UNDERSTANDING MULTILINGUAL LEARNERS' MATHEMATICAL EXPERIENCES AND MEANING MAKING IN A CANADIAN EDUCATIONAL SETTING

FATIMA ASSAF

Thesis submitted to the University of Ottawa
in partial Fulfillment of the requirements for the
degree of Doctor of Philosophy in Education

Faculty of Education
University of Ottawa

Acknowledgments

First and foremost, I thank Allah (subhanahu wa ta'ala) for providing me with support and patience to complete a Ph.D. in Teaching, Learning and Evaluation, in the Faculty of Education, at the University of Ottawa.

Acknowledgment is most due to my supervisor, Dr. Chris Suurtamm, for her immaculate words of wisdom, encouragement, and support throughout the process of my Ph.D. Working with Chris changed my life around and helped me believe that I could obtain my Ph.D.—Once was a dream. Today became a reality. Honestly, there are no words that could ever explain or show my gratitude for having her as my supervisor. I will never forget her patience and generosity with me. She treated me as a colleague, broadening my perspectives and challenging my abilities in order to flourish. Her constant encouragement to present at conferences and workshops enlightened my experience, expanded my thinking, and provided me the opportunity to meet fellow graduate students and researchers, from whom I have gained invaluable knowledge and insights. Thank you, Chris!

I also acknowledge with deep appreciation the support of my committee members Dr. Richard Barwell, Dr. Ruth Kane, and Dr. Raymond LeBlanc for their support and guidance in my research study. Thank you to Núria Planas for her contributions as my external examiner.

Finally, I would like to express my deepest appreciation to my dear friend Alison Goss for her wisdom, kind-heartedness, and for always being there for me when I needed her most. To my husband, Fadl Assaf, who I am eternally grateful to than he will ever know. Without exception, he provided the encouragement I needed to fuel my ambition and helped me to surpass all the challenges and difficulties I faced. To my children J’afar, whose sense of responsibility, words of encouragement, and kindness are incomparable; and Hawraa, whose motherly sense of caring, kindness, and compassion are an inspiration to me. Thank you to my mother, my father, and my brothers, for their prayers and support throughout my academic career. You all looked up to me and treated me as an expert in everything—Although I always feel I still have lots to learn.

Table of Contents

Abstract

This is an ethnographic study designed to form an in-depth description and understanding of multilingual learners' mathematical experiences and meaning-making in a plurilingual educational setting. I assumed a sociocultural perspective that draws from Vygotsky's theory of learning and development. A sociocultural perspective offers a promising epistemological conceptualization of children, their learning, and language development as mediated by social, cultural, and historical contexts. One grade 2/3 classroom with 18 students from Eritrea, Nepal, Somalia, Sudan, and Syria participated in the study. The data included observations, video recordings of students working on mathematics activities, copies of students' work, and interviews with students. The results of the study revealed the teacher's pedagogical practices as a significant influence on students' mathematical meaning-making and learning experiences, which in turn influenced students' individual identities as mathematics learners in the grade 2/3 classroom. There were also institutional and sociopolitical aspects that influenced the teacher's practices, and in turn, influenced students' mathematics experiences. Hence, the influences on students' learning of mathematics take the form or shape of a reciprocal formation where one influence is connected to and influences the other. The findings of the analysis also show that it was the students' interactions with one another that were at the heart of their meaning making. Students' interactions were significant to their meaning making as they were constantly learning from one another. Little meaning making would have happened without these interactions. These interactions encouraged students to develop a collection of resources to share their thinking and ideas through verbal, visual, and written mathematical communications. Hence, utilizing language to make meaning and to negotiate their mathematical understanding. Ultimately, the descriptions of multilingual learners' mathematics experiences and meaning making may inform research and practices to support other multilingual learners' experiences in mathematics education. This study also contributes to what is still a very limited body of literature on multilingual students' mathematical experiences and meaning in a Canadian educational setting.

List of Figures

List of Tables

Appendices

CHAPTER 1: INTRODUCTION

In the last 6 years, Canada has welcomed more than 1, 200, 000 immigrants and refugees; about half of them are school aged children and young adults (Citizenship and Immigration Canada, 2017). Refugee children are among the most vulnerable, as many may have experienced trauma, forced displacement, separation from families, loss of loved ones, and extensive periods in refugee camps and resettlement facilities, (Anderson et al., 2004; Kaprielian-Churchill, 1996). The effects of these experiences on the children are further challenged by the burden of adapting to a new context and a new school, including adaptation to the ways of the host society—its values, behavioral patterns, and language (Anderson et al., 2004; Kaprielian-Churchill, 1996). The challenges experienced by immigrant and refugee children at school extend well beyond the mastery of curriculum content and requirements (Rummens & Dei, 2010). Newly arrived immigrant and refugee children grapple with numerous resettlement stresses at school, such as non-recognition of prior schooling, differential educational levels, lack of familiarity with the school system and practices, conflicts between home-school cultural values, and unwelcoming school environments (Rummens & Dei, 2010, p. 4). In addition, newly arrived immigrant and refugee children enter our educational systems (i.e., Canadian in this case) with a wide variety of previous experiences. Some have had interrupted or inconsistent schooling, some have been home schooled, and still others have never attended school before. When these students come to our schools, they are confronted with a whole new experience, and with a new language as the means through which they interact with mathematics.

Language is recognized to play a crucial role in the teaching, learning, and doing of mathematics, an idea that is widely acknowledged by mathematics education researchers (Barwell, 2014a; Morgan et al., 2014; Moschkovich, 2017; Planas et al., 2018). The crucial role of language is even more important to consider in plurilingual[1] classrooms found in many Canadian schools. In such classrooms, the teaching and learning of mathematics is carried out in languages other than the student's home language. In the case of immigrants and refugees, where Arabic, Nepali, or Somali is often the newcomer's first language, English is often the language

---

[1] I have taken plurilingual to imply any classroom with the presence of students who speak different languages, whether these languages are used or not, may be referred to as a plurilingual classroom.

of instruction.

There are different views regarding multilingual[2] learners' success in such situations. Some believe that poor performance of multilingual learners in mathematics is attributed to the learner's limited proficiency in the language of instruction—suggesting that fluency in that language is necessary for the development of mathematical competency (Clarkson, 2006; Cummins, 2001; Halai, 2007, 2009). A review of the literature in this area challenges the idea that students need to have mastered the official language of instruction prior to learning mathematics (Barwell, 2005; Civil, 2008; Moschkovich, 2007). These researchers demonstrate that the knowledge of the language of instruction is only one aspect of becoming competent in mathematics and leads us to consider the multiple resources multilingual learners use to construct knowledge, negotiate meanings, and to communicate mathematically (Moschkovich, 2002, 2007). According to Gorgorió and Planas (2001), knowledge of the language of instruction is not simply or only a matter of reaching a survival competency in that language, but also attaining knowledge of the beliefs, values, and culture predominant in the host society. These learners may experience issues linked to conflicting social, cultural, linguistic, and political barriers, which are very often at odds with those previously experienced. Thus, learning and mathematical achievement emerge from a wide array of interrelated influences. I define influence as the capacity to affect someone's behaviour, actions, or views in a direct or indirect way, but which leads to the transformation of an experience. Some researchers claim "poor performance by multilingual learners thus cannot be attributed to the learners' limited language proficiencies in isolation from the wider social, cultural and political factors that infuse schooling" (Setati et al., 2008, p. 16).

Understanding learning is a complicated matter, as learning is influenced by many factors, including

> individual experiences and personal idiosyncrasies; biological differences; family motifs and identities; cultural values; ethnic, racial, and other identities with varying social status; school-based behaviors, and practices; the relationships

[2] For present purposes, I use the definition of multilingual as articulated by Barwell (2003) to refer to any person in the presence of more than one language, even if they are not used, may be referred to as multilingual.

> between students and their teachers; and societal ideologies, among others.
> (Nieto, 2010, p. 49)

A key point in Nieto's account is that if educators or researchers were to focus on just one or two of these factors, they might lose sight of the larger picture because "learning needs to be understood in the broader context of the sociocultural and sociopolitical lives of students, teachers, and schools" (Nieto, 2010, p. 49). The factors included in Nieto's account impact Canada at a very national level, both as a multicultural country, or as Vertovec (2007) would say a country of 'superdiversity' where, individuals may be informed by several ethnicities, histories, races, classes, religions, and may speak a mixture of several languages (Barwell, 2016; Gollnick & Chin, 2008; Nieto, 2010; Vertovec, 2007), and also, as a plurilingual country with two official languages- English and French- and the use of over 200 other languages within the population—including Indigenous languages and languages of immigrant populations, based on the 2016 census language data. These individuals "are encouraged to maintain their cultural distinctiveness, and to participate in the daily life of the larger society" (Sam, 2006, p. 20).

My interest in immigrant and refugee children and in these ideas, in general, are motivated by my experience working with newly arrived immigrant and refugee children. Since 2016, I have volunteered at an Elementary School to assist in supporting Syrian-Canadian children in an English Literacy Development classroom and in mainstream mathematics classrooms for grades 2-5. As a volunteer and an Arabic and English speaker, I supported and interacted with many students, especially many of the Syrian children, on a one-on-one basis or during small-group work. I often worked with small groups of students on various language-related activities or mathematics problem-solving activities. The students would often ask me for help or involve me in their conversations. By listening to and talking with children about the tasks at hand, or about matters concerning their daily activities, I gained a lot of knowledge and understanding about their learning experiences. For example, when working with a group of newly arrived Syrian students on a mathematical task that required computation with a three-digit number using a branching diagram, one of the students became really frustrated with this strategy and insisted on using a traditional algorithm. She explained to me that in Syria and in Jordan they only used algorithms to solve problems, and also when she works with her dad on her homework, they use an algorithm, which she sees as a lot easier and less complicated. This brief, yet, very informative conversation I had with this student pulled me into her past experiences of doing mathematics in both Syria and in Jordan, shed light on her father as someone who supports her with her mathematics homework, and also, revealed her point of view and attitude towards algorithms. Such experiences, and the presence of many immigrant and refugee children in our schools motivated the current study that formed an in-depth description and understanding of multilingual learners mathematical learning experiences and meaning making through the sociocultural and sociopolitical setting in which students learn. Moreover, this research provided a better understanding of the role of language in relation to the teaching and learning of mathematics.

Research Questions

The objective of my study is to form an in-depth description and understanding of multilingual learners' mathematical experiences and meaning making through the sociocultural and sociopolitical setting in which students learn. The study addresses the following research questions:

What are the mathematical experiences of grade 2/3 students in a plurilingual classroom in Ontario, Canada?

a) What are the influences on their mathematics experiences?
b) How do they negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

Thesis Outline

The remainder of this thesis is divided into six chapters. Chapter 2 is my literature review and where I situate my study in the literature. I first discuss current perspectives on the learning of mathematics and then discuss these in relation to the Ontario Mathematics Curriculum, the curriculum used by Ontario teachers. Next, I introduce recent studies on multilingual students' learning of mathematics as it relates to students' access to communication in mathematics. Then, I address studies that explore a range of influences on multilingual students' learning of mathematics to understand students' experiences learning mathematics in an additional language. Also, to understand the role of language in relation to the teaching and learning of mathematics in various educational settings.

In chapter 3, I discuss the sociocultural theory of learning and development including the notions of internalization and the zone of proximal development. These ideas set out the theoretical perspectives that underpin the theoretical basis of my study, which also guide my observations and analysis.

In chapter 4, I discuss detailed information concerning the methodology of my study. First, I explain the rationale for my choice of an ethnographic study and its potential for engaging children participants in human inquiry to understand their experiences and social worlds as they do. Next, I explain my research design. I first describe the recruitment and procedures, followed by a description of the research setting and the participants in order to convey the context of the study. I then explain the data collected for this study, including my observations of the mathematics block, video recordings of students engaged in mathematics problem-solving activities, and follow-up interviews with the students. The chapter concludes with a description of the data analysis methods. For the data analysis, I used two analytical frameworks, one came from my analysis of the research literature that prompted me to organize the influences drawn from research into four categories including individual, pedagogical, institutional, and sociopolitical features of mathematics classrooms. The other analytical framework came from Gee's (2005, 2011) construct of building tasks (i.e., significance, activity—mathematical actions, identity, relationships, politics, and sign systems or knowledge) which I translated to the context of mathematics education. The chapter concludes with a discussion about the role of the researcher and the trustworthiness of the study.

In Chapters 5 and 6, I address the two sub research questions by presenting the results for each question separately. Presenting the results for each sub research question in two different chapters allows me to explicitly address each question in detail. For the first sub question: What are the influences on students' learning of mathematics? I structured the presentation of the results in terms of the four influences on students' learning of mathematics which came from the organization of my literature review, including individual, pedagogical, institutional and sociopolitical features of the grade 2/3 mathematics classroom. The findings were a result of the analysis of my observation notes. Then, based on the evidence presented by the results on the four influences, I explicitly answer the first sub question (i.e., what are the influences on students' learning of mathematics?) by providing a general summary of the characteristics based on the findings.

Next, in Chapter 6, I present the results for the second sub research question: How do students negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting? I chose four video recordings of students working on mathematics problem-solving activities to analyze in detail. Each of these video recordings focuses on students participating in one of the following mathematical problem-solving activities: Activity #1: Add 'Em Up; Activity #2: Times Tussle; Activity #3: Books in a Classroom; and Activity #4: Ameer's Coins. I present each activity individually. For each activity, I provide a detailed description of the entire activity in chronological order. At the end of the discussion of the four activities, I look across my analysis of the four activities and discuss this cross-analysis in order to explicitly answer the second sub research question: How do students negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

All this then leads to Chapter 7, the discussion chapter, where I consider the results of the first and second sub research question to address my main research question: What are the experiences of multilingual learners? As I address my main research question, I ground the discussion in a sociocultural understanding of learning and development. Considering the results of the first and second sub research question, as well as, situating the discussion in a sociocultural understanding of learning and development allowed me to provide a detailed account and description of students' mathematics learning experiences.

Chapter 8 is the concluding chapter where I provided a general summary of findings for the two sub research questions and the main research question. Then, I discuss the significance of the study, contributions of the study, and implications for practice.

CHAPTER 2: LITERATURE REVIEW

This literature review is divided into four main sections. I begin by introducing current perspectives on the learning of mathematics, and then, I describe how learning is viewed in the Ontario Ministry of Education Mathematics Curriculum. I have included current perspectives on learning mathematics and connect it to the Ontario Mathematics Curriculum as it is important to 1) show how research on the learning of mathematics have shifted in their perspective and 2) to set the context for my study. Next, I introduce recent studies on multilingual students' learning of mathematics as it relates to students' access to communication in mathematics. Following that, I address studies that explore a range of influences on multilingual students' learning of mathematics, including the influences of individual, pedagogical, institutional, and sociopolitical features of mathematics classrooms. Reviewing these four influences provides an understanding of students' experiences of learning mathematics in an additional language as informed by the various influences, and also, of the role of language in relation to the teaching and learning of mathematics in various educational settings. In particular, leaning on this review of the research literature offers an account of the current state of the research on language diversity and mathematics education. The literature review concludes with a summary of the research literature leading into the rationale for theory and methodology choice.

Current Perspectives on the Learning of Mathematics

Current perspectives on the learning of mathematics illustrate that learning is not just individual but that it is a social process, and it involves students interacting and communicating with one another to come to a consensus on a given matter (Barwell, 2014b, 2014c; Barwell et al. 2016; Chronaki & Planas, 2018; Hunter et al., 2018; Lerman, 2000; McGraw & Rubinstein, 2008; Moschkovich et al., 2018; Planas et al., 2018; Sfard, 2006). Hence, understanding current perspectives allows us to see the shift in thinking and what the focus is on, which could ultimately inform our views. For instance, much of the early research in mathematics education situated learning within an individualist psychological framework—through which cognition operated as the core principle. In this individualist psychological framework, learning was considered to be the transmission of knowledge from the teacher to the student, and the individual's mind was perceived as the primary source of development (Sfard, 2006, p. 156). With this view, students acquired various skills implicitly through repeated observation and participation (Moyer, 2001). The emphasis is on explicit teacher instruction, memorization of rules, and repeated practice with procedures. That is, the teacher explains the relevant lesson by demonstrating procedures for solving mathematical problems, and students then repeatedly write out problems and solutions on worksheets or the board with minimal discussion (Kohn, 1999). According to Sfard (1998), such perspectives lead us to view "the human mind as an empty container to be filled with certain materials and about the learner as becoming an owner of these materials" (p. 5). Such a view of the human mind suggests that there is an end point in the process of learning (Sfard, 1998).

More recently, however, the view that learning mathematics is a social process was more prevalent in research on mathematics education (Barwell et. al., 2016; Halai & Clarkson, 2016; Lerman, 2000; Moschkovich et al., 2018; Planas et al., 2018; Sfard, 2006). This shift, according to Sfard (2006), focused on the idea that mathematics is a form of discourse. With this view, mathematics learning originates in communication with others through social interactions, and language is viewed as a contributing factor (Barwell, 2014b; Sfard, 2006). Some researchers refer to these social interactions as "math-talk learning community" (Hufferd-Ackles et al., 2004). These researchers define math-talk learning community as "a classroom community in which the teacher and the students use discourse to support the mathematical learning of all participants (p. 86). In an important way, "language mediates between individuals and society, acting as a 'cultural tool' through which shared ways of thinking can be taken up by individuals and used to make meaning and solve problems" (Barwell, 2014c, p. 78). This suggests that students, through participating in mathematical discussions take, as well as give, a set of interesting and promising ideas that not only help to elaborate students' arguments, but that incorporate other students' reasoning in order to co-construct new knowledge (Francisco, 2013; Inagaki et al., 1998). For example, McGraw and Rubinstein (2008) state, that when students are provided with an opportunity to discuss, analyze, and explore their own and the reasoning of others—"they can become powerful mathematical thinkers" (p. 164). Viewed in this way, interaction and participation in meaningful mathematical activities are mediating factors for students' learning—understood as a description of and a means for the development of agency (Bergem & Pepin, 2013; Daniels, 2001; Langer-Osuna et al. 2016; Wagner & Andersson, 2018).

The importance of participation and communication has surfaced in many policy documents in mathematics education. For instance, in the Ontario Mathematics Curriculum Grades 1-8

(OME, 2005), there is clear emphasis on students' communication in the mathematics classroom. Students' communication is regarded as an essential process in learning mathematics, through which students' mathematical thinking and understanding is shared.

> Through communication, students are able to reflect upon and clarify their ideas, their understanding of mathematical relationships, and their mathematical argument. (OME, 2005, p. 17)

In addition, fostering and encouraging students' communication and discussion in mathematical problem-solving activities provides

> students with the opportunity to ask questions, make conjectures, share and clarify ideas, suggest and compare strategies, and explain their reasoning. As they discuss ideas with their peers, students learn to discriminate between effective and ineffective strategies for problem solving. (OME, 2005, p. 25)

Consequently, many Ontario mathematics classrooms have shifted from mainly teacher-dominated instruction, where silent and individual activities were considered the norm, to greater participation in mathematical discourse practices (Suurtamm & Graves, 2009). This focus on communicating mathematically, both orally and in writing, is similar to what researchers suggest (e.g., Forman, 1996; Moschkovich, 2002, 2007). However, some researchers may advise that the focus on communication may be a challenge for students learning an additional language, while others consider a description of the resources and competencies in mathematical communication (Barwell, 2014c; Forman, 1996; Moschkovich, 2002, 2007). Such emphasis on communication highlights the important role of language in relation to learning mathematics in an additional language.

Multilingual Students' Learning of Mathematics

A review of the research literature on multilingual students' learning of mathematics suggests that the construction of mathematical knowledge through participation and communication is central to learning and teaching mathematics in various educational settings (Moschkovich et al., 2018; Planas et al., 2018). The underlying focus is centered on the recognition that mathematics learning is a process of participation and socialization into mathematical discourse (Barwell, 2020). By mathematical discourse, I imply the opportunities that permit and promote students to share mathematical thinking and "ideas through classroom discussions, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29). Researchers who focus on multilingual learners emphasize the role of communication plays in mathematics. For instance, Moschkovich (2018) states that 'communicating to learn mathematics' is pivotal because it supports students' conceptual understanding which involves explanations, arguments, and justifications that students construct as they engage in their own or other students' reasoning. Central to this idea of communicating to learn mathematics for multilingual learners is a movement between mathematical (formal) language and everyday (informal) language to orchestrate and promote meaningful mathematics learning and discussion (Moschkovich, 2018; Setati & Adler, 2000; Webb & Webb, 2016). Mathematical language refers to the standard terminologies used to describe and discuss mathematics. Everyday language refers to words or terms used in our day-to-day encounters or conversations to express mathematical understanding and ideas. It has also been argued that a movement from students' everyday (informal) language to (formal) mathematics language is very beneficial for multilingual students and should not be treated as dichotomous because everyday language may connect to students' everyday lives, which could help them identify and make sense of the mathematics they are working on (Barwell, 2016; Moschkovich, 2002, 2008, 2018; Prediger et al., 2016; Tshabalala & Clarkson, 2016). In sum 'communicating to learn mathematics' is not simply or mainly a matter of learning vocabulary or pronouncing certain words correctly but it is also providing multilingual students opportunities "to describe patterns, make generalizations, and representations to support their claims, all central mathematical practices" (Moschkovich, 2018, p. 17).

Several researchers demonstrate the importance of engaging multilingual learners in mathematical discourse and highlight the important role that a teacher can play (Civil & Hunter, 2015; Hunter & Hunter, 2018). These researchers also draw our attention to the dissonance found between students' cultures and those found in the school especially when certain practices (i.e., explaining, justifying, or inquiring into the reasoning of others) do not coincide with students' home environments or within their communities. For example, in New Zealand, Hunter and Hunter (2018) drew on the *Communication and Participation Framework* (CPF) designed by one of the researchers to support teachers in gradually shifting students toward participation and the use of mathematical discourse practices (see Hunter, 2008, for more detail on the design of CPF). In particular, the researchers focused on the actions teachers take to engage students (i.e., predominantly Pāsifika) in mathematical talk that builds from their cultural norms and values, as a way to shape their participation within collaborative inquiry and argumentation. The researchers share that "Pāsifika students often interpret their role in the mathematics classroom as a passive process of looking and listening not as an active process of sense-making where they are listening, looking and asking questions (albeit, sometimes silently in their heads)" (p. 6). In this study, teachers built on students' home listening practices by emphasizing the need for collaboration, sense-making, and following up with explanations and additional questions (p. 7). Also, by relying on the Pāsifika values of collaboration, teachers relied on cultural contexts that students were familiar with from their home experiences (e.g., creation of siapo (fine cloth) by a group of people collectively working together) to help them understand the need for multiple skills and expertise when working together. In this way, the researchers argue, teachers show that all students' contributions are equally valid and valued. The researchers further discuss other teacher actions that are pertinent to students' engagement in equitable opportunities to learn mathematics such as, showing an ethic of care; using tasks and problems set within known cultural and social contexts; preparing students for a challenge in mathematics; showing students a belief in their ability; encouraging students to take intellectual risks in their mathematical reasoning; modelling ways students could explain their mathematical reasoning and ways they could question, agree, disagree and challenge politely; emphasizing the need for explanations to be structured; and encouraging the monitoring of group members sense-making. According to Hunter and Hunter (2018), for students to have equitable opportunities to engage and participate in a range of mathematical practices, "a teacher needs to develop a respectful learning environment that intentionally builds on the lived cultural and social values of the students through deliberate pedagogical actions" (p. 16).

Other researchers focus on the role of language in relation to the teaching and learning of mathematics in multilingual classrooms (e.g., Barwell, 2020; Moschkovich et. al., 2018; Planas, 2018; Planas & Ngoepe, 2019; Takeuchi, 2015). In particular, there is a focus on the link between learning mathematics and language learning as both are conveyed as being the same process and involve socialization (Barwell, 2020). Drawing on the view of learning mathematics and language as socialization and the Bakhtinian notion of centripetal and centrifugal language forces, Barwell (2020) explored mathematics learning in four different second-language contexts to identify common patterns of socialization across second-language settings, including a mainstream classroom, a sheltered classroom for Indigenous students, a welcome classroom for new immigrants, and a French-immersion classroom. The results of the study revealed seven socialization events and the associated socialization practices across four classrooms. The socialization events and practices were identified by instances "in which participants make some explicit reference to language or discourse norms or to language or discourse diversity" (p. 159). These include, 1) students' use of their home languages (e.g., teachers instructing students to use the language of instruction or students using their home language with peers but the language of instruction with their teacher); 2) the occurrence of nonstandard accents, pronunciation, or orthography (e.g., teacher correcting students' pronunciations and practicing words with them); 3) explicit attention to features of mathematical discourse (i.e., the teacher presenting or explaining vocabulary, morphology, syntax, and grammar); 4) encounters with mathematics classroom genres (i.e., word problems, definitions, worksheets, quizzes, and teacher explanations, among many others); 5) use of gestures in mathematical interactions (e.g., teachers and students using gestures to describe shapes or angles); 6) explaining mathematical thinking—although students found it challenging, they had little difficulty with the mathematics required and they drew on a range of sources of meaning such as, the teacher's utterances and gestures; and 7) moments of reduced participation—these situations were sometimes marked by the teacher commenting or students explicitly avoiding active interaction (p.161). Moreover, a comparison of the four classrooms led to a distinction between two groups: language positive classrooms and language neutral classrooms. In terms of students' use of home languages, the researcher explains that in language positive classrooms, there was a clear discussion and attention on students' knowledge of other languages, but an emphasis on learning the language of instruction was still evident (p. 174). In language neutral classrooms, attention was mostly given to the use of a single language—preferably, the language of instruction. According to Barwell, "the presence of students' home languages (or other forms of expression) is not in itself sufficient; their productive use depends on effective socialization practices" (p. 174). The researcher further argues that although the socialization practices may seem controlled or influenced by the teacher, "the socialization events are initiated by students or involve students in the joint production of knowledge...making clear the active role of students in their socialization into mathematics and language" (p. 174).

Although the above studies offer significant findings in understanding the importance of the teacher's actions in ensuring multilingual students' access to discursive communication in mathematics, as well as, the view of learning mathematics as socialization into mathematical discourse, their findings cannot be easily translated across settings. As explicated in Barwell's (2020) research, the four second-language contexts portrayed similar and different aspects of language use in mathematics. As such, it may be too complex to argue that multiple language use should be considered a resource for the teaching and learning of mathematics or, a move towards culturally responsive practices in mathematics education is ideal for ensuring the success of multilingual learners without considering the differences among settings, languages, and students. Thus, the role of language and students' experiences in mathematics educations are highly complex and are linked to a wide array of interrelated influences on students' learning.

Influences on Multilingual Students' Learning of Mathematics

I conducted an extensive review of the research literature in the area of language diversity and mathematics education which revealed a range of influences on multilingual students' learning of mathematics in various settings. This analysis prompted me to organize the influences that this research found into four categories: individual, pedagogical, institutional, and sociopolitical features of mathematics classrooms. These influences are important to understand, as we come to understand multilingual students' mathematics learning experiences in terms of 1) the learner's individual characteristics and what they bring into the classroom; 2) teachers' pedagogical practices and views and their influence on students' learning; 3) the institution's influence on the use and role of language in learning, and 4) provincial policies that influence the school policies and teachers' pedagogy which in turn influence students' learning. Although I describe each of these influences separately, they are highly interconnected and form a nested system of influences on students' experiences learning mathematics in an additional language.

Individual Influences

Research on individual influences focuses on the characteristics of the individual learner that influence his or her mathematics learning. Much of this research has examined the role of the students' proficiency with the language of instruction as influencing their development of mathematical understanding. When multilingual learners join a mathematics classroom that is taught in a language that is foreign to them, students often experience new words and new meanings that "can turn into a wide variety of cultural conflicts and disruptions of the learning process" (Gorgorió & Planas, 2001 p. 12). As well as reaching a survival competency in the language of instruction, attaining knowledge of the social and cultural communicative tools is also required (Gorgorió & Planas, 2001).

Some researchers view students' poor performance in mathematics as attributed to the learners' limited proficiency in the language of instruction (Clarkson, 2006; Cummins, 2001; Parvanehnezhad & Clarkson, 2008). These researchers further suggest that attaining proficiency in more than one language could have positive effects on learners' mathematics achievement. Using Cummins's work as a basis, Parvanehnezhad and Clarkson (2008) in Australia used a qualitative approach to test the threshold hypothesis in relation to second language learners' mathematics attainment. According to the threshold hypothesis, "the level of linguistic competence attained by bilingual children may act as an intervening variable in mediating the effects of bilingualism on their cognitive and academic development" (Cummins, 2001, p. 74). What follows are the notions that Cummins (2001) addresses in relation to the learners' language proficiency and cognitive effects:

- high levels of proficiency in both languages leads to positive cognitive effects;
- native-like proficiency in one of the languages offers neither negative nor positive cognitive effects;
- low levels of proficiency in both their languages lead to negative cognitive effects

Parvanehnezhad and Clarkson (2008) reported on 16 Iranian students in grades 4 and 5 with Farsi as their first language (L1), and English as their second language (L2) (p. 58). The findings with respect to Cummins's threshold hypothesis suggested a connection between the students' language competency and their mathematical competency. The results illustrated that "most students with high/high language competency also had high mathematics competency, and most students with low/ low language competencies had low mathematics competency" (Parvanehnezhad & Clarkson, 2008, p. 66). However, Barwell (2005) believes that such conclusions "should be treated with caution, [because] linguistic proficiency is unlikely to be the only factor in such students' attainment" (p. 331), suggesting that the language of instruction should not be the only attribution of students' difficulties. Other factors might have an effect on students' achievements, including individual, socio-political, and socio-economic factors that impact students' learning of mathematics (Barwell et al., 2017). Thus, the development of students' language proficiency in more than one language per se "is only a one-dimensional approach to dealing with the complexity of teaching and learning mathematics" (Essien, 2010, p. 169).

Another focus of research on individual influences has been on the use of language switching[3] (see for example, Clarkson, 2006; Grosjean, 1999; Moschkovich, 2005, 2019; Parvanehnezhad & Clarkson, 2008; Planas & Setati, 2009; Setati, 1998; Ramirez & Milk, 1986). For some researchers, switching from one language to another, for example, switching from the language of instruction to one's first (or even second) language has been seen as a strategy to compensate for language difficulty in the language of instruction (Clarkson, 2006; Grosjean, 1999; Ramirez & Milk, 1986). Other researchers, adopt a different, more positive stance with respect to language switching and consider the way in which language switching is influenced by the students' experiences and the physical environments in which they live (Moschkovich, 2005; Planas & Setati, 2009). These researchers view students' use of their home language(s) as a resource for learning mathematics, and not a challenge.

More recent research suggests that rather than solely focusing on language switching, we need to consider how broader categories such as translanguaging (García & Wei, 2014) and hybrid language practices (Moschkovich, 2019) provide resources for mathematics teaching and learning. García and Wei (2014), for instance, describe translanguaging as the use of multiple discursive practices to communicate effectively and to make sense of a mathematical situation (García & Wei, 2014, p. 22). Hence, it is not a mere switch or shift between languages, but the "use of original and complex interrelated discursive practices ... that make up the speakers' complete language repertoire" (García & Wei, 2014, p. 22). Translanguaging, in this regard, alludes to disrupt earlier notions that view a speaker's use of their first, second, or third language as separate, and suggests the dynamic and interactive aspect involved in the use of more than one language in any given context (García & Wei, 2014). In a similar vein, Moschkovich (2019) interprets hybrid language practices

> as providing resources in multiple ways to participate in mathematical discourse practices, from elaborating on a point that is repeated without repeating the

---

[3] By language switching, I mean switching from one language to another in the course of a discussion or conversation. While Moschkovich (2005) defines language switching as the "use of two languages during solitary" activity, I have chosen to use it to encompass both individual and social interaction.

original utterances, to providing words and phrases from the mathematics register in two languages, to managing the conversational floor and face-saving. (Moschkovich, 2019, p. 106)

Drawing on a sociocultural perspective and an integrated view of *academic literacy in mathematics*,[4] Moschkovich (2019) argues that instead of looking for deficits and merely acknowledging the challenges students face, we must identify the mathematical discourse practices evident in students' contributions. In order to understand how bilingual students participated in academic literacy mathematics and how hybrid language practices supported students' participation in mathematical practices, Moschkovich (2019) examined a mathematical conversation between two ninth-grade students to explain their mathematical understanding. Moschkovich's (2019) findings show that the students used hybrid language practices to participate in several valued mathematical practices such as, stating assumptions explicitly, connecting a claim to a representation, and attending to the precision of a claim (p. 99). The findings also show that the students used two languages (i.e., Spanish and English) "as a resource for elaborating ideas while expanding, repeating or adding information for another speaker" (p.102). Moschkovich's findings suggest that while some students may sometimes use their home language to compensate for missing words, "...other students will use their home language to explain an idea, justify an answer, describe mathematical situations or elaborate, expand and provide additional information" (p. 108). These findings caution us against focusing on deficit models of multilingual students to recognizing students' participation in valued mathematical concepts and mathematical practices.

Several researchers have also demonstrated that enhancing language acquisition and mathematical vocabulary does not necessarily improve mathematics learning and thus the links are more complex than that (e.g., Barwell et al., 2017; Gorgorió & Planas, 2001; Setati et al., 2008). Gorgorió and Planas's (2001) qualitative study explored the experiences and social

[4] According to Moschkovich (2019) *academic literacy in mathematics*, "includes not only cognitive aspects of mathematical activity—activities that happen in one's mind such as mathematical reasoning, thinking, concepts, and metacognition—but also social and cultural aspects—activities that happen with other people, such as participation in the eight Common Core State Standards (CCSS) for mathematical practices—and discourse aspects—activities that happen when using mathematical language, reading, writing, listening, or talking about mathematics" (p. 18).

interactions of 15 and 16-year-olds in Barcelona. The results indicated that even though some of the students eventually acquired a reasonable understanding of the Catalan language, they still experienced difficulties understanding the language register of their mathematics teacher (Gorgorió & Planas, 2001). Evidently, acquiring a reasonable understanding of the language of instruction does not imply that multilingual learners will overcome the linguistic barriers that arise in the new culture. In addition, some students who belong to cultures with other alphabets and numbers, experience difficulties adapting to new ones, as they do not align with students' previous knowledge (Gorgorió & Planas, 2001). These researchers also documented instances where not knowing everyday language interfered with work on mathematical activities. Students became disturbed by the context of the problem and were unable to complete the activity. These findings illustrate that the conflicts experienced by multilingual learners are not always due strictly to a language obstacle, but are often linked to the distance found between students' social and cultural frames of reference and the implicit ones within the school (Gorgorió & Planas, 2001, p. 10).

More current research has shown a shift away from thinking of the multilingual learner as disadvantaged by being less fluent in the language of instruction towards valuing the multiple resources that the individual learner brings to the mathematics classroom from their home and previous learning experiences (Barwell, 2005, 2017; Barwell et al., 2017; Barwell et al., 2018; Moschkovich, 2014, 2017; Planas, 2014, 2018; Planas & Ngoepe, 2019; Setati, 2005; Takeuchi, 2015). In particular, such perspectives recognize students as valuable contributors of knowledge in the mathematics classroom, and also, recognize what influences students' mathematics learning (Elbers & de Haan, 2005; Gorgorió & Planas, 2001; Moschkovich, 2002). Some studies demonstrate that multilingual students draw on multiple resources to clarify their descriptions of mathematical objects and to communicate their mathematical knowledge. Moschkovich (2002), for example, working in a bilingual (Spanish-English) classroom of sixth-grade through eighth-grade students in the US, found that although students may use words that do not exist in either Spanish or English when describing a rectangle, it was clear that the students were referring to a rectangle (p. 201). The students used gestures, concrete objects, drawings of rectangles, and their home language to clarify their description and to illustrate what they meant (Moschkovich, 2002). Similarly, in Utrecht, the Netherlands, Elbers and de Haan (2005) have shown that students do not treat their difficulties with words as mere problems of vocabulary, but as "part of the process of mathematical exploration", which influence their use of multiple resources to clarify their understanding and meaning (Elbers & de Haan, 2005, p. 57). Hence, the languages used by these students to express their thinking, the symbolic expression, and any material used, influence students' mathematical knowledge and experiences.

The emphasis on students' engagement in mathematical discourse as influencing their mathematics experiences is further recognized as a description of and a means for the development of mathematics learning identities (Barwell et al., 2017; Langer-Osuna, 2018; Norén, 2011). Using Norén's (2011) definition, identity or identities is "something we communicate and experience in interactions with others, it is not a fixed entity or a set of qualities, and it is a temporary, slippery and shifting formation" (p. 98). Hence, as students engage in various discourses, they are constantly contributing to discussions about their roles as mathematics learners, and what it means to know and do mathematics (Norén, 2011). In general, some researchers argue that students develop their identities as "doers of mathematics" as they participate or resist engagement in mathematics, and in the process they are contributing to the ongoing regeneration of their identities as established in their classroom (Cobb et al., 2008). In particular, various researchers recognize learners' agency and mathematics authority as contributing to their mathematics learning identities (Langer-Osuna et al., 2016; Langer-Osuna, 2017; Norén, 2011; Norén & Andersson, 2016). Agency, according to Langer-Osuna et al., (2016) "is understood as individuals' intentional, self-initiated action directed toward accomplishing an explicit or implicit goal" (p. 169). Consequently, students recognize agency by exercising control over their engagement in the mathematics activity, by relying on various resources, and by identifying themselves as participants and doers of mathematics. Even the ways in which mathematical authority is distributed among students influences and contributes to their mathematics learning experiences (Langer-Osuna, 2018). Several researchers have argued that a key component of collaborative mathematics classrooms is the distribution of mathematical authority (Amit & Fried, 2005; Cobb et al., 2008, 2009; Langer-Osuna, 2017). The distribution of authority concerns the degree to which students are offered opportunities to be involved in making mathematical decisions, taking ownership of their ideas, and exercising agency in the classroom (Cobb et al., 2008, 2009; Langer-Osuna et al. 2016; Langer-Osuna, 2017). Hence, learning and effective schooling, according to latter views, entail that we view individual students' languages and cultures as important resources in the learning and teaching of mathematics (Barwell, 2005; Civil, 2008; Gorgorió & Planas, 2001; Planas & Ngoepe, 2019; Moschkovich, 2002, 2007, 2017; Razfar, 2013) and, therefore, have significant influences on learners' mathematical knowledge, agency, and mathematical authority (Langer-Osuna et al. 2016; Langer-Osuna, 2017).

Pedagogical Influences

A second key influence on students' learning of mathematics is characterized by teachers' pedagogical practices, as well as, their views and experiences that shape and influence student's learning of mathematics. A review of the research literature suggests that the constraints and affordances that students experience in the classroom depend on the pedagogical practices of their teachers (Adler & Ronda, 2017; Kaur, 2013). For instance, Adler and Ronda (2017) argue that, "learners' access to doing and talking mathematics is through their teacher" (p. 66). Hence, teachers play a fundamental role in students' access and engagement in mathematical discourse (Hunter & Hunter, 2018). Similar to the teacher actions outlined earlier by Hunter and Hunter (2018), Moschkovich (2018) suggests several instructional practices for teachers of English learners. These instructional practices include,

- Knowing students' previous instructional experience.
- Maintaining high commitment to students' academic success.
- Having high expectations for all students.
- Having autonomy to change curriculum and instructions to meet the specific needs of students.
- Rejecting models that position students as intellectually disadvantaged.
- Focusing on mathematical concepts and connections among those concepts.
- Using high cognitive mathematical tasks.
- Promoting students' conceptual understanding.
- Supporting students' participation in mathematical discussions as they learn English.
- Focusing on mathematical practices such as reasoning and justifying, not accuracy in using individual words, and address much more than vocabulary.
- Treating home and everyday language as resource, not deficits.

- Drawing on multiple resources available in classrooms—such as objects, drawings, graphs, and gestures—as well as home languages and experiences outside of school. (Moschkovich, 2018, pp. 15-16)

Such practices are reflective of what research has identified as effective teaching strategies (e.g., Anthony & Walshaw, 2009; NCTM, 2014; Towers & Proulx, 2013) that are "necessary to promote deep learning of mathematics" (NCTM, 2014, p. 9). In their work, for example, Cao, Guo, Ding and Mok (2013) found that, in all classrooms, when teachers encourage discussion, communication, and expression of students' own thoughts and ideas, they provide all students with higher efficiency in learning and in constructing their own understanding (p. 63). In this way, students are no longer being provided with knowledge, but instead are constructors of their own knowledge (p. 63). Teachers, the researchers state, "should seize every opportunity to guide and encourage the students, who can thus actively explore and build their own knowledge system leading to an improved cognitive structure of mathematics learning" (pp. 63-64). Interestingly, in their work, Novotná and Hospešová (2013) compare the mathematics classroom situation to that of "an orchestra with a conductor and musicians" (p. 140), arguing, for instance, that similar to the roles of the conductor and the musicians in the orchestra being inseparable and necessary, so too, are the roles of the teacher and his/her students in the classroom. According to Novotná and Hospešová (2013), the conductor's role could clearly resemble the role of the teacher in the classroom, and even the student's role could, to some degree, resemble the role of the conductor to his/her classmates (p. 140). In a similar vein, Planas (2012) draws on the metaphor of *orchestration*. Her idea of orchestration refers to "the manner in which various voices are individually used to produce coordinated results" (p. 333). She explains, "as in the case with instruments in an orchestra, the final production requires successful individual practices with instruments that successfully interact with each other" (p. 333). Hence, it is not just the conductor's role that is essential, it is also the relationships of the players who facilitate the orchestration that is important as well. As such, orchestration alludes to the important and contributing necessity of everyone's engagement and involvement in the classroom, while too, being mindful of the fact that "all contexts are socially and politically charged in such a way that the orchestration cannot be attributed to a single person" (Planas, 2012, p. 334). Suggesting, an awareness of the significant roles other agents may possess that may determine the value of specific classroom practices (Planas, 2012). This said, it becomes possible to argue that both the teacher's pedagogical practices and students' role in the mathematics classroom are important because without the presence and contribution of all participants, neither learning nor teaching may be endorsed effectively.

Another influence on students' learning of mathematics is characterized by teachers' views and experiences that shape their pedagogical practices, which in turn influence student's learning of mathematics. There is a great deal of research that examines teachers' views on the significance of students' cultural and ethnic background on their mathematics learning and subsequently how these views impact teacher practice (e.g., Abreu, 2005; Abreu & Gorgorió, 2007; César & Favilli, 2005). Some suggest that when teachers are confronted with a more diverse classroom, they tend to minimize the cultural differences among their students and may have the belief that mathematics knowledge is culture-free and universal (Abreu, 2005; Abreu & Gorgorió, 2007; César & Favilli, 2005). Therefore, teachers may not recognize that students have different cultural ways of approaching mathematics. As a result, teachers may not question whether mathematics in their school is the same or different in other places of the world, and may not even consider other mathematical representations (Abreu & Gorgorió, 2007). Some researchers have demonstrated that teachers often see mathematics as universal. For example, César and Favilli (2005) argued that while teachers had a positive and accepting attitude toward culturally diverse students' integration in school, the teachers' experiences and views about mathematics acted as a barrier to effective inclusion (p. 1158). For instance, some teachers viewed mathematics as universal, explaining that it is mostly a matter of computation, which is "international" (César & Favilli, 2005, p. 1159). In such situations, teachers' understanding of mathematics as universal prevents them from inviting and accepting alternate ways of solving problems, which can cause problems for children learning mathematics.

Other researchers have found that teachers often believe that students must have mastery of the language of instruction before learning mathematics. For instance, in several qualitative studies, researchers found that teachers tend to assume that when multilingual students learn the host culture language, they will learn mathematics in the same way as the rest of the class (Abreu & Gorgorió, 2007; Gorgorió & Planas, 2001). As one teacher explained: "I don't think it is a good idea to have children with no Catalan proficiency in math class…you know, math requires a sophisticated use of language"; "as soon as they learn our language, there are no more significant differences…maybe, in the social sciences, but certainly not in the mathematics classroom" (Gorgorió & Planas, 2001, p. 20-21). The teacher's arguments are supported by the example she gives regarding a student who knew how to do subtraction, but used a different algorithm. When the teacher solved the problem using another algorithm and explained her solution, the student assumed that he had done it wrong. In the teacher's view, if he had a better understanding of Catalan, this would not have been the case. According to Favilli and Tintori (2002) students enter the mathematics classroom with their own

> set of mathematical knowledge which could appear non-standard (with respect to the rest of the class); they possibly use computing algorithms which are different from the ones used by their classmates (and which sometimes are not accepted by the teachers). (Favilli & Tintori, 2002, p. 12)

Teachers often interpret these differences as evidence of the students' limited mathematical knowledge or limited proficiency in the language of instruction.

Another aspect of research has shown that the language that the teacher views as more privileged influences what language the students use and for what purposes. Setati (2005), working in South Africa, found that while teachers may view multiple language use to contribute to students' mathematics learning, the teachers' pedagogical practices influence the students' choice of language and their choice to use specific discourses—mathematical and nonmathematical. Setati's (2005) observations of a grade four mathematics lesson taught by a multilingual teacher demonstrated that the teacher and her students shared the same home language (i.e., Setswana), however, English, as Setati refers to it, is the *language of learning and teaching* (LoLT). During the lesson, the teacher's instruction involved language switching and was accompanied by a switch in mathematical discourse. These language practices were reflected in students' discussions and interactions. Like their teacher, the students used English to articulate the steps taken to solve the mathematical problem (procedural discourse), yet Setswana was used as the language of solidarity (conceptual discourse) where "learners articulate, share, discuss, reflect upon, and refine their understanding of the mathematics" (p. 449). These findings raise the more general issue of how students are often trained to become proficient in doing mathematics in a language that has social and economic power (Setati, 2005, 2008).

Institutional Influences

One additional strand of research on multilingual students' learning of mathematics focuses on institutional influences. Research in this area identify the school and the classroom as the institutions, and where, institutional influences impact mathematics teaching and learning. These institutional influences may legitimize the use of certain languages and disregard others (Barwell, 2012, 2018; Chronaki & Planas, 2018; Moschkovich, 2005; Norén & Andersson, 2016; Planas & Setati, 2009, Planas & Valero, 2016; Setati, 2005). According to Barwell (2012), "the pressure comes from a preference for a single school language, a standardized mathematical register, national and institutional language policies and the high status accorded to some natural languages and social languages" (p. 327). In an important way, diverse educational settings bring about differences in how languages are viewed and used in mathematics classrooms (Barwell et al., 2016; Moschkovich, 2014).

Several researchers have demonstrated that although the use of multiple languages in the mathematics classroom could support students' learning, multiple language use is not a straightforward matter, as varying language infrastructures in and around the school creates different demands on students and their mathematics teachers. In South Africa, for example, the language-in-education policy recognizes 11 official languages and encourages multiple language use in mathematics classrooms. However, Setati and Adler (2000) have found that the context in which the mathematics lesson takes place influences which language the students and their teachers use and for what purposes. For instance, in rural classrooms in South Africa, the teaching and learning of mathematics are carried out in a context where there is very limited English language infrastructure. This puts pressure on teachers to constantly model and encourage English, as the classroom is the only context in which learners have this exposure (p. 255). In contrast, in more urban environments, there is a considerable amount of English language infrastructure in and around the school. As a result, the students and their teachers are more supportive of English as the language of learning and teaching.

The few studies that have explored the use of multiple languages in second language learning settings in the UK, Australia and Canada – clearly show that the teaching and learning of mathematics is exclusively in English because English is the language used by the majority of teachers and learners at schools (Barwell, 2005, 2014c; Clarkson, 2006). As a result, students' home languages are rarely heard in mathematics classrooms. However, researchers have found that students in second language learning settings are not necessarily prohibited from using their home languages during mathematical activities, but when and how it should be used is controlled by the teacher (Barwell, 2014c; Parvanehnezhad & Clarkson, 2008). For instance, in Australia, Parvanehnezhad and Clarkson (2008) reported on students using their home language (i.e., Farsi) during solitary activities. In Quebec, Canada, Barwell (2014c) reported on Cree students in grades 5-6 that were identified by the school as needing additional language support (p. 914). The use of Cree among students was used when they were invited to explain to another some aspect of the mathematics work that they understood. However, the students' teacher would constantly remind and encourage them to speak in English. Such control is also reflected in a small printed notice mounted on one of the classroom walls that reads 'Remember to speak in English' (Barwell, 2014c). Interestingly, when the students were asked about the notice, they reminded the researcher of the time when one of the students in the class wrote on the blackboard 'Remember to speak in Cree'. According to Barwell (2014c), the notice represents at least two voices: "the institutional voice represented by the notice and representative of the institutional status of English, and the student's voicing of different intentions" (p. 918). This work clearly reflects a tension amid institutional influences—that of policymaking and that of classroom interactions. The school legitimizes the use of a particular language, while the students or teachers might prefer to use their home languages as well. In certain situations, the teacher may not allow its use, while in others s/he might limit its use for certain activities, but may repeatedly encourage students to speak in the language of instruction—a practice understood to improve students' competencies in the language of instruction.

Sociopolitical Influences

An additional and more recent strand of research on multilingual students' learning of mathematics focuses on sociopolitical influences. Sociopolitical influences relate to government or public affairs of a country or province or the policies that govern it. Setati (2005) suggests two categories of sociopolitical influences, each at a different level. One is at the macro level of provincial policy, and the second is at the micro level of the school or even classroom interactions. In either case, relations exist within and outside an educational setting that influence the actions, behaviors or beliefs of those within that setting, including school boards, schools, teachers, and students. Thus, when we talk about sociopolitical influences we are dealing with issues related to educational policies which have an influence on teacher practice and, in turn, influence students' participation and attainment in mathematics education (Barwell et al. 2016; Chronaki & Planas, 2018; Civil & Planas, 2011; Halai & Clarkson, 2016; Setati, 2005; Planas, 2018; Setati & Planas, 2012). For instance, research in this area has found that while students' home languages may be accepted, and certain pedagogies have supported language-as-resource; the use of students' language(s) is politically mediated (Planas & Civil, 2013). Students learning an additional language in plurilingual classrooms face "a complex relationship between, on the one hand, the different value given to different languages, and on the other hand, the students' language choices in their mathematical practices" (Civil & Planas, 2011, p. 1). This apprehension, according to Setati and Planas (2012), "is not just about language and/or mathematics; it is also about social relations, the political role of language, and the context in which the mathematics is taught and learned" (p. 167-168).

Several researchers have demonstrated the effects of the political role of language on mathematics education (Civil & Planas, 2011; Setati, 2005, 2008; Setati & Planas, 2012). In South Africa, for example, Setati (2008) found that English as an international language emerged as a dominant cultural model that shaped teachers' and students' language choice in mathematics classrooms (p. 106). For instance, all 6 intermediate mathematics teachers interviewed in this study expressed "ideological and pragmatic reasons for their preferences to teach mathematics in English" (p. 106). Their reasons for the notion that English is an international language were due to the fact that textbooks, examinations, and higher education are all in English (p. 106). With respect to the learners, they too preferred English as it rendered more status at a national and institutional level. According to Setati (2008), "learners who position themselves in relation to English are concerned with access to social goods and positioned by the social and economic power of English" (p. 103). Here, the students and teachers do not focus on epistemological access, rather, they argue for English as more useful for "getting on in the world" (Barwell, 2012). Language, then, is much more than just means for thinking and doing mathematics; "it is also political and has implications for how social goods are or ought to be distributed" (Barwell, Barton, Setati, 2007, p. 118).

Other research has found that privileging the language of instruction may lead to the exclusion or inclusion of certain individuals in classroom discussions and in decision-making processes (Planas & Civil, 2013; Setati, 2005). In Barcelona and in Tucson, Planas and Civil

(2013) found that schools tend to privilege the language of instruction (i.e., Catalan or English) over other languages. As a result, the students whose home language is not the language of instruction did not participate in whole-class discussions (Planas & Civil, 2013). According to Planas and Civil (2013), the political issue here rests on the value given to the language of instruction during classroom interactions, which has a significant influence on the development and continuity of non-dominant speakers' mathematical participation. As such, ignoring the political role of language assumes that power relationships do not exist in society (Gutiérrez, 2010; Setati, 2005).

Summary

The reviewed studies report on the multiple resources that the multilingual learner draws on to describe mathematical objects and to communicate their mathematical knowledge, suggesting that the use of multiple resources "can provide access both to mathematical ideas and powerful ways of thinking and speaking" (Morgan, 2007, p. 241). The studies also explored the development of students' language proficiency and found that exclusive emphasis on enhancing language acquisition does not necessarily imply that multilingual learners will overcome the linguistic barriers that arise in the new culture. In addition, teachers' perspectives regarding the demand to learn the language of instruction prior to attending mathematics classrooms, and their attitudes regarding the universality of mathematics may ultimately hinder the students learning of mathematics (Barwell, 2005; Civil, 2008; Moschkovich, 2007). Other research suggests that it is too complex to implement the flexible use of two or more languages in the mathematics classroom, as there are issues related to different language infrastructures in and around the schools that shape and influence the pedagogical and language practices of an educational setting (Setati & Adler, 2000). According to Setati (2005), "decisions about which language to use, how to use it, and for what purposes, are thus both pedagogic and political" (p. 451). Hence, the findings from a study in one educational setting might not be easily applied in other settings, and should be treated with caution (Setati & Barwell, 2006).

These studies have broadened my thinking in terms of the varied ways in which the influences of individual, pedagogical, institutional, and sociopolitical features of mathematics classrooms could influence students' learning of mathematics in plurilingual settings. These influences are highly interconnected. For instance, a dominant cultural model that sees a single language to provide academic and social access is influenced by a national and institutional language policy, which in turn influences the teachers' pedagogical and language practices in the mathematics classroom, which then influences students' language choice and the choice of discourses used in a plurilingual mathematics classroom.

Although this literature review on multilingual learners and mathematics suggests that researchers have observed many children in countries spanning the globe, there is still a small number of research in Canada (e.g., Barwell, 2014c, 2017, 2018, 2020; Takeuchi, 2015, 2016) that specifically focus on multilingual learners' mathematical experiences in a plurilingual educational setting. Hence, an objective of my research study is to build on this research to provide an in-depth description and understanding of multilingual learners mathematics experiences in a plurilingual educational setting, in Canada.

CHAPTER 3: THEORETICAL PERSPECTIVES

In this chapter, I set out the theoretical perspectives that underpinned the basis of my study. It also guided my observation and analysis to understand multilingual learners' experiences and meaning making in a plurilingual educational setting. I assumed a sociocultural perspective that views learning and development as embedded in cultural as well as social contexts. The predominant rationale for this interest is centered on the recognition that only by seeing mathematics classrooms from students' experiences and interaction in mathematical discourse can researchers understand the social processes nested and created in students' learning, participation, and mathematics experiences (Clarke, 2013). Hence, a sociocultural perspective that draws from Vygotsky's theory offers a promising epistemological conceptualization of children, their learning and language development as mediated by social, cultural, and historical contexts. In the following sections, I provide a detailed overview of sociocultural theory and discuss the stance by which my views and my study are framed.

Thinking and Learning—A Brief Historical Overview

For decades, children's thinking and learning were perceived to be the mere transmission of knowledge from an adult to the child (Kozulin, 2012). However, when Jean Piaget's theory of cognitive development reached the West and other English-speaking communities, we witnessed a major swing from the notion of transmission to the notion of construction (Kozulin, 2012). Despite this prominent shift, some scholars continued to perceive the latter notion in individualistic terms because for Piaget, the individual's mind is perceived as the primary source of their development (Sfard, 2006, p. 156). This notion was inspired by approaches in psychology that viewed learning as a biological-like process of growth (Alrø & Skovsmose, 2002; Kozulin, 2012). Viewed in this way, it becomes natural to consider development as preceding learning, and as a result, teaching would need to be modified to meet the child's development, as each stage manifests specific learning opportunities (Alrø & Skovsmose, 2002). These earlier views focused on establishing cross-situational commonalities rather than differences by way of assuming that children's learning processes are universal across contexts (Sfard, 2006, p. 156).

According to Kozulin (2012), it was "only when multiculturalism became an undeniable empirical reality of the European and American classrooms did educators and researchers finally discover the ever-present phenomenon of culture in children's thinking and learning" (p. x). Many scholars found support in Lev Vygotsky's ideas (1896-1934) as he was among the first modern psychologists to suggest that "development is embedded in cultural as well as social contexts and these cultural contexts affect learning in fundamental ways" (Forman, 1989, p. 33). Vygotsky rejected the dominant view of universalistic psychology, and considered knowledge to be socially constructed (Ford & Forman, 2006). That is, the sociocultural forces evident in the child's life from the very beginning shape their development. Hence, it becomes possible to argue, that cognition is situated in specific social, cultural, historical, and institutional settings, and "that these settings shape and provide cultural tools that are mastered by individuals to form this functioning" (Wertsch, 1994, p. 204). This argument shifts the focus on learning and development from something that takes place exclusively inside a person's mind to something that originates in social interactions and culturally organized activities (Ford & Forman, 2006; Lave, 1991; Wells, 1999). To this end, I first provide a brief clarification in the terminology used to illustrate affinity to Vygotsky's theory. Second, an overview of the central features of this theory and their more recent formulations are presented.

Illustrating Affinity to Vygotsky's Theory

There are ample debates about the terminology used by researchers to illustrate their affinity to Vygotsky's theory. This was actually one of my own personal dilemmas in trying to discern the various terms and to decide on which I would adopt. These terms include, 'neo-Vygotskian', 'cultural-historical', 'sociocultural', 'situated cognition', and 'activity theory'. Van Oers (1998) has argued that utilizing the various terms associated with Vygotsky's theory, simply addresses a different focus within the development of his theory (p. 475). It should be made clear, however, that Vygotsky himself preferred to use 'cultural psychology' or 'cultural-historical psychology' when referring to his theory, and rarely, if ever used the term 'sociocultural' (Van Oers, 1998). Wertsch (1985; 1991) holds much credit for coining the term 'sociocultural' to capture and explicate the view that human mental functioning is situated in social, cultural, historical, and institutional settings. He adds, "that these settings shape and provide cultural tools that are mastered by individuals to form this functioning" (Wertsch, 1994, p. 204). In order for this theory to be of greatest benefit, I will pleasingly follow this convention by using the term sociocultural, which invokes a connection to the work of Vygotsky and the more recent research devoted to sociocultural studies (Forman et al., 1993; Lantolf, 2006; Lave & Wenger, 1991; Rogoff, 1995; 2003; Wells, 1999; 2000; Wertsch, 1991).

Sociocultural Theory of Learning and Development

This research adopts a sociocultural understanding of learning and development that considers “learning, thinking, and knowing…[as] relations among people engaged in activity in, with, and arising from the socially and culturally structured world” (Lave, 1991, p. 67). An account of Vygotsky’s theory of human development identifies three key features that form the core of his theory (Wells, 1999, 2000; Wertsch, 1991). The first is reliance on a genetic method. That is, to understand individual development, it is important to consider the influence of the history and the wider community or institution on the individual’s participation and attainment in an activity. Vygotsky considered four basic lines in the development of behaviour: development in the species as a whole (phylogenesis); development over time in the wider culture of which the individual was becoming a member (cultural history); development through interactions in specific sociocultural settings (microgenesis); and finally, development over the life of an individual (ontogenesis).

Rogoff (1995) elaborates on this notion of development on multiple levels by focusing on the last three basic lines in the development of behaviour. In her work, she suggests analytical tools and processes to observe development within culturally valued activities. She proposes three planes of analysis: apprenticeship, guided participation, and participatory appropriation—corresponding to three different aspects of focus in sociocultural activity: community/ institutional, interpersonal, and personal (Rogoff, 1995). The apprenticeship or community/ institutional plane focuses on understanding the historical/ institutional context of an activity. In other words, it “focuses attention on the specific nature of the activity involved, as well as on its relation to practices and institutions of the community in which it occurs – economic, political, spiritual, and material” (p. 142). The guided participation or interpersonal plane stresses the mutual involvement of individuals and their social partners as they engage in the events and with materials of everyday life through communication and coordination (p. 146-147). These engagements “may be tacit or explicit, face-to-face or distal, involved in shared endeavors with specific familiar people or distant unknown individuals or groups – peers as well as experts, neighbors as well as distant heroes, siblings as well as ancestors” (p. 147). The participatory appropriation or personal plane refers "to the process by which individuals transform their understanding of and responsibility for activities through their own participation" (p.151). Although these planes of analysis could be viewed as different focuses, they are mutually constituted and cannot stand alone in the analysis of development within sociocultural activity because to understand one would require the involvement of the others (Rogoff, 1995).

A second key feature of Vygotsky's theory pertains to the role of cultural tools, which are both material and symbolic. These include, and are not limited to "language, various systems of counting, mnemonic techniques, algebraic symbol system, work of art, writing, schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs; and so on" (Vygotsky, 1981a, p. 137; as cited in Wertsch, 1985*,* p. 79). From a Vygotskian perspective, these tools "are culturally produced and represent cultural innovations and changes, [and] as we internalize their use, our thought is undeniably cultural in nature" (Nasir & Hand, 2006, p. 461). Vygotsky proposed that language is one of the most powerful and pervasive cultural tools connecting intrapsychological and interpsychological functioning (Lantolf & Thorne, 2000; Wertsch, 1991). Language acts to mediate the relationship between the individual and the social world (Lantolf & Thorne, 2000). Language simultaneously serves a dual role in human functioning: it is a means of communication, and it mediates higher mental functions (Nasir & Hand, 2006, p. 461).

Finally, Vygotsky views higher mental functions, such as voluntary attention, logical memory, problem solving and the formation of concepts, as created social processes. Vygotsky (1981) emphasizes what is socioculturally specific. That is, the intellectual development of the individual should not be understood without acknowledging a person's interaction with other people in their social environment (Wells, 1999). As Vygotsky (1981) stated, "the very mechanism underlying higher mental functions is a copy from social interaction; all higher mental functions are internalized social relationships" (p. 164).

In an important way, Vygotsky's (1981) "genetic law of cultural development" combines these three features as a way to illustrate that the individual and the society in which s/he participates are mutually constituted. The vastly cited formulation is as follows:

> Any function in the child's cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category. This is equally true with regard to

> voluntary attention, logical memory, the formation of concepts, and the development of the volition…[I]t goes without saying that internalization transforms the process itself and changes its structure and functions. Social relations or relations among people genetically underlie all higher functions and their relationships. (p. 163)

This law includes several strong aspects that are worth highlighting. The first asserts that higher mental functions are not only attributed to individuals, but can equally occur between groups on the interpsychological plane (Wertsch, 1985). The second addresses the link between interpsychological and intrapsychological functioning. According to Wertsch (1985), "instead of merely claiming that individuals somehow learn by participating in interpsychological functioning, Vygotsky's formulation means that there is an inherent connection between the two planes of functioning" (p. 61). Vygotsky's statements with regards to the genetic law of cultural development and the social origin of higher mental functions are surfaced throughout his writings. Here, I will consider two fundamental constructs that have been widely considered by sociocultural researchers; internalization and the zone of proximal development. According to Wertsch (1985), "when analyzing these and other phenomena, the underlying claim is always that in order to understand higher mental functioning on the intrapsychological plane, one must conduct a genetic analysis of its interpsychological precursors" (p. 61).

Internalization

Vygotsky viewed internalization "as a process whereby certain aspects of patterns of activity that had been performed on an external plane come to be executed on an internal plane" (Wertsch, 1985, p. 61). Vygotsky defined external activity in terms of "semiotically mediated social processes" (Wertsch, 1985, p. 62), which are not limited to speech—though Vygotsky (1978) argued, speech plays an essential role in the organization of higher mental functions, but also includes cultural tools (1981a, p. 137; as cited in Wertsch, 1985, p. 79). These socially mediated processes provide an understanding of the emergence of internal functioning (Wertch, 1985, p. 62). Vygotsky viewed "social reality as playing a primary role in determining the nature of internal intrapsychological functioning" (Wertch, 1985, p. 63). In other words, the internal mental processes are created as a result of one's exposure to social processes (Wertch, 1985, p. 63). In this way, any changes that happen on an external plane are inherently linked to changes that occur in the internal plane. The emphasis, according to Vygotsky (1981), is on the capacity of cultural tools to transform higher mental functioning.

It is important to note, however, that internalization has become one of the most criticized facets of Vygotsky's theory. We note, for example, the theoretical claim that Vygotsky was suggesting a process that involves a simple transfer model of social phenomena from the external, interpsychological plane to the internal, intrapsychological plane (Wertsch, 1985, p. 63). Social constructivist critics of Vygotsky's theory, such as Cobb and Yackel (1996), are among the researchers who interpreted internalization in terms of a transfer model. Arguing, that this aspect of Vygotsky's theory suggests that learners simply "inherit the cultural meanings that constitute their intellectual bequest from prior generations" (p. 186). With respect to mathematics education, they argued that this would entail that students simply "imitate culturally established mathematical practices" as they interact with others (Cobb & Yackel, 1996, p. 185). In a similar vein, Rogoff (1995), who in many ways is a follower of Vygotsky, claims that the internalization perspective "views development in terms of a static, bounded "acquisition" or "transmission" of pieces of knowledge (either by internal construction or by internalization of external pieces of knowledge)" (p. 153). Still, for others, they claim that the mere use of the term 'internalization' creates the impression of focusing solely on the individual's mind in the construction of knowledge (Lave & Wenger, 1991).

I recognize the contribution to be made by these researchers, but in my view, I do not think that Vygotsky was claiming that children are merely importing an external process to the internal plane, or that by using the term internalization will entail a focus on the individual learner. I would agree with John-Steiner and Mahn (1996) that interpreting sociocultural theory along these lines may lead to a condensed and inadequate understanding of the sociocultural aspect of Vygotsky's theory. That is, we basically simplify the importance of the active, mutually constitutive nature of learning and its social and intergenerational dynamic (John-Steiner & Mahn, 1996). Although Rogoff (1990, 1991, 1995) criticizes the internalization perspective, we can agree that since the social aspects of children's functioning are an integral part to their individual aspects, then what is practiced in social interaction is never on the outside of a barrier. Children are already participants in social activities because they are actively observing and working with others because it is through participation that children transform their understanding and skill (Rogoff, 1990; 1991). Consequently, the sociocultural theory of internalization is not a matter of transfer, but entails a mutual, dynamic, and active process in the coconstruction of knowledge (John-Steiner & Mahn, 1996; Lantolf & Thorne, 2000). This said, the use of the term internalization should not impose misinterpretations because viewed from a sociocultural perspective; cognition is situated in specific cultural, historical, and institutional settings. As a result, there is a clear emphasis on the sociocultural nature of human activity and development (Daniels, 2001). I am also consoled by Leont'ev's reminder that "the process of internalization is not the *transferal* of an external activity to a pre-existing, internal 'plane of consciousness': it is the process in which this plane is *formed*" (as cited in Wertsch & Stone, 1985, p. 163). Viewed in this way, the emphasis is on the inherent relationship between the two planes of activity, and where internalization is addressed in consideration of the social origins of individual activity—involves transformation rather than an identical replication (Wertsch, 1985). Moreover, I find the notion of internalization as holding a direct correlation with identity. Identity, according to Holland et al. (1998), Roth (2004), and Wenger (2008), develops through a person's interaction in social communities, and where "the focus must be on the process of their mutual constitution" (Wenger 2008, P. 146). In this way, identity is not stable but constantly re/produced, individually and collectively (Roth, 2004). Hence, one could argue that learning and identity are inseparable because learning transforms who a person is and what they are capable of doing (Lave & Wenger, 1991; Wenger, 2008). On this view, "learning involves the whole person; it implies not only a relation to specific activities, but a relation to social communities – it implies becoming a full participant, a member, a kind of person" (Lave & Wenger, 1991, p. 53). In other words, learning implies a process of constant transformation or regeneration of a person's identity as they engage in various opportunities enabled by socially mediated experiences. Using Gee's (2005) words, as we engage in conversations with others, we are "co-constructing socioculturally situated identities" (p. 9).

Zone of Proximal Development

In contrast to the Piagetian positioning that learning should be matched with the child's development, Vygotsky (1978) argued, "the only good learning is that which is in advance of development" (p. 89). According to Vygotsky (1978), "learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (p. 90). It was in support of this perspective that Vygotsky (1978) introduced the construct of the zone of proximal development. He argued that in order to understand the relationship between development and learning, we must determine at least two developmental levels. The first is the actual developmental level determined by what has already been established—"only those things that children can do on their own" (p. 85). The second is the potential level of development, which as suggested by the definition of the zone of proximal development, is "the distance between the [child's] actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). This clearly suggests that learning and interacting with others precedes and shapes development (Lantolf & Thorne, 2000, p. 207).

From a Vygotskian perspective, what children learn should never be determined by cause and effect; rather, it should be motivated through purposefully designed learning environments, which stimulates developmental changes (Lantolf & Thorne, 2000). Viewed in this way, the zone of proximal development is not only a model of the developmental process, but also a conceptual tool that allows educators to understand what already has been achieved by the learner, and what is in the process of maturation (Lantolf & Thorne, 2000; Vygotsky, 1978). When used proactively, what children can do with assistance today, s/he will be able to do on their own at a later time (Vygotsky, 1978). Also significant is that from a Vygotskian approach to learning, this does not depend on a one-way flow of knowledge from teacher to learner. Other students, through collaborative learning, can also provide one another assistance (Wells, 1999). Although Vygotsky specified "more capable peers", it has become apparent for many researchers that it is not necessary for there to be a person who is more capable than others (Forman & McPhail, 1993; Wells, 1999). During collaborative learning, any participant when working on joint initiated tasks, whether during whole-class discussions, in small groups, or dyads can possibly make a contribution that helps towards a solution (Wells, 1999). For learning to occur in the zone of proximal development, Wells (1999) argues "it is not so much a more capable other that is required as a willingness on the part of all participants to learn with and from each other" (p. 324). Also, in their work, Brown et al. (1996) argue that in the zone of proximal development, it is not necessarily exclusive to the collaboration of "people, adults and children…but it can also include artifacts such as books, videos, wall displays, scientific equipment, and a computer environment", or anything of that matter that is intended to support learning (p. 191). Thus, providing assistance in the zone of proximal development has a purpose, not of role or status, but which could potentially assist others through collaboration itself (Wells, 1999, p. 308). For one thing, Moll (1990) suggests that the focus “should be on the creation, development and communication of meaning through the collaborative use of mediational means rather than on the transfer of skills from the more to less capable partner” (as cited in Daniels, 2001, p. 60). These understandings shifted the focus on learning from something that takes place exclusively inside a person’s head to something that originates in social interactions and culturally organized activities (Ford & Forman, 2006; Lave, 1991; Wells, 1999). In what follows, I describe how a sociocultural understanding of human development informed research into the teaching and learning of mathematics, as well as, how it could inform my own research.

Mathematics Education and Sociocultural Theory

Research in mathematics education witnessed a vast interest in the social and cultural elements involved in the teaching and learning of mathematics (Lerman, 2000; 2001). These shifts were partly what Lerman (2000) referred to as the *social turn* in mathematics education research. In his account of the social turn, Lerman (2000) states that the social turn was “intended to signal something different; namely, the emergence into the mathematics education research community of theories that see meaning, thinking, and reasoning as products of social activity” (p. 23). In order to understand this phenomenon, many mathematics researchers found support in a sociocultural understanding of learning and development (Vygotsky, 1978) that considers “the use of discourse and some of its contents (norms, values, valorizations) as crucial mediating tools in order to interpret the mathematical learner in context” (Gorgorió & Planas, 2004, p. 21).

Several researchers have argued that interactions and participation in mathematical discourse practices encourage students’ mathematical thinking (Assaf & Graves, 2019; Inagaki et al., 1998; Forman, 1996, 2003; Francisco, 2013; Hunter et al., 2018; Moschkovich et al., 2018; Planas et al., 2018). Although children might disagree, or they might have discrete interpretations or understandings, the discussion that these children are involved in enables them to share their views and to develop an answer or understanding that supports the problem at hand (Assaf & Graves, 2019). On the contrary, students who do not openly participate in whole class discussions, nevertheless internalize many elements of the discussions that take place (Inagaki et al., 1998), and what we may consider to be an individual mathematical activity, from a Vygotskian perspective, is still to be understood as social given that the other may still serve to guide the individual's actions (Daniels, 2001). Though they are not present physically, their voice may still be present, which results in the coconstruction of knowledge (Daniels, 2001). According to Inagaki et al. (1998) "the secret for the success of whole-class discussion can be found in students' active and socially sensitive minds" (p. 523).

Drawing on the transformative view of internalization, it becomes possible to argue that it is through the negotiation of meaning during mathematical interactions that learning and development occur, as each learner's individual mental functions mediate (and at the same time are mediated by) the constitutive interpsychological processes of the group (Chang-Wells & Wells, 1993, p. 86). This further highlights the significant role of social interaction where students develop their mathematical and logical thinking from their experiences, which is constantly developing as they interact with others (Schliemann & Carraher, 2002). Rogoff (1990) argues that since the social aspects of children's functioning are an integral part of their individual aspects, then what is practiced in social interaction is never on the outside of a barrier. Children are already participants in social activities because they are actively observing and working with others. It is through participation that children transform their understanding and skill (Rogoff, 1990; 1991). Viewed in this way, the focus is on the "creation, development and communication of meaning through the collaborative use of mediational means" (Daniels, 2000, p. 60), rather than on the transfer of knowledge from the head of the teacher to that of the student. Therefore, focusing on the social and mediated nature of an activity shifts the focus of learning from the individual to seeing learners "as differentially-positioned members of social and historical collectivities" (Norton & Toohey, 2011, pp. 418-19).

Hence, turning to sociocultural theories, we shift from what multilingual learners cannot do, and instead consider the multiple resources multilingual children use to communicate mathematically (Moschkovich, 2007; 2017). In Vygotskian terms, the focus is on "the potential for progress in what learners already know and do" (as cited in Moschkovich, 2012, p. 91). With respect to learners' competencies and resources, Moschkovich (2007, 2017) includes mathematical Discourses.[5] For Moschkovich (2007), "mathematical Discourses include not only

---

[5] Moschkovich relies on the distinction outlined in Gee's (2005) work concerning "Discourse" with a "big D" and "discourse" with a "little d". In his work, Gee (2005) notes that "discourse"

ways of talking, acting, interacting, thinking, believing, reading, and writing but also communities, values, beliefs, points of view, objects, and gestures" (p. 95). Hence, participating in classroom mathematical Discourse is a process of becoming a member of a community and acquiring the practices of that community (Sfard, 1998, 2006).

In his work, Gee (2005) talks about "situated meanings". He implies that meaning is grounded in actual practices and experiences. When we consider the meanings of words in their actual context of use, these words are neither stable nor general. Instead, words have different meanings in different contexts of use, and are based on our understanding of that context and on our past experiences. In addition, situated meanings "are *negotiated* between people in and through communicative social interaction (Gee, 2005, p. 94). Bearing this view, Gee (2005) states, allows us to "see that the meanings of words are also integrally linked to and vary across different social and cultural groups" (p. 53).

Adopting a sociocultural perspective that views learning and development as embedded in cultural as well as social contexts guided my observation and analysis to understand multilingual learners' experiences and meaning making in plurilingual educational setting. The rational for this interest is centered on the recognition that students develop their mathematical thinking and understanding from their experiences, which are constantly changing and developing as they interact with others. Also, the language they use to express their thinking, the symbolic expression, and any material they use all provide important information to expand and enhance their mathematical knowledge. In addition, I find a sociocultural perspective compatible with the way I am thinking about the influences that impact students' learning of mathematics in plurilingual classrooms. For example, when we talk about cultural tools, I would consider what influence does the student's use of gestures, drawings, concrete objects, and more specifically, language have on the learning situation. From a sociocultural perspective, language is viewed to have meaning only in and through social interactions (Gee, 2005).

---

with a "small d" implies the use of language to enact activities and identities. However, when language is used with non-language "stuff" to enact identities and activities, then he notes that "big D" Discourses are involved.

CHAPTER 4: METHODOLOGY

This is an ethnographic study of multilingual learners' meaning making and mathematical experiences. I find ethnography compatible with my theoretical perspective and the way I think about learning. In the sense that I anticipated learning and understanding more fully how multilingual learners experience and make sense of mathematics in a plurilingual educational setting. Furthermore, ethnography provides a setting and approach where I could understand and describe how multilingual learners negotiate mathematical meaning, and can examine the influences that impact students' learning of mathematics.

In this chapter, I provide detailed information concerning the methodology of my study. I first highlight what I have taken ethnography to mean. Then, I give prominence to ethnography and its potential for engaging children participants in human inquiry to understand their experiences and social worlds as they do. I also draw on some of the issues and responsibilities researchers could face when working with a vulnerable research population. Then, I include an account of how ethnography connects to my research study. Next, I describe my recruitment and procedures, followed by a description of the research setting and the participants. I then explain the data collection instruments used for this study. The section concludes with the data analysis methods that I used.

Defining Ethnography

With the increase of researchers adopting an ethnographic methodology (Gordon et al., 2001; Green & Bloome, 2004), defining what counts as an ethnography can be a complex endeavor due to the many forms, approaches, and disciplines that come under 'ethnography' or 'ethnographic' (Green & Bloome, 2004, p. 183). Historically, ethnography is known to be rooted within the discipline of anthropology where the use of theories of culture is central (Geertz, 1973). In the early days of ethnography, the concern was on *what ethnographers do* (Geertz, 1973; Wolcott, 1987), and more recently, there has been an emphasis on the process of ethnography—*how it is done* (LeCompte & Schensul, 2010; Wolcott, 2008). Hence, to think of what ethnography stands for is to posit what various researchers have taken ethnography to mean. For instance, doing ethnography, as described by Geertz (1973), is an interpretive act of 'thick description' of what a group of people is up to. Others consider ethnography "as a systemic approach to learning about the social and cultural life of communities, institutions, and other settings" (LeCompte & Schensul, 2010, p. 1). Still for others, ethnographic fieldwork involves living with a group of people for an extended period of time and documenting and interpreting their way of life, beliefs, and values (Atkinson & Hammersly, 2007, p. 1). In short, there is no single, standard meaning of ethnography that could capture all of its meanings in all contexts, as there are multiple approaches to ethnography (Atkinson & Hammersly, 2007). In referring to the multiple approaches of ethnography, Agar (2006) argues,

> They will be different because of different combinations of ethnographer and community, different ways that a study moves, different choices and different contingencies along the way, different events in the world around the study—any or all of these can change the trajectory of a study over time. More than one path is possible. (p. 5)

With the existing literature in mind, I turn to a paper by Wolcott (1987) entitled *On Ethnographic Intent.* Although Wolcott wrote this text at least three decades ago, what has particularly resonated with me is what Wolcott (1987) calls ethnographic intent. He explains that however ethnography is to be defined; it must be oriented towards cultural interpretation—namely to describe and interpret cultural behaviour, which Wolcott (1987) states is the 'essence' to the entire ethnographic endeavor. Hence, it is not simply the borrowing of ethnographic techniques, such as the time spent in a field of study, or providing clear and detailed descriptions of a context, or keeping good relationships, bonds, and understandings of participants and their problems or resolutions that results in better ethnography or assures it to be ethnographic (Wolcott, 1987). It is the researcher's prevailing concern that is to be geared towards the nature of culture itself that makes a study ethnographic (Wolcott, 1987, p. 45). In other words, a researcher should focus on "discerning cultural patterning in the behavior they observe", while also, engaging in "an ongoing dialogue dealing with the nature of culture itself" (Wolcott, 1987, p. 45). Moreover, culture "is best revealed within what people *do*, what they *say* (and *say they do*), or within an uneasy tension between what they *really do* and what they *say they ought to do*" (Wolcott, 1987, p. 45). Put even more modestly, culture, according to Abu-Lughod (1991) "is an account of particular social processes as practiced by particular people in particular settings" (as cited in Wolcott, 2008, p. 241). Hence, culture is not waiting to be discovered; rather it can only be "inferred from the words and actions of members of the group under study" (Wolcott, 1987, p. 41). Suggesting that it is not through mere observations that we could perceive culture; rather it is through participants' words that researchers can understand a culture. My goal then, as an ethnographic researcher, is to learn and to understand how things work for a group of students, in a particular social system, from a particular perspective, along and in collaboration with the students. Whether the issues or concerns that the students encounter are social, cultural, or political, they are best revealed through their experiences, which will ultimately enable me to describe and to interpret their cultural behaviors.

A Child-Centered Ethnographical Approach

Ethnography, as a research methodology is recognized as one of the main research methods in exploring children's social worlds (James, 2007; James & Prout, 2015; Maguire, 2005a). Noteworthy is that some researchers may have adopted the prominent changes within the discipline of sociology that have come to recognize children as "not passive receptors of socialization but are active social agents managing their own experiences and negotiating around adult controls" (Greene & Hogan, 2005, p. 4). Levine (2007) argues that an ethnographic study that involves children

> is a descriptive account, based on field observations and interviews, of the lives, activities, and experiences of children in a particular place and time, and of the contexts—social, cultural, institutional, economic—that make sense of their behavior there and then. (p. 247)

Levine (2007) further suggests "there would be no need for such descriptive accounts if childhood and children in ideal and practice were uniform across human populations and historical periods" (p. 247). Hence to closely monitor the multiple ways in which children see their childhood in particular and the world in general, we must focus on understanding children's voices as they speak about things that concern them (Christensen & James, 2008b; White et al. 2010). Drawing on the work of Bakhtin, researchers Maguire and Graves (2001) conceptualize *voice* as "the speaking personality that is recognized, heard, or valued in an utterance or text in a particular context" (p. 564). Whereas *listening*, according to Tangen (2008) refers to "an active process of communication that involves hearing and/ or reading, interpreting and constructing understanding, and the understanding of the child that results from listening to its voice therefore is contextual and interactional" (p. 159). Tangen (2008) adds that listening is not limited to spoken or written work—"most children will benefit from having opportunities to express themselves in a variety of ways, including play, creative and aesthetic activities" (Tangen, 2008, p. 159). Most famously, this is demonstrated by Vivian Gussin Paley's work (1986; 1988) work that describes the richness embedded in caring more about what children say and think, as it is in listening to them that we are capable of discerning, recognizing, and uncovering children's secrets, thinking, and experiences (Paley, 1988). She adds, it is in breaking away from our own conventionalities that we come to realize young children's distractions as pure sounds of their thinking (Paley, 1986, p. 122). According to Paley (1986) "no matter what the age of the student; someone must be there to listen, respond, and add a dab of glue to the important words that burst forth" (p. 127).

Moreover, James (2007) argues, "giving voice to children is not simply or only about letting children speak; it is about exploring the unique contribution to our understanding of and theorizing about the social world that children's perspectives can provide" (p. 262). Children, are people who need to be provided with respectful places and spaces for expressing their voices in their own right, as they can make a contribution to their social worlds (Alderson, 1995; Corsaro, 2017; Maguire, 2005). Hence, a child-centered epistemological stance (Alderson, 1995; Corsaro, 2017) shifts the focus of much of the research from depicting an oversimplified portrayal of children's social worlds from the perspective of 'others' to leading many researchers to consider students' voices as a new approach to research with children (Christensen & James, 2008; Prosser & Burke, 2008; James, 2001, 2007; Maguire, 1999; Tangen, 2008; White et al., 2010). While researchers may recognize the influences children could have on any inquiry, they cannot avoid the methodological and ethical issues when conducting research with children (James, 2007)—a key topic which I now discuss.

Responsibilities and Issues for Ethnographic Researchers Working with Children

Since all of this research was conducted with children, I felt it was necessary to review some of the responsibilities and issues described in the literature concerning working with children, as these have guided many of the choices I made throughout this research. A review of research literature suggests that researchers face additional responsibilities and issues when working with a vulnerable research population (James, 2007; Maguire, 2005a; Prosser & Burke, 2011). According to James (2007), although researchers may be providing opportunities for children to express their voices, "this is not in of itself sufficient to ensure that children's voices and views are heard" (p. 262). Also important are the major methodological and ethical issues faced by ethnographic researchers. Here, the inherent issues of authenticity, clumping together children as members of a category, the nature of children's participation in an inquiry, their language rights, and their voices and choices in mainstream policy documents, are of particular importance.

