The Impact of Population Ageing in Canada:

An OLG General Equilibrium Model Analysis

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Abstract

Over the next few decades Canada will be challenged by the issue of population ageing. In this paper, I use an overlapping-generation general equilibrium model to gauge the economic, fiscal, and social security impacts of population ageing during the 21st century. The simulation results demonstrate that population ageing will severely affect the Canadian economy and the standard of life of the Canadians. However, the paper also illustrates that results are critically dependent on the projected demographic variants. To do this I analyze four variants: the high, low, constant, and medium fertility scenarios projected by the United Nations.

1. Introduction

Similarly to other OECD countries, Canada faces a serious issue of population ageing. The 2011 Canadian census shows that the population, at just over 30 million, had grown from 29 million in 1995 (Census Program 2011). Meanwhile, the structure of the population has changed greatly. In 1995, the population of elderly people (over 65 years old) was about 3.5 million. Two decades later, this number has jumped to almost 5.8 million in 2015 (U.N. Population Division, 2015). In addition, the fertility rate in Canada is rather low, at about 1.6 children per woman. That is, the number of the elderly in increasing faster than children are being born.

The origin of the population issue that Canada is now facing is partly due to the baby boom after World War II when the number of babies born in Canada increased dramatically in the post-war years from 1946 to 1965. Data from Statistics Canada shows that the fertility rate was 3.01 children per woman in 1945. It continued to increase and peaked at 3.94 in 1959 and remained high at least until 1965 (U.N. Population Division, 2015). These cohorts have eventually stimulated the economy through increased consumption; however, it also had some other effects on the society.

After these babies grew up, the young age dependency ratio dropped dramatically in 1975. Compared to 56.4 and 48.7 per 100 workers in 1965 and 1970, respectively, the young age dependency ratio in 1980 was only 40 per 100 workers. With a decline in the fertility rate, this ratio kept dropping; it went from 33.6 per 100 workers in 1980 to 23.5 per 100 workers in 2015. The low fertility rate is one important cause of the population aging and its ensuing problem on the economy. That is, because fewer babies are born each year, there are not enough new workers to enter the labor market to keep up the production level. In addition, the government and the workers have to contribute more to support the consumption of the elderly. And indeed, the old age dependency ratio rose to 23.8 per 100 workers in 2015 compared to only 17.6 per 100 workers 20 years before. This ratio is expected to increase to 38.5 in 2030 when all of the baby-boomers have retired and are out of the workforce (U.N. Population Division, 2015).

Population aging could lead to a reduction in labor force participation which will cause, at least for some time, a rise in real wages. The change in national savings is also expected to have an impact on interest rates. Pension plans will also be affected. For example, the contribution rate of the Canada Pension Plan was only 1.8% per employee in 1967. In the paper, "Housing Costs of Elderly Families", Chawla and Wannell state that "over the following decade, the rate rose by 0.1 percentage points per year, reaching 2.8% in 1996. Larger increases followed. In 2003, the rate reached a new plateau, 4.95%" (Chawla and Wannell 2004). The federal government made policy changes regarding pension claims. For instance, it is reducing pension benefits for people who redeem benefits at age 60 and it is granting more benefits if people agree to collect their benefits at age 70. In addition, the government created an investment board in 1998 to manage and invest the pension funds. The Canada Pension Plan (CPP) will transition from a completely pay-as-you-go system (PAYG) to a hybrid system that is between PAYG and a fully-funded system.

The degree to which demographic changes will affect the Canadian economy is unknown in part because of the uncertainty underlying the demographic projection. However, we can at least take different scenarios of demographic changes and try to gauge their impacts on the economy. The Conference Board of Canada recently reported research on a long-term view of Canada's changing demographics (Adès et al, 2016). The report concludes that "a larger Canadian population cannot completely offset the effects of an aging population on the economy. However, it does soften the impact" (Adès et al, 2016). Population aging slows labor force growth, which affects household spending and government revenue. Even though it is impossible to reverse the impact of population aging, increasing levels of immigration and fertility rates can generate an higher economic growth and relieve part of the government budget pressure.

In this paper, I will use an overlapping-generation (OLG) computable general equilibrium model (OLG-CGE model) to analyze how population aging will affect the Canadian economy. The methodological approach used for this analysis is based on two frameworks. The first framework is an overlapping-generation model, which was first introduced by Samuelson (1958). In this paper we create 21 age groups to depict 21 overlapping generations. The model will help us analyze the actions of individuals at different stages of their life. The second framework is an applied general equilibrium model by which decisions of agents are fully modeled based on optimizing behaviors (profit maximization, utility maximization) under technical and budget constraints and where every markets (goods, labor, and capital), clear at all time.

The overall idea of this paper is to examine and explain the impact of population aging on the Canadian economy. The paper consists of six sections. The next section is a very brief selected review of the literature on population ageing issues. In Section Three, I will introduce some demographic scenarios projected by the United Nations' Population Division. In Section Four, I describe the OLG-CGE model used for this problem. In Section Five, I will show the simulation results and, lastly, Section Six concludes and provides caveats and areas for future research.

2. Brief Selected Review of the Literature

This section reviews some issues on population ageing. Population ageing is a world-wide phenomenon. Many countries are beginning to suffer the issue of an ageing population. This will impact not only their economy, but also the political and social conditions during the 21st century. The population division of the United Nations has published a report (2002) describing global trends in population ageing. The report shows that population ageing "has profound implications for many facets of human life", and it will continue to grow more rapidly. (U.N. Population Division, 2002)

The conventional view is that population ageing causes a drop in labor participation so that the support ratio (number of workers over total population) decreases. Tinker (2002) shows that "An ageing population will have some clear social implications such as changing the support ratios with resulting effects on the resources of the country and individuals but some are more speculative." Börsch-Supan (2003) states that "the decline in the working population as a share of population as a whole will 'eat up' around one third of long-term productivity growth". He also mentions "shift in the age structure will not bring about a secular increase in earnings of a significant magnitude." Fehr et al. (2011) made several models for Germany, closed and open economy models, and with alternative assumptions on pension systems. According to them, the model "predicts a fall in future interest rates due to ageing, since the reduction in the work force dampens effective labor growth more than capital accumulation. Due to the consideration of additional social security systems, however, payroll taxes increase much more in the multi-country model. As a consequence, savings and capital accumulation is dampened much more than labor growth so that world interest rates increase during the transition" (Fehr et al 2011).

National saving will be affected because of a longer life expectancy and smaller chorts of workers. Fougère et al (2009) suggest that "population ageing may contribute to lower personal savings by close to 50%". Loayza et al. (2000) provide empirical evidence for the negative long-run effect of population ageing on saving. Börsch-Supan and Winter (2001) get similar results to the one described later on in this paper, that saving rates increase in the short term and decline thereafter. In a closed economy, national savings go directly to domestic investment and the capital stock is affected. Börsch-Supan et al. (2006) demonstrate that "an increase in national savings leads to an increase in the capital stock and thereby to a decrease in the rate of return to capital, which then crowds out further savings". The price of capital will also decrease due to population ageing, but they sates that, "its level is higher in the demographically younger regions."

Population ageing may lead to a pension system crisis and to avoid such a crisis social security system reforms have been progressively introduced Börsch-Supan made an analysis of the impact of population aging on pension system and states that "As the public pay-as-you-go pension systems of the aging industrialized countries are likely to become seriously strained under the growing dependency burden, the question arises whether a society should rely on private savings to finance old-age consumption" (Börsch-Supan, A., 1995). He also notes that "in an economy with a shrinking labor force, funding a pension system cannot work as an escape route from a rising burden of dependency. Faltering rates of return prohibit this mechanism." Boeri, Börsch-Supan and Tabellini (2001) state that European citizens will be willing to opt out the pay-as-you-go pension systems to partially funded system when they will realize the problem of population ageing. Börsch-Supan et al. (2006) also suggest that population ageing "may be amplified when a pension reform, induced by the demographic change, shifts old-age provision from pure pay-as-you-go towards more pre-funding."

Börsch-Supan et al. (2006) indicate that “while the patterns of population ageing are similar in most countries, their timing and initial conditions differ substantially. Hence, to the extent that capital is internationally mobile, population ageing will induce capital flows between countries.” It would therefore seem that a multi-country OLG-CGE model would eventually be required to fully capture the impact of population aging across the world.

Lee and Mason (2009) draw a key idea that “it is insufficient to focus on the relative number of people in age groups. The productivity of those individuals also matters. Because investment in human capital and fertility are closely connected, the total amount produced by a cohort will not”. Börsch-Supan (2003) suggests that “labor productivity will need to increase over and above this mechanism in order to compensate for the impact of population aging on domestic production. Hence, we will need more education and training to speed up human capital formation.”

3. U.N. Demographic Projections

Because future trends in fertility are uncertain, their economic and fiscal impacts are hard to determine. This section presents diverse demographic projections from the United Nations' (U.N.) population division, in particular four scenarios referred to as low, medium, high, and constant fertility variants.

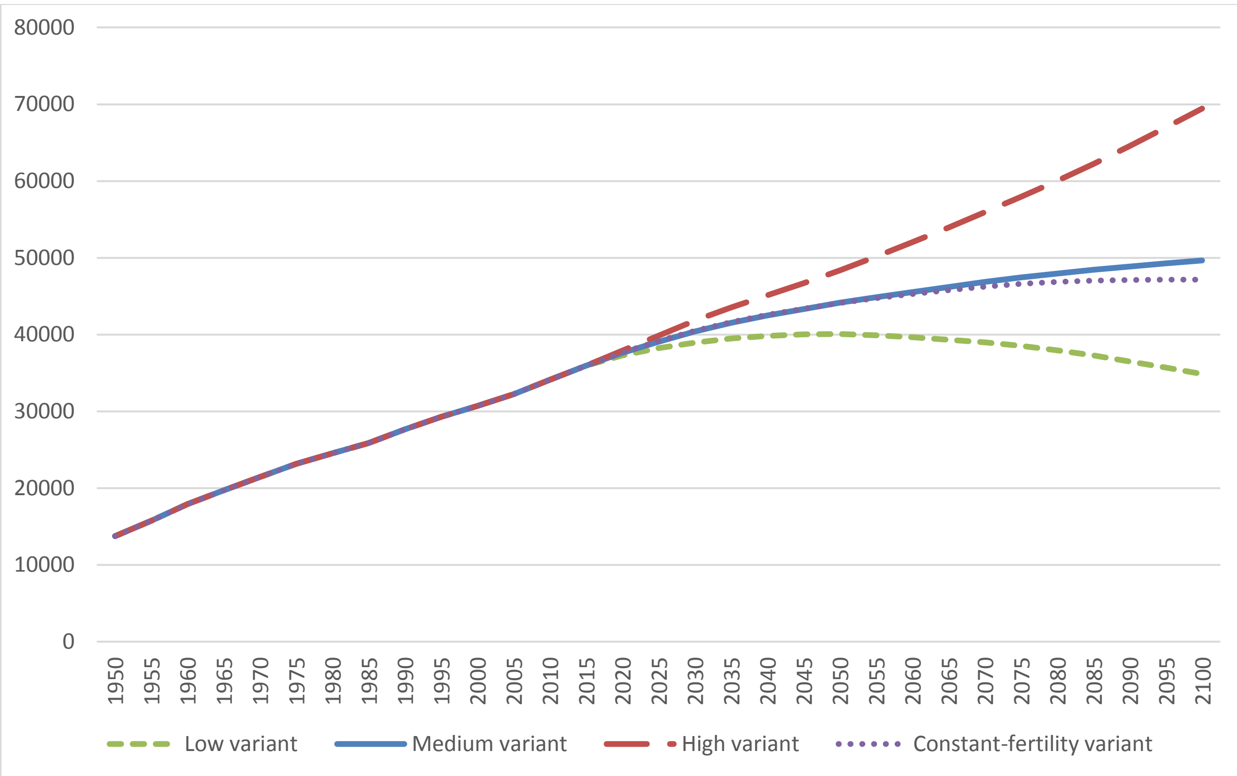

Fig 1. U.N. demographic projections of four fertility variants

Fig 1. gives a basic view of how these four fertility variants would impact the population size of Canada. The population of Canada is currently about 35.9 million. By 2100, it could reach 69.4 million in the high fertility variant, 34.9 million in the low variant, 47.1 million in the constant variant, in comparison to 49.7 million in the medium variant. These projections depend on assumptions made with respect to the total fertility rate, immigration rates, life expectancy, and mortality, to which we turn.

