformation of the order λ.

The nuclear shape can also be expressed in a body-fixed coordinate system:

$$R(\theta,\phi) = R_0 \left[1 + \sum_{\ell,m} a_{\ell m} y_{\ell m}(\theta,\phi)\right] \qquad (2.4)$$

The $a_{\ell m}$'s, analogous to $\alpha_{\lambda\mu}$'s, also satisfy the following condition

$$a_{\ell,-m} = (-1)^m \, a_{\ell,m} \tag{2.5}$$

Expressions (2.1) and (2.4) are related to each other by a rotation of the coordinate axes. We write the following transformation relations (Rose 1957):

$$y_{\ell m}(\theta',\phi') = \sum_{m'} D^{\ell *}_{mm'}(\theta_i) y_{\ell m'} \, (\theta,\phi), \tag{2.6}$$

$$a_{\ell m} = \sum_{m'} D^{\ell *}_{m'm} \, \alpha_{\ell m'} \tag{2.7a}$$

and

$$\alpha_{\ell m} = \sum_{m'} D^{\ell}_{mm'} \, a_{\ell m'} \tag{2.7b}$$

where the $D^{\ell}_{mm'}$ are the elements of the D-matrix, which for each ℓ, constitute a $(2\ell+1)$-dimensional representation of the three dimensional rotational group. θ_i's are three Euler angles (these are α, β, and γ in Rose's notation). We note here that the above transformations are unitary in character and the D-functions are also eigenfunctions of the quantum mechanical symmetric top.

A satisfactory description of the shape of the nucleus, very different from a sphere, would then require a large number of terms in a series including higher order spherical harmonics. For a quadrupole deformation λ or $\ell = 2$. In a body-fixed coordinate system when the axes coincide with the principal axis of the deformed sphere we have

the following conditions:

$$a_{2,0} \neq 0; \quad a_{2,\pm 1} = 0; \quad a_{2,2} = a_{2,-2} \neq 0$$

that is, the shape of the nucleus, under these situations, can be described by two independent parameters plus the threeEuler angles. The kinetic energy for such a system can be written as:

$$T = \underset{\text{vibration}}{T(\dot{a}_{2,m})} + \underset{\text{rotation}}{T(a_{2,m}, \dot{\theta}_i)} \tag{2.8}$$

In expanded form the vibrational kinetic energy for the quadrupole case is,

$$T_{vib} = \frac{1}{2} B_2 \sum_m \dot{a}^2_{2,m} = \frac{1}{2} B_2 \, (\dot{a}^2_{2,0} + 2\dot{a}^2_{2,2}) \tag{2.9}$$

From the mechanics of the rotating rigid body (Goldstein 1950) the kinetic energy for rotation of the nucleus in terms of the angular momentum can be written as

$$T_{rot} = \frac{1}{2} \sum_K \mathcal{I}_K \, \omega^2_K \tag{2.10}$$

with the moment-of-inertia $\mathcal{I}_K$ (for the K axis) defined as,

$$\mathcal{I}_K = B_2 \sum_{m,m'} a_{2,m} \, a_{2,m'} \, \langle 2Mm | I^2_K | 2Mm' \rangle \tag{2.11}$$

and $\omega_K (= \dot{\phi}_K)$ being the components of the angular velocity in the body-

fixed system (K = 1, 2, 3). I_K's are the angular momentum operators. The expectation value of these operators involves the normalized rotor eigenfunctions $|IMK\rangle$, which can be expressed in terms of the properly symmetrized combination of the D-functions,

$$|IMK\rangle = \left[\frac{(2I+1)}{16\pi^2(1+\delta_{K0})}\right]^{1/2} (D^I_{M,K} + (-1)^I D^I_{M,-K}) \qquad (2.12)$$

where I represents the total angular momentum of the nucleus, M, its projection on the spacefixed z-axis and K, the projection on the body-symmetry axis.

Bohr and Mottelson have introduced the deformation parameter β and the non-axiality parameter γ in the following manner:

$$a_{2,0} = \beta \cos\gamma; \quad a_{2,\pm 2} = \frac{1}{\sqrt{2}} \beta \sin\gamma \qquad (2.13)$$

Eq. (2.9) then becomes

$$T_{vib} = \frac{1}{2} B_2 (\dot{\beta}^2 + \beta^2\dot{\gamma}^2) \qquad (2.14)$$

Upon evaluating the matrix elements (Eq. (2.11)) explicitly, the expression for the moment-of-inertia due to Bohr (1952), can be written as

$$\mathcal{I}_K = 4B_2\beta^2 \sin^2(\gamma - \tfrac{2}{3}\pi K); \quad K = 1, 2, 3 \qquad (2.15)$$

Having obtained the classical kinetic energy, we shall now proceed to quantize this. The procedure adopted is due to Pauli (1958), which

essentially amounts to expressing the kinetic energy operator as $-\frac{\hbar^2}{2}\nabla^2$, in the given coordinate system (the mass parameter can be absorbed in it). The details of mathematical procedure for obtaining the operator ∇^2 in any coordinate system can be found in the text books on tensor analysis e.g. Willis (1958).

The kinetic energy is expressed as the change in time of the "line element". In a space with generalized coordinates x_i, the line element is written as

$$ds = [G_{\lambda\mu}\, dx^{\lambda}\, dx^{\mu}\,]^{1/2} \tag{2.16}$$

and the kinetic energy as,

$$T = \frac{1}{2}\left(\frac{ds}{dt}\right)^2 = \frac{1}{2}\sum_{\lambda\mu} G_{\lambda\mu}\, d\dot{x}^{\lambda}\, d\dot{x}^{\mu} \tag{2.17}$$

where $G_{\lambda\mu}$ is the covariant matric tensor and can be expressed as

$$G_{\lambda\mu} = \begin{pmatrix} R_{\lambda} & 0 \\ 0 & D_{\lambda} \end{pmatrix} \tag{2.18}$$

with D_{λ} representing the diagonal matrix associated with the vibrational degrees of freedom and R_{λ}, the 3 x 3 rotational kinetic energy matrix:

$$R_{\lambda} = \sum_{K=1}^{3} J_K^{\lambda}\, \omega_K^2$$

Expressing the angular velocity components ω_K, in terms of the time derivative of the Euler angles $\dot{\theta}_i$, in the matrix form,

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} -\sin\theta_2 \cos\theta_3 & \sin\theta_3 & 0 \\ \sin\theta_2 \sin\theta_3 & \cos\theta_3 & 0 \\ \cos\theta_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \qquad (2.19)$$

The Laplacian operator in the generalized coordinate system, adopted here, can be written as (Pauli 1958),

$$\nabla^2 = \frac{1}{\sqrt{G_{\lambda\mu}}} \frac{\partial}{\partial x^\lambda} \left(\sqrt{G_{\lambda\mu}}\, G^{\lambda\mu} \frac{\partial}{\partial x^\mu}\right) \qquad (2.20)$$

where $G_{\lambda\mu}$ can be obtained from the determinant of Eq. (2.18) and $G^{\lambda\mu}$ is its inverse. In the present problem the variables x^λ are: β, γ, θ_1, θ_2 and θ_3. In the case considered here ($\lambda = 2$), $G_{\lambda\mu}$ turns out to be

$$G_{\lambda\mu} = B_2^2 \beta^2 \sin^2\theta_2 \mathcal{I}_1 \mathcal{I}_2 \mathcal{I}_3 \qquad (2.21)$$

It is now a lengthy but straightforward calculation to obtain the quantum mechanical operators of the vibrational as well as rotational kinetic energies. Eq. (2.20) combined with Eqs. (2.14) and (2.21) yields

$$T_{vib} = \frac{-\hbar^2}{2B_2}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\left(\beta^4 \frac{\partial}{\partial\beta}\right) + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\left(\sin 3\gamma \frac{\partial}{\partial\gamma}\right)\right] \qquad (2.22)$$

The expression for T_{rot} has the form of the rotational kinetic energy operator for an asymmetric top,

$$T_{rot} = \frac{\hbar^2}{2} \sum_K \frac{I_K^2}{\mathcal{I}_K} \tag{2.23}$$

with the moments-of-inertia $\mathcal{I}_K$ given by Eq. (2.15).

The ideas outlined above, were first developed by Bohr and Mottelson yielding a phenomenological description of the deformed nucleus. In order to simplify the calculations they assumed the nucleus to be axially symmetric (i.e. $\gamma = 0$) and that in the zeroth order the nuclear Hamiltonian can be written as a sum of two separate terms representing rotational and intrinsic motions, In their "adiabatic approximation", the rotational motion is assumed to be slow enough so that the intrinsic structure is not disturbed. The complete Hamiltonian is then written as

$$H = T_{rot} + T_{vib} + V \tag{2.24}$$

where V is the potential energy and for small oscillations, this has the following form:

$$V = \frac{1}{2} c (\beta - \beta_0)^2 \tag{2.25}$$

where β_0 is the equilibrium deformation and c is the restoring force parameter.

The Schrodinger equation can now be set up with this Hamiltonian and the eigenfunctions ($<nIMK|$) as given in Eq. (2.12), with an additional quantum number n, to take into account the vibrational modes of excitation. The quantum number n, is denoted as $n_\beta(=1)$ for the β-vibrational state and as $n_\gamma(=1)$ for the γ-vibrational state in nuclei. The symmetry requirements of the wave function allow the following energy states to appear in the spectrum (see Fig. (2.1):

a) The ground-state with $K = 0$, $n_\beta = 0$, $n_\gamma = 0$, $I = 0+, 2+, 4+, 6+, \ldots$ etc.

b) The β-vibrational state with $K = 0$, $n_\beta = 1$, $n_\gamma = 0$, $I = 0+, 2+, 4+, \ldots$ etc.

c) For γ-vibrational state $K = 2$, $n_\beta = 0$, $n_\gamma = 1$, $I = 2+, 3+, 4+, 5+, \ldots$ etc. (The + sign denotes the positive parity states).

In the axially symmetric case $\mathcal{J}_1 = \mathcal{J}_2 = \mathcal{J} = 3B_2\beta^2$ and $\mathcal{J}_3 = 0$, For the ground state ($K = 0$) the energy equation is then written as

$$E_I = \frac{\hbar^2}{2\mathcal{J}} I(I+1) \qquad (2.26)$$

There are experimental evidences for the existence of purely rotational spectra which satisfy the $I(I+1)$ rule (Eq. 2.26), (e.g. ^{238}Pu). As the frequency of rotation increases with the higher angular momentum, the centrifugal force distorts the nuclear shape and perturbations due

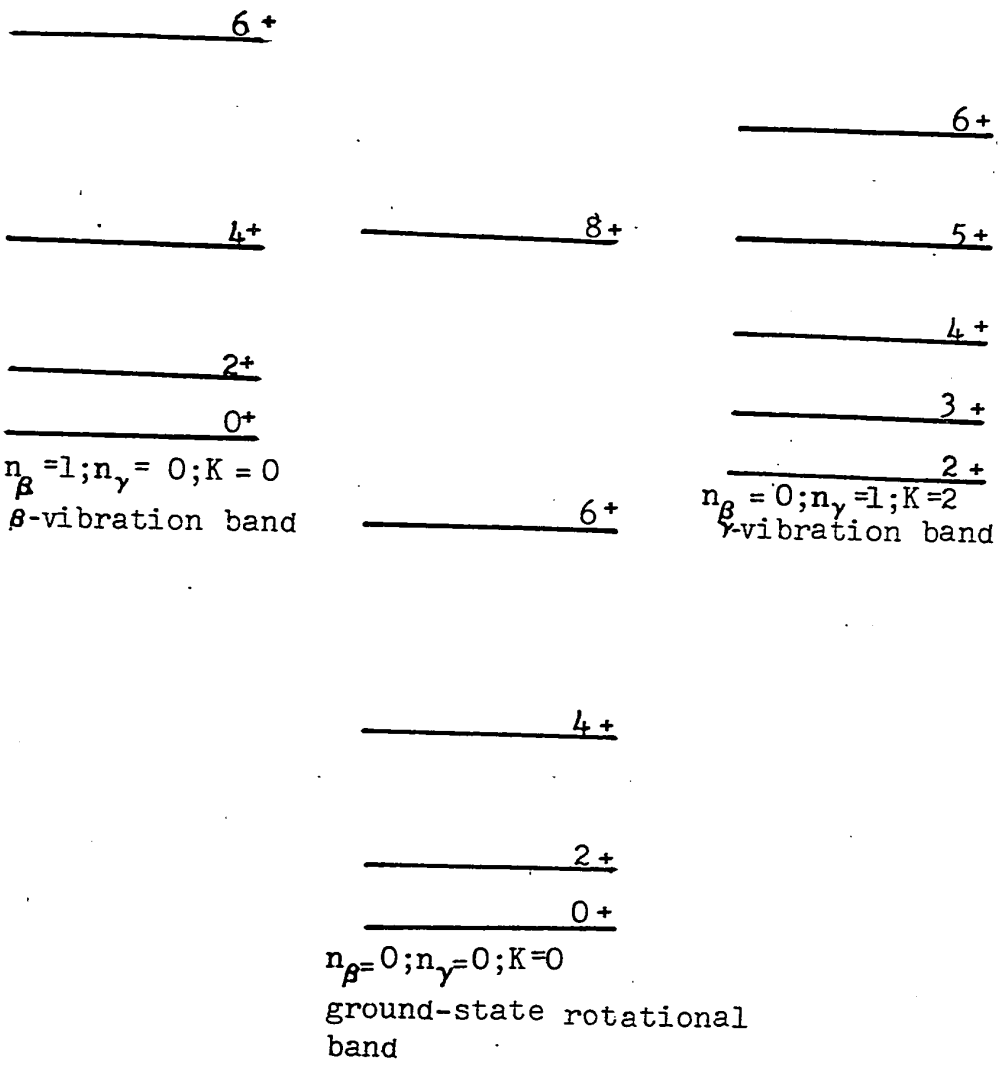

Fig (2.1): Predicted energy level scheme of the Deformed Even-Even Nucleus.

to rotation-vibration interactions become prominent. Calculations, using the perturbation due to band-mixing, rotation-vibration interactions, centrifugal stretching etc. have been carried out by many workers e.g. Hansen et al (1959), Nathan and Nilsson (1965), Bohr and Mottelson (1953) and Marshalek (1967). Bohr and Mottelson have shown that by introducing the first order correction term to the energy equation it becomes:

$$E_I = \frac{\hbar^2}{2\mathcal{I}} I(I+1) + BI^2 (I+1)^2 \qquad (2.27)$$

where B is another parameter characterizing non-adiabaticity in the rotational mode. Limitations of (2.27) have been pointed out in Chapter I.

Nilsson (1955) has first introduced the coupling of the nucleons in the outer shell (in the Shell model picture) to the nucleons in the core causing deviations from spherical symmetry in the nucleus. The core, which in this model is spherically symmetric in the beginning, becomes "polarized" by the external particles and this aspherical arrangement possesses an energy minimum. Nilsson has set up a Hamiltonian incorporating all these effects and the solution for each state for various values of deformation parameter has been shown in Nilsson-diagrams.

The nuclear wave-functions that are deduced from the Bohr-Mottelson model calculations were used to calculate the transition probabilities between various excited states. It turns out that the E(2)

transitions that occur in the first excited state and the ground-state of even-even nuclei have a shorter lifetime, by a factor up to 100 than the single-particle transitions of the same multipole character. This coincided with the experimental values of large quadrupole moments (or shorter half-lives for transitions) found in the deformed nuclei.

2) Rotation-Vibration Model

This model has been proposed by Faessler and Greiner (1964). They have included the perturbations due to non-adiabaticity and band-mixing in the Bohr-Mottelson model. Faessler and Greiner treated the rotation-vibration interaction in an exact manner and have carried out numerical diagonalization of the various matrix elements of the Hamiltonian which included these effects from the first principle consideration. The four parameters occurring in their expression are ;(i) the reciprocal moment-of-inertia (found from energy spacings in the ground-state rotational band), (ii) the γ-vibrational energy, (iii) the β-vibrational energy and (iv) the equilibrium deformation β_0. Tables of level energies and amplitudes of wave functions for the various levels of the ground-state, β, and γ bands are given by Faessler, Greiner and Sheline (1965). This model is reasonably successful for nuclei which have the values of the energy ratio E_4/E_2 $(=R_4)$ in the range ~ 3 to 3.33.

3) Harris Model

Harris (1964) has proposed a new approach to the calculation of the rotational bands of deformed even-even nuclei with observed high spin states. Extending the cranking model* (Inglis 1954) to higher powers in ω (nuclear angular velocity) and solving the equations by making use of the perturbation theory Harris has given expression for energy which contains two parameters $\mathcal{J}_0$ and C:

$$E = \frac{\hbar^2}{2\mathcal{J}_0} I(I+1) \{1 - x + 4x^2 - 24x^3 + \ldots\} \qquad (2.28)$$

where $x \equiv \frac{C}{\mathcal{J}_0^3} I(I+1)$. In the above equation if $C = 0$ or if $C \neq 0$ but x very small, one obtains the original Bohr-Mottelson equation; if the terms upto x are retained, Eq. (2.28) resembles Bohr-Mottelson equation (Eq. (2.27)) which contains the first order correction term. The two parameters, $\mathcal{J}_0$ and C were fitted by least-squares procedure for each nucleus.

In a subsequent paper Harris (1965), from the self-consistency approach (Feynman 1939), has given the following two equations for calculating energies, which contain even higher order terms in ω:

$$E_{rot} = \frac{1}{2} \omega^2 (\mathcal{J}_0 + 3C\omega^2 + 5 D\omega^4 + 7F\omega^6 + \ldots) \qquad (2.29)$$

and $$[I(I+1)]^{1/2} = \omega(\mathcal{J}_0 + 2C\omega^2 + 3D\omega^4 + 4F\omega^6 + \ldots) \qquad (2.30)$$

* The Cranking model has been described later in this chapter with the microscopic models.

where D and F are extra parameters. In principle ω may be eliminated from the above two equations leaving one equation for E_{rot} as a function of I. Harris has obtained better fit to the observed energy levels using upto three parameters in the Eqs. (2.29) and (2.30).

4) Moszkowski's (1966) and Sood's (1968) models

Since both the above models are similar we describe them under one heading. Both have used the hydrodynamic assumption (as in the Bohr-Mottelson model) in order to write the expression for the moment-of-inertia, which assumes it to be proportional to β^2. On applying the equilibrium condition ($\frac{dE}{d\beta} = 0$) the following two equations are obtained:

$$E_I = \frac{1}{2\mathcal{J}_0} \frac{HV(V+1)}{(1-V)^2} \qquad (2.31)$$

and

$$I(I+1) = \frac{HV}{(1-V)^4} \qquad (2.32)$$

where V is another parameter, which could be eliminated between these two equations. The energy equation is used to plot E_I/E_2 versus E_4/E_2. The experimental points for rare-earth nuclei are found to lie close to the theoretical curves in the region $R_4 \simeq 3$ to $R_4 = 3.33$.

5) Variable Moment-of-Inertia (VMI) model

Recently Mariscotti, Scharff-Goldhaber and Buck (MSB) (1969) have proposed a two parameter formula that gives a very good fit to the rotational and quasirotational states of the ground-state band of deformed even-even nuclei. Their energy expression is given by

$$E_I(\mathcal{I}_I) = \frac{1}{2} C(\mathcal{I}_I - \mathcal{I}_0)^2 + \frac{I(I+1)}{2\mathcal{I}_I} \qquad (2.33)$$

where $\mathcal{I}_I$ (in units of $\hbar^2$) is the moment-of-inertia for the spin I state and is obtained from the equilibrium condition

$$\frac{\partial E(\mathcal{I})}{\partial \mathcal{I}} = 0 \qquad (2.34)$$

The two parameters, $\mathcal{I}_0$ and C, are defined as the "ground-state moment-of-inertia" and the "stiffness parameter" respectively. The semiclassical equation for energy in terms of the deformation β (Eq. (1) of Diamond et al 1964) can be compared with Eq. (2.33). In the VMI model the deformation parameter β is replaced by a general variable x and it is assumed that the moment-of-inertia $\mathcal{I} \approx x^n$. MSB have found that the best fits for strongly deformed nuclei as well as for the near spherical nuclei, are obtained by putting n = 1. Mariscotti (1970) and Scharff-Goldhaber and Goldhaber (1970) in their subsequent publications have allowed for the parameter $\mathcal{I}_0$ to have negative values as well. This has extended the lower limit of the range of applicability of

the VMI model from $R_4 \geq 2.23$ to $R_4 \geq 1.82$ and could include very "soft" nuclei (vibrational nuclei) to be described within the VMI model framework. For example, the nuclei ^{110}Sn, $^{120-126}Te$, etc. for which R_4 is less than 2.23. A physical significance for negative $\mathcal{J}_0$ is given by Scharff-Goldhaber and Goldhaber (1970), who have assumed a phase change taking place in these nuclei above the spin 2+ level from a "superconducting" to a "normal" state. These authors have associated this process with the symmetry breaking effect in the pair excitation.

MSB have also shown mathematical equivalence of their model with the Harris model, although the latter does not allow for the negative values of the parameter $\mathcal{J}_0$. VMI model provides a simple empirical description of the ground-state bands for a wide range of even-even nuclei. Attempts have also been made to construct microscopic models in support of the VMI model. da Providencia and Urbano (1970) have provided a theoretical foundation to the VMI model by proposing a new method for the determination of the excited rotational states which emerged out of a critical analysis of the Villars (1957) theory of nuclear collective rotation. In a subsequent paper, the same authors (1972) have proposed a theoretical justification to the VMI model by working out a theory based upon the generator coordinates method. Thieberger (1970) has proposed a simple mechanical model as a classical analog to the VMI model, which could also account for the negative values of $\mathcal{J}_0$. Das et al (1970) have proposed a theoretical interpretation and extension of the VMI model. Their model was applied for a simultaneous

analysis of the ground-state rotational and β-vibrational or γ-vibrational bands. Morinigo's (1970) attempt to derive the VMI energy equation on theoretical grounds have been described in Chapter IX. of this thesis.

6) Draper's model

Draper (1970) and McCauley and Draper (1971)have assumed a phenomenological choice for the functional form of the moment-of-inertia. In terms of the deformation parameter β, it is written as

$$\mathcal{J} = \alpha\beta^n \tag{2.35}$$

This approximates to a variety of physically reasonable shapes of the nucleus. In eq. (2.35), n is any number and α is a parameter. The case n = 1 resembles the VMI model. The energy equation is given as

$$E_I = \frac{1}{2} C_\beta (\beta - \beta_0)^2 + \frac{I(I+1)}{2\alpha\beta^n} \tag{2.36}$$

where C_β is another parameter and β_0 represents the ground-state deformation. The equilibrium condition determining the energy minimum is given by

$$\partial E/\partial\beta = 0 \tag{2.37}$$

These authors have carried out a least-squares fit to their energy equation for ~ 88 nuclei ranging from A = 96 to 248. The values for parameter n, are found to be between 2.8 for ^{168}Yb and 0.7 for ^{128}Ce. n is close to 1 for nuclei near the magic mass number.

7) Governor model

Trainor and Gupta (1971) have proposed this model which is based on the following assumptions:

(a) The centrifugal stretching in the rotating nucleus is controlled by harmonic restoring forces and

(b) there exists a rotationally invariant core in the deformed nuclei.

Since this model uses dynamic nuclear stretching which reduces the speed of rotation (and hence the rotational energy) for a given angular momentum, it is called the "governor model". The total energy is given by

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}_s} + \frac{1}{2} k (r_s - r_o)^2 \qquad (2.38)$$

where k is the (empirical) stiffness constant and $\mathcal{J}_s$, the moment-of-inertia for any stretching s, is given by

$$\mathcal{J}_s = \frac{2}{5} M_s (a^2 + ab + b^2) \qquad (2.39)$$

where a and b are the semimajor and semiminor axes respectively of the rotating deformed nuclei (assumed to be an ellipsoid). M_s is the effective mass of that portion of the nucleus which is taking part in rotation and is evaluated from the following equation

$$M_s = \frac{4\pi}{3} \rho_{eff} b^2 (a-b) \qquad (2.40)$$

with ρ_{eff}, the effective density, considered as one of the parameters of the equation. In Eq. (2.38), (r_s-r_o) is the displacement from equilibrium of the mass centers of the two parts of the effective mass of the ellipsoidal nucleus which lies outside the non-rotating spherical core. The degree of stretching at any moment is given by the ratio, $(b_o-b)/b_o$, with b_o as the semimajor axis of the nucleus in the ground-state.

The stretching parameter s_I corresponding to angular momentum I, is then given by the minimization condition

$$\partial E_I/\partial r_s = 0 \qquad (2.41)$$

Eq. (2.38) of this model bears some resemblance to the energy equation of the VMI model.

Trainor and Gupta have obtained good agreement with experiment for those nuclei, which fall in the category of "rigid-rotators" and also for some neutron deficient nuclei..

(B) Asymmetric-Rotor Models:

General remarks:

Marty (1956 , 1957) in his investigations on the collective excitations of nuclei has shown that all nuclei cannot be considered to be purely axially symmetric; more realistically, asymmetry in the

shape should be taken into consideration (from the first principle) while describing nuclear collective motions. In this section we shall describe some of the asymmetric rotator models in which the nucleus is no longer assumed to be symmetric about the body fixed symmetry axis and the moment-of-inertia about each axis is different. A general description of the nuclear collective motions has been given in the previous section under Bohr-Mottelson model.

From equations (2.22) to (2.25) a general form of the Hamiltonian for nuclear collective motion can be written as

$$H = \frac{-\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\left(\beta^4\frac{\partial}{\partial\beta}\right) + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\left(\sin 3\gamma\frac{\partial}{\partial\gamma}\right) - \frac{1}{4\beta^2}\sum_{k=1}^{3}\frac{I_k^2}{\sin^2\left(\gamma - \frac{2\pi}{3}k\right)}\right] + \frac{1}{2}C(\beta-\beta_0)^2 + \frac{1}{2}C_\gamma(\gamma-\gamma_0)^2 \qquad (2.42)$$

The Schrodinger equation can then be formed with the above Hamiltonian and the eigenfunctions given in Eq. (2.26). The solution to this equation is quite complicated. Different models use different approximations to solve this Schrodinger equation.

1) Davydov-Filippov model

Davydov and Filippov (1958) have assumed fixed values of β and γ for the rotating nucleus, that is the intrinsic structure of the nucleus remained unchanged during rotation (called the "adiabatic approximation").

The Schrodinger equation in this case can be written as

$$\frac{\hbar^2}{8B\beta^2}\left[\sum_{k=1}^{3}\frac{I_k^2}{\mathrm{Sin}^2(\gamma-\frac{2\pi}{3}k)}\right]\psi = E\psi \qquad (2.43)$$

here, k denotes the nuclear fixed axis (1, 2, 3). For the asymmetric-rotor K is not a good quantum number. The eigenfunctions can then be written as a sum over the various values of K,

$$\psi_{IM} = \sum_{K} g^{I}_{Ki}(\gamma)\,|KMI\rangle \qquad (2.44)$$

where $g_{Ki}(\gamma)$'s are the various amplitudes. For a given I, K runs over all even integers less than or equal to I, except for the odd I, where K = 0 is excluded. Consequently, there are no I = 1 states and I = 3 has a fixed value for K, namely 2. The subscript i indicates that for other values of I there are more than one states, corresponding to different allowed values of K. Thus, there is one 0+ state, two 2+ states, one 3+ state, three 4+ states, two 5+ states, and so on.

Solving for the angular momentum matrix elements in explicit form the following equation arises and its solutions give the energies

for the various states:

$$g^{I}_{K+2}\ (\mathcal{I}_2-\mathcal{I}_1)\mathcal{I}_3[(I-K)(I-K-1)(I+K+1)(I+K+2)]^{1/2}$$

$$+\ g^{I}_{K}\ [2(\mathcal{I}_1+\mathcal{I}_2)\mathcal{I}_3\ (I^2+I-K^2) + 4\mathcal{I}_1\mathcal{I}_2K^2 - 8\ (\frac{E}{\hbar^2})\mathcal{I}_1\mathcal{I}_2\mathcal{I}_3]$$

$$+\ g^{I}_{K-2}\ (\mathcal{I}_2-\mathcal{I}_1)\mathcal{I}_3[(I+K)(I+K-1)(I-K+1)(I-K+2)]^{1/2} = 0 \qquad (2.45)$$

where $\mathcal{I}_1$, $\mathcal{I}_2$ and $\mathcal{I}_3$ represent the moments-of-inertia for the three axes. The determinant of the coefficients of g_K's must be zero for a solution. For I = 2, the following equation emerges which has two roots giving two energies for this spin value,

$$(\frac{E}{\hbar^2})^2 - 2\ (\frac{E}{\hbar^2})\ (\mathcal{I}_1^{-1}+\mathcal{I}_2^{-1}+\mathcal{I}_3^{-1}) + \frac{3}{8}\ [\mathcal{I}_1^{-2}+\mathcal{I}_2^{-2}+6\mathcal{I}_1^{-1}\mathcal{I}_2^{-1}$$

$$+\ 8\ (\mathcal{I}_1^{-1}+\mathcal{I}_2^{-1})\mathcal{I}_3^{-1}] = 0 \qquad (2.46)$$

Fig. (2-2) shows the various values for energies for different states as a function of γ from 0^0 to 30^0 (taken from Davydov-Filippov (1958)). $\gamma = 0$, corresponds to the axially symmetric case and $\mathcal{I}_1 = \mathcal{I}_2 = \mathcal{I}$ and $\mathcal{I}_3 = 0$ for this case.The roots of Eq. (2.46) for this situation

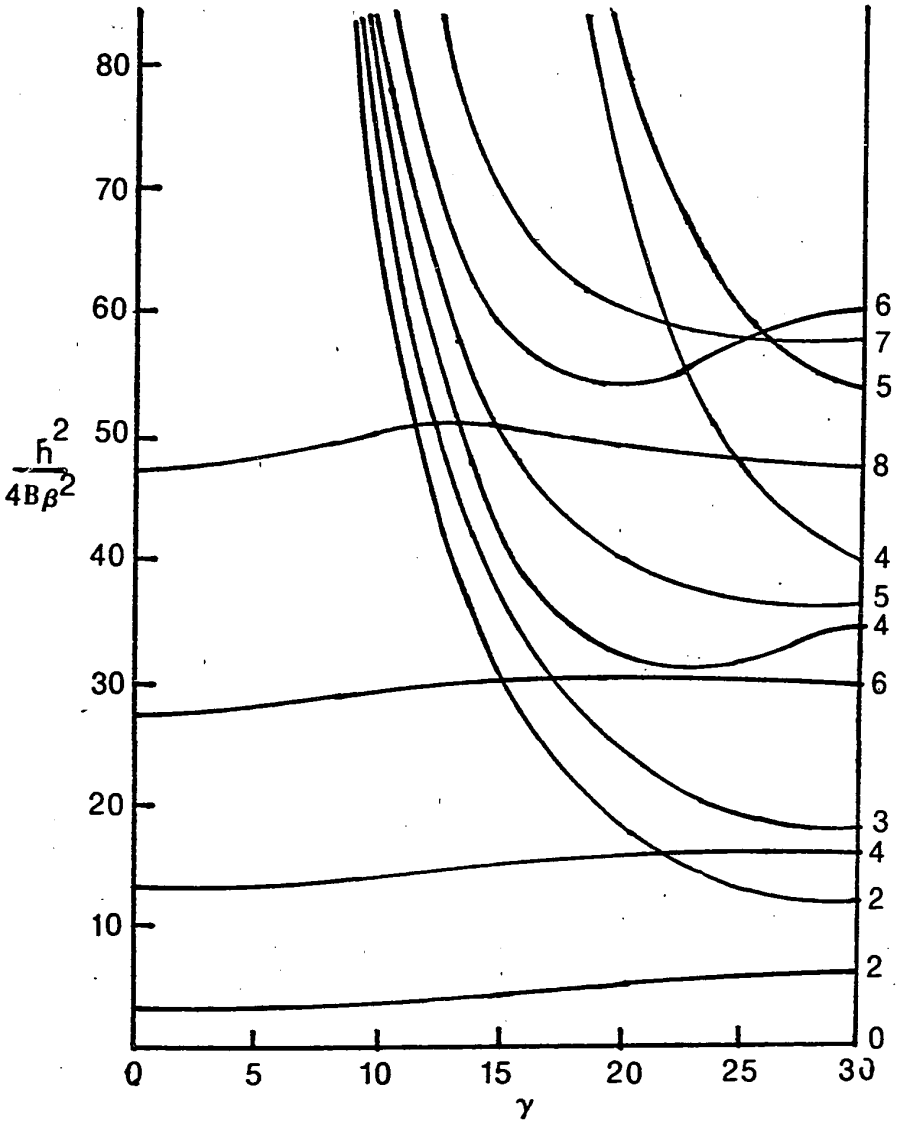

Fig (2.2) Energy levels of the Adiabatic Asymmetric-Rotor as a function of the asymmetry parameter γ. The energies are in units of $\hbar^2/4B\beta^2$ and γ in degrees. The spin value of each level is indicated at the right hand side.

turns out to be of the form:

$$E_I = \frac{\hbar^2}{2\mathfrak{J}} I(I+1) \qquad \text{for } K = 0.$$

Further details of this model appear in Chapter III. Davydov-Filippov (1958) and Davydov and Rostovsky (1959) have given expressions for the reduced transition probabilities for the various quadrupole transitions between the levels of the asymmetric rotator.

2) Davydov-Chaban model

In 1960 Davydov and Chaban proposed to modify the previous model of Davydov and Filippov by letting the deformation parameter β vary with the angular momentum. That is, the effective value of β in the Davydov-Chaban model is different for each excited state but the parameter γ remains fixed throughout. In this case the Hamiltonian from Eq. (2.42) is of the following form:

$$H = \frac{-\hbar^2}{2B}\left[\frac{1}{\beta^3}\frac{\partial}{\partial\beta}\left(\beta^3\frac{\partial}{\partial\beta}\right) - \frac{1}{4\beta^2}\sum_{k=1}^{3}\frac{I_K^2}{\sin^2\left(\gamma - \frac{2\pi}{3}k\right)}\right] + \frac{1}{2}C(\beta-\beta_0)^2 \qquad (2.47)$$

Davydov and Chaban showed that the energy of the collective nuclear excitations which are described by the operator (2.47) is determined

from the following expression:

$$E_{I\tau\nu} = \hbar\omega_0 \{ (\nu + 1/2) [1 + \frac{3}{2} (\frac{\mu}{p_{I\tau}})^4 E_\tau(I)]^{1/2} + \frac{1}{4} (\frac{\mu}{p_{I\tau}})^2 E_\tau(I) + \frac{1}{2} (p_{I\tau} - 1)^2 \mu^{-2} \} \quad (2.48)$$

where ω_0 ($= \sqrt{C/B}$) and μ are the new parameters of this theory; $E_\tau(I)$ are the rotational state energies (in units of $\hbar^2/(4B\beta_0^2)$) of the adiabatic theory of non-axial nuclei (Davydov-Filippov model). $p_{I\tau}$ is determined for each value of the quantum numbers I and τ from the solution of the equation

$$(p_{I\tau} - 1) \; p_{I\tau}^3 = \frac{1}{2}\mu^4 E_\tau(I) \; . \quad (2.49)$$

Thus $p_{I\tau}$ depends on the value of the parameters μ and γ; ν is one of the roots (ν_0, ν_1, ν_2...) of the transcendental equation

$$H_\nu (-\mu^{-1} p_{I\tau} [4 - 3/p_{I\tau}]^{1/4}) = 0 \quad (2.50)$$

where

$$H_\nu(x) = \frac{1}{2\Gamma(-\nu)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (2x)^k \Gamma(\frac{k-\nu}{2})$$

is of the form of Hermite function for non-integral values of ν.

Detailed tables of the Davydov-Chaban model parameters and energy ratios corresponding to these parameters were compiled by Day et al (1960). Williams and Davidson (1962) have given general methods for calculating energy ratios as well as transition probabilities for the quadrupole and the octupole vibrations in the framework of Davydov-Chaban model.

3) Davydov, Rostovsky and Chaban model

A general phenomenological theory of quadrupole excitations would be complete if both β and γ vibrations are simultaneously included in the energy calculation. The separation of variables required to solve this problem becomes a complicated affair. Davydov (1961) and Davydov, Rostovsky and Chaban (1961) have introduced approximations in solving such a Hamiltonian. They have first separated the β and γ variables in the Hamiltonian (2.42) without including the potential energy due to γ-vibrations. Then the γ-dependent potential energy in its modified form,

$$V(\gamma) = \frac{BC_\gamma}{\hbar^2} \beta_0^4 (\gamma-\gamma_0)^2 \qquad (2.51)$$

is added to the equation. Later Davydov and Rostovsky (1964), following a similar method, have treated the problem in a more exact manner. Expressions for monopole matrix elements are also given by these authors.

4) Krutov's model

Krutov (1968a,b) has presented a new approach to the description of the collective motion of deformed nuclei in terms of the change in the density distribution of the nuclear matter with time. The Hamiltonian for the rotational motion has been written by assuming the nucleus to behave as an axially asymmetric body. From the ordinary formulas of rigid body mechanics it is written as

$$H_{rot} = \frac{\hbar^2}{2} \sum_{\nu=1}^{3} \frac{I_\nu^2}{\mathcal{J}_\nu} \qquad (2.52)$$

where ν indicates the three body-fixed axis, and $\mathcal{J}_\nu$, the moment-of-inertia about the ν-axis. Assuming a uniform density distribution, the moment-of-inertia has been computed. For $\nu = 3$ axis it is written as

$$\mathcal{J}_3 = \mathcal{J}_{rs} \left(\frac{15}{\pi}\right)^{1/2} \gamma.\beta \left(1 - \frac{4}{7}\sqrt{\frac{5}{\pi}}\ \beta + ...\right) \qquad (2.53)$$

where β and γ are the two parameters, defined as the total mass deformation and non-axiality parameters respectively, (similar in some respects with those used in the Davydov-Filippov model) and $\mathcal{J}_{rs}$ is the moment-of-inertia of the rigid sphere ($\mathcal{J}_{rs} = \frac{2}{5} M_A R^2$). Krutov and Krutov and Zackrevsky (1969a,b) have computed expressions for level energies and also expressions for B(E2) values for transitions between various low lying rotational states in the non-axial nuclei. The details of this model appear in Chapter IV of this thesis.

PART II. Microscopic Models

General remarks:

The models discussed in Part A are all macroscopic in nature. From the microscopic point of view the nucleus is a many body system and hence the many body theory as applied to atomic physics, solid-state physics, elementary particle physics etc, has also been applied in the study of the collective excitations in nuclei; for example, the Hartree-Fock theory of atomic physics, the pairing force theory for superconductivity in solid-state physics, the ideas of superfluidity from the theory of liquid helium and the field quantization methods from the elementary particle theory.

The importance of pairing force for the description of the deformed nuclei has been shown in many ways. Bohr, Mottelson and Pines (1958) were the first to draw attention to the fact that the methods of superconductivity and superfluidity could be applied to study the properties of nuclear matter. The experimentally observed energy gap, the lowering of the effective moment-of-inertia below the rigid-rotator value and the spherical shapes of the closed shell nuclei are the outcome of the pairing correlation that exists between the nucleons. The coriolis force acts in a direction which tends to decouple the pairing correlation. This causes a decrease in the energy gap parameter, Δ. The effective moment-of-inertia increases in this process and

a correction to the energy equation becomes necessary. At high rotational frequencies the coriolis force becomes comparable in magnitude to the pairing force and eventually destroys this. This brings a change in the phase of the nucleus from the superfluid to the normal state. Mottelson and Valatin (1960) have worked out the above theory which was later experimentally verified by Johnson et al (1971). In Chapter VI we have further discussed this process.

Many different models have been developed from the basic assumptions of pairing and multipole-multipole type of interactions in nuclei. In the following we describe some of the microscopic models.

1) The Superfluid model

Applying the mathematical methods developed by Bogolyubov (1958 a,b) for the superfluidity and superconductivity theories, Belyaev (1959) and Solov'ev (1958) elaborated a model for the nucleus called the "superfluid" model. In these approaches a refinement in the shell model is introduced by taking into account the residual interactions leading to pair correlations of the superconducting type and multipole-multipole interactions, whereby it becomes possible to describe states in nuclei which were formerly described phenomenologically as nuclear surface oscillations. The interaction Hamiltonian is written as a sum of three separate terms

$$H = H_{av} + H_{pair} + H_{coll} \qquad (2.54)$$

where H_{av} is the self-consistent field of the nucleus, H_{pair} is for the

pairing interactions and H_{coll} represents the multipole-multipole type interactions. In solving this Belyaev has used the technique of canonical transformations and Solov'ev the variational principles proposed by Bogolyubov.

From 1963 onwards Solov'ev and his coworkers have done a series of calculations for the even-even nuclei, by taking into account the long range quadrupole-quadrupole forces and short range pairing interactions and have interpreted the non-rotational collective excitations as quadrupole and octupole vibrational states. The details of these investigations are given in a review article by Solov'ev (1965). The weaknesses of this model lie in the use of a crude and oversimplified Hamiltonian and a large number of parameters considered to be constants e.g. the pairing interaction constants for neutrons and protons, quadrupole-quadrupole interaction constants and the energies of many single-nucleon states.

2) The Quasiparticle model of Migdal

Migdal (1965) has introduced another trend in the microscopic theory of nuclei. He has considered the nucleus as a gas composed of two types of interacting quasiparticles situated in a potential well. The effective interaction between these quasiparticle is characterized by several constants, which are considered to be the same for all the intermediate and heavy nuclei. The quasiparticles have certain effective mass and momentum corresponding to the elementary low energy excitations.

This effective mass is determined by compairing theory with the experiment. In order to describe the excitations of this system, the other parameters required are related to the effective potential well in which these quasiparticles move. For a system which can be described by short range forces, the above mentioned parameters are: the shape of the nucleus, the depth of the potential well and the width of the layer through which the density of the nuclear matter passes from its value inside the nucleus to zero outside. Although these parameters vary from nucleus to nucleus, they are chosen as approximately constant for a comparatively large group of nuclei.

