



UNIVERSITÉ D'OTTAWA
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STATIC AND DYNAMIC DESIGN OF
LONGITUDINALLY STIFFENED SIMPLY SUPPORTED
ORTHOTROPIC BRIDGE DECKS.

by

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PREFACE

The application of orthotropic plate theory to the design of longitudinally stiffened highway bridge decks has received attention in recent years. This investigation deals with the three aspects of design of such bridges namely static, frequency and forced vibration analysis and design of longitudinally stiffened, simply supported deck systems based on idealization of the bridge deck as a homogeneous orthotropic plate of constant thickness. The study provides new design concepts including geometric and structural optimization which ultimately leads to minimum weight design. For the purpose of rapid analysis of frequencies of vibrations, the theoretical procedure is supplemented by an empirical formulation. Forced vibration analysis of the plate structure is based on equivalent and interconnected beam idealization. A combined programme is developed for the three aspects of design and analysis and the final results are shown as a series of design tables and charts covering practical ranges of deterministic and stochastic parameters of the problem. Theoretical results are supplemented by extensive experimental investigations.

This study was undertaken to investigate the accuracy of orthotropic idealization for static and dynamic design of longitudinally stiffened bridge decks. Both deterministic and stochastic design parameters are investigated in detail. Simplifying assumptions are

introduced which are seen to reduce the complexity of mathematical procedures and making the method more suitable for design purposes. The interaction of design parameters is studied and governing parameters are treated for optimisations of final design.

Important features of this investigation are:

- a) Developing combined programme for deflections, moments, frequencies and amplification factors based on closed form mathematical analysis of orthotropic plate including the effect of Poisson's ratio.
- b) Introduction of new concepts of geometric and structural optimization for economical design in each of the three phases.
- c) Accounting plate action by introducing new concepts of interconnected and equivalent beams and computing amplifications and dynamic deflections under and away from vehicle runs.
- d) Empirical formulation for rapid computations of frequencies of fundamental and higher modes.
- e) Simultaneous treatment of deterministic and stochastic parameters of both bridge and vehicle, and to study their interaction for assessing their relative importance for their accurate determination in design processes.
- f) Presenting results of investigations in the form of design tables and charts covering practical range of parameters for optimum design.

The accuracy of simplifying assumptions on the final results is investigated by extensive experimentation. The important features of the experimental programme include

- a) Accuracy of orthotropic idealization in static, frequency and forced vibration analysis by tests on models.
- b) Accuracy of results by empirical methods.
- c) Accuracy of concept of equivalent and interconnected beam idealization of plate structure in a range of plate to beam transition.
- d) Accuracy of results based on simulated initial conditions of vibrations of vehicle. This is done by comparing deflections and strains at and away from centre (17 model tests), frequencies of vibrations (96 values) and amplification factors under and away from vehicle run (86 tests).

From these theoretical and experimental investigations it is seen that orthotropic flexural rigidities have comparatively more influence on deflections, frequencies and therefore needs careful considerations. Optimum solution based on frequency and mass maximisation gives economical design both under static and dynamic loads. The dynamic amplification factors for these optimum sections are seen to be consistently less for most combinations of design parameters. These optimum sections can be obtained by using geometric relationship between

dimensions of the slab and stiffeners. Empirical method for determining frequencies of vibration gives sufficiently accurate results, and can be used for replacing trial and error solution to transcendental frequency equations. The concept of interconnected beam idealisation considerably simplifies the complexity of forced vibration analysis of two dimensional slab structure and gives results sufficiently accurate for design purposes. The results which are shown as design tables and charts take into consideration practical ranges of both stochastic and deterministic parameters and can be used for direct design and optimisation

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NOTATIONS

a	Span of bridge.
A_m, B_m, C_m, D_m	Constants to be determined from boundary conditions - in static analysis.
$A_{mn}, B_{mn}, C_{mn}, D_{mn}$	Constants to be determined from boundary condition - in frequency analysis.
A_F	Amplification factor.
A_R	Aspect ratio b/a .
A_S	Equivalent area per unit width of deck.
b	Width of bridge deck.
B	Maximum fractional variation in applied load.
c	Rigidity parameter $b/a (D_x/D_y)^{1/4}$.
C_1	Constant in torsional rigidity of stiffener.
D	Rigidity of plate, $Eh^3/(12(1-\nu^2))$.
D_1	Coupling rigidity (νD) of orthotropic plate.
D_x, D_y	Flexural rigidities of orthotropic plate in x and y direction.
D_{xy}	Torsional rigidity of orthotropic plate.
e_x	Depth of centre of gravity of T section from top of slab.
E_x, E_y	Elastic modulus of orthotropic plate in x and y direction.
E	Elastic modulus of slab material.
f_B	Frequency of the equivalent bridge in cycles per second $\sqrt{\frac{K_B}{M_B}}$.
f_C	Crossing frequency $\pi v/a$.
f_R	Frequency ratio f_B/f_v .
f_v	Frequency of the idealised vehicle in cycles per second $\sqrt{\frac{K_v}{M_v}}$.
$F(t)$	Force at time t .

g	Acceleration due to gravity.
G_{xy}	Shear modulus.
h	Thickness of slab; uniform thickness of orthotropic plate.
H	Height of T section.
H_1	Torsional rigidity ($D_{1+2} D_{xy}$).
I	Moment of inertia of repeating T section about common neutral axis.
K	$\frac{P_a}{2\pi^2 \sqrt{D_x D_y}}$
K_1	Variable in empirical equations.
K_B	Spring constant of the equivalent bridge system.
K_D	Coefficient of deflection of optimum sections.
K_F	Coefficient of frequencies of optimum sections.
K_M	Coefficient of mass ratio of optimum sections.
K_V	Spring constant of the idealised vehicle
m	Number of terms in series solution.
	Number of half sine waves in x direction
M_B	Total mass of the equivalent bridge system.
M_R	Mass ratio M_V/M_B .
M_V	Total mass of vehicle.
M_x, M_y, M_{xy}	Moments (P_a^2/ab), per unit width of element.
n	Number of half sine waves in y direction.
p	term $-\{(\alpha/\beta)(A_m+1) + (\beta/\alpha)(B_m+1)\}$.
$p(x,y)$	External loads on element.
P	Idealised vehicle load at centre of plate.

q	$(A_m - B_m):$
Q_x, Q_y	Shear per unit width of element.
r	$\{(\alpha/\beta)C_m + (\beta/\alpha)D_m\}.$
R	Frequency ratio $\omega_{mn}/(\frac{m^2\pi^2}{a^2} \sqrt{\frac{D_x}{P_0}}).$
R_1	$H_1/D_y.$
R_2	$D_x/D_y.$
S	$C_m + D_m$
s	c/c spacing of stiffeners.
t	Time.
t	Thickness of stiffeners.
v	Speed of vehicle.
w_{xy}	Deflection function of plate.
w_1	Deflection function of strip of span a .
w_2	Deflection function of plate.
w_c	Deflection of bridge, under vehicle at centre of bridge.
w_c	Deflection $(Pa^4/abD_x) \times 10^{-1}$ at centre.
w_{cz}	Deflection at centre of z th beam.
w_{st}	Deflection under static load $(M_v g/K_B).$
w_{vm}	Initial distortion of vehicle spring.
w_v	Deflection of vehicle mass after time t .
w_z	Deflection at z th beam any time t .
x	System of axis.
y	System of axis.
$y_{vm}(y)$	Function of y in free vibration analysis.
z_1	$-\{(\alpha/\beta) - (\beta/\alpha) - (D_1/D_y) \cdot (\frac{\pi}{a})^2\}.$

z_2	$\{(\alpha/\beta) - (\beta/\alpha) - (\frac{H_1 + 2D_{xy}}{D_y})(\frac{\pi}{a})^2 \cdot \alpha\beta\}$
α	$a \frac{(D_y/D_x)^{1/4}}{\pi} \cdot (\frac{2}{1-u})^{1/2}$
α_s	$h/H.$
β	$a \frac{(D_y/D_x)^{1/4}}{\pi} (\frac{2}{1+u})^{1/2}.$
β_s	$t/s.$
γ_{xy}	Shearing strain.
δ_c	Deflection of bridge under load P at center.
Δv	Distortion of vehicle spring.
ϵ_x, ϵ_y	Unit elongation in x and y.
ξ_s	ratio $\frac{t}{H}.$
θ	Variable in empirical equations.
μ	Torsion parameter $\frac{H_1}{(D_x D_y)^{1/2}}.$
ν_x, ν_y	Poisson's ratios of orthotropic plate in x and y.
ν	Poisson's ratio of isotropic plate.
ρ	Density per unit volume of material of slab and stiffener.
ρ_o	Mass density per unit area of orthotropic plate.
σ_x, σ_y	Components of normal stress in x and y.
$\tau_{yx}, \tau_{xy}, \tau_{yz}, \tau_{xz}$	Components of tangential and shearing stress.
ψ_{mn}	Frequency coefficients.
ϕ_{mn}	Frequency coefficients.
ϕ	Phase angle in initial condition of vehicle.
ω_{mn}	Circular frequency of bridge in cycles per second.

CHAPTER 1

INTRODUCTION

1.1.0 Object and Scope

The object of this study is to investigate the static, free and forced vibration design and analysis of simple span, longitudinally stiffened highway bridge decks using the theory of orthotropic plate formulation. The investigation also provides new design concepts including geometric and structural optimization which ultimately leads to minimum weight design. For the purpose of rapid analysis of frequencies of vibration, the theoretical procedure is supplemented by empirical formulation and finally a forced vibration analysis corresponding to two dimensional plate behaviour is provided using the concept of interconnected beam idealization. A combined programme is developed for the complete three phase design analysis and final results are shown in a series of design tables and charts covering a practical range of parameters. Also, theoretical results are supplemented by extensive experimental investigations and good agreements are found.

The static analysis of a bridge deck commences with the determination of geometric dimensions namely those of slab and stiffeners to satisfy the requirements of deflections and moments under standard loading conditions, while dynamic analysis leads to natural frequencies of vibrations, mode shapes and finally amplification factors under forced vibrations caused by passing vehicles. A

parametric study of the problem reveals that the results of static and free vibration analysis can serve advantageously as an input to forced vibration analysis which involves interaction of both vehicle and plate parameters.

1.2.0 Literature Survey

Some research results dealing with the static and dynamic behaviour of stiffened simply supported orthotropic bridge decks or similar structures are available. Inglis (28) analyzed a prismatic beam subjected to rolling mass and pulsating loads at constant speeds and he introduced the concept of impact factors. Biggs and others (4,6,16,35,50 and 52) investigated the dynamic behaviour of bridges idealized as beams and conducted experiments on some beam models. He concluded that vertical oscillation of vehicles is a major factor of impact on such bridges. Valetsos and others (53,54) studied the distribution of loads to girders in slab and girder bridges and verified the theoretical results by field tests. More recently, Valetsos and Huang (51) reported a general method of treatment of the dynamic response of continuous and cantilever bridges idealized as simple beams. Walker (50) considered a more realistic formulation to represent the moving load and studied the effects of such formulation on bridges idealized as simple beams. Csagoly (10) studied the variation of frequency of vibration with dimensions of longitudinal stiffeners. Richardson (43,40) stressed the need for further research and verification by field tests from design considerations.

Plate action was taken into consideration by Valetsos and Yamada (49), who obtained frequencies of vibration by treating the deck as i) a plate continuous over the beam and ii) an orthotropic plate. They neglected the effect of Poisson's ratio and no experimental results were reported to compare with those obtained analytically. Iyenger and others (28,29,30) also studied the free vibration of orthotropic plates neglecting the effect of Poisson's ratio and suggested modifications. Heins and Looney (20) presented the static analysis of orthotropic plates using finite difference techniques. No experimental work was reported. Most of the work reported on forced vibration analysis is in the form of numerical studies (23,24,25,26,27,49,50,52), laboratory tests (28,29,30,36) and field tests (14,42,54). Timoshenko and Krieger (47) noted that the formulation for determining rigidity parameters of equivalent orthotropic plates can be based on the distribution of stiffnesses of longitudinal beams and such parameters should be verified by model tests. They observed that lateral contraction of such plates has negligible effects on the deflections. Troitsky (48) supplemented these suggestions and discussed the methods of determining the rigidity parameters of orthotropic plates. Huffington (19) presented a theoretical analysis for the determination of the rigidity parameters of symmetrically stiffened orthotropic plates and suggested modifications for extending the analysis to other boundary conditions. Hoppmann and others (17,18) studied the bending of orthogonally stiffened plates. No experimental work is reported in these works. As part of the present work, Ng and Kulkarni (38,39,40) provided empirical formulation for determining frequencies of

orthotropic and isotropic plates and obtained maximum amplification factors under critical combinations of three bridge initial conditions.

1.3.0 Limitations and Scope

All research work cited above have their own boundaries of limitations. Each research work emphasized a few aspects of the total problem of static and dynamic design to the exclusion of others and some have dealt with only one phase of the three-phase analysis required in design. For example, the distribution of amplification factors at points away from the vehicle received very little attention. The bridge deck which is stiffened by longitudinal beams is assumed equivalent to an homogeneous orthotropic plate of constant thickness. The validity of equating the equivalence in deflections, strains and frequencies of an actual stiffened bridge deck with an orthotropic plate of constant thickness needs careful consideration. While the deflections under static loads are inversely proportional to the rigidities of orthotropic plates the frequencies of vibration are directly proportional to rigidities of an equivalent orthotropic plate. This opposite degree of dependency of deflections and frequencies on stiffness parameters of orthotropic plate may not give the same accuracy in results in the static and dynamic analysis. Thus the formulation giving reasonably accurate results in static analysis may not give the same accuracy in frequency analysis. This is of particular importance as no exact theoretical method is suitable for calculating the orthotropic stiffness parameters of plates stiffened only from one side and therefore reliance of obtaining such parameters

must be based on model tests. Further, as the three-phase design and analysis refer to the same plate structure, experimental verification needs to be performed on the same model. Equivalent beam idealization in forced vibration analysis may give close representation of the actual behaviour when the aspect ratio of the plate (width/span) is small and therefore, plate action is relatively insignificant. For relatively large aspect ratios, the accuracy of the equivalent beam idealization of the plate is expected to deteriorate and therefore, experimental verification by tests on models designed to cover plate to beam transition becomes important. In many of the investigations, the bridge was assumed at rest before a vehicle entered the span. This may lead to underestimation of the amplification factors and therefore in order to evaluate the maximum amplification factors, careful consideration must be given to the initial conditions of the bridge. The sequence of design is usually to determine the dimensions of slab and stiffeners which would satisfy the deflection and moment requirement under static load. The adequacy of the section thus finalized is then investigated by performing frequency and forced vibration analysis. However, once the section is finalized to meet the requirements under static loads, resulting constant values of frequencies and mass parameters which enter into dynamic analysis, do not lead to any alternative to improve the resistance of the section for achieving smaller amplification factors. To achieve this, simultaneous treatment of common parameters involved in the three aspects of design and analysis is necessary. Such simultaneous treatment of parameters may result in sections of maximum structural efficiency in

each of the three design phases of the plate. These points appear to have received little attention in previous investigations.

1.4.0 Present Study

Present investigation deals with

- 1) Simultaneous treatment of parameters involved in three-phase design and analysis, and developing a combined programme suitable for design purposes.
 - 2) the accounting of deterministic and stochastic parameters and their relative importance from design considerations.
 - 3) the study of interaction of parameters.
 - 4) Determination of sections with maximum structural efficiency in static and dynamic behaviour - optimum design.
 - 5) Presentation of empirical formulation for rapid analysis of frequencies of vibration.
 - 6) Introduction of interconnected beam concept which considerably reduces the complexity of two dimensional forced vibration analysis of plates and comparing the accuracy of distribution of amplification factors thus obtained, with experimental results.
- Theoretical analysis includes, as basis, the critical combinations of bridge initial conditions to give limiting values of amplification in individual cases.
- 7) Presentation of ready reference design charts and tables for deflections, frequencies and amplifications covering a wide range of practical cases.

8) The experimental programme includes

a) Comparing accuracy of orthotropic formulation by static and dynamic tests on models having identical boundary conditions and comparing deflections, strains and frequencies.

b) Comparing accuracy of empirical formulation by comparing values with theoretical and experimental frequencies.

c) Comparing concepts of equivalent and interconnected beam idealization by dynamic tests on models covering a wide range of plate beam transition. This is achieved by comparison of the distribution of amplification factors under and away from the vehicle.

The investigation is presented in the following order:

Theoretical Analysis - Chapter 2

Interaction of Parameters - Chapter 3

Optimum Design - Chapter 4

Experimental Programme - Chapter 5

Comparisons and Discussions - Chapter 6

Conclusions - Chapter 7

References - Appendix I

Programme - Appendix II

Figures - Appendix III

Tables - Appendix IV

Tests - Results - Appendix V

CHAPTER 2

THEORETICAL ANALYSIS

2.1.0 Introduction

This Chapter deals with the static, frequency and forced vibration analysis of the bridge deck comprising deck slab and symmetrically located longitudinal stiffeners located on one side of the slab. The important assumption introduced is that the slab-stiffener system can be represented by an equivalent orthotropic plate of constant thickness having elastic properties coinciding with the axes of the system. The analysis is presented in the following stages:

- a) Derivation of the governing differential equation based on equilibrium, moment, shears and rigidities.
- b) Three phase analysis of orthotropic plate and introduction of the concept of interconnected beam idealization of the plate for obtaining distribution of amplification factors.
- c) Computation of orthotropic rigidity parameters of a typical section under consideration.
- d) Combined programme.

2.1.1 Equilibrium of an Orthotropic Plate Element

2.1.2 Assumptions Following original treatment by Timoshenko (47) -

- 1) The deflection of the orthotropic plate is small and does not exceed one-fourth of the plate thickness.
- 2) The plate material is perfectly elastic, continuous,

homogeneous, obeys Hooke's law and possesses different elastic properties in orthogonal directions.

3) Thickness of plate is small as compared to the dimensions of the plate.

4) Body forces of the plate material can be neglected in the derivation of the governing differential equations.

5) There is no deformation in the middle plane of the plate, i.e., the middle plane of the plates remains unstretched during bending.

6) Linear elements remain perpendicular to the middle plane of the plate before bending remains straight and normal to the deflection surface of the plate after bending.

7) The normal stresses transverse to the plane of the plate are neglected.

2.1.5 Stress Distribution

The distribution of normal, tangential and shearing stresses (i.e., σ_x , σ_y , τ_{yx} , τ_{xy} , τ_{yz} and τ_{xz}) acting on an element of dimensions dx and dy of the orthotropic plate of constant thickness h is shown in Fig. 1. The plane Oxy is taken as the middle plane of the element and as such coincides with the middle plane of the system.

The internal stresses vary throughout the plate. At any point dx and dy from the origin, the increments are $\frac{\partial \sigma}{\partial x} dx$ and $\frac{\partial \sigma}{\partial y} dy$. The stress distribution and resulting moments and shears are shown in Tables 1 and 2, and Fig. 2. These moments and shears acting on the element are in equilibrium with the external loading $p(x,y)$ acting on the element.

2.1.4 Equations of Equilibrium

Projecting all the forces acting on the element onto the z axis and neglecting quantities of higher order (48) :

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = -p(x,y) \quad (1)$$

Taking moments of all forces with respect to the x and the y axis and neglecting quantities of higher order

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \quad (2)$$

and

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = 0 \quad (3)$$

Differentiating Eq. 2 and Eq. 3 with respect to y and x and using Eq. 1,

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -p(x,y) \quad (4)$$

2.1.5 Orthotropic Elastic Constants

If the element in Fig. 1 having its sides parallel to the orthotropic axis be subjected to the action of σ_x , σ_y and τ_{xy} uniformly distributed over the sides the resultant components of deformations ϵ_x , ϵ_y and γ_{xy} are given by (48) :

$$\begin{aligned} \epsilon_x &= \frac{\sigma_x}{E_x} - \nu_y \frac{\sigma_y}{E_y} \\ \epsilon_y &= \frac{\sigma_y}{E_y} - \nu_x \frac{\sigma_x}{E_x} \end{aligned} \quad \text{and} \quad \gamma_{xy} = \frac{\tau_{xy}}{\sigma_{xy}} \quad (5)$$

where E_x, E_y - ν_x, ν_y rigidity modulus and Poisson's ratios in x and y direction, and G_{xy} - shear modulus.

Using Bett's reciprocal theorem,

$$\nu_x E_y = \nu_y E_x \quad (6)$$

Expressing deflections of the element at a distance z from neutral surface as $w(x,y)$, unit strains ϵ_x, ϵ_y and γ_{xy} can be written i.e.

$$\begin{aligned} \epsilon_x &= -z \frac{\partial^2 w}{\partial x^2} & \text{and } \gamma_{xy} &= -2z \frac{\partial^2 w}{\partial x \partial y} \\ \epsilon_y &= -z \frac{\partial^2 w}{\partial y^2} \end{aligned} \quad (7)$$

2.1.6 Moments, shears, Orthotropic Flexural, Torsional Rigidities and Differential Equation of Equilibrium of Bent Orthotropic Plate

Eliminating ϵ_x, ϵ_y and γ_{xy} from Eq. 5 and Eq. 7, and using the resulting expressions for σ_x, σ_y and τ_{xy} the moments and shears can be obtained as

$$\begin{aligned} M_x &= -D_x \left(\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right) \\ M_y &= -D_y \left(\frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2} \right) \\ M_{xy} &= -2 D_{xy} \frac{\partial^2 w}{\partial x \partial y} \\ Q_x &= -\frac{\partial}{\partial x} \left(D_x \frac{\partial^2 w}{\partial x^2} + H_1 \frac{\partial^2 w}{\partial y^2} \right) \\ Q_y &= -\frac{\partial}{\partial y} \left(D_y \frac{\partial^2 w}{\partial y^2} + H_1 \frac{\partial^2 w}{\partial x^2} \right) \end{aligned} \quad (8)$$

in which

$$D_x = \frac{E_x h^3}{12(1-\nu_x \nu_y)} \quad \text{Flexural rigidity of orthotropic plate in x direction}$$

$$D_y = \frac{E_y h^3}{12(1-\nu_x \nu_y)} \quad \text{Flexural rigidity of orthotropic plate in y direction}$$

$$D_{xy} = G_{xy} \frac{h^3}{G} \quad \text{Torsional rigidity of plate} \quad (9)$$

$$2H_1 = D_{xy} \nu_y + \nu_x D_y + 4D_{xy}$$

and from Eq. 6. $D_{xy} \nu_y = D_y \nu_x = D_1$

Using Eq. 8 in Eq. 4

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H_1 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x,y) \quad (10)$$

which is the differential equation of equilibrium for the bending of orthotropic plates.

2.2.0 A. Static Analysis

2.2.1 Dimensions and System of Axis

A typical bridge deck of width b and span a with symmetrically placed beams at c/c spacing s is simply supported at $x = 0$ and $x = a$, and subjected to point load P at ξ and η (Figs. 3, 4 and 5). The system of axis is so chosen that at $x = 0$ the free edges are located at $y = \eta_1$ and $(-\eta_2)$. A complete monolithic action between slab and stiffeners is assumed.

2.2.2 Singular Solutions for Bent Orthotropic Strip and Plate

The solution for deflection, moments under the load and at any point on the slab is obtained by solving the governing differential equation of equilibrium Eq. 10. It should also satisfy the simply supported and free boundary conditions of the system. The closed form solution is obtained by superimposing the singular solution for the bending of an infinitely long simply supported orthotropic strip of span a having deflection function w_1 on that of the finite orthotropic plate of dimensions b and a and having deflection w_2 so that,

$$w = w_1 + w_2 \quad (11)$$

The function w_1 is taken in the form

$$w_1 = \sum_{m=1}^{\infty} y_m(y) \sin m\pi \frac{x}{a} \quad (12)$$

in which $y_m(y)$ is a function of y only and w_1 satisfies simply supported boundary conditions

$$\begin{aligned} w_1 &= 0 & \text{at } x &= 0 \\ \frac{\partial^2 w_1}{\partial x^2} &= 0 & \text{and } x &= a \end{aligned} \quad (13)$$

Using Eq. 12 in Eq. 10

$$\frac{D_x}{D_y} \left(\frac{m\pi}{a}\right)^4 y_m(y) - \frac{2H_1}{D_y} \left(\frac{m\pi}{a}\right)^2 y_m^{II}(y) + y_m^{IV}(y) \quad (14)$$

the roots of Eq. 14 are

$$\pm \frac{m\pi}{a} \{R_1 \pm (R_1^2 - R_2)^{1/2}\}^{1/2} \quad (15)$$

in which

$$R_1 = \frac{H_1}{D_y} \quad \text{and} \quad \mu = \frac{H_1}{(D_x D_y)^{1/2}} \quad (16)$$

$$R_2 = \frac{D_x}{D_y}$$

The roots given by Eq. 15 are

- i) real and unequal, i.e., $R_1 > R_2$, $\mu > 1$
 - ii) real and equal, i.e., $R_1 = R_2$, $\mu = 1$
 - iii) imaginary, i.e., $R_1 < R_2$, $\mu < 1$
- (16a)

As most of the flexible beam and slab type bridges correspond to case (iii) of Eq. 16a, solutions are only obtained for this case.

The deflection functions w_1 and w_2 corresponding to case (iii) and carrying load P at ξ (Fig. 3) are taken in the form

$$w_1 = K \sum_{m=1}^{\infty} \frac{1}{m^3} \left(\alpha \sin \frac{m\eta}{\alpha} + \beta \cos \frac{m\eta}{\alpha} \right) e^{-\frac{m\eta}{\beta}} \sin \frac{m\pi\xi}{a} \sin \frac{m\pi x}{a} \quad (17a)$$

and

$$w_2 = K \sum_{m=1}^{\infty} \frac{1}{m^3} \left[\left\{ \alpha A_m \sin \frac{m\eta}{\alpha} + \beta B_m \cos \frac{m\eta}{\alpha} \right\} e^{-\frac{m\eta}{\beta}} \right. \\ \left. + \left\{ \alpha C_m \sin \frac{m\eta}{\alpha} + \beta D_m \cos \frac{m\eta}{\alpha} \right\} e^{\frac{m\eta}{\beta}} \right] \sin \frac{m\pi\xi}{a} \sin \frac{m\pi x}{a} \quad (17b)$$

in which

A_m, B_m, C_m, D_m - constant to be determined from boundary conditions for each load position

$$\alpha = \frac{a(D_v/D_x)^{1/4}}{\pi} \left(\frac{2}{1-u}\right)^{1/2}$$

$$\beta = \frac{a(D_v/D_x)^{1/4}}{\pi} \left(\frac{2}{1+u}\right)^{1/2} \quad (18)$$

and

$$\kappa = \frac{Pa}{2\pi^2 \sqrt{D_x D_y}}$$

2.2.3 Boundary Conditions

The deflection functions w_1 and w_2 have to satisfy simultaneously the boundary conditions along the free edges, i.e., $y = \eta_1$ and $y = \eta_2$ (Fig. 4). In other words, the algebraic sum of moments M_y and shears Q_y obtained from w_1 and w_2 must be equal to zero, i.e.,

$$M_y \left| \begin{array}{c} w_2 @ \\ y = \eta_1 \\ y = \eta_2 \end{array} \right| = - M_y \left| \begin{array}{c} w_1 @ \\ y = \eta_1 \\ y = \eta_2 \end{array} \right|$$

and

$$Q_y \left| \begin{array}{c} w_2 @ \\ y = \eta_1 \\ y = \eta_2 \end{array} \right| = - Q_y \left| \begin{array}{c} w_1 @ \\ y = \eta_1 \\ y = \eta_2 \end{array} \right| \quad (19)$$

From Eqs. 8 and 9, Eq. 19 can be written as

$$\left(\frac{\partial^2 w_2}{\partial y^2} + \frac{D_1}{D_y} \frac{\partial^2 w_2}{\partial x^2} \right) = - \left(\frac{\partial^2 w_1}{\partial y^2} + \frac{D_1}{D_y} \frac{\partial^2 w_1}{\partial x^2} \right)$$

and

(20)

$$\frac{\partial}{\partial y} \left\{ \frac{\partial^2 w_2}{\partial y^2} + \frac{(H_1 + 2D_{xy})}{D_y} \frac{\partial^2 w_2}{\partial x^2} \right\} = - \frac{\partial}{\partial y} \left\{ \frac{\partial^2 w_1}{\partial y^2} + \frac{(H_1 + 2D_{xy})}{D_y} \frac{\partial^2 w_1}{\partial x^2} \right\}$$

Using Eqs. 17(a) and 17(b), the second order derivatives in Eq. 20 are derived in Table 3. Each of Eq. 20 is now rewritten, i.e.

$$\begin{aligned} & K \sum_{m=1}^{\infty} \frac{1}{m} \left[\left[\frac{A_m}{\beta} \{ Z_1 \sin \frac{mv}{\alpha} - 2 \cos \frac{mv}{\alpha} \} e^{-\frac{mv}{\beta}} + \frac{B_m}{\alpha} \{ Z_1 \cos \frac{mv}{\alpha} + 2 \sin \frac{mv}{\alpha} \} e^{-\frac{mv}{\beta}} \right. \right. \\ & \quad + \frac{C_m}{\beta} \{ Z_1 \sin \frac{mv}{\alpha} + 2 \cos \frac{mv}{\alpha} \} e^{\frac{mv}{\beta}} \\ & \quad \left. \left. + \frac{D_m}{\alpha} \{ Z_1 \cos \frac{mv}{\alpha} - 2 \sin \frac{mv}{\alpha} \} e^{\frac{mv}{\beta}} \right] \sin \frac{m\pi\xi}{a} \sin \frac{m\pi x}{a} \right. \\ & = - K \sum_{m=1}^{\infty} \frac{1}{m} \left[- \left\{ \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} \right) (\beta \cos \frac{mv}{\alpha} - \alpha \sin \frac{mv}{\alpha}) \right\} e^{-\frac{mv}{\beta}} \right. \\ & \quad \left. - \left\{ \frac{D_1}{D_y} \left(\frac{\pi}{a} \right)^2 (\alpha \sin \frac{mv}{\alpha} + \beta \cos \frac{mv}{\alpha}) \right\} e^{-\frac{mv}{\beta}} \right] x \sin \frac{m\pi\xi}{a} \sin \frac{m\pi x}{a} \end{aligned} \quad (21)$$

in which

$$Z_1 = \frac{\alpha}{\beta} - \frac{\beta}{\alpha} - \frac{D_1}{D_y} \left(\frac{\pi}{a} \right)^2 \alpha \beta \quad (22)$$

and

$$\begin{aligned}
 & K \sum_{m=1}^{\infty} \left[\frac{A_m}{b} \left| -\frac{1}{b} \{Z_2 \sin \frac{mv}{a} - 2 \cos \frac{mv}{a}\} + \frac{1}{a} \{Z_2 \cos \frac{mv}{a} + 2 \sin \frac{mv}{a}\} \right| e^{-\frac{mv}{b}} \right. \\
 & + \frac{B_m}{a} \left| -\frac{1}{b} \{Z_2 \cos \frac{mv}{a} + 2 \sin \frac{mv}{a}\} + \frac{1}{a} \{-Z_2 \sin \frac{mv}{a} + 2 \cos \frac{mv}{a}\} \right| e^{-\frac{mv}{b}} \\
 & + \frac{C_m}{b} \left| \frac{1}{b} \{Z_2 \sin \frac{mv}{a} + 2 \cos \frac{mv}{a}\} + \frac{1}{a} \{Z_2 \cos \frac{mv}{a} - 2 \sin \frac{mv}{a}\} \right| e^{\frac{mv}{b}} \\
 & + \frac{D_m}{a} \left| \frac{1}{b} \{Z_2 \cos \frac{mv}{a} - 2 \sin \frac{mv}{a}\} + \frac{1}{a} \{-Z_2 \sin \frac{mv}{a} - 2 \cos \frac{mv}{a}\} \right| e^{\frac{mv}{b}} \\
 & \sin m\pi \frac{\xi}{a} \sin m\pi \frac{x}{a} \\
 & = -K \sum_{m=1}^{\infty} \left[-\left(\frac{1}{a^2} + \frac{1}{b^2}\right) \left\{ \left(\frac{a}{b} \sin \frac{mv}{a} - \cos \frac{mv}{a}\right) + \left(-\frac{b}{a} \sin \frac{mv}{a} - \cos \frac{mv}{a}\right) \right\} e^{-\frac{mv}{b}} \right. \\
 & - \frac{H_1 + 2D_{xy}}{D_y} \left(\frac{\pi}{a}\right)^2 \left\{ \left(-\frac{a}{b} \sin \frac{mv}{a} - \cos \frac{mv}{a}\right) + \left(\cos \frac{mv}{a} - \frac{b}{a} \sin \frac{mv}{a}\right) \right\} e^{-\frac{mv}{b}} \right] \\
 & \sin m\pi \frac{\xi}{a} \sin m\pi \frac{x}{a} \tag{23}
 \end{aligned}$$

in which

$$Z_2 = \left\{ \frac{a}{b} - \frac{b}{a} - \left(\frac{\pi}{a}\right)^2 \left(\frac{H_1 + 2D_{xy}}{D_y}\right) ab \right\} \tag{24}$$

Equations 21 to 24 give four simultaneous equations corresponding to $y = \eta_1$ and $y = \eta_2$ (Fig. 4), thus determining constants A_m , B_m , C_m , and D_m for each load position $P(\xi, \eta)$.

2.2.4 Deflections, Moments and Shears

The deflection function w for determining the deflections, moments and shears at any point of the orthotropic plate is then

obtained by superimposing individual solutions w_1 and w_2 , i.e.

$$w = \frac{Pa}{2\pi^2 (D_x D_y)^{1/2}} \sum_{m=1}^{\infty} \frac{1}{m^3} \left| \{ \alpha(A_m + 1) \sin \frac{my}{a} + \beta(B_m + 1) \cos \frac{my}{a} \} e^{-\frac{mv}{b}} \right. \\ \left. + \{ \alpha C_m \sin \frac{my}{a} + \beta D_m \cos \frac{my}{a} \} e^{\frac{mv}{b}} \right| \sin \frac{m\pi x}{a} \sin \frac{m\pi y}{a} \quad (25)$$

Moments M_x , M_y , M_{xy} and shears Q_x , Q_y are obtained from Eq. 8 by using second order derivatives of function w derived in Table 4.

2.3.0 Frequencies of Vibration

2.3.1 Equation of Dynamic Equilibrium

The differential equation of motion of a transversely vibrating plate of homogeneous orthotropic material is obtained from Eq. 10 by replacing the term $p(x,y)$ which corresponds to the external load by a "inertia force" term $-\rho_o \frac{\partial^2 w}{\partial t^2}$ in which deflection w is a function of time, and ρ_o is the mass density per unit area of the plate, i.e.

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H_1 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho_o \frac{\partial^2 w}{\partial t^2} = 0 \quad (26)$$

Deflection function $w(x,y,t)$ is expanded in the plate eigen function $y_{mn}(y)$ and harmonic function of time containing circular frequency ω_{mn} , i.e.

$$w(x,y,t) = \sum_m \sum_n y_{mn}(y) \sin \frac{m\pi x}{a} \sin \omega_{mn} t \quad (27)$$

Equation (27) satisfies the simply supposed boundary conditions of the bridge.

in which $y_{mn}(y)$ is a function of y only and m, n - number of half sine waves in the x and y direction, respectively.

2.3.2 Boundary Conditions

Taking the origin at the centre of one of the simply supported edges (Fig. 3), the boundary conditions simulating the bridge boundaries are

- i) Deflection $w_{mn}(x, y) = 0$ at simply supported edges
- ii) Moment $M_x = 0$ at $x = 0$ and $x = a$
- iii) Moment $M_y = - \left[D_y \frac{\partial^2 w_{mn}(x, y)}{\partial y^2} + D_1 \frac{\partial^2 w_{mn}(x, y)}{\partial x^2} \right] = 0$ at free edge
- iv) Shear $Q_y = - \left[D_y \frac{\partial^3 w_{mn}(x, y)}{\partial y^3} + (2H_1 - D_1) \frac{\partial^3 w_{mn}}{\partial x^2 \partial y} \right] = 0$ at $y = \pm \frac{b}{2}$

Expression 27 identically satisfies the simply supported boundary conditions given in Eq. 28.

Using Eq. 27 in Eq. 29,

$$\begin{aligned} \frac{\partial^2 y_{mn}(y)}{\partial y^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 y_{mn}(y) &= 0 \quad \text{at } y = \pm \frac{b}{2} \\ \frac{\partial^3 y_{mn}(y)}{\partial y^3} - \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \frac{y_{mn}}{\partial y} &= 0 \end{aligned} \quad (30)$$

Complete solution to the problem is obtained by satisfying the boundary conditions given by Eq. 28 and Eq. 30 by Eq. 27 which with Eq. 26 gives characteristic equations the roots of which lead to natural frequencies of vibration.

Using Eq. 27 in Eq. 26,

$$D_y y_{mn(y)}^{IV} - 2H_1 \left(\frac{m\pi}{a}\right)^2 y_{mn}^{II} + \left[\left(D_x \left(\frac{m\pi}{a}\right)^4 - \rho_o \omega_{mn}^2\right) y_{mn(y)} \right] = 0 \quad (31)$$

The roots of Eq. 31 are

$$\pm \left(\frac{m\pi}{a}\right) \left(\frac{D_x}{D_y}\right)^{1/4} \left[\frac{H_1}{\sqrt{D_x D_y}} \pm \left\{ \frac{H_1^2}{D_x D_y} - 1 + \frac{\omega_{mn}^2}{\left(\frac{m\pi}{a}\right)^4 \frac{D_x}{\rho_o}} \right\}^{1/2} \right]^{1/2} \quad (32)$$

2.3.3 Frequency Ratio

The term $\frac{m\pi}{a} \frac{D_x}{\rho_o}$ in Eq. 32 represents square of mth natural frequency in cycles per second of a simply supported beam of span a and having mass density ρ_o . Therefore the last term in Eq. 32 is the ratio of the square of circular frequencies of a plate of span a to a beam of span a.

Introducing

$$R^2 = \frac{\omega_{mn}^2}{\left(\frac{m\pi}{a}\right)^4 \frac{D_x}{\rho_o}} \quad (33)$$

and
$$\mu = \frac{H_1}{\sqrt{D_x D_y}}$$

the roots given by Eq. 31 are rewritten as

$$\pm \left(\frac{m\pi}{a}\right) \left(\frac{D_x}{D_y}\right)^{1/4} \left[\mu \pm \left\{ \mu^2 - 1 + R^2 \right\}^{1/2} \right]^{1/2} \quad (34)$$

which give four different solutions each corresponding to symmetric and antisymmetric modes for each frequency ratio R greater or less than unity.

2.3.4 General Solution - Symmetric and Antisymmetric Modes

The general solutions $y_{mn}(y)$ for $R > 1$ and $R < 1$ is taken in the form

$$y_{mn}(y) = A_{mn} \cosh \psi_{mn} \frac{y}{b} + B_{mn} \cos \phi_{mn} \frac{y}{b} + C_{mn} \sinh \psi_{mn} \frac{y}{b} + D_{mn} \sin \phi_{mn} \frac{y}{b} \quad (35)$$

in which

$$\text{for } R > 1, \text{ i.e., } \mu < (\mu^2 - 1 + R^2)^{1/2}$$

$$\psi_{mn}^2 = (m\pi c)^2 [(\mu^2 - 1 + R^2)^{1/2} + \mu]$$

$$\phi_{mn}^2 = (m\pi c)^2 [(\mu^2 - 1 + R^2)^{1/2} - \mu] \quad (36)$$

$$\text{and } \psi_{mn}^2 - \phi_{mn}^2 = 2\mu(m\pi c)^2$$

$$\text{for } R < 1, \text{ i.e., } \mu > (\mu^2 - 1 + R^2)^{1/2}$$

$$\psi_{mn}^2 = (m\pi c)^2 [\mu - (\mu^2 - 1 + R^2)^{1/2}]$$

$$\phi_{mn}^2 = (m\pi c)^2 [\mu + (\mu^2 - 1 + R^2)^{1/2}] \quad (37)$$

$$\text{and } \psi_{mn}^2 + \phi_{mn}^2 = 2\mu(\pi mc)^2$$

$$c = \frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4}$$

$A_{mn}, B_{mn}, C_{mn}, D_{mn}$ = constants to be determined from boundary conditions.

The first two terms, namely hyperbolic cosine and cosine terms in Eq. 35, correspond to plate eigen functions for symmetric modes and the remaining two terms namely the hyperbolic sine and sine terms give antisymmetric modes. Therefore, the general solutions for all the four cases are -

Case 1, $R > 1$, i.e., $\mu < (\mu^2 - 1 + R^2)$; symmetric mode

$$y_{mn}(y) = A_{mn} \cosh \psi_{mn} \frac{y}{b} + B_{mn} \cos \phi_{mn} \frac{y}{b} \quad (38)$$

Case 2, $R > 1$, i.e., $\mu < (\mu^2 - 1 + R^2)$; antisymmetric mode

$$y_{mn}(y) = C_{mn} \sinh \psi_{mn} \frac{y}{b} + D_{mn} \sin \phi_{mn} \frac{y}{b} \quad (39)$$

Case 3, $R < 1$, i.e., $\mu < (\mu^2 - 1 + R^2)$; symmetric mode

$$y_{mn}(y) = A_{mn} \cosh \psi_{mn} \frac{y}{b} + B_{mn} \cos \phi_{mn} \frac{y}{b} \quad (40)$$

Case 4, $R < 1$, i.e., $\mu > (\mu^2 - 1 + R^2)$; antisymmetric mode

$$y_{mn}(y) = C_{mn} \sinh \psi_{mn} \frac{y}{b} + D_{mn} \sin \phi_{mn} \frac{y}{b} \quad (41)$$

2.3.5 Transcendental Frequency Equations

The functions $y_{mn}(y)$ given in Eq. 38 to Eq. 40 together with the boundary conditions given by Eq. 30 give two equations in each of the cases given in Eq. 38 to Eq. 41. The constants $A_{mn} \dots D_{mn}$, are eliminated by setting the determinant of these two equations to zero, which lead to transcendental equations giving frequencies of vibrations. By eliminating one constant, the plate eigen function required to determine the mode shape in the unitized form are obtained. This is done in the following stages.

First, second and third order derivatives of $y_{mn}(y)$ appearing in Eq. 30, are evaluated for cases 1 to 4 and are given in Table 5. Using these expressions, two equations satisfying the boundary conditions given in Eq. 30 are derived for each case and are given in Table 6. These simultaneous equations cannot be solved

directly to evaluate constants $A_{mn} \dots D_{mn}$. However, by setting the determinants of these equations to zero, transcendental equations giving frequencies of vibration can be obtained. Thus, using $y = \pm \frac{b}{2}$, the determinants of equations in Table 6 are set to zero, and the resulting frequency equations for case 1 to 4 are

Case 1

$$\tanh \frac{\psi_{mn}}{2} \left[\frac{\frac{\psi_{mn}}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] = - \tan \phi_{mn} \frac{b}{2} \left[\frac{\frac{\phi_{mn}}{b} \left\{ \frac{\phi_{mn}^2}{b^2} + \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\phi_{mn}^2}{b^2} + \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] \quad (42)$$

Case 2

$$\coth \frac{\psi_{mn}}{2} \left[\frac{\frac{\psi_{mn}}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] = \cot \frac{\phi_{mn}}{2} \left[\frac{\frac{\phi_{mn}}{b} \left\{ \frac{\phi_{mn}^2}{b^2} + \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\phi_{mn}^2}{b^2} + \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] \quad (43)$$

Case 3

$$\tanh \frac{\psi_{mn}}{2} \left[\frac{\frac{\psi_{mn}}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] = \tanh \frac{\phi_{mn}}{2} \left[\frac{\frac{\phi_{mn}}{b} \left\{ \frac{\phi_{mn}^2}{b^2} - \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\phi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] \quad (44)$$

Case 4

$$\coth \frac{\psi_{mn}}{2} \left[\frac{\frac{\psi_{mn}}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] = \coth \frac{\phi_{mn}}{2} \left[\frac{\frac{\phi_{mn}}{b} \left\{ \frac{\phi_{mn}^2}{b^2} - \frac{2H_1 - D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}}{\left\{ \frac{\phi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\}} \right] \quad (45)$$

To eliminate the term $\frac{2H_1}{D_y}$ in Eqs. 42 to 45, the relationship between ψ_{mn} and ϕ_{mn} given in Eqs. 36 and 37 is rewritten using $\mu = \frac{H_1}{\sqrt{D_x D_y}}$ and $c = \frac{b}{a} \sqrt{\frac{D_x}{D_y}}$

i.e.

$$\psi_{mn}^2 - \phi_{mn}^2 = \frac{2H_1}{D_y} \left(\frac{m\pi b}{a}\right)^2 \text{ for } R > 1 \quad (46)$$

and

$$\psi_{mn}^2 + \phi_{mn}^2 = \frac{2H_1}{D_y} \left(\frac{m\pi b}{a}\right)^2 \text{ for } R < 1$$

Using Eq. 46, the transcendental frequency equations given by Eqs. 42 to 45, are given by

Case 1, $R > 1$, $\mu < (\mu^2 - 1 + R^2)^{1/2}$ Symmetric modes

$$-\frac{\tan \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = \frac{\psi_{mn}}{\phi_{mn}} \frac{\left[\left(\frac{\phi_{mn}}{c}\right)^2 + \frac{D_1/D_y}{\sqrt{D_x/D_y}} \right]^2}{\left[\left(\frac{\psi_{mn}}{c}\right)^2 - \frac{D_1/D_y}{\sqrt{D_x/D_y}} \right]} \quad (47)$$

Case 2, $R > 1$, $\mu < (\mu^2 - 1 + R^2)^{1/2}$ Antisymmetric modes

$$\frac{\tan \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = \frac{\phi_{mn}}{\psi_{mn}} \frac{\left[\left(\frac{\psi_{mn}}{c}\right)^2 - \frac{D_1/D_y}{\sqrt{D_x/D_y}} \right]^2}{\left[\left(\frac{\phi_{mn}}{c}\right)^2 + \frac{D_1/D_y}{\sqrt{D_x/D_y}} \right]} \quad (48)$$

Case 3, $R < 1$, $\mu > (\mu^2 - 1 + R^2)^{1/2}$ Symmetric modes

$$\frac{\tanh \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = \frac{\psi_{mn}}{\phi_{mn}} \frac{\left[\left(\frac{\phi_{mn}}{c}\right)^2 - \frac{D_1/D_y}{\sqrt{D_x/D_y}} \right]^2}{\left[\left(\frac{\psi_{mn}}{c}\right)^2 - \frac{D_1/D_y}{\sqrt{D_x/D_y}} \right]} \quad (49)$$

Case 4, $R < 1$, $\mu > (\mu^2 - 1 + R^2)^{1/2}$ Antisymmetric modes

$$\frac{\tanh \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = \frac{\phi_{mn}}{\psi_{mn}} \frac{\left[\frac{\psi_{mn}}{c} \right]^2 - \frac{D_1/D_v}{\sqrt{D_x/D_v}}}{\left[\frac{\phi_{mn}}{c} \right]^2 - \frac{D_1/D_v}{\sqrt{D_x/D_v}}} \quad (50)$$

2.3.6 Plate Eigen Functions $y_{mn}(y)$ - Mode Shapes

These functions are obtained by evaluating one constant, say B_{mn} and D_{mn} for symmetric and antisymmetric modes, in terms of A_{mn} and C_{mn} from the simultaneous equations given in Table 5, and substituting in the expressions $y_{mn}(y)$ in Eqs. 38 to 41. The resulting equations are given in Table 7, for all the four cases.

2.3.7 Frequency Equations and Eigen Functions When Poisson's Ratio $\nu=0$

The frequency equations and plate eigen functions for the case $\nu = 0$, are obtained by equating coupling rigidity $D_1 = 0$ in the equations in Table 7 and in Eqs. 47 to 50. The resulting expressions thus obtained are given in Tables 8 and 9.

2.4.0 Forced Vibration Analysis

2.4.1 Idealization and Slope

The formulation is based on the principle of beam behaviour of plate structure. The plate is divided into series of interconnected beams parallel to the path of the moving vehicle. The formulation presented is for computing the dynamic deflections and strains at the

centre of the beam under the vehicle and also at mid sections of the beams away from the vehicle. This is a comparatively simple approach as compared to the two dimensional analysis of the plate structure.

2.4.2 Assumptions

1) The bridge vibrates in its first mode and each beam is represented by a single degree of freedom system corresponding to its deformation in the fundamental mode. The peak amplitudes of centre-deflections of these beams are unequal but in the same phase.

2) The resulting vibrations of the bridge follow closely the forcing frequency of the vehicle.

3) The vehicle is represented by a one degree of freedom system. Actually, a multiple axle vehicle has many degrees of freedom as the mass tends to vibrate on a combined spring action of springs and tires. But important vehicle motion with respect to the bridge vibration occurs when they are in the phase.

4) The spacing of the axles are small as compared to dimensions of the bridge.

5) Unsprung mass is small and total mass of the vehicle is assumed to be at its centre of gravity.

6) Vehicle is moving at constant speed while it is on the span.

7) Damping in the system is neglected.

Assumptions 3 to 7 result in higher input values of vehicle dynamic load and resulting bridge deflections and strains tend to be of higher magnitude. Assumption 5 is justified by the results in

Table 10 where distribution of unsprung and sprung masses are given for typical medium and heavy vehicles. In all the cases, it is seen that average distribution of sprung mass exceeds 85% of the total mass of the vehicle.

2.4.3 Dynamic Deflections and Amplification Factors Under the Vehicle

Let the bridge of span a and width b be divided into a set of series of parallel beams 0, 1, 2, ... as shown in Fig. 6. Let the vehicle speed travelling on beam say 0, be v ft per sec and it enters the plate from the left hand support when $t = 0$. It is assumed that the effect of vehicle mass being small the deflected shape of the beam is sinusoidal at all times and the bridge deflection w under the vehicle after it has travelled a distance of $x = vt$ is given by

$$w = w_c \sin f_c t \quad (51)$$

in which

w_c - deflection at centre of beam

f_c - crossing frequency $\frac{v}{a}$

t - time taken by the vehicle to reach the distance $x = vt$, from the left hand support.

Idealized vehicle and bridge systems under these idealizations are shown in Fig. 7(a,b, and c). During time t ; let the deflection of the vehicle mass M_v be denoted by w_v so that the resulting distortion Δv of the vehicle spring is

$$\Delta v = w_v - w$$

and from Eq. 51

$$\Delta v = w_v - w_c \sin f_c t \quad (52)$$

in which Δv - distortion of the vehicle spring

K_V - stiffness of the vehicle spring

The force $F(t)$ in the vehicle spring due to distortion Δv and mass M_V is

$$F(t) = K_V \Delta v + M_V \cdot g \quad (53)$$

If the vehicle acceleration is denoted by $\frac{\partial^2 w_V}{\partial t^2}$, the inertia force is $M_V \frac{\partial^2 w_V}{\partial t^2}$ and the equation of motion of the vehicle mass is then written by equating the inertial force to the force in the spring,

$$\text{i.e.} \quad M_V \frac{\partial^2 w_V}{\partial t^2} = -K_V (w_V - w_C \sin f_C t)$$

$$\text{i.e.} \quad M_V \frac{\partial^2 w_V}{\partial t^2} + K_V (w_V - w_C \sin f_C t) = 0 \quad (54)$$

To apply the vehicle force to the bridge it is necessary to transform the bridge into an equivalent single degree of freedom system using generalised coordinates.

The resultant dynamic forces on the bridge at any time t is

$$- F(t) \sin f_C t + K_B w_C \quad (55)$$

in which K_B is the stiffness of the bridge.

The equation of motion of the bridge is written by equating the inertia force $M_B \frac{\partial^2 w_C}{\partial t^2}$ to Eq. 55,

$$\text{i.e.} \quad M_B \frac{\partial^2 w_C}{\partial t^2} = -K_B w_C + F(t) \sin f_C t$$

i.e. using Eq. 53

$$M_B \frac{\partial^2 w_c}{\partial t^2} + K_B w_c - \{M_{vg} + K_V (w_v - w_c \sin f_c t)\} \sin f_c t = 0 \quad (56)$$

Equations 54 and 56 represent the dynamic equilibrium of the vehicle and the bridge systems respectively. These simultaneous equations are solved to find w_c as a function of time.

Dividing both sides of Eqs. 54 and 56 by M_V and M_B respectively,

$$\frac{\partial^2 w_v}{\partial t^2} + \frac{K_V}{M_V} (w_v - w_c \sin f_c t) = 0 \quad (57)$$

and

$$\frac{\partial^2 w_c}{\partial t^2} + \frac{K_B}{M_B} w_c \frac{M_{vg}}{M_B} + \frac{K_V}{M_B} (w_v - w_c \sin f_c t) \sin f_c t = 0 \quad (58)$$

The ratios $\frac{K_V}{M_V}$, $\frac{K_B}{M_B}$, $\frac{M_{vg}}{M_B}$ and $\frac{K_V}{M_B}$ in Eqs. 57 and 59, are defined as follows

$$\frac{K_V}{M_V} = f_v^2, \text{ where } f_v = \text{frequency of vehicle in rad/sec}$$

$$\frac{K_B}{M_B} = f_B^2, \text{ where } f_B = \text{frequency of bridge in rad/sec}$$

(59)

$$\frac{M_V}{M_B} = M_R, \text{ ratio of masses of vehicle and bridge}$$

$$\frac{f_B}{f_v} = f_R, \text{ ratio of frequencies of bridge and vehicle}$$

and

$$\frac{M_{vg}}{M_B} = \frac{M_{vg}}{K_B} \cdot \frac{K_B}{M_B} = w_{st} f_b^2$$

where $\frac{M_v g}{K_B} = W_{ST}$ - static deflection of bridge under load $M_v g$

and

$$\frac{K_v}{M_B} = \frac{K_v}{M_v} \frac{M_v}{M_B} = M_R f_v^2 \quad (60)$$

Using Eqs. 59 and 60, Eqs. 57 and 58 are rewritten as

$$\frac{\partial^2 w_v}{\partial t^2} + f_v^2 (w_v - w_c \sin f_c t) = 0 \quad (61)$$

and

$$\frac{\partial^2 w_c}{\partial t^2} + f_v^2 \{ (f_R^2 (w_c - w_{st} \sin f_c t) - M_R (w_v - w_c \sin f_c t) \sin f_c t \} = 0 \quad (62)$$

Equations 61 and 62 are solved by numerical integration for w_c as a function of time and dividing the dynamic deflection w_c by static deflection w_{st} , amplification factors $(\frac{w_c}{w_{st}})$ are obtained.

2.4.4 Distribution of Dynamic Deflections and Amplification Factors away from Vehicle Run

The computations for distribution of dynamic deflections and their corresponding amplification factors are based on the following assumptions:

1) All the points on the plate are vibrating with the same frequency and as such have the same frequency and mass ratio.

2) The central deflections w_c for any position of the load follow the static behaviour and as such is a function of rigidity parameters and dimensions of the plate.

3) Frequency ratio is other than unity and cross modes are absent.

Let the plate be divided into a series of interconnected imaginary beams 0, 1, 2, ... z-1, z as shown in Fig. 6. Let $w_c, w_{c1}, w_{c2}, \dots, w_{cz-1}, w_{cz}$ be the deflection at the centre of these beams for any load position $x = vt$. The magnitudes of these deflections are in the same ratio corresponding to their static values and are in the same phase since the plate is assumed to have no cross modes.

The deflections $w, w_1, w_2, \dots, w_z$, corresponding to any load position $x = vt$ are obtained using Eq. 51

i.e. for zth beam

$$w_z = w_{cz} \sin f_c t \quad (63)$$

in which f_c is common for all beams as these beams are of span a.

Considering each beam in turn and following the identical procedure through equations similar to Eq. 52 to Eq. 60, the equations of dynamic equilibrium for each beam are obtained. Thus for zth beam these equations of equilibrium are

$$\frac{\partial^2 w_v}{\partial t^2} + f_v^2 (w_v - w_{cz} \sin f_c t) = 0 \quad (64)$$

and

$$\frac{\partial^2 w_{cz}}{\partial t^2} + f_v^2 \{ f_R^2 (w_{cz} - w_{stz} \sin f_c t) - M_R (w_v - w_{cz} \sin f_c t) \sin f_c t \} = 0 \quad (65)$$

which are solved by numerical integration.

2.4.5 Initial Condition of the Vehicle

The initial vibration of the vehicle at the time it enters the span is accounted for in the analysis as initial distortion of

vehicle spring measured as a fraction of the total vehicle mass m_v . The nature of distortion whether tension or compression is accounted for by introducing the concept of the phase angle ϕ . The vehicle is assumed to have only vertical oscillations and as such the angle ϕ can take only two possible initial values giving tension or compression in the springs. This system is shown in Fig. 7.

w_{vm} is the magnitude of possible maximum distortion caused by vehicle mass M_v and measured as a fraction of its maximum theoretical value,

i.e.

$$w_{vm} = B \frac{M_{vg}}{K_v} \quad (66)$$

B = reduction factor and is always < 1 . When $\phi = 0$, the spring is in neutral position. The tension and compression correspond to the phase angle $\phi = \pi/2$ and $\phi = 3\pi/2$, respectively.

Rewriting Eq. 66

$$B = w_{vm} \frac{K_v}{M_{vg}} \quad (67)$$

To express B , in terms of bridge parameters, the numerator and denominator of the right hand side of Eq. 67 are divided by K_B , i.e.

$$B = w_{vm} \frac{K_v/K_B}{M_{vg}/K_B} \quad (68)$$

Now $\frac{M_{vg}}{K_B} = w_{st}$, static deflection of the bridge and from Eq. 59

$$K_v/K_B = \frac{M_R}{f_R^2} \quad (69)$$

Using Eq. 69 in Eq. 68

$$B = \frac{w_{vm}}{w_{st}} \frac{M_R}{(f_R)^2} \quad (70)$$

Also

$$w_{vi} = w_{vm} \sin \phi \quad (71)$$

where w_{vi} - initial distortion of vehicle spring (Fig. 7).

Equations 70 and 71 define the initial vehicle conditions just before it enters the span.

2.5.0 Numerical Integration

The differential equations of dynamic equilibrium (Eqs. 61 and 62) are solved using numerical integration (3). Starting from time $t = 0$, when the displacements w_v and w_c of the vehicle and bridge are presumably known the time scale is divided into small intervals and computations are carried out in steps by successively extrapolating the displacements from one time interval, say s to $s+1$ or $s-1$ to s . This is shown in Fig. 7 in which the time acceleration curve is plotted against time t . The displacement $w_{c(s+1)}$ at point $(s+1)$ is computed knowing the displacement at the preceding point $w_{c(s)}$ and the average velocity between points s and $s+1$, i.e.

$$w_{c(s+1)} = w_{c(s)} + \left(\frac{\partial w_c}{\partial t} \right) \Delta t \quad (72)$$

in which Δt - time interval between s and $(s+1)$ and average velocity

$$\frac{\partial w_c}{\partial t} \text{ is } \left(\frac{\partial w_c}{\partial t} \right)_{AV} = \frac{w_{c(s)} - w_{c(s-1)}}{\Delta t} + \frac{\partial^2 w_c}{\partial t^2} \Delta t \quad (73)$$

The term $\frac{\partial^2 w_c}{\partial t^2} \Delta t$ in Eq. 73 is the average acceleration during the period of time. This approximation is shown in Fig. 7(c) where the time acceleration plot is shown as the sum of the small additions of successive intervals shown by horizontal lines.

Eliminating $\frac{\partial w_c}{\partial t}$ from Eqs. 72 and 73

$$w_{c(s+1)} = 2w_{c(s)} - w_{c(s-1)} + \frac{\partial^2 w_c}{\partial t^2} \Delta t^2 \quad (74)$$

To calculate the value $w_{c(1)}$ at $t = 0$ when $w_{c(s-1)}$ is unknown; acceleration was assumed constant during the first time interval and equal to its initial value. Using this approximation

$$w_{c(1)} = \frac{1}{2} \frac{\partial^2 w_c}{\partial t^2} (\Delta t)^2 \quad (75)$$

2.6.0 Rigidities of Orthotropic Plate

The flexural and torsional rigidities of the orthotropic plate given by Eq. 9 are determined by distributing the stiffnesses per unit width of the plate. This is done by introducing the following assumptions:

2.6.1 Assumptions

- 1) The reinforcing effects of the ribs which are arranged parallel to the x axis (Fig. 15), in the direction of y is negligible as the influence of reinforcing effects of lateral rigidity of the ribs on the distribution of forces in the orthotropic plate is negligible.
- 2) In the absence of diaphragms, the equivalent thickness of an homogeneous orthotropic plate can be replaced by thickness h of the plate.

3) The plate and stiffeners are of the same material and a perfect bond exists between them.

4) The neutral surface for the stresses in the direction of x is parallel to the middle plane of the plate and is located at a distance e_x from the top surface of the plate (Fig. 3). The distance e_x is determined from the condition that the resultant force acting on the repeating section must be zero.

5) The rigidity moduli E_x , E_y and Poisson's ratios ν_x , ν_y in the orthogonal direction are very close to their corresponding values of E and ν of the material of slab, i.e.

$$\begin{aligned} \text{i)} \quad E_x &\approx E_y \approx E \\ \text{ii)} \quad \nu_x &= \nu_y = \nu \end{aligned} \quad (76)$$

6) Flexural rigidity D_x and torsional rigidity H_1 of the plate stiffener combination can be calculated as the algebraic sum of flexural and torsional rigidities of slab and stiffener respectively.

2.6.2 Rigidity D_y

$$D_y = E_y \frac{h^3}{12(1 - \nu_x \nu_y)}$$

Using (6)

$$D_y = E \frac{h^3}{12(1 - \nu^2)} \quad (77)$$

2.6.3 Rigidity D_x

This is obtained as the sum of rigidities of the plate and stiffener about common neutral axis -

Rigidity of plate:

$$D_x \Big|_{\text{plate}} = \frac{E_x h^3}{12(1-\nu_{xy})} + \frac{E_x}{(1-\nu_{xy})} t h \left(e_x - \frac{h}{2}\right)^2 \quad (78)$$

in which

e_x = depth of C.G. of repeating T section from top of slab

i.e.

$$e_x = \frac{1}{2} \left\{ \frac{Sh^2 + t(H-h)^2}{Sh + t(H-h)} \right\}$$

Using Eqs. 76 and 77 in Eq. 78,

$$D_x \Big|_{\text{plate}} = D \left\{ 1 + 12 \left(\frac{e_x}{h} - \frac{1}{2} \right)^2 \right\} \quad (79)$$

Rigidity of stiffener

$$D_x \Big|_{\text{stiffener}} = \frac{EI}{S} \quad (80)$$

in which I is moment of inertia of stiffener about common neutral axis

S = C/C spacing of stiffeners

$$\text{i.e. } D_x \Big|_{\text{stiffener}} = \frac{E}{S} \left\{ \frac{t(h-h)^3}{12} + t(H-h) \left(\frac{H-h}{2} - e_x \right)^2 \right\} \quad (81)$$

$$D_x \Big|_{\text{orthotropic plate}} = D_x \Big|_{\text{plate}} + D_x \Big|_{\text{stiffener}} \quad (82)$$

2.6.4 Torsional Rigidity

This is calculated as the sum of torsional rigidities of plate and stiffener.

From Eq. 9

$$H_1 \Big|_{\text{plate}} = \frac{1}{2} (D_{xy} + \nu_y D_{yx} + 4D_{xy}) \quad (83)$$

Using Eqs. 9 and 76

$$\nu_y D_{yx} + \nu_x D_{xy} = 2\nu D$$

and

$$2D_{xy} = G_{xy} \frac{h^3}{6} \quad (84)$$

Using $G = \frac{E}{2(1+\nu)}$ in Eq. 84, Eq. 83 is written

i.e.

$$H_1 \Big|_{\text{plate}} = \frac{1}{2} \{2\nu D + 2(1-\nu)D\} = \nu D \quad (86)$$

Similarly

$$H_1 \Big|_{\text{stiffener}} = \frac{G_{xy} T}{2} \quad (87)$$

in which T is geometric torsional rigidity of stiffener per unit width of plate, i.e.

$$T = \frac{\tau(H-h)^3}{C_1 S} \quad (88)$$

where C_1 - constant depending on dimension of stiffener and is given by

$$C_1 = 2.6[1.80 + e^{1.65/\sqrt{t}/(H-h)}] \text{ for } \frac{t}{H-h} < 1 \quad (89)$$

$$= 2.6[1.80 + e^{1.65/(t)/(H-h)}] \text{ for } \frac{t}{H-h} > 1$$

From Eqs. 86 and 87,

$$H_1 \left| \begin{array}{c} \text{orthotropic plate} \end{array} \right. = H_1 \left| \begin{array}{c} \text{plate} \end{array} \right. + H \left| \begin{array}{c} \text{stiffener} \end{array} \right.$$

2.7.0 Programme

A combined programme was written for the purpose of numerical analysis. The input parameters were basic dimensions and elastic properties of the bridge and vehicle, namely:

Bridge - span, aspect ratio, Poisson's ratio, modulus of elasticity, dimensions of plate and stiffeners.

Vehicle - speed, initial conditions B and ϕ , frequency, weight.

The programme output is deflections, moments, frequencies, dynamic deflections, strains and amplification factors. It evaluates rigidity and mass parameters and solves Eqs. 21 and 23 to obtain the constants A_m to D_m for given load position $P(\xi, n)$. Two separate subroutines for matrix multiplications and inversions are used to solve the 4x4 matrices corresponding to unknowns A_m D_m . It evaluates deflections and moments using Eqs. 25 and 8 respectively and seven terms in the series. Frequency of vibrations are obtained by solving transcendental frequency Eqs. 47 to 50 corresponding to symmetric and antisymmetric modes. Solution is obtained by subdividing either ψ_{mn}

or ϕ_{mn} into small intervals of 0.001 and equating the left and right side of these equations up to an accuracy of 0.5% or less. Using frequency and mass of the bridge and utilizing the vehicle input of frequency, mass, speed and initial conditions B and ϕ , the programme evaluates dynamic amplifications by solving differential equations of equilibrium 61 and 62 by numerical integration. Total time T for vehicle on span was evaluated using speed and span, and time interval Δt corresponding to $T/200$ was used for integration purposes. Forced vibration analysis was carried out for 3 passages of vehicles. The initial conditions of the bridge were corresponding to the last time interval, i.e. 200 of the previous stage. This is achieved by putting do loops to account for the last stage of the bridge deflections and velocity to be fed as initial values of the following stages.

The programme statements for a typical case are given in Appendix II

CHAPTER 3

INTERACTION OF PARAMETERS

3.1.0 Introduction

This chapter deals with the study of the relative importance of bridge and vehicle parameters and numerical solutions using the three stage orthotropic analysis presented in Chapter 2. This is done in the following steps:

- 1) Identification of bridge and vehicle parameters.
 - 2) Evaluation of practical range of bridge and vehicle parameters and presentation of results in the form of design tables.
 - 3) Interpretation of relative influence of each parameter on final results in its extreme ranges by typical graphs.
 - 4) Parametric influence leading to empirical formulation of frequencies and comparison of theoretical results.
 - 5) Presenting results in a series of tables for design purposes.
- 

3.2.0 Bridge and Vehicle Parameters

Bridge and vehicle parameters both in individual and common form are given in Table 11. Dimensionless form of these parameters, their practical range considered for numerical solution and results obtained, are given in Table 12.

3.3.0 Physical Interpretation of Parametric Ranges

3.3.1 Plate Beam Transition - Static Analysis

The variation of aspect ratio $\frac{b}{a}$ from 1.0 to 0.5 and rigidity ratio $\frac{D_y}{D_x}$ from 0.01 to 0.5 was considered sufficient for plate beam transition of equivalent orthotropic plates. Unlike isotropic plates, the plate beam transition is a function of aspect and rigidity ratios and when $\frac{D_y}{D_x} = 1$ and $\mu = 1$, the values obtained correspond to isotropic plates. Smallest values of rigidity ratio $\frac{D_y}{D_x} = .01$ indicates that stiffener sections are heavier or spacing of stiffeners is very close, as compared to width of the plate.

3.3.2 Frequency Analysis

Frequency analysis is based on the assumption that minimum value of rigidity parameter $mc = m \frac{b}{a} \left(\frac{D_x}{D_y}\right)^{1/4}$ does not exceed 0.25. This is due to the fact that when $\frac{D_x}{D_y} > 1$, and taking minimum values of $m = 1$ (first symmetric mode) the minimum value of $\frac{b}{a}$ covered is 0.25. For most practical cases, the parameter mc usually exceeds 2.0 and therefore $mc = 0.25$ is considered the lowest possible value for theoretical analysis. The maximum value of torsional parameter $\mu = \left(\frac{H_1}{\sqrt{D_x D_y}}\right)$ is 0.9999 and therefore frequency values for the first three symmetrical and antisymmetrical modes are obtained for $mc = 0.25$ to $mc = 10$ and $\mu = 0.1$ to 0.9 through an interval of 0.1. The effect of Poisson's ratio ν is studied by varying the ratio $(D_1/D_y)/\sqrt{D_x/D_y}$ from 0 to 0.1 through an interval of 0.025, i.e., $D_x/D_y = \alpha$ to 9 for $\nu = 0.3$ which was found sufficient to investigate the effect of Poisson's ratio ν on frequencies of vibration.

3.3.3 Forced Vibration Analysis

The treatment of parameters in the forced vibration analysis is considered in two separate groups, namely, (a) deterministic parameters, i.e., speed of vehicle, frequencies and masses of bridge and vehicle, and (b) stochastic parameters, i.e., initial conditions of vibrations of bridge and vehicle.

Bridge frequencies (f_B) are considered from 1.5 cycles per second to 9 cycles per second through six intervals. Crossing frequency ($\frac{\pi v}{a}$) is taken from $f_c = 0.5\pi$ min to $f_c = 2.5\pi$ max., through six intervals.

Assuming the variation of vehicle speeds from 30 mph to 70 mph, the span ranges covered lie between 20' to 200'. The frequency ratio $f_R = \left(\frac{f_B}{f_v}\right)$ varies from 0.4 min to 6 maximum through eight intervals which include resonance conditions $f_R = 1$. This gives the frequency band of the vehicle between 0.25 to 22.5 cycles per second. Mass ratios ($\frac{M_v}{M_B}$) are considered between .01 minimum to 0.2 max. These numerical values and their range of variation are assumed to cover the possible practical cases for the purpose of design.

The parameters which cannot be directly determined on an accountable basis include the magnitude of initial vibrations of the vehicle B and ϕ as the reasons causing initial vehicle vibrations cannot be clearly defined to form a mathematical formulation. Similar difficulty is seen when accounting for the initial vibrations of a bridge as it is a function of density of the traffic. In the present study, bridge initial conditions are included on an idealized basis

to obtain the optimum amplification factors under three passes of the same vehicle ($B = 0.3$ and $\phi = 1.5\pi$) which theoretically is considered to present the most critical combination of these parameters.

In the first stage, the bridge was assumed at rest, and maximum amplification factors for the same initial conditions of the vehicle were obtained.

In the second stage, the final values of the bridge dynamic deflections in stage one were chosen as the initial conditions of the bridge and maximum amplification factors were obtained.

In the third stage, the initial conditions of the bridge were assumed corresponding to the maximum dynamic deflections in stage two and maximum amplification factors are obtained.

The amplification factors of the third stage can be taken as giving the upper limit of amplification factors for the particular combination of the bridge and vehicle.

3.4.0 Relative Parametric Influence on Final Results

3.4.1 Presentation

Numerical analysis was based on the parameters as given in Table 12. In the first stage of analysis, relative influence of the parameters on the final results was studied. This was done by varying one parameter at a time and calculating the results corresponding to limiting ranges of the remaining parameters. The variation is presented in the form of curves and tables in the following sections.

3.5.0 Static Analysis

3.5.1 Effect of Rigidity Ratio D_y/D_x on Deflections and

Moments for Constant Values of μ

Deflection and moment coefficients for aspect ratio $\frac{b}{a} = 1.0$ to 0.5 and $\mu = 0.1$ are plotted against D_y/D_x in Figs. 8(a) and 8(b). Deflection coefficients (w_c) are calculated as a multiplier to $\frac{Pa^4}{abD_x} \times 10^{-1}$ and the moment coefficients M_x and M_y as multipliers to $\frac{Pa^2}{ab}$, under load P at the centre of the plate. Similar curves for torsion parameter $\mu = 0.9$ are plotted in Figs. 9(a) and 9(b). Assuming coefficient w_c , M_x and M_y as unity at $\frac{D_y}{D_x} = 0.01$, corresponding values at $\frac{D_y}{D_x} = .1$ and 0.5 for $\frac{b}{a} = 1$ and 0.5 are given in the table below for $\mu = 0.1$ and 0.9 .

Variation of Coefficients w_c , M_x and M_y with $\frac{D_y}{D_x}$

μ	D_y/D_x	$A_R = 1.0$			$A_R = 0.5$		
		w_c	M_x	M_y	w_c	M_x	M_y
.1	.01	1.0	1.0	1.0	1.0	1.0	1.0
	.1	.590	.620	1.94	.63	.635	1.70
	.5	.392	.455	3.20	.59	.575	2.70
.9	.01	1.0	1.0	1.0	1.0	1.0	1.0
	.1	.590	.615	1.86	.76	.725	1.77
	.5	.455	.48	3.01	.71	.625	2.78

Both deflection and moment coefficients decrease with an increase in D_y/D_x , but M_y increases with an increase in D_y/D_x , the trend

being more predominant between $D_y/D_x = .01$ to $.1$ for w_c and M_x , and between $D_y/D_x = .1$ to $.5$ for M_y . Similar trends of variation of w_c , M_x and M_y with D_y/D_x are seen when $\mu = 0.9$. For smaller values of b/a , w_c and M_x tend to be independent of D_y/D_x between $D_y/D_x = .1$ and $.5$ and M_y between $D_y/D_x = .01$ and $.1$. Absolute values of w_c , M_x and M_y tend to be smaller with increase in the torsion parameter μ but comparing their variation with D_y/D_x , it can be concluded that, both deflection and moment parameters are less sensitive to change in μ than to the rigidity ratio D_y/D_x .

3.5.2 Effect of Torsion Parameter μ on Deflection and Moments

for Constant Values of D_y/D_x

Variation of deflection coefficient for constant values of $D_y/D_x = .01$ and $.1$ is plotted in Fig. 10(a) and 10(b), for aspect ratio $b/a = 1.0$ to $.5$. Similar curves for M_x and M_y are given in Figs. 11(a), 11(b) and 12(a), 12(b), respectively. It can be seen that both deflection and moment coefficients decrease with an increase in μ and for w_c and M_x , the trend of variation is similar for the same range of b/a and D_y/D_x . As compared to moments, deflection is less sensitive to μ , and after $\mu = 0.4$, the variation is linear. This trend is shown in the table below where the values corresponding to $\mu = 0.1$, are taken as unity and corresponding ratios for $\mu = 0.4$ and 0.9 are given for $D_y/D_x = .01$ and 0.1 .

Variation of Coefficient w_c , M_x and M_y with μ

$\frac{D_y}{D_x}$	μ	$A_R = 1.0$			$A_R = .5$		
		w_c	M_x	M_y	w_c	M_x	M_y
.01	.1	1.0	1.0	1.0	1.0	1.0	1.0
	.4	.88	.89	.895	.87	.875	.845
	.9	.76	.76	.77	.785	.77	.715
.1	.1	1.0	1.0	1.0	1.0	1.0	1.0
	.4	.86	.86	.86	.97	.93	.88
	.9	.76	.75	.73	.95	.86	.75

For values of $D_y/D_x = .1$ and $b/a = .5$, the variation of w_c , M_x and M_y with μ is very small for all practical purposes.

Comparing Figs. 10, 11 and 12, it is seen that the rigidity ratio D_y/D_x has more influence as compared to the torsion parameter on values of deflection and moment coefficients for a wide range of combination of aspect ratios b/a and as such any approximation involved in computing torsion parameter μ will have smaller effects on accuracies of deflections and moments.

3.5.3 Effect of D_y/D_x and b/a on Distribution of Deflection

at Centre

The effect of common parameters D_y/D_x and b/a on distribution of deflection at the centre is shown in Fig. 13(a) and 13(b) in which the ratio of deflections w/w_c is plotted at $y/b = 0$ to $y/b = 0.5$, $x/a = 0.5$, and $D_y/D_x = .05, .01, .05, 0.1, 0.5$ and 1.0 . This is done for $\mu = 0.3$ in Fig. 13(a) and 13(b) and similar graphs are plotted in Fig. 14(a) and 14(b) for $\mu = 0.9$. It is seen that deflection ratios

w/w_c for the same range of b/a are not the same and vary with the rigidity ratio D_y/D_x . This deflection profile is of particular interest while considering the plate as a series of interconnected beams for forced vibration analysis. Variation of deflection ratios across width at centre cross section of plates, is a function of both rigidity and aspect ratios and needs simultaneous consideration in equivalent beam idealization of orthotropic plates.

3.6.0 Frequencies of Vibration

3.6.1 Variation of Frequency Ratio R with Rigidity Parameter mc

$$(m b/a (D_x/D_y))^{1/4}$$

The variation of frequency ratio R with rigidity parameter mc is plotted on log-log scale in Fig. 15(a) and 15(b), for two cases namely $\mu = 0.1$ and 0.9 . The ratios correspond to frequencies of vibration for the first three symmetrical ($n = 1, 3, 5$) and anti-symmetrical ($n = 2, 4, 6$) modes, and Poisson's ratio $\nu = 0$. The rigidity parameter was varied from $.1$ to 10 .

It is seen that in both cases of $\mu = 0.1$ and 0.9 , the frequency ratio decrease with increase in mc and tend to asymptote to $R = 1$, for large value of mc . For lower modes, i.e., $n = 2$, frequency ratios appear to be more sensitive up to $mc = 1$ and $mc = 3$ for $\mu = 0.1$ and 0.9 , and thereafter are almost independent of mc . This range of independency of R on mc seems to be between $mc = 2$ to $mc = 5$ for $n = 3$ to $n = 6$, $\mu = 0.1$. The variation of frequency ratio with mc appears to be almost linear on log-scales for values of $R = 2.0$ and more.

3.6.2 Variation of Frequency Ratio R with Torsion Parameter μ

The variation of frequency ratio R with torsion parameter μ is shown in Fig. 16 for $n = 2$ and for five typical values of mc .

Similar curves for $n = 6$ are given in Fig. 17. It is seen that for the lower modes ($n = 2$), frequency ratio increases with the torsion parameter μ . This trend is more predominant for lower values of $mc = 0.25$ and diminishes with an increase in μ . In the higher range of $mc = 3$ and more, the frequency ratio is almost independent of μ . Assuming frequency ratio $R = 1$ for $\mu = 0.1$, the frequency ratios for $mc = .25, .5, .75, 1.0$ and 2.0 can be shown to be 2.72, 2.22, 1.83, 1.58 and 1.19 respectively. However, for $n = 6$, Fig. 17, the frequency ratio seems to be almost independent of μ for any value of mc .

3.6.3 Variation of Frequency Ratio R with Poisson's Ratio

$$\text{Parameter } D_1/D_y / \sqrt{D_x/D_y}$$

The variation of frequency ratio R with Poisson's ratio parameter $D_1/D_y / \sqrt{D_x/D_y}$ is shown in Fig. 18, in which $D_1/D_y / \sqrt{D_x/D_y}$ is varied from 0 to .1 and corresponding values of R are plotted for $n = 2$, $mc = 0.25$ and $\mu = 0.1$ to .9. It is seen, that R decreases with an increase in Poisson's ratio and the effect is more predominant in the lower ranges of μ . Assuming the frequency ratio equals unity for $D_1/D_y / \sqrt{D_x/D_y} = 0$, the variation of R is shown in the table below for $n = 2, 3, 4, 5, 6$ and for typical values of mc and μ .

Variation of Frequency Ratio R with
Poisson's Ratio Parameter $(D_1/D_y)/\sqrt{D_x/D_y}$

n	$\frac{D_1/D_y}{D_x/D_y}$	$\mu = 0.1$			$\mu = 0.9$		
		mc .25	1.0	5.0	.25	1.0	5.0
2	0	1.0	1.0	1.0	1.0	1.0	1.0
	.1	.5550	.9000	.9900	.9440	.9705	.9972
3	0	1.0	1.0	1.0	1.0	1.0	1.0
	.1	.9966	.9455	.9867	.9962	.9771	.9964
4	0	1.0	1.0	1.0	1.0	1.0	1.0
	.1	.9992	.9848	.9788	.9984	.9899	.9945
5	0	1.0	1.0	1.0	1.0	1.0	1.0
	.1	.9994	.9944	.9748	.9994	.9950	.9932
6	0	1.0	1.0	1.0	1.0	1.0	1.0
	.1	.9996	.9973	.9758	1.0	.9975	.9926

It can be seen that in the range of variation considered, the frequency ratio decreases with an increase in Poisson's ratio although the dependency reduces with an increase in mc and μ . However, for values of $n = 6$ and more, R is almost independent of $(D_1/D_y)/\sqrt{D_x/D_y}$.

It is therefore concluded that for lower values of mc and μ , the frequency ratio R is not independent of Poisson's ratio and should be accounted for in the analysis.

3.7.0 Comparative Study of the Nature of Variation of Deflection, Moments and Frequencies on Orthotropic Rigidity Parameters

D_x , D_y and μ

Deflection and moment coefficients w_c and M_x decrease with an increase in rigidity ratio D_y/D_x for constant values of μ and b/a , Fig. 8(a) and 8(b), while frequency ratio R decreases with an increase in parameter $m \frac{b}{a} \left(\frac{D_x}{D_y}\right)^{1/4}$ for constant values of μ . Assuming the values of m and $\frac{b}{a}$ are constant, the decrease in frequency ratio R , Fig. 15(a) and 15(b), can be taken as due to an increase in $\left(\frac{D_x}{D_y}\right)^{1/4}$. It is therefore seen that static coefficients w_c and M_x and frequency ratio R have opposite degrees of dependency on orthotropic rigidity parameter $\frac{D_y}{D_x}$. Similarly, comparing the increase of w_c and M_x in Figs. 10, 11 and 12 and the decrease of frequency ratio R in Fig. 16, both with an increase in torsion parameter μ , it is seen that static and frequency coefficients are inversely proportional to the same degree of variation of orthotropic torsion parameter μ . This leads to the conclusion that the approximate formulation based on stiffness distribution for determining orthotropic rigidity parameters may not give the same degrees of accuracies in static and dynamic analysis. The accuracy of concepts in rigidity formulation must be, therefore, verified by static and frequency test on models.

3.8.0 Empirical Formulation

3.8.1 Empirical Relationship for Computing Frequency Ratio R .

Interaction of Parameters D_x , D_y , μ , b/a , a , m and n ,

Method of Formulation

The important parameters affecting the frequency ratio R are D_x , D_y , μ , b/a , m and n . It is intended to combine these parameters to produce one closed term equation giving frequency ratio R , sufficiently accurate for all practical purposes.

The method is essentially a trial-error approach and consists of studying the variation in frequency ratio R by treating one parameter at a time for constant values of the remaining parameters, (Figs. 8,9,10,11,12,15,16,17 and 18). Individual algebraic expressions governing the nature of variations are developed by using curve fitting techniques and using Eq. 36 the final expression is written by superimposing these individual expressions to give a closed form relationship between frequency ratio R and the parameters entering into the formulation. The results thus obtained are compared with those by solving transcendental frequency equations and accuracy of the final results is compared.