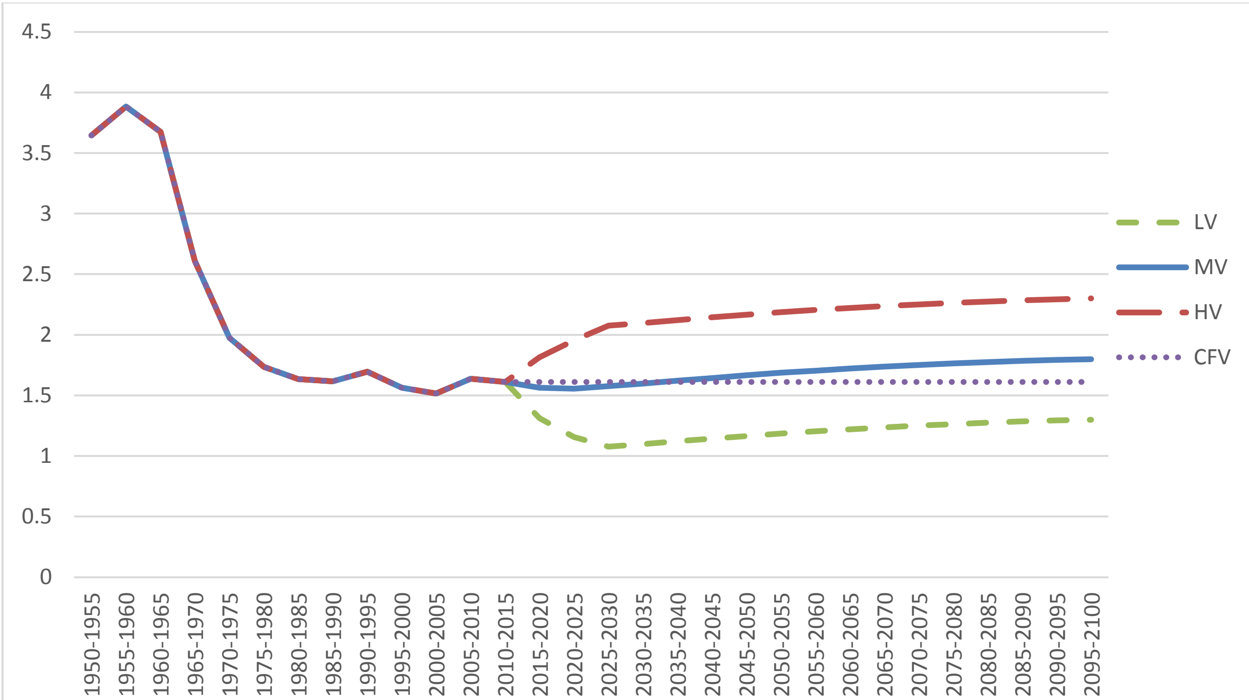

Fig 2. Total fertility rate with four demographic projections

U.N. demographic projections use a probabilistic method to estimate total fertility. The U.N. then defines low, high, and constant fertility, which differ from the medium variant only in their projected levels of total fertility. Fig 2.shows both historical and prospective rates of fertility in Canada. U.N. demographic projections assume that "Under the high variant, fertility is projected to remain 0.5 children per woman above the fertility in the medium variant over most of the projection period." According to the U.N. Population Revision (2015), to insure a smoother transition with the common baseline period (2010–2015), fertility in the high variant is initially +0.25 child in the first projection period (2015–2020), +0.4 child in the second projection period (2020–2025), and +0.5 child thereafter. Similarly, under the low variant, fertility is 0.5 child per woman below the fertility in the medium variant. Initially, the statistics are −0.25 child per woman in 2015– 2020, −0.4 child per woman in 2020–2025, and −0.5 child per woman in 2025–2100. As Fig. 2 shows, fertility is 1.61 children per woman in the baseline period (i.e., last period observed). Under the medium variant, it decreases to 1.5633 and 1.555 children per woman in 2015–2020 and 2020–2025, respectively. For these same periods, under the high variant, fertility is 1.8133 and 1.955 children per woman, respectively. Under the low variant, it is 1.3133 and 1.155 children per woman, respectively. By 2025–2030, total fertility in Canada reaches 2.0765 children in the high variant and 1.0765 children in the low variant, compared to 1.5765 children in the medium variant. Under the constant fertility variant, the level of children per woman remains constant from 2015 to 2100 at the baseline period fertility of 1.61 children per woman.[1]

The U.N. assumes that life expectancy, under-five mortality, and migration rates are the same across medium, high, low and constant fertility variants. While I will not go through the normal mortality assumptions in detail, the basic idea is that the under-five mortality rate is calculated by initially determining life expectancy at birth by the probabilistic method. However, to make the projection simple, the U.N. assumes that there exists only one variant of future mortality trends, i.e., each variant scenario faces the same mortality effect. On the other hand, as a result of incomplete data and rapid geographic movement of people, population change due to international migration is difficult to project. Therefore, the U.N. defines the net migration rate as "the difference between the number of immigrants and the number of emigrants for a particular country and period of time to show the net effect of international migration on the respective population" (U.N. Population Revision, 2015). Fig 3. gives the estimated data and projections about these three rates. Life expectancy at birth rose gradually (left axis). It was 81.78 years in the baseline period (2010–2015). By the end of the century, it is projected to rise

[1] The 2015 U.N. population report states that over the very long run, "Fertility varies slightly over the projection period in such a way that the net reproduction rate always remains equal to one, thus ensuring the replacement of the population over the long run." (U.N. Population Revision, 2015).

to 92.24 years. The under-five mortality (left axis) is projected to decline from 45 deaths per 1000 in 1950 to 1 death per 1000 in 2100, compared to 5 deaths per 1000 in the baseline year. The net migration rate has changed dramatically (right axis). It fell from 8.3 per 1000 in 1950-1955 to 6.7 in 2010–2015. From then on, it is assumed to decrease progressively to converge to 2.

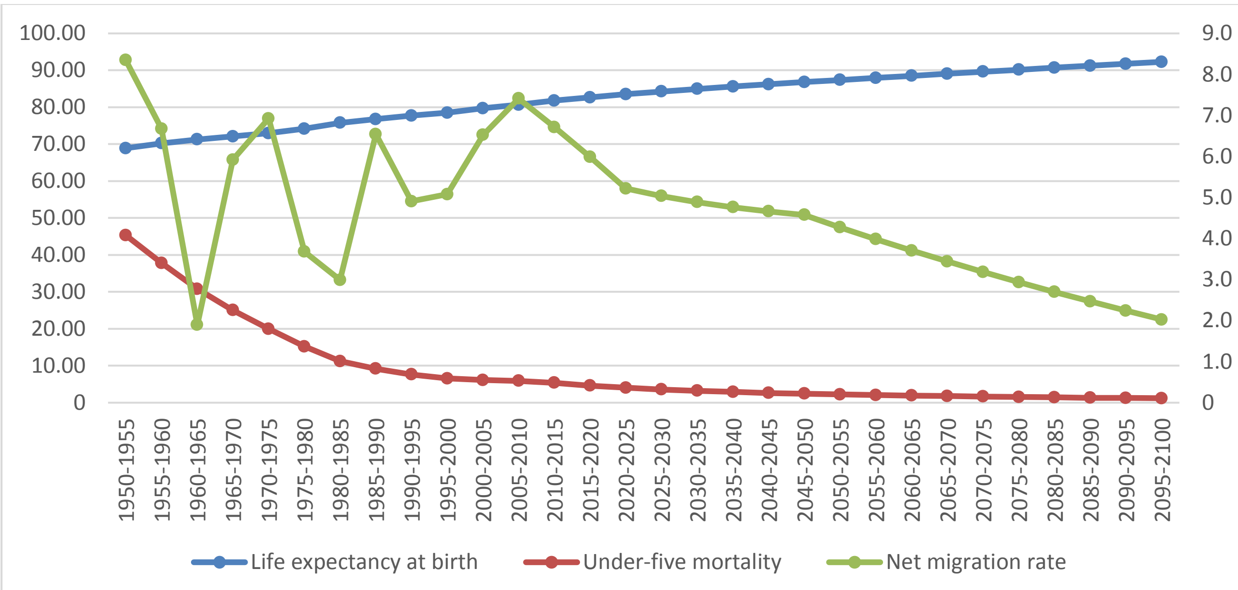

Fig 3. Rates of life expectancy at birth, under five mortality and net migration

Since we are interested in the effects of population aging, it is important to understand the structure of de population per age groups and its projection over the century, not just the size of population (as was shown in Fig. 1). Fig 4. shows the implications of the varying shares of the young (0–14 years), the medium (15–64 years), and the elderly (65 years and above) in the total population using three different colors. In each age group, variants are distinguished by four line types. The U.N. projections show that the share of the young (share_Y) in the total population fell from almost 30% in 1950 to 16% in 2015. By the end of the 21st century, under low, medium, high, and constant fertility variants, this share will drop to 10.2%, 14.8%, 19.4%, and 13.2%, respectively. The share of the elderly (share_O) went up from 7.7% in 1950 to 16.1% in

2015. Under each of the fertility variants, it will reach 38.1%, 30.5%, 24.5%, and 32.3% by 2100, respectively. The share of the 15–64 year age group (share_W) has changed from 62.6% in 1950 to 67.9% in 2015. Under each of the fertility variants, its share will decrease to 51.7%, 54.7%, 56.1%, and 54.5%, respectively, by 2100.

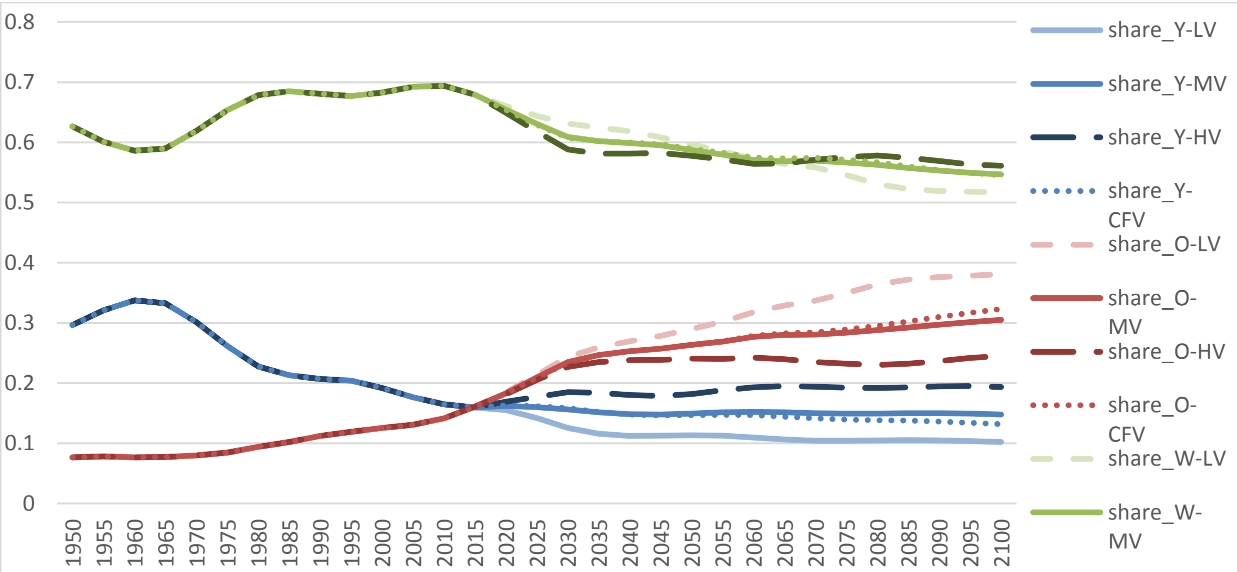

Fig 4. Shares of young(Y), old (O) and 15-64 years old (W) generations in total population

Finally, I will discuss the UN projections for the age dependency ratio, which is the ratio (for both sexes) that compares the non-working population to the working population. The total dependency ratio (YOADR) is defined as the number of people aged 15–64 divided by those aged 0–14 plus those aged 65+. In addition, the total dependency ratio is composed of the young age dependency ratio (YADR) and the old age dependency ratio (OADR). The YADR is the number of people aged 0-14 divided by the number of people aged 15-64 while the OADR is the number of people aged 65+ divided by the number of people aged 15-64.

Fig 5. shows the trends of the YADR, the OADR, and the YOADR. Between 1950 and 2015, the YADR dropped from 47.4% to 23.5%. Under the low, medium, high, and constant fertility variants, the YADR will continue to decrease to 19.8%, 27.1%, 34.5%, and 24.2%, respectively, till the end of century. On the other hand, since 2015, the increase in the OADR has exceeded the decrease in the YADR. By 2100, the ratios will be 73.7%, 55.8%, 43.7%, and 59.3%, respectively. Also, the projections for the YOADR show a dramatic inflection point starting 2010. Under the medium variant, the YOADR goes up with an increasing OADR, even if the YADR stays relatively constant at a low level.

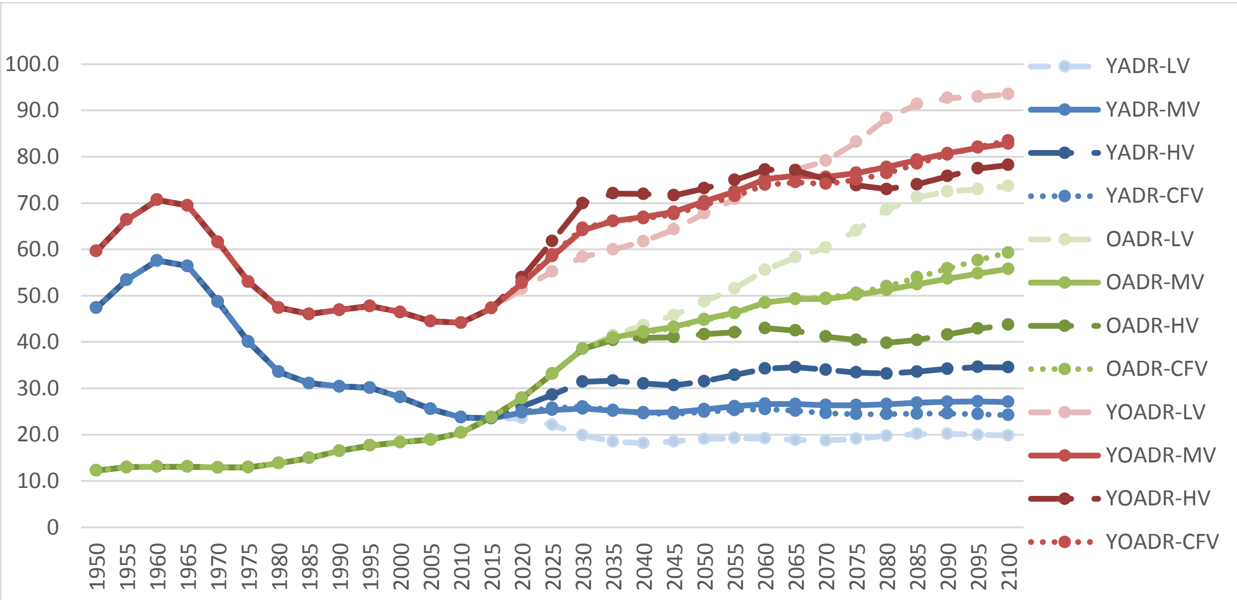

Fig 5. Dependency ratios

The economic impact of population aging will depend in part on its initial impact on the labor supply. Demographics affect firms' production, households' decisions on consumption and saving, the government's budget as well as the sustainability of the pension plan system. Although we could simply speculate qualitatively on the potential impacts of ageing, here we want to quantify the economic and fiscal impacts. To do this, we use an applied overlapping-generations computable general equilibrium model (OLG-CGE). The equations of the model provide a framework of analysis that improve understanding of the channels through which demographics affect economic agents. The OLG general equilibrium model is described in Section 4 and the simulations' results are analyzed in Section 5. Section 6 concludes and provides caveats to the analysis as well as future research.