Migdal's theory predicts the frequencies and intensities of transitions, the magnetic and electric quadrupole moments of nuclei to an accuracy of 10-20% when compared with their observed values.

3) Marshalek's model

Marshalek (1967) has assumed the pairing plus quadrupole forces in order to set up the nuclear Hamiltonian, and using the technique of Hartree-Fock-Bogolyubov (HFB) he has derived expression for the moment-of-inertia. He has then solved the time dependent HFB equation taking the coriolis force as perturbation and has obtained expression for the ground-state band level energies, (similar to the Bohr-Mottelson Eq. (2.27)), upto the second order term in I(I+1). Marshalek has shown that the coefficient of the $I^2(I+1)^2$ term (denoted as 'B' in the Bohr-

Mottelson case) contains contributions from the following (a) the centrifugal stretching of the self-consistent quadrupole field (B_β and B_γ), (b) the changes in pairing correlation (B_Δ), (c) the coriolis force contribution (B_c), and (d) the condition required to correct the number of particles (B_λ). Thus B is written as

$$B = B_\beta + B_\gamma + B_\lambda + B_\Delta + B_c \qquad (2.55)$$

We may note here that this B coefficient in Bohr-Mottelson equation represents a correction due to rotation-vibration interaction. Marshalek's calculations show that B_c term is much greater than the B_β term (indicating centrifugal stretching).

Marshalek has also evaluated the E2 transition probabilities among members of the β and γ vibrational bands. He has given numerical estimates of the band-mixing parameters and branching ratios.

Marshalek's model is a kind of microscopic analog to the classical macroscopic model formulated by Diamond et al (1964) and Moszkowski (1966) where the authors consider the phenomenon of nuclear centrifugal stretching as the main perturbing factor.

4) Bes' Model

In the previous model Marshalek has solved the time dependent HFB equations using "adiabatic approximation" of slow, large amplitude vibrations. Bes (1963) has carried out calculations in the non-adiabatic

limit by making use of the mathematical technique called the random-phase approximation (RPA). Here the amplitude is considered small for harmonic collective vibrations. In another publication Bes et al (1965) have calculated properties of γ-vibrational states including band-mixing effects and coriolis interaction of second-order.

Bes , Landowne, and Mariscotti (1968) have carried out calculations for the energies of the ground-state rotational levels. Their Hamiltonian has again been set up with quadrupole force fields and pairing-interaction. The energy equation for calculating rotational energy has the following form

$$E = E_{pair}\ (\beta,\ \Delta p,\ \Delta n) + E_{coul}\ (\beta) + \frac{\hbar^2 I(I+1)}{2\mathcal{I}(\beta,\ \Delta p,\ \Delta n)} \qquad (2.56)$$

where E_{pair} is the ground-state pairing energy, E_{coul} is the Coulomb energy and $\mathcal{I}$ is the moment-of-inertia expressed as a function of the deformation β and the pairing gap parameters Δp and Δn. The energy equation has been minimized for each value of the angular momentum with respect to the parameters β, Δp and Δn and the energy value is normalized to the experimental energy of the first 2+ state. This model is, in a sense, an extension of the classical centrifugal stretching model of Diamond et al (1964). In Bes' model the expression for moment-of-inertia is a function of the gap parameter whereas Diamond et al take it to be $\approx 3B\beta^2$.

5) Cranking Model

First suggested by Inglis (1954) this microscopic model is most often applied to large deformed nuclei. In this thesis we have referred to the Cranking model before and we shall now describe it to some extent.

Inglis has assumed that the nucleons are moving in a deformed average field, U, and also have some residual interaction V. The Schrodinger equation for such a system is therefore

$$H\psi(o) = E(o)\ \psi(o) \tag{2.57a}$$

or

$$E(o) = \langle\psi(o)|H|\psi(o)\rangle \tag{2.57b}$$

The Hamiltonian according to Inglis is written as

$$H = \frac{p^2}{2m} + \sum_i U_i + \sum_{i<j} V_{ij} \tag{2.58}$$

If we suppose that due to some external cranking the deformed field rotates around some axis, (x-axis), with an angular velocity ω, the Hamiltonian, H and the Schrodinger equation become time dependent

$$H' = e^{-i/\hbar\, J_x\omega t}\ H\ e^{i/\hbar\, J_x\omega t} \tag{2.59}$$

and

$$i\hbar \frac{\partial}{\partial t}\ \Phi = H'\Phi \tag{2.60}$$

The energy for this rotating system becomes

$$E = \langle\Phi|H'|\Phi\rangle \tag{2.61}$$

and the rotational energy is defined as the difference between this rotating energy (Eq. (2.61)) and the energy of the stationary system (Eq. (2.57b))

$$E_{rot}(\omega) = \langle\Phi|H'|\Phi\rangle - \langle\psi(o)|H|\psi(o)\rangle \tag{2.62}$$

Eq. (2.60) can be reduced to a stationary system if the laboratory frame of reference is transformed to a frame of reference which rotates around the x axis with the same ω. We then have the following transformation relations:

$$\Phi \rightarrow \psi(\omega,t) = e^{i/\hbar\, J_x\omega t}\Phi \tag{2.63a}$$

$$\frac{\partial}{\partial t} \rightarrow e^{i/\hbar\, J_x\omega t}\ \frac{\partial}{\partial t}\, e^{-i/\hbar\, J_x\omega t} \tag{2.63b}$$

and

$$H' \rightarrow e^{i/\hbar\, J_x\omega t}\ H'\ e^{-i/\hbar\, J_x\omega t} = H \tag{2.63c}$$

On substituting the above equations to Eq. (2.60) the Schrodinger equation in the final form can be written as

$$i\hbar\ \frac{\partial\psi(\omega,t)}{\partial t} = (H - \omega J_x)\ \psi(\omega,t) \tag{2.64}$$

Since in the above equation $(H - \omega J_x)$ is stationary, $\psi(\omega,t) = e^{-i/\hbar\, E(\omega)t}\psi_{(\omega)}$. The cranking model formula is then expressed as

$$(H - \omega J_x)\ \psi(\omega) = E(\omega)\psi(\omega) \qquad (2.65)$$

In the above equation we recognize $\omega.J$ as the coriolis term resulting from the introduction of the rotating frame of reference.

Next we proceed to write the rotational energy expression for an arbitrary value of ω from Eq. (2.62),

$$E_{rot}(\omega) = \langle\psi(\omega)|H|\psi(\omega)\rangle - \langle\psi(o)|H|\psi(o)\rangle \qquad (2.66)$$

In order to compare the theoretical values with the experimental ones, Inglis has given a semiclassical relation between the angular momentum I and ω,

$$\langle\psi(\omega)|J_x|\psi(\omega)\rangle = \sqrt{I(I+1)} \qquad (2.67)$$

The moment-of-inertia can then be written using Eq. (2.67) and Eq. (2.26) as follows

$$\frac{2\mathcal{J}(\omega)}{\hbar^2} = \frac{\langle\psi(\omega)|J_x|\psi(\omega)\rangle^2}{E_{rot}(\omega)} \qquad (2.68)$$

The observed spacings between rotational energy levels, when compared with the cranking model equations, were found to lie between the cranking model values and that of the rigid-rotator. Harris (1964) has extended the cranking model by considering higher orders of ω. The Harris model has already been described in the first part of this chapter.

CHAPTER III

SOME FEATURES OF THE DAVYDOV-FILIPPOV MODEL

ABOUT THE MODEL

Within its limited range of applicability the Davydov-Filippov (1958) model is one of the more successful models for deformed even nuclei. As described in Chapter II the Hamiltonian for the rotational motion in the framework of Davydov-Filippov model is written as

$$H_{rot}(\theta) = \frac{\hbar^2}{8B\beta^2_{eff}} \sum_{\lambda=1}^{3} \frac{I_\lambda^2}{\sin^2(\gamma_{eff} - \frac{2\pi\lambda}{3})} \qquad (3.1)$$

where $\lambda = 1, 2, 3$ refer to the three nuclear axes and β_{eff} and γ_{eff} are the two parameters referring to the effective values for β and γ respectively. The expressions for the moment-of-inertia ($\mathcal{I}$) are written as

$$\mathcal{I}_\lambda = 4B\beta^2_{eff} \sin^2(\gamma_{eff} - \frac{2\pi\lambda}{3}) \qquad (3.2)$$

If the energies are now written in the units of $\hbar^2/4B\beta^2_{eff}$ the rotational Hamiltonian can be expanded as follows

$$H_{rot} = \frac{a+b}{4}[I^2 - I_3^2] + \frac{c}{2} I_3^2 + \frac{a-b}{4}[I_1^2 - I_2^2] \qquad (3.3)$$

where

$$a^{-1} = \mathrm{Sin}^2\left(\gamma - \frac{2\pi}{3}\right),\ b^{-1} = \mathrm{Sin}^2\left(\gamma + \frac{2\pi}{3}\right),\ \text{and}\ c^{-1} = \mathrm{Sin}^2\,\gamma \qquad (3.4)$$

and also $\gamma \equiv \gamma_{eff}$. The angular momentum operators I_1, I_2, and I_3 satisfy the following commutation relation

$$[I_1, I_2] = iI_3 \qquad (3.5)$$

The wave function for the rotational Hamiltonian (3.1) is written as

$$\phi_{IM}(\theta) = \sum_{K\geq 0} |IMK\rangle\ A_K^I \qquad (3.6)$$

where $|IMK\rangle$'s are the eigenfunctions of a symmetric top and A_K's are the coefficients of expansion depending upon the asymmetry parameter γ. K's are the projection quantum numbers of I on the 3-axis of the nucleus (Davidson 1968). For even I, K takes on even values as 0, 2, 4,.. up to I and for odd I, it takes values as 2, 4, 6,... up to (I-1). M represents the quantum numbers associated with the projection of I on the z-axis. The $|IMK\rangle$'s can again be expressed in terms of the familiar D-functions, which belong to the completely symmetric representation of group D_2 (Rose 1957)

$$|IMK\rangle = \left(\frac{2I+1}{16\pi^2(1+\delta_{0K})}\right)^{1/2} \{D^I_{MK} + (-1)^I D^I_{M,-K}\}^{*} \qquad (3.7)$$

* The Kronecker δ appears because both parts of the wave function are the same for K = 0.

The rotational energies, E(I) and the wave functions (3.6) are determined from a solution of the system of equations

$$\sum_{K\geq 0} \{\langle IMK|H_{rot}|IMK\rangle - E(I)\,\delta_{KK'}\}\,A_{IK} = 0 \qquad (3.8)$$

The following interesting features of Eq. (3.8) which arise due to the symmetry properties of the wave function are very useful in writing out the explicit forms for the rotational energy for various states:

(a) The matrix elements occurring in (3.8) connect states differing in values of K by zero and two units,

$$\langle IMK|H_{rot}|IMK\rangle = \frac{a+b}{4}\{I(I+1) - K^2\} + \frac{1}{2}cK^2$$

and

$$\langle IMK|H_{rot}|IMK+2\rangle = \frac{a-b}{8(1+\delta_{oK})}\{1 + (-1)^I\delta_{oK}\} \times$$

$$\{(I-K)(I-K-1)(I+K+1)(I+K+2)\}^{1/2}.$$

(b) Williams (1965) has shown that sum of all the roots $E_\tau(I)$ of Eq. (3.8) satisfies the following simple sum rule for a fixed value of I:

$$\frac{12}{I(I+1)} \sum_{\tau=1}^{n_I} E_\tau(I) = \begin{cases} (I+2)(a+b+c) \text{ if I is even,} \\ (I-1)(a+b+c) \text{ if I is odd.} \end{cases}$$

From the above equations there follows another important sum rule for the rotational energies of the nucleus, which makes it possible to connect the sum of the energies of levels having even spin with the

sum of the energies of levels with the subsequent odd spin.

$$\sum_{\tau=1}^{n_I} E_\tau(I) = \sum_{\tau=1}^{n_{I+1}} E_\tau(I+1) \quad (I\text{-even})$$

In particular, for example,

$$E_1(2) + E_2(2) = E(3).^*$$

We may note here that the above sum rules do not depend upon the values of a, b, and c

(c) A relationship between a, b, and c can be written as

$$\frac{a+b+c}{9} = \frac{ab+bc+ac}{24} = \frac{abc}{18} = \frac{1}{\mathrm{Sin}^2(3\gamma)}$$

The solutions of Eq. (3.8) can be analytically expressed as follows:

For I=2, we have two levels, denoted by subscript $\tau(=1,2)$;

$$E_\tau(2) = \frac{3[3 + (-1)^\tau \sqrt{9-8\ \mathrm{Sin}^2(3\gamma)}]}{\mathrm{Sin}^2(3\gamma)} . \qquad (3.9)$$

For I = 3 and 5 the energy equations are

$$E(3) = \frac{18}{\mathrm{Sin}^2(3\gamma)} \qquad (3.10)$$

* In this work we have only considered levels with positive parity.

and

$$E_\tau(5) = \frac{9[5 + (-1)^\tau \sqrt{9-8\ \mathrm{Sin}^2(3\gamma)}]}{\mathrm{Sin}^2(3\gamma)} \qquad (3.11)$$

where $\tau = 1,2$.

For I = 4, 6, and 8, analytical forms of the energy equations cannot be obtained, instead, numerical values of energies are obtained by solving third, fourth and fifth degree algebraic equations respectively. For example the three spin 4 energy levels are the roots of the following equation:

$$[E(4)]^3 - \frac{90}{\mathrm{Sin}^2(3\gamma)}[E(4)]^2 + \frac{48}{\mathrm{Sin}^4(3\gamma)}\{27+26\ \mathrm{Sin}^2(3\gamma)\}[E(4)]$$

$$- \frac{640}{\mathrm{Sin}^4(3\gamma)}\{27+7\ \mathrm{Sin}^2(3\gamma)\} = 0 \qquad (3.12)$$

The equations for obtaining energy values of levels with higher spins are quite lengthy and involved and since we do not need them here for our purpose, they are not presented. In fig. (2.2) we have reproduced the graph of energy values as function of the parameter γ for various spins, shown in order. We note here that for $\gamma = 0$ the energy spectrum is identical to that of an axially-symmetric nucleus.

The energy spectrum of the asymmetric rotor has been calculated by various workers including Davydov and Rostovsky (1959), DeMille et al (1959), and Moore and White (1960) up to levels with spin 24 and more for the range of γ between 0^0 and 30^0. (The nucleus is considered symmetric between these two limits).

Davydov-Filippov model thus involves two parameters, A_{DF}* ($=\hbar^2/4B\beta^2$) and γ in the calculations of energy spectra . The parameter γ is essentially a measure of the deviation of the shape of the nucleus from axial symmetry and β is the deformation from the spherical shape. While calculating energies the simplifying assumption in this model is that the parameters β and γ do not vary during rotation i.e. the nucleus rotates without altering its shape ("adiabatic approximation").

THE PRESENT WORK

In this chapter we have carried out the following investigations:

(a) Compared two methods of obtaining the asymmetry parameter γ,

(b) presented systematics of γ and A_{DF},

(c) re-examined a relationship for γ proposed by Sheline (1962) and

(d) proposed a new correlation between γ/A and A_{DF}.

COMPARISON OF TWO METHODS FOR OBTAINING γ

There are two methods for obtaining the parameter γ:

(i) From the ratio $R_\gamma(2)$, where

$$R_\gamma(2) = \frac{E_\gamma(2+)}{E_1(2+)} = \frac{3+\sqrt{9-8\sin^2(3\gamma)}}{3-\sqrt{9-8\sin^2(3\gamma)}} \qquad (3.13)$$

and the quantity $E_\gamma(2+)$ represents the energy of the 2+ state in the γ-vibration band; frequently this is identified with the second 2+ state.

* The quantity is usually denoted by A; to differentiate it from the mass number we have added the subscript DF.

As γ varies from 30^0 to 0^0 this ratio varies from 2 to infinity.

(ii) From the ratio $R(4) = E_1(4+)/E_1(2+)$, where $E_1(4+)$ is the first 4+ excited state. This ratio varies from $\frac{10}{3}$ at $\gamma = 0^0$ to $\frac{8}{3}$ at $\gamma = 30^0$.

Method (i) has been frequently used by workers in this field (Davydov and Filippov 1958, Sheline 1962, Hiura and Suekane 1960, Davydov 1968) presumably because it is simpler of the two. Eq. (3.13) can be easily solved for γ. On the other hand the use of method (ii) involves solution of a cubic equation (3.12) for $E_1(4+)$. Tables for R(4) exist (Moore and White 1960) but their use requires interpolation and hence are not very reliable. Consequently method(ii) has been seldom employed. However, method (i) has one important shortcoming. There are many nuclei for which $R(4) < \frac{8}{3}$ though $R_\gamma(2) > 2$. For such nuclei Eq. (3.13) would still give a real γ, though strictly speaking such nuclei are outside the range of applicability of the Davydov-Filippov (DF) model. The DF model has been compared with experimental data by Van Patter (1959/60) and Grigorev and Avotina (1960) amongst others. An inspection of the figures given in these two references shows that a great majority of nuclei which fall far from DF predictions have $R(4) < \frac{8}{3}$. We conclude that the DF model should be applied to only such nuclei for which R(4) lies between $\frac{10}{3}$ and $\frac{8}{3}$ (such nuclei would be called DF nuclei). We emphasize this point here because sometimes it is not fully appreciated [e.g. Petry et al 1968].

In addition there is the problem of the proper identification of the correct 2+ state to be used. The Davydov-Fillipov model does not

account for the beta-vibrational band. In those few nuclei for which beta and gamma bands have been identified, the 2+ state of beta band lies below the 2+ state of gamma band; but it is by no means a general rule and for a nucleus for which beta and gamma bands have not been identified, one cannot be certain as to which is the proper 2+ state of the gamma band.

Even when the correct identification is available, the results obtained for other energy levels are not altogether satisfactory. We illustrate this point with ^{154}Gd for which beta and gamma bands have been identified. In fig. (3.1), on the extreme left we show the experimental energy levels as recently obtained by Varnell et al (1969). The quantity γ was obtained from $E_\gamma(2+)/E_1(2+)$ and the other levels were calculated and are shown in the middle of fig. (3.1). It would be noticed that the calculated values for other energy levels, both for the ground band and for the gamma band are not in very good agreement with the experiment. Mallmann and Kerman (1960) have attempted to explain such discrepancies by taking into account the interaction of the rotation with beta-vibration. It can be shown that when rotation-vibration interaction is taken into account, in first order approximation the energy of the rotational states is given by:

$$E_b(^nI,\gamma,\beta) = A_{DF}[\varepsilon(^nI,\gamma) - b\{\varepsilon(^nI,\gamma)\}^2] \qquad (3.14)$$

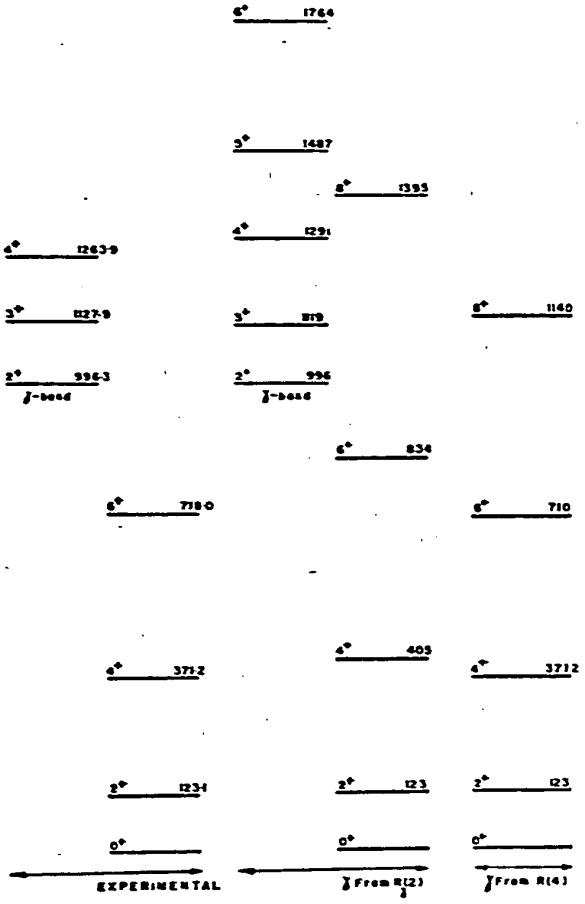

Fig (3.1). Experimental and calculated values of energies of ^{154}Gd.

where b is a constant and $\varepsilon(^{n}I,\gamma)$ are eigenvalues in units of A_{DF} for the DF model without the interaction. However, such a correction would not work for ^{154}Gd. We notice that the discrepancies between calculated and observed values are in opposite directions for 3+ and 4+ states of the gamma-band while the above equation implies that the deviations should be in the same direction for all the energy levels. Of course one can think of adding another term, a cubic in $\varepsilon(^{n}I,\gamma)$, to the right hand side of Eq. (3.14). But in view of the fact that the effect of beta vibration has been incorporated in a better way in the model of Davydov and Chaban (1960) the addition of a cubic does not seem to be worthwhile. On the extreme right we show the energy levels for the ground state obtained by using values of γ calculated from R(4). We find that the calculated values thus obtained are in better agreement with the experimental values than those obtained previously.

In Table (3.I) we show values of γ determined by the two methods for DF nuclei. The values of γ recorded in column 6 were obtained from R(4) disregarding the experimental uncertainties. The ranges shown in column 7 were obtained from extreme values of R(4) when experimental uncertainties are taken into account. Using method (i) γ was obtained from both the second and the third 2+ excited states; the γ values thus obtained are seen to be not too different. Further it would be noticed that for a great number of nuclei there is an appreciable difference between the γ values determined by the two methods. This difference is quite often larger than the experimental uncertainty. Values by method (i) are usually smaller than those by method (ii). We infer that method (ii) should be preferred over method (i) if one wishes to seek good agreement with the ground state band.

TABLE (3.I). Experimental data for energy levels and calculated values of γ by the two methods. Most of the experimental values in columns 2, 3 and 4 are from Lederer et al (1967), Mariscotti et al (1969) and Nuclear Data Sheets. An asterisk after a value indicates that its spin and/or parity assignment is uncertain.

	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_2(2+)$ (keV)	$E_3(2+)$ (keV)	γ from R(4) (degrees)	Range of γ from R(4) (degrees)	γ from $R_2(2)$ (degrees)	γ from $R_3(2)$ (degrees)
^{12}C	4430	14080*	16110		11.0		20.3	
^{24}Mg	1370	4120	4230	7350*	22.1		21.9	16.8
^{106}Ru	94	313*			7.7			
^{124}Ba	229.5±1.0	650.6 ±2.0			25.0	24.7 - 25.4		
^{126}Ba	256.1 ±1.0	711.6 ±2.0			26.1	25.7 - 26.4		
^{128}Ce	207.3	607.3			23.5			
^{130}Ce	254.1	710.7			25.7			
^{150}Nd	132	397	840	1060	22.1		15.5	13.9
^{152}Sm	121.78±0.05	366.4±0.3	811	1087	22.1	22.0 - 22.1	15.2	13.2
^{154}Sm	81.99±0.05	267±1	1210*	1440	15.9	15.2 - 16.4	10.4	9.5
^{154}Gd	123.07±0.05	371.2±0.2	816	999	22.0	21.9 - 22.0	15.2	13.8
^{156}Gd	88.967±0.005	288.16±0.05	1154	1168.2	16.6	16.5 - 16.6	11.1	11.0
^{158}Gd	79.51±0.01	261.45±0.05	1185	1521*	14.0	13.9 - 14.1	9.8	9.2

TABLE (3.I) continued

	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_2(2+)$ (keV)	$E_3(2+)$ (keV)	γ from R(4) (degrees)	Range of γ from R(4) (degrees)	γ from $R_2(2)$ (degrees)	γ from $R_3(2)$ (degrees)
^{160}Gd	75.3 ±0.5	247±2	1010		14.6	8.1 - 16.8	10.8 - 10.9	
^{156}Dy	138±3	403±6			23.6	21.7 - 25.4		
^{158}Dy	99±1	317±3	946	1083	17.8	15.4 - 19.5	12.7 - 12.9	12.0 - 12.1
^{160}Dy	86.7 ±0.1	284 ±1	966.1		14.8	13.8 - 15.7	11.9	
^{162}Dy	81.0 ±0.3	266 ±1	890		14.3	12.2 - 15.7	12.0	
^{164}Dy	73.39±0.05	242.2 ± 0.1	761.8	1987*	13.1	12.7 - 13.4	12.3	7.7
^{158}Er	192.7±1.0	528.4 ±2.0			26.8	26.3 - 27.4		
^{160}Er	126.2±1.0	390.5 ±2.0			20.5	19.6 - 21.3		
^{162}Er	101±1	327±3			16.6	13.1 - 18.6		
^{164}Er	91±1	298±3	858	1308	14.9	0 - 17.7	12.8 - 13.0	10.5 - 10.6
^{166}Er	80.60±0.05	264.9 ±0.2	787	2295	14.1	13.8 - 14.4	12.7	7.5
^{168}Er	79.8 ±0.5	264.0 ±0.5	822		12.2	0 - 14.5	12.3 - 12.4	
^{170}Er	79.0 ±0.5	261±2	930		12.8	0 - 15.8	11.5 - 11.6	
^{162}Yb	166.5 ±1.0	486.7 ±2.0			23.5	23.0 - 24.0		

TABLE (3.I) continued

	$E_1(2^+)$ (keV)	$E_1(4^+)$ (keV)	$E_2(2^+)$ (keV)	$E_3(2^+)$ (keV)	γ from R(4) (degrees)	Range of γ from R(4) (degrees)	γ from $R_2(2)$ (degrees)	γ from $R_3(2)$ (degrees)
^{164}Yb	122.5±0.4	384 ±1			19.6	19.2 - 20.0		
^{166}Yb	101.8 ±0.4	329.7 ±1.0			16.6	15.6 - 17.4		
^{168}Yb	87±1	284±3	986	1233	15.5	0 - 18.1	11.7 - 11.8	10.5 - 10.7
^{170}Yb	84.2±0.1	277.7 ±1.0			13.2	11.6 - 14.4		
^{172}Yb	78.7 ±0.5	260.3 ±1.0	1468*		12.3	0 - 14.9	9.2 - 9.3	
^{174}Yb	76.5 ±0.5	252±3			13.6	0 - 16.8		
^{176}Yb	82.1±0.5	270±3	1270		14.0	0 - 16.8	10.1 - 10.2	
^{166}Hf	158.7±0.4	470.7±1.5			22.8	22.5 - 23.1		
^{168}Hf	123.9±0.4	385±1			20.2	19.8 - 20.6		
^{170}Hf	100.0±0.3	320.6±1.0			17.7	17.1 - 18.3		
^{172}Hf	94.5±0.3	307.9±1.0			15.8	14.6 - 16.6		
^{174}Hf	90.9	298	899		14.7		12.6	
^{176}Hf	88.0±0.5	290±1			13.5	9.2 - 15.4		
^{178}Hf	93.2±0.1	306.8±0.2	1269	1421*	13.8	13.3 - 14.2	10.8	10.2

TABLE (3.I) continued

	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_2(2+)$ (keV)	$E_3(2+)$ (keV)	γ from R(4) (degrees)	Range of γ from R(4) (degrees)	γ from $R_2(2)$ (degrees)	γ from $R_3(2)$ (degrees)
^{180}Hf	93.33±0.05	308.6±0.2			12.4	12.0 - 12.9		
^{172}W	122.9±0.4	376.9±1.0			21.0	20.7 - 21.4		
^{174}W	111.9±0.3	355±1			18.7	18.2 - 19.1		
^{176}W	108.7±0.3	384.5±1.0			17.7	17.1 - 18.2		
^{178}W	104±5	342±7			14.0	0 - 20.9		
^{180}W	102±5	336±7	1008		13.6	0 - 20.9	12.3 - 12.9	
^{182}W	100.10±0.05	329.40±0.05	1222		13.9	13.7 - 14.0	11.4	
^{184}W	111.2	364.0	904	1270*	15.0		13.8	11.8
^{186}W	122.5	399.0	730		15.8		16.0	
^{178}Os	131.6±0.3	397.7±1.0			21.8	21.6 - 22.1		
^{180}Os	132.2±0.3	408.5±1.0			20.6	20.3 - 20.9		
^{182}Os	126.9±0.3	400.2±1.0			19.2	18.8 - 19.6		

TABLE (3.I) continued

	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_2(2+)$ (keV)	$E_3(2+)$ (keV)	γ from R(4) (degrees)	Range of γ from R(4) (degrees)	γ from $R_2(2)$ (degrees)	γ from $R_3(2)$ (degrees)
^{184}Os	119.8±0.3	383.6±0.4	924		17.9	17.5 - 18.2	14.2	
^{186}Os	137.20±0.05	433.9±0.1	767.4		19.0	18.9 - 19	16.5	
^{188}Os	155.0±0.1	477.9±0.1	633	1305*	20.7	20.7 - 20.8	19.2	13.6
^{190}Os	186.7±0.1	547.8±0.1	557.9		23.3	23.3 - 23.4	22.3	
^{192}Os	205.79	580.4*	489.1		25.3		25.2	
^{182}Pt	153.7±0.4	416.2±1.0			27.7	27.3 - 28.1		
^{184}Pt	162.1±0.4	434.8±1.0			28.6	28.1 - 29.4		
^{220}Ra	177*	474*			28.8			
^{224}Ra	84.4	253.0*			22.3			
^{226}Ra	67.8	210.0			20.4			
^{228}Ra	59	185*			19.6			
^{226}Th	72.13	226.5*			19.5			
^{228}Th	57.5±0.1	186.6±0.2	969	1620	16.3	15.9 - 16.7	9.7	7.6
^{230}Th	53.3	174.2	676	783	15.3		11.2	10.4

TABLE (3.I) continued

	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_2(2+)$ (keV)	$E_3(2+)$ (keV)	γ from R(4) (degrees)	Range of γ from R(4) (degrees)	γ from $R_2(2)$ (degrees)	γ from $R_3(2)$ (degrees)
^{232}Th	49.8±0.1	163±1	774	788	15.0	13.2 - 16.3	10.1	10.0
^{230}U	51.7	174			0			
^{232}U	47.6±0.1	156.6±0.2	735	867	14.0	13.0 - 14.7	10.1 - 10.2	9.4
^{234}U	43.50±0.05	143.5±0.2	852*	922*	13.2	12.4 - 13.9	9.0 - 9.1	8.7
^{236}U	45.28±0.05	148.7±0.5	953*		14.3	13.2 - 15.2	8.7	
^{238}U	44.7±0.1	148±1	1035	1058	12.0	0 - 14.5	8.3	8.2
^{236}Pu	44.6	147			13.5			
^{238}Pu	44.11±0.05	146.0±0.5	984	1030	12.1	9.4 - 13.5	8.5	8.3
^{240}Pu	42.88±0.05	141.7±0.5	903	945	12.7	10.4 - 14.0	8.7	8.5
^{242}Cm	42.2±0.1	139±3			13.6	0 - 17.4		
^{244}Cm	42.9±0.1	142.3±0.5			11.1	0 - 13.3		
^{246}Cm	42.9	142.4			10.7			
^{248}Cm	43.4±0.1	143.6±0.5			12.2	8.5 - 14.0		

TABLE (3.I) continued

	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_2(2+)$ (keV)	$E_3(2+)$ (keV)	γ from R(4) (degrees)	Range of γ from R(4) (degrees)	γ from R(2) (degrees)	γ from $R_3(2)$ (degrees)
^{250}Cf	42.2	140.4	1032		8.8		8.1	
^{254}Fm	44.9	149.6*	693		7.8		10.2	

SYSTEMATICS OF γ AND A_{DF}

The quantity A_{DF} was calculated using the value of γ obtained by method (ii). In figs. (3.2) and (3.3), γ and A_{DF} are shown as functions of the neutron numbers N. Systematics of γ were studied several years ago by Hiura and Suekane (1960), however they determined γ by the method (i) and were led to include many non-DF nuclei in their study. Also during the last few years a lot of new experimental data have become available, which are incorporated in fig. (3.2). It would be noticed that the behaviour of γ and A_{DF} is somewhat similar. For a given element both show a regular variation with N. In some cases there are some erratic points, but these could be due to uncertainties in the energy levels. Away from the closed shells, these systematics could be used for interpolation and extrapolation purposes. For example, for ^{222}Ra, we find $\gamma \approx 25^{o}$; combined with the experimental value of $E_1(2+) = 111.2$ keV, this gives $E_1(4+) = 315$ keV, $E_1(6+) = 594$ keV, etc. We can also surmise that γ for $E_2(2+)$ would be somewhat smaller, say $\gamma \simeq 22^{o}$ corresponding to this value for γ, we estimate $E_2(2+) = 341$ keV. We may add here that $E_2(2+)$ is very sensitive to small changes in γ and the estimated value of $E_2(2+)$ here is subject to much greater uncertainties than the ones for the ground state band.

SHELINE'S RELATION

A few years ago Sheline (1962) plotted the product of the mass number A and the energy of the first 2+ excited state of even nuclei

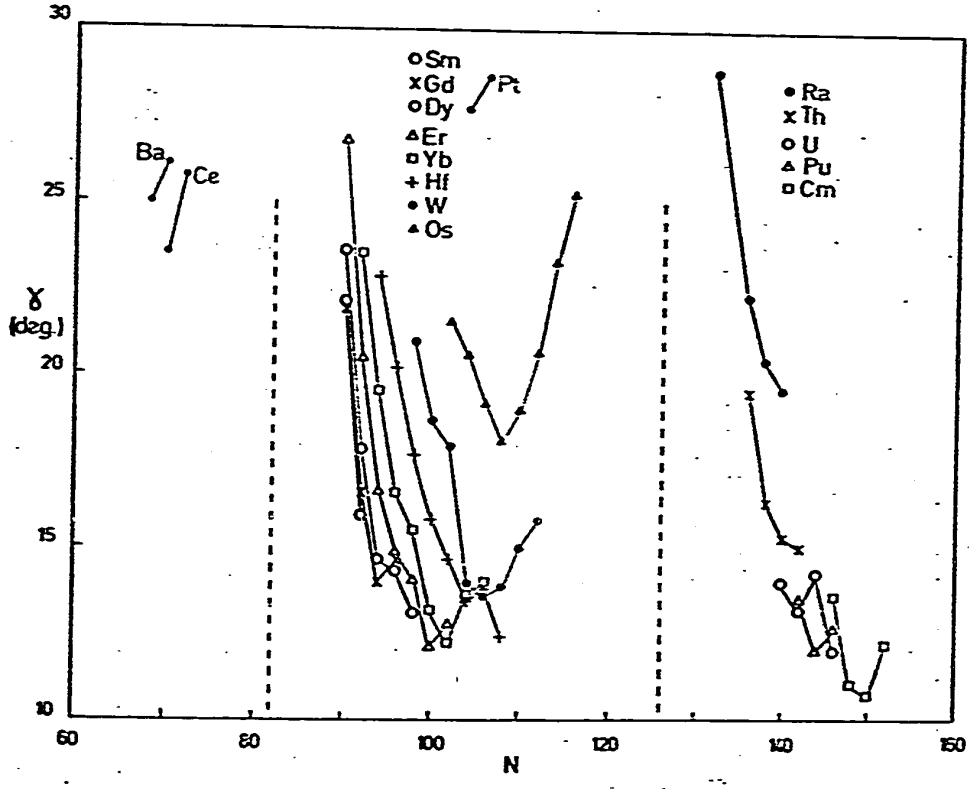

Fig (3.2). γ, determined by the method (ii) as a function of the neutron number N. Points for the same atomic number have been joined by straight lines.

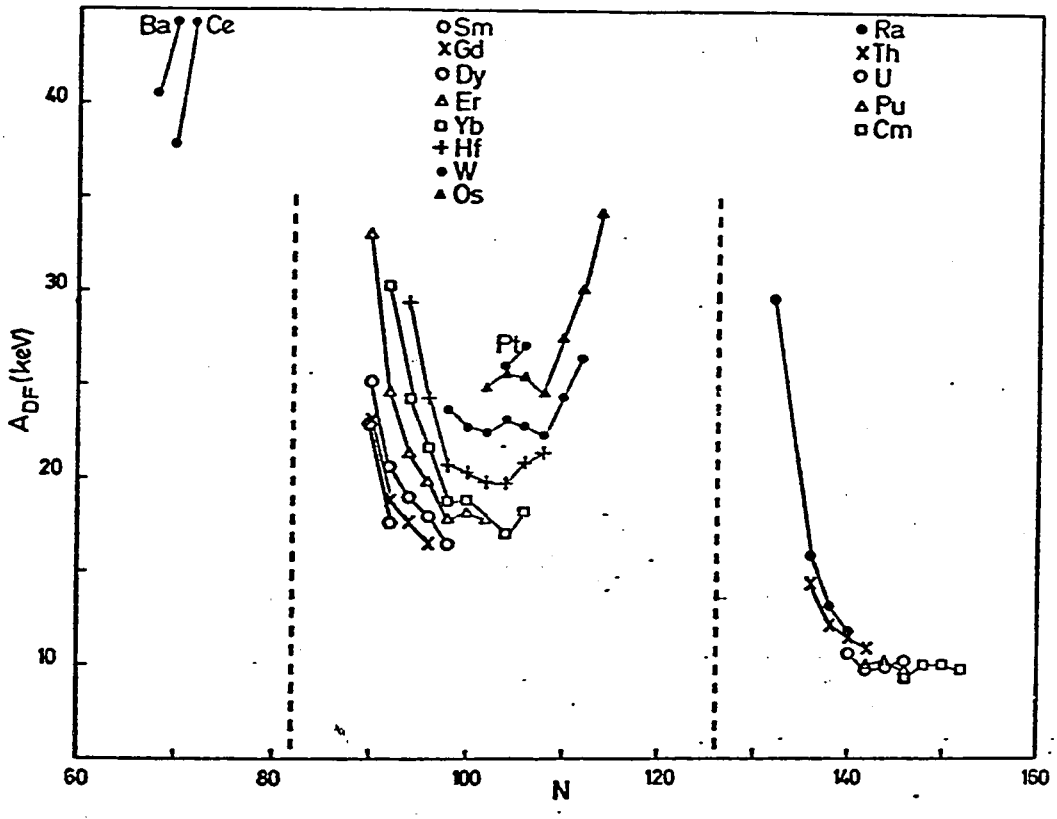

Fig (3.3). A_{DF} as a function of the neutron number N. Points for the same atomic number have been joined by straight lines.

against the parameter γ; γ being apparently determined by the method (i). He found an approximately linear relationship for $\gamma < 24^0$. For $\gamma > 24^0$ most of the nuclei fell far from the linear relation. On examination we found that practically all of the nuclei which belong to this category have $R(4) < \frac{8}{3}$. This suggested that if one were to restrict oneself to DF nuclei ($\frac{10}{3} > R(4) > \frac{8}{3}$), probably a linear relationship will hold. Also a lot of new experimental data had become available. So a re-examination of Sheline's relation was considered worthwhile.

In fig. (3.4) we show $(E_{2+})A$ versus γ for all such DF nuclei. The area enclosed in the small rectangle is shown on an enlarged scale in fig. (3.5). To avoid confusion, experimental uncertainties have not been shown. Near R(4) = 10/3, γ is very sensitive to small changes in R(4) and the points having $\gamma \approx 10^0$ have considerable uncertainties.

It would be noticed from fig. (3.4) that there is a broad band (width $\approx 8^0$) of linear correlation* between (E_{2+}) A and γ though one cannot speak of a linear relation between the two.

A NEW CORRELATION

We have plotted γ/A versus A_{DF} in fig. (3.6). The points are seen to fall in a band of linear correlation*.

* After the publication (Varshni and Bose 1970) of the above work Abecasis and Femenia (1971) have reexamined these correlations and have analysed them from a statistical point of view. The parameters were obtained by an overall least-squares fitting procedure. They found the above relationships to be equivalent at a 95% level of significance which led them to assume that there exists a strong linear dependence between the variables considered in Sheline's relationship as well as between γ/A and A_{DF}.

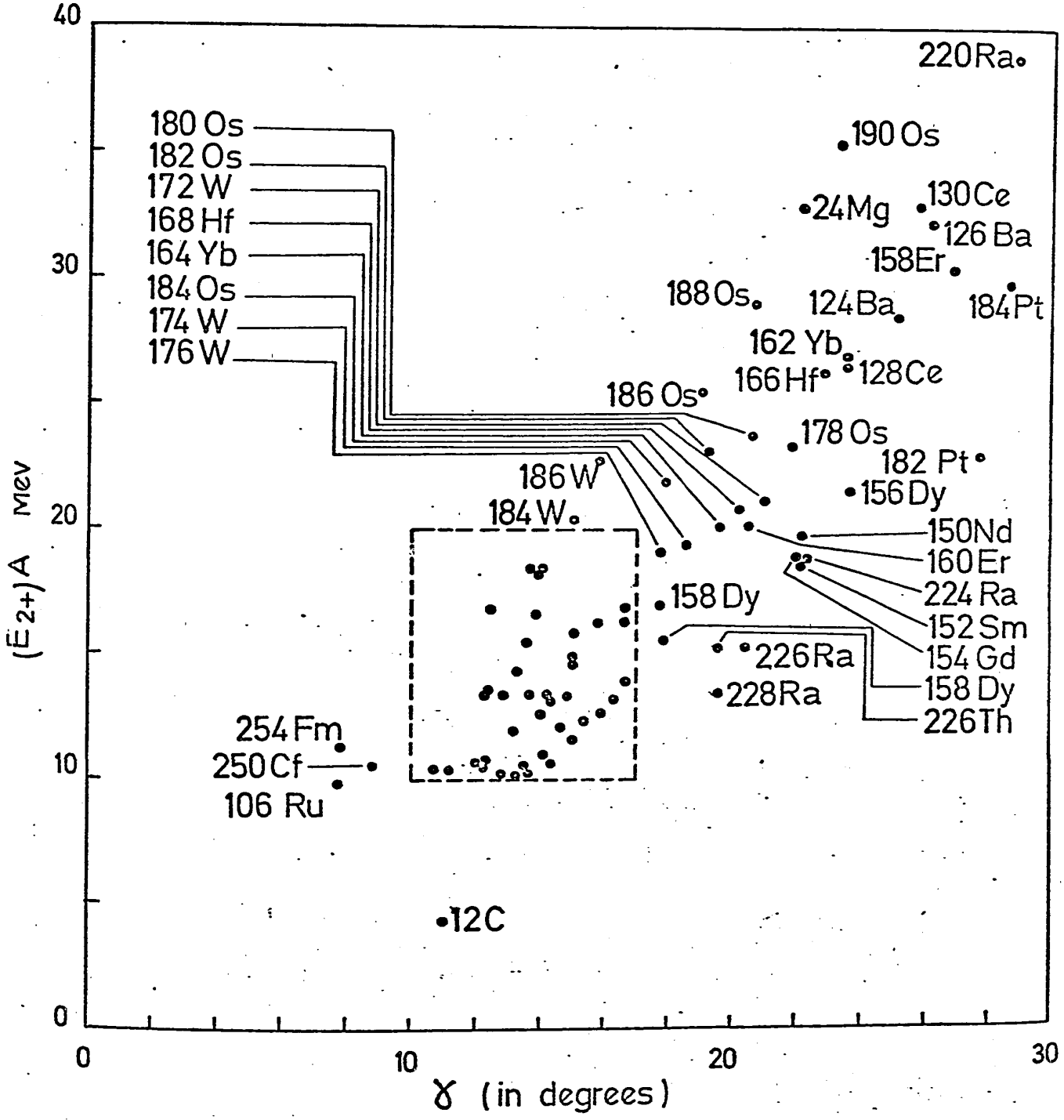

Fig (3.4). $(E_{2+})A$ versus γ. To minimize confusion, experimental uncertainties have not been shown.