In her work, James (2007) describes three interlocking themes that constitute problems for research with children. The first deals with matters of authenticity, and James (2007) maintains that children's original words, thoughts, and so forth are gained first hand and are an accurate record of what children have said. However, she states, "it remains the case that the words and phrases have been chosen by the researcher and have been inserted into the text to illustrate an argument or underline a point of view" (James, 2007, p. 265). The words and phrases are selected by the researcher, not the child. James argues that quoting children deserves more critical attention, which would entail re-examining the socially constructed character of childhood and to acknowledge the cultural context of their production (p. 265). The second theme highlights a hidden danger in clumping children together as members of a category. Here, James (2007) refers to the use of conceptualizations such as "the voices of children" or "children's voices", which risks glossing over the diversity of children's own voices (p. 262). James emphasizes that children are not identical and their experiences can vary tremendously based on their historical, social, and cultural contexts. The third theme "involves questioning the nature of children's participation in the research process" (p. 262). Here, James draws on the shift in "dominant paradigms of practice from research *with* children toward research *by* and *for* children" (Prosser & Burke, 2011, p. 259). She focuses on encouraging children to conduct their own research into matters that concern them. She also argues that if children were invited to conduct their own research, this could help to "redress the power imbalance between adults and children during the research process" (p. 267). Also, it could "produce interesting and worthwhile outcomes that may have a more powerful practical and policy impact than the more traditional kinds of research done by adults that is simply informed by what children say" (p. 268). All things considered, children's perspectives, points of view, and experiences must be regarded as places from which any analysis begins, rather than conclusive descriptions of an observed phenomenon informed by children's words (James, 2007, p. 269).

However, James (2007) does not consider the issues that researchers face when conducting research *with*, for, or *by* children living and learning in diverse language and cultural contexts.

Maguire (2005a) argues that there should always be considerations of multilingual children's voices, their language rights, and their choices in mainstream policy documents and research that engage children's participation. Maguire further argues that policy makers and researchers are required to critically examine their own discursive policies and practices in research that involves children in general and children who live and learn in multiple language contexts in particular (p. 1). Maguire breaks away from an oversimplified portrayal of children who are learning and living in multiple language contexts and offers a pertinent conceptualization for understanding children's participation, their experiences, views, perspectives, interests, and needs, "especially from the perspective of children themselves" (Maguire, 2005a, p. 2). Yet, Maguire (2005a) admits that with this new view of children as social actors and in focusing on their agency brings about responsibilities and issues for policy makers and researchers working with children (p. 4). For instance, the role that adults play in many research contexts reflects "one of power and authority over children typically mediated, manifested, and negotiated through signed parental or guardian informed consent forms in one or both of Canada's major languages (English or French)" (Maguire, 2005a, p. 5). This conveys another important issue regarding whose voice is allowed to speak and is to be represented, heard, and recognized (Maguire, 2005a). It also sends a message to the child that as a researcher, s/he cannot speak their language, and that their language is not valued (Interview, March 18, 2015). Another key point addressed by Maguire (2005a) encourages one to question the lack of attention given to multilingual children's voices in Canadian ethical documents, given that Canadian legislation such as the *Charter of Rights and Freedoms and Multicultural Act* "values two official languages and embraces the concept of linguistic and cultural diversity as a national resource in contributing to "Canada's heritage" (p. 8). Yet, the political issue here rests on the value given to the two official languages- English and French- embraced by the Canadian legislation, which has considerable influence on the development and continuity of all speakers' participation in school activities, in research, and ethical policy documents. Maguire (2005a) states that "children's informed understanding of what an inquiry is about is essential" and it should be communicated in a clear manner and in languages that can support children's understanding (p. 12). Moreover, Maguire (2005a) suggests that researchers embrace other mediational means, such as gestures, symbols, diagrams, or even pictures to support children's understanding. These mediational means, she further states, are not neutral and must be suitable for all ages, culturally sensitive, and linguistically appropriate (p. 12). Maguire (2005a) takes “a strong epistemological stance with regard to the importance of listening to their [children’s] voices and making a commitment to understand their perspectives and social worlds as they do” (p. 4).

James (2007) and Maguire (2005a) offer steps in a promising direction for qualitative researchers working with children in general and children who are living and learning in diverse language and cultural contexts in particular. Moreover, being aware of the issues researchers working with children face can ultimately enable researchers to adopt a particular epistemological positioning to support participants’ unique qualities or characteristics, in order to understand their perspectives and social worlds as they do (Maguire, 2005a).

My Epistemological Positioning

My interest in students’ learning experiences sparked in 2015 when I took a course on Qualitative Research as part of my Ph.D. course requirements. At that time, I came across a reflective piece by Maguire (2005a) entitled, *What if you talked to me? I could be interesting!* This reflective piece and a sociocultural perspective motivated me to rethink my epistemological assumptions about engaging with children and to reconsider the possible contributions children can and do make to their social environments. In reflecting back on my role as a parent and educator prior to reading Maguire’s work, my role as a parent appeared to be more an authority figure who controlled how my children are to act, behave, and to some broad extent, what they should or should not know. I also saw myself instantaneously jumping in with my own perspectives on, for example, what they should wear to school, why they should attend Saturday morning Arabic classes, why they should learn to play the piano and engage in martial arts rather than for instance, learn to play the guitar and soccer. I went as far as sometimes controlling the procedures and strategies they should use to solve their mathematics homework without really providing my children much space to express their opinions and perspectives about matters at hand. It was as though I assumed my role was to shape their perspectives and experiences to match my own beliefs, worldviews, and values. However, Maguire’s work (1999; 2005a; 2005b; 2009) encouraged me to see children as intelligent social actors who have a sense of their personal agency and have very good social radar for and about their social environments and worlds. I now acknowledge that children not only know they have the power to act, but they know that they can act, and are aware of their sociocultural environments and the mediators within them. In line with this view, I argue that by listening to children's voices and not jumping to our own conclusions, we, as educators and researchers, may be better equipped at understanding children's intentionality, and also, are capable of helping children acquire a sense of personal agency.

Children's perspectives and experiences are not to be underestimated, as there is much in what children say deliberately or even incidentally that we can utilize to appreciate and understand their experiences and meet their needs. If we continue to claim that we should strive for equal and quality education for all, then I believe it is in our best interest to include children in the dialogue and take their perspectives and experiences seriously. James and Prout (2015) argue that this radical shift in thinking is achieved through the adoption of an ethnographic methodology, as "it advocated placing children's role, experiences and activities as central…[which also] requires direct participation and observation of the worlds that children inhabit and create" (p. x).

Hence, adopting an ethnographic methodology guided my participation and observation to understand how multilingual students experience and make sense of mathematics in a plurilingual setting. Furthermore, ethnography provided a setting and approach where I am able to see and describe the influences that impacted students' learning of mathematics. Also, what the children say about those influences is important and relevant to this study. For example, the children might reflect that they prefer to use English when working on mathematical activities with others who speak their home language, although their teacher might encourage them to speak in their home language.

Research Design

Given my sociocultural perspective of learning and development, I wanted to conduct an ethnographic study to understand multilingual learners' mathematics experience and meaning making in a Canadian educational setting. Hence, the study took place in a grade 2/3 classroom over a period of 7 months. Within this time frame, I was in the classroom 4-5 times a week. Each of my visits to the classroom included observations and video recordings of students working on mathematics activities. Also, closer to the end of data collection each student was invited to engage in one audio recorded interview.

The observations included notes of the ways the students and their teacher engaged with one another and with mathematical problem-solving activities. The observation notes were analyzed to answer the first sub research question: What are the influences on multilingual learners' mathematics experiences?

The second component of the study focused on video recorded data of students engaged in mathematics problem solving activities. These video recorded data were analyzed to answer the second sub research question: How do multilingual learners negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

The third component of research included one-on-one interviews with students to better understand their experiences with and views of mathematics. The interviews were used for contextual information when answering the main research question (i.e., what are the experiences of multilingual learners), as well as, the two sub research questions.

All this then feeds into the discussion chapter where I consider the analysis of my observations, video recordings, as well as, the one-on-one interviews I had with the students to answer my main research question: What are the mathematical experiences of multilingual learners?

Recruitment and Procedures

Upon approval from the ethics committee at the University of Ottawa, I requested permission from a schoolboard to conduct my research project (see Appendix A). My choice for conducting this research study in the school system, as opposed to youth clubs, community centers, or random dwelling sampling, is because the school is the main educational setting where students develop their mathematical knowledge and experiences, as it is there "where mathematics is formally encountered as a discipline" (Grootenboar & Marsham, 2016, pp. 1-2). Hence, the school is the most naturalistic setting where students are formally exposed to the teaching and learning of mathematics. As such, it is only by observing and interacting with students in mathematics that researchers and educators are better equipped to understand and describe how multilingual learners' mathematical experiences unfold in a plurilingual educational setting.

Once the school board granted me permission, the chair of the research review committee at the schoolboard contacted the principals of two preferred elementary schools that fit the research purpose. The preferred elementary schools welcomed several multilingual learners into their schools in the past couple of years. In addition, these particular schools invested in the development of an English as a Second Language (ESL) and an English Literacy Development (ELD) program for students whose first language is other than the language of instruction and require support.

Upon receiving a response from the chair informing us of a principal's interest in participating in the research, my supervisor and I officially emailed the principal and invited their school to participate in the research (see Appendix B, Appendix C, Appendix D). The principal approved and she indicated that I will be working with a grade 2/3 teacher. My supervisor and I then scheduled a time to meet the principal and teacher to review the research plan, and also, to answer any questions they may have regarding the research. Following that meeting, I arranged a suitable time with the teacher to meet the students in her classroom.

On Friday November 2nd, 2018, I visited the grade 2/3 classroom for the first time. I introduced myself to the students as both a volunteer and a graduate student interested in their mathematics learning. As a volunteer, I visited the classroom 4-5 times a week from the first week of November 2018 until December 8th, 2018. With the teacher's permission, I supported and interacted with many students, especially many of the multilingual learners, on a one-on-one basis or during small-group work. I often worked with small groups of students on various language-related activities and mathematics problem solving activities. It did not take long for students to feel comfortable with my presence in their classroom given that I was there on a regular basis. The students would often ask me for help or involve me in their conversations. My presence in the school helped me build a strong rapport among both students and teachers that I worked with.

Then, on December 4th, 2018 the classroom teacher and I met with the school's Multicultural Liaison Officers (MLOs).[6] The intention of this meeting was to review the research plan with the MLOs so they could communicate that information to students' parents, and should the parents have any questions or concerns the MLOs would address those accordingly. Following that meeting, I invited all the students to participate in the research project. I explained to them what the research was about, what their participation would entail, and

---

[6] The general goal of MLOs is to help ensure the successful integration of newcomer children and their families in schools. They also act to bridge any language barriers between families and the school as they usually speak the family's home language (i.e., Arabic, Nepali, & Somali).

addressed any questions or concerns they had (see Appendix E). I also explained that I would need their parent's approval for them to participate by signing a consent form, and once I receive their parents' permission, I will ask them to sign a form as well. All information letters, consent forms, and assent texts were translated in the languages spoken by the students in the classroom, including Arabic, Nepali, and Somali (see Appendix F, Appendix G).

I received parental consent and student assent from all students in the classroom, except from one student who chose to participate in the research, but did not want to be audio or video recorded. The student's involvement in the classroom remained the same, but was not video or audio recorded. From then, I was collecting data from January 2019 until mid-April 2019. As a researcher, I continued to be seen as a volunteer to assist the students. Also, following data collection, I continued to assist as a volunteer once a week till the end of May 2019.

Research Setting and Participants

All students in the classroom were invited to participate in the research project. Although some students moved and others arrived, ultimately, there were 18 students who participated in the project including, the student who moved close to the end of data collection, the newly arrived student from Somalia, and the one student who chose to participate but did not want to be video or audio recorded. There were fourteen students in grade 3 and four students in grade 2. All students were invited to participate because I did not intend on excluding any of the children in the classroom. Also, all students play an integral role in developing mathematical discussion and in co-constructing mathematical knowledge. In addition, my intention was to maintain a natural setting and to not interfere with the classroom dynamics or structure of that setting. Hence, all students who agreed and who obtained parental consent were considered participants in the research study. **Table 1** which follows includes the students who participated in the study, their grade and age, where their families migrated from, where they were born, what they celebrate, the languages they spoke, how old they were when they came to Canada, and their previous schooling experiences.

Most of the background information about the students was gathered from the informal conversations and follow-up interviews I had with them, and also from the "*All About Me*" poster that hung on one of the classroom walls (**Figure 1**). The poster was created early on in the school year when each student was asked to complete an information card about themselves, and each card was then connected to their home countries or where they came from on a world map. Interestingly, the students were almost always observed referring to the poster whether to new visitors, their friends, parents, or their teachers as it characterised who they are and where they are from.

Figure 1

All About Me Poster

All students' parents were either refugees or have immigrated to Canada. Also, 70% of the students were born outside of Canada, in Eritrea, Nepal, Somalia, Sudan, and Syria (**Figure 2**). All students spoke a first language other than English at home, including Arabic, Chin, Hindi, Nepali, and Somali (**Figure 3**). For students born outside of Canada, four came to Canada when they were between the age of 1 and 4 years old, and eight students arrived within the past 18 months and were between the ages of 5 and 6. There was also 1 student from Somalia who arrived to Canada and joined the classroom in February 2019.

Figure 2

Students' Country of Birth

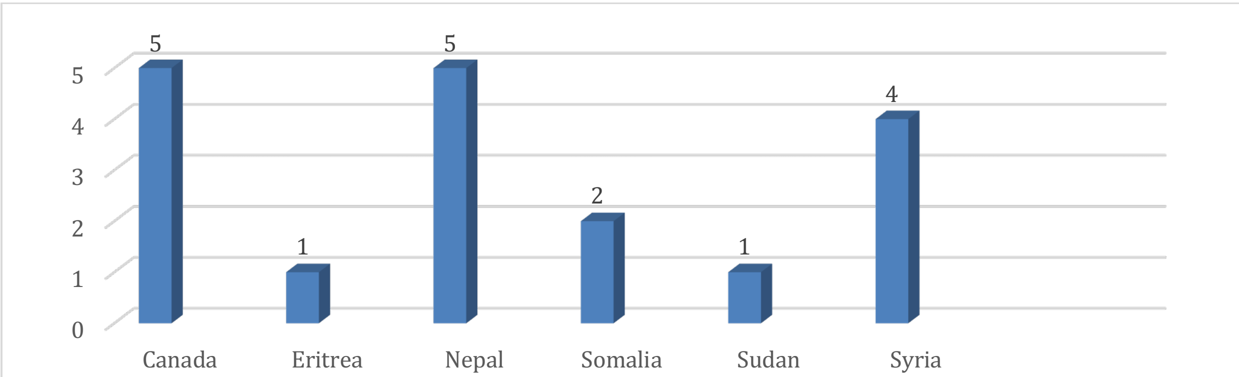
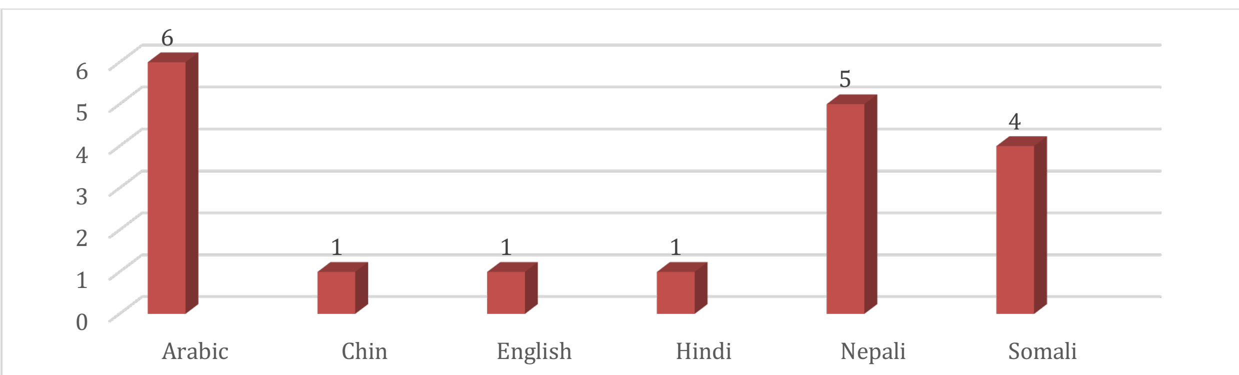


Figure 3

Home Languages Spoken by Students



Given the diversity of students in the classroom, I felt I was fortunate to work in the grade 2/3 mathematics classroom. I was familiar with this age group and the mathematical ideas taught in these grades through my volunteer work in similar classrooms. Also, at this age, the children were capable of articulating their mathematical thinking orally, in writing, or through the use of other resources. Furthermore, I have chosen to work with immigrant and refugee children because of the degree of contrast between their home countries and that of the host society. Also, the emotional and physical trauma that refugees or immigrants have experienced can affect their psychological and emotional wellbeing during the resettlement process which may indirectly affect their education (Loewen, 2004, p. 36). In addition, the educational systems that the children may have been exposed to present key differences, especially in terms of the school structure, the curriculum, and the language of instruction (Anderson et al., 2004; Loewen, 2004). I understand that there are challenges in including immigrant and refugee children in a research project. However, I have made myself aware of the need to be mindful and sensitive to children's experiences prior to resettlement and to the diversity of those and other experiences, as they could vary tremendously based on their social, cultural and historical frames of reference (James, 2007; Maguire, 2005a). In addition, I am aware of matters of authenticity, where children's original words, thoughts, and so on are an accurate record of what children have said (James, 2007). Also, I have taken into account children's language rights, and ensured that language is not a barrier to children's involvement in the research project (Maguire, 2005a).

Table 1

Background Information About the Students in the Grade 2/3 Classroom

Pseudonym	I am in grade…	I am… years old	My family comes from…	I was born in…	My family celebrates …	Languages I can speak are…	I came to Canada when I was…	Previous Schooling Experience
Ahmed	3	8	Somalia	Canada	Eid	English	Born in Canada	NA
Abed	3	8	Sudan	Sudan	Eid Halloween Birthday & Ramadan	Arabic	6 (almost 7)	Some Previous School Experiences
Hayat	3	8	Jordan	Canada	Birthday & Halloween	Arabic & English	Born in Canada	NA
Ikbal	3	8	Somalia	Eritrea	Eid, Birthday, and Ramadan	Somali & English	5	No previous schooling experiences

Joud	2	7	Canada & Somalia	Canada	Ramadan, Eid	Somali & English	Born in Canada	NA
Khadija	3	8	Syria	Syria	Eid & Ramadan	Arabic	5	No Previous School Experiences
Mustapha	2	7	Syria	Syria	Birthday & Ramadan	Arabic	6	No Previous School Experiences
Niraj	3	8	India & St. Vincent	Canada	Diwali, Basakichris, & Thanksgiving	Hindi & English	Born in Canada	NA
Pooja	3	8	Nepal	Nepal	Christmas & Halloween	Nepali	4	No Previous School Experiences
Rachet	3	8	Bhutan	Nepal	Christmas	Nepali	5	Some Previous School Experiences

Raj	3	8	Bhutan	Nepal	Christmas	Nepali, Hindi, English, & French	5	No Previous School Experiences
Sabina	3	7	Nepal	Nepal	Birthday	Nepali & English	6	Some Previous School Experiences
Sahar	3	8	Syria	Syria	Everything	Arabic & English	3	No Previous School Experiences
Samartha	3	8	Nepal	Nepal	Birthdays & celebrates special stuff	Nepali, English, & French	1	NA
Sajan	3	8	Myanmar	Canada	Christmas, New Year & Thanksgiving	Chin & English	Born in Canada	NA
Suhail	3	8	Somalia	Somalia	Eid & Ramadan	Somali & Arabic	8	Little to No Previous School Experiences

Tamam	2	7	Syria	Syria	Eid & Ramadan	Arabic & English	5 (almost 6)	Little to No Previous School Experiences
Yaser	2	7	Somalia	Somalia	Eid, Birthday, Ramadan	Somali	4	No Previous School Experiences

Who was Involved in the Video Recorded Problem-Solving Activities?

I will also take advantage of this space to provide some background information about the students involved in the video recorded activities. The information shared was gathered from my observations in the classroom, from informal conversations and follow-up interviews I had with the students, and from stories or comments the teacher shared.

Abed:

Abed was in grade 3 and he was 8 years old. He was born in Sudan and came to Canada when he was 6 years old. He shared that he comes from a family of 6—father, mother, and four siblings, and he is the oldest in his family. Abed mentioned that because he was the oldest, he had to help his brothers and sisters with homework and more specifically with reading. He appeared very quiet, and would share his answers to questions posed by the teacher only when he was sure they were correct. At times, he would share his answers with me, and once I confirmed they were correct, he would raise his hand to share them with the rest of the class. In working with Abed, I did not notice any major setbacks or difficulties that he countered learning mathematics especially since he transitioned from doing mathematics exclusively in Arabic to English. I did notice that he preferred that we (i.e., his teacher or myself) read the mathematical task for him, which helped him make sense of what is required of him. Abed also preferred to work on his own—something he might have been used to doing in his home country. Yet, that slowly changed as their teacher continued to encourage students to work together.

Joud:

Joud was in grade 2, she was 7 years old, and she was born in Canada. Joud shared that her mother and father came from Somalia and that their home language is Somali. She uses Somali when speaking to her mom and dad, but uses English when communicating with her siblings. Joud mentioned that she comes from a large family including her father, mother, and 6 siblings. She explained that she was considered the youngest in her family until her mother gave birth to her new baby brother, which she says, “changed everything”. It was hard to believe that Joud was in grade 2 as her physical build hinted that she could have been in grade 3. My data suggests that she was self-driven and rarely required support from her teachers. Also, she was very clever and a little too talkative at times.

Khadija:

Khadija was in grade 3 and had recently turned 8. She was born in Syria and came to Canada when she was 5 years old. Her teacher shared that Khadija had no prior schooling experience before coming to Canada. She was the oldest in her family and had three younger siblings. Khadija shared that she acted as her family's interpreter. I observed her on several occasions with her father at the school, where she communicated her father's comments, questions, or concerns to her teachers and to the office administrator. Khadija described Arabic as the language she uses to communicate with her parents and siblings. She mentioned that her parents encourage her and her siblings to speak Arabic at home because they believe that their home language defines who they are and connects them with their extended families abroad. Generally, my observations suggest that she was very quiet, spoke only when spoken to, and preferred not to share her thinking unless called upon or put on the spot.

Mustapha:

Mustapha was in grade 2 and was 7 years old. He was born in Syria and came to Canada close to a year before I met him. His teacher mentioned that he had no prior schooling experience before coming to Canada. When he first arrived to her classroom, he was physically aggressive and impulsive if he did not get his way. During lunch, he would take other students' snacks (e.g., Oreo cookies, chips, juice, gummies, bear paws, etc.). Also, his teacher shared that Mustapha became reluctant to engage with any of his classmates. The teacher explained that his reluctance was due to his frustration of not knowing how to speak English. Early on, he attempted to speak with his classmates in Arabic, and those who shared a similar language would try to speak and interact with him, but he was not interested because most, if not all, were girls. The teacher explained that Mustapha spent a lot of time on his own on the carpet and would be observed working with the hundreds chart that hung on the blackboard. His teacher explained that Mustapha taught himself how to count through his continuous interaction with the hundred's chart. His negative attitude towards being in the classroom subsided with the support of his teacher and in learning some English words.

My observations of Mustapha suggest that he was very energetic, active, and very aware of his surroundings. Mustapha seemed to have no fear to make mistakes, and would be observed rechecking his answer to understand where he went wrong—if not on his own, with the support of the teacher or myself. He interacted with everyone in the classroom and was well-liked by many. Mustapha loved to cheat on games as he was very competitive and took great pride in winning. However, he was supportive of those who were losing and guided them to win a turn by helping them change their strategy. Also, Mustapha took part in all classroom activities with perseverance, but despised using scissors as he was still learning how to use them and found them frustrating.

Mustapha's teacher shared that mathematics was his strengths, especially computation (i.e., adding and subtracting numbers). He was capable of calculating numbers in his head and with the help of his hands. Although it was a little difficult for him to verbally explain how he got an answer, he would visually share the strategies he used. These strategies resembled those they learned in class, such as parts of 10, counting by 2's, 5's and 10's, and decomposing numbers.

Interestingly, there was one student (i.e., Sajan) in the classroom that Mustapha referred to as "my brother". I noticed Mustapha constantly observing and imitating Sajan in terms of how he spoke and acted. Also, there was a song that Sajan sang and that Mustapha replicated. I asked Sajan for the name of the song, but he did not recall what it was or who sang it, but shared that it was a new song and was a mix of Spanish and English. I dedicated my morning and afternoon commutes listening to the radio to try to figure out what song it was. I eventually managed to find that it was "Taki Taki" by DJ Snake, Selena Gomez, Ozuna and Cardi B. This song became part of their interactions on a day-to-day basis.

Rachet:

Rachet was in grade 3 and was 8 years old. He was born in Nepal, but came to Canada when he was 5. Rachet shared that he attended school in Nepal prior to coming to Canada and where he learned mathematics in Nepali and English. In Canada, he lived with his mother and siblings, and his father had to stay in Nepal for work related purposes. Rachet repeatedly shared that he favored mathematics of all school subjects because as he explained, " it's very easy, especially multiplication". He also enjoyed attending the afterschool homework club because that's where "we did so much math". Rachet's eagerness and willingness to explore multiplication led the teacher to introduce it to the rest of the students in his class. He was observed paying close attention to the computational queries posed by the teacher or other students and was devoted to solve them. My observations show that Rachet was a well-behaved student who was eager to learn and to share the strategies he used to solve various tasks.

Raj:

Raj was also in grade 3 and he was 8 years old. He was born in Nepal and came to Canada when he was 5. He said that his father and mother were from Bhutan and moved to Nepal before he was born. Raj shared that he did not attend school in Nepal and did not know any English prior to coming to Canada. My observations suggest that Raj was very cheerful and interacted with everyone in class. He was actually one of the first few students to approach me and to include me in his conversations. It did not take him long to share that he liked having me in his class. He would almost always welcome me into the classroom with a hug.

Interestingly, Rachet and Raj shared that they were cousins. Rachet's family came to Canada a couple of months following Raj's family. Rachet and Raj were always seen together in and outside the classroom. They were mostly observed communicating using their home language "Nepali". When asked about speaking Nepali together, Rachet explained that "when there is something that Raj or me don't understand in English, we say it in Nepali". They also said that they used Nepali when they did not want anyone to understand what they were saying, like "when we're speaking about you [myself]". They further explained that they sometimes used Nepali without thinking about it—something they were used to doing growing up together and interacting outside the grade 2/3 classroom.

Sabina:

Sabina was 7 years old and was in grade 3. She was born in Nepal and came to Canada when she was 6 years old. Sabina said that she attended school in Nepal but did not learn any English prior to coming to Canada. My observations suggest that Sabina valued speaking to students who shared the same home language (i.e., Nepali), but only during their nutrition break. For instance, she would include herself in other students' conversations, and if they were speaking in English, she would switch to Nepali and they would as well. However, when working on classroom related activities they almost always interacted in English. Also, the interview data revealed that Sabina did not enjoy mathematics a lot because she found it difficult. The teacher shared that Sabina had some processing delays, and required repetition and a slower pace for learning. Otherwise, my observations show that Sabina was very energetic, kind at heart, and loved to make jokes. She almost always had something negative but funny to share about her lunch, which gave her an excuse to not eat it.

Sahar:

Sahar was 8 years old and was in grade 3. She was born in Syria and came to Canada when she was 3 years old. The interview data suggest that Sahar loved mathematics and was eager to engage in it. Sahar did share that she preferred to have me close by when working on mathematics, she explained, “I like to be beside you, so you can help me if I’m stuck. I want to get everything right”. My observations suggest that Sahar loved to participate in class. However, she was reluctant to share when she did not understand or know how to do something. For example, she found telling time a bit complicated and would try to avoid raising her hand to share. I approached her to practice telling time and she immediately said, “I know my numbers. Why do I always get the wrong time?”. After some practice, she understood how to tell time and she was very happy. From then on, she would come up to me and whisper, “I know what time it is. It’s eleven o’clock [or whatever time it was]”—it became our catchphrase.

Sajan:

Sajan was in grade 2 and he was 8 years old. He was born in Canada and his family came from Myanmar. He speaks Chin and English, but shared that he used mostly English with his younger sibling and parents. My observations suggest that Sajan was very well liked by all the students in his class and was considered the “cool kid”. He was one of the few students who was shy, a little reserved, and would try not to make eye contact with me. However, my familiarity with some of the YouTube shows, movies, games, and consoles that the students were interested in helped to defuse the shyness he felt towards me. I would intentionally include myself in their conversations just so Sajan could feel comfortable with my presence in the classroom. As time went on, Sajan asked if we could have our own handshake, which meant the world to me. He became a lot more open and would approach me for help when needed. In general, my observations show that he was a curious, energetic and hard-working student. However, he did not always favor sharing his thinking out loud and preferred to share only one-on-one, with his teacher or myself.

Samartha:

Samartha was in grade 3 and was 8 years old. She was born in Nepal and came to Canada when she was a year old. Samartha shared that her home language is Nepali, and it is the language she used to communicate with her mother, father, younger sister, and most importantly her maternal grandmother—who she mentioned does not know how to speak English but with whom she had a strong relationship with and have valued speaking to her in Nepali. I observed that Samartha often volunteered in class and was eager to share her mathematical thinking and understanding during whole class discussions or during small group work. I observed her rely on the various strategies she learned, or even those she invented to share her thinking.

Suhail:

Suhail was 8 years old and was in grade 3. At the time of data collection, she was a newly arrived student from Somalia and her home language was Somali. Students' reactions towards her arrival and presence in the grade 2/3 classroom were interesting. In general, the students were very welcoming, and most, if not all students approached Suhail to introduce themselves. Several of them were curious to know if she spoke any English and deliberately asked, "do you speak English?" Suhail did not respond as she was immediately introduced to Joud who spoke Somali and was asked to explain to her the mathematics activity they were working on. At that time, the students were working on an activity called "Symmetry Art: Snowman". However, Mustapha, who did not speak Somali, was readily available to offer his support. I was quite taken by his generosity and keenness to help Suhail in any way he could. For instance, Mustapha stepped outside of his comfort zone to help Suhail cutout a hat for her snowman—especially since he did not favor using scissors. Then during the language block, Mustapha helped her read a book by encouraging her to repeat after him. The book was a little challenging for Mustapha but he did not give up and seemed very confident. Other students enacted the role of the teacher; in that, they assessed Suhail in terms of her knowledge of the English alphabet. One student, Samartha, got a clipboard with a paper mounted on top and explained to Suhail, "I want to see what letters in English you know. I will write a letter and you say it". According to Samartha's assessment, Suhail was capable of articulating the letters up to the letter 'O'. Samartha eagerly shared the results with her teacher. Although the examples shared may seem intimidating to some students who recently arrived, the welcoming atmosphere that was evident among the students cultivated a sense of belonging for Suhail in the classroom.

In terms of Suhail's educational experiences, she shared that she grew up in a refugee camp in Somalia and had no formal schooling experience. However, she shared that during her stay at the refugee camp, she joined other children to learn the alphabet and how to count in Somali and English. My observations suggest that Suhail was very quiet and preferred working on her own, but was very perceptive of her surroundings and of what others were doing. The students in the classroom were generally very supportive and were readily available to offer a helping hand when they felt Suhail needed support or did not appear engaged. Generally, the students spoke to her in English and would visually explain what was expected of her. For instance, if a task required that she finds the symmetry of a shape, they would visually illustrate to her how it could be done. Suhail used very little language when interacting with her classmates. On various occasions, Suhail would communicate to me in Somali. She would even refer to, read, or count numbers in Arabic. Hence, when working on mathematics, we would alternate between Arabic, Somali and English alongside the use of visuals and gestures to communicate with one-another. Although I did not speak Somali, I was capable of somewhat making sense what she was referring to. My observations also show that despite Suhail's limited use of English, she did not resort to copying or using other student's solutions, but tried to make sense of the activities and to find solutions using the resources available. She was not afraid of making mistakes and would share her thinking using gestures, concrete tools, and the languages she knew. She would even evaluate other students' answers and would offer alternate solutions to the tasks.

Tamam:

Tamam was 7 years old and was in grade 2. She was born in Syria and came to Canada when she was 6 years old. Tamam was another student who avoided making eye contact with me during my first couple of visits to the classroom. My observations suggest that Tamam was very quiet and preferred not to share in class. I recall my very first interaction with Tamam was when Mrs. Holland asked me to go through the hundreds chart, so I could explore some understanding of her mathematics knowledge. When Tamam was asked to join me at the round table, she was very reluctant and seemed a bit nervous, as though I was going to interrogate her. However, when we started to talk about the things she noticed and what she wondered about when she looked at the chart, the tension subsided a bit. After that brief interaction, Tamam became a lot more comfortable with my presence and was no longer reluctant to approach me should she need anything.

Data Collection

In order to address the research questions, this study consisted of three types of data: in class observations; video recordings of students engaged in mathematical problem-solving activities; and follow-up interviews with the students. All data sets (i.e., observation notes, video recordings, and interviews) were used to address the main research question: What are the mathematical experiences of multilingual learners. However, specific data sets were used to address the two sub research questions: a) What are the influences on students' mathematics experiences? and b) How do students negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting? In what follows I describe each data set and which of the sub questions it addressed. The video and audio recordings for all data sets ranged in length from 5 to 30 minutes (See Appendix H).

My Observations of the Mathematics Block

My data consisted of detailed observation notes written in the form of journal entries for each day I visited and interacted with the students in the mathematics classroom. Understanding students' mathematics experiences from a sociocultural perspective provoked me to note the ways the students and their teacher engaged with one another; the ways they interacted around mathematical problem-solving activities; the ways they produced oral or written explanations of their mathematical thinking; and of the ways they used the language(s) they know and other resources during mathematical discussions (see Appendix I). My initial intention was to record my observations as I was interacting with the students during the mathematics block. However, it became a little distracting for the students, as they were curious about who and what I was writing about. Hence, I decided to retain my observations and to write about them after I left the classroom each day.

With the teacher's permission, I also made 18 audio recordings of whole class mathematics discussions during which students were introduced to a new term, skill, strategy, reviewed a lesson, or received an explanation of a particular mathematics activity. These recordings supplemented my observation notes and helped to provide background information in order to better understand the context of the mathematics activities. The observations helped to answer the first sub research question: What are the influences on students' mathematics experiences?

Video Recordings of Students Engaged in Mathematics Problem-solving Activities

Part of my work was to focus on students' discussions during mathematics problem-solving activities. Recognizing that learning, according to a sociocultural perspective, is socially mediated, I knew that it was important to use video or audio recordings to capture students' interactions and thinking through their mathematical discussions and explanations as they worked together on mathematical problem-solving activities, as well as, the ways they hesitated, made eye contact, used multiple resources, and gestures. The data consisted of 13 video or audio recordings of student-student interactions on mathematics activities. These activities acted as a review of what students learned including patterning, perimeter, area, transformation, symmetry, and number sense. I also have 10 video or audio recordings of whole class math talks as the students and their teacher worked together on problem-solving activities. There are also 24 video or audio recordings of students working in their mathematics centers as they engaged in mathematics games, collaborated on various symmetry and pentomino activities, or worked on word problems. There are also 6 audio recordings of students engaged in extra or other self-directed mathematics activities. Data also includes photographs of student work.

All the video and audio recordings that I collected as data informed my work in developing an understanding of students' experiences and how they negotiate meaning making during mathematics problem solving activities. After reviewing all the data multiple times, I chose four recordings to analyze in great detail. I felt these four video recordings represented the kinds of interactions and activities that students engaged with during the mathematics block, and also, provide the reader with a detailed analysis and understanding of multilingual students' meaning making. These recordings captured students' meaning making through their mathematical discussions and explanations as they engaged in mathematics tasks, as well as the ways they used strategies, concrete tools, and gestures. I also gained an understanding and account of the students' experiences in listening to and talking with them about the mathematical tasks at hand, about what they already know, about how they make sense of new ideas, and about matters concerning their daily activities. Also, photographs of students' work were representations of their work. In particular, the four recordings and photographs helped me form an in-depth description and understanding of the second sub research question: How do multilingual learners negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

Follow-up Interviews with the Students

Closer to the end of data collection, sometime in May 2019, each student was invited to engage in one brief audio recorded interview on a one-on-one basis (see Appendix J). These interviews lasted approximately 5-10 minutes. A sociocultural perspective prompted me to develop an understanding of students' previous and new mathematical learning experiences, and also, their reflections towards their participation in the mathematics classroom. In this way, I endeavored to acknowledge students' histories and the wider community as influencing their mathematics experiences in the grade 2/3 classroom. Hence, the kinds of questions asked were meant to explore students' perspectives, conceptualizations of and experiences learning mathematics at school, at the afterschool homework club, in their home countries, and at home with their family. These questions also provided students with an opportunity to reflect on their own participation in mathematics and in engaging with others. In particular, the follow-up interviews were designed for contextual information pertaining to students' views and perspectives, and their use of language(s) when learning mathematics. These interviews supported me in developing a detailed understanding of students' mathematics experiences and assisted me in answering the main research question as well as the two sub research questions.

The reader might notice that I did not interview the teacher. The reason for this is because I was in the classroom for 7 months and I was constantly having informal conversations with her. Such natural conversations helped me form a much more precise understanding of the instructional practices that the teacher used to promote and facilitate mathematics learning in the classroom. For instance, on various occasions, we talked about things like her choice of certain activities, how she organized math centres, how she grouped students to work together, or how she differentiated work for some students. All these informal conversations informed my understanding of her pedagogical practices in the mathematics classroom, which to a certain extent substituted for a formal interview.

Table 2 provides an overview of the data collection methods, participants, and the research question(s) each data collection method addressed. Although, to some degree, all of the data sources informed answering all research questions.

Table 2

Data Collection Methods, Participants, and Research question(s) Addressed

Data Collection Method	Participants	Research Question(s) Addressed
Observation Notes	Students & Teacher	Main Question & First Sub Research Question

Video Recordings	Students or Students-Teacher	Main Question & Second Sub Research Question
Interviews	Students	Main question & Two Sub Research Questions

Data Analysis

Recognizing that learning and development are socially mediated, I used a microanalytic approach to generate a detailed description and understanding of grade 2/3 students' mathematics experiences in a plurilingual classroom. As Bloome and Golden (1982) write

> Microanalytic approaches focus on smaller units of context, such as that of a classroom, and examine the patterns of behavior of individuals in these smaller contextual units. Microanalytical approaches are often based on the underlying assumptions (1) that smaller units of context are embedded in larger units of context-i.e., show-and-tell time is embedded in a classroom, which is embedded in a school, which is embedded in a community/ ideological context and (2) that such smaller units of context can be "pulled out" of the larger contexts in which they are embedded without a loss of integrity. (pp. 208-209)

On this view, a microanalytic approach allows for a detailed description of the social interaction within an immediate context (i.e., the grade 2/3 mathematics classroom), and also the wider social processes of the community within which the school was situated. In this way, I was able to focus on the socially mediated experience as established in the grade 2/3 classroom and informed by the practices and policies outside and within the educational setting. Hence, to develop an in-depth description and understanding of the socially mediated experience of students in the grade 2/3 classroom, I used two analytical frameworks, one comes from my analysis of the research literature that prompted me to organize the influences that the research found into four categories, and the other comes from Gee's (2005, 2011) construct of building tasks to analyze "language-in-use", which I translated to the context of mathematics education. Although my theoretical perspective was not my analytical framework per se, the analytical frameworks I chose and the microanalytic approach I used closely aligned with sociocultural theory. For example, analyzing in detail the individual, pedagogical, institutional, and sociopolitical influences on students' learning of mathematics prompted me to not only consider what is taking place in the grade 2/3 classroom but to also consider the influences of the wider community on students' mathematics learning experiences. Also, analyzing "language-in-use" using Gee's building tasks closely connects to Vygotsky's view that language is one of the most powerful and pervasive cultural tools connecting intrapsychological and interpsychological functioning (Lantolf & Thorne, 2000; Wertsch, 1991).

Analysis of My Observations of the Mathematics Block

One aspect of the analysis I focused on was my observation notes, which were used to answer and give me an understanding of the first sub research question: What are the influences on multilingual learners' mathematics experiences? I analyzed and presented the data using the four influences on students' learning of mathematics including, individual, pedagogical, institutional and sociopolitical features of the mathematics classroom. These four influences came from the organization of my literature review. Before I organized where things went within these four categories, I read through my observation notes without making reference to or using the four influences as my categories. As I read my notes, I generated various categories specific to what I noticed in the data. These categories included certain things that were reflective of students' identities, the use of language and other resources, mathematical strategies, student-student and student-teacher interactions, particular things related to the teacher's instructional practices, as well as the context of the school and provincial policies. I assigned each category with a colour using sticky notes. I then went through my notes and organized the data within the four influences.

First, I concentrated on individual influences, with a focus on the role of students' individual identities as contributing to their mathematics learning experiences. Here, I found it convenient to lean on Gibson's (1988) work on what she refers to as the students' strategy of acculturation. In her work, Gibson (1988) uses the term acculturation to describe "a process of culture change and adaptation which results when groups with different cultures come in contact. The end result may not be rejection of old traits or their replacement. Acculturation may be an additive process or one in which old and new traits are blended (Haviland 1985: 628-29)" (as cited in Gibson, 1988, pp. 24-25). The strategy of acculturation is similar to what Barwell (2020) has most recently come to call "socialization" in his work in mathematics education. In particular, Barwell

(2020) sees learning mathematics and learning a second language as processes of socialization. Learning mathematics is seen “in terms of students’ socialization into mathematical discourses, where this discourse involves various signs (words, symbols, diagrams, gestures, etc.) and forms of argumentation (posing problems, conjecturing, reasoning, etc.)” (p. 153). Learning a second language, Barwell argues could also be seen as a process of socialization into the proficient use of the language practices of the classroom language (p. 153). I analyzed the role of students’ individual identities on their mathematics learning experiences with a view to the processes of acculturation and socialization.

The second key influence focused on teachers’ pedagogical practices. My observational notes revealed 8 pedagogical moves that the teacher used to promote and facilitate mathematics learning in the classroom, including 1) the set-up of the physical environment of the classroom, 2) the design of the mathematics block, 3) the use of interactive and engaging mathematical activities, 4) the use of mathematical discourse practices, 5) the use of multiple strategies, 6) the use of concrete learning tools, 7) the act of making mathematical connections, and 8) differentiating instruction and work. Hence, I organized and analyzed the data as it appeared.

One additional strand I concentrated on is institutional influences, with a focus on the language infrastructures around and, in the school, that influenced the dominance of English and created different demands on students and their teacher in the grade 2/3 classroom. My analysis revealed three layers of language infrastructures that influenced the dominance of English: 1) the language infrastructure surrounding the school, 2) the language infrastructure in the school, and 3) the classroom within which mathematics education unfolded. The analysis formed a general description of the context in which mathematics is taught and learned, and of the role that language may play in that context.

In terms of sociopolitical influences, there were two important provincial policies that I focused on as they have a strong impact on teaching in Ontario. One is the Ontario mathematics curriculum and a supporting document, and the other is the Education Quality and Accountability Office (EQAO) provincial assessment of mathematics. These two provincial policies seemed most significant and relevant to mathematics teaching and learning in the grade 2/3 classroom. Hence, it is in considering these two provincial policies that I analyzed and presented the data for sociopolitical influences.

The analysis formed a general description of the role of students’ identities, the teacher’s pedagogical moves, the context in which mathematics is taught and learned and the role that language may play in that context, as well as, provincial policies that influence mathematics teaching and learning. It is in this manner that I analyzed my observation notes based on individual, pedagogical, institutional and sociopolitical influences, which helped me address the first sub research question: What are the influences on multilingual learners' mathematics experiences?

Analysis of Students' Engagement in Mathematics Problem-solving Activities

In order to address the second sub question (i.e., How do multilingual learners negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?), I adapted Gee's (2005) analytical framework of discourse analysis. Drawing on Gee's (2005) definition, discourse analysis "involves asking questions about how language, at a given time and place, is used to construe the aspects of the situation network as realized at that time and place and how the aspects of the situation network simultaneously give meaning to that language" (p. 110). Gee (2005), who in many ways is a follower of Vygotsky, argues that language has meaning only in and through social interactions. Gee's view of language is directly correlated with Vygotsky's understanding of language as mediating the connection between the individual and the social world (Lantolf & Thorne, 2000). Hence, language has no meaning without people's interactions with one another, as it is through such interactions that gives language its meaning. On reflection, various researchers in mathematics education have drawn on Gee's (2005, 2011) work to offer insightful discussions about the role of language in relation to the teaching and learning of mathematics (e.g., Moschkovich 2007; 2017; Setati et al., 2018; Setati 2005, 2008). My study is intended to build on this research to form an in-depth description and understanding of multilingual learners' meaning making experiences.

For this analysis, I specifically focused on the video recordings of students' participation in mathematical problem-solving activities. First, I transcribed each video recording using Transana (see Appendix K). Next, I divided the entire transcript into stanzas. Each stanza contains portions of the transcript, but taken together, the entire transcript of the problem solving session is included. Stanzas, according to Gee (2005), are "clumps" of utterances that deal with a unitary topic or perspective and which appear (from various linguistic details) to have been planned together. In this case, the stanzas are interactively produced (p. 107). These stanzas reflected situations when the students were interacting with mathematical problem-solving activities, and more specifically, with specific terms, the rules of the game, with one-another, with various tools and strategies, and turn taking when playing a mathematics game. Then, for each of the stanzas, I drew on Gee's (2005, 2011) construct of building tasks in order to analyze "language-in-use". Language-in-use, according to Gee (2011) "is a tool, not just for saying or doing things, but also, used alongside other non-verbal tools, to build things in the world" (p. 88). Hence, whenever we speak or write, we are building or designing one of seven *things* in the world or in reality. These *things* are what Gee (2005, 2011) refers to as the "seven building tasks" of language, which include significance, activity, identity, relationships, politics, sign systems or knowledge, and connections. For the purpose of this research, I have translated the building tasks to the context of mathematics education. In particular, I exclusively focused on the first 6 building tasks as the building task concerning "connections" was imbedded as part of the task on "activity". In *activity*, I focused on mathematical actions and within which *connections* was considered as one of the actions. **Table 3** describes the 6 building tasks of language in use and how they correspond to mathematics education.

Table 3

Analytical Framework for the Six Building Tasks of "Language in Use" and their Connection to Mathematics Education

Building Tasks	Language-in-use	Mathematical Connection	Seven different questions about any piece of language-in-use
Significance	We use language to make things significant in order to give them meaning or value.	I considered the act of giving importance or attention to a particular term, idea or understanding.	How is this piece of language being used to make certain things significant or not and in what ways?
Activities	We use language to carry out actions and to get recognized as engaging in a certain kind of activity.	I considered mathematical actions, and I used the mathematical processes in NCTM (2000) to define mathematical actions which highlight ways of using and acquiring mathematical knowledge including, ***Problem-solving***- the act of engaging in a task where the solution to a problem is unknown, and where, students draw on their knowledge to reason their way to a solution. ***Reasoning and proof***- the act of students justifying their decisions, explaining their choices, negotiating	What activity or activities is this piece of language being used to enact (i.e., get other to recognize as going on)?

		understanding, and making mathematical conjectures. ***Communication-*** the act of sharing and clarifying mathematical understandings and ideas visually, orally and in writing —"Through communication, ideas become objects of reflection, refinement, discussion, and amendment" (NCTM, 2000, p. 60). ***Representation-*** the act of capturing a mathematical understanding, concept, or relationship using concrete objects, drawings, diagrams, and symbolic expressions. ***Connections-*** the act of seeing the connection between mathematical ideas, mathematical topics, and among procedures and concepts.	

Identities	We use language to get recognized as taking on a certain identity or role, that is, to build an identity here and now.	How do students see themselves as mathematics learners and how do they come to understand what it means to do mathematics as it is realized in the classroom (Cobb et al., 2008, 2009)? In particular, is the student identifying with the mathematics activity, merely cooperating with the teacher, or are they resisting engagement in the activity (Cobb et al., 2008, 2009)?	What identity or identities is this piece of language being used to enact (i.e., get others to recognize as operative)?
Relationships	We use language to build relationships with other people, groups and institutions.	I observed the relationship that students built with one another, with the teacher, the activity, and with the resources they used as mathematics learners.	What sort of relationship or relationships is this piece of language seeking to enact with others (present or not)?
Politics	We use language to build or destroy a perspective on the distribution of social goods (politics).	I focused on sociomathematical norms, which is the understanding of what counts as an acceptable or legitimate mathematical explanation and justification (Yackel & Cobb, 1996, p. 461). At the same time, I looked at the	What perspective on social goods is this piece of language communicating?

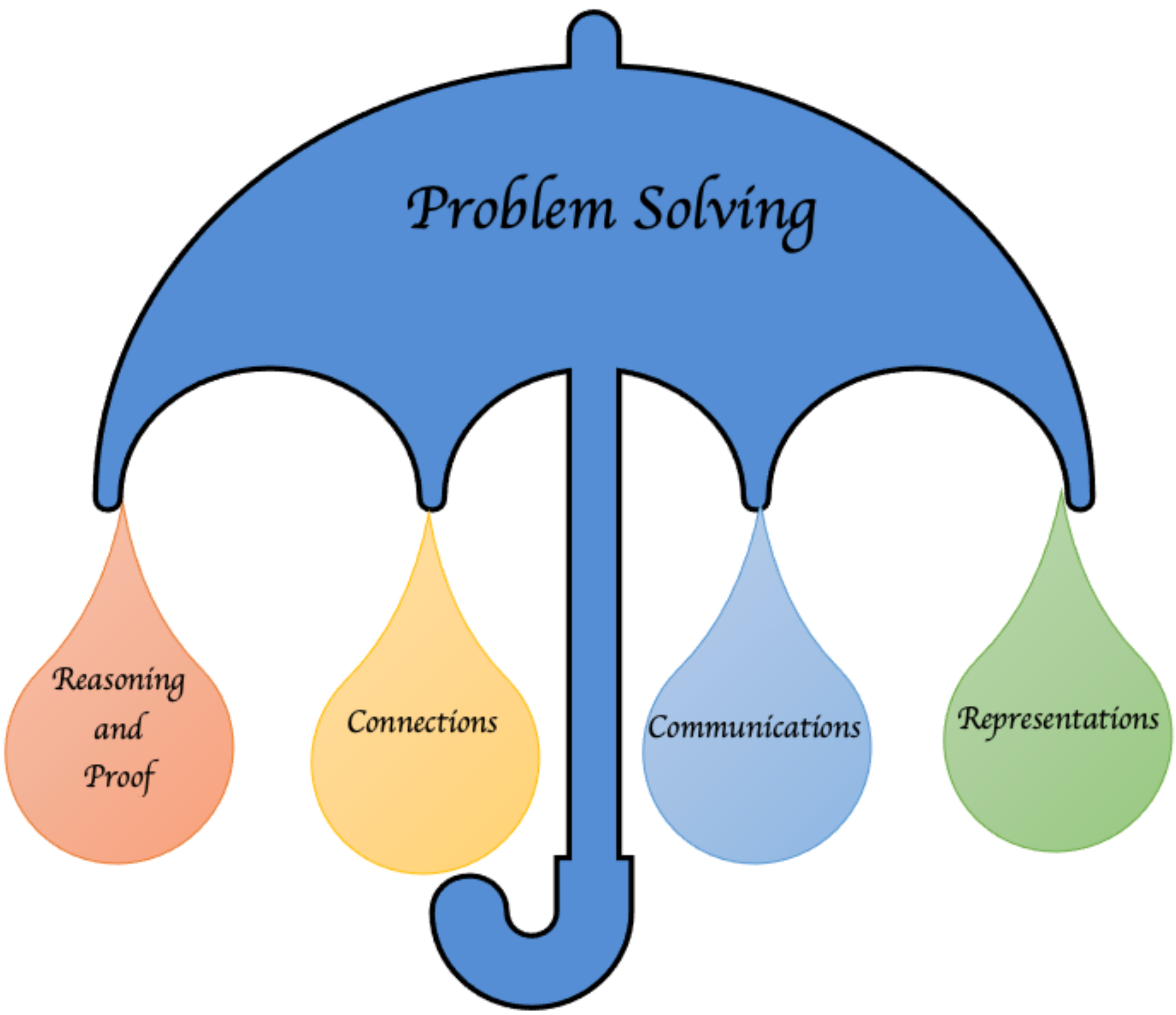
		distribution of mathematical authority to understand who is in charge to legitimize certain aspects of the mathematics activity and implicitly sanction others (Cobb et al., 2008, 2009; Langer-Osuna, 2017, 2018; Yackel & Cobb, 1996).	
Sign Systems or Knowledge	We use language to build privilege or prestige for one sign system or knowledge claim over another.	I focused on how children in this context are using the languages they know, everyday language (informal) vs. mathematical language (formal), and mathematical and world knowledge to work through mathematical problem-solving activities.	How does this piece of language privilege or deprivilege specific sign systems or different ways of knowing and believing or claims to knowledge and belief?

According to Gee's building tasks of language in use, he states that we use language to make things **significant** in order to give them meaning or value. Within a mathematical context, I took significance to imply giving importance or attention to a particular term, idea or understanding *(Building task 1)*.

We use language to carry out actions and to get recognized as engaging in a certain kind of **activity**. Here, I considered mathematical actions, and I used the mathematical processes as defined by NCTM (2000) as the mathematical actions through which students develop mathematical knowledge. These processes include: *problem solving*, *reasoning and proof*, *communication, representation, and connections*, and they are similar to the mathematical processes in the Ontario mathematics curriculum. *Problem solving* is the act of engaging in a task where the solution to a problem is unknown, and where, students draw on their knowledge to reason their way to a solution. R*easoning and proof* focuses on the students act of justifying their decisions, explaining their choices, negotiating understanding, and making mathematical conjectures. *Communication* is a way of sharing and clarifying mathematical understandings and ideas visually, orally and in writing— "Through communication, ideas become objects of reflection, refinement, discussion, and amendment (NCTM, 2000, p. 60). *Representation* is the act of capturing a mathematical concept or relationship using concrete objects, drawings, diagrams, and symbolic expressions*). Connections* is the act of seeing the link between mathematical ideas, mathematical topics, and among procedures and concepts. In my work, I viewed problem-solving as an umbrella within which the other mathematical actions come into play (NCTM, 2000) (**Figure 4**). Hence, as students engage in problem-solving, they reason their way to a solution by communicating their understanding and ideas by creating representations that capture their understandings, and by making or seeing connections between mathematical ideas and concepts. Taken together, these mathematical actions are interconnected and feed off one another. For example, when interacting with the students on a mathematics game, the students reasoned their way to a solution by communicating their understanding and ideas through creating various representations and making connections between what they already know and a new understanding. At one point, Sahar used her fingers to create a representation and to signify a connection between what she already knows about 5 times 5 and adding a group of 5 to get the product for 6 and 5 (i.e., 30). Sahar held up 5 fingers on one hand as a representation of the number 25, then she held up her thumb on the other hand as a representation of an additional group of 5. She then brought her hands together creating three representations: 1) a representation of addition, 2) a representation of a number sentence, and 3) a representation of the product 30—(i.e., 25 + 5 = 30). In this brief example, Sahar used and acquired mathematical knowledge by applying the mathematical processes.

Figure 4

Problem Solving Umbrella



Moreover, Gee argues that we use language to get recognized as taking on a certain **identity** or role, that is, to build an identity here and now. For example, one can have various roles (i.e., a student, a friend, and a son or daughter) and speak and act as is required by that role. My concern looked at how students see themselves as mathematics learners and how they come to understand what it means to do mathematics as it is realized in the classroom (Cobb et al., 2008, 2009). In particular, is the student identifying with the mathematics activity, merely cooperating with the teacher, or are they resisting engagement in the activity (Cobb et al., 2008, 2009) (*Building task 3)*.

We use language to build **relationships** with other people, groups and institutions. More specifically, I observed the relationship that students built with one another, with the teacher, the activity, and with the tools they used as mathematics learners (*Building task 4)*.

We use language to build or destroy a perspective on the distribution of social goods (**politics**). Here, I focused on sociomathematical norms—by which I imply, the understanding of what counts as an acceptable or legitimate mathematical explanation and justification (Yackel & Cobb, 1996, p. 461). At the same time, I looked at the distribution of mathematical authority to understand who is in charge to legitimize certain aspects of the mathematics activity and implicitly sanction others (Cobb et al., 2008, 2009; Langer-Osuna, 2017; Yackel & Cobb, 1996) (*Building task 5)*.

We use language to build privilege or prestige for one **sign systems or knowledge** claim over another. Here, I analyzed student's choice of language (i.e., the terms used), their developing sense of the mathematical problem-solving activity, and the tools they used or adapted (*Building task 6)*.

Furthermore, Gee (2005) states that since we use language to build things in the world, a discourse analyst can ask several different questions about any piece of language-in-use. Below, I list the questions that guided my analysis of the six building tasks I used:

1. How is this piece of language being used to make certain things significant or not and in what ways? (Building task 1)
2. What activity or activities is this piece of language being used to enact (i.e., get other to recognize as going on)? (Building task 2)
3. What identity or identities is this piece of language being used to enact (i.e., get others to recognize as operative)? (Building task 3)
4. What sort of relationship or relationships is this piece of language seeking to enact with others (present or not)? (Building task 4)
5. What perspective on social goods is this piece of language communicating (i.e.,

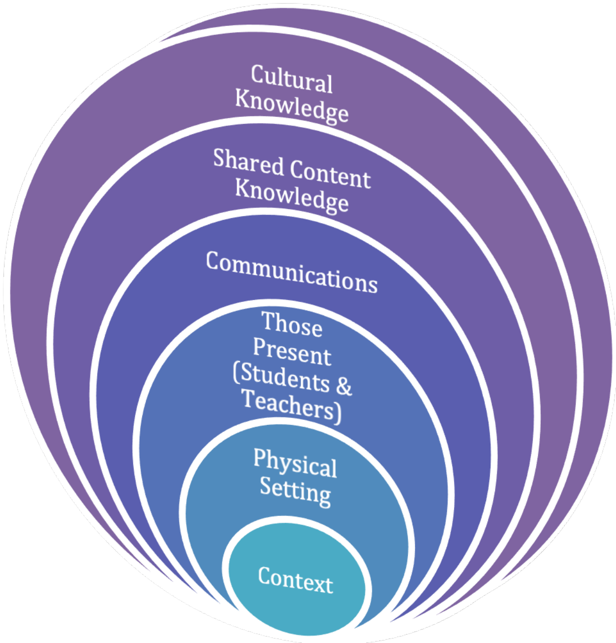
what is being communicated as to what is taken to be "normal," "right," "good," "correct," "proper," "appropriate," "valuable," "the ways things are," "the way things ought to be," "high status or low status," "like me or not like me," and so forth)? (Building task 5)

6. How does this piece of language privilege or deprivilege specific sign systems (e.g., Spanish vs. English, technical language vs. everyday language, words vs. images, words vs. equations) or different ways of knowing and believing or claims to knowledge and belief? (Building task 6)

For example, as I read through the transcript, I relied on these questions to identify places in the transcript where students used language to build any of the 6 tasks in their interactions. Hence, using Gee's analytical framework of discourse analysis allowed me to form an understanding of how students negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational context. Gee (2011) suggests that context is not static or something just there, rather it has a spiral effect on the very makeup of context itself (**Figure 5**)—that is, the physical setting, the communications that takes place— "…how we speak—what we say and how we say it", and everything or anything that may be relevant and helps create that very context, including those present, their bodies, eye gaze, gestures, movements, shared knowledge, and cultural knowledge (Gee, 2011, p. 84). As I read through and analyzed the transcript, besides looking at it line by line, I considered the situation or context in which the students were involved. Thus, I considered everything that happened before, everything that came after, and everything that was going on during students' interactions.

Figure 5

The Spiral Effect on the Context



Analysis of Follow-up Interviews with the Students.

The follow-up interviews were used for explanatory and contextual information for the main research question: What are multilingual learners' mathematical experiences? As well as, the two sub questions: a) What are the influences on multilingual learners' mathematics experiences? and b) How do multilingual learners negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting? I paid particular attention to students' conceptualizations of "what is mathematics" or "what it means to do mathematics", as well as, their experiences learning mathematics at school, at the afterschool homework club, and at home. The analysis develops an in-depth understanding of students' previous and new mathematical learning experiences, and also, their reflections towards their participation in the mathematics classroom.

My Role as Researcher

When I think of my role as a researcher, I feel that who I am and where I have come from served me very well in this project. For example, I shared a similar home language (i.e., Arabic) with several students in the classroom. Although students did not necessarily communicate with me in Arabic, they seemed pleased that we spoke the same language. Also, the way I dressed was similar to what some students wore to school, especially in terms of the hijab that acted as a symbol or representation of my religion. Without question, students who shared the same religion knew that I was familiar with what they could or could not eat, and appeared sure that I was fasting during the holy month of Ramadan because they would ask me about my practices, how my children were doing, and the food that we will be having for Iftar.

I also shared some of the students' experiences in terms of fleeing a war-torn country, attending school in my home country, and learning mathematics using my home language. Then, having come to Canada and experiencing a whole new school system, language, and culture. I recall feeling oblivious to my surroundings because it was a very different context than what I was used to. For instance, in Canada, the desks were organized in groups, we had time for gathering at the carpet, we had two nutrition and one lunch break, students raised all 5 fingers to signal that they wanted to answer a question, and people were of various ethnic and cultural backgrounds—all these experiences of mine were similar to what many students in the grade 2/3 classroom reflected on. I believe I was fortunate to have been exposed to such experiences as I could relate to many of the students in the classroom. In other words, I was part of that lens which helped me to understand what the students were going through.

Also, when I continue to think of my role as the researcher, I feel that the manner in which I approached this research project helped to dismiss any tension between the teacher, the students, and myself. By that I mean, when I visited the classroom in November, I did not collect data right away. I took it upon myself to help the students and their teacher get used to my presence in the classroom so that they could continue to go about their daily routines as they normally would. In so doing, I gradually built a relationship with the students and their teacher which helped to disrupt the formality of me, being the researcher, and they, the research participants. Hence, this relationship we built led the students and their teacher to treat me as one of them. I recall, at one point, one of the students asked their teacher if they could add my name to their cubbies so I could have my own space to place my coat and bag. Several students would even welcome me into their classrooms with a hug. Even my birthday was added to the classroom calendar.

Lastly, I related well to the classroom context given my previous experience as a volunteer working with multilingual students and their teachers at other schools for 6 years, and thus, I understood the students' roles, feelings, and experiences learning mathematics in an additional language. Also, the conversations I had with the students happened much more easily and a lot sooner than I had anticipated. All this, I believe, was because of who I was, what I brought to the classroom, and my cultural experiences which coincided with those present in the grade 2/3 classroom.

Trustworthiness

The idea of establishing trustworthiness in qualitative research is often understood or associated with the term credibility. Shenton (2004), for instance, suggests that to ensure the credibility of research, the researcher should provide a "thick description of the phenomenon under scrutiny" (p. 69). He further argues that "providing a detailed description can be an important provision for promoting credibility as it helps to convey the actual situations that have been investigated and, to an extent, the contexts that surround them" (p. 69). In this study, building thick description generally came from my data. In particular, the trustworthiness of my study comes from the length of time that I spent in the classroom, the amount of data that I collected, my detailed data analysis, as well as using students' original words and working with my supervisor.

My ongoing presence in the grade 2/3 classroom, as well as, the ongoing conversations I had with the students and with the teacher allowed me to confirm my observations. I was in the grade 2/3 classroom for 7 months and I was interacting with students on a regular basis. During my time there, I was constantly observing the ways the students and their teacher engaged with one another; the ways they interacted around mathematical problem-solving activities; the ways they produced oral or written explanations of their mathematical thinking; and of the ways they used the language(s) they know and other resources during mathematical discussions. I also had many conversation with the classroom teacher and the students, as well as audio recordings of whole class mathematics discussions that supplemented my observation notes and helped to provide background information in order to better understand the context of the mathematics activities. The conversations and audio recordings helped me to clarify any misinterpretations or understandings I had.

The extensive amount of data I collected adds to the trustworthiness of this study. The data consisted of 1) 18 audio recordings of whole class mathematics discussions; 2) 13 video or audio recordings of student-student interactions on mathematics activities; 3) 10 video or audio recordings of whole class math talks; 4) 24 video or audio recordings of students working in their mathematics centers; 5) 6 audio recordings of students engaged in extra or other self-directed mathematics activities; 6) many photographs of student work; and 7) audio recordings of one-on-one interviews with each student in the classroom. All data collected informed my work in developing an understanding of students' experiences and how they negotiate meaning making during mathematics problem-solving activities.

In terms of the data analysis, I used two analytical frameworks, one came from my analysis of the research literature that prompted me to organize the influences that the research found into four categories: individual, pedagogical, institutional, and sociopolitical features of mathematics classrooms. The other analytical framework came from Gee's (2005, 2011) construct of building tasks which I translated to the context of mathematics education. There are also other studies that influenced how I analyzed the data such as, Moschkovich's (2007) work that encouraged me to move away from what students cannot do and to consider the multiple resources they use to communicate mathematically. I also used the mathematical processes in NCTM (2000) to define mathematical actions which highlight ways of using and acquiring mathematical knowledge including, *problem solving*, *reasoning and proof*, *communication, representation, and connections*. Also, the work of Cobb et al., (2008; 2009) was of great significance in examining students' mathematics identities. The work of Cobb et al., (2008), Langer-Osuna (2017), and Yackel and Cobb (2014) were also pertinent for examining sociomathematical norms and mathematical authority. Hence, by relaying on the work of these researchers my findings are warranted and supported by their work.

In addition, students' original words, thoughts, and so forth are an accurate record of what children have said. I have included 1) a detailed description of the entire interaction of each video recorded activity, 2) several block quotes of what the students said, and 3) included copies of students' work. I have included a detailed description of the entire interaction for each activity because I wanted to provide the reader with a detailed account and understanding of multilingual students' meaning making and its context. If I included only specific segments of the transcript, they would have appeared out of context and would not show the development of students' mathematics thinking and meaning making. I also wanted to stay true to what the students said by using their own words as reflected by the full transcripts of their interactions and not rely solely on my interpretations. In this way, my interpretations can be evaluated by other researchers.

Lastly, trustworthiness comes from working with my supervisor. Together, we went over my observations as well as all the transcripts; we talked through them and we unpacked the influences on students' learning, as well as, the mathematical ideas involved, which promoted their credibility and trustworthiness. Also, considering the analysis for the first and second sub research questions and the one-on-one interviews I had with each student, led to a re-examination of the results with my thesis supervisor to meaningfully address the main research question.

CHAPTER 5: RESULTS - INFLUENCES ON LEARNING

Influences on Students' Learning of Mathematics

In this section of the presentation of the results, I focus on addressing the first sub research question: What are the influences on students' mathematics learning experiences? I structure the presentation of the results in terms of four influences on students' learning of mathematics that came from the organization of my literature review, individual, pedagogical, institutional and sociopolitical features of the mathematics classroom. The findings are a result of the analysis of my observation notes. First, I present the individual influences on students' learning experiences. Second, I present the influence of the teacher's pedagogical practices. Next, I concentrate on institutional influences. Then, I focus on sociopolitical influences. This section provides a description and understanding of the influences on students' mathematics experiences. The results in this section will generally focus on the data gathered from my observation notes and interviews.

Individual Influences

My analysis demonstrates the role of students' individual identities as having a strong influence in their mathematics learning experiences. As detailed in the analysis section of the methodology chapter, when I explored students' identities, it was useful to lean on Gibson's (1988) work, in particular the processes of acculturation. Acculturation implies a process of adaptation with the host society which results in the balancing of two cultures. This could result in social and cultural change but may not result in the rejection or replacement of old values, traits, and habits. Talking about acculturation in the context of the mathematics classroom has a great deal to do with students' identities. Students' identities are not static but constantly changing and evolving as reflected in, for example, the language(s) they know and their social and cultural repertoires, which are important resources for their learning of mathematics. In what follows, I discuss the students' process of acculturation in a Canadian educational setting, then I discuss the students' acculturation in the mathematics classroom.

Acculturation in a Canadian Educational Setting

The analysis of the data suggests that most students experienced acculturation with the host society in an attempt to embrace the traits, values, or habits of the dominant culture, which resulted in the balancing of two cultures—that is, the dominant culture and their cultures. Although some of the traits, values, or habits embraced by newly arrived students may not directly relate to students' mathematics learning experiences, they informed who the students were as individuals in the grade 2/3 classroom. The traits, values or habits of the dominant culture were particularly evident in the way students were assigned birth dates, the way their names were pronounced, how they dressed, the food they brought for lunch, the way they adjusted to the norms and values of the classroom, their imitation of others in the classroom and even, in how they used the languages they know. For instance, some students seemed to have experienced a social or cultural change by being assigned a birth date upon their arrival to Canada. As the classroom teacher explained, most students who came from Somalia did not have dates of birth for unknown reasons. So, it tended to be a little more complicated to know a student's age which was necessary for determining students' grade levels in Canada. Students with no birth dates were given the same month and day (i.e., January first). However, their years of birth differ and depend on the limited information the family reception center received from students' parents or through the literacy or mathematics assessments students take upon their arrival.

I also noticed that students did not necessarily wear traditional clothing to school, except for three students who wore hijabs. For instance, Suhail wore the long-styled Somali hijab and would sometimes use her hijab as a way to hide or cover her face, especially when others teased her such as, when one student imitated how Suhail used her fingers to count. In general, students dressed for school in ways that were similar to other students. Students wore hoodies and shoes that were considered "cool". Also, those who had some prior schooling in their home countries shared that they wore uniforms to school, but here, as Sabina from Nepal shared, "I wear whatever I want".

One particular conversation I had with some students that resonated with me was when they shared that their names were pronounced differently at school than at home or in their home language. The conversation started with two students sharing that their aunt and mother shared the name "Fatima" but stressed the Arabic pronunciation of the name. This led other students to share how their names were pronounced by family at home or in their home language. Some students appeared bothered by it, but accepted it because they felt it was inappropriate to correct their teachers. Also, there are certain letters in some languages that are difficult for others to pronounce. For example, in Arabic, some of the most difficult letters to pronounce are 'haa', 'khaa', 'ghayn', '3ayn', and 'qaaf'. Such letters, if not pronounced correctly could alter the accuracy of saying someone's name. Still, some names might not consist of these letters, but how a name is pronounced in a different language might be pronounced differently than in the home language.

I also noted that the lunches of students who recently arrived from their home countries were quite traditional and culturally oriented, and as time passed, they became less so. For instance, students whose families recently came from Syria had lunches that included a traditional cheese, zaatar (thyme), or labneh (strained yogurt) and cucumber sandwich on Arabic pita bread. Students who came from Somalia had basmati rice with either chicken or meat, and students from Nepal had soup or rice/noodles with vegetables, chicken or meat. After a while, students' lunches gradually started to resemble that of the dominant culture and included, toast, bagels, burgers, salads, cheese and crackers. Some students even started to bring lunchmate packs, juice boxes, chips, and granola bars—things that they were not familiar with early on.

I also noted that newly arrived students were very cooperative and observant of their surroundings and gradually learned to pick up on the classroom norms and values, including, trying to engage in all classroom activities, trying their best, being prepared, working hard, asking for support from the teacher or other students, raising their hand before talking, asking for permission to use the rest room, walking and not running in the hallway, eating only during nutrition breaks, respecting other students and their different points of view, being kind to one another, believing in each other, keeping ones' hands to themselves and respecting each other's space. Acculturation in the classroom norms were also evident in the way students were taught or told to raise their hand to signal that they had a question or comment. For instance, one student reflected on how in Syria students raised their hand holding up one finger, whereas in Canada, students raised their hand holding up all five fingers. The student shared an urge to imitate or to adapt their classmate's way of raising their hand, as accepted by or acted by the host society, leading them to change a trait they once had.

There was also a constant urge reflected in some students' behavior to be like others in the class by imitating them in order to fit in. For instance, Mustapha, holding on to his identity, proudly identified himself by his country of birth, Syria, and as an Arabic speaker. However, I noticed his keenness to be like others in his class, namely Sajan. Mustapha would refer to Sajan as his ‘brother’—a term he used to reflect their friendship. Mustapha was frequently observed looking at Sajan and trying to imitate the way he acted by walking and talking in a similar way. He even picked up on a song that Sajan would sing, and when asked about the song, Mustapha would say “this is Sajan and my song”. The song was actually a mix of Spanish and English called, “Taki Taki” by DJ Snake, Selena Gomez, Ozuna and Cardi B. Mustapha also picked up on certain dance moves that Sajan would do when expressing excitement and winning, such as “the shoot dance” and “the hype dance”. These dance moves were very popular at the time and most of the boys in the class replicated them. Such experiences became part of Mustapha’s identity, which was constantly changing as he interacted with his classmates and adapted to his new context.

The students also appeared to have valued the use of English in the grade 2/3 classroom as opposed to using their home language with those who shared their language. This seemed to be determined by the dominance of the English language in the classroom. As one student shared, “In Canada everyone speaks English not Arabic like Syria. I have to speak English.” Although the students valued the use of English, they still recognized their home languages as vehicles to connect with their families in Canada and abroad. For instance, some students considered their home language as their only means to communicate with their parents and younger siblings, as one student said, “My mom and dad and my little brother don’t know English, so I have to speak Nepali”. I also observed students communicating with their parents in their home languages, and also, acting as their parent’s interpreters by accompanying them to school interviews or other functions. Their role was not unfamiliar, as many immigrant and refugee children I have worked with shared that they take on that role involuntarily because their parents do not know how to speak English. Such roles reflect and give students a sense of agency as they are operating within this Canadian context as the spokespeople for their parents. Other students saw their home languages as valuable for connecting and communicating with their families back home, as one student shared, “I need Arabic because I can talk with my family in Syria”. I also noted students reflecting their parents’ perspectives on the importance of maintaining their home languages by enrolling their children in heritage language classes or in Quranic studies, or by encouraging its use at home and with friends who share a similar language. Khadija shared,

> I speak Arabic. And that's our language my mom says. When I go to my friends, they keep talking in English. And my mom said don't talk in English because we don't want to lose our language…If you speak English then you can learn [it]. If our dads didn't speak English, it's good to speak a little English. Yeah, it's good to speak. But Arabic is your language. So that's your language. It is more important to speak Arabic…It is not good if we lost our language.

While students appeared to have adapted to the dominant culture of the host society by embracing the traits, values, or habits of that culture, still, the students identified with their countries of birth, the languages they spoke, having religious observance (e.g., wearing a hijab, fasting, praying etc.), and following dietary practices.

Acculturation in the Mathematics Classroom

As we see students try to adapt or adjust their values, traits, and habits to the dominant culture, my analysis illustrates that they were also trying to observe how other students were doing mathematics, as they may come to realize that the way they used to do mathematics is not the accepted way of doing mathematics in this context. However, in the process of acculturation in the mathematics classroom, multilingual students relied on various resources that influenced their mathematics learning experiences in the grade 2/3 classroom including, the influence of their home languages on communicating mathematics, students reading and writing mathematics, as well as, the influence of their experiences learning mathematics elsewhere. All this then influences students' views of mathematics.

The Influence of Students' Home Languages on Communicating Mathematics. The influence on students' home languages on communicating mathematics were reflected in the adjustment that a student makes when they come into the classroom, in shunning the home language even when others speak it, and in some students' use of their home language to explain a mathematics activity or for translating purposes.

In the mathematics classroom, language was not viewed as an obstacle by the students, as all students were regarded by their teacher as learners of language and mathematics despite their limited use of English. The classroom teacher, and myself included, supported students' use of their home languages, but there was little space for its use, especially since most students who shared similar languages preferred not to make use of them during mathematical interactions. I noted that when a new student came into the classroom and they were speaking a language other than English, they quickly came to realize that the language they grew up with was neither the language used to do mathematics nor the language used by the majority in the classroom. For instance, when Suhail first arrived in the classroom she relied on her first language, Somali, to communicate with students and her teachers. However, neither her teachers nor most of her classmates shared a similar language, and for those who spoke Somali, they appeared unwilling to use it when interacting with Suhail. From then on, Suhail rarely, if ever, spoke Somali with others, leading her to use her limited English vocabulary along with the use of gestures to communicate with others. At times, she appeared a little withdrawn from the rest of her classmates as a result of her limited use of English, but then she gradually used other means to communicate mathematically with others.

I also noticed that when I spoke Arabic with those who shared a similar language, they almost immediately responded in English without making use of the Arabic language. At one point, a student appeared in utter shock that I was speaking in Arabic, and deliberately asked me to speak in English, "why you speaking in Arabic? Tell me in English". Thus, to fit in with rest of the students in the class, or of fear of being left out or rejected by other students, students shared an urge to learn and communicate with others in English.

While several students shunned the idea of using their home language even when others speak it, some students made use of their home language to explain a mathematics activity or to translate something to someone who shared a similar language. On various occasions, I observed Raj and Rachet communicating together in Nepali. This often occurred when Raj and Rachet were working together, and especially when Raj would express confusion or seek clarification from Rachet regarding what was expected in a mathematical task. Rachet would use his home language to either translate the question or to simply tell Raj what to do. When I would approach them, they would immediately switch to English, as though they feared being judged for using another language. When asked about what they were saying, Raj would respond, "Nothing. We're just talking", and on occasion, Rachet would explain to me what it was they were saying. They also used Nepali to gossip about others, leading other students to question their use of Nepali in class. Another student, Mustapha, communicated using his home language, Arabic, only when I could not make sense of his answer to a mathematical problem, leading him to translate his answer (i.e., the number) to Arabic, "Talateen [Thirty]". Also, when Suhail arrived to the classroom, she relied on Arabic and Somali to count and make mathematical computations. At one point in our interactions, we compared numbers in all three languages (i.e., Somali, Arabic, and English). Suhail had a good understanding of her numbers in Arabic and Somali, and she was capable of counting in English from 1-10, but confused the numbers that followed by relying on the Arabic and Somali number naming system. For example, when saying the number thirty-two in Arabic or Somali, she would say the digit in the ones place followed by the conjunction (i.e., and) then the digit in the tens place (e.g., two-and-thirty). I noticed the same thing with Mustapha and Khadija who would say or write the number using English numerals, but would rely on the Arabic number naming system. They would even write their numbers starting with the digit in the ones place followed by the digit in the tens place. For example, when writing the number 45, they would first write 5 then write 4. At times, they would confuse the order of the numbers and write 54 as opposed to 45.

Students Reading and Writing Mathematics. Also evident was the influence of students reading and interpreting mathematics text as well as writing mathematics symbols and representations. The findings suggest that some students had difficulty reading and interpreting mathematics text, so the teacher and I relied on alternate methods to help students make sense of an activity. For instance, students who had limited use of English would seek the support of their teacher or my support, and we would read and visually model to students the task using concrete learning tools. At one point, I was working with Suhail on trying to find what 4 tens plus 3 ones equal to. I relied on the use of base-ten-blocks[7] (i.e., rods as a representation of tens and units as a representation of ones) to create a visual representation of the number sentence, and to then add what they equaled to by counting by ones or by tens and ones. By working on one together, Suhail was capable of carrying on her own for the next 3 questions. Other students who have been in the classroom awhile longer would either reread the activity a couple of times or would ask others (i.e., teachers or students) to read the activity for them in an attempt to make sense of it.

The influence of writing mathematically was particularly evident in their writing of narrative texts and in writing mathematics symbols. Concerning the writing of narrative texts, at the beginning of every month, students were asked to “write about the weather graph” by describing

---

[7] Learning tools used to help students learn mathematical concepts in number including, place value, addition, subtraction, multiplication and division.

what they see using mathematical terms such as more, less, equal, or the same as. Just as an example, students would write, "there are more snowy days than rainy days", or "snowy is more than rainy". However, newly arrived students experienced some reluctance expressing themselves in writing, and in most cases, were either exempt from doing it or, encouraged to orally talk about the graph while a teacher wrote what they were saying. Regarding writing mathematics symbols, this was particularly evident when some students expressed confusion about certain numbers not resembling what they were taught in their home country, at home, or even in other classrooms in Canada. For instance, when I worked with Tamam on comparing the numbers on the hundreds chart, our conversation started with me asking what she notices or wonders about. One thing that she shared was how she noticed that she does not write the number 9 or 2 the same way as on the hundred's chart, and that it confused her because she sometimes was not sure what the number was. It was not that she did not know her numbers but because they were written differently that confused her. I also noted the same concern with other students especially when I wrote the number "2" in a way that they were not familiar with like, "". They too expressed confusion because they were not sure what the number was.

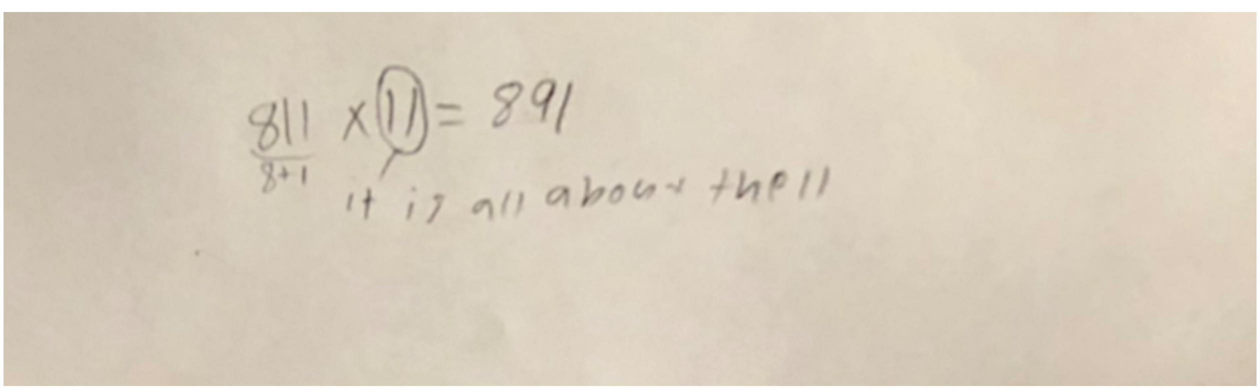
The Influence of Students' Experiences Learning Mathematics Elsewhere. There were also particular things that the students were exposed to at the afterschool homework club, in their home countries, and at home that influenced their mathematics learning in the grade 2/3 classroom. Concerning the afterschool homework club, several students in the grade 2/3 classroom shared that they attended the homework club twice a week, on Tuesday and Thursday. The language of teaching and learning at the club was English, and the focus appeared to be on repeated practice of multiplication facts through worksheets that emphasized procedures. One student shared that at the homework club, "we do the same thing over, and over, and over". Still, another student said that "sometimes I need to practice even more. And then when I practice and practice and practice, I get it and I know".

Also, the students were well aware of the difference between how their teacher taught such things as multiplication versus how it was taught at the homework club. For instance, when the classroom teacher introduced students to multiplication, she encouraged students to use various strategies (e.g., array models, equal groups, or repeated addition) and concrete learning tools (e.g., hundreds charts, counters, drawings, or their fingers). Students who attended the homework club were keen on quickly sharing the answer without showing their work or justifying how they got the answer. The students then appeared to face a sort of struggle, in that they tried to maintain or make use of what they learned at the homework club, but still felt an urge or need to respect their teacher's expectations—that is, to share the steps, strategies, and tools used to find the product. For example, Raj who was well aware of Mrs. Holland's expectations shared that in the homework club, "they don't let us to count. They can't let us to use the hundred's chart"—strategies and tools highly encouraged in Mrs. Holland's classroom but not at the homework club. Other students commented on how at the homework club, they were encouraged to work on "harder" or "bigger" numbers, whereas in Mrs. Holland's classroom "it was easy". One student, Rachet, shared a strategy or as he referred to it, "a magic trick" for multiplying by 11 that he was taught at the homework club. At first, it was not clear how he got the answer, as all I could see on the provided piece of paper was Rachet writing, "81 x 11", then he wrote 8 plus 1 underneath the 81 and concluded that the answer would be "891" **(Figure 6**). I asked him to explain how he got the answer, and he shared,

> If it's 8 and 1, you have to add these [8 + 1] together. And it makes 9. And then I put the 9 in the middle [he wrote the number 9 on the paper]. And then I put the '1' on the side [wrote 1 on the right side of the number 9]. And 8 here [wrote 8 on the left side of the number 9]. It is all about the 11.

