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Avoiding Blindness to Health Status: A New Class of Health Achievement and Inequality Indices

Paul Makdissi* and Myra Yazbeck†

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* CIRPÉE and Department of Economics, University of Ottawa, 55 Laurier E. (10125), Ottawa, Ontario, Canada, K1N 6N5; Email: paul.makdissi@uottawa.ca.
† CIRPÉE and Department of Epidemiology Biostatistics and Occupational Health, Purvis Hall, McGill University, Montreal, Canada, H3A 1A2; Email: myra.yazbeck@mcgill.ca.

Abstract

This paper argues that health transfers from an individual at a lower rank in the health distribution to a person at a higher rank may decrease the concentration index if the former has a slightly higher income. The concentration index, being mainly focused on the socio-economic dimension of health inequality, can produce such counter-intuitive results that overlook the pure health inequality aversion of the planner. Building on Atkinson (1970), Yitzhaki (1983) and Wagstaff (2002), this paper presents a simple new class of health achievement and health inequality indices that overcomes the above mentioned problem.

Key words: *Health inequality, Health Achievement.*

JEL Classification: D63, I10.

Résumé

Cet article propose argue qu'un transfert de santé d'un individu vers un autre situé à un rang plus élevé de la distribution de santé peut faire diminuer l'indice de concentration si le premier individu a un revenu légèrement supérieur. Cet indice, en se concentrant exclusivement sur la dimension socioéconomique des inégalités de santé peut produire ce genre de résultat contre-intuitif puisqu'il est totalement insensible aux inégalités pures de santé. Cet article construit sur les apports d'Atkinson (1970), Yitzhaki (1983) et Wagstaff (2002) afin de proposer une nouvelle classe d'indices d'accomplissement et d'inégalité de santé qui corrige le problème mentionné.

Mots clés: *Inégalité de santé, accomplissement en santé.*

Classification JEL: D63, I10.

1 Introduction

A large body of the health inequality measurement literature is based the accumulated knowledge in income inequality measurement. Early contributions to health inequality measurement by Le Grand (1989) and Le Grand and Rabin (1987) proposed the well known Gini coefficient as measure of *pure* inequality in mortality. Yet, because the social planner may often be interested in the socioeconomic dimension of health inequalities (rather than pure inequalities), the use of the concentration index was found to be more appropriate (see Wagstaff, van Doorslaer and Paci, 1989; and Wagstaff, Paci and van Doorslaer, 1991). Consequently, most of the literature adopted the concentration index as a widely accepted measure of health inequality.

The concentration index (or any other relative inequality measure) presents three well-known measurement problems. First, it does not capture variations in the average level of health of the population considered (for more details see Wagstaff, 2002). As a result, a policy that improves the average level of health, while keeping its relative distribution constant (i.e., a translation of distribution), will be erroneously deemed a neutral policy. Second, the concentration index has a poor performance when discrete variables are used. The ordinal nature of the information allows the analyst, at best, to rank individuals. This, in turn, makes the value of the concentration index somewhat arbitrary (Erreygers, 2006 and Zheng, 2008).[1] The last measurement problem is the *mirror problem* as pointed out by Clarke *et al.* (2002). It stresses on the inconsistency in the rankings produced by health attainment and those produced by health shortfalls. More specifically, the concentration index of health was proved to be equal to the concentration index of shortfall in health multiplied by the ratio of average shortfall in health to average health status (see Erreygers, 2009).

To address the first problem, Wagstaff (2002) suggests the use of an achievement index that captures simultaneously the average level of health and the socioeconomic inequality of its distribu-

[1] Most of the available health status information is given in the form of categorical variables.

tion. A real solution to the second problem has not yet been offered in the literature. A common practice is to assume that there exist a reliable ratio scaled variable (e.g., the Health Utility Index (HUI)) and use it to replace the information contained in the categorical variables. We will adopt this solution. To account for the third measurement problem, Erreygers (2009) has proposed a modified version of the concentration index. Yet, recently Lambert and Zheng (2011) showed that there exist no index of relative inequality that can avoid the *mirror problem*. For this reason, we focus on inequality in health statuses and consider inequality in shortfall as a contributor to total health inequality.

