Essays in Monetary Economics

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Abstract

Chapter 1.—This chapter addresses model specification uncertainty using the Bayesian Generalized Method of Moments (GMM). Employing Canadian data, I estimate 64 hybrid New Keynesian models which differ in their lag specification, and use a modified GMM quadratic function to produce model posteriors. I compute optimal discretionary policies for each model and then derive a posterior-weighted policy and loss. My results show that i) policy should respond more to the output gap than inflation, ii) a more aggressive policy is prescribed for the period of stagflation in the 1970s and early 1980s and iii) a relatively light-touch policy is recommended during the Great Moderation, and produces better outcomes. This last result supports the hypothesis of 'good luck' over 'good policy'.

Chapter 2.—In this chapter I develop an inverse control procedure to recover the underlying preferences of a monetary authority engaged in discretionary policymaking. I adjoin the first-order condition (FOC) of the optimal interest rate rule-setting derived under discretion to the usual least squares moment conditions during the GMM procedure. Using Monte Carlo simulations, I show that the preferences on output gap stabilization and interest rate smoothing may be recovered. Robustness reveals that recovering the preference on the output gap is dependent upon policy actions having sufficient effect on the macroeconomy. Further testing indicates that the procedure functions for alternative starting values, may be adapted to different lag specifications of the underlying model, and is able to recover different sets of policy preferences.

Chapter 3.—This chapter tests the hypothesis that the monetary authorities of Canada, the United States and the United Kingdom have exhibited similar preferences over stabilizing the output gap and smoothing the interest rate, by way of an inverse control algorithm (FOC-based GMM) for a discretionary policymaker. For the sample period covering 1968:1-2006:4, the FOC-based provides comparable structural estimates to a benchmark specification using an instrument-based GMM. The data suggest no role for output stabilization in any country, but a large and significant concern for interest rate smoothing is observed in Canada. Measures of fit reject optimality in the United States for baseline specification sample, but do not preclude it in any country when sample periods are restricted to the current mandates. Policymakers' reaction functions are shown to be sensitive to the underlying policy preferences, though decreasingly so at high levels of interest rate smoothing. Robustness is seen with respect to starting values and fixed policy coefficients.

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General Introduction

This thesis consists of three essays on the use of the generalized method of moments (GMM) in monetary economics. The first chapter is a self-contained essay which shows how the GMM score can be used to derive a pseudo-likelihood measure and thus be used to evaluate specification uncertainty in a New Keynesian framework. It then proceeds to apply this to the Canadian context. The second and third chapter consider the empirical assessment of policymakers' preferences vis-à-vis macroeconomic outcomes, namely inflation and output stabilization, as well as the smoothing of the path of interest rates over time. In the second chapter, I demonstrate how the first-order condition implied by an optimizing discretionary policymaker can be joined to the least-squares normal equations of the GMM system to provide to recover his concern over stabilizing output and smoothing the path of the interest rate. The third chapter is an empirical application of this method, which provides estimates of the policy preferences of the central banks of the United States, Canada, and the United Kingdom. This introductory section provides a brief overview of the three chapters.

Chapter 1 is concerned with model uncertainty. Econometric analysis which is conditioned on a given model, i.e., takes a model as a granted, potentially falls into the trap highlighted as early as Brainard (1967) of advocating ineffective or, more worryingly, counterproductive policy actions. Without accounting for the fact of competing representations of the economic environment, even a well-fitted model could lead a policymaker astray.

Much progress has been made in respect of accounting for model uncertainty; first, in articulating the dimensions in which it exist, local or global, where the former concerns models which lie within a small ‘radius’ of a central specification, and the latter allowing non-nested models to be considered together; second, in designing policies which are robust to misspecification, by calculating their outcome dispersion and action dispersion (Brock et al., 2007). However, in considering the popular empirical New Keynesian model, limitations persist in obtaining data on private agents’ expectations, which form a touchstone of the model’s dynamics, but also in selecting the appropriate lag specification.

It is known that the GMM procedure, under the assumption of rational expectations, can instrument for expectations, by exploiting the orthogonality between these and the errors in prediction. The novelty in chis chapter is to borrow from Yin (2009) constructing a pseudo-likelihood (i.e., a measure of a model’s fit within a space of candidate models) based on the score function of the GMM procedure, and to therefore allow the procedure to accommodate model uncertainty. As an application, I use Canadian data for the period spanning 1962:2-2013:4, and evaluate the posterior probability of 64 empirical New Keynesian models which differ in their lag specification. My results show that, while the candidate models are relatively similar in their probabilities, they entail different dynamics and, therefore, different optimal policies for a discretionary policymaker. I thus present a weighted policy that is portable across the model space. I go on to show that, depending on the period under consideration, the weighted policy is either more assertive (during the era of stagflation) or light-touch (over the span of the Great Moderation), and that the losses incurred by the policymaker are, respectively, larger and smaller over these periods. These findings are robust to a different measure of the interest rate, output gap, and when the policymaker is operating under pure inflation targeting.

The assumption of the policymaker’s preferences over managing the economy, i.e., whether they load on stabilizing inflation, or the output gap, smoothing the path of the interest rate, or some combination of these, need not be taken as a conditioning element of analysis either. To this end, Chapter 2 extends the literature on inverse control, meaning the *simultaneous* estimation of structural and preference parameters, subject to an optimizing policymaker.

Previous work by Givens and Salemi (2008) had adapted the GMM framework to do such simultaneous estimation for an optimizing policymaker operating with simple Taylor-type rule. Instead, I retain the assumption a discretionary policymaker. When behaving optimally, the policymaker acts according to a well-defined first-order condition (FOC), namely the interest rate rule to which he is bound. When adjoining this FOC to the least squares normal equations of the equilibrium system, it becomes possible to retrieve his preference profile. This occurs because in arriving at the optimal interest rate rule, the vector of coefficients which lay out the interest rule carries through information regarding the preference parameters.

I demonstrate the ability of this FOC-based GMM procedure to recover preference profiles by way of a battery of Monte Carlo simulations and robustness tests. I show the estimates converge to the true values of the structural and policy preferences as the sample size increases. The size of the policy coefficient, which governs the ability of the policymaker to influence the macroeconomy, is inversely related to the bias on the output gap stabilization parameter, but does not impair the ability to retrieve the preference for interest rate smoothing. The procedure is also not dependent on a particular starting values (though care must be taken to allow the search algorithm to find the saddle path of the equilibrium system) and can accommodate different lag specifications of the empirical New Keynesian model. Finally, the procedure can distinguish between different policy preference sets, such as the 'unipolar' mandate of strict inflation management, or the 'bipolar' mandate which places equal concern on stabilizing inflation and the output gap.

The 'unipolar' and 'bipolar' mandates, and those somewhere in between, are exemplified by the mandates of the Bank of Canada, the Federal Reserve, and the Bank of England, respectively. And yet these countries have followed a similar similar macroeconomic evolution with respect to their experience with inflation, the output gap and interest rate, since the late 1960s. With the procedure just derived, it is possible to formally test whether or not these central banks exhibited identical policy preference profiles over this period. This forms the concern of Chapter 3.

I begin by presenting benchmark estimates of the structural estimates of each country's New Keynesian system using the method of Chapter 1. I then repeat this exercise with the FOC-based method of Chapter 2, which prove comparable, and provide estimates of the policy preference parameters. My results suggest that none of these countries exhibited a concern for output gap stabilization over the period spanning 1968:1-2006:4. However, Canada displays a large and significant concern for interest rate smoothing which is not shared by the United States or the United Kingdom. Moreover, optimality on the part of the discretionary policymaker is not rejected for Canada or the United Kingdom. Robustness tests indicate that the interest rate rule, which forms the FOC of interest, varies substantially with different underlying preference parameters, confirming that the procedure should be able to reveal the preferences of these monetary authorities. The results are robust to different starting values and using the (higher) policy coefficient estimates obtained with the instrument-based method. Finally, a formal comparison of the stated preferences of the policymakers with what the inverse control procedure reveals is effected by limiting the sample periods to those which begin at the time of adoption of the current mandates. The results are still broadly similar: no concern for output gap stabilization is detected among the three countries, which is directly at odds with the mandate communicated by the Federal Reserve, and the Bank of Canada is estimated to have a large point estimate on its interest-rate smoothing preference parameter.

Throughout these chapters, I have maintained the assumption that the policymaker under scrutiny is one of the discretionary type. Of course, the monetary economics literature presents both theoretical and empirical arguments for a different type of policymaking strategy, namely commitment, and the debate over which style better fits the data remains unresolved (Givens, 2012). To this end, further work to develop a comparable GMM procedure to retrieve the policy preferences of an optimizing commitment-type policymaker would prove valuable, and I intend to pursue this research in the future.

Chapter 1

Model Uncertainty and Weighted Policy using GMM

1.1 Introduction

In standard econometric analysis, a model is chosen ex-ante based on theoretical considerations and fit to available data. Testable implications of a given model are rejected or kept based on the goodness of fit; policy rules may be decided against this same criterion. In this context, the model is effectively a conditioning element of the analysis, with important implications for policymaking. If, for instance, the monetary authority computes the optimal policy for a misspecified — but well-fitted — model, there is a risk of underestimating the losses from putting such a policy into motion. Consider a model whose optimal policy suggests that small changes in the interest rate are likely to produce a large response in the inflation rate. Taking the model as a conditioning element for granted leaves open the possibility that, in fact, a far more forceful intervention is necessary to generate the desired macroeconomic response.

In reality, the policymaker is not aware of the true data-generating process, but is equipped to evaluate a number of candidate models. The Bank of Canada, for instance, employs many researchers to compute the probable effect of different policies under different models. The monetary authority ultimately acts on its levers with an evaluation of what outcome it considers most likely. Bayesian model averaging is a tool to formally weight competing models as the true data-generating process, and so provide policy guidance that accounts for this uncertainty.

In this paper, I demonstrate how the generalized method of moments (GMM) estimator may be used in a Bayesian context to compute model posteriors and thus account for model uncertainty. This is useful, in particular, when instrumenting for forward-looking expectations, if such data is missing or lacking, as in Canada. While GMM is a popular method for estimating the parameters of New Keynesian models, particularly those of the Phillips curve, it has not to my knowledge been employed in the context of specification uncertainty.[1] As an application of this method, I explore model uncertainty as faced by the monetary authority in Canada by evaluating 64 semi-structural hybrid New Keynesian models of the type used by Rudebusch (2002). Each candidate models differs from the next in terms of its lag specification. Under the assumption of rational expectations, the GMM estimator grants the ability to instrument for expectations of inflation and the output gap, data which is not as complete in Canada as it is in the United States. Unrestricted by the availability of data on expectations, I am able to evaluate the candidate models over longer periods of time and under different subsamples.

Presupposing the monetary authority engages in discretionary policymaking, I compute the optimal policy for each model using the method of Söderlind (1999). These policies are combined according to the models' posteriors, thereby producing a weighted policy that accounts for each candidate model's unique reduced-form dynamics. My results show that a posterior-weighted policy entails a strong reaction to the output gap relative to inflation, along with a significant interest rate smoothing term. I also investigate the weighted policy

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[1] Mavroeidis et al. (2014) provide a comprehensive review of papers employing GMM estimators for the New Hybrid Phillips Curve (similar to the Phillips Curve form used in this paper), as well as the challenges of identification and weak instrumentation.

under two subsamples: one for the stagflation era and another for the Great Moderation. In the former, a much more aggressive optimal policy is found using the Bayesian GMM procedure. In the latter, an aggressive weighted policy emerges as well, but with the benefit of delivering the lowest average losses across the three samples; this last result is suggestive of 'good luck' over 'good policy'. My findings are robust to an alternative measure of the interest rate, using a linear trend to estimate potential GDP and the output gap, and when the policymaker operates under pure inflation targeting.

The rest of this chapter is organized as follows. Section 2 surveys the literature on model uncertainty and its applications to monetary economics, and the empirical literature on GMM estimation of New Keynesian Phillips curves. In section 3, I describe how model uncertainty is treated formally, the New Keynesian models to be estimated and how the GMM estimator can be used to produce model posteriors. I describe the data and instrument set used for the Bayesian GMM procedure in section 4. Section 5 discusses the results: model posteriors and weighted-optimal policies. In Section 6, I examine the results under two subsamples and provide robustness checks. Section 7 concludes.

1.2 Literature Review

In this section I sketch the history of model uncertainty in monetary economics. I also discuss the use of the GMM estimator for hybrid New Keynesian models, specifically in estimating the Phillips curve.

Concerns about model uncertainty in monetary economics are not new. Brainard (1967) provides a theoretical overview of the difficulties associated with model uncertainty, in the case where the reference model is correct but there is uncertainty regarding the effect of policy and of exogenous variables. Brainard shows that robust policy must account for more than the expected value of the parameters of interest, and will be less aggressive than optimal policy in the case of certainty equivalence.

Attempts to quantify robust policy followed. McCallum (1988) prescribes a policy rule that targets 3 percent growth of nominal GNP — the non-inflationary target implied for the U.S. by the natural rate hypothesis — while controlling for changes in velocity and deviations from the target path. This rule is assessed in several competing models and evaluated based on the deviation of nominal GNP from the target path throughout the simulation period. This policy rule is found to perform similarly well across VAR, classical and Keynesian models. Taylor (1993) provides what is perhaps the best-known simple rule designed to perform well across diverse specifications, requiring the policymaker to set the interest to one-and-a-half times the inflation rate plus half the output the gap of the current period; this rule remains a benchmark against which other policies are assessed.

However, these exercises provide no formal framework for evaluating model uncertainty and computing robust policies. Broadly-speaking, uncertainty may be ‘local’ or ‘global’. In the former case, a reference model is surrounded by other candidates which differ in terms of their parameter values, shocks or non-parametric entropy (i.e., distance from the reference model). The minimax criterion is often advocated to determine robust policy in this framework. Conceptually, the minimax represents a zero-sum game between two players, Nature and the policymaker. Nature selects the model which generates the maximum losses, while the policymaker chooses the policy which minimizes loss, given Nature’s choice. This can be understood as a ‘worst-case scenario’ policymaking. Examples of this approach to model uncertainty include Onatski and Stock (2002), Giannoni (2007) and Hansen and Sargent (2008).

Levin and Williams (2003) extend the discussion of uncertainty to a ‘global’ context, wherein models are discrete, non-nested specifications. Their candidate models comprise a forward-looking New Keynesian model, a purely backwards-looking model and a hybrid specification that permits inflation to respond to expectations while displaying inertia, a common empirical finding. They apply a global policy rule to three competing models, each granted a one-third weight. Their results suggest that policies which place weight on both inflation and output stabilization perform well across discrete specifications.

Brock et al. (2003, 2007) employ Bayesian Model Averaging methods to formally weight candidate models according to their posterior probabilities. They investigate the Rudebusch (2002) formulation of the New Keynesian model under specification uncertainty. Their model space includes both backward-looking and hybrid forms of the model, differing in terms of lag specification. Two criteria for evaluating robust policies are suggested: outcome dispersion, which measures how the losses of a given policy vary across the models under consideration; and action dispersion, which measures the change in optimal policy when moving between models. Brock et al. show that the Taylor (1993) rule performs well across the suite of models, although their robust policy, weighted according to each candidate's posterior probability, is more aggressive.

This approach has gained traction. Cogley and Sargent (2005) model the monetary authority as a robust decision-maker that reassess the probabilities of its competing models of the economy in each period, in light of new data. Three candidate models are considered: the first allows both a short- and long-run trade-off between inflation and unemployment, the second a short-run exploit, while the third is a natural rate model where only unexpected inflation matters for unemployment (the Lucas-Sargent model). They find that, while the data indicate that the inflation expectations model was highly likely to be the correct one by the early 1970s (entailing a zero inflation optimal policy) the downside risk of misspecification meant that robust policy required maintaining positive inflation and gradual disinflation throughout the late 1970s and early 1980s, so as to avoid high rates of unemployment implied by the other models. Similarly, Cogley et al. (2011) use Bayesian methods to evaluate four models with UK data, while explicitly accounting for parameter uncertainty as well as model uncertainty. Their results suggest that backwards-looking models have lower posterior probabilities, while forward-looking models exhibit less fault tolerance, i.e., they generate large losses when policies designed for backwards models are applied.

As the purpose of this paper is to apply the GMM estimation method in a Bayesian context, it is worth mentioning that GMM has been used extensively to assess the fit of New Keynesian Phillips curves (NKPC). Since these specifications include a forward-looking term, they cannot be estimated by OLS, but can be estimated by GMM using moment conditions. Among the most important contributions is the paper by Galí and Gertler (1999), who allow a role for inflation inertia to resolve the poor fit of forward-looking NKPC curves. They find that forward-looking behaviour dominates but with a non-negligible role for inflation persistence. Galí et al. (2001) reach similar conclusions looking at Euro area data, though they find evidence for relatively stronger forward-looking component to inflation dynamics. A detailed survey on empirical evidence for the New Keynesian Phillips curve is provided by Mavroeidis et al. (2014), who discuss the implementation of GMM. These authors raise concerns about weak instrumentation and identification. The moment condition requires that the inflation forecast error be uncorrelated to the information set, which itself should have predictive content for next period inflation. Lagged data is used to instrument, yet under a rational expectations framework, agents' forecasts should fully incorporate available data. Thus the instruments, while plausibly exogenous, may lead to weak identification. As noted by Stock et al. (2002), to the extent that identification is weak, GMM estimators will be biased to their OLS counterparts. Nevertheless, the GMM approach remains decidedly popular.

1.3 Bayesian GMM evaluation of model uncertainty

1.3.1 Bayesian model averaging

In a monetary framework, the central bank wishes to analyze the effect of a policy p on some macroeconomic outcome of interest θ, such as the output gap or realized inflation. Employing a model m and data set d, the conditional probability measure $p(\theta|p, m, d)$ summarizes the response of the variable(s) of interest to a given policy. However, this assumes a model: m is treated as a conditioning element. The policymaker does not, in practice, have access to the data-generating process for θ. If the model is misspecified, the actual effect of implementing p may be unexpected. To account for model uncertainty, m must be integrated out as a conditioning element, and treated as an unobservable like parameters or innovations (Brock et al., 2007). The probability measure of interest becomes:

$$p(\theta|p,d) = \sum_m p(\theta|p,m,d)p(m|d) \tag{1.1}$$

Thus the effect of a policy p on outcome θ is the sum of outcomes across all models, weighted by the posterior probability attached to each model, $p(m|d)$. Bayes' rule is used to determine the posteriors, according to:

$$p(m|d) \propto p(d|m)p(m) \tag{1.2}$$

1.3.2 Generalized Method of Moments (GMM) and Likelihood

The generalized method of moments was introduced by Hansen (1982). Given a model $Y_i = X_i'\beta + u_i(\beta)$, we may rewrite it in terms of its error $u_i(\beta) = Y_i - X_i'\beta$. We suspect some endogeneity in the Xs, i.e. that they are correlated with the error term u_i, so that $E(u_i(\beta)X_i) \neq 0$. We choose a set of instruments Z_i which are correlated with the Xs but uncorrelated with our error term:

$$E(u_i(\beta)Z_i) = 0 \tag{1.3}$$

Then we can define $g_i(\beta) = u_i(\beta)Z_i$ which yields $E(g_i(\beta_0)) = 0$ at the true value of the parameter, β_0. This is the moment-matching (or orthogonality) condition. Given that we are working with samples, this condition is replaced with the sample average:

$$h(Y_i, X_i; \beta) = T^{-1}\sum_{i=1}^{n} g_i(\beta) \tag{1.4}$$

As there are typically more equations in $h(Y_i, X_i; \beta)$ than parameters to estimate, it is necessary to minimize the weighted sum-of-squares of the above, Q, instead of solving for each parameter explicitly:

$$\hat{\beta}_n = \underset{\beta}{\operatorname{argmin}}\, Q(\beta) = \underset{\beta}{\operatorname{argmin}} \left[\sum_{i=1}^{n} h(Y_i, X_i; \beta)\right]' S^{-1} \left[\sum_{i=1}^{n} h(Y_i, X_i; \beta)\right] \tag{1.5}$$

where S^{-1} is the optimal weighting matrix. The two-step GMM procedure is adopted: the minimization is initially performed with an identity weighting matrix, and in a second step, S^{-1} is redefined as the inverse of the moment covariance matrix derived from the first step estimates.

Yin (2009) shows that, when evaluated at $\hat{\beta}$, the objective function of the GMM follows a chi-squared distribution and behaves like $-2\log\{p(d|\beta_0)\}$. From this a pseudo-likelihood is easily retrieved and posteriors can be constructed:

$$\tilde{L}(d|\hat{\beta}) \propto \exp\left\{-\frac{1}{2}Q(\hat{\beta})\right\} \tag{1.6}$$

Thus, a higher (pseudo-)likelihood is assigned when the objective function is closer to zero at the estimated value of the parameters. Model posteriors are then generated according to

$$p(\beta^j|d) \propto \tilde{L}(d|\beta^j)p(\beta^j) \tag{1.7}$$

and policy effects weighted according to (1.1).

1.3.3 New Keynesian model space

To illustrate the use of the GMM estimator in the context of model uncertainty, I evaluate sixty-four empirical New Keynesian model specifications according to the method described in the previous section. These models are IS-Phillips curve systems that follow the Rudebusch

(2002) representation used by Brock et al. (2007), with forward-looking expectations in the IS curve as in Castelnuovo et al. (2006):

$$y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)\sum_{j=1}^{n} a_j y_{t-j} - b(\bar{i}_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + \eta_t \tag{1.8}$$

$$\pi_t = \alpha E_{t-1}\bar{\pi}_{t+3} + (1-\alpha)\sum_{j=1}^{n} c_j \pi_{t-j} + \sum_{j=1}^{n} \kappa_j y_{t-j} + \varepsilon_t \tag{1.9}$$

$$\text{subject to } \lambda + (1-\lambda)\sum_{j=1}^{n} a_j = 1 \text{ and } \alpha + (1-\alpha)\sum_{j=1}^{n} c_j = 1.$$

The first equation (the IS curve) relates the output gap, y_t, to expectations of the output gap in the next period, lags of the output gap and the difference between the average interest rate in the previous four quarters, $\bar{i}_{t-1}$ and the average expected inflation rate over the next four quarters, $E_{t-1}\bar{\pi}_{t+3}$. The second equation (the PC curve) relates inflation, π_t, to expected inflation over the next four quarters and lags of inflation and the output gap.

In the formulation above, the policymaker can only influence the output gap insofar as there is a difference between expected inflation and the interest rate. The parameters λ and α control the relative contribution of forward-looking expectations against inertia.[2] The theoretical models of price-setting behaviour which give rise to the New Keynesian Phillips curve (Taylor, 1980; Rotemberg, 1982; Calvo, 1983) imply $\alpha \simeq 1$, though many argue for the existence of inflation persistence due to frictions in price-setting, or overlapping price and wage contracts.[3] Lagged output terms in the IS curve are justified on empirical grounds and can be thought of as habit formation in consumption (Fuhrer and Rudebusch, 2004).

Model uncertainty, more precisely specification uncertainty, is introduced by allowing variation in the number of lags of the output gap that enter in (1.8) and (1.9), as well as the lags of inflation in (1.9). For each variable, there may be up to four lags, for a total

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[2] The first restriction, $\lambda + (1-\lambda)\sum_{j=1}^{n} a_j = 1$, is an extension of that found the habit-formation literature, e.g. Svensson (1999) Fuhrer (2000). The second restriction denies the possibility of a long-run trade-off between inflation and the output gap.

[3] Rudebusch (2002) and Mavroeidis et al. (2014) provide discussions on the role of inertia in the Phillips Curve.

of 64 candidate models. Different formulations of the lag structure are used to i) capture any serial correlation in the model's errors and ii) to identify the extent of output gap and inflation inertia. If models with more lags of the output gap or inflation generate larger posteriors, that will provide evidence for greater inertia in the economy's dynamics.

The models may be collected in a space:

$$M = m_1, m_2, ..., m_j \text{ for } j = 1, ..., 64$$

where each m_j corresponds to a different version of the model above, in terms of its lag structure. Table 1.5 in Appendix A identifies each model according to the number of y lags in the IS equation, and the number of y lags and π lags in the Phillips Curve equation.

As an alternative to using survey-based data, I proceed by way of GMM estimation, under the assumption of rational expectations (RE). Under RE, expected inflation is decomposed into realized inflation and the inflation expectation error. The error-in-forecast becomes part of the equation system's error terms. Then, (1.8) and (1.9) may be rewritten as:

$$\begin{aligned} y_t = {} & \lambda y_{t+1} + (1-\lambda)\sum_{j=1}^{n} a_j y_{t-j} + b(\bar{i}_{t-1} - \bar{\pi}_{t+3}) \\ & + \underbrace{\eta_t + \lambda E_{t-1} y_{t+1} - \lambda y_{t+1} - bE_{t-1}\bar{\pi}_{t+3} - b\bar{\pi}_{t+3}}_{h_{IS,t}} \end{aligned} \tag{1.10}$$

$$\pi_t = \alpha\bar{\pi}_{t+3} + (1-\alpha)\sum_{j=1}^{n} c_j \pi_{t-j} + \sum_{j=1}^{n} \kappa_j y_{t-j} + \underbrace{\varepsilon_t + \alpha E_{t-1}\bar{\pi}_{t+3} - \alpha\bar{\pi}_{t+3}}_{h_{PC,t}} \tag{1.11}$$

The composite errors $h_{IS,t}$ and $h_{PC,t}$ now include the expectation errors. Under rational expectations, agents' forecast error at t should be uncorrelated with the information set available at t, I_t. Likewise, the GMM moment conditions require that the inflation forecast error be uncorrelated with a set of instruments Z_t with predictive power for next period inflation. Z_t will include a subset of data from the information set at time t such that $Z_t \in I_t$. Instruments are drawn from lagged data on inflation, interest rates, wage inflation, and so on. In a multiple-equation setting, let

$$g_t(Z_t) = \begin{bmatrix} h_{IS,t}(Z_t) \\ h_{PC,t}(Z_t) \end{bmatrix} \tag{1.12}$$

such that $E[g_t(Z_t)] = 0$ is satisfied in each period t.

The models m_j defined above can be uniquely identified by their respective parameter vectors. Starting from the model with maximum lags,

$$\beta^{64} = [\lambda, a_1, a_2, a_3, a_4, b, \kappa_1, \kappa_2, \kappa_3, \kappa_4, \alpha, c_1, c_2, c_3, c_4] \tag{1.13}$$

every other model in the space can be obtained by setting the relevant restrictions on the components of β, i.e., constraining the value of certain lags to 0. Thus, β^j can replace m_j in uniquely identifying the models. These restrictions also affect the minimization of each model's objective function, generating unique pseudo-likelihoods which can be used to derive model posteriors, as described in the previous section.

The motivation to adopt the Bayesian GMM procedure in this case comes from the lack of conventional quarterly data series on expectations in Canada. To be clear, survey-based measures are available: the Conference Board of Canada publishes an inflation expectations series which goes back to 1970. In each quarter, participants are asked to forecast the annual inflation rate for the year. Dufour et al. (2006) produce quarterly inflation expectations from this series as follows: the first quarter annual measure is divided by four, the difference between the second quarter annual measure and the realized first quarter inflation is divided by three, and so on. However, this assumes agents believe each quarter's inflation rate to be uniform over the remaining quarters.

1.4 Data

The data are quarterly and span 1962:2-2013:4. My measure of inflation is the annualized quarterly percent change in the GDP deflator. As described above, I decompose inflation expectations into realized inflation and an error-in-forecast. The interest rate i_t is the bank rate from 1962:2 to 1999:1, and the target for the overnight rate from 1999:2 to 2014:3.[4] To compute the output gap, I split real GDP into a trend and cyclical component using the Hodrick-Prescott (HP) filter. However, as noted by Dufour et al. (2006), the HP filter detrends using information from the entire sample, which is at odds with the GMM requirement that the residual at time t be orthogonal to the information set at time t. As a fix, I generate the GDP trend series for each period t using data ending in this same period (i.e., using a one-sided HP filter).[5,6]

The GMM instrument set comprises four lags of inflation, three lags of the output gap, two lags of CPI inflation and a lag of wage inflation. This set was chosen to guarantee convergence in the 2-step GMM across all models, and is broadly similar to those used in other papers (see Mavroeidis et al., 2014). The null of an optimal weight matrix is not rejected by Hansen's test of overidentifying restrictions in any of the models, providing confidence that the instrument set is appropriate and that the GMM estimator is consistent.

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[4] The midpoint of the Bank of Canada's operating band, and twenty-five basis points below the bank rate. From June 1994 onwards, the bank began to use the overnight rate as its key monetary policy instrument. In February 1999, the Large Value Transfer System was introduced and the emphasis on the target for the overnight was made clear. This rate is more appropriate for international comparisons.

[5] Compared to the two-sided filter, the one-sided filter did not yield substantially different parameter estimates of the models, but produced large losses in the stagflation subsample in specifications with four lags of the output gap in the IS equation and model 25. Losses generated when using the 2-sided filter, while not reported, were comparable to those obtained using the linear trend (see the section on robustness).

[6] Ideally, real-time data vintages should be used to ensure no future information (i.e., newer estimates of GDP in earlier periods) influences the data, but at the time of writing these were not available for periods prior to late 2012.

1.5 Weighted policy under specification uncertainty

1.5.1 The policymaker's optimization problem

For each candidate model, the policymaker chooses the interest rate so as to minimize a loss function R that accounts for the asymptotic variance in the output gap, inflation rate and changes in the interest rate between two periods (indicating a preference for smoothing the path of i_t over time).

$$\min_{i_t} \qquad R = \lambda_y var(y_\infty) + var(\pi_\infty) + \lambda_i var(\Delta i_\infty) \tag{1.14}$$

$$\text{subject to} \qquad S_{t+1} = AS_t + Bi_t + v_{t+1} \tag{1.15}$$

where (1.15) is the system of equations in state-space representation.[7] The coefficients λ_y and λ_i indicate the weight placed on output gap and interest rate stabilization, relative to inflation stabilization. I assume $\lambda_y = 1$ and $\lambda_i = 0.1$.

Solving the minimization problem above for a given model yields an optimal policy of the form:

$$i_t = f_{y1}y_t + ... + f_{\pi 1}\pi_t + ... + f_i i_{t-1} \tag{1.16}$$

The full optimal policy of each model is recorded in Tables 1.6-1.8 (according to sample) of Appendix D. The optimal policy for a given model is a function of all of the lags of the target variables (the output gap, the inflation rate and the previous period's interest rate), therefore models with more lags will feature more terms in the optimal policy rule. Following Castelnuovo et al. (2006), I calculate the sum of the policy coefficients on the lags of y_t and π_t to permit direct comparisons across models (see Tables 1.9-1.11). The objects of comparison are therefore simple rules which measure the total response to the output gap, inflation and

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[7] See Appendix C for details on how to set up the state-space representation and how to compute optimal policies and losses for each candidate model.

change in interest rate:

$$f_y = \sum_{j=1}^{n} f_{yj},\ f_\pi = \sum_{j=1}^{n} f_{\pi j} \text{ and } f_i, \text{ where } n \leq 4.$$

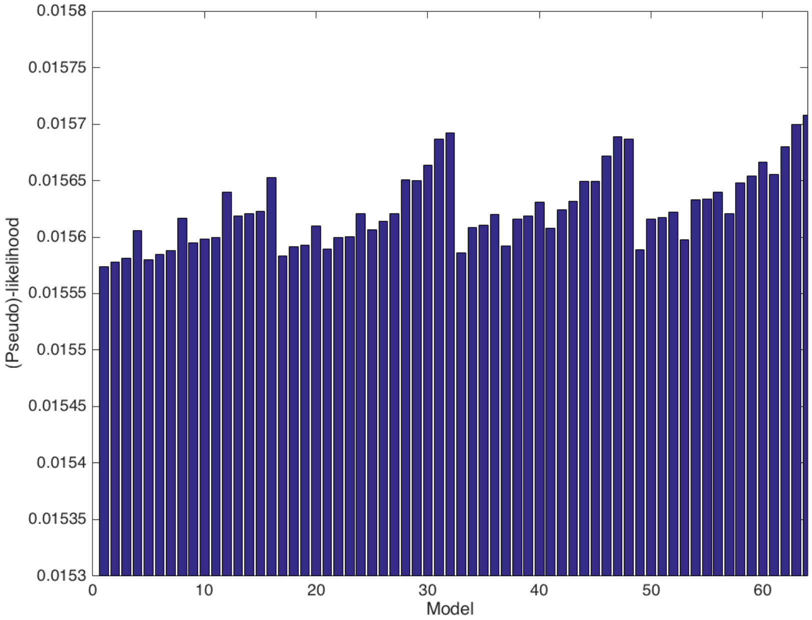
In Brock et al. (2007), the weighted policy rule represents a true robust policy — one that minimizes the average loss across the model space. However, this requires that individual model policies not be optimal, only responding to the first lags of the output gap, inflation and the interest rate. This ensures the portability of the weighted policy across the model space. By contrast, in this paper I draw on the fully optimal policies for each candidate model for the weighted policy. Since this requires different numbers of coefficients in i_t depending on the model, there is no guarantee that the weighted policy, expressed through f_y, f_π and f_i, minimizes losses on average.

1.5.2 Model posteriors and common coefficients

Adopting the methodology of Yin (2009), each model is estimated via two-step GMM. The value of the score function $Q(\hat{\beta})$ is modified according to (1.7) to obtain a pseudo-likelihood. The pseudo-likelihood, in turn, is multiplied by a uniform prior, 1/64, to obtain posteriors. These are re-scaled to obtain probabilities that add up to 1. Figure 1.1 shows each model's posteriors side-by-side. The posteriors are tightly bound, close to the uniform prior of 0.0156, and slightly increasing in model complexity. Therefore there is no reason to reject any model out-of-hand.

I begin by discussing the estimated coefficients on the common model coefficients: λ, the share of forward-looking output gap on current period output gap; b, the policy effectiveness coefficient; and α, the share of forward-looking inflation expectations on current period inflation. A scatter-plot of the estimated common coefficients can be found in Figure 1.4 (Appendix B). In the full sample, the coefficients on λ lie between 0.5 and 0.6, with a posterior-weighted mean coefficient is 0.554. Coefficients are clustered in groups of four, highest when the number of lags of the output gap in the PC equation is one or four, and lower when there are two or three lags. That is, specifications with one or four lags of the output gap in (1.4) exhibit a larger role for forward-looking expectations, while those with two or three lags show a more even contribution of expectations and inertia on the current period's output gap.

Figure 1.1: Model posteriors, full sample



For b, the policy parameter, the range of estimated coefficients is relatively small, lying between 0.039 and 0.065, while the posterior-weighted mean is 0.051. Policy effectiveness is somewhat decreased in models with one or four lags of the output gap in the IS equation (models 4-8, 12-16, 20-24, 28-32), and increased in models with two or three lags.

The plot of estimated coefficients on α — the share of forward-looking inflation expectations on current period inflation — shows values above 0.5 in all specifications, indicating that forward-looking expectations on inflation are a stronger determinant of π_t than inflation inertia. The posterior-weighted mean coefficient is 0.623. Models 1-16 and 17-32's coefficients are governed entirely by the the number of lags of inflation in the PC equation (one and two, respectively). Models 49-64, which have four lags of inflation in (1.3), show a decreasing value of α according to the number of lags of the output gap in the PC equation.

1.5.3 Weighted policy

The weighted policy is calculated by averaging the models' optimal policies (where the lagged coefficients have been summed) according to their respective posterior probabilities. Averaging the optimal policies in this manner yields the following weighted policy:

$$f_y = 1.723,\ f_\pi = 0.224,\ f_i = 0.406$$

Relative to the rule proposed by Taylor (1993), the posterior-weighted policy prescribes a stronger response to the output gap compared to inflation and incorporates a significant interest rate smoothing term.[8] At first glance, the high coefficient on the output gap appears to be due to the sample period being more more focused on the mid 1980s onwards, a period characterized by stability in inflation, and more recently, extraordinarily loose monetary policy to stimulate consumer demand. However, as this coefficient remains high even in the era of stagflation (see below), the more likely explanation is that there is strong dependence between output gap and inflation volatility, requiring the policymaker to address the former more aggressively to affect the latter.