3.8.2 Empirical Equations

These equations are presented for two cases

a) $\frac{D_x}{D_y} > 1$ and $\mu < 0.999$ (slab and stiffener)

b) $D_x = D_y$ and $\mu = 1.0$ (slab only)

a) $\frac{D_x}{D_y} > 1$ and $\mu < 0.9$:

Frequency ratio R is given by

$$R = [1 + \{(\frac{\theta}{\pi m_c})^2 + \mu\}^2 - \mu^2]^{1/2} \quad (90)$$

in which

$$\theta = N\pi + \frac{1}{k_1} \log e \{1 + \pi\mu(m_c)^2\} \quad (91)$$

where

$N = 0.5, 2.5, 4.5, \dots$ for first, second, third antisymmetrical modes ($n = 2, 4, 6$)

$N = 1.5, 3.5, 5.5, \dots$ for second, third, fourth symmetrical modes ($n = 3, 5, 7$) (92)

$K_1 = 1, 2, 3, \dots$ for first, second and third modes.

b) $D_x = D_y$ and $\mu = 1.0$

Frequency ratio R is given by

$$R = 1.0 + \left(\frac{(a/b)(n-1)}{2m} \right)^2 \pi. \quad (92a)$$

3.8.3 Comparison of Results

a) $\frac{D_x}{D_y} > 1$ and $\mu < .999$:

The frequency ratios R are calculated using Eq. 90 for $\mu = 0.1, 0.3, 0.5, .9$; $mc = 0.5, 0.75, 1.0, 2.0, 3.0, 5.0$ and 10.0 ; and $n = 2, 3, 4, 5, 6$. These values are given in Table 13 and are compared with the corresponding values computed using the theoretical method. It is seen that the empirical values are higher for $n = 4, 5, 6$ for any mc and smaller for $n = 2$ and 3 in the range $mc < 1.0$. Maximum error (12%) is seen for $n = 2$ and $mc = 0.5$. However, the error reduces to less than 4% for $mc = 1$ and more. For higher values of n , the results are in very good agreement with the theoretical values for all parametric combinations.

b) $D_x = D_y$ and $\mu = 1.0$:

Frequency coefficients $\frac{\omega_a^2}{m^2 \pi^3} \sqrt{\frac{\rho_o}{D_x}}$ are computed using relationships given by Eq. 92, and plotted in Fig. 19(a), (b), (c) and (d) for

$\frac{b}{a} = 1.0, .8, .6$ and $.4$ and $m = 1$ to 6 and are compared with corresponding theoretical curves. A very good agreement between theoretical and empirical values is found for all combinations of m and n .

Thus it is concluded that these empirical equations 90 and 92 give frequencies of vibrations sufficiently accurate for practical design purposes.

3.9.0 Forced Vibrations

3.9.1 Presentation

The relative parameter influence on variation of amplifications for single and three passage of vehicles are presented for typical cases. The case of passing of a single vehicle represents normal conditions of design while the passage of three vehicles under assumed optimum conditions give critical case which represents optimum limit in actual practice. However, it indicates the theoretical upper limit for the particular combinations of parameters. The influence of parameters, frequency ratio, mass ratio, crossing frequency and vehicle initial conditions on variations of amplifications is discussed for typical cases and are then treated for concept of arriving at optimum section during the design process. The second phase of computations include design tables giving amplification factors which it is believed, covers the practical ranges of parameters going into the design process.

3.9.2 Effect of Frequency Ratio on Amplification

The effect of frequency ratio is shown in Fig. 20(a) to 20(e) and Fig. 21(a) to 21(e), in which results for normal optimum cases are

given for constant values of M_v/M_B and v/a . For frequency ratio f_R greater than 2.0, the curves are smooth and relatively stable and amplifications are seen between 1.20 to 1.40 in normal cases and 2.2 to 2.5 for optimum conditions. There is a trend of a decrease of amplifications with $f_B/f_v = 2.0$ and more. For ratios (f_R) less than 2.0, the curves are more irregular but generally increase to maximum 1.75 to 2.0 for normal and 3.25 to 4.0 for optimum cases. In the range of f_B/f_v 0 to 1, the curves increase rapidly. It may be noted that absolute values of f_v and f_B are not of much importance but ratio f_B/f_v is significant. Assuming the amplifications in a normal case (Stage I) as unity the values of amplifications corresponding to optimum cases (Stage III) are given in the table below.

Variation of relative Amplification Factors Stage III -
with bridge frequency and mass ratios.

$$(A_f(I) = 1.0, f_c = 0.5\pi)$$

M_v/M_B	$f_B = 3\pi(1.5 \text{ cps})$				$f_B = 18\pi(9 \text{ cps})$			
	$f_R = .4$	1.0	2.0	6.0	$f_R = .4$	1.0	2.0	6.0
.01	1.7945	1.5208	1.9947	1.8712	2.0707	1.0042	1.9103	2.0146
.03	1.7944	1.3946	1.9936	1.8713	2.1879	1.1872	1.9080	2.0146
.05	1.7844	1.3817	1.9927	1.8713	2.0336	1.3488	1.9380	2.049
.08	1.8166	1.4546	1.9912	1.8718	2.1578	1.4640	1.8958	2.0142
.1	1.7899	1.3759	1.9902	1.8716	2.1162	1.3449	1.8914	2.0182
.2	1.6993	1.3717	1.9828	1.8722	1.9952	1.4070	1.8305	2.0159

There is an increase (max. 2.188) in amplifications for given combinations of f_B and M_v/M_B . The maximum increment appears when $f_B/f_v \gg 1$ and the minimum is when $f_B/f_v = 1$. This trend is opposite

to the variation with f_B/f_v as seen in Figs. 20 and 21, in which the peaks appear at $f_B/f_v = 1.0$. This can be explained as follows.

The peaks at $f_B/f_v = 1$, are caused by beat effects which occur at low frequency ratios. The relative oscillations of vehicle almost disappear at one or more points along the bridge. At these points the bridge vibration greatly increases because most of the energy originally contained in the vehicle oscillations is transferred to the bridge.

The possible reason for reduced rate of increment when $f_B/f_v = 1$, can be given as initially high values which are taken as the basis with respect to which the increments are calculated.

3.9.3 Effect of Mass Ratio on Amplifications

The effect of mass ratio M_v/M_B is shown in Fig. 22(a) to 22(f) for constant values of $\pi v/a$, f_B/f_v and six values of bridge frequency f_B . These are shown for normal and critical conditions of the passage of vehicles. For high frequency ratios, the value of mass ratio has practically no influence on amplifications. The effect is apparent when frequency ratio approaches unity. Amplifications increase with smaller M_v/M_B when f_B/f_v approaches unity. Assuming the amplifications for $M_v/M_B = 0.01$ unity, the amplifications for $M_B/M_v = 0.1$ are given below for typical cases in Stage I and Stage III.

Amplifications for Mass Ratio 0.1 Stage I and III

(A_F for $M_V/M_B = .01$ is unity)

f_c	f_B	Stage I				Stage III			
		$r_{R.8}$	1.0	2.0	6	$r_{R.8}$	1.0	2.0	6.0
0.5	3π	.8750	.8542	.9877	.9985	.7652	.7716	.9855	.9907
	12π	.8649	.4639	.9814	.9997	.8418	.6115	.9769	.9987
	18π	.8922	.4799	.9853	.9986	.8874	.6400	.9755	.9992
2.5	3π	.9595	.9676	.7298	1.002	.8774	.9140	.9760	.9972
	12π	.8360	.8136	.9839	1.00	.8165	.7587	.9623	.9977
	18π	.8602	.7433	.9906	.9979	.9988	.6568	.9866	.9982

It is seen from the table above that an increase in the mass ratio M_B/M_V causes a reduction in amplification. The same is true for all ranges of frequency ratio but variation is predominant when the frequency ratio is close to unity. Reduction in amplification as low as 0.4799 for $f_B/f_V = 1$ is seen when mass ratio increases from .01 to 0.1. For an increase in bridge frequency from 3π to 18π , the reduction in amplification for $M_V/M_B = .01$ to 0.1 for $f_B/f_V = 1.0$ is almost close to 50% for $f_c = 0.5$ and 25% for $f_c = 2.5$. The variation of amplification with mass ratio M_V/M_B is shown in Fig. 23 in which initial compression B is varied from 0.0 to 0.858. The variation is linear but amplification reduces with an increase in mass ratios. Linear variation is explained by the fact that B is a multiplier and magnitudes of deflection increase linearly with an increase in the dynamic force.

3.9.4 Effect of Crossing Frequency f_c and Phase Angle ϕ

These parameters cause the change in location of the

maximum amplifications on span. The number of vehicle-oscillations while it is on span are governed by the crossing frequency f_c . This is shown in Fig. 24(a), (b), (c) and (d). For the same span, the crossing frequency in Fig. 24(b) and (d) is 1-1/2 times more than that shown in Fig. 24(a) and (c). This results in six and nine peaks as shown in Fig. 24. The effect of the initial phase angle ϕ is to change the position of the peaks on span. Figure 24(a) is shown when the vehicle travels with the initial phase angle $\phi = 1.5\pi$ and Fig. 24(c) the angle ϕ is 0.5π . This results in a minimum peak at centre of the span in Fig. 24(a), while that in Fig. 24(c) is a maximum peak at centre, which corresponds to a phase difference of 180° . Similar cases are shown in Fig. 24(b) and (d). The parameter ϕ can be therefore accounted for by assuming that the location of maximum positive amplification is at the centre of span. This gives upper bound values of dynamic deflections and takes into consideration the optimum effects of parameter ϕ .

3.10.0 Table of Results - Design Data

3.10.1 Contents

Numerical data covering deflections, moments, frequencies of vibrations and amplification factors is presented in a series of tables. The selection of parameters was made on its influence and the ease of its determination at any stage of design and range of parameters was decided so as to cover a wide range of possible practical cases.

Parameters which have relatively small influence (e.g. Poissons's ratio on frequencies) are given in the form of correction tables

while those having an important influence, but not deterministic (e.g. initial conditions of bridge and vehicle) are treated as individual cases. Tabulation is as detailed below.

3.10.2 Static Analysis (deflections and moments)

Coefficients of deflections and moments are tabulated for combinations of rigidity ratio D_y/D_x , aspect ratio b/a , and torsion parameter μ . The deflection coefficients are given as a multiplier to $\frac{Pa^4}{ab} \times 10^{-1}$ and moment coefficients M_x, M_y as a multiplier to $\frac{Pa^2}{ab}$. The values correspond to under load P at the centre of the plate. The intervals and range of parameters are as given below:

- i) Rigidity ratio $\frac{D_y}{D_x}$: .01 to .1, 10 intervals of .01
.1 to 0.2, 4 intervals of .15
- ii) Aspect ratio $\frac{b}{a}$: 1.0 to 0.5, 6 intervals of 0.1
- iii) Torsion parameter : 0.1 to 0.9, 9 intervals of 0.1.

Deflection coefficients are given in Table 14 while moment coefficients M_x and M_y are given in Tables 15 and 16, respectively.

3.10.3 Frequencies of Vibration (frequency ratios)

Frequency ratios are tabulated for combinations of $\frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4}$, torsion parameter μ and cross mode n for only two cases of Poisson's ratio parameter $(D_1/D_y)/\sqrt{D_x/D_y}$. The absolute values of frequencies in cycles per second can be found by multiplying the ratios by $\frac{m \pi^2}{a^2} \sqrt{\frac{D_x}{\rho_0}}$. The intervals and range of parameters are:

- i) $m_c \left(m \frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4} \right)$: 0.25 to 2, 8 intervals of 0.25,
2.0 to 5.0, 7 intervals of 0.5
5.0 to 10.0, 5 intervals of 1.0.
- ii) Torsional parameter μ : 0.1 to 0.9, 9 intervals of 0.1
- iii) Cross mode n : 2 to 6, two symmetrical ($n = 3, 5$) and three antisymmetrical modes.
- iv) Poisson's ratio parameter $(D_1/D_y)/\sqrt{D_x/D_y}$: two cases, 0 and 0.1.

The range of variation of frequency ratio R with $(D_1/D_y)/\sqrt{D_x/D_y}$ between 0.0 and 0.10 is covered by correction coefficients. The frequency ratios for $n = 2, 3, 4, 5$ and 6 are given in Tables 17, 18, 19, 20 and 21 respectively. The correction coefficients for variation of frequency with Poisson's ratio parameter $(D_1/D_y)/\sqrt{D_x/D_y}$ between 0.0 and 0.1 and $n = 2$ to 6 are given in Table 22.

3.10.4 Forced Vibrations - Amplification Factors

Computations of amplification factors involve simultaneous treatment of coupled independent bridge and vehicle systems having a discrete range of parameters in each of them. From the bridge design point of view, the vehicle parameters are considered independent and as such there can be almost an infinite combination of parameters involving both the systems. Attention is therefore paid to the range of these parameters which cover most of the practical cases. The range of common parameters, mass ratio M_v/M_b , frequency ratio f_b/f_v , crossing frequency f_c and bridge frequencies f_b which cover most of the practical cases are given in the table below.

Range of Parameters

Parameter	Range							
f_c	0.5π	π	1.4π	1.8π	2.2π	2.5π	-	-
M_R	0.01	0.3	0.05	0.08	0.10	0.20	-	-
f_B	3π	6π	9π	12π	15π	18π	-	-
f_R	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0

The number of practical cases which combine these parameters exceed 1760. Amplification factors for these actual combinations are computed. The intermediate ranges not covered by these cases can be computed by interpolation.

Important parameters which include initial bridge conditions are treated for three typical cases.

i) Maximum Amplification factors for passing of one vehicle are given for all combinations of the above parameters in Table 23(a) to (f) for individual six crossing frequencies $\pi v/a$. These correspond to normal magnitudes of amplifications for use in the design.

ii) Maximum amplifications for passing of two vehicles in succession are given for the above cases in Table 24(a) to (f) for six individual cases of crossing frequencies $\pi v/a$. These figures can be treated as applicable to cases where the above average traffic density is applicable.

iii) Maximum amplifications for passing of three vehicles under optimum conditions are given in Table 25(a) to (e) for six individual crossing

frequencies. These can be treated as theoretical upper limits possible under assumed optimum conditions.

For all combinations of the parameters, the vehicle initial conditions were assumed to be $B = 0.3$ and $\phi = 1.50$. To get the optimum effects corresponding to ϕ , the amplifications given in Tables 23, 24 and 25 can be assumed to be at the centre for finding the magnitude of dynamic deflections. If the effect of B is to be accounted for in cases other than $B = 0.3$, the values of amplification can be obtained by linearly increasing or decreasing in the proportion of increment or decrement in B . This can be seen in Fig. 23 where the effect of variation in B on amplification is plotted for constant values of the remaining parameters, and change in amplification appears to be linear for all possible values of B .

CHAPTER 4

OPTIMUM DESIGN

4.1.0 Introduction

This Chapter deals with the method of optimum design of a plate subjected to static and dynamic loads. The conventional design procedure gives sections which satisfy the deflection and stress levels under static loads. Once the relative dimensions of slabs and stiffeners are finalized to satisfy these requirements, the resulting values of frequencies and mass densities become fixed and therefore cannot offer any alternatives to minimize amplifications under dynamic loads. This is overcome in the proposed method of analysis. The approach is based on minimizing the amplifications under dynamic loads in the first stage of analysis and the section thus finalized is then made to satisfy the deflection and stress requirements under static loading conditions. The approach is based on the minimum weight concept, which aims at obtaining relationships between geometric dimensions of slab and stiffener for given dimensions of width and span. This is presented in the following stages:

1. Concept and definition of optimum design.
2. Parameters considered.
3. Relationship of parameters for optimum sections.
4. Design charts - data.
5. Relative importance of parameters for interpretation in analysis.
6. Comparative accuracy.

4.2.0 Optimum Design Requirements

The bridge designed on the maximum weight basis has the following properties:

1. Maximum strength/weight ratio;
2. Satisfies design requirements under static loads;
3. When combined with vehicle parameters, the resulting combinations tend to minimize amplification factors under dynamic loads;
4. Parameters governing optimum solution should be accountable in the initial stages of design.

The cross sectional dimensions of the plate satisfying these requirements are defined as ideal in the analytical process. However, sections can be designed close to these theoretically ideal properties by proper combinations of the parameters.

4.3.0 Selection of Parameters for Optimum Design

A minimum weight design results in smaller mass density (ρ_0) per unit area of the plate. This reduction in mass density lowers magnitude of the bridge mass M_B and causes higher values of bridge frequencies (f_B) since frequency is inversely proportional to the mass per unit area of the plate. These higher values of frequencies (f_B) and lower values of mass M_B of the bridge result in a higher frequency ratio (f_B/f_v) and a higher mass ratio (M_v/M_B) for a given vehicle having frequency f_v and mass M_v . Thus with higher frequency ratio it is possible to minimize the amplification factors by

- (i) avoiding resonant conditions; and

(ii) resulting amplifications in the range $f_R > 1$, are less sensitive to increase in f_R and further decrease with increase in f_R (Figs. 20 and 21, Section 3.9.2).

Similarly, higher mass ratios result in decrease in amplifications (Fig. 22, Section 3.9.3) and give economical sections of lighter weight. Further, the parameter mass ratio M_V/M_B can be accommodated in design requirements under static loads. As frequency is the ratio of stiffness and mass, higher values of frequencies accompany higher stiffnesses with minimum cross sectional areas of the section. This leads to higher values of strength to weight ratios.

Thus, the resulting solution closely satisfies the requirements 1 to 4 of an ideal section in 4.2.0. Therefore, it is concluded that minimum weight design can be based on

1) Maximization of frequencies of bridge

or,

2) Maximization of mass ratios M_V/M_B

or

both.

4.4.0 Optimization Definitions

The fundamental frequency of bridge (f_B) of span a , rigidity D_x and mass density ρ_o is given by

$$f_B = \frac{m^2 \pi^2}{a^2} \sqrt{\frac{D_x}{\rho_o}} \quad (93)$$

in which $m = 1$. The fundamental frequency is assumed to be independent of the aspect ratio b/a . As higher frequencies are multiples

of the fundamental frequencies, it is sufficient to maximize the fundamental frequency f_B in Eq. 93. This is done by adjusting relative geometric dimensions of slab and stiffeners to give maximum value of the ratio D_x/ρ_o for given values of the span. As this process relates to maximizing geometric dimensions of the cross section of the bridge, it is termed geometrical optimization. Maximization of the mass ratio M_v/M_B to produce the same deflections relates to the structural behaviour under external loads and is termed structural optimization.

4.4.1 Geometric Optimization

The cross section of a typical beam-slab bridge is shown in Fig. 5.

The following dimensionless parameters are introduced:

$$\begin{aligned}\alpha_s &= \frac{h}{H} \\ \beta_s &= \frac{t}{s} \\ \xi_s &= \frac{t}{h}\end{aligned}\tag{94}$$

in which h - thickness of slab

H - total thickness of section

t - width of stiffener

s - centre to centre spacing of stiffeners

Assuming the plate and stiffener are made of the same material having modulus of elasticity E and density ρ , the expression $\frac{D_x}{\rho_o}$ is computed using Eqs. 78 to 82. The expressions for calculating the ratio $\frac{D_x}{\rho_o}$ in terms of the dimensionless parameters

α_s , β_s and ξ_s (Eq. 94), are derived in Table 26.

The relationship between α_s , β_s and ξ_s for maximization value of D_x/ρ_s is obtained by varying the value of one parameter while keeping the values of the remaining parameters constant.

4.4.2 Structural Optimization

The deflection δ_c of a bridge of span a , aspect ratio b/a under point load P at the centre can be given by

$$\delta_c = \frac{Pa^4}{ab\sqrt{D_x D_y}} \cdot K_D \quad (96)$$

where K_D is the coefficient of deflections.

Rewriting Eq. 96,

$$\frac{\delta_c}{\rho A_s} = \frac{Pa^4}{\rho A_s ab\sqrt{D_x D_y}} \cdot K_D \quad (97)$$

in which

ρ = density of the material of slab and stiffener

A_s = equivalent cross sectional area per unit width of section and is given by

$$A_s = \frac{1}{s} \{s \times h + (H-h)t\}$$

and from Eq. 94

$$A_s = H \{ \alpha_s + \beta_s (1 - \alpha_s) \} \quad (98)$$

The term $\rho A_s ab$ in Eq. 97 is the total weight of the bridge.

Assuming that the load P represents the vehicle, the ratio $P/\rho A_s ab$ represents the mass ratio M_v/M_b of the system, i.e.

$$M_R = \frac{P}{\rho A_s g b} \quad (99)$$

Using Eq. 97, Eq. 99 is rewritten

$$M_R = \frac{\delta_c \sqrt{D_x D_y}}{a \rho A_s K_D} \quad (100)$$

Using $D_y = \frac{E H^3}{12(1-\nu^2)} \alpha_s^3$, Eq. 98 and D_x from Table 26, Eq. 100 is expressed in terms of dimensionless parameters α_s , β_s , ξ_s and K_D . Maximum values of M_R are then obtained by varying the value of one parameter while keeping the values of a , ρ and δ_c constant. The term δ_c/a is then included in Eq. 100 to obtain deflection per unit span.

Thus in both cases, it is seen that maximization of frequency and mass ratios includes treatment of parameters ξ_s , β_s and α_s .

4.5.0 Range of Dimensionless Parameters α_s , β_s and ξ_s .

This is chosen so as to include the possible practical ranges and are given below:

$$\begin{aligned} \alpha_s &= 0.15 \text{ to } 0.50 \text{ through } 0.05 \\ \beta_s &= 0.10 \text{ to } 0.9 \text{ through } 0.10 \\ \xi_s &= 1.0 \text{ to } 3.5 \text{ through } 0.5 \end{aligned} \quad (101)$$

4.6.0 Relationship Between α_s , β_s and ξ_s for Maximum Frequency

and Mass Ratios

The variation of frequency with α_s and β_s and mass ratios with α_s , β_s and $\xi_s = 1$ and 3 is given in Tables 27 and 28 respectively.

The frequency coefficients are obtained as multiplier to $H \sqrt{\frac{E_s m \pi^2}{\delta a^2}}$ and mass coefficients as multiplier to $\frac{\delta c}{a^4 \rho} \times 10^{10}$. The frequency coefficients are obtained for $\xi_s = 1$ and mass coefficients are given for two values of ξ_s , i.e., $\xi_s = 1$ and $\xi_s = 3$. The values of β_s at which the maximum frequencies and mass ratios are obtained, are shown in the last two columns of Tables 27 and 28. The variation of mass ratios and frequencies against β_s is plotted in Figs. 24(a,b,c,d) to 27(a,b,c,d) for six values of α_s and $\xi_s = 1.0$.

It can be seen from Figs. 24 and 27, that both mass and frequency coefficients increase with increase in β_s , attain a maximum value after which decrease with increase in β_s . The magnitude of β_s where maximization is seen depends on magnitudes of α_s , but appears to be higher with higher values of α_s . This is clear from Figs. 28 and 29(a,b), in which curves in Figs. 24 to 27 are plotted on the same scale. The peaks corresponding to maximum frequencies occur at higher values of α_s with increase in β_s (Fig. 28). A similar trend is observed in Fig. 29(a,b) for $\xi_s = 1$ and $\xi_s = 3$ for maximum mass ratios. In both cases, absolute peak values of frequencies and mass coefficients decrease with increase in α_s and β_s . The physical interpretation of increase peak and decrease of these coefficients with increase in β_s is of importance from design considerations. For constant α_s , increase in β_s increases the area and stiffness of the section. Increase in values of coefficients indicate that rate of increase in stiffness is more than that in area. Peaks indicate optimum combination of stiffness and area while decrease indicates that the rate of increase in dead weight is more than that in stiffness. Peak frequency coefficients are

less sensitive to α_s and β_s as compared to mass ratios as they reduce from 0.315 to 0.295 for a change in α_s from 0.15 to .45. The variation in peak mass ratios reduces from 1.6 to 0.42 for $\xi_s = 1.0$ and 2.0 to .45 for $\xi_s = 3.0$. peak mass coefficients increase with increase in ξ_s and are less sensitive to β_s for $\alpha_s = 0.35$ and more. The peaks in Figs. 28 and 29 appear to lie on a smooth curve which indicates that the relationship between α_s and β_s follow a definite pattern in the region of peak values.

The values of β_s which correspond to peaks are plotted against α_s in Fig. 30(a,b). These were taken from the last two columns of Tables 27 and 28. It is seen that the relationship in Fig. 30(a,b) follow a linear variation and can be represented by a straight line as shown. The equation of these straight lines are

$$\begin{aligned} \beta_s &= 2\alpha_s \text{ for maximum frequencies} \\ \beta_s &= \{1.4 + 0.15 (\xi_s - 1)\}\alpha_s \text{ for maximum mass ratios} \end{aligned} \quad (102)$$

The accuracy of this straight line representation in Eq. 102 is compared in Table 29, in which the peak coefficients given in the last two columns of Table 27 and 28 are compared to those using Eq. 102. It is seen from Table 29 that the peak values of mass and frequency coefficients using relationship between α_s , β_s and ξ_s compared well with peaks in Tables 27 and 28. It is therefore concluded that the coefficients α_s , β_s and ξ_s can be calculated by linear relationship given by Eq. 102 for obtaining peak coefficients of mass and frequencies.

4.7.0 Limitations

These two criteria of geometric and structural optimizations given by Eq. 102 cannot be satisfied simultaneously by a given optimum section. Table 30 gives frequency coefficients computed using frequency and mass maximization. The frequency coefficients in Table 30 calculated using structural optimization are consistently less as compared to those using geometric optimization. The same trend in coefficients of mass ratios calculated using these concepts is also noted. This indicates that a given section can be optimized either to give maximum frequencies or maximum mass ratios. However, in actual design practice, these two criteria can be considered as two boundaries including an area which contains sections close to both.

4.8.0 Applications to Design

This section deals with obtaining deflections, frequency and mass coefficients of optimized sections for known values of α_s and ξ_s . The range of parameters were chosen to include practical ranges as given below:

Thickness of slab (h) = 6, 7, 8, 9, 10, 11 inch

Aspect ratio (b/a) = 1.0, 0.9, 0.8, 0.7, 0.6 and 0.5

α_s (h/H) = 0.10, 0.15, 0.2, 0.25, 0.3, 0.35

ξ_s (t/h) = 1.0, 1.5, 2.0, 2.5, 3.0, 3.5

(103)

Optimum sections for each of the above combinations were determined by using Eq. 102 to determine β_s for both cases.

The deflection, mass and frequency coefficients are obtained as multiplier to common parameters as given below:

Coefficient	Multiplier	
1) Deflection K_D	$\frac{Pa^4}{ab \sqrt{D_x D_y}} \times 10^{-2}$	
2) Frequency K_F	$H \sqrt{\frac{E_D}{\rho}} \times \frac{\pi^2}{a^2}$	(104)
3) Mass ratio K_M	$\frac{\delta_c \sqrt{D_x D_y}}{a^4 \rho H} \times 10^3$	

Rigidities D_x and D_y in Eq. 104 of optimum sections corresponding to combinations of parameters given in Eq. 103 are given in Table 31 for section giving maximum mass ratio and in Table 32 for sections giving maximum frequencies. Using these values of rigidities, absolute values of deflections, frequencies and mass ratio of optimum sections can be obtained.

4.9.0 Limitations of Deflections from Mass Ratio

The multiplier to mass ratio coefficient given in Eq. 104 is $\frac{\delta_c \sqrt{D_x D_y}}{a^4 \rho H} \times 10^3$. The term δ_c can be expressed to suit the design specifications 3.3.5 of CSA Standard S6-1966 which specifies that the ratio deflection to span shall not exceed 1/600. The optimum mass ratio can be therefore expressed as

$$M_R = \frac{1}{600a^3} \cdot \frac{\sqrt{D_x D_y}}{\rho H} \times 10^3 \times K_M \quad (105)$$

where K_M is the coefficient of mass ratio.

4.10.0 Design Charts - Mass ratios, Frequencies, Deflections and Influence Surfaces - of Optimum Sections

4.10.1 Mass Ratios, Frequencies and Deflection Coefficients

These coefficients are obtained as multipliers given in Eq. 104 and are presented for combinations of parameters given in Eq. 103. The coefficients K_M , K_D and K_F of optimum sections were obtained for both criteria of optimizations and are presented in a total of 38 units of individual combinations of parameters as given in Table 33. The charts were prepared for variations of one parameter for constant values of remaining parameters and are given in Figs. 31 to 37 which are detailed in Table 33.

4.10.2 Influence Surfaces of Deflections

The influence surfaces of deflections of optimum sections for load at centre are given for the following combinations:

$$A_R = 1.0, \text{ and } 0.5, \text{ through interval of } 0.1$$

$$\xi_s = 1.0 \text{ to } 3.0 \text{ through interval of } 0.5$$

$$\alpha_s = 0.25 \text{ to } 0.50 \text{ through interval of } 0.05.$$

These are given as detailed in Tables 34(a) and 34(b).

Deflections of optimum sections were obtained using mass maximization and results are presented for quarter plate in each of the above 60 individual cases. The combinations not covered can be obtained by interpolation using central deflections in Figs. 35 to 37. Influence surfaces are given in Fig. 38 to Fig. 48 and individual combinations are detailed in Table 34(a) and 34(b). The coefficients are obtained as

a multiplier to $\frac{Pa^4}{ab \sqrt{D_x D_y}} \times 10^{-2}$ and therefore absolute values can be obtained by using $\sqrt{D_x D_y}$ which are given in Table 31 for depth of slab 6" to 11" given in six intervals.

4.11.0 Relative Influence of Parameters α_s and ξ_s on Mass, Frequency and Deflections (K_M , K_F and K_D) Coefficients of Optimum Sections

4.11.1 Influence of ξ_s on K_M , K_F and K_D : (Figs. 31, 34, and 35)

The coefficient K_M and K_F increases with increase in ξ_s , the rate of increment being more sensitive in lower ranges of α_s . Coefficient K_D decreases with increase in ξ_s . Thus, increase in mass ratios and frequencies results in a decrease in deflection coefficients.

4.11.2 Influence of A_R on K_M , K_F and K_D : (Figs. 32 and 36)

Coefficient K_M decreases with an increase in aspect ratio b/a . The reverse trend is seen for variation K_D with an increase in b/a . This indicates that longer spans are accompanied by higher coefficient of mass ratios which result in smaller relative deflections. The coefficient K_D is independent of A_R as fundamental frequency is assumed independent of aspect ratio b/a .

4.11.3 Effect of α_s on K_M , K_F and K_D : (Figs. 33, 34 and 37)

This has relatively more influence on coefficients K_M , K_F and K_D as compared to ξ_s and A_R . The coefficients K_M and K_F decrease rapidly in lower ranges $\alpha_s = 0.25$ and thereafter the rate is smaller. The selection of a particular mass ratio or frequency can be very

easily obtained by varying this single parameter α_s as compared to ξ_s . The coefficient K_D increases with an increase in α_s . This indicates that higher frequencies and mass ratios follow smaller deflections and are obtained in lower values of α_s .

4.12.0 Magnitudes of Coefficients K_M , K_F and K_D by Two Criteria of Optimizations (Figs. 31 to 37)

The magnitude of coefficient K_M is more than K_F when calculated by mass maximization. This is seen in Figs. 31 to 33 in which solid lines corresponding to mass maximization and are always above the dotted lines which are for frequency maximization. Reverse trend is seen when frequency maximization is used and is seen in Fig. 34(b) where frequencies shown by dotted lines are above solid ones. The area contained by these two lines decrease with an increase in α_s , and ξ_s .

Comparing coefficient K_D in Fig. 35 to Fig. 37, it is seen that smaller deflections are obtained using frequency maximization for same combinations of parameters. This can be taken as a measure of the comparative economy achievable by the two criteria of optimizations. It is therefore concluded that frequency maximization is preferable to follow as smaller deflections are obtained for the same combinations of the remaining parameters.

CHAPTER 5

EXPERIMENTAL PROGRAMME

5.1.0 Introduction

The experimental investigations were basically designed to verify the accuracy of

- a) Orthotropic formulation for static and dynamic analysis of plate having slab and longitudinal stiffeners;
- b) Empirical formulation for computing frequencies of orthotropic plates;
- c) Concepts of equivalent and interconnected beam idealization of two dimensional behaviour of orthotropic plate through plate to beam transition.

The accuracy of orthotropic formulation is evaluated by comparing deflections, strains under static loads and frequencies of vibrations by tests on the same models having identical boundary conditions. The accuracy of empirical formulation of frequencies was evaluated by comparing theoretical and experimental values with those obtained using empirical equations. The concepts of equivalent and interconnected beam idealization of two dimensional orthotropic plate was verified by comparing distribution of amplification factors under and away from vehicle runs. Models were fabricated to cover a wide range of orthotropic rigidities for (a), (b) and (c) and aspect ratio b/a to cover sufficient range of plate beam transition which was considered important especially in case (c).

This is presented in two stages; namely, experimental programme which is given in this chapter and comparison of results are given in Chapter 6.

5.2.0 Experimental Programme

Experimental investigations were designed to obtain deflections, strains, frequencies of vibration and dynamic amplification factors. Models were fabricated in four different series to cover a wide range of rigidity ratio D_x/D_y and each series included five models to cover a wide range of aspect ratio b/a . The test set up was designed to simulate the hinged and roller boundary conditions of the bridge, using fixed pedestal and roller bearing at the ends of the models. Tests were carried out on models and deflections, strains, frequencies of vibrations and dynamic strains were obtained under static and dynamic loads. The plate was divided into imaginary beams 1-1, 2-2, and 3-3 (Fig. 60), and the vehicle was made to run on beams 1-1 and 2-2. The initial conditions of the vehicle were simulated to present tension and compression in the springs corresponding to $\phi = 1.5\pi$ and 0.5π , thus giving four vehicle tests for each plate. Dynamic strains were obtained at three pre-selected locations at the centre of beams 1-1, 2-2 and 3-3 (Fig. 60) at each of these four vehicle runs on the plate. The velocity of vehicle was maintained constant for all tests. The results thus obtained were later used for comparisons.

5.3.0 Details of Models

Models were fabricated using aluminium alloy 6061-T6 having the following properties:

- 1) Tensile strength - 38,000 psi
- 2) Yield strength (0.2%) - 35,000 psi
- 3) Shear strength - 30,000 psi
- 4) Modulus of elasticity - 10×10^6 psi
(Tension and compression)
- 5) Modulus of elasticity in shear - 3.8×10^6 psi
- 6) Poisson's ratio - $1/3$
- 7) Weight - 0.098 lbs/cu.in.

Aluminium plates $1/16"$ and $1/8"$ thick were used to represent slab and stiffeners were made of aluminium rectangular bars i) $1/4" \times 1/4"$, ii) $1/4" \times 1/2"$ and iii) $3/4" \times 1/4"$. Models were fabricated in four different series A, B, C and D each having different rigidity ratio D_x/D_y and each of five models in one series were of different aspect ratios b/a . The details of these models are given in Tables 35(a) and (b). Models corresponding to series A, B and C were chosen to have aspect ratios 1.0, 0.8, 0.67, 0.57 and 0.5 and those for series D were 1.0 and 0.5. The rigidity ratios (D_x/D_y) covered are 9.25 minimum for series B to 154.56 maximum for series C in four intervals. The width of all models for series A and B was 20 in. for all five models in one series and width of models in series C and D was 12 in.

5.4.0 Fabrication

5.4.1 Fabrication of Models

The plates were first cut to the dimensions and five symmetrically placed stiffeners were fastened to the plate using 10/32" dia., 7/8" long flat slot headed brass machine screws placed at 3" centre to centre. Alignment of stiffeners was first done by fixing them to the plate by epoxy glue and holes were then drilled for passing the brass screws through the plate and stiffeners. The arrangement of screws holding the plate and stiffener was staggered in such a way that at any cross section of the plate, only two screws appear on the section. Preliminary tests indicated that only the glue was insufficient to hold the stiffener in position when the plate was vibrated by an electromagnetic device. However, the glue alone was sufficient to hold the plate and stiffener together for static tests. Models corresponding to Series C and D were used only for static tests and as such no screws were used to strengthen the bond between the plate and stiffeners. Models A and B were mainly designed for frequency and dynamic tests in addition to tests under static loads. Additional precautions, therefore, were taken to tighten the screws using 10/32 in steel hexagonal nuts using tooth lock washers. Countersunk holes were drilled from the top of the plates so that the top surface of the plate is left sufficiently smooth after the screws were tightened. This was done to create sufficiently smooth paths for passage of vehicles during the forced vibration tests. The remaining portion of the screws projecting from the lower sides of the stiffeners was used to apply

concentrated static loads at selected points of the plate. Typical models are shown in photos in Figs. 48(a) to 48(d).

5.4.2 Fabrication of Test Frame

✓ The test frame was built to give clear inside dimensions 7'0" long, 24" wide and 42" high using 3" x 6" x 3/16" steel box sections. Horizontal runners of 7'0" long were fixed to the columns using steel angles 4" x 4" x 1/4" from the inside and 6" x 6" x 1/4" steel plates from the outside. Two cross pieces of the box sections were mounted on the runners using 1" dia., 18" long steel studs. One of the cross pieces was kept fixed at 12" from one end of the frame and the other was adjustable to accommodate the model of a given span length. The frame selected was comparatively heavy so that it provides relatively solid supports to models and have negligible vibrations during frequency tests of the models. Inside dimensions of 24" x 84" of the frame were designed to accommodate the model of 20" x 40" (max.). The clear length of 44" remaining on either side of the model was designed to provide road and stopping arrangements for the vehicle during forced vibration tests. The plan and end elevation of the test frame are shown in Figs. 49 and 50, and the photo in Fig. 51.

5.4.3 Simulation of Boundary Conditions

Simply supported boundary conditions which represent hinged and roller ends of the deck in the fields were identically simulated using fixed pedestal and roller bearings respectively in the model tests. End beams consisting of 3/4" x 3/4" steel square bars were fixed to the

ends of the models. The length of square sections was made equal to the width 20" of models and was provided with grooves to house the stiffeners so that a uniform contact was maintained between the plate and the beams at all points (Fig. 52). The plate was fixed to the edge beams using six screws and nuts to avoid relative slipping during bending of the plate. The end portions of the square beams were turned to a 5/8" dia. on both sides which in turn hold the bearing (5/8" x 1-1/2") at their ends. The height, 1-1/2", of bearings was designed to give 3/8" clear distance between the top of the cross piece and the lower side of edge beams. The bearings were supported on a cross piece so that rotation and horizontal movement of the edge of model was possible when loads were applied to it. This side of the model represented the roller end. To provide stability to the roller end during free vibration tests, loads were applied to the edge beams at three locations using wires. These wires pass through the slits made in the cross pieces, and carry loads at their ends. This arrangement made rotation and horizontal movement of the roller end possible without lifting.

Similar arrangements of fixing the plate to the edge beam were made except the roller bearings were replaced by pedestal bearings and pedestals were fixed to the cross piece of the test frame. This ensured only rotation avoiding the horizontal travel and was considered to represent the hinged end. This system worked very well during static, free and forced vibration tests. Each time the model was replaced by separating it from the edge beams without disturbing preset boundary conditions. The details of the free end are given in Fig. 52 and the photos in Fig. 53.

5.5.0 Fabrication of Vehicle

The essential features of a vehicle trolley are given in Figs. 54 and 55 and the photo in Fig. 56. It consists of

1. A base frame and wheels
2. Vehicle springs of known stiffness
3. Calibration and anchoring device and
4. Sprung mass

5.5.1 Base Frame with Wheels

The base frame consisting of 1" x 1/8" wide longitudinal beams, 1" x 1/2" wide cross beams with 2" x 2" supporting block at the rear ends was made by chipping excess material from the 6" long, 4" wide and 1" deep solid aluminium block. The 2" x 2" solid block was made to support the cylinder containing the spring and anchoring device as shown in Fig. 54(a) and (b) in plan & front elevation. At the four corners of the frame, 1/3" thick solid portion was left to support screws supporting the wheels aligned on a 4" wide, 5" long rectangle. Wheels were made up of 7/8" dia roller bearings and were fixed to the frame leaving a gap of 1/16" between the outer surface of the frame and inner side of the bearings. This ensured a smooth rotation of the bearings without locking. The mounting, which holds the sprung mass, was located at 1" from the rear end, which on span of 5", resulted in 20% and 80% distribution of the total weight on the front and rear axles.

5.5.2 Vehicle Spring

The vehicle spring in Fig. 55(a) and (b) was made from standard spring steel with eight coils arranged in 1-1/2" height. The spring was mounted between the lower end of the plunger and above the lower surface of the trolley frame. The top and bottom coils of the spring were soldered to the projecting ends of the plunger and the anchoring bolt as shown in Fig. 55. This avoided the relative slipping of the spring during oscillation of sprung mass, mounted on the top of the plunger. The lower screw holding the spring was positioned in such a way that resulting oscillations of sprung mass are always vertical.

5.5.3 Calibration of Vehicle Spring

The spring was calibrated for determining its stiffness characteristics by putting loads on top of the plunger. The resulting compression of the spring was measured using 1/1000 dial gauges. The load deflection curve is shown in Fig. 57. The average unit load deflection was 18.5×10^{-3} in. and corresponding stiffeners was 54.0 lbs per inch. The load compression tests of the spring were repeated and a fair consistency in the behaviour of the spring was seen.

5.5.4 Calibration and Anchoring Device

The vehicle output and initial conditions of the vehicle were measured using a perspex cantilever beam of 6" long, 3/4" wide and 1/16" thick, Figs. 42 and 43. The cantilever was mounted on an aluminium column 3/4" x 1/6" x 5" high, fixed near the front end of the trolley. The free end of the cantilever was anchored to the plunger using a 1/2" dia. mild steel stud as shown in Fig. 55(a) and

(b). A straining gauge was mounted near the fixed end of the cantilever and output of the strain gauge was calibrated against tension and compression of the spring for a given vehicle load. This is shown in Fig. 58. The details are given in the photos in Fig. 56. Load deflection and load strain calibration data for the vehicle is given in Table 36. An anchoring device consisting of a steel lever 3-3/4" long, 3/8" wide and 1/8" thick mounted on an aluminium column 3/4" x 1/4" and 2-3/4" fixed near the oscillating plunger, was used to adjust the initial tension or compression of the spring to the required level, (Fig. 55(a) and (b)). Horizontal notches were made in the top portion of the plunger and using the front end of the steel lever, anchoring of the plunger to the required height before it entered the plate gave the necessary level of initial tension or compression in the spring. Unlocking the plunger was done by a horizontal piece fixed to the frame and near the edge of the model. This rotates the steel lever, thus releasing the anchor and setting the vibrations from the required level of compression and tension in the vehicle spring.

5.5.5 Sprung Mass

The sprung mass of the vehicle was made up of the plunger of 1/2" dia and 2" long which in turn was supporting weights of known magnitudes to get the required frequency range of the vehicle. These weights were firmly positioned over top of the plunger using 1/4" steel studs fixed to its top using aluminium packing plates. The cylinder which houses the plunger shown in Fig. 55(a) and (b), was 1-1/2" inside diameter and 2-1/4" in height. This was fixed to

the trolley frame. There were 10 vertical lines of ball bearings symmetrically arranged at an angular distance of 36° inside the walls of the housing cylinder. The outer diameter of these bearings was 1-1/2". This ensured smooth and frictionless vertical oscillations of the plunger and the supporting weights.

5.6.0 Instrumentation

The instrumentation was designed to measure:

- 1) deflection, strains under static loads,
- 2) frequencies of vibrations imposed by electromagnetic device, and
- 3) dynamic strains during forced vibration tests.

5.6.1 Deflections and Strains

Standard 1/1000" dial gauges were used to measure the deflections at quarter points and along the width of the model. Magnetic bases of the gauges were supported on an independent frame. Static and dynamic strains were obtained using the strain gauges of the following type:

Gauge type: EA-13-125AP-120

single arm self temp. compensating

Resistance in ohms: $120 \pm 0.15\%$

Gauge factor: $2.09 \pm 0.5\%$

Gauge size: 1/8" x 3/8" coil size

Temp. range: 400°F - static

500°F - dynamic

Strain range: $\pm 30,000$ to $\pm 50,000$ micro strains.

The gauges were fastened to the stiffeners using M-200 bond adhesives. Separate two-point terminals were used for each gauge to secure stability of the gauge during vibration of models.

5.6.2 Electromagnetic Device

The standard electromagnetic device in two separate units was used for setting up vibrations in the plate. The electromagnet was mounted on the frame and the arrangement for adjusting the gap between the model and the magnet was as shown in Fig. 51(a,b). The unit was provided with a reading dial of 0 to 100 cps with a multiplier switch of 10 for reading frequencies in the higher modes. The unit is shown in Fig. 59.

5.6.3 Digital Data Acquisition System

The SY256 data acquisition system was used to measure static and dynamic strains output of the models. The system is essentially a modular remote monitoring equipment designed specifically for monitoring strain gauges. It is equipped with a capacity for inputs for 1,2,4 active arm gauges to be read directly in micro strains. The strain output can be either from an output indicator or in the form digital output from the printer which synchronizes with the operation of the system. The printing speed can be adjusted to read 20 gauges in one second or 20 readings of one gauge in one second. The former was used for taking strain outputs under static loads while the latter was used for print-output of dynamic strains of models subjected to action of moving vehicles. The sequence of operation involve

- 1) calibration of each channel to minimize error,
- 2) balancing of each channel for no load - zero reading.

The system is equipped for giving repeat output of the same gauge which helps to give stabilized strains for higher accuracies. The system can be used for strain output from 60 gauges with a capacity to accommodate more channels if needed.

5.7.0 Testing

The test programme was divided into the following stages:

- a) Load deflection strains under point load at centre and distribution along longitudinal cross sections at centre and quarter points of the plate.
- b) Frequencies of vibration.
- c) Influence lines travelling point loads and vehicle static load.
- d) Dynamic strains under moving vehicles.

The programme of testing is detailed in Table 37 giving models, instrumentation in static frequency and forced vibration tests.

5.7.1 Load Deflections-Strains Under Static Load

Tests were carried out as per columns 2, 3, 4 and 5 of the programme schedule given in Table 37. Strains and deflections were measured at the centre, quarter and end points as given in Fig. 60. Tests on typical models are given in the photos in Fig. 61. Data are presented in Tables A1 to A7 and individual test data are detailed in Table 41.

Models corresponding to Series A and B were tested and

deflection at 9 points and strain at the centre under point load at centre are obtained. Models C1 and C5 were tested for deflection at 9 and 15 points respectively. For both C1 and C5, strains were obtained at 9 points. Models C2, C3 and C4 were tested only for deflection at the centre. Models D1 and D5 were tested for deflections at 15 points while strains were obtained at 9 points for D5. For all the models, incremental loads were applied at the centre, using loading pans suspended by a wire tied to projecting screws. Tests were repeated twice to see the consistency and readings were taken both during loading and unloading cycles. The deflections inside the plate were obtained using 1/1000" gauges and those along the free edges were obtained using 1/1000" gauges. Strains were directly obtained using the data acquisition system.

5.7.2 Frequencies of Observation

The models corresponding to Series A and B were tested for frequencies of vibration. This was considered on the basis of dimensions. Models corresponding to Series C and D were smaller and difficulty was experienced in mounting the models and making them stable during vibration tests. The stiffeners were glued to plates for models C and D and in preliminary tests, it was seen that the glue was not strong enough to hold the stiffener and the plate together. The test schedule is given in columns 6 and 7 of Table 37 and photos of typical mode shapes are given in Fig. 62(a) to 62(d). The numerical values of frequencies of fundamental and higher modes are given in Table 48 (a) and (b) for models in Series A and B respectively.

Models were made ready by fastening small galvanized iron

plates 3" x 5" near the hinged ends and on the centre line of models. This was found necessary for inducing vibrations to the aluminium plate using an electromagnetic device. The electromagnet was mounted on a cross piece fixed to box sections of the main frame, and is shown in the photos in Fig. 59 and Fig. 62. The magnet was adjusted over the iron plate in such a way that a small gap was left between the magnet and the plate. The model was then made stable by suspending weights from the roller end. The plate was then vibrated at continuously increasing frequencies and magnitude was made steady when the accompanying amplitude was excessive with synchronizing noise. The sand was then spread on the model to find the accompanying mode shape. The numerical values were read from the oscillator dial corresponding to each mode shape. The magnitude of fundamental frequency was verified by excessive amplitudes of vibrations which was accompanied by a large synchronizing noise. Frequencies as high as $m = 6$ and $n = 4$ were obtained and modes thus obtained are given in Table 38(a) and (b).

5.7.3 Influence Lines

These tests were done for calibrating of beams 1-1, 2-2 and 3-3 for forced vibration analysis. The tests schedule is given in columns 8 and 9 in Table 37. The strain-deflection data, both under travelling static load and vehicle, was collected as per detailed test schedule given in Table 39. The travelling static loads were applied at three selected points on line 1-1, 2-2 and 3-3 and strain deflection at points CC2, CL2, CLL2, CC3, CL3 and CLL3 (Fig. 60) were obtained for models in Series A and B. This data are given in Table A8 for

model B and in Table A9 for model A. Strain output from these points was obtained for static vehicle position at $a/4$, $a/2$ and $3a/4$ on line 1-1 and 2-2 and is given in Tables A10 and A11 for models B and A. Details of the data given in Tables A10 and A11 are given separately in Table A2.

5.7.4 Forced Vibration Tests

Forced vibration tests were done on models in Series A and B. The vehicle initial conditions were set to $\phi = 0.5\pi$ and 1.5π and $B = 0.50$ in both cases. The vehicle load was 20.5 lbs. The vehicle frequency f_v was obtained from stiffeners characteristic of the vehicle spring given in Fig. 57 and was 5 cps. The magnitude of strain on the calibrating beam for $B = 0.5$ was obtained from Fig. 58. This was adjusted to ± 1700 micro strains for initial tension and compression which correspond to 0.5 times the vehicle load. The vehicle was set to these initial conditions by adjusting the lever position. The vehicle vibrations starting from these initial values were set by releasing this lever by a cross arm which was fixed to the frame and above the roller end of model A. A ramp 16-1/2" long was made near the roller end of the model. Slope of 1 vertical to 5 horizontal was adjusted on the ramp and each time the vehicle was released from the preset position on the ramp. The vehicle position at the entrance was so adjusted that the centre line of vehicle mass lies vertically above the stiffeners marked 1-1 and 2-2 in Fig. 60. For each vehicle run, time dependent strain output from 3 strain gauges on stiffeners at centre sections, was obtained. This was repeated for two vehicle runs on 1-1 and 2-2 and for each run with initial condition

for $\phi = 0.5\pi$, $\phi = 1.5\pi$ and $B = 0.5$. The schedule of these test programmes is given in Table 40. The test set up is shown in the photos in Fig. 63(a) and (b). During the initial tests, the vehicle run was guided by two horizontal beams clamped at the ends of the models. But this was discontinued in the later stages as the vehicle run was found sufficiently straight and without any deviations. The vehicle speed was maintained constant for each run by releasing it from the same position on the ramp. For each model, accurate timings were determined by time dependent digital output from the data acquisition system. These tests were repeated as travelling load experiments by anchoring the plunger and time taken by the vehicle to give maximum strain at the centre was found. For smaller models 0/20" span, the fifth reading was maximum. Similar experiments were conducted on all the models. The velocity of the vehicle thus determined was 3.33 ft/sec. This gave minimum 2-1/2 cycles of vehicle oscillation for 20" span to 5 cycles for 40" span. Forced vibration tests data is given in Tables A12 to A20 and in Appendix 5 and the details of these is given in Table 43.

5.8.0 Mass and Frequency Ratios of the Models

The maximum frequency of models in Series A and B was 71.50 and 75 cps respectively. This frequency was for the models with aspect ratio $b/a = 1$. The minimum frequency was for $b/a = 0.5$ and was 17.88 and 18.75 cps for Series A and B respectively. This rate of frequency of models was very high as compared to vehicle frequency 5 cps. The frequencies of models were therefore reduced by tying weights uniformly from the lower side of the models. The resulting frequencies

and mass ratios for models in A and B are given in Table 44. These values were used to obtain theoretical distribution of amplification factors. The values of frequencies and mass ratio before weights were tied are given in Table 44.

5.9.0 Theoretical Results - Models

5.9.1 Deflections and Strains

The deflections 10^{-3} inch under unit load at the centre were calculated for models in Series A, B, C and D. These were obtained at $y/b = 0$ and $x/a = 0.1, 0.2, 0.3, 0.4$ and 0.5 ; and $y/b = 0.5, y/b = 0.2$ and 0.4 . The strains $\times 10^{-6}$ under unit load at the centre were calculated for models in Series A, B, C and D. Deflections and strains are given in Table 45(a) and (b) and Table 46.

5.9.2 Frequencies of Vibrations

Frequencies of vibrations for the first six symmetrical and antisymmetrical modes ($m = 1$ to 6 and $n = 1$ to 6) were calculated using empirical and theoretical methods. The distribution of weights per unit area was 0.0114 lb/in^2 for models in Series A and 0.01765 lb/in^2 for models in Series B. Using the rigidities from Table 35(b) and mass densities, the frequencies in cycles per second were obtained by multiplying frequency ratios R by $\frac{m^2 \pi^2}{a^2 2\pi} \sqrt{\frac{D}{\rho_0 x}}$. These were obtained using theoretical and empirical methods and are given in Tables 47, 48, 49 and 50.

5.9.3 Amplification Factors

The distribution of amplification factors along the span are given in Figs. 64 to 67. These are as given below:

- 1) Models B_1 to B_5 , $\phi = 1.5\pi$ - Fig. 64
- 2) Models A_1 to A_5 , $\phi = 1.5\pi$ - Fig. 65
- 3) Models B_1 to B_5 , $\phi = 0.5\pi$ - Fig. 66
- 4) Models A_1 to A_5 , $\phi = 0.5\pi$ - Fig. 67.

The mass and frequency ratios were taken from Table 44. The vehicle speed was 3.33 ft/sec. For theoretical analysis, the time span was divided into 200 intervals and thus the shortest time interval was 1/400 sec for 20" span and 1/200 sec for 40" span. For the vehicle frequency of 5 cps, the number of peaks as seen from Fig. 64(a) is 2-1/2 for B_1 and Fig. 64(e) is 5 for model B_5 . The peaks for models B_2 , B_3 and B_4 are between these values. The vehicle was assumed to enter the model from the left hand side.

These theoretical results are compared with experimental values and comparison is given in Chapter VI.

CHAPTER 6

COMPARISONS AND DISCUSSION

6.1.0 Introduction

This chapter deals with the comparison of experimental results and discussion of accuracy and concepts introduced in the theoretical investigation. This is presented in the following stages:

- 1) Comparison of deflections and strains under static loads.
- 2) Comparison of frequency ratios and frequencies by theoretical, empirical and experimental methods.
- 3) Comparison of dynamic deflections and strains under and away from moving vehicle.
- 4) Discussions of accuracy of results.
- 5) Discussions on applications.

6.2.0 Comparisons

6.2.1 Comparison of Deflections at Centre

Theoretical and experimental deflections under incremental loads applied at the centre are shown in Fig. 68 for five models of Series A. Similar variations for five models of Series B and C, and two models of Series D are shown in Figs. 69, 70 and 71 respectively. Experimental values were taken from Table A1 to A7 and theoretical results given in Table 45(a) were used for obtaining the deflections for given loads.

In general, close agreement between theoretical and experimental deflections is found for all 17 models in Series A, B, C and D.

A good degree of linearity and consistency was observed during repeated loading and unloading tests for these models. Experimental deflections are found to be closer to the theoretical values for all 17 models and are closer especially for models A4 and A5. Theoretical deflections for all models in Series B and D (Figs 69 and 71) are higher than experimental values. This trend is not seen for Series A and C (Figs. 68 and 70). Theoretical deflections for models A1, A2, A5, C1 and C5 are higher than the experimental, and for the remaining models, i.e., A3, A4, C2, C3 and C5 theoretical values are less. Assuming theoretical values as 1, the percent variation in experimental deflections for 17 models is given in the table below.

Percent Degree of Accuracy in Experimental
Deflection at Centre (Figs. 68 and 70) - $\left(\frac{\text{Exp.} - \text{Theory}}{\text{Theory}} \right)$

Series	Model No.				
	1	2	3	4	5
A	7.50	5.0	6.00	1.00	1.00
B	15.00	12.50	8.00	8.50	12.50
C	7.50	12.5	4.00	9.00	9.00
D	20.00	-	-	-	20.00

6.2.2 Comparison of Strains at Centre

Theoretical and experimental strains under incremental loads at the centre are shown in Fig. 72(a) to (d) for five models in Series A.

A similar comparison for models in Series B is shown in Fig. 73(a) to (d) and for three models in Series C and D in Fig. 73A. Close agreement between theoretical and experimental strains is seen for most cases and in particular for models A5, B1, B4 and C5. Experimental strains in the case of four models, i.e., A3, B1, B5 and D5 seems less than the theoretical strains while the remaining 9 models have higher experimental strains. Maximum deviation is seen for models A3 and C5. Experimental strains plotted in Figs. 72, 73 and 73A are calculated for standard load of 10 lbs for 13 models and are given in the table below.

Comparison of Theoretical and Experimental Strains ($\times 10^6$)
at Centre
Load 10 lbs

Series	Model									
	1		2		3		4		5	
	The.	Expt.	The.	Expt.	The.	Expt.	The.	Expt.	The.	Expt.
A	80	95	90	120	105	85	130	150	165	170
B	45	40	55	70	70	80	80	83	90	95
C	70	85	-	-	-	-	-	-	123	130
D	-	-	-	-	-	-	-	-	130	150

It can be seen that variation in experimental strains is consistent with model dimensions for most models except A3. Experimental strains for one series seem to vary in the same proportion with those of the theoretical values. Minimum error between theoretical and experimental strains for one model (A4) is 3.76%. Maximum error of 22% is for model C1 and the average error is between 6% and 15%.

6.2.3 Comparisons of Influence Lines of Deflections at Centre

Sections of Plate

The influence lines of theoretical and experimental deflections at centre sections ($x/a = 0$ and $y/b = 0, 0.2$ and 0.4) are shown in Fig. 74(a) to 74(c) and Fig. 75(a) to (c), for five models in Series A and B respectively. Influence lines of theoretical and experimental deflections at centre are shown in Fig. 76(a) to 76(d) for models C1, C5, D1 and D5 respectively. In these cases in Figs. 74, 75 and 76 unit load was applied between $x/a = 0$ to $x/a = 0.5$ and along $y/b = 0$. At the three sections under consideration, it is seen that experimental deflections closely follow theoretical values for any position of the load. The ratio of experimental deflections at $x/a = 0.25$ and $x/a = 0.50$ for models in Series A and B is given in the table below.

Ratio of Deflections at $x/a = 0.25$ and 0.5

at $y/b = 0, 0.2$ and 0.4

(curves in Figs. 74, 75 and 76)

Models	y/b	Models				
		1	2	3	4	5
A	0	0.73	0.72	0.68	0.71	0.65
	0.2	0.73	0.69	0.72	0.70	0.80
	0.4	0.70	0.73	0.76	0.73	0.70
B	0	0.77	0.71	0.67	0.77	0.73
	0.2	0.70	0.65	0.74	0.74	0.73
	0.4	0.80	0.70	0.71	0.72	0.74
C	0	0.77	-	-	-	0.68
D	0	0.68	-	-	-	0.69

From the values of ratios of deflections given in the table above, it is seen that the deflection pattern across sections $y/b = 0$, 0.2 and 0.4 for load travelling on $y/b = 0$ are similar and can be represented by the same equations. This supplements the basic assumption in interconnected beam idealization, that plate deflection can be represented by the same equations, which in this case is assumed sinusoidal.

Comparing the ratios in the above table with the theoretical ratio of deflections for the sine curve which is 0.707, it is seen that experimental values are close to the theoretical one and sufficiently close representation of deflection of plate is achieved by assuming the plate as a series of interconnected parallel beams.

6.2.4 Deflection on Free Ends (Unit load at centre)

The magnitudes of theoretical and experimental deflections on free ends at $x/a = 0.5$ and $y/b = 0.5$ are given in the table below. The percentage error assuming theoretical deflection as 100% for models in Series A and B is calculated and is given in the table below.

Comparison of Deflections on Free Ends

$x/a = 0.5$ and $y/b = 0.5$ - CLE2

Models	Defl. $\times 10^{-3}$ in	Model				
		1	2	3	4	5
A	Theory	0.358	0.843	1.852	2.866	4.5
	Expt	0.34	0.800	1.800	3.1	5.0
	% Error	5.00	5.1	3.56	8.2	11.10
B	Theory	0.203	0.586	1.233	2.183	3.475
	Expt	0.2	0.575	1.25	2.05	3.20
	% Error	1.45	1.83	1.38	6.10	7.9

The percent error in deflections at $x/a = 0.5$ and $y/b = 0.5$, for models C5 and D5 was seen to be 2.78% and 9.7%. It is seen that the experimental deflections on free ends, are very close to theoretical values and errors in higher orders are found in models having aspect ratios $b/a = 0.57$ and 0.5 . However, the maximum error is of the order of 11% for one model and for the remaining the errors are seen to be between 1% and 8%.

6.2.5 Comparison of Strains Away From Centre

Theoretical and experimental strains at sections $x/a = 0.5$ and $y/b = 0.2$ and 0.4 are given in the table below for models in Series B. These are calculated for load 10 lbs at centre and locations CL2 and CLL2 in Fig. 60. Experimental strains are taken from influence line tests given in Tables A8 and A9.

Comparisons of Strains $\times 10^{-6}$ Away From
Centre - load 10 lbs at centre

Locations $x/a=0.5$, $y/b=0.2$ (CL2), $y/b=0.4$ (CLL2)

y/b Models		Models									
		1		2		3		4		5	
		The.	Expt	The	Expt	The	Expt	The	Expt	The	Expt
0.2	B	31.0	26.5	46.58	- 55.10	48.0	65.18	60.70	74.50	66.0	
0.4	B	17.64	16.75	27.78	29.5	37.69	34.3	47.24	44.0	56.48	56.0

It is seen that experimental strains for models in Series B are less than the theoretical strains at both locations away from the centre. As seen for strains at the centre, a definite consistency in magnitude of strains with dimensions of models is seen for strains away

from the centre. The decrease in experimental strains with increase in distance from centre is seen in agreement with theoretical variation. The percent error in experimental strains at point $y/b = 0.2$ and 0.4 varies between 6% to 15% for five models and is almost negligible for model B5 and at $y/b = 0.4$. The average error is 9.40%. Experimental strains for models in Series A were obtained for $y/b = 0.4$ and were 21.70, 31.0, 58.0 and 67.0 for models A1, A2, A4 and A5 respectively for load 10 lbs at centre. The maximum error of 20% was seen for strains in model A1 and average error for all four models was 14.88%.

6.3.0 Accuracy of Deflections and Strains in Static Tests

Close agreement between theoretical and experimental deflections and strains, is seen for all tests. The percentage error calculated is of the order of 10% and less for most models. This trend is seen for deflections and strains at centre, free ends and quarter span both under print loads at centre and influence lines tests. However in some cases, error as high as 20% to 30% is seen. The possible reason may be adjustments of models at simply supported edges to simulate hinged and roller end conditions. It was observed during experimentation, that a slight overtightening of screws between plate and edge beam significantly affected the rotation and horizontal moments. This, in turn, was giving semi-fixed edge conditions. On the other hand, loose screws can cause slipping from edge beams. Before starting each test, bearings were checked and set in order to avoid stiffening effect at edges. Weights suspended from roller end were also observed to create similar effects.

However, in most tests, the results were seen to be sufficiently accurate for all practical purposes.

6.4.0 Frequencies of Vibration

Theoretical frequencies of vibration are calculated by solving i) transcendental frequency equations, ii) empirical equations, and these frequencies are compared with the experimental values.

6.4.1 Comparison of Theoretical Frequencies

Frequency ratios R and frequencies of vibrations for models in Series A and B are calculated using both methods and are given in Tables 47 to 50. Frequency ratios and frequencies in cycles per second for the first 6 symmetrical and antisymmetrical modes for 10 models in Series A and B are calculated by empirical method and are given in Tables 47 and 48; and those by theoretical method are given in cycles per second in Tables 49 and 50.

It is seen from the results of frequency ratios in Tables 47 and 50, that the frequency ratios calculated using the empirical method, are higher in all combinations of longitudinal and cross modes. This is so for all 10 models in Series A and B. However, the difference in magnitudes is small in the lower modes and the difference goes on diminishing and is almost negligible in higher values of m and n . Assuming the magnitudes of theoretical frequency ratios as 100%, the degree of accuracy of frequency ratios obtained by using the empirical method are given below for typical cases in Series A and B.

Comparison of Accuracy Percentage of Frequency Ratios Theoretical
and Empirical Methods
(based on values in Tables 47 and 48)

m	n	Model A			Model B		
		1	3	5	1	3	5
1	2	6.0	6.4	7.0	7.6	2.02	4.0
	4	2.54	6.70	3.94	7.2	2.03	6.0
	6	3.70	1.45	1.37	5.1	4.3	3.22
6	2	2.16	2.4	7.4	1.97	2.95	5.1
	4	1.43	1.78	2.84	1.34	2.10	2.8
	6	0.17	0.46	3.5	0.6	0.95	1.08

It is seen from the table above that maximum error is 8% in both cases. Empirical formulation gives comparatively more accurate results in higher modes and for $m = 6$ and $n = 6$, an average error between 0.2% to 1% is seen between the empirical and theoretical methods. In lower values of m , the error seems to reduce with increase in n . In both the methods, fundamental frequency is assumed to be equal to the corresponding beam frequencies and as such the frequency ratios for $m = 1$ and $n = 1$ equals unity.

6.4.2 Comparison of Experimental Frequencies (in cps) with Theoretical and Empirical Frequencies

A three-way comparison of theoretical, empirical and experimental frequencies of vibrations in cycles per second is shown in Table 51 for four models in Series A. A similar comparison is given in Table 52 for five models in Series B. It is seen that, experimental frequencies are close to both theoretical and empirical values for most

combinations. Empirical frequencies in cycles per second are higher than the theoretical for all combinations of longitudinal and cross modes. Comparing experimental frequencies in models A and B, it is seen that frequency in model A are close to the theoretical results than in model B. However, in both cases, experimental results are sufficiently accurate and close to both theoretical and empirical values.

6.4.3 Degree of Accuracy in Experimental Frequencies

Percent errors in experimental and empirical frequencies are given in Table 53 for four models in Series A. Similar values for five models in Series B are given in Table 54.

It is seen that maximum errors in experimental frequencies for models A1, A2, A4 and A5 are 4.84%, 8.04%, 10.78% and 7.95% respectively. The corresponding maximum errors in empirical values are 6.00%, 16.06%, 6.45% and 3.39%. Maximum errors in experimental frequencies for models B1, B2, B3, B4 and B5 are seen to be 7.16%, 8.19%, 12.47%, 16.67% and 13.88%, respectively. The corresponding maximum errors in empirical values are 4.15%, 5.05%, 8.33%, 11.47% and 7.24%, respectively. The average errors in experimental and empirical values for models in A and B are given in the table below.

Average Errors in Frequency by Experimental and Empirical Methods

Series	Error in	1	2	3	4	5
A	Emp.	3.30	7.10	-	3.0	2.6
	Expt.	3.40	5.00	-	4.90	4.30
B	Emp.	3.0	3.00	4.20	7.00	3.30*
	Expt.	5.0	5.40	9.70	7.40	7.70

Errors less than 1% were not accounted for calculating the above table. It is seen that maximum average errors in empirical and experimental values are 7.10 and 5.00 for Series A and 7.00 and 9.70 for Series B.

6.4.4 Comparison of Experimental Frequency Ratios

Theoretical, empirical and experimental values of frequency ratios are plotted against n in Fig. 77(a), (b) and (c) for models A1, A2 and A4, respectively. Similar curves for models A5, B1, B2 and B3, B4, B5 are given in Figs. 78(a), (b), (c) and 79(a), (b), and (c) respectively. Theoretical and empirical values are shown by solid and dotted lines while experimental values are shown by points plotted on the same scale. It can be seen that experimental points and trend of variation are close to theoretical and empirical curves. Frequency ratios are seen sensitive to both m and n but for $m = 1$, variation is sensitive in lower values of n and reaches almost infinity after $n = 3$ and more. This trend is predominant in models having aspect ratio $b/a = 1$. In higher ranges of $m = 6$ and more, frequency ratios, are less sensitive to values of n and tend to be asymptotic to beam frequencies.

It is thus seen that the experimental frequencies of vibrations are sufficiently close to theoretical frequencies. The method of empirical equations give frequencies of fundamental and higher modes which are close to theoretical and experimental frequencies.

6.5.0 Forced Vibrations

6.6.0 Theoretical Computations

6.6.1 Amplification Factors

The distribution of theoretical amplification factors for two initial conditions of vehicles, namely $B = 0.5$ and $\phi = 1.5\pi$ and 0.5π are calculated and are given in Figs. 64 and 66 for five models in Series B. Similar variation of theoretical amplifications for four models in Series A are given in Figs. 65 and 67. Theoretical values of mass and frequency ratios used for calculating amplifications, are taken from Table 40 and correspond to the models with weight tied uniformly from the lower side of models.

6.6.2 Dynamic Deflections

Dynamic deflections at three points located at the centre sections, namely $x/a = 0.5$ and $y/b = 0, 0.2, \text{ and } 0.4$ (CC2, CL2, CLL2 in Fig. 60) were obtained for five models in Series B and CC2 and CLL2 for four models in Series A. These were obtained for vehicles travelling on 1-1 and 2-2 (Fig. 60) and for two initial conditions of vehicle for each run. Influence lines of deflections at these points (CC2, CL2 and CLL2) are plotted in Fig. 80(a) to (d) for load travelling on 1-1 and in Fig. 80 (a') to (d') for load travelling on 2-2 for models in Series A. Similar influence lines for five models in Series B are given in Fig. 81(a) to (e) and Fig. 81 (a') to (e'). Dynamic deflections were then obtained using amplification in Figs. 64 to 67, influence lines of deflections in Figs. 80 and 81 and static weight of vehicle. Numerical values of dynamic deflections at any desired location can be obtained by product

of amplification and unit load deflection at that location and multiplying the product by the weight of vehicle. These are calculated at locations where positive and negative peaks of amplifications are seen and are given in the following tables:

Table 55(a) - Peak dynamic deflections at CC2, CLL2 for A1, A2, A4 and A5 - $B = 0.5$, $\phi = 1.5\pi$ and $\phi = 0.5\pi$

Table 55(b) - Peak dynamic deflections at CC2, CLL2 for A1, A2, A4 and A5 - $\phi = 0.5\pi$

Table 56(a) - Peak dynamic deflections at CC2, CL2, CLL2 for B1, B2, B3, B4 and B5 - $B = 0.5$ and $\phi = 1.5\pi$

Table 56(b) - Peak dynamic deflections at CC2, CL2, CLL2 for B1, B2, B3, B4 and B5 - $\phi = 0.5\pi$

6.6.3 Dynamic Strains

Dynamic strains corresponding to dynamic deflections in Tables 55 and 56 were obtained using vehicle crawl strains given in Fig. 82(a) to (d) and Fig. 82(a') to (d') for vehicle travel on 1-1 and 2-2. These correspond to models in Series A. Similar curves for models in Series B are given in Fig. 83. Using amplification factors given in Figs. 64 to 67, corresponding dynamic strains were obtained for vehicle travel on 1-1 and 2-2 for both models in Series A and B. The values were used for comparison with the experimental strains obtained during forced vibration tests.

6.7.0 Comparison of Dynamic Strains

There were nine models tested for 86 comparisons of dynamic strains, as per test schedule in Table 40. Comparisons of dynamic strains in plate is presented for two combinations, namely,

1. Dynamic strains in plate under vehicle run - 28 tests.
2. Dynamic strains in plate away from vehicle run - 58 tests.

6.7.1 Comparison of Dynamic Strains under Vehicle Run

These are presented for six typical combinations of two vehicle runs on 1-1 and 2-2 for two initial conditions of vehicles, i.e., $\phi = 1.5\pi$ and $\phi = 0.5\pi$ for models in Series A and B. Strain outputs from points CC2 and CL2 for five models in Series B and from CC2 and CLL2 in four models in Series A are compared in Figs. 84 to 89 as given below.

Comparison of Dynamic Strains under Vehicle
(Fig. 60 for symbols)

Fig. No.	Models	Output From	Vehicle Run	
84(a) to 84(e)	B1 to B5	CC2	1-1	1.5π
85(a) to 85(e)	B1 to B5	CC2	1-1	0.5π
86(a) to 86(d)	B1, B3, B4, B5	CL2	2-2	1.5π
87(a) to 87(d)	B1, B3, B4, B5	CL2	2-2	0.5π
88(a) to 88(d)	A1, A2, A4, A5	CC2	1-1	1.5π
89(a) to 89(d)	A1, A2, A4, A5	CC2	1-1	0.5π

Vehicle was assumed to enter the plate from the lefthand support. Experimental data output of dynamic strains given in Tables A12 to A20 was used for plotting in Figs. 84 to 89.

Theoretical values of dynamic strains are shown by dotted lines and corresponding experimental strains are plotted by solid curves. Theoretical curves were obtained by plotting strains at positive and negative peaks shown in Figs. 64 to 67 and joining these peaks by smooth curves. Experimental strains were plotted at time intervals obtained from

printed data output and by joining these points by smooth parallel curves. There is close agreement between the theoretical and experimental values of dynamic strains shown in Figs. 84 to 89. Trend, magnitude and distribution of dynamic strains are found to agree closely for most combinations. Theoretical values appear to be higher in the centre portion and in particular for long spans, the reason being damping effect is not considered in theoretical analysis and duration of vehicle on plate is higher for longer spans. Experimental negative peaks near left hand support in Fig. 84(b) is seen higher than theoretical values. This is attributed to impact effect felt by the plate when the vehicle just enters the plate. However, this is not seen in the remaining tests. There is consistent phase difference between positive and negative peaks of theoretical and experimental values. This trend is seen for most combinations given in Figs. 84 to 89. These phase differences are seen as experimental investigations were based on digital output of dynamic strains at a time interval between two successive readings of 0.05 sec. The fraction of this time interval occurring in theoretical distribution was not possible to consider in experimental results. The resulting error appears as positive and negative peaks not occurring at the same location of the plate. Experimental strains near the right hand supports are seen higher than theoretical values. This trend is seen for most combinations and in particular for models B2 and B4 in Fig. 84(b) and (d). The possible reason for this opposite trend is the possibility of secondary vibrations set up in weights tied to the model and its effect being felt by the model when vehicle effects near the three quarter spans are comparatively smaller than at the centre.

However, comparing trend, distribution and magnitudes of strains it can be seen that reasonable accuracy in theoretical results is possible when plates having aspect ratios as small as 1 are treated as equivalent beam for obtaining dynamic deflections and strains under the path of vehicle. Same accuracy is found for all models having aspect ratios ranging from 1 to 0.5 covering reasonable range of transition of plate to beam.

6.7.2 Comparisons of Dynamic Strains away from Vehicle

Comparison of theoretical and experimental dynamic strains in plate away from vehicle run are given for two vehicle runs 1-1 and 2-2, two vehicle initial conditions $\phi = 1.5\pi$ and 0.5π and strains obtained at 3 locations CC2, CL2 and CLL2 for five models in Series B and four models in Series A. These are given in fourteen combinations in Figs. 90 to 103 as detailed below.

Comparison of Dynamic Strains in Plate Away From Vehicle Run
(Fig. 60 for symbols)

Figure No.	Models	Vehicle Run	Output From	ϕ
90(a) to (d)	B1,B3,B4,B5	1-1	CL2	1.5π
91(a) to (e)	B1 to B5	1-1	CLL2	1.5π
92(a) to (d)	B1,B3,B4,B5	1-1	CL2	0.5π
93(a) to (e)	B1 to B5	1-1	CLL2	0.5π
94(a) to (e)	B1 to B5	2-2	CC2	1.5π
95(a) to (e)	B1 to B5	2-2	CLL2	1.5π
96(a) to (e)	B1 to B5	2-2	CC2	0.5π
97(a) to (e)	B1 to B5	2-2	CLL2	0.5π
98(a) to (d)	A1,A2,A4,A5	2-2	CC2	1.5π
99(a) to (d)	A1,A2,A4,A5	2-2	CLL2	1.5π
100(a) to (d)	A1,A2,A4,A5	2-2	CC2	0.5π
101(a) to (d)	A1,A2,A4,A5	2-2	CLL2	0.5π
102(a) to (d)	A1,A2,A4,A5	1-1	CLL2	1.5π
103(a) to (d)	A1,A2,A4,A5	1-1	CLL2	0.5π

The method of plotting experimental and theoretical strains was similar to that followed in plotting strains under vehicle run. It can be seen that the trend of variation of both static and dynamic strains is similar to those under vehicle run in Figs. 84 to 89. Initial impact effects at the time the vehicle just enters the plate is seen by higher negative peaks near the lefthand supports in Figs. 90(c), (d); 91(b), (d); 92(a), (d); 93(a), (b) and 97(b) for models in Series B; and in Figs. 98(c) and 103(b) for models in Series A. Theoretical peak strains at centre are higher for most combinations. Experimental strains are seen higher in 3/4 span near righthand supports for the majority of combinations given in Figs. 90 to 103. The possible reasons for these due to the deviations effect of vehicle damping or secondary vibrations set in due to weights tied to the plate as discussed before. The magnitudes of dynamic strains away from vehicle are smaller than those under vehicle when vehicle runs on 1-1. When vehicle run is closer to free edges, the strains near the free edges are seen more than the strains below the vehicle. The experimental strains for combination given in Figs. 90 to 103, are seen close to theoretical strains and a reasonable degree of consistency in trend, magnitudes and distribution of peaks is observed in both experimental and theoretical strains.

6.7.3 Accuracy of Experimental Dynamic Strains Under and Away From Vehicle Run

The trend of variation of theoretical and experimental dynamic strains was seen to be in close agreement in all the 86 tests given in Figs. 84 to 103. The magnitudes of both theoretical and experimental

strain were seen maximum when the vehicle is near the centre region of the plate, and are important from design consideration. The accuracy of dynamic strains is, therefore, calculated for positive peaks where maximum strains are seen. The phase difference between the theoretical and experimental peaks is neglected and only absolute maximum values of strains are given in Table 57 for these 86 combinations given in Figs. 84 to 103. Accuracy is expressed in terms of percent error taking theoretical values as 100%. It may be noted that when the vehicle run is on 1-1 and 2-2 locations CC2 and CL2 respectively are under the vehicle. It can be seen from Table 57 that magnitudes of peak strains are consistently in agreement for most combinations. The distribution of percent errors in dynamic strains under and away from vehicle run is as follows.

Distribution of Percent Errors in Dynamic Strains
Under and Away From Vehicle

Error Distribution	Under vehicle run		Away from vehicle run	
	No. of Tests	%	No. of Tests	%
0 - 5%	4	14.30	9	16.20
5 - 10%	8	28.60	20	35.80
10-15%	11	39.00	20	35.80
15 - 20%	4	14.30	5	8.90
above 20	1	3.80	2	3.30
Total Tests.	28	-	56	-
Max. error %	23.0		27%	

It is seen from the above tabulation that 80% of the total tests have an error between 0 to 15%. About 4% of the tests give

errors between 20 to 30%.

It is therefore seen that the present method of dynamic analysis based on equivalent and interconnected beam idealization of two dimensional plate structure, gives reasonably accurate results. A good degree of consistency in accuracy is observed for all models in both Series A and B which covers a reasonably wide range of plate to beam transition. Overall accuracy of the experimental programme is presented in Table 58 in which distribution of errors in experimental results is given for static frequency and forced vibration tests. Accuracy of frequencies computed by the empirical method is also given in Table 58. It is seen that the results are sufficiently accurate and the range of variation of accuracy in test results is within the acceptable range of accuracy for the design purposes.

6.8.0 Discussion of Methods and Suitability of Application to Design

A reasonable degree of accuracy is seen in deflections and strains at and away from the centre for all models in Series A, B, C and D. Experimental results were obtained by static tests on seventeen models covering a wide range of rigidity and aspect ratios. Theoretical analysis is based on orthotropic simulation and rigidity parameters were obtained by distributing stiffness across the width of the plate. From the results it is seen that orthotropic idealization based on stiffness distribution gives both deflections and strains of consistent accuracy. Frequencies of vibrations were obtained by tests on 9 models and results compared with those obtained using orthotropic theory. A good degree of accuracy is seen in frequencies of vibrations in fundamental and higher

modes. It was shown that deflection, strain and frequencies have opposite degrees of dependencies on orthotropic rigidity parameters. These static and dynamic tests on the same models indicate that the orthotropic simulation gives reasonably accurate results of deflections, strains and frequencies of vibrations.

Experimental frequencies are compared with frequencies computed by theoretical and empirical methods. It is observed that these results are in very good agreement. Empirical method of frequency analysis is seen simple as compared to the theoretical method which involves solution to transcendental frequency equations by trial and error. Accuracy of empirical approach is seen consistent for most combinations of rigidity parameters and two extreme ranges of orthotropy. This method can be used for computing frequencies reasonably accurately for all

Concept of equivalent and interconnected beam idealization is seen to give reasonably accurate results in forced vibration analysis of the plate. Two dimensional plate structures can be analyzed by introducing simple interconnected beam concepts and distribution of dynamic deflections and strains at any point of plate can be obtained. Exact theoretical analysis of computing distribution of dynamic strains and deflections is complex and cannot be easily incorporated in design analysis. The alternate method based on interconnected beam concepts gives sufficiently accurate results and can be easily incorporated in design process. The accuracy of this method is verified by 86 tests on models covering a wide range of rigidity and aspect ratios, and is seen to give accuracies sufficient for design purposes.

CHAPTER 7

CONCLUSIONS

This investigation deals with static free and forced vibration design analysis of simply supported longitudinally stiffened bridge decks. The analysis is based on orthotropic idealization in which rigidity parameters were computed using the principle of stiffness distribution. New concepts are introduced which considerably simplifies the complexity of analysis and also gives solution for economic design. These concepts are:

- 1) Geometric and structural optimization leading to minimum weight solution giving cross section of bridge, economical in both static and dynamic analysis, for given dimensions of span and width.
- 2) Development of empirical method for frequency computations of bridges in any range of orthotropy.
- 3) Forced vibration analysis by treating the plate as a series of interconnected beams.

The resulting solution is presented through a series of tables and charts, the interpretation of which, it is believed will be, of help for the static and dynamic design of such bridges. A combined programme covering static, free and forced vibration analysis is developed and is given in Appendix II. Theoretical results thus obtained are verified by extensive experimentation. The degree of accuracy was compared and good agreement between theoretical and experimental results is found. Based on these investigations the following conclusions are drawn:

1) The method of calculating orthotropic flexural and torsional rigidity parameters using the principle of stiffness distribution, gives sufficiently accurate results for design purposes. Other methods of computing orthotropic stiffeners need experimental verification by tests on models as results of static and frequency analysis have opposite degrees of dependencies on flexural and torsional rigidities of orthotropic plate.

2) Orthotropic flexural rigidities in orthogonal directions have more influence as compared to torsional rigidity of the plate. Therefore, more accurate presentation of flexural rigidities is of importance in orthotropic idealization.

3) The design based on principle of mass and frequency maximization tend to give sections close to minimum weight sections. The amplification factors for such sections are consistently less for any vehicle parameters. This gives a good control on dynamic effects while design is still in the static stage. Static deflection requirements, as per code, can be accommodated in the design process.

4) Frequency maximization is a function of two dimensionless parameters, namely

a) β_s , a ratio $\frac{\text{width of stiffener}}{\text{spacing of stiffeners}}$

and

b) α_s , a ratio $\frac{\text{thickness of slab}}{\text{total height of section}}$

and are related by $\beta_s = 2\alpha_s$ for maximum frequency for given width and span.