The objective of this paper is to shed light on a fourth measurement problem that has not been acknowledged in the literature on concentration indices. More specifically we point out the indices' *blindness to health status*. We show that any index that belongs to Wagstaff's class of health achievement indices or extended concentration indices reacts favourably when a health transfer is made from an individual at a lower rank in the health distribution to a person at a higher rank, provided that the former has a slightly higher income. This measurement problem makes the reliability of these indices questionable, especially if they are used for health policy performance evaluation. To address this issue, we first propose a more general class of indices that allows for the presence of an arbitrage between the health status and the socio-economic status. We then develop the axiomatic framework that describes the desired ethical properties of the proposed health achievement index as well as the associated socio-economic health inequality index.

The remaining of the paper unfolds as follows. The next section presents the theoretical measurement framework followed by the proposed parametric class of health achievement and inequality indices. Section 3 presents a brief empirical illustration using the Joint Canada/United States Surveys of Health 2004 and the Canadian Community Health Survey 2007-2008. The last section summarizes our results and suggests few possible extensions.

2 Measurement Framework

In what follows, we give a brief description of Wagstaff's health achievement index and the extended concentration index and discuss the possible issues that may result from the use of this index. To overcome these issues, we first propose a new generalized class of health achievement and inequality indices. We then propose a set of normative axioms that characterizes this class of indices. Finally, we present a parametric sub-class of this generalized class of indices that can be used in empirical work.

2.1 Wagstaff's Health Achievement Indices and Health Concentration Indices

To measure socioeconomic health inequality, individuals are ranked by their socio-economic status from lowest to highest. Let y represent individual income (or any other indicator of the socioeconomic status) and $F(y)$ be the cumulative distribution of income such that $p = F(y)$ is the socioeconomic rank of an individual with income y. Let $h(p)$ is be the health status of an individual at socioeconomic rank p, then Wagstaff's achievement indices can be written as follows:

$$A(\nu) = \int_0^1 \nu(1-p)^{\nu-1} h(p)dp, \ \nu > 1, \tag{1}$$

where ν can be interpreted as a parameter of socio-economic health inequality aversion (Yitzhaki, 1983). Wagstaff's class of extended concentration indices, $I(\nu)$, can be associated to the achievement indices defined in (1) using $I(\nu) = 1 - \frac{A(\nu)}{\mu}$, where $\mu = \int_0^1 h(p)dp$ is the average level of population's health. The exact expression for those extended concentration indices is

$$I(\nu) = \frac{1}{\mu} \int_0^1 \nu(1-p)^{\nu-1}(\mu - h(p))dp. \tag{2}$$

When $\nu = 2$, equation (2) gives the standard health concentration index that is widely used in the health inequality literature. Any index that belongs to Wagstaff's class of health achievement (concentration) indices may increase (decrease) when a health transfer is made from an individual at a lower rank in the health distribution to a person at a higher rank, provided that the former has a slightly higher income. These indices can exhibit such an erratic behaviour as by construction the impact of a marginal increase in the level of health status $h(p)$ on the achievement index $A(\nu)$ (i.e., $\nu(1-p)^{\nu-1}$) is independent of the health status and decreasing in the socio-economic rank p. This is why, in some cases, it can be misleading to use this class of indices for health policy evaluation.