4. The Model

The model used here is based on a smaller version of the model by Georges and Seçkin (2016). Georges and Seçkin have built an OLG-CGE model, itself in the Auerbach and Kotlikoff (1987) tradition with endogenous labour supply and they introduce age-specific mortality, following Börsch-Supan et al. (2006), with a perfect annuity market, through which unintentional bequests are implicitly distributed. The theoretical description of this approach was first presented in Yaari (1965). Several OLG models have been developed and extended in many directions over the last 20 years. For a survey of these advances, see Fehr et al. (2013) and references therein. The model for this paper simplifies the Georges and Seçkin (2016) model by assuming a one-sector model with exogenous labour supply and within a closed economy framework. These simplifications are not necessarily realistic to analyze the economic and fiscal impact of Canadian demographic projections, but it permits to have a simple framework that can be extended in the future. Yet, even this simple model allows precise replication of the population structure from the UN population projections given in Section 3. Furthermore, the model is particularly well suited to a comparison of fertility scenarios since its OLG structure and calibration explicitly allows for the incorporation of ageing effects related to age-specific productivity differences and age-specific government expenditures. In the following, I draw heavily on the presentation of the model by Georges and Seçkin (2016).

4.1. Production sector

The production sector is characterized by a representative firm which produces a single good using a Cobb–Douglas technology. $Y_{j,\tau}$ represents the (aggregate) output (GDP) in each period τ (five-year) for country j (Canada). A is total factor productivity. In this model, it is kept constant across time. This means that the model does not attempt to capture a realistic growth process but focuses exclusively on the impact of demographic pressures. *K* and *L* represent capital stock and effective labor force, respectively. α_j, is the share of physical capital in GDP (*Y*). There is no intermediary good. The following is the Cobb–Douglas production function:

(1) $$Y_{j,\tau} = A_{j,\tau} K_{j,\tau}^{\alpha_j} L_{j,\tau}^{1-\alpha_j} \ , 0 < \alpha_j < 1, \ j \in J = \{\text{Canada}\}$$

First order condition to maximize profit provides the rental rate of capital ($re_{j,\tau}$) and the wage rate per unit of effective labor ($w_{j,\tau}$):

(2) $$re_{j,\tau}/P_{j,\tau} = \alpha_j A_{j,\tau}(K_{j,\tau}/L_{j,\tau})^{\alpha_j - 1} \ ; P_{j,\tau} = 1$$

(3) $$w_{j,\tau}/P_{j,\tau} = (1-\alpha_j) A_{j,\tau}(K_{j,\tau}/L_{j,\tau})^{\alpha_j}$$

4.2. Household behavior

We classify adult generations into seventeen age groups (i.e., age groups G5 = 20-24 years old, …, G21 = 100-104). Younger generations (age groups G1 = 0-4,…, G4 = 15-19) are fully dependent on their parents and play no active role in the model, but they influence the age dependent components of public expenditure (health and education) and the population structure. The model introduces age-specific mortality with a perfect annuity market, through which unintentional bequests are implicitly distributed.

Individuals' utilities follow a CES type intertemporal utility function of consumption ($Con_{j,t+k,g+k}$) and leisure ($TLeis_{j,t+k,g+k}$) in Georges and Seçkin (2016) so that labor supply is endogenous. However, here, we assume that there is no choice between work and leisure. Therefore labor supply is exogenous. In the overlapping-generation model, each individual lives for (at most) 21 different periods and there are 21 age groups: G1 = 0–4 age old, G2 = 5–9…G20 = 95–99, and G21 = 100-104. θ_j, is the inverse of the intertemporal elasticity of substitution, which represents the decision of individuals with respect to consumption and saving. $\frac{SR_{j,t+k,g+k}}{(1+\rho_j)}$ indicates the subjective discount factor, where $SR_{j,t+k,g+k}$ represents the survival rate at any specific age. It is assumed to be 0 for the last age group (100-104), which means that if people did not die before 104, then at the end of that year they would die with probability 1. For all other age groups, the probability of survival is less than 1 and depends on age. This means that a specific share of the population of that (specific) age dies. The utility function for a representative household born at time *t* is:

(4) $$U_j = \frac{1}{1-\theta_j} \sum_{k=0}^{16} \{ (\prod_{k-0}^{k} \frac{SR_{j,t+k,g+k}}{(1+\rho_j)}) \left(Con_{j,t+k,g+k}\right)^{1-\theta_j} \}$$

The household intertemporal (dynamic) budget constraint can be written as follows:

(5) $$HA_{j,\tau+1,\gamma+1} = 1/SR_{j,\tau,\gamma} \{ \left(1 - \tau x_{j,\tau}^{L} - Ctr_{j,\tau}\right) Y_{j,\tau,\gamma}^{L} + \left[1 + \left(1 - \tau x_{j,\tau}^{K}\right) ri_{j,\tau}\right] HA_{j,\tau,\gamma} + Pens_{j,\tau,\gamma} - (1 + \tau x_{j,\tau}^{C}) Con_{j,\tau,\gamma} \}$$

$\tau = t + k;\ \gamma = g + k; k = 0, \dots 16.$

$Y_{j,\gamma}^{L}$ is gross labour income, $HA_{j,\tau,\gamma}$ represents the individual assets at the start of period τ, $Pens_{j,\tau,\gamma}$ is pension benefits accruing to retired individuals, τx^{L} is the effective tax rate on labour income, *Ctr* is the worker contribution to the PAYG pension system, τx^{K} is the effective tax rate on asset income where *ri* is the rate of return on assets (defined later on in Eq. (17) with respect to the rental price of capital *re* and the depreciation rate of capital) and τx^{C} is the effective tax rate on consumption spending. It is assumed that people are not altruist. They do not plan to leave bequests. However, as they do not know when they will die, there could be unplanned bequests. To avoid this, they buy an annuity to an insurance company while the unplanned bequest go to the insurance company. [2] This is the same framework as the one developed in Yaari (1965) (Case

[2] We assume the implicit existence of an actuarially fair insurance company. The term 1/*SR* reflects how accidental bequests are dissipated through the annuity market.

B). As in his case, this setting permits to eliminate the survival rate from the Euler equation (Eq. 9). Based on equation (5), we arrive at the household's lifetime budget constraint:

(6)

$$\sum_{k=0}^{k=16}\left\{\frac{\prod_{k=0}^{k} SR_{j,t+k,g+k}}{\prod_{\tau=t}^{t+k}[1+(1-\tau x_{j,\tau}^{K})ri_{j,\tau}]}\left[Y_{j,t+k,g+k}^{L}\left(1-\tau k_{j,t+k}^{L}-Ctr_{j,t+k}\right)+Pens_{j,t+k,g+k}-\left(1+\tau x_{j,t+k}^{C}\right)Con_{j,t+k,g+k}\right]\right\}=0$$

Gross labor income per capita, $Y_{j,\gamma}^{L}$ (i.e., per representative household of generation γ), is defined in Eq. (7) as the standardized wage rate $w_{j,\gamma}$, (the wage rate of a new adult) multiplied by $EP_{j,\gamma}$, an age-dependent productivity (earnings) profile reflecting the individual's change in earning capacity over time, and defined as a quadratic function of age (Eq. 8). LS is the individual exogenous supply of potential non-effective units of labour. $PART$ is an age-dependent, time-invariant labour participation rate.

(7) $$Y_{j,\tau,\gamma}^{L}=w_{j,\tau}EP_{j,\gamma}LS_{j,\gamma}PART_{j,\gamma};\tau=\mathrm{t}+\mathrm{k};\gamma=\mathrm{g}+\mathrm{k};\mathrm{k}=0\ldots16.$$

(8) $$EP_{j,\tau,\gamma}=\mathrm{c}+(\lambda)(\mathrm{kw}+1)-(\psi)(kw+1)^{2},c,\lambda,\psi\geq0\quad kw=0,\ldots8.$$

Equation (9) describes the intertemporal first-order condition for lifetime consumption. Since there is no preference for leisure in our model, the equation only depends on the tax rates on capital and consumption, interest rate of return on assets, time preference, and the intertemporal elasticity of substitution:

(9) $$\frac{Con_{j,t+k+1,g+k+1}}{Con_{j,t+k,g+k}}=\left[\frac{1+\left(1-\tau x_{j,t+k+1}^{K}\right)ri_{j,t+k+1}}{1+\rho_{j}}\frac{1+\tau x_{j,t+k}^{C}}{1+\tau x_{j,t+k+1}^{C}}\right]^{\frac{1}{\theta_{j}}}$$

4.3. Government sector and pension plans

Public expenditure on health and education (Eqs. 11-12) are age-dependent (i.e., fixed per person of a specific age). Other types of public expenditures (Eq. 13) are assumed to be age invariant, that is, they are fixed per capita, so that other total expenditure depends only on the size of the total population, not on its age structure. In this version of the model, public debt is assumed to remain constant per capita while the government must respect its budget constraint (Eq.10). Total government spending, including interest on the public debt, must be financed each period by tax revenues from labour, capital and consumption taxes. Here, the tax rate on labour income $\tau x^{L}_{j,\tau}$, is the endogenous variable that fulfils the government budget constraint. In simulation results the path for this tax rate will therefore be indicative of the fiscal pressure of ageing in Canada.

(10) $$Bond_{j,\tau+1} - Bond_{j,\tau} = Gov_{j,\tau} + GovE_{j,\tau} + GovH_{j,\tau} + ri_{j,\tau}Bond_{j,\tau} - \sum_{\gamma=g}^{g+k} Pop_{j,\tau,\gamma}\,\{\tau x_{j,\tau}^{C}\left(Con_{j,\tau,\gamma}\right) + \tau x_{j,\tau}^{L}\left(w_{j,\tau}EP_{j,\gamma}LS_{j,\gamma}\right) + \tau x_{j,\tau}^{K}(ri_{j,\tau}HA_{j,\tau,\gamma})\}$$

To reiterate, government expenditures on health and education are age-dependent. For each age group, there are variable costs of health and education. We denote the remaining government expenses as $Gov_{j,\tau}$. In each period, age structure does not affect the total amount of these other spending; however, they are affected by total population size:

(11) $GovH_{j,\tau} = \sum_{\gamma=g-4}^{g+ka} Pop_{j,\tau,\gamma}HEAC_{j,\gamma}$; ka=-4,…16.

(12) $GovE_{j,\tau} = \sum_{\gamma=g-4}^{g+ka} Pop_{j,\tau,\gamma}EDUC_{j,\gamma}$; ka=-4,…16.

(13) $Gov_{j,\tau} = TPop_{j,\tau}GEPC_{j}$

We assume a PAYG pension plan separated from the government budget constraint so as to gauge the impact of the pension pressure due to ageing independently from the rest of the budget. Retirement age is at 65 years of age. The pension benefits depend on life-time labor income and the pension replacement rate (or pension benefit rate) as follows:

(14)

$$Pens_{j,\tau,g+kr} = PensR_j \times \frac{1}{9}\sum_{kw=0}^{8} w_{j,\tau-kr+kw,g+kw} \times EP_{j,g+kw}LS_{j,g+kw}PART_{j,g+kw} \quad ;$$

kr=9…16.

The contribution rate, *Ctr*, applied on gross labour income, is determined endogenously so as to permit the financing of total pension benefits (Eq. 15). Hence, endogenous increases in the contribution rate will be indicative of future pension plan tensions due to population ageing (separately from other fiscal pressures)

(15) $$\sum_{kr=9}^{kr=16} Pop_{j,\tau,g+kr}Pens_{j,\tau,g+kr} = Ctr_{j,\tau}\sum_{kw=0}^{8} Pop_{j,\tau,g+kw}Y_{j,\tau,g+kw}^{L}$$

4.4 Investment and asset returns

The accumulation of Canada's capital stock (*Kstock*) is subject to depreciation (Eq 16), where *Inv* represents investment and *δ* the depreciation rate of capital. Physical capital (*Kstock*) and government bonds (*Bond*) are assumed to be perfectly substitutable. Hence the expected rate of return on bonds *ri* is equal to the rate of return on physical assets defined as the rental price of capital minus the depreciation rate (Eq 17).