5) Mass maximization is a function of three dimensionless parameters, namely, β_s , α_s and ξ_s where

$$\xi_s = \frac{\text{thickness of stiffener}}{\text{thickness of slab}}$$

and are related by

$$\beta_s = [1.4 + 0.15 (\xi_s - 1)] \alpha_s$$

6) Sections giving simultaneous mass and frequency maximization can be obtained from these two independent relations and are simultaneously satisfied when $\xi_s = 5$, corresponding sections for which are seen impractical. However, these two optimization ranges can be assumed enclosing a zone and design can be based to lie in a zone close to both ranges.

7) Fundamental frequencies of these types of bridges can be closely represented by the corresponding beam frequency given by

$$\omega_{11} = \frac{\pi^2}{a^2} \sqrt{\frac{D_x}{\rho_0}} \text{ rad/sec}$$

8) The effect of Poisson's ratio parameter $(D_1/D_y) / D_x/D_y$ is more important for computing first antisymmetric mode ($n=2$). The frequencies corresponding to higher modes are less dependent on Poisson's ratio.

9) For higher modes, i.e., $m=6$ and above, the plate frequency is less dependent on n .

10) Frequencies are more dependent on orthogonal flexural rigidities than torsional rigidities. This trend is similar to deflections in static analysis.

11) Frequencies increase with an increase in torsional parameter, while deflections decrease with an increase in torsional parameters.

12) The empirical method gives sufficiently accurate values of fundamental and higher frequencies of vibration of orthotropic plates. This method is comparatively simple, and can be used even where access to computer is not available.

13) The equivalent beam action and distribution of deflection on width of orthotropic plate is a function of rigidity and aspect ratios. This trend should be noted while considering a plate as a series of interconnected parallel beams.

14) A single sprung mass idealization of theoretically multidegree of freedom system gives close approximation and is sufficiently accurate in results for design purposes.

15) A single degree idealization of multidegree plate system gives close approximation and plate dynamic strains under vehicle can be obtained by treating the plate as an equivalent beam for even a high degree of plate action. Results are consistent for wide range of plate beam transition.

16) Interconnected beam idealization gives sufficiently accurate results of distribution of dynamic deflection and strain at any point of the plate. This idealization is found to give sufficiently accurate results for plate structure where severe plate action is expected. The same accuracy in results of distribution of dynamic strains is seen for a wide range of plate to beam transition. This comparatively simple approach can therefore be successfully used in place of theoretically

exact two dimensional analysis which is complex and not always suitable for design.

17) Maximum amplifications can be assumed to be at centre to give optimum dynamic deflections. This design simplification is necessary, as vehicle initial conditions which partly govern the location of maximum amplifications cannot be precisely accounted for in the design.

18) Initial conditions of bridge (and vehicle) are not a predictable parameter in the design. These are, therefore, computed for individual cases. The proper values are to be selected depending on the density of traffic in the area. These are given for three cases, i.e., normal, intermediate and severe and corresponding amplification factors for more than 1700 practical cases are given in Stage I, II and III for design purposes.

19) Frequency optimization is seen to give lesser deflections as compared to mass maximization and as such is preferable from design considerations.

20) Design analysis simultaneously satisfying both criteria, namely, frequency and mass maximization, is seen to give section which are impractical in actual practice. However, by use of charts and tables, it is possible to finalize the design which is close to both the criteria of optimization.

21) The effect of critical combination of crossing frequency and vehicle phase angle on amplification, can be accounted for by assuming the maximum amplification is always at midspan.

APPENDIX I

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APPENDIX II

PROGRAMME

STATIC AND DYNAMIC ANALYSIS OF LONGITUDINALLY STIFFENED SIMPLY-
SUPPORTED ORTHOTROPIC DECKS OF HIGHWAY BRIDGES

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THIS PROGRAMME CALCULATES DEFLECTIONS, MOMENTS, FREQUENCIES OF
VIBRATIONS AND AMPLIFICATIONS OF SIMULATED ORTHOTROPIC DECK UNDER
STATIC AND DYNAMIC LOADS. THIS PROGRAMME IS DEVELOPED AS PART OF
THE PROJECT. IT CALCULATES FREQUENCIES OF VIBRATIONS FOR FIRST THREE
SYMMETRICAL AND ANTISYMMETRICAL MODES. DYNAMIC DEFLECTIONS USING
VEHICLE INPUT DATA OF FREQUENCY, SPEED AND INITIAL CONDITIONS OF
VIBRATIONS.

BASIS OF CALCULATIONS

STATIC ANALYSIS--SUPERPOSITION OF INDIVIDUAL SOLUTIONS OF
INFINITE ORTHOTROPIC STRIP AND FINITE ORTHOTROPIC PLATE.

FREE VIBRATION ANALYSIS--SOLUTIONS TO TRANSCENDENTAL FREQUENCY EQUATIONS
AND EMPIRICAL FREQUENCY EQUATIONS FOR EACH SYMMETRICAL AND
ANTISYMMETRICAL MODES. SOLUTION TAKES INTO ACCOUNT POISSONS RATIO
FORCED VIBRATION ANALYSIS--INTERCONNECTED AND EQUIVALENT BEAM
IDEALISATION OF ORTHOTROPIC PLATE.

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0018 DEN=0.10
0019 RHO=((TS)+((TR*WR)/S))*(DEN/384.4)
0020 ARWID/PNS
0021 AREA=((S*TS)+(TR*WR))/S
0022 AREA=AREA/(TS+TR)
0023 TR3=TR*TR*TR
0024 NU2=NU*NU
0025 TS2=Y*TS
0026 TS3=TS*TS
0027 DY=(E*TS3)/(12.0*(1.0-NU2))
0028 CG=((S*TS2*0.50)+(TR*WR*(TS+(0.50*TR)))/((S*TS)+(TR*WR))
0029 W=((WR*TR3/12.0)+(WR*TR*(TS+(0.50*TR)-CG))*2)
0030 XBAR2=12.0*((CG/TS)-0.50)**2)
0031 DX=(DY*(1.0+XBAR2))+(E*W/S)
0032 CNTT=((DX/(E*AREA))*0.50)/(TS+TR)
0033 CFRE=CNTT*(TS+TR)
0034 CD2=WR/TR
0035 IF((CD2).LE.1.0) GO TO 220
0036 GO TO 221
0037 C1=((1.80)+(EXP(1.65/((CD2)**0.50))))
0038 GO TO 223
0039 C1=((1.80)+(EXP(1.65/CD2))
0040 GO TO 223
0041 H=((0.60+E/(2.0*(1.0+NU)))*(WR*TR3/(C1*S))
0042 H=H+DY
0043 NU=H/((DX*DY)**0.50)
0044 D1=NU*DY
0045 DXV=(H-D1)/2.0
0046 RR1=NU
0047 RR2=(H+(2.0*DXV))/DY
0048 SPN=1.0
0049 TP11=(2.0/(1.0-HU))
0050 TP12=ABS(TP11)
0051 TP1=((DY/DX)**0.25)/PAY)*(TP12)**0.50)
0052 ALP=SPN*TP1
0053 TP2=((DY/DX)**0.25)/PAY)*(2.0/(1.0+HU))**0.50
0054 BTA=SPN*TP2
0055 T=1.0/(2.0*PAY2*(DX*DY)**0.50)
0056 R=SPN*T
0057 B=1.0
0058 DD 40 J=1.7
0059 IB=B
0060 PY=B*AR/TP1
0061 QV=B*AR/TP2
0062 CWLO=0.50
0063 PY1=CWLO*PY
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0064 PY2=(CWLO-1.0)*PY
0065 QY1=CWLO*QY
0066 QY2=(CWLO-1.0)*QY
0067 B3=RSIN(0.50*B*PAY)/B
0068 B2=B3*B
0069 TP3=RR1*PAY2 *ALP*BTA/(SPN*SPN)
0070 TP4=RR2*PAY2 *ALP*BTA/(SPN*SPN)
0071 Z1=(ALP/BTA)-(BTA/ALP)-TP3
0072 Z2=(ALP/BTA)-(BTA/ALP)-TP4
0073 BT2=1.0/(BTA*BTA)
0074 AP2=1.0/(ALP*ALP)
0075 ABT=1.0/(ALP*BTA)
0076 D(18.1)=
1 (B3/BTA)*(EXP(-QY1))*(Z1*SIN(PY1))-(2.0*COS(PY1))
D(18.2)=(B3/ALP)*(EXP(-QY1))*(Z1*COS(PY1))+(2.0*SIN(PY1))
D(18.3)=
1 (B3/BTA)*(EXP(-QY2))*(Z1*SIN(PY2))-(2.0*COS(PY2))
D(18.4)=(B3/ALP)*(EXP(-QY2))*(Z1*COS(PY2))+(2.0*SIN(PY2))
D(18.5)=-D(18.3)
D(18.6)=D(18.4)
D(18.7)=-D(18.1)
D(18.8)=D(18.2)
D(18.9)=
1 (B2/BTA)*(EXP(-QY1))*((-1.0/BTA)*((Z2*SIN(PY1))-(2.0*
1 COS(PY1)))+(1.0/ALP)*((-Z2*COS(PY1))+(2.0*SIN(PY1)))
D(18.10)=
1 SIN(PY1)))+(1.0/ALP)*((-1.0/BTA)*((Z2*COS(PY1))+(2.0*
1 SIN(PY1)))+(1.0/ALP)*((-Z2*SIN(PY1))+(2.0*COS(PY1)))
D(18.11)=(B2/BTA)*(EXP(-QY2))*((-1.0/BTA)*((Z2*SIN(PY2))-(2.0*
1 COS(PY2)))+(1.0/ALP)*((-Z2*COS(PY2))+(2.0*SIN(PY2)))
D(18.12)=
1 (B2/ALP)*(EXP(-QY2))*((-1.0/BTA)*((Z2*COS(PY2))+(2.0*
1 SIN(PY2)))+(1.0/ALP)*((-Z2*SIN(PY2))+(2.0*COS(PY2)))
D(18.13)=D(18.11)
D(18.14)=-D(18.12)
D(18.15)=D(18.9)
D(18.16)=-D(18.10)
RNI=(AP2+BT2)*((BTA*COS(PY1))-(ALP*SIN(PY1)))
RHT=(RR1*PAY2)*((ALP*SIN(PY1))+(BTA*COS(PY1)))
RH2=(1.0/(SPN*SPN))*RHT
D(18.17)=(B3)*(EXP(-QY1))*(RH1+RH2)
D(18.18)=D(18.17)
RH3=((ALP/BTA)*SIN(PY1))-COS(PY1)
RH4=((BTA/ALP)*SIN(PY1))-COS(PY1)
RH5=((ALP/BTA)*SIN(PY1))-COS(PY1)
RH6=(COS(PY1))-((BTA/ALP)*SIN(PY1))

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0101  D(IB,19)=(B2)*(EXP(-QV1))*(((AP2+BT2)*(RH3+RH4)))+(RR2*PAY2/(SPN*
0102  1SPN))*(RH5+RH6)))
0103  D(IB,20)=-D(IB,19)
0104  B=B+1.0
0105  40 CONTINUE
0106  A(1,1)=D(1,1)
0107  A(1,2)=D(1,2)
0108  A(1,3)=D(1,3)
0109  A(1,4)=D(1,4)
0110  A(2,1)=D(1,5)
0111  A(2,2)=D(1,6)
0112  A(2,3)=D(1,7)
0113  A(2,4)=D(1,8)
0114  A(3,1)=D(1,9)
0115  A(3,2)=D(1,10)
0116  A(3,3)=D(1,11)
0117  A(3,4)=D(1,12)
0118  A(4,1)=D(1,13)
0119  A(4,2)=D(1,14)
0120  A(4,3)=D(1,15)
0121  A(4,4)=D(1,16)
0122  BB(1)=D(1,17)
0123  BB(2)=D(1,18)
0124  BB(3)=D(1,19)
0125  BB(4)=D(1,20)
0126  CALL MATINV(A,4)
0127  M=1.0
0128  M2=M*M
0129  M3=M*M*M
0130  ALP1=M*CW*AR/TP1
0131  ALP2=M*CW*AR/TP2
0132  S1=SIN(ALP1)
0133  C1=COS(ALP1)
0134  SS1=SIN(M*PAY*CL)
0135  SS2=SIN(M*PAY*CLL)
0136  E1=EXP(-ALP2)
0137  E2=EXP(ALP2)
0138  AB=ALP/BTA
0139  BA=BTA/ALP
0140  TEMP1=((AB+C(1))*S1)+((1.0+C(2))*C1))*EI
0141  TEMP2=((AB+C(3))*S1)+(C(4)*C1))*E2
0142  DEF(1)=(BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0143  DAF(1)=DEF(1)
0144  D2X=-((M2*PAY2/(SPN*SPN))*((BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0145  S6=(((-AB*(C(1)+1.0))-(BA*(C(2)+1.0)))*(C1/ALP)

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0146 S7=((C(1)-C(2))*(-S1/ALP)
0147 S8=(((-AB*(C(1)+1.0))- (BA*(C(2)+1.0))))*S1
0148 S9=((C(1)-C(2))*C1
0149 S10=((AB*(C(3))- (BA*(C(4))))*(C1/ALP)
0150 S11=((C(3)+C(4))*(-S1/ALP)
0151 S12=((AB*(C(3))- (BA*(C(4))))*S1
0152 S13=((C(3)+C(4))*C1
0153 S14=((((S6+S7))+ (S8+S9))*(-1.0/BTA))))*E1
0154 S15=((((S10+S11))+ (S12+S13))*(-1.0/BTA))))*E2
0155 D2Y=(R/M)*((S14+S15)*SS1*SS2
0156 FEX(1)=-((DX*D2X)+(D1*D2Y))
0157 FEY(1)=FEY(1)
0158 FEY(1)=-((DY*D2Y)+(D1*D2X))
0159 FAY(1)=FEY(1)
0160 A(1.1)=D(1.1)+D(2.1)
0161 A(1.2)=D(1.2)+D(2.2)
0162 A(1.3)=D(1.3)+D(2.3)
0163 A(1.4)=D(1.4)+D(2.4)
0164 A(2.1)=D(1.5)+D(2.5)
0165 A(2.2)=D(1.6)+D(2.6)
0166 A(2.3)=D(1.7)+D(2.7)
0167 A(2.4)=D(1.8)+D(2.8)
0168 A(3.1)=D(1.9)+D(2.9)
0169 A(3.2)=D(1.10)+D(2.10)
0170 A(3.3)=D(1.11)+D(2.11)
0171 A(3.4)=D(1.12)+D(2.12)
0172 A(4.1)=D(1.13)+D(2.13)
0173 A(4.2)=D(1.14)+D(2.14)
0174 A(4.3)=D(1.15)+D(2.15)
0175 A(4.4)=D(1.16)+D(2.16)
0176 BB(1)=D(1.17)+D(2.17)
0177 BB(2)=D(1.18)+D(2.18)
0178 BB(3)=D(1.19)+D(2.19)
0179 BB(4)=D(1.20)+D(2.20)
0180 CALL MATINV(A,4)
0181 M=2.0
0182 M2=M*M
0183 M3=M2*M
0184 ALP1=M*CM*AR/TP1
0185 ALP2=M*CM*AR/TP2
0186 S1=SIN(ALP1)
0187 C1=COS(ALP1)
0188 SS1=SIN(M*PAY*CL)
0189 SS2=SIN(M*PAY*CLL)
0190 E1=EXP(-ALP2)
0191

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0192 E2=EXP(ALP2)
0193 AB=ALP/BTA
0194 BA=BTA/ALP
0195 TEMP1=((AB+C(1))*SI)+((1.0+C(2))*CI))*E1
0196 TEMP2=((BTA/M3)*R*(TEMP1+TEMP2))*SS1*SS2
0197 DEF(2)=DEF(1)+DEF(2)
0198 D2X=-((M2*PAY2/(SPN*SPN))*((BTA/M3)*R*(TEMP1+TEMP2))*SS1*SS2
0199 S6=(((-AB*(C(1)+1.0))-BA*(C(2)+1.0))*CI/ALP)
0200 S7=(((-AB*(C(1)+1.0))-BA*(C(2)+1.0))*SI
0201 S8=(((-AB*(C(1)+1.0))-BA*(C(2)+1.0))*SI
0202 S9=((C(1)-C(2))*CI
0203 S10=((AB+C(3))*SI)-((BA*C(4))*CI/ALP)
0204 S11=((C(3)+C(4))*SI/ALP)
0205 S12=((AB+C(3))-BA*(C(4))*SI
0206 S13=((C(3)+C(4))*CI
0207 S14=((S6+S7)+((S8+S9)*(-1.0/BTA))*E1
0208 S15=((S10+S11)+((S12+S13)*(-1.0/BTA))*E2
0209 D2Y=(R/M)*((S14+S15))*SS1*SS2
0210 FAX(2)=FEX(1)+FEX(2)
0211 FEY(2)=FEX(1)+FEX(2)
0212 FEY(2)=FEX(1)+FEX(2)
0213 FEY(2)=FEX(1)+FEX(2)
0214 FEY(2)=FEX(1)+FEX(2)
0215 A(1,1)=D(1,1)+D(2,1)+D(3,1)
0216 A(1,2)=D(1,2)+D(2,2)+D(3,2)
0217 A(1,3)=D(1,3)+D(2,3)+D(3,3)
0218 A(1,4)=D(1,4)+D(2,4)+D(3,4)
0219 A(2,1)=D(1,5)+D(2,5)+D(3,5)
0220 A(2,2)=D(1,6)+D(2,6)+D(3,6)
0221 A(2,3)=D(1,7)+D(2,7)+D(3,7)
0222 A(2,4)=D(1,8)+D(2,8)+D(3,8)
0223 A(3,1)=D(1,9)+D(2,9)+D(3,9)
0224 A(3,2)=D(1,10)+D(2,10)+D(3,10)
0225 A(3,3)=D(1,11)+D(2,11)+D(3,11)
0226 A(3,4)=D(1,12)+D(2,12)+D(3,12)
0227 A(4,1)=D(1,13)+D(2,13)+D(3,13)
0228 A(4,2)=D(1,14)+D(2,14)+D(3,14)
0229 A(4,3)=D(1,15)+D(2,15)+D(3,15)
0230 A(4,4)=D(1,16)+D(2,16)+D(3,16)
0231 BB(1)=D(1,17)+D(2,17)+D(3,17)
0232 BB(2)=D(1,18)+D(2,18)+D(3,18)
0233 BB(3)=D(1,19)+D(2,19)+D(3,19)
0234 BB(4)=D(1,20)+D(2,20)+D(3,20)
0235 CALL MATINV(A,4)
0236 CALL MATMPY(A,4,4, BB,1,C)
0237 M=3.0

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0238 M2=M*M
0239 M3=M2*M
0240 ALP1=M*CW*AR/TP1
0241 ALP2=M*CM*AR/TP2
0242 SI=SIN(ALP1)
0243 CI=COS(ALP1)
0244 SS1=SIN(M*PAY*CL)
0245 SS2=SIN(M*PAY*CLL)
0246 E1=EXP(-ALP2)
0247 E2=EXP(ALP2)
0248 AB=ALP/BTA
0249 BA=BTA/ALP
0250 TEMP1=((AB+C(1))*SI)+((1.0+C(2))*CI))*E1
0251 TEMP2=((AB+C(3))*SI)+(C(4)*CI))*E2
0252 DEF(3)=(BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0253 DAF(3)=DEF(1)+DEF(2)+DEF(3)
0254 D2X=- (M2*PAY2/(SPN*SPN))*((BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0255 S6=(((-M2*(C(1)+1.0))-BA*(C(2)+1.0))*CI/ALP)
0256 S7=(C(1)-C(2))*(-SI/ALP)
0257 S8=(((-AB*(C(1)+1.0))-BA*(C(2)+1.0))*SI
0258 S9=(C(1)-C(2))*CI
0259 S10=((AB+C(3))-BA*(C(4)))*(CI/ALP)
0260 S11=(C(3)+C(4))*(-SI/ALP)
0261 S12=((AB+C(3))-BA*(C(4)))*SI
0262 S13=(C(3)+C(4))*CI
0263 S14=((S6+S7)+((S8+S9)*(-1.0/BTA)))*E1
0264 S15=((S10+S11)+((S12+S13)*(1.0/BTA)))*E2
0265 D2Y=(R/M)*(S14+S15)*SS1*SS2
0266 FEX(3)=-((DX+D2X)+D1*D2Y)
0267 FAX(3)=FEX(1)+FEX(2)+FEX(3)
0268 FEY(3)=-((DY+D2Y)+D1*D2X)
0269 FAY(3)=FEY(1)+FEY(2)+FEY(3)
0270 A(1.1)=D(1.1)+D(2.1)+D(3.1)+D(4.1)
0271 A(1.2)=D(1.2)+D(2.2)+D(3.2)+D(4.2)
0272 A(1.3)=D(1.3)+D(2.3)+D(3.3)+D(4.3)
0273 A(1.4)=D(1.4)+D(2.4)+D(3.4)+D(4.4)
0274 A(2.1)=D(1.5)+D(2.5)+D(3.5)+D(4.5)
0275 A(2.2)=D(1.6)+D(2.6)+D(3.6)+D(4.6)
0276 A(2.3)=D(1.7)+D(2.7)+D(3.7)+D(4.7)
0277 A(2.4)=D(1.8)+D(2.8)+D(3.8)+D(4.8)
0278 A(3.1)=D(1.9)+D(2.9)+D(3.9)+D(4.9)
0279 A(3.2)=D(1.10)+D(2.10)+D(3.10)+D(4.10)
0280 A(3.3)=D(1.11)+D(2.11)+D(3.11)+D(4.11)
0281 A(3.4)=D(1.12)+D(2.12)+D(3.12)+D(4.12)
0282 A(4.1)=D(1.13)+D(2.13)+D(3.13)+D(4.13)
0283 A(4.2)=D(1.14)+D(2.14)+D(3.14)+D(4.14)

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0284 A(4,3)=D(1,15)+D(2,15)+D(3,15)+D(4,15)
0285 A(4,4)=D(1,16)+D(2,16)+D(3,16)+D(4,16)
0286 BB(1)=D(1,17)+D(2,17)+D(3,17)+D(4,17)
0287 BB(2)=D(1,18)+D(2,18)+D(3,18)+D(4,18)
0288 BB(3)=D(1,19)+D(2,19)+D(3,19)+D(4,19)
0289 BB(4)=D(1,20)+D(2,20)+D(3,20)+D(4,20)
0290 CALL MATINV(A,4)
0291 CALL MATMPY(A,4,4,BB,1,C)
0292 H=A,0
0293 M2=M*N
0294 M3=M2*M
0295 ALP1=M*CM*AR/TP1
0296 ALP2=M*CM*AR/TP2
0297 S1=SIN(ALP1)
0298 C1=COS(ALP1)
0299 S2=SIN(M*PAY*CL)
0300 S2=SIN(M*PAY*CLL)
0301 E1=EXP(-ALP2)
0302 E2=EXP(ALP2)
0303 AB=ALP/BTA
0304 BA=BTA/ALP
0305 TEMP1=((AB+C(1))*S1)+((1,0+C(2))*C1))*E1
0306 TEMP2=((AB+C(3))*S1)+((C(4)*C1))*E2
0307 DEF(4)=(BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0308 DAF(4)=DEF(1)+DEF(2)+DEF(3)+DEF(4)
0309 D2X=-M2*PAY2/(SPN*SPN))*((BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0310 S6=((AB+C(1))+1,0)-((BA*(C(2)+1,0))*C1/ALP)
0311 S7=((C(1)-C(2))*(-S1/ALP)
0312 S8=((AB*(C(1))+1,0))-((BA*(C(2)+1,0))*S1
0313 S9=((C(1)-C(2))*C1
0314 S10=((AB*(C(3)))-((BA*(C(4))*C1/ALP)
0315 S11=((C(3)+C(4))*(-S1/ALP)
0316 S12=((AB*(C(3)))-((BA*(C(4))*S1
0317 S13=((C(3)+C(4))*C1
0318 S14=((S6+S7)+((S8+S9)*(-1,0/BTA)))*E1
0319 S15=((S10+S11)+((S12+S13)*(-1,0/BTA)))*E2
0320 D2Y=(R/M)*((S14+S15)*SS1*SS2
0321 FAX(4)=FEX(1)+FEX(2)+FEX(3)+FEX(4)
0322 FEY(4)=FEX(1)+FEX(2)+FEX(3)+FEX(4)
0323 FAY(4)=FEY(1)+FEY(2)+FEY(3)+FEY(4)
0324 A(1,1)=D(1,1)+D(2,1)+D(3,1)+D(4,1)+D(5,1)
0325 A(1,2)=D(1,2)+D(2,2)+D(3,2)+D(4,2)+D(5,2)
0326 A(1,3)=D(1,3)+D(2,3)+D(3,3)+D(4,3)+D(5,3)
0327 A(1,4)=D(1,4)+D(2,4)+D(3,4)+D(4,4)+D(5,4)
0328 A(2,1)=D(1,5)+D(2,5)+D(3,5)+D(4,5)+D(5,5)

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0330 A(2,2)=D(1,6)+D(2,6)+D(3,6)+D(4,6)+D(5,6)
0331 A(2,3)=D(1,7)+D(2,7)+D(3,7)+D(4,7)+D(5,7)
0332 A(2,4)=D(1,8)+D(2,8)+D(3,8)+D(4,8)+D(5,8)
0333 A(3,1)=D(1,9)+D(2,9)+D(3,9)+D(4,9)+D(5,9)
0334 A(3,2)=D(1,10)+D(2,10)+D(3,10)+D(4,10)+D(5,10)
0335 A(3,3)=D(1,11)+D(2,11)+D(3,11)+D(4,11)+D(5,11)
0336 A(3,4)=D(1,12)+D(2,12)+D(3,12)+D(4,12)+D(5,12)
0337 A(4,1)=D(1,13)+D(2,13)+D(3,13)+D(4,13)+D(5,13)
0338 A(4,2)=D(1,14)+D(2,14)+D(3,14)+D(4,14)+D(5,14)
0339 A(4,3)=D(1,15)+D(2,15)+D(3,15)+D(4,15)+D(5,15)
0340 A(4,4)=D(1,16)+D(2,16)+D(3,16)+D(4,16)+D(5,16)
0341 BB(1)=D(1,17)+D(2,17)+D(3,17)+D(4,17)+D(5,17)
0342 BB(2)=D(1,18)+D(2,18)+D(3,18)+D(4,18)+D(5,18)
0343 BB(3)=D(1,19)+D(2,19)+D(3,19)+D(4,19)+D(5,19)
0344 BB(4)=D(1,20)+D(2,20)+D(3,20)+D(4,20)+D(5,20)
0345 CALL MATINV(A,4)
0346 CALL MATMPY(A,4,4,BB,1,C)
0347 M=B,0
0348 M2=M*M
0349 M3=M2*M
0350 ALP1=M*CW*AR/TP1
0351 ALP2=M*CW*AR/TP2
0352 S1=SIN(ALP1)
0353 C1=COS(ALP1)
0354 SS1=SIN(M*PAY*CL)
0355 SS2=SIN(M*PAY*CLL)
0356 E1=EXP(-ALP2)
0357 E2=EXP(ALP2)
0358 AB=ALP/BTA
0359 BA=BTA/ALP
0360 TEMP1=((AB+C(1))*S1)+((1.0+C(2))*C1))*E1
0361 TEMP2=((AB+C(3))*S1)+(C(4)*C1))*E2
0362 DEF(5)=(BTA/H3)*R*(TEMP1+TEMP2)*SS1*SS2
0363 DAF(5)=DEF(1)+DEF(2)+DEF(3)+DEF(4)+DEF(5)
0364 D2X=-((M2*PAY2/(SPN*SPN))*((BTA/H3)*R*(TEMP1+TEMP2)*SS1*SS2
0365 S6=(((-AB*(C(1)+1.0))-BA*(C(2)+1.0)))*(C1/ALP)
0366 S7=((C(1)-C(2))*(-S1/ALP)
0367 S8=(((-AB*(C(1)+1.0))-BA*(C(2)+1.0)))*S1
0368 S9=((C(1)-C(2))*C1
0369 S10=((AB+C(3))-BA*(C(4)))*(C1/ALP)
0370 S11=((C(3)+C(4))*(-S1/ALP)
0371 S12=((AB+C(3))-BA*(C(4)))*S1
0372 S13=((C(3)+C(4))*C1
0373 S14=((S6+S7)+((S8+S9)*(-1.0/BTA)))*E1
0374 S15=((S10+S11)+((S12+S13)*(-1.0/BTA)))*E2
0375 D2Y=((R/M)*((S14+S15)*SS1*SS2

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0376 FEX(5)=-((DX+D2X)+(D1+D2Y))
0377 FAX(5)=FEX(1)+FEX(2)+FEX(3)+FEX(4)+FEX(5)
0378 FEY(5)=-((DY+D2Y)+(D1+D2X))
0379 FAY(5)=FEY(1)+FEY(2)+FEY(3)+FEY(4)+FEY(5)
0380 A(1.1)=D(1.1)+D(2.1)+D(3.1)+D(4.1)+D(5.1)+D(6.1)
0381 A(1.2)=D(1.2)+D(2.2)+D(3.2)+D(4.2)+D(5.2)+D(6.2)
0382 A(1.3)=D(1.3)+D(2.3)+D(3.3)+D(4.3)+D(5.3)+D(6.3)
0383 A(1.4)=D(1.4)+D(2.4)+D(3.4)+D(4.4)+D(5.4)+D(6.4)
0384 A(2.1)=D(1.5)+D(2.5)+D(3.5)+D(4.5)+D(5.5)+D(6.5)
0385 A(2.2)=D(1.6)+D(2.6)+D(3.6)+D(4.6)+D(5.6)+D(6.6)
0386 A(2.3)=D(1.7)+D(2.7)+D(3.7)+D(4.7)+D(5.7)+D(6.7)
0387 A(2.4)=D(1.8)+D(2.8)+D(3.8)+D(4.8)+D(5.8)+D(6.8)
0388 A(3.1)=D(1.9)+D(2.9)+D(3.9)+D(4.9)+D(5.9)+D(6.9)
0389 A(3.2)=D(1.10)+D(2.10)+D(3.10)+D(4.10)+D(5.10)+D(6.10)
0390 A(3.3)=D(1.11)+D(2.11)+D(3.11)+D(4.11)+D(5.11)+D(6.11)
0391 A(3.4)=D(1.12)+D(2.12)+D(3.12)+D(4.12)+D(5.12)+D(6.12)
0392 A(4.1)=D(1.13)+D(2.13)+D(3.13)+D(4.13)+D(5.13)+D(6.13)
0393 A(4.2)=D(1.14)+D(2.14)+D(3.14)+D(4.14)+D(5.14)+D(6.14)
0394 A(4.3)=D(1.15)+D(2.15)+D(3.15)+D(4.15)+D(5.15)+D(6.15)
0395 A(4.4)=D(1.16)+D(2.16)+D(3.16)+D(4.16)+D(5.16)+D(6.16)
0396 BB(1)=D(1.17)+D(2.17)+D(3.17)+D(4.17)+D(5.17)+D(6.17)
0397 BB(2)=D(1.18)+D(2.18)+D(3.18)+D(4.18)+D(5.18)+D(6.18)
0398 BB(3)=D(1.19)+D(2.19)+D(3.19)+D(4.19)+D(5.19)+D(6.19)
0399 BB(4)=D(1.20)+D(2.20)+D(3.20)+D(4.20)+D(5.20)+D(6.20)
0400 CALL MATINV(A.4)
0401 CALL MATMPY(A.4,4,BB.1,C)
0402 H=6.0
0403 M2=M*M
0404 M3=M2*M
0405 ALP1=M*CW*AR/TP1
0406 ALP2=M*CW*AR/TP2
0407 SI=SIN(ALP1)
0408 CI=COS(ALP1)
0409 SS1=SIN(M*PAY*CL)
0410 SS2=SIN(M*PAY*CLL)
0411 E1=EXP(-ALP2)
0412 E2=EXP(ALP2)
0413 AB=ALP/BTA
0414 BA=BTA/ALP
0415 TEMP1=((AB+C(1))*SI)+((1.0+C(2))*CI))*E1
0416 TEMP2=((AB+C(3))*SI)+(C(4)*CI))*E2
0417 DEF(6)=(BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0418 DAF(6)=DEF(1)+DEF(2)+DEF(3)+DEF(4)+DEF(5)+DEF(6)
0419 D2X=-((M2*PAY2/(SPN*SPN))*BTA/M3)*R*(TEMP1+TEMP2)*SS1*SS2
0420 S6=(((-AB*(C(1)+1.0))-((BA*(C(2)+1.0))*CI/ALP)
0421 S7=(C(1)-C(2))*((-SI/ALP)

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0422 S8=(-AB*(C(1)+1.0))-((BA*(C(2)+1.0))) *SI
0423 S9=(C(1)-C(2)) *CI
0424 S10=((AB*(C(3)))-(BA*(C(4))))*(C*ALP)
0425 S11=(C(3)+C(4)) *(-SI/ALP)
0426 S12=((AB*(C(3)))-(BA*(C(4)))) *SI
0427 S13=(C(3)+C(4)) *CI
0428 S14=((S6+S7)+((S8+S9)*(-1.0/BTA))) *E1
0429 S15=((S10+S11)+((S12+S13)*(-1.0/BTA))) *E2
0430 D2V=(R/M)*(S14+S15) *SS1*SS2
0431 FEX(6)=-((OX+D2X)+(D1+D2Y))
0432 FAX(6)=FEX(1)+FEX(2)+FEX(3)+FEX(4)+FEX(5)+FEX(6)
0433 FEY(6)=-((DY+D2Y)+(D1+D2X))
0434 FAY(6)=FEY(1)+FEY(2)+FEY(3)+FEY(4)+FEY(5)+FEY(6)
0435 A(1.1)=D(1.1)+D(2.1)+D(3.1)+D(4.1)+D(5.1)+D(6.1)+D(7.1)
0436 A(1.2)=D(1.2)+D(2.2)+D(3.2)+D(4.2)+D(5.2)+D(6.2)+D(7.2)
0437 A(1.3)=D(1.3)+D(2.3)+D(3.3)+D(4.3)+D(5.3)+D(6.3)+D(7.3)
0438 A(1.4)=D(1.4)+D(2.4)+D(3.4)+D(4.4)+D(5.4)+D(6.4)+D(7.4)
0439 A(2.1)=D(1.5)+D(2.5)+D(3.5)+D(4.5)+D(5.5)+D(6.5)+D(7.5)
0440 A(2.2)=D(1.6)+D(2.6)+D(3.6)+D(4.6)+D(5.6)+D(6.6)+D(7.6)
0441 A(2.3)=D(1.7)+D(2.7)+D(3.7)+D(4.7)+D(5.7)+D(6.7)+D(7.7)
0442 A(2.4)=D(1.8)+D(2.8)+D(3.8)+D(4.8)+D(5.8)+D(6.8)+D(7.8)
0443 A(3.1)=D(1.9)+D(2.9)+D(3.9)+D(4.9)+D(5.9)+D(6.9)+D(7.9)
0444 A(3.2)=D(1.10)+D(2.10)+D(3.10)+D(4.10)+D(5.10)+D(6.10)+D(7.10)
0445 A(3.3)=D(1.11)+D(2.11)+D(3.11)+D(4.11)+D(5.11)+D(6.11)+D(7.11)
0446 A(3.4)=D(1.12)+D(2.12)+D(3.12)+D(4.12)+D(5.12)+D(6.12)+D(7.12)
0447 A(4.1)=D(1.13)+D(2.13)+D(3.13)+D(4.13)+D(5.13)+D(6.13)+D(7.13)
0448 A(4.2)=D(1.14)+D(2.14)+D(3.14)+D(4.14)+D(5.14)+D(6.14)+D(7.14)
0449 A(4.3)=D(1.15)+D(2.15)+D(3.15)+D(4.15)+D(5.15)+D(6.15)+D(7.15)
0450 A(4.4)=D(1.16)+D(2.16)+D(3.16)+D(4.16)+D(5.16)+D(6.16)+D(7.16)
0451 BB(1)=D(1.17)+D(2.17)+D(3.17)+D(4.17)+D(5.17)+D(6.17)+D(7.17)
0452 BB(2)=D(1.18)+D(2.18)+D(3.18)+D(4.18)+D(5.18)+D(6.18)+D(7.18)
0453 BB(3)=D(1.19)+D(2.19)+D(3.19)+D(4.19)+D(5.19)+D(6.19)+D(7.19)
0454 BB(4)=D(1.20)+D(2.20)+D(3.20)+D(4.20)+D(5.20)+D(6.20)+D(7.20)
0455 CALL MATINV(A.4)
0456 CALL MATMPY(A.4,4,BB.1,C)
0457 N=7.0
0458 M2=M*N
0459 M3=M2*M
0460 ALP1=M*CW*AR/TP1
0461 ALP2=M*CW*AR/TP2
0462 S1=SIN(ALP1)
0463 C1=COS(ALP1)
0464 SS1=SIN(M*PAY*CLL)
0465 SS2=SIN(M*PAY*CLL)
0466 E1=EXP(-ALP2)
0467 E2=EXP(ALP2)

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RGD=0.0
RRHQ=0.01765/386.40
MU2=MU*MU
CQ=1.0
WID=20.0
SPN=40.0
SPN2=SPN*SPN
AR=WID/SPN
M=1.0
DO 91 J=1.6
ZZZ=M*AR*((DX/DY)**0.25)
ZZZ=ZZZ*ZZ
C2=ZZZ
TC=PAY2*RGD*C2
M2=M*M
N=3.0

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N=3 AND 5 FOR SECOND AND THIRD SYMMETRICAL MODES.
N= 2, 4 AND 6 FOR FIRST, SECOND AND THIRD ANTISYMMETRICAL MODES.

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2233445566778899
5566778899001122
0000000000000000

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DD 101 JJ=1.2
AAC=((N-2.0))*0.000128
DD 30 K=1.7996
PHE=AAC*PAY*CO
PHE2=PHE*PHE
LHS=TAN(0.50*PHE)
AAC2=AAC*AAC
AC=((2.0*HU+C2)+(AAC2))*0.50
SE=(ABS(AC))*PAY
SE2=SE*SE
TEMP=(PHE2+YC)/(SE2-TC)
TEMP2=TEMP*TEMP
RHS=-((SE/PHE)*TEMP2*TANH(0.50*SE))
```

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 U U U U U U U U

NOTE- FOR ANTISYMMETRICAL MODES REPLACE TEMP AND RHS. BY
TEMP=(SE2-TC)0(PHE2+TC)
RHS=(PHE/SE)*TEMP2*TANH(0.50*SE)

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21

FORTRAN IV G LEVEL

C*****

```

0606 WRITE(3,51)
0607 FORMAT(1H1.2X,'INTERVAL'.7X,'TIME'.10X,'AMPLIFICATION'./)
0608 FB=17.60*PAY
0609 FV=10.0*PAY
0610 FR=FB/FV
0611 TIME=1.0
0612 FC=PAY/TIME
0613 NR=0.32
0614 B=0.50
0615 PME=1.50*PAY
0616 FR2=FR*FR
0617 R1=(B/MR)*FR2
0618 Z(1)=R1*SIN(PHE)
0619 Y(1)=0.0
0620 FB2=FB*FB
0621 DT=(0.005)*(PAY/FC)
0622 P=DT*DT
0623 ZA(1)=-(FB2/FR2)*Z(1)
0624 Z(2)=Z(1)*P*ZA(1)
0625 ZA(2)=-(FB2/FR2)*Z(2)
0626 ALP=FC*DT
0627 A1=FB2*SIN(ALP)
0628 R2=MR*FB2/FR2
0629 A2=R2*Z(2)*SIN(ALP)
0630 YA(2)=A1+A2
0631 Y(2)=(1.0/6.0)*YA(2)*P
0632 DT2=DT*DT
0633 WRITE(3,64)
0634 FORMAT(1H'.5X'.PARAMETERS')
0635 WRITE(3,65)
0636 FORMAT(1H'.5X'.MR=.9X,'FB=.9X,'FR=.9X,'FC=.9X,'B=.9X,'PHE=')
0637 WRITE(3,75)NR,FB,FR,FC,B,PHE
0638 FORMAT(1H'.6X'.FB.4.3X.FB.4.3X.FB.4.7X.FB.4)
0639 WRITE(3,999)
0640 FORMAT(1H'.5X'.INITIAL-VEHICLE ENTERING SPAN')
0641 INT=1
0642 DT1=0.0
0643 WRITE(3,52)INT,DT1,Y(1)
0644 FORMAT(1H'.5X'.13.5X.F7.3.5X.E12.6)
0645 WRITE(3,83)
0646 FORMAT(1H'.5X'.INTERMEDIATE PEAKS - VEHICLE ON SPAN')
0647 DO 32 I=3,200
0648 Z(I)=((2.0)*Z(I-1))-(Z(I-2))+(P*ZA(I-1))
0649 Y(I)=((2.0)*Y(I-1))-(Y(I-2))+(P*YA(I-1))

```

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FORTRAN IV G LEVEL 21