2.2 An Axiomatic Approach to the Measure of Health Achievement and Inequality

In this section we propose a more general class of indices that allows the analyst to overcome the indices' *blindness to health status* pointed out earlier. We also characterize the associated social preferences using an axiomatic approach. The class of indices that we will be using is derived from social preferences over the health distribution that allows for dependence between the health status and the socio-economic status. It has the following form[2]:

$$S = \int_0^1 w(p)u(h(p))dp, \tag{3}$$

where $u(h(p))$ represents the social evaluation of the quality of life that a person derives from a health status h and $w(p)$ represents the socioeconomic weight. It is assumed that $w(p) \geq 0$ and $\int_0^1 w(p)dp = 1$. We first use an equally equivalent distributed level of health A as a health achievement index. It represents the level of health equally distributed such that the social planner will be indifferent between the level of S and $u(A)$.[3] Formally A is defined implicitly by:

$$u(A) = \int_0^1 w(p)u(h(p))dp, \tag{4}$$

[2] This mathematical form is inspired from the literature on income inequality (Berrebi and Silber, 1981), it was transposed in pure health inequality by Lambert and Zheng (2011) who used health status as a ranking variable. The form that we use is different from the one in Lambert and Zheng (2011) since we use the socioeconomic status as the ranking variable instead of health status.

[3] Note that $\int_0^1 w(p)u(A)dp = u(A)$.

which can be re-written as:

$$A = u^{-1}\left[\int_0^1 w(p)u(h(p))dp\right]. \tag{5}$$

It is well-known that when the social preferences display pure or socioeconomic health inequality aversion, a health inequality index can be associated to it (Atkinson, 1970):

$$I = 1 - \frac{A}{\mu}. \tag{6}$$

This is a measure of the relative loss resulting from unequal distribution of health. The health inequality index and the health achievement index are linked to the social preferences as laid in equation (3). For this reason, understanding the ethical principles underlying these classes of indices requires an axiomatic approach that characterizes the social preferences in equation (3). In what follows, we present these axioms and discuss the conditions under which they are satisfied.

Axiom 1 ***Monotonicity (M) :*** *Given other things, an increase of the health status of one individual must increase the index of social preferences over the health distribution.*

Lemma 1 *A satisfies **M** if* $\frac{du(h)}{dh} > 0$.

To account for pure health inequalities, the achievement index and the health inequality index must satisfy (all proofs are in appendix)

Axiom 2 ***Pure Health Transfer (PHT) :*** *Given other things, a pure transfer of health from a person to any other person with the same socioeconomic status but a lower health status must increase (decrease) the health achievement (inequality) index.*

Lemma 2 *A and I satisfy **PHT** if* $\frac{d^2u(h)}{dh^2} < 0$.

The Lorenz dominance condition proposed by Le Grand (1989) and Le Grand and Rabin (1987) obeys $\boldsymbol{PHT}$. However, Wagstaff's achievement indices $A(\nu)$ do not satisfy $\boldsymbol{PHT}$ since they are linear in the health status. If one wants to take into account the pure health dimension of health inequalities then, another class of indices is needed.

In order to account for socioeconomic health inequalities, the achievement index must satisfy

Axiom 3 ***Socioeconomic Health Transfer (SHT) :*** *Given other things, a pure transfer of health from a person to any other person with the same health status but a lower socioeconomic status must increase (decrease) the health achievement (inequality) index.*

Lemma 3 *A and I satisfy* $\boldsymbol{SHT}$ *if* $\frac{dw(p)}{dp} < 0$.

A health achievement or inequality index that satisfies $\boldsymbol{PHT}$ but does not satisfy $\boldsymbol{SHT}$ will be blind to socioeconomic status. Also, a health achievement or inequality index that satisfies $\boldsymbol{SHT}$ but does not satisfy $\boldsymbol{PHT}$ will be blind to health status. From a social choice perspective, it is natural to impose both ethical principles (i.e., $\boldsymbol{SHT}$ and $\boldsymbol{PHT}$) on health achievement (inequality) indices. Whilst Wagstaff's achievement indices $A(\nu)$ and the extended health concentration indices $I(\nu)$ do not satisfy $\boldsymbol{PHT}$, they do satisfy $\boldsymbol{SHT}$.

