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DESIGN OF AN OPTIMAL FEEDBACK CONTROLLER FOR A SIGNALIZED TRAFFIC INTERSECTION

by

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To my parents, my wife

and my family.

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ABSTRACT

In this thesis a probabilistic model of a signalized traffic intersection is considered. The development of queues is described by a controlled Markov chain with the state of the signal as the control variable. The model can accept general arrival distributions. An optimal feedback control is obtained by Dynamic Programming. The controller is synthesized by piece-wise linear machines. Results of computer simulation based on actual traffic data of an intersection in Ottawa are given. In addition a review of existing optimization procedures for traffic control is presented.

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CHAPTER 1

INTRODUCTION

Many models have been described in the literature for the delays that vehicles experience in a signalized traffic intersection. A review will be given in Chapter 2 where the theory background and existing optimization procedures for traffic control are presented. In general we can classify the control schemes proposed in two categories, fixed-cycle and vehicle-actuated control. For fixed-cycle control explicit formulas for the traffic delay have been given. In most cases the arrival distributions were assumed to be Poisson, since for more general distributions the analytic treatment is rather complicated. For vehicle-actuated control of an isolated intersection, delay expressions have been obtained for some specialized controls and arrival distributions.

One of the first contributions in this area was made by Dunne and Potts [1] who proposed a heuristic control algorithm for a traffic-actuated intersection. Grafton and Newell [2] considered the transient behaviour of a traffic-actuated light and posed the problem of optimizing the operation of the traffic light by minimizing an integral performance criterion. Both the above papers however deal with deterministic traffic only. A general approach for a good sub-optimal control of a complex intersection, based on principles of stochastic control, was given by van Zijverden and Kwakernaak [8]. Martin-Löff [5] proposed a very interesting model for the computation of an optimal feedback control. This model has the major advantage of producing optimal controls, with respect to some criterion, for general arrival distributions.

In Chapter 3 of this thesis, the model of Martin-Löff is considered. By modifying appro pri ately the state transition probabilities, the optimal control minimizing the expected delay over a long time period is computed. The method used is Dynamic Programming. The actual traffic data for an intersection in Ottawa were used. The optimal control at a given time is obtained as a function of the previous control and queue lengths in the two directions of the intersections. The optimal feedback controller is then synthesized using piece-wise linear discriminant functions. Results of computer simulation of the optimally controlled intersection are finally given in order to study the signal-cycle lengths produced by this method.

In the conclusion of the thesis, the results of the optimal control of the actual traffic data for an intersection in Ottawa are discussed.

CHAPTER 2

THEORY BACKGROUND AND EXISTING OPTIMIZATION PROCEDURES FOR TRAFFIC CONTROL

2.1 INTRODUCTION.

The general objectives of traffic control are to improve safety and to increase the comfort and convenience of drivers. This general statement of noble intention must first be reduced to quantifiable objectives. Ideally one would like to set up the traffic control problem as a system optimization problem, by defining an objective function and seeking an optimum (maximum or minimum) for this function. This function will generally depend on certain controllable parameters of the system; for example, the light cycle, the split (allocation of green time to different traffic streams), offset (time phase of cycles) of different lights, and so on.

Many criteria of performance have been suggested in traffic control, among which are the following :

1. Average delay per vehicle,
2. Maximum individual delay,
3. Percentages of cars that are stopped as they go through the system,
4. Average numbers of stops before a vehicle goes through the system (or through an intersection)
5. Throughput of the system (or individual intersections)
6. Travel time through the system (average or maximum).

Criterion 1 is the most widely used, and it is not always compatible with criterion 2. It is sensible to use criterion 1 as a primary objective function with 2 used by way of a constraint on the maximum individual delay. This is not only for humanitarian reasons, but also for a very pra$_{c}$tical one : Any design that penalizes a few drivers very much may induce changes in their driving habits, which in turn may destroy the design. Criteria 3 and 4 are dictated by the known aversion of drivers to stops. Stops are both a nuisance and an expense because of excessive gasoline consumption and wear and tear on cars. In addition, in the past few years we have become increasingly aware that stops tend to increase car-induced air pollution. Criterion 5 is particularly important during period of peak traffic, when it is important to maximize the utilization of the traffic system. Finally, criterion 6 is very much related to criterion 1, and in fact it is frequently used in lieu of that criterion.

Sometimes an objective function is not explicitly related to the preceding criteria. An example is the through-band (or green band) design of synchronization of traffic lights. In that design, one tries to align the green phases of a sequence of traffic lights with such time offsets as to allow uninterrupted flow through as many of them as possible to as many drivers as possible. The through-band design more or less maximizes the opportunity of driving through a string of traffic lights without stopping. Indirectly, it tends to reduce stops and delays sometimes.

Among the traffic systems that have been investigated for the purpose of optimizing their operation are the following :

1. An isolated signalized intersection,
2. A system of a few intersections (2 to 5),
3. An urban street network controlled by traffic lights,
4. Critical traffic links such as tunnels, brigdes, and sections of freeways.

For all these systems, one would like to have a methodology for optimizing their operation with respect to a given objective function, in response to observations concerning traffic inputs. A general theory of optimization of traffic systems, particularly with realistic stochastic traffic inputs, has been rather elusive. Exact optimization has been obtained only for a few simple systems with special traffic inputs. For most traffic systems we have relied heavily on heuristic control schemes coupled with suboptimization schemes for idealized versions of the real systems, or portions thereof. Let us briefly outline the general priciples of operation which have been suggested for the four types of optimizing problems encountered in each case.

The single intersection operating below saturation levels is discussed in a paper by McNeil and Weiss [3 , chapter 2]. In that chapter it is shown how one can select a fixed cycle that minimizes delay when the traffic input is a stationary stochastic process. In the same book [3 , chapter 3] Gazis discusses the case of an oversaturated intersection. What distinguishes the oversaturated from the undersaturated case is the time dependence of traffic inputs plus the fact that the instantaneous information concerning traffic inputs is not sufficient for optimization. Residual long queues propagate the delay from one light cycle to another. Thus the optimization of the operation of the traffic light of an oversaturated intersection entails minimizing the delay to all the traffic which crosses the intersection during the rush period; that is, the period of oversaturation. A similar approach is discussed in [3, chapter3] for systems of oversaturated intersections, and a few other special oversaturated systems.

The methodology of control of urban street networks has borrowed heavily from the traditional methodologies of progression and traffic actuation. Thus, a typical modern control system works on the following principle : one tries to synchronize as many traffic lights as possible in order to provide the opportunity of uninterrupted flow to as many cars as possible. The synchronization scheme is selected for average traffic conditions, and changes relatively infrequently, sometimes no more than three or four times during the day. In any case, the time scale of switching from one synchronization scheme to another is of the order of minutes or even hours. This type of control has been referred to as macro-control (or major loop control). The problem associated with macro-control is to find a good synchronization scheme for given traffic conditions.

Over and above macrocontrol, we frequently need to pay special attention to critical intersections. This is done usually at time increments of the order of one second, and has been referred to as microcontrol (or minor loop control). The problem associated with microcontrol is to find an algorithm for adjusting the traffic lights in response to variations of traffic demand, in order to minimize the delay in the neighborhood of the critical intersection. Both macrocontrol and microcontrol coexist in most modern, computerized traffic control systems. However, most systems in operation today depend largely on macrocontrol schemes. The degree of utilization of microcontrol schemes is limited by our lack of experience concerning the inner workings of traffic. Since microcontrol is expensive in instrumentation requirements, it is unwise to invest too much in it without reliable information about its efficacy.

The fourth class of traffic systems usually involves the need of maintaining free flow over critical traffic links, such as tunnels, bridges, and freeways, by regulating the traffic input into these critical links. Much of the methodology in this area has been developed empirically. Some models of input control have been cast in the framework of mathematical programming and control theory.

2.2 ANALYTICAL METHODS.

During the past years much effort has been concentrated on developing procedures for applying a more appropriate objective function to the problem of offset optimization. A great deal has been accomplished in the way of more realistic analysis of traffic flow, however only a very few number of schemes are pratical enough or have been developed to the point where they may be applied on a city wide basis.

Vehicular delay has come to be the most generally accepted index of performance for signal timing schemes. Delay at an isolated signal is defined as follows : First, consider a point on the approach to an intersection so far from the signal, that traffic here has not yet begun to slow down when the light is red. Now think of a second point, sufficiently far downstream of the signal, that vehicles which were required to stop have ceased to accelerate. Delay to a particular vehicle is defined as the difference between the time it actually takes to travel between these two points and the time it would take if the approach always had the right of way. Total network delay is simply the sum of the delay experienced by all the vehicles in the system under specified flow conditions.

In 1967, Newell[4] reported on a rigorously theoretical analysis of delay in a very simplified network. Newell 's model assumed a one way arterial street crossed at regular intervals by other one way streets. Traffic flow approaching that first signal on the arterial is assumed steady, as are the flows on all the cross streets. The signals are considered to be perfect on-off switches and traffic becomes equivalent to a fluid moving along the artery with constant velocity. No turns are allowed in the network. Newell shows that even in the case of such a simplified model that maximum through bands do not result in minimum delay. In fact it is demonstrated that minimum delay is coincident with maximum through bands only if equal green splits are employed at each intersection. Newell goes on to outline a procedure he has developed for determining offsets which minimize delay, however his method is of little practical value due to obvious over simplification of the model upon which he bases his analysis.

Several years earlier in 1963, Newell [6] had studied the operation of signalized arterials under heavy flow conditions. He assumed a certain degree of platoon dispersion and performed a graphical analysis of flow between two intersections. Newell demonstrated that delay is minimized when adjacent signals are phased so that the last car in a platoon just clears the intersection before the signal turns red. This usually implies that the first few vehicles in the platoon must be stopped. Newell seems to have abandoned this approach as he could not support his conclusions with a rigorous analytical model. This is particularly unfortunate, because, as will be seen later, it was just at this time that empirical work was under way which would form the basis for surprisingly reliable mathematical platoon dispersion models.

Since Newell 's initial work, a great deal of effort has been concentrated on developing an analytical expression for delay at a signalized intersection. The more realistic of these expressions have been critically reviewed in a recent article by Allsop [7]. It is usual to divide the signal cycle into short intervals of time, and the number of vehicles arriving during each one of these intervals is assumed to be a random variable distributed according to a Poisson or Binomial distribution (or more complex derivative of either of these). Vehicles are generally assumed to depart at a constant rate provided a queue is present, however some more complex models have been derived. By combining these vehicle arrival and departure models, elaborate analytical expressions for delay at a signalized intersection have been developed. However, as Allsop [7, page 267] says, the mathematical fascination of the more complex variations of these models has '' caused them to be pursued to a level of intricacy far greater than is likely to find practical application ''.

Probably the most useful and widely accepted of these analytical expressions is that developed by Webster [9]. Assuming Poisson arrivals and a constant rate departure model, Webster simulated the traffic flow through an isolated intersection on a computer. The result was an empirical expression for delay which agrees very closely with actual delays observed at a number of sites. In fact, Webster 's formulae are so well established that Hutchinson [10] has used them as a comparis on standard in a numerical evaluation of several delay expressions.

Another delay formula worthy of mention is that developed by Newell [11] who used a constant service departure model but assumed that arrival headways were distributed identically and independently, and subject to a minimum of 1/s (s = saturation flow rate). Newell 's model compares favourably with Webster 's.

It may be concluded, that mathematical models have been developed to predict vehicular delay at signalized intersections which are realistic, and yet sufficiently simple that they may be applied to real life situations. However, it must be remembered that all of these models represent traffic flow at isolated intersections with random vehicle arrivals. Clearly, this doesnot represent the case of a signal located within a network of signalized intersections, each having an arrival pattern which is not random but is profoundly influenced by other signals in the network. However, as will be seen later, much of what has been learned in the analysis of isolated signals may be applied in the study of flow within a network of signals.

One of the first analytical attempts to relate delay experienced at a particular signal to the arrival of a platoon of vehicles release from an upsteam intersection was done by Allsop [12]. His model assumes that the vehicle arrival rate is constant throughout the platoon. Traffic leaves the intersection at the saturation flow rate, as long as there is a queue, and passes undelayed when there is no queue. Delay is defined as the integral of queue length over time. Allsop derives relations for delay in terms of platoon duration, arrival time of lead vehicle, arrival rate throughout the platoon, saturation flow rate, cycle length and green time split. A sensitivity analysis showed that small variations in the arrival rate from one platoon to the next leads to increased average delay per cycle. Also it was found that small variations in platoon duration, given a fixed arrival time for the lead car, may or may not lead to increased or decreased average delay per cycle. Although Allsop's work represents a significant advance in the analytical approach to traffic flow research, there are serious limitations which prevent it from being put to pratical use. First, is his assumption that the input to a traffic signal is essentially a square wave. In actual fact traffic flow varies all the way from a pure square wave, to pratically random input. Secondly, Allsop assumes that the arrival time of the initial vehicle is known and he provides no means of relating this to the relative phasing of the upstream signal. Finally, Allsop's equations are somewhat cumbersome to apply, in that there are a number of different possible situations for which different formulae must be used.

2.3 SIMULATION METHODS.

It became apparent that the analytical approach to the problem of synchronizing signals to minimize delay, didnot appear promising to those responsible for the control of actual traffic networks. As a result many traffic researchers turned to the digital computer and employed simulation techniques in order that some of the restrictions of the simplistic analytical models might be relaxed. Some of the first works in this area was done by Saleeb and Hartley [13] who developed a computer model to simulate traffic through a linked-pair of signals. The most important feature of their model was that it allowed for platoon dispersion between the signals.

In 1965 Pacey [14] had studied platoon diffusion and proposed a theory which was later refined by Grace and Potts [15]. Basically it is assumed that the speeds of vehicles released from an intersection are normally distributed, and that the speed of any car is unchanged as it progresses down the link. This model has been shown to adequately reproduce the dispersion of a platoon travelling in medium traffic with little interference. Recently, Robertson [16] has applied a smoothing factor, which is a function of journey time, to improve the fit between platoons calculated by Pacey 's model and observed platoons. However, a comparative analysis by Seddon [17] has shown that the improvement produced by this refinement is marginal, and that either Pacey 's or Robertson 's method may be considered adequate representations of platoon dispersion.

It appears that Hartley has used Pacey 's model in his simulation. Traffic is assumed to arrive randomly at the upstream intersection according to a Poisson distribution. When the signal is green, vehicles are released at a rate up to the simulation flow rate. Hartley runs his simulation model for an hour of real time before it is assumed that steady state has been reached. More recent work seems to indicate that this is unnecessary. The simulation is run several times while the relative phasing between the two intersections is adjusted. The result is a plot of delay versus offset which yields the optimum phasing for the link. In order to assess the effect of considering platoon diffusion, simulations were performed in which diffusion was ignored and it was demonstrated that the resulting error in delay is considerable, in fact approximately 40%, in the area of the optimum offset. Hartley also observed that optimum offset does not seem to be significantly sensitive to traffic volume. This conclusion was also arrived at by many traffic researchers at this time. Hillier and Rothery [18], at the British Road Research Laboratory, performed a series of empirical test which also demonstrated the stability of optimum offset. Hartley also investigated the response of his simulation model to transient traffic conditions. He experimented with step input, impulse input and wave input, which is an accurate representation of rush hour traffic demand. The results of these transient state simulations are very interesting. Unfortunately it appears that little further research has been done in this area. Recently, Hartley [19] has reported on further experiments with his simulation model. Using first Pacey 's dispersion model and later Robertson 's, Hartley calculated a travel time probability distribution for the link being simulated. Employing a Monte Carlo type simulation technique, he produced delay offset curves from each of these distributions and found that they were pratically identical. Next, he tried a rectangular travel time distribution and produced another delay curve which was very similar to the first two. Hartley thus concluded that the delay-offset relation for any traffic link is not sensitive to the shape of the assumed travel time distribution. This is a very significant discovery, because in terms of the computer hardware required, it is much more economical to generate events from a square distribution than from any other probability curve. Unfortunately, Hartley does not appear to have ~~done~~ any field studies to validate his simulations.