```

0650 IF(ABS(Y(1)).GT.10000.0)GO TO 41
0651 GO TO 97
0652 SE=FC*DT2
0653 TEMP1=Z(I)-((Y(I))*SIN(SE))
0654 ZA(I)=-(FB2/FR2)*TEMP1
0655 TEMP2=SIN(SE)-Y(I)
0656 YA(I)=-(FB2*TEMP2)+((R2)*((TEMP1)*SIN(SE))
0657 IF(ABS(Y(1)).GT.0.0000010)GO TO 99
0658 GO TO 42
0659 WRITE(3,48)I,DT2,Y(I)
0660 48 FORMAT(1H,5X,13.5X,F7.4,5X,E12.6)
0661 42 DT2=DT2+DT
0662 32 CONTINUE
0663 INTF=200
0664 DTF=PAY/FC
0665 WRITE(3,102)
0666 102 FORMAT(1H,5X,FINAL-VEHICLE JUST LEFT SPAN)
0667 WRITE(3,45)INTF,DTF,Y(200)
0668 45 FORMAT(1H,5X,13.5X,F7.4,5X,E12.6)
0669 RETURN

```

APPENDIX III

FIGURES 1 TO 103

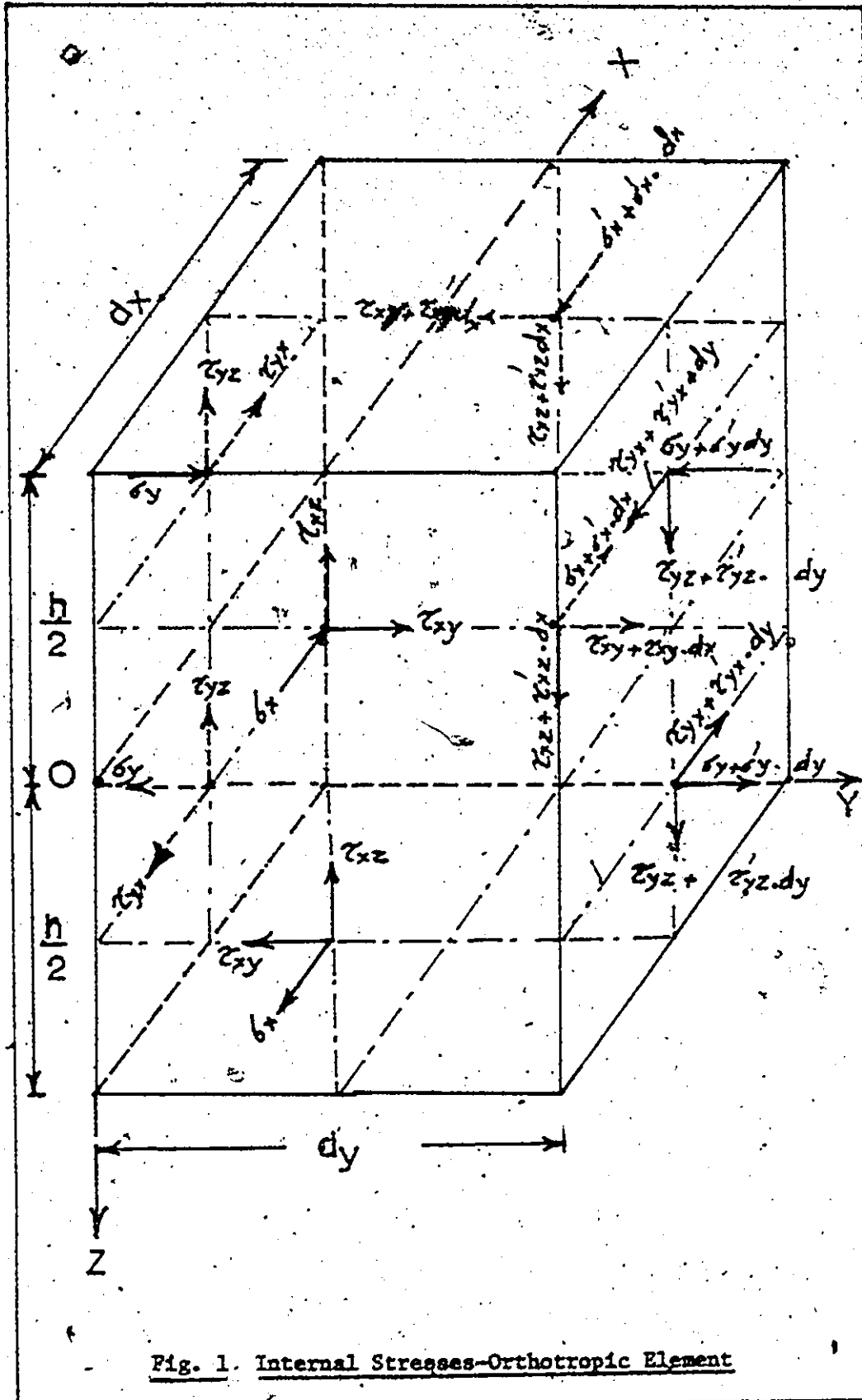


Fig. 1. Internal Stresses-Orthotropic Element

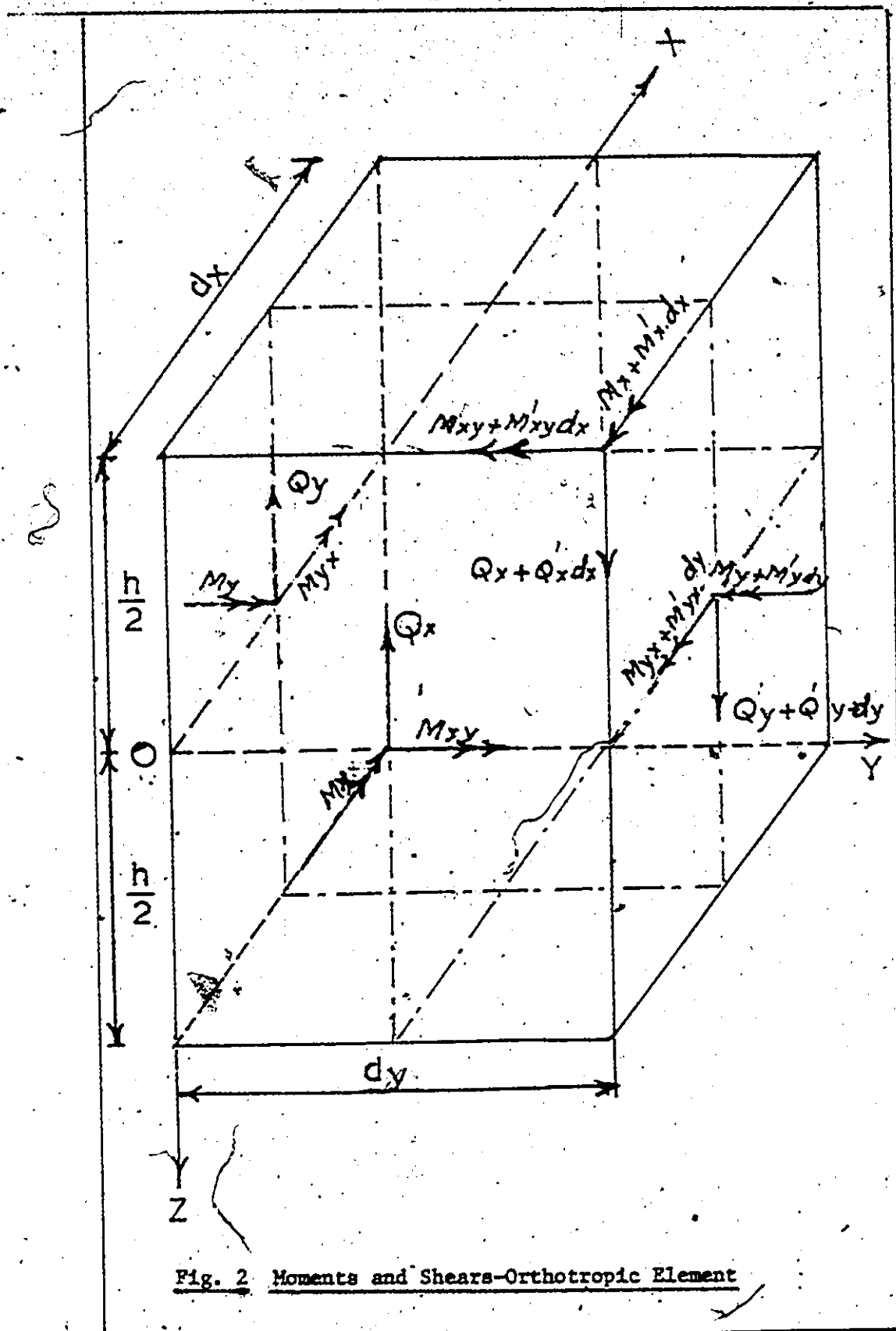


Fig. 2 Moments and Shears-Orthotropic Element

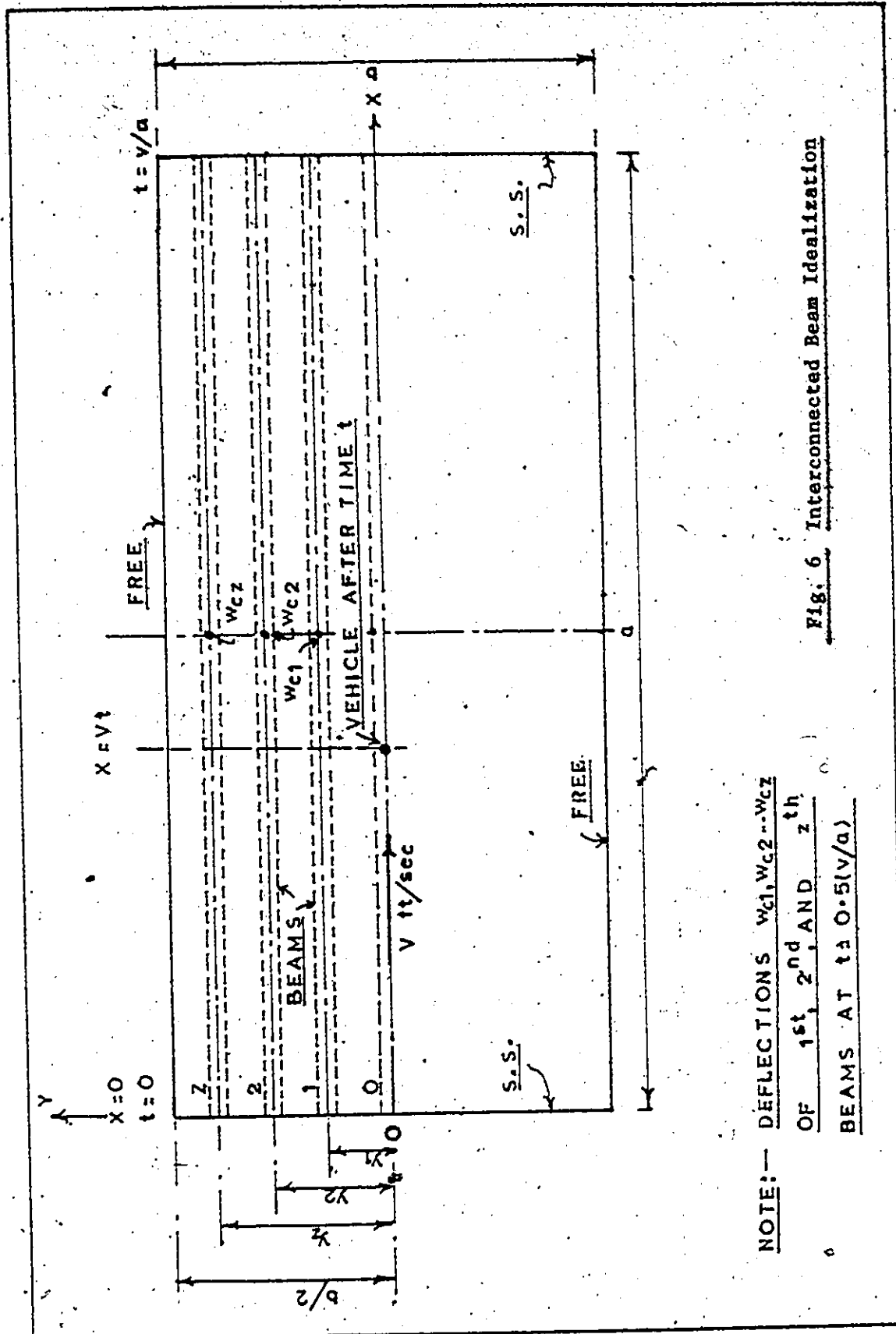


Fig. 6. Interconnected Beam Idealization

NOTE:— DEFLECTIONS w_{c1}, w_{c2}, w_{cz}
OF 1st, 2nd, AND 3rd
BEAMS AT $t = 0.5(v/a)$

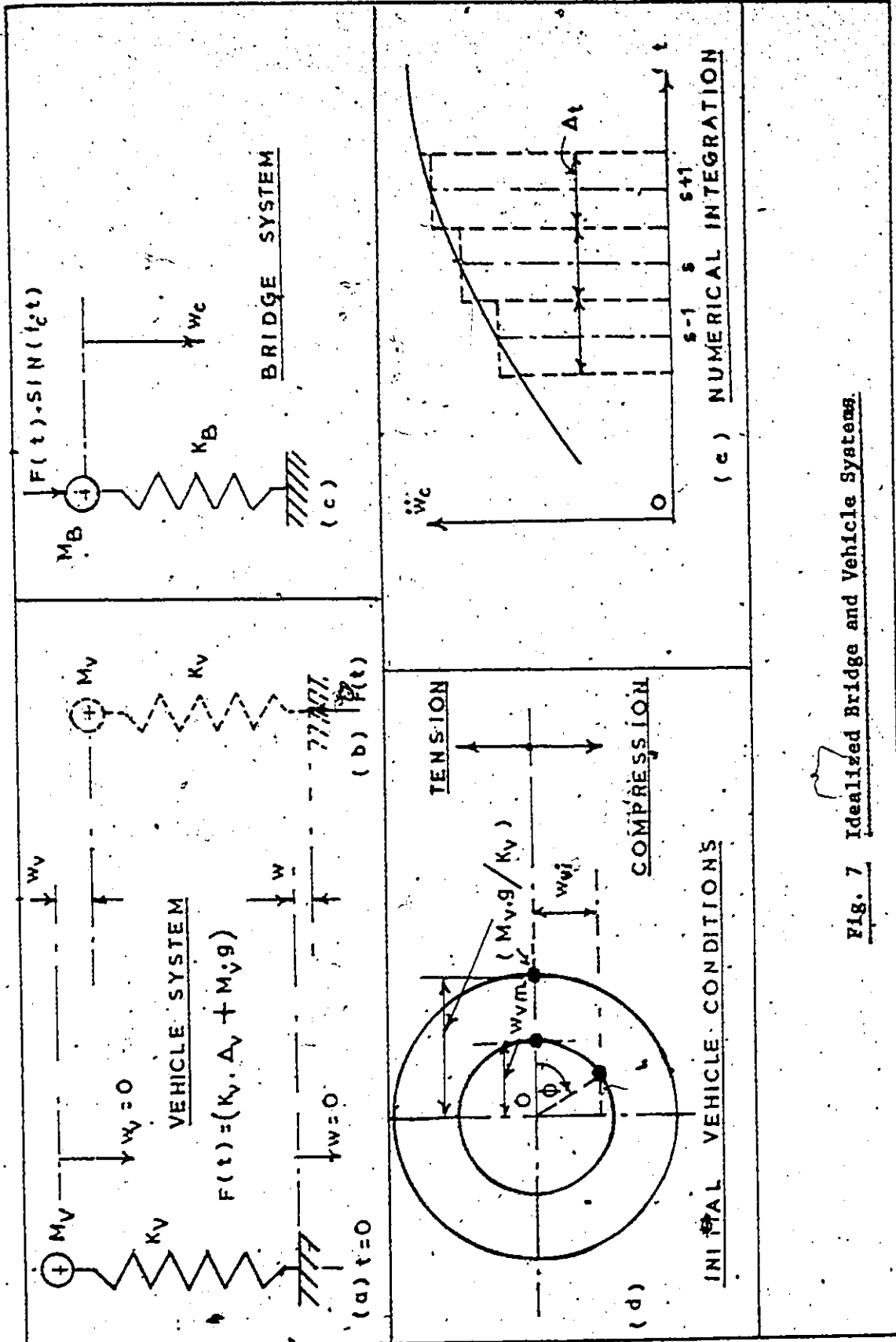


Fig. 7 Idealized Bridge and Vehicle Systems

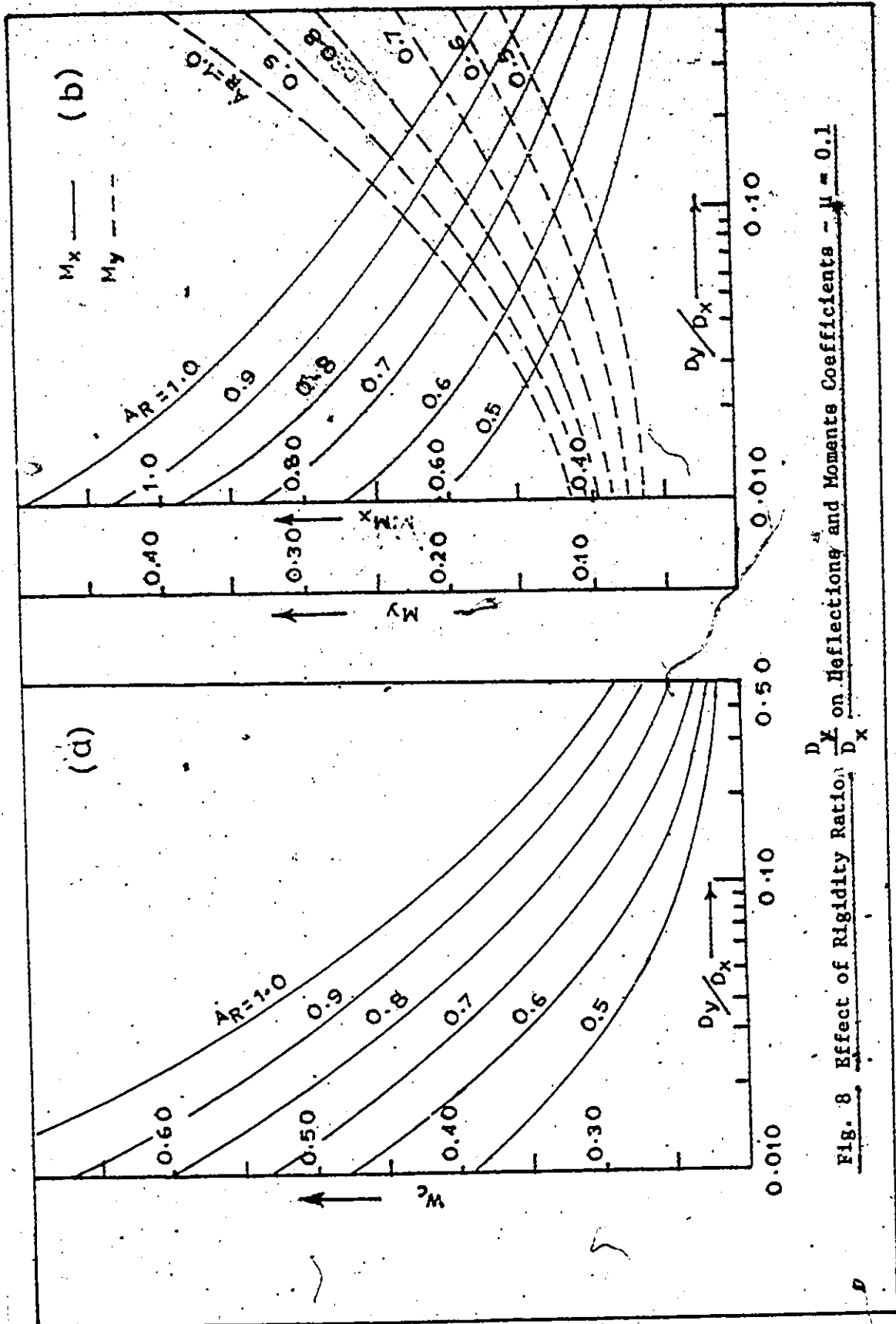


Fig. 8 Effect of Rigidity Ratio $\frac{D_y}{D_x}$ on Deflection and Moments Coefficients - $\mu = 0.1$

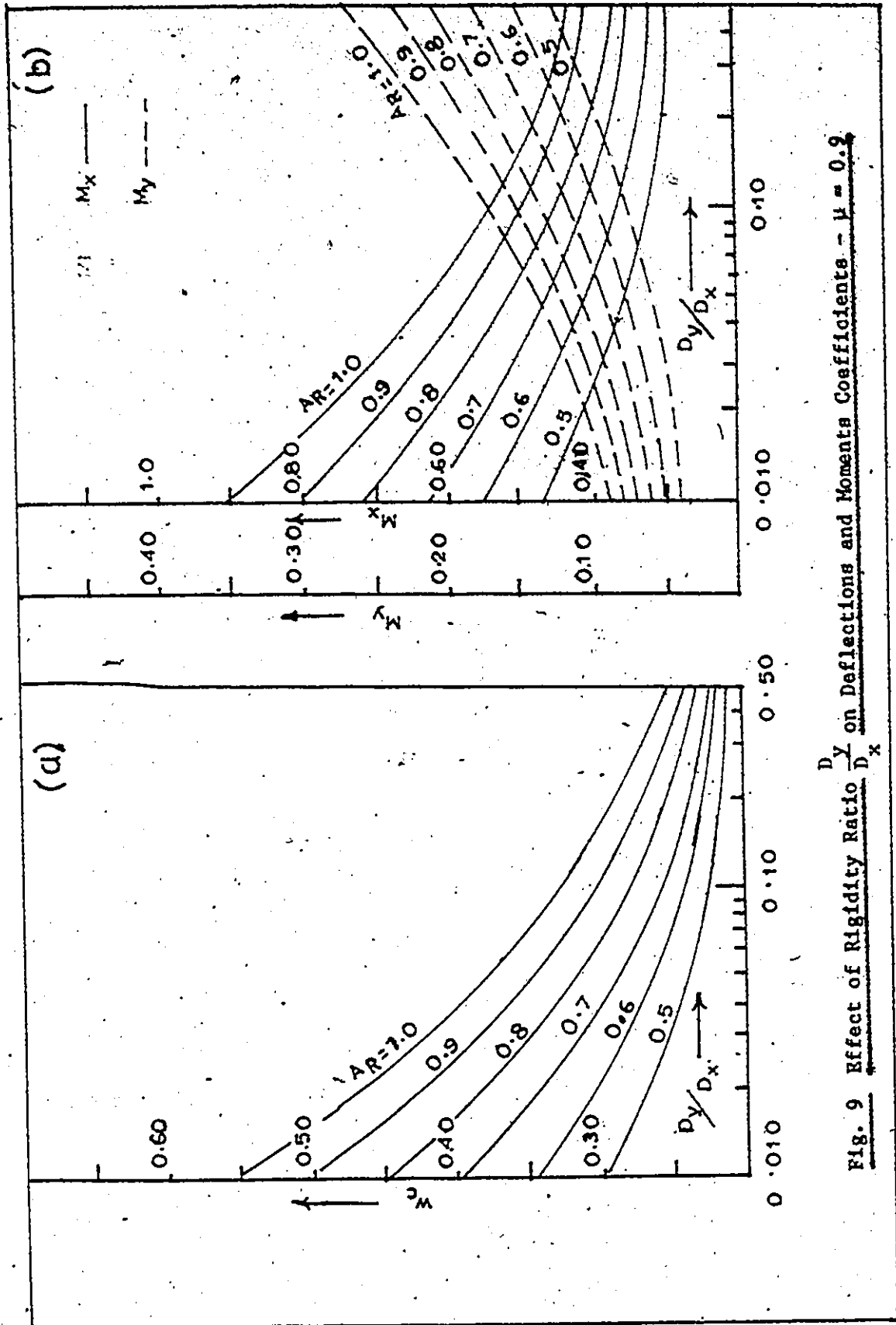
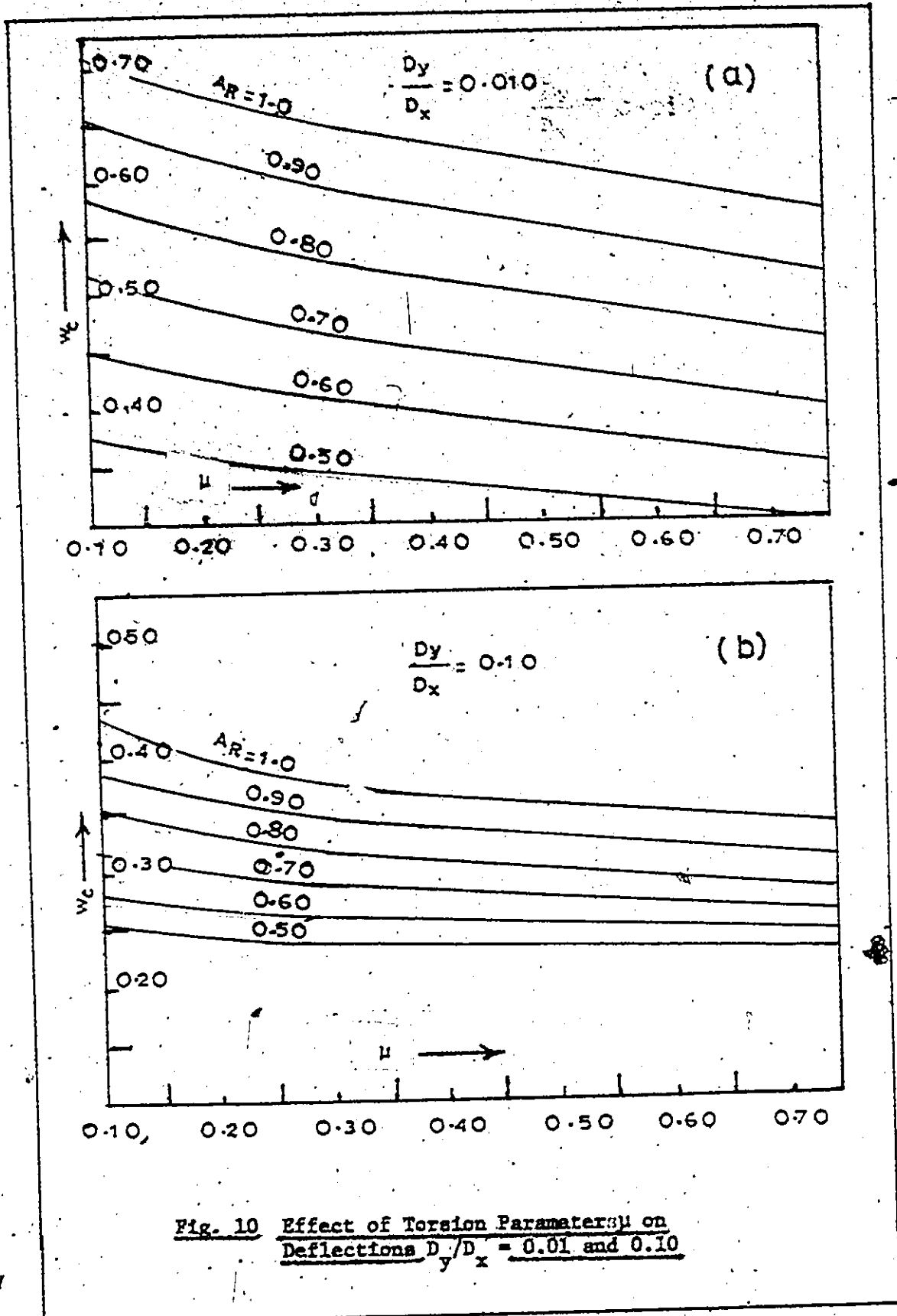


Fig. 9 Effect of Rigidity Ratio $\frac{D_y}{D_x}$ on Deflections and Moments Coefficients - $\mu = 0.2$



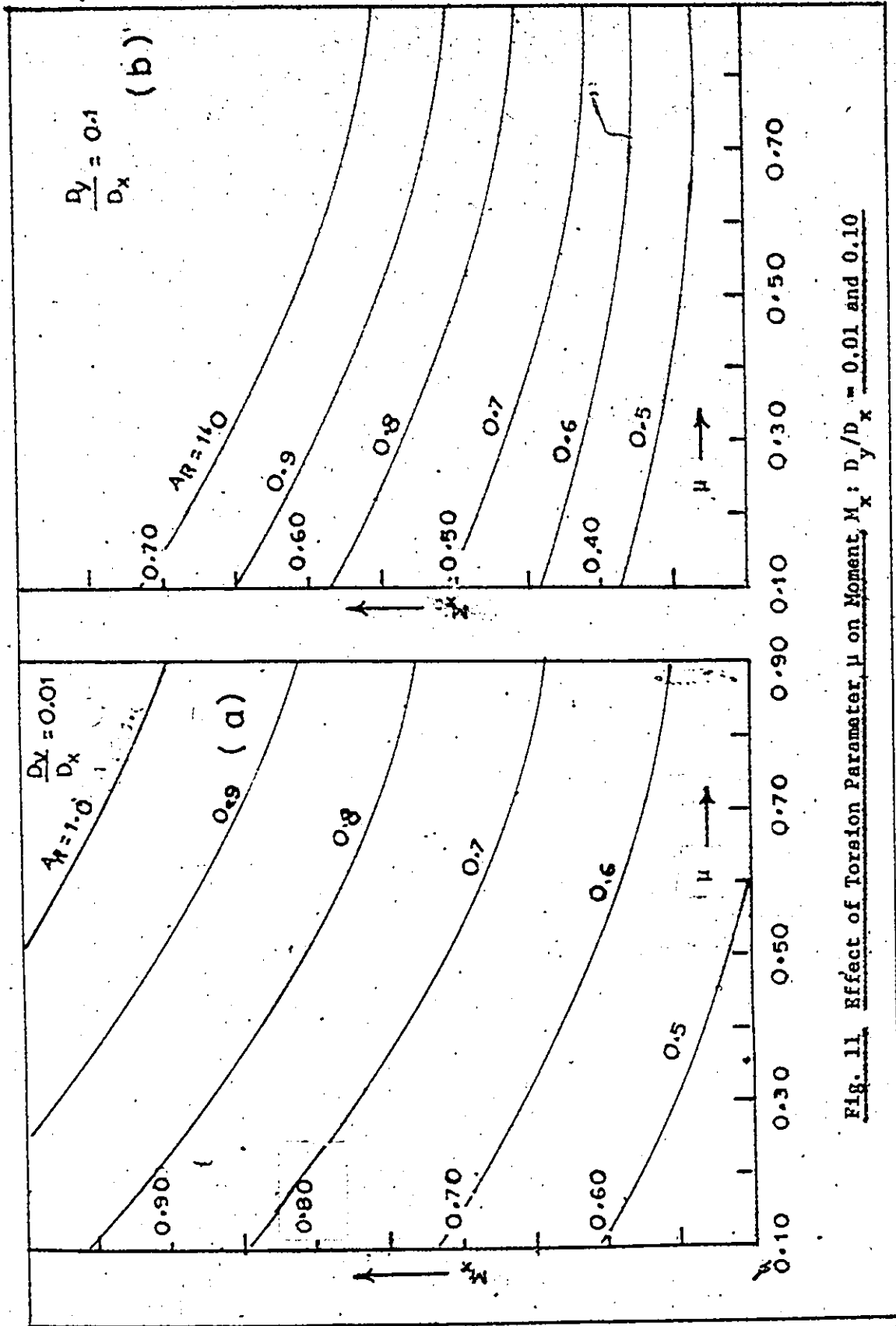


Fig. 11. Effect of Torsion Parameter μ on Moment M_x : $D_y/D_x = 0.01$ and 0.10

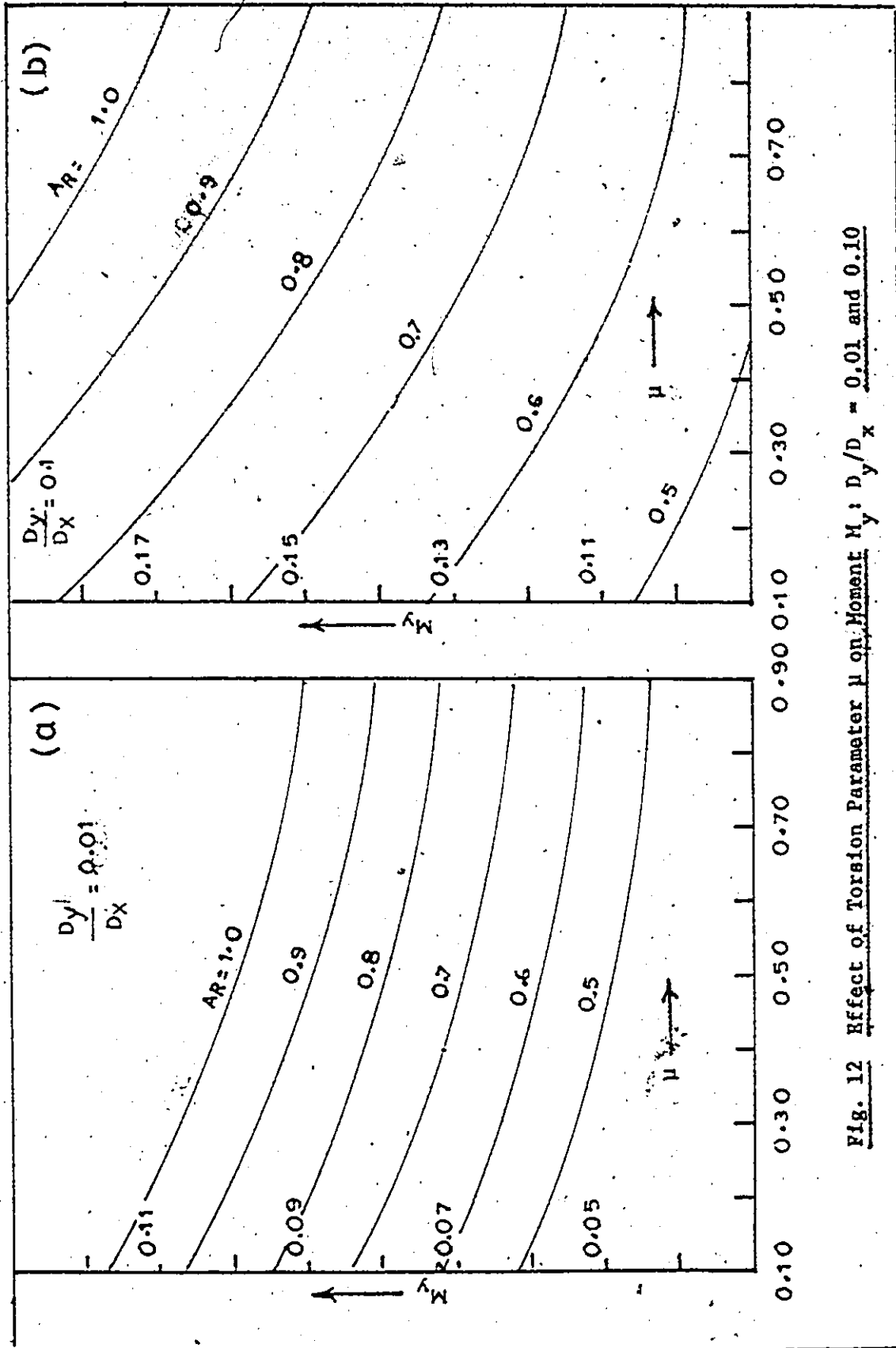


Fig. 12 Effect of Torsion Parameter μ on Moment M_y ; $D_y/D_x = 0.01$ and 0.10

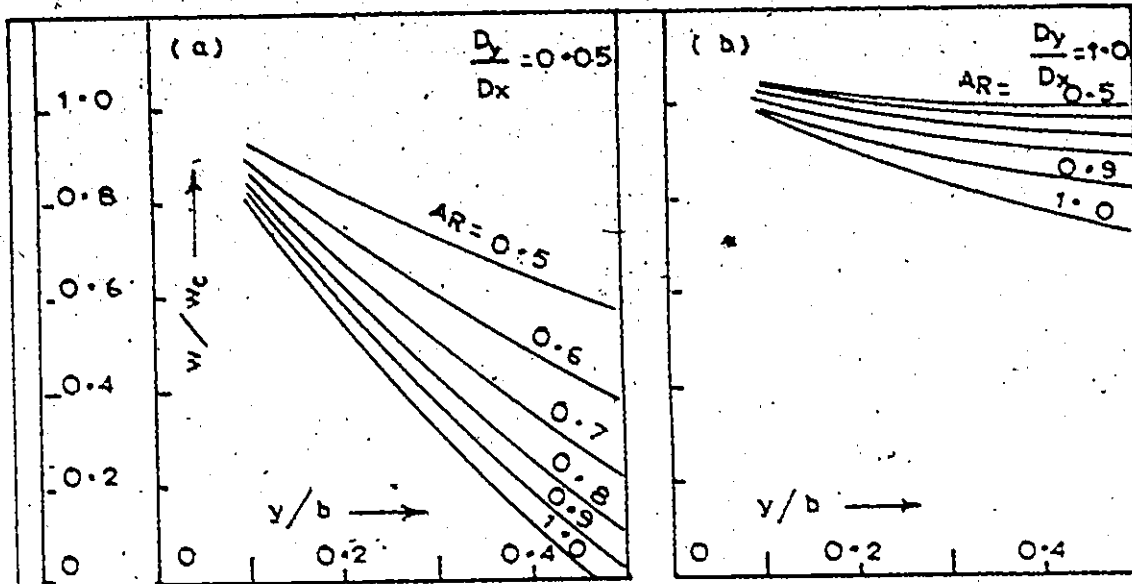


Fig. 13 Deflection Ratio w/w_c at $x/a = 0.5$ and $y/b = 0$ to 0.5 ; $\mu = 0.3$

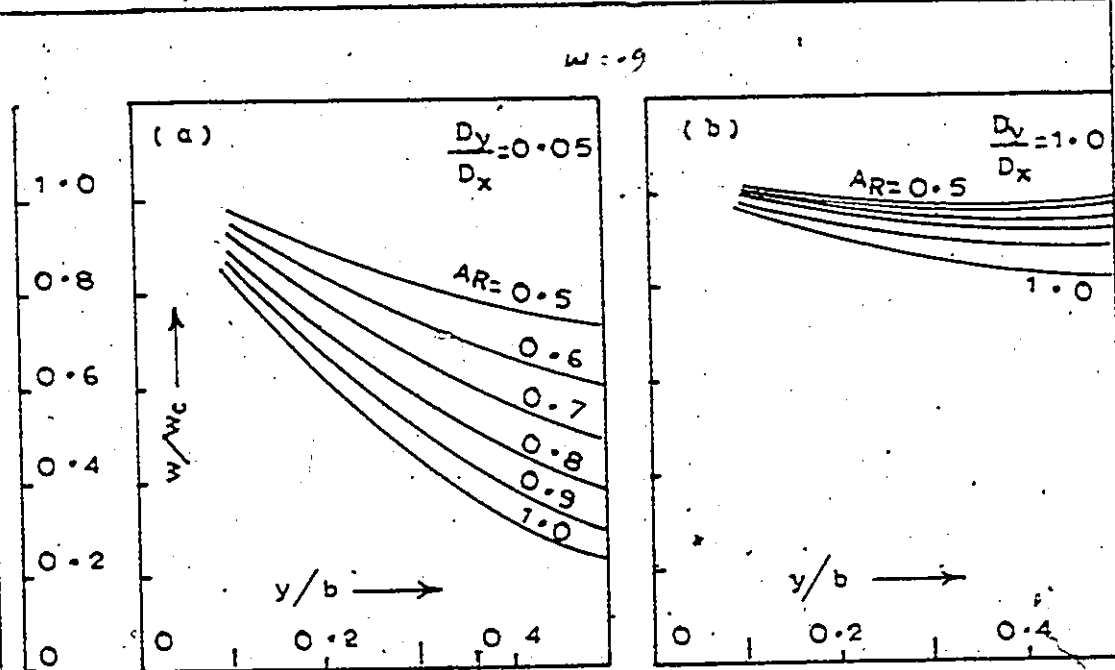
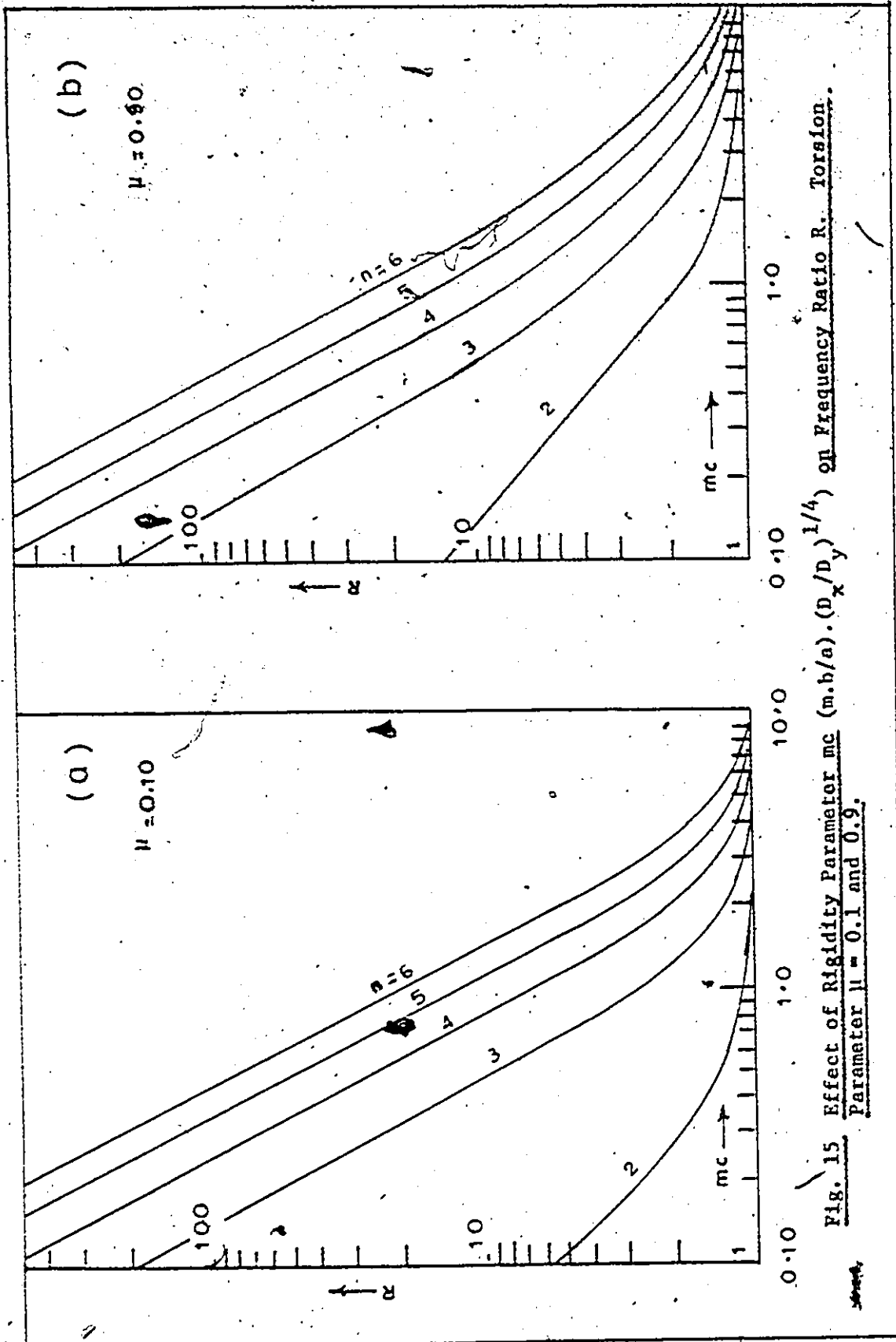


Fig. 14 Deflection ratio w/w_c at $x/a = 0.5$ and $y/b = 0$ to 0.5 ; $\mu = 0.9$



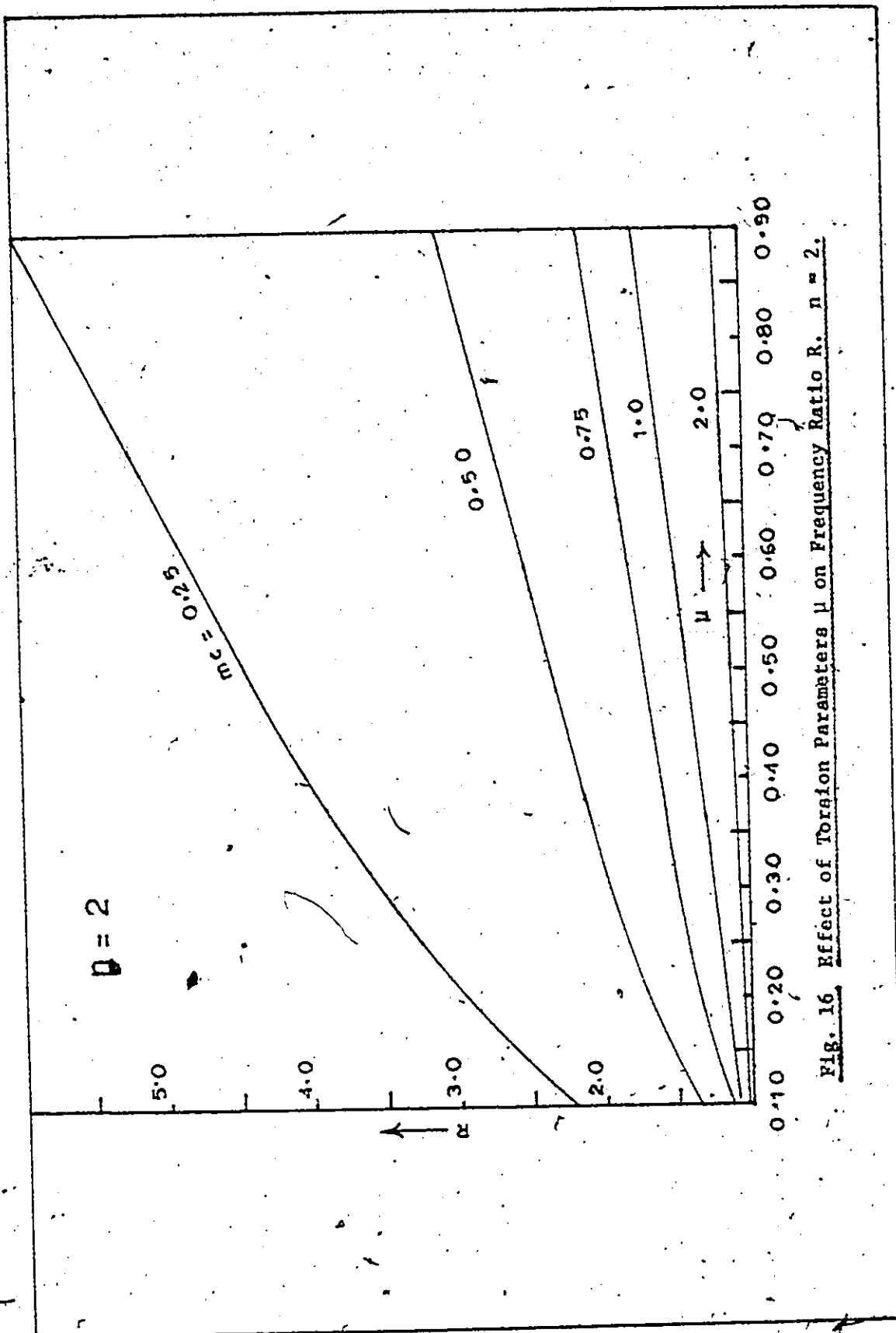


Fig. 16 Effect of Torafon Parameters μ on Frequency Ratio R . $n = 2$.

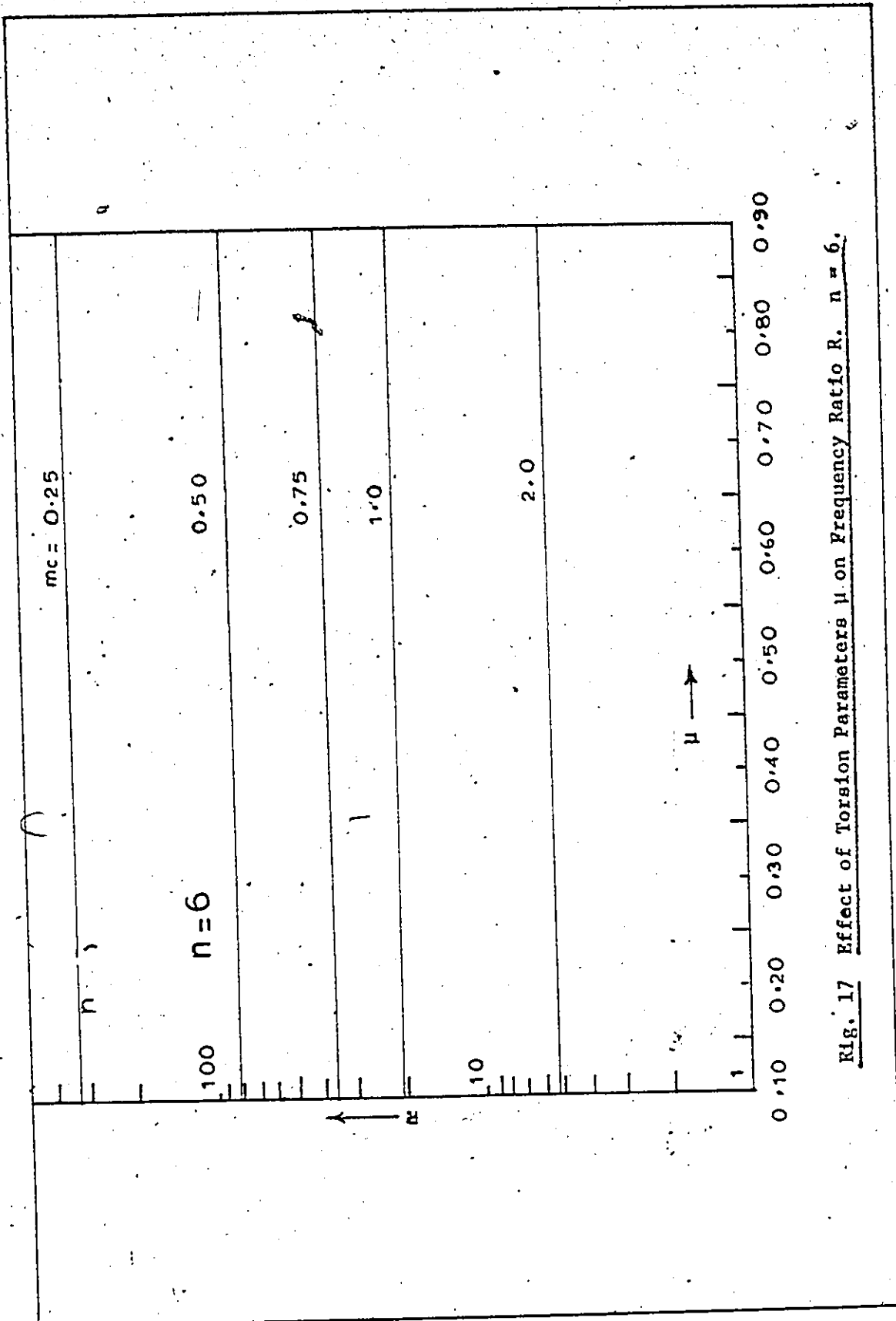
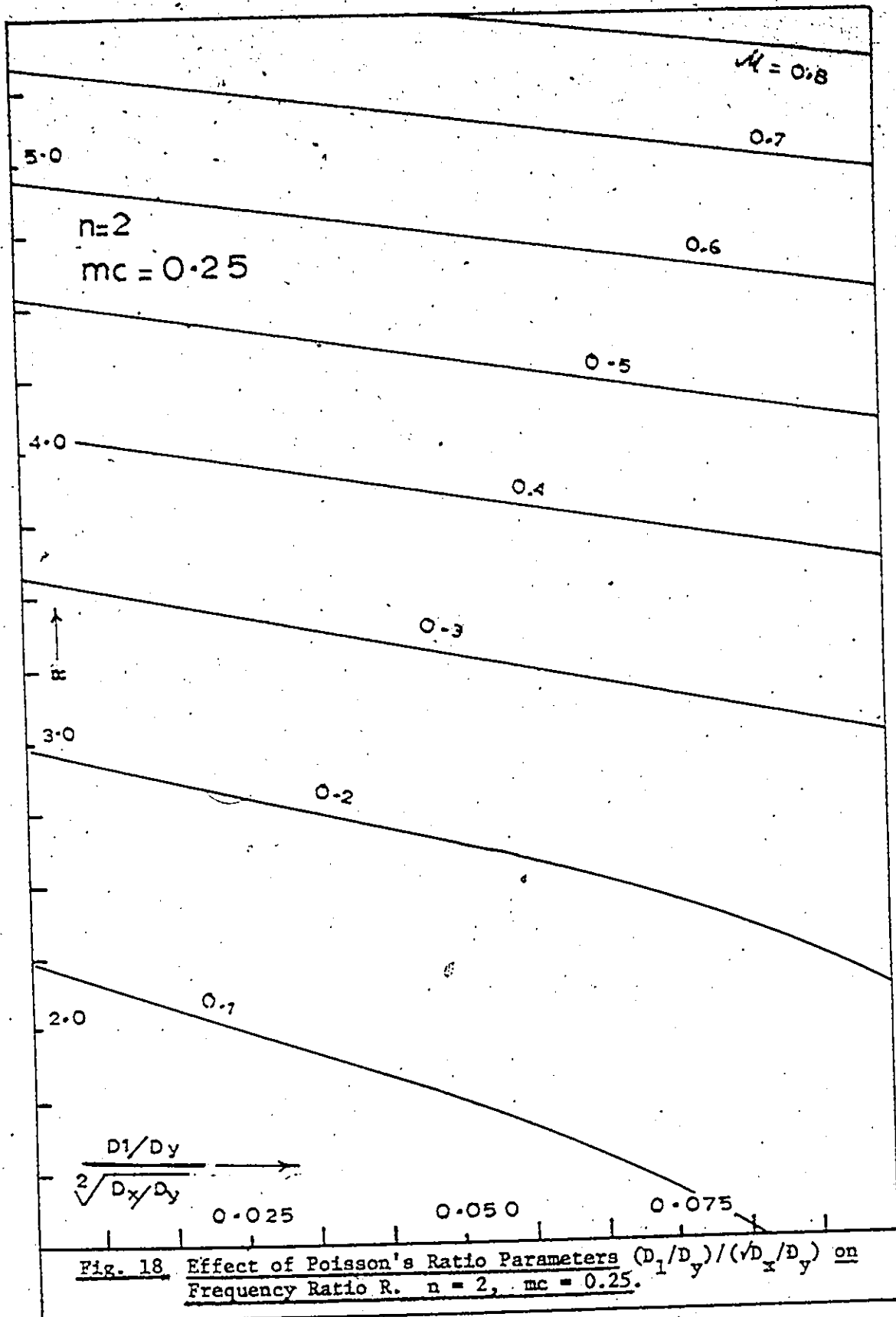


Fig. 17 Effect of Torsion Parameters μ on Frequency Ratio R . $n = 6$.



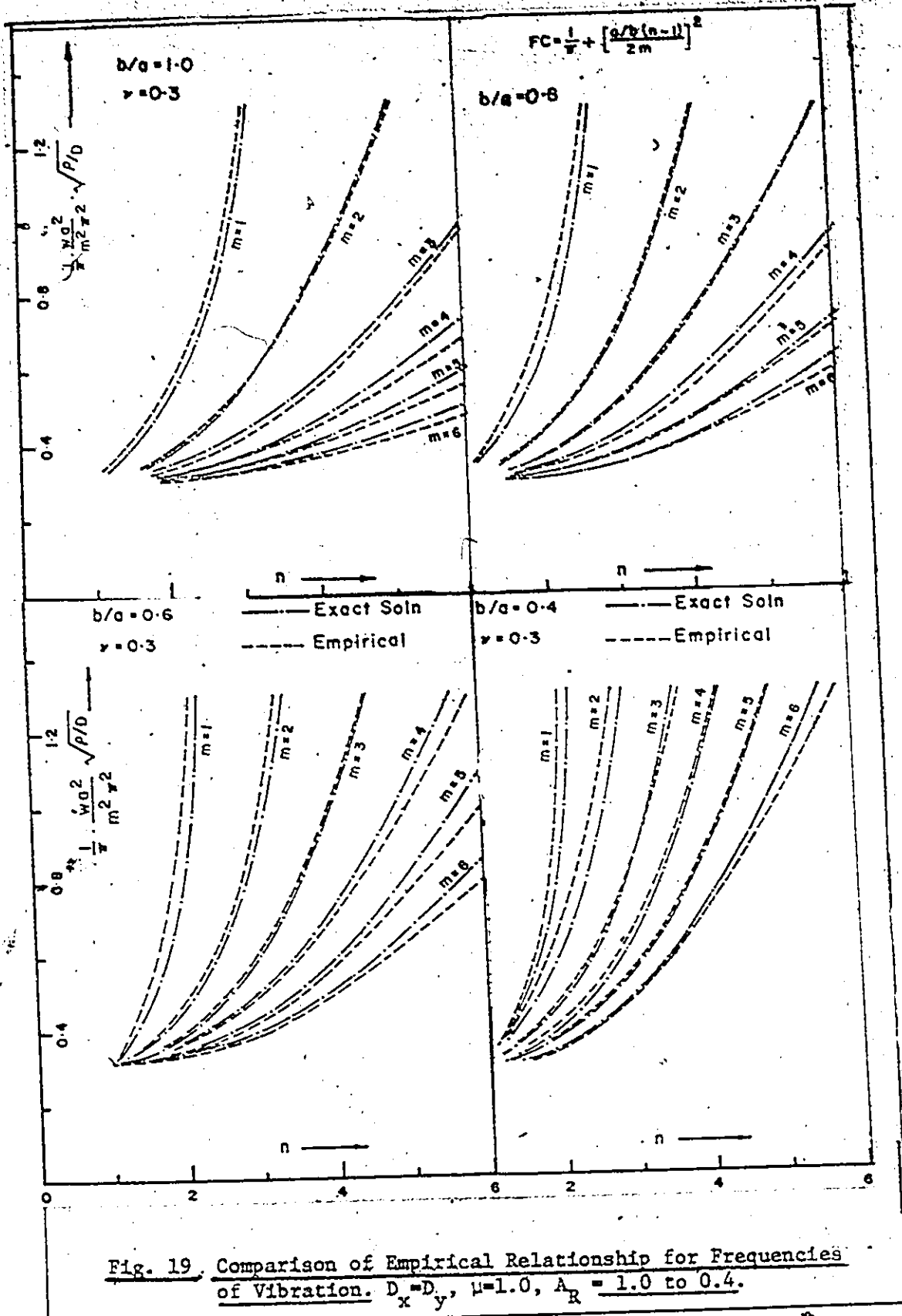


Fig. 19. Comparison of Empirical Relationship for Frequencies of Vibration. $D_x = D_y$, $\mu = 1.0$, $A_R = 1.0$ to 0.4 .

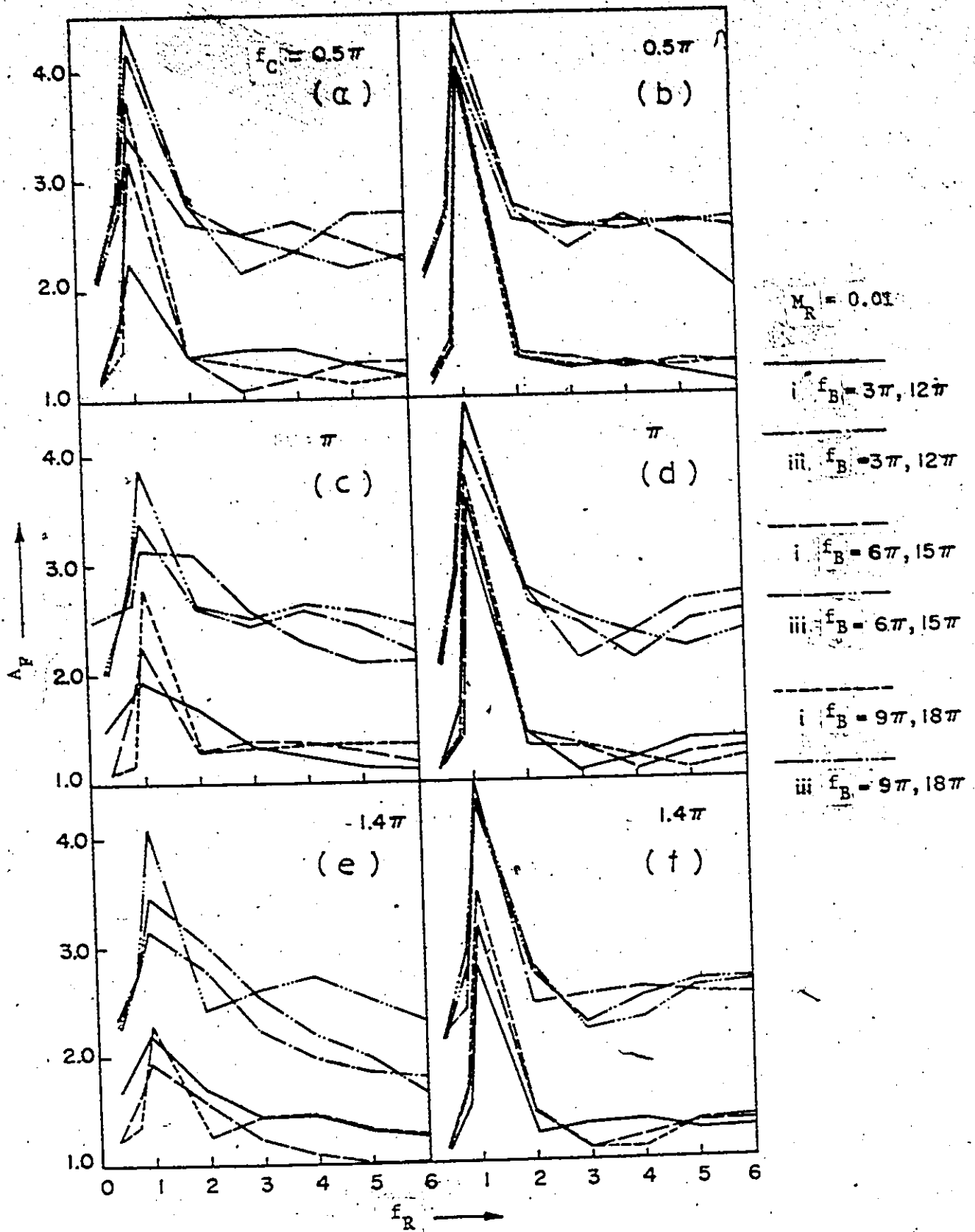


Fig.20 Effect of Frequency Ratio (f_R) On Amplification(A_F)

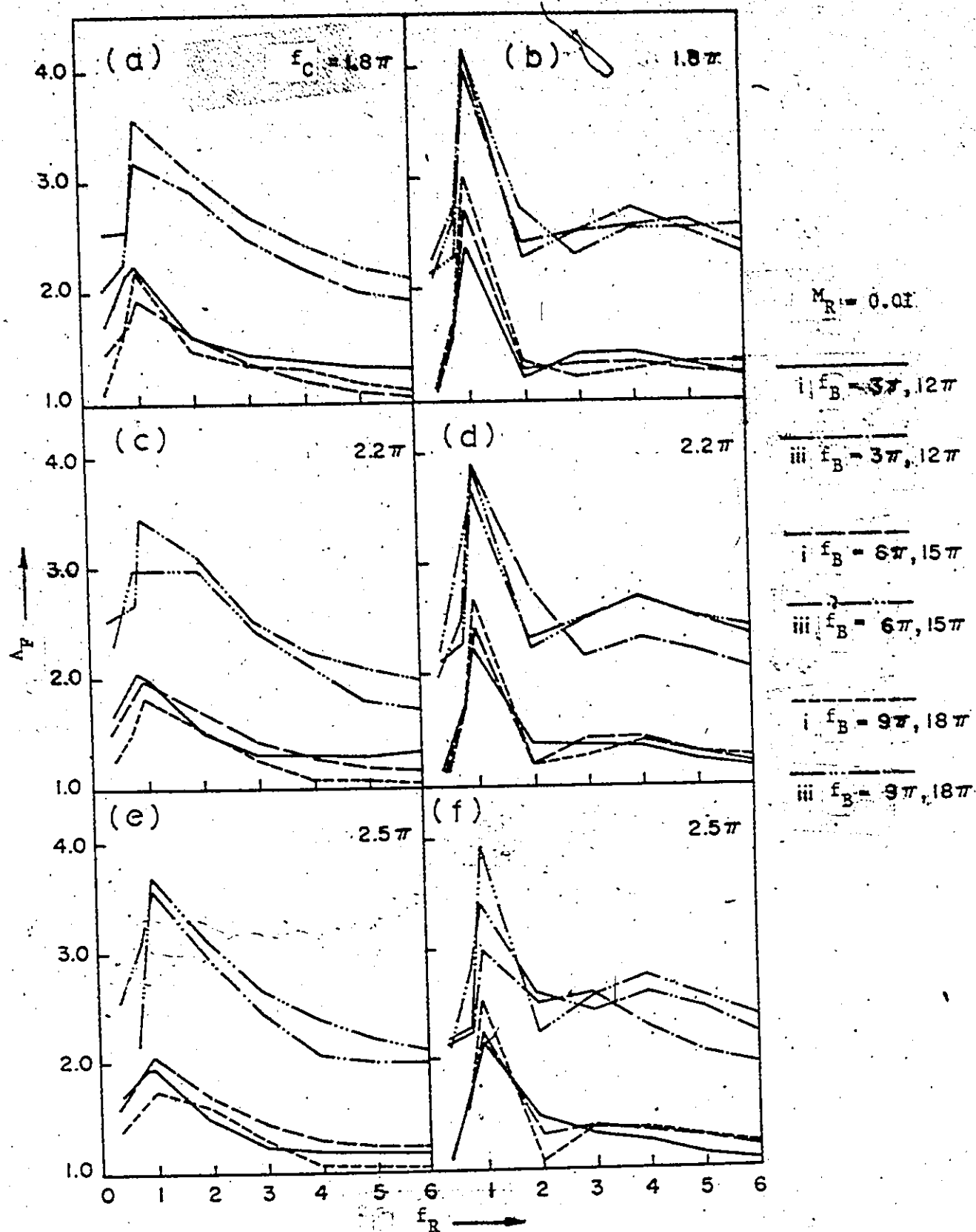


Fig. 21 Effect of Frequency Ratio (f_R) on Amplifications (A_T)

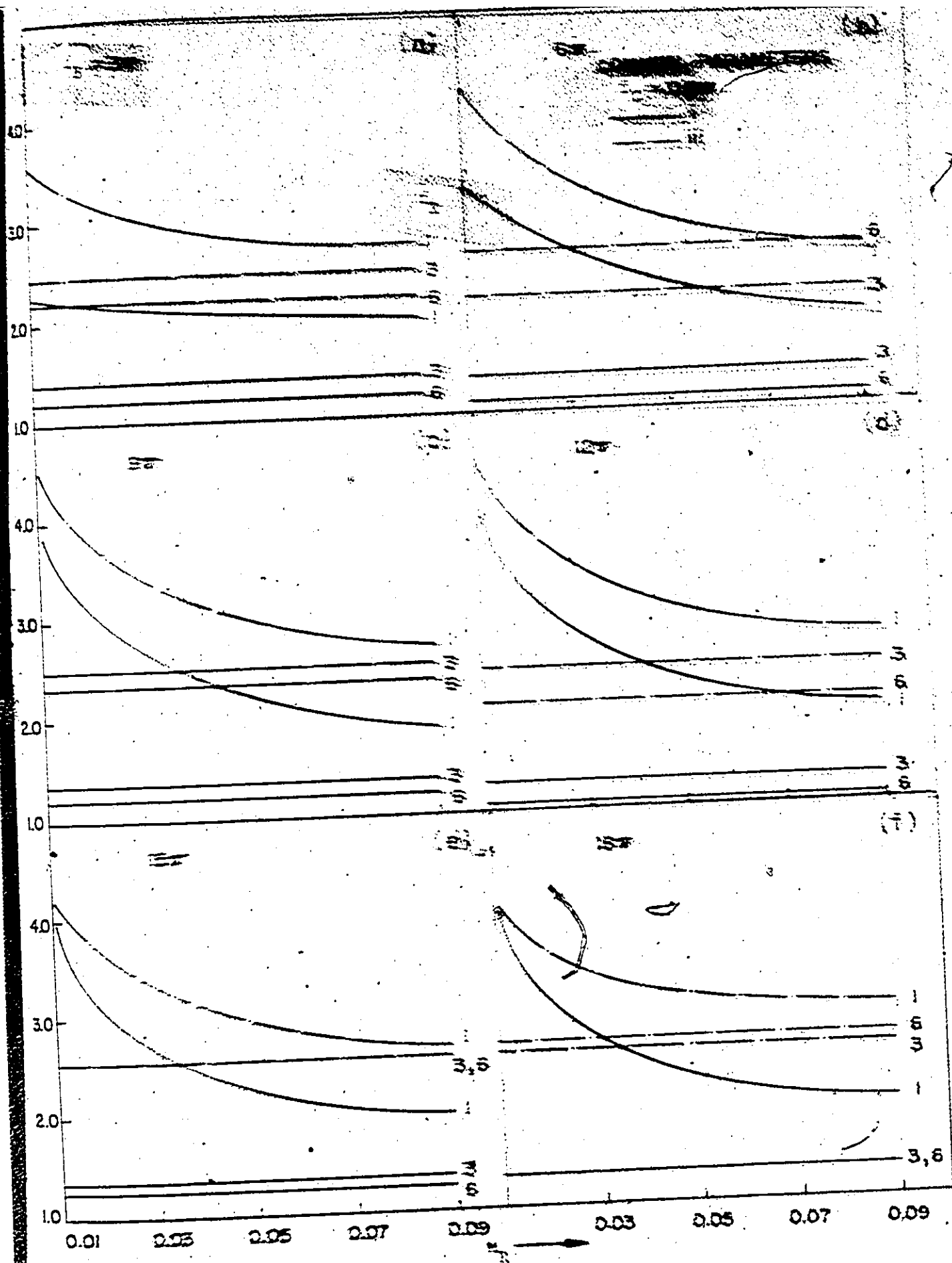


Fig. 22 Effect of Mass Ratio (M_p) on Amplifications (A_p)

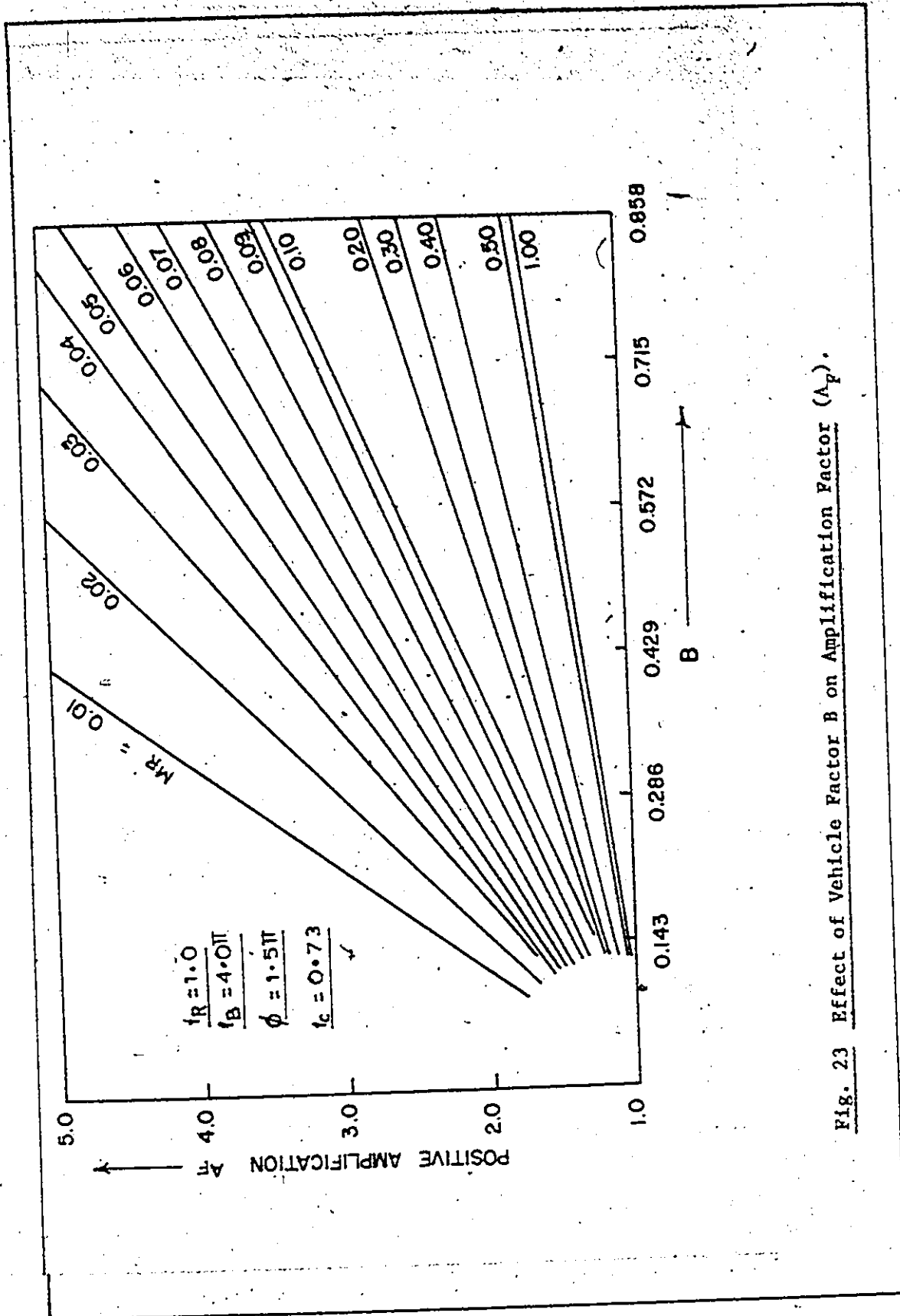


Fig. 23 Effect of Vehicle Factor B on Amplification Factor (A_F).

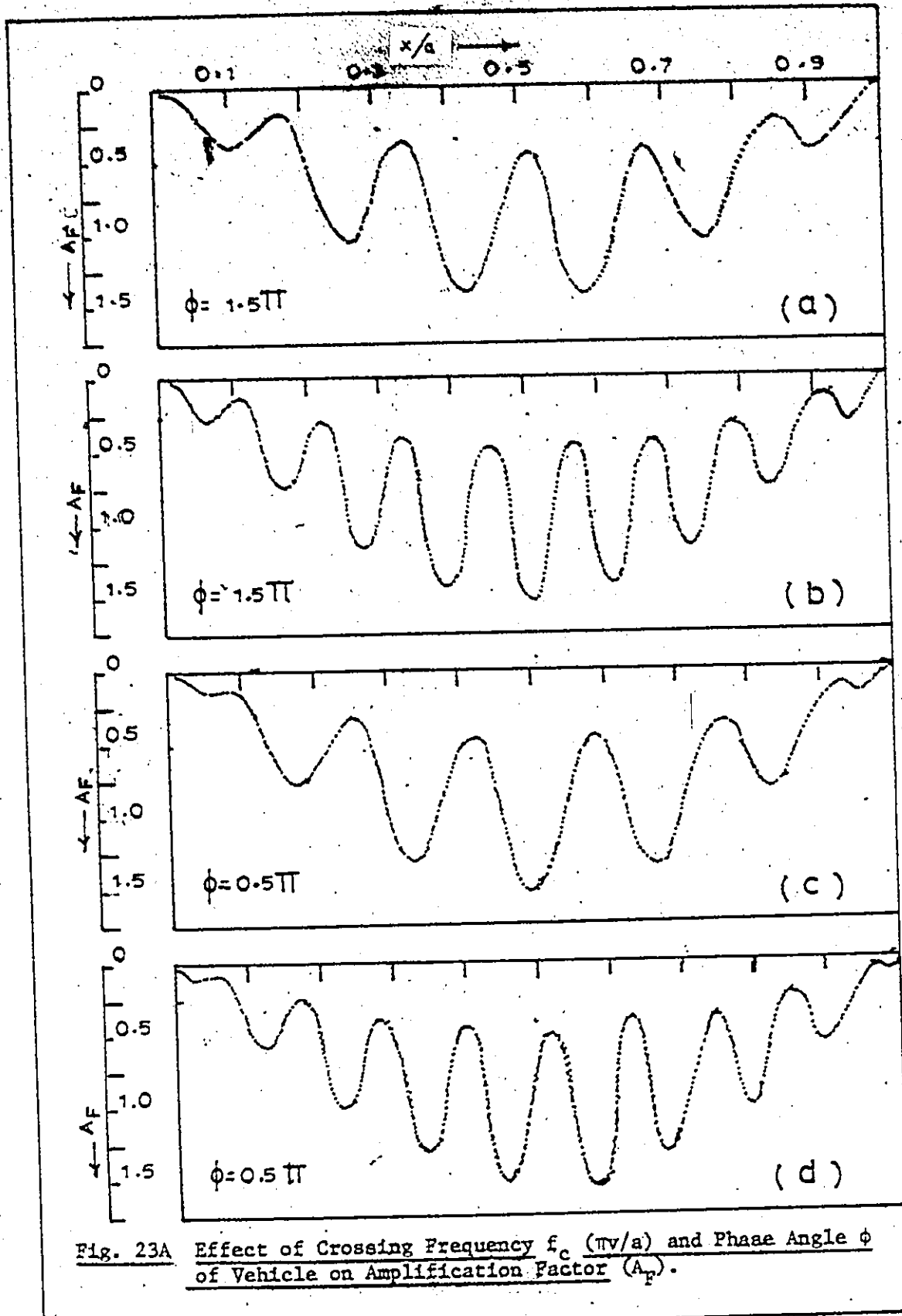


Fig. 23A Effect of Crossing Frequency f_c ($\pi v/a$) and Phase Angle ϕ of Vehicle on Amplification Factor (A_F).

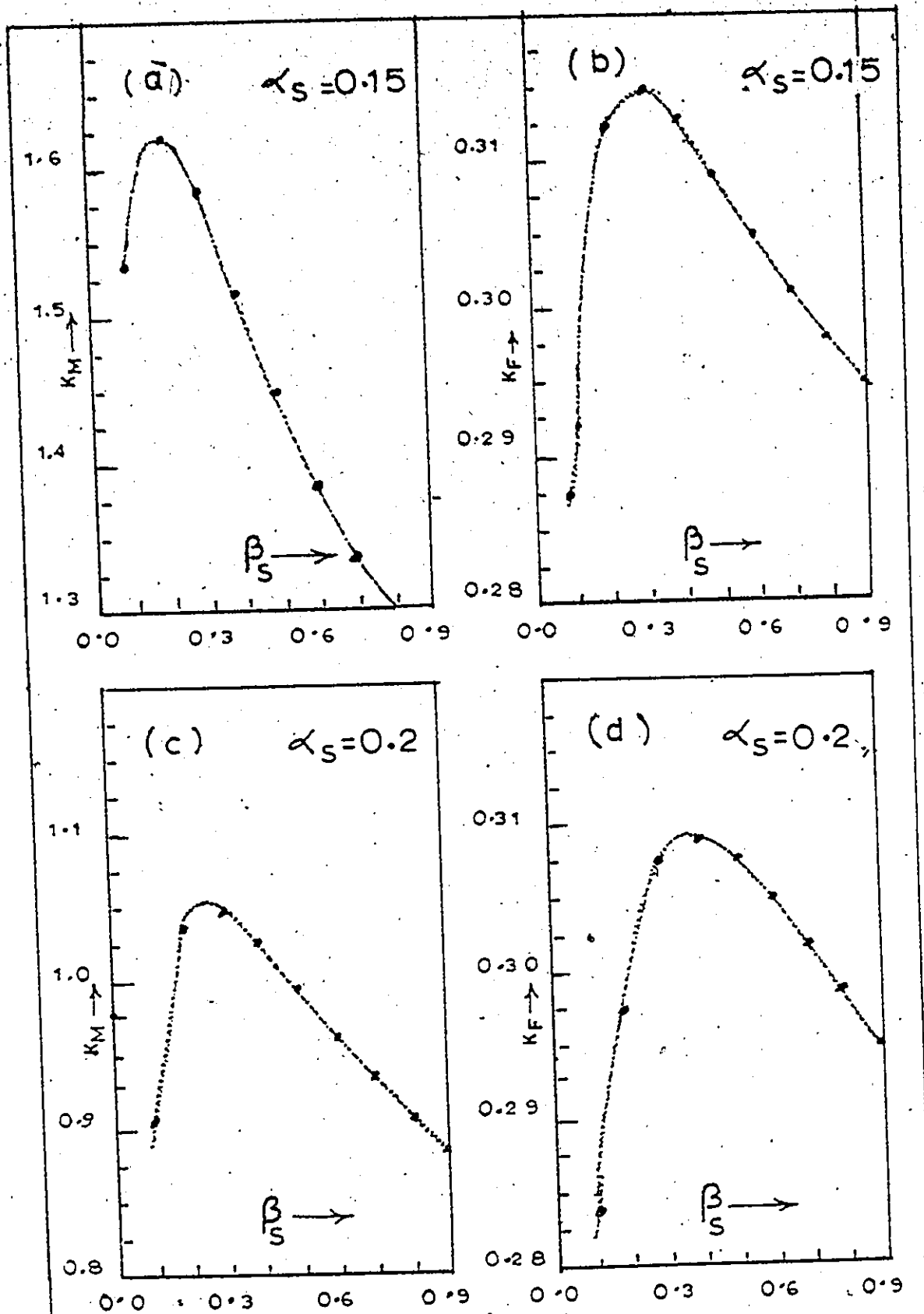


Fig. 24 Effect of β_s on Coefficients of Frequency K_F and Mass Ratio K_M , $\alpha_s = 0.15$ and 0.20 .

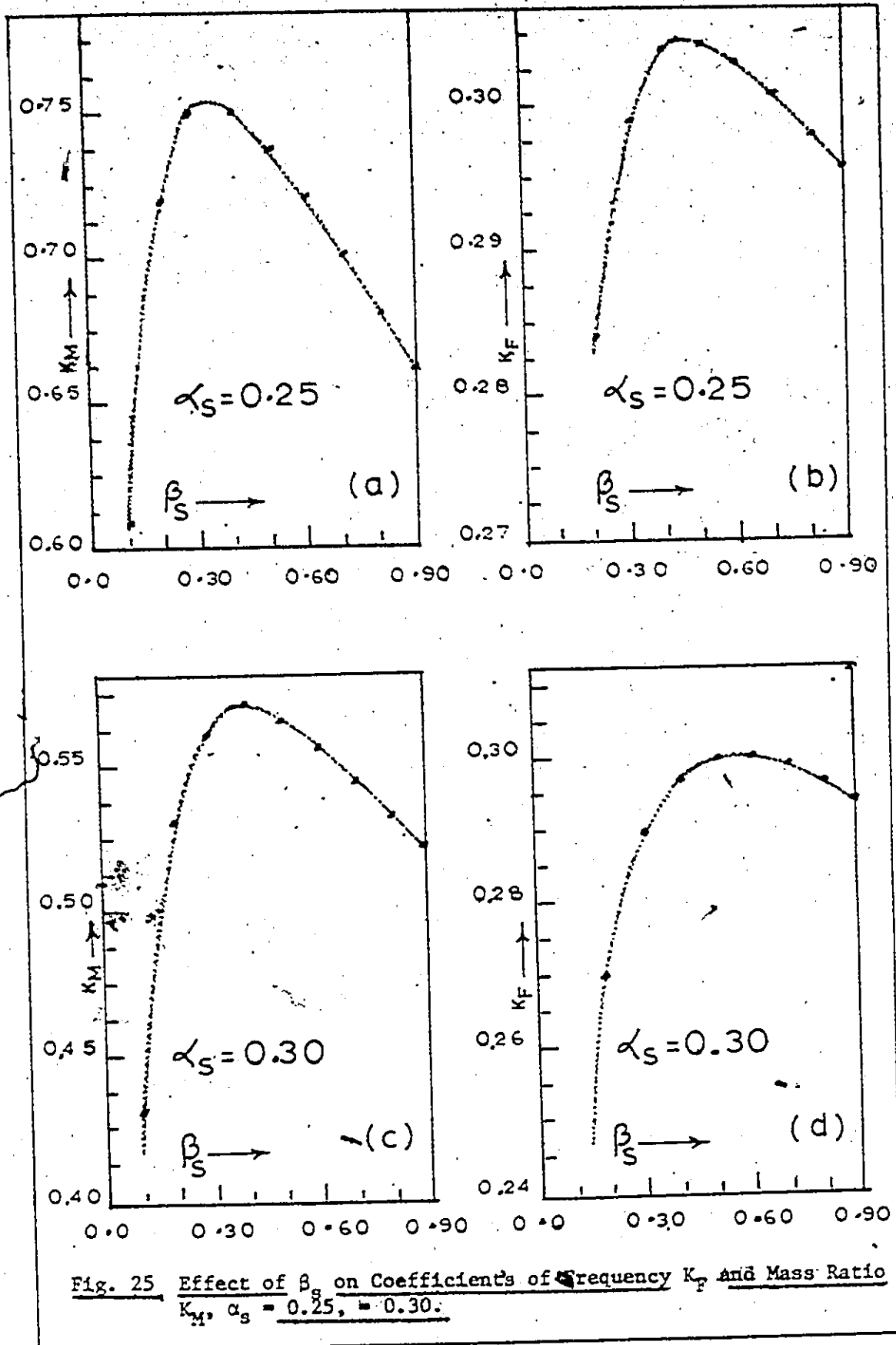
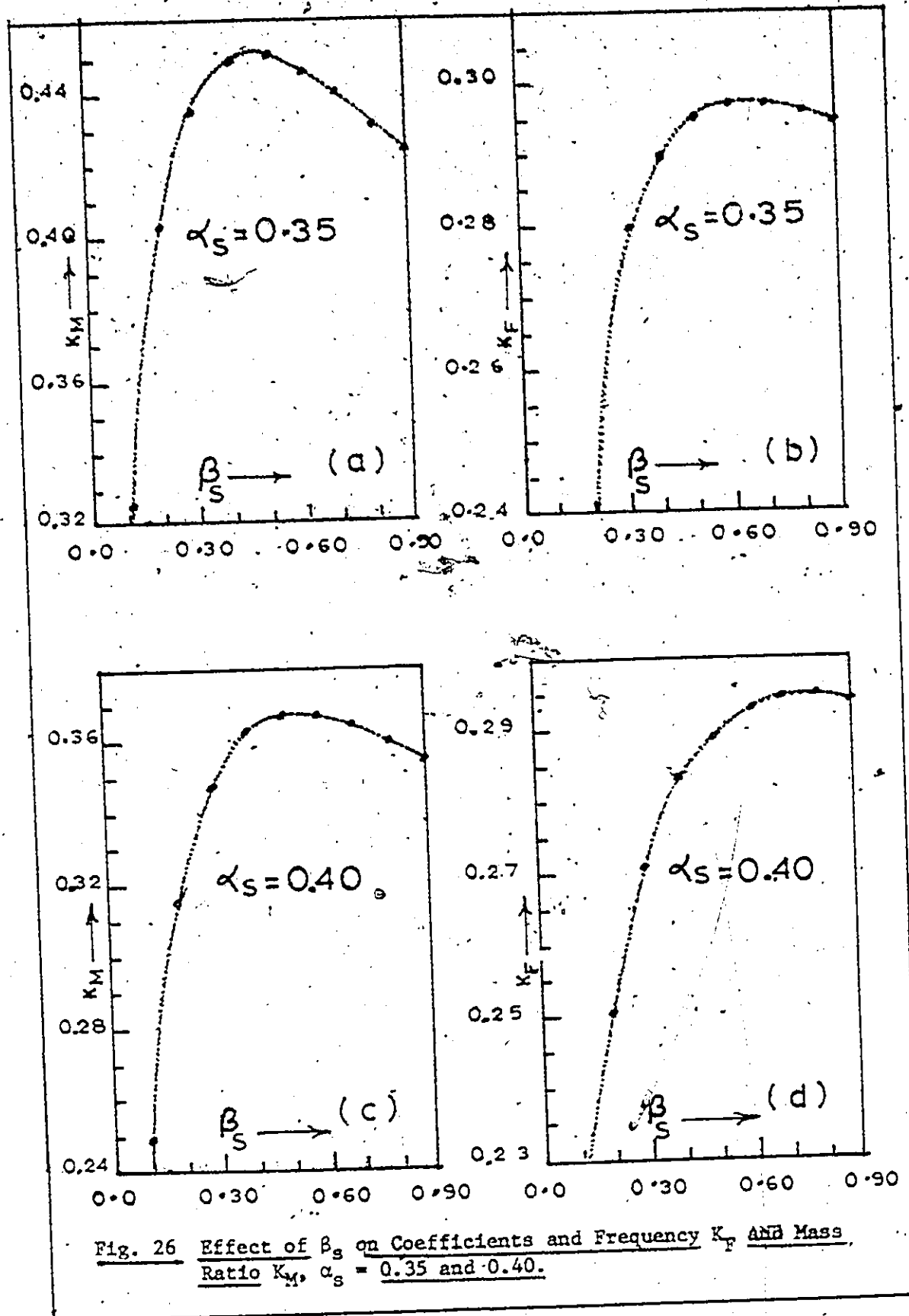
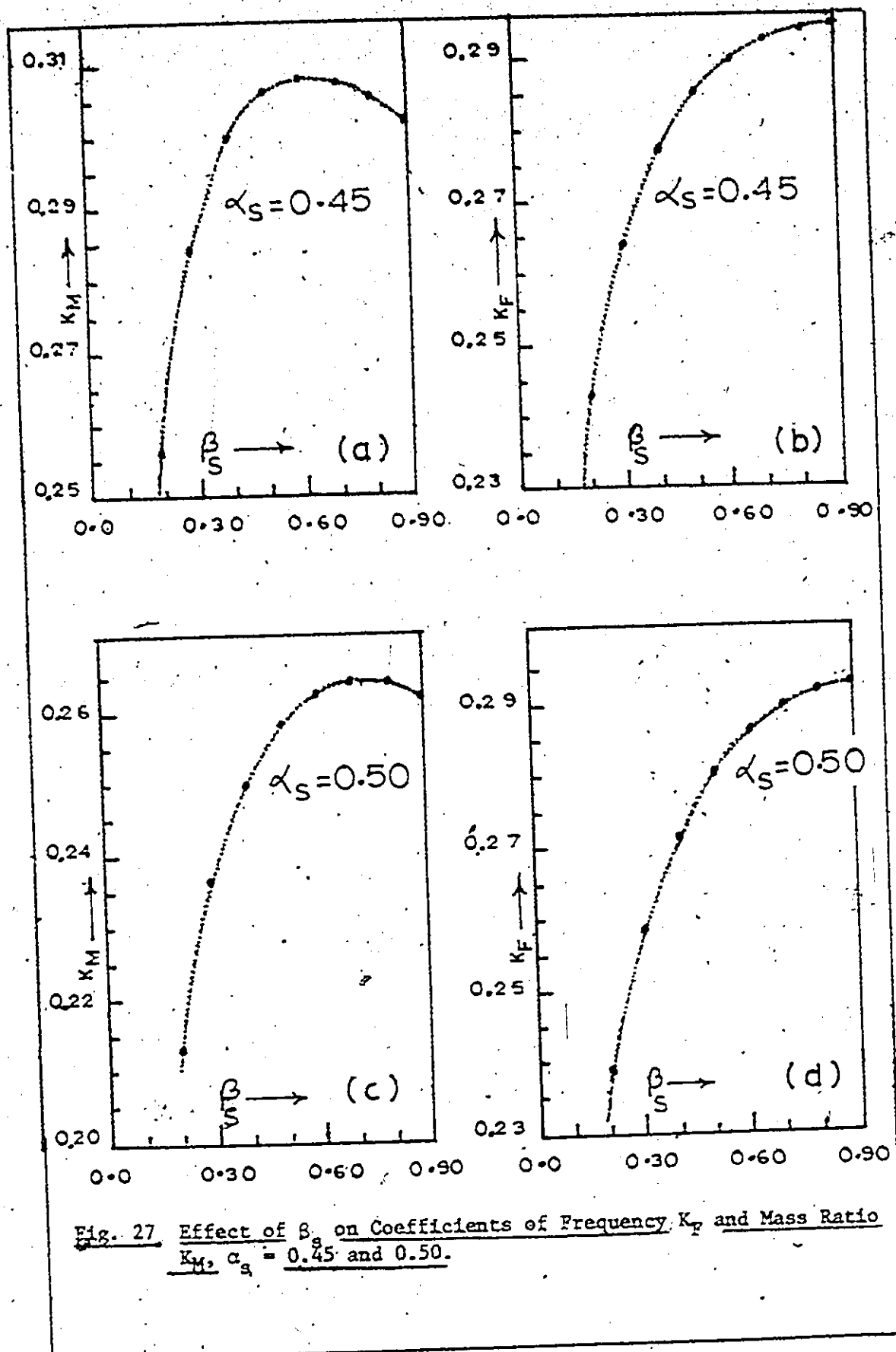


Fig. 25 Effect of β_s on Coefficients of Frequency K_F and Mass Ratio K_M , $\alpha_s = 0.25, 0.30$.





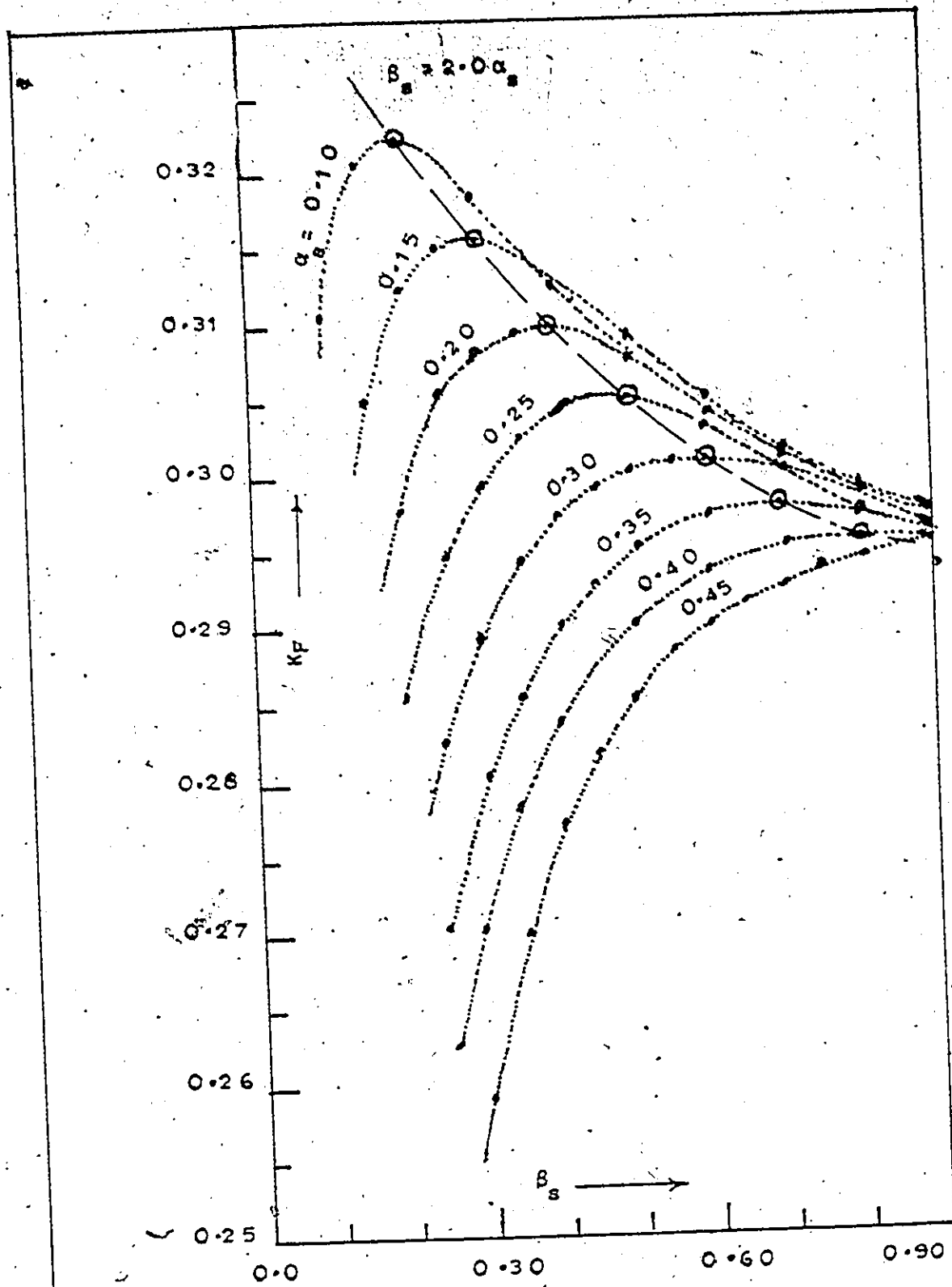


Fig. 28. Effect of β_s on Maximization of Coefficients of Frequency K_p , $\alpha_s = 0.10$ and 0.45 .

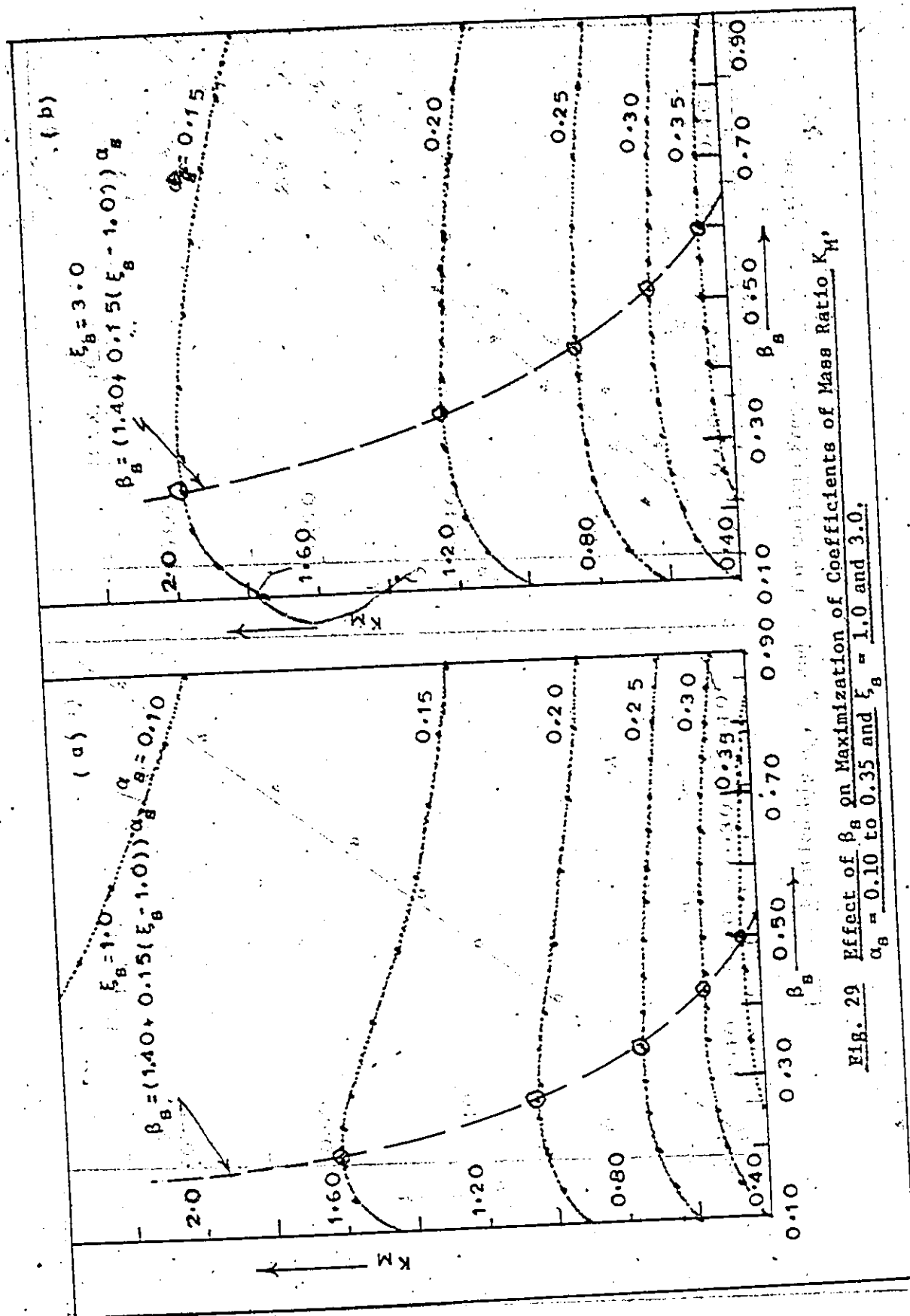


Fig. 29 Effect of β_B on Maximization of Coefficients of Mass Ratio K_M ,
 $\alpha_B = 0.10$ to 0.35 and $\epsilon_B = 1.0$ and 3.0 .

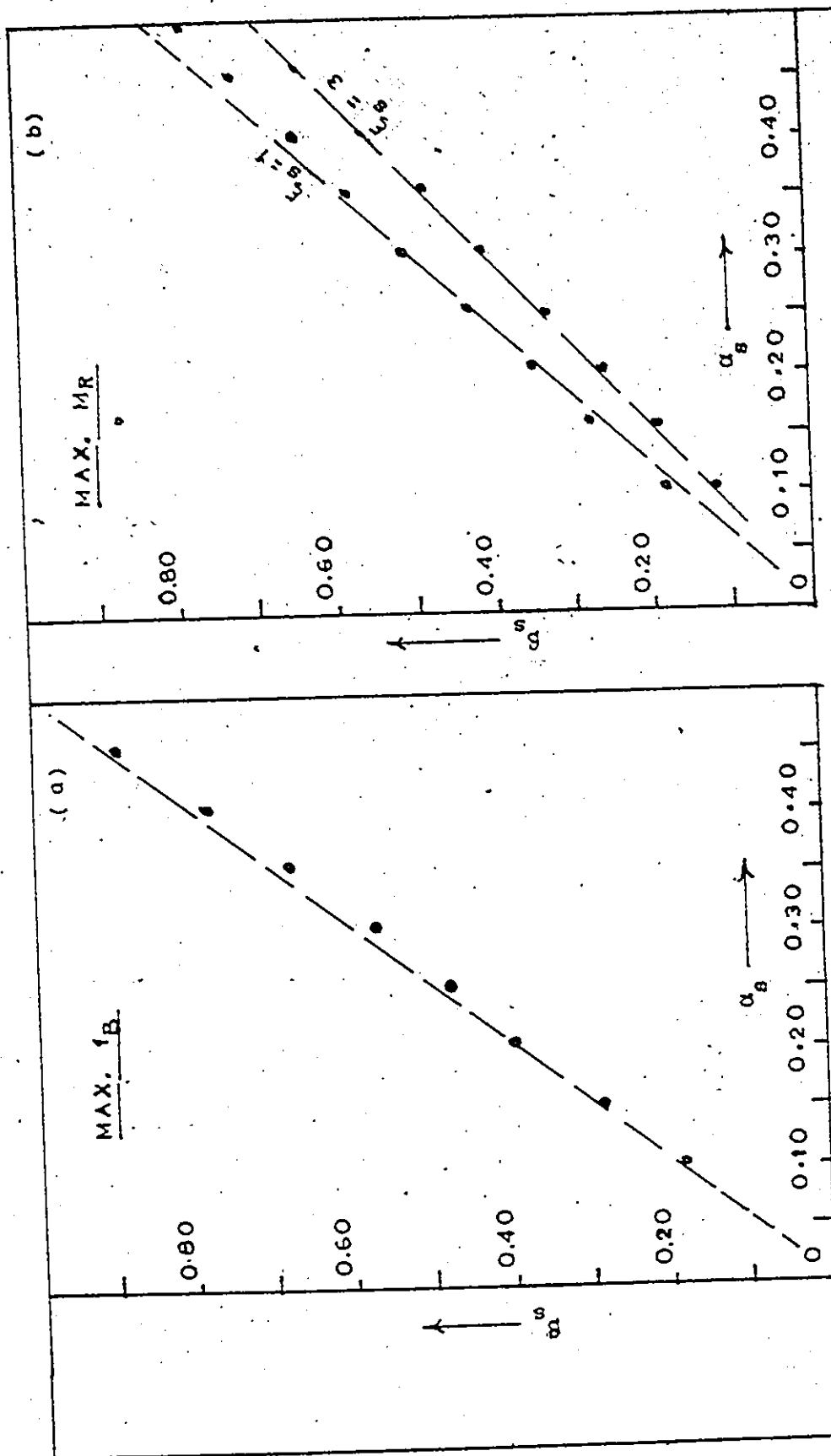
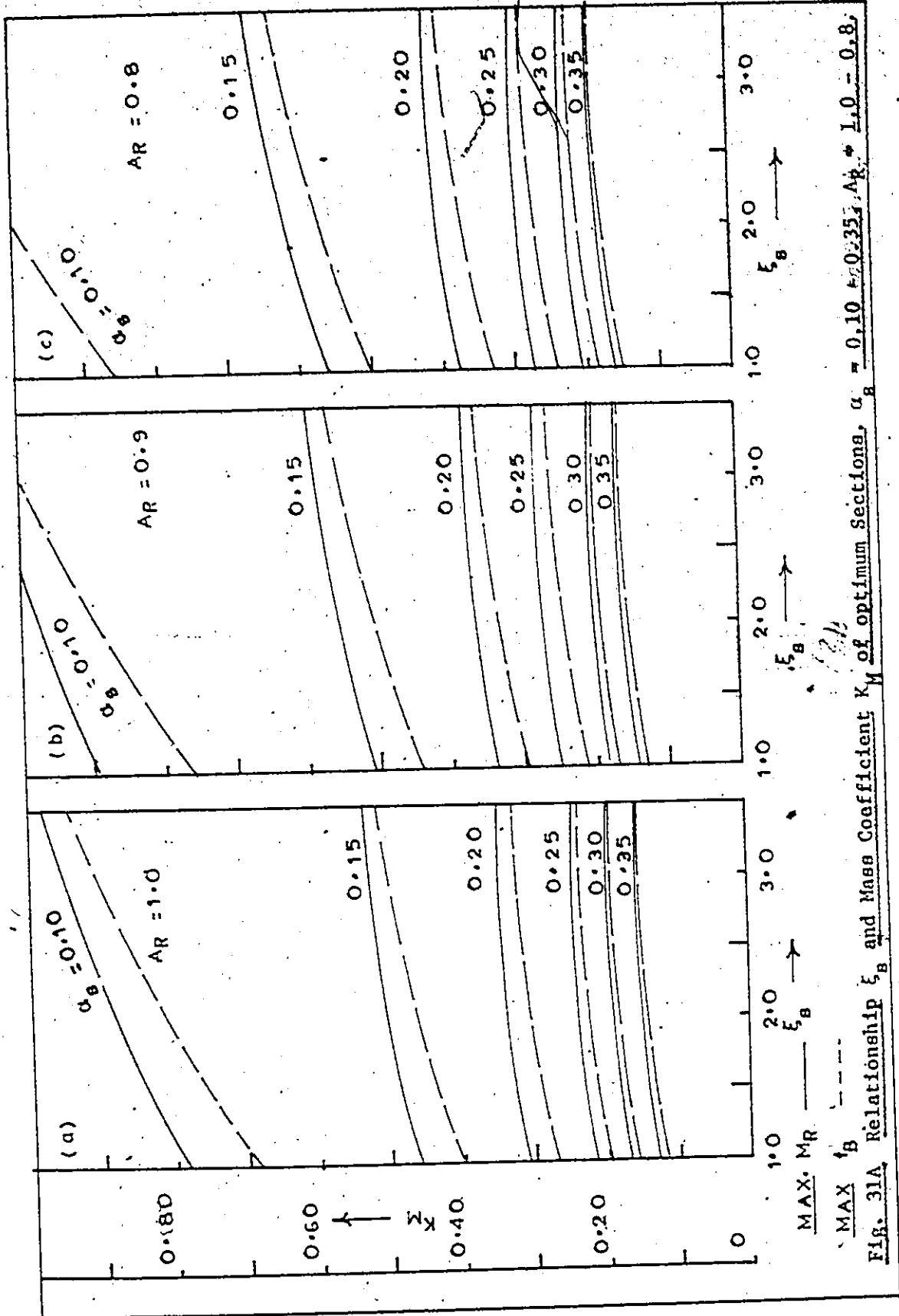


Fig. 30 Relationship α_B , β_B and ζ_B for Optimum Frequency and Mass Ratios.



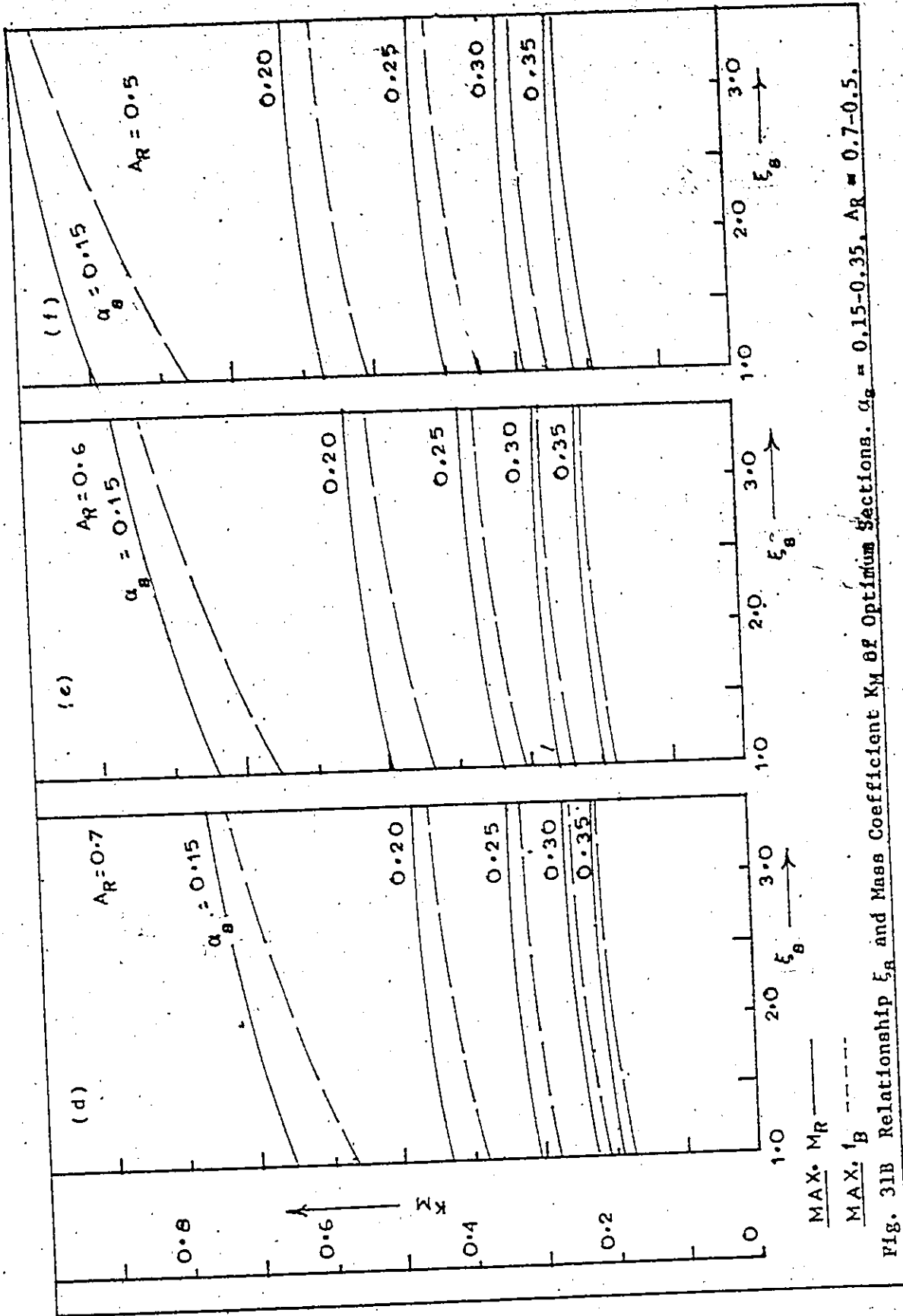
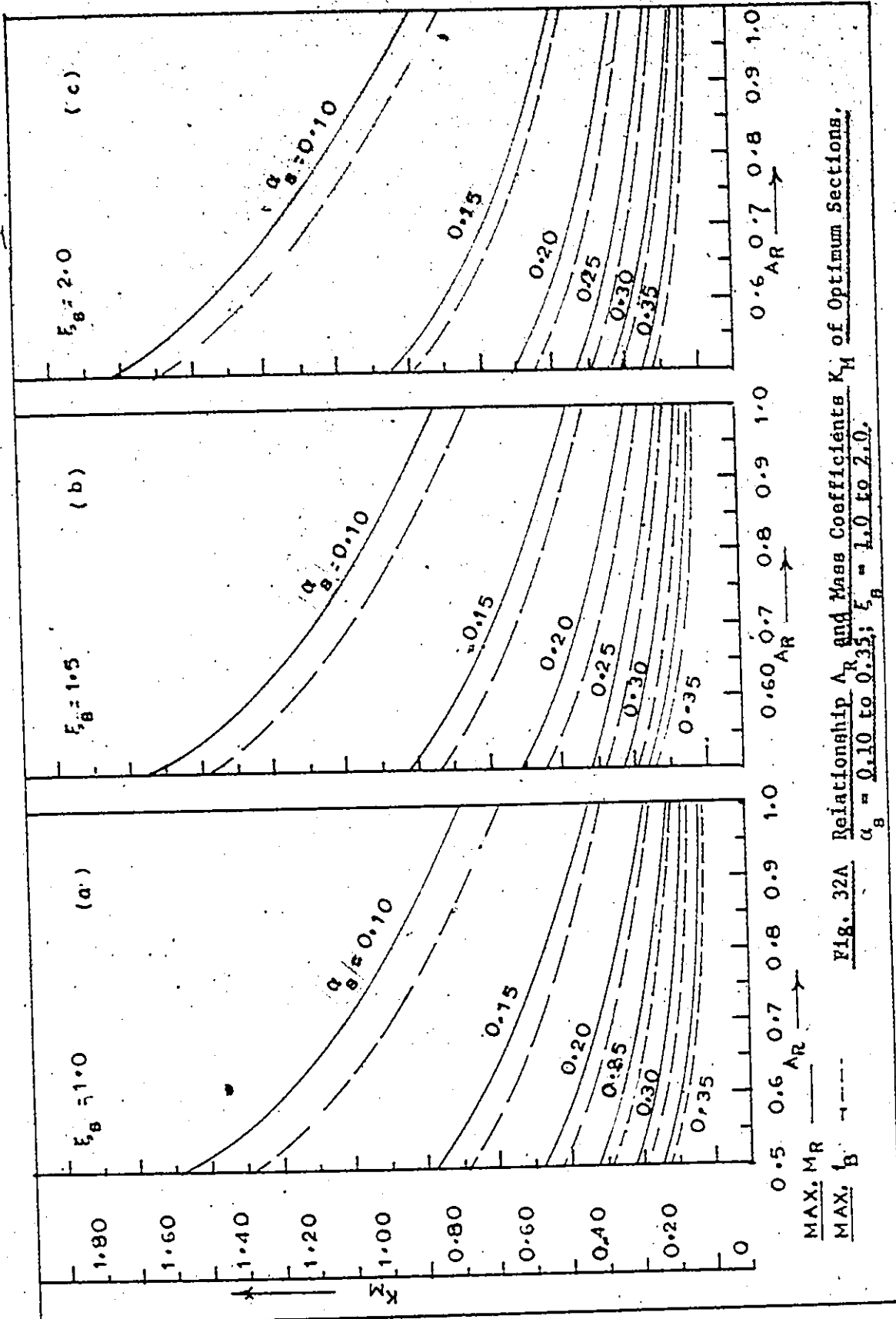
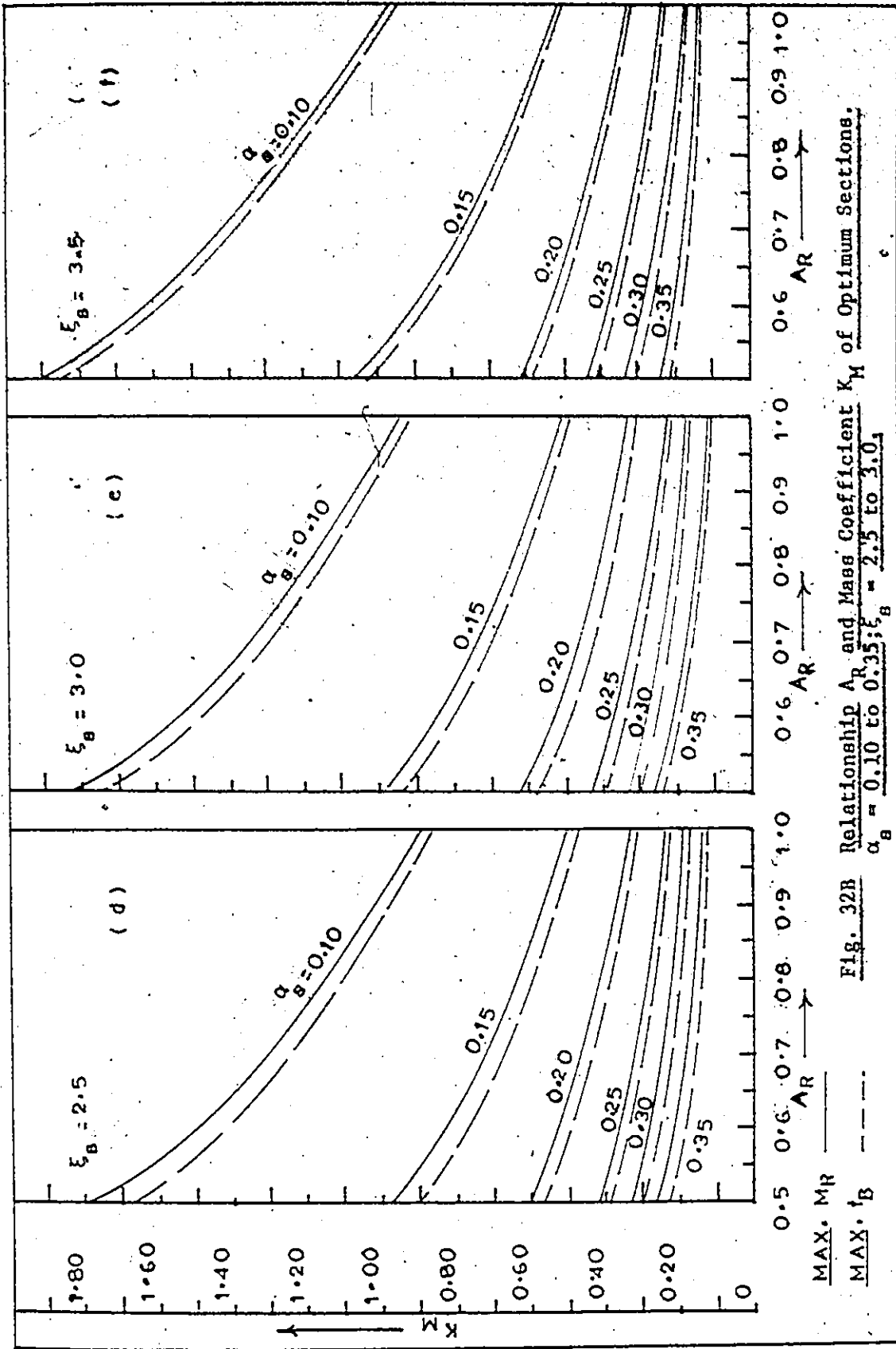
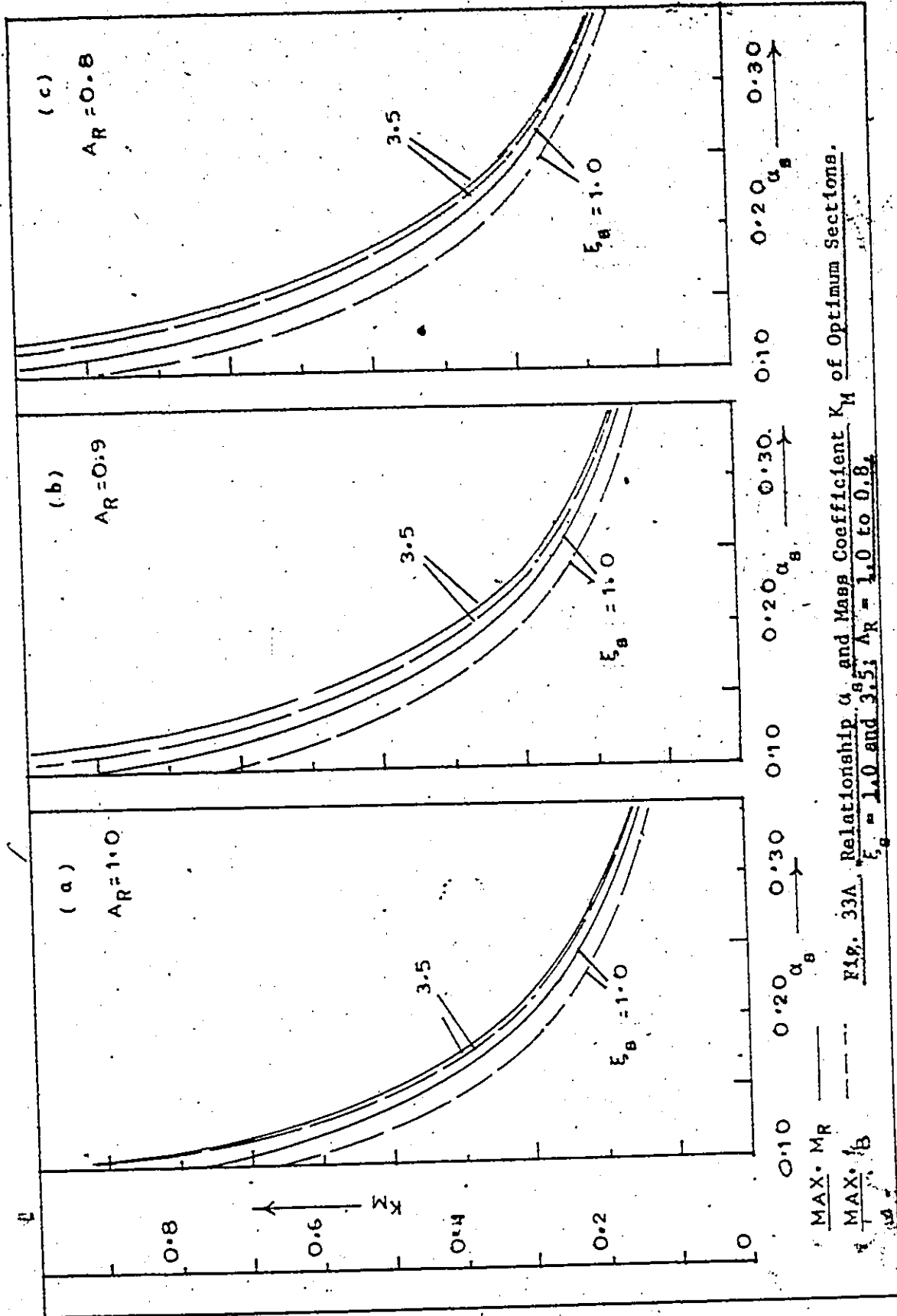


Fig. 31B Relationship ξ_B and Mass Coefficient K_M of Optimum Sections. $\alpha_B = 0.15-0.35$, $A_R = 0.7-0.5$.







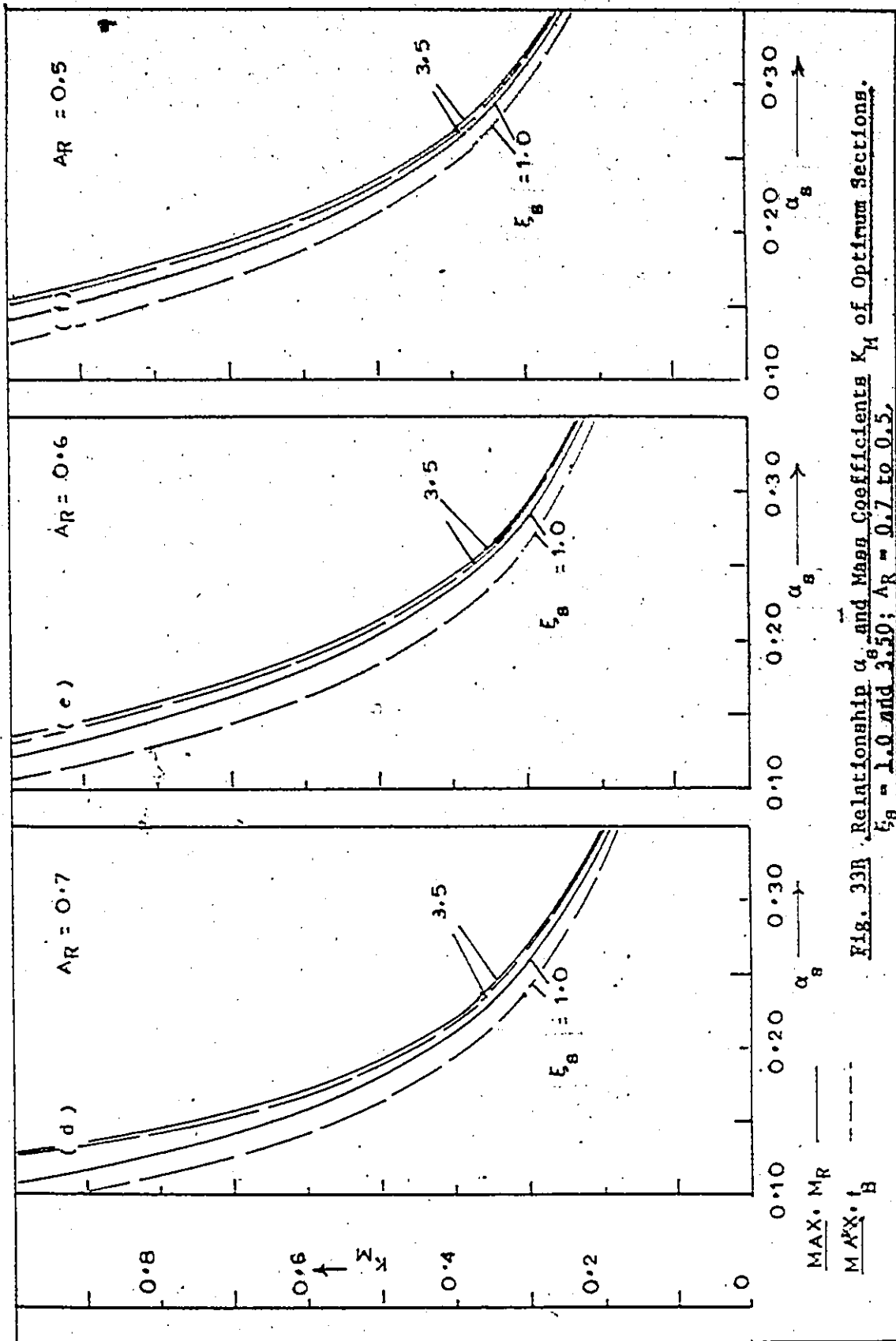


Fig. 33B. Relationship α_B and Mean Coefficients ζ_B of Optimum Sections.
 $\zeta_B = 1.0$ and 3.50 ; $AR = 0.7$ to 0.5 .

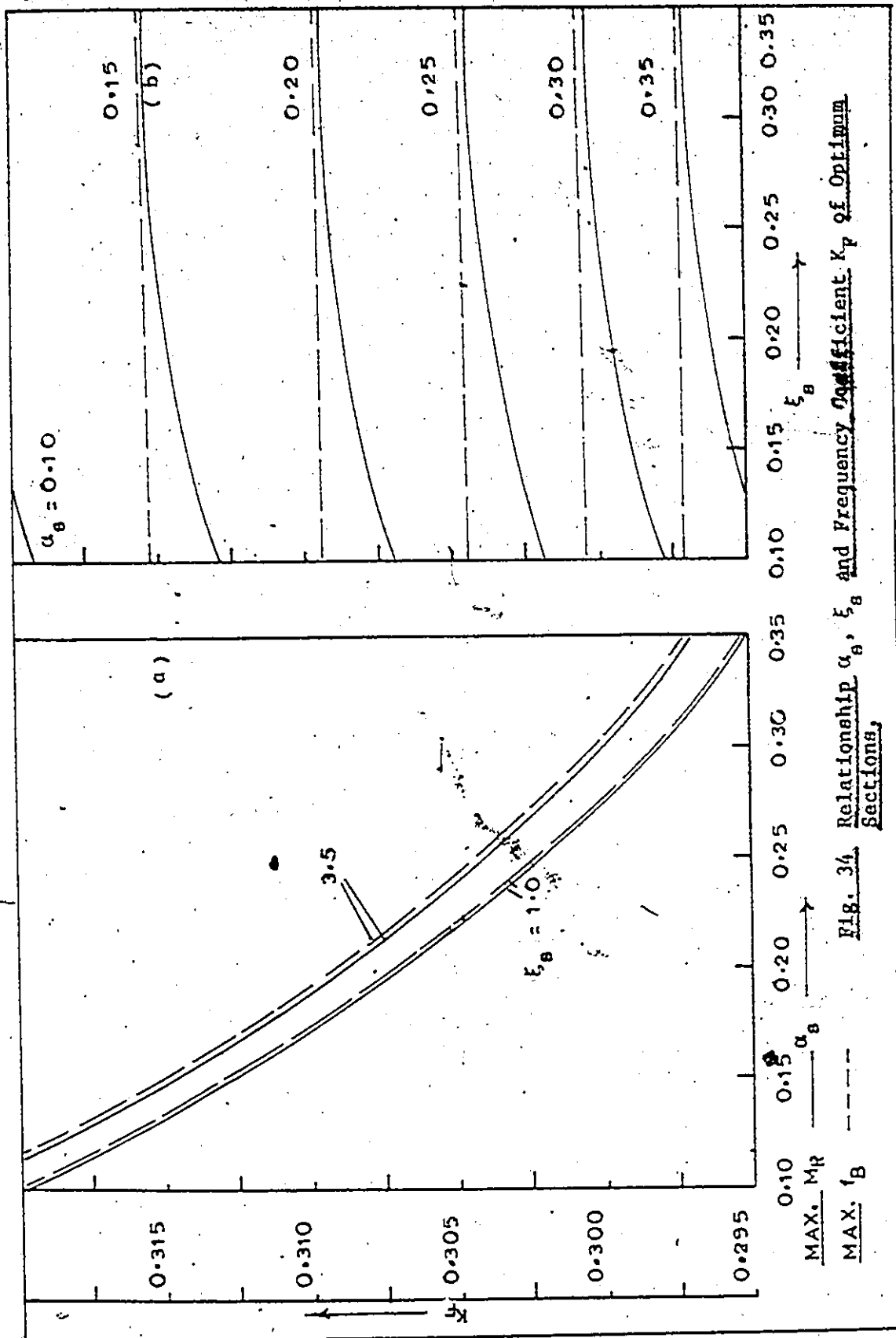
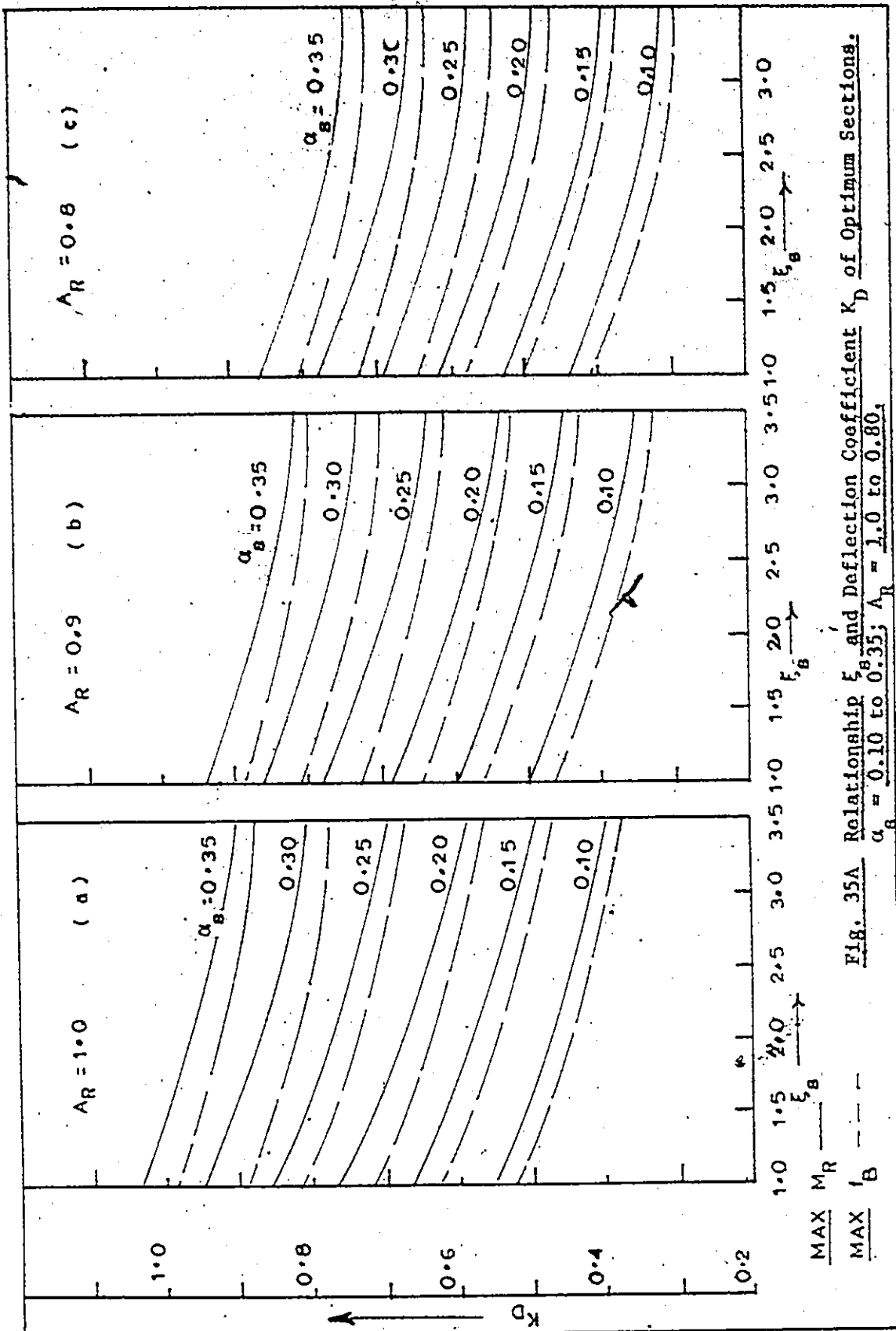
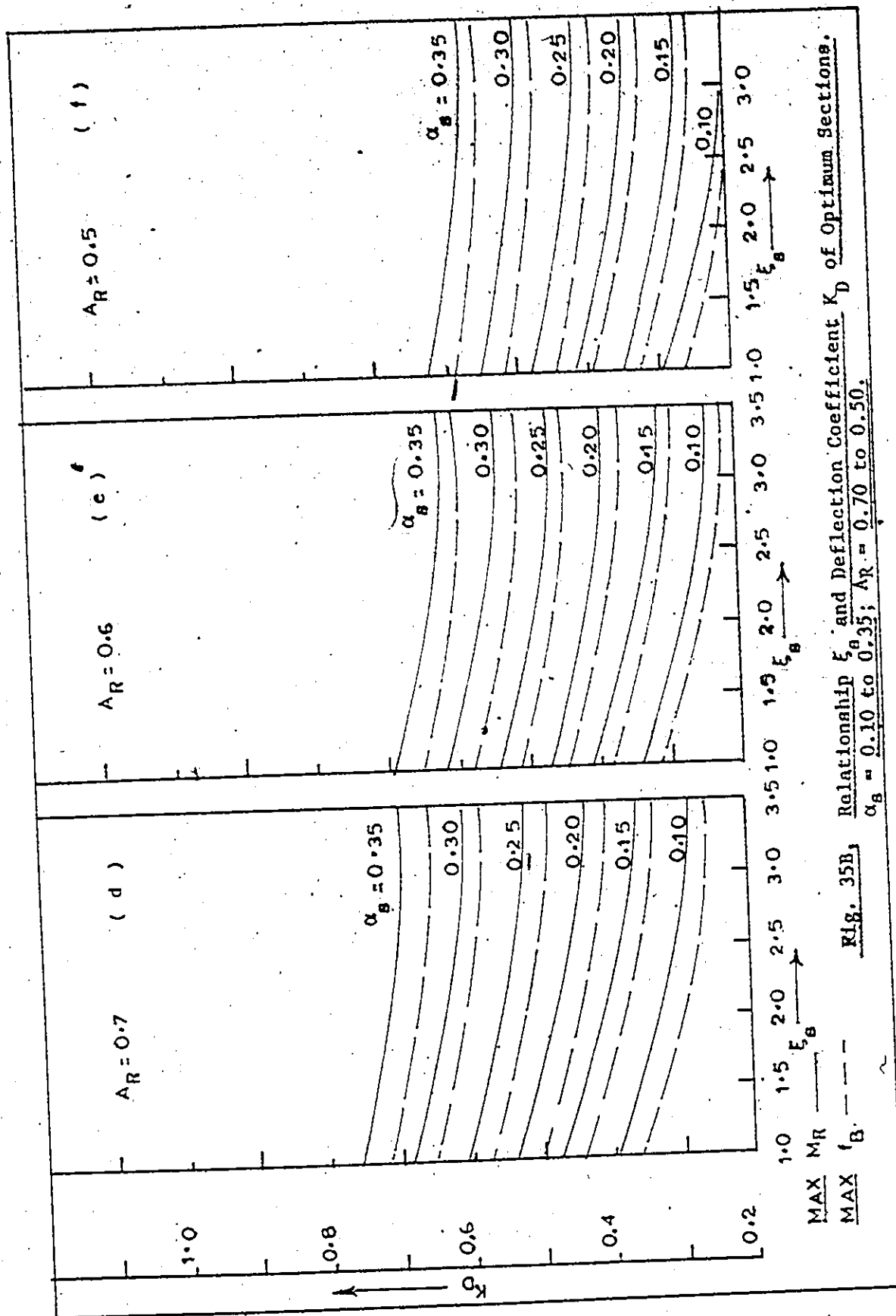


FIG. 34 Relationship α_B , ξ_B and Frequency Coefficient K_p of Optimum Sections,





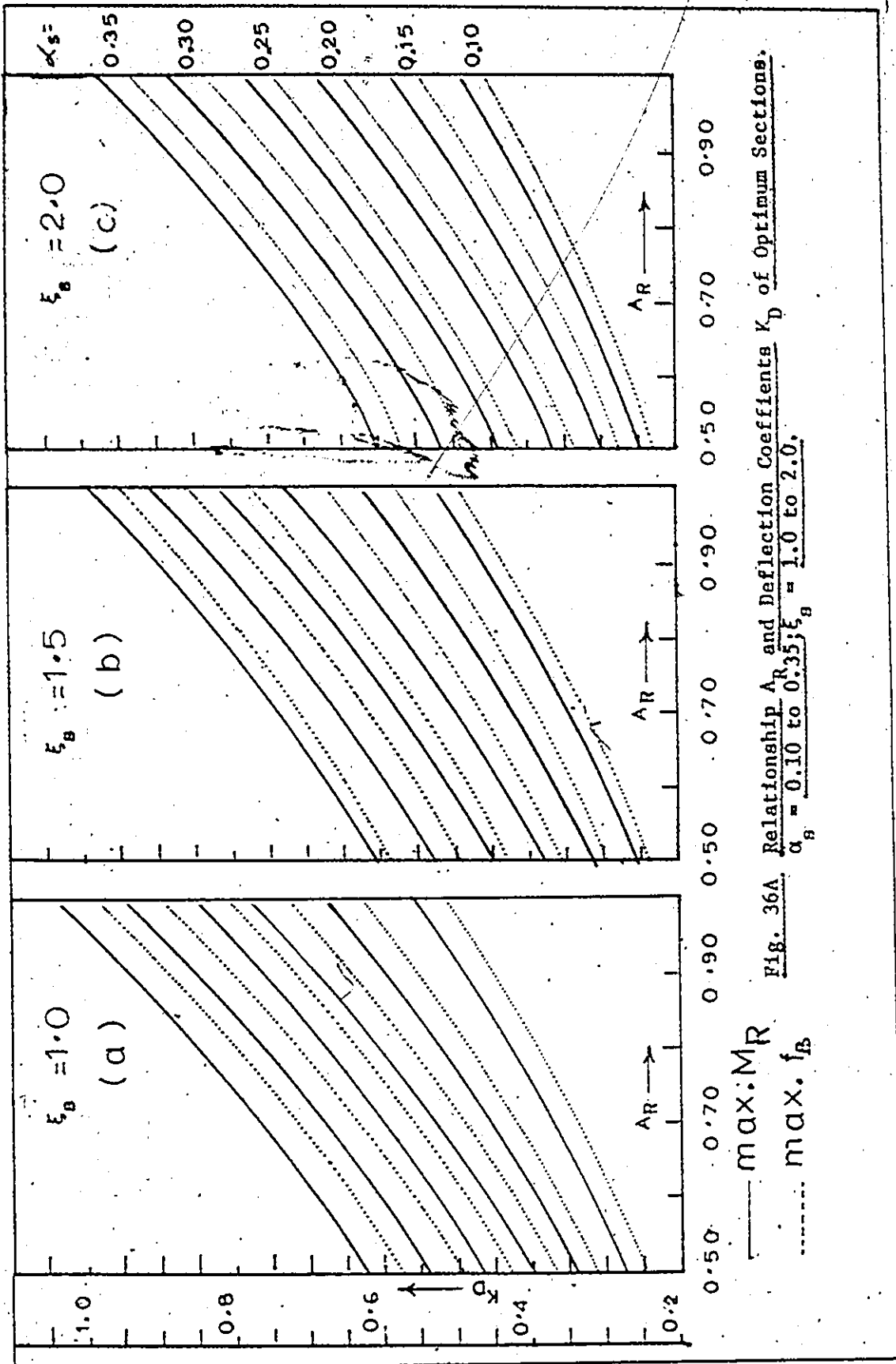
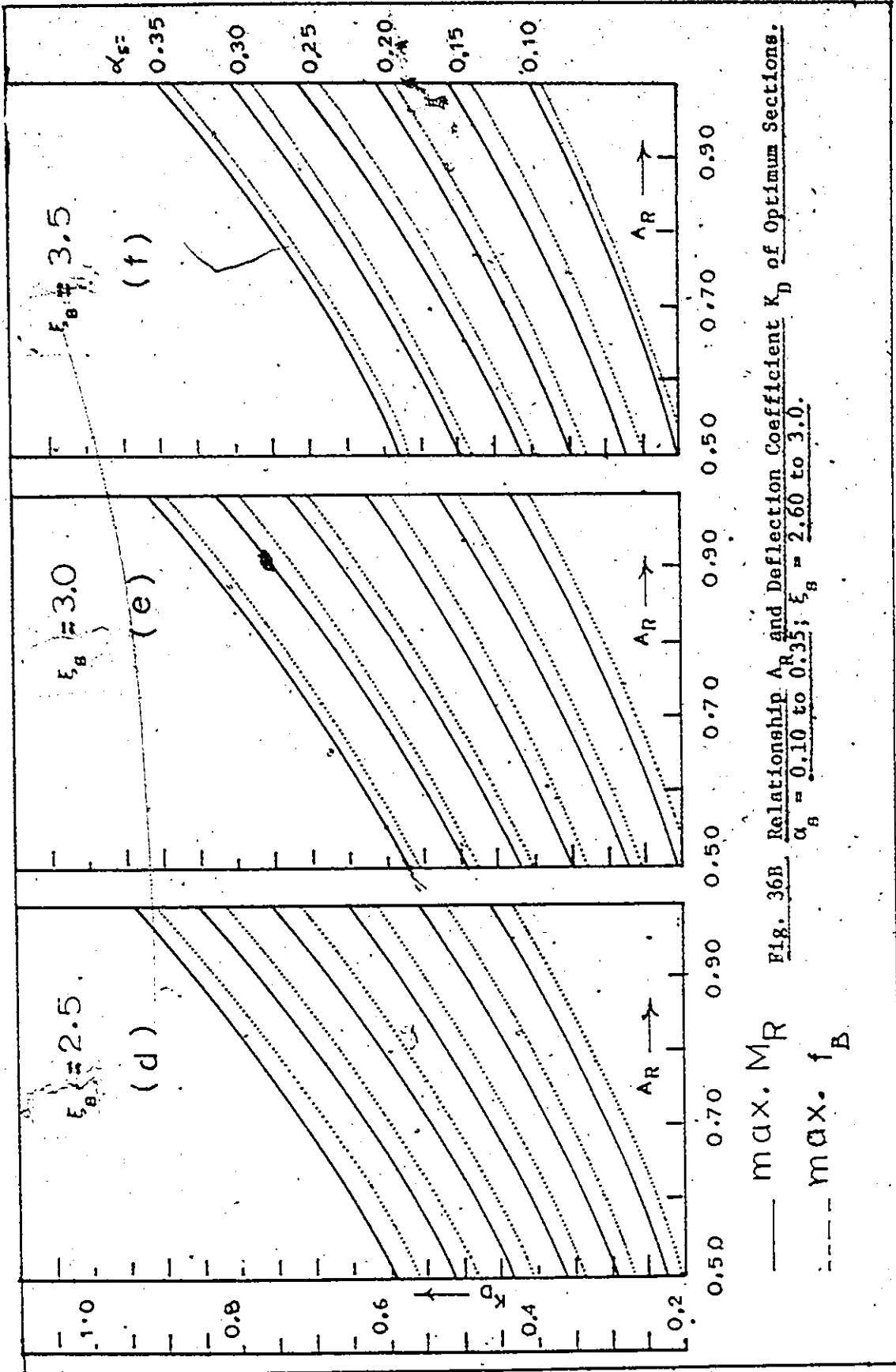
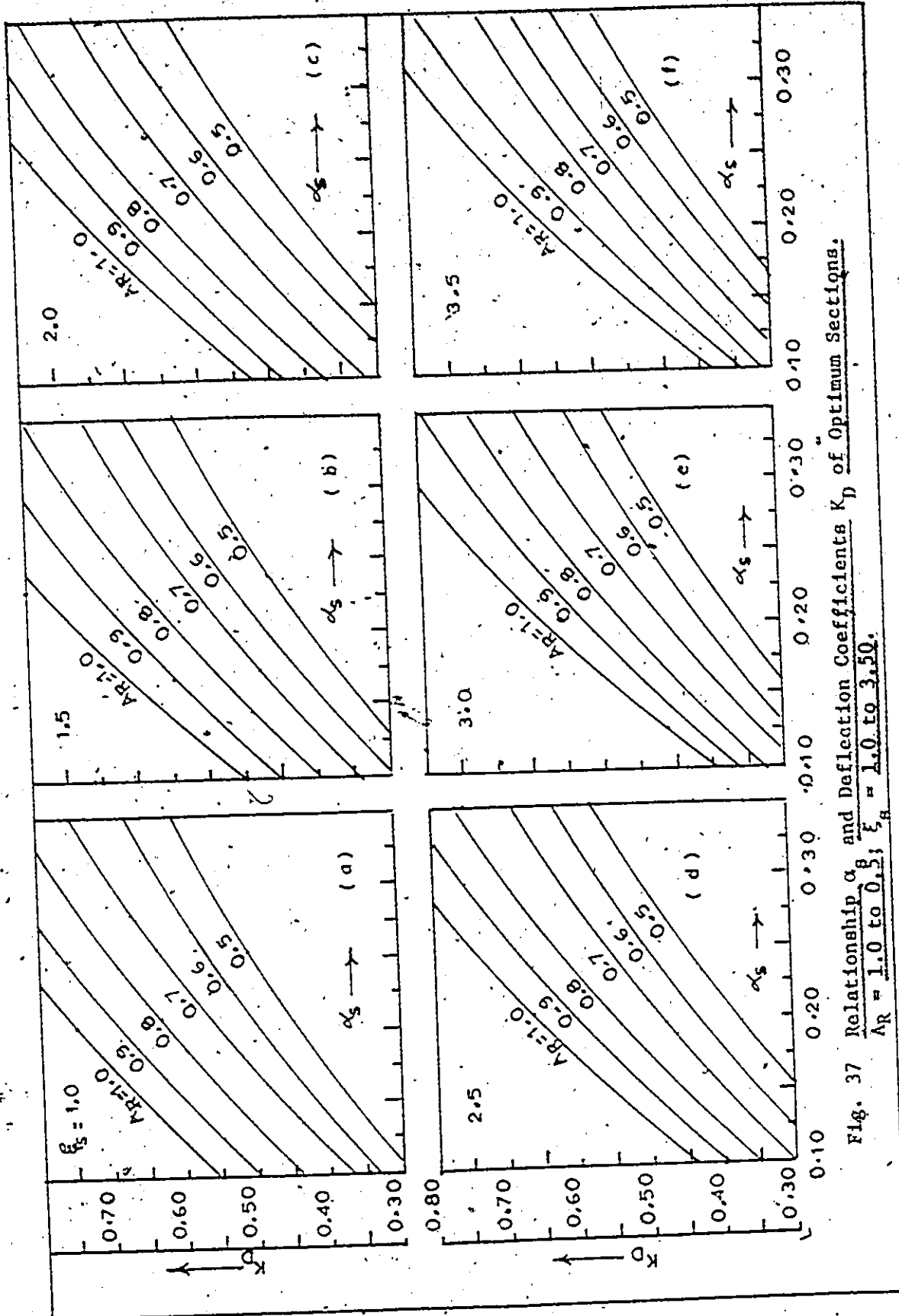


Fig. 36A. Relationship A_R and Deflection Coefficients K_D of Optimum Sections.
 $\alpha_B = 0.10$ to 0.35 ; $\xi_B = 1.0$ to 2.0 .





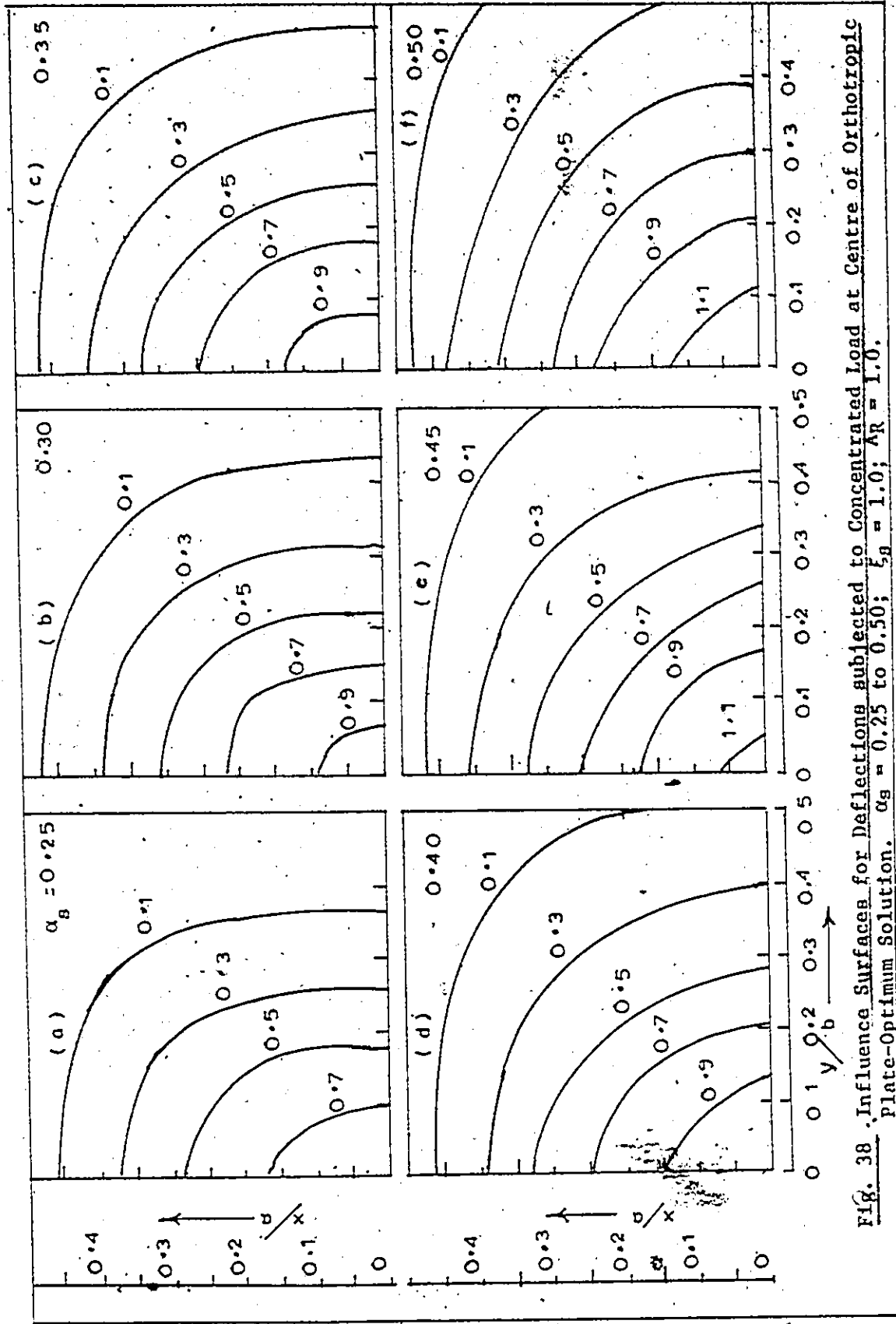
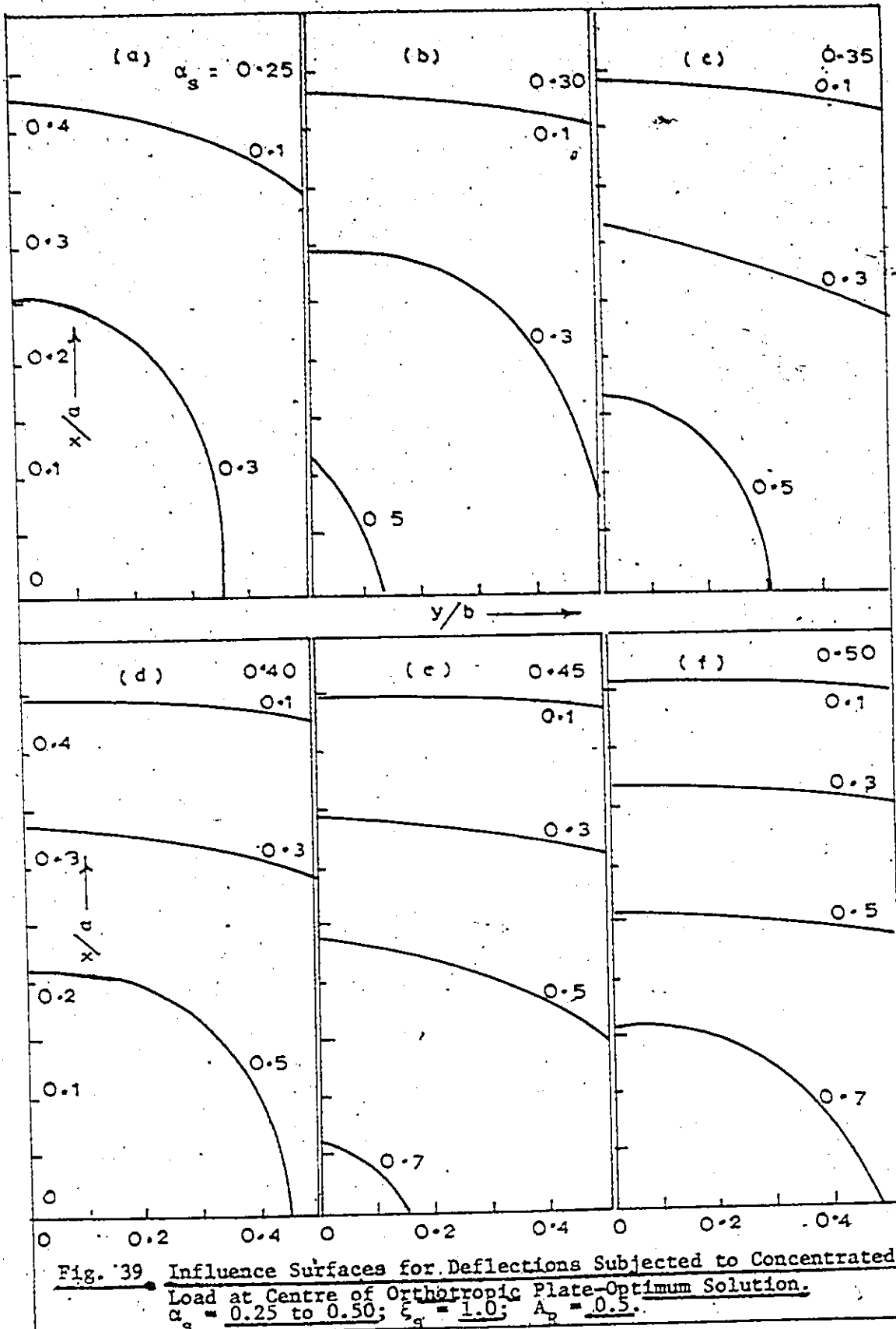


Fig. 38. Influence Surfaces for Deflections subjected to Concentrated Load at Centre of Orthotropic Plate-Optimum Solution. $\alpha_g = 0.25$ to 0.50 ; $\xi_g = 1.0$; $\lambda_R = 1.0$.



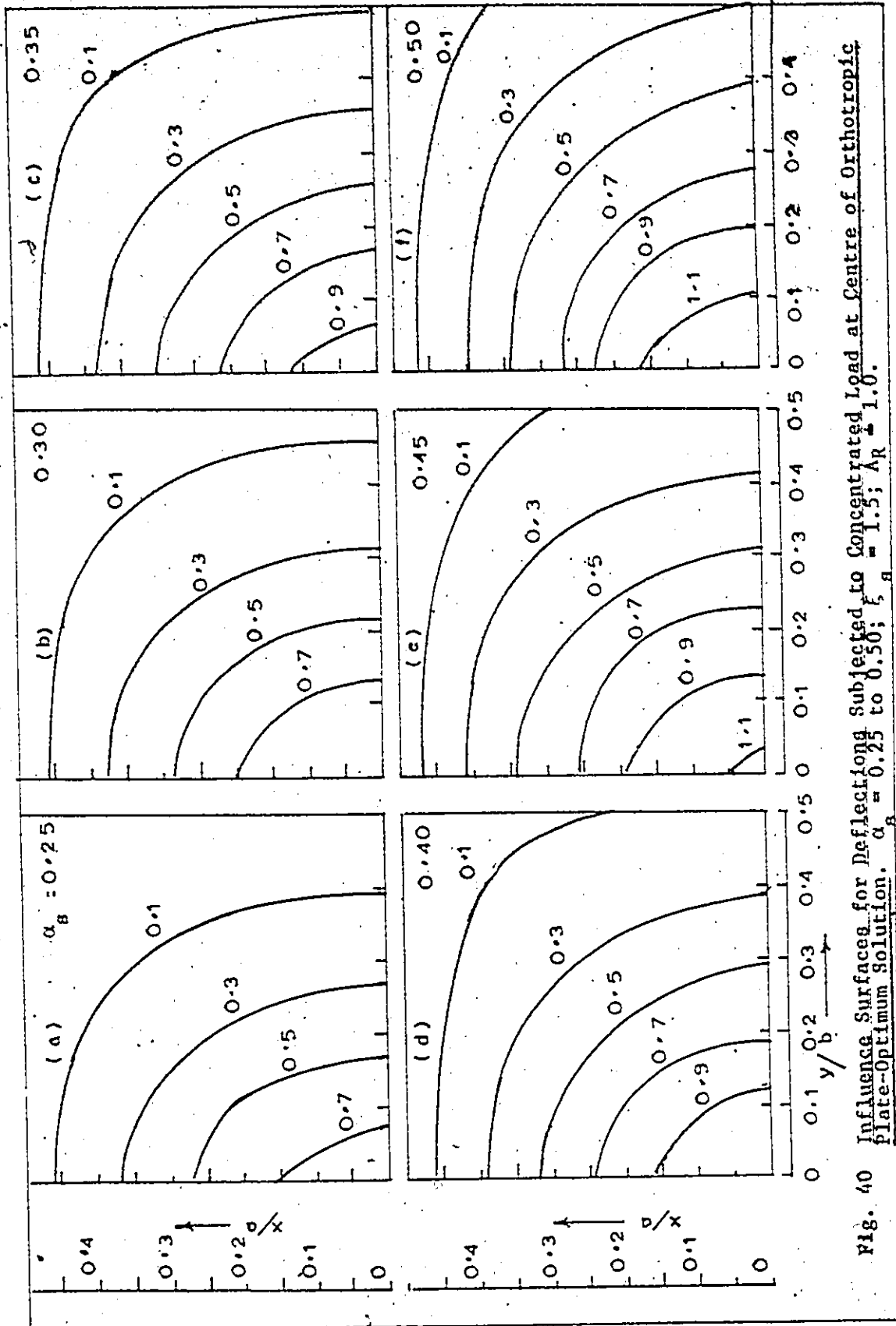


Fig. 40 Influence Surfaces for Deflection Subjected to Concentrated Load at Centre of Orthotropic Plate-Optimum Solution. $\alpha_B = 0.25$ to 0.50 ; $\xi_B = 1.5$; $A_B = 1.0$.

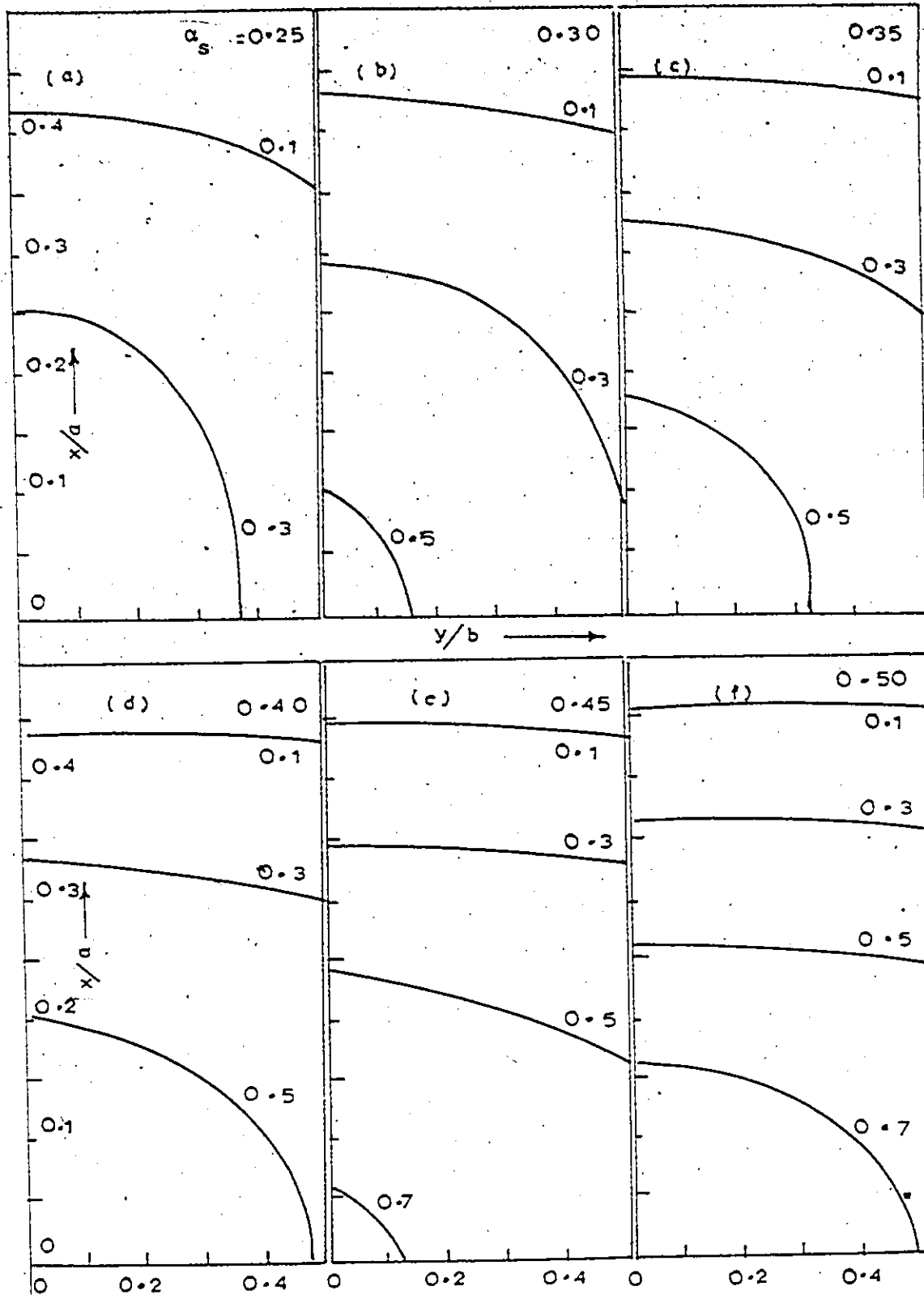
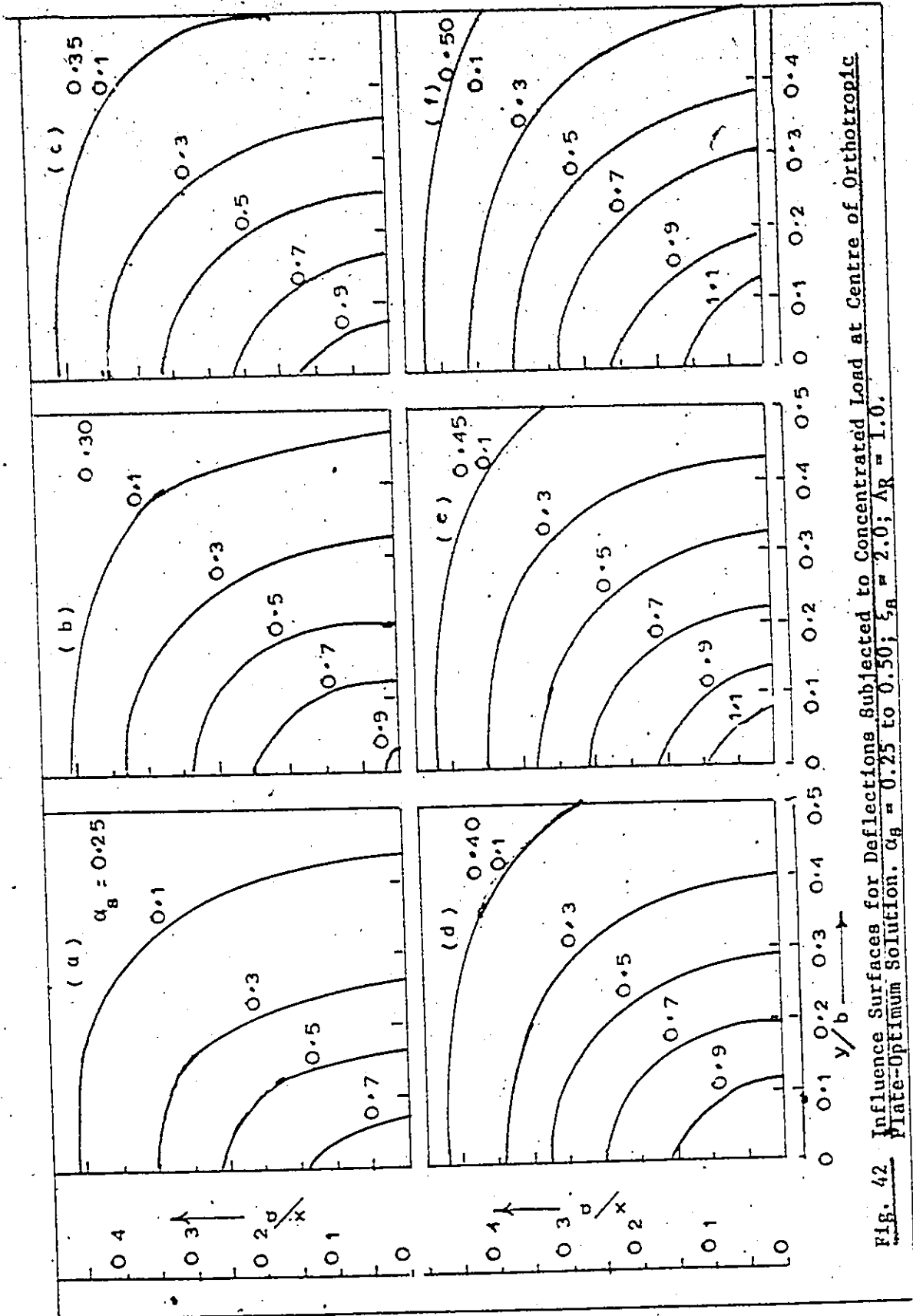


Fig. 41 Influence Surfaces for Deflections Subjected to Concentrated Load at Centre of Orthotropic Plate-Optimum Solution. $\alpha_s = 0.25$ to 0.50 ; $\xi_s = 1.5$, $A_R = 0.5$.



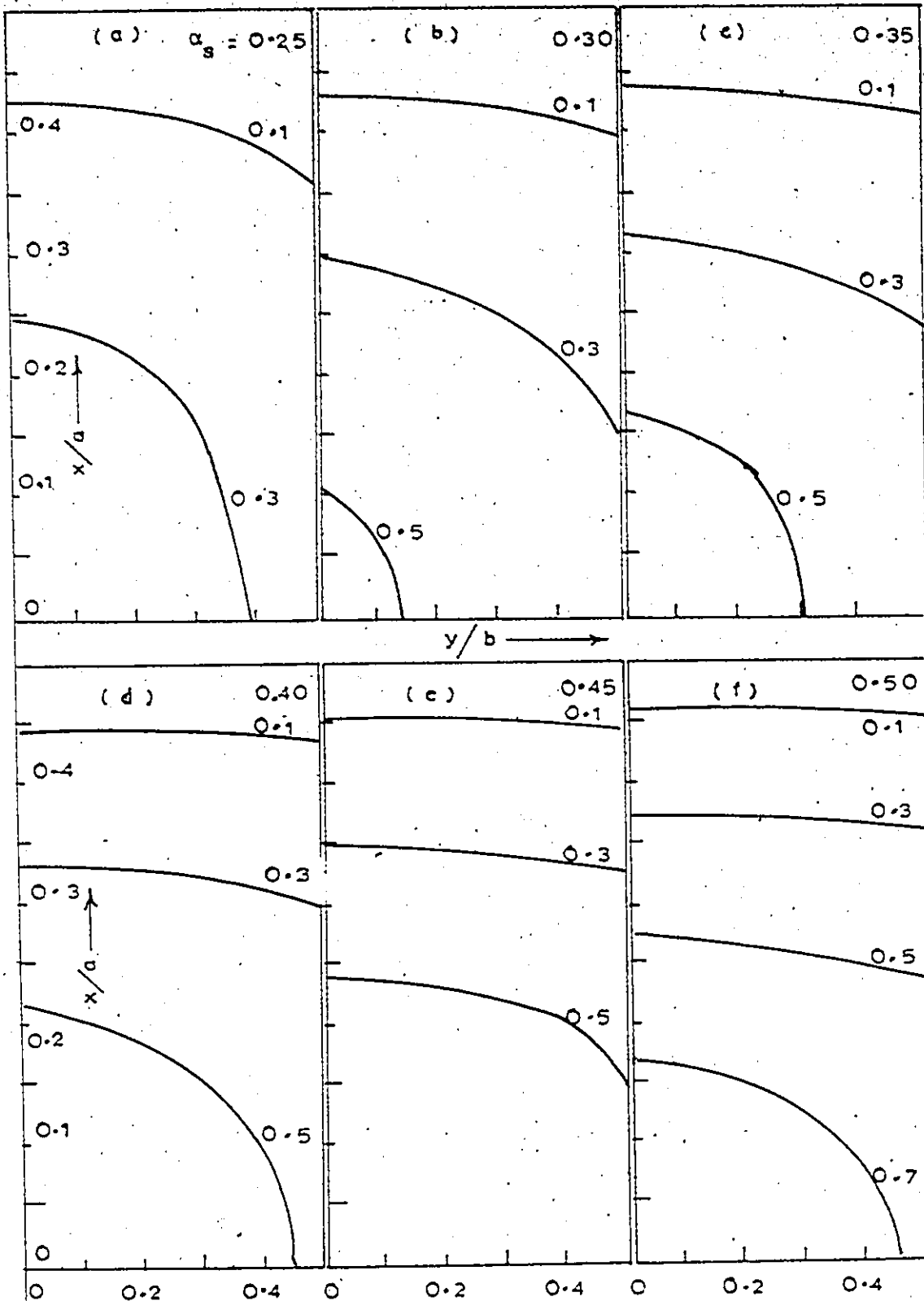


Fig. 43 Influence Surfaces for Deflections Subjected to Concentrated Load at Centre of Orthotropic Plate-Optimum Solution.
 $\alpha_s = 0.25$ to 0.50 ; $\xi_s = 2.0$; $A_R = 0.5$.

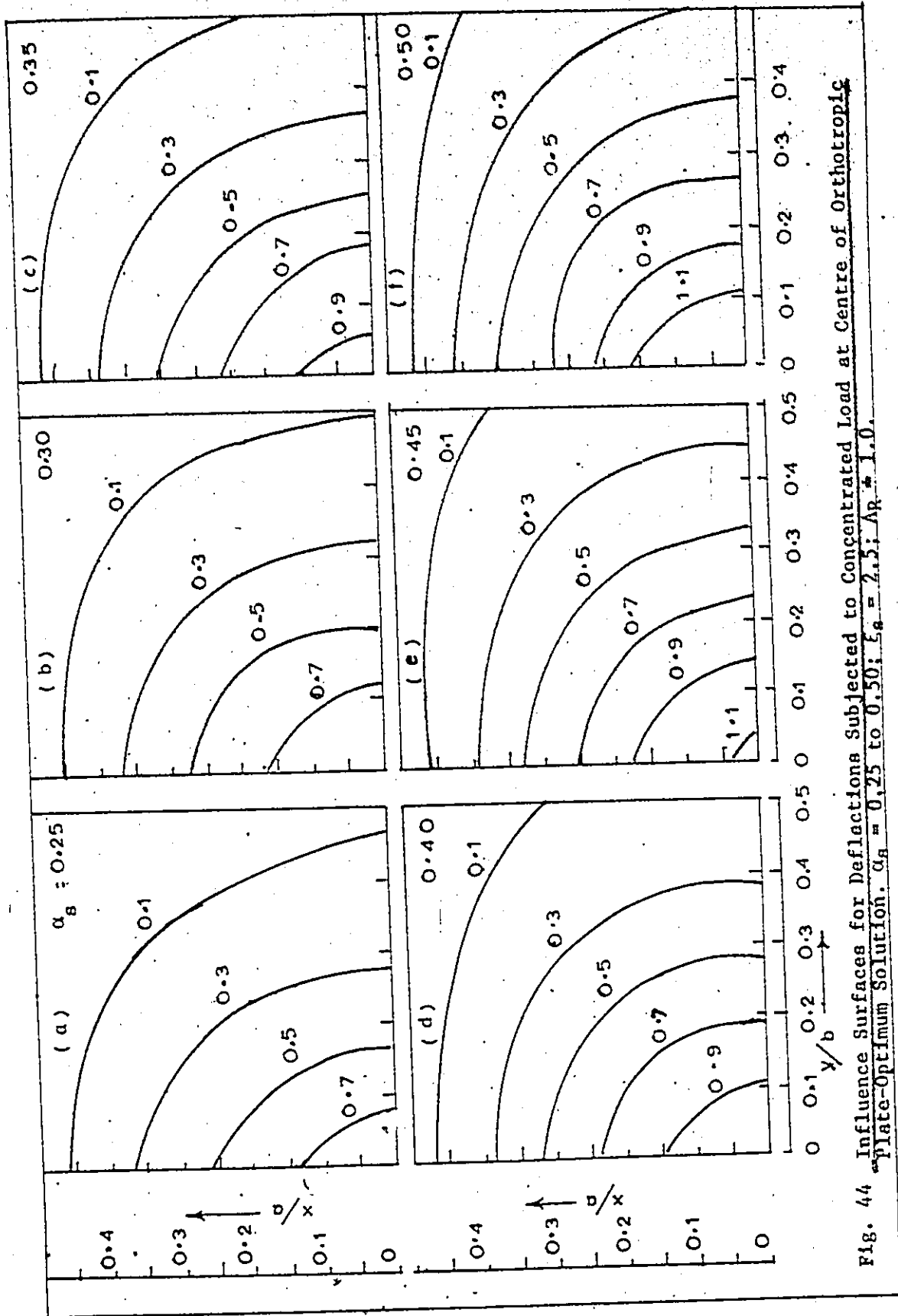


Fig. 44 Influence Surfaces for Deflections Subjected to Concentrated Load at Centre of Orthotropic Plate-Optimum Solution. $\alpha_s = 0.25$ to 0.50 ; $\epsilon_s = 2.5$; $\mu_s = 1.0$.

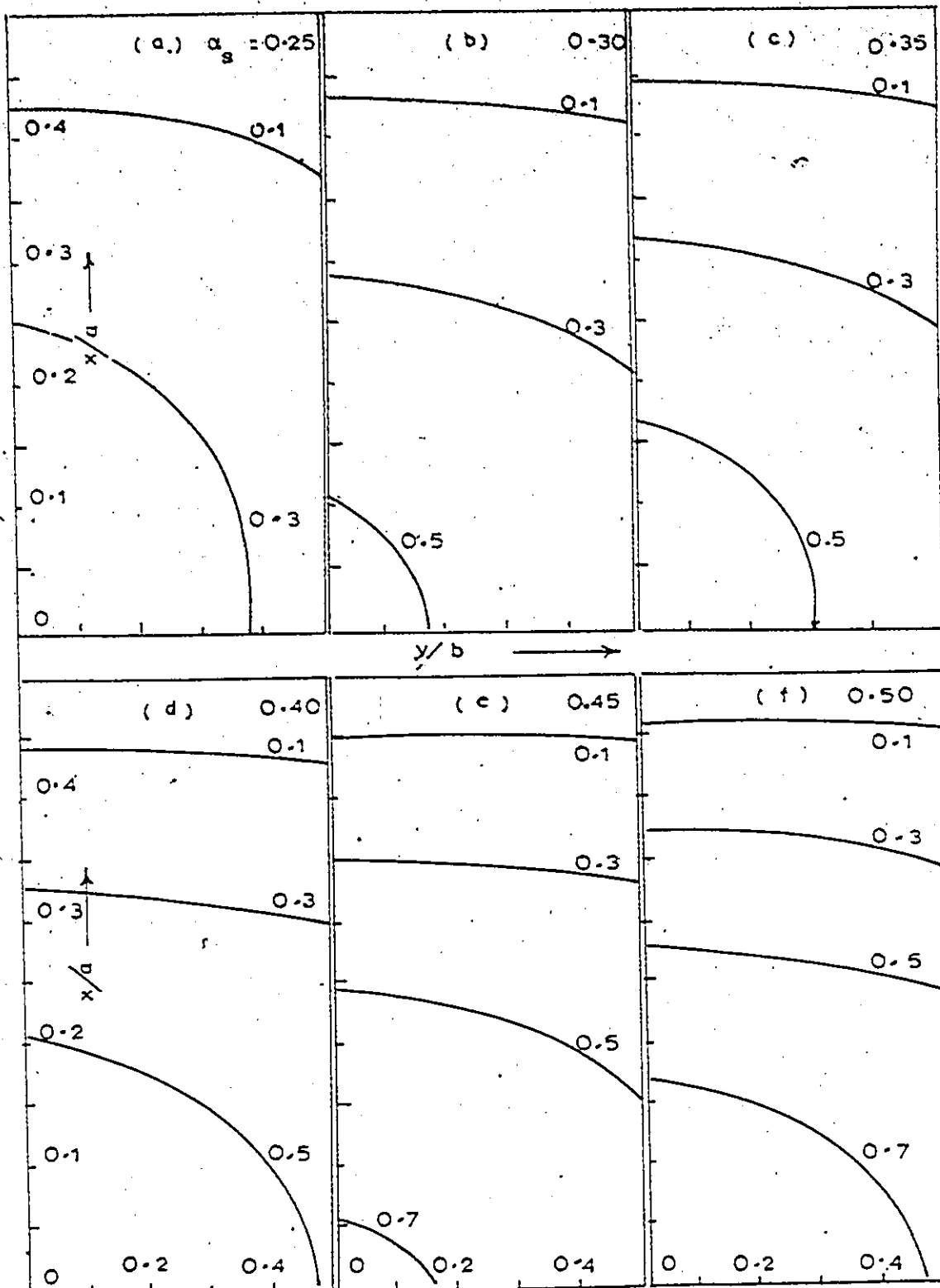


Fig. 45 Influence Surfaces for Deflections Subjected to Concentrated Load at Centre of Orthotropic Plate-Optimum Solution.
 $\alpha_s = 0.25$ to 0.50 ; $\xi_s = 2.5$; $A_R = 0.5$.

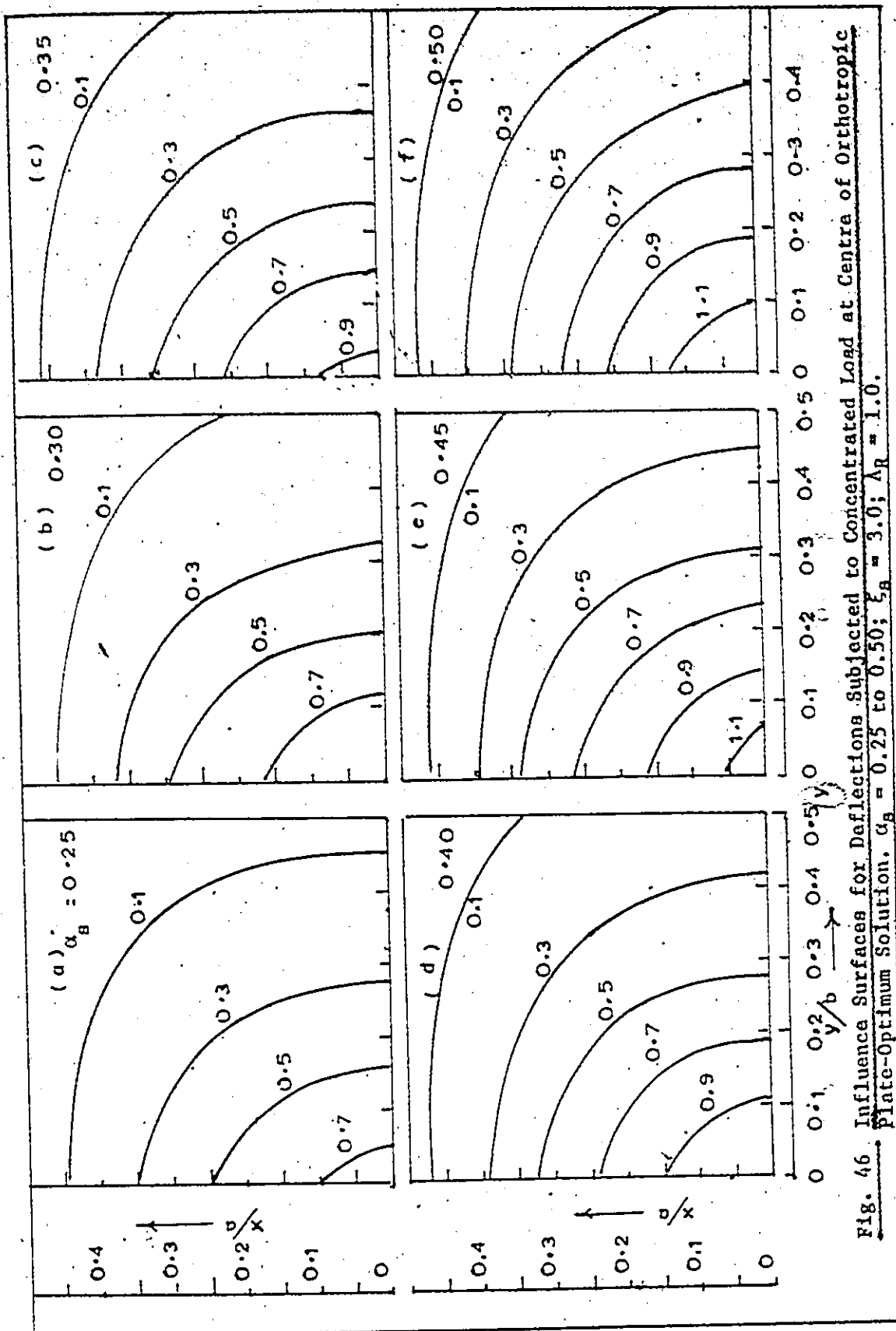
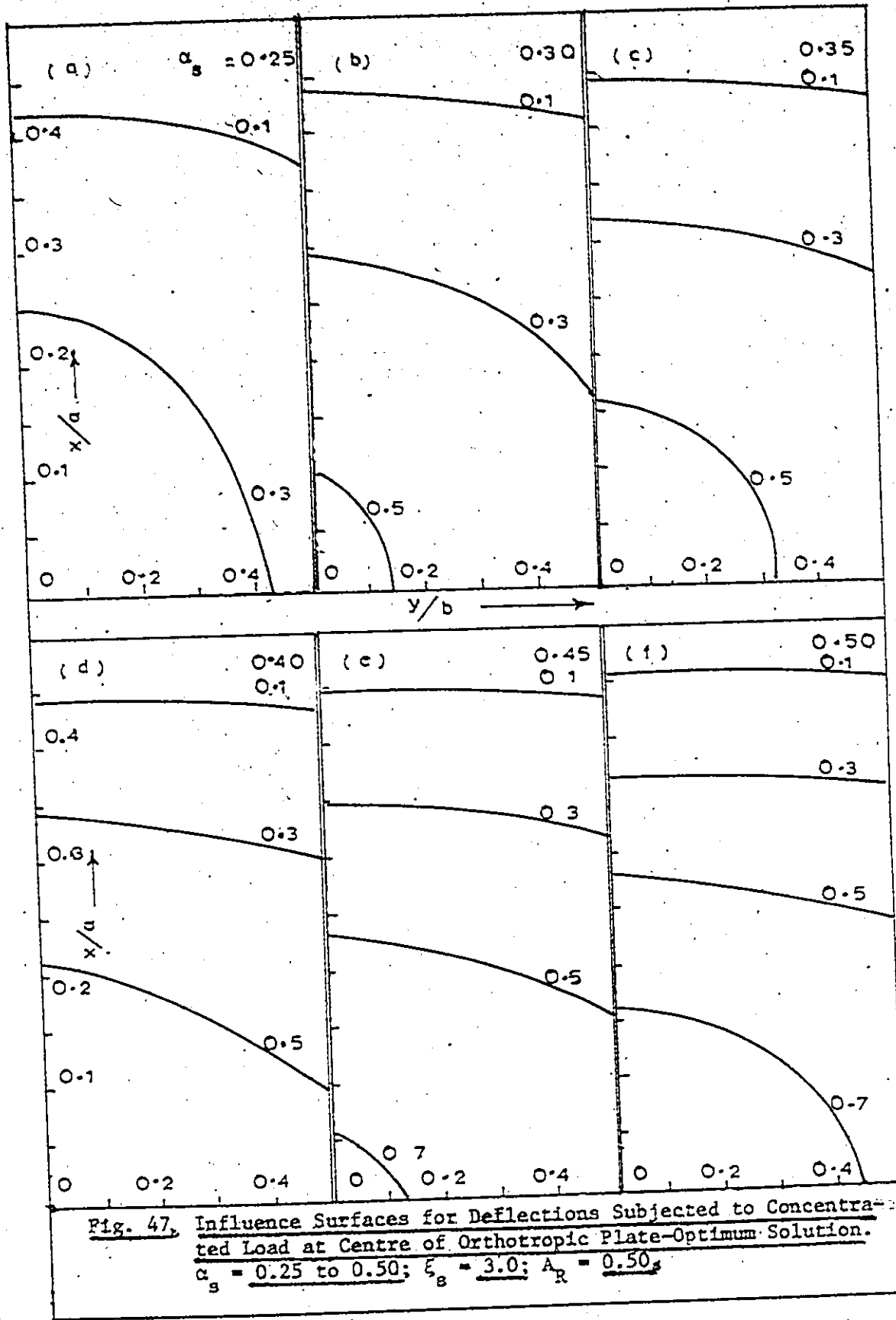


Fig. 46 Influence Surfaces for Deflections Subjected to Concentrated Load at Centre of Orthotropic Plate-Optimum Solution. $\alpha_B = 0.25$ to 0.50 ; $\xi_B = 3.0$; $\lambda_B = 1.0$.



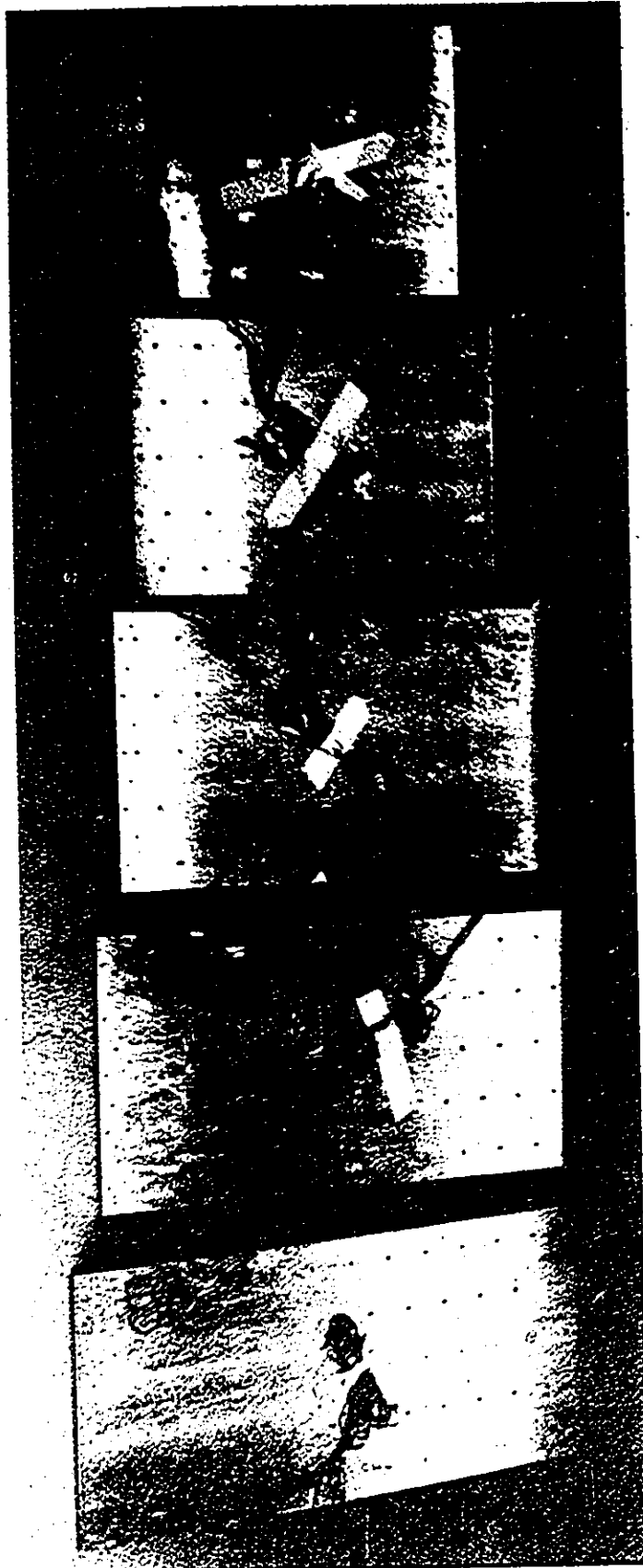
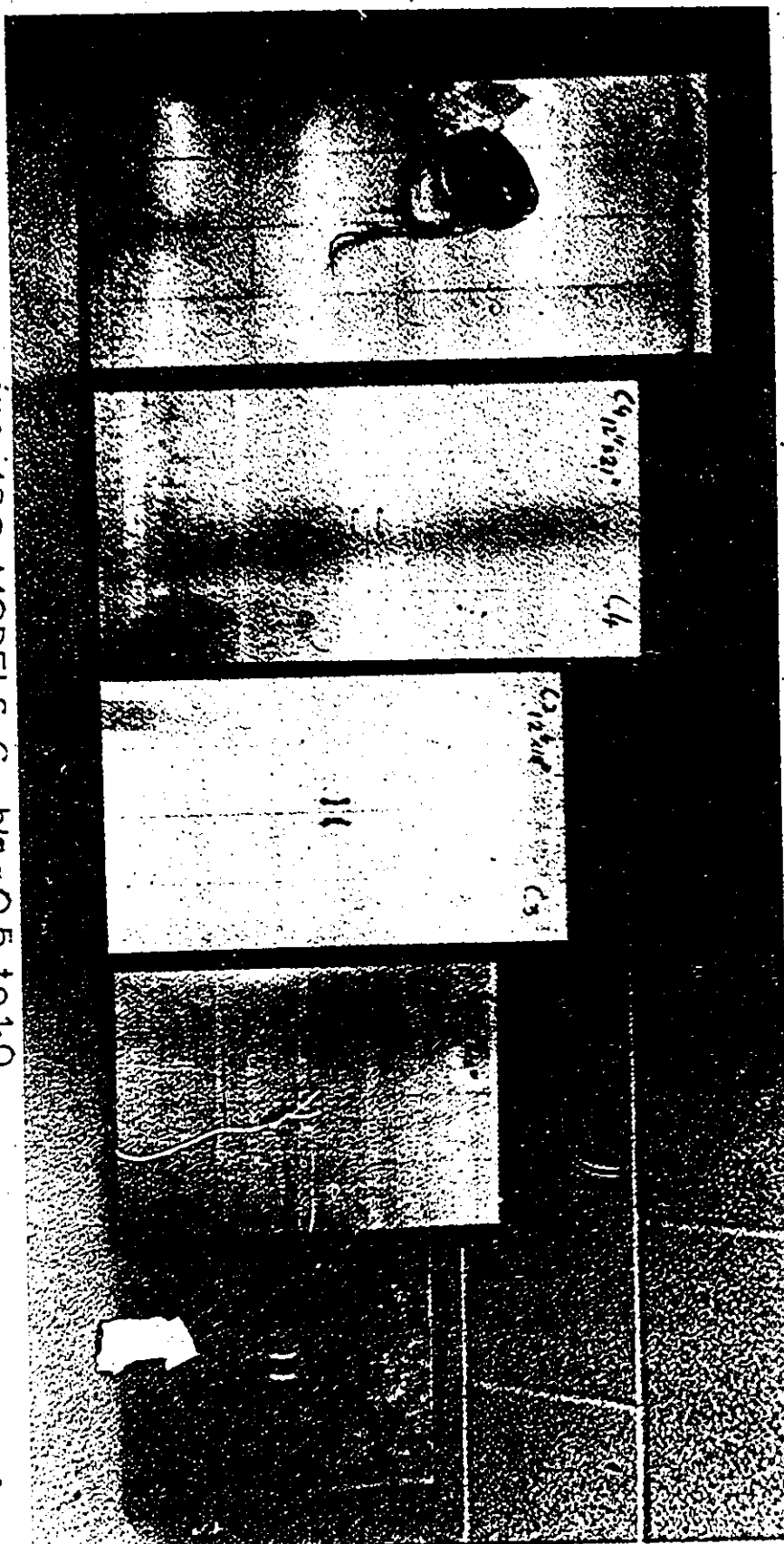


FIG. 48A. MODELS B. $b/a=1.0$ to 0.5



FIG. 48B. MODELS B (bottom). $b/a=1.0$ to 0.5

FIG. 48C. MODELS C $b/a = 0.5$ to 1.0



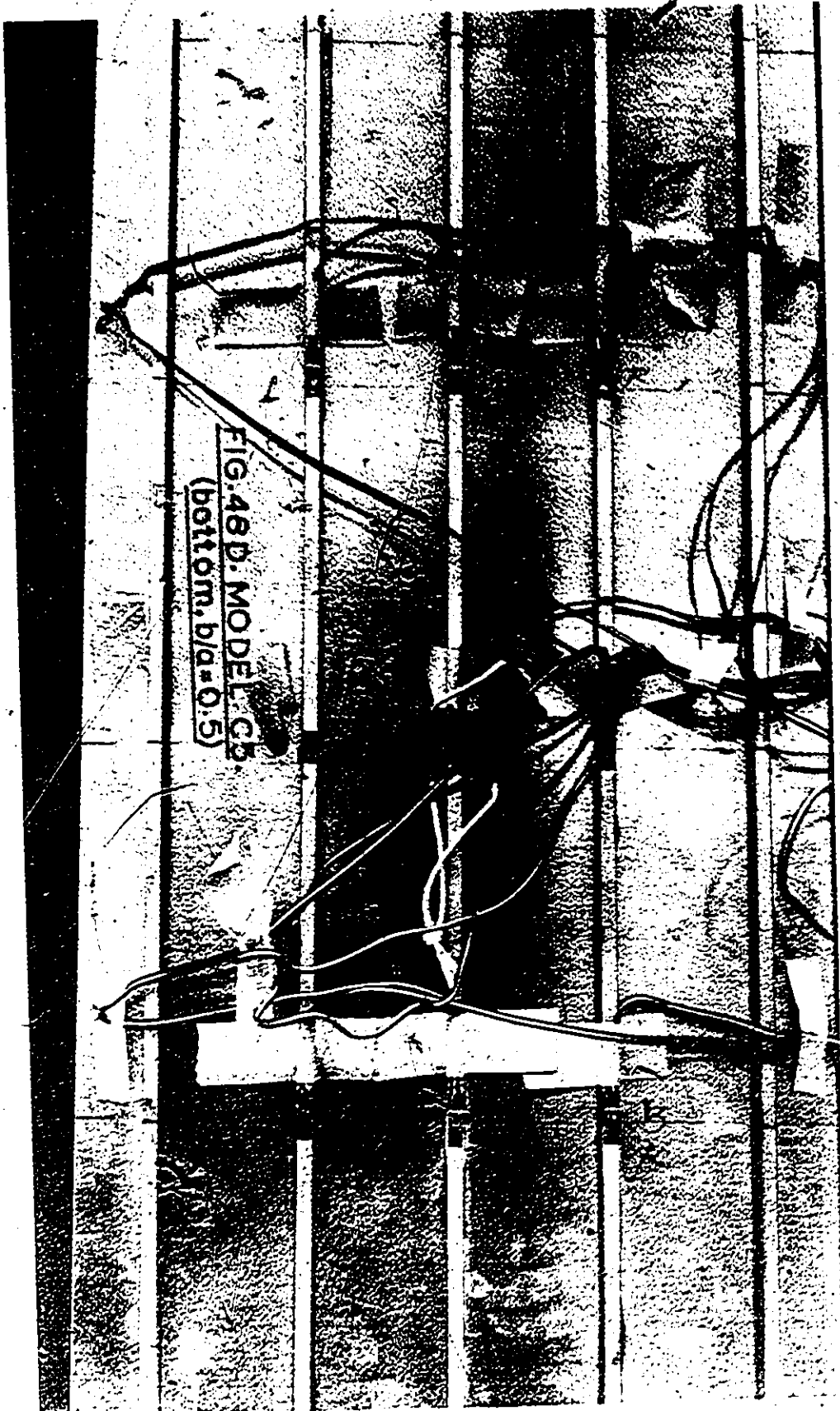


FIG. 48D, MODEL C5
(bottom, b/a=0.5)

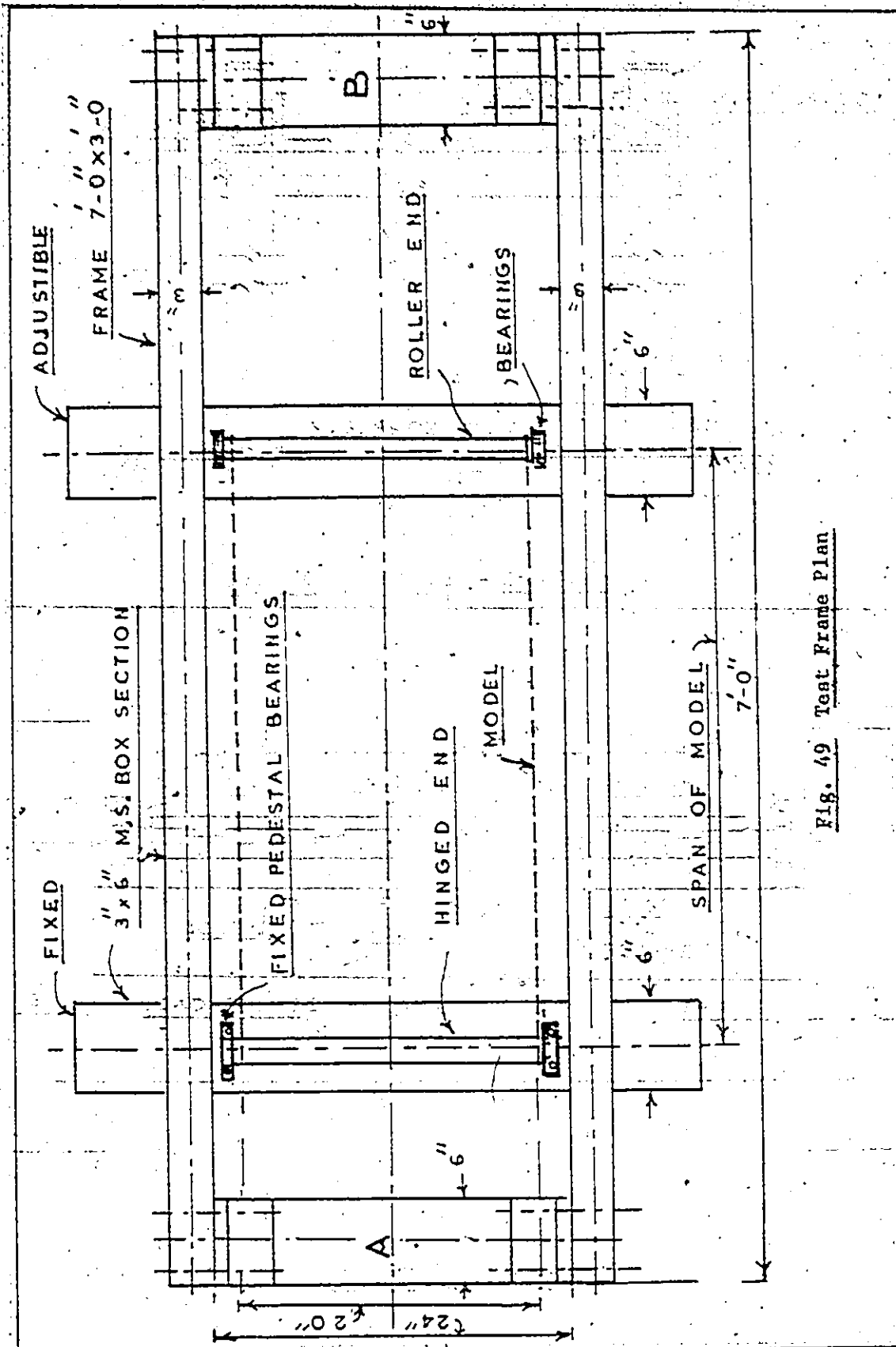


Fig. 49 Test Frame Plan

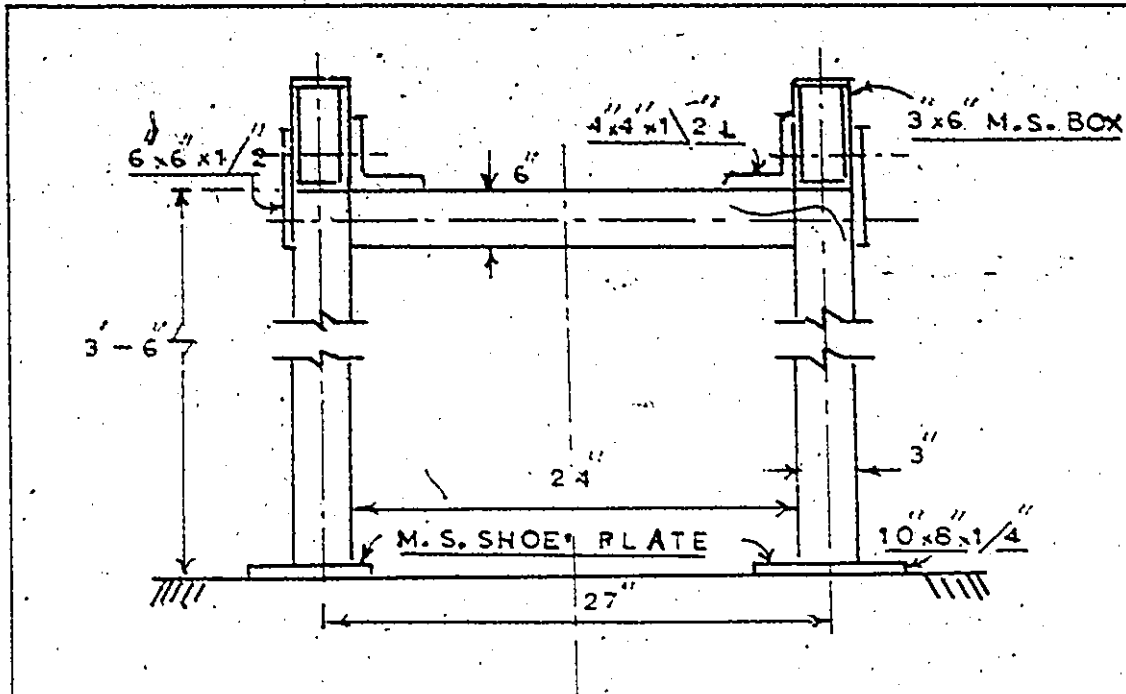


Fig. 50 Test Frame - End Elevation

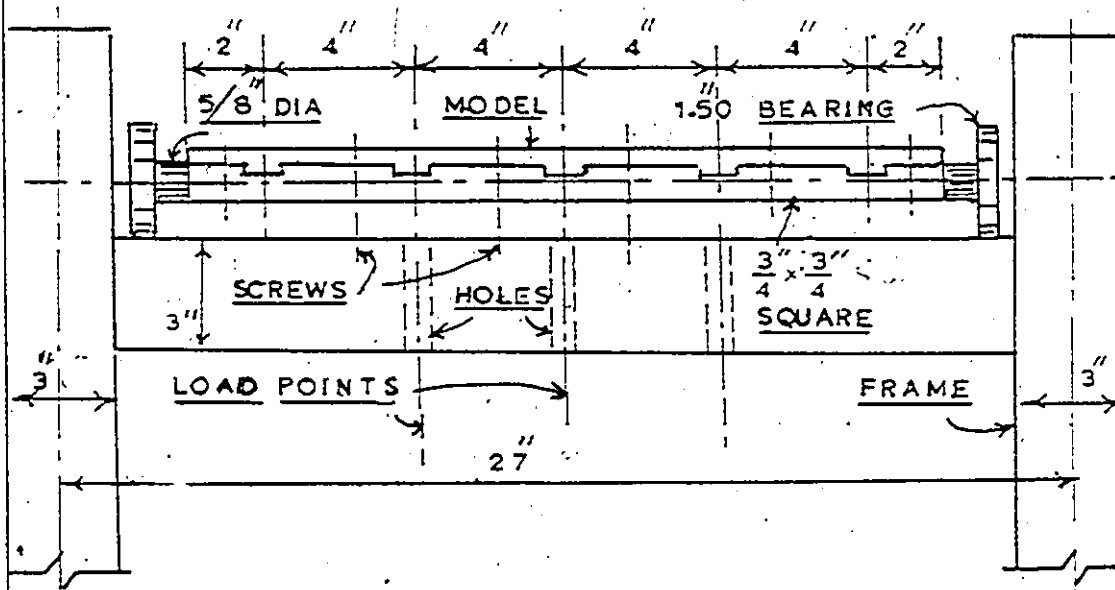
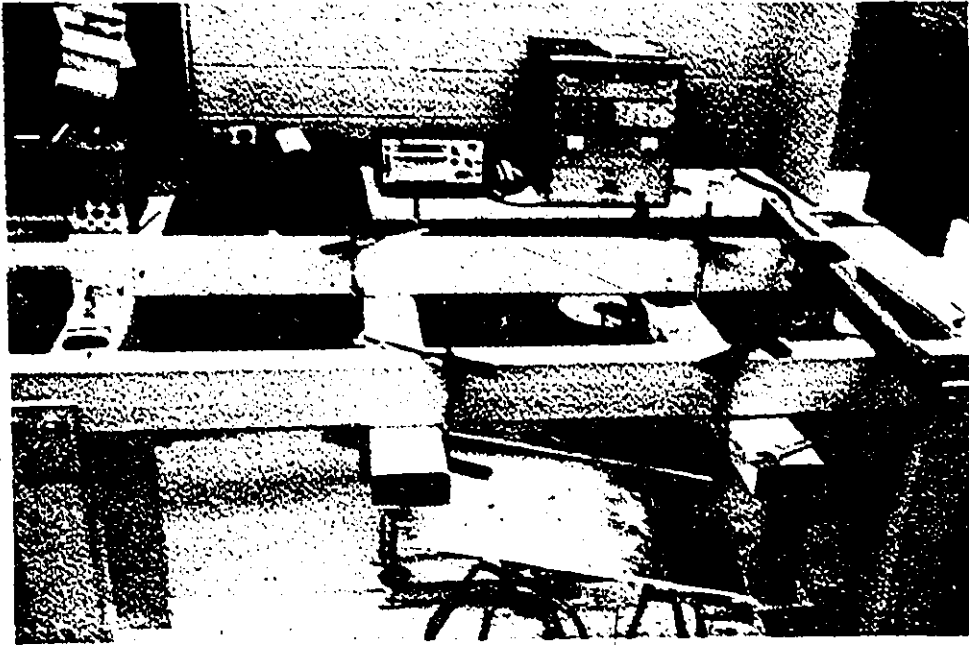


Fig. 52 Roller End

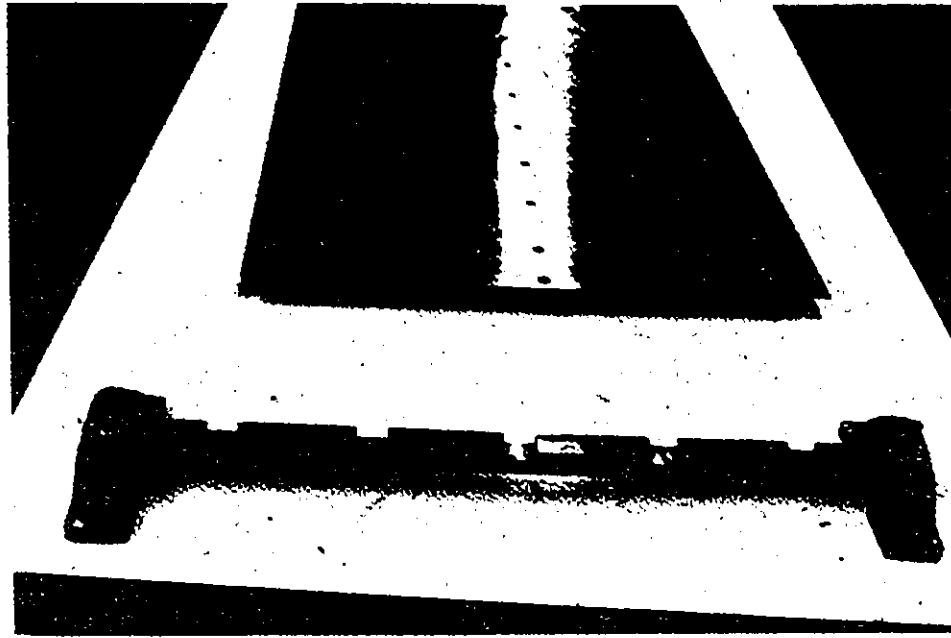


(a)



(b)

Fig. 51 (a,b). TEST FRAME AND EQUIPMENT



HINGED
END

ROLLER
END

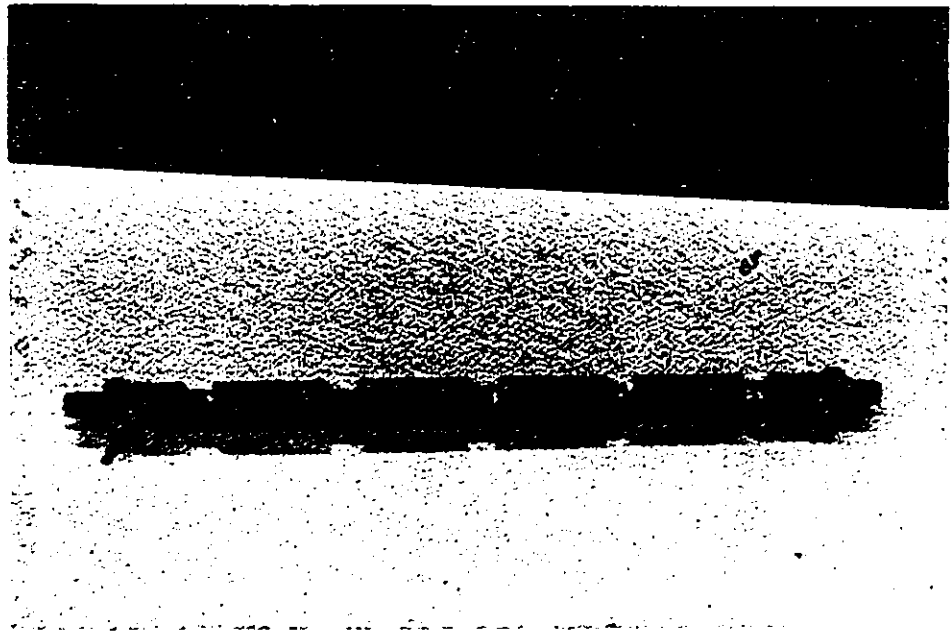
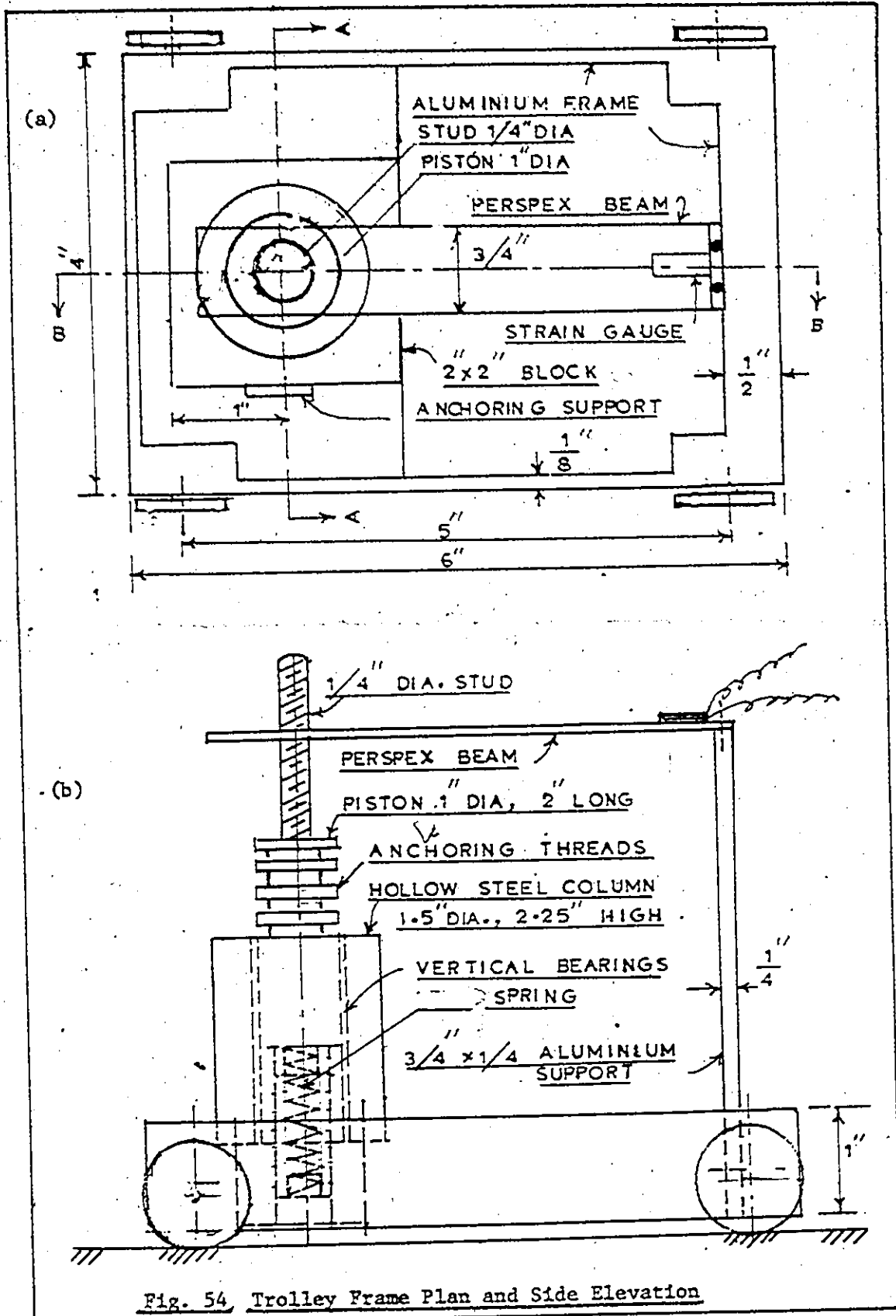