Losses for each model given its optimal policy are recorded in Table 1.1 and range from a minimum of 13.689 in model 17 to a maximum of 35.986 in model 64. The posterior-weighted mean loss is 25.895.

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[8] The Taylor rule reads as $i_t = 0.5y_t + 1.5\pi_t$. It does not include a smoothing term on the interest rate.

Table 1.1: Losses, by model, 1962:2-2013:4 sample

Model	Loss	Model	Loss	Model	Loss	Model	Loss
1	15.531	17	13.689	33	15.213	49	14.337
2	14.314	18	13.923	34	13.95	50	14.061
3	14.257	19	14.057	35	14.08	51	14.183
4	13.927	20	13.83	36	13.861	52	19.069
5	31.494	21	31.302	37	31.557	53	30.532
6	31.854	22	34.542	38	31.085	54	31.609
7	32.114	23	34.783	39	30.955	55	31.571
8	31.546	24	33.713	40	29.613	56	35.937
9	25.843	25	25.582	41	25.732	57	24.939
10	26.161	26	28.583	42	26.781	58	27.697
11	26.249	27	28.732	43	26.672	59	27.604
12	26.017	28	28.153	44	25.478	60	31.614
13	28.07	29	28.362	45	27.793	61	27.435
14	28.561	30	31.688	46	29.809	62	29.906
15	28.617	31	32.975	47	31.09	63	31.501
16	28.443	32	32.143	48	29.653	64	35.986

Note: Losses are computed by solving the minimization problem stated in (1.14) and (1.15) for each model. See Appendix C for details on model specifications and solution procedure.

1.6 Sub-sample stability and robustness

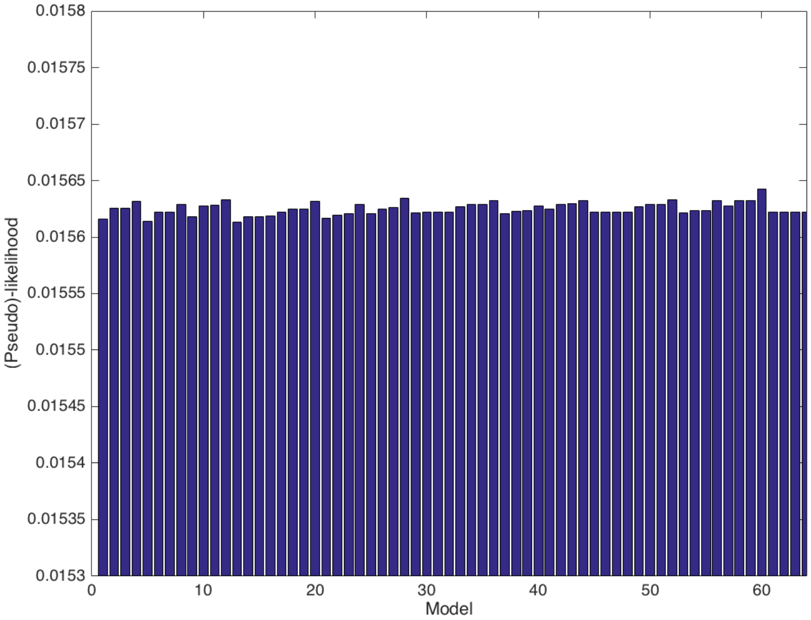
1.6.1 Stagflation: 1971:1-1983:4

The first sub-sample of interest spans the period of high inflation and high unemployment of the 1970s and early 1980s. Figure 1.2 shows the model posteriors after obtaining the pseudo-likelihood from the GMM estimation procedure. Once again, the posteriors are close to each other and there is no strong evidence for or against any particular model.

In this sample, the weighted coefficient on $\hat{\lambda}$ is 0.412, suggesting that output gap inertia is somewhat more significant determinant of the present output gap than expectations. This is true, in particular, of models with two or three lags of the output gap in the IS equation. Models with four instead display a greater role for output gap expectations, while models with one weigh the effects of expectations and inertia almost evenly.

The estimated policy response coefficients $\hat{b}$ are lower than in the full sample, with a posterior mean of 0.042. This indicates that a given policy i_t will have a somewhat weaker effect on the output gap than in the full sample, consistent with the idea of the stickiness of high inflation and high unemployment during this era. Models with one or four lags of the output gap have an estimated coefficient of approximately zero, while the specifications with two lags have $\hat{b}$ around 0.06. Models with three lags display the largest estimated coefficients on the policy response parameter, with $\hat{b}$ at approximately 0.1.

Figure 1.2: Model posteriors, 1971:1-1983:4 sample



The estimated coefficients on α show a pattern of decreasing value with the number of lags of inflation in the PC equation. The posterior weighted coefficient is 0.483, lower than in the full sample and suggestive of a greater role for inflation inertia during this period.

Summing the coefficients for lags of the output gap and inflation and applying the posterior probabilities yields the following weighted policy rule:

$$f_y = 2.868,\ f_\pi = 0.863,\ f_i = 0.412$$

Table 1.2: Losses, by model, 1971:1-1983:4 sample

Model	Loss	Model	Loss	Model	Loss	Model	Loss
1	16.778	17	65.977	33	19.193	49	21.316
2	16.913	18	16.926	34	14.893	50	15.533
3	16.915	19	17.48	35	14.742	51	15.359
4	16.712	20	38.343	36	13.98	52	14.005
5	55.934	21	248.687	37	52.781	53	58.673
6	75.254	22	61.654	38	45.824	54	48.4
7	74.889	23	63.422	39	44.282	55	47.098
8	73.313	24	78.716	40	46.332	56	46.844
9	35.093	25	2.8e3	41	38.688	57	42.229
10	58.179	26	45.594	42	37.687	58	39.326
11	58.047	27	46.907	43	37.172	59	38.902
12	66.028	28	59.576	44	38.441	60	39.191
13	9.0e3	29	2.3e3	45	2.0e3	61	4.0e3
14	3.6e3	30	3.7e3	46	2.6e3	62	4.7e3
15	1.3e3	31	1.3e4	47	2.9e3	63	3.4e3
16	1.2e3	32	1.2e4	48	6.1e5	64	1.1e6

Note: Losses are computed by solving the minimization problem stated in (1.14) and (1.15) for each model. See Appendix C for details on model specifications and solution procedure.

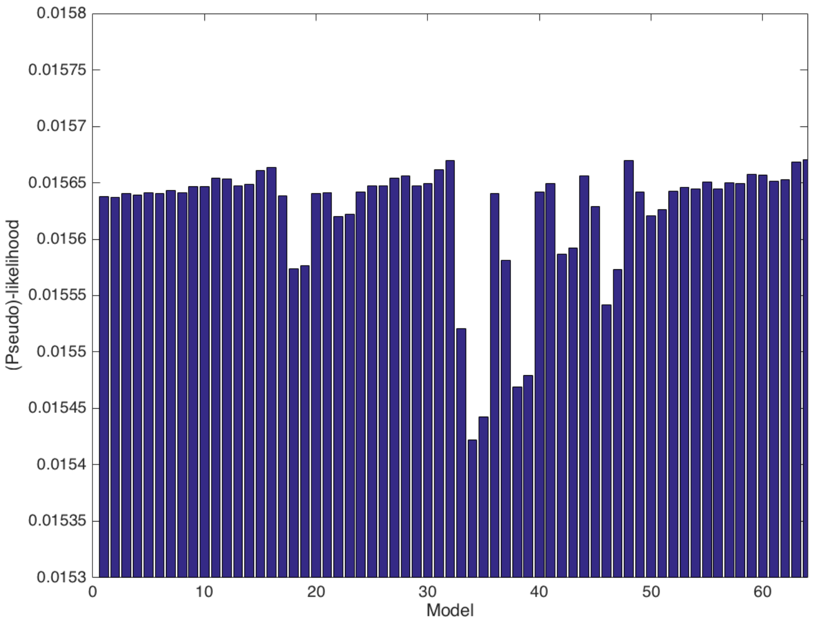
In contrast to its full sample counterpart, this rule is much more aggressive, prescribing stronger reactions to both the output gap and inflation. f_y remains more than three times as large as f_π, indicating that output gap stabilization in Canada over this period was crucial to minimizing overall volatility in the policymaker's objective function. Table 1.2 lists the losses for each model when the optimal policy is applied. The posterior-weighted loss is very high at 2.7e5; however, these large losses are driven by models 13-16, 25, 29-32, 45-48 and 61-64, in which the optimal discretionary rule does little to address the asymptotic variance of the target variables.

1.6.2 Great Moderation: 1984:1-2006:4

The second sub-sample I examine spans the Great Moderation, a period characterized by lower volatility in the output gap and inflation, and which lasted from the mid 1980s to the mid 2000s. The posteriors suggest the data provide some evidence against models 34, 35, 38, 39, 46 and 47, all of which have three lags of inflation in the PC equation. Aside from this, no model is strongly favoured.

Returning to Figure 1.4, there is a distinctive clustering of the coefficients on λ according to the number of output gap lags in the PC equation. The weighted coefficient is 0.575, higher than in the other samples and indicating the dominance of forward-looking output gap expectations on the output gap. For b, the weighted estimated coefficient is -0.001, the lowest among the three samples. Thus, during the Great Moderation, the data indicate little effect of policy on the output gap; this holds true for every specification. Finally, the weighted estimated coefficient on α is 0.626, close to the full sample coefficient and showing a relatively stronger effect of expectations over inflation inertia on current period inflation (though this is not true for all models). Models 48-64, which have four lags of inflation in the Phillips curve equation, display the largest coefficients on inflation expectations.

Figure 1.3: Model posteriors, 1984:1-2006:4 sample



For this sample, the weighted policy rule is:

$$f_y = 4.629,\ f_\pi = 0.342,\ f_i = 0.603$$

In contrast with the stagflation era sample, this weighted policy is the less aggressive in terms of responding to inflation. However, the response to the output gap and the interest rate smoothing term is higher than in the other samples, i.e., the policymaker's focus should fall on movements in output as opposed to inflation, and she can afford to adjust the path of i_t more incrementally from period to period. It should be noted that the high coefficient on f_y results from the weak effect of policy, given the estimated b coefficients, requiring the policymaker to respond more aggressively in order to get traction. Table 1.3 provides the losses for each model. What is most striking is that the weighted loss is 14.693, just over half the value in the full sample. That the policymaker fares better in this sample despite i) being able to adjust the interest rate more incrementally and ii) the weak effect of policy as measured by the weighted coefficient on b, is suggestive of 'good luck' over 'good policy'; in other words, these results suggest that volatility in the policymaker's variables of interest declined over this period.

Table 1.3: Losses, by model, 1984:1-2006:4 sample

Model	Loss	Model	Loss	Model	Loss	Model	Loss
1	8.568	17	10.797	33	15.901	49	9.158
2	8.51	18	8.705	34	8.49	50	10.397
3	8.55	19	8.732	35	8.56	51	10.671
4	12.798	20	22.007	36	22.192	52	19.437
5	14.447	21	17.358	37	24.925	53	16.239
6	14.267	22	14.282	38	13.83	54	18.063
7	14.297	23	14.313	39	13.909	55	18.447
8	18.878	24	26.805	40	27.041	56	28.461
9	11.56	25	13.962	41	18.941	57	13.582
10	10.882	26	10.707	42	9.987	58	15.216
11	10.933	27	10.741	43	9.944	59	15.997
12	14.996	28	21.324	44	21.679	60	24.083
13	11.21	29	13.517	45	18.115	61	13.12
14	10.915	30	10.97	46	10.515	62	14.068
15	10.897	31	10.908	47	10.381	63	14.405
16	13.395	32	17.443	48	17.694	64	18.942

Note: Losses are computed by solving the minimization problem stated in (1.14) and (1.15) for each model. See Appendix C for details on model specifications and solution procedure.

1.6.3 Robustness

In this section, I consider three alternatives to the benchmark scenario. First, I test the sensitivity of the results to using the 3-month Treasury bill yield rate as the measure of i_t. I then consider the effect of pure inflation targeting in the policymaker's loss function, which corresponds more closely to the Bank of Canada's stated objective on monetary policy.[9] Finally, I provide an alternative measure of the output gap by computing potential GDP using a quadratic trend and recalculate the weighted policies. These results are summarized in Table 1.4.

For the first exercise, I substitute in the rate on 3-month Treasury bills to proxy for i_t. The results here are broadly similar to those in the benchmark scenario, with the stagflation sample weighted policy prescribing the most aggressive response to inflation, while The

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[9] Under this scenario, I still allow for interest rate smoothing.

moderation era sample again yields the lowest losses (15.001). With the 3-month Treasury bill, losses in the stagflation era are two orders of magnitude lower than in the benchmark scenario, albeit still highest among the three samples considered.

Table 1.4: Summary of posterior-weighted common coefficients, weighted policy and losses, by procedure and sample

Procedure	Sample period	Common coefficients			Weighted policy			Loss
		λ	b	α	f_y	f_π	f_i	
Baseline	1962:2-2013:4	0.554	0.051	0.623	1.723	0.224	0.406	25.950
	1971:1-1983:4	0.412	0.042	0.483	2.868	0.863	0.412	2.7e4
	1984:1-2006:4	0.543	0.006	0.558	4.629	0.342	0.603	14.693
Treasury 3-month bill	1962:2-2013:4	0.559	0.047	0.623	1.681	0.222	0.415	25.926
	1971:1-1983:4	0.407	0.039	0.483	2.294	0.786	0.431	101.012
	1984:1-2006:4	0.545	0.004	0.557	4.726	0.356	0.642	15.001
Inflation targeting	1962:2-2013:4	0.554	0.051	0.623	1.421	0.187	0.448	26.416
	1971:1-1983:4[1]	0.413	0.042	0.483	2.205	0.934	0.438	2.8e4
	1984:1-2006:4	0.543	0.006	0.558	4.462	0.414	0.619	14.805
Linear trend	1962:2-2013:4	0.565	0.058	0.563	1.635	0.429	0.416	28.367
	1971:1-1983:4	0.408	0.04	0.488	1.857	0.714	0.427	79.359
	1984:1-2006:4	0.605	0.029	0.534	0.232	-0.09	0.54	16.636

Note: Common coefficients are calculated as the posterior-weighted average of each model's estimated coefficients. The weighted policy coefficients f_y, f_π and f_i are obtained by summing coefficients on output gap lags and inflation lags in each model's optimal rule. Losses are computed by solving the minimization problem stated in (1.14) and (1.15) for each model and weighting these by the model posteriors. See Appendix C for details on model specifications and solution procedure..

[1] Model 24 could not be stabilized.

Returning to the benchmark configuration, I examine the results of pure inflation targeting by setting $\lambda_y = 0$ in (1.14). The previous pattern of strong policy recommendations for the era of stagflation reappear, this time with a slightly larger response coefficient on inflation. Focused exclusively on stabilizing variance in inflation, the weighted policy unsurprisingly suggests a weaker response to deviations in the output gap in the full and stagflation samples, although still substantial. The policymaker's losses in each sample are very similar to those in the initial configuration. The slightly higher losses in the full sample (26.189) are driven by models 57 and 61, which tilt the weighted average up. This demonstrates the interdependence between the variation in the output gap and inflation in the Great Moderation era: even without explicitly having output gap stabilization as a goal, the policymaker must respond to movements in this variable to affect the variance in inflation effectively.

Finally, the models were re-estimated and solved using a quadratic linear trend to estimate potential GDP.[10] As before, it is necessary to produce the series iteratively for each period t using only data up to this period so as to respect the orthogonality condition of (1.3). Nevertheless, the basic results hold when computing weighted policies. An aggressive interest rate rule appears again in the stagflation subsample, while a very light-touch policy yields the lowest losses in the era of moderation. In particular, the weighted policy rule in this era is almost completely insensitive to inflation.

1.7 Concluding remarks

In this paper, I evaluated model uncertainty in the framework of an empirical New Keynesian model with varying lag structures. To circumvent the difficulty in obtaining survey data on inflation and output gap expectations, I applied the generalized method of moments for the estimation procedure. Using the method developed by Yin (2009), I generated pseudo-likelihoods for each model by modifying the GMM objective criterion, when evaluated at the parameter estimates. This allowed the construction of model posteriors and the computation of a weighted policy rule and loss for the model space.

In the samples considered, the pseudo-likelihoods reveal that the candidate models are all relatively likely. However, as summarized in the baseline results of Table 1.4, the optimal discretionary policies and losses are markedly different when moving between models and sample periods; in this sense, there is a significant degree of outcome and action dispersion. In the era of stagflation, a stronger response to movements in both the output gap and the

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[10] It was not possible to obtain estimates for all models using the benchmark instrument set, so a fourth lag of the output gap was used in this scenario.

inflation rate emerges. In addition, the weighted loss is much larger, due to several models in which the policymaker cannot stabilize the variables of interest. In the period spanning the Great Moderation, on the other hand, a policy focused on combating volatility in the output gap produces the lowest losses across the samples, suggesting that the macroeconomic volatility had been muted during this time. In addition, all samples include a significant role for interest rate smoothing, and this is somewhat more prominent in the 1984:1-2006:4 sample. This sensitivity to the sample period under consideration was anticipated by Brock et al. (2007), and highlights the type of uncertainty first described by Brainard (1967), i.e., that of a change in the effect of a given policy on the central bank's targets over time. I find that the results are also robust to changes in the measures of the interest rate and output gap, and when operating under pure inflation targeting.

The sensitivity of the results in this paper to the chosen estimation procedure (i.e. 2-step versus a continually updating estimator with a given convergence criterion) and instruments remains to be explored. In addition, the critique levied by Mavroeidis et al. (2014) also presents a challenge: under a rational expectations framework, agents base their expectations on all the available data, thus instrumenting using this very same data is of limited usefulness as a proxy. Nevertheless, the GMM procedure remains useful when data on expectations is unavailable or of limited quality.

Appendix A: Models, by lag structure

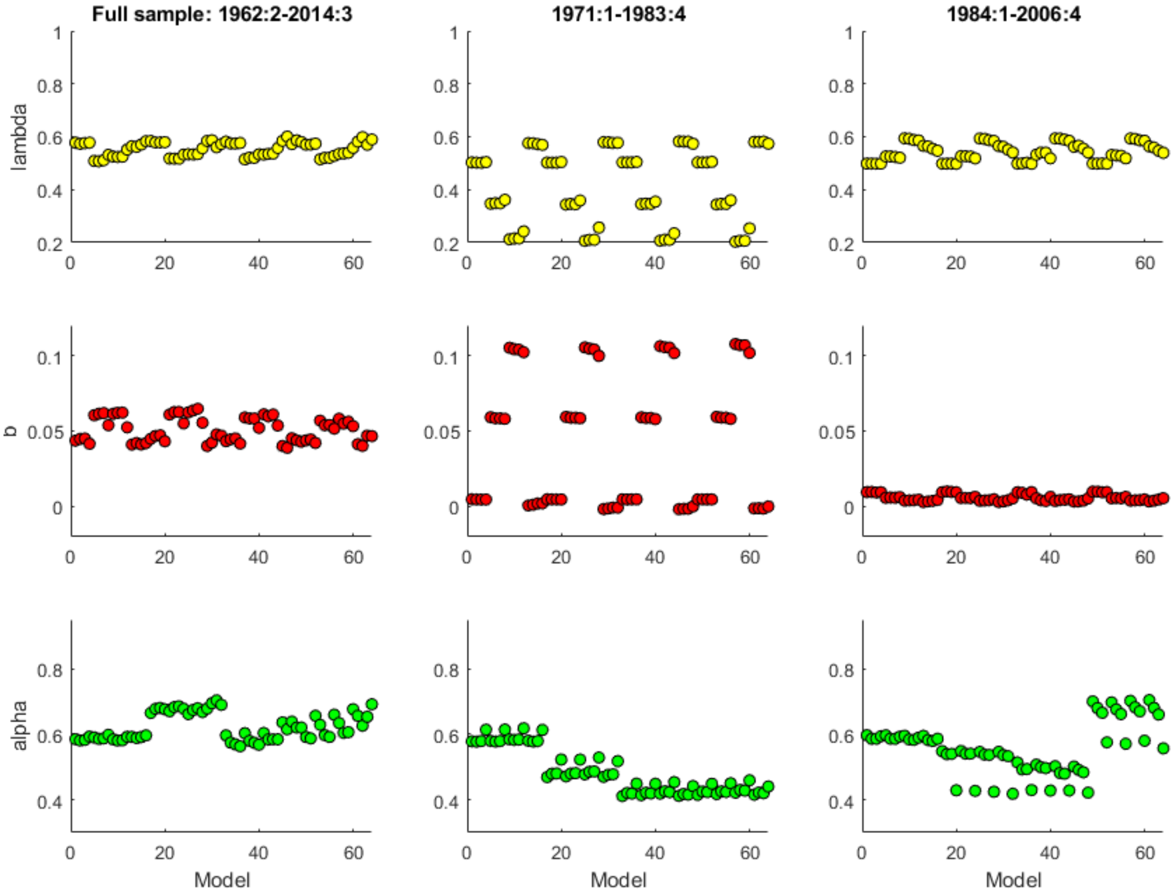
Table 1.5: Models, by lag structure

Model	Number of y lags in IS, y & π lags in PC	Model	Lags	Model	Lags	Model	Lags
1	111	17	112	33	113	49	114
2	121	18	122	34	123	50	124
3	131	19	132	35	133	51	134
4	141	20	142	36	143	52	144
5	211	21	212	37	213	53	214
6	221	22	222	38	223	54	224
7	231	23	232	39	233	55	234
8	241	24	242	40	243	56	244
9	311	25	312	41	313	57	314
10	321	26	322	42	323	58	324
11	331	27	332	43	333	59	334
12	341	28	342	44	343	60	344
13	411	29	412	45	413	61	414
14	421	30	422	46	423	62	424
15	431	31	432	47	433	63	434
16	441	32	442	48	443	64	444

Table 1.5 associates a model number to each specification, according to its lag structure in the IS and PC equation system defined by (1.3) and (1.4). The first number in the Lags column represents how many output gap lags are in the IS equation. The second and third numbers indicate the number of output gap and inflation lags in the PC equation, respectively.

Appendix B: Common model coefficients, by sample

Figure 1.4: Common model coefficients, by sample



Appendix C: Solving the models

This section explains how the models are recast into a state-space representation and solved, which enables the computation of the asymptotic variances of y_t, π_t and i_t and, finally, losses under a given policy. This material is adapted from Söderström et al. (2005) and Castelnuovo et al. (2006).

1. State-space representation

To obtain a discretionary solution, the models considered in this Chapter must be written in state-space form:

$$S_{t+1} = AS_t + Bi_t + v_{t+1} \tag{1.C1}$$

where S_{t+1} is a state vector, A,B contain the model parameter estimates and v_{t+1} is a vector of error terms.

As an example, consider model 5 (see Appendix A), whose lag structure is also used in Chapters 2 and 3:

$$y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)(ay_{t-1} + (1-a)y_{t-2}) + b(\bar{i}_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + \eta_t \tag{1.C2}$$

$$\pi_t = \alpha E_{t-1}\bar{\pi}_{t+3} + (1-\alpha)\pi_{t-1} + \kappa y_{t-1} + \varepsilon_t \tag{1.C3}$$

where superfluous subscripts on a and κ have been omitted for presentation.

Forward equations (1.C2) and (1.C3) by one period. Take expectations as of t and rewrite to express the forward-looking variables $E_t y_{t+2}$ and $E_t\bar{\pi}_{t+4}$ in terms of the predetermined variables:

$$\lambda E_t y_{t+2} + \frac{b}{4} E_t \pi_{t+4} = E_t y_{t+1} - (1-\lambda) a_1 y_t - (1-\lambda) a_2 y_{t-1} + b i_t$$
$$- \frac{b}{4} E_t \left(\pi_{t+3} + \pi_{t+2} + \pi_{t+1}\right) \tag{1.C4}$$
$$\frac{\alpha}{4} E_t \pi_{t+4} = (1 - \frac{\alpha}{4}) E_t \pi_{t+1} - \frac{\alpha}{4} E_t \left(\pi_{t+3} + \pi_{t+2}\right)$$
$$- (1-\alpha)\pi_t - \kappa y_t \tag{1.C5}$$

The innovations return via the predetermined variables

$$\pi_{t+1} = E_t \pi_{t+1} + u_t \tag{1.C6}$$
$$y_{t+1} = E_t y_{t+1} + v_t \tag{1.C7}$$

and are captured in

$$v_{1t+1} = \begin{bmatrix} u_t & 0 & v_t & 0 \end{bmatrix}$$

Define the vector $x_{t+1} = [x_{1t+1}, x_{2t+1}]$, partitioned into predetermined and jump variables

$$x_{1t+1} = \begin{bmatrix} y_{t+1}, y_t, \pi_{t+1}, i_t \end{bmatrix}' \tag{1.C8}$$
$$x_{2t+1} = \begin{bmatrix} E_t y_{t+2}, E_t \pi_{t+4}, , E_t \pi_{t+3}, , E_t \pi_{t+2} \end{bmatrix}' \tag{1.C9}$$

and let

$$v_{t+1} = \begin{bmatrix} v_{1t+1} & 0_{4x1} \end{bmatrix}$$

Then, the model may be expressed in the following state-space form:

$$\begin{bmatrix} x_{1t+1} \\ \Omega E_t x_{2t+1} \end{bmatrix} = A_1 \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B_0 i_t + v_{t+1} \tag{1.C10}$$

Then, the matrices Ω, A_1 and B are given by:

$$\Omega = \begin{bmatrix} \lambda & b/4 & 0 & 0 \\ 0 & \alpha/4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.C11}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -a & -(1-\lambda)(1-a) & 0 & 0 & 1 & -\frac{b}{4} & -\frac{b}{4} & -\frac{b}{4} \\ -\beta & 0 & -(1-\alpha) & 0 & 0 & \frac{-\alpha}{4} & \frac{-\alpha}{4} & 1-\frac{\alpha}{4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{1.C12}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 1 & b & 0 & 0 & 0 \end{bmatrix}' \tag{1.C13}$$

while the errors return in v_{t+1}.

One can premultiply both sides of (1.C10) by

$$A_0 = \begin{bmatrix} I_{4x4} & 0_{4x2} \\ 0_{2x4} & \Omega \end{bmatrix}^{-1}$$

to obtain the usual state-space representation:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bi_t + v_{t+1} \tag{1.C14}$$

where $A = A_0^{-1} A_1$ and $B = A_0^{-1} B_0$.[11]

2. Optimal discretionary policy

The next step is to solve the optimal regulator problem for a discretionary, time-consistent solution. Defining the vector of instruments targeted by the central bank for its loss function to be:

$$\begin{aligned} z_t &= \begin{bmatrix} \pi_t & y_t & \Delta i_t \end{bmatrix}' \\ &= C_s S_t + C_i i_t \end{aligned} \tag{1.C15}$$

where

$$\begin{aligned} C_s &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ C_i &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}' \end{aligned}$$

Then, the quadratic losses at time t can be expressed as:

$$L_t = z_t' K z_t \tag{1.C16}$$

where K provides the weights for the target variables in the loss function (relative to infla-

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[11] The upper block of A_0^{-1} is the identity matrix, which ensures that v_{t+1} is unaffected by premultiplication.

tion):

$$K = \begin{bmatrix} \lambda_y & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda_i \end{bmatrix} \tag{1.C17}$$

L_t can be further decomposed into:

$$\begin{aligned} L_t &= \begin{bmatrix} S_t' & i_t' \end{bmatrix} \begin{bmatrix} C_s' \\ C_i' \end{bmatrix} K \begin{bmatrix} C_s & C_i \end{bmatrix} \begin{bmatrix} S_t \\ i_t \end{bmatrix} \\ &= S_t'C_s'KC_sS_t + S_t'C_s'KC_ii_t + i_t'C_i'KC_sS_t + i_t'C_i'KC_ii_t \\ &= S_t'QS_t + S_t' + S_t'Ui_t + i_t'U'S_t + i_t'Ri_t \end{aligned} \tag{1.C18}$$

where $Q = C_s'KC_s$, $U = C_s'KC_i$, $R = C_i'KC_i$.

Using (1.C18), the optimal control problem of the central bank is given by the following intertemporal loss function:

$$\min_{i_t} J_t = (1-\delta)E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t}(S_t'QS_t + S_t' + S_t'Ui_t + i_t'U'S_t + i_t'Ri_t) \tag{1.C19}$$

subject to the law of motion of the economy, given by (1.C1), and a discount factor δ. The optimal discretionary rule for i_t will take the form

$$i_t = Fx_{1t} \tag{1.C20}$$

The corresponding reduced-form of the system is expressed as:

$$x_{1t+1} = Mx_{1t} + v_{1t+1} \tag{1.C21}$$

$$x_{2t} = Gx_{1t} \tag{1.C22}$$

The full solution to the standard optimal linear regulator problem, which yields the optimal discretionary rule in (1.C20), is described in more detail in Appendix A of Chapter 2.

3. Unconditional variance and loss

As the discount factor δ approaches 1, the intertemporal loss function asymptotes to the sum of the unconditional variances of the target instruments, i.e.

$$R = var(\pi_\infty) + \lambda_y var(y_\infty) + \lambda_i var(i_\infty)$$

thereby permitting the representation of the problem in (1.14) and (1.15). Moreover, the reduced-form of the system given by (1.C21) allows the unconditional variance-covariance matrix of the predetermined variables — which contain the elements of the loss function — to be found by solving the Lyapunov equation:

$$\Sigma_{x1} = M\Sigma_{x1}M' + \Sigma_{v1} \tag{1.C23}$$

where Σ_{v1} is the covariance matrix of the errors v_{1t+1}.

The residuals obtained via the GMM procedure, $h_{IS,t}$ and h_{PC_t}, include expectation errors. To compute Σ_{v1}, the simple residuals, η_t and ε_t, must be retrieved by removing the expectation errors (written as the difference between expected and realized values). Expected values are estimated by using an approximating model, wherein the agent determines his expectations via least squares using the same instrument set described in section 4 as regressors. In each period t, the agent employs data in the instrument set up to $t-1$ to predict the current period's output gap and inflation.

Appendix D: Optimal discretionary rules

Table 1.6: Optimal discretionary rules, by model, 1962:2-2013:4

Model	f_{y1}	f_{y2}	f_{y3}	f_{y4}	$f_{\pi 1}$	$f_{\pi 2}$	$f_{\pi 3}$	$f_{\pi 4}$	f_i
1	0.774	0	0	0	-0.052	0	0	0	0.562
2	1.588	0.208	0	0	0.442	0	0	0	0.463
3	1.598	0.21	-0.006	0	0.44	0	0	0	0.461
4	1.814	0.103	-0.198	-0.298	0.429	0	0	0	0.445
5	0.979	0.055	0	0	-0.047	0	0	0	0.511
6	1.833	0.283	0	0	0.438	0	0	0	0.407
7	1.835	0.285	-0.004	0	0.436	0	0	0	0.405
8	2.067	0.141	-0.244	-0.383	0.425	0	0	0	0.394
9	0.984	0.008	-0.082	0	-0.038	0	0	0	0.498
10	1.763	0.211	-0.11	0	0.438	0	0	0	0.404
11	1.768	0.203	-0.131	0	0.439	0	0	0	0.403
12	2.165	0.01	-0.49	-0.503	0.426	0	0	0	0.379
13	3.011	0.066	-1.11	-1.843	0.072	0	0	0	0.372
14	3.89	0.408	-1.122	-2.11	0.365	0	0	0	0.331
15	4.02	0.41	-1.206	-2.228	0.358	0	0	0	0.327
16	3.779	0.285	-1.203	-2.112	0.368	0	0	0	0.327
17	0.766	0	0	0	-0.06	0.018	0	0	0.557
18	1.8	0.256	0	0	0.243	-0.105	0	0	0.436
19	1.826	0.201	-0.091	0	0.24	-0.115	0	0	0.428
20	1.931	0.136	-0.197	-0.158	0.259	-0.099	0	0	0.43
21	0.963	0.053	0	0	-0.057	0.017	0	0	0.508
22	2.01	0.326	0	0	0.234	-0.103	0	0	0.386
23	2.034	0.279	-0.08	0	0.231	-0.111	0	0	0.381
24	2.167	0.175	-0.235	-0.236	0.261	-0.092	0	0	0.384
25	0.967	0.006	-0.081	0	-0.055	0.015	0	0	0.495
26	1.944	0.255	-0.115	0	0.246	-0.101	0	0	0.383
27	1.969	0.152	-0.282	0	0.232	-0.125	0	0	0.37
28	2.291	-0.001	-0.565	-0.354	0.275	-0.105	0	0	0.36
29	3.159	0.046	-1.238	-2.002	-0.009	-0.003	0	0	0.359
30	4.148	0.486	-1.2	-2.258	0.186	-0.103	0	0	0.313
31	3.693	0.386	-1.073	-1.837	0.153	-0.145	0	0	0.314
32	3.604	0.32	-1.1	-1.762	0.188	-0.133	0	0	0.317

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Table 1.6: Optimal discretionary rules, by model, 1962:2-2013:4 (continued from previous page)

Model	f_{y1}	f_{y2}	f_{y3}	f_{y4}	$f_{\pi 1}$	$f_{\pi 2}$	$f_{\pi 3}$	$f_{\pi 4}$	f_i
33	0.76	0	0	0	-0.018	0.003	-0.003	0	0.563
34	2.154	0.338	0	0	0.439	0.016	0.083	0	0.428
35	2.215	0.265	-0.152	0	0.44	0.014	0.091	0	0.417
36	1.952	0.265	-0.084	0.165	0.45	0.034	0.097	0	0.448
37	0.961	0.052	0	0	-0.021	0.003	-0.004	0	0.514
38	2.372	0.408	0	0	0.434	0.015	0.082	0	0.378
39	2.443	0.338	-0.15	0	0.437	0.015	0.091	0	0.369
40	2.187	0.29	-0.127	0.073	0.448	0.037	0.095	0	0.402
41	0.959	0.005	-0.08	0	-0.022	0.004	-0.003	0	0.499
42	2.259	0.335	-0.105	0	0.421	0.003	0.074	0	0.376
43	2.323	0.221	-0.33	0	0.408	-0.016	0.079	0	0.361
44	2.106	0.113	-0.392	-0.036	0.408	-0.006	0.075	0	0.389
45	3.143	0.041	-1.237	-1.99	0.038	-0.001	0.004	0	0.36
46	4.479	0.521	-1.308	-2.388	0.312	-0.028	0.053	0	0.318
47	4.004	0.454	-1.169	-1.951	0.272	-0.084	0.047	0	0.312
48	3.493	0.331	-1.05	-1.565	0.294	-0.063	0.051	0	0.333
49	0.77	0	0	0	-0.06	0.005	-0.006	0.004	0.564
50	2.247	0.359	0	0	0.441	-0.003	0.058	-0.027	0.423
51	2.297	0.289	-0.148	0	0.446	-0.006	0.066	-0.027	0.413
52	2.078	0.843	0.899	0.88	0.167	-0.089	-0.053	-0.073	0.422
53	0.974	0.049	0	0	-0.062	0.005	-0.005	0.005	0.518
54	2.539	0.431	0	0	0.448	0.001	0.051	-0.039	0.379
55	2.585	0.371	-0.134	0	0.455	0	0.058	-0.038	0.372
56	2.263	0.89	0.873	0.869	0.181	-0.088	-0.057	-0.079	0.386
57	0.979	-0.011	-0.095	0	-0.075	0.007	-0.003	0.007	0.503
58	2.407	0.336	-0.134	0	0.436	-0.017	0.035	-0.045	0.378
59	2.45	0.228	-0.354	0	0.43	-0.039	0.038	-0.044	0.363
60	1.882	0.618	0.53	0.723	0.118	-0.115	-0.068	-0.068	0.388
61	2.89	0.011	-1.127	-1.772	-0.013	-0.006	-0.004	0	0.371
62	4.243	0.481	-1.203	-2.133	0.33	-0.036	0.032	-0.024	0.321
63	3.792	0.407	-1.098	-1.713	0.287	-0.105	0.018	-0.028	0.314
64	3.174	0.899	0.015	-1.17	0.051	-0.114	-0.057	-0.049	0.327

Note: Optimal rules are computed by solving the minimization problem stated in (1.11) and (1.12) for each model. Rules are of the form: $i_t = f_{y1} y_{t-1} + ... + f_{\pi 1} \pi_{t-1} + ... + f_i i_{t-1}$. See appendices A and C for details on the solution procedure and model characteristics.