(16) $$Kstock_{j,\tau+1} = Inv_{j,\tau} + (1-\delta_j)Kstock_{j,\tau}$$

(17) $$ri_{j,\tau} = re_{j,\tau} - \delta_j$$

4.5 Market and aggregation conditions

We assume that Canada is a closed economy; therefore, there are no net exports in our model. The equilibrium in the good market and in the factor markets are given as follows:

(18) $$Y_{j,\tau} = \sum_{\gamma=g}^{g+k} Pop_{j,\tau,\gamma}Con_{j,\tau,\gamma} + Inv_{j,\tau} + Gov_{j,\tau} + GovH_{j,\tau} + GovE_{j,\tau}$$

(19) $$L_{j,\tau} = \sum_{\gamma=g}^{g+k} Pop_{j,\tau,\gamma}LS_{j,\gamma}EP_{j,\gamma}PART_{j,\gamma}$$

(20) $$K_{j,\tau} = Kstock_{j,\tau}$$

4.6 Demographic process

In this paper, a demographic shock is simulated under different demographic variants. Following Georges and Seckin (2016), a demographic process (Eq. 21), superimposed on the OLG model, provides the exogenous shock behind the simulations results. The first law of motion simply implies that the number of children (age group $g+ka$ for ka=-4, i.e. age group 0-4) born at time τ is equal to the size of the preceding generation multiplied by the "birth rate", BR, in period τ-1. The second law of motion gives the size of any age group $g+ka$ beyond the first generation, as the size of this age group a period ago, multiplied by the age specific conditional "survival" rate, SR. In this model survival rates vary across time and age. For the final generation (ka=16), the age group 100-104, the conditional survival rate is zero. This means that for the oldest age group, everyone dies with certainty at the end of the period.

$$(23) \qquad Pop_{j,\tau,g+ka} = \begin{cases} Pop_{j,\tau-1,g+ka} BR_{j,\tau-1} & for\ ka = -4 \\ Pop_{j,\tau-1,g+ka-1}\left(SR_{j,\tau-1,g+ka-1}\right) & for\ ka \in [-3, \dots 16] \end{cases}$$

4.7 Model calibration

For model calibration purposes, we initially assume a stationary population at the start of 2015 and total population is normalised at 1. Based on the UN population data per age group for year 2015, I follow the method by Georges and Seçkin (2016) and compute birth rates and age variable fertility rates so that the population structure per age group would replicate itself periods after periods as shown in Table 1a. This is the population structure that is assumed in the calibration part of the model and which is used together with the model to replicate the Social Accounting Matrix of Tables 3a/b. Then, from 2015 onwards, I apply birth rates and variable fertility rates that are consistent with the UN population data projections, as shown in Table 1b.

Hence, from 2020 onwards, the population structure and population size vary.[3] This procedure allows precise modelling of the population size and structure within the model and replicates the UN demographic projections given in Section 2 from 2020 onwards.

Table 1 Computing "birth" and "survival" rates from UN projections for Canada*

a) UN population structure per age group in 2015 and stationary population assumption from 2015 onwards

Years/Generations	G1 (0-4)	G2 (5-9)	…	G21 (100+)	Total Pop
2015	1942	1947		8.	35940
2020	1942	1947		8.	35940
…					
2100	1942	1947		8.	35940

b) Constructing implied birth rates and survival rates from UN population data from 2010 onwards

Years/Generations	G1 (0-4)	G2 (5-9)	…	G21 (100+)	Total Pop
2015	1942	1947		8.	35940
2020	2013	2033		10	37600
…					
2100	2402	2448		359	49668

* Numbers in cells correspond to historical (2015) and projected (UN medium variant) population sizes (in thousands) by years and generations (relabeled here G1 to G21). The "birth rates" are computed "vertically" for consecutive time periods on the basis of G1 population sizes (e.g., BR=2013/1942= 1.036265). "Survival rates" are computed "diagonally" across consecutive age groups and time periods (e.g., SR=2033/1942= 1.046595). *Source: Compiled using UN Population Division data – World Population Prospects, the 2015 Revision (esa.un.org/unpd/wpp/unpp/panel_indicators.htm)*

A Social Accounting Matrix (SAM) is a square matrix that describes the flow of all transactions in Canada's economy by different institutional agents (See Table 3a/3b). Each cell of the SAM represents the payment from the column account to the row account; in other words, the rows are where revenues are reported and the columns are for expenditures. In the OLG general equilibrium model, we assume that the SAM represents the economy's initial equilibrium called the benchmark equilibrium. The data used in the benchmark SAM is based on CANSIM, a socioeconomic database by Statistics Canada. However, by proposing a closed economy model, we need to adjust different elements so as to maintain an initial equilibrium. Table 3b provides the same number as a proportion of GDP. The firm produces a good and its value is identical to the spending on this good: consumption spending, government spending classified by education,

[3] Technically, the model is simulated from 1925 to 2260. It starts in 1925 for welfare analysis purposes and ends in 2200 to also ensure a stationary sate.

health, and others; and investment spending. We can also represent this spending as total demand for the commodity. As the tables show, total expenditure represents the GDP of Canada at market price. Since we assume that the capital share of output (revenue or GDP) α is 0.3 and the labor share of output $1-\alpha$ will be 0.7, we can allocate the total output to the two production factors: capital and labor, which are also the total amount of revenues accruing to capital and labor factors, respectively. Capital revenue include capital revenues paid to the owners of capital, taxes on capital, and depreciation of capital. As we mentioned above, capital costs should match the total capital revenues obtainable on the market. In the labor market, total revenue from labor depends on firm payments to wage earners, including wage taxes and pension plan contributions. Households receive their revenue from the incomes of both labor and capital, pension benefits from the government, and interest revenues on the public debt. These revenues are equal to the money they can spend on final consumption expenditure. The government sector finances its spending by collecting taxes and borrowing money. Thus, the sum of expenditures on education, health, and other spending and interest payments on debt be financed by taxes on capital revenues, labor revenues, and consumption spending. Besides, households' savings will flow into investment. In a stationary state saving is just equal to the investment needed to offset the depreciation of the capital stock.

I will define some values for the benchmark parameters (see Table 2). Fougère et al. (2009) mention that for the Canadian economy, α_j =0.3, $\delta_j = 5.1\%$ and $PensR_j$ (the pension replacement rate) = 0.3 have been widely used. In the study by Carey and Rabesona, effective tax rates on capital and labor are 29.6% and 13.9%, respectively (Carey and Rabesona 2002). Because of complications in determining capital taxation, Baylor (2005) sets the effective tax rate on capital at 45% (Baylor 2005). Due to numerical issues, I also had to assume different taxes rate on wage, consumption and capital. Those are 24.96%, 12% and 28.82%, respectively. The contribution rate is set at 16.8%. In addition, we use 0.25 as the value of the intertemporal elasticity of substitution.

Table 2

Some behavioral parameter values.

Parameters	Description	Values
αj	Production share of capital	0.3
δ_j	Depreciation rate of capital	5.1%
$PensR_j$	Pension replacement rate	0.3
$\tau x_{j,\tau}^{L}$ (=WTxR)	Effective tax rate on labour	24.96%
$\tau x_{j,\tau}^{C}$	Effective tax rate on consumption	12%
$\tau x_{j,\tau}^{K}$	Effective tax rate on capital	28.82%
$\theta_j = 1/Sig_j$	Inverse of the constant inter-temporal elasticity of substitution	1/0.25
$Ctr_{j,\tau}$	Contribution rate	16.8%

Table 3a

	FRIM	CAP ITAL	LABOUR	CON	GOV	GOVH	GOVE	INV	PEN	TOTAL
FIRM				1,070.899	263.107	126.104	102.42	143.003		1,705.533
CAPITAL	511.66									511.66
LABOUR	1,193.873									1,193.873
HOUSEHOLDS		221.194	695.192		82.279				200.741	1,199.407
TAX		147.463	297.94	128.508	32.912					606.822
SAVING		143.003								143.003
CONTR			200.741							200.741
TOTAL	1,705.533	511.66	1,193.873	1,199.407	378.298	126.104	102.42	143.003	200.741	

Table 3b

	FRIM	CAP ITAL	LABOUR	CON	GOV	GOVH	GOVE	INV	PEN	TOTAL
FIRM				0.62	0.15	0.07	0.06	0.08		1.00
CAPITAL	0.30									0.30
LABOUR	0.70									0.70
HOUSEHOLDS		0.13	0.41		0.05				0.12	0.70
TAX		0.09	0.17	0.08	0.02					0.36
SAVING		0.08								0.08
CONTR			0.12							0.12
TOTAL	1.00	0.30	0.70	0.70	0.22	0.07	0.06	0.08	0.12	

5. Simulation results from the model

In this section I analyze population aging by comparing simulation results for the different fertility variants introduced in Section 2. In Section 5.1, I describe results for the private sector (firms and consumers). Results pertaining to the government sector and pension plans are given in section 5.2.

5.1 Private sector (firms and consumers)

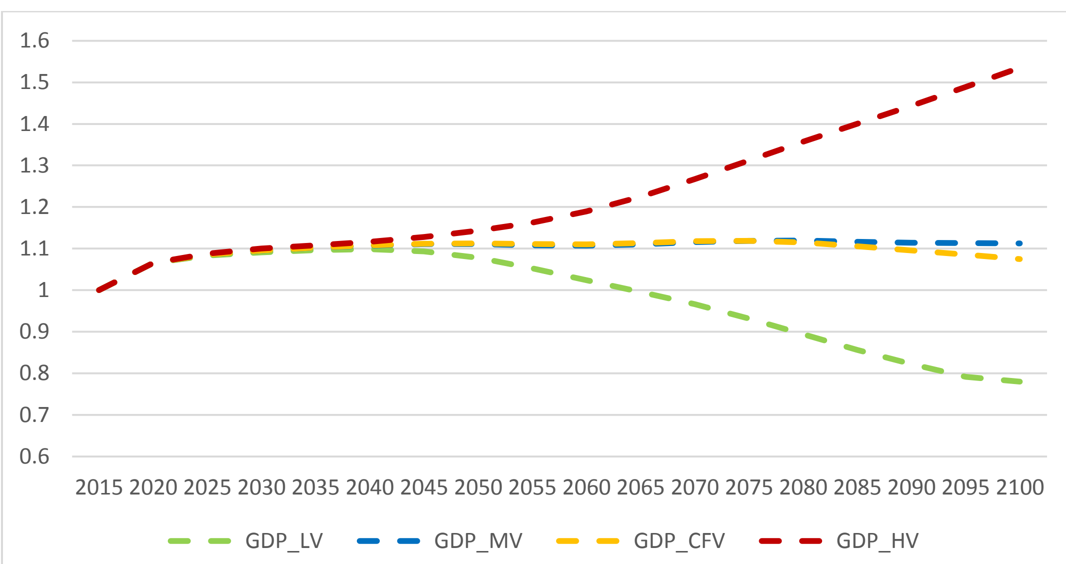

Fig. 7 GDP

Fig. 7 shows the growth of GDP due to the demographic projections associated to the four variants. Note that our results do not include total factor productivity growth in order to focus exclusively on the impact of demography. Hence, simulation results should not be viewed as a forecast for GDP over the 21st century, but as representing the pure effect of demographics. As mentioned before, GDP is normalised to 1 in 2015 so that results are easily interpreted in terms of growth rates over the century. Also it is assumed that the economy and the population was at a steady state in 2015, just before the UN population demographic projections revision. In a sense, agents realise the shock at the end of 2015 and start to change their behavior accordingly. In all four scenarios GDP increases with a similar growth rate until 2035, and then GDP paths start to diverge. The results indicate that, from 2015 onwards, GDP increases only in the high fertility variant; this is because additional young people enter the labour market (around 2040) under the high fertility scenario. In the constant fertility and medium fertility variants, GDP stays roughly constant after 2040 until 2100. In the low fertility variant, however, GDP experiments a strong decline because of the small cohort of workers entering the job market after 2040. At the end of the century we find that the high fertility scenario increases GDP from 1 (in 2015) to 1.53, while the low fertility scenario decreases GDP to 0.78. From 2015 to 2100, the effective GDP growth rate of low, medium, constant, and high fertility variants are: -22%, 11.2%, 7.5%, and 53.5%, respectively. Of course, a larger labour force under the high fertility scenario can boost the size of the economy. However, it is more important to focus our attention on living standards, and in particular on output per capita (GDP/POP).

In Fig. 8, we see that GDP per capita drops significantly after 2020 in all scenarios. The GDP per capita tends to converge toward a similar level (across all scenarios) at the end of the century, representing roughly a drop by 20% with respect to 2015. There is however a notable difference. In the high fertility scenario – the decrease is up-fronted (i.e., earlier in the century), while the GDP per capita's decrease in the low variant scenario is not as fast in the first part of the century

To understand these results further, GDP per capita can be decomposed in two ratios, *Y/Pop= Y/L*L/Pop*, that is total output per population is assumed to be equal to the output per worker (labour productivity) and the support ratio (the number of efficient workers, *L*, per consumers, *POP*). Note that *Y/L* is a direct function of the capital-labor ratio (*K/L*), so that changes in *K/L* should explain the changes in *Y/L*.

