CRUSTAL MOVEMENT AROUND THE GREAT LAKES

by

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ABSTRACT

A time series analysis was used to determine rates of vertical crustal movement within the Great Lakes region of North America. This vertical movement of the earth's crust has, over a long period of time, a major effect on shoreline structures such as protection works, marinas, wharves and harbours.

A statistical analysis was carried out on time series of mean monthly lake elevations as recorded at five gauges around Lake Superior. It was found that these time series are composed mainly of autoregressive and periodic components.

A set of new time series was generated by subtracting mean monthly lake elevations between pairs of gauges around Lake Superior. It was found that these new series were composed mainly of first order linear trends and small periodic components.

Assuming that the results obtained for Lake Superior are typical of the Great Lakes as a whole, first order linear trends in time series of differences in elevations were computed for each of the lakes. It was then assumed that the first order linear trends represent rates of relative crustal movement. The computed relative rates of movement were converted to absolute rates of movement using the Nipissing zero isobase as a datum.

The significant results of this study are that rates of vertical crustal movement have been computed for all major harbours on the Great Lakes and that a datum has been established common to all of the lakes. The crustal movement rates can be used in the preliminary design of shoreline structures while the datum line will be of use to geologists in extending ancient beach lines.

The significant contributions in this study include the use of time series analysis on lake levels and differences in lake levels, the use of a circular distribution of crustal movement within lake basins and the confirmation by lake levels of rates of crustal movement determined from other geophysical phenomena.

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NOTATIONS

The following abbreviations and symbols are used in this study.

Abbreviations

$\text{cov}(R_t, R_{t+k})$	Sample autocovariance of stochastic component at lag k
$\text{var}\ P$	Sample variance of periodic component P_t
$\text{var}\ P_m$	Sample variance due to periodicity in the monthly means of periodic component P_t
$\text{var}\ P_{sd}$	Sample variance due to periodicity in the monthly standard deviation of periodic component P_t
$\text{var}\ R_t$	Sample variance of the stochastic component R_t
$\text{var}\ Y_t$	Sample variance of time series Y_t
$\text{var}\ \varepsilon$	Sample variance of independent random time series ε_t
$\text{var}\ \Delta_t^r$	Sample variance of the differenced time series, Δ_t^r

Greek Symbols

α	Confidence level
α_1, α_2	Coefficients in linear autoregressive model
γ	Degrees of freedom in confidence limit determination

Δ_t^r	rth order series of differences in trend component, T_t
δ_t	Independent random portion of trend component, T_t
ε_t	Independent random portion of autoregressive component
θ_j	Phase angle of jth harmonic of periodic component, P_t
μ_t	Population expected mean value of time series, Y_t
ρ_k	kth order population autocorrelation coefficient
$\sigma_1, \sigma_2, \sigma_3$	Population standard deviations used in randomness tests
σ	Population variance of time series, Y_t
σ_δ^2	Population variance of independent random time series, δ_t
σ_ε^2	Population variance of independent random time series, ε_t
σ_T^2	Population variance of trend component T_t
σ_{reg}^2	Population variance explained by linear regression analysis
σ_{res}^2	Population variance not explained by linear regression analysis
ϕ	Angular frequency of autocorrelation
τ	Lag used in autocorrelation

χ^2	Chi-square test statistic
ψ	Angular phase difference of autocorrelation
ω	Frequency of spectral density estimate

Other Symbols

a_o, a_1, a_2, a_3	Linear regression coefficients
A_j, B_j	Fourier coefficients used in periodic component, P_t
c_1, c_2, c_3	Constants in normal equations
C_j	Periodogram amplitude for jth harmonic of periodic component, P_t
$C(k)$	kth order sample autocovariance function
CL_α, CL'_α	Confidence limits at α percent for sample autocorrelation function and mean estimated spectrum
d_1, d_2, d_3	Constants in normal equations
e_1, e_2, e_3	Observation errors for rates of relative crustal movement
E	Sum of the squares of weighted observation errors
E_j	Expected number of values of time series in jth class interval of χ^2 test
$E(Y_t)$	Expected value of Y_t
F	F test value, ratio of variances
$F(\varepsilon)$	Value of ε from sample distribution
$F_o(\varepsilon)$	Value of ε from theoretical population distribution

g_j	Fisher test statistic for jth harmonic of periodogram
G_1, G_2, G_3	Number of runs detected in randomness tests
h_j	jth order harmonic of periodogram
k	Number of harmonics in periodic component P_t
K_1, K_2, K_3	Standard normal variates
l	Number of autocovariances computed
m	Number of categories used in randomness tests
m_1, m_2, m_3	Computed rates of relative crustal movement
M_1, M_2, M_3	Population means used in randomness tests
n	Weighting factor in spectral density estimates
n_α	Standard normal deviate used in computing confidence levels
N	Total number of observations of time series Y_t
O_j	Observed number of values of time series in jth class interval of χ^2 test
p	Coefficient in theoretical correlogram of second order linear autoregressive model
P_g	Probability function of Fisher test statistic g
P_t	Periodic component of time series Y_t
$\overline{P}_i$	Mean value of periodic component P_t in ith month
$\overline{\overline{P}}$	Overall mean of periodic component P_t
q_1, q_2	Computed rates of relative crustal movement
r	Order of variate difference Δ_t^r

r_1, r_2	True rates of relative crustal movement
R	First order linear correlation coefficient
R^2	Coefficient of determination
R_t	Stochastic component of time series Y_t
S_i	Sample standard deviation of periodic component P_t in ith month
$\overline{S}$	Mean standard deviation of periodic component P_t
SE	Standard error of estimate of polynomial regression
t	Unit of time, one month
T_t	Trend component of time series Y_t
$\hat{T}_t$	Sample estimate of trend component from linear regression
$\overline{T}$	Mean value of component T_t
u(k)	Smoothed value of variance density function at kth frequency
v(k)	Raw variance density function at kth frequency
W	Number of positive or negative values of independent random variable ε_t
Y_t	Time series of mean monthly lake elevations
$\overline{Y}$	Mean of time series Y_t

CHAPTER 1

INTRODUCTION

1.1 Motivation

It is well known that the earth's crust is not stable; earthquakes and fault movements are reported almost daily. Available evidence (4)* indicates that movements of the crust have been taking place ever since it was formed. The geological record shows an alternation of cycles of emergence followed by erosion and of inundation followed by sedimentation. Mountain ranges have been uplifted and eroded, the eroded material being deposited possibly in areas that were subsiding. At three different times high mountain ranges have been uplifted in the northern Great Lakes region (58).

Less well known but nonetheless important is a slow long-term vertical movement of the earth's crust. At the present time this movement of the crust is being observed at many places (32). The Atlantic coast of North America is subsiding between Saint John, New Brunswick, and Key West, Florida. The northern shore of the Gulf of Mexico is subsiding; the western shore is rising. The Pacific coast of North America is subsiding except for the area north of

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*Numerals in parentheses refer to corresponding references in the Bibliography.

Seattle, which is rising slightly. In South America, the Pacific coast between Peru and Cape Horn is rising, while to the east, Brazil is subsiding. In Europe, the Baltic area is rising while France and the Mediterranean area are subsiding. Britain is tilting, with the north rising and the south subsiding. Japan and the Philippine Islands are rising; the west coast of India is subsiding. These facts are deduced from long-term observations of tide gauges at points along the coasts of the regions mentioned.

In the Great Lakes region, the effects of vertical crustal movement have been observed since the middle of the nineteenth century. Before the present large-scale industrialization of the Great Lakes Basin began, the effects of crustal movement were of very little importance to man. Today, however, when the lakes are used for interests such as hydropower, navigation, sewage disposal and recreation, the long-term effects of crustal movement have become of much greater importance.

As examples of the economic effects of crustal movement, harbours on Lake Superior's western shoreline are estimated (25) to have lost over 1/2 foot in navigable depth over a period of 50 years. This is of major concern to shipping interests on the Great Lakes who normally load their vessels to the maximum depth possible. Similarly other effects such as the changing patterns of erosion and inundation have major effects on today's urbanization of the shoreline.

The motivation for this study was, therefore, to determine a method of forecasting the magnitudes and directions of vertical crustal movement within the Great Lakes Basin so that, in future, projects likely to be affected by this phenomenon can be designed to accommodate its effects.

1.2 Literature Review

Absolute rates of crustal movement can be determined in areas containing large lakes by comparison of lake elevations recorded at different gauges around the circumference of the lakes. The statistical technique of time series analysis has been used to determine the various components which make up time series of lake elevations and differences in lake elevations. Theoretically computed rates of vertical crustal movement often need adjusting to account for the influence of other factors on the lake elevations. These factors include both natural phenomena and man-induced changes in lake regime. The rates of movement computed from lake elevations can be compared with rates of movement computed from other natural phenomena.

1.2.1 Determination of Rates of Crustal Movement from Water Elevations of Lakes

The first recorded investigation of vertical crustal movement in the Great Lakes region using lake elevations was that of Gilbert in 1896 (28). He drew up plans for a measuring system to cover the whole of the Great Lakes Basin and derived methods of measurement designed to eliminate the various sources of error such as solar and lunar tides, wind effects, pressure differential effects and errors in the measuring equipment. Through lack of equipment, Gilbert worked with only four pairs of lake elevation gauges which he placed so that the lines joining the pairs were in the estimated direction of the maximum crustal movement since the Nipissing epoch of the upper lakes.

Freeman (25) stated in 1926 that he was led to study "this matter of tilting of the earth in relation to lake levels" while seeking an explanation for the apparent change in the relative elevations of Lake Huron and Lake Erie. He chose twenty pairs of gauges with long accurate records, each pair of gauges measuring water elevation on either side of one of the Great Lakes. Using monthly mean and yearly mean elevations, as compared to Gilbert's (28) daily data, Freeman derived rates of crustal movement in feet per hundred miles per hundred years and wrote,

> "Whatever the cause, continuous progressive tilting upward toward the north at the rate of about half a foot per one hundred miles per century in the southern part of the Great Lakes region, with indications of double this rate over some parts of the lake system, is proved beyond all doubt."

With only twenty pairs of gauges available, Freeman followed Gilbert's assumption that each lake basin was tipping in its entirety at the same rate without consideration of local topography and variation in gravity.

Moore (57) wrote in 1922 that his investigations into crustal movement were prompted by the discovery in the winter of 1919-1920 that the elevations of Lake Erie as recorded at Port Colborne were between 0.10 and 0.20 foot lower than the corresponding elevations recorded at Cleveland, although the two gauges had been carefully set to the same datum by water level transfer in 1875. Further investigation showed that the gauge at Harbor Beach was giving lower elevations than that at Milwaukee and that the Marquette gauge was indicating lower elevations than the gauge at Duluth. Moore then checked other gauges and found that in every case gauges to the north and east were giving lower readings.

> "The only logical conclusion," Moore (57) wrote, "appeared to be that there was in progress a movement of the earth's crust over the entire region, a tilting movement that was causing a relative rise in the land to the north and east."

Having chosen reliable gauges at widely spaced points with records covering long periods of time, Moore calculated rates of vertical movement between the gauges. Using the five summer months, June to October, to reduce interference from wind, he plotted the annual differences between each pair of gauges and drew in best-fitting straight lines. These gave the rates of crustal movement between the pairs of gauges. On each lake, Moore separated these rates into a north-south component of movement and an east-west component. Using a least squares method, he obtained one representative north-south (vertical) movement and one representative east-west (vertical) movement for each lake; resolving these two gave one resultant rate of movement for the shoreline of each lake together with the direction of movement for each lake. For example, Moore's results gave an average rate of movement for Lake Erie of 0.46 foot per 100 miles per 100 years in a direction 31° north of west. Unlike Gilbert (28) and Freeman (25), Moore recognized that the rate of movement was not uniform over the whole Great Lakes Basin.

In 1948, Moore (58) gave the results of further investigations that he had carried out on ninety-one long-term gauges on the Great Lakes. In this study he plotted the annual differences in five monthly mean summer elevations between each pair of long-term gauges on each lake and drew best-fitting straight lines through the points. The rates of movement indicated by these lines were then adjusted for harmony using a least-squares technique. The average correction required was 0.03 foot per 100 years and the maximum correction to any rate was 0.06 foot per 100 years. With the rates at the principal gauges on each lake established, the movement at other gauges around the lakes was obtained by comparison with at least two of the principal gauges. Then he attempted to determine the rate of movement between each of the lakes by comparing the results of land level lines run at widely separated times. Compared to levelling by water, land levels are relatively inaccurate but this method may give an indication of the between-lake movement. Even with rates of movement determined over the whole of the Great Lakes Basin, the measurements are still only relative and so Moore referred them to present day sea level (that is, 1948 sea level) using level lines run from Oswego to New York. This showed that the land on the U.S. side of Lake Superior was subsiding with respect to mean sea level at rates amounting to 0.09 foot per 100 years at Point Iroquois, 0.62 foot per 100 years at Marquette, and 1.03 foot per 100 years at Duluth. Partly because of this investigation by Moore, the P-5 rule curve for regulating Lake Superior was replaced by the so-called 1949 Rule, which was designed to reduce the frequency of high elevations on the lake and minimize the damages experienced at U.S. harbors due to high lake elevations. The line of zero crustal movement derived by Moore passed through Newfoundland; St. John, New Brunswick; Kingston, Ontario; and across the top of the Great Lakes. He considered that areas north of this line are rising relative to sea level, those south of the line are subsiding. Moore (58) stated,

> "If this line of zero movement is considered as the arc of a circle, which it approximates, the focus is about 200 miles east of James Bay. If the circle is continued it will pass close to Churchill, where the records of the tide gauge, when properly interpreted, show zero movement."

Lilly (49) gave examples of the evidence for tilting in the Great Lakes region based on his study of elevation gauges around the lakes. He stated that,

> "It is definitely established that the relationship between the earth's crust and the direction defined by the water surface is undergoing change ....."

He also pointed out that this change has usually been ascribed to a tilting of the earth's crust but that the same results would be observed if the crust were rigid and the direction of gravity were changing.

Price (61) reported in 1954 on the assessment of crustal movement on Lake Ontario. He took five gauges, Toronto, Kingston, Cape Vincent, Oswego and Port Dalhousie, and used the mean elevation during the four summer months June to September to eliminate possible discrepancies due to ice, floods, spring and autumn gale conditions, etc. Price used a method of adjusting the resulting rates of crustal movement based on the assumption that at all points equidistant from some straight hinge-line the rates of movement must all be equal.

In May 1957, The Vertical Control Subcommittee of the Co-ordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data published its first interim report on crustal movement in the Great Lakes region (74). This report presented the findings of a comparison carried out between the methods and results of Moore (58) and Price (61). Several differences in technique and basic data were found between these two studies, and the Subcommittee related these differences to the differences in the calculated rates of crustal movement determined by each author. They were of the opinion that the differences between the two investigations were too large to reconcile and so they decided to carry out an independent study utilizing a method which combined the best features of both reports. The method finally adopted was to use four-month mean elevations, June to September, a least squares regression line to fit the data points, and a least squares triangulation adjustment between main gauging stations. Gauges having shorter periods of record were then joined to the triangulation net. Two reports were issued for each of the Great Lakes (74-81). The first of the two reports for each lake gave the calculated rates of crustal movement between principal harbours around the lake, and the second report gave the calculated movement at the smaller gauges. Of the 118 gauges used by the Subcommittee only 31 had periods of record exceeding 20 years, which must be considered the very minimum period of significance. In 11 of the sets of gauges for which the Subcommittee computed regression lines there were only two years of record common to both gauges. This means that, in these cases, the regression line was fitted to only two data points. These data are of little value for computing long-term trends from time-series for which it is known that there are highly significant short-term components due to meteorological effects, wind set-up, instrument and operator errors, gauge location peculiarities, etc.

Maclean (52) contended that the previously measured rates of crustal movement merely represented the change in the net difference of the accumulated effects and errors, because lake elevation gauge records are not corrected for meteorological effects, wind set-up, or instrument and operator errors. Maclean also noted that geologists have found that the hinge-line of the ancient Nipissing beach lies in the northern sector of the Great Lakes, and concluded that

> "it is reasonable to assume that the value of the rates of modern crustal movement, even the existence of modern crustal movement around the Great Lakes, is in doubt."

The Vertical Control Subcommittee, together with the Lake Levels Subcommittee, published a review of Maclean's report in 1964 (82). Their main criticism of Maclean's report was his interpretation of the effect of meteorological conditions on long-term gauge records,

> "without doubt meteorological effects are the most significant errors in water level transfers. They do not, however, have any effect on the determination of crustal movement rates over long periods ... . There are many plots of gauge differences covering periods of 50 to 100 years that show a pronounced slope in the best-fitting line drawn through them. This slope can only be explained by relative movement of the land at the gauge sites. The normal line in such a plot demonstrating only average net wind set-up between the two points would be a horizontal line at a fixed distance above or below the zero axis equal to the amount of the average net set-up. To obtain a sloping line the wind would have to increase or decrease progressively over a long period, and this is not physically true."

The report concluded that the crustal movement was present in the Great Lakes region and that the only satisfactory method of measuring it was by differences in lake elevations.

The Inland Waters Branch, Department of Energy, Mines and Resources (now Water Planning and Management Branch, Department of the Environment) began a systematic study of the crustal movement around the Great Lakes in 1967 (40). The first step in this study was to update the figures derived by the Vertical Control Subcommittee in 1956 using the same method of averaging the elevations over the four summer months, utilizing the same gauges, but employing a digital computer to calculate least-squares linear regression lines on each set of elevation differences. A second program was written to work from basic data and compute rates of crustal movement for all main gauges. This program was used to analyse data from only those pairs of gauges having continuous common periods of record greater than 20 years. From the results of the two methods, two types of graph were obtained. For the first type of graph, lines joining one gauge to each of the other gauges on the lake were drawn on a map and divided according to the rates of crustal movement between the stations. From these, lines of equal rates of movements were drawn on the map after the style of contour lines. The second method was to plot rates of crustal movement at gauges around a lake relative to one reference gauge in feet per 100 miles per 100 years versus the whole circle bearings of the gauges relative to the reference gauge. This method was used to produce graphs for all the Great Lakes and it was found in each case that some form of circular curve was present, implying that the measured crustal movement was basin-wide at least.

1.2.2 Time Series Analysis

If the observations of a given hydrologic phenomenon such as discharge or water level are recorded sequentially in time the resulting sequence is called a time series (53). In nearly all cases, the observations are associated with points in time equally spaced, or each observation is associated with an interval in time, where the length of each interval is the same.

Statistical techniques have long been applied in the study of hydrologic phenomena, but little attention had been paid to the sequential order of their occurrence. In recent years there has been an increasing awareness of the need to consider the time distribution characteristics of hydrologic events. The investigations of these characteristics are referred to collectively as time series analysis.

Hydrologic time series may be considered to be the sums of deterministic and stochastic components. A deterministic component is said to exist (91) when observations separated in time are dependent. Deterministic components include trends, jumps and periodicities. Trend is a slow gradual change of a variable over the period of record.

Trends might be caused, for example, by filling of lakes by sediment, by gradual urbanization of a watershed or by vertical crustal movement. Jumps, that is positive or negative slippages, in hydrologic time series might be caused by landslides (91) or forest fires or by man made changes such as dredging river channels (6,46). Periodicities have been defined by Granger and Hatanaka (29) as "a constant fluctuation with set period." The presence of periodic functions in certain hydrologic time series is attributed by Matalas (53) to various astronomical events which are themselves periodic. The phases of the moon are directly associated with ocean tides, the annual rotation of the earth about the sun produces seasonal variations in winds, precipitation and evapotranspiration as does the daily rotation of the earth on its own axis.

The stochastic component of a time series is that portion which cannot be predicted exactly. Stochastic components are generally considered (91) to consist of persistence and independent random elements. Persistence means that successive members of a time series are linked among themselves in some persistent manner. Examples in hydrology include the tendency for wet years to occur in groups, similarly for dry years, and the generally high correlation between river flow on one day and flow on the following day during flood recession periods.

In the analysis of a hydrologic time series several approaches (91) can be employed. Usually to facilitate analysis the deterministic components present (if any) are detected and removed from the time series (53) and the remaining stochastic component is then analysed separately.

Trend components may conveniently be removed by either the method of moving averages (91) in which a series of overlapping means of successively weighted values are calculated or by regression analysis which detects linear relationships between the variable and time. Regression analysis is to be preferred because of the so-called Slutzky-Yule effect associated with the moving average process. Yule (92) showed that this effect consists of an induced long-period oscillation together with the removal of oscillations with periods comparable to the extent of the moving average process.

Method of detecting and removing periodicities have been used for a considerable time by economists (29) and meteorologists (92). The most common method is the Schuster's periodogram (68) in which a series of sine curves of given period and determined amplitude are subtracted from the time series. The drawback to the periodogram approach is that one must know the basic periodicities which one wishes to remove. In hydrology, however, this is less of a problem since virtually all periodicity consists of the annual cycle and its harmonics.

After the removal of trends, jumps and periodicities the remaining portion of the time series is the stochastic component. The element of persistence in the stochastic component can be measured using an autocorrelation coefficient in a manner analogous to that used to measure the degree of linear association between two sets of observations (53). Yevjevich (91) found that the autocorrelation between annual values of discharge of the St. Lawrence River is about 0.7 for a lag of one year. This high persistence is due to the storage effect of the Great Lakes. The remaining portion of the stochastic component is assumed to be an independent random variable (53) due to non-systematic errors in recording, transmitting and digitising the data as well as to other interacting random processes.

1.2.3 Factors Affecting Water Elevations of Lakes

At any instant in time a number of factors affect the elevation of a lake water surface including barometric pressure, wind, local physiographic features, ice cover in winter, and the influence of man. Over a long period of time the natural influences are assumed to be eliminated. Man-made changes such as dredging and diversions do have long term effects, however. These effects vary from lake to lake.

1.2.3.1 Lake Superior

Two man-induced factors affect the long term elevations of Lake Superior; namely the diversion of water into Lake Superior at Long Lake and Ogoki and the regulation of Lake Superior outflows at Sault Ste. Marie. Because of the complete regulation of lake outflows, and hence elevations, the diversions can be considered to have no effect on Lake Superior elevations although they will still have an effect on the downstream lakes.

The control structure across the outlet of Lake Superior was completed in August 1922 under the orders of approval issued by the International Joint Commission in 1914 (unpublished papers, International Lake Superior Board of Control). No formal regulation plan was followed for the first six years; all major interests agreed on each change in outflow. On July 31, 1928, the Lake Superior Board of Control adopted an operating rule curve known as "Tentative Rule Curve D". Due to the Long Lake diversion into Lake Superior a new rule curve "P5" was adopted by the Board in 1941. A further diversion, Ogoki, and the study of crustal movement around Lake Superior by Moore (58) led to a further curve "1949 Rule" being initiated in May, 1951. At the time that the "1949 Rule" was introduced it was decided (58) that the gauges at Marquette, Duluth, Port Arthur and Michipicoten would be corrected for crustal movement by -0.08, -0.14, -0.06 and -0.10 respectively. Preliminary daily elevations for the gauges at Duluth and Marquette are corrected for crustal movement by the Lake Survey Centre, U.S. Department of Commerce while published mean daily elevations for these two gauges are not corrected. The present (summer, 1972) corrections used are -0.12 foot for Duluth and -0.08 foot for Marquette. The Canadian gauges at Port Arthur (now Thunder Bay) and Michipicoten Harbour are not corrected for crustal movement.

1.2.3.2 Lake Michigan-Huron

Rowe (65) has estimated that the recession of the lake elevation since 1860 is 3.40 feet at the Harbor Beach gauge, 3.10 feet at the southern tip of Lake Huron and the southern half of Lake Michigan and 3.59 feet along the most northerly shore of Lake Huron. Rowe (65) used 5-year mean elevations to develop a relationship between lake elevation and time. This is, in effect, use of a moving average technique with its associated problems.

This overall lowering of lake elevation is due to a combination of the Chicago diversion out of Lake Michigan, the retardation of outflow due to ice in the St. Clair and Detroit Rivers each winter and the effects of man's modifications of the natural stage-discharge relationship in the outlet channel.

It has been estimated (unpublished report by the International Great Lakes Levels Board) that the effect of the diversions into Lake Superior at Long Lake and Ogoki would produce a net rise in elevation of Lake Michigan-Huron of around 0.37 foot. This is counter-balanced to some extent by an increasing consumptive use of water which would lower the elevation by around 0.1 foot. It has been estimated (25) that the present maximum allowable Chicago diversion of 3,200 cfs lowers the elevation of Lake Michigan-Huron by 0.23 foot. In 1928, however, the diversion was as high as 10,000 cfs and so its effect has not been constant.

The retarding effect of winter ice in the St. Clair and Detroit Rivers has been estimated (5) to range between 0.18 foot to 0.87 foot (although due to dredging and pollution this retardation has recently been much reduced) and the effect has been to raise Lake Michigan-Huron elevations by up to 0.5 foot since 1900.

A report of the Canadian Interdepartmental Committee on Compensating Sills in the St. Clair River (6) has disclosed that the total lowering of Lake Michigan-Huron in the period 1890-1970, due to dredging in the St. Clair and Detroit Rivers was between 1.5 and 1.8 feet. Dredging for sand and gravel in the St. Clair River accounted for 0.4-0.5 foot lowering in the period 1890-1900 and 0.4-0.5 foot lowering in the period 1904-1924. Dredging for navigation channels in the St. Clair River accounted for lowerings of 0.2-0.3 foot in the period 1933-1938 and 0.1 foot in the period 1960-1961. In addition, dredging in the Detroit River is estimated to have caused an additional lowering of 0.3 foot in the period 1870-1957 and 0.12 foot in the years 1960 and 1961.

Brunk (5) estimated that the effect of the dredging in the St. Clair and Detroit Rivers has been to lower Lake

Michigan-Huron by about 2 feet since the 1880's. Another source (6) has indicated that the lowering has been roughly 1 foot since 1900. Since 1930 over 50 million cubic yards of material have been dredged from the St. Clair and Detroit Rivers, not including routine maintenance dredging. Some of this material taken from navigation channels has been subsequently dumped back into the rivers in such positions as to partially compensate for the original channel changes.

1.2.3.3 Lake Erie

Lake Erie mean lake elevations have been affected over the last hundred years by several factors. It has been computed (unpublished report, International Great Lakes Levels Board) that the Long Lake and Ogoki diversions have raised the elevation by 0.23 foot, the Chicago diversion has lowered the lake by 0.14 foot, and the Welland Canal and New York State Barge Canal lowered the lake by 0.33 foot. Some small effect on the mean lake elevation may be expected from the operation since 1964 of the ice boom in the Niagara River.

Observation of graphs of differences in mean monthly elevations versus time has shown (40) that, for a few pairs of gauges on Lake Erie, certain portions of the plots seem to show no trend in differences in elevations. These periods of zero or very low trend appear to be separated by jumps or abrupt changes in the record, so that the graph shows a series of step-like changes rather than a smooth trend. These steps in the record could be due to movement at a fault line(s) located between the pair of gauges used (40) or the jumps could be due to gauge relocation or other gauge disturbances not corrected for in the original gauge records (46).

The significance of those data sequences with zero trend can be checked by examining statistically the differences between the mean elevations for each flat sequence. This is necessary because of the large random component present in these time series, particularly on Lake Erie. Assuming a statistical significance is verified, then an explanation of the jumps in elevation can be sought in geological records, gauge histories and records of harbour construction. Korkigian (46) has shown that for the time series Cleveland minus Buffalo the jumps correspond with the dates of gauge relocations at the two sites (12). He, however, used only one elevation per year at each gauge (made up of the mean of the four monthly means, June through September).

1.2.3.4 Lake Ontario

The control works in the St. Lawrence River were completed in 1959 and so are not yet an important factor in the long-term lake elevations. No additional factors affect the elevations of Lake Ontario.

1.2.4 Determination of Rates of Crustal Movement by Other Methods

The most obvious method of computing rates of crustal movement is from simple observation of the changing physical characteristics of an area. Stuntz (71), a land surveyor of Wisconsin in 1853, observed that the river beds at the western end of Lake Superior were growing deeper, indicating a tilting of the area with the western end of the lake subsiding. From questioning Indians living near the St. Louis River at the western end of Lake Superior, he inferred "the gradual rise of water at this (west) end of the lake and the falling of the same at the east." This type of evidence enabled investigators to perform rough calculations regarding the effects of the movement. Thus, Spencer (69) was able to tell the American Association for the Advancement of Science in 1894 that:

> "the end of the (Niagara) Falls seems destined, if we read the future by the past, to be effected, not by the erosion expending itself on the rock, but by terrestrial deformation turning the drainage of all the upper lakes into the Mississippi, by way of Chicago, just as the Huron waters were lately turned from the Ottawa into the Niagara drainage; and at the recent rate it would seem that about 5,000 or 6,000 years at the most would be needed."

Farrand (20) undertook a study of former shorelines in the western and northern Lake Superior region. By visual location of such features as wave-cut cliffs, cobble beach ridges and other continuous morphological evidence he derived maps showing elevations of previous shorelines from which isobases or lines of equal elevation were drawn. Farrand's derivation of the isobases belonging to the glacial Lake Nipissing are of particular interest in this study. From a knowledge of the total uplift to which the shoreline at a particular location has been subjected and a knowledge of the time over which this uplift has occurred, a rate of vertical movement can be derived.

By re-levelling several level lines covering the triangular area, Quebec - Lac St. Jean - La Malbaie, Frost and Lilly (26) determined that there had been changes in elevation at points within this area at rates of up to 1.96 feet per century. The movement detected by Frost and Lilly appears to be of two types. There is a circular area in the centre of Laurentide Park which is sinking relative to Quebec, while in the Lac St. Jean area the land is rising relative to Quebec. While two measurements up to 50 years apart cannot be considered as reliable as a continuous record, this study shows a way of extending the study of crustal movement away from the lake and sea coasts.

Gale (27) published a further account of the detection of vertical crustal movement in the Quebec - Lac St. Jean - La Malbaie area, with a spur westwards to Senneterre, by comparison of geodetic surveys run at different times. His results are in general agreement with those of Frost and Lilly (26), although a slightly different computation method was used and an additional survey, run in 1966, was available. Gale found that the maximum rate of vertical movement (which occurred at Senneterre) was 54.6 cms/50 years (3.58 feet/100 years). It is also reported in this study that re-levelling of the Trans-Canada line (begun in 1966) shows no evidence of significant movement between Vancouver and Calgary, Toronto and Montreal or in the New Brunswick area, although all the data had not then been analyzed.

Moore (58) computed rates of crustal movement in the Great Lakes region relative to mean sea level. The level lines that he employed were taken at different times within the period 1857 to 1941. Moore used levels from New York to Rensselaer to Oswego, Cleveland, Harbor Beach and Sault Ste. Marie. The most uncertain section of the levelling is that between Lakes Erie and Huron although other sections have been subject to criticism (52). He computed that the movement of the gauge at Point Iroquois, on Lake Superior, with respect to mean sea level at New York was -0.09 foot/100 years.

Walcott (84) reviewed recently published data on vertical crustal movement in North America from observations of lake elevations in the Great Lakes region, tidal records along the Atlantic Coast and at Churchill, on Hudson's Bay, geodetic re-levelling, and gravity measurements. He compared these results with radio-carbon dating of material found in positions related to past lake or sea levels, such as seashells, wood, and other organic materials found on old strand-lines or delta foreset beds. The author concluded that the available evidence for recent vertical movement is consistent with the broad uplift of the land that has occurred over at least the last 7,000 years and that this movement is postglacial rebound. Of particular interest in this study is Walcott's research into the elastic yield of the earth caused by fluctuating water elevations in the Great Lakes. Because of this elastic yield, the measured value of a lake elevation will comprise both the actual water elevation and a deformation of the ground caused by that depth of water. It has been found (84) that the maximum contribution of elastic yield to the measured rates of crustal movement is around ten per cent.

Lewis (48) investigated sediment sequences around Manitoulin Island in Lake Huron. By correlating the elevations of small lakes on the island with the radio-carbon dates of organic sediments (gyttja) in the lakes he inferred a rate of uplift of 2.2 mm per year (0.72 feet/100 years).

Innes (37) and Innes and Argun-Weston (38) investigated the interrelationship between gravity anomalies and isostasy in northern Ontario. Innes stated that

> "the evidence for this region of the Shield, however, suggests conditions analogous to those in Scandinavia and strongly supports the hypothesis that uplift in these formerly glaciated regions is isostatic and due primarily to the deficiency of mass in the subcrustal layer following removal of the ice."

He also concluded that the gravity data indicate that the crust has yet to rise about 800 feet before isostatic balance is restored.

Recent work by Walcott (84) indicated that the region of maximum uplift is associated with an extensive free air anomaly.

Milne and Davenport (55,56) have prepared maps showing lines of equal acceleration as a per cent of g, the gravitational attraction. These are normally given in terms of that per cent of g with a return period equal to 100 years.

Other investigations resulting in isolines include those of Andrews (2) using postglacial recovery and rebound curves and Walcott (83) studying the isostatic response to crustal loading.

1.3 Objectives

The review of the literature has shown that (1) previously computed rates of crustal movement in the Great Lakes region were determined in a rather crude manner from lake elevations, (2) the time series analysis has reached the stage of development at which it can be used in practice, (3) the other factors affecting lake elevations are known, and (4) rates of crustal movement computed from other geophysical phenomena are available.

The literature has also shown that the previous investigations using lake elevations to detect crustal movement had four major disadvantages, namely (1) only data from four months of the year were used, (2) the assumption was made, without any evidence, that time series of differences in lake elevations could be represented by first order linear trends, (3) no realistic datum was used to convert relative rates of movement to absolute rates, and (4) no realistic datum was used to draw together the results for each lake.

The objectives of this study are therefore (1) to make a complete statistical analysis of time series of mean monthly lake elevations and time series of differences in mean monthly lake elevations in order to identify the important components and test the hypothesis that a first order linear trend can adequately represent time series of differences in lake elevations, (2) to find a common datum for measurements of relative crustal movement on all the Great Lakes, and (3) to compare rates of crustal movement computed in this study with results computed by other methods.

CHAPTER 2

THEORETICAL DEVELOPMENTS

Time series analysis was used to analyse two sets of time series in order to determine their significant statistical components. The two types of time series analysed were (a) mean monthly lake elevations, and (b) differences between mean monthly lake elevations as recorded at different gauges on a lake's circumference.

The analyses were used to validate the assumption that time series of differences in lake elevations can be represented by first order linear trends. A second assumption was made that these trends are estimates of relative rates of vertical crustal movement. The computed relative movements were then adjusted for compatibility by triangulation and circular adjustment, converted to absolute rates of movement by reference to a stable datum and finally computed in their most useful form as rates of movement of land relative to the mean lake surface.

2.1 Development of Relative Rates of Crustal Movement

The same form of time series analysis was performed for both of the types of time series described above. However, only the linear trends resulting from the analysis of the second type of time series were adjusted and used in later analyses.

2.1.1 General Model of Time Series

It was assumed that a time series, Y_t, can be represented by a linear additive model of the form

$$Y_t = T_t + P_t + R_t \quad 2.1$$

where T_t is a linear trend component, P_t is a periodic component and R_t is a stochastic component. Linearity in the trend component is used here in the sense of linearity of the regression coefficients (91). The process of determining the importance of each component involves (a) detection and removal of trends, (b) detection and removal of periodicities, and (c) analysis of stochastic component. At each step of the procedure (Figure 2.1) the presence of different components was detected using both time and frequency domain techniques.

2.1.1.1 Linear Trend Component

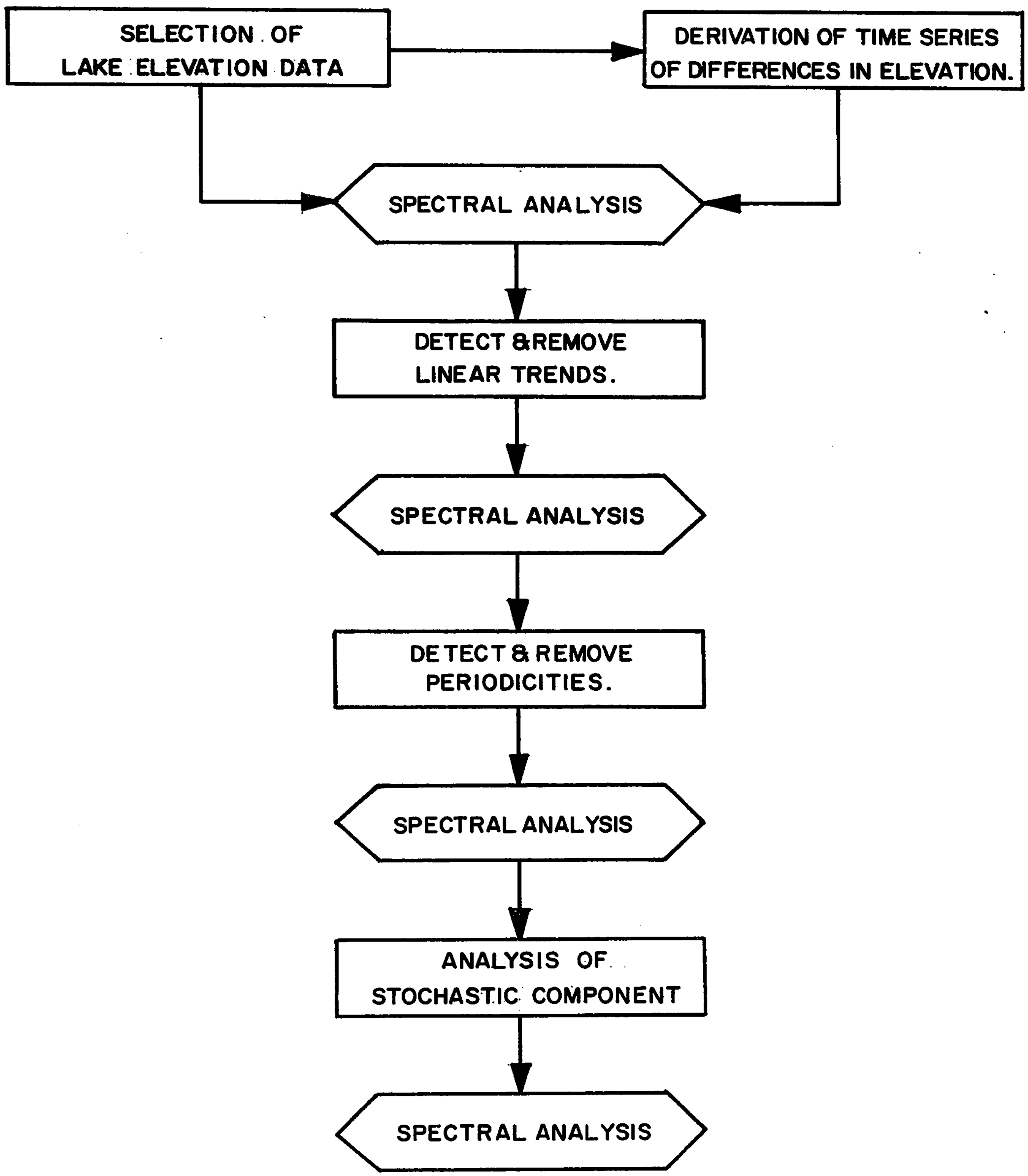
Linear trends were detected by the variate difference technique and removed by polynomial regression.

The variate difference method was applied to detect the linear trends present in the time series, and to determine the degree of a polynomial required to fit the trend.

A proposed model for a trend, T_t, is assumed to be composed of a linear trend (polynomial), $a_o + a_1t$, and on independent random variable, δ_t, as

FIGURE 2.1

ANALYSIS OF TIME SERIES



$$T_t = a_o + a_1 t + \delta_t \qquad 2.2$$

where a_o and a_1 are constants. Such a model gives rise to a method which allows elimination of a trend component by differencing. A successive differencing, $\Delta_t^1 = T_2 - T_1$, $T_3 - T_2$, ...; ....; $\Delta_t^r = T_t - T_1$, $T_{r+1} - T_2$, ...; will generate new series Δ_t^r, and therefore eliminate entirely any element which is a polynomial in time.

The variance of the differenced time series, var Δ_t^r is related to the variance of the independent random component (91), σ_δ^2, by

$$\text{var } \Delta_t^r = \sigma_\delta^2 \binom{2r}{r} \qquad 2.3$$

Equation 2.3 shows that the variance of the differenced time series is the product of a function of the order of the difference and the variance of the independent random component. By successively generating differences until the function var $\Delta_t^r / \binom{2r}{r}$ is constant for all further differences, r, the order of linear trend present can be determined as the number of values of var$\Delta_t^r / \binom{2r}{r}$ significantly different from the constant value σ_δ^2.

A polynomial regression is then used to remove any significant linear trend since this technique (in contrast to the moving average technique) does not alter the statistical properties of the other components of time series Y_t. The general form of polynomial used is

$$\hat{T}_t = \sum_{j=0}^{k} a_j t^j \qquad 2.4$$

where $\hat{T}_t$ is the computed value of the trend component at time t and a_j are linear regression coefficients. The first step in the polynomial regression procedure is to approximate the trend component with a first order linear model

$$\hat{T}_t = a_o + a_1 t \qquad 2.5$$

where $\hat{T}_t$ is the t-th value of the trend component computed from the regression equation.

The variance explained by this regression equation, σ^2_{reg}, is equal to the total variance of T_t, σ^2_T, minus the variance of the residuals, σ^2_{res}, where

$$\sigma^2_T = \frac{1}{N-1} \sum_{t=1}^{N} (T_t - \overline{T})^2 \qquad 2.6$$

and

$$\sigma^2_{res} = \frac{1}{N-2} \sum_{t=1}^{N} (T_t - \hat{T}_t)^2 \qquad 2.7$$

where N is the total number of observations of T_t.

The polynomial regression technique continues to add successive terms to the approximation to the trend component such as

$$\hat{T}_t = a_o + a_1 t + a_2 t^2 \qquad 2.8$$

and

$$\hat{T}_t = a_o + a_1 t + a_2 t^2 + a_3 t^3 \qquad 2.9$$

until the variance explained by the regression polynomial, σ^2_{reg}, no longer increases as j increases. At this point the regression is stopped and the fitted polynomial $\hat{T}_t$ is subtracted from the time series T_t.

2.1.1.2 Periodic Component

After removal of linear trends the time series is analysed for periodicity using the periodogram approach (68). The periodic component, P_t, is considered to be composed only of periodic sine and cosine functions of various amplitudes and phases arranged so that

$$P_t = \overline{P} + \sum_{j=1}^{k} (A_j \cos \frac{2\pi jt}{N} + B_j \sin \frac{2\pi jt}{N}) \qquad 2.10$$

where $\overline{P}$ is the mean value of the periodic component, P_t, k is the number of harmonics, N is the total number of points in the component, P_t, and A_j and B_j, The Fourier coefficients, are defined as

$$A_j = \frac{1}{N} \sum_{t=1}^{N} P_t \cos \frac{2\pi jt}{N} \qquad 2.11$$

and

$$B_j = \frac{1}{N} \sum_{t=1}^{N} P_t \sin \frac{2\pi jt}{N} \qquad 2.12$$

Once A_j and B_j have been computed for each harmonic, the amplitudes, C_j, and phases, θ_j, of the harmonics can be found from

$$C_j = (A_j^2 + B_j^2)^{1/2} \quad 2.13$$

$$\theta_j = \mathrm{Sin}^{-1} (A_j/C_j) \quad 2.14$$

A plot of C_j^2 versus harmonic, j, is called a periodogram.

2.1.1.3 Stochastic Component

After removal of the linear trend component, T_t, and periodic component, P_t, from the original time series, Y_t, the remaining portion, R_t, is assumed to be a stochastic component. Furthermore, it is assumed that the stochastic component can be represented by a linear autoregressive model (Markov model) expressed as

$$R_t = \sum_{j=0}^{k} \alpha_j R_{t-j} + \varepsilon_t \quad 2.15$$

where α_j are coefficients, and ε_t is an independent random variable having mean equal zero and variance equal σ_ε^2.

For practical purposes it is frequently sufficient to use only a few terms in the above equation to approximate the sequence. It is assumed that first-order and second-order approximations, respectively, would approximate the sequence sufficiently, that is

$$R_t = \alpha_1 R_{t-1} + \varepsilon_t \quad 2.16$$

and

$$R_t = \alpha_1 R_{t-1} + \alpha_2 R_{t-2} + \varepsilon_t \quad 2.17$$

Under the assumption of stationarity, when Eq. 2.16 is multiplied by R_{t-1}, and the expectation operation is performed, the result

$$\rho_k - \rho_k \, \rho_{k-1} = 0, \qquad k > 0 \qquad 2.18$$

obtains, where $E(R_t, R_{t-1}) = C(k)$ is the autocovariance and $\rho_k = C(k)/C(o)$ is the autocorrelation function. From Eq. 2.18 it can be shown that the theoretical correlogram is given by

$$\rho_k = \rho_1^k \qquad 2.19$$

in which ρ_1 is estimated by the sample first autocorrelation coefficient.

Similarly, for the second-order autoregressive model, the theoretical correlogram is given by

$$\rho_k = \frac{p^k \sin(k\phi + \psi)}{\sin \psi} \qquad 2.20$$

where $-1 \leqslant \alpha_1 < 0$ and $\alpha_1^2 + 4\alpha_2 < 0$, and $p = \sqrt{-\alpha_2}$, $\cos\phi = a_1/2\sqrt{-\alpha_2}$, and $\tan\psi = [(1 + p^2)/(1 - p^2)] \tan\phi$.

The suitability of first- or second-order models are tested based on the χ^2 tests. Whereas, the independent random variable in the Markov model is tested (7) whether it might have normal or log-normal distributions.

2.1.1.4 Spectral Analysis

Two related techniques, correlation analysis and spectral analysis can be used as an aid in distinguishing the various components present in a time series. Correlation analysis shows, in a time domain, to what extent and by what process points separated in the time series are related to each other. Spectral analysis shows the same relationships transformed to the frequency domain.

The theory of variance spectrum analysis postulates that the time series under examination is a sample from a population series characterized by a continuous spectrum (39). That is, the observed time series is a random sample of a process in time (or space) that is made up of oscillations of all possible frequencies. A variance spectrum partitions the variance into a number of intervals or bands of frequency and is related to the autocorrelation function, ρ_k, by

$$v(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_k \exp(-ikw)dk \qquad 2.22$$

which, since ρ_k is an even function, becomes

$$v(w) = \frac{1}{\pi} \int_{0}^{\infty} \rho_k \cos(wk)dk \qquad 2.23$$

A plot of v(w) versus w, the frequency, is called a variance density spectrum. The integral of the spectrum is equal to the normalized variance of the time series

$$\rho_o = \int_{0}^{\infty} v(w)dw = 1 \qquad 2.24$$

The statistical properties of the spectrum are superior to those of the correlogram since both the estimation problem and the sampling properties of the spectrum are more simply and exactly treated (39).

2.1.2 Adjustment of Rates of Relative Crustal Movement

The first order linear trends derived in the time series analyses of differences in the mean monthly lake elevations are assumed to be rates of relative crustal movement. Because of the different periods of gauge record available these rates of movement were adjusted to provide overall compatibility.

2.1.2.1 Adjustment by Triangulation

The relative rates of movement (first order linear trends) were adjusted for each lake separately using a least-squares triangulation technique in which each rate of movement between two gauges was assigned a "weight" or measure of relative reliability. These weights were computed for each rate of movement as R^2N where R^2 is the coefficient of determination (91) for the regression equation relating differences in mean monthly elevations between two gauges to time, and N is the number of months of record common to both gauges. In this way the applied weight increases as the proportion of the time series explained by the linear trend increases and also increases as the period of common record increases. The larger the explained variance and the longer the period of common record the greater is the reliability that can be attached to the corresponding rate of relative movement and so the higher the weighting factor used in the adjustment. This method assumes that the accuracy of the method of determining a rate of movement is independent of the distance between gauges.