Figure 6

Rachet's Magic Trick for Multiplying by 11



In this example, Rachet added the digits of the number [81] and then placed the sum between the original two digits, giving him 891. Also, his comment that "it is all about the 11" implies that this “magic trick” could be used with any two-digit number multiplied by 11, which is true. The trick Rachet used did not seem to coincide with the strategies the students were encouraged to use in Mrs. Holland’s classroom, especially those that encouraged students to show how they got the answer. However, in asking Rachet to explain his solution, I was able to understand the strategy he used to find the product. In so doing, I may have welcomed new or alternate ways of finding the product for multiples of 11.

The students’ experiences in their home countries also influenced their mathematics learning. These mathematics experiences in their home countries are connected to the language of instruction, the mathematics they learned, and how mathematics is taught and perceived. Students who attended school in their home countries shared that they were taught predominantly in the official language of that country (e.g., Arabic, Nepali, or Somali) but some were introduced to English to do mathematics. For instance, Suhail shared that in Somalia mathematics was mostly taught in Somali but teachers used English for translating numbers or mathematics terminology. Rachet said that, in Nepal, they used both English and Nepali when working with numbers. He explained that they wrote numbers using the English numerical system, but used the Nepali naming system when referring to the numbers.

In most instances, when the students talked about the mathematics they learned in their home country, they talked about calculations, computations, and number, similar to the work with number that students learned in the grade 2/3 classroom. For example, Abed shared that, in Sudan, he learned number operation (i.e., “plus” and “minus”), and did calculations similar to those he did with Mrs. Holland, although he did them in a different way. He also mentioned that he preferred the mathematics taught in the grade 2/3 classroom, He said, “Math here is better. Because in Sudan, we don’t do times, but here we do times”. The students are recognizing that they are doing similar operations. However, when they talk about how they learn mathematics, there appeared to be some differences.

The students talked about how mathematics was taught by referring to the materials they used, which in most cases, in their home country, was a blackboard, or individual blackboards to write their own solutions. None of the students talked about previous experience doing mathematics collaboratively. In general, how mathematics was taught in their home countries placed an emphasis on individual and explicit teacher instruction—that is, the teacher explains the relevant lesson or demonstrates procedures for solving mathematical problems on the board, and the students then solve problems with minimal discussion. For instance, Abed shared that, in Sudan, each student had their own mini portable chalkboard that they used to individually solve questions posed by their teacher, and then they were called upon to share their answers. Suhail also shared that the school she attended was different than the one in Canada. She explained, "It's different. In Somalia, we have a blackboard and the teacher uses it. And he asks you a question, and you can raise your hand and answer it" (Translation).

There also was a punitive aspect expressed by students when they got something wrong or did not do certain things. Sabina reflected on the punishment they endured for not doing their homework.

> We did do it [math] in Nepal. But if we didn't do [homework], the teacher will hit us with the stick or the belt. My brother forgot to do his homework because he was too busy, then the teacher say he didn't do homework, so he slapped him with a stick. That time I was in grade 1. I didn't get hit because I always do my homework.

Similar experiences were emphasized by students who did not attend school in their home country but who talked about their parents' experiences. For instance, Samartha shared her mom's experience learning mathematics in Nepal, she said, "they do math. And if you get it wrong, they [teachers] do very bad things…they slap your hands with sticks". Contrary to these experiences, students in the grade 2/3 classroom in this study were not penalized or punished for not doing their homework or getting the wrong answer. Instead, students were exposed to a welcoming and nurturing environment that fostered inclusion and was respectful of students' differences and learning needs. Students recognized that their experiences in their home country were different from what they were currently experiencing.

Students also worked on mathematics at home. When students were asked about doing mathematics at home, they talked about the people they worked with, the language(s) they used, and the mathematics they worked on. Some students did not work on mathematics at home because their parents were unfamiliar with either mathematics or English. One student shared that she did not work with her parents because "my parents doesn't know [math]".

Students who worked on mathematics at home stated that they either worked on their own, with their mother or father, or worked with their older siblings. For instance, Khadija said that "when I have some math. Like when I was with Mrs. Hodges…She gave me math sheets and I do it at home by myself. But sometimes I ask my mom. She help me a little". Suhail also shared that she practiced mathematics at home with her siblings and that she felt it was easier to work with them than to work on mathematics at school. She said, "My siblings teach me sometimes math…It is easy when they teach me" (Translation).

The students said that they mostly used their home language when they worked with their parents or siblings on mathematics. For instance, when Suhail worked with her siblings they used Somali, which could explain why she felt it was easier to work with them. Another student, Khadija, acknowledged that she practiced reading and writing numbers in Arabic with her mom.

Mustapha shared that he worked on mathematics at home by playing "Mad City in Roblox". As part of the game, the player defeats a boss and robs banks to gain money (i.e., in game currency). The player then needs to accumulate enough money to buy the desired vehicle or equipment. Hence, this game deals with money amounts, which was something the students and their teacher were working with at the time, and Mustapha might have made that connection. However, in general, students' experiences doing mathematics at home seemed to have a focus on repeated practice of procedures. One student, Rachet, said that he usually practiced his multiplication using the strategy or "magic trick" he learned at the homework club. Sabina also said that her older brother provided her with repeated practice of addition, subtraction, and multiplication questions to solve, she said,

> My Brother. He give takeaway and adding. And um, the what it called? It's like the cross [Multiplication]. Yeah, he give me that but I don't do it because I just leave it like that. And my brother says, "why didn't you do it?" And I said, "I don't know." And he said, "I'll help you".

Ikbal shared that she practiced adding with her father using an algorithm or strategy "different" from the one used in Mrs. Holland's classroom (**Figure 7**). As an example, Ikbal drew a line, wrote the number 24 on top, the number 20 on the bottom, and a plus sign on the left. Then to add the numbers, she started at twenty-four and counted up by tens using her fingers, giving her 44, which she wrote on the right-side of the algorithm. Interestingly, the strategy Ikbal used to add the numbers together was similar to that used in Mrs. Holland's classroom, but how she presented the addition algorithm was quite unique. Prior to this interview, Ikbal had not referred to this algorithm or made use of it when working on addition problems. It could be that she dismissed its use given that it was not similar to the strategies used by the teacher or other students in the classroom.

Figure 7

Ikbal's Use of a Unique Algorithm

Students' Views of Mathematics. Students' conceptualizations of "what is mathematics" and "what it means to do mathematics were influenced by their mathematics experiences in and outside the mathematics block. In particular, the mathematics activities that students engaged with in the classroom, at home, and at the homework club not only shaped their understanding of mathematics, but also, informed students' mathematical meaning-making and learning experiences. I first share students' conceptualizations of what is mathematics or what it means to do mathematics, as well as their understanding of what is regarded as mathematics and not mathematics. Next, I talk about what the students liked about doing mathematics. Understanding multilingual students' conceptualizations of "what is mathematics" and "what it means to do mathematics" allows for a better understanding of how students view mathematics learning, as well as the extent to which their mathematics experiences could influence their conceptualizations.

From students' perspectives, mathematics was mostly understood as numbers, counting, and number operations (i.e., addition, subtraction, multiplication, and division). In fact, for several students, when they were asked what mathematics is, they talked about learning tools such as the hundreds chart, ten frames, and Lego (i.e., for multiplication), but all of these are still related to numbers. Although I asked what is mathematics, some of the students did not respond with a conceptualization of what mathematics is, instead, they reflected on what it means to do mathematics. For example, some students shared that when they heard the word "math", they felt a need “to get ready for doing math”, “to do math”, and to “learn math”. Raj, on the other hand, saw mathematics as writing mathematical solutions, which he appeared to have enjoyed.

Raj: You can write. And I like writing the most.

Fatima: And what do you write?

Raj: You write numbers.

Notice that in this quote even though Raj is talking about what it means to do mathematics and he talks about writing mathematical solutions, he is still referencing numbers. Students' understanding of mathematics as only numbers is consistent with the results of research that underline how students' conceptualizations of mathematics might be considered too narrow and stereotypical (Grootenboer & Marshman, 2016; Hall, 2013).

Some students also seemed to have formed certain conceptualizations of what may be regarded as mathematics and what is not. Students' understanding of what may be regarded as mathematics and not mathematics seemed to be influenced by students' previous learning experiences. For example, when Khadija was working on symmetry and transformations with another student, she looked at me and said, “this is boring. This is not math. I want to do math”. Perhaps since such activities did not involve numbers and computations, Khadija did not consider them related to mathematics. After explaining to her that these activities were mathematics, she said “I want to do a different kind of math”. Also, some students were not keen on sharing their work because they regarded the final answer as most important. As one student shared, “With my sister, I just have to write the answer. Here, Mrs. Holland wants me to show everything”. This focus that the students had on getting the correct answer sometimes created a fear of making mistakes. Still others, were shy to show they were using their fingers to make mathematical computations. I noticed that when working with Pooja, she would either pull her hands underneath the table or make little movement with her fingers so that I would not see that she was using them. Such conceptualizations could be influenced by students' previous experiences where certain cultures may value the final answer as more important than the strategies or steps taken to solve it.

When the students were asked about “what they liked about doing mathematics”, they shared their views towards mathematics, as well as the importance of mathematics for what Barwell (2012) would call “getting on in the world” (p. 326). In terms of students' views towards mathematics, students shared things they liked or disliked about the subject. Some students liked the act of doing mathematics because they found it “fun”, and when asked how or why they saw mathematics as “fun”, they shared things like, “you can write”, “you can talk”, “you can play [math] games”, and “you can add or takeaway”. Others liked particular topics in mathematics such as, counting, addition, subtraction, multiplication, or division. When students were asked how or why they saw mathematics as “fun”, they shared things like, “you can write”, “you can talk”, “you can play [math] games”, and “you can add or takeaway”. Other students, Sabina and Khadija, shared things they disliked about mathematics. They talked about how writing in mathematics made them bored or tired because there was a lot of writing involved. Sabina shared, “sometimes I don’t like [math challenges] because my hand gets tired from writing”. In general students were very positive towards mathematics. It was only particular aspects of mathematics such as writing that they did not enjoy.

A few students also talked about the importance of mathematics for getting a job or pursuing higher education. For instance, Khadija shared that “if you know all the math, you can be a scientist”, which is something she considered a necessity for becoming “a baby doctor”. Although multilingual learners talked about the importance of mathematics for getting a job or pursuing higher education, it also seemed to be emphasized by the other students in the classroom who talked about different kinds of occupations where mathematics would be required (e.g., a cashier, an accountant, a doctor, and a teacher). In her work, in South Africa, Setati (2008) finds that students mostly focus on the power of “English” for gaining access to social goods. She further argued that “learners who position themselves in relation to English are concerned with access to social goods and positioned by the social and economic power of English” (p. 103). However, the results of my study suggest that the students regarded mathematics as a necessity for future employment and higher education similar to Grootenboer and Marshman’s (2016) work. Hence, students in my study saw mathematics as their access to social goods.

Pedagogical Influences

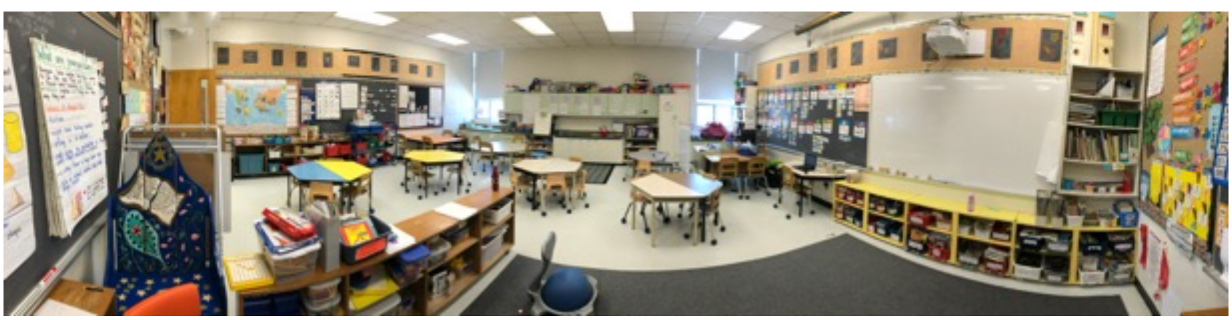
My analysis shows that the teacher had a variety of pedagogical moves which influenced students’ mathematics learning experiences. By pedagogical moves, I mean the instructional practices or strategies that the teacher used to promote and facilitate mathematics learning in the classroom. Her range of pedagogical moves to support and influence students’ mathematics learning included 1) the set-up of the physical environment of the classroom, 2) the design of the mathematics block, 3) the use of interactive and engaging mathematical activities, 4) the use of mathematical discourse practices, 5) the use of strategies, 6) the use of concrete learning tools, 7) the act of making mathematical connections, and 8) differentiating instruction and work. Although these pedagogical moves could be viewed as having different focuses, they are interrelated and form a nested system of factors that influences students' learning and engagement in mathematics.

The Set Up of the Physical Environment of the Classroom

The physical set up of the classroom disrupts the so called 'traditional classroom' that fosters individual seating arrangement where desks are in rows and a teacher at the front, but instead, inspires a collaborative learning classroom that is welcoming, engaging, and a feel good to be at school atmosphere (**Figure 8**). It would appear that the classroom set up of children sitting in groups with easy access to materials encourages interactions and collaborations among the students as they engage in mathematics. The classroom includes student tables with 3-4 student chairs, the teacher's desk off to the side of the room, carpeted space, storage shelves filled with materials for students to use, and vividly decorated classroom walls.

Figure 8

The Grade 2/3 Classroom



The set up of hexagon-shaped student tables invites students to be seated together, which encourages student collaboration. An orange square table and a large circular table with a number of chairs were also present and these became our 'working space', where the students and I interacted on various mathematics related activities. When the teacher asked students to work with me, they would immediately move to one of those tables. This set up encouraged interactions and discussions when working on mathematics activities.

The teacher's desk was located in the far-right corner of the classroom and the teacher is rarely seen sitting at her desk. Rather, the teacher's desk was used as students working space, and at times, students would place a cushion underneath the teacher's desk, turning it into a cozy and quiet space for students to work in pairs or individually. There is also a horseshoe table located close to the teacher's desk. The teacher uses this space to interact with a small group of students, encouraging hands-on learning and discussion between the students and the teacher.

There is also a large open carpeted space that serves as a community gathering spot for the morning calendar, entry activities, lessons, math talks, readings, announcements, and celebrations. The teacher also uses this space to work with a small group of students on mathematics or other activities. This space is surrounded with books organized by genres (i.e., science fiction, action and adventure, fantasy, comics, travel etc.), and on the opposite side sits a 6-section storage cabinet that contains various learning tools including, linking cubes, base ten blocks, counters, pentomino tiles, 2-Dimentional and 3-Dimentional shapes, calculators, hundreds charts, and portable mini-clocks.

The walls of the classroom are very decorative, and reflect students' work, their day-to-day schedules, posters related to mathematics and other subject areas, a huge hundreds chart, keyword lists, learning skills and work habits (e.g., responsibility, organization, initiative, collaboration etc.). Students refer to and rely on the use of the posters and charts for various activities in the class. There is also a blackboard, whiteboard, and an overhead projector where the teacher presents most of the lessons and activities.

Overall, the classroom is set up in a way that fosters collaboration between the students, their teacher, and learning resources are readily available for student use. Also, the set-up encourages students to take ownership of their classroom space by engaging with others and the resources available. In this context, the students were regarded as active participants in learning and not passive recipients of knowledge.

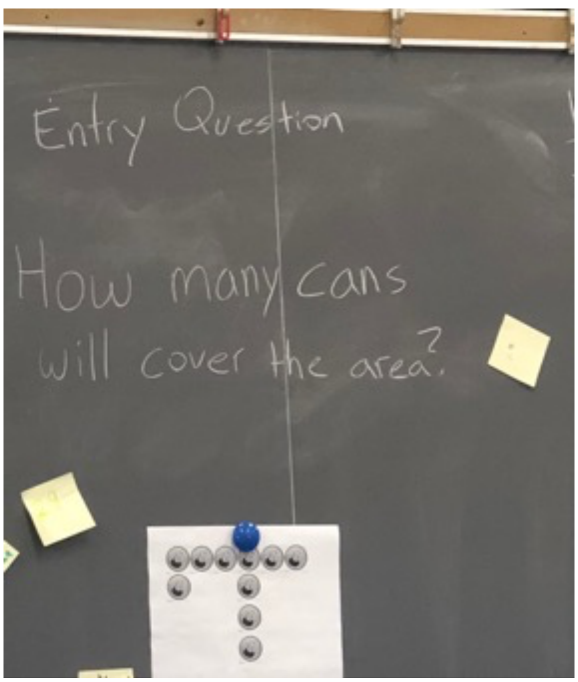
The Design of the Mathematics Block

The mathematics block took place during the first time period of each day for about 60 minutes. The mathematics block included a mathematics entry activity, a math talk related to the entry activity, the morning calendar, or catching-up on mathematics activities, followed by a mini-lesson, math centres or an additional math talk.

Upon students' entry to the classroom, they engaged in a mathematics entry activity set up on the whiteboard or chalkboard (**Figure 9**). The students read the activity, or had someone read it to them, and then used individual whiteboards or sticky notes to work on the activity. Some students decided to pair-up, others chose to work independently, and some sought one-on-one support from their teacher or myself. Once all the students had an opportunity to work on the activity, they met at the carpet for a math talk related to the entry activity or to tie it into the mathematics lesson.

Figure 9

Example of Entry Activity

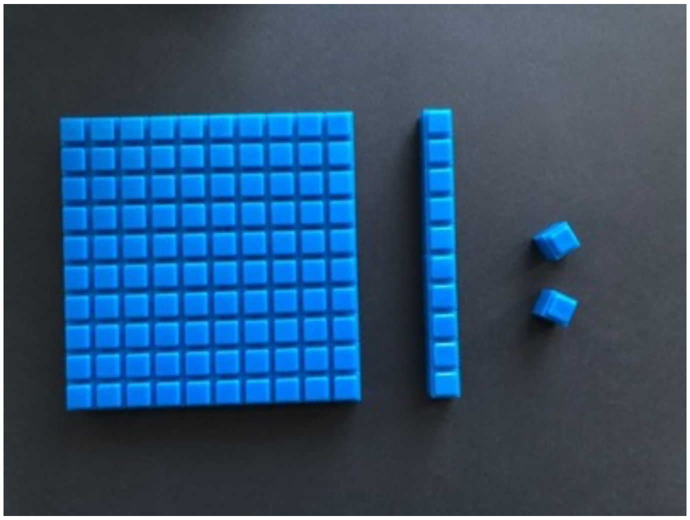


In this context, math talk means mathematical discussion. The intention behind such discussions were to elaborate on students' answers, deepen understanding, and make learning connections. This could help students see the multiple strategies that could be used to develop an answer, making them aware that there may not be only one solution to a problem. For the math talk, the teacher starts by reviewing what the activity was about, and she either shares or invites the students to share some of the strategies they used to solve the activity. The mathematics lessons were at times part of the math talk, and where, the teacher reviewed a mathematics skill, introduced a new topic, or explicitly taught a new concept. The teacher is observed constantly facilitating discussions by probing students to share their mathematical thinking (i.e., strategies, approaches, or solutions), and by encouraging or guiding them to make connections between mathematical ideas and concepts.

Following the entry activity, the students and their teacher engaged in the morning calendar routine. The morning calendar activity reflected various mathematics concepts such as, number sense and numeration, measurement, patterning, data management, and probability. An example of a calendar routine included students creating a pattern for the school days of each month using picture cut-outs (i.e., circles, squares, rectangles, leaves, hearts, snowmen, etc.) and using actions like clapping or tapping. For each school day, students added one tick mark to the tally chart that acted as a representation of the number of days they have been in school. They then added the total number of tick marks by skip counting by fives. Students also created a representation of the number of days they have been in school using base ten blocks (e.g., For 112 days in school, students used one flat (100), one rod (10), and two units, equalling to 112) (**Figure 10**). The students also used a bar graph to display the weather data for each school day for the duration of one month (e.g., sunny, windy, rainy, stormy, snowy, etc.). Once they completed an entire month, the teacher created a mathematics activity related to the bar graph such as asking students to show the data using tally marks and to provide a description of the graph using mathematical terms such as more, less, equal or the same. There was also a line plot for displaying precipitation. They also described probability as a measure of the likelihood it will rain or snow using mathematical terms such as likely, unlikely, equally likely, more likely, and impossible. The students then used a portable clock to represent the time of sunrise and sunset of each school day. During the calendar routine, the teacher might also ask some students to review or catch-up on missed mathematics activities with the support of volunteers or myself.

Figure 10

Representing Number of Days in School Using Base Ten Blocks



Once the morning calendar routine was completed, the teacher invited all students to join her back on the carpet for a mathematics lesson where, she introduced a new skill or strategy, or reviewed a previously taught skill or strategy. Following that, the teacher would either introduce another math talk related to an open-ended mathematics activity, or she might divide the students to work in math centers. The teacher usually had 4-5 math centres set-up with 3-5 students at each center. Each group spent approximately 15 minutes at each centre with 4-5 rotations through different math centres. Some of the activities were repeated on subsequent days in order to provide the students with an opportunity to engage in all activities. Also, each volunteer or teacher present at the time was assigned to a math centre. For instance, a day of math centers would always include a teacher led, small group, hands-on lesson. The objective of these teacher led lessons was to target instruction with a group of students in order to meet their needs. At one point, students worked with their teacher on a lesson using Lego (i.e., brick pieces) to understand multiplication using a variety of strategies including, area or array models, repeated addition, and equal groups (**Figure 11**). The teacher supported students to explore such strategies by differentiating instruction for each group depending on their needs.

Figure 11

Multiplication Through the Use of Lego Pieces

$2 + 2 + 2 + 2 = 8$ or

4 groups of 2 ($4 \times 2 = 8$) or,

2 groups of 4 ($2 \times 4 = 8$)

Another group of students might be working on a mathematics challenge activity or a math quest activity such as *Alf's Adventure*. The objective of this particular activity was to help the Alf get to the castle by overcoming a few adventures. These adventures included problem-solving activities, such as figuring out the pattern and the next couple numbers in a set (e.g., 1, 3, 5, 7, _, _, _.), finding the difference between two set of numbers, representing data on a bar graph, finding the perimeter and area of various shapes, drawing a shapes transformation, and solving word problems (e.g., You are collecting jewels. You find 5 bags of jewels in a cave. There are 8 jewels in each bag. How many jewels did you find?). After the students surpassed all the adventures (i.e., problem-solving activities), the Alf makes it to the castle and that completes the activity.

A third group of students might be working on an activity that involved the use of coins to create an image or picture. The students used a tally chart to represent the number of each coin they used. Then, they calculated how much money they used in total. As for the fourth group, they might be working on an activity related to pentominoes. The students had to predict which pentomino image would make a box with an open lid. They were then provided with cut outs of pentominoes and asked to compare their observations with their predictions. Finally, the fifth group, might engage in a line of symmetry activity. The students were provided with half an image created with pentominoes and they were to create the other half using the same pentominoes.

This brief description regarding the design of the mathematics block illustrated how mathematics learning unfolds in this classroom, and how the students and their teacher maneuver through each of the activities. The students engaged in a variety of activities including, entry activities, math talk related to the entry activities and tied into minilessons, morning calendar routines that reflected various mathematics concepts and skills, space to catch-up on mathematics activities, and math centres that provided students with an opportunity to practice and apply various strategies and skills. These activities encouraged students to be active participants in the classroom through hands-on and engaging problem-solving activities.

The Use of Interactive and Engaging Mathematical Activities

Another pedagogical move that supported or influenced students' mathematics learning experiences was the use of interactive and engaging mathematical activities. The data shows that the teacher provided students with various tasks that were generally open-ended. By open-ended, I mean mathematical tasks that allowed for different solutions or multiple ways to arrive at a solution. Such tasks promoted reasoning and problem-solving, and encouraged students to use a variety of strategies to achieve a solution. In general, the teacher focused on the process and students' reflections in problem-solving activities rather than on their answers. And although some tasks appeared challenging for some students, they provided students with opportunities to explore mathematical ideas, concepts, and to use multiple strategies and concrete learning tools. The students engaged in a variety of problem-solving activities including, mathematics games, mathematics challenges, and math talk.

The games generally focused on practicing certain procedures or strategies, and with space for students to invent their own strategies. The students usually worked with others in groups of 2 or 3. The types of games that students engaged with were 1) *Slides and Ladders* with a focus on adding doubles; 2) *Spin and Double it* also focused on adding doubles, and at times, students were to add doubles plus 1 depending on where the spinner lands; 3) *Building 10 with dice* focused on adding several dice together that equaled 10; 4) *Add 'Em Up* focused on adding the numbers on two rolled dice; 5) *First to 100* focused on adding or subtracting two-digit numbers; 6) *Pick & Choose* focused on subtracting the numbers 10, 20, 30, 40, 50, or 60 from three digit numbers; 7) *Take-off* focused on either adding or subtracting; 8) *Checkers/ Chess* also focused on either adding or subtracting, 9) *Times Tussle* focused on multiplying a digit by 5 or 10 to make a product, and 10) *Symmetry/ Transformation* games focused on either matching two cards together, or using tetras tiles or pentominoes to match an image on one side.

The mathematics challenge activities acted as a review or practice of what students learned including, but not limited to addition, subtraction, multiplication, and division; patterns and relations; perimeter and area; symmetry and transformations; collecting data and graphing; and addition and subtraction of money amounts. Such activities were a lot like the math quest activity (i.e., *Alf's adventure*) that I described earlier. These tasks promoted reasoning and problem solving, as students developed an understanding that supported their mathematics learning based on the information provided and that they needed to solve.

In terms of the math talk, these were generally open-ended and focused on word problems and 3-Act tasks. The word problems are similar to those shared in a subsequent section of the results on students' meaning making in mathematical problem-solving activities, such as *Books in a Classroom* and *Ameer's Coins*. As for the 3-Act tasks, these were either adapted from Graham Fletcher's (2019) website or created by the teacher. Such tasks took the form of a story and consisted of three acts. In act one, the teacher introduced the central conflict of the task or story by sharing a video or an image and discussed it. In act two, students reasoned their way to a solution by relying on the information they had or were seeking to find. In act three, the students and their teacher discussed or shared the strategies used, and then concluded the solution to the task. In what follows, I share a 3-Act task that the students and their teacher worked on entitled "The Cookie Monster" by Graham Fletcher (2019). I have chosen to provide a detailed account of this task as I observed the teacher facilitating it, and it reflected the types of open-ended tasks the students and their teacher engaged in. The general objective of this task was to work on addition and subtraction within 50 by figuring out the number of cookies the monster ate.

Act one—The teacher introduced the math talk by sharing that they were going to watch a video about a cookie monster, and she encouraged students to observe, she said, "Get ready to observe: 'what are you hearing, noticing and wondering?'". The teacher wrote "Observe/Notice & Wonder" on the board, and played the video. In the video, the students saw the hand of a cookie monster grabbing a box of cookies from a grocery bag, opening it, and then eating several cookies. The students asked to see the video again, and the teacher replayed the video one additional time.

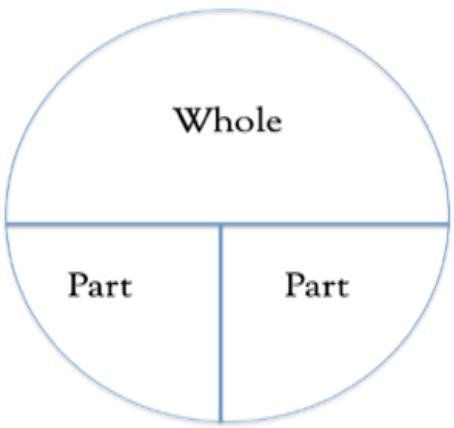
In order to encourage and generate discussion, the teacher asked students what they noticed, and then what they wondered. The teacher asked, "who can tell me something they have heard or saw [Observed/ Noticed]?", then she wrote students' responses on the board. The students shared things like, "Eating lots of cookies; A furry hand; Oreos/ cookies; An open bag; and Crunching sounds". The teacher then asked, "what did you wonder about?" One student said, "How many were eaten?" As soon as this question popped-up, students shouted out some numbers, "24, 35, 6, 10, 7, 8, 14, 20, 12, and 9". The teacher explained that "a lot of the estimates are between 12-

20, and one person is thinking 35".

Moving on, the teacher then asked, "Do we need a little bit more information? What might we need to know, what might help?" The students shared, "Who was the furry hand?, How many were left?, and How many cookies were in the tray?" The teacher confirmed that they were to try and figure out, 1) How much the monster ate? and 2) How many cookies were in the tray?" The teacher then drew a diagram on the board that prompted students to see what they had in terms of parts and whole, and what they were still missing (**Figure 12**).

Figure 12

A Part/ Part/ Whole diagram



Act two—The teacher showed the students an image of a full tray of Oreo cookies (**Figure 13**), and she asked, "What should we figure out?" Several students immediately got up on the board and counted the cookies. Interestingly, one student said, "some cookies are fatter and others skinnier, it won't be the same"—suggesting that there will not be the same number of cookies in each row. The teacher assured her that she counted the cookies and they were all the same in each row. Then, one of the students counting said, "there's 16 in this row". A number of students made the connection that there must be 16 cookies in each row, "so then there's three 16". The teacher asked, "So if there's 16 cookies in each row. How many are there in total? Grab the whiteboards, markers, hundreds' chart. Whatever you need. And show me your work". Some students worked together, others worked on their own, and some worked with their teacher, myself, or other volunteers in the classroom.

Figure 13

A Full Tray of Oreo Cookies

Act three—The students and their teacher reconvened at the carpet, and shared their solutions and some of the strategies they used. The students agreed that there were 48 cookies in total, and the teacher wrote the number 48 (i.e., the whole) in the diagram. She then shared a picture of the tray with the missing cookies (**Figure 14**).

Figure 14

An Oreo Tray with Missing Cookies

Two students went up to the board and counted the cookies in each row one-by-one, and the teacher wrote the number of cookies in each row (**Figure 15**).

Figure 15

A Written Representation of the Number of Cookies in Each Row

One student then shared the total number of cookies left in the tray, and the teacher asked the student to explain how they got the answer.

Student: 26. There's 26 cookies left.

Mrs. Holland: How did you get that?

Student: Easy. I took the 1 from the 11, and put it here [pointing to the 9]. 9 + 1 = 10. 10 + 10 = 20. 20 + 6 = 26.

Mrs. Holland: Did anyone use a different strategy?

Student: I just counted. 11, 12, 13, 14, 15, 16, 17, 18, 19 [Using their fingers, they started at 11 and counted up 9]. 20, 21, 22, 23, 24, 25, 26 [Then, counted up 6 more].

The teacher repeated what the students shared, then she prompted them to tell her what they were still missing, she said,

Mrs. Holland: So, we know the whole is 48, which is the number of cookies in the tray. We know that there's 26 cookies left in the tray. What are we missing?

Student: How many cookies the monster ate?

Mrs. Holland: So, how do we figure that out?

Student: 48 – 26.

Mrs. Holland: Do we all agree?

Students: Yeah.

Mrs. Holland: Okay, let's do the math.

The teacher then gave the students some space to find the solution and to then share their strategies. Using the hundreds chart, one student shared that they started at 48 and counted backwards 26. Another student shared, "Start at 48. Take away 10 that's 38. Then another 10 that's 28, then 6 more equals 22". Others did it mentally—the same strategies used with the hundreds chart, but in their head and when asked, they shared, "first I take away 20. So that's 28. Then I take away another 6. So, 22."

Finally, the teacher provided the students with a quick recap of what they solved and reflected on some of the strategies that students used to solve the activity. This 3-Act task and others similar provide students with space to explore mathematical problem-solving skills and to apply various strategies in order to reason their way to a solution, which in turn influenced students' mathematics learning experiences.

The Use of Mathematical Discourse Practices

The data shows that the teacher provided students with multiple opportunities to engage in mathematical discourse practices during the mathematics block. By mathematical discourse practices, I refer to the opportunities provided by the teacher that permitted and promoted students to share mathematical thinking and "ideas through classroom discussions, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29). The teacher also relied on mathematical and non-mathematical discourses—that is, the use of mathematical (formal) language and everyday (informal) language to orchestrate and promote meaningful mathematics learning and discussion. Mathematical language refers to the standard terminologies used to describe and discuss mathematics. Everyday language refers to words or terms used in our day-to-day conversations to express mathematical understanding and ideas.

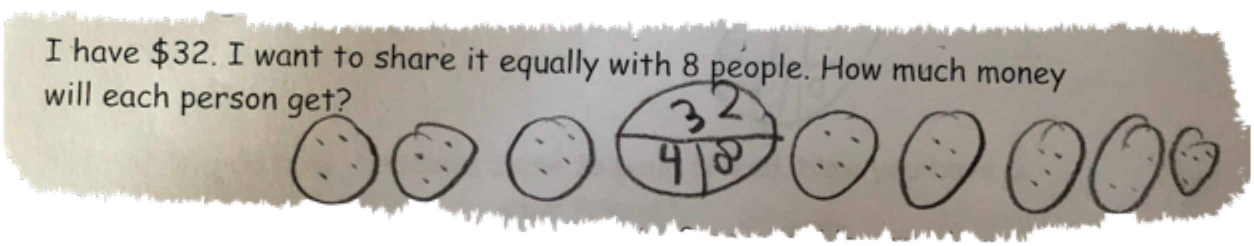
The use of mathematical and non-mathematical discourses were evident throughout the mathematics block, influencing students mathematical thinking and understanding. For instance, when Khadija and Samartha were working on the following word problem,

> *I have $32. I want to share it equally with 8 people. How much money will each person get?*

Khadija was not sure what was meant by the word "equally", she asked, "what is this word equally?" Samartha explained, "equally means 'the same' or 'the same thing'. Like, I get one, you get one. I get two, you get two. It's like that". Samartha's explanation appeared to have been influenced by the concept or idea that their teacher shared with them, and that is, splitting or sharing objects into equal parts or groups. Samartha's explanation helped Khadija comprehend what the question implied, leading Khadija to draw eight circles and to divide the $32 among 8 people. Khadija counted by ones up to $32 by placing a dot for each number she counted in each circle, giving her a total of $4 for each person (**Figure 16**). She also used the part/part/ whole diagram as a representation of her thinking. This said, the use of everyday words helped Khadija make sense of the word problem, and to then reason her way to a solution.

Figure 16

Khadija's Visual Representation of her Solution



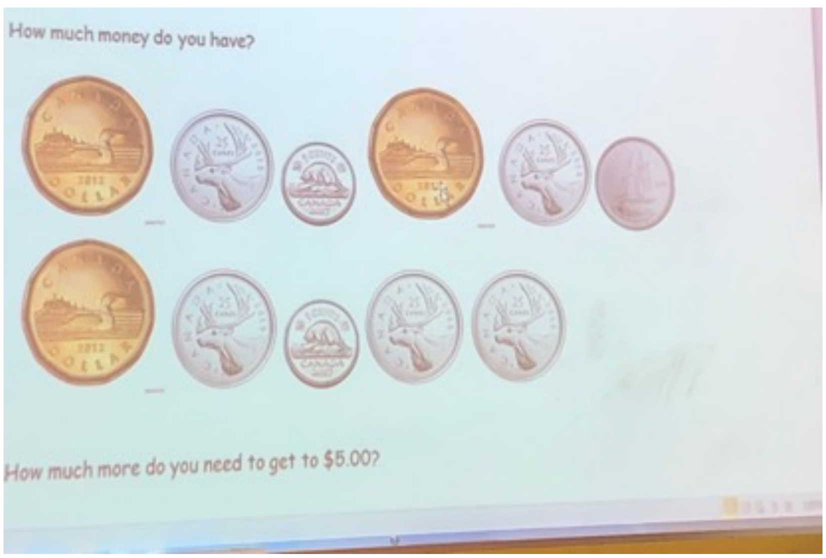
The data also revealed different types of interactions where students engaged in mathematical and non-mathematical discourses, 1) during student-student interactions; 2) during student-teacher interactions; and 3) during whole-class math talk or mini lessons. These different types of interactions occurred differently on different days and provided students with opportunities to engage in mathematics learning that fostered communication with others to share and engage in their own and other students' thinking.

Student-Student Interactions. My analysis shows that student-student interactions took on a variety of different formats including working independently, collectively, or collaboratively. Students who worked independently took ownership of the activity and required limited to no assistance from others. Students who worked collectively, they were seated in the same space, paying attention, and sometimes imitating what one another was doing, but not necessarily feeding off of one another's thinking or ideas to come up with one solution. Students who worked collaboratively and not collectively, they jointly or mutually developed understanding of the activity and worked together to reason their way to a solution.

Noteworthy is that when I first arrived to the classroom, I noticed that students mostly worked independently or collectively. The teacher did share that the students were not quite used to working collaboratively on mathematical problem-solving activities without her directive or lead. But as students took part in more open-ended activities, the more I saw students collaborating and building off of one another's thinking to come up with a solution. For instance, in one task, the teacher shared an image with several coins and a two-part question. The students first needed to figure out how much money they had, then to find out how much more they needed to get to $5.00 (**Figure 17**).

Figure 17

Multi-Step Mathematics Task



The students or their teacher first read the task, then students grabbed individual whiteboards or sticky notes to work on it. Some students worked in pairs, others choose to work on their own, and some asked for one-on-one support from their teacher or myself. Rachet and Abed were among the students who collaborated on the task by building off of one another's thinking or ideas to come up with one solution. The interaction started with Rachet and Abed placing the coins in groups and adding how much they equaled.

Rachet: Okay so these are 3 dollar [Referred to the three loonies].

Abed: Yeah 3 dollar. And these 25 and 25-, [Referred to the quarters]

Rachet: 50. And 25 [Attempted to add an additional quarter].

Abed: 75, 100, 125 [Counted up by 25 to add the quarters].

Rachet: No.

Abed: Yeah look. There's 1, 2, 3, 4, 5 twenty-five cents [Opened five fingers]. 25, 50, 75, 100-, [Counted up by 25 using his fingers]

Rachet: 125. And this 5. 130 [Added one nickel]. 135. [Added another nickel]. And that's-, [Referred to the dime]

Abed: Plus 10. 145.

Rachet: 145 and 3 dollar is-,

Abed: 4 dollar and 45. Done.

Rachet: No. How much to 5 dollar Mrs. Holland said.

Abed: Oh.

Rachet: 55, 65, 75, 85, 95, 100. [Rachet got a sticky note and wrote the total amount of coins they got—that is, 4.45. Then, he counted up by tens (5 dimes) and added one 5 (1 nickel) to get a dollar, equaling to $5.00].

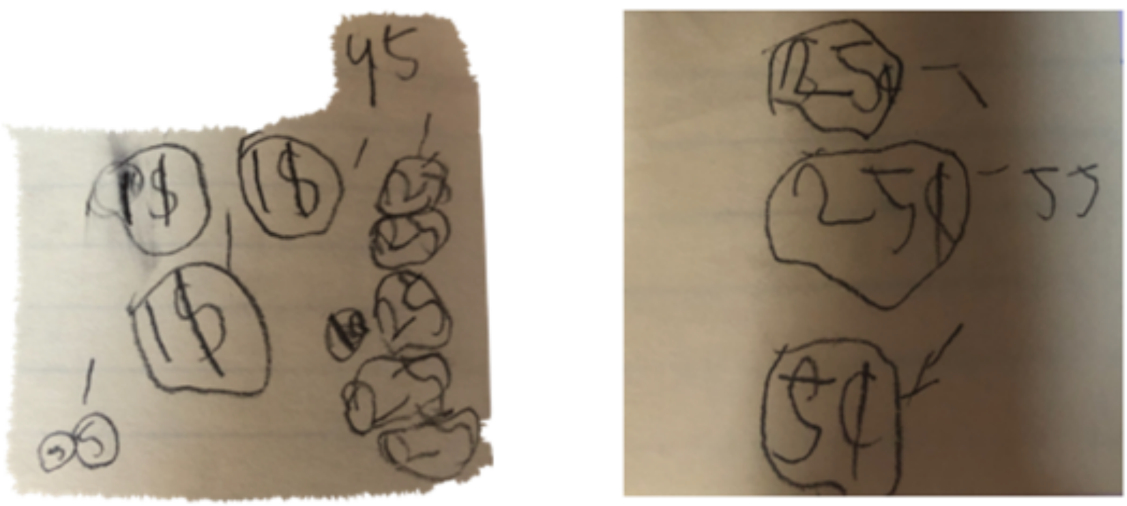
In this student-student interaction, Rachet and Abed agreed that the 3 loonies equaled three dollars, then they added the quarters by counting up by 25. There was a slight disagreement about what the quarters equaled, but once Abed explained that there were 5 quarters in total and created a representation of the number of coins on his fingers, Rachet understood or saw how the 5 quarters equaled to 125 or 1 dollar and 25 cents. They then added 2 nickels and 1 dime, giving them 1 dollar and 45 cents. Then, they added the $1.45 to $3.00, giving them $4.45. Once they figured out the total amount of money they had, they tried to find how much more they needed to get to $5.00. Here, Rachet counted up 5 dimes plus 1 nickel, equaling to 55 cents—the amount needed to get to 5 dollars. Also, notice how Abed and Rachet referred to each coin by their value rather than referring to them by their names. It may be because they were trying to find the total amount of money they had and felt a need to work with how much each coin was worth, or they were simply more comfortable referring to the coins by their value instead of their name (i.e., quarter and dime). They even used the word "and" instead of "plus" to indicate adding. Students' use of various words did not appear as a language obstacle rather it portrayed their interpretation and understanding of how much each coin was worth and that they needed to add them together to find the solution.

Also, noteworthy, is that during student-student interactions, the teacher supported students' use of their home languages. As mentioned in the section on *individual influences*, Raj and Rachet were among the few students that communicated together in their home languages during the mathematics block. Other students who shared similar languages (e.g., Abed, Mustapha, Tamam, Khadija, and Sahar) were a lot more reluctant to make use of their home languages when interacting on mathematics activities. The teacher's support of students' use of their home language(s) promoted a learning environment that valued what students bring with them to the classroom, and that their languages contribute to their mathematics learning experiences.

Student-Teacher Interactions. Concerning student-teacher interactions, these had noticeable influence on students' engagement in mathematical discussions, especially when students were encouraged to share their thinking with their teacher, myself, or other teachers and volunteers. Student-teacher interactions happened throughout the mathematics block, and more so, one-on-one or in small groups. For instance, when the students either worked with their teacher to solve the money task or approached her to share their solutions to the task, she encouraged them to share their thinking and ideas by asking, "show me how you know that?" or "show me how you got that?". In general, students appeared reluctant to show their work or the strategies they used. Hence, these questions encouraged students to share their mathematical understandings and ideas by relying on oral, written, or visual representations of their thinking. At one point, Ahmed approached the teacher and verbally shared that there were $4.45 worth of coins. He also had a sticky note where, he drew some of the coins and included how much each was worth. He also had the numbers 45 and 55 written on there as well (**Figure 18**). Although Ahmed shared the correct answer for the first part of the problem, the teacher appeared a lot keener to know the strategies and tools he used to reason his way to a solution, she asked "show or tell me how you got $4.45?". Ahmed explained that he first added the "dollars" together, which equaled $3.00. Then, he referred to 4 quarters and shared that they equaled $1.00, giving him $4.00 in total. Interestingly, Ahmed focused on only 4 and not all 5 quarters. It could be that he relied on his prior knowledge that 4 quarters equaled a $1.00—something they discussed in a previous lesson. Ahmed then shared that he added the two nickels plus one dime, giving him 20 cents, plus the remaining 25 cents, equaling to 45 cents, and that all together the coins equaled $4.45. The teacher confirmed his answer, then she asked him to explain how he got 55 cents for the second part of the task. He grabbed another sticky note to create a visual representation of the coins he needed to get to $5.00, and by verbally communicating that 25 plus 25 is 50, plus 5 more equaled to 55 cents—the amount needed to get to $5.00 (**Figure 18**). The teacher confirmed his answers and applauded the work he did. In this example, the teacher did not simply accept Ahmed's answers, but provided him with space to explain, reflect on, and justify his thinking, which in turn provided her with information about how he interpreted the task, what he knows, and the strategies and tools he used to reason his way to a solution.

Figure 18

Ahmed's Solution to Part 1 & 2



The teacher appeared well aware of the fact that several students were still not used to sharing their mathematical thinking, or were not sure how to share their solutions. In such cases, students when asked to share how they got their answer, they would either not respond, point to their answer, or share that they "counted" or "added". However, the teacher was constantly observed encouraging students to rely on the use of verbal, written, and concrete tools to model and share their mathematical thinking and the strategies they used to find a solution.

Whole-Class Math Talk or Discussion. Once all the students had an opportunity to solve the activity and shard their solutions with their teacher or myself, we all met at the carpet for a math talk. The intention behind the whole-class math talk were to elaborate on students' answers, deepen understanding, and make learning connections. During this math talk, the teacher was particularly keen on sharing or giving students space to share the strategies they used to come to a solution. This helped students see the multiple strategies that could be used to develop an answer, making them aware that there may not be only one solution to a problem.

The teacher started the math talk by reviewing what the task was about. The teacher then wrote "$1.00" on the board and asked the students to tell her how many cents were in a dollar? Most responded by sharing "100" or "100 cents". The teacher wrote "$5.00 = ___", and then asked, "so, if I am trying to get to $5.00. I can say $5.00 is that same as how many cents?". The students responded by sharing, "500 cents". The teacher confirmed their answer and shared that this brief review should help them figure out the solution to the task.

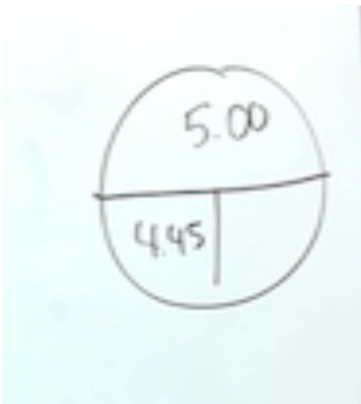
Moving on, the teacher shared the strategy that most students used to solve the first part of the task, and that was, grouping the coins together. The teacher grouped the coins by drawing a circle around the 3 loonies and another circle around 4 quarters, then she asked the students to share what each group equalled. The students agreed that there were $3.00 worth of loonies and the four quarters equalled $1.00. One student then shared that 3 dollars plus 1 dollar equalled $4.00. The teacher acknowledged the student's answer, then she referred to the remaining coins.

Mrs. Holland:	So, then we have these left-over. We have 25 cents, plus 10 cents, plus 5 cents, plus 5 cents. What's 25 plus 10?
Students:	25 plus 10 is 35. Plus 5 is 40. Plus 5 is 45.
Mrs. Holland:	Good Job. So, how much money do we have now?
Students:	4 dollars and 45 cents.
Mrs. Holland:	We know we have 4 dollars and 45 cents. And that's the same thing as saying 445 cents. So that's all the work we did for our first part. Did we all know how much money we have?
Students:	445. [Or] 4 dollars and 45 cents.
Mrs. Holland:	Either 445 cents or 4 dollars and 45 cents.

The teacher then referred to the part/ part/ whole diagram to help students see what information they had and what they were missing or needed to find. She explained that they were trying to get to $5.00, which she wrote as the whole. Then, she referred to the total amount of coins they had which was $4.45 and she wrote that as one of the parts (**Figure 19**).

Figure 19

Part/ Part/ Whole diagram



The teacher then referred to the hundreds chart (**Figure 20**), which was used by several students to find how many more coins they needed to get to $5.00. One student was quick to share that they counted by tens from 45 to 90 and then added 5 more, giving them 55 cents, which is the amount of money they needed to get to $5.00: “Yeah, I counted. 10, 20, 30, 40, 50. And 1, 2, 3, 4, 5. 55”. The teacher then shared that Samartha counted by fives to get to 100, also giving her 55 cents. Here too, the teacher made a connection to multiplication by explaining that Samartha used 11 groups of 5, which were equivalent to 55. The teacher also gave Ahmed some space to share what he did. Ahmed explained, “I did 25 and 25. 50. And 5 more”. Ahmed used two 25 cent coins and one 5 cent coin, equaling to 55 cents. Once again, the teacher made a connection to multiplication in terms of the two groups of 25. Then, she referred to the strategy that Rachet and Abed used, which she explained was equivalent to 5 groups of 10 plus a 5-cent coin, equaling to 55 cents. The teacher concluded the activity by sharing another strategy that they could have used, and that was, 55 pennies or 55 groups of 1.

Figure 20

Hundreds Chart

Most whole-class mathematics talks that the students and their teacher engaged in unfolded in a similar manner. The teacher took advantage of the math talk to discuss and share some of the strategies students used to solve the mathematics activity, and also, to elaborate on their answers, deepen understanding, and make learning connections.

The Use of Multiple Strategies

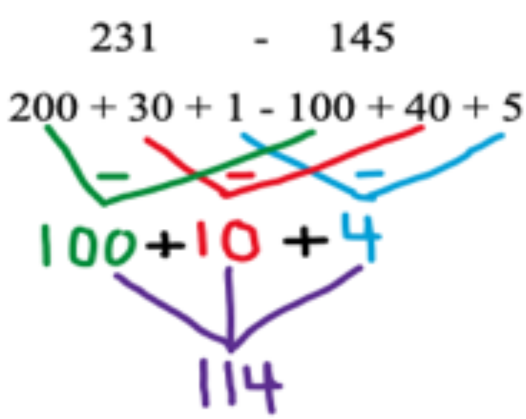
The data also show that the teacher encouraged the use of multiple strategies by modelling their use for students to reason their way to a solution. For instance, when the teacher introduced students to subtracting two-digit and three-digit number, she relied on the use of various strategies including subtracting using counters or base ten blocks; counting backwards using their fingers, the number line, or the hundreds chart; branching by decomposing numbers; regrouping; and using the standard algorithm. The teacher also encouraged students to make use of these strategies by giving them space to share or reflect on how they got the solution, influencing students' mathematical experiences.

The data also revealed that at times, the teacher and myself included, encouraged students to use specific strategies based on our own experiences, which were sometimes rejected by students, especially when they found an alternate strategy. At one point, the students were asked to find the solution to 30 – 4, and the strategy they chose to use varied depending on their experiences. For instance, one student placed 30 in their head, and using their fingers they counted backwards 4—that is, 29, 28, 27, 26, giving them 26 as their answer. Another student relied on the use of the hundreds chart and counted backwards from 30, crossing off the 30, 29, 28, 27, leaving them with the answer 26. The same strategy was used using their fingers as well. However, when the classroom teacher and most students in the class relied on the use of the first strategy, the student appeared obligated to adopt that same strategy. I personally encouraged the use of the second strategy where, I would subtract the numbers starting with 30 then 29, 28, 27, and the number that follows (i.e., 26) would be the answer. This was actually the strategy that I was taught growing up, and I encouraged the student to make use of it, but they were reluctant to do so, given that their teacher and the rest of the students in the class have done it differently.

Another instance that resonated with me regarding teacher versus students' use of strategies was when the classroom teacher asked me to work with Ahmed on subtracting by counting backwards using the number line. The teacher shared that Ahmed was relying on the branching strategy—a method of decomposing numbers to either add or subtract. Most students relied on the branching strategy because they used it for addition, and then for subtraction but with numbers that did not require regrouping. And given that at the time the students were not yet introduced to regrouping, they made incorrect use of the branching strategy (**Figure 21**). In figure 21, the student started by subtracting the digits in the ones place, followed by the digits in the tens place, and then the digits in the hundreds place but without realizing that their answer was incorrect.

Figure 21

Incorrect Use of the Branching Strategy for Subtraction



Hence, the use of the branching strategy may not always be ideal. As such, the teacher shared that it would be better for students to rely on the number line instead of branching. Most students were familiar with the number line as the teacher repeatedly used it in her lessons and math talk, and encouraged students to make use of it when subtracting. When working with Ahmed, I provided him with the following subtraction problem 93 - 27, and I suggested he uses the number line that I drew on the whiteboard. Ahmed took a small pause then he shared his answer.

Ahmed:	74
Fatima:	How did you get?
Ahmed:	Easy. 7-3 is 4. And 90 – 20 is 70. So, it's 74.

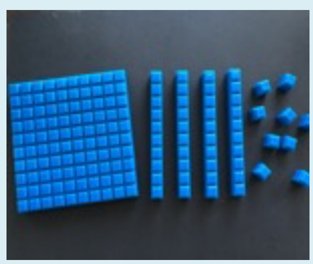
It appeared that Ahmed used the branching strategy to get his answer without using the number line. I then proposed that he checks his answer using the number line. He seemed a little reluctant, so I initiated the counting backwards and Ahmed continued with the counting, giving him 66. I provided him with another problem and encouraged him to use the number line, but Ahmed was keener on using the branching strategy, which again led him to get the incorrect answer. I then drew a number line and asked him to use it, but he became really infuriated with me and said, "stop talking about this stupid number line". He then refused to do any more problems. However, on the following day, Ahmed shared his answer to a subtraction problem that the teacher had on the board (i.e., 155 - 65). When asked how he got the answer, he counted backwards by tens and ones, he said, "So I took 60. 145, 135, 125, 115, 105, 95 [Subtracted 60 by counting backwards by tens]. Then I took 5. 94, 93, 92, 91, 90 [Subtracted 5 by counting backwards by ones]". Here, Ahmed did not rely on branching but on counting backwards—a strategy reflective of the number line. Also, by asking Ahmed how he got the answer, the teacher provided him with space to communicate his thinking and the strategy he used. In this way, students' ideas are valued, and it allowed students to see the multiple strategies they could use to reason their way to a solution. Yet, the interaction I had with Ahmed made me feel as though I was enforcing the use of a specific strategy when clearly Ahmed preferred to use another. I wonder if it would have been better to introduce him to regrouping using the branching strategy instead of encouraging the use of the number line to count backwards, especially when he initially did not favor its use.

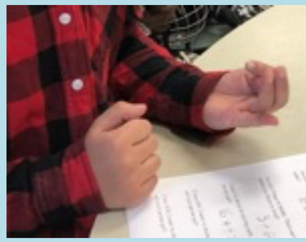
The Use of Concrete Learning Tools

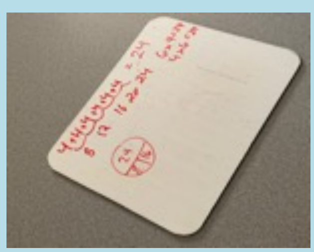
Concrete learning tools were used in a variety of ways. The teacher used them in modelling mathematical ideas and concepts, they were available to students to use in problem solving situations, and the students used them in a variety of ways. **Table 4** describes the kind of tools used by the teacher and students during the mathematics block.

Table 4

The Types of Tools Used by the Teacher and Students

Name of concrete learning tool	Image of tool	What might it be used for?
Hundreds charts		Demonstrating strategies for various operations such as addition, subtraction, multiplication, and division.
Base ten blocks		Used for counting, adding, subtracting, and understanding place value.(i.e., hundreds [flat], tens [rods], and ones [units]).
Counters		Demonstrating counting and patterning. Also, used for creating representations of mathematical computations including, addition, subtraction, multiplication and division.

Tiles		Modelling ways to find the perimeter and area of a shape. Also, used as array or area models for multiplication and division.
Fingers		Used as visual aids and representations for understanding number, counting, skip counting, counting forwards and backwards.
2D and 3D shapers		Modelling symmetry and transformations.
Pentomino or Tetris shapes		Modelling symmetry and transformations.
Bodies		Used for creating representations of symmetry and transformations.

Coins		Concrete play-based coins, images, or drawings of coins were used to represent, add, and subtract money amounts. Also, used for understanding probability in terms of the likelihood that students will get heads or tails using a quarter.
Rulers		Demonstrating how to measure various objects in the classroom. Also used for measuring the perimeter of a shape.
Snacks		Snacks (e.g., candies & popcorn) used to create representations of groups of 10 and a 100.
Classroom carpet		The classroom *carpet* was used as a representation of perimeter and area (e.g., the teacher would ask students to sit at the perimeter or area of the carpet).
Individual whiteboards		Creating representations of their thinking and solutions.

Drawings		Drawings as representations of mathematical computations or representing fractions.

The data show that the teacher encouraged students to use these various tools by engaging them in tasks that invited them to explore a mathematical idea or a concept in a hands-on and interactive way. For instance, when the teacher introduced students to symmetry, she created many hands-on activities that students engaged with to understand the concept. For one activity, she provided each student with an uppercase letter printed on a transparency sheet. The teacher modelled how students could look for a line of symmetry by folding the letter vertically, horizontally, and diagonally. However, such terms appeared quite foreign to some students, so the teacher compared the fold of a vertical line as a representation of a hotdog bun, and the fold of a horizontal line as a representation of a hamburger bun. Interestingly, students who were not familiar with hotdogs could not make the connection, leading the teacher to draw representations of what these terms implied. The students then explored how many lines of symmetry they could find for their letters and organized them into three groups: line of symmetry, no line of symmetry, and maybe a line of symmetry. The students appeared extremely engaged and guided one another.

Also, the use of concrete tools were relevant for all students, and more specifically, to those who were relatively new and with limited use of English. For instance, when the teacher introduced students to finding the perimeter and area of shapes, she used drawings of 2D squares and the tiles as counters to find the perimeter and area. Suhail was observed attentively listening and looking at what the teacher was doing. Once the teacher provided students with space to find the area and perimeter of a square, Suhail imitated what the teacher did. In so doing, Suhail was capable of finding the perimeter and area of a shape, and also, participating in the activity like the rest of the students in her class. Interestingly, students not only imitated the teacher's use of the tools but invented their own strategies using the tools to represent their thinking and solutions. At one point, I was working with Khadija on problem-solving activities involving division. Khadija made use of the hundreds chart to reason her way to a solution. At first, she appeared to have relied on a strategy modelled by the teacher which consisted of separating a number into equal parts or groups (e.g., *There are 15 skittles and 5 students want to share them*. Students using the hundreds would group the skittles by fives, giving them 3 groups of 5). However, as Khadija became familiar with the process of division, she invented her own strategies using the hundreds chart. For one task, it read,

> I have $40. I want to share it equally with 10 people. How much money will each person get?

Khadija first asked me to read the question, then she grabbed the hundreds chart and started counting by twos by placing two fingers vertically on the chart covering the 1 and 11, then the 2 and 12, and all the way to 10 and 20, leading her to realize that she did not share the $40 equally. Then, she counted by threes by placing three fingers vertically on the chart, which also led her to realize that she still did not share the $40 equally. She then used four of her fingers by placing them vertically on 4 numbers (i.e., 1, 11, 21 and 31) all the way to 10, 20, 30, and 40. She then confirmed that each person would get $4 based on her use of four fingers to get to 40. There may be other strategies that Khadija could have used, or by relying on the teacher's strategy, but she invented her own strategy based on the task she was working on. Her strategy may not work for all tasks but it was one that she used to reason her way to a solution for a specific task. That being said, how the students decide on using the concrete learning tools could communicate their mathematical thinking and understandings.

The Act of Making Mathematical Connections

The data also show that in her lessons and during math talk, the teacher made connections between mathematical ideas to broaden students' mathematical understanding of concepts and procedures. The teacher made connections between 1) number and quantity; 2) counting forwards and adding; 3) counting backwards and subtracting; 4) skip counting, repeated addition, and multiplying; 5) skip counting, equal groups, and division; 6) quantities in terms of "more than", "less than", and "the same amount as"; 7) probability in terms of the likelihood that an event will occur (e.g., unlikely, equally likely, more likely, & less likely); 8) number or quantity and place value; 9) a number name, a number symbol, and a representation of quantity; 10) one-to-one correspondence between a number and a concrete tool; 11) an operation and a solution; 12) rearranging grouping of ones into tens, or groupings of tens into hundreds; and between 13) mathematics and world knowledge. These connections were imbedded throughout the teacher's lessons and math talk, and during small group or one-on-one interactions with students. For instance, when the teacher introduced students to multiplication, she demonstrated how it could be thought of in terms of repeated addition (i.e., 5 + 5 + 5 + 5 + 5 = 25), skip counting (i.e., 5, 10 15, 20, 25), counting forwards or backwards (i.e., they know that 5 times 5 is 25, plus or minus 5 would be the product for the number before or after), and equal groups (i.e., 5 groups of 5). She also made the connection between finding the area of a shape as a representation of an array model for multiplication. In this way, students were capable of relying on the use of various strategies to reason their way to a solution when working with multiplication.

Connections were also relevant in terms of how students transferred the use of a particular strategy across mathematical ideas. For instance, at one point, a student was able to make the connection between doubling 5, 5 plus 5, and 2 times 5. The student also created representations of these which helped them see 2 times 5 in terms of 2 groups of 5. Moreover, as students endeavoured in the process of problem solving, they made connections between mathematical strategies or concepts. These connections were influenced by their teacher, as they seemed to have relied on their prior knowledge to see or make connection, influencing their mathematical meaning making. In other words, the knowledge that they have of the relationship between strategies or concepts influenced students' next steps.

Differentiating Instruction and Work

My data shows that the teacher appeared aware of the student's diverse levels of expertise and learning experiences, and would make accommodations and modifications that supported students' learning. For instance, as part of the mathematics center rotations, the classroom teacher organized for a group of students to work with her. The teacher would modify her instructions depending on the knowledge and skills the group was to build or acquire. She would even modify instructions within groups on a one-on-one basis to maximize students' participation and attainment in mathematics. At one point, the teacher was working with students on solving multi-digit addition problems using regrouping. Some students did not appear ready to work with regrouping. As such, the teacher provided a task suitable for students to work with regrouping (e.g., 399 + 289), and a parallel task for students to work with multi-digit addition problems using smaller numbers (e.g., 234 + 212). The differentiated work was intended to target students' specific needs by providing them with space and time to understand the mathematics. At most times, differentiated work was unnoticeable among students, but at certain times, some students were aware that their tasks differed from other students' work. Students would pinpoint or tease others for having to work on something that appeared easier than what they were working on. As one student said to another, "you have an easier one. Ours [referring to himself and his friends] is more harder". This actually led the student to move far away from the group and to work independently, and at another time, it led the student to ask their teacher if they could do a task similar to other students' work, challenging themselves in order to fit in.

Moreover, students who were relatively new and required language and mathematics support were usually grouped together and received synchronous support from their classroom teacher and the ESL/ ELD teacher. The ESL/ ELD teacher visited the classroom during the mathematics block and interacted with the students on various problem-solving activities, math games, or math challenges—some modified to meet students' strengths. At times, and although very rarely, the ESL/ELD teacher invited multilingual learners to her classroom to focus on developing their understanding of a mathematical concept or skill. She also brought in some mathematics activities that were included as part of the math centers, such as the activity related to the use of coins to create an image (i.e., toonies, loonies, quarters, dimes, nickels and pennies), then to use a tally chart to represent the number of coins used of each kind and what each equaled to, and then to calculate the amount of money used in total. Interestingly, how the ESL/ ELD teacher introduced the activity to various groups differed depending on their knowledge and experience working with coins and adding money. Students who had experience working with coins did not require much guidance. The teacher simply explained what the students needed to do and they engaged with the activity on their own. The students approached the teacher only to share their solutions or to confirm instructions. However, when the teacher worked with ESL/ELD students—Suhail, Sabina, Tamam, and Yaser, she relied on visuals and by modelling what they needed to do. For instance, when she explained to the students that they needed to create an image using the coins, she shared several examples to help students make sense of the kinds of images or pictures they could create. As for completing the tally chart, the teacher modelled how they were to use tick marks as representations of the number of coins they used. Some students were capable of proceeding on their own, while others required the teacher's guidance in counting and keeping track of the tick marks. For adding money amounts for each coin, the teacher encouraged students to either count along or count with some guidance by ones, twos, fives, or tens, depending on the coins they used and their knowledge of skip counting. The teacher also wrote the amount for them or encouraged them to write it on their own. Then for adding the total amount of money they used, the teacher either did the computations for them or guided them through it. Tamam, who used 40 pennies and 10 nickels, she counted the amount she had of each coin by counting by ones for the pennies and by fives for the nickels. The teacher then wrote down the amount of money Tamam had of each coin and asked her to add them together. Tamam used the hundreds chart to add them, giving her a total of $0.90. Suhail, Sabina, and Yaser required a lot more support, and the teacher modelled how to they could do the computations. In general, the manner in which either teacher (i.e., classroom teacher and ESL/ELD teacher) made accommodations and modifications to support students' learning needs and strengths influenced students' mathematical learning experiences.

Institutional Influences

By institutions, I imply the society or educational settings where people live, work, and interact with others. In this study, the school and the grade 2/3 classroom are regarded as the institutions, and where, institutional influences impact mathematics teaching and learning. I talked about institutional influences in other sections when I talked about things like students' identities and pedagogical moves, but here, I am going to focus very specifically on the influence of the institution on the dominance of English in the mathematics block. A review of the research literature suggests that the context or the institution in which the teaching and learning of mathematics takes place influences language use as varying language infrastructures around and, in the school, create different demands on students and their teachers (Setati & Adler, 2000). My analysis revealed three layers of language infrastructures that influenced the dominance of English in the grade 2/3 classroom: 1) the language infrastructure surrounding the school, 2) the language infrastructure in the school, and 3) the classroom within which mathematics education unfolds.

The Language Infrastructure Surrounding the School

The school was in Canada, a plurilingual country with two official languages- English and French- and the use of over 200 other languages within the population—including aboriginal languages and languages of immigrant and refugee populations. By provincial law, most students in Ontario attend school in either English-language or French-language schools (with some instruction to learn the second official language). Generally, students' home languages, if other than French or English, have no official status provincially or federally.

The school in my study was located in an urban environment of a highly multicultural population with a large number of new immigrants and refugees. It was surrounded by many businesses that were culturally oriented, with people speaking English and other languages. Nonetheless, there was considerable English language use in these businesses and in the neighborhood, given that English was used in government and other related services including, public transit, hospitals, healthcare, welfare programs, police protection, banking, grocery stores, shopping centers etc. Thus, use of the English language had high status in the context.

The Language Infrastructure in the School

Over 70% of the student population spoke a language other than English, and required English as a second language (ESL) or Early literacy development (ELD) support. As a result, the school had an ESL and ELD program for students whose first language is other than English. Students who required ESL support had little to no previous knowledge of the English language or limited educational experiences. ELD students, on the other hand, had limited to no access to school and had significant gaps in their education.

In addition to the ESL/ ELD program, the school had Multicultural Liaison Officers (MLOs). The general goal of the MLOs was to help ensure the successful integration of newcomer children and their families to schools. They also act to bridge language barriers between families and the school as they usually speak the family's home language. The MLOs at the school of my study spoke Arabic, Nepali, and Somali—as these languages represented the languages of the student population at the school. Their role was vital for my research as well, as they communicated with students or their parents to help clarify any concerns or to act as an interpreter, particularly when I introduced the study and sought parental consent. At one point, one of the MLOs acted as an interpreter for the interview I had with one of the students, Suhail, as she was new and in the process of learning English.

Moreover, the school operated in a single language (i.e., English), and the principal, the administrative staff, and most teachers regularly spoke English. The vice-principal also spoke

Arabic and interacted with many students and their parents in Arabic. English was also the language of teaching and learning and was used across all subjects. French or core French was the one subject that students engaged in using a language other than English. There was also a bulletin board inside the school building with the word “welcome” written in multiple languages. When celebrations or activities happened at the school, such as the “multicultural potluck” or the “math night”, posters or signs in English, Arabic, Nepali, and Somali were posted on the school’s front door. Also, most letters or forms from the school or the school board were made available in all four languages. In general, the use of students’ home languages was supported and encouraged in the school.

The Grade 2/3 Classroom

In terms of the grade 2/3 classroom in this study, everything in the classroom was in English including, the language of instruction, the mathematics activities, tasks, and games, the posters, charts, signs, images, books, stories, iPads, and computers. Also, students who required ESL/ ELD support worked in their regular classroom for most of the day and received support from their teacher and the ESL/ ELD teacher simultaneously. The ESL/ELD teacher visited the classroom during the mathematics block and interacted with students on various mathematics activities in English. The classroom teacher and other teachers that visited the classroom communicated in English, and very rarely did a teacher share a student’s home language. Even if a teacher shared the student’s home language, students preferred to use English as opposed to their home languages. For instance, the students’ French teacher spoke Arabic but rarely, if ever, were the students who spoke Arabic communicating with him in Arabic. Also, when I first arrived in the classroom, I tried to communicate with Arabic speakers in Arabic when working on mathematics, but they would almost always respond in English. The students were happy to know we shared a similar language as they eagerly shared that information with their teacher and classmates, but they would not speak Arabic when we were working together.

Not all students in the classroom shared a similar home language, and if they did, they appeared reluctant or refused to use it in the mathematics block. Although their teacher supported its use in the classroom, most students preferred not to use their home language. For instance, when Suhail first arrived in the classroom, the teacher asked Joud, who spoke Somali, to help Suhail get familiar with the classroom routines, and to introduce her to the mathematics activity they were working on. Joud began by speaking in Somali but immediately switched to English, and made very little use of her home language after that. Joud might have been more comfortable speaking in English given that she was born in Canada and had been at the school since she was in kindergarten. Suhail, who was new and had no knowledge of English spoke mainly in Somali, and Joud would respond in English. Joud did not appear keen on using her home language, yet Suhail was highly dependent on it as her only means of communication. Suhail even used Somali to communicate with her teachers and other students in her class. The teacher shared that this was very common among newly arrived students, especially during the first few days. Suhail, who realized that neither her teachers nor most of the students in her class shared her language, no longer made use of Somali when interacting with others, leading her to initially withdraw from engaging in any student-student or whole-class discussions, but then she became aware of other means to communicate mathematically.

Students were also well aware of other students' home languages and their mathematical knowledge, and they sometimes used that as an excuse to not work with them. For instance, when Mustapha, Yaser, and Suhail were grouped together to work on a mathematics game, Mustapha did not want to play with Suhail and suggested that she works with Yaser given that they both spoke Somali. He also suggested that we (Mustapha and I) work together. It may be because we both spoke Arabic, even though we rarely made use of it when working together. In another example, Yaser expressly shared that he did not want to work with Suhail because she did not know how to speak English and did not know how to do mathematics, he said "I don't want to play with Suhail because she doesn't know how to play and doesn't speak English". Other students were open and deliberate in asking others to not use their home language in class. For instance, one student said, "here you speak English because that's what we all speak". Another student shared, "Don't speak in your language because I don't understand what you're saying". Students preferred that they all use English and not their home languages, as it was a common language between them and they could understand. In this context, students' home language appeared to be used against them in such a way that they (i.e., the students) were either rejected by others, or they felt forced to stop making use of their home language as imposed by the power and dominance of the English language in the classroom. The dominance of the English language could portray the voice of the institution—that is, the language legitimized by the school and the status accorded to that language in this context, leading students to want to learn and speak English in order to fit in, to be like the rest of the students in the class, and to interact and communicate with them using the language of the majority. Unlike the study by Barwell (2014c) with Cree students in grades 5-6 in Quebec, Canada, where the students preferred to use their home language even though the school legitimized the use of a particular language, in my study, the students felt an urge to learn and speak in English to fit in.

Sociopolitical Influences

Sociopolitical influences relate to a combination of social and political influences. By social, I imply the society or educational setting where people live, work, or interact with others in an organized way. Political relates to government or public affairs of a country or province or the policies that govern it. In either case, relations exist within and outside educational settings that influence the actions, behaviors or beliefs of those within that setting, including school boards, schools, teachers, and students. In this context, I have considered provincial policy which relates to educational policies that have an influence on teacher practice and, in turn, influence students' mathematics learning experiences. There were two important provincial policies that I would like to focus on as they have a strong impact on teaching in Ontario. One is the Ontario mathematics curriculum and a supporting document, and the other is the Education Quality and Accountability Office (EQAO) provincial assessment of mathematics. These two provincial policies seemed most significant and relevant to mathematics teaching and learning in the grade 2/3 classroom. I will first outline each policy and then after each policy, discuss the influence of each on mathematics teaching and learning in this context, through the analysis of my data.

The Ontario Mathematics Curriculum and Supporting Document

In this section, I provide an overview of the *Ontario Curriculum: Mathematics* (2005) as it is the document that teachers use to guide their planning and instruction in mathematics. I also describe a document that is referenced in the mathematics curriculum, *Supporting English Language Learners: A practical guide for Ontario educators grades 1 to 8* (2008), as it is very relevant to the context of this study. I then demonstrate how these two documents were reflected in and thus influenced the teacher's pedagogical moves, and in turn, student learning.

The structure of the *Ontario Curriculum: Mathematics* (2005) includes frontmatter that acts as the foundation of the curriculum and is divided into five sections: 1) *Introduction*; 2) *The*

Program in Mathematics; 3) *The Mathematical Processes*; 4) *Assessment and Evaluation of Student Achievement*; and 5) *Some Considerations for Program Planning in Mathematics*. The introduction provides an overview of the importance of mathematics, the principles underlying the Ontario mathematics curriculum, and the roles and responsibilities of students, parents, teachers, and principals. The program in mathematics section provides an overview of the organizational structure of "the knowledge and skills students are expected to acquire, demonstrate, and apply in their classwork" (p. 7). In particular, it explains that overall and specific curriculum expectations are at the core of the document and are organized by grade. Each grade is divided into the same five strands: number sense and numeration, measurement, geometry and spatial sense, patterning and algebra, and data management and probability. In each strand, there are overall and specific expectations. Overall expectations are the knowledge and skills that students are expected to demonstrate by the end of each grade (p.7). Specific expectations describe particular aspects of the overall expectation associated with it, reflecting in greater detail the "big ideas" of mathematics in the five strands (p. 7). For example, an overall expectation for number sense and numeration states, that "by the end of Grade 3, students will solve problems involving the addition and subtraction of single- and multi-digit whole numbers, using a variety of strategies, and demonstrate an understanding of multiplication and division" (p. 55). There were several specific expectations associated with this, the following is one of them: "By the end of grade 3, students will add and subtract money amounts, using a variety of tools (e.g., currency manipulatives, drawings), to make simulated purchases and change for amounts up to $10" (p. 56).

In addition to the overall and specific expectations, the Ontario mathematics curriculum (2005) outlines seven mathematical process expectations through which students acquire and apply mathematical knowledge and skills including, problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. **Table 5** describes the mathematical process expectations outlined for grades 1, 2 and 3.

Table 5

Mathematical Process Expectations in the Ontario Mathematics Curriculum Grades 1-3

Mathematical Processes	Expectations In grades 1, 2, and 3, students will:
Problem Solving	Apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving	Apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others);
Reflecting	Demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by explaining to others why they think their solution is correct);
Selecting tools and computational strategies	Select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
Connecting	Make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts;

Representing	Create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems;
Communicating	Communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations. (OME, 2005, p. 32, 42, & 54)

The frontmatter also includes key messages related to assessment and evaluation of student achievement of the curriculum expectations and includes an Achievement Chart that outlines the criteria for different levels of achievement. The section on considerations for program planning in mathematics provides guidance about Ministry of Education priorities and policies. It also points to various resources that are either produced or supported by the Ministry of Education, and that teachers are encouraged to use when planning instruction. One particular discussion noted in this section is specific to multilingual learners who require English as a second language (ESL) or Early literacy development (ELD) support. The discussion outlines several instructional and assessment strategies for addressing the needs of multilingual learners. These include,

- Modification of some or all of the curriculum expectations, based on the student's level of English proficiency;
- Use of a variety of instructional strategies (e.g., extensive use of visual cues, manipulatives, pictures, diagrams, graphic organizers; attention to the clarity of instructions; modelling of preferred ways of working in mathematics; previewing of textbooks; pre-teaching of key specialized vocabulary; encouragement of peer tutoring and class discussion; strategic use of students' first languages);
- Use of a variety of learning resources (e.g., visual material, simplified text, bilingual dictionaries, culturally diverse materials);
- Use of assessment accommodations (e.g., granting of extra time; use of alternative forms of assessment, such as oral interviews, learning logs, or portfolios; simplification of

language used in problems and instructions). (Ontario Mathematics Curriculum, 2005, p. 28)

Teachers are also encouraged to refer to The Ontario Curriculum, Grades 1–8: English As a Second Language and English Literacy Development – A Resource Guide (2001). This resource guide has been replaced by Supporting English Language Learners: A practical guide for Ontario educators grades 1 to 8 (2008). Both guides offer detailed information about understanding multilingual learners, about accommodating and modifying expectations of the Ontario curriculum for multilingual learners, and provide assessment and evaluation strategies that focus on meeting the needs of multilingual learners and improving their learning experiences. In general, the accommodations and modifications are intended to support students' learning by providing them with space and time to experience success in learning all program areas, based on each student's stage of second-language acquisition and literacy development. The guides document descriptions of four stages for ESL students and four stages for ELD students as they progress towards proficiency in English. They also provide detailed descriptions of the language skills and knowledge that multilingual learners are expected to acquire at each stage, based on listening, oral expression and language knowledge, writing, and orientation (see Appendix L, Appendix M). It is also noted that the rate of students' progression from one stage to the another could vary among students, and students could demonstrate skills associated with more than one stage (2001, p. 11). Hence, these stages are not fixed and teachers should not expect a steady progression between the stages, as students could associate with more than one stage at a given time.

The guides (2001, 2008) also provide sample adaptations of teaching units for integrating language and content instruction to meet the expectations of the Ontario curriculum. The sample provided for mathematics in the resource guide (2001) addresses adaptation of expectations for a measurement strand for Grade 4 (see Appendix N). The sample also contains modified language expectations that support students in developing reading, writing, and oral or visual communication skills in mathematics. For example, an expectation for grade 4 measurement states that "students will select the most appropriate standard unit (millimetre, centimetre, decimeter, metre, or kilometre) to measure linear dimensions and the perimeter of regular polygon" (p. 79). The modified expectation for students in stage 1 states, that "students will demonstrate an understanding of key vocabulary words (millimetre, centimetre, decimetre, metre, perimeter, polygon) and their application" (p. 79). As for the modified expectation for students in stage 2, it states that "students will usually select the most appropriate standard unit (millimetre, centimetre, decimetre, metre, or kilometre) to measure linear dimensions and the perimeter of regular polygons" (p. 79). In terms of the expectation for reading and measurement, it states that students will "understand the vocabulary and language structures". For the modified expectation, it states that students in stage 1 will "understand some of the vocabulary appropriate to the measurement unit" (p. 79), and in stage 2 students will "understand most vocabulary and some of the language structures appropriate to the measurement unit" (p. 79). Notice how the wording of the expectations changes from selecting the most appropriate standard unit and understanding the vocabulary, to demonstrating some understanding in stage 1, and to usually selects and understands most vocabulary in stage 2. These modifications are intended to support students' integration and attainment in mathematics based on their stage of language development.

The Ontario Curriculum and the Grade 2/3 Classroom

In the section on pedagogical influences, we see ideas from the Ontario mathematics curriculum (2005) as well as the resource or practical guide for English language learners (2001; 2008) reflected in the teacher's pedagogical moves. My observations indicate that the teachers' instructional strategies and mathematics content presented aligned with the curriculum expectations. Furthermore, she encouraged the students to develop and apply the mathematical processes.

For instance, several activities aligned with the above examples of the overall and specific expectations as the data shows that the grade 2/3 students engaged in the subtraction of multi-digit whole numbers with money amounts. In one activity, students worked on the following task:

> You have $49. You would like to have $65. How much more money do you need?

Working together, the teacher encouraged students to use mathematical processes such as *reasoning* to find a solution, *communicating* and r*eflecting* on their mathematical understandings and ideas, and creating *representations* of their thinking using a variety of *tools and strategies*.

For example, when the students approached their teacher to share their solutions, they were encouraged to show her how they got the answer, thus communicating and reflecting. They also reasoned and created representations of their thinking using various tools and strategies. One student, Khadija, showed her teacher how she used her fingers to count up by ones from $49 to $65, giving her $16. Another student, Sahar, shared how she used the hundreds chart to count backwards by ones from $65 to $49, giving her $16. Raj, who also used the hundreds chart, shared that he started at $49, added $10, then added $5, plus $1, giving him $65. He then added $10 plus $5 plus $1, equaling to $16—the amount he needed to get $65. Hayat, on the other hand, shared that she added $20 to $49 giving her $69. Then, she counted backwards to $65 giving her $4, and concluded that $20 minus $4 equaled the amount of money she needed to get $65 (i.e., $16). Some students also relied on their previous experiences to make or see *connections* between mathematical ideas or concepts. For example, one student shared that counting forwards from 49 to 65 to find the difference is like adding 16 to 49. Noteworthy is that throughout the mathematics block, whether during whole class lessons, math talk, or small group guided instruction, the teacher actively engaged students in applying the mathematical processes by probing their thinking and encouraging them to share their understandings and ideas.

The use of the mathematical processes was also evident in a subsequent section of the analysis on students' meaning making in mathematical problem-solving activities. The data brought forth several mathematical actions including, reasoning and proving, communicating, representing, and making connections. I noticed that many of those mathematical actions that I used in organizing the data to understand how multilingual learners negotiate mathematical meaning during mathematical problem-solving activities align with the mathematical processes outlined in the curriculum. In my analysis, the mathematical actions came primarily from engaging in mathematical problem-solving activities where students reasoned their way to a solution by communicating their understanding and ideas by creating representations that capture their understandings, and by making or seeing connections between mathematical ideas—contributing to their mathematical understanding of concepts and procedures. In this way, the teacher's pedagogical moves reflected the curriculum expectations, and supported students to apply and engage in the mathematical processes in meaningful ways.

My data also suggests that the information about addressing the needs of multilingual learners from the mathematics curriculum (2005) and the resource/ practical guides (2001, 2008) were reflected in the way the teacher accommodated to the needs of students to support their integration and success in mathematics. The teacher incorporated multimodal teaching and learning strategies that were similar to those suggested in the mathematics curriculum and the resource/ practical guides, including, but not limited to, 1) regarding students' home languages as a resource for learning mathematics; 2) allocating more time for students to complete tasks; 3) using simplified text and instructions that supported students' needs; 4) using everyday (informal) language to teach the formal language of mathematics; 5) using various visual learning tools (e.g., concrete tools, pictures, diagrams, gestures, etc.) so that students can easily adapt and understand ideas and concepts; and 6) using culturally relevant materials that are accessible and attainable to all students. For instance, during small group guided instruction, the teacher worked with a group of multilingual learners on measuring using a ruler. In her lesson, the teacher simplified her instructions and relied on the use of visual learning tools to demonstrate how the students could use the ruler to measure things. The teacher first described the features of a ruler and encouraged students to practice saying the words "centimetres" and "millimetres". She then showed them how they could read and measure using the ruler. They also talked about different types of rulers such as the meter stick. The teacher also modelled a few examples and provided students with space and time to measure various objects in the classroom. She also encouraged them to share what and how they were measuring—the students appeared very confident and proud of what they could do. The teacher's use of various instructional strategies and visual learning resources supported and influenced students' mathematics learning experiences, and cultivated a learning environment that made mathematics learning accessible to all students despite of their limited use of English.

Education Quality and Accountability Office (EQAO)

The EQAO assessment is a provincial policy and testing program that has an impact on Ontario teachers, particularly those teachers teaching grades 3 and 6 in elementary school as these are years when students participate in the assessment. In what follows, I provide a brief description of the EQAO assessments, followed by a discussion of how I saw the assessment reflected in some of the teacher's pedagogical moves.

The *Education Quality and Education Office* (EQAO) is an agency of the Government of Ontario that creates and administers large-scale assessments for assessing students' literacy (reading and writing) and mathematics skills. In grades 3 and 6, students are assessed in reading, writing, and mathematics. In grade 9, students are assessed in mathematics. In grade 10, students take a literacy test referred to as, the *Ontario Secondary School Literacy Test*, which is a high school graduation requirement. It is noted that EQAO assessments align with the Ontario curriculum and are developed by educators in Ontario (Education Quality and Education Office, 2020). According to EQAO's news releases (2018), "schools' large-scale assessment results can be used in conjunction with other school and classroom information to identify strengths, highlight areas of growth and support student achievement".