Fig. 53. ROLLER AND HINGED ENDS.



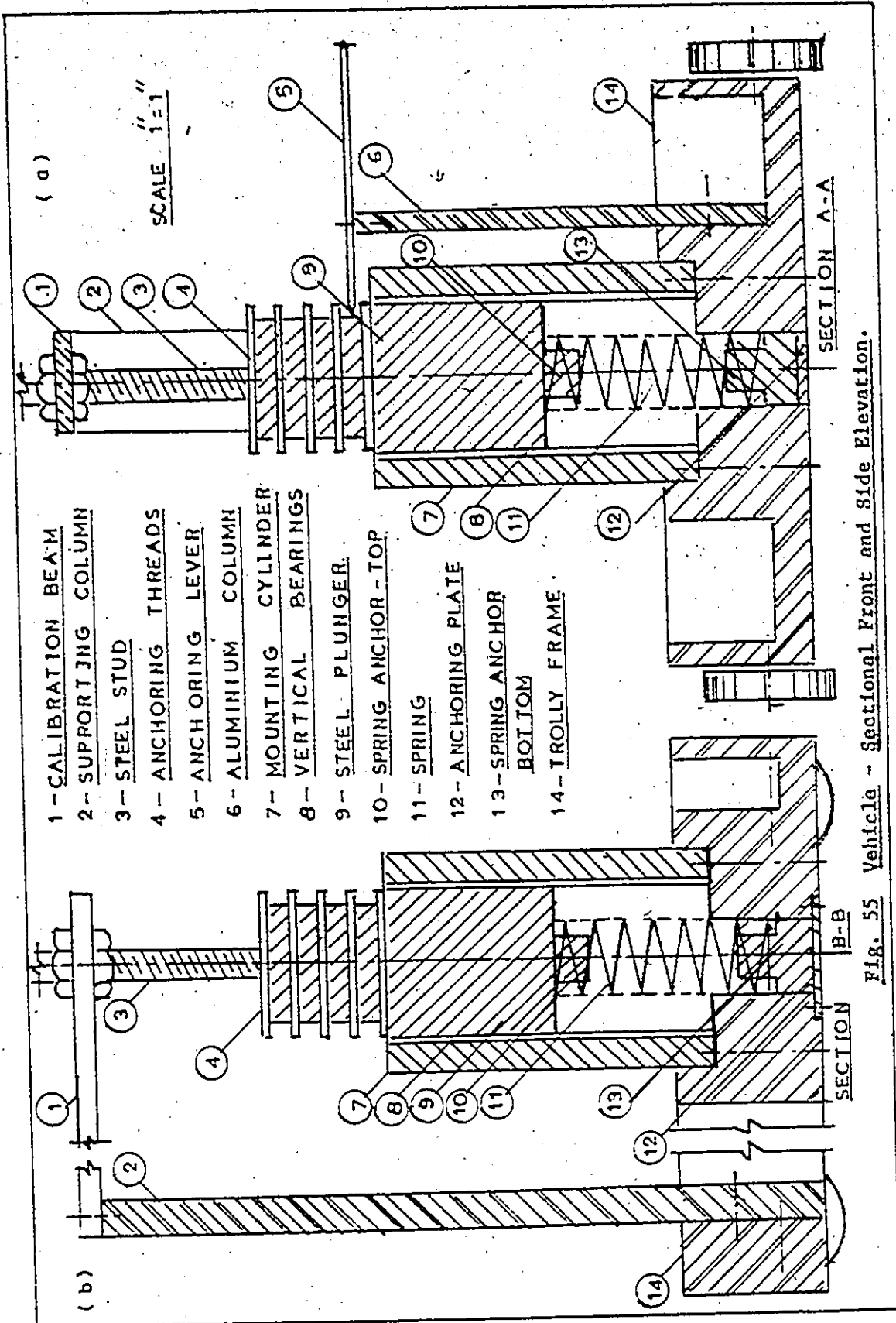
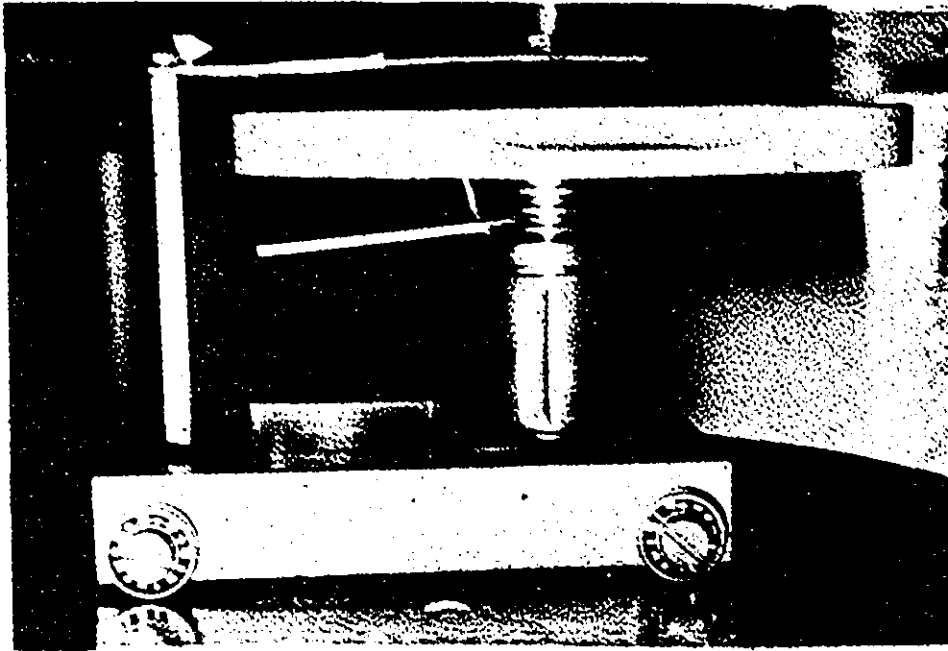


Fig. 55 Vehicle - Sectional Front and Side Elevation.



(b)

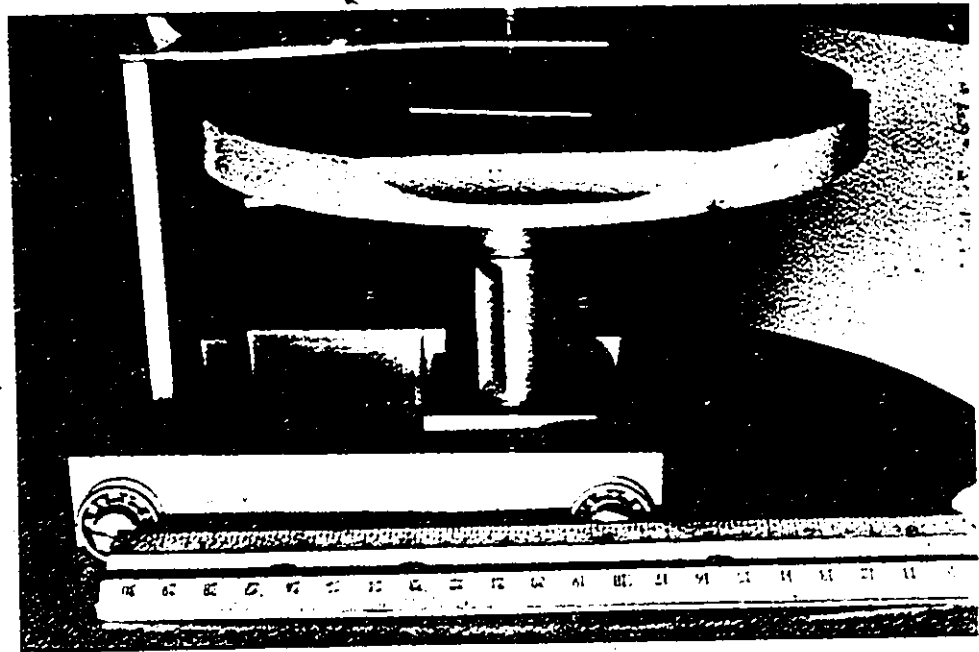
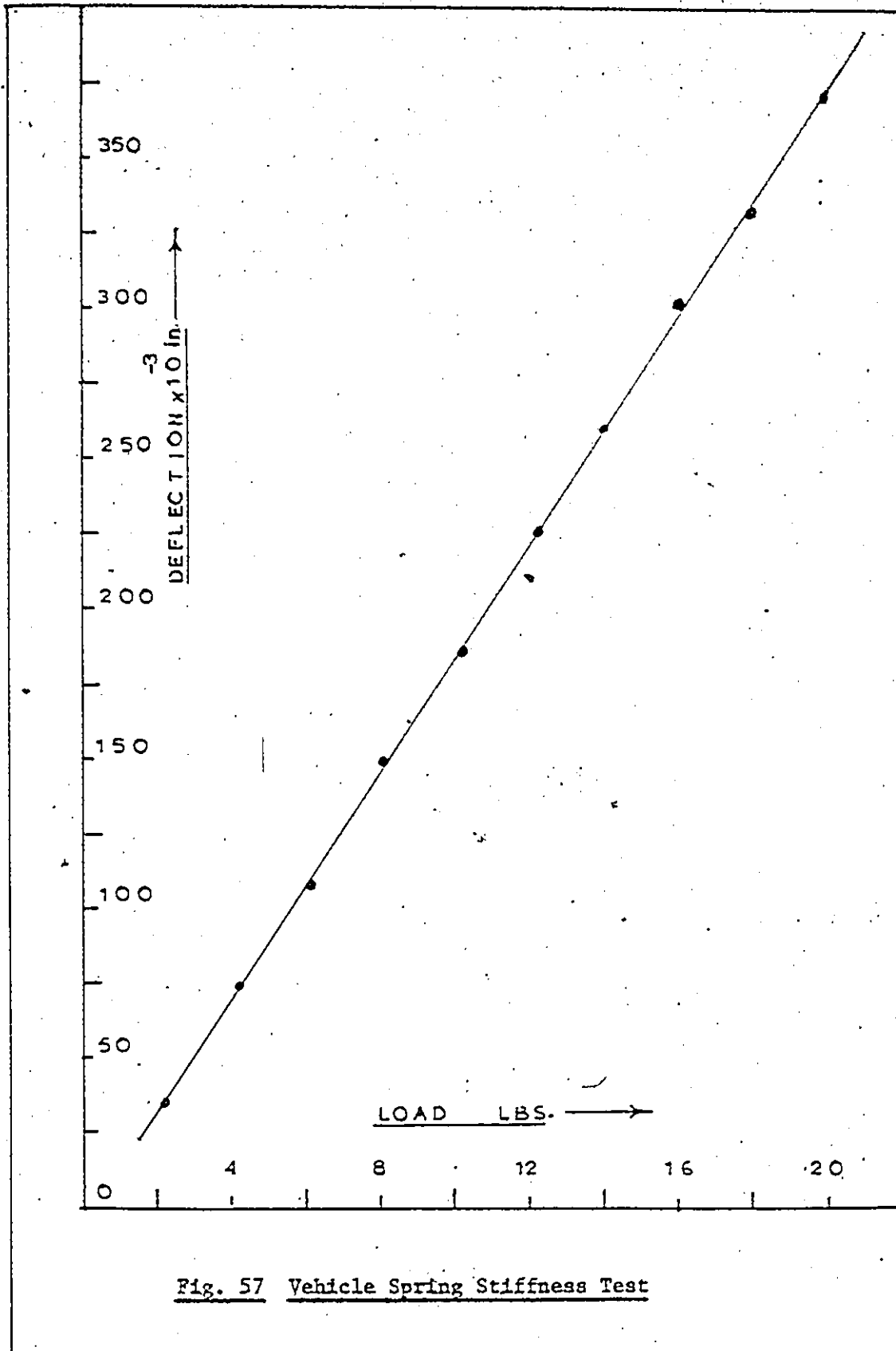
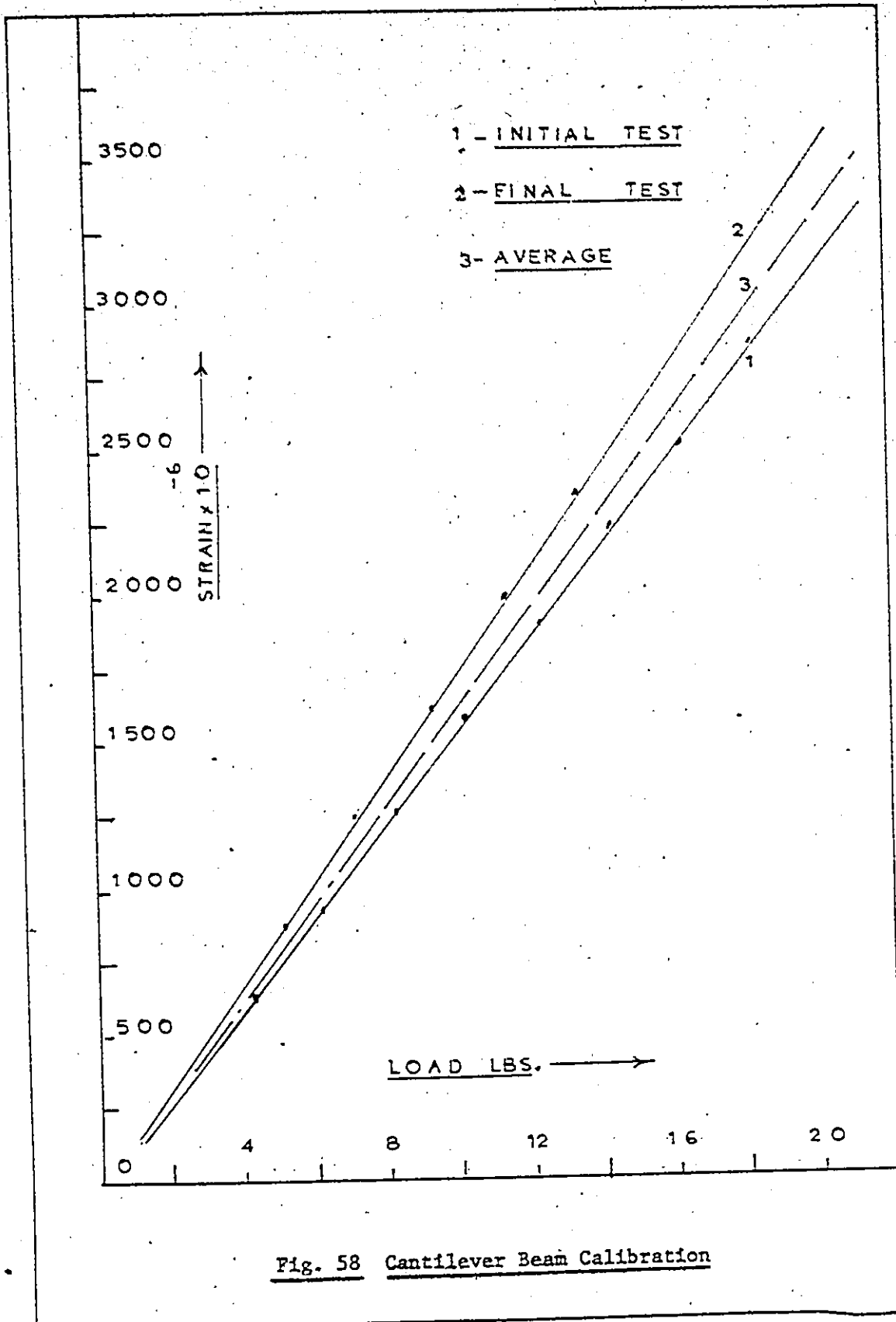


Fig. 56(a,b). VEHICLE TROLLEY. DETAILS





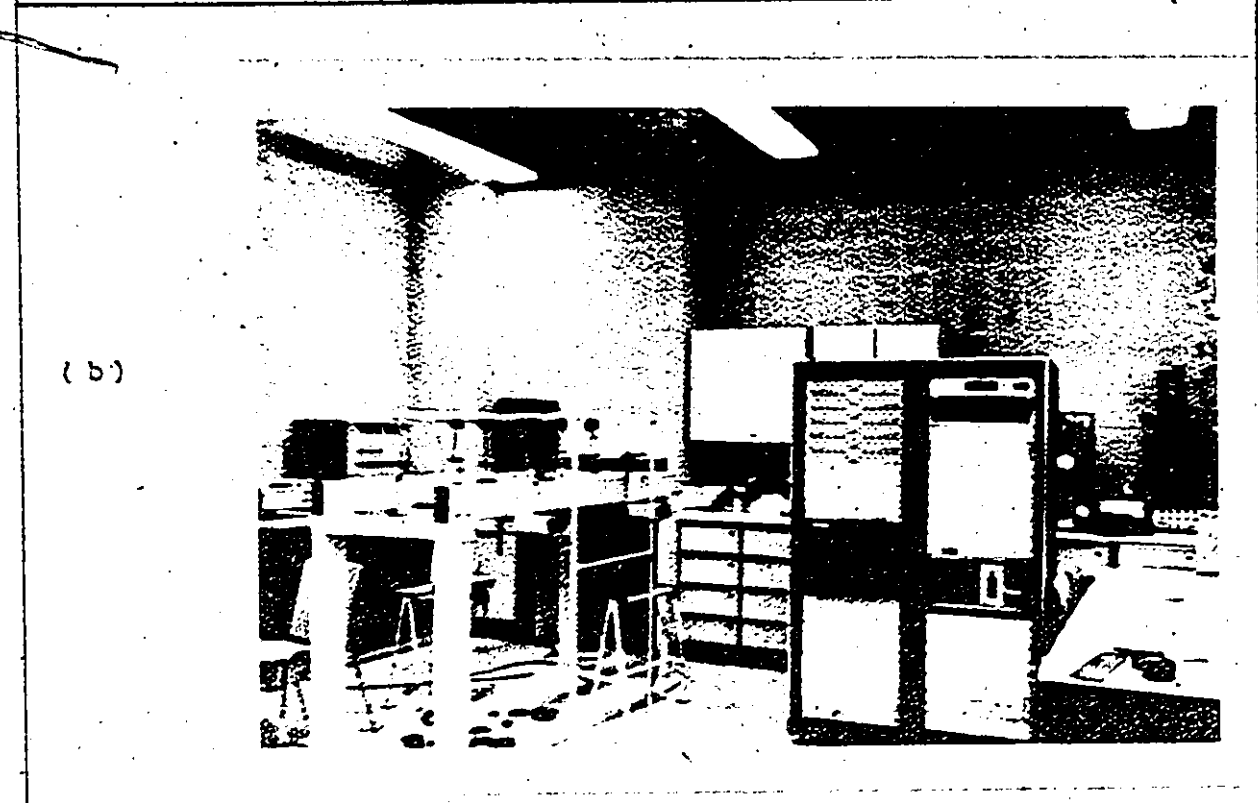
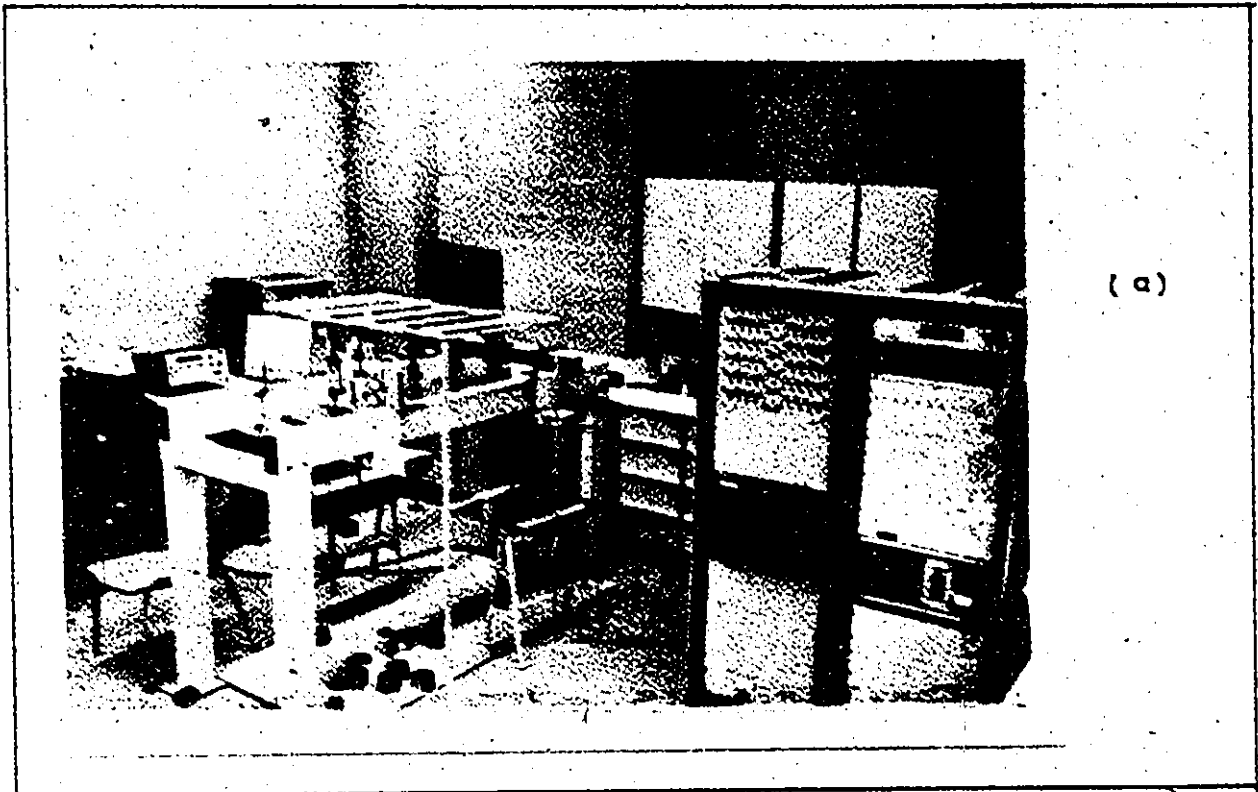


Fig. 59(a,b). EQUIPMENT.
DATA ACQUISITION SYSTEM
ELECTROMAGNETIC DEVICE

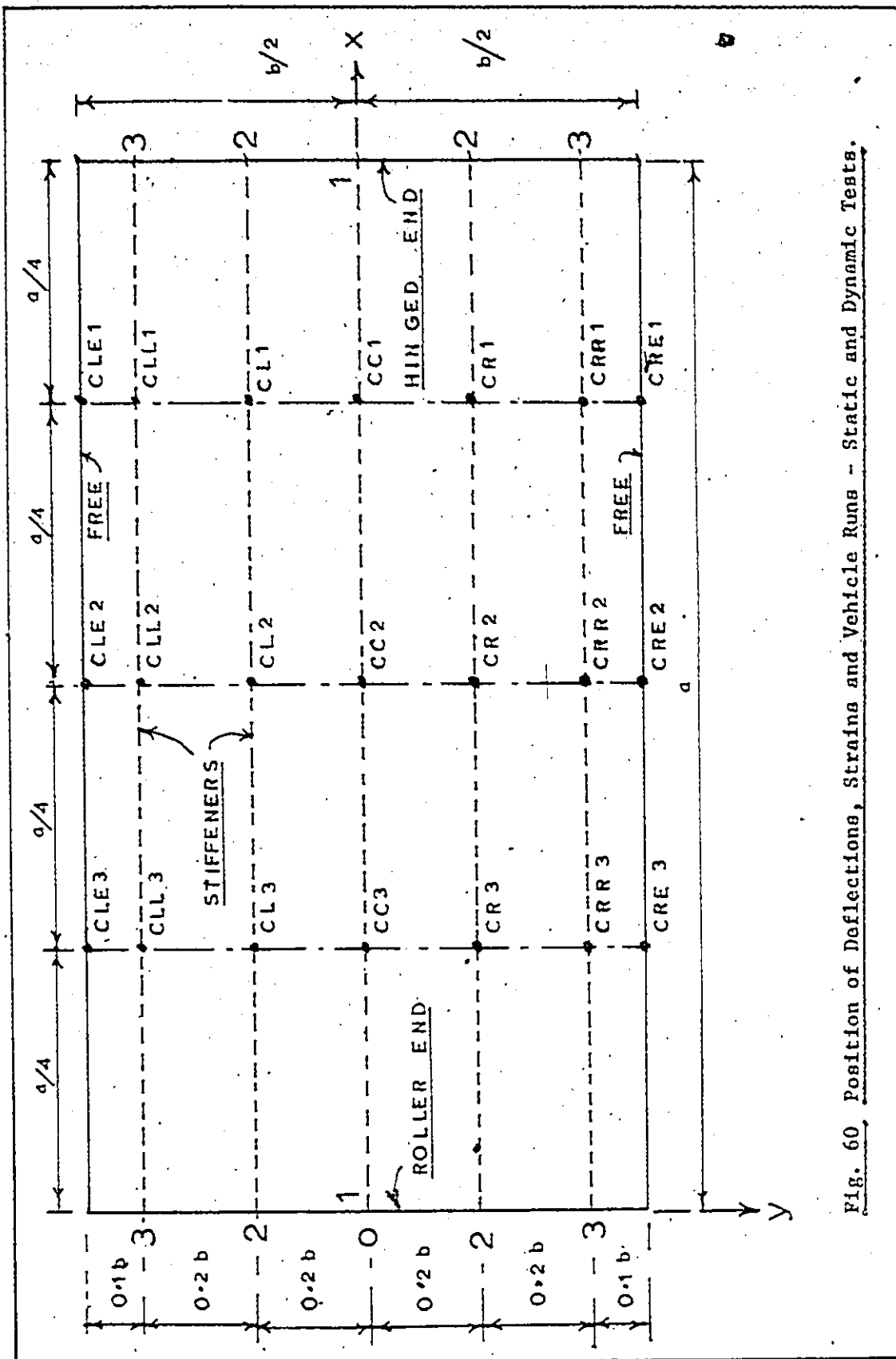
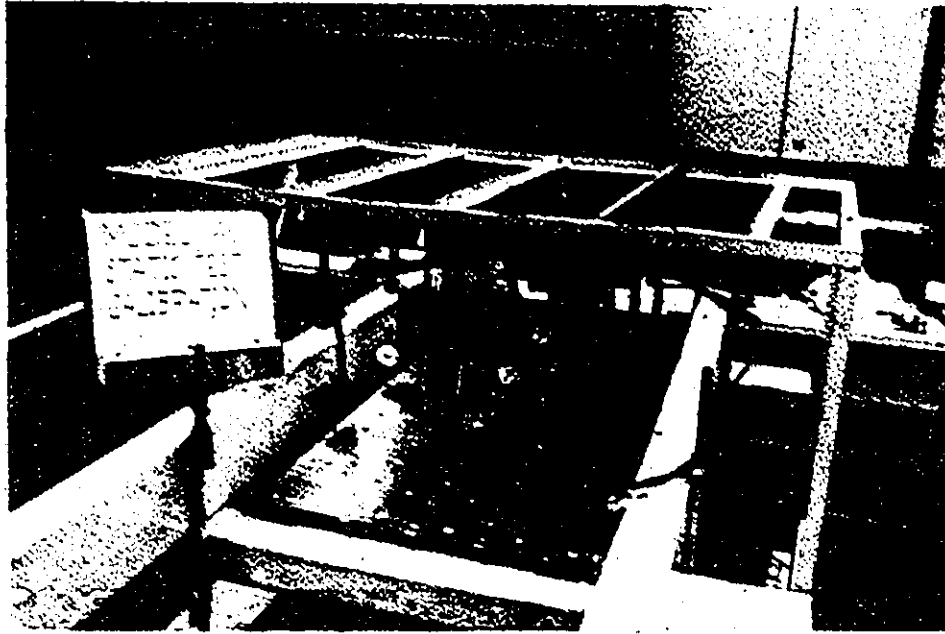
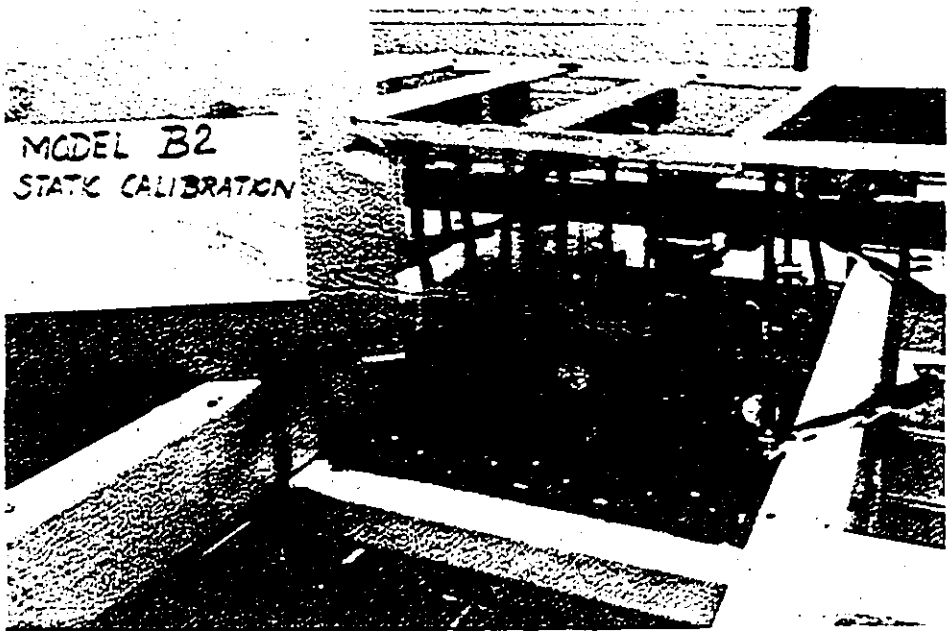


Fig. 60, Position of Deflections, Strains and Vehicle Runs - Static and Dynamic Tests.



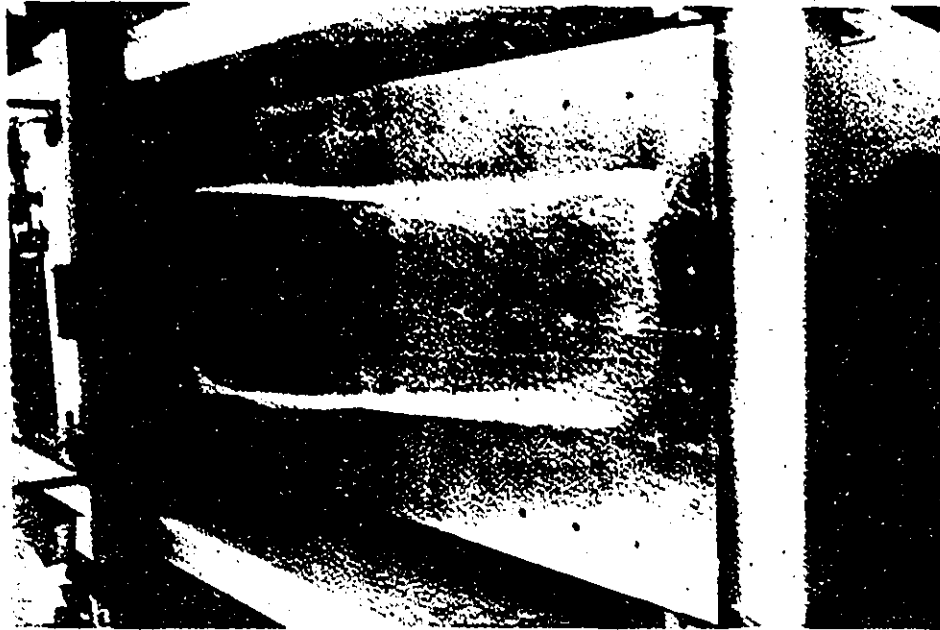
MODEL-A5



MODEL-B2

MODEL B2
STATIC CALIBRATION

Fig. 61. STATIC LOAD- DEFLECTION/ STRAINS TESTS.



$m = 1$

$n = 3$

MODEL-A 5

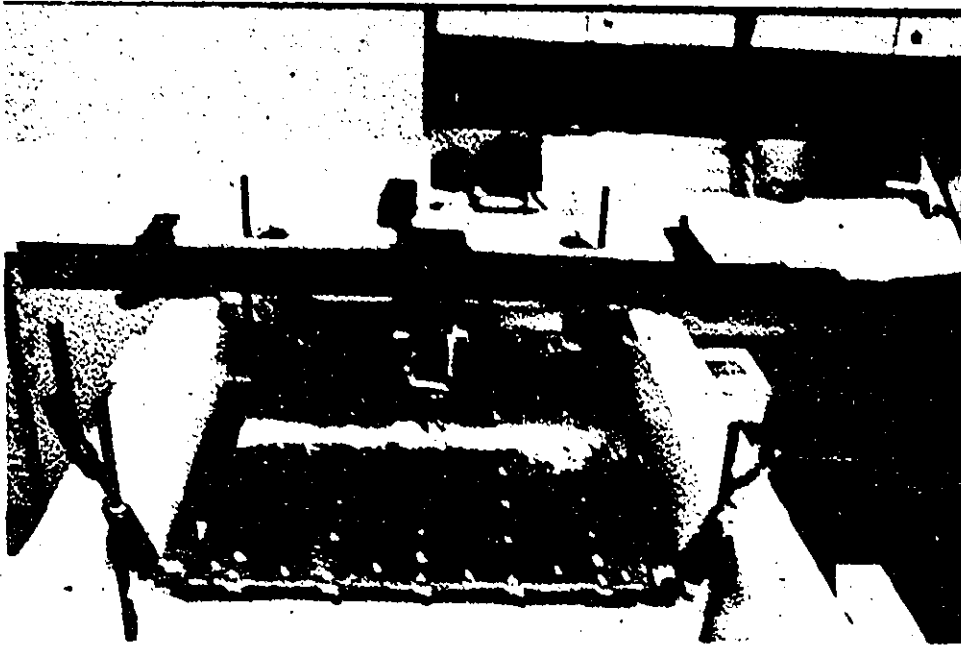


$m = 1$

$n = 5$

MODEL-A 2

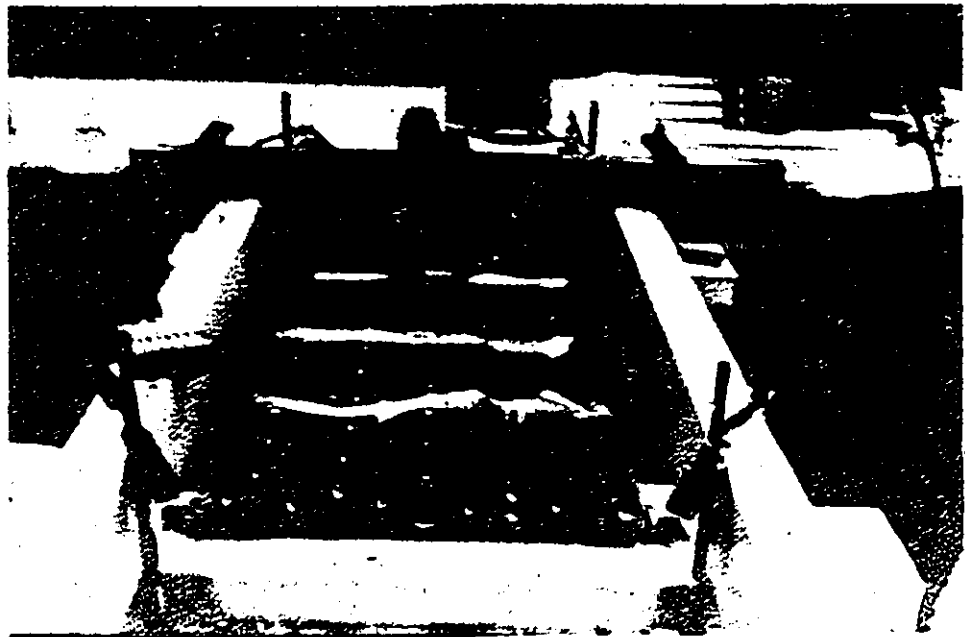
Fig.62(a). MODE SHAPES
 $m=1, n=3,5$



$m = 2$

$n = 1$

MODEL-B2



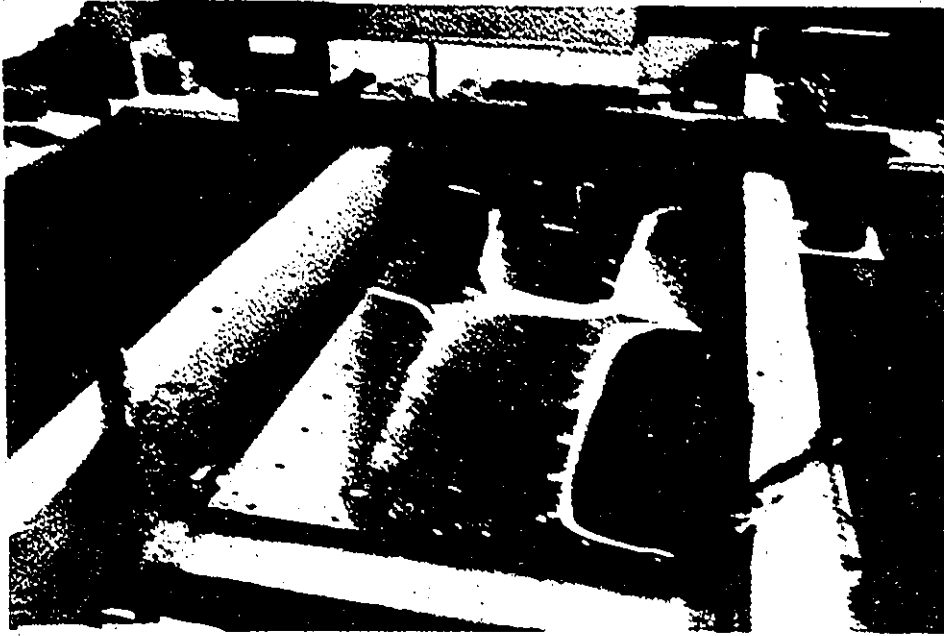
$m = 4$

$n = 1$

MODEL-B4

Fig.62(b). MODE SHAPES

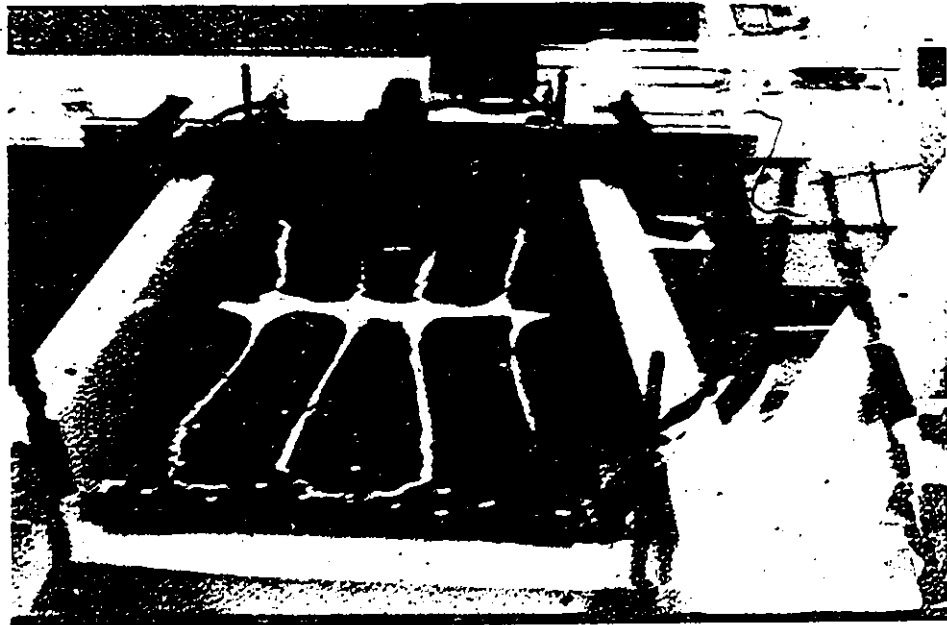
$m=2,4, n=1$



$m = 2$

$n = 3$

MODEL-A5



$m = 2$
 $n = 5$

MODEL-B4

Fig.62(c). MODE SHAPES

$m=2, n=3, 5$

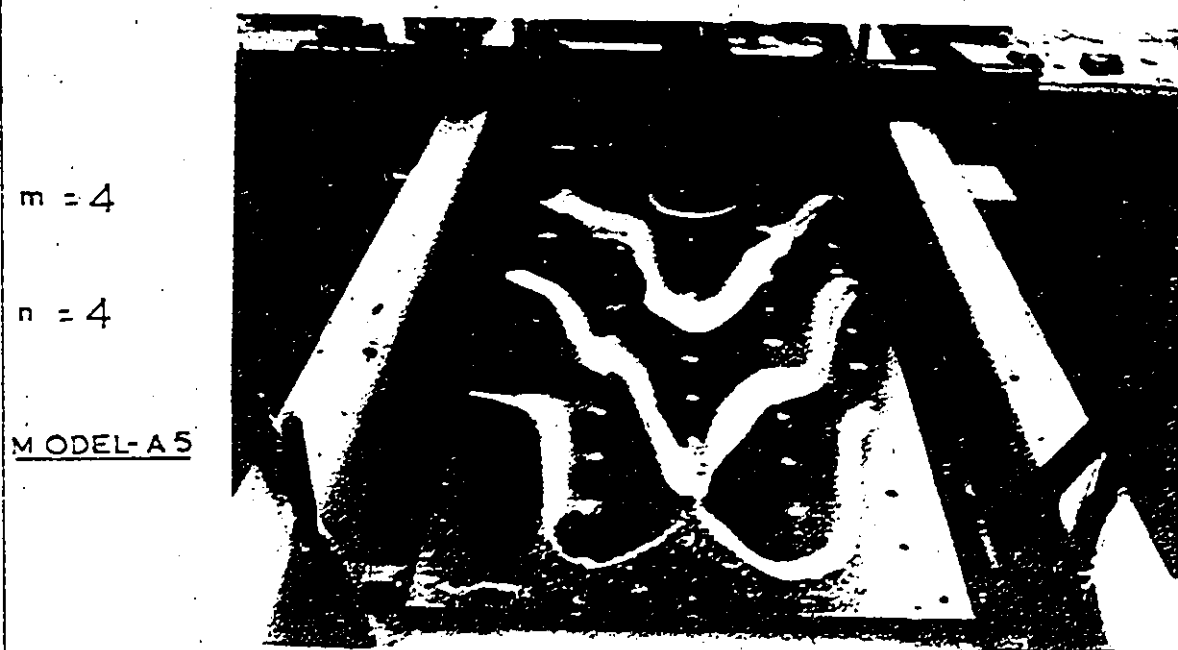
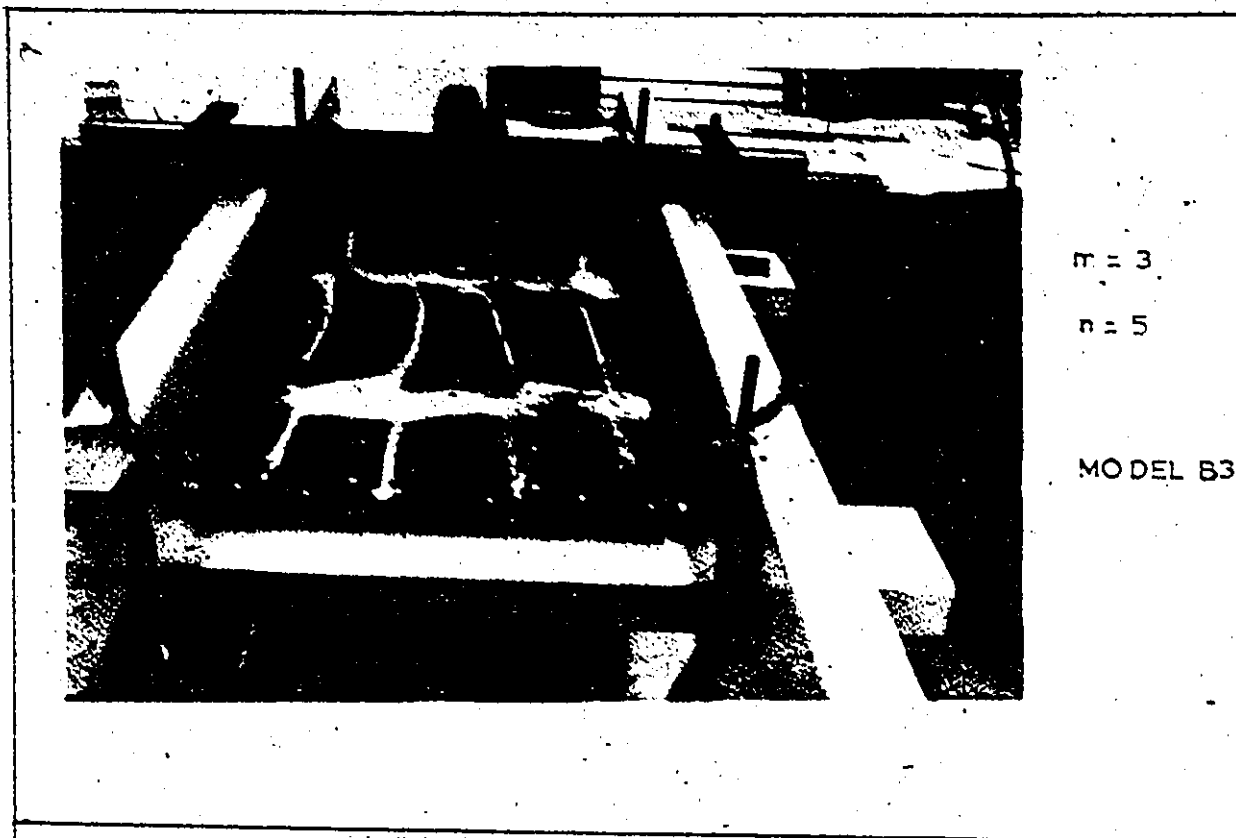
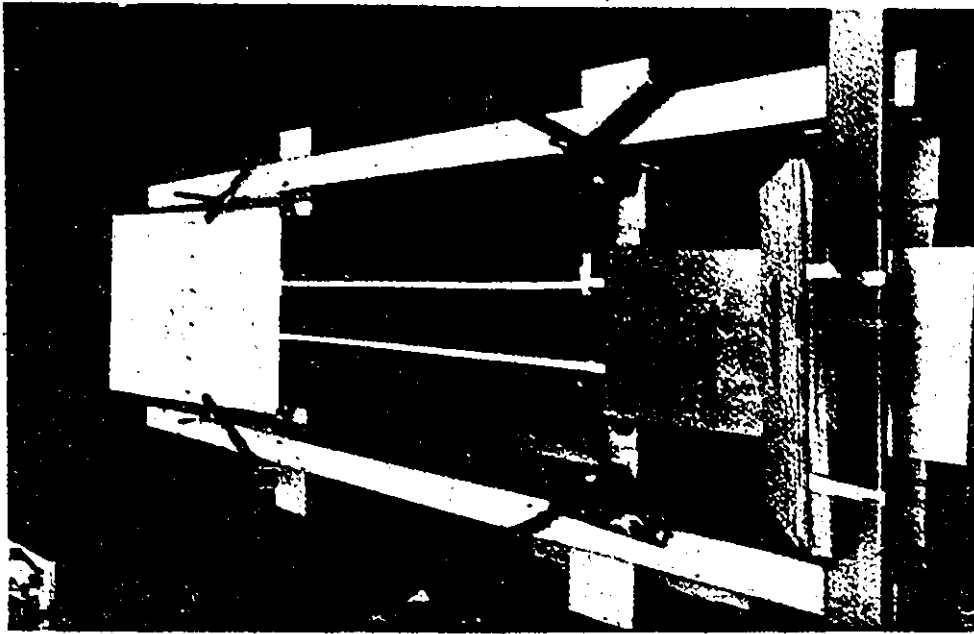
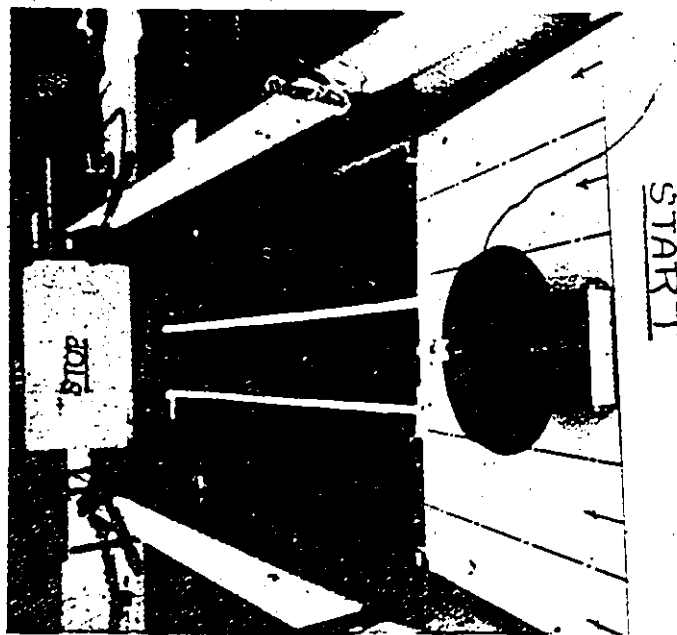


Fig.62(d). MODE SHAPES
 $m=3,4, n=4,5$



(a)

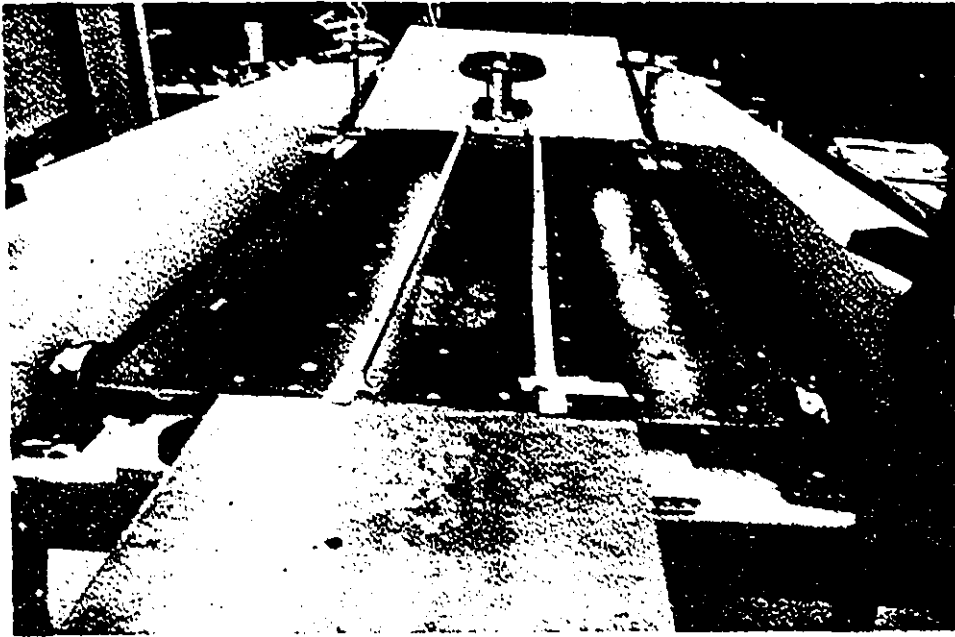


(b)

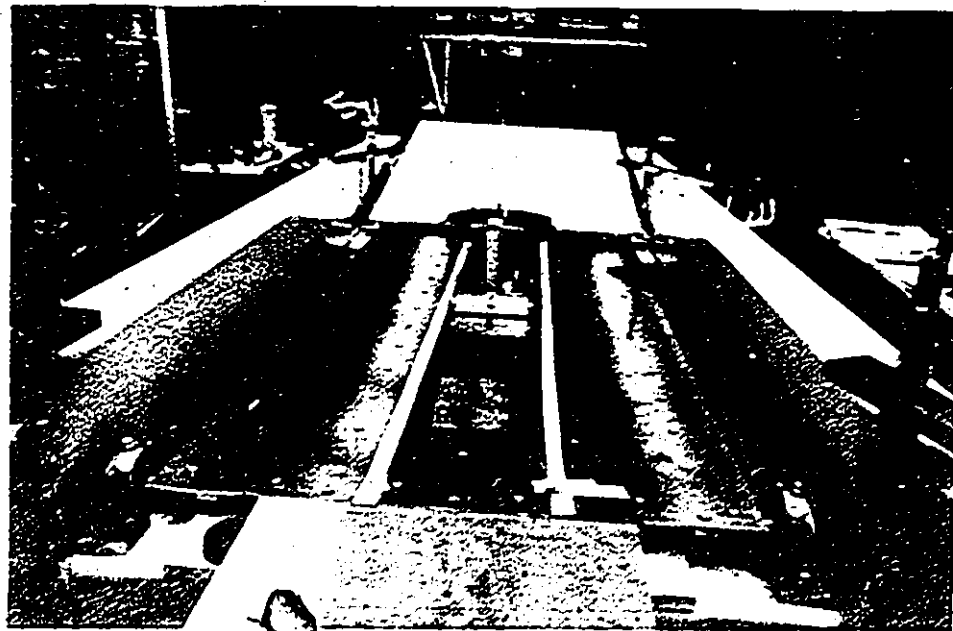
Fig. 63 A. FORCED VIBRATION TESTS, SET UP.

(a). MODEL AND APPROACHES.

(b). VEHICLE AT START.



(a)



(b)

Fig. 63B. FORCED VIBRATION TESTS, SET UP
(a) VEHICLE ENTERING SPAN
(b) VEHICLE AT MIDDLE OF RUN

AMPLIFICATION FACTOR A_F

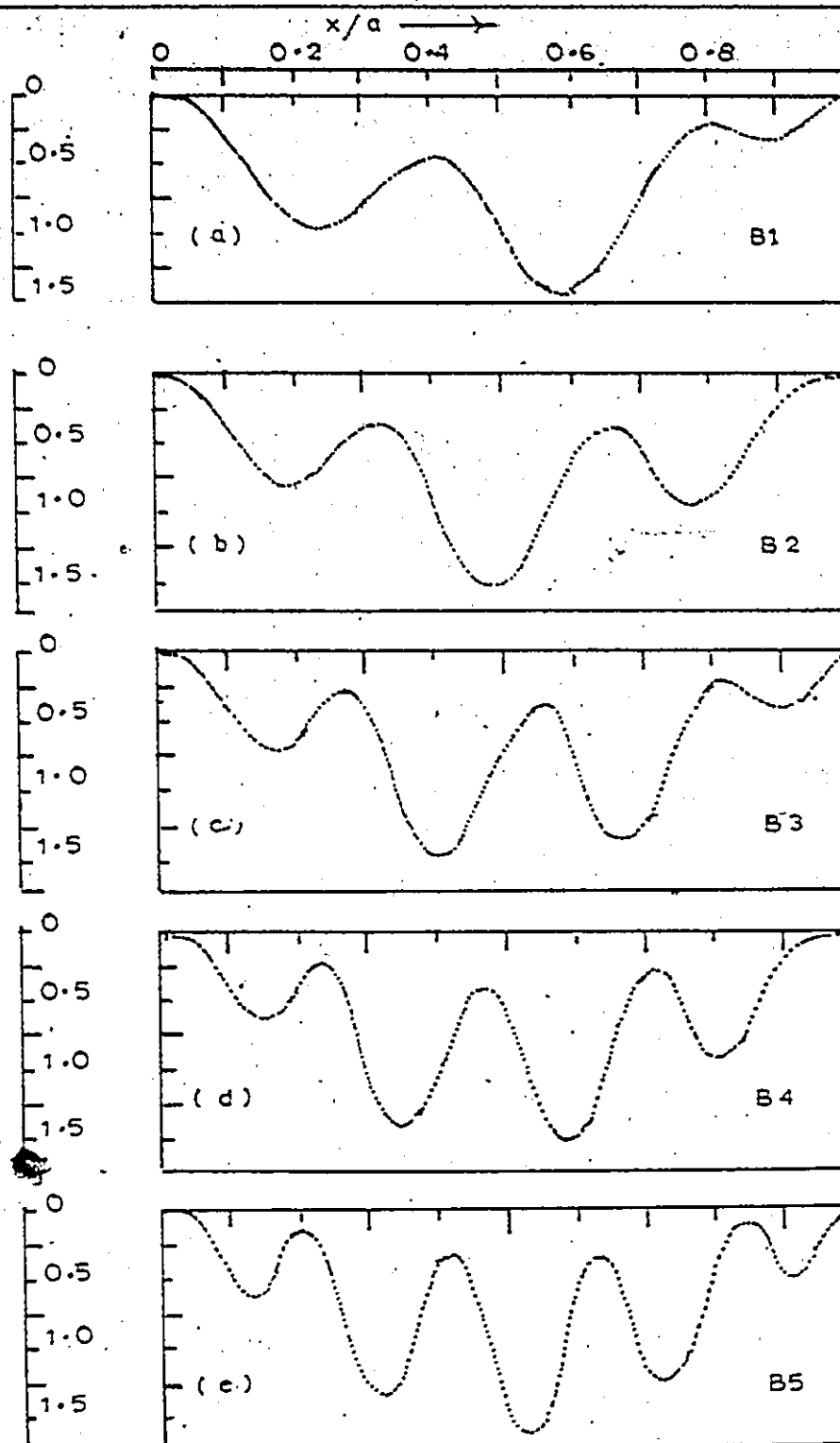


Fig. 64 Distribution of Amplification Factors.
Model B, Initial Conditions of Vehicle - $\beta = 0.5$;
 $\phi = 1.5\pi$.

AMPLIFICATION FACTOR AF

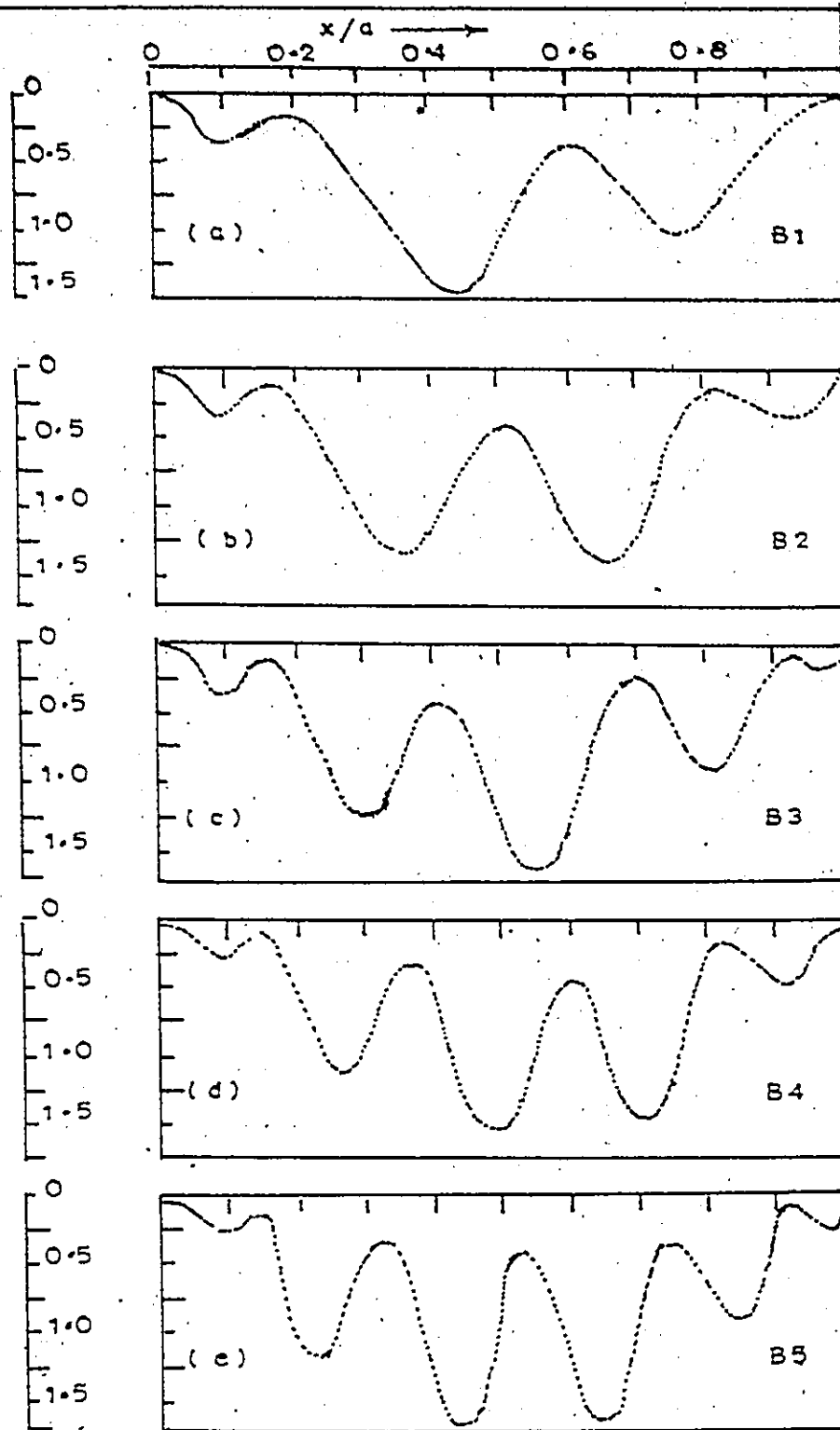


Fig. 65 Distribution of Amplification Factors. Model B, Initial Conditions of Vehicle, $\beta = 0.5; \phi = 0.5\pi$.

AMPLIFICATION FACTOR AF

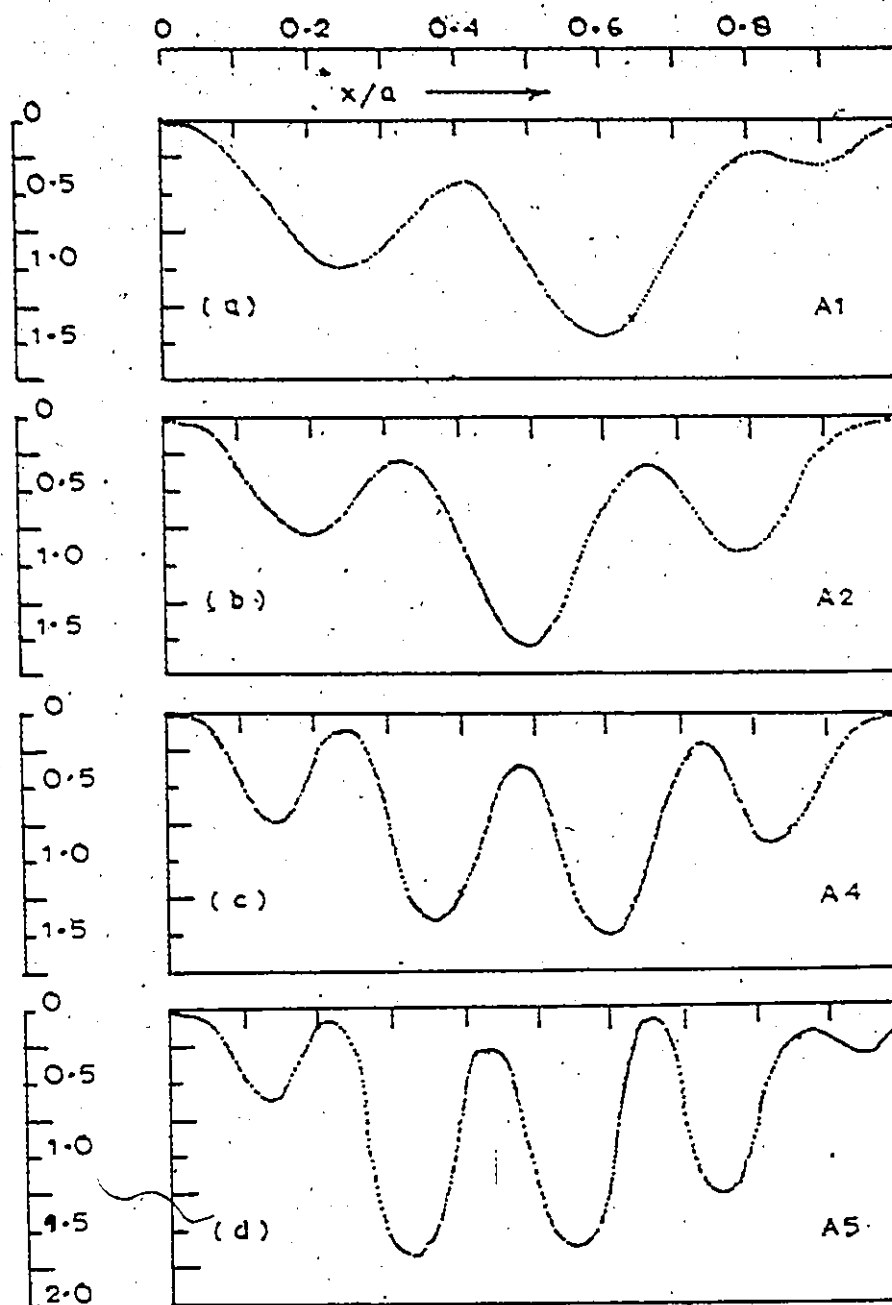


Fig. 66 Distribution of Amplification Factors.
Model A, Initial Condition of Vehicle, $\beta = 0.5$;
 $\phi = 1.5\pi$.

AMPLIFICATION FACTOR A_F

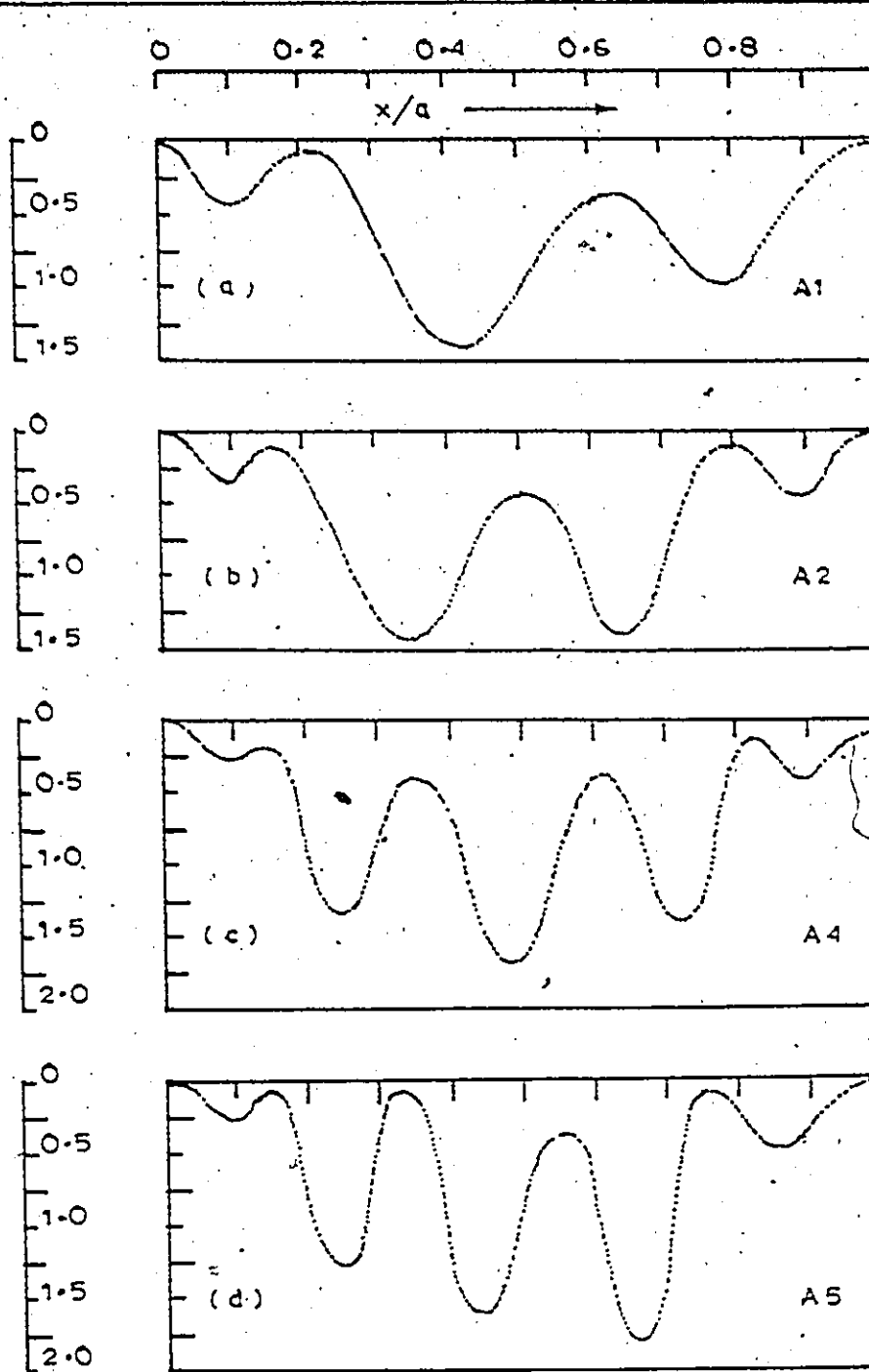


Fig. 67 Distribution of Amplification Factors. Model A.
Initial Conditions of Vehicle, $\beta = 0.5$; $\phi = 0.5\pi$.

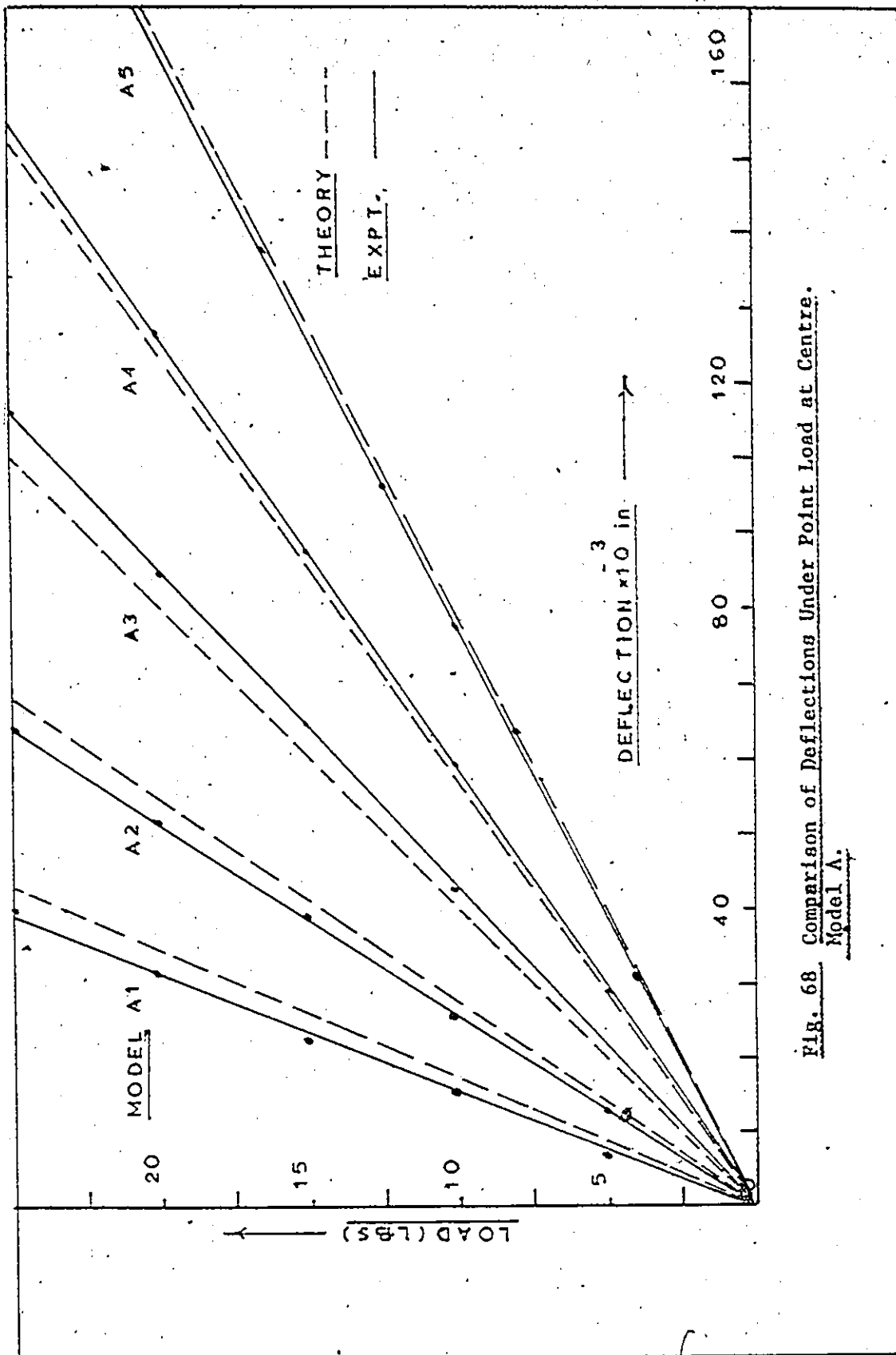


Fig. 68 Comparison of Deflections Under Point Load at Centre.
Model A.

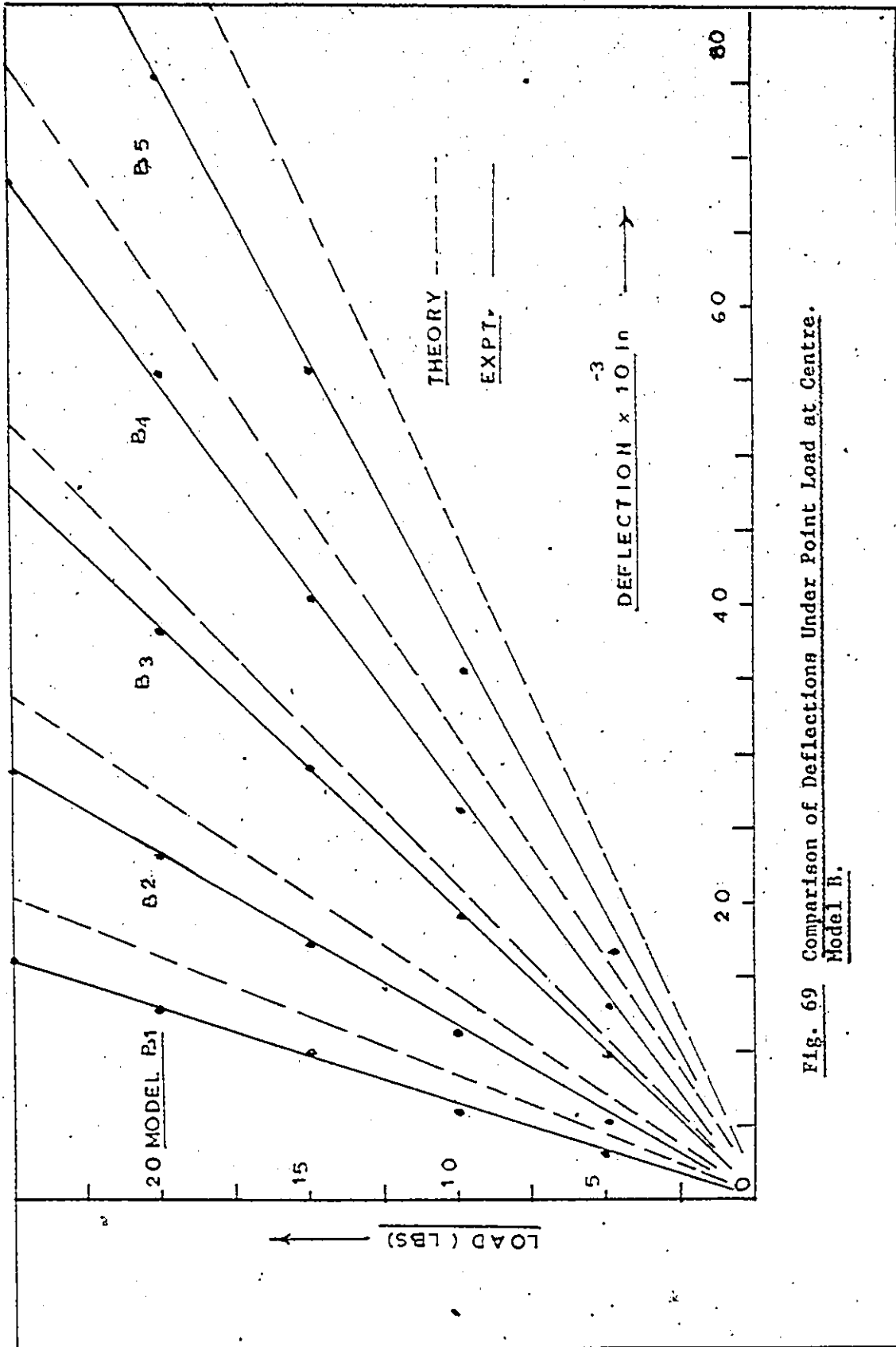


Fig. 69 Comparison of Deflections Under Point Load at Centre.
Model B.

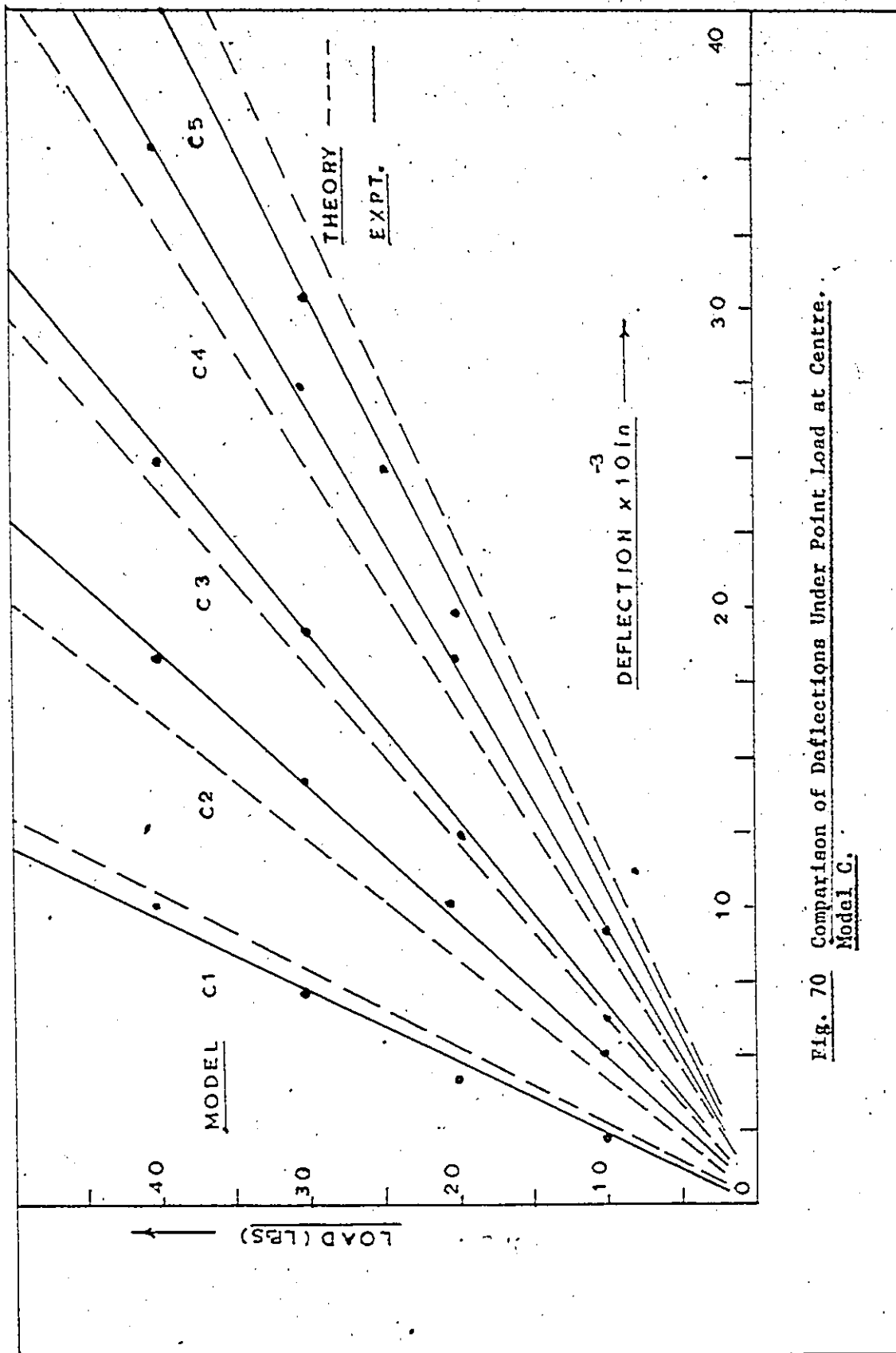


Fig. 70 Comparison of Deflections Under Point Load at Centre.
Model C.

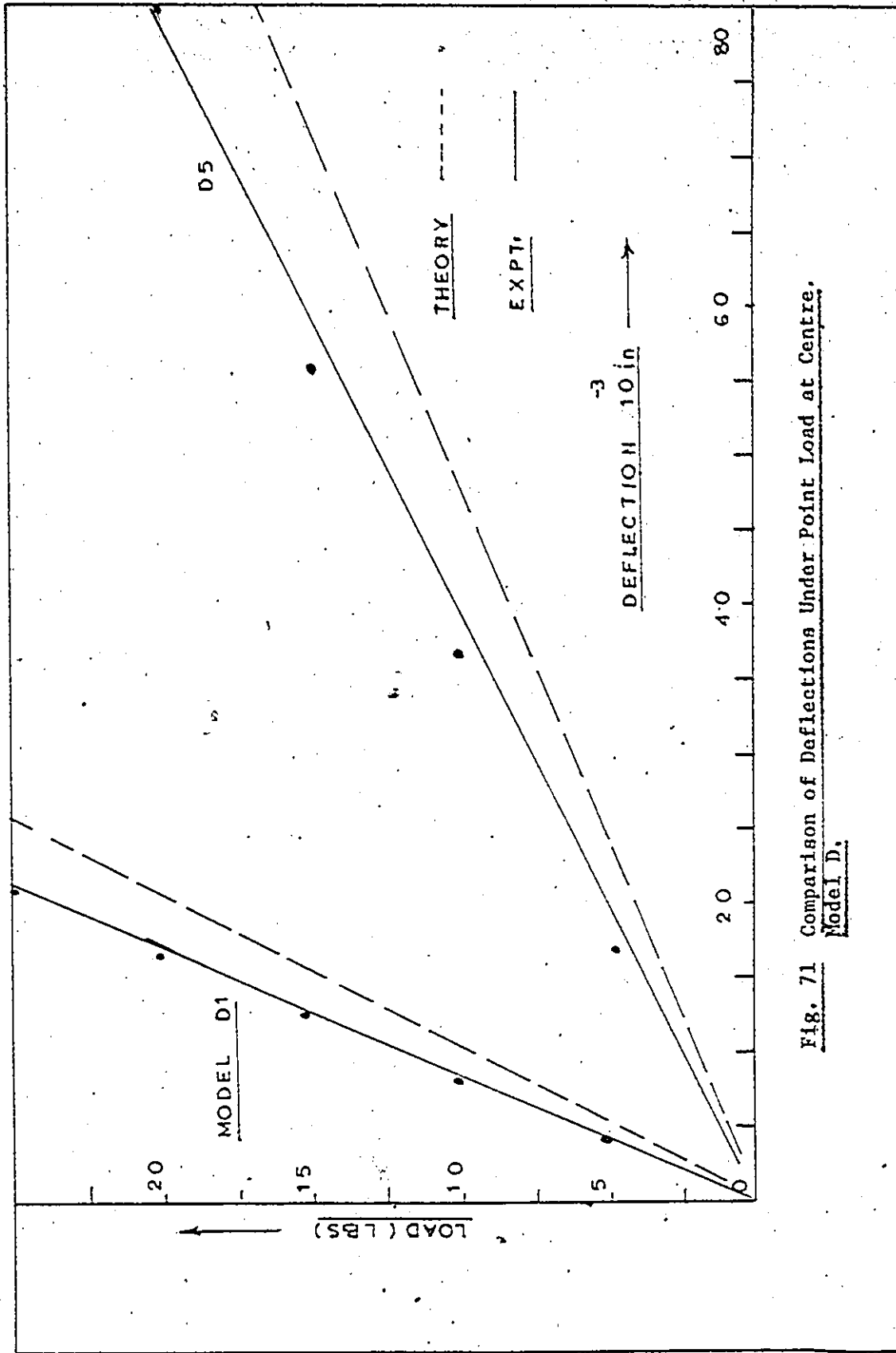
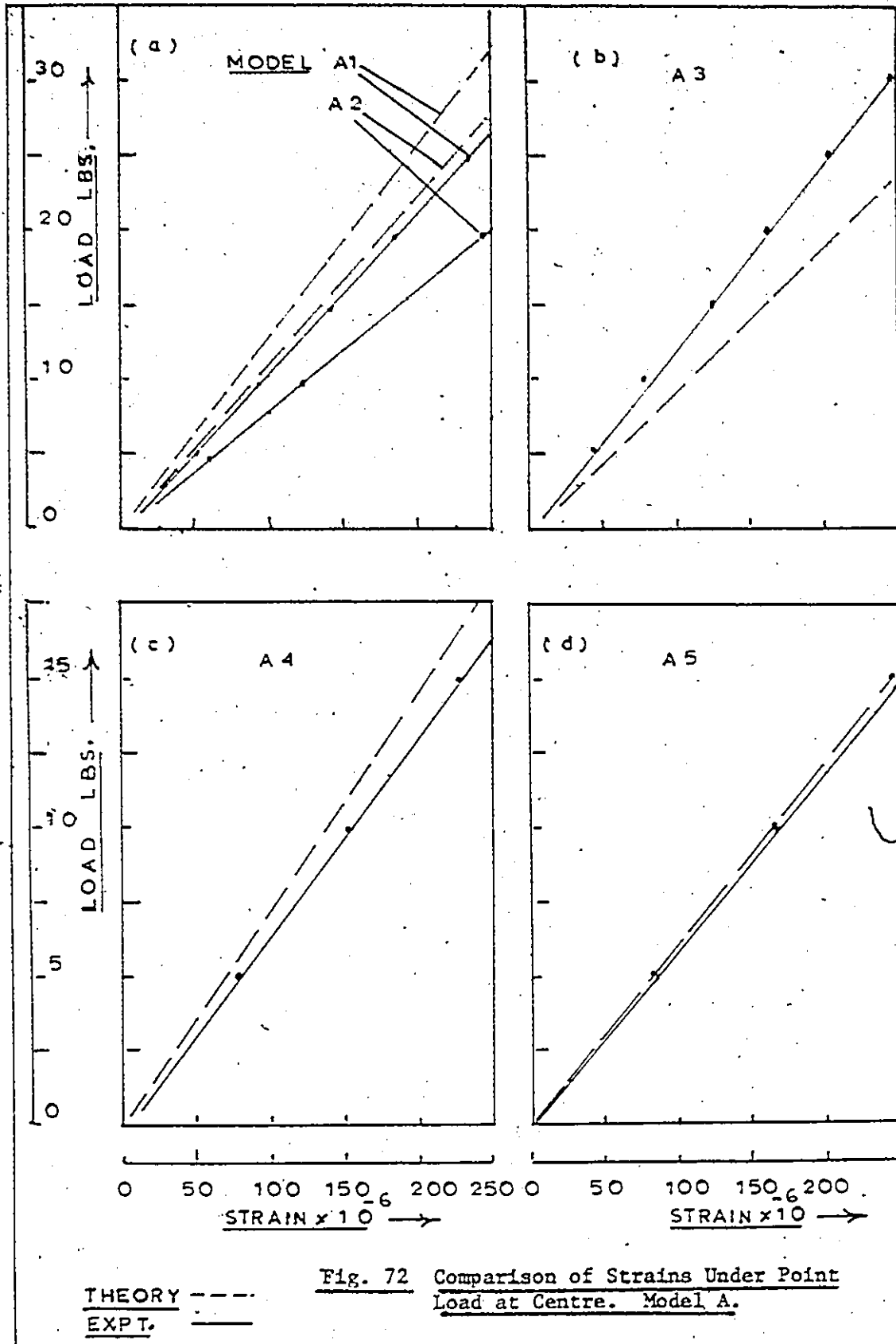
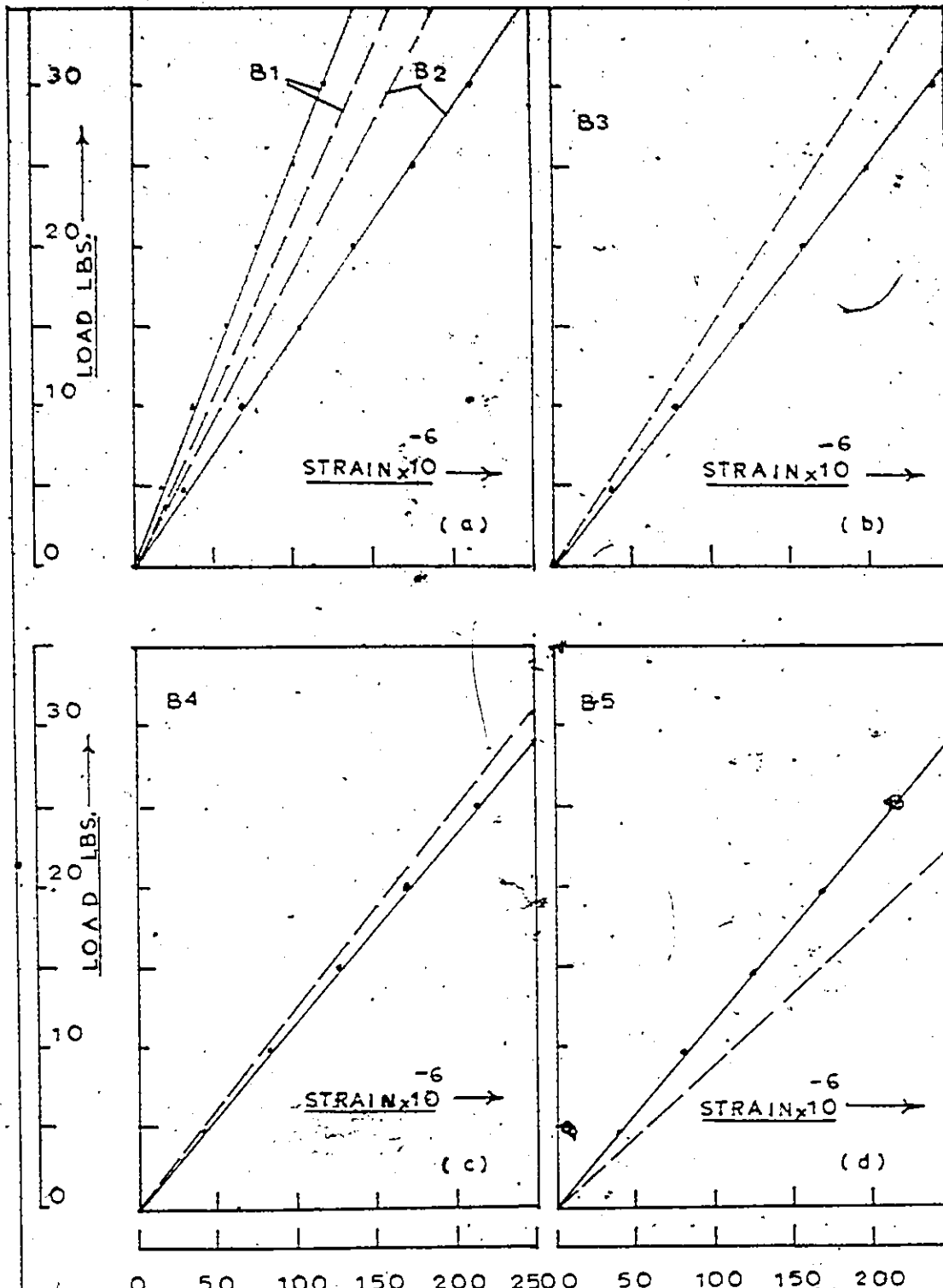


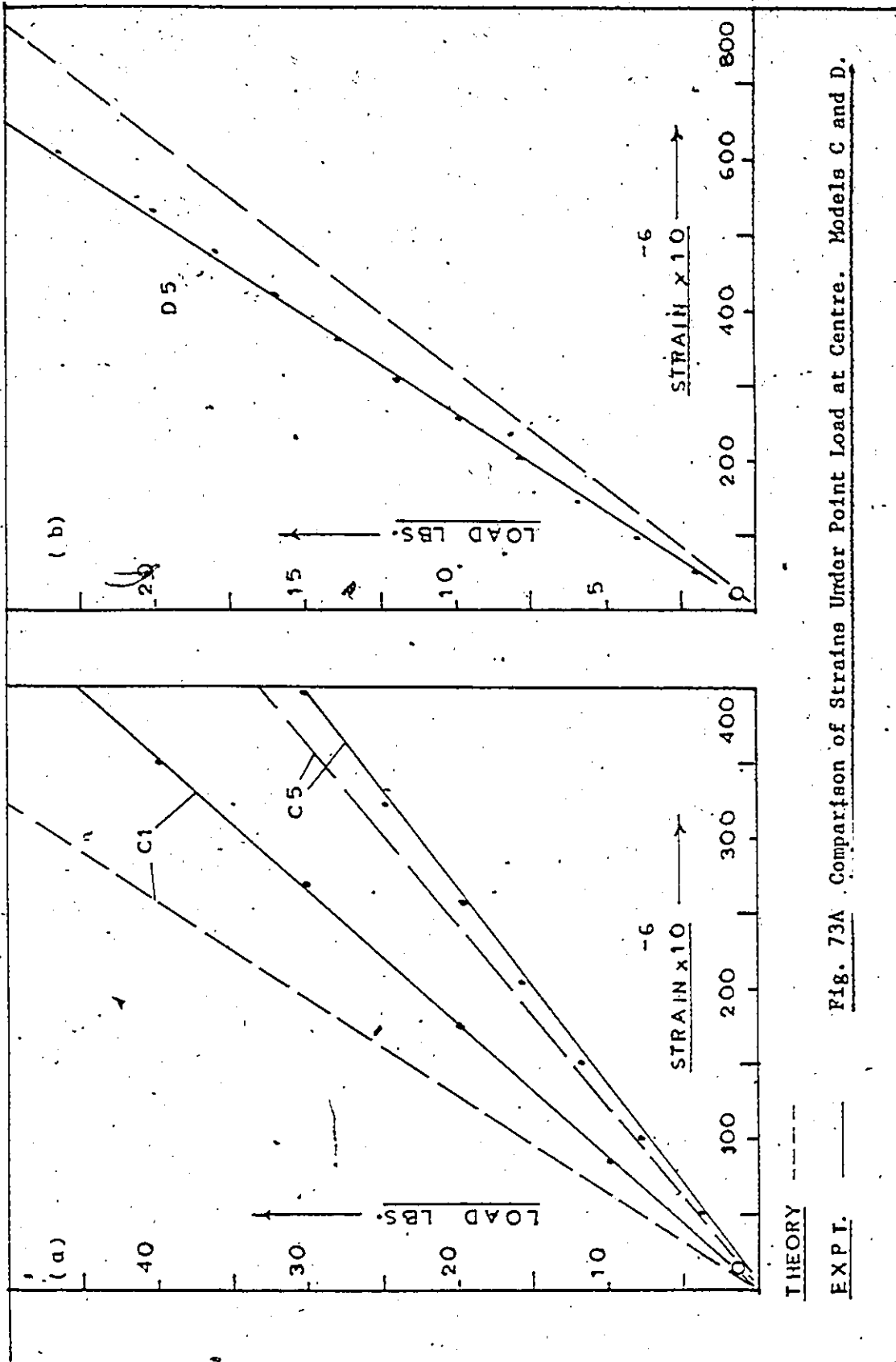
Fig. 71 Comparison of Deflections Under Point Load at Centre.
Model D.





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Fig. 73 Comparison of Strains Under Point Load at Centre. Model B.



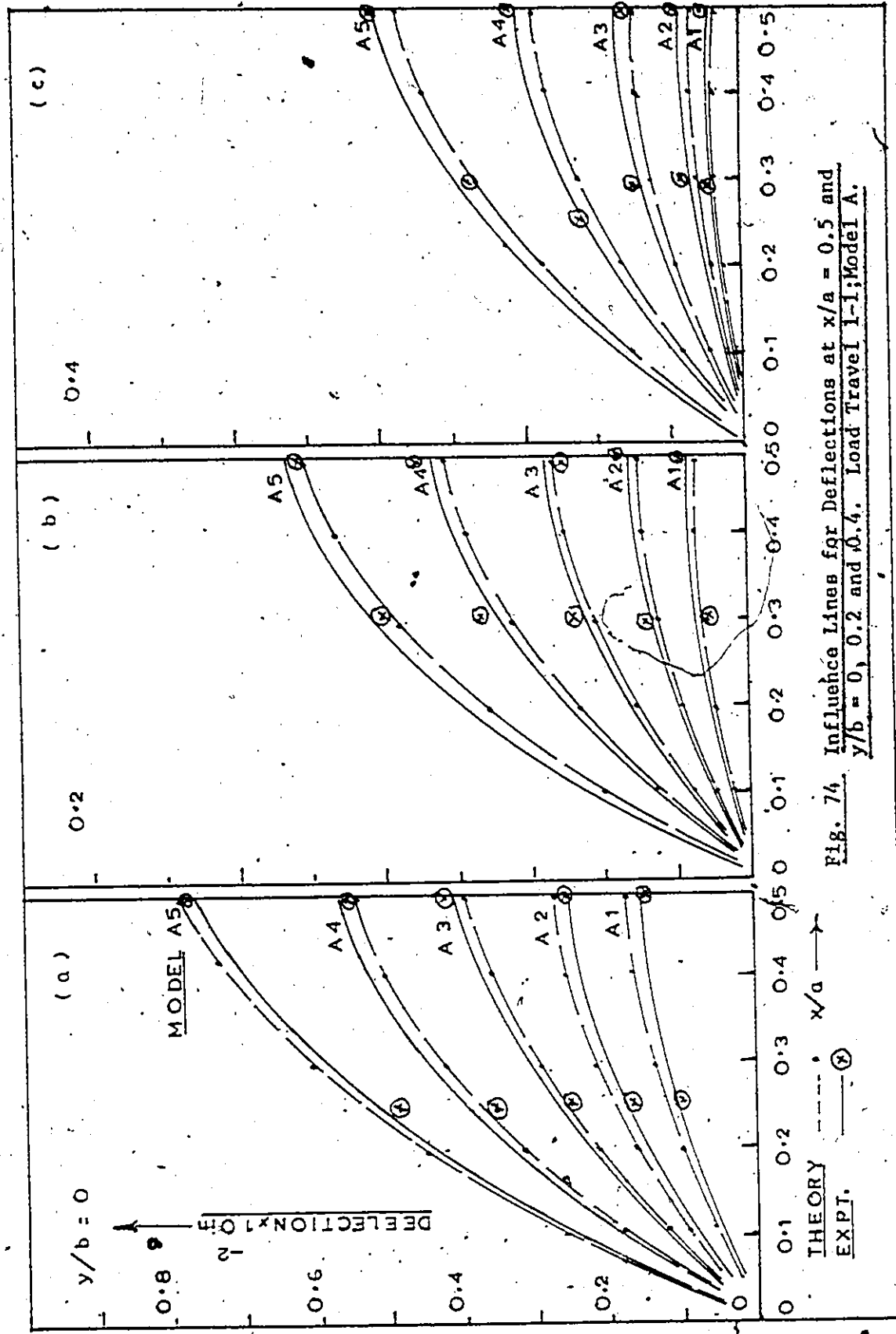
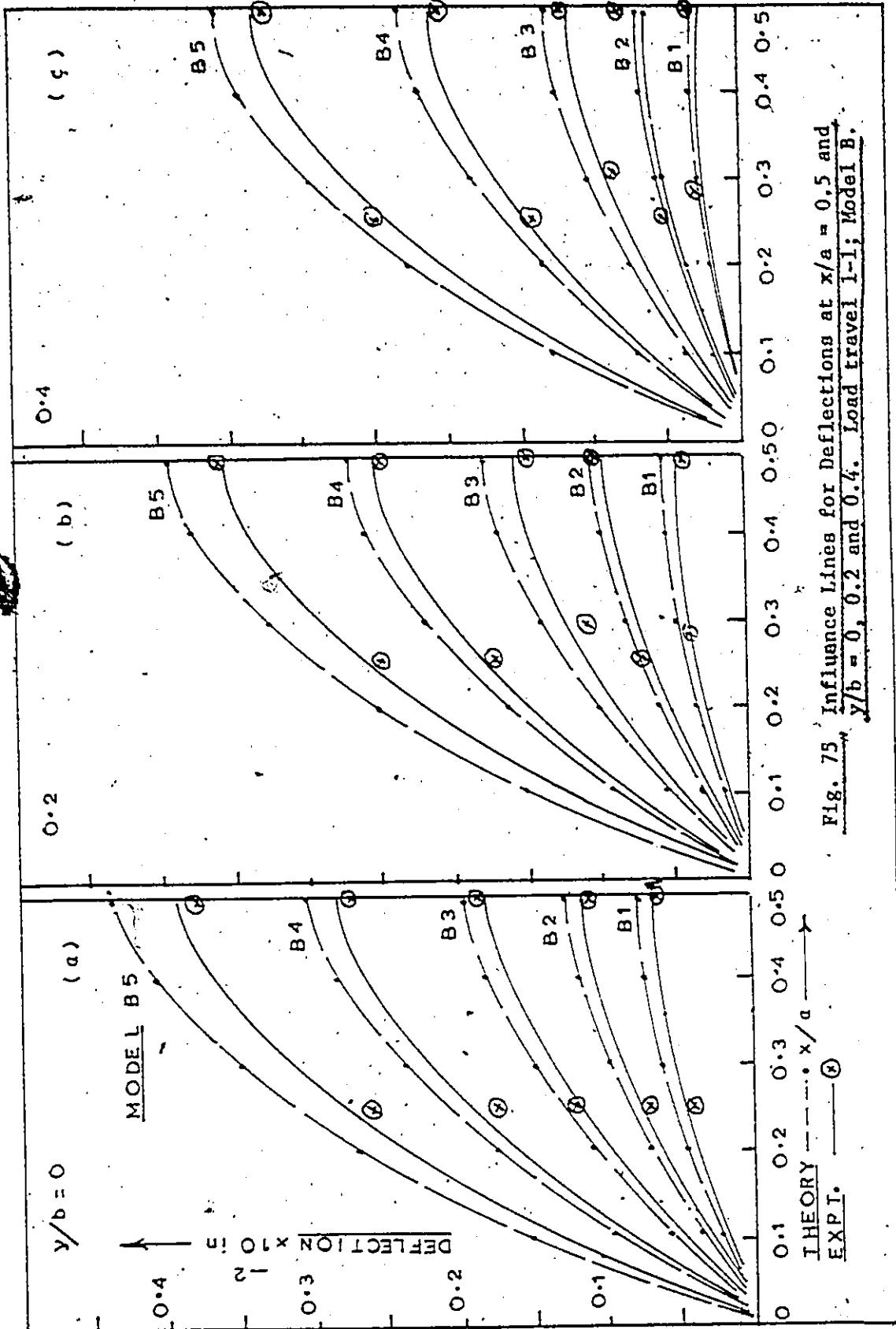


Fig. 74. Influence Lines for Deflections at $x/a = 0.5$ and $y/b = 0, 0.2$ and 0.4 . Load Travel 1-1; Model A.



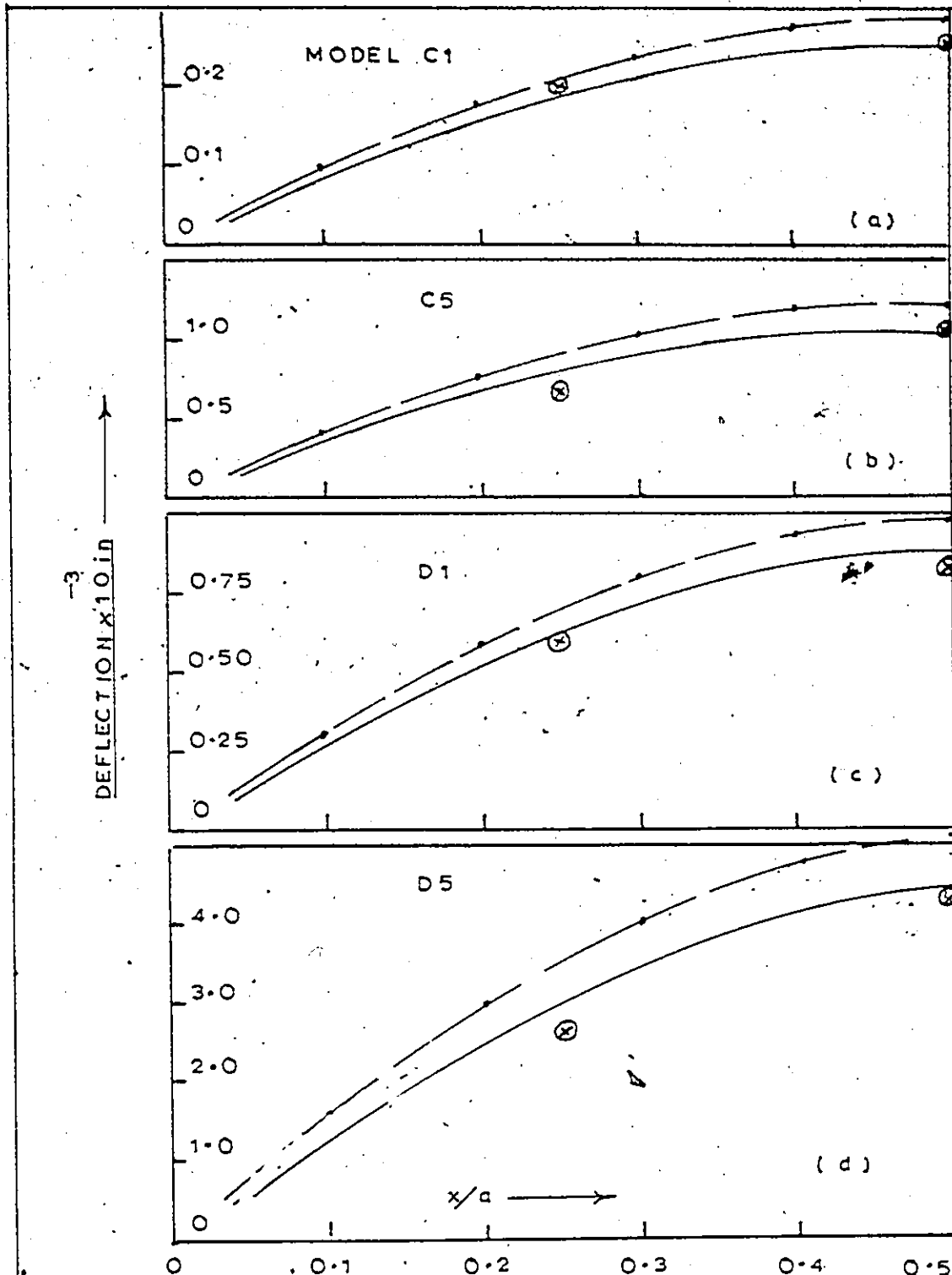


Fig. 76 Influence Lines for Deflections $x/a = 0.5$ and $y/b = 0$.
Load Travel 1-1; Models C and D.

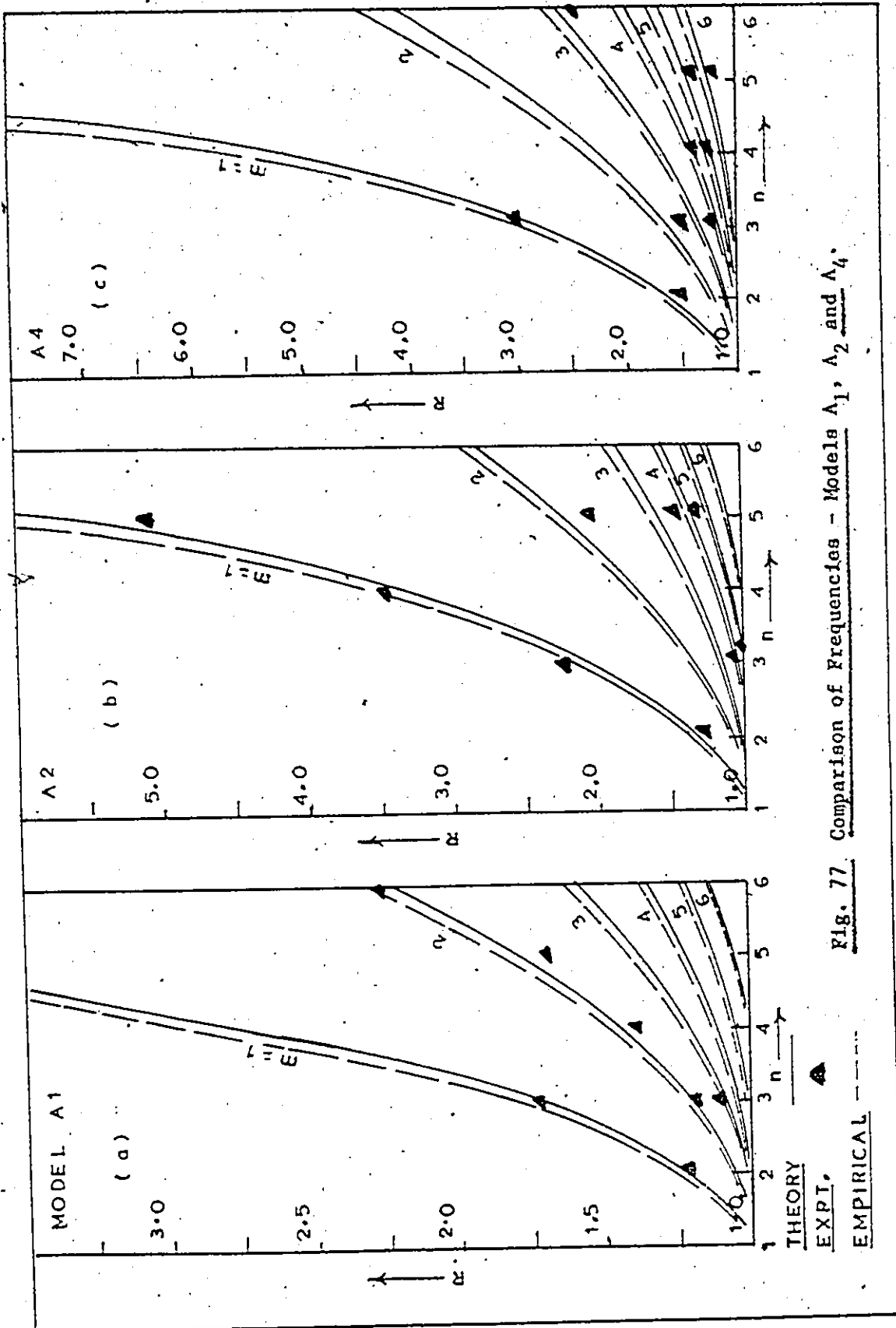


Fig. 77 Comparison of Frequencies - Models Λ_1 , Λ_2 and Λ_4 .

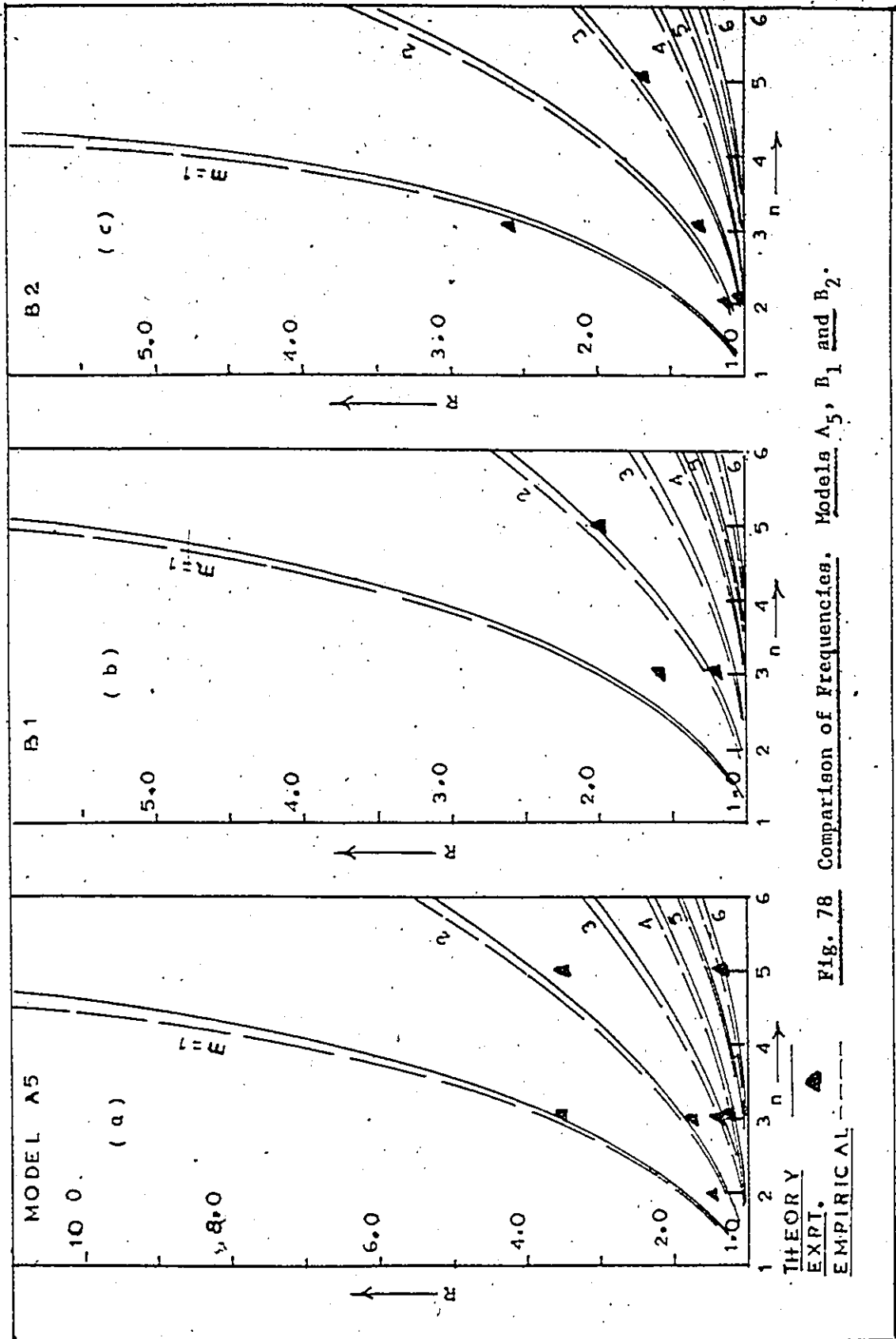


Fig. 78 Comparison of Frequencies. Models A_5 , B_1 and B_2 .

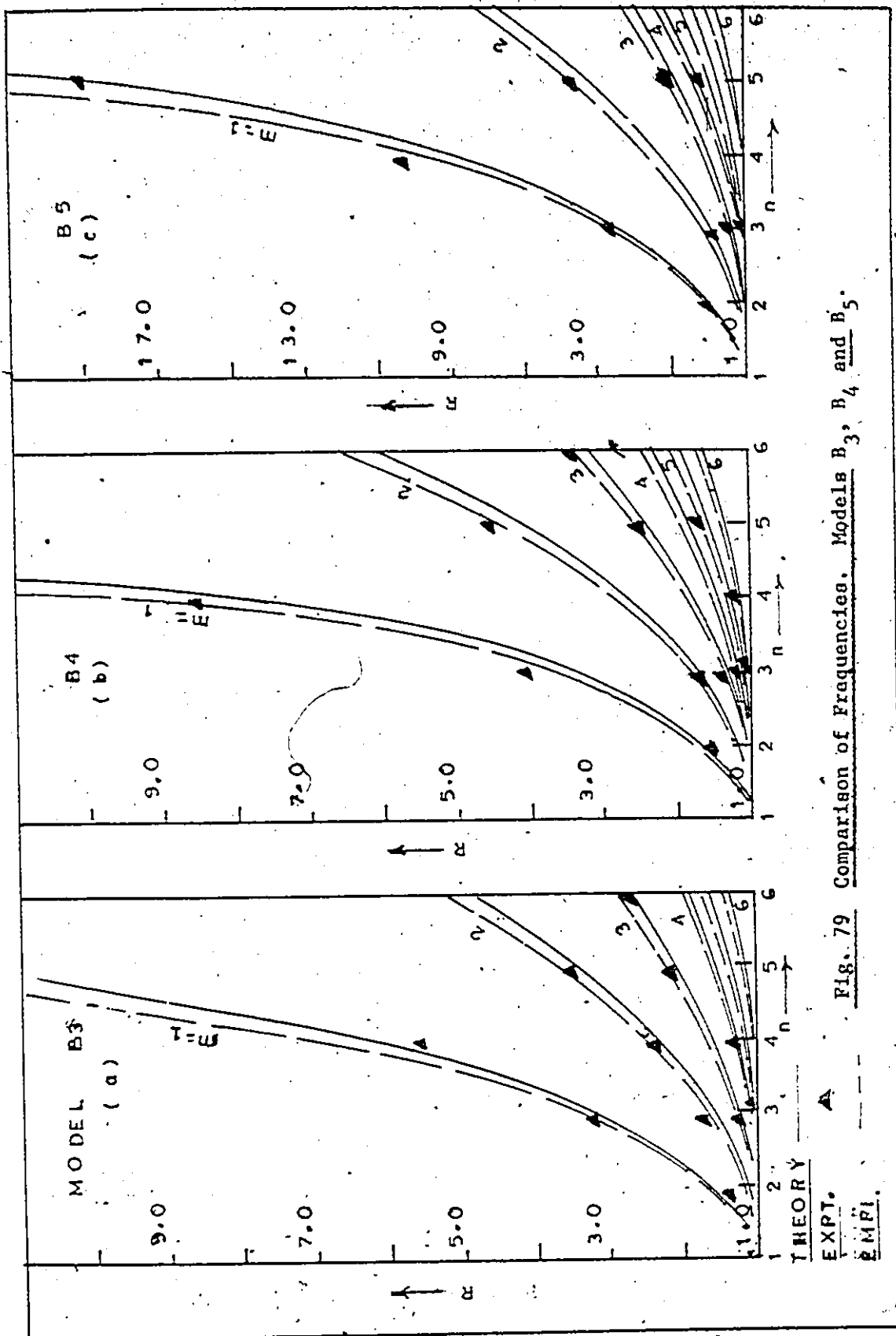
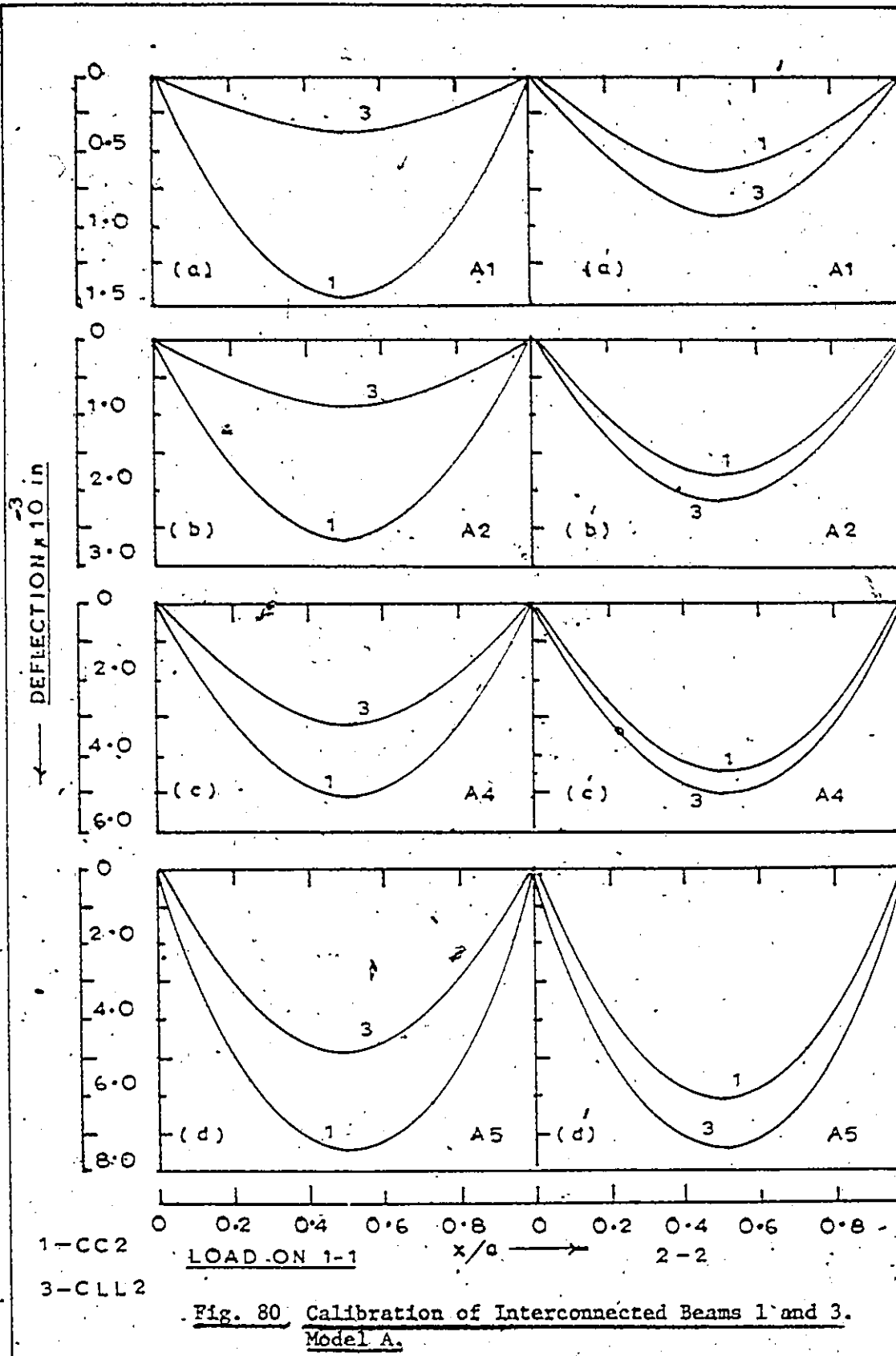
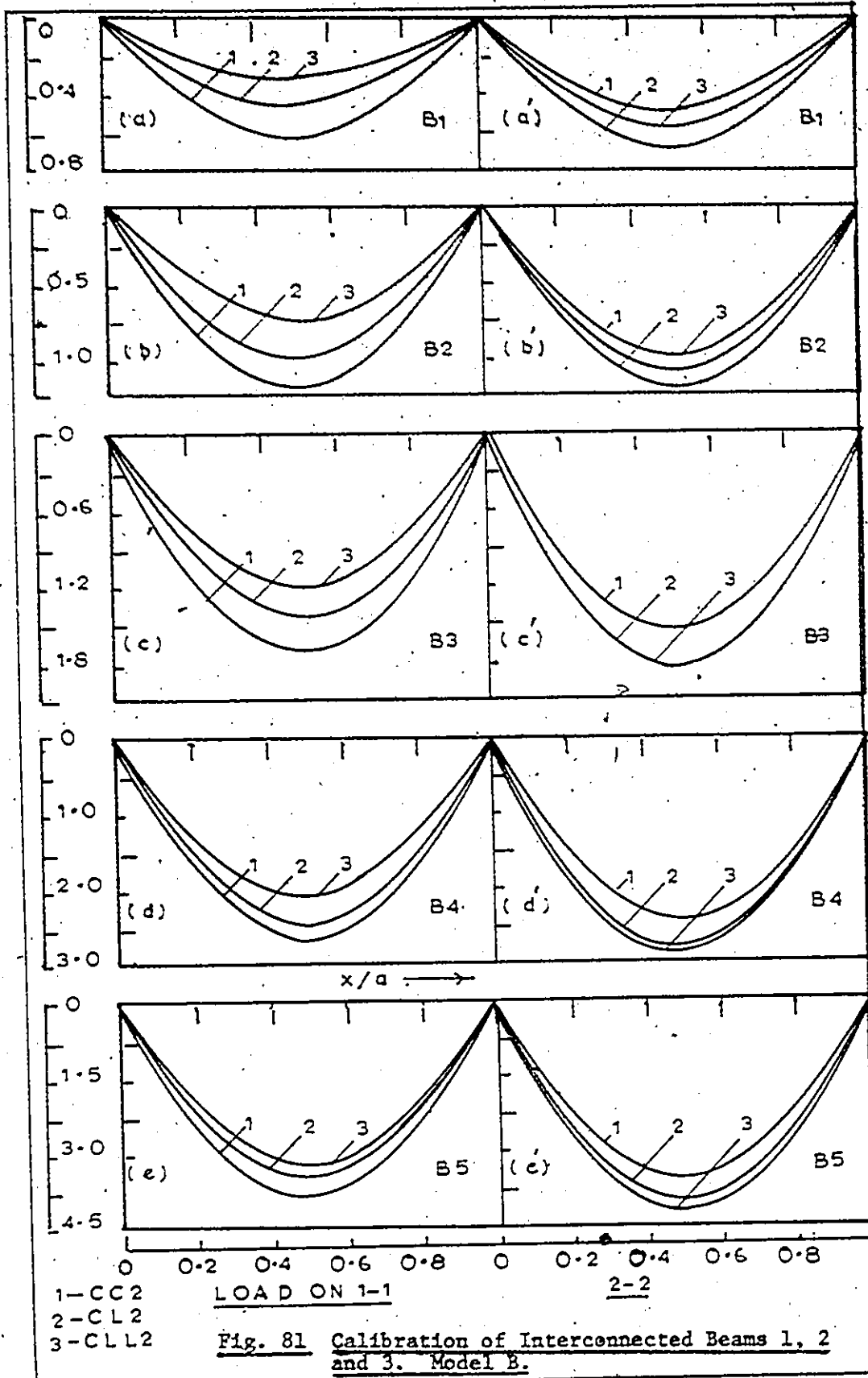
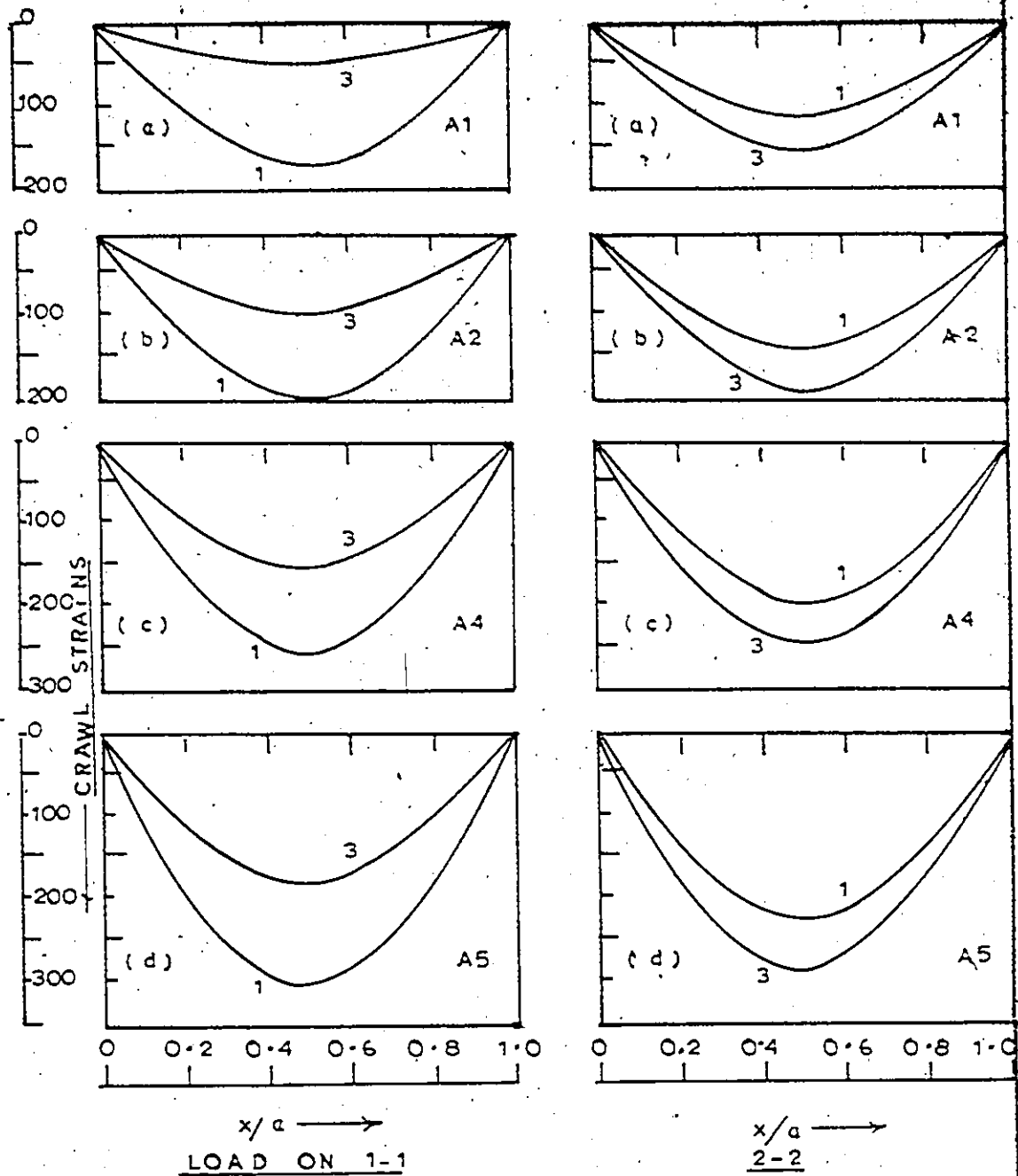


Fig. 79 Comparison of Frequencies. Models B_3 , B_4 and B_5 .

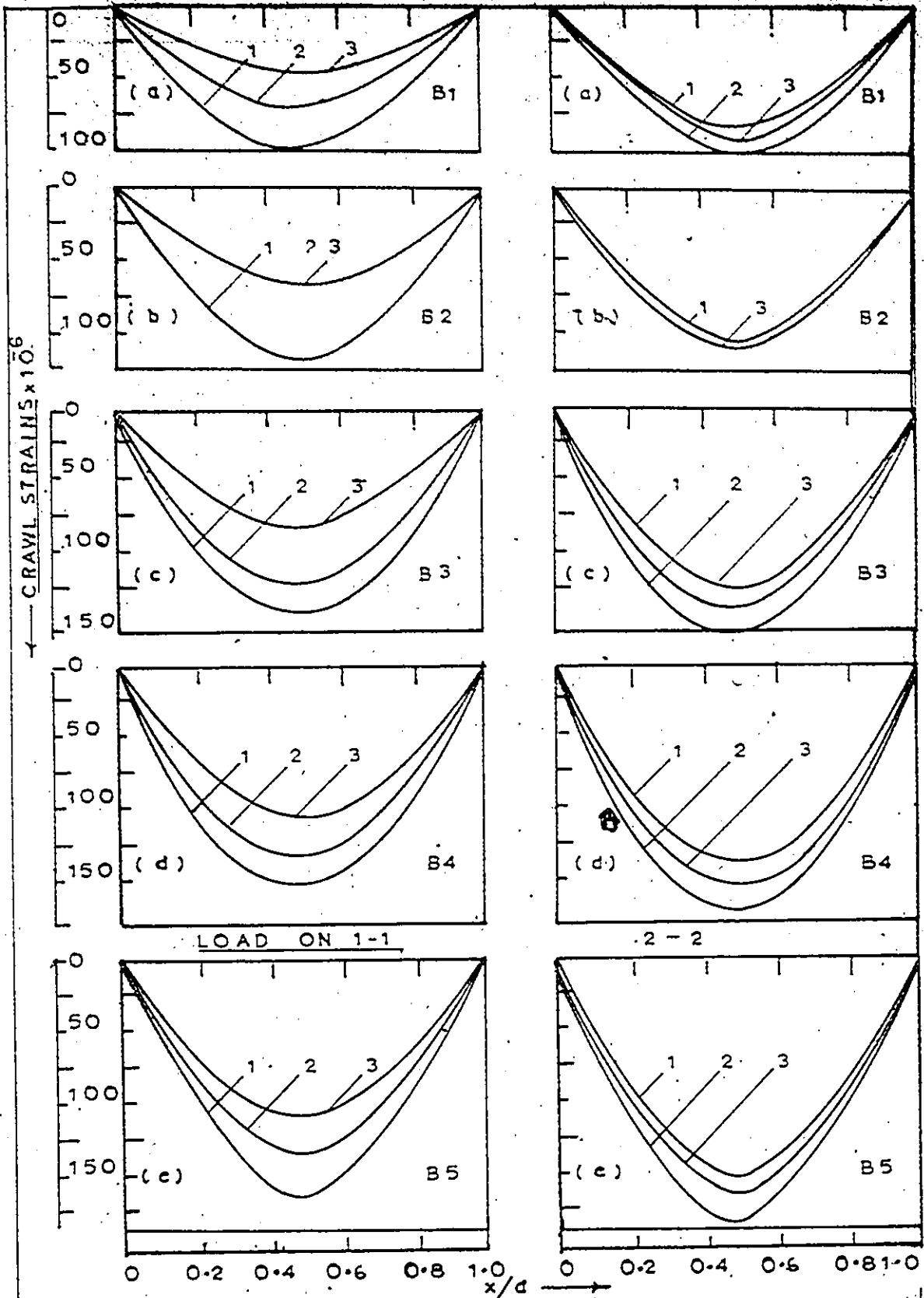






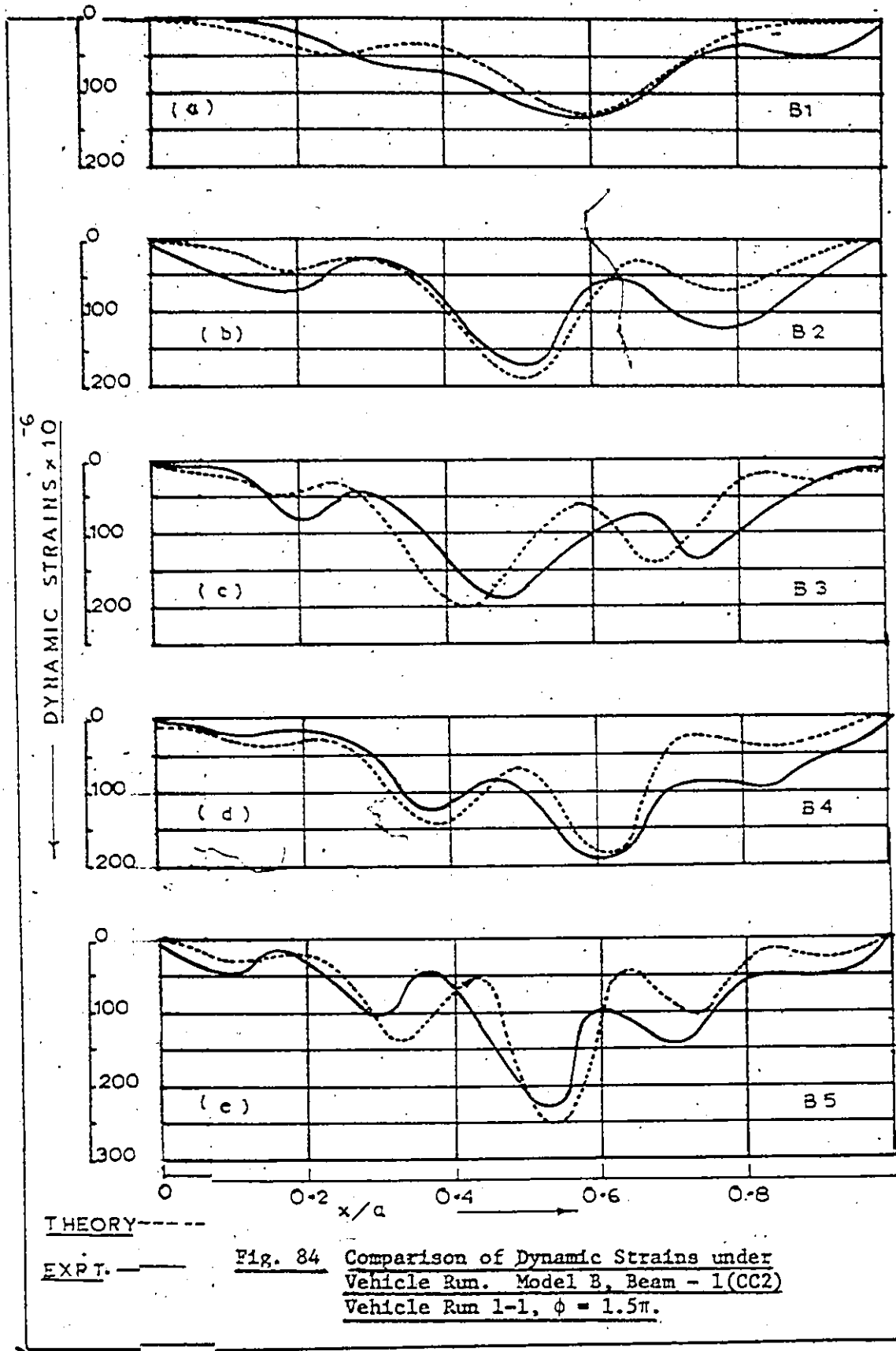
1 - CC2
3 - CLL2

Fig. 82 Crawl Strains - Interconnected Beams 1 and 3.
Model A.



1-CC 2
2-CL 2
3-CLL 2

Fig. 83 Crawl Strains - Interconnected Beams 1, 2 and 3.
Model B.



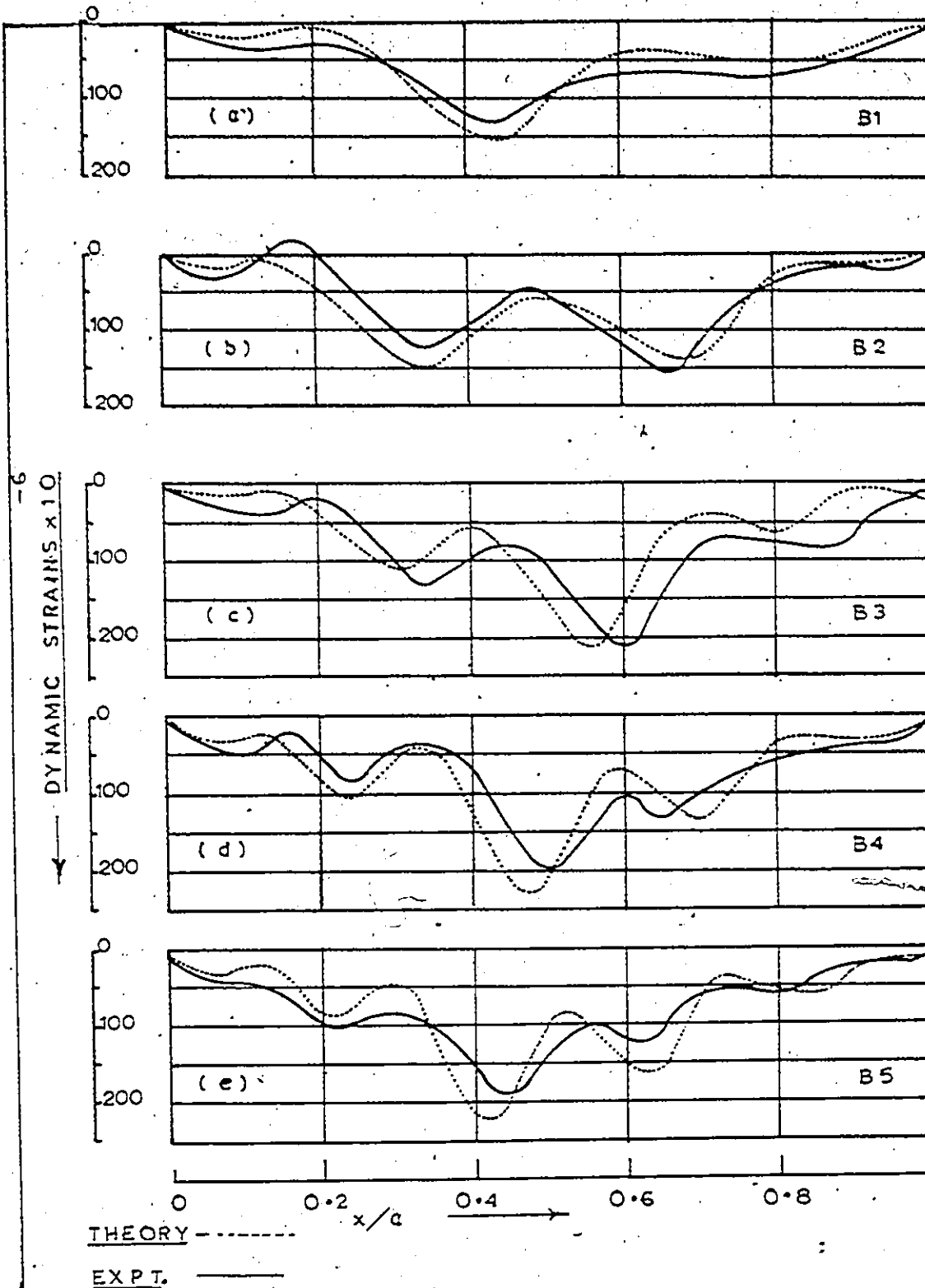
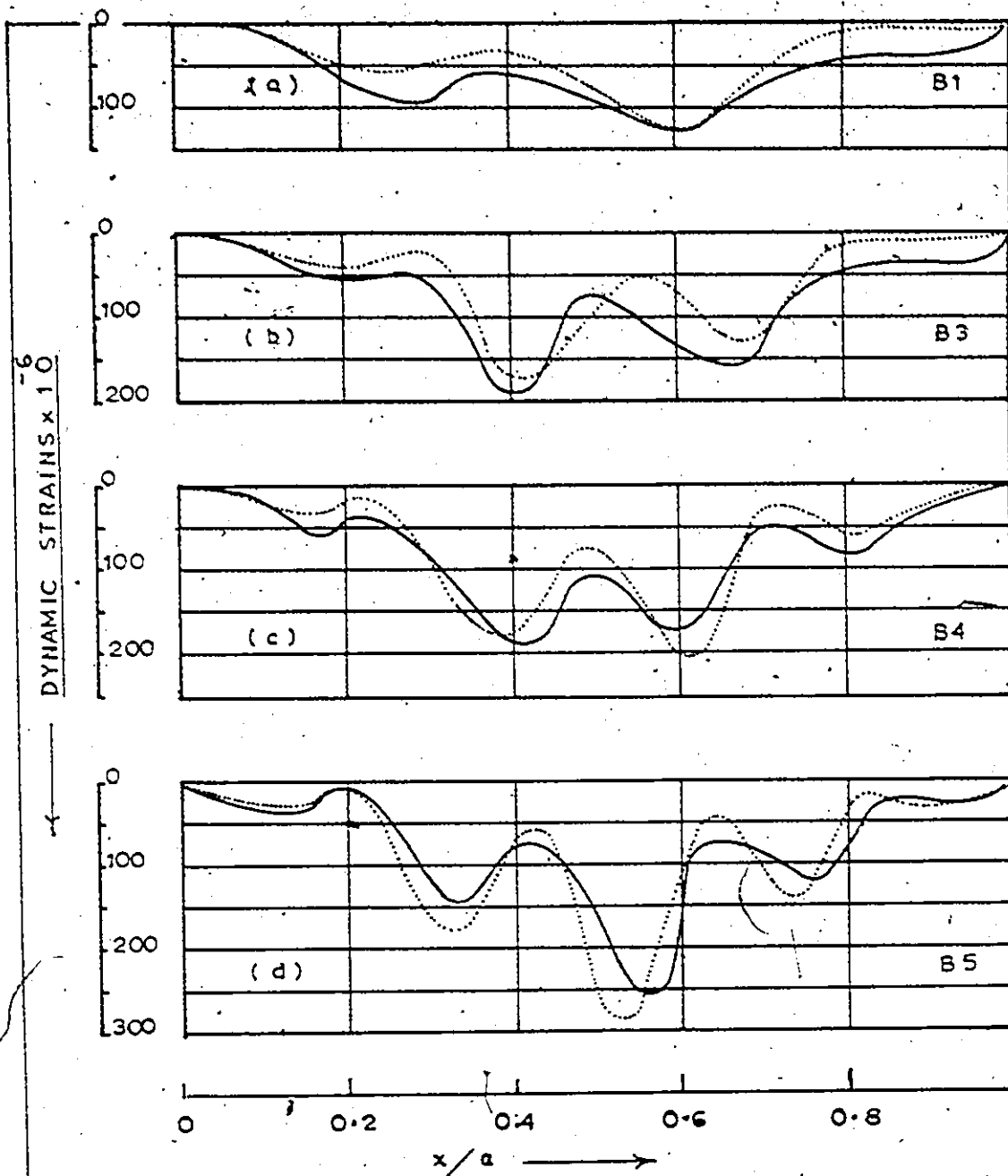


Fig. 85 Comparison of Dynamic Strains under Vehicle Run.
 Model B, Beam - 1(CC2) Vehicle Run 1-1, $\phi = 0.5\pi$.



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Fig. 86 Comparison of Dynamic Strains under Vehicle Run,
Model B, Beam 2 (CL2); Vehicle Run 2-2, $\phi = 1.5\pi$.

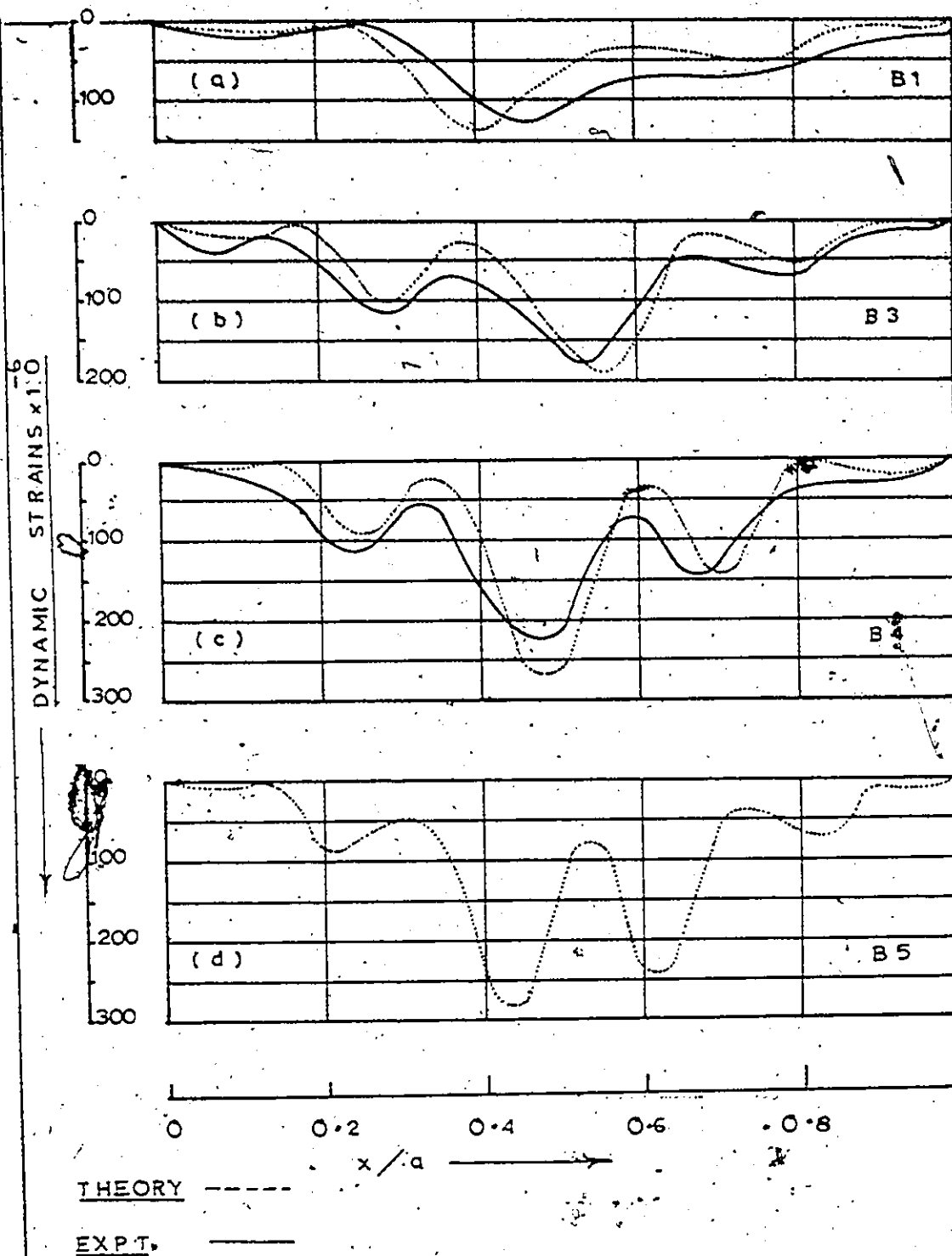


Fig. 87 Comparison of Dynamic Strains under Vehicle Run.
 Model B, Beam 2 (CL2); Vehicle Run 2-2, $\phi = 0.5\pi$.

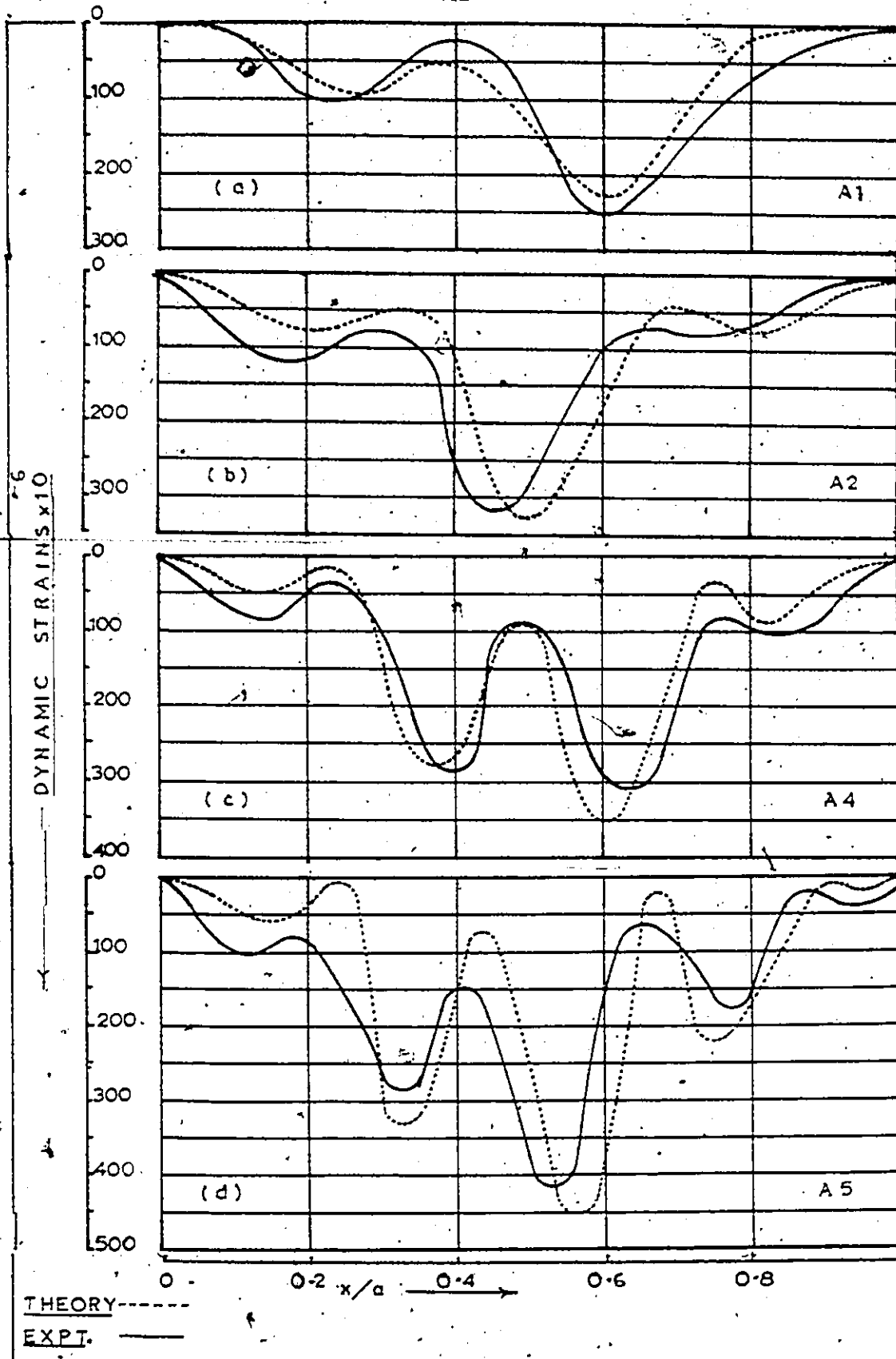


Fig. 88 Comparison of Dynamic Strains under Vehicle Run.
Model A, Beam 1 (CC2); Vehicle Run 1-1, $\phi = 1.5\pi$.

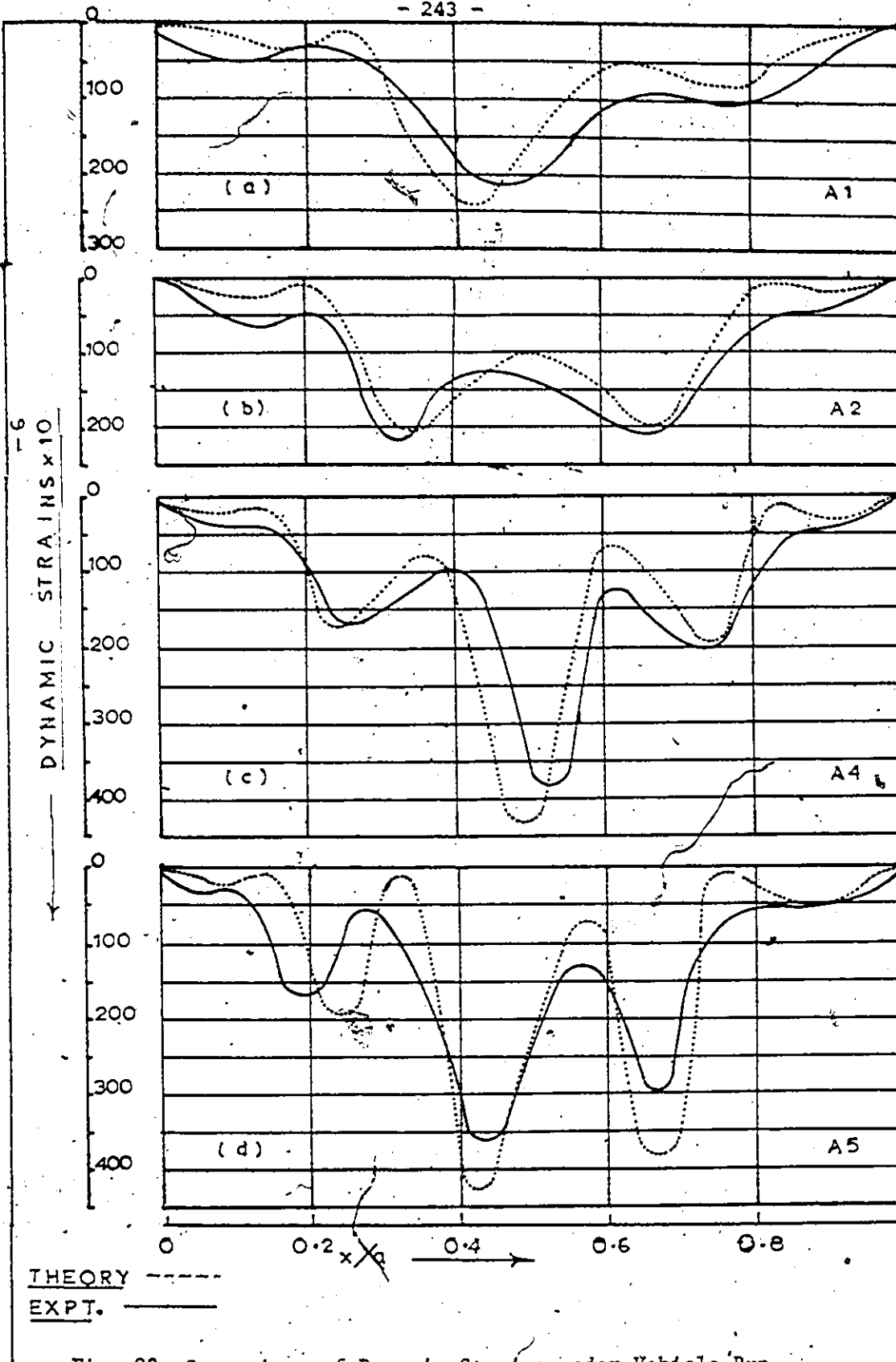
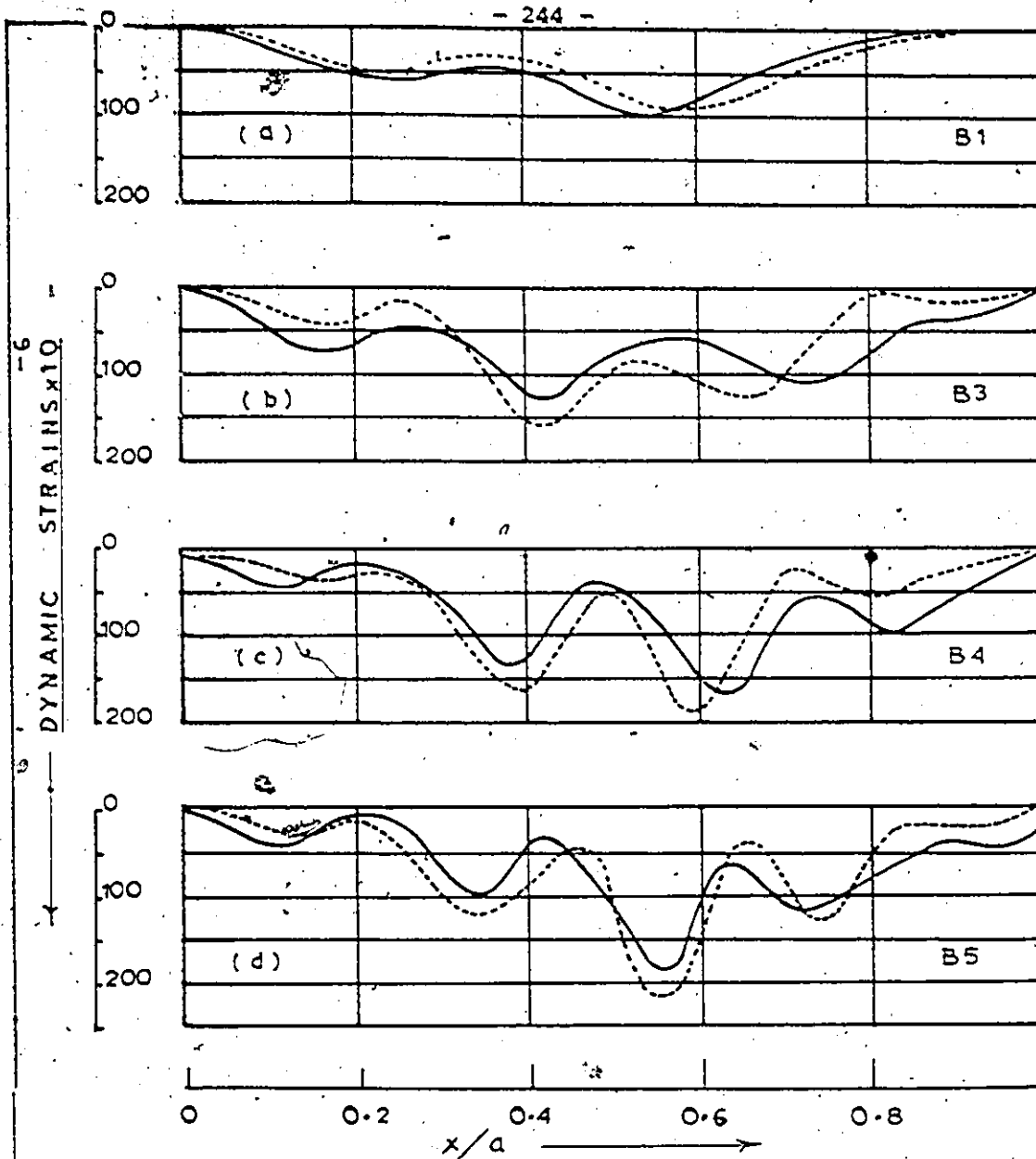


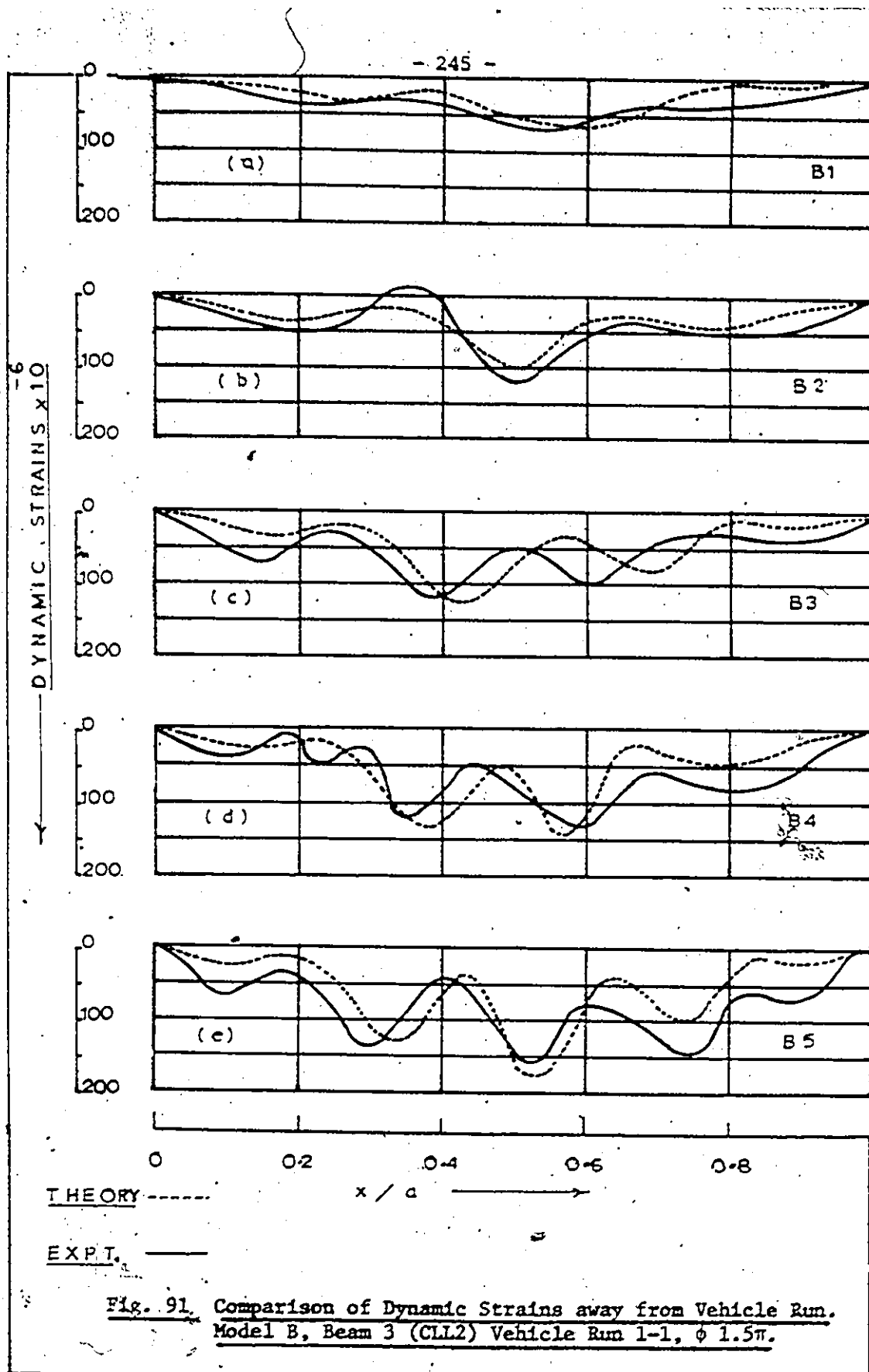
Fig. 89 Comparison of Dynamic Strains under Vehicle Run.
Model A, Beam 1 (CC2); Vehicle Run 1-1, $\phi = 0.5\pi$.

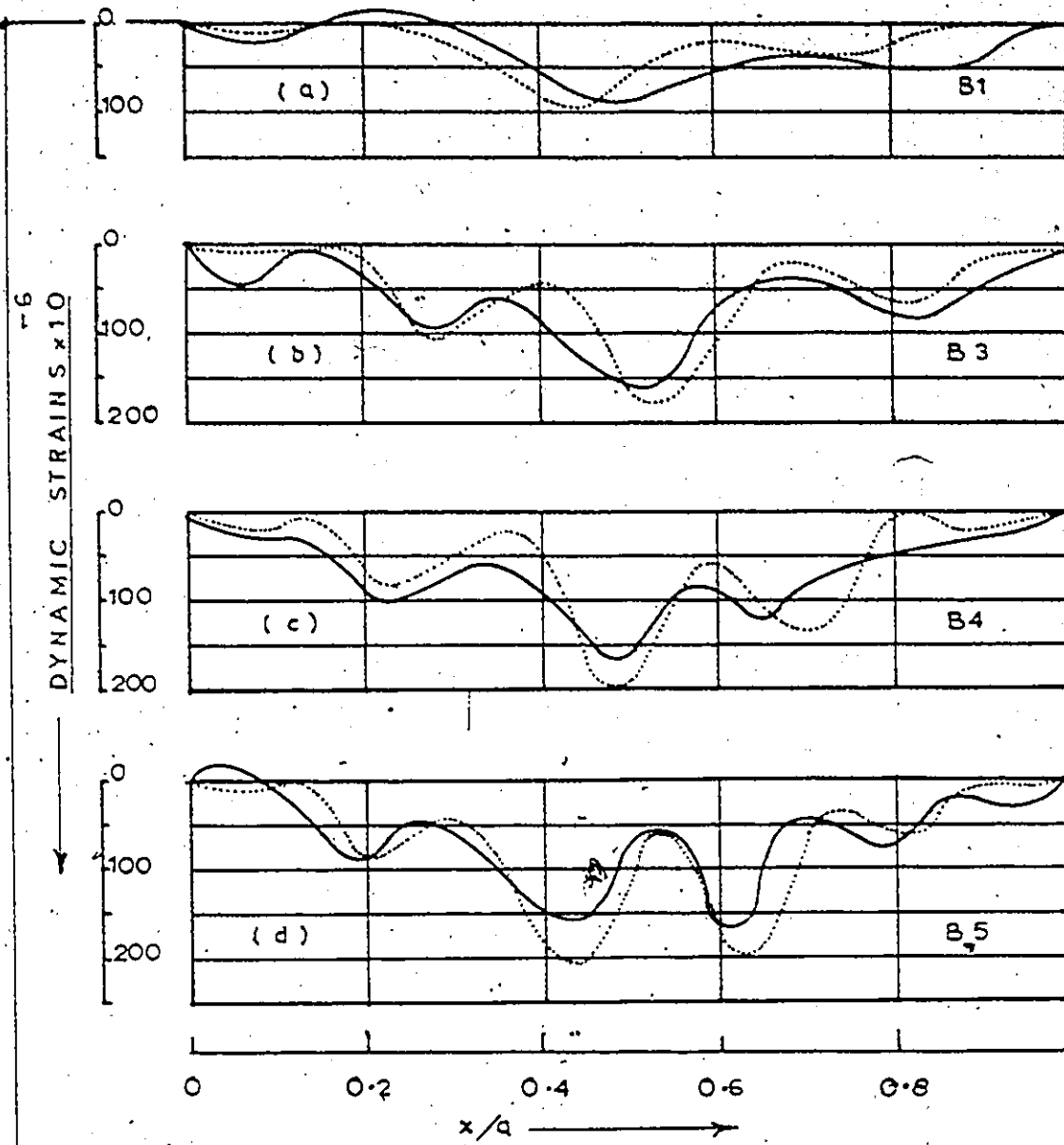


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Fig. 90. Comparison of Dynamic Strain away from Vehicle Run.
Model B, Beam 2 (CL2); Vehicle Run 1-1, $\phi = 1.5\pi$.





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Fig. 92 Comparison of Dynamic Strains away from Vehicle Run.
Model B, Beam 2 (CL2); Vehicle Run 1-1; $\phi = 0.5\pi$.

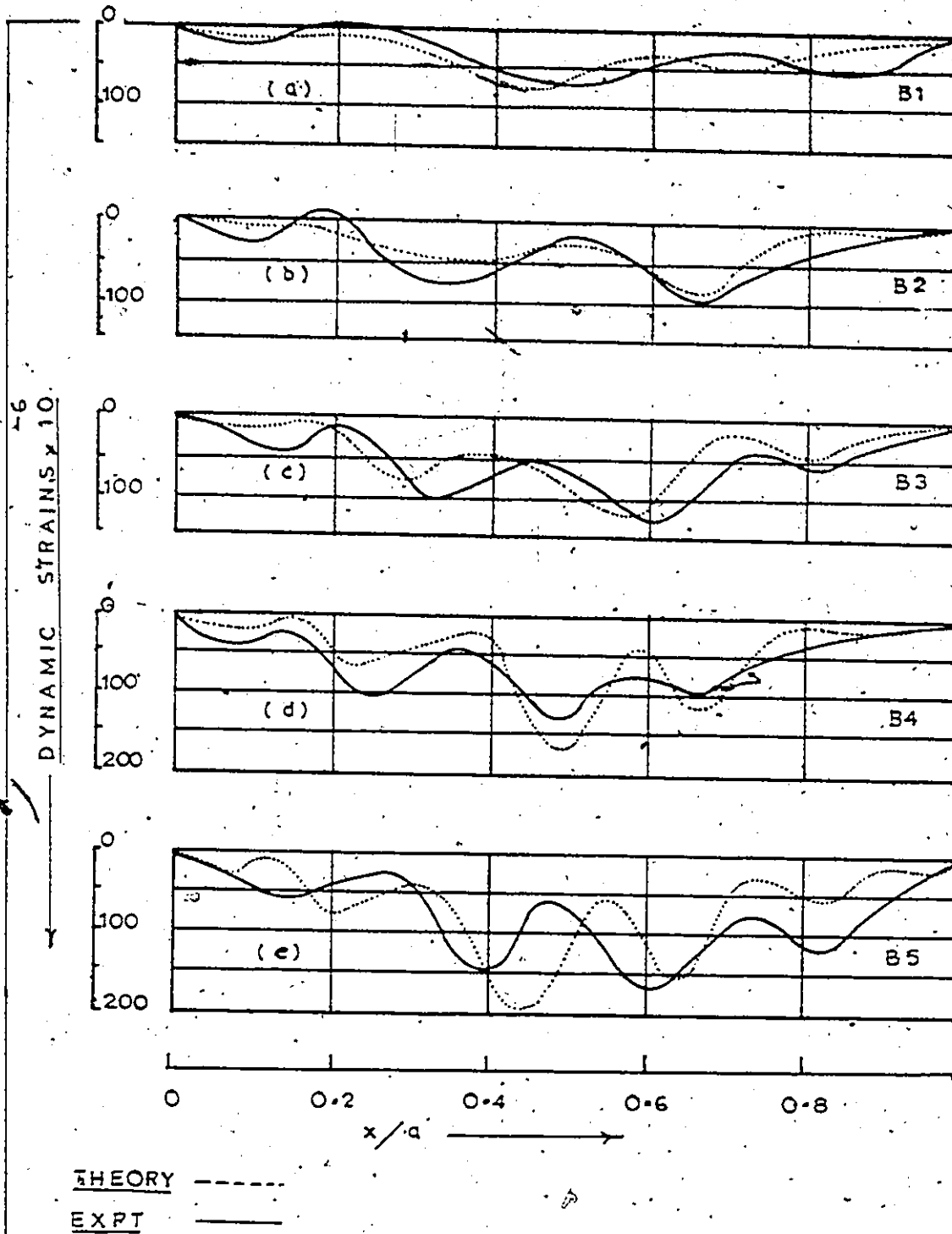
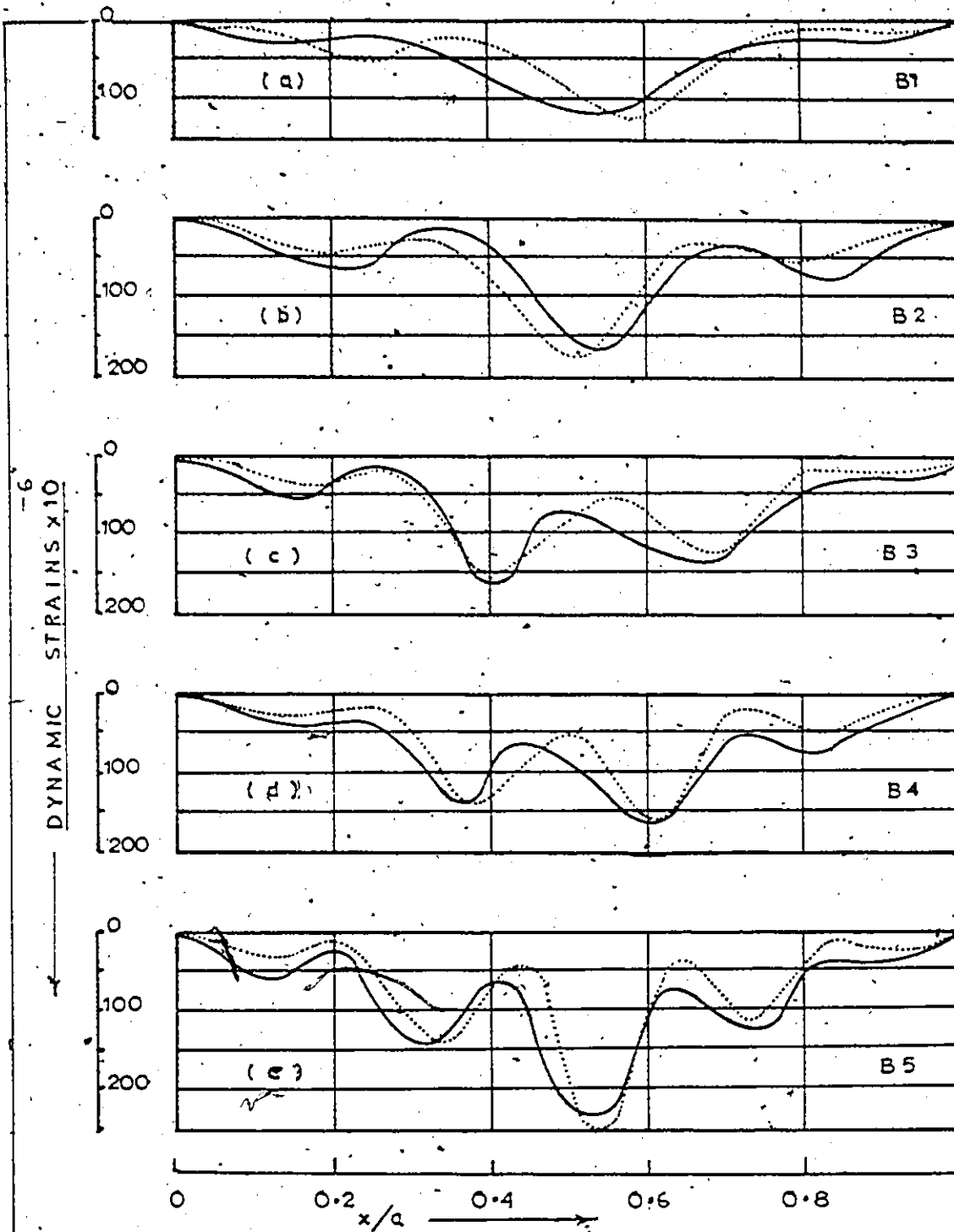


Fig. 93 Comparison of Dynamic Strains away from Vehicle Run.
Model B, Beam 3 (CLL2) Vehicle Run 1-1, $\phi = 0.5\pi$.



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Fig. 94 Comparison of Dynamic Strains away from Vehicle Run.
Model B, Beam 1 (CC2); Vehicle Run 2-2, $\phi = 1.5\pi$.

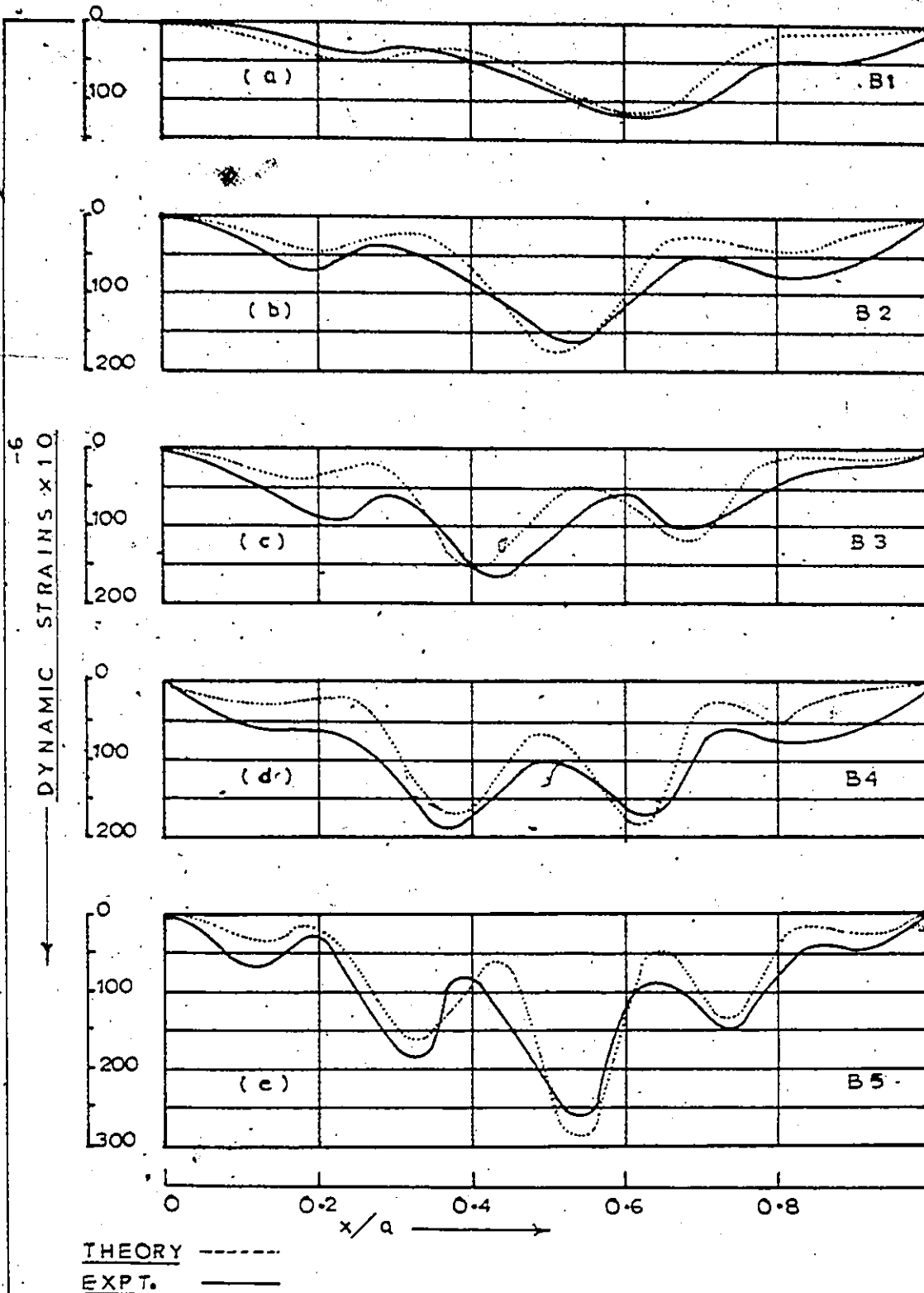


Fig. 95 Comparison of Dynamic Strains away from Vehicle Run.
Model B, Beam 3 (CLL2); Vehicle Run 2-2, $\phi = 1.5\pi$.

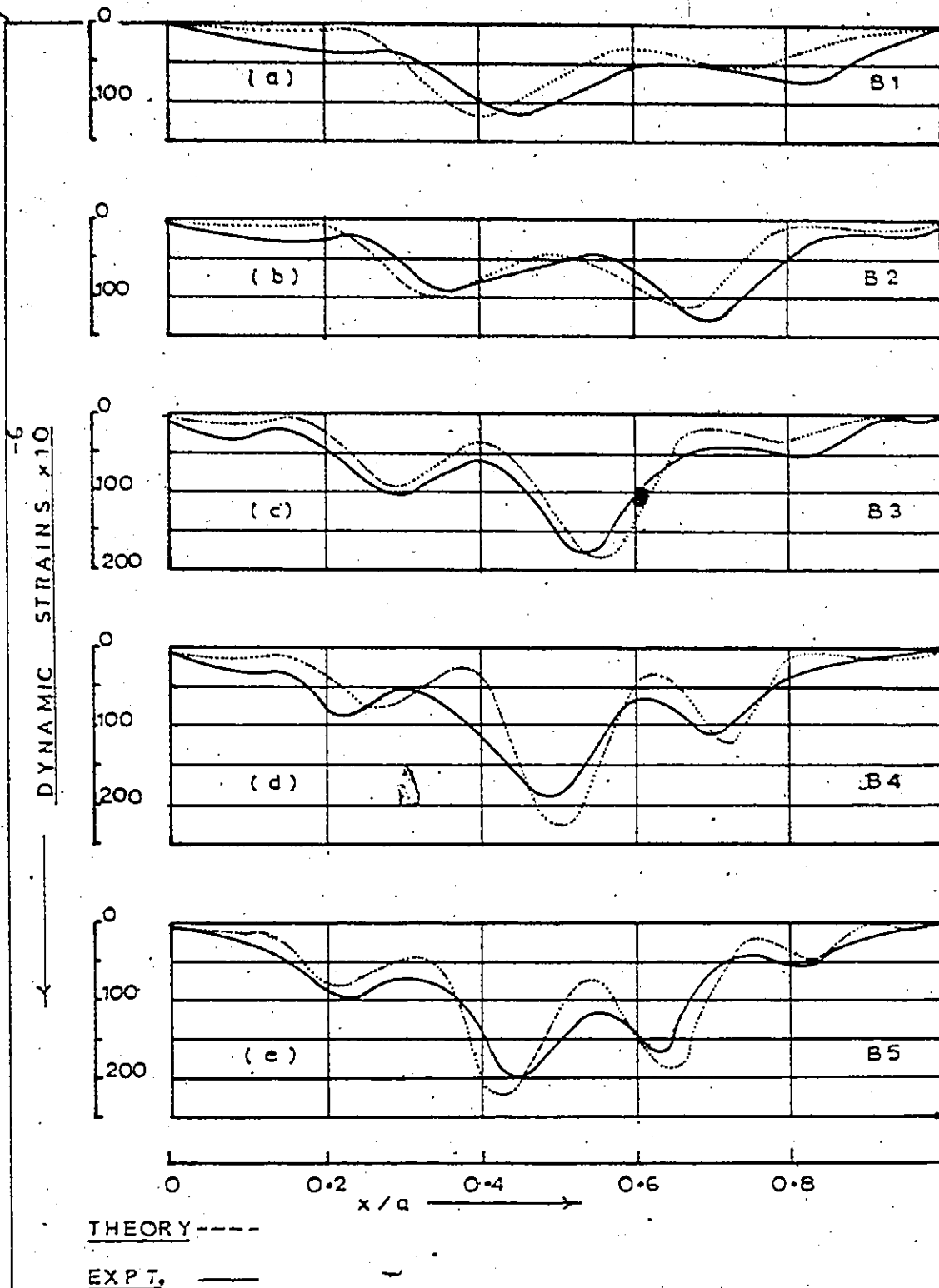


Fig. 96 Comparison of Dynamic Strains away from Vehicle Run.
Model B, Beam 1 (CC2); Vehicle Run 2-2, $\phi = 0.5\pi$.

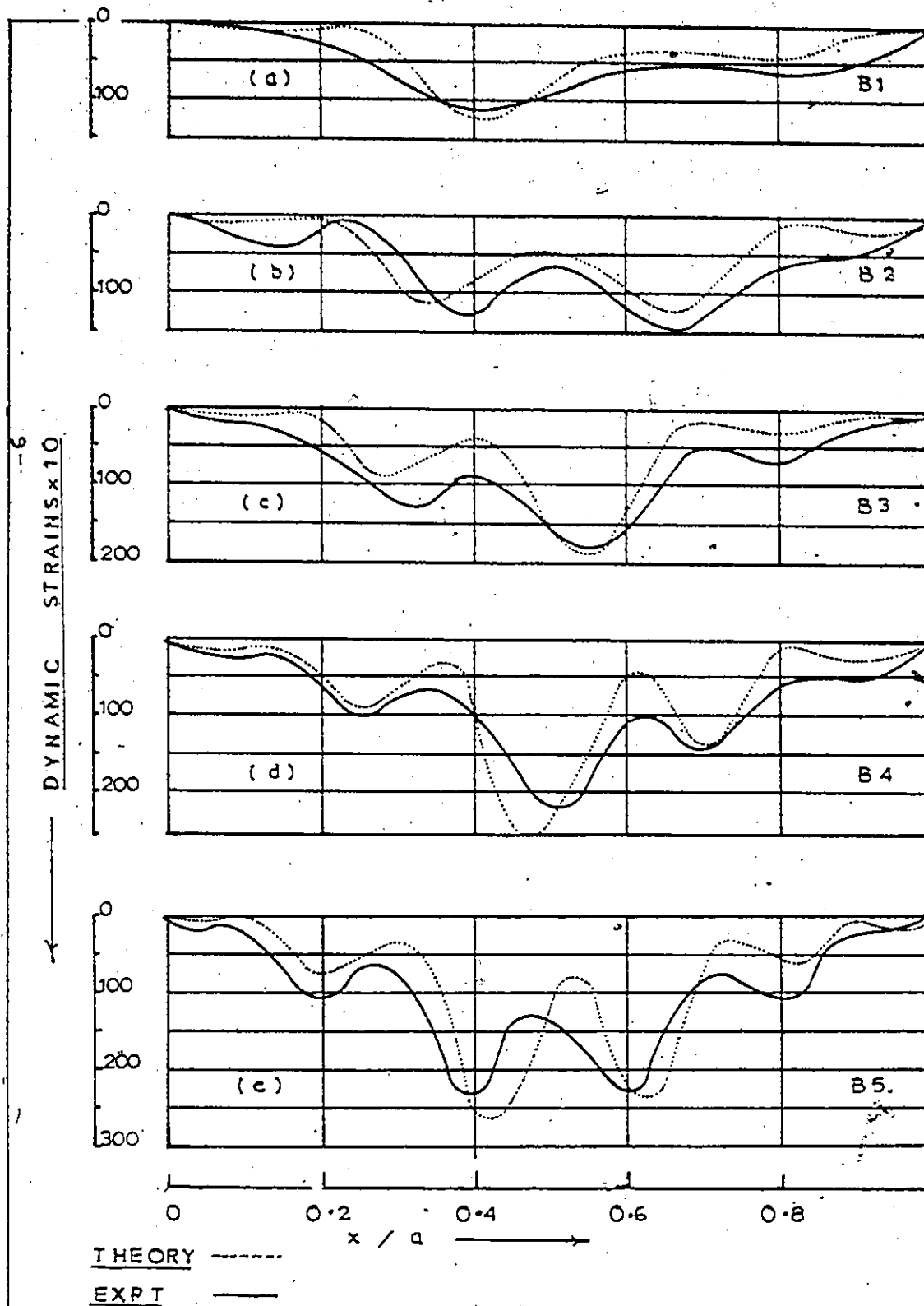


Fig. 97 Comparison of Dynamic Strains away from Vehicle Run.
Model B, Beam 3 (CLL2); Vehicle Run 2-2, $\phi = 0.5\pi$.

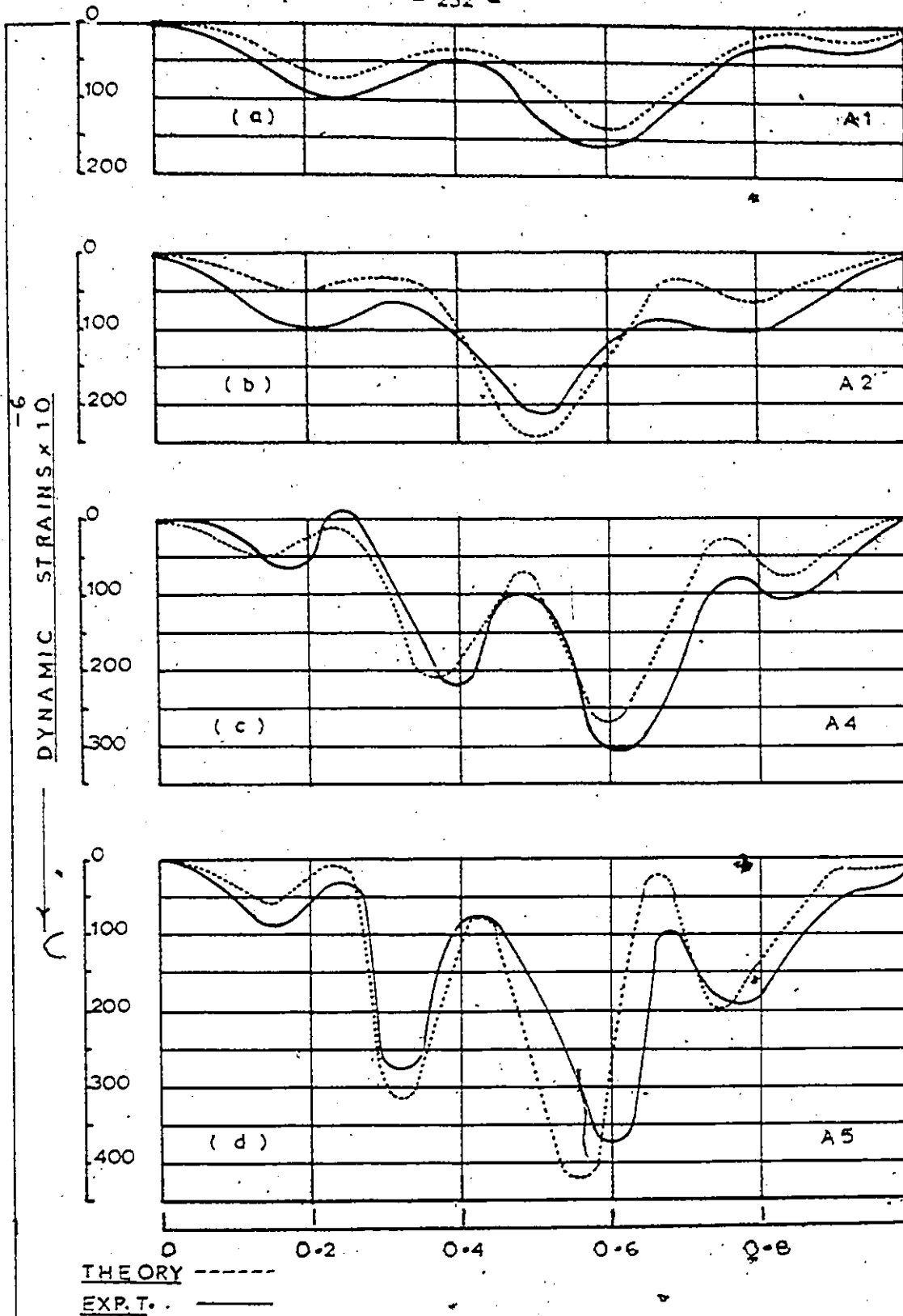


Fig. 98 Comparison of Dynamic Strains away from Vehicle Run.
 Model A, Beam 1 (CC2); Vehicle Run 2-2, $\phi = 1.5\pi$.

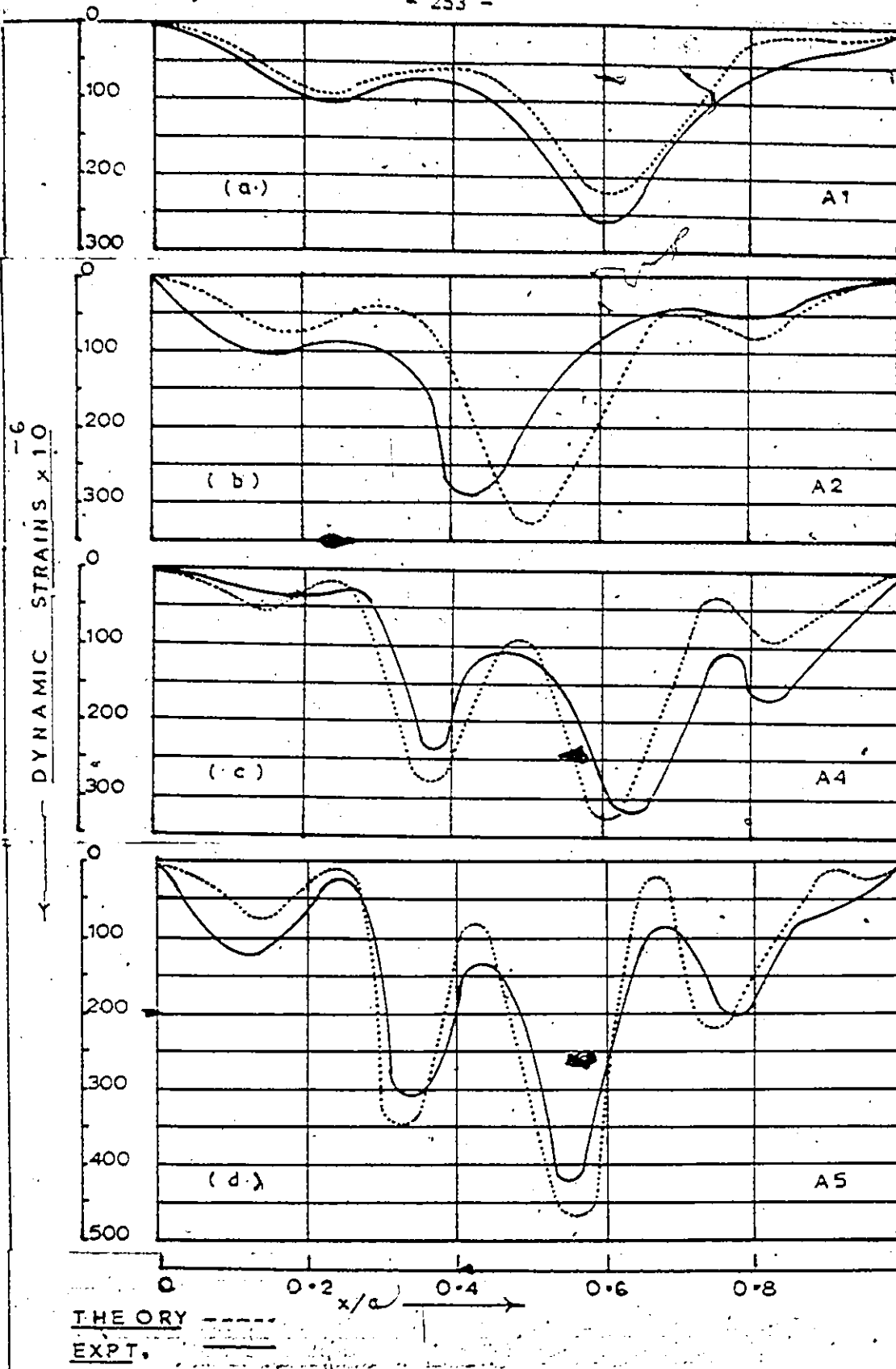


Fig. 99 Comparison of Dynamic Strains away from Vehicle Run.
Model A, Beam 3 (CLL2); Vehicle Run 2-2, $\phi = 1.5\pi$.

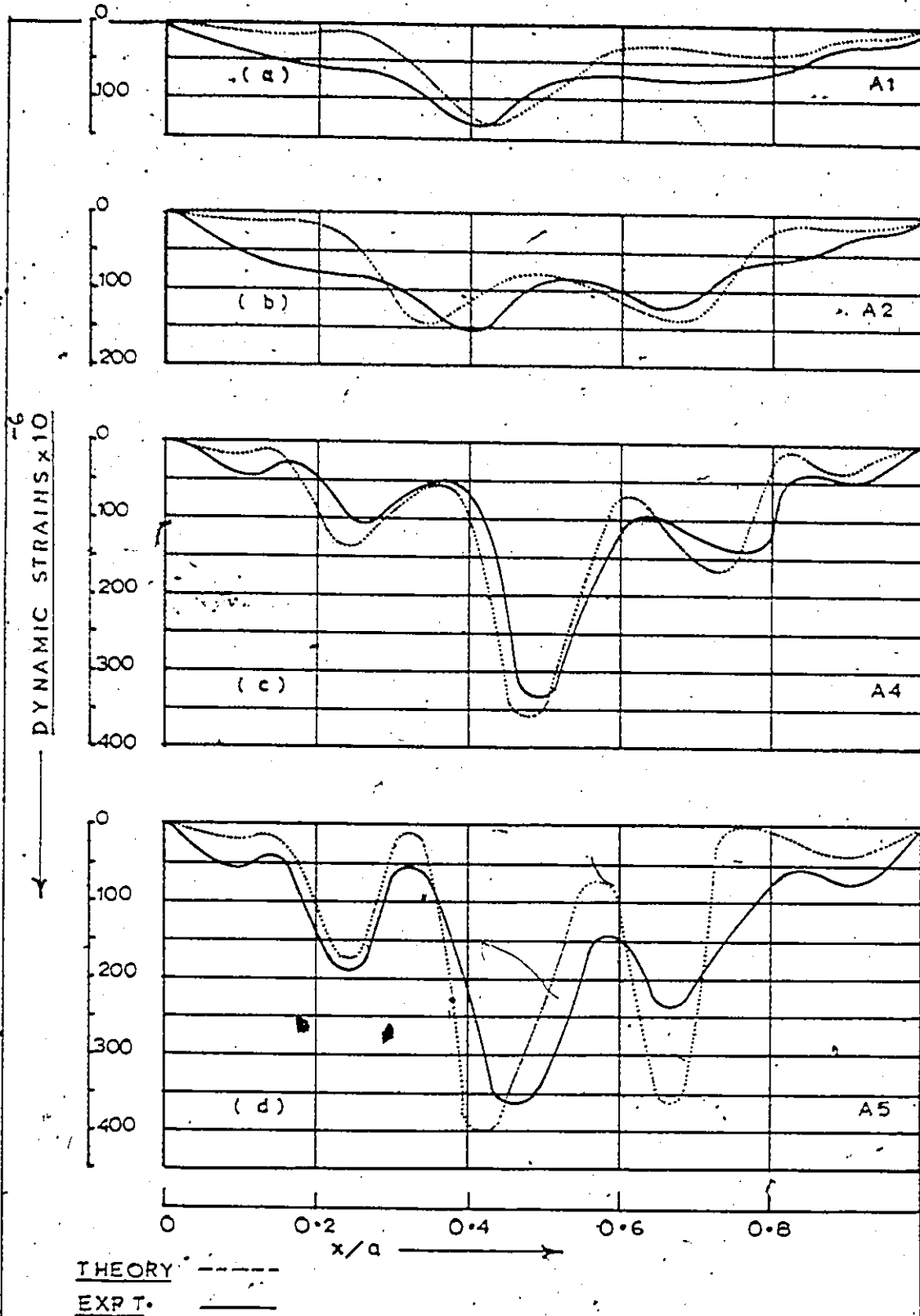


Fig. 100 Comparison of Dynamic Strains away from Vehicle Run.
Model A, Beam 1 (CC2); Vehicle Run 2-2, $\phi = 0.5\pi$.

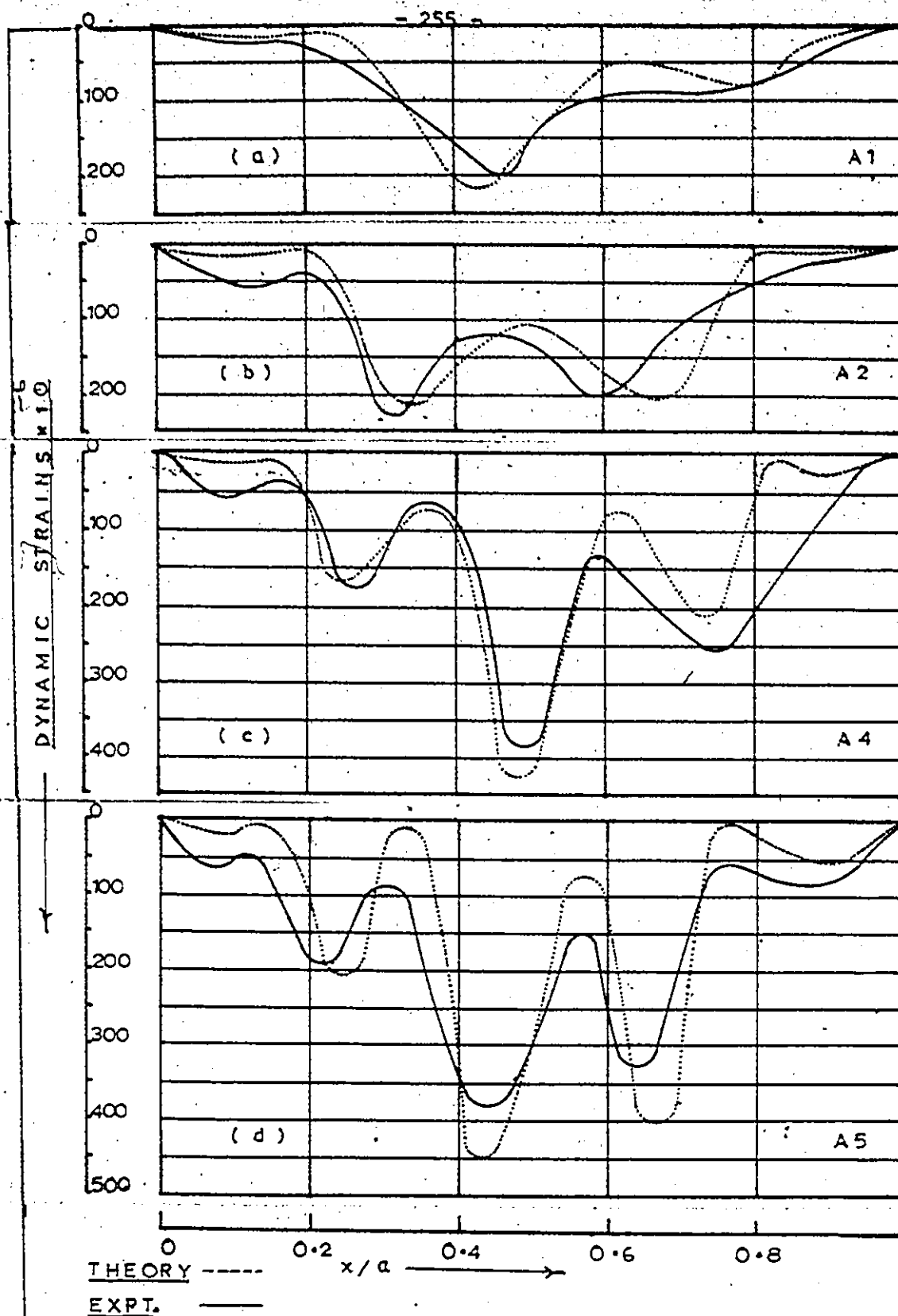
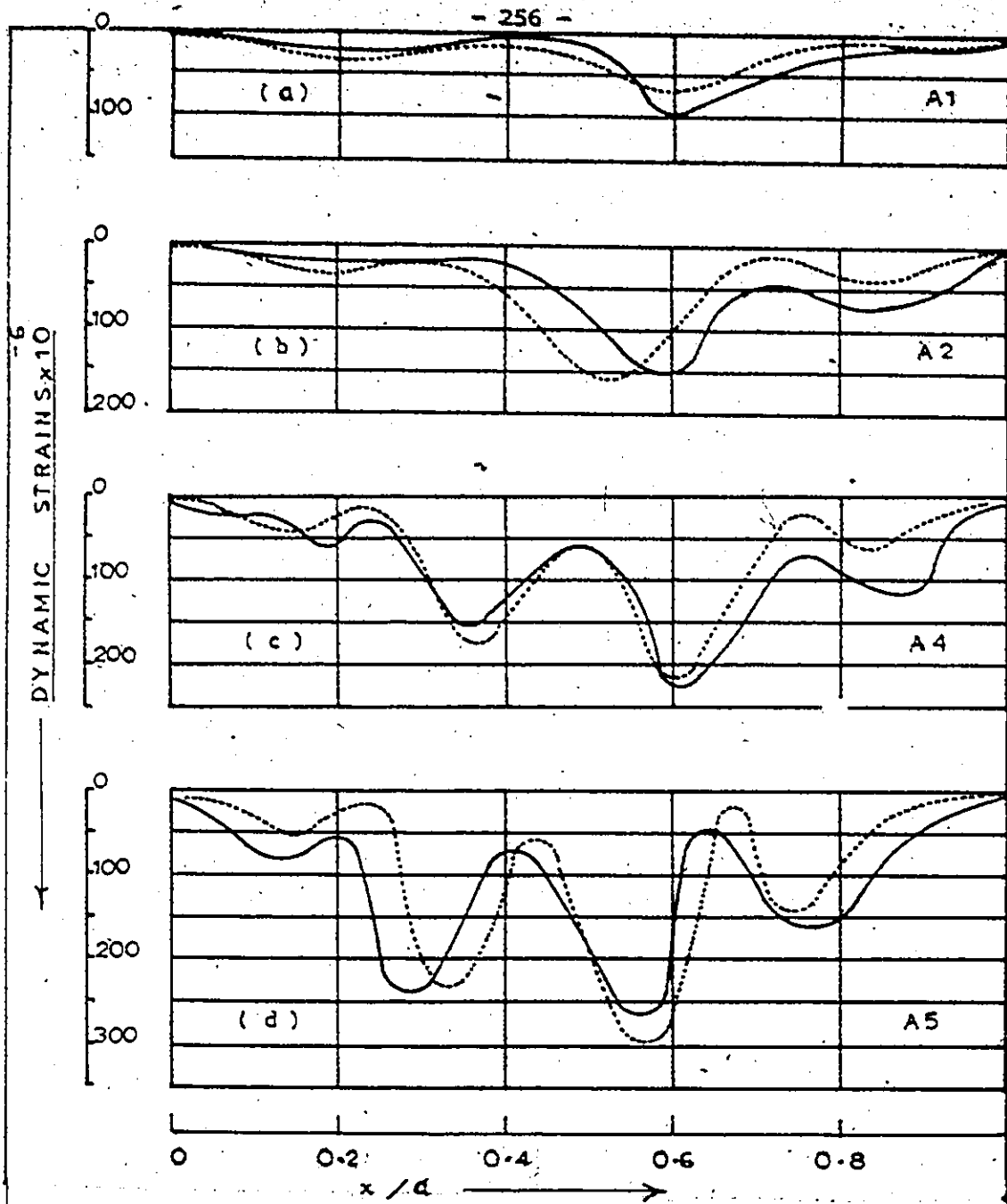
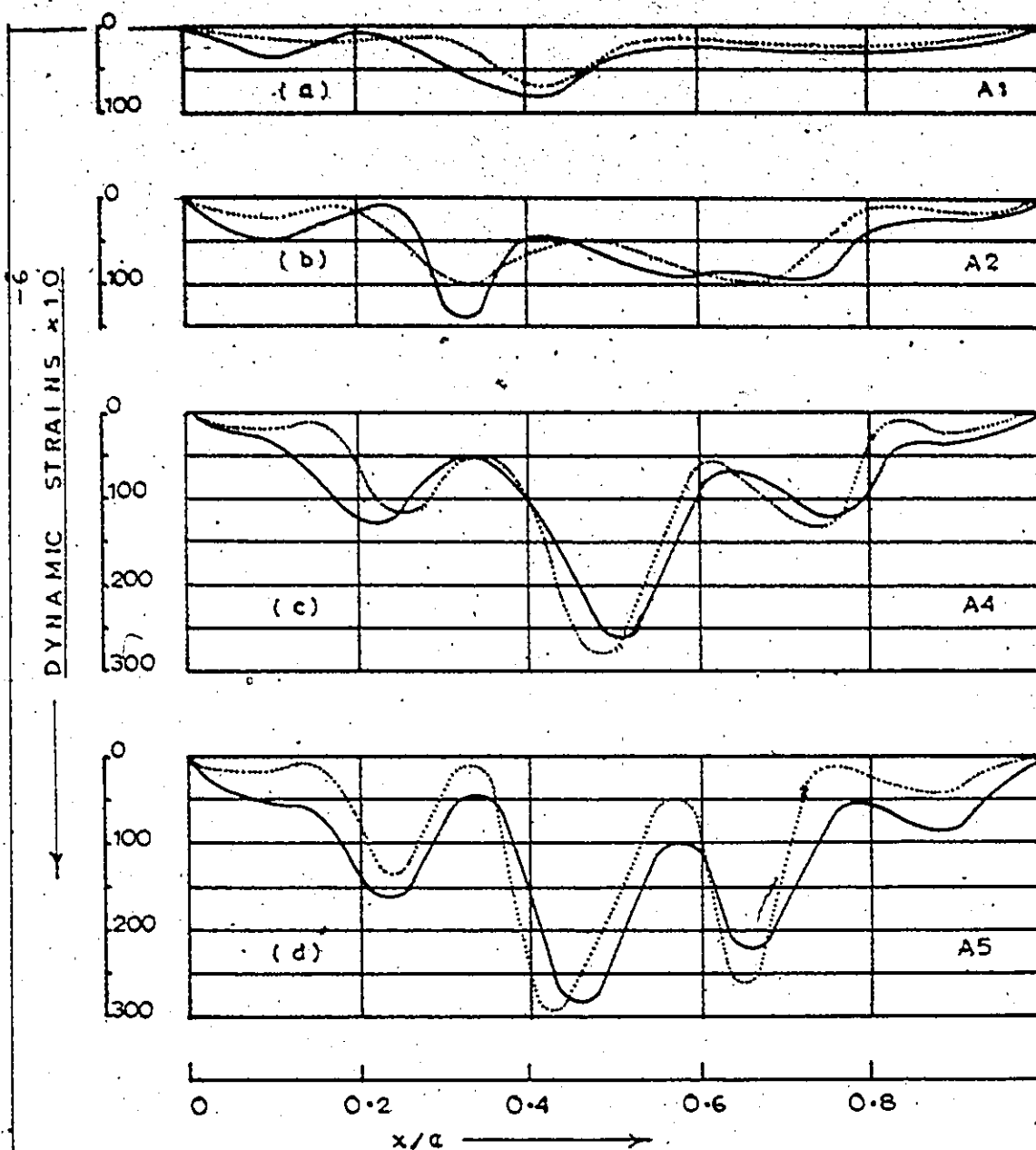