The value of the weighting factor R^2N was computed for all pairs of gauges, and those three pairs having the three highest weights; $(R^2N)_j$, $j = 1,2,3$, (say between gauges A, B, and C) were used in the initial triangulation. The residual or observation errors for these three rates of relative movement can be written as

$$e_1 = r_1 - m_1 \qquad 2.25$$

$$e_2 = r_2 - m_2 \qquad 2.26$$

and

$$e_3 = (r_2 - r_1) - m_3 \qquad 2.27$$

where r_1 and r_2 are the true rates of relative movement between gauges A and B and gauges A and C respectively and m_1, m_2 and m_3 are the computed rates of movement between A and B, A and C, and B and C respectively. The sum of the squares of the weighted observation errors is

$$E = \frac{\sum_{j=1}^{3} (R^2N)_j \; e_j^2}{\sum_{j=1}^{3} (R^2N)_j} \qquad 2.28$$

Simplifying and differentiating equation 2.28 with respect to r_1 and r_2 yields two normal equations:

$$c_1r_1 + c_2r_2 + c_3 = 0 \qquad 2.29$$

and

$$d_1r_1 + d_2r_2 + d_3 = 0 \qquad 2.30$$

where c_1, c_2, c_3, d_1, d_2 and d_3 are constants. Solving these two equations simultaneously the true rates of relative movement, r_1, r_2, and r_1-r_2 were determined. Using these three adjusted rates of relative movement the other computed rates of relative movement (first order linear trends in time series of differences in mean monthly elevations) were adjusted to agreement within any given triangulation.

Once adjusted rates of relative crustal movement were obtained between all pairs of gauges around a particular lake, then isolines or lines of equal rate of relative movement were plotted on a map of the lake.

2.1.2.2 Adjustment by Circular Distribution

Assume that crustal movement is acting on a lake basin as an entire unit and the movement is in only one direction, say the northern side of the basin is rising relative to the southern side. Then from a position A, in the centre of the lake basin, the relative movement at gauges on the circumference of a circle with centre A would, when plotted against angular position, form a sine curve. In practice gauges are not available at the lake centres, and the gauges which are available are not distributed around the circumference of a circle. The modification used is to plot rates of crustal movement as measured at gauges around the lake relative to one reference gauge on the lake, in feet per 100 miles per 100 years, versus the whole circle bearings of the gauges relative to the reference gauge.

2.2 Development of Absolute Rates of Crustal Movement

Relative rates of vertical crustal movement are restricted, in a sense that comparison cannot be made between relative rates on one lake and relative rates on another lake. This could be overcome by using sea level as a common datum and transferring the datum by land levelling. Alternately a form of geologic datum could be used to convert relative rates of movement to absolute rates.

2.2.1 Geodetic Relevelling as a Means of Transferring a Datum

In order to obtain absolute rates of crustal movement the relative rates should be referred to some constant datum. The most obvious datum to use is mean sea level. Changes in sea level can, however, be caused by factors such as uplift of areas now under the sea, sedimentation, changes in volume of glaciers and polar ice caps, spreading of the continents, long-term variations in temperature or pressure, etc., (30). Over geologic time mean sea level has undoubtedly changed many times, notably as the volume of water locked up in the glaciers has changed. However, no great change in the volume of water in the oceans has taken place in the last hundred years or so (66). Tidal gauges at various locations do show a change with time in the relationship between a gauge datum on land and the mean sea level, but the rate of change is not sufficiently well defined (30,32,34,54,66) that sea level in one particular year could be used as a datum.

From some mean sea level, land levelling could be used to obtain the relative elevation of a point in an area suspected of movement. Comparing this elevation with an elevation obtained from a similar set of land levels at some other time, an estimate of the rate of crustal movement could be obtained. The process would be complicated in practice because of the numerous possible errors. Among other factors, the possible error in the rate of movement determined in this manner is proportional to the number of years between levelling surveys; the greater the time difference, the less will be the uncertainty in the result. However, using sets of levels taken many years ago introduces the problem of unequal errors; level lines run today using modern equipment and techniques are far more accurate than level lines run in the nineteenth century.

2.2.2 Nipissing Zero Isobase as a Datum

If, as assumed, the movement being measured is the result of isostatic adjustment following the last glacial retreat, then the maximum uplift would be in the northeast (assuming that the maximum thickness of the last glacial advance was over eastern Hudson's Bay). There would be no current subsidence in the Great Lakes region and, somewhere along the southern boundary, a line of zero movement would exist establishing the southern limit of the, presumably diminishing, circle of influence of the uplift. If this line of zero movement could be established then all rates of crustal movement presently determined could be referred to this line.

One method of locating this line of zero movement is by examining the remaining shorelines of lakes formed during the period of maximum depression of the land surface. The Nipissing Shoreline, formed between 6,000 and 4,000 years ago (60), is the most recent prominent shoreline of the ancestral Great Lakes, (20). It follows fairly closely the shoreline of the present Lakes Superior and Michigan-Huron. An isobase is a form of contour-line joining points of equal elevation along the geologic shoreline, and the zero isobase is a contour-line joining points at which the gradient of the shoreline changes. South of the zero isobase, the beaches are horizontal; north of the zero isobase, the beaches are warped upward. The Nipissing zero isobase has been traced from the Bayfield peninsula on Lake Superior in an east-south-east direction crossing Lake Michigan near Escanaba and Lake Huron near Alpena and Port Elgin; its projection eastward would pass just north of Toronto (52).

To locate a line of zero movement from lake elevations, the computed values of relative movement are examined and, where two stations exhibit zero relative movement, it can be assumed they are both to the south of some zero isobase. The results can then be compared with the geologic evidence regarding the isobase.

From the viewpoint of the present uncertainty of levelling to mean sea level, and the geologic evidence, it was decided to use the Nipissing zero isobase as a datum from which to compute absolute rates of crustal movement.

2.3 Development of Rates of Movement of Land Relative to Lake Surface

The procedure followed in developing rates of movement of land relative to the mean lake surface depends upon the nature of the lake. Lakes Superior and Ontario are presently regulated, that is the lake discharges are controlled by regulatory structures. Lakes Michigan-Huron and Erie, on the other hand, are not regulated.

If any overall movement of the surface of an unregulated lake occurs (and it is expected that there would be a rise in lake elevations as the land around the lake rises) then this would show up as a trend in each gauge record; that is, if a gauge shows a trend of q_1 ft/100 years and has a computed absolute rate of movement of q_2 ft/100 years, then the mean lake surface is moving at a rate of q_2-q_1 ft/100 years.

Assuming that a simple weir relationship controls the outflow of a lake and that the outlet area is rising relative to some other parts of the lake due to crustal movement, then the increase in outlet elevation will decrease the lake outflows. The outflow will remain decreased until the elevation of the lake has risen sufficiently to restore outlet conditions. Thus, to flow the same long-term mean outflow, which is required by continuity considerations, the mean lake elevation must be continuously rising to counteract the effects of crustal movement. In practice the crustal movement effect will act continuously and the mean lake elevation will respond continuously, without any significant change in outflow being noticed. This means that the mean elevation of such a lake must be rising at a rate equal in magnitude and direction to the rate at which the land around the lake outlet is rising.

A second approach possible for unregulated lakes is to measure linear trends in the lake inflow and outflow over as long a period as possible and then compute the resulting change in storage. If it is found that there has been no long-term change in storage then the result of tipping the lake by crustal movement can be determined. Taking the longest possible cross-section of the lake at right angles to the isolines of rate of movement, it is clear that an upward movement of the land by one foot over a given time period at one end of the section relative to the land at the other end of the section must result in a rise in mean lake elevation of one-half foot over the same period of time. This assumes that the lake basin moves as a complete unit.

In the cases of Lakes Superior and Ontario, the same procedure can be followed provided that it is assumed that the regulation process has not introduced any long-term trend into lake elevations.

Combining the absolute rates of movement derived in Section 2.2 with the rate of movement of the mean lake elevation derived in Section 2.3 will result in rates of movement of gauges around the lake relative to the mean lake elevation.

CHAPTER 3

GREAT LAKES SYSTEM

3.1 General Description of System

The Great Lakes basin consists of four principal lakes (Superior, Michigan-Huron, Erie and Ontario) and the peripheral lands of the tributary areas around the shores (87). These vary in width from less than 10 miles to more than 100 miles. For hydrologic purposes, Lakes Michigan and Huron are considered to be one lake since there is no change in elevation at the Straits of Mackinac. The connecting channels of the lakes are the St. Marys River between Lake Superior and Lake Michigan-Huron, the St. Clair River, Lake St. Clair and Detroit River system between Lake Michigan-Huron and Lake Erie, the Niagara River between Lake Erie and Lake Ontario and the St. Lawrence River between Lake Ontario and the Atlantic Ocean.

Surrounding the basin of the Great Lakes are the Appalachian mountains on the south, the Adirondacs on the east, the Precambrian Shield on the north and the Mississippi Valley in the west. The surface of the basin shows the effects of the glacial movements of the Ice Age. In the northern portions of the basin is the Precambrian Shield consisting of rock knobs which have been denuded of their soil and ground down by glacial action, resulting in rough wooded country.

In the southern areas, the country is generally rolling upland which slopes to the flatter lowlands around the lakes. Some of these lower areas have coverings of sand and silt which were deposited over the glacial tills during the high lake elevations of the post-glacial period.

The climate of the Great Lakes region is dominated by the easterly movement of air masses. The region lies in the path of the prevailing westerly winds and is affected by major storms moving out of the south. The easterly movement of air results in moderated air temperatures and extremely high snowfalls in the lee of the lakes. The mean annual temperature over the basin varies from 31°F, with an average length of frost-free period of 75 days north of Lake Superior, to 48°F and an average length of frost-free period of 180 days at the western end of Lake Erie in the south.

Table 3.1 shows some of the physical data used in describing the basin (87). The important physical characteristics of the basin are the ratio of water to land, column (2) vs (3), and the large storage capacity available in the lakes, column (4). The magnitude of water quantity changes is shown in the total of column (4), where one foot depth over all the lakes is equivalent to the mean flow of the St. Lawrence River for about four months. The average elevation above mean sea level, column (5), indicates that there is about 22 feet of fall between Lake Superior and Lake Michigan-Huron, about 8 feet between Lake Michigan-Huron and Lake Erie,

TABLE 3.1

PHYSICAL DATA PERTAINING TO THE GREAT LAKES[1]

Lake	Drainage Area: Land Area (Square Miles)	Drainage Area: Water Surface Area (Square Miles)	Storage Capacity Per Foot of Elevation of Lake (cfs for one month)	Average Elevation (above m.s.l.)	Range of Monthly Mean Elevation (ft.)	Outlet River	Mean Outflow: (cfs)	Mean Outflow: (Inches on Total Drainage Basin)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Superior	49,000	31,800	337,000	600.4	4.1	St. Marys	75,000	12.6
Michigan-Huron	46,600 49,400	22,400 23,000	481,000	578.8	6.2	St. Clair Lake St. Clair Detroit	189,000	11.6
Erie	29,400	9,900	103,000	570.4	5.4	Niagara	205,000	10.6
Ontario	27,100	7,500	80,000	244,6	6.6	St. Lawrence	241,000	11.1
TOTALS	201,500	94,600	1,003,000					

1 From Witherspoon (87)

2 Period of record 1860 - 1954

about 326 feet from Lake Erie to Lake Ontario and about 245 feet difference in elevation from Lake Ontario to the Atlantic Ocean. This demonstrates the importance of these large reservoirs to the development of hydro-electric power since the major portion of this head is utilized. In column (6) the range of lake elevation during the period from 1860 to 1954 is shown. These figures, when combined with column (4), show the large amount of storage available in the lakes between high and low water. The last two columns of the Table show the relative contribution of each lake to the outflow and the average annual depth of runoff from each basin. These characteristics of the Great Lakes basin make it a unique hydrologic system.

3.2 Mechanism of Vertical Crustal Movement

It is assumed that the cause of the vertical movement measured in the Great Lakes area is isostatic recovery following the retreat of the last continental glaciation.

From the study of earthquakes (55) a great deal of information about the structure of the earth has been gained. The seismic evidence, in agreement with geologic indications, shows that the earth has a crust composed of several distinct layers. The outermost layer is of granitic composition and is covered locally by sedimentary rocks of various thickness. It rests on a layer whose composition approximates vitreous basalt, and this in turn rests on an ultrabasic rock, peridotite, at a depth some 30 km below the surface. The average thickness of the granitic and basaltic layers are about 10 and 20 km, respectively. The sedimentary layer is discontinuous, and the granitic layer is not present everywhere below the oceans. From a consideration of the density of the various layers, more information can be obtained about the earth's core. The density of the granitic layer is about 2.67 and the density of the basalt and peridotite not much greater. The average density of the entire earth, however, is 5.5 and therefore the core must be composed of heavier material than the crustal material in order to compensate for the lighter layers. Calculations (55) give a density of approximately 8 for the core material. This figure is slightly greater than the density of iron and slightly less than the density of nickel, so the core probably consists of a mixture of these two elements and, for this reason, is called nifel from the chemical symbols.

Similarly, the granitic and sedimentary layers are called sial from the initial letters of the preponderant oxides, silica and alumina; the basaltic layer is called sima from silica and magnesia. The sial is crystalline and rigid but the sima lacks rigidity and will yield slowly to long-continued stresses. The marked break at the base of the crust, between the sima and the mantle, is known as the Mohorovicic discontinuity.

Geologists, having the above concept of the earth's structure, visualize continents as large sial "rafts" floating on a "sea" of sima. Since the density of sima (nearly 3.0) is not much greater than that of sial (about 2.7), the "rafts" are largely submerged.

Large topographical features such as mountains, which rise above the general level of the continent, have a correspondingly greater depth of sial beneath them; while shallow features such as continental shelves penetrate only a small distance into the sima. That state of balance which is maintained between adjacent columns of matter in the crust and supported by the basaltic substratum, is called isostasy (4). It appears probable that very large topographical features on the earth's surface are bounded by faults and supported by the upward pressure of the substratum, that is, they are isostatically compensated on a regional scale. The Alps and the Rockies, for example, are essentially balanced in this way. Smaller local features, however, may be supported not by isostasy but by the rigidity of the crust.

Considering an individual column of the earth's crust, an analogy can be made to a weighted rod floating vertically in water. The ratio of the total length of the rod to its height above the water surface is a constant, depending on the density of the rod material. If the rod is pushed deeper into the water, a volume of water will be displaced and a force will be felt attempting to push it back to its original position. It is postulated (4) that this occurs to the earth's crust. As a glacier advances over a continent it pushes the sial "raft" down into the sima, displacing the latter and creating a balancing up-thrust force. Later, as the glacier retreats, the forces will again be out of equilibrium and the up-thrust will push the sial "raft" back to its original position. Due to the fluid nature of the sima and the rigidity of the sial, this upward movement is likely to be a series of short fast rises superimposed on a long steady rise.

It is probable that in most cases the degree of restoration will not be complete due to the weight of surface water and glacially deposited drift. In the case of the Great Lakes region, the drift is commonly believed to be several hundred feet thick in places (60). The opposite situation will, of course, occur in the Canadian Shield, the source region for the drift deposited to the south. There, since material was removed during glaciation, uplift probably proceeded to a greater degree although not sufficiently to return the area to its original elevation.

In the Great Lakes region, retreat of the Labrador sector of the Laurentide Ice Sheet from its point of farthest advance, about 150 miles south of Lake Michigan, began some 17,000 years ago (60). At first the glacial lakes, formed of meltwater between the basin rims and the ice front, discharged southwards into the Mississippi River. Subsequent advances and retreats of the ice front together with associated isostatic activity produced a complex system of lakes with discharges varying in direction with time. Lakes approximately occupying the present Lake Huron basin, for example, discharged at different times through the Mattawa-Ottawa Rivers, the Kirkfield-Fenelon Falls and Trent Valley river systems, and various other outlet channels as well as the present St. Clair-Detroit Rivers.

3.3 Water Elevation Data Selection

Great Lakes water elevations are recorded by the Lake Survey Center, U.S. Department of Commerce and by the Water Survey of Canada.

From the records kept by these organizations forty gauges were selected on the basis of location and length of continuous record. The location criterion was used to select gauges situated as evenly as possible around all four lakes. Gauges with periods of record less than fifteen years were eliminated since this is considered to be the minimum period for detection of long term trends.

The periods of continuous record used for the gauges are shown on Figure 3.1 and the locations of the gauges used are given on Figure 3.2.

The selection of monthly mean elevations is a compromise between the additional information content of daily data with its correspondingly high stochastic component and

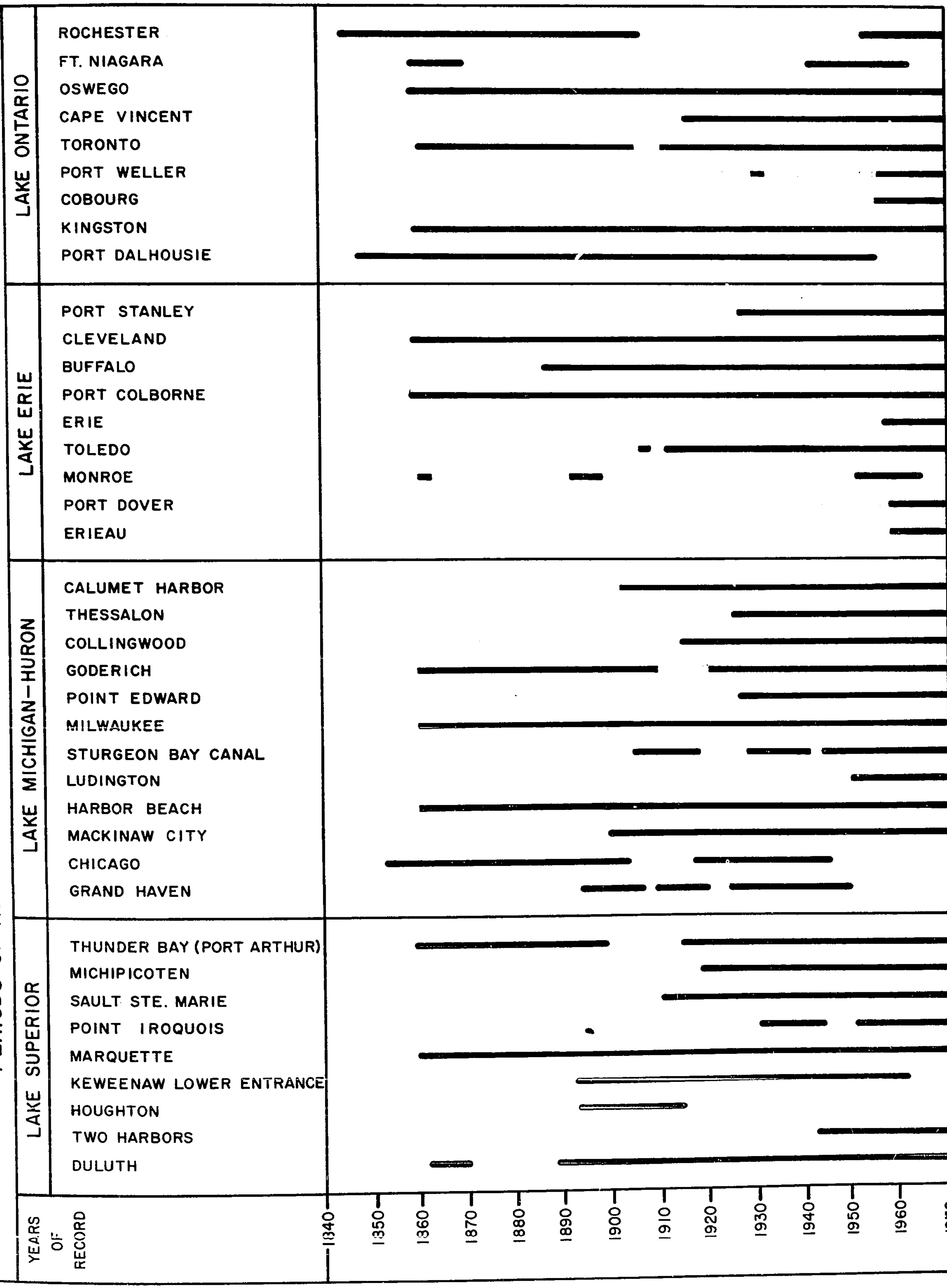
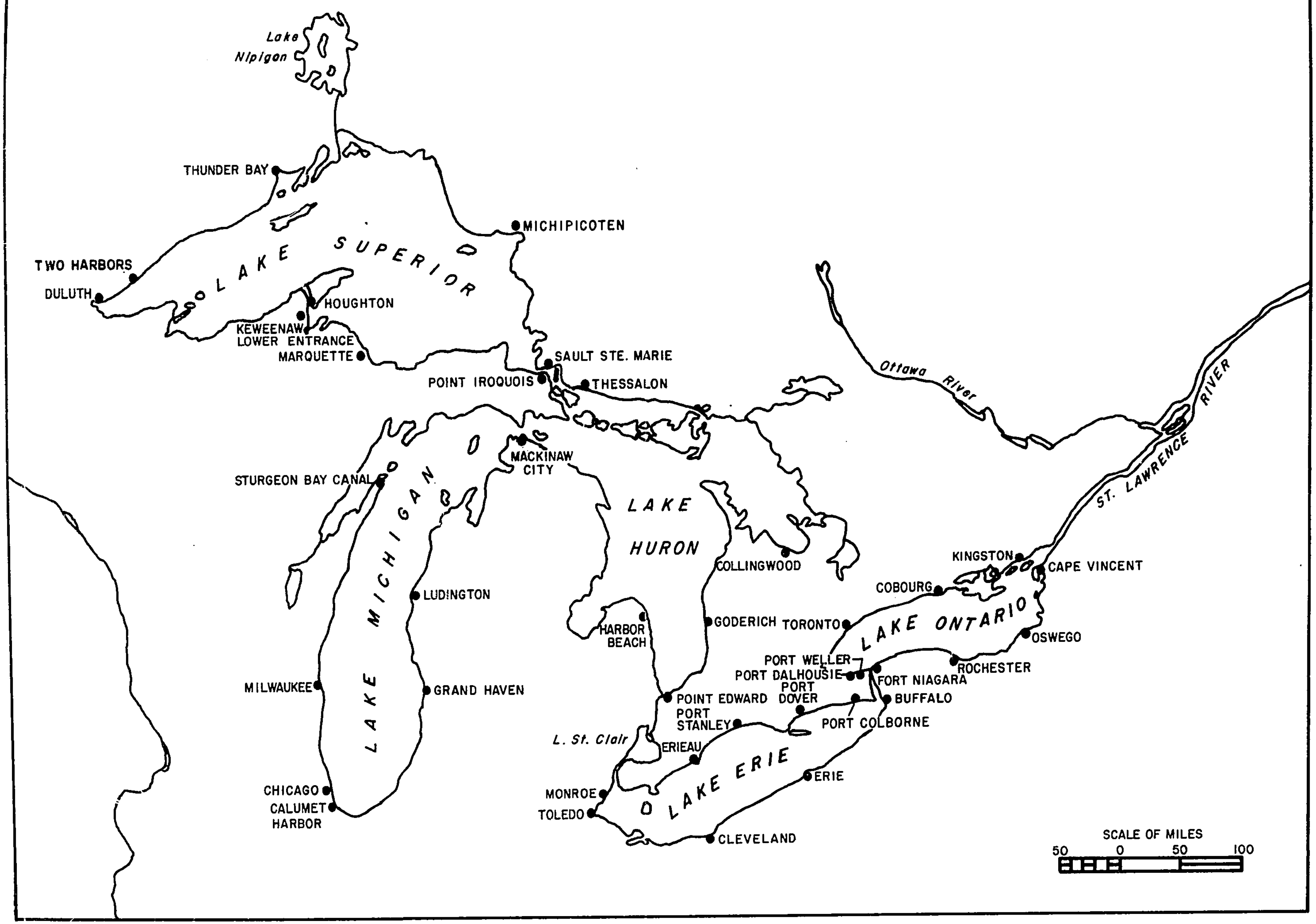


FIGURE 3·1

PERIODS OF RECORD OF WATER ELEVATION GAUGES FOR THE GREAT LAKES

FIGURE 3·2

LOCATIONS OF WATER ELEVATION GAUGES FOR THE GREAT LAKES



vastly increased computation time and at the other extreme, annual or 5-year mean elevations with their reduced information content and smoothing effects.

The forty gauges selected have a total period of record of 31,060 months. Rather than conduct time series analyses on all this data it was decided to carry out experimental analyses for Lake Superior data only. From an examination of the data it was decided to use mean monthly lake elevations at Thunder Bay, Michipicoten, Sault Ste. Marie, Marquette and Duluth for the period 1915 to 1970 for these experimental analyses.

CHAPTER 4

NUMERICAL PROCEDURE

Time series of mean monthly lake elevations and differences in mean monthly lake elevations were analysed for trend, periodic and stochastic components. Autocorrelation and spectral analyses were used to indicate statistically significant components present in the time series.

4.1 Linear Trend Component

It was assumed that any trends present in the time series were linear. The order of linear trend was determined by using the variate difference technique. Successive series of differences were taken

$$\Delta_1^1 = T_2 - T_1 \qquad 4.1$$

$$\Delta_2^1 = T_3 - T_2 \qquad 4.2$$

and

$$\Delta_1^2 = \Delta_2^1 - \Delta_1^1 \qquad 4.3$$

$$\Delta_2^2 = \Delta_3^1 - \Delta_2^1 \qquad 4.4$$

and so on. For each of these series the variances var Δ_t^1, var Δ_t^2, ..., var Δ_t^r were computed and the values var $\Delta_t^r/\binom{2r}{r}$ were plotted versus r for values of r from 1 to 10. The principle of the variate difference technique is that from an initially large value, var $\Delta_t^r/\binom{2r}{r}$, will diminish as r increases until the value remains constant for any further increase in r. The number of values of var $\Delta_t^r/\binom{2r}{r}$ which are significantly different from the constant value is an indication of the order of linear trend present.

Linear trends were removed by a polynomial regression analysis. The procedure used was to generate a series of variables, t^2, t^3 and t^4. The means and variances of these variables were computed and their matrix of intercorrelations with the dependent variable was inverted using the standard Gauss-Jordan Method. The inverse matrix was then employed in a standard multiple linear regression analysis for each stage of the polynomial. The analysis ceased when the addition of another order of polynomial produced no reduction in the residual sum of squares.

Considering one particular step in the polynomial regression,

$$T_t = a_o + a_1 t + a_2 t^2 \qquad 4.5$$

the results of the method include the intercept, a_o, the regression coefficients a_1 and a_2, the multiple correlation coefficient, R, where

$$R^2 = \frac{\text{sum of squares due to regression}}{\text{total sum of squares}} \qquad 4.6$$

$$R^2 = \frac{\sum_{t=1}^{N} (\hat{T}_t - \overline{T})^2}{\sum_{t=1}^{N} (T_t - \overline{T})^2} \qquad 4.7$$

where T_t is the recorded value of the time series, $\overline{T}$ is the mean of T_t, t = 1, 2, ..., N and $\hat{T}_t$ is the predicted value from Equation 4.5.

Also given in the results of the regression are the standard error of estimate, which is the square root of the difference between the total sum of squares and the sum of squares due to regression divided by n-3 degrees of freedom, and the F value, which is the sum of squares due to regression divided by the standard error of estimate.

$$SE = \left[\frac{\sum_{t=1}^{N} (T_t - \hat{T}_t)^2}{N-3} \right]^{1/2} \qquad 4.8$$

The F value, computed as

$$F = \frac{\sum_{t=1}^{N} (\hat{T}_t - \overline{T})^2}{(SE)^2} \qquad 4.9$$

was used as a test of the hypothesis $H_o : a_1 = 0$, that is as a test of the significance of the regression equation.

Finally the improvement in terms of sum of squares, that is the sum of squares due to regression at one stage minus the sum of squares due to regression at the previous stage, was used as an indicator of the significance of each step in the regression.

4.2 Periodic Component

After removal of linear trends the remaining time series was analysed for periodicities using a periodogram technique. A table of monthly means and standard deviations of the time series was used as a preliminary indication of the presence of any periodicity.

The theoretical equations (Equations 2.10, 2.11 and 2.12) for the periodogram (68) were modified (91) to give a simpler numerical procedure to

$$A_j = \frac{1}{12} \sum_{i=1}^{12} (\overline{P}_i - \overline{\overline{P}}) \cos \frac{2\pi ij}{12} \qquad 4.10$$

and

$$B_j = \frac{1}{12} \sum_{i=1}^{12} (\overline{P}_i - \overline{\overline{P}}) \sin \frac{2\pi ij}{12} \qquad 4.11$$

where $\overline{P}_i$ is the long term mean for the ith period, i = 1, 2, ..., 12, $\overline{\overline{P}}$ is the long term mean, and j is the harmonic number. That is, in a monthly time series, $\overline{\overline{P}}$ is the annual mean, while $\overline{P}_i$ are the 12 monthly means.

If var P is the total variance of the periodic component then

$$\text{var } P = \text{var } P_m + \text{var } P_{sd} + \text{var } R \qquad 4.12$$

where var P_m is the variance due to periodicity in the monthly means, var P_{sd} is the variance due to periodicity in the monthly standard deviations and var R is the variance of the residual stochastic component. But

$$\text{var } P_m = \frac{1}{12} \sum_{j=1}^{12} (\overline{P}_i - \overline{\overline{P}})^2 \qquad 4.13$$

and

$$\text{var } P_{sd} = \frac{1}{12} \sum_{j=1}^{12} (S_i - \overline{S})^2 \qquad 4.14$$

where S_i and $\overline{S}$ are the monthly and annual standard deviations of the time series P_t. The results of the harmonic analysis then divide var P_m and var P_{sd} into the proportions explained by the various harmonics of the annual periodicity and express these as percentages of var P_m and var P_{sd}.

The significance of the amplitude of each harmonic, C_j^2, was tested (Equation 2.13) using Fisher's g statistic (91)

$$g_j = \frac{C_j^2}{\sum_{j=1}^{k} C_j^2} \qquad 4.15$$

where k is the total number of harmonics.

4.3 Stochastic Component

The residual left from time series, Y_t, after removal of trend component, T_t, and periodic component, P_t, is assumed to be a stochastic component. The further assumption that the stochastic component consists of a first or second order linear autoregressive model was tested using χ^2 tests on the sample and theoretical autocorrelation coefficients.

The sample autocorrelation coefficients were computed from

$$r(k) = \frac{\text{cov }(R_t,\ R_{t+k})}{(\text{var } R_t \text{ var } R_{t+k})^{1/2}} \qquad 4.16$$

while the theoretical autocorrelation coefficients for first and second order linear autoregressive models were computed from Equations 2.19 and 2.20 using sample estimates of ρ_1 and ρ_1 and ρ_2 from

$$\alpha_1 = \rho_1 \qquad 4.17$$

for the first order model, and

$$\alpha_1 = \frac{\rho_1\ (1 - \rho_2)}{1 - \rho_1^2} \qquad 4.18$$

and

$$\alpha_2 = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} \qquad 4.19$$

for the second order model respectively.

Confidence limits were computed for the sample autocorrelation coefficients using Anderson's method (1)

$$CL_\alpha = \frac{-1 \pm \eta_\alpha \sqrt{N-k-2}}{N-k-1} \qquad 4.20$$

where N is the number of observed values in the time series, k is the order of the autocorrelation coefficient and η_α is the standard normal deviate for a two-tail test at a significance level of α percent.

The chi-square test (24), used to test the goodness of fit of the sample to the two autoregressive models, is based on the principle that the total range of the sample distribution being tested is divided into k mutually exclusive and exhaustive class intervals, the jth class containing O_j observations and having an expected number of observations E_j. A test statistic is defined as

$$\chi^2 = \sum_{j=1}^{k} \frac{(O_j - E_j)^2}{E_j} \qquad 4.21$$

which is distributed as chi-square with k-1 degrees of freedom.

The independent random variable in the autoregressive models was tested against normal and log-normal distributions using χ^2 and Kolmogorov-Smirnov (45) tests. The Kolmogorov statistic, D, is based on the deviations of the sample distribution function $F(\varepsilon)$ from the completely specified continuous hypothetical distribution function $F_o(\varepsilon)$ such that

$$D = \max |F(\varepsilon) - F_o(\varepsilon)| \qquad 4.22$$

The test requires that the value of D computed from the sample distribution be less than the tabulated value of D at the required confidence level.

Probability density and cumulative probability distributions of the independent random variable in the autoregressive models were computed and a set of non-parametric randomness tests were performed.

In the first test the time series is tested for any clustering tendency, that is to say any tendency of the independent random variable in the autoregressive models towards groupings in order of magnitude. This is done by classifying the observations into a few broad categories by quantities and counting the number of runs of increasing or decreasing magnitude in each category. The population mean of the time series is defined as

$$M_1 = \frac{N(N+1) - \sum_{i=1}^{m} n_i^2}{N} \qquad 4.23$$

where N is the total number of observations and n_i is the number of observations in the ith category and m is the number of categories. The population standard deviation, σ_1, is defined (7) as

$$\sigma_1 = \left[\frac{\sum_{i=1}^{m} n_i^2 \left[\sum_{i=1}^{m} n_i^2 + N(N+1) \right] - 2N \sum_{i=1}^{m} n_i^3 - N^3}{N^2 \, (N-1)} \right]^{1/2} \qquad 4.24$$

while the standard normal variate on which tests may be made is

$$K_1 = \frac{(G_1 + 0.5 - M_1)}{\sigma_1} \qquad 4.25$$

where G_1 is the number of runs counted.

The second non-parametric test checks whether the directions of movement in the independent random variable in the autoregressive models tends to cluster. This would indicate that the directions of movement tend to persist. The population mean and standard deviation for this test are given by

$$M_2 = \frac{2N-1}{3} \qquad 4.26$$

and

$$\sigma_2 = \left[\frac{16N-29}{90}\right]^{1/2} \qquad 4.27$$

so that K_2, the standard normal variate is

$$K_2 = \frac{G_2 + 0.5 - M_2}{\sigma_2} \qquad 4.28$$

where G_2 is the number of up or down runs detected.

The third test shows if there is any predominance of upward or downward changes. The test for trend in direction is done by counting the number of plus and minus signs in the independent random variable in the autoregressive models. The number of positive and negative signs should be equal if there is no predominating movement. The mean is given by

$$M_3 = \frac{N-1}{2} \qquad 4.29$$

the standard deviation is given by

$$\sigma_3 = \left[\frac{N+1}{12}\right]^{1/2} \qquad 4.30$$

and the standard normal variate is given by

$$K_3 = \frac{M_3 - G_3 - 0.5}{\sigma_3} \qquad 4.31$$

where G_3 is the smaller number of either positive or negative signs.

The sampling distribution for all three tests is sufficiently well approximated by a normal distribution for a relatively large number of observations (7). The normal probabilities corresponding to the deviations of the number in the run from the expected value (expressed in terms of the standard normal variable) indicate the probability of this value occurring in a series of random independent observations. Probabilities below 0.01 are used as an arbitrary indication of randomness.

4.4 Spectral Analysis

Sample autocorrelation coefficients (91) were computed from Equation 4.16, while confidence limits for the autocorrelation function were calculated using Anderson's method (1) as in Equation 4.21.

The raw spectral density was derived from the autocovariances, $C(\ell)$, by

$$v(k) = \frac{n}{\ell}\left[C(o) + C(\ell)\cos k\pi + 2\sum_{j=1}^{\ell-1} C(j)\cos\frac{kj\pi}{\ell}\right] \qquad 4.32$$

where n is a weighting factor equal to 0.5 when $k = 1$ and ℓ and equal to 1.0 for $1 < k < \ell$ where ℓ is total number of autocovariances computed.

The raw spectral density v(k) was then smoothed using a Hamming (39) filter

$$u(k) = 0.23\ v(k-1) + 0.54\ v(k) + 0.23\ v(k+1) \qquad 4.33$$

for k = 2, 3 ...., ℓ, while for k = 1

$$u(1) = 0.54\ v(1) + 0.46\ v(2) \qquad 4.34$$

and for k = ℓ

$$u(\ell) = 0.46\ v(\ell-1) + 0.54\ v(\ell) \qquad 4.35$$

Confidence limits for the spectral density (39) were computed as

$$CL_{\alpha}(N,k) = \frac{\chi^2_{100-\alpha}(\gamma)}{\gamma} \qquad 4.36$$

and

$$CL^1_{\alpha}(N,k) = \frac{\chi^2_{\alpha}(\gamma)}{\gamma} \qquad 4.37$$

where CL_{α} and CL^1_{α} are factors by which the mean estimated spectrum is multiplied, $\gamma = 2N/k$ is defined as the equivalent degrees of freedom where N is the number of observed values and k is the lag time, α is the required confidence level and $\chi^2_{\alpha}(\gamma)$ is the α% value of the chi-square distribution with γ degrees of freedom.

The spectral estimates were plotted on a logarithmic scale for two reasons; (a) It has been found (91) that for most natural time series low frequencies are of more importance than high frequencies and a logarithmic scale brings out this difference, and (b) confidence limits can be plotted more easily on a logarithmic scale since this involves only adding a constant factor to the mean spectral estimate.

A test was made of the spectral density computations by determining whether or not the sum of the spectral density estimates, $\sum_{k=1}^{\ell} v(k)$ was equal to the original variance of the time series, var Y_t.

CHAPTER 5

RESULTS AND DISCUSSION

Time series analyses were carried out on an experimental basis for mean monthly Lake Superior elevations as recorded at five different gauges. Time series analyses were also carried out on the four sets of differences in mean monthly lake elevations. Following these experimental analyses, linear trends were computed for the remaining gauges around the Great Lakes. These linear trends were adjusted and related to a datum.

5.1 Time Series Analysis for Lake Superior

5.1.1 Analysis of Mean Monthly Lake Elevations

The results of the time series analyses of mean monthly Lake Superior elevations at Marquette, Duluth, Michipicoten, Sault Ste. Marie and Thunder Bay for the period 1915 to 1970 are given in Table 5.1.

Table 5.1 indicates that the most important components of the mean monthly lake elevations are an autoregressive component (first order linear) and an annual periodicity in the mean. This is to be expected of such a time series, since each elevation is highly dependent on previous elevations with a natural 12-month cycle superimposed. The importance of the various time series components

TABLE 5.1

STATISTICAL COMPONENTS OF MEAN MONTHLY

ELEVATIONS OF LAKE SUPERIOR

Component	Variance explained by time series components (%)				
	Marquette	Duluth	Michipicoten	Sault Ste. Marie	Thunder Bay
I. Deterministic					
(a) Linear Trends					
1st order	3.26	8.21	0.35	0.15	0.16
2nd order	3.94	3.25	7.25	5.43	2.89
3rd order	0.90	0.62	0.10	0.53	0.65
4th order	-	-	-	-	-
Total	8.10	12.08	7.70	6.11	3.70
(b) Periodicity in the Mean					
12 month cycle	33.69	34.79	35.94	42.57	37.88
6 month cycle	0.89	0.42	0.58	1.66	0.73
4 month cycle	0.05	0.04	0.05	0.01	0.09
3 month cycle	0.02	0.02	0.02	0.03	0.02
2.4 month cycle	0.01	0.01	0.02	0.05	0.02
2 month cycle	0.01	0.01	0.01	-	0.01
Total	34.67	35.29	36.62	44.32	38.75
(c) Periodicity in the Variance					
12 month cycle	0.17	0.21	0.62	0.27	0.24
6 month cycle	0.05	0.04	0.03	-	0.02
4 month cycle	0.01	-	0.01	-	-
3 month cycle	-	-	-	0.01	-
2.4 month cycle	-	-	-	0.02	-
2 month cycle	-	-	-	-	-
Total	0.23	0.25	0.66	0.30	0.26
II. Stochastic					
(a) Autoregression	51.91	48.13	49.24	43.87	53.84
(b) Independent Random	5.09	4.25	5.78	5.40	3.45

remains relatively constant in all five time series; that is, the percent of the total variance of the time series explained by the autoregressive model varies only from 43.87% for Sault Ste. Marie to 53.85% for Thunder Bay (Port Arthur). In all cases the periodicity in the variance is of very little importance compared to the periodicity in the mean.

An example of the spectra obtained at each step of the analysis is shown in Fig. 5.1 for Marquette, 1915-1970.

5.1.2 Analysis of Differences in Mean Monthly Lake Elevations

The results of the time series analyses on the four series of differences in mean monthly elevations, that is, Marquette minus Duluth, Michipicoten, Sault Ste. Marie, and Thunder Bay, respectively, are given in Table 5.2.

From Table 5.2 it can be seen that for time series made up of differences in mean monthly lake elevations between pairs of gauges, a first order trend is generally the most important component. This, however, is not so in the case of Marquette minus Sault Ste. Marie where the independent random component controls a larger percentage of the total variance than does the first order linear trend. This is explained by the fact that the Sault Ste. Marie gauge is not situated on the open lake but is affected by backwater

FIGURE 5.1
SPECTRAL ANALYSIS OF MEAN MONTHLY ELEVATIONS,
MARQUETTE (1915-70), SHOWING UPPER 95% CONFIDENCE LIMIT.

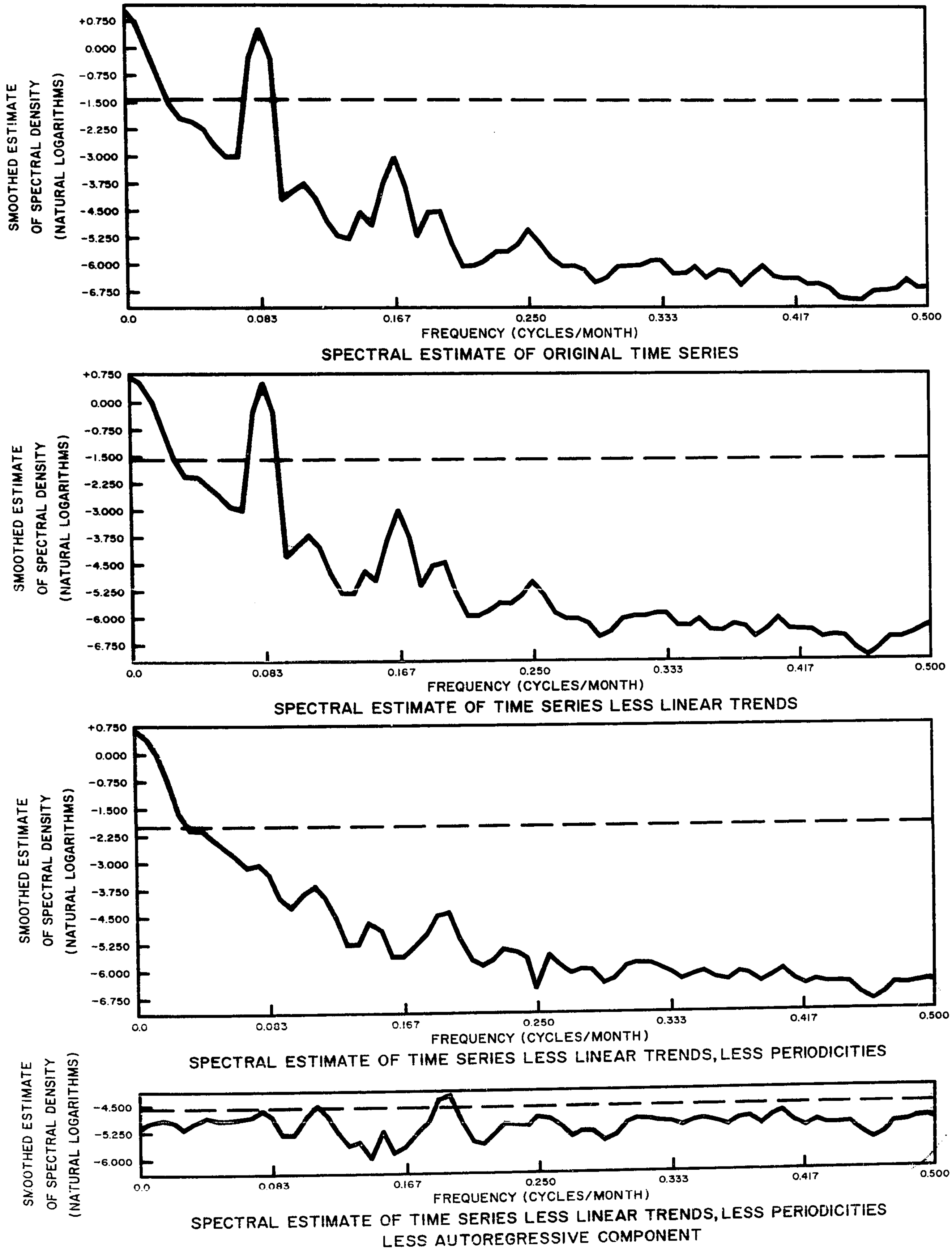


TABLE 5.2

STATISTICAL COMPONENTS OF DIFFERENCES IN MEAN MONTHLY ELEVATIONS OF LAKE SUPERIOR

Component	Variance explained by time series components (%)			
	Marquette Minus Duluth	Marquette Minus Michipicoten	Marquette Minus Sault Ste. Marie	Marquette Minus Thunder Bay
I. Deterministic				
(a) Linear Trends				
1st order	31.79	50.03	28.43	49.83
2nd order	0.33	4.22	0.72	2.17
3rd order	0.40	3.51	0.91	0.55
4th order	-	-	-	-
Total	32.52	57.76	30.06	52.55
(b) Periodicity in the Mean				
12 month cycle	17.54	0.39	2.56	14.15
6 month cycle	3.21	0.29	2.96	0.24
4 month cycle	0.02	0.13	0.35	0.24
3 month cycle	0.08	-	0.03	0.03
2.4 month cycle	-	0.12	0.25	0.08
2 month cycle	-	0.01	0.04	-
Total	20.85	0.94	6.19	14.74
(c) Periodicity in the Variance				
12 month cycle	7.80	3.98	1.34	3.78
6 month cycle	1.12	0.17	0.76	2.47
4 month cycle	1.20	0.25	0.58	1.35
3 month cycle	3.13	0.16	0.28	0.84
2.4 month cycle	2.06	0.20	0.23	0.35
2 month cycle	0.54	0.08	0.06	0.06
Total	15.85	4.84	3.25	8.85
II. Stochastic				
(a) Autoregression	0.25	10.98	29.45	2.69
(b) Independent Random	30.58	25.48	31.05	21.17

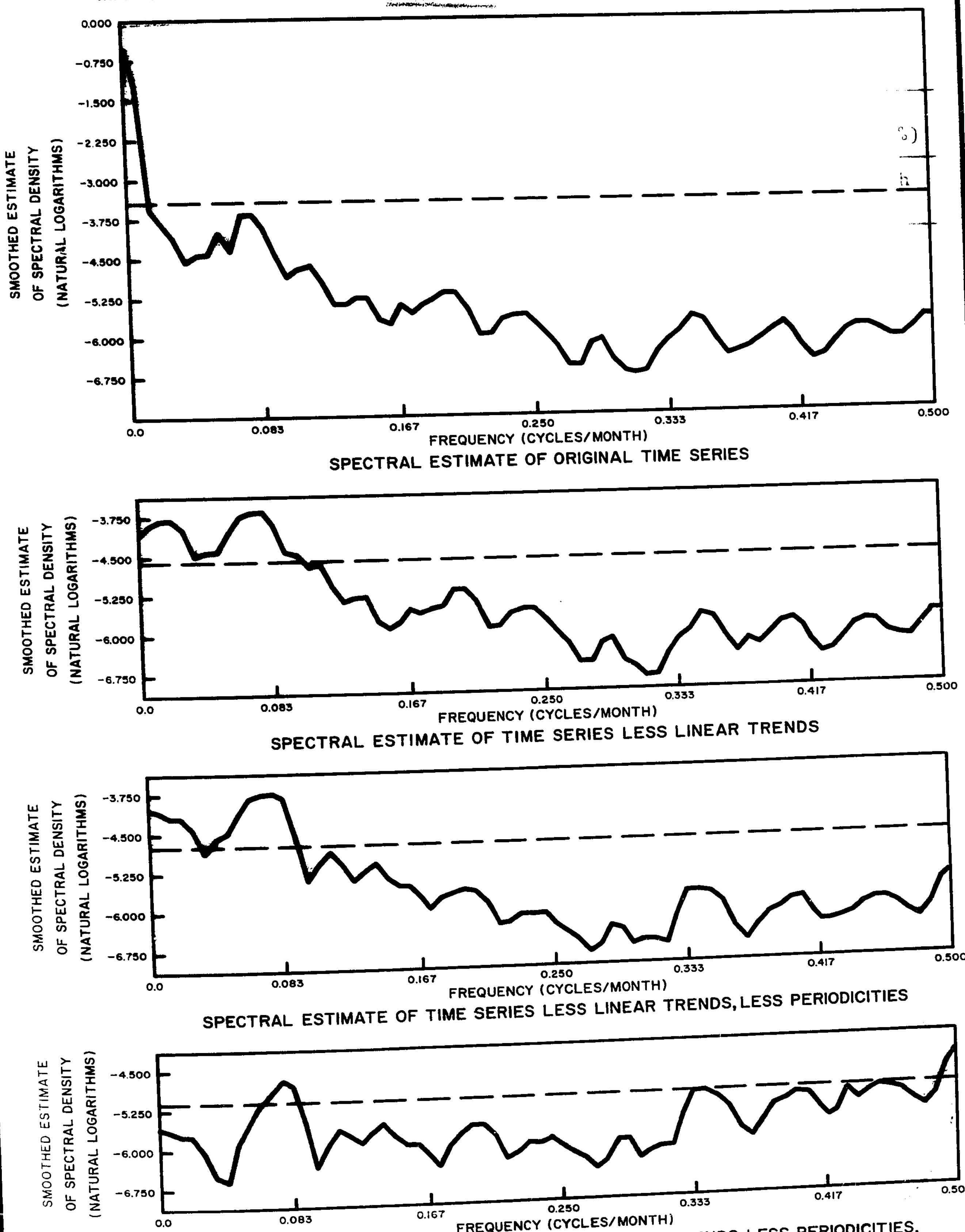
from the Lake Superior regulation control structure in the St. Mary's River. Such a factor introduces a larger autoregressive component into time series made up of differences between this gauge and other gauges, thus depressing the relative importance of linear trends. The importance of periodic components as well as autoregressive components varies greatly between pairs of gauges. Marquette minus Duluth and Marquette minus Thunder Bay have large periodic components and small autoregressive components while Marquette minus Michipicoten and Marquette minus Sault Ste. Marie have smaller periodic components and larger autoregressive components. In all cases the periodicity in the variance is of the same order of magnitude as the periodicity in the mean. For Marquette minus Michipicoten the periodicity in the variance actually exceeds the periodicity in the mean.

