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**LA THÈSE A ÉTÉ  
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**TWO ASPECTS OF DATA TRANSMISSION**

by

**J.S. Jawanda**

A thesis submitted to  
the School of Graduate Studies  
of the University of Ottawa  
in partial fulfillment of the requirements  
for the degree of Master of Applied Science

Department of Electrical Engineering  
Faculty of Science and Engineering  
University of Ottawa

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ABSTRACT

This thesis studies two problems in block oriented data transmission systems. The first is the evaluation of the Mean-Error-Free Interval, in blocks, between two blocks in error. We establish a relation between the Mean-Error-Free Interval and the mean block error rate.

The second aspect of data transmission considered in this research is the selection of an optimal packet (Block) length. The problem of determining the optimal message packet (Block) length, which minimises the network operational overheads over some real data channel and data channel models, while using different transmission strategies for error control, is analysed. Examples of a typical distribution of optimal block lengths, as a function of several parameters of interest, are given; the question of channel efficiency is also discussed.

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CHAPTER 1

INTRODUCTION

The field of Computer-Communications and its applications have grown rapidly during the past decade. Remote terminals are now commonly connected to computers for use in time sharing systems, bank credit checking inventory systems, business information systems, etc., to achieve resource sharing of hardware, software and data bases. The resource sharing concept has added new dimensions to the capabilities of the user. Under this new environment, he can now benefit from the versatility and power of the interlinking of computers and terminals. In all these shared computing systems, the modern telecommunications provide the essential data communication services.

Viewed in this light, the evaluation of the quality of data communication and the efficient utilisation of a transmission facility becomes an essential part of planning and design of such systems. The quality of data communication is evaluated in terms of its impact on the

whole system. "Data service should generally be deemed excellent when the capability of a data processing system is not diminished perceptibly by the insertion of data communication".[1]. A quantitative description of data communication efficiency is necessary to guide the efficient use of service and to use such terms which are meaningful to the user, the design engineer and the forces which maintain the systems.[2].

In computer-communication, the error control technique of automatic repeat request (ARQ) is the most commonly used to deliver error free messages from some message source to some message sink or destination. Typically the messages are divided into information blocks, with header identification and error detection parity bits being appended to them. The blocks are then transmitted over a communication channel and at the message destination they are tested for errors. If errors are detected, retransmission of those blocks in error is requested from the source. This process continues until all blocks are delivered error free. The traditional approach to the selection of the message block length for data transmission in a computer-communication network is dependent on the

optimal channel throughput. By this approach optimal message block lengths, which use the maximum possible fraction of available channel capacity in the presence of retransmissions, acknowledgement delays and block overhead, have been chosen.

In chapter two of this thesis a quantitative performance parameter (Mean-Error-Free Interval) is considered as a descriptor of the quality of data transmission in a block oriented data communication system. A relationship is established between the Mean-Error-Free Interval and the mean block error rate. The case of transmission through a random error channel is studied in particular and results for some real noisy data channels are also presented.

In chapter three, types of error control techniques available in the literature are described briefly in the introduction, with emphasis on the automatic-repeat-request error control techniques. A brief review of the present literature on the question of optimal block length, which maximises channel efficiency, is presented, with a detailed exposure of two recent papers in which the question of

optimal block length has been addressed. We then extend those results for two channels of practical interest, to determine the effect of error rate, block (Packet) length, transmission delay and error-recovery mechanism on the efficiency of the communication channel.

Finally, in chapter four, conclusions and comments are presented.

CHAPTER 2

MEAN-ERROR-FREE INTERVAL OF A NOISY  
DATA CHANNEL

2.1 MEAN-ERROR-FREE INTERVAL

Traditionally, the Bit Error Rate has been used as a criterion to quantify the performance of a digital facility. Efficient transmission of data over a digital facility is, however, dependent on both the exact structure of the error pattern and the (average) Bit Error Rate. Commonly, data is transmitted in packets and in all such applications the throughput obtained is sensitive to the correlation among bit errors. For data transmission, the Bit Error Rate criterion has been changed in favour of a criterion that expresses error-free transmission as the percentage of all one second intervals which are error-free.[1,2]. This criterion can be used relatively easily to obtain a bound on the over all throughput efficiency for packet oriented, Automatic Repeat Request (ARQ) based transmission systems for most packet lengths presently used.

A related performance measurement parameter, which has proven useful in data transmission applications is the mean interval between two successive errors. For packet-oriented transmission, this will mean the mean number of error-free packets between two packets in error and will be defined as Mean-Error-Free Interval (MEFI) .

## 2.2 STATEMENT OF THE PROBLEM

We consider a data string of  $N$  bits composed of packets of length  $L$  bits each, including overheads. This string is transmitted through a random-error channel with Bit Error Rate (BER), i.e. probability of a bit being in error,  $p$ . A packet will be considered to be in error when at least one of its bits is in error. We define an Error-Free Interval (EFI) as the number of consecutive error-free packets between two packets in error. The problem, then, is to evaluate the Mean-Error-Free Interval (MEFI), measured in packet lengths, bits or seconds and see how it is affected by values of  $N, L$  and  $p$ .

### 2.3 CONDITIONAL MEAN-ERROR-FREE INTERVAL

We will first calculate the conditional MEPI assuming that out of the  $B=N/L$  packets we know that  $i$  packets are in error,  $i=0,1,2,\dots,B$ . Here  $B$  is assumed to be an integer.

There are  $\binom{B}{i}$  possible arrangements of the  $i$  packets in error among the total  $B$ . In those  $\binom{B}{i}$  combinations we observe various EFI's, of length  $K$  ranging from 1 to  $B-i$ . The following result will be proved.

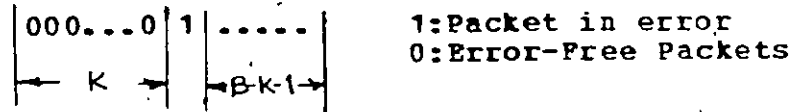
Proposition 1: For given  $B$  and  $i$ , the number of EFI's of length  $K$  is given by:

$$N(K) = (i+1) \cdot \binom{B-K-1}{i-1} ; K=1,2,\dots,B-i \quad \text{-- (1)}$$

For  $0 < i < B$

Proof: An EFI of length  $K$  may occur at either (i) one of the ends of the data string, or (ii) the interior of the data string.

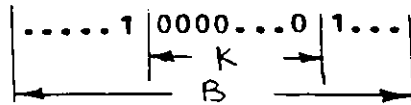
In case (i) the EFI's of length  $K$  packets have the following pattern:



The number of those patterns is equal to the number of combinations of the remaining (i-1) error packets in (B-K-1) positions,

$$2 \binom{B-K-1}{i-1} \quad \text{-- (2)}$$

The pattern of EPI's of type (ii) is:



and their number is equal to the number of the combinations of the remaining (i-2) errors in (B-K-2) positions, i.e.,

$$(B-k-1) \binom{B-K-2}{i-2} \quad \text{-- (3)}$$

Therefore, the total number of EPI's of length K is:

$$\begin{aligned}
 N(K) &= 2 \binom{B-K-1}{i-1} + (B-K-1) \binom{B-K-2}{i-2} \\
 &= 2 \binom{B-K-1}{i-1} + (B-K-1) \frac{(B-K-2)!}{(i-2)! (B-K-i)!}
 \end{aligned}$$

$$\begin{aligned} &= 2 \binom{B-K-1}{i-1} + (i-1) \frac{(B-K-1)(B-K-2)!}{(i-1)(i-2)!(B-K-i)!} \\ &= 2 \binom{B-K-1}{i-1} + (i-1) \binom{B-K-1}{i-1} \\ &= (i+1) \binom{B-K-1}{i-1} \quad \text{for } K > 1 \quad \text{--- (4)} \end{aligned}$$

We can now compute the conditional mean EPI. The result is surprisingly simple.

Proposition 2: The Mean Error Free Interval  $E(B, i)$  given that  $i$  packets out of  $B$  are in error, is:

$$\begin{aligned} E(B, i) &= \frac{B}{i+1} ; \text{ for } i=0, 1, 2, \dots, B-1 \\ &= 0 ; \text{ for } i=B \quad \text{--- (5)} \end{aligned}$$

Proof: The number of EPI's of length  $K$  are, by proposition one:

$$N(K) = (i+1) \binom{B-K-1}{i-1}$$

Then

$$E(B, i) = \frac{\sum_{K=1}^{B-i} K N(K)}{\sum_{K=1}^{B-i} N(K)}$$

We will compute separately the denominator and numerator of the above expression, making use of the following combinatorial identity:

$$\binom{n-1}{m} = \binom{n}{m} + \binom{n-1}{m-1} + \binom{n-2}{m-2} + \dots + \binom{n-m}{0}$$

for  $n > m$

Now

$$\begin{aligned} \sum_{K=1}^{B-i} \binom{B-K-1}{i-1} &= \sum_{K=1}^{B-i} \binom{B-K-1}{B-i-K} \\ &= \binom{B-2}{B-i-1} + \binom{B-3}{B-i-2} + \dots + \binom{i-1}{0} \\ &= \binom{B-1}{B-i-1} \\ &= \binom{B-1}{i} \end{aligned}$$

also:

$$\begin{aligned} \sum_{K=1}^{B-i} K \binom{B-K-1}{i-1} &= \sum_{K=1}^{B-i} K \binom{B-K-1}{B-K-i} \\ &= \binom{B-2}{B-i-1} + 2 \binom{B-3}{B-i-2} + \dots + B-i \binom{i-1}{0} \\ &= \left[ \binom{B-2}{B-i-1} + \binom{B-3}{B-i-2} + \dots + \binom{i-1}{0} \right] \end{aligned}$$

$$\begin{aligned}
 & + \left[ \dots + \binom{B-3}{B-i-2} + \dots + \binom{i-1}{0} \right] + \\
 & + \dots + \dots + \dots + \binom{i-1}{0} \\
 & = \left[ \binom{B-1}{B-i-1} \right] + \left[ \binom{B-2}{B-i-2} \right] + \dots + \left[ \binom{i-1}{0} \right] \\
 & = \binom{B}{B-i-1} = \binom{B}{i+1}
 \end{aligned}$$

Therefore:

$$\begin{aligned}
 E(B, i) &= \frac{\sum_{K=1}^{B-i} K N(K) \binom{B}{i+1}}{\sum_{K=1}^{B-i} N(K) \binom{B}{i}} \\
 &= \frac{B! / (i+1)! (B-i-1)!}{B! (i)! (B-i-1)!} \\
 &= \frac{B}{i+1}
 \end{aligned}$$

It is to be noted here that the above results for the conditional Error-Free Interval are independent of the type of the channel and thus, valid not only for the random-error channel, but for any type of channel noise as long as the packets in error are independently distributed.

2.4 UNCONDITIONAL MEAN-ERROR-FREE INTERVAL

Concentrating now on the random-error channel we can calculate the unconditional MEFI.

First, the probability of a packet being in error, i.e., having at least one bit in error, is:

$$P = 1 - (1-p)^L \quad \text{-- (6)}$$

where  $p$  is BER and  $L$  is the length of the packet in bits.

Proof

Prob. of a bit being in error =  $p$

Prob. of a bit not being in error =  $1-p$

Prob. of  $L$  bits not being in error =  $(1-p)^L$

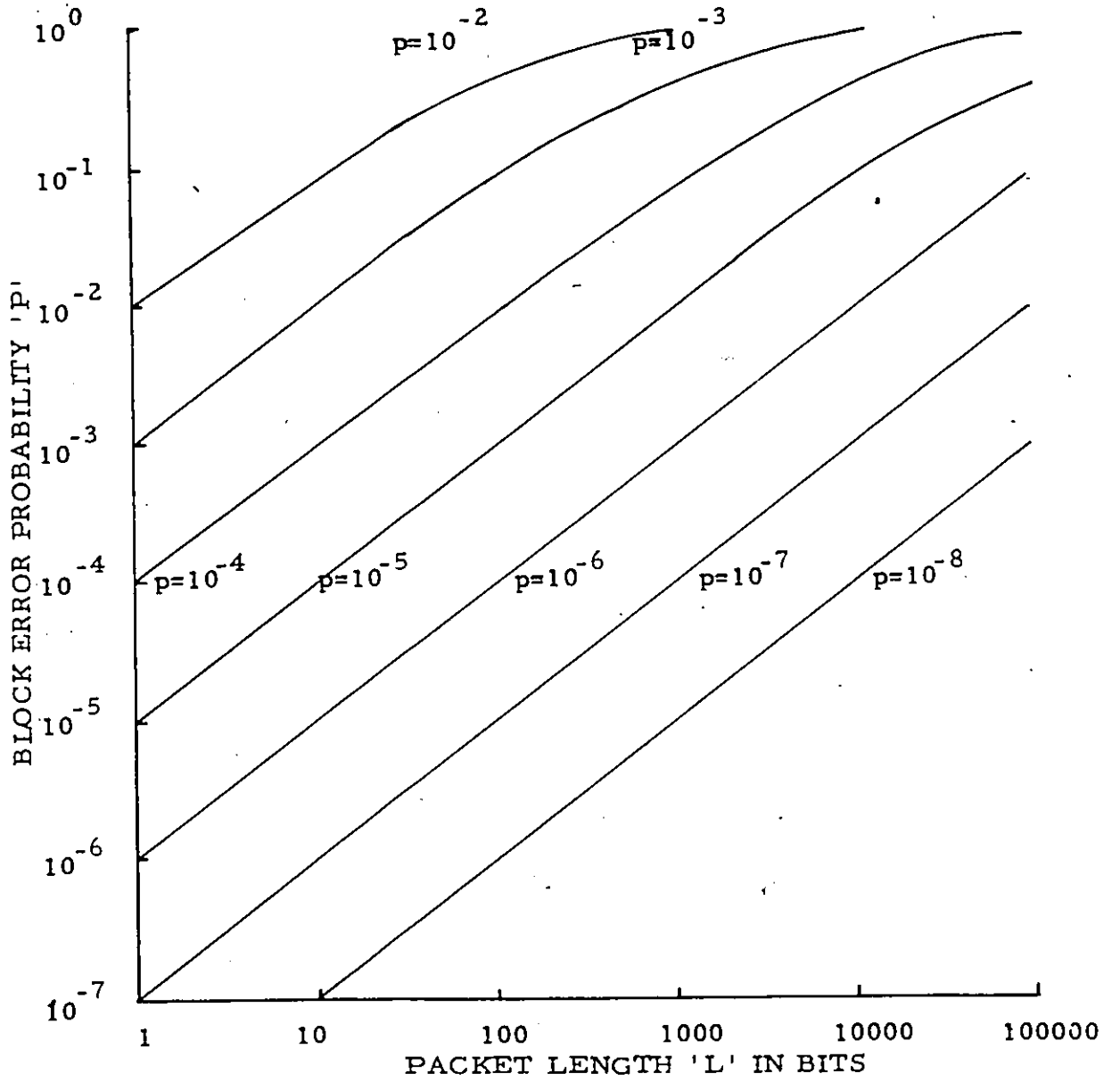
Prob. of packet not in error =  $(1-p)^L$

Thus: Prob. of a packet having at least one error =  $1 - (1-p)^L$

$$\text{or } P = 1 - (1-p)^L$$

If the bit errors are positively correlated the situation will be better than what is indicated by (6).

Fig. 1 shows a plot of  $P$  Vs  $L$ , for various BER's  $p$ .



P Vs L  
FIG. 1

The probability of  $i$  packets being in error is given by the binomial distribution:

$$(PR)_i = \binom{B}{i} P^i (1-P)^{B-i} \quad \text{-- (7)}$$

The following proposition gives the unconditional Mean EFI:

Proposition 3: In a data string of  $B$  packets, the Mean Error-Free Interval  $\bar{E}$ , measured in packet lengths, is:

$$\bar{E}(B) = \frac{B}{P(B+1)} [1 - P^{B+1} - (1-P)^{B+1}] \quad \text{-- (8)}$$

Proof:

$$\begin{aligned} \bar{E}(B) &= \sum_{i=0}^B E(B,i) (PR)_i \\ &= \sum_{i=0}^{B-1} \frac{B}{i+1} \binom{B}{i} P^i (1-P)^{B-i} \\ &= \frac{B}{(B+1)P} \sum_{i=0}^{B-1} \binom{B+1}{i+1} P^{i+1} (1-P)^{B-i} \\ &= \frac{B}{(B+1)P} \sum_{j=1}^B \binom{B+1}{j} P^j (1-P)^{B+1-j} \\ &= \frac{B}{(B+1)P} [1 - P^{B+1} - (1-P)^{B+1}] \end{aligned}$$

Remark: Relation (8) is also valid for non random error channels, whenever (7) is valid, i.e., where the packets in error are statistically independent.

A case of practical interest arises when the data string is very long, i.e.,  $B \rightarrow \infty$

Taking the limit in (8) we obtain:

Corollary 1: For a very long data string, the Mean-Error-Free Interval, measured in packet lengths, is:

$$\begin{aligned} \bar{E}_1(\infty) &= \frac{1}{P} ; \text{ for } P < 1 \\ &= 0 ; \text{ for } P = 1 \end{aligned} \quad \text{-- (9)}$$

The apparent discontinuity of  $\bar{E}_1(\infty)$  as a function of the packet error rate  $P$ , and at  $P=1$ , can easily be explained. If  $P=1$ , all packets are in error and thus  $\bar{E}_1(\infty) = 0$ . But if, for example  $P=0.9999$ , and  $B = \infty$ , 1 packet out of every 10,000 is error-free thus  $\bar{E}_1(\infty)$  is close to the value  $1/P=1$ .