Fig. 101. Comparison of Dynamic Strains away from Vehicle Run.
Model A, Beam 3 (CLL2); Vehicle Run 2-2, $\phi=0.5\pi$.



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Fig. 102 Comparison of Dynamic Strains away from Vehicle Run.
Model A, Beam 3 (CLL2); Vehicle Run 1-1, $\phi = 1.5\pi$.



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Fig. 103 Comparison of Dynamic Strains away from Vehicle Run.
Model A, Beam 3 (CLL2); Vehicle Run 1-1, $\phi = 0.5\pi$.

APPENDIX IV

TABLES 1 to 58

Table 1 -

Stresses in an element of orthotropic plate.

<u>DISTRIBUTION OF STRESSES AT</u>			
<u>X = 0</u>	<u>Y = 0</u>	<u>X = X + dx</u>	<u>Y = Y + dy</u>
σ_x	σ_y	$\sigma_x + \frac{\partial \sigma_x}{\partial x} \cdot dx$	$\sigma_y + \frac{\partial \sigma_y}{\partial y} \cdot dy$
τ_{xy}	τ_{yx}	$\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} \cdot dx$	$\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} \cdot dy$
τ_{xz}	τ_{yz}	$\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x} \cdot dx$	$\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y} \cdot dy$

Table 2 -

Moments and shears -

<u>MOMENTS</u>	<u>SHEARS</u> ✓
$M_x = \int_{-h/2}^{+h/2} \sigma_x \cdot z \cdot dz$	$Q_x = \int_{-h/2}^{+h/2} \tau_{xz} \cdot dz$
$M_y = \int_{-h/2}^{+h/2} \sigma_y \cdot z \cdot dz$	
$M_{xy} = \int_{-h/2}^{+h/2} \tau_{xy} \cdot z \cdot dz$	$Q_y = \int_{-h/2}^{+h/2} \tau_{yz} \cdot dz$
$M_{yx} = \int_{-h/2}^{+h/2} \tau_{yx} \cdot z \cdot dz$	

Table 3 Second Order Derivatives of Deflection Functions, w_1, w_2

F_{xy}	F''_{xy}	EXPRESSION
w_1	$\frac{\partial^2 w_1}{\partial x^2}$	$-K \sum_{m=1}^{\infty} \frac{1}{m^3} \left(\frac{m\pi}{a} \right)^2 \left\{ \alpha \sin \frac{m\pi y}{\alpha} + B \cos \frac{m\pi y}{\alpha} \right\} e^{\frac{-m\pi y}{\beta}} \cdot \sin \frac{m\pi x}{a}$
w_1	$\frac{\partial^2 w_1}{\partial y^2}$	$-K \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{1}{\beta^2 + \frac{1}{\alpha^2}} \right) \left\{ B \cos \frac{m\pi y}{\alpha} - \alpha \sin \frac{m\pi y}{\alpha} \right\} e^{\frac{-m\pi y}{\beta}} \cdot \sin \frac{m\pi x}{a}$
w_2	$\frac{\partial^2 w_2}{\partial x^2}$	$-K \sum_{m=1}^{\infty} \frac{1}{m^3} \left(\frac{m\pi}{a} \right)^2 \left[\left\{ A_m \alpha \sin \frac{m\pi y}{\alpha} + B_m \cdot B \cos \frac{m\pi y}{\alpha} \right\} e^{\frac{-m\pi y}{\beta}} \right. \\ \left. + \left\{ C_m \alpha \sin \frac{m\pi y}{\alpha} + D_m B \cos \frac{m\pi y}{\alpha} \right\} e^{\frac{m\pi y}{\beta}} \right] \sin \frac{m\pi x}{a}$
w_2	$\frac{\partial^2 w_2}{\partial y^2}$	$-K \sum_{m=1}^{\infty} \frac{1}{m} \left(\left[\left\{ \left(\frac{\alpha}{\beta} - \frac{B}{\alpha} \right) \sin \frac{m\pi y}{\alpha} - 2 \cos \frac{m\pi y}{\alpha} \right\} \frac{A_m}{\beta} + \left\{ \left(\frac{\alpha}{\beta} - \frac{B}{\alpha} \right) \cos \frac{m\pi y}{\alpha} + 2 \sin \frac{m\pi y}{\alpha} \right\} \frac{B_m}{\alpha} \right] e^{\frac{-m\pi y}{\beta}} \right. \right. \\ \left. \left. + \left[\left\{ \left(\frac{\alpha}{\beta} - \frac{B}{\alpha} \right) \sin \frac{m\pi y}{\alpha} + 2 \cos \frac{m\pi y}{\alpha} \right\} \frac{C_m}{\beta} + \left\{ \left(\frac{\alpha}{\beta} - \frac{B}{\alpha} \right) \cos \frac{m\pi y}{\alpha} - 2 \sin \frac{m\pi y}{\alpha} \right\} \frac{D_m}{\alpha} \right] e^{\frac{m\pi y}{\beta}} \right] \cdot \sin \frac{m\pi x}{a} \right)$

Table 4 Second Order Derivatives of Function w

F_{xy}	F''_{xy}	EXPRESSION
	$\frac{\partial^2 w}{\partial x^2}$	$\frac{P}{2a(D_x D_y)^{1/2}} \sum_{m=1}^{\infty} \frac{1}{m} \left[\left\{ \alpha(A_m+1) \sin \frac{m\gamma}{\alpha} + B(B_m+1) \cos \frac{m\gamma}{\alpha} \right\} e^{\frac{-m\gamma}{\beta}} + \left\{ \alpha C_m \sin \frac{m\gamma}{\alpha} + B D_m \cos \frac{m\gamma}{\alpha} \right\} e^{\frac{+m\gamma}{\beta}} \right] \sin \frac{m\pi x}{a} \sin \frac{m\pi y}{a}$
w	$\frac{\partial^2 w}{\partial y^2}$	$\frac{Pa}{2\pi^2(D_x D_y)^{1/2}} \sum_{m=1}^{\infty} \frac{1}{m} \left[\left\{ -\left(\frac{p}{\beta} + \frac{q}{\alpha}\right) \sin \frac{m\gamma}{\alpha} + \left(\frac{p}{\alpha} - \frac{q}{\beta}\right) \cos \frac{m\gamma}{\alpha} \right\} e^{\frac{-m\gamma}{\beta}} + \left\{ \left(\frac{x}{\beta} - \frac{s}{\alpha}\right) \sin \frac{m\gamma}{\alpha} + \left(\frac{x}{\alpha} + \frac{s}{\beta}\right) \cos \frac{m\gamma}{\alpha} \right\} e^{\frac{+m\gamma}{\beta}} \right] \sin \frac{m\pi x}{a} \sin \frac{m\pi y}{a}$
	$\frac{\partial^2 w}{\partial x \partial y}$	$\frac{P}{2\pi(D_x D_y)^{1/2}} \sum_{m=1}^{\infty} \left[\left\{ p \sin \frac{m\gamma}{\alpha} + q \cos \frac{m\gamma}{\alpha} \right\} e^{\frac{-m\gamma}{\beta}} + \left\{ x \sin \frac{m\gamma}{\alpha} + s \cos \frac{m\gamma}{\alpha} \right\} e^{\frac{+m\gamma}{\beta}} \right] \sin \frac{m\pi x}{a} \sin \frac{m\pi y}{a}$

Table 5 First, Second and Third Order Derivatives of Functions $y_{mn}(y)$ for Case 1, 2, 3, and 4

Case	Function	Expression
1	$y_{mn}(y)$	$\left\{ A_{mn} \cosh \psi_{mn} \frac{y}{b} + B_{mn} \cos \phi_{mn} \frac{y}{b} \right\}$
	$y'_{mn}(y)$	$\left\{ \frac{\psi_{mn}}{b} A_{mn} \sinh \psi_{mn} \frac{y}{b} - \frac{\phi_{mn}}{b} B_{mn} \sin \phi_{mn} \frac{y}{b} \right\}$
	$y''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^2}{b^2} A_{mn} \cosh \psi_{mn} \frac{y}{b} - \frac{\phi_{mn}^2}{b^2} B_{mn} \cos \phi_{mn} \frac{y}{b} \right\}$
	$y'''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^3}{b^3} A_{mn} \sinh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}^3}{b^3} B_{mn} \sin \phi_{mn} \frac{y}{b} \right\}$
2	$y_{mn}(y)$	$\left\{ C_{mn} \sinh \psi_{mn} \frac{y}{b} + D_{mn} \sin \phi_{mn} \frac{y}{b} \right\}$
	$y'_{mn}(y)$	$\left\{ \frac{\psi_{mn}}{b} C_{mn} \cosh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}}{b} D_{mn} \cos \phi_{mn} \frac{y}{b} \right\}$
	$y''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^2}{b^2} C_{mn} \sinh \psi_{mn} \frac{y}{b} - \frac{\phi_{mn}^2}{b^2} D_{mn} \sin \phi_{mn} \frac{y}{b} \right\}$
	$y'''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^3}{b^3} C_{mn} \cosh \psi_{mn} \frac{y}{b} - \frac{\phi_{mn}^3}{b^3} D_{mn} \cos \phi_{mn} \frac{y}{b} \right\}$
3	$y_{mn}(y)$	$\left\{ A_{mn} \cosh \psi_{mn} \frac{y}{b} + B_{mn} \cosh \phi_{mn} \frac{y}{b} \right\}$
	$y'_{mn}(y)$	$\left\{ \frac{\psi_{mn}}{b} A_{mn} \sinh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}}{b} B_{mn} \sinh \phi_{mn} \frac{y}{b} \right\}$
	$y''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^2}{b^2} A_{mn} \cosh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}^2}{b^2} B_{mn} \cosh \phi_{mn} \frac{y}{b} \right\}$
	$y'''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^3}{b^3} A_{mn} \sinh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}^3}{b^3} B_{mn} \sinh \phi_{mn} \frac{y}{b} \right\}$
4	$y_{mn}(y)$	$\left\{ C_{mn} \sinh \psi_{mn} \frac{y}{b} + D_{mn} \sinh \phi_{mn} \frac{y}{b} \right\}$
	$y'_{mn}(y)$	$\left\{ \frac{\psi_{mn}}{b} C_{mn} \cosh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}}{b} D_{mn} \cosh \phi_{mn} \frac{y}{b} \right\}$
	$y''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^2}{b^2} C_{mn} \sinh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}^2}{b^2} D_{mn} \sinh \phi_{mn} \frac{y}{b} \right\}$