Table 1.7: Optimal discretionary rules, by model, 1971:1-1983:4

Model	f_{y1}	f_{y2}	f_{y3}	f_{y4}	$f_{\pi 1}$	$f_{\pi 2}$	$f_{\pi 3}$	$f_{\pi 4}$	f_i
1	1.705	0	0	0	-0.309	0	0	0	0.731
2	5.373	0.462	0	0	0.444	0	0	0	0.612
3	5.372	0.488	0.046	0	0.438	0	0	0	0.613
4	3.924	-0.65	-0.929	-1.428	0.373	0	0	0	0.615
5	1.537	0.348	0	0	-0.244	0	0	0	0.516
6	3.788	1.207	0	0	0.506	0	0	0	0.34
7	3.786	1.23	0.05	0	0.502	0	0	0	0.342
8	2.928	0.487	-0.476	-0.834	0.212	0	0	0	0.363
9	1.335	0.516	0.233	0	-0.229	0	0	0	0.468
10	3.208	1.457	0.436	0	0.51	0	0	0	0.281
11	3.216	1.481	0.487	0	0.507	0	0	0	0.282
12	2.357	0.745	0.018	-0.911	0.225	0	0	0	0.289
13	325.821	2.444	-138.446	-196.906	0.228	0	0	0	0.198
14	188.847	1.739	-80.052	-113.946	0.261	0	0	0	0.217
15	108.736	0.89	-45.964	-65.469	0.271	0	0	0	0.231
16	114.387	2.257	-47.536	-70.034	0.219	0	0	0	0.202
17	1.232	0	0	0	0.597	0.211	0	0	0.762
18	3.657	0.263	0	0	0.746	0.254	0	0	0.662
19	3.725	0.357	0.188	0	0.751	0.261	0	0	0.664
20	3.925	0.671	0.472	0.747	-0.375	-0.115	0	0	0.682
21	1.409	0.335	0	0	0.744	0.264	0	0	0.524
22	2.823	0.826	0	0	0.796	0.274	0	0	0.398
23	2.853	0.911	0.188	0	0.808	0.284	0	0	0.401
24	3.171	1.321	0.33	0.403	-0.205	-0.059	0	0	0.385
25	1.196	0.494	0.221	0	0.094	0.036	0	0	0.483
26	2.45	1.066	0.355	0	0.822	0.285	0	0	0.34
27	2.49	1.152	0.552	0	0.838	0.296	0	0	0.342
28	2.696	1.609	0.744	0.253	-0.128	-0.033	0	0	0.305
29	-161.964	-2.266	68.697	97.732	1.029	0.368	0	0	0.182
30	-182.059	0.67	80.369	111.833	0.631	0.213	0	0	0.233
31	-355.361	-0.468	154.667	216.794	0.719	0.249	0	0	0.221
32	-360.775	-3.804	153.672	218.135	0.341	0.111	0	0	0.197

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Table 1.7: Optimal discretionary rules, by model, 1971:1-1983:4 (continued from previous page)

Model	f_{y1}	f_{y2}	f_{y3}	f_{y4}	$f_{\pi 1}$	$f_{\pi 2}$	$f_{\pi 3}$	$f_{\pi 4}$	f_i
33	1.901	0	0	0	0.736	0.385	0.188	0	0.729
34	3.62	0.223	0	0	0.731	0.373	0.192	0	0.672
35	3.623	0.199	-0.062	0	0.727	0.371	0.196	0	0.671
36	3.133	-0.08	-0.281	-0.555	0.678	0.331	0.163	0	0.672
37	1.908	0.411	0	0	0.849	0.451	0.218	0	0.47
38	2.948	0.797	0	0	0.772	0.403	0.206	0	0.396
39	2.955	0.767	-0.079	0	0.766	0.401	0.21	0	0.394
40	2.736	0.532	-0.309	-0.637	0.716	0.357	0.178	0	0.392
41	1.654	0.603	0.272	0	0.904	0.486	0.235	0	0.416
42	2.571	1.045	0.368	0	0.803	0.426	0.219	0	0.333
43	2.577	1.01	0.284	0	0.796	0.423	0.222	0	0.331
44	2.352	0.767	0.027	-0.727	0.735	0.369	0.186	0	0.328
45	-144.505	-0.595	62.837	87.732	0.87	0.456	0.221	0	0.206
46	-152.316	0.963	67.767	93.5	0.706	0.36	0.18	0	0.231
47	-160.925	0.919	71.406	98.689	0.711	0.363	0.186	0	0.23
48	2.8e3	45.385	-1.2e3	-1.6e3	-0.139	-0.057	-0.026	0	0.183
49	1.815	0	0	0	0.772	0.386	0.166	-0.038	0.732
50	3.763	0.251	0	0	0.772	0.372	0.168	-0.044	0.667
51	3.761	0.227	-0.061	0	0.767	0.371	0.173	-0.043	0.666
52	3.149	-0.065	-0.268	-0.537	0.681	0.332	0.163	-0.002	0.673
53	1.839	0.402	0	0	0.892	0.453	0.195	-0.04	0.475
54	2.997	0.833	0	0	0.81	0.401	0.181	-0.042	0.392
55	3.002	0.806	-0.073	0	0.804	0.398	0.185	-0.042	0.391
56	2.708	0.534	-0.299	-0.665	0.707	0.353	0.178	0.004	0.391
57	1.58	0.589	0.267	0	0.948	0.49	0.214	-0.039	0.423
58	2.598	1.082	0.375	0	0.838	0.423	0.195	-0.04	0.329
59	2.603	1.057	0.305	0	0.832	0.419	0.198	-0.039	0.327
60	2.27	0.751	0.019	-0.86	0.705	0.353	0.182	0.009	0.323
61	-208.604	-1.409	89.92	126.221	0.973	0.488	0.21	-0.046	0.2
62	-207.898	0.875	91.842	127.202	0.776	0.374	0.164	-0.041	0.227
63	-174.753	0.998	77.535	107.187	0.746	0.361	0.163	-0.039	0.231
64	3.7e3	62.653	-1.5e3	-2.2e3	-0.159	-0.067	-0.03	0.001	0.181

Note: Optimal rules are computed by solving the minimization problem stated in (1.11) and (1.12) for each model. Rules are of the form: $i_t = f_{y1} y_{t-1} + ... + f_{\pi 1} \pi_{t-1} + ... + f_i i_{t-1}$. See appendices A and C for details on the solution procedure and model characteristics.

Table 1.8: Optimal discretionary rules, by model, 1984:1-2006:4

Model	f_{y1}	f_{y2}	f_{y3}	f_{y4}	$f_{\pi 1}$	$f_{\pi 2}$	$f_{\pi 3}$	$f_{\pi 4}$	f_i
1	1.205	0	0	0	0.073	0	0	0	0.707
2	5.324	0.486	0	0	0.463	0	0	0	0.524
3	5.452	0.457	-0.076	0	0.464	0	0	0	0.525
4	9.975	0.452	-0.789	-2.36	0.463	0	0	0	0.427
5	1.001	-0.016	0	0	0.063	0	0	0	0.755
6	5.279	0.411	0	0	0.45	0	0	0	0.579
7	5.419	0.383	-0.071	0	0.452	0	0	0	0.578
8	10.095	0.236	-0.818	-2.336	0.46	0	0	0	0.481
9	0.387	-0.145	-0.105	0	0.065	0	0	0	0.809
10	3.356	-0.175	-0.547	0	0.452	0	0	0	0.641
11	3.568	-0.234	-0.672	0	0.457	0	0	0	0.632
12	8.23	-0.85	-2.054	-2.153	0.451	0	0	0	0.521
13	0.595	0.009	0.046	0.164	0.064	0	0	0	0.826
14	4.361	0.395	0.131	0.945	0.445	0	0	0	0.654
15	4.68	0.415	0.1	1.158	0.46	0	0	0	0.639
16	7.73	-0.149	-0.664	0.01	0.476	0	0	0	0.553
17	1.228	0	0	0	0.2	0.03	0	0	0.704
18	5.396	0.531	0	0	0.589	0.082	0	0	0.522
19	5.528	0.517	-0.063	0	0.587	0.08	0	0	0.522
20	12.294	1.359	-0.602	-4.799	0.725	0.293	0	0	0.399
21	1.012	-0.016	0	0	0.182	0.027	0	0	0.754
22	5.384	0.444	0	0	0.575	0.082	0	0	0.587
23	5.512	0.432	-0.053	0	0.575	0.08	0	0	0.584
24	12.575	1.016	-0.619	-4.783	0.732	0.293	0	0	0.457
25	0.39	-0.144	-0.105	0	0.18	0.027	0	0	0.809
26	3.286	-0.139	-0.528	0	0.565	0.082	0	0	0.652
27	3.504	-0.178	-0.634	0	0.567	0.079	0	0	0.641
28	10.07	-0.205	-2.074	-4.037	0.646	0.26	0	0	0.508
29	0.616	0.011	0.048	0.169	0.195	0.029	0	0	0.823
30	4.419	0.429	0.132	0.957	0.58	0.082	0	0	0.657
31	4.722	0.447	0.084	1.156	0.595	0.076	0	0	0.634
32	9.317	0.191	-0.823	-1.637	0.749	0.293	0	0	0.516

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Table 1.8: Optimal discretionary rules, by model, 1984:1-2006:4 (continued from previous page)

Model	f_{y1}	f_{y2}	f_{y3}	f_{y4}	$f_{\pi 1}$	$f_{\pi 2}$	$f_{\pi 3}$	$f_{\pi 4}$	f_i
33	1.25	0	0	0	0.404	0.102	0.038	0	0.702
34	5.242	0.527	0	0	0.659	0.2	0.086	0	0.546
35	5.513	0.487	-0.167	0	0.654	0.197	0.089	0	0.554
36	12.369	1.381	-0.607	-4.836	0.728	0.291	-0.004	0	0.397
37	0.997	-0.018	0	0	0.435	0.116	0.047	0	0.754
38	5.182	0.441	0	0	0.643	0.18	0.076	0	0.632
39	5.456	0.409	-0.138	0	0.646	0.181	0.081	0	0.636
40	12.666	1.034	-0.625	-4.824	0.734	0.291	-0.005	0	0.455
41	0.393	-0.139	-0.101	0	0.406	0.112	0.045	0	0.813
42	3.22	-0.068	-0.502	0	0.626	0.174	0.083	0	0.661
43	3.395	-0.112	-0.642	0	0.624	0.172	0.084	0	0.649
44	10.221	-0.203	-2.115	-4.104	0.648	0.257	-0.006	0	0.505
45	0.699	0.012	0.052	0.188	0.511	0.14	0.057	0	0.808
46	4.449	0.489	0.136	1.008	0.667	0.165	0.074	0	0.665
47	4.89	0.538	0.051	1.392	0.687	0.155	0.078	0	0.643
48	9.42	0.186	-0.85	-1.669	0.75	0.289	-0.007	0	0.514
49	1.231	0	0	0	-0.018	0.089	0.087	0.062	0.703
50	5.964	0.494	0	0	0.264	-0.507	-0.522	-0.397	0.481
51	6.866	0.495	-0.194	0	0.422	-0.46	-0.508	-0.429	0.463
52	14.158	1.306	-0.861	-5.182	0.864	0.06	-0.312	-0.302	0.365
53	0.991	-0.018	0	0	0.002	0.092	0.084	0.055	0.759
54	5.724	0.355	0	0	0.151	-0.498	-0.479	-0.342	0.546
55	6.751	0.33	-0.174	0	0.288	-0.477	-0.485	-0.379	0.527
56	15.241	0.961	-1.049	-5.312	0.888	0.041	-0.329	-0.31	0.409
57	0.412	-0.15	-0.109	0	0.013	0.109	0.097	0.061	0.804
58	4.495	-0.425	-0.774	0	0.135	-0.665	-0.623	-0.42	0.556
59	5.987	-0.675	-1.246	0	0.357	-0.687	-0.687	-0.511	0.512
60	14.341	-0.418	-3.009	-5.346	0.942	0.05	-0.349	-0.331	0.436
61	0.65	0.01	0.051	0.181	0.018	0.103	0.09	0.056	0.815
62	4.304	0.298	0.16	1.005	0.048	-0.51	-0.463	-0.308	0.615
63	5.646	0.318	0.067	1.491	0.252	-0.526	-0.513	-0.386	0.562
64	11.913	0.033	-1.227	-1.767	0.944	0.051	-0.323	-0.311	0.458

Note: Optimal rules are computed by solving the minimization problem stated in (1.11) and (1.12) for each model. Rules are of the form: $i_t = f_{y1}y_{t-1} + ... + f_{\pi 1}\pi_{t-1} + ... + f_i i_{t-1}$. See appendices A and C for details on the solution procedure and model characteristics.

Table 1.9: Sum of coefficients of lags, optimal discretionary rule, by model, 1962:2-2013:4

Model	f_y	f_π	f_i	Model	f_y	f_π	f_i
1	0.774	0.562	-0.018	33	-0.052	0.76	0.563
2	1.796	0.463	0.538	34	0.442	2.493	0.428
3	1.802	0.461	0.545	35	0.44	2.328	0.417
4	1.421	0.445	0.581	36	0.429	2.299	0.448
5	1.034	0.511	-0.022	37	-0.047	1.014	0.514
6	2.116	0.407	0.531	38	0.438	2.78	0.378
7	2.116	0.405	0.542	39	0.436	2.632	0.369
8	1.58	0.394	0.58	40	0.425	2.423	0.402
9	0.911	0.498	-0.021	41	-0.038	0.885	0.499
10	1.864	0.404	0.498	42	0.438	2.489	0.376
11	1.84	0.403	0.471	43	0.439	2.215	0.361
12	1.182	0.379	0.476	44	0.426	1.791	0.389
13	0.124	0.372	0.041	45	0.072	-0.043	0.36
14	1.066	0.331	0.336	46	0.365	1.304	0.318
15	0.995	0.327	0.235	47	0.358	1.338	0.312
16	0.749	0.327	0.282	48	0.368	1.209	0.333
17	0.766	0.557	-0.057	49	-0.042	0.77	0.564
18	2.056	0.436	0.469	50	0.138	2.606	0.423
19	1.936	0.428	0.478	51	0.124	2.438	0.413
20	1.712	0.43	-0.048	52	0.16	4.699	0.422
21	1.015	0.508	-0.057	53	-0.04	1.023	0.518
22	2.336	0.386	0.461	54	0.131	2.969	0.379
23	2.233	0.381	0.475	55	0.12	2.823	0.372
24	1.871	0.384	-0.043	56	0.169	4.894	0.386
25	0.892	0.495	-0.063	57	-0.04	0.873	0.503
26	2.085	0.383	0.41	58	0.145	2.61	0.378
27	1.839	0.37	0.385	59	0.106	2.324	0.363
28	1.371	0.36	-0.132	60	0.17	3.753	0.388
29	-0.035	0.359	-0.024	61	-0.011	0.002	0.371
30	1.177	0.313	0.301	62	0.084	1.389	0.321
31	1.169	0.314	0.172	63	0.009	1.388	0.314
32	1.062	0.317	-0.17	64	0.055	2.918	0.327

Note: The simplified rules in tables E4-E6 are obtained by summing coefficients on output gap lags and inflation lags in each model's optimal rule. Optimal rules are computed by solving the minimization problem stated in (1.11) and (1.12) for each model, and are of the form: $i_t = f_{y1}y_{t-1} + ... + f_{\pi 1}\pi_{t-1} + ... + f_i i_{t-1}$. See appendices A and C for details on the solution procedure and model characteristics.

Table 1.10: Sum of coefficients of lags, optimal discretionary rule, by model, 1971:1-1983-4

Model	f_y	f_π	f_i	Model	f_y	f_π	f_i
1	1.705	-0.309	0.731	33	1.901	1.309	0.729
2	5.836	0.444	0.612	34	3.843	1.296	0.672
3	5.906	0.438	0.613	35	3.76	1.294	0.671
4	0.917	0.373	0.615	36	2.217	1.172	0.672
5	1.885	-0.244	0.516	37	2.319	1.518	0.47
6	4.995	0.506	0.34	38	3.745	1.381	0.396
7	5.065	0.502	0.342	39	3.643	1.376	0.394
8	2.106	0.212	0.363	40	2.322	1.251	0.392
9	2.084	-0.229	0.468	41	2.529	1.626	0.416
10	5.101	0.51	0.281	42	3.984	1.447	0.333
11	5.184	0.507	0.282	43	3.871	1.441	0.331
12	2.209	0.225	0.289	44	2.42	1.289	0.328
13	-7.087	0.228	0.198	45	5.468	1.546	0.206
14	-3.411	0.261	0.217	46	9.913	1.245	0.231
15	-1.807	0.271	0.231	47	10.089	1.26	0.23
16	-0.926	0.219	0.202	48	-25.179	-0.222	0.183
17	1.232	0.808	0.762	49	1.815	1.285	0.732
18	3.92	1	0.662	50	4.014	1.269	0.667
19	4.269	1.012	0.664	51	3.927	1.268	0.666
20	5.815	-0.491	0.682	52	2.278	1.174	0.673
21	1.744	1.008	0.524	53	2.241	1.5	0.475
22	3.65	1.07	0.398	54	3.83	1.35	0.392
23	3.952	1.092	0.401	55	3.735	1.346	0.391
24	5.225	-0.264	0.385	56	2.278	1.242	0.391
25	1.91	0.13	0.483	57	2.437	1.613	0.423
26	3.87	1.107	0.34	58	4.055	1.415	0.329
27	4.194	1.135	0.342	59	3.964	1.41	0.327
28	5.303	-0.161	0.305	60	2.18	1.249	0.323
29	2.199	1.396	0.182	61	6.128	1.625	0.2
30	10.814	0.843	0.233	62	12.021	1.274	0.227
31	15.632	0.968	0.221	63	10.967	1.23	0.231
32	7.227	0.452	0.197	64	-29.888	-0.254	0.181

Note: The simplified rules in tables E4-E6 are obtained by summing coefficients on output gap lags and inflation lags in each model's optimal rule. Optimal rules are computed by solving the minimization problem stated in (1.11) and (1.12) for each model, and are of the form: $i_t = f_{y1}y_{t-1} + ... + f_{\pi 1}\pi_{t-1} + ... + f_i i_{t-1}$. See appendices A and C for details on the solution procedure and model characteristics.

Table 1.11: Sum of coefficients of lags, optimal discretionary rule, by model, 1984:1-2006:4

Model	f_y	f_π	f_i	Model	f_y	f_π	f_i
1	1.205	0.073	0.707	33	1.25	0.544	0.702
2	5.81	0.463	0.524	34	5.769	0.945	0.546
3	5.834	0.464	0.525	35	5.833	0.94	0.554
4	7.278	0.463	0.427	36	8.308	1.014	0.397
5	0.986	0.063	0.755	37	0.979	0.598	0.754
6	5.69	0.45	0.579	38	5.622	0.899	0.632
7	5.731	0.452	0.578	39	5.727	0.908	0.636
8	7.177	0.46	0.481	40	8.251	1.021	0.455
9	0.136	0.065	0.809	41	0.153	0.563	0.813
10	2.634	0.452	0.641	42	2.65	0.883	0.661
11	2.662	0.457	0.632	43	2.641	0.88	0.649
12	3.173	0.451	0.521	44	3.799	0.899	0.505
13	0.814	0.064	0.826	45	0.951	0.707	0.808
14	5.833	0.445	0.654	46	6.083	0.905	0.665
15	6.353	0.46	0.639	47	6.871	0.92	0.643
16	6.927	0.476	0.553	48	7.088	1.032	0.514
17	1.228	0.231	0.704	49	1.231	0.221	0.703
18	5.927	0.671	0.522	50	6.458	-1.162	0.481
19	5.982	0.667	0.522	51	7.167	-0.975	0.463
20	8.251	1.018	0.399	52	9.42	0.31	0.365
21	0.995	0.209	0.754	53	0.973	0.233	0.759
22	5.828	0.657	0.587	54	6.079	-1.168	0.546
23	5.891	0.655	0.584	55	6.907	-1.054	0.527
24	8.189	1.025	0.457	56	9.841	0.29	0.409
25	0.141	0.207	0.809	57	0.152	0.28	0.804
26	2.62	0.647	0.652	58	3.296	-1.572	0.556
27	2.692	0.647	0.641	59	4.066	-1.528	0.512
28	3.755	0.906	0.508	60	5.568	0.312	0.436
29	0.843	0.224	0.823	61	0.892	0.267	0.815
30	5.936	0.662	0.657	62	5.768	-1.234	0.615
31	6.41	0.671	0.634	63	7.522	-1.173	0.562
32	7.047	1.042	0.516	64	8.952	0.361	0.458

Note: The simplified rules in tables E4-E6 are obtained by summing coefficients on output gap lags and inflation lags in each model's optimal rule. Optimal rules are computed by solving the minimization problem stated in (1.11) and (1.12) for each model, and are of the form: $i_t = f_{y1}y_{t-1} + ... + f_{\pi 1}\pi_{t-1} + ... + f_i i_{t-1}$. See appendices A and C for details on the solution procedure and model characteristics.

Chapter 2

A GMM Approach to Inverse Control Under Discretion

2.1 Introduction

In the optimal control literature, monetary policy evaluation typically follows the two-step approach described by Salemi (2006): subject to an estimated model and calibrated loss function (step 1), the performance of a monetary policy rule is examined (step 2). The assumed loss function, calibrated exogenously, formalizes policy goals such as minimizing variance in inflation and the output gap. With inverse control, however, the two steps are merged: both model and loss function are estimated simultaneously, subject to the restriction that the policymaker is behaving optimally. By providing estimates of the loss function parameters, inverse control answers the following question: "If we presume the policymaker is acting optimally and we see these outcomes, what policy preferences were guiding his behaviour?"

In this paper, I extend the application of inverse control. I demonstrate how the generalized method of moments (GMM) can be used to estimate the policy preferences of a monetary authority engaged in discretionary policymaking. I build on Givens and Salemi

(2008), in which the standard GMM moment conditions are augmented with the optimality conditions arising when a simple Taylor-type rule is adopted. By way of Monte Carlo simulations in which an empirical New Keynesian model is adopted, I reveal that the procedure is able to identify both the structural parameters and the policymaker's preferences.

The logic of the procedure is as follows. In a standard GMM setup, the moment conditions implied by the reduced-form of the model are used to estimate its structural parameters. The moment conditions are simply the least-squares normal equations, which set orthogonality between the error terms and the state variables. Under inverse control, the moment conditions are augmented by the unconditional expectation of the first-order conditions (FOCs) implied by the type of policymaking being considered. At the optimum, a certain combination of structural and preference parameters set the FOCs to zero. These FOCs carry information about the relationship between the structural parameters and the policymaker's preferences. In the form of an additional moment, this information is exploited in the GMM procedure to estimate the policymaker's preference parameters.

The results of paper confirm that the GMM procedure is able to recover the underlying policy preferences of a discretionary monetary authority. The Monte Carlo experiments reveal that the FOC-augmented GMM procedure outperforms the standard GMM procedure in this way, and also assists in recovering the remaining structural parameters with greater accuracy. Robustness checks offer four refinements. First, the true value of the policy coefficient in the IS equation must be sufficiently large for the procedure to recover the policymaker's preference over the output gap. This can be understood as a requirement that the policymaker have some measurable influence over the economy for this preference to be detected in the data. Second, the procedure is robust to changes in the starting value of the parameters. Third, the procedure succeeds when adjusting the lag specification of the basic model, but in all cases, estimates of the structural parameters of the PC equation are returned with greater precision than those of the IS equation. Fourth, the procedure can discriminate between different sets of policy preferences, such as pure inflation targeting or a dual mandate to stabilize the output gap and inflation. However, the FOC-augmented system's ability to outperform the standard GMM system is related to the size of the preference over the output gap.

The contributions of this paper are twofold. First, it extends the application of the FOC-augmented GMM procedure to the framework of discretionary policymaking, which, along with commitment strategies, incorporates rational expectations. Previously, the procedure was established to work when the monetary authority applies simple Taylor-type rules (Givens and Salemi, 2008) or using backwards-looking models (Favero and Rovelli, 2003). Second, on an empirical level, the FOC-augmented procedure circumvents the need to instrument for inflation expectations using data which, in a rational expectations framework, are only somewhat informative.[1]

The rest of the paper is organized as follows. Section 2 summarizes the literature on inverse control in its different formulations, with emphasis on earlier attempts to exploit moment conditions. Section 3 outlines the model to be estimated and the adopted GMM procedure in detail. Section 4 summarizes the main results of the Monte Carlo simulations. Section 5 provides four tests of robustness. A conclusion follows.

2.2 Literature Review

In the optimal control literature, the loss function of the policymaker is typically calibrated exogenously. The weights on the volatility in the output gap or smoothing changes in the interest rate are chosen relative to the overarching concern for the variance in inflation; see, for instance, Jensen (2002), Walsh (2003), or McCallum and Nelson (2004). More weight is usually granted to the goal output gap stabilization over that of interest rate smoothing (e.g. Rudebusch and Svensson, 1999; Brock et al., 2007), though Giannoni and Woodford (2004) argue that for the sake of maintaining low average interest rates, the policymaker may choose to accept greater variability in inflation and the output gap. Yet, there is a burgeoning

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[1] See Mavroeidis et al. (2014) for a thorough review and discussion of this literature.

literature on inverse control, in which the policy parameters are estimated, which I relate here.

Sack (2000) estimates a vector autoregression for which the objective function accounts for volatility in inflation and unemployment, and provides an estimate of the unemployment parameter of 0.79. Sack does not let interest rate smoothing appear as an explicit goal in the loss minimization function; instead, the graduality in the path of this variable is explained in part by the Federal Reserve not knowing the structure of the economy with certainty. Dennis (2006), using a quasi-full information maximum likelihood technique, finds point estimates of 2.941 and 4.517 for output gap stabilization and interest rate smoothing, respectively, during the Volcker-Greenspan era, though the former is not found to be statistically significant. Using Bayesian methods, Ilbas (2012) establishes the existence of a break in the conduct of monetary policy at the start of the Volcker–Greenspan period, moving from an empirical feedback rule to an explicit optimization framework under commitment. The estimates reveal that the policy preferences of the Federal Reserve during the latter era may be ranked, in order, as inflation stabilization, interest rate stabilization, interest rate smoothing and output growth.[2]

Givens (2012) estimates a simple New Keynesian model using a maximum likelihood method with Kalman filtering algorithms. Estimation is performed under both commitment and discretion to determine which style of policymaking is preferred empirically. The estimates under commitment indicate a strong preference for interest rate smoothing ($\lambda_{\Delta i} = 2.5581$) over output gap stabilization ($\lambda_y = .01351$). However, in the discretion model, both estimated parameters are close to zero ($\lambda_{\Delta i} = 0.0579$ and $\lambda_y = 0.0987$). Nevertheless, appealing to the Bayesian Information Criterion, the discretion model is preferred in terms of fit with the data.

Söderström et al. (2005) adopt a minimum-distance-to-moments approach to reveal the policymaker's preferences in the context of discretionary policymaking. The theoretical

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[2] This last term is computed as the distance between output from its natural level and the volatility of output growth in the previous period.

moments are defined as the standard deviations and a set of autocorrelations implied by the model's parameterization, and there are corresponding empirical moments from the data. To estimate the parameters of the model, the quadratic distance between these is minimized, subject to a weighting by the inverse matrix of standard errors of the estimated moments in the data. Their estimates reveal no preference for output gap stabilization and a preference for interest rate smoothing which exceeds that of inflation stabilization. Castelnuovo et al. (2006) reprises this strategy to estimate the weight on interest rate smoothing, estimated at 0.55, while exogenously setting the parameter on output gap stabilization to 0.5.

Castelnuovo and Surico (2004) estimate the policy preferences of the Federal Reserve under discretion by exploiting the relationship between the optimal interest rate rule and the state variables. Under discretionary policymaking, the optimal interest rate rule follows a vector of the state variables which depends on the convolutions of both the structural parameters of the economy and the preferences of the policymaker. First, a backwards-looking model is estimated, followed by a stochastic version of the optimal interest rate rule implied by a discretionary policy framework. Finally, the preference parameters λ_y and $\lambda_{\Delta i}$ are calibrated so as to minimize the sum of squared deviations over time between the optimal policy and the fitted path of interest rates.

The procedure I adopt in this paper is most similar in spirit to Favero and Rovelli (2003) and Givens and Salemi (2008), who also employ a GMM approach augmented with FOCs to reveal the policy preferences of the Federal Reserve. The model of Favero and Rovelli (2003) is a backwards-looking specification, which is estimated jointly with the Euler equation that solves the monetary authority's intertemporal optimization problem at the optimum. For the sample period spanning 1980:3–1998:3, estimates of λ_y and $\lambda_{\Delta i}$ are 0.00125 and 0.0085, respectively. In the Monte Carlo simulations of Givens and Salemi (2008), the monetary authority is presumed to adopt a Taylor-type rule and the FOCs which adjoin the standard GMM moments are derived from this assumption. The authors report no weight accorded to either output gap stabilization or interest rate smoothing for their forward-looking model.

Unlike the former paper, I employ a model that allows for forward-looking behaviour and, in contrast with the latter paper, I assume the monetary authority exploits private sector expectations and reoptimizes in each period, i.e., operates under discretion.

2.3 Description of method

In this section, I describe the Monte Carlo simulations used to demonstrate the efficacy of the FOC-augmented GMM procedure in recovering the preference parameters of the policymaker. First, an empirical New Keynesian model with specified parameter values is chosen and solved for its equilibrium law of motion under the assumption of discretionary policymaking. The reduced-form of the model acts as the data-generating process. I recover the parameters of the underlying model by employing a GMM procedure based on the least-squares normal equations derived from the reduced-form of the model, to which I add the FOC which holds at the discretionary solution to the policymaker's minimization problem. One hundred trials of sample data are generated at different sample sizes, and I return the average parameter estimates and the standard errors in each case. I test the hypothesis that adjoining this FOC will return more accurate estimates of the policy preference parameters of the data-generating process by comparing the results under a GMM augmented by the FOC to a GMM procedure exclusive of it.

2.3.1 Data-generating process and first-order condition

The model used here belongs to the class of empirical forward-looking New Keynesian models and follows the presentation in Söderström et al. (2005):

$$y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t \tag{2.1}$$

$$\pi_t = \alpha E_{t-1}\bar{\pi}_{t+3} + (1-\alpha)\pi_{t-1} + \kappa y_{t-1} + v_t \tag{2.2}$$

where $\bar{\pi}_t = 1/4 \sum_{j=0}^{3} \pi_{t-j}$ is annual inflation. The true parameter values are noted in Table 2.1 and are similar to those of Givens and Salemi (2008). Appendix B showcases impulse response functions for a 1% decrease in output, a 1% increase in inflation and a combined shock on the paths of the output gap, inflation rate and interest rate.

The first equation describes the output gap at time t in terms of its next period expectation (formed one period earlier) and two lags, the latter adding inertia and justified on the grounds of habit formation in consumption (see Svensson, 1999 or Fuhrer, 2000). The output gap is also related to the difference between the interest rate at t and the expectation of inflation in the following period; here, monetary policy exercises its influence on the output gap with a one-period lag. The weights on the parameters of the output gap are constrained to sum to one as in the Monte Carlo exercise of Fuhrer and Rudebusch (2004). This restriction follows from assuming that households derive utility from consumption over and above a habit based on consumption in the previous two periods. It implies that output does not respond in the long-run to shifts in the real interest rate.[3]

Equation (2.2) is a Phillips curve which relates inflation at t to the expectation of inflation one period ahead and to its one-period lag, as well as to the lagged output gap. Staggered price-setting behaviour, modeled in the manner of Calvo (1983), provides for the forward-looking component. The presence of inflation inertia can be justified on theoretical grounds of indexation (Christiano et al., 2005), rule-of-thumb price setting (Galí and Gertler, 1999), or as an empirical construct to circumvent "jump" dynamics, as evidenced in Fuhrer and Moore (1995). The coefficients on expected and lagged inflation sum to one to ensure no long-run trade-off between inflation and the output gap.

The monetary authority adjusts the interest rate i_t to minimize a quadratic loss function R subject to equations (2.1) and (2.2):

$$R = E_t(1-\delta) \sum_{j=0}^{\infty} \delta^j \left\{ \pi_{t+j}^2 + \lambda_y y_{t+j}^2 + \lambda_{\Delta i}(i_{t+j} - i_{t+j-1})^2 \right\} \tag{2.3}$$

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[3] That is, the only real interest rate value consistent with the steady state output gap is zero.

The loss function R reveals the monetary authority's concern for volatility in the output gap and inflation, as well as a desire to smooth the path of the interest rate over time. δ denotes the discount factor, while λ_y and $\lambda_{\Delta i}$ weigh the relative concerns for output gap stabilization and interest rate smoothing over inflation volatility, respectively. The targets for the output gap and inflation are normalized to zero, and I set $\lambda_y = 0.2$ and $\lambda_{\Delta i} = 0.3$, similar to Givens and Salemi (2008). The formulation of the loss function in (2.3) is familiar in the literature on optimal monetary policy (see, e.g., Giannoni and Woodford, 2004; Dennis, 2006; Givens, 2012). As Svensson (1999) points out, the mandates of central banks can be mapped accurately to a loss function which penalizes deviations from an inflation and output gap target.

When solving rational expectations models, one must make an a priori determination about the style of policymaking employed by the monetary authority. In what follows, I assume the central bank reoptimizes in each period, taking as given expectations of inflation and the output gap. That is, I presume the monetary authority is operating under discretion.[4] A discretionary policymaker retains the flexibility to change their behaviour as conditions evolve, independently of any prior engagements. Reneging on a previous course of action can yield improvements in the performance of the economy (Barro and Gordon, 1983). However, the policymaker cannot continually surprise private agents characterized by rational expectations; it must be that current behaviour does not encode a future change in incentives. The discretionary policymaker thus faces the additional constraint of not changing policy in a predictable manner. Discretion can be thought of as a style of learning, in which the underlying model is taken as given, but with the preference for flexibility suggested

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[4] Instead, the policymaker could apply some form of precommitment to a given policy, tying its hands as innovations to the economy are realized over time. Under commitment, the monetary authority can more readily affect the expectations of the private sector. On the other hand, a discretionary policymaker cannot fully anchor expectations, as the private sector is aware of the ad-hoc nature of the monetary authority's designs. By this argument, discretion yields worse stabilization outcomes (e.g., Clarida et al., 1999a) and a commitment-type strategy — along with its reputational concerns — would be preferable. The empirical determination of the style of policymaking actually used, however, remains under-developed (Givens, 2012) and is beyond the scope of this paper.

by Kreps (1979).[5]

Under discretion, the equilibrium law of motion for the predetermined variables $x_{1t} = \begin{bmatrix} y_t, y_{t-1}, \pi_t, i_t \end{bmatrix}'$ is given by:

$$x_{1t+1} = Mx_{1t} + v_{1t+1} \tag{2.4}$$

Equation (2.4) is the data-generating process used in testing. Deriving the above requires the first-order condition in equilibrium, which states that the interest rate rule at t is equal to some linear combination of the vector of state variables in the same period, to hold:

$$i_{t+1} = Fx_{1t} \tag{2.5}$$

The vector F is comprised of a number of matrices which carry information about the relationships between the structural parameters and the policymaker's preferences. Appendix A details how the model's reduced-form solution is obtained, along with the construction of F. I next describe the FOC-augmented GMM procedure used to the underlying parameters of (2.1) and (2.2).