From (6) and (9) we obtain the expression:

$$\bar{E}_1(\infty) = \frac{1}{1-(1-p)^L} \quad \text{-- (10)}$$

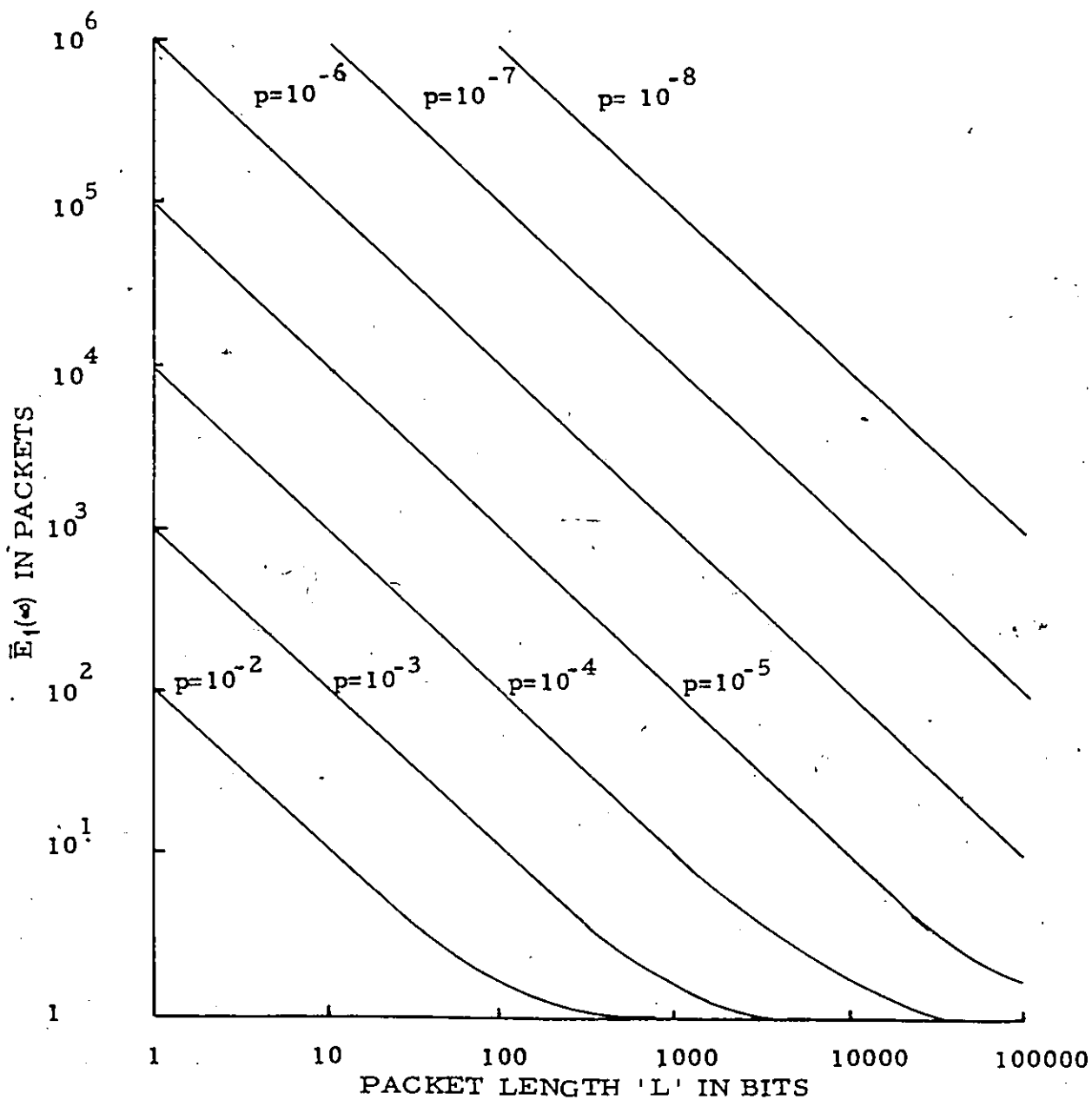
which gives the Mean-Error-Free Interval, measured in packet lengths, as a function of the Bit Error Rate and of the packet length.

Fig. 2a shows the plot of  $\bar{E}_1(\infty)$  Vs L for various BER's p for the random error channel.

If R bits/sec is the data rate, we can obtain the value of  $\bar{E}_1(\infty)$  in sec from (10) as:

$$\bar{E}_1(\infty) = \frac{L/R}{1-(1-p)^L} \quad \text{(sec)} \quad \text{-- (11)}$$

Fig. 2b shows the plot of  $\bar{E}_1(\infty)$  in sec Vs L for various BER's p for random error channel.



$\bar{E}_1(\infty)$  Vs L  
FIG. 2a RANDOM CHANNEL

2.5 MEAN-ERROR-FREE INTERVAL FOR A LONG STRING  
OF DATA ON A BALKOVIC CHANNEL

The Mean-Error-Free Interval for a long string of data, when packets in error are statistically independent, is given by:

$$\bar{E}_1(\infty) = \frac{1}{P}$$

where P is the probability that a packet will have at least one error. In the telephone channels reported by Balkovic et al [3], this probability is related to packet length by the following relations for different rates, which are derived from the curves given in [4].

For 1200 b/s

$$P = .0000139(L)^{.80626} \quad L > 10 \quad \text{-- (12)}$$

For 2000 b/s

$$P = .0000088(L)^{.811036} \quad L > 10 \quad \text{-- (13)}$$

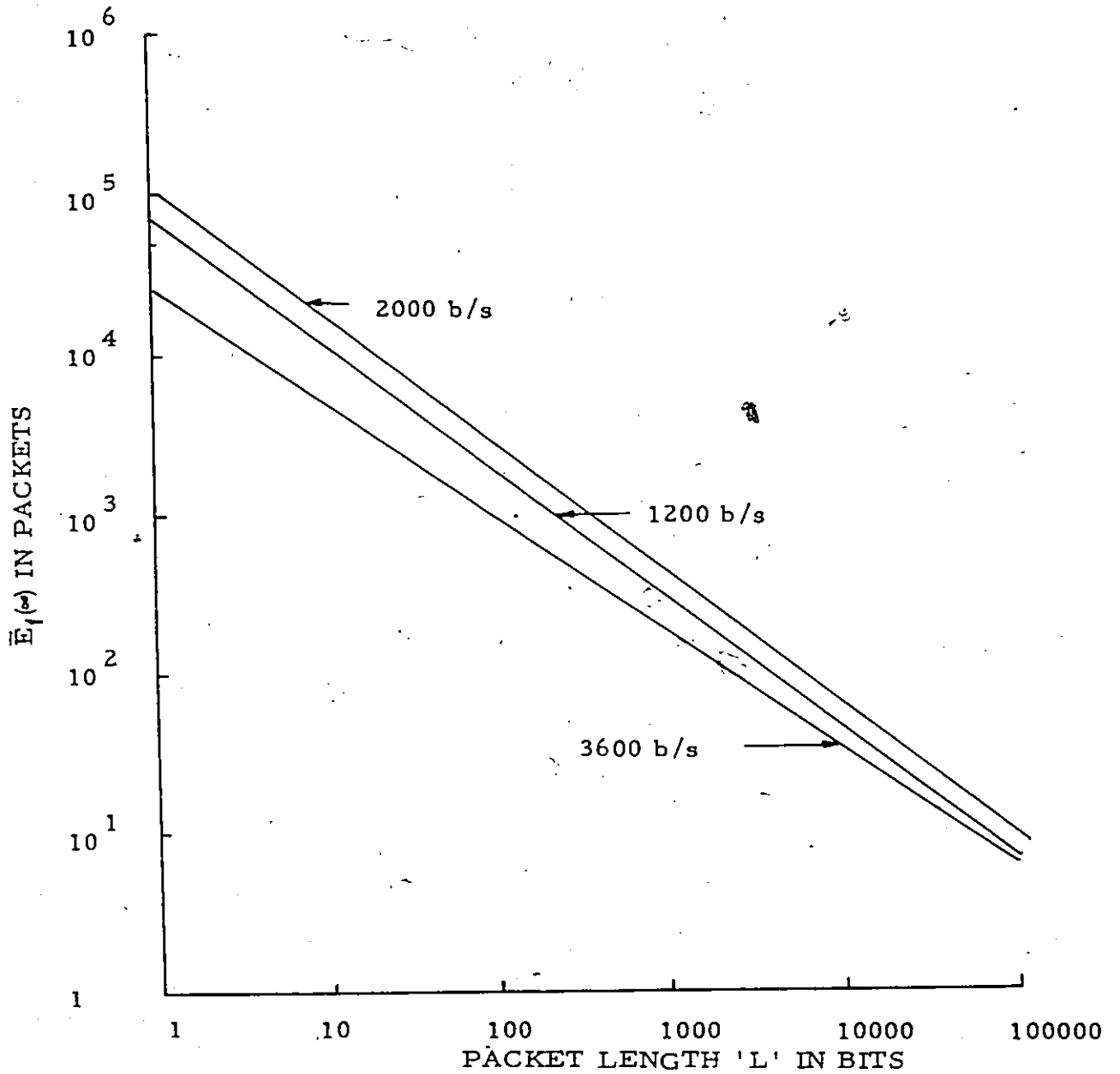
For 3000 b/s

$$P = .0000037(L)^{.72695} \quad L > 10 \quad \text{-- (14)}$$

where L represents the block size in bits.

Using the above relations in the equation (9) for P, we obtain a relationship for the Mean-Error-Free Interval measured in packets, for different data rates.

Fig. 3 shows a plot of  $\bar{E}_1(\omega)$  Vs L for various data rates of the Balkovic channel.



$\bar{E}_1(s)$  Vs L  
FIG. 3 BALKOVIC CHANNEL

2.6 MEAN-ERROR-FREE INTERVAL FOR A LONG STRING OF DATA ON A DNPL MICROWAVE CHANNEL.

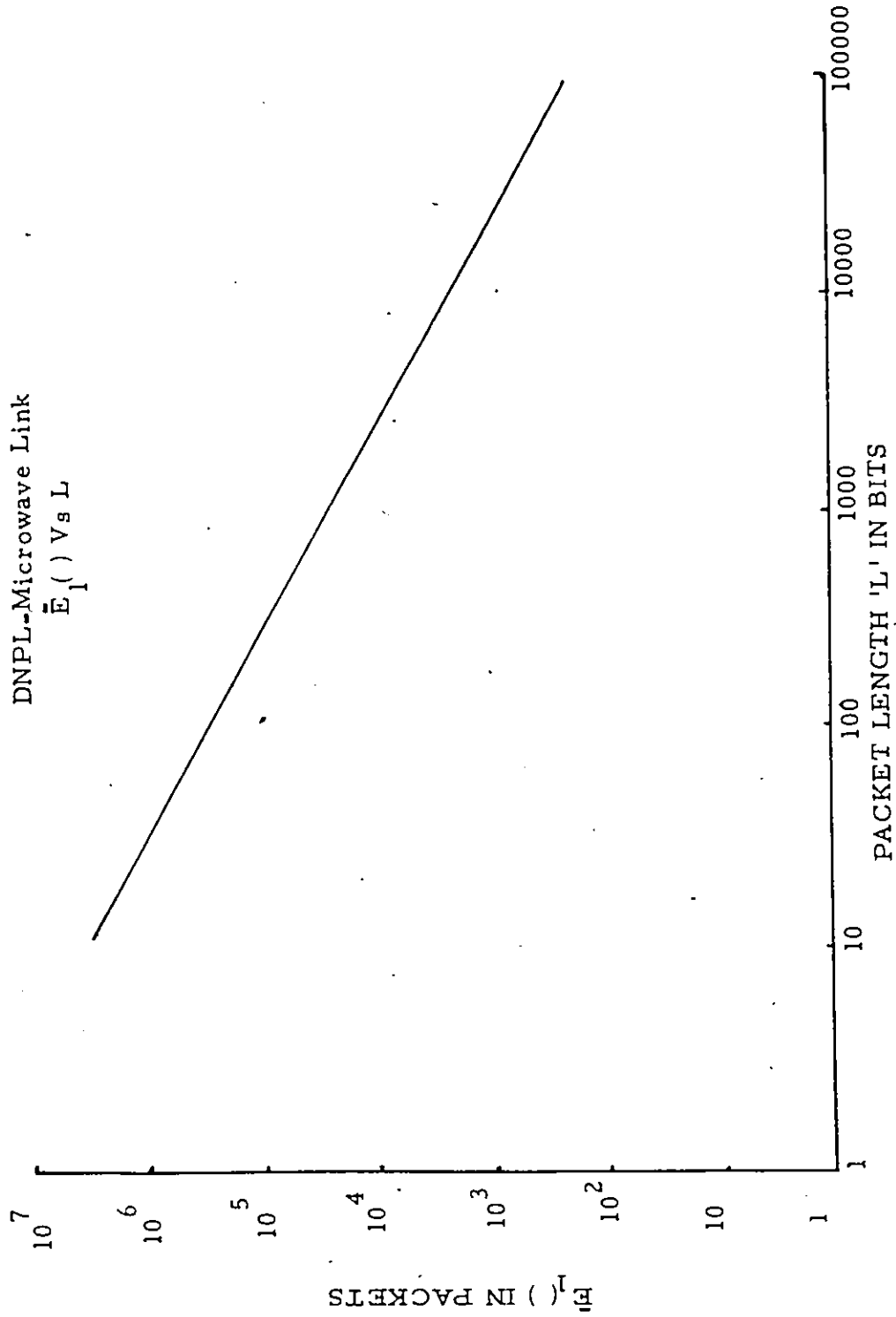
The actual error characteristic of the DNPL microwave channel include both random and burst errors.[5]. The error probability of a packet having at least one error, determined by experiments, is found to be

$$P = (10)^{a_0 + a_1 \log_{10} L} \quad \text{--(15)}$$

where  $a_0 = -6.5$ ,  $a_1 = 1.07$  and  $L$  is the packet size.

Now using relation (15) for  $P$  in the equation (9) we get the expression for MEFI in packets on a DNPL microwave channel.

Fig. 4 shows a plot of  $\bar{E}_1(\infty)$  Vs  $L$  for a DNPL microwave channel.



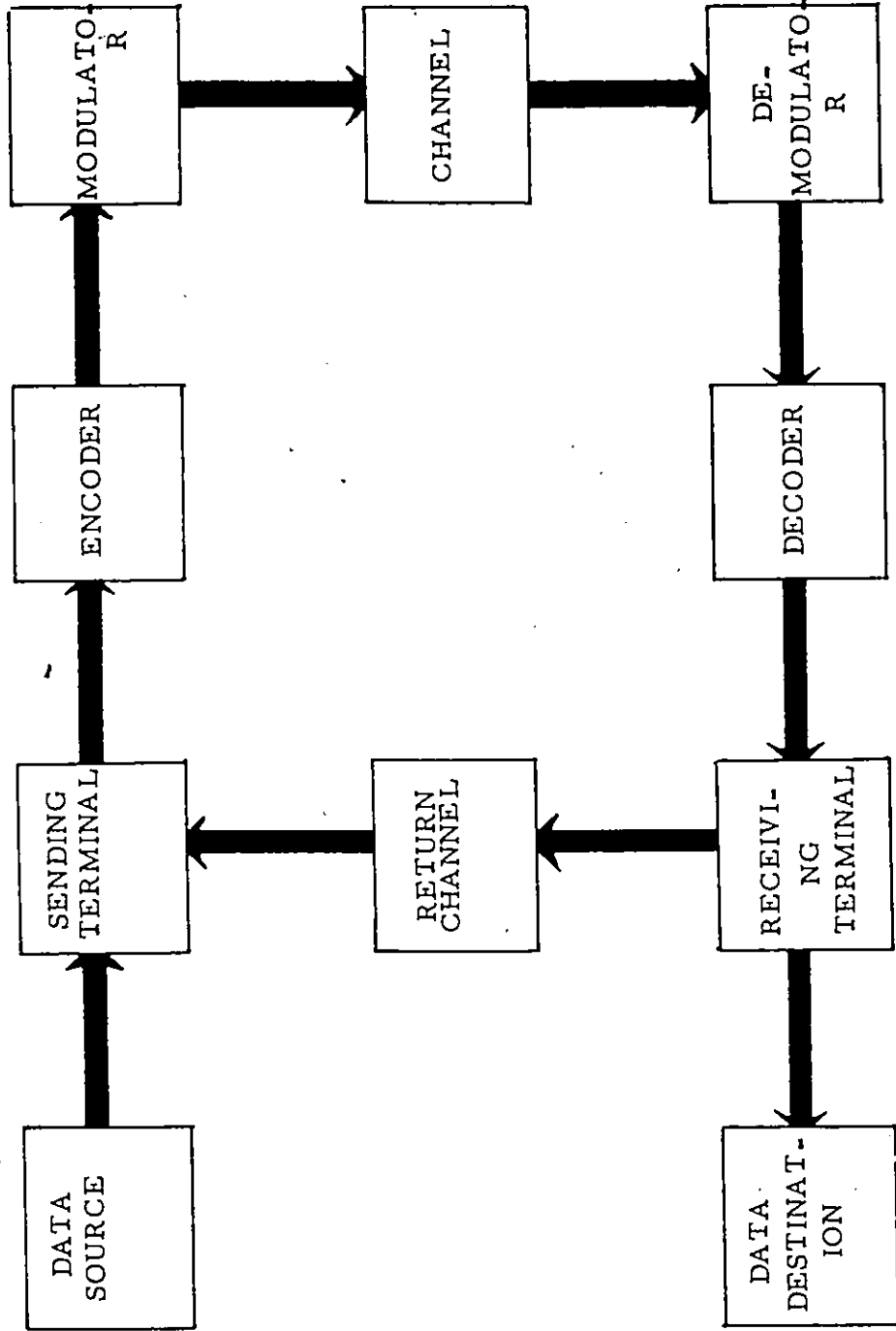
CHAPTER 3

OPTIMAL BLOCK SIZE ANALYSIS

3.1 INTRODUCTION

Communication channels are becoming a more and more important feature of modern day computer-communication networks. In practical implementation of these networks, the channels are found to be invariably noisy. So there is always a finite probability of an error occurring in the transmission of a message. Consequently, it becomes necessary not only to employ some kind of error correction and detection scheme but also to partition messages into fixed size blocks.[7]. Though the principle purpose of partitioning messages is to enable reliable and efficient data communication, it also assists in memory management.