One of the first simulation models actually put to practical use, is the one which was used in determining optimal fixed time offsets for the San Jose computer controlled traffic system. The model employed is described by A. Chang [20] and also by Gazis and Bermant [21] all from IBM. It is assumed that vehicles travelling from one intersection to the next all travel at the same speed. When joining a queue of stopped cars, vehicles are assumed to decelerate instantly to zero speed. Unlike other models described to this point the simulation takes account of queue length. Cars leave intersections at a constant rate and accelerate in zero time to the speed at which all vehicles on the link are assumed to be travelling. Again, delay is defined as the integral of queue length over time. An iterative search procedure involving repetitive simulations, while signals parameters are varied, determines the values of offsets and green splits which minimize delay. Although the traffic model employed in the San Jose simulation is obviously over simplified, Chang [20] claims that signal settings derived by this method have yielded a 10% reduction in total network delay over settings obtained by conventional means.

A few very complex traffic simulation models which accurately reproduce every detail of traffic flow have been developed, although primarily as research tools. One such model described by Klinjhout [22] simulated the passage of vehicles through a fully actuated signalized intersection with muti-lane approaches and two loop detectors in each lane. The approach and departure of every vehicle is described by calculations based on information generated from vehicle acceleration and deceleration distributions. Movement of each vehicle through a queue is modelled in all its intricate detail. Even driver reaction to the amber signal is determined by sampling from a probability distribution, however, it was shown that this latter refinement had negligible influence on the end result.

Another simulation model of equal sophistication has been developed to describe traffic flow on a grid consisting of four intersections by R. Sagen [23]. Vehicles are generated at the fringes of the network randomly from an exponential distribution at an average representative of the hour of the day being simulated. Traffic conditions on the link are evaluated before the car is allowed to enter and it may be delayed until congestion is clear. Traffic flow on links between the intersections is described by a complex car following theory, which assumes that drivers attempt to travel at a desired speed, which is ofcourse generated from an appropriate distribution. The influence of other vehicles in the traffic stream is accounted for by calculations involving desired spacing and minimum stopping distance as a function of speed. Vehicles are assumed to react to a red signal as soon as the stopping distance reach up to or beyond the stop line, from then on the car following theory takes care of the queuing process. Left turning vehicles are forced to stop and search for a gap in opposing traffic, and then decide, according to a gap acceptance distribution, if they have enough space to make their manoeuver. By means of a digital to analog converter, Sagen has arranged that the instantaneous traffic situation on the grid can be displayed on an oscilloscope. By having the oscilloscope repeat the display at short intervals of time, and coupling it with a camera and projector, Sagen has been able to produce on-line movies of his simulation.

Detailed simulation models, such as the two described above, are excellent tools for theoretical traffic researchers to use in experimenting with new traffic control strategy, such as the logic details for advanced traffic responsive equipment. However they do not represent pratical means for determining optimal timing parameters for existing fixed time signal networks. The task of setting up such a detailed model to represent an entire city network and collecting the data to calibrate this model would be awesome. Also, Klijnhout recommends that his model of a signalized intersection be run for two hours of real time, before steady state is reached.

2.4 AVAILABLE SIGNAL OPTIMIZATION PACKAGES.

One of the currently available computerized signal optimization packages is called the Combination Method. It was developed by the Greater London Council in the mid sixties, and was one of the methods tested in a five years experiment performed at Glasgow, to evaluate several different strategies of area traffic control. Basically the method is as follows. A crude simulation calculates the delay-offset relation for each link in the system. It assumes that the delay on any link is a function only of the offset between the signals at either end of that link. Once the delay-offsetrelation for each link has been determined, a routine is set into motion which reduces all of these delay-offset tables to a single delay-offset relation for the whole network. This is done by systematically combining the offset relations for adjacent links. After the final relation is determined, the offset producing minimum delay for this hypothetical link is selected, and the calculation sequence is retraced to determine the optimum offset for each link in the system. For those readers who are interested, the method is described in more detail by Hillier [24] .

The simulation used to determine the delay-offset relation for an individual link in the Glasgow experiment was somewhat over simplified. As Holroy [25] points out, it was assumed that traffic was discharged from the upstream signal at a constant rate during the green period, and no account is taken of platoon dispersion. However, since that time Robertson 's platoon diffusion model has been incorporated into the model, and it is now assumed that the discharge rate at the upstream signal is heavier during the beginning of the green. Nevertheless the simulation takes no account of turning vehicles nor the degree of platoon dispersion on the upstream link, both of which will influence the arrival pattern at the down-stream signal. The Combination Method is available as a computer package, or as a consulting service for urban area not having computer facilities, and, according to Huddart [26], it has been applied sucessfully in cities in the U.K., Australia and the U.S..

Another important method of optimizing signal settings, which was tested during Glasgow experiment, is the TRANSYT routine. developed by Robertson [27] while he was employed by Plessey Electronics. Rather than minimum total delay, the objective function of the TRANSYT routine is minimum average journey time . This objective function is sought by minimizing the total average number of stopped vehicles in the network at one time. According to Robertson on a purely economic basis this goal seems to be more desirable[28].

In priciple, TRANSYT is similar to the Combination Method, in that it consists of a traffic model and an optimization routine, however, there are significant differences beyond this point. In order to make the amount of computer time required realistic, a deterministic traffic model is used so that average traffic is represented. The main advantage that the TRANSYT model has over the Combination Method is that it is not restricted by the assumption that the delay on a particular link is a function of the offset on that link only, but also depends on the offset and traffic conditions on the upstream link. This is accomplished by simulating traffic on the whole network at once. In effect the passage of individual vehicles is traced through the network while the prohibitive computational detail of Sagen 's model is avoided. This continuous type of network modelling means that vehicle turns can be accounted for, and also that the shape of the discharge profile at the upstream intersection in any particular link does not have to be assumed. Platoon dispersion is accounted for by Robertson 's smoothing factor method. Figure 1* [29] is a comparison of observed platoons and those calculated by Robertson 's method. The F in the diagrams is the smoothing factor and is a function of travel time. The routine output histograms representing arrival and departure profiles at each stopbar; a sample of these is shown in Figure 2* [30]. The heavy dark arrows indicate turning movements.

Since the model is deterministic the arrival platoons represent average traffic behaviour. In reality the actual number of stops that would be observed would be greater than predicted. This is due to the random component of the traffic arrivals, which would occasionally cause the formation of overflow queues, even during times when the intersection was under saturated. Based on a platoon dispersion study, Robertson has derived an analytical relationship between random delay and degree of saturation. He used this relation in TRANSYT. Signal parameters are adjusted by using an iterative

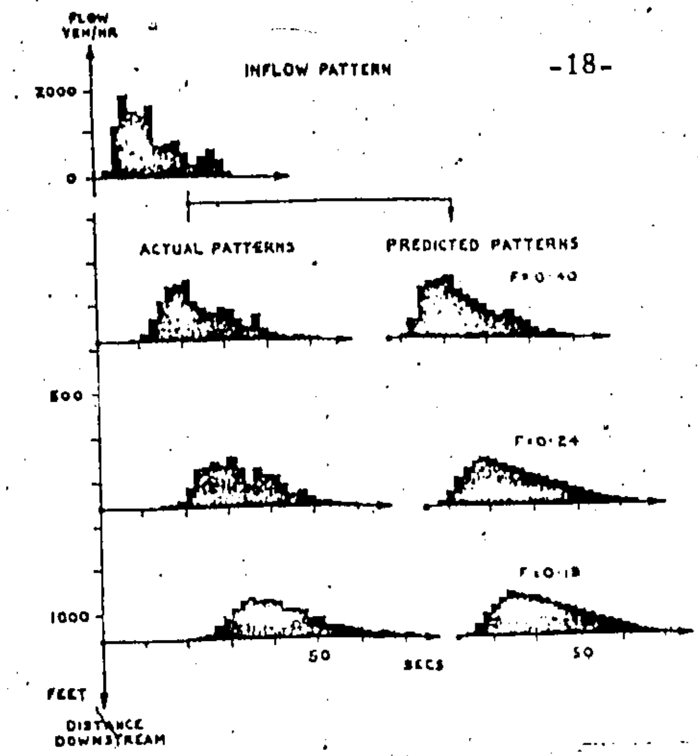


Figure 1* Platoon dispersion patterns (after Robertson, 1969)

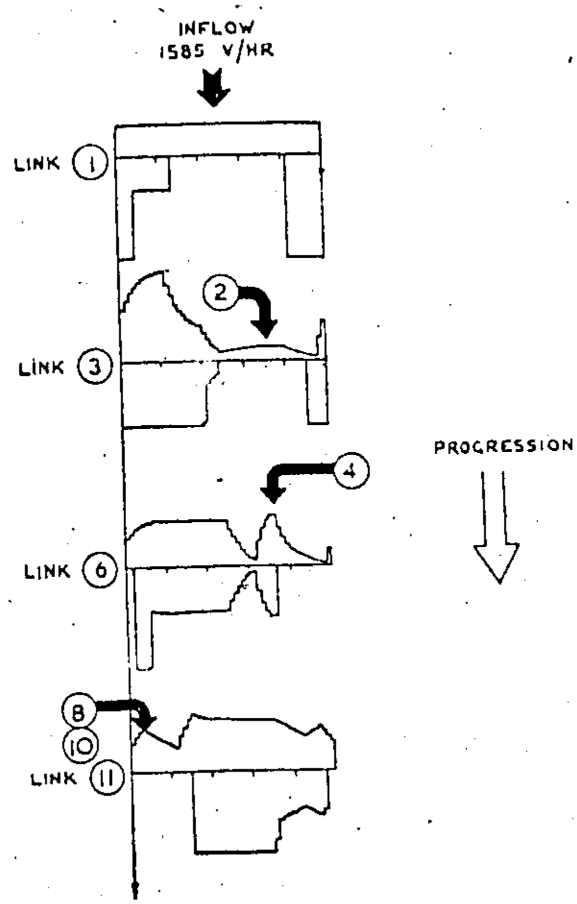


Figure 2* Intersection arrival and discharge patterns predicted by Transyt (after Robertson, 1969)

process called hill climbing until network simulation results in the minimum value of the performance index.

In 1966 the British Road Research Laboratory conducted an experiment on Cromwell Road in London. Network performance under first,manual signal settings obtained by conventional traffic engineering methods, and later, signal settings obtained using TRANSYT, were compared. TRANSYT reduced the total vehicle hours spent in the system per hour from 177.4 to 150.0 (12.9%) even though there had been an increase in flow of 5%. TRANSYT had predicted that vehicle hours in the system would fall from 176.7 to 150.8.

In the Glasgow experiment [31], the TRANSYT method was shown to reduce average journey time by 4% compared with settings produced by the Combination Method. This is not surprising due to more realistic traffic model which TRANSYT used.

According to Robertson [32], optimum settings, including green split and cycle time, for a network of 50 signals can be obtained in 12 minutes, however, if offsets only are required the computational time may be reduced by 30 to 40 per cent. The first non-experimental application of TRANSYT was in the City of Berne in Switzerland, many others are planned.

2.5 EMPIRICAL METHODS.

To this point in the discussion, traffic arrival profiles, upon which calculations of the delay offset relation are based, have been produced by mathematical models. Some traffic researcher have shown interest in deriving delay-offset relations from vehicle arrival profiles observed in the field. Hillier and Rothery [18] were among the first to derive delay offset relations from arrival profiles which were actually observed and recorded in the field. These arrival profiles were measured at four locations downstream of a signal light in London, England. From these arrival histograms, delay offset relations were generated by what was essentially a graphical technique. For a particular offset, cumulative demand and service functions were plotted. The area between the two functions represents vehicular delay. Shifting these period functions relative to each other is equivalent to changing the offset between the two signals. Thus by shifting the curves and noting the resulting area between them the delay offset relation was generated.

After extensive data collection and reduction, Hillier and Rothery arrived at the following conclusions.

(1) Optimal offset is not significantly affected by traffic volume

(2) Delay is approximately minimized by using a progression speed equal to the mean speed.

(3) Random delay increases as the degree of saturation approaches 100%.

Their latter conclusion is somewhat obvious as the probability of an overflow queue increases greatly as the signal becomes saturated.

One of the other few traffic researchers to deal directly with observed signal profiles, in the study of optimum offset between two adjacent signals, was Wormleighton [33], [34] of the University of Toronto. Wormleighton has equated the downstream traffic signal to a bulk service queue with non-homogeneous Poisson input. Using queuing theory, he was thus able to derive expressions for mean queue length and mean delay. Information collected from induction loop vehicle detectors, by the Toronto traffic control computer, allowed Wormleighton to construct histograms representing average arrival profiles at locations he wished to study. These average arrival profiles were used as an estimate of the varying Poisson intensity function of the queing model. After investigating delay offset relations produced by this method, from observations made at only two intersections, Wormleighton made the following conclusions.

(1) The travel time of the leading edge of the platoon does not depend on volume.

(2) Optimal offset increases with volume by about 1 second for each 100 vehicles/hour.

(3) Offsets within 5 seconds of the best one give mean delays within one second of the minimum.

(4) Errors in estimates of saturation flow rates and expected overflow queue will result in error in delay estimates but will not effect the location of the optimum offset.

(5) The offset which minimizes the number of stops is not necessarily the one which minimizes delay.

Recently, Millar [35] proposed a signal optimization package entitled SIGRID. SIGRID is an iterative routine which seeks to minimize the following network utility function.

$$Y = \sum_{i=1}^{n} V.IF.(D-A)^2$$

where

n : total number of links in the network

V : the volume of the link i

IF : the importancefactor of the link i. This is meant to represent the relative weighting that should be given to link i when determining the optimal compromise of offsets between link i and its neighbors.

D : the offset desired on link i

A : the actual offset which SIGRID will assign to link i

The desired offset, D, is an input to SIGRID, and must be determined for each link i, in order to maximize some objective function on that link. Millar has chosen minimum delay as a suitable objective function. The actual traffic data to the City of Ottawa 's 238 signalized intersections were studied under traffic conditions representative of morning, evening, and off peak hours and the SIGRID method was applied.

This SIGRID package is essentially identical to SIGOP [36]. It seems appropriate to point out at this time that SIGOP appeared equally as effective as TRANSYT in the Glasgow experiment.

What has been said to this point, by no means represents a complete survey of research in the area of determining optimal settings for fixed time traffic signals. However, the author feels that it does constitute a representative cross section of work that has been done, and that it is essential to understand the relevance and significance of studies which he has undertaken.

2.5 SUMMARY OF APPROACHES TO TRAFFIC SIGNAL RESEARCH.

After reviewing the various books, projects, studies, and reports, which have been mentioned above, it is possible to make the following general comments. In order that they may be solved, analytical models of traffic flow, which have been developed to date, are so oversimplified and unrealistic that they do not at all relate to actual traffic behaviour. As an alternative, traffic researchers have constructed traffic simulation models. Some of these models are so detailed that they reproduce every intricacy of traffic flow. However there is little hope that they may be applied to entire city networks. Nevertheless, these models should not be regarded merely as toys of the theoretician, as they constitute valuable research tools, from which much can be learned about the intricacies of traffic flow and its response to different control strategies.

Although the inadequacies of the time space method of co-ordinating fixed time traffic signals are widely acknowledged, very few of the alternatives, which have been suggested, exist in a form which would allow them to be applied on a city wide basis. Probably the best currently available routine for signal network optimization is TRANSYT, by virtue of its simplied yet apparently accurate traffic simulation model. Some may argue that it is closely rivaled by a routine called SIGOP [36].

The empirical approach to signal optimization is attractive as it eliminates the uncertainty associated with mathematical simulation. Also, a refined empirical optimization routine could yield substantial savings in terms of professional time and computing costs. Both the work done by Wormleighton [33] and Hillier and Rothery [37], demonstrated that representative and stable arrival profiles can be collected in the field. It appears that an effort should be made to develop a routine, whereby signal settings would be optimized according to traffic behaviour observed in the field rather than simulated on a computer.

CHAPTER 3

A VEHICLE-ACTUATED OPTIMAL FEEDBACK CONTROLLER FOR A SIGNALIZED TRAFFIC INTERSECTION.

3.1 INTRODUCTION.

As stated before, in general the control schemes can be classified in two categories, fixed-cycle and vehicle-actuated control. In this chapter, the latter one is studied by using the model of Martin-Löff [5]. Modifying appropriately the state transition probabilities, the optimal control minimizing the expected delay over a long time period is computed. The method used is Dynamic Programming. The actual traffic data in the morning, evening and off peak hours of an intersection in Ottawa were used. The optimal control at a given time is obtained as a function of the previous control and queue lengths in the two directions of the intersection. The optimal feedback controller is then synthesized using piecewise linear discriminant functions. Results of computer simulation of the optimally controlled intersection are finally given in order to study the signal cycle lengths produced by this method. The results of this chapter were reported in [38].