2.3.2 Moments for the GMM procedure

From (2.4), the least-squares normal equations may be defined as $E(v_{1t+1}x'_{1t}) = 0$. In the model adopted here, this amounts to eight equations that form the basis of the GMM procedure. The unconditional expectation of the FOC in (2.5) provides the theoretical ninth moment of interest. Collect the structural parameters of the model, including the underlying preferences of the monetary authority, into a vector $\rho = \{\lambda, a_1, b, \alpha, \kappa, \lambda_y, \lambda_{\Delta i}\}$. Let $g(\rho, v_{t+1})$ hold both the corresponding sample moments implied by (2.4) and the theoretical moment embodied in (2.5). The GMM criterion to minimize, Q, is then given by

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[5] See, e.g., Cogley and Sargent (2005) for learning environments in which the policymaker reassesses, over time, the probabilities accorded to competing candidate models.

$g(\rho, v_{t+1})' S^{-1} g(\rho, v_{t+1})$, with S^{-1} the optimal weighting matrix. A two-step procedure is adopted: the minimization is initially performed with an identity weighting matrix, and in a second step, S^{-1} is redefined as the inverse of the moment covariance matrix derived from the first step estimates.

A computational concern noted in Givens and Salemi (2008) is that the least-squares normal equation moments and FOC-derived moments may differ substantially in their numerical scales as different parameter values are tested, yielding biased estimates. For this reason, the authors re-express the moments derived from the normal equations as correlations. This did not occur when adopting the discretionary framework and its implied FOC: the numerical scales of the two moment sets did not diverge meaningfully, and the estimates proved more accurate when leaving the equations in their usual form. However, other combinations of FOC and normal-equation moments may prove sensitive to this scaling issue.

2.3.3 Search algorithm

Using the data generated by the underlying model, a search algorithm moves through values of the vector ρ to minimize the GMM objective function.[6] The algorithm relies on two types of moves. First, an exploratory step searches locally for an increase or decrease in each parameter value which improves the value of the objective function. Then, the algorithm takes larger and larger steps in the overall direction determined in the first step, i.e., it follows the established pattern. In cases where the algorithm fails to yield a stable equilibrium under the discretionary solution algorithm, or when the estimates are highly implausible, the current trial is discarded and the next one begins.[7] I perform 100 draws of sample data, for sample sizes of 100, 250, 500 and 1,000 data points from the data-generating procedure, which are passed through the GMM procedure to yield estimates of the underlying

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[6] The search algorithm, PATERN, is part of the GQOPT numerical computation package developed by Richard Quandt, and is based on an algorithm developed by Hooke and Jeeves (1961).

[7] Implausible estimates are defined as those where at least one parameter estimate has a value of 100 or greater — well outside any reasonable range, and which would warrant re-starting the algorithm.

parameters.[8] This is effected both with and then without the FOC imposed over the course of estimation. The same random number generator seeds are used for both the FOC-augmented and standard GMM procedures for comparability.

2.4 Main Results

The model described in (2.1) and (2.2) implies eight least-squares normal equations which form the basis of the GMM procedure. The hypothesis to be tested is whether the addition of a ninth moment condition, formulated as the unconditional expectation of (2.5), helps recover the underlying preference parameters of the policymaker, λ_y and $\lambda_{\Delta i}$.

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[8] The starting values are given by the vector $\rho_s = \big[0.05, 0.11, 0.05, 0.37, 0.05, 0.05, 0.05\big]$.

Table 2.1: Results of Monte Carlo simulations, by conditions imposed and sample size

		FOC imposed				FOC not imposed			
Param.	True value	Sample size				Sample size			
		100	250	500	$1,000$	100	250	500	$1,000$
λ	0.300	0.299	0.289	0.293	0.293	0.290	0.317	0.342	0.396
		(0.026)	(0.022)	(0.018)	(0.017)	(0.029)	(0.024)	(0.021)	(0.018)
a	0.650	0.666	0.669	0.648	0.641	0.675	0.669	0.631	0.613
		(0.021)	(0.010)	(0.008)	(0.006)	(0.022)	(0.011)	(0.010)	(0.007)
b	0.200	0.225	0.215	0.211	0.208	0.215	0.190	0.191	0.163
		(0.017)	(0.014)	(0.010)	(0.008)	(0.020)	(0.014)	(0.012)	(0.009)
α	0.500	0.506	0.504	0.500	0.502	0.504	0.503	0.499	0.500
		(0.005)	(0.003)	(0.002)	(0.002)	(0.005)	(0.004)	(0.002)	(0.002)
κ	0.150	0.144	0.149	0.150	0.148	0.150	0.150	0.149	0.146
		(0.008)	(0.004)	(0.003)	(0.002)	(0.008)	(0.004)	(0.003)	(0.002)
λ_y	0.200	0.423	0.273	0.239	0.208	0.381	0.210	0.187	0.110
		(0.102)	(0.052)	(0.034)	(0.019)	(0.097)	(0.051)	(0.031)	(0.016)
$\lambda_{\Delta i}$	0.300	1.156	0.418	0.335	0.316	1.009	0.539	0.404	0.391
		(0.606)	(0.050)	(0.017)	(0.012)	(0.250)	(0.115)	(0.028)	(0.023)
Q	—	2.611e-2	1.059e-2	5.140e-3	2.281e-3	2.131e-2	8.861e-3	4.190e-3	1.810e-3
Successful trials		95	98	100	100	91	96	100	100

Note: The Monte Carlo estimates reported above are for the following model: $y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \kappa y_t + v_t$. "FOC imposed" estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = -F X_t$ which holds at the discretionary rational expectations solution. This condition is omitted in the calculation of the "FOC not imposed" estimates. Estimated values represent a simple average across successful trials (out of a maximum possible 100) for four sample sizes, with standard errors denoted in parentheses. Q represents the average value of the GMM score function at the estimated parameter values across all successful trials.

Table 2.1 summarizes the results of the Monte Carlo simulations, while detailed estimates may be found in Tables 2.8-2.15 of Appendix C. The left panel provides estimates for the parameters of the model when the first-order condition in (2.5) is imposed as an additional moment in the GMM procedure; this is omitted for the estimates of the right panel, which represent the control scenario. The FOC-augmented procedure yields estimates of the structural and preference parameters which converge to the true values as the sample size increases. In particular, the estimates of the underlying preference parameters of the policymaker, λ_y and $\lambda_{\Delta i}$, are more accurate when the first-order condition is imposed in the course of estimation. In the right panel, by contrast, the estimate for $\lambda_{\Delta i}$ remains substantially above its true value, even at the 1,000 sample size. Moreover, other parameter estimates are sharpened when adjoining the first-order condition, particularly a_2 and κ; this can be seen in the total deviation difference between the two panels for given sample sizes, which favours the FOC-augmented estimates at all but the 250 sample size estimates. Finally, the number of successful trials, on base 100, is increased when imposing the first-order condition. For all sample sizes, however, GMM criterion Q, i.e., the minimized weighted quadratic of the moments, was closer to zero when omitting the FOC.

In both the FOC-augmented and control scenarios, there is an excess of moments over the number of parameters to be estimated (two and one, respectively). The Sargan-Hansen test allows the verification of these additional restrictions. The null hypothesis is that the overidentifying restrictions implied by the model are valid. For the null to hold, the GMM criterion Q, evaluated at the parameter estimates and scaled by the number of observations, must be sufficiently close to zero. Formally, the following statistic is asymptotically chi-squared distributed with $k - l$ degrees of freedom:

$$Q \times T \sim \chi^2_{k-l} \tag{2.6}$$

where k denotes the number of moments and l the number of parameters being estimated. A priori, the test should not reject the overidentifying restrictions more frequently when running the FOC-augmented GMM system, as the additional restriction has a true value of zero.

Table 2.2 presents the percentage of trials for which the overidentifying restrictions were rejected by the Sargan-Hansen J-test, according to the chosen procedure and sample size. The rejection region is allowed to vary in each case, as a test of sensitivity. For either procedure, the probability of rejection of the overidentifying restrictions falls as the sample size or the test size increases. Moreover, for given sample and test sizes, the proportion of trials where the overidentifying restrictions are rejected is typically lower when imposing the

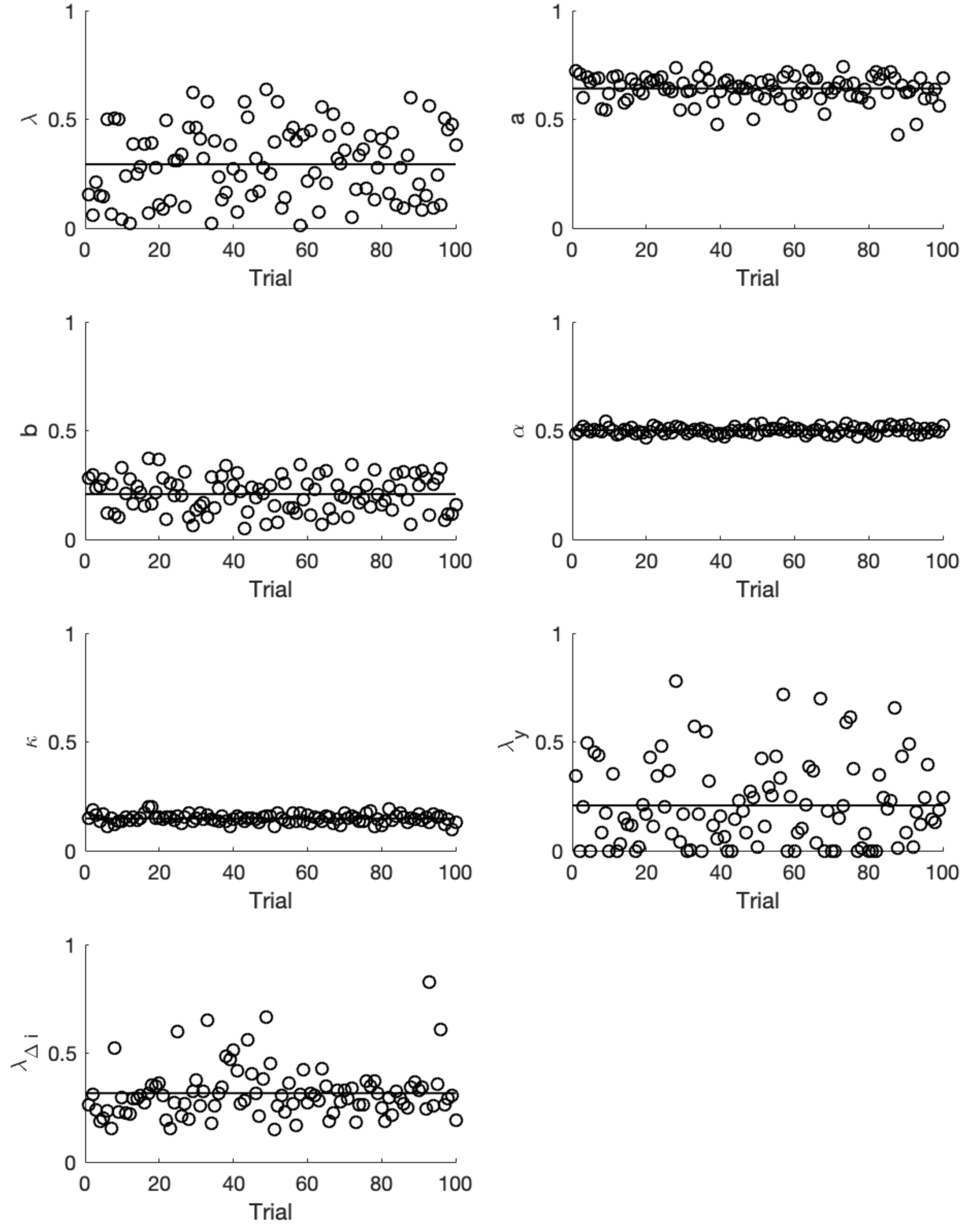
FOC, confirming the prediction.

Table 2.2: Percentage of trials leading to rejection of the overidentifying restrictions, by procedure and sample size

Procedure	Sample size	Test size				
		0.75	0.90	0.95	0.975	0.99
FOC not imposed	100	61.5	27.5	16.5	5.5	3.3
	250	53.1	24.0	15.6	11.5	4.2
	500	48.0	28.0	21.0	13.0	4.0
	1000	51.0	28.0	10.0	3.0	2.0
FOC imposed	100	34.7	14.7	5.3	5.3	2.1
	250	33.7	20.4	7.1	5.1	5.1
	500	33.0	19.0	13.0	7.0	1.0
	1000	35.0	11.0	4.0	1.0	1.0

A scatter plot of the parameter estimates when imposing the FOC (for a sample size of 1,000) is provided in Figure 1, along with the mean value as represented by a horizontal line. The structural parameters relative to the Phillips curve — α and κ — are tightly estimated, while those of the IS curve display a greater dispersion. The preference parameters — λ_y in particular — are also more scattered than those of the Phillips curve.

Figure 2.1: Parameter estimates when imposing FOC, by trial, 1000 sample size



Why does the addition of the first-order condition as a moment improve the estimates? As hinted at in the previous section, the vector F carries information about the relationships between the structural parameters and the policymaker's preference parameters; it is a convolution of these elements. Thus, when searching over the parameter space, the GMM minimization algorithm looks for parameter values which imply a candidate F that closely matches the observed values of i_t to a linear combination of the remaining state variables. The restrictions arising from the least-squares normal equations must be met in parallel with the theoretical moment. In the case of a simple policy rule described in Givens and Salemi (2008), the GMM approach fully bypasses the need to nest the optimal control exercise under the econometrician's estimation procedure. The optimal policy coefficients are free parameters jointly estimated with the structural and policy preference coefficients. In the case of discretion, however, it is not possible to circumvent the the optimal control exercise, as the vector elements of the FOC are embedded in the reduced-form matrix M. Nevertheless, the FOC-augmented estimation procedure requires the FOC moment to be jointly satisfied with the least squares normal equations, anchoring parameter estimates more closely to their true values compared to the control (no FOC) procedure.

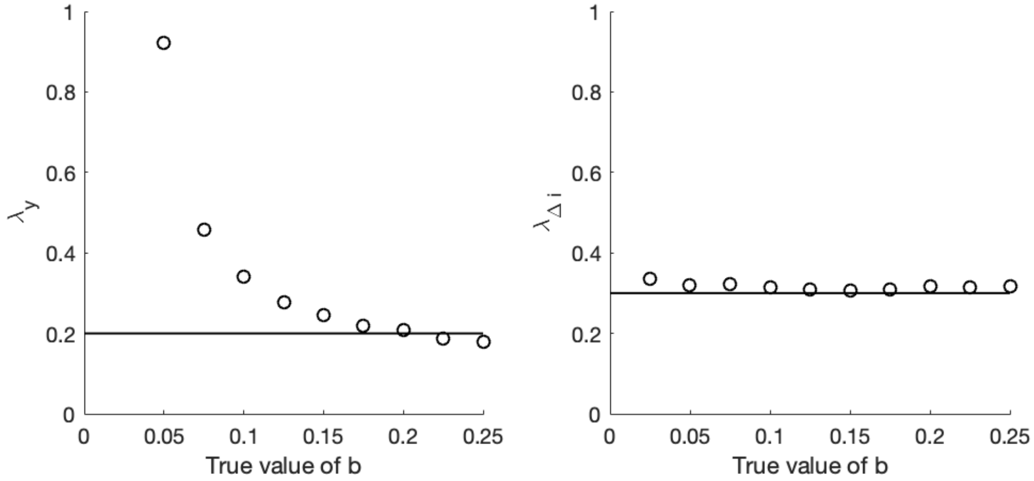
2.5 Robustness

2.5.1 Adjusting the true value of the policy coefficient

In this class of model, the policymaker is able to affect the macroeconomy through the difference in the interest rate from expected inflation. The effect size is governed by the parameter b. But suppose the policymaker could not meaningfully affect the macroeconomy through its lever, that is, a situation where the true b value is particularly low. Then, no matter the policy preferences of the monetary authority, any policy action will have no observable effect on the output gap or inflation, and one might question whether it is possible to back up said preferences from the data. It is therefore sensible to examine the sensitivity of the preference parameter estimates to changes in the value of b.

I rerun the FOC-augmented procedure, varying the true value of the parameter b through a grid from 0.025 to 0.250 in steps of 0.025. Figure 2 displays how the mean value of the policy parameters λ_y and $\lambda_{\Delta i}$ change as a function of b, when imposing the FOC and for a sample size of 1,000. At low levels of the policy parameter, the policymaker's preferences over stabilizing the output gap are not revealed as manipulations of the interest rate have no traction on output. As b increases, the mean estimated values of λ_y converge to the true value and the standard error decreases. On the other hand, the preference parameter for smoothing the interest rate is estimated with accuracy at all levels of b. Meanwhile, the other structural parameters receive no benefit in their estimated values from increasing the true underlying value of b, with the exception of λ. Table 2.16 in Appendix D details the recovered parameter estimates for each true value of b.

Figure 2.2: Policy preference parameter estimates for increasing value of b, 1,000 sample



2.5.2 Varying the starting values of the parameters

In this section, I examine the sensitivity of the GMM model to the starting values of the parameters during the search procedure. I reset the starting values of the IS equation and policy preference parameters either 10% above or 10% below their values in the initial specification. These alternative starting value scenarios are nested one in another, for a total of 32 alternatives at the 1,000 sample level.[9] With each change in starting values of the parameters, 100 trials are attempted at the 1,000 sample size, and the mean estimates and standard errors reported.

Table 2.3: Results of Monte Carlo simulations, variable starting values

Param.	True value	1000 sample size	
		b varying	b fixed
λ	0.200	0.299	0.290
		(0.016)	(0.016)
a	0.650	0.640	0.643
		(0.006)	(0.006)
b	0.200	0.206	0.210
		(0.009)	(0.008)
α	0.500	0.502	0.502
		(0.002)	(0.002)
κ	0.150	0.148	0.148
		(0.002)	(0.002)
λ_y	0.200	0.198	0.205
		(0.020)	(0.020)
$\lambda_{\Delta i}$	0.300	0.315	0.311
		(0.011)	(0.011)
Q	—	5.624e-04	6.102e-04

The results of this experiment are reported in the left column of Table 2.3. Parameter estimates at the 1,000 sample size are close to those obtained in the benchmark specification, though the mean estimates of $\lambda_{\Delta i}$ and b are somewhat higher and lower, respectively. Given the sensitivity of parameter estimates to b, as discussed in the previous section, the experiment was rerun with the starting value of this parameter fixed. The right column of Table 2.3 reports that there is no sizeable difference when this condition is imposed. Broadly, the test reveals that the GMM procedure is not dependent on a particular set of starting values. This should not be taken to mean, however, that any starting values will return accurate

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[9] The starting values for the parameters of the PC equation were left unchanged due to precision of the estimates returned across trials.

estimates. Care should be taken to choose starting values which are reasonably close to a region of the parameter space where a stable discretionary solution occurs.[10]

2.5.3 Different lag specifications

I have previously considered model uncertainty with respect to the lag specification of the New Keynesian system using GMM. I showed that it is possible to transform the objective criterion Q, when evaluated at the parameter estimates, into a pseudo-likelihood. From this it is possible to calculate the posterior probabilities of various lag specifications of the basic model. My results showed that none of the representations are particularly favoured in terms of model posteriors. This implies that the dynamics of each model must be accounted for when designing a weighted policy. Although beyond the scope of this paper, it should be possible for the procedure developed here to accommodate different degrees of inertia in the IS-PC equation system, so as produce a posterior-weighted estimate of the policymaker's preferences. As an indication of the flexibility of the FOC-augmented procedure, I put forward two alternative specifications. I also consider whether adjusting the number of estimated parameters affects their precision, given the disparity in standard errors between those of the IS and PC equation (as highlighted in Figure 1).

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[10] This is remarked in an appendix to Givens and Salemi (2008).

Table 2.4: Results of Monte Carlo simulations, by conditions imposed and sample size

		FOC imposed				FOC not imposed			
Param.	True value	Sample size				Sample size			
		100	250	500	5000	100	250	500	5000
λ	0.300	0.296	0.290	0.302	0.306	0.290	0.298	0.317	0.348
		(0.023)	(0.021)	(0.017)	(0.015)	(0.026)	(0.023)	(0.019)	(0.016)
b	0.200	0.219	0.211	0.202	0.194	0.266	0.223	0.191	0.171
		(0.018)	(0.014)	(0.012)	(0.009)	(0.028)	(0.019)	(0.013)	(0.011)
α	0.500	0.518	0.505	0.505	0.503	0.513	0.506	0.503	0.502
		(0.006)	(0.003)	(0.002)	(0.002)	(0.007)	(0.004)	(0.002)	(0.002)
κ	0.150	0.148	0.146	0.146	0.148	0.148	0.145	0.147	0.147
		(0.006)	(0.004)	(0.002)	(0.002)	(0.006)	(0.004)	(0.002)	(0.002)
λ_y	0.200	1.088	0.243	0.197	0.182	1.017	0.252	0.176	0.145
		(0.505)	(0.031)	(0.023)	(0.014)	(0.571)	(0.037)	(0.024)	(0.012)
$\lambda_{\Delta i}$	0.300	2.729	0.563	0.381	0.316	5.809	0.708	0.435	0.359
		(0.796)	(0.129)	(0.026)	(0.011)	(1.511)	(0.136)	(0.034)	(0.020)
Q	—	1.736e-2	5.824e-3	2.968e-3	1.211e-3	1.063e-2	3.992e-3	1.849e-3	7.812e-4
Successful trials		94	99	99	100	90	96	100	100

Note: The Monte Carlo estimates reported above are for the following model: $y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)y_{t-1} - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \kappa y_{t-1} + v_t$. "FOC imposed" estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = -FX_t$ which holds at the discretionary rational expectations solution. This condition is omitted in the calculation of the "FOC not imposed" estimates. Estimated values represent a simple average across successful trials (out of a maximum possible 100) for four sample sizes, with standard errors denoted in parentheses. Q represents the average value of the GMM score function at the estimated parameter values across all successful trials.

In the first case, the IS equation is restricted to one parameter governing the weight of expectations against the lag of the output gap. Table 2.4 presents the results of the GMM procedure for this specification, both in the FOC augmented case and without the additional constraint imposed. The results are broadly similar to those arising from the base specification. The policy preference parameters are estimated with greater accuracy when imposing the FOC, and estimates converge to their true values as the sample size increases. Examining the standard errors reveals that, as before, the estimates on the PC equation are returned with more precision than those of the IS equation.

In a second alternative specification, I add a second lag of the output gap to the Phillips curve equation and rerun the Monte Carlo simulations. The estimates are provided in Table 2.5, and the same patterns are revealed here as in the preceding case. These results indicate the GMM procedure is suitable for different specifications of the New Keynesian system. However, there is no evidence that the number of lags affects the difference in the precision of parameter estimates of the IS and PC equations.

Table 2.5: Results of Monte Carlo simulations, by conditions imposed and sample size

Param.	True value	FOC imposed Sample size				FOC not imposed Sample size			
		100	250	500	5000	100	250	500	5000
λ	0.300	0.307	0.294	0.333	0.321	0.308	0.311	0.347	0.378
		(0.027)	(0.024)	(0.020)	(0.018)	(0.030)	(0.025)	(0.021)	(0.020)
a	0.650	0.695	0.677	0.649	0.642	0.706	0.685	0.645	0.627
		(0.022)	(0.011)	(0.010)	(0.006)	(0.024)	(0.012)	(0.010)	(0.007)
b	0.200	0.229	0.211	0.194	0.197	0.220	0.186	0.194	0.171
		(0.018)	(0.013)	(0.010)	(0.009)	(0.023)	(0.015)	(0.014)	(0.009)
α	0.500	0.507	0.507	0.501	0.503	0.504	0.503	0.498	0.501
		(0.006)	(0.003)	(0.002)	(0.002)	(0.006)	(0.003)	(0.002)	(0.002)
κ_1	0.150	0.129	0.122	0.133	0.143	0.144	0.135	0.145	0.146
		(0.010)	(0.007)	(0.006)	(0.004)	(0.010)	(0.007)	(0.005)	(0.003)
κ_2	0.100	0.113	0.121	0.112	0.103	0.102	0.116	0.105	0.101
		(0.008)	(0.006)	(0.005)	(0.003)	(0.007)	(0.006)	(0.004)	(0.003)
λ_y	0.200	0.908	0.254	0.204	0.181	0.513	0.185	0.126	0.070
		(0.368)	(0.044)	(0.039)	(0.026)	(0.200)	(0.036)	(0.034)	(0.017)
$\lambda_{\Delta i}$	0.300	2.039	0.585	0.386	0.322	2.118	0.638	0.438	0.356
		(0.883)	(0.108)	(0.036)	(0.013)	(0.916)	(0.123)	(0.043)	(0.018)
Q	—	2.283e-2	9.735e-3	5.221e-3	1.931e-3	1.608e-2	5.825e-3	2.434e-3	9.881e-4
Successful trials		99	98	98	100	93	96	100	100

Note: The Monte Carlo estimates reported above are for the following model: $y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)\sum_{j=1}^{2} a_j y_{t-j} - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \kappa_1 y_{t-1} + \kappa_2 yt - 2 + v_t$. "FOC imposed" estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = -FX_t$ which holds at the discretionary rational expectations solution. This condition is omitted in the calculation of the "FOC not imposed" estimates. Estimated values represent a simple average across successful trials (out of a maximum possible 100) for four sample sizes, with standard errors denoted in parentheses. Q represents the average value of the GMM score function at the estimated parameter values across all successful trials.

2.5.4 Inflation targeting, dual mandate

Central banks differ in the statements they communicate to the public about the aim of monetary policy. The Bank of Canada, for instance, has a unipolar mandate of inflation targeting: “[t]he objective of monetary policy is to preserve the value of money by keeping inflation low, stable and predictable...” (Bank of Canada, 2011). By contrast, the Federal Reserve of the United States puts forward a dual mandate with respect to output and inflation, stating that the Federal Open Market Committee shall act “to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates” (Federal Reserve, 2016). In this section, I adjust the underlying preference parameters of the data-generating process to represent, in turn, the unipolar and bipolar mandates, and in each case rerun the Monte Carlo simulations to determine whether the GMM procedure is able to retrieve these preference sets. Throughout, I maintain the assumption that the policymaker also seeks to smooth the path of the interest rate, $\lambda_{\Delta i}$.

Table 2.6: Results of Monte Carlo simulations, by conditions imposed and sample size

		FOC imposed				FOC not imposed			
Param.	True value	Sample size				Sample size			
		100	250	500	1000	100	250	500	1000
λ	0.300	0.255	0.269	0.235	0.234	0.253	0.274	0.254	0.227
		(0.025)	(0.022)	(0.018)	(0.017)	(0.026)	(0.023)	(0.020)	(0.019)
a	0.650	0.679	0.667	0.660	0.657	0.692	0.658	0.660	0.657
		(0.022)	(0.010)	(0.007)	(0.005)	(0.022)	(0.011)	(0.008)	(0.006)
b	0.200	0.242	0.220	0.239	0.233	0.225	0.216	0.219	0.232
		(0.016)	(0.013)	(0.011)	(0.009)	(0.019)	(0.014)	(0.012)	(0.010)
α	0.500	0.508	0.504	0.505	0.503	0.507	0.504	0.504	0.503
		(0.006)	(0.003)	(0.002)	(0.002)	(0.006)	(0.004)	(0.002)	(0.002)
κ	0.150	0.145	0.149	0.146	0.149	0.145	0.148	0.147	0.150
		(0.007)	(0.004)	(0.003)	(0.002)	(0.007)	(0.004)	(0.003)	(0.002)
λ_y	0.000	0.432	0.143	0.092	0.033	0.411	0.121	0.061	0.023
		(0.154)	(0.030)	(0.018)	(0.006)	(0.129)	(0.030)	(0.018)	(0.006)
$\lambda_{\Delta i}$	0.300	0.580	0.490	0.344	0.313	0.704	0.930	0.363	0.319
		(0.087)	(0.074)	(0.021)	(0.014)	(0.119)	(0.391)	(0.030)	(0.018)
Q	—	2.839e-2	1.121e-2	5.865e-3	2.764e-3	2.416e-2	8.781e-3	5.003e-3	2.371e-3
Successful trials		95	97	99	99	93	95	100	100

Note: The Monte Carlo estimates reported above are for the following model: $y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)\sum_{j=1}^{2} a_j y_{t-j} - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t\pi_{t+1} + (1-\alpha)\pi_{t-1} + \kappa_1 y_{t-1} + \kappa_2 yt - 2 + v_t$. "FOC imposed" estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = -FX_t$ which holds at the discretionary rational expectations solution. This condition is omitted in the calculation of the "FOC not imposed" estimates. Estimated values represent a simple average across successful trials (out of a maximum possible 100) for four sample sizes, with standard errors denoted in parentheses. Q represents the average value of the GMM score function at the estimated parameter values across all successful trials.

The structural and preference parameter estimates under a pure inflation targeting regime are provided in Table 2.6. In this scenario, the true value of λ_y is set to zero. As before, preference parameter estimates converge to their true values as the sample size increases, but the imposition of the FOC only outperforms the regular system in recovering of $\lambda_{\Delta i}$ at lower sample sizes. In addition, there is no longer any benefit to the accuracy of the structural parameter estimates when running the augmented system over the standard GMM routine. Moreover, a downward bias appears in the estimates of λ appear in both the FOC-augmented and standard systems, in contrast with the benchmark scenario.

These results may be contrasted with those presented in Table 2.7, which reflect the dual mandate case (i.e., where $\lambda_y = 0$). The FOC-augmented GMM procedure returns more accurate estimates of the underlying parameters of the data-generating process, and estimates appear to converge to their true values as the sample size increases. By contrast, the standard system is does not provide unbiased estimates of the policymaker's preferences, even at the highest sample sizes. In addition, estimates of the parameters of the IS equation are poor, whereas the FOC-augmented system returns them with increasing accuracy.

Table 2.7: Results of Monte Carlo simulations, by conditions imposed and sample size

Param.	True value	FOC imposed				FOC not imposed			
		Sample size				Sample size			
		100	250	500	1000	100	250	500	1000
λ	0.300	0.417	0.384	0.324	0.323	0.426	0.493	0.453	0.532
		(0.029)	(0.026)	(0.024)	(0.019)	(0.029)	(0.026)	(0.024)	(0.021)
a	0.650	0.651	0.597	0.624	0.637	0.585	0.545	0.591	0.569
		(0.061)	(0.016)	(0.012)	(0.007)	(0.030)	(0.021)	(0.013)	(0.010)
b	0.200	0.179	0.177	0.204	0.191	0.196	0.139	0.152	0.105
		(0.016)	(0.013)	(0.013)	(0.010)	(0.023)	(0.015)	(0.014)	(0.011)
α	0.500	0.519	0.505	0.500	0.501	0.514	0.502	0.497	0.496
		(0.005)	(0.003)	(0.002)	(0.002)	(0.005)	(0.003)	(0.002)	(0.002)
κ	0.150	0.142	0.139	0.149	0.146	0.147	0.140	0.146	0.142
		(0.008)	(0.004)	(0.003)	(0.003)	(0.008)	(0.004)	(0.003)	(0.002)
λ_y	1.000	2.545	0.830	0.840	0.912	2.858	0.546	0.629	0.506
		(0.755)	(0.091)	(0.058)	(0.045)	(0.926)	(0.080)	(0.070)	(0.050)
$\lambda_{\Delta i}$	0.300	2.284	0.603	0.543	0.330	2.610	0.896	0.633	0.582
		(0.899)	(0.108)	(0.106)	(0.017)	(0.641)	(0.235)	(0.054)	(0.040)
Q	—	2.446e-2	8.450e-3	4.406e-3	2.530e-3	1.955e-2	7.579e-3	3.644e-3	1.818e-3
Successful trials		95	100	100	100	91	97	100	100

Note: The Monte Carlo estimates reported above are for the following model: $y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)\sum_{j=1}^{2} a_j y_{t-j} - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \kappa_1 y_{t-1} + \kappa_2 yt - 2 + v_t$. "FOC imposed" estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = -FX_t$ which holds at the discretionary rational expectations solution. This condition is omitted in the calculation of the "FOC not imposed" estimates. Estimated values represent a simple average across successful trials (out of a maximum possible 100) for four sample sizes, with standard errors denoted in parentheses. Q represents the average value of the GMM score function at the estimated parameter values across all successful trials.

Two facts emerge when considering these results alongside those of the unipolar mandate and the benchmark scenario. First, the parameters of the IS equation are more difficult for the standard GMM system to estimate when the policymaker exhibits a preference over limiting variance in the output gap, and the biases in the estimates are increasing in the strength of this preference. Second, absent this preference (the unipolar mandate case), the FOC-augmented system performs better only at lower sample sizes. A tentative explanation is as follows. Less constrained by the unipolar policymaker, movements in the output gap are sufficient for the standard GMM system to identify the parameters of (2.1) and the preference over smoothing the path of the interest rate, but only given enough data points. However, when the policymaker is determined to limit fluctuations in the output gap, there is no longer sufficient variation in this variable for the standard system to exploit. The FOC, however, encodes additional information about the relationship between the output gap and the interest rate set by a discretionary policymaker in light of her preferences. When imposing the FOC, it becomes again possible to retrieve accurate estimates of the IS equation parameters, as well as the policymaker's preference over variance in the output gap.

It should be stressed that imposing the FOC is never a disadvantage: even if the policymaker does not, in reality, care about the output gap, this extra constraint helps in estimating the preference over the interest rate smoothing, at least at lower sample sizes. Thus, an econometrician who is agnostic about the true data-generating process and preference parameters would choose to impose the FOC. Overall, these results provide some confidence that the procedure developed in the paper can discriminate between data-generating processes with substantively different underlying preference parameters.

2.6 Conclusion

In this paper I have extended the GMM approach to recovering policy preference parameters to the case where the monetary authority is operating under discretion. This is done by adding to the usual least squares normal equations the FOC derived under a rational expectations model, which relates the optimal current period interest rate to a linear combination of the state variables. The hypothesis that the FOC aids in the recovery of the weights placed on output gap stabilization and interest rate smoothing is tested by attempting to recover the model parameters under the usual GMM procedure, which precludes the FOC, to one augmented by the unconditional expectation of this optimality condition.

The power of the procedure is demonstrated through Monte Carlo simulations, in which an empirical New Keynesian model acts as the data-generating process. A search algorithm minimizes the GMM criterion built from the least squares normal equations (and FOC) and delivers the parameter estimates which most closely sets the criteria to zero. My results indicate that when adjoining the FOC, not only are the policy preference parameters recovered with accuracy, but other structural parameters of the model are sharpened.

Further testing demonstrates that the procedure requires a suitably large policy coefficient in the IS equation for the monetary authority's preference over the output gap to be retrieved without upward bias in the generated data. Moreover, the GMM procedure is able to recover the policy parameters using variable starting values. A third robustness check reveals that the procedure continues to return accurate estimates under alternative specifications, but that the estimated structural parameters of the IS equation continue to exhibit more variance than those of the PC equation. Finally, the procedure is able to discriminate between different policy preference regimes, such as inflation targeting and a dual mandate. However, this last set of tests reveals that the ability of the augmented system to outperform the standard GMM procedure is increasing in the policymaker's preference over movements in the output gap.

As described by Givens (2012), discretion forms but one-half of the monetary policy divide in the framework of rational expectations. The GMM procedure described in this paper could be extended further to recover policy preference parameters in the case of commitment-type policymaking, which, combined with the results of Givens and Salemi (2008), would encompass the range of approaches to policymaking typically described in the literature. While commitment and discretion are non-nested, other methods for comparing fit between the two models, such as the Bayesian Information Criterion, could be assessed under GMM by considering the pseudo-log-likelihood transformation I have described in chapter 1.

The FOC-augmented procedure developed in this paper has straightforward empirical applications. Estimates of the policy preference parameters can be used to test whether the stated mandates of monetary authorities match what is recovered from the data using inverse control. In Chapter 3, I compare the mandates of the central banks of Canada, the United States and the United Kingdom using the tool developed here. A similar profile of macroeconomic outcomes is suggestive that they may, in fact, have had identical policy preferences despite communicating differing mandates.

Appendix A: State-space and discretionary solution

In this section, I identify the relationship between the policymaker's preferences and the first-order condition in (2.5) . This material builds on what was presented in Appendix C of Chapter 1 and closely follows Givens (2012).