The partitioning of messages into blocks, makes it essential to adopt a transmission strategy for error correction.[7]. Basically there are two types of error detection and correction strategies.[4].



AUTOMATIC REPEAT REQUEST SYSTEM (ARQ)

FIG. 5

1.) Forward-error correction (FEC), in which each message block transmitted incorporates redundant bits appropriately coded, so that errors in transmission are corrected at the receiver with the help of the code (which is agreed in advance between receiver and transmitter).

2.) Automatic-Repeat-Request (ARQ), in which each message block transmitted also incorporates redundant bits, but for error detection only. Each block is transmitted separately to the receiver. On the reception of a message block the receiver sends an acknowledgement to the transmitter. A positive acknowledgement is sent if the received message block is free of transmission errors, otherwise a negative acknowledgement is sent. At the transmitter, if the acknowledgement received is positive then the next block is transmitted, otherwise the same message block is retransmitted. A schematic diagram of an ARQ system is given in fig. 5.

ARQ systems have been extensively compared with FEC systems on some real channels [8] and on some theoretical channel models [9-14]. In every instance the results have exhibited that ARQ schemes are superior to FEC schemes on

the basis of reliability and the relative insensibility of this reliability to conditions on the channel. Thus despite very active research work done on FEC schemes, ARQ schemes are still commonly used in computer-communications for error control on the transmission channels. [15].

Furthermore, ARQ schemes are subdivided into two types of schemes.[10,11].

a.) The simplest and by far the most widely used ARQ scheme is the so called Stop-and-wait transmission strategy.[16]. In this type of scheme, the sending end terminal after transmitting a block of data waits for positive or negative acknowledgement from the receiving end terminal, before transmitting a new data block or retransmitting the same data block respectively. The Stop-and-wait scheme is very simple and easy to implement, but suffers from inherent inefficiency due to idle time spent waiting for acknowledgement for each transmitted block.

b.) The second type of ARQ scheme is the so called continuous transmission strategy, which is implemented in

two ways. 1.) The sending end terminal transmits data blocks continuously without waiting for acknowledgement and retransmits the unconfirmed data block only. 2.) The sending end terminal transmits data blocks continuously without waiting for the acknowledgement. But when negative acknowledgement is received, the receiver pulls back and retransmits the unconfirmed data block along with all the blocks that followed it.

### 3.2 THE EFFECT OF DATA BLOCK SIZE ON ARQ SYSTEMS

The key figure of merit in evaluation of ARQ systems is the transmission efficiency or the throughput [4,19], which is defined as the ratio of the average number of information bits accepted by the receiving terminal per unit time to the bit rate of the channel. In the context of this thesis, ARQ systems are defined as the systems which employ ARQ schemes for error controlling over a transmission channel. Usually, in a given communication system, such quantities as the block error probability for a given block length, the acknowledgement delay and the number of parity bits per block size are all fixed. This means that once an ARQ scheme has been defined, the only uncertain variable


that effects the throughput or the channel efficiency is the block size. In fact, if the block size is increased, the error probability and the overhead due to the last unfilled message block increases. On the other hand, if the block size is decreased, the overhead due to header and parity bits increases. These increase in overheads decreases the throughput. Thus there is usually an optimum block size for each given set of conditions. But this choice of optimum block size for a particular communication system requires a compromise to balance system complexity, sensitivity to errors and other factors like retransmission delays, acknowledgement delays, etc.

### 3.3 REVIEW OF THE PRESENT LITERATURE FOR OPTIMAL BLOCK SIZE

The problem of determining an optimal message block size in an ARQ system has been studied by many authors. Usually the basic approach adopted is to maximise channel efficiency, which is generally defined as a ratio of the time required for transmission of a message (unblocked) to the total amount of time required for the transmission of message block by block from transmitter to receiver. For

example Kucera [20] Wood [21] and Kirilin [22] have studied the effect of the message block size on the transmission efficiency. In thorough analysis of a particular situation, Chu [23] has derived the optimal block size, but he assumed a special distribution of the message length found in typical computer time-sharing systems. However, in all these papers, the analysis has been restricted to relatively slow data rates, where overhead due to the acknowledgement is not dominant. Balkovic et al [24] have studied satellite communication systems where the overhead due to acknowledgement is quite dominant.

None of these papers mentioned above give any method of finding the optimal block size under a continuous transmission scheme, except by Chu [23], which derives it for a particular situation only. Zacharov [7] studied the question of the optimal block size for one particular continuous transmission strategy and demonstrates that optima exist under a range of practical conditions encountered in high-speed data communication. Field [25] considers the effect of error rate, block length, transmission delay and error recovery mechanism on the efficiency of the communication channel used in



computer-communication. The criterion in Field's analysis is to maximise channel utilisation, which has been defined as the ratio of the effective computer-computer communication capacity of the channel used in communication. Recent studies conducted by Shamim et al [26] consider the transmission of exponentially distributed message lengths through a random-error channel employing stop-and-wait ARQ strategy. In [26] the main objective is to minimise the overhead factor involved in transmission of the message.

In the next section, a detailed mathematical analysis of two recent papers [23] by Chu and [26] by Shamim et al is presented with an intent to study the effect of error rate, block (Packet) length, transmission delay and error-recovery mechanism on the efficiency of the communication channels of practical interest, as for example the telephone channel by Balkovic et al [3] and the DNPL microwave channel by Peatfield et al [5].

### 3.4 CHU'S ANALYSIS FOR OPTIMAL BLOCK SIZE

Chu in [23] developed a mathematical model to analyse the problem of optimal block length for an ARQ system, i.e., the block length which maximises channel efficiency. The model treats two types of ARQ schemes. 1.) Stop-and-wait; 2.) Continuous transmission; Furthermore the model considers two types of error channels, a.) Random error channel; b.) Burst error channel;

We shall now give the formulation of the mathematical model.

The message length  $l$  is a random variable and can be characterised by a probability distribution  $P_L(l)$ . When the message is partitioned in fixed size blocks of size  $B$  bits per block, the expected number of blocks per message is given by:

$$N(B) = \sum_{n=1}^{\infty} n \cdot P_L \{ (n-1)B < l < nB \} \quad l=1,2,\dots \quad \text{-- (16)}$$

In [23],  $l$  is assumed to be geometrically distributed. The average number of blocks per message in this case can be expressed as:

$$N(B) = \frac{B-1}{(1-q)} \quad \text{-- (17)}$$

The structure of a message block is composed of an address of  $b_1$  bits in the front, an information block of  $B$  bits in the middle, and a checking code of  $b_2$  bits in the end. The sum  $b$  of these extra bits ( $b=b_1+b_2$ ) is called the block overhead.

Stop-and-wait transmission strategy

In the Stop-and-wait transmission strategy we first consider that the last unfilled message block is filled with dummy information. It is assumed that message block errors during transmission and retransmission are independent. In this case the expected acknowledgement overhead for a message block of size  $(B+b)$  on a channel with a transmission rate  $R$  bits/s can be expressed as [23]:

$$A(B+b) = A + \sum_{i=1}^{\infty} [E(B+b)]^i \left( A + \frac{B+b}{R} \right) \quad \text{-- (18)}$$

where  $A$  is the acknowledgement delay associated with each message block and the term  $\left[ \sum_{i=1}^{\infty} [E(B+b)]^i \right]$  represents the cumulative probability of error occurrence in transmitting a message block of size  $(B+b)$ ;  $E(B+b)$

represents the probability that a block of size (B+b) will have at least one error on transmission over a channel.

The expected wasted time due to acknowledgement and retransmission, in transmitting a message in fixed sized blocks,  $\bar{W}_1(B)$ , which is the product of the expected number of blocks (of size B+b) per message and the expected acknowledgement overhead can be given as:

$$\bar{W}_1(B) = (1-q)^{B-1} \cdot \left\{ A + \sum_{i=1}^{\infty} [E(B+b)]^i \left( A + \frac{B+b}{R} \right) \right\}$$

Since  $0 < E(B+b) < 1$ ,  $B > 0$  and  $b > 0$

$$\sum_{i=1}^{\infty} [E(B+b)]^i = \frac{E(B+b)}{1-E(B+b)}$$

Thus:

$$\bar{W}_1(B) = (1-q)^B \left\{ A + \frac{E(B+b)}{1-E(B+b)} \cdot \left( A + \frac{B+b}{R} \right) \right\} \quad \text{--- (19)}$$

The expected wasted time due to block overhead and the last unfilled partitioned block,  $\bar{W}_2(B)$ , is equal to the difference between the time to transmit a blocked message and the time to transmit the unblocked message.

$$\bar{W}_2(B) = N(B) \cdot \frac{B+b}{R} - \frac{1+b}{R} \quad \text{--- (20)}$$

where  $b'$  is the overhead for the unblocked message and  $l$  is the average message length.

Thus the total expected time wasted in transmitting a message in fixed sized blocks,  $\bar{W}(B)$ , is equal to the sum of the wasted time due to block overhead and unfilled message block plus the wasted time due to acknowledgement and retransmission. Consequently,  $\bar{W}(B)$  can be expressed as:

$$\bar{W}(B) = (1-q)^{B-1} \left[ A + \frac{E(B+b)}{1-E(B+b)} \cdot \left( A + \frac{B+b}{R} \right) + \frac{B+b}{R} \right] - \left( \frac{1+b}{R} \right) \quad \text{--- (21)}$$

The minimum of  $\bar{W}(B)$ , which exists for the case of the ARQ schemes analysed in [22], represents the optimal block length which minimises the channel wasted time. The necessary condition for the existence proven in the analysis of [23], is that

$$E''(B+b) > -[E'(B+b)]^2$$

Where  $E'(B+b)$  &  $E''(B+b)$  represent the first order and second order partial derivatives of  $E(B+b)$ .

In case an end-of-message character is used to designate the end of the message block in the last unfilled message block, then the  $\bar{W}(B)$  function reduces to:

$$\bar{W}(B) = N(B) \frac{b}{R} - \frac{b'}{R} \quad \text{-- (22)}$$

Thus the total expected wasted time in transmitting a message in fixed sized blocks reduces to:

$$\bar{W}(B) = (1-q)^{B-1} \left[ A + \frac{E(B+b)}{1-E(B+b)} \left( A + \frac{B+b}{R} \right) + \frac{b}{R} \right] - \frac{b'}{R} \quad \text{-- (23)}$$

The minimum solution of  $\bar{W}(B)$  in this case is obtained as before, using numerical analysis techniques, and it also represents the optimal block length for the case where an end-of-message character is used.

Continuous transmission strategies

In continuous transmission strategies there are two cases to be considered.

The first case treated is that message blocks are transmitted continuously without waiting for an acknowledgement and each unconfirmed message block is retransmitted. When a reverse channel is used for the acknowledgement, the acknowledgement time is zero and the overhead is the fixed cost of the reverse channel, which is independent of block size. If a reverse channel is not used, the acknowledgement overhead is still considered to be zero because the acknowledgement signal is very short as compared to the length of the message block.

Incorporating this in the equation for  $\bar{W}(B)$  derived for the case of the Stop-and-wait strategy with an end-of-message character, we get the following equation for the total average time wasted to transmit a message block:

$$\bar{W}(B) = (1-q)^{B-1} \left[ \frac{E(B+b)}{1-E(B+b)} \left( \frac{B+b}{R} + \frac{b}{R} \right) - \frac{b^2}{F} \right] \quad (24)$$

The minimum of the above function if it exists, will represent the optimal block length.

In the second case of the continuous transmission strategy, message blocks are transmitted continuously also without waiting for acknowledgement and the transmitter retransmits unconfirmed message blocks together with all those message blocks that followed it. In this case the acknowledgement time is the sum of the time required to detect an error message block and the time required to send the acknowledgement signal. The expected acknowledgement overhead is expressed as:

$$A(B+b) = \sum_{i=1}^{\infty} \left( A + \frac{B+b}{R} \right) E(B+b)^i \quad \text{-- (25)}$$

where  $A = t_1 + t_2$

$t_1$  = Time required to detect an error block

$t_2$  = Time required to generate an acknowledgement signal

The total expected wasted time to transmit a message block with an end-of-message character becomes

$$W(B) = (1-q)^{B-1} \left[ \frac{E(B+b)}{1-E(B+b)} \left( A + \frac{B+b}{R} \right) + \frac{b}{R} + \frac{b'}{P} \right] \quad \text{-- (26)}$$

The minimum of  $\bar{W}(B)$ , which exists, represents the optimal block length for this case.

For the random-error channel, the value of  $E(B+b)$  is obtained as follows:

$$E(B+b) = 1 - (1-C)^{B+b} \quad \text{-- (27)}$$

where  $C$  is the bit error rate. For the burst error channel, the  $E(B+b)$  turns out to be a complex polynomial, which is obtained by employing curve fitting techniques to the curves shown in [28] for 2000 b/s and 4800 b/s channels.

Substituting the above mentioned values for  $E(B+b)$  respectively in the expression for  $\bar{W}(B)$  (Total time wasted in transmitting a message block), derived for all the four cases of ARQ schemes, and obtaining their minimum solution respectively, will give the optimal block length for the respective channel and ARQ case under consideration.

The minimum of  $\bar{W}(B)$  is obtained, for the four cases analysed above, by differentiating  $\bar{W}(B)$  w.r.t.  $B$  and equating to zero and numerically solving the thus obtained equation. The necessary condition for the existence of a

minimum is that the second order partial derivative of  $\bar{W}(B)$  w.r.t.  $B$  should be positive.

### 3.5 SHAMIM ET AL'S ANALYSIS FOR OPTIMAL PACKET SIZE

Shamim et al in [26] also developed a mathematical model to determine the optimal packet (Block) size. The system studied in their analysis is a Stop-and-wait ARQ with a random error channel. The basic approach used in this analysis is to minimise the total overhead factor, which is the sum of the overhead factor, involved in transmission of messages packet by packet [27], i.e., the network operational overhead factor, and the overhead factor due to blank padding. The network operational overhead factor is defined as the ratio of the overheads (in bits) incurred in transmission to the amount (in bits) of the useful information, and can be written as [26]:

$$F = \frac{\left\{ \frac{1}{N} \sum_{i=1}^N I + \frac{1}{N} \sum_{i=1}^N Y \right\} k(B)}{np \left\{ \frac{1}{N} \sum_{i=1}^N I + \frac{1}{N} \sum_{i=1}^N \alpha_i \right\} B} \quad \text{--- (28)}$$

Where  $I =$  Integral number of packets in  $i$

ith message

$\alpha_i$  = Fractional number,  $0 < \alpha_i < 1$

$Y_i$  = Indicator function which takes on values 0 or 1;  $Y_i = 0$ , only if  $\alpha_i = 0$

$N$  = Number of messages transmitted

$B$  = message block size in bits

$k(B)$  = Network operational overheads per packet  
(in bits)

In the analysis by Chu [23], the network operational overheads per packet  $k(B)$ , has been assumed to be constant and independent of packet size. In the present analysis, it is assumed to be a function of packet size.

Next we present the evaluation of the network operational overheads by evaluating various factors individually, which contribute to network operational overheads.

Overheads due to extra bits to be transmitted with each packet

The message length is assumed to be a random variable and is described by a probability distribution with a mean message length  $l$  in bits per message, which can be written as:

$$l = \frac{1}{N} \sum_{i=1}^N M_i \quad \text{-- (29)}$$

where  $M_i (i=1,2,3,\dots,N)$  is the  $i$ th message length

Each message is segmented into fixed size packets or blocks of  $B$  bits. The structure of these packets consists of an address of  $b_1$  bits in the front, information bits in the middle and a check code of  $b_2$  bits at the end of the message packet. These extra bits ( $b=b_1+b_2$ ), are called the block overhead which is transmitted with each block (Packet).

Overheads due to transmission errors and acknowledgement delays.

The Stop-and-wait strategy is used over a channel in half duplex mode for the error control. It assumed that the routing strategy is fixed and the acknowledgement overhead over channel having a transmission rate of R bits/sec is given by:

$$F_{na} = AR \text{ bits}$$

Futhermore, in case a negative acknowledgement is received due to the occurrence of transmission errors, then it will cause retransmission and reacknowledgement of the packet and thereby produce futher delays. Thus the time wasted in retransmission and acknowledgement [23] is given as:

$$T_{wra} = \sum_{i=1}^{\infty} [E(B+b)]^i \left( A + \frac{B+b}{R} \right) \text{ sec.}$$

where  $E(B+b)$  represents the probability that a packet of size  $(B+b)$  bits will have at least one error on transmission. Thus the term  $\left[ \sum_{i=1}^{\infty} [E(B+b)]^i \right]$  is the cumulative probability of the error occurrence on transmission in

packets of size (B+b) bits each.