In the income inequality literature, it is common to impose more ethical principles to characterize higher levels of inequality aversion. Kolm (1976) argues that transfers that occurs in the bottom of the income distribution may be deemed more important from the social planner's point of view. This idea can be transposed in the dimension of pure health inequality with the following axiom:

Axiom 4 ***Pure Health Transfer Sensitivity(PHTS) :*** *If a pure health transfer takes place from a person with health status h to another person with the same socioeconomic status but a lower* *health status $h-\iota$, then the magnitude of the increase (decrease) the health achievement (inequality) index must be larger for a lower h.*

Lemma 4 *A and I satisfy **PHTS** if $\frac{d^3u(h)}{dh^3} > 0$.*

Note that since they do not satisfy ***PHT***, Wagstaff's achievement indices $A(\nu)$ do not satisfy ***PHTS*** for the same reason: the linearity of those indices in health statuses. Intuitively this is trivial since ***PHTS*** is a more stringent requirement on pure health inequality aversion than ***PHT***.

The principle of health transfer sensitivity can also be applied to socioeconomic health inequalities by adapting Zoli's (1999) positional transfer sensitivity principle:

Axiom 5 ***Socioeconomic Health Transfer Sensitivity(SHTS) :*** *If a pure transfer of health takes place from a person at socioeconomic rank p to another person with the same health status but at a lower socioeconomic status $p-\varrho$, then the magnitude of the increase (decrease) the health achievement (inequality) index must be larger for lower p.*

Lemma 5 *A and I satisfy **SHTS** if $\frac{d^2w(p)}{dp^2} > 0$.*

Wagstaff's achievement indices $A(\nu)$ and the generalized health concentration indices $I(\nu)$ both satisfy ***SHTS*** when $\nu > 2$.

It would be possible to develop higher order ethical principles by adapting Fishburn and Willig (1984) generalized transfer principles to pure health inequalities as well as Aaberge (2009) and Makdissi and Mussard (2008) positional generalized transfer principles to socioeconomic health inequalities but this is beyond the scope of this paper.

2.3 A Parametric Class of Health Achievement and Inequality Indices

In this section we provide a parametric class of health achievement (and its corresponding health inequality indices) that respects the general class presented in the previous section. The social preferences associated with this parametric class of indices will take the following form:

$$S(\nu,\varepsilon)=\begin{cases}\int_0^1 \nu(1-p)^{\nu-1}\frac{h(p)^{1-\varepsilon}}{1-\varepsilon}dp & \text{for } \nu\geq 1,\ \varepsilon\geq 0 \text{ and } \varepsilon\neq 1\\ \int_0^1 \nu(1-p)^{\nu-1}\ln h(p)dp & \text{for } \nu\geq 1,\ \varepsilon=1\end{cases} \tag{7}$$

The corresponding class of health achievement index is given by:

$$A(\nu,\varepsilon)=\begin{cases}\left[\int_0^1 \nu(1-p)^{\nu-1}h(p)^{1-\varepsilon}dp\right]^{\frac{1}{1-\varepsilon}} & \text{for } \nu\geq 1,\ \varepsilon\geq 0 \text{ and } \varepsilon\neq 1\\ e^{\int_0^1 \nu(1-p)^{\nu-1}\ln h(p)dp} & \text{for } \nu\geq 1,\ \varepsilon=1\end{cases} \tag{8}$$

and the class of health inequality index takes the following form:

$$I(\nu,\varepsilon)=\begin{cases}1-\frac{1}{\mu}\left[\int_0^1 \nu(1-p)^{\nu-1}h(p)^{1-\varepsilon}dp\right]^{\frac{1}{1-\varepsilon}} & \text{for } \nu\geq 1,\ \varepsilon\geq 0 \text{ and } \varepsilon\neq 1\\ 1-\frac{1}{\mu}e^{\int_0^1 \nu(1-p)^{\nu-1}\ln h(p)dp} & \text{for } \nu\geq 1,\ \varepsilon=1\end{cases} \tag{9}$$

As mentioned earlier, we can interpret ν as a parameter of socioeconomic health inequality aversion (Yitzhaki, 1983). The term ε can be interpreted as a parameter of pure health inequality aversion (Atkinson, 1970). When ε is set to zero, we obtain Wagstaff's class of achievement and extended concentration indices. Further, when ε is set to zero and ν is set to two, then $I(2,0)$ is the widely used health concentration index.