3.2 THE MODEL [38]

We consider a traffic intersection of two directions controlled by a two-phase signal. The yellow phase is incorporated in the effective loss time, which is chosen as time unit for the model. Considering thus the discrete time $i = 1, 2, 3, \ldots$, and assuming that the signal state is constant in every interval $(i, i+1)$, we define the state variables :

$x_1(i)$ = queue length in direction 1 at time i

$x_2(i)$ = queue length in direction 2 at time i

and the control function (state of the signal) :

$$r(i) = \begin{cases} 1 \text{ if the signal is red in direction 1 in } (i-1,\ i) \\ 2 \text{ if the signal is red in direction 2 in } (i-1,\ i) \end{cases}$$

The system evolution is described by the following sets of transition probabilities :

$$\begin{aligned} P_k(n;m) &= P[x_k(i+1)=m \mid x_k(i) = n] \text{ when direction k is } \underline{\text{closed}} \\ Q_k(n,m) &= P[x_k(i+1)=m \mid x_k(i) = n] \text{ when direction k is } \underline{\text{open}} \end{aligned} \qquad (1)$$

(k = 1,2)

P_k are determined by the arrival distributions and Q_k by the arrival and departure distributions.

A direction is said to be <u>open</u> in the interval (i,i+1) if and only if the signal shows red in the other direction in both interval (i-1,i) and (i,i+1). This definition incorporates into the model the time losses.

We can consider now the system as a Markov Chain in the space of points (x_1 , x_2 , r) with transition probabilities depending on the binary control variable r(i). The one-step transition probabilities from time i to i+1 are :

$$\left.\begin{aligned} P_{(1)}(n_1,n_2,1;m_1,m_2,r) &= P_1(n_1,m_1)Q_2(n_2,m_2)\delta_{r,1} \\ P_{(1)}(n_1,n_2,2;m_1,m_2,r) &= P_1(n_1,m_1)P_2(n_2,m_2)\delta_{r,1} \end{aligned}\right\} \ \underline{\text{if } r(i+1) = 1}$$

$$(2)$$

$$\left.\begin{aligned} P_{(2)}(n_1,n_2,1;m_1,m_2,r) &= P_1(n_1,m_1)P_2(n_2,m_2)\delta_{r,2} \\ P_{(2)}(n_1,n_2,2;m_1,m_2,r) &= Q_1(n_1,m_1)P_2(n_2,m_2)\delta_{r,2} \end{aligned}\right\} \ \underline{\text{if } r(i+1) = 2}$$

where

$$\delta_{i,j} = 1 \text{ if } i = j \text{ and } 0 \text{ if } i \neq j$$

The model is thus a controlled Markov Chain in the 3-dimensional space (x_1, x_2, r).

3.3 OPTIMIZATION BY DYNAMIC PROGRAMMING.

Let $C(x_1, x_2)$ be the cost per unit time of having queues x_1, x_2 in directions 1 and 2, respectively. The optimal control function $r(.)$ will be determined in such a way that the following conditional expected cost is minimized :

$$J = E\left\{\sum_{s=t}^{T} C(x_1(s), x_2(s)) \mid x_1(t)=n_1, x_2(t)=n_2, r(t)=r\right\} \quad (3)$$

where T is the ''horizon'' of optimization and t is the present time. n_1 and n_2 are the present queue sizes, varying from 0 to N.

Let the minimum conditional expected cost be :

$$v(n_1, n_2, r, t) = \min_{\substack{r(.)\\ [t,T]}} E\left\{\sum_{s=t}^{T} C(x_1(s), x_2(s)) \mid x_1(t)=n_1, x_2(t)=n_2, r(t)=r\right\} \quad (4)$$

We can determine $v(n_1, n_2, r, t)$ by Dynamic Programming as follows :

$$v(n_1, n_2, 1, t-1) = \min_{\substack{r(.)\\ [t-1,T]}} E\left\{\sum_{s=t-1}^{T} C(x_1(s), x_2(s)) \mid x_1(t-1)=n_1, x_2(t-1)=n_2, r(t-1)=1\right\} =$$

$$= C(n_1, n_2) + \min_{\substack{r(.)\\ [t-1,T]}} E\left\{\sum_{s=t}^{T} C(x_1(s), x_2(s)) \mid x_1(t-1)=n_1, x_2(t-1)=n_2, r(t-1)=1\right\} =$$

$$= C(n_1, n_2) + \min_{\substack{r(.)\\ [t-1,t]}} \sum_{m_1=0}^{N} \sum_{m_2=0}^{N} \sum_{i=1}^{2} v(m_1, m_2, i, t) P_{(r)}(n_1, n_2, 1; m_1, m_2, i)$$

$$= C(n_1,n_2) + \min \Big\{ \sum_{m_1=0}^{N} \sum_{m_2=0}^{N} v(m_1,m_2,1,t)P_1(n_1,m_1)Q_2(n_2,m_2), \qquad (5)$$
$$\sum_{m_1=0}^{N} \sum_{m_2=0}^{N} v(m_1,m_2,2,t)P_1(n_1,m_1)P_2(n_2,m_2) \Big\}.$$

Similarly ,

$$v(n_1,n_2,2,t-1)= C(n_1,n_2)+\min \Big\{ \sum_{m_1=0}^{N} \sum_{m_2=0}^{N} v(m_1,m_2,1,t)P_1(n_1,m_1)P_2(n_2,m_2), \qquad (6)$$
$$\sum_{m_1=0}^{N} \sum_{m_2=0}^{N} v(m_1,m_2,2,t)Q_1(n_1,m_1)P_2(n_2,m_2) \Big\}$$

From (4) it is obvious that :

$$v(n_1,n_2,r,T) = 0 \qquad \text{for all } n_1 , n_2 , r \qquad (7)$$

The optimal control can now be obtained by solving recursively the equations (5) and (6) with terminal condition (7). The control is obtained as a function of the previous control and previous queue sizes , i.e. ,

$$r(t) = f(x_1(t-1) , x_2(t-1) , r(t-1) , t) \qquad (8)$$

If the traffic data are such that the transition probabilities (1) do not depend on time and the horizon T is very large, then there exists a steady state control minimizing the cost (3) , i.e. ,

$$r(t) = g(x_1(t-1) , x_2(t-1) , r(t-1)) \qquad (9)$$

In this thesis we concentrate on the synthesis aspects of the optimal feedback controller given by (9) and use of it in computer simulation of the overall system. We take $C(n_1,n_2) = n_1 + n_2$, so that v is proportional to the delay .

3.4 EXAMPLE : AN INTERSECTION IN OTTAWA.

We considered the intersection of Bank and Sunnyside Streets in Ottawa South.

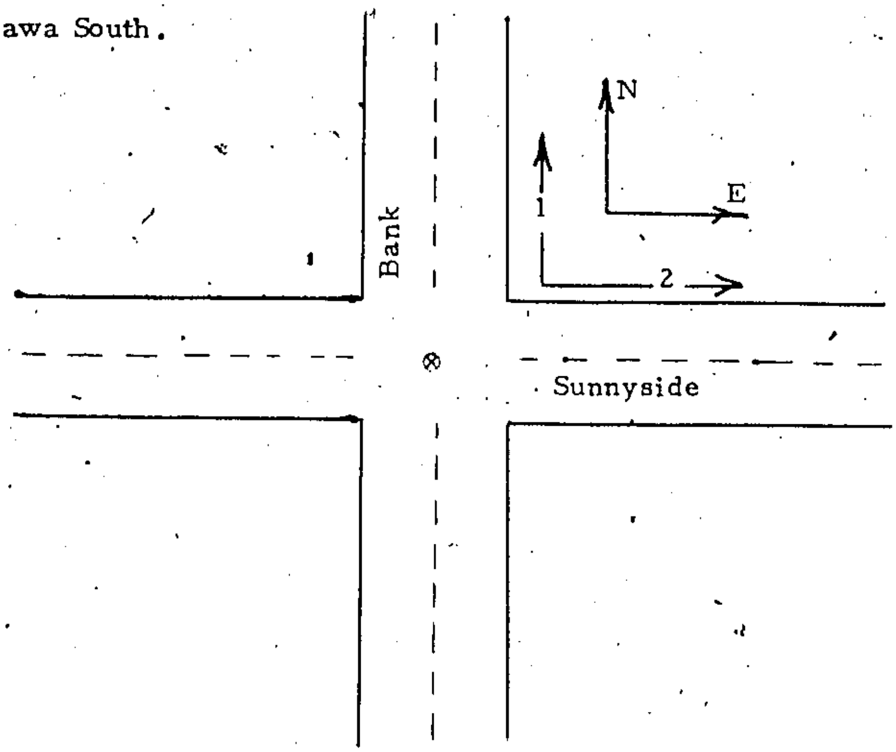


Figure 1

Arrival data were obtained from the Traffic Systems Control section of the Regional Government of Ottawa-Carleton.

Northbound and Southbound arrivals on Bank street and Eastbound, Westbound arrivals on Sunnyside were measured every 5 seconds during the morning and afternoon peak hours as also an off-peak hour. From these data and considering as unit of time 5 seconds we obtained the arrival probability distributions shown in Table 1. Although the model can function with general empirical arrival distributions we decided to approximate these distributions as shown in Table 2.

ARRIVAL PROBABILITY DISTRIBUTIONS						
No. of Arrivals (per unit time)	BANK			SUNNYSIDE		
	Morning Peak	Off Peak	Afternoon Peak	Morning Peak	Off Peak	Afternoon Peak
0	.32222	.40317	.30952	.76666	.87857	.70952
1	.36190	.37936	.35238	.21904	.11904	.23333
2	.20476	.20000	.25097	.00952	.00238	.05714
3	.08888	.01587	.06825	.00476	0	0
4	.01746	.00158	.01746	0	0	0
5	.00476	0	.00158	0	0	0

Table 1

APPROXIMATE ARRIVAL DISTRIBUTIONS P_n					
BANK			SUNNYSIDE		
Morning Peak	Afternoon Peak	Off Peak	Morning Peak	Afternoon Peak	Off Peak
$\binom{5}{n}\left(\frac{1}{5}\right)^n\left(\frac{4}{5}\right)^{5-n}$	$\binom{4}{n}\left(\frac{1}{4}\right)^n\left(\frac{3}{4}\right)^{4-n}$	$\binom{4}{n}\left(\frac{1}{5}\right)^n\left(\frac{4}{5}\right)^{4-n}$	$\binom{2}{n}\left(\frac{1}{8}\right)^n\left(\frac{7}{8}\right)^{2-n}$	$\binom{2}{n}\left(\frac{1}{8}\right)^n\left(\frac{7}{8}\right)^{2-n}$	$\binom{2}{n}\left(\frac{1}{16}\right)^n\left(\frac{15}{16}\right)^{2-n}$

Table 2

In order to have a model with a finite number of states we assumed that the queue lengths in both directions are bounded. The bound was chosen as $N = 14$.

The discharge rates of the intersection during a unit time interval (5 sec.) of the green phase were taken as

$\nu_1 = 4$ vehicles , for direction 1 (Bank).

$\nu_2 = 2$ vehicles , for direction 2 (Sunnyside)

3.5 TRANSITION PROBABILITIES.

The transition probability

$P_k(n,m) = P\,(\,x_k(i+1) = m \mid x_k(i) = n\,)$ when direction k is closed is easily calculated from the arrival statistics, since it is equal to the probability of (m-n) arrivals.

If the arrival distribution in direction k is of type :

$$p_n^{(k)} = \binom{K}{n} p^n (1-p)^{K-n} \qquad (10)$$

then

$$P_k(n,m) = \binom{K}{m-n} p^{m-n} (1-p)^{K-m+n} \qquad (11)$$

The calculation of the transition probabilities $Q_k(n,m) = P\,(x_k(i+1) = m \mid x_k(i) = n\,)$ when direction k is open is more difficult.

Let ρ_k be the traffic intensity in direction k during green periods. The probability that exactly n queued items will clear a random-arrival, constant - service queue beginning with r in the queue before the queue first vanishes is given by the Borel-Tanner [39] probabilities :

$$b_k(n,r) = A(n,r)\, e^{-\rho_k n} \rho_k^{\,n-r} \quad , n = r,\ r+1\ ,\dots. \qquad (12)$$

where

$$A(n,r) = \frac{r}{(n-r)\,!}\, n^{\,n-r-1} \qquad (13)$$

The transition probabilities $Q_k(n,m)$ can now be calculated from the expressions given in [39] :

$$Q_k(n,m) = e^{-\nu_k \rho_k} \rho_k^{\,\nu_k+m-n} \sum_{j=1}^{m} B(m,j) A(\nu_k+j\ ,\ n), \text{ for } n \le \nu_k\ ,\ m > 0$$

$$Q_k(n,m) = \sum_{j=n}^{\nu_k} b_k(j,n) \qquad \text{for } n \leq \nu_k \;;\; m = 0$$

..... (14)

$$= \binom{K}{m-n+\nu_k} p^{m-n+\nu_k} (1-p)^{K-m+n-\nu_k} \qquad \text{for } n > \nu_k$$

$$= 0 \qquad \text{for } m-n > \nu_k$$

where the coefficients $B(m,j)$ are calculated recursively from the expression

$$B(m,n) = -\sum_{j=n}^{m-1} A(m,j)B(j,n) \quad \text{for } m > n \qquad (15)$$

with $B(m,m) = 1$

The traffic intensity ρ_k is calculated from the expression :

$$\rho_k = \frac{\text{Average No. of arrivals per unit interval}}{\text{Average No. of departures per unit Green interval}}$$

3.6 COMPUTATION OF THE OPTIMAL CONTROL.

Using the Dynamic Programming equations (5),(6),(7) with the transition probabilities obtained from (11),(12),(13),(14),(15) and starting with T greater than 30 units of time , we found that steady state solution is obtained in less than 24 time units. The results,which include the total cummulative delay over 25 time units, are shown in Table A ,and the computer programs in Appendix B.

The steady-state optimal control of the form (9) is given, for the morning peak hour, by the figures 2 and 3. Similar results were obtained for the afternoon peak and off peak hours. (Figures 4,5,6,7).