The overheads due to retransmission and reacknowledgement of a message packet can be expressed as:

$$\begin{aligned}
 P_{\text{rat}} &= \sum_{i=1}^{\infty} [E(B+b)]^i \left( A + \frac{B+b}{R} \right) R \text{ bits} \\
 &= \frac{E(B+b)}{1-E(B+b)} \left( A + \frac{B+b}{R} \right) R \text{ bits} \quad \text{-- (30)}
 \end{aligned}$$

Thus the total network operational overheads per message block incurred in transmission of a message block over a channel, which are equal to the sum of the overheads due to block overhead, due to first acknowledgement and the overheads due to the expected retransmission and reacknowledgement, is given as:

$$k(B) = (b+AR) + \frac{E(B+b)}{1-E(B+b)} (B+b+AR) \text{ bits}$$

Thus the network operational overhead factor can be written as:

$$\begin{aligned}
 P_{\text{np}} &= \frac{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N Y_i \right\}}{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N \alpha_i \right\} B} \left( b+AR \right) + \frac{E(B+b)}{1-E(B+b)} (B+b+AR) \quad \text{-- (31)}
 \end{aligned}$$

Overhead factor due to dummy information

As the message length was assumed to be a random variable, the probability of the message length being an integer multiple of the packet size is very remote. Thus the last unfilled message block is filled with dummy information. Which give rise to the overhead factor and can be written as:

$$F = \frac{(1 - \frac{1}{N} \sum_{i=1}^N \alpha_i) B}{nb - \sum_{i=1}^N M_i} \quad \text{--- (32)}$$

Thus the total overhead factor can be expressed as:

$$F_T = F_{nb} + F_{np} = \frac{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N Y_i \right\} (b+AR) + \frac{E(B+b)}{1-E(B+b)} (B+b+AR)}{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N \alpha_i \right\} B + \frac{(1 - \frac{1}{N} \sum_{i=1}^N \alpha_i) B}{nb - \sum_{i=1}^N M_i}} \quad \text{--- (33)}$$

On the basis that the message lengths are long, i.e.,  $M_i \gg \alpha_i B$ , it is assumed that:

$$\frac{1}{N} \sum_{i=1}^N Y_i = \frac{1}{N} \sum_{i=1}^N \alpha_i + \frac{1}{2}$$

Thus the expression for  $F$  reduces to:

$$F = \frac{T}{1 - \frac{B}{2L}} \left[ m + \frac{E(B+b)}{1 - E(B+b)} (B+m) \right] + \frac{B}{2L} \quad \text{--- (34)}$$

where  $m = H + AR$

For a random error channel, the probability that a message block (Packet) of size  $(B+b)$  bits will have at least one error on transmission is given by:

$$E(B+b) = 1 - (1-C)^{B+b} \quad \text{--- (35)}$$

When this value of  $E(B+b)$  is substituted in the expression for  $F_T$ , an expression for the total overhead factor, incurred in transmission of a message packet by packet over a random error channel employing the Stop-and-wait strategy for error control, is obtained. It can be expressed as:

$$F_{TS} = \frac{B^2 + B(m+2l) + 2lm}{2lB(1-C)(B+b)} - 1 \quad \text{--- (36)}$$

It is assumed in the analysis that errors in the blank padding do not cause retransmissions.

To obtain an optimal packet size that minimises the total overhead factor  $F_{TS}$ , the expression for  $F_{TS}$  is differentiated with respect to  $B$  and equated to zero. This gives rise to a polynomial of the form:

$$xB^3 + yB^2 + zB + w = 0 \quad \text{--- (37)}$$

where  $x = \ln(1-C)$

$$y = (m+2l)\ln(1-C) - 1$$

$$z = 2lm \cdot \ln(1-C)$$

$$w = 2lm$$

It can be shown that the second order partial derivative of  $F_{TS}$  with respect to  $B$  is positive definite and the above polynomial has only one positive real root which represents the required  $B_{opt}$  for the random error channel using the Stop-and-wait strategy for error control.

3.5.1 EXTENSION TO CONTINUOUS TRANSMISSION  
STRATEGY

In this section, the analysis of Shamim et al [26] for the case of the Stop-and-wait strategy, shall be extended to cover two continuous transmission strategies. The basic approach - analogous to the case of the Stop-and-wait transmission strategy in [26]- is to minimise here again the total overhead factor. Which is- as defined in the previous section - the sum of the network operational overhead factor and the overhead factor due to blank padding as defined in [27] and calculated as follows:

First, the message length is assumed to be a random variable with mean  $l$  bits expressed as;

$$l = \frac{1}{N} \sum_{i=1}^N M_i$$

where  $M_i (i=1,2,\dots,N)$  is the message length of the  $i$ th message

Messages are segmented into fixed sized blocks or packets of size  $B$  bits each. The structure of each message

block consists of an address of  $b_1$  bits in the front, an information block of size  $B$  bits in the middle, and a checking code of  $b_2$  bits at the rear of the message packet [23,24,25,26,27]. Thus:

Overheads due to extra bits transmitted with each packet

$$F_{bo} = b = b_1 + b_2 \text{ bits}$$

Overheads due to transmission errors and acknowledgement delays

We shall first evaluate the overheads for the case of a continuous strategy, in which message blocks are transmitted continuously without waiting for acknowledgement and each unconfirmed message block is retransmitted. In case a reverse channel is used for acknowledgement  $AR=0$ , since acknowledgement overheads is the fixed cost of the reverse channel, which is independent of the packet size. If a reverse channel is not used then the acknowledgement overhead will be the time required to transmit those acknowledgement signals for which message blocks were in error. Since the acknowledgement signal is very short compared to the length of message block [23], that is:

$$AR \ll B+b$$

We can assume  $AR = 0$

In that case the network operational overheads per packet or block can be expressed as [25]:

$$k(B) = b + \frac{E(B+b)}{1-E(B+b)} (B+b) \text{ bits} \quad \text{-- (38)}$$

where  $E(B+b)$  represents the probability of a message block having at least one error on transmission over the channel.

Thus the network operational overhead factor for this case of continuous transmission strategy can be expressed as [25]:

$$F_{np} = \frac{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N Y_i \right\} \left[ b + \frac{E(B+b)}{1-E(B+b)} (B+b) \right]}{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N \alpha_i \right\} B} \quad \text{-- (39)}$$

where  $I_i$  = Integral number of packets in  
 $i$ th message

$\alpha_i$  = Fractional number,  $0 < \alpha_i < 1$

$Y_i$  = Indicator function which takes on values 0 or 1;  $Y_i = 0$ , only if  $K_i = 0$   
 $N$  = Number of messages transmitted  
 $B$  = message block size in bits

Now we shall evaluate overheads for the Second case of the continuous transmission strategy, in which message blocks are transmitted continuously without waiting for acknowledgement and transmitter retransmits the unconfirmed message block together with those message blocks that follow it. Thus when an error block is detected, the whole duration of acknowledgement time is a waste. For this type of the transmission strategy the overheads due to reacknowledgement and retransmission can be given as [23]:

$$P_{\text{rat}} = \sum_{i=1}^{\infty} \left( A + \frac{B+b}{R} \right) [E(B+b)]^i R \text{ bits} \quad \text{-- (40)}$$

where  $A = t_1 + t_2$

$t_1$  = Time required to detect an error block

$t_2$  = Time required to generate an acknowledgement signal

Thus the total network operational overheads per

message block can be written as:

$$k(B) = b + \frac{E(B+b)}{1-E(B+b)} (A R + B + b)$$

For this type of continuous transmission strategy the network operational overhead factor can be expressed as [26]:

$$P_{np} = \frac{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N Y_i \right\}}{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N \alpha_i \right\} B} \left\{ b + \frac{E(B+b)}{1-E(B+b)} (B+b+A R) \right\} \quad \text{--- (41)}$$

Overhead due to blank padding

The overhead due to blank padding for both the cases of continuous transmission strategies is the same as in the case of the Stop-and-wait strategy in [26]. Thus the total overhead factor for both the cases of continuous transmission strategies can be expressed respectively as:

$$P_{TCI} = \frac{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N Y_i \right\}}{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N \alpha_i \right\} B} \left\{ b + \frac{E(B+b)}{1-E(B+b)} (B+b) \right\} + \frac{B}{21} \quad \text{--- (42)}$$

and

$$\begin{aligned}
 F_{TCII} = & \frac{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N Y_i \right\}}{\left\{ \frac{1}{N} \sum_{i=1}^N I_i + \frac{1}{N} \sum_{i=1}^N \alpha_i \right\} B} \left\{ b + \frac{E(B+b)}{1-E(B+b)} (B+b+A R) \right\} \\
 & + \frac{B}{2l} \quad \text{--- (43)}
 \end{aligned}$$

On the basis that the message lengths are long enough, i.e.,  $M_1 \gg \alpha_1 B$ , it is assumed that:

$$\frac{1}{N} \sum_{i=1}^N Y_i = \frac{1}{N} \sum_{i=1}^N \alpha_i + \frac{1}{2}$$

Using the above equation and the expression for  $l$ , the expression for  $F_{TCI}$  and  $F_{TCII}$  reduces to:

$$F_{TCI} = \frac{2l+B}{2lB} \left\{ b + \frac{E(B+b)}{1-E(B+b)} (B+b) \right\} + \frac{B}{2l} \quad \text{--- (44)}$$

and

$$F_{TCII} = \frac{2l+B}{2lB} \left\{ b + \frac{E(B+b)}{1-E(B+b)} (B+b+A R) \right\} + \frac{B}{2l} \quad \text{--- (45)}$$

In order to find the optimal packet (Block) size, we need to find  $F'_{TCI} = 0$  and  $F'_{TCII} = 0$  when  $F''_{TCI} > 0$  and  $F''_{TCII} > 0$  respectively. Which we shall obtain in the next sections for particular channel characteristics.

### 3.6 OPTIMAL MESSAGE BLOCK SIZE ANALYSIS

In this section we shall obtain the optimal message block size under both the Stop-and-wait transmission strategy and the continuous transmission strategies for different channel characteristics.

#### 3.6.1 RANDOM ERROR CHANNEL

For a random error channel, the probability that a message block of size  $(B+b)$  bits will have at least one error is given by:

$$E(B+b) = 1 - (1-C)^{B+b} \quad \text{-- (46)}$$

#### Stop-and-wait strategy

The optimal block length for the case of the Stop-and-wait transmission strategy has already been derived in [26]. So it would not be considered here again.

Continuous transmission strategies

We shall obtain the optimal block length for two cases of continuous transmission strategies, as follows:

On substituting  $E(B+b) = 1 - (1-C)^{B+b}$  in (44) and (45), we obtain expressions for the total overhead factor as defined in [26] for two continuous transmission strategies, in case of the random error channel, respectively. This can be expressed as:

$$F_{TCI} = \frac{2l+B}{2lB} \left\{ b \frac{1 - (1-C)^{B+b}}{B+b} \right\} + \frac{B}{2l} \quad \text{--- (47)}$$

and

$$F_{TCII} = \frac{2l+B}{2lB} \left\{ b + \frac{1 - (1-C)^{B+b}}{B+b} (B+b+A R) \right\} + \frac{B}{2l} \quad \text{--- (48)}$$

Furthermore, to determine the optimal block lengths, we evaluate  $F'_{TCI} = 0$  and  $F'_{TCII} = 0$  when  $F''_{TCI} < 0$  and  $F''_{TCII} < 0$  as follows:

$$F_{TCI} = xB^3 + yB^2 + zB + w = 0 \quad \text{-- (49)}$$

Where  $x = \ln(1-C)$

$$y = (b+2l)\ln(1-C) - 1$$

$$z = 2lb \cdot \ln(1-C)$$

$$w = 2lb$$

and

$$F_{TCII} = xB^3 + yB^2 + zB + w = 0 \quad \text{-- (50)}$$

where  $x = \ln(1-C)$

$$y = (b+A_1R+2l)\ln(1-C) - 1$$

$$z = (2lb+2lA_1R)\ln(1-C)$$

$$w = 2lb+2lA_1R$$

Also

$$F_{TCI} = \frac{B[-3B^2 \ln(1-C) - 2B(b+2l)\ln(1-C) + 2B - 2lb \ln(1-C)] + [B \ln(1-C) + 1][B^2 \ln(1-C) + B(b+2l)\ln(1-C) - B + B(2lb \ln(1-C))] + 2lb}{21B^2(1-C) \quad B+b}$$

Which on evaluation turns out to be a positive quantity.

$$\begin{aligned}
 F_{TCII}^n &= B \left[ \frac{-3B \ln(1-C) - 2B[(b+A_1R+2l) \ln(1-C)] + 2B}{B+b} \right. \\
 &\quad - (2lb+2lA_1R) \ln(1-C) - 2lA_1R(1-C) \ln(1-C) \\
 &\quad + [B \ln(1-C) + 2] [B \ln(1-C) + B(b+2l+A_1R) \\
 &\quad \left. \frac{\ln(1-C) - B + B(2lb+2lA_1R) \ln(1-C) + 2lb+2lA_1R}{21B(1-C)} \right]
 \end{aligned}$$

Which on evaluation also turns out to be a +ve quantity for values of B ranging between 200 < B < 4000 bits.

Consequently, the numerically obtained solutions of equations (49) and (50) -using Newton Raphson's method [29]- represent the required optimal block or packet sizes which minimise the total overhead factor in a random error channel, while using continuous transmission strategies respectively.

Fig.6 and fig.8 illustrate the behaviour of the solutions of equations (49) and (50) for different values of mean message length and acknowledgement overhead for two continuous transmission strategies respectively.