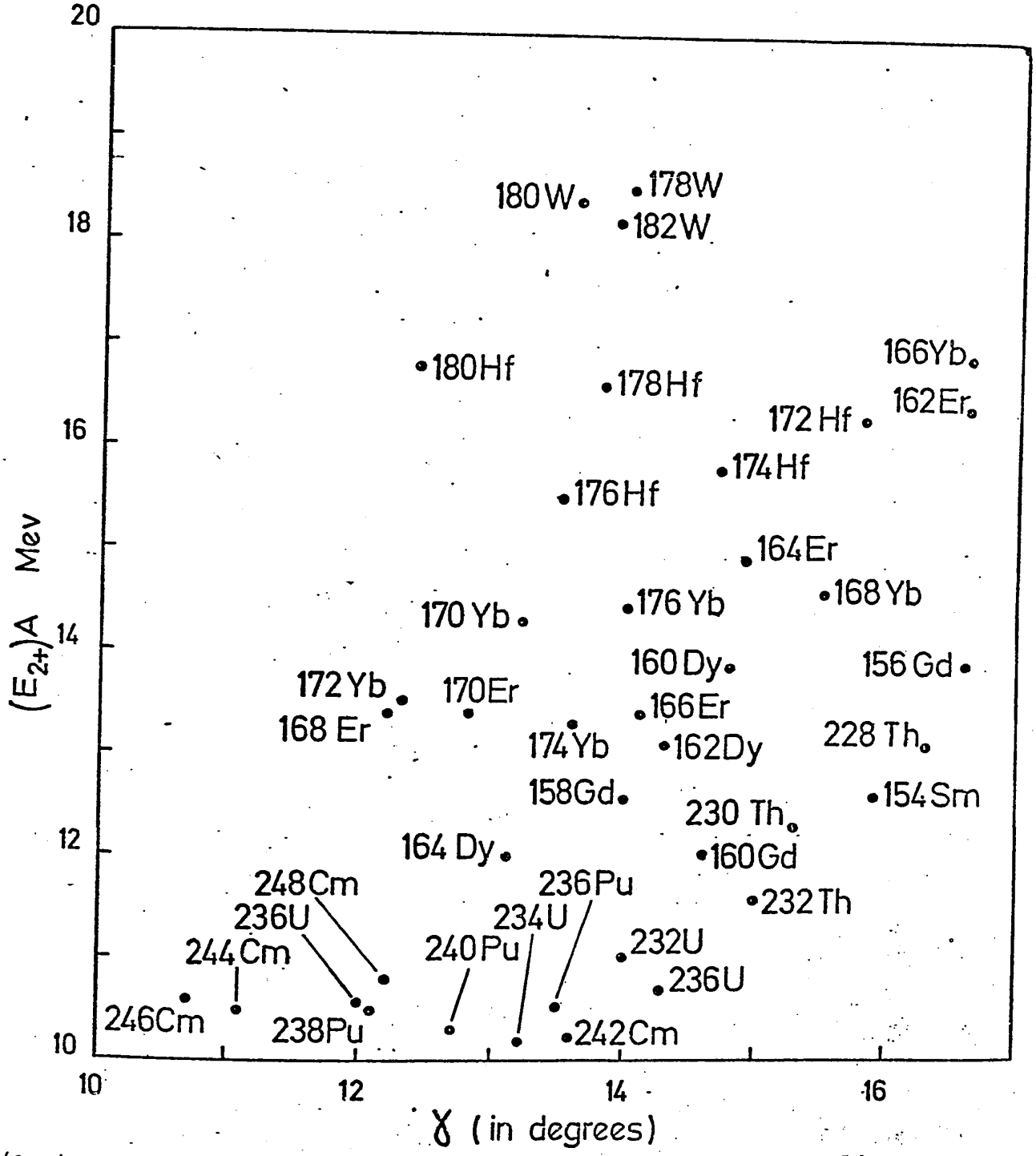

Fig (3.5). Enlarged version of the area enclosed by broken line rectangle in Fig (3.4).

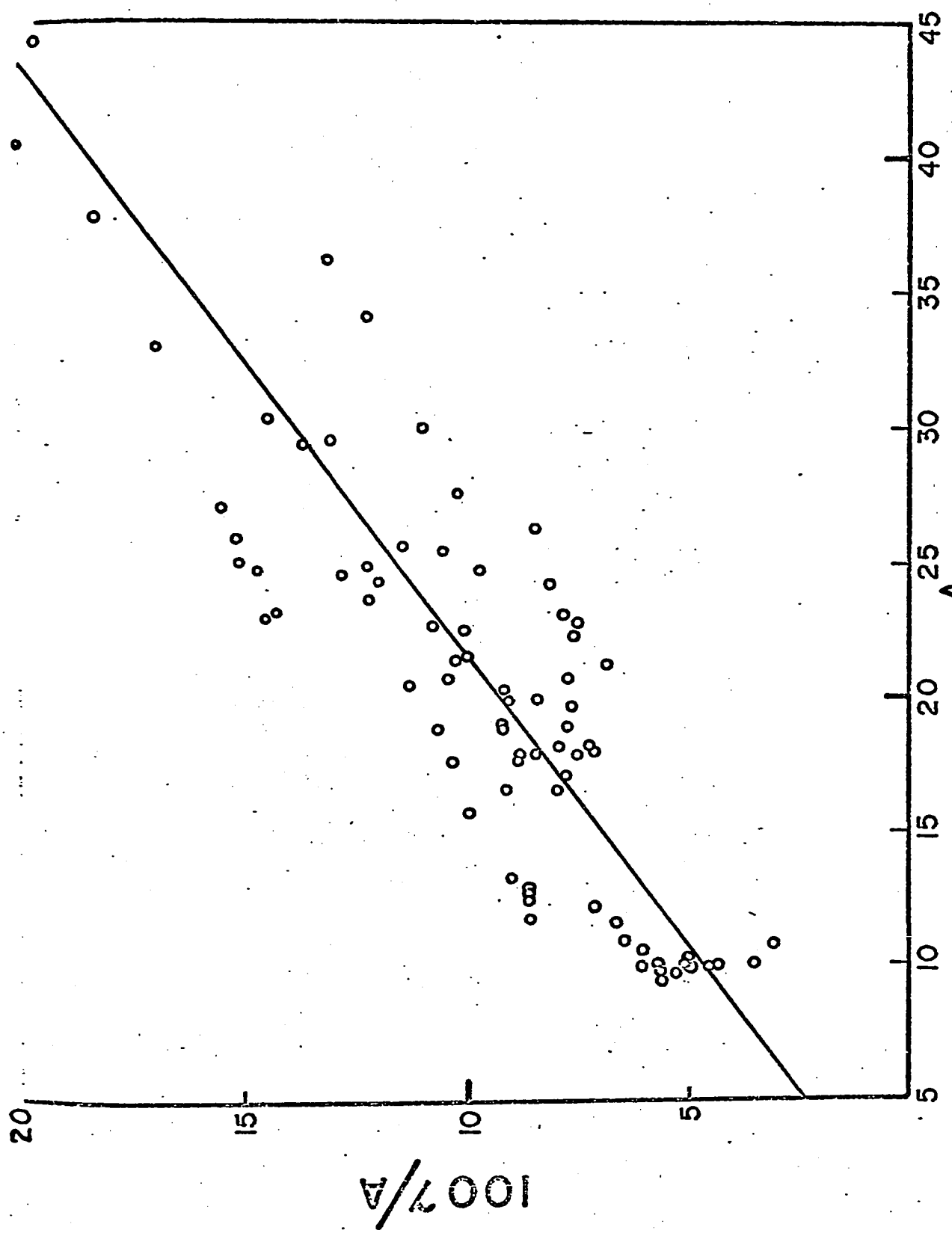

Fig (3.6). 100γ/A versus A_{DF}.

CHAPTER IV

ON THE KRUTOV MODEL FOR DEFORMED EVEN-EVEN NUCLEI

ABOUT THE MODEL

The starting point of the Bohr-Mottelson (1953) description of the collective motion in nuclei is its analogy with the dynamics of the "liquid drop". Davydov and Filippov (1959) have adopted the same analogy to obtain the Hamiltonian for the rotational motion of the deformed even-even nuclei. There are other groups of models (Inglis 1954, Belyaev 1959, Migdal 1959, Nilsson and Prior 1961) which start with Inglis' approach that the change in the intrinsic energy of the nucleus due to the rotation of the non-spherical potential is just its rotational energy. Recently Krutov (1968 a,b) has suggested a new approach to describe the rotational Hamiltonian of the deformed nucleus. He describes the collective motions in nuclei as the change in the density distribution of the nuclear matter with time.

Krutov (1968) and Krutov and Zackrevsky (1969 a,b) have applied this approach to nuclei having a non-axial equilibrium shape. Krutov limits himself to the case when the coupling between the rotation and intrinsic motion can be neglected, i.e. an asymmetric rotator. The rotational Hamiltonian of the non-axial nucleus is then equal to

$$H_{rot} = \frac{\hbar^2}{2} \sum_{\nu=1}^{3} \frac{I_\nu^2}{F_\nu} \qquad (4.1)$$

where I_ν is the angular momentum projection on the ν axis of the nucleus-fixed system; F_ν is the effective moment of inertia under the rotation around the ν axis.

According to the model suggested by Krutov

$$F_\nu = \int \tilde{\rho}_\nu \{ r^2 - (x'_\nu)^2 \} \, d\vec{r}' \tag{4.2a}$$

and

$$\tilde{\rho}_\nu (r') = \rho(r') - \{ \rho_{min} (r') \}_\nu \tag{4.2b}$$

where $\rho(r')$ is the nuclear mass density distribution, $\{\rho_{min} (r')\}_\nu$ is the minimum density at the point $\vec{r}'$ under the rotation of the nucleus around the ν axis, $\vec{r}'$ or $x_{\nu = 1,2,3}$ represents the coordinates in the nucleus-fixed system.

Assuming a uniform distribution of the density $\rho(r')$ and using equations (4.2) Krutov obtains

$$F_{1,2} = F_{rs} \frac{3}{2} \left(\frac{5}{\pi}\right)^{1/2} \beta \{ 1 - \frac{31}{112} \left(\frac{5}{\pi}\right)^{1/2} \beta + \frac{1.43}{\pi} \beta^2 \pm \frac{1}{\sqrt{3}} \gamma \ldots \} \tag{4.3a}$$

$$F_3 = F_{rs} \left(\frac{15}{\pi}\right)^{1/2} \beta \gamma \{ 1 - \frac{4}{7} \left(\frac{5}{\pi}\right)^{1/2} \beta + \ldots \} \tag{4.3b}$$

where β is the total deformation parameter of the nucleus, γ is the non-axiality parameter and F_{rs} is the moment of inertia of a rigid sphere possessing the nuclear mass and radius ($F_{rs} = \frac{2}{5} M_A R^2$, $R = 1.216\ A^{1/3}$fm). Higher order terms in the expansion (4.3a) and (4.3b) were neglected, since the values of β and γ for most of the nuclei considered in this model, render this to be a valid approximation. It is to be noted here that F_3 is a function of γ whereas in the Davydov-Filippov model, it is a function of γ^2(See Eq. (3.4)). Such functional relationship in the Davydov-Filippov description arises due to their basic assumption in treating the nuclear motions analogous to the surface oscillations of the "liquid drop".

Like the Davydov-Filippov (DF) (1968) model, Krutov's model also gives rise to a ground state band with spin sequence I = 0+,2+,4+,6+ etc., and a gamma band with the spin sequence 2+, 3+, 4+, 5+,..etc. For the Hamiltonian (4.1) it is possible to obtain analytical expressions for $E_1(2+)$ and $E_\gamma(2+)$ in terms of F_1, F_2, F_3. As mentioned in Chapter III $E_1(2+)$ is the energy of the 2+ level of the ground state band and $E_\gamma(2+)$ that of the 2+ level in the gamma band. Krutov and Zackrevsky (1969 a,b) have used the experimental values of the two 2+ states to calculate the parameters β and γ for 23 even-even nuclei. They restricted themselves to such even-even nuclei for which the following relation is closely satisfied:

$$E_1(2+) + E_\gamma(2+) = E_\gamma(3+) \qquad (4.4)$$

For an asymmetric rotator, equation (4.4) is exactly satisfied (Davidson 1968). The degree to which this relation is satisfied can be used as a criterion for ignoring the coupling between the rotation and intrinsic motion.

Krutov and Zackrevsky (1969 a,b) have also calculated the reduced probabilities of E2 transitions between two rotational levels of the non-axial nucleus, the magnetic moments of the lowest rotational states and the probabilities of magnetic dipole transitions between these states. These results are expressed in terms of β_e (the parameter of total charge deformation) and γ_e (the charge non-axiality parameter). Because of the difference between the mass and the charge distributions, these two parameters are different* from β and γ. The calculated values of the transition probabilities and gyromagnetic factors are in reasonable accord with the experimental values, lending support to the applicability of Krutov's model to such nuclei. These results show the consistency of the assumptions involved in the parameters β_e and γ_e, but do not say anything about β and γ.

OUTLINE OF THE PRESENT PROJECT

A satisfactory nuclear model should also be able to reproduce the observed energy levels. In the present chapter we have obtained the energies of the levels of the ground-state band of the doubly even nuclei applying Krutov's model. There are certain similarities between the DF and the Krutov models. Both are asymmetric rotor models, both disregard the

* The separation of charge and mass parameters requires a renormalization of the moment of inertia expressions in terms of β_e and γ_e.

vibration - rotation interaction and both have two parameters involved in the calculation of energy levels. A comparison of the two is thought to be appropriate. Here, we have also obtained the ground band energies for the DF model and the calculated values from the two models have been compared with the experimental values.

PROCEDURE FOR THE CALCULATION OF ENERGY LEVELS

In chapter III we have given expressions for the Hamiltonian of nuclear rotational motion and the energy levels for spin I = 2+, 3+, 4+ and 5+ for the DF model. Similar analytical expressions for energies can be written in the case of Krutov's model. The only difference lies in the form of the moment of inertia used in the above two cases. For Krutov's model, Eqs. (4.3a) and (4.3b) are in the form of a series in terms of the parameters β and γ and hence the analytical form of the energy equations are quite lengthy to handle. Nevertheless for lower spin values we have used the analytical forms for calculating the level energy. For example the energy equation for the 2+ levels in terms of the moments-of-inertia F_1, F_2, and F_3 is given by

$$E_{1,\gamma}(2+) = \hbar^2\left[\sum_{\nu=1}^{3}\frac{1}{F_\nu} \mp \left\{\left(\sum_{\nu=1}^{3}\frac{1}{F_\nu}\right)^2 + \sum_{\nu<k}\frac{1}{F_\nu F_k}\right\}^{1/2}\right] \quad (4.5)$$

where the negative sign gives the 2+ level for the ground state band and the positive sign is for the same level in the gamma band.

Even for the DF model, the expressions for level energy beyond the 4+ level become quite complicated (see Ch. III). In order to obtain the numerical values for level energies with higher spins, we have adopted a procedure analogous to the one used in the calculations of energy spectrum of the asymmetric top molecules.

Fairly elaborate calculations have been carried out by various authors (Witmer 1927, Wang 1929, Kramers and Ittmann 1929, Klein 1929, Ray 1932) to obtain the energy spectrum of the asymmetric top molecules. The method we have used here has been suggested by Ray (1932) and extensively used by King, Hainer and Cross (1943). Following their notation the energy expression for the level with spin I can be written as

$$E(I_\tau) = \frac{1}{2}\ (A+C)I(I+1) + \frac{1}{2}\ (A-C)\ E_\tau(\kappa) \qquad (4.6)$$

where κ is defined as the asymmetry parameter and is a function of A, B, and C;

$$\kappa = \frac{2[B - \frac{1}{2}\ (A+C)]}{A-C} \qquad (4.7)$$

here A, B, and C are the reciprocals of the moments-of-inertia;

$$A = \frac{\hbar^2}{2F_1}\ ;\ B = \frac{\hbar^2}{2F_2}\ ;\ C = \frac{\hbar^2}{2F_3}$$

and τ is a number which has (2I+1) values for a fixed value of the spin I.

The parameter κ has values between -1 and +1 which correspond to the prolate and the oblate shapes of the nucleus respectively. Day and Mallmann (1960) have published tables for various values of κ each corresponding to a value of γ (the DF asymmetry parameter)

In Eq. (4.6), $E_\tau(\kappa)$ is the quantity that replaces the term with K^2 in the case of a symmetric rotor. The procedure of calculating $E_\tau(\kappa)$ has been discussed by Allen and Cross (1963) and it was also followed here

RESULTS

In the case of the asymmetric rotor two-parameter models, the values of the parameters may be obtained by either of the following two methods, as described in the previous chapter

(i) From $E_1(2+)$ and $E_\gamma(2+)$. For Krutov's model, Krutov and Zackrevsky (1969) have calculated β and γ by this method and these are reproduced in Table (4.I). To compare the results obtained from these parameters with those of the DF model, the parameter γ_{DF}* occuring in the DF model was also calculated from $E_1(2+)$ and $E_\gamma(2+)$ and the resulting values are shown in Table (4.II). No value is shown for ^{194}Pt, as for this nucleus $E_\gamma(2+)/E_1(2+)$ is less than 2, which is below the DF range. The experimental data employed in these calculations are shown in Table (4.III.). The values of the energy levels tbr the ground state band of these 23 nuclei as calculated by using the DF as well as the Krutov model

* To distinguish between the two γ's a subscript DF has been added to the γ values for the DF model.

Table (4-I)

Parameters of Krutov's model as calculated by methods (i) and (ii).

Nucleus	Method (i)		Method (ii)	
	β	γ (degrees)	β	γ (degrees)
^{150}Nd	0.239	3.567	0.2210	29.08
^{152}Sm	0.221	3.167	0.2351	29.04
^{154}Sm	0.331	1.783	0.3402	17.20
^{154}Gd	0.218	4.000	0.2273	28.61
^{156}Gd	0.298	2.450	0.3069	18.14
^{158}Gd	0.327	2.117	0.3336	14.46
^{160}Gd	0.338	2.333	0.3459	15.30
^{160}Dy	0.293	2.867	0.2997	16.02
^{162}Dy	0.308	2.884	0.3147	13.72
^{164}Dy	0.349	2.884	0.3387	13.23
^{164}Er	0.229	4.567	0.2732	15.46
^{166}Er	0.286	3.383	0.3025	14.56
^{168}Er	0.293	3.100	0.2975	11.93
^{170}Er	0.289	2.717	0.2950	12.56
^{172}Yb	0.285	1.683	0.2899	11.93
^{176}Yb	0.263	2.050	0.2682	14.20
^{182}W	0.202	2.633	0.2059	13.78
^{186}W	0.158	5.583	0.1618	16.39
^{188}Os	0.123	8.467	0.1262	25.09
^{190}Os	0.100	12.0		
^{192}Os	0.089	14.834		
^{194}Pt	0.055	21.0		
^{232}Th	0.272	2.017	0.2804	15.65

TABLE (4.II)

Parameters of the Davydov-Filippov model as calculated by methods (i) and (ii)

Nucleus	Method (i) γ_{DF} (degrees)	Method (ii) γ_{DF} (degrees)
^{150}Nd	13.91	22.15
^{152}Sm	13.23	22.13
^{154}Sm	9.54	15.87
^{154}Gd	13.84	22.00
^{156}Gd	11.05	16.62
^{158}Gd	10.33	14.07
^{160}Gd	10.87	14.60
^{160}Dy	11.90	15.22
^{162}Dy	11.94	13.53
^{164}Dy	12.30	13.11
^{164}Er	12.88	14.93
^{166}Er	12.67	14.18
^{168}Er	12.35	12.29
^{170}Er	11.58	12.76
^{172}Yb	9.26	12.37
^{176}Yb	10.15	14.04
^{182}W	11.38	13.89
^{186}W	16.03	15.84
^{188}Os	19.16	20.75
^{190}Os	22.28	23.39
^{192}Os	25.19	25.33
^{194}Pt		
^{232}Th	10.03	15.03

TABLE (4.III)

Experimental data employed in the determination of parameters. Most of the data are from the compilations of Lederer et al. (1967) and Mariscotti et al. (1969). Ground state band values for ^{160}Dy and ^{162}Dy are from Ewan and Andersson (1968). For purposes of identification, $E_\gamma(2+)$ was taken to be the energy of that 2+ state which most closely satisfies relation [4.4]

Nucleus	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_\gamma(2+)$ (keV)	y	100 ρ
^{150}Nd	132	397	1060	0.326	
^{152}Sm	121.78 ± .05	366.4 ± .3	1087	0.325	-2.17
^{154}Sm	81.99 ± .05	267 ± 1	1440	0.077	-0.13
^{154}Gd	123.07 ± .05	371.2 ± .2	999	0.317	-0.80
^{156}Gd	88.967 ± .005	288.16 ± .05	1154	0.094	-0.65
^{158}Gd	79.51 ± .01	261.45 ± .05	1185	0.045	-0.28
^{160}Gd	75.3 ± .5	247 ± 2	1010	0.053	
^{160}Dy	86.8	283.8	966.1	0.064	+0.35
^{162}Dy	80.6	265.6	890	0.038	
^{164}Dy	73.39 ± .05	242.2 ± .1	761.8	0.033	+0.84
^{164}Er	91 ± 1	298 ± 3	858	0.059	+0.32
^{166}Er	80.6 ± .05	264.9 ± .2	787	0.047	+0.88
^{168}Er	79.8 ± .5	264 ± .5	822	0.025	+0.53
^{170}Er	79 ± 5	261 ± 2	930	0.055	
^{172}Yb	78.7 ± .5	260.3 ± 1	1468	0.026	-0.21
^{176}Yb	82.1 ± .5	270 ± 3	1270	0.045	
^{182}W	100.1 ± .05	329.4 ± .05	1222	0.043	-0.67
^{186}W	122.5	399	730	0.076	
^{188}Os	155 ± .1	477.9 ± .1	633	0.250	-0.23

(cont'd ..)

Table (4.III) cont...

Nucleus	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_\gamma(2+)$ (keV)	y	100 ρ
^{190}Os	186.7 ± .1	547.8 ± .1	557.9	0.399	-1.53
^{192}Os	205.79	580.4	489.1	0.513	+0.65
^{194}Pt	328.5 ± 1	811.1 ± 2	622.1	0.864	
^{232}Th	49.8 ± .1	163 ± 1	788	0.060	

are compared with the experimental values in figures (4.1a) and (4.1b).

(ii) From $E_1(2+)$ and $E_1(4+)$. In chapter III we have discussed that for the DF model this method gives better results for the ground state bands of a number of nuclei than those obtained by the method (i). The parameter γ_{DF} in the DF model was determined from the ratio $R(4)=E_1(4+)/E(2+)$ and the values obtained are shown in Table (4.II). No value is shown against ^{194}Pt as for this nucleus $R(4)<\frac{8}{3}$ (the limiting value for the DF model). The parameters β and γ for the Krutov model were also determined from $E_1(2+)$ and $E_1(4+)$ by an iterative process with the constraints $0<\beta<1$ and $0<\gamma<30^0$. The calculated values thus obtained are shown in Table (4.1). For ^{190}Os, ^{192}Os and ^{194}Pt, values of β and γ could not be obtained within the stipulated ranges.

Using these parameters, energy levels for the ground-state band were again calculated for the two models and the results are shown, along with the experimental values, in figures (4.2a) and (4.2b).

ANALYSIS OF THE RESULTS

a) Results by method (i)

Generally speaking, results by both the models are poor, the Davydov-Filippov ones being the better of the two. We may note here that the calculated values of 2+ levels of ^{150}Nd and ^{164}Er by the Krutov model do not coincide with the observed ones; presumably Krutov and Zackrevsky used experimental values for these two energy levels different from those given in Table (4.III).

The pattern of results obtained by the two models is similar; if for a nucleus the DF model gives, relatively speaking, good results, so does the Krutov model. Similarly if the DF model gives very poor results for some nucleus, so does the Krutov one. This can be attributed to the fact that both are pure rotator models and the degree of agreement would depend on how closely a given nucleus corresponds to this picture.

For a rigid symmetric rotator, the following ratio (Davidson 1968) should be equal to zero:

$$\rho = \frac{E_1(2+) + E_\gamma(2+) - E_\gamma(3+)}{E_1(2+) + E_\gamma(2+)} \tag{4.8}$$

The degree of deviation of this ratio from zero can give an idea of the influence of the rotation-vibration interaction. In column 6 of Table (4.III) we have recorded the values of 100ρ for such nuclei for which data are available for the three energy levels. By definition we have considered here only such nuclei which closely satisfy equation (4.4), consequently, even small uncertainties in the individual energy levels lead to a very large percentage uncertainty (sometimes of the order of 100%) in the value of ρ. This makes a correlation of this ratio with the results of figures (4.1a) and (4.1b) somewhat difficult and inconclusive.

We can, however, turn to another criterion, which is based on the Bohr-Mottelson model. For a rigid axially symmetric rotator, the following quantity

$$y = \frac{10}{3} - \frac{E_1(4+)}{E_1(2+)} \tag{4.9}$$

may be considered to be an approximate measure of the strength of the rotation-vibration interaction. This quantity is tabulated in column 5 of Table (4.III).A perusal of Table (4.III) and figures (4.1a) and (4.1b) shows that the degree of agreement between theory and experiment is closely correlated with the value of y. As an illustration we may quote here three cases:

^{164}Dy agreement good, $y = 0.033$

^{152}Sm agreement poor, $y = 0.325$

^{194}Pt no agreement, $y = 0.864$

Thus it appears highly likely that the differences between the observed and the calculated values are due to neglect of the β-vibration in the two models. For the Davydov-Filippov case, an improved model which incorporates the β-vibration does exist - the Davydov-Chaban (1960) model. For the Krutov model, to a first degree of approximation the effect of β-vibration can be allowed by expressing the energy of a level as

$$E(I) = E_K(I) - b\{E_K(I)\}^2 \qquad (4.10)$$

where $E_K(I)$ represents the energy of the level given by the Krutov model for the rigid-rotator case, and b is a parameter .

(b) Results by method (ii)

At first sight it may appear that this method would almost certainly lead to an improvement in the calculated values. This however is not necessarily so in all cases. As an illustration, in figure (4.3)

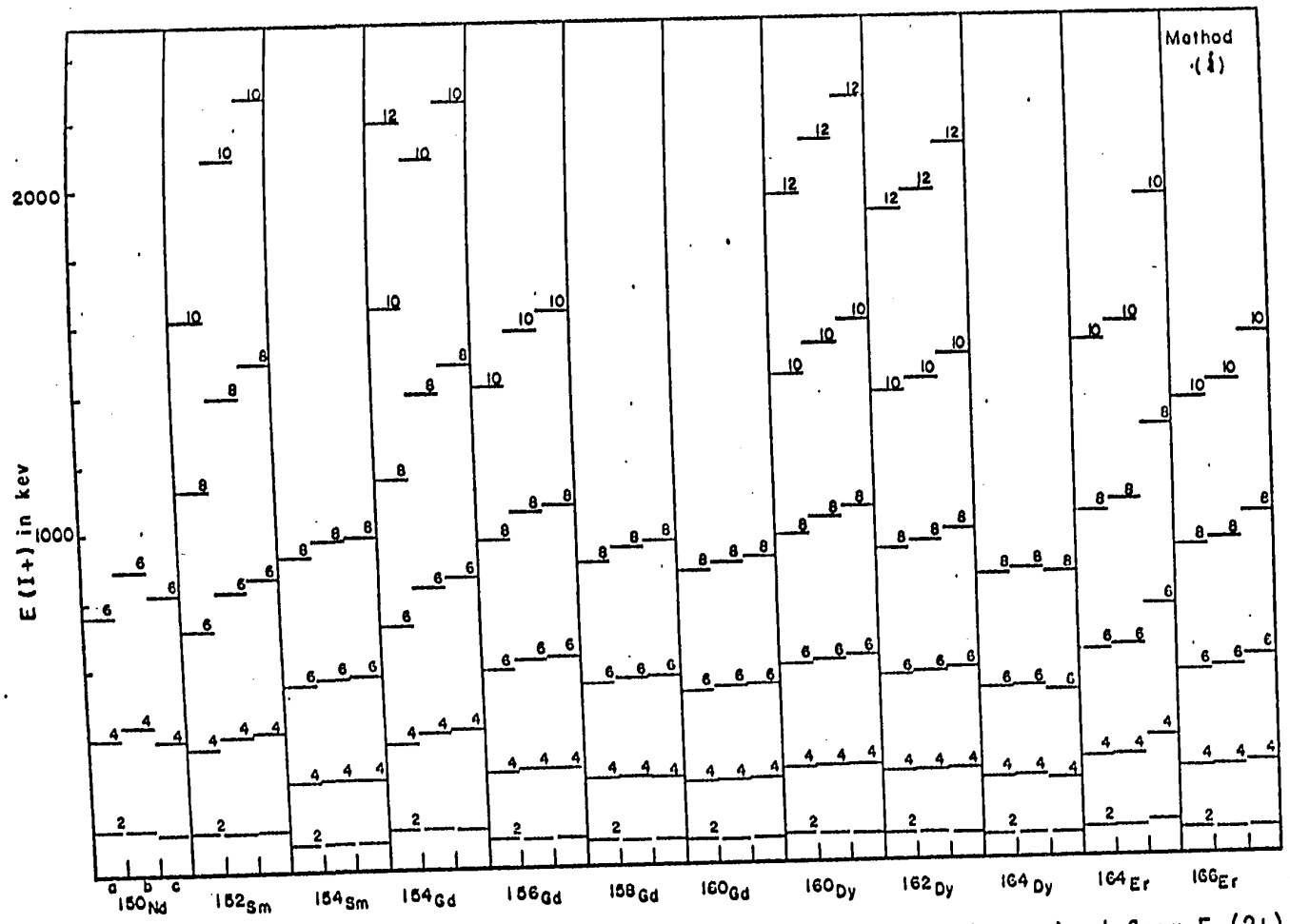

Fig. (4-1a) Experimental and calculated energy levels. The parameters were determined from $E_1(2+)$ and $E_\gamma(2+)$. There are three columns for each nucleus, see e.g. ^{150}Nd- a, shows the experimental energy levels, b, those calculated from the Davydov-Filippov model and c, those calculated from the Krutov model. Spins have been identified; parity in all cases is positive.

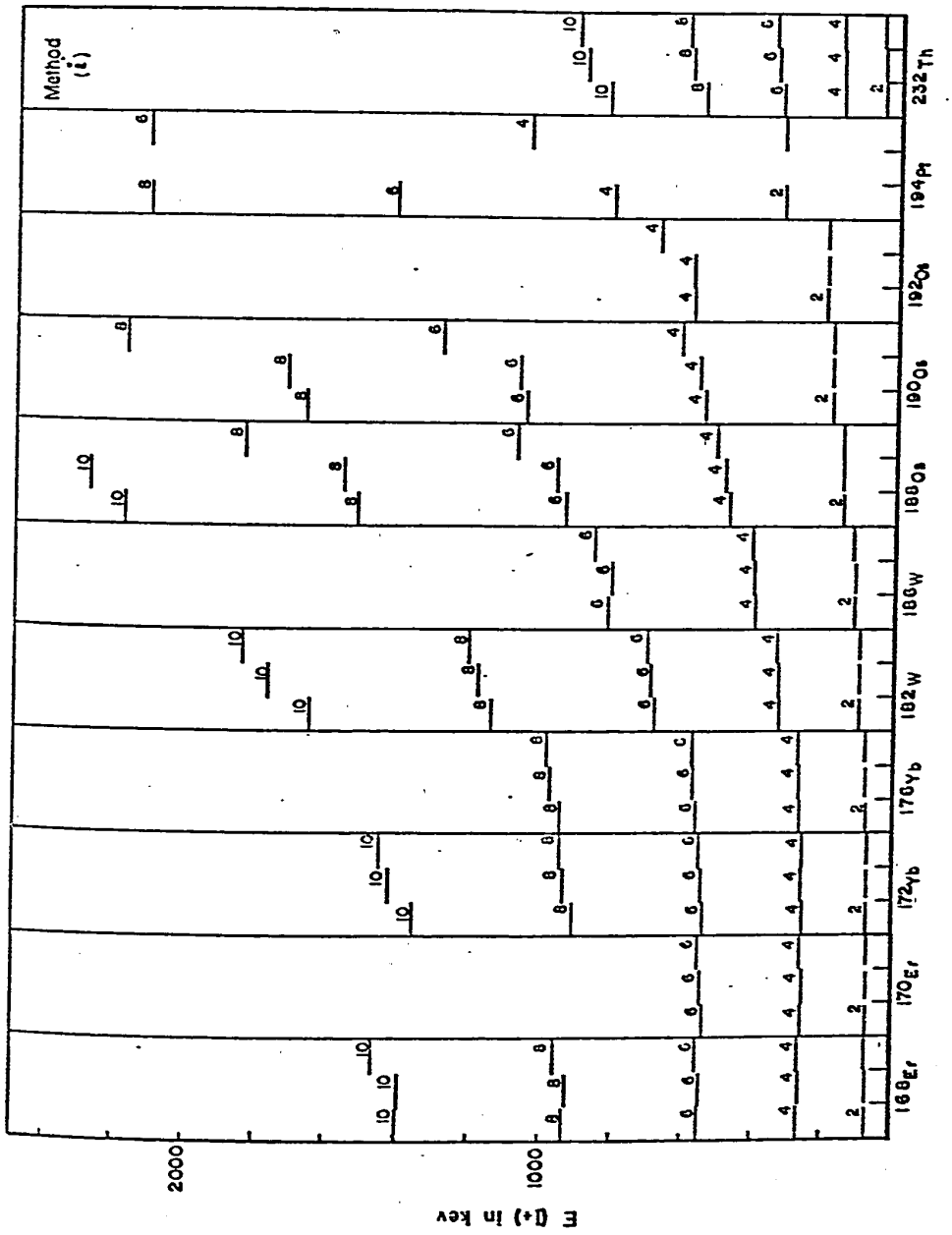

Fig. (4-1b) See explanation below Fig. (4-1a)

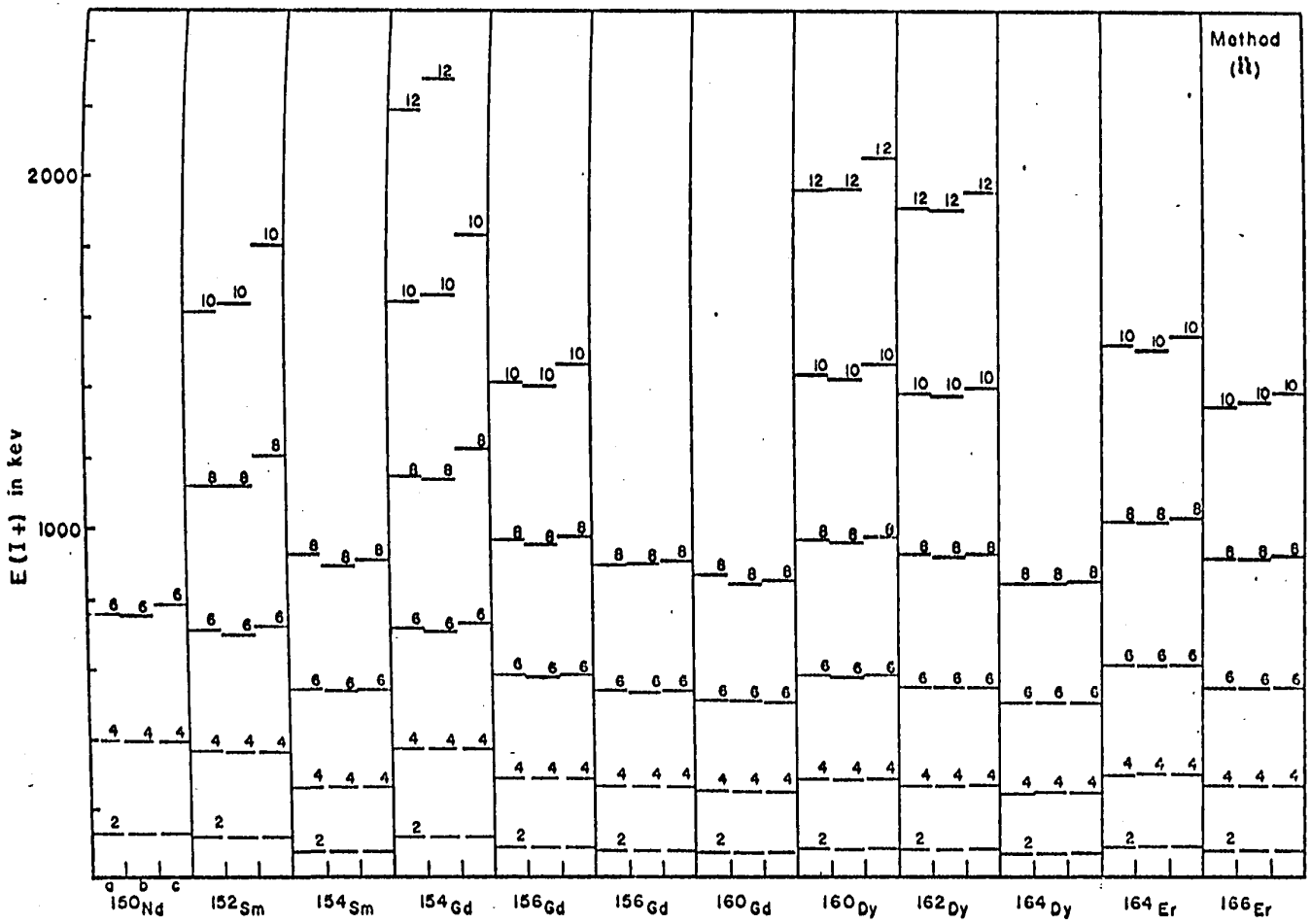

Fig. (4-2a) Experimental and calculated energy levels. The parameters were determined from $E_1(2+)$ and $E_1(4+)$ There are three columns for each nucleus, see e.g. ^{150}Nd- a, shows the experimental energy levels, b, those calculated from the Davydov-Filippov model and c, those calculated from the Krutov model. Spins have been identified; parity in all cases is positive.