Table A

		Previous Control = 1						Previous Control = 2					
		Next Control			Expected Minimum Delay Over 25 Time Units			Next Control			Expected Minimum Delay Over 25 Time Units		
n_1	n_2	MP	OP	AP	MP	OP	AP	MP	OP	AP	MP	OP	AP
0	0	2	2	2	31.6	15.3	31.6	2	2	2	30.5	14.4	30.4
0	1	1	1	1	33.4	17.3	33.3	1	1	1	37.0	20.4	37.0
0	2	1	1	1	35.1	18.6	35.0	1	1	1	40.7	23.2	40.6
0	3	1	1	1	39.0	22.4	38.9	1	1	1	46.5	28.7	46.4
0	4	1	1	1	42.7	25.3	42.6	1	1	1	52.3	33.2	52.2
0	5	1	1	1	48.5	30.7	48.4	1	1	1	60.0	40.3	60.0
0	6	1	1	1	54.3	35.2	54.2	1	1	1	67.7	46.4	67.7
0	7	1	1	1	61.9	42.2	61.9	1	1	1	77.1	55.0	77.1
0	8	1	1	1	69.7	48.3	69.6	1	1	1	86.7	62.8	86.7
0	9	1	1	1	79.1	57.0	79.1	1	1	1	97.6	72.9	97.7
0	10	1	1	1	88.7	64.8	88.7	1	1	1	108.7	82.4	108.9
0	11	1	1	1	99.6	74.9	99.7	1	1	1	120.5	94.0	120.7
0	12	1	1	1	110.7	84.4	110.9	1	1	1	128.1	105.1	128.3
0	13	2	1	2	111.1	96.0	111.1	2	1	2	110.1	117.0	110.1
0	14	2	1	2	60.3	107.1	60.3	2	2	2	59.6	109.6	59.6
1	0	2	2	2	33.8	17.3	33.7	2	2	2	31.4	15.3	31.4
1	1	1	1	1	36.8	20.5	36.8	2	2	2	38.3	22.0	38.2
1	2	1	1	1	38.8	22.0	38.7	2	2	2	43.5	26.1	43.4
1	3	1	1	1	43.9	26.9	43.8	2	2	2	50.4	32.5	50.3
1	4	1	1	1	48.0	29.9	47.9	2	2	2	57.6	38.3	57.6
1	5	1	1	1	54.8	36.5	54.8	2	2	2	66.4	46.3	66.3
1	6	1	1	1	61.1	41.2	61.0	2	2	2	75.6	53.7	75.5
1	7	1	1	1	69.6	49.3	69.5	1	1	1	85.8	63.3	85.8
1	8	1	1	1	77.8	55.8	77.8	1	1	1	95.7	71.5	95.8
1	9	1	1	1	87.8	65.3	87.8	1	1	1	106.8	82.4	107.0
1	10	1	1	1	97.7	73.5	97.8	1	1	1	118.0	92.3	118.2
1	11	1	1	1	108.8	84.4	109.0	1	1	1	129.5	104.5	129.8
1	12	1	1	1	120.0	94.3	120.2	2	1	2	135.2	116.1	135.4
1	13	2	1	2	113.2	106.5	113.2	2	1	2	111.1	127.5	111.1
1	14	2	2	2	62.2	112.3	62.2	2	2	2	60.6	110.6	60.6
2	0	2	2	2	36.2	19.6	36.1	2	2	2	32.5	16.4	32.5

2	1	1	1	1	40.7	24.0	40.6	2	2	2	39.5	23.0	39.4
2	2	1	1	1	42.9	25.6	42.8	2	2	2	44.7	27.3	44.7
2	3	1	1	2	49.1	31.7	49.0	2	2	2	51.7	33.7	51.6
2	4	1	1	1	53.7	35.0	53.6	2	2	2	59.0	39.5	58.9
2	5	1	1	1	61.4	42.5	61.4	2	2	2	67.8	47.5	67.7
2	6	1	1	1	68.2	47.7	68.1	2	2	2	77.1	55.0	77.0
2	7	1	1	1	77.3	56.5	77.3	2	2	2	87.6	64.6	87.6
2	8	1	1	1	85.9	63.5	85.9	2	2	2	98.6	73.7	98.6
2	9	1	1	1	96.1	73.7	96.2	2	2	2	110.6	84.8	110.7
2	10	1	1	1	106.1	82.4	106.3	2	2	2	122.9	95.5	123.0
2	11	1	1	1	117.1	93.9	117.3	2	2	2	134.3	108.1	134.5
2	12	1	1	1	127.9	104.3	128.3	2	2	2	136.6	120.2	136.7
2	13	2	1	2	115.5	117.0	115.5	2	2	2	112.2	130.3	112.2
2	14	2	2	2	64.2	114.3	64.2	2	2	2	61.6	111.7	61.6
3	0	2	2	2	39.0	22.1	38.9	2	2	2	33.8	17.6	33.8
3	1	1	1	1	44.9	27.9	44.8	2	2	2	40.9	24.4	40.8
3	2	1	1	1	47.4	29.6	47.3	2	2	2	46.4	28.8	46.4
3	3	1	1	1	54.7	36.8	54.6	2	2	2	53.6	35.3	53.5
3	4	1	1	1	59.7	40.4	59.6	2	2	2	61.2	41.3	61.1
3	5	1	1	1	68.1	48.8	68.1	2	2	2	70.2	49.5	70.2
3	6	1	1	1	75.3	54.3	75.3	2	2	2	79.7	57.1	79.7
3	7	1	1	1	84.7	63.9	84.8	2	2	2	90.4	66.8	90.4
3	8	1	1	1	93.4	71.4	93.6	2	2	2	101.5	76.0	101.6
3	9	1	1	1	103.6	82.2	103.8	2	2	2	113.6	87.3	113.7
3	10	1	1	1	113.3	91.5	113.6	2	2	2	125.9	98.1	126.1
3	11	1	1	1	123.8	103.3	124.1	2	2	2	137.2	110.8	137.4
3	12	1	1	1	134.1	114.2	134.4	2	2	2	138.8	123.0	138.9
3	13	2	1	2	118.1	127.0	118.1	2	2	2	113.4	132.8	113.4
3	14	2	2	2	66.3	116.6	66.3	2	2	2	62.8	112.8	62.8
4	0	2	2	2	42.2	25.1	42.1	2	2	2	35.6	19.2	35.5
4	1	1	1	1	49.4	32.1	49.3	2	2	2	42.9	26.2	42.8
4	2	1	1	1	52.2	34.0	52.1	2	2	2	49.0	31.1	49.0
4	3	1	1	1	60.4	42.1	60.3	2	2	2	56.7	38.0	56.6
4	4	1	1	1	65.7	46.0	65.7	2	2	2	64.8	44.5	64.7
4	5	1	1	1	74.7	55.2	74.7	2	2	2	74.3	53.1	74.3
4	6	1	1	1	82.0	61.2	82.1	2	2	2	84.3	61.2	84.3
4	7	1	1	1	91.4	71.4	91.6	2	2	2	95.4	71.2	95.4
4	8	1	1	1	99.9	79.5	100.1	2	2	2	106.8	80.7	106.9
4	9	1	1	1	109.6	90.6	109.8	2	2	2	119.2	92.4	119.3
4	10	1	1	1	118.6	100.4	119.0	2	2	2	131.6	103.4	131.8
4	11	1	1	1	128.3	112.4	128.7	2	2	2	142.5	116.4	142.7
4	12	1	1	1	137.8	123.4	138.1	2	2	2	142.4	128.7	142.6

4	13	2	1	2	121.1	136.1	121.1	2	2	2	115.1	137.7	115.1
4	14	2	2	2	68.7	119.2	68.7	2	2	2	64.4	114.3	64.4
5	0	2	2	2	45.7	28.4	45.6	2	2	2	37.8	21.3	37.7
5	1	1	1	1	54.3	36.6	54.2	2	2	2	45.4	28.7	45.3
5	2	1	1	1	57.2	38.6	57.1	2	2	2	52.1	34.2	52.0
5	3	1	1	1	66.0	47.4	65.9	2	2	2	60.3	41.6	60.2
5	4	1	1	1	71.7	51.9	71.7	2	2	2	69.0	48.7	68.9
5	5	1	1	1	80.6	61.6	80.7	2	2	2	79.0	57.7	78.9
5	6	1	1	1	87.8	68.3	88.0	2	2	2	89.6	66.4	89.6
5	7	1	1	1	96.7	78.9	96.9	2	2	2	101.2	76.9	101.2
5	8	1	1	1	104.5	87.4	104.8	2	2	2	113.2	87.0	113.2
5	9	1	1	1	113.3	98.8	113.5	2	2	2	126.0	99.0	126.1
5	10	1	1	1	121.5	108.7	121.7	1	2	1	136.7	110.7	136.9
5	11	1	1	1	130.2	120.5	130.4	1	2	1	145.3	124.0	145.5
5	12	1	1	1	138.7	131.3	138.9	2	2	2	146.6	136.7	146.7
5	13	2	1	2	124.3	143.5	124.3	2	2	2	117.2	144.3	117.2
5	14	2	2	2	71.2	122.0	71.2	2	2	2	66.2	116.3	66.2
6	0	2	2	2	49.6	31.9	49.5	2	2	2	40.2	23.6	40.1
6	1	2	2	2	58.5	40.4	58.4	2	2	2	48.0	31.1	47.9
6	2	1	1	1	62.3	43.4	62.3	2	2	2	55.0	36.7	54.9
6	3	1	1	1	71.2	52.6	71.2	2	2	2	63.4	44.3	63.3
6	4	1	1	1	76.9	58.1	77.0	2	2	2	72.4	51.5	72.3
6	5	1	1	1	85.2	68.0	85.4	2	2	2	82.7	60.7	82.6
6	6	1	1	1	91.7	75.4	92.0	2	2	2	93.5	69.6	93.5
6	7	1	1	1	99.6	86.1	99.8	2	2	2	105.4	80.3	105.4
6	8	1	1	1	106.5	94.8	106.7	2	2	2	117.9	90.8	117.9
6	9	1	1	1	114.1	105.9	114.3	1	2	1	127.1	103.0	127.2
6	10	1	1	1	121.4	115.5	121.5	1	2	1	134.8	115.1	134.9
6	11	1	1	1	129.1	126.7	129.2	1	2	1	142.7	128.7	142.8
6	12	1	1	1	136.8	136.8	136.9	1	2	1	147.6	141.5	147.7
6	13	2	1	2	127.5	148.0	127.5	2	2	2	119.5	148.2	119.5
6	14	2	2	2	73.9	124.9	73.8	2	2	2	68.2	118.3	68.2
7	0	2	2	2	53.8	35.8	53.7	2	2	2	43.0	26.1	42.9
7	1	2	2	2	63.0	44.6	62.9	2	2	2	51.0	33.9	50.9
7	2	1	1	1	67.1	48.6	67.1	2	2	2	58.5	39.9	58.4
7	3	1	1	1	75.4	58.1	75.5	2	2	2	67.3	47.7	67.2
7	4	1	1	1	80.3	64.3	80.5	2	2	2	76.8	55.4	76.7
7	5	1	1	1	87.4	74.4	87.6	2	2	2	87.4	64.8	87.3
7	6	1	1	1	92.9	82.0	93.0	2	2	2	98.7	74.1	98.7
7	7	1	1	1	99.4	92.4	99.5	1	2	1	109.9	85.0	109.9
7	8	1	1	1	105.3	100.7	105.4	1	2	1	116.3	95.7	116.4
7	9	1	1	1	111.9	111.0	111.9	1	2	1	123.2	108.2	123.2
7	10	1	1	1	118.3	119.8	118.4	1	2	1	130.2	120.6	130.2

7	11	1	1	1	125.2	129.9	125.2	1	2	1	137.5	134.4	137.5
7	12	1	1	1	132.2	138.9	132.2	1	2	1	143.1	147.2	143.1
7	13	2	1	2	129.6	148.8	129.8	2	2	2	122.1	153.0	122.1
7	14	2	2	2	76.6	127.8	76.6	2	2	2	70.3	120.6	70.3
8	0	2	2	2	58.1	40.1	58.1	2	2	2	46.2	29.1	46.1
8	1	2	2	2	67.2	49.1	67.2	2	2	2	54.5	37.1	54.5
8	2	1	1	1	70.2	54.0	70.3	2	2	2	62.5	43.7	62.4
8	3	1	1	1	76.9	63.7	77.0	2	2	2	71.8	51.8	71.7
8	4	1	1	1	80.7	70.1	80.8	2	2	2	81.7	60.0	81.6
8	5	1	1	1	86.1	79.9	86.2	2	2	2	92.8	69.8	92.7
8	6	1	1	1	90.6	86.9	90.6	1	2	1	99.1	79.5	99.1
8	7	1	1	1	95.9	96.4	95.9	1	2	1	104.8	90.9	104.8
8	8	1	1	1	101.1	103.7	101.1	1	2	1	110.7	102.0	110.6
8	9	1	1	1	106.8	112.7	106.8	1	2	1	116.9	114.8	116.8
8	10	1	1	1	112.7	120.2	112.6	1	2	1	123.3	127.6	123.2
8	11	1	1	1	118.9	128.9	118.8	1	2	1	130.0	141.7	129.9
8	12	1	1	1	125.3	136.7	125.2	1	1	1	135.8	151.3	135.7
8	13	2	1	2	128.7	145.3	128.7	2	1	2	125.1	157.0	125.1
8	14	2	2	2	79.2	129.3	79.2	2	2	2	72.7	123.2	72.7
9	0	2	2	2	61.4	44.6	61.5	2	2	2	49.7	32.4	49.6
9	1	1	2	1	68.2	53.9	68.3	2	2	2	58.4	40.7	58.3
9	2	1	1	1	70.0	59.1	70.1	2	2	2	66.8	47.6	66.7
9	3	1	1	1	74.3	68.5	74.3	2	2	2	76.4	56.1	76.3
9	4	1	1	1	77.2	74.1	77.2	1	2	1	83.1	64.6	83.0
9	5	1	1	1	81.3	82.8	81.2	1	2	1	87.6	74.7	87.5
9	6	1	1	1	85.1	88.6	85.0	1	2	1	92.3	84.9	92.2
9	7	1	1	1	89.6	96.5	89.5	1	2	1	97.4	96.5	97.3
9	8	1	1	1	94.3	102.5	94.2	1	2	1	102.8	108.1	102.6
9	9	1	1	1	99.4	109.9	99.3	1	2	1	108.4	121.2	108.3
9	10	1	1	1	104.8	116.3	104.6	1	1	1	114.2	128.3	114.1
9	11	1	1	1	110.4	123.5	110.3	1	1	1	120.2	135.5	120.1
9	12	1	1	1	116.2	130.3	116.1	1	1	1	125.7	142.6	125.6
9	13	1	1	1	122.2	137.5	122.1	1	1	1	122.4	147.6	122.4
9	14	2	2	2	80.5	126.2	80.6	2	2	2	75.2	126.0	75.2
10	0	2	2	2	61.2	49.2	61.1	2	2	2	53.6	35.9	53.5
10	1	1	2	1	63.8	58.5	63.7	2	2	2	62.5	44.4	62.4
10	2	1	1	1	65.2	62.5	65.1	1	2	1	68.3	51.6	68.1
10	3	1	1	1	67.8	70.5	67.6	1	2	1	71.6	60.4	71.4
10	4	1	1	1	70.3	74.5	70.1	1	2	1	75.1	69.2	74.9
10	5	1	1	1	73.6	81.2	73.4	1	2	1	79.1	79.6	78.9
10	6	1	1	1	77.1	85.5	76.9	1	2	1	83.4	90.0	83.2
10	7	1	1	1	81.1	91.6	80.9	1	1	1	88.0	100.4	87.9

10	8	1	1	1	85.4	96.5	85.2	1	1	1	92.8	105.8	92.6
10	9	1	1	1	90.0	102.4	89.8	1	1	1	97.8	111.8	97.6
10	10	1	1	1	94.8	107.8	94.6	1	1	1	102.9	117.6	102.7
10	11	1	1	1	99.8	113.8	99.6	1	1	1	108.1	123.8	107.9
10	12	1	1	1	104.9	119.6	104.7	1	1	1	113.0	129.9	112.8
10	13	1	1	1	110.1	125.8	109.9	1	1	1	111.1	134.3	111.0
10	14	2	2	2	78.7	116.8	78.7	2	1	2	77.9	116.8	77.8
11	0	1	2	1	54.3	52.2	54.0	1	2	1	54.5	39.8	54.3
11	1	1	1	1	55.3	59.9	55.0	1	2	1	56.6	48.6	56.3
11	2	1	1	1	56.5	61.5	56.3	1	2	1	59.0	56.2	58.7
11	3	1	1	1	58.6	66.8	58.3	1	2	1	62.0	65.3	61.7
11	4	1	1	1	61.0	69.4	60.7	1	2	1	65.2	74.5	65.0
11	5	1	1	1	64.0	73.9	63.7	1	1	1	68.8	79.9	68.6
11	6	1	1	1	67.2	77.4	67.0	1	1	1	72.7	84.1	72.4
11	7	1	1	1	70.8	81.9	70.6	1	1	1	76.7	88.8	76.5
11	8	1	1	1	74.7	86.1	74.4	1	1	1	80.8	93.4	80.6
11	9	1	1	1	78.7	90.8	78.5	1	1	1	85.0	98.3	84.8
11	10	1	1	1	82.8	95.4	82.6	1	1	1	89.3	103.3	89.1
11	11	1	1	1	87.0	100.3	86.8	1	1	1	93.7	108.4	93.4
11	12	1	1	1	91.3	105.3	91.1	1	1	1	97.7	113.5	97.4
11	13	1	1	1	95.7	110.4	95.4	1	1	1	97.1	117.3	96.8
11	14	2	2	2	73.0	103.8	73.0	1	1	1	73.0	103.8	73.0
12	0	1	2	1	43.7	49.7	43.4	1	2	1	43.9	44.1	43.6
12	1	1	1	1	44.7	52.6	44.4	1	2	1	45.9	53.1	45.6
12	2	1	1	1	45.9	53.8	45.6	1	1	1	48.1	56.8	47.9
12	3	1	1	1	47.9	56.6	47.6	1	1	1	50.8	59.9	50.5
12	4	1	1	1	50.1	58.8	49.9	1	1	1	53.7	62.9	53.4
12	5	1	1	1	52.8	61.9	52.5	1	1	1	56.7	66.4	56.5
12	6	1	1	1	55.7	64.9	55.4	1	1	1	59.9	69.9	59.6
12	7	1	1	1	58.7	68.4	58.5	1	1	1	63.2	73.6	62.9
12	8	1	1	1	61.9	71.9	61.6	1	1	1	66.5	77.4	66.2
12	9	1	1	1	65.2	75.6	64.9	1	1	1	69.8	81.4	69.5
12	10	1	1	1	68.5	79.4	68.2	1	1	1	73.2	85.3	72.8
12	11	1	1	1	71.8	83.4	71.5	1	1	1	76.6	89.3	76.2
12	12	1	1	1	75.2	87.3	74.8	1	1	1	79.8	93.4	79.4
12	13	1	1	1	78.6	91.3	78.2	1	1	1	80.1	96.5	79.8
12	14	2	2	2	64.1	87.4	64.0	1	1	1	64.1	87.4	64.0
13	0	1	1	1	32.0	38.4	31.6	1	1	1	32.2	38.5	31.8
13	1	1	1	1	33.0	39.4	32.6	1	1	1	33.9	40.4	33.6
13	2	1	1	1	34.2	40.5	33.8	1	1	1	35.8	42.3	35.5
13	3	1	1	1	35.9	42.4	35.6	1	1	1	38.0	44.6	37.6
13	4	1	1	1	37.8	44.3	37.5	1	1	1	40.2	47.0	39.8
13	5	1	1	1	40.0	46.6	39.6	1	1	1	42.5	49.6	42.1