	$y'''_{mn}(y)$	$\left\{ \frac{\psi_{mn}^3}{b^3} C_{mn} \cosh \psi_{mn} \frac{y}{b} + \frac{\phi_{mn}^3}{b^3} D_{mn} \cosh \phi_{mn} \frac{y}{b} \right\}$

Table 6 Equations for Determining Constants $A_{mn}, \dots, D_{mn}$ for Cases 1 to 4

Case	Equations
1	$A_{mn} \left[\cosh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} - B_{mn} \left[\cosh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^2}{b^2} + \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0 \right.$ $A_{mn} \left[\sinh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^3}{b^3} - \frac{2H_1 - D_1}{D_y} \cdot \frac{\psi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] + B_{mn} \left[\sinh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^3}{b^3} + \frac{2H_1 - D_1}{D_y} \cdot \frac{\phi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0$
2	$C_{mn} \left[\sinh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} \right] - D_{mn} \left[\sinh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^2}{b^2} + \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0$ $C_{mn} \left[\cosh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^3}{b^3} - \frac{2H_1 - D_1}{D_y} \cdot \frac{\psi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] - D_{mn} \left[\cosh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^3}{b^3} + \frac{2H_1 - D_1}{D_y} \cdot \frac{\phi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0$
3	$A_{mn} \left[\cosh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} \right] + B_{mn} \left[\cosh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0$ $A_{mn} \left[\sinh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^3}{b^3} - \frac{2H_1 - D_1}{D_y} \cdot \frac{\psi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] + B_{mn} \left[\sinh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^3}{b^3} - \frac{2H_1 - D_1}{D_y} \cdot \frac{\phi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0$
4	$C_{mn} \left[\sinh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} \right] + D_{mn} \left[\sinh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^2}{b^2} - \frac{D_1}{D_y} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0$ $C_{mn} \left[\cosh \psi_{mn} \frac{y}{b} \left\{ \frac{\psi_{mn}^3}{b^3} - \frac{2H_1 - D_1}{D_y} \cdot \frac{\psi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] + D_{mn} \left[\cosh \phi_{mn} \frac{y}{b} \left\{ \frac{\phi_{mn}^3}{b^3} - \frac{2H_1 - D_1}{D_y} \cdot \frac{\phi_{mn}}{b} \left(\frac{m\pi}{a} \right)^2 \right\} \right] = 0$

Table 7. Plate Eigen Functions $y_{mn}(y)$ for Cases 1 to 4

Case	Functions
1	$\frac{y_{mn}(y)}{A_{mn}} = \frac{\cosh \psi_{mn} \frac{y}{b}}{\cosh \frac{\psi_{mn}}{2}} + \left[\left(\frac{\psi_{mn}}{c} \right)^2 - \frac{D_1/D_2}{(D_1/D_2)^{1/2}} \right] \frac{\cos \phi_{mn} \frac{y}{b}}{\cos \frac{\phi_{mn}}{2}}$
2	$\frac{y_{mn}(y)}{C_{mn}} = \frac{\sinh \psi_{mn} \frac{y}{b}}{\sinh \frac{\psi_{mn}}{2}} + \left[\left(\frac{\psi_{mn}}{c} \right)^2 - \frac{D_1/D_2}{(D_1/D_2)^{1/2}} \right] \frac{\sinh \phi_{mn} \frac{y}{b}}{\sinh \frac{\phi_{mn}}{2}}$
3	$\frac{y_{mn}(y)}{A_{mn}} = \frac{\cosh \psi_{mn} \frac{y}{b}}{\cosh \frac{\psi_{mn}}{2}} - \left[\left(\frac{\psi_{mn}}{c} \right)^2 - \frac{D_1/D_2}{(D_1/D_2)^{1/2}} \right] \frac{\cosh \phi_{mn} \frac{y}{b}}{\cosh \frac{\phi_{mn}}{2}}$
4	$\frac{y_{mn}(y)}{C_{mn}} = \frac{\sinh \psi_{mn} \frac{y}{b}}{\sinh \frac{\psi_{mn}}{2}} - \left[\left(\frac{\psi_{mn}}{c} \right)^2 - \frac{D_1/D_2}{(D_1/D_2)^{1/2}} \right] \frac{\sinh \phi_{mn} \frac{y}{b}}{\sinh \frac{\phi_{mn}}{2}}$

Table 8 Table of Results - Transcendental Frequency Equations for Symmetric and Antisymmetric modes for Cases Poisson's Ratio $\nu = 0$ and $\nu = 0$

<u>Case</u>	<u>Equations</u>
1	$\frac{\tan \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = - \left(\frac{\phi_{mn}}{\psi_{mn}} \right)^3$
2	$\frac{\tan \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = \left(\frac{\psi_{mn}}{\phi_{mn}} \right)^3$
3	$\frac{\tanh \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = \left(\frac{\phi_{mn}}{\psi_{mn}} \right)^3$
4	$\frac{\tanh \frac{\phi_{mn}}{2}}{\tanh \frac{\psi_{mn}}{2}} = \left(\frac{\psi_{mn}}{\phi_{mn}} \right)^3$

Table 9 Table of Results - Plate Eigen Functions $y_{mn}(y)$ for Symmetric and Antisymmetric Modes for Cases Poisson's Ratio $\nu = 0$ and $\nu = 0.3$

Case	Functions
1	$\frac{y_{mn}(y)}{A_{mn}} = \left[\frac{\cosh \psi_{mn} \frac{y}{b}}{\cosh \frac{\psi_{mn}}{2}} + \left(\frac{\psi_{mn}}{\phi_{mn}} \right)^2 \frac{\cosh \phi_{mn} \frac{y}{b}}{\cosh \frac{\phi_{mn}}{2}} \right]$
2	$\frac{y_{mn}(y)}{C_{mn}} = \left[\frac{\sinh \psi_{mn} \frac{y}{b}}{\sinh \frac{\psi_{mn}}{2}} + \left(\frac{\psi_{mn}}{\phi_{mn}} \right)^2 \frac{\sinh \phi_{mn} \frac{y}{b}}{\sinh \frac{\phi_{mn}}{2}} \right]$
3	$\frac{y_{mn}(y)}{A_{mn}} = \left[\frac{\cosh \psi_{mn} \frac{y}{b}}{\cosh \frac{\psi_{mn}}{2}} - \left(\frac{\psi_{mn}}{\phi_{mn}} \right)^2 \frac{\cosh \phi_{mn} \frac{y}{b}}{\cosh \frac{\phi_{mn}}{2}} \right]$
4	$\frac{y_{mn}(y)}{C_{mn}} = \left[\frac{\sinh \psi_{mn} \frac{y}{b}}{\sinh \frac{\psi_{mn}}{2}} - \left(\frac{\psi_{mn}}{\phi_{mn}} \right)^2 \frac{\sinh \phi_{mn} \frac{y}{b}}{\sinh \frac{\phi_{mn}}{2}} \right]$

Table 10

Distribution of sprung and unsprung wts

<u>Make-Type</u>	<u>Gross Wt.</u> <u>lbs</u>	<u>Sprung Wt</u> <u>% of Total</u>	<u>Unsprung</u> <u>% of Total</u>	<u>Remarks</u>
6 Cylinder E100 F	4,300	85.30	14.70	105" WB Ford
6 Cylinder E100 F	4,300	85.30	14.70	123" WB Ford
6 Cylinder U100 F	4,300	81.40	18.60	92" WB Ford
6 Cylinder F100 F	4,550	83.50	16.50	131" WB Ford
6 Cylinder E100 F	4,800	83.80	16.20	123" WB Ford
6 Cylinder U100 F	4,900	83.00	17.00	92" WB Ford
6 Cylinder F100 F	5,200	82.50	17.50	131" WB Ford
6 Cylinder E200 F	5,250	86.60	13.40	105" WB Ford
6 Cylinder E200 F	5,250	86.60	13.40	123" WB Ford
V-8-F100 F	5,500	87.30	12.70	131" WB Ford
6 Cylinder E200 F	6,000	88.30	11.70	123" WB Ford
6 Cylinder E300 F	6,050	83.50	16.50	105" WB Ford
6 Cylinder E300 F	6,200	83.80	16.20	123" WB Ford
6 Cylinder F250 F	6,200	86.70	13.30	131" WB Ford
6 Cylinder F250 F	7,100	83.20	16.80	131" WB Ford
V-8-F250 F	8,100	88.50	11.50	131" WB Ford
6 Cylinder F350 F	8,300	84.10	15.90	135" WB Ford
6 Cylinder E300 F	8,300	87.30	12.70	123" WB Ford
6 Cylinder F350 F	10,000	86.70	13.30	135" WB Ford
6 Cylinder F350 F	10,000	86.70	13.30	159" WB Ford
Medium GMC	13,800	86.80	13.20	167" WB GMC
Heavy Duty GMC	50,580	82.90	11.10	235" WB GMC

Table 11 -

Bridge and vehicle parameters.

<u>Bridge</u>	<u>Vehicle</u>	<u>Common</u>
1) Weight or Mass - M_B	1) Weight or Mass - M_v	1) Crossing frequency $f_c = \pi \frac{v}{a}$
2) Width b and aspect ratio $A_R = \frac{b}{a}$	2) Speed - v ft / sec.	2) The time in seconds.
3) Longitudinal, lateral and torsional rigidities - D_x , D_y and H_1	3) Natural frequency - f_v	Vehicle is on span = a/v 3) Mass ratio - $M_R = \frac{M_v}{M_B}$
4) Elasticity modulus E and poisson's ratio ν .	4) Initial conditions - ϕ , B	4) Frequency ratio = $f_R = \frac{f_B}{f_v}$
5) Thickness of slab - h dimensions and spacing of stiffener - t , $(H-h)$, s		
6) Fundamental and higher frequencies of vibrations - f_B , m , n .		
7) Initial velocity and deflections -		