It is expected that all students attending publicly funded elementary and secondary schools in Ontario take part in the assessments, including multilingual learners and students with special education needs (Administration and Accommodation Guide: Primary and Junior Assessments, 2019/2020). In certain situations, multilingual learners and students with special education needs could be exempted, or they could receive accommodations (Administration and Accommodation Guide: Primary and Junior Assessments, 2020). Section 2 of *Supporting English Language Learners: A practical guide for Ontario educators grades 1 to 8* (2008) states, "English language learners should participate in the Grade 3 and Grade 6 provincial assessments in reading, writing, and mathematics…when they have acquired the level of proficiency in English required for success" (p. 41). In other words, multilingual learners could be exempt from participating in EQAO if they have not acquired the level needed to successfully participate in the assessment. For instance, it is noted that if mathematics terms have to be defined for multilingual learners, then a student must be exempted from participating in the mathematics assessment (Administration and Accommodation Guide, 2019/2020, p. 27).

Concerning accommodations for multilingual learners, the *Administration and Accommodation Guide* (2019/2020) did not necessarily include any accommodations specific to multilingual learners. It does state, however, that "English language learners in the **early stages** or **steps** of English-language acquisition are eligible for permitted accommodations" (p. 26). The permitted accommodations are similar to or the same as those listed for students with special education needs and "it is assumed that these students require accommodations for classroom assessments throughout the school year" (p. 26). Some of the permitted accommodations include, the use of sign language or an oral interpreter, unified English Braille (UEB) for both language and mathematics, large-print booklets, coloured-paper versions (regular or large print), MP3 audio (includes descriptive text) plus tactiles and regular or larger print booklets, use of a computer, word processor, or assistive devices and technology, and audio recording of student responses (Administration and Accommodation Guide: Primary and Junior Assessments, 2020, p. 21-22). Hence, multilingual learners that are not eligible for accommodations must adhere to the assessment procedures that are applicable to all grade 3 and 6 students. The assessment procedures that are specific to mathematics include,

- Allow[ing] primary students to choose to use a calculator, mathematics manipulatives (including virtual manipulatives) or math applications that are non-instructional **after** questions 1–7 in Section 1 and for all of Section 2 of the math component.
- Allow[ing] junior students to choose to use a calculator, mathematics manipulatives (including virtual manipulatives) or math applications that are non-instructional throughout the math component.
- Reading the instructions and/or questions **only** to students who request it.
- Allow[ing] French Immersion students to access the French Immersion Glossary.
- **Not** allow[ing] the use of calculators or any type of mathematics manipulatives for questions 1–7 in Section 1 of the primary assessment.
- Not [to] define, translate, explain or review mathematical terms.
- Not [to] allow the use of a dictionary or a thesaurus (this includes electronic dictionaries and translators).
- Not [to] influence the students' answers or draw attention to a specific part of the assessment.

(Administration and Accommodation Guide: Primary and Junior Assessments, 2019/2020, p. 4)

These assessment procedures indicate that all students in the classroom excepts for those in the early stages or steps of English-language acquisition must take part in the assessment without much accommodations.

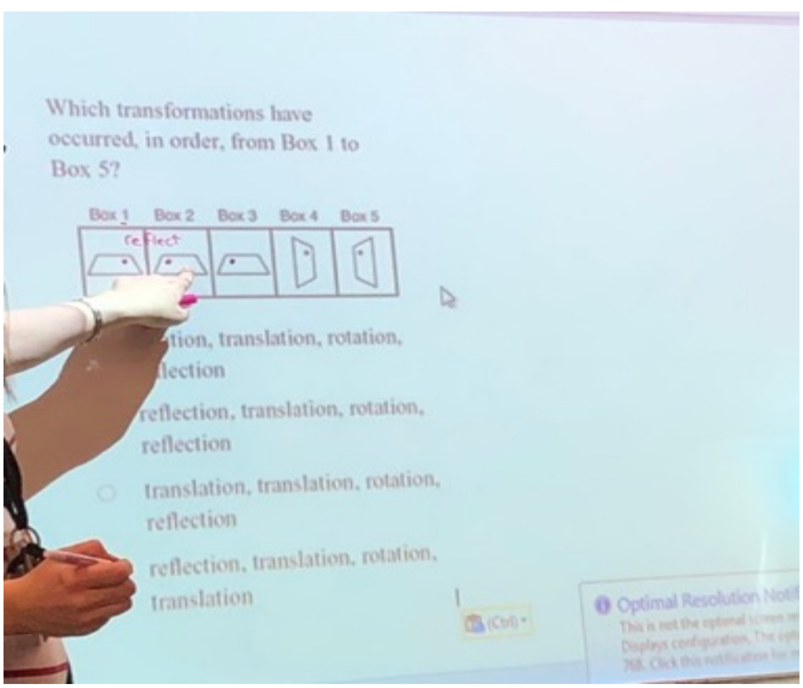
Education Quality and Accountability Office (EQAO) and the Grade 2/3 Classroom

In terms of EQAO's influence on the teacher's pedagogical moves, my analysis revealed three main influences. The EQAO appeared to have influenced 1) some of the mathematics content in the classroom, 2) some of the language (e.g., mathematics terminology) that the teacher used, and 3) time spent talking about test taking strategies. In what follows, I share a sample task that the teacher engaged students with, and then I discuss how EQAO was reflected within the classroom and its influence on students' mathematics experiences.

During the mathematics block, the teacher shared several tasks similar to those found in the EQAO assessment booklets. The teacher might have used such tasks to practice and familiarize students with what to expect when taking part in the mathematics assessment. In one task, the students needed to find the transformation that occurred, in order, from Box 1 to Box 5. The students were provided with an image of 4 transformations followed by multiple answer options (**Figure 22**). The teacher engaged students in a math talk to solve the task. The teacher first reviewed the mathematics terminology for transformations by providing students with everyday words associated with each term. The teacher explained that the words, "rotate, translate, and reflect" are mathematics words, and the words "turn, slide, and flip" are words we use in our everyday lives. She then shared a trick that students could use to remember what the mathematical terms mean when they see them in EQAO—that is, rotate has the letter "t" and so does turn, translate has the letters "sl" and so does slide, and reflect has the letters "fl" and so does the word flip.

Figure 22

Sample Task Adapted from EQAO Assessment Booklet



The teacher also shared test taking strategies including, reading the question carefully, understanding what the image is showing, and the process of elimination for multiple choice questions. The teacher suggested that students read the question carefully to understand what it is asking. She then helped students make sense of the image by explaining that they needed to look at the shape and to pay attention to the position of the dot and the lines of the shape in each box. The teacher then asked students to share what they think happened to the shape in each box, and to explain their choices or answers. When students shared the incorrect transformation, the teacher drew their suggested transformation on the board to visually show them what it looked like. Interestingly, as much as the teacher encouraged the use of the mathematical terminology, the students either used everyday terms or created visual representations of the transformation without verbally communicating the term. The teacher, however, would either ask students to restate the transformation using the mathematical term or she would just say it. Here, I think that although the curriculum supported the use of everyday terms for grade 3 students, the teacher might have wanted to familiarize students with the mathematics terminology which were reflective of those found in EQAO so that students could aptly or confidently participate in EQAO.

Once the students and their teacher labeled each box with its matching transformation, the teacher taught them the "process of elimination" to choose the correct answer. She encouraged them to look at each option and to see if the first word matched the transformation in box 2. Then, to look at the 3rd, and 4th word and to ensure that they matched the order of transformation in each box. She also suggested that they go through other options to confirm that they had the correct answer.

This task was representative of several tasks that the teacher used from the sample EQAO assessment booklets and that appeared to have influenced some of the pedagogy in the classroom. Also, the fact that EQAO is only in English (or French) seemed to have privileged the English language given that it is the language in which students take part in the assessment. Furthermore, the fact that EQAO only assesses language and mathematics promotes language and mathematics as the most important subjects to students, teachers, and parents.

Addressing the First Sub Research Question

In order to address the first sub research question concerning the influences on students' mathematical learning experiences, I specifically focused on data gathered from my observation notes. In the previous sections, I presented a detailed analysis of four influences including, individual, pedagogical, institutional, and sociopolitical influences. The first section of the results focused on individual influences on students' mathematics learning experiences. The second section focused on the teacher's pedagogical practices which in turn influenced students' learning of mathematics. The third section focused on institutional influences and their impact on mathematics teaching and learning. The fourth section focused on sociopolitical influences with a particular focus on educational policies that have an influence on teacher practice and, in turn, influence students' mathematics learning experiences. In an important way, various themes started to emerge within each of the influences. Now, based on the findings presented by the analysis, I will turn to explicitly answer the first sub research question (i.e., What are the influences on students' mathematics experiences?) by providing a general summary of the findings for each of the four influences.

Individual Influences

The analysis of the data brought forth the role of students' identities as having a strong influence on their mathematics learning experiences. Students' identities were not static but constantly changing and evolving as reflected in, for example, the language(s) they know and their social and cultural repertoires, which appeared as important resources for their learning of mathematics. The findings show that in order to fit in with the rest of the students in the class, or of fear of being left out or rejected by others in the classroom, students shared an urge to embrace the traits of their social group and to learn and communicate with others in English. Although the students privileged using English in the classroom, some students still relied on their home language to communicate mathematically, especially when clarifying understanding to one another or when counting or writing numbers. The fact that students may feel that the language in which they grew up using in their day-to-day interactions or at school is all of a sudden like a forbidden language in the classroom, makes a difference to their processing of mathematics. This is also tied to the culture to how one does mathematics, and the manner in which students approached doing mathematics might be a shifting culture. As I observed students try to change, adapt or adjust their ways and the language they use to do mathematics, their identities were constantly changing and evolving. To this end, I argue that the language(s) spoken by students are part of their identities, and it is in this community or context (i.e., their classroom and the students within the classroom) that affirms their identities. Moreover, students' conceptualizations of "what is mathematics" and "what it means to do mathematics" were influenced by their mathematics experiences in and outside the mathematics block. In particular, the mathematics activities that students engaged with in the grade 2/3 classroom, at the afterschool homework club, in their home countries, or at home with their family, not only shaped their understanding of mathematics, but also, informed students' mathematical meaning-making and learning experiences.

Pedagogical Influences

All too often, when we think of teaching and learning, we think of them as separate entities. For instance, we think teaching is somebody telling and explaining, and the student is absorbing all that telling and explaining. However, in this grade 2/3 mathematics classroom, learning and teaching were jointly connected because the teaching was really about the classroom environment that the teacher sets up for the students to learn and engage in mathematics. Hence, in this study, teaching and learning are intrinsically linked because students' mathematics experiences were influenced by the teacher's pedagogical moves which shaped students' learning and engagement in mathematics. The findings show that the classroom set-up of children sitting in groups with easy access to materials encouraged interactions and collaborations among the students, their teacher, and the learning resources which were readily available for student use. Overall, the classroom set-up inspired students to take ownership of their classroom space. The findings also show that the students engaged in a variety of activities including, entry activities used at the beginning of class to engage students in mathematics; math talk related to the entry activities and tied into minilessons to introduce a new skill or strategy or to review a previously taught skill or strategy; morning calendar routines that reflected various mathematics concepts and skills; math centers that provided students with an opportunity to practice and apply various strategies and skills; and some space for students to catch-up on mathematics activities. These activities illustrated the design of the mathematics block by demonstrating how mathematics learning unfolded in the classroom and how the students and their teacher maneuvered through these activities. The students also engaged in various tasks that were generally open-ended and allowed for different solutions or multiple ways to arrive at a solution, such as mathematics games, mathematics challenges, and math talk. The games focused on practicing certain procedures or strategies and with space for students to invent their own strategies. The mathematics challenge activities generally acted as a review or practice of what students learned including, subtraction, multiplication, and division; patterns and relations; perimeter and area; symmetry and transformations; collecting data and graphing; and addition and subtraction of money amounts. The math talk focused on word problems and 3-Act tasks where the students and their teacher worked together to find a solution. Also, the teacher engaged students in mathematical discourse practices that promoted students to share their mathematical thinking and ideas through verbal, visual, and written communications. The teacher relied on mathematical and non-mathematical discourses to orchestrate and promote meaningful mathematics learning 1) during student-student interactions; 2) during student-teacher interactions; and 3) during whole-class math talk or mini lessons. Student-student interactions took on a variety of different forms including working independently, collectively, or collaboratively. As students engaged in more open-ended activities, the more students collaborated and build off of one another's thinking to come up with a solution. Student-teacher interactions had noticeable influence on students' engagement in mathematical discussions, especially when students were encouraged to share their mathematical understandings and ideas. As for whole-class math talk or discussions, these were generally intended for elaborating on students' answers, deepening understanding, and making learning connections. The findings also show that the teacher encouraged the use of multiple strategies by modelling their use for students to reason their way to a solution, and where students imitated or invented their own strategies. The teacher also incorporated the use of various learning tools to represent mathematical ideas and to support students' mathematical thinking and understanding. The teacher encouraged students to use these tools by engaging them in activities that invited them to explore a mathematical idea in a hands-on and interactive way. The use of various learning tools were relevant to all students in general, and to those who were relatively new and have not yet developed the vocabulary to share their thinking verbally. Whether it was through imitating the teacher's use of the tools or students inventing their own strategies using various tools, the students made use of concrete learning tools to develop and represent their mathematical ideas.

The findings also show the teacher made connections between mathematical ideas to broaden students' mathematical understanding of concepts and procedures. The connections were imbedded throughout the teacher's lessons and math talk, and during small group or one-on-one interactions with students. Connections were also relevant in terms of how students transferred the use of a particular strategy across mathematical ideas. Also, as students endeavored in the process of problem solving, they made connections between mathematical strategies or concepts. These connections were influenced by their teacher, as they relied on their prior knowledge to see or make connection, influencing their mathematical meaning making. In other words, the knowledge that they have of the relationship between strategies and concepts influenced students' next steps.

Also, the teacher appeared well aware of the student's diverse levels of expertise and learning experiences, and would make accommodations and modifications that supported students' learning. The teacher modified her instructions depending on the knowledge and skills the group was to build or acquire. She would even modify instructions within groups on a one-on-one basis to maximize students' participation and attainment in mathematics. At most times, differentiated work was unnoticeable among students, but if noticed, students challenged themselves by asking their teacher to do the same tasks as other students. In general, the manner in which the classroom teacher or the ESL/ELD teacher made accommodations and modifications to support students' learning needs and strengths influenced students' mathematical learning experiences.

To this end, I argue that the teacher did so many things to give the students rich mathematics experiences. Similar to the findings of McClain and Cobb (2001), which suggested "the sequences of instructional activities and the general classroom social norms established in the classroom contributed significantly to the students' [mathematical] growth" (p. 242), in my study, the teacher's pedagogical moves provided opportunities for students to explore mathematics through the use of mathematical discourse practices which provided them with opportunities to participate in mathematics learning that fostered communication with others to share and engage in their own and other students' thinking, to explore problem-solving skills, to develop an understanding of mathematical terminology and associated concepts, and to apply and use various strategies and learning tools to find a solution. Hence, influencing their mathematics learning experiences.

Institutional Influences

The findings also show that there were institutional influences that impacted mathematics teaching and learning, and more specifically, the influence of the dominance of English in the grade 2/3 classroom. The findings revealed three layers of language infrastructures that influenced the dominance of English, 1) the language infrastructure surrounding the school, 2) the language infrastructure in the school, and 3) the classroom within which mathematics education unfolds. The language infrastructure surrounding the school portrayed the dominance of the English language. The school was in Canada and students' home languages, if other than French or English, had no official status provincially or federally. There was also considerable English language use in neighbouring businesses, as well as in government and other related services in which the school is located. Thus, use of the English language had high status in the context. In terms of the language infrastructure in the school, the school operated in a single language (i.e., English) and the principal, the administrative staff, and most teachers regularly spoke English. English was also the language of teaching and learning and was used across all subjects. French or core French is the one subject that students engaged in using a language other than English. However, the use of students' home languages was supported and encouraged in the school as reflected by the posters, signs, letters or forms that were made available in multiple languages. Concerning the grade 2/3 classroom in the study, everything in the classroom was in English. The classroom teacher and other teachers that visited the classroom communicated in English, and very rarely did a teacher share a student's home language. Even if a teacher shared the student's home language, students preferred to use English as opposed to their home language. Also, not all students in the classroom shared a similar home language, and if they did, they appeared reluctant or refused to use it during the mathematics block. Although their teacher supported the students' use of their home language in the classroom, most students preferred not to use it. Only newly arrived students made use of their home language but that quickly changed once they realized that not all students or their teacher shared a similar language. Taken together, these language infrastructures suggest that the context in which the teaching and learning of mathematics occurred influenced the dominance and continuity of the English language by the students in the classroom.

Sociopolitical Influences

Sociopolitical aspects that contributed to and influenced students' mathematics learning experiences were reflected in how provincial policies and institutional influences impacted mathematics teaching and learning. Provincial policies relate to educational policies that have an influence on teacher practice and, in turn, influence students' mathematics learning experiences. Institutional influences relate to the school and the grade 2/3 classroom in which the teaching and learning of mathematics took place and where varying language infrastructures around and in the school, create different demands on students and their teachers.

There were two important provincial policies that I focused on and were relevant to mathematics teaching and learning in the grade 2/3 classroom. One was the *Ontario Curriculum: Mathematics* (2005) and a supporting document, and the other was the Education Quality and Accountability Office (EQAO) provincial assessment of mathematics. The findings show that the ideas from the Ontario mathematics curriculum (2005) as well as the resource or practical guide for English language learners (2001; 2008) were reflected in the teacher's pedagogical moves. For example, the teachers' instructional strategies and mathematics content presented aligned with the curriculum expectations. The teacher also encouraged students to develop and apply the mathematical processes outlined in the curriculum, such as *reasoning* to find a solution, *communicating* and r*eflecting* on their mathematical understandings and ideas, and creating *representations* of their thinking using a variety of *tools and strategies*. The findings also show that the information about addressing the needs of multilingual learners from the mathematics curriculum (2005) and the resource/ practical guides (2001, 2008) were reflected in the way the teacher accommodated the needs of students to support their integration and success in mathematics. The teacher used various instructional strategies and visual learning resources that supported and influenced students' mathematics learning experiences, and cultivated a learning environment that made mathematics learning accessible to all students despite of their limited use of English. Concerning the Education Quality and Accountability Office (EQAO) provincial assessment of mathematics, it also had an impact on the teacher's pedagogical moves, particularly because she had grade 3 students in her classroom and they were to participate in the assessment. The teacher used various tasks from the sample EQAO assessment booklets which appeared to have influenced and privileged some of the pedagogy in the classroom including 1) some of the mathematics content in the classroom, 2) some of the language (e.g., mathematics terminology) that the teacher used; and 3) time spent talking on test taking strategies. Also, the fact that EQAO is only in English (or French) seemed to have privileged the English language given that it is the language in which students take part in the assessment. Furthermore, the fact that EQAO only assesses language and mathematics promotes language and mathematics as the most important subjects to students, teachers, and parents.

Concluding Remarks

In addressing the first sub research question, I argue that individual, pedagogical, institutional, and sociopolitical influences are interconnected and form a nested system of factors that influenced students' mathematics learning experiences. These four influences cannot stand alone without considering the connection between them because to consider one influence requires an understanding of its influence on the others. Consider for instance the results of the study that revealed how the teacher's pedagogical moves appeared the strongest influence on students' mathematics experiences in the grade 2/3 classroom. As a result of the teacher's moves, the students engaged in mathematical discourse practices through hands-on, interactive, and engaging mathematical activities, they used a variety of strategies to solve problems, made use of concrete learning tools, and made connections between mathematical ideas. Hence, the teacher's pedagogical moves provided the context for students' rich mathematics experiences in the grade 2/3 classroom, and it is there where they developed their identities as mathematics learners within the context of a new culture. We also notice that how the teacher dealt with the Ontario mathematics curriculum emerged as a positive influence on her teaching and on students learning because the curriculum is supportive of the use of mathematical discourse practices. As such, the influence of the curriculum is a positive as opposed to a negative influence. Also, the teacher took it upon herself to deal with EQAO by incorporating it into her normal practice as opposed to changing her practices. In particular, the teacher used sample EQAO tasks to engage students in math talks and to help students understand that there might be different kinds of mathematics terminology. In addition, the dominance and power of the English language in the curriculum, the EQAO, as well as in and around the school did not result in the teacher rejecting students' home language(s), instead, she encouraged and supported its use in the grade 2/3 classroom. Hence, the influences on students' learning of mathematics take the form or shape of a reciprocal formation, but where the teacher appeared to have the strongest influence as she helped the students react to individual, institutional, and sociopolitical influences through all that she did in the environment she created. It is in this way that I have explicitly addressed the first sub research question based on the evidence that has been presented in this section of the results concerning the *Influences on Students' Learning of Mathematics*.

CHAPTER 6: RESULTS – MEANING MAKING

Students' Meaning Making in Mathematical Problem-solving Activities

In this section of the presentation of the results, I focus on addressing the second sub research question: How do students negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting? I chose four video recordings of students working on mathematics problem-solving activities to analyze in detail. Each of these video recordings focuses on students participating in one of the following mathematical problem-solving activities: Activity #1: Add 'Em Up; Activity #2: Times Tussle; Activity #3: Books in a Classroom; and Activity #4: Ameer's Coins. I present each activity individually.

For each activity, I provide a general description of the activity, followed by a summary of what happened that day in the mathematics block preceding the activity. Next, I provide a detailed description of the entire activity in chronological order, divided into stanzas. Each stanza contains portions of the transcript, but taken together, the entire transcript of the problem solving session is provided. Each stanza begins with a brief overview of the situation, followed by the portion of the transcript, and then a short discussion to unpack the interactions in that portion of the transcript. These stanzas provide insights into students' interactions with the activity, and more specifically, with specific terms, the rules of the game, with one another, with various tools and strategies, and turn taking when playing a mathematics game. I have included a detailed description of the entire interaction for each activity because I wanted to provide the reader with a detailed account and understanding of multilingual students' meaning making and its context. If I included only specific segments of the transcript, they would have appeared out of context and would not show the development of students' mathematics thinking and meaning making. I also wanted to stay true to what the students said by using their own words as reflected by the full transcripts of their interactions and not rely solely on my interpretations.

At the end of the full set of stanzas that describe the students' interaction with the activity, I discuss my analysis of children's meaning-making within that activity by drawing on Gee's (2005) analytical framework of discourse analysis specifically the construct of building tasks of "language in use" as described in **Table 3** in my methodology chapter. These building tasks include, significance, activity (i.e., mathematical actions), identity, relationships, politics, and sign systems or knowledge, which were all part of students' mathematical meaning making.

Within each activity, I considered particular things that appeared **significant** to students' meaning making such as giving importance or attention to a particular term, idea or understanding. For **activity**, as explained in **Table 3**, I considered mathematical actions and I used the mathematical processes in NCTM to define mathematical actions which highlight ways of using and acquiring mathematical knowledge including, problem solving, reasoning and proof, communication, representation, and connections (NCTM, 2000). In terms of **identity**, I considered how students see themselves as mathematics learners and how they come to understand what it means to do mathematics (Cobb et al., 2008, 2009). In particular, are the student identifying with the mathematics activity, merely cooperating with the teacher, or are they resisting engagement in the activity (Cobb et al., 2008, 2009). I also analyzed the **relationships** that students built with one another, with the teacher, the activity, and with the tools they used as mathematics learners. As for **politics**, I looked at the distribution of mathematical authority to understand who is in charge to legitimize certain aspects of the mathematics activity and implicitly sanction others (Cobb et al., 2008, 2009; Langer-Osuna, 2017; Yackel & Cobb, 1996). Finally, I considered what **sign systems or knowledge** were privileged as the students interacted on the activities (see **Table 3** for a full description of the 6 building tasks and how they correspond to mathematics education). At the end of the four activities, I look across my analysis of the four activities and discuss this cross-analysis in order to answer the second sub research question: How do students negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

Activity #1: "Books in a Classroom"

<u>What was the activity?</u> I shared several mathematics activities with the classroom teacher, and one activity the teacher decided to share with the students was "Books in a Classroom" (**Figure 23**). As the title of the activity suggests, there were three bins of books in a classroom. The number of books in each bin were 23, 37, and 56. The student [Fatima] takes all the books out of the bins and puts them into piles of 10. The question then asks, how many piles of 10 can Fatima make with all the books?

The mathematics activity was related to what students were learning at the time, such as composing and decomposing numbers into hundreds, tens, and ones, counting forward by tens, and solving problems that involve the addition of two-and three-digit numbers, using a variety of tools and strategies. We felt this activity would challenge students' thinking and encourage mathematical discussions. Also, we were interested in seeing what prior knowledge and strategies students would rely on to solve this activity.

Figure 23

Books in a Classroom Activity Handout

Name(s)___________________________________

Math Problem Solving

There are 3 bins of books in a classroom. The number of books in each bin is shown in this picture.

Fatima takes all the books out of the bins and puts them into piles of 10.

How many piles of 10 can Fatima make with all the books?

Show and tell how you know.

<u>What happened that day?</u> The morning routine unfolded as it normally would with a mathematics entry activity, followed by students catching up on work they missed or had not yet completed, while the rest of the students and their teacher engaged in the morning calendar. Once the morning calendar was completed, the teacher invited all students to join her on the carpet. The teacher then read the activity to the students, and one student [Sahar] suggested they use the actual books in the classroom. The teacher's initial response was "yes!", but she then realized that the number of books required is quite large and it would be impractical for the

students to use them. The teacher suggested they use other tools in the classroom without identifying what they could use. The teacher then drew attention to the term “piles”—a term some students may have not encountered before. She particularly asked them to consider another word or a mathematical term that they would use, she asked: “what is another way we can we say ‘piles’, or what is the math term or word that we use?” Some students responded by sharing the term, “groups” and “groups of”—a term the students and their teacher discussed previously during their math talks in some preliminary work on multiplication.

The teacher then shared the activity handout with the students. I was speaking to her about my plan to video record 2-3 students working on the activity. Samartha and Joud overheard our conversation and volunteered to be part of the video recorded group. I also invited Khadija to join us. Raj and Mustapha voluntarily joined us a little later, followed by Abdul who had been working on his own but checking or sharing his answers with me. Mustapha, Raj and Abdul were not video recorded, but could be heard through the video recording.

<u>Stanza 1</u>

This stanza shows the start of the activity with the group. It begins with me asking the students to read the activity and Samartha volunteered to do so. Interestingly, as Samartha read the activity, she switched the name Fatima with her name, Samartha.

Samartha:	Math solving. Wait. Math problem solving, right?
Fatima:	Yes.
Samartha:	There are three bins of books in the classroom. The number of the books in each bin is sh-, sh-,
Fatima:	Shown
Samartha:	Shown in this picture. I will show this to your camera.
Khadija:	Can you see it?
Samartha:	There you go people. There you go, you can see it right? Can you see it? I'm sorry because it's upside down.
Fatima:	Okay, I think they can see it. Keep going.
Samartha:	Fatima takes. Samartha takes all the books out of the bin and puts them into piles of 10. How many piles of 10 can Samartha make with all the books? Show and tell how you know.

Fatima:	Show and tell how you know.
Samartha:	Instead of Fatima, it's Samartha.

Stanza 2

As the reader will see in the next portion of the transcript, stanza 2, after Samartha read the activity, Joud shared her thinking by reasoning her way to a solution. Then, Samartha shared her solution, but by displaying a different understanding of the activity.

Fatima:	Alright. So how many-,
Joud:	I think it's 10 groups of 10. See 2 plus 5 [Referring to the digit in the tens place of the first and third bin of books] [Pause-Distracted by other students]. 2 plus 3 [Referring to the digit in the tens place for the first and second bin]. 20 plus 30 equals 50 [She added the digit in the tens place of the first and second bin together]. 50 plus 50 equals a 100 [Adds the sum of the addend of the first and second bin to the third bin]. And that's 10 tens.
Fatima:	Show me your work.
Samartha:	So, you're supposed to add 10 in ten piles. So-,
Joud:	Where do we write it?
Fatima:	You can write it right here [Pointed to the bottom of the page]. Or you can write it at the back.
Samartha:	/Wait, wait right here, wait right here Mrs. Fatima./ [She got up and walked towards the math manipulative's storage shelves]

As we look more carefully at this segment of the transcript, we can see that Joud shared her answer by indicating that she thinks "it's 10 groups of 10". Then, she explained her thinking by focusing on the digits in the tens place for the number of books in each bin. Notice how Joud referred to the number in the tens place as 2, 5, 3 then she switched it to 20, 30 and 50, still, solidifying her understanding of place value. She indicated that she had 2 tens for the first bin, 3 tens for the second bin, and 5 tens for the third bin. As we can see, she then added the tens together, equaling a 100, and which she shared was equivalent to "10 tens". Also notice how Joud used the term "groups" and not "piles". She appeared to have privileged the teacher's vocabulary given that the teacher discussed it with them. This is evidence that Joud picked up on the term and it may have played into her meaning making. In picking up on the term, Joud made the term and the teacher significant, establishing the teacher's mathematical authority in the classroom.

Next, we see that Samartha shared her understanding by explaining that they are "supposed to add 10 in ten piles." It may not be completely clear what she intended, but I think she was suggesting that each bin had 10 books and that they needed to add them together. I also wonder if her explanation and solution were based on what Joud said concerning "10 groups of 10" or "10 tens". Samartha may not have taken into account Joud's entire explanation and only retained what made sense to her. In so doing, Samartha developed and expressed agency by intentionally building her own understanding of the activity to come to a solution. She built her own understanding based on 1) her interpretation of the problem and 2) her interpretation of Joud's explanation. Also, we notice that, unlike Joud, Samartha used the term "piles" and not "groups", privileging the term used on the activity handout. Samartha then got up and walked towards the manipulatives' storage shelves.

<u>Stanza 3</u>

In the next segment of the transcript, stanza 3, Khadija appeared disengaged and I asked her to think about the activity. Samartha returned with a handful of rods and units, and she shared the reasoning of her solution. In so doing, Samartha demonstrated autonomy by taking control and responsibility for her learning through her selection of tools to use.

Fatima:	Come on Khadija think about it?
Khadija:	What is it?
Samartha:	So, Mrs. Fatima. There's 10 groups of te-, 10 books. Each of-, each of the 3 bins there are 10, 10 and 10 which makes 30 [Referring to a rod in her hand and another two on the table].
Khadija:	But how I do it? [To me-asking how she would solve the activity]
Samartha	So, it's 10 plus 10 plus 10.
Joud:	I already said it. 3 groups and add the 2 groups that makes 50. And add 5 tens.
Samartha:	Shush shush:: So, 2. So 3 groups of 10 is 30. So-,

In this stanza we notice that by using one rod in her hand and another two on the table, Samartha explained that in each of the three bins there were 10 books, and together they equaled 30. In using the rods, she created a representation of the number of books in each bin. The fact that Samartha used the rods as a representation of the number of books she thought were in each bin is important because it reflected her understanding of the value of the rod as a representation of 10. Also, notice how this time Samartha used the term "groups" and not "piles" and seemed comfortable alternating between the two terms, reflecting her confidence and understanding of the terms.

Khadija then asked how she could solve the problem. Joud reexplained her solution by sharing that 3 groups of 10, plus 2 groups of 10 equal 50, and that by adding the additional 5 groups of 10 it would equal the answer. Samartha was not interested in hearing Joud's explanation, so she tried to quiet her, reinforcing Samartha's mathematical authority. This could also reflect the relationship between Samartha and Joud. Samartha then repeated her solution by sharing that 3 groups of 10 is 30.

Stanza 4

In the following part of the transcript Khadija developed some understanding of what might be expected of her to solve the activity. We also see Samartha reexplaining her understanding of the problem by insisting on her initial solution.

Khadija:	Mrs. Fatima, do we like we add all these numbers together? [She pointed to the number of books for each bin]
Fatima:	You have to tell me how many groups of 10 you can make with these [To Khadija].
Samartha:	So, there's 3 groups of 10.
Fatima:	How do you know there's 3 groups of 10? [To Samartha]
Samartha:	Because there's 3 books [bins] and you have to add 3. So, these are-, These are 3 [Referring to three rods in her hand]. So, 3 groups of 3 is-, [Placed one rod for each bin of books]. 3 groups of 10 is 30 [Brought rods together].
Fatima:	Okay, but there's 23 books in this one, 37 books in this one, and 56 books in this one.

Samartha:	Wait right here [Got up and walked towards the manipulatives shelving area]
Khadija:	So how, how do it?
Fatima:	[Pointing to the number of books] So how are we going to do these in groups of 10?
Abed:	It's 10 groups of 10.
Khadija:	What 10 groups of 10? [Pause] So, we add the tens together?
Fatima:	So, add the tens together and see how many tens there are?
Khadija:	Oh! 2 tens [Referring to the first bin by 2 tens]. 1, 2, 3. Excuse me. 1, 2, 3, 4, 5 [Continued by just counting the rest].

As we can see in this part of the transcript, Khadija, who appeared to be listening to Joud and Samartha's explanations and solutions developed some understanding of what might be expected of her. She pointed to the number of books for each bin and asked if she needed to add all the numbers together. My response neither confirmed nor answered her question but I simply restated the question of the problem. Also, notice how I refer to the term "groups" and not "piles", picking up on the term Joud and Samartha used, which reinforces their and the teacher's mathematical authority.

We also see Samartha still insisting on her initial explanation that there were three groups of 10. I referred to the actual number of books for each bin, and Samartha got up and walked towards the math manipulative's storage shelves.

We also notice that once again Khadija asked how she could solve the problem. I referred to the handout and repeated the question of the problem. Abed then enthusiastically shared the answer "10 groups of 10". It seems that Abed also focused on the digit in the tens place of each bin and ignored the digit in the ones place. Khadija appeared confused by Abed's claim, and with intonation she asked, "What 10 groups of 10?" She took a small pause, then she asked if she should add the tens together. It seemed as though Khadija merged all that she heard this far and developed some understanding of what might be expected of her. I acknowledged that she could add the tens together and to see how many tens there were. Khadija then grabbed some rods and created a representation of the number of books in each bin by only focusing on the digit in the tens place of all three bins. She grabbed 2 rods for the first bin, 3 rods for the second bin, and 5 rods for the third bin. I noticed that Khadija's use of the rods reflected her understanding of the value of the rod, helping her make one-to-one correspondence between the rods and the digit in the tens place for each bin.

Stanza 5

In the following part of the transcript Samartha came back with a handful of units. However, she was disappointed to see Khadija and Joud using the rods she got. Samartha tried to bring all the rods towards her, ruining Khadija's representation.

Samartha:	Hey! Go bring your own tiles people.
Khadija:	Hey:: [To Samartha who tried to take her rods]
Samartha:	These are-, I bring these.
Khadija:	I'm counting it. You're ruining it.
Samartha:	I need some.
Khadija:	And I. You ruined it.
Samartha:	[Mocking Khadija]
Khadija:	1, 2 [Counting the rods for first bin of books]. 1, 2, 3 [Counting the rods for the second bin of books]. 1, 2, 3, 4 [Counting rods for the third bin of books and realizes that she's missing a rod]. I don't want yours. I'm getting my own one's [She passed all the rods towards Samartha. Then, she got up to get her own rods].
Samartha:	[Mocking Khadija] Wait, wait, wait. [To Joud, who tried to get some rods from her].
Joud:	[Showed a thumb down to the camera].
Samartha:	[Reorganizing her rods as she referred to the handout. She checked that she had 2 rods and 3 units for the first bin. For the second bin, she checked that had 3 rods and counted 7 units] 1, 2, 3, 4, 5, 6, 7. [For the third bin, she grabbed 5 rods and counted 6 units] 1, 2, 3, 4, 5, 6.
Joud:	You can have the extra one [To Samartha. Referring to the rod]
Samartha:	No, it's okay you can have that.
Joud:	Are these extra yours? [To Samartha. Referring to the rods]
Samartha:	Yup, you can have some.

[Khadija returned with some rods]

Khadija: 1, 2, 3 [She hesitated and recounted]. 1, 2. [Counted 2 rods for the first bin]. 1, 2, 3, 4 [She incorrectly counted 4 rods for the second bin and hesitated]. 1, 2. [Recounted for first bin]. 1, 2, 3. 1. [Recounted for second bin. Then, she pulls away the extra rod, leaving 3 rods for the second bin]. 1, 2, 3, 4, 5 [Counted 5 rods for the third bin].

Samartha: Mrs. Fatima.

[Stop for announcement]

Joud: Samartha. It's all stuck together [Referred to the 3 rods she linked together]

Samartha: Shush:: [To Joud]

Khadija: 1, 2, 3, 4, 5 [Counted the rods for the third bin of books]. 1, 2, 3, 4, 5 [Recounted the rods for third bin of books]. 1, 2, 3 [Counted the rods for the second bin of books]. 1, 2 [Counted the rods for the first bin of books]. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [Counted all the rods together, then she brings them all together].

As we can see in this part of the transcript, after Samartha ruined Khadija's representation, Khadija regrouped the rods by focusing on the digit in the tens place of each bin of books. She then realized that she was missing a rod for the third bin, so she passed the rods over to Samartha and got up to get her own.

We also see Samartha creating a representation of the number of books in each bin but by focusing on both the digits in the tens and ones place. However, when Khadija passed the rods to Samartha, her representation was disarranged. Samartha cleared her working space and reorganized the rods and units for each bin of books. She got 2 rods and 3 units for the first bin, 3 rods and 7 units for the second bin, and 5 rods and 6 units for the third bin. Again, making one-to-one correspondence between the rods and the digits in the tens place, and between the units and the digits in the ones place, in order to create a representation of the number of books in each bin. Also, Samartha's use of the units reflected her understanding of the value of the units.

Joud finally managed to get some rods from Samartha. Joud started with the second bin with 37 books—she got 3 rods and linked them together. Joud tried to share with Samartha what she did, but Samartha quieted her—either because Samartha was too focused and did not want to be distracted, or she did not want to hear what Joud had to share. Again, this could also reflect the relationship between them.

Khadija then came back with a handful of rods. She placed them on the table and counted the rods for each bin. She hesitated a couple of times as she would compare them to the number on the activity handout. Again, making a representation and one-to-one correspondence between rods and the digit in the tens place of the number of books in each bin. Khadija then counted all the rods together, equaling 10.

Joud then tried to take some rods from Khadija, but Khadija did not let her take any. Khadija recounted her rods to ensure that she still had the correct number. It was interesting how Khadija checked the representation of bins from right to left, but where earlier, she created her representation from left to right. Khadija then counted the rods one-by-one without separating them into their separate bins.

Stanza 6

As seen in this portion of the transcript and the one that follows, I asked the students to show me what they did. In this stanza of the transcript, Khadija shared her solution.

Fatima:	Alright show me what you did.
Khadija:	100 tens. 100 tens.
Samartha:	Why are you screaming? [To Khadija]
Mustapha:	No, 100 zero.
Khadija:	Stop [To Mustapha].
Mustapha:	Zero, zero, zero, zero [To Khadija.]
Khadija:	Now the one's. 3. 7. 6.

As we see, in response to my question Khadija referred to the sum of the digit in the tens place of all three bins of books as "100 tens". She appeared to have developed an understanding of what the sum of the addends equaled to by relying on her counting of the rods—that is, 10 rods equal a 100 or as she referred to it as "100 tens". Khadija then got up to go grab some units but immediately returned to check how many she needed by referring to the handout. She held up 3 fingers as a representation of the first bin, then she walked away. Notice how Khadija understood that she needed to represent the digit in the ones place for all three bins without any lead or instruction from me. She did appear observant of what others were doing and sharing, so I she might have imitated what Samartha was doing.

<u>Stanza 7</u>

After Samartha completed her representation using the rods and units for each bin of books, she wrote and drew her solution on the activity handout. We can see that in this part of the transcript, I asked her to share what she did. Samartha was reluctant at first, then expressed that she was bored. She was bored, or tired, of the commotion that resulted from other students surrounding our group and who needed help with the activity. This situation reflects the day-to-day setting of a classroom where students are working on an activity but seek the teacher's attention for assistance or clarification. Although Samartha was a little reluctant at first, she then shared what she did.

Fatima:	Okay Samartha go. Show me what you did.
Samartha:	I did it.
Fatima:	Show me what you did.
Samartha:	I did it.
Fatima:	What did you do?
Samartha:	I'm bored.
Fatima:	Huh?
Samartha:	I'm bored of all of these people here. These two people are talking so loud and-

[Stopped for announcements]

Samartha:	Okay so-,
Fatima:	So.
Samartha:	One is 23. 37. And 56. [Pointing to the rods and units she created for each bin.]
Fatima:	Okay.
Samartha:	And all-, when you take all of these te-, little ones. Um the ones [Referring to the units as she gathered them together. Then, she separated the units for each bin]. And you add 2. No, 3. And then you add 1, 2, 3, 4, 5, 6, 7. And you add 6. So, um 3 plus 7 is 10. So, it's-, So, this is 16.

And this is 20 plus 30 is 50 [Referring to the rods for the first and second bin of books by holding them in her hand]. And um fif-, This is 50 tens with 20 and 30 [Referring to the first and second bin]. And then you add um 60, 70, 80, 90, 100 [Adding the third bin by counting up]. And when you add them altogether it's 116.

As we see, in response to my question, Samartha first shared the number of books for each bin by referring to the representation she created using the rods and units. Then, she separated the units from her initial representation so that they resemble the digit in the ones place of each bin of books, and she shared that we needed to add them together. It seemed as though Samartha separated the units for two reasons, 1) she might have wanted to ensure that she had the correct number of units, and 2) she wanted to explain how she added the digit in the ones place of all three bins together. Interestingly, she was capable of just subitizing what the number of units were for the first and third bin of books, but she needed to count the units one-by-one for the second bin.

In order to add the units together, Samartha added the 3 and 7 then she added 6 more, equalling 16. In adding the 3 and 7 together first, she might have relied on the strategy they learned called “parts of 10” or “anchors of 10” for making 10, which helps to simplify the addition process by creating a friendly number. Samartha then added the digit in the tens place of all three bins of books. First, she added the first bin (20) with the second bin (30) and referred to the sum as “50 tens”. Then, she added the third bin by counting up by 10 using the 5 rods, giving her a 100. A viewing of the video shows that she first had 5 rods in her hand, then she added the rods one-by-one as she counted up by 10. Samartha explained that by adding the rods and units together, they equaled a 116.

As Samartha was explaining her thinking and the representation she created, Joud grabbed two rods from Khadija's representation. Joud now had a total of 10 rods and moved to work on the units.

Stanza 8

In the following segment of the transcript, I asked Samartha to tell me how many tens there were in a 116.

Fatima: So how many tens are in a 116?

Samartha:	One hundred. 10. 10. [Pause] 10.
Mustapha:	100.
Samartha:	100 [Repeated Mustapha's claim].
Mustapha:	A 100.
Fatima:	You said altogether you have 116. So how many tens are in a 116?
Mustapha:	106.
Samartha:	6.
Fatima:	Look at them and count how many they are? [Referring to the rods]
Samartha:	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [Counting the rods].
Fatima:	What about these? [Pointing to the units]
Samartha:	16 [Samartha grabbed the units and started to link or connect them together].
Joud:	Mrs. Fatima. I organized them.
Khadija:	[Returned with a handful of units. She placed them on the table and counted them.] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16. 16 ones. Mrs. Fatima. There's 16 ones.
Fatima:	16 ones?
Khadija:	Yeah altogether.
Joud:	Mrs. Fatima, I organized mine.
Khadija:	Count them [Hands Fatima the units]. 16 ones. 16 ones. 16 ones.

As we see, in response to my question Samartha focused on the number of tens in a 100 without considering the 16 ones. Mustapha also responded to my question and shared the number, "100", and Samartha repeated his answer. I then referred to rods and units on the table and I asked them about how many tens there were in total. Mustapha responded by sharing the number "106" and Samartha shared the number "6". It was not clear from both of their answers what they intended, and I think it might have not been clear to them what I was asking. I then asked Samartha to count how many rods there were. Samartha counted 10 rods, and then I referred to the units and asked, "What about these [the units]?" Samartha responded with total number of units, then she grabbed the units and started to connect them together.

Also, during this time Joud had the rods and units organized into their separate bins. In so doing, she too created a representation of the number of books in each bin, and also, made one-to-one correspondence between the rods and the digit in the tens place, and between the units and the digit in the ones place.

Next, Khadija came back with a handful of units, placed them on the table, and we see her quietly counting them, “1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16”. It appeared that she brought back the exact number of units. I think she might have added how many she needed on her way to grab the units. Khadija then shared that there were 16 one’s, and with intonation, I responded, “16 one’s?” She confirmed that together they equaled 16 and she handed them to me, asking me to count them.

Stanza 9

The bell rang for recess, but the students and I decided to stay inside to continue working on the activity. In continuing to work on the activity, I thought it would be best to revisit the problem from the beginning to make sure students understand what was being asked. After Khadija read the activity, in the following part of the transcript, I asked her what she thought we were supposed to do to solve the activity.

Fatima:	Okay. Khadija, what are we going to do?
Khadija:	I don’t know. Tell someone else.
Samartha:	You can use my strategy.
Fatima:	Sure, do you want to show her?
Samartha:	So, you add. So, there's 2 groups [She got two rods]. There's 3 groups [She got three units]. And there's 23 [Confirmed the total number of books in the first bin]. And there's 30 [She got three rods]. 37 [Referred to the total number of books in the second bin]. 1, 2, 3, 4, 5, 6, 7 [She counted 7 units]. And then um there's 50 [She got 5 rods]. 1, 2, 3, 4, 5, 6 [She counted 6 units]. And there's 6 [Confirmed the units for the third bin of books]. And then you add [Referred to adding the units together]. And you-, these two-, All of this is 16 [Referred to sum of the units]. And all of this is a one hun-., is 100 [Referred to sum of the rods]. And you-, And I add them all together. And I found the question. You should-, you should add all these together.

Joud:	I explained it to you before.

As seen in this portion of the transcript, Khadija was unwilling to share her thinking and she suggested I ask someone else because by does not know. In so doing, Khadija develops and expresses agency by reflecting on her learning and on what she does not know or what she was unsure of. Samartha then suggested that Khadija could use her strategy, which reinforces Samartha's mathematical authority. Samartha explained to Khadija that she needed to "add", and she created a visual representation of the number of books in each bin using the rods and units. For the first bin of books, she grabbed two rods and three units. For the second bin, she grabbed 3 rods and counted 7 units. For the third bin, she grabbed 5 rods and counted 6 units. Samartha then explained that she needed to add the units together, so she gathered the units into one pile and said that they equaled 16. She also gathered the rods together and said they equaled a hundred. Then, she shared that they need to add the 10 rods and 16 units together. Samartha then passed the rods and units to Khadija and asked her to add them together. Joud then shared that she explained her answer to me before, reinforcing her mathematical authority by sharing that she already solved the activity.

<u>Stanza 10</u>

In the next part of the transcript, I began by asking Samartha to share the total number of books she got. Then, we focused on organizing the total number of books into groups of 10.

Fatima:	Okay. So, Samartha said that there's-. Samartha, how many did you say altogether?
Khadija:	100 [Referred to the sum of the digit in the tens place of all three bins.]
Samartha:	100 [Repeated after Khadija.]
Joud:	There's 100 and 16 [Referred to the total number of books in all three bins.]
Fatima:	116 what? What are these? 100 and 16 what?
Khadija:	Yeah, I was think-, I was thinking of that.
Samartha:	16 Groups. 100 and 16 um groups of 100? Groups of 10? Groups of 1?
Khadija:	So, 100 [Pointed to the 10 rods on the table]. We take a 10 of the 16 [Referred to the 16 units and suggested we take away 10 units, but

without taking them away]. /And we add a 10 /[She grabbed a rod and placed it next to the other rods, making 11 rods].

Joud: /Just put them in piles of 10/.

Khadija: So, it's one hundred and sixteen.

Fatima: Okay, so how are you going to do that? Show me.

Khadija: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [Took away ten units]. 1, 2, 3, 4, 5 [Counted remaining units]. This is not 16. It needs one more [She realized that she was missing a unit].

In this stanza we see, in response to my question, Khadija shared the sum of the digits in the tens place of all three bins of books, and Samartha repeated Khadija's answer. Joud then shared the total number of books in the three bins, which equaled 116. I then asked, "16 what? What are these? 100 and 16 what?" Khadija shared that she was thinking about that, then she pointed to the rods and confirmed that there were a 100 in total. Then, she suggested we replace 10 units with a rod, and confirmed that they would all equal 116. Joud then suggested to put the total number of books in piles of 10.

Next, I asked Khadija to show me how she was going to replace the units with the rods. Khadija took away 10 units and counted the remaining units. She realized that the total number of units did not equal 16, so she added an additional unit. She then added the extra rod to the 10 rods, making 11 rods and 6 units. The fact that Khadija replaced the 10 units with a rod is important because it reflects here understanding of rearranging grouping of ones into tens. Also, Khadija developed and expressed agency by building her own understanding and solving the activity based on all the observations and discussions that unfolded in our interaction.

Stanza 11

As seen in this portion of the transcript, I asked the students to tell me how many tens there were in total. Khadija focused on the sum of 11 rods, and Joud referred to the total number of rods.

Fatima: So how many tens are there now?

Khadija: Sixtee-, um 110.

Joud: /11/.

Fatima: What did you say Joud?

Joud: 11 tens.

Fatima: How? Can you show me how there's 11 tens?

Joud: Because if um Samartha got the ones [units] to make it into a 10 and she added 6 other ones. And what Khadija added another 10 [rod]. But before she added another 10 [rod], there were 10. Because 10 groups of 10 makes a 100. And she [Khadija] added another 10 to make it 11 groups of 10. To make it one hundred and ten. And she added 6 more [units].

In this stanza we see that I asked Joud to show me how there were 11 tens. Joud explained her thinking by giving credit to what Samartha and Khadija had done by explaining that Samartha used the units to create 10 plus 6, and that Khadija added an additional rod to the 10 rods. She also said that before Khadija added the additional rod, they were 10 rods and equaled a 100. And when Khadija added the additional rod, they equaled 11 groups of 10, which was a 110 plus 6 units. In reflecting on what Samartha and Khadija had done, Joud expressed agency by sharing her own understanding of what they have done, and also, taking responsibly for her learning.

<u>Stanza 12</u>

In the following segment of the transcript, I restated the activity question to ensure that the students understood how Joud got “11 tens” or “11 groups of 10”. Also, notice how I first stated the question using the word pile but then switched it to the word groups, privileging the term the teacher used as well as that the students used.

Fatima: Okay, so how many <u>piles</u> of 10 can Fatima make with all the books? How many <u>groups</u> of 10 did Fatima make with all the books?

Joud: 11.

Fatima: 11. Did you see that Samartha?

Samartha: No.

Fatima: Okay, do you want to show Samartha what you guys did?

Joud: We-, um. We show what Khadija did? [Took away the additional rod they added to the 10 rods].

Fatima: Samartha, do you want to look and see what they did?

Khadija: Um Samartha.

Joud: Samartha, do you know what Khadija did?

Khadija: Samartha [Trying to get Samartha's attention].

Joud: Samartha [Trying to get Samartha's attention].

Khadija: She's not listening. Take off her pencil [Tried to grab her pencil but Samartha pulled it back].

Fatima: Samartha. I want you to listen to see what they did. You did it right, but I want to show you how they got the groups of 10.

Joud: And how we got the answer right.

Khadija: We got it right.

Fatima: Samartha also got it right but she got the total number of books.

Joud: Samartha got the total, but we got the group total.

Fatima: You got the group total.

Joud: She got the 100 and 16.

Fatima: I think she'll hear you. So, why don't you go ahead.

Joud: Khadija added another group of 10 to make 11. 11 piles of ten [She added a rod to the 10 rods, making 11 rods]. And she [Khadija] took away 10 of your ones to make it into a 6 [Pointed to the units]. And she added, and she-, as I said she added another 10 to make it a 100 and 16.

In this stanza we notice that Joud responded to my question by sharing the answer, "11". I then asked Joud and Khadija to show Samartha what they did, but Samartha seemed not interested. I insisted that they share what they did in hope that Samartha would hear them. Joud then explained that Khadija added an additional rod to make a total of 11 rods, and that Khadija also took away 10 from the 16 units, which still equaled 116.

Stanza 13

In the following part of the stanza, I refer to Samartha's activity handout where she drew 10 rods close together in addition to another rod and 6 units. In particular, we focus on counting the number of rods she drew on her activity handout,

Fatima: Okay. Samartha! If you count your groups of 10, how many groups of 10 do you have?

Samartha: Here [Pointing to the rods she drew on her activity handout].

Fatima: Yeah. How many are there? Let's count them. One-,

Samartha: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Fatima: And then what about this one? [Pointed to the additional rod she had].

Samartha: This one is um-,

Khadija: 11.

Samartha: 11 [Repeated after Khadija].

Fatima: So, how many groups of 10 did Fatima make?

Khadija: 11.

Samartha: 11[Again, repeated after Khadija].

Fatima: 11 groups of 10.

As we see in this transcript, I initiated the counting of the rods in order to help her see the rods in terms of their quantity or groups of 10. Samartha counted 10 rods without including the additional rod she had. I pointed to the additional rod and asked her about it. It seemed as though Samartha did not want to associate the additional rod with the other rods—still seeing them as 10 rods plus 10 and 6 ones. Worthy to note is that earlier on in the activity, Samartha linked 10 units together which formed into one rod, which is where I think she got the idea to draw the additional rod.

Notice that during this time Khadija was attentively listening to our conversation, and she responded by sharing the number "11"—implying that there were 11 rods or 11 groups of ten. Samartha repeated Khadija's answer and then I asked, "so how many groups of 10 did Fatima make?". Khadija said "11" and Samartha appeared to have repeated Khadija's answer without making sense of why the answer was 11.

<u>Stanza 14</u>

In the next segment of the transcript, stanza 15, we focused on how many books were leftover or were not used.

Fatima: And then how many books were leftover?

Samartha: 11 [Referred to the total number of rods].

Khadija: 10 [Referred to the units or the total number of rods].

Fatima: How many books were leftover that we-, we didn't use?

Khadija: 6 [The number of books leftover].

Samartha: 11[Referred to the total number of rods].

Khadija: 6 [Corrected Samartha's answer].

Notice how in this transcript there were various interpretations to the question I posed. For instance, Samartha responded by sharing the number "11", which I believe referred to the total number of rods and was a repeat of the number that we just discussed in stanza 14. Khadija on the other hand, shared the number "10", which I think either referred to the 10 units we exchanged for the rod or the total number of rods we got when adding the digits in the tens place of all three bins. I also think the word 'leftover' might have been confusing for them because when I switched the wording of my question to include, "how many books that we didn't use?" Khadija responded with the answer, "6"—which was the number of books leftover. However, Samartha insisted on her initial answer "11" which led Khadija to correct Samartha's answer, reinforcing Khadija's mathematical authority.

Stanza 15

As we will see in the next portion of the transcript, Samartha developed an understanding of the difference between the sum of the addends and groups of 10. In so doing, Samartha develops agency by reflecting on her learning and building on her own understanding.

Samartha: Oh, I found something. So, this is one hun-. So, this is elev-, It's showing it's 11 groups of 10. And this is 6. So, when you add them together, it's 100 and 16.

Fatima: And how many tens are there in total?

Samartha: 11.

Fatima: And how many ones are left over or not used?

Samartha: 6.

Fatima: That's perfect.

As we look more carefully at this segment of the transcript, we can see that Samartha realized the difference between the total number of books in all three bins and how many groups of 10 there were in total. To explain her thinking, Samartha referred to the rods drawn on her handout to imply 11 groups of 10, then she explained that when we add the rods and units together, they equal 116 and are equivalent to 11 groups. This then completed the interaction I had with Samartha, Khadija, and Joud.

Stanza 16

In the following stanza I focused on the conversation I had with Mustapha and Raj. Mustapha and Raj were not video recorded at the time but could be heard speaking to me as I interacted with Samartha, Khadija, and Joud. I decided to separate the conversation into this stanza as it did not play into the other students' thinking or discussion. As you will see in this portion of the transcript, our conversation started with Mustapha asking me to read the activity. After I read the activity, Mustapha shared the number 30, but it was not clear what he was implying or referring to. Then, there was a long pause as the boys appeared distracted by others. We reconvened when Raj shared how he stacked the rods, leading me to focus their attention on how many groups of 10 we could make with all the books.

Mustapha:	Can you read this?
Fatima:	There are 3 bins of books in the classroom. The number of books in each bin is shown in this picture. There's 23 books in this one, 37 in this one, and 56 in this one. Fatima takes all the books out of the bins and puts them into piles of 10. How many piles of 10 can Fatima make with all the books? Show and tell how you know.
Samartha:	Mrs. Fatima, I know you would help them.
Mustapha:	30 [Not clear what he implied or was referring to].
[Pause]	
Raj:	Look I built it. Look, I can stack it. [Raj stacked the rods on top of each other]
Fatima:	Oh wow! Okay, let's look at how many groups of 10 we can make with the books.
Raj:	100.
Fatima:	How do you know that?
Raj:	Guess.
Fatima:	Did you just hear Abed's answer?
Raj:	No, guess.

Fatima:	I don't want you to guess. I want you to look at these [Pointed to the three bins], and then tell me how many groups-,
Raj:	These are all 100.
Fatima:	How do you know that?
Raj:	This is 80 [Referring to the sum of the digits in the ten's place of the second and third bin of books]. That's 20 [Referring to the digit in the tens place of the first bin]. 100 [Sum of the digits in the tens place of all three bins].
Fatima:	Okay, so how many tens are in a hundred?
Mustapha:	20 [Ten's place for first bin]. 30 [Ten's place for second bin]. 130 [Estimated guess for the sum of the addends?].
Raj:	No. Um:: It's 10. 10 tens.
Fatima:	10 tens. Show me your work. Show me how you got that.
Raj:	Oh!
Fatima:	And remember there may be more than 10 tens.
Raj:	You know how I got it? Look. 50 [Referring to the digit in the ten's place of the third bin of books]. 50 plus 30 is 80. [Repeated the number 50 and added the digit in the ten's place of the second bin (30), equaling 80]
Mustapha:	No, 50 plus 50 is 100 [To Raj].
Raj:	No look! 50 plus 30 is 80. You add 2 more [20] is a 100. 20.
Mustapha:	100 [Mustapha agreed].
Raj:	100.
Fatima:	Okay, but what happens to my 3, 7, and my 6?
Raj:	100. 6 plus 3 is-, 6 plus 3 is 9.

[STOP for announcements]

Rachet:	Fatima, I got it. Fatima, I got it.
Raj:	It's 100 and something. [To Rachet]
Mustapha:	Is it 150? [To Rachet]

Raj: 50. 30 and 20 equals 100. [For adding the units, he held up 7 fingers and added 3 more, equaling 10. And he counted up 6 more]. 10, 11, 12, 13, 14, 15, 16. It's 116.

As we can see, in response to my question about how many groups of 10 we could make with all the books, Raj shared the number 100. I asked him to explain how he knew the answer is a 100. Raj referred to the handout and shared his reasoning by focusing on the digits in the tens place and adding them together. First, Raj added the digit in the tens place of the second and third bin together, and then he added the remaining digit in the tens place of the first bin, equaling a 100.

I then asked, "how many tens are in a hundred?" Mustapha responded by decomposing the number of books for the first and second bin into tens. Then, it seemed as though Mustapha tried to estimate the total number of books for all three bins by sharing the number "130". Raj evaluated Mustapha's response by indicating that there were 10 tens in a hundred.

Next, we see that I asked Raj to show me how he got 10 tens, and I also hinted that there were more than 10 tens. In response to my question, Raj reexplained how he added the digit in the tens place of all three bins together. Mustapha who appeared to be paying close attention to what Raj was saying disagreed with him and said, "No, 50 plus 50 is 100." Mustapha might have assumed that Raj miscalculated given that he heard him say the number 50 twice. Raj disagreed and reexplained how he added the digit in the tens place of all three bins together. Also, notice how Raj said "2 more" when referring to the digit in the tens place for the first bin, then he corrected himself and said "20", establishing a sense of agency by modifying what he was saying.

I then referred to the digit in the ones place for all three bins and I asked, "what happens to my 3, 7, and my 6?" Raj used his fingers as a tool to add the digit in the ones place of all three bins together. First, he held up 7 fingers as a representation of the ones digit for the second bin, then he added 3 more and counted up 6 all through the use of his fingers. He then concluded that the answer is 116.

This then concluded the discussion I had with Raj and Mustapha as the bell rang and they were asked to get ready for recess. Although, they partially answered the activity question, they relied on various strategies to share their thinking, and they were capable of 1) concluding that the sum of the digits in the tens place for all three bins was a 100, 2) they agreed that there were 10 tens in a hundred, and 3) they added the digits in the ones place together and determined that there were 116 books in total.

Summary of Main Ideas: Activity #1 "Books in a Classroom"

In what follows, I discuss children's meaning-making by drawing on Gee's (2005) analytical framework of discourse analysis as described in **Table 3** in the methodology chapter, specifically the construct of building tasks of "language in use" including, significance, activity (i.e., mathematical actions), identity, relationships, politics, and sign systems or knowledge.

Significance. This activity brought forth several things that appeared to become significant to the students' mathematics meaning-making. These include their understanding of the mathematical term "groups", the use of specific strategies, and various tools that appear significant to their understanding of mathematics.

It seems that some students relied on the discussion they had with their teacher regarding the term "piles" to develop a sense of what it implies (i.e., groups). There is evidence that Joud, Samartha, Raj, and Abed picked up on the term, and it may have played into their meaning-making, or more specifically, how they made sense of the activity. In picking up on the term, the students made the term and their teacher significant. Noteworthy is that Samartha referred to both terms 'piles' and 'groups' interchangeably, reflecting her confidence and understanding of both terms. Khadija and Mustapha, on the other hand, rarely, if ever, used either term. It was only when Khadija repeated after Abed to inquire about what he said that she used the term "groups". At one point, I picked up on the term "groups" as well, and in so doing, I not only made the term significant but made the students and presumably their teacher's use of the term significant. Hence, our use of the term groups signifies that we have internalized the term based on other students' use of it.

Particular strategies appeared to be significant in students' meaning-making such as, decomposing numbers into their respective tens and ones, making computations based on groupings of tens and ones, rearranging grouping of ones into tens, as well as, counting, skip counting, and counting-on. The transcript provides evidence that all students involved in the activity had an understanding of the place-value system, which included an understanding of base-ten groupings with emerging computational strategies. For instance, when students were trying to find the sum of all three bins of books, they decomposed the number into tens and ones. In so doing, the students did not see the number as single units rather they understood the number as a representation of tens (groups of 10) and ones (single units), portraying an important understanding of place value. Counting the rods and units, or counting their fingers as representations of rods, were also significant strategies that students relied on to find the sum of the digits in the tens and ones places.

It was evident that the students relied on concrete materials that appeared to be meaningful to them. These include the use of rods and units, their fingers, the handout, and their drawings. The rods and units appeared to be one of the most significant tools, as these were used to model the number of books in each bin using the place value system and assisted them in finding the sum of the books using computational strategies with an understanding of grouping. Their fingers were most pertinent when they were trying to figure out how many units they needed or when adding the digits in the one's place. The activity handout was used when reflecting on the number of books in each bin, which also seemed to have helped the students see the numbers as representations of tens and ones. Drawings were mainly used to illustrate their solutions and to reflect their understanding of the activity on the handout.

Activities: Mathematical Actions. In analyzing the transcript, I looked for the evidence of mathematical actions as described in **Table 3** that portrayed the students' use and development of mathematical thinking skills. These mathematical actions include the use of reasoning and proving skills, communication of ideas, representation of ideas using various tools, and making connections between ideas or concepts. I will talk about each in turn.

Reasoning and Proving. My analysis of this data shows that the students approached the activity or reasoned their way to solution in very different ways, and yet, at times, they seemed dependent on what others said or done. As such, I will consider each of the participants as a case in point in order to describe how they reasoned their way to a solution. We notice that at the beginning of our interaction the students seemed to have partially solved the activity. However, as the student interacted with one another, the interpretations or understandings they developed, the discussions that unfolded between them, and the representations they created, all portrayed their mathematical reasoning and proving skills.

For instance, Joud was quick to share her answer by indicating that she thinks "it's 10 groups of 10". Then, she explained her thinking by adding the digits in the ten's place solidifying her conceptual understanding of place value. She added 20 plus 30, giving her 50, then she added 50 plus 50, giving her a 100, and which she then shared is equivalent to “10 tens”. Notice how Joud focused on only the digits in the ten’s place while disregarding the digits in the one’s place. Although Joud did not fully answer the activity question, she was capable of concluding that there were “10 tens” in a 100, reflecting her understanding of grouping.

Samartha shared a solution to the activity by displaying a different understanding or interpretation. She considered each bin to have 10 books each, and she added them together, giving her 30. Here, I wondered if her explanation and solution were based on what Joud said concerning “10 groups of 10” or “10 tens”. Samartha might have not taken into account Joud’s entire explanation and only retained what made sense to her. However, once the exact number of books in each bin were brought to her attention, Samartha created representations of each bin using the rods and units. In order to do so, she decomposed the number of each bin into tens and ones, and dealt with each respectively. Then, she shared her solution by first adding the units together and then she added the tens together, equaling 116. When asked how many groups of 10 are in a 116, Samartha responded by sharing that there are 10 tens. Once I referred to the units, Samartha grabbed the units and linked or connected them together, creating one rod. On the activity handout, Samartha drew 10 rods plus 1 rod and 6 units, but she continued to insist that there were 10 tens.

Khadija appeared to have merged all the discussions that unfolded and developed an understanding that she needed to add the tens together. After she received my confirmation, she created representations of the digits in the ten’s place using the rods. When asked to share what she did, Khadija referred to the sum the of the rods as “100 tens”. I think she might have inferred that since the sum of the addends equaled a 100 and were grouped by tens, they could be referred to as “100 tens”. Khadija then brought 16 units and shared the total units she had.

It was only in the second recording that the students considered the sum of the digits in the ones place to determine how many groups of 10 Fatima could make with all the books. Specifically, Khadija, from looking at the rods and units realized that she could rearrange 10 units into a rod—that is, she takes 10 units away and replaces them with a rod, giving her 11 rods and 6 units.

However, Samartha was still unsure of the number of groups there were in total, and she seemed to confuse the groups of 10 with the sum of the digits in the tens place. Samartha would simply repeat Khadija's answer without making a connection between what she is saying and what it might imply. It was only near the end that Samartha realized the difference between the sum of the addends and the group's total.

In a similar vein, Raj and Mustapha confused the sum of the digits in the tens place of all bins to imply the groups of 10 total without taking into account the digits in the ones place. And although, they partially answered the activity question, they were capable of 1) coming to a consensus that the sum of the tens for all three bins was a 100, 2) they agreed that there were 10 tens in a hundred, and 3) they added the ones together and determined that there were 116 books in total.

Communication. The students relied on oral and visual communication to share their mathematical understandings, ideas, and to enact the use of their strategies. For example, the students communicated when they 1) responded to a question, asked a question, or posed a concern; 2) read or asked me to read the problem-solving activity; 3) quietened one another; 4) made a link between them and the activity; 5) repeated the teacher's use of the term "groups"; 6) referred to the number of books on the handout as a written representation of what they were, or to ask if they should add the digits in the tens place, or even add all three bins together; 7) shared their thinking by reasoning their way to a solution; 8) displayed a different understanding or interpretation of the activity; 9) decomposed the number of books in each bin into the tens and ones place; 10) stressed the value of a specific term by constantly using it; 11) explained or reexplained a mathematical understanding or idea; 12) expressed disengagement or confusion; 13) captured a mathematical idea or understanding using concrete tools; 14) shared their representations; 15) estimated the total number of books in the three bins; 16) made computations based on groupings of tens and ones; 17) shared how many groups or piles of 10 Fatima made with all the books; 18) counted the number of rods or units; 19) shared the sum of the rods and units; 20) when they repeated each other's answers or claims; 21) made one-to-one correspondence between the rods and the digits in the tens place, and between the units and the digits in the ones place; 22) rearranged grouping of ones into tens; 23) evaluated, corrected, or confirmed each other's answers; 24) when they shared or explained to one another their solutions; and 25) wrote or drew their solution on the activity handout. In this interaction, communication was not only the spoken word, but included various tools that the students relied on to communicate their thinking.

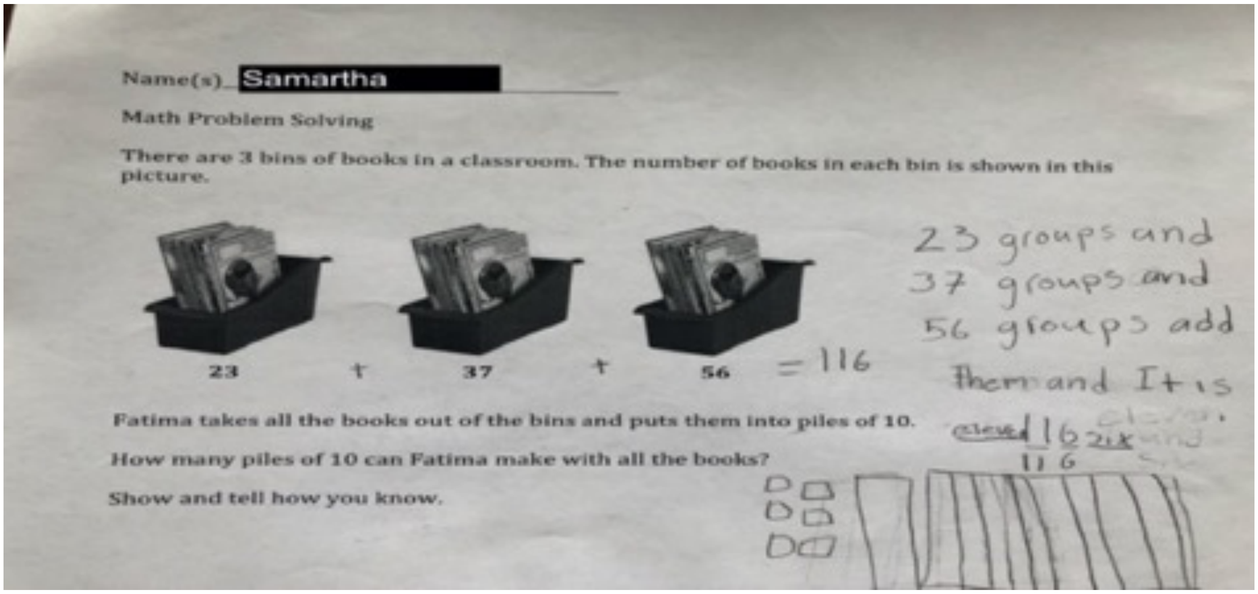
Representation. It was evident that the students relied on concrete tools to represent their mathematical understandings and ideas by using their fingers, base ten blocks, the activity handout, and drawings. In terms of their fingers, Raj and Khadija used their fingers as a representation for adding the digits in the ones place together. For instance, Raj held up 7 fingers as a representation of the one's digit for the second bin, then he added 3 more as a representation of the one's digit for the first bin, and then counted up 6 which was the representation of the one's digit for the third bin.

One of the most significant representation appeared to be the base ten blocks (i.e., rods and units) which were used to create or build a representation of the number of books in each bin, and that also acted as a representation of their thinking and ideas. For instance, the rods were used as representations of the digits in the tens place, and the units were used as representations of the digits in the ones place. The students also appeared to have developed a conceptual understanding of the digits in the tens place as representing groups of 10. The students also used the rods and units as a representation of a number sentence. At one point, Samartha placed a rod for each bin as a representation of ten books for each bin, and she referred to them by communicating a number sentence, she said "10 plus 10 plus 10…So 3 groups of 10 is 30". The fact that Samartha used the rods as a representation of the number of books she thought were in each bin is important because it reflected her understanding of the value of the rod as a representation of 10. The rods and units were repeatedly used by the students to assist them in finding the sum of all three bins, and then to rearrange grouping of ones into tens.

Concerning the activity handout, the students referred to the number of books on the handout as a written representation of what they were, or to ask if they should add the digits in the tens place, or even add all three bins together. With respect to drawings, Samartha used the activity handout to draw a representation of her solution and thinking (**Figure 24**). On her handout, we see drawings of rods and units as a representation of the sum of the digits in all three bins of books. She also created a number sentence using the addition operation (i.e., +) written between the number of books for each bin, and she included an equal sign (i.e., =) to indicate the sum of the three bins equaled 116. We also see that she wrote "23 groups and 37 groups and 57 groups add them and it is eleven 116 six"—meaning that the three bins added together equal 116 and are equivalent to 11 groups of 10 and 6 units remaining. This drawing not only illustrated her solution to the problem but was a visual or modelled representation of her thinking and understanding of the problem.

Figure 24

Samartha's Solution



Connections. The students also made connections between mathematical ideas or concepts. These included making a connection between 1) the term ‘piles’ and ‘groups’, especially when Samartha alternated between the two terms by considering them to mean the same thing; 2) the rods and the digits in the tens place, and between the units and the digits in the ones place—more so in terms of the base ten blocks acting as a representation of a digit in the tens or ones place; 3) quantity and place value—particularly when the students added the tens and ones separately based on their value in a number; 4) a number name, a number symbol, and a representation of quantity (e.g., the written digit 7, the spoken word “seven”, and a representation of a quantity (7) using the units or their fingers); 5) the operation of addition and a solution to the activity—here, the connection made when the students developed an understanding of needing to “add” or group the number of books by tens to satisfy the tasks objective; and 6) rearranging grouping of ones into tens—this, took place closer to end of the activity and when Khadija suggested that they regroup the ten ones unit a tens rod.

Identities. As the students became immersed in the activity, it was apparent that the students enacted various roles and developed their individual identities as mathematics learners. These identities framed their roles in the activity and how they perceived or made sense of the activity. Analysis of the data suggests that students recognized agency by exercising control over their engagement in the mathematics activity, and by identifying themselves as participants and doers of mathematics. The students expressed or developed agency by using their own individual strategies, by modelling their own solutions using concrete tools or their fingers, and by verifying that their representations matched the number of books in each bin. In term of students' individual identities, Joud established a mathematical identity that portrayed her as building her own understanding and by attempting to solve the solution on her own. She also exercised control over the strategies she used and by clearly sharing them with the rest of the group. However, her perseverance and determination in the beginning faded away after she thought that she found the solution to the activity. Then on, she merely cooperated with the students and myself, and would repeatedly justify that she found or shared the solution first. At one point, she takes the responsibility for explaining or clarifying to Samartha how she could find or determine the number of groups/ piles Fatima made with all the books.

Samartha established a mathematical identity that portrayed her as taking ownership of her learning by reflecting on and self-directing her understanding of the activity through her self-initiated use of the rods. She was also determined to use her name in the activity rather than the one given. This reminded me of what Gutiérrez (2011) referred to as the mirror/window metaphor. In her work, Gutiérrez (2011) states that students should be able to use mathematics to look out at the world but should also be able to see and recognize themselves in the mathematics they are learning. Gutiérrez might have been referring to or implying the cultural and social characteristics of an individual should be recognized and imbedded in students' learning. However, Samartha's use of her name helps her see or provokes the impression that she was part or included in the activity. Also, by manipulating or representing the books in piles of 10, she becomes part of the activity through her use of the rods and units to illustrate her understanding and thinking.

Khadija established a mathematical identity that depicted her as first resisting engagement in the activity, and to later identifying with the activity. At first, Khadija acknowledged unfamiliarity with activity and of what is expected of her as a way to describe her disengagement. This was particularly evident when she asked for clarification in how she should approach the activity, she asked, "what is it?"; "But how I do it?"; "So how-, how do it?"

However, as she continued to observe and listen in on the contributions shared by others, this led her to build her own understanding and to imitate Samartha's use of the rods. She was also capable of monitoring her own progress and checking her own work with some guidance from me.

Raj developed a mathematical identity that portrayed his eagerness to find the solution to the problem. He exercised control over the strategies he used to find the sum of the addends and how many piles/groups there were. Raj also developed and expressed agency by correcting or verifying his own work or that shared by Mustapha, and by attempting to find a solution to the activity by reflecting on his own understanding.

Mustapha established a mathematical identity that portrayed him as merely cooperating with the activity. He attempted to build his own understanding as he attentively listened and engaged in the discussions that unfolded between Raj and myself. Mustapha also took responsibility of evaluating or confirming Raj's answers.

Relationships. There are several relationships at play or that evolved and that may have influenced the students' mathematical meaning-making. These include the relationship between the students and the problem-solving activity, the relationship between the students and concrete tools, the relationship between the students and myself, and the relationships between the students as they interacted.

Concerning the relationship between the students and the activity, it was apparent that the students portrayed various levels of confidence in terms of finding a solution. Joud portrayed confidence in the activity as was reflected by the solution she shared early on in our interaction. And although she did not fully answer the activity question, Joud clearly described her ideas and thinking. Samartha, who early on in our interaction misinterpreted the number of books in each bin, still portrayed confidence in the solution she shared based on her own interpretation and understanding of the activity. And once she realized the exact number of books in each bin, she portrayed high levels of confidence by reflecting on and sharing her thinking. Khadija who appeared a little confused at first, portrayed confidence after she developed some understanding of what was expected of her. Raj's level of confidence in the activity was quite prevalent, as reflected in his enthusiastic behavior to find a solution. Mustapha seemed to have relied on the discussions that unfolded to generate ideas of what was expected of him, and to then share his response to questions I posed, both reflecting and generating his confidence.

My analysis of this transcript shows that the previous experiences and the relationships the students built with the base ten blocks before this activity helped shape how they used the rods and units to create a representation and a solution to the problem. In the activity, the students were capable of easily representing the tens digit as groups of 10 using the rods, and representing the ones digit as groups of 1 using the units based on their understanding of what they are and what they could be used for. Hence, their understanding and familiarity with the base ten blocks helped students use them to reason their way to a solution.

The relationship between the students and I depended on the level of support they each needed to make sense of the activity. Joud did not require much support given that she developed some understanding of the activity. Samartha required clarification and guidance after she misinterpreted the number of books in each bin. Khadija looked to me quite often because she was not sure what was expected of her, and to then ensure she was on the right track. As for Raj and Mustapha, they mainly relied on me to confirm their answers and move there thinking forward. Also, our relationship seemed to change throughout the activity. In the beginning, they looked to me quite often for guidance, but once they understood or made sense of what may be expected of them, I no longer intervened. It was only when I referred to the sum of the digits in the ones place in an attempt to help them make a connection to the number of groups that could be made that they required some guidance and clarification.

In terms of the relationship between the students as they interacted, the students, more so Samartha and Joud, appeared rather competitive with an urge to be the first to find and share their solution to the activity. As a result, the students first approached the activity individually by developing their own understanding or interpretation of the activity, then created representations of the number of books in each bin to find a solution. However, a detailed review of the video and transcript revealed that the students did rely on one another to form some understanding of what was expected of them to work their way to a solution. This was particularly evident when 1) Khadija relied on the discussions to build an understanding of the activity; 2) Khadija and Joud imitated Samartha's use of the rods and units to create a representation of the books in the bins; 3) Khadija and Joud collaborated on trying to rearrange the grouping of ones into tens (one rod); 4) Joud tried to explain to Samartha how they found the total number of groups; and 5) Mustapha would correct or confirm Raj's answers to build an understanding of the activity when Raj was explaining and sharing his thinking. One other instance that could reflect the relationship between Samartha and Joud was particularly evident when Samartha quietened Joud when she tried to share her thinking and when sharing how she stacked three rods together. In both instances, Samartha may have appeared uninterested either because she was too keen on finding the solution on her own, or she was just too focused on what she was doing and did not want any distractions.

Politics. There are several political aspects in the activity that portrayed the distribution of mathematical authority among the participants and that may influence students' understanding and engagement in the activity. Some of these demonstrate where the mathematics authority resides in the classroom. Other aspects portrayed my role as a mathematical authority. Still other aspects we can see students assuming mathematical authority.

In reviewing the transcript, I noticed that the classroom teacher portrayed mathematical authority as reflected in students' use of the term she used. For instance, Joud, and at times, Samartha appeared to have honored their teacher's term of "groups" as opposed to the term on the handout "piles" in their discussions. I also picked up on the term "groups", and thus, I may have reinforced both the students' mathematical authority and their teacher's mathematical authority by using the term after they did.

I also portrayed mathematical authority by regulating the students' behavior and confirming or challenging their responses. As an example, I probed their thinking by asking them to reflect or elaborate on a solution they posed. I also noticed that I purposefully distributed mathematical authority. For instance, at a certain point I asked Joud and Khadija to show Samartha how they got the groups of ten total.

In many instances, we can see that various students took on mathematical authority. For instance, Samartha demonstrated a strong sense of mathematical authority by 1) having determination in being the first to find the solution to the activity; 2) developing her own interpretation of the activity; 3) bringing rods and units to model her solution, 4) interrupting or quieting Joud either because she was not interested in hearing what Joud had to say, or she was just too keen on sharing her own solutions, and 5) suggesting to others to use her strategies.

Joud portrayed mathematical authority by depicting a reasonable understanding of the activity and by quickly sharing her solution to the problem. Although Joud did not fully answer the activity question, she was capable of sharing a solution that focused on the addends of the digits in the tens place of all three bins, reflecting her understanding of the activity.

Khadija exercised mathematical authority by trying to make sense of the activity based on her understanding and interpretation of the discussions that unfolded. However, she was too keen on getting my conformation before she could proceed, recognizing my authority and complying with it.

Raj also demonstrated mathematical authority by correcting Mustapha and justifying his thinking. He too would readily seek my confirmation to ensure that he had the correct answer. As for Mustapha, he portrayed mathematical authority by frequently monitoring the contributions shared by others.

Sign Systems or Knowledge. With respect to what sign systems or knowledge are privileged, the use of English, the students' use of specific terms or numbers, their interpretation of the activity, and the use of certain tools were privileged or made significant by those seen with more mathematical authority or had a strong sense of agency.

In this interaction, the use of English was privileged because the activity was presented in English and all our communications were in English. Hence, the privileging of English might explain Khadija's hesitation to find the solution to the activity because she might have had trouble understanding what to do. Also, students' use of the term "groups" appeared to have privileged the teacher's vocabulary, and the use of the term "piles" privileged the term in the activity. We also notice students use of "50 tens" and "100 tens" when referring to groupings of 10. The students might have inferred that since the number of books they are referring to is 50 or 100 and are grouped by tens, they could refer to them as 50 tens or 100 tens. Notice also how some students referred to the number in the tens place as 2, 5, 3 then switched to saying it correctly 20, 30 and 50. These examples privilege a particular way of referring to something which influences their meaning making.

Concerning their developing sense of the mathematical problem-solving activity, early on in the activity, the students developed distinct interpretations of the activity. These interpretations may be founded on either their own understanding of the activity, from what their teacher said, or from picking up on one another's phrasing. However, the manner in which the students shared their thinking and their continued reiteration of it appeared to have unconditionally privileged their ways of knowing. Notice, for instance, how Joud, Abed, and Raj focused on only the digits in the tens place when adding the books for all three bins. They might have modelled what the teacher said or even developed their own interpretation of what "piles of ten" or "groups of 10" means. As for Samartha's early speculations, these appeared dependent on her understanding or interpretation of Joud's explanation, which might have led her to assume that there were only 10 books in each bin and she needed to add them together. The fact that Samartha constantly repeated her solution using the rods; she not only attempted to privilege her understanding and solution but privileged the use of the rods as means to find a solution to the activity, leading others to adapt their use and to find much meaning in the base ten blocks.

Students' understandings or interpretations reflected their ways of knowing, and also, exposed their mathematics thinking. However, as their involvement in the activity progressed, the students continued to build on each other's ideas, influencing their next steps.

Activity #2: "Ameer's Coins"

What was the activity? "Ameer's Coins" was another mathematics problem solving activity that the students engaged in, and that the teacher felt it would be thought-provoking for students to engage in (**Figure 25**). The activity was related to what students were learning at the time, including adding and subtracting money amounts, and also, representing and comparing money amounts (e.g., 1 toonie, 2 loonies, 2 quarters, 4 dimes and 2 nickels is $5.00), using various strategies and tools such as coins, drawings, and the hundreds chart.