13	6	1	1	1	42.2	49.0	41.8	1	1	1	44.8	52.3	44.4
13	7	1	1	1	44.5	51.6	44.1	1	1	1	47.2	55.0	46.7
13	8	1	1	1	46.8	54.3	46.4	1	1	1	49.5	57.8	49.1
13	9	1	1	1	49.2	57.0	48.7	1	1	1	51.9	60.6	51.4
13	10	1	1	1	51.5	59.8	51.1	1	1	1	54.3	63.5	53.8
13	11	1	1	1	53.9	62.6	53.4	1	1	1	56.7	66.3	56.1
13	12	1	1	1	56.3	65.5	55.8	1	1	1	59.0	69.1	58.4
13	13	1	1	1	58.7	68.3	58.1	1	1	1	59.9	71.5	59.4
13	14	2	2	2	51.6	66.7	51.3	1	1	1	51.6	66.7	51.3
14	0	1	1	1	20.8	23.7	20.4	1	1	1	20.9	23.7	20.5
14	1	1	1	1	21.8	24.7	21.4	1	1	1	22.2	25.1	21.8
14	2	1	1	1	22.9	25.7	22.5	1	1	1	23.6	26.6	23.2
14	3	1	1	1	24.2	27.1	23.8	1	1	1	25.0	28.2	24.6
14	4	1	1	1	25.6	28.6	25.2	1	1	1	26.5	29.8	26.0
14	5	1	1	1	27.0	30.2	26.6	1	1	1	27.9	31.4	27.5
14	6	1	1	1	28.5	31.8	28.0	1	1	1	29.4	33.1	29.0
14	7	1	1	1	29.9	33.4	29.5	1	1	1	30.9	34.7	30.4
14	8	1	1	1	31.4	35.1	31.0	1	1	1	32.4	36.4	31.9
14	9	1	1	1	32.9	36.7	32.4	1	1	1	33.9	38.1	33.3
14	10	1	1	1	34.4	38.4	33.9	1	1	1	35.4	39.8	34.8
14	11	1	1	1	35.9	40.1	35.3	1	1	1	36.8	41.5	36.3
14	12	1	1	1	37.4	41.8	36.8	1	1	1	38.3	43.2	37.7
14	13	1	1	1	38.8	43.5	38.3	1	1	1	39.4	44.7	38.8
14	14	2	2	2	37.3	43.7	36.9	1	1	1	37.3	43.7	36.9

Table A

Note :

MP : Morning-Peak hour
OP : Off-peak hour
AP : Afternoon-peak hour.

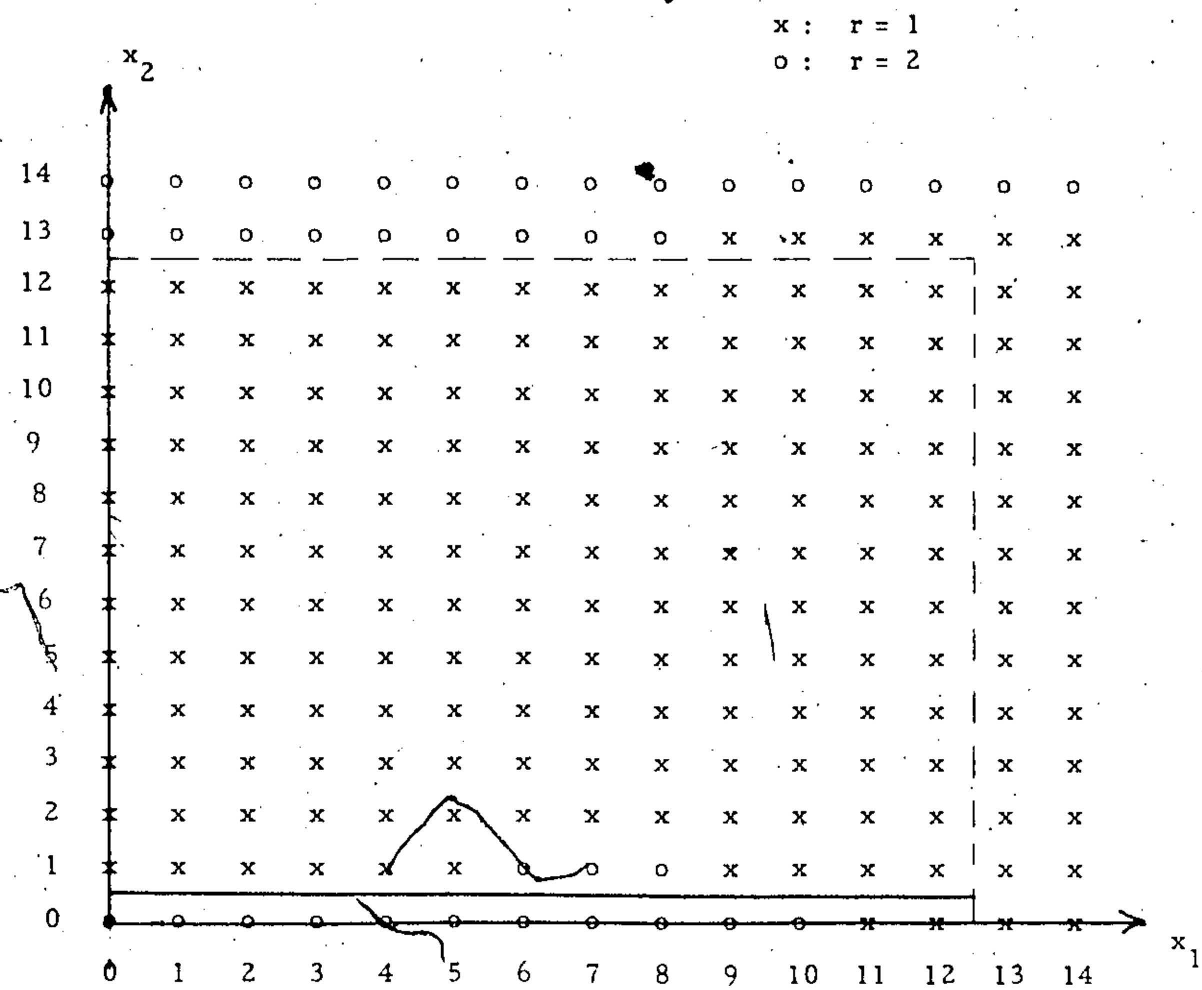


Fig. 2 Optimal control $r(i) = g(x_1(i-1), x_2(i-1), r(i-1) = 1)$

(morning-peak hour)

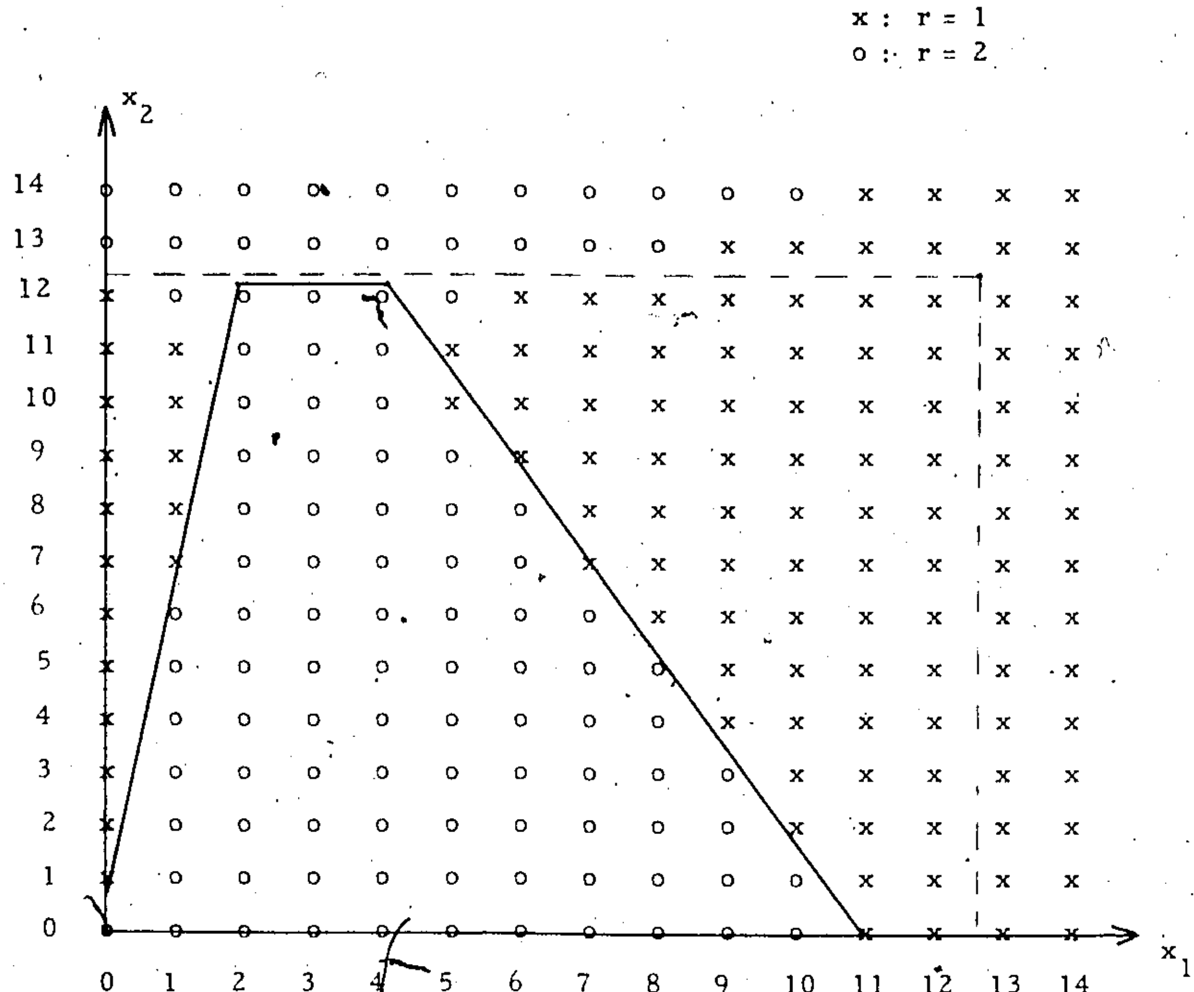


Fig. 3 Optimal control $r(i) = g(x_1(i-1), x_2(i-1), r(i-1) = 2)$

(Morning-peak hour)

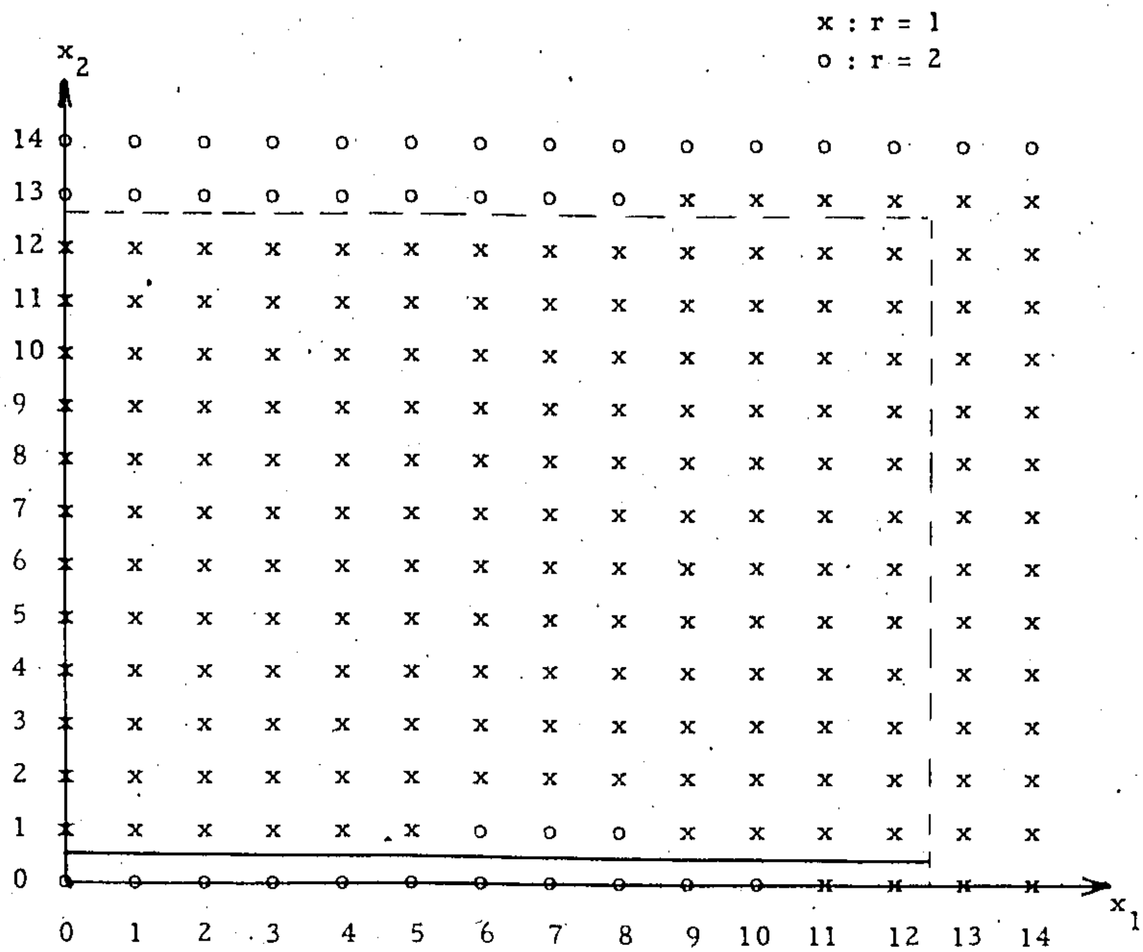


Fig. 4 Optimal control $r(i) = g(x_1(i-1), x_2(i-1), r(i-1) = 1)$

(Afternoon-peak hour)