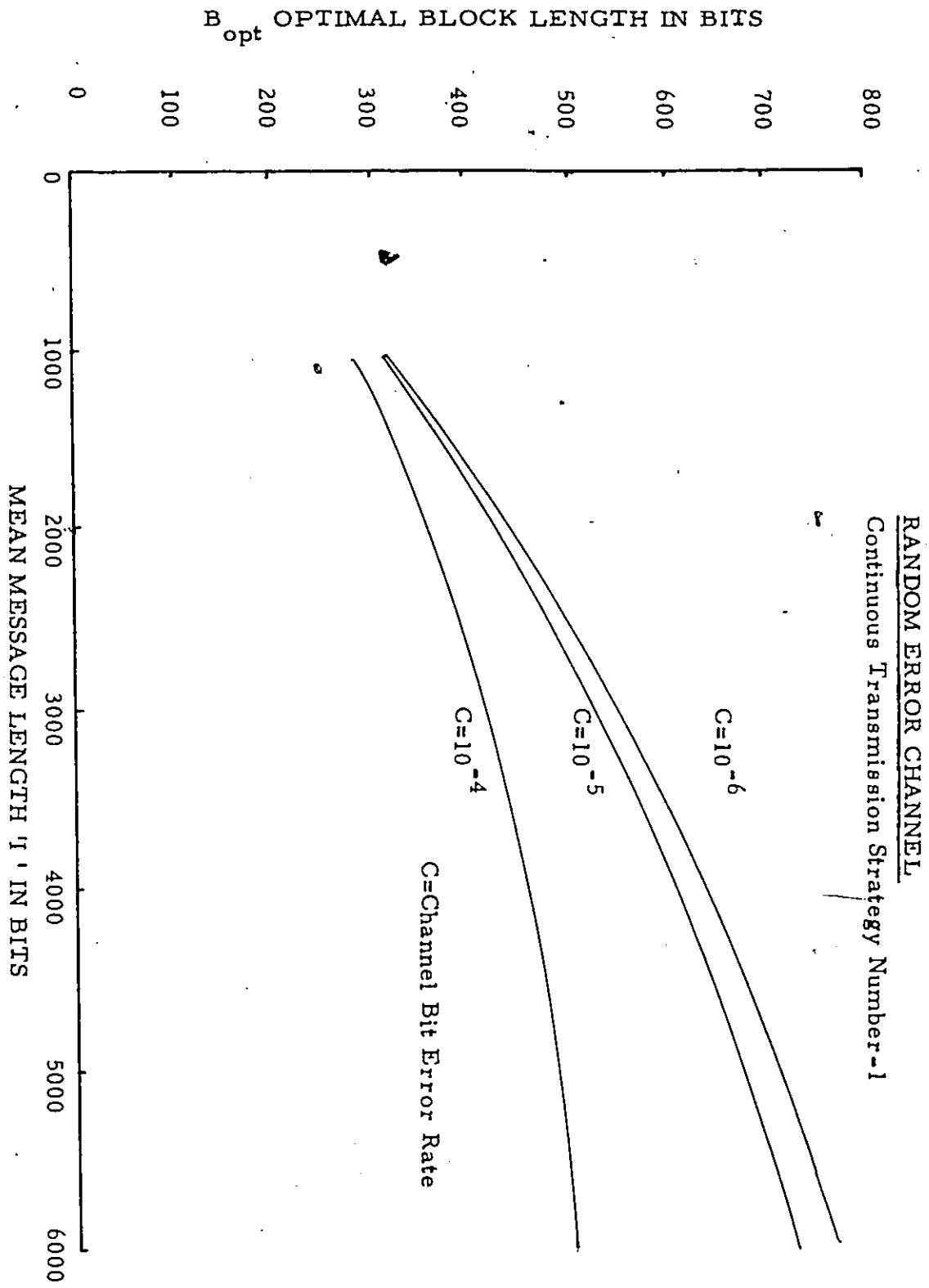


FIG. 6  
MEAN MESSAGE LENGTH 'M' IN BITS

$F_{TCI}$  TOTAL OVERHEAD FACTOR

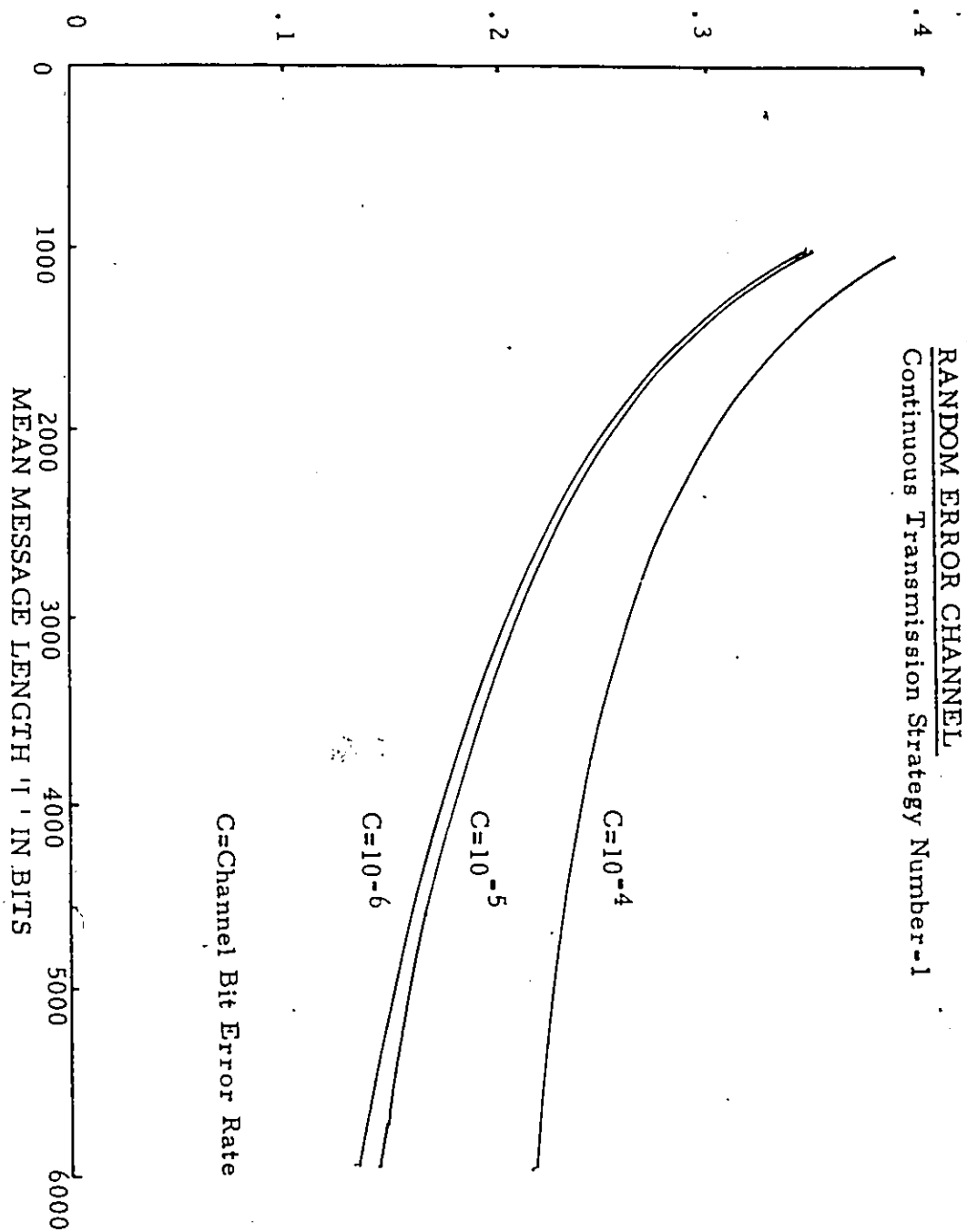
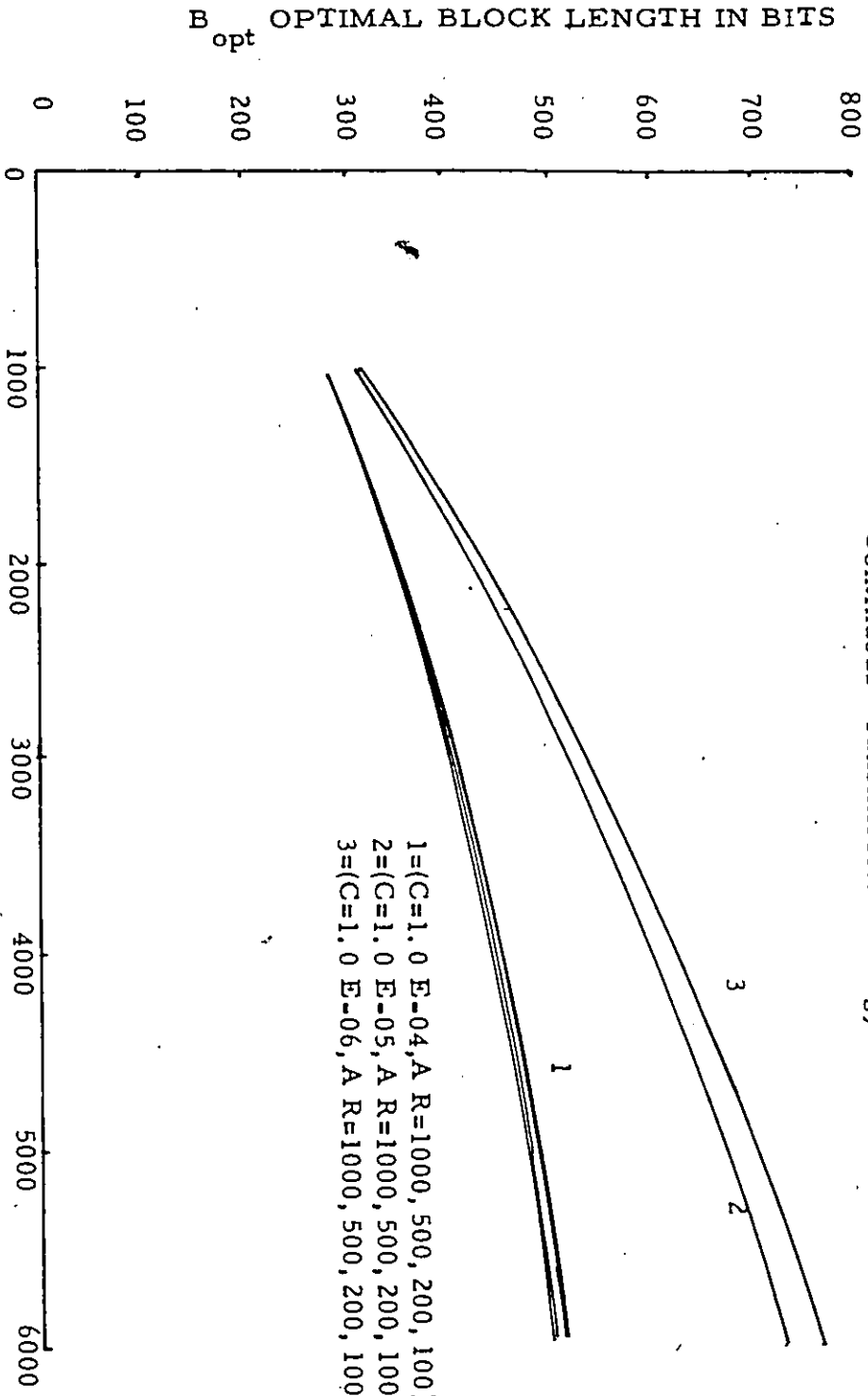


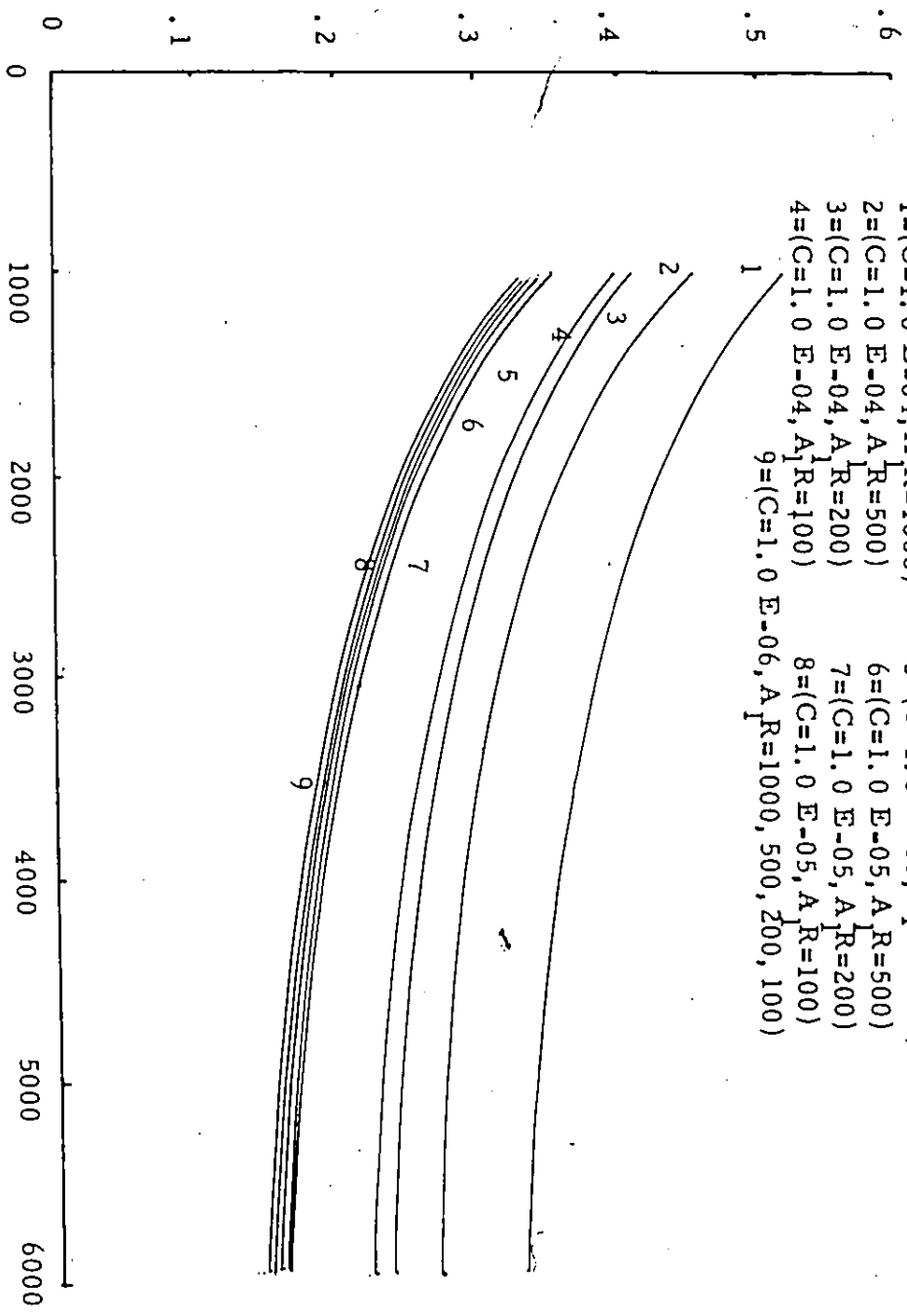
FIG. 7

RANDOM ERROR CHANNEL  
Continuous Transmission Strategy Number-2



MEAN MESSAGE LENGTH  $l$  IN BITS  
FIG. 8

$F_{TCH}$  TOTAL OVERHEAD FACTOR



- 1=(C=1.0 E-04, A<sub>1</sub>R=1000)
- 2=(C=1.0 E-04, A<sub>1</sub>R=500)
- 3=(C=1.0 E-04, A<sub>1</sub>R=200)
- 4=(C=1.0 E-04, A<sub>1</sub>R=100)
- 5=(C=1.0 E-05, A<sub>1</sub>R=1000)
- 6=(C=1.0 E-05, A<sub>1</sub>R=500)
- 7=(C=1.0 E-05, A<sub>1</sub>R=200)
- 8=(C=1.0 E-05, A<sub>1</sub>R=100)
- 9=(C=1.0 E-06, A<sub>1</sub>R=1000, 500, 200, 100)

RANDOM ERROR CHANNEL  
 Continuous Transmission Strategy Number-2

MEAN MESSAGE LENGTH 'l' IN BITS  
 FIG. 9

Similarly fig.7 and fig.9 demonstrate the behaviour of the total overhead factor, calculated using the optimal value of block size, i.e.,  $B_{opt}$  for different mean message length and acknowledgement overhead respectively.

### 3.6.2 TELEPHONE CHANNELS BY BALKOVIC ET AL.

For the telephone channels reported by Balkovic et al [3], the probability of a packet having at least one error can be expressed as [4]:

$$E(B+b) = a(B+b)^c \quad \text{--(51)}$$

where  $a$  and  $c$  are parameters, the values of which depend on channel data rates ( $R$  bits/sec). From the curves given in [4], we obtain the following values for the above mentioned parameters:

$$\begin{aligned} R = 1200 \text{ b/s} & \quad a = .0000139 \quad c = 0.80626 \\ R = 2000 \text{ b/s} & \quad a = .0000088 \quad c = 0.81103 \\ R = 3000 \text{ b/s} & \quad a = .0000370 \quad c = 0.72695 \end{aligned} \quad \text{--(52)}$$

Stop-and-wait transmission strategy

We obtain an expression for the total overhead factor in the case of the Stop-and-wait strategy by substituting equation (51) into (34)

$$F_{TS} = \frac{2l+B}{2lB} \left[ m + \frac{a(B+b)^c}{1-a(B+b)} (m+B) \right] + \frac{B}{2l} \quad \text{-- (53)}$$

In order to minimise  $F_{TS}$  the total overhead factor, we differentiate (53) with respect to B and equate to zero:

$$\begin{aligned} \frac{dF_{TS}}{dB} = & \frac{2lB(1-a(B+b))}{2} \left[ m + 2B + 2lBac(B+b)^{c-1} + 2la(B+b)^c \right] \\ & - \left[ \frac{2lm+mB+B}{2} + 2lBa(B+b) \right] \left[ 2l(1-a(B+b)) - 2lcB \right. \\ & \left. a(B+b)^{c-1} \right] \\ & \frac{B}{2lB(1-a(B+b))^2} = 0 \end{aligned} \quad \text{-- (54)}$$

Futhermore, we evaluate the second order partial derivative of  $F_{TS}$ ,

$$\begin{aligned} \frac{d^2F_{TS}}{dB^2} = & \frac{2lB(1-a(B+b))}{2} \left[ 4lB + 4l^2Bac(B+b)^{c-1} + 4l^2a(B+b)^c \right. \\ & - 4lBac(B+b)^{c-1} - 8lBa(B+b)^{c-2} + 4l^2mac(B+b)^{c-1} \\ & + 2lBca(B+b)^{c-1} + 4lBa(B+b)^{c-2} + 4lmBa(c-1)c(B+b)^{c-2} \\ & \left. + 4lmac(B+b)^{c-1} + 2lmBac(c-1)(B+b)^{c-2} + 4lmBca \right] \end{aligned}$$

$$\begin{aligned}
 & + (B+b)^{c-1} + 21B^3 ca (B+b)^{c-2} + 61B^2 ca (B+b)^{c-1} ] \\
 & - [ [ 41B^2 (1-a(B+b))^2 (21(1-a(B+b))^c - 21Bca(B+b)^2) ] \\
 & [ 21B^2 + 41Ba(B+b)^2 - 21Ba(B+b)^2 - 41B^2 a(B+b)^2 \\
 & - 41Ba + 41Ba(B+b)^{c-1} + 21Ba(B+b)^2 + 21B^2 a(B+b)^2 \\
 & + 41Ba(B+b)^{c-1} + 21B^2 ca(B+b)^{c-1} + 21B^3 ca(B+b)^{c-1} ] \\
 & \hline
 & [ 21B^4 (1-a(B+b))^4 ]
 \end{aligned}$$

It is obvious that the expression for  $F_{ms}''$ , in the case of the Stop-and-wait strategy over Balkovic et al telephone channels, is a complex function. Thus we need to evaluate the function  $F_{ms}''$ . We find that for the ranges of interest for the variables used in the above expression, is a positive quantity. Therefore the numerically obtained solution of (54) represents the required B which minimises the total overhead factor in this case.

In fig.10,  $B_{opt}$  the numerically obtained solution of equation (54) is plotted as a function of the mean message length for various values of the acknowledgement overhead (AR) and for different data rates over Balkovic et al channels.