Using lemmas 1, 2, 3, 4 and 5, we can describe the ethical properties of this class of indices:

Proposition 1 *The health achievement index $A(\nu,\varepsilon)$ satisfies **M**. Furthermore, the health achievement index $A(\nu,\varepsilon)$ and the health inequality index $I(\nu,\varepsilon)$ satisfy:*

1. ***PHT** and **PHTS** if $\varepsilon>0$,*
2. ***SHT** if $\nu>1$,*
3. ***SHTS** if $\nu>2$.*

From Proposition 1, it is clear that neither Wagstaff's achievement index, nor the extended health concentration index obey $\boldsymbol{PHT}$. This explains why they are blind to health status. Thus, a policy maker who wishes to account for health status and socioeconomic status simultaneously should choose a positive value for ε and a value for ν that exceeds one.

3 Empirical Illustration

In this section, we present a brief illustration using the parametric class introduced in the previous section. We also provide empirical evidence that this parametric class may, in some cases, rank distributions differently if compared to the ranking given by Wagstaff's class of indices. In a first step, we compare health inequalities between the U.S. and Canada. We show how the health achievement and the health inequality rankings between Canada and the U.S. seem robust to a change in socioeconomic or pure health inequality aversion. In a second step, we focus on health inequalities in Canada, more precisely in the Greater Montréal region. We divide the extended Montréal region into four administrative subregions. This allows us to illustrate how a change in socioeconomic and/or pure health inequality aversion can change the ranking between some of these subregions when the indices' blindness to health status is accounted for.

3.1 Comparing health distribution in the U.S. and Canada

To compare health achievement and health inequality in Canada and in the U.S., we use the Joint Canada/United States Surveys of Health (JCUSH) 2004. This survey entails 8,688 observations of which 3,505 are Canadian residents and 5,183 are U.S. residents. It covers individuals between the age 18 and 85 years and information about their clinical condition as well as their demographic characteristics and their socioeconomic status. We use information on household income to infer the socioeconomic rank of the individual. The health status of the individual is based on the Health Utility Index Mark 3 (HUI3), which covers eight attributes: vision, hearing, speech, ambulation, dexterity, emotion, cognition, and pain. Each attribute has five or six levels and each attribute utility score ranges from 0 (for instance blind for vision) to 1 (perfect vision). The HUI3 ranges from -0.36 to 1, the negative values are there to express a state that is worse than death whereas values of 0 reflect death.

Table 1: Indices for Canada and the U.S.A.

	$A(\nu=2,\varepsilon)$			$I(\nu=2,\varepsilon)$		
	$\varepsilon=0$	$\varepsilon=1$	$\varepsilon=2$	$\varepsilon=0$	$\varepsilon=1$	$\varepsilon=2$
Canada	0.841327	0.759429	0.349726	0.03528	0.129189	0.598981
USA	0.825597	0.725244	0.284505	0.043362	0.159644	0.670338
	$A(\nu,\varepsilon=1)$			$I(\nu,\varepsilon=1)$		
	$\nu=1$	$\nu=2$	$\nu=3$	$\nu=1$	$\nu=2$	$\nu=3$
Canada	0.810074	0.759429	0.729261	0.071116	0.129189	0.163782
USA	0.789987	0.725244	0.683903	0.084625	0.159644	0.207547

Looking at Table 1 we notice that Canada has higher health achievement indices and lower health inequality indices and that these results are robust to all assigned values of ν and ε. More specifically, these rankings are all similar to Wagstaff's achievement index $A(2,0)$ and to the standard health concentration index $I(2,0)$.[4] This means that, in this particular case, all the estimated indices would have ranked Canada above the U.S regardless of whether they are blind to health status. Thus, at this point, the added value of using our class of indices remains mainly theoretical as it is not empirically verified.