An example of the spectra obtained at each step of the analysis is given on Fig. 5.2 for Marquette minus Michipicoten, 1915 to 1970.

To verify the effect of the length of the time series on the distribution of the variance among the trend, periodicity, autoregressive and independent random components, Table 5.3 shows the results of the time series analysis for mean monthly lake elevations at Marquette for the period of record, 1860-1970, and for Marquette minus Duluth for the period of common record, 1888 to 1970. For both time series

FIGURE 5.2

SPECTRAL ANALYSIS OF DIFFERENCES IN MEAN MONTHLY ELEVATION, MARQUETTE-MICHIPICOTEN (1915-70), SHOWING UPPER 95% CONFIDENCE LIMIT.



SPECTRAL ESTIMATE OF ORIGINAL TIME SERIES

SPECTRAL ESTIMATE OF TIME SERIES LESS LINEAR TRENDS

SPECTRAL ESTIMATE OF TIME SERIES LESS LINEAR TRENDS, LESS PERIODICITIES

SPECTRAL ESTIMATE OF TIME SERIES LESS LINEAR TRENDS, LESS PERIODICITIES, LESS AUTOREGRESSIVE COMPONENT

TABLE 5 .3

RESULTS OF USING LONGER LENGTH OF DATA IN DETERMINING STATISTICAL COMPONENTS OF MEAN MONTHLY ELEVATIONS AND DIFFERENCES IN MEAN MONTHLY ELEVATIONS OF LAKE SUPERIOR

Component	Variance explained by time-series components (%)	
	Marquette (1860-1970	Marquette Minus Duluth (1888-1970)
I. Deterministic		
(a) Linear Trends		
1st order	10.81	48.06
2nd order	1.33	0.10
3rd order	0.09	0.32
4th order	-	-
Total	12.23	48.48
(b) Periodicity in the Mean		
12 month cycle	31.17	14.47
6 month cycle	0.50	2.73
4 month cycle	0.06	0.01
3 month cycle	0.01	0.08
2.4 month cycle	0.01	-
2 month cycle	0.01	0.03
Total	31.76	17.32
(c) Periodicity in the Variance		
12 month cycle	0.09	1.58
6 month cycle	0.03	0.70
4 month cycle	-	0.26
3 month cycle	-	0.79
2.4 month cycle	-	0.66
2 month cycle	-	0.64
Total	0.12	4.63
II. Stochastic		
(a) Autoregression	50.48	0.93
(b) Independent Random	5.40	28.64

the differences in results are insignificant between the record length used in Tables 5.1 and 5.2, 1915 to 1970, and the longer periods used in Table 5.3. For lake elevations at Marquette the longer data length produces slightly larger linear trend and independent random components with a corresponding decrease in the percent of original variance explained by periodicity. For the time series of differences in elevations at Marquette and Duluth the longer data length results in an increased linear trend and decreased periodicity with a slight decrease in independent random component.

5.2 Linear Trend Analysis

5.2.1 Lake Superior

Table 5.4 shows the first order linear trends computed between gauges around Lake Superior. Marquette and Duluth, the gauges with the longest records, were used as major gauges from which all others were subtracted. The first column in Table 5.4 gives the correlation coefficient for the first order linear regression equation relating differences in mean monthly lake elevations to time. The second column shows the slope of this regression line expressed as a rate of vertical movement in feet per 100 years. This rate is then shown after adjustment in the third column. The fourth column gives the number of months of common record of the two gauges, that is, the number of data upon which the regression is based. These four columns of results are

TABLE 5.4

COMPUTED AND COMPUTED-ADJUSTED RATES OF RELATIVE CRUSTAL MOVEMENT AROUND LAKE SUPERIOR

GAUGE B	GAUGE A: Marquette — Correlation Coefficient (1)	Marquette — Unadjusted Rate, ft/100 years (2)	Marquette — Adjusted Rate, ft/100 years (3)	Marquette — No. of Months of Common Record (4)	Duluth — Correlation Coefficient (1)	Duluth — Unadjusted Rate, ft/100 years (2)	Duluth — Adjusted Rate, ft/100 years (3)	Duluth — No. of Months of Common Record (4)
	Rates of Movement A-B							
Port Arthur	0.7396	0.7088	0.7413	1210	0.8940	1.1639	1.1392	1089
Michipicoten	0.8306	0.9902	1.0134	653	0.8827	1.4345	1.4113	653
Sault Ste. Marie	0.5755	0.5525	**	742	0.7727	0.9913	**	742
Point Iroquois	0.5261	0.3314	0.3659	408	0.6871	0.7983	0.7638	408
Marquette					0.6856	0.3597	0.3979	1211
Keweenaw L.E.	0.0897*	0.0393	0.0393	829	0.7233	0.4371	0.4371	829
Houghton	0.1607*	-0.0826	-0.0111	295	0.5856	0.4583	0.3868	295
Duluth	0.6856	-0.3597	-0.3979	1211				
Two Harbors	0.3978	-0.5394	-0.4134	350	0.1441*	0.1106	0.0154	350

* Statistically non-significant relationships between differences in elevations and time

** Figures for Sault Ste. Marie not considered reliable because of gauge location then repeated in the next four columns to the right for the second combination of gauges.

From Table 5.4, it can be seen that the only station not having a significant rate of movement relative to Duluth is Two Harbors and so the line of zero movement or datum is taken to be somewhere between Two Harbors and the next nearest station, which is Keweenaw Lower Entrance.

Table 5.5 shows the results of determining first order linear trends in records of gauge elevations at Marquette for different periods of record in order to detect any rate of movement of the mean lake surface.

It was determined from Table 5.5 that the mean elevation of Lake Superior is rising relative to Duluth at a rate of 0.80 foot/100 years. Based on this rate the rates of movement of land around the lake were determined relative to mean lake elevations and are listed in Table 5.7.

Fig. 5.3 shows the isolines of computed rates of relative crustal movement for the Lake Superior basin and Fig. 5.4 shows the circular distribution of these rates using Port Arthur (Thunder Bay) as the gauge to which rates of movement are referred.

Table 5.6 compares the relative rates of crustal movement computed in this study with the results of previous studies (31,32,58,77,81). The agreement is very good for those gauges having long periods of record. Any differences are probably due to mainly the different lengths of record

TABLE 5.5

DETERMINATION OF TREND IN WATER ELEVATION FOR LAKE SUPERIOR

Station	Length of Record Used (Months)							
	1211	1210	829	742	653	408	350	295
	The upper figure of each pair is the linear trend in feet per 100 years, the lower figure of each pair is the corresponding correlation coefficient.							
Marquette	0.70 0.34	0.69 0.36	0.38 0.12	0.82 0.23	0.63 0.16	-0.55 0.13	-2.07 0.31	-0.15 0.03
Duluth	1.06 0.47							
Port Arthur		-0.01 0.00						
Keweenaw L.E. *			0.34 0.11					
Sault Ste. Marie				0.26 0.08				
Michipicoten					-0.35 0.09			
Point Iroquois						-0.88 0.21		
Two Harbors							-1.53 0.23	
Houghton*								-0.06 0.01

*These periods of record do not extend to 1970.

FIGURE 5.3

CRUSTAL MOVEMENT ON LAKE SUPERIOR
LINES OF EQUAL RATE OF ABSOLUTE MOVEMENT FT/100 YEARS

NOTE: TO OBTAIN RATES OF MOVEMENT OF LAND RELATIVE TO LAKE SURFACE, SUBTRACT 0·80 FT/100 YEARS.

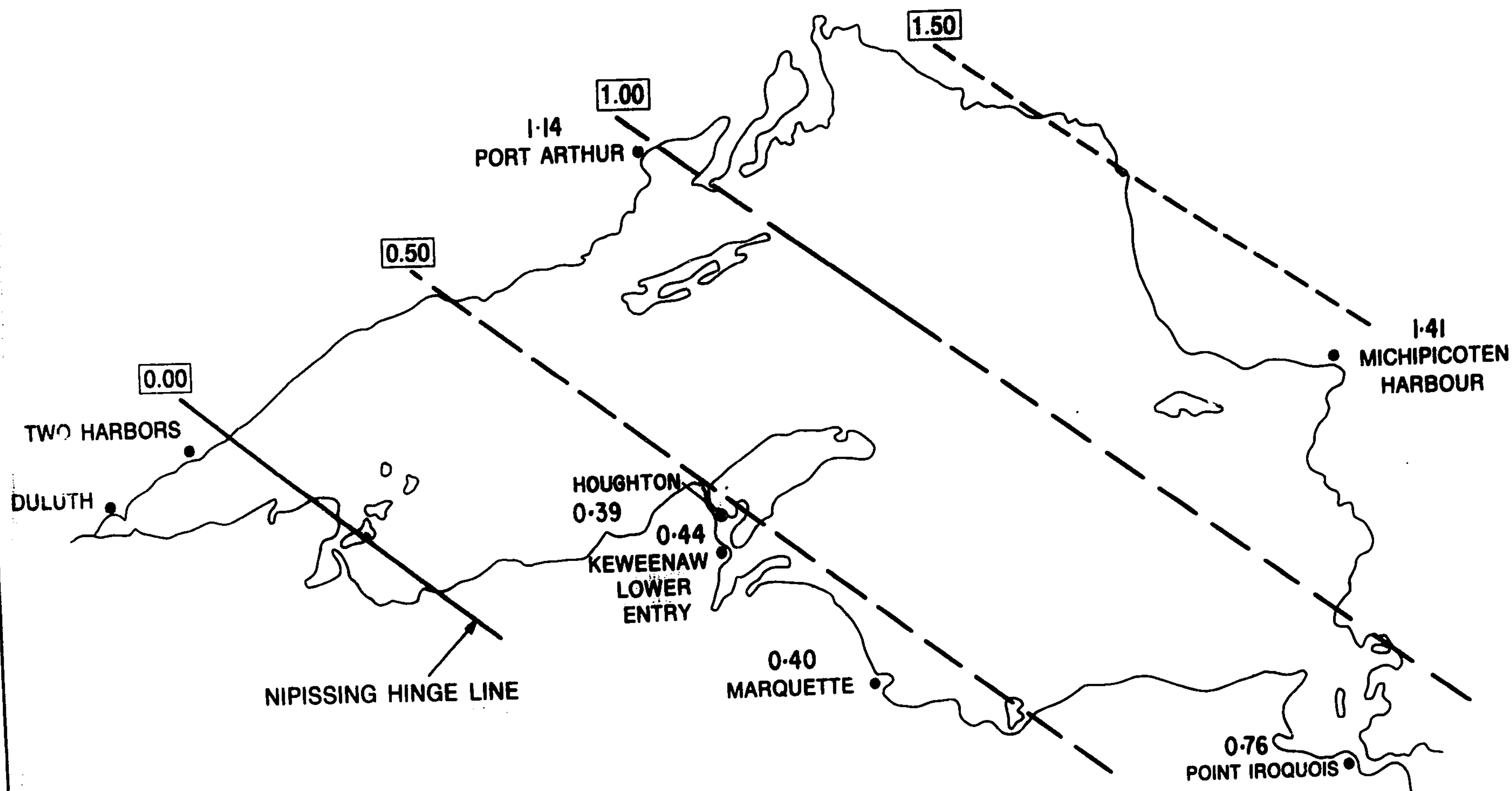


FIGURE 5.4
CIRCULAR DISTRIBUTION OF CRUSTAL MOVEMENT
AROUND LAKE SUPERIOR

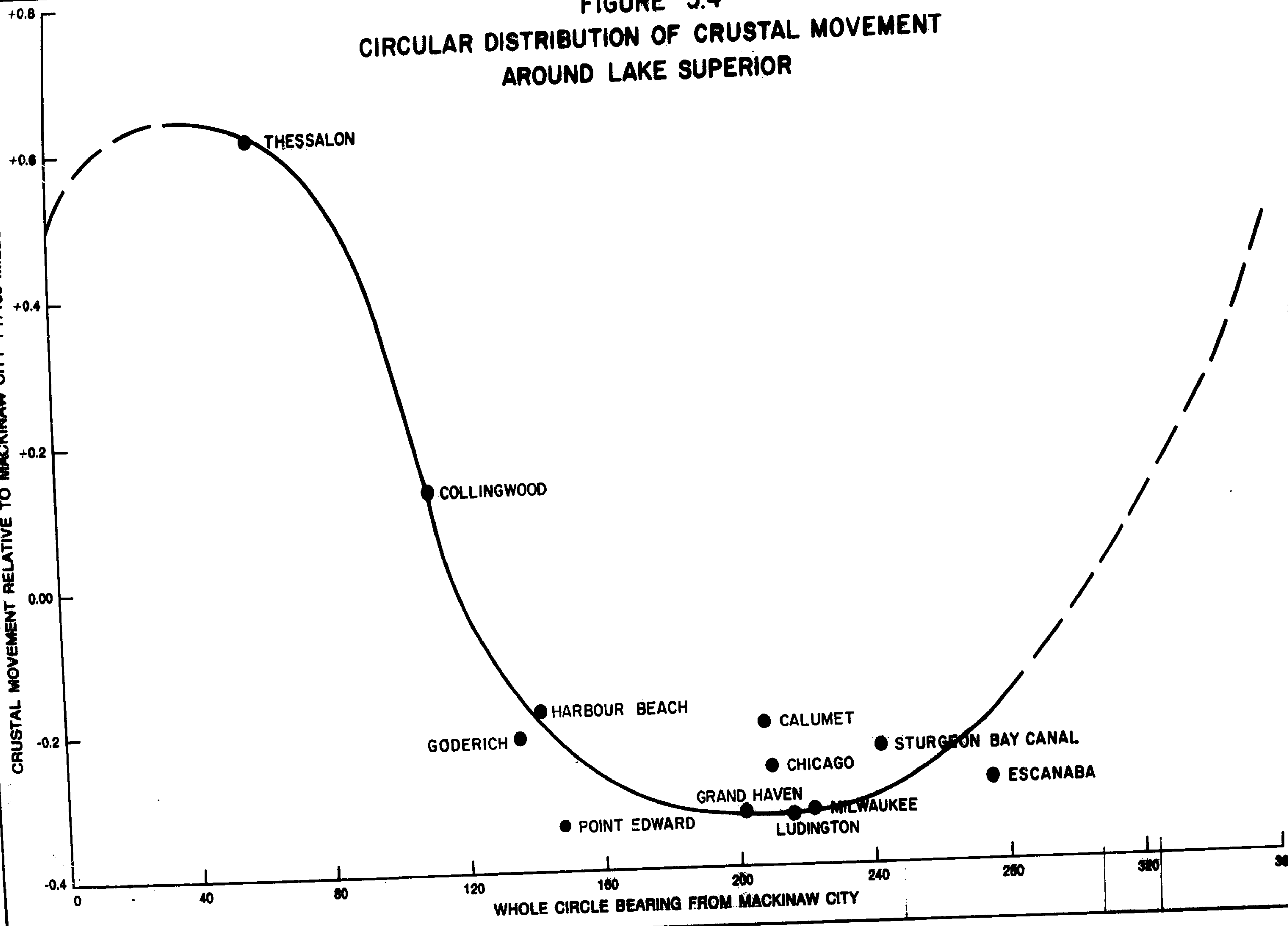


TABLE 5.6

CRUSTAL MOVEMENT AROUND LAKE SUPERIOR IN FEET PER 100 YEARS RELATIVE TO MARQUETTE

Gauge	Author							
	Gutenberg (31)	Gutenberg (32)	Moore (58)	V.C.S., (77,81) unadjusted	V.C.S., (77, 81) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
Arthur	+0.97	+0.65	+0.68	+0.60	+0.65	+0.71	+0.74	1210
ipicoten Harbor	+0.70	+0.49	+0.55	+1.01	+1.04	+0.99	+1.01	653
t Iroquois			+0.53	+0.39	+0.27	+0.33	+0.36	408
efish Point				-0.43	-0.42			
d Marais				+0.20	+0.13			
sing			-0.46	+0.46	+0.42			
que Isle				+0.04	-0.01			
enaw L.E.				-0.06	-0.05	+0.04	+0.04	829
hton			+0.05	-0.08	+0.03	-0.08	+0.01	295
enaw U.E.				+0.14	+0.09			
d Traverse Bay				+0.28	+0.30			
er Harbor			+0.36	+0.33	+0.31			
nagon			-0.08	-0.04	-0.16			
e Harbor				+0.06	+0.14			
k River				-0.89	-0.87			
land			-0.41	-0.63	-0.62			
ucopia				-0.40	-0.44			
t Wing			-0.24	-0.26	-0.32			
th	-0.59	-0.39	-0.40	-0.34	-0.31	-0.36	-0.40	1211
Harbors			-0.21	-0.14	-0.28	-0.54	-0.41	350

TABLE 5.6 (continued)

Gauge	Author							
	Gutenberg (31)	Gutenberg (32)	Moore (58)	V.C.S., (77,81) unadjusted	V.C.S., (77,81) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
..ver Bay				+0.06	+0.08			
..tsen				-0.67	-0.61			
..nd Marais			+0.25	+0.20	+0.09			
..k Harbor				+0.51	+0.47			
..shington Harbor				+0.33	+0.33			
..ult Ste. Marie	+0.39					+0.55	(3)	742
..le Royal			+0.39					

..tes: (1) + = rate of movement indicates a rise of the land adjacent to the gauging station with respect to Marquette

(2) V.C.S. = Vertical Control Subcommittee

(3) Figure for Sault Ste. Marie is not considered reliable because of gauge location.

TABLE 5.7

RATE OF MOVEMENT OF THE LAND AROUND LAKE SUPERIOR
RELATIVE TO THE LAKE SURFACE, FEET PER 100 YEARS

Port Arthur	+0.34
Michipicoten	+0.61
Point Iroquois	-0.04
Marquette	-0.40
Keweenaw L.E.	-0.36
Houghton	-0.41
Duluth	-0.80
Two Harbors	-0.80

available to the investigators and to the more sophisticated techniques of analysis used in the present study.

Fig. 5.5 shows the positions of Nipissing isobases around Lake Superior determined from observation of raised shorelines (20). The similarity of the pattern of isolines in Fig. 5.5 with those in Fig. 5.3, determined in this study, is immediately apparent. Since the directions of vertical crustal movement indicated from raised shorelines and from lake elevation gauges are similar, then it is a reasonable assumption that the same tilting process has been operating throughout post-Nipissing time. Average rates of vertical uplift can be computed from Fig. 5.5 by dividing the total uplifts of the Nipissing isobases by the length of time over which the uplift has occurred. Table 5.8 provides a comparison between rates of vertical movement computed from Nipissing isobases (20) and rates of movement computed from lake elevation gauges in the present study. Considering that one set of figures results from 5,500 years of record (60) and the other from less than 100 years of record, the agreement is quite satisfactory and suggests that uplift has proceeded uniformly in the Lake Superior basin at these approximate rates for several thousand years.

5.1.1 Lake Michigan-Huron

Table 5.9 shows the results of determining first order linear trends in differences in mean monthly elevations

FIGURE 5.5
NIPISSING ISOBASES FOR LAKE SUPERIOR (after W.R. Farrand, 1960)

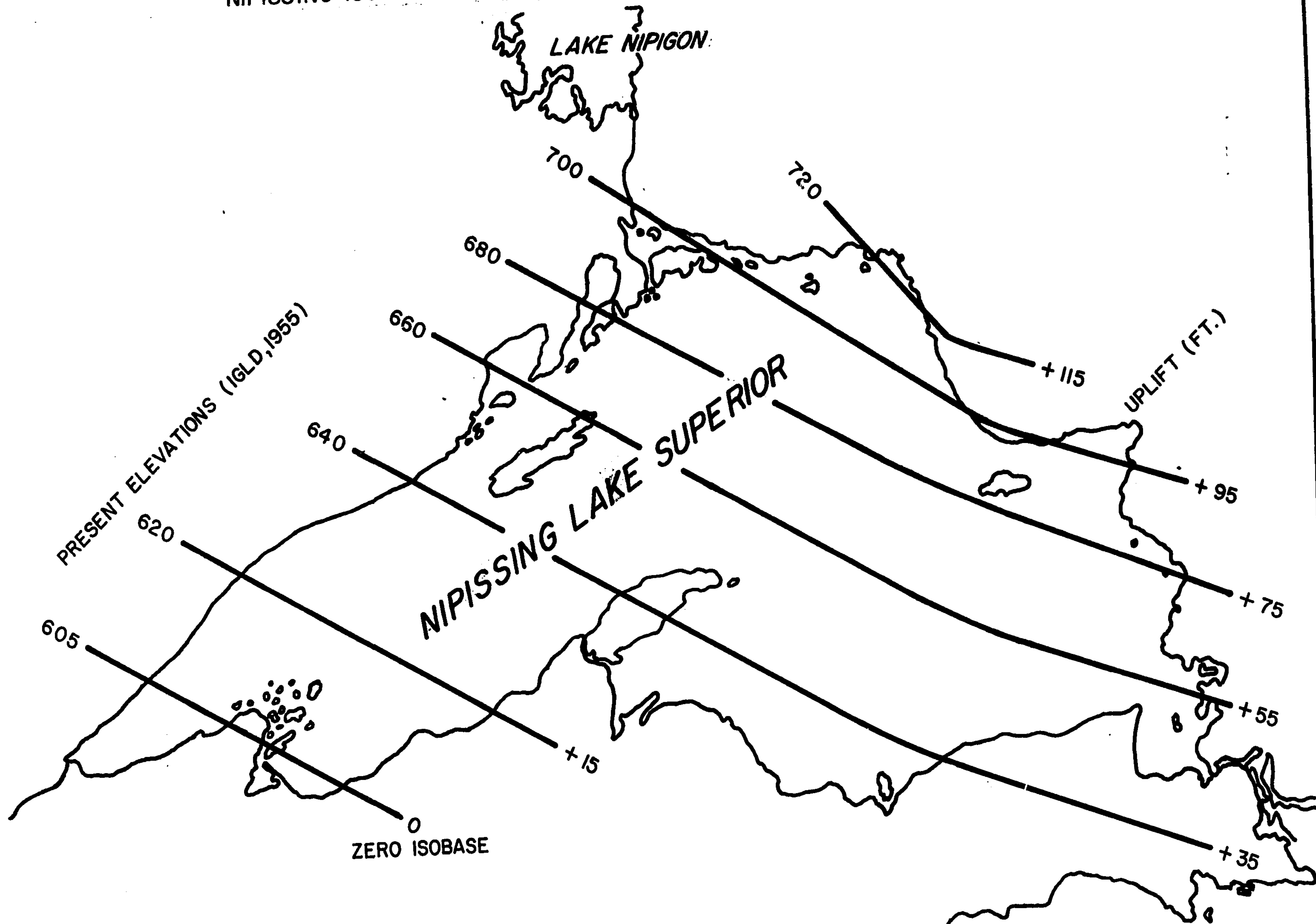


TABLE 5.8

COMPARISON OF RATES OF CRUSTAL MOVEMENT, LAKE SUPERIOR

Location	Isobase Elevation* (IGLD,1955)	Rate of Uplift** (Ft/100 yrs)	Rate of Uplift*** (Ft/100 yrs)
Port Arthur	660	1.00	1.14
Michipicoten	710	2.00	1.41
Point Iroquois	647	0.76	0.76
Marquette	630	0.45	0.40
Keweenaw L.E.	628	0.42	0.44
Houghton	630	0.45	0.39
Duluth	605	0	0
Two Harbors	605	0	0

* After Farrand (20)

** Derived from Farrand (20) using a zero isobase elevation of 605 ft. and a time span since Nipissing of 5500 years (Prest, (60)).

*** Present study results

TABLE 5.9

COMPUTED AND COMPUTED-ADJUSTED RATES OF RELATIVE CRUSTAL MOVEMENT AROUND LAKE MICHIGAN-HURON

GAUGE B	GAUGE A							
	Milwaukee				Harbor Beach			
	Correlation Coefficient (1)	Rate of Movement A-B Unadjusted (2)	Rate of Movement A-B Adjusted (3)	No. of Months of Common Record (4)	Correlation Coefficient (1)	Rate of Movement A-B Unadjusted (2)	Rate of Movement A-B Adjusted (3)	No. of Months of Common Record (4)
Milwaukee					0.7155	-0.4748	-0.4877	1318
Sturgeon Bay Canal	0.6300	0.4722	0.4662	663	0.0369*	-0.0276	-0.0216	669
Escanaba	0.5568	0.4674	0.4499	450	0.0940	-0.0554	-0.0379	446
Thessalon	0.8083	1.2538	1.1912	519	0.8283	0.6409	0.7035	525
Collingwood	0.8087	1.1019	1.0833	708	0.8652	0.5795	0.5956	714
Goderich	0.5298	0.2928	0.3004	1282	0.4963	-0.1797	-0.1873	1280
Point Edward	0.5744	1.1287	-	513	0.3591	0.5111	-	519
Harbor Beach	0.7155	0.4748	0.4877	1318				
Mackinaw City	0.7268	0.7881	0.7798	840	0.4507	0.2838	0.2921	846
Ludington	0.5741	0.3154	0.2849	303	0.4762	-0.2332	-0.2027	306
Grand Haven	0.0981*	0.0817	0.1176	644	0.3859	-0.3341	-0.3700	644
Calumet Harbor	0.2264	0.1536	0.1418	805	0.4429	-0.3577	-0.3459	811
Chicago	0.0628*	-0.0498	-0.0390	909	0.5337	-0.5158	-0.4876	885

*Statistically non-significant relationship between differences in elevations and time for Lake Michigan-Huron. Gauges at Milwaukee, on Lake Michigan, and Harbor Beach, on Lake Huron, were chosen as gauges from which all other gauge records would be subtracted. The choice of Milwaukee and Harbor Beach was based on the long periods of record available (135 and 111 years respectively) and the fact that the gauge locations have remained relatively stable over these years (32). Calumet Harbor, at the southern end of Lake Michigan, would have been a good gauge to use were it not for the large number of changes in the gauge location that have occurred over the period of record (32).

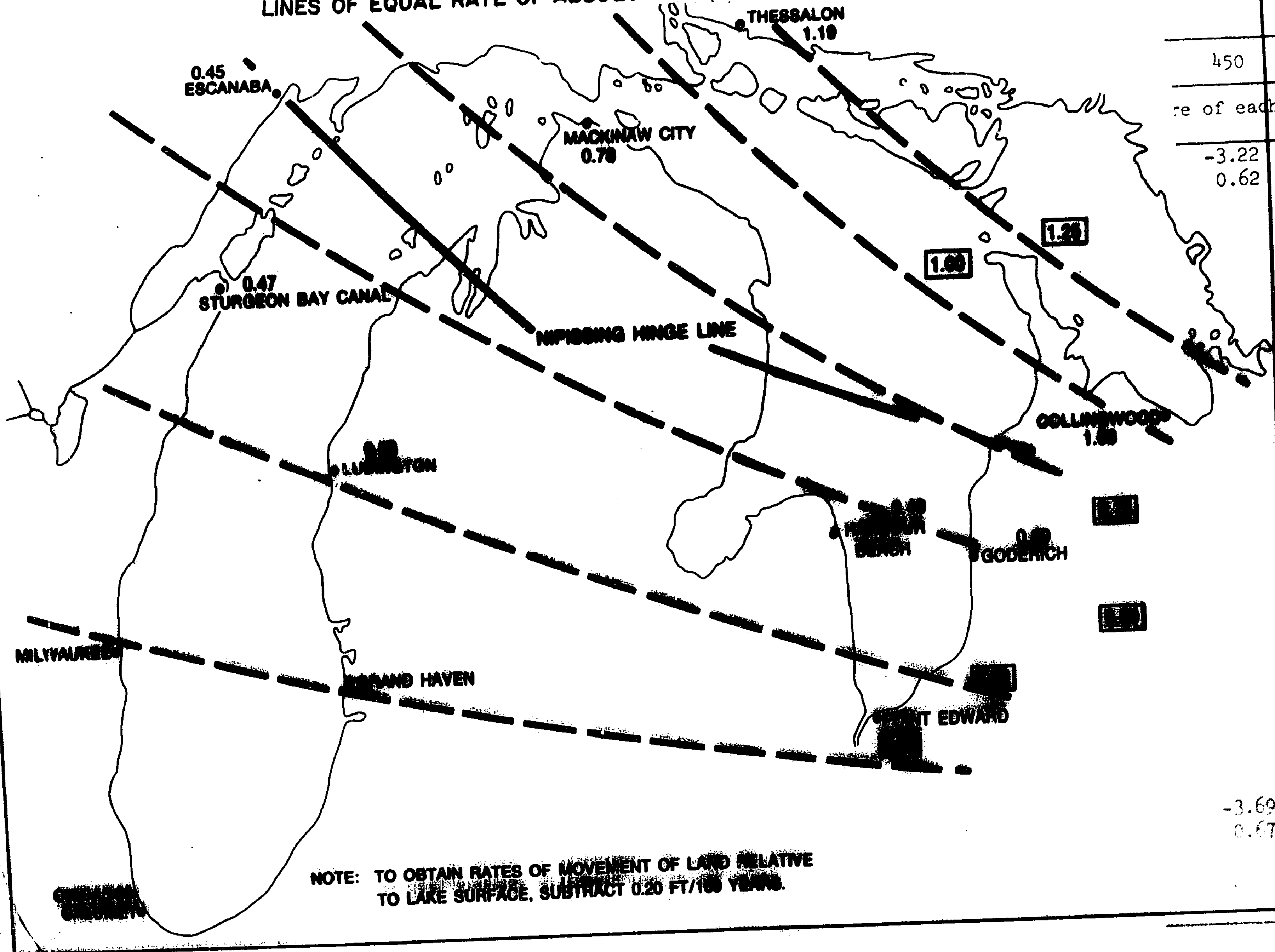
Table 5.9 shows that there are no significant correlations between Milwaukee and any of the gauges located south of Ludington. The line of zero rate of relative crustal movement has therefore been assumed to pass through Milwaukee and to the south of Ludington (Fig. 5.6).

First order linear trend in Lake Michigan-Huron elevations at Milwaukee is given in Table 5.10, as approximately -2.35 feet per 100 years, that is a lowering of lake elevation. Table 5.11 compares the results of the present study with those of earlier studies (28,25,31,32,58,76,80) using elevations of Lake Michigan-Huron. Table 5.12 presents rates of crustal movement relative to mean Lake Michigan-Huron elevation after taking into account the many factors affecting lake elevations. Fig. 5.7 shows circular distribution of crustal movement around Lake Michigan-Huron.

FIGURE [illegible]

CRUSTAL MOVEMENT ON LAKE MICHIGAN-HURON

LINES OF EQUAL RATE OF ABSOLUTE MOVEMENT FEET/100 YEARS



NOTE: TO OBTAIN RATES OF MOVEMENT OF LAND RELATIVE TO LAKE SURFACE, SUBTRACT 0.20 FT/100 YEARS.

450	303
re of each pair	
-3.22	-1.47
0.62	0.22

- 90 -

-3.69
0.67

-1.79
0.27

DETERMINATION OF TREND IN ELEVATION FOR LAKE MICHIGAN-HURON

The upper figure of each pair is the linear trend in feet per 100 years, the lower figure of each pair is the corresponding correlation coefficient.

Station	Length of Record Used (Months)											
	1318	1282	909	840	805	708	663	644	519	513	450	303
Milwaukee	-2.35 0.57	-2.36 0.57	-3.65 0.71	-0.89 0.16	-0.83 0.14	-0.41 0.06	-0.91 0.15	-1.90 0.31	1.99 0.21	1.68 0.17	-3.22 0.62	-1.47 0.22
Harbor Beach	-2.82 0.64											
Goderich		-2.65 0.62										
Chicago*			-3.60 0.71									
Mackinaw City				-1.68 0.29								
Calumet Harbor					-0.98 0.16							
Collingwood						-1.50 0.22						
Sturgeon Bay Canal							-1.38 0.22					
Grand Haven*								-1.98 0.33				
Thessalon									0.74 0.08			
Point Edward										0.56 0.06		
Escanaba*											-3.69 0.67	
Ludington												-1.79 0.27

* These periods of record do not extend to 1970.

TABLE 5.11

CRUSTAL MOVEMENT AROUND LAKE MICHIGAN-HURON
IN FEET PER 100 YEARS RELATIVE TO MILWAUKEE

Gauge Site	Author								
	Gilbert (28)	Freeman (25)	Gutenberg (31,32)	Moore (58)	VCS (76,80) unadjusted	VCS (76,80) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
ssalon				0.91	1.34	1.22	1.25	1.19	519
lingwood			0.92	0.81	1.00	1.02	1.10	1.08	708
erich			0.33	0.32	0.46	0.47	0.29	0.30	1282
eport					2.59[t]	2.33[t]			
ington				0.40[t]	0.44[t]	0.40[t]			
t Sanilac					1.38[t]	1.39[t]			
bor Beach		0.57	0.36	0.36	0.42	0.47	0.47	0.49	1318
t Austin	1.01	0.71		0.65[t]	0.70[t]	0.63[t]			
City		0.53		0.40[t]	-0.58[t]	-0.36[t]			
ena				0.50[t]	0.67[t]	0.76[t]			
sque Isle				0.64[t]	0.76[t]	0.81[t]			
boygan				0.89[t]	0.82[t]	0.75[t]			
kinac Island					1.06[t]	0.95[t]			
kinaw City		0.93	0.66	0.73	0.78	0.82	0.79	0.78	840
oskey				0.69	0.76	0.78			
rlevoix				0.64	0.63	0.72			
verse City					0.02	0.24			

TABLE 5.11 (continued)

CRUSTAL MOVEMENT AROUND LAKE MICHIGAN-HURON
IN FEET PER 100 YEARS RELATIVE TO MILWAUKEE

Gauge Site	Author								
	Gilbert (28)	Freeman (25)	Gutenberg (31,32)	Moore (58)	VCS (76,80) unadjusted	VCS (76,80) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
ukegan				0.05	-0.13	-0.14			
nosha				0.13	0.29	0.28			
cine				-0.31	-0.21	-0.17			
rt Washington				0.09	0.11	0.10			
eboygan				0.20	0.19	0.25			
nitowac				0.24	0.09	0.16			
o Rivers				-0.10	0.04	0.09			
waunee				-0.07	0.09	0.02			
goma				0.06	0.14	0.10			
urgeon Bay Canal		0.61		0.40	0.49	0.50	0.47	0.47	663
troit Harbor					0.23	0.30			
ckson Harbor					0.03	0.06			
een Bay				-0.31	0.21	0.18			
amico					-0.12	-0.08			

TABLE 5.11 (continued)

CRUSTAL MOVEMENT AROUND LAKE MICHIGAN-HURON
IN FEET PER 100 YEARS RELATIVE TO MILWAUKEE

Gauge Site	Author							
	Gilbert (28)	Freeman (25)	Gutenberg (31,32)	Moore (58)	VCS (76,80) unadjusted	VCS (76,80) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
and					0.30	0.46			
nkfort				0.12	0.20	0.22			
tage Lake				0.28	0.40	0.40			
istee				-0.20	-0.10	-0.14			
ington				0.42	0.34	0.35	0.32	0.28	303
twater				0.08	0.00	0.13			
te Lake				0.18	0.27	0.31			
kegon				0.21	0.14	0.20			
nd Haven				0.03	0.15	0.19	0.08	0.12	644
lland				0.16	0.08	0.18			
gatuck				-0.07	-0.06	-0.01			
th Haven				0.23	0.19	0.22			
Joseph				0.35	0.37	0.42			
higan City				0.25	0.54	0.48			
iana Harbor					0.17	0.13			
cago				-0.09	-0.10	-0.06	-0.05	-0.04	909
umet Harbor			0.09	0.00	0.07	0.10	0.15	0.14	805

TABLE 5.11 (continued)

CRUSTAL MOVEMENT AROUND LAKE MICHIGAN-HURON
IN FEET PER 100 YEARS RELATIVE TO MILWAUKEE

Gauge Site	Author								
	Gilbert (28)	Freeman (25)	Gutenberg (31,32)	Moore (58)	VCS (76,80) unadjusted	VCS (76,80) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
nto					0.19	0.14			
nominee				0.15	0.30	0.32			
canaba	0.86	0.82	0.75	0.54	0.59	0.59	0.47	0.45	450
nistique				0.24	0.50	0.47			
ubinway				0.73	0.70	0.80			
. Ignace				0.36	0.75[t]	0.76[t]			
tour				0.84	0.93[t]	0.98[t]			
. James				0.27	0.87	0.82			
rt Huron		0.42							
int Edward							1.13		513

TES:

(1) t = derived by triangulation from the author's results

(2) + = rate indicates a rise of the land with respect to Milwaukee.

(3) VCS = Vertical Control Subcommittee.

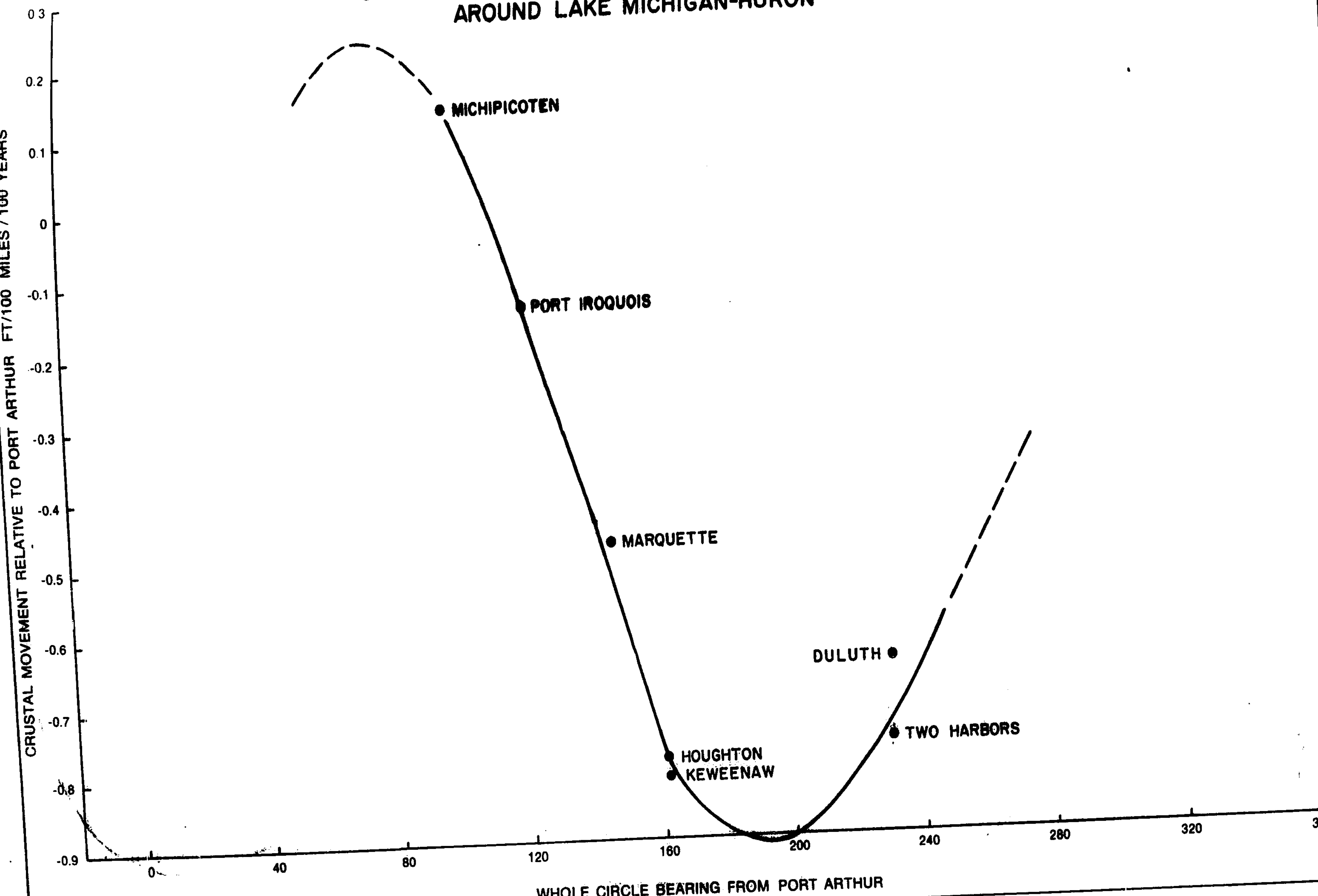
TABLE 5.12

RATE OF MOVEMENT OF THE LAND AROUND LAKE MICHIGAN-HURON RELATIVE TO THE LAKE SURFACE, FEET PER 100 YEARS

Milwaukee	-0.20
Sturgeon Bay Canal	0.27
Escanaba	0.25
Thessalon	0.99
Collingwood	0.88
Goderich	0.10
Point Edward	*
Harbor Beach	0.29
Mackinaw City	0.58
Ludington	0.08
Grand Haven	-0.20
Calumet Harbor	-0.20
Chicago	-0.20

*Rate of movement is not certain

FIGURE 5.7
CIRCULAR DISTRIBUTION OF CRUSTAL MOVEMENT
AROUND LAKE MICHIGAN-HURON



It is interesting to note the good agreement between the results in Table 5.12 and those given by Lewis (48) using sediment sequences on Manitoulin Island in Lake Huron.

It is also of interest to note that the divide separating the Great Lakes Drainage Basin from the Mississippi River Drainage Basin is less than 8 feet above the mean surface elevation of Lake Michigan-Huron, so that at a rate of increase in elevation of 0.20 foot per 100 years it would take about 4,000 years for the upper Great Lakes to change their outflow from the St. Lawrence River to the Mississippi River System. This computation assumes, of course, no further lowering of lake elevations due to other causes.

5.2.3 Lake Erie

Lake Erie is the shallowest of the Great Lakes and is subject to rapid short-term fluctuations in elevation from one end of the lake to the other (seiches). This causes a large independent random component in time series of differences in elevations between two gauges which, coupled with a shortage of reliable gauges, makes the evaluation of crustal movement around Lake Erie very uncertain.

There are only three gauges on Lake Erie with long periods of record; namely, Port Colborne, Buffalo and Cleveland. Of these, the gauges at Port Colborne and Cleveland were used as main gauges from which other gauge records were subtracted. Table 5.13 lists the computed first order linear trends in differences in elevations and shows that generally correlation coefficients between time and differences in elevation on Lake Erie are low (the highest value is only 0.5988). For three gauges out of nine (Erieau, Port Dover and Erie) it was impossible to arrive at a reasonable adjusted rate of crustal movement and for two other gauges (Toledo and Monroe) the adjusted rates are extremely doubtful, due to a very low correlation coefficient for Toledo and a short period of record for Monroe. At this stage in the analysis it was evident that only results from the four gauges Port Colborne, Buffalo, Cleveland and Port Stanley could be considered further.

Korkigian (46) using mean monthly elevations for the four months June to September has shown that gauge records on Lake Erie contain non-homogeneities. Using 12 months of data per year non-homogeneities proved to be statistically significant in only one case, Port Colborne. The time series of Port Colborne minus Buffalo shows the following first order linear trends for different periods: (1) for 1860-1970 the trend is -0.1345 ft/100 years, (2) for 1860-1925 the trend is -0.0537 ft/100 years, (3) for 1926-1950 the trend is +0.0090 ft/100 years, and (4) for 1951-1970 the trend is +0.0089 ft/100 years. Between 1925 and 1926 there is a jump in the record of +0.02 foot and between 1950 and 1951 there

TABLE 5.13

COMPUTED AND COMPUTED-ADJUSTED RATES OF RELATIVE CRUSTAL MOVEMENT AROUND LAKE ERIE

GAUGE B	GAUGE A							
	Cleveland				Port Colborne			
	Rates of Movement A-B							
	Correlation Coefficient (1)	Unadjusted Rate ft/100 years (2)	Adjusted Rate, ft/100 years (3)	No. of Months of Common Record (4)	Correlation Coefficient (1)	Unadjusted Rate, ft/100 years (2)	Adjusted Rate, ft/100 years (3)	No. of Months of Common Record (4)
Erieau	0.2199	-0.4013	-	155	0.0446*	-0.0782	-	158
Port Stanley	0.4960	0.3197	0.2530	558	0.3196	-0.2701	-0.2034	561
Port Dover	0.3918	1.7739	-	143	0.5098	1.6560	-	146
Port Colborne	0.5988	0.4557	0.4564	1329				
Buffalo	0.4294	0.3250	0.3233	1071	0.3196	-0.1345	-0.1332	1071
Erie	0.1903*	0.6089	-	149	0.3399	0.6340	-	149
Cleveland					0.5988	-0.4557	-0.4564	1329
Toledo	0.1490	0.1601	0.2476	722	0.0691*	-0.1212	-0.2087	722
Monroe	0.5366	0.2200	0.1993	289	0.3514	-0.2778	-0.2571	289

*Statistically non-significant relationships between differences in elevations and time is a jump in the record of -0.10 foot. No physical causes of these jumps can be found in the gauge histories (12).

Table 5.14 shows an overall trend in Lake Erie elevation of approximately -0.75 foot per 100 years. This estimate is subject to a large uncertainty as is shown by the small correlation coefficients in Table 5.14.

Referring the computed relative rates of movement to a datum is not simple on Lake Erie since there is no Nipissing zero isobase (52). The solution taken was to accept the southernmost gauge, Cleveland, as a datum.

One of the gauges with a long period of record is Buffalo, located near the outlet of Lake Erie. Since Lake Erie is unregulated, the mean lake elevation rises at the same rate as the outlet. The Buffalo gauge is shown, on Table 5.13, to be rising, relative to Cleveland, at a rate of 0.3250 foot/100 years.

Figs. 5.8 and 5.9, respectively, show lines of equal rates of absolute movement and the circular distribution of crustal movement around Lake Erie. Table 5.15 compares the results of this study for Lake Erie with the results from other earlier studies (28,25,31,32,75,79). Table 5.16 shows rates of crustal movement relative to the mean lake elevation.