For this activity, the students were presented with an image of 4 quarters (25 cents) that belonged to a student named Ameer. The objective of the activity was to determine what coins Abdul and Raja had, given that Abdul and Raja had the same amount of money as Ameer, but Abdul had more than 4 coins and Raja had less than 4 coins. The activity was a multi-step problem that required students to make several computations including figuring out the amount of money Ameer had, and the number of coins Abdul and Raja had, while ensuring that both Abdul and Raja had the same amount of money as Ameer.

Figure 25

Ameer's Coins Activity Handout

Names: ______________________________

Ameer has 4 coins shown below.

Abdul has more than 4 coins. Raja has less than 4 coins. Abdul and Raja both have the same amount of money as Ameer.

What coins do they each have?

What happened that day? The teacher told the students that they will be working on several mathematics challenge activities. The students' reactions were quite welcoming as they were eager to take part in mathematics activities that were considered challenging. The teacher then provided the students with a brief introduction to the activity. She explained that the activity would require them to think about 'parts of a whole' and how to make an 'equal part of a whole'. She then shared an example using the number 18 and asked students to reflect on ways they could make 18 by adding two numbers. As an example, she asked, "So let's say I have the whole 18. I know that 9 plus 9 is 18. That's one way to make it. What is another way to make 18? Can anyone think of another way? Show me when you know. Like this [thumb down-do not know], or like this [Thumb up-I know]". One of the students shared, "10 plus 8", another student shared multiple solutions, such as "17 plus 1; 16 plus 2; or 15 plus 3". Then, one student suggested subtracting by sharing, "19 minus 1". The teacher also asked the students to do some really careful thinking and talking in their groups.

Following this brief introduction, the teacher organized the students into small groups. Prior to this day, I asked if I could work with Rachet, Abed, and Mustapha, and so she organized the groups accordingly. The students and I decided to work in the school library as the classroom could get really loud during math centers. We also took some markers and individual whiteboards to write or draw on, and the students were provided with a copy of the activity handout.

Stanza 1

I started the mathematics activity by asking the students to read the problem. Rachet noticed Mustapha's and Abed's interest in reading, and so he suggested taking turns reading the activity, establishing his mathematical authority. Ultimately, they all collaborated on reading the activity together. As you will see in this portion of the transcript, after the students read the activity, they were quick to share their understanding by focusing on specific statements or parts of the activity.

Rachet:	2 dollar. 1 dollar [Not clear what he was referring to].
Mustapha:	No. 4. 4. 4. Is 8 [Referring to the number of coins they each had. Then, he added the total number of coins that Raja and Abdul had].
Abed:	It's 1 dollar. Everybody has 1 dollar [Referring to the amount of money they each had].
Rachet:	Raja has less than 4 dollar [coins] [Rachet turns to Mustapha].
Mustapha:	He have-,
Rachet:	So, it's 3, 2, or 1 [Referring to the number of coins Raja could have].
Mustapha:	Okay. [Pause]. Mrs. Fatima, it's 3 [Held up 3 fingers]?
Fatima:	Think about it. Let's see.
Mustapha:	2 [Mustapha held up 2 fingers]?
Fatima:	What's 3? What's 2?
Mustapha:	This is 3. [Mustapha pointed to the sentence, "Raja has less than 4 coins"].

As we see in this transcript, at first, it was not completely clear what Rachet was implying when he said, "1 dollar. 2 dollar". Then, the second time he spoke, he focused on the statement, *Raja has less than 4 coins*, and speculated that Raja had either 1, 2, or 3 coins. Also, notice how Rachet referred to the word 'coins' as 'dollar' when explaining to Mustapha that Raja had "less than 4 dollar [coins]". It appears that Rachet was not differentiating between the two terms, and presumably thinking they are one and the same. Also, Rachet developed agency by correcting Mustapha's claim based on the understanding of the statement in the activity, which was evidence for his claim.

On the other hand, Mustapha and Abed focused on the statement, *Abdul and Raja both have* *the same amount of money as Ameer*. However, they interpreted the statement from different viewpoints. For instance, Mustapha focused on the <u>number of coins</u> they each had, then he added the number of coins Abdul and Raja had together, giving him 8. Yet, Abed focused on the <u>amount of money</u> they each had, which was a dollar each. Also, when Abed was sharing his thinking, he looked at me for confirmation, but I did not respond as I did not want to intervene. In seeking my confirmation, Abed recognized my mathematical authority.

In response to Rachet's clarification regarding *Raja has less than 4 coins*, Mustapha held up 3 fingers as he shared the number 3. I asked him to think about it, but he took my response to imply that he was wrong. Mustapha then held up 2 fingers and asked if it was 2. In so doing, Mustapha recognized my mathematical authority by trying to seek my confirmation. I then asked, "what's 3? What's 2?" Mustapha responded by pointing to the statement, *Raja has less than 4 coins*, making a connection between what he said and the statement of the problem—that is, *Raja has less than 4 coins*, and Mustapha thought he had 3 or 2 coins. It seems that Mustapha saw the activity handout as being a strong authority as he referred to it as evidence for his answers or claims. Also, in sharing their understanding by focusing on specific statements or parts of the activity, the students developed and expressed agency by building their own understanding or by building on each other's understanding.

<u>Stanza 2</u>

In the following part of the transcript Abed moved forward and elaborated on his thinking concerning "Everybody has 1 dollar" based on his interpretation of the statement, *Abdul and Raja both have the same amount of money as Ameer*.

Abed:	Everybody gets 1 dollar.
Fatima:	How?
Abed:	Because 4 coins. And every 4 coins make 1 dollar.
Mustapha:	No. 1, 2, 3, 4 [To Abed. Presumably referring to the fact that they can't all have 4 coins each].
Fatima:	Do all 4 coins, or any 4 coins make a dollar?
Abed:	That's 25 cents-,
Rachet:	/Is it 11 dollar?/
Abed:	That's-, Only 25 cents does.

Fatima: 25 cents?

Abed: Does only 1 dollar [Finishing my sentence].

Mustapha: /Does 1 dollar actually/.

Abed: Four 25 cents does 1 dollar.

Fatima: Okay.

As we can see in this part of the transcript, Abed's reasoning or solution seems to suggest that he interpreted the statement he was referring to, to imply that Abdul and Raja had the same amount of money as well as the same number of coins as Ameer. Again, developing agency by building on his understanding and reflecting on his thinking. Mustapha expressed disagreement and created a representation of the number 4 as he counted on his fingers. Mustapha's disagreement might be based on Rachet's earlier clarification regarding Raja not having 4 coins because he had *less* than 4 coins. As we see in the transcript, I asked Abed, if any 4 coins make a dollar. Abed responded by sharing that only if the four coins were 25 cents, then they would equal a dollar. Mustapha confirmed Abed's answer, reinforcing their mathematical authority.

Notice that during this time Rachet quietly counted on his fingers, and asked if the answer was 11 dollars. It seems that Rachet moved from thinking about the number of coins that Raja could have, to presumably thinking about the number of coins or the amount of money the students had in total.

<u>Stanza 3</u>

We continued to unpack the activity, as shown in the transcripts in this stanza and the one that follows, by focusing on each of the four statements. In this stanza of the transcript, we focused on Ameer's coins. However, there was a confusion over whether students were looking at the value of the money or the number of coins involved when considering "how much" money Ameer had.

Fatima: So, how much does Ameer have?

Rachet: Ameer. 4 dollar (coins) [In correctly referred to the number of coins Ameer had as dollars].

Abed: No, 1 dollar [Referring to the amount of money Ameer had].

Mustapha: No, 4 dollar (coins) [In correctly referred to the number of coins Ameer had as dollars].

Abed: 1 dollar because-,

Rachet: Ameer has 4 coins [To Abed].

Abed: No, these are the 4 coins, but make-, [To Rachet]

Rachet: But /it's one-,/ it's 1 dollar [Referring to the amount of money].

Abed: Yeah, it's 1 dollar.

As we see, in response to my initial question in the transcript, Rachet and Mustapha focused on the number of coins that Ameer had (4 coins), but by incorrectly referring to them as dollars. Abed focused on the amount of money that Ameer had (1 dollar). Rachet then looked at Abed, pointed to Ameer's coins, and said that Ameer had 4 coins. Notice that Rachet correctly referred to Ameer's coins as 'coins' and not 'dollars'. Abed evaluated Rachet's response by pointing to Ameer's coins, referring to them as 4 coins, and attempting to say what they equaled, but was interrupted by Rachet who shared that the coins equaled a dollar. Abed then confirmed Rachet's answer of one dollar. It is evident here that Rachet and Abed both saw themselves as having autonomy in sharing their thinking and in correcting each other's answers based on their understanding of the statement, which reinforces their mathematical authority.

Stanza 4

In the next part of the transcript, we moved on to the second statement of the problem. I read it and then the students shared their thinking.

Fatima: Okay. And then it says here that Abdul has more than 4 coins.

Rachet: Maybe he has 5.

Fatima: Maybe he has 5.

Mustapha: No, maybe he has one b-.,

Abed: Maybe he has 6.

Fatima: Maybe he has 6.

Mustapha: Wait, or 1 billion.

Rachet: /or 7, or 8,/ or, 9 or, 10.

Mustapha: 1 Billion.

Rachet and Abed shared the possibilities of the number of coins that Abdul could have. Some of their initial suggestions included 5 and 6 coins. Then, Rachet shared that Abdul could have 7, 8, 9, or 10 coins, and Mustapha shared that Abdul could have one billion coins. The fact that the students shared all these numbers is significant as it showed that there are many, many possibilities.

Stanza 5

As seen in this portion of the transcript, we continued our discussion, focusing on the number of coins Raja could have as he has less than 4.

Fatima:	Alright, let's keep going. Raja has less than 4 coins.
Abed:	So, he has like 3 or 2.
Rachet:	Or 1.
Fatima:	Yes, and then-,
Mustapha:	I think 2.

As we see in this transcript, the students shared all the possibilities given that Raja had less than three coins. Also, the viewing of the video showed that Mustapha wrote the number 2 followed by the number 4 on the whiteboard as a representation of Raja's and presumably Ameer's coins.

Stanza 6

In the next part of the transcript, we focused on the statement *Abdul and Raja both have the same amount of money as Ameer*. Rachet and Mustapha continued to think about the coins in terms of their quantity and not their value. Abed, on the other hand, thought about the coins the boys had in terms of their value and by adding the amount of money each had in total.

Fatima:	Abdul and Raja both have the same amount of money as Ameer.
Abed:	So, 3 dollars [Referring to the amount of money they had in total].
Mustapha:	/4. 8/ [Referring to the total number of coins Abdul and Raja had].
Rachet:	No, it's 8, 9, 10, 11, 12, 13 [Counted up from where Mustapha left off using his fingers, giving him 13]. It's 12 dollar [Coins]. [He then counted backwards 1, giving him 12—referring to the number of coins they had in total].
Mustapha:	Can I have eraser? [Asked for whiteboard eraser to erase the 2 and 4]
Fatima:	Where did we get 12 dollars from?

Mustapha:	It's 4, 8, 12. Done.
Abed:	It's 3 dollars [Holds up 3 fingers].

As we see in this transcript, Abed was quick to share that Ameer, Raja, and Abdul had 3 dollars in total. His solution was based on his understanding that the boys had the same amount of money as Ameer, which was 1 dollar each. However, Mustapha interpreted the same amount of money to imply the same number of coins as Ameer. He thought that since Ameer had 4 coins, Raja and Abdul would have 4 coins each as well, and together, Raja and Abdul had 8 coins. Rachet's interpretation was more aligned with Mustapha's understanding, as he too focused on the number of coins the boys had in total. However, Rachet added Ameer's 4 coins, giving him 12 coins in total. Also, notice how here he referred to the coins as dollars.

Next, we see that I asked Rachet, "where did we get 12 dollars from?" Mustapha responded by counting up by 4 to 12, which suggested that the boys each had 4 coins and equaled 12. However, Abed insisted on his initial answer and held up 3 fingers as a representation of the number he was referring to.

<u>Stanza 7</u>

As seen in this portion of the transcript, I asked the students to consider the type of coins each person had based on Abed's understanding that the boys had a dollar each and equaled 3 dollars in total.

Fatima:	If Abed thinks they all have 3 dollars. What coins do they each have?
Rachet:	4 [Referring to the number of coins each person had].
Fatima:	But Abdul has more than 4 coins. What are the coins he has?
Mustapha:	1 dollar [Referring to the amount of money Abdul had].
Rachet:	4 [Repeated his initial answer].
Abed:	Like dollars?
Mustapha:	All have 4. 8 [Referring to each person as having 4 coins, then he held up his thumb and quietly said the number 8, as if he was skip counting by 4]. 8 [Repeated the number 8].
Fatima:	How? Where did you get 8 from?
Rachet:	It's 7.

Abed: Yeah, yeah you get 8. /It's because he [Ameer] has 4. Abdul gets 4. And then Raja gets 2 or 3/. So, it's 8.

Rachet: /It’s 7. It's 7. It's 7. It's 7. It's 7 or-,/ It's 7 or 6.

Mustapha: Or 8. 8.

As we see in this transcript, in response to my question, Rachet shared that each person had 4 coins, which is still based on his understanding that Abdul and Raja had the same number of coins as Ameer. However, I reminded them that Abdul had more than 4 coins and asked them to tell me what might his coins be. Mustapha responded by referring to the amount of money Abdul had (1 dollar). Rachet still suggested that they each had 4 coins, and Abed asked “like dollars?” Mustapha then shared that the boys had 4 coins each, and he held up his thumb and said the number 8, as if he was skip counting by 4 or adding 4 more. I then asked Mustapha to explain how or where he got the 8 from but he did not respond. Rachet then shared the number 7. Abed then explained that Ameer had 4 coins, Abdul had 4 coins, and Raja had 2 or 3 coins, and together they had 8. The number 8 might refer to the number of coins Ameer and Abdul had in total. Rachet then repeated the number 7 a few times but then concluded with “7 or 6”—here, I think her was referring to Abdul as having 4 coins and Raja having 2 or 3 coins, and together, they equaled 7 or 6.

Stanza 8

As you will see in the next segment of the transcript, we focused on the number of coins Raja had, but then we shifted to Ameer’s coins after Abed suggested that one of the coins Raja had could be 25 cents.

Fatima: Let’s look at the coins that each person has. So, who do you guys want to start with? Do you want to start with Abdul, or do you guys want to start with Raja?

Mustapha: /Raja/.

Abed: /Raja/.

Rachet: /Raja/.

Fatima: Okay. So, if Raja has less than 4 coins, what are the coins he has?

Rachet: 1, or 2, or 3 [Referring to the number of coins Raja could have].

Abed: /25 cents/ [Referring to the type of coin Raja could have].

Fatima: But remember Abdul and Raja have the same amount of money as Ameer, which is how much?

Mustapha: It's easy.

Rachet: 4 dollar [Referring to the number of coins Ameer had].

Abed: It's 12. It's cause 4, 4, 4 [Referring to the total number of coins they had, and considers each person to have 4 coins].

Mustapha: 4, 8, 12 [Counting up by 4].

Rachet: I already said 12. It's not 12.

Fatima: Let's look at how many coins Ameer has [Referred to the handout].

Rachet: 4 coins.

Abed: /4 coins/.

Mustapha: /4 coins/.

Fatima: And how much are these 4 coins? How much money are they?

Abed: 1 dollar [Referred to the amount of money Ameer had]. 25 cents I mean [Then he referred to what each coin was worth].

Fatima: Altogether. Altogether how much are they?

Abed: 1 dollar.

Fatima: 1 dollar. So, if Ameer has a dollar, and you have the same amount of money as Ameer, how much money would you have?

Abed: 1 dollar.

Fatima: So how much money is Abdul gonna have?

Abed: 1 dollar.

Mustapha: /1 dollar/.

Fatima: How much money is Raja gonna have?

Rachet: 1 dollar.

Mustapha: /1 dollar/.

Fatima: 1 dollar.

Mustapha: 25, 50, 75. This 100. 1 dollar [Referring to Ameer's coins. He counted up by 25, giving him 100 and is equivalent to 1 dollar].

As we see in this transcript, the students decided to start with Raja, and I asked them about what coins he had. Rachet focused on the number of coins that Raja could have, and Abed focused on the type of coin Raja had by suggesting a 25 cent coin, which was not possible given that Raja had less than 3 coins and equaled a dollar. As such, I reminded them that Abdul and Raja both had the same amount of money as Ameer, and then I asked them to tell me how much money Ameer had. In response to my question, Rachet referred to the number of coins Ameer had (4 coins). Then, Abed referred to the number of coins they all had in total (12—based on his understanding that each had 4 coins). Mustapha then counted up by 4 to confirm or check Abed's answer, reinforcing Mustapha's mathematical authority. Rachet disagreed by sharing that he already said 12 and it was not right, reinforcing Rachet's mathematical authority

Next, we see that I focused their attention on the number of coins Ameer had. They agreed that Ameer had 4 coins in total. We then considered the amount of money Ameer had. Abed responded by sharing that Ameer had a dollar, but then he referred to what each coin was worth (25 cents). I rephrased my question to imply the total amount of money Ameer had, and Abed responded with a dollar. I confirmed his answer and then I asked, "if Ameer has a dollar, and you have the same amount of money as Ameer, how much money would you have?" It was clear from Abed's response that he understood that Ameer would have a dollar. I then asked them about Abdul and Raja, and they agreed that each person would have a dollar.

The viewing of the video showed that Mustapha took the whiteboard and wrote 25, 25, 25, 25. He added them together by counting up by 25 and confirmed that a hundred was equivalent to 1 dollar. He then wrote another set of 4 twenty-fives without making any reference to them.

Stanza 9

In the following part of the transcript, it emerged that up to this point, Rachet was unaware of how much Ameer's coins were worth.

Rachet:	This is 25 cents [Pointed to a coin on the activity sheet]?
Abed:	Yeah.
Fatima:	Yes. 25 cents, plus 25 cents, plus 25 cents, plus 25 cents. Is how much [Pointed to each of the 4 coins on the handout]?
Rachet:	Oh.
Mustapha:	25 plus 25-,
Abed:	25 cents plus-. 1 dollar.

Rachet:	This is-, /It's 50 plus 50/. [Pointed to two coins, then the other two].
Fatima:	Which is how much?
Rachet:	One hundred.
Mustapha:	/100/.
Fatima:	So that's what Ameer has, right?
Rachet:	Ameer has 1 dollar now?
Fatima:	Yes, 1 dollar.
Rachet:	Ok.

In this stanza we notice that Rachet pointed to one of Ameer's coins on the handout and asked if it was 25 cents. Abed and I confirmed that it was 25 cents. Here, I wondered if Mustapha's representation on the whiteboard influenced Rachet's awareness of what Ameer's coins were. Also, in asking what the coin was worth, Rachet develops agency by enquiring about what he does not know.

Next, we see that I referred to each coin and I asked about how much they equaled to. Mustapha attempted to add the coins, but Abed was quick to share the answer (1 dollar). The viewing of the video showed Rachet grouping the quarters into two groups by pointing to them and adding them together (i.e., 25 + 25 = 50 and 25 + 25= 50). I asked Rachet about how much they equaled to, both Rachet and Mustapha responded by sharing a "100". With intonation, Rachet then asked, "Ameer has 1 dollar now?". I confirmed his answer. The fact that Rachet grouped the quarters by 2 to acquire the sum is important because it reflected his understanding of grouping addends to simplify the addition process.

Stanza 10

We continued to unpack the activity, and as shown in the transcript in this stanza, we focused on Abdul's coins. However, I started by counting Ameer's coins as a comparison to the statement *Abdul has more than 4 coins*.

Fatima:	Okay, Abdul has more than 4 coins. And here Ameer has coin number 1, number 2, number 3, and number 4. [Pointing to the coins on the activity sheet].
Rachet:	Abdul 5. 5 [Referring to the number of coins Abdul had].
Fatima:	You're thinking 5? (0:07:48.0)

Rachet: Yeah.

Abed: Yeah.

Fatima: Ok. 1, 2, 3, 4, and 5. Right? [I drew 5 coins on the whiteboard].

Rachet: /Twen-, wait. 25 cents. 25, 25, 25, 25/. It's 125. [Wrote 25 in each of the coins drawn on the whiteboard, giving him 5 quarters].

Abed: /It's, it's, it's/ 4 dollars and 25 cents.

Rachet: It's 1 dollar and-,

Fatima: But it says here that Abdul and Raja both have the same amount of money as Ameer.

Rachet: Then it's 1 dollar and 25 cents

Abed: Oh, it means that we add all of them together.

As we see in this transcript, Rachet and Abed considered Abdul to have 5 coins. I drew 5 circles on the whiteboard as a representation of 5 coins. Rachet pulled the whiteboard towards him and wrote 25 in each of the circles, representing 5 quarters. He then shared that the total would be 125. Notice how Rachet considered Abdul to have the same amount of coins as Ameer (i.e., four 25 cent coins), but also had an additional coin given that he had *more than 4 coins*. Abed then shared, “it’s 4 dollars and 25 cents”. However, it was not clear what he was referring to. I repeated the statement, *Abdul and Raja both have the same amount of money as Ameer*, in an effort to help them realize that their answers did not match the amount Ameer had. Rachet still suggested that Abdul had a dollar and 25 cents, and Abed shared that we needed to add them together. Here, I think Abed was suggesting we add Abdul and Raja’s coins together based on his interpretation of the statement *Abdul and Raja have the same amount of money as Ameer.*

Stanza 11

As seen in this portion of the transcript, we continued our discussion, focusing again on Ameer’s coins in an attempt to help them make the connection between the amount of money Ameer had and the fact that Abdul and Raja had the same amount of money as Ameer.

Fatima: You all have the same amount of money as Ameer. So, if you have the same amount of money as Ameer how much do you have?

Abed: I have 1 dollar and 25 cents.

Fatima:	How much does Ameer have? How much money does Ameer have?
Abed:	1 dollar and 25 cents.
Fatima:	1, 2, 3, 4 [Counted the coins on the paper].
Mustapha:	1 dollar.
Abed:	1 dollar.
Rachet:	1 dollar.
Fatima:	1 dollar. So how much money will you have if you had the same as Ameer?
Abed:	1 dollar.
Fatima:	Does that look like a dollar to you? [Pointed to the whiteboard]
Rachet:	No.
Abed:	No.
Rachet:	It's 1 dollar and 25 cents.
Fatima:	So how can we make that a dollar?
Abed:	1 dollar [Erased the 25 in one of the coins and wrote 1] .

In this stanza we see that I started off by asking the students to tell me how much money they would have given they had the same amount of money as Ameer. Abed responded by referring to the amount on the whiteboard that equaled 1 dollar and 25 cents. I then asked them about how much money Ameer had by referring to the handout and counting Ameer's coins. The students all agreed that Ameer had "1 dollar". I confirmed their response and then I asked them about how much money they would have if they had the same amount of money as Ameer. Abed responded with, "1 dollar". I then pointed to the coins on the whiteboard and asked if the 5 coins equaled a dollar. Both Rachet and Abed agreed that the coins did not equal a dollar, and Rachet confirmed that they equaled 1 dollar and 25 cents. Abed then erased the 25 in one of the coins on the whiteboard and wrote the number 1, and proceeded to erase the rest. I think Abed wrote a dollar as a representation of the total amount of money Abdul and Raja had which is the same amount of money that Ameer had.

Stanza 12

In the next segment of the transcript, I focused on visually explaining to the students how Ameer's coins equaled a dollar. Then, I asked them to focus on the 5 coins drawn on the whiteboard and to think of how they could also equal a dollar.

Fatima:	Okay, boys look. How much did you say Ameer has?
Abed:	1 dollar.
Fatima:	Okay. So, these 4 coins equal-, [Referred to Ameer's coins on the handout]
Rachet:	One dollar.
Abed:	/One dollar/.
Fatima:	A dollar. That's right. So, let's put a dollar here [Wrote $1.00 next to Ameer's coins].
Rachet:	25 cents [Pointed to Ameer's coin].
Fatima:	Yes, each one is 25 cents and together they equal-, how much?
Rachet:	One hundred.
Fatima:	One hundred. Which is the same thing as a dollar. Right?
Rachet:	[Nods]
Fatima:	And you're thinking that Abdul has 5 coins. But these 5 coins have to equal-,
Rachet:	One dollar.
Abed:	/A dollar/.
Fatima:	A dollar. Can I have the marker? Okay, they have to equal one dollar. [Wrote an equal sign (=) and $1.00 next to Abdul's coins on the whiteboard]
Mustapha:	/5, 10, 15, 20, 25/ [Skip counting by 5].
Fatima:	How can I make them equal 1 dollar?
Rachet:	Um, it's 125 cents.
Abed:	Just put 4 coins.
Mustapha:	No, it's 100.
Rachet:	No, it's 1 dollar 25 cents.
Abed:	Or 2 dollar and 25 cents.

Rachet: I mean 1 doll-,

Fatima: It has to be a dollar guys. But it has to equal one dollar [Referring to Abdul’s coins as I pointed to the whiteboard].

Abed: It's 3-,

Fatima: No, no, leave it Mustapha [Mustapha tried to erase the coins on the whiteboard and I asked him to leave them].

In this stanza we notice that the students and I agreed that Ameer had 4 coins and each was worth 25 cents, which equaled a hundred and was equivalent to one dollar. I then referred to the coins I drew on the whiteboard and explained that when we add the coins together, they should equal 1 dollar.

Next, I asked the students how I could make the 5 coins equal a dollar. Rachet responded by sharing that Abdul had 125 cents. It still seemed that Rachet was considering the 4 quarters (25 cents) that Ameer had plus an additional 25 cents as a representation of Abdul’s coins. Abed then suggested we use 4 coins as a representation of Abdul’s coins, which are the same as Ameer’s coins. Mustapha responded by sharing that “it’s 100”—perhaps suggesting that Abdul’s coins should equal 100 cents. Rachet repeated that Abdul had 1 dollar and 25 cents, and Abed suggested that it could be 2 dollars and 25 cents—implying, Ameer’s 4 quarters plus Abdul’s 5 quarters would equal 2 dollars and 25 cents. I then repeated that Abdul’s coins had to equal a dollar, and Abed said, “it is 3”— suggesting they all had 3 dollars in total.

Stanza 13

As you will see in the next portion of the transcript, two students (Raj and Sajan) walked into the library proudly sharing that they have completed the activity and were waiting for the second activity.

Raj: We're waiting for the next one, come on [Two students walked into the library].

Rachet: It's hard bro.

Raj: It was so easy.

Rachet: No, it's hard.

Raj: We're done.

Rachet:	Mrs. Holland help you.
Raj:	Un, un [no, no]. Mrs. Holland didn't help me. I know what is it.
Fatima:	What is it Raj? Do you want to tell them what it is?
Raj:	Why?
Fatima:	So how many coins does Abdul have?
Raj:	You have to put 10 cents and 1 dollar.
Fatima:	10 cents and 1 dollar?
Raj:	Mrs. Holland told me that and I'm done.
Rachet:	You see Mrs. Holland helped you.
Abed:	1 dollar and 10 cents [Repeated Raj's answer].
Fatima:	No, it has to be a dollar. It has to be a dollar.
Rachet:	/Mrs. Holland helped you./
Raj:	Yeah, you have to put like 10 cents up to like 10. Wait. 1, 2, 3. Wait, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 [Quietly skip counted by 10 up to 100].

As we see, Rachet, who was not impressed by Raj's comment, explained that the activity was hard, except the other student (Raj) was quick to share that he found it easy. Rachet then suggested that Mrs. Holland helped them. Notice how the students expressed agency by sharing how they felt about the activity, reinforcing their mathematical identity. The students continued to argue, but then I asked Raj to share how he solved the activity. He responded by sharing that we have to put 10 cents and 1 dollar, and Abed repeated after Rachet. It was not clear to me what Raj intended by 10 cents and 1 dollar, and I repeated what he said with intonation. Raj shared that Mrs. Holland confirmed his answer and he was done, reinforcing the teacher's mathematical authority.

Next, we see that I explained that Abdul's coins had to equal a dollar. Raj then shared that we have to put 10 cents up to a 100, and he quietly skip counted by ten on his fingers. It was not clear to me at the time what Raj was doing because I was distracted by the other students I was working with. However, the viewing of the video suggests that Raj was referring to the coins that Abdul could have, and that's ten 10 cent coins.

<u>Stanza 14</u>

In the following part of the transcript, I focused on the 5 coins drawn on the whiteboard by asking the students to give me 5 coins that equaled a dollar.

Fatima:	Give me 5 coins that equals a dollar?
Mustapha:	25 plus 25, plus 25, plus 25 equal 1 dollar.
Fatima:	But Abdul has more than 4 coins. And we've decided to start with 5 coins.
Mustapha:	5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100. [Mustapha quietly skip counted up by 5 up to 20 times].
Abed:	Oh, 9 dollars.
Fatima:	Where are you getting 9 dollars from?
Rachet:	Is it 125 take away-,
Fatima:	It all has to be the same as Ameer's. Ameer has a dollar, so Abdul is going to have a dollar. But with these coins. So, what are Abdul's coins?
Mustapha:	It's 100 [Said that in connection to his skip counting by 5. Then, he got up and walked towards the other students].

As we see, in response to my initial question, Mustapha shared that 4 quarters equal a dollar. I responded by clarifying that Abdul had more than 4 coins and that we agreed on 5 coins. Mustapha then skip counted by 5 up to a 100. It was not clear to me what Mustapha intended by skip counting by 5.

Next, we see that Abed responded by sharing 9 dollars. I asked him where he got 9 dollars from, but he only smiled. Again, it was not clear what he intended, but I wonder if he added the number of coins he assumed each person to have had. Rachet then suggested we subtract the fifth coin that Abdul had to make it 4 coins and equal to Ameer's coins. Notice, how Rachet was still thinking about the same amount of money to imply the same type of coins and the same number of coins as Ameer. I then explained that since Ameer had a dollar then Abdul's coins should also add up to a dollar.

Stanza 15

As you will see in the next portion of the transcript, we continued to focus on the type of coins Abdul could have.

Fatima:	What are Abdul's coins? What coins does he have? Let's see.

Rachet:	25 cents.
Fatima:	Okay, 25 [I wrote 25 on the whiteboard]. What else does he have?
Rachet:	25.
Fatima:	Ok [Wrote another 25].
Abed:	That's all 1 dollar and 25 cents.
Fatima:	They're not all going to be the same as Ameer's coins. Some of them can be different.
Abed:	It's 2 dollars.
Fatima:	It's 2 dollars together. But these one's [pointed to the coins on the whiteboard] don't have to all be 25 cents. They can be different from what Ameer has. So, if I have 50 cents now. How much more do I need to make a dollar?
Raj:	/No. Put 10, 10, 10, 10 up to 100. Put 10, 10, 10, 10 up to 100/.
Fatima:	/How much more do I need to make a dollar if I have to fill these coins?/
Rachet:	125 take away 25.
Fatima:	All the coins we use have to equal just a dollar.
Rachet:	I'm so confused.
Mustapha:	Is it 110?
Fatima:	Okay. Abdul has more than 4 coins. And we've decided to start with 5 coins. So far, we have 25 plus 25. And then here I will add another 25. So right now, I have 75 cents, right?
Abed:	Yeah.
Fatima:	What other two coins can we use to make a dollar?
Abed:	/A dollar/.
Raj:	Two 10 cents.
Abed:	Two 10 cents.
Fatima:	Okay, if I put 10 here [Wrote a 10 in one of the coins]. That's 75 plus 10 is how much?
Abed:	80-, 85.
Fatima:	Plus, another 10?
Abed:	95

Fatima: That's 95.

Rachet: 1 dollar and. Wait. Plus 5. That's a dollar. So, we need 5 more. That's a dollar.

Fatima: Yeah and that's a dollar. You see [Drew an extra circle and wrote the number 5].

Rachet: Yeah.

Fatima: So, how many coins does Abdul have?

Rachet: 1, 2, 3, 4, 5, 6. 6 coins.

Abed: 6 coins.

Fatima: Yeah that's how many coins Abdul has [Pointed to whiteboard].

As we can see in this part of the transcript, we started to fill each coin on the whiteboard with an amount. Rachet suggested 25 cents, then he suggested another 25 cents, and I wrote them on the whiteboard. I then asked them about what other coins we could use, but Abed shared that the coins will add up to a dollar and 25 cents—assuming we will continue to fill the remaining coins with 25 cents.

Next, we see that I explained to the students that Abdul's coins will not necessarily be the same as Ameer's coins and that they could be different. Abed then shared that it will be 2 dollars. Here, I think Abed was referring to the amount of money Ameer and Abdul will have in total—considering Ameer had a dollar and Abdul had a dollar. I confirmed that together, Abdul and Ameer's coins would equal 2 dollars. Then, I referred to Abdul's coins on the whiteboard and explained that Abdul's coins do not all have to be 25 cents and they could be different from Ameer's coins. I then combined the two 25 cent coins together which equaled 50 cents, and I asked what other coins could Abdul have that would add up to or make a dollar.

The viewing of the video showed that Raj did not agree with our use of the 5 coins and suggested we put 10 tens up to a hundred. At the time, I missed what Raj said as I was too focused on asking Rachet and Abed about other coins they could use to fill the those on the whiteboard.

Notice then how Rachet suggested we takeaway 25 from 125. Here, Rachet returned to his earlier claim, and that is, if we wrote 25 cents in all 5 coins, it would equal 125, but since they need to equal a dollar, Rachet suggested we take away 25 cents. Rachet then shared that he was confused, and Mustapha asked if the answer was a hundred and ten.

I then reminded them of what we had agreed on in terms of Abdul having more than 4 coins, and that we thought he could have 5 coins and so far we considered two of his coins to be 25 cents. I then wrote an additional 25 in one of the coins which equaled 75 cents. Abed agreed and I asked about what other coins we could use and that will add up to a dollar. Raj suggested we use "Two 10 cents", and Abed repeated Raj's suggestion. I then wrote 10 cents in the two coins, and they concluded that the total would 95 cents. Notice how Rachet realized that we needed to add 5 more to get a dollar. I drew another coin and wrote 5. Rachet then counted Abdul's coins and concluded that he had 6 coins in total. Abed agreed and I confirmed their answer.

<u>Stanza 16</u>

After we dealt with Abdul's coins, we focused on Raja's coins as we can see in the transcript below.

Abed:	What was the question?
Fatima:	We have one more. Raja has less than 4 coins. So how many coins does Raja have?
Rachet:	3, 2, or 1.
Fatima:	Okay let's say he has 3 [I drew three coins on the whiteboard]. What 3 coins will give me a dollar?
Abed:	25 cen-, Uh, 25 cents.
Rachet:	50 [Pointing to one of the coins on the whiteboard].
Fatima:	Is there a coin that's 50 cents?
Abed:	10. 10. 10. [Referring to each coin as being worth 10 cents]. 10, 20, 30 [Skip counting by 10 to add the three coins together]. Oh, doesn't work.
Fatima:	But it has to equal a dollar.
Rachet:	50 and 25. 25. [Referred to one coin as 50 cents and the two coins each worth 25 cents].
Abed:	Oh yeah! 50 and 25. 25. It makes a dollar.
Fatima:	How much?

Abed:	50 and 25 and 25 makes a dollar.
Rachet:	50 cents plus 25 cents and 25 cents.
Abed:	It makes a dollar because 50 plus 25 is 75 plus 25 is 100.
Fatima:	That does work. But is there a coin that is 50 cents?
Rachet:	No.
Abed:	Yeah 1 dollar and 50 cents.

As we see in this transcript, the discussion started with Abed asking me what the question of the activity was. I responded by sharing that we have Raja's coins left to work with. Rachet started by sharing that Raja could have 1, 2, or 3 coins. I drew three circles on the whiteboard and asked what three coins equaled a dollar. First, Abed suggested we use 25 cents, but quickly changed his mind and considered each coin as a representation of 10 cents. He then counted up by tens and realized that 3 ten cent coins did not add up to a 100 or a dollar. I reminded him that the coins had to equal a dollar.

The viewing of the video showed Rachet pointing to one of the coins on the whiteboard and then suggesting we use 50 cents. I asked if there actually was a 50 cent coin. He then pointed to each coin and suggested we use "50, 25, and 25". Abed expressed agreement by reiterating Rachet's suggestion and confirmed that they would equal a dollar. Rachet responded by emphasizing the need to add them together. Abed added them together, equaling a 100. Although their solution does not satisfy the activities criteria, the fact that they chose these three numbers is important because it shows their understanding of these three addends equaling a dollar

I then wrote down the numbers they shared in the coins on the whiteboard and I confirmed that the numbers they chose equaled a dollar or 100 cents. But then I asked if there was a coin worth 50 cents. Rachet acknowledged that it does not exist, but Abed confirmed that it does and shared an example, "yeah, 1 dollar and 50 cents". Here, I think Abed might have relied on his broader mathematical or world knowledge to conjecture that a 50 cent coin does exist such as, when we make a purchase or refer to an amount we owe.

<u>Stanza 17</u>

As the reader will see in the next portion of the transcript, Rachet realizes that using 3 coins will not work for Raja's coins.

Rachet:	We can have like 10 cents, 5 cents and 25 cents.

Fatima:	And how much does that equal to?
Rachet:	10 plus 5 is 15. 15 plus 25 is 5 plus 5 [Decomposed the 15 and 25 into ones and tens].10 [Added the digits in the ones place together]. 10 and 20 [Added the digits in the tens place together]. 30 plus 10 [Added the sum of the digits in the tens place and the sum of the digits in the ones place, giving him 40]. Yeah, not 100.

In this stanza we notice that Rachet suggested we use 10, 5 and 25 cent coins as he held up 3 fingers as a representation of three coins. I then asked Rachet how much these coins would equal to. In response, Rachet added 10 plus 5, giving him 15. Next, he decomposed the 15 and 25 into ones and tens (15 was decomposed into 10 plus 5, and 25 was decomposed into 20 plus 5). Then, he added the digits in the ones place (5 plus 5), giving him 10. He then added the digits in the tens place (10 plus 20), giving him 30. After that, he added the sum of the digits in the tens place (10) and sum of the digits in the ones place (30), giving him 40. The sum of the three coins led Rachet to realize that they do not equal 100 or $1.00.

<u>Stanza 19</u>

In the next part of the transcript, I asked the students to suggest other coins that Raja could have.

Fatima:	So, if that doesn't work, what else could it be?
Raj:	One dollar.
Fatima:	What did you say Raj?
Rachet:	One dollar
Raj:	/One dollar/.
Fatima:	How?
Raj:	1 dollar. 1 coin.
Rachet	Yeah. He has 1 dollar because less than 4 coins.
Fatima:	4 coins. So how many coins does Raja have?
Rachet:	1. 1 dollar, 1 dollar, 1 dollar, 1 dollar.
Abed:	/1 dollar, 1 dollar, 1 dollar, 1 dollar/.
Fatima:	And how many coins does Abdul have?
Rachet:	6 or 1 dollar.

Fatima: Do you get it?

Abed: Yeah.

Rachet: [Nodded].

Notice that during this time Raj was attentively listening to our conversation, and in response to my question, he shared that Raja had one dollar. Rachet repeated after him, and then Raj explained that a dollar is one coin. Rachet and Abed agreed that Raja had one coin and equaled a dollar. We also agreed that Abdul had 6 coins and equaled a dollar. A viewing of the video showed that Mustapha was distracted by the other student (Sajan) and moved away from our group. This then concluded our interaction and we joined the rest of the class for a math talk related to the activity.

For the math talk, the teacher focused on asking the students to share the strategies they used to solve the activity. The students' choice of coins were quite diverse. For example, one group shared "8 dimes and 4 nickels", another group shared "10 dimes", still another shared "20 nickels", another had "100 pennies", and another had "4 dimes, 2 nickels, and 2 quarters".

When it was our groups turn to share, Rachet and Abed volunteered to do so. They shared that Abdul had 6 coins and identified the coins he had, they said "we got three 25 cent, two 10, and one 5. And it's 100. One dollar". The teacher asked them to explain how they added the coins together. Rachet explained, "25 plus 25. 50. And 25 is 75. Plus 10. 85. Plus 10. 95. Add 5 is 100. Or 1 dollar". As for Raja's coins, most, if not all groups agreed that Raja would have one coin (a loonie or one dollar) given that he had less than 4 coins and equaled a dollar. The fact that the students clearly shared their solution with the rest of the class, demonstrated their understanding of the activity.

Summary of Main Ideas: Activity #2 "Ameer's Coins"

In what follows, I discuss children's meaning-making by using my analytical framework that draws on Gee's (2005) building tasks of "language in use" as described in **Table 3** in the methodology chapter. These building tasks include, significance, activity (i.e., mathematical actions), identity, relationships, politics, and sign systems or knowledge.

Significance. Several different aspects in the transcript appear to become significant to the students' mathematical meaning-making. These include the student's developing sense of mathematical concepts, the strategies they used, and concrete tools that appear significant to their understanding of mathematics in general, and more specifically, to their understanding of the activity.

My analysis of this data shows that the students appeared to be developing a sense of what the concepts “more than”, “less than”, and “the same amount of money as” mean as a way to compare and understand quantity relationships. Hence, students’ understanding of the difference between “more than”, “less than”, and the “the same amount of money as” were very significant in this activity. For the most part, we see that the students know that Raja has less than 3 coins; and they know their choices are 1, 2, or 3 coins. They know that Abdul has more than 4 coins; and they know their choices are 5, 7, 8, 9, 10, or even one billion coins. However, when focusing on the statement, *Abdul and Raja both have the same amount of money as Ameer*, they would confuse “the same amount as Ameer” to imply the same ‘number’ and ‘type’ of coins as Ameer. For instance, at one point, Abed acknowledged that the boys each had a dollar given that Ameer had a dollar, but by also assuming that the boys had the same coins as Ameer—that is, 4 quarters each. Thereafter, it appears that they understand the concept, but then they misunderstand it. This speaks to what Moschkovich (2008) refers to as “multiple interpretations”, which draws our attention to the fact that when the children get confused, we should expect that they are not looking at, talking about, referring to, or imagining the same things (Moschkovich, 2008). These ambiguous and shifting meanings did not prove to interrupt the mathematical discussion, but rather were significant to their understanding of the activity.

It is evident that the students relied on certain strategies which appeared to be meaningful to them. These include the use of counting, skip counting, counting forward and backward, adding and subtracting money amounts coupled with an understanding of number and quantity, as well as, compare money amounts, decompose numbers into ones and tens, and use everyday situations as contexts for making sense of the activity. The transcript provides evidence that the students could add the total number of coins and the amount of money by relying on skip counting, or decomposing the numbers to simplify the addition process. They also appeared to have developed a quantifiable or countable understanding of 25 and 50 when adding the amount of money that they assumed Abdul had. The use of everyday situations or world knowledge was one other strategy that seemed significant to students’ meaning making. For instance, when we were dealing with Raja’s coins, Rachet suggested we use a 50-cent coin plus 25 plus 25, which equaled a dollar. Then, there was a discussion of whether a 50-cent coin existed in real life. Abed confirmed that it does and shared an example, “yeah, 1 dollar and 50 cents”. Here, I think Abed might have relied on his broader mathematical knowledge to conjecture that a 50 cent coin does exist in our day-to-day encounters with money.

Particular concrete tools appeared to be significant, such as their fingers, the activity handout, and the individual whiteboard. Their fingers were used to represent what they were saying, as well as for pointing, counting, skip counting by 5 or 10, and adding and subtracting money amounts. The activity handout was consistently used to refer to Ameer’s coins, and also, as evidence for their claims. For instance, at a certain point, Rachet pointed to the statement *Raja has less than 4 coins* to clarify to Mustapha that Raja could not have 4 coins because he had less than 4 coins. In terms of the individual whiteboard, the writing of numbers or drawings made on it were significant. For instance, the students used it to represent a number or the quantity of coins they were referring to. It was also used to draw coins as a representation of the coins Abdul and Raja had.

Activities: Mathematical Actions. The analysis of the transcript brings forth the evidence of mathematical actions as described in **Table 3** that portrayed the students’ use and development of mathematical thinking skills. These mathematical actions include the use of reasoning and proving skills, communication of ideas, representation of ideas using various tools, and making connections between ideas or concepts. I will talk about each in turn.

Reasoning and Proving. In terms of reasoning and proving, the transcript provides evidence that the students reflected on their own solutions and those shared by others by justifying their answers, explaining their choices, negotiating understanding, and making mathematical conjectures to determine whether or not their solutions satisfied the activities objectives. For instance, when we first began working on the activity, the students were quick to share their understanding or solutions by focusing on specific statements of the activity. However, how students individually made sense of certain statements differed significantly among the students leading them to explain and justify their understanding. Just as an example, at one point, Abed claimed that “everybody gets 1 dollar. Because 4 coins. And every 4 coins make 1 dollar”. Mustapha, on the other hand, focused on the number of coins the boys had in total—reasoning that they each had 4 coins and equaled 12 coins in total. As for Rachet, he considered Abdul to have 4 coins and Raja to have 2 or 3 coins, and suggested they equaled either “6 or 7” coins—depending on the total coins Raja had.

Similar ideas and reasonings continued to circulate in our discussion, even more so, when we focused on Abdul's coins. It was only when we gradually started filling the coins on the whiteboard with various amounts that the students saw how the coins could add up to a dollar and not necessarily be the same as Ameer's coins but equal the same amount. The same was also true for Raja's coins. They continued to negotiate understanding by reflecting on the coins they could use and what they add up to using various strategies.

Communication. The students also relied on oral and visual communication to share their mathematical understandings, ideas, and to use various strategies. For example, the students communicated when they 1) responded to a question, asked a question, or posed a concern; 2) collaborated on reading the activity; 3) shared their interpretations of the different statements in the activity—with a focus on "less than", "more than", and "the same as"; 4) shared the total number of coins the boys had; 5) shared the total amount of money the boys had; 6) used various strategies to express their thinking or ideas (i.e., adding, subtracting, counting, counting up, skip counting, etc.); 7) justified their answers, explained their solutions, negotiated understanding, and made mathematical conjectures; 8) elaborated on or repeated what someone else said; and 9) when they relied on the handout as evidence for their claims, or to refer to Ameer's coins.

Representation. The analysis of the transcript brought forth several tools that students relied on to represent their mathematical thinking including, their fingers, the small whiteboard, and the coins on the activity handout. In terms of their fingers, the students used their fingers to create a representation of the number they were referring to, or as a representation of the number of coins they needed. Just as an example, Abed held up 3 fingers as a representation of three dollars. Their fingers were also used as a representation for counting, counting forward, and for skip counting by 10 and 5. At a certain point, Mustapha used his fingers to skip count by 5 up to a 100, and where, each finger was a representation of 5. The students also used the small whiteboards to write representations of the numbers they were referring to. For instance, Mustapha wrote two sets of four 25 cents (i.e., 25, 25, 25, 25) as a representation of Ameer and Abdul's coins. I also initiated the use of the whiteboard to draw coins, to represent either Abdul or Raja's coins as well as money amounts up to a dollar. The activity handout was used as a representation of Ameer's coins, and also, as a representation for their claims.

Connections. It was also evident that the students made connections between mathematical ideas. Some of these included making a connection between 1) a number name, a number symbol, and a representation of quantity (e.g., 25 [Spoken], 25 [written], 25 + 25 + 25 + 25 = 100 or $1); 2) adding and subtracting (i.e., the increase or decrease in quantity)—this was evident when Rachet suggested that they take away one of Abdul's 5 coins to match them with Ameer's 4 coins; 4) repeated addition and skip counting—here, the students considered each of the three boys to have 4 coins, and concluded that 4 plus 4 plus 4 is 4, 8, 12 by skip counting; 5) the concept of quantity and an understanding of the amounts represented by the coins, for example, the student recognized that 4 quarters or 4 twenty-five cents equaled a dollar or 100 cents; 6) connections between quantities in terms of "more than", "less than", and "the same amount as"—this was especially so when the students were making sense of the number of coins each person could have based on the terms noted in the task; 7) their everyday experiences and the activity, for example, when Abed suggested that one of Raja's coins could be 50 cents based on his understanding of something we buy or owe being worth 50 cents; 8) between what they are saying and the statements of the activity, for example, when I questioned Mustapha's response, he responded by making a connection to a statement in the task to back his claim or answer; and finally 9) the students made connections between what they or others have stated in order to elaborate and reiterate given information, such as when the students agreed that the boys had 4 coins each and explained their solutions by repeating or elaborating on what another student said.

Identities. As the students engaged in the activity, we can see the different roles that the students play and the individual identities that were built. These identities continued to reflect the roles they enacted and how they developed mathematical ideas and understandings of the activity. Overall, the students expressed autonomy and agency in various ways by developing their own interpretations of various statements in the activity, by reflecting and directing their own learning, and through the use of their own strategies. It also seemed as though their identities changed during the activity. In the beginning, they appeared to have identified with the activity and were eager to share their solutions, but as the discussion progressed and they started sensing confusion, they merely cooperated with me and appeared a little disengaged. For instance, Rachet established a mathematical identity that recognizes his autonomy in trying to build his own understanding and in attempting to develop a solution to the activity. He also developed agency by enquiring about what he does not know. At one point, Rachet acknowledged that he was confused and that the activity was hard, and although he would appear disengaged, he was readily determined to find a solution and did not give up.

Abed developed a mathematical identity that portrayed him as taking responsibility for his learning by also trying to reason his way to a solution. He appeared to identify with the activity by guiding, correcting, and explaining to others his interpretation of the problem. At one point, he too appeared a little disengaged, but once he developed some understanding, he was eager to reflect on his thinking.

Mustapha expressed a mathematical identity that portrayed him as building and developing his own understanding dependent on his interpretation of the problem and the discussions that unfolded. He was also enthusiastic to share his thinking and answers to questions I posed. In general, he identified with the mathematics activity up until he was distracted by the presence of the students who joined us in the library. Mustapha also demonstrated agency through his self-initiated use of the whiteboard and his fingers to create representations of what he or others were saying.

Relationships. Analysis of this transcript also revealed several different relationships that were prominent in students' interactions and may influence their mathematical meaning-making. These included the relationship between the students and the problem-solving activity, the relationship between the students and myself, the relationship between the students and their classroom teacher, and the relationships between the students as they interacted.

By far, the activity appeared a little challenging for the students. However, students' levels of confidence correlated with the manner in which they engaged in the activity. In general, the students portrayed high levels of confidence as reflected by their eagerness to find a solution to the activity. They worked hard and continued to try longer despite the confusion they experienced. For instance, Rachet portrayed confidence by reflecting on and correcting other students' answers based on his interpretations of the activity. Abed reflected confidence in the activity by justifying his answers when others questioned or negated them. Mustapha portrayed confidence by not simply relying on other's answers but by negotiating his mathematical understanding. Raj portrayed confidence in explaining his solution for Abdul's coins by skip counting by 10.

The relationship the students and I developed during our interactions was a peculiar one because as much as I tried not to intervene, the ambiguous and shifting meanings they continued to develop, prompted me to get involved in an attempt to guide, clarify, or focus the discussion for the purpose of furthering understanding.

In terms of the relationship between the students and their classroom teacher, this was particularly evident when Rachet questioned Raj for having finished the activity, claiming that Mrs. Holland helped him find the answer. And, at a certain point, Raj does validate or back his solution by sharing, “Mrs. Holland told me that and I’m done”. In so doing, Raj portrays his teacher as accessible, caring, the one in charge, and possess mathematical authority.

The data also revealed the relationship that was apparent between the students as they interacted. This was particularly evident in how they communicated with one another. I noticed that as the students communicated their interpretations, ideas, or their thinking, they were constantly helping one another build their understanding of the activity. They did not appear at all competitive, rather it seemed as though they shared responsibility for each other’s learning by trying to discuss, justify, and explain their thinking to one another. All in all, they were deliberate in working together to find a solution to the activity.

Politics. I also noticed that there were several political occurrences in terms of the distribution of mathematical authority and that contributed to students’ meaning-making in the problem-solving activity. One demonstrates my role as a mathematical authority, another establishes their classroom teacher’s mathematical authority, still another we can see that various students took on mathematical authority, and another we can see the activity handout as a strong authority.

Concerning my role as a mathematical authority, I noticed that I portrayed mathematical authority by asking students to reflect on their answers by connecting it to the criteria of the task, as well as, to guide, clarify, or focus the discussion for the purpose of furthering their understanding. For example, at a certain point, I created a visual representation of the number of coins they assumed Abdul had and together we reflected on the types of coins he could have, in order to help them see Abdul’s coins as different and more than Ameer’s coins. In terms of their teacher’s mathematical authority, at one point, Raj confirmed his answer to be correct by referring to the fact that the teacher approved it or told him the answer.

There were also many instances where we can see that the students took on mathematical authority. For instance, Abed established his mathematical authority by trying to share his understanding of the activity and reasoning his way to a solution. Mustapha exercised mathematical authority by frequently monitoring the contributions shared by Abed and Rachet.

Rachet often portrayed mathematical authority by evaluating or confirming other students' answers or interpretations of the activity. Raj reflected mathematical authority by sharing his solutions to the activity.

The students also seemed to regard the activity handout as being a strong authority. This was particularly evident when the students would refer to the handout as evidence for their claims. It also seemed as though the handout would structure or refocus their discussions based on certain statements of the activity.

Sign Systems or Knowledge. Concerning what sign systems or knowledge were privileged, these were typically reflected in students' developing sense of specific words such as "dollars vs. coins", as well as, in students' use of certain strategies, and in their interpretation of specific statements. The knowledge of those who are seen with mathematical authority and have developed a strong sense of agency may be privileged over the knowledge of others. I noticed that for a good portion of our interaction Rachet would refer to the number of coins a person had as 'dollars' instead of 'coins', and Mustapha seemed to have picked up on the term. In so doing, their use of the word 'dollars' might have privileged that term and it may influence what the term implies or refers to. In terms of the strategies used, when Mustapha initiated the counting on his fingers to add the number of coins the boys had in total, Rachet as well as Abed imitated Mustapha by using their fingers to count or add the number of coins the boys had in total. I also noticed that students relied on someone else's interpretation of certain statements, ideally by those who had the most agency, Mustapha and Rachet. For instance, when Abed came to the consensus that each of the boys had a dollar, he was quick to change his mind by picking up on Rachet and Mustapha's solution and treating it as the solution to the activity.

Activity #3: "Times Tussle"

What was the activity? "Times Tussle"[8] was one of several mathematics games the students engaged in during their math centers (**Figure 26**). The objective of this game was to practice multiplying the numbers 2-10 by 5 and 10 using multiple strategies. Times Tussle is designed for groups of 2-4 students. *Materials:* Each group of students required a Times Tussle game board and numeral cards (4 copies, numbered from 2-10). Each player will need several markers (a

---

[8] Origo Education

different colour for each player). *Directions:* The students are asked to shuffle the numeral cards and place them face down in a stack. The first player draws a card and decides whether to multiply the number by 5 or 10 to make a product on the gameboard. For example, Sahar draws the number 8. She can multiply 8 x 5 or 8 x 10, and she can claim the product 40 or 80 on the gameboard by covering it with a marker. Although some of the numbers appear more than once on the gameboard, Sahar may only claim one number for each turn. If the two possible products are unavailable, the player misses a turn and the card is returned to the bottom of the stack. The first player to make a 2 x 2 square or a line of four adjacent markers (horizontal, vertical or diagonal) is the winner.

Figure 26

Times Tussle Gameboard and Numbered Cards

Times Tussle Game Board

20	50	25	50	10	30
70	30	10	90	45	80
35	40	25	40	15	45
80	15	50	100	90	35
45	25	20	40	50	100
45	25	30	20	30	15
70	60	35	60	20	40

What happened that day? The mathematics block unfolded as it normally would with a mathematics entry activity, followed by the morning calendar, and a brief lesson that acted as a review to the *Times Tussle* game. For the lesson, the teacher discussed and modelled various ways to find the product of 5 x 6 and 10 x 6. The teacher then divided the students into small groups. The groups consisted of 3-4 students, and the teacher assigned each group with a mathematics problem-solving activity.

On the previous day, I worked with Tamam and Suhail on *Times Tussle*, and I asked the classroom teacher if I could record them working together at a later time. I was quite impressed by how Tamam and Suhail were working together to reason their way to a solution. I was also interested in gathering more data working with Suhail and Tamam given that they were relatively new in the classroom. Mrs. Holland agreed and she asked Tamam and Suhail to work with me that day. Other students were interested to join our group, and we ended up with a group of 4 students including, Tamam, Suhail, Sabina, and Sahar. Sabina was also relatively new to the classroom. We decided to work in the school library as the classroom could get really loud during math centers. Each player had 4 turns in total and the activity lasted for approximately 15 minutes.

Stanza 1

This stanza shows the start of the activity with the group. It begins with me asking the students to get 5 square tiles each to be used as markers to play the game.[9] The students first decided on a colour for their markers, then picked tiles from the bin.

Fatima:	You can get 5 tiles each.
Sabina:	We only need 5. [Addressing the group]
Fatima:	5 each.
Tamam:	But I said I was going to be blue.
Sabina:	5. [Repeating what I said]
Fatima:	Tamam is blue girls. Okay?
Sahar:	I'm yellow.
Sabina:	Um I'll be red.
Sahar:	Or green because Suhail is red.
Sabina:	I'll be green. Green is a nice colour too.
Tamam:	1, 2, 3, 4, 5. [Counting her tiles].

[9] The tiles were to be placed on the gameboard where the number equals the product of two numbers—that is, the number drawn from the stack of cards and then multiplied by 5 and 10.

Sabina:	3. [Incorrectly referring to the number of counters she had (As there were two in her hand)].
Sahar:	4. [Referring to the number of counters she has].
Sabina:	4. 3. No. 2. [And then she grabbed a tile from the bin making 3 in her hand] 4. No. 3. [Grabbed another tile from the bin, making 4 in her hand] 4, 5. Where's 5? [Grabbed a 5th tile from the bin] 5.
Tamam:	Wait. I have 6. 1, 2, 3, 4, 5, 6 [Confirmed the number of tiles she had and counted them. She returns the 6th tile to the bin].
Suhail:	1, 2, 3, 4, 5. [Counting her markers]
Fatima:	Or 6. Or you could have 6. Get 6.
Sahar:	I got-, I got-. I don't know.
Fatima:	1, 2, 3, 4, 5, 6. Here Suhail you have 6. [Counting Suhail's tiles]
Tamam:	Wait. I want 6.
Sabina:	No, you could only get 5.
Fatima:	Or 6.
Sabina:	Really?
Fatima:	Here 6 [I passed Sabina a 6th tile].
Tamam:	Uhm wait. I can't find one. Oh, right there. 1, 2, 3, 4, 5, 6 [Grabbed an extra tile and counted them].

In this stanza we notice that Sabina repeated that they needed 5 tiles each, recognizing that she is paying attention to my authority and the rules. There was also some discussion about who will have the blue tiles. The students then picked the tiles from the bin, they counted a loud, or communicated the number of tiles they had. They all seemed to count their 5 tiles in different ways, displaying a different understanding of counting and number.

Notice that Tamam counted the tiles one-by-one as she picked them out of the bin and placed them in her hand. She realized that she had 6 tiles and decided to recount. She returned the extra tile to the bin. I then shared that they could each have 6 tiles. Tamam grabbed the tile back from the bin and recounted her tiles. I noticed that in counting and recounting, Tamam counted in a stable order and seemed to recognize that the last number she said tells her the quantity.

Sabina who decided to go with green tiles, grabbed 2 tiles from the bin and placed them in her hand. She looked at the tiles and tried to make sense of the number she had without counting. She incorrectly referred to the number of tiles she had, then she identified the number she had by referring to them as two. She then grabbed a tile from the bin and attempted to count up, making 3 in her hand. She incorrectly identified the number she had as 4, then she identified the number she had as 3. She reached back into the bin and got an additional tile and counted up, making 4 in her hand. In counting up, she went up to 5 and she realized that she did not have the 5th tile, she reached into the bin and grabbed another tile, making 5 in her hand. It appears that Sabina tried to make one-to-one correspondence between the tiles she had and a number, as the tiles were a representation of a number.

Suhail grabbed red tiles from the bin and placed them on the table. She then looked at the other students and she imitated them by counting her tiles. She picked the tiles off the table and counted them one by one, making 5 tiles in total. When she heard me say that they could each have 6 tiles, she reached into the bin and grabbed one tile. The fact that she grabbed just one tile is important because it illustrated her understanding of "movement is magnitude"—that is the idea that by increasing the number of tiles to 6, the quantity of tiles increased by only 1, making the connection between the number she had and the increase in quantity.

As for Sahar, she grabbed yellow tiles from the bin and stacked them in her hand. Then, she organized them side by side on the table and counted them one-by-one up to 8. When I said that they could each have 6 tiles, Sahar did not admit the number she had. It seems that she did not want to give-up any of her tiles. Sahar appears to see herself as having agency by not sharing how much she had, reinforcing her mathematics authority.

<u>Stanza 2</u>

In the next segment of the transcript, stanza 2, I asked the students to teach the group how to play the game.

Fatima:	Who's going to teach the group what we're supposed to do?
Sabina:	I don't know.
Sahar:	Um so. We have to um. / Is it groups? /
Fatima:	/So, Tamam, you know how to play this game/. You want to tell them what were supposed to do?

Sahar:	I think. Oh, it's groups.
Fatima:	What is groups? Groups of what?
Sahar:	I remember I did this game with Mrs. Holland.
Fatima:	So how do you play it?
Sahar:	So, um you. You can do this by tens and by fives. So, you pick a number and then it shows 9. You can do 90 [Holds up 9 fingers]. Or you can count by fives [Holds up 5 fingers]. All with your hands.
Fatima:	Yeah. And if we want to think about it in terms of groups, what do we say?
Sahar:	Um. Groups of.
Fatima:	Groups of. So, let's say I got 8. What do I say?
Sahar:	8 groups of 10. No 5.
Fatima:	Ok. Or 8 groups of-?
Sahar:	10.

In this segment of the transcript, we notice that Sabina shared that she does not know how to play, recognizing agency by reflecting on her knowledge or understanding of the game. In fact, Sabina was the only student who was not familiar with the game because she was absent when the classroom teacher introduced it to the class.

A viewing of the video showed that Sahar asked if this game is about "groups". I missed her question as I was asking Tamam to share with the group how to play the game. Tamam did not respond, then Sahar shared that she thinks it is groups. In response I asked, "what is groups? Groups of what?", and Sahar responded by sharing that she remembered playing this game with Mrs. Holland, again recognizing agency by reflecting on her knowledge of the game.

I then asked Sahar to explain how to play the game, and she said, "you can do this by tens and by fives. So, you pick a number and then it shows 9. You can do 90 [Holds up 9 fingers]. Or you can count by fives [Holds up 5 fingers]. All with your hands". It was interesting that she automatically said 90 when she held up 9 fingers. It is not clear whether she was skip counting in her head, or she just knew that 9 times 10 is 90, or if she even referred to what their teacher taught them with respect to their times 10 multiplication facts—and that is, any number multiplied by 10 is shifted to the left and you add a zero, or she used some other strategy. What is important here is that Sahar made a connection between 9 and 10 and the product 90. She also seemed to be using her fingers in two very different ways 1) she held up 9 fingers to represent 90, and 2) she held up 5 fingers to represent counting by 5. The use of her fingers seemed to help with the representation of a number and her mathematical meaning making.

Next, we see that Sahar and I had a discussion about the term "groups". I used the number 8 as an example and I asked, "let's say I got 8. What do I say?". In response Sahar suggested 8 groups of 10, but quickly changed her mind and said "No 5"—implying 8 groups of 5. I confirmed her answer and encouraged her to restate her initial response. Sahar completed my sentence by sharing the number 10—implying 10 groups of 5. In reviewing the transcript, I noticed that by drawing on the **concept** of "groups", I gave prominence and attention to that particular term, and the students may come to see that term as important, or it could define what we are doing in the game.

Stanza 3 (Turn 1 Tamam)

As shown in the transcript in this stanza and the transcripts that follow, each student played their turn for a total of four turns each. In this stanza, I started by asking the students who wanted to go first. Then, Tamam played her first turn.

Fatima:	Alright. Who wants to go first?
Tamam:	Can I?
Sahar:	Ok and then me.
Fatima:	Then Sahar, then Sabina, and then Suhail.
Sabina:	/Suhail/.
Sahar:	/Suhail/.
Tamam:	10 um. [Grabbed a card to play her turn. Then, read the number]
Fatima:	So, what did you get Tamam?
Tamam:	10.
Sahar:	5, 10, 15, 20, 25, 30, 35, 40, 45, 50. [Counted by 5]
Fatima:	So, are you going to do times 5 or times 10?
Tamam:	Wait.
Sahar:	Oh. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. [Recounted by 5]
Tamam:	5, 10, 15, 20, 25, 30, 35, 40, 45, 50. [Counted by 5]

Sahar: By tens or by fives? [Speaking to Tamam].

Tamam: Tens [Responds to Sahar].

Sahar: Ok. 10, 20-,10, 20-, [Counted by tens and stopped]

Fatima: Okay. So, you're going to do 10 times 10?

Sahar: 10 times 10 is a hundred. One hundred.

Tamam: One hundred.

Fatima: One hundred. Yeah.

As we can see in this part of the transcript, Tamam drew a card and shared it with the group. As she was still looking at her card, I asked if she wanted to do times 5 or times 10. Tamam then started counting by 5 using her fingers. Notice that during this time Sahar also quietly counted by 5 on her fingers. Sahar and Tamam's use of their fingers were a representation of skip counting by 5, and each finger was a representation of a group of 5 (i.e., 10 groups of 5). Also, by counting by 5 they made the connection between 5 and 10 and the product 50. Hence, the use of their fingers to find the product of 5 and 10 is significant because it illustrated their understanding and reasoning.

Sahar then looked at Tamam and asked, "By tens or by fives?". Notice how I said "times 5 or times 10" and Sahar said "by tens or by fives". It could be that Sahar was translating what I said into their terminology because in their mind they might not see what they were doing as multiplying but as skip counting. Also, Sahar prompted Tamam to share which multiple she wanted to use, which could indicate her confidence and knowledge in this situation, and could be connected to her identity as a learner. Tamam then responded, "tens", and I asked, "so you're going to do 10 times 10? Sahar then started to count on her fingers by tens but then stopped and said, "10 times 10 is a hundred. One hundred". Here, Sahar made a connection between 10 times 10 and the product 100. Again, I wonder if Sahar used the strategy of adding a zero when multiplying by 10, or she knew that 10 times 10 is 100, or used some other strategy. Tamam then repeated the number "100" as she looked for it on the gameboard. She then claimed it on the gameboard, connecting the oral and written representation of the number.

<u>Stanza 4 (Turn 1 Sahar)</u>

As seen in the following part of the transcript Sabina appeared a bit distracted and I asked her to pay attention. Then, Sahar played her turn.

Fatima: Sabina, I want you to pay attention, okay? So, you can learn how to play the game.

Sahar: 3. [Grabbed a card to play her turn. Then, read the number]

Sabina: I don't even know times.

Sahar: 30. Or 5, 10. 5, 10, 15. 30 or 15 [Shared the product of 3 and 10. Then, counted by 5].

Fatima: So, what do you want 30 or 15?

Sahar: 30. [Claimed the product 30 on the gameboard]

Fatima: 30. Alright.

Sahar: I remember me and Pooja we did it and I got um. 4 um. I won 4 times, but we still um, we still played it.

As we see, in response to my comment, Sabina shared, "*I* don't even know times", as a way to describe her disengagement. Sabina seemed to demonstrate agency by acknowledging what she does not know.

Next, we see that Sahar grabbed Tamam's card and returned it to the stack of cards, reinforcing the rules of the game. Then, she drew a card to play her turn and read the number on the card. Sahar shared the product of 3 and 10, which is 30, again making a connection between 3 and 10 and the product 30. She then used her fingers as a representation to skip count by 5, making a connection between 3 and 5 and the product 15. Sahar then confirmed the products and claimed the number 30 on the gameboard. Sahar then reflected on her experience playing this game by sharing that she played it with Pooja and she won 4 times, illustrating her relationship with the game.

Stanza 5 (Turn 1 Sabina)

In the next part of the transcript, it was Sabina's turn but she did not seem completely sure what is expected of her. She appeared to be merely cooperating with the group and the organizational skill of turn-taking where, she sat quietly, awaited her turn, and played her turn when called upon.

Fatima: Alright. Selina your turn.

Sabina: 10. [Grabbed a card to play her turn. Then, read the number]

Fatima:	10. Do you want to do 10 times 5, or 10 times 10?
Suhail:	100. [Shared the product of 10 and 10]
Sabina:	10 times 10.
Fatima:	Which is how much?
Suhail:	100.
Sabina:	Is 200?
Suhail:	No. [Shook her head]
Sabina:	100?
Fatima:	100.
Sabina:	There's no 100. [Looking at the gameboard]
Fatima:	Yeah, look.
Tamam:	Yes, there's one here. [Pointed to a 100 on the gameboard]
Sahar:	/Yes, there is/.
Suhail:	[Pointed to a 100 on the gameboard]
Sabina:	Oh.
Sahar:	I think there's one more. [Referring to a 100 on the gameboard]

As we see in this transcript Sabina grabbed a card and read it. I asked if she wanted to do 10 times 5 or 10 times 10, but she was quiet. Then, Suhail shared the product of 10 and 10, making a connection between 10 and 10 and the product 100. Here, I wonder whether Suhail drawn on the answer shared earlier, or used some other strategy because she did not appear to be using skip counting. Also, Suhail portrayed agency by sharing the product of 10 and 10 for Sabina's turn, reinforcing her mathematics authority.

Next, we see that Sabina decided to go with 10 times 10, and I asked, "which is how much?" Again, Suhail responded with the product of 10 and 10, and Sabina shared with intonation, "is 200?". Interestingly, Suhail took responsibility of evaluating Sabina's answer by conveying disagreement. In evaluating Sabina's answer, Suhail appears to demonstrate agency, which reinforces her mathematical authority. Sabina then shared with intonation, "100?". Once I confirmed that the product of 10 and 10 is 100, Sabina looked at the gameboard and concluded that there were no more hundreds to claim. Sahar shared that there is one more, and Tamam and Suhail pointed to one on the gameboard, which helped Sabina make a connection between the oral and written representation of the number 100. Sabina then claimed the 100 on the gameboard.

It seemed as though Sabina was a bit reluctant to accept Suhail's answer, which to some extent could portray their relationship. In that Sabina might not take into account Suhail's answer as legitimate given that she was considered relatively new to the classroom and with limited knowledge of English. This could also reflect the relationship between Sabina and myself where, she accepted the answer after I confirmed it.

<u>Stanza 6 (Turn 1 Suhail)</u>

As seen in this portion of the transcript, it was Suhail's turn to play. First, she added Sabina's card to the stack of cards, reinforcing the rules of the game. Then, she drew a card to play her turn.

Fatima:	Alright Suhail. What did you get?
Suhail:	90. [Grabbed a card. Then, shared the product of 9 and 10]
Fatima:	Ok, you got 9. So, do you want to do 9 times 10 or 9 times 5?
Suhail:	[Counted on her finger]
Sabina:	[Laughing]
Sahar:	What? [To Tamam and Sabina]
Sabina:	She did this. [Gestured with her hand what Suhail did when she counted on her fingers]
Sahar:	So?
Suhail:	30. [Then, continued to count on her fingers]
Sahar:	5, 10, 15, 20, 25, 30, 35, 40, 45. 45. [Counted by 5]
Fatima:	45. Do you want to show her [Suhail] how we can count? Suhail look at how Sahar is counting.
Sahar:	5, 10, 15, 20, 25, 30, 35, 40, 45, 50. No 45. [Counted by 5]
Suhail:	/5, 10, 15, 20, 25, 30, 35, 40, 45, 50/. [Counted along]
Tamam:	/10, 15, 20, 25, 30, 35, 40, 45, 50. 45/ [Counted along]
Sabina:	/35, 40, 45, 50, 55/ [Counted along]
Tamam:	No, 50, 55. [Laugh]
Fatima:	Yeah, we need 45. So, do you want to go 9 times 10 or 9 times 5?

Suhail:	55?
Tamam:	45. There's one here, one there. [Pointed to numbers 45 on the gameboard]
Suhail:	[Claimed the product 90]
Fatima:	Oh, you want to do 90?
Suhail:	Yes. [Nodded]
Fatima:	Okay.
Tamam:	Sabina.
Sahar:	She blocked you. [To Sabina]
Sabina:	I'm blocked.

As we see, in response to my question regarding what card she got, Suhail responded by sharing the product of 9 and 10, making a connection between 9 and 10 and the product 90. It was still not clear if she relied on a specific strategy, or she just knew the answer. Suhail then mumbled and it was hard to hear what she was saying, but it appeared as though she was counting by 5 on her fingers. At one point, she held up 6 fingers and said the number 30. She then realized that she needed to hold up 3 more fingers, which could be a representation of the product 45. The fact that she held up 6 then later held up an additional 3 fingers is important because it reflected her understanding of skip counting by 5 for 9 groups.

Next, we see that Sahar started to count by 5 using her fingers, and then she shared her answer with the group. Sahar portrayed agency by finding the product for Suhail's turn, which also reinforces Sahar's mathematics authority. I then asked Sahar if she wanted to show Suhail how she counted. In reviewing the transcript, I notice that I made the manner in which Sahar was counting significant, which could establish where the mathematical authority exists. I also noticed I did not give space for Suhail to share how she was counting and quickly asked Sahar. In doing so, I may have portrayed Suhail as someone who does not know how to count. This too could influence their meaning making as they may recognize the way Sahar counted as the correct and only way to count.

In response to my request, Sahar counted by 5 using her fingers, and the students counted along. Tamam looked at Sahar and imitated her use of fingers to count, which appeared different than how Tamam usually used her fingers to count. Suhail also counted a long but used a different way to count on her fingers. Sabina only started counting at 35 and counted up to 55. She too used her fingers but without making any one-to-one connection between the number she was counting and her fingers.

A viewing of the video showed that the students used their fingers in different ways. One used the thumb as their 6th, another used their pointer as their 6th. I wonder whether that interfered with their interactions because they represent numbers in different ways. Also, it seemed that when Sahar and the students were skip counting by 5, they might have relied on rote counting, leading them to surpass the number needed. However, the use of their fingers which was a representation of the number of times they need to count by 5 (i.e., 9 times) could have led them to know or realize that they have surpassed the number they needed. Sahar then corrected herself by closing her pinky and sharing the product 45. Tamam also realized that they exceeded the counting they needed, and I confirmed that the answer is 45.

I then asked Suhail if she wanted to go with 9 times 10 or 9 times 5. Suhail shared the last number she heard the students count, and that was 55. Tamam pointed to the correct product on the gameboard, which helped Suhail make a connection between the oral and written representation of the number, which also portrayed Tamam's mathematics authority. Suhail then claimed the number 90 in order to block Sabina. Suhail's intuitive awareness of the gameboard helped her strategize where she wanted to place her marker.

Stanza 7 (Turn 2 Tamam)

The students continued to take turns playing the game, In the next segment of the transcript, Tamam played her second turn.

Fatima:	Ok Tamam your turn.
Tamam:	40. Or 5, 10, 15, 20. [Grabbed a card. Then, she said the product of 4 and 10, and she counted by 5]
Sahar:	/5, 10, 15, 20/ [Counted by 5]
Tamam:	40 or 20. [Shared the products of (4 and 10) and (4 and 5)]
Sahar:	No. 40 or 5, 10, 15, 20. Oh 20. 40 or 20. [Expressed disagreement. Counted by 5. Then, shared the products]
Tamam:	Wait. Put this one in there. [Asked Suhail to place card back in stack]

Sahar:	Oh, what if the card that you got, the one's that you flipped over you can keep them, or you just leave them together?
Tamam:	I choosed 40 [Claimed the product 40 on the gameboard]
Fatima:	Um you can do that. Yeah sure. [Responding to Sahar]
Tamam:	I choosed 40.
Fatima:	You chose 40?
Tamam:	[Nodded]

As we see in this transcript, Tamam got the number 4 and she shared the product of 4 and 10, which was 40. Then, she counted by 5 on her fingers to get the product of 4 and 5. Notice how Sahar also counted along as she looked at Tamam's fingers. Here, Tamam's fingers appeared as a tool for Sahar to count by 5, and where together, they created a representation of the product 20. Tamam then confirmed the two products, but Sahar evaluated Tamam's answer by recounting by 5 using her fingers, reinforcing Sahar's mathematics authority. Here too, Sahar appeared to recognize agency by taking responsibility of evaluating and checking Tamam's answer, which may have been influenced by the context of the activity. For instance, from the beginning of our interaction, Sahar portrayed herself as more knowledgeable given that 1) she explained to the group how they should play the game, 2) she referred to the fact that she had experience playing this game with Mrs. Holland and with her friend, Pooja, and 3) I had specifically asked her to show Suhail how to count by 5. Interestingly, how Sahar portrayed herself in this activity may play into the students' meaning making as they may recognize her as more knowledgeable and they need to comply with her.

Tamam then realized that Suhail still had the card she flipped earlier, and she asked Suhail to put it back with the rest of the cards, again reinforcing the rules of the game. Tamam then claimed the product 40 on the gameboard.

<u>Stanza 8 (Turn 2 Sahar)</u>

In the next part of the transcript Sahar played her turn.

Fatima:	Alright, Sahar.
Sahar:	7. 70. Or 5, 10, 15, 20, 25, 30, 35. 70 or 35. [Grabbed a card. She read the card and shared the product of 7 and 10. Then, she counted by 5, and shared the two products]

Tamam: /Suhail/ Suhail/

Sabina: There's someone um back. [Referring to the librarian who passed behind her]

Fatima: So, what did you choose?

Sahar: 70. [Claimed the product 70 on the gameboard]

As we see in this transcript, Sahar first confirmed the number she got and she shared the product of 7 and 10, which is 70. Next, Sahar counted by 5 using her fingers, creating a representation of the product 35. Sahar confirmed the two products, then she claimed the number 70.

Stanza 9 (Turn 2 Sabina)

In the following part of the transcript, Sabina grabbed a card, read the number on the card, and appeared to be thinking about the number she will multiply it by.

Fatima: Alright who's turn is it?

Sahar: Sabina.

Sabina: Okay.

Tamam: Wait, did she even put one? [Asked if Suhail claimed a number]

Sahar: Yeah. [Pointed to the marker on the gameboard]

Fatima: Yup.

Sabina: 5. I'll do 5 times, 5 times-, [Grabbed a card. Then, read the number on the card]

Sahar: Can we do times?

Fatima: Isn’t that what we're doing?

Sahar: Wait, but we're doing 5's and 10's.

Fatima: Yeah, so it's 5 times 5 or 5 times 10.

Sahar: Oh.

Fatima: Yeah.

Sabina: 5 time-, 5 time-, 5 time-, 5.

Tamam: 5, 10, 20. Wait 5, 10, 15, 20, 25. [Counted by 5. Then, recounted by 5]

Sabina: /5 times/

Tamam: 50 or 25. [Shared the products]

Sahar: What? What did you say? [To Tamam]

Tamam: 50 or 25 [Repeated products]

Sabina: What? [To Tamam]

Tamam: 50 or 25. [Repeated products]

Sabina: 50 or 25. I'll block Sahar. [Shared products. Then, claimed the product 25 on the gameboard]

Fatima: Oh.

In this stanza, we also notice that Sahar asked if we could do times. The fact that Sahar asked that is important because it appeared to have reflected her understanding of the game—that is, we are counting by 5 and 10, and she might not have considered what we were doing as multiplying or times, reflecting how she made sense of the game this far.

As we look more carefully at this segment of the transcript, we can see that Tamam was counting and recounting by 5 using her fingers, creating a representation of the product 25. Tamam seemed to have developed agency by correcting herself through recounting, and also, by solving or figuring out the products for Sabina's turn.

Tamam then confirmed the two products, and Sahar and Sabina asked Tamam to repeat the products. Sabina then added the card back to the stack and reiterated the products as she looked for them on the gameboard. Sabina claimed the product 25 in order to block Sahar. Sabina's intuitive awareness of the gameboard helped her strategize where she wanted to place her marker. Notice how most of the numbers they claimed on the gameboard were based on who they wanted to block or in an effort to win.

Stanza 10 (Turn 2 Suhail)

In the next part of the transcript Suhail played her turn.

Fatima: Alright now it's-, um.

Sabina: Suhail.

Fatima: Suhail's turn.

Suhail: 20 [Grabbed a card. Then, shared the product of 2 and 10]

Fatima: Yeah.

Sabina:	She said 20.
Sahar:	Or 5, 10. [Counted by 5]
Sabina:	/10/, 20, 30, 40, 50, 60, 70, 80, 90, 100. [Counted by 10]
Tamam:	20. Wait 5, 10. 10 or 20. 10 or 20. [Shared the product of 2 and 10. Then, counted by 5, and confirmed the two products]
Suhail:	25? [Counted on her fingers. Then, said 25 with intonation]
Fatima:	How can we do 2 times 5?
Tamam:	5, 10 done. [Responded to my question]
Sahar:	/5, 10/. [Responded to my question]
Fatima:	5, 10. Or 10, 20. [I counted by 5. Then, counted by 10]
Tamam:	20 or um 10. [Shared the two products]
Suhail:	5, 10. 10, 20. [Counted by 5. Then, counted by 10]
Fatima:	So, what do you want Suhail?
Suhail:	20 [Claimed the product 20 on the gameboard]
Fatima:	20. Okay.
Sahar:	Hey, you blocked me [To Sabina].
Tamam:	I know what Sabina is doing.
Sahar:	What? They all blocked me.

As we see in this transcript, Suhail grabbed a card and shared the product of 2 and 10, which is 20. She then appeared to count on her fingers, but it was not clear what she was counting.

Notice that during this time Sahar counted by 5 using her fingers, creating a representation of the product 10. Sabina looked at Sahar and tried to imitate what she was doing. However, Sabina did not appear to be imitating exactly what Sahar was doing, but was counting by ten. Although Sabina did not participate a lot when it was her turn or try to figure out the products for her turn, the fact that she counted by 10 here was important because it reflected her understanding of counting by 10 and that she was capable of doing it correctly.

Next, we see that Tamam looked at Suhail's card and shared the product of 2 and 10, which is 20. Then, she counted by 5, creating a representation of the product 10. Tamam then looked at Suhail and shared the two products.

Despite the fact that Sahar and Tamam shared the product of 2 and 5, notice that Suhail did not rely on their answers, but continued to count on her fingers. She then shared with intonation the number 25. It appeared that she was keen on trying to find the product on her own.

I then asked the students how we can do 2 times 5. In response, Sahar and Tamam counted by 5 using their fingers. Then, they held up 2 fingers as a representation of the number 10. Interestingly, Sahar appeared to alternate between how she used her fingers to count, but Tamam held on to one specific way for counting on her fingers. It was only when Tamam was imitating Sahar that she counted like Sahar.

I confirmed their answer by counting by 5 and then by counting by 10 using my fingers. Tamam repeated the two products, and Suhail imitated my counting by quietly counting on her fingers. By imitating my counting, Suhail appeared to be making sense of what I was doing in terms of counting by 5 and counting by 10 to get the product, which could play a role in her meaning making. Suhail then claimed the product 20 on the gameboard which blocked Sahar. Again, Suhail's intuitive awareness of the gameboard and Sahar's tiles on there helped Suhail strategize where she wanted to place her marker.

Stanza 11 (Turn 3 Tamam)

As we will see in the next portions of the transcript, before Tamam flipped her card, she reflected on the strategy she thought Sabina might think of in order to build a line of 4 adjacent markers.