3.2 Comparing health distribution within the Greater Montréal region

We next concentrate on health achievement and inequality within the Greater Montréal region in Canada and use the Canadian Community Health Survey (CCHS) 2007-2008. This survey is cross-sectional, it covers 131,061 Canadians aged 12 and above. It provides information related to their health status, their clinical conditions, their health care utilization as well as health determinants. We have 8,572 observations for the Greater Montréal region. As in the previous example, we use information on household income to infer the socioeconomic rank of the individual and the HUI3 as indicator of the individual health status. Four subregions are considered: Montréal (located on

[4] We take the case where $\nu=2$ since it is associated with the standard health concentration index. We acknowledge that Wagstaff's class of indices allows for ν to take any value larger than one.

an island in the St-Lawrence river), Laval (located on an island adjacent to Montréal), Montérégie (suburbs located on the south shore of the river) and Laurentides (suburbs located on its north shore).

Table 2: Indices for the Greater Montréal region.

	$A(\nu = 2, \varepsilon)$			$I(\nu = 2, \varepsilon)$		
	$\varepsilon = 0$	$\varepsilon = 1$	$\varepsilon = 2$	$\varepsilon = 0$	$\varepsilon = 1$	$\varepsilon = 2$
Montréal	0.866472	0.796713	0.37618	0.022059	0.100793	0.575426
Montérégie	0.867005	0.810485	0.486994	0.025344	0.088882	0.452539
Laval	0.873196	0.819217	0.49288	0.021047	0.081564	0.447425
Laurentides	0.869551	0.812145	0.309894	0.027358	0.09157	0.653367
	$A(\nu, \varepsilon = 1)$			$I(\nu, \varepsilon = 1)$		
	$\nu = 1$	$\nu = 2$	$\nu = 3$	$\nu = 1$	$\nu = 2$	$\nu = 3$
Montréal	0.834493	0.796713	0.775435	0.058153	0.100793	0.124808
Montérégie	0.846776	0.810485	0.7902	0.048085	0.088882	0.111686
Laval	0.851641	0.819217	0.80155	0.045213	0.081564	0.10137
Laurentides	0.850999	0.812145	0.789155	0.04811	0.09157	0.117285

The computed achievement and inequality indices reported Table 2 indicate that the health achievement and health inequality rankings for the Greater Montréal region varies for different values of pure and socioeconomic health aversion parameters. Let us focus on the lower panel of Table 2 and start by looking at the health inequality index, $I(1,1)$. Proposition 1 indicates that there is no socioeconomic health inequality aversion when $\nu = 1$. The corresponding inequality ranking is: Laval, Montérégie, Laurentides and Montréal. If we introduce socioeconomic health inequality aversion (i.e. we set $\nu = 2 > 1$) and socioeconomic health transfer sensitivity (i.e. we set $\nu = 3 > 2$), the inequality ranking for this particular example remains unchanged. As for the health achievement index imposing no socioeconomic health inequality aversion, $A(1,1)$, results in the following ranking: Laval, Laurentides, Montérégie and Montréal. Increasing ν to 2 does not change this ranking. However, introducing socioeconomic health transfer sensitivity by increasing ν to 3 changes the ranking to: Laval, Montérégie, Laurentides and Montréal. These results are in line with Wagstaff's (2002) argument that taking into account the average health status and changing the level of socioeconomic health inequality aversion can modify the rankings provided by the health concentration index.

Let us turn to the upper panel of the table. It is clear that, by introducing pure health inequality aversion (i.e., $\varepsilon > 0$), the rankings provided by $A(2, \varepsilon = 0)$ and $I(2, \varepsilon = 0)$ change in most of the cases. This provides empirical support to our theoretical argument. If we consider health inequality indices $I(2, \varepsilon)$ and analyse how the rankings produced by these indices change with a variation of the parameter ε, then, for $\varepsilon = 0$, the subregions are ranked as follows (from lowest to highest inequality): Laval, Montréal, Montérégie and Laurentides. For this ranking, only socioeconomic health inequality aversion is taken into account since the social planner has no pure health inequality aversion (i.e. $\varepsilon = 0$). If we introduce pure health inequality aversion by increasing ε to 1, the ranking changes to: Laval, Montérégie, Laurentides and Montréal. Further, increasing pure health inequality to $\varepsilon = 2$ changes the ranking to: Laval, Montérégie, Montréal and Laurentides. As for the health achievement indices, consider the case where there are no pure health inequality aversion, $A(2, \varepsilon = 0)$. For this index, the ranking (from highest to lowest achievement) is: Laval, Laurentides, Montérégie and Montréal. The ranking stays the same when we increase ε to 1 but changes to: Laval, Montérégie, Montréal and Laurentides, when we consider $\varepsilon = 2$. This variation in the rankings provides an empirical evidence of the relevance of the theoretical argument regarding the importance of addressing the indices' blindness to health status.