It has been shown that the presence of crustal movement around Lake Erie is open to some question but, at

TABLE 5.14

DETERMINATION OF TREND IN WATER ELEVATION FOR LAKE ERIE

Station	1329	1071	722	549	289	155	149	143
	The upper figure of each pair is the linear trend in feet per 100 years, the lower figure of each pair is the corresponding correlation coefficient.							
Cleveland	-0.75 0.25	-0.43 0.12	0.63 0.12	1.43 0.20	-0.60 0.21	8.64 0.36	10.16 0.41	10.35 0.40
Port Colborne	-1.21 0.39							
Buffalo		-0.76 0.22						
Toledo			0.47 0.08					
Port Stanley				1.12 0.16				
Monroe*					-0.82 0.27			
Erieau						9.04 0.39		
Erie							9.55 0.40	
Port Dover								8.57 0.35

* This period of record does not extend to 1970.

FIGURE 5.8

CRUSTAL MOVEMENT ON LAKE ERIE

LINES OF EQUAL RATE OF ABSOLUTE MOVEMENT FEET/100 YEARS

NOTE: TO OBTAIN RATES OF MOVEMENT OF LAND RELATIVE TO LAKE SURFACE, SUBTRACT 0.32 FT/100 YEARS.

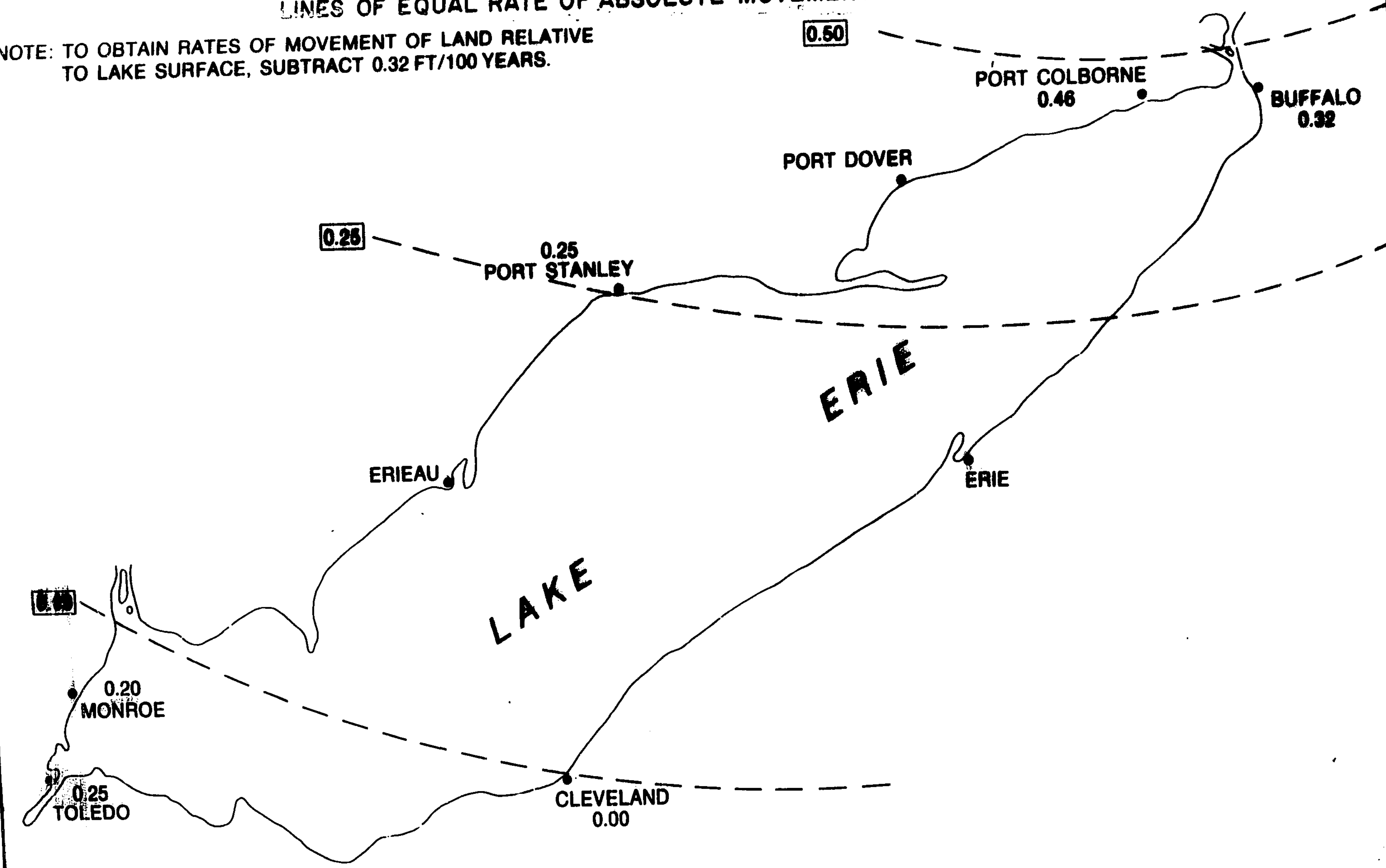


FIGURE 5.9

CIRCULAR DISTRIBUTION OF CRUSTAL MOVEMENT AROUND LAKE ERIE

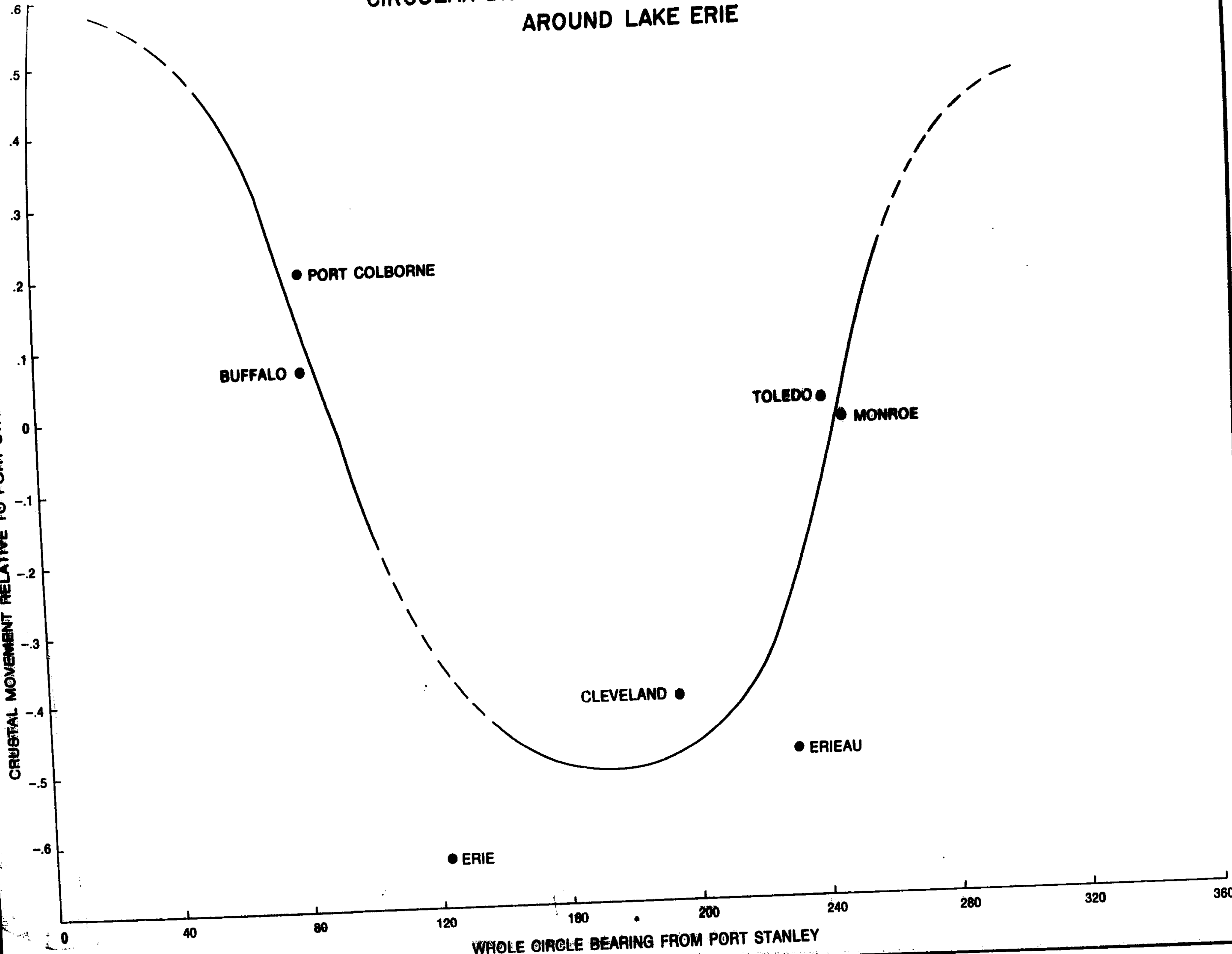


TABLE 5.15

CRUSTAL MOVEMENT AROUND LAKE ERIE
IN FEET PER 100 YEARS RELATIVE TO PORT COLBORNE

Gauge Site	Author								
	Gilbert (28)	Freeman (25)	Gutenberg (31,32)	Moore (58)	VCS (75,79) unadjusted	VCS (75,79) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
ort Dover							1.66	-	146
ort Stanley			-0.55	-0.25	-0.07	-0.18	-0.27	-0.20	561
uffalo		-0.04[t]	-0.07[t]	-0.06[t]	-0.04	0.00	-0.13	-0.13	1071
ackawanna					+0.72	+0.80			
unkirk				-0.13	-0.12	-0.13			
rie				-0.46	-0.63	-0.57	-0.63	-	149
onneaut				-0.57	-0.47	-0.47			
shtabula				-0.27	-0.41	-0.35			
airport				-0.10[t]	-0.58	-0.47			
leveland	-0.71	-0.49	-0.67	-0.33	-0.43	-0.37	-0.46	-0.46	1329
ocky River					-0.61	-0.67			
orain				-0.43	-0.59	-0.48			
ermilion				-0.18	-0.44	-0.35			
uron					-0.39	-0.29			
andusky				-0.25	-0.12	-0.02			
ort Clinton				-0.57	-0.45	-0.48			
oledo				-0.18	+0.01	-0.12	-0.12	-0.21	722
onroe				-0.09	-0.21	-0.18	-0.28	-0.26	289
rieau							-0.42	-	158

OTES: (1) t = derived by triangulation from the author's results.
(2) + = rate indicates a rise of the land with respect to Port Colborne.
(3) VCS = Vertical Control Subcommittee.

TABLE 5.16

RATES OF MOVEMENT OF THE LAND AROUND LAKE ERIE

RELATIVE TO THE LAKE SURFACE, FEET PER 100 YEARS

Erieau	*
Port Stanley	-0.07
Port Dover	*
Port Colborne	+0.14
Buffalo	0.00
Erie	*
Cleveland	-0.32
Toledo	*
Monroe	*

*Rate of movement is not certain

the present time, the evidence is not sufficient to provide a definite answer to the problem, and so, the rates of crustal movement derived from first order linear trends in water elevation differences will be retained in this study, although with serious reservations.

5.2.4 Lake Ontario

Lake Ontario is the second of the Great Lakes presently regulated. Four gauging stations, Port Dalhousie, Toronto, Kingston and Oswego have over 1,000 months of record and Rochester has 953 months. Toronto and Oswego were chosen as gauges from which differences would be taken, because of their long periods of record and since they are at opposite ends of the lake. In addition, the gauge at Oswego has special significance since its elevations are referred to in the regulation of Lake Ontario.

Table 5.17 shows the first order linear trends computed in differences in lake elevations. The range of correlation coefficients in Table 5.17 is very large, varying from 0.0527 to 0.8057 and generally being proportional to the length of common record between the gauging stations concerned. The only exceptions to this being between Toronto and Port Weller, and Oswego and Port Weller. Table 5.17 indicates a rise of the eastern end of Lake Ontario relative to the western end of 0.7826 foot per 100 years. No additional information is provided by the trends in elevations

TABLE 5.17

COMPUTED AND COMPUTED-ADJUSTED RATES OF RELATIVE CRUSTAL MOVEMENT AROUND LAKE ONTARIO

Gauge B	Gauge A							
	Oswego				Toronto			
	Rates of Movement A-B							
	Correlation Coefficient (1)	Unadjusted Rate, ft/100 years (2)	Adjusted Rate, ft/100 years (3)	No. of Months of Common Record (4)	Correlation Coefficient (1)	Unadjusted Rate, ft/100 years (2)	Adjusted Rate, ft/100 years (3)	No. of Months of Common Record (4)
Port Weller	0.1547	-0.0760	-0.2182	202	0.7056	0.2289	0.3711	205
Port Dalhousie	0.2279	-0.4943	-0.4833	1193	0.0527 *	0.1170	0.1060	1124
Toronto	0.7826	-0.5917	-0.5893	1291				
Cobourg	0.1122	-0.1663	-0.2371	171	0.2279	0.2814	0.3522	174
Kingston	0.2537	0.0737	0.0741	1326	0.8057	0.6659	0.6634	1294
Cape Vincent	0.4433	0.1418	0.0008	650	0.7342	0.4491	0.5901	650
Oswego					0.7826	0.5917	0.5893	1291
Rochester	0.4694	-0.1997	-0.2186	828	0.7112	0.3497	0.3686	767
Fort Niagara	0.7020	-0.3597	-0.3521	385	0.5698	0.2449	0.2373	373

*Statistically non-significant relationships between differences in elevations and time

(Table 5.18). If the outlet of the lake is rising at approximately 0.75 foot per 100 years, then the mean lake elevation is also rising at this rate, since regulation of Lake Ontario has been in effect only since 1958 and is not yet an important factor in long-term lake elevations.

Table 5.19 compares the results of the present study for Lake Ontario with those of earlier studies (25, 31,58,61,74,78) using lake elevations. Table 5.20 lists rates of crustal movement relative to the Lake Ontario mean level. Figs. 5.10 and 5.11 indicate respectively lines of equal rate of crustal movement and the circular distribution of crustal movement around Lake Ontario.

The effect of crustal movement on the regulation of Lake Ontario is interesting. The regulation limits for Lake Ontario adopted by the Canadian and U.S. Governments were defined (11) in terms of the fixed land to water elevation relationship at the Oswego gauge. However, Fig. 5.10 shows that the land in the area of the regulation structures (International Rapids Section of the St. Lawrence) is rising relative to the land at Oswego by around 0.16 foot per 100 years while the land at the western end of the lake is sinking relative to Oswego at a rate of over 0.50 foot per 100 years.

Assuming a simple outflow relationship for Lake Ontario this means that elevations at Oswego are rising

TABLE 5.18

DETERMINATION OF TREND IN WATER ELEVATION FOR LAKE ONTARIO

Station	Length of Record Used (Months)							
	1326	1291	1193	828	650	385	202	171
	The upper figure of each pair is the linear trend in feet per 100 years, the lower figure of each pair is the corresponding correlation coefficient.							
Oswego	-0.65 0.18	-0.62 0.17	-0.63 0.15	-0.73 0.25	0.86 0.11	-0.64 0.24	-1.61 0.19	3.72 0.17
Kingston	-0.72 0.20							
Toronto		-0.02 0.01						
Port Dalhousie*			-0.13 0.03					
Rochester				-0.53 0.18				
Cape Vincent					0.71 0.09			
Fort Niagara*						-0.28 0.11		
Port Weller							-1.54 0.18	
Cobourg								3.89 0.17

* These periods of record do not extend to 1970

TABLE 5.19

CRUSTAL MOVEMENT AROUND LAKE ONTARIO IN FEET PER 100 YEARS RELATIVE TO TORONTO

Gauge Site	Author								
	Freeman (25)	Gutenberg (31)	Moore (58)	Price (61)	VCS (74,78) unadjusted	VCS (74,78) adjusted	Present study, unadjusted	Present study, adjusted	Period of common record, months
ort Weller							+0.23	+0.37	205
obourg					+0.17	+0.20	+0.28	+0.35	174
ingston		+0.59[t]	+0.81	+0.66	+0.71	+0.69	+0.67	+0.66	1294
ort Dalhousie	-0.45	+0.16	-0.19	-0.10	+0.16	+0.12	+0.12	+0.11	1124
ibbetts Point					+1.00	+0.76			
acketts Harbor			+0.59		+0.56	+0.45			
ort Ontario					+0.50	+0.36			
ittle Sodus Bay					+0.01	+0.10			
odus Bay					-0.24	-0.15			
ochester			+0.33		+0.36	+0.24	+0.35	+0.37	767
ak Orchard					+0.43	+0.29			
lcott			+0.28		+0.41	+0.23			
ilson					+0.32	+0.18			
ort Niagara			+0.28		+0.24	+0.10	+0.24	+0.24	373
swego	+0.23	+0.64	+0.58	+0.45	+0.62	+0.56	+0.59	+0.59	1291
ape Vincent		+1.35	+0.64	+0.66	+0.45	+0.58	+0.45	+0.59	650
amilton									

otes: (a) t= derived by triangulation from the author's results.
+ = rate indicates a rise of the land with respect to Toronto.
VCS = Vertical Control Subcommittee.

TABLE 5.20

RATES OF MOVEMENT OF THE LAND AROUND LAKE ONTARIO RELATIVE TO THE LAKE SURFACE, FEET PER 100 YEARS

Port Weller	-*
Port Dalhousie	*
Toronto	-0.75
Cobourg	-0.40
Kingston	-0.09
Cape Vincent	-0.16
Oswego	-0.16
Rochester	-0.38
Fort Niagara	-*

*Rate of movement is not certain

FIGURE 5.10

CRUSTAL MOVEMENT ON LAKE ONTARIO

LINES OF EQUAL RATE OF ABSOLUTE MOVEMENT, FEET/100 YEARS

NOTE: TO OBTAIN RATES OF MOVEMENT OF LAND RELATIVE TO LAKE SURFACE, SUBTRACT 0.75 FT/100 YEARS.

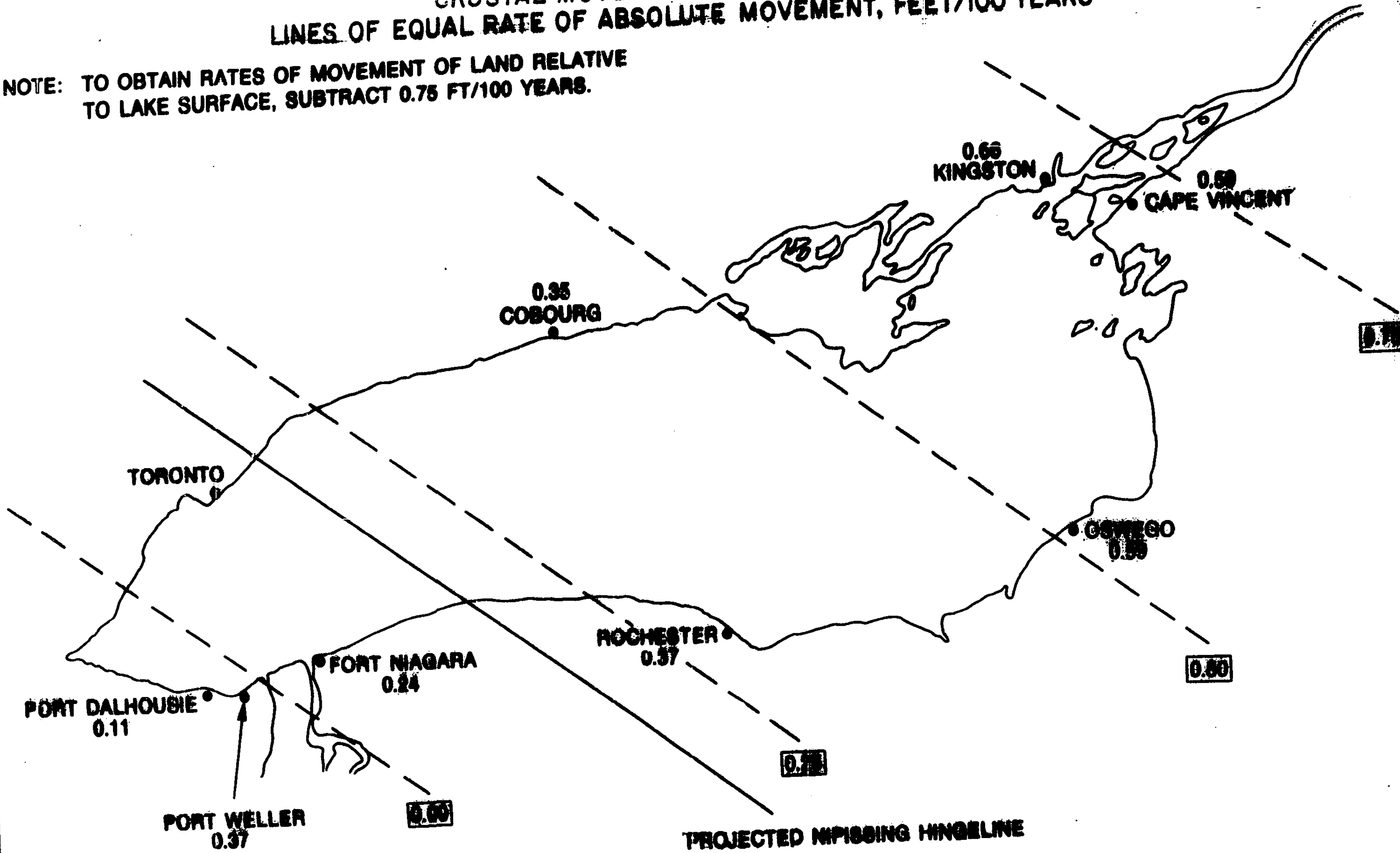
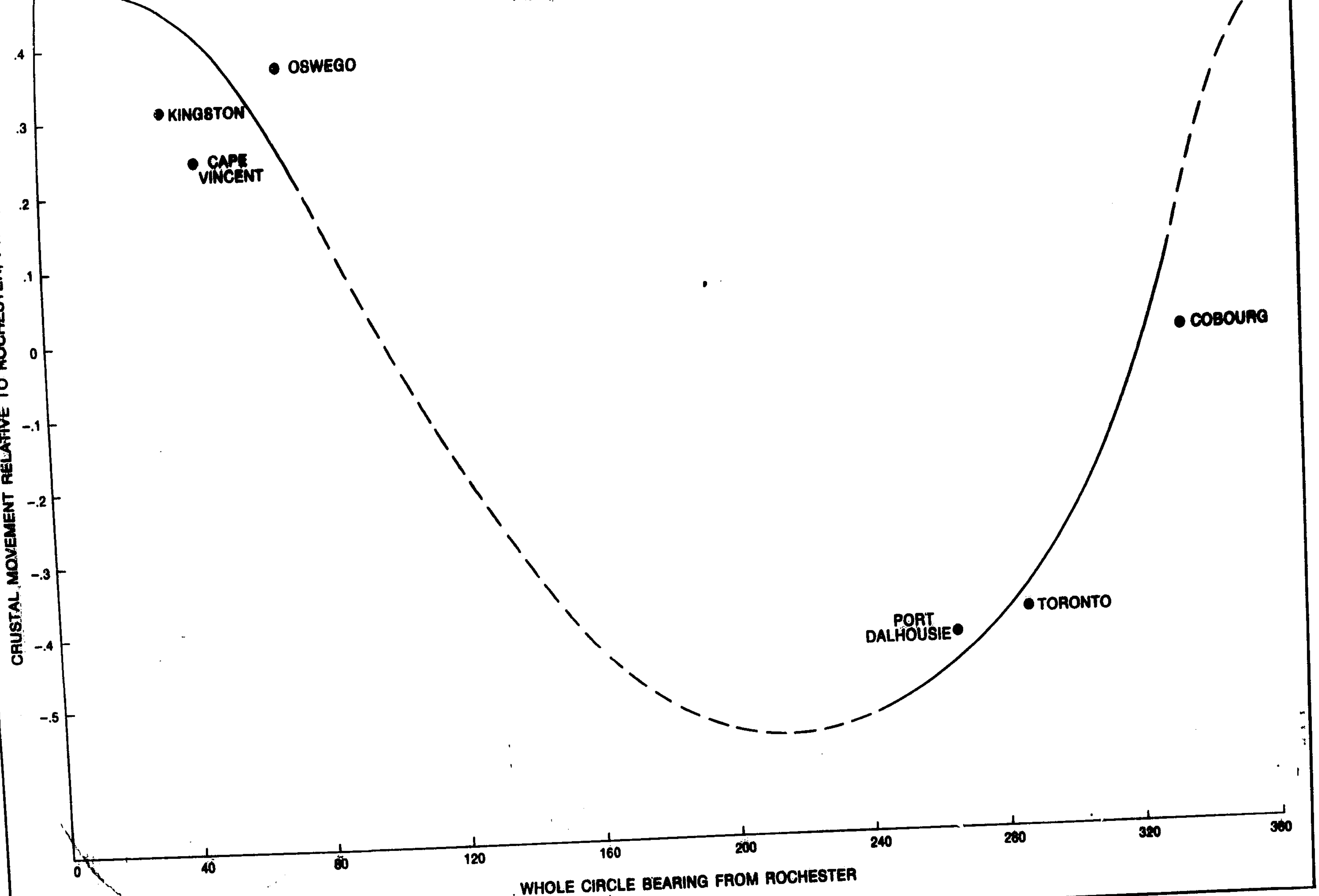


FIGURE 5.11
CIRCULAR DISTRIBUTION OF CRUSTAL MOVEMENT
AROUND LAKE ONTARIO



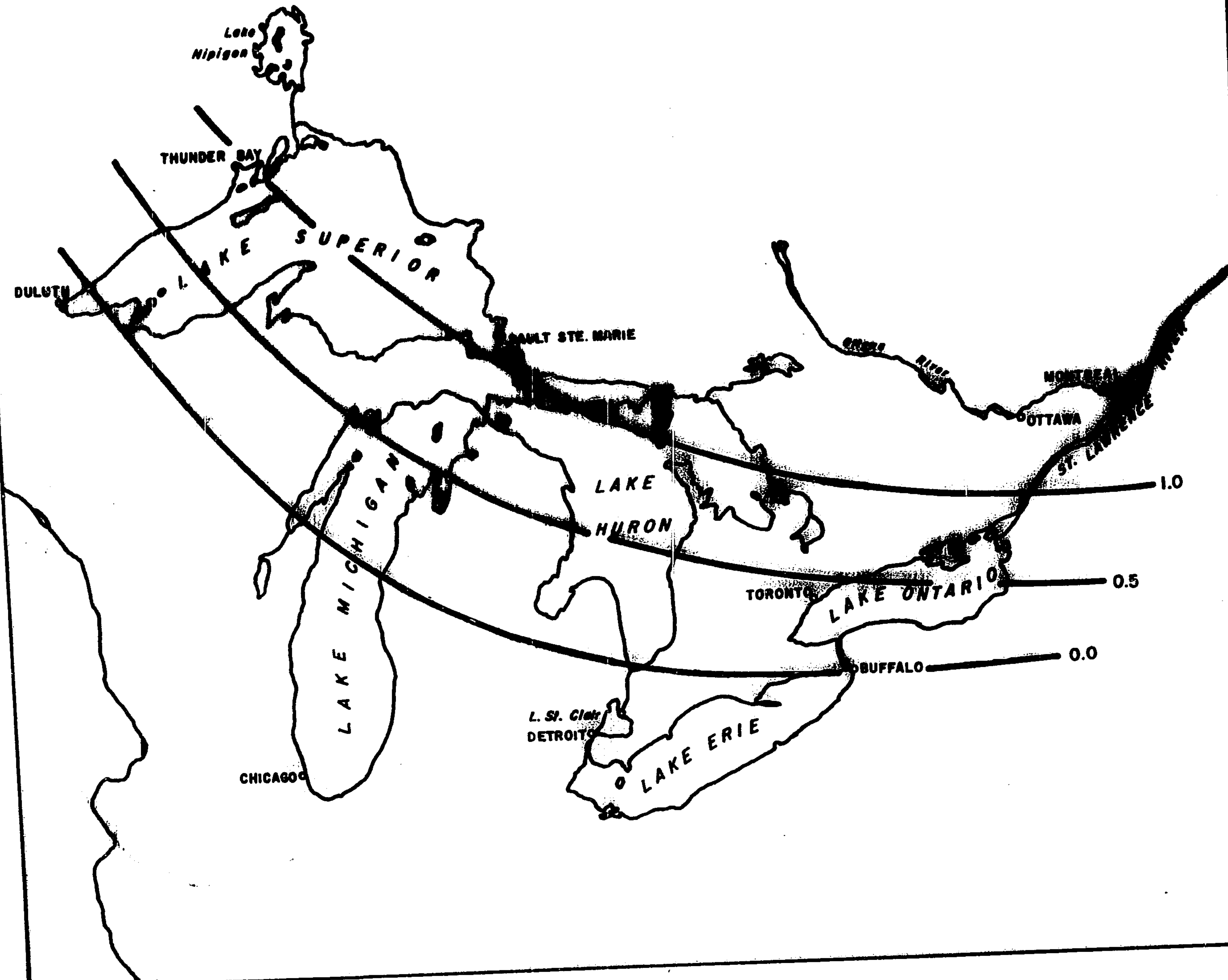
relative to the land at a rate only one-third the rate at which elevations at the western end of the lake are rising. The present arrangement is, therefore, a compromise. If the lake was regulated using elevations at the outlet, flooding would eventually occur at the western end of the lake. If, on the other hand, regulation of Lake Ontario was carried out using elevations measured at the western end of the lake, this would reduce the flooding risk but would necessitate increased channel excavation to compensate for the naturally decreasing cross-section of the St. Lawrence River.

5.3 Crustal Movement Around the Great Lakes

Fig. 5.12 shows lines of equal rates (isolines) of crustal movement for the Great Lakes region. These were derived by combining isolines from Figs. 5.3, 5.6, 5.8, and 5.10. The datum used approximates the previously determined zero lines on Lakes Superior and Michigan-Huron but is not in agreement with the zero lines determined individually for Lakes Erie and Ontario.

The isolines on Fig. 5.12 confirm the opinions held by many investigators (25,31) that Lakes Superior and Ontario are affected more by crustal movement than the other two lakes. The shape of the lines also agrees generally with isolines derived by other investigators using other methods (2,38,83).

FIGURE 5.12
CRUSTAL MOVEMENT WITHIN THE GREAT LAKES BASIN
RATES OF MOVEMENT IN FEET PER 100 YEARS



It may be noted from Fig. 5.12 that no crustal movement is indicated around Lake Erie. This results from the large uncertainty in computed areas of movement around this lake together with the fact that the trend in isolines based on the other three lakes is to pass to the north of Lake Erie.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

A time series analysis of five records of mean monthly Lake Superior elevations showed that the dominant statistical components are autoregressive and periodic. A similar analysis of time series of differences in mean monthly elevations showed first order linear trends to be the dominant components.

Assuming that such time series are representative of the Great Lakes as a whole, only first order linear trends in differences in elevations were determined for the other lakes. It was also assumed that these first order linear trends represent rates of relative crustal movement.

The Nipissing zero isobase was used as a datum to convert relative rates of movement to absolute rates of vertical crustal movement.

Rates of movement computed in this study agree very well with rates of movement computed in other studies using lake elevations, beach elevations and sediment deposits.

The study shows that in general, the Great Lakes Basin as a unit is tilting, with the north-eastern area rising at a rate of approximately 1.50 feet per 100 years relative to the south-western area.

6.2 Recommendations

The study has shown one specific area in which more research is needed, that is, the status of vertical crustal movement around Lake Erie. Research in this area should be concentrated on detailed examination of gauge histories with the objective of producing statistically consistent records of lake elevations.

It is also recommended that, when they become available, the geodetic levels presently being run from Father Point, Quebec, be used to verify the results of this study. By using the existing first-order levelling (1953-1958) together with the levels presently being run it will be possible to compute absolute changes in elevation of points around the lakes.

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APPENDIX A

TABLES SHOWING DETAILED RESULTS OF TIME SERIES ANALYSIS OF MEAN MONTHLY LAKE SUPERIOR ELEVATIONS AT MARQUETTE FOR THE PERIOD 1860 to 1970

APPENDIX A

LIST OF TABLES

TABLE A1

MONTHLY MEAN ELEVATIONS AT MARQUETTE FOR THE PERIOD OF RECORD 1860-1970

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1860	599.60	599.57	599.88	599.97	600.13	600.40	600.39	600.37	600.48	600.39	600.30	599.93
1861	599.64	599.62	599.59	599.62	600.29	600.70	600.62	600.58	600.53	600.49	600.17	600.11
1862	600.01	599.56	599.43	599.38	599.80	599.93	599.97	600.18	600.23	600.27	600.03	599.82
1863	599.57	599.41	599.25	599.13	599.22	599.27	599.40	599.68	599.84	599.83	599.92	599.57
1864	599.30	599.17	599.05	598.95	599.30	599.37	599.47	599.62	599.64	599.47	599.17	599.28
1865	599.04	598.66	598.67	599.07	599.63	599.96	600.23	600.23	600.21	600.16	599.68	599.41
1866	598.93	598.52	598.71	599.20	599.40	599.69	599.88	600.01	599.96	600.03	599.92	599.35
1867	599.08	599.34	599.44	599.47	599.81	599.72	599.95	600.16	600.09	600.13	600.08	599.48
1868	599.31	598.74	599.26	599.67	599.58	599.58	599.68	599.70	599.87	599.87	599.83	599.83
1869	599.31	599.09	598.67	599.21	599.61	599.62	600.00	600.44	600.55	600.55	600.44	599.79
1870	599.56	599.33	599.34	599.44	599.58	600.39	600.58	600.59	600.74	600.59	600.40	599.48
1871	599.39	598.79	599.21	599.70	600.24	600.37	600.43	600.49	600.59	600.53	600.46	599.90
1872	599.60	599.49	599.37	599.27	599.92	600.30	600.57	600.74	600.90	600.80	600.65	600.35
1873	600.25	600.06	600.09	600.12	600.44	600.74	601.03	601.31	601.37	601.32	601.13	600.85
1874	600.59	600.26	600.22	600.32	600.39	600.59	600.97	601.06	601.16	601.22	601.04	600.73
1875	600.41	600.37	600.41	600.41	600.63	600.99	600.98	601.07	601.30	601.15	601.01	600.81
1876	600.61	600.40	600.31	600.34	600.88	601.56	601.95	602.06	601.95	601.62	601.46	601.18
1877	600.82	600.58	600.32	600.24	600.23	600.49	600.82	600.88	600.72	600.72	600.51	600.44
1878	600.32	600.44	599.67	599.64	599.91	600.22	600.29	600.17	600.00	600.07	599.87	599.55
1879	599.49	599.61	599.91	599.52	599.16	599.39	599.63	599.75	599.64	599.73	599.65	599.21
1880	599.02	598.99	598.89	598.90	599.65	600.45	600.60	600.59	600.59	600.54	600.48	600.22
1881	599.96	599.86	599.77	599.68	599.99	600.43	600.49	600.54	600.77	601.11	601.04	600.76
1882	600.41	600.16	600.05	599.97	600.13	600.15	600.43	600.61	600.61	600.45	600.42	600.24
1883	600.01	599.72	599.72	599.97	599.98	600.13	600.30	600.36	600.31	600.16	600.01	599.86
1884	599.81	599.64	599.54	599.35	599.61	599.69	599.89	599.93	600.25	600.58	600.44	600.24
1885	600.01	599.83	599.75	599.70	600.12	600.37	600.62	600.71	600.63	600.44	600.31	600.02
1886	599.65	599.52	599.46	599.53	599.79	599.92	600.01	599.99	599.97	600.09	600.00	599.83
1887	599.46	599.48	599.79	599.96	599.69	599.92	600.23	600.27	600.11	600.13	599.85	599.67
1888	599.54	599.53	599.46	599.46	599.93	600.68	600.90	601.07	601.01	600.89	600.71	600.35
1889	600.16	599.88	599.65	599.65	600.02	600.19	600.40	600.55	600.68	600.53	600.23	599.87
1890	599.78	599.66	599.42	599.39	599.70	600.05	600.35	600.50	600.63	600.60	600.39	600.03
1891	599.63	599.49	599.46	599.42	599.63	599.63	599.67	599.77	599.66	599.83	599.65	599.56
1892	599.56	599.28	599.15	599.16	599.49	599.87	599.90	600.02	600.07	599.97	599.80	599.52
1893	599.24	599.15	599.20	599.30	599.80	600.33	600.62	600.68	600.59	600.56	600.40	600.17
1894	599.99	599.81	599.90	600.05	600.83	601.02	601.08	601.21	601.13	601.15	601.13	600.91
1895	600.61	600.39	600.22	600.12	600.49	600.89	601.01	601.06	601.20	601.25	600.96	600.64
1896	600.45	600.25	600.05	600.14	600.79	601.10	601.23	601.25	601.08	600.76	600.83	600.70
1897	600.53	600.30	600.22	600.24	600.56	600.89	601.19	601.31	601.25	601.05	600.75	600.32
1898	599.94	599.76	599.58	599.57	599.81	600.29	600.70	600.83	600.90	600.87	600.67	600.44
1899	600.07	599.87	599.90	599.87	600.58	601.07	601.32	601.51	601.67	601.48	601.38	601.16
1900	600.78	600.59	600.37	600.27	600.44	600.51	600.72	601.08	601.60	601.68	601.65	601.27
1901	600.93	600.62	600.42	600.36	600.65	600.75	601.22	601.35	601.18	601.21	601.14	600.82
1902	600.46	600.25	600.11	600.16	600.48	600.77	601.02	601.03	601.07	600.97	600.99	600.82
1903	600.51	600.33	600.14	600.30	600.71	601.07	601.25	601.43	601.42	601.59	601.37	601.07
1904	600.65	600.46	600.38	600.30	600.60	600.90	600.99	601.08	601.21	601.36	601.27	600.87
1905	600.60	600.26	600.17	600.38	600.62	600.80	601.10	601.23	601.46	601.46	601.30	601.09
1906	600.85	600.56	600.35	600.28	600.61	600.91	601.03	601.06	601.08	600.97	600.79	600.58
1907	600.35	600.19	600.07	600.07	600.23	600.68	600.83	601.05	601.29	601.27	601.00	600.65
1908	600.21	599.98	599.83	599.74	600.14	600.68	600.96	601.03	600.88	600.70	600.36	600.12
1909	599.82	599.59	599.48	599.42	599.75	600.07	600.31	600.53	600.53	600.42	600.39	600.39
1910	600.14	599.87	599.64	599.75	599.88	600.03	600.61	600.10	600.09	600.05	599.81	599.54
1911	599.18	599.03	598.77	598.66	598.95	599.42	599.76	600.23	600.33	600.32	600.16	600.02
1912	599.90	599.67	599.55	599.57	600.03	600.33	600.48	600.67	600.79	600.74	600.59	600.35
1913	600.03	599.72	599.66	599.79	600.21	600.50	600.73	600.88	600.93	601.15	601.01	600.83

TABLE A1 (CONTINUED)

1914	600.53	600.34	600.05	599.97	600.36	600.59	600.81	600.88	600.93	600.85	600.57	600.21
1915	599.94	599.81	599.59	599.43	599.72	600.03	600.36	600.47	600.51	600.84	600.94	600.82
1916	600.72	600.53	600.27	600.46	601.08	601.55	601.72	601.81	601.93	601.76	601.58	601.26
1917	600.87	600.51	600.44	600.37	600.48	600.64	600.73	600.82	600.85	600.76	600.56	600.25
1918	600.84	599.78	599.65	599.57	599.81	600.24	600.37	600.48	600.64	600.56	600.64	600.56
1919	600.37	600.18	600.00	600.11	600.34	600.55	600.65	600.67	600.64	600.51	600.58	600.46
1920	600.13	600.02	599.99	600.32	600.47	600.77	601.01	601.00	600.84	600.71	600.53	600.30
1921	600.11	599.81	599.67	599.82	600.17	600.46	600.64	600.75	600.74	600.57	600.28	600.04
1922	599.71	599.50	599.35	599.56	600.04	600.26	600.56	600.71	600.78	600.58	600.38	600.14
1923	599.90	599.73	599.59	599.51	599.70	599.77	599.99	600.11	600.16	600.12	600.01	599.85
1924	599.72	599.41	599.18	599.13	599.26	599.35	599.44	599.70	599.92	599.96	599.80	599.55
1925	599.19	599.02	598.87	598.97	599.05	599.32	599.53	599.62	599.55	599.49	599.17	598.94
1926	598.58	598.37	598.32	598.23	598.30	598.63	598.99	599.15	599.46	599.76	599.82	599.77
1927	599.57	599.46	599.43	599.53	600.00	600.38	600.77	600.89	600.79	600.78	600.67	600.48
1928	600.23	599.96	599.83	599.95	600.27	600.58	600.96	601.11	601.39	601.55	601.37	601.03
1929	600.72	600.54	600.58	600.69	600.67	600.79	600.99	600.99	600.95	600.97	600.88	600.64
1930	600.41	600.25	600.19	600.05	600.37	600.70	601.02	601.03	600.87	600.86	600.70	600.51
1931	600.22	599.95	599.79	599.64	599.83	600.07	600.36	600.43	600.42	600.51	600.54	600.48
1932	600.32	600.21	600.21	600.03	600.32	600.54	600.81	601.00	601.03	600.72	600.59	600.37
1933	600.19	600.08	599.95	600.01	600.49	600.81	600.92	601.00	600.98	600.99	600.93	600.72
1934	600.55	600.37	600.25	600.30	600.65	600.91	600.98	600.90	600.99	601.11	601.06	600.96
1935	600.69	600.55	600.44	600.40	600.63	600.84	601.10	601.23	601.17	600.96	600.85	600.60
1936	600.47	600.27	600.21	600.29	600.78	601.04	601.04	600.95	600.98	600.81	600.55	600.22
1937	600.13	600.08	600.02	599.99	600.50	600.75	600.88	601.08	601.12	601.04	601.01	600.77
1938	600.62	600.45	600.28	600.54	601.15	601.49	601.52	601.50	601.38	601.10	601.01	600.77
1939	600.69	600.59	600.48	600.50	600.82	601.29	601.60	601.56	601.45	601.17	600.83	600.60
1940	600.33	599.99	599.77	599.68	600.07	600.61	600.87	600.95	600.90	600.72	600.68	600.47
1941	600.27	600.04	599.77	599.82	600.25	600.58	600.73	600.84	601.11	601.46	601.33	601.06
1942	600.74	600.52	600.40	600.40	600.82	601.08	601.13	601.16	601.26	601.26	601.30	601.17
1943	600.93	600.80	600.67	600.62	601.00	601.56	601.81	601.81	601.64	601.36	601.13	601.77
1944	600.36	600.12	599.95	599.90	600.39	600.96	601.36	601.66	601.73	601.49	601.21	600.95
1945	600.64	600.43	600.41	600.86	600.92	601.01	601.24	601.32	601.45	601.31	601.11	600.93
1946	600.69	600.52	600.36	600.57	600.67	600.86	601.15	601.21	601.21	601.20	601.25	601.05
1947	600.74	600.55	600.31	600.34	600.76	601.35	601.65	601.57	601.57	601.34	601.07	600.75
1948	600.42	600.09	599.90	600.06	600.50	600.56	600.68	600.84	600.90	600.65	600.60	600.50
1949	600.30	600.16	599.92	599.94	600.35	600.60	601.05	601.25	601.17	601.14	601.05	600.74
1950	600.64	600.45	600.30	600.40	601.03	601.64	601.89	602.02	601.92	601.87	601.75	601.47
1951	601.69	600.90	600.91	601.14	601.53	601.64	601.82	601.82	601.90	601.93	601.77	601.50
1952	601.28	601.04	600.82	600.91	601.11	601.22	601.71	602.00	601.87	601.40	600.92	600.70
1953	600.43	600.29	600.13	600.27	600.51	601.05	601.41	601.51	601.41	601.09	600.69	600.53
1954	600.29	600.01	599.92	599.91	600.64	601.01	601.21	601.12	600.89	600.76	600.53	600.30
1955	600.03	599.77	599.75	599.85	600.19	600.41	600.48	600.72	600.67	600.81	600.76	600.49
1956	600.25	600.05	599.74	599.75	599.91	600.23	600.65	600.81	600.78	600.56	600.45	600.20
1957	599.91	599.66	599.58	599.65	600.05	600.34	600.73	600.76	600.78	600.62	600.52	600.33
1958	600.09	599.96	599.73	599.66	599.74	599.96	600.35	600.55	600.76	600.68	600.60	600.45
1959	600.19	599.94	599.84	599.80	600.16	600.54	600.72	600.90	601.23	601.33	601.07	600.70
1960	600.47	600.25	599.99	600.01	600.76	601.05	601.07	601.01	600.93	600.68	600.55	600.33
1961	600.05	599.84	599.75	599.83	600.16	600.38	600.48	600.50	600.57	600.57	600.51	600.34
1962	600.10	599.83	599.62	599.65	600.02	600.35	600.47	600.55	600.76	600.71	600.36	600.18
1963	599.87	599.65	599.55	599.76	600.01	600.36	600.59	600.70	600.66	600.55	600.43	600.22
1964	599.92	599.73	599.53	599.52	600.19	600.55	600.79	600.89	600.99	600.92	600.73	600.44
1965	600.15	599.92	599.75	599.79	600.24	600.49	600.62	600.67	600.62	600.73	600.55	600.50
1966	600.36	600.11	600.07	600.14	600.42	600.76	600.82	601.00	600.96	600.77	600.63	600.45
1967	600.24	600.09	599.89	600.13	600.51	600.69	600.87	600.99	600.85	600.67	600.58	600.35
1968	600.07	599.94	599.68	599.95	600.32	600.85	601.31	601.62	601.82	601.77	601.53	601.24
1969	601.08	600.94	600.71	600.68	601.03	601.10	601.15	601.15	600.99	600.84	600.68	600.42
1970	600.21	600.01	599.80	599.78	600.42	600.88	601.19	601.27	601.21	601.19	601.41	601.21

TABLE A2

SPECTRAL DENSITY OF MEAN MONTHLY LAKE ELEVATIONS

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.OF SPECTRUM
.13320E+04	0.	.10000E+01	.36149E+01	.12851E+01
.14400E+03	.69444E-02	.92694E+00	.23476E+01	.85341E+00
.72000E+02	.13889E-01	.77458E+00	.87368E+00	-.13504E+00
.48000E+02	.20833E-01	.58810E+00	.38487E+00	-.95486E+00
.36000E+02	.27778E-01	.40940E+00	.17113E+00	-.17653E+01
.28800E+02	.34722E-01	.27630E+00	.11667E+00	-.21484E+01
.24000E+02	.41667E-01	.21535E+00	.11591E+00	-.21549E+01
.20571E+02	.48611E-01	.23681E+00	.81440E-01	-.25079E+01
.18000E+02	.55556E-01	.33099E+00	.62123E-01	-.27786E+01
.16000E+02	.62500E-01	.47046E+00	.70949E-01	-.26458E+01
.14400E+02	.69444E-01	.61874E+00	.65722E-01	-.27223E+01
.13091E+02	.76389E-01	.72914E+00	.75081E+00	-.28660E+00
.12000E+02	.83333E-01	.76363E+00	.16669E+01	.51095E+00
.11077E+02	.90278E-01	.70621E+00	.70566E+00	-.34862E+00
.10286E+02	.97222E-01	.57440E+00	.18218E-01	-.40053E+01
.96000E+01	.10417E+00	.40369E+00	.22645E-01	-.37878E+01
.90000E+01	.11111E+00	.23814E+00	.24196E-01	-.37216E+01
.84706E+01	.11806E+00	.11376E+00	.18849E-01	-.39713E+01
.80000E+01	.12500E+00	.59534E-01	.12557E-01	-.43774E+01
.75789E+01	.13194E+00	.84742E-01	.10069E-01	-.45983E+01
.72000E+01	.13889E+00	.18013E+00	.96636E-02	-.46394E+01
.68571E+01	.14583E+00	.32027E+00	.11741E-01	-.44446E+01
.65455E+01	.15278E+00	.46753E+00	.95584E-02	-.46503E+01
.62609E+01	.15972E+00	.57970E+00	.21342E-01	-.38471E+01
.60000E+01	.16667E+00	.61854E+00	.39179E-01	-.32396E+01
.57600E+01	.17361E+00	.56786E+00	.19961E-01	-.39140E+01
.55385E+01	.18056E+00	.44086E+00	.57826E-02	-.51529E+01
.53333E+01	.18750E+00	.27276E+00	.85964E-02	-.47564E+01
.51429E+01	.19444E+00	.10706E+00	.95261E-02	-.46537E+01
.49655E+01	.20139E+00	-.14779E-01	.57496E-02	-.51586E+01
.48000E+01	.20833E+00	-.66580E-01	.46068E-02	-.53802E+01
.46452E+01	.21528E+00	-.38150E-01	.46251E-02	-.53763E+01
.45000E+01	.22222E+00	.60879E-01	.48467E-02	-.53294E+01
.43636E+01	.22917E+00	.20530E+00	.47503E-02	-.53495E+01
.42353E+01	.23611E+00	.35882E+00	.40671E-02	-.55048E+01
.41143E+01	.24306E+00	.47654E+00	.50343E-02	-.52915E+01
.40000E+01	.25000E+00	.52065E+00	.77280E-02	-.48629E+01
.38919E+01	.25694E+00	.46985E+00	.59587E-02	-.51229E+01
.37895E+01	.26389E+00	.34346E+00	.38133E-02	-.55693E+01
.36923E+01	.27083E+00	.18113E+00	.39666E-02	-.55299E+01
.36000E+01	.27778E+00	.25686E-01	.38532E-02	-.55589E+01
.35122E+01	.28472E+00	-.86898E-01	.29413E-02	-.58289E+01
.34286E+01	.29167E+00	-.12853E+00	.25771E-02	-.59611E+01
.33488E+01	.29861E+00	-.91897E-01	.30509E-02	-.57923E+01
.32727E+01	.30556E+00	.15315E-01	.30323E-02	-.57984E+01
.32000E+01	.31250E+00	.16867E+00	.23646E-02	-.60471E+01