2. Discretionary solution: the optimal linear regulator problem

Recall from Appendix C in Chapter 1 that the model in (2.1) and (2.2) may be expressed in state-space form:

$$\begin{bmatrix} x_{1t+1} \\ \Omega E_t x_{2t+1} \end{bmatrix} = A_1 \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B_0 i_t + v_{t+1} \tag{2.A1}$$

The losses faced by the policymaker at time t take the form $L_t = z_t' K z_t$, where $z_t = \begin{bmatrix} \pi_t & y_t & \Delta i_t \end{bmatrix}'$ are the target variables of the policymaker, and where

$$K = \begin{bmatrix} \lambda_y & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda_i \end{bmatrix}$$

provides the weights for the target variables in the loss function (relative to inflation).

The intertemporal loss function is therefore:

$$L_0 = E_0(1-\delta)\sum_{t=0}^{\infty} \delta^t z_t' W z_t \tag{2.A2}$$

with δ as the discount factor.

The decision problem is to choose i_t in period t to minimize the intertemporal loss function

(2.A2) under discretion, i.e, subject to (2.A1), x_{1t} given, and

$$i_{t+1} = F_{t+1}x_{1t+1} \tag{2.A3}$$

$$x_{2t+1} = G_{t+1}x_{1t+1} \tag{2.A4}$$

where F_{t+1} and G_{t+1} are determined by the decision problem at $t+1$. The forward-looking variables x_{2t} must be rewritten in terms of the predetermined variables x_{1t} and the control variable i_t to recast the system as a standard optimal linear regulator problem.

Take the conditional expectation of (2.A1) as of t:

$$\begin{bmatrix} I_{4x4} & 0_{4x2} \\ 0_{2x4} & \Omega \end{bmatrix} \begin{bmatrix} E_t x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} i_t \tag{2.A5}$$

where A_1 and B_1 have been partitioned conformably with x_{1t} and x_{2t}. Combine (2.A4) with the upper block of (2.A5) to yield

$$E_t x_{2t+1} = G_{t+1}E_t x_{1t+1} = G_{t+1}(A_{11}x_{1t} + A_{12}x_{2t} + B_1 i_t) \tag{2.A6}$$

Multiply (2.A6) by Ω and set equal to the lower block of (2.A5) to express x_{2t} in the following form:

$$x_{2t} = \tilde{A}_t x_{1t} + \tilde{B}_t i_t \tag{2.A7}$$

where

$$\tilde{A}_t \equiv (A_{22} - \Omega G_{t+1}A_{12})^{-1}(\Omega G_{t+1}A_{11} - A_{21})$$
$$\tilde{B}_t \equiv (A_{22} - \Omega G_{t+1}A_{12})^{-1}(\Omega G_{t+1}B_1 - B_2)$$

Substitute (2.A7) into the upper block of (2.A5) and reinsert the non-zero block of innovations:

$$x_{1t+1} = A_t^* x_{1t} + B_t^* i_t + v_{1t+1} \tag{2.A8}$$

where

$$\begin{aligned} A_t^* &\equiv A_{11} + A_{12}\tilde{A}_t \\ B_t^* &\equiv B_1 + A_{12}\tilde{B}_t \end{aligned}$$

Then, z_t may be written in terms of x_{1t} and i_t, with the corresponding one-period loss expressed as

$$z_t' W z_t = \begin{bmatrix} x_{1t} \\ i_t \end{bmatrix}' \begin{bmatrix} Q_t & N_t \\ N_t' & R_t \end{bmatrix} \begin{bmatrix} x_{1t} \\ i_t \end{bmatrix} \tag{2.A9}$$

where

$$\begin{aligned} Q_t &\equiv \tilde{W}_{XX} + \tilde{A}_t'\tilde{W}_{Xx}' + \tilde{W}_{Xx}\tilde{A}_t + \tilde{A}_t'\tilde{W}_{xx}\tilde{A}_t \\ N_t &\equiv \tilde{W}_{Xx}\tilde{B}_t + \tilde{A}_t'\tilde{W}_{xx}\tilde{B}_t + \tilde{W}_{Xi} + \tilde{A}_t'\tilde{W}_{xi} \\ R_t &\equiv \tilde{B}_t'\tilde{W}_{xx}\tilde{B}_t + \tilde{W}_{xi}'\tilde{B}_t + \tilde{B}_t'\tilde{W}_{xi} + \tilde{W}_{ii} \end{aligned}$$

To formulate (2.A9) as above, it is necessary to define the auxiliary matrix

$$\tilde{W} = D'WD = \begin{bmatrix} \tilde{W}_{XX} & \tilde{W}_{Xx} & \tilde{W}_{Xi} \\ \tilde{W}_{Xx}' & \tilde{W}_{xx} & \tilde{W}_{xi} \\ \tilde{W}_{Xi}' & \tilde{W}_{xi}' & \tilde{W}_{ii} \end{bmatrix}$$

conformable with x_{1t} (X), x_{2t} (x) and i_t (i).

Given that the model is linear-quadratic, a solution in $t+1$ yields an optimal value function of quadratic form, $x'_{1t+1}V_{t+1}x_{1t+1} + u_{t+1}$, and a linear relation between the forward-looking variables and the state variables $x_{2t+1} = G_{t+1}x_{1t+1}$. Thus the problem at t must satisfy the Bellman equation:

$$\begin{aligned}(1-\delta)[x'_{1t}V_t x_{1t} + v_t] = (1-\delta)\min_{i_t}\{&z'_t W z_t \\ &+ \delta E_t[x'_{1t+1}V_{t+1}x_{1t+1} + u_{t+1}]\}\end{aligned} \tag{2.A10}$$

subject to (2.A8), (2.A9) and x_{1t} given.

The first-order condition with respect to i_t is

$$N'_t x_{1t} + R_t i_t + \delta B^{*\prime}_t V_{t+1}(A^*_t X_t + B^*_t i_t) = 0 \tag{2.A11}$$

Solving for i_t yields

$$i_t = F_t x_{1t} \tag{2.A12}$$

where

$$F_t \equiv -(R_t + \delta B^{*\prime}_t V_{t+1})^{-1}(N'_t + \delta B^{*\prime}_t V_{t+1} A^*_t)$$

Substitute (2.A12) into (2.A7) to re-express x_{2t} as

$$x_{2t} = G_t x_{1t}$$

where

$$G_t \equiv \tilde{A}_t + \tilde{B}_t F_t$$

Combining (2.A12) and (2.A10),

$$V_t \equiv Q_t + N_t F_t + F_t' N_t' + F_t' R_t F_t + \delta(A_t^* + B_t^* F_t)' V_{t+1}(A_t^* + B_t^* F_t) \tag{2.A13}$$

The system of equations for $\tilde{A}_t$, $\tilde{B}_t$, A_t^*, B_t^*, Q_t, N_t, R_t, F_t, G_t, and V_t comprises a mapping from $(F_{t+1}, G_{t+1}, V_{t+1})$ to (F_t, G_t, V_t). The solution to this mapping is a fixed point (F, G, V), at the limit of (F_t, G_t, V_t) when $t \to -\infty$.

Given stability, the discretion equilibrium is formulated as

$$x_{1t+1} = Mx_{1t} + v_{1t+1} \tag{2.A14}$$

$$\begin{bmatrix} x_{2t} \\ i_t \end{bmatrix} = Gx_{2t} \tag{2.A15}$$

where $M = A^* + B^* F$ and $G = [G' F']'$.

Examining the expression for F given in (2.A12), I note the presence of the matrices N_t and R_t, which are themselves functions of the auxiliary matrix $\tilde{W}$. $\tilde{W}$ contains the weights ascribed by the monetary authority to its various optimization objectives. This information is carried through to the first-order condition, and so the presence of (2.A12) in its equilibrium as a theoretical moment for the augmented GMM procedure aids in recovering the preferences of the policymaker.

Appendix B: Impulse response functions

Figure 2.3: Impulse response — 1% decrease in output

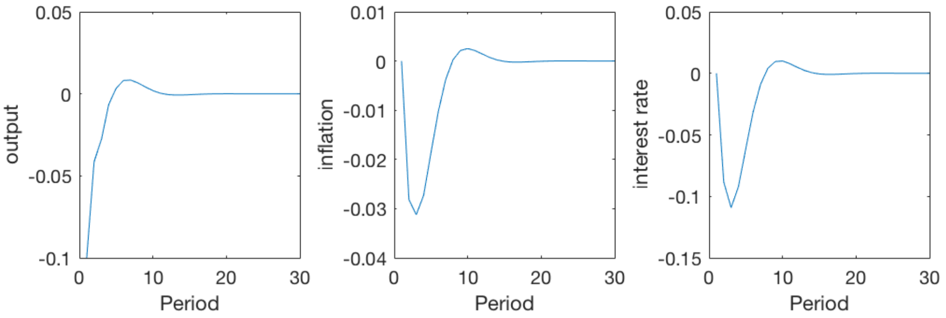


Figure 2.4: Impulse response — 1% increase in inflation

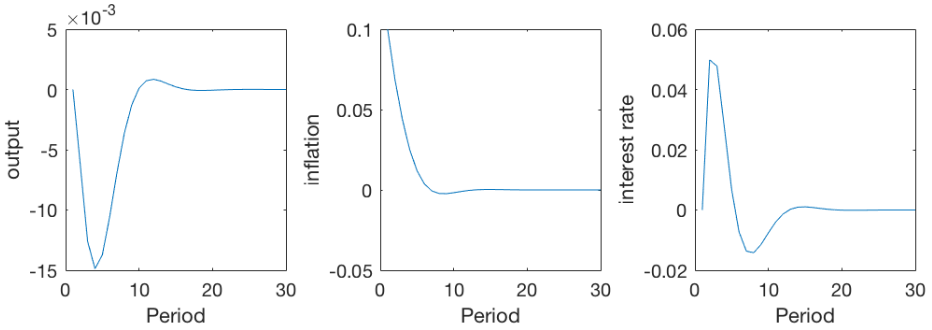
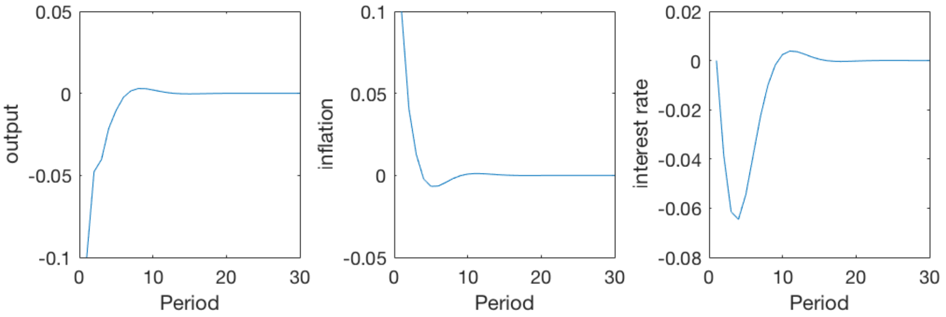


Figure 2.5: Impulse response — combined 1% output and inflation shock



Appendix C: Detailed simulation results

Table 2.8: Results of Monte Carlo simulations, FOC imposed, 100 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.631	0.634	0.029	0.538	0.108	0.136	0.150	51	0.258	0.574	0.163	0.561	0.036	0.171	0.019
2	0.046	0.669	0.373	0.469	0.117	1.147	0.368	52	0.662	0.470	0.043	0.554	0.103	0.228	0.493
3	0.000	0.925	0.392	0.406	0.271	0.000	0.306	53	0.169	0.740	0.514	0.523	0.115	0.834	0.916
4	0.465	0.836	0.095	0.490	0.223	0.338	0.780	54	0.072	0.562	0.525	0.578	0.019	0.120	0.044
5	0.638	0.928	0.012	0.485	0.078	0.000	0.033	55	0.719	0.633	0.052	0.661	0.101	0.025	0.018
6	0.002	0.722	0.383	0.460	0.198	0.000	0.301	56	0.063	0.750	0.221	0.435	0.212	0.000	0.235
7	0.000	0.694	0.392	0.461	0.190	1.107	2.280	57	0.371	0.678	0.256	0.460	0.241	0.303	0.204
8	0.530	0.538	0.190	0.477	0.226	0.000	1.297	58	0.610	0.084	0.052	0.550	0.074	0.834	0.468
9	0.352	0.682	0.279	0.516	0.195	0.000	0.994	59	0.266	0.683	0.244	0.481	0.250	1.455	0.256
10	0.489	0.773	0.103	0.579	0.060	0.378	0.058	60	0.123	0.693	0.176	0.531	0.078	0.102	0.025
11	0.024	0.628	0.510	0.480	0.120	0.000	1.394	61	0.102	0.777	0.230	0.526	0.106	0.012	0.047
12	0.641	0.399	0.054	0.471	0.242	0.000	0.771	62	0.456	0.515	0.104	0.573	0.202	0.000	0.247
13	0.067	0.652	0.377	0.523	0.098	0.020	0.205	63	0.485	0.983	0.052	0.473	0.123	0.000	0.215
14	0.770	0.236	0.051	0.452	0.110	0.139	2.399	64	0.031	0.622	0.409	0.471	0.070	0.000	0.512
15	0.633	1.040	0.023	0.503	0.143	0.058	0.264	65	0.355	0.702	0.145	0.506	0.127	1.966	0.278
16	0.542	1.056	0.043	0.527	0.109	0.000	0.093	66	0.052	0.666	0.484	0.423	0.220	1.109	0.900
17	0.156	0.441	0.242	0.526	0.052	0.000	0.093	67	0.334	0.706	0.103	0.426	0.175	0.009	0.092
18	0.052	0.745	0.380	0.536	0.081	0.014	0.256	68	0.708	0.743	0.013	0.597	0.091	0.531	0.066
19	0.632	0.694	0.019	0.603	0.065	0.000	0.159	69	0.025	0.687	0.359	0.500	0.159	0.000	0.290
20	0.376	0.965	0.119	0.511	0.147	0.652	0.445	70	0.055	0.651	0.331	0.523	0.204	0.000	0.562
21	0.000	0.584	0.424	0.443	0.184	0.000	0.543	71	0.116	0.726	0.500	0.492	0.184	0.000	0.829
22	0.622	0.661	0.072	0.478	0.221	0.452	0.632	72	0.063	0.691	0.210	0.452	0.189	0.000	0.205
23	0.642	0.533	0.026	0.449	0.158	0.000	0.022	73	0.562	0.406	0.104	0.439	0.130	0.000	0.905
24	0.504	0.561	0.119	0.499	0.108	0.000	0.962	74	0.059	0.824	0.124	0.574	0.119	1.014	0.133
25	0.503	0.855	0.087	0.572	0.088	0.740	1.085	75	0.330	0.715	0.218	0.472	0.056	0.138	0.215
26	0.512	0.322	0.100	0.484	0.067	0.561	0.557	76	0.699	0.060	0.084	0.583	0.004	0.013	0.068
27	0.002	0.779	0.011	0.452	0.180	0.240	0.000	77	0.023	0.590	0.385	0.531	0.109	0.000	0.232
28	0.022	0.710	0.293	0.588	0.272	2.105	0.515	78	0.080	0.876	0.259	0.510	0.113	0.076	0.148
29	0.674	0.709	0.058	0.492	0.216	0.642	3.531	79	0.066	0.655	0.386	0.438	0.215	0.412	0.483
30	0.658	0.660	0.015	0.629	0.220	0.096	0.141	80	0.001	0.642	0.667	0.510	0.057	0.144	1.920
31	0.185	0.764	0.248	0.511	0.210	0.033	0.312	81	0.085	0.903	0.294	0.574	0.134	0.068	0.095
32	0.026	0.537	0.520	0.484	0.227	0.000	0.682	82	0.397	0.701	0.230	0.493	0.216	0.000	0.504
33	0.453	0.473	0.129	0.477	0.118	0.011	0.392	83	0.002	1.008	0.305	0.524	0.181	0.000	0.070
34	—	—	—	—	—	—	—	84	0.058	0.458	0.595	0.494	0.042	0.043	0.290
35	—	—	—	—	—	—	—	85	0.204	0.488	0.247	0.452	0.147	0.000	0.507
36	0.318	0.452	0.236	0.531	0.016	0.117	0.070	86	0.440	0.889	0.113	0.362	0.252	0.000	0.648
37	0.180	0.564	0.207	0.513	0.162	0.000	0.753	87	—	—	—	—	—	—	—
38	0.039	0.544	0.377	0.500	0.200	0.000	0.266	88	0.667	0.902	0.010	0.542	0.192	0.000	0.325
39	0.130	0.690	0.325	0.460	0.311	0.106	0.614	89	0.505	1.018	0.051	0.465	0.166	0.000	0.175
40	0.042	0.786	0.332	0.499	0.122	0.620	0.336	90	0.013	0.657	0.529	0.439	0.180	0.177	1.589
41	0.079	0.704	0.389	0.401	0.201	0.000	0.428	91	0.520	0.797	0.118	0.507	0.209	0.041	0.296
42	0.096	0.710	0.327	0.495	0.219	0.000	0.626	92	0.533	0.595	0.072	0.479	0.049	1.378	0.321
43	0.740	0.070	0.029	0.502	0.003	0.009	0.221	93	0.364	0.921	0.117	0.500	0.121	0.363	0.104
44	0.016	0.783	0.426	0.544	0.178	8.084	7.204	94	0.008	0.621	0.304	0.519	0.118	1.262	0.371
45	0.193	0.568	0.213	0.609	0.043	0.000	0.060	95	—	—	—	—	—	—	—
46	0.737	0.030	0.011	0.477	0.058	2.888	57.442	96	0.622	1.100	0.003	0.575	0.000	0.471	0.068
47	0.190	0.698	0.237	0.565	0.133	2.980	1.071	97	0.013	0.534	0.549	0.565	0.144	0.000	0.293
48	0.601	0.902	0.058	0.445	0.221	0.547	0.502	98	0.053	0.497	0.320	0.493	0.046	0.235	0.177
49	—	—	—	—	—	—	—	99	0.267	0.669	0.229	0.553	0.099	0.207	0.231
50	0.054	0.656	0.413	0.541	0.327	1.678	1.020	100	0.450	0.832	0.090	0.548	0.231	0.586	0.108

Table 2.9: Results of Monte Carlo simulations, FOC imposed, 250 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.262	0.667	0.239	0.560	0.087	0.583	0.220	51	0.197	0.689	0.158	0.575	0.046	0.151	0.022
2	0.016	0.739	0.396	0.439	0.198	0.000	0.296	52	0.317	0.762	0.240	0.485	0.159	0.428	0.463
3	0.603	0.810	0.034	0.484	0.123	0.002	0.089	53	0.470	0.675	0.121	0.583	0.139	0.111	0.123
4	0.194	0.546	0.228	0.501	0.152	0.000	0.344	54	0.646	0.614	0.102	0.498	0.210	0.104	0.962
5	0.323	0.722	0.166	0.535	0.184	0.000	0.292	55	0.346	0.689	0.138	0.472	0.173	0.726	0.325
6	0.418	0.644	0.127	0.493	0.142	0.038	0.274	56	0.554	0.616	0.082	0.528	0.125	0.000	0.142
7	0.015	0.709	0.289	0.506	0.208	0.000	0.283	57	—	—	—	—	—	—	—
8	0.759	0.417	0.032	0.523	0.053	0.279	0.066	58	0.052	0.641	0.268	0.473	0.140	0.188	0.296
9	0.126	0.725	0.284	0.530	0.172	0.000	0.269	59	0.524	0.832	0.069	0.481	0.131	0.108	0.307
10	0.694	0.527	0.019	0.525	0.061	0.231	3.123	60	0.173	0.681	0.198	0.489	0.153	0.170	0.161
11	0.090	0.603	0.313	0.489	0.144	0.238	0.368	61	0.193	0.673	0.216	0.525	0.142	0.195	0.145
12	0.191	0.722	0.303	0.496	0.157	0.230	0.291	62	0.520	0.712	0.057	0.516	0.137	0.000	0.395
13	0.552	0.824	0.039	0.545	0.153	0.000	0.143	63	0.111	0.709	0.267	0.507	0.107	1.060	0.294
14	0.002	0.675	0.370	0.531	0.110	0.000	0.217	64	0.236	0.615	0.369	0.454	0.161	0.173	1.262
15	0.207	0.674	0.234	0.437	0.138	0.000	0.331	65	0.466	0.867	0.089	0.549	0.112	0.689	0.131
16	0.247	0.829	0.128	0.527	0.162	0.000	0.089	66	0.213	0.648	0.226	0.478	0.226	0.191	0.313
17	0.437	0.697	0.121	0.459	0.178	0.000	0.422	67	0.087	0.602	0.409	0.501	0.156	0.903	0.617
18	0.123	0.675	0.372	0.479	0.156	0.000	0.492	68	0.081	0.786	0.367	0.560	0.133	0.646	0.267
19	0.046	0.699	0.305	0.470	0.193	0.565	0.352	69	0.489	0.512	0.111	0.471	0.125	0.070	0.449
20	0.241	0.536	0.527	0.485	0.145	0.590	0.719	70	0.601	0.529	0.065	0.503	0.138	0.059	1.048
21	0.078	0.544	0.492	0.471	0.159	0.000	1.304	71	0.386	0.683	0.157	0.469	0.190	0.000	0.249
22	0.599	0.872	0.001	0.494	0.178	0.452	0.000	72	0.168	0.647	0.239	0.487	0.175	0.437	0.354
23	0.508	0.652	0.080	0.521	0.102	0.559	0.499	73	0.210	0.662	0.241	0.454	0.120	0.193	0.360
24	0.274	0.749	0.094	0.508	0.128	0.584	0.028	74	0.003	0.536	0.351	0.522	0.111	0.000	0.323
25	0.033	0.725	0.392	0.530	0.220	0.174	0.520	75	0.155	0.723	0.280	0.495	0.196	0.000	0.302
26	0.047	0.679	0.299	0.532	0.128	0.068	0.228	76	0.078	0.580	0.465	0.509	0.099	0.171	0.507
27	0.015	0.783	0.238	0.539	0.208	0.461	0.129	77	0.065	0.752	0.261	0.477	0.166	0.000	0.185
28	0.176	0.673	0.210	0.485	0.134	0.586	0.191	78	0.000	0.945	0.012	0.511	0.139	0.730	0.000
29	0.095	0.553	0.541	0.529	0.164	0.078	0.476	79	0.232	0.804	0.134	0.586	0.150	0.142	0.052
30	0.621	0.698	0.093	0.511	0.119	0.046	0.553	80	0.596	0.661	0.036	0.507	0.168	0.000	0.073
31	0.334	0.726	0.165	0.506	0.234	0.085	0.228	81	0.392	0.712	0.159	0.531	0.156	0.000	0.186
32	0.317	0.555	0.214	0.478	0.121	0.063	0.342	82	0.147	0.767	0.255	0.497	0.142	0.000	0.133
33	0.440	0.622	0.173	0.481	0.234	0.000	0.288	83	0.466	0.568	0.105	0.448	0.130	0.000	0.198
34	—	—	—	—	—	—	—	84	0.001	0.693	0.212	0.402	0.259	0.000	0.182
35	0.085	0.588	0.300	0.512	0.198	0.000	0.442	85	0.297	0.704	0.193	0.477	0.153	0.000	0.303
36	0.033	0.693	0.313	0.514	0.097	0.512	0.292	86	0.172	0.632	0.296	0.464	0.194	0.285	0.471
37	0.233	0.623	0.271	0.478	0.169	1.158	0.749	87	0.003	0.626	0.538	0.524	0.130	0.283	0.598
38	0.087	0.561	0.709	0.492	0.170	0.638	1.079	88	0.539	0.706	0.084	0.483	0.180	0.000	0.251
39	0.167	0.691	0.301	0.509	0.174	0.048	0.380	89	0.422	0.790	0.146	0.527	0.147	0.317	0.145
40	0.581	0.424	0.089	0.504	0.199	0.106	0.253	90	0.063	0.749	0.382	0.514	0.085	0.816	0.490
41	0.202	0.712	0.482	0.465	0.201	0.000	2.012	91	0.638	0.438	0.097	0.513	0.151	1.372	1.714
42	0.174	0.657	0.193	0.505	0.062	0.891	0.119	92	0.007	0.791	0.299	0.535	0.151	0.000	0.144
43	0.200	0.631	0.241	0.501	0.151	0.000	0.301	93	0.475	0.349	0.154	0.558	0.164	4.293	1.056
44	0.289	0.581	0.156	0.521	0.113	0.022	0.169	94	0.634	0.563	0.054	0.548	0.125	0.091	0.529
45	0.334	0.739	0.224	0.479	0.119	0.049	0.482	95	0.487	0.609	0.095	0.519	0.112	0.191	0.115
46	0.077	0.670	0.243	0.507	0.191	0.250	0.196	96	0.238	0.777	0.202	0.449	0.250	0.075	0.172
47	0.478	0.770	0.101	0.485	0.154	0.000	0.205	97	0.667	0.544	0.037	0.576	0.143	0.000	0.082
48	0.647	0.753	0.007	0.510	0.140	0.000	0.022	98	0.069	0.725	0.341	0.504	0.098	1.169	0.611
49	0.603	0.776	0.059	0.538	0.106	0.274	0.128	99	0.052	0.632	0.299	0.530	0.125	0.000	0.210
50	0.539	0.584	0.127	0.516	0.121	0.000	2.594	100	0.569	0.599	0.061	0.530	0.109	0.353	0.395

Table 2.10: Results of Monte Carlo simulations, FOC imposed, 500 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.036	0.695	0.407	0.480	0.143	0.424	0.328	51	0.403	0.681	0.157	0.519	0.098	0.573	0.174
2	0.495	0.767	0.072	0.486	0.163	0.066	0.135	52	0.441	0.609	0.145	0.525	0.147	0.000	0.249
3	0.000	0.729	0.371	0.507	0.199	0.000	0.621	53	0.452	0.607	0.131	0.479	0.201	0.491	0.250
4	0.005	0.712	0.334	0.495	0.184	0.000	0.290	54	0.102	0.661	0.299	0.525	0.182	0.119	0.271
5	0.545	0.499	0.102	0.518	0.158	0.229	0.269	55	0.098	0.681	0.247	0.488	0.160	0.440	0.222
6	0.299	0.652	0.185	0.511	0.109	0.759	0.251	56	0.448	0.693	0.150	0.505	0.148	0.191	0.396
7	0.480	0.742	0.106	0.507	0.136	0.361	0.199	57	0.500	0.565	0.089	0.519	0.133	0.675	0.324
8	0.429	0.560	0.143	0.485	0.122	0.000	0.185	58	0.483	0.603	0.136	0.516	0.193	0.805	0.308
9	0.320	0.652	0.187	0.503	0.158	0.000	0.296	59	0.219	0.628	0.254	0.508	0.107	0.243	0.357
10	0.511	0.560	0.142	0.495	0.116	0.008	0.552	60	0.423	0.391	0.255	0.510	0.194	0.653	1.069
11	0.406	0.604	0.158	0.489	0.140	0.179	0.349	61	0.114	0.652	0.250	0.505	0.147	0.027	0.228
12	0.405	0.819	0.092	0.505	0.142	0.268	0.075	62	0.276	0.627	0.217	0.469	0.137	0.386	0.412
13	0.268	0.594	0.179	0.452	0.131	0.000	0.260	63	0.458	0.648	0.113	0.523	0.128	0.153	0.233
14	0.265	0.752	0.172	0.499	0.180	0.000	0.333	64	0.566	0.532	0.114	0.474	0.220	0.163	0.568
15	0.423	0.657	0.186	0.483	0.162	0.007	0.407	65	0.217	0.636	0.263	0.515	0.139	0.338	0.217
16	0.024	0.667	0.370	0.508	0.141	0.315	0.285	66	0.512	0.634	0.071	0.478	0.136	0.018	0.253
17	0.123	0.536	0.328	0.520	0.142	0.000	0.562	67	0.456	0.823	0.079	0.485	0.147	0.434	0.247
18	0.324	0.569	0.192	0.504	0.099	0.093	0.211	68	0.422	0.677	0.261	0.455	0.203	0.042	0.694
19	0.300	0.596	0.205	0.495	0.164	0.066	0.337	69	0.149	0.734	0.303	0.514	0.126	0.511	0.264
20	0.004	0.648	0.386	0.524	0.145	0.000	0.371	70	0.490	0.569	0.144	0.516	0.138	0.070	0.392
21	0.021	0.615	0.357	0.479	0.184	0.016	0.377	71	0.364	0.622	0.159	0.485	0.155	0.000	0.285
22	0.331	0.611	0.164	0.495	0.100	0.381	0.364	72	0.389	0.673	0.130	0.485	0.116	0.136	0.279
23	0.177	0.717	0.226	0.509	0.195	0.166	0.216	73	0.248	0.687	0.231	0.492	0.172	0.014	0.304
24	0.480	0.653	0.097	0.539	0.170	0.000	0.205	74	0.230	0.667	0.276	0.486	0.156	0.082	0.371
25	0.198	0.560	0.256	0.502	0.124	0.248	0.297	75	0.206	0.839	0.179	0.537	0.097	0.473	0.070
26	0.519	0.706	0.115	0.503	0.135	0.206	0.224	76	0.446	0.612	0.127	0.527	0.169	0.528	0.288
27	0.200	0.669	0.216	0.486	0.129	0.386	0.230	77	0.476	0.617	0.152	0.484	0.159	0.191	0.386
28	0.036	0.742	0.351	0.516	0.162	0.919	0.432	78	0.542	0.532	0.064	0.510	0.121	0.017	0.311
29	0.155	0.617	0.340	0.456	0.162	0.013	0.477	79	0.446	0.595	0.152	0.505	0.111	0.725	0.212
30	0.335	0.609	0.204	0.491	0.121	0.361	0.220	80	0.633	0.555	0.038	0.469	0.151	0.000	0.360
31	0.325	0.661	0.184	0.508	0.182	0.000	0.341	81	0.374	0.777	0.164	0.523	0.174	0.000	0.175
32	0.010	0.671	0.330	0.483	0.169	0.121	0.172	82	0.439	0.627	0.154	0.429	0.147	0.000	0.361
33	0.361	0.557	0.166	0.506	0.165	0.000	0.404	83	0.002	0.750	0.333	0.495	0.175	0.000	0.224
34	0.152	0.683	0.311	0.488	0.140	0.559	0.539	84	0.083	0.651	0.320	0.489	0.155	0.000	0.456
35	0.571	0.497	0.198	0.523	0.167	1.681	0.927	85	0.597	0.706	0.047	0.501	0.136	0.063	0.290
36	0.370	0.568	0.173	0.494	0.168	0.020	0.339	86	0.268	0.737	0.181	0.525	0.114	0.439	0.147
37	0.207	0.671	0.238	0.509	0.157	0.000	0.274	87	0.002	0.674	0.399	0.531	0.164	0.181	0.517
38	0.233	0.662	0.215	0.532	0.136	0.014	0.221	88	0.430	0.605	0.140	0.509	0.165	0.054	0.263
39	0.472	0.702	0.091	0.492	0.158	0.003	0.192	89	0.523	0.623	0.082	0.491	0.180	0.000	0.192
40	0.197	0.697	0.264	0.504	0.159	0.464	0.323	90	0.155	0.557	0.327	0.519	0.113	0.137	0.401
41	0.063	0.670	0.477	0.445	0.173	0.032	0.737	91	0.380	0.628	0.176	0.511	0.126	0.253	0.344
42	0.029	0.675	0.291	0.489	0.150	0.000	0.427	92	0.181	0.580	0.247	0.551	0.113	2.000	0.489
43	0.122	0.695	0.277	0.501	0.128	0.000	0.247	93	0.269	0.605	0.246	0.493	0.122	0.009	0.219
44	0.357	0.750	0.158	0.509	0.192	0.000	0.276	94	0.498	0.576	0.078	0.478	0.175	0.000	0.120
45	0.599	0.578	0.043	0.514	0.147	0.000	0.218	95	0.008	0.611	0.447	0.516	0.126	0.445	0.857
46	0.019	0.732	0.387	0.504	0.181	0.208	0.634	96	0.317	0.653	0.175	0.532	0.162	0.874	0.357
47	0.097	0.638	0.335	0.516	0.167	0.000	0.283	97	0.090	0.681	0.214	0.458	0.158	0.142	0.184
48	0.229	0.732	0.204	0.527	0.170	0.796	0.372	98	0.008	0.808	0.393	0.515	0.096	0.732	0.251
49	0.051	0.656	0.264	0.492	0.143	0.112	0.226	99	0.296	0.551	0.243	0.496	0.188	0.000	0.338
50	0.599	0.713	0.046	0.495	0.153	0.544	0.386	100	0.017	0.627	0.294	0.501	0.130	0.061	0.428