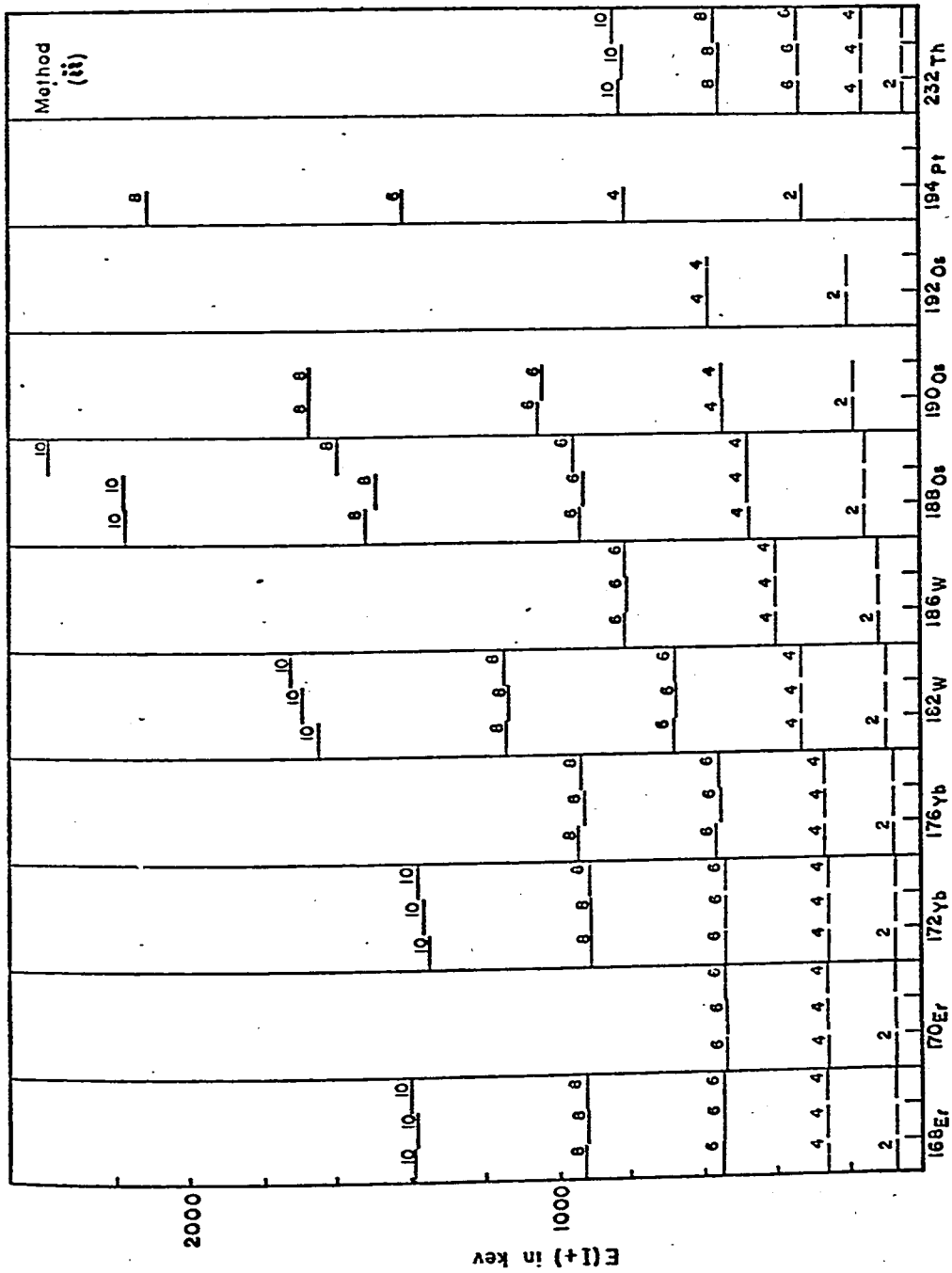

Fig. (4-2b) See explanation below Fig. (4-2a)

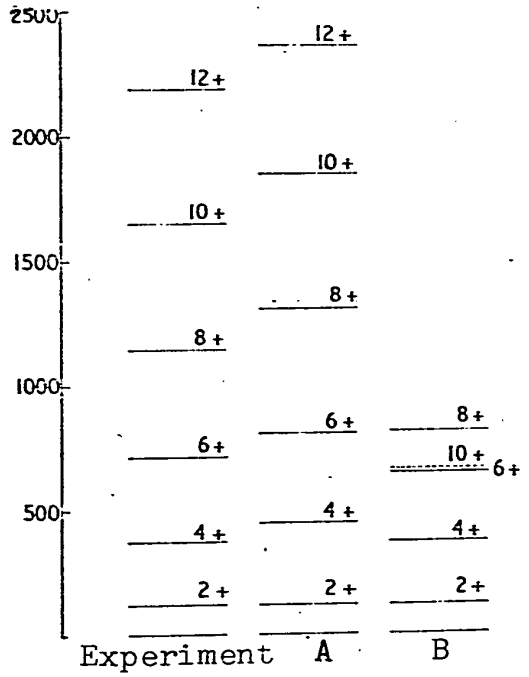

Fig.(4.3) Energy levels of ^{154}Gd. A, Parameters of the Bohr-Mottelson model determined from $E_1(2+)$ and $E_\gamma(2+)$. B, Parameters determined from $E_1(2+)$ and $E_1(4+)$.

we show the results obtained from a two parameter Bohr-Mottelson equation:

$$E = AI(I+1) + BI^2(I+1)^2 \qquad (4.11)$$

In the case indicated in figure (4.3) A, and B were obtained from $E_1(2+)$ and $E_\gamma(2+)$ (contribution of E_β to B was not taken into account). In the second case, figure (4.3)B, A and B were determined from $E_1(2+)$ and $E_1(4+)$. It would be noticed that in B the agreement has lessened.

The results shown in figures (4.2a) and(4.2b) for Krutov and DF models show a very marked improvement over those obtained by method (i). For 6+ and 8+ levels, the results by both the models are of comparable agreement with the experimental data. However, for higher spin levels, i.e. 10+ and 12+, the Davydov-Filippov model gives better results than the Krutov one.

A comparison of the results by the two methods for the DF model shows an interesting feature. In method (i) we find that the differences between the observed and the calculated energy levels are in the same direction. In method (ii), by determining the parameters from $E_1(2+)$ and $E_1(4+)$ one is sort of partially correcting for the rotation-vibration interaction. In figures (4.2a) and (4.2b) it should be noticed that in several cases the sign of the (calculated-observed) tends to change as one goes to higher spin levels, ^{152}Sm and ^{154}Gd are good examples.

A comparison of γ_e and γ (method (i)) as determined by Krutov and Zackrevsky (1969) indicates that the charge distribution is more asymmetric than the mass distribution in most of the nuclei considered here. However, if we compare γ_e and γ (method (ii)), the reverse appears to be the case. We must add here though, that the latter comparison is not very meaningful because γ_e was obtained by a method which would correspond to method (i). It would, of course, be better to compare γ(ii) with γ_e obtained from the ground state transition probabilities, however, no such calculations are available.

For the Krutov model (Table (4.I)), it would be noticed that the values of β obtained by the two methods are very close, but the γ values are very different, being rather large and somewhat unrealistic in the second case.

The difference between the γ_{DF} as obtained by the two methods are usually much less than those for the γ of the Krutov model. On intuitive grounds, the behaviour of γ_{DF} appears to be closer to the physical situation; the rotation-vibration interaction is after all not very large and it should not drastically change the asymmetry of the nucleus.

CHAPTER V

HIGHER-ORDER TERMS IN THE VARIABLE MOMENT-OF-INERTIA MODEL

ABOUT THE VMI MODEL

In chapter II of this thesis we have briefly described the "Variable Moment-of-Inertia"(VMI) model proposed by Mariscotti, Scharff-Goldhaber and Buck (1969). The energy expression in this model is given as a sum of a kinetic and a potential energy term written as

$$E_I = \frac{I(I+1)}{2\mathcal{J}_I} + \frac{1}{2} C (\mathcal{J}_I - \mathcal{J}_0)^2, \tag{5.1}$$

where C is called the restoring force constant and $\mathcal{J}_0$ is the ground state moment-of-inertia. The equilibrium condition which determines the moment-of-inertia $\mathcal{J}_I$ (in units of $\hbar^2$) for a state with spin I is given by

$$\frac{\partial E(\mathcal{J})}{\partial \mathcal{J}} = 0 \tag{5.2}$$

Eq. (5.1) combined with Eq. (5.2) yields the following equation

$$E_I = [I(I+1)/2\mathcal{J}_I] \{1 + [I(I+1)/4C\mathcal{J}_I^3]\} \tag{5.3}$$

The two parameters $\mathcal{J}_0$ and C characterize each nucleus defining its moment-of-inertia and energy for each state with angular momentum I for the ground-state rotational band. Both E_I and $\mathcal{J}_I$ are increasing functions of I. The "softness" i.e. the relative increase of the moment-of-inertia with the angular momentum I (defined by Morinaga (1966)

as $\frac{1}{\mathcal{J}_I}\ \frac{\Delta\mathcal{J}}{\Delta I}$) can be expressed as

$$\mathcal{J}^{-1}(d\mathcal{J}/dI) = [(2I+1)/2C^{2}(3\mathcal{J}-2\mathcal{J}_0)] \qquad (5.4)$$

and for the particular case I = 0,

$$\sigma = [\mathcal{J}^{-1}(d\mathcal{J}/dI)]_{I=0} = \frac{1}{2C\mathcal{J}_0^3} ,$$

where σ is the "softness parameter" which characterizes the nucleus into various categories depending upon its value for the particular nucleus. $\sigma = 0$ represents a hard nucleus and sets the "adiabatic limit" for the rotating motion whereas for those nuclei which are very soft $\sigma \to \infty$. Within these two limits the VMI model gives the range for $R_4 (= E_4/E_2)$ between 3.33 and 2.23 respectively which determines the limits of validity of this model. It was further shown by MSB that these two parameters vary smoothly with N and Z numbers of the nucleus: $\mathcal{J}_0$ reaches highest and σ the lowest values at the stability line for nuclei farthest removed from magic proton and neutron numbers.

An interesting feature of the VMI model is its mathematical equivalence with the Harris (1965) model. Harris has shown that an extension of the cranking model to the next order of perturbation theory in the angular velocity ω leads to a very good agreement with the experimental data on rotational bands of even-even nuclei in the rare-earth region. This model is based on the two following equations

$$E_I' = \frac{1}{2}\ \omega^2\ (\mathcal{J}_0' + 3C'\omega^2) \qquad (5.5a)$$

and

$$[I(I+1)]^{1/2} = \omega(\mathcal{J}_0' + 2C'\omega^2) \tag{5.5b}$$

Here the parameters are $\mathcal{J}_0'$ and C'. By eliminating ω from the above two equations an energy expression in terms of $\mathcal{J}_0'$ and C' can be derived. As Mariscotti et al (1969) have pointed out, if the moment of inertia $\mathcal{J}_I$ is defined as

$$\mathcal{J}_I = [I(I+1)]^{1/2}/\omega \tag{5.6}$$

one obtains, from Eq. (5.5a)

$$\mathcal{J}_I = \mathcal{J}_0' + 2C'\omega^2 = \mathcal{J}_0' + 2C'[I(I+1)/\mathcal{J}_I^2] \tag{5.7}$$

If the parameters C and $\mathcal{J}_0$ of the VMI model are made equal to $\frac{1}{4C'}$ and $\mathcal{J}_0'$ respectively of the Harris model, one obtains by using Eq. (5.7) the energy E_I' written as

$$E_I' = \frac{1}{2}\omega^2 (\mathcal{J}_I + C'\omega^2)$$

Substituting ω from Eq. (5.6), one obtains

$$E_I' = \frac{1}{2}\{\mathcal{J}_I + C'[I(I+1)/\mathcal{J}_I^2]\}[I(I+1)/\mathcal{J}_I^2]$$

or

$$E_I' = I(I+1)/2\mathcal{J}_I \{1 + C'[I(I+1)/\mathcal{J}_I^3]\} \tag{5.8}$$

Although the two parameter Harris model and the VMI description appear to be completely unrelated, both lead, surprisingly to the same energy expression (Eqs. (5.8) and (5.3)).

MSB have compared the experimental and theoretical values for a large number (~88) of even-even nuclei and have shown that the agreement between the two is quite good. However, a closer analysis of the available data shows that in quite a few cases (e.g. ^{120}Xe, ^{122}Xe, ^{124}Xe, ^{126}Xe, ^{190}Os etc.) the rms percent deviation of the VMI calculated energies from the experimental ones are substantially greater than the experimental uncertainties.

IN THIS PROJECT: INTRODUCTION

Here we wish to show that the deviations between the VMI calculated and the experimental energies can arise from the higher order terms in the potential energy component $(\frac{1}{2} C(\gamma-\gamma_0)^2)$ of Eq. (5.1). We have extended the VMI energy equation by adding an anharmonic term to the potential energy. We have then applied this new model, called VMI23, to calculate the ground-state bands of 122 even-even nuclei. We have shown that the inclusion of the anharmonic term improves agreement with the experimental data in many cases and provides a sensitive means of probing the potential energy surfaces of nuclei.

ANALYSIS OF THE POTENTIAL ENERGY SURFACES

In fig. (5.1a) we have sketched the potential energy as a function of the deformation β. The general shape of the curve was first discussed by Alder et al (1956). Belyaev (1958) and Szymanski (1970) have shown that such a potential surface can be obtained from the superfluid model of the nucleus. Myers and Swiatecki (1966) have proposed the following analytical form for $V(\beta)$ from a semi-empirical theory of nuclear masses and deformations:

$$v(\beta) = A\beta^2 - B\beta^3 + Se^{-d\beta^2} \tag{5.9}$$

where A, B, S, and d are parameters. Any expression representing $V(\beta)$ can be expanded about the equilibrium value β_0 by Taylor's theorem as follows:

$$V(\beta) = V(\beta_0) + V''(\beta_0)\frac{(\beta-\beta_0)^2}{2!} + V'''(\beta_0)\frac{(\beta-\beta_0)^3}{3!}$$

$$+ V^{IV}(\beta_0)\frac{(\beta-\beta_0)^4}{4!} + \ldots \text{ etc.} \tag{5.10}$$

We have examined the behaviour of the potential energy surface for a large number of even-even nuclei using Myers-Swiatecki parameters. It was found that in those cases (~110) where the resulting curve is of the "conventional" form, by which we mean the type of the curve shown by solid line in fig. (5-1a), the nuclei can be classified into three broad categories with regard to the effect of the higher order terms. These categories are based on the behaviour of the curve in the

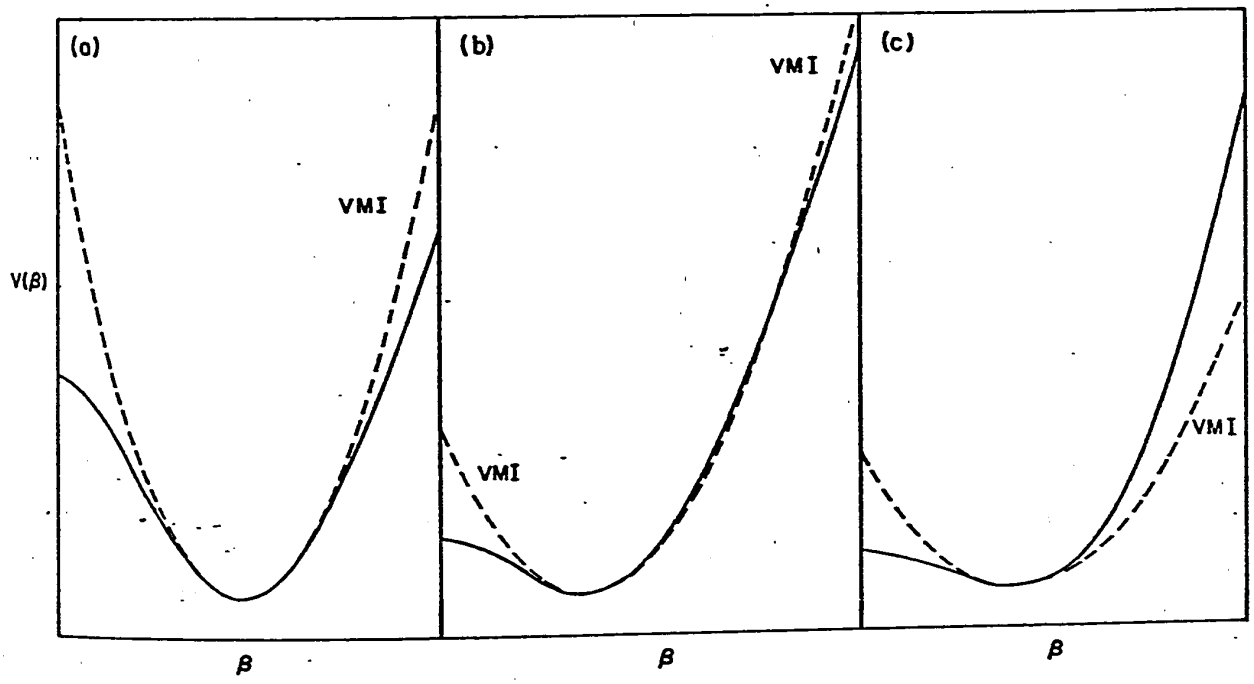

Fig (5.1a,b,c). Realistic potential energy surfaces and the VMI approximation. Three different situations as explained in the text, are shown. The solid line curves were obtained from Eq.(5.9); the broken line curves represent the contribution of the first two terms in Eq.(5.10).

region $\beta > \beta_0$ and are shown schematically in Figs.(5-1(a), 1(b) and 1(c)). The broken line curves refer to Eq. (5.10) with only the first two terms taken into account; with the identification $\beta \sim \beta$, these correspond to the VMI model. In the region $\beta < \beta_0$ there are variations from these three types, but these are not of interest here, because the VMI model probes the region $\beta > \beta_0$ only. The characteristics of these three types for $\beta > \beta_0$ are described below together with some examples of representative nuclei.

(a) The VMI curve lies above the Myers-Swiatecki (MS) curve. Usually both $V'''(\beta_0)$ and $V^{iv}(\beta_0)$ are negative. In some cases $V^{iv}(\beta_0)$ is positive but small. About 75 cases were found; some examples are ^{154}Gd, ^{160}Er, ^{164}Yb, and ^{184}Pt.

(b) Initially the MS curve is above the VMI curve, but there is a crossover at a certain value of β and VMI goes above the MS curve, $V'''(\beta_0)$ is positive but $V^{iv}(\beta_0)$ is negative. Examples: ^{122}Xe, ^{124}Xe, ^{134}Ce, ^{190}Os ($\sim$ 30 cases).

(c) The MS curve lies above the VMI curve. $V'''(\beta_0)$ and $V^{iv}(\beta_0)$ are both positive. Only two cases were encountered: ^{108}Cd and ^{128}Xe.

VMI23 MODEL

In the present investigation, for the ground-state rotational bands in even-even nuclei, we study the effect of the addition of the first order anharmonic term in the potential energy of the VMI model. The expression for energy in this model, called VMI23, can be

written as

$$E_I(\mathcal{J}) = \frac{1}{2} c_2(\mathcal{J}-\mathcal{J}_0)^2 + \frac{1}{6} c_3(\mathcal{J}-\mathcal{J}_0)^3 + \frac{I(I+1)}{2\mathcal{J}} \qquad (5.11)$$

where c_2, c_3, and $\mathcal{J}_0$ are the parameters; the condition for equilibrium being the same as in Eq. (5.2). Making use of Eq. (5.2) and (5.11), we derive the following relations:

$$r_I^4 - (2 - \sigma_3/\sigma_2)\, r_I^3 + (1-\sigma_3/\sigma_2)\, r_I^2 - \sigma_3 I(I+1) = 0 \qquad (5.12)$$

and

$$\mathcal{J}_0 E_I(\mathcal{J}) = \frac{(r_I-1)^2}{4\sigma_2} + \frac{(r_I-1)^3}{6\sigma_3} + \frac{I(I+1)}{2r_I} \qquad (5.13)$$

where $r_I = \mathcal{J}_I/\mathcal{J}_0$ and the quantities σ_2 and σ_3 are defined as

$$\sigma_2 = \frac{1}{2c_2\mathcal{J}_0^{\,3}} \qquad (5.14)$$

and

$$\sigma_3 = \frac{1}{c_3\mathcal{J}_0^{\,4}} \qquad (5.15)$$

The "softness" measure, can be derived from Eq. (5.12) and is found to be equal to σ_2. It is of interest to note that it is independent of σ_3.

For positive $\mathcal{J}_0$, the limiting energy ratios that are obtained from this model are the same as those from the VMI model, i.e. R_I $(=E_I/E_2)$ can vary between $[I(I+1)/6]$ and $[I(I+1)/6]^{2/3}$. Here we

have considered the case of positive $\mathcal{J}_0$ only. Assumption of a negative $\mathcal{J}_0$ can generate R_I below $[I(I+1)/6]^{2/3}$ as is the case for the VMI model (Scharff-Goldhaber and Goldhaber 1970).

CALCULATIONS AND RESULTS

A least-squares fit was performed, using Eqs. (5.12) and (5.13), to obtain the parameters for 122 even-even nuclei. Each energy value was weighted by the square of its inverse. Eq. (5.11) and (5.13) are, as such, non-linear and the determination of c_2, c_3 and $\mathcal{J}_0$ is quite involved. Eq. (5.12) admits of at least two real roots for all positive values of c_2 and c_3. However, when c_3 is negative care has to be exercised; if c_3 becomes too large, all the four roots become complex. From the theory of equations it may be shown that the limiting value of c_3 for a given value of c_2 may be obtained from the following equation

$$3(\sigma_3/\sigma_2)^3 - 8(\sigma_3/\sigma_2)^2 + 8(\sigma_3/\sigma_2) + 64\sigma_2 I(I+1) = 0 \qquad (5.16)$$

In the computer program a constraint to this effect was incorporated.

In order to compare our results with those obtained from the VMI model, calculations were also carried out for the VMI model. We may note here that for some 60 nuclei we have taken the data from MSB and results from the VMI model are available for these nuclei in

their paper. However, we also wanted to calculate the rms deviations, and for the sake of uniformity, a recalculation for these nuclei was considered worthwhile. In Table (5.1) we have recorded the parameters for the VMI and VMI23 models together with the values of σ_2 and σ_3. Though the same symbol, $\mathcal{J}_0$, has been used for the ground state moment-of-inertia in the two models, it is not expected to cause any confusion. For three nuclei c_3 is practically zero, and for 25 others its value is very small and an accurate determination of its value for these cases is not feasible with the present data. Quantitatively for such nuclei, the ratio of the cubic to the square term, i.e. $c_3(\mathcal{J}-\mathcal{J}_0)/3c_2 < 1 \times 10^{-6}$ for 2+ levels. These small values are shown in parentheses and these are to be considered as approximate estimates only; in some cases the uncertainty may be as high as a factor of 100.

The results are presented in Table (5.II). For each nucleus the first row shows the experimental energies with the reported errors. The values shown in parentheses indicate values reported as doubtful and not taken into account, except when they refer to 4+ and 6+ levels, in the least-squares fit and in calculating rms% deviations. An asterisk following a value indicates that the spin of the level is uncertain. The rms deviation in the first row, if shown, refers to the $\pm$ uncertainties in the experimental values. The second and third rows give the energies calculated from the VMI and VMI23 models, respectively. In the case of the VMI23 model, when c_3 is negative, the energies become complex beyond a certain level. Such levels are shown by four asterisks in the third row.

TABLE (5-I). Values of the parameters occurring in the VMI and VMI23 models. A greater number of significant figures has been retained than is warranted by the accuracy of the data to ensure exact reproductibility of the calculated values shown in Table (5-II). For convenience of reading, the mass number of nucleus has been shown by the side of its symbol. The same symbol ($\mathcal{J}_0$) has been used for the ground state moment of inertia in the two models, however, it is not expected to cause any confusion. In the last four columns, the numbers following E give powers of 10 by which the figure to the left of E is to be multiplied. Thus 0.1709E 07 stands for 0.1709×10^7. There are considerable uncertainties in the values of c_3 shown in parentheses.

		VMI model		VMI23 model				
		$\mathcal{J}_0$	C	$\mathcal{J}_0$	c_2	c_3	σ_2	σ_3
		(10^{-3}keV^{-1})	(10^6keV^3)	(10^{-3}keV^{-1})	keV^3	keV^4		
24 Mg	a,b	2.122	3066.9982	1.881	0.1709E 07	0.5023E 13	0.440E 02	0.159E-01
76 Se	c	1.969	10.49	2.044	0.1088E 08	-0.5908E 08	0.538E 01	-0.970E 03
78 Se	c	1.878	14.65	1.878	0.1465E 08	(0.14E 04)	0.515E 01	0.566E 08
102 Zr	d	19.36	7.930	18.27	(0)	0.2916E 10	very large	0.308E-02
104 Mo	d	14.36	4.281	12.00	0.6250E-01	0.7481E 09	0.463E 07	0.645E-01
106 Mo	d	16.78	5.921	15.16	(0)	0.1487E 10	v. large	0.127E-01
102 Ru	e,f	1.823	5.494	1.484	0.4805E 07	0.8681E 08	0.318E 02	0.238E 04
106 Ru	g	31.91	189.1999	31.89	(0)	0.2980E 13	v. large	0.324E-06
110 Ru	d	10.51	4.181	7.431	0.8211E 01	0.5684E 09	0.148E 06	0.577E 00
102 Pd	h	0.8069	7.108	1.236	0.8548E 07	-0.2266E 09	0.310E 02	-0.189E 04
106 Pd	i	2.320	8.496	2.499	0.9325E 07	-0.1505E 09	0.344E 01	-0.170E 03
108 Pd	j	2.863	5.383	3.225	0.6339E 07	-0.1531E 09	0.235E 01	-0.604E 02

(continued)

		δ_0	C	δ_0	c_2	c_3	σ_2	σ_3
114 Pd	d	5.890	4.588	5.205	0.3179E 07	0.2220E 09	0.112E 01	0.614E 01
106 Cd	k	1.087	12.03	1.087	0.1203E 08	(0.13E 04)	0.324E 02	0.560E 09
108 Cd	k	1.785	15.34	1.317	0.1165E 08	0.6579E 09	0.188E 02	0.505E 03
112 Cd	k	1.031	10.53	0.1220	0.6617E 07	0.5897E 09	0.416E 05	0.765E 07
120 Xe	j	5.584	3.375	0.4649	0.2470E 05	0.2739E 09	0.201E 06	0.782E 05
122 Xe	j	5.856	4.248	1.228	0.3944E 03	0.3868E 09	0.685E 06	0.114E 04
124 Xe	1	5.340	4.991	0.5080	0.4400E 03	0.4309E 09	0.867E 07	0.348E 05
126 Xe	1	4.178	5.046	1.414	0.1392E 07	0.3650E 09	0.127E 03	0.685E 03
128 Xe	1	1.884	4.377	1.760	0.4184E 07	0.2295E 08	0.219E 02	0.454E 04
124 Ba	j	11.47	4.752	8.821	0.2251E 05	0.7379E 09	0.324E 02	0.224E 00
126 Ba	j	10.29	6.188	7.415	0.5885E 03	0.8807E 09	0.208E 04	0.376E 00
130 Ba	m	5.463	5.472	2.751	0.1077E 07	0.5121E 09	0.223E 02	0.341E 02
142 Ba	n	3.226	2.814	0.0005	0.7295E 06	0.1919E 09	0.547E 13	0.831E 17
144 Ba	n	11.78	1.631	7.858	0.2276E 06	0.1214E 09	0.453E 01	0.216E 01
128 Ce	j	13.35	5.480	12.60	0.2778E 07	0.5817E 09	0.900E-01	0.682E-01
130 Ce	o	10.33	5.898	8.955	0.1946E 07	0.6922E 09	0.358E 00	0.225E 00
132 Ce	o	6.888	6.106	6.094	0.3777E 07	0.3801E 09	0.585E 00	0.191E 01
134 Ce	o	5.039	9.420	3.496	0.3278E 07	0.9686E 09	0.357E 01	0.691E 01
136 Ce	o	1.950	9.865	2.091	0.1063E 08	-0.1431E 09	0.514E 01	-0.366E 03

(continued)

		S_0	C	S_0	c_2	c_3	σ_2	σ_3
146 Ce	n	7.671	2.223	7.756	0.2304E 07	-0.1110E 08	0.465E 00	-0.249E 02
148 Ce	n	16.48	1.548	16.38	0.1464E 07	0.1381E 08	0.777E-01	0.101E 01
150 Ce	n	30.08	1.752	29.88	0.1442E 07	0.7100E 08	0.130E-01	0.177E-01
134 Nd	m	7.859	5.059	7.506	0.4117E 07	0.1576E 09	0.287E 00	0.200E 01
152 Nd	n	39.22	1.464	37.18	0.4498E 03	0.2772E 09	0.216E 02	0.189E-02
154 Nd	n	41.12	2.496	39.86	0.8041E 03	0.7581E 09	0.982E 01	0.523E-03
150 Sm	p	1.769	1.644	2.700	0.2112E 07	-0.4692E 08	0.120E 02	-0.401E 03
152 Sm	j,q	23.44	1.698	22.70	0.1067E 07	0.9188E 08	0.401E-01	0.410E-01
154 Sm	j	36.54	4.302	35.57	(0)	0.1700E 10	large	0.367E-03
156 Sm	n	39.30	4.935	39.19	0.3031E 07	0.1897E 10	0.274E-02	0.223E-03
158 Sm	n	41.11	6.115	40.97	0.3383E 07	0.2378E 10	0.215E-02	0.149E-03
154 Gd	j	23.29	1.845	22.29	0.9462E 06	0.1325E 09	0.477E-01	0.306E-01
156 Gd	j,r	33.30	2.921	33.29	0.2899E 07	0.6469E 07	0.467E-02	0.126E 00
158 Gd	j	37.44	4.106	37.44	0.4106E 07	(0.26E 03)	0.232E-02	0.199E 04
160 Gd	j	39.77	4.690	39.01	0.1189E 05	0.2311E 10	0.708E 00	0.187E-03
156 Dy	j	20.06	1.591	17.99	0.4192E 06	0.1496E 09	0.205E 00	0.638E-01
158 Dy	s,t	29.78	2.840	29.77	0.2795E 07	0.1136E 08	0.670E-02	0.112E 00
160 Dy	u	34.09	3.620	34.09	0.3620E 07	(0.26E 03)	0.349E-02	0.289E 04
162 Dy	t	37.00	5.308	37.00	0.5307E 07	(0.26E 03)	0.186E-02	0.208E 04

(continued)

		δ_0	C	δ_0	c_2	c_3	σ_2	σ_3
164 Dy	j	40.62	4.199	40.62	0.4199E 07	(0.18E 04)	0.178E-02	0.201E 03
156 Er	j	2.403	2.031	0.4922	0.1167E 07	0.7027E 08	0.359E 04	0.242E 06
158 Er	j	13.08	2.046	12.56	0.1649E 07	0.4255E 08	0.153E 00	0.944E 00
160 Er	j	22.91	2.741	22.90	0.2738E 07	0.4557E 06	0.152E-01	0.798E 01
162 Er	s	29.19	3.759	29.44	0.4887E 07	-0.2738E 09	0.401E-02	-0.486E-02
164 Er	s	32.47	4.911	32.62	0.5862E 07	-0.2566E 09	0.246E-02	-0.344E-02
166 Er	j	36.94	4.182	36.94	0.4182E 07	(0)	0.237E-02	v. large
168 Er	v,j	37.45	8.867	37.45	0.8866E 07	(0.12E 05)	0.107E-02	0.414E 02
170 Er	j	37.85	7.553	37.80	0.5727E 07	0.2502E 10	0.162E-02	0.190E-03
158 Yb	j	1.949	2.145	2.814	0.2779E 07	-0.7052E 08	0.807E 01	-0.226E 03
160 Yb	j	9.182	2.482	7.645	0.1237E 07	0.1432E 09	0.905E 00	0.204E 01
162 Yb	j	16.45	2.598	16.31	0.2393E 07	0.3164E 08	0.482E-01	0.447E 00
164 Yb	j	23.69	2.981	23.72	0.3046E 07	-0.1392E 08	0.123E-01	-0.227E 00
166 Yb	j	28.97	3.822	29.09	0.4368E 07	-0.1345E 09	0.465E-02	-0.104E-01
168 Yb	s	33.78	3.537	33.90	0.4205E 07	-0.2218E 09	0.305E-02	-0.341E-02
170 Yb	v,j	35.37	6.002	35.37	0.6002E 07	(0)	0.188E-02	v. large
172 Yb	j	37.85	5.882	37.85	0.5882E 07	(0.12E 05)	0.157E-02	0.396E 02
174 Yb	j	39.20	9.213	38.78	0.1605E 05	0.8193E 10	0.534E 00	0.540E-04
176 Yb	j	36.44	7.909	36.44	0.7909E 07	(0.12E 05)	0.131E-02	0.461E 02

(continued)

		δ_0	C	δ_0	c_2	c_3	σ_2	σ_3
166 Hf	j	17.34	2.523	17.34	0.2523E 07	(0.77E 03)	0.380E-01	0.144E 05
168 Hf	j	23.28	2.617	23.28	0.2617E 07	(0.20E 01)	0.151E-01	0.170E 07
170 Hf	j	28.94	2.147	28.94	0.2147E 07	(0.77E 03)	0.961E-02	0.186E 04
172 Hf	j	31.16	3.546	31.16	0.3546E 07	(0.16E 02)	0.466E-02	0.663E 05
174 Hf	j	32.59	4.178	32.59	0.4178E 07	(0.16E 02)	0.346E-02	0.554E 05
176 Hf	j	33.87	6.154	33.83	0.5544E 07	0.3084E 09	0.233E-02	0.248E-02
178 Hf	j	31.98	7.387	31.98	0.7387E 07	(0.41E 04)	0.207E-02	0.233E 03
180 Hf	j	32.05	13.87	32.03	0.1195E 08	0.2372E 10	0.127E-02	0.401E-03
172 W	j	22.74	1.631	23.23	0.2061E 07	-0.4957E 08	0.194E-01	-0.693E-01
174 W	j	25.96	2.640	25.96	0.2640E 07	(0.17E 02)	0.108E-01	0.130E 06
176 W	j	26.89	3.218	26.89	0.3218E 07	(0.10E 04)	0.799E-02	0.187E 04
178 W	j	28.05	4.495	28.53	0.6085E 07	-0.3470E 09	0.354E-02	-0.435E-02
180 W	j,x	28.47	4.850	28.47	0.4850E 07	(0.12E 04)	0.447E-02	0.132E 04
182 W	x	29.85	10.77	29.80	0.9343E 07	0.9421E 09	0.202E-02	0.135E-02
184 W	j	26.79	9.856	26.71	0.7255E 07	0.2601E 10	0.362E-02	0.755E-03
186 W	j	24.29	10.72	24.11	0.5423E 07	0.4614E 10	0.658E-02	0.641E-03
172 Os	y	9.282	1.814	10.06	0.2506E 07	-0.1000E 09	0.196E 00	-0.976E 00
174 Os	y	14.77	0.8630	14.77	0.8630E 06	(0.64E 02)	0.180E 00	0.328E 06
176 Os	y	20.16	1.329	19.73	0.1084E 07	0.2958E 08	0.601E-01	0.223E 00

(continued)

		δ_0	c	δ_0	c_2	c_3	σ_2	σ_3
178 Os	j	21.18	1.743	21.39	0.1938E 07	-0.2514E 08	0.264E-01	-0.190E 00
180 Os	j	21.27	2.280	21.27	0.2280E 07	(0.77E 03)	0.228E-01	0.636E 04
182 Os	j	22.80	3.430	23.12	0.4590E 07	-0.2613E 09	0.881E-02	-0.134E-01
184 Os	j	24.70	5.554	24.42	0.3388E 07	0.8587E 09	0.101E-01	0.327E-02
186 Os	j	21.46	6.159	21.07	0.3248E 07	0.1036E 10	0.165E-01	0.490E-02
188 Os	j	18.70	5.121	18.17	0.2688E 07	0.6678E 09	0.310E-01	0.137E-01
190 Os	j	14.96	4.358	13.20	0.7546E 06	0.6294E 09	0.288E 00	0.523E-01
192 Pt	j	16.46	1.025	10.78	0.7714E 03	0.7416E 08	0.517E 03	0.999E 00
194 Pt	j	15.26	1.062	8.783	0.7614E 03	0.6674E 08	0.969E 03	0.252E 01
196 Pt	j	11.62	1.122	4.453	0.5586E 03	0.6460E 08	0.101E 05	0.394E 02
198 Pt	j	7.050	2.112	2.357	0.1230E 06	0.1674E 09	0.310E 03	0.194E 03
190 Pt	j	6.450	2.768	4.616	0.1376E 07	0.1343E 09	0.369E 01	0.164E 02
192 Pt	j	5.711	3.287	1.887	0.4548E 06	0.2571E 09	0.164E 03	0.307E 03
194 Pt	j	5.153	3.301	3.474	0.1653E 07	0.1779E 09	0.721E 01	0.386E 02
196 Pt	z	4.379	3.781	4.173	0.3455E 07	0.4620E 08	0.199E 01	0.714E 02
190 Hg	A	3.821	6.307	4.496	0.8956E 07	-0.5024E 09	0.614E 00	-0.487E 01
192 Hg	B	3.789	6.671	4.388	0.9171E 07	-0.4806E 09	0.645E 00	-0.561E 01
194 Hg	B	3.523	6.394	4.310	0.9558E 07	-0.6000E 09	0.653E 00	-0.483E 01
196 Hg	B	3.488	6.213	4.308	0.9403E 07	-0.6000E 09	0.665E 00	-0.404E 01

(continued)

		$\mathcal{J}_0$	C	$\mathcal{J}_0$	C_2	C_3	σ_2	σ_3
228 Th	j	51.47	0.7805	51.51	0.8292E 06	-0.1625E 08	0.441E-02	-0.874E-02
232 Th	j	59.84	0.8315	59.84	0.6040E 06	0.9146E 08	0.392E-02	0.155E-02
232 U	j	62.51	0.8243	62.51	0.8243E 06	(0.13E 03)	0.246E-02	0.500E 03
234 U	d	60.73	0.8539	60.73	0.8539E 06	(0)	0.180E-02	large
236 U	j	61.97	0.1790	65.73	(0)	0.1340E 10	v. large	0.397E-04
238 U	D	66.50	1.118	66.58	0.1210E 07	-0.1943E 08	0.140E-02	-0.262E-02
238 Pu	j	67.82	1.601	67.80	0.1529E 07	0.4560E 08	0.105E-02	0.104E-02
240 Pu	j	69.88	1.821	69.38	0.1250E 00	0.1428E 10	0.120E 05	0.302E-04
242 Pu	D	67.22	1.558	67.20	0.1497E 07	0.2078E 08	0.110E-02	0.236E-02
242 Cm	j	70.52	0.6007	70.52	0.6007E 06	(0.10E 03)	0.237E-02	0.400E 03
244 Cm	j	69.73	1.717	69.74	0.1771E 07	-0.3834E 08	0.832E-03	-0.110E-02
246 Cm	E	69.67	1.567	69.67	0.1567E 07	(0.11E 03)	0.944E-03	0.400E 03
248 Cm	j	69.04	2.099	68.60	0.6254E 04	0.1860E 10	0.248E 00	0.243E-04
250 Cf	F	70.78	1.525	70.78	0.1525E 07	(0.39E 03)	0.925E-03	0.101E 03
254 Fm	G	66.53	2.001	66.53	0.2001E 07	(0.50E 03)	0.849E-03	0.101E 03

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TABLE (5-II). Experimental and calculated energies (in keV) of levels of ground state bands by the VMI and VMI23 models. For each nucleus the first row shows the experimental energies with the reported errors. The values shown in parentheses indicate values reported as doubtful and not taken into account, except when they refer to 4+ or 6+ levels, in the least-squares fit and in calculating, rms % deviations. An asterisk following a value indicates that the spin of the level is uncertain. The rms deviation in the first row, if shown, refers to the ± uncertainties in the experimental values. The second and third rows give the energies calculated from the VMI and VMI23 models, respectively. In the case of the VMI23 model, when c_3 is negative, the energies become complex beyond a certain level. Such levels are shown by four asterisks in the third row.