P

BALKOVIC TELEPHONE CHANNELS  
Stop-And-Wait Transmission Strategy

- 1, 4, 7, 10=(A=.139 E-04, C=.80626)
- 2, 5, 8, 11=(A=.88 E-05, C=.81103)
- 3, 6, 9, 12=(A=.37 E-06, C=.72995)

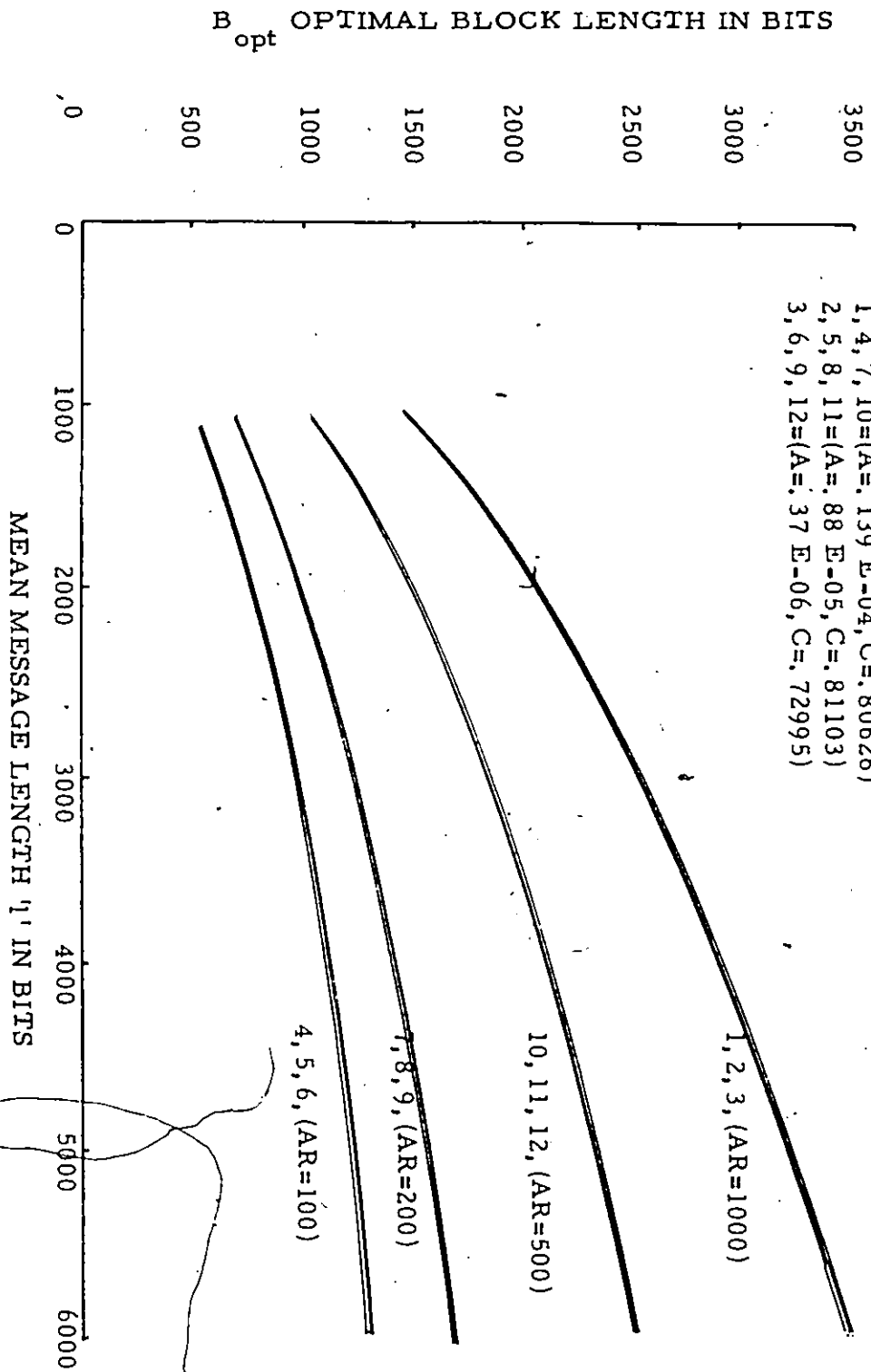


FIG. 10

BALKOVIC TELEPHONE CHANNELS  
Stop-And-Wait Transmission Strategy

- 1, 4, 7, 10=(A=, 139 E-04, C=, 80626)
- 2, 5, 8, 11=(A=, 88 E-05, C=, 81103)
- 3, 6, 9, 12=(A=, 37 E-06, C=, 72695)

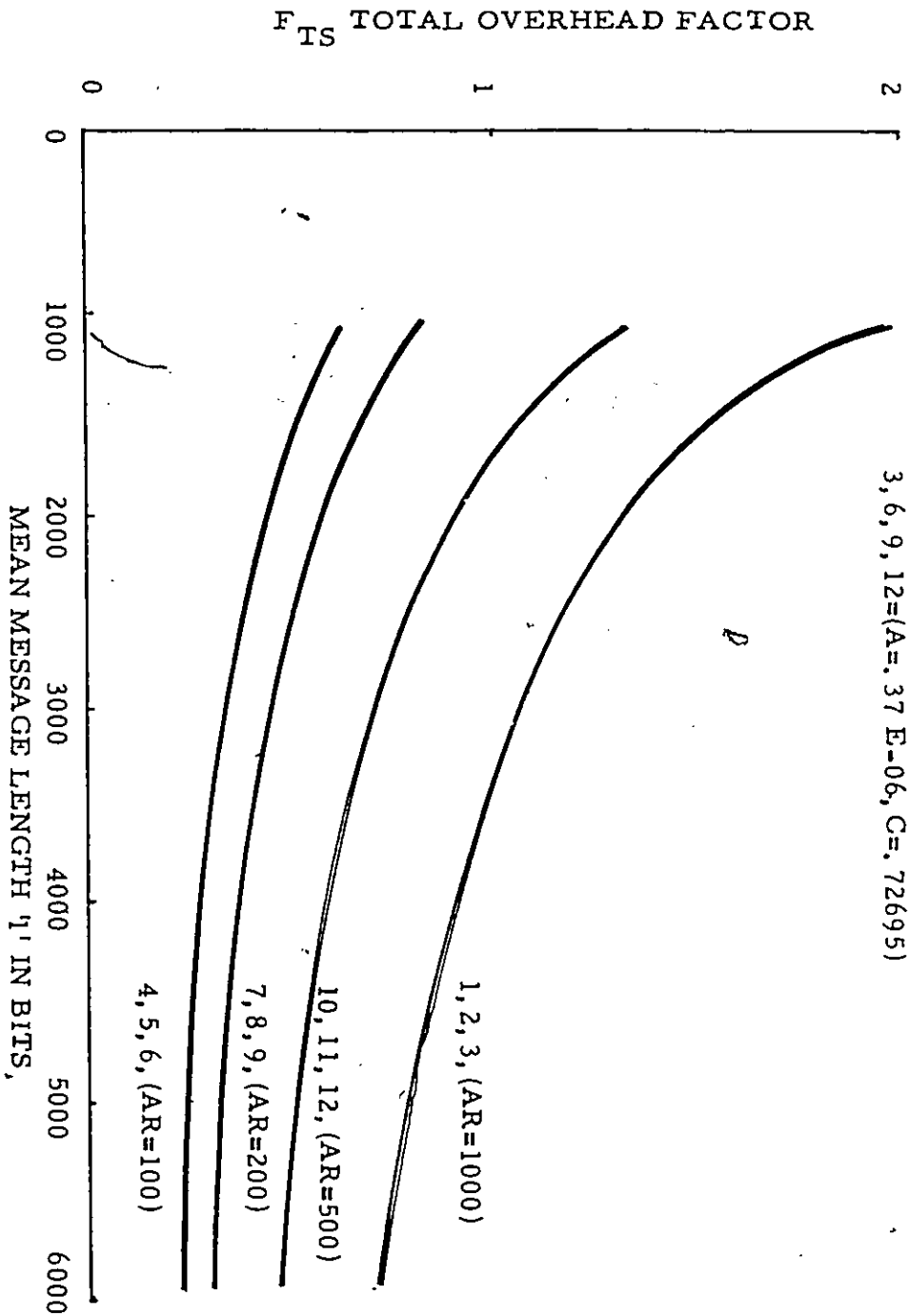


FIG. 11


Fig. 11 shows the plot of the total overhead factor - calculated using the optimal value of the packet length ( $B_{opt}$ ) - against mean message length for various values of the acknowledgement overhead (AR)

### Continuous transmission strategies

As mentioned earlier, there are two cases to be considered here.

In the first the case, if we compare (44), the expression for the total overhead factor in the case of continuous transmission strategy, with (34), the total overhead factor in the case of the Stop-and-wait transmission strategy, the two equations are exactly similar if the variable  $m$  has value equal to  $b$  (The block overhead). So the analysis for the Stop-and-wait strategy is applicable to the first case of the continuous transmission strategy.

In the second case of the continuous transmission strategy, the expression for the total overhead factor for a Balkovic et al telephone channel can be obtained by substituting (51) into (45):



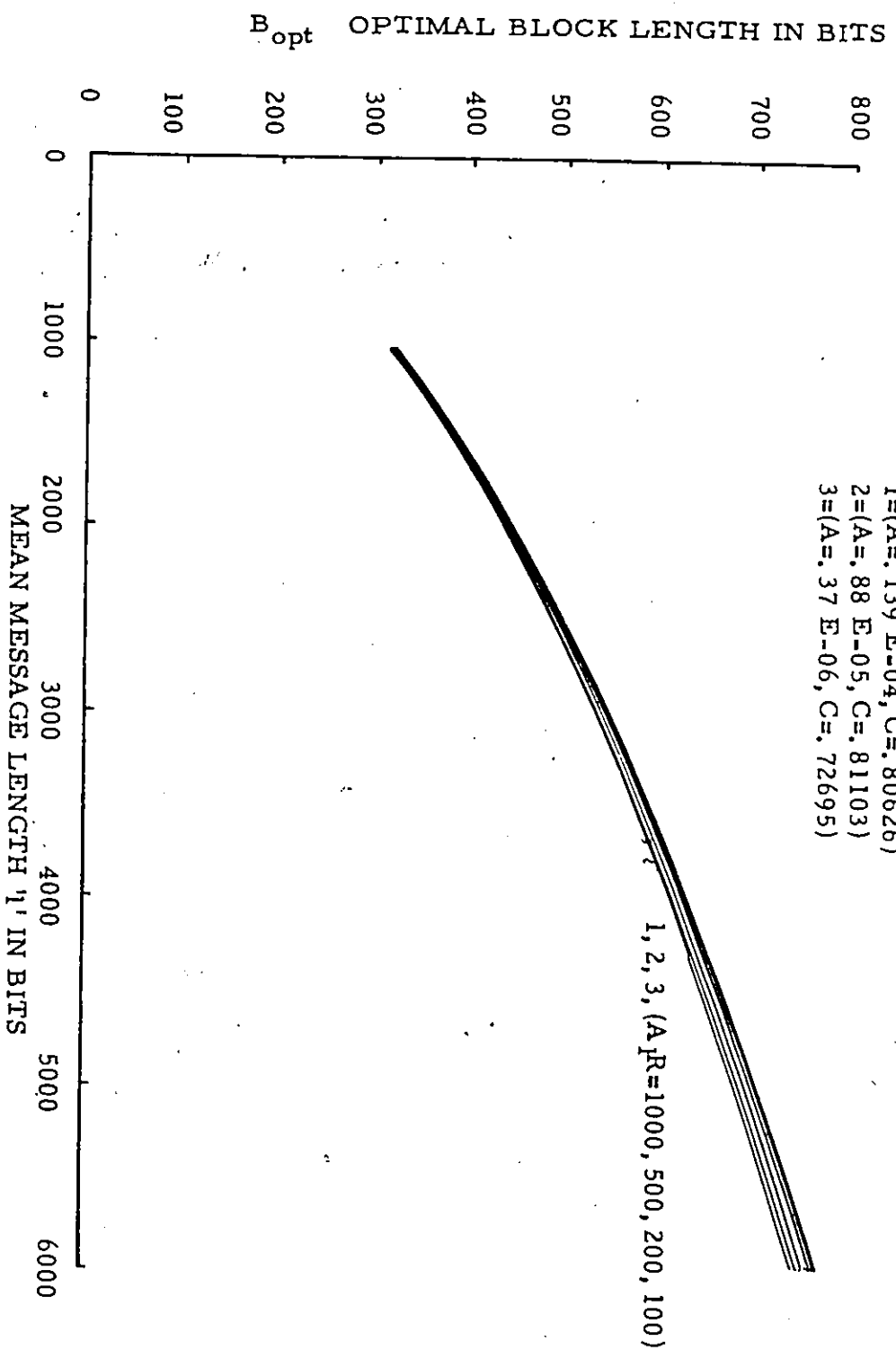


$$\begin{aligned}
 & \frac{ac(B+b)^{c-1}}{2} + 41B - 21B \frac{ac(B+b)^2}{c-1} - 41Ba(B+b)^{c-1} \\
 & - 41 \frac{A_1 R ac(B+b)^2}{c-1} + 41 \frac{acb(B+b)^2}{c-2} + 81 \frac{A_1 R}{c-2} \\
 & a \frac{c(B+b)^{c-1}}{c-1} + 41 \frac{Bbac(c-1)(B+b)^{c-2}}{2} + 41 \frac{bac}{c-2} \\
 & (B+b)^3 + 21acB \frac{b(c-1)(B+b)^{c-2}}{2} + 41acBb(B+b)^{c-1} \\
 & + 21B \frac{ac(c-1)(B+b)^c}{c} + 61B \frac{ac(B+b)^{c-1}}{c} ] - [ [ 41B \\
 & (1-a(B+b))^2 ] [ 21 - 21Bac \frac{(B+b)^2}{c-1} - 21a(B+b)^{c-1} ] \\
 & [ 41 \frac{BA_1 R ac(B+b)^2}{c-1} + 41 \frac{acB(B+b)^2}{c} + 21 \frac{A_1 R}{c} \\
 & B \frac{ac(B+b)^c}{c} + 21B \frac{-21B a(B+b)^2}{c} - 41 \frac{b-41 A_1 R}{2c} \\
 & a(B+b)^{c-1} + 41 \frac{ab(B+b)^2}{c-1} + 41 \frac{A_1 Ra(B+b)^3}{c-1} + 41 \frac{Bb}{c-1} \\
 & ac(B+b)^2 + 21acB \frac{b(B+b)^{c-1}}{2} + 21B \frac{ac(B+b)^{c-1}}{2} ] ] \\
 & \hline
 & [ 21B(1-a(B+b))^c ]^4 \qquad \qquad \qquad -- (57)
 \end{aligned}$$

We evaluate (57), in order to check the sign of the magnitude of  $F''_{\pi c II}$ . We find that for the ranges of interest of the variables used in equation (57) i.e.  $200 < (B+b) < 4000$ ,  $100 < A_1 R < 200$ ,  $1000 < l < 6000$  and  $c$  and  $a$  as defined by equation (52),  $F''_{\pi c II}$  is a positive quantity. Thus the numerically obtained solution of (56) using the Newton-Ralphson method [29] represents the required  $B_{opt}$ .

BALKOVIC TELEPHONE CHANNELS  
Continuous Transmission Strategy Number -2

- 1=(A=.139 E-04, C=.80626)
- 2=(A=.88 E-05, C=.81103)
- 3=(A=.37 E-06, C=.72695)



1, 2, 3, (A, R=1000, 500, 200, 100)

FIG. 12

BALKOVIC TELEPHONE CHANNELS  
Continuous Transmission Strategy Number-2

- 1=(A=, 139 E-04, C=, 80626)
- 2=(A=, 88 E-05, C=, 81103)
- 3=(A=, 37 E-06, C=, 72695)

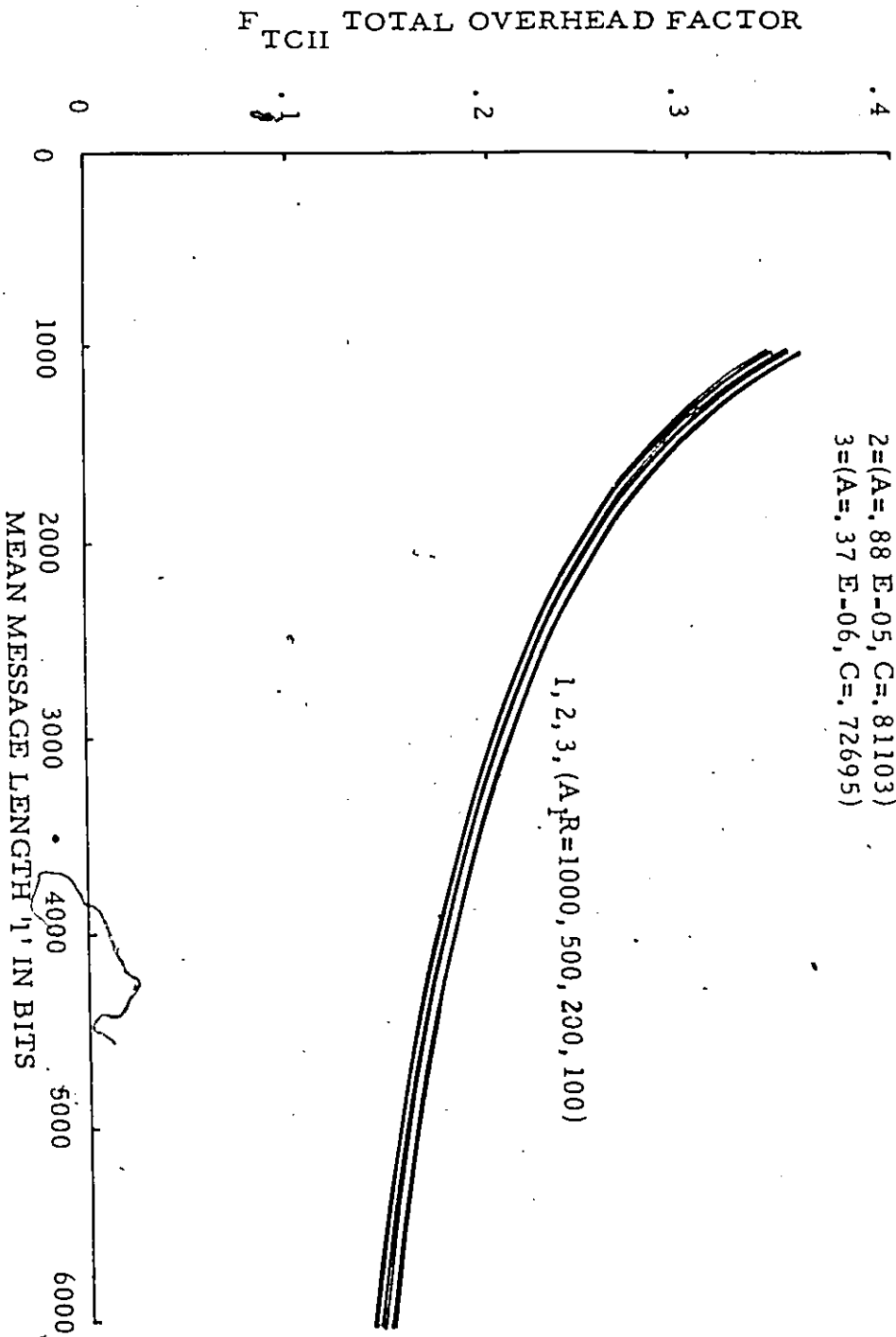


FIG. 13

Fig. 12 shows a plot of  $B_{opt}$  as a function of mean message length, for various values of the acknowledgement overhead and for three data rates over Balkovic et al telephone channels.