4 Conclusion

In this paper we point out a new problem that arises when using the concentration index in the analysis of socio-economic health inequalities: the indices' *blindness to health status*. We then propose a new class of indices that accounts for this problem by allowing for dependence between health status and socio-economic rank to be present in the underlying social welfare function. Since the health achievement index and the corresponding inequality index are linked to the social welfare function, we characterize the ethical principles it should obey and provide the conditions under which they are satisfied. To illustrate how the proposed indices provide robust rankings we provide two empirical illustrations. Evidence seems to corroborate our theoretical argument that when the concentration index fails to provide robust rankings the new proposed index performs better. While this paper solves one of the measurement problems encountered when using concentration indices, some other measurement issues remain unsolved namely the *mirror problem* and the erroneous classification in the presence of categorical data. Further research in this direction is required.

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A Proofs

Proof of Lemma 2. Consider a finite transfer of health, $\delta > 0$, between two individuals with the same socioeconomic status p but two different health status: h^- and h^+ ($h^+ > h^-$). $\boldsymbol{PHT}$ will be satisfied if:

$$
\begin{aligned}
u(h^- + \delta) + u(h^+ - \delta) &> u(h^-) + u(h^+), && (10)\\
u(h^- + \delta) - u(h^-) &> u(h^+) - u(h^+ - \delta). && (11)
\end{aligned}
$$

Dividing both side by δ and taking the limit when $\delta \to 0$ yields

$$
\begin{aligned}
\lim_{\delta \to 0} \frac{u(h^- + \delta) - u(h^-)}{\delta} &> \lim_{\delta \to 0} \frac{u(h^+) - u(h^+ - \delta)}{\delta}, && (12)\\
\frac{du(h^-)}{dh} &> \frac{du(h^+)}{dh}, && (13)
\end{aligned}
$$

thus, $\frac{d^2u(h)}{dh^2} < 0$ for all $h \in \Re$. ■

Proof of Lemma 3. Consider a finite health transfer $\delta > 0$ from a person with socioeconomic rank p to another at a lower socioeconomic rank $p - \varrho$ and that both have the same level of health h. $\boldsymbol{SHT}$ will be satisfied if

$$
\begin{aligned}
w(p)u(h - \delta) + w(p - \varrho)u(h + \delta) &> w(p)u(h) + w(p - \varrho)u(h), && (14)\\
w(p - \varrho)\left[u(h + \delta) - u(h)\right] &> w(p)\left[u(h) - u(h - \delta)\right]. && (15)
\end{aligned}
$$

Dividing both side by δ and taking the limit when $\delta \to 0$ yields

$$
\begin{aligned}
w(p - \varrho) \lim_{\delta \to 0} \frac{u(h + \delta) - u(h)}{\delta} &> w(p) \lim_{\delta \to 0} \frac{u(h) - u(h - \delta)}{\delta}, && (16)\\
w(p - \varrho)\frac{du(h)}{dh} &> w(p)\frac{du(h)}{dh}. && (17)
\end{aligned}
$$

Since $\frac{du(h)}{dh} > 0$, this implies that $w(p - \varrho) > w(p)$. This, in turn, is satisfied if $\frac{dw(p)}{dp} < 0$ for all $p \in [0, 1]$. ■