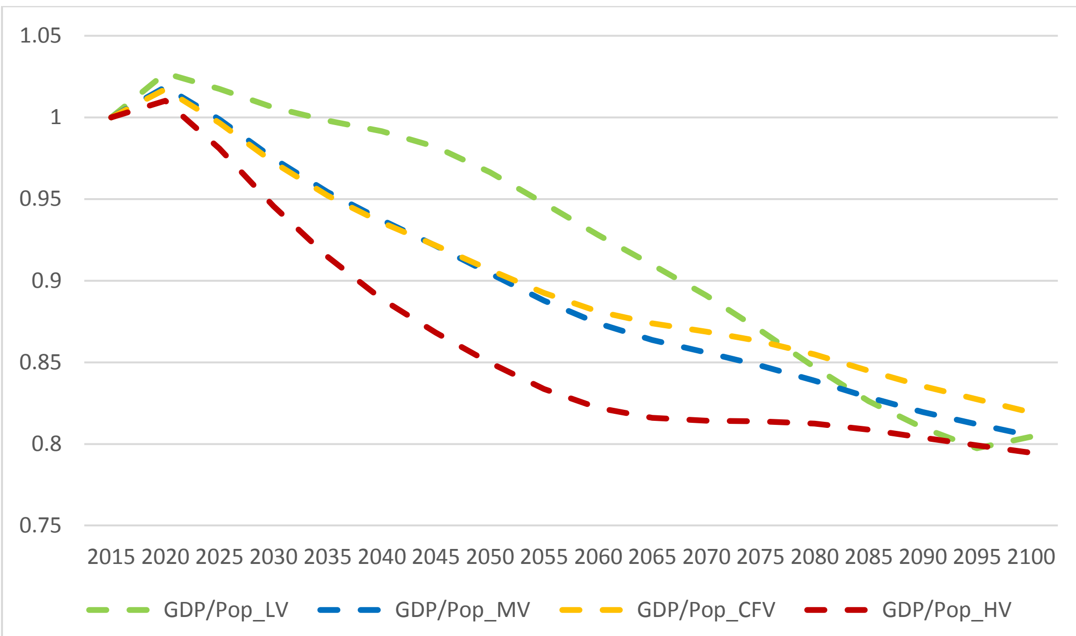

Fig. 8 GDP per Capita

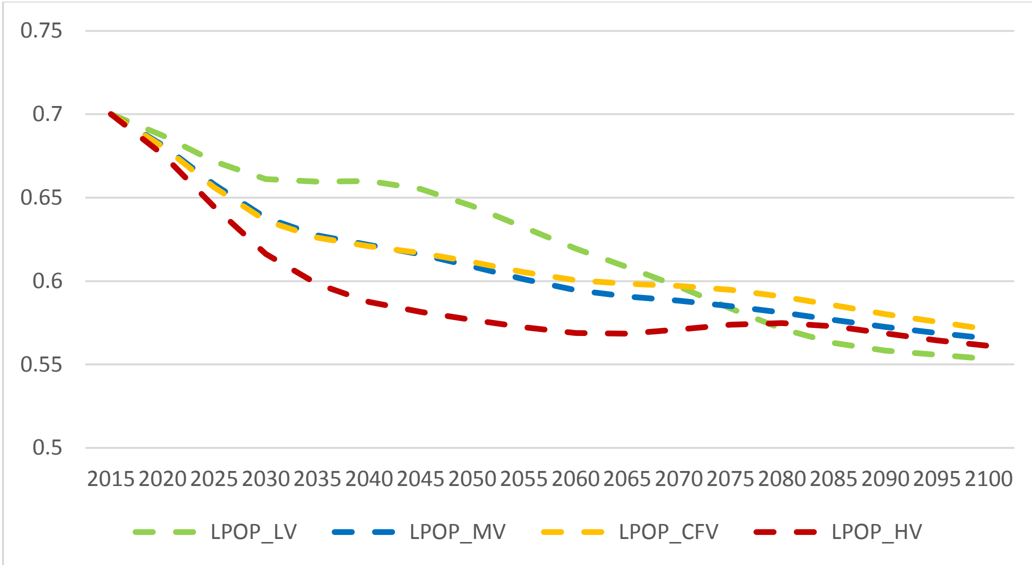

Fig. 9 Support Ratio (L/POP)

Let us first focus our attention on the support ratio (L/POP) in Fig. 9. Fig. 9 indicates a fall in the support ratio over the 21st century. It begins at 0.7, and by the end of the century, it has decreased to roughly 0.58 in all scenarios that is a fall of roughly 20%. However, the timing of the fall is quite different across scenarios. Since there are more babies (more consumers) born at the start of 2020 under a high fertility scenario, the ratio L/POP declines more rapidly in this scenario than under alternative demographic variants. However, this will be followed, 20 years later (around 2040), by a larger cohort of workers, which should reduce the dramatic fall in the support ratio under the high fertility scenario. Then, the support ratio remains at a roughly stable level. In the low fertility scenario, however, the support ratio falls gentler at the beginning as there are less babies (POP does not increase as much). However, it is followed by a steeper decrease starting 2040. In essence, under a low fertility variant, the size of the labour market shrinks dramatically. The evolution in the support ratio may explain why GDP per capita starts to fall more dramatically after 2040 under the low fertility variant while on the other hand we observe a slowdown in the fall of the living standard in the high fertility variant.

However, to understand further the evolution in the GDP per capita over the 21st century, we must also understand the impact of the demographic scenarios on the capital-labor ratio and therefore on labour productivity. The capital to (efficient) labor ratio is given in Fig. 10. Efficient labor, which partly determines the support ratio (Fig. 9), will also affect the denominator of the K/L ratio. For example, a higher fertility scenario implies a larger cohort of workers by 2040, which tends to push down the K/L ratio further than under the low variant scenario (with a smaller cohort of workers by 2040). However, this will be reversed by 2085 when these respective cohorts start to retire. Then, the high fertility ratio will see a large decline in the labor force, which might sustain the K/L ratio in contrast to what would happen in the low fertility scenario. All these changes in K/L due to changes in L will affect the labour productivity (Y/L) *ceteris paribus* (i.e., for a given amount of capital). Therefore, given the decomposition *Y/Pop= Y/L*L/Pop*, we should expect that these changes in the size of the working population will affect the GDP per capita. For example, for an unchanged stock of capital, if the labor force is larger under the high fertility scenario starting 2040, we should then expect a decline in labor productivity (Y/L) and therefore a larger decline in Y/POP as showed in Fig. 8.

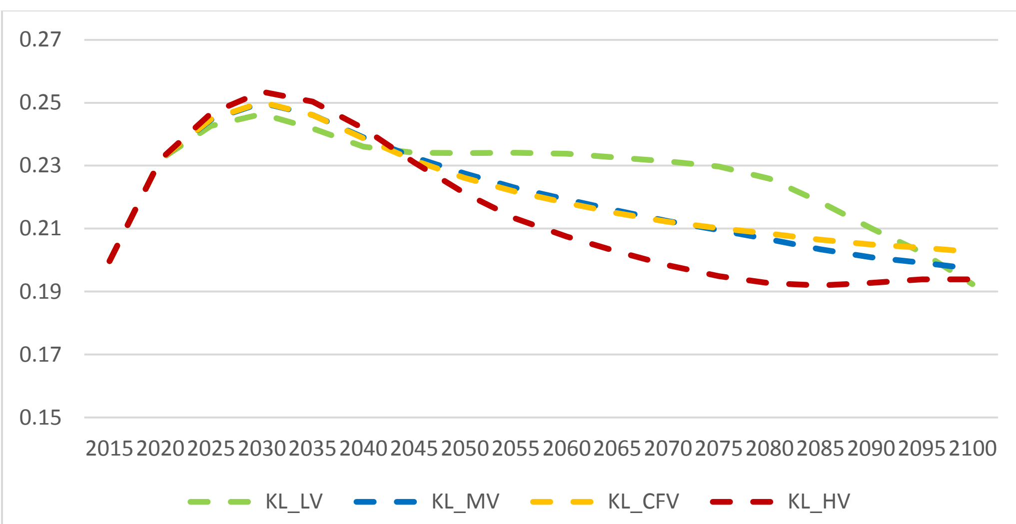

Fig. 10 Ratio of Capital per Worker (K/L)

However, to fully understand what happens to the *K/L* ratio and therefore the *Y/L and Y/POP* ratios, we also need to understand the impact of demographics on the stock of capital. The most appropriate way to understand the evolution in the capital stock is by observing the net aggregate saving rate in the economy, as given in Fig. 11. First, the saving rates increases as people are surprised in 2015 by the magnitude of the shock, suggesting to them that they will leave longer than expected, so that they need to save more. However, baby boomers (born 1945-1965) have already started taking their retirement (by 2010). They now live on their capital income and will progressively dissave. Indeed, Fig. 11 shows that saving rates decrease after 2020 but remain positive. By 2030, however, all baby-boomers will be retired and this starts generating massive dissaving by these generations in order to sustain their consumption levels. Additionally, this is not offset by the savings of the baby-bust generations (born 1965 and after) as their cohorts are smaller. Interestingly, however, we also see that the high fertility variant eventually push back the saving rates to positive values by mid-century because, in this scenario, larger cohorts of workers are entering the labour market, eventually entirely offsetting the dissaving of the baby boomers generations. In 2100, the saving rate will be negative under all scenarios, except the high fertility scenario.

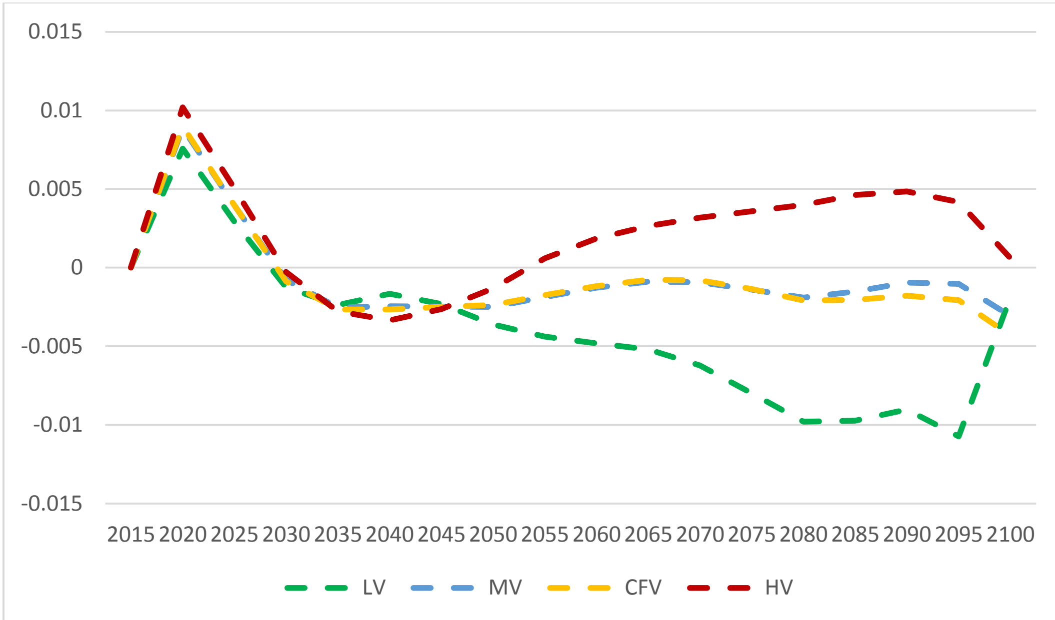

Fig. 11 Net Saving Rate

Because net saving rates are positive until 2030 across all scenarios, we see a general increase in the *K/L* ratios in Fig. 10. After 2030, we observe a decrease in the *K/L* ratio in all scenarios as net saving rates turn negative. The high fertility scenario benefits from a positive saving rates and therefore an increase in the stock of capital (and labour productivity) in the second part of the century. However, this is offset by a larger working age population (larger *L*) after 2040, until about 2080, after which results are reversed.

Observe that there is a direct relation between the *K/L* ratio (Fig. 10) and labour productivity *Y/L* (Fig. 12).[4] Hence, all these effects (changes in support ratios *L/POP* and changes in the *K/L* – and thus *Y/L* –ratios) largely explain what happens to GDP per capita in Fig. 8, given that

$Y/Pop = Y/L * L/Pop$.

[4] Note that simulations under the low fertility variant generated one infeasibility, which probably explains the end of period jump in labour productivity in Fig. 12).

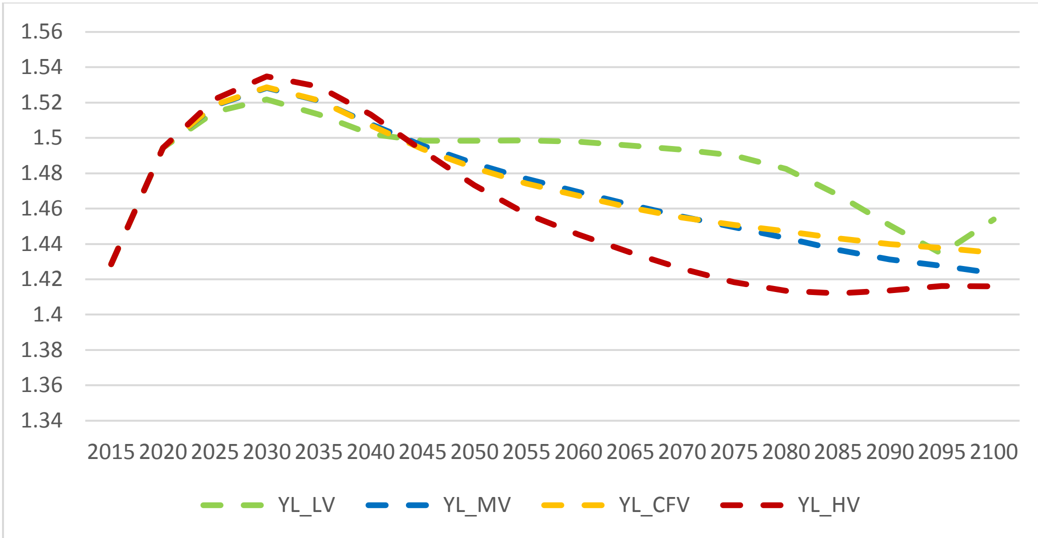

Fig. 12 Labour productivity (Y/L)

Finally, observe the impact of the *K/L* ratio (or *Y/L* ratio) on factor prices (Fig. 13 and Fig 14). Consistent with theory, when the *K/L* ratio increases (until 2030 in Fig. 10) we observe an increase in labour productivity (Fig. 12), which increases the wage rate (Fig. 13). This is the well know capital-deepening effect of a progressively ageing society. Smaller cohort of workers have access to more capital per worker, which make them more productive and permit to sustain higher wages.