TABLE A2 (CONTINUED)

.31304E+01	.31944E+00	.33032E+00	.20985E-02	-.61665E+01
.30638E+01	.32639E+00	.45403E+00	.22537E-02	-.60952E+01
.30000E+01	.33333E+00	.49888E+00	.26168E-02	-.59458E+01
.29388E+01	.34028E+00	.44922E+00	.19948E-02	-.62172E+01
.28800E+01	.34722E+00	.32396E+00	.19343E-02	-.62480E+01
.28235E+01	.35417E+00	.16005E+00	.21747E-02	-.61309E+01
.27692E+01	.36111E+00	.20904E-03	.20906E-02	-.61703E+01
.27170E+01	.36806E+00	-.11778E+00	.19210E-02	-.62549E+01
.26667E+01	.37500E+00	-.16651E+00	.21076E-02	-.61622E+01
.26182E+01	.38194E+00	-.13497E+00	.23028E-02	-.60736E+01
.25714E+01	.38889E+00	-.31577E-01	.19045E-02	-.62635E+01
.25263E+01	.39583E+00	.11849E+00	.15559E-02	-.64657E+01
.24828E+01	.40278E+00	.27599E+00	.15877E-02	-.64455E+01
.24407E+01	.40972E+00	.39757E+00	.16609E-02	-.64004E+01
.24000E+01	.41667E+00	.44485E+00	.18751E-02	-.62791E+01
.23607E+01	.42361E+00	.39880E+00	.16431E-02	-.64112E+01
.23226E+01	.43056E+00	.27520E+00	.15303E-02	-.64823E+01
.22857E+01	.43750E+00	.10939E+00	.16073E-02	-.64332E+01
.22500E+01	.44444E+00	-.54653E-01	.16454E-02	-.64098E+01
.22154E+01	.45139E+00	-.17336E+00	.12650E-02	-.66727E+01
.21818E+01	.45833E+00	-.21859E+00	.87639E-03	-.70397E+01
.21493E+01	.46528E+00	-.18178E+00	.94206E-03	-.69674E+01
.21176E+01	.47222E+00	-.72671E-01	.14528E-02	-.65343E+01
.20870E+01	.47917E+00	.81824E-01	.15460E-02	-.64721E+01
.20571E+01	.48611E+00	.24319E+00	.14610E-02	-.65287E+01
.20282E+01	.49306E+00	.36667E+00	.17054E-02	-.63740E+01
.20000E+01	.50000E+00	.41492E+00	.20663E-02	-.61820E+01

NATURAL LOGARITHM OF THE MEAN SPECTRUM IS, -.18576E+01

CHECK SUM OF POWER SPECTRAL ESTIMATES IS .41812E+00AND
SHOULD BE .41812E+00

TABLE A3

RESULTS OF VARIATE DIFFERENCE TECHNIQUE FOR DETECTING LINEAR TRENDS

DIFFERENCE NUMBER R	MEAN OF DIFFERENCES	VARIANCE OF DIFFERENCES	VARIANCE DIVIDED BY (2R) (R)
1	.12096E-02	.60751E-01	.30376E-01
2	-.12782E-03	.55626E-01	.92710E-02
3	-.57186E-03	.12966E+00	.64832E-02
4	-.75301E-04	.40534E+00	.57906E-02

VARIANCE OF ORIGINAL SERIES IS .41812E+00

TABLE A4

RESULTS OF POLYNOMIAL REGRESSION

NUMBER OF OBSERVATIONS 1332

POLYNOMIAL REGRESSION OF NUMBER 1

INTERCEPT	587.67437
REGRESSION COEFFICIENT 1	.00664
MULTIPLE CORRELATION COEFFICIENT	.32881
STANDARD ERROR OF ESTIMATE	.61113

ANALYSIS OF VARIANCE FOR 1 DEGREE POLYNOMIAL

SOURCE OF VARIATION	DEGREE OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F VALUE	IMPROVEMENT IN TERMS OF SUM OF SQUARES
DUE TO REGRESSION	1	60.21385	60.21385	161.22642	60.21385
DEVIATION ABOUT REGRESSION	1330	496.72019	.37347		
TOTAL	1331	556.93404			

POLYNOMIAL REGRESSION OF NUMBER 2

INTERCEPT	290.24777
REGRESSION COEFFICIENT 1	.31728
REGRESSION COEFFICIENT 2	-.00008
MULTIPLE CORRELATION COEFFICIENT	.34840
STANDARD ERROR OF ESTIMATE	.60679

ANALYSIS OF VARIANCE FOR 2 DEGREE POLYNOMIAL

SOURCE OF VARIATION	DEGREE OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F VALUE	IMPROVEMENT IN TERMS OF SUM OF SQUARES
DUE TO REGRESSION	2	67.60027	33.80014	91.79906	7.38643
DEVIATION ABOUT REGRESSION	1329	489.33376	.36820		
TOTAL	1331	556.93404			

TABLE A4 (CONTINUED)

POLYNOMIAL REGRESSION OF NUMBER 3

INTERCEPT	-5067.94687
REGRESSION COEFFICIENT 1	8.71213
REGRESSION COEFFICIENT 2	-.00446
REGRESSION COEFFICIENT 3	.00000
MULTIPLE CORRELATION COEFFICIENT	.34973
STANDARD ERROR OF ESTIMATE	.60670

ANALYSIS OF VARIANCE FOR 3 DEGREE POLYNOMIAL

SOURCE OF VARIATION	DEGREE OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F VALUE	IMPROVEMENT IN TERMS OF SUM OF SQUARES
DUE TO REGRESSION	3	68.11793	22.70598	61.68688	.51766
DEVIATION ABOUT REGRESSION	1328	488.81610	.36808		
TOTAL	1331	556.93404			

POLYNOMIAL REGRESSION OF NUMBER 4

NO IMPROVEMENT

TABLE A5

SPECTRAL DENSITY OF MEAN MONTHLY LAKE ELEVATIONS AFTER REMOVAL OF LINEAR TRENDS

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.OF SPECTRUM
.13320E+04	0.	.10000E+01	.23924E+01	.87229E+00
.14400E+03	.69444E-02	.91761E+00	.17976E+01	.58643E+00
.72000E+02	.13889E-01	.74501E+00	.86990E+00	-.13938E+00
.48000E+02	.20833E-01	.53305E+00	.39234E+00	-.93561E+00
.36000E+02	.27778E-01	.32987E+00	.17338E+00	-.17522E+01
.28800E+02	.34722E-01	.17847E+00	.11889E+00	-.21295E+01
.24000E+02	.41667E-01	.10892E+00	.11557E+00	-.21579E+01
.20571E+02	.48611E-01	.13310E+00	.80054E-01	-.25251E+01
.18000E+02	.55556E-01	.23993E+00	.60994E-01	-.27970E+01
.16000E+02	.62500E-01	.39789E+00	.72761E-01	-.26206E+01
.14400E+02	.69444E-01	.56599E+00	.67556E-01	-.26948E+01
.13091E+02	.76389E-01	.69116E+00	.74833E+00	-.28991E+00
.12000E+02	.83333E-01	.73040E+00	.16603E+01	.50699E+00
.11077E+02	.90278E-01	.66538E+00	.70260E+00	-.35296E+00
.10286E+02	.97222E-01	.51577E+00	.17638E-01	-.40377E+01
.96000E+01	.10417E+00	.32196E+00	.21802E-01	-.38258E+01
.90000E+01	.11111E+00	.13407E+00	.23934E-01	-.37324E+01
.84706E+01	.11806E+00	-.76363E-02	.18646E-01	-.39821E+01
.80000E+01	.12500E+00	-.69911E-01	.12649E-01	-.43702E+01
.75789E+01	.13194E+00	-.41615E-01	.10361E-01	-.45697E+01
.72000E+01	.13889E+00	.66655E-01	.98810E-02	-.46171E+01
.68571E+01	.14583E+00	.22579E+00	.11378E-01	-.44760E+01
.65455E+01	.15278E+00	.39308E+00	.90637E-02	-.47035E+01
.62609E+01	.15972E+00	.52093E+00	.20859E-01	-.38700E+01
.60000E+01	.16667E+00	.56536E+00	.38718E-01	-.32514E+01
.57600E+01	.17361E+00	.50801E+00	.19647E-01	-.39298E+01
.55385E+01	.18056E+00	.36441E+00	.56527E-02	-.51756E+01
.53333E+01	.18750E+00	.17428E+00	.84336E-02	-.47755E+01
.51429E+01	.19444E+00	-.13033E-01	.92169E-02	-.46867E+01
.49655E+01	.20139E+00	-.15098E+00	.54191E-02	-.52178E+01
.48000E+01	.20833E+00	-.20945E+00	.43828E-02	-.54301E+01
.46452E+01	.21528E+00	-.17680E+00	.44496E-02	-.54149E+01
.45000E+01	.22222E+00	-.64254E-01	.47079E-02	-.53585E+01
.43636E+01	.22917E+00	.99752E-01	.45008E-02	-.54035E+01
.42353E+01	.23611E+00	.27392E+00	.39417E-02	-.55361E+01
.41143E+01	.24306E+00	.40775E+00	.50247E-02	-.52934E+01
.40000E+01	.25000E+00	.45803E+00	.78562E-02	-.48465E+01
.38919E+01	.25694E+00	.40064E+00	.59172E-02	-.51299E+01
.37895E+01	.26389E+00	.25745E+00	.36197E-02	-.56214E+01
.36923E+01	.27083E+00	.73619E-01	.36763E-02	-.56059E+01
.36000E+01	.27778E+00	-.10224E+00	.35714E-02	-.56348E+01
.35122E+01	.28472E+00	-.22923E+00	.27947E-02	-.58800E+01

TABLE A5 (CONTINUED)

.34286E+01	.29167E+00	-.27550E+00	.25213E-02	-.59830E+01
.33488E+01	.29861E+00	-.23285E+00	.29933E-02	-.58114E+01
.32727E+01	.30556E+00	-.11023E+00	.30108E-02	-.58056E+01
.32000E+01	.31250E+00	.64564E-01	.22344E-02	-.61038E+01
.31304E+01	.31944E+00	.24869E+00	.19643E-02	-.62326E+01
.30638E+01	.32639E+00	.38960E+00	.21005E-02	-.61656E+01
.30000E+01	.33333E+00	.44117E+00	.24534E-02	-.60103E+01
.29388E+01	.34028E+00	.38551E+00	.18644E-02	-.62848E+01
.28800E+01	.34722E+00	.24398E+00	.18776E-02	-.62778E+01
.28235E+01	.35417E+00	.58649E-01	.21034E-02	-.61642E+01
.27692E+01	.36111E+00	-.12181E+00	.20478E-02	-.61910E+01
.27170E+01	.36806E+00	-.25500E+00	.18991E-02	-.62664E+01
.26667E+01	.37500E+00	-.30946E+00	.20972E-02	-.61672E+01
.26182E+01	.38194E+00	-.27261E+00	.23087E-02	-.60711E+01
.25714E+01	.38889E+00	-.15426E+00	.19430E-02	-.62435E+01
.25263E+01	.39583E+00	.17085E-01	.15504E-02	-.64692E+01
.24828E+01	.40278E+00	.19705E+00	.15311E-02	-.64818E+01
.24407E+01	.40972E+00	.33656E+00	.15747E-02	-.64537E+01
.24000E+01	.41667E+00	.39139E+00	.18499E-02	-.62926E+01
.23607E+01	.42361E+00	.34026E+00	.16746E-02	-.63922E+01
.23226E+01	.43056E+00	.20130E+00	.15339E-02	-.64800E+01
.22857E+01	.43750E+00	.14331E-01	.15504E-02	-.64692E+01
.22500E+01	.44444E+00	-.17118E+00	.15646E-02	-.64601E+01
.22154E+01	.45139E+00	-.30571E+00	.12089E-02	-.67180E+01
.21818E+01	.45833E+00	-.35697E+00	.87040E-03	-.70466E+01
.21493E+01	.46528E+00	-.31517E+00	.90088E-03	-.70121E+01
.21176E+01	.47222E+00	-.19112E+00	.14126E-02	-.65623E+01
.20870E+01	.47917E+00	-.15588E-01	.15266E-02	-.64847E+01
.20571E+01	.48611E+00	.16782E+00	.14582E-02	-.65306E+01
.20282E+01	.49306E+00	.30876E+00	.17067E-02	-.63732E+01
.20000E+01	.50000E+00	.36445E+00	.20848E-02	-.61731E+01

NATURAL LOGARITHM OF THE MEAN SPECTRUM IS, -.20279E+01

CHECK SUM OF POWER SPECTRAL ESTIMATES IS .36698E+00 AND
SHOULD BE .36698E+00

TABLE A6

MONTHLY MEANS AND STANDARD DEVIATIONS OF MEAN MONTHLY LAKE ELEVATIONS

MONTH	LONG TERM MEAN	STANDARD DEVIATION
JAN	-.24342	.47321
FEB	-.45270	.46086
MAR	-.56145	.44793
APR	-.51505	.45615
MAY	-.18515	.48461
JUN	.10981	.51732
JUL	.32314	.51678
AUG	.42224	.50735
SEP	.44963	.50133
OCT	.39468	.47532
NOV	.24909	.48050
DEC	.00919	.47941

TABLE A7

RESULTS OF PERIODIC ANALYSIS

TOTAL VARIANCE OF THE PERIODIC SERIES	.3670E+00
VARIANCE OF THE MEANS	.1328E+00
VARIANCE OF THE STANDARD DEVIATIONS	.4886E-03
PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED BY THE VARIANCE OF THE MEANS	.3619E+02
PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED BY THE VARIANCE OF THE STANDARD DEVIATIONS	.1332E+00
PERCENT OF THE TOTAL VARIANCE WHICH IS NOT EXPLAINED BY THE VARIANCE IN THE MEAN AND STANDARD DEVIATION	.6367E+02

TABLE A8

PERIODOGRAM COMPONENTS
FOR THE MEAN

HARMONIC	A	B	C SQUARED	PERIOD	PHASE
1	-.3297E-01	-.5095E+00	.2607E+00	.1200E+02	-.6462E-01
2	.6301E-01	.1526E-01	.4203E-02	.6000E+01	-.1333E+01
3	-.2257E-01	.1713E-02	.5122E-03	.4000E+01	.1495E+01
4	.1794E-02	-.1070E-01	.1177E-03	.3000E+01	.1661E+00
5	.5228E-02	.5683E-02	.5962E-04	.2400E+01	-.7437E+00
6	-.1062E-01	-.1054E-13	.1127E-03	.2000E+01	-.1571E+01

HARMONIC VARIANCES IN THE MEAN

HARMONIC	VARIANCE	PERCENT OF TOTAL VARIANCE IN THE MEANS	SIGNIFICANT AT 95 PER CENT CONFIDENCE L. (FISHER TEST)
1	.1303E+00	.9812E+02	YES
2	.2102E-02	.1582E+01	NO
3	.2561E-03	.1928E+00	NO
4	.5887E-04	.4431E-01	NO
5	.2981E-04	.2244E-01	NO
6	.5635E-04	.4242E-01	NO

TABLE A9

PERIODOGRAM COMPONENTS FOR THE STANDARD DEVIATION

HARMONIC	A	B	C SQUARED	PERIOD	PHASE
1	-.1548E-01	-.2167E-01	.7090E-03	.1200E+02	-.6202E+00
2	.1253E-01	.8895E-02	.2361E-03	.6000E+01	-.9535E+00
3	-.1765E-02	.2321E-02	.8503E-05	.4000E+01	.6503E+00
4	.3098E-02	-.1713E-02	.1253E-04	.3000E+01	.1066E+01
5	-.1711E-02	-.2711E-02	.1028E-04	.2400E+01	-.5629E+00
6	-.1324E-02	.5680E-15	.1753E-05	.2000E+01	.1571E+01

HARMONIC VARIANCES IN THE STANDARD DEVIATIONS

HARMONIC	VARIANCE	PERCENT OF TOTAL VARIANCE IN THE STANDARD DEVIATION	SIGNIFICANT AT 95 PER CENT CONFIDENCE L. (FISHER TEST)
1	.3545E-03	.7248E+02	YES
2	.1181E-03	.2414E+02	NO
3	.4252E-05	.8693E+00	NO
4	.6267E-05	.1281E+01	NO
5	.5139E-05	.1051E+01	NO
6	.8764E-06	.1792E+00	NO

TABLE A10

SPECTRAL DENSITY OF MEAN MONTHLY LAKE ELEVATIONS AFTER REMOVAL OF LINEAR TRENDS AND PERIODICITIES

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.OF SPECTRUM
.13320E+04	0.	.10000E+01	.23897E+01	.87116E+00
.14400E+03	.69444E-02	.95070E+00	.17993E+01	.58738E+00
.72000E+02	.13889E-01	.89351E+00	.87218E+00	-.13676E+00
.48000E+02	.20833E-01	.84402E+00	.39278E+00	-.93450E+00
.36000E+02	.27778E-01	.80019E+00	.17340E+00	-.17521E+01
.28800E+02	.34722E-01	.76034E+00	.11558E+00	-.21578E+01
.24000E+02	.41667E-01	.72308E+00	.11155E+00	-.21933E+01
.20571E+02	.48611E-01	.69081E+00	.76337E-01	-.25726E+01
.18000E+02	.55556E-01	.66280E+00	.61548E-01	-.27879E+01
.16000E+02	.62500E-01	.63795E+00	.72514E-01	-.26240E+01
.14400E+02	.69444E-01	.61958E+00	.61173E-01	-.27940E+01
.13091E+02	.76389E-01	.60079E+00	.53949E-01	-.29197E+01
.12000E+02	.83333E-01	.57908E+00	.42216E-01	-.31650E+01
.11077E+02	.90278E-01	.55635E+00	.19905E-01	-.39168E+01
.10286E+02	.97222E-01	.53405E+00	.21723E-01	-.38294E+01
.96000E+01	.10417E+00	.51222E+00	.22705E-01	-.37852E+01
.90000E+01	.11111E+00	.49209E+00	.23973E-01	-.37308E+01
.84706E+01	.11806E+00	.46717E+00	.18431E-01	-.39937E+01
.80000E+01	.12500E+00	.44119E+00	.12242E-01	-.44029E+01
.75789E+01	.13194E+00	.41544E+00	.10500E-01	-.45564E+01
.72000E+01	.13889E+00	.38986E+00	.10552E-01	-.45514E+01
.68571E+01	.14583E+00	.36671E+00	.11393E-01	-.44748E+01
.65455E+01	.15278E+00	.34608E+00	.82682E-02	-.47953E+01
.62609E+01	.15972E+00	.33063E+00	.83191E-02	-.47892E+01
.60000E+01	.16667E+00	.31548E+00	.12811E-01	-.43574E+01
.57600E+01	.17361E+00	.30336E+00	.10243E-01	-.45812E+01
.55385E+01	.18056E+00	.29078E+00	.67014E-02	-.50054E+01
.53333E+01	.18750E+00	.27610E+00	.88973E-02	-.47220E+01
.51429E+01	.19444E+00	.25830E+00	.94500E-02	-.46617E+01
.49655E+01	.20139E+00	.24011E+00	.56187E-02	-.51817E+01
.48000E+01	.20833E+00	.22100E+00	.46040E-02	-.53808E+01
.46452E+01	.21528E+00	.20292E+00	.47241E-02	-.53551E+01
.45000E+01	.22222E+00	.18455E+00	.50614E-02	-.52861E+01
.43636E+01	.22917E+00	.16942E+00	.47730E-02	-.53448E+01
.42353E+01	.23611E+00	.16065E+00	.39218E-02	-.55412E+01
.41143E+01	.24306E+00	.15566E+00	.37736E-02	-.55797E+01
.40000E+01	.25000E+00	.15101E+00	.49855E-02	-.53012E+01
.38919E+01	.25694E+00	.13908E+00	.47681E-02	-.53458E+01
.37895E+01	.26389E+00	.12656E+00	.35142E-02	-.56509E+01
.36923E+01	.27083E+00	.12073E+00	.36333E-02	-.56176E+01

TABLE A10 (CONTINUED)

.36000E+01	.27778E+00	.11933E+00	.36362E-02	-.56168E+01
.35122E+01	.28472E+00	.11708E+00	.29384E-02	-.58299E+01
.34286E+01	.29167E+00	.11579E+00	.26800E-02	-.59219E+01
.33488E+01	.29861E+00	.11225E+00	.31910E-02	-.57474E+01
.32727E+01	.30556E+00	.10869E+00	.31649E-02	-.57556E+01
.32000E+01	.31250E+00	.10928E+00	.23313E-02	-.60613E+01
.31304E+01	.31944E+00	.11571E+00	.20596E-02	-.61852E+01
.30638E+01	.32639E+00	.12221E+00	.19131E-02	-.62590E+01
.30000E+01	.33333E+00	.12102E+00	.17877E-02	-.63268E+01
.29388E+01	.34028E+00	.11448E+00	.15741E-02	-.64540E+01
.28800E+01	.34722E+00	.10696E+00	.18755E-02	-.62789E+01
.28235E+01	.35417E+00	.10016E+00	.21424E-02	-.61458E+01
.27692E+01	.36111E+00	.92146E-01	.20891E-02	-.61710E+01
.27170E+01	.36806E+00	.79763E-01	.19654E-02	-.62321E+01
.26667E+01	.37500E+00	.64289E-01	.21410E-02	-.61465E+01
.26182E+01	.38194E+00	.49494E-01	.24287E-02	-.60204E+01
.25714E+01	.38889E+00	.37100E-01	.20939E-02	-.61687E+01
.25263E+01	.39583E+00	.31045E-01	.16306E-02	-.64188E+01
.24828E+01	.40278E+00	.29409E-01	.15554E-02	-.64660E+01
.24407E+01	.40972E+00	.32839E-01	.14485E-02	-.65372E+01
.24000E+01	.41667E+00	.36157E-01	.15275E-02	-.64841E+01
.23607E+01	.42361E+00	.37270E-01	.16132E-02	-.64296E+01
.23226E+01	.43056E+00	.35477E-01	.16271E-02	-.64209E+01
.22857E+01	.43750E+00	.28030E-01	.16503E-02	-.64068E+01
.22500E+01	.44444E+00	.14515E-01	.15954E-02	-.64406E+01
.22154E+01	.45139E+00	.30620E-02	.12414E-02	-.66915E+01
.21818E+01	.45833E+00	-.50273E-02	.91645E-03	-.69950E+01
.21493E+01	.46528E+00	-.11051E-01	.97534E-03	-.69327E+01
.21176E+01	.47222E+00	-.14860E-01	.15012E-02	-.65015E+01
.20870E+01	.47917E+00	-.15221E-01	.16138E-02	-.64292E+01
.20571E+01	.48611E+00	-.11972E-01	.14912E-02	-.65082E+01
.20282E+01	.49306E+00	-.66725E-02	.14487E-02	-.65371E+01
.20000E+01	.50000E+00	-.24717E-02	.14040E-02	-.65684E+01

NATURAL LOGARITHM OF THE MEAN SPECTRUM IS, -.24108E+01

CHECK SUM OF POWER SPECTRAL ESTIMATES IS .23367E+00 AND
SHOULD BE .23367E+00

TABLE A11

THEORETICAL AND SAMPLE CORRELATION COEFFICIENTS FOR FIRST AND SECOND ORDER LINEAR AUTOREGRESSIVE MODELS

(1) LAG
(2) SAMPLE AUTOCORRELATION COEFFICIENT
(3) THEORETICAL FIRST ORDER LINEAR AUTOREGRESSIVE AUTOCORRELATION COEFFICIENTS
(4) THEORETICAL SECOND ORDER LINEAR AUTOREGRESSIVE AUTOCORRELATION COEFFICIENTS

(1)	(2)	(3)	(4)
1	.950	.950	.950
2	.893	.903	.893
3	.844	.859	.838
4	.800	.816	.786
5	.760	.776	.738
6	.722	.737	.692
7	.690	.701	.649
8	.661	.666	.609
9	.636	.633	.571
10	.618	.602	.536

CONFIDENCE LIMITS ON AUTOCORRELATION COEFFICIENTS

(1) ORDER
(2) 1 PCT. UPPER LIMIT
(3) 5 PCT. UPPER LIMIT
(4) SAMPLE AUTOCORRELATION COEFFICIENT
(5) 5 PCT. LOWER LIMIT
(6) 1 PCT. LOWER LIMIT

(1)	(2)	(3)	(4)	(5)	(6)
1	.063	.044	.950	-.046	-.064
2	.063	.044	.893	-.046	-.065
3	.063	.044	.844	-.046	-.065
4	.063	.044	.800	-.046	-.065
5	.063	.044	.760	-.046	-.065
6	.063	.044	.722	-.046	-.065
7	.063	.044	.690	-.046	-.065
8	.063	.044	.661	-.046	-.065
9	.063	.044	.636	-.046	-.065
10	.063	.044	.618	-.046	-.065

TABLE A12

RESULTS OF CHI-SQUARE TESTS OF FIRST AND SECOND ORDER LINEAR AUTOREGRESSIVE MODELS

(A) FIRST ORDER LINEAR MODEL
COMPUTED VALUE OF CHI-SQUARE IS 27.30144

DEGREES OF FREEDOM ARE 9

TABULATED VALUES OF CHI-SQUARE ARE

99 PCT	95 PCT	5 PCT	1 PCT
2.088	3.325	16.919	21.666

TOTAL VARIANCE .234
VARIANCE NOT EXPLAINED BY 1ST ORDER MODEL, .023

PERCENT OF TOTAL VARIANCE EXPLAINED IS 90.34

(B) SECOND ORDER LINEAR MODEL
COMPUTED VALUE OF CHI-SQUARE IS 11.24416

DEGREES OF FREEDOM ARE 8

TABULATED VALUES OF CHI-SQUARE ARE

99 PCT	95 PCT	5 PCT	1 PCT
1.646	2.733	15.507	20.090

TOTAL VARIANCE .234
VARIANCE NOT EXPLAINED BY 2ND ORDER MODEL, .213

PERCENT OF TOTAL VARIANCE EXPLAINED IS 8.84

TABLE A13

SPECTRAL DENSITY OF MEAN MONTHLY LAKE ELEVATIONS AFTER REMOVAL OF LINEAR TRENDS, PERIODICITIES AND AUTOREGRESSIVE MODEL

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.OF SPECTRUM
.13310E+04	0.	.10000E+01	.73980E-02	-.49065E+01
.14400E+03	.69444E-02	.10197E+00	.73901E-02	-.49076E+01
.72000E+02	.13889E-01	-.47882E-01	.69644E-02	-.49669E+01
.48000E+02	.20833E-01	-.34183E-01	.64193E-02	-.50484E+01
.36000E+02	.27778E-01	-.20602E-01	.50347E-02	-.52914E+01
.28800E+02	.34722E-01	-.66690E-02	.62185E-02	-.50802E+01
.24000E+02	.41667E-01	-.29292E-01	.74867E-02	-.48946E+01
.20571E+02	.48611E-01	-.21913E-01	.63770E-02	-.50551E+01
.18000E+02	.55556E-01	-.90897E-02	.71385E-02	-.49423E+01
.16000E+02	.62500E-01	-.49360E-01	.10770E-01	-.45310E+01
.14400E+02	.69444E-01	.17265E-01	.10835E-01	-.45250E+01
.13091E+02	.76389E-01	.43822E-01	.11811E-01	-.44388E+01
.12000E+02	.83333E-01	.24990E-01	.10313E-01	-.45744E+01
.11077E+02	.90278E-01	.85958E-02	.59289E-02	-.51279E+01
.10286E+02	.97222E-01	.81264E-02	.77694E-02	-.48576E+01
.96000E+01	.10417E+00	-.59599E-02	.91014E-02	-.46993E+01
.90000E+01	.11111E+00	.55592E-01	.10709E-01	-.45367E+01
.84706E+01	.11806E+00	.20362E-01	.90118E-02	-.47092E+01
.80000E+01	.12500E+00	.63548E-02	.68463E-02	-.49840E+01
.75789E+01	.13194E+00	.47166E-02	.67962E-02	-.49914E+01
.72000E+01	.13889E+00	-.14078E-01	.74360E-02	-.49014E+01
.68571E+01	.14583E+00	-.16982E-01	.86475E-02	-.47505E+01
.65455E+01	.15278E+00	-.43434E-01	.65879E-02	-.50225E+01
.62609E+01	.15972E+00	.72203E-02	.76763E-02	-.48696E+01
.60000E+01	.16667E+00	-.19808E-01	.12385E-01	-.43913E+01
.57600E+01	.17361E+00	.11990E-01	.10371E-01	-.45688E+01
.55385E+01	.18056E+00	.26721E-01	.72645E-02	-.49248E+01
.53333E+01	.18750E+00	.39148E-01	.10493E-01	-.45571E+01
.51429E+01	.19444E+00	.13571E-01	.11476E-01	-.44675E+01
.49655E+01	.20139E+00	.19120E-01	.72216E-02	-.49307E+01
.48000E+01	.20833E+00	-.25604E-04	.63625E-02	-.50573E+01
.46452E+01	.21528E+00	.11504E-01	.69915E-02	-.49631E+01
.45000E+01	.22222E+00	-.26609E-01	.78336E-02	-.48493E+01
.43636E+01	.22917E+00	-.54590E-01	.77617E-02	-.48586E+01
.42353E+01	.23611E+00	-.36356E-01	.66329E-02	-.50157E+01
.41143E+01	.24306E+00	.15849E-02	.70312E-02	-.49574E+01
.40000E+01	.25000E+00	.74502E-01	.95843E-02	-.46476E+01
.38919E+01	.25694E+00	.90266E-02	.94302E-02	-.46638E+01
.37895E+01	.26389E+00	-.61924E-01	.72495E-02	-.49268E+01
.36923E+01	.27083E+00	-.43173E-01	.78780E-02	-.48437E+01

TABLE A13 (CONTINUED)

.36000E+01	.27778E+00	.88375E-02	.79912E-02	-.48294E+01
.35122E+01	.28472E+00	-.59795E-02	.68306E-02	-.49863E+01
.34286E+01	.29167E+00	.24634E-01	.64550E-02	-.50429E+01
.33488E+01	.29861E+00	.80821E-03	.79378E-02	-.48361E+01
.32727E+01	.30556E+00	-.37372E-01	.78889E-02	-.48423E+01
.32000E+01	.31250E+00	-.50876E-01	.60352E-02	-.51101E+01
.31304E+01	.31944E+00	.40320E-04	.53503E-02	-.52306E+01
.30638E+01	.32639E+00	.79871E-01	.52054E-02	-.52581E+01
.30000E+01	.33333E+00	.56406E-01	.50943E-02	-.52796E+01
.29388E+01	.34028E+00	.13206E-01	.48017E-02	-.53388E+01
.28800E+01	.34722E+00	-.44261E-02	.56537E-02	-.51754E+01
.28235E+01	.35417E+00	.11757E-01	.66446E-02	-.50140E+01
.27692E+01	.36111E+00	.43918E-01	.66488E-02	-.50133E+01
.27170E+01	.36806E+00	.33849E-01	.63913E-02	-.50528E+01
.26667E+01	.37500E+00	-.77234E-02	.69173E-02	-.49737E+01
.26182E+01	.38194E+00	-.21818E-01	.81188E-02	-.48136E+01
.25714E+01	.38889E+00	-.62448E-01	.70381E-02	-.49564E+01
.25263E+01	.39583E+00	-.42006E-01	.58149E-02	-.51473E+01
.24828E+01	.40278E+00	-.47507E-01	.54648E-02	-.52094E+01
.24407E+01	.40972E+00	.65391E-03	.50940E-02	-.52797E+01
.24000E+01	.41667E+00	.24704E-01	.53982E-02	-.52217E+01
.23607E+01	.42361E+00	.33185E-01	.59117E-02	-.51308E+01
.23226E+01	.43056E+00	.55495E-01	.58866E-02	-.51351E+01
.22857E+01	.43750E+00	.62014E-01	.60380E-02	-.51097E+01
.22500E+01	.44444E+00	-.22318E-01	.58774E-02	-.51366E+01
.22154E+01	.45139E+00	-.34512E-01	.47744E-02	-.53445E+01
.21818E+01	.45833E+00	-.22340E-01	.34851E-02	-.56593E+01
.21493E+01	.46528E+00	-.22746E-01	.37427E-02	-.55880E+01
.21176E+01	.47222E+00	-.31939E-01	.56616E-02	-.51740E+01
.20870E+01	.47917E+00	-.36211E-01	.62618E-02	-.50733E+01
.20571E+01	.48611E+00	-.21510E-01	.58429E-02	-.51425E+01
.20282E+01	.49306E+00	.10054E-01	.57232E-02	-.51632E+01
.20000E+01	.50000E+00	.67775E-02	.53394E-02	-.52326E+01

NATURAL LOGARITHM OF THE MEAN SPECTRUM IS, -.49368E+01

CHECK SUM OF POWER SPECTRAL ESTIMATES IS .22584E-01 AND SHOULD BE .22584E-01

TABLE A14

CUMULATIVE PROBABILITY DISTRIBUTION OF INDEPENDENT RANDOM COMPONENT

Z	PERCENT PROBABILITY OF THE TIME SERIES BEING LESS THAN OR EQUAL TO Z
-.3247E+00	.1578E+01
-.2810E+00	.1878E+01
-.2530E+00	.2630E+01
-.2318E+00	.3681E+01
-.2143E+00	.4583E+01
-.1994E+00	.5409E+01
-.1862E+00	.6612E+01
-.1743E+00	.7889E+01
-.1634E+00	.9391E+01
-.1533E+00	.1037E+02
-.1438E+00	.1142E+02
-.1349E+00	.1345E+02
-.1265E+00	.1540E+02
-.1184E+00	.1668E+02
-.1106E+00	.1796E+02
-.1031E+00	.1983E+02
-.9592E-01	.2141E+02
-.8891E-01	.2314E+02
-.8209E-01	.2442E+02
-.7544E-01	.2630E+02
-.6893E-01	.2810E+02
-.6255E-01	.3065E+02
-.5628E-01	.3261E+02
-.5011E-01	.3411E+02
-.4402E-01	.3629E+02
-.3801E-01	.3854E+02
-.3206E-01	.4087E+02
-.2616E-01	.4222E+02
-.2030E-01	.4395E+02
-.1448E-01	.4576E+02
-.8675E-02	.4778E+02
-.2890E-02	.5026E+02
.2890E-02	.5259E+02
.8675E-02	.5515E+02
.1448E-01	.5702E+02
.2030E-01	.5808E+02
.2616E-01	.5988E+02
.3206E-01	.6213E+02
.3801E-01	.6431E+02

TABLE A14 (CONTINUED)

.4402E-01	.6619E+02
.5011E-01	.6784E+02
.5628E-01	.6957E+02
.6255E-01	.7175E+02
.6893E-01	.7446E+02
.7544E-01	.7603E+02
.8209E-01	.7776E+02
.8891E-01	.7986E+02
.9592E-01	.8122E+02
.1031E+00	.8249E+02
.1106E+00	.8355E+02
.1184E+00	.8460E+02
.1265E+00	.8580E+02
.1349E+00	.8685E+02
.1438E+00	.8753E+02
.1533E+00	.8873E+02
.1634E+00	.8963E+02
.1743E+00	.9068E+02
.1862E+00	.9174E+02
.1994E+00	.9256E+02
.2143E+00	.9339E+02
.2318E+00	.9512E+02
.2530E+00	.9602E+02
.2810E+00	.9632E+02
.3247E+00	.9752E+02

TABLE A15

PROBABILITY DENSITY DISTRIBUTION OF INDEPENDENT RANDOM COMPONENT

LOW POINT	HIGH POINT	MID POINT	PROBABILITY DENSITY
	-.3247E+00		.1578E+01
-.3247E+00	-.2810E+00	-.3029E+00	.3005E+00
-.2810E+00	-.2530E+00	-.2670E+00	.7513E+00
-.2530E+00	-.2318E+00	-.2424E+00	.1052E+01
-.2318E+00	-.2143E+00	-.2231E+00	.9016E+00
-.2143E+00	-.1994E+00	-.2069E+00	.8264E+00
-.1994E+00	-.1862E+00	-.1928E+00	.1202E+01
-.1862E+00	-.1743E+00	-.1803E+00	.1277E+01
-.1743E+00	-.1634E+00	-.1688E+00	.1503E+01
-.1634E+00	-.1533E+00	-.1583E+00	.9767E+00
-.1533E+00	-.1438E+00	-.1486E+00	.1052E+01
-.1438E+00	-.1349E+00	-.1394E+00	.2029E+01
-.1349E+00	-.1265E+00	-.1307E+00	.1953E+01
-.1265E+00	-.1184E+00	-.1224E+00	.1277E+01
-.1184E+00	-.1106E+00	-.1145E+00	.1277E+01
-.1106E+00	-.1031E+00	-.1069E+00	.1878E+01
-.1031E+00	-.9592E-01	-.9953E-01	.1578E+01
-.9592E-01	-.8891E-01	-.9242E-01	.1728E+01
-.8891E-01	-.8209E-01	-.8550E-01	.1277E+01
-.8209E-01	-.7544E-01	-.7877E-01	.1878E+01
-.7544E-01	-.6893E-01	-.7218E-01	.1803E+01
-.6893E-01	-.6255E-01	-.6574E-01	.2554E+01
-.6255E-01	-.5628E-01	-.5942E-01	.1953E+01
-.5628E-01	-.5011E-01	-.5320E-01	.1503E+01
-.5011E-01	-.4402E-01	-.4707E-01	.2179E+01
-.4402E-01	-.3801E-01	-.4102E-01	.2254E+01
-.3801E-01	-.3206E-01	-.3503E-01	.2329E+01
-.3206E-01	-.2616E-01	-.2911E-01	.1352E+01
-.2616E-01	-.2030E-01	-.2323E-01	.1728E+01
-.2030E-01	-.1448E-01	-.1739E-01	.1803E+01
-.1448E-01	-.8675E-02	-.1158E-01	.2029E+01
-.8675E-02	-.2890E-02	-.5782E-02	.2479E+01
-.2890E-02	.2890E-02	.1200E-14	.2329E+01
.2890E-02	.8675E-02	.5782E-02	.2554E+01
.8675E-02	.1448E-01	.1158E-01	.1878E+01
.1448E-01	.2030E-01	.1739E-01	.1052E+01
.2030E-01	.2616E-01	.2323E-01	.1803E+01
.2616E-01	.3206E-01	.2911E-01	.2254E+01
.3206E-01	.3801E-01	.3503E-01	.2179E+01

TABLE A15 (CONTINUED)

.3801E-01	.4402E-01	.4102E-01	.1878E+01
.4402E-01	.5011E-01	.4707E-01	.1653E+01
.5011E-01	.5628E-01	.5320E-01	.1728E+01
.5628E-01	.6255E-01	.5942E-01	.2179E+01
.6255E-01	.6893E-01	.6574E-01	.2705E+01
.6893E-01	.7544E-01	.7218E-01	.1578E+01
.7544E-01	.8209E-01	.7877E-01	.1728E+01
.8209E-01	.8891E-01	.8550E-01	.2104E+01
.8891E-01	.9592E-01	.9242E-01	.1352E+01
.9592E-01	.1031E+00	.9953E-01	.1277E+01
.1031E+00	.1106E+00	.1069E+00	.1052E+01
.1106E+00	.1184E+00	.1145E+00	.1052E+01
.1184E+00	.1265E+00	.1224E+00	.1202E+01
.1265E+00	.1349E+00	.1307E+00	.1052E+01
.1349E+00	.1438E+00	.1394E+00	.6762E+00
.1438E+00	.1533E+00	.1486E+00	.1202E+01
.1533E+00	.1634E+00	.1583E+00	.9016E+00
.1634E+00	.1743E+00	.1688E+00	.1052E+01
.1743E+00	.1862E+00	.1803E+00	.1052E+01
.1862E+00	.1994E+00	.1928E+00	.8264E+00
.1994E+00	.2143E+00	.2069E+00	.8264E+00
.2143E+00	.2318E+00	.2231E+00	.1728E+01
.2318E+00	.2530E+00	.2424E+00	.9016E+00
.2530E+00	.2810E+00	.2670E+00	.3005E+00
.2810E+00	.3247E+00	.3029E+00	.1202E+01
.3247E+00			.2479E+01

TABLE A16

TESTS OF DISTRIBUTION OF INDEPENDENT RANDOM COMPONENT AGAINST NORMAL AND LOG-NORMAL DISTRIBUTIONS

```
COMPUTED VALUE OF CHI-SQUARE BASED ON THE NORMAL
DISTRIBUTION IS                              .1813E+03
NUMBER OF DEGREES OF FREEDOM IS                     62


COMPUTED VALUE OF KOLMOGOROV-SMIRNOV PARAMETER
BASED ON THE NORMAL DISTRIBUTION IS          .7557E-01
SAMPLE SIZE IS                                    1331


EXPECTED VALUE OF THE SAMPLE IS              .1204E-14
STANDARD DEVIATION OF THE SAMPLE IS          .1503E+00



THE LARGEST NEGATIVE VALUE IN THE ARRAY IS    -1.107578

HENCE THE ABSOLUTE VALUE OF IT WILL BE ADDED TO  EACH NUMBER

IN THE ENTIRE ARRAY BEFORE THE NATURAL LOG IS TAKEN
COMPUTED VALUE OF CHI-SQUARE BASED ON THE LOG -
NORMAL DISTRIBUTION IS                       .1330E+04
NUMBER OF DEGREES OF FREEDOM IS                     62


COMPUTED VALUE OF KOLMOGOROV-SMIRNOV PARAMETER
BASED ON THE LOG-NORMAL DISTRIBUTION
IS                                           .2213E+00
SAMPLE SIZE IS                                    1331


EXPECTED VALUE OF THE MEAN OF THE
LOGARITHMS OF THE SAMPLE IS                  .8659E-01
STANDARD DEVIATION OF THE LOGARITHMS
OF THE SAMPLE IS                             .2912E+00
```

TABLE A17

RESULTS OF RANDOMNESS TESTS

TEST NO.	STANDARD NORMAL VARIABLE	RECORDED VALUE	RECORDED MEAN	RECORDED STANDARD DEVIATION
1	-5.43	913.00	999.24	15.78
2	54.74	1728.00	887.00	15.37
3	1.47	649.00	665.00	10.54

NOTE.

THE STANDARD NORMAL VARIABLE (K) INDICATES THE NUMBER OF STANDARD DEVIATIONS THAT THE TEST VALUE IS BELOW THE EXPECTED (MEAN) VALUE IF THE OBSERVATIONS ARE INDEPENDENT. VALUES OF K LESS THAN -2.33 OCCUR WITH A PROBABILITY OF LESS THAN 0.01 FOR A SEQUENCE OF INDEPENDENT OBSERVATIONS (USE K GREATER THAN 2.58 FOR TEST 3 - 2-TAIL PROBABILITY).

SPECIFICALLY, SUCH VALUES OF K INDICATE -

TEST 1 - A TENDENCY FOR OBSERVATIONS TO CLUSTER INTO THE RESPECTIVE CATEGORIES.

TEST 2 - A TENDENCY FOR DIRECTIONS OF MOVEMENT TO PERSIST (IE. - CLUSTERING NOT EXCLUSIVELY DUE TO A FEW LARGE CHANGES.)

TEST 3 - AN UPWARD OR DOWNWARD TREND DUE TO A RELATIVE SHORTAGE OF - OR + SIGNS RESPECTIVELY (TREND DUE TO LARGER MOVEMENTS IN ONE DIRECTION IS NOT DETECTED BY THIS TEST).