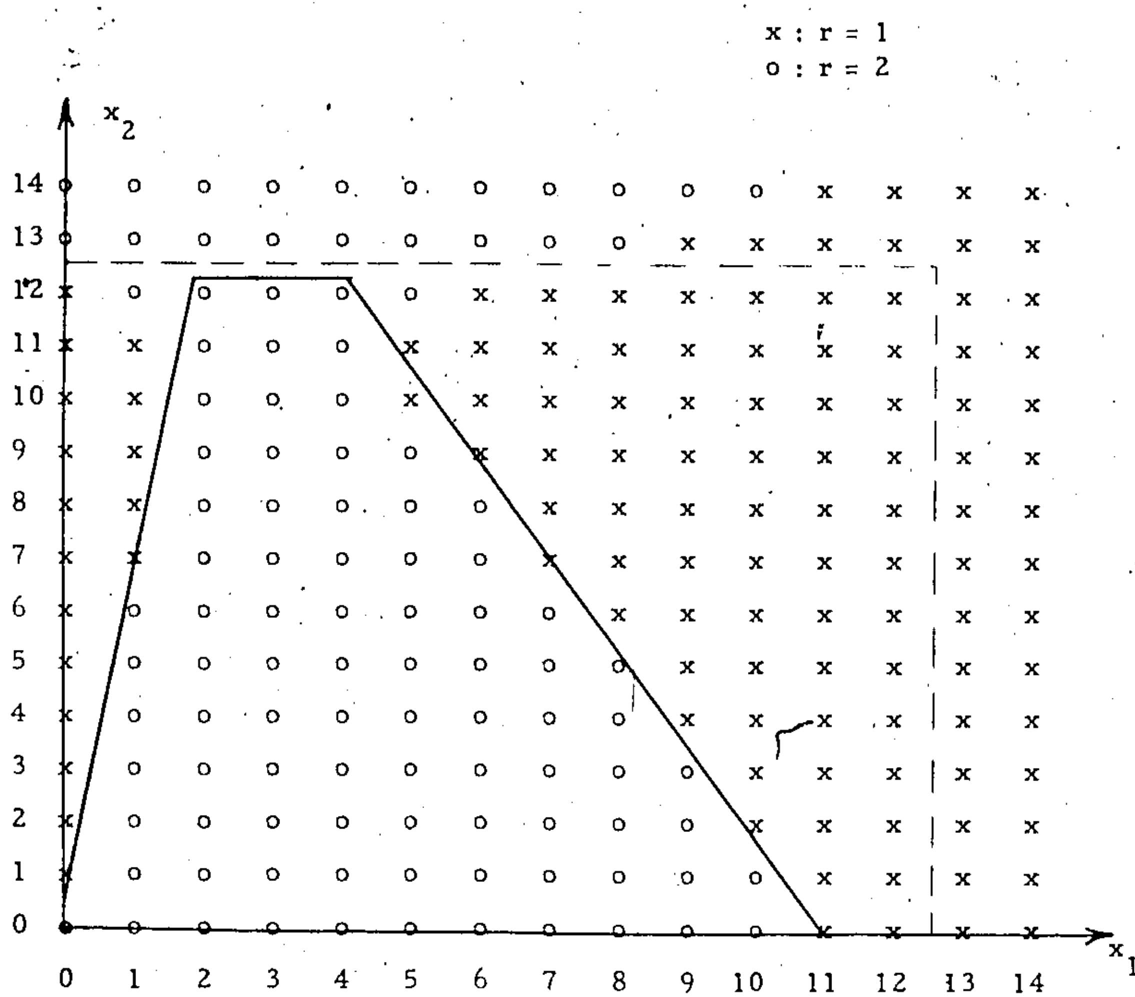


Fig. 5 Optimal control $r(i) = g(x_1(i-1), x_2(i-1), r(i-1) = 2)$

(Afternoon-peak hour)

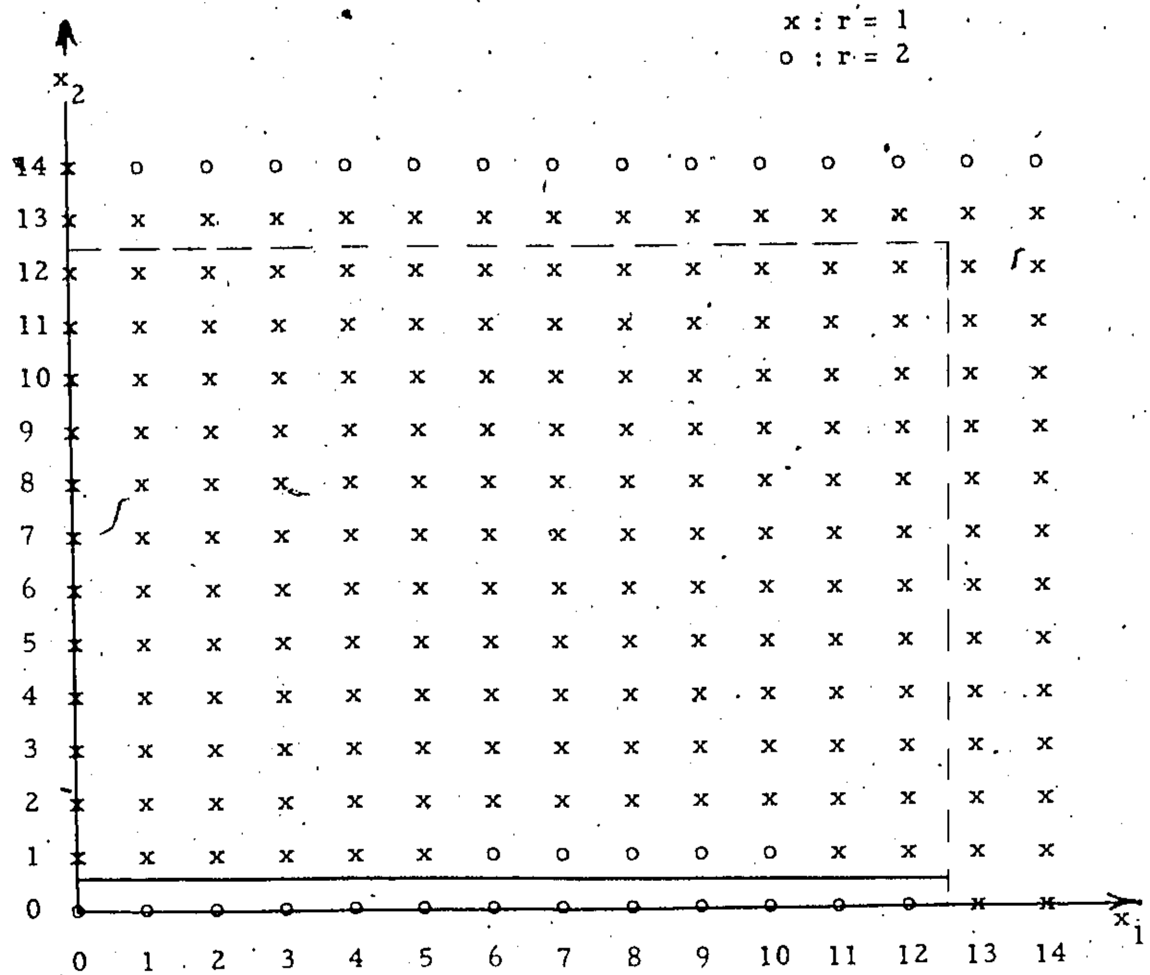


Fig. 6 Optimal control $r(i) = g(x_1(i-1), x_2(i-1), r(i-1) = 1)$

(Off-peak hour)

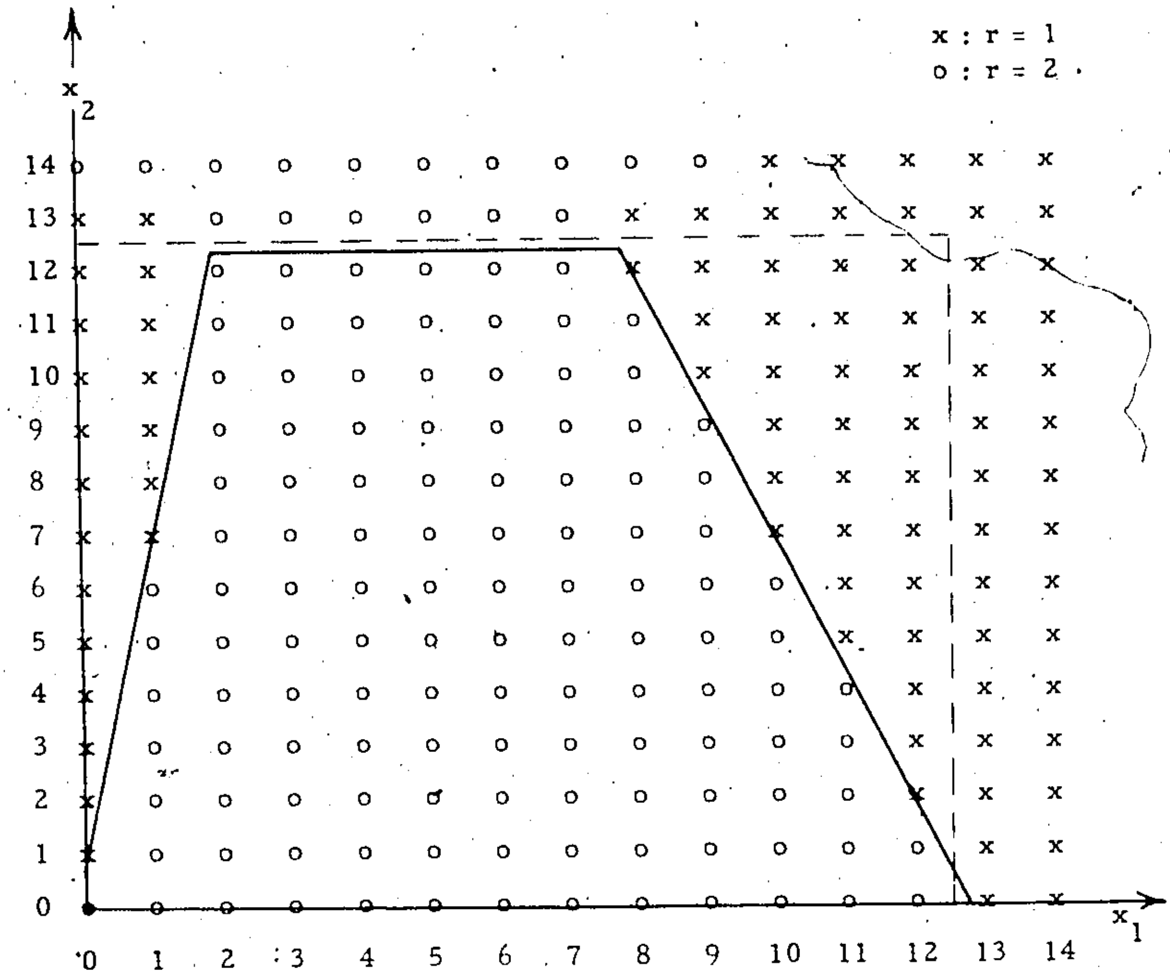


Fig. 7 Optimal control $r(i) = g(x_1(i-1), x_2(i-1), r(i-1) = 2)$

(Off-peak hour)

3.7 SYNTHESIS OF THE OPTIMAL FEEDBACK CONTROLLER.

Since the control function r(t) takes only two values we attempted to synthesize the optimal feedback controller (9) using a Learning-Machine [40] in the pattern space of the 450 points (x_1, x_2, r), as shown in figure 8 .

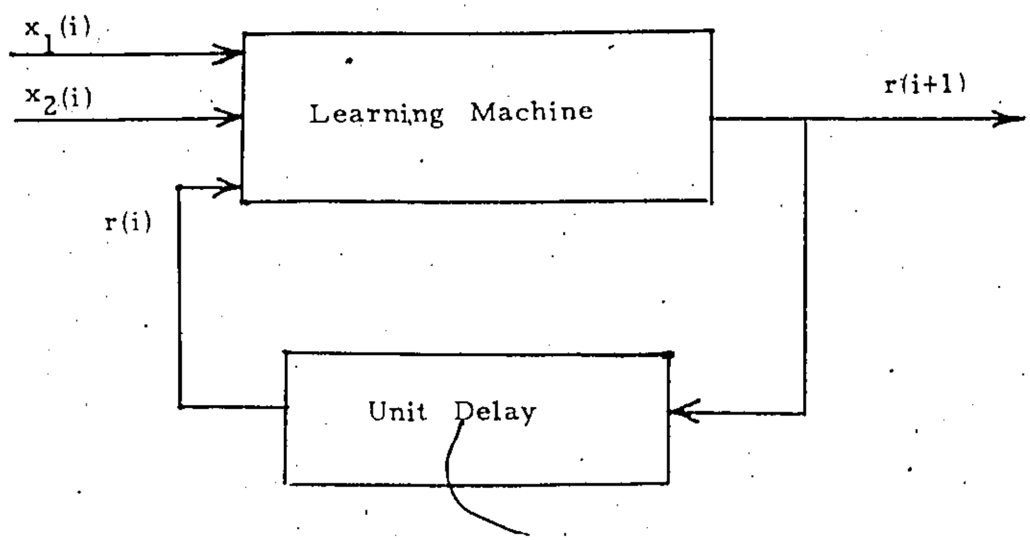


Fig. 8 The optimal controller as a single learning machine.

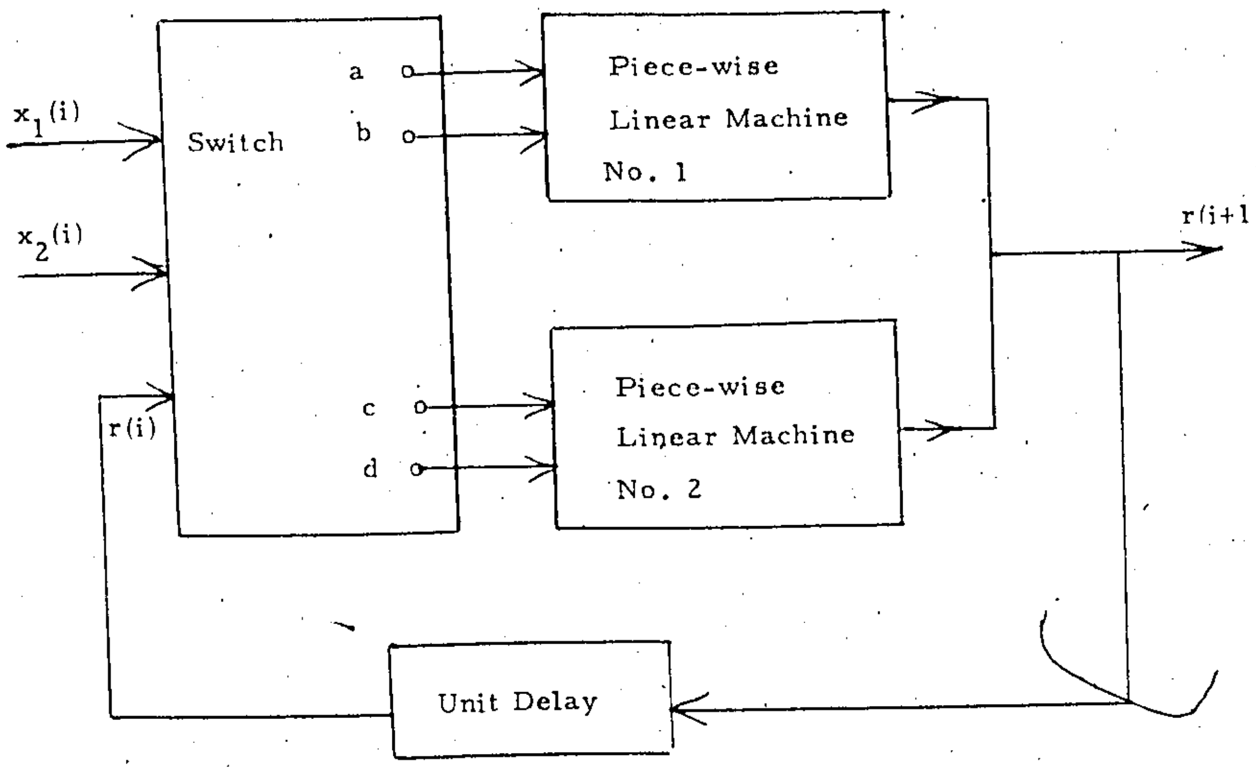
Despite many attempts using either a Threshold Logic Unit (TLU) or a Commitee Machine [40] with various pattern encoding schemes [41] we failed to obtain a Learning Controller with an acceptable error in pattern classification. It appears that the function $g(x_1, x_2, r)$ in (9) cannot be expressed in the form :

$$g(x_1, x_2, r) = g_1(x_1) + g_2(x_2) + g_3(x_3) \qquad (16)$$

for which Linear separability of the appropriately encoded patterns

(x_1, x_2, r) is guaranteed [41].

We then decided to synthesize the optimal feedback controller (9) as a logic combination of two Piece-wise Linear Learning Machines [40], one corresponding to $r(t-1) = 1$ and the other to $r(t-1) = 2$. The result is shown in Figure 9.



If $r(i) = 1$ then $x_1(i), x_2(i)$ are connected to terminals a, b.

If $r(i) = 2$ then $x_1(i), x_2(i)$ are connected to terminals c, d.

Fig. 9 The optimal controller as a combination of two piece-wise linear machines.

In this way not only the dimensionality of the pattern space is reduced from 3 to 2, but also the separation can be done very easily, by inspection of the pattern space.