I=	2	4	6	8	10	12	14	16	Rms % deviation
24 Mg	1370.	4120	8109.	13206.					
	1352.86	4196.48	8168.23	13027.77	18619.87	24838.41	31606.62	38866.40	1.36
	1363.13	4154.40	8127.14	13130.60	19060.37	25838.58	33404.20	41707.79	0.58
76 Se	560	1332	2264	3272	4403				
	560.01	1334.06	2252.34	3281.79	4403.40	5604.62	6876.41	8211.92	0.28
	559.81	1335.40	2254.14	3282.25	4400.31	5595.50	6858.60	8182.61	0.27
78 Se	613	1499	2539	3575					
	617.87	1478.15	2500.38	3647.30	4897.52	6236.93	7655.41	9145.25	1.50
	617.87	1478.17	2500.41	3647.33	4897.56	6236.99	7655.47	9145.33	1.50
102 Zr	151.9	478.5	964.5	(1551)					
	151.28	482.17	960.36	1559.41	2260.16	3048.67	3914.53	4849.71	0.56
	151.73	479.76	963.05	1586.66	2339.28	3211.99	4197.46	5289.60	0.19
104 Mo	192.3	561.0	1081.0						
	191.31	568.03	1072.35	1673.92	2354.89	3103.52	3911.47	4772.50	0.91
	191.95	563.77	1077.67	1712.83	2455.58	3296.05	4226.67	5241.40	0.35
106 Mo	171.7	522.5	(1034.3)						
	170.77	528.58	1027.13	1636.18	2336.33	3114.34	3960.66	4868.08	0.84
	171.38	525.22	1031.27	1670.68	2430.26	3300.09	4272.39	5340.80	0.36

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
102 Ru	475.4 ±0.6	1106	1872.5±0.5	2703					0.06
	474.62	1112.22	1864.05	2704.31	3618.06	4595.37	5629.10	6713.81	0.37
	475.11	1109.82	1862.69	2709.45	3635.86	4632.39	5692.12	6809.77	0.34
106 Ru	94.0	313.*	657.*						
	93.99	313.13	656.98	1124.90	1716.02	2429.24	3263.31	4216.79	0.02
	93.99	313.05	656.90	1125.27	1717.87	2434.43	3274.67	4238.32	0.01
110 Ru	240.8	663.9	1240.0	(1947.7)					
	239.71	672.82	1229.21	1878.41	2603.28	3392.72	4238.88	5135.89	0.96
	240.36	667.36	1235.77	1922.70	2713.74	3598.72	4569.98	5621.48	0.37
102 Pd	558	1279	2111						
	558.41	1275.92	2114.48	3047.44	4059.14	5139.10	6279.75	7475.30	0.17
	557.99	1278.99	2110.99	3023.20	3996.98	5019.27	6079.99	7170.67	0.00
106 Pd	511.8	1228.9	2076.4*						
	512.00	1227.41	2078.17	3033.07	4074.25	5189.89	6371.53	7612.74	0.09
	511.80	1228.90	2076.40	3020.97	4043.43	5131.02	6274.49	7466.73	0.00
108 Pd	433.8 ± 1.	1047.5±1.5	1770.0 ±5.						0.23
	434.10	1045.05	1772.77	2590.23	3482.00	4437.88	5450.57	6514.53	0.17
	433.79	1047.50	1770.01	2570.37	3430.38	4337.53	5282.26	6256.70	0.00
114 Pd	332.9	853.6	1503.0						
	332.53	856.71	1499.35	2231.70	3037.72	3906.95	4831.95	5807.14	0.26
	332.91	853.62	1503.03	2256.16	3098.42	4020.02	5013.77	6074.13	0.00
106 Cd	633	1495	2494	3504					
	637.25	1477.02	2463.35	3563.49	4758.37	6035.27	7385.02	8800.63	1.25
	637.26	1477.04	2463.37	3563.53	4758.42	6035.33	7385.08	8800.71	1.25
108 Cd	633	1505	2538	3734					
	632.02	1508.42	2548.84	3715.64	4987.18	6349.17	7791.34	9305.90	0.35
	633.24	1502.24	2545.48	3729.05	5032.90	6443.52	7951.03	9547.82	0.19
112 Cd	617	1417	2377						
	616.10	1423.01	2369.56	3424.68	4570.21	5794.05	7087.44	8443.73	0.32
	617.01	1417.02	2372.54	3455.84	4649.95	5943.19	7326.91	8794.36	0.11

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
120 Xe	321.8 ± 1	794.4 ± 2	1396 ± 3	2097 ± 4	(2870)				0.25
	319.41	807.13	1399.82	2072.29	2810.42	3604.96	4449.35	5338.62	1.07
	321.30	797.38	1395.03	2093.74	2880.76	3747.24	4686.51	5693.38	0.22
122 Xe	331.1 ± 1	828.6 ± 2	1467 ± 3	2217 ± 4	(3036)				0.24
	328.43	842.68	1471.94	2188.37	2976.41	3825.90	4729.63	5682.19	1.15
	330.43	832.38	1466.86	2211.48	3052.35	3979.81	4986.61	6067.08	0.28
124 Xe	357.1 ± 0.2	878.9±0.3	1548.2±0.3	2330.4±0.4	3223.7±0.4				0.03
	352.05	898.89	1566.51	2325.78	3160.35	4059.59	5015.92	6023.65	1.58
	355.63	885.27	1551.06	2330.02	3207.91	4174.76	5223.15	6347.23	0.44
126 Xe	389 ± 1.0	943 ±1.4	1636± 2.4	2437± 3.2	3317 ± 3.7				0.17
	385.50	957.59	1647.69	2427.82	3282.17	4200.41	5175.15	6200.79	1.00
	388.79	945.02	1633.27	2430.96	3324.14	4303.10	5360.62	6491.04	0.19
128 Xe	443 ± 1.0	1033 ±1.4	1737 ±1.7	2513 ±2.6					0.15
	442.73	1035.37	1733.67	2513.82	3362.00	4269.04	5228.33	6234.83	0.15
	442.89	1034.56	1733.20	2515.55	3368.07	4281.75	5250.09	6268.15	0.14
124 Ba	229.5 ± 1	650.6 ± 2	1223 ± 3	(1857)					0.34
	228.71	656.86	1215.49	1872.58	2609.93	3415.71	4281.54	5201.15	0.69
	229.38	651.61	1221.82	1916.79	2721.68	3625.96	4621.65	5702.42	0.11
126 Ba	256.1 ± 1	711.6 ± 2	1333 ± 3	2090 ± 4	(2919)				0.28
	253.44	725.24	1339.23	2060.47	2869.13	3752.32	4700.94	5708.16	1.32
	255.30	716.14	1333.50	2082.34	2946.82	3915.74	4980.63	6134.80	0.40
130 Ba	357.2	901.7	1592.8	2396					
	355.12	912.94	1596.13	2374.30	3230.48	4153.61	5135.82	6171.19	0.83
	357.16	902.40	1590.70	2397.64	3308.17	4311.82	5400.76	6568.87	0.08
142 Ba	359.7	835.4	1467.4						
	357.16	853.94	1444.09	2106.15	2827.80	3600.89	4419.58	5279.44	1.63
	357.55	848.90	1451.73	2147.27	2923.80	3773.17	4689.26	5667.29	1.17
144 Ba	199.4	530.4	962.1	1472					
	198.07	537.39	964.53	1457.78	2005.05	2598.50	3232.59	3903.15	0.89
	199.38	530.80	960.87	1472.99	2056.90	2705.40	3413.14	4175.88	0.08

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
128 Ce	207.3	607.3	1157.8	1820.0	(2573)				
	206.18	613.15	1158.70	1809.96	2547.53	3358.65	4234.25	5167.55	0.62
	207.27	608.41	1154.89	1822.49	2596.12	3465.23	4421.89	5459.87	0.17
130 Ce	253.2	709.6	1323.5	2052.5					
	251.56	718.32	1324.84	2036.74	2834.55	3705.60	4640.95	5633.89	0.80
	253.14	710.77	1319.92	2055.29	2901.17	3846.60	4883.31	6004.84	0.17
132 Ce	325.6	859.1	1543.6	2332.6	(3208)				
	324.34	866.50	1543.29	2321.54	3182.79	4115.08	5109.93	6160.97	0.53
	325.52	860.52	1539.67	2335.53	3231.98	4218.12	5285.96	6429.32	0.16
134 Ce	409.4	1049.3	1864.1	(2811)					
	407.23	1061.19	1867.31	2788.47	3803.96	4900.33	6068.02	7299.84	0.75
	409.34	1050.21	1861.36	2813.04	3887.13	5071.09	6355.61	7733.38	0.09
136 Ce	551.7	1313.4	(2212)						
	551.89	1312.19	2213.51	3223.59	4323.84	5502.00	6749.23	8058.84	0.07
	551.64	1313.26	2211.76	3213.22	4298.09	5453.37	6669.70	7939.91	0.01
146 Ce	258.6	668.7	1171						
	258.63	668.34	1171.36	1745.00	2376.62	3057.97	3783.19	4547.89	0.04
	258.60	668.71	1171.02	1742.25	2369.38	3043.94	3759.89	4512.69	0.00
148 Ce	158.7	454.4	840.9						
	158.66	454.76	840.53	1293.96	1802.53	2358.12	2954.97	3588.79	0.05
	158.71	454.41	840.90	1297.21	1811.40	2375.71	2984.63	3634.01	0.00
150 Ce	97.1	306.1	606.8	983.2					
	96.93	306.87	607.49	981.56	1417.04	1905.34	2440.10	3016.46	0.18
	97.08	306.20	606.46	983.32	1426.94	1930.27	2488.04	3096.13	0.03
134 Nd	294.0	788.6	1419.6	2147					
	293.37	792.41	1419.07	2141.82	2943.11	3811.57	4739.14	5719.78	0.29
	293.92	789.49	1417.09	2148.65	2968.25	3865.40	4832.53	5863.90	0.11
152 Nd	75.9	240.6	487.9	810.0					
	75.27	243.01	490.17	804.60	1176.65	1598.94	2065.75	2572.60	0.77
	75.72	241.54	488.11	808.50	1197.31	1650.18	2163.49	2734.17	0.25

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
154 Nd	72.8 72.35 72.67	235.2 236.87 235.97	483.9 485.37 483.82	812.0 808.69 811.20	1198.40 1213.90	1647.33 1688.31	2149.51 2231.29	2700.04 2840.10	0.54 0.19
150 Sm	334 334.39 334.00	773. 770.32 772.99	1278.* 1281.24 1277.98	1850.49 1827.91	2468.33 2409.41	3128.28 3012.13	3825.63 3626.57	4556.81 4241.84	0.26 0.00
152 Sm	121.78±0.05 120.95 121.69	366.4±0.3 369.87 367.57	712 ± 3. 712.31 707.86	1122 ±10 1127.34 1126.49	1615± 15 1601.96 1613.42	2158 2127.43 2161.65	2697.49 2765.94	3307.43 3422.22	0.61 0.67 0.35
154 Sm	81.99±0.05 81.53 81.87	267 ± 1 267.70 266.84	545 ±5 550.44 548.84	927 ±20 920.37 922.75	1368.49 1384.21	1886.90 1929.39	2468.87 2554.96	3108.74 3257.93	1.19 0.69 0.43
156 Sm	76 75.96 76.00	250.2 250.46 250.20	518 517.80 518.01	870.86 875.12	1302.31 1317.73	1805.30 1842.47	2373.75 2446.35	3002.31 3126.65	0.07 0.00
158 Sm	72.8 72.72 72.81	240.3 240.51 240.25	498.5 499.29 498.65	844.5 843.65 844.44	1267.71 1274.43	1765.61 1785.71	2331.90 2375.66	2961.63 3041.91	0.12 0.02
154 Gd	123.07±0.05 122.09 123.02	371.2±0.2 374.56 371.57	718.1 ± 1. 723.02 717.42	1146 ±10 1146.21 1145.19	1644± 15. 1630.79 1645.05	(2189) 2167.79 2210.02	2750.75 2834.83	3374.81 3515.29	0.57 0.72 0.08
156 Gd	88.967±.005 88.90 88.92	288.16±.05 288.42 288.44	584.5±0.5 584.80 584.81	966 ± 5 964.49 964.55	1415.5* 1416.25 1416.53	1931.20 1932.00	2502.37 2504.07	3124.18 3127.28	0.23 0.09 0.09
158 Gd	79.51±0.01 79.59 79.59	261.45±.05 261.40 261.40	539.03±.05 537.73 537.74	898.2±0.5 899.55 899.55	1338.14 1338.15	1845.82 1845.82	2416.02 2416.03	3043.22 3043.23	0.03 0.15 0.15
160 Gd	75.3 ± 0.5 75.06 75.28	247 ± 2 247.45 246.78	509 ± 5 511.46 510.19	863 ±10. 860.00 861.72	1285.77 1298.00	1781.99 1816.03	2342.61 2413.12	2962.38 3086.81	0.92 0.35 0.15

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
156 Dy	138 ± 3	403 ± 6	766 ±10	1212 ±15					1.60
	137.07	407.29	769.25	1201.18	1690.21	2227.92	2808.31	3426.87	0.80
	137.98	402.95	765.95	1211.89	1731.05	2316.51	2963.00	3666.33	0.01
158 Dy	98.94	317.26	637.87	1044.02	1519.4	(2047.7)			
	98.87	317.75	637.98	1043.01	1520.14	2059.85	2654.91	3299.64	0.09
	98.87	317.67	637.76	1042.80	1520.35	2061.12	2658.08	3305.70	0.09
160 Dy	86.8	283.8	581.5	967.5	1429.2	1952	2515.5	3092.2	
	87.12	284.30	580.28	963.09	1422.13	1948.68	2535.64	3177.27	0.44
	87.12	284.30	580.29	963.09	1422.14	1948.69	2535.66	3177.29	0.44
162 Dy	80.6	265.6	548.4	921.0	1374.7	1900.9			
	80.64	265.58	548.23	920.51	1374.28	1902.08	2497.34	3154.43	0.04
	90.64	265.58	548.23	920.50	1374.27	1902.06	2497.31	3154.38	0.04
164 Dy	73.39±0.05	242.2±0.1	501.3±0.5	839 ± 5					0.30
	73.47	242.09	500.07	840.24	1255.33	1738.61	2284.13	2886.75	0.16
	73.47	242.09	500.07	840.24	1255.33	1738.61	2284.14	2886.76	0.15
156 Er	344.4 ± 1	797.3± 2	1340.5 ± 4	1950.7 ± 6					0.29
	343.36	802.53	1343.43	1947.68	2604.58	3307.02	4049.90	4829.33	0.47
	344.41	797.25	1340.67	1958.61	2641.26	3381.83	4175.30	5017.77	0.01
158 Er	192.7 ± 1	528.4 ±2	972.2 ±3	1496.0 ± 4	2075.7 ±7	2684.4±10			0.37
	191.32	535.19	975.98	1489.72	2062.95	2686.96	3355.57	4064.18	0.69
	192.19	532.66	971.01	1486.11	2065.98	2702.74	3390.76	4125.74	0.56
160 Er	126.2 ± 1	390.5 ±2	766.8 ± 3	1231.4 ± 4	1763.5 ± 6	2342.9 ±8			0.48
	125.85	392.45	767.10	1227.31	1758.31	2349.91	2994.71	3687.09	0.32
	125.89	392.56	767.27	1227.54	1758.63	2350.35	2995.31	3687.90	0.32
162 Er	100.7	327.4	664.1	1094.0	1589.9	2150.9			
	101.22	327.27	661.13	1086.71	1591.08	2164.24	2798.43	3487.52	0.47
	100.72	327.40	663.96	1091.99	1593.24	2149.28	****	****	0.12
164 Er	91.39	299.47	614.34	1024.34	1517.9	2082.4	2699.5		
	91.60	299.75	613.82	1022.06	1513.71	2079.67	2712.40	3405.71	0.25
	91.30	299.53	614.78	1024.83	1517.13	2079.71	2701.29	3370.99	0.08

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
166 Er	80.6±0.05	264.9±0.2	545 ± 1	910 ± 5	(1340)				0.29
	80.65	264.82	544.57	910.62	1354.10	1867.19	2443.23	3076.63	0.06
	80.65	264.82	544.57	910.63	1354.11	1867.20	2443.24	3076.64	0.06
168 Er	79.8 ± .5	264 ±0.5	549 ±0.5	933	1393				0.30
	79.85	264.27	549.11	928.82	1397.26	1948.24	2575.91	3274.93	0.25
	79.85	264.27	549.11	928.83	1397.26	1948.23	2575.90	3274.91	0.25
170 Er	79 ± 0.5	261 ± 2	542 ± 3						0.66
	78.97	261.12	541.86	915.19	1374.56	1913.54	2526.15	3207.00	0.04
	79.01	261.02	542.04	917.92	1384.84	1939.35	2578.35	3299.01	0.01
158 Yb	357.9 ± 2	833.9 ± 4	1382.2 ± 6						0.49
	358.30	830.75	1385.71	2004.76	2677.15	3395.70	4155.27	4951.91	0.27
	357.88	833.86	1382.13	1979.01	2609.28	3260.91	3922.19	4577.92	0.01
160 Yb	243.0 ± 1	638.3 ± 3	1147.1 ± 5	1735.8 ± 7					0.43
	241.86	644.89	1147.47	1725.11	2364.15	3055.75	3793.64	4573.12	0.65
	242.94	639.29	1144.21	1737.96	2408.74	3148.49	3951.22	4812.29	0.16
162 Yb	166.5 ± 1	486.7 ± 2	922.9 ± 4	1444.1 ± 6	2013.5± 8				0.46
	165.93	490.09	922.15	1436.27	2017.36	2655.52	3343.71	4076.68	0.43
	166.19	489.35	920.74	1436.09	2021.26	2666.97	3366.60	4115.20	0.41
164 Yb	122.5 ±0.4	384 ± 1	758.± 2.5	1219 ± 4	1748 ± 5	(2322)	(2928)		0.31
	122.44	384.83	757.06	1217.23	1750.50	2346.48	2997.56	3697.95	0.15
	122.37	384.86	757.24	1217.22	1749.56	2343.55	2991.32	3686.88	0.14
166 Yb	101.8 ±0.4	329.7 ± 1	667.1 ± 2	1097.0 ± 4	1604 ± 5	2172 ± 6	(2774.4)	(3402.2)	0.33
	101.98	329.67	665.88	1094.36	1602.08	2178.98	2817.23	3510.68	0.20
	101.75	329.83	667.36	1096.82	1602.90	2172.81	2795.84	3462.70	0.04
168 Yb	87.73	286.55	585.30	970.05	1425.10				0.18
	87.87	286.49	584.11	968.39	1428.57	1955.83	2543.06	3184.49	0.03
	87.71	286.66	585.34	969.72	1425.26	****	****	****	
170 Yb	84.4	277.5	573.4	962.9	1436.2	(1986)			0.06
	84.35	277.76	573.27	962.37	1436.50	1987.84	2609.51	3295.60	0.06
	84.35	277.76	573.28	962.38	1436.51	1987.85	2609.52	3295.61	

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
172 Yb	79.7 ± 0.5	260.3 ±1	540.0 ±3	910 ± 5	1352 ± 8				0.55
	70.09	260.30	538.58	906.62	1357.05	1882.86	2477.77	3136.28	0.29
	70.09	260.30	538.58	906.63	1357.05	1882.86	2477.78	3136.30	0.29
174 Yb	76.5 ± 0.5	252 ± 3	527 ± 5	892 ± 8					0.94
	76.33	252.00	526.27	891.03	1344.28	1870.15	2480.18	3169.40	0.22
	76.46	252.49	525.40	892.99	1352.79	1902.77	2540.96	3265.55	0.18
176 Yb	82.1 ± 0.5	270 ± 3	564 ± 5	947± 10					0.94
	82.01	271.01	561.97	940.35	1423.10	1979.38	2610.88	3311.96	0.27
	82.01	271.01	561.97	948.35	1423.10	1979.39	2610.89	3311.97	0.27
166 Hf	158.7 ± 0.4	470.7±1.5	897.6 ±3	1407.0 ±4	1971 ± 6	2565 ± 10	(3178)		0.32
	158.03	472.58	893.35	1395.76	1964.84	2590.74	3266.45	3986.72	0.60
	158.03	472.58	893.35	1395.76	1964.85	2590.75	3266.47	3986.75	0.60
168 Hf	123.9 ± 0.4	385 ± 1	736.1 ±3	1212 ±4	1734 ± 5	2304 ± 10	(2910)		0.34
	123.86	386.27	735.06	1208.11	1730.86	2313.29	2948.11	3629.79	0.27
	123.86	386.27	735.06	1208.11	1730.87	2313.31	2948.13	3629.80	0.27
170 Hf	100.0 ± 0.3	320.6 ± 1	641.1 ± 3	1041.0 ±4	1503. ± 6	2013 ± 8	2564 ±10	3147± 20	0.42
	100.97	320.68	636.66	1031.07	1491.25	2008.07	2574.77	3186.14	0.77
	100.97	320.69	636.66	1031.08	1491.26	2008.08	2574.78	3186.16	0.77
172 Hf	94.5± 0.3	307.9 ± 1	627 ± 3	1036 ± 4	1519 ± 6	2063 ± 8	2651 ± 10	(3273)	0.38
	95.00	308.17	624.76	1030.27	1512.67	2062.49	2672.27	3336.08	0.47
	95.00	308.17	624.76	1030.27	1512.68	2062.50	2672.28	3336.09	0.47
174 Hf	91.01	297.45	608.37	1009.42	1485.6	(2026.4)			0.15
	91.13	297.46	607.24	1008.01	1488.70	2040.19	2655.04	3327.24	0.15
	91.14	297.46	607.25	1008.01	1488.71	2040.20	2655.06	3327.26	
176 Hf	88.± 0.5	290 ± 1.	596 ± 5	998 ±10	1492.				0.66
	88.03	289.53	596.61	999.83	1489.91	2058.48	2698.32	3403.28	0.14
	88.08	289.42	596.08	999.25	1490.82	2063.81	2712.35	3431.50	0.12
178 Hf	93.2 ±0.1	306.8±0.2	632.5±0.5	1059 ± 3					0.16
	93.24	306.70	632.09	1059.48	1579.06	2181.99	2860.64	3608.48	0.05
	93.24	306.70	632.09	1059.48	1579.06	2182.00	2860.65	3608.50	0.05

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
180 Hf	93.33±0.05	308.6±0.2	641.1±0.3	1084.9±0.5					0.05
	93.30	308.74	641.38	1084.66	1631.29	2274.01	3005.95	3820.82	0.04
	93.32	308.59	640.97	1084.79	1634.46	2284.85	3031.35	3869.86	0.01
172 W	122.9 ±0.4	376.9 ±1	727.2± 3	1147 ± 4	1616 6	2129 ± 8	2677 ±10.	(3253)	0.36
	123.86	376.20	721.03	1137.27	1612.01	2136.67	2705.08	3312.62	0.69
	122.86	377.42	726.60	1145.55	1617.66	2130.84	2675.45	3242.91	0.10
174 W	111.9 ±0.3	355 ± 1	704.2 ±3	1137. ± 4.	1635 6	2186 ± 8.	(2780)		0.35
	112.22	354.85	701.72	1132.86	1634.37	2196.37	2811.59	3474.45	0.31
	112.22	354.85	701.72	1132.86	1634.38	2196.38	2811.61	3474.47	0.31
176 W	108.7 ±0.3	348.5 ±1.	699.4 ±3.	1140. ± 4.	1648 6	2206 ± 8	(2801)	(3425)	0.35
	109.12	348.74	696.41	1133.28	1645.44	2222.68	2857.33	3543.45	0.46
	109.12	348.74	696.41	1133.29	1645.45	2222.69	2857.34	3543.47	0.46
178 W	104 ± 5.	342 ±7	697 ±10.	1152.±15	1679 ±20.	2264± 25	2894 ± 30		2.23
	105.42	341.35	690.66	1136.85	1666.51	2269.17	2936.67	3662.53	1.02
	104.07	339.21	690.29	1139.61	1669.51	2262.65	2899.96	****	0.67
180 W	103.58	337.68	688.54	1138.4	1667 ±20.	2252 ±25			0.67
	104.03	337.79	685.53	1131.57	1662.82	2268.87	2941.51	3674.17	0.48
	104.03	337.79	685.53	1131.58	1662.83	2268.88	2941.52	3674.18	0.48
182 W	100.1	329.4	680.4	1144.0	1711.6				
	99.99	329.53	680.85	1144.31	1710.08	2369.04	3113.12	3935.31	0.07
	100.08	329.48	680.32	1143.79	1711.69	2376.81	3133.03	3975.09	0.02
184 W	111.2	364.0	748.2						
	111.12	364.32	747.76	1247.97	1852.30	2549.82	3331.37	4189.32	0.07
	111.21	364.04	748.29	1254.59	1875.07	2603.15	3433.21	4360.44	0.01
186 W	122.5	399.0	818.0						
	122.35	399.76	817.14	1358.13	2008.08	2754.76	3588.15	4500.09	0.15
	122.51	399.03	818.07	1370.06	2047.14	2842.65	3750.93	4767.03	0.01
172 Os	227.7	606.1	1054.5						
	228.40	600.47	1061.12	1588.65	2170.97	2800.24	3470.89	4178.74	0.67
	227.93	604.11	1056.24	1548.26	****	****	****	****	0.22

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
174 Os	158.5 158.92 158.92	434.5 432.16 432.16	776.8 776.56 776.57	1170.7 1174.53 1174.54	1616.1 1616.26 1616.27	2095.39 2095.41	2607.44 2607.45	3149.02 3149.04	0.31 0.31
176 Os	135.2 134.79 135.24	395.4 396.75 395.26	742.3 744.91 742.32	1157.3 1158.57 1158.34	1633.6 1625.64 1632.77	2138.24 2158.59	2690.75 2730.77	3278.99 3345.47	0.34 0.05
178 Os	131.6 ± 0.3 131.91 131.51	397.7 ± 1 397.53 398.22	760.8 ± 2 757.82 760.01	1193.7± 3 1190.82 1193.02	1681.7 ± 4 1683.31 1682.46	2218.5 ±5 2226.51 2218.33	2814.18 2793.34	3441.60 3401.90	0.24 0.26 0.08
180 Os	132.2 ±0.3 133.31 133.31	408.5 ± 1 407.73 407.74	795.1 ± 2 785.33 785.33	1257.3 ±3 1243.01 1243.02	1767.5 ± 4 1766.44 1766.45	2308.5 ±6 2345.98 2345.99	(2874.9) 2974.72 2974.73	3647.45 3647.47	0.24 1.01 1.01
182 Os	126.9 ±0.3 127.31 126.58	400.2 ± 1 400.57 401.89	793.9 ± 2 788.72 793.97	1276.9 ±3 1268.99 1272.91	1809.6 ± 5 1825.90 1811.74	2448.57 ****	3129.04 ****	3861.22 ****	0.25 0.59 0.27
184 Os	119.8 ± 0.3 119.42 119.78	383.6±0.4 384.99 383.77	773.9±0.6 775.40 773.41	1274.6±0.7 1271.12 1274.89	(1871.2) 1856.83 1877.99	2520.88 2574.80	3254.29 3359.02	4050.05 4225.45	0.14 0.29 0.04
186 Cs	137.2±0.05 136.64 137.15	433.9±0.1 436.32 434.22	868.7±0.1 870.52 867.57	1420.5±0.3 1415.57 1421.09	(2068.1) 2054.07 2083.23	2773.31 2845.20	3563.74 3700.09	4418.00 4642.21	0.02 0.40 0.08
188 Cs	155.0 ± 0.1 154.27 154.96	477.9±0.1 481.41 478.45	939.8±0.3 941.51 938.15	1513.6±0.5 1506.99 1514.74	(2169.5) 2159.70 2195.28	2887.08 2970.34	3680.03 3832.67	4531.62 4776.48	0.04 0.50 0.11
190 Os	186.7 ± 0.1 185.34 186.69	547.8±0.1 554.67 548.57	1050 ± 5 1052.49 1047.80	1662 ±10 1648.60 1663.96	2325.06 2383.71	3069.98 3197.45	3874.93 4097.85	4733.56 5079.07	0.38 0.84 0.14
182 Pt	153.7 ± 0.4 152.01 153.04	416.2 ± 1 425.17 419.67	771.4 ±2 775.31 771.99	1202.4 ±3 1183.38 1195.95	(1695.4) 1638.69 1682.75	(2238.4) 2134.33 2226.23	2665.39 2821.72	3228.22 3465.56	0.25 1.47 0.54

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
184 Pt	162.1 ± 0.4 160.20 162.07	434.8 ± 1 443.17 436.18	797.3 ± 2 803.42 794.55	1228.9 ± 3 1221.82 1223.16	1704.7 ± 4 1687.65 1713.31	(2201.3) 2194.00 2258.89	(2723) 2735.97 2855.31	(3726) 3309.88 3498.97	0.24 1.18 0.37
186 Pt	191.1 ± 0.6 188.45 190.50	489.6±1.5 500.22 492.44	876.8± 2 888.11 878.91	1341.1 ± 3 1333.41 1335.64	1855.7 ± 5 1825.69 1853.81	(2407) 2358.20 2427.25	2926.15 3051.35	3525.94 3722.50	0.27 1.50 0.37
188 Pt	265.9 ± 0.6 265.25 265.90	671.3 ±2 676.50 671.44	1184.6± 3 1178.34 1184.52	(1782) 1748.92 1786.95	(2436) 2376.00 2467.46	3051.60 3218.21	3770.04 4035.33	4527.03 4908.19	0.26 0.56 0.01
190 Pt	292 ± 5 290.13 292.11	733 ±10 740.08 732.45	1283 ±15 1289.18 1280.01	1903 ± 20 1913.52 1915.29	2636 ±25 2599.69 2626.62	3338.95 3405.98	4125.11 4247.48	4953.47 5146.50	1.28 0.87 0.35
192 Pt	317 ± 1 315.00 316.84	785 ± 1 797.26 787.93	1388 ±10 1383.72 1379.03	2063 ± 20 2049.38 2069.80	2780.18 2847.64	3566.95 3703.77	4403.18 4631.63	5283.92 5626.08	0.63 0.92 0.41
194 Pt	328.5 ± 1 327.15 328.45	811.1 ±2 818.53 811.83	1411.6± 3 1413.09 1409.51	2099.4 ± 5 2086.25 2100.91	2824.16 2873.63	3617.75 3719.17	4460.56 4631.21	5347.72 5604.87	0.25 0.59 0.09
196 Pt	355.7 355.60 355.69	878* 878.96 877.98	1510* 1508.96 1509.96	2220.43 2227.63	2999.10 3017.29	3835.63 3869.91	4723.34 4779.02	5657.18 5739.70	0.08 0.00
190 Hg	416.4 417.24 416.41	1041.5 1034.86 1041.53	1771.7 1779.41 1771.73	2620.81 2557.29	3542.10 3350.35	4532.13 ****	5582.98 ****	6688.62 ****	0.46 0.00
192 Hg	423 423.73 423.01	1058. 1052.05 1058.02	1803 1809.83 1803.03	2666.38 2611.45	3604.37 3446.64	4612.45 ****	5682.53 ****	6808.46 ****	0.40 0.00
194 Hg	428 429.09 428.09	1065 1056.29 1064.44	1800 1810.03 1800.53	2660.50 2578.24	3590.82 ****	4589.92 ****	5649.86 ****	6764.67 ****	0.59 0.04

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
196 Hg	426	1062	1785						0.79
	427.44	1050.29	1798.24	2641.87	3564.49	4555.16	5606.04	6711.21	0.21
	426.50	1058.76	1788.06	2553.85	****	****	****	****	
228 Th	57.5 ± 0.1	186.6±0.2	378 ± 1						0.19
	57.51	186.51	378.05	623.30	915.01	1247.41	1616.02	2017.25	0.03
	57.50	186.61	378.01	621.85	909.32	1232.90	1586.15	1963.26	0.00
232 Th	49.8 ± 0.1	163 ± 1	333 ± 3	555 ± 5	828 ± 8				0.77
	49.73	162.87	333.91	556.62	825.25	1134.88	1481.41	1861.46	0.24
	49.84	162.69	333.09	555.95	827.10	1143.13	1501.23	1899.07	0.13
232 U	47.6 ± 0.1	156.6±0.2	321. ± 1.						0.23
	47.64	156.35	321.24	536.74	797.50	1098.89	1436.96	1808.43	0.12
	47.64	156.35	321.25	536.74	797.51	1098.89	1436.96	1808.43	0.12
234 U	43.4	143.1	295.5	496.1					0.04
	43.42	143.04	295.41	496.25	741.23	1026.37	1348.14	1703.50	0.04
	43.42	143.04	295.41	496.25	741.23	1026.37	1348.14	1703.50	
236 U	45.28 ±0.05	148.7±0.5	312 ± 1						0.28
	47.18	148.62	292.90	471.60	678.95	910.89	1164.46	1437.37	4.29
	45.18	149.39	311.28	529.58	803.11	1130.76	1511.47	1944.26	0.33
238 U	44.92	148.04	306.75	517.13	774.83	1074.2	1412.		
	44.91	148.22	306.79	516.67	773.69	1073.92	1413.81	1790.21	0.09
	44.87	148.18	306.92	517.08	774.26	1074.05	1412.26	1785.02	0.07
238 Pu	44.11 ±0.05	146.0±0.5	303.6 ±1.	514 ± 5.					0.54
	44.10	146.03	303.62	513.97	773.81	1079.83	1428.88	1818.03	0.01
	44.11	146.03	303.59	514.05	774.42	1081.84	1433.65	1827.47	0.01
240 Pu	42.88 ±0.05	141.7±0.5	296 ± 5						1.00
	42.83	141.99	295.75	501.72	757.15	1059.13	1404.83	1791.54	0.14
	42.86	141.85	295.85	503.78	764.63	1077.45	1441.34	1855.43	0.07
242 Pu	44.50	147.25	305.95	517.67	778.75	1086.7			
	44.49	147.26	306.02	517.73	778.98	1086.36	1436.62	1826.80	0.02
	44.50	147.25	305.94	517.56	778.84	1086.58	1437.82	1829.86	0.01

(continued)

I=	2	4	6	8	10	12	14	16	rms % deviation
242 Cm	42.2 ± 0.1	139 ± 3	285 ± 5						1.61
	42.25	138.72	285.25	476.99	709.28	978.02	1279.74	1611.49	0.14
	42.25	138.72	285.25	476.99	709.28	978.03	1279.74	1611.50	0.14
244 Cm	42.9 ± 0.1	142.3±0.5	296 ± 5	502 ± 10					1.32
	42.91	142.22	296.08	501.99	757.04	1058.25	1402.69	1787.62	0.04
	42.91	142.22	296.11	501.95	756.63	1056.74	1398.79	1779.31	0.03
246 Cm	42.9	142.4	295.9	501					
	42.94	142.23	295.87	501.16	755.03	1054.35	1396.09	1777.45	0.08
	42.94	142.23	295.87	501.16	755.03	1054.35	1396.09	1777.45	0.08
248 Cm	43.4 ± 0.1	143.6±0.5	300 ±10						1.94
	43.36	143.82	299.80	509.08	769.03	1076.90	1429.93	1825.48	0.11
	43.39	143.71	299.91	511.03	776.11	1094.27	1464.67	1886.49	0.05
250 Cf	42.2	140.4	291						
	42.27	140.02	291.33	493.58	743.77	1038.86	1375.89	1752.11	0.19
	42.27	140.02	291.33	493.58	743.77	1038.86	1375.89	1752.11	0.19
254 Fm	44.9	149.5*	(310)						
	44.98	149.07	310.38	526.29	793.80	1109.78	1471.18	1875.14	0.21
	44.98	149.07	310.39	526.30	793.80	1109.78	1471.19	1875.15	0.21

DISCUSSION

A. ANALYSIS OF RESULTS

It will be noticed from Table (5-II) that for all nuclei, the rms% deviations from the VMI23 model are either less than or equal to those obtained from the VMI model. In quite a few cases (e.g. ^{120}Xe, ^{122}Xe, ^{124}Xe, ^{126}Xe, ^{126}Ba, ^{134}Ce, and Pt isotopes), the improvement is quite substantial. However, one can argue that this alone is not enough to indicate that the c_3 term is necessary. After all, by increasing the number of parameters, one expects to obtain a better fit to the experimental data. To examine this point, a more demanding criterion due to Gauss (Worthing and Geffner 1960) can be employed. This criterion which depends on the number of parameters can be expressed for our case as follows:

$$\Omega = \frac{1}{n-m} \sum \frac{(E_{calc}-E_{obs})^2}{E_{obs}^2} = \text{minimum} \qquad (5.17)$$

where n is the number of points and m the number of free parameters.

Those nuclei for which c_3 is zero or else very small can clearly be left out of consideration. In these cases the rms% deviations by the two models are more or less the same and the inclusion of the cubic term is not essential. We also have to leave out those nuclei for which only three levels are available, because the Gauss criterion can not be used when n = m. The problem of fitting 3 levels to a three-parameter model is, however, non-trivial; for a given value

of R_4, the VMI23 model admits of only a narrow range for R_6; unless R_6 lies in this range it is not possible to obtain an exact fit. For the remaining nuclei, in Table (5-III), we show the values of Ω for the two models. Except for the following 10 nuclei: ^{76}Se, ^{102}Ru, ^{128}Xe, ^{156}Gd, ^{158}Dy, ^{164}Yb, ^{174}Yb, ^{176}Hf, ^{238}Pu, ^{244}Cm, the VMI23 Ω's are seen to be better than the VMI Ω's for the nuclei listed in the table. It will also be noticed that for the 10 exceptions, VMI Ω's are only slightly less than those for VMI23 , but amongst the other 50 nuclei there are many cases for which Ω(VMI23) is smaller that Ω(VMI) by very large factors. We believe this test establishes that for these 50 nuclei the inclusion of the cubic term is essential.

Amongst these 50 nuclei, there are many cases for which there are marked differences between the VMI and VMI23 predictions at high spin levels. The measurement of these levels would be of considerable interest. It is quite likely that the effect of other higher terms will show up.

In the VMI23 model, when c_3 is negative the energies become complex beyond a certain level. This result, however, does not take into account the effect of quartic and other higher terms. The same result would be obtained, though, if the net effect of cubic and other higher order terms is negative. This would provide a natural cut-off to the rotational band.

TABLE (5-III) The Gauss criterion sum, Ω, eq. (5.17), for the VMI and VMI23 models. The values have been multiplied by 10^6.

Nucleus	$\Omega \times 10^6$ VMI model	$\Omega \times 10^6$ VMI23 model
24 Mg	368.29	132.5
76 Se	12.63	17.88
102 Ru	27.48	45.44
108 Cd	24.98	13.96
120 Xe	229.12	19.41
122 Xe	266.01	31.11
124 Xe	415.69	48.37
126 Xe	165.29	9.21
128 Xe	4.72	8.16
126 Ba	348.51	64.02
130 Ba	137.83	2.83
144 Ba	158.82	2.67
128 Ce	76.59	11.57
130 Ce	126.47	11.94
132 Ce	55.84	10.83
134 Ce	111.90	3.46
150 Ce	6.73	0.47

TABLE (5-III) (cont'd)

Nucleus	$\Omega \times 10^6$ VMI model	$\Omega \times 10^6$ VMI23 model
134 Nd	16.95	5.05
152 Nd	117.49	24.28
154 Nd	57.65	15.07
152 Sm	74.83	30.75
154 Sm	94.46	73.02
158 Sm	2.74	0.14
154 Gd	85.73	1.48
156 Gd	1.45	2.15
160 Gd	24.55	8.57
156 Dy	128.15	0.04
158 Dy	1.38	1.96
156 Er	44.25	0.02
158 Er	71.86	62.00
162 Er	32.63	2.81
164 Er	8.76	1.01
160 Yb	83.25	10.37
164 Yb	3.53	5.04
166 Yb	6.02	0.39
168 Yb	5.24	0.18
174 Yb	9.70	13.67

TABLE (5-III). (cont'd)

Nucleus	$\Omega \times 10^6$ VMI model	$\Omega \times 10^6$ VMI23 model
176 Hf	3.04	3.51
180 Hf	0.27	0.08
172 W	67.39	1.60
178 W	144.97	78.01
182 W	0.91	0.07
176 Os	19.32	0.64
178 Os	10.21	1.26
182 Os	57.88	17.73
184 Os	17.18	0.67
186 Os	31.94	2.50
188 Os	49.36	5.03
190 Os	140.53	7.75
182 Pt	430.79	117.32
184 Pt	233.33	34.69
186 Pt	374.84	33.49
190 Pt	125.94	30.27
192 Pt	168.39	66.80
194 Pt	70.60	3.55

TABLE (5-III) (cont'd)

Nucleus	$\Omega \times 10^6$ VMI model	$\Omega \times 10^6$ VMI23 model
232 Th	9.96	4.17
238 U	1.23	0.75
238 Pu	0.03	0.04
242 Pu	0.08	0.02
244 Cm	0.25	0.48

Myers and Swiatecki (1966) have determined the parameters occurring in their mass equation. The parameters A, B, S, and d can be expressed in terms of these. Myers and Swiatecki have estimated the accuracy of their shell-correction parameters to be approximately 10% - 20%. From this the uncertainty in A, B, S, and d comes out to be 20% - 40%. Admittedly, one cannot expect to obtain accurate results for individual nuclei from the parameters given by Myers and Swiatecki. It would hardly be fair to expect any sort of numerical agreement between $V'''(\beta_0)$ calculated from the MS equation and c_3 obtained here. However, we thought it would be of some interest to compare the signs of the two. For the nuclei listed in Table (5-III), it was found that in 29 cases the signs of $V'''(\beta_0)$ and c_3 are the same, while in 30 cases they are opposite; one case, ^{24}Mg, is outside the applicability of the MS equation.

B. LIMITING CASES

When $c_3 = 0$, Eq. (5.11), ofcourse, reduces to the energy equation for the VMI model. The other limiting case, $c_2 = 0$, is also of considerable interest. Eq. (5.11) can then be written as (Varshni 1971)

$$E_I(\mathcal{I}) = \frac{1}{6} c_3 (\mathcal{I}-\mathcal{I}_0)^3 + \frac{I(I+1)}{2\mathcal{I}} \qquad (5.18)$$

The model represented by this equation shall be called the VMI3 model. Combined with the equilibrium condition (5.2), Eq. (5.18) leads

to an analytical expression for the energy,

$$E_I = (p/3\hbar_0)[-p-3y \pm (p+2y)(1+2y/p)^{1/2}] \qquad (5.19)$$

where $y = [I(I+1)]^{1/2}$ and $p = \hbar_0^2 c_3^{1/2}/2$.

As we have mentioned before that the energy equations of the VMI model and that of the Harris (1965) model were shown to be equivalent by MSB, we shall also show here that an energy expression equivalent to Eq. (5.19) can be obtained from a Harris type of approach, but with a different type of dependence of the energy and angular momentum on the angular velocity.

We write the energy in the laboratory system in the form (Varshni 1971a)

$$E = E(o) + \frac{1}{2}\omega^2 \left(\sum_{n=0}^{\infty} a_n \omega^n \right) \qquad (5.20)$$

and the angular momentum as

$$\langle \phi | J_x | \phi \rangle = \omega \left(\sum_{n=0}^{\infty} b_n \omega^n \right) \qquad (5.21)$$

Here ω is the nuclear angular velocity and n is an integer. a_n's and b_n's are parameters. In equations (5.20) and (5.21) we differ from Harris (1964 and 1965) in an important respect: here n can be both, odd or even. Harris considered the case of n even only because higher order corrections to the cranking model lead to an expression for E in even powers of ω. However, here, we have adopted the philosophy

that equations (5.20) and (5.21) express the dependence of the moment-of-inertia on the degree of rotation in general terms, provided these are self-consistent. We also recognize that Eqs. (5.20) and (5.21) are non-invariant with respect to time-reversal; such a situation, though inelegant, is, however, not impossible. Bryan and Gersten (1971) have discussed the possibility of time-reversal non-invariance in nuclear forces.

Making use of the Hellmann-Feynman theorem (Hellmann 1937, Feynman 1939) and the self-consistency requirement, we obtain

$$E = E(0) + \frac{1}{2}\omega^2(\mathcal{J}_0 + 4\lambda_1\omega + 6\lambda_2\omega^2 + 8\lambda_3\omega^3 + 10\lambda_4\omega^4 + \ldots) \quad (5.22)$$

and

$$\langle\phi|J_x|\phi\rangle = \omega(\mathcal{J}_0 + 3\lambda_1\omega + 4\lambda_2\omega^2 + 5\lambda_3\omega^3 + 6\lambda_4\omega^4 + \ldots) \quad (5.23)$$

Here $\mathcal{J}_0$, λ_1, λ_2.., etc. are parameters. $\mathcal{J}_0$ may be considered to be the moment-of-inertia in the ground state. Here we restrict ourselves to the first two (n = 0 and 1) terms. Then we have

$$E_I = \frac{1}{2}\omega^2(\mathcal{J}_0 + 4\lambda_1\omega) \quad (5.24)$$

and

$$[I(I+1)]^{1/2} = \omega(\mathcal{J}_0 + 3\lambda_1\omega). \quad (5.25)$$

The parameter ω may be eliminated from these, leading to the following equation for the energy E_I:

$$E_I = \frac{p}{3\mathcal{J}_0}\{-y + (p+2y)[-1 \pm (1 + 2y/p)^{1/2}]\} \quad (5.26)$$

where $p = \hbar^2_0/6\lambda_1$ and $y = [I(I+1)]^{1/2}$. Eq. (5.26) is seen to be identical with Eq. (5.19). The solution with the + sign before the square root spans the region $R_4 = 2.467$ to $R_4 = 3.333$. With the negative sign before the square root, one has to assume that $\hbar_0$ is negative (Scharff-Goldhaber and Goldhaber 1970). E_0 is different from zero in this case and the level energy is $E'_I = E_I - E_0$. The range of this solution is between $R_4 = 1.826$ and $R_4 = 2.467$. As is the case with the VMI model (Scharff-Goldhaber and Goldhaber 1970, Mariscotti 1970) the VMI3 model has a natural limit at $R_4 = 1.826$.