TABLE A18

SUMMARY OF TIME SERIES ANALYSIS OF
MEAN MONTHLY LAKE ELEVATIONS

COMPONENT	PERCENT OF TOTAL VARIANCE EXPLAINED BY THIS COMPONENT
LINEAR TRENDS	12.230883
PERIODICITIES	31.882185
AUTOREGRESSION	50.485637
RESIDUAL	5.401295

APPENDIX B

TABLES SHOWING DETAILED RESULTS OF TIME SERIES ANALYSIS OF DIFFERENCES BETWEEN MEAN MONTHLY LAKE SUPERIOR ELEVATIONS AT MARQUETTE AND AT DULUTH FOR THE PERIOD 1888 to 1970

APPENDIX B

LIST OF TABLES

TABLE B1

DIFFERENCES IN MONTHLY MEAN LAKE ELEVATIONS

MARQUETTE MINUS DULUTH

1888-1970

(100.0 SIGNIFIES ORIGINAL DATA IS MISSING AND NO DIFFERENCE IS POSSIBLE)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1860	100.00	100.00	100.00	.32	.25	.35	.38	.31	.44	.31	.39	100.00
1861	100.00	.51	-.38	.24	.28	.54	.30	.30	.34	.27	.29	.61
1862	.86	.60	.44	.33	.07	.21	.28	.32	.25	.36	.45	.51
1863	.45	.42	.43	.27	.23	.36	.35	.33	.25	.30	.65	.51
1864	.47	.55	.36	.24	.43	.35	.36	.50	.37	.46	.35	.61
1865	.65	.28	.42	.38	.45	.37	.32	.24	.21	.37	.42	.46
1866	.27	.07	.26	.30	.25	.35	.25	.15	.37	.42	.64	-.04
1867	-.04	.33	.62	.43	.77	.08	-.02	.31	.16	.22	.60	.32
1868	.31	.33	.49	.71	.22	.31	.19	.29	.33	.36	.16	.59
1869	.29	.31	.34	.30	.30	.30	.31	.29	-.45	.07	.30	.30
1870	.32	.30	.30	.30	-.69	.31	.31	.32	.30	.31	.30	.31
1871	.31	.31	.31	.30	.31	.32	.31	.31	.31	.32	.32	.50
1872	100.00	100.00	100.00	100.00	100.00	.27	.30	.19	.24	.29	.30	.29
1873	.37	.30	.30	.29	.37	.28	.30	.29	.47	100.00	100.00	100.00
1874	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1875	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1876	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1877	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1878	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1879	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1880	100.00	100.00	100.00	100.00	.26	.31	.31	.23	.43	.24	.24	100.00
1881	100.00	100.00	100.00	.41	.17	.18	.34	.30	.12	.30	.28	100.00
1882	100.00	100.00	100.00	.37	.05	.06	.24	.24	.16	-.05	-.06	100.00
1883	100.00	100.00	100.00	100.00	.39	.30	.20	.25	.08	.20	100.00	100.00
1884	100.00	100.00	100.00	100.00	.28	-.21	100.00	100.00	.21	.27	.30	.39
1885	.36	100.00	100.00	100.00	.16	.13	.19	.23	.31	.28	.24	.45
1886	100.00	100.00	100.00	.27	.24	.23	.19	.19	.27	.25	.29	100.00
1887	100.00	100.00	100.00	100.00	.13	.16	.18	.14	.15	.35	.31	.21
1888	.41	.49	.35	.30	.15	.05	.05	.38	.20	.16	.19	.43
1889	.47	.35	.28	.29	.19	.22	.24	.25	.23	.22	.28	.24
1890	.44	.49	.34	.28	.54	.20	.18	.19	.22	.14	.25	.34
1891	.29	.34	.34	.23	.27	.12	.14	.15	.10	.22	.31	.30
1892	.59	.37	.37	.25	.20	.26	.25	.32	.30	.18	.39	.46
1893	.47	.44	.46	.29	.32	.26	.40	.40	.34	.27	.45	.40
1894	.37	.42	.41	.17	.22	.27	.30	.32	.38	.26	.42	.44
1895	.46	.46	.39	.22	.21	.27	.24	.26	.28	.37	.29	.34
1896	.30	.36	.29	.10	.09	.18	.27	.29	.29	.24	.28	.31
1897	.37	.24	.25	.16	.21	.11	.05	.22	.18	.21	.28	.36
1898	.30	.28	.24	.21	.22	.16	.17	.22	.30	.27	.27	.42
1899	.29	.29	.28	.17	.13	.10	.18	.09	.20	.09	.25	.38
1900	.32	.39	.29	.21	.23	.22	.21	.06	.19	.09	.33	.36
1901	.38	.38	.23	.19	.21	.10	.16	.23	.18	.28	.34	.33
1902	.33	.35	.10	.26	.15	.25	.21	.26	.25	.29	.26	.30
1903	.37	.47	.25	.17	.08	.21	.27	.23	.15	.26	.31	.40
1904	.29	.29	.23	.21	.19	.12	.16	.20	.14	.12	.22	.20
1905	.39	.29	.18	.26	.08	.07	.09	.06	-.02	.17	.18	.24
1906	.25	.24	.22	.10	.12	.10	.14	.08	.10	.20	.12	.18
1907	.24	.28	.23	.15	.13	.11	.17	.18	.24	.20	.27	.25
1908	.32	.28	.23	.16	.11	.09	.16	.21	.16	.07	.26	.25
1909	.24	.23	.22	.10	.09	.11	.16	.05	.15	.20	.04	.19
1910	.30	.28	.23	.10	.20	.16	.73	.14	.13	.18	.34	.38

TABLE B1 (CONTINUED)

1911	.29	.25	.20	.12	.11	.10	.21	.15	.10	.18	.29	.16
1912	.43	.29	.22	.13	.03	.15	.17	.13	.15	.25	.32	.32
1913	.36	.38	.29	.17	.16	.21	.21	.15	.09	.21	.27	.30
1914	.26	.41	.22	.16	.09	.08	.15	.15	.08	.13	.34	.37
1915	.28	.19	.25	.13	.08	.09	.12	.13	.07	.14	.28	.20
1916	.33	.31	.13	.11	.10	.10	.06	.11	.17	.19	.28	.29
1917	.32	.32	.19	.13	.21	.14	.15	.15	.15	.21	.23	.34
1918	1.15	.20	.19	.13	.12	.24	.11	.07	.25	.15	.25	.15
1919	.29	.26	.15	.07	.14	.09	.13	.25	.18	.16	.22	.34
1920	.22	.22	.12	.25	.10	.04	.14	.02	.04	.10	.20	.19
1921	.17	.18	.16	.04	.07	.02	.10	.18	.17	.22	.20	.33
1922	.28	.18	.10	.09	.05	.09	.08	.06	.05	.07	.09	.23
1923	.15	.36	.25	.19	.07	.08	.08	.15	.05	.16	.14	.16
1924	.32	.19	.15	.09	.13	.08	.05	.06	.08	.01	.22	.31
1925	.19	.16	.12	.10	.15	.06	.12	.11	-.02	.21	.20	.29
1926	.21	.15	.22	.21	.06	.16	.10	.03	-.04	.11	.18	.19
1927	.25	.27	.20	.06	.03	.08	.10	.18	.14	.14	.17	.23
1928	.30	.20	.24	.12	.15	.11	.09	.04	.11	.11	.17	.20
1929	.31	.17	.09	.08	.17	.07	.15	.14	.07	.12	.22	.18
1930	.25	.12	.24	.12	-.01	.15	.10	.12	.14	.15	.15	.20
1931	.16	.15	.15	.12	.06	.03	.08	.07	.10	.13	.22	.07
1932	.11	.16	.33	.03	0.00	.01	.09	.05	.17	.12	.07	.14
1933	.08	.25	.17	.07	-.01	.04	-.02	.06	.01	.12	.18	.21
1934	.21	.18	.15	.07	.02	.07	0.00	.06	.03	.06	.06	.15
1935	.15	.21	.03	.01	-.01	.04	.01	-.02	.05	.07	.07	.11
1936	.16	.21	.11	.12	.01	.03	.01	-.01	-.03	.15	.19	.10
1937	.12	.19	.17	-.01	.07	.06	.06	0.00	.05	.17	.20	.16
1938	.26	.15	.12	.06	-.02	.05	.06	.11	.05	-.08	.11	.22
1939	.14	.12	.14	.08	-.02	.04	.08	.04	.02	.14	.15	.22
1940	.38	.15	.17	.07	.11	.09	.05	.05	.10	.05	.16	.19
1941	.17	.29	-.85	-.02	.03	.02	.05	.06	-.05	.10	.16	.12
1942	.20	.19	.10	.02	.01	.01	.04	.02	.09	.09	.15	.15
1943	.15	.18	.10	.05	-.01	.03	.06	.11	.15	.07	.20	1.26
1944	.17	.19	.05	.01	-.03	-.04	.13	.08	.03	.09	-.02	.23
1945	.19	.11	.04	.07	.08	-.04	.02	.06	0.00	.16	.06	.15
1946	.18	.13	.02	.03	.03	-.03	.04	.09	.01	.04	.11	.12
1947	.12	.33	.23	-.09	-.03	-.03	.06	-.03	.04	-.01	.10	.11
1948	.11	.10	.11	.03	.06	.03	.05	-.01	-.04	.01	.02	.10
1949	.06	.10	.04	-.02	-.03	-.04	-.04	-.03	.11	.07	.12	.08
1950	.12	.09	.04	-.01	-.07	.06	.01	.04	-.03	-.01	.12	.07
1951	.74	.07	.06	.03	.04	-.02	.03	-.01	.02	-.03	.07	.08
1952	.07	.02	-.01	.03	.05	0.00	.02	.02	.12	.23	.09	.13
1953	.13	.18	.06	.03	-.06	-.02	.03	-.04	.14	.01	.06	.15
1954	.08	.07	.14	0.00	.01	-.08	.01	.06	-.02	.08	.05	.10
1955	.12	.03	.15	-.07	-.04	0.00	-.04	-.01	-.02	.03	.11	.01
1956	-.01	.11	-.01	.02	-.07	-.03	.03	.02	.02	-.06	.08	.06
1957	.17	.08	0.00	-.05	-.07	-.04	-.03	-.05	0.00	-.01	.09	.07
1958	.01	.17	.02	-.02	-.04	-.02	-.05	.06	-.04	0.00	.04	.10
1959	.14	.09	0.00	-.02	-.08	-.04	-.05	-.10	-.10	.03	.10	-.02
1960	.09	.08	.05	-.04	-.07	-.02	.03	-.11	-.07	.03	.07	.12
1961	.11	0.00	-.07	-.05	-.08	-.02	-.02	.01	-.04	-.04	.03	.08
1962	.22	0.00	-.18	-.10	-.14	-.08	-.04	-.09	-.02	-.01	-.03	.11
1963	.12	.01	-.15	-.14	-.11	-.10	-.04	-.04	-.12	-.08	.07	.07
1964	.02	.06	-.04	-.16	-.10	-.09	-.08	-.07	-.06	.02	0.00	-.01
1965	.06	.04	-.08	-.13	-.12	-.10	-.03	-.08	-.14	0.00	-.03	-.06
1966	.08	-.01	-.18	-.12	-.10	-.08	-.09	-.09	.03	0.00	.02	.02
1967	-.04	.01	-.09	-.18	-.11	-.16	-.04	-.04	-.07	-.06	.03	.04
1968	-.04	.22	-.04	-.12	-.12	-.12	-.06	-.05	-.10	-.08	-.06	-.08
1969	-.04	-.06	-.03	-.18	-.12	-.09	-.09	-.03	-.06	-.05	0.00	-.03
1970	.04	.04	-.08	-.20	-.17	-.13	-.07	-.01	-.05	-.12	-.02	-.05

TABLE B2

SPECTRAL DENSITY OF DIFFERENCES IN MEAN MONTHLY LAKE ELEVATIONS

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.OF SPECTRUM
.99600E+03	0.	.10000E+01	.24536E+00	-.14050E+01
.14400E+03	.69444E-02	.65789E+00	.11387E+00	-.21727E+01
.72000E+02	.13889E-01	.56182E+00	.89331E-02	-.47180E+01
.48000E+02	.20833E-01	.47815E+00	.44232E-02	-.54209E+01
.36000E+02	.27778E-01	.41863E+00	.27612E-02	-.58921E+01
.28800E+02	.34722E-01	.38889E+00	.24298E-02	-.60199E+01
.24000E+02	.41667E-01	.37228E+00	.35350E-02	-.56450E+01
.20571E+02	.48611E-01	.37412E+00	.35814E-02	-.56320E+01
.18000E+02	.55556E-01	.41021E+00	.27987E-02	-.58786E+01
.16000E+02	.62500E-01	.44985E+00	.20076E-02	-.62108E+01
.14400E+02	.69444E-01	.54963E+00	.20049E-02	-.62122E+01
.13091E+02	.76389E-01	.61862E+00	.18590E-01	-.39851E+01
.12000E+02	.83333E-01	.66412E+00	.41396E-01	-.31846E+01
.11077E+02	.90278E-01	.63101E+00	.19288E-01	-.39483E+01
.10286E+02	.97222E-01	.52937E+00	.15025E-02	-.65006E+01
.96000E+01	.10417E+00	.44791E+00	.21007E-02	-.61655E+01
.90000E+01	.11111E+00	.39556E+00	.24418E-02	-.60150E+01
.84706E+01	.11806E+00	.35831E+00	.13838E-02	-.65829E+01
.80000E+01	.12500E+00	.35462E+00	.17853E-02	-.63282E+01
.75789E+01	.13194E+00	.35656E+00	.26579E-02	-.59302E+01
.72000E+01	.13889E+00	.38490E+00	.22887E-02	-.60798E+01
.68571E+01	.14583E+00	.44888E+00	.18497E-02	-.62927E+01
.65455E+01	.15278E+00	.53793E+00	.18186E-02	-.63097E+01
.62609E+01	.15972E+00	.61385E+00	.54712E-02	-.52083E+01
.60000E+01	.16667E+00	.64040E+00	.97270E-02	-.46329E+01
.57600E+01	.17361E+00	.60746E+00	.53103E-02	-.52381E+01
.55385E+01	.18056E+00	.51791E+00	.20310E-02	-.61992E+01
.53333E+01	.18750E+00	.42461E+00	.23874E-02	-.60375E+01
.51429E+01	.19444E+00	.35792E+00	.22947E-02	-.60772E+01
.49655E+01	.20139E+00	.32947E+00	.19755E-02	-.62269E+01
.48000E+01	.20833E+00	.32606E+00	.19891E-02	-.62201E+01
.46452E+01	.21528E+00	.33408E+00	.17949E-02	-.63228E+01
.45000E+01	.22222E+00	.36155E+00	.18518E-02	-.62916E+01
.43636E+01	.22917E+00	.36676E+00	.30640E-02	-.57880E+01
.42353E+01	.23611E+00	.52526E+00	.31591E-02	-.57575E+01
.41143E+01	.24306E+00	.58677E+00	.18742E-02	-.62796E+01
.40000E+01	.25000E+00	.61728E+00	.21599E-02	-.61377E+01
.38919E+01	.25694E+00	.58326E+00	.31201E-02	-.57699E+01
.37895E+01	.26389E+00	.51725E+00	.25936E-02	-.59547E+01
.36923E+01	.27083E+00	.41528E+00	.15581E-02	-.64643E+01
.36000E+01	.27778E+00	.33150E+00	.16247E-02	-.64224E+01
.35122E+01	.28472E+00	.30126E+00	.24330E-02	-.60186E+01
.34286E+01	.29167E+00	.32020E+00	.22092E-02	-.61151E+01
.33488E+01	.29861E+00	.33234E+00	.18552E-02	-.62897E+01
.32727E+01	.30556E+00	.34678E+00	.15542E-02	-.64668E+01
.32000E+01	.31250E+00	.41944E+00	.25212E-02	-.59830E+01

TABLE B2 (CONTINUED)

.31304E+01	.31944E+00	.49483E+00	.28870E-02	-.58475E+01
.30638E+01	.32639E+00	.60164E+00	.19250E-02	-.62528E+01
.30000E+01	.33333E+00	.60635E+00	.16100E-02	-.64315E+01
.29388E+01	.34028E+00	.54844E+00	.16245E-02	-.64225E+01
.28800E+01	.34722E+00	.47493E+00	.19070E-02	-.62622E+01
.28235E+01	.35417E+00	.39002E+00	.20219E-02	-.62037E+01
.27692E+01	.36111E+00	.32244E+00	.16642E-02	-.63984E+01
.27170E+01	.36806E+00	.31228E+00	.14751E-02	-.65190E+01
.26667E+01	.37500E+00	.30379E+00	.27213E-02	-.59066E+01
.26182E+01	.38194E+00	.30742E+00	.29772E-02	-.58168E+01
.25714E+01	.38889E+00	.34427E+00	.21955E-02	-.61213E+01
.25263E+01	.39583E+00	.39284E+00	.20756E-02	-.61775E+01
.24828E+01	.40278E+00	.46030E+00	.24642E-02	-.60059E+01
.24407E+01	.40972E+00	.54375E+00	.25488E-02	-.59721E+01
.24000E+01	.41667E+00	.57261E+00	.18775E-02	-.62778E+01
.23607E+01	.42361E+00	.54315E+00	.13684E-02	-.65941E+01
.23226E+01	.43056E+00	.43717E+00	.14276E-02	-.65517E+01
.22857E+01	.43750E+00	.38481E+00	.20826E-02	-.61741E+01
.22500E+01	.44444E+00	.31194E+00	.22929E-02	-.60779E+01
.22154E+01	.45139E+00	.26854E+00	.12474E-02	-.66867E+01
.21818E+01	.45833E+00	.27856E+00	.11371E-02	-.67793E+01
.21493E+01	.46528E+00	.29077E+00	.21330E-02	-.61502E+01
.21176E+01	.47222E+00	.30431E+00	.25319E-02	-.59788E+01
.20870E+01	.47917E+00	.37741E+00	.17485E-02	-.63490E+01
.20571E+01	.48611E+00	.46331E+00	.16432E-02	-.64111E+01
.20282E+01	.49306E+00	.52585E+00	.24723E-02	-.60026E+01
.20000E+01	.50000E+00	.55831E+00	.27300E-02	-.59035E+01

NATURAL LOGARITHM OF THE MEAN SPECTRUM IS, -.47878E+01

CHECK SUM OF POWER SPECTRAL ESTIMATES IS .21123E-01AND
SHOULD BE .21123E-01

TABLE B3

RESULTS OF VARIATE DIFFERENCE TECHNIQUE FOR DETECTING LINEAR TRENDS

DIFFERENCE NUMBER R	MEAN OF DIFFERENCES	VARIANCE OF DIFFERENCES	VARIANCE DIVIDED BY (2R) (R)
1	-.46231E-03	.14384E-01	.71918E-02
2	-.11066E-03	.39247E-01	.65411E-02
3	.90634E-04	.12774E+00	.63868E-02
4	-.61492E-03	.44299E+00	.63284E-02

VARIANCE OF ORIGINAL SERIES IS .21123E-01

TABLE B4

RESULTS OF POLYNOMIAL REGRESSION

NUMBER OF OBSERVATIONS 996

POLYNOMIAL REGRESSION OF NUMBER 1

INTERCEPT	8.24132
REGRESSION COEFFICIENT 1	-.00421
MULTIPLE CORRELATION COEFFICIENT	.69324
STANDARD ERROR OF ESTIMATE	.10485

ANALYSIS OF VARIANCE FOR 1 DEGREE POLYNOMIAL

SOURCE OF VARIATION	DEGREE OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F VALUE	IMPROVEMENT IN TERMS OF SUM CF SQUARES
DUE TO REGRESSION	1	10.11076	10.11076	919.66105	10.11076
DEVIATION ABOUT REGRESSION	994	10.92804	.01099		
TOTAL	995	21.03880			

POLYNOMIAL REGRESSION OF NUMBER 2

INTERCEPT	-25.71436
REGRESSION COEFFICIENT 1	.03100
REGRESSION COEFFICIENT 2	-.00001
MULTIPLE CORRELATION COEFFICIENT	.69399
STANDARD ERROR OF ESTIMATE	.10480

ANALYSIS OF VARIANCE FOR 2 DEGREE POLYNOMIAL

SOURCE OF VARIATION	DEGREE OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F VALUE	IMPROVEMENT IN TERMS OF SUM OF SQUARES
DUE TO REGRESSION	2	10.13261	5.06630	461.28323	.02185
DEVIATION ABOUT REGRESSION	993	10.90619	.01098		
TOTAL	995	21.03880			

TABLE B4 (CONTINUED)

POLYNOMIAL REGRESSION OF NUMBER 3

INTERCEPT	5425.36357
REGRESSION COEFFICIENT 1	-8.44613
REGRESSION COEFFICIENT 2	.00438
REGRESSION COEFFICIENT 3	-.00000
MULTIPLE CORRELATION COEFFICIENT	.69628
STANDARD ERROR OF ESTIMATE	.10453

ANALYSIS OF VARIANCE FOR 3 DEGREE POLYNOMIAL

SOURCE OF VARIATION	DEGREE OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F VALUE	IMPROVEMENT IN TERMS OF SUM CF SQUARES
DUE TO REGRESSION	3	10.19971	3.39990	311.16132	.06710
DEVIATION ABOUT REGRESSION	992	10.83909	.01093		
TOTAL	995	21.03880			

POLYNOMIAL REGRESSION OF NUMBER 4

NO IMPROVEMENT

TABLE B5

SPECTRAL DENSITY OF DIFFERENCES IN MEAN MONTHLY LAKE ELEVATIONS AFTER REMOVAL OF LINEAR TRENDS

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.OF SPECTRUM
.99600E+03	0.	.10000E+01	.48277E-02	-.53334E+01
.14400E+03	.69444E-02	.33970E+00	.60739E-02	-.51038E+01
.72000E+02	.13889E-01	.15778E+00	.53523E-02	-.52302E+01
.48000E+02	.20833E-01	-.52244E-03	.34651E-02	-.56650E+01
.36000E+02	.27778E-01	-.11414E+00	.27858E-02	-.58832E+01
.28800E+02	.34722E-01	-.17329E+00	.25885E-02	-.59567E+01
.24000E+02	.41667E-01	-.20751E+00	.33673E-02	-.56936E+01
.20571E+02	.48611E-01	-.20477E+00	.32265E-02	-.57364E+01
.18000E+02	.55556E-01	-.12910E+00	.26907E-02	-.59179E+01
.16000E+02	.62500E-01	-.49008E-01	.21155E-02	-.61585E+01
.14400E+02	.69444E-01	.14481E+00	.21106E-02	-.61608E+01
.13091E+02	.76389E-01	.27709E+00	.18533E-01	-.39882E+01
.12000E+02	.83333E-01	.36794E+00	.41210E-01	-.31891E+01
.11077E+02	.90278E-01	.30828E+00	.19128E-01	-.39566E+01
.10286E+02	.97222E-01	.11295E+00	.15096E-02	-.64959E+01
.96000E+01	.10417E+00	-.43425E-01	.20412E-02	-.61942E+01
.90000E+01	.11111E+00	-.14292E+00	.22991E-02	-.60752E+01
.84706E+01	.11806E+00	-.21541E+00	.12783E-02	-.66622E+01
.80000E+01	.12500E+00	-.22108E+00	.17147E-02	-.63685E+01
.75789E+01	.13194E+00	-.21545E+00	.25584E-02	-.59684E+01
.72000E+01	.13889E+00	-.15780E+00	.22224E-02	-.61092E+01
.68571E+01	.14583E+00	-.30124E-01	.17795E-02	-.63314E+01
.65455E+01	.15278E+00	.14314E+00	.17161E-02	-.63677E+01
.62609E+01	.15972E+00	.29254E+00	.53134E-02	-.52375E+01
.60000E+01	.16667E+00	.34506E+00	.95084E-02	-.46556E+01
.57600E+01	.17361E+00	.28641E+00	.51689E-02	-.52651E+01
.55385E+01	.18056E+00	.11841E+00	.19649E-02	-.62323E+01
.53333E+01	.18750E+00	-.59077E-01	.23375E-02	-.60587E+01
.51429E+01	.19444E+00	-.18558E+00	.22768E-02	-.60850E+01
.49655E+01	.20139E+00	-.23421E+00	.19915E-02	-.62188E+01
.48000E+01	.20833E+00	-.24001E+00	.20087E-02	-.62103E+01
.46452E+01	.21528E+00	-.22278E+00	.18222E-02	-.63077E+01
.45000E+01	.22222E+00	-.16761E+00	.18973E-02	-.62673E+01
.43636E+01	.22917E+00	-.15512E+00	.31098E-02	-.57732E+01
.42353E+01	.23611E+00	.15196E+00	.32254E-02	-.57367E+01
.41143E+01	.24306E+00	.26758E+00	.19252E-02	-.62527E+01
.40000E+01	.25000E+00	.32976E+00	.21427E-02	-.61457E+01
.38919E+01	.25694E+00	.26423E+00	.30610E-02	-.57890E+01
.37895E+01	.26389E+00	.13773E+00	.25638E-02	-.59663E+01
.36923E+01	.27083E+00	-.56790E-01	.15489E-02	-.64702E+01
.36000E+01	.27778E+00	-.21773E+00	.16073E-02	-.64332E+01
.35122E+01	.28472E+00	-.27463E+00	.23668E-02	-.60462E+01
.34286E+01	.29167E+00	-.23873E+00	.21558E-02	-.61396E+01
.33488E+01	.29861E+00	-.21327E+00	.18575E-02	-.62885E+01
.32727E+01	.30556E+00	-.18415E+00	.15824E-02	-.64488E+01
.32000E+01	.31250E+00	-.41520E-01	.25225E-02	-.59825E+01

TABLE B5 (CONTINUED)

.31304E+01	.31944E+00	.10679E+00	.28746E-02	-.58518E+01
.30638E+01	.32639E+00	.31572E+00	.19229E-02	-.62539E+01
.30000E+01	.33333E+00	.32731E+00	.16079E-02	-.64328E+01
.29388E+01	.34028E+00	.22140E+00	.16208E-02	-.64248E+01
.28800E+01	.34722E+00	.81273E-01	.19030E-02	-.62643E+01
.28235E+01	.35417E+00	-.80553E-01	.20129E-02	-.62082E+01
.27692E+01	.36111E+00	-.21148E+00	.16486E-02	-.64078E+01
.27170E+01	.36806E+00	-.22946E+00	.14649E-02	-.65260E+01
.26667E+01	.37500E+00	-.24309E+00	.27235E-02	-.59058E+01
.26182E+01	.38194E+00	-.23354E+00	.30039E-02	-.58079E+01
.25714E+01	.38889E+00	-.15783E+00	.22303E-02	-.61056E+01
.25263E+01	.39583E+00	-.59290E-01	.20560E-02	-.61870E+01
.24828E+01	.40278E+00	.74964E-01	.24244E-02	-.60222E+01
.24407E+01	.40972E+00	.24067E+00	.25390E-02	-.59760E+01
.24000E+01	.41667E+00	.29988E+00	.18701E-02	-.62818E+01
.23607E+01	.42361E+00	.24895E+00	.13417E-02	-.66138E+01
.23226E+01	.43056E+00	.48370E-01	.14015E-02	-.65702E+01
.22857E+01	.43750E+00	-.48342E-01	.20756E-02	-.61775E+01
.22500E+01	.44444E+00	-.18508E+00	.23008E-02	-.60745E+01
.22154E+01	.45139E+00	-.26528E+00	.12298E-02	-.67009E+01
.21818E+01	.45833E+00	-.24388E+00	.11012E-02	-.68114E+01
.21493E+01	.46528E+00	-.21421E+00	.21177E-02	-.61574E+01
.21176E+01	.47222E+00	-.18151E+00	.25398E-02	-.59757E+01
.20870E+01	.47917E+00	-.34054E-01	.17618E-02	-.63414E+01
.20571E+01	.48611E+00	.13596E+00	.16621E-02	-.63997E+01
.20282E+01	.49306E+00	.26150E+00	.24802E-02	-.59994E+01
.20000E+01	.50000E+00	.32735E+00	.27152E-02	-.59089E+01

NATURAL LOGARITHM CF THE MEAN SPECTRUM IS, -.56641E+01

CHECK SUM CF POWER SPECTRAL ESTIMATES IS .10883E-01AND

SHOULD BE .10883E-01

TABLE B6

MONTHLY MEANS AND STANDARD DEVIATIONS OF DIFFERENCES IN MEAN MONTHLY ELEVATIONS

MONTH	LONG TERM MEAN	STANDARD DEVIATION
JAN	.10933	.13385
FEB	.08141	.06606
MAR	.00433	.12328
APR	-.05286	.04695
MAY	-.06693	.05846
JUN	-.06689	.04988
JUL	-.03590	.08199
AUG	-.03900	.06084
SEP	-.04246	.06708
OCT	-.01628	.06327
NOV	.04206	.06073
DEC	.08318	.13592

TABLE B7

RESULTS OF PERIODIC ANALYSIS

TOTAL VARIANCE OF THE PERIODIC SERIES	.1088E-01
VARIANCE OF THE MEANS	.3660E-02
VARIANCE OF THE STANDARD DEVIATIONS	.9780E-03
PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED BY THE VARIANCE OF THE MEANS	.3363E+02
PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED BY THE VARIANCE OF THE STANDARD DEVIATIONS	.8987E+01
PERCENT OF THE TOTAL VARIANCE WHICH IS NOT EXPLAINED BY THE VARIANCE IN THE MEAN AND STANDARD DEVIATION	.5739E+02

TABLE B8

PERIODOGRAM COMPONENTS FOR THE MEAN

HARMONIC	A	B	C SQUARED	PERIOD	PHASE
1	.7479E-01	.2292E-01	.6119E-02	.1200E+02	-.1273E+01
2	.1534E-01	.3029E-01	.1153E-02	.6000E+01	-.4689E+00
3	-.1152E-02	-.1759E-02	.4423E-05	.4000E+01	-.5798E+00
4	-.5459E-02	-.1914E-02	.3347E-04	.3000E+01	-.1234E+01
5	.1402E-02	-.1281E-02	.3605E-05	.2400E+01	.8304E+00
6	-.3481E-02	.7226E-15	.1212E-04	.2000E+01	.1571E+01

HARMONIC VARIANCES IN THE MEAN

HARMONIC	VARIANCE	PERCENT OF TOTAL VARIANCE IN THE MEANS	SIGNIFICANT AT 95 PER CENT CONFIDENCE L. (FISHER TEST)
1	.3059E-02	.8353E+02	YES
2	.5764E-03	.1574E+02	NO
3	.2211E-05	.6038E-01	NO
4	.1673E-04	.4569E+00	NO
5	.1803E-05	.4922E-01	NO
6	.6060E-05	.1655E+00	NO

TABLE B9

PERIODOGRAM COMPONENTS FOR THE STANDARD DEVIATION

HARMONIC	A	B	C SQUARED	PERIOD	PHASE
1	.2395E-01	.1190E-01	.7150E-03	.1200E+02	-.1110E+01
2	.7398E-02	.1636E-01	.3223E-03	.6000E+01	-.4248E+00
3	.1075E-01	-.1100E-02	.1168E-03	.4000E+01	.1469E+01
4	.1501E-01	.1154E-01	.3587E-03	.3000E+01	-.9154E+00
5	.8323E-02	.1510E-01	.2973E-03	.2400E+01	-.5037E+00
6	-.1708E-01	.6880E-15	.2917E-03	.2000E+01	.1571E+01

HARMONIC VARIANCES IN THE STANDARD DEVIATIONS

HARMONIC	VARIANCE	PERCENT OF TOTAL VARIANCE IN THE STANDARD DEVIATION	SIGNIFICANT AT 95 PER CENT CONFIDENCE L. (FISHER TEST)
1	.3575E-03	.3402E+02	NO
2	.1611E-03	.1533E+02	NO
3	.5839E-04	.5556E+01	NO
4	.1793E-03	.1707E+02	NO
5	.1487E-03	.1415E+02	NO
6	.1458E-03	.1388E+02	NO

TABLE B10

SPECTRAL DENSITY OF DIFFERENCES IN MEAN MONTHLY ELEVATIONS AFTER REMOVAL OF LINEAR TRENDS AND PERIODICITIES

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.CF SPECTRUM
.99600E+03	0.	.10000E+01	.50663E-02	-.52851E+01
.14400E+03	.69444E-02	.16333E+00	.65790E-02	-.50239E+01
.72000E+02	.13889E-01	.98714E-01	.53662E-02	-.52276E+01
.48000E+02	.20833E-01	.10980E+00	.31765E-02	-.57520E+01
.36000E+02	.27778E-01	.97294E-01	.30117E-02	-.58053E+01
.28800E+02	.34722E-01	.76566E-01	.25484E-02	-.59723E+01
.24000E+02	.41667E-01	.29191E-01	.25271E-02	-.59807E+01
.20571E+02	.48611E-01	.15703E-01	.26967E-02	-.59157E+01
.18000E+02	.55556E-01	.73500E-01	.26248E-02	-.59427E+01
.16000E+02	.62500E-01	.89598E-02	.20595E-02	-.61853E+01
.14400E+02	.69444E-01	.57201E-01	.23470E-02	-.60546E+01
.13091E+02	.76389E-01	.40047E-01	.36647E-02	-.56090E+01
.12000E+02	.83333E-01	.61509E-01	.32289E-02	-.57356E+01
.11077E+02	.90278E-01	.77656E-01	.21019E-02	-.61649E+01
.10286E+02	.97222E-01	.34476E-01	.13382E-02	-.66165E+01
.96000E+01	.10417E+00	.34715E-01	.14933E-02	-.65067E+01
.90000E+01	.11111E+00	.57149E-01	.19498E-02	-.62400E+01
.84706E+01	.11806E+00	.27292E-01	.15747E-02	-.64537E+01
.80000E+01	.12500E+00	.20905E-02	.16913E-02	-.63823E+01
.75789E+01	.13194E+00	-.63096E-02	.19806E-02	-.62243E+01
.72000E+01	.13889E+00	-.24371E-01	.17927E-02	-.63241E+01
.68571E+01	.14583E+00	.23229E-01	.14732E-02	-.65203E+01
.65455E+01	.15278E+00	.31891E-01	.13885E-02	-.65796E+01
.62609E+01	.15972E+00	.22035E-01	.17292E-02	-.63601E+01
.60000E+01	.16667E+00	.18424E-01	.17965E-02	-.63219E+01
.57600E+01	.17361E+00	.53068E-01	.17335E-02	-.63576E+01
.55385E+01	.18056E+00	.13579E-01	.17616E-02	-.63416E+01
.53333E+01	.18750E+00	.29150E-01	.20592E-02	-.61854E+01
.51429E+01	.19444E+00	-.76307E-02	.19200E-02	-.62554E+01
.49655E+01	.20139E+00	-.32189E-01	.14244E-02	-.65540E+01
.48000E+01	.20833E+00	-.20296E-01	.15820E-02	-.64491E+01
.46452E+01	.21528E+00	-.12130E-01	.16045E-02	-.64349E+01
.45000E+01	.22222E+00	-.33425E-01	.16710E-02	-.63943E+01
.43636E+01	.22917E+00	-.87093E-01	.25749E-02	-.59619E+01
.42353E+01	.23611E+00	.13909E-01	.26720E-02	-.59249E+01
.41143E+01	.24306E+00	-.98663E-03	.17476E-02	-.63495E+01
.40000E+01	.25000E+00	.14657E-01	.18800E-02	-.62765E+01
.38919E+01	.25694E+00	-.12784E-02	.23544E-02	-.60515E+01
.37895E+01	.26389E+00	.47139E-01	.20213E-02	-.62040E+01
.36923E+01	.27083E+00	.80719E-02	.14371E-02	-.65451E+01
.36000E+01	.27778E+00	-.32270E-01	.15587E-02	-.64639E+01

TABLE B10 (CONTINUED)

.35122E+01	.28472E+00	-.79598E-01	.20717E-02	-.61794E+01
.34286E+01	.29167E+00	.22205E-02	.17154E-02	-.63681E+01
.33488E+01	.29861E+00	-.14715E-01	.15393E-02	-.64764E+01
.32727E+01	.30556E+00	-.78686E-01	.13371E-02	-.66172E+01
.32000E+01	.31250E+00	-.33591E-01	.17600E-02	-.63425E+01
.31304E+01	.31944E+00	-.77443E-01	.18985E-02	-.62667E+01
.30638E+01	.32639E+00	-.78297E-03	.16257E-02	-.64218E+01
.30000E+01	.33333E+00	-.42089E-01	.14798E-02	-.65159E+01
.29388E+01	.34028E+00	-.89077E-01	.10825E-02	-.68285E+01
.28800E+01	.34722E+00	-.41667E-01	.11183E-02	-.67960E+01
.28235E+01	.35417E+00	-.27776E-01	.16545E-02	-.64043E+01
.27692E+01	.36111E+00	-.35210E-01	.18736E-02	-.62799E+01
.27170E+01	.36806E+00	-.10514E-01	.16073E-02	-.64332E+01
.26667E+01	.37500E+00	-.45787E-01	.22984E-02	-.60755E+01
.26182E+01	.38194E+00	-.54523E-01	.21863E-02	-.61256E+01
.25714E+01	.38889E+00	-.20682E-01	.17083E-02	-.63723E+01
.25263E+01	.39583E+00	-.55205E-01	.18946E-02	-.62688E+01
.24828E+01	.40278E+00	-.10495E+00	.18998E-02	-.62660E+01
.24407E+01	.40972E+00	-.76754E-01	.16608E-02	-.63885E+01
.24000E+01	.41667E+00	-.94979E-01	.13986E-02	-.65723E+01
.23607E+01	.42361E+00	-.10843E-01	.13916E-02	-.65773E+01
.23226E+01	.43056E+00	-.10457E+00	.11313E-02	-.67844E+01
.22857E+01	.43750E+00	.34447E-01	.12880E-02	-.66547E+01
.22500E+01	.44444E+00	-.83469E-02	.16681E-02	-.63961E+01
.22154E+01	.45139E+00	-.55125E-01	.10153E-02	-.68926E+01
.21818E+01	.45833E+00	-.18582E-01	.86613E-03	-.70515E+01
.21493E+01	.46528E+00	.11407E-01	.13736E-02	-.65903E+01
.21176E+01	.47222E+00	-.47532E-01	.19745E-02	-.62274E+01
.20870E+01	.47917E+00	-.17803E-01	.17435E-02	-.63519E+01
.20571E+01	.48611E+00	-.11483E-01	.15044E-02	-.64993E+01
.20282E+01	.49306E+00	-.35508E-01	.18131E-02	-.63127E+01
.20000E+01	.50000E+00	-.30887E-01	.17059E-02	-.63737E+01

NATURAL LOGARITHM OF THE MEAN SPECTRUM IS, -.62111E+01

CHECK SUM OF POWER SPECTRAL ESTIMATES IS .62451E-02AND
SHOULD BE .62451E-02

TABLE B11

THEORETICAL AND SAMPLE CORRELATION COEFFICIENTS FOR FIRST AND SECOND ORDER LINEAR AUTOREGRESSIVE MODELS

(1) LAG
(2) SAMPLE AUTOCORRELATION COEFFICIENT
(3) THEORETICAL FIRST ORDER LINEAR AUTOREGRESSIVE AUTOCORRELATION COEFFICIENTS
(4) THEORETICAL SECOND ORDER LINEAR AUTOREGRESSIVE

(1)	(2)	(3)	(4)
1	.163	.163	.163
2	.099	.027	.099
3	.110	.004	.027
4	.097	.001	.011
5	.077	.000	.004
6	.029	.000	.001
7	.016	.000	.000
8	.074	.000	.000
9	.009	.000	.000
10	.058	.000	.000

CONFIDENCE LIMITS ON AUTOCORRELATION COEFFICIENTS

(1) ORDER
(2) 1 PCT. UPPER LIMIT
(3) 5 PCT. UPPER LIMIT
(4) SAMPLE AUTOCORRELATION COEFFICIENT
(5) 5 PCT. LOWER LIMIT
(6) 1 PCT. LOWER LIMIT

(1)	(2)	(3)	(4)	(5)	(6)
1	.073	.051	.163	-.053	-.075
2	.073	.051	.099	-.053	-.075
3	.073	.051	.110	-.053	-.075
4	.073	.051	.097	-.053	-.075
5	.073	.051	.077	-.053	-.075
6	.073	.051	.029	-.053	-.075
7	.073	.051	.016	-.053	-.075
8	.073	.051	.074	-.053	-.075
9	.073	.051	.009	-.053	-.075
10	.073	.051	.058	-.053	-.075

TABLE B12

RESULTS OF CHI-SQUARE TEST OF FIRST AND SECOND ORDER LINEAR AUTOREGRESSIVE MODELS

(A) FIRST ORDER LINEAR MODEL

COMPUTED VALUE OF CHI-SQUARE IS 28.07699

DEGREES OF FREEDOM ARE 9

TABULATED VALUES OF CHI-SQUARE ARE

99 PCT	95 PCT	5 PCT	1 PCT
2.088	3.325	16.919	21.666

TOTAL VARIANCE .006

VARIANCE NOT EXPLAINED BY 1ST ORDER MODEL, .006

PERCENT OF TOTAL VARIANCE EXPLAINED IS 2.57

(B) SECOND ORDER LINEAR MODEL

COMPUTED VALUE OF CHI-SQUARE IS 19.88707

DEGREES OF FREEDOM ARE 8

TABULATED VALUES OF CHI-SQUARE ARE

99 PCT	95 PCT	5 PCT	1 PCT
1.646	2.733	15.507	20.090

TOTAL VARIANCE .006

VARIANCE NOT EXPLAINED BY 2ND ORDER MODEL, .006

PERCENT OF TOTAL VARIANCE EXPLAINED IS 3.13

TABLE B13

SPECTRAL DENSITY OF DIFFERENCES IN MEAN MONTHLY ELEVATIONS AFTER REMOVAL OF LINEAR TRENDS, PERIODICITIES AND AUTOREGRESSIVE MODEL

PERIOD	FREQUENCY	AUTOCORRELATION COEFFICIENT	SMOOTHED SPECTRAL ESTIMATE	NATURAL LOG.OF SPECTRUM
.99400E+03	0.	.10000E+01	.28255E-02	-.58691E+01
.14400E+03	.69444E-02	-.19965E-01	.36538E-02	-.56120E+01
.72000E+02	.13889E-01	-.48923E-01	.29582E-02	-.58232E+01
.48000E+02	.20833E-01	.66199E-01	.18001E-02	-.63199E+01
.36000E+02	.27778E-01	.68379E-01	.17278E-02	-.63609E+01
.28800E+02	.34722E-01	.52519E-01	.14432E-02	-.65409E+01
.24000E+02	.41667E-01	-.44932E-02	.14526E-02	-.65344E+01
.20571E+02	.48611E-01	-.57464E-02	.16069E-02	-.64334E+01
.18000E+02	.55556E-01	.69358E-01	.16119E-02	-.64303E+01
.16000E+02	.62500E-01	-.12711E-01	.12869E-02	-.66555E+01
.14400E+02	.69444E-01	.38534E-01	.15115E-02	-.64947E+01
.13091E+02	.76389E-01	.15187E-01	.23784E-02	-.60413E+01
.12000E+02	.83333E-01	.39752E-01	.21312E-02	-.61511E+01
.11077E+02	.90278E-01	.61269E-01	.14737E-02	-.65200E+01
.10286E+02	.97222E-01	.63554E-02	.10011E-02	-.69067E+01
.96000E+01	.10417E+00	.14061E-01	.11249E-02	-.67900E+01
.90000E+01	.11111E+00	.48424E-01	.14721E-02	-.65211E+01
.84706E+01	.11806E+00	.17549E-01	.12326E-02	-.66986E+01
.80000E+01	.12500E+00	-.41579E-02	.13967E-02	-.65737E+01
.75789E+01	.13194E+00	-.79957E-02	.16859E-02	-.63854E+01
.72000E+01	.13889E+00	-.30780E-01	.15742E-02	-.64540E+01
.68571E+01	.14583E+00	.23417E-01	.13124E-02	-.66359E+01
.65455E+01	.15278E+00	.29103E-01	.12499E-02	-.66847E+01
.62609E+01	.15972E+00	.73584E-02	.16219E-02	-.64242E+01
.60000E+01	.16667E+00	.11339E-02	.17348E-02	-.63569E+01
.57600E+01	.17361E+00	.47024E-01	.17215E-02	-.63646E+01
.55385E+01	.18056E+00	.20009E-03	.17878E-02	-.63268E+01
.53333E+01	.18750E+00	.20248E-01	.21505E-02	-.61420E+01
.51429E+01	.19444E+00	-.40831E-02	.20644E-02	-.61829E+01
.49655E+01	.20139E+00	-.29964E-01	.15841E-02	-.64477E+01
.48000E+01	.20833E+00	-.96112E-02	.18016E-02	-.63191E+01
.46452E+01	.21528E+00	.75755E-02	.18546E-02	-.62901E+01
.45000E+01	.22222E+00	-.16943E-01	.19722E-02	-.62286E+01
.43636E+01	.22917E+00	-.87604E-01	.31023E-02	-.57756E+01
.42353E+01	.23611E+00	.29924E-01	.32886E-02	-.57173E+01
.41143E+01	.24306E+00	.45848E-02	.22236E-02	-.61086E+01
.40000E+01	.25000E+00	.10264E-01	.24085E-02	-.60288E+01
.38919E+01	.25694E+00	-.12294E-01	.29642E-02	-.58211E+01
.37895E+01	.26389E+00	.51668E-01	.25515E-02	-.59711E+01
.36923E+01	.27083E+00	.13361E-01	.18603E-02	-.62870E+01

TABLE B13 (CONTINUED)

.36000E+01	.27778E+00	-.25190E-01	.20397E-02	-.61949E+01
.35122E+01	.28472E+00	-.78531E-01	.26702E-02	-.59256E+01
.34286E+01	.29167E+00	.29849E-01	.21559E-02	-.61395E+01
.33488E+01	.29861E+00	.10149E-01	.19265E-02	-.62520E+01
.32727E+01	.30556E+00	-.68997E-01	.17093E-02	-.63717E+01
.32000E+01	.31250E+00	-.10643E-01	.22890E-02	-.60797E+01
.31304E+01	.31944E+00	-.64742E-01	.24662E-02	-.60051E+01
.30638E+01	.32639E+00	.27364E-01	.20824E-02	-.61742E+01
.30000E+01	.33333E+00	-.18810E-01	.18559E-02	-.62894E+01
.29388E+01	.34028E+00	-.80588E-01	.13654E-02	-.65963E+01
.28800E+01	.34722E+00	-.19295E-01	.14286E-02	-.65511E+01
.28235E+01	.35417E+00	-.71706E-02	.21063E-02	-.61628E+01
.27692E+01	.36111E+00	-.24495E-01	.23346E-02	-.60599E+01
.27170E+01	.36806E+00	.96245E-02	.19753E-02	-.62270E+01
.26667E+01	.37500E+00	-.35305E-01	.28371E-02	-.58650E+01
.26182E+01	.38194E+00	-.42605E-01	.26791E-02	-.59223E+01
.25714E+01	.38889E+00	.12585E-01	.21093E-02	-.61614E+01
.25263E+01	.39583E+00	-.28803E-01	.23527E-02	-.60522E+01
.24828E+01	.40278E+00	-.81521E-01	.23012E-02	-.60743E+01
.24407E+01	.40972E+00	-.45556E-01	.20041E-02	-.62125E+01
.24000E+01	.41667E+00	-.68660E-01	.16726E-02	-.63934E+01
.23607E+01	.42361E+00	.21322E-01	.16727E-02	-.63933E+01
.23226E+01	.43056E+00	-.10653E+00	.13611E-02	-.65995E+01
.22857E+01	.43750E+00	.58555E-01	.15044E-02	-.64993E+01
.22500E+01	.44444E+00	.60884E-02	.19273E-02	-.62517E+01
.22154E+01	.45139E+00	-.62152E-01	.11751E-02	-.67464E+01
.21818E+01	.45833E+00	-.11051E-01	.10089E-02	-.68989E+01
.21493E+01	.46528E+00	.28411E-01	.15893E-02	-.64444E+01
.21176E+01	.47222E+00	-.46206E-01	.22563E-02	-.60940E+01
.20870E+01	.47917E+00	-.10884E-01	.19508E-02	-.62395E+01
.20571E+01	.48611E+00	.41769E-02	.16555E-02	-.64037E+01
.20282E+01	.49306E+00	-.28090E-01	.20256E-02	-.62019E+01
.20000E+01	.50000E+00	-.22898E-01	.19357E-02	-.62473E+01

NATURAL LOGARITHM OF THE MEAN SPECTRUM IS, -.62493E+01

CHECK SUM OF POWER SPECTRAL ESTIMATES IS .60496E-02AND
SHOULD BE .60496E-02

TABLE B14

CUMULATIVE PROBABILITY DISTRIBUTION OF INDEPENDENT RANDOM COMPONENT

Z	PERCENT PROBABILITY OF THE TIME SERIES BEING LESS THAN OR EQUAL TO Z
-.1639E+00	.1308E+01
-.1409E+00	.2515E+01
-.1260E+00	.3521E+01
-.1147E+00	.4829E+01
-.1054E+00	.6137E+01
-.9740E-01	.6942E+01
-.9031E-01	.7948E+01
-.8391E-01	.8853E+01
-.7802E-01	.1056E+02
-.7255E-01	.1258E+02
-.6742E-01	.1368E+02
-.6257E-01	.1509E+02
-.5794E-01	.1730E+02
-.5352E-01	.1871E+02
-.4926E-01	.2062E+02
-.4514E-01	.2233E+02
-.4115E-01	.2404E+02
-.3726E-01	.2646E+02
-.3347E-01	.2827E+02
-.2975E-01	.3099E+02
-.2610E-01	.3310E+02
-.2251E-01	.3511E+02
-.1897E-01	.3843E+02
-.1546E-01	.4155E+02
-.1199E-01	.4427E+02
-.8549E-02	.4658E+02
-.5121E-02	.4920E+02
-.1706E-02	.5080E+02
.1706E-02	.5322E+02
.5121E-02	.5573E+02
.8549E-02	.5825E+02
.1199E-01	.5956E+02
.1546E-01	.6207E+02
.1897E-01	.6408E+02
.2251E-01	.6579E+02
.2610E-01	.6861E+02
.2975E-01	.7103E+02
.3347E-01	.7294E+02
.3726E-01	.7485E+02

TABLE B14 (CONTINUED)

.4115E-01	.7767E+02
.4514E-01	.7918E+02
.4926E-01	.7988E+02
.5352E-01	.8159E+02
.5794E-01	.8360E+02
.6257E-01	.8501E+02
.6742E-01	.8662E+02
.7255E-01	.8783E+02
.7802E-01	.8954E+02
.8391E-01	.9085E+02
.9031E-01	.9205E+02
.9740E-01	.9346E+02
.1054E+00	.9427E+02
.1147E+00	.9487E+02
.1260E+00	.9618E+02
.1409E+00	.9708E+02
.1639E+00	.9839E+02

TABLE B15

PROBABILITY DENSITY DISTRIBUTION OF
INDEPENDENT RANDOM COMPONENT

LOW POINT	HIGH POINT	MID POINT	PROBABILITY DENSITY
	-.1639E+00		.1308E+01
-.1639E+00	-.1409E+00	-.1524E+00	.1207E+01
-.1409E+00	-.1260E+00	-.1334E+00	.1006E+01
-.1260E+00	-.1147E+00	-.1204E+00	.1308E+01
-.1147E+00	-.1054E+00	-.1101E+00	.1308E+01
-.1054E+00	-.9740E-01	-.1014E+00	.8048E+00
-.9740E-01	-.9031E-01	-.9386E-01	.1006E+01
-.9031E-01	-.8391E-01	-.8711E-01	.9054E+00
-.8391E-01	-.7802E-01	-.8096E-01	.1710E+01
-.7802E-01	-.7255E-01	-.7529E-01	.2012E+01
-.7255E-01	-.6742E-01	-.6999E-01	.1107E+01
-.6742E-01	-.6257E-01	-.6499E-01	.1408E+01
-.6257E-01	-.5794E-01	-.6025E-01	.2213E+01
-.5794E-01	-.5352E-01	-.5573E-01	.1408E+01
-.5352E-01	-.4926E-01	-.5139E-01	.1911E+01
-.4926E-01	-.4514E-01	-.4720E-01	.1710E+01
-.4514E-01	-.4115E-01	-.4315E-01	.1710E+01
-.4115E-01	-.3726E-01	-.3921E-01	.2414E+01
-.3726E-01	-.3347E-01	-.3537E-01	.1811E+01
-.3347E-01	-.2975E-01	-.3161E-01	.2716E+01
-.2975E-01	-.2610E-01	-.2793E-01	.2113E+01
-.2610E-01	-.2251E-01	-.2431E-01	.2012E+01
-.2251E-01	-.1897E-01	-.2074E-01	.3320E+01
-.1897E-01	-.1546E-01	-.1722E-01	.3119E+01
-.1546E-01	-.1199E-01	-.1373E-01	.2716E+01
-.1199E-01	-.8549E-02	-.1027E-01	.2314E+01
-.8549E-02	-.5121E-02	-.6835E-02	.2616E+01
-.5121E-02	-.1706E-02	-.3413E-02	.1610E+01
-.1706E-02	.1706E-02	-.1978E-15	.2414E+01
.1706E-02	.5121E-02	.3413E-02	.2515E+01
.5121E-02	.8549E-02	.6835E-02	.2515E+01
.8549E-02	.1199E-01	.1027E-01	.1308E+01
.1199E-01	.1546E-01	.1373E-01	.2515E+01
.1546E-01	.1897E-01	.1722E-01	.2012E+01
.1897E-01	.2251E-01	.2074E-01	.1710E+01
.2251E-01	.2610E-01	.2431E-01	.2817E+01
.2610E-01	.2975E-01	.2793E-01	.2414E+01
.2975E-01	.3347E-01	.3161E-01	.1911E+01
.3347E-01	.3726E-01	.3537E-01	.1911E+01

TABLE B15 (CONTINUED)

.3726E-01	.4115E-01	.3921E-01	.2817E+01
.4115E-01	.4514E-01	.4315E-01	.1509E+01
.4514E-01	.4926E-01	.4720E-01	.7042E+00
.4926E-01	.5352E-01	.5139E-01	.1710E+01
.5352E-01	.5794E-01	.5573E-01	.2012E+01
.5794E-01	.6257E-01	.6025E-01	.1408E+01
.6257E-01	.6742E-01	.6499E-01	.1610E+01
.6742E-01	.7255E-01	.6999E-01	.1207E+01
.7255E-01	.7802E-01	.7529E-01	.1710E+01
.7802E-01	.8391E-01	.8096E-01	.1308E+01
.8391E-01	.9031E-01	.8711E-01	.1207E+01
.9031E-01	.9740E-01	.9386E-01	.1408E+01
.9740E-01	.1054E+00	.1014E+00	.8048E+00
.1054E+00	.1147E+00	.1101E+00	.6036E+00
.1147E+00	.1260E+00	.1204E+00	.1308E+01
.1260E+00	.1409E+00	.1334E+00	.9054E+00
.1409E+00	.1639E+00	.1524E+00	.1308E+01
.1639E+00			.1610E+01

TABLE B16

TESTS OF DISTRIBUTION OF INDEPENDENT RANDOM COMPONENT AGAINST NORMAL AND LOG-NORMAL DISTRIBUTIONS

COMPUTED VALUE OF CHI-SQUARE BASED ON THE NORMAL
DISTRIBUTION IS .1321E+03
NUMBER OF DEGREES OF FREEDOM IS 54

COMPUTED VALUE OF KOLMOGOROV-SMIRNOV PARAMETER
BASED ON THE NORMAL DISTRIBUTION IS .7491E-01
SAMPLE SIZE IS 994

EXPECTED VALUE OF THE SAMPLE IS -.1984E-15
STANDARD DEVIATION OF THE SAMPLE IS .7778E-01

THE LARGEST NEGATIVE VALUE IN THE ARRAY IS -.626760

HENCE THE ABSOLUTE VALUE OF IT WILL BE ADDED TO EACH NUMBER

IN THE ENTIRE ARRAY BEFORE THE NATURAL LOG IS TAKEN
COMPUTED VALUE OF CHI-SQUARE BASED ON THE LOG -
NORMAL DISTRIBUTION IS .1239E+04
NUMBER OF DEGREES OF FREEDOM IS 54

COMPUTED VALUE OF KOLMOGOROV-SMIRNOV PARAMETER
BASED ON THE LOG-NORMAL DISTRIBUTION
IS .2448E+00
SAMPLE SIZE IS 994

EXPECTED VALUE OF THE MEAN OF THE
LOGARITHMS OF THE SAMPLE IS -.4818E+00
STANDARD DEVIATION OF THE LOGARITHMS
OF THE SAMPLE IS .3004E+00

TABLE B17

RESULTS OF RANDOMNESS TESTS

TEST NO.	STANDARD NORMAL VARIABLE	RECORDED VALUE	RECORDED MEAN	RECORDED STANDARD DEVIATION
1	-1.58	724.00	746.05	13.63
2	52.79	1363.00	662.33	13.28
3	.55	491.00	496.50	9.11

NOTE.