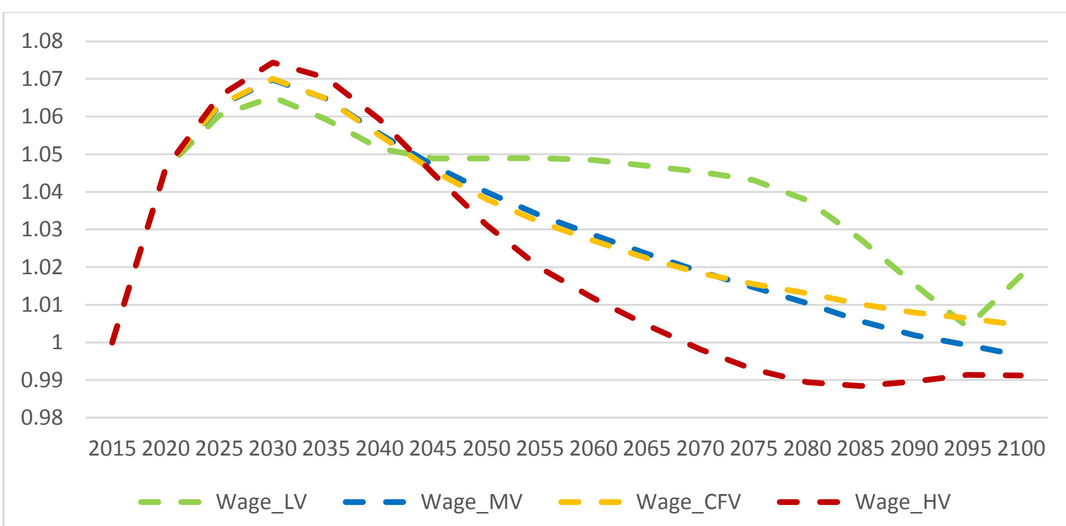

Fig. 13 Wage Rate

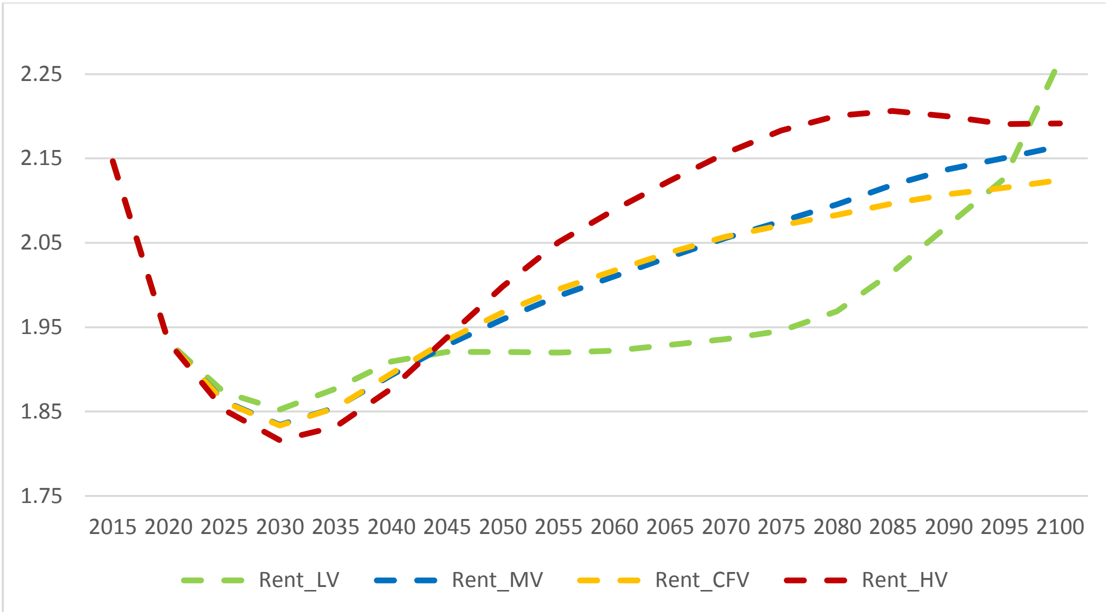

Fig. 14 Rental rate of Capital

Yet, starting 2030 all baby boomers are retired, and massive dissaving decrease the stock of capital. At that moment, capital becomes the rare factor (K/L decreases in Fig. 10) and this will generate upward pressures on the interest rate (the return to capital). This is consistent with the asset meltdown assumption according to which the price of assets could start to tumbling down when baby boomers will start to sell their assets 'en masse'. As there is an inverse relationship between assets prices (that fall when we sell them) and interest rates, we should therefore see an increase in market interest rates to attract potential savers (i.e., potential buyers of the assets sold by the baby boomers).

Although this same general pattern is observed across all scenarios, observe however that in the high fertility scenario the labour force will be larger by 2040, and this would negatively affect the wage rate and positively affect the interest rate. On the other hand, the low fertility scenario should favor wage earners over capital rentiers (compare both scenarios in Fig. 13 and 14).

5.2 Government sector

Before discussing government expenditure, it is important to note the age profiles for government expenses on health and education per age groups. Government spends more money for young people on education than on the elderly, and spends more for old people on health. Thus, as Fig. 15 shows, it is assumed that age groups G2-G5 (ages 5-10 to 20-25) receive most of education resources per capita. However, the government spends more on health on a per capita basis, on the elderly. People tend to remain relatively healthy until 45 years old. After that, the health expenses increase, reaching its peak at G18 (85-94 year old).

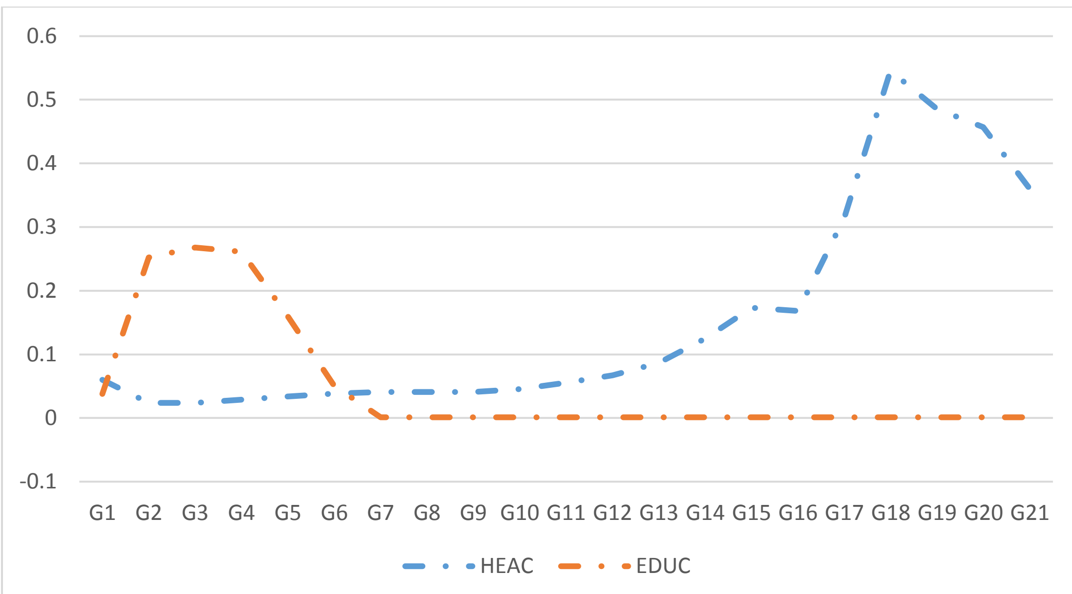

Fig. 15 Age Profile of Government Expenditure on Health (HEAC) and Education (EDUC)

Demographic changes, by changing the population structure over time, will therefore have an impact on education and health spending and fiscal consequences. Fig. 16.1 illustrates the pressures in healthcare cost due to population ageing. From 2015 to 2100, the growth rates in these healthcare costs for the low, medium, constant, and high fertility variants are 89.7%, 122.8%, 119.8%, and 162.9%, respectively. Health expenditures increase across all variants although, as expected, the upward pressure is higher in the high fertility variant as the population size is also bigger under this scenario. However, because the GDP will also be higher under this scenario, it is worth projecting these costs as a ratio of GDP, as illustrated in Fig. 16.2. Interestingly, we now get the reversed result. In fact, under a high fertility scenario, the ratio of expenditure on health to current GDP rise at a really low level, compared to other scenarios.

In Fig. 17.1, we see that the low fertility variant which lead to smaller size cohorts, also lead, as expected, lower government expenditures on education. On the other hand, the high fertility variant that generate larger cohort sizes will lead to higher levels of education expenditures. The growth rate from 2015 to 2100 is -35.6% under low fertility scenario, and they are 1.9%, 6.3%, and 109.7%, respectively, under the other scenarios. In Fig. 17.2, we also show the profiles in terms of education expenditures per GDP.

Finally, for other age government spending, it was assumed that they remain constant per capita (identical for each age group). Figure 17.1 shows these projected paths. For medium, constant, and high fertility, 'other' government spending increases by 32.1%, 25.2%, and 83.1%, respectively. For the low variant it falls by 6.5%. Observe that these paths are the same as those projected for the size of population under the four variants (Fig. 1). It is normal because we assume that these government spending remain constant per capita, so that when population grows, these government spending grow proportionally. Finally Fig. 17.2 shows these paths as a ratio of GDP. As Fig. 17.2 shows, under a high fertility scenario, the ratio of 'other' government spending to GDP remains generally above the same ratio for other variants. However, around 2090, the ratio of 'other' government spending to GDP under the low fertility scenario will start to overcome the same ratio for the high fertility case. Therefore, if Canada remains at a low fertility rate, the population size in Canada will get smaller, allowing for smaller government spending, but GDP will fall even more, so that the spending in terms of GDP will eventually increase.