For five nuclei, namely, ^{102}Zr, ^{106}Mo, ^{106}Ru, ^{154}Sm, and ^{236}U, the parameter c_2 is almost zero and in these cases the VMI3 model is applicable. It appears that the potential energy surface of these nuclei is so steep, that the c_3 term, instead of being just a correction term, becomes the major term. It may be pointed out that the VMI model or its extensions only probe the region $\hbar > \hbar_0$ and consequently the negative behaviour of the c_3 term for $\hbar < \hbar_0$ does not create any problem.

GRAPHICAL COMPARISON

Several years ago it was shown by Mallmann (1959) with the then available data, that the energy ratios R_6 and R_8, plotted against R_4, lie on two "universal" curves. In those cases, where the energy levels are a function of two parameters, such a plot of R_I and R_4 is a convenient method for comparing theory with experiment and has been frequently employed (Sood 1967, 1968 , Stephens et al 1968, Wold 1969, Scharff-Goldhaber and Goldhaber 1970, Mariscotti 1970).

Such a plot is, however, not very sensitive to small differences; it would be difficult to discern a difference of $\leq 1\%$ in the energy in such a plot of convenient size. Varshni (1971b) has suggested another plot which is very sensitive to small differences in the region $1.826 < R_4 < 3.333$. It employs either of the following functions (Varshni 1968):

$$G_I = \frac{E_I/I - E_2/2}{E_4/4 - E_2/2} \tag{5.27}$$

or

$$Ga_I = \frac{E_I/\sqrt{I(I+1)} - E_2/\sqrt{6}}{E_4/\sqrt{20} - E_2/\sqrt{6}} \tag{5.28}$$

The G function is convenient when a theoretical expression gives the behaviour E-I in the vibrational region while the Ga function is useful if $E \sim [I(I+1)]^{1/2}$ in the vibrational region. Here we shall employ the Ga function.

In fig. (5-2) we show a plot of Ga_6 versus R_4. Theoretical curves obtained from the VMI and VMI3 models are shown together with the experimental points. To orientate the reader, we also show the results obtained from the two-parameter Bohr-Mottelson (1953) and those from the Davydov-Filippov model (1958). Ga_6 becomes increasingly sensitive as we go from $R_4 = 10/3$ to $R_4 = 1.826$. A 1% uncertainty in each of E_2, E_4 and E_6 can lead to $\sim 3.5\%$ uncertainty in Ga_6 at $R_4 = 3.33$, and to $\sim 7\%$ uncertainty at $R_4 \simeq 2.4$. On a 25cm x 23cm version of Fig. (5-2),

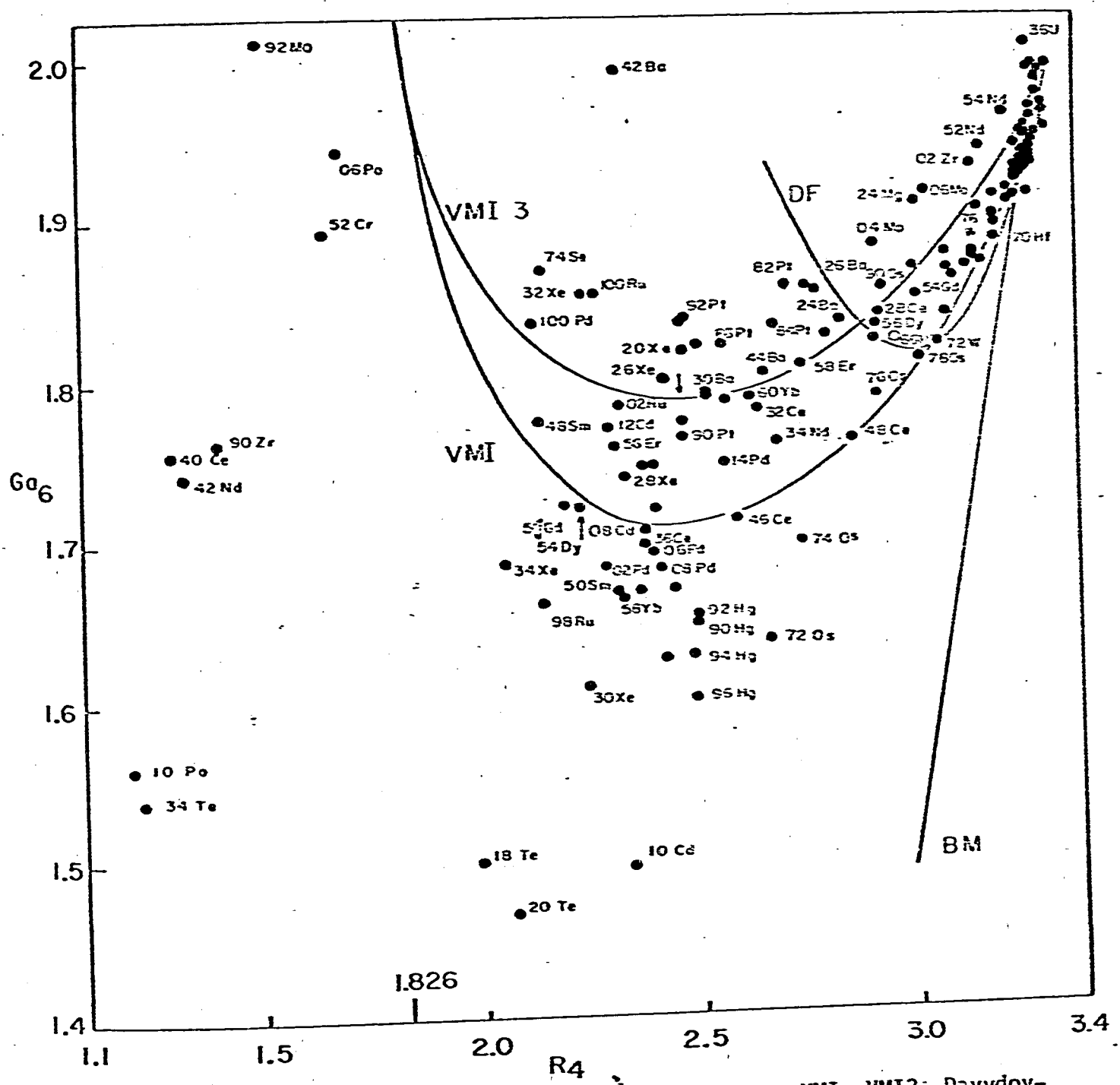

Fig. (5.2) Plot of Ga_6 versus R_4. Calculated values from VMI, VMI3, Davydov-Filippov (DF) and Bohr-Mottelson (BM) models are shown by solid lines. Experimental values for each nucleus are shown by dots which are identified by the last two digits of the mass-number and the chemical symbol. Near R_4 = 3.333 there is much overlapping of the experimental points and hence these could not be identified legibly.

one can compare experiment and theory to ~0.1%. This method is essentially equivalent to fixing the two parameters in the theoretical expression from E_2 and E_4 and then comparing the observed and calculated values of E_I. In some ways it is a more critical test than the numerical least-squares fit discussed earlier, which it complements.

To minimize confusion, the error bars for the experimental points are not shown in fig. (5-2). However, even if we allow for the maximum errors, it is not possible to have a single curve passing through all the points. As a matter of fact, from fig. (5-2), in conjunction with the experimental errors given in Table (5-II), we can estimate that in the region $2.23 < R_4 < 3.333$, the best theoretical two-parameter expression cannot give a fit better than 0.5% for all of the nuclei. If we require a fit better than 0.5% for all the nuclei having $R_4 > 2.23$, we certainly need a third parameter.

It will be noticed from fig. (5-2) that in the neighbourhood of $R_4 = 3.333$, a good number of points lie on the VMI curve or very close to it. This accounts for the spectacular success of the VMI model for such nuclei. As we go away from the $R_4 = 10/3$, there is an increasing spread in the experimental points. The effect of adding the c_3 term can now be discussed, assuming that the fit is being made with just three levels namely the 2+, 4+ and 6+ levels. Nuclei having c_3 positive lie above the VMI curve, and those having c_3 negative lie below it. The VMI3 curve sets the limit up to which an exact fit can be made for positive c_3. If a nucleus lies above the VMI3 curve it is not possible to obtain an exact fit for the VMI23 model though there are

three parameters to be determined from 3 levels. For negative c_3 it is not possible to draw a similar limiting curve because the limiting value of c_3 depends on the value of c_2 (see eq. (5.16)). However, an approximate measure of the extent of negative c_3 region can be given. Its boundary lies approximately the same distance below the VMI curve as the VMI3 curve lies above the VMI curve. This statement is based on the data for nuclei which have $R_4 > 2.25$, and which lie below the VMI curve.

In the foregoing we have discussed the (Ga_6, R_4) plot only. Broadly speaking similar patterns in the distribution of points are seen in the (Ga_8, R_4), (Ga_{10}, R_4) and (Ga_{12}, R_4) plots also. There are, however, differences in details which appear to be due to the quartic and other higher order terms in eq. (5.10).

CONCLUSIONS

In conclusion, we have shown that the inclusion of the $(\mathcal{I}-\mathcal{I}_0)^3$ term leads to a better description of the rotational and quasi-rotational ground state bands of even-even nuclei. The present analysis provides a sensitive means of obtaining information about the shape of the potential energy surfaces of nuclei. The c_3 values determined here represent the first reliable estimates of the magnitudes of the anharmonic terms in the potential energy.

CHAPTER VI

POTENTIAL ENERGY SURFACES OF "BACK-BENDING" NUCLEI

INTRODUCTION

Recently, Johnson, Ryde and Sztarkier (1971) observed an interesting and anomalous feature in the ground-state rotational band of ^{160}Dy. On plotting the nuclear moment-of-inertia defined by

$$2\mathcal{I}/\hbar^2 = [dE/dI(I+1)]^{-1} \tag{6.1}$$

as a function of the square of the rotational frequency, ω; they found a sharp sudden increase of $\mathcal{I}$ at higher angular momenta states. Here ω is defined by

$$\hbar\omega = dE/d\sqrt{I(I+1)} \tag{6.2}$$

Subsequent investigations by the Stockholm group (Johnson et al 1972) and by others (Thieberger et al 1972, Lieder et al 1972, Beuscher et al 1972, and Taras et al 1972), have confirmed and extended these observations to other nuclei. It has been found that besides the sudden increase in $\mathcal{I}$ in the $(\mathcal{I},\omega^2)$ plot, sometimes there is a back-bending behaviour resulting in an S-shaped curve. This behaviour is often referred to as "back-bending" in the literature on this subject. For information retrieval purposes, we feel that a more scientific looking name is called for.

We propose to call it the "S effect" after the S-shaped curve in the ($\mathcal{J}$,ω^2) plot.

The S effect is also seen quite readily in a plot of the transition energy (E_I-E_{I-2}) versus I. Such a plot for a number of nuclei is shown in Figs. (6.1a) and (6.1b) The qualitative behaviour of the curve for a nucleus on such a plot is similar to the corresponding curve for that nucleus on an (ω^2,$\mathcal{J}$) plot; the effect is, however, more pronounced in the latter plot.

A possible explanation of the S effect is the Coriolis anti-pairing (CAP) effect, first predicted by Mottelson and Valatin (1961). These authors pointed out that because of the CAP effect the strength of the pairing force will become progressively weaker with increasing rotational frequency and at a critical value of the spin, I_c, these pairing correlations will disappear. Above I_c, the moment-of-inertia is expected to approach the rigid rotor value. Subsequent investigations (Chan and Valatin 1966, Krumlinde 1968), showed that the transition may take place in two stages, since the transition frequency for neutrons and protons may be different.

Recently, several new theoretical attempts to explain the S effect have appeared. Krumlinde and Szymanski (1971) have examined a model of a rotor coupled to valence particles which are distributed over two degenerate levels and interact through a pairing force. An alternative interpretation has been proposed by Stephens and Simon (1972), who investigate the Coriolis effects at high angular momentum on particles in

various j-shells, principally the $i_{13/2}$ shell, in a rotational nucleus. They find that at moderately high values of the spin ($I \geq 10$) a pair of these particles tend to decouple from the core and align their angular momenta, $2j-1$, with the rotational angular momentum of the core. The resulting quasi-particle states mix with the ground-state band and this leads to an S effect in the ground band. Both of the investigations are model investigations and so far have not been applied to any real nucleus.

Molinari and Regge (1972) have taken the viewpoint that the observed experimental effects are due to the hybridization of bands and have carried out calculations on ^{162}Er. For best results they need to mix three bands (K=0, K=2 and K=3). We also note here a phenomenological attempt due to Wahlborn and Gupta (1972) to represent S-shaped curves in $(\mathcal{J},\omega^2)$ plots by a formula.

The nuclear potential energy surface is a subject of considerable interest for many nuclear processes. It plays a vital role for collective states and for the fission reaction. A better determination of the surface would not only improve our understanding of these processes; but also provide a test of the theoretical models which attempt to calculate the surface.

In a previous study (Ch. V of this thesis) we have extended the variable moment-of-inertia (VMI) model of Mariscotti, Scharff-Goldhaber and Buck (1969) for the ground state rotational bands in even-even nuclei by adding an anharmonic term to the potential energy. It was shown that the inclusion of the anharmonic term improves agreement with the experimental

data in many cases, and provides a sensitive means of probing the potential energy surfaces of nuclei. In this chapter we investigate the behaviour of the potential energy surface of nuclei which show the S-effect by employing a polynomial in $(\mathcal{J}-\mathcal{J}_0)$ for the potential energy.

In the VMI model the energy is expressed as the sum of the potential energy and the kinetic energy. We represent the potential energy by the following expression.

$$V(\mathcal{J}) = \frac{1}{2} c_2 (\mathcal{J}-\mathcal{J}_0)^2 + \frac{1}{6} c_3 (\mathcal{J}-\mathcal{J}_0)^3 + \frac{1}{24} c_4(\mathcal{J}-\mathcal{J}_0)^4$$

$$+ \frac{1}{120} c_5 (\mathcal{J}-\mathcal{J}_0)^5 + \frac{1}{720} c_6 (\mathcal{J}-\mathcal{J}_0)^6 \qquad (6.3)$$

where $\mathcal{J}_0$ is the ground-state moment-of-inertia and c_2, c_3, c_4, c_5 and c_6 are parameters. Thus the total energy for a state with spin I is given by

$$E_I(\mathcal{J}) = \frac{I(I+1)}{2\mathcal{J}} + \frac{1}{2} c_2 (\mathcal{J}-\mathcal{J}_0)^2 + \frac{1}{6} c_3 (\mathcal{J}-\mathcal{J}_0)^3$$

$$+ \frac{1}{24} c_4(\mathcal{J}-\mathcal{J}_0)^4 + \frac{1}{120} c_5 (\mathcal{J}-\mathcal{J}_0)^5$$

$$+ \frac{1}{720} c_6(\mathcal{J}-\mathcal{J}_0)^6 \qquad (6.4)$$

The condition for equilibrium which determines the moment-of-inertia $\mathcal{I}_I$ (in units of $\hbar^2$) for each state is given by

$$\frac{\partial E_I(\mathcal{I})}{\partial \mathcal{I}} = 0. \tag{6.5}$$

Condition (6.5) yields the following relation for the ratio $r_I (= \mathcal{I}_I/\mathcal{I}_o)$

$$-I(I+1)/2r_I^2 + (r_I-1)/p_2 + (r_I-1)^2/p_3 + (r_I-1)^3/p_4$$

$$+ (r_I-1)^4/p_5 + (r_I-1)^5/p_6 = 0 \tag{6.6}$$

where $p_2 = 1/c_2\mathcal{I}_o^3$,

$p_3 = 2/c_3\mathcal{I}_o^4$,

$p_4 = 6/c_4\mathcal{I}_o^5$,

$p_5 = 24/c_5\mathcal{I}_o^6$,

and $p_6 = 120/c_6\mathcal{I}_o^7$,

In terms of r_I, the expression for the energy may be more conveniently written as

$$\mathcal{I}_o E_I(\mathcal{I}) = I(I+1)/2r_I + (r_I-1)^2/2p_2 + (r_I-1)^3/3p_3$$

$$+ (r_I-1)^4/4p_4 + (r_I-1)^5/5p_5 + (r_I-1)^6/6p_6. \tag{6.7}$$

We shall call this model the VMI6P model (6P for the sixth degree polynomial).

CALCULATIONS AND RESULTS

In this study we have considered the following nuclei: ^{132}Ce, ^{158}Dy, ^{160}Dy, ^{158}Er, ^{162}Er, ^{164}Yb, ^{166}Yb, ^{168}Yb, ^{170}Yb, ^{168}Hf and ^{172}Hf. In eight of these, namely ^{132}Ce, ^{158}Dy, ^{160}Dy, ^{158}Er, ^{162}Er, ^{164}Yb, ^{166}Yb and ^{168}Hf, the S-effect has been observed. High precision results on the other three nuclei have become available recently and these were included here for comparison purposes and to show the behaviour of a series of Yb isotopes.

The six parameters entering in our model were determined by means of a least squares fitting procedure involving all the experimentally known level energies. All energies were given the same weight. The difference on this point from our previous study (Varshni and Bose 1972) may be noted here. The parameters thus determined are shown in Table (6.I). The calculated level energies are compared with the observed ones in Table (6-II). The first line against each nucleus shows the experimental values, the second, the experimental errors; the rms experimental error is shown in the last column of the second line. The third line gives the calculated values and the fourth one, the difference between the calculated and experimental values, the rms deviation between the two values is shown in the last column of the fourth line. Experimental errors in the level energies were not available for ^{170}Yb and ^{172}Hf. For six of the nuclei under consideration, namely ^{158}Dy, ^{160}Dy, ^{162}Er, ^{168}Yb, ^{170}Yb and ^{172}Hf, Wahlborn and Gupta (1972) have fitted their formula; results obtained by

TABLE (6-I) Values of the parameters occurring in the VMI6P model obtained from the least-squares fit.

Nucleus	Source of Expt. Data. Reference No.	g_0 (10^{-2}keV^{-1})	c_2 (10^{6}keV^{3})	c_3 (10^{8}keV^{4})	c_4 (10^{10}keV^{5})	c_5 (10^{13}keV^{6})	c_6 (10^{14}keV^{7})
^{132}Ce	a	0.754248	9.44513	-16.3741	51.4670	-9.42462	66.1149
^{158}Dy	b	3.02176	3.70676	-2.58828	5.19155	-1.17119	10.4485
^{160}Dy	c	3.42633	3.91554	2.94421	-17.2956	2.69192	-16.6799
^{158}Er	d	0.979774	1.6500×10^{-3}	2.79919	-0.320868	-0.324238	2.24767
^{162}Er	c	2.90725	3.82001	3.20769	-15.3910	1.99324	-10.000
^{164}Yb	e	2.35151	2.44609	3.13863	-7.57962	0.538877	-0.999511
^{166}Yb	d	2.92031	4.85175	-2.86549	3.77042	-1.0000	8.83056
^{168}Yb	c	3.39458	4.33941	-2.39242	-1.50826	0.510069	-3.44354
^{170}Yb	f	3.54329	6.01786	5.87050	-32.0875	-4.99057	354.341
^{168}Hf	e	2.32280	2.15720	2.75129	-5.95600	0.392000	-0.778904
^{172}Hf	f	3.11671	3.85632	1.34796	-10.4211	1.82001	-13.7026

a Taras et al (1972), b Thieberger et al (1972), c Johnson et al (1972)
d Beuscher et al (1972), e Lieder et al (1972), f Wahlborn and Gupta (1972).

TABLE(6-II). Experimental and calculated energies (in keV) of levels of ground-state bands. For each nucleus the first row shows the experimental energies (E), the second shows the errors in these values (Er) (for ^{170}Yb and ^{172}Hf these values are not available), the third shows the energies calculated (T) using the VMI6P model and the fourth row gives the differences (D) between the calculated and the experimental energies. The level energies as calculated by Wahlborn and Gupta (WG) are shown in the fifth row for those nuclei where they are available. For the nucleus ^{162}Er, the sixth row gives the energy values as calculated by Molinari and Regge (MR). Such cases, where the value of r_I from Eq(6.4) is not positive, are indicated by two asterisks.

Nucleus	I =	2	4	6	8	10	12	14	16	18	20	22	Rms deviation
^{132}Ce	E	325.4	857.6	1540.0	2326.8	3154.5	3724.2	4236.4	4935.0	5758.4			
	Er	0.33	0.62	0.92	1.21	1.47	1.58	1.66	1.80	1.98			1.39
	T	320.54	864.80	1541.05	2315.56	3161.67	3681.55	4281.70	4963.69	5726.22	6568.12	7488.28	
	D	-4.86	7.20	1.05	-11.24	7.17	-42.65	45.30	28.69	-32.18			25.79
^{158}Dy	E	98.8	317.0	637.6	1043.9	1519.9	2049.2	2612.6	3190.7	3781.7	4407.5	5085.6	
	Er	0.30	0.42	0.52	0.60	0.67	0.74	0.79	0.85	0.90	0.95	1.00	0.74
	T	97.88	316.55	638.21	1044.50	1519.44	2048.28	2613.79	3190.28	3780.54	4409.57	5084.65	
	D	-0.92	-0.45	0.61	0.60	-0.46	-0.92	1.19	-0.42	-1.16	2.07	-0.95	1.00
	WG	96.11	313.6	638.4	1052	1533	2062	2620	3197	3788	4407		
^{160}Dy	E	86.79	283.81	581.21	967.21	1428.91	1951.71	2515.21	3091.91	672.41			
	Er	0.01	0.02	0.20	0.28	0.35	0.46	0.55	0.63	0.69			0.42
	T	86.76	283.84	581.21	967.17	1428.96	1951.68	2515.22	3091.91	3672.41	**	**	
	D	-0.03	0.03	0.00	-0.04	0.05	-0.03	0.01	0.00	0.00			0.03
	WG	85.86	282.3	580.8	969.6	1434	1958	2519	3095	3676	4279		
^{158}Er	E	192.0	527.1	970.2	1493.2	2072.1	2680.2	3190.2	3663.0	4229.3			
	Er	0.30	0.42	0.52	0.60	0.67	0.74	0.79	0.85	0.90			0.67
	T	190.88	528.93	971.10	1491.75	2070.09	2682.45	3182.47	3677.05	4222.58	4819.54	5467.71	
	D	-1.12	1.83	0.90	-1.45	-2.01	2.25	-7.73	14.05	-6.72			5.95
^{162}Er	E	102.08	329.63	666.76	1096.76	1602.86	2165.06	2745.66	3292.26	3846.46			
	Er	0.10	0.14	0.17	0.27	0.33	0.45	0.54	0.67	0.84			0.46
	T	101.67	329.27	666.72	1097.15	1603.19	2164.48	2745.89	3292.15	3846.44	**	**	
	D	-0.41	-0.36	-0.04	0.39	0.33	-0.58	0.23	-0.11	-0.02			0.33
	WG	100.0	327.1	667.8	1104	1615	2175	2747	3296	3850	4447		
	MR	101.38	329.66	667.51	1096.24	1601.87	2166.71	2744.52	3293.18	3846.09			

continued.....

Table(6-II) continued...

Nucleus	I=	2	4	6	8	10	12	14	16	18	20	22	rms deviation
^{164}Yb	E	123	385	759	1221	1751	2328	2898	3388				2.12
	Er	1.00	1.41	1.73	2.00	2.24	2.45	2.65	2.83	3917.49	4491.66	5111.48	
	T	122.74	385.25	758.95	1220.91	1751.07	2327.99	2898.00	3388.01				0.14
	D	-0.26	0.25	-0.05	-0.09	0.07	-0.01	0.00	0.01				
^{166}Yb	E	102.2	330.3	668.0	1098.2	1605.9	2175.6	2779.4	3273.9	3782.9	4371.7		0.82
	Er	0.30	0.42	0.52	0.60	0.67	0.74	0.79	0.85	1.31	1.35	4992.79	
	T	101.50	329.70	668.29	1099.33	1606.20	2173.09	2780.97	3268.65	3792.44	4367.14		3.88
	D	-0.70	-0.60	0.29	1.13	0.30	-2.51	1.57	-5.25	9.54	-4.56		
^{168}Yb	E	87.73	286.55	585.30	970.05	1425.45	1935.95	2488.55	3073.05	3686.85			0.46
	Er	0.01	0.02	0.07	0.12	0.24	0.55	0.63	0.70	0.80	4330.46	5004.63	
	T	87.61	286.52	585.40	970.05	1425.33	1936.08	2488.48	3073.07	3686.85			0.08
	D	-0.12	-0.03	0.10	0.00	-0.12	0.13	-0.07	0.02	0.00			
	WG	87.50	286.4	585.9	971.2	1426	1936	2488	3074	3687	4327		
^{170}Yb	E	84.25	277.4	573.3	963.4	1438	1983	2580					
	Er	- -	Not available		- -								
	T	84.21	277.45	573.26	963.42	1437.99	1983.00	2580.00	3232.48	3950.32	4736.36	5591.35	0.03
	D	-0.04	0.05	-0.04	0.02	-0.01	0.00	0.00					
	WG	83.97	277.2	573.7	964.7	1439	1984	2581	3211				
^{168}Hf	E	124	385	756	1212	1734	2303	2854					2.00
	Er	1.00	1.41	1.73	2.00	2.24	2.45	2.65					
	T	123.56	385.47	755.86	1211.85	1734.12	2302.97	2853.99	3291.25	3769.98	4291.64	4856.34	0.26
	D	-0.44	0.47	-0.14	-0.15	0.12	-0.03	-0.01					
^{172}Hf	E	95.17	309.2	628.2	1037	1521	2064	2654	3277	3919			
	Er	- -	Not available		- -								
	T	95.09	309.16	628.26	1037.13	1520.69	2064.26	2653.88	3277.02	3919.00	**	**	0.15
	D	-0.08	-0.04	0.06	0.13	-0.31	0.26	-0.12	0.02	0.00			
	WG	94.22	308.2	629.4	1041	1526	2067	2654	3278	3941			

them are shown in fifth line against the respective nucleus. Molinari and Regge's (1972) results on ^{162}Er are given in the sixth line against this nucleus. Though calculated values by the VMI6P model are shown up to 22+ state in all cases, the reliability of these values rapidly decreases as one goes farther from the highest observed level.

A limitation of the polynomial expression for the potential energy may be noted here. The region that we are exploring is far away from the fission hump and one expects, that, except for local irregularities, the potential energy should continue to rise with β up to the fission hump. This asymptotic condition is not built into the polynomial expression and for r_I greater than that for the highest observed level it is not satisfactory for extrapolation purposes. For example, for ^{160}Dy, the potential energy surface shows an inflexion point after the 18+ level and after reaching a low maximum the energy begins to go down. Consequently no positive solution for r_I could be obtained for I>18. A similar situation occurs for ^{162}Er and ^{172}Hf. Such levels are indicated by two asterisks in Table (6-II).

The expression for the potential energy, eq. (6.3), looks like a Taylor expansion. A word of caution regarding the significance of the parameters is necessary. In the course of least-squares fitting it was found that for most of the nuclei (the exceptions being ^{168}Yb and ^{170}Yb) in our list, for high spin levels, $(r_I-1) \geq 1$. Consequently the reader is warned that for such nuclei the c_n's in eq. (6.3) should not be interpreted as, or compared with, derivatives of the potential energy surface.

A difference on this point with a previous investigation made in Chapter V of this thesis , is emphasized; in that study it was legitimate to consider c_3 as a derivative for most of the nuclei.

At this point the reader may well ask why a sixth degree polynomial was chosen, why not fifth, seventh or for that matter any other degree. In making this choice we have been governed by making a compromise between obtaining agreement with the experimental data on one hand and computational problems on the other. Too few terms lead to poor agreement with experimental data, too many lead to computational difficulties in the non-linear least-squares fitting. There are large cancellations between contributions from the different terms in Eq. (6.6) and unless the parameters are kept under certain constraints, it is not possible to find meaningful solutions for r_I. This difficulty is in addition to other well known difficulties associated with non-linear least-squares fits which are discussed in books on computer methods and numerical analysis. As discussed below, with the sixth-degree polynomial we have been able to obtain reasonable results for all nuclei in our list except one.

DISCUSSION

A comparison of the rms experimental errors and the rms deviations of the calculated values from the experimental ones shows that the agreement between the calculated and observed values is excellent for ^{158}Dy, ^{160}Dy, ^{162}Er, ^{164}Yb, ^{168}Yb, ^{170}Yb, ^{168}Hf and ^{172}Hf, reasonable for ^{158}Er

and ^{166}Yb, and very poor for ^{132}Ce. The degree of agreement appears to depend on the strength of the dip in the plot of transition energies versus I. A satisfactory representation of the potential energy surface of ^{132}Ce will clearly require more than six parameters if a polynomial expression in $(\mathcal{J}-\mathcal{J}_0)$ is used.

The rms deviations, however, do not tell the whole story. A study of the differences between the observed and calculated values for ^{158}Er and ^{166}Yb shows that the maximum differences occur in the S effect region; for low values of the angular momentum, the agreement is quite satisfactory.

For ^{162}Er, we note that our calculated values with six-parameters are in slightly better agreement with the experimental data than those by Molinari and Regge (1972) with seven parameters. The empirical expression of Wahlborn and Gupta (1972) uses a total of five parameters. In all cases the results obtained in this study are better than those obtained by Wahlborn and Gupta, specially for those nuclei which show the S effect.

Another way to compare experiment and theory is through the $(\mathcal{J},\omega^2)$ plot. This is shown for three nuclei in Fig. (6.3). In making this plot we have used the expression for $\hbar\omega$ as given by Johnson et al (1972). The expression for $\hbar\omega$ given by Thieberger et al (1972) gives slightly different results. The agreement between the experimental points and theoretical curves is seen to be satisfactory in Fig. (6.3).

The gradual change in the behaviour of the isotopes of Yb may be seen in Fig. (6.1b). The curvature in the region before the onset of the S effect is seen to decrease as we go from ^{164}Yb to ^{170}Yb. A similar behaviour can also be noticed in ^{168}Hf and ^{172}Hf. For the few nuclei in which the S effect has been so far observed, the following rule appears to be followed: In a series of isotopes, the softer the nucleus, the more pronounced is the effect.

The potential energy surfaces obtained from the derived parameters are shown in Fig. (6.2a) and (6.2b). The VMI model only probes the region $\delta > \delta_0$, hence only this region has been shown. A comparison of the surfaces for "normal" nuclei (^{168}Yb, ^{170}Yb and ^{172}Hf) with those for S effect nuclei shows that in the latter there is an arch like protuberance. In a sense the obtained potential energy surfaces are "experimental" ones and it would be of interest to compare these with those calculated from theory. In recent years there have been a number of attempts to calculate the potential energy surfaces of nuclei to varying degrees of sophistication. A semiempirical method for describing the influence of single-particle shell effects on nuclear ground-state masses and deformations was introduced a few years ago by Myers and Swiatecki (1967) and they suggested an analytical formula for the nuclear potential energy surface. Soon after this development Strutinsky (1967, 1968) took an important step and proposed a general method of calculating the nuclear potential energy surface. This method is sometimes denoted the Strutinsky prescription or shell-correction method. Strutinsky calculated the microscopic contribution using single-particle levels

obtained from a one-body potential. The difference between the sum of the exact single-particle energies and an integral over an average density of levels is considered to be the shell-correction term. This is then added to the appropriate liquid-drop model energy. A foundation to the shell correction approach has been recently presented by Bunatian, Kolomietz and Strutinsky (1972). A good number of calculations on the potential energy surfaces of nuclei have been carried out in recent years (Nilsson et al 1969, Johansson et al 1970, Metag et al 1971, Mosel and Scharnweber 1970, Mosel and Schmitt 1971, Brack et al 1972, Gotz et al 1972, Moller 1972 and Bolsterli et al 1972), using the Strutinsky prescription. However, most of them are on heavy nuclei; no results are available for nuclei which have been considered in this study. We may note here that the aforesaid investigations do not take into account any Coriolis effects in the calculation of the nuclear potential energy surface. It would be of interest, even in a model calculation, to see if the inclusion of Coriolis effects leads to the type of protuberance that we have found.

A suggestion due to Taras et al (1972) may also be discussed here. These authors have plotted the level excitation energy as a function of I(I+1) for S effect nuclei. These authors claim that the points fall on two intersecting straight lines. If this suggestion were correct to the degree of accuracy of the experimental data, one would expect that in the plot of transition energy versus I, the points should fall on two lines except for one point near the intersection of the two lines. As can be readily noticed from Figs. (6.1a) and (6.1b), this is not true. We conclude that the suggested relation is very approximate.

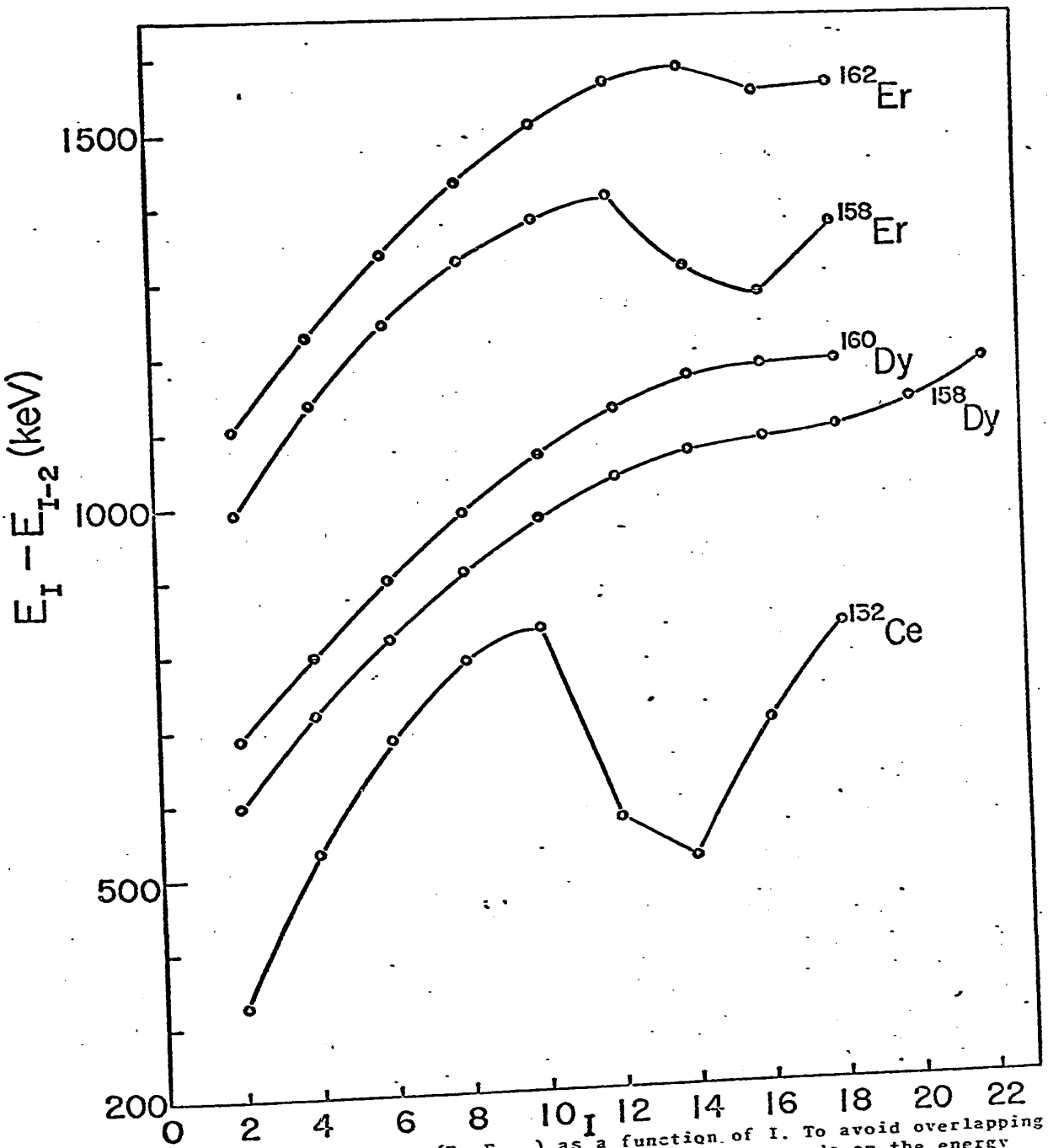

Fig. 6.1a Transition energy ($E_I - E_{I-2}$) as a function of I. To avoid overlapping points for various nuclei have been shifted upwards on the energy scale by the following amounts (in keV): ^{158}Dy(500), ^{160}Dy(600), ^{158}Er(800) and ^{162}Er(1000).

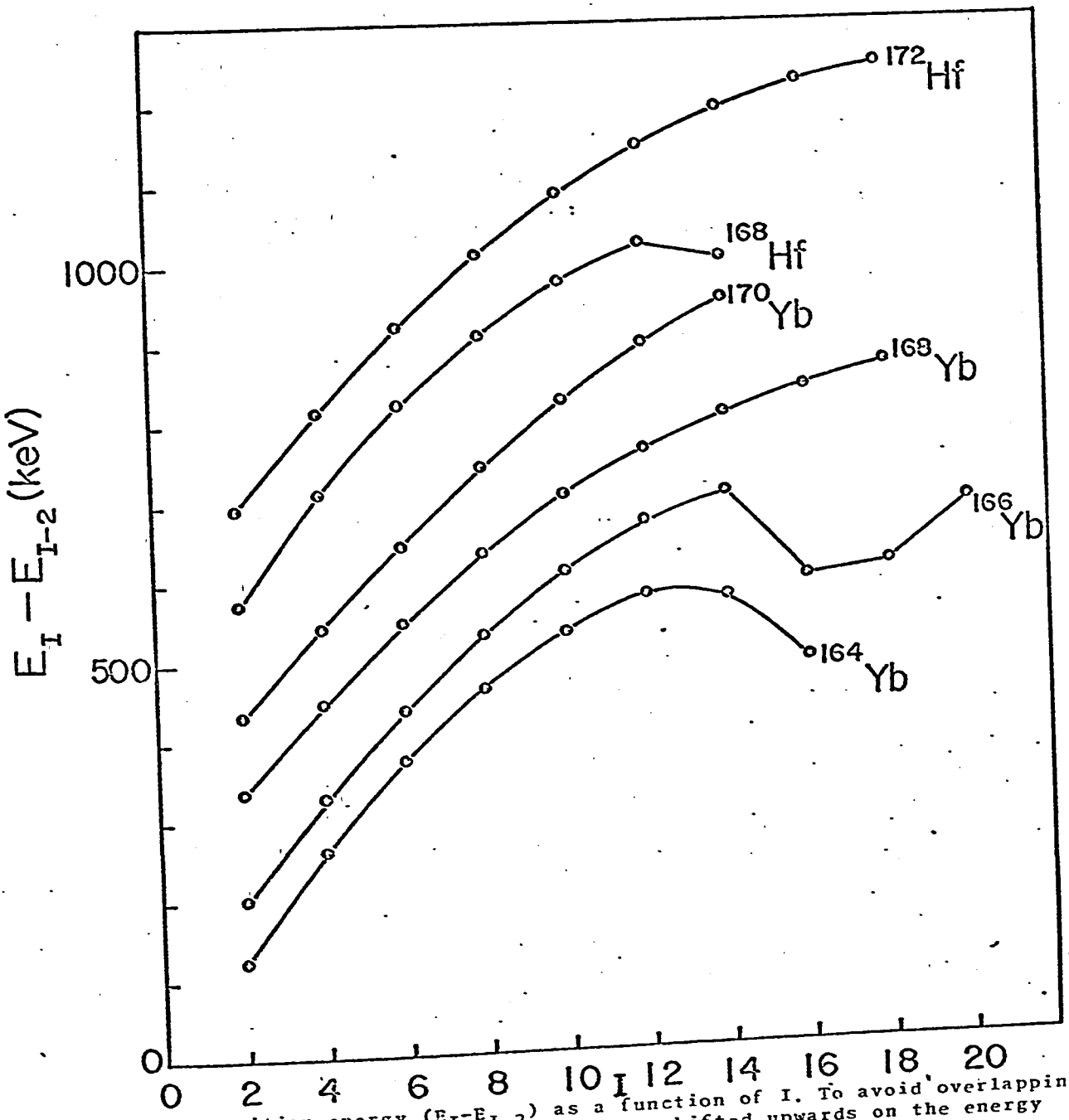

Fig. 6.1b Transition energy ($E_I - E_{I-2}$) as a function of I. To avoid overlapping, points for various nuclei have been shifted upwards on the energy scale by following amounts (in keV): ^{168}Hf(450), ^{172}Hf(600). ^{166}Yb(100), ^{168}Yb(250), ^{170}Yb(350),

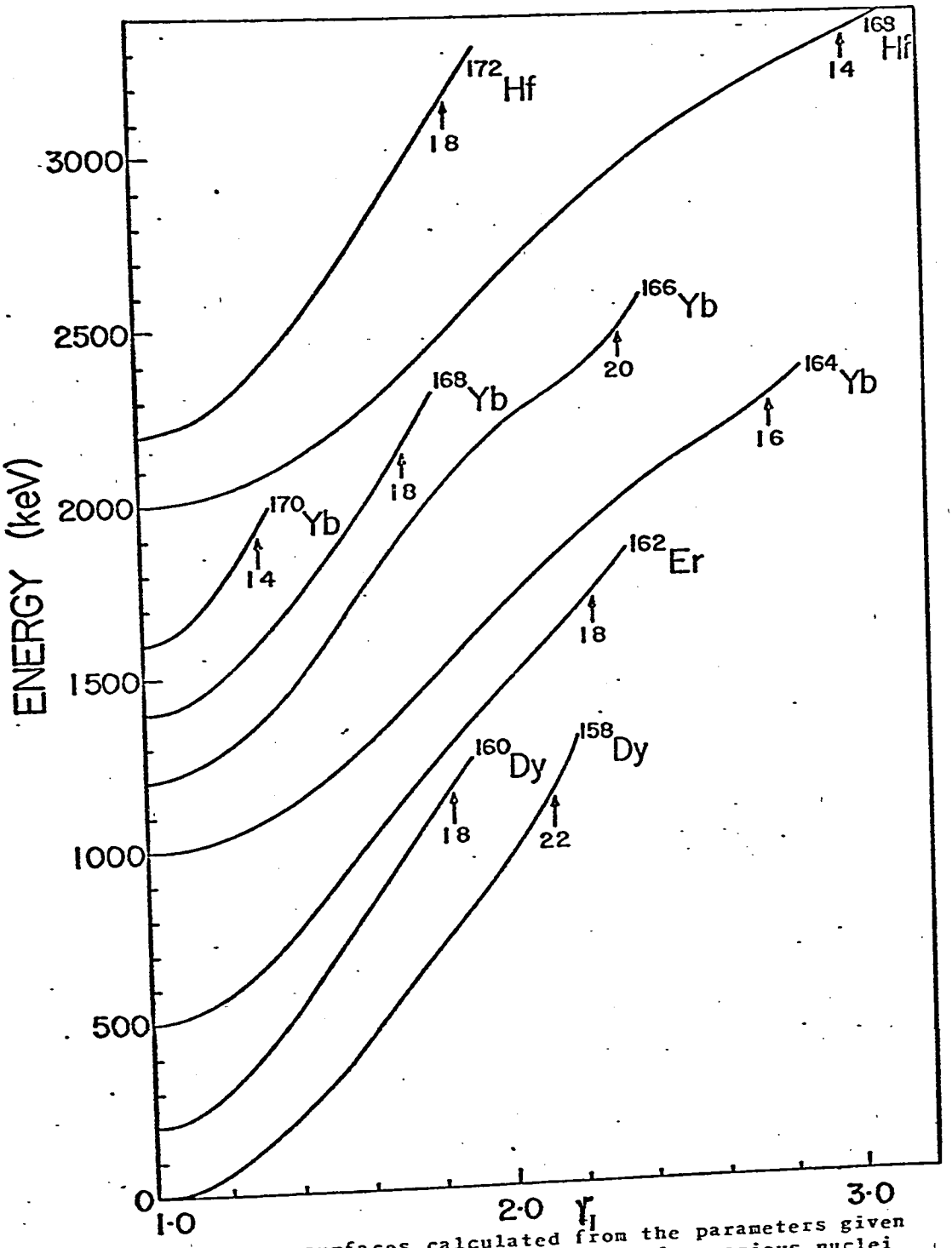

Fig. 6.2a Potential energy surfaces calculated from the parameters given in Table 6.I. To avoid overlapping, curves for various nuclei have been shifted upwards on the energy scale by the following amounts (in keV): ^{160}Dy(200), ^{162}Er(500), ^{164}Yb(1000), ^{166}Yb(1200), ^{168}Yb(1400), ^{170}Yb(1600), ^{168}Hf(2000), and ^{172}Hf(2200). Only the region $\mathcal{J} \geq \mathcal{J}_0$ is shown. The position of the highest observed level has been indicated by an arrow; the number below the arrow gives the spin.