Table 2.11: Results of Monte Carlo simulations, FOC imposed, 1000 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.154	0.722	0.283	0.487	0.150	0.346	0.265	51	0.394	0.669	0.155	0.535	0.112	0.425	0.150
2	0.059	0.706	0.297	0.502	0.186	0.000	0.312	52	0.580	0.596	0.078	0.500	0.174	0.110	0.259
3	0.213	0.601	0.232	0.519	0.161	0.203	0.237	53	0.094	0.682	0.303	0.500	0.147	0.289	0.308
4	0.148	0.693	0.244	0.505	0.133	0.493	0.189	54	0.139	0.656	0.256	0.510	0.141	0.252	0.231
5	0.146	0.673	0.276	0.497	0.168	0.000	0.203	55	0.429	0.626	0.143	0.509	0.130	0.433	0.361
6	0.502	0.686	0.121	0.507	0.111	0.452	0.233	56	0.463	0.595	0.145	0.506	0.174	0.332	0.268
7	0.065	0.690	0.254	0.499	0.152	0.439	0.153	57	0.399	0.694	0.121	0.533	0.135	0.716	0.170
8	0.503	0.548	0.116	0.496	0.119	0.081	0.522	58	0.011	0.717	0.344	0.494	0.171	0.000	0.311
9	0.502	0.543	0.101	0.542	0.134	0.171	0.229	59	0.427	0.563	0.181	0.515	0.133	0.250	0.424
10	0.041	0.618	0.330	0.516	0.136	0.000	0.296	60	0.215	0.699	0.252	0.502	0.163	0.000	0.272
11	0.238	0.693	0.212	0.502	0.154	0.354	0.224	61	0.447	0.618	0.113	0.508	0.127	0.084	0.317
12	0.021	0.699	0.276	0.479	0.139	0.000	0.220	62	0.253	0.642	0.231	0.502	0.154	0.100	0.306
13	0.389	0.658	0.162	0.487	0.154	0.032	0.292	63	0.075	0.622	0.300	0.478	0.150	0.213	0.280
14	0.247	0.576	0.242	0.507	0.139	0.152	0.290	64	0.558	0.723	0.068	0.492	0.133	0.387	0.429
15	0.282	0.590	0.214	0.500	0.147	0.121	0.304	65	0.208	0.689	0.315	0.500	0.157	0.366	0.348
16	0.386	0.686	0.152	0.513	0.174	0.117	0.273	66	0.423	0.690	0.142	0.505	0.152	0.034	0.186
17	0.071	0.661	0.371	0.484	0.202	0.000	0.313	67	0.523	0.595	0.096	0.522	0.126	0.697	0.225
18	0.390	0.634	0.162	0.494	0.200	0.017	0.353	68	0.322	0.525	0.247	0.497	0.146	0.000	0.331
19	0.278	0.618	0.215	0.490	0.150	0.210	0.350	69	0.298	0.644	0.202	0.480	0.116	0.181	0.278
20	0.106	0.693	0.369	0.467	0.155	0.166	0.364	70	0.359	0.623	0.194	0.516	0.172	0.000	0.328
21	0.087	0.673	0.284	0.496	0.145	0.430	0.307	71	0.460	0.642	0.100	0.475	0.150	0.000	0.289
22	0.494	0.682	0.093	0.523	0.155	0.111	0.191	72	0.052	0.671	0.344	0.490	0.158	0.150	0.340
23	0.125	0.678	0.257	0.516	0.152	0.345	0.156	73	0.176	0.743	0.216	0.506	0.149	0.208	0.182
24	0.310	0.693	0.201	0.500	0.140	0.479	0.271	74	0.333	0.656	0.170	0.532	0.136	0.588	0.261
25	0.309	0.636	0.247	0.487	0.159	0.203	0.601	75	0.364	0.609	0.186	0.497	0.133	0.615	0.261
26	0.339	0.641	0.203	0.512	0.128	0.367	0.211	76	0.183	0.666	0.249	0.519	0.173	0.377	0.374
27	0.096	0.629	0.309	0.491	0.153	0.079	0.270	77	0.426	0.607	0.149	0.471	0.180	0.000	0.348
28	0.464	0.739	0.101	0.518	0.171	0.782	0.198	78	0.132	0.601	0.320	0.509	0.113	0.012	0.373
29	0.623	0.543	0.065	0.513	0.133	0.038	0.325	79	0.275	0.636	0.205	0.511	0.145	0.077	0.313
30	0.461	0.666	0.135	0.503	0.147	0.169	0.377	80	0.408	0.574	0.158	0.497	0.115	0.000	0.250
31	0.411	0.628	0.152	0.485	0.171	0.000	0.259	81	0.349	0.697	0.176	0.487	0.137	0.000	0.189
32	0.320	0.634	0.169	0.494	0.144	0.002	0.323	82	0.160	0.716	0.242	0.475	0.192	0.000	0.297
33	0.580	0.546	0.102	0.504	0.166	0.571	0.649	83	0.437	0.683	0.135	0.521	0.142	0.350	0.217
34	0.019	0.701	0.287	0.500	0.147	0.169	0.180	84	0.105	0.706	0.303	0.518	0.159	0.244	0.323
35	0.400	0.648	0.143	0.508	0.140	0.000	0.258	85	0.275	0.625	0.226	0.502	0.172	0.192	0.292
36	0.233	0.736	0.234	0.489	0.137	0.548	0.317	86	0.091	0.719	0.310	0.530	0.152	0.228	0.268
37	0.132	0.681	0.292	0.500	0.159	0.320	0.344	87	0.336	0.689	0.181	0.518	0.129	0.655	0.248
38	0.164	0.579	0.338	0.475	0.134	0.117	0.486	88	0.598	0.430	0.069	0.502	0.150	0.014	0.346
39	0.383	0.476	0.187	0.486	0.113	0.056	0.472	89	0.128	0.655	0.308	0.526	0.144	0.433	0.369
40	0.273	0.626	0.247	0.485	0.145	0.159	0.513	90	0.203	0.625	0.251	0.498	0.167	0.082	0.329
41	0.074	0.673	0.304	0.473	0.159	0.064	0.420	91	0.081	0.627	0.314	0.530	0.139	0.488	0.344
42	0.241	0.682	0.221	0.492	0.150	0.000	0.269	92	0.151	0.653	0.280	0.480	0.154	0.016	0.244
43	0.582	0.651	0.052	0.501	0.134	0.000	0.280	93	0.563	0.475	0.112	0.512	0.130	0.176	0.827
44	0.508	0.596	0.127	0.517	0.148	0.146	0.563	94	0.094	0.691	0.254	0.481	0.169	0.120	0.260
45	0.148	0.651	0.240	0.502	0.151	0.229	0.403	95	0.245	0.595	0.281	0.508	0.154	0.245	0.358
46	0.321	0.642	0.190	0.496	0.144	0.182	0.317	96	0.106	0.641	0.326	0.491	0.157	0.397	0.610
47	0.169	0.643	0.231	0.505	0.130	0.084	0.211	97	0.505	0.599	0.088	0.511	0.119	0.147	0.261
48	0.278	0.675	0.209	0.489	0.156	0.272	0.381	98	0.453	0.636	0.117	0.498	0.146	0.131	0.291
49	0.638	0.500	0.070	0.529	0.158	0.244	0.664	99	0.476	0.561	0.116	0.495	0.096	0.186	0.306
50	0.250	0.609	0.249	0.481	0.159	0.015	0.451	100	0.380	0.691	0.160	0.524	0.131	0.245	0.194

Table 2.12: Results of Monte Carlo simulations, FOC not imposed, 100 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.712	0.502	0.009	0.534	0.099	0.000	13.458	51	0.459	0.472	0.114	0.566	0.022	0.095	0.017
2	0.023	0.575	0.737	0.474	0.090	0.634	1.264	52	0.653	0.463	0.047	0.551	0.102	0.042	0.385
3	0.039	1.018	0.112	0.416	0.272	0.000	0.019	53	0.049	0.742	0.620	0.524	0.115	0.783	0.919
4	0.098	0.855	0.303	0.497	0.221	1.380	1.042	54	0.749	0.160	0.074	0.572	0.018	0.112	0.217
5	0.632	1.008	0.003	0.488	0.079	0.000	0.001	55	0.743	0.641	0.059	0.677	0.116	0.000	0.040
6	0.000	0.815	0.037	0.445	0.218	0.000	0.003	56	0.004	0.756	0.223	0.426	0.204	0.000	0.201
7	0.049	0.651	0.516	0.478	0.185	1.402	5.189	57	0.610	0.618	0.116	0.450	0.236	0.000	0.260
8	0.420	0.551	0.278	0.481	0.228	0.000	1.008	58	0.660	0.094	0.020	0.536	0.091	0.576	0.122
9	0.051	0.720	0.611	0.544	0.203	0.000	1.372	59	0.657	0.628	0.033	0.457	0.252	0.000	0.102
10	0.552	0.767	0.078	0.572	0.062	0.377	0.072	60	0.478	0.621	0.081	0.528	0.074	0.042	0.035
11	0.000	0.625	0.543	0.478	0.119	0.000	1.438	61	0.006	0.783	0.303	0.536	0.115	0.000	0.062
12	0.186	0.618	0.073	0.467	0.271	0.000	0.036	62	0.717	0.259	0.025	0.565	0.182	0.000	4.624
13	0.062	0.638	0.413	0.525	0.092	0.027	0.241	63	0.241	1.016	0.052	0.467	0.155	0.000	0.019
14	0.774	0.202	0.053	0.444	0.119	0.011	2.650	64	0.004	0.625	0.430	0.475	0.067	0.000	0.514
15	0.622	1.127	0.007	0.465	0.157	0.000	0.011	65	0.652	0.730	0.011	0.494	0.129	0.444	0.063
16	0.483	1.067	0.066	0.534	0.110	0.159	0.062	66	0.005	0.671	0.529	0.426	0.214	1.096	0.992
17	—	—	—	—	—	—	—	67	0.366	0.733	0.092	0.423	0.192	0.100	0.089
18	0.012	0.753	0.330	0.506	0.072	0.000	0.123	68	0.690	0.871	0.002	0.599	0.101	0.193	0.000
19	0.040	0.804	0.010	0.603	0.086	0.159	0.000	69	0.121	0.646	0.434	0.504	0.152	0.000	0.662
20	0.532	0.965	0.045	0.503	0.143	0.211	0.674	70	0.014	0.674	0.316	0.522	0.210	0.000	0.417
21	0.010	0.550	0.568	0.483	0.172	0.000	1.098	71	0.078	0.729	0.536	0.494	0.184	0.000	0.845
22	0.626	0.665	0.070	0.479	0.219	0.568	0.643	72	0.002	0.690	0.236	0.452	0.193	0.000	0.236
23	0.070	0.704	0.038	0.457	0.175	0.480	0.002	73	0.553	0.406	0.108	0.436	0.128	0.000	0.893
24	0.504	0.564	0.113	0.499	0.111	0.000	0.844	74	0.465	0.827	0.022	0.565	0.117	0.511	0.039
25	0.445	0.818	0.131	0.573	0.091	0.652	1.048	75	0.259	0.770	0.194	0.483	0.050	0.165	0.072
26	0.359	0.578	0.013	0.492	0.070	0.823	0.002	76	0.031	0.529	0.602	0.555	0.046	0.103	0.203
27	0.464	0.733	0.013	0.449	0.171	0.000	0.002	77	0.110	0.566	0.386	0.536	0.117	0.000	0.338
28	0.041	0.701	0.296	0.588	0.268	1.818	0.488	78	0.579	0.708	0.077	0.521	0.082	0.000	2.871
29	0.677	0.685	0.059	0.485	0.215	0.435	4.113	79	0.062	0.647	0.409	0.439	0.217	0.403	0.548
30	0.021	0.730	0.177	0.655	0.253	5.491	0.327	80	—	—	—	—	—	—	—
31	0.058	0.740	0.352	0.508	0.215	0.037	0.415	81	0.138	0.943	0.189	0.571	0.134	0.066	0.040
32	0.100	0.515	0.486	0.482	0.224	0.000	0.694	82	0.467	0.698	0.185	0.496	0.218	0.000	0.538
33	0.437	0.496	0.117	0.473	0.119	0.011	0.263	83	0.000	1.100	0.046	0.521	0.200	0.000	0.001
34	0.021	0.882	0.285	0.464	0.169	0.000	0.113	84	0.125	0.423	0.547	0.496	0.040	0.037	0.306
35	—	—	—	—	—	—	—	85	0.012	0.579	0.166	0.442	0.168	0.000	0.106
36	0.745	0.000	0.009	0.526	0.010	0.000	16.980	86	—	—	—	—	—	—	—
37	0.045	0.578	0.278	0.516	0.160	0.000	0.903	87	0.011	0.693	0.162	0.433	0.281	0.000	0.054
38	0.121	0.526	0.299	0.498	0.197	0.000	0.205	88	—	—	—	—	—	—	—
39	0.003	0.686	0.454	0.466	0.300	0.000	0.876	89	0.492	1.048	0.043	0.463	0.175	0.000	0.097
40	0.610	0.710	0.050	0.485	0.110	0.025	0.988	90	0.063	0.649	0.535	0.441	0.176	0.074	1.894
41	0.030	0.706	0.434	0.402	0.201	0.000	0.471	91	0.505	0.783	0.132	0.506	0.212	0.000	0.323
42	0.476	0.638	0.184	0.505	0.210	0.000	2.072	92	0.699	0.523	0.011	0.471	0.044	0.706	1.427
43	0.099	0.618	0.364	0.464	0.042	0.203	0.046	93	0.022	0.919	0.269	0.503	0.120	0.415	0.117
44	0.260	0.753	0.250	0.536	0.176	5.088	4.404	94	0.021	0.638	0.267	0.514	0.120	1.211	0.271
45	—	—	—	—	—	—	—	95	—	—	—	—	—	—	—
46	—	—	—	—	—	—	—	96	0.012	1.138	0.101	0.573	0.000	2.441	0.024
47	0.055	0.694	0.313	0.571	0.128	3.409	1.397	97	0.418	0.381	0.331	0.566	0.135	0.000	0.532
48	0.638	0.832	0.057	0.440	0.217	0.000	1.435	98	0.006	0.513	0.334	0.494	0.044	0.238	0.154
49	—	—	—	—	—	—	—	99	0.623	0.524	0.063	0.544	0.084	0.000	0.871
50	0.015	0.669	0.355	0.536	0.334	1.444	0.664	100	0.558	0.807	0.050	0.541	0.232	0.000	0.115

Table 2.13: Results of Monte Carlo simulations, FOC not imposed, 250 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.369	0.642	0.190	0.559	0.085	0.560	0.237	51	0.044	0.699	0.210	0.574	0.044	0.144	0.023
2	0.182	0.764	0.201	0.433	0.205	0.016	0.116	52	0.135	0.771	0.355	0.486	0.160	0.435	0.467
3	0.605	0.805	0.036	0.483	0.122	0.000	0.106	53	0.548	0.673	0.079	0.580	0.137	0.000	0.112
4	—	—	—	—	—	—	—	54	0.661	0.596	0.092	0.499	0.200	0.000	1.282
5	0.055	0.702	0.378	0.547	0.191	0.000	0.586	55	0.514	0.661	0.074	0.468	0.169	0.442	0.359
6	0.250	0.681	0.213	0.500	0.140	0.266	0.293	56	0.529	0.719	0.045	0.523	0.135	0.000	0.032
7	0.034	0.719	0.237	0.503	0.211	0.000	0.188	57	0.002	0.673	0.435	0.560	0.176	0.248	0.759
8	0.791	0.596	0.000	0.519	0.055	0.018	0.000	58	0.054	0.642	0.270	0.472	0.140	0.186	0.304
9	0.144	0.716	0.288	0.530	0.172	0.000	0.297	59	0.549	0.823	0.061	0.479	0.130	0.000	0.398
10	0.697	0.519	0.017	0.523	0.059	0.000	3.285	60	0.350	0.644	0.136	0.485	0.153	0.076	0.175
11	0.503	0.473	0.126	0.485	0.138	0.055	0.527	61	0.054	0.685	0.265	0.524	0.144	0.176	0.137
12	0.008	0.730	0.429	0.496	0.161	0.240	0.338	62	0.652	0.678	0.014	0.513	0.117	0.000	10.119
13	0.174	0.864	0.044	0.539	0.178	0.000	0.005	63	0.084	0.719	0.263	0.506	0.107	1.032	0.249
14	0.057	0.691	0.278	0.521	0.104	0.000	0.117	64	0.169	0.637	0.411	0.459	0.169	0.158	1.176
15	0.196	0.682	0.226	0.436	0.141	0.000	0.292	65	0.133	0.886	0.204	0.551	0.115	0.745	0.099
16	0.277	0.832	0.106	0.528	0.163	0.000	0.070	66	0.497	0.571	0.120	0.477	0.220	0.000	0.468
17	0.167	0.774	0.096	0.453	0.199	0.000	0.047	67	0.039	0.608	0.443	0.501	0.154	0.895	0.637
18	0.023	0.686	0.440	0.479	0.157	0.000	0.503	68	0.048	0.771	0.400	0.565	0.133	0.584	0.293
19	0.604	0.595	0.061	0.464	0.178	0.000	0.564	69	0.035	0.661	0.209	0.464	0.139	0.210	0.134
20	0.638	0.324	0.175	0.480	0.139	0.451	0.918	70	0.621	0.503	0.058	0.503	0.136	0.000	1.289
21	0.060	0.539	0.512	0.474	0.157	0.000	1.320	71	0.251	0.716	0.178	0.466	0.198	0.000	0.157
22	0.554	0.869	0.002	0.497	0.177	0.588	0.000	72	0.535	0.546	0.087	0.481	0.169	0.061	0.457
23	0.622	0.591	0.039	0.515	0.091	0.112	1.289	73	0.250	0.678	0.192	0.454	0.119	0.222	0.238
24	0.642	0.689	0.022	0.498	0.122	0.015	0.059	74	0.031	0.522	0.364	0.524	0.116	0.000	0.421
25	0.050	0.723	0.386	0.530	0.219	0.149	0.529	75	0.211	0.678	0.330	0.498	0.188	0.000	0.564
26	0.069	0.685	0.289	0.535	0.126	0.123	0.222	76	0.025	0.590	0.493	0.510	0.101	0.152	0.495
27	0.463	0.761	0.067	0.528	0.210	0.000	0.083	77	0.097	0.756	0.221	0.474	0.162	0.000	0.141
28	0.564	0.648	0.033	0.460	0.140	0.081	0.065	78	0.562	0.909	0.007	0.497	0.131	0.000	0.006
29	0.163	0.536	0.497	0.529	0.165	0.079	0.493	79	0.427	0.788	0.079	0.584	0.145	0.053	0.059
30	0.629	0.695	0.087	0.511	0.119	0.019	0.604	80	0.359	0.710	0.098	0.514	0.173	0.484	0.059
31	0.439	0.716	0.121	0.504	0.231	0.000	0.239	81	0.352	0.734	0.151	0.529	0.158	0.000	0.120
32	0.339	0.623	0.108	0.464	0.124	0.066	0.077	82	—	—	—	—	—	—	—
33	0.051	0.732	0.130	0.472	0.248	0.000	0.027	83	0.304	0.646	0.091	0.435	0.147	0.036	0.050
34	0.000	0.580	0.299	0.507	0.109	0.000	0.321	84	0.055	0.697	0.176	0.394	0.271	0.000	0.150
35	0.044	0.590	0.322	0.510	0.194	0.000	0.440	85	0.002	0.776	0.138	0.475	0.168	0.000	0.035
36	0.380	0.595	0.206	0.512	0.091	0.393	0.642	86	0.427	0.566	0.168	0.459	0.196	0.000	0.537
37	0.466	0.558	0.164	0.477	0.164	1.249	1.014	87	0.055	0.621	0.501	0.524	0.130	0.301	0.599
38	—	—	—	—	—	—	—	88	0.424	0.740	0.108	0.484	0.191	0.060	0.135
39	0.481	0.657	0.135	0.506	0.167	0.000	0.464	89	0.560	0.782	0.079	0.523	0.144	0.146	0.177
40	0.512	0.598	0.030	0.501	0.212	0.087	0.012	90	0.088	0.748	0.368	0.514	0.086	0.810	0.487
41	0.233	0.698	0.453	0.467	0.200	0.000	2.028	91	0.682	0.372	0.068	0.508	0.147	1.028	2.254
42	0.708	0.523	0.010	0.492	0.054	0.094	0.161	92	0.031	0.785	0.302	0.535	0.150	0.000	0.160
43	—	—	—	—	—	—	—	93	0.528	0.310	0.131	0.557	0.163	4.145	1.081
44	0.232	0.594	0.174	0.521	0.114	0.032	0.159	94	0.656	0.520	0.049	0.547	0.121	0.000	0.829
45	0.385	0.720	0.200	0.479	0.120	0.000	0.572	95	0.101	0.681	0.233	0.522	0.117	0.303	0.097
46	0.224	0.634	0.198	0.498	0.190	0.000	0.221	96	0.377	0.770	0.140	0.447	0.249	0.000	0.167
47	0.410	0.800	0.094	0.479	0.165	0.000	0.088	97	0.595	0.644	0.023	0.582	0.154	0.099	0.009
48	0.646	0.755	0.007	0.510	0.144	0.000	0.020	98	0.010	0.727	0.376	0.503	0.100	1.160	0.631
49	0.628	0.860	0.014	0.533	0.113	0.115	0.013	99	0.017	0.573	0.462	0.541	0.113	0.000	0.614
50	0.526	0.569	0.140	0.516	0.121	0.000	2.343	100	0.380	0.648	0.159	0.539	0.115	0.949	0.499

Table 2.14: Results of Monte Carlo simulations, FOC not imposed, 500 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.313	0.662	0.250	0.478	0.142	0.411	0.336	51	0.623	0.625	0.056	0.513	0.095	0.367	0.290
2	0.519	0.765	0.063	0.485	0.163	0.000	0.136	52	0.458	0.605	0.138	0.525	0.147	0.000	0.256
3	0.013	0.710	0.417	0.511	0.199	0.000	0.865	53	0.605	0.548	0.069	0.474	0.195	0.091	0.330
4	0.027	0.722	0.279	0.493	0.186	0.000	0.204	54	0.023	0.660	0.333	0.522	0.187	0.000	0.277
5	0.599	0.439	0.091	0.517	0.154	0.125	0.426	55	0.046	0.685	0.272	0.488	0.160	0.450	0.233
6	0.597	0.551	0.074	0.512	0.103	0.667	0.605	56	0.051	0.730	0.389	0.513	0.152	0.300	0.398
7	0.552	0.745	0.067	0.505	0.134	0.240	0.188	57	0.633	0.497	0.042	0.513	0.128	0.215	0.515
8	0.442	0.551	0.142	0.485	0.119	0.000	0.198	58	0.682	0.467	0.058	0.510	0.179	0.076	0.925
9	0.152	0.657	0.293	0.503	0.160	0.000	0.359	59	0.579	0.515	0.080	0.503	0.100	0.020	0.633
10	0.506	0.562	0.144	0.495	0.116	0.011	0.544	60	0.425	0.385	0.262	0.512	0.190	0.705	1.158
11	0.027	0.658	0.354	0.490	0.145	0.231	0.359	61	0.327	0.620	0.160	0.503	0.142	0.000	0.231
12	0.561	0.807	0.045	0.501	0.135	0.010	0.109	62	0.490	0.555	0.131	0.467	0.132	0.251	0.632
13	0.133	0.631	0.200	0.452	0.140	0.000	0.183	63	0.318	0.675	0.169	0.527	0.128	0.250	0.190
14	0.195	0.779	0.161	0.498	0.184	0.000	0.187	64	0.555	0.524	0.132	0.476	0.222	0.195	0.704
15	0.424	0.651	0.187	0.483	0.162	0.000	0.411	65	0.447	0.587	0.154	0.513	0.136	0.296	0.247
16	0.313	0.627	0.221	0.506	0.138	0.292	0.284	66	0.509	0.626	0.076	0.478	0.135	0.000	0.285
17	0.100	0.538	0.341	0.520	0.142	0.000	0.559	67	0.573	0.821	0.037	0.481	0.141	0.000	0.380
18	0.385	0.564	0.152	0.503	0.099	0.073	0.183	68	0.230	0.690	0.461	0.461	0.202	0.108	0.849
19	0.464	0.549	0.133	0.495	0.159	0.000	0.399	69	0.420	0.689	0.192	0.516	0.123	0.586	0.437
20	0.010	0.648	0.383	0.525	0.146	0.000	0.376	70	0.548	0.539	0.115	0.514	0.135	0.000	0.470
21	0.003	0.611	0.394	0.474	0.185	0.000	0.467	71	0.359	0.623	0.161	0.485	0.155	0.000	0.283
22	0.582	0.516	0.064	0.491	0.091	0.147	0.691	72	0.455	0.673	0.097	0.485	0.114	0.143	0.269
23	0.366	0.704	0.131	0.504	0.194	0.061	0.170	73	0.219	0.686	0.247	0.491	0.174	0.000	0.315
24	0.036	0.736	0.198	0.542	0.188	0.095	0.085	74	0.381	0.647	0.182	0.479	0.150	0.000	0.344
25	0.560	0.404	0.097	0.496	0.118	0.000	0.447	75	0.534	0.837	0.051	0.532	0.094	0.333	0.061
26	0.609	0.658	0.078	0.499	0.130	0.000	0.411	76	0.566	0.563	0.083	0.524	0.165	0.337	0.402
27	0.570	0.578	0.070	0.480	0.123	0.068	0.389	77	0.027	0.669	0.515	0.497	0.157	0.344	0.595
28	0.215	0.709	0.300	0.516	0.159	0.973	0.614	78	0.418	0.589	0.099	0.513	0.127	0.211	0.213
29	0.454	0.531	0.203	0.460	0.155	0.000	0.799	79	0.690	0.488	0.038	0.497	0.105	0.161	0.402
30	0.583	0.519	0.089	0.487	0.116	0.171	0.318	80	0.099	0.678	0.233	0.479	0.163	0.847	0.241
31	0.247	0.677	0.211	0.507	0.185	0.000	0.300	81	0.450	0.774	0.130	0.522	0.169	0.000	0.197
32	0.399	0.631	0.112	0.473	0.162	0.009	0.083	82	0.306	0.694	0.133	0.416	0.165	0.002	0.115
33	0.023	0.634	0.268	0.504	0.177	0.000	0.256	83	0.103	0.761	0.235	0.492	0.175	0.000	0.141
34	0.159	0.681	0.310	0.489	0.140	0.572	0.552	84	0.020	0.655	0.364	0.490	0.156	0.000	0.505
35	0.565	0.499	0.202	0.523	0.167	1.665	0.918	85	0.543	0.756	0.044	0.501	0.145	0.111	0.090
36	0.328	0.686	0.064	0.486	0.185	0.064	0.028	86	0.423	0.712	0.124	0.523	0.113	0.383	0.179
37	0.208	0.670	0.238	0.509	0.156	0.000	0.277	87	0.020	0.641	0.473	0.528	0.158	0.104	0.884
38	0.261	0.649	0.216	0.530	0.135	0.000	0.276	88	0.602	0.522	0.086	0.512	0.146	0.019	0.594
39	0.461	0.705	0.093	0.492	0.159	0.000	0.179	89	0.478	0.637	0.097	0.492	0.182	0.054	0.171
40	0.547	0.504	0.165	0.504	0.147	0.402	1.522	90	0.065	0.573	0.375	0.520	0.113	0.147	0.398
41	0.280	0.633	0.325	0.443	0.172	0.000	0.768	91	0.306	0.648	0.210	0.511	0.128	0.283	0.318
42	0.166	0.673	0.198	0.485	0.150	0.000	0.292	92	0.702	0.339	0.030	0.544	0.101	1.475	1.067
43	0.016	0.682	0.395	0.503	0.123	0.000	0.385	93	0.335	0.590	0.217	0.493	0.120	0.000	0.236
44	0.021	0.794	0.223	0.504	0.199	0.000	0.112	94	0.464	0.598	0.078	0.478	0.180	0.014	0.094
45	0.581	0.642	0.025	0.511	0.150	0.000	0.046	95	0.193	0.561	0.370	0.511	0.120	0.318	1.040
46	0.404	0.646	0.213	0.497	0.170	0.000	1.149	96	0.568	0.579	0.077	0.526	0.156	0.592	0.553
47	0.214	0.618	0.281	0.517	0.165	0.000	0.300	97	0.366	0.633	0.122	0.454	0.155	0.000	0.209
48	0.063	0.710	0.352	0.533	0.165	0.871	0.748	98	0.076	0.813	0.352	0.516	0.096	0.733	0.243
49	0.023	0.660	0.281	0.493	0.147	0.115	0.240	99	0.071	0.586	0.396	0.501	0.195	0.000	0.453
50	0.453	0.790	0.059	0.499	0.167	0.865	0.077	100	0.077	0.608	0.287	0.501	0.130	0.000	0.513

Table 2.15: Results of Monte Carlo simulations, FOC not imposed, 1000 sample size

Iteration	λ	a_1	b	α	κ	W_y	W_i	Iteration	λ	a_1	b	α	κ	W_y	W_i
1	0.496	0.670	0.112	0.481	0.144	0.154	0.303	51	0.560	0.628	0.082	0.529	0.110	0.299	0.187
2	0.078	0.702	0.290	0.503	0.186	0.000	0.315	52	0.533	0.614	0.098	0.502	0.176	0.243	0.235
3	0.527	0.492	0.108	0.517	0.155	0.019	0.349	53	0.054	0.678	0.346	0.502	0.148	0.310	0.374
4	0.617	0.589	0.052	0.498	0.124	0.053	0.346	54	0.215	0.652	0.212	0.509	0.141	0.262	0.204
5	0.039	0.682	0.330	0.498	0.169	0.000	0.208	55	0.552	0.579	0.090	0.507	0.126	0.273	0.493
6	0.647	0.643	0.047	0.499	0.106	0.041	0.421	56	0.397	0.594	0.204	0.509	0.174	0.420	0.359
7	0.576	0.588	0.061	0.487	0.141	0.016	0.189	57	0.633	0.637	0.038	0.526	0.127	0.213	0.313
8	0.532	0.529	0.103	0.494	0.118	0.000	0.581	58	0.074	0.716	0.312	0.492	0.170	0.000	0.311
9	0.530	0.532	0.089	0.541	0.133	0.134	0.236	59	0.581	0.480	0.107	0.511	0.127	0.078	0.682
10	0.406	0.516	0.190	0.518	0.127	0.000	0.544	60	0.260	0.679	0.255	0.504	0.161	0.000	0.354
11	0.519	0.641	0.089	0.492	0.150	0.109	0.263	61	0.464	0.614	0.102	0.505	0.129	0.003	0.297
12	0.072	0.695	0.253	0.479	0.138	0.000	0.216	62	0.252	0.639	0.233	0.501	0.155	0.084	0.317
13	0.419	0.642	0.156	0.487	0.152	0.000	0.347	63	0.221	0.599	0.226	0.476	0.148	0.181	0.258
14	0.507	0.461	0.136	0.505	0.134	0.001	0.484	64	0.592	0.732	0.045	0.485	0.131	0.070	0.340
15	0.473	0.555	0.110	0.496	0.144	0.000	0.255	65	0.555	0.607	0.116	0.493	0.151	0.136	0.491
16	0.435	0.670	0.135	0.511	0.173	0.040	0.309	66	0.483	0.685	0.113	0.503	0.150	0.000	0.196
17	0.061	0.649	0.428	0.489	0.208	0.000	0.437	67	0.664	0.511	0.044	0.518	0.120	0.317	0.483
18	0.426	0.607	0.165	0.495	0.198	0.000	0.495	68	0.284	0.534	0.272	0.498	0.147	0.000	0.342
19	0.155	0.644	0.255	0.482	0.154	0.174	0.298	69	0.542	0.572	0.094	0.476	0.111	0.000	0.447
20	0.512	0.620	0.133	0.459	0.147	0.000	0.450	70	0.419	0.614	0.165	0.516	0.170	0.000	0.344
21	0.257	0.642	0.222	0.497	0.143	0.425	0.371	71	0.439	0.644	0.109	0.475	0.151	0.015	0.288
22	0.442	0.701	0.100	0.523	0.161	0.135	0.135	72	0.405	0.582	0.193	0.485	0.154	0.000	0.526
23	0.307	0.653	0.186	0.515	0.151	0.346	0.167	73	0.507	0.704	0.083	0.502	0.143	0.002	0.239
24	0.420	0.675	0.149	0.499	0.138	0.438	0.285	74	0.616	0.555	0.058	0.526	0.127	0.212	0.546
25	0.528	0.563	0.129	0.483	0.153	0.000	0.888	75	0.658	0.475	0.054	0.490	0.124	0.107	0.561
26	0.615	0.542	0.074	0.506	0.121	0.081	0.374	76	0.192	0.664	0.248	0.520	0.173	0.383	0.386
27	0.135	0.634	0.250	0.486	0.155	0.058	0.191	77	0.402	0.612	0.157	0.471	0.182	0.000	0.329
28	0.573	0.726	0.054	0.513	0.169	0.452	0.204	78	0.224	0.581	0.274	0.509	0.112	0.000	0.394
29	0.638	0.514	0.065	0.513	0.131	0.000	0.452	79	0.558	0.531	0.102	0.511	0.135	0.000	0.881
30	0.543	0.644	0.094	0.501	0.144	0.035	0.446	80	0.381	0.588	0.163	0.495	0.117	0.000	0.219
31	0.101	0.691	0.229	0.484	0.182	0.000	0.137	81	0.073	0.732	0.274	0.486	0.146	0.000	0.139
32	0.287	0.641	0.182	0.494	0.145	0.000	0.314	82	0.125	0.730	0.228	0.475	0.195	0.000	0.222
33	0.174	0.649	0.336	0.514	0.174	0.921	0.560	83	0.573	0.639	0.078	0.518	0.137	0.153	0.315
34	0.489	0.629	0.098	0.495	0.140	0.000	0.228	84	0.423	0.620	0.180	0.513	0.152	0.076	0.607
35	0.375	0.658	0.144	0.507	0.142	0.000	0.218	85	0.261	0.634	0.225	0.504	0.173	0.238	0.269
36	0.591	0.535	0.119	0.484	0.122	0.271	2.012	86	0.073	0.717	0.322	0.530	0.152	0.226	0.276
37	0.520	0.595	0.112	0.493	0.152	0.088	0.512	87	0.652	0.582	0.047	0.508	0.118	0.109	0.815
38	0.145	0.581	0.359	0.479	0.133	0.135	0.514	88	0.597	0.437	0.066	0.502	0.150	0.000	0.310
39	0.399	0.472	0.177	0.485	0.112	0.046	0.466	89	0.015	0.665	0.382	0.529	0.145	0.497	0.428
40	0.437	0.578	0.166	0.483	0.142	0.034	0.631	90	0.120	0.639	0.274	0.497	0.169	0.068	0.289
41	0.103	0.664	0.292	0.471	0.161	0.000	0.433	91	0.289	0.590	0.217	0.528	0.138	0.465	0.354
42	0.136	0.698	0.255	0.490	0.151	0.000	0.225	92	0.344	0.615	0.200	0.479	0.147	0.000	0.288
43	0.560	0.673	0.049	0.499	0.138	0.000	0.166	93	0.224	0.595	0.303	0.519	0.138	0.385	0.565
44	0.574	0.562	0.096	0.514	0.143	0.026	0.749	94	0.409	0.652	0.117	0.477	0.164	0.000	0.244
45	0.451	0.580	0.116	0.497	0.145	0.007	0.513	95	0.273	0.590	0.264	0.508	0.154	0.237	0.352
46	0.500	0.616	0.094	0.488	0.140	0.000	0.277	96	0.194	0.626	0.284	0.490	0.156	0.389	0.627
47	0.407	0.588	0.139	0.504	0.126	0.000	0.268	97	0.275	0.685	0.132	0.514	0.126	0.315	0.104
48	0.532	0.596	0.103	0.485	0.149	0.000	0.681	98	0.532	0.615	0.085	0.497	0.143	0.034	0.344
49	0.651	0.491	0.060	0.526	0.157	0.000	0.627	99	0.572	0.536	0.066	0.488	0.093	0.009	0.297
50	0.383	0.580	0.184	0.481	0.155	0.000	0.486	100	0.572	0.634	0.077	0.519	0.126	0.000	0.309

Appendix D: Robustness tables

Table 2.16: Mean parameter estimates across 100 trials for increasing true value of b, 1,000 sample size

	True value of b									
	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250
λ	0.135	0.173	0.226	0.247	0.250	0.261	0.278	0.293	0.296	0.320
	(0.015)	(0.014)	(0.016)	(0.017)	(0.016)	(0.016)	(0.016)	(0.017)	(0.018)	(0.017)
a	0.663	0.663	0.657	0.653	0.651	0.649	0.645	0.641	0.639	0.635
	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.006)
b	0.039	0.069	0.092	0.116	0.144	0.167	0.188	0.208	0.233	0.246
	(0.001)	(0.002)	(0.003)	(0.005)	(0.005)	(0.006)	(0.007)	(0.008)	(0.010)	(0.011)
α	0.502	0.503	0.502	0.502	0.502	0.502	0.502	0.502	0.502	0.501
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
κ	0.155	0.153	0.151	0.150	0.150	0.149	0.149	0.148	0.147	0.147
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
λ_y	1.194	0.921	0.457	0.342	0.278	0.245	0.220	0.208	0.188	0.180
	(0.316)	(0.269)	(0.077)	(0.048)	(0.034)	(0.028)	(0.022)	(0.019)	(0.017)	(0.016)
$\lambda_{\Delta i}$	0.335	0.320	0.322	0.313	0.310	0.306	0.309	0.316	0.315	0.318
	(0.021)	(0.018)	(0.021)	(0.014)	(0.013)	(0.011)	(0.012)	(0.012)	(0.012)	(0.010)

Chapter 3

Revealing Central Bank Preferences Under GMM

3.1 Introduction

The management of expectations is a primary concern of central banks, since these substantially affect the trajectory of the macroeconomy (Woodford, 2005). Clear communication and transparency by monetary authorities allows private agents to anticipate policy actions, but also detect any discrepancy between what is signalled and what is implemented. When actions are seen to be in accordance with policy announcements, central bank credibility is increased, and this in turn strengthens their ability to guide expectations. To this end, modern policymakers make explicit the frameworks by which they operate. Central bank mandates are a cornerstone of communication, which relate the objectives of monetary policy and the levers used to influence the economy.

In this chapter, I examine what the data reveal about the underlying preferences of monetary authorities, using the procedure developed in Chapter 2, and to what extent this maps to their stated preferences. The motivation for this exercise stems from two observations. First, not all central bank mandates are alike: some identify a 'unipolar' mandate, namely of combating inflation; others describe an equal concern for managing output and inflation (the 'bipolar' mandate); and still others lie somewhere in between. Consider the Bank of Canada, the Federal Reserve of the United States, and the Bank of England, which respectively fit these roles. Second, a a cursory glance at the trajectories of the output gap, the inflation rate, and the interest rate of these countries, as shown in Figures 3.1, 3.2, and 3.3, reveals that they have followed similar macroeconomic trajectories: increasing inflation and interest rates throughout the 1970s, sharp declines in the early to mid 1980s, a long period of low volatility through the mid-2000s, and finally a push of interest rates against the lower bound in response to a large and sustained output gap during the Great Recession.