THE STANDARD NORMAL VARIABLE (K) INDICATES THE NUMBER OF STANDARD DEVIATIONS THAT THE TEST VALUE IS BELOW THE EXPECTED (MEAN) VALUE IF THE OBSERVATIONS ARE INDEPENDENT. VALUES OF K LESS THAN -2.33 OCCUR WITH A PROBABILITY OF LESS THAN 0.01 FOR A SEQUENCE OF INDEPENDENT OBSERVATIONS (USE K GREATER THAN 2.58 FOR TEST 3 - 2-TAIL PROBABILITY).

SPECIFICALLY, SUCH VALUES OF K INDICATE -

TEST 1 - A TENDENCY FOR OBSERVATIONS TO CLUSTER INTO THE RESPECTIVE CATEGORIES.

TEST 2 - A TENDENCY FOR DIRECTIONS OF MOVEMENT TO PERSIST (IE. - CLUSTERING NOT EXCLUSIVELY DUE TO A FEW LARGE CHANGES).

TEST 3 - AN UPWARD OR DOWNWARD TREND DUE TO A RELATIVE SHORTAGE OF - OR + SIGNS RESPECTIVELY (TREND DUE TO LARGER MOVEMENTS IN ONE DIRECTION IS NOT DETECTED BY THIS TEST).

TABLE B18

SUMMARY OF TIME SERIES ANALYSIS OF DIFFERENCES IN MEAN MONTHLY ELEVATIONS

COMPONENT	PERCENT OF TOTAL VARIANCE EXPLAINED BY THIS COMPONENT
LINEAR TRENDS	48.480400
PERIODICITIES	21.954378
AUTOREGRESSION	.925540
RESIDUAL	28.639683