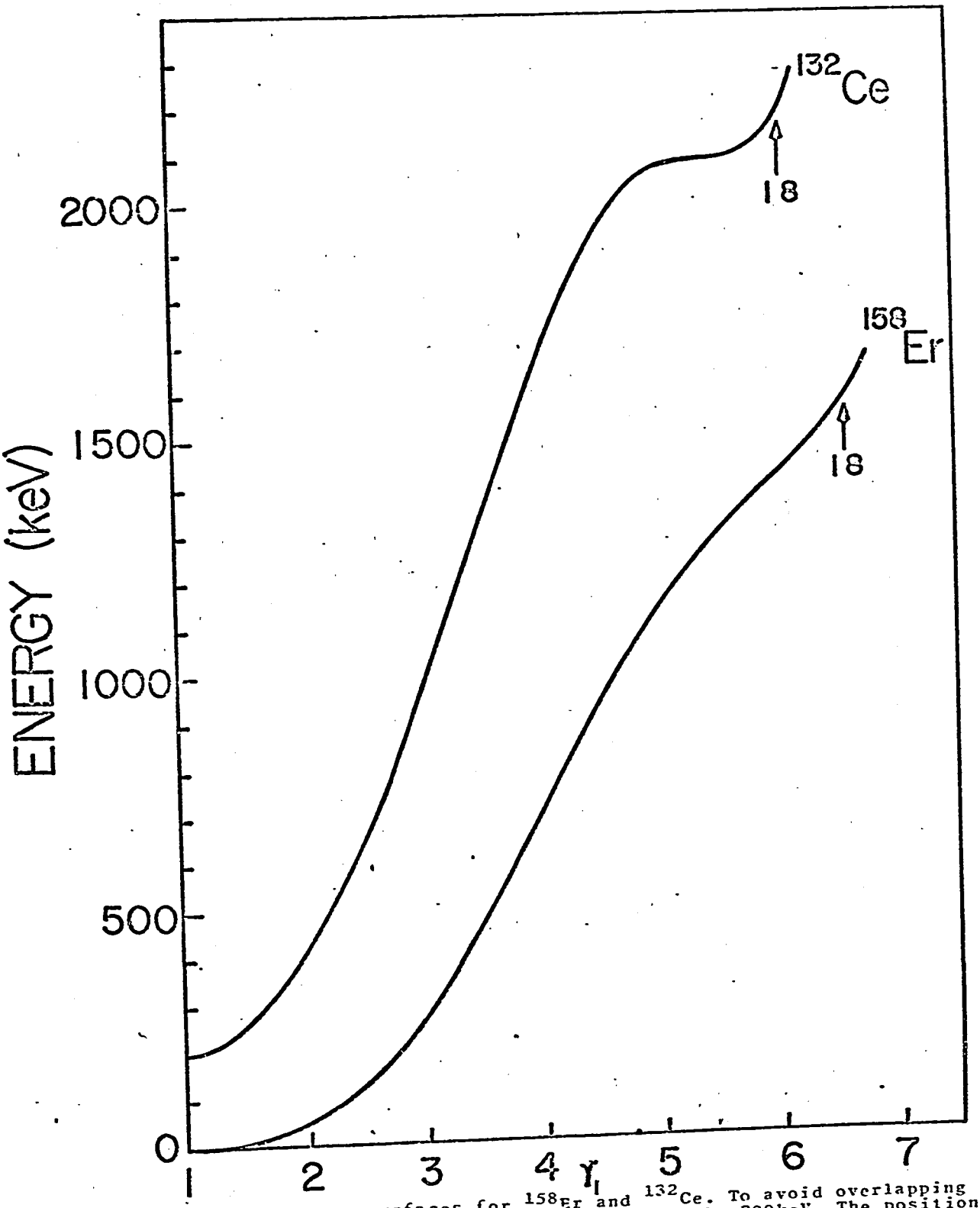

Fig.6.2b- Potential energy surfaces for ^{158}Er and ^{132}Ce. To avoid overlapping the curve for ^{132}Ce has been shifted upwards by 200keV. The position of the highest observed level is shown by an arrow; number indicates spin.

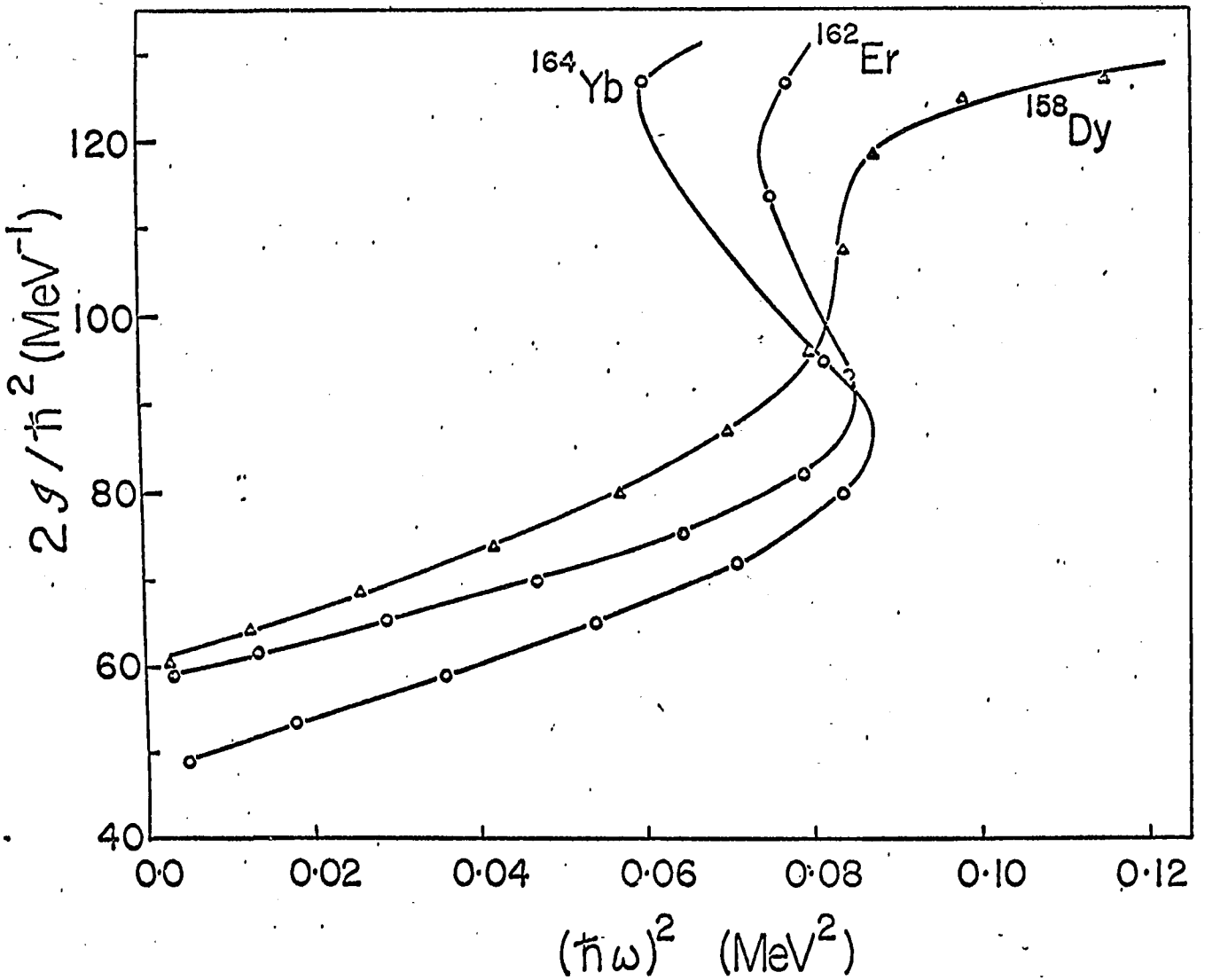

Fig. 6.3 $2\mathcal{I}/\hbar^2$ versus $(\hbar\omega)^2$ for three representative nuclei, which show the S effect. The experimental points are shown by open circles, closed circles and triangles. The solid line curves show the results obtained from the VMI6P model.

CHAPTER VII

TRANSITION QUADRUPOLE MOMENTS AND THE VMI MODEL FOR EVEN-EVEN NUCLEI

INTRODUCTION

The Variable-Moment-of-Inertia (VMI) model, proposed by Mariscotti, Scharff-Goldhaber and Buck (MSB) (1969) has been described in Chapters II and V of this thesis. The VMI model suggests that the intrinsic quadrupole moment of the higher spin states of deformed even-even nuclei may be larger than that of the ground-state. MSB have also found an empirical relationship between the transition quadrupole moment Q_{02} and the transition moment-of-inertia, $\mathcal{I}_{02}$, defined by them as

$$\mathcal{I}_{02} = \frac{1}{2}(\mathcal{I}_0 + \mathcal{I}_2) \tag{7.1}$$

Their proposed relation is

$$Q_{02} = k\,\mathcal{I}_{02}^{1/2} \tag{7.2}$$

where k is a constant.

The transition quadrupole moments, Q_{02} are obtained from the reduced transition probability, B(E2), and for a transition between $0^+ \leftrightarrow 2^+$ it is:

$$Q_{02}^2 = \frac{16\pi}{5}\, B(E2;\ 0^+ \leftrightarrow 2^+) \tag{7.3}$$

Here, we have suggested an alternative expression for the transition quadrupole moment in terms of the mass number and the moments of inertia of the initial and final states. Experimental results for the transitions $0+ \leftrightarrow 2+$, $2+ \leftrightarrow 4+$, $4+ \leftrightarrow 6+$ and $6+ \leftrightarrow 8+$ are compared with those obtained from the proposed relation.

FORMULATION

We write the transition moment-of-inertia $(\mathcal{I}_I, \mathcal{I}_{I+2})$ in the following form:

$$\mathcal{I}^2_{I,I+2} = (\mathcal{I}^2_I + \mathcal{I}^2_{I+2})/2 \tag{7.4}$$

and the proposed expression for the transition quadrupole moment as

$$Q_{I,I+2} = KA^{1/4} \mathcal{I}^{1/2}_{I,I+2} \tag{7.5}$$

where K is a constant. The introduction of mass number is not completely arbitrary. Some justification of the mass number dependence can be obtained from the Bohr-Mottelson model. If we combine the following two equations

$$\mathcal{I}_{irrot} = \frac{2}{5} AMR_0^2\beta^2 [0.89 + o(\beta^2)] \tag{7.6}$$

and

$$Q_0 \propto ZR_0^2\beta \ , \tag{7.7}$$

we obtain

$$Q_0 \propto (ZR_0/A^{1/2}) \mathcal{I}^{1/2}_{irrot} \tag{7.8}$$

which suggests the possibility of using a factor A^n. Trial and error showed n ~0.25 and this value was adopted.

RESULTS

In fig. (7.1) we have shown Q_{02} versus $A^{1/2}\vartheta_{02}$ for 52 nuclei; the smooth curve represents relation (7.5) with $K = 10.5 \times 10^{-24} cm^2 keV^{1/2}$. The data employed are the same as that used by MSB except for the following changes and additions:

^{192}Pt: A more recent value (Schwarzchild 1966) for Q_{02} has been shown.

^{152}Sm, ^{172}Yb, ^{174}Yb, and ^{176}Yb: Q_{02} values obtained from the recent measurements of Diamond et al (1969a) and Sayer et al (1970) have also been shown in addition to those given by MSB.

^{156}Er, ^{158}Er, and ^{160}Er: Q_{02} data have now become available (Diamond et al 1969b) for these nuclei, and these are included in fig. (7.1).

DISCUSSION

It would be noticed from fig. (7.1) that Eq. (7.5) is in satisfactory accord with the experimental data. Further a comparison of this figure with fig. 9 of MSB shows that the relation proposed here gives a better agreement with the experimental Q_{02} values for heavy nuclei than the one proposed by MSB. For the others, the agreement is comparable.

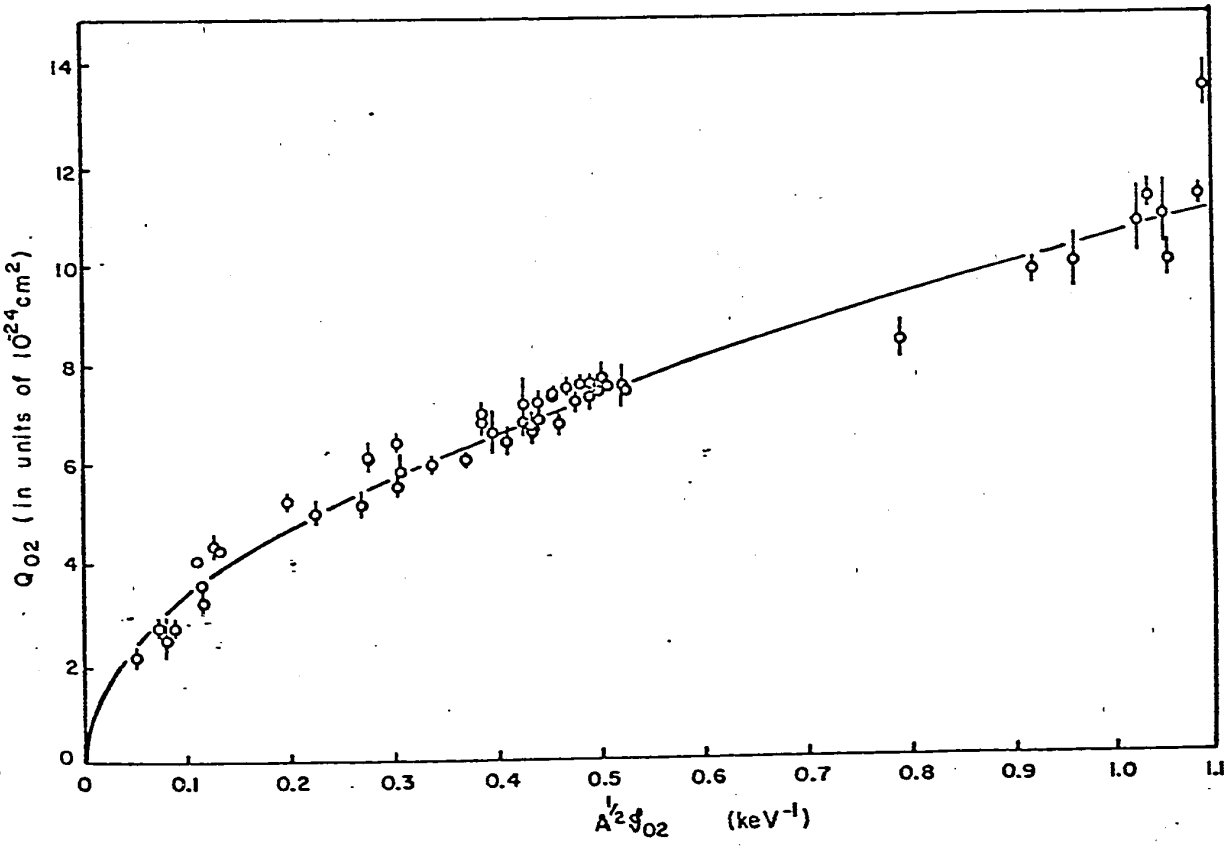

Fig. (7-1) Transition quadrupole moment Q_{02} versus $A^{1/2}f_{02}$. The curve represents eq. (7.5).

Groups of investigators at Berkeley and at Oak Ridge have recently succeeded in obtaining B(E2) values for transitions between levels having $I \geq 2$ in a number of even-even nuclei. Diamond et al (1969a and 1969b) have measured the half-lives of several transitions in the ground-state collective bands of ^{160}Er, ^{158}Er, ^{156}Er, and ^{152}Sm and thus obtained B(E2; I+2→I) for I as high as 8. Sayer et al (1970) have obtained B(E2; I→I+2) for a number of states of ^{152}Sm, ^{166}Er, and $^{172, 174, 176}Yb$ from multiple Coulomb excitations.

In order to examine the validity of Eq. (7.5) for 2+ ↔4+ and higher transitions, $Q_{I,I+2}$ values were obtained from the above mentioned B(E2) values using the following relationships between $Q_{I,I+2}$ and B(E2) for the rotational model:

$$B(E2;\ I \to I+2) = \frac{15}{32\pi}\ e^2 Q^2_{I,I+2}\ \frac{(I+1)(I+2)}{(2I+1)(2I+3)} \qquad (7.9)$$

and for transitions I+2→I it is

$$B(E2;\ I+2 \to I) = \frac{15}{32\pi}\ e^2 Q^2_{I,I+2}\ \frac{(I+1)(I+2)}{(2I+3)(2I+5)} \qquad (7.10)$$

These $Q_{I,I+2}$ values are plotted versus $A^{1/2} g_{I,I+2}$ in fig. (7.2). The curve represents Eq. (7.5) with the same value for K as in fig. (7.1).

It will be noticed, that if we make full allowance for experimental uncertainties, the proposed relation is in accord with the experimental points to about the same degree as in fig. (7.1). More careful scrutiny, however, shows that all the six points for 6+ ↔ 8+

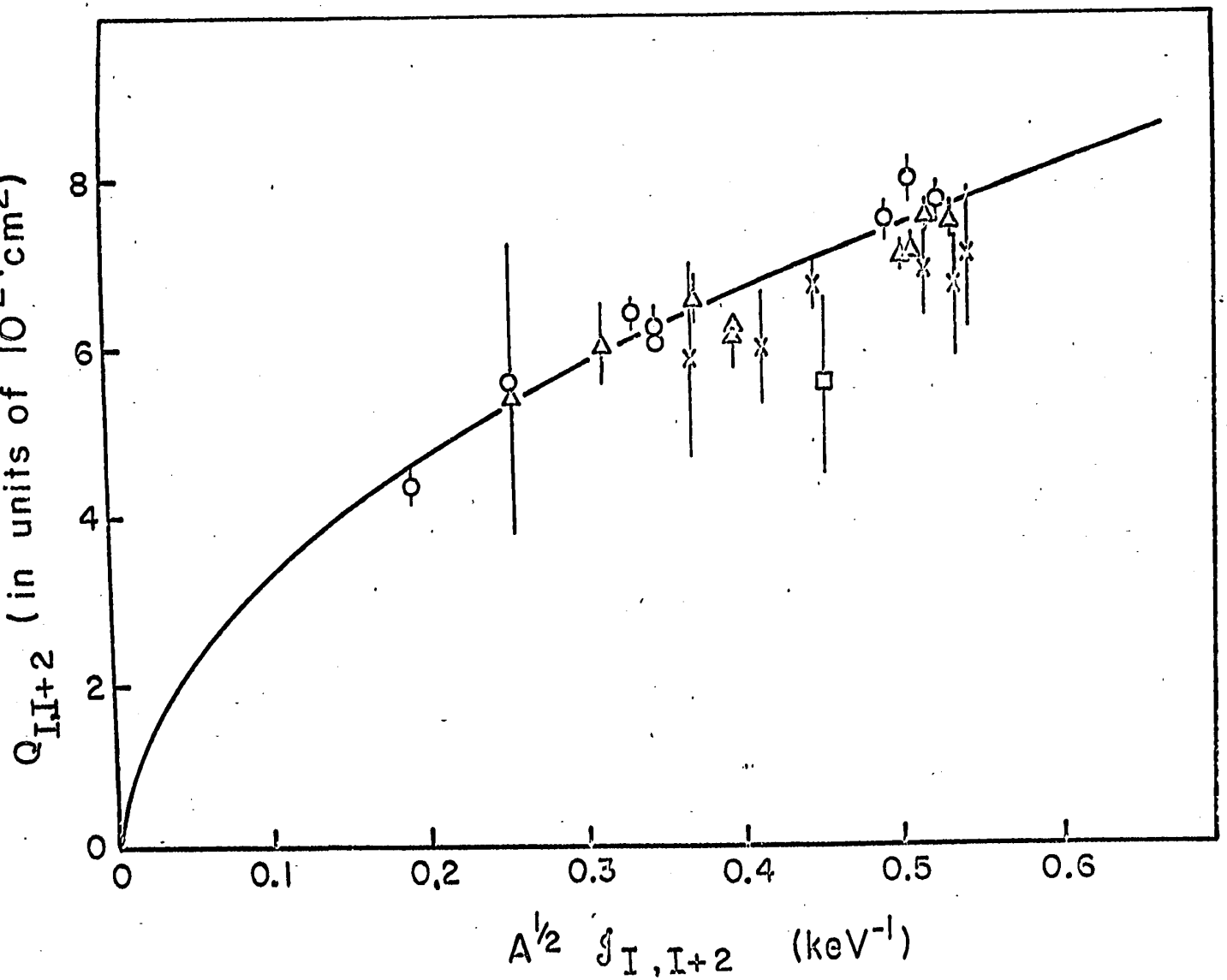

Fig. (7-2) $Q_{I,I+2}$ versus $A^{1/2}\mathcal{J}_{I,I+2}$ for I = 2, 4, 6, and 8. Data points (o) 2+↔4+, (Δ) 4+↔6+, (X) 6+↔8+ and (□) 8+↔10+ transitions. The curve represents eq. (7.5)

transitions lie below the curve, on the average, by ~ 8%. A similar result has been found by Scharff-Goldhaber and Goldhaber (1970) for the expression proposed by MSB; the calculated values being too high by ~ 15%.

In conclusion we find that for nuclei which can be described by the VMI model, the relation (7.5) can be used for correlating and predicting $Q_{I,I+2}$ values for I = 0, 2, 4, and 6.

CHAPTER VIII

VMI MODEL FOR ODD NUCLEI BANDS WITH K = 1/2

INTRODUCTION

The occurrence of low-lying rotational bands with each band being built on a particular intrinsic configuration is a well known characteristic feature of the deformed odd-A nuclei. The extension of the collective models to the odd-A nuclear systems can be made by assuming that the odd-proton (or neutron) is coupled to the even-even core, which may be considered to be deformed and axially symmetric in its simplest form. The internal motion of the nucleons in such axially symmetric core can be characterized by a projection of the total angular momentum on the symmetry axis of the nucleus and is denoted by K. Due to the interaction of the rotation with internal motion, K is not a good quantum number in general. When the rotational energy levels are much lower than the energy difference between single-particle levels, the influence of the rotation on the single particle motion can be neglected (Bohr and Mottelson 1953).

Volkov (1972) has developed and theoretically justified a form of the VMI energy equation which is applicable to the odd-A nuclei when coriolis coupling effects are considered to be small. Gregory and Taylor (1972) have applied Volkov's equation to calculate the rotational energy levels of the 7/2 + [404] band of ^{171}Lu and 11/2 - [505] band of ^{153}Gd and have found a satisfactory agreement with the observed values. In this chapter we have studied the applicability of the VMI model in calculating rotational energy levels of the K = 1/2 band of odd-A nuclei. An energy

equation has been formulated under the VMI model framework. Calculations for 34 bands of odd-A nuclei with $K = 1/2$ have been carried out and the calculated values are compared with the exprimental ones.

For $K = 1/2$ band the matrix element for the rotation and particle coupling Hamiltonian has non-vanishing diagonal terms (Davidson and Feenberg 1953). The reason for considering these bands alone is to include in our study the effect of such coupling terms in the framework of the VMI model treatment of the energy spectrum. We have elaborated this point later in this study.

FORMULATIONS

If the nucleus is symmetric, it is reasonable to suppose that the core will be too and therefore will behave as a symmetrically deformed even-even nucleus. The rotational energy operator, in the symmetric rotor model (Bohr and Mottelson 1953), in terms of the moment-of-inertia $\mathcal{I}$ and rotational angular momentum $\vec{R}$ is simply written as

$$H_{rot} = \frac{\hbar^2}{2\mathcal{I}} \vec{R}^2 \qquad (8.1)$$

The odd particle with its angular momentum $\vec{j}$ couples with the rotational angular momentum $\vec{R}$, of the core to form the total angular

momentum, $I(= R + j)$. The rotational Hamiltonian then becomes

$$H_{rot} = \frac{\hbar^2}{2\mathcal{J}} (I-j)^2 = \frac{\hbar^2}{2\mathcal{J}} I^2 - \frac{\hbar^2}{2\mathcal{J}} (I.j + j.I) + \frac{\hbar^2}{2\mathcal{J}} j^2$$

or

$$H_{rot} = \frac{\hbar^2}{2\mathcal{J}} (I(I+1) - 2I_z j_z + j_z^2) - \frac{\hbar^2}{2\mathcal{J}} (I_- j_+ + I_+ j_-)$$

$$+ \frac{\hbar^2}{4\mathcal{J}} (j_- j_+ + j_+ j_-) \qquad (8.2)$$

Here the operators I_z and j_z may be replaced with their eigenvalues which are usually assumed equal and denoted with K. Furthermore, the last term in Eq. (8.2) is small and depends only on the intrinsic coordinates, hence it can be incorporated in the intrinsic Hamiltonian. The second term on the right hand side of Eq. (8.2) is of the form $(\vec{\omega} \cdot \vec{j})$ that classically would give rise to the Coriolis forces, hence it is called the Coriolis term. Here this term arises due to the partial decoupling of the intrinsic motion from the instantaneous collective field caused by the rotation of the nuclear symmetry axis. The Coriolis term has diagonal matrix elements for the case where $K = 1/2$ (Davidson and Feenberg 1953). This follows from the fact that the Coriolis term which connects only collective states for which K-values differ by one and since the nuclear wave function contains states with both $\pm K$, it directly contributes to

the energy states with K = 1/2, only. The eigenvalue for this term is written as

$$<H_{coriolis}> = <|- \frac{\hbar^2}{2\mathcal{I}} (I_- j_+ + I_+ j_-)|>$$

$$= \frac{\hbar^2}{2\mathcal{I}} a(-1)^{I+1/2}(I+1/2) \qquad (8.3)$$

where a is called the 'decoupling parameter'. The decoupling parameter can have very strong influence on the level order of the K = 1/2 band. The Coriolis term has been studied in detail by several workers (Nilsson 1955, Prior 1958, Mottelson and Nilsson 1959, Kerman 1958, Feifrlik 1966, and Dzhelepov and Dranitsyna 1967).

Thus in the simplest rotor model (Bohr and Mottelson 1953) assuming the nuclear moment-of-inertia to be constant, the energy of nuclear states for the odd-A, axially symmetric nucleus can be written as

$$E_{I,K} = \epsilon + (\frac{\hbar^2}{2\mathcal{I}}) [I(I+1) - 2K^2 + \delta_{K,1/2} a(-1)^{I+1/2}(I+1/2)] \qquad (8.4)$$

where ϵ is the intrinsic energy.

We see therefore, that a rotational band is built on each particle state; furthermore, K is entirely due to the particle angular momentum since collective angular momentum has no component on the body symmetry axis.

Subsequent investigations and analysis show that Eq. (8.4) is not adequate to explain the rotational spectrum of odd-A nuclei. The perturbations caused by vibration-rotation interaction and band-mixing, affect the energy level spacings. Furthermore the decoupling parameter has also been found to vary with the spin and its values are significantly different for the one and the same band in different nuclei. The inclusion of higher order terms in angular momentum becomes necessary to include the above mentioned effects in the energy equation.

In the VMI model the energy of a level is expressed as a sum of the kinetic and the potential energy terms . Thus in the case of odd-A nuclei for the K = 1/2 band, the simple VMI energy equation can be written as

$$E_I = E_{1/2} + \frac{1}{2\mathcal{I}_I}\left(I(I+1) - 2\left(\frac{1}{2}\right)^2 + a(-1)^{I+1/2}(I+1/2)\right)$$

$$- \frac{1}{2\mathcal{I}_{1/2}}\left(\frac{1}{2}\left(\frac{1}{2}+1\right)-2\left(\frac{1}{2}\right)^2 -a\right) + \frac{1}{2}c(\mathcal{I}_I-\mathcal{I}_{1/2})^2 \qquad (8.5)$$

where $E_{1/2}$ is the energy of I = 1/2 level on which the particular rotational band is built, and the last term is the potential energy part. c is again the restoring force constant and $\mathcal{I}_{1/2}$ the moment-of-inertia of the I = 1/2 state. The equilibrium condition which determines the moment-of-inertia $\mathcal{I}_I$ (in units of $\hbar^2$) for any state with spin I is given by

$$\frac{\partial E_I}{\partial \mathcal{I}} = 0 \qquad (8.6)$$

Subsequent analysis of the observed rotational bands of odd-A nuclei with Eq. (8.6) showed that it is necessary to take into account the dependence of the decoupling parameter on the spin. As a first approximation, we take this dependence to be of the same form as in the Bohr-Mottelson model. The final expression for the energy can be written as

$$E_I = E_{1/2} + \frac{1}{2\mathcal{I}_I}[I(I+1) - 2k^2 + a(-)^{I+1/2}(I+1/2) + b(-1)^{I+1/2}I(I+1)(I+1/2)] - \frac{1}{2\mathcal{I}_{1/2}}[\tfrac{1}{2}(\tfrac{1}{2}+1) - 2(\tfrac{1}{2})^2 - a - \tfrac{3}{4}b] + \tfrac{1}{2}c(\mathcal{I}_I - \mathcal{I}_{1/2})^2 \qquad (8.7)$$

where b is another parameter incorporating the higher order correction to the Coriolis term.

CALCULATIONS AND RESULTS

The application of the equilibrium condition (8.6) on the energy equation (8.7) gives the following relation in terms of r_I $(= \mathcal{I}_I/\mathcal{I}_{1/2})$

$$r_I^3 - r_I^2 - \sigma[I(I+1) - \tfrac{1}{2} + a(-)^{I+1/2}(I+1/2) + b(-)^{I+1/2}I(I+1)(I+1/2)] = 0 \qquad (8.8)$$

where $$\sigma = \frac{1}{2c\mathcal{I}_{1/2}^3} \qquad (8.9)$$

is called the "softness parameter". The cubic equation (8.8) has one real root for any finite positive value of $\mathcal{I}_{1/2}$ and c. The energy, Eq. (8.7)

can also be written in terms of r_I:

$$\mathcal{J}_{1/2}\,[E_I - E_{1/2}] = \frac{1}{2r_I}\,[I(I+1) - \frac{1}{2} + a\,(-)^{I+1/2}(I+1/2)$$

$$+ b(-)^{I+1/2} I(I+1)\,(I+1/2)]$$

$$+ \frac{(r_I - 1)^2}{4\sigma} - \frac{1}{2}\,(\frac{1}{4} - a - \frac{3}{4}\,b) \qquad (8.10)$$

The above equation involves four parameters which can be fitted by least-squares procedure. As such equation (8.10) is non-linear and fitting $\mathcal{J}_{1/2}$, a, b, and σ is quite involved. The parameters are listed in Table (8-I), and in Table (8-II) we compare the observed and calculated energies.

DISCUSSION:

The rms percentage deviation between observed and calculated values of energies, reported in Table (8-II), show the over-all fit by Eq. (8.10) is satisfactory. Out of the 34 bands of odd-A nuclei, studied here, the following seven cases have high rms percentage deviation (between 1 and 3%): ^{155}Sm[521], ^{157}Gd[521], ^{175}Yb[521], ^{159}Gd[660], ^{185}Os[510], ^{223}Ra[640], and ^{181}Hf[?].

For some cases the poor fit to the energy levels may be due to two main limitations. Firstly for many bands only four observed levels are available; since each energy value is weighted by the square of its inverse, an uncertainty in the lower observed level can easily disturb the four

Table 8-I. VMI model parameters for odd-A nuclei and neighbouring even-even nuclei

Nucleus	Nilsson orbital	Source of exptl. data	Odd nucleus					Neighbouring even-even nucleus	
			$\mathcal{J}_{1/2}$ (10^{-3}keV^{-1})	c (10^{6}keV^{3})	a	b	σ	$\mathcal{J}_{o}$ (10^{-3}keV^{-1})	c (10^{6}keV^{3})
^{161}Ho	[411]	a	38.51	4.958	-8.070	8.492×10^{-5}	1.766×10^{-3}		
^{167}Tm	$[411]_G$	b	39.84	3.936	-7.238	2.007×10^{-3}	2.009×10^{-3}	36.94	4.182
^{169}Tm	$[411]_G$	c	40.07	8.642	-7.792	1.456×10^{-3}	8.993×10^{-4}	37.45	8.867
^{171}Lu	[411]	d	37.10	5.968	-7.305	4.033×10^{-3}	1.641×10^{-3}		
^{177}Lu	[411]	c	38.08	3.0×10^{7}	-7.747	-8.704×10^{-4}	3.018×10^{-10}		
^{155}Gd	[530]	e	57.96	1.089	0.721	5.247×10^{-3}	2.358×10^{-3}		
^{157}Gd	[530]	e	42.41	0.3971	-1.969	3.469×10^{-3}	1.651×10^{-2}		
^{155}Sm	[521]	c	32.58	3.0×10^{9}	0.455	1.657×10^{-2}	4.819×10^{-12}		
^{155}Gd	[521]	e	35.34	2.364	3.902	-4.149×10^{-3}	4.792×10^{-3}		
^{157}Gd	[521]	e	40.72	3.0×10^{6}	0.944	7.985×10^{-3}	2.468×10^{-9}		
^{159}Gd	[521]	e	42.71	2.484	4.932	-1.891×10^{-3}	2.584×10^{-3}		
^{161}Gd	[521]	e	45.99	8.338	2.540	-7.139×10^{-4}	6.165×10^{-4}		
^{165}Dy	[521]	c	46.90	7.145	5.768	4.253×10^{-4}	6.783×10^{-4}		
^{169}Er	$[521]_G$	f	42.46	10.77	8.281	-1.430×10^{-9}	6.065×10^{-4}	37.45	8.867
^{169}Yb	[521]	g	42.34	2.898	7.763	1.198×10^{-3}	2.273×10^{-3}		

Table 8-I. continued...

Nucleus	Nilsson orbital	Source of Exptl. data	Odd-nucleus					Neighbouring even-even nucleus	
			$\mathcal{J}_{1/2}$ (10^{-3}keV^{-1})	c (10^{6}keV^{3})	a	b	σ	$\mathcal{J}_{0}$ (10^{-3}keV^{-1})	c (10^{6}keV^{3})
^{171}Yb	$[521]_G$	c	41.46	8.956	8.485	-1.124×10^{-4}	7.834×10^{-4}	35.37	6.002
^{173}Yb	[521]	h	41.08	7.815	7.566	-3.122×10^{-3}	9.229×10^{-4}		
^{175}Yb	[521]	h	36.04	5.015	6.904	2.648×10^{-3}	2.130×10^{-3}		
^{173}Hf	$[521]_G$	c	38.87	4.881	8.136	2.980×10^{-4}	1.744×10^{-3}	31.16	3.546
^{175}Hf	[521]	c	37.08	7.795	7.485	-2.126×10^{-4}	1.258×10^{-3}		
^{179}W	[521]	i	32.82	8.352	8.211	-1.831×10^{-4}	1.693×10^{-3}		
^{159}Gd	[660]	e	6.551	0.3224	-0.6152	-3.336×10^{-2}	5.516		
^{171}Yb	[510]	h	34.71	0.6468	2.298	-1.357×10^{-2}	1.849×10^{-2}		
^{173}Yb	[510]	h	42.41	3.0×10^{6}	1.937	-1.210×10^{-9}	2.185×10^{-9}		
^{175}Yb	[510]	h	42.17	1.532	1.551	2.403×10^{-3}	4.352×10^{-3}		
^{177}Yb	[510]	c	40.21	2.452	2.430	-2.967×10^{-3}	3.137×10^{-3}		
^{177}Hf	[510]	j	36.76	0.1132	2.653	-5.369×10^{-2}	8.892×10^{-2}		
^{183}W	$[510]_G$	c	38.38	1.650×10^{6}	1.999	-2.731×10^{-3}	5.360×10^{-9}	29.85	10.77
^{185}Os	$[510]_G$	k	38.98	3.0×10^{6}	-0.2555	3.553×10^{-3}	2.814×10^{-9}	24.70	5.554
^{223}Ra	$[640]_G$	l	56.73	9.782×10^{2}	1.972	-2.051×10^{-3}	2.800×10^{-2}		

Table 8-I. continued....

Nucleus	Nilsson orbital	Source of exptl. data	Odd nucleus					Neighbouring even-even nucleus	
			$\mathcal{I}_{1/2}$ (10^{-3}keV^{-1})	c (10^{6}keV^{3})	a	b	σ	$\mathcal{I}_{0}$ (10^{-3}keV^{-1})	c (10^{6}keV^{3})
^{235}U	[631]	c	82.22	13.80	-2.978	3.909×10^{-3}	6.519×10^{-5}		
^{239}Pu	$[631]_G$	c	79.75	2.160	-5.847	4.881×10^{-4}	4.564×10^{-4}	67.82	1.601
^{251}Cf	$[620]_G$	m	77.76	9.380×10^{4}	3.009	-1.643×10^{-3}	1.134×10^{-8}	70.78	1.525
^{181}Hf	[?]	j	51.94	3.0×10^{6}	0.4771	1.139×10^{-2}	1.189×10^{-9}		

A subscript G after Nilsson Orbital indicates the ground state

a) K.G. Rensfelt, Nucl. Phys. A156, 534 (1970)
b) H. Noma, T. Shibata and Y. Yoshizawa, J. Phys. Soc. Japan 28, 546 (1970)
c) B.S. Dzhelepov and G.F. Dranitsyna, Bull. Acad. Sci. USSR, Phys. Series 31, 146 (1967).
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f) T.J. Mulligan, R.K. Sheline, M.E. Bunker and E.T. Jurney, Phys. Rev. C. 2, 655 (1970).
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i) B. Harmatz and T.H. Handley, Nucl. Phys. A121, 481 (1968).
j) F.A. Rickey, Jr. and R.K. Sheline, Phys. Rev. 170, 1157 (1968).
k) Nuclear Data B1 1-85 (1966).
l) C. Vieu, C. Briancon, G. Bastin, F.C. Leang, J. Treherne and R.J. Wallen, Bull. Acad. Sci. USSR, Phys. Series 31, 90 (1967).
m) Nuclear Data B3-2-77 (1969).

Table 8-II. Experimental and calculated energies (in keV) for the K = 1/2 band. The energies are reckoned from I = 1/2 level starting from column 4. Against each nucleus the first row shows the observed values, the second, calculated values from the VMI model. Upto I = 15/2, these values are written in the first two rows. If for a nucleus levels are available beyond I = 15/2, the same are carried over to the third and fourth rows for the experimental and calculated values respectively from the fifth column for I = 17/2 to the 11th column for I = 29/2 levels. Energies, which are doubtful are shown within parenthesis.

Note: for ^{161}Ho [411] the alternate levels from I = 5/2 are not observed and the Nilsson orbital for ^{181}Hf is uncertain.