Is it merely a coincidence that their outcomes have been so similar for such a long period of time? Perhaps the idiosyncrasies of each country's economic environment were such that heterogeneous preferences were necessary to obtain similar cross-country outcomes. But whether the monetary authorities exhibited common preferences can be tested. I hypothesize that, in reality, these central banks were operating with similar preferences since the late 1960s. This also implies that, as of the 1990s, at least two of the three central banks were not acting in accordance with their mandate.

In the previous chapter, I developed an inverse control program for discretionary policymakers, in the spirit of Givens and Salemi (2008). Inverse control allows for the simultaneous estimation of structural and preference parameters, under the assumption of optimality. I put this procedure to the test to estimate the policy preferences of the Bank of Canada, the Federal Reserve of the United States, and the Bank of England. The primary contribution of this paper, then, is to provide estimates for each country using this procedure. Moreover, to my knowledge, there have been no previous attempts to estimate of the policy preferences of the Bank of Canada and the Bank of England.

The results of the chapter may be summarized as follows. For the sample period covering 1968:1-2006:4, the inverse control method provides comparable structural estimates to a benchmark specification using instrument-based GMM method of Chapter 1. The main estimates suggest none of the countries exhibit a concern for output stabilization, but interest rate smoothing appears as a preference for the Bank of Canada. Measures of fit reject optimality in the United States for the baseline specification sample. Robustness tests indicate that the results are broadly stable with respect to different starting values or fixed policy coefficients. In addition, variability is seen in the policymakers' reaction function over alternative policy preference profiles, but this is decreasing at high levels of interest rate smoothing. When using subsamples respecting the beginning of each country's current mandate, a similar profile of preferences is detected, and optimality cannot be rejected for any of the central banks.

The chapter is organized as follows. Section 2 provides a discussion of the empirical literature on the estimated central bank preferences. It also provides an overview of the techniques used to establish reference targets for the inflation and interest rates. Section 3 describes the empirical model that is to be estimated, and briefly outlines the two GMM procedures employed: an instrument-based method to provide baseline estimates, and the FOC-based method which also generates policy preference estimates. In section 4, the data used for the estimation procedure is detailed. Section 5 provides the main results, and robustness tests are featured in Section 6. Section 7 considers the parameterization of central bank preferences in respect of their current mandates, and provides estimates for subsamples in line with the adoption of the mandates. Concluding remarks follow.

3.2 Literature review

This section briefly summarizes the main findings in respect of central bank preferences. It also discusses the literature surrounding the choice of inflation and interest rate targets.

Givens (2012) examines the preferences of the Federal Reserve under both commitment and discretion for the period spanning 1982 to 2008. In the former case, the estimates suggest a strong preference for interest rate smoothing ($\lambda_{\Delta i} = 2.5581$) over output gap stabilization ($\lambda_y = .01351$). In the discretion model, both estimated parameters are not significantly different from zero. The author finds, however, that this second model is preferred in terms of fit. Söderström et al. (2005), under the assumption of discretion and using data for 1968 to 1996, find that interest rate smoothing is a greater concern than inflation stabilization ($\lambda_{\Delta i} = 1.109$). In Favero and Rovelli (2003), for the sample period spanning 1980:3–1998:3, estimates of λ_y and $\lambda_{\Delta i}$ are 0.00125 and 0.0085, respectively — approximately nil relative to inflation management, but significant. Estimates provided by Ilbas (2012) order the policy preferences of the Federal Reserve during the Volcker–Greenspan era as inflation stabilization, interest rate stabilization, interest rate smoothing and output growth. Solving for an optimal Taylor-type rule in a backwards-looking specification, Dennis (2006) finds a statistically significant estimate for interest rate smoothing ($\lambda_{\Delta i} = 4.517$) but not for output stabilization over the same period. However varied, none of the estimates coincide with the stated mandate of the Federal Reserve, pointing instead to a concern for interest rate smoothing which is equal or super-ordinate to controlling inflation.

It is typical in the literature to assume constant inflation and interest rate targets (Clarida et al., 1998, 1999b). This is explicitly stated in Givens (2012); Söderström et al. (2005); Dennis (2006); Ilbas (2012), and appears to be the case in formulating the loss function for Favero and Rovelli (2003). However, significant attention has been devoted to the possibility that the failure by central banks to manage the rising tide of inflation in the 1970s can be explained by an adjustment of targets. The contention is that the Federal Reserve's performance in this era resulted from adopting a progressively higher target, and as such became too sanguine about inflation prior to the appointment of Volcker.

A number of approaches to estimating the targets are found in the literature. Ireland (2007) proposes a Taylor rule in which the inflation target may respond to supply shocks. Using a New Keynesian model in the spirit of Clarida et al. (1999b), the author's estimated inflation target roughly tracks movements in actual inflation: rising through the 1960s and

1970s to approximately 8 percent before falling back in the 1980s and hovering around 2 percent from the 1990s onward. Thus, in this formulation, inflation never deviated from its target by more than a few percentage points, and the rise and fall of inflation are substantively explained by adjustments in the target. The argument is put forward that supply shocks underlie the change in targets, but from the data, the possibility of purely random changes cannot be rejected. Leigh (2008) also relaxes the assumption of a constant implicit inflation target for the Federal Reserve in the specification of the Taylor rule. The target is modelled as a random walk and a Kalman filter approach is adopted for estimation purposes. Estimates of the implicit inflation target are provided for the Volcker-Greenspan era, and are shown to lie between 3 and 4 percent until 1992 and dropping to 1-2 percent thereafter.

Belaygorod and Dueker (2005) augment the Woodford (2003) DSGE model with an interest smoothing equation that accounts for discrete changes in the the target level of the federal funds rate, and incorporate a time-varying inflation target. In their estimations for the period spanning 1984 to 2004, the target varies from a low of 0.3 percent in the mid 1980s to a high of 4.7 percent at the start of the 1990s, with an unconditional mean of 2.5 percent and high persistence of deviations. However, the authors note that the inclusion of the discreteness adjustment in the interest smoothing equation substantially reduces the importance of the time-varying inflation target.

Kozicki and Tinsley (2005) consider a backwards-looking model in which there are shifts in the inflation target and where private agents, endowed with a constant-gain learning mechanism, may fail to correctly perceive the target. The inflation target is allowed to vary from its previous value through regime change, partial accommodation of supply shocks, and through exogenous shocks. Their estimates suggest that the time-varying target peaked near 8 percent in in the late 1970s, dropping to nil in 1982-1983, to 4 percent through the mid-80s and mid 90s and hovered around 2 percent thereafter. Moreover, the time-varying target leads inflation in the 1960s, suggesting that policymakers were amenable to higher rates of inflation in this decade. On the other hand, inflation leads the target in the 1970s, and the target only follows due to (presumably) some accommodation. The drop in the target in the 1980s is attributed to regime change, i.e., the appointment of Volcker. Private agents, for their part, would appear to have underestimated the target through the 1970s and strongly overestimated it in the early to mid 80s; but by the late 1980s, their perception became essentially correct.

Recent results from Milani (2017) show that the alternatives of time-varying and constant targets may hinge on whether the monetary authority is learning. In contrast to Kozicki and Tinsley (2005), the author adopts a New Keynesian framework with forward-looking behaviour, and allows for both the Federal Reserve and private agents to be endowed either with rational expectations or subjective expectations formed through a constant-gain learning algorithm. When the Federal Reserve sets policy under rational expectations, there are large movements in the inflation rate target, which peaks at eight percent in the 1970s and descends to two percent by the early 2000s. Relaxing the rational expectations assumption and introducing the learning algorithm results in an inflation target that is stable over most of the post-war period, ranging from 1.91 to 2.81%.

3.3 Empirical Model Framework and GMM procedures

3.3.1 New Keynesian model

The model employed for the empirical analysis in this paper is the same I have used previously to test the FOC-augmented GMM routine. It is an empirical forward-looking New Keynesian model which follows the timing of Söderström et al. (2005) and the lag structure employed by Givens (2012):

$$y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)(ay_{t-1} + (1-a)y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t \tag{3.1}$$

$$\pi_t = \alpha E_{t-1}\bar{\pi}_{t+3} + (1-\alpha)\pi_{t-1} + \beta y_{t-1} + v_t \tag{3.2}$$

where $\bar{\pi}_t = 1/4\sum_{j=0}^{3}\pi_{t-j}$ represents annual inflation.

Equation (3.1) relates the output gap at time t to its next period expectation and two lags.[1] The output gap is also related to the difference between the interest rate at t and the expectation of inflation in the following period. In this model, monetary policy can affect the output gap with a one-period lag. This assumption is borrowed from Rudebusch (2002), who argues that it is necessary to account for delays due to the processing of information by agents when using data of a relatively high frequency (e.g., quarterly). Equation (3.2) is a Phillips curve which relates inflation at t to the expectation of inflation one period ahead and to its one-period lag, as well as to a lag of the output gap. The model dispenses with the possibility of a long-run trade-off between inflation and the output gap by imposing that the coefficients on expected and lagged inflation sum to one.

3.3.2 Instrument-based GMM

As a first step, I reprise the GMM procedure adopted in Chapter 1; these estimates will provide reference point for those obtained with the new, FOC-augmented procedure described in Chapter 2. The first approach to estimating the system described in (3.1) and (3.2) relies on exploiting the rational expectations hypothesis which underlies the model, i.e., of orthogonality between the innovations (inclusive of the forecast error) in each period t and the information set I_t available up to this period. This requires the creation of composite error terms, $h_{IS,t}$ and $h_{PC,t}$, which include the expectation error. These are set against instruments

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[1] This specific formulation is employed by Givens (2012), and the two-lag structure can be derived from a representative household which values consumption over and above a habit that depends on average consumption in the preceding two periods.

Z_t which are subsets of I_t:

$$g_t(Z_t) = \begin{bmatrix} h_{IS,t}(Z_t) \\ h_{PC,t}(Z_t) \end{bmatrix} \tag{3.3}$$

such that a weighted quadratic of the sample analogue of $E[g_t(Z_t)] = 0$ is minimized.[2] The instrument set used in this paper is comprised of four lags of the inflation rate and four lags of the output gap.

3.3.3 FOC-augmented GMM

To implement the procedure described in Chapter 2, it is necessary to describe the optimization problem faced by the policymaker. I assume the monetary authority adjusts the interest rate i_t to minimize a standard quadratic loss function:

$$R = E_t(1-\delta)\sum_{j=0}^{\infty}\delta^j\left\{\pi_{t+j}^2 + \lambda_y y_{t+j}^2 + \lambda_{\Delta i}(i_{t+j} - i_{t+j-1})^2\right\} \tag{3.4}$$

subject to equations (3.1) and (3.2).

Of interest in (3.3) are parameters for the weights on controlling volatility in the output gap (λ_y) and the change in interest rates from period to period ($\lambda_{\Delta i}$). The weights are relative to that which is placed on controlling inflation volatility, such that a value greater than one on λ_y would indicate a greater concern for smoothing output than stabilizing inflation. δ^j is the discount factor applied to the procession of period j losses.

Given the assumption of a discretionary policymaker, the equilibrium law of motion for the predetermined variables $x_{1t} = \begin{bmatrix} y_t, y_{t-1}, \pi_t, i_t \end{bmatrix}'$ is of the form:

$$x_{1t+1} = Mx_{1t} + v_{1t+1} \tag{3.5}$$

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[2] A complete description of the method is provided in Chapter 1.

In addition, the optimizing discretionary policymaker will set interest rates according to the following first-order condition (FOC):

$$i_{t+1} = Fx_{1t} \tag{3.6}$$

where F, I have argued in the Appendix to Chapter 2, codes information about the policymaker's preferences and the structural parameters of the model.

The procedure is built upon on the eight least-squares normal equations $E(v_{1t+1}x'_{1t}) = 0$, which result from (3.5), to which the unconditional expectation of the FOC in (3.6) is added as a ninth moment of interest. The structural parameters of the model, and the underlying preferences of the monetary authority, are collected into a vector $\rho = \{\lambda, a_1, b, \alpha, \beta, W_y, W_i\}$. Let $g(\rho, v_{t+1})$ hold both the corresponding sample moments of the least-squares normal equations and FOC, and whose weighted quadratic is minimized via a search procedure. [3]

3.4 Data

Estimating the system described in (3.1) and (3.2) requires data on the output gap, the inflation rate and the interest rate for Canada, the United States,and the United Kingdom. I follow the same variable construction method as Salemi (2006) for the United States (described next), and make the appropriate modifications for Canada and the United Kingdom.

The raw data span 1965:1 to 2012:3 and are sourced from *FRED*, the Federal Reserve Bank of St. Louis' online database. Output is measured as seasonally-adjusted real GDP in (billions of) chained 2009 dollars. Reprising Salemi (2006), I construct a measure of per-capita GDP by dividing this value by the interpolated population estimates taken from the 2013 *Economic Report of the President*, which is then transformed into logarithms. I then estimate potential output using a linear time trend and construct the output gap as

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[3] The baseline starting values are given the vector: $\rho^s = \big[0.05, 0.05, 0.05, 0.37, 0.05, 0.05, 0.05\big]$.

the difference between the log-transformed series and the trend.[4,5] The inflation rate is the compounded annual rate of change of the seasonally-adjusted GDP implicit price deflator. The interest rate measure is the annualized secondary market yield of the 3-month Treasury Bill series.

For the FOC-augmented method, I designate target values for inflation and interest rates by fitting continuous piecewise linear trends, where kinks occur at 1980:I (corresponding to the inauguration of Volcker's as Chairman of the Federal Reserve) and 1986:I. The deviation from target is taken as the difference between the realized inflation rate or interest rate and the trend line. I constrain the slopes of the piecewise linear trends to be the same for inflation and the interest rate. The target paths for inflation and the interest rate are therefore increasing in the period spanning 1965:1 to 1979:4, rapidly decreasing between 1980:1 and 1985:4, and decreasing more gently thereafter.

The data points for Canada and the United Kingdom are constructed in the same fashion, but the population counts are obtained from the OECD. For Canada, the interest rate measure is the bank rate through the fourth quarter of 1998, and the target for the overnight rate thereafter; for the United Kingdom, I use the discount rate. I constrain the kinks for these datasets to occur at the same time as those as the US.[6] While it would be desirable to estimate the target values alongside the other parameters of interest in the model, that exercise is beyond the scope of this paper. I instead side with Salemi (2006) and Ireland (2007) in assuming there was an accommodation of rising inflation in the 1970s and a reversal of course thereafter. In addition, to formally control for the problem of target uncertainty, I provide a robustness exercise for subsamples limited to when the current mandates were

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[4] While the Congressional Budget Office provides estimates of the output gap, this is not available for Canada or the United Kingdom. Using the method described here ensures consistency of construction across countries.

[5] As in the first chapter, for the instrument-based method, the linear trend was constructed iteratively to help respect the orthogonality condition between the residuals and the information set, though this did not yield estimates markedly different to those obtained with a full-sample trend. It remains, however, that only real-time vintages of GDP data — as opposed to revised data — would guarantee no future information is influencing the estimate of the trend, irrespective of the way it is constructed.

[6] Figures 1, 2 and 3 in the Appendix display the data points and target lines for each country.

implemented.

3.5 Main results

Table 3.1 provides the results for both the instrument-based ('Inst.') and FOC-based ('FOC') GMM procedures for the sample period spanning 1968:1 to 2006:4. I begin with an examination of the estimates produced with the method of the first chapter. For all three countries, expectations and lags of the output gap take on roughly equal roles in the determination of the present output gap; for the United States and United Kingdom, the tilt is slightly in favour of expectations, while the reverse is true for Canada. Estimates of α are more variable, with both Canada and the United Kingdom showing a dominant role for expectations in determining inflation. The forward- and backward-looking split on inflation is more even for the United States, with a point estimate comparable to that of Givens (2012). In the United States, the policy coefficient, b, is similar in magnitude to that reported by Dennis (2006), while in Canada and the United Kingdom the estimates are lower. Estimates of κ are also generally small.

Table 3.1: Empirical estimates by country and procedure, 1968:1 - 2006:4

Param.	Canada		United States		United Kingdom	
	Inst.	FOC	Inst.	FOC	Inst.	FOC
λ	0.515	0.432	0.512	0.412	0.569	0.446
	(0.05)	(0.07)	(0.02)	(0.11)	(0.04)	(0.22)
a	1.09	1.23	0.925	1.26	0.864	1.07
	(0.08)	(0.05)	(0.05)	(0.05)	(0.07)	(0.15)
b	0.013	0.010	0.038	0.012	0.014	0.025
	(0.01)	(1.6)	(0.01)	(1.4)	(0.01)	(1.4)
α	0.662	0.675	0.399	0.551	0.864	0.758
	(0.04)	(0.08)	(0.06)	(0.08)	(0.07)	(0.10)
κ	0.017	0.007	-0.008	0.011	0.014	0.033
	(0.03)	(1.0)	(0.02)	(0.56)	(0.05)	(0.92)
λ_y	—	0.000	—	0.000	—	0.000
	—	(2.6e4)	—	(0.00)	—	(1.9e4)
$\lambda_{\Delta i}$	—	15.9	—	0.930	—	11.0
	—	(6.9)	—	(1.6)	—	(9.4)
Q	3.8e-2	2.9e-2	4.1e-2	6.2e-2	2.5e-2	3.8e-2
$Q \times T$	6.1	4.4	6.4	9.6	3.89	5.9
$Q \times T$; p=	0.870	0.109	0.844	0.008	0.920	0.052

Notes: The empirical estimates reported above for the following model: $y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$. 'Inst' estimates are based on the rational expectations hypothesis that the error-in-expectation at t is uncorrelated to the information set at t; the procedure is outlined in more detail in Chapter 1. 'FOC' estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = FX_t$ which holds at the discretionary rational expectations solution. Standard errors, computed using the delta method, are denoted in parentheses. Q represents the value of the GMM score function at the estimated parameter values. $Q \times T$ is the Sargan-Hansen J-test statistic for the model's overidentifying restrictions, where T represents the number of observations.

Turning to the FOC-based procedure results reveals a strong concordance between the estimates obtained via each method. Minor differences can be noted, however. First, the point estimates of λ are somewhat depressed relative to those of the instrument-based GMM. Point estimates of α are higher for the United States, and lower for the United Kingdom, while very similar in Canada. The largest differences are seen in the estimates of a, which are uniformly increased across countries. Estimates of b on the other hand, are not markedly different for Canada nor the United Kingdom, but lower for the United States. The novelty of the FOC-based procedure is, of course, is in retrieving estimates of the preference parameters, λ_y and $\lambda_{\Delta i}$. The preference on output stabilization, λ_y, is never found to be different from zero. As I have noted, values not significantly different from zero were also obtained by Dennis (2006), Givens and Salemi (2008), Givens (2012) and Favero and Rovelli (2003) for the United States. Moreover, though the standard error on these coefficients is large, I have shown in the previous chapter that at lower sample sizes the bias on the point estimate would be upward. Turning to the smoothing parameter, $\lambda_{\Delta i}$, its estimates are different from zero, significantly so for Canada. Although others have found estimated values for $\lambda_{\Delta i}$ well in excess of one (Dennis, 2006; Givens, 2012) the magnitudes obtained here are larger still for Canada and the United Kingdom. I present a closer examination of why this occurs in the following section, but in short, it appears to owe to the lack of differentiation between optimal reaction functions at high levels of the smoothing parameter.

At the bottom of the table I report the results of the Sargan-Hansen test of overidentifying restrictions, constructed as the value of the score function at the point estimates scaled by the number of observations. In the instrument-based GMM method, the overidentifying restrictions are never rejected. When running the FOC-based procedure, the overidentifying restrictions — which includes the the optimal interest rule, or reaction function — are not rejected for Canada or the United Kingdom. In other words, the data do not reject optimality on the part of the discretionary policymaker in these countries, but this is not supported for the United States. Overall, the results only sustain the common-preferences hypothesis with respect to the desire for output gap stabilization between Canada and the United Kingdom.

Table 3.2: Empirical estimates, by country and procedure, 1968:1 - 2006:4

Param.	Canada		United States		United Kingdom	
	No FOC	FOC	No FOC	FOC	No FOC	FOC
λ	0.411	0.432	0.000	0.412	0.233	0.446
	(0.34)	(0.07)	(0.00)	(0.11)	(6.7)	(0.22)
a	1.23	1.23	1.30	1.26	1.06	1.07
	(0.07)	(0.05)	(0.08)	(0.05)	(0.16)	(0.15)
b	0.024	0.010	0.114	0.012	0.121	0.025
	(3.2)	(1.6)	(0.51)	(1.4)	(5.6)	(1.4)
α	0.681	0.675	0.529	0.551	0.782	0.758
	(0.08)	(0.08)	(0.09)	(0.08)	(0.10)	(0.10)
κ	0.007	0.007	0.016	0.011	0.056	0.033
	(1.5)	(1.0)	(0.54)	(0.56)	(1.0)	(0.92)
λ_y	0.000	0.000	0.000	0.000	0.000	0.000
	(190.2)	(2.5e4)	(0.00)	(0.00)	(2.0e4)	(1.9e4)
$\lambda_{\Delta i}$	2.55	15.9	0.067	0.930	2.27	11.0
	(6.7)	(6.8)	(0.95)	(1.6)	(8.4)	(9.4)
Q	4.0e-2	2.9e-2	5.2e-2	6.2e-2	3.7e-2	3.8e-2
$Q \times T$	6.23	4.44	8.01	9.58	5.75	5.93
$Q \times T$; p=	0.013	0.109	0.005	0.008	0.016	0.052
$\mathscr{L}$	-91.73	-90.83	38.92	36.56	-227.14	-226.47

Notes: The empirical estimates reported above for the following model: $y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$. 'FOC' estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = F X_t$ which holds at the discretionary rational expectations solution. This condition is omitted in the calculation of the 'No FOC' estimates. Standard errors, computed using the delta method, are denoted in parentheses. Q represents the value of the GMM score function at the estimated parameter values. $Q \times T$ is the Sargan-Hansen J-test statistic for the model's overidentifying restrictions, where T represents the number of observations. $\mathscr{L}$ is the pseudo-log-likelihood of the model, calculated as $\mathscr{L} = T/2 \ln(|\Phi|)$, where $|\Phi|$ is the determinant of the residual error covariance matrix.

More can be gleaned about the results of the FOC procedure by rerunning it without imposing (3.6) and observing the change in the model fit. Those results are presented in Table 2 in the 'No FOC' column. In Canada, the structural parameter estimates are very close regardless of the imposition of the FOC. In the United States, the unaugmented procedure does not find a role for expectations in driving the output gap, and produces a far stronger estimate of the policy coefficient b. The United Kingdom also shows a reduced role for expectations in the IS equation, and a much larger role for the policy coefficient. To assess model fit, I present the results of the Sargan-Hansen test again, but also a pseudo-likelihood measure $\mathscr{L}$ derived from the residual error covariance matrix. Across all countries, imposing the FOC works to increase the value of $Q \times T$, and in Canada and the United Kingdom, this is enough to move from rejection to non-rejection of the overidentifying restrictions. Similarly, the pseudo-likelihood measure indicates a preference for the FOC-augmented system in both of these countries, bolstering the claim that discretion cannot be rejected as the style of policymaking over this period. By contrast, the standard system is preferred for the United States: imposing discretionary optimality worsens the fit of the data, and does not improve the results of the overidentifying restrictions test.

3.6 Robustness

3.6.1 Relating reaction functions and preference parameters

The reaction function for the model, which also doubles as the overidentifying restriction for the FOC-augmented system, is written out as:

$$i_t = f_{y1} y_{t-1} + f_{y2} y_{t-2} + f_{\pi} \pi_{t-1} + f_i i_{t-1} \tag{3.7}$$

i.e., the optimal interest rate rule followed by the discretionary policymaker responds to the vector of pre-determined variables. Table 3 shows the reaction function implied by the parameter estimates provided by the FOC-augmented procedure. For Canada, the combination of a high preference for smoothing the interest rate and a relatively low estimated policy coefficient are such that the implied reaction function loads heavily on maintaining the previous period's interest rate, with only a marginal concern for adjusting in response to fluctuations in the output gap or inflation. By contrast — taking the point estimates on λ_y and $\lambda_{\Delta i}$ as given — there is considerably more reaction to these variables in the cases of the United States and United Kingdom, though the smoothing coefficient f_i remains predominant.[7]

Table 3.3: Implied reaction function, by country, 1968:1 - 2006:4

Country	Est. parameter		Reaction function coefficient			
	λ_y	$\lambda_{\Delta i}$	f_{y1}	f_{y2}	f_π	f_i
Canada	0.0	15.9	0.034	-0.010	0.009	0.882
United States	0.0	0.93	0.193	-0.064	0.117	0.803
United Kingdom	0.0	11.0	0.116	-0.009	0.010	0.788

Notes: The above reports the coefficients of optimal rules for a discretionary policymaker given the following model: $y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$, given the estimated parameters obtained using the 'FOC' method (see Table 3.1). The optimal rules are of the form $i_t = f_{y1} y_{t-1} + f_{y2} y_{t-2} + f_\pi \pi_{t-1} + f_i i_{t-1}$. It is assumed throughout that the preference for inflation management is set to one.

Since the FOC in (3.5) is the overidentifying restriction used to obtain estimates of λ_y and $\lambda_{\Delta i}$, it should display a sufficient amount of variability as a function of the underlying preference parameters in order for the procedure to settle on its estimates. Table 4 in the appendix shows the effect of varying these parameters on the implied reaction function of the model, given the structural parameter estimates. Starting from a benchmark value of $\lambda_{\Delta i}$ commonly seen in the literature (0.3), the results suggest there is quite a bit of variation in the optimal policy rule as λ_y moves from zero to one, i.e. moving towards a bipolar mandate. Fixing λ_y at 0 and then increasing $\lambda_{\Delta i}$ up to 10 — thereby approaching the main estimates — yields more variation in the rule, but at a decreasing rate. In other words, it becomes difficult for the procedure to select between the optimal policy rule at high levels of the smoothing parameter. Nevertheless, the reaction functions at these levels of $\lambda_{\Delta i}$ are considerably different to those seen for low values (e.g., 0.3, 0.5). If λ_y is then increased while keeping $\lambda_{\Delta i}$ at 10, there is still variation in the rule. Thus, the procedure should be able to

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[7] The reaction function for the United States is comparable to that obtained by Dennis (2006).

pick up non-zero values of λ_y based on the variation in the optimal policy rule. The least amount of variation is seen in the rule applicable to the United States, which may explain why the FOC as an overidentifying restriction is rejected in that case.

3.6.2 Fixing the policy coefficient

Chapter 2 provided a robustness exercise in which the true underlying value of the policy coefficient b was varied to see how this affected the FOC-augmented procedure's ability to retrieve the policy preference parameters. This was done under the hypothesis that the policymaker's ability to meaningfully affect the macroeconomy might be a necessary condition to retrieving his preferences. While estimates of $\lambda_{\Delta i}$ are insensitive to b, there is an inverse relationship between the (upward) bias on estimates of λ_y — along with the size of its standard error — and the policy coefficient, as predicted.

Since the estimates of b differ somewhat depending on the method used, I consider what happens under the FOC procedure if b is fixed to the values obtained using the instrument method. This implies an increase in the value of the policy coefficient for Canada and, more substantially, the United States. If an upward bias is to be expected for low values of b, it should be expected that estimates of λ_y will remain grounded at zero for these countries when fixing this parameter to a higher value. The results of this test are presented in Table 4 and are consistent with the prediction. In no case do the estimated λ_y's differ from zero. Point estimates of $\lambda_{\Delta i}$, on the other hand, are reduced compared to the baseline specification. However, looking at the model fit, as calculated via $\mathscr{L}$, reveals that the fixed b specification is not preferred over the 'FOC' estimates of Table 1 for Canada and the United Kingdom, nor the 'No FOC' estimates for the United States (given that the overidentifying restrictions are rejected in this case).

Table 3.4: Empirical estimates, b free or fixed, by country, 1968:1 - 2006:4

Param.	Canada		United States		United Kingdom	
	b free	b fixed	b free	b fixed	b free	b fixed
λ	0.432	0.422	0.412	0.352	0.446	0.461
	(0.07)	(0.08)	(0.11)	(0.11)	(0.22)	(0.14)
a	1.23	1.24	1.26	1.26	1.07	1.08
	(0.05)	(0.07)	(0.05)	(0.06)	(0.15)	(0.14)
b	0.010	0.013	0.012	0.038	0.025	0.014
	(1.6)	—	(1.4)	—	(1.4)	—
α	0.675	0.674	0.551	0.533	0.758	0.730
	(0.08)	(0.08)	(0.08)	(0.08)	(0.10)	(0.09)
κ	0.007	0.004	0.011	0.010	0.033	0.005
	(1.0)	(1.3)	(0.56)	(0.67)	(0.92)	(2.6)
λ_y	0.000	0.000	0.000	0.000	0.000	0.000
	(2.6e4)	(411.4)	(0.00)	(207.3)	(1.9e4)	(3.3e3)
$\lambda_{\Delta i}$	15.9	8.62	0.930	0.219	11.0	7.73
	(6.9)	(6.3)	(1.6)	(1.1)	(9.4)	(11.2)
Q	2.9e-2	3.5e-02	6.2e-2	6.7e-02	3.8e-2	4.6e-02
$Q \times T$	4.44	5.37	9.58	10.39	5.93	7.11
$Q \times T$; p=	0.109	0.147	0.008	0.016	0.052	0.068
$\mathscr{L}$	-90.83	-92.08	36.56	38.20	-226.47	-226.95

Notes: The empirical estimates reported above for the following model: $y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$. Estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = F X_t$ which holds at the discretionary rational expectations solution. 'b free' estimates allow the policy coefficient to be estimated as part of the procedure, while 'b fixed' estimates use the point estimate of b obtained from the instrument-based method (see Table 3.1). Standard errors, computed using the delta method, are denoted in parentheses. Q represents the value of the GMM score function at the estimated parameter values. $Q \times T$ is the Sargan-Hansen J-test statistic for the model's overidentifying restrictions, where T represents the number of observations. $\mathscr{L}$ is the pseudo-log-likelihood of the model, calculated as $\mathscr{L} = T/2 \ln(|\Phi|)$, where $|\Phi|$ is the determinant of the residual error covariance matrix.

3.6.3 Varying starting values

In Chapter 2, I also presented a robustness exercise in which I allowed some of the starting values of the parameters to be adjusted upwards or downwards by 10% to see if the FOC-augmented procedure could still recover the policy preferences with accuracy. I echo this exercise here by making the same adjustment to the starting values of structural parameters of the IS equation and the policy preference parameters, namely increasing and then decreasing each by 10%. The full results are presented in Tables 3.7-3.9 in the Appendix.

Some general observations can be made about these specifications. In the case of Canada, the parameters appear sensitive to a number of changes in the starting values, often converging to a system characterized by little role for expectations in determining the output gap, high effectiveness of policy, and marginal concern for interest rate smoothing. However, where the estimates differ substantially from those of the baseline specification, the pseudo-loglikelihood and overidentifying restrictions suggest a worse fit to the data. Givens and Salemi (2008), using a GMM procedure for policymaker operating with a simple Taylor-type rule, noted that starting values must be chosen such that the search algorithm remains on the saddle path. It appears that for Canada it is easier to stray off into a region of the parameter space that is problematic. By comparison, for the United States and United Kingdom, both structural and policy preference estimates are stable under alternative starting values. In particular, point estimates of the interest rate smoothing parameter $\lambda_{\Delta i}$ lie around 1 for the United States and around 10 for the United Kingdom in those specifications with the best fit, values in keeping with the main estimates.[8]

3.7 Comparison with Current Mandates

In this section, I describe the current mandates of the central banks of the United States, Canada, and the United Kingdom in the context of their recent history. I suggest what

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[8] Although unreported here, two further the exercises were conducted, in which the start and end dates of the sample were extended by one year, respectively; these did not yield substantively different results.

these mandates would be expected to yield in the parameterization of the loss function given in (3.3), and then rerun the FOC-based algorithm for subsamples which begin at the time of adoption of the current mandates. The timing of these mandates varies: for the United States, it appears in 1977, but those of Canada and the United Kingdom only emerge in the 1990s. Thus, a direct comparison of the mandates with what the data reveals (via the inverse control procedure) is limited to these later periods. The trade-off is that in exchange for abstracting from the uncertainty regarding targets, the number of data points is reduced.

3.7.1 Bank of Canada

As stated on the Bank of Canada website: "The objective of monetary policy is to preserve the value of money by keeping inflation low, stable and predictable... Canada's monetary policy framework consists of two key components that work together: the inflation-control target and the flexible exchange rate. This framework helps make monetary policy actions readily understandable, and enables the Bank to demonstrate its accountability to Canadians." The wording of the mandate makes explicit not only the unipolar concern for inflation management, but also the desire to effectively communicate such policy preferences to private agents. The unipolar mandate of price stability is elsewhere described as being in place since 1991, with an inflation target of 2 percent, and a control range of 1 to 3 per cent around the target.

In terms of preference parameters, the Bank of Canada mandate's is unambiguous and would be parameterized by setting $\lambda_y = 0$ and $\lambda_{\Delta i} = 0$.

3.7.2 Federal Reserve

The Federal Reserve's mandate reflects the goals set for it by Congress. It is stated in the Federal Reserve Act, as amended in 1977, that the Board of Governors and the Federal Open Market Committee should act "so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates." The nascent understanding that no trade-off between unemployment and inflation exists in the long-run, however, meant that for these goals not to be in competition, maximum employment should be interpreted to mean full employment (Judd and Rudebusch, 1999). It is, in fact, the Humphrey-Hawkins Act of 1978 that clarifies that price stability and full employment are national economic objectives, and which is interpreted to set the dual mandate on the Federal Reserve (Thornton, 2012).

Modelling the preferences of the Federal Reserve in the loss function displayed in (3.3) would be achieved by setting $\lambda_y = 1$, reflecting the twin concerns for output management and inflation stabilization. To be clear, while the Federal Reserve Act makes note of moderate long-term interest rates, this is not accounted for in the interpretation seen in Federal Reserve communiqués, so it appears reasonable to assume the dual mandate leaves $\lambda_{\Delta i} = 0$.

3.7.3 Bank of England

Per Howells (2013), five monetary policy regimes characterize the post-war experience of the United Kingdom. The last of these, namely the inflation targeting era, began in 1992 as the United Kingdom withdrew from the Exchange Rate Mechanism. Shortly thereafter, the Bank gained statutory independence and an explicit mandate. The Bank of England Act (1998) states: "In relation to monetary policy, the objectives of the Bank of England shall be — (a) to maintain price stability, and (b) subject to that, to support the economic policy of Her Majesty's Government, including its objectives for growth and employment."[9] While the Bank of England Act granted the Bank independence in setting interest rates, price stability is defined by the government's inflation target, currently 2 percent, and is announced each

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[9] Wording very similar to this was available on the Bank of England website on the "Monetary Policy Framework" page prior to a December 2017 redesign, along with a third sentence: "The remit recognises the role of price stability in achieving economic stability more generally, and in providing the right conditions for sustainable growth in output and employment." This phrasing highlights the complementarity of the objectives but does not clarify whether the Bank of England regards price stability as either a necessary or sufficient condition. On the other hand, in the Monetary Policy Remit of Autumn 2017, the Chancellor affirmed that it is the government's belief that "Price and financial stability are *essential pre-requisites* for strong, sustainable and balanced growth in all regions and sectors of the economy." (emphasis mine)

year by the Chancellor of the Exchequer in the annual Budget statement.[10]

The Bank of England's mandate is the most complex of the three considered in the paper. As the wording "subject to [price stability]" suggests, the Bank of England may act so as to support prevailing government objectives around growth and employment, provided these do not interfere with the goal of managing inflation. The wording does not preclude, for example, allowing excess volatility in output (via heavily contractionary monetary policy) if this is necessary to achieve inflation stabilization. The standard loss function described by (3.3) could be parameterized in respect of the Bank of England's mandate by setting $1 \geq \lambda_i > 0$, indicating that the concern for smoothing fluctuations in output forms part of its objective function, but that it is no greater than that of inflation stabilization. As with the the Bank of Canada and the Federal Reserve, there is no indication of a preference for smoothing the path of the interest rate, so it would be expected that $\lambda_{\Delta i} = 0$.