APPENDIX C

LISTING OF COMPUTER PROGRAM USED FOR TIME SERIES ANALYSIS

```
      PROGRAM SAMPLE(INPUT,OUTFUT,TAPE5=INPUT,TAFE6=OLTPUT)
C
C
C     PROGRAM SAMPLE
C
C     PROGRAM TO ANALYSE A TIME SERIES AND DETERMINE THE PERCENTAGES
C     OF THE INITIAL VARIANCE EXPLAINED BY LINEAR TRENDS, FERIODIC
C     COMPONENTS, AUTOREGRESSIVE COMPONENT AND RESIDUAL.
C
C     LINEAR TRENDS ARE DETECTED BY A VARIATE DIFFERENCE TECHNIQUE
C     AND REMOVED BY  A STEPWISE LEAST SQUARES REGRESSION.
C
C     PERIODICITIES ARE DETECTED AND REMOVED BY A PERIODOGRAM ANALYSIS
C     USING STUMMS,MODIFICATION. THE PROGRAM IS SET UP TO DETERMINE
C     6 HARMONICS OF THE ANNUAL CYCLE FOR MONTHLY DATA. AUTOREGRESSIVE
C     COMPONENT IS TESTED FOR USING CHI-SQUARE AND KOLMOGOROV-SMIRNOV
C     CHECKS AGAINST THEORETICAL FIRST AND SECOND ORDER LINEAR
C     MARKOV MODELS. THE PROGRAM IS SET UP TO USE 10 AUTOCORRELATION
C     COEFFICIENTS.
C
C     THE RESIDUAL IS ANALYSED FOR DISTRIBUTION BY CHECKING AGAINST
C     NORMAL AND LOG-NORMAL, PRINTING OUT PROBABILITY MASS AND
C     PROBABILITY DENSITY DISTRIBUTIONS AND BY PERFORMING NON-PARAMETRIC
C     RANDOMNESS TESTS. THE NUMBER OF CLASS INTERVALS USED IN THE
C     PROBABILITY DISTRIBUTION DETERMINATIONS CAN EITHER BE SPECIFIED
C     OR WILL BE COMP UTED IN THE PROGRAM TO A MAXIMUM OF 100.
C
C     AT EACH STEP OF THE ANALYSIS AN AUTOCOVARIANCE AND SPECTRAL
C     DENSITY ANALYSIS IS PERFORMED AND THE RESULTS PLOTTED TO ENABLE
C     VISUAL DETECTION OF REMAINING SIGNIFICANT COMPONENTS.
C
C     THE LAST PAGE OF PRINT-OUT IS A SUMMARY TABLE OF THE ANALYSIS GIVI
C     THE PERCENTAGES OF THE ORIGINAL VARIANCE EXPLAINED BY EACH COMPONE
C
C     THE PROGRAM WAS ORIGINALLY WRITTEN FOR MONTHLY HYDROLOGIC TIME
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C     SERIES AND WILL REQUIRE MODIFICATIONS FOR DATA CF OTHER SPACING
C     (E.G. DAILY OR HOURLY DATA).
C
C     INPUT DATA
C
C     TITLE - ALPHANUMERIC TITLE OF UP TO 80 CHARACTERS ON A SINGLE CARD
C     N -  NUMBER OF OBSERVATIONS IN THE INPUT TIME SERIES.
C     N1 - ORDER OF LINEAR TREND TO BE ANALYSED FOR -- THIS PROVIDES AN
C          INITIAL CONTROL ON THE STEPWISE REGRESSION.
C     N2 - NUMBER OF LAGS REQUIRED FOR SPECTRAL ANALYSIS, USE N/10
C          AS AN INITIAL APPROXIMATION OR ELSE DETERMINE N2 FROM THE
C          FREQUENCY RANGE DESIRED.
C     N3 - IN DETECTING LINEAR TRENDS IN A TIME SERIES A TIME ORIGIN IS
C          REQUIRED TC PROVIDE A BASE FOR THE INDEPENDENT VARIABLE.
C          THIS PROGRAM USES AS BASE THE INITIAL YEAR OF THE TIME
C          SERIES AND FROM THAT CREATES A FIRST ORDER LINEAR TIME
C          SERIES.
C     X -  TIME SERIES TO BE ANALYSED.
C
C     VARIABLE DIMENSIONS - ONLY THE FOLLOWING VARIABLES, ALL LOCATED
C     IN THE MAIN PROGRAM, WILL NEED RE-DIMENSIONING
C     IN THE EVENT OF CHANGES IN N, N1, OR N2.
C
C         X(N), Y(N), Z(N),
C         CL(N2), U(N2), V(N2), F(N2), XLOG(N2)
C         XX(N*(N1+1)), P(N*3),
C         D((N1+2)*(N1+1)/2, DI(N1*N1),
C         B(N1), E(N1), SB(N1), T(N1),
C         XBAR(N1+1), STD(N1+1), COE(N1+1),
C         SUMSQ(N1+1), ISAVE(N1+1)
C
C     NOTE THAT SUBROUTINES POLY, GDATA, ORDER, MINV, MULTR AND CORRE AR
C     ADAPTED FROM IBM SCIENTIFIC SUBROUTINE PACKAGE.
C
      COMMON /BK/ TITLE(80),J0,TITLE1(8)
```

```
      COMMON/BL/DI(100),D(66),B(10),E(10),SB(10),T(10),XBAR(11),STD(11),
     1COE(11),SUMSQ(11),ISAVE(11)
      DIMENSION F(75),XLOG(75),CL(75),U(75),V(75),XX(6660),P(4000)
      DIMENSION X(1332),Y(1332),Z(1332)
      JI=5
      JO=6
      SIGNIF=0.0001
C     SIGNIF ACCURACY TO WHICH VARIANCES DETERMINED IN SPECTRAL ANALYSIS
C     ARE TO BE TESTED.
      READ (JI,150) TITLE
      READ (JI,170) N,N1,N2,N3
      WRITE (JO,200)
      WRITE (JO,160) TITLE
      READ (JI,180) (X(J),J=1,N)
      WRITE (JO,190) (X(J),J=1,N)
      WRITE (JO,200)
      DO 10 J=1,N
   10 Y(J)=N3+(J-1)/12.0
      DO 20 J=1,N
   20 Z(J)=X(J)
      TITLE1(1)=10H
      TITLE1(2)=10H
      TITLE1(3)=10HORIGINAL T
      TITLE1(4)=10HIME SERIES
      TITLE1(5)=10H
      TITLE1(6)=10H
      TITLE1(7)=10H
      TITLE1(8)=10H
      CALL SPECTRA (N,N2,Z,TEST,CL,U,V,F,XLOG)
      DO 30 J=1,N
   30 Z(J)=X(J)
      CALL VARIAT (N,N1,Z,OVAR)
      VARO=TEST
      TABS=ABS(OVAR-TEST)
      IF (TABS.GT.SIGNIF) GO TO 90
```

```
      DO 40 J=1,N
40    Z(J)=X(J)
      CALL POLY (N,Y,Z,N1,1,XX,P)
      DO 50 J=1,N
50    X(J)=Z(J)
      TITLE1(2)=TITLE1(3)
      TITLE1(3)=TITLE1(4)
      TITLE1(4)=10H LESS LINE
      TITLE1(5)=10HAR TRENDS
      CALL SPECTRA (N,N2,Z,TEST,CL,U,V,F,XLOG)
      VAR1=TEST
      CALL STUMM (12,6,N,X,RVAR,Y)
      DO 60 J=1,N
60    Z(J)=X(J)
      TITLE1(1)=TITLE1(2)
      TITLE1(2)=TITLE1(3)
      TITLE1(3)=TITLE1(4)
      TITLE1(4)=TITLE1(5)
      TITLE1(5)=10HLESS PERIO
      TITLE1(6)=10HDICITIES
      CALL SPECTRA (N,N2,Z,TEST,CL,U,V,F,XLOG)
      VAR2=TEST
      TABS=ABS(RVAR-TEST)
      IF (TABS.GT.SIGNIF) GO TO 90
      CALL MARKOV (N,X,IFLAG,F,Y,Z)
      NOUT=1
      IF (IFLAG.EQ.2) NOUT=2
      NT=N-NOUT
      DO 70 J=1,NT
70    Z(J)=X(J)
      CALL STANDX (NT,Z,A1,A2)
      TITLE1(1)=10HRESIDUALS
      TITLE1(2)=10HAFTER REMO
      TITLE1(3)=10HVAL OF LIN
      TITLE1(4)=10HEAR TRENDS
```

```
      TITLE1(5)=10H, PERICDIC
      TITLE1(6)=10HITIES + AU
      TITLE1(7)=10HTORES COMP
      TITLE1(8)=10H
      CALL SPECTRA (NT,N2,Z,TEST,CL,U,V,F,XLOG)
      VAR3=TEST
      TABS=ABS(A2*A2-TEST)
      IF (TABS.GT.SIGNIF) GO TO 90
      CALL DIST (NT,0,Z)
      DO 80 J=1,N
80    Z(J)=X(J)
      CALL RAND (NT,X,Z)
      P1=(VARO-VAR1)/VARO*100.0
      P2=(VAR1-VAR2)/VARO*100.0
      P3=(VAR2-VAR3)/VARO*100.0
      P4=VAR3/VARO*100.0
      WRITE (JO,110) TITLE
      WRITE (JO,120)
      WRITE (JO,130) P1,P2,P3,P4
      GO TO 100
90    WRITE (JO,140)
100   CONTINUE
      STOP
C
110   FORMAT (1H1,//,1X,80A1)
120   FORMAT (/,40X,"SUMMARY OF TIME SERIES MAKE-UP",//,30X,"COMPONENT",
     131X,"PERCENT OF ORIGINAL VARIANCE",/,30X,"---------",31X,"EXPLAINE
     2D BY THIS COMPONENT",/,70X,"--------------------------",//)
130   FORMAT (//,30X,"LINEAR TRENDS",36X,F10.6,////,30X,"PERIODICITIES",
     136 XF10.6,////,30X,"AUTOREGRESSION",35X,F10.6,////,30X,"RESIDUAL",
     241X,F10.6)
140   FORMAT (1H1, /////, 1X, "ERROR IN COMPUTED VARIANCE")
150   FORMAT (80A1)
160   FORMAT (//,9X,80A1)
170   FORMAT (4I5)
```

```
180    FORMAT (11X,12F5.2)                                   A1790
190    FORMAT (1X,12F8.2)                                    A1800
200    FORMAT (1H1)                                          A1810-
       END
```

```
C       SUBROUTINE SPECTRA
C
        SUBROUTINE SPECTRA (N,M,X,CHKSUM,CL,U,V,F,XLOG)
        COMMON /BK/ TITLE(80),JO,TITLE1(8)
        DIMENSION X(1), CL(1), V(1), U(1)
        DIMENSION F(1), XLOG(1)
        MM=M+1
        XM=M
        XMM=MM
        CALL AUTO (N,M,X,CL)
        CALL RAWS (M,CL,V)
        CALL SMOO (M,V,U,CHKSUM)
        WRITE (JO,110)
        WRITE (JO,70) TITLE
        WRITE (JO,60) TITLE1
        WRITE (JO,80)
        DO 40 I=1,MM
        XI=I-1
        IF (I.EQ.1) GO TO 10
        T=2.0*XM/XI
        F(I)=1.0/T
        ATCO=CL(I)/CL(1)
        GO TO 20
   10   ATCO=1.0
        T=N
        F(I)=0.0
   20   CONTINUE
        IF (I.EQ.47) WRITE (JO,110)
        IF (U(I).GE.0.) GO TO 30
        XLOG(I)=10HNO. LOG
        WRITE (JO,120) T,F(I),ATCO,U(I),XLOG(I)
        GO TO 40
   30   XLOG(I)=ALOG(U(I))
        WRITE (JO,90) T,F(I),ATCO,U(I),XLOG(I)
   40   CONTINUE
```

```
      SUM=0.0
      DO 50 I=1,MM
   50 SUM=SUM+U(I)
      UBAR=SUM/XMM
      UBAR=ALOG(UBAR)
      WRITE (JO,100) UBAR
      WRITE (JO,130) CHKSUM,CL(1)
      WRITE (JO,110)
      CALL GRAF (MM,F,XLOG)
      RETURN
C
   60 FORMAT (//,24X,8A10)
   70 FORMAT (//,9X,80A1)
   80 FORMAT (//,64X,"SMOOTHED",4X,"NATURAL",/,48X,"AUTOCORRELATION",1X,
     1"SPECTRAL",4X,"LOG. CF",/,28X,"PERIOD",5X,"FREQUENCY",2X,"COEFFICIE
     2NT",3X,"ESTIMATE",4X,"SPECTRUM",/)
   90 FORMAT (24X,5E12.5)
  100 FORMAT (//,26X,"NATURAL LOGARITHM OF THE MEAN SPECTRUM IS,",E12.5,
     1//)
  110 FORMAT (1H1)
  120 FORMAT (24X,4E12.5,2X,A10)
  130 FORMAT (//,26X, "CHECK SUM OF POWER SPECTRAL ESTIMATES IS",E12.5,"
     1AND SHOULD BE",E12.5)
      END
```

```
      SUBROUTINE GRAF (MM,X,Y)
      COMMON /BK/ TITLE(80),JO,TITLE1(8)
      DIMENSION X(MM,1), Y(MM,1), NPNTS(1)
      WRITE (JO,10) TITLE1
      N=1
      IGRID=2
      IWIDTH=101
      JLNGTH=51
      NSBVL=20
      NSBHL=10
      IPRNTR=JO
      CALL IAS002 (IGRID,IWIDTH,JLNGTH,NSBVL,NSBHL,IPRNTR)
      NPNTS(1)=MM
      IOUT=JO
      CALL IAS003 (N,X,Y,NPNTS,IOUT,IPRNTR)
      RETURN
C
10    FORMAT (//,24X,8A10,//,10X,"PLOT CF NATURAL LOG. OF SPECTRUM VS FR
     1EQUENCY")
      END
```

```
      SUBROUTINE AUTO (N,M,X,R)
      DIMENSION X(1), R(1)
      XN=N
      MM=M+1
      SUM=0.0
      DO 10 I=1,N
 10   SUM=SUM+X(I)
      XBAR=SUM/XN
      DO 20 I=1,N
 20   X(I)=X(I)-XBAR
      DO 40 IP=1,MM
      IY=N-IP+1
      XY=IY
      SUM=0.0
      DO 30 J=1,IY
      K=J+IP-1
 30   SUM=SUM+X(J)*X(K)
 40   R(IP)=(1.0/XY)*SUM
      RETURN
      END
```

```
C      SUBROUTINE RAWS
C
       SUBROUTINE RAWS (M,CL,V)
       DIMENSION CL(1), V(1)
C
C      COMPUTE RAW SPECTRAL DENSITY ESTIMATE FOR BMD02T TYPE SPECTRUM
C
       PI=(ATAN(1.0))*4.0
       XM=M
       MM=M+1
       DO 20 J=1,MM
       LJ=J-1
       XJ=LJ
       SUM1=0.0
       DO 10 K=1,MM
       LK=K-1
       XK=LK
       A=(XJ*XK*PI)/XM
       E=1.0
       IF (K.EQ.1.OR.K.EQ.MM) E=0.5
   10  SUM1=SUM1+E*CL(K)*CCS(A)
   20  V(J)=(2.0/PI)*SUM1
       RETURN
       END
```

```
C     SUBROUTINE SMOO
C
      SUBROUTINE SMOO (M,V,U,CHKSUM)
      DIMENSION V(1), U(1)
C
C     APPLY HAMMING WEIGHTS TC OBTAIN SMOOTHED SFECTRUM
C
      MM=M+1
      U(1)=0.54*V(1)+0.46*V(2)
      U(MM)=0.46*V(M)+0.54*V(MM)
      DO 10 J=2,M
   10 U(J)=0.23*V(J-1)+0.54*V(J)+0.23*V(J+1)
      XM=M
      PI=(ATAN(1.0))*4.0
      SUM=0.0
      DO 20 J=2,M
   20 SUM=SUM+U(J)
      SUM=SUM+0.5*(U(1)+U(MM))
      CHKSUM=SUM*(FI/XM)
      RETURN
      END
```

```
C     SUBROUTINE POLY
C
C
C     SUBROUTINE TO PERFORM A POLYNOMIAL REGRESSION
C
C
C     PURPOSE...CALL SUBRCUTINES TO PERFORM A POLYNOMIAL REGRESSION,
C     PRINT THE REGRESSION COEFFICIENTS AND ANALYSIS OF VARIANCE FOR
C     SUCCESSIVELY INCREASING DEGREES, AND OPTIONALLY PRINT A TABLE
C     OF RESIDUALS.
C
C
C     NOTE...THE NUMBER OF OBSERVATIONS, N, MUST BE GREATER THAN M+1,
C     WHERE M IS THE HIGHEST DEGREE POLYNOMIAL SPECIFIED. IF THERE IS NO
C     REDUCTION IN THE RESIDUAL SUM OF SQUARES BETWEEN TWO SUCCESSIVE
C     DEGREES OF POLYNOMIAL THE PROGRAM TERMINATES THE PROBLEM BEFORE
C     COMPLETING THE ANALYSIS TC THE HIGHEST ORDER POLYNOMIAL SPECIFIED
C
C
C     N       NUMBER OF DATA POINTS WITHIN THE ARRAY
C     TEMP1   INDEPENDENT VARIABLE
C     TEMP2   DEPENDENT VARIABLE
C     TITLE1  IDENTIFICATICN
C     M       ORDER OF POLYNOMIAL REQUIRED
C     NORES   RESIDUAL INDICATOR ,IF 1 NO RESIDUALS
C             WILL BE PRINTED
C
C
      SUBROUTINE POLY (N,TEMP1,TEMP2,M,NORES,X,P)
      COMMON /BK/ TITLE(80),JO,TITLE1(8)
      COMMON/BL/DI(100),D(66),B(10),E(10),SB(10),T(10),XBAR(11),STD(11),
     1COE(11),SUMSQ(11),ISAVE(11)
      DIMENSION TEMP1(1), TEMP2(1), ANS(10)
      DIMENSION X(1), P(1)
      WRITE (JO,140)
```

```
      WRITE (JO,150) TITLE
      WRITE (JO,120) TITLE1
      WRITE (JO,160) N
      L=N*M
      DO 10 I=1,N
      J=L+I
      X(I)=TEMP1(I)
   10 X(J)=TEMP2(I)
      CALL GDATA (N,M,X,XBAR,STD,D,SUMSQ)
      MM=M+1
      SUM=0.0
      NT=N-1
      DO 60 I=1,M
      ISAVE(I)=I
      CALL ORDER (MM,D,MM,I,ISAVE,DI,E)
      CALL   MINV(DI,I,DET,B,T)
      CALL MULTR (N,I,XBAR,STD,SUMSQ,DI,E,ISAVE,B,SB,T,ANS)
      WRITE (JO,170) I
      SUMIP=ANS(4)-SUM
      IF (SUMIP) 20,20,30
   20 WRITE (JO,250)
      GO TO 70
   30 WRITE (JO,180) ANS(1)
      DO 40 J=1,I
   40 WRITE (JO,190) J,B(J)
      WRITE (JO,280) ANS(2)
      WRITE (JO,290) ANS(3)
      WRITE (JO,200) I
      WRITE (JO,210)
      SUM=ANS(4)
      WRITE (JO,220) I,ANS(4),ANS(6),ANS(10),SUMIP
      NI=ANS(8)
      WRITE (JO,230) NI,ANS(7),ANS(9)
      WRITE (JO,240) NT,SUMSQ(MM)
      COE(1)=ANS(1)
```

```
      DO 50 J=1,I
50    COE(J+1)=B(J)
      LA=I
60    CONTINUE
70    CONTINUE
      NP3=N+N
      DO 80 I=1,N
      NP3=NP3+1
      P(NP3)=COE(1)
      L=I
      DO 80 J=1,LA
      P(NP3)=P(NP3)+X(L)*COE(J+1)
80    L=L+N
      N2=N
      L=N*M
      DO 90 I=1,N
      P(I)=X(I)
      N2=N2+1
      L=L+1
90    P(N2)=X(L)
      IF (NORES.EQ.1) GO TO 100
      WRITE (JO,140)
      WRITE (JO,170) LA
      WRITE (JO,260)
100   NP2=N
      NP3=N+N
      DO 110 I=1,N
      NP2=NP2+1
      NP3=NP3+1
      RESID=P(NP2)-P(NP3)
      TEMP2(I)=RESID
      IF (NORES.EQ.1) GO TO 110
      WRITE (JO,130) I,P(I),P(NP2),P(NP3),RESID
110   CONTINUE
      WRITE (JO,270)
```

```
      RETURN
C
120   FORMAT (26X,8A10,/)
130   FORMAT (8X,I6,F18.5,F14.5,F17.5,F15.5)
140   FORMAT (1H1,43X,21HFOLYNCMIAL REGRESSION,//)
150   FORMAT (9X,80A1)
160   FORMAT (7X,23H NUMBER OF OBSERVATIONS,I6//)
170   FORMAT (7X,32H POLYNOMIAL REGRESSION OF NUMBER,I3/)
 0180 FORMAT(8X,*INTERCEPT*,23X,F20.5)
190   FORMAT (8X,*REGRESSION COEFFICIENT*,I3,15X,F12.5)
200   FORMAT (5X,30X,24HANALYSIS OF VARIANCE FOR,I4,19H  DEGREE POLYNOMI
     1AL/)
210   FORMAT (5X,5X,19HSOURCE OF VARIATION,7X,9HDEGREE OF,7X,6HSUM OF,9X
     1,4HMEAN,10X,1HF,9X,20HIMPROVEMENT IN TERMS/37X,7HFREEDOM,8X,7HSQUA
     2RES,7X,6HSQUARE,7X,5HVALUE,8X,17HOF SUM OF SQUARES/)
220   FORMAT (8X,*DUE TO REGRESSION*,12X,I6,5X,F12.5,2X,F12.5,1X,F12.5,
     18X,F12.5)
230   FORMAT (8X,*CEVIATICN ABOUT REGRESSION*,3X,I6,5X,F12.5,2X,F12.5)
240   FORMAT (8X,*TOTAL*,24X,I6,5X,F12.5,//////)
250   FORMAT (5X,17H  NO IMPROVEMENT )
260   FORMAT (5X,27X,18HTABLE OF RESIDUALS//5X,16H OBSERVATION NO.,5X,7H
     1X VALUE,7X,7HY VALUE,7X,10HY ESTIMATE,7X,8HRESIDUAL/)
270   FORMAT (1H1)
280   FORMAT (8X,*MULTIPLE CORRELATION COEFFICIENT*,8X,F12.5,/)
290   FORMAT (8X,*STANDARD ERROR OF ESTIMATE*,14X,F12.5,//)
      END
```

```
C
      SUBROUTINE GDATA
C
C
C        GENERATE INDEPENDENT VARIABLES UP TO THE M-TH POWER (THE
C        HIGHEST DEGREE POLYNOMIAL SPECIFIED) AND COMPUTE MEANS,
C        STANDARD DEVIATIONS, AND CORRELATION COEFFICIENTS.  THIS
C        SUBROUTINE IS NORMALLY CALLED BEFORE SUBROUTINES ORDER,
C        MINV AND MULTR IN THE PERFORMANCE OF A POLYNOMIAL
C        REGRESSION.
C
C     USAGE
C        CALL GDATA (N,M,X,XBAR,STD,D,SUMSQ)
C
C     DESCRIPTION OF PARAMETERS
C        M     - THE HIGHEST DEGREE POLYNOMIAL TO BE FITTED.
C        N     - NUMBER OF OBSERVATIONS.
C        X     - INPUT MATRIX (N BY M+1) .  WHEN THE SUBROUTINE IS
C                CALLED, DATA FOR THE INDEPENDENT VARIABLE ARE
C                STORED IN THE FIRST COLUMN OF MATRIX X, AND DATA FOR
C                THE DEPENDENT VARIABLE ARE STORED IN THE LAST
C                COLUMN OF THE MATRIX.  UPON RETURNING TO THE
C                CALLING ROUTINE, GENERATED POWERS OF THE INDEPENDENT
C                VARIABLE ARE STORED IN COLUMNS 2 THROUGH M.
C        XBAR  - OUTPUT VECTOR OF LENGTH M+1 CONTAINING MEANS OF
C                INDEPENDENT AND DEPENDENT VARIABLES.
C        STD   - OUTPUT VECTOR OF LENGTH M+1 CONTAINING STANDARD
C                DEVIATIONS OF INDEPENDENT AND DEPENDENT VARIABLES.
C        D     - OUTPUT MATRIX (ONLY UPPER TRIANGULAR PORTION OF THE
C                SYMMETRIC MATRIX OF M+1 BY M+1) CONTAINING CORRELA-
C                TION COEFFICIENTS.  (STORAGE MODE OF 1)
C        SUMSQ - OUTPUT VECTOR OF LENGTH M+1 CONTAINING SUMS OF
C                PRODUCTS OF DEVIATIONS FROM MEANS  OF INDEPENDENT
C                AND DEPENDENT VARIABLES.
C
C     REMARKS
```

```
C
C          N MUST BE GREATER THAN M+1.
C          IF M IS EQUAL TO 5 OR GREATER, SINGLE PRECISION MAY NOT BE
C          SUFFICIENT TO GIVE SATISFACTORY COMPUTATIONAL RESULTS.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE
C
C       METHOD
C          REFER TO B. OSTLE, "STATISTICS IN RESEARCH", THE IOWA STATE
C          COLLEGE PRESS, 1954, CHAPTER 6.
C
      SUBROUTINE GDATA (N,M,X,XBAR,STD,D,SUMSQ)
C     DOUBLE PRECISION X,XBAR,STD,D,SUMSQ,T1,T2
      DIMENSION X(1), XBAR(1), STD(1), D(1), SUMSQ(1)
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHCULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT AND ABS IN
C        STATEMENT 180 MUST BE CHANGED TO DSQRT AND DABS.
C
C     GENERATE INDEPENDENT VARIABLES
C
      IF (M-1) 30,30,10
   10 L1=0
      DO 20 I=2,M
      L1=L1+N
      DO 20 J=1,N
```

```
      L=L1+J
      K=L-N
20    X(L)=X(K)*X(J)
C
C     CALCULATE MEANS
C
30    MM=M+1
      DF=N
      L=0
      DO 50 I=1,MM
      XBAR(I)=0.0
      DO 40 J=1,N
      L=L+1
40    XBAR(I)=XBAR(I)+X(L)
50    XBAR(I)=XBAR(I)/DF
C
      DO 60 I=1,MM
60    STD(I)=0.0
C
C     CALCULATE SUMS OF CROSS-FRODUCTS OF DEVIATIONS
C
      L=((MM+1)*MM)/2
      DO 70 I=1,L
70    D(I)=0.0
      DO 80 K=1,N
      L=0
      DO 80 J=1,MM
      L2=N*(J-1)+K
      T2=X(L2)-XBAR(J)
      STD(J)=STD(J)+T2
      DO 80 I=1,J
      L1=N*(I-1)+K
      T1=X(L1)-XBAR(I)
      L=L+1
80    D(L)=D(L)+T1*T2
```

```
      L=0
      DO 90 J=1,MM
      DO 90 I=1,J
      L=L+1
90    D(L)=D(L)-STD(I)*STD(J)/DF
      L=0
      DO 100 I=1,MM
      L=L+I
      SUMSQ(I)=D(L)
100   STD(I)=SQRT(ABS(D(L)))
C
C     CALCULATE CORRELATICN COEFFICIENTS
C
      L=0
      DO 110 J=1,MM
      DO 110 I=1,J
      L=L+1
110   D(L)=D(L)/(STD(I)*STD(J))
C
C     CALCULATE STANDARD DEVIATIONS
C     SUBROUTINE  CORRE
C
C
      DF=SQRT(DF-1.0)
      DO 120 I=1,MM
120   STD(I)=STD(I)/DF
      RETURN
      END
```

```
C     SUBROUTINE ORDER
C
C
C        CONSTRUCT FROM A LARGER MATRIX OF CORRELATION COEFFICIENTS
C        A SUBSET MATRIX OF INTERCORRELATIONS AMONG INDEPENDENT
C        VARIABLES AND A VECTOR OF INTERCORRELATIONS OF INDEPENDENT
C        VARIABLES WITH DEPENDENT VARIABLE.  THIS SUBROUTINE IS
C        NORMALLY USED IN THE PERFORMANCE OF MULTIPLE AND POLYNOMIAL
C
C
C     USAGE
C        CALL ORDER (M,R,NDEP,K,ISAVE,RX,RY)
C
C     DESCRIPTION OF PARAMETERS
C        M     - NUMBER OF VARIABLES AND ORDER OF MATRIX R.
C        R     - INPUT MATRIX CONTAINING CORRELATION COEFFICIENTS.
C                THIS SUBROUTINE EXPECTS ONLY UPPER TRIANGULAR
C                PORTION OF THE SYMMETRIC MATRIX TO BE STORED (BY
C                COLUMN) IN R.  (STORAGE MODE OF 1)
C        NDEP  - THE SUBSCRIPT NUMBER OF THE DEPENDENT VARIABLE.
C        K     - NUMBER OF INDEPENDENT VARIABLES TO BE INCLUDED
C                IN THE FORTHCOMING REGRESSION. K MUST BE GREATER
C                THAN OR EQUAL TO 1.
C        ISAVE - INPUT VECTOR OF LENGTH K+1 CONTAINING, IN ASCENDING
C                ORDER, THE SUBSCRIPT NUMBERS OF K INDEPENDENT
C                VARIABLES TO BE INCLUDED IN THE FORTHCOMING REGRES-
C                SION.
C                UPON RETURNING TO THE CALLING ROUTINE, THIS VECTOR
C                CONTAINS, IN ADDITION, THE SUBSCRIPT NUMBER OF
C                THE DEPENDENT VARIABLE IN K+1 POSITION.
C        RX    - OUTPUT MATRIX (K X K) CONTAINING INTERCORRELATIONS
C                AMONG INDEPENDENT VARIABLES TO BE USED IN FORTH-
C                COMING REGRESSION.
C        RY    - OUTPUT VECTOR OF LENGTH K CONTAINING INTERCORRELA-
C                TIONS OF INDEPENDENT VARIABLES WITH DEPENDENT
C                VARIABLES.
C
```

```
C
C        REMARKS
C           NONE
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           FROM THE SUBSCRIPT NUMBERS OF THE VARIABLES TO BE INCLUDED
C           IN THE FORTHCOMING REGRESSION, THE SUBROUTINE CONSTRUCTS THE
C           MATRIX RX AND THE VECTOR RY.
C
C
      SUBROUTINE ORDER (M,R,NDEP,K,ISAVE,RX,RY)
      DIMENSION R(1), ISAVE(1), RX(1), RY(1)
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION R,RX,RY
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C     COPY INTERCORRELATIONS OF INDEPENDENT VARIABLES
C     WITH DEPENDENT VARIABLE                  9
C
      MM=0
      DO 70 J=1,K
      L2=ISAVE(J)
      IF (NDEP-L2) 10,20,20
   10 L=NDEP+(L2*L2-L2)/2
      GO TO 30
   20 L=L2+(NDEP*NDEP-NDEP)/2
```

```
30    RY(J)=R(L)
C
C     COPY A SUBSET MATRIX OF INTERCORRELATIONS AMONG
C     INDEPENDENT VARIABLES
C
      DO 70 I=1,K
      L1=ISAVE(I)
      IF (L1-L2) 40,50,50
40    L=L1+(L2*L2-L2)/2
      GO TO 60
50    L=L2+(L1*L1-L1)/2
60    MM=MM+1
70    RX(MM)=R(L)
C
C     PLACE THE SUBSCRIPT NUMBER OF THE DEPENDENT
C     VARIABLE IN ISAVE(K+1)
C
      ISAVE(K+1)=NDEP
      RETURN
      END
```

```
C     SUBROUTINE MINV
C
C
C        INVERT A MATRIX
C
C     USAGE
C        CALL MINV(A,N,D,L,M)
C
C     DESCRIPTION OF PARAMETERS
C        A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY
C            RESULTANT INVERSE.
C        N - ORDER OF MATRIX A
C        D - RESULTANT DETERMINANT
C        L - WORK VECTCR OF LENGTH N
C        M - WORK VECTOR OF LENGTH N
C
C     REMARKS
C        MATRIX A MUST BE A GENERAL MATRIX
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C        NONE
C
C     METHOD
C        THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT
C        IS ALSC CALCULATED. A DETERMINANT OF ZERO INDICATES THAT
C        THE MATRIX IS SINGULAR.
C
      SUBROUTINE MINV(AA,N,DD,L,M)
      DOUBLE PRECISION A,D,BIGA,HOLD
      DIMENSION A(100),L(1),M(1)
      DIMENSION AA(1)
      NN = N**2
      DO 5 K = 1,NN
    5 A(K) = AA(K)
C
```

```
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION A,C,BIGA,HOLD
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  ABS IN STATEMENT
C        10 MUST BE CHANGED TO DABS.
C
C        SEARCH FOR LARGEST ELEMENT
C
      D=1.0
      NK=-N
      DO 180 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
      IF (DABS(BIGA) - DABS(A(IJ))) 10,20,20
   10 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
C
C        INTERCHANGE ROWS
```

```
C
      J=L(K)
      IF (J-K) 50,50,30
30    KI=K-N
      DO 40 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
40    A(JI)=HOLD
C
C        INTERCHANGE COLUMNS
C
50    I=M(K)
      IF (I-K) 80,80,60
60    JP=N*(I-1)
      DO 70 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
70    A(JI)=HOLD
C
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C        CONTAINED IN BIGA)
C
80    IF (BIGA) 100,90,100
90    D=0.0
      RETURN
100   DO 120 I=1,N
      IF (I-K) 110,120,110
110   IK=NK+I
      A(IK)=A(IK)/(-BIGA)
120   CONTINUE
C
```

```
C           REDUCE MATRIX
C
        DO 150 I=1,N
        IK=NK+I
        HOLD=A(IK)
        IJ=I-N
        DO 150 J=1,N
        IJ=IJ+N
        IF (I-K) 130,150,130
  130   IF (J-K) 140,150,140
  140   KJ=IJ-I+K
        A(IJ)=HOLD*A(KJ)+A(IJ)
  150   CONTINUE
C
C           DIVIDE ROW BY PIVOT
C
        KJ=K-N
        DO 170 J=1,N
        KJ=KJ+N
        IF (J-K) 160,170,160
  160   A(KJ)=A(KJ)/BIGA
  170   CONTINUE
C
C           PRODUCT OF PIVOTS
C
        D=D*BIGA
C
C           REPLACE PIVOT BY RECIPROCAL
C
        A(KK)=1.0/BIGA
  180   CONTINUE
C
C           FINAL ROW AND COLUMN INTERCHANGE
C
        K=N
```

```
190     K=(K-1)
        IF (K) 260,260,200
200     I=L(K)
        IF (I-K) 230,230,210
210     JQ=N*(K-1)
        JR=N*(I-1)
        DO 220 J=1,N
        JK=JQ+J
        HOLD=A(JK)
        JI=JR+J
        A(JK)=-A(JI)
220     A(JI)=HOLD
230     J=M(K)
        IF (J-K) 190,190,240
240     KI=K-N
        DO 250 I=1,N
        KI=KI+N
        HOLD=A(KI)
        JI=KI-K+J
        A(KI)=-A(JI)
250     A(JI)=HOLD
        GO TO 190
    260 DO 270 K = 1,NN
    270 AA(K) = A(K)
        DD = D
        RETURN
        END
```

```
C
C     SUBROUTINE MULTR
C
C
C        PERFORM A MULTIPLE LINEAR REGRESSION ANALYSIS FOR A
C        DEPENDENT VARIABLE AND A SET OF INDEPENDENT VARIABLES.  THIS
C        SUBROUTINE IS NORMALLY USED IN THE PERFORMANCE OF MULTIPLE
C        AND POLYNOMIAL REGRESSION ANALYSES.
C
C     USAGE
C        CALL MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)
C
C     DESCRIPTION OF PARAMETERS
C        N     - NUMBER OF OBSERVATIONS.
C        K     - NUMBER OF INDEPENDENT VARIABLES IN THIS REGRESSION.
C        XBAR  - INPUT VECTOR OF LENGTH M CONTAINING MEANS OF ALL
C                VARIABLES. M IS NUMBER OF VARIABLES IN OBSERVATIONS.
C        STD   - INPUT VECTOR OF LENGTH M CONTAINING STANDARD DEVI-
C                ATIONS OF ALL VARIABLES.
C        D     - INPUT VECTOR OF LENGTH M CONTAINING THE DIAGONAL OF
C                THE MATRIX OF SUMS OF CROSS-PRODUCTS OF DEVIATIONS
C                FROM MEANS FOR ALL VARIABLES.
C        RX    - INPUT MATRIX (K X K) CONTAINING THE INVERSE OF
C                INTERCORRELATIONS AMONG INDEPENDENT VARIABLES.
C        RY    - INPUT VECTOR OF LENGTH K CONTAINING INTERCORRELA-
C                TIONS OF INDEPENDENT VARIABLES WITH DEPENDENT
C                VARIABLE.
C        ISAVE - INPUT VECTOR OF LENGTH K+1 CONTAINING SUBSCRIPTS OF
C                INDEPENDENT VARIABLES IN ASCENDING ORDER.  THE
C                SUBSCRIPT OF THE DEPENDENT VARIABLE IS STORED IN
C                THE LAST, K+1, POSITION.
C        B     - OUTPUT VECTOR OF LENGTH K CONTAINING REGRESSION
C                COEFFICIENTS.
C        SB    - OUTPUT VECTOR OF LENGTH K CONTAINING STANDARD
C                DEVIATIONS OF REGRESSION COEFFICIENTS.
C        T     - OUTPUT VECTOR OF LENGTH K CONTAINING T-VALUES.
```

```
C       ANS    - OUTPUT VECTOR OF LENGTH 10 CONTAINING THE FOLLOWING
C                INFORMATION..
C                ANS(1)  INTERCEPT
C                ANS(2)  MULTIPLE CORRELATION COEFFICIENT
C                ANS(3)  STANDARD ERROR OF ESTIMATE
C                ANS(4)  SUM OF SQUARES ATTRIBUTABLE TO REGRES-
C                        SION (SSAR)
C                ANS(5)  DEGREES OF FREEDOM ASSOCIATED WITH SSAR
C                ANS(6)  MEAN SQUARE OF SSAR
C                ANS(7)  SUM OF SQUARES OF DEVIATIONS FROM REGRES-
C                        SION (SSDR)
C                ANS(8)  DEGREES OF FREEDOM ASSOCIATED WITH SSDR
C                ANS(9)  MEAN SQUARE OF SSDR
C                ANS(10) F-VALUE
C
C     REMARKS
C        N MUST BE GREATER THAN K+1.
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C        NONE
C
C     METHOD
C        THE GAUSS-JORDAN METHOD IS USED IN THE SOLUTION OF THE
C        NORMAL EQUATICNS.  REFER TO W. W. COOLEY AND P. R. LOHNES,
C        "MULTIVARIATE PROCEDURES FOR THE BEHAVIORAL SCIENCES",
C        JOHN WILEY ANC SONS, 1962, CHAPTER 3, AND B. OSTLE,
C        "STATISTICS IN RESEARCH", THE IOWA STATE COLLEGE PRESS,
C        1954, CHAPTER 8.
C
      SUBROUTINE MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)
      DIMENSION XBAR(1), STD(1), D(1), RX(1), RY(1), ISAVE(1), B(1), SB(
     11), T(1), ANS(1)
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
```

```
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION XBAR,STD,D,RX,RY,B,SB,T,ANS,RM,BO,SSAR,SSDR,SY,FN
C    1,FK,SSARM,SSDRM,F
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT AND ABS IN
C        STATEMENTS 122, 125, AND 135 MUST BE CHANGED TO DSQRT AND DABS.
C
      MM=K+1
C
C        BETA WEIGHTS
C
      DO 10 J=1,K
   10 B(J)=0.0
      DO 20 J=1,K
      L1=K*(J-1)
      DO 20 I=1,K
      L=L1+I
   20 B(J)=B(J)+RY(I)*RX(L)
      RM=0.0
      BO=0.0
      L1=ISAVE(MM)
C
C        COEFFICIENT OF DETERMINATION
C
      DO 30 I=1,K
      RM=RM+B(I)*RY(I)
C
C        REGRESSION COEFFICIENTS
C
```

```
      L=ISAVE(I)
      B(I)=B(I)*(STD(L1)/STD(L))
C
C        INTERCEPT
C
30    BO=BO+B(I)*XBAR(L)
      BO=XBAR(L1)-BO
C
C        SUM OF SQUARES ATTRIBUTABLE TO REGRESSION
C
      SSAR=RM*D(L1)
C
C        MULTIPLE CORRELATION COEFFICIENT
C
      RM=SQRT(ABS(RM))
C
C        SUM OF SQUARES OF DEVIATIONS FROM REGRESSION
C
      SSDR=D(L1)-SSAR
C
C        VARIANCE OF ESTIMATE
C
      FN=N-K-1
      SY=SSDR/FN
C
C        STANDARD DEVIATICNS OF REGRESSION COEFFICIENTS
C
      DO 40 J=1,K
      L1=K*(J-1)+J
      L=ISAVE(J)
      SB(J)=SQRT(ABS((RX(L1)/D(L))*SY))
C
C        COMPUTED T-VALUES
C
40    T(J)=B(J)/SB(J)
```

```
C
C      STANDARD  ERROR OF ESTIMATE
C
      SY=SQRT(ABS(SY))
C
C       F  VALUE
C
      FK=K
      SSARM=SSAR/FK
      SSDRM=SSDR/FN
      F=SSARM/SSDRM
C
      ANS(1)=BO
      ANS(2)=RM
      ANS(3)=SY
      ANS(4)=SSAR
      ANS(5)=FK
      ANS(6)=SSARM
      ANS(7)=SSDR
      ANS(8)=FN
      ANS(9)=SSDRM
      ANS(10)=F
      RETURN
      END
```

```
C     SUBROUTINE VARIAT
C
      SUBROUTINE VARIAT (N,N1,X,OVAR)
C
      COMMON /BK/ TITLE(80),JO,TITLE1(8)
      DIMENSION VAR(10), XVAR(10), XBAR(10), X(1)
C
      XN=N
      SUM1=0.0
      SUM2=0.0
      DO 10 J=1,N
      SUM1=SUM1+X(J)
      SUM2=SUM2+X(J)**2
   10 CONTINUE
      OVAR=(SUM2/XN)-(SUM1/XN)**2
      DO 50 K=1,N1
      N2=N-K
      XN=N2
      SUM1=0.0
      SUM2=0.0
      DO 20 J=1,N2
      X(J)=X(J+1)-X(J)
      SUM1=SUM1+X(J)
      SUM2=SUM2+X(J)**2
   20 CONTINUE
      XBAR(K)=SUM1/XN
      XVAR(K)=SUM2/XN
      IF (XVAR(K).LE.0.0) GO TO 60
      Z1=1.0
      K2=2*K
      DO 30 L=1,K2
      XL=L
      Z1=Z1*XL
   30 CONTINUE
      Z2=1.0
```

```
      DO 40 L=1,K
      XL=L
      Z2=Z2*XL
40    CONTINUE
      Z3=Z1/(Z2**2)
      VAR(K)=XVAR(K)/Z3
50    CONTINUE
60    WRITE (JO,80)
      WRITE (JO,90) TITLE
      WRITE (JO,100)
      WRITE (JO,110)
      DO 70 K=1,N1
      WRITE (JO,120) K,XBAR(K),XVAR(K), VAR(K)
70    CONTINUE
      WRITE (JO,130) OVAR
      RETURN
C
80    FORMAT (1H1)
90    FORMAT (//,9X,80A1)
100   FORMAT (///,44X,28HVARIATE DIFFERENCE TECHNIQUE,/)
110   FORMAT(29X,"DIFFERENCE      MEAN          VARIANCE       VARIANCE",/,
     131X"NUMBER          OF              OF           DIVIDED BY",/,33X"R
     2    DIFFERENCES   DIFFERENCES      ( 2R )",/,72X,"(  R )",//)
120   FORMAT (32X,I2,7X,3(E12.5,2X),//)
130   FORMAT (34X,"VARIANCE OF ORIGINAL SERIES IS ",E12.5,//)
      END
```

```
C     SUBROUTINE  STUMM
C
      SUBROUTINE STUMM (N,N1,N2,Y,RVAR,TEMP)
C     PROGRAM TO USE STUMMS METHOD TO DERIVE THE PERIODIC COMPONENTS
C     OF THE MEAN AND VARIANCE OF A TIME SERIES
C
C     INPUT IS
C
C     Y     EQUALLY SPACED DISCRETE OBSERVATIONS, USUALLY MONTHLY,
C     REPLACED WITHIN THE SUBROUTINE BY AN ARRAY OF RESIDUALS,FOLLOWING
C     REMOVAL OF ALL HARMCNICS OF THE BASE PERIOD.
C     N     BASIC PERIOD FOR WHICH HARMONICS ARE TO BE COMPUTED
C           USUALLY 12 MONTHS
C     N1    NUMBER OF HARMONICS TO BE COMPUTED, FOR 12 MONTHS THERE IS A
C           MAXIMUM OF 6 HARMONICS
C     N2    TOTAL NUMBER CF INPUT OBSERVATIONS
C     THE PROGRAM IS DIMENSIONED FOR UP TO 12 HARMONICS
C
C     RVAR VARIANCE OF RESIDUALS(AFTER REMOVAL OF PERIODIC COMPONENTS)
C
      COMMON /BK/ TITLE(80),JO,TITLE1(8)
      DIMENSION X(12), A(12), B(12), C(12), F(12), VAR(12), THETA(12), P
     1(12), SD(12), XMON(12), PTEST(12), INDX(12)
      DIMENSICN Y(1), TEMF(1)
      XMON(1) =3HJAN
      XMON(2) =3HFEE
      XMON(3) =3HMAR
      XMON(4) =3HAPR
      XMON(5) =3HMAY
      XMON(6) =3HJUN
      XMON(7) =3HJUL
      XMON(8) =3HAUG
      XMON(9) =3HSEP
      XMON(10)=3HOCT
      XMON(11)=3HNOV
```

```
      XMON(12)=3HDEC
      IF (N.NE.12) WRITE (JO,490)
      FISHER=0.61615
      CALL STANDX (N2,Y,A1,A2)
      TV=A2**2.0
      PI=(ATAN(1.0))*4.0
      XN=N
      XN1=N1
      NPRD=N2/N
      DO 20 K=1,N
      IC=K
      DO 10 J=1,NPRD
      TEMP(J) =Y(IC)
      IC=IC+N
10    CONTINUE
      CALL STANDX (NPRD,TEMP,A1,A2)
      X(K)=A1
      SD(K) =A2
20    CONTINUE
      DO 40 K=1,N
      I=K
      DO 30 J=1,NPRD
      Y(I)=(Y(I)-X(K))/SD(K)
30    I=K+J*N
40    CONTINUE
      SUM1=0.0
      SUM2=0.0
      DO 50 K=1,N
      SUM1=SUM1+X(K)
      SUM2=SUM2+SD(K)
50    CONTINUE
      XBAR=SUM1/XN
      SDBAR=SUM2/XN
      SUM1=0.0
      SUM2=0.0
```

```
      DO 60 K=1,N
      SUM1=SUM1+(X(K)-XBAR)**2
      SUM2=SUM2+(SD(K)-SDBAR)**2
60    CONTINUE
      VARM=SUM1/XN
      VARSD=SUM2/XN
      VAR1=VARM
      ICOUNT=0
70    ICOUNT=ICOUNT+1
      SUM=0.0
      DO 90 J=1,N1
      XJ=J
      SUM1=0.0
      SUM2=0.0
      DO 80 K=1,N
      XK=K
      TEMP1=COS(2.0*PI*XJ*XK/XN)
      TEMP2=SIN(2.0*PI*XJ*XK/XN)
      TEMP1=TEMP1*(X(K)-XBAR)
      TEMP2=TEMP2*(X(K)-XBAR)
      SUM1=SUM1+TEMP1
      SUM2=SUM2+TEMP2
80    CONTINUE
      A(J)=(2.0/XN)*SUM1
      B(J)=(2.0/XN)*SUM2
      C(J)=A(J)**2+B(J)**2
      F(J)=XN/XJ
      THETA(J)=ATAN(-A(J)/B(J))
      VAR(J)=C(J)/2.0
      SUM=SUM+VAR(J)
90    CONTINUE
      P1=(VARM*100.0)/TV
      P2=(VARSD*100.0)/TV
      P3=100.0-P1-P2
      DO 100 J=1,N1
```

```
      P(J)=VAR(J)*100.0/SUM
100   CONTINUE
      SPTE=0.0
      DO 110 K=1,N1
110   SPTE=SPTE+C(K)
      DO 120 K=1,N1
      PTEST(K)=C(K)/SPTE
      INDX(K)=3H NO
120   IF (PTEST(K).GT.FISHER) INDX(K)=3HYES
      IF (ICOUNT.GT.1) GO TO 140
      WRITE (JO,290)
      WRITE (JO,300)
      WRITE (JO,320) TITLE
      WRITE (JO,310)
      DO 130 J=1,12
130   WRITE (JO,480) XMON(J),X(J),SD(J)
      WRITE (JO,290)
      WRITE (JO,300)
      WRITE (JO,320) TITLE
      WRITE (JO,350)
      WRITE (JO,360) TV
      WRITE (JO,370) VARM
      WRITE (JO,380) VARSC
      WRITE (JO,390) P1
      WRITE (JO,400) P2
      WRITE (JO,410) P3
140   CONTINUE
      WRITE (JO,290)
      WRITE (JO,300)
      WRITE (JO,320) TITLE
      GO TO (150,160), ICCUNT
150   WRITE (JO,440)
      GO TO 170
160   WRITE (JO,450)
170   CONTINUE
```

```
      WRITE (JO,330)
      DO 180 J=1,N1
      WRITE (JO,340) J,A(J),B(J),C(J),F(J),THETA(J)
180   CONTINUE
      WRITE (JO,300)
      GO TO (190,200), ICCUNT
190   WRITE (JO,460)
      GO TO 210
200   WRITE (JO,470)
210   CONTINUE
      GO TO (220,230), ICCUNT
220   WRITE (JO,420)
      GO TO 240
230   WRITE (JO,430)
240   CONTINUE
      DO 250 J=1,N1
      WRITE (JO,500) J,VAR(J),P(J),INDX(J)
250   CONTINUE
      IF (ICOUNT.GT.1) GO TO 270
      DO 260 K=1,N
      X(K)=SD(K)
260   CONTINUE
      XBAR=SDBAR
      VAR1=VARSD
      GO TO 70
270   CONTINUE
      RVAR=(P3*TV/100.0)
      DO 280 I=1,N2
280   Y(I)=Y(I)*SQRT(RVAR)
      WRITE (JO,290)
      RETURN
C
290   FORMAT (1H1)
300   FORMAT (/////)                                                    DEV
310   FORMAT (47X,29HMONTH     LONG TERM  STANDARD,/,59X,18HMEAN
```

```
                                                                              M1760
                                                                              M1770
     1IATION)                                                                 A1530
320   FORMAT (9X,80A1,//)                                        PERIOD
330   FORMAT (7X,"HARMONIC       A         B        C SQUARED                 M1790
     1  PHASE",//)
340   FORMAT (9X,I2,4X,5(1X,E10.4),/)                                         A157
350   FORMAT (25X,"ANALYSIS OF VARIANCE",//)
360   FORMAT (5X,"TOTAL VARIANCE OF THE PERIODIC SERIES",18X,E10.4,/)
370   FORMAT (5X,"VARIANCE OF THE MEANS",34X,E10.4,/)                         A1590
380   FORMAT (5X,"VARIANCE OF THE STANDARD DEVIATIONS",20X,E10.4,/)           A1600
390   FORMAT (5X,"PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED",/,5X
     1,"BY THE VARIANCE OF THE MEANS",27X,E10.4,/)                            A1620
400   FORMAT (5X,"PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED",/,5X
     1,"BY THE VARIANCE OF THE STANDARD DEVIATIONS",13X,E10.4,/)              A1640
410   FORMAT (5X,"PERCENT OF THE TOTAL VARIANCE WHICH IS NOT EXPLAINED",      A1650
     1/,5X,"BY THE VARIANCE IN THE MEAN AND STANDARD DEVIATION",5X,E10.4
     2,//)                                                                    M1900
420   FORMAT (7X,64HHARMONIC          VARIANCE        PERCENT OF TOTAL        M1910
     1 SIGNIFICANT,/,39X,36HVARIANCE IN THE        AT 95 PER CENT,/,43X,5     M1920
     2HMEANS,12X,13HCONFICENCE L.,/,60X,13H(FISHER TEST),//)                  M1930
430   FORMAT (7X,64HHARMONIC          VARIANCE        PERCENT OF TOTAL        M1940
     1 SIGNIFICANT,/,39X,36HVARIANCE IN THE        AT 95 PER CENT,/,39X,3     M1950
     25HSTANDARD DEVIATION     CONFIDENCE L.,/,60X,13H(FISHER TEST),//)
440   FORMAT (20X,"PERIODOGRAM COMPONENTS FOR THE MEAN",//)                   A1710
450   FORMAT (13X,"PERIODOGRAM COMPONENTS FOR THE STANDARD DEVIATION",//
     1)                                                                       A1740
460   FORMAT (22X,"HARMONIC VARIANCES IN THE MEAN",//)                        M2000
470   FORMAT (16X,"HARMONIC VARIANCES IN THE STANDARD DEVIATIONS",//)         M2010
480   FORMAT (48X,A3,5X,F10.5,2X,F10.5)                                       M2020
490   FORMAT (1H0,35HNOTE. FISHER TEST HAS TO BE CHANGED)                     M2030-
500   FORMAT (9X,I2,10X,E10.4,8X,E10.4,15X,A3)
      END
```

```
C     SUBROUTINE  MARKOV
C
      SUBROUTINE MARKOV (N,X,IFLAG,XR,RES,RE2)
C
C     SUBROUTINE TO INVESTIGATE THE HYPOTHESES THAT A STATIONARY TIME
C     SERIES MAY BELONG TC EITHER A FIRST ORDER LINEAR MARKOV OR A
C     SECOND ORDER LINEAR MARKOV MODEL
C
C     SAMPLE AND THEORETICAL SERIAL CORRELATION COEFFICIENTS ARE
C     COMPUTED AND CONFIDENCE LIMITS AT 5 AND 1 PERCENT
C
C     CHI-SQUARE VALUES ARE COMPUTED AND COMPARED TO THE TABULATED
C     VALUES
C
C     N = SAMPLE SIZE
C     X = SAMPLE OF STATICNARY TIME SERIES, REPLACED BY RESIDUALS FROM
C     FIRST OR SECOND ORDER MARKOV PROCESS
C     IFLAG - VARIABLE RETURNED TO MAIN PROGRAM TO  INDICATE WHETHER
C           FIRST OR SECOND ORDER MARKOV IS A BETTER FIT.
C
C     REFERENCES ..
C
C     (A) CONFIDENCE LIMITS
C         ANDERSON, R.L., DISTRIBUTIONS OF THE SERIAL CORRELATION
C         COEFFICIENTS  ANN. MATH. STAT. VOL.8, NO.1,1-13, 1941
C         YEVEVICH, V.M., FLUCTUATION OF WET AND DRY YEARS,
C         HYDROLOGY PAPER NO.4, COLORADO STATE UNIVERSITY
C     (B) CHI-SQUARE TEST
C         MATALAS, N.C., TIME SERIES ANALYSIS, WATER RESOURCES RESEARCH
C         VOL.3, NO.3, 817-829, 1967
C
      INTEGER RDR,PTR,DF1,DF2
      COMMON /BK/ TITLE(80),PTR,TITLE1(8)
      DIMENSION X(1), XR(1), RES(1), RE2(1)
      DIMENSION RN(10), RHO1(10), RHO2(10), XBAR(2), STD(2), RX(4), RY(4
```

```
     1), B(2), D(2), T(2)
C
C     PRINT HEADINGS
C
      WRITE (PTR,190) TITLE
C
C     COMPUTE INITIAL VARIANCE
C
      CALL STANDX (N,X,EXEAR,XSTD)
      OVAR=XSTD*XSTD
C
C     COMPUTE SAMPLE AUTOCORRELATION COEFFICIENTS RN(K)
C
      DO 20 K=1,10
      N1=N-K
      DO 10 J=1,N1
      N2=J+K
      XR(J)=X(J)
      XR(N1+J)=X(N2)
   10 CONTINUE
      CALL CORRE (N1,2,XR,XBAR,STD,RX,RY,B,D,T)
      RN(K)=RY(2)
   20 CONTINUE
C
C     COMPUTE THEORETICAL AUTOCORRELATION COEFFICIENTS FOR A FIRST
C     ORDER LINEAR MARKOV (AUTOREGRESSIVE) MODEL
C
      ONE=RN(1)
      DO 30 K=1,10
      RHO1(K)=ONE**K
   30 CONTINUE
C
C     COMPUTE THEORETICAL AUTOCORRELATION COEFFICIENTS FOR A SECOND
C     ORDER LINEAR MARKOV (AUTOREGRESSIVE) MODEL
C
```

```
      B12=(RN(1)-RN(1)*RN(2))/(1.0-RN(1)**2)
      B13=(RN(2)-RN(1)**2)/(1.0-RN(1)**2)
      RHO2(1)=RN(1)
      RHO2(2)=RN(2)
      DO 40 K=3,10
      RHO2(K)=B12*RHO2(K-1)+B13*RHO2(K-2)
   40 CONTINUE
C
C     COMPUTE THE CHI-SQUARE STATISTIC FOR FIRST ORDER LINEAR
C     MARKOV MODEL
C
      CHI1=0.0
      DO 50 K=1,9
      TEMP=1.0
      IF (K.NE.1)  TEMP=RN(K-1)
      R=RN(K+1)-2.0*RN(1)*RN(K)+RN(1)**2*TEMP
      R=R**2
      CHI1=CHI1+R
   50 CONTINUE
      SIGMA=(1.0/N)*((1.0-RN(1)**2)**2)
      CHI1=CHI1/SIGMA
      DF1=9
C
C     COMPUTE THE CHI-SQUARE STATISTIC FOR SECOND ORDER LINEAR
C     MARKOV MODEL
C
      CHI2=0.0
      DO 90 K=1,8
      IF (K.NE.1)  GO TO 60
      TEMP1=1.0
      TEMP2=0.0
      GO TO 80
   60 IF (K.NE.2)  GO TO 70
      TEMP1=RN(1)
      TEMP2=1.0
```

```
      GO TO 80
70    TEMP1=RN(K-1)
      TEMP2=RN(K-2)
80    CONTINUE
      R=RN(K+2)-2.0*B12*RN(K+1)+(B12**2-2.0*B13)*RN(K)+2.0*B12*B13*TEMP1
     1+B13**2*TEMP2
      R=R**2
      CHI2=CHI2+R
90    CONTINUE
      SIGMA=(1.0/N)*((1.0+B12**2-B13**2-2.0*B12*RN(1))**2+2.0*(RN(1)-B12
     1-B13*RN(1))**2)
      CHI2=CHI2/SIGMA
      DF2=8
C
C     PRINT THEORETICAL AND SAMPLE SERIAL CORRELATION COEFFICIENTS
C
      WRITE (PTR,200)
      DO 100 K=1,10
      WRITE (PTR,210) K,RN(K),RHO1(K),RHO2(K)
100   CONTINUE
C
C     PRINT CONFIDENCE LIMITS ON SERIAL CORRELATION COEFFICIENTS
C
      WRITE (PTR,180)
      WRITE (PTR,220)
      DO 110 K=1,10
      TEMP1=2.326*(N-K-1)**0.5
      TEMP2=1.645*(N-K-1)**0.5
      XU1=(-1.0+TEMP1)/(N-K)
      XU5=(-1.0+TEMP2)/(N-K)
      XL5=(-1.0-TEMP2)/(N-K)
      XL1=(-1.0-TEMP1)/(N-K)
      WRITE (PTR,230) K,XL1,XU5,RN(K),XL5,XL1
110   CONTINUE
C
```

```
C     DETERMINATION OF HYFOTHESES
C
      C1=1.646
      C2=2.733
      C3=15.507
      C4=20.090
      C5=2.088
      C6=3.325
      C7=16.919
      C8=21.666
C
C     COMPUTE RESIDUALS
C
      NONE=N-1
      DO 120 J=2,N
  120 RES(J-1)=X(J)-RHO1(1)*X(J-1)
      CALL STANDX (NONE,RES,EXBAR,XSTD)
      RVAR1=XSTD*XSTD
      NTWO=N-2
      DO 130 J=3,N
  130 RE2(J-2)=X(J)-RHO2(1)*X(J-1)-RHO2(2)*X(J-2)
      CALL STANDX (NTWO,RE2,EXBAR,XSTD)
      RVAR2=XSTD*XSTD
      PCENT1=(OVAR-RVAR1)*100.0/OVAR
      PCENT2=(OVAR-RVAR2)*100.0/OVAR
C
C     FINAL OUTPUT
C
      IF (PCENT1.LT.PCENT2) GO TO 150
      DO 140 J=1,NONE
  140 X(J)=RES(J)
      IFLAG=1
      GO TO 170
  150 DO 160 J=1,NTWO
  160 X(J)=RE2(J)
```

```
      IFLAG=2
  170 CONTINUE
      WRITE (PTR,190) TITLE
      WRITE (PTR,240)
      WRITE (PTR,250) CHI1
      WRITE (PTR,270) DF1
      WRITE (PTR,280) C5,C6,C7,C8
      WRITE (PTR,300) OVAR,RVAR1
      WRITE (PTR,290) PCENT1
      WRITE (PTR,260) CHI2
      WRITE (PTR,270) DF2
      WRITE (PTR,280) C1,C2,C3,C4
      WRITE (PTR,310) OVAR,RVAR2
      WRITE (PTR,290) PCENT2
C
C     INPUT / OUTPUT FORMATS
C
      RETURN
C
  180 FORMAT (////)
  190 FORMAT (1H1,//,9X,80A1,/)
  200 FORMAT (20X,54HTHEORETICAL AND SAMPLE SERIAL CORRELATION COEFFICIE
     1NTS,//,25X,9H(1) ORDER,/,25X,41H(2) SAMPLE SERIAL CORRELATION COEF
     2FICIENT,/,25X,41H(3) THEORETICAL FIRST ORDER LINEAR MARKOV,/,29X,3
     31HSERIAL CORRELATION COEFFICIENTS,/,25X,42H(4) THEORETICAL SECOND
     4ORDER LINEAR MARKOV,/,29X,31HSERIAL CORRELATION COEFFICIENTS,//,22
     5X,3H(1),13X,3H(2),11X,3H(3),11X,3H(4),/)
  210 FORMAT (22X,I3,7X,F10.3,4X,F10.3,4X,F10.3)
  220 FORMAT (20X,52HCONFIDENCE LIMITS ON SERIAL CORRELATION COEFFICIENT
     1S,//,25X,9H(1) ORDER,/,25X,22H(2) 1 PCT. UPPER LIMIT,/,25X,22H(3)
     25 PCT. UPPER LIMIT,/,25X,41H(4) SAMPLE SERIAL CORRELATION COEFFICI
     3ENT,/,25X,22H(5) 5 PCT. LOWER LIMIT,/,25X,22H(6) 1 PCT. LOWER LIMI
     4T,//,22X,3H(1),7X,3H(2),5X,3H(3),5X,3H(4),5X,3H(5),5X,3H(6),/)
  230 FORMAT (22X,I3,3X,5F8.3)
  240 FORMAT (20X,56HRESULT OF CHI-SQUARE TEST AGAINST FIRST AND SECOND
```

```
      1ORDER,/,20X,27HLINEAR MARKOV DISTRIBUTIONS,//)                   N2110
250    FORMAT (20X,36H(A) FIRST ORDER LINEAR MARKOV MODEL ,/,24X,31HCOMPU N2120
      1TED VALUE OF CHI-SQUARE IS,9X,F10.5,/)                            N2130
260    FORMAT (20X,36H(B) SECOND ORDER LINEAR MARKOV MODEL,/,24X,31HCOMPU N2140
      1TED VALUE OF CHI-SQUARE IS,9X,F10.5,/)                            N2150
270    FORMAT (24X,22HDEGREES OF FREEDOM ARE,25X,I3,///)                 N2160
280    FORMAT (24X,34HTABULATED VALUES OF CHI-SQUARE ARE,//,25X,6H99 PCT, N2170
      15X,6H95 PCT,6X,5H5 PCT,6X,5H1 PCT,//,20X,4(4X,F7.3),////)         N2180
290    FORMAT (24X,38HPERCENT OF TOTAL VARIANCE EXPLAINED IS,15X,F5.2,/// N2190
      1//)                                                               N2200
300    FORMAT (24X,14HTOTAL VARIANCE,34X,F10.3,/,24X,43HVARIANCE NOT EXPL N2210
      1AINED BY 1ST ORCER MARKOV,6X,F9.3,/)                              N2220
310    FORMAT (24X,14HTOTAL VARIANCE,34X,F10.3,/,24X,43HVARIANCE NOT EXPL N2230
      1AINED BY 2ND ORDER MARKOV,6X,F9.3,/)                              N2240
       END                                                               N2250-
```

```
      SUBROUTINE CORRE (N,M,X,XBAR,STD,RX,R,B,D,T)
C
C     COMPUTE MEANS, STANDARD DEVIATIONS, SUMS OF CROSS-PRODUCTS OF
C     DEVIATIONS, AND CORRELATION COEFFICIENTS
C
C     CALL CORRE (N,M,X,XBAR,STD,RX,R,B,D,T)
C
C     N    = NUMBER OF OBSERVATIONS (MUST BE GE 2)
C     M    = NUMBER OF VARIABLES (MUST BE GE 1)
C     X    = ARRAY CONTAINING DATA
C     XBAR = OUTPUT VECTOR OF LENGTH M CONTAINING MEANS
C     STD  = OUTPUT VECTOR OF LENGTH M CONTAINING STANDARD DEVIATIONS
C     RX   = OUTPUT MATRIX (M * M) CONTAINING SUMS OF CROSS-PRODUCTS
C            OF DEVIATIONS FROM MEANS
C     R    = OUTPUT MATRIX (ONLY UPPER TRIANGULAR PORTION OF THE
C            SYMMETRIC MATRIX OF M BY M) CONTAINING CORRELATION
C            COEFFICIENTS (STORAGE MODE OF 1)
C     B    = OUTPUT VECTOR OF LENGTH M CONTAINING THE DIAGONAL OF THE
C            MATRIX OF SUMS OF CROSS-PRODUCTS OF DEVIATIONS FROM MEANS
C
      DIMENSION X(1), XBAR(1), STD(1), RX(1), R(1), B(1), D(1), T(1)
C
C     IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C     C IN COLUMN 1 MUST BE REMOVED FROM THE DOUBLE PRECISION STATEMENT
C
C     DOUBLE PRECISION XBAR,STD,RX,R,B,T
C
C     THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C     APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS  ROUTINE
C
C     THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C     CONTAIN DOUBLE PRECISION FUNCTIONS DSQRT AND DABS
C
      DO 10 J=1,M
      B(J)=0.0
```

```
      T(J)=0.0
10    CONTINUE
      K=(M*M+M)/2
      DO 20 I=1,K
      R(I)=0.0
20    CONTINUE
      FN=N
      L=0
      DO 40 J=1,M
      DO 30 I=1,N
      L=L+1
      T(J)=T(J)+X(L)
30    CONTINUE
      XBAR(J)=T(J)
      T(J)=T(J)/FN
40    CONTINUE
      DO 60 I=1,N
      JK=0
      L=I-N
      DO 50 J=1,M
      L=L+N
      D(J)=X(L)-T(J)
      B(J)=B(J)+D(J)
50    CONTINUE
      DO 60 J=1,M
      DO 60 K=1,J
      JK=JK+1
      R(JK)=R(JK)+D(J)*D(K)
60    CONTINUE
C
C     CALCULATE MEANS
C
      JK=0
      DO 70 J=1,M
      XBAR(J)=XBAR(J)/FN
```

```
C
C     ADJUST SUMS OF CROSS-PRODUCTS OF DEVIATIONS FROM TEMPORARY MEANS
C
      DO 70 K=1,J
      JK=JK+1
      R(JK)=R(JK)-B(J)*B(K)/FN
   70 CONTINUE
C
C     CALCULATE CORRELATICN COEFFICIENTS
C
      JK=0
      DO 80 J=1,M
      JK=JK+J
      STD(J)=SQRT(ABS(R(JK)))
   80 CONTINUE
      DO 110 J=1,M
      DO 110 K=J,M
      JK=J+(K*K-K)/2
      L=M*(J-1)+K
      RX(L)=R(JK)
      L=M*(K-1)+J
      RX(L)=R(JK)
      IF (STD(J)*STD(K)) 100,90,100
   90 R(JK)=0.0
      GO TO 110
  100 R(JK)=R(JK)/(STD(J)*STD(K))
  110 CONTINUE
C
C     CALCULATE STANDARD CEVIATIONS
C
      FN=SQRT(FN-1.0)
      DO 120 J=1,M
      STD(J)=STD(J)/FN
  120 CONTINUE
C
```

```
C     COPY THE DIAGONAL OF THE MATRIX OF SUMS OF CROSS-PRODUCTS OF
C     DEVIATIONS FROM MEANS
C
      L=-M
      DO 130 I=1,M
      L=L+M+1
      B(I)=RX(L)
  130 CONTINUE
      RETURN
      END
```

```
C     SUBROUTINE  DIST
C
      SUBROUTINE DIST (N,K,X)
C
C     SUBROUTINE TO COMPUTE THE PROBABILITY DENSITY FUNCTION,
C     CUMULATIVE PROBABILITY DISTRIBUTION AND PERFORM THE CHI
C     -SQUARE AND KOLMOGOROV-SMIRNOV TESTS AGAINST A NORMAL
C     AND A LOG-NORMAL DISTRIEUTION
C
C
C      N   NUMBER OF POINTS IN THE SUBJECT ARRAY
C      K   NUMBER OF CLASS INTERVALS TO BE USED
C          MUST BE AN ODD INTEGER
C          IF K IS NOT SPECIFIED ( I.E.IS 0 ) THEN K WILL BE
C          COMPUTED USING A SIMPLIFIED VERSION OF EQUATION 30.72
C          PAGE 438, THE ADVANCED THEORY OF STATISTICS, VOL. 2
C          KENDALL AND STUART, 1961.
C      X   SUBJECT ARRAY
C
C
      INTEGER A
      COMMON /BK/ TITLE(80),A,TITLE1(8)
      DIMENSION Y(100), Z(100), DEL(100), POS(100), W(100), D(100), S(10
     10), X(1)
      XN=N
      XBAR=0.0
      STDX=0.0
      CALL STANDX (N,X,XBAR,STDX)
      IF (K.GT.0) GO TO 20
      IF (N.LT.450) GC TO 10
      K=2.0*(0.8622*(N-1))**0.4
      K=K*2-1
      IF (K.GE.100) K=99
      GO TO 20
   10 K=15
```

```
20    CONTINUE
      XK=K
      KH=K/2
      AXK=1.0/XK
      C0=2.515517
      C1=0.802853
      C2=0.010328
      D1=1.432788
      D2=0.189269
      D3=0.001308
      DO 30 J=1,KH
      XJ=J
      Q=XJ*AXK
      B=1.0/Q**2
      B=SQRT(ALOG(B))
      C=B-(C0+C1*B+C2*B**2)/(1.0+D1*B+D2*B**2+D3*B**3)
      Z(J)=C
30    CONTINUE
      KH1=KH+1
      KH2=K-1
      JJ=KH
      DO 40 J=KH1,KH2
      Z(J)=Z(JJ)
      JJ=JJ-1
40    CONTINUE
      DO 50 J=1,KH
50    Z(J)=-Z(J)
      DO 60 J=1,KH2
      S(J)=Z(J)
      Z(J)=XBAR+Z(J)*STDX
60    CONTINUE
      DO 80 J=1,KH2
      Y(J)=0.0
      DO 70 I=1,N
      IF (X(I).LE.Z(J)) Y(J)=Y(J)+1.0
```

```
 70     CONTINUE
 80     CONTINUE
        Y(K)=XN
        WRITE (A,280)
        WRITE (A,310) TITLE
        WRITE (A,300)
        WRITE (A,320)
        DO 90 J=1,K
 90     W(J)=Y(J)*100.0/XN
        DO 100 J=1,KH2
        IF (J.EQ.40) WRITE (A,280)
        IF (J.EQ.80) WRITE (A,280)
 100    WRITE (A,330) Z(J),W(J)
        DEL(1)=W(1)
        DO 110 J=2,KH2
        DEL(J)=W(J)-W(J-1)
        POS(J)=(Z(J)+Z(J-1))/2.0
 110    CONTINUE
        DEL(K)=100.0-W(KH2)
        WRITE (A,280)
        WRITE (A,310) TITLE
        WRITE (A,340)
        WRITE (A,350)
        WRITE (A,360) Z(1),DEL(1)
        DO 120 J=2,KH2
        IF (J.EQ.40) WRITE (A,280)
        IF (J.EQ.80) WRITE (A,280)
        WRITE (A,370) Z(J-1),Z(J),POS(J),DEL(J)
 120    CONTINUE
        WRITE (A,380) Z(KH2),DEL(K)
        SUM=0.0
        DO 130 J=1,K
        DEL(J)=(DEL(J)*XN)/100.0
        SUM=SUM+DEL(J)**2
 130    CONTINUE
```

```
      CHI=((XK/XN)*SUM)-XN
      IDF=K-3
      DO 140 J=1,K
      XJ=J
      W(J)=W(J)/100.0
      G=XJ/XK
      F=W(J)-G
      D(J)=ABS(F)
  140 CONTINUE
      CALL SORTX (K,D)
      SK=D(1)
      WRITE (A,280)
      WRITE (A,290)
      WRITE (A,400)
      WRITE (A,310) TITLE
      WRITE (A,390) CHI,IDF,SK,N
      WRITE (A,410) XBAR,STDX
C     CHECK IF NEGATIVE VALUE EXISTS IN THE ARRAY X.
      DO 150 I=1,N
      IF (X(I).LT.0.0) GO TO 180
  150 CONTINUE
C     CHECK IF THE SMALLEST VALUE IS ZERO, IF SO, REPLACE IT WITH 0.0001
      DO 170 I=1,N
      IF (X(I).EQ.0.0) GO TO 160
      GO TO 170
  160 X(I)=0.0001
  170 CONTINUE
      GO TO 210
C     FIND THE LARGEST NEGATIVE NUMBER IN THE ARRAY.
  180 SMALL=0.0
      DO 190 I=1,N
      IF (X(I).LT.SMALL) SMALL=X(I)
  190 CONTINUE
      WRITE (6,440) SMALL
      SMALL=ABS(SMALL)
```

```
      DO 200 I=1,N
      X(I)=X(I)+SMALL
      IF (X(I).EQ.0.0) X(I)=0.0001
      X(I)=ALOG(X(I))
200   CONTINUE
      GO TO 230
210   DO 220 I=1,N
      X(I)=ALOG(X(I))
220   CONTINUE
230   CALL STANDX (N,X,XBAR,STDX)
      DO 240 J=1,KH2
240   Z(J)=(XBAR+S(J)*STDX)
      DO 250 J=1,KH2
      Y(J)=0.0
      DO 250 I=1,N
      IF (X(I).LE.Z(J)) Y(J)=Y(J)+1.0
250   CONTINUE
      Y(K)=XN
      DO 260 J=1,K
      XJ=J
      W(J)=Y(J)/XN
      G=XJ/XK
      F=W(J)-G
260   D(J)=ABS(F)
      CALL SORTX (K,D)
      SK=D(1)
      W(1)=Y(1)
      SUM=W(1)**2
      DO 270 J=2,K
      W(J)=Y(J)-Y(J-1)
270   SUM=SUM+W(J)**2
      CHI=((XK/XN)*SUM)-XN
      WRITE (A,420) CHI,IDF,SK,N
      WRITE (A,430) XBAR,STDX
      WRITE (A,280)
```

```
                                                                        P1760
                                                                        P1770
      RETURN                                                            P1780
C                                                                       P1790
280   FORMAT (1H1)
290   FORMAT (////)
300   FORMAT(15X,"CUMULATIVE PROBABILITY DISTRIBUTION OF A ",
     1"TIME SERIES",//)
310   FORMAT (9X,80A1,/)
320   FORMAT(24X,"Z",21X,"PERCENT PROBABILITY",/,
     146X,"OF THE TIME SERIES",/,
     246X,"BEING LESS THAN OR",/,
     346X,"EQUAL TO Z",//)
330   FORMAT (19X,E10.4,22X,E10.4)
340   FORMAT(15X,"PROBABILITY DENSITY DISTRIBUTION OF A TIME SERIES",//)
350   FORMAT(10X,"LOW POINT       HIGH POINT      MID PCINT          PROB
     1ABILITY",/,61X,"DENSITY",/)
360   FORMAT (25X,E10.4,25X,E10.4)
370   FORMAT (10X,E10.4,5X,E10.4,5X,E10.4,10X,E10.4)
380   FORMAT (10X,E10.4,40X,E10.4)
390   FORMAT(15X,"COMPUTED VALUE OF CHI-SQUARE BASED ON THE NORMAL  ",/,
     1        15X,"DISTRIBLTION IS                          ",E10.4,/,
     2        15X,"NUMBER CF DEGREES OF FREEDOM IS          ",I10,///,
     3        15X,"COMPUTEC VALUE OF KOLMOGOROV-SMIRNOV PARAMETER   ",/,
     4        15X,"BASED ON THE NORMAL DISTRIBUTION IS  ",E10.4,/,
     5        15X,"SAMPLE SIZE IS                           ",I10,///)
400   FORMAT(31X,"RESULTS OF TESTS",//)
410   FORMAT(15X,"EXPECTEC VALUE OF THE SAMPLE IS       ",E10.4,/,
     1        15X,"STANDARD DEVIATION OF THE SAMPLE IS  ",E10.4,///)
420   FORMAT(15X,"COMPUTEC VALUE OF CHI-SQUARE BASED ON THE LOG -   ",/,
     1        15X,"NORMAL CISTRIBUTION IS                   ",E10.4,/,
     2        15X,"NUMBER CF DEGREES OF FREEDOM IS          ",I10,///,
     3        15X,"CCMPUTEC VALUE OF KOLMOGOROV-SMIRNOV PARAMETER    ",/,
     4        15X,"BASED ON THE LOG-NORMAL DISTRIBUTION ",/,
     5        15X,"IS                                       ",E10.4,/,
     6        15X,"SAMPLE SIZE IS                           ",I10,///)
430   FORMAT(15X,"EXPECTEC VALUE OF THE MEAN OF THE    ",/,
```

```
                                                          ",E10.4,/,
            15X,"LOGARITHMS OF THE SAMPLE IS
     1      15X,"STANDARC DEVIATION OF THE LOGARITHMS ",/,
     2      15X,"OF THE SAMPLE IS                       ",E10.4,/,1H1)
     3
 440  FORMAT (//, 20X, "THE LARGEST NEGATIVE VALUE IN THE ARRAY IS",
     1F12.6, //, 20X, "HENCE THE ABSOLUTE VALUE OF IT WILL BE ADDED TO
     2EACH NUMBER ", //, 20X, "IN THE ENTIRE ARRAY BEFORE THE NATURAL LO
     3G IS TAKEN")                                                  P2100-
      END
```

```
C     SUBROUTINE  SORTX
C
      SUBROUTINE SORTX (N,X)
C
C     SUBROUTINE TO SORT AN ARRAY OF REAL VARIATES IN
C     DESCENDING ORDER
C
      DIMENSION X(1)
      K=(N-1)
      DO 10 L=1,K
      M=(N-L)
      DO 10 J=1,M
      IF (X(J).GE.X(J+1)) GO TO 10
      XTEMP=X(J)
      X(J)=X(J+1)
      X(J+1)=XTEMP
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STANDX (N,X,Y,Z)
C
C     SUBROUTINE  STANDX
C
C     SUBROUTINE TO COMPUTE THE ARITHMETIC MEAN AND STANDARD
C     DEVIATION OF A SERIES
C
      DIMENSION X(1)
      XN=N
      SUM1=0.0
      SUM2=0.0
      DO 10 J=1,N
      XSQ=X(J)**2
      SUM1=SUM1+X(J)
      SUM2=SUM2+XSQ
   10 CONTINUE
      Y=SUM1/XN
      Z=((SUM2/XN)-(Y**2))**0.5
      RETURN
      END
```

```
C     SUBROUTINE  RAND
C
      SUBROUTINE RAND (N,S,SS)
C
C     PURPOSE ..
C     RANDOMNESS TESTS
C
C     REFERENCE- STATISTICAL METHODS IN HYDROLOGY -
C                PROCEEDINGS OF HYDROLOGY SYMPOSIUM NO.5
C                P. 249-251
C
C
      INTEGER F1,F2,F3,F4,FIRST,SECOND
      COMMON /BK/ TITLE(80),NOPUT,TITLE1(8)
      DIMENSION S(1), SS(1)
C
C
      T=N
C
C     TEST NO. 1 BEGINS
C     SERIES IS TESTED FOR ANY CLUSTERING TENDENCY
C     THIS IS DONE BY CLASSIFYING THE OBSERVATIONS INTO 4 QUARTILES
C     THE NUMBER OF RUNS IN EACH CATEGORY IS COUNTED ( A RUN HERE IS DEF
C     INED AS A GROUP OF CONSECUTIVE OBSERVATIONS IN ANY CATEGORY )
C
      ICOUNT=1
      CALL SORTX (N,SS)
      K=N/2
      L=N/4
      M=(3*N)/4
      F050=SS(K+1)
      FF025=SS(L+1)
      FF075=SS(M+1)
      AVGFF=((FF025-F050)+(F050-FF075))/2.0
      F025=F050+AVGFF
```

```
      F075=F050-AVGFF
      F1=0.0
      F2=0.0
      F3=0.0
      F4=0.0
      DO 80 I=1,N
      IF (S(I)-F025) 10,60,60
 10   IF (S(I)-F050) 20,50,50
 20   IF (S(I)-F075) 30,40,40
 30   FIRST=4
      F4=F4+1.0
      GO TO 70
 40   FIRST=3
      F3=F3+1.0
      GO TO 70
 50   FIRST=2
      F2=F2+1.0
      GO TO 70
 60   FIRST=1
      F1=F1+1.0
 70   CONTINUE
      IF (I.EQ.1) GO TO 80
      IF (FIRST.NE.SECOND) ICOUNT=ICOUNT+1
 80   SECOND=FIRST
      R1=ICOUNT
      SMSQ=F1**2+F2**2+F3**2+F4**2
      SMCU=F1**3+F2**3+F3**3+F4**3
      RM1=(T*(T+1.)-SMSQ)/T
      STDV1=SQRT((SMSQ*(SMSQ+T*(T+1.))-2.*T*SMCU-T**3)/((T**2)*(T-1.)))
      STNV1=((R1+0.5)-RM1)/STDV1
C
C     TEST NO. 2 BEGINS
C     SERIES IS TESTED FOR ANY CLUSTERING TENDENCY IN DIRECTION OF MOVEM
C     ENT THE RUNS UP AND DOWN ARE COUNTED IN THIS TEST
C
```

```
C
C
      SN=0.0
      SP=0.0
      DO 120 I=2,N
      K=I-1
      IF (S(I)-S(K)) 100,110,90
 90   SN=SN+1.0
      FIRST=-1
      GO TO 110
 100  SP=SP+1.0
      FIRST=1
 110  CONTINUE
      IF (I.EQ.2) GO TO 120
      IF (FIRST.NE.SECOND) ICOUNT=ICOUNT+1
 120  SECOND=FIRST
      R2=ICOUNT
      RM2=(2.*T-1.)/3.0
      STDV2=SQRT((16.*T-29.)/90.)
      STNV2=((R2+0.5)-RM2)/STDV2
C
C     TEST NO. 3 BEGINS
C     THIS TEST SHOWS IF THERE IS A PREDOMINANCE OF UPWARD OR DOWNWARD C
C     HANGES
C
C
C
      IF (SN-SP) 130,140,140
 130  SMIN=SN
      GO TO 150
 140  SMIN=SP
 150  CONTINUE
      RM3=(T-1.)/2.0
      STDV3=SQRT((T+1.)/12.)
      STNV3=(RM3-(SMIN+0.5))/STDV3
```

```
      WRITE (NOPUT,270)
      WRITE (NOPUT,260) TITLE
      WRITE (NOPUT,160)
      WRITE (NOPUT,240)
      WRITE (NOPUT,220) F025,F050,F075,FF025,FF075,AVGFF
      WRITE (NOPUT,250) F1,F2,F3,F4
      WRITE (NOPUT,230) SN,SP,SMIN
      WRITE (NOPUT,170)
      I=1
      WRITE (NOPUT,180) I,STNV1,R1,RM1,STDV1
      I=I+1
      WRITE (NOPUT,180) I,STNV2,R2,RM2,STDV2
      I=I+1
      WRITE (NOPUT,180) I,STNV3,SMIN,RM3,STDV3
      WRITE (NOPUT,190)
      WRITE (NOPUT,200)
      WRITE (NOPUT,210)
      RETURN
C
  160 FORMAT (9X,78HTHREE SERIAL CORRELATION TESTS USING R NS ,OR CLUSTE
     1RING, OF LIKE OBSERVATIONS/)
  170 FORMAT (20X,12HTEST RESULTS/1H ,20X,12H,,,,,,,,,,,,/1H ,8HTEST NO.
     1,12X,24HSTANDARD NORMAL VARIABLE,9X,7HR,VALUE,10X,6HR,MEAN,10X,20H
     2R,STANDARD DEVIATION)
  180 FORMAT (1H ,3X,I1,24X,F6.2,16X,F10.2,6X,F10.2,12X,F10.2)
  190 FORMAT (1H ,5X,5HNOTE./1H ,5X,5H,,,,,/1H ,5X,122HTHE STD. NORMAL V
     1ARIABLE(K) INDICATES THE NUMBER OF STD. DEVIATIONS THAT THE TEST V
     2ALUE IS BELOW THE EXPECTED (MEAN) VALUE/1H ,125HIF THE OBSERVATION
     3S ARE INDEPENDENT. VALUES OF K LESS THAN -2.33 OCCUR WITH A PROBAB
     4ILITY OF LESS THAN 0.01 FOR A SEQUENCE OF)
  200 FORMAT (1H ,83HINDEFENDENT OBSERVATIONS (USE K GREATER THAN 2.58 F
     1OR TEST 3 - 2-TAIL FROBABILITY)./1H ,5X,40HSPECIFICALLY, SUCH VALU
     2ES OF K INDICATE-/1H ,78HTEST 1- A TENDENCY FOR OBSERVATIONS TO CL
     3USTER INTO THE RESPECTIVE CATEGORIES./1H ,77HTEST 2- A TENDENCY FO
     4R DIRECTIONS OF MOVEMENT TO PERSIST (IE.- CLUSTERING NOT)
```

```
210   FORMAT (1H ,40HEXCLUSIVELY DUE TO A FEW LARGE CHANGES.)/1H ,122HTE
     1ST 3- AN UPWARD OR DOWNWARD TREND DUE TO A RELATIVE SHORTAGE OF -
     2OR + SIGNS RESPECTIVELY (TREND DUE TO LARGER MOVEMENTS/1H ,47HIN O
     3NE DIRECTION IS NOT DETECTED BY THIS TEST).)
220   FORMAT (1H ,31X,4HF025,11X,4HF050,11X,4HF075,10X,5HFF025,10X,5HFF0
     175,10X,5HAVGFF/1H ,21X,6F15.2)
230   FORMAT (1H ,8HCHANGES.,5X,F5.1,9H=NEGATIVE,5X,F5.1,9H=POSITIVE,5X,
     1F5.1,8H=MINIMUM)
240   FORMAT (1H ,32X,17HCATEGORY BOUNDARY/1H ,31X,34H-----------------
     1-----------------)
250   FORMAT (1H ,12HCATEGORY NO.,13X,1H1,14X,1H2,14X,1H3,14X,1H4/1H ,9H
     1OCCURANCE,3X,4F15.0)
260   FORMAT (9X,80A1,//)
270   FORMAT (1H1)
      END
```