Nucleus	Nilsson orbital	$E_{1/2}$ (ground level)	I= 1/2	3/2	5/2	7/2	9/2	11/2	13/2	15/2	rms % deviation
^{161}Ho	[411]	211.1	0	9.9		140.9		370.9		693.9	
				7.50	123.54	140.84	345.02	371.69	657.42	692.98	
					1098.9		(1573.9)				
				1052.97	1096.99	1524.18	1576.33	2064.27	2124.36		
^{167}Tm	[411]	0	0	10.4	116.5	142.3	325	370	620	689	
				10.57	116.62	142.57	325.74	370.03	619.15	686.26	0.54
				991	1087	1421	1549				
				987.98	1083.95	1424.28	1556.16	1920.85	2096.80		0.54
^{169}Tm	[411]	0	0	8.40	118.17	138.91	331.8	367.6	636.7	690.6	
				8.40	118.20	138.90	331.95	367.51	636.55	690.87	0.03
^{171}Lu	[411]	208.1	0	11.3	124.4	155.6	349.6	403.0			
				11.31	124.98	154.71	348.50	404.36	661.68	755.09	0.39
^{177}Lu	[411]	569.6	0	8.78	124.43	144.56	357.99	388.87	699.48		
				8.78	125.68	145.37	357.22	385.89	695.18	729.82	0.62
^{155}Gd	[530]	(423)	0	28	66	133	194				
				28.00	66.00	133.01	194.01	304.17	377.02	537.03	0.00
^{157}Gd	[530]	809	0	28	92	153	(251)				
				28.00	92.00	153.00	251.00	342.24	459.85	582.43	0.00

Table 8-II continued..

Nucleus	Nilsson orbital	$E_{1/2}$ (ground level)	I=1/2	3/2	5/2	7/2	9/2	11/2	13/2	15/2	rms % deviation
^{155}Sm	[521]	824	0	50	113	252	339				
				50.24	114.90	249.91	334.25	596.77	645.87	1103.02	1.19
^{155}Gd	[521]	556	0	58	102	228	310				
				58.00	102.00	228.00	310.00	486.34	612.12	818.37	0.00
^{157}Gd	[521]	704	0	41	91	199	(284)				
				41.31	93.29	196.54	277.75	459.41	548.68	834.64	1.82
^{159}Gd	[521]	507	0	52	82	199	254				
				52.00	81.99	199.98	253.98	427.08	509.10	727.25	0.01
^{161}Gd	[521]	356	0	41	82	173	248	397			
				40.79	81.46	175.53	249.10	393.87	501.00	692.71	0.65
^{165}Dy	[521]	108.16	0	50.43	72.77	189.53	229.01	412.33			
				50.40	72.72	189.86	229.11	411.98	466.65	713.91	0.10
^{169}Er	[521]	0	0	64.5	74.6	224.1	242.1	475.0			
				64.50	74.59	224.19	242.20	474.90	500.60	813.52	0.03
^{169}Yb	[521]	24.3	0	62.7	75.1	219.4	240.0	462.4	487.7	783.1	
				62.74	75.32	219.66	239.97	462.02	486.53	782.35	0.20
				809.4	1174.2	1193.3	1626.0	1630.5			
				806.67	1173.29	1192.44	1628.24	1636.85	2141.60		0.20
^{171}Yb	[521]	0	0	66.73	75.88	230.57	246.88	486.00			
				66.75	75.90	230.38	246.83	486.20	509.98	830.23	0.05

Table 8-II contined....

Nucleus	Nilsson orbital	$E_{1/2}$ (ground level)	I=1/2	3/2	5/2	7/2	9/2	11/2	13/2	15/2	rms % deviation
^{173}Yb	[521]	398	0	64 63.70	80 79.72	222 224.41	257 257.60	477 474.25	531.92	807.24	0.62
^{175}Yb	[521]	913	0	71 70.34	91 90.36	248 253.79	283 284.42	547 540.15	573.09	921.66	1.31
^{173}Hf	[521]	0	0	69.75 69.76	91.50 91.51	241.88 241.90	262.10 262.12	508.29	535.59	861.66	0.01
^{175}Hf	[521]	125.90	0	70.52 70.52	87.46 87.46	249.85 249.85	280.20 280.20	529.79	573.40	904.00	0.00
^{179}W	[521]	221.7	0	82.9 82.90	96.5 96.51	286.7 286.72	311.0 311.02	601.76	636.58	1019.33	0.01
^{159}Gd	[660]	780	0	75 75.00	219 214.11	277 277.89	502 511.17	498.38	876.07	720.17	1.45
^{171}Yb	[510]	(945)	0	50 49.99	107 106.99	199 198.99	309 306.99	407.05	587.83	655.46	0.01
^{173}Yb	[510]	1031	0	42 42.22	89 89.75	190 188.26	275 273.82	428.62	552.20	763.30	0.71
^{175}Yb	[510]	511	0	41 41.00	89 89.01	183 183.01	262 262.01	406.93	507.36	702.70	0.01
^{177}Yb	[510]	323	0	45.8 45.80	93.3 93.30	194.7 194.70	281.5 281.50	427.40	556.48	733.05	0.00

Tabel 8-II continued....

Nucleus	Nilsson orbital	$E_{1/2}$ (ground level)	I=1/2	3/2	5/2	7/2	9/2	11/2	13/2	15/2	rms % deviation
^{177}Hf	[510]	(567)	0	41 41.00	98 98.01	136 136.01	272 272.03	239.51	509.90	333.53	0.01
^{183}W	[510]	0	0	46.48 46.60	99.08 99.92	207.01 206.17	308.94 306.62	466.54	621.81	826.01	0.62
^{185}Os	[510]	0	0	37.4 37.87	97.3 102.11	198.1 193.67	317.8 303.56	456.45	602.15	828.43	3.58
^{223}Ra	[640]	0	0	29.90 30.14	61.51 62.15	124.90 121.40	174.65 172.33	247.10 250.10	315.80 317.63	405.20 406.72	1.38
^{235}U	[631]	0.08	0	13.00 13.01	51.70 51.65	83.80 83.64	149.70 150.14	204.80 205.05	294.70 294.20	378.18	0.17
^{239}Pu	[631]	0	0	7.83 7.83	57.26 57.27	75.69 75.67	163.75 163.73	193.00 193.02	318.17	358.85	0.01
^{251}Cf	[620]	0	0	25.0 25.01	47.8 47.84	105.5 105.45	148.0 147.88	236.32	300.63	417.11	0.06
^{181}Hf	[?]	1745	0	31 31.16	72 73.30	155 153.68	219 215.71	363.74	421.98	666.60	1.27

parameter least-squares fitting. Secondly there is a built-in limitation of the VMI model calculation. Unless the value of r_I for a particular nucleus lies within a certain range, no acceptable solution can be obtained from eq. (8.8) [e.g. see discussion in Chapter V and VI].

For some of the nuclei, the softness parameter, σ, is very low (e.g. ^{155}Sm[521], ^{157}Gd[521], ^{159}Gd[660], ^{185}Os[510], and ^{181}Hf[?]) indicating thereby that the variation in the moment-of-inertia with the angular momentum is insignificant. For the majority of bands in this study no information is available about the errors in the experimental data. Hence a comparison of the rms % deviation in the experimental uncertainties cannot be made with the rms % deviation between calculated and observed values of energies.

In Table (8-I) we have also shown, where available, the values of the parameters $\mathcal{J}_0$ and c for the neighbouring even-even nuclei (Varshni and Bose 1972). It is to be noticed here that such comparison can only be made for the ground-state bands, since $\mathcal{J}_0$ and c for the neighbouring nuclei are only for this state. The $\mathcal{J}_{1/2}$ values for odd-A nuclei are seen to be all larger than the corresponding value of the even-even nuclei. This is in agreement with the previous discussion of this problem (Prior 1958, Mottelson and Nilsson 1959). The correction to the moment-of-inertia due to the last odd particle comes from the second order of the Coriolis force. The values of the parameter, c for the following three bands are rather large as compared to the respective neighbouring nucleus:

^{183}W[510], ^{185}Os[510], and ^{251}Cf[620].

At present we do not have a plausible explanation for this large value.

In the VMI model the variation in the moment-of-inertia implies the variation of the deformation which in turn will lead to a change in the strength of the interaction between the valence particle and the rotating deformed core. For K = 1/2 bands such changes are considerable and the Coriolis forces cannot be adequately represented by one term alone. It can be noticed in Table (8-I) that both the parameters a and b contribute significantly to the calculation of the energy values.

CHAPTER IX

APPLICATION OF MORINIGO'S MODEL TO THE GROUND-STATE BANDS OF EVEN-EVEN NUCLEI

INTRODUCTION

The remarkable success of the empirical 'Variable Moment-of-Inertia' (VMI) model proposed by Mariscotti, Scharff-Goldhaber, and Buck (1969), in the prediction of rotational energy levels of the ground-state bands of even-even nuclei, has led theoretical physicists to attempt to provide a more fundamental basis to this model. One such attempt is due to Morinigo (1970). He has suggested the use of the elements of the mass quadrupole tensor, $m\underline{Q}$, as the nonredundant generalized coordinates for the collective description of the nucleus.

ABOUT MORINIGO'S MODEL

Morinigo (1970) has defined $\underline{Q}$ as a product of the 3 x n order matrix of $\underline{X}$ and its transpose, $\underline{X}^T$ which are constructed from the Cartesian components (x_i, y_i, z_i) of the position vectors r_i for the ith particle in a nuclear system of n particles each having the same mass m. The potential energy, V, in terms of the above defined quantities is then written as

$$V = \frac{1}{2} m\omega^2 \sum_i^n (x_i^2 + y_i^2 + z_i^2) = \frac{1}{2} m\omega^2 \, \mathrm{Tr} \{\underline{X}\underline{X}^T\} \qquad (9.1)$$

where it has been assumed that the particles are placed in a harmonic-

oscillator well such that the classical frequency of oscillations is ω. (The symbol Tr denotes the trace of the matrix)

Morinigo has further, taken into account the perturbation in the nuclear system caused by the two-body forces associated with the quadrupole potentials V', written as

$$V' = \frac{1}{2} a \, \mathrm{Tr} \{\underline{X}^T \underline{X}\} \, \mathrm{Tr} \{\underline{X}^T \underline{X}\} - \frac{3}{2} a \, \mathrm{Tr}\{\underline{X}^T \underline{X} \, \underline{X}^T \underline{X}\} \qquad (9.2)$$

The expression for the kinetic energy, T, as derived by Morinigo, is,

$$T = \frac{1}{2} \sum_i^3 [(I_{ii})^{-1} \vec{J}_i^2 + \frac{(I_{ii} - M_{ii})}{I_{ii} M_{ii}} \vec{k}_i^2] + \frac{1}{2} m \, \mathrm{Tr} \{\dot{\underline{R}}_0 \, \dot{\underline{R}}_0\} \qquad (9.3)$$

where I_{ii} are the diagonal elements of the moment-of-inertia tensor, $\vec{J}_i$ denotes the ith component of the total angular momentum and similarly $\vec{k}_i$ is the intrinsic angular momentum component.

The quantities M_{ii} are defined as follows,

$$M_{11} = I_{11} - (I_{22} - I_{33})^2 / I_{11} \quad (123 \text{ cyclic}) \qquad (9.4)$$

and $\dot{\underline{R}}_0$ is the time derivative of the diagonal matrix whose elements are the square roots of the diagonal quadrupole tensor $\underline{Q}_0$. The complete Hamiltonian for the nucleus is then written as

$$H = \sum_i^3 \frac{1}{2} [m\omega^2 \rho_i^2 + m^{-1} \pi_i^2 + (I_{ii})^{-1} \vec{J}_i^2 + F_{ii} \vec{k}_i^2 + a \sum_j \rho_i^2 \rho_j^2 - 3a\rho_i^4] \qquad (9.5)$$

In eq. (9.5) the three diagonal elements of $\underline{R}_0$ are denoted by the symbols (ρ_1,ρ_2,ρ_3) and the momentum operators conjugate to them by (π_1, π_2, π_3); I_{ii} and F_{ii} are functions of the coordinates ρ_i only.

In order to obtain the expectation value for the Hamiltonian (9.5), Morinigo has made the following propositions:

a) The first two terms on the right hand side in Eq. (9.5) are quite large compared to the rest and hence a perturbation treatment is in order.

b) The quantum numbers associated with the ρ_i description are large enough that a classical description is a meaningful approximation.

He then arrives at the following energy expression:

$$E(J) - E(o) = \frac{\hbar^2 J(J+1)}{2I} + \frac{9\hbar^2\lambda^2(I-I_o)^2}{I(4I-3I_o)(3I_o-2I)} - \frac{9a}{m^2}(I-I_o)^2 \qquad (9.6)$$

where $\hbar^2\lambda^2$ represents the expectation value of the operator $(\vec{k}^2-\vec{k}_3^2)$ and the last term on the right hand side appears due to V' defined by Eq. (9.2). The quantity I_o is defined as: $I_o = \frac{1}{3}(I_{11} + I_{22} + I_{33})$ and $I = I_{11} = I_{22}$ in the case of a symmetric-rotor. The best value for the level energy is then given by Eq. (9.6) evaluated at the value of I for which E(J) is a minimum,

$$\frac{\partial E(J)}{\partial I} = 0 \qquad (9.7)$$

The physically meaningful values of I must in all cases lie in the range between $0.75I_0$ and $1.5I_0$, which correspond to the extremes of a flat disc nucleus and a line nucleus, respectively. Morinigo has shown that under certain approximations, the energy expression (9.6) can reduce to the VMI model energy expression.

It is of interest to examine the applicability of expression (9.6) itself to the level energies of the ground-state bands of even-even nuclei. In the present chapter we report results of such an investigation.

CALCULATIONS

We shall measure I in units of $\hbar^2$, and introduce three auxiliary parameters as follows

$$t = 9\lambda^2 \tag{9.8a}$$

$$A = \frac{9a\hbar^4}{m^2} \tag{9.8b}$$

and

$$s = AI_0^3 \tag{9.8c}$$

For the ratio $r_J = I/I_0$, the equilibrium condition (9.7) yields the following equation

$$r_J^7\,[-256s] + r_J^6\,[1408s] + r_J^5\,[-3024s]$$

continued.....

$$+ r_J^4 [3168s + 16t - 64J(J+1)]$$

$$+ r_J^3 [-1620s - 64t + 288 J(J+1)]$$

$$+ r_J^2 [324s + 102t - 468J (J+1)]$$

$$+ r_J [-72t + 324 J(J+1)] + 18t - 81J(J+1) = 0 \qquad (9.9)$$

In terms of r_J the energy can be written as

$$I_o E(J) = \frac{J(J+1)}{2r_J} + \left[\frac{t}{(-8r_J^3 + 18r_J^2 - 9r_J)} - s\right] (r_J - 1)^2 \qquad (9.10)$$

To initiate a non-linear least-squares calculation, one needs to know the approximate values of the parameters. The calculations carried out towards this end, yielded some interesting results concerning the signs of the parameters in the Morinigo theory. We summarize these findings here.

At the first instance, it is natural to assume that s and t are both positive. However, it was found that this assumption generates R_4 values which are always greater than 10/3. Consequently other sign combinations of s and t were tried. In Table (9-I) we list and label some typical combinations. Thus A type of combinations refer to s and t both positive, B combinations have s negative but t positive,

C have both s and t negative, and D have s positive and t negative. In Figs. (9-1a) and (9-1b) we show R_4 as a function of t for A and B types of combinations respectively; similarly Figs (9-2a) and (9-2b) show R_4 as function of (-t) for C and D types of combinations. The curves shown in Figs. (9-1a), (9-1b), (9-2a) and (9-2b) are of course only for a certain range of values of s and t; extension of these ranges does not change the qualitative behaviour of these patterns. For the purpose of broad conclusions that we shall draw here, these figures are quite adequate. It will be noticed from Figs. (9-1a) and (9-1b) that the ratio R_4 is always ≥ 3.333 while for real nuclei $R_4 \leq 3.333$; thus A and B types of combinations have to be ruled out for fitting the Morinigo model to real nuclei. Figs. (9-2a) and (9-2b) show that if t is negative, it is possible to generate $R_4 \leq 3.333$. To determine whether one or both of C and D types of combinations are applicable, the results obtained from a few such combinations were plotted on a Mallmann plot (R_6 vs R_4). It was found that D types of combinations give curves which are too low as compared to the experimental points except in the vicinity of $R_4 = 3.333$. C types of combinations gave reasonable results and in subsequent analysis only such combinations were used.

Table (9-I) Some typical combinations (A, B, C, and D types) between the parameters s and t.

Type of combination	s	t
A1	1×10^{-6}	1×10^{3} to 1×10^{8}
A2	1×10^{3}	1×10^{3} to 1×10^{8}
A3	1×10^{5}	1×10^{3} to 1×10^{8}
B1	-1×10^{-6}	1×10^{3} to 1×10^{8}
B2	-1×10^{3}	1×10^{3} to 1×10^{8}
B3	-1×10^{5}	1×10^{3} to 1×10^{8}
C1	-1×10^{-6}	-1×10^{3} to -1×10^{8}
C2	-1×10^{3}	-1×10^{3} to -1×10^{8}
C3	-1×10^{5}	-1×10^{3} to -1×10^{8}
D1	1×10^{-6}	-1×10^{3} to -1×10^{8}
D2	1×10^{3}	-1×10^{3} to -1×10^{8}
D3	1×10^{5}	-1×10^{3} to -1×10^{8}

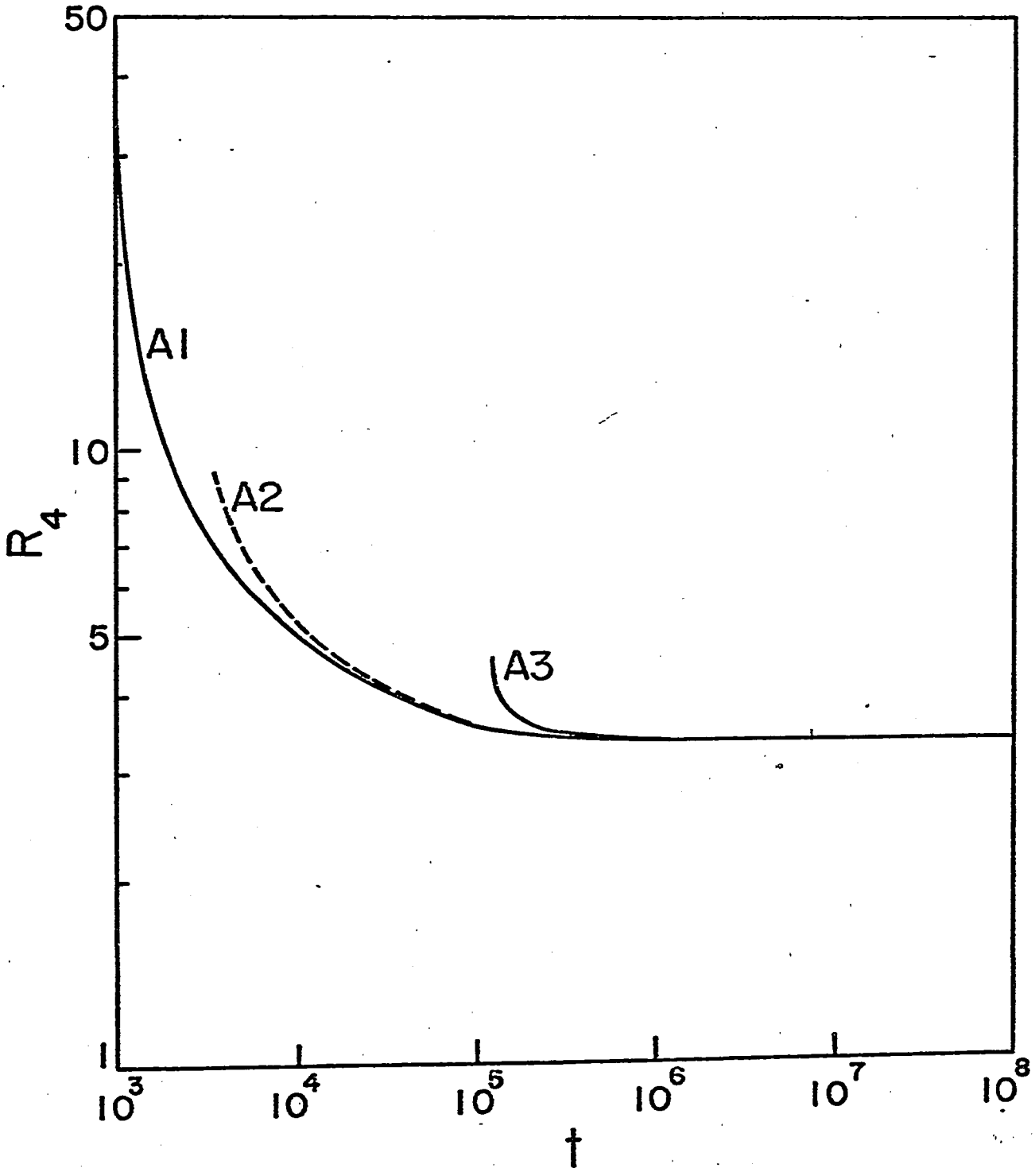

Fig. (9-1a) R_4 as a function of t for positive values of s (A-type combinations as discussed in the text). A1, A2, and A3 are explained in Table (9-I).

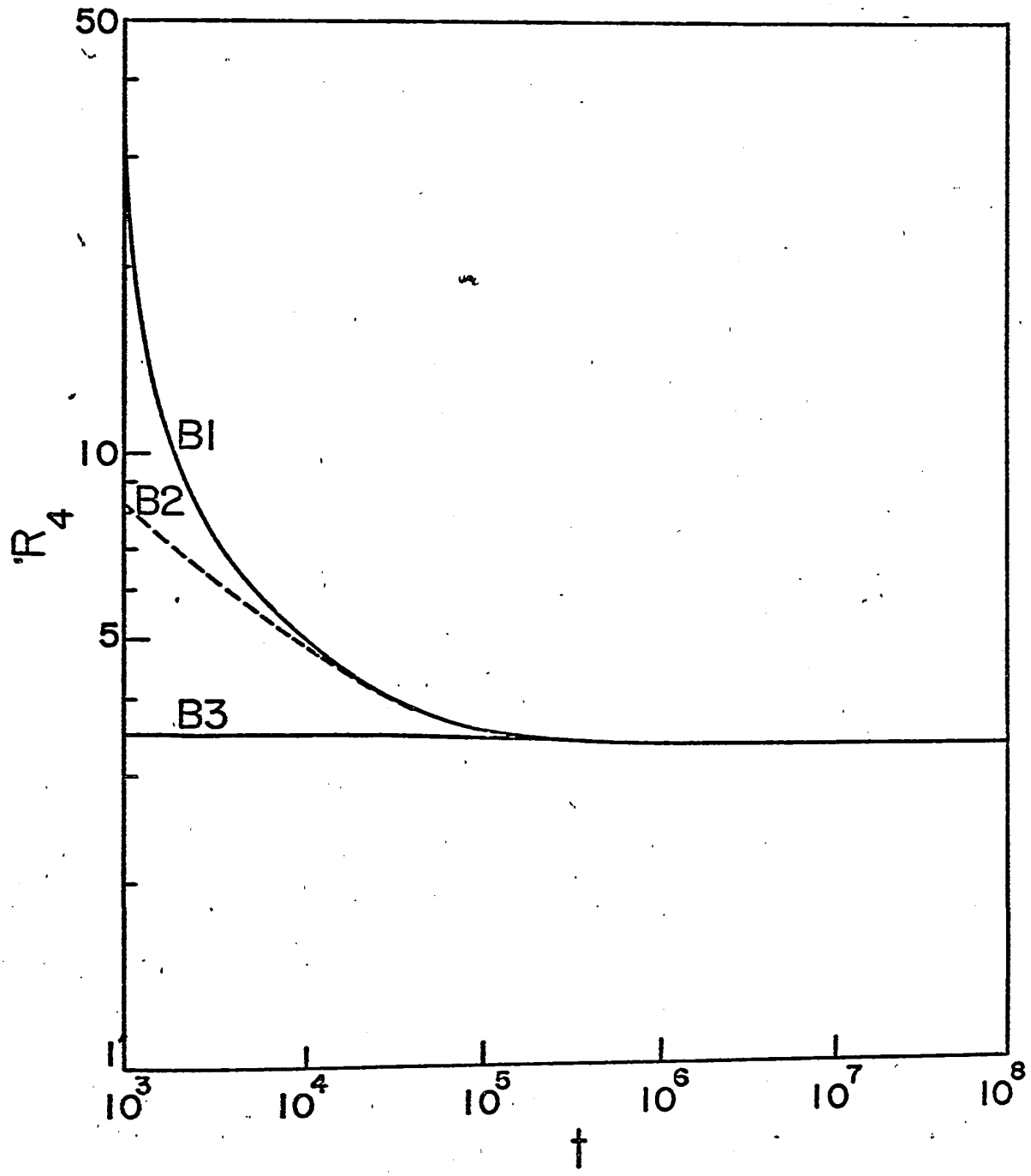

Fig. (9-1b) R_4 as a function of t for negative values of s (B-type combinations as discussed in text). B1, B2, and B3 are explained in Table (9-I).

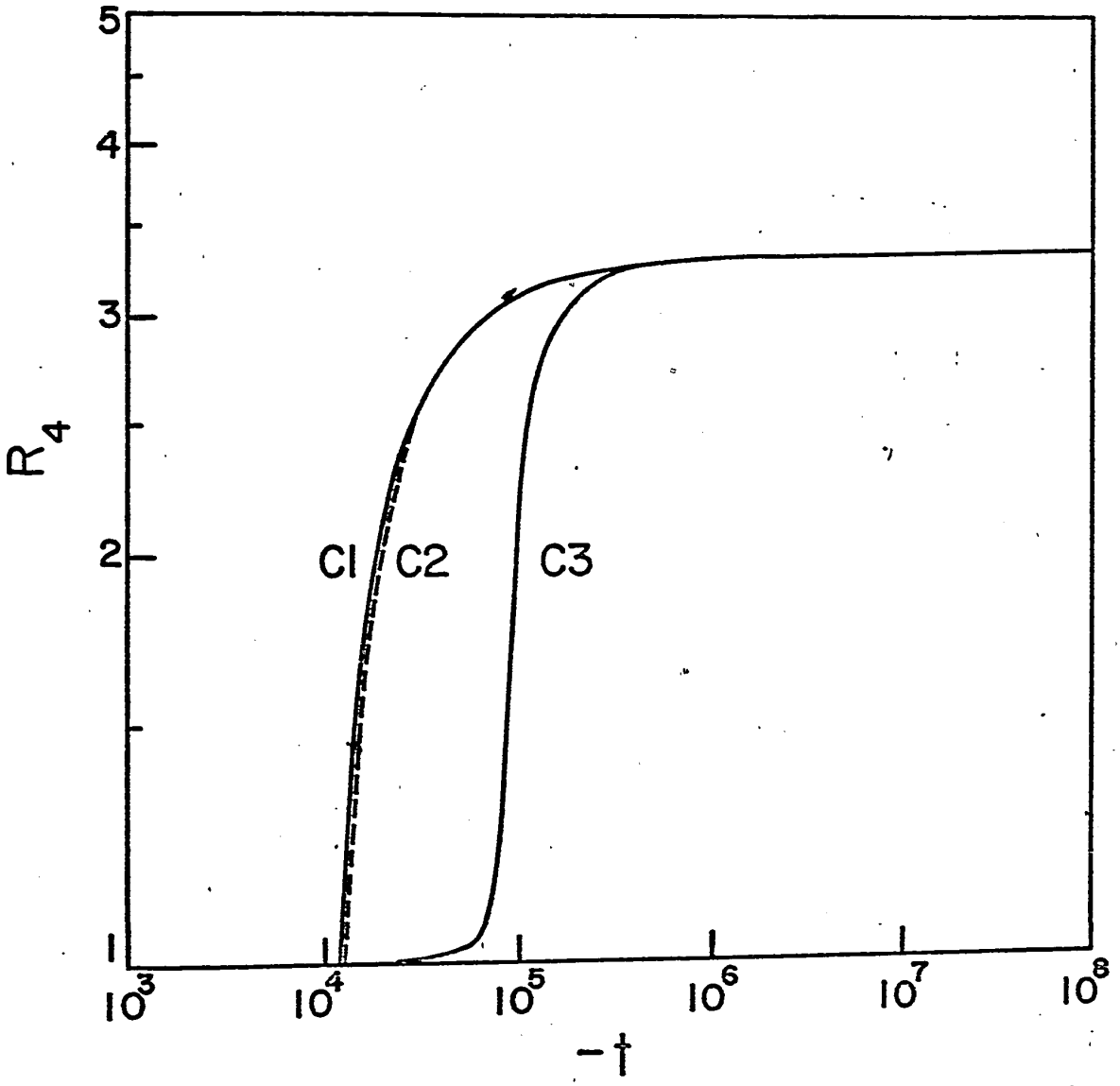

Fig.(9-2a) R_4 as a function of (-t) for negative values of s (C-type combinations as discussed in the text). C1, C2, and C3 are explained in Table (9-I)

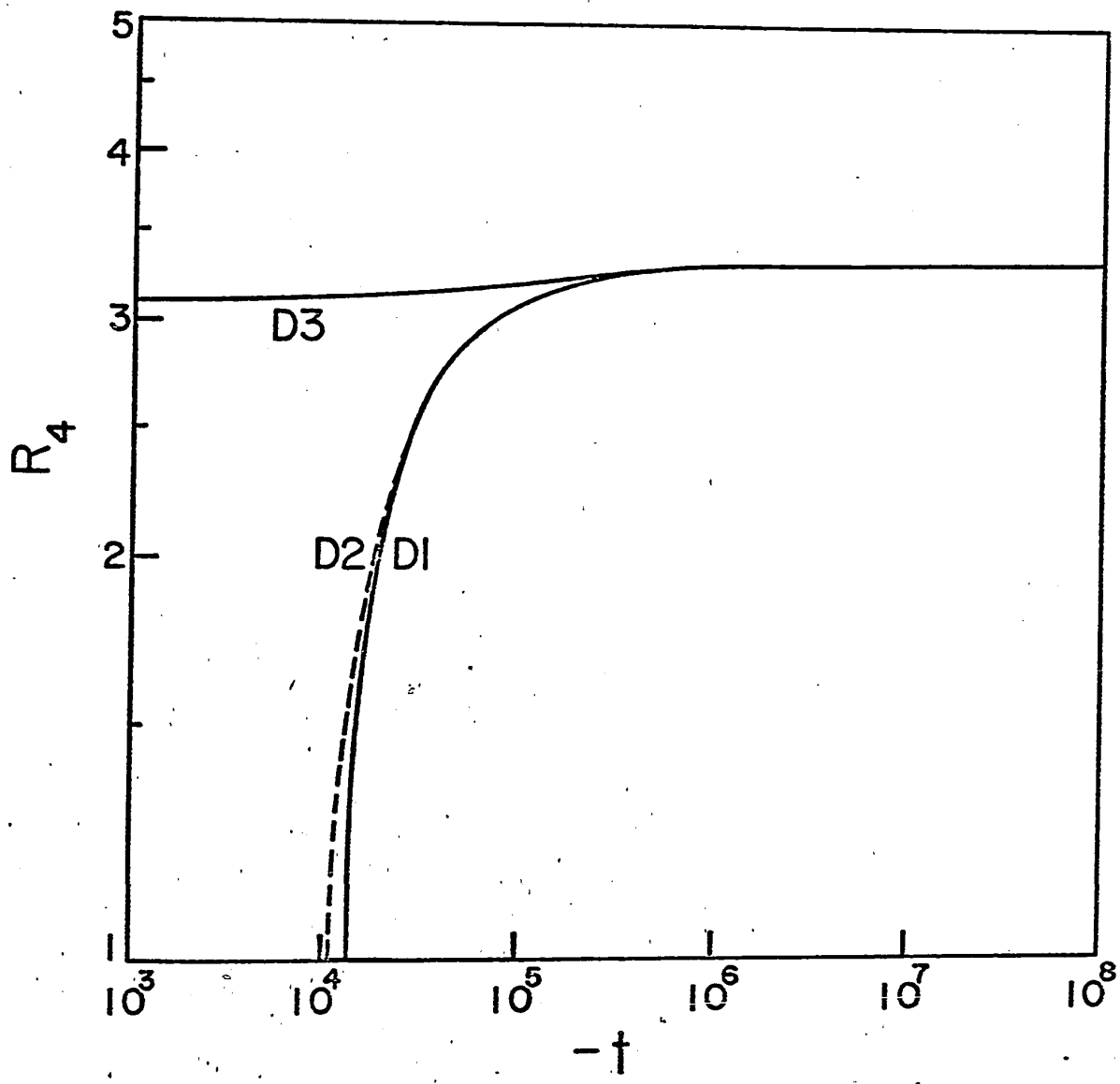

Fig. (9-2b) R_4 as a function of (-t) for positive values of s (D-type combinations as discussed in the text). D1, D2 and D3 are explained in Table (9-I).

RESULTS

In Table (9-II) we give the parameters for 12 representative nuclei determined by non-linear least-squares method. Each energy value was weighted by the square of its inverse. The nuclei in Table (9-II) represent the whole range of pattern of energy levels - from very soft nuclei to very hard nuclei. The ratio R_4 varies monotonically with the nuclear softness. As column 3 of Table (9-II) shows, the selected nuclei have their R_4 values more or less evenly distributed between the interval ≈ 2.1 to ≈ 3.33. Attempt was made to select such nuclei for which a good number of levels are available.

The results are shown in Table (9-III). The Morinigo model and the VMI23 model are both three-parameter models and a comparison between the results from the two was considered appropriate. Against each nucleus, the first line shows the experimental values, the second, the calculated values from the Morinigo model and the third, calculated values from the VMI23 model. For ^{100}Pd and ^{152}Gd, R_4 is less than 2.23 and no results are shown for the VMI23 model for these two nuclei.

DISCUSSION

For the VMI model, for $R_4 < 2.23$, it becomes necessary to assume that the parameter $\mathcal{J}_0$ (defined as the ground-state moment-of-inertia) is negative (Scharff-Goldhaber and Goldhaber 1970). However, for the Morinigo model, we notice from Table (9-II) that I_0 (analogous to $\mathcal{J}_0$ of VMI model) remains positive even below $R_4 = 2.23$. The negative

values for A and t are however not satisfactory and their justification is an open question.

It will be noticed from Table (9-III) that the VMI23 results are better than the Morinigo results except in the region near the rigid rotor limit, where the results by the two models are comparable.

In conclusion we find that subject to a justification for negative A and t, the Morinigo model can be applied to the ground-state bands of even-even nuclei, but it is less satisfactory than the VMI23 model.

Table (9-II) The parameters occurring in the Morinigo model (redefined as in eqs. (9-8a), (9-8b), and (9-8c)) obtained from the least-squares fit. The calculated energies depend on differences of large terms involving s and t, consequently I_o, t and A are given to six-significant figures to ensure reproducibility of results given in Table (9-III).

Nucleus	Sources of exptl. data	R_4	I_o (10^{-2} keV^{-1})	t (10^6)	A (10^{11} keV^3)
^{100}Pd	a	2.129	0.765569	-0.485197	-11.5569
^{152}Gd	b	2.194	1.36886	-0.441670	- 1.83726
^{128}Xe	c	2.332	0.954640	-0.497368	- 6.04860
^{126}Xe	c	2.424	1.09537	-0.816920	- 6.51884
^{186}Pt	d	2.562	2.05370	-1.04537	- 1.25587
^{184}Pt	d	2.682	2.21883	-1.08923	- 1.03239
^{130}Ce	e	2.803	1.29281	-1.00929	- 4.81280
^{176}Os	f	2.925	2.32752	-1.22500	- 0.996042
^{154}Gd	d	3.016	2.49462	-1.85590	- 1.21828
^{168}Hf	d	3.107	2.36025	-2.00959	- 1.54843
^{166}Yb	d	3.239	2.90375	-4.14801	- 1.62349
^{242}Pu	g	3.309	6.71930	-20.9596	-0.664301

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Table (9-III) Experimental and calculated energies (in keV) of levels of ground-state bands. For each nucleus the first row shows the experimental energies, the second shows the energies calculated using Morinigo model and the third row shows the calculated energies from the VMI23 model. For ^{100}Pd and ^{152}Gd ($R_4<2.23$) no results are shown for the VMI23 model. The rms % deviation in the first row, if shown, refers to ± uncertainties in the experimental values. The experimental energies shown in parenthesis are reported as doubtful and are not included in calculating rms deviations..

Nucleus/J=	2	4	6	8	10	12	14	16	Rms % deviation
^{100}Pd	664	1414	2293	3283					
	665.33	1400.01	2322.84	3265.91	4060.85	4534.08	4504.28	3780.26	0.86
^{152}Gd	344.26	755.4	1227.4	1747					
	344.96	747.48	1244.32	1737.43	2127.76	2312.67	2184.38	1628.70	0.91
^{128}Xe	443±1.0	1033±1.4	1737±1.7	2513±2.6					0.15
	443.66	1024.67	1754.80	2502.86	3136.35	3518.40	3506.00	2948.53	0.69
	442.89	1034.56	1733.20	2515.55	3368.07	4281.75	5250.09	6268.15	0.14
^{126}Xe	389±1.0	943±1.4	1636±2.4	2437±3.2	3317±3.7				0.17
	390.23	928.68	1650.09	2465.60	3286.95	4024.45	4586.12	4876.95	1.03
	388.79	945.02	1633.27	2430.96	3324.14	4303.10	5360.62	6491.04	0.19

Table (9-III) cont.

Nucleus/J=	2	4	6	8	10	12	14	16	Rms % deviation
^{186}Pt	191.1±0.6	489.6±1.5	876.8±2	1341.1±3	1855.7±5	(2407)			0.27
	191.64	483.72	884.10	1351.10	1843.46	2319.46	2736.53	3051.05	0.80
	190.50	492.44	878.91	1335.64	1853.81	2427.25	3051.35	3722.50	0.37
^{184}Pt	162.1±0.4	434.8±1	797.3±2	1228.9±3	1704.7±4	(2201.3)	(2723)	(3726)	0.24
	162.34	431.73	802.03	1235.86	1695.93	2144.51	2543.11	2852.31	0.54
	162.07	436.18	794.55	1223.16	1713.31	2258.89	2855.31	3498.97	0.37
^{130}Ce	253.2	709.6	1323.5	2052.5					
	253.34	707.84	1323.54	2049.28	2803.63	3524.08	4141.66	4585.55	0.24
	253.14	710.77	1319.92	2055.29	2901.17	3846.60	4883.31	6004.84	0.17
^{176}Os	135.2	395.4	742.3	1157.3	1633.6				
	135.47	391.40	746.64	1167.98	1622.09	2075.20	2492.96	2840.23	0.74
	135.24	395.26	742.32	1158.34	1632.77	2158.59	2730.77	3345.47	0.05
^{154}Gd	123.07±0.05	371.2±0.2	718.1±1	1146±10	1644±15	(2189)			0.57
	123.19	369.23	720.40	1151.60	1637.74	2153.53	2673.39	3171.39	0.39
	123.02	371.57	717.42	1145.19	1645.05	2210.02	2834.83	3515.29	0.08
^{168}Hf	123.9±0.4	385.0±1	756.1±3	1212±4	1734±5	2304±10	(2910)		0.34
	124.02	383.61	756.12	1216.16	1738.25	2296.73	2865.67	3418.81	0.26
	123.86	386.27	755.06	1208.11	1730.87	2313.31	2948.13	3629.80	0.27
^{166}Yb	101.8±0.4	329.7±1	667.1±2	1097±4	1604±5	2172±6	(2774.4)	(3402.2)	0.33
	101.81	329.64	667.07	1097.20	1603.97	2171.88	2785.75	3430.55	0.01
	101.75	329.83	667.36	1096.82	1602.90	2172.81	2795.84	3462.70	0.04

Table (9-III) cont.

Nucleus/J=	2	4	6	8	10	12	14	16	Rms % deviation
^{242}Pu	44.50	147.25	305.95	517.67	778.75	1086.7			
	44.50	147.25	305.95	517.59	778.92	1086.61	1437.43	1828.23	0.01
	44.50	147.25	305.94	517.56	778.84	1086.58	1437.82	1829.86	0.01

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POST ORAL DISCUSSION

The following points which emerged from the oral examination are noteworthy:

(1) In Chapter II: It was pointed out by Professor Y. Nogami that the equation (2.39) as used by Trainor and Gupta (1971) in their calculation for the moment-of-inertia of the nucleus, is in error. The correct expression is as follows

$$\mathcal{I}_S = \frac{1}{5} M_S (a^2 + ab + 2b^2)$$

(2) In Chapters V and VI: Some doubts were raised regarding the equivalence of the potential energy (P.E.) surfaces as obtained from the VMI model energy equation and those obtained from the deformation energy considerations (e.g. Myers and Swiatecki (1967), Strutinsky (1967), Krumlinde (1968), Mosel and Scharnweber (1970), and Moller (1972)).

If one follows the assumption, $\mathcal{I} \sim \beta$, within the framework of the VMI model, it can then be argued that the equivalence of the above two are justified in representing the true nuclear deformation energy. On the other hand if the moment-of-inertia, $\mathcal{I}$, depends, in general, also on other nuclear properties, such as, pairing, coriolis effect, etc., the equivalence of the two P.E. surfaces seems to be doubtful and the VMI P.E. surface may

not be the true representation of the deformation energy. Nevertheless, there is little doubt that there is an intimate connection between the two P.E. surfaces.

(3). In Chapter VII: Recent observations of the lifetimes of levels upto spin 18+ of the nucleus ^{158}Er by Ward et al* (1973) shows that their results are consistent with the rotational model description inspite of the severe "back bending" phenomenon (see ch. VI) observed in this nucleus. The question was raised whether the relationships between transition quadrupole moment and the moment-of-inertia as proposed by Mariscotti et al (1969) [Eq. (7.2)] and the alternative expression [Eq. (7.5)] as suggested by us, are able to give satisfactory values of quadrupole moments in this case.

We have carried out calculations using the quadrupole moments observed by Ward et al (1973). Our results show that the above two relations [Eqs. (7.2) and (7.5)] are not satisfactory for this nucleus for transitions involving levels beyond spin 10+. This is due to the fact that the VMI model description involves large changes in the values of the moment-of-inertia with spin whereas the observations of Ward et al (1973) for ^{158}Er indicate very little variation in it (since it fits the rotational model description).

* Ward, D., Andrews, H.R. Geiger, J.S., Graham, R.L. and Sharpey-Schafer, J.F., 1973, Phys. Rev. Letters, 30, 493.

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