Fatima:	Alright Tamam.
Tamam:	I know what Sabina is doing. She's trying to go this way. [Pointed to gameboard]
Sabina:	No.
Fatima:	Tamam your turn.
Sahar:	Yeah:: It's 4 in a row.
Tamam:	Um, I got 10. 10. 5, 20. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. 50. [Grabbed a card. Then, she showed the card to the camera, read the number and counted by 5]
Sahar:	Or a hundred. 50 or one hundred. [Shared the product of 10 and 10. Then, shared the two products]

Sahar:	There's no more 100. So, you have to pick 50. Don't go 50 near mine [Referring to the gameboard]
Tamam:	There's no 50 either. [Referred to the gameboard]
Sahar:	Yeah, there is. 50 right here. 50 right here. And 50 right here. 50-, [Pointed to the 50 on the gameboard]
Tamam:	Uh wait. [Fixed tiles on gameboard] Okay, then I'll just go here. [Tamam claimed the product 50 on the gameboard]
Sahar:	Oh, I didn't see that. [Referred to the product Tamam claimed]

In this part of the transcript, we see that Tamam used two fingers to represent a diagonal line of 4 adjacent markers, and Sahar confirmed that it is 4 in a row. Although Tamam did not use the word 'diagonal', Tamam's use of her fingers and the phrases used (i.e., go this way and 4 in a row), illustrated their interpretation of Sabina's strategy to form a diagonal line 4 adjacent markers and reinforced the rules of the game.

Next, we see that Tamam grabbed a card and shared the number she got. Interestingly, Tamam did not share the product of 10 and 10, instead she counted by 5 on her fingers, creating a representation of the product 50. Notice that Sahar shared the product of 10 and 10, while also confirming the product of 10 and 5. Sahar portrayed agency by confirming Tamam's answer and sharing the product of 10 and 10, which also reinforces Sahar's mathematics authority.

Sahar then shared that there were no more hundreds on the gameboard, and Tamam shared that there were no fifties either. However, Sahar pointed to the fifties she saw on the gameboard, making a connection between the oral and representation of the number for Tamam. Tamam then claimed the product 50 on the gameboard.

Stanza 12 (Turn 3 Sahar)

This stanza shows Sahar playing her turn.

Sahar:	70. Or 5, 10, 15, 20, 25, 30, 35. 70 or 35. I don't know um. [Grabbed a card. She shared the product of 7 and 10. Then, she counted by 5 and confirmed the two products]
Sabina:	But you can block:: [Referred to who Sahar could block on the gameboard]

Sahar: I'm gonna go to-, I'm going to go to 35. Blocked you [to Tamam]. [Claimed the product 35]

Sabina: Who? [To Sahar]

Tamam: What? [To Sahar]

Sahar: I blocked Tamam. So, she does not go this [Pointed to the 15], to here [Pointed to the 35], not get to a 100 and 40. So, she is not gonna get 4 in a row [Referred to the numbers Tamam still needed to win].

Fatima: Oh.

Tamam: I'm not doing that. I'm doing a different one.

Fatima: Sabina. We're gonna go one more turn and that's it.

Sahar: I think you're going this way. Like that. That's what you're trying to do? [Referred to the shape of a 4 square]

As we see in this transcript, Sahar got the number 7. She shared the product of 7 and 10, which is 70. Then, she counted by 5 using her fingers, creating a representation of the product 30. Sahar then claimed the number 35 and confessed that she blocked Tamam. Sahar's intuitive awareness of what 4 in a row would look like led her to claim the 35, which prevented Tamam from forming a horizontal line of 4 adjacent markers. Tamam disagreed to having planned to make a horizontal line, and after some thought, Sahar concluded that Tamam was trying to make a 4 square. Again, Sahar's use of the words "this way" and "like that" and her use of her fingers were a representation of the line or shape that she assumed Tamam was creating.

Stanza 13 (Turn 3 Sabina)

In the next part of the transcript, Sabina grabbed a card, read the number on it, and appeared to be thinking of the number she wanted to multiply it by.

Fatima: Come on Sabina.

Sabina: Fiv-, I mean 4. [Grabbed a card. Then, she read the card]

Fatima: So, what do you want to do?

Sabina: Okay 4 times-,

Suhail: 40? [Shared the product of 4 and 10 with intonation]

Fatima: Yeah. 40 or,

Sahar:	70 and 70 [Held up the two cards she had with the number 7 on each. Then, she shared the product of 7 and 10 for each card]
Sabina:	Or, or 70. [Repeated after Sahar]
Fatima:	Let's count. 5, 10, 15, 20.
Sabina:	/5, 10, 15, 20. / Yay 20.
Fatima:	So, 40 or 20.
Sabina:	40 or 20. 40 or 20. 40 or 20.
Sabina:	Umm. I'm gonna block Sahar. Okay Never mind. What did you say? 40 or-,
Fatima:	40 or 20.
Sabina:	40 or 20. [Claimed the product 40]

In this stanza we notice that Suhail and not Sabina shared the product of 4 and 10, which is 40. Suhail portrayed agency by sharing the products for Sabina's turn, which also reinforces Suhail's mathematics authority. I confirmed her answer and I asked for the other product.

Next, we that Sahar held up the two cards she got from turn 2 and turn 3 (i.e., 7 & 7) and shared their product for 7 and 10 which is 70. Interestingly, Sabina, who assumed that Sahar was referring to her card repeated the number Sahar shared. I then initiated the counting by 5 using my fingers which prompted Sabina to count along, creating a representation of the product 20. Sabina did not only count along but tried to imitate my use of my fingers to count. Sabina then repeated the number 20 as she looked for it on the gameboard. I then shared that the products are either 40 or 20, and Sabina repeated the products as she continued looking at the gameboard. Sabina then claimed the number 40 on the gameboard, making a connection between the oral and representation of a number.

<u>Stanza 14 (Turn 3 Suhail)</u>

As seen in this portion of the transcript, it was Suhail's turn to play.

Fatima:	Alright, Suhail's turn.
Suhail:	50 [Grabbed a card. Then, she shared the product of 5 and 10]
Tamam:	50 or 5, 10, 15-. 5, 10, 15, 20, 25. 25. [Shared the product of 5 and 10. Then, she counted 5 groups of 5]

Suhail:	/5, 10, 15, 20, 25/, 30. 25, 25? [Counted 5 groups of 5. Then, she shared the product 25 with intonation]
Fatima:	Yup, 5, 10, 15, 20, 25. [Counted by 5 to confirm]
Tamam:	50 or 25. [Shared the two products]
Sahar:	5, 10, 15, 20, 25, 30, 35, 40, 45, 50. [Counted 10 groups of 5]
Tamam:	25, 30, 35, 40, 45, 50, 55, 40, 45, 50, 55, 60-, [Counted by 5]

In this stanza we see that Suhail grabbed a card and shared the product of 5 and 10, which is 50. Tamam then looked at Suhail's card and responded by confirming the product of 5 and 10, and that is 50. Then, she counted by 5, creating a representation of the product 25. Tamam portrayed agency by confirming Suhail's answer and helping her find the product for 5 and 5, which also reinforced Tamam's mathematics authority.

Suhail also counted by 5 but she went as far as 6 times, then she counted back one group of 5. The fact that Suhail went back just one group of 5 is important because it reflected her understanding of movement is magnitude—that is, she counted up one additional group of 5 but understood that she needed to move backwards by only one group of 5. Suhail then looked at me and with intonation shared the product of 5 and 5. I confirmed her answer by counting by 5, reinforcing my mathematical authority. Tamam appeared to be counting a long but she continued to count by 5 up to 16 times, reflecting her knowledge and understanding of counting by 5 to that vast of a number.

Sahar also tried to find the product of 5 and 10, but instead of counting 5 groups of 10 or sharing the product like she did previously, she counted 10 groups of 5 using her fingers, creating a representation of the product 50. In doing so, Sahar explored an alternate grouping arrangement to find the product of 5 and 10 by counting 10 groups of 5. Suhail then claimed the number 25 on the gameboard.

Stanza 15 (Turn 4 Tamam)

In the following part of the transcript, Tamam played her turn.

Tamam:	70. 70 or 5, 10, 15, 20, 25, 30, 35. [Grabbed a card. Then, she shared the product of 7 and 10, and counted by 5]
Fatima:	So, what are you going to pick Tamam?

Tamam: Um I got no way to win. Um I put it on here's [On Sabina's marker]. I put it on 35. [Claimed the product 35]

Fatima: Yeah because 7 times 5 is 35.

In this stanza, we see that Tamam grabbed a card and shared the product 70 for 7 and 10. Then, she counted by 5 using her fingers, creating a representation of the product 35. Tamam then claimed the product 35 on the gameboard, and I confirmed her answer.

Stanza 16 (Turn 4 Sahar)

As seen in this portion of the transcript, Sahar drew a card and got the number 3. She then moved on to share her solution.

Sahar: I can break her heart together. [Grabbed a card as she sang]

Fatima: Ok what is it? [Referring to the card]

Sahar: 30 [Shared the product of 3 and 10. Then, she claimed the product 30 on the gameboard]

Fatima: Okay, you want to go 30. Okay.

Tamam: I can still win.

Sahar: No, you can't.

Fatima: Come on Sabina your turn.

Tamam: Yes.

Sahar: No, you can't.

Tamam: That's it [Pointed to gameboard]

Fatima: You guys are good blockers.

Sabina: Um-,

Sahar: I can break your heart-, [Sang]

As we can see in this part of the transcript, Sahar shared the product 30 for 3 and 10. Sahar then claimed the product 30 on the gameboard, blocking Tamam from building a four square. Sahar's intuitive awareness of Tamam's tiles led her to strategize which space she wanted to claim which blocked Tamam from making a 4 square. Next, we see that Tamam pointed to two empty spaces where she could make a horizontal line of 4 adjacent markers to win . Tamam then placed her last two markers in the two spaces, giving her a horizontal line of 4 adjacent markers.

Stanza 17 (Turn 4 Sabina)

In the following segment of the transcript, Sabina grabbed the number 6 and attempted to count up by ones.

Fatima:	Okay Sabina's turn.
Sabina:	6, 7-, [Grabbed a card and counted up by one]
Sahar:	60. Or 5, 10, 15, 20, 25, 30, 35. [She shared the product of 6 and 10. Then, she counted by 5]
Sabina:	/5, 10, 15, 20, 25, 30, 35/. [Counted along]
Sabina:	60 or 35. [Shared the two products]
Fatima:	Count again.
Sahar:	5, 10, 15, 20, 25, 30. 30. [Recounted by 5]
Fatima:	30 yeah.
Tamam:	Mine are all done. [Referred to the tiles]
Sabina:	30 okay. [Repeated product]
Sahar:	Yeah, it would be possible because this is 25-, [Held up 5 fingers].
Tamam:	/Mrs. Fatima mines are all done/. [Referred to the tiles]
Fatima:	It's our last turn.
Tamam:	Wait [She looked at me and pointed to the two tiles on the gameboard]
Sahar:	This is 25, and then you a-, Wait. This is 25 and then you add another one. [She held up 5 fingers on one hand, then held up her thumb on the other hand]
Fatima:	No. [To Tamam]
Sahar:	Wait, this is 25 and then you add another one, it's gonna be 30. [Again, she held up 5 fingers on one hand, then held up her thumb on the other hand]
Fatima:	Exactly. [To Sahar]
Tamam:	This one was here. This one was here. [Returned one tile back to the gameboard]
Fatima:	Alright Sabina go.

Tamam:	I got one more to do. [Referred to the space on the gameboard]
Sabina:	30. There's no 30. [Repeated product. Then, referred to gameboard]
Fatima:	There's a 30 here. Here's a 30.
Sabina:	I want another one, um.
Tamam:	There's a 30. [Pointed to gameboard]
Fatima:	Oh.
Sabina:	Why cou-, why couldn't it be right here and here. [Pointed to gameboard]
Fatima:	It's okay [To Sabina]
Sahar:	Can you block her? [To Sabina]
Tamam:	No.
Sahar:	Mrs. Mrs.

[Sabina claimed the product 30 on the gameboard]

As we see in this transcript Sahar looked at Sabina's card and shared the product 60 for 6 and 10. Then, she counted by 5 using her fingers, and Sabina counted along but by looking at Sahar's fingers. Notice that Sahar counted up to 35—a viewing of the video shows that as Sahar was counting, she did not make one-to-one correspondence between her fingers and the number she was counting—that is, her 6th finger corresponded with 35 rather than 30. Sabina did not realize that Sahar miscounted and confirmed the products are 60 or 35. I asked them to recount, and Sahar recounted by 5 using her fingers. Sabina did not count along but looked at Sahar as she counted. Sahar shared the answer 30 and I acknowledged it. Sabina repeated the product 30 as she looked for it on the gameboard. Notice that Sahar portrayed agency by helping Sabina find the products, which also reinforced Sahar's mathematics authority.

Next, we see that Sahar looked at me and shared, "Yeah, it would be possible because this is 25 [Held up 5 fingers on her right hand]. And then you add another one [Held up her thumb on her left hand]. It's gonna be 30 [Brought hands side by side]". Here, Sahar appeared to connect what she already knew about 5 times 5 equaling 25 and then adding an additional group of 5 to get 30, which is the product of 6 and 5. Notice that Sahar held up 5 fingers as a representation of the number 25, and where each finger was a representation of a group of 5, and together they equaled 25. Then, she held up her thumb as a representation of an additional group of 5 and the number 30. Also, by bringing her hands together she created a representation of addition—that is, 25 plus 5 equals 30.

Tamam then shared that she does not have any more tiles. Tamam looked at me and pointed to the two markers she added to the gameboard which created a horizontal line of four adjacent markers. I expressed disagreement and Tamam removed the two markers she added. A viewing of the video shows that Tamam returned one of the tiles—Tamam placed a marker on a space she had not played as an urge to win.

Sabina then shared that there were no more thirties close to her already claimed tiles. Tamam suggested a space that Sabina could claim, but Sahar asked Sabina to block Tamam. However, Sabina decided to claim a space that was far from all the claimed spaces.

Stanza 18 (Turn 4 Suhail)

This stanza shows the last turn played in the game. It begins we me sharing that Suhail will play the last turn.

Fatima:	Alright Suhail will play the last turn. Let's go.
Sahar:	Mrs. Um-,
Tamam:	No, the last turn is me. Suhail then me and we're done.
Sahar:	What?
Tamam:	Because I wanna win this thing.
Fatima:	Alright, Suhail.
Sahar:	No, then me, then Sabina and then that's all.
Tamam:	No::
Fatima:	Come on Suhail. Okay. What did Suhail get? [Suhail grabbed a card]
Suhail:	60 [Shared the product of 6 and 10]
Fatima:	60. Or 5, 10, 15, 20, 25, 30 [Initiated the counting by 5]
Suhail:	/5, 10, 15, 20, 25, 30
Sahar:	/20, 25, 30/
Tamam:	/20, 25, 30/
Fatima:	30. So 60 or 30? [To Suhail]
Suhail:	30
Tamam:	There's a 30, here. There's only one 30. [Pointed to gameboard]
Suhail:	25? [Asked with intonation]

Fatima:	Huh?
Suhail:	25
Sahar:	No, not 25 because 5, 10, 15, 20, 25, 30. [Corrected Suhail. Then, she counted by 5]
Tamam:	30 [Pointed to the 30 on the gameboard]
Fatima:	So, 30 or 60.
Sahar:	Oh, here, here, here, here, here [Pointed to a 60 that could block Tamam from winning]
Tamam:	No, stop telling here.
Sabina:	There's 2. [Showed camera that she had 2 tiles left]
Sahar:	Please, please, please, here, here, here. [To Suhail and pointed to gameboard]
Tamam:	Stop telling here [To Sahar]
Sahar:	No, here, here. [To Suhail and pointed to gameboard]
Tamam:	Stop. [To Sahar]
Fatima:	Ok guys stop. Suhail will pick on her own.
Suhail:	[Suhail claimed the product (60) on the gameboard]
Fatima:	Alright
Sahar:	She blocked you. She blocked you. [To Tamam]
Sabina:	Stop it [To Tamam who was rearranging the markers on the gameboard]
Fatima:	And then we're done.
Sahar:	Yup we're done.
Sabina:	We're done quickly
Sahar:	Bye bye.
Sabina:	Bye.

As seen in this portion of the transcript ,Suhail grabbed a card and shared the product 60 for 6 and 10. I confirmed her answer and I initiated the counting by 5 using my fingers. Suhail, Sahar and Tamam counted along. Tamam then pointed to a 30 on the gameboard, creating a connection between the oral and written representation of a number for Suhail. It seemed as though Tamam was trying to distract Suhail from seeing the 60 that could block her from winning. Suhail then shared the number 25. Sahar evaluated Suhail's answer, then she counted by 5 using her fingers, getting the product 30. Here too, Sahar portrayed agency by evaluating Suhail's answer, which reinforced Sahar's mathematics authority.

Sahar and Sabina urged and begged Suhail to claim the 60 which could block Tamam from winning. Tamam appeared upset and asked Sahar to not tell Suhail which space to claim. Tamam pointed to another 60 that Suhail could claim. Suhail decided to count between the two sixties, which Sahar referred to as "Eeny, meeny, miny, moe"—a children's rhythm used to help in choosing a person or something in a game. Suhail stopped counting at the space that would block Tamam from winning. Still, Suhail did not claim that space so I initiated a countdown to speed things up a bit. Suhail then claimed the 60 that blocked Tamam from winning, and that concluded the game.

Summary of Main Ideas: Activity #3 "Times Tussle"

In what follows, I discuss children's meaning-making by drawing on Gee's (2005) analytical framework of discourse analysis as described in **Table 3** in the methodology chapter, specifically the construct of building tasks of "language in use" including, significance, activity (i.e., mathematical actions), identity, relationships, politics, and sign systems or knowledge.

Significance. This transcript brings forth particular things that appear to become significant to the students' mathematical meaning-making. These include their developing mathematical terminologies, the use of specific strategies, and various tools that appear significant to their understanding of mathematics.

In terms of mathematical terminology, the students seemed to be developing a sense of what the words "times" means as a way to talk about multiplication as well as thinking about multiplication as making "groups". For instance, when we first began the game, Sahar referred to the term "groups" and I repeated the term and drew attention to it by asking them to explain what it meant. I notice that I was also emphasizing the word "times" as a way for them to see that what they were doing was multiplication but often they would refer to it as counting "by tens or by fives", reflecting their use of skip counting. For instance, at one point, Sahar used the term "times" when sharing, "10 times 10 is a hundred", and at another point, she asked, "can we do times?", hence, starting to pick up the terminology for multiplication that I used. Consequently, the use of particular terms may play a role in their meaning-making in that 1) the students may come to see these terms as important, and 2) these terms could define what they were doing in the game.

It is evident that the students also relied on particular strategies which appeared to be meaningful to them. These include the use of counting, skip counting, counting forward and backward, adding a zero when multiplying by 10, and prior mathematical knowledge in terms of their understanding of what 10 times 10 equals to—something they discussed in class previously and which seemed to have emerged as known facts. Their counting strategies demonstrated important characteristics such as stable order, one-to-one correspondence, cardinality, and understanding of movement as magnitude, which is the idea that as one moves up or down in the counting sequence the quantity increases or decreases. The transcript provides evidence that all students involved in the game had an understanding of stable order in counting, and recognized that the last number said represented the quantity of objects. Skip counting seemed to be a significant strategy that the students connected with multiplication and they appeared to have some fluency with this. For instance, Sahar explored an alternate grouping arrangement to find the product of 5 and 10 by counting 10 groups of 5 instead of counting 5 groups of 10. And although students appeared fluent in skip counting by 10, they often relied on the strategy of adding a zero when multiplying by 10.

Particular concrete tools appeared to be significant in their meaning making such as the tiles and their fingers. The tiles were used for counting as well as to mark their place on the game board. One of the most significant tools was their fingers, as these were constantly used to assist them with skip counting, their main strategy for multiplication. It was interesting that different students used their fingers to count in very different ways. For instance, one might use the thumb as their 6th, another might use their pointer as their 6th. I wonder whether that interferes with their interactions because they represent numbers in a variety of ways.

Activities: Mathematical Actions. In analyzing the transcript, we notice that there was evidence of students' use of mathematical actions as described in **Table 3**, which contributed to their meaning making and portrayed the students' use and development of mathematical thinking skills. These mathematical actions include the use of reasoning and proving skills, communication of ideas, representation of ideas using various tools, and making connections between ideas or concepts. I will talk about each in turn.

Reasoning and Proving. In terms of reasoning and proving, the transcript provides evidence that the students talked through their claims by relying on their prior mathematical knowledge, skip counting, or adding a zero when multiplying by 10 to justify and explain a product. They also recognized that by counting and recounting on their fingers, they could evaluate or check their answers or each other's answers. For example, when Suhail shared the product 25 for 6 times 5, Sahar questioned Suhail's answer by skip counting by 5 and sharing the correct product, 30. In skip counting by 5, Sahar visually justified her answer. Also, the students appeared to rely on their intuitive awareness of the gameboard and tiles claimed on the gameboard to strategize which space they wanted to claim to gain the most points. I noticed that most of the numbers they claimed were based on their desire to win and who they wanted to block. They would even reflect on each other's plans to win by thinking about which line of 4 adjacent markers or a 4 square they tried to claim.

Communication. The analysis demonstrates that the students used oral and visual communication as a way of thinking aloud and to communicate with others to share their mathematical understandings, ideas, and to demonstrate their strategies. For example, the students communicated when they 1) counted or recounted the number of tiles they had,; 2) shared their understanding of the game; 3) reflected on their experiences playing the game; 4) translated something I said into their terminology; 5) responded to a question, asked a question, or posed a concern; 6) evaluated, corrected, or confirmed each other's answers 5) read the number they flipped or the number they claimed; 7) skip counted by 5 using their fingers; 8) shared the product(s); 9) used everyday language and pointed to illustrate a 4 square or a line of 4 adjacent markers; 10) used concrete tools to demonstrate their thinking; and 11) referred to a number on the gameboard as a written representation of the product.

Representation. The students also relied on concrete tools to represent their mathematical understandings and ideas by using the tiles, their fingers, and the gameboard. In terms of the tiles, the students collected tiles from a bin to be used as markers to claim a number on the gameboard. However, as the students collected their tiles, they also counted and recounted their tiles, and in so doing, the tiles became a representation of a number.

One of the most frequent representations the students relied on was their fingers. They used their fingers as a representation of skip counting by 5. They also relied on each other's fingers for skip counting such as when Sahar used Tamam's fingers to count along as she skip counted by 5. Also, their fingers were used to represent groupings. For example, Sahar held up 9 fingers to represent the product 90 (i.e., 9 groups of 10), and Suhail held up 9 fingers to represent the product 45 (9 groups of 5). Their fingers were also used to signify a connection between what they already knew and a new understanding or knowledge. For instance, Sahar created a representation to make a connection between what she already knows about 5 times 5 and adding a group of 5 to get the product for 6 and 5 (i.e., 30). Sahar held up 5 fingers on one hand as a representation of the number 25, then she held up her thumb on the other hand as a representation of an additional group of 5. She then brought her hands together creating three representations: 1) a representation of addition, 2) a representation of a number sentence, and 3) a representation of the product 30—(i.e., 25 + 5 = 30). In addition, the students used their fingers as a representation of a line of 4 adjacent markers or a 4 square. For example, Tamam used her fingers to represent a diagonal line of 4 adjacent markers on the gameboard while Sahar used her fingers to represent a 4 square on the gameboard.

In terms of the gameboard, the students would point to a number on the gameboard to make a link between the oral and written representation of a number.

Connections. The students made several connections between mathematical ideas or concepts. Some of these included making a connection between 1) number and quantity, such as when Tamam counted in a stable order and seemed to recognize that the last number she said tells her the quantity or the number of tiles I asked them to get; 2) multiplying by 5 and skip counting by 5, like when students recognized that when multiplying 5 they could skip count by 5 to find the product, 3) between oral and written representations of a number, for example, when Sabina shared that there were no more hundreds on the gameboard, the other students helped her make a connection between the oral and written representation of the number 100 by pointing it on the gameboard; and 4) one-to-one correspondence between a number and an object, similar to when they were skip counting by 5 using their fingers, they would make a one-to-one connection between the number they are counting and their fingers.

Identities. As the game evolved, we can see the different roles that the students played and the individual identities that were constructed. These identities as mathematics learners influenced students' roles in the activity and how they made sense of the activity. In general, the students developed or expressed agency in multiple ways such as by correcting their answers, solving the products on their own, and through the use of their own individual strategies. For instance, Sahar established a mathematical identity that portrayed her energetic and interactive engagement in the game and the mathematics involved in the game. She appeared to identify with the activity and as someone who has experience and knowledge playing it. She also seemed to take on the role of the teacher by guiding and explaining to others how they could play the game and how they could skip count by 5. Also, Sahar exercised control over the strategies she used to find the products and took responsibility for correcting her work.

Sabina seemed to demonstrate agency by reflecting on her knowledge or understanding of the game. At one point, she acknowledged what she does not know as a way to describe her disengagement. This was particularly evident when she shared, "*I* don't even know times". Sabina was not familiar with the game, and as such, she merely cooperated with the activity and the organizational skill of turn-taking where, she sat quietly, awaited her turn, and played her turn when called upon. However, she appeared to recognize the instructions of the game by complying with them. She would also rely on others to find the products but would attempt to imitate what others were doing, reflecting her identity as a mathematics learner.

Suhail established a mathematical identity that portrayed her as directing and taking responsibility for her learning. Although Suhail would continuously observe others to make sense of what is expected of her, she exercised control over the strategy she wanted to use without relying on others. This is critical because as she continued to develop familiarity with the game and with skip counting, she took control over how she made sense of the numbers and counted using her fingers.

Tamam developed a mathematical identity that recognizes her autonomy by directing and taking responsibility for her learning. She would solve the products on her own, correct her own work, and built her own understanding of multiplication. She too took on the role of the teacher by guiding others to find the products and by encouraging them to comply with the rules of the game.

Relationships. Several different relationships were at play and evolved during these interactions that may influence the students' mathematical meaning-making. These include the relationship between the students and the activity, the relationship between the students and myself, and the relationships between the students as they interacted.

The students had various experiences playing the game and it felt as though these previous experiences and their relationship with the game and the mathematics in the game influenced their level of confidence. For instance, Sahar portrayed confidence by sharing her understanding of the game and by finding the products on her own. Tamam reflected confidence in the game by sharing the products for the numbers she flipped and those flipped by Sabina and Suhail. Suhail portrayed confidence by not relying on others to find the products but by attempting to do it on her own. Sabina, however, appeared disengaged and would rely on the answers shared by others without attempting to find the products on her own.

The relationship between the students and I seemed to change during the game. In the beginning, they looked to me quite often for direction, but as the game progressed and they became familiar with the game, I did not have to intervene as much and they did not look to me for direction.

My analysis of this data shows that there were three main characteristics of the relationship between the students as they interacted, 1) collaboration, 2) imitation, and 3) trust. The students were collaborative rather than competitive, and rather than let one another struggle, they would constantly help one another. It appeared as though they developed trust in one another, and the authority shifted from Sahar and me to the whole group and they started to check with one another, evaluate, or even imitate one another. Also, the relationship between the students helped them model and imitate one another's strategies, which may be significant to their meaning making because they were constantly learning from one another.

Politics. Political aspects that contribute to meaning making include the ways in which mathematical authority was distributed between the students and myself, between student-to-student, and between the students and the rules of the game. In terms of mathematical authority, there are instances that demonstrate my role as a mathematical authority and other instances where we can see that some of the students assumed mathematical authority. In reviewing the transcript, I noticed that I portrayed mathematical authority in multiple ways through my use of terminology, reinforcing the rules of the game, regulating their behavior, and confirming or challenging their responses. For instance, I would probe students' thinking by asking them to consider and or reflect on the term "groups" by sharing an example.

I also noticed that at times I purposefully distributed mathematical authority. As an example, at a certain point I asked Sahar to show Suhail how she skip counted by 5. In so doing, I may have established Sahar's mathematical authority by asking her to demonstrate a strategy. And in many instances we can see that various students took on mathematical authority. For instance, Suhail often portrayed mathematical authority by evaluating or confirming the products shared by Sabina. Tamam exercised mathematical authority by frequently monitoring the contributions shared by Sabina and Suhail. Sahar demonstrated a strong sense of mathematical authority by 1) explaining to the group how they should play the game, 2) by reflecting on her experiences playing this game, and 3) by monitoring the contributions shared by Tamam, Suhail, and Sabina. In exercising this mathematical authority none of them actually seemed to do it in an evaluative sense.

The students seemed to see the rules of the game as being a strong authority. For instance, the students always returned the card to the bottom of the stack before playing their turn as they saw this as one of the games rules. Also, they would work hard to meet the goal of the game which was to strategically choose a number that would give them either a 4 square or a line of 4 adjacent markers. The goal of the game would structure their decisions and their discussion to make the most strategic move.

Sign Systems or Knowledge. Concerning what sign systems or knowledge are privileged, these were mostly revealed in students' use of certain terminologies, certain strategies, and certain ways to play the game. What was privileged seemed to be the kinds of things that were used by those who had the most agency, Sahar, Tamam, as well as myself. For instance, Sahar's and Tamam's ideas seemed to be taken up by others. I also noticed that our use of certain mathematical terminology such as, "groups" and "times", might have privileged these terms, which in turn, may have influenced the students' understanding of the game and their interpretation of these terms. Concerning the strategies used, Sahar's strategies seem to be privileged, and sometimes a strategy became privileged because I recognized it. For instance, I noticed that I privileged how Sahar was using her fingers to count by asking her to demonstrate her strategy. At one point, Sabina was observed imitating Sahar's use of fingers, and I wonder if it was due to my asking Sahar to demonstrate her strategy. Also, the students imitated certain strategies used by Sahar and Tamam by continuously imitating their use to find the products such as, the use of their fingers to skip count by 5 and adding a zero when multiplying by 10.

Activity #4: "Add 'Em Up"

<u>What was the activity?</u> "Add 'Em Up" is a mathematics game that the students engage in during their math centers as extra review or for extra practice of certain strategies (**Figure 27**). The objective of this game is to practice adding single-digit numbers. Add 'Em Up is designed

for groups of 2-4 students. *Materials*: Each group of students needs a gameboard numbered from 2-12 with several numbers appearing more than once, and two six-sided dice—one is numbered from 1-6 and the other has dots arranged in patterns representing the numbers 1 through 6. Also, each player requires several markers (a different colour for each player). *Directions:* In Add 'Em Up, players attempt to make a 2 x 2 square or a line of four adjacent markers (horizontal, vertical or diagonal). This is done by placing a tile (used to mark their space) on a number on the gameboard that is equal to the sum of the two rolled dice. For example, if Samartha rolls 2 and 6, she adds the two numbers together using her own strategies. Samartha can then place a tile on an "8" on the gameboard. Although some of the numbers appear more than once on the gameboard, Samartha may only claim one number for each turn. If the sum is unavailable, the player misses a turn. However, during this interaction, I tweaked the rules of the game slightly so that the players do not miss a turn but instead jointly claim a number on the gameboard.

Figure 27

Add 'Em Up Game Board and Dice

Add 'Em Up

2	6	10	8	4
7	4	8	7	6
5	9	7	5	9
10	8	6	9	8
12	5	11	6	3

What happened that day? On Tuesday February 26th, the classroom teacher asked me to practice counting-on with Suhail using the mathematics game 'Add 'Em Up'. Counting-on is a strategy of when adding two numbers, the student starts to count from the biggest number and adds the second number to it by counting-up. For example, if a student wants to add 7 and 3, the student is encouraged to start with the largest number "7" in their head, and then count-up by three ones to the number 10, "8, 9, 10". This strategy encourages students to move away from counting-all such as "1, 2, 3, 4, 5, 6, 7... 8, 9, 10", and to count-on to find the sum.

Initially, I was supposed to work with Suhail on playing this game, but Samartha offered to work with Suhail rather than do the morning calendar with the rest of the class, so Samartha joined us. We had 10 turns in total; Samartha played three turns—one turn in the beginning and two turns close to the end of the game. She missed the turns in the middle due to a minor injury, during which I took over her turns.

Stanza 1

This stanza shows the start of the activity with Suhail and Samartha. I begins we me sharing the objective of our interaction, which was to practice counting-on. The students then picked square tiles from a bin to be used as markers to play the game. The tiles were to be placed on the gameboard where the number equals the sum of two rolled dice.

Fatima:	We're going to practice how Suhail is going to hold a number in her head and count forward. I can't see you. I can't see you [To Samartha who was seated outside the camera lens].
Suhail:	1. [Counting the tiles]
Samartha:	Ha-ha. I'm so close to her. [Close to Suhail]
Suhail:	2. [Counting the tiles]
Fatima:	This one? [Referring to the red counter]
Suhail:	Yeah.
Samartha:	Hey people in the future [Looking at the iPad and speaking to the potential viewers of the research video].
Suhail:	3. [Counting the tiles]
Fatima:	What colour do you want to do? [To Samartha]
Samartha:	I want to do these colours? [Samartha grabs a handful of different coloured tiles]

[Samartha and Suhail laugh]

Fatima:	They have to be the same colour.
Samartha:	I just told Mrs. Holland, “can I just play with Suhail because I didn't want to do the calendar”. [Laughs] I made an excuse.
Suhail:	Yellow. Oh. [Referring to Samartha’s tiles]
Fatima:	There's no more. Why don't you use this one Suhail because there's not enough from that one? [Referring to the solid red counter instead of the clear one.]
Samartha:	She doesn't say no. She's a good old jelly belly. 20, 21, 22, 23, 24, 25, 26 [Referring to Mrs. Holland. Then, she counts along with the students on the carpet.]
Fatima:	No more, no more. [Telling Suhail that there are no more clear red tiles]
Samartha:	Wait, there's more in the other one. [Referring to another bin of tiles]
Suhail:	3, 4, 5, si-; 5.
Fatima:	Sure. 5. Okay.
Suhail:	1, 2, 3, 4, 5. [Counting Samartha’s tiles then passes over the extras]
Fatima:	5. I think 5 is enough for now. We'll get more if we need them.
Samartha:	Ouch, Ouch [Touches her neck]
Suhail:	1, 2, 3, 4, 5 [Counts her tiles]
Fatima:	5, 6, 7, 8 [I give Suhail 3 additional tiles] 1, 2, 3, 4, 5, 6, 7 [I count Samartha’s tiles and give her 3 additional tiles]
Suhail:	/6, 7/, 8
Samartha:	Ouch it hurts. It hurts. [Referring to her neck]

As we look more carefully at this segment of the transcripts, we can see that in explaining to the students the objective of our interaction, I used the term ‘count forward’ and **<u>not</u>** ‘count-up’ or ‘count-on’. In reviewing the transcript, I noticed that by privileging the teacher’s vocabulary, I made the term and the teacher significant, which established where the mathematical authority existed in the classroom. There is no evidence that the students picked up on the term. Yet, it may be something that plays into their meaning making.

As we also see in this stanza, Suhail picked square tiles from the bin without any directions because she knew that she needed tiles as markers to play the game. Hence, the use of tiles appeared significant because for many mathematics activities, the students use tiles as markers, counters, or to measure with. Also, as Suhail gathered her tiles from the bin, she counted aloud, or communicated the number of tiles as she picked them out one-by-one, she counted "1", "2", "3". Suhail is not only counting but appears to be representing a number using the tiles.

I asked Samartha which colour tiles she wanted to use, and in response, Samartha grabbed a handful of tiles that were different colours, placed them on the table, and said, "*I* want to do these colours". I reminded her that they all have to be the same colour. Samartha then grabbed several yellow tiles, and placed them on the table without counting them. She went along with what I said possibly because 1) she knows the rules of the game and 2) she sees me as an authority figure so she complied. This could be another instance of recognizing authority in terms of myself and the rules of the game.

Here too, Samartha referred to herself using the pronoun "I", making a personal connection to herself as she shared what she felt and thought. For example, at one point she shared "*I'm* so close to her"—implying that she is sitting close to Suhail, which is in response to my concern that she was seated outside the camera lens. In another example, Samartha shared, "*I* just told Mrs. Holland, can *I* play with Suhail because *I* don't want to do the calendar. *I* made an excuse". Samartha seems to see herself as having personal agency by being able to make decisions within this classroom, which she sees as part of her identity.

Also evident in this stanza is an instance that could reflect Samartha's relationship with her classroom teacher. Samartha shared that "She [Mrs. Holland] doesn't say no. She's a good old jelly belly". Even though I might see the teacher as an authority figure, Samartha seems to have established a very positive relationship with her teacher and feels comfortable to share her feelings and opinions towards her.

Notice that during this time Suhail is still heard communicating the number of tiles she had in her hand, she looked at her hand and said, "3". The fact that she knows they are 3 is important because she is making sense of how many she has. She then picked additional tiles from the bin and is heard counting-up, "…4, 5". Suhail then looked at me and repeated the total number of tiles she had, she said, "5". I repeated the number of tiles to acknowledge that the number she had was 5, "Sure. 5. Okay", again stepping in as the mathematical authority.

Suhail then counted Samartha's tiles. She pointed to each counter one-by-one using her index finger and counted 5 tiles, "1, 2, 3, 4, 5". Suhail then grabbed the extra tiles and returned them to the bin—leaving Samartha with 5 tiles in total. Interestingly, Samartha had not counted her yellow tiles when she took them out of the bin and Suhail does the counting for her, which could illustrate to Samartha that Suhail has more mathematical authority. I may have set that relationship up from the beginning when I said Mrs. Holland asked me to work with Suhail, which could create power differential between them.

I repeated the number of tiles Samartha had and I shared, "I think 5 is enough for now. We'll get more if we need them". Suhail then recounted the tiles she had in her hand by taking them out one-by-one, she counted, "1, 2, 3, 4, 5". The fact that Suhail recounted her tiles is important because she makes a connection between the total number of tiles they each have by comparing that they both have the same number of tiles.

I then decided to give the students 3 more tiles each to avoid wasting time afterwards trying to find more. I gave Suhail 3 additional tiles, and as I counted Samartha's tiles, Suhail is heard counting along. I stopped counting at 7 and Suhail counted up to 8. In confirming the number of tiles that they each have, I am perhaps reinforcing myself as the authority, which may influence their meaning making. At the same time, Samartha touched the bandage on the back of her neck and shared that it hurts.

<u>Stanza 2</u>

As you will see in the next portion of the transcript, I asked Samartha to show Suhail how to play Add Em' Up. Samartha responded by communicating her understanding of the instructions of the game and by addressing Suhail.

Fatima:	Do you want to show Suhail how to play this game?
Samartha:	Suhail. This is how you play the game. So, when you roll the dice and whe-. You roll the dice two times [meaning both dice]. And whatever number you get you count it.
Fatima:	/You have to add/.
Samartha:	And then whatever number-. And then whatever number you counted, you have to-, you put it on here [points to the gameboard]. And you always have to remember where it goes okay. Done.
Fatima:	Ok, so Suhail first.

Suhail: 3 [Places counter on gameboard].

Fatima: No, No, first-,

Samartha: No:: [Removes counter off gameboard].

Suhail: Oh sorry [Attempts to remove the counter off the gameboard but Samartha had already done so].

In this this stanza we notice that by explaining to Suhail how to play the game, Samartha takes on the role of the teacher. I may have 1) portrayed Samartha as more knowledgeable, and 2) characterized Samartha as in charge of legitimizing the instructions of the game, which reflected the identity I gave her. Also, Samartha freely expressed her understanding of the game and appeared to identify herself as someone who has experience playing it. Hence, the roles that Samartha and I played in this interaction could portray mathematical authority as distributed.

Also notice how Samartha's use of the words "count it" and "counted" reflected her choice of language and her understanding of how to play Add 'Em Up. Samartha might have not seen what she was doing as adding but as counting-on, and she might have seen it as her only strategy to play the game, which may influence her meaning making and privileges that particular sign system (i.e., counting). Although Samartha's use of the words "count it" or "counted" are clearly mathematical terms, I privileged the use of the word "add", which could privilege one sign system over any other.

As Samartha was explaining the game, Suhail grabbed the numbered die and rolled the number 3. Rather than rolling two dice and adding them up, she placed a marker on 3. Suhail communicated the number she claimed, she said, "3". She makes a connection between the oral and written representation of a number. In response, Samartha grabbed the dice and removed the marker off the 3, and I said, "no, no", which indicated to Suhail that something is not right. Suhail apologized, she said "oh, sorry", and then reached over to remove the marker, but Samartha had already done so. In apologizing, Suhail recognized authority and complied with it. Also, Suhail tried to figure out what she should do, but Samartha and I jumped in to indicate that she had done it wrong, which reinforces our mathematical authority.

Stanza 3 (Turn 1 Suhail)

In the next segment of the transcript, I showed Suhail how she should roll the dice, and then she was observed playing her first turn.

Fatima:	Wait, wait, you go like this. You roll the dice [Showing Suhail how to roll the dice]. 2 plus 2 [Pointing to the dice]. Okay? Go.
Suhail:	Okay.
Fatima:	3 plus 3 [Pointing to the dice]. 3 [Pointing to my head].
Suhail:	Plus 3 [Finishes my number sentence].
Samartha:	Is 6.
Fatima:	N-, [Hesitant]
Suhail:	No sixeths?
Fatima:	6. But I want Suhail to do it, okay?
Suhail:	3 [Tapped her head]
Samartha:	Ouch, Ouch, Ouch [Refers to her neck]
Suhail:	4-, [She attempts to count-up but is interrupted]
Fatima:	Ok, where's 6? [Based on Samartha's answer]
Suhail:	Sixeths [Repeats the number as she looks for it on the gameboard].
Samartha:	Ouch. [Refers to her neck]
Fatima:	Are you ok? [To Samartha]
Suhail:	Sixeths? [Lifts counter and points to the number on gameboard]
Fatima:	Six. Yes. Six.

We can see in this stanza that I showed Suhail how she should roll the dice by holding them, shaking them, and rolling them onto the table. I also indicated that she needed to add them together by referring to the word plus in my number sentence. I think my use of the addition operation (i.e., plus) influenced her meaning making in terms of knowing that she needed to combine the two dice.

In response to my explanation, Suhail nodded her head, picked the dice off the table, and rolled the dice by imitating what I did—holding the dice, shaking them, and rolling them onto the table. In imitating me, Suhail recognized authority and complied with it. Suhail rolled two threes, and she arranged them together side-by-side. I then communicated the numbers on the dice, I said "3 plus 3". I repeated the number "3" as I pointed to Suhail's head, which is a gesture the teacher used to indicate holding a number in her head, and Suhail finished my number sentence, she said "plus 3". I then started to try counting-up with Suhail by holding my hand with my thumb out, but Samartha interrupts and says the answer “6”. I hesitated which might have given Suhail the impression that the answer is wrong, and she asked with intonation, “no sixeths?”. I confirmed the answer is 6 and explained to Samartha that, “I want Suhail to do it”, which again reinforced my role as the authority figure as I legitimized turn taking and portrayed Suhail as someone who needs to practice and is capable of acting on her own.

Next, we see that Suhail attempted to count-up on her own. She made a fist, tapped her forehead as she said the number 3, then opened up her index finger and counted the number “4”, but was interrupted by me. I pointed to the gameboard and asked, “where’s 6?”—based on Samartha’s answer. I had not realized at the time that Suhail was trying to count-up. The fact that she imitated me by tapping her head and used her fingers to count-up is important because she tried to make sense of what I was doing or what she is supposed to do to combine the dice together. Interestingly, Suhail used her index finger to begin counting-up where as I started with my thumb. This could be an instance related sign systems or knowledge with respect to how we use our fingers to count. This could influence their meaning making in that they may consider how I used my fingers to count as the correct way.

Suhail then communicated the number 6 as she looked for it on the gameboard. Suhail appeared to make a connection between the number she verbally articulated and the written representation of the number on the gameboard. Suhail placed a counter on top of the number 6, but then lifted the counter and communicated the number with intonation, she said “sixeths?” In response, I repeated the number twice in order to 1) confirm the number she claimed is correct, and 2) to share the correct pronunciation of the number. However, by repeating the number 6, I privilege the Canadian dialect for pronouncing that number, when it may be her way or how she was taught to pronounce it. This could be an instance of privileging one sign system or knowledge over another.

Stanza 4 (Turn 1 Samartha)

After Samartha was a little distracted by her neck and trying to find a Kleenex, she returned to her chair to play her turn. She grabbed the dice, lifted her hand, and dropped the dice onto the table. Samartha then brought the dice together and read the numbers on the dice.

Samartha:	6 plus 5.
Suhail:	No [Grabs the dice from Samartha's hand]

Samartha: I'm doing it.

Suhail: Okay [Nods]

Samartha: Oh, that's um:: [Samartha rolls the dice once more]

[Pause]

Suhail: 3 [Reads numbered die]

Samartha: 6 [Reads number on dotted die]

Suhail: 6. 7, 8, 9. [Counting up]

Samartha: /7, 8, 9/ [Counting up]

As seen in this portion of the transcript, following Samartha's first attempt to roll the dice, she appeared to make a connection between the numbers rolled and the operation of addition, she said "6 plus 5". This act of bringing the dice together may be regarded as a representation of a mathematical number sentence (i.e., 6 + 5). Also, the fact that Samartha articulates the operation of addition is important because it reflected her understanding of combining the two numbers together.

Suhail then grabbed the dice from Samartha's hand and said, "no". Suhail held the dice, shook her hand, and rolled them onto the table, based on how I showed her earlier. This could be another instance of reinforcing my mathematical authority as she repeated what I did, and she may see it as the only way they should roll the dice. Here too, Suhail takes on the role of the teacher by recognizing autonomy and exercising control over how Samartha should roll the dice.

In response to what Suhail did, Samartha said, "I'm doing it". Suhail nodded her head and said, "okay". It appears that Samartha did not make sense of what Suhail was trying to do, and might have assumed that Suhail thinks it is still her turn. Samartha then grabbed the dice, lifted her hand, and dropped the dice onto the table. She rolled the number 3 and 6 dots. Samartha arranged them together side-by-side and took a small pause. Suhail looked at the dice and communicated the number "3". Samartha then used the dots as a representation of the number, and she said, "6".

Suhail then initiated counting-up. She made a fist, tapped her forehead, and repeated the number, "6". Then together, they counted-up 3 more using their index, middle, and ring finger, they counted "7, 8, 9". It appears that she was imitating what I was doing. Also, Suhail appeared to know that she could put the number 6 in her head and use her fingers to count-up. The use of their fingers act as a tool in creating a visual representation for counting 3 more, and where, each finger represents a number—index finger (1) = 7, middle finger (2) = 8, and ring finger (3) = 9.

As seen in this stanza, Samartha and Suhail collaboratively engaged in finding the solution to the sum of the addends, which played a role in their meaning making. Also, the dice appeared to play a significant role in helping Suhail and Samartha acquire the sum of the addends—that is, one die indicated the number they needed to count-up from and the other die indicated how much more they needed to count.

Samartha then claimed the number 9 that is close to Suhail's 6. It seemed that Samartha chose to block Suhail from making a vertical line of 4 adjacent squares. Also evident is her intuitive awareness of the gameboard and the numbers on it, which helped her strategize where the counter should go.

Stanza 5 (Turn 2 Suhail)

In the following part of the transcript, Samartha realized that the scratch on her neck started to bleed. She asked me to play her turn and went to the office to get another bandage.

Samartha:	Oh, I'm bleeding so bad. Okay can you go for my turn?
Fatima:	Sure. [Pause-fixing iPad]. Okay, I'll go for Samartha's turn.
Suhail:	No Samartha.
Fatima:	It's your turn.
Suhail:	Yeah.
Fatima:	Yes.
Suhail:	5. [Reads numbered die]. 5, 6, 7, 8, 9, 10, 11 [Counts-up].

A viewing of the video showed that Suhail rolled the dice and arranged them together side-by-side. I then reached over for the dice thinking it was Samartha's return, but Suhail shared, "No, Samartha"—indicating that it is her turn. I passed her the dice and she played her turn.

As we can see, Suhail rolled the dice and got the number 5 and 6 dots. Suhail arranged them together side-by-side and read the number "5". She then took a brief pause and looked at the dice. She then reiterated the number "5" and pointed to each dot one-by-one to count-up, she said "6, 7, 8, 9, 10, 11". Interestingly, when Suhail was combining the dice, she chose the die with the symbol first which is a representation of the number she could read, then she counted-on the dots one by one. There is no evidence that she knows the dots represent the number 6 because she does not say the number. However, the fact that she knows she can use the dots to count-up is important as it reflected her understanding of what she is supposed to do. Also, it reflected autonomy in the sense that she exercised control over how she wanted to combine the numbers on the dice.

Suhail claimed the number 11 on the gameboard, which sat next to the first counter she claimed. She expressed astonishment, “Wow”—she seemed particularly pleased that she has two red tiles next to each other, which allows her to build a 4 square or a horizontal line of 4 adjacent tiles.

Stanza 6 (Turn 2 Fatima)

In the next segment of the transcript, it was my turn and I applied the strategy of counting-up using my head and fingers, which reinforced mathematical authority in terms of using the teachers’ strategy to count-up. I started by rolling the dice and I rolled the number 5 and 5 dots.

Fatima: My turn. 5 [Pointing to my head].

Suhail: Plus 5 [Finishes my number sentence by referring to the numbered die]

Fatima: 5. 6, 7, 8, 9, 10. [Counting-up]

Suhail: /6, 7, 8, 9, 10/. [Counting along]

Fatima: Where's 10 Suhail?

[Suhail points to the first 10 she sees, then gently taps her head as she realizes there's another 10 and points to it]

Fatima: There's 2 tens. Where do you think I should put it [the counter] Suhail?

Suhail: 2 [Repeats that there are two tens].

Fatima: So where should I put it? On this one, or this one? [Referring to the two tens on the gameboard]

[Suhail points to the 10 closer to the top center]

As we see in this transcript, I communicated the number “5” as I pointed to my head. Suhail brought the two dice together and communicated the addition operation and the number, she said “plus 5”. When Suhail looked at the dice, she first referred to the die that has the number on it. This is important because Suhail recognized the symbol as a specific number but does not appear to name the number that is associated with the dots. However, she knew that she needed to count-up and used the dots to do so. This is critical because she is playing this game and trying to model what Samartha and I were doing, but she does it in her own way, which reflected her identity. I then repeated the number 5, pointed to my head, and I counted-up using my fingers. I saw the 5 dots as 5 and counted-up using the number 5. Suhail looked at my fingers and counted along, she counted “6, 7, 8, 9, 10”.

I then asked Suhail to show me the 10 on the gameboard, and Suhail pointed to two tens, making a connection between the oral and the written numbers. I acknowledged that there were two tens and asked, “where do you think I should put it [the marker] Suhail?” Suhail repeated the number “2” and pointed to the 10 that is closer to the top center. It appeared that she chose the 10 that is farthest away from her tiles. Her intuitive awareness of the gameboard helped her make sense where my counter should go. This could also be another instance of recognizing autonomy, although influenced by my actions, Suhail exercised control over which 10 I should claim on the gameboard.

Stanza 7 (Turn 3 Suhail)

As you will see in the next portion of the transcript, Suhail did not use her fingers to count-up, but used the dice. Suhail recognized autonomy by exercising control over which strategy she wanted to use.

Fatima: Ok Suhail your turn.

[Suhail rolls the dice and takes a pause]

Fatima: So, what's that?

Suhail: 2 [Reads numbered die]. 3, 4, 5 [Counts-up].

[Suhail claims the number 5 on the board game]

Fatima: 5. Oh::

Suhail: Yeah.

In this stanza we notice that Suhail rolled the dice and got the number 2 and 3 dots. She took a small pause and looked at the dice. I then asked, “so what’s that?”. Suhail responded by communicating the number “2” and counting-up using the dots, she counted “3, 4, 5”. Again, when Suhail looked at the dice, she referred to the die that has the number on it and used the dots to count-up. It appeared that Suhail tried to legitimize the use of the dots as a tool to count-up despite being taught to use her fingers. The use of the dots may appear significant to Suhail as it is the strategy she created and she may find it easier to use. Also, the use of the dots could create a connection between the dots and the number they each represent—one dot = 6, another dot = 7, and another = 8. However, in using her fingers, Suhail had to make a connection between the number she is counting-up (i.e., 3), her fingers (i.e., 3), and the number that each finger represents (i.e., index = 6, middle finger = 7, and ring finger = 8). In this manner, the process involved in counting-up makes the use of the dots on the dice appear easier.

Suhail claimed the number 5 on the gameboard. There were three spaces that she could choose from. However, she chose the 5 that sat next to her previously claimed tiles. It appeared that her intuitive awareness of what 4 tiles would look like horizontally, helped her to reason and make sense of which space she wanted to claim.

Stanza 8 (Turn 3 Fatima)

As seen in this portion of the transcript, I rolled the dice and I got the number 1 and 5 dots.

Fatima:	Let's see. 5 [Pointing to the dice]. 5 [Pointing to my head].
Suhail:	6 [Counts-up]
Fatima:	6. Oh:: 6. There we go. [I claim the number 6 on the gameboard]

As we see, I pointed to the die with the dots and said the number "5". Then, I repeated the number 5 and I tapped my head to indicate that it is there. Suhail then looked at the dice and shared her answer, she said "6". Since I articulated the number represented by the dots, Suhail relied on that representation to make sense of what the dots are and to then count-up using the numbered die. Although it was my turn, Suhail recognized autonomy by providing her solution to the sum of the two rolled dice. I then held up my hand, opened my thumb to indicate plus 1, and I said "6", which confirmed Suhail's answer and reinforced my role as having mathematical authority.

For this turn, I decided to claim my own space as I was interested to see how Suhail would react if I was to block the numbers she claimed. After claiming the number 6, Suhail gasped. She seemed to have recognized that she could no longer create a 4 square or a vertical line of 4 adjacent tiles.

Stanza 9 (Turn 4 Suhail)

In the following stanza, Suhail made use of her fingers, but only to create a representation of the numbered die.

Suhail: 1, 2, 3 [Holds right hand up and counts]. 4, 5 [Looks at the dice and counts-up]

Fatima: 3 [Points to head] 4, 5 [Uses fingers to count-up]

Suhail: /4, 5/. 5. 5. [Repeating it as she looks for the 5 on the gameboard]

As we can see in this part of the transcript, Suhail rolled the number 3 and 2 dots. She brought the dice together, looked at them, and counted three fingers, she said "1, 2, 3"—creating a representation of the numbered die. Suhail then used the dots to combine the dice, she counted "4, 5". Here, Suhail recognized autonomy by freely providing her solution and using her own strategy to find the sum of the addend, which influenced her meaning making in terms of making sense of quantity and counting-on.

I then pointed to my head as I said the number "3", and I opened my fingers one at a time and together we counted, "4, 5". This could be an instance of reinforcing mathematical authority where, I used my strategy to confirm her answer.

Suhail then looked for the number 5 on the gameboard. She tapped her head and repeated the number 5. There were 2 fives, and she chose the one that blocked my 9, which prevented me from creating a vertical line of 4 adjacent squares or a 4 square.

Stanza 10 (Turn 4 Fatima)

As you will see in this portion of the transcript, I started off by praising Suhail, which I have noticed is something that I have very specifically done, and that could play into her meaning making in terms of confirming that she is on the right track. Then, I rolled the dice and I got the number 4 and 3 dots.

Fatima: Good job Suhail. Let's see. 4 [I point to the die and we both read the number]

Suhail: /4./ 5, 6, 7.

Fatima: /5, 6, 7/.

Suhail: Yeah

Fatima: 7. Where's my 7? I'll put it here.

As we see, I pointed to the die with the bigger number, and we both read the number "4". Suhail then counted-up using each dot, she counted "5, 6, 7". I then claimed the number 7 on the gameboard which blocked Suhail from creating a horizontal line of 4 adjacent tiles or a 4 square. She gasped and tapped her head—which appeared to show that she is not happy that I blocked her.

Stanza 11 (Turn 5 Suhail)

In this stanza, Suhail rolled the dice and got the number 6 and 4 dots. She arranged the dice side-by-side and read the numbered die, she said, "sixeths". Suhail then counted-up using the dots, she counted "7, 8, 9, 10". Suhail then placed her marker on 10, but hesitated as she picked it up and looked at the gameboard. Suhail does not just claim a space, but appears to check to see if there is another space she could claim.

Stanza 12 (Turn 5 Fatima)

For my turn, I rolled the dice and I got the number 6 and 1 dot. I read the number "6", touched my head, and I held up my hand to count, but Suhail looked at the dice and said the answer "7". Suhail then pointed to each die—she read the number "6" and counted-up one more using the dot, she said "7". This could be an instance of reasoning and proof as she appeared to have checked her answer. I then praised Suhail by saying "Good job", which could confirm her answer and reinforce mathematical authority. I then claimed the number 7 on the gameboard.

Stanza 13 (Turn 6 Suhail)

In the next segment of the transcript, Suhail rolled the dice and got the number 5 and 6 dots.

Suhail:	5. [Reads the number on the die] 6, 7, 8, 9, 10, 11. [Counts-up using the dots on the die]
Fatima:	Uh 11? Let's see is there an 11 here somewhere?
Suhail:	Um:: No 11.
Fatima:	Oh, no. 11? Yeah, 11 here.
Suhail:	Oh.
Fatima:	You can put it over here. Put it on top look.
Suhail:	Oh.
Fatima:	You have two on top of each other now.

Suhail:	Nods.

In this part of the transcript, Suhail read the number 5 and counted-up 6 more as she pointed to each dot, giving her 11. Suhail then claimed the number 12 on the gameboard and expressed excitement because she formed a horizontal line of 4 adjacent tiles. It was not clear why Suhail claimed a 12 after having counted correctly. Then, with intonation I said, "Uh 11?", and I looked for an 11 by lifting the tiles one-by-one. Suhail looked at the gameboard and said, "no, 11". I found one that was already claimed and I showed it to Suhail. I then asked her to reclaim that space. Suhail appeared surprised, which could suggest that she knew she needed to miss a turn since there was no space to claim. I did tweak the rules of the game thinking I did not want to upset Suhail by telling her that she had to miss a turn. However, she did not question or object to it given that 1) she knows that I am in charge and could legitimize certain rules and ignore others, and 2) she might have not been comfortable expressing her understanding of the rules since I am older—which could be a cultural behaviour.

Stanza 14 (Turn 6 Fatima)

As you will see in this portion of the transcript I played my turn.

Fatima:	My turn. 3 [Reading numbered die, and pointing to dotted die].
Suhail:	3 [Reads numbered die]. 2. 3, 4 [Counts-all?].
Fatima:	4. Where's my 4? Oh, right here.
Suhail:	Gasps

In this stanza we see that I rolled the number 3 and 1 dot. I articulated the number 3, pointed to the other die, and waited for Suhail to answer. Suhail looked back and forth at the two dice and said, "3. 2, 3, 4". She started with what I said, "3", and then it seemed that she started with the number 1 and counted-on three more. This could be another instance of recognizing autonomy in that she exercised control over how she made sense of the numbers and to count-on.

I then claimed the number 4 on the gameboard, but Suhail gasped as she might have been aware that I am one tile away from forming a diagonal line of 4 adjacent tiles.

Stanza 15 (Turn 7 Suhail)

In the next part of the transcript, Suhail rolled the dice and got the number 2 and 2 dots.

Fatima:	How much is that?
Suhail:	2 [Hold up two fingers]. 3, 4, 5. 5. [Counts the dots].
Fatima:	Or 3 [Point to head]. 4, 5 [On fingers]. Right?
Suhail:	Nods.

In this stanza, Suhail looked at the dice and took a small pause. I then asked, "how much is that?" In response, Suhail started with the numbered die, she said "2", and created a representation of that number using her index and middle finger. She then counted-up using the dots on the die, she counted "3, 4, 5". Suhail tapped her head and repeated the number "5" as she looked for it on the gameboard. Suhail then claimed the last available 5 on the gameboard.

I pointed to my head and said the number "3". Then, I opened my fingers one at a time to count-up two more, and Suhail counted along " 4, 5". Again, this act of counting-up using my head and fingers could influence her meaning making in terms of getting proof that her answer is correct, which could reinforce my mathematical authority.

Stanza 16 (Turn 7 Fatima)

As seen in this portion of the transcript, for my turn, I rolled the number 5 and 3 dots and I asked Suhail to try it.

Fatima:	You do it.
Suhail:	5, Sixeths, [Using dice] 7, 8 [Using my fingers to count-up]
Fatima:	Good job. There we go [I claim the number 8 on the game board]. Suhail your turn.

As we can see in this part of the transcript, Suhail read the number "5" on the numbered die, and started to count-up using the dots on the die, she counted, "5, 6", but then switched to my fingers and finished her counting, she counted "7, 8". Suhail tried to use the dots to count up but it seemed that my use of my fingers to count a long prompted Suhail to use them. I continued to establish myself as the authority by prioritizing the use of my fingers to count-up, and Suhail complied with it. I then claimed the number 8 on the gameboard.

Stanza 17 (Turn 8 Suhail)

Still enforcing the use of our head and fingers, I deliberately said to Suhail, "Let's do this one in your head", privileging counting-up using that particular strategy, as we can see in the transcript below.

Fatima:	Let's do this one in your head. 5 [Point to my head and open one finger].
Suhail:	5. Sixeths, 7 [She uses her fingers and counts 2 more].
Fatima:	Good Job Suhail.

As seen in this portion of the transcript, Suhail rolled the dice and got the number 5 and 3 dots. I pointed to my head and I said, "5". I then opened my thump to initiate the counting and Suhail used her fingers to count-up, she counted, "5. Sixeths, 7". I then praised her by saying, "Good Job, Suhail". In praising her and continuing to enforce the use of our head and fingers to count-up might build the impression that I value the use of our fingers to count-up which legitimizes that particular strategy over any other. Suhail then claimed the number 7 on the gameboard.

Stanza 18 (Turn 8 Fatima)

In the next part of the transcript, I rolled the dice and I got the number 6 and 4 dots. Again, I asked Suhail to try it.

Fatima:	My turn. Try it Suhail. Try it. 6.
Suhail:	7, 8, 9, 10. [Using her fingers].
Fatima:	10. Oh! 10 are all gone. 10 here. I can put one here.

As we see in this transcript, I read the number 6 and held up my hand. Suhail then used her fingers to count-up, she counted "7, 8, 9, 10". I then shared that all the tens are gone and I claimed a 10 that was already claimed.

Stanza 19

In this stanza, Samartha returned from the office. She looked at the gameboard and shared, "I just need one more" and pointed to the empty space. Samartha then grabbed the dice to play a turn, but Suhail expressed disagreement, she said "no, it's-," and attempted to get the dice from Samartha. Samartha explained that it is her turn since she just returned. I then intervened by explaining to Samartha that I just played her turn. Samartha then passed the dice to Suhail. This could be another instance of recognizing mathematical authority in terms of myself and the rules of the game, and Samartha complied with it.

Stanza 20 (Turn 9 Suhail)

In the next part of the transcript Suhail played her turn. First, she rolled the dice and got the number 6 and 4 dots. Notice then how Suhail used her fingers to count without me initiating it.

Suhail:	7, 8, 9, 10. [Counting-up using her fingers]
Fatima:	10 again. Is there a 10 here? Let's see is there another 10 somewhere, somewhere, somewhere. [Looking for a ten on the gameboard]
Suhail:	One 10.
Fatima:	One 10?
Suhail:	Yeah.
Samartha:	If there's no 10, you skip-,
Fatima:	Oh, there's a 10 here. You can go here or here [I point to the two tens already claimed].
Samartha:	Wait. You skip a turn if there's no 10.
Fatima:	We're just going on top.
Samartha:	Oh my God. This is not how you originally play the game.
Fatima:	I changed the rules a bit.

As seen in this part of the transcript, Suhail did not read the numbered die, but counted-up 4 more using her fingers, she counted "7, 8, 9, 10". I then asked, "Is there another 10 somewhere?"—given that we claimed a ten twice. Suhail responded, "one ten". I then looked for another ten space by lifting the counters off the gameboard. I found one and said to Suhail that she could reclaim either space. Samartha then explained that we needed to skip a turn if there is no space to claim, and I said that we are just going on top. Samartha was not impressed, she said, "Oh my God. This is not how you originally play the game". I shared that I changed the rules of the game a bit, but she still did not seem too keen. In opposing to how Suhail and I were playing the game, Samartha exhibited active agency as she shared her knowledge and understanding of the rules of the game, which reinforced mathematics authority as distributed—that is, Samartha relied on her understanding of the rules to dispute what we were doing, and I exercised authority in terms of sanctioning the rules of the game.

Stanza 21 (Turn 9 Samartha)

In the following part of the transcript, Samartha rolled the dice and one fell to the ground. She picked it up, placed it on the table, and shared the numbers she rolled.

Samartha:	5 plus 1. It's easy. It's 6.
Suhail:	No 5 plus 1. [Shows me the die].
Samartha:	It's 6.
Fatima:	This is a 5? [Showing Samartha the die with one dot]
Samartha:	Yeah, it was 5.
Fatima:	This one. Is it this one right here? [Referring to the die with the dot].
Samartha:	Yeah that was 5. But it rolled. It was 5.
Suhail:	1.
Samartha:	It was 5.
Fatima:	Oh, it was 5. She says it was 5. So, it was like 5 and then 1. So, she chose to use 5. It's okay. Keep going. Suhail's turn.

As we see in this transcript, Samartha used the addition operation in a number sentence to indicate that she got 5 plus 1. She then shared the answer "6". However, Suhail looked at the dice and expressed disagreement, "No 5 plus 1". She grabbed the die and showed me that it was 1 dot, which would suggest that Samartha rolled 1 plus 1. I looked at the die and asked Samartha, "this is a 5?" Samartha explained, "Yeah, it was 5". To clarify, I asked "is it this one right here", and Samartha responded, "Yeah that was 5. But it rolled. It was 5". Suhail disagreed and confirmed that it was "1"—based on what she saw when it was on the table. I then explained to Suhail that the die was 5 dots but it rolled to 1 on the table, and Samartha decided to go with 5 dots.

In showing and referring to the dice rolled on the table, Suhail exercised mathematics authority by questioning Samartha's actions and relaying on the dice on the table as proof, which also reflects her identity. Also, in agreeing or taking Samartha's side, I reinforced where the mathematical authority resides in this interaction, and Suhail complied with it. Samartha then claimed the number 6 leaving her with one space to build a vertical line of 4 adjacent tiles or a 4 square.

Stanza 22 (Turn 10 Suhail)

When it was Suhail's turn to play, she grabbed the dice and rolled the number 4 and 4 dots, as seen in the transcript below.

Suhail:	4. [Reads numbered die] 5, 6, 7, 8 [Uses fingers to count-up 4 more]
Samartha:	/Is that 4 plus 4 is [pause] 8/. Oh::
Fatima:	Good job.
Samartha:	No. No wait. Um ok you can put it.
Fatima:	Okay.
Samartha:	I'm not going outside today because yesterday I started to cough because of the wind.
Fatima:	Okay.

In this part of the transcript, Suhail read the die with the number 4 and used her fingers to count-up, she counted, "5, 6, 7, 8". As Suhail was counting, Samartha asked, "is that 4 plus 4 is [pause] 8?" Samartha did not share the answer until Suhail reached the number and together they said, "8". In response, I said, "good job", which acted to confirm their answer, again reinforcing my mathematical authority.

Also, as Suhail was claiming the number 8 on the gameboard, Samartha expressed disagreement, she said "No:: No::". This led Suhail to lift the tile off the gameboard. Samartha then said, "um okay you can put it", and Suhail placed the tile back on the 8. This could reflect the relationship that existed between them in that Suhail was attentive to what Samartha said and did not ignore her given that she was portrayed as someone who knows how to play the game, which to some extent reflected their identities.

Stanza 23 (Turn 10 Samartha)

In the following part of the transcript, Samartha did not read or refer to the numbers she rolled. It appeared as though she did the math in her head and then claimed the number 8. However, Suhail attempted to make sense of what Samartha had done by referring to the dice.

Suhail:	4 + 4. 8 [Reads dice and refers to the number Samartha claimed on the gameboard]
Fatima:	8.
Samartha:	Oh:: 8. Four in a row!
Fatima:	Samartha won. 1, 2, 3, 4, 5.

Samartha: I was like this one, no this one.

As we see in this transcript, Suhail built a mathematical sentence using the numbers that Samartha rolled (4 and 4), then she read the number Samartha claimed, "8". I repeated the number "8" to confirm Samartha's answer.

As Samartha continued to look at the gameboard, she realized that there is another 8 that she could claim, which would give her a horizontal line of 4 adjacent tiles, she said, "Oh:: 8. Four in a row!". I responded by acknowledging that Samartha won and I counted the number of tiles Samartha had in a row. It turned out that Samartha had 5 in a row and not 4, but she did not consider the two tiles on top of each other to count as one. Samartha seemed to believe that when the rules of the game are set, they should not be altered—recognizing mathematical authority in terms of the rules of the game. This then concluded our interaction in this game.

Summary of Main Ideas: Activity #4 "Add 'Em Up"

In this summary, I draw on Gee's (2005) analytical framework of discourse analysis to discuss children's meaning making. This analytic framework, described in **Table 3** in the methodology chapter, specifically focuses on the construct of building tasks of "language in use" including, significance, activity (i.e., mathematical actions), identity, relationships, politics, and sign systems or knowledge.

Significance. There were several instances that appear to be significant to students' mathematical meaning-making. Some of them involved the use of specific mathematical terminologies, others involved the use of strategies, and others portrayed the use of concrete tools that emerged as important to their understanding of the mathematics involved in the game.

Concerning mathematical terminology, I noticed that my use of the term "count forward" and not 'count-up' or 'count-on' as a means to talk about adding made the term and the teacher appear significant. There is no evidence that the students picked up on the term. Yet, it may be something that plays into their meaning making. Also, Samartha's use of the terms "count it" and "counted" reflected her understanding of the game. The fact that Samartha referred to what she was doing as counting is important because she might have not seen what she was doing as adding but as counting-on, and she might have seen it as her only strategy to play the game, portraying her knowledge and experience of the game.

There also seemed to be specific strategies that the students relied on and were meaningful to them including counting and counting-up. As in the previous analysis of the children playing a game, their counting strategies demonstrated important characteristics such as stable order, one-to-one correspondence, cardinality, and understanding of movement as magnitude that is the idea that as one moves up or down in the counting sequence the quantity increases or decreases. The transcript for this activity provides evidence that Suhail and Samartha had an understanding of stable order in counting, and recognized the idea that the last number in a count represented the quantity. Also, counting-up seemed to be a significant strategy that the students used and connected with adding. At one point, I pointed to Suhail's head which is a gesture the teacher uses to indicate holding a number in their head and then counting-up using their fingers. Suhail picked up on the use of this strategy by tapping her own forehead as she said the number 3, then opened up her index finger and counted up. The fact that she imitated me is important because she tried to make sense of what I was doing or what she is supposed to do to combine the dice.

In terms of concrete tools, the tiles, the dice, and their fingers appeared significant. The tiles were used for counting as well as markers on the gameboard. For instance, Suhail counted the tiles to make sense of how many they each had to play the game—reflecting her mathematical understanding of counting and quantity. One of the most important tools appeared to be the dice as Suhail and Samartha used them to calculate the sums. For example, the students began with the number on one die and counted-up using the other die. For Suhail, she chose the die with the symbol firs, then she counted-up the dots one by one. This is important because Suhail recognizes the symbol as a specific number but does not appear to name the number that is associated with the dots, and may not know what number the dots represent because she does not say the number. Also, Suhail knows she can use the dots to count-up which reflects her understanding of what she is required to do, and also, it is a strategy she created and relied on to add the dice. As for Samartha, she chose the die with the bigger number and added the second number to it. Samartha would arrange the dice together side-by side and would articulate the operation of addition. Also, the use of their fingers for counting appeared significant to students' meaning making. They used their fingers to imitate the strategy of holding a number in their head and to count-up on their fingers, or when counting along. Interestingly, students reflected different ways of using their fingers to count. This might interfere with their interactions because they represented numbers in a variety of ways, and they might have considered the way I used my fingers to count as the correct and only way.

Activities: Mathematical Actions. In analyzing the transcript, we also notice that Samartha and Suhail carried out mathematical actions as described in **Table 3** of my analytical framework that contributed to their meaning making. These mathematical actions include the use of reasoning and proving skills, communication of ideas, representation of ideas using various tools, and making connections between ideas or concepts.

Reasoning and Proving. The act of reasoning and proving was evident in several places and contributed to their meaning making in that it reflected how students negotiated understanding to derive to a solution. One place where it was quite prevalent was when Samartha and Suhail were combining the dice to determine the sum. They would reason their way to a solution by first articulating the numbers rolled on the dice in a number sentence or not, then they used their fingers, the dots on the die or a combination of both, or used mental strategies to determine the sum of the rolled dice.

Also, the act of checking the sum shared by either student could be evidence for reasoning and proving. For instance, when Suhail questioned the sum of the dice rolled by Samartha, she grabbed the die to show me that Samartha rolled 1 plus 1 and not 5 plus 1—suggesting that Samartha should claim the number 2 and not 6. Suhail, through the use of gestures and some words, was capable of negotiating and explaining her thinking by relaying on the dice rolled on the table as proof. This was also evident when Samartha rolled the dice and claimed the sum without reading or referring to the numbers she rolled. Samartha appeared to have done the math in her head and claimed a number on the gameboard. However, Suhail checked the numbers rolled and then confirmed the sum.

Also evident was their awareness of the gameboard and the numbers on it, which helped them reason and strategize which space they wanted to claim on the gameboard. They were not simply claiming a number but reasoning where they needed to place their counters in order to 1) build a line of 4 adjacent markers; and 2) block one another from winning.

Communication. We can see that the students relied on oral and visual communications to share their mathematical understandings, ideas, and to enact the use of their strategies. For example, the students communicated when they 1) counted or recounted the tiles they had; 2) articulated their understanding of the instructions of the game; 3) responded to a question or posed a concern; 4) read or referred to the numbers on the die or on the gameboard; 5) shared a number sentence; 6) completed someone else's number sentence; 7) counted-up using the dots or their fingers; and 8) shared the sum of the two rolled dice.

Representation. There were also several instances where Samartha and Suhail used concrete tools to create representations of their mathematical understandings and ideas using the tiles, the dice, their fingers, and the gameboard. These tools helped students make connections between written and spoken words for numbers, and between number and quantity. Concerning the tiles, as was evident in the previous game, the students collected tiles from a bin to be used for claiming a number on the gameboard. However, as the students collected their tiles, they counted and recounted their tiles, and in so doing, the tiles became a representation of a number and quantity.

The students also relied on the dice as representations for adding. For instance, Samartha would arrange the dice together side-by-side creating a representation of a mathematical number sentence, then she would add the dice together mentally or by counting up on her fingers. As for Suhail, she would choose the die with the symbol first as a representation of the number she could read, then she would count-up the dots one by one. At times, Suhail would rely on the representation that Samartha or I articulated for the dotted die to make sense of what the dots represent and to then count-up using the numbered die.

The use of their fingers also acted as a tool for creating visual representations for counting-up and where each finger represented a number. For instance, when adding 6 plus 3, they would start at 6 and count-up 3 more using their fingers—index finger (1) = 7, middle finger (2) = 8, and ring finger (3) = 9. Suhail also made use of her fingers to create a representation of the numbered die and then used the dots to combine the dice, influencing her meaning making in terms of making sense of quantity and then counting-on.

Connections. The students also made connections between mathematical ideas or concepts. These were particularly evident when the students made connections between 1) number and quantity, for example, when they saw that the number of tiles they had equaled the total amount of tiles I asked them to get—that is, 1, 2, 3, 4, 5 tiles are 5 tiles in total; 2) number and addition, such as when they recognized that the two numbers rolled on the dice, when added together, equaled the sum, 3) more than and less than, especially when Suhail recognized that Samartha had more than 5 tiles and she returned the extras to the bin; 4) they also made connections between oral and written representation of a number like, when Suhail would communicate the sum of the addends as she claimed the number on the gameboard ; and 5) between a number and a concrete tool (e.g., one-to-one correspondence between the dots on the die and the number counted).

Identities. Also evident in this activity are the identities that Samartha and Suhail built as mathematics learners. Although Suhail does not verbally communicate much in this activity, she establishes a mathematical identity that portrayed her as engaged, and continuously making sense of the rules of the game and how to count-up as she played her turn. Suhail appeared to identify with the activity by recognizing autonomy and exercising control over which strategy she wants to use to combine the numbers on the dice. This is critical because she is playing this game and trying to model what Samartha and I were doing, but she does it in her own way, which also reflects her identity and influences her understanding of quantity and counting-up. She also appeared to take on the role of the teacher by questioning Samartha's computations especially when it was not clear to her how Samartha got the sum of the addends. These examples are important because they reflected Suhail's identity in terms of her recognizing the instructions explained to her, then complying with them and sharing them with Samartha.

Samartha established a mathematical identity that portrayed her as having personal agency by being able to make decisions within this classroom, which she sees as part of her identity. In communicating with Suhail and myself, Samartha referred to herself using the pronoun "I", making a personal connection to herself as she shared what she felt and thought. For example, at one-point Samartha shared, "*I* just told Mrs. Holland, can *I* play with Suhail because *I* don't want to do the calendar. *I* made an excuse". This example and others similar illustrate Samartha's identity and the sense of autonomy she exercises in the classroom and in this activity in particular. Samartha also takes on the role of the teacher by showing Suhail how to play the game. I asked Samartha to show Suhail how to play the game, and in so doing, I may have 1) portrayed Samartha as more knowledgeable and 2) characterized Samartha as in charge of legitimizing the instructions of the game, which could reflect the identity I gave her. However, Samartha freely expressed her understanding of the game and appeared to identify herself as someone who has experience playing it, which exhibited her sense of agency.

Relationships. Relationships came out in a variety of ways during these interactions and that influenced the students' mathematical meaning-making. One was the relationship between the students and the activity, another was the relationship between Samartha and the classroom teacher, still, another was the relationship between the students and myself, and another was the relationship between Samartha and Suhail.

The students had different experiences playing the game which influenced their relationships with the game and their level of confidence. For instance, Samartha portrayed high levels of confidence in the game by clearly articulating to Suhail how they should play the game and in reflecting on the rules of the game. Suhail, although new to this game, once she gained some understanding of how to play the game, she reflected confidence by not relying on others but relying on her own strategies to find the sum of the addends.

The transcript also reflected Samartha's relationship with her classroom teacher. For instance, at a certain point, Samartha shared that "She [Mrs. Holland] doesn't say 'no'. She's a good old jelly belly". Even though I might see the teacher as an authority figure, Samartha seems to have established a very positive relationship with her teacher and feels comfortable sharing her feelings and opinions towards her.

The relationship between the students and I appeared to change during the game. Initially, I was to work with Suhail on playing this game and she would look to me for guidance or direction. Once Samartha volunteered to work with Suhail, Suhail looked to Samartha for direction, and I simply guided them. Then, when I took over Samartha's turn, I noticed that I had two roles 1) to play Samartha's turns and 2) to teach and encourage Suhail to use her fingers to count-up based on their teacher's instructions.

As for the relationship between the students, Samartha and Suhail had a particular relationship, and I noticed that their relationship had two main characteristics 1) a teacher-student role and 2) competitiveness. The relationship between Samartha and Suhail seemed to change during the game. In the beginning, Suhail appeared to have more mathematical authority by taking charge of counting Samartha's tiles. I may have setup that relationship from the beginning when I said, "Mrs. Holland asked me to practice how Suhail will hold a number in her head and count forward", which could illustrate to Suhail that she had more mathematical authority. But as the game progressed, I noticed that Suhail started to look to Samartha for direction. I wonder if this had anything to do with me asking Samartha to show Suhail how to play the game, which could have portrayed Samartha as having experience playing the game and more capable of explaining the instructions of the game, portraying her as having more mathematical authority. This also could have led Samartha to ignore Suhail when she attempted to show her how she should roll the dice, or even when Suhail questioned the sum of the addends that Samartha shared. Samartha did not take into account what Suhail shared by ignoring her. On the contrary, Suhail appeared to continue looking to Samartha for direction. For instance, when Suhail was claiming the number 8 on the gameboard, Samartha expressed disagreement, leading Suhail to lift the tile off the gameboard. Then, when Samartha said, “um okay you can put it”, Suhail placed the tile back on the 8. Here too, Samartha appears competitive rather than collaborative in that she did not want Suhail to claim a space that could either lead her to win or to block Samartha from winning.

Politics. Analysis of this data revealed that there were political aspects in those instances where I might have illustrated where the mathematical authority resides. Some instances portrayed their classroom teacher’s role as an authority, other aspects portrayed my role as a mathematical authority, and in several instances, we see students assuming mathematical authority. The distribution of mathematics authority influenced their participation and mathematical attainment in the activity.

In reviewing the transcript, I noticed that I acknowledged the classroom teacher by deliberately referring to her as the one who asked me to work with Suhail and to practice counting forward. I also noticed that I privileged the teacher’s vocabulary by using the term she used “count forward” and the strategy she uses to count-up. These instances establish the classroom teacher’s authority and portray her as the one in charge. There were also some instances that establish my role as a mathematical authority. I portrayed mathematical authority by reinforcing the rules of the game, regulating their behavior, and confirming or challenging their responses. Hence, influencing their meaning making.

At other times students assumed mathematical authority. For instance, Suhail exercised mathematical authority by doing the counting for Samartha who had not counted her yellow tiles when she took them out of the bin. Also, Suhail exercised control over how Samartha should roll the dice. When Samartha rolled the dice by lifting her hand and dropping the dice onto the table, Suhail grabbed the dice, shook her hand, and rolled them onto the table, based on how I showed her to do it. As for Samartha, she portrayed mathematical authority by freely explaining to Suhail how she should play the game, and by reinforcing the rules of the game and sharing that they should not be altered. Also, Samartha exercised authority in terms of jumping in to indicate to Suhail that she had not played her turn correctly when Suhail was actually trying to figure out what she should do.

Sign Systems or Knowledge. In terms of what sign systems or knowledge were privileged, these were mostly reflected in students' use of certain methods or ways to play the game, in their use of certain mathematical terms, and in students' pronunciation of certain words. It emerged that these were the kinds of things that were used by those who were seen with mathematical authority and had the most agency, Samartha, and myself. For instance, I noticed that the manner in which I asked Suhail to roll the dice became significant as she imitated its use and expected Samartha to do so as well, but Samartha refused and privileged her way of the rolling the dice. Also, Samartha's objection to altering the rules of the game privileged the actual rules as opposed to those I made which became significant in how she played the game.

Regarding students' use of certain mathematical terms, Samartha's use of the words "count it" and "counted" not only reflected her understanding of how to play Add 'Em Up, but portrayed her choice of language and privileged a particular strategy (i.e., counting). And although Samartha's use of the words "count it" or "counted" are mathematical terms, I noticed that I focused on the use of the term "add". In so doing, I privileged that particular term and it may have formed their understanding of what the game is about.

There was also an instance that portrayed how the pronunciations of certain words were privileged. For instance, Suhail would refer to the number 6 as "sixths". At one point, I repeated the number twice to confirm her answer and more explicitly to share the correct pronunciation of the number 6. By repeating the number 6, I might have privileged the Canadian dialect over other dialects for pronouncing that number, rather than recognizing her way of pronouncing it. At times, Suhail would model my pronunciation, reinforcing my mathematical authority, and complying with it.

Addressing the Second Sub Research Question

In order to address my second sub research question concerning how students negotiate mathematical meaning during mathematical problem-solving activities, I specifically focused on data gathered from four video and audio recordings of grade 2/3 students interacting on various mathematics activities. In the previous section, I presented a detailed analysis of children's meaning-making by drawing on Gee's (2005) analytical framework of discourse analysis, specifically the construct of building tasks of "language in use" including, significance, activity (i.e., mathematical actions), identity, relationships, politics, and sign systems or knowledge.

These building tasks were all part of students' mathematical meaning making. In an important way, various themes started to emerge within each of the building tasks, which helped me to understand how the students negotiated meaning-making during mathematics activities. Now, based on the findings presented by the analysis, I turn to explicitly answer my second sub research question by providing a general synthesis of the results based on each of the building tasks for the four episodes.

Significance

Throughout students' interactions there were particular things that appear to become significant to their mathematical meaning-making. These include their understanding of particular mathematical terminologies, the use of specific strategies, and various tools that appear significant to their understanding of mathematics in general, and more specifically, to their understanding of the mathematics activity.