3.7.4 Results

Based on the above, I rerun the FOC-augmented procedure for the following periods: 1991:1-2006:4 for Canada, 1980:1-2006:4 for the United States, and 1998:2-2006:4 for the United Kingdom.[11] As the results in Table 5 show, for each country, the FOC-augmented results are preferred in terms of fit and provide substantial reduction in the the standard errors over the system based on the least squares normal equations alone. Point estimates for the structural parameters are still reasonable: λ appears lower compared to the baseline specification, suggesting that expectations are a less potent factor in explaining the output gap in these later sample periods; correspondingly, the point estimate a is a little bit larger.

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[10] In this sense, the Bank of England still enjoys less independence than its Canadian and American counterparts. To wit, the legislation affords the British government authority to instruct the Bank of England on interest rates for a limited period in extraordinary circumstances.

[11] While inflation targeting began in late 1992 for the United Kingdom, the remit regarding output stabilization only begins with the adoption of the Bank of England Act of 1998.

Table 3.5: Empirical estimates for current monetary policy mandates, by country and procedure

Param.	Canada		United States		United Kingdom	
	No FOC	FOC	No FOC	FOC	No FOC	FOC
λ	0.024	0.309	0.411	0.306	0.000	0.278
	(132.0)	(0.69)	(0.26)	(2.5)	(416.5)	(3.2)
a	1.43	1.35	1.30	1.29	1.54	1.39
	(0.47)	(0.10)	(0.07)	(0.12)	(0.15)	(0.28)
b	0.168	0.054	0.003	0.022	0.000	0.017
	(7.5)	(1.9)	(5.7)	(6.1)	(3.8e4)	(7.4)
α	0.896	0.884	0.488	0.476	0.881	0.881
	(0.14)	(0.14)	(0.07)	(0.07)	(0.12)	(0.12)
β	0.139	0.118	0.005	0.004	0.000	0.000
	(0.72)	(0.56)	(1.6)	(1.5)	(0.00)	(0.00)
λ_y	0.000	0.004	0.000	0.000	0.010	0.000
	(1.4e4)	(2.3e3)	(7.9e4)	(1.5e3)	(3.7e4)	(2.9)
$\lambda_{\Delta i}$	2.24	4.33	1.49	0.078	0.000	0.265
	(15.8)	(19.2)	(7.9)	(4.0)	(3.9e4)	(2.9)
Q	8.9e-03	7.6e-03	4.7e-02	4.3e-02	6.9e-02	5.9e-02
$Q \times T$	0.56	0.48	5.01	4.64	2.34	2.02
$Q \times T$; p=	0.45	0.79	0.03	0.10	0.13	0.36
$\mathscr{L}$	7.3	7.5	82.2	83.5	1.1	1.8

Notes: The empirical estimates reported above for the following model: $y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$. The sample periods run from the beginning of current monetary policy mandates and end prior to the Great Recession, and are specifically: 1991Q1-2006Q4 for Canada, 1980Q1-2006Q4 for the United States, 1998Q2-2006Q4 for the United Kingdom. "FOC" estimates are produced by augmenting the least-squares normal equations in the GMM estimator with the first-order condition $i_t = F X_t$ which holds at the discretionary rational expectations solution. This condition is omitted in the calculation of the "No FOC" estimates. Standard errors, computed using the delta method, are denoted in parentheses. Q represents the value of the GMM score function at the estimated parameter values. $Q \times T$ is the Sargan-Hansen J-test statistic for the model's overidentifying restrictions, where T represents the number of observations. $\mathscr{L}$ is the pseudo-log-likelihood of the model, calculated as $\mathscr{L} = T/2 \ln(|\Phi|)$, where $|\Phi|$ is the determinant of the residual error covariance matrix.

With respect to the preference parameters, $\lambda_{\Delta i}$ is lowered throughout, but remains well above one for Canada, though without being significantly different from zero; the loss of significance may be explained by the restricted number of data points available. On the other hand, he overidentifying restrictions are no longer rejected in any case, and the FOC-augmented systems are preferred in terms of fit. Thus, the data would indicate that in the period of time during which the current mandate was communicated, discretion cannot be dismissed as an account of the style of policymaking. Yet nowhere does a concern for smoothing fluctuations in the output gap appear, which is directly at odds with the stated mandate of the Federal Reserve. For the United Kingdom, it may have been that output stabilization was either never necessary, or that the management of inflation precluded acting in a manner to stabilize employment. In Canada, an estimated λ_y of zero is consistent with the stated aim of monetary policy.

3.8 Conclusion

I have previously demonstrated how GMM can be employed to retrieve estimates of policymakers' preference parameters when they are operating under discretion — an application of the inverse control procedure. In short, this involves adjoining the first-order condition implied by an optimizing discretionary policymaker as a moment for estimation in addition to the usual least squares normal equations. In this paper, the procedure is put to the test by estimating an empirical New Keynesian model for Canada, the United States and the United Kingdom. At the outset, I maintain the hypothesis that all three central banks exhibited the same preference profile over the period under consideration. This hypothesis rests on an examination of the path of key macroeconomic variables spanning the era of stagflation and the Great Moderation, and implies that at least two of the three central banks were not operating according to their stated mandates as of the 1990s.

The empirical analysis is conducted under a number of alternative scenarios and specifications. As a benchmark, I provide estimates of the structural parameters of the system using the instrument-based GMM procedure that underpins Chapter 1. In pursuing the

FOC-based method Chapter 2, I assume that the policymakers adjusted their inflation and interest rate targets, first upwards as a kind of ratchet effect resulting from the experience of the 1970s, then downwards from the 1980s. In this scenario, no preference for output stabilization is detected. Interest rate smoothing, however, appears as a distinct concern for the Bank of Canada. Moreover, discretionary optimality is not rejected for either Canada or the United Kingdom. The optimal interest rate rule, which acts as the key overidentifying restriction, varies with respect to different underlying preferences, but less for very strong preferences over interest rate smoothing. Further robustness checks reveal the results are stable under alternative starting values and for values of the policy coefficient fixed at the levels found in the instrument-based procedure. Finally, I present an exercise in which the inverse control procedure is put to the test for sample periods spanning beginning at the point of adoption of current mandates. As in the baseline specification, no concern for output gap stabilization is detected, and the point estimate on the smoothing parameter remains large for Canada. Moreover, optimality is no longer rejected for the United States in this restricted sample.

Generally speaking, the empirical evidence does not reject the possibility that monetary authorities were acting optimally in the discretionary framework (excepting the United States in the main sample). However, this does not represent a formal test of whether the style of policymaking is better described as discretionary or commitment-like, the latter of which has been argued by some (e.g., Ilbas, 2012) to be a more plausible framework for how modern central banks conduct policy. But as the debate remains unsettled Givens (2012), it would be valuable to adapt the the GMM procedure described in Chapter 2 to retrieve the preferences of a policymaker operating under the commitment framework.

Appendix A: Figures

Figure 3.1: Canada: Macroeconomic Variables, 1965-2012

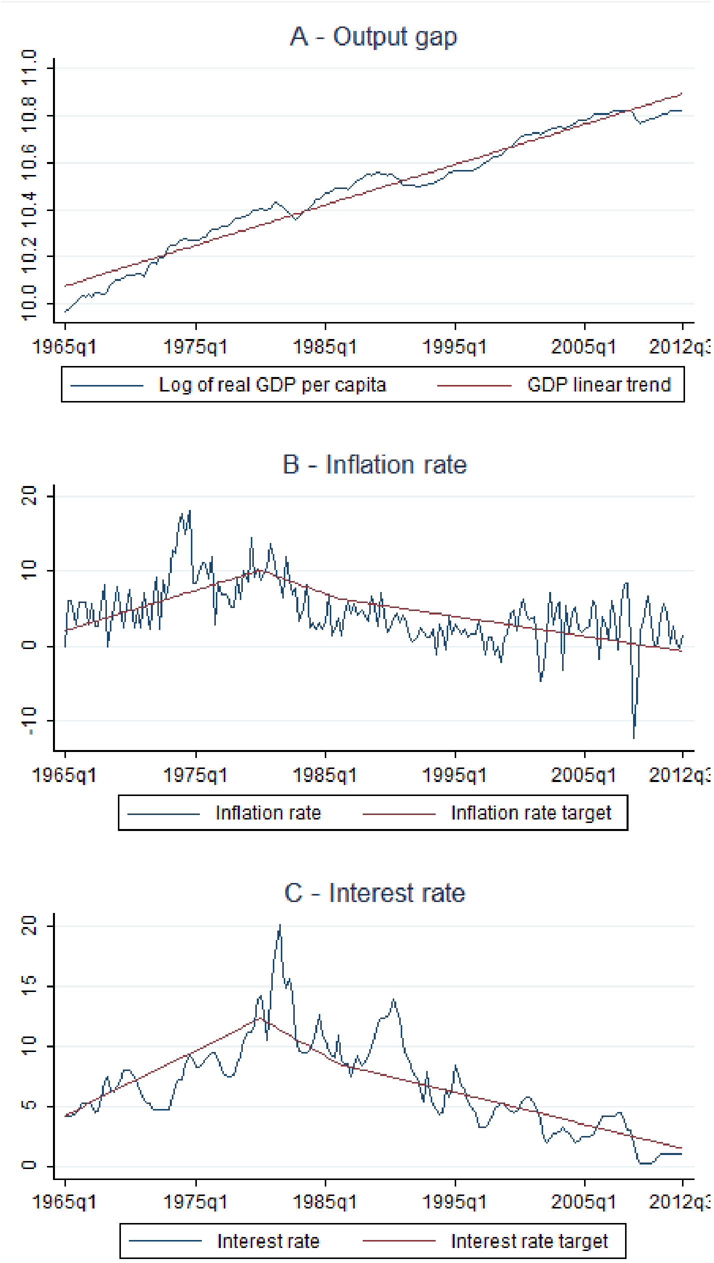

Figure 3.2: United States: Macroeconomic Variables, 1965-2012

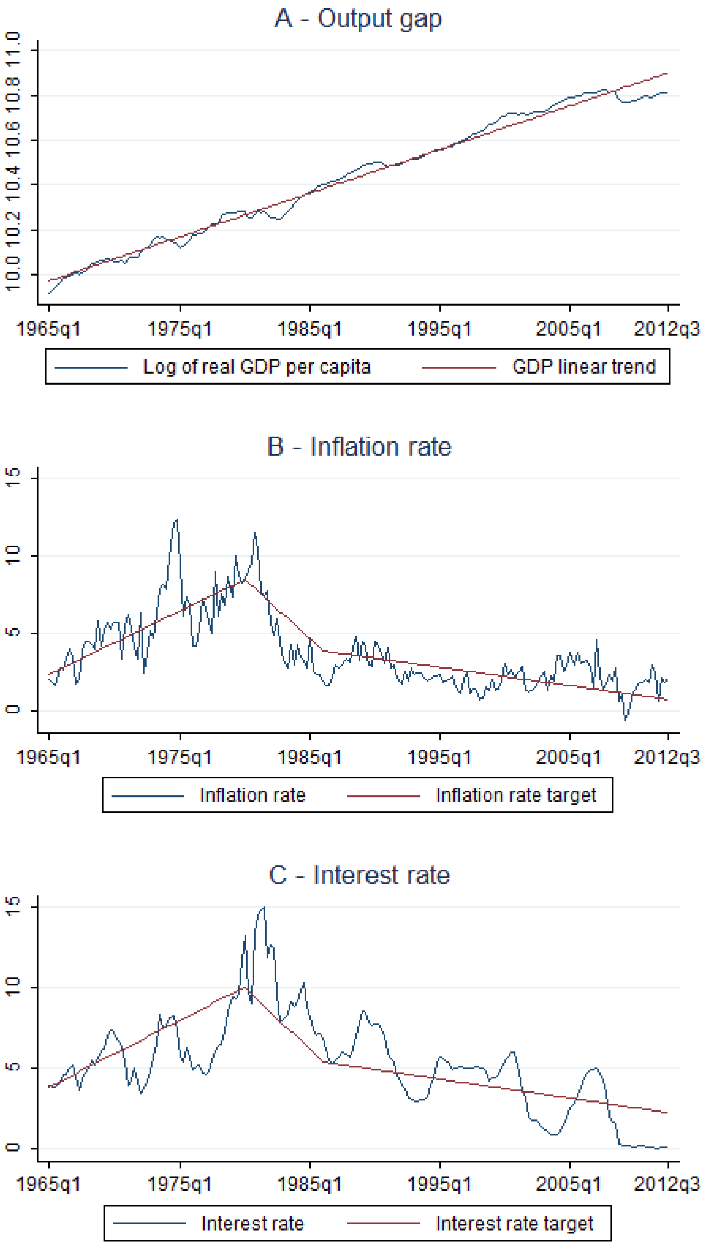

Figure 3.3: United Kingdom: Macroeconomic Variables, 1965-2012

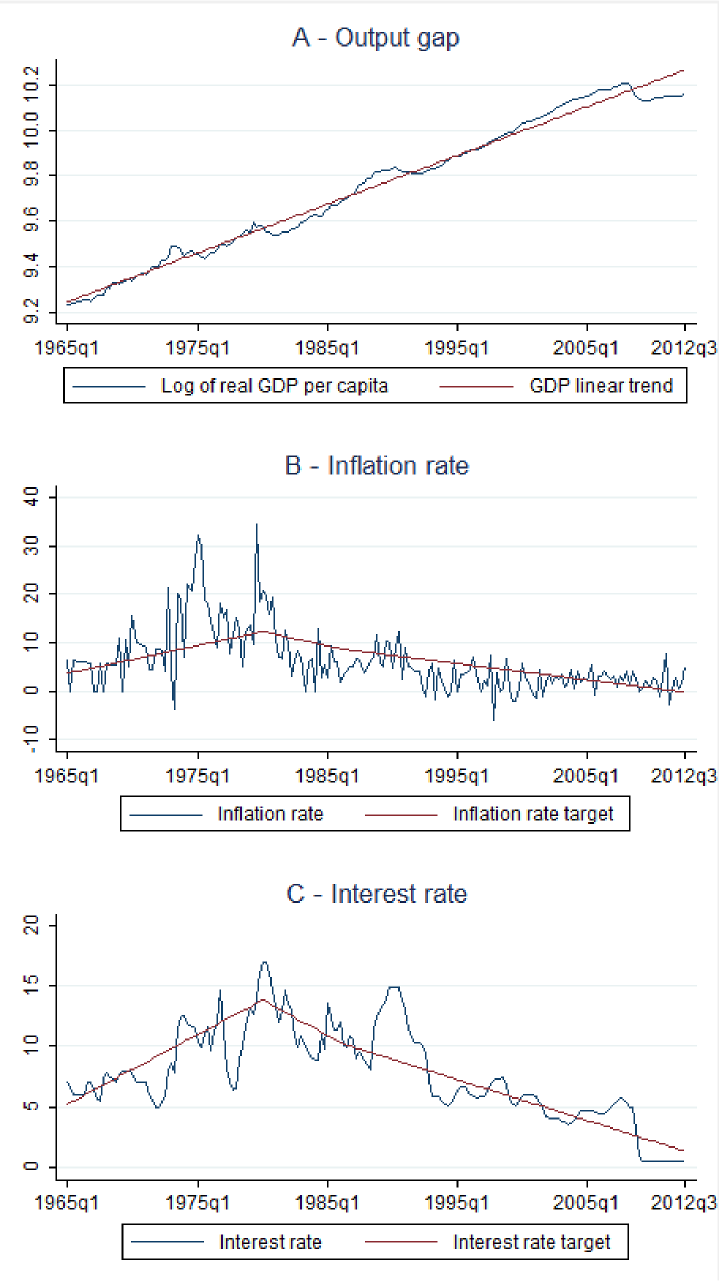

Table 3.6: Optimal discretionary policy rules for varying preference parameters, by country

λ_y	$\lambda_{\Delta i}$	Canada				United States				United Kingdom			
		f_{y1}	f_{y2}	f_π	f_i	f_{y1}	f_{y2}	f_π	f_i	f_{y1}	f_{y2}	f_π	f_i
1.0	0.3	1.086	-0.152	0.058	0.719	0.503	0.048	-0.018	0.645	0.583	0.072	-0.030	0.787
0.0	0.3	0.508	-0.070	0.113	0.765	0.129	0.010	-0.361	0.766	0.271	0.031	-0.057	0.821
0.0	0.5	0.439	-0.061	0.085	0.775	0.104	0.009	-0.270	0.782	0.217	0.025	-0.042	0.833
0.0	1.0	0.364	-0.051	0.057	0.787	0.077	0.006	-0.181	0.803	0.160	0.019	-0.027	0.849
0.0	2.0	0.307	-0.043	0.038	0.797	0.056	0.005	-0.120	0.824	0.117	0.014	-0.018	0.864
0.0	5.0	0.251	-0.036	0.023	0.806	0.036	0.003	-0.068	0.851	0.076	0.009	-0.010	0.883
0.0	10.0	0.223	-0.032	0.016	0.810	0.025	0.002	-0.043	0.871	0.054	0.006	-0.006	0.897
0.5	10.0	0.257	-0.037	0.013	0.811	0.049	0.004	-0.016	0.849	0.075	0.009	-0.005	0.891
1.0	10.0	0.292	-0.042	0.012	0.808	0.069	0.006	0.003	0.832	0.093	0.011	-0.005	0.886
2.0	10.0	0.352	-0.050	0.010	0.800	0.101	0.010	0.031	0.807	0.125	0.015	-0.004	0.876

Notes: The above reports the coefficients of optimal rules for a discretionary policymaker given the following model: $y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)(ay_{t-1} + (1-a)y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$, and using the point estimates shown in Table 1, i.e., derived using the method described in Chapter 1. The optimal rules are of the form $i_t = f_{y1}y_{t-1} + f_{y2}y_{t-2} + f_\pi \pi_{t-1} + f_i i_{t-1}$ and are reported for varying values of λ_y and $\lambda_{\Delta i}$, respectively the preferences for output gap stabilization and smoothing the interest rate. It is assumed throughout that the preference for inflation management is set at $\lambda_\pi = 1$.

Table 3.7: FOC-based estimates for Canada, varying starting values, 1968:1,2006:4

Param.	λ		a		b		λ_y		$\lambda_{\Delta i}$	
	-	+	-	+	-	+	-	+	-	+
λ	0.091	0.436	0.018	0.315	0.093	0.434	0.084	0.170	0.443	0.071
	(15.9)	(0.10)	(131.3)	(1.2)	(18.1)	(0.11)	(21.6)	(6.8)	(0.07)	(24.8)
a	1.29	1.21	1.29	1.27	1.30	1.20	1.29	1.28	1.19	1.29
	(0.14)	(0.07)	(0.19)	(0.10)	(0.19)	(0.08)	(0.18)	(0.15)	(0.06)	(0.16)
b	0.177	0.012	0.216	0.073	0.185	0.017	0.186	0.142	0.012	0.189
	(3.8)	(1.7)	(5.2)	(2.7)	(4.5)	(1.7)	(4.7)	(3.9)	(1.2)	(4.4)
α	0.651	0.675	0.649	0.665	0.649	0.678	0.651	0.652	0.677	0.651
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
κ	0.003	0.006	0.003	0.005	0.004	0.008	0.004	0.003	0.010	0.004
	(2.4)	(1.1)	(2.2)	(1.5)	(2.0)	(0.97)	(2.1)	(2.2)	(0.79)	(2.1)
λ_y	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(6.6)	(1.1e4)	(104.3)	(26.0)	(2.7e3)	(2.2e3)	(494.0)	(33.0)	(9.3e4)	(272.4)
$\lambda_{\Delta i}$	0.078	12.9	0.075	0.282	0.090	8.61	0.084	0.094	21.9	0.081
	(3.9)	(7.3)	(4.1)	(3.5)	(3.8)	(6.7)	(4.1)	(4.1)	(6.1)	(3.9)
Q	4.6e-2	3.0e-2	4.3e-2	4.6e-2	4.4e-2	3.2e-2	4.3e-2	4.4e-2	2.9e-2	4.3e-2
$Q \times T$	7.08	4.59	6.69	7.08	6.86	4.97	6.71	6.83	4.47	6.69
$Q \times T$; p=	0.029	0.101	0.035	0.029	0.032	0.083	0.035	0.033	0.107	0.035
$\mathscr{L}$	-92.83	-90.76	-92.93	-92.99	-93.25	-90.84	-93.06	-92.97	-90.12	-92.84

Notes: The above reports the estimated parameters for the following mdoel: $y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)(ay_{t-1} + (1-a)y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$, derived using the method described in Chapter 2 with the FOC imposed. Estimates are provided for differing starting values of the parameter indicated at the top of each column, where "-" indicates that a decrease of 10% in the starting value relative to the baseline specification, and "+" an increase of 10%. The baseline starting values are given the vector: $\rho^s = \left[\lambda^s, a^s, b^s, \alpha^s, \kappa^s, \lambda_y^s, \lambda_{\Delta i}^s\right] = \left[0.05, 0.05, 0.05, 0.37, 0.05, 0.05, 0.05\right]$.

Table 3.8: FOC-based estimates for the United States, varying starting values, 1968:1,2006:4

Param.	λ		a		b		λ_y		$\lambda_{\Delta i}$	
	-	+	-	+	-	+	-	+	-	+
λ	0.413	0.420	0.409	0.407	0.404	0.395	0.412	0.406	0.402	0.422
	(0.11)	(0.11)	(0.14)	(0.15)	(0.14)	(0.18)	(0.13)	(0.14)	(0.17)	(0.10)
a	1.25	1.25	1.26	1.26	1.26	1.26	1.26	1.26	1.26	1.26
	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
b	0.013	0.010	0.014	0.015	0.016	0.019	0.013	0.016	0.015	0.008
	(1.3)	(2.0)	(1.8)	(1.7)	(1.4)	(1.4)	(1.9)	(1.4)	(1.8)	(2.1)
α	0.552	0.554	0.550	0.552	0.549	0.546	0.551	0.551	0.546	0.555
	(0.08)	(0.08)	(0.09)	(0.09)	(0.08)	(0.08)	(0.09)	(0.08)	(0.09)	(0.09)
κ	0.011	0.011	0.011	0.012	0.011	0.011	0.011	0.011	0.010	0.011
	(0.55)	(0.59)	(0.56)	(0.56)	(0.57)	(0.58)	(0.57)	(0.56)	(0.61)	(0.57)
λ_y	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.00)	(2.3e3)	(680.8)	(494.2)	(0.00)	(0.00)	(518.4)	(0.00)	(779.6)	(4.5e3)
$\lambda_{\Delta i}$	0.921	1.22	0.884	0.824	0.674	0.432	0.939	0.774	0.582	1.45
	(1.6)	(4.5)	(3.5)	(3.3)	(1.5)	(1.4)	(3.6)	(1.5)	(2.9)	(5.0)
Q	6.2e-2	6.3e-2	6.2e-2	6.3e-2	6.4e-2	6.4e-2	6.2e-2	6.4e-2	6.3e-2	6.2e-2
$Q \times T$	9.68	9.71	9.59	9.79	9.90	9.96	9.55	9.85	9.79	9.58
$Q \times T$; p=	0.008	0.008	0.008	0.007	0.007	0.007	0.008	0.007	0.007	0.008
$\mathscr{L}$	36.91	36.63	36.66	37.01	37.00	37.28	36.65	37.01	36.77	36.25

Notes: The above reports the estimated parameters for the following mdoel: $y_t = \lambda E_{t-1}y_{t+1} + (1-\lambda)(ay_{t-1} + (1-a)y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$, derived using the method described in Chapter 2 with the FOC imposed. Estimates are provided for differing starting values of the parameter indicated at the top of each column, where "-" indicates that a decrease of 10% in the starting value relative to the baseline specification, and "+" an increase of 10%. The baseline starting values are given the vector: $\rho^s = \left[\lambda^s, a^s, b^s, \alpha^s, \kappa^s, \lambda_y^s, \lambda_{\Delta i}^s\right] = \left[0.05, 0.05, 0.05, 0.37, 0.05, 0.05, 0.05\right]$.

Table 3.9: FOC-based estimates for the United Kingdom, varying starting values, 1968:1,2006:4

Param.	λ		a		b		λ_y		$\lambda_{\Delta i}$	
	-	+	-	+	-	+	-	+	-	+
λ	—	0.446	0.449	0.445	0.445	0.426	0.451	0.425	0.442	0.438
	(—)	(0.24)	(0.21)	(0.23)	(0.24)	(0.29)	(0.21)	(0.28)	(0.26)	(0.29)
a	—	1.06	1.07	1.07	1.07	1.08	1.07	1.07	1.06	1.06
	(—)	(0.15)	(0.15)	(0.15)	(0.14)	(0.14)	(0.15)	(0.15)	(0.15)	(0.14)
b	—	0.029	0.024	0.027	0.025	0.033	0.024	0.036	0.029	0.031
	(—)	(1.4)	(1.4)	(1.4)	(1.5)	(1.6)	(1.4)	(1.4)	(1.5)	(1.6)
α	—	0.765	0.755	0.758	0.758	0.759	0.756	0.765	0.761	0.761
	(—)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
κ	—	0.042	0.031	0.033	0.033	0.033	0.032	0.041	0.035	0.034
	(—)	(0.81)	(0.95)	(0.91)	(0.94)	(0.90)	(0.94)	(0.78)	(0.90)	(0.92)
λ_y	—	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(—)	(2.5e4)	(1.9e4)	(2.7e4)	(1.7e4)	(1.4e4)	(1.7e4)	(2.4e4)	(3.3e4)	(3.5e4)
$\lambda_{\Delta i}$	—	10.9	11.5	10.5	10.8	6.16	12.1	7.15	10.1	8.65
	(—)	(8.8)	(9.5)	(9.3)	(9.3)	(8.2)	(9.6)	(8.0)	(9.0)	(8.8)
Q	—	3.9e-2	3.8e-2	3.8e-2	3.8e-2	3.9e-2	3.8e-2	3.9e-2	3.8e-2	3.8e-2
$Q \times T$	—	5.99	5.93	5.97	5.90	6.02	5.92	6.06	5.92	5.94
$Q \times T$; p=	—	0.050	0.051	0.051	0.052	0.049	0.052	0.048	0.052	0.051
$\mathscr{L}$	—	-226.36	-226.44	-226.40	-226.51	-226.64	-226.33	-226.63	-226.31	-226.22

Notes: The above reports the estimated parameters for the following mdoel: $y_t = \lambda E_{t-1} y_{t+1} + (1-\lambda)(a y_{t-1} + (1-a) y_{t-2}) - b(i_{t-1} - E_{t-1}\bar{\pi}_{t+3}) + u_t$, $\pi_t = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \beta y_t + v_t$, derived using the method described in Chapter 2 with the FOC imposed. Estimates are provided for differing starting values of the parameter indicated at the top of each column, where "-" indicates that a decrease of 10% in the starting value relative to the baseline specification, and "+" an increase of 10%. The baseline starting values are given the vector: $\rho^s = \left[\lambda^s, a^s, b^s, \alpha^s, \kappa^s, \lambda_y^s, \lambda_{\Delta i}^s\right] = \left[0.05, 0.05, 0.05, 0.37, 0.05, 0.05, 0.05\right]$. For the United Kingdom, the model could not be stabilized when increasing λ^s by 10%.

Bibliography

Bank of Canada (2011). Monetary Policy. http://www.bankofcanada.ca/core-functions/monetary-policy. Last accessed on July 3, 2017.

Barro, R. J. and Gordon, D. B. (1983). Rules, discretion and reputation in a model of monetary policy. *Journal of monetary economics*, 12(1):101–121.

Belaygorod, A. and Dueker, M. (2005). Discrete monetary policy changes and changing inflation targets in estimated Dynamic Stochasic General Equilibrium models. *Federal Reserve Bank of St. Louis Review*, 87(6):719–733.

Brainard, W. C. (1967). Uncertainty and the effectiveness of policy. *American Economic Review*, 57(2):411–425.

Brock, W. A., Durlauf, S. N., and West, K. D. (2003). Policy evaluation in uncertain economic environments. Technical report, National Bureau of Economic Research.

Brock, W. A., Durlauf, S. N., and West, K. D. (2007). Model uncertainty and policy evaluation: Some theory and empirics. *Journal of Econometrics*, 136(2):629–664.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Castelnuovo, E. et al. (2006). The Feds preference for policy rate smoothing: Overestimation due to misspecification? *Topics in Macroeconomics*, 6(2):1416.

Castelnuovo, E. and Surico, P. (2004). Model uncertainty, optimal monetary policy and the preferences of the Fed. *Scottish Journal of Political Economy*, 51(1):105–126.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45.

Clarida, R., Galı, J., and Gertler, M. (1998). Monetary policy rules in practice: Some international evidence. *European Economic Review*, 42(6):1033–1067.

Clarida, R., Galí, J., and Gertler, M. (1999a). The science of monetary policy: A new Keynesian perspective. *Journal of Economic Literature*, 37(4):1661.

Clarida, R., Gali, J., and Gertler, M. (1999b). The science of monetary policy: a new keynesian perspective. *Journal of economic literature*, 37(4):1661–1707.

Cogley, T., De Paoli, B., Matthes, C., Nikolov, K., and Yates, T. (2011). A Bayesian approach to optimal monetary policy with parameter and model uncertainty. *Journal of Economic Dynamics and Control*, 35(12):2186–2212.

Cogley, T. and Sargent, T. J. (2005). The conquest of US inflation: Learning and robustness to model uncertainty. *Review of Economic dynamics*, 8(2):528–563.

Dennis, R. (2006). The policy preferences of the US Federal Reserve. *Journal of Applied Econometrics*, 21(1):55–77.

Dufour, J.-M., Khalaf, L., and Kichian, M. (2006). Inflation dynamics and the New Keynesian Phillips curve: an identification robust econometric analysis. *Journal of Economic Dynamics and Control*, 30(9):1707–1727.

Favero, C. A. and Rovelli, R. (2003). Macroeconomic stability and the preferences of the Fed: A formal analysis, 1961-98. *Journal of Money, Credit, and Banking*, 35(4):545–556.

Federal Reserve (2016). Purposes & Functions. https://www.federalreserve.gov/aboutthefed/pf.htm. Last accessed on July 3, 2017.

Fuhrer, J. and Moore, G. (1995). Inflation persistence. *Quarterly Journal of Economics*, 110(1):127–159.

Fuhrer, J. C. (2000). Habit formation in consumption and its implications for monetary-policy models. *American Economic Review*, pages 367–390.

Fuhrer, J. C. and Rudebusch, G. D. (2004). Estimating the Euler equation for output. *Journal of Monetary Economics*, 51(6):1133–1153.

Galí, J. and Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics*, 44(2):195–222.

Galí, J., Gertler, M., and Lopez-Salido, J. D. (2001). European inflation dynamics. *European Economic Review*, 45(7):1237–1270.

Giannoni, M. and Woodford, M. (2004). Optimal inflation-targeting rules. In *The Inflation-Targeting Debate*, pages 93–172. University of Chicago Press.

Giannoni, M. P. (2007). Robust optimal monetary policy in a forward-looking model with parameter and shock uncertainty. *Journal of Applied Econometrics*, 22(1):179–213.

Givens, G. E. (2012). Estimating central bank preferences under commitment and discretion. *Journal of Money, Credit and Banking*, 44(6):1033–1061.

Givens, G. E. and Salemi, M. K. (2008). Generalized method of moments and inverse control. *Journal of Economic Dynamics and Control*, 32(10):3113–3147.

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica: Journal of the Econometric Society*, pages 1029–1054.

Hansen, L. P. and Sargent, T. J. (2008). *Robustness*. Princeton University Press.

Hooke, R. and Jeeves, T. A. (1961). "Direct search" solution of numerical and statistical problems. *Journal of the ACM (JACM)*, 8(2):212–229.

Howells, P. (2013). The US Fed and the Bank of England: Ownership, structure and 'independence'. *UWE Economics Working Papers*.

Ilbas, P. (2012). Revealing the preferences of the US Federal Reserve. *Journal of Applied Econometrics*, 27(3):440–473.

Ireland, P. N. (2007). Changes in the Federal Reserve's inflation target: Causes and consequences. *Journal of Money, Credit and Banking*, 39(8):1851–1882.

Jensen, H. (2002). Targeting nominal income growth or inflation? *American Economic Review*, 92(4):928–956.

Judd, J. and Rudebusch, G. D. (1999). The goals of U.S. monetary policy. *FRBSF Economic Letter*, 99-04.

Kozicki, S. and Tinsley, P. A. (2005). Permanent and transitory policy shocks in an empirical macro model with asymmetric information. *Journal of Economic Dynamics and Control*, 29(11):1985–2015.

Kreps, D. M. (1979). A representation theorem for" preference for flexibility". *Econometrica: Journal of the Econometric Society*, pages 565–577.

Leigh, D. (2008). Estimating the Federal Reserve's implicit inflation target: A state space approach. *Journal of Economic Dynamics and Control*, 32(6):2013–2030.

Levin, A. T. and Williams, J. C. (2003). Robust monetary policy with competing reference models. *Journal of Monetary Economics*, 50(5):945–975.

Mavroeidis, S., Plagborg-Møller, M., and Stock, J. H. (2014). Empirical evidence on inflation expectations in the New Keynesian Phillips curve. *Journal of Economic Literature*, 52(1):124–188.

McCallum, B. T. (1988). Robustness properties of a rule for monetary policy. In *Carnegie-Rochester Conference Series on Public Policy*, volume 29, pages 173–203. North-Holland.

McCallum, B. T. and Nelson, E. (2004). Targeting vs. instrument rules for monetary policy. Technical report, National Bureau of Economic Research.

Milani, F. (2017). Learning and the evolution of the Fed's inflation target. Technical report, mimeo, University of California, Irvine.

Onatski, A. and Stock, J. H. (2002). Robust monetary policy under model uncertainty in a small model of the US economy. *Macroeconomic Dynamics*, 6(1):85–110.

Rotemberg, J. J. (1982). Sticky prices in the United States. *Journal of Political Economy*, 90(6):1187–1211.

Rudebusch, G. and Svensson, L. E. (1999). Policy rules for inflation targeting. In *Monetary Policy Rules*, pages 203–262. University of Chicago Press.

Rudebusch, G. D. (2002). Assessing nominal income rules for monetary policy with model and data uncertainty. *The Economic Journal*, 112(479):402–432.

Sack, B. (2000). Does the Fed act gradually? A VAR analysis. *Journal of Monetary Economics*, 46(1):229–256.

Salemi, M. K. (2006). Econometric policy evaluation and inverse control. *Journal of Money, Credit and Banking*, 38(7):1737–1764.

Söderlind, P. (1999). Solution and estimation of RE macromodels with optimal policy. *European Economic Review*, 43(4):813–823.

Söderström, U., Söderlind, P., and Vredin, A. (2005). New-Keynesian models and monetary policy: A re-examination of the stylized facts. *The Scandinavian Journal of Economics*, 107(3):521–546.

Stock, J. H., Wright, J. H., and Yogo, M. (2002). A survey of weak instruments and weak identification in generalized method of moments. *Journal of Business & Economic Statistics*, 20(4):518–529.

Svensson, L. E. (1999). Inflation targeting: some extensions. *The Scandinavian Journal of Economics*, 101(3):337–361.

Taylor, J. B. (1980). Aggregate dynamics and staggered contracts. *Journal of Political Economy*, 88(1):1–23.

Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester Conference Series on Public Policy*, volume 39, pages 195–214. Elsevier.

Thornton, D. L. (2012). The dual mandate: Has the Fed changed its objective? *Federal Reserve Bank of St. Louis Review*, 94(2):117–133.

Walsh, C. E. (2003). Speed limit policies: The output gap and optimal monetary policy. *American Economic Review*, 93(1):265–278.

Woodford, M. (2003). Optimal interest-rate smoothing. *Review of Economic Studies*, 70(4):861–886.

Woodford, M. (2005). Central bank communication and policy effectiveness. Technical report, National Bureau of Economic Research.

Yin, G. (2009). Bayesian generalized method of moments. *Bayesian Analysis*, 4(2):191–207.