In Figures 2 and 3 the patterns corresponding to $r(t-1)=1$ and $r(t-1)=2$ respectively, are shown. These results correspond to the morning-peak hour. Similar results are obtained for the afternoon-peak and off-peak hours (Figures 4,5,6,7). As shown in Figure 3 the pattern separation by a piece-wise linear surface of only 3 linear segments produces an error of $5/169 = 3\%$. The separation of patterns in Figure 2 is accomplished by a single plane with a resulting error of $3/169 = 1.8\%$. The above results are for the morning-peak hour.

For the afternnon-peak hour, when the previous control $r(t-1) = 1$, the patterns are shown in Figure 4. And again only one plane is needed for separation with an error of $3/169 = 1.8\%$. When the previous control $r(t-1) = 2$, the patterns are shown in Figure 5. A piece-wise linear surface of three linear segments separate the patterns with an error of $5/169 = 3\%$.

Finally, for the off-peak hour, the results are shown in Figures 6 and 7. Similarly the errors are of $5/169 = 3\%$ and $4/169 = 2.3\%$ corresponding to the previous controls of $r(t-1) = 1$ and $r(t-1)=2$ respectively.

Ofcourse we can reduce these errors to zero but the complexity of the controller increases significantly.

Figures 10 and 11 give details of the structure of the Piece-wise Linear Machines of Figure 9, which correspond to the Morning-peak hour. Similarly, corresponding to the Afternoon-peak and Off-peak hours, Piece-wise Linear Machines number 3,4,5,6 are drawn in Figures 12,13,14,15 respectively.

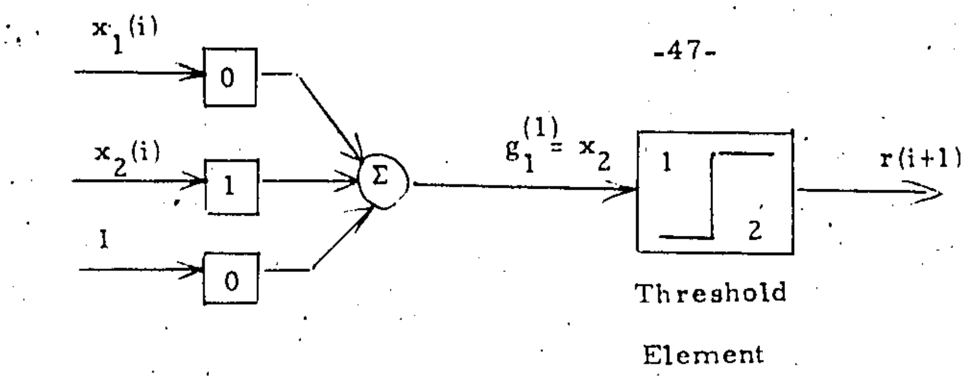


Fig. 10 Piece-wise Linear Machine No. 1

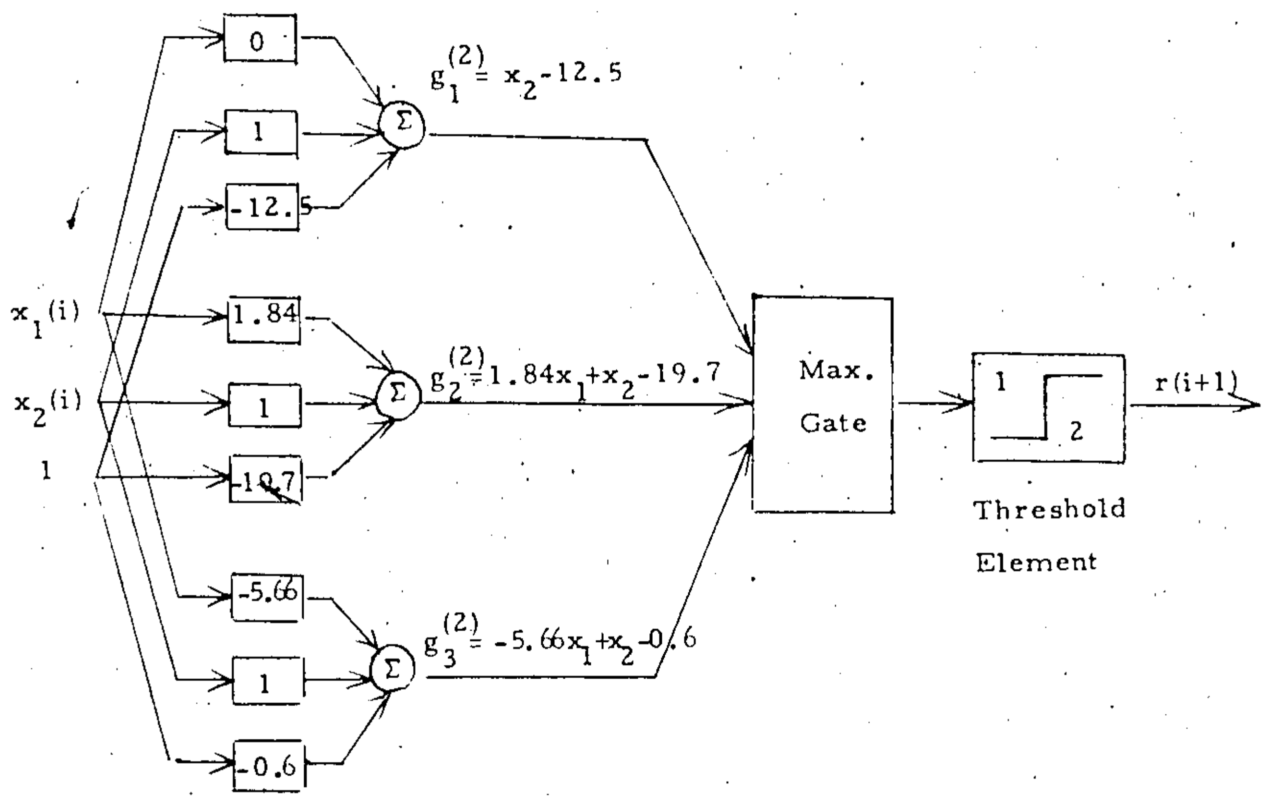


Fig. 11 Piece-wise Linear Machine No. 2

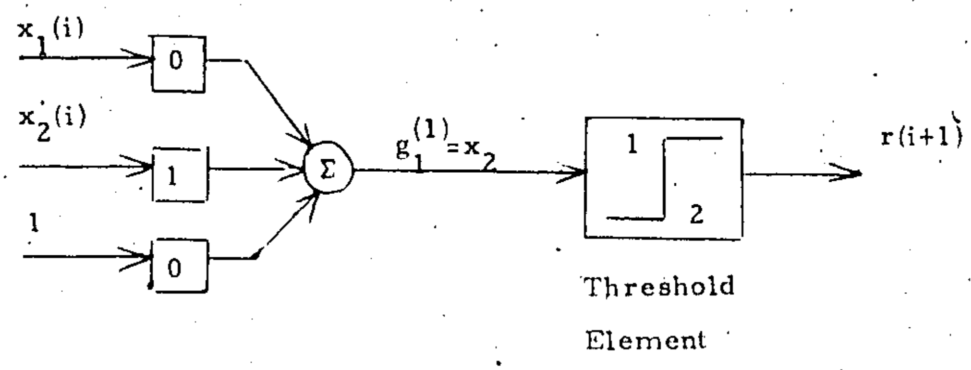


Fig. 12 Piece-wise Linear Machine No. 3

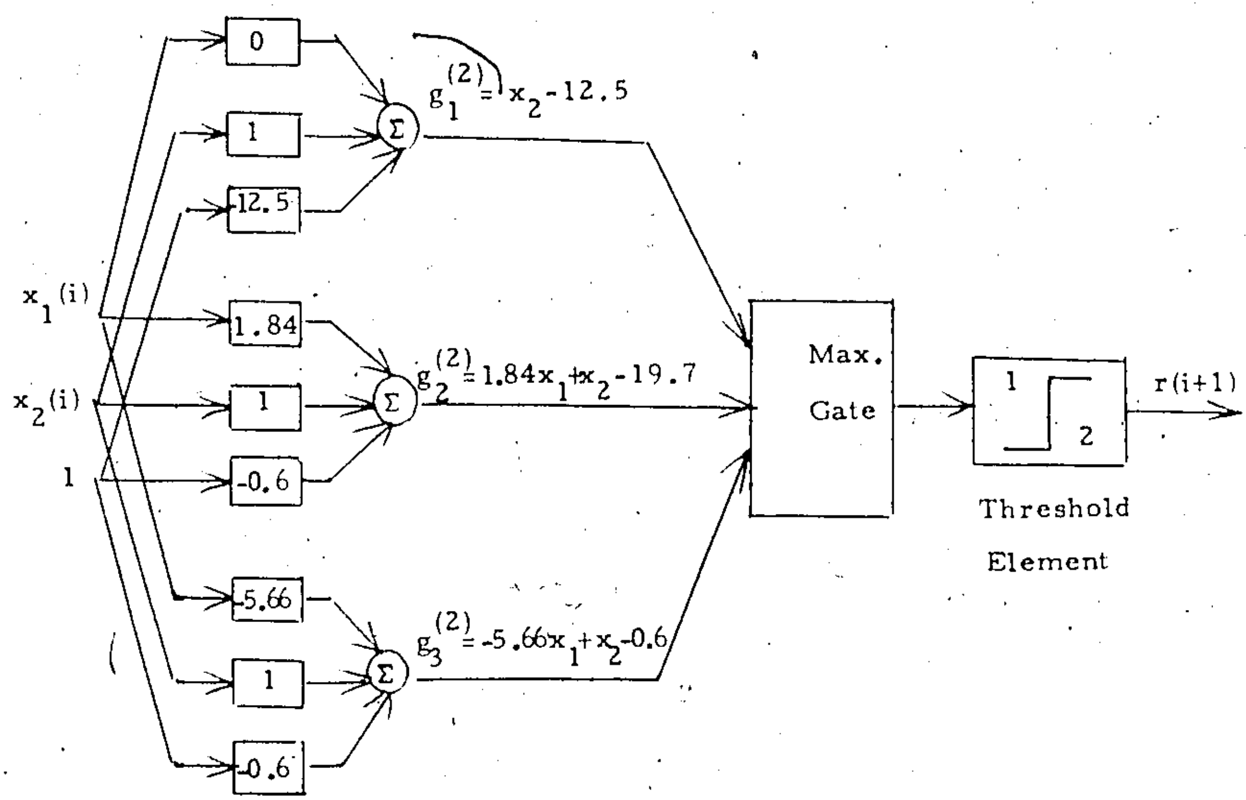


Fig. 13 Piece-wise Linear Machine No. 4

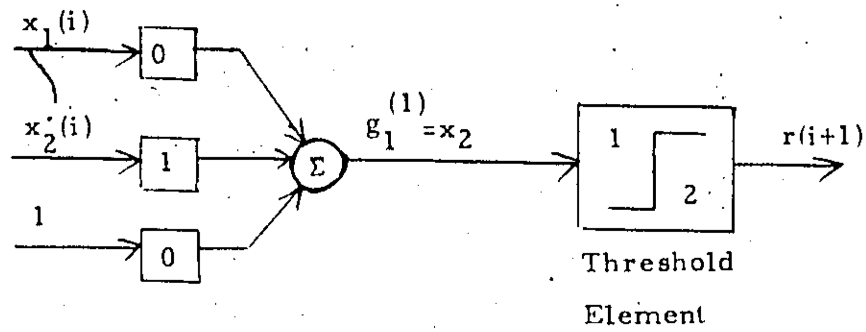


Fig. 14 Piece-wise Linear Machine 5

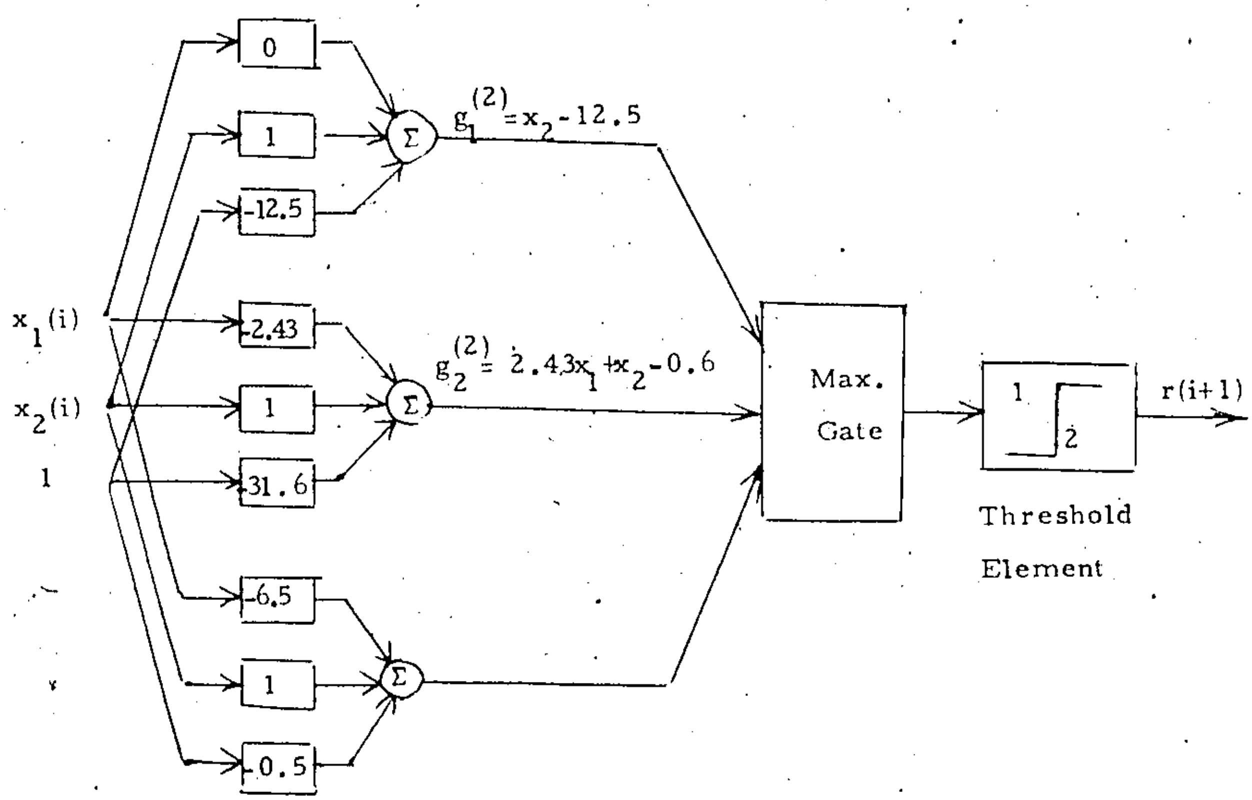


Fig. 15 Piece-wise Linear Machine No. 6

The following table is the summary of the results of the synthesis of the optimal feedback controller.

Hour	Patterns' separation requirement		Error
Morning Peak	$r(t-1) = 1$	one plane	1.8%
	$r(t-1) = 2$	3 linear segments	3%
Afternoon Peak	$r(t-1) = 1$	one plane	1.8%
	$r(t-1) = 2$	3 linear segments	3%
Off Peak	$r(t-1) = 1$	one plane	3%
	$r(t-1) = 2$	3 linear segments	2.3%

Table 3 Results of the synthesis of the optimal feedback controller.

3.8 RESULTS OF COMPUTER SIMULATION.

Having obtained the structure of the optimal feedback controller for the **various** hours , as shown in Figures 9, 10, ..., to 15, we simulated the overall system according to the equations :

$$\begin{cases} x_1(i+1) = x_1(i) + q_1(i+1) - 4\,(\,g(x_1(i), x_2(i), r(i)) - 1\,) \\ x_2(i+1) = x_1(i) + q_2(i+1) - 2\,(\,2 - g(x_1(i), x_2(i), r(i))\,) \\ r(i+1) = g\,(\,x_1(i), x_2(i), r(i)\,) \end{cases} \quad (17)$$

with $0 \leq x_1(i) \leq 12$, $0 \leq x_2(i) < 12$

The initial conditions $x_1(0), x_2(0), r(0)$ were taken arbitrarily.

The random binomial arrivals $q_1(i)$ and $q_2(i)$ were produced by a binomial numbers random generator which we developed specially for that purpose (Appendix A). The simulation was carried in the IBM 360/65.

The upper bound of queues was lowered from 14 to 12 because the consideration of a finite number of states in the Markovian model introduces errors at the boundary of the state space.

From the digital simulation the basic statistics of the light cycle were obtained. They are shown in table 4.