Fig.13 shows the plot of the total overhead factor, calculated as a function of the mean message length for various data rates over Balkovic et al telephone channels. It is found that curves are insensitive to changes in the reacknowledgement and retransmission overheads ( $A_1 R$ ).

3.6.3 PEATFIELD ET AL'S DNPL MICROWAVE CHANNEL

From the experimental measurement on Daresbury Nuclear Physics Laboratory (DNPL) microwave link as reported in [5], the actual probability of a packet having at least one error is found to be [7]:

$$\log_{10} \{E(B+b)\} = a_0 + a_1 \log_{10}(B+b)$$

where  $a_0 = -6.5$  and  $a_1 = 1.07$

Which can also be expressed as:

$$E(B+b) = (10)^{a_0 + a_1 \log_{10}(B+b)} \quad \text{-- (58)}$$

Stop-and wait transmission strategy

In this case the general expression for the total overhead factor as derived in [27], becomes:

$$F_{TS} = \frac{2l+B}{2lB} \left\{ m + \frac{10^A}{1-10^A} (m+B) \right\} + \frac{B}{2l} \quad \text{-- (59)}$$

where  $A = a_0 + a_1 \log_{10}(B+b)$  and  $DA = a_1 \log_{10}(e) / (B+b)$

As before, the minimum solution of equation (59) is found by differentiating it with respect to B, equating to zero, then solving the resultant equation - which is expressed below - numerically using the Newton-Ralphson method [29]. Thus:

$$F_{TS} = \left\{ \left[ \frac{2}{2lB} - \frac{4lm+4l}{2l^2B^2} + \frac{A}{2l^2B} + \frac{A}{2l^2B} \right] - \frac{4l}{2l^2B} + \frac{4l}{2l^2B} \right. \\ \left. + 2lmB - 2lB + 2lB \right\} \cdot 10 \ln(10) DA \} = 0 \quad \text{-- (60)}$$

Here again, we have to check the sign of the second order partial derivative of equation (59), in order to ascertain that the numerically obtained solution of equation

(60) represents the actual minimum solution. The second order partial derivative of equation (59) is as shown below:

$$\begin{aligned}
 F_{TS}'' &= \left[ \frac{21B(1-10^{-A})}{2A} \right] \left\{ 41B + 41 \frac{10^{-2A}}{A} + 41 \frac{B10^{-2A}}{A} \ln(10) \frac{DA}{2} \right. \\
 &\quad + 41 \frac{m10^{-A}}{A} \ln(10) \frac{DA}{2} - \left[ 81 \frac{B^2}{2} + 41 \frac{m^2}{2} + 41 \frac{m-41B+61B}{2} \right] \\
 &\quad 10 \ln(10) \frac{DA}{A} - \left[ (41 \frac{B}{2} + 41 \frac{mB+21mB}{A} - 21B + 21B) \right. \\
 &\quad \left. 10 (\ln(10) \frac{DA}{A} - 10 \ln(10) \frac{DA(B+b)}{A}) \right] \\
 &\quad - \left[ \left( 41B(1-10^{-A}) \right) \left( 21 - 2L10^{-A} - 21B10^{-A} \ln(10) \frac{DA}{2} \right) \left( 21B \right. \right. \\
 &\quad \left. \left. - 41m + 41 \frac{B10^{-3A}}{2} + 41 \frac{m10^{-A}}{A} \right) - (41 \frac{B}{2} + 41 \frac{mB+21mB}{2} \right. \\
 &\quad \left. \left. - 21B + 21B \right) 10 \ln(10) \frac{DA}{A} \right] \\
 &\quad \left. \right\} \\
 &\quad \text{-----} \\
 &\quad \left[ 21B(1-10^{-A}) \right]^4 \qquad \qquad \qquad \text{-- (61)}
 \end{aligned}$$

Obviously function (61) is complex. Thus we need to compute it. It is found that for the ranges of interest for the variables used in function (61), i.e.,  $1000 < l < 6000$ ,  $200 < B < 4000$  and  $100 < m < 200$ , the second order partial derivative of function (60) is a positive quantity. Thus the numerically obtained solution of function (60) is the required optimal packet size ( $B_{opt}$ ), which minimises the total overhead factor while using the Stop-and-wait transmission strategy.

DNPL-MICROWAVE CHANNEL  
Stop-And-Wait Transmission Strategy

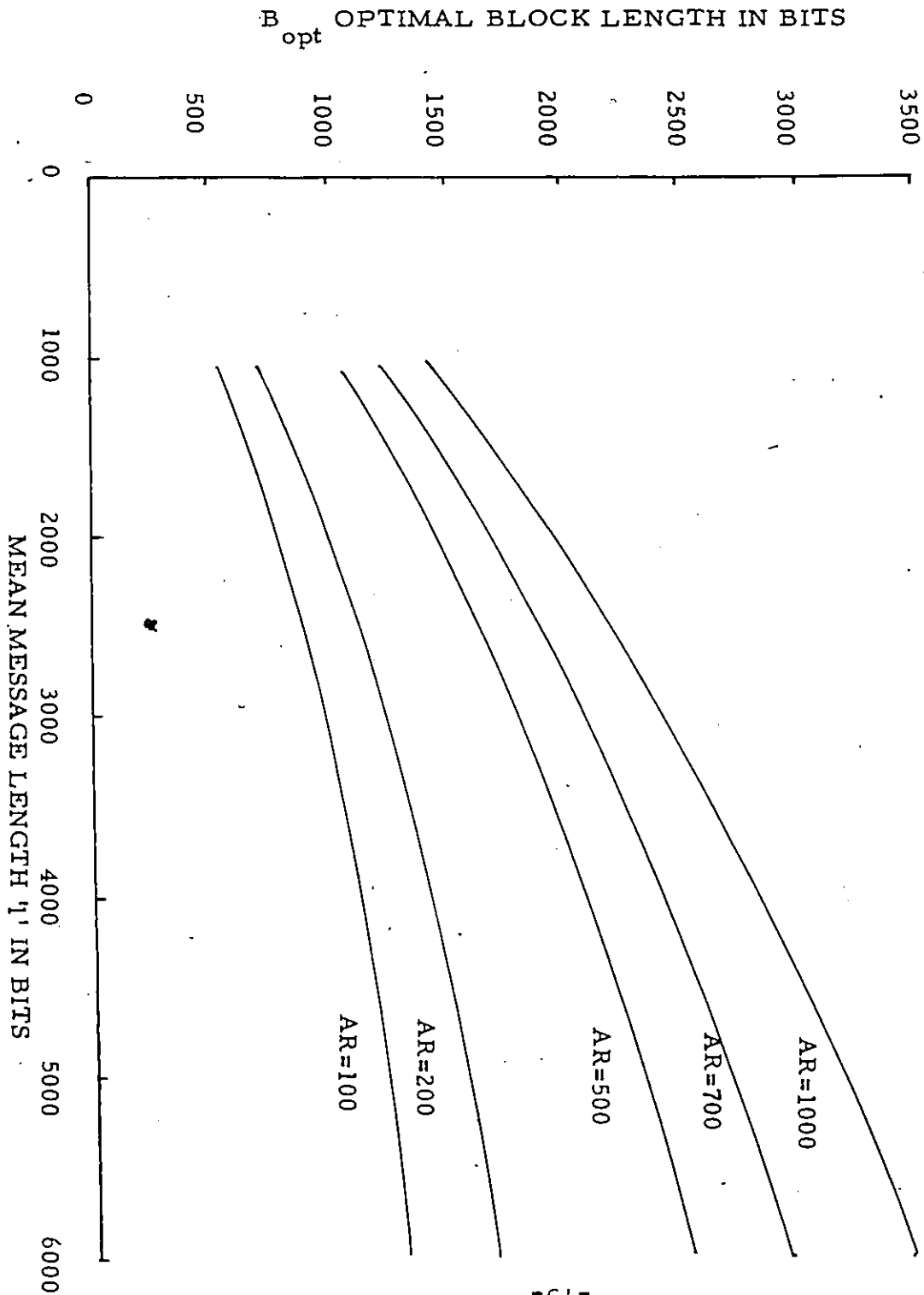


FIG. 14

DNPL-MICROWAVE CHANNEL  
Stop-And-Wait Transmission Strategy

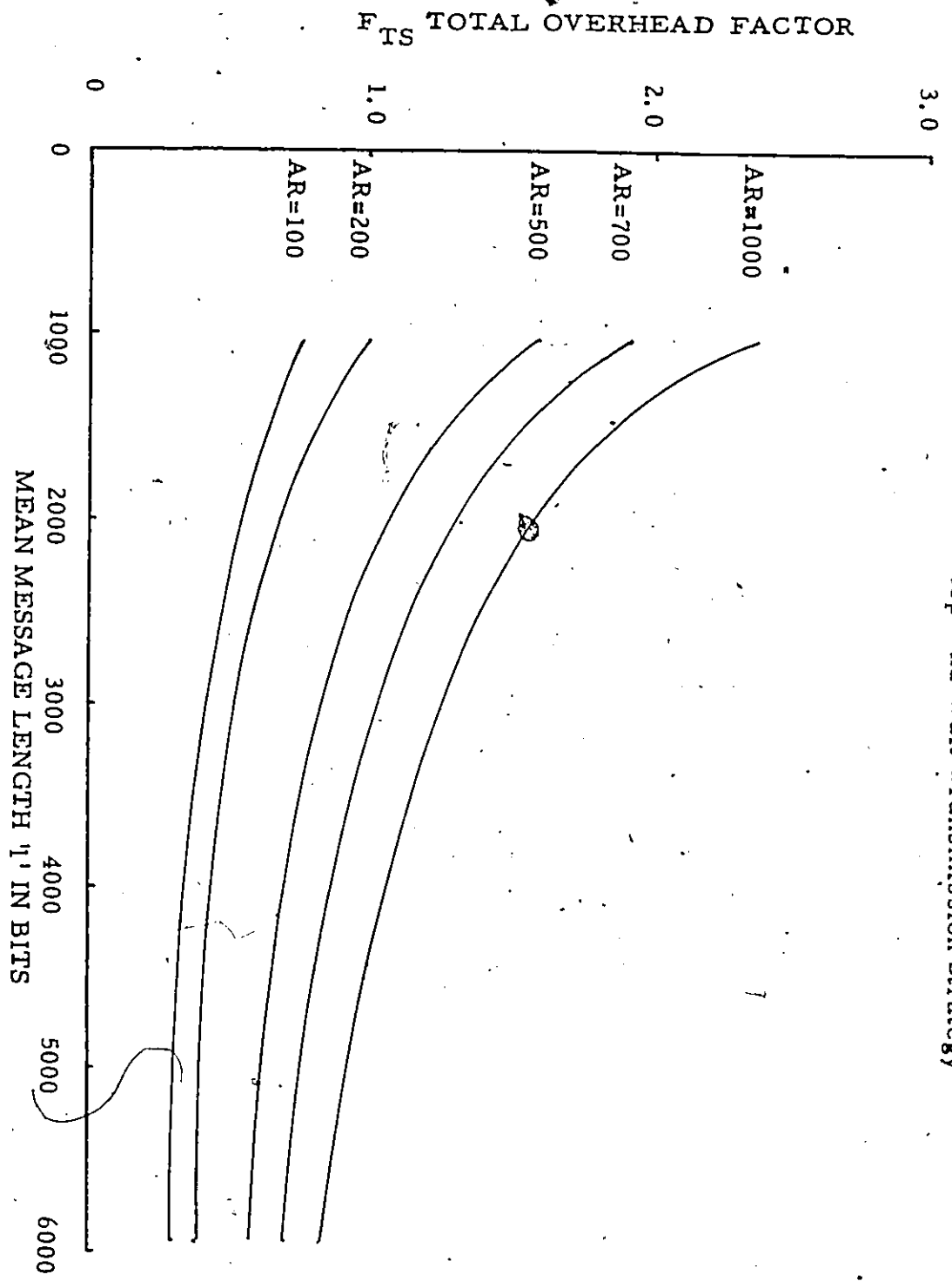


FIG. 15

Fig.14 shows the numerically obtained result of function (60), namely  $B_{opt}$ , as a function of the mean message length  $(l)$  for the various values of the acknowledgement overhead  $(AR)$ .

Fig.15 shows a plot of the total overhead factor - using the optimal packet size - as a function of the mean message length for the various values of the acknowledgement overhead  $(AR)$ .

#### Continuous transmission strategy

As before, we shall be considering the second type of continuous transmission strategy only, since we have already mentioned that the analysis derived for the case of the Stop-and-wait strategy is also applicable to the first type of the continuous transmission strategy.

The expression for the total overhead factor in the second case of the continuous transmission strategies can be arrived at by substituting equation (58) in (45). Thus:

$$F_{TCII} = \frac{21+B}{21B} \left\{ b + \frac{10^A}{1-10^A} (B+b+A_1 R) \right\} + \frac{B}{21} \quad \text{--- (62)}$$

Next we differentiate  $F_{TCII}$  with respect to B and equate to zero. Thus:

$$\begin{aligned} F_{TCII}' &= \left\{ [21B(1-10^A)] \left[ \frac{10^A}{A} (b+2B+2110^A + A_1 R 10^A + (21A_1 R + 21B+BA_1 R) 10^A \ln(10) DA) \right] - [21-2110^A - 21B10^A \ln(A) DA] \left[ \frac{10^A}{A} (21b+bB+B + 21A_1 R 10^A + 21B10^A + BA_1 R 10^A) \right] \right\} \\ &\quad \text{-----} = 0 \\ &\quad [21B(1-10^A)]^2 \quad \text{--- (63)} \end{aligned}$$

where  $A = a + a \log_{10}(B+b)$  and  $DA = a \log_{10}(e) / (B+b)$

In order to check for the existence of the minimum solution of equation (63), we obtain the second order partial derivative of equation (63) with respect to B and check the sign of the magnitude of this second order partial derivative. Thus:

$$\begin{aligned} F_{TCII}'' &= [21B(1-10^A)]^2 \left[ 41B-41B10^A + (81 B+81 b+41BA_1 R + 41bB+81B) 10^A \ln(10) DA + 81 A_1 R 10^A \ln(10) DA + (41 A_1 RB+41 B + 21B A_1 R+41 bB+21bB + 21B) 10^A \right] \end{aligned}$$

$$\begin{aligned}
 & \frac{(\ln(10)DA)^2}{2} - \left[ \frac{(2lB - 2lB^2) \ln(10)DA}{2} - \frac{2lB^2 \ln(10)DA}{2} \right] \\
 & \frac{(2lB - 4l b)^2}{2} - \frac{(4l A_1 R - 4l b)^2}{2} + \frac{4l A_1 R^2}{2} + \frac{4l A_1 R b}{2} + \frac{4l B^2}{2} + \frac{4l B^3}{2} \\
 & \frac{4l A_1 R B}{2} + \frac{4l B^2}{2} + \frac{4l A_1 R^2}{2} + \frac{4l b^2}{2} + \frac{4l B^2}{2} + \frac{4l B^3}{2} \\
 & \frac{\ln(10)DA}{2} \\
 & \hline
 & [2lB(1 - 10^{-A})]^4 \quad \text{--- (64)}
 \end{aligned}$$

On computing function (64), it is found that  $F^*$  is positive for all values of the variables  $B, l, A_1 R$  and  $b$  lying in the ranges of interest, e.g.,  $200 < B < 4000$ ,  $1000 < l < 6000$ ,  $100 < A_1 R < 200$  and  $b = 50$ . Thus the numerically obtained solution of equation (63) represents the required  $B_{opt}$ .