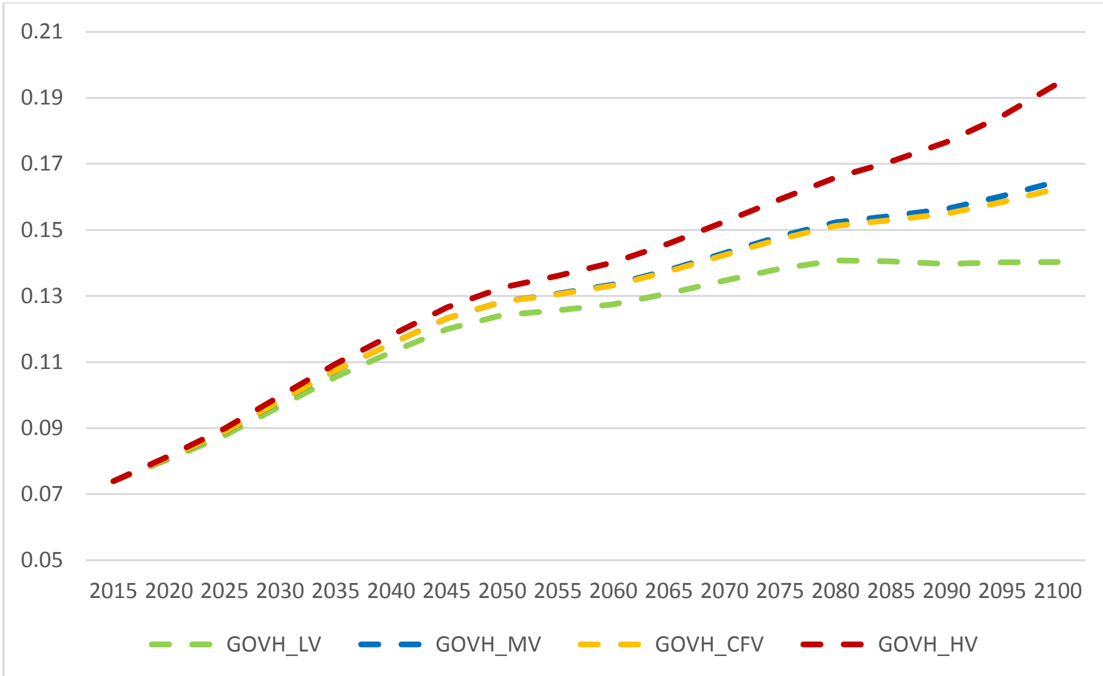

Fig. 16.1 Government Expenditure on Health

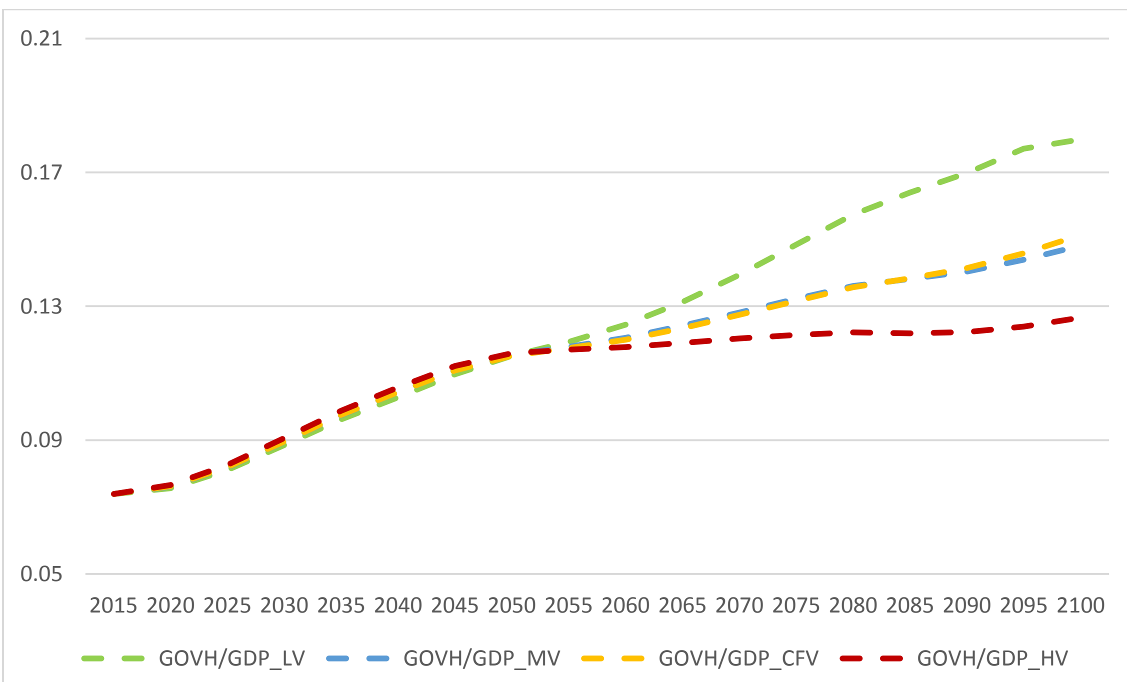

Fig. 16.2 Government Expenditure on Health to Current GDP

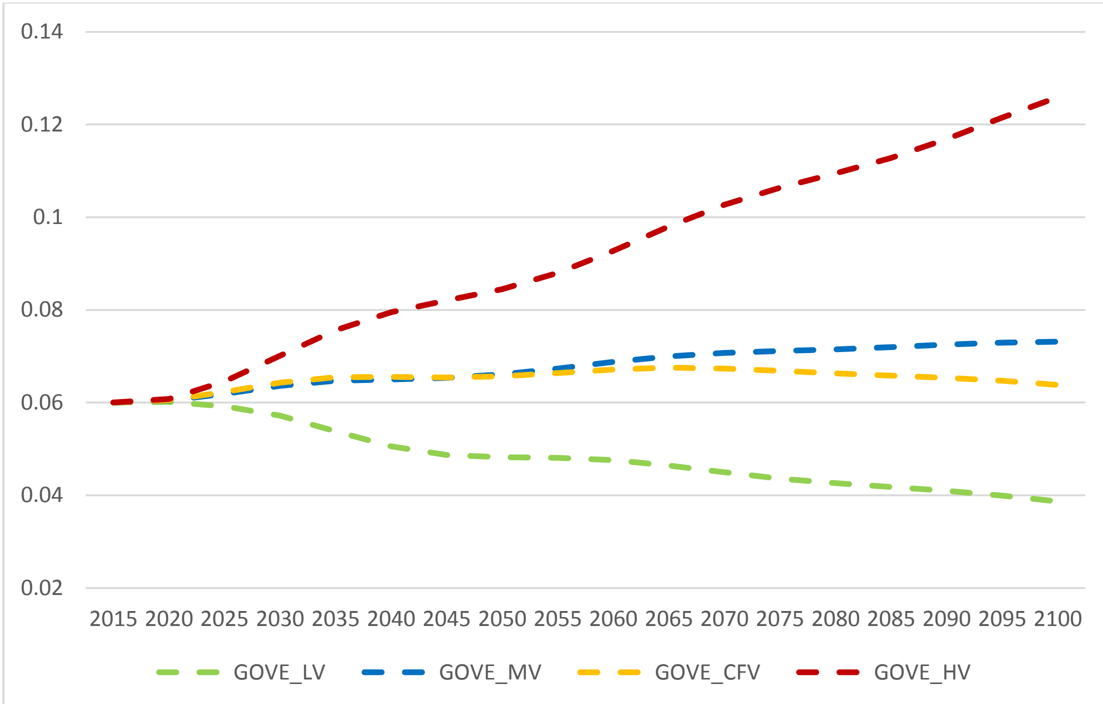

Fig. 17.1 Government Expenditure on Education

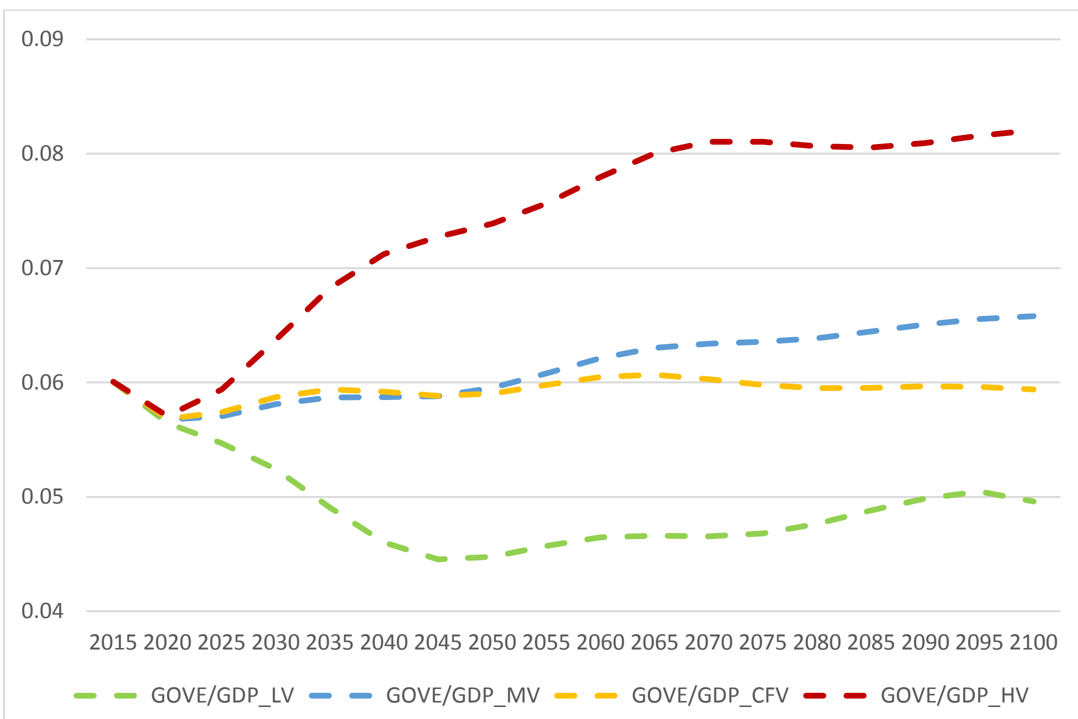

Fig. 17.2 Government Expenditure on Education to Current GDP

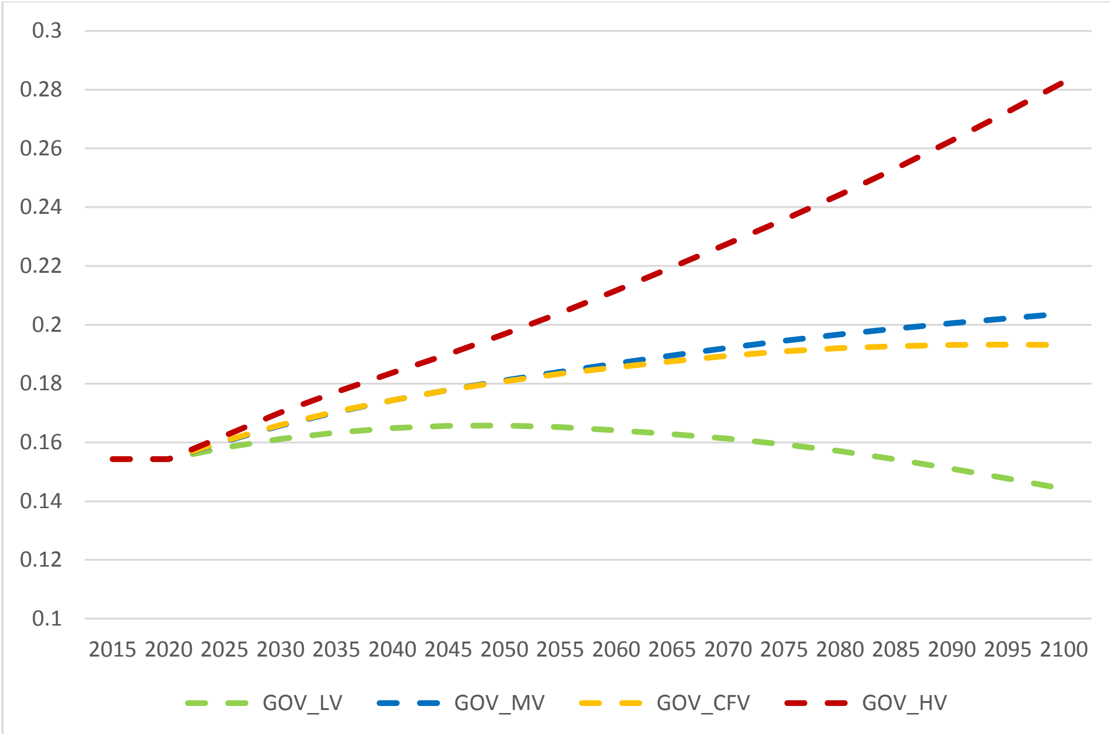

Fig. 18.1 Government Expenditure on Others

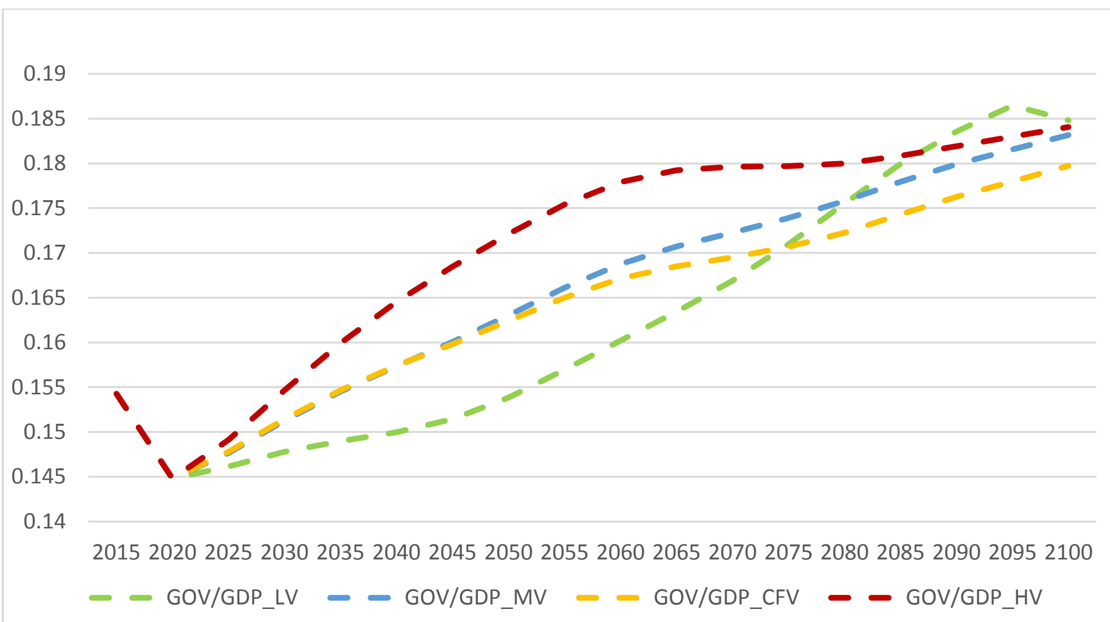

Fig. 18.2 Government Expenditure on Others to Current GDP

We now move to the pay-as-you-go pension plan projections. Fig. 19 shows that total pension benefits are expected to increase due to population aging. From 2015 to 2080, growth rates of total pension benefits for all scenarios are around 148%. Note that these paths reflect the explicit commitment of a pay-as-you-go system where the defined benefit rate is known in advance (whereas the contribution rate is an endogenous variable that adjusts so as to make the pension plan sustainable).[5] By 2080, however, the high fertility variant implies a larger cohort of retiring workers, (when they reach 65 years old) so that total pension benefits start to increase further in this scenario (relative to the other variants). From 2080 to 2100, pension benefits for the low fertility variant reduce by 4.8%; medium, constant, and high fertility variants increase by 7.6%, 8.4%, and 20%, respectively.

It would be a mistake, however, to draw the conclusion that the lower fertility variant relieves the pressures on pension benefits. In fact Fig. 20 shows an opposite result when we take the ratio of pension benefits to current GDP (pension benefits/GDP). In Fig. 20, all of the scenarios have similar growth rates from 2015 to 2040. However, after 2040 different cohort sizes (depending on the fertility variants) will join the labour market, which influences the GDP accordingly. From 2040 to 2100, the ratio of pension benefits to current GDP for low, medium, constant, and high fertility variants increase by 77.2%, 40%, 45.9%, and 12.9%, respectively. This ratio indicates that even though pension benefits under a high fertility scenario are eventually higher than under other scenarios, workers under this scenario will have less pressure for taking care of the elderly. To gain a deeper understanding of this effect, it is important to consider the changes in the pension contribution rates needed to ensure a sustainable pension system.

[5] Note that pension benefits depend on a constant (and promised) pension benefit rates applied to an average life-time labour income. The average life time labour income is slightly lower under the high fertility variant because of the pattern for the wage rate given in Fig. 13. This explains why aggregate pension benefits are slightly lower in 2070 and 2080 (This concerns people born before 2015 but that will be affected at some point in the labour market by the fertility variants and therefore by the different sizes of future (and younger) cohorts of workers.