In terms of terminologies and concepts, the students appeared to be developing a sense of what various words mean as a way to talk about their understanding or interpretation of the activity. In so doing, the students internalized these terms as they made use of them when working with others. For instance, in *Books in a* Classroom, some students relied on the discussion they had with their teacher regarding the term "piles" meaning "groups" to calculate the total number of books by focusing on the digits in the tens place and to create groupings of tens. In *Ameer's Coins*, the students appeared to be developing a sense of what the concepts "more than", "less than", and "the same amount of money as" mean as a way to compare and understand quantity relationships. In *Times Tussle* the word "times" was used as a way to talk about multiplication as well as thinking about multiplication as "groups". In *Add 'Em Up*, students' use of the words "count it" and "counted" reflected their understanding of how they could combine the dice, and that is, by counting-on and not adding per se. Consequently, the use of particular terms may play a role in their meaning-making in that 1) the students may come to see these terms as important, and 2) these terms could define what they were doing in the game. Although students might have ambiguous and shifting meanings, these did not prove to be obstacles to the mathematical discussion, but rather were significant to their understanding of the activity.

It is evident that the students also relied on particular strategies which appeared to be meaningful to them. These include, counting, skip counting, counting forward and backward, adding a zero when multiplying by 10, decomposing numbers into their respective tens and ones, making computations based on groupings of tens and ones, rearranging grouping of ones into tens, comparing money amounts, and relaying on prior mathematical or world knowledge. Their counting strategies demonstrated important characteristics such as stable order, one-to-one correspondence, cardinality, and understanding of movement as magnitude. The data provide evidence that the students involved in the two games had an understanding of stable order in counting, and recognized that the last number said represented the quantity of objects. Skip counting seemed to be a significant strategy that the students connected with multiplication as well as adding, and they appeared to have some fluency with this. Also, counting-forwards was one other strategy that students used and connected with adding, and most students had confidence with this. Counting the rods and units, or counting their fingers as representations of rods, were also significant strategies that students relied on to find the sum of the digits in the tens and ones place, portraying an important understanding of place values. The understanding of place values was also apparent when students were finding the total amount of coins a person had by decomposing the numbers to simplify the addition process. In general, students' understanding of the place-value system encompassed an understanding of base-ten groupings and various computational strategies. Students also appeared to have developed a quantifiable understanding of 25 and 50 when combing the amount of money that a person had. Concerning prior mathematical or world knowledge, these were particularly evident when students appeared to have relied on something they learned such as adding a zero when multiplying by 10, or "parts of 10" or "anchors of 10" when adding, which helps to simplify the addition process, or when students reflected on their world knowledge to share that a 50 cents coin does exist in or day-to-day encounters. Taken together, these examples and others similar reflected their understanding of the activity and the mathematics involved in the activity, which informed their meaning-making.

Particular concrete tools appeared to be significant in their meaning making such as, the tiles, their fingers, the dice, rods and units, the handout, drawings, and the individual whiteboard. In both games, the tiles were used for counting as well as to mark their place on the game board, which also reflected their mathematical understanding of counting and quantity. One of the most significant tools used in all activities appeared to be their fingers. For instance, the students used their fingers to assist them with creating a representation of a number, for counting, skip counting, counting up, adding, subtracting, to count along, as well as for pointing or gesturing to something as evidence for their claims. It was interesting that different students used their fingers to count in very different ways, which could inform or interfere with their interactions as students represent numbers in a variety of ways.

Other tools that the students used and were meaningful to them were specific to certain activities. For instance, in *Books in a Classroom* the rods and units appeared to be a significant tool that students used to model the number of books in each bin using the place value system, and also, assisted them to find the sum of the books in all three bins, reflecting their use of computational strategies with an understanding of grouping. In *Add 'Em Up*, the most significant tool appeared to be the dice as it is through which the students acquired the sum of the addends by either beginning with the number on one die and counting-up the dots on the other die, or by choosing the die with the bigger number and adding the second number to it using their fingers or through mental computations. Students' use of the dice demonstrated an understanding of adding in terms of counting, quantity, and number relationships.

The activity handout was mostly used for reflecting on the problem and the images included on the handout (i.e., the number of books or Ameer's 4 quarters). In *Ameer's* Coins, the handout was also used as evidence for their claims. Students would refer to certain statements on the handout to back or clarify what they are implying. Drawings were mainly used in the *Books in the Classroom* activity to illustrate their solutions and to reflect their understanding of the activity. In terms of the individual whiteboard, it was mainly used in *Ameer's Coins* to represent a number or the quantity of coins that students were referring to. In general, the use of particular terminologies or concepts, strategies, and tools appeared to help clarify students' understanding as they engaged in the activities or solved a problem.

Activities—Mathematical Actions

The analysis of the data brought forth several mathematical actions that portrayed the ways of using and developing mathematical knowledge and skills. We notice that mathematical actions came primarily from engaging in problem-solving activities where students reasoned their way to a solution by communicating their understanding and ideas, by creating representations that capture their understandings, and by making or seeing connections between mathematical ideas and concepts—contributing to their mathematical attainment and demonstrating their understanding through what they say, do, and represent.

Reasoning and Proving

In terms of reasoning and proving, an analysis of the data provides evidence that students' interactions in the activities, the interpretations or understandings they developed, the discussions that unfolded between them, and the representations they created, all portrayed their mathematical reasoning and proving skills. In general, the students reflected on their own solutions and those shared by others by justifying their answers, explaining their choices, negotiating understanding, and making mathematical conjectures to determine whether or not their solutions satisfied the activities objectives. And in the process, they relied on various strategies including, but limited to counting, skip counting, adding, subtracting, making computations based on groupings of tens and ones, rearranging grouping of ones into tens, their mathematical or world knowledge, as well as concrete tools and each other—all to reason their way to a solution and to evaluate or reflect on their own or each other's solutions.

Also evident was their intuitive awareness of the gameboard and the numbers on it, which helped them reason and make sense of which space they wanted to claim on the gameboard. They were not simply claiming a number but reasoning and making sense of which space they wanted to claim to block their opponent or gain a line of 4 adjacent markers or a 4 square.

Communication

In analyzing the frequency of speech turns in each activity, I noted that the students spoke more than I did as approximately 70% of the speech turns were produced by the students and 30% of the speech turns were produced by me. This demonstrates that during these activities the students did the bulk of the communicating. It is evident that the students relied on oral and visual communications to share their mathematical understandings, ideas, and to enact the use of certain strategies and tools. In general, the students relied on communication for sharing and developing mathematical thinking. For instance, students communicated when they 1) responded to a question, asked a question, or posed a concern; 2) shared their understanding of the activity or read the activity; 3) reflected on their experiences or recognized themselves in the activity; 4) translated something into their terminology; 5) used mathematical terms or concepts; 6) expressed disengagement or confusion; 7) evaluated, corrected, or confirmed each other's answers; 8) displayed a different understanding or interpretation of the activity; 9) referred to the handout as evidence for their claims; 10) counted, skip counted, added, subtracted, or decomposed numbers; 11) shared the product(s) or the sum of the addends; 12) shared a number sentence or completed someone else's number sentence; 13) captured a mathematical idea or understanding using concrete tools; 14) shared or modelled their representations; 15) made one-to-one correspondence; 16) referred to a number on the gameboard as a written representation of a number; 17) elaborated on or repeated what someone else said; 18) justified their answers, explained their solutions, negotiated understanding, and made mathematical conjectures; 19) wrote or drew their solution on the activity handout or whiteboard; and 20) used everyday language to reflect on their mathematical understanding.

Representation

The students also relied on concrete tools to represent their mathematical understandings and ideas by using the tiles, their fingers, the rods and units, the dice, the gameboard, activity handout, and drawings. The use of these tools encouraged understanding and oral or visual communications. In terms of the tiles, as the students collected their tiles, they counted and recounted their tiles, and in so doing, the tiles became a representation of a number, reflecting their understating of number and quantity.

One of the most meaningful representations the students relied on was the use of their fingers. Their fingers were used as a visual aid that also influenced their understanding of number and quantity. For instance, the students used their fingers to create a representation of the sum of the addends, the product(s), and the number of coins or units they needed. They also used their fingers to make mathematical computations (e.g., adding, subtracting and multiplying) through the use of various strategies including, counting, skip counting, counting forwards and backwards. Students also used their fingers to create representations of a number sentence, or a representation of a line of 4 adjacent markers or a 4 square. Their fingers were also used to capture a relationship between what they already knew and a new understanding or knowledge. They also relied on each other's fingers as a representation of a number or for skip counting.

Another significant representation appeared to be the use of the rods and units in the base ten blocks, which portrayed a representation of their thinking and ideas. For instance, the rods and units were used to 1) model the number of books in each bin; 2) add the total number of books in all three bins; 3) represent a number sentence; and 4) to rearrange grouping of ones into tens. The rods and units also seemed to help students to develop a conceptual understanding of the digits in the tens place as representation of groups of 10.

One other representation used was the dice, which students used as a representation for adding or counting-on. Also, the students would bring the dice together side-by-side creating a representation of a mathematical number sentence. In terms of the gameboard, the students would refer to the gameboard to make a link between the oral and written representation of a number. As for the activity handout, the students used the images on the handout as a representation of number or quantity and to support their claims. With respect to drawings, some students used the activity handout or the whiteboard to draw rods or write numbers as a representation of their solution or thinking. I also initiated the use of the whiteboard to draw coins, which students used as representations of money amounts up to a dollar.

Connections

It was also evident that the students made connections between mathematical ideas, broadening their mathematical understanding of concepts and procedures. For instance, students made connections between 1) number and quantity; 3) counting forwards and adding; 4) counting backwards and subtracting; 5) skip counting, repeated addition, and multiplying; 6) quantities in terms of "more than", "less than", and "the same amount as"; 7) number or quantity and place value; 4) a number name, a number symbol, and a representation of quantity; 8) one-to-one correspondence between a number and a concrete tool; 9) an operation and a solution; 10) rearranging grouping of ones into tens; 11) mathematics and world knowledge; and 12) between each other's claims in order to elaborate, reiterate, or evaluate given information.

Identities

As students interacted in various mathematics activities, we can see the different roles that the students play and the individual identities that are constructed. These identities as mathematics learners influenced their roles in the activity and how they made sense of the activity. In general, the students developed or expressed agency in multiple ways as portrayed by their sense of autonomy in the classroom and in the activity, as well as, by identifying with the activity, reflecting and directing their own learning, taking responsibility or ownership for their learning, building their own understanding, correcting their own answers, checking their own work, monitoring their own progress, modelling their own solutions using concrete tools or their fingers, and making use of their individual strategies.

Most students identified with the mathematics activities by either recognizing themselves in the activities or actively engaging in the mathematics involved in the activities. Those who merely cooperated or resisted engagement in an activity portrayed unfamiliarity with the activity or acknowledged what they do not know, and at times, they would confess that they were confused as a way to describe their disengagement.

Also, several students appeared to be taking on the role of the teacher by guiding and explaining to others how they could play the game, use a strategy, or by encouraging others to comply with the rules of the game. At times, I noticed that by asking students to share their understandings of the game or their solutions to the activities, I may have portrayed them as more knowledgeable and in charge of legitimizing certain aspects of the activity, which could reflect the identities I gave them.

Relationships

Several different relationships were at play or evolved during these interactions that were meaningful and may influence the students' mathematical meaning-making. These include the relationship between the students and the activity, the relationship between the students and concrete tools, the relationship between the students and their classroom teacher, the relationship between the students and myself, and the relationships between the students as they interacted.

The students had various experiences engaging in the mathematics activities and these experiences influenced their level of confidence. Generally, students' levels of confidence were reflected in their enthusiastic or energetic behavior towards the activity, in their understanding of the activity, and in trying to find a solution to the activity. If students experienced confusion, they seemed to work hard and continued to try longer to come to an understanding of what was expected.

The previous experiences and the relationships the students built with the concrete tools before engaging in the activities helped shape their use of the tools in these activities. If students were not familiar with their use, they would either imitate what others were doing, or develop their own approaches to using the tools. In so doing, they develop their own understanding and take responsibility for their learning.

The students' relationship with their classroom teacher was mainly reflected in what students said about their teacher. This was particularly evident in two activities *Add 'Em Up* and *Ameer's Coins*. In *Add 'Em Up*, there was an instance that appeared to reflect Samartha's relationship with her teacher. Even though I might see the teacher as an authority figure, Samartha seems to have established a very positive relationship with her teacher and feels comfortable to share her feelings and opinions towards her. In *Ameer's Coins*, the students appeared to have portrayed their teacher as accessible, the one in charge, and possess mathematical authority especially when determining or backing the reliability of their answers, which could be a reflection of their learning.

The relationship between the students and I seemed to change during our interactions and depended on the support they needed. In the beginning, the students looked to me quite often for direction, but as they progressed and became familiar with the mathematics activities, I did not have to intervene as much and they did not look to me for direction. However, *Ameer's coins* seemed a bit challenging for the students, which prompted me to be more involved in an attempt to guide, clarify, or focus the discussion for the purpose of furthering understanding.

In terms of the relationship between the students, these varied depending on the activity and who was involved in the activity which in turn influenced their interactions and participation in the activities. For instance, in *Books in the Classroom*, certain students appeared rather competitive and had the urge to be the first to find a solution. It also seemed as though the students approached the activity individually. However, the data revealed the students indirectly or implicitly relied on one another to form some understanding of what was expected of them in terms of working their way to a solution. In *Ameer's coins,* I noticed that as the students communicated their interpretations, ideas, or their thinking, they were constantly helping one another build an understanding of the activity. They did not appear at all competitive, rather it seemed as though they shared responsibility for each other's learning by trying to discuss, justify, and explain their thinking to one another. All in all, they were deliberate in working together to find a solution to the activity. In the *Times Tussle* game, the students appeared collaborative rather than competitive, and rather than let one another struggle, they would constantly help one another. It appeared as though they developed trust in one another, and they would check with one another, evaluate, or even imitate one another. In *Add 'Em Up*, I noticed that that Suhail quite often looked to Samartha for direction. However, Samartha appeared to ignore Suhail and give little credit or attention to what she was saying or doing. It may be that Samartha regarded Suhail as relatively new to the classroom with not much experience. It was also apparent that Samartha seemed rather competitive and had an urge to win and would attempt to control which numbers Suhail could claim. In general, students' interactions helped them model and imitate one another's strategies, which may be significant to their meaning making as they were constantly learning from one another.

Politics

Political aspects that contribute to meaning making were reflected in the ways in which mathematical authority was distributed. There were instances that demonstrate a discussion of where the mathematics authority resides in the classroom, other instances portrayed my role as a mathematical authority, other instances we can see that some of the students assumed mathematical authority, and still, other instances students seemed to see the rules of the game or the activity handout as being a strong authority.

In reviewing the results, I noticed that the classroom teacher inevitably represented mathematical authority in students' interactions. This was particularly evident when students honored their teacher's terminology by picking up on the term she used as opposed to that written in the activity. I also noticed that at times, I established the classroom teacher's authority and portrayed her as the one in charge by privileging her vocabulary and the strategies she uses. In this way, the students could come to see the term or their teacher's strategy as the most accurate, and it may be something that plays into their meaning making.

I also portrayed mathematical authority through my use of specific terminology, by reinforcing the rules of the game, regulating their behavior, and by confirming or challenging their responses. I also noticed that at times I purposefully distributed mathematical authority by asking a student to show another student how to play a game, to share a strategy, or to share their solutions, reinforcing students' mathematical authority and legitimizing their role.

It was also evident that various students took on mathematical authority. Students' mathematical authorities were reflected in the ways in which they evaluated or confirmed each other's solutions; explained to others how they should play the game or how they should find a solution; by frequently monitoring the contributions shared by others; by reflecting on their experiences playing the game; by developing their own understanding and interpretation of the activity or the discussions that unfolded; by depicting a reasonable understanding of the activity; by modelling their own solutions using concrete tools; and by finding the solutions for each other's turns. In exercising mathematical authority, they created power differential between one another, influencing their participation in the mathematics activity in terms of what is learned and how it is learned.

The students often saw the rules of the game and the activity handout as being a strong authority. The rules and goals of the game would structure their decisions and their discussion to make the most strategic move. The students also seemed to regard the activity handout as being a strong authority. This was particularly evident when the students would refer to the handout as evidence for their claims. It also seemed as though the handout would structure or refocus their discussions based on certain statements of the activity.

These examples established where the mathematical authority resides in the classroom in general, and more specifically illustrated the distribution of mathematical authority among the participants in the activities. Also, these examples influenced their meaning making in terms of understanding who is in charge to legitimize certain aspects of the activity including and not limited to the terms used, the rules of the game, the strategies and tools used.

Sign Systems or Knowledge

The discussion about what sign systems or knowledge are privileged is somewhat connected to the ideas presented in the sections on politics and identity. Talking about whose knowledge or ideas are privileged has a great deal to do with mathematical authority and the identities that were constructed in the students' sense of agency. Those who are seen with mathematical authority and have developed a strong sense of agency their knowledge might be privileged over the knowledge of others. Also, certain terminologies, certain pronunciations, certain strategies, and certain ways to play the game or solve the problem-solving activities were privileged, and these seemed to be the kinds of things that were used by those who had the most agency.

I noticed that our use of certain terminologies such as, "groups", "times", "piles", "count it or counted", "dollars vs. coins" portrayed students' choice of language and could have privileged these terms, which in turn, may have influenced the students' understanding of the activity and their interpretation of these terms. These understandings may be founded on the discussion they had with their classroom teacher about certain terms, their own interpretation of the activity, or from what someone said. However, the manner in which the students shared their thinking and their continued reiteration of it appeared to have unconditionally privileged their ways of knowing, giving them voice and choice in their learning. There was also an instance that portrayed how the pronunciations of certain words were privileged by those who appeared to have the most authority, myself, such as when I encouraged the use of the Canadian dialect for saying numbers. Concerning the strategies used, certain strategies used by students who seemed to have the most mathematical authority were privileged and imitated by others, and sometimes a strategy became privileged because of how I recognized it. Regarding the manner in which students played the games or solved the activities, these were reflected by students imitating or relying on the use of certain ways of doing something (mathematical or non-mathematical), and in their interpretations or from someone else's interpretations of the activity to illustrate their mathematical thinking and understanding. These examples focused on how students in the activities used their mathematical or world knowledge, specific terms and pronunciations of words, and their unique way to count on their fingers to illustrate their understanding and to work through the activity, which to some broad extent privileged one sign system or knowledge over another.

Concluding Remarks

In addressing the second sub research question, the findings of the analysis show that students' interactions with one another appeared at the heart of their meaning making. These interactions encouraged students to develop a collection of resources to share their thinking and ideas through verbal, visual, and written mathematical communications. Hence, the students utilized language to make meaning and to negotiate their mathematical understanding. Drawing on the analysis of each of the building tasks (i.e., significance, activity, identity, relationships, politics, sign systems or knowledge), students' meaning making appeared complex, generative, and dynamic as it involved the use of language to 1) make certain terminologies, strategies, and tools **significant**; 2) to carry out mathematical actions (i.e., **activity**) which portrayed students' ways of using and developing mathematical knowledge and skills; 3) to take on a certain **identity** as students engaged in the classroom and the mathematical activities; 4) to build

relationships with other students, their teacher, or with mathematics; 5) to distribute mathematical authority (i.e., **politics**); and 6) to privilege or deprivilege one **sign system or knowledge** over another. All of these things were part of students' mathematical meaning making which seemed to revolve around students' interactions—making us aware of the fundamental role of language to make meaning and to negotiate mathematical understanding. This said, it is in providing students with space to work on mathematics and to communicate their thinking or ideas, using various tools and strategies, that we as educators are better equipped at discerning their mathematical thinking, meaning making, and experiences. It is in this way that I have explicitly addressed the research question based on the evidence that has been presented in this section of the results concerning students' meaning making in mathematical problem-solving activities.

CHAPTER 7: DISCUSSION

In the previous chapters, I did an in-depth analysis to respond to the first and second sub research questions:

a) What are the influences on students' mathematics experiences?
b) How do students negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

In this discussion chapter, I address my main research question: What are the mathematical experiences of grade 2/3 students in a plurilingual classroom in Ontario, Canada? To address my main research question, I went back to the results of the two-sub research questions and I looked across them to see what were the main topics that described the mathematical experiences of multilingual learners. I felt that the data fell into four main topics, which emerged from my review of the results for the two sub research questions. The first topic addresses students' mathematics experiences in their home country and with their parents and siblings. The second topic relates to students' experiences in a Canadian mathematics setting. The third topic focuses on students' social interactions as they experienced mathematics. The fourth topic addresses students developing mathematical identities through their mathematics experiences. The reader might notice that two of these topics are connected to the setting of students' experiences, and the other two are looking at the interactions and how students are building their identities.

As I discuss these four topics, I reflect on my grounding of this thesis from a sociocultural understanding of learning and development. At the beginning of the thesis, in chapter 3, I discussed sociocultural theory at length because it is the theoretical basis of this study. In my discussion, I considered three key features that form the core of Vygotsky's theory including, a genetic method, cultural tools, and social interactions, as well as two fundamental constructs that have been widely considered by sociocultural researchers: internalization and the zone of proximal development. As part of the genetic method, Vygotsky acknowledges the influence of the history and the wider community or institution on the individual's participation in an activity. In particular, Vygotsky considered four basic lines in the development of behaviour: development in the species as a whole (phylogenesis); development over time in the wider culture of which the individual was becoming a member (cultural history); development through interactions in specific sociocultural settings (microgenesis); and finally, development over the life of an individual (ontogenesis). The second feature pertains to cultural tools, which Vygotsky notes "are culturally produced and represent cultural innovations and changes, [and] as we internalize their use, our thought is undeniably cultural in nature" (Nasir & Hand, 2006, p. 461). Commonly cited examples of cultural tools include "language, various systems of counting, mnemonic techniques, algebraic symbol system, work of art, writing, schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs; and so on" (Vygotsky, 1981a, p. 137; as cited in Wertsch, 1985*,* p. 79). The third feature relates to the idea that the individual and the society in which s/he participates are mutually constituted.

In terms of the two fundamental constructs; internalization and the zone of proximal development, Vygotsky viewed internalization as a process whereby certain aspects of patterns of activity that had been performed on the external (social) plane become executed on the internal (individual) plane. Vygotsky further argued that internal mental processes are created as a result of one's exposure to social processes. Hence, the emphasis is on the inherent relationship between the two planes of activity, and where internalization is addressed in consideration of the social origins of individual activity, which involves transformation rather than an identical replication (Wertsch, 1985). As for the zone of proximal development, Vygotsky (1987) argued that in order to understand the relationship between development and learning, we must determine at least two developmental levels. The first is the actual developmental level determined by what children can do on their own. The second is the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (p. 86). Although Vygotsky specified "more capable peers", many researchers have argued that it is not necessary for there to be a person who is more capable than others because any person interacting with others could make a contribution that helps towards a solution (Forman & McPhail, 1993; Wells, 1999). Other researchers also add that such interactions are not necessarily exclusive to another human being but could include various artifacts or other things that support learning (Brown et al. 1996).

My grounding of this thesis and this discussion from a sociocultural understanding of learning and development stems from my belief in this theory, which I believe shaped who I am as a person. In particular, sociocultural theory shaped my thinking and understanding about how to act and interact with those surrounding me and it shaped how I approached this research study as a whole—from the research questions I posed, to what data I chose to collect, to how I interacted with the students and their teacher in the grade 2/3 classroom, to how I analyzed the data, all those stemmed from my belief in a sociocultural understanding of learning and development which grounded this entire thesis. For instance, understanding students' mathematics experiences from a sociocultural perspective provoked me to make observational notes of the ways the students and their teacher engaged with one another; the ways they interacted around mathematical problem-solving activities; the ways they produced oral or written explanations of their mathematical thinking; and of the ways they used the language(s) they know and other resources during mathematical discussion. In so doing, I took account of the role of social interactions, the role of cultural tools (i.e., language and other resources), and the role of the wider community, which cultivated my understanding of the influences on students' mathematics experiences. A sociocultural perspective also encouraged me to video or audio record instances of students working together on various activities to capture their mathematical discussions and explanations, as well as, the ways they hesitated, made eye contact, used multiple resources, and gestures. Here too, my consideration of the socially mediated experience, the use of cultural tools, as well as, the role of internalization and the zone of proximal development guided my understanding of how students negotiate mathematical meaning during problem solving activities. Additionally, a sociocultural perspective prompted me to develop an understanding of students' previous and new mathematical learning experiences, and also, their reflections towards their participation in the mathematics classroom. In this way, I endeavored to acknowledge students' histories and the wider community or institution as influencing their mathematics experiences in the grade 2/3 classroom. Even in my analysis of the data, a sociocultural perspective encouraged me to consider the analytical frameworks I chose including, the four influences on students' learning of mathematics that came from the organization of my literature review, as well as, Gee's (2005, 2011) construct of building tasks to analyze "language-in-use". Although my theoretical perspective was not my analytical framework per se, the analytical frameworks I chose closely aligned with sociocultural theory. Just as an example, when I considered the students' individual influences, one aspect I focused on was the influence of students' experiences learning mathematics elsewhere; in their home countries, at home with their parents and siblings, and at the afterschool homework club. Considering students' experiences learning mathematics elsewhere is closely related to Vygotsky's discussion of the genetic method, more preciously, the fourth line of development concerning development over the life of an individual. By taking into account students' experiences in these diverse sociocultural settings, I was able to understand the influence of all those experiences on students' mathematics conceptualizations and their experiences in the grade 2/3 classroom. Also, analyzing "language-in-use" using Gee's building tasks closely connects to Vygotsky's view that language is one of the most powerful and pervasive cultural tools connecting intrapsychological and interpsychological functioning (Lantolf & Thorne, 2000; Wertsch, 1991).

Thus, bringing forth sociocultural theory in my discussion will help me to form an in-depth response and understanding of my main research question: What are the mathematical experiences of grade 2/3 students in a plurilingual classroom in Ontario, Canada? In what follows, I organize the discussion using the four topics noted above. I ground the discussion in a sociocultural understanding of learning and development, and, I relate the findings to other relevant research when appropriate.

Students' Experiences Outside a Canadian Mathematics Setting

The results of the study show that as well as mathematics experiences in a Canadian classroom, the students experienced learning mathematics in contexts situated outside a Canadian setting. They experienced mathematics in their home countries, and experienced mathematics at home with their parents and siblings. Such experiences were part of students' mathematics learning histories. By history, I mean a series of events that a person experiences over a particular period of time and that could influence their present experiences. Vygotsky emphasizes the role of students' histories on their participation and development in specific sociocultural settings (Wells, 1999, 2000; Wertsch, 1991). In this context, students' histories led them to form certain conceptualizations and understandings of mathematics, which influenced their mathematics experiences in the grade 2/3 classroom.

Students' Mathematics Experiences in their home countries

The students had some experiences doing mathematics in their home countries and when they talked about their experiences, they talked about the mathematics they learned, the languages they used, and how mathematics was taught. Students' reflections often focused on specific cultural tools (e.g., calculations, language) they used to learn mathematics.

Sociocultural theory highlights the role of cultural tools as a representation of cultural innovation. In this regard, students' experiences in their home countries were culturally specific. That is, their description of doing mathematics in their home country provided insights into the meaning of mathematics, how mathematics was represented, and how mathematics was taught in their home culture. For instance, the students saw the mathematics they learned in their home country, as focused on calculations, computations, and number. Some of the students talked about doing some mathematics in English, even in their home country. While most of them did mathematics in their home language, they saw English being used in the mathematics classroom. However, English was predominantly used to translate numbers or mathematics terminology, or to write numbers using the English numerical system. Also, how the students described the mathematics taught in their home countries reflected explicit emphasis on teacher instruction with minimal discussion or interactions. That is, the teacher explained the relevant lesson or demonstrated procedures for solving mathematical problems on the board, and the students then solved problems with minimal discussion. There also was a punitive aspect expressed by students when they got something wrong or did not do certain things. Students' experiences in their home countries signify a culturally specific way of teaching and learning mathematics. This is contrary to the experiences that students in the grade 2/3 classroom were exposed to—a welcoming and nurturing environment that fostered interactions, inclusion, and was respectful of students' differences and learning needs.

Students' Mathematics Experiences with their Parents and Siblings

Students also experienced doing mathematics in their home, in Canada, and when they talked about it, they talked about the people they worked with, the language(s) they used, and the mathematics they worked on. The focus in their discussions appeared to be on their interactions with others as they worked on mathematics. Vygotsky's work emphasizes a person's interactions with others in specific sociocultural settings as influencing learning and development. Generally, students who worked on mathematics at home stated that they worked with their mother or father, or with their older siblings. Students mainly used their home language when working with their parents or siblings on mathematics. Also, the mathematics that the students described doing at home focused on repeated practice of procedures of mathematics operations. Some students even used algorithms or strategies at home that appeared different from the ones used in the grade 2/3 classroom. Also, at home, the students were not required to show steps or strategies they used to solve questions. There was a focus on the final answer rather than the steps or strategies used to solve a question. Students' use of their home languages and the use of certain algorithms or strategies emphasizes the use of certain cultural tools to do mathematics at home and with their parents and siblings—showcasing how mathematics is taught and learned in that particular setting.

The Influence of Students' Outside Experiences on their Grade 2/3 Mathematics Experiences

Students recognized that their mathematics experiences in their home country and at home were different from what they were experiencing in the grade 2/3 classroom. Still, students' mathematics learning histories in specific sociocultural settings appeared to have informed some of their conceptualizations and understandings of mathematics in the grade 2/3 classroom. This is because students' learning histories came to be their internalized social realities that informed their mathematics learning experiences. In chapter 5, I included some examples of the *Influence of Students' Experiences Learning Mathematics Elsewhere*. For instance, some students did not view activities related to symmetry and transformation as mathematics given that the activities did not involve numbers and calculations. Other students seemed to have dismissed the use of certain strategies they learned in their home countries or at home because they were different from those used by the teacher or other students in the grade 2/3 classroom. Such conceptualizations, understandings, and even behaviors appeared to be influenced by students' mathematics learning histories that were not compatible with those they experienced in the grade 2/3 classroom—leading some students to 1) question whether certain activities were related to mathematics, 2) dismiss the use of certain strategies or tools (e.g., specific algorithms), and 3) value the final answer as more important than the steps or strategies used. The influence of students' previous experiences on their experiences in the grade 2/3 classroom concurs with Abreu and Gorgorió (2007), Favilli and Tintori (2002), and Gorgorió and Planas's (2001) work where they discuss that students may experience cultural conflict when they enter the mathematics classroom because what they bring with them (i.e., algorithms, strategies, or tools) may appear different and could interfere with their learning of mathematics. Hence, the cultural tools that the students used in their home countries or at home appeared socioculturally specific, and at times, not compatible with those used in the grade 2/3 classroom—leading some students to dismiss their use and to adapt their teacher's or classmates' ways.

Students' Experiences in a Canadian Mathematics Setting

The students had mathematics experiences in two different settings in their school in Canada. The main setting was the Grade 2/3 classroom with Mrs. Holland. The other setting was a homework club which they attended only occasionally but in which they also received mathematics instruction. The children's experiences in these two settings were somewhat different. In one setting, the Grade 2/3 classroom, they explored mathematics through the use of mathematical discourse practices. By mathematical discourse practices, I refer to the opportunities provided by the teacher that encouraged students to share mathematical thinking and ideas through verbal, visual, and written communication (NCTM, 2014). In the homework club, their mathematical experiences were very similar to those from their home country or in working with parents or siblings at home. The wider community also influences students' mathematics experiences in a Canadian setting. By wider community, I mean the practices and policies outside and within the educational setting that influence the actions, behaviors, or beliefs of those within that setting. Within these settings, students interacted with others and with other things in their social environment which were part of their mathematics learning experiences. Sociocultural theory acknowledges a person's interaction with other people or other things in their social environment which precedes and shapes their development (Lantolf & Thorne, 2000). In this study, such interactions helped students build a 7-month history of experiences within the grade 2/3 classroom which I observed during my time there. Although I did not necessarily observe them in the afterschool homework club or the wider community per se, the influence of these two settings was evident in their mathematics experiences in the grade 2/3 classroom through their interactions and behaviours.

Students' Experiences in the Grade 2/3 Classroom

In this research study, the structure of the grade 2/3 classroom and the teacher's pedagogical moves played a significant role in students' mathematical meaning-making experiences. In the section on *Pedagogical Influences* in chapter 5, I provided a detailed account of the teacher's instructional practices in the grade 2/3 classroom. These practices included the set-up of the physical environment of the classroom, the design of the mathematics block, the use of interactive and engaging mathematical activities, the use of mathematical discourse practices, the use of strategies, the use of concrete learning tools, making mathematical connections, and differentiating instruction and work. Her practices are reflective of what research has identified as effective teaching strategies (e.g., Anthony & Walshaw, 2009; NCTM, 2014) that are "necessary to promote deep learning of mathematics" (NCTM, 2014, p. 9). As a result of the teacher's practices, the students developed reasoning and proving skills through mathematical activities, developed an understanding of mathematical terminology and associated concepts, used a variety of strategies to solve problems, and made use of concrete learning tools to develop and represent their mathematical ideas. This interest in the teacher's practices as motivating students' participation in the mathematics classroom aligns with sociocultural theorists speaking about the zone of proximal development. In the grade 2/3 mathematics classroom, learning in the zone of proximal development was inspired by the teacher's purposefully designed mathematics block that extended children's prior learning and embraced interactions with the teacher, other students, and with cultural tools. Cultural tools, in this context, are reasoning and proving skills, terminologies and concepts that students developed, the strategies students used, and the concrete learning tools they used. All of these cultural tools were introduced by the teacher and informed students' mathematics learning experiences in the grade 2/3 classroom. Thus, it was in students' interactions with others and in their use of such tools that students internalized particular ways of doing mathematics. Sociocultural theorists would suggest that this socially and culturally mediated internalization is a result of the students' engagement with cultural tools (Lantolf & Thorne, 2000; Wertsch, 1991). These cultural tools helped to shape students' mathematics experiences. I will talk about each in turn.

Experiences Developing Reasoning and Proving Skills Through Mathematical Activities

Students were engaged in working together in various activities that were open-ended and that promoted students' engagement in reasoning and proving. These activities allowed for different solutions or multiple ways to arrive at a solution and included mathematics games, mathematics challenges, and math talks. The students' interactions in such activities, the interpretations or understandings they developed, the discussions that unfolded between the students, and the representations they created, all portrayed their mathematical reasoning and proving skills. In reasoning and proving with others, the students reflected on their own solutions and those shared by others by listening to one another, repeating and elaborating on what another said, justifying their answers, explaining their choices, and negotiating understanding. In the process, the students determined whether or not their solutions satisfied the activities' objectives. From a Vygotskian perspective, these are all created social process. Hence, it is in students working together on open-ended mathematical activities that they developed various reasoning and proving skills to come to a consensus on a given matter. These findings align with the studies of researchers like Assaf and Graves (2019), Elbers and de Haan (2005), Graves and Zack (1997), McGraw and Rubinstien (2008) and Weber et al., (2010). In their work, these researchers discuss how children's reasoning develops as they work and reason together on open-ended mathematical activities which encourage different solutions or multiple ways to arrive at a solution. According to Rasmussen and Marongelle (2006), providing the students with activities that are open-ended and generative, encourages students to discuss and justify their answers in order to determine if they are correct or not.

Experiences Developing an Understanding of Mathematical Terminology and Associated Concepts

Students developed mathematical terminology and deepened their understanding of concepts through the activities they worked on. While interacting with the students, the teacher used both mathematical and non-mathematical discourses—that is, the use of mathematical (formal) language and everyday (informal) language. Often the students used formal and informal language as they worked together. For instance, when one student was not sure about what a mathematics term meant, other students would rely on their teacher's explanation with everyday language to help their classmates make sense of the term, leading them then reason their way to a solution. From a Vygotskian perspective, language serves a dual role in human functioning: it is a means of communication, and it mediates higher mental functions (Nasir & Hand, 2006, p. 461). In my study, the dual role of language was evident in students' communications through discussion but with others' language, which mediated their meaning making and higher mental functions. Also, students' use of others' language leads to the emergence of internalization because what the students learned from their teacher, they used with one another to make meaning and to reason their way to a solution. My findings also concur with the work of Setati and Adler (2000) in South Africa that discussed the language practices of teachers in primary multilingual mathematics classroom. The researchers found that mathematics communication (i.e., either written or spoken) was linked to a movement between informal language and formal language, and "where the one practice enables the other" (Setati & Adler, 2000, p. 265). It has also been argued that a movement between students' everyday (informal) language to (formal) mathematics language is very beneficial for students because everyday language may connect to students' everyday lives which could help them identify or make sense of the mathematics they were working on (Barwell, 2016; Moschkovich 2002, 2008; Tshabalala & Clarkson, 2016).

Students' Experiences Using Various Strategies

The results also show how students' experiences involved using a variety of strategies—a very different experience than the one they had in their home country or working with their family at home. These strategies, such as skip counting, adding a zero when multiplying by 10, or grouping tens and ones, appeared meaningful to students' learning as they incorporated their use in the games and problem-solving activities. Even their counting strategies were varied and demonstrated that they recognized stable order, one-to-one correspondence, cardinality, and understanding of movement as magnitude. As they became experienced with these strategies, they applied them to their experiences with other processes such as using skip counting when adding or multiplying. In applying these strategies, they have internalized their use because they became part of their experiences. These experiences of using different strategies are also related to what Vygotsky refers to as 'various systems of counting' that are culturally produced. The systems they used in their classroom may have been different from those they had used prior to coming to that classroom. Thus, the students' experiences of using different strategies transformed their thinking and understanding.

Given that their teacher encouraged a variety of strategies, their experiences with various strategies encouraged some students to invent their own strategies. For instance, when the students made use of the hundreds chart to find a solution, they not only relied on a strategy modelled by the teacher, they extended it to invent their own strategies. Although the strategies they invented were specific to the tasks they were working on, it was what they used to reason their way to a solution. According to Carpenter et al., (1997) and Sahin et al., (2020), when students are provided with the space to invent their own strategies for solving problems involving addition and subtraction, they develop an understanding of mathematical concepts and procedures. Hence, students' use of invented strategies may allow them to understand numbers and counting in ways that are not permitted through the use of standard algorithms (Carpenter et. al, 1997; Sahin et al., 2020).

Students' Experiences with Concrete Learning Tools

The students experienced a variety of different mathematics activities that prompted them to use different tools, such as their fingers, tiles, dice, base ten blocks, and drawings. Sociocultural theory emphasizes the use of tools or cultural tools—understood as a means and a development of cultural innovations and changes, and their use makes one's thoughts undeniably cultural in nature (Nasir & Hand, 2006; Wertsch, 1985). Hence, the use of cultural tools alludes to the sociocultural nature of an interaction with others and other things. Notably, students' experiences with concrete tools in the grade 2/3 classroom are a bit different from their prior mathematical experiences where they were not encouraged to use a variety of tools. The use of concrete learning tools in the grade 2/3 classroom were embedded in the activities rather than being optional. In this way, students started to see these tools as part of their everyday mathematics experiences or what it means to do mathematics. This was true even for those students who were new and could not make sense of what the teacher was saying. These students may have experienced explanations with the use of objects, and minimal words, and were encouraged to use various tools to express their thinking and to reason their way to a solution visually, even when they have not yet developed the vocabulary to do so verbally. This really starts to show that their repertoire of cultural tools to express and explore mathematics is changing. Other researchers' work also underlines the important ways students' use of various tools mediates students' mathematics learning and the development of their thinking (Barwell, 2005; Elbers & de Haan, 2005; Gorgorió & Planas, 2001; Moschkovich, 2005, 2008; Webb & Webb, 2016).

Students' Experiences at the Afterschool Homework Club

During classroom interactions, the students would reflect on the mathematics they learned at the afterschool homework club that many attended, and it was apparent that there was a different approach to mathematics at the homework club than the students experienced in the classroom. As argued earlier, Vygotsky emphasizes the role of students' interactions in specific sociocultural settings. In this regard, students' experiences at the afterschool homework were critical because the understandings they built, such as those about multiplication, were transferred into the grade 2/3 classroom, which informed their mathematics experiences. As discussed in chapter 5, in the homework club, the students were introduced to multiplication through worksheets and repeated practice of procedures. Interestingly, the students' eagerness to do multiplication, which came from the homework club, led their teacher to introduce it to the rest of the students in the class. Students who attended the homework club appeared keen on quickly sharing the answer and when asked how they got the answer, they would simply say that they just know, or share the strategy of counting backward or forwards from a known multiplication fact. In the grade 2/3 classroom, the classroom teacher encouraged students to use various strategies (e.g., array models, equal groups, or repeated addition) and concrete learning tools (e.g., hundreds charts, counters, drawings, or their fingers) to help them conceptualize multiplication rather than see it as memorized facts. Students who attended the afterschool club appeared reluctant to do so because they relied on memorization of multiplication facts and did not want to use such strategies and tools. They appeared to have experienced conflict about how to approach mathematics – wrestling with the idea that it is a procedure to be applied versus a process to be understood. So, as the students tried to maintain and make use of what they learned at the homework club, they still felt a need to respect their teacher's expectations by sharing the steps, strategies, or tools used to find the product.

The Wider Community's Influence on Students' Mathematics Experiences

The students' experiences in a Canadian mathematics setting also reflected what they experienced in the wider community that they lived in. Vygotsky emphasizes the influence of the wider community or institution on the individual's participation in an activity. In particular, the focus is on "the specific nature of the activity involved, as well as on its relation to practices and institutions of the community in which it occurs" (Rogoff, 1995, p. 142). In this discussion, I consider the dominance of the English language in the wider community, as well as, provincial education policies, as impacting the actions, behaviors, or beliefs, and experiences of the students within the grade 2/3 classroom.

Dominance of the English Language on Students' Mathematics Experiences

The results of the study revealed three layers of language infrastructures that influenced the dominance of English in the grade 2/3 classroom: 1) the language infrastructure within the community within which the school was situated, 2) the language infrastructure in the school, and 3) the language infrastructure in the classroom within which mathematics education unfolds. Although the school was in an urban environment of a highly multicultural population with many new immigrants and refugees who spoke many different languages, English was used in businesses and in the neighbourhood in general. English was also predominantly used in government and other related services. In the school, over 70% of the student population spoke a language other than English, and required English as a second language (ESL) or Early literacy development (ELD) support, but the school operated in a single language (i.e., English). English was the language of teaching and learning and was used across all subjects except for French or core French. Also, everything in the classroom was in English. The classroom teacher and other teachers who visited the classroom communicated in English, and very rarely did a teacher share a student's home language. Even the students in the classroom did not all share similar home languages and, if they did, they appeared reluctant or refused to use it in the classroom. Thus, the use of the English language had high status in this context and was part of the students' experience of doing mathematics. These findings concur with the work of Setati and Adler (2000) in South Africa who found that varying language infrastructures in and around the school effects the dominance or use of a particular language in the mathematics classroom.

Provincial Policies Stimulus on the Teaching and Learning of Mathematics

Two educational policies relevant to mathematics teaching and learning were reflected in the teacher's pedagogical moves, and in turn, influenced student's mathematics experiences in the grade 2/3 mathematics classroom. These educational policies included the Ontario mathematics curriculum and a supporting document, as well as, the Education Quality and Accountability Office (EQAO) provincial assessment of mathematics. The dominance of the English language was also evident in educational policies, as they were strictly in English.

The teacher's implementation of the Ontario mathematics curriculum emerged as a positive influence on students' learning because of the focus on students' communication and discussion in mathematical problem-solving activities. Research suggests that communication and discussion of mathematical ideas is an essential process in learning mathematics, as it is through which students' mathematical thinking and understanding is shared (Barwell, 2014b, 2014c; Barwell et al. 2016; Chronaki & Planas, 2018; Hunter et al., 2018; Lerman, 2000; McGraw & Rubinstein, 2008; Moschkovich et al., 2018; Planas et al., 2018; OAME, 2005; Sfard, 2006). The results chapters provided many examples where students were actively engaged in activities that aligned with the overall and specific expectations noted in the curriculum. The students were also encouraged to develop and apply the mathematical processes (i.e., reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing and communicating) that were explicit in the curriculum. The use of mathematical processes were also evident in chapter 6 on *Students' Meaning Making in Mathematical Problem-solving Activities*. As shown in that chapter, the students engaged in mathematical actions (i.e., reasoning and proving, communications, representations, and connections) that align with the mathematical processes outlined in the curriculum. Also, as a result of the information provided in the mathematics curriculum (2005) and the resource/ practical guides (2001, 2008) about addressing the needs of multilingual learners, the students had extra time to complete tasks; they used their home language as a resource for learning mathematics; they used various strategies and concrete tools to share their mathematics thinking; and they made sense of mathematics terminology using a movement between informal language and formal language.

Although the EQAO influenced some of the mathematics content in the classroom, some of the mathematics terminology the teacher used, and time spent talking about test taking strategies, the students experienced these as woven into regular mathematics activities. The teacher incorporated various tasks similar to those found in the EQAO assessment booklets, but she did so by engaging students in interactive and engaging mathematics discussion to help students develop an understanding of what to expect when taking part in the mathematics assessment, and to align it with their usual mathematics experiences in the classroom.

The Role of Social Interactions in Students' Mathematics Experiences

Children experienced a variety of social interactions as part of their mathematics experience. The classroom set up of children in groups with access to materials encouraged interactions and collaboration among the students, their teacher, and the learning resources. Also, the teacher provided students with multiple opportunities to engage in mathematical discourse practices.

Thus, students shared mathematical thinking and ideas through verbal, visual, and written communication. Social interaction and the use of discourse practices in the mathematics classroom is related to what sociocultural researchers refer to as socially mediated experience. According to Schliemann and Carraher (2002), it is through social interaction that students develop their mathematical and logical thinking, which is constantly developing as they interact with others. Hence, students' intellectual development should not be understood without acknowledging a person's interaction with other people in their social environment (Wells, 1999). Generally, the results of this study revealed two types of interactions where students engaged in mathematical discourse practices, during student-teacher interactions and student-student interactions. Such interactions are also related to the construct of the zone of proximal development. From a sociocultural approach to learning, learning does not depend on a one-way flow of knowledge from teacher to learner. Rather, other students, through collaborative learning, can be situated within the zone of proximal development extending one another's prior learning and providing one another assistance. It is not necessary for there to be a person who is more capable than others for learning to occur (Forman & McPhail, 1993; Wells, 1999).

Student-Teacher Interactions

Student-teacher interactions happened throughout the mathematics block in one-on-one, small group, or whole-class discussions. In these interactions, the teacher encouraged students to develop and apply the mathematical processes such as reasoning to find a solution, communicating and reflecting on their mathematical understandings and ideas, and creating representations of their thinking using tools and strategies. In this way, the students experienced space to discuss and determine the soundness of their solutions as they spoke with the teacher. Similar to the findings of McClain and Cobb (2001), which suggested "the sequences of instructional activities and the general classroom social norms established in the classroom contributed significantly to the students' [mathematical] growth" (p. 242), in my study, the student-teacher interactions provided opportunities for students to reflect on their own solutions and those shared by others by listening to one another, repeating and elaborating on what another said, justifying their answers, explaining their choices, and negotiating understanding. In the process, the students developed various reasoning and proving skills to come to a consensus on a given matter.

The students experienced the teacher interacting with them in a variety of ways, always with a view to make their mathematical thinking visible. For instance, if the teacher saw that a student was uncomfortable with sharing their mathematical thinking verbally, due to their recent arrival or limited use of English, she encouraged them to use other means to communicate mathematically. This might be through gestures (e.g., their fingers), their bodies (e.g., for creating representations of symmetry or transformations), drawings, and various objects or concrete tools. In this way, the students made use of a variety of cultural tools to communicate their thinking. The students were thus all able to share their mathematical ideas. According to NCTM (2014), when students are provided with space to share their ideas and representations, they are regarded as "active members of the discourse community as they explain their reasoning and consider the mathematical explanations and strategies of their classmates" (p. 35). Having access to the use of different tools or strategies helps students to see or identify "how different approaches to solving a task are the same and how they are different" (p. 35).

In general, the students seemed to have experienced a very positive relationship with their teacher and felt comfortable to share their views towards her. The students also portrayed their teacher as accessible, the one in charge, and possessing mathematical authority, especially when confirming their answers or the rules of a mathematics game. Students also reflected their teachers' positive attitude and positive energy towards mathematics as depicted by their own views and experiences learning mathematics. According to Anthony and Walshaw (2009), the interactions that students have with their teacher "become a resource for developing students' mathematical competencies and identities" (p. 150), and also, influence how the students see and think of themselves in the classroom

Student-Student Interactions

The results revealed two very different ways the students experienced working together on mathematics activities. Students were observed working collectively and collaboratively on multiple occasions. As discussed in chapter 5, students who worked collectively, they were seated in the same space, paying attention, and sometimes imitating what one another was doing, but not necessarily feeding off of one another's thinking or ideas to come up with one solution. Students who worked collaboratively and not collectively, they jointly or mutually developed understanding of the activity and worked together to reason their way to a solution. In chapters 5 and 6, I provided several examples where students worked collectively and collaboratively. For instance, in *Books in the Classroom*, Khadija, by relying on the discussion that unfolded and by imitating what Joud and Samartha were doing was capable of making sense of the activity and to then reason her way to a solution—thus, working collectively. In *Ameer's Coins*, Mustapha, Abed, and Rachet constantly helped one another build an understanding of the activity. It also seemed as though they shared responsibility for each other's learning by trying to discuss, justify, and explain their thinking to one another. They were also deliberate in working together to find a solution to the activity—thus, working collaboratively. By drawing on Vygotsky's transformative view of internalization, I argue that it was through students' interactions, whether collectively or collaboratively, that learning and development occurred, as each student's thoughts mediate and are mediated by their socially structured interactions.

The findings also revealed that what students experienced during student-teacher interactions was visible in student-student interactions. The students' use of the mathematical actions that they saw and used became an integral part of students' discussions and interactions, which is what sociocultural theorists refer to as the transformative aspect of internalization. As discussed in chapter 6, students engaged in mathematical actions as they interacted with one another. Their experiences included engaging in mathematical problem-solving activities where they *reasoned* their way to a solution by *communicating* their understanding and ideas by creating *representations* that capture their understandings, and by making or seeing *connections* between mathematical ideas—contributing to their mathematical understanding of concepts and procedures. The students use of mathematical actions as they interacted with one another portrayed their ways of using and developing mathematical knowledge and skills as they engaged and interacted in the mathematics problem-solving activities. Also, when the students worked together, there were instances where they disagreed or had different interpretations and understandings. However, their further discussion enabled them to share their views and to develop new understandings. This is similar to a situation reported by Takeuchi's (2016) ethnographic study that examined English language learners' opportunities to learn in a multilingual mathematics classroom in a Canadian elementary school. The researcher found that students tend to gain access to a wider variety of work practices when working with their classmates as they are able to express their disagreements and negotiate their understandings. According to Yackel and Cobb (1996), when students are provided with opportunities to share and reflect on their own or other students' solutions, it could significantly contribute to their mathematical learning experiences.

Interestingly, when the students were asked if they liked to work with others when doing mathematics, some talked about how it was fun while others expressed frustration. For instance, some students felt that mathematics was a lot more fun when they worked together because they could talk and help each other. Other students felt frustrated and preferred working on their own because they were bothered by their classmates copying their work or questioning their answers. Students' views were built from their experiences working with others on mathematics in the grade 2/3 classroom. Students' experiences corresponded with the findings of researchers like Attard (2009) who explored grade 6 students' views on mathematics teaching and learning, and Grootenboer and Marshman (2016) work that explored middle school students' beliefs and attitudes about mathematics education. These researchers found that students' perceptions portray their mathematics education experiences within the classroom. Thus, students' experiences learning mathematics informed their perceptions. My findings would concur.

Students Developing Mathematical Identities Through Their Mathematics Experiences

The results of my study brought forth the relationship between students' identities and their mathematics learning experiences. Students' identities were not static but constantly changing and evolving, as they interacted with others and engaged in mathematics learning. Noteworthy is that students' interactions and engagements in mathematics were not limited to their experiences in the grade 2/3 classroom, but embraced students' mathematics experiences in their home countries, at home, and in the afterschool homework. It was as though the students were bridging one experience to the other, which appeared to transform their identity as they participated in mathematics learning in specific sociocultural settings. Hence, students' mathematics learning histories were critical to the development of their identities in the grade 2/3 classroom. Identity, as argued in chapter 3, holds a direct correlation with the transformative view of internalization. As the development of our thoughts is a result of one's exposure to social processes, so too is the development of our identities that develop through one's interactions with others or other things in an activity (Lave & Wenger, 1991; Wenger, 2008). Within the context of the grade 2/3 classroom, students' mathematical identities were built in a variety of ways through their experiences as reflected in, for example, 1) their actions and the individual identities that were constructed, 2) the distribution of mathematical authority, 3) the process of acculturation in the mathematics classroom, and 4) in becoming more “Canadian”.

Students’ Actions and their Individual Identities

Through students’ mathematical experiences, they constructed identities as mathematics learners. These identities influenced how they perceived or made sense of mathematics, and how they identified themselves as participants and “doers of mathematics”. This finding aligns with the work of Cobb and colleagues (2008, 2009) who analyzed the identities middle school students develop in mathematics classrooms. In their work, they discuss that students come to understand what it means to do mathematics as it is realized in the classroom and the extent to which they come to identify with that activity. Hence, they argue that “the identities that students are developing as doers of mathematics…[reflects] the ways that students are coming to think about themselves in relation to mathematics and the extent to which they are developing a commitment to and are coming to see value in mathematics as it is realized in the classroom” (Cobb et al., 2008, p. 1).

Through their mathematics experiences in the grade 2/3 classroom, the students developed their identities and expressed agency and autonomy in multiple ways. The results demonstrate that the students’ mathematics experiences included reflecting and directing their own learning, taking responsibility or ownership for their learning, building their understanding, correcting their answers, checking their own work, monitoring their own progress, modelling their solutions using concrete tools or their fingers, and using their individual strategies. Also, most students in the classroom considered themselves as a mathematics learner in a variety of ways. Some immediately and actively engaged in the activities, and others may have merely cooperated or resisted engagement in an activity due to such things as unfamiliarity with the activity. These identities shifted over time. For instance, some students may have first resisted engagement in the activity, then later, identified with the activity after they understood what was expected of them. Other students established a mathematical identity that demonstrated autonomy which demonstrated their willingness to work on a solution to the activity, and although at times they might appear discouraged and disengaged at points, they were determined to find a solution. Still, other students established a mathematical identity as directing and taking responsibility for their learning. Although some students would observe others to make sense of what is expected of them, they ultimately determined the strategy they used without relying on others. These identities varied and changed over time as students continued to experience mathematics and engaged with others. From a Vygotskian perspective, students' participation in sociocultural interactions transforms who they are and what they are capable of doing (Lave & Wenger, 1991; Wenger, 2008). Hence, what students could not do on their own before, they were capable of doing on their own at a later time, enabled by students' socially mediated experiences which contributed to the development of their identities. According to Cobb et al., (2008), whether students participate or resist engagement in mathematics, they are constantly developing and contributing to the ongoing redevelopment of their identities as established in their classroom. My findings would concur.

Students' Sense of Mathematical Authority

The ways in which mathematical authority was distributed influenced and contributed to students' meaning-making and mathematics learning experiences. Several researchers have argued that a key component of collaborative mathematics classrooms is the distribution of mathematical authority (Amit & Fried, 2005; Cobb et al., 2008, 2009; Langer-Osuna, 2017). The distribution of authority concerns the degree to which students are offered opportunities to be involved in making mathematical decisions, taking ownership of their ideas, and exercising agency in the classroom (Cobb et al., 2008, 2009; Langer-Osuna, 2017). The teacher's valuing of all students' mathematical thinking demonstrated the emphasis on shared mathematical authority within the classroom. However, as students interacted, mathematical authority emerged in a variety of ways.

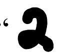
As students interacted and different students took on different roles or demonstrated different mathematical identities, some students' thinking became privileged, not so much by the teacher, but by the students. My findings suggest that the knowledge of students who were seen to have mathematical authority and a strong sense of agency was privileged over the knowledge of others. Their use of certain terminologies, strategies, or ways to play a game or solve the activities often were imitated by other students. For instance, in *Times Tussle*, Sahar and Tamam's use of certain mathematical terminology such as, "groups" and "times", seemed to have privileged these terms, which in turn, may have influenced students' understanding of the game, these terms, and multiplication itself. Also, the students privileged certain strategies that were used by Sahar and Tamam, continuously using those strategies to find the products. It was not only Sahar and Tamam who had mathematics authority in the room. This privileging of mathematics authority occurred with respect to other students in many other situations. From a transformative view of internalization, it could be argued that when the students adapted other students' use of strategies, tools, terms, or interpretations, they were working together to build and transform shared meanings and understandings. This aligns with the work of Yackel and Cobb (1996) who argue that "the development of individuals' reasoning and sense-making processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematical meanings" (p. 460). Students experienced mathematical meaning making during their interactions and communications with others, rather than a transfer of tools or strategies from one student to another.

In other instances, several students exercised mathematical authority by taking on the role of the teacher as they guided and explained to others how to play a game, solve an activity, use a strategy, or enforce the rules of a game. According to Yackel and Cobb (1996), taking on the role of teacher is only done when students have built their own ways of judging "when it is appropriate to make a mathematical contribution and what constitutes an acceptable mathematical contribution" (p. 473). By taking on the role of the teacher, I argue the students utilized the zone of proximal development by developing some understanding of what their peers have done and then guiding them to develop a solution that matches the activity's objective, and in the process, they are learning with and from each other—influenced by how the teacher set up mathematics learning experiences for the students. In chapter 6, I provided several examples where students took on the role of the teacher. For example, some students took on the role of the teacher by questioning other students' computations, particularly when the computation method was not clear. Other students took on the role of the teacher by evaluating or confirming other students' answers or interpretations of the activity. In exercising mathematical authority, the students authored their ideas, justified their thinking, and engaged in mathematical discussions that influenced students' mathematical meaning-making experiences. According to Langer-Osuna (2017), "as students author ideas, decide and justify whether particular mathematical ideas are reasonable or correct, and press one another for explanations, they take on the forms of intellectual authority that fuel the collaborative mathematics classroom" (p. 238). In general, students experienced the distribution of mathematics authority in the grade 2/3 classroom in my study which influenced students' participation and engagement in mathematics, led students to develop understanding, and helped students to identify with the mathematics activities.

Students' Experiences of Acculturation in the Mathematics Classroom

Part of newly arrived students' mathematics experiences included acculturation in the mathematics classroom. Acculturation can be understood as a process of experiencing change as individuals of one culture come into continuous contact with another culture (Bishop, 2002; Gibson, 2001; Sam & Berry, 2006). In his work on mathematics education, Bishop (2002) argues that "all mathematics education is a process of acculturation, and that every learner experiences cultural conflict in that process" (p. 195). He further argues that students either choose to adopt the dominant culture's ways or continue to be an outsider (p. 197). The students in the classroom in this study arrived to a whole new context and a new language as the means through which they were to experience mathematics. In their home countries, students experienced mathematics in particular ways, with direct instruction as the teacher explained the lesson on the chalkboard and the students engaged in silent and individual activities with little discussion. They are currently experiencing mathematics in a classroom where they are expected to use language to communicate their mathematical understandings and ideas, and to use a variety of strategies and tools. Vygotsky proposed that language is one of the most powerful and pervasive cultural tools that mediates the relationship between the individual and the social world (Lantolf & Thorne, 2000; Wertsch, 1991), and that language has meaning only in and through social interactions (Gee, 2005). In the grade 2/3 classroom, the students used language to engage in mathematics learning that fostered communication with others in order to share their own and other students' thinking and to develop mathematical understanding. My study shows that the students used language to 1) share their understanding or solution to the mathematics activity; 2) reflect on their own or other students' claims or solutions; 3) evaluate, correct, or confirm each other's answers; 4) refer to evidence for their claims; 5) use mathematical terms and everyday language; 6) count; 7) compute; 8) share or model their representations using concrete tools; 9) make one-to-one correspondence between concrete tools and numbers; and 10) refer to a number on the gameboard as a written representation of the number. My results on students enhancing the way they communicate mathematically aligns with the work of Moschkovich (2002) in a bilingual (Spanish-English) classroom of sixth-grade through eighth-grade students in the US. She found that classroom conversations that include mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments (p. 193), as well as, the use of gestures, concrete learning tools, and the student's first language could support students in learning to communicate mathematically (p. 208).

Part of students' acculturation was also moving to English as their language of communication. Although the students privileged using English in the classroom, some students occasionally relied on their home language to communicate mathematically, especially when clarifying understanding to one another or when counting or writing numbers. Two students , Raj and Rachet, communicated together using both their home language and English, particularly when one expressed confusion regarding what was expected of them in a mathematical activity. The student who was more proficient in English either translated the question or told the other student what to do using both their home language and English. Students' use of their home language and English is consistent with García and Wei's (2014) notion of translanguaging—that is, students use multiple discursive practices to communicate and make sense of their mathematical situation (p. 22). These researchers further argue that students' use of more than one language reflects the "use of original and complex interrelated discursive practices...that make up the speakers' complete language repertoire" (García & Wei, 2014, p. 22). Raj and Rachet are cousins living in close proximity to one another, which may influence their use of both languages when interacting together—something they are used to doing and transferred to the context of the classroom.

At times, some students relied on their home language when counting or writing numbers. For example, some students would rely on the Arabic or Somali number naming system to count or write numbers. Using English numerals, the students would say the digit in the ones place, with or without the conjunction (i.e., and), followed by the digit in the tens place (e.g., two-and-thirty or two-thirty). Also, by relying on the Arabic or Somali number naming system, the students would sometimes transpose their numbers when writing them in English (e.g., 32 would be written as 23). Also, several students expressed confusion because certain numbers were written in ways that did not resemble how they were written in their home country, at home, or in Canada. For instance, some students shared that they do not write the number 9 or 2 the same way as it was written on the hundreds chart, and they sometimes were confused because they were not sure what the number was. This confusion also arose when I wrote the number "2" in a way that the students were not used to like, “”. They were not sure what that number was. These examples of students’ experiences with number are consistent with the work of Gorgorió and Planas (2001) with newly arrived immigrants in Barcelona. The researchers found that when students join a mathematics classroom that is taught in a language that is foreign to them, they often experience new numbers that “can turn into a wide variety of cultural conflicts and disruptions of the learning process” (p. 12).

Experiencing Becoming More Canadian

In order to fit in with rest of the students, or for fear of being left out by others, students wanted to identify with their social group in the grade 2/3 classroom by 1) learning and communicating with others in English and 2) embracing the traits, values, or habits of their social group. From a Vygotskian perspective, who we are is a result of our interactions with others and with whom we identify and become a collective (Roth, 2004). Hence, through students’ interactions with one another and their eagerness to identify with others in the grade 2/3 classroom, they transformed their ways to fit in and to build their histories as a collective over the 7-month period that I observed. Chapter 5 provided many examples of how students identified with their social group. For instance, when a new student came into the classroom and was speaking a language other than English, they realized that neither their teachers nor most of their classmates shared a similar language, leading them to use their limited English vocabulary along with the use of gestures to communicate with others. Although the classroom teacher supported students’ use of their home language, most students who shared similar languages chose not to use them during mathematical interactions. Some students were quite deliberate in asking others to not use their home language in the classroom. Even when I spoke Arabic to help students make sense of a mathematical idea, some almost immediately responded in English rather than using Arabic, other students questioned my use of Arabic and insisted I spoke to them in English. Unlike the findings found in Barwell’s (2020) research where teachers requested or instructed students to use the language of instruction, in my research, the students themselves were requesting and instructing others to use English even when their teacher welcomed the use of other languages in the grade 2/3 classroom. Hence, to try and distinguish the grade 2/3 classroom as language positive[10] or language neutral[11] in relation to students' use of their home languages, as described in Barwell's (2020) research, I argue, may be too complex because there are many factors neither controlled by the teacher or the students that influence which language is used and for what purposes.

The findings also show that some students embraced the traits, values, or habits of others to identify with their social group, as well as, to build their histories in the grade 2/3 classroom. These were evident in the way students dressed, the food they had for lunch, and even, in the way they behaved. As discussed in chapter 5, some students dressed for school to look similar to other students in their classroom, rather than the traditional clothing of their home culture. Also, students' lunches gradually changed to resemble that of the dominant culture. Some students even transformed the way they raised their hand to signal that they had a question or comment to match their classmates' way of raising their hand. Some students also went as far as transforming the way they walked, talked, sang, and danced just to be like others in the classroom. Hence, through the students' experiences and observations, they transformed their ways to identify with their social group by gradually building on their identities—compliant with the transformative view of internalization.

Concluding Remarks

The main purpose of the discussion chapter was to reflect on my findings, the sociocultural underpinnings of this study, and other research to form an in-depth response and understanding of my main research question: What are the mathematical experiences of grade 2/3 students in a plurilingual classroom in Ontario, Canada? In my response to the main research question, I argue that multilingual students' mathematics experiences were shaped by their previous experiences, their interactions with one another, their use of language, and their teacher's pedagogical moves.

Students' previous experiences learning mathematics at the afterschool homework club, at

---

[10] In language positive classrooms, there is a clear discussion and attention on students' knowledge of other languages, but an emphasis on learning the language of instruction is still evident (Barwell, 2020, p. 174).

[11] In language neutral classrooms, attention is mostly given to the use of a single language—preferably, the language of instruction (Barwell, 2020, p. 174).

home, and in their home countries informed their understanding of mathematics and what it means to do mathematics. Thus, all that the students experienced elsewhere became part of their repertoire as they engaged in the grade 2/3 mathematics classroom as they leaned on those experiences to make meaning and express their mathematical thinking. In this way, students' histories or previous experiences in specific sociocultural settings influenced their learning and development.

In the grade 2/3 classroom, students' interactions with one another strongly impacted their mathematics learning experiences. In working together, the students made sense of their own and each other's interpretations, understandings, strategies, and tools to reason their way to a solution. The students were constantly negotiating their understanding by sharing their varied understandings and ways of thinking. In this way, the students accessed, shared, and reflected on each other's solutions, which contributed to their mathematics learning experiences. From a Vygotskian perspective, through social interactions students' learning and development occur as their thinking is mediated by their socially structured experiences. As seen in my research, the experiences that students had during their interactions with others transformed their thinking and understanding of mathematics.

Students' mathematics experiences were also shaped by their use of language. Language in this regard was a tool not only for saying or doing things but used alongside other non-verbal tools to build mathematical meaning. Students' use of language emphasized the use of certain cultural tools to engage in mathematics in the grade 2/3 classroom and elsewhere. Whether students used English, their home languages, mathematical language, or non-verbal tools, they utilized language to make meaning and to communicate their mathematical thinking. Thus, I argue that students' use of language in the mathematics classroom had meaning through their interactions in specific sociocultural settings. Vygotsky's theory would concur.

Finally, the teacher's pedagogical moves provided the context for students' mathematics experiences to evolve in the grade 2/3 classroom. In this context, the teachers moves appeared closely aligned with a sociocultural perspective of learning and development, as reflected by the instructional practices or strategies that the teacher used to extend students' prior learning, and to embrace social interactions with the teacher, other students, and with cultural tools (e.g., reasoning and proving skills, strategies, concrete learning tools). It was through such opportunities to engage in mathematics that the students were continuously building their identities as they participated in mathematics learning. These identities as mathematics learners influenced students' roles in mathematics and how they perceived or made sense of mathematics—all contributing to their mathematics learning experiences.

It is in taking the results of the two sub research questions as well as grounding the discussion in a sociocultural understanding of learning and development and other research that allowed me to provide a detailed account and description of the mathematical experiences of grade 2/3 students in a plurilingual classroom in Ontario, Canada.

CHAPTER 8: CONCLUSION

This chapter begins with a general summary of the study. Then, I discuss the significance of the study, contributions of the study, and implications for practice.

Summary of the Study

This ethnographic study was designed to better understand multilingual learners' mathematical experiences and meaning-making in a plurilingual educational setting. The study addressed the following research questions:

What are the mathematical experiences of grade 2/3 students in a plurilingual classroom in Ontario, Canada?

a) What are the influences on their mathematics experiences?

b) How do they negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

In this study, I assumed a sociocultural perspective that views learning and development as embedded in cultural as well as social contexts. As such, the study took place over 7 months, with visits 4-5 times a week in a grade 2/3 classroom in Ontario, Canada with 18 students from Eritrea, Nepal, Somalia, Sudan, and Syria. Data included observations, video recordings of students working on mathematics activities, copies of students' work, and interviews with students. In my analysis of the data, I relied on two analytical frameworks, one came from my analysis of the research literature that prompted me to organize the influences that the research found into four categories, and the other came from Gee's (2005, 2011) construct of building tasks to analyze "language-in-use", which I translated to the context of mathematics education. In response to the first sub research question, I argued that teacher's pedagogical moves were significant to students' mathematics learning and the development of their identities. Through the teacher's pedagogical moves, the students engaged in mathematics learning that fostered the use of various strategies, concrete learning tools, and communication with others to reason their way to a solution. Hence, students' identities were constantly changing and evolving as they engaged with the mathematical practices of the grade 2/3 classroom. In my response to the second sub research question, I used the detailed analysis of several video recorded sessions of students engaged in mathematics activities and argued that as the students interacted with one another they utilized language to make meaning and to negotiate their mathematical understanding. Through the use of language, the students made certain terminologies, strategies, and tools significant; they portrayed their ways of using and developing mathematical knowledge and skills; they took on certain identities as they engaged with mathematics activities; they built relationships with other students, their teacher, or with mathematics; they distributed mathematical authority; and they privileged certain sign systems or knowledge over others. All of these revolved around students' interactions and their use of language to make meaning and to negotiate their mathematical understanding. Then based on the results of the two sub research questions as well as grounding the discussion in a sociocultural understanding of learning and development I addressed my main research question. In my response to the main research question, I argued that students' histories or previous experiences in specific sociocultural settings became part of their repertoire which informed their mathematics learning in the grade 2/3 classroom. I also argued that students' interactions with one another strongly impacted their mathematics learning experiences as they were constantly negotiating their understanding by sharing their varied understandings and ways of thinking. Also, students' mathematics experiences were shaped by their use of language which emphasized the use of certain cultural tools to engage in mathematics in the grade 2/3 classroom and elsewhere. Finally, I reiterated the important role of the teacher's pedagogical moves which closely aligned with a sociocultural perspective of learning and development and provided the context for the students' mathematics experiences. Ultimately, the detailed descriptions of multilingual learners' mathematics experiences and meaning making build on the small number of research in Canada (e.g., Barwell, 2014c, 2017, 2018, 2020; Takeuchi, 2015, 2016) that specifically focuses on multilingual learners' mathematical experiences in a plurilingual educational setting.

Significance of The Study

My study has been built on various researchers' findings in the area of language diversity and mathematics education. The results of my study extend these findings by explicitly forming an in-depth understanding of the significant role of language, a teacher's pedagogical moves, and students' interactions on their mathematics experiences.

In terms of language, several researchers have challenged the idea that students need to have mastered the official language of instruction prior to learning mathematics (Barwell, 2005; Civil, 2008; Moschkovich, 2007). These researchers suggest that the knowledge of the language of instruction is only one aspect of becoming competent in mathematics, and lead us to consider the multiple resources multilingual learners use to construct knowledge, negotiate meanings, and to communicate mathematically (Moschkovich, 2002, 2007). According to Moschkovich, if all we see are students who

> ...mispronounce English words, or don't know vocabulary, instruction will focus on these deficiencies. If, instead, we learn to recognize the mathematical ideas these students express in spite of their accents, code-switching, or missing vocabulary, then instruction can build on students' competencies and resources. (Moschkovich 2007, p. 3)

Hence, Moschkovich (2007) argues that we need to shift our focus from what multilingual learners cannot do, and instead consider the multiple resources multilingual children use to communicate mathematically. I agree with those who have argued that language is not a barrier and that we need to focus on the resources students use to communicate mathematically. As evident in my study, the students drew on a variety of resources to make mathematical meaning, and their meaning making was not strongly dependent on proficiency in the language of instruction. Whether students were relatively new, had little prior schooling experiences, or had little to no knowledge of the English language, they engaged in all aspects of mathematics activities. Thus, rather than proficiency in the language of instruction being a factor, it was the opportunities the teacher provided for the students to develop their identities as mathematics learners, which promoted their mathematics learning experiences.

The underlying focus on the role of the teacher as shaping the mathematics experiences of multilingual students affirms the work of researchers who argue that the constraints and affordances that students experience in the classroom depend on the pedagogical practices of their teachers (Adler & Ronda; Kaur, 2013). For instance, Adler and Ronda (2017) argue that, "learners' access to doing and talking mathematics is through their teacher" (p. 66). In my study, the findings demonstrate that students' experiences in the grade 2/3 mathematics classroom were influenced by multiple teacher moves that supported students' mathematics communication and interactions with others, including how she organized the classroom, facilitated mathematical discussions, encouraged collaboration and communication between students, engaged students with open-ended mathematics tasks, provided tools for the students to work with, and encouraged the use of multiple strategies when solving problems. It was in providing documentation over 7 months that all these teacher moves became clear and illustrated an environment which was supportive of mathematics learning. Thus, the range of pedagogical moves that emerged in the grade 2/3 classroom is significant to this study.

Moreover, the findings of researchers demonstrate the role of students' interactions and communication with others as pivotal in supporting the development of students' conceptual understanding. This development occurs through the explanations, arguments, and justifications that students construct as they engage with other students on mathematical problem solving activities (Moschkovich et al., 2018). My study extended these findings through its rich description of the various ways that students' mathematics learning and meaning making originated in their interactions with the teacher or other students. Through these interactions students developed multiple resources to communicate their thinking and ideas. These multiple resources included various objects, drawings, students' bodies, gestures, facial expressions, tone of voice, mathematical processes (e.g., reasoning and proving), their experiences of the world, their mathematical knowledge and mathematical language, the languages they know, and other students' ideas and interpretations. Although previous research has identified the use of similar resources during students' interaction with others, such similarities leads to a degree of uniformity or convergence in the research literature in the area of language diversity and mathematics education.

Contributions of The Study

This study contributes to what is still a very limited body of literature on multilingual students' mathematical experiences in a plurilingual setting in Canada. Hence, results from this study add to the body of literature that could support educators as they plan mathematics learning for multilingual learners and provide insights to researchers in the area of mathematics learning in plurilingual settings. In particular, this study contributes to research in terms of 1) the use of a microanalytic approach to generate a detailed description, 2) the development of an analytic framework regarding influences on learning and the adaptation of Gee's framework to a mathematics setting, 3) being an in-depth ethnographic study, 4) providing voice for students and a forum to see the mathematics identity of these students as individuals engaged in mathematics, and 5) demonstrating the complexity and sophistication of students' thinking and meaning making.

One contribution of the study is that it adopted a microanalytic approach. Adopting a microanalytic approach allowed for a detailed description of the research setting and the participants involved in the research. It also allowed for a thorough description of the influences on students' learning of mathematics, as well as, a thorough account of how the students negotiated mathematical meaning during mathematical problem-solving activities. Such meticulous depictions portrayed the socially mediated experience as established in the grade 2/3 classroom and informed by the practices and policies outside and within the educational setting. Hence, allowing for an in-depth understanding of grade 2/3 students' mathematics experiences in a plurilingual educational setting, in Canada.

Also, my use of two analytical frameworks allowed for the detailed analysis and descriptions to emerge in this study. For instance, I developed the analytic framework of the influences on learning based on my analysis of the research literature. Analyzing in detail the individual, pedagogical, institutional, and sociopolitical influences on students' learning of mathematics provoked me to consider 1) the role of students' individual identities on their mathematics learning experiences, 2) the instructional practices or strategies that the teacher used to promote and facilitate mathematics learning in the classroom, 3) the society or educational settings where people live, work, and interact with others, and 4) to consider the relations that exist within and outside an educational setting. In considering these four influences, I not only reflected on what is taking place in the grade 2/3 classroom, but I also considered the influences of the wider community on students' mathematics learning experiences. In a similar vein, my translation of Gee's (2005) construct of building tasks of "language in use" to the context of mathematics education led to a detailed description and understanding of how students use language to make meaning and negotiate their mathematical thinking. Although various researchers in mathematics education have drawn on Gee's (2005, 2011) work to offer insightful discussions about the role of language in relation to the teaching and learning of mathematics (e.g., Moschkovich 2007; 2017; Setati et al., 2018; Setati 2005, 2008), my study builds on this research by providing a more thorough and detailed account of the construct of building tasks of "language in use" in the grade 2/3 mathematics classroom. Thus, my use of these two analytical frameworks is a key contribution.

Another contribution of the study is that it was an ethnography that took place over 7 months. I was fortunate to observe students develop and build their identities as mathematics learners over time. In some cases, from the moment they arrived until they have been there for a few months. For example, I saw students come to the classroom with very limited school experience and little knowledge of the English language, and to then engage with other students on mathematics using oral and visual communications to share their mathematical understandings, ideas, and to use a variety of strategies and tools. I also saw how some students valued the use of English in the grade 2/3 classroom while other students relied on their home language to communicate mathematically, especially when clarifying understanding to one another or when counting or writing numbers. I even got to see them come with their lunch of traditional food to having granola bars. I also saw them mimic other students' behaviors in order to fit in. I also got to observe how students picked up on the classroom norms and values. While most students appeared to have experienced acculturation with the host society by embracing the traits, values, or habits of that culture, I still saw students identifying with their countries of birth, the languages they spoke, having religious observance (e.g., wearing a hijab, fasting, praying etc.), and following dietary practices. My understandings of students' experiences learning mathematics in a plurilingual educational setting, using a sociocultural perspective, along and in collaboration with the students would have not formed such a novel and informative understanding without my presence in the grade 2/3 classroom for that extended period of time.

The study also provides a voice for multilingual students and a forum to see the mathematics identity of these students as individuals engaged in mathematics. For instance, students' conceptualizations of "what is mathematics" or "what it means to do mathematics" were influenced by students' experiences in and outside the mathematics block. In particular, the mathematics activities that students engaged with in their home countries, in the grade 2/3 classroom, at home, and at the homework club not only shaped their understanding of mathematics but also, informed students' mathematical meaning-making and learning experiences. Also, the findings brought forth the relationship between students' identities and their mathematics learning experiences. Students' mathematical identities were built in a variety of ways within the context of the classroom as reflected in, for example, 1) the different roles that the students play and the individual identities that were constructed; 2) how mathematical authority was distributed; 3) the process of acculturation in the mathematics classroom in terms of their use of oral and visual communications to share their mathematical understandings, and 4) in their sense of becoming more "Canadian". Hence, students' identities were not static, but constantly evolving and changing as they interacted with mathematics, their teachers, and other students.

This study also shows the novel way in which the students negotiated mathematical meaning but that too demonstrated the complexity and sophistication of students' thinking. As students worked on a solution to various mathematics activities, they relied on language to build and negotiate mathematical meaning. The students used language to interact with the mathematical problem-solving activity, and more specifically, with one another (i.e., the classroom teacher and other students), with specific terms, the rules of the game, and with various learning tools and strategies. In the process, particular things became significant to their mathematical meaning-making such as the term "groups", partitioning numbers into their respective tens and ones, or the use of their fingers. They also relied on several mathematical actions that portrayed their ways of using and developing mathematical knowledge and skills. Also, they played different roles and constructed their individual identities as doers of mathematics. The students also built several different relationships (i.e., with the classroom teacher or between the students). They also discussed where the mathematical authority resides in the classroom, and they privileged specific sign systems and knowledge of those with mathematical authority—all contributing to their experiences and mathematical meaning making.

Implications for Practice

Many school boards across Canada continue to welcome multilingual learners into their educational systems. This research study broadens our knowledge of multilingual learners' meaning making and experiences learning mathematics in a plurilingual setting, which may inform practice to support other multilingual learners in terms of, 1) disrupting notions that speak to student' proficiency in the language of instruction as a necessity for learning mathematics, and 2) considering various practices that teachers could incorporate in their teaching to help support multilingual learners in mathematics. The section then concludes with plans for knowledge mobilization.

The findings from the study indicate that students' proficiency in the language of instruction did not prove to be a necessity for learning mathematics. In the grade 2/3 classroom, language was not viewed as an obstacle by the students or their teacher as all students were regarded as learners of language and mathematics. The students were encouraged to engage in all aspects of the mathematics block despite their limited use of English or limited prior schooling experiences. Although the dominance of English in the classroom appeared to have initially led newly arrived students to withdraw from engaging in student-student or whole-class mathematics discussions, the students became aware of other means to communicate mathematically. In this context, language was more than the spoken word. There were several communicative resources that students used to negotiate their mathematical ideas and understandings. These communicative resources went beyond verbal or oral means, and involved written and visual means that students used to share and create representations of their mathematical thinking including, various objects; drawings; pictures; gestures; their own bodies; facial expressions; their experiences of the world; their mathematical knowledge and mathematical language; other students' ideas and interpretations; and the languages they know. Students used anything that they perceived as useful to negotiate mathematical meaning in relation to a mathematical task.

When we look at the teacher's pedagogical moves, we see that the teacher made a significant difference. Her range of teaching practices helped support multilingual learners in mathematics, as the students were capable of mathematical meaning making even when they were not proficient in the language of instruction. As a result of the teacher's moves, the students interacted with one another, developed reasoning and proving skills through mathematical activities, developed an understanding of mathematical terminology and associated concepts, used a variety of strategies to solve problems, and made use of concrete learning tools to develop and represent their mathematical ideas. In other words, their sense of agency and their sense of identity was built because of the environment the teacher created and because of the kinds of situations the teacher allowed the students to have. According to Yackel and Cobb (1996) "the teacher plays a central role in establishing the mathematical quality of the classroom environment and in establishing norms for mathematical aspects of students' activity" (p. 475). Hence, when students are regarded as active participants in the mathematics classroom and are encouraged to share their mathematical thinking using various tools and strategies that make sense to them, they will not feel segregated or withdrawn from participating in mathematics and become powerful thinkers and doers of mathematics.

Importantly, this research will be made accessible to practitioners and the school board as it is envisioned to enhance the teaching and learning of mathematics and to support multilingual learners' mathematical experiences in a plurilingual educational setting. A final written report was shared with the school board and will be shared with others who might be interested in multilingual learners' mathematical experiences. I have also conducted various workshops with teacher education students, and I am also prepared to do a presentation at a professional learning event for current teachers and the school board. I also intend to develop a research to practice monograph for the Ministry of Education such as those included in "What Works? Research into Practice". Since the focus is on how language and mathematics intersect, information from this study adds to the body of literature that could support educators as they plan mathematics learning for all learners in mathematics education.

REFERENCES

Abreu, G. de (2005). Cultural identities in the multiethnic mathematical classroom. In M. Bosch, M. (Ed.), *Proceedings of the fourth Congress of the European Society for Research in Mathematics Education* (pp. 1131-1140). FUNDEMI IQS, Universitat Ramon Llull.

Abreu, G., de, & Gorgorió, N. (2007). Social representations and multicultural mathematics teaching and learning. In D. Pitta-Pantazi & G. Philippou (Eds.*),* In *Proceedings of the fifth congress of the European society for research in mathematics Education* (pp. 1559-1566). Larnaca, Cyprus: Department of Education, University of Cyprus.

Adler, J., & Ronda, E. (2017). Mathematical instruction in discourse matters. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 64-81). CRC Press.

Agar, M. (2006). An ethnography by any other name. *Forum: Qualitative Social Research*, *7*(4), 1-24.

Alderson, P. (1995). *Listening to children: Children, ethics and social research*. Barnardo's.

Alrø, H., & Skovsmose, O. (2002). Dialogue and learning. In H. Alrø & O. Skovsmose (Eds.), *Dialogue and learning in mathematics education: Intention, reflection, critique* (pp. 113-136). Kluwer Academic Publishers.

Amit, M., & Fried, M. N. (2005). Authority and authority relations in mathematics education: A view from an 8th grade classroom. *Educational Studies in Mathematics, 58*, 145-168.

Anderson, A., Hamilton, R., Moore, D., Loewen, S. & Frater-Mathieson, K. (2004). Education of refugee children: Theoretical perspectives and best practices. In R. Hamilton & D. Moore (Eds.), *Educational interventions for refugee children: Theoretical perspectives and implementing best practice* (pp. 1-11). Routledge Falmer.