Proof of Lemma 4. Consider two finite transfers of health $\delta > 0$. The first takes place between two individuals with the same socioeconomic status p but two health status h^- and $h^- - \iota$. The second takes place between two individuals with the same socioeconomic status p but two health status h^+ ($h^+ > h^-$) and $h^- - \iota$. ***PHTS*** will be satisfied if

$$u(h^- - \delta) + u(h^- - \iota + \delta) - u(h^-) - u(h^- - \iota) > u(h^+ - \delta) + u(h^+ - \iota + \delta) - u(h^+) - u(h^+ - \iota). \quad (18)$$

Dividing both side by δ and taking the limit when $\delta \to 0$ yields

$$\lim_{\delta \to 0} \frac{u(h^- - \iota + \delta) - u(h^- - \iota)}{\delta} - \lim_{\delta \to 0} \frac{u(h^-) - u(h^- - \delta)}{\delta} >$$
$$\lim_{\delta \to 0} \frac{u(h^+ - \iota + \delta) - u(h^+ - \iota)}{\delta} - \lim_{\delta \to 0} \frac{u(h^+) - u(h^+ - \delta)}{\delta} \quad (19)$$
$$\frac{du(h^- - \iota)}{dh} - \frac{du(h^-)}{dh} > \frac{du(h^+ - \iota)}{dh} - \frac{du(h^+)}{dh}. \quad (20)$$

Dividing both side by ι and taking the limit when $\iota \to 0$ yields

$$\lim_{\iota \to 0} \frac{\frac{du(h^- - \iota)}{dh} - \frac{du(h^-)}{dh}}{\iota} > \lim_{\iota \to 0} \frac{\frac{du(h^+ - \iota)}{dh} - \frac{du(h^+)}{dh}}{\iota} \quad (21)$$
$$-\frac{d^2u(h^-)}{dh^2} > -\frac{d^2u(h^+)}{dh^2}, \quad (22)$$

thus, $\frac{d^3u(h)}{dh^3} > 0$ for all $h \in \Re$. ■

Proof of Lemma 5. Let consider two finite transfers of health $\delta > 0$. The first takes place between two individuals with the same health status h but two different socioeconomic status p^- and $p^- - \varrho$. The second takes place between two individuals with the same health status h but two different socioeconomic status p^+ and $p^+ - \varrho$. ***SHTS*** will be satisfied if

$$w(p^- - \varrho)\left[u(h + \delta) - u(h)\right] - w(p^-)\left[u(h) - u(h - \delta)\right] >$$
$$w(p^+ - \varrho)\left[u(h + \delta) - u(h)\right] - w(p^+)\left[u(h) - u(h - \delta)\right]. \quad (23)$$

Dividing both sides by δ and taking the limit when $\delta \to 0$ yields

$$w(p^- - \varrho) \lim_{\delta \to 0} \frac{u(h + \delta) - u(h)}{\delta} - w(p^-)\frac{u(h) - u(h - \delta)}{\delta} >$$
$$w(p^+ - \varrho) \lim_{\delta \to 0} \frac{u(h + \delta) - u(h)}{\delta} - w(p^+)\frac{u(h) - u(h - \delta)}{\delta} \quad (24)$$
$$\left[w(p^- - \varrho) - w(p^-)\right] \frac{du(h)}{dh} > \left[w(p^+ - \varrho) - w(p^+)\right] \frac{du(h)}{dh}. \quad (25)$$

Dividing both sides by ϱ and taking the limit when $\varrho \to 0$ yields

$$-\frac{du(h)}{dh}\lim_{\varrho\to 0}\frac{w(p^-)-w(p^- - \varrho)}{\varrho} > -\frac{du(h)}{dh}\lim_{\varrho\to 0}\frac{w(p^-)-w(p^- - \varrho)}{\varrho} \tag{26}$$

$$\frac{dw(p^-)}{dp} < \frac{dw(p^+)}{dp}. \tag{27}$$

Thus $\frac{d^2w(p)}{dp^2} > 0$ for all $p \in [0,1]$. ■