Hour	Average cycle of traffic light	Standard deviation
Morning peak	24.15 seconds	16.43 seconds
Afternoon peak	28.90 seconds	17.69 seconds
Off peak	49.24 seconds	20.78 seconds

Table 4

CONCLUSION.

In this thesis a review of existing optimization schemes for traffic control was first given. This review will give to the reader an idea of the various problems encountered in traffic optimization and methods for their solution. The main emphasis of the thesis was in the design of a vehicle-actuated optimal controller which minimizes the delay associated with a single intersection. A probabilistic mathematical model of the intersection was used. This model has the major advantage of not needing the usual Poisson-arrivals assumption. It utilizes a finite, controlled Markov Chain. The transition probabilities of the Chain were obtained from the arrival and service data. The method of Dynamic Programming was used for obtaining the optimal feedback control. Because of the finite-state space assumption the results of Dynamic Programming have to be constrained in a subset of the state space. Errors occur in the boundary of the state space. The optimal controller was synthesized using a combination of piece-wise linear machines. This method is very simple and very appropriate for the case under consideration. It also leads to a very simple and inexpensive design. The parameters of the optimal controller can easily be adjusted for the three traffic hours considered. An example of an intersection in Ottawa was considered. Real traffic data were used.

It is left for further research to modify the model for including the possibility of turns in the intersection. This case was not considered because of lack of traffic data.

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APPENDIX A

GENERATION OF BINOMIAL RANDOM VARIABLES [A3]

Let Y be a discrete random variable with point probabilities $p_i = \Pr.[\,Y = y_i\,]$ for i = 1,2,.... The direct way to generate Y is to generate U and put $Y = y_i$ if

$$p_1 + p_2 + \ldots + p_{i-1} < U < p_1 + p_2 + \ldots + p_i$$

However, this method requires complicated machine programs.

An alternative way due to Marsaglia [A1] is simple, fast, and seems to be well suited to high-speed computations. Let p_i for $i = 1, 2, 3, \ldots, n$ be expressed by k decimal digits as :

$$p_i = .\,\delta_{1i}\,\delta_{2i}\ldots\delta_{ki} \quad \text{where the } \delta\text{'s are decimal digits.}$$

(If the domain of the random variable is infinite, it is necessary to truncate the probability distribution at p_n). Define

$$P_o = 0\ ,\ P_r = 10^{-r}\sum_{i=1}^{n}\delta_{ri} \quad \text{for } r = 1,2,\ldots,k \qquad \text{and}$$

$$\Pi_s = \sum_{r=0}^{s} 10^{r}\, P_r\ , \quad s = 1,2,\ldots,k.$$

Number the computer memmory locations by $0, 1, 2, \ldots, \Pi_k - 1$. The memory locations are divided into k mutually exclusive sets such that the s th set consists of memory location $\Pi_{s-1}, \Pi_{s-1} + 1, \ldots, \Pi_s - 1$. The information stored in the memory locations of the s th set consists of y_1 in δ_{s1} locations, y_2 in δ_{s2} locations, ..., y_n in δ_{sn} locations.

Denote the decimal expansion of the uniform deviates generated by the computer by $u = .d_1 d_2 d_3 \ldots$ and finally let a(m) be the content of the memory location m. Then if :

$$\sum_{i=0}^{s-1} P_i \leq U < \sum_{i=0}^{s} P_i$$

put

$$y = a\,(.d_1 d_2 \ldots d_s + \Pi_{s-1} - 10^s \sum_{i=1}^{s-1} P_i)$$

This method is perhaps the best all-around method for generating random deviates from a discrete distribution. In order to illustrate this method consider the problem of generating deviates from a binomial distribution with point probabilities

$$p_i = \binom{n}{i} p^i (1-p)^{n-i}$$

for $n=5$ and $p = 0.20$. The point probabilities to 4D are :

Value of Random Variable	Point Probabilities
0	$p_0 = 0.3277$
1	$p_1 = 0.4096$
2	$p_2 = 0.2048$
3	$p_3 = 0.0512$
4	$p_4 = 0.0064$
5	$p_5 = 0.0003$

and thus $P_0=0$, $P_1 = 0.9$, $P_2 = 0.07$, $P_3 = 0.027$, $P_4 = 0.0030$ from which $\Pi_0 = 0$, $\Pi_1 = 9$, $\Pi_2 = 16$, $\Pi_3 = 43$, $\Pi_4 = 73$. The 73 memory locations are divided into 4 mutually exclusive sets such that

Set	Memory locations
1	0,1,2,....,8
2	9,10,11,....,15
3	16,17,....,42
4	43,44,....,72

Among the nine memory locations of Set 1 , zero is stored δ_{10} =3times, 1 is stored δ_{11} = 4 times, 2 is stored δ_{12} = 2 times ; the seven locations of Set 2 store 0 δ_{20} = 2 times and 3 δ_{23} = 5 times;etc...

A summary of the memory locations is set out below :

	Value of Random Variable					
	0	1	2	3	4	5
Frequency (Set 1)	3	4	2	0	0	0
Frequency (Set 2)	2	0	0	5	0	0
Frequency (Set 3)	7	9	4	1	6	0
Frequency (Set 4)	7	6	8	2	4	3

Then to generate the random variables, if

$0 \le u \le 0.9$	put $y = a(d_1)$
$.9 \le u \le 0.97$	put $y = a(d_1 d_2 - 81)$
$.97 \le u \le 0.997$	put $y = a(d_1 d_2 d_3 - 954)$
$.997 \le u \le 1.000$	put $y = a(d_1 d_2 d_3 d_4 - 9927)$

Note · The decimal expansion of the uniform deviates generated by the computer is denoted by

$$u = .d_1 d_2 d_3 \ldots.$$

This can be done by using Subroutine RANDU [A2] as shown below :

```
      SUBROUTINE RANDU ( IX , IY , YFL )
      IY = IX * 65539
      IF ( IY ) 5,6,6
    5 IY = IY + 2147483647 + 1
```

```
6 YFL = IY
  YFL = YFL * 0.4656613E - 9
  RETURN
  END
```

Description of Parameters

IX For the first entry this must contain any odd integer number with nine or less digits. After the first entry, IX should be the previous value of IY computed by this Subroutine.

IY A resultant integer random number required for the next entry to this Subroutine. The range of this number is between 0 and 2**31.

YFL The resultant uniformly distributed, floating point, random number in the range 0 to 1.0

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APPENDIX B

```
OPTIMAL FEEDBAGK CONTROL BY DYNAMIC PROGRAMMING ( MORNING-PEAK HOUR )
FORTRAN IV G LEVEL  21            MAIN            DATE = 75130       12/01/27        PAGE 00
      DIMENSION V1(15,15,25),V2(15,15,25),AA(18,14),BB(14,14)
      INTEGER RV1,RV2,T,A,B,C,D
      IOUT=3
      DO 441 L=1,15
      DO 442 K=1,15
      V1(L,K,1)=0.0
      V2(L,K,1)=0.0
  442 CONTINUE
  441 CONTINUE
      RO1=0.25
      RO2=0.125
      DO 199 I=1,18
       K=1
 1000 CALL ACAL(I,K,AA)
      K=K+1
      IF(K.LE.14) GO TO 1000
  199 CONTINUE
      DO 299 I=1,14
      AA(I,I)=1.0
  299 BB(I,I)=1.0
      BB(2,1)=-AA(2,1)*BB(1,1)
      DO 98 M=3,14
      N=1
  300 I=N
      SUM=0.0
  200 SUM=SUM+AA(M,I)*BB(I,N)
      I=I+1
      IF(I.LE.(M-1)) GO TO 200
      BB(M,N)=-SUM
      N=N+1
      IF(N.LE.(M-1)) GO TO 300
   98  CONTINUE
      T=2
  887 N1=0
  889 N2=0
  888 BANG=0.0
      DUNG=0.0
      P1Q2V1=0.0
      P1P2V2=0.0
      P1P2V1=0.0
      Q1P2V2=0.0
      DO 100 I1=1,15
      M1=I1-1
      DO 2031 J1=1,15
      M2=J1-1
      K1=M1-N1
      L1=M2-N2
      K2=M1-N1+4
      L2=M2-N2+2
      A=0
      IF(K1.EQ.0)A=1
      IF(K1.EQ.1)A=5
      IF(K1.EQ.2)A=10
      IF(K1.EQ.3)A=10
      IF(K1.EQ.4)A=5
      IF(K1.EQ.5)A=1
      B=0
      IF(L1.EQ.0)B=1
```

```
      IF(L1.EQ.1)B=2
      IF(L1.EQ.2)B=1
      D=0
      IF(L2.EQ.0)D=1
      IF(L2.EQ.1)D=2
      IF(L2.EQ.2)D=1
      C=0
      IF(K2.EQ.0)C=1
      IF(K2.EQ.1)C=5
      IF(K2.EQ.2)C=10
      IF(K2.EQ.3)C=10
      IF(K2.EQ.4)C=5
      IF(K2.EQ.5)C=1
      P1=(1./5.)**(K1)*(4./5.)**(5-M1+N1)*A
      P2=(1./8.)**(L1)*(7./8.)**(2-M2+N2)*B
      IF(N1+4.LT.M1) GO TO 800
      IF(N1.GT.4) GO TO 801
      IF(M1.EQ.0) GO TO 802
      GO TO 803
800   Q1=0.0
      GO TO 400
801   Q1=(1./5.)**(K2)*(4./5.)**(5+N1-M1-4)*C
      GO TO 400
802   IF(N1.EQ.0) GO TO 3030
      SUMQ1=0.0
      J=N1
290   SUMQ1=SUMQ1+AA(J,N1)*EXP(-RO1*J)*(RO1**(J-N1))
      J=J+1
      IF(J.LE.4) GO TO 290
      Q1=SUMQ1
      GO TO 400
3030  Q1=1.0
      GO TO 400
803   IF(N1.EQ.0) GO TO 800
      SUMQ11=0.0
      L=1
111   SUMQ11=SUMQ11+BB(M1,L)*AA(4+L,N1)
      L=L+1
      IF(L.LE.M1) GO TO 111
      Q1=EXP(-4*RO1)*(RO1**(4+M1-N1))*SUMQ11
400   IF(N2+2.LT.M2) GO TO 401
      IF(N2.GT.2) GO TO 402
      IF(M2.EQ.0) GO TO 403
      GO TO 404
401   Q2=0.0
      GO TO 600
402   Q2=(1./8.)**(L2)*(7./8.)**(2+N2-M2-2)*D
      GO TO 600
403   IF(N2.EQ.0) GO TO 4040
      SUMQ2=0.0
      J=N2
490   SUMQ2=SUMQ2+AA(J,N2)*EXP(-RO2*J)*(RO2**(J-N2))
      J=J+1
      IF(J.LE.2) GO TO 490
      Q2=SUMQ2
      GO TO 600
4040  Q2=1.0
      GO TO 600
```

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```
404   IF(N2.EQ.0) GO TO 401
      SUMQ22=0.0
      L=1
444   SUMQ22=SUMQ22+BB(M2,L)*AA(2+L,N2)
      L=L+1
      IF(L.LE.M2) GO TO 444
      Q2=EXP(-2*RO2)*(RO2**(2+M2-N2))*SUMQ22
600   P1Q2V1=P1Q2V1+P1*Q2*V1(I1,J1,T-1)
      P1P2V2=P1P2V2+P1*P2*V2(I1,J1,T-1)
      P1P2V1=P1P2V1+P1*P2*V1(I1,J1,T-1)
      Q1P2V2=Q1P2V2+Q1*P2*V2(I1,J1,T-1)
2031  CONTINUE
100   CONTINUE
      IF(P1Q2V1-P1P2V2)65,75,75
65    BANG=BANG+P1Q2V1
      RV1=1
      GO TO 81
75    BANG=BANG+P1P2V2
      RV1=2
81    V1(N1+1,N2+1,T)=BANG+N1+N2
      IF(P1P2V1-Q1P2V2)415,415,416
415   DUNG=DUNG+P1P2V1
      RV2=1
      GO TC 417
416   DUNG=DUNG+Q1P2V2
      RV2=2
417   V2(N1+1,N2+1,T)=DUNG+N1+N2
      WRITE(IOUT,133)N1,N2,RV1,RV2,V1(N1+1,N2+1,T),V2(N1+1,N2+1,T)
133   FORMAT(' ',I3,4X,I3,4X,I3,4X,I3,15X,F10.6,20X,F10.6)
      N2=N2+1
      IF(N2.GT.14) GO TO 777
      GO TO 888
777   N1=N1+1
      IF(N1.GT.14) GO TO 222
      GO TO 889
222   T=T+1
      IF(T.LT.25) GO TO 887
      STOP
      END
```

SYSTEM COMPUTER SIMULATION (MORNING - PEAK HOUR)

```
$JOB    ACCT-NUM,'NGUYEN   ',TIME=30,PAGES=20
        DIMENSION ML1(100),ML2(100),R(1000),X1(1000),X2(1000),Q1(1000),
       1Q2(1000)
         INTEGER Y,R,X1,X2,Q1,Q2
        IIN=5
        IOUT=6
        DO 199 I=1,72
        READ(IIN,27)ML1(I)
27      FORMAT(I1)
199     CONTINUE
        DO 299 I=1,54
        READ(IIN,37)ML2(I)
37      FORMAT(I1)
299      CONTINUE
        X1(1)=0
        X2(1)=0
        R(1)=2
        IY1=4561
        IY2=7879
        DO 160 I=1,999
        IF(X1(I).GE.13) GO TO 30
        IF(X2(I).GE.13) GO TO 30
        IF(R(I).EQ.1) GO TO 25
        GO TO 26
25      IF(X2(I).GT.0) GO TO 20
        GO TO 21
20      R(I+1)=1
        GO TO 30
21      R(I+1)=2
        GO TO 30
26      IF(((X2(I)-12.5).LT.0.0).AND.((X2(I)+1.84*X1(I)-19.7).LT.0.0).AND.
       1((X2(I)-5.66*X1(I)-0.6).LT.0.0)) GO TO 66
        R(I+1)=1
        GO TO 30
66      R(I+1)=2
30      IX1=IY1
        CALL RANDU(IX1,IY1,YFL1)
        U=YFL1
        IF(U.GE.0.0.AND.U.LT.0.9) GO TO 1001
        IF(U.GE.0.9.AND.U.LT.0.97) GO TO 1002
        IF(U.GE.0.97.AND.U.LT.0.997) GO TO 1003
        IF(U.GE.0.997.AND.U.LT.1.0) GO TO 1004
1001     LY=U*10
        IF(LY.NE.0) GO TO 100
        Y=0
        GO TO 500
100     Y=ML1(LY)
        GO TO 500
1002    LY=U*100
        Y=ML1(LY-81)
        GO TO 500
1003    LY=U*1000
        Y=ML1(LY-954)
        GO TO 500
1004     LY=U*10000
        Y=ML1(LY-9927)
500     Q1(I+1)=Y
        IX2=IY2
        CALL RANDU(IX2,IY2,YFL2)
        U=YFL2
```

```
      IF(U.GE.0.0.AND.U.LT.0.9) GO TO 2001
      IF(U.GE.0.9.AND.U.LT.0.98) GO TO 2002
      IF(U.GE.0.98.AND.U.LT.0.998) GO TO 2003
      IF(U.GE.0.998.AND.U.LT.1.0) GO TO 2004
 2001 LY=U*10
      IF(LY.NE.0) GO TO 400
      Y=0
      GO TO 600
 400  Y=ML2(LY)
      GO TO 600
 2002  LY=U*100
      Y=ML2(LY-81)
      GO TO 600
 2003  LY=U*1000
      Y=ML2(LY-963)
      GO TO 600
 2004  LY=U*10000
      Y=ML2(LY-9945)
 600  Q2(I+1)=Y
      X1(I+1)=X1(I)+Q1(I+1)-4*(R(I+1)-1)
      IF(X1(I+1).LE.0) X1(I+1)=0
      IF(X1(I+1).GE.13) R(I+2)=2
      X2(I+1)=X2(I)+Q2(I+1)-2*(2-R(I+1))
      IF(X2(I+1).LE.0) X2(I+1)=0
      IF(X2(I+1).GE.13) R(I+2)=1
      WRITE(IOUT,10) X1(I+1),X2(I+1),R(I),R(I+1),I,Q1(I+1),Q2(I+1)
 10   FORMAT(' ',7I15)
 160  CONTINUE
      STOP
      END
```

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