Anthony, G. & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the west. *Journal of Mathematics Education, 2*(2), 147-164.

Assaf, F., & Graves, B. (2019). “It’s not a calculator thingy because a calculator can’t answer the question”: Multilingual children’s mathematical reasoning. *Research in Mathematics Education, 21*(2), 1-20.

Atkinson, P., & Hammersley, M. (2007). *Ethnography: Principles in practice* (3rd Ed.) Routledge.

Attard, C. (2009). “My favourite subject is maths. For some reason no-one really agrees with me”: What year 6 students say about mathematics. In R. Hunter, B, Bicknell, & T. Burgess (Eds.), *Crossing divide*s*: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1). MERGA.

Baker, C & Hornberger, H. N. (Eds.) (2001). *An introductory reader to the writings of Jim Cummins.* Multilingual Matters LTD.

Barwell, R. (2003). Patterns of attention in the interaction of a primary school mathematics student with English as an additional language. *Educational Studies in Mathematics, 53*, 35-59.

Barwell, R. (2005). Working on arithmetic word problems when English is an additional language. *British Educational Research Journal, 31*(3), 329-348.

Barwell, R. (2012). Heteroglossia in multilingual mathematics classrooms. In H. Forgasz & F. Rivera (Eds.), *Toward equity: Gender, culture, and diversity: Advances in mathematics education* (pp. 315– 332). Springer.

Barwell, R. (2014a). Mathematics and language. In P. Andrews, Rowland, T., & S. Brindley (Eds.), *Masterclass in Mathematics Education: International Perspectives on Teaching and Learning* (pp. 75-86). Bloomsbury Publishing.

Barwell, R. (2014b). Language background in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 331-336). Springer.

Barwell, R. (2014c). Centripetal and centrifugal language forces in one elementary school second language mathematics classroom. *ZDM Mathematics Education, 46*, 911-922.

Barwell, R. (2016). Mathematics education, language, and super diversity. In A. Halai & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms: Issues for policy, practice and teacher Education* (pp. 25-39). Sense Publishers.

Barwell, R. (2017). Mathematics texts, alterity and the expropriation of mathematical discourse in second language mathematics classrooms. In J. Langman & H. Hansen-Thomas (Eds.), *Discourse analytic perspectives on STEM education: Exploring interaction and learning in the multilingual classroom* (pp. 119-137). Springer.

Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *Journal of Mathematical Behavior, 50,* 155-168.

Barwell, R. (2020). Learning mathematics in a second language: Language positive and language neutral classrooms. *Journal for Research in Mathematics Education, 51*(2), 150-178.

Barwell, R., Barton, B., & Setati, M. (2007). Multilingual issues in mathematics education: Introduction. *Educational Studies in Mathematics, 64*(2), 113-119.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J. N., Planas, N., Phakeng, M., Valero, P., Villavicencio Ubillús, M. (Eds.) (2016). *Mathematics education and language diversity: The 21st ICMI study*. Springer International Publishing.

Barwell, R., & Kaiser, G. (2005). Mathematics education in culturally diverse classrooms. *Analysis, 37*(2), 61-63.

Barwell, R., Kubota-Zarivnij, K., Culotta, C. (2018). Learning mathematics when students are new to schooling and new to English. In A. Kajander, J. Holm & E. J. Chernoff (Eds.), *Teaching and Learning Secondary School Mathematics*: *Canadian perspectives in an international context* (pp.101-116). Springer International Publishing.

Barwell, R., Moschkovich, J., & Phakeng, M. (2017). Language diversity and mathematics: Second language, bilingual, and multilingual learners. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 583–606). National Council of Teachers of Mathematics.

Bergem, K. O., & Pepin, B. (2013). Developing mathematical proficiency and democratic agency through participation - An analysis of teacher-student dialogues in a Norwegian 9th grade classroom. In B. Kaur, A. Glenda, M. Ohtani, & D. Clarke (Eds.), *Student voice in mathematics classrooms around the world* (pp. 143-160). Sense Publishers.

Bloome, D., & Golden, C. (1982). Literacy learning, classroom processes, and race: A microanalytic study of two desegregated classrooms. *Journal of Black Studies*, *13*(2), 207–226.

Brown, L. A., Ash, D., Rutherford, M., Nakagawa, K., Gordon, A., & Campione C. J. (1996). Distributed expertise in the classroom. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 188-228). Cambridge University Press.

Cao, Y., Guo, K., Ding, L., & Ah Chee Mok, I. (2013). Students at the front: Examples from a Beijing Classroom. In B. Kaur, G. Anthony, M. Ohtani & D. Clarke (Eds.), *Student voice in mathematics classrooms around the world* (pp. 65-88). Sense Publishers.

Carpenter, P. T., Franke, L. M., Jacobs, R. V., Fennema, E., Empson, B. S. (1997). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*(1), 3-20.

César, M., & Favilli, F. (2005). Diversity seen through teachers' eyes: Discourse about Multicultural classes. In M. Bosch (Ed.), *Proceedings of the fourth Congress of the European Society for Research in Mathematics Education* (pp. 1153-1164). FUNDEMI IQS, Universitat Ramon Llull.

Chang-Wells, G. L. M., & Wells, G. (1993). Dynamics of discourse: Literacy and the construction of knowledge. In E. A. Forman, N. Minick, C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 58-90). Oxford University Press.

Christensen, P., & James, A. (2008b). Childhood diversity and commonality: Some methodological insights. In P. Christensen & A. James, (Eds.), *Research with children: Perspectives and practices,* (pp. 156-172). Falmer Press.

Chronaki, A., & Planas, N. (2018). Language diversity in mathematics education research: A move from language as representation to politics of representation. *ZDM – International Journal on Mathematics Education, 50*(2), 1101-1111.

Citizenship and Immigration Canada (2017, September 12). Welcome Refugees: Key figures. http://www.cic.gc.ca/english/refugees/welcome/milestones.asp

Civil, M. (2008). Mathematics teaching and learning of immigrant students: An overview of the research field across multiple settings. In O. Skovsmose & B. Greer (Eds), *Opening the cage: Critique and politics of mathematics education* (pp. 127-142). Sense Publishers.

Civil, M., & Hunter, R. (2015). Participation of non-dominant students in argumentation in the mathematics classroom. *Intercultural Education, 26*(4), 296-312.

Civil, M., & Planas, N. (2011). Whose language is it? Reflections on mathematics education and language diversity from two contexts. In S. Mukhopadhyay & W. Roth (Eds.), *Alternative* *forms of knowing (in) mathematics: Celebrations of diversity of mathematical practices* (pp. 71–89). Sense Publishers.

Clarke, D. (2013). Appendix: The LPS research design. In B. Kaur, G. Anthony, M. Ohtani & D. Clarke (Eds.), *Student voice in mathematics classrooms around the world* (pp. 227-242). Sense Publishers.

Clarkson, P. C. (2006). Australian Vietnamese students learning mathematics: High ability bilinguals and their use of their languages. *Educational Studies in Mathematics*, *64*, 191-215.

Cobb, P., Gresalfi, M., & Hodge, L. l. (2008). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education, 39*(1), 1-29.

Cobb, P., Gresalfi, M., & Hodge, L. l. (2009). A design research perspective on the identities that students are developing in mathematics classrooms. In B. Schwarz, T. Dreyfus, & R. Hershkowitz (Eds.), *Transformation of knowledge through classroom interaction* (pp. 223–243). Routledge.

Cobb, P., & Yackel, B. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*(3/4), 175-190.

Corsaro, W. A. (2017). *The sociology of childhood* (5th ed.). SAGE Publications Inc.

Daniels, H. (2001). *Vygotsky and Pedagogy.* Routledge Falmer.

Elbers, E. & de Haan, M. (2005). The construction of word meaning in a multicultural classroom. Mediational tools in peer collaboration during mathematics lessons. *European Journal of Psychology of Education*, *20*(1), 45-59.

Education Quality and Accountability Office (EQAO). (2020, September 13). https://www.eqao.com/en

Education Quality and Accountability Office's (EQAO) news releases (2018, September 13). https://www.eqao.com/about-eqao/media-room/2018-school-board-results/

Essien, A. A. (2010). Investigating proficiency in the language of instruction as means of improving mathematical proficiency in a multilingual classroom. *Education As Change, 14*(2), 169-185.

Favilli, F., & Tintori, S. (2002). Teaching mathematics to foreign pupils in Italian compulsory schools: Findings from a European project. In P. Valero & O. Skovsmose (eds.). *Proceedings of the 3rd International MES Conference* (pp. 1-14). Centre for Research in Learning Mathematics.

Fletcher, G. (2019, September 15). 3-Act Tasks. https://gfletchy.com/3-act-lessons

Ford, M. J., & Forman, E. A. (2006). Redefining disciplinary learning in classroom contexts. In J. Green & A. Luke (Eds.), *Review of educational research* (Vol. 30, pp. 1–32). American Education Research Association.

Forman, A. E. (1989). Contextual constraints and empower children. In L. T. Lucien (Ed.,), *Social Interaction and the Development of Children's Understanding* (pp. 32-33). Ablex.

Forman, A. E. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. Steffe, P. Nesher, P. Cobb, B. Sriraman & B. Greer (Eds.), *Theories of mathematical learning* (pp. 115-130). Lawrence Erlbaum Associates.

Forman, A. E. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, G. Martin, & D. Schifter, (Eds.), *A Research Companion to Principles and Standards for Mathematics* (pp. 333-352). National Council of Teachers of Mathematics.

Forman, A. E., & McPhail, J. (1993). Vygotskian perspective on children's collaborative problem-solving activities. In E. A. Forman, N. Minick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 213-229). Oxford University Press.

Forman, A. E., Minick, N., & Stone, A.C. (1993). *Contexts for Learning: Sociocultural dynamics in children's development.* Oxford University Press.

Francisco, M. J. (2013). Learning in collaborative settings: Students building on each other's ideas to promote their mathematical understanding. *Educational Studies of Mathematics, 82,* 417-438.

García, O., & Wei, L. (2014). *Translanguaging: Language, bilingualism and education*. Palgrave MacMillan.

Gee, J. (2005). *An introduction to discourse analysis: Theory and method* (2nd edition). Routledge.

Gee, J. P. (2011). *How to do discourse analysis: A Toolkit*. Routledge.

Geertz, C. (1973). *Interpretation of cultures*. Basic Books.

Gibson, M. A. (1988). *Accommodation without assimilation: Sikh immigrants in an American high school*. Cornell University Press.

Gordon, T., Holland, J. & Lahelma, E. (2001). Ethnographic research in educational settings. In P. Atkinson, A. Coffey, S. Delamont, J. Lofland & L. Lofland (Eds.), *Handbook of ethnography* (pp. 188-203). SAGE Publications Ltd.

Gorgorió, N., & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics, 47*, 7-33.

Graves, B., & Zack, V. (1997). Collaborative mathematical reasoning in an inquiry classroom. In E. Pehkonen (Ed.), *Proceedings of the 21st conference of the international group for the psychology of mathematics education*, (Vol. 3, pp. 17–24). Gummerus.

Green, J. & Bloome, D. (2004). Ethnography and ethnographers of and in education: A situated perspective. In J. Flood, S. B. Heath & D. Lapp (Eds.), *Handbook of research on teaching literacy through the communicative and visual arts* (pp. 181-202). Macmillan Publishers.

Greene, S., & Hogan, D. (2005). Ethnographic research methods with children and young people. In S. Greene & D. Hogan (Eds.), *Researching children's experience,* (pp. 124-140). Sage Publications Ltd.

Gutiérrez, R. (2010). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 41,* 1-32.

Gutiérrez, R. (2011). Context matters: How should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp.17-34). Springer.

Grootenboer, P., & Marshman, M. (2016). *Mathematics, affect and learning: Middle school students' beliefs and attitudes about mathematics education*. Springer.

Grosjean, F. (1999). Individual bilingualism. In B. Spolsky (Ed.), *Concise encyclopedia of educational linguistics* (pp. 284–290). Elsevier

Halai, A. (2007). Learning mathematics in English medium classrooms in Pakistan: Implications for policy and practice. *Bulletin of Education & Research, 29*(1), 1-15.

Halai, A. (2009). Politics and practice of learning mathematics in multilingual classrooms: Lessons from Pakistan. In R. Barwell (Eds.), *Multilingualism in mathematics classrooms: Global perspectives* (pp. 47-62). Cromwell press group.

Halai, A., & Clarkson, P. (Eds.) (2016). *Teaching and learning mathematics in multilingual classrooms: Issues for policy, practice and teacher education*. Sense Publishers.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116.

Hunter, R. (2008). Facilitating communities of mathematical inquiry. In M. E. Goos, R. Brown & K. Maker (Eds.), *Navigating currents and charting directions*: *Proceedings of the 31st annual conference of the mathematics education research group of Australasia* (Vol. 1, pp. 31–39). MERGA.

Hunter, R., Civil, M., Herbel-Eisenmann, B., Planas, N., & Wagner, D. (Eds.). (2018). *Mathematical discourse that breaks barriers and creates space for marginalized learners*. Sense Publishers.

Hunter, R., & Hunter, J. (2018). Opening the space for all students to engage in mathematical practices within collaborative inquiry and argumentation. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas & D. Wagner (Eds.). *Mathematical discourse that breaks barriers and creates space for marginalized* learners (pp. 1-22). Sense Publishers.

Inagaki, K., Hatano., & Morita, E. (1998). Construction of mathematical knowledge through whole-class discussion. *Learning and Instruction, 8*(6), 503-526.

James, A. (2001). Ethnography in the study of children and childhood. In P. Atkinson, A. Coffey, S. Delamont, J. Lofland & L. Lofland (Eds.), *Handbook of ethnography* (pp. 246-257). SAGE Publications Ltd.

James, A. (2007). Giving voice to children's voices: Practices and problem, pitfalls and potentials. *American Anthropologist, 109*(2), 261-272.

James, A., & Prout, A. (Eds.). (2015). *Constructing and reconstructing childhood: Contemporary issues in the sociological study of childhood* (3rd Ed.). Routledge.

John-Steiner, V. & Mahn, H. (1996). Sociocultural approaches to learning and development: A Vygotskian framework. *Educational Psychologist, 31*(3/4), 191-206.

Kaprielian-Churchill, I. (1996). Refugees and education in Canadian schools. *International Review of Education, 42*(4), 349-366.

Kaur, B. (2013). Participation of students in content-learning classroom discourse: A study of two grade 8 mathematics classes in Singapore. In B. Kaur, G. Anthony, M. Ohtani, & D. Clarke. (Eds.), *Student voice in mathematics classrooms around the world* (pp. 65-88). Brill - Sense.

Kohn, A. (1999). What works better than traditional math instruction: Why the basics just don't add up. In A. Kohn (Ed.), *The schools our children deserve: Moving beyond traditional classrooms and "tougher standards"* (pp.159-182). First Houghton Mifflin (Trade).

Kozulin, A. (2012). Forward. In L. S.Vygotsky (1986). *Thought and language* (pp. ix-xxiii), MIT Press.

Kozulin, A. (2012). Vygotsky in context. In Vygotsky, L. S. (1986). *Thought and language* (pp. xxv-lxxvii), MIT Press.

Langer-Osuna, M. J. (2017). Authority, identity, and collaborative mathematics. *Journal for Research in Mathematics education, 48*(3), 237-247.

Langer-Osuna, M, J. (2018). Exploring the central role of student authority relations in collaborative mathematics. *ZDM Mathematics Education, 50,* 1077-1087.

Langer-Osuna, M. J., Moschkovich, J., Norén, E., Powell, B. A., & Vazquez, S. (2016). Student agency and counter-narratives in diverse multilingual mathematics classrooms: Challenging deficit perspectives. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, M. Setati-Phakeng, P. Valero & M.V. Ubillús (Eds.), *Mathematics education and language diversity: The 21st ICMI study* (pp. 163-173). Springer International Publishing.

Lantolf, P. J. (2006). Sociocultural theory and L2: State of the art. *Studies in second language acquisition, 28*(1), 67-109.

Lantolf, P. J., & Thorne, L. S. (2000). Sociocultural theory and second language learning. In P. J. Lantolf (Ed.), *Sociocultural theory and second language learning* (pp. 197-221). Oxford University Press.

Lave, J. (1991). Situating learning in communities of practice. In L. B. Resnick, J. M. Levine & S. B. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 63-82). American Psychological Association.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge University Press.

LeCompte, M. D, & Schensul, J. J. (2010). *Designing and conducting ethnographic research: An introduction* (2nd ed.). AltaMira Press.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Ablex.

Lerman, S. (2001). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics, 46,* 87-113.

Levine, A. R. (2007). Ethnographic studies of childhood: A historical overview. *American Anthropologist, 109*(2), 247-260.

Loewen, S. (2004). Second language concerns for refugee children. In R. Hamilton, & D. Moore (Eds.), *Educational interventions for refugee children: Theoretical perspectives and implementing best practice* (pp. 35-52). Routledge Falmer.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education, 32*(3), 236-266.

Maguire, M. H. (1999). A bilingual child's choices, and voices: Lessons in noticing, listening, and understanding. In F. Elizabeth (Ed.), *Reading and writing in more than one language* (pp.116-140). TESOL.

Maguire, M. H. (2005). What if you talked to me? I could be interesting! Ethical research considerations in engaging with bilingual/ multilingual child participants in human inquiry. *Forum: Qualitative Social Research, 6*(1), 1-17.

Maguire, M. H. (2005b). Identity and agency in primary trilingual children's multiple cultural worlds: Third space and heritage languages. In J. Cohen, K. T. McAlister, K. Rolstad & J. MacSwan (Eds.), *Proceedings of the 4th International Symposium on Bilingualism* (pp. 1423-1445). Cascadilla Press.

Maguire, M. H. (2009). Whose literacy learning landscapes matter? Learning from children's disruptions. *Learning Landscape, 3*(1), 189-205.

Maguire, M. H., & Graves, B. (2001). Speaking personalities in primary school children's L2 writing. *TESOL QUARTERLY, 35*(4), 561-593.

McGraw, R., & Rubinstein-Avila, E. (2008). Middle school immigrant students developing mathematical reasoning in Spanish and English. *Bilingual Research Journal, 31,* 147-173.

Morgan, C. (2007). Who is not multilingual? *Educational Studies in Mathematics, 64*, 239-242.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: An overview of the research in the field. *ZDM Mathematics Education, 46,* 843- 853.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2&3), 189–212.

Moschkovich, J. (2005). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*, 121-144.

Moschkovich, J. (2007). Bilingual mathematics learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 89-104). Teachers College Press.

Moschkovich, J. (2008). "I went by twos, he went by one": Multiple interpretations of inscriptions as resources for mathematical discussions. *The Journal of the Learning Sciences, 17*, 551-587.

Moschkovich, J. (2012). How equity concerns lead to attention to mathematical discourse. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education* (pp. 147-163). Springer.

Moschkovich, J. (2014). Bilingual/ Multilingual issues in learning mathematics. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 57-61). Springer.

Moschkovich, J. (2017). Revisiting early research on early language and number names. *EURASIA Journal of Mathematics Science and Technology Education, 13*(7b), 4143-4156.

Moschkovich, J. (2018). Talking to learn mathematics with understanding: Supporting academic literacy in mathematics for English learners. In L. A. Bailey, A. C. Maher & C. L. Wilkinson (Eds.), *Language, literacy, and learning in the STEM disciplines: How language counts for English learners* (pp. 14-34). Routledge.

Moschkovich, J. (2019). Codeswitching and mathematics learners: How hybrid language practices provide resources for student participation in mathematical practices. In J. MacSwan & Faltis, C. J. (Eds.). *Codeswitching in the classroom: Critical perspectives on teaching, learning, policy, and ideology* (pp. 88-113). Routledge.

Moschkovich, J., Wagner, D., Bose, A., Rodrigues Mendes, J., & Schutte, M. (Eds.). (2018). *Language and communication in mathematics education: International perspectives*. Springer International Publishing.

Moyer, P. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*, 175-197.

Nasir, S. N., & Hand, M. V. (2006) Exploring sociocultural perspectives on race, culture, and learning. *Review of Educational Research, 76*(4), 449-475.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics: An overview*. National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

Nieto, S. (2009). *The light in their eyes: Creating multicultural learning communities* (10th ed.). Teachers College Press.

Norén, E. (2011). Students' mathematical identity formations in a Swedish multilingual mathematics classroom. *Nordic Studies in Mathematics Education*, *16*(1-2), 95–113.

Norén, E., & Andersson, A. (2016). Multilingual students' agency in mathematics classrooms. In A. Halai & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms* (pp. 109-124). Sense Publishers.

Norton, B., & Toohey, K. (2011). Identity, language learning, and social change. *Language Teaching, 44*(4), 412-446.

Novotná, J., & Hospešová, A. (2013). Students and their teacher in a didactical situation: A case study. In B. Kaur, G. Anthony, M. Ohtani & D. Clarke (Eds.), *Student voice in mathematics classrooms around the world* (pp. 65-88). Brill - Sense.

Ontario Ministry of Education. (2005). *The Ontario curriculum grades 1-8: Mathematics revised*. Queens Printer for Ontario.

Paley, V. (1986). On listening to what the children say. *Harvard Educational Review, 56*(2), 122-131.

Paley, V. (1988). *Bad guys don't have birthdays: Fantasy play at four*. University of Chicago Press.

Parvanehnezhad, Z., & Clarkson, P. (2008). Iranian bilingual students reported use of language switching when doing mathematics. *Mathematics Education Research Journal, 20*(1), 52-81.

Planas, N. (2012). Commentary on the chapter by Richard Barwell, "Heteroglossia in multilingual mathematics classrooms": Heteroglossia and "orchestration" in multilingual mathematics classrooms. In H. Forgasz & F. Rivera (Eds.), *Towards equity in mathematics education*: *Advances in mathematics education* (pp. 333-338). Springer-Verlag Berlin Heidelberg.

Planas, N. (2014). One speaker, two languages: Learning opportunities in the mathematics classroom. *Educational Studies in Mathematics, 86,* 306-320.

Planas, N. (2018). Language diversity builds mathematics learning as much as mathematics learning builds language diversity. In U. Gellert, C. Knipping & H. Straehler-Pohl (Eds), *Inside the mathematics class (Advances in mathematics education)* (pp. 103-118). Springer International Publishing.

Planas, N., & Civil, M. (2013). Language-as-resource and language-as-political: Tensions in the bilingual mathematics classroom. *Mathematics Research Group of Australasia, 25,* 361-378.

Planas, N., Morgan, C. & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp. 196-210). Routledge.

Planas, N., & Ngoepe, M. (2019). Right to learns mathematics: From language as right to language as mathematically relevant to resource. In P. Rogers & G. Kaiser (Eds.), *Equity in Mathematics Education: Influences of feminism and Culture* (pp. 93-110). Information Age Publishing.

Planas, N., & Setati, M. (2009). Bilingual students using their languages in the learning of mathematics. *Mathematics Education Research Journal, 21*(3), 36-59.

Planas, N., & Valero, P. (2016). Tracing the socio-cultural-political axis in understanding mathematics education. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of the psychology of mathematics education: The journey continues* (pp. 447–479). Sense Publishers.

Prediger, S., Clarkson, P., & Bose, A. (2016). Purposefully relating multilingual registers: Building theory and teaching strategies for bilingual learners based on an integration of three traditions. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, M. Setati-Phakeng, P. Valero & M.V. Ubillús (Eds.), *Mathematics education and language diversity: The 21st ICMI study* (pp. 193–215). Sense Publisher.

Prediger, S., & Schüler-Meyer, A. (Eds.) (2017). Fostering the mathematics learning of language learners: Introduction to trends and issues in research and professional development. *Eurasua Journal of Mathematics, Science and Technology Education, 13*(7), 4049-4056.

Prosser, J., & Burke, C. (2008). Image-based educational research: Childlike perspectives. In J. G. Knowles & A. L. Cole, (Eds.)*, Handbook of the arts in qualitative research: Perspectives, methodologies, examples and issues* (pp. 407–419). Sage Publications Inc.

Ramirez, G. A., & Milk, R.D. (1986). Notions of grammaticality among teachers of bilingual pupils. *TESOL Quarterly, 20*(3), 495-513.

Rasmussen, C. & Marrongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education*, *37*(5), 388–420.

Razfar, A. (2013). Multilingual Mathematics: Learning through contested spaces of meaning making. *International Multilingual Research Journal, 7*, 175-196.

Rogoff, B. (1990). *Apprenticeship in thinking.* Oxford University Press.

Rogoff, B. (1991). Social interaction as apprenticeship in thinking: Guidance and participation in spatial planning. In L. B. Resnick, J. M. Levine & St. D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 349-364). American Psychological Association.

Rogoff, B. (1995). Observing sociocultural activity on three planes: Participatory appropriation, guided participation, and apprenticeship. In J. V. Wertsch, P. D. Rio & A. Alvarez (Eds.), *Sociocultural studies of mind* (pp. 139-164). Cambridge University Press.

Roth, W-M. (2004). Identity as dialectic: Re/making Self in urban school. *Mind, Culture, and Activity, 11*(1), 48-69.

Rummens, J. A. & Dei, G. (2010, November 24). Including the excluded: De-marginalizing immigrant/refugee and racialized students. *Education Canada. https://www.edcan.ca/articles/including-the-excluded-de-marginalizing-immigrantrefugee-and-racialized-students/*

Sahin, N., Dixon, K. J., & Schoen, C. B. (2020). Investigating the association between students' strategy use and mathematics achievement. *School Science and Mathematics*, *1*20(6), 325-332.

Sam, D. L. (2006). Acculturation: Conceptual background and core components. In D.L. Sam & J.W. Berry, *The Cambridge handbook of acculturation psychology* (pp. 11-26). Cambridge University Press.

Schliemann, A. D., & Carraher, D. W. (2002). The evolution of mathematical reasoning: Everyday versus idealized understandings. *Developmental Review*, *22*(2), 242-266.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27*(2), p. 4-13.

Sfard, A. (2006). Participationist discourse on mathematical learning. In J. Maass., & W. Schloeglmann (Eds.), *New mathematics education research and practice* (pp. 153-170). Sense Publishers.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for research in Mathematics Education, 36*(5), 447-466.

Setati, M. (2008). Access to mathematics versus access to the language of power: The struggle in multilingual mathematics classrooms. *South African Journal of Education*, *28*, 103–116.

Setati, M. (2016). Mathematics education and language diversity. In A. Halai, A., & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms* (pp. 11-23). Sense Publishers.

Setati, M., & Adler, J. (2000). Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*, 243-269.

Setati, M., Molefe, T., & Langa, M. (2008). Using language as a transparent resource in the teaching and learning of mathematics in a grade 11 multilingual classroom. *Pythagoras, 67*, 14-25.

Setati, M., & Planas, N. (2012). Mathematics education across two language contexts: A political perspective. In O. Skovsmose & B. Greer. (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 167–186). Sense Publishers.

Setati (Phakeng), M., Planas, N., Bose. A., & Njurai, E. (2018) Teaching and learning mathematics in trilingual classrooms. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized* learners (pp. 277-293). Sense Publishers.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information, 22*(2), 63-75.

Suurtamm, C., & Graves, B. (2009). Mathematics education reform: The role of coherence within the complexity of change. http://www.qucosa.de/fileadmin/data/qucosa/documents/8098/Proceedings-636pages-Dresden2009_535-539.pdf

Takeuchi, A. M. (2015). The situated multiliteracies approach to classroom participation: English language learners' participation in classroom mathematics practices. *Journal of Language Identity & Education, 14*(3), 159-178.

Takeuchi, A. M. (2016). Friendships and group work in linguistically diverse mathematics classrooms: Opportunities to learn for English language learners. *Journal of the Learning Sciences, 25*, 411-437.

Tangen, R. (2008). Listening to children's voices in educational research: some theoretical and methodological problems. *European Journal of Special Needs Education, 23*(2), 157-166.

Towers, J. & Proulx, J. (2013). An enactivist perspective on teaching mathematics: Reconceptualizing and expanding teaching actions. *Mathematics Teacher Education and Development, 15*(1), 5-28.

Tshabalala, L., & Clarkson, P. (2016). Mathematics teacher's language practices in a Grade 4 multilingual class. In A. Halai & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms: Issues for policy, practice and teacher education* (pp. 211-226) (Series). Brill - Sense.

Van Oers, B. (1998). From context to contextualizing. *Learning and Instruction, 8*(6), 473-488.

Vertovec, S. (2007). Super-diversity and its implications. *Ethnic and Racial Studies*, *30*(6), 1024–1054.

Vygotsky, L. (1978). *Mind in society*. Harvard University Press.

Vygotsky, L. S. (1981). The genesis of higher mental functions. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp.144-188). M.E. Sharpe.

Wagner, D., & Andersson (2018). Intersecting language repertoires when 4-year-olds count. In J. Moschkovich, D. Wagner, A. Bose, J. R. Mendes & M. Schütte (Eds.), *Language and communication in mathematics education: International perspectives* (pp. 105-118). Springer International Publishing.

Web, L., & Web, P. (2008) Introducing discussion into multilingual mathematics classrooms: An issue of code switching? *Pythagoras, 67,* 26-32.

Weber, K., Radu, I., Mueller, M., Powell, A., & Maher, C. (2010). Expanding participation in problem solving in a diverse middle school mathematics classroom. *Mathematics Education Research Journal, 22*(1), 91–118.

Wolcott, F. H. (1987). On ethnographic intent. In G. Spindler & L. Spindler (Eds.), *Interpretative ethnography of education: At home and abroad* (pp. 37-57). Erlbaum.

Wolcott, F. H. (2008). *Ethnography a way of seeing.* AltaMira Press.

Wells, G. (1999). *Dialogic inquiry towards a sociocultural practice and theory of education*. Cambridge University Press.

Wells, G. (2000). Dialogic inquiry in education. In D. C. Lee & P. Smagorinsky, (Eds.), *Vygotskian perspectives on literacy research: Constructing meaning through collaborative inquiry* (pp. 51-85). Cambridge University Press.

Wenger, E. (2008). *Communities of practice: Learning, meaning, and identity.* Cambridge University Press.

Wertsch, J. V. (1985). *Vygotsky and the social formation of mind.* Harvard University Press.

Wertsch, J. V. (1991). *Voices of the mind: A sociocultural approach to mediated action.* Harvard University Press.

Wertsch, J. V. (1994). The primacy of mediated action in sociocultural studies. *Mind, Culture, and Activity, 1*, 202-208.

Wertsch, J. V., & Stone, C. A. (1985). The concept of internalization in Vygotsky's account of the genesis of higher mental functions. In J. V. Wertsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* (pp. 162-179). Cambridge University Press.

White, A., Bushin, N., Carpena-Mendez, F., & Laoire, N. C. (2010). Using visual methodologies to explore contemporary Irish childhoods. *Qualitative Research, 10*(2), p. 143-158.

Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in Mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Appendix A Recruitment Letter for School Board

Hi,

My name is Fatima Assaf and I am a PhD candidate under the supervision of Dr. Christine Suurtamm. The intention of this letter is to invite your school board to participate in my research study. The objective of this research study is to understand English language learners' mathematical experiences in a multilingual educational setting. Understanding English language learners' mathematical experiences may help children's success in such situations, as many school boards across Canada continue to welcome English language learners into their educational systems.

This qualitative study addresses the following research questions:

What are the mathematical experiences of English language learners?

a) How do English language learners negotiate mathematical meaning during mathematical problem solving activities in a multilingual educational setting?
b) How do the previous and new mathematics learning experiences and meaning making of English language learners emerge and evolve in a multilingual educational setting?

The research will invite the students and their teacher to participate in the following:

- <u>Observations of the mathematics classroom.</u> I would like to visit a mathematics classroom once a week for one and a half months. The mathematics classroom will run as usual. For my study, I will make notes of the ways the students engage with one another; the ways they interact around mathematical problem solving activities; and the ways they produce oral or written explanations of their mathematical thinking. With the teacher's permission, I will make audio recordings of the mathematics activity and discussion during the mathematics block. These audio recordings will only be used as sound files and will serve as supplement to my observation notes.
- <u>Mathematical problem solving activities.</u> As part of students' regular mathematical activity in the classroom, I would like to focus on 2-3 instances of students working together on mathematical problem solving to make more in-depth observations and recordings of students' discussions. With parental permission, student-student interactions may be video and/ or audio recorded, and students' work may be photographed.
- <u>Follow-up conversations:</u> I would like to invite students to engage in 1-2 brief audio-recorded informal conversations during or following the problem-solving activities, which will last approximately 5-10 minutes each. The topics will focus on their thoughts towards the mathematics problem-solving activity and their experiences with mathematics at school and outside of school.

The research will take place during school time and in the presence of the classroom teacher. The involvement of the teacher in the research and in the mathematics classroom will not change from her/ his role as teacher. In addition, all students who agree and who obtain parental consent would be considered participants in the research study. Also, should the parent or child decline to participate in the study, the child's involvement in the classroom remains the same, but the child will not be video or audio recorded.

In the event that a student or their teacher is uncomfortable, I will reassure them that they do not have to continue. Also, to safeguard the identity of the participants, I will use pseudonyms to remove identifiers, and the school and the school board will not be identified. Furthermore, a final written report and a one-page summary will be sent to the school board and the school.

If your school board grants me permission to conduct my research, I would like to send a recruitment letter to several elementary school principals whose schools have welcomed English language learners. Once a school agrees to participate, information will be forwarded to grade 3, 4, 5, 6 teachers, who may then contact me if they wish to participate. If more than one school and/or teacher is interested in being a participant, a decision will be made taking into account the selection criteria. Also, upon selection of a school and classroom, a letter or email will be sent to other principals and teachers who responded thanking them for their interest and informing them that a selection has been made.

If you have any questions about this study please feel free to contact me, or my supervisor, Dr. Christine Suurtamm. If, however, you have any questions regarding the ethical conduct of this study, you may contact the Office for Ethics in Research, University of Ottawa, Tabaret Hall, 550 Cumberland Street, Room 154, Ottawa, ON K1N 6N5 Tel: [613) 562-5387 Email: ethics@uottawa.ca

Thank you,

Mrs. Fatima Assaf
PhD Candidate
Faculty of Education
University of Ottawa

Dr. Christine Suurtamm
Vice-Dean Research and
Professional Development
Faculty of Education
University of Ottawa

Appendix B Recruitment Letter for Principal

Dear [Name of Principal],

My name is Fatima Assaf and I am a PhD candidate under the supervision of Dr. Christine Suurtamm. The intention of this letter is to invite your school to participate in my research study. The objective of this research study is to understand English language learners' mathematical experiences in a multilingual educational setting. Understanding English language learners' mathematical experiences may help children's success in such situations, as many school boards across Canada continue to welcome English language learners into their educational systems.

This qualitative study addresses the following research questions:

What are the mathematical experiences of English language learners?

c) How do English language learners negotiate mathematical meaning during mathematical problem solving activities in a multilingual educational setting?
d) How do the previous and new mathematics learning experiences and meaning making of English language learners emerge and evolve in a multilingual educational setting?

The research will invite the students and their teacher to participate in the following:

- <u>Observations of the mathematics classroom.</u> I would like to visit a mathematics classroom once a week for one and a half months. The mathematics classroom will run as usual. For my study, I will make notes of the ways the students engage with one another; the ways they interact around mathematical problem solving activities; and the ways they produce oral or written explanations of their mathematical thinking. With the teacher's permission, I will make audio recordings of the mathematics activity and discussion during the mathematics block. These audio recordings will only be used as sound files and will serve as supplement to my observation notes.
- <u>Mathematical problem solving activities.</u> As part of students' regular mathematical activity in the classroom, I would like to focus on 2-3 instances of students working together on mathematical problem solving to make more in-depth observations and recordings of students' discussions. With parental permission, student-student interactions may be video and/ or audio recorded, and students' work may be photographed but not identify students.
- <u>Follow-up conversations:</u> I would like to invite students to engage in 1-2 brief audio-recorded informal conversations during or following the problem-solving activities, which will last approximately 5-10 minutes each. The topics will focus on their thoughts towards the mathematics problem-solving activity and their experiences with mathematics at school and outside of school.

The research will take place during school time and in the presence of the classroom teacher. The involvement of the teacher in the research and in the mathematics classroom will not change from her/ his role as teacher. In addition, all students who agree and who obtain parental consent would be considered participants in the research study. Also, should the parent or child decline to participate in the study, the child's involvement in the classroom remains the same, but the child will not be video or audio recorded.

In the event that a student or their teacher is uncomfortable, I will reassure them that they do not have to continue. Also, to safeguard the identity of the participants, I will use pseudonyms to remove identifiers, and the school and the school board will not be identified. Furthermore, a final written report and a one-page summary will be sent to the school board and the school.

If you agree to participate in this research study, please pass on the enclosed recruitment letters and posters to grade 3, 4, 5 and 6 teachers at your school. If teachers wish to participate they can then contact me. If more than one school and/or teacher is interested in being a participant, a decision will be made taking into account the selection criteria. Upon a teacher's approval, I will arrange a time to meet the teacher and the students.

If you have any questions about this study please feel free to contact me, or my supervisor, Dr. Christine Suurtamm. If, however, you have any questions regarding the ethical conduct of this study, you may contact the Office for Ethics in Research, University of Ottawa, Tabaret Hall, 550 Cumberland Street, Room 154, Ottawa, ON K1N 6N5
Tel.: [613) 562-5387
Email: ethics@uottawa.ca

Thank you,

Mrs. Fatima Assaf
PhD Candidate
Faculty of Education
University of Ottawa

Dr. Christine Suurtamm
Vice-Dean Research and
Professional Development
Faculty of Education
University of Ottawa

Appendix C Recruitment letter for Teacher

Hi,

My name is Fatima Assaf and I am a PhD candidate under the supervision of Dr. Christine Suurtamm. The intention of this letter is to invite your classroom to participate in my research study. The objective of this research study is to understand English language learners' mathematical experiences in a multilingual educational setting. Understanding English language learners' mathematical experiences may help children's success in such situations, as many school boards across Canada continue to welcome English language learners into their educational systems.

This qualitative study addresses the following research questions:

What are the mathematical experiences of English language learners?

a) How do English language learners negotiate mathematical meaning during mathematical problem solving activities in a multilingual educational setting?
b) How do the previous and new mathematics learning experiences and meaning making of English language learners emerge and evolve in a multilingual educational setting?

The research invites the students and their teacher to participate in the following:

- Observations of the mathematics classroom. I would like to visit a mathematics classroom once a week for one and a half months. The mathematics classroom will run as usual. For my study, I will make notes of the ways the students engage with one another; the ways they interact around mathematical problem solving activities; and the ways they produce oral or written explanations of their mathematical thinking. With your permission, I will make audio recordings of the mathematics activity and discussion during the mathematics block. These audio recordings will only be used as sound files and will serve as supplement to my observation notes.
- Mathematical problem solving activities. As part of students' regular mathematical activity in the classroom, I would like to focus on 2-3 instances of students working together on mathematical problem solving to make more in-depth observations and recordings of students' discussions. With parental permission, student-student interactions may be video and/ or audio recorded, and students' work may be photographed but not identify students.
- Follow-up conversations: I would like to invite students to engage in 1-2 brief audio-recorded informal conversations during or following the problem-solving activities, which will last approximately 5-10 minutes each. The topics will focus on their thoughts towards the mathematics problem-solving activity and their experiences with mathematics at school and outside of school.

The research activities will take place during school time and in your presence as the mathematics teacher. Your involvement in the research and in the mathematics classroom will not change from your role as teacher. In addition, all students who agree and who obtain parental consent would be considered participants in the research study. Also, should the parent or child decline to participate in the study, the child's involvement in the classroom remains the same, but the child will not be video or audio recorded.

In the event that you or any of your students are uncomfortable, we do not have to continue. Also, to safeguard the identity of the participants, I will use pseudonyms to remove identifiers, and the school and the school board will not be identified. Furthermore, a report summarizing the findings will be sent to your school.

If you have any questions about this study please feel free to contact me, or my supervisor, Dr. Christine Suurtamm. If, however, you have any questions regarding the ethical conduct of this study, you may contact the Office for Ethics in Research, University of Ottawa, Tabaret Hall, 550 Cumberland Street, Room 154, Ottawa, ON K1N 6N5
Tel.: [613) 562-5387
Email: ethics@uottawa.ca

If you agree to participate in this research study, please contact me. If more than one teacher is interested in being a participant, a decision will be made taking into account the selection criteria. I will then arrange a time to meet with you and the students.

Thank you,

Mrs. Fatima Assaf
PhD Candidate
Faculty of Education
University of Ottawa

Dr. Christine Suurtamm
Vice-Dean Research and
Professional Development
Faculty of Education
University of Ottawa

Appendix D Information Letter and Consent form for Teacher

Hi,
You have been invited to participate in a research project conducted by Fatima Assaf under the supervision of Dr. Christine Suurtamm as part of Fatima Assaf's thesis project, in Doctorate of Philosophy in Education, at the University of Ottawa.

The purpose of this study is to understand how English language learners come up with explanations of their mathematical thinking using the languages they know and other resources. Since the focus is on how language and mathematics intersect, this information will benefit educators to support all learners in mathematics education.

The Ottawa-Carleton Research and Evaluation Advisory Committee has granted approval for this study. The school Principal has also given permission for this study to be carried out in your school.

In participating in this study, your involvement in the research and in the mathematics classroom will not change from your role as teacher. With your permission, I will visit your classroom once a week for one and a half months to make notes as students work together during the mathematics lesson. My notes will focus on the ways students use the languages they know and other resources during mathematical discussions. I will also make audio recordings of the mathematics activity and discussion during the mathematics block, which will serve to supplement my notes. With your and parents' permission, I will video and/ or audio record student's working together on mathematical problem-solving activities to make more in-depth observations and analysis of students' mathematical discussions, and your interaction with a child may be recorded. I will also take photographs of students' work and your work, such as a problem on the board. I may also ask questions to the students about doing mathematics. These questions might include finding out more about their experiences with mathematics at school and outside of school. The project will occur at your school and in your mathematics classroom.

Participation in this research is completely voluntary, and you may decide to withdraw from the research at any time. All data gathered from you until the time of withdrawal will be destroyed.

All collected information will remain confidential, and your name, the students' names, the school, and the school board will not be identified. The data will be used for the completion of the thesis project as well as in research journals and presentations, but no names will be mentioned.

I will keep my notes, the video and audio recordings, and photographs in a locked filing cabinet at my home during the research, and when I am done the project any written records and photographs will be stored in a locked filing cabinet at my home. After ten years, any written records and photographs will be shredded and the video and audio recordings will be securely deleted from my computer.

Please indicate on the attached form whether you wish to participate in this study. Your cooperation will be greatly appreciated.

If you have further questions or concerns, please do not hesitate to contact me or my supervisor Dr. Christine Suurtamm.

If you have any questions regarding the ethical conduct of this study, you may contact the Office for Ethics in Research, University of Ottawa, Tabaret Hall, 550 Cumberland Street, Room 154, Ottawa, ON K1N 6N5 Tel.: (613) 562-5387 Email: ethics@uottawa.ca
Sincerely,

Fatima Assaf

The information collected for this project is confidential and protected under the Municipal Freedom of Information and Protection of Privacy Act.
I have read and understood the request to participate in the study of *Children's Mathematical Experiences in a Multilingual Educational Setting*. I have discussed it with Fatima Assaf and:

☐ I give permission to participate.

☐ I give permission to be audio recorded.

☐ I give permission to be video recorded.

☐ I give permission for my work to be photographed. Identifiable information will be removed.

☐ The video-recordings may be used for analysis and presentations.

☐ The video-recordings may be used **only** for analysis.

This form is to be completed and returned **ONLY** if I consent to participate in this research.
Teacher's Name: *(please print)*__________________ *Date*: ________________
Signature of Teacher: ________________________________

Appendix E Verbal Information Script for Students

The following is an information script was verbally presented to the participating class:

Hello everyone,

My name is Fatima Assaf and I am studying at the University of Ottawa. My teacher is Dr. Christine Suurtamm. Today, I would like to talk to you about my research project. My research is called *Children's Mathematical Experiences in a Multilingual Educational Setting.* It is about children learning mathematics and speaking more than one language. This project will help me to understand how children come up with explanations of their mathematical thinking using the languages they know and other resources.

For this project, you and your teacher will not be asked to do anything different from what you would normally do while doing mathematics. I will visit your class once a week for one and a half months to make notes as you work together in the mathematics classroom. I will make notes about the ways you use the languages you know and other resources during mathematical discussions. I would also like to video and/ or audio record you working together on mathematical activities, and we will talk about doing mathematics. I may also take photographs of your schoolwork.

I will keep my notes, the video and audio recordings, and photographs in a locked filing cabinet at my home during the research, and when I am done the project they will be stored on my password protected computer.

This research project will not affect your schoolwork or your grades and will not appear in any school records. Also, your names will not appear in any research related documents/ papers. If you choose to stop participating in the study, you can stop at any time with no consequences.

Before I begin this research project, I need to get your parents approval for you to participate by signing the following form. The form talks about the project and it tells the parents what is involved. It is very similar to what I have talked about now. If your parents do not want you to participate in this project, nothing changes and you stay in the classroom and work on the mathematics activities, and will not be video or audio recorded. If your parents give you permission to participate, I will ask you to sign a form as well.

Does anyone have any questions, or is there anything you would like me to explain? If at any time you have any questions you can always email me, or my supervisor Dr. Christine Suurtamm.

Appendix F Information letter and Consent form for Parents

(Date)

Dear Parent/Guardian:

I am a researcher studying English language learners' mathematical experiences in a multilingual educational setting. This project will help to understand how children come up with explanations of their mathematical thinking using the languages they know and other resources. Since the focus is on how language and mathematics intersect, this information will benefit educators to support all learners in mathematics education.

The Ottawa-Carleton Research and Evaluation Advisory Committee has granted approval for this study. The school Principal has also given permission for this study to be carried out in your son/daughter's school.

In order to conduct this study, I will visit your child's class once a week for one and half months to make notes as students work together during the mathematics lesson. I will make notes about the ways they use the languages they know and other resources during mathematical discussions. I would also like to video and/ or audio record them working together on mathematical problem-solving activities to make more in-depth observations and recordings of students' mathematical discussions. I may also ask questions to the students about doing mathematics. These questions might include finding out more about their experiences with mathematics at school and outside of school. I may also take photographs of students' work. The project will occur in your child's classroom with their teacher during the time set aside to do mathematics. The students will see my role as one of observer of their learning and classroom volunteer.

Participation in this research is completely voluntary, and if you agree that your child can participate, your child may withdraw from the research at any time. Also, should you or your child decline to participate in the study, your child's learning is not disrupted in any way. They will take part in the usual classroom activities but will not be video or audio recorded.

The research will have no impact on your child's attendance in class or on his/her grades. All collected information will remain confidential, and the student's name, the teacher's name, the school, and the school board will not be identified. The data will be used for the completion of the thesis project as well as in research journals and presentations, but no names will be mentioned.

I will keep my notes, the video and audio recordings, and photographs in a locked filing cabinet at my home during the research, and when I am done the project any written records and photographs will be stored in a locked filing cabinet at my home. After ten years, any written records and photographs will be shredded and the video and audio recordings will be securely deleted from my computer.

Please indicate on the attached form whether you permit your son/daughter to participate in this study. Your cooperation will be greatly appreciated.
If you have further questions or concerns, please do not hesitate to contact me or my supervisor Dr. Christine Suurtamm.

If you have any questions regarding the ethical conduct of this study, you may contact the Office for Ethics in Research, University of Ottawa, Tabaret Hall, 550 Cumberland Street, Room 154, Ottawa, ON K1N 6N5 Tel.: (613) 562-5387 Email: ethics@uottawa.ca
Sincerely,

Fatima Assaf

The information collected for this project is confidential and protected under the Municipal Freedom of Information and Protection of Privacy Act.
I have read and understood the request for my child to participate in the study of *Children's Mathematical Experiences in a Multilingual Educational Setting*. I have discussed it with my child and:

☐ I give permission for my child to participate.

☐ I give permission for him/her to be audio recorded.

☐ I give permission for him/her to be video recorded.

☐ I give permission for his/her work to be photographed. Identifiable information will be removed.

☐ The video-recordings may be used for analysis and presentations.

☐ The video-recordings may be used **only** for analysis.

This form is to be completed and returned to the school **ONLY** if I consent to my child participating in this research.
Name of Student: *(please print)*__________________ *Date*: ________________
Name of Parent/Guardian: *(please print)*____________________________
Signature of Parent/Guardian: ________________________________

Appendix G Assent Text for Students

☐ I understand what this research is about.

☐ I agree to participate in the study about *Children's mathematical experiences in a multilingual educational setting.*

☐ If I have any questions about this study, I know that I can ask Fatima Assaf or her teacher Dr. Christine Suurtamm.

Fatima Assaf Dr. Christine Suurtamm

Participant's Signature: Name: Date:

Researcher's Signature: Name: Date:

Appendix H Complete List of Data Collected

Activity	Grouping	Materials	Focus	Research Equipment	Date
Math Lessons *Introduce a new math topic/ strategy to use, review a lesson/ topic for adding, subtracting, multiplication & division*					
	Whole-class	Chalk board	Introduce a new topic: Adding equal groups	Audio recorded & took pictures	Jan 17, 2019
	Whole-class	Math challenge activity	Explaining math challenge activity	Audio recorded & took pictures	Jan 18, 2019
	Whole-class	Chalk board & base ten blocks	Subtracting using various strategies: Number line, branching, base ten blocks, & subtraction algorithm	Audio recorded & took pictures	Jan 21, 2019

	Whole-class	Chalk board & base ten blocks	Subtracting with base ten blocks & number line	Audio recorded & took pictures	Jan 22, 2019
	Whole-class	Tiles & rectangle shape on paper	Measurement-Perimeter & Area	Audio recorded & took pictures	Jan 23, 2019
	Whole-class	Whiteboard with EQAO math activity	Transformations-Rotation, Translation, & Reflection-Multiple choice--Product of elimination	Audio recorded & took pictures	Jan 29, 2019
	Whole-class	Tiles & rectangle shape on paper	Area, multiplication (grouping), & division	Audio recorded & took pictures	Feb 1, 2019
	Whole-class	Tiles & rectangle shape on paper	Area, Perimeter, multiplication (grouping), & division	Audio recorded & took pictures	Feb 8, 2019

	Whole-class	Chalk board & base ten blocks	Groups of 10-Base 10 blocks-Tens & ones—10 groups of 10, &/ or 100 ones	Audio recorded & took pictures	Feb 8, 2019
	Whole-class	Chalk board & base ten blocks	Groups of 10-Base 10 blocks-Tens & ones—10 groups of 10, &/ or 100 ones or 10x10 = 100	Audio recorded & took pictures	Feb 11, 2019
	Whole-class	Chalk board & pentomino tiles	Transformations-Rotation, Translation, & Reflection	Audio recorded & took pictures	Feb 12, 2019
	Whole-class	Chalk board, students' pictures, pentomino tiles, math challenge activities	Review equal groups & transformations. Intro. to two Math challenge activities Activity #1: Rotation, Translation, & Reflection Math challenge Activity #2: Word problems	Audio recorded & took pictures	Feb 20, 2019

	Whole-class	Whiteboard & EQAO math activity	Patterning & Number Sentences- Product of elimination	Audio recorded & took pictures	Feb 22, 2019
	Whole-class	Whiteboard & math activity	Money-Naming & Adding them in cents & dollars	Audio recorded & took pictures	Feb 25, 2019
	Whole-class	Chalk board & counters, tiles, square shape on paper, on carpet	Review/ introduce a new topic for various strategies to do Multiplication: equal groups, repeated addition, area, array. Introduce students to math centres	Audio recorded & took pictures	March 4, 2015
	Whole-class	Various manipulative for each math center	Instructions for math centers	Audio Recorded	March 5, 2019
	Whole-class	Chalk board & counters, tiles, square sheet, on carpet	Review, introduce a new topic for various strategies to do multiplication—figuring out area &	Audio recorded & took pictures	March 6, 2019

			modelling for multiplication		
	Whole-class	Chalk board & counters, tiles, square shape on paper	Equal groups for multiplications	Audio recorded & took pictures	March 8, 2019
Calendar *Morning Routine/ Entry*					
	One student & whole-class participation	Calendar, bar graph & line graph, hundreds chart, base ten blocks, patterning pictures, & clock.	Bar graph (weather: sunny, windy, stormy, etc.), line graph (precipitation), counting days of school using tally marks, base ten blocks, hundreds chart, telling time (sunrise & sunset)	Video Recorded	Feb 11, 2019
Math Challenge Activities *May involve what students learned in class: adding, subtracting, graphs, patterning, word*					

problems, perimeter, area, & symmetry).					
	Interactive & Individual	Graphing activity paper	Graphing: Creating a graph based on the information they collected on Jan 15th. What is your favourite………?	Pictures of students' work	Jan 17, 2019
	Group #1: Two students Group #2: Three students	Math challenge activity, pencils, erasers, clear ruler & pencil crayons	Math Challenge Activity-Word problem, patterning, addition, subtraction, perimeter & area	Audio recorded & photocopied students' work	Jan 18, 2019
	ELD Teacher with group of two students	Math challenge activity: Question #1	Math Challenge-Word problem: Patterning	Video recorded	Jan 18, 2019
	Individual/ or with me: Group of four students	Math challenge activity, pencils, erasers, clear ruler & pencil crayons	Continued…Math Challenge Activity Word problem, patterning, addition, subtraction, perimeter & area	Audio recorded & photocopied students' work	Jan 21, 2019 Continued from Jan 18, 2019

	Group of three students	Math challenge activity, pencils, erasers, & clear ruler	Math Challenge Activity: Graph, word problems (adding), area, perimeter, & transformation	Video recorded & copies of students' work	Feb 1, 2019
	Group of two students	Math challenge activity, pencils, erasers, & clear ruler	Continued…Math Challenge Activity Word problems (adding), area, perimeter, & transformation	Audio recorded & photocopied students' work	Feb 8, 2019 Continued from Feb 1, 2019
	Group of two students	Math challenge activity, pencils & erasers, base ten blocks, & hundreds chart	Math Challenge Activity-Grouping (Who am I?) & Patterning	Pictures of students working	Feb 12, 2019
	Group #1: Three students Grou p #2: Three students Group #3: Three students	Math challenge activity, pencils & erasers, base ten blocks, & hundreds chart	Math Challenge Activity - Adding & Subtracting/ Try some multiplication & Division	Boys group: Audio recorded Girl s group: Video recorded Girl s group: Video recorded Cop ies of students' work	Feb 20, 2019
	Group #1: Two students Group #2: Two students	Math challenge activity, pencils & erasers, base ten blocks, & hundreds chart	Math Challenge Activity - Adding & Subtracting/ Try some multiplication & Division	Video recorded & took pictures/ copies of students' work	Feb 22, 2019- Continued from Feb 20, 2019

	Group #1: Three students & teacher Group #2: Four students	Math challenge activity, pencils & erasers, & hundreds chart	Math Challenge Activity - Adding & Subtracting/ Try some multiplication & division	Audio recorded & copies of students' work	Feb 25, 2019- Continued from Feb 22, 2019
	Group of three students	Math challenge activity, pencils & erasers, & hundreds chart	Math Challenge Activity - Try some Multiplying & Dividing word Problems (Extra)	Pictures of students' work	Feb 26, 2019
	Group of two students	Math challenge activity, pencils & erasers, base ten blocks, & hundreds chart	Math Challenge Activity - Try some Multiplying & Dividing word Problems (Extra)	Audio recorded/ pictures & copies of students' work	Feb 28, 2019
	Group of five students	Math challenge activity, pencils & erasers, base ten blocks, & hundreds chart	Math Challenge Activity - Graphing, subtracting, adding, grouping (Who am I?)	Took some pictures—not a lot	Mar 1, 2019
Math Talk *Problem solving activities, word problems, or 3 Act Task videos from G. Fletcher or Teacher*					

Math Talk #1	Whole-class/ Individual/ Interacting with Others	Small whiteboards & markers, & hundreds chart	Cookie Monster-Multidimensional Problem solving 3-Act Task: G. Fletcher Video & Pictures Problem solving activity-Adding & subtracting	Pictures of students' work	Jan 10, 2019
Math Talk #2	Whole-class/ Individual/ Interacting with Others	Whiteboard & magnetic shapes	Symmetry - Geometry-spatial sense or reasoning	Video/ audio recorded	Jan 14, 2019
Math Talk #3	Whole-class/ Individual/ Interacting with Others	Whiteboard, math sketchbook, pencils, & erasers	Word Problem: Beads for Lizards Adding or subtracting	Video/ audio recorded & pictures of students' work	Jan 14, 2019
Math Talk #4	Whole-class/ Individual/ Interacting with Others	Small whiteboards & markers	Cracker Monster-Multidimensional Problem solving 3-Act task: Teacher Video & Pictures Adding & subtracting	Video/Audio recorded & Pictures of students' work	Jan 30, 2019

Math Talk #5	Whole-class/ Individual/ Interacting with Others	Small whiteboards & markers	Word Problem: Mr. Ruth & Chocolates (Extra work from math talk #4) Adding &/ or subtracting	Video/Audio recorded & Pictures of students' work	Jan 30, 2019
Math Talk #6	Individual/ Interacting with others-sharing	Math Sketchbook, pencils, erasers, & pencil crayons	Money & Shapes: How much are my shapes worth $? Problem solving activity- Adding/ Multiplying	Pictures of students' work	Feb 8, 2019
Math Talk #7	Whole-class/ Individual/ Interacting with Others	Small white/ chalk boards, markers, & hundreds chart	Skittles-Multidimensional Problem-solving activity: Teacher Video & Pictures - Multiplying, adding, counting, grouping	Audio recorded & took pictures of students' work	Feb 26, 2019
Math Talk #8	Whole-class/ Individual/ Interacting with Others	Small white/ chalk boards, markers, & hundreds chart	Word Problem: Skittles - multiplication, adding, counting, grouping	Audio recorded & took pictures of students' work	Feb 28, 2019-a review of students' work from Feb 26, 2019
Math Talk #9	Whole-class/ Individual/ Interacting with Others	Small whiteboards & markers	Word problem: Money (Entry Activity) Adding money	Audio recorded & took pictures of students' work	March 6, 2019

Math Talk #10	Whole-class/ Individual/ Interacting with Others	Small whiteboards, markers, & hundred's chart	Word problem: Money (Entry Activity) Adding money	Audio recorded & took pictures of students' work. Sticky notes of students' work	March 8, 2019
Math Centres *Games, Symmetry activities, Pentomino activities, Fatima's Math Challenges*					
	Individual/ Group of two or three students & Video recorded one student	Geometric 2D shapes using handout/ Chart Paper & Pentomino tile shapes	Symmetry Geometry—creating symmetry on opposite side—spatial sense/ reasoning	Audio recorded/ pictures of students' work	Jan 15, 2019
	Group of two students	Geometric 2D shapes using handout	Symmetry Geometry—creating symmetry on opposite side—spatial sense/ reasoning	Video Recorded/Audio recorded	Jan 17, 2019
	Groups of three or four students	Matching Game-Symmetry cards	Matching Game-Symmetry Geometry—Matching symmetry game—spatial sense/ reasoning	Audio Recorded/ Pictures	Jan 22, 2019

	ELD/ESL teacher with two students	Tiles & triangle shapes on paper	Measurement-Perimeter & Area	Video Recorded	Jan 23, 2019
	Group of two or four students	Matching Game-Symmetry cards	Geometry—Matching symmetry game—spatial sense/ reasoning	Audio Recorded	Jan 29, 2019
	Group of three students	Math challenge activity & base ten blocks (Rods & Units)	Math Challenge Activity: Books in a Classroom Word Problem: Adding & Grouping in 10s	Video Recorded/ I also have pictures of others students' work	Feb 6, 2019
Shape Series	Individual/ Group of two or three students, or five for 100 treats activity/ puzzle	100 Piece Puzzle, 'my hundreds chart of candy'—lots of it too, 100 drops of water (water container, mini jar, medicine dropper & ruler), A quarter & whiteboard to keep track of tally marks,	Shape Series (Patterning with linking cubes) "How do you see the shapes growing?" What do you notice/ wonder about? Answer is entirely visual. Counting to hundred with different activities & utensils	Pictures of students' work	Feb 11, 2019

	Group #1: Three students, plus one on his own Group #2: Two students Gr oup #3: Three students	Math challenge activity, linking cubes, & small whiteboards	Shape Series (Patterning with linking cubes) "How do you see the shapes growing?" What do you notice/ wonder about? Answer is entirely visual. Patterning, Number sense-counting	Video Record & Took Pictures of students' work	Feb 12, 2019
	Group #1: Three students, then one additional student joins, & one other but wasn't much involved.	Math challenge activity & linking cubes	Shape Series (Patterning with linking cubes) "How do you see the shapes growing?" What do you notice/ wonder about? Answer is entirely visual. Patterning, Number sense-counting	Video Record & Took Pictures of students' work	Feb 19, 2019
	Group of two students	Game board & dice	Game: Add 'Em up Adding	Video Recorded	Feb 26, 2019

	Group of five students	Pentomino tiling puzzles, plastic cube, & shape cut outs	Pentomino Geometry: spatial sense/ reasoning	Audio recorded & took pictures of students’ work	March 4, 2019
	Group of six students & teacher	Lego, paper, pencils, & hundreds chart	Lego - Multiplication/ Modelling	Video & audio record. Took pictures of students’ work.	March 4, 2019
	Group of four students & ELD teacher	Coins, blank paper for image, & handout for tallying	Money-Make an Image with coins on blank piece of paper. Handout for tallying their money & then calculating/ adding the total number of coins used	Pictures of students’ work	March 4, 2019
	Group #1: Group of five students with teacher Group #2: Group of 4 students with teacher	Lego, paper, pencils, & hundreds chart	Lego- Multiplication/ Modelling	Video & audio record. Took pictures of students’ work.	March 5, 2019
	Group #1: Group of four students & ELD/ESL teacher	Coins, blank paper for image, & handout for tallying	Money- Make an Image with coins on blank piece of paper. Handout for tallying their money & then calculating/	Pictures of students’ work	March 5, 2019

	Group #2: Group of five students with ELD teacher		adding the total number of coins used		
	Group #1: Group of five students Group #2: Group of four students	Pentomino tiling puzzles, plastic cube, & shape cut outs	Pentomino Geometry—spatial sense & reasoning	Took pictures of students' work	March 5, 2019
	Group of five students with teacher	Lego, paper, pencils, & hundreds chart	Lego-Multiplication/ Modelling	Video & audio recorded. Took pictures of students' work.	March 6, 2019
	Group of four students with ELD teacher	Coins, blank paper for image, & handout for tallying	Money- Make an Image with coins on blank piece of paper. Handout for tallying their money & then calculating/ adding the total number of coins used	Audio recorded & took pictures of students' work	March 6, 2019
	Group of four students with teacher	Lego, paper, pencils, & hundreds chart	Lego-Multiplication/ Modelling	Video & audio recorded. Took pictures of students' work.	March 8, 2019

	Group of five students with ELD teacher	Coins, blank paper for image, & handout for tallying	Money- Make an Image with coins on blank piece of paper. Handout for tallying their money & then calculating/ adding the total number of coins used	Audio recorded & took pictures of students' work	March 8, 2019
	Group of two students	Game board, game cards & tiles.	Math Game: Times Tussle (x10) (x5) Multiplying by 5 & 10 from 1-9.	Video recorded with iPad	April 3, 2019
	Group of four students	Game board, game cards & tiles.	Math Game: Times Tussle (x10) (x5) Multiplying by 5 & 10 from 1-9.	Video recorded with iPad	April 5, 2019
	Group of three students	Math challenge activity, small whiteboard with marker, eraser cloth, pencils & erasers	Math Challenge Activity: Ameer's Coins Word Problem: Money, adding various coins to make $1.	Video recorded & I also have copies of students' work	May 8, 2019

	Group of three students	Math challenge activity, small whiteboard with marker, eraser cloth, & pencils & erasers	Math Challenge Activity: Find the Fraction Word problem: Fractions-adding &/ or Subtracting to find the remaining fraction	Video recorded & I also have pictures/copies of others students' work	May 8, 2019
Other Activities (Extra/ Side work students interacted with)					
	Individual	Chalk Board	Rachet-53x11 Multiplication	Audio Recorded	Jan 17, 2019
	Group of two students	Tiles & rectangle shape on paper	Perimeter & Area	Audio recorded & took pictures of students' work	Jan 18, 2019
	Group of two students	White/ Smart Board & magnetic shapes	Symmetry Geometry—creating symmetry on opposite side—spatial sense/ reasoning	Pictures of students' work	Jan 18, 2019

	Individual	Small whiteboards & markers	Subtracting using number line	Audio recorded & took pictures of students' work	Jan 18, 2019
	Individual	Small whiteboards & markers	Subtracting with number line & hundreds chart	Pictures of students' work	Jan 22, 2019
	Individual	Back of patterning worksheet or on small whiteboards	Extra Math problem solving questions: Adding/ Multiplication	Pictures of students' work	Feb 6, 2019

Appendix I Observational Protocol

Date:______________________________

Timing	Description of activity	Who is talking?	What are they saying?	Language(s) being used	Reflections and emerging questions

Appendix J Follow-up Interview Questions with Students

Interview Questions:

1. The interview will begin with questions specific to mathematics activity (i.e., what did you think of the math activity; is there another way to solve this problem; what did you notice; etc.)

2. What do you like about doing math?

3. Do you like to work with others when doing math?

4. What languages do you use when you are doing math?

5. Is doing math in this class similar to how you learned math before?
    a. How is it the same/ different?

6. Do you do any math at home?
    a. With parents?
    b. Talk about doing math at home.

Appendix K Transcription conventions

(The glosses given below are mostly taken from ten Have P. Doing Conversational Analysis: a Practical Guide. London, Thousand Oaks, New Delhi: SAGE Publications, 1999. p. 213 - 214)

SYMBOL	MEANING
/ /	*Slashes* indicate overlapping speech (spoken at the same time). They indicate the beginning point of overlap as well as the point at which an utterance terminates another utterance. The following shows two speakers whose talk overlaps. Ruby: I thought you wanted to know /what that is./ Mary: /Which are?/
=	*Equal signs*, indicate 'latching,' that is, two utterances that follow one another without any perceptible pause. One is put at the end of one line, and one at the beginning of another line to indicate that there is no "gap" between the two lines.
(7.1)	*Number in parentheses*, indicates silence or pause (of seven seconds and one tenth of a second)
[ ]	Square brackets contain transcriber's descriptions additional to transcription
::	*Colons*, indicate prolongation of sound
-,	*A dash with a comma,* indicates a cut-off or a false start
.	*A period*, indicates a stopping fall in tone
,	*A comma,* indicates a continuing intonation, like when you are enumerating things
?	*A question mark,* indicates a rising intonation
(( ))	*Empty double parentheses,* used if a single word or short phrase is completely unintelligible.
((text))	*Double parentheses,* contains transcriber's best attempt at transcribing a difficult passage

Appendix L Stages of Second-Language Acquisition and Literacy Development for ESL Students

Table A1.1: ESL, Grades 1 to 3 – Listening

Stage 1 Students understand basic spoken English. They:	**Stage 2** Students understand key information presented in highly supported contexts in a variety of settings. They:	**Stage 3** Students understand social English, but They: require contextual support to understand academic language. They:	**Stage 4** Students understand spoken English in most contexts. They:
– follow simple directions with support from visual cues – respond to clear, short, simple questions – respond briefly to short, simple stories, songs, and poems – respond to familiar conversational topics using single words and short phrases – respond to gestures, courtesies, and tones of voice, and follow classroom routines	– participate in conversations on familiar topics – understand key vocabulary and concepts related to a theme/topic – request clarification when necessary – respond to direct questions, frequently used commands, courtesies, and some humour – respond to non-verbal signals in familiar contexts – begin to respond to unseen speakers (e.g., on the radio, on the telephone, over the school public-address system) – identify main ideas in visually supported oral presentations containing familiar vocabulary	– respond to discussions and conversations – identify key information in most contexts, with the aid of some repetition – respond appropriately to body language, non-verbal signals, tone of voice, pauses, stress, and intonation – respond to unseen speakers (e.g., over the school public-address system, on the radio, on the telephone) – follow a series of simple instructions	– participate in most social and academic discussions – respond to complex sentences – understand age-appropriate expressions and idioms

Table A1.2: ESL, Grades 1 to 3 – Speaking

Stage 1 Students speak English for basic communication. They:	**Stage 2** Students speak English with increasing spontaneity and accuracy. They:	**Stage 3** Students initiate conversations and participate in discussions and presentations using a variety of strategies. They:	**Stage 4** Students speak English accurately in most situations. They:
– use short, patterned questions to seek information – share personal information (e.g., name, address) – express basic needs using single words – identify familiar names, objects, and actions – speak with sufficient clarity for teacher comprehension – begin to use (with assistance) subject–predicate order, simple verb tenses, negatives, questions, plurals, pronouns, adjectives, adverbs, common contractions, and basic prepositions of location and direction – imitate some English stress and intonation patterns – use everyday gestures and courtesies to convey meaning – participate in short, prepared role plays and dialogues	– ask simple questions – participate in social discussions using short phrases and short sentences – participate, with prompting, in academic discussions, using short phrases and short sentences – initiate and maintain face-to-face conversations – recount familiar events, stories, and key information – give simple directions or instructions and communicate simple observations – express personal opinions and emotions – speak with sufficient clarity and accuracy for listener comprehension – speak at almost the pace of first-language speakers, showing some control of stress, timing, and rhythm – express meaning with growing competence, using present and past verb tenses when explaining causes and results, direction, and time	– initiate and maintain conversations – participate in discussions based on classroom themes – make short, effective oral presentations in an academic context – speak with clear pronunciation and enunciation – begin to self-correct simple grammatical errors – use voice to indicate emphasis through pacing, volume, intonation, and stress	– use most language structures appropriate to the grade level – speak with fluency and clarity in group situations – self-correct common grammatical errors – make academic presentations – use idiomatic and colloquial language appropriately

Table A1.3: ESL, Grades 1 to 3 – Reading

Stage 1 Students read and comprehend simple written English. They:	**Stage 2** Students read for specific purposes when background knowledge and vocabulary are familiar. They:	**Stage 3** Students demonstrate increasing independence in a variety of reading tasks, with ongoing support. They:	**Stage 4** Students demonstrate control of grade-appropriate reading tasks. They:
– recognize the alphabet in print – know the direction of English print – read pictures and use picture clues – begin to use phonetic and context clues and sight recognition to understand simple texts (e.g., pattern books, chart stories, songs, chants, rhymes) – recognize familiar words and repeated phrases in plays, poems, stories, and environmental print – participate in shared reading activities, choral reading, and rehearsed reading in a small group – select appropriate reading materials, with assistance	– use reading strategies to assist in deriving meaning from text (e.g., predicting; rereading; phonics; recognition of cueing systems, repetition, and word families) – understand familiar vocabulary in age-appropriate stories, poems, scripts, environmental print, and computer text – select main ideas in short, familiar passages from a variety of genres – use some correct phrasing and rhythm when reading familiar material aloud – use the school library, with assistance, to find personal reading materials for enjoyment and information	– begin to follow written instructions – describe story components (e.g., character, plot, setting) – read and understand grade-appropriate text, with minimal assistance – use grade-appropriate resources that provide some visual or contextual support (e.g., graphic organizers, class word lists, theme-book collections, environmental print, picture dictionary, table of contents)	– respond independently to written instructions – recall and retell a written story – figure out meaning in text that may be unfamiliar, unsupported by visual context, and contain challenging vocabulary and sentence structures – read a variety of print material – begin to use independent research skills in the classroom and school library – choose and enjoy material for personal reading similar in scope and difficulty to that being read by peers

Table A1.4: ESL, Grades 1 to 3 – Writing

Stage 1 Students begin to write simple English structures. They:	**Stage 2** Students write in a variety of contexts using simple English structures. They:	**Stage 3** Students write English in a variety of contexts with increasing independence and accuracy. They:	**Stage 4** Students write English for a variety of purposes using appropriate conventions. They:
– begin to dictate labels, phrases, and sentences to a scribe – print the English alphabet in upper- and lower-case letters – copy written information, following left-to-right and top-to-bottom progression – complete sentence patterns based on familiar and meaningful context and vocabulary – add words to sentence openers to complete a thought – write some personally relevant words – express ideas through drawing, writing in the first language, and labelling – write personal information (e.g., name, address) – participate in shared writing activities in small groups – participate in a variety of prewriting activities – begin to use computers for writing activities	– compose short, simple, patterned sentences based on learned phrases and classroom discussion – write some common and personally relevant words – use capital letters and final punctuation – begin to use basic sentence structures (e.g., statements, questions) – use appropriate formats to write for a variety of purposes (e.g., lists, signs, labels, captions, cards, stories, letters, journals) – use the writing process, with assistance (e.g., participate in structured prewriting activities; make some changes between the initial and final draft) – use computers to begin to develop word-processing skills	– write short compositions, making some use of appropriate verb tenses, prepositions, simple and compound sentences, and descriptions, and beginning to use new vocabulary and idioms – use conventional spelling for most common and personally relevant words – write to record personal experiences and thoughts, to narrate a story, and to convey information – begin to write independently in all subject areas – use the stages of the writing process, with support (e.g., prewriting, producing drafts, and publishing) – write collaboratively with peers	– begin to write competently in all subject areas – contribute to cooperative class writing – use a variety of forms of writing – write short, original compositions using all stages of the writing process – observe most conventions of punctuation

Table A1.5: ESL, Grades 1 to 3 – Orientation

Stage 1 Students begin to adapt to the new environment. They:	**Stage 2** Students demonstrate understanding of and adaptation to the new environment. They:	**Stage 3** Students demonstrate increasing understanding of and involvement in the new environment. They:	**Stage 4** Students demonstrate growing awareness, understanding, and appreciation of their own and others' cultural heritage as part of the Canadian context. They:
– find personally relevant school locations independently – begin to adapt to a variety of teaching strategies used in a Canadian classroom – begin to respond to social situations appropriately – begin to work with a partner on a common academic task – call some classmates and staff by name – communicate critical needs to school staff and peers – develop connections with some staff and peers in the school – follow some classroom and school routines and schedules – rely on the home language and culture to think, communicate, and process new experiences	– ask for assistance and communicate needs – continue to use and take pride in the home language – follow school routines, behaviour expectations, and emergency procedures – interact with peers outside own linguistic or cultural group – participate actively in regular class programs, with modifications – participate in controlled, directed group work (e.g., simple research projects) – participate in most classroom and some school activities (e.g., field trips, sports, clubs) – respond appropriately in most social situations	– continue to use and take pride in the home language – understand and follow school routines, behaviour expectations, and emergency procedures – state basic information about the neighbourhood – actively participate in the daily life of the school – respond appropriately to most teaching approaches (e.g., active learning, the informal classroom atmosphere) – show increasing initiative in cooperative group activities – teach new arrivals key locations in the school	– contribute fully in small, cooperative groups – accept and respect similarities and differences between self and peers – demonstrate pride in own heritage and language – identify and discuss characteristics of the various cultures that make up the community

Appendix M Stages of Second-Language Acquisition and Literacy Development for ELD Students

Table B1.1: ELD, Grades 1 to 3 – Oral Expression and Language Knowledge

Stage 1 Students begin to use standard Canadian English in appropriate contexts. They:	**Stage 2** Students demonstrate increasing use of standard Canadian English in appropriate contexts. They:	**Stage 3** Students demonstrate independence in using standard Canadian English in appropriate contexts. They:
– request clarification or confirmation when necessary – describe personal experiences – retell simple stories – participate in chants and/or choral speaking	– participate in classroom and group discussions – share personal experiences and opinions – retell stories with some detail – present puppet plays – use different varieties of spoken English (e.g., standard Canadian English, regional languages, interlanguages) in appropriate contexts	– contribute to classroom and group discussions – discuss and interpret stories, movies, and news events – participate in role-playing activities

Table B1.2: ELD, Grades 1 to 3 – Reading

Stage 1 Students read and comprehend simple written English. They:	**Stage 2** Students read for specific purposes when background knowledge and vocabulary are familiar. They:	**Stage 3** Students demonstrate increasing independence in a variety of reading tasks, with support as needed. They:	**Stage 4** Students demonstrate control of grade-appropriate reading tasks. They:
– understand concepts of print (e.g., progression from left to right, top to bottom) – recognize the alphabet in print – use alphabetical order – read pictures and use picture clues – recognize frequently used words (e.g., the, went, in) – begin to use phonics, context clues, and sight recognition for comprehension in pattern books, chart stories, songs, chants, and rhymes – select and read, with assistance, print material appropriate to their reading ability, interests, and age – begin to use primary dictionaries	– recognize the alphabet in script – scan for details such as letter sounds or specific words – select main ideas in short, familiar passages – begin to use reading strategies to derive meaning from text (e.g., predicting, deducing, rereading, phonics, recognizing word families) – begin to show fluency in oral reading, using some correct phrasing and rhythm – use the school library, with assistance, to select reading material for personal enjoyment and information – use primary dictionaries	– understand grade-appropriate text, with assistance – select main ideas in short passages from a variety of sources – extend their academic/technical vocabulary in curriculum subject areas – choose a variety of personal reading materials – begin to use grade-appropriate resources such as graphic organizers, class word lists, theme-book collections, environmental print, and tables of contents	– understand grade-appropriate text that may be unfamiliar and unsupported by visual context clues, and that may contain complex sentence structures – use research skills in the classroom and school library – begin to choose personal reading material

Table B1.3: ELD, Grades 1 to 3 – Writing

Stage 1 Students begin to write using basic structures. They:	**Stage 2** Students write for a variety of purposes, with support. They:	**Stage 3** Students write in a variety of contexts with increasing independence and accuracy. They:	**Stage 4** Students write for a variety of purposes, applying knowledge of the conventions of written English appropriately. They:
– print the alphabet in upper- and lower-case letters – copy words, phrases, and sentences – write personal information (e.g., name, address) – write about personal experiences or classroom discussion, using patterned sentences – participate in a variety of prewriting activities such as dramatic play, drawing, and talk – spell some personally relevant words – begin to use computers for word processing	– write invitations, thank-you notes, and personal stories – write short paragraphs based on classroom discussion – use the writing process (e.g., participate in structured prewriting activities; make some changes between the initial and the final draft) – spell most common and personally relevant words – use computers for word processing	– write about personal experiences, thoughts, feelings, stories, and information with some fluency – use a range of vocabulary and sentence structures – demonstrate an awareness that the writing process involves prewriting, drafting, and publishing – apply knowledge of the conventions of standard Canadian English in their writing, with increasing accuracy (e.g., use capital letters and periods; use conventional spelling for common and personally relevant words) – begin to write independently in all subject areas	– use a variety of writing formats – write short, original compositions using all steps of the writing process, including publication – use correct punctuation and spelling most of the time – write with some competence in all subject areas, with a clear focus, coherent organization, and varied vocabulary

Table B1.4: ESL, Grades 1 to 3 – Orientation

Stage 1 Students begin to adapt to new environments, both personal and academic. They:	**Stage 2** Students demonstrate understanding of and adaptation to new environments, both personal and academic. They:	**Stage 3** Students demonstrate increasing understanding of and involvement in new environments, both personal and academic. They:	**Stage 4** Students demonstrate awareness of self and others as part of the Canadian context. They:
– take pride in and respect their own culture – locate key school locations (e.g., washrooms) – begin to understand and follow essential school norms, schedules, routines, and emergency procedures – ask for assistance and communicate needs to appropriate school personnel and/or peers – begin to relate information about Canada (e.g., climate, holidays, safety) to their own activities and interests – work with a partner on a common academic task – begin to adapt to a variety of teaching approaches and strategies used in a Canadian classroom	– respect other cultures – participate with increasing comfort and confidence in classroom activities – demonstrate understanding of basic information about the community and about Canada – participate in directed group work such as simple research projects – respond with increasing confidence to a variety of teaching approaches and strategies (e.g., an informal classroom atmosphere, active learning, the use of games as a learning activity, activities that involve asking questions of a teacher)	– show interest in other cultures – explain school norms, routines, behaviour expectations, and emergency procedures to new students – discuss some current events – show increasing initiative in cooperative group activities such as research projects – respond appropriately to most teaching approaches and strategies (e.g., an informal classroom atmosphere, active learning, use of games as a learning activity, activities that involve asking questions of a teacher)	– identify and appreciate the contributions of various cultures within Canada – contribute fully in a small, cooperative group to create a final product or presentation – learn effectively from a variety of teaching approaches and strategies (e.g., an informal classroom atmosphere, active learning, the use of games as a learning activity, activities that involve asking questions of a teacher)

Appendix N Modifications and Expectations for a Grade 4 Measurement Strand

Grade 4, Mathematics
Measurement

This unit adaptation addresses expectations for Grade 4 from the curriculum policy documents for mathematics and language.

To motivate students and engage their interest in mathematics, teaching should highlight activities that are creative and highly relevant to students' lives. The culminating project in this unit adaptation is about classroom design – a topic of interest to students and one that builds on their experiences and prior knowledge.

ESL and ELD Descriptors

Modifications of the mathematics and language expectations should be based on the language proficiency of ESL and ELD students as indicated by the stages of proficiency and the ESL and ELD descriptors outlined in the Grade 4 to 6 tables in Part 2 of this guide. Teachers should use these descriptors to align teaching and learning strategies with students' proficiency in Canadian English. The descriptors should also be used to help determine learning expectations and assessment and evaluation strategies that are appropriate for the ESL or ELD student at each stage.

Although this unit adaptation focuses on students who are at Stages 1 and 2, students at Stages 3 and 4 will also require some adaptations or accommodations to help them learn effectively. The "Teaching Strategies" section addresses some of these adaptations.

ELD Considerations

ELD students will need considerable support from teachers and peers in this unit. Some ELD students will not have had previous experiences in school classrooms. The measurement strands in Grades 1 to 3 provide useful suggestions for program adaptation, although specific ELD modifications and rubrics should be based on the Grade 4 program. Many of the activities that other students attempt independently will need to be broken into discrete steps with accompanying oral explanations of their purpose and function. Detailed descriptors for the relevant stages and skills are provided in the Grade 4 to 6 ELD tables in Part 2.

Expectations and Modifications for ESL and ELD Students

Expectations	Modified Expectations for Stage 1 Students	Modified Expectations for Stage 2 Students
Mathematics		
Students will:		
– draw items given specific lengths	– no modification	– no modification
– select the most appropriate standard unit (millimetre, centimetre, decimetre, metre, or kilometre) to measure linear dimensions and the perimeter of regular polygons	– demonstrate an understanding of key vocabulary words (*millimetre, centimetre, decimetre, metre, kilometre, perimeter, polygon*) and their applications	– usually select the most appropriate standard unit (millimetre, centimetre, decimetre, metre, or kilometre) to measure linear dimensions and the perimeter of regular polygons
– use linear dimensions and perimeter and area measures with precision to measure length, perimeter, and area	– measure length, perimeter, and area, with support	– measure length, perimeter, and area with precision, with minimal support
– explain the difference between perimeter and area and indicate when each measure should be used	– demonstrate an understanding of the difference between perimeter and area by showing when each should be used	– explain, with teacher support, the difference between perimeter and area and indicate when each measure should be used
– select the most appropriate standard unit (e.g., millilitre, litre) to measure the capacity of containers	– usually select the most appropriate standard unit (e.g., millilitre, litre) to measure the capacity of containers	– no modification
Language – Reading		
Students will:		
– understand the vocabulary and language structures appropriate for this grade level	– understand some of the vocabulary appropriate to the measurement unit	– understand most vocabulary and some of the language structures appropriate to the measurement unit
Language – Writing		
Students will:		
– communicate ideas and information for a variety of purposes and to specific audiences	– communicate ideas and information in short, coherent, patterned sentences	– communicate ideas and information about a central idea using simple sentences
Language – Oral and Visual Communication		
Students will:		
– contribute and work constructively in groups	– work constructively in a group in a limited role	– contribute and work constructively in groups in a limited role
– present information to their peers in a focused and organized form on a topic of mutual interest	– with guidance and using visuals, present basic information to peers (e.g., about their classroom floor plan)	– with some guidance and using visuals, present key information to peers in a focused and organized form (e.g., about their classroom floor plan)