Fig. 17 shows a plot of the optimal packet size as a function of the mean message length, for various values of the reacknowledgement and retransmission overheads ( $A_1 R$ ). It is obvious from the plot that  $B_{opt}$  is almost insensitive to changes in  $A_1 R$ .

Fig. 18 shows the result of the total overhead factor ( $F_{MCH}$ ) for various values of the mean message length at different  $B_{opt}$ . Here also  $F_{MCH}$  seemed to be insensitive to changes in  $A_1 R$ .

DNPL-MICROWAVE CHANNEL  
Continuous Transmission Strategy Number-2

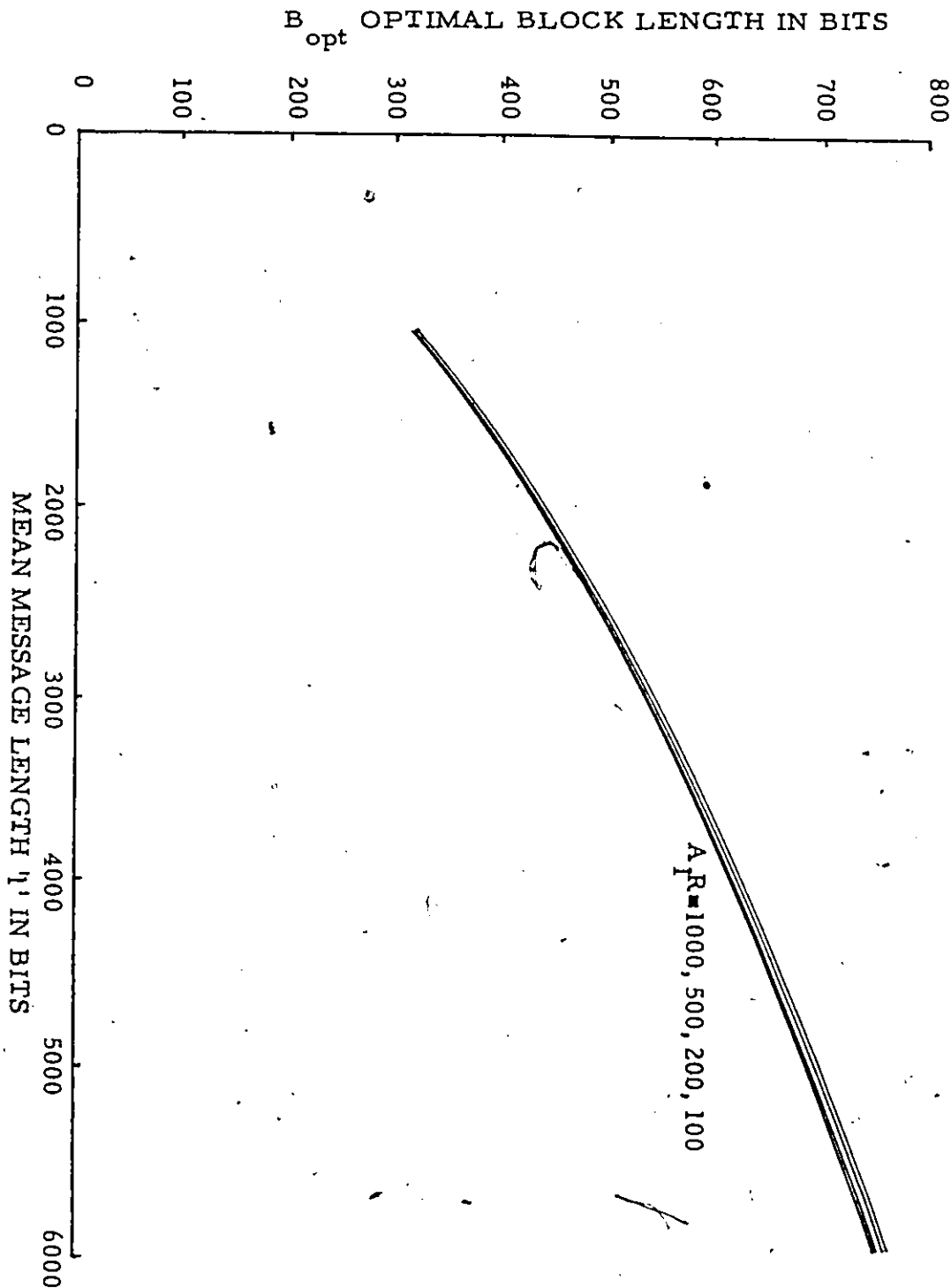


FIG. 16

DNPL-MICROWAVE CHANNEL  
Continuous Transmission Strategy Number-2

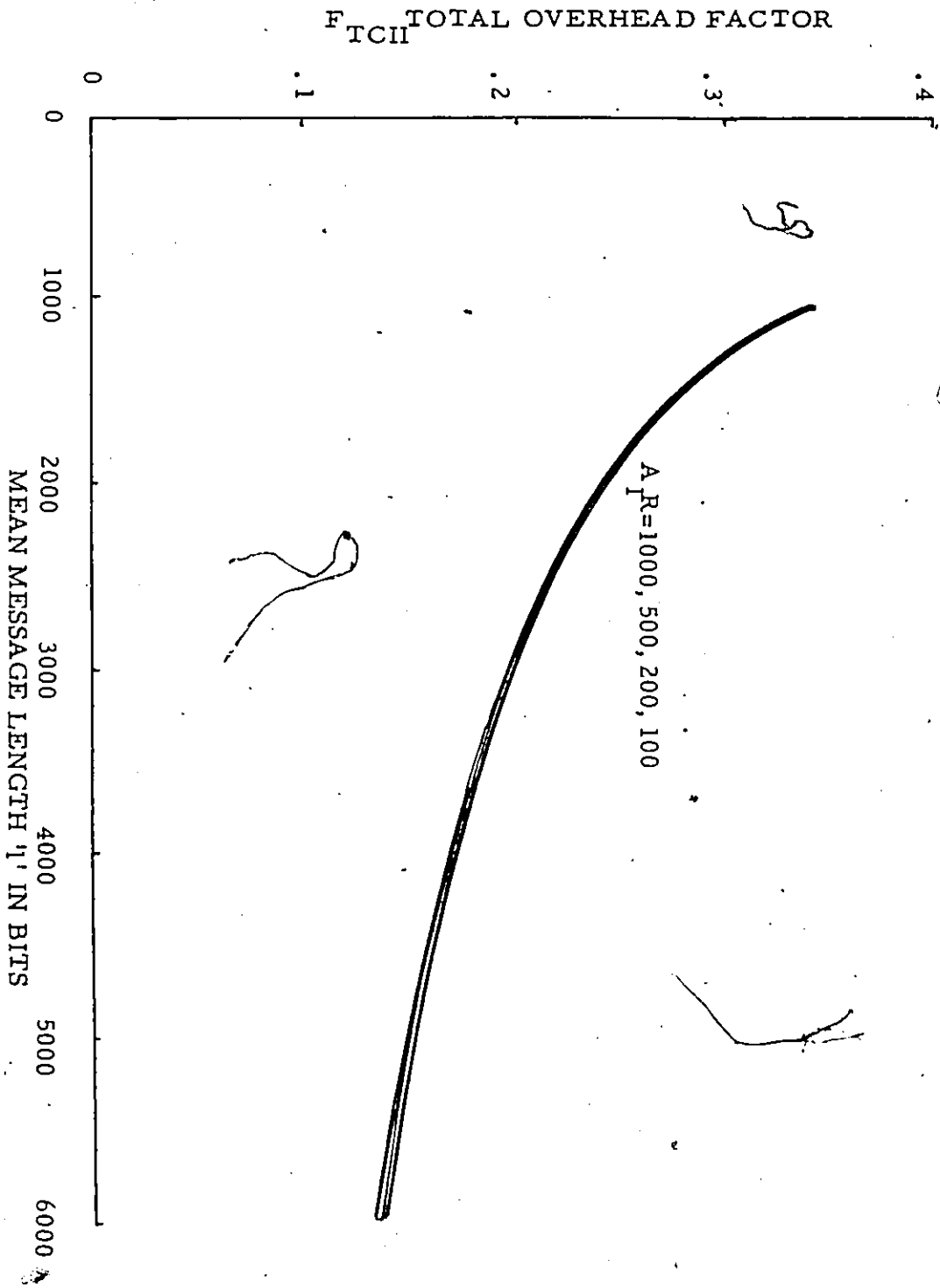


FIG. 17

CHAPTER 4

CONCLUSION

We have considered two aspects of data communication. First the evaluation of performance of a block oriented data transmission facility in terms of the Mean-Error-Free Interval. We derived certain general expressions for the Mean-Error-Free Interval. The results were then applied to a random error channel, specific telephone channels and a microwave channel. The MEFI is an important measure of performance and it is believed that the results obtained will be useful to the design of data transmission systems.

The second aspect of data communication that we have addressed in this thesis is the question of optimal block or packet size, which minimises the channel overhead or maximises the channel utilisation. Some general expressions were derived for the total overhead factor for channels of practical interest, e.g., the random error channel, the Balkovic et al telephone channel and a DNPL microwave channel, while using two types of transmission strategies.

The observation we make from the results obtained is that the Stop-and-wait strategy results into higher total,

overhead factor as compared to continuous transmission strategies for all channel characteristics considered so far. Also, of the two continuous transmission strategies considered, the strategy number one gives higher total overhead factor for all three channels considered in this thesis. It is also observed from the graphs for the total overhead factor that as the mean message length ( $l$ ) increases the graphs show an exponential decay, which implies that the total network operational overhead incurred in transmitting messages with large  $l$  values is less than the one for messages with small  $l$ . Correspondingly, the curves for optimal block length show an exponential increase as  $l$  increases, which is because the increase in the mean message length causes the total overhead factor to decrease and since large blocks mean smaller overheads, the optimal block length increases with an increase in  $l$ .

Futhermore the optimal block lengths corresponding to continuous transmission strategies are insensitive to changes in  $A$  &  $P$  for all three channels considered, as seen from the graphs. In the case of the Stop-and-wait transmission strategy, the larger the value of  $AR$  the higher the value of the total overhead factor and thus the smaller

the values for the optimal block length. In the case of the random error channel an increase in bit error rate causes an increase in total overhead factor and thus decrease in the optimal block length. If  $P$  is the message block size in bits, without overheads, and  $F_T$  is the total overhead factor, then the expected channel efficiency, when using fixed size message blocks with ARQ strategies for error control, can be defined as  $\frac{P}{F_T + P}$ . Thus minimising of  $F_T$  also maximises channel efficiency.

It is believed the results presented in this thesis will be useful in the selection of optimal fixed message block sizes for data communications.

REFERENCES

- [1] Mahoney Jr. J.J., Mansell J.J., and Matlack R.C., "User's View of the Network", Bell System Tech. Journal, May-June, Vol. 54, No. 5, pp 833-844, 1975
- [2] Horton D.J., and Bowie P.G., "An Overview of Data Route", Proceedings of 1974 ICC Minneapolis, June 1974 pp 2A-1 to 2A-5.
- [3] Balkovic M.D., Klancer H.W., Klare S.W., and McGruther W.G., "High-Speed Voiceband Data Transmission Performance on the Switched Telecommunication Network", Bell System Tech. Journal, April 1971, Vol. 50, No. 4, pp 1349-1384
- [4] Burton H.O., and Sulliman D.D., "Error and Error Control", Proceedings IEEE, Vol 60, Nov. 1972, pp 1293-1301
- [5] Peatfield A., Sherman H., and Zarcharov B., "Error Characteristics of Fast Data Links",  
( To be Published )
- [6] Georganas N.D., Shiva S.G.S., Verma P.K., and Jawanda J.S., "Evaluation of the Mean Error Free Interval over a Noisy Data Channel", IEEE Trans. on Communicatins, Vol. COM-26, No. 1, pp 185-187, January 1978.
- [7] Zacharov B., "Transmission Strategy and Optimal Block Size in High-Speed Date Communications", Proceedings IEE,

Vol. 120, No. 8, Aug. 1973

- [8] Fontaine A.B., and Gallager R.G. "Error Statistics and Coding for Binary Transmission over Telephone Circuits", Proceedings IRE, Vol. 49, pp 1059-1065 June 1961.
- [9] Metzner J.J., and Morgan K.C., "Coded Binary Decision Feedback Communication Systems", IFF Trans. on Communication Systems, Vol. CS5-8, pp 101-113, June 1960
- [10] Benice J.R., and Frey Jr. A.H., "An Analysis of Transmission Systems", IEEE Trans Communication Tech., COM-12, pp 135-145, December 1964.
- [11] Benice J.R., and Frey A.H., "Comparison of Error Control Techniques", IEEE Trans. on Communication Tech., Vol. COM-12, pp 146-154, December 1964.
- [12] Metzner J.J., and Morgan K.C., "Coded Feed-Back Communication Systems", Proceedings Nat. Electronic Conference Chicago, Illinois, Vol. 16, pp 250-255, October 1960.
- [13] Metzner J.J., and Morgan K.C., "Reliable Fail-Safe Binary Communication", Proceedings IRE Wescon Conference, pp 192-206, August 1960
- [14] Cowell W.F., and Burton H.O., "Computer Simulation of the Use of a Group Code with Retransmission on Gilbert Burst Channel", AIEE Trans on Com., pt. 1 no. 58, pp

577-580, January 1962.

- [15] Kretzner E.R., "The New Look in Data Communications", Bell Lab. Record, pp 259-265, October 1973.
- [16] Eisenbies J.L., "Conventions for Digital Communication Link Design", IBM System Journal, Vol. 6, No. 4, pp 267-302, 1967
- [17] Reiffen B., Schmidt W.G., and Yudkin H.L., " The Design of an Error-Free Data Transmission Systems for Telephone Circuits", AIEE Trans. Comm. Electron, Vol. 80, pt 1, pp 224-231, July 1961
- [18] Frochlich F.F., and Anderson R.P., "Data Transmission over Self Contained Error Detection and Retransmission Channel", Bell System Tech. Journal, Vol 43, pp 375-398, January 1964.
- [19] McGruther W.G., "Thruput of High-Speed Data Transmission Systems Using Block Petransmission Error Control Schemes over Voiceband Channels", Proceeding ICC Philadelphia, Pennsylvania, pp 15-19--15-2, June 1972.
- [20] Kucera J.J., "Transfer Rate of Information bits", Computer Design, June 1968, pp 56-59.
- [21] Wood F.B., "Optimum Block Length for Data Transmission with Error Checking", Communications and Electron , Vol. 40, pp 855-861, 1959.

- [22] Kirlin P.L., "Variable Block Length and Transmission Efficiency", IEEE Trans. on Comm. Tech., CT-17, No.3, pp 350-354, June 1969.
- [23] Chu W.W., "Optimal message Block Size for Computer Communications with Error Detection and Retransmission Strategies", IEEE Trans. on Comm., Vol COM-22, No. 10, pp 1516-1525, October 1974.
- [24] Balkovic M.D., and Muench P.E., "Effect of Propagation Delay caused by Satellite Circuits on Data Communication Systems that use Block Retransmission for Error Correction", IEEE International Conference on Communications, Boulder, Colorado, 1969, 69CP367 COM, pp 31-36.
- [25] Field J.A., "Efficient Computer-Computer Communications", Presented at The Canadian Communication and Power Conference, pp 20-21, Montreal 1974.
- [26] Shamim A.A., Majithia J.C., and Mark J.W., "Study of Overheads in Packet-Switched Data Networks", Proceedings IEE, Vol. 122, No. 11, November 1975, pp 1203-1206.
- [27] Majithia J.C., and Bhar P., "Analysis of Overheads in Packet Switched Data Networks", Proceedings IEE, Vol. 121, No.11, November 1974, pp 1375-1376.
- [28] Gilbert E.N., "Capacity of a Burst Noise Channel", Bell System Tech. Journal, Vol. 39, September 1960,

pp 1253-1265.

[29] Hamming R.W., Numerical Methods for Scientist and  
Engineers, New-York, McGraw Hill, 1962.

APPENDIX

BALKOVIC TELEPHONE CHANNEL [3]

Bell Laboratories conducted in 1969-70 a connection survey of the Bell system Switched Telecommunications Network. Error performance measurements were made on approximately 600 toll connections dialed from 12 receiving to 92 transmitting sites in the United States and Canada.

Using this source data, M.D. Balkovic, H.W. Klancer, S.W. Klare, and W.G. McGruther presented estimates of data transmission error performance on a bit, burst and block basis, at data rates of 1200, 2000, 3600 and 4800 b/s in their paper 'High-Speed Voiceband Data Transmission Performance on the Switched Telecommunications Network', in The Bell System Tech. Journal, April 1971.

DARESBUY NUCLEAR PHYSICS LABORATORY  
MICROWAVE CHANNEL (DNPL) [5]

The Daresbury Nuclear Physics Laboratory has set up an experimental microwave link, operating over a distance of

70 km and including a repeater. This link operates at 7 GHz and attains rates of up to 10 b/s in both duplex channels. The link terminates at central 360/165 computer.

Measurements of the noise characteristics of these links have been made and presented by Peatfield A., Sherman H., and Zarcharov B. in their paper 'Error Characteristics of Fast Data Links', [5].