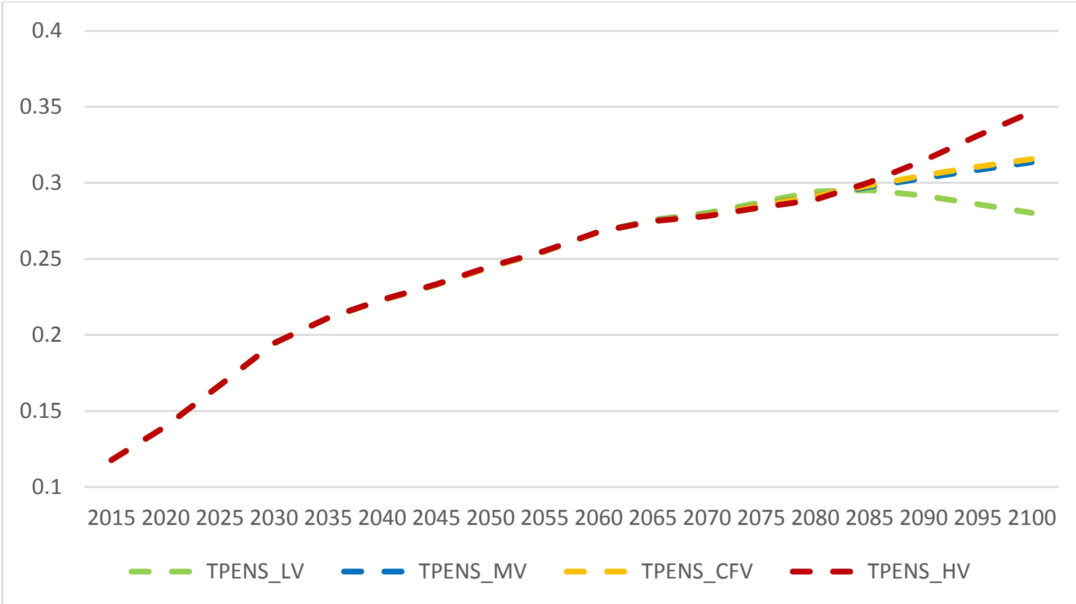

Fig. 19 Total Pension benefits

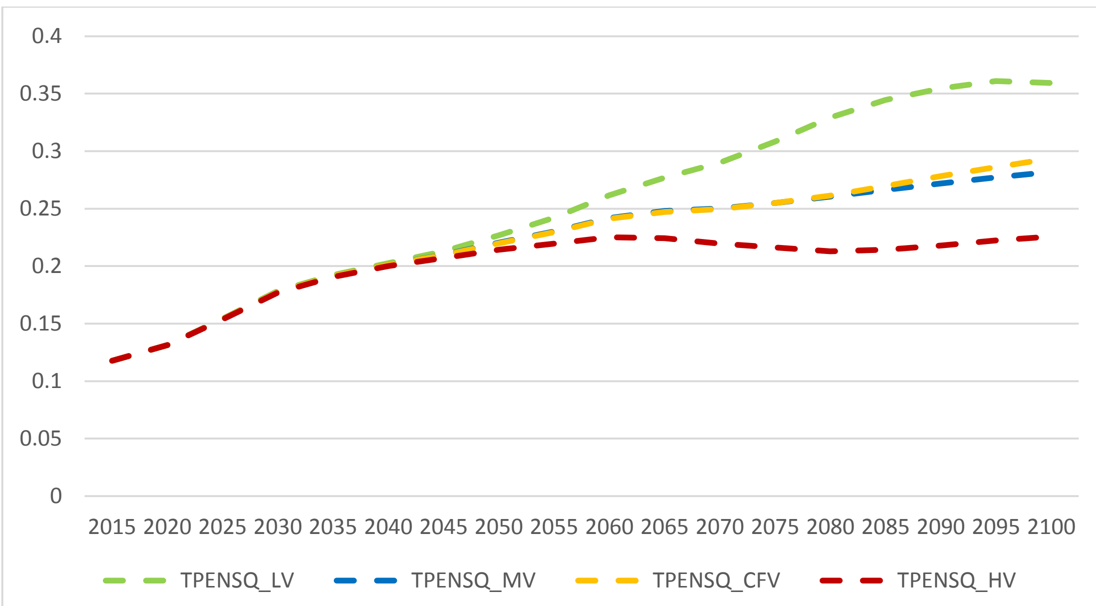

Fig. 20 Total Pension benefits to Current GDP

Fig. 21 represents the evolution of the (endogenous) contribution rates for the four fertility scenarios. As the number of old people increases, the contribution rate rises as well. In 2100, the contribution rate of each scenario increases by 205.1%, 139.4%, 149.5%, and 91.8%, respectively. Clearly, a high fertility variant that leads to larger sizes of workers (starting 2040) will alleviate the pressures on the contribution rates that these workers must pay to finance the promised pension benefits of the elderly. From 2040 to 2100, the contribution rate of the low fertility variant increases by 77.2%, while that of the high fertility variant rises by only 12.9%. What is more, between 2060 and 2100, the contribution rate of the low fertility variant increases from a rate of 37% to 51%, compared to that of high fertility variant, which stays between the range of 30% and 32%. The pressure on the contribution rate stays at a low level under a high fertility scenario compared to other scenarios in the long-term, even though the total pension benefits of the high fertility scenario increases to a higher level after 2080 (see Fig.19). This reveals that a high fertility scenario might mitigate the workers' pension contribution burden. However, a contribution rate of 30% (of the labour income) for households is still an issue for Canadian society.

As noted before, government expenditures will increase dramatically. Due to population aging, health expenses will rise. The way for government to finance this is through taxation. Fig. 22 shows the rate of labor income tax. We observe a significant increase in the wage tax rate over the century. The rate increases by 98%, 81.7%, 78.9%, and 75.2% under low, medium, constant, and high fertility variants, respectively. The increase in the wage tax rate arrives sooner under the high fertility scenario because there is an extra increase in government expenditure on education while the other government spending (that remains constant per capita) must increase with the size of the population. However, after 2085, the labour income tax rate in the low fertility scenario exceeds the rate in the high fertility scenario. Thus, this reveals that even if the low fertility scenario is associated with smaller government spending and seems to relieve pressure on the labour income tax for households, eventually, this will be paid back in the future.

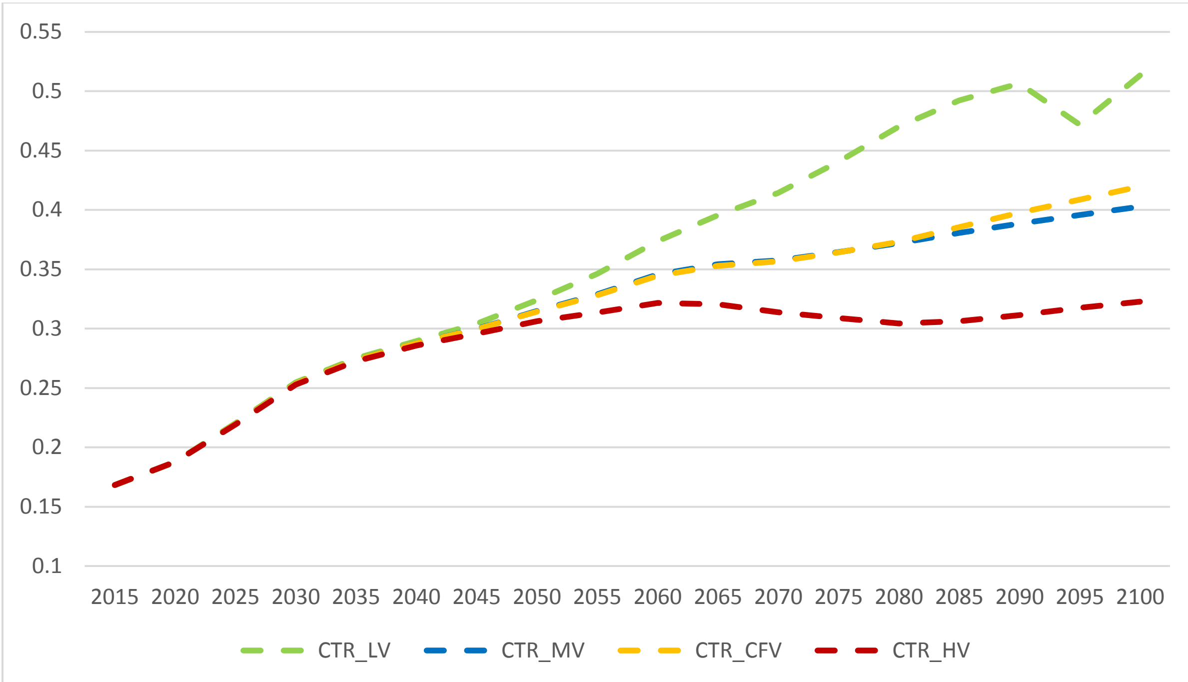

Fig. 21. Contribution Rate of Pension Plans

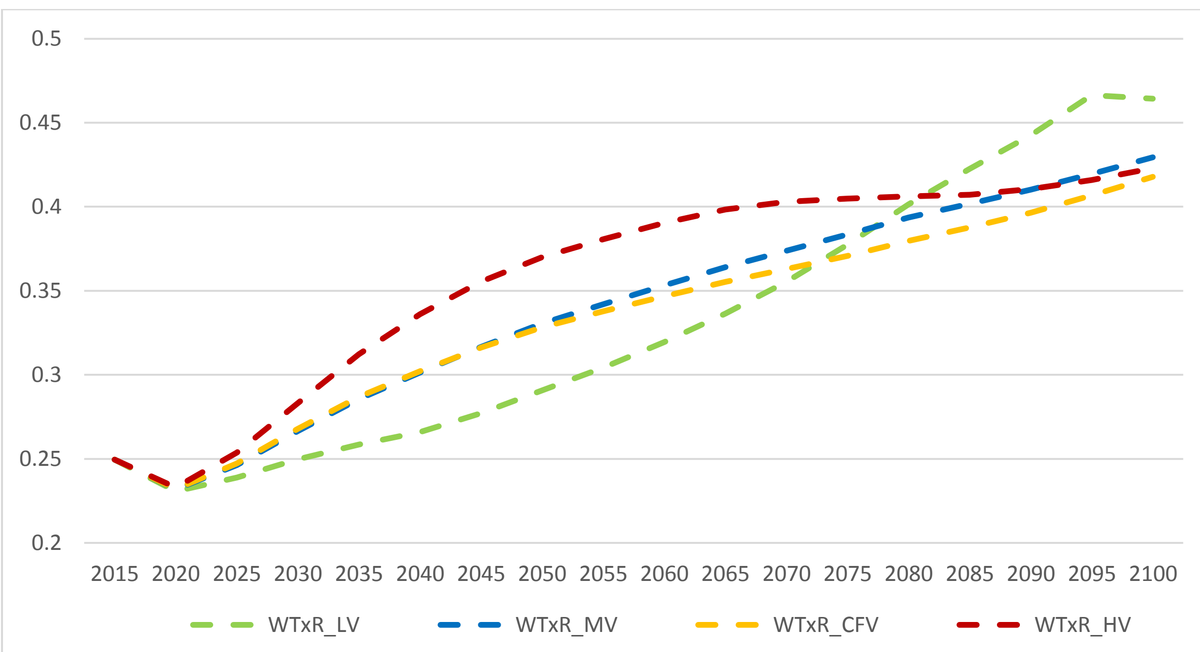

Fig. 22 Wage Tax Rate

6. Conclusion, caveats and future research

The paper attempts to evaluate the impacts of demographic trends on the Canadian economy. Because demographic projections are uncertain, economic projections are also uncertain. This is why it was decided to analyse four demographic projections provided by the United Nations (the medium, high, low, and constant fertility variants).

The simulation results outlined in this paper illustrate that population aging in Canada affects the economy in various ways. First, the wage rate will rise in the short term, while it will drop to the original level (or below) in the long term. Second, when people learn about the demographic shock (or more correctly, about the demographic revision) people realize that their life expectancy has increased. Although this is a good news for them, it also triggers a desire to increase their saving to provide for their retirement. Accordingly our model predicts an increase in net saving rates, as well as domestic investment. However, after a temporary upsurge, the net saving rate usually goes back and even becomes negative. Third, from the trends of total pension benefits under different fertility scenarios, we can find that population aging challenges the Canadian pension system. The contribution rate of pension plans is expected to increase over the next 20 years. The situation will be getting worse with a lower fertility rate. For a long-term solution, instead of attempting to increase the fertility rate, policy makers can set alternative policies, such as, for example, postponing retirement or increasing immigration. Finally, government expenditure will increase under all fertility scenarios; therefore, the tax on the wage rate will increase dramatically.

The model that we have used to provide these projections is rather simple, so that there are several caveats that need to be mentioned. First, Canada is assumed to be a closed economy. Yet international trade plays a significant role in today's economy. Introducing an open economy may typically mitigate the issue on population aging. It can bring more labor force and stimulate consumption and investment. In a closed economy model national saving is equal to domestic investment. However in an open economy, there are foreign funds that are flowing in and out the country, which would affect the current account of the economy and mitigate the impacts that ageing can have on factor returns.

Second, it was assumed that labor supply is exogenous. The households do not make decision on work and leisure. Yet we know that household is likely to do this in reality, to some extent, so that labour supply should be endogenous and dependent on age. Also, it is well-known that an endogenous labour supply setting provides a more accurate age-dependent consumption profile over the life-cycle.

Third, we assumed a pay-as-you-go system. Yet, the pension system in Canada is a hybrid of a PAYG and fully funded system. That is, the pension system might not be as bad as our model projects. Further study is needed to explore pension reforms, including reductions in the pension benefit replacement rate.

Finally, the model used assumes a perfect annuity markets (as was also modeled in Börsch-Supan et al. (2006) and in Georges and Seçkin (2016). In this setting, people do not leave intentional bequest. But unintentional bequest are transferred to an insurance company in return for an annuity over their retirement life-time. Hansen and Imrohoro˘glu (2008) have introduced an incomplete annuity market and note that "when the annuity market is shut down, consumption displays a hump shape where consumption peaks well before retirement." In fact "consumption might increase early in life when survival probabilities are high and the effective rate of discount is low. As survival probabilities fall, the slope of the consumption profile may become negative." Hence, the consumption path over the life cycle is more realistic than under a model with exogenous labour supply and perfect annuity market whereby the consumption profile tends to increase over the lifecycle.

On a last policy note, increasing the fertility rate is not the only method the government can use to boost the economy under an aging population issue. There are other ways that might be more effective to achieve the same goal. For example, applying a more aggressive immigration policy by importing more skilled workers into the Canadian economy can change the age structure. However, it is important to note that instead of population aging, the restructuring of age groups through immigration could cause even more complex issues. Thus, policy makers must pay close attention to the social consequences of their policies with respect to population ageing.

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