Table 12 Dimensionless Form and Practical Range of Bridge and Vehicle Parameters in Static, Free and Forced Vibration Analysis

Static	Frequency	Forced Vibration
1. Aspect ratio $b/a = 1.0$ to 0.5 through an interval of 0.1.	1. Rigidity parameter $m.b/a.(D_x/D_y)^{1/4} = 0.25$ to 10.0 through an interval of 0.25 and 1.0.	1. Frequency ratio $f_B/f_Y = 0.4, 0.8, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0$.
2. Rigidity ratio $D_y/D_x = 0.01$ to 0.5 through an interval of 0.01.	2. Torsion parameter $\mu = 0.1$ to 0.9 through an interval of 10.1.	2. Mass ratio $M_y/M_B = 0.01, 0.03, 0.05, 0.08, 0.10$ and 0.2.
3. Torsion parameter: μ ($M_1/\sqrt{D_x D_y}) = 0.1$ to 0.9.	3. Cross mode $n = 2, 3, 4, 5, 6$.	3. Bridge frequency $f_B = 3\pi, 6\pi, 9\pi, 12\pi, 15\pi$, and 18π .
4. Poisson's ratio $= 0.3$ through an interval of 0.1.	4. Poisson's ratio parameter $(D_1/D_y)/\sqrt{(D_x/D_y)} = 0$ and 0.1.	4. Crossing frequency $f_C(\pi v/a) = 0.5\pi, \pi, 1.4\pi, 1.8\pi, 2.2\pi$, and 2.5π .
5. Load P.		5. Vehicle passes - 3.
		6. Vehicle conditions - $\beta = 0.3$ $\phi = 1.5\pi$.
Results	Results	Results
Deflection (Pa^4/abD_x) , moments M_x and M_y (Pa^2/ab) for combinations of parameters in 1 to 4 above.	Frequency ratio obtained for combinations of parameters in 1 to 4 above. Correction coefficients for Poisson's ratio are given.	Amplification factors for combinations of parameters 1 to 5 above.

Table 13. Comparison of frequency ratio R by theoretical and empirical method.

$$c = \frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4}, \quad \mu = H / \sqrt{D_x \cdot D_y}$$

$mc = 0.5$ to 10.0 , $n = 2$ to 6

Empirical method					Theoretical method			
$\mu =$					$\mu =$			
mc	0.1	0.3	0.5	0.9	0.1	0.3	0.5	0.9
$n = 6$								
0.50	81.3950	82.1157	82.7747	83.9560	81.1681	81.4396	81.7106	82.2874
0.75	36.3905	37.0380	37.5928	38.5364	36.1572	36.4361	36.7139	37.12668
1.00	20.6357	21.2104	21.6789	22.4562	20.4167	20.6956	20.9728	21.5133
2.00	5.4502	5.8210	6.1093	6.5940	5.2989	5.5619	5.8151	6.2828
3.00	2.6858	2.9533	3.1682	4.0083	2.5887	2.8239	3.0375	3.4186
5.00	1.4074	1.5620	2.1500	1.9312	1.3731	1.5274	1.6635	1.9003
10.00	1.0505	1.1003	1.1481	1.2393	1.0473	1.0964	1.1420	1.2274
$n = 5$								
0.50	49.3350	49.9400	50.4960	51.5030	49.1580	49.4695	49.7800	50.3710
0.75	22.1151	22.6630	23.1380	23.9560	21.9590	22.2568	22.5529	23.1410
1.00	12.5930	13.0830	13.4890	14.1740	12.4440	12.7452	13.0370	13.5997
2.00	3.4610	3.7820	4.0360	4.4680	3.3650	3.6328	3.8806	4.3272
3.00	1.8620	2.0840	2.2661	2.5820	1.8086	2.0238	2.2137	2.5426
5.00	1.1930	1.3040	1.4050	1.5860	1.1787	1.2943	1.3962	1.5757
10.00	1.0258	1.0590	1.0910	1.1540	1.0251	1.0570	1.0873	1.1446
$n = 4$								
0.50	25.3607	25.9955	26.5796	27.6342	25.1942	25.5326	25.8902	26.5417
0.75	11.4857	12.0588	12.5548	13.4055	11.3325	11.6709	11.9981	12.6284
1.00	6.6442	7.1534	7.5736	8.2751	6.5004	6.8272	7.1406	7.7227
2.00	2.0817	2.3876	1.6292	3.0333	1.9941	2.2502	2.4732	2.8566
3.00	1.3539	1.5389	1.6936	1.9632	1.3130	1.4795	1.6236	1.8709
5.00	1.0816	1.1613	1.2351	1.3753	1.0712	1.1426	1.2070	1.3239
10.00	1.0136	1.0360	1.0589	1.1051	1.0119	1.0299	1.0413	1.0814
$n = 3$								
0.50	9.2980	9.7550	10.1800	10.9590	9.3427	9.7558	10.1556	10.9029
0.75	4.3540	4.7650	5.1290	5.7650	4.3588	4.7492	5.1062	5.7457
1.00	2.6740	3.0320	3.3370	3.8610	2.6721	3.0169	3.3230	3.8583
2.00	1.2540	1.4290	1.5820	1.8540	1.2520	1.4287	1.5799	1.8366
3.00	1.0820	1.1730	1.2600	1.4220	1.0822	1.1715	1.2509	1.3914
5.00	1.0230	1.0600	1.0980	1.1740	1.0225	1.0551	1.0858	1.1440
10.00	1.0050	1.0160	1.0236	1.0521	1.0046	1.0125	1.0203	1.0361
$n = 2$								
0.50	1.4577	1.8520	2.1485	2.7316	1.5043	1.9737	2.4110	3.0973
0.75	1.1837	1.3949	1.6250	2.0921	1.1959	1.5094	1.7635	2.1789
1.00	1.0898	1.2489	1.4323	1.8929	1.1141	1.3082	1.4735	1.7498
2.00	1.0249	1.1004	1.1929	1.3919	1.0291	1.0816	1.1298	1.2199
3.00	1.0140	1.0609	1.1180	1.2433	1.0126	1.0357	1.0575	1.1254
5.00	1.0073	1.0315	1.0606	1.1221	1.0044	1.0142	1.0208	1.0390
10.00	1.0029	1.0120	1.0230	1.0478	1.0012	1.0039	1.0053	1.0091

Table 14(a) Deflection Coefficients $\frac{Pa^4}{abD} \times 10^{-1}$

$\frac{D_y}{D_x}$	Torsion Parameter μ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN) = 1.0									
0.01	0.723	0.691	0.664	0.639	0.617	0.598	0.580	0.564	0.549
0.02	0.614	0.585	0.560	0.539	0.520	0.504	0.489	0.475	0.463
0.03	0.560	0.532	0.508	0.488	0.471	0.456	0.443	0.431	0.421
0.04	0.525	0.497	0.475	0.456	0.440	0.426	0.414	0.403	0.394
0.05	0.500	0.472	0.450	0.432	0.417	0.404	0.393	0.383	0.374
0.06	0.481	0.453	0.431	0.414	0.400	0.388	0.377	0.368	0.360
0.07	0.464	0.437	0.416	0.399	0.386	0.374	0.364	0.356	0.348
0.08	0.451	0.424	0.403	0.387	0.374	0.363	0.354	0.346	0.339
0.09	0.439	0.412	0.392	0.377	0.364	0.354	0.345	0.337	0.331
0.10	0.428	0.402	0.383	0.368	0.356	0.346	0.337	0.330	0.324
0.12	0.410	0.385	0.367	0.353	0.342	0.333	0.325	0.318	0.312
0.14	0.395	0.371	0.354	0.341	0.331	0.322	0.315	0.309	0.303
0.16	0.383	0.360	0.343	0.331	0.321	0.313	0.307	0.301	0.296
0.18	0.371	0.350	0.334	0.323	0.313	0.306	0.300	0.295	0.290
0.20	0.362	0.341	0.326	0.315	0.307	0.300	0.294	0.289	0.285
ASPECT RATIO (WIDTH/SPAN) = 0.9									
0.01	0.653	0.624	0.598	0.576	0.556	0.538	0.522	0.508	0.495
0.02	0.558	0.529	0.506	0.486	0.469	0.455	0.441	0.430	0.419
0.03	0.510	0.482	0.460	0.441	0.426	0.413	0.401	0.391	0.382
0.04	0.478	0.451	0.430	0.413	0.398	0.386	0.376	0.367	0.359
0.05	0.455	0.428	0.408	0.392	0.378	0.367	0.358	0.349	0.342
0.06	0.436	0.410	0.391	0.376	0.363	0.353	0.344	0.336	0.330
0.07	0.421	0.396	0.377	0.363	0.351	0.341	0.333	0.326	0.320
0.08	0.408	0.384	0.366	0.352	0.341	0.332	0.324	0.317	0.312
0.09	0.397	0.373	0.356	0.343	0.332	0.324	0.317	0.310	0.305
0.10	0.387	0.364	0.348	0.335	0.325	0.317	0.310	0.304	0.299
0.12	0.370	0.349	0.334	0.322	0.313	0.306	0.300	0.294	0.290
0.14	0.356	0.336	0.322	0.312	0.304	0.297	0.291	0.286	0.282
0.16	0.344	0.326	0.313	0.303	0.296	0.290	0.284	0.280	0.276
0.18	0.334	0.317	0.305	0.296	0.289	0.283	0.279	0.275	0.271
0.20	0.325	0.309	0.298	0.290	0.283	0.278	0.274	0.270	0.267

Table 14(b) Deflection Coefficients $\frac{Pa^4}{abD_x} \times 10^{-1}$									
Torsion Parameter μ									
$\frac{D_y}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN)= 0.8									
0.01	0.586	0.558	0.534	0.513	0.495	0.480	0.466	0.453	0.442
0.02	0.502	0.474	0.453	0.435	0.420	0.407	0.395	0.385	0.377
0.03	0.458	0.432	0.412	0.395	0.382	0.371	0.361	0.353	0.345
0.04	0.429	0.404	0.385	0.370	0.358	0.348	0.340	0.332	0.326
0.05	0.407	0.384	0.366	0.352	0.341	0.332	0.324	0.318	0.312
0.06	0.390	0.368	0.351	0.339	0.328	0.320	0.313	0.307	0.302
0.07	0.376	0.355	0.339	0.327	0.318	0.310	0.304	0.298	0.294
0.08	0.364	0.344	0.329	0.318	0.310	0.303	0.297	0.291	0.287
0.09	0.353	0.334	0.321	0.311	0.303	0.296	0.290	0.286	0.281
0.10	0.344	0.326	0.314	0.304	0.297	0.290	0.285	0.281	0.277
0.12	0.329	0.313	0.302	0.293	0.287	0.281	0.276	0.273	0.269
0.14	0.316	0.302	0.292	0.285	0.279	0.274	0.270	0.266	0.263
0.16	0.306	0.294	0.285	0.278	0.272	0.268	0.264	0.261	0.258
0.18	0.297	0.286	0.278	0.272	0.267	0.263	0.260	0.257	0.254
0.20	0.290	0.280	0.273	0.267	0.263	0.259	0.256	0.253	0.251
ASPECT RATIO (WIDTH/SPAN)= 0.7									
0.01	0.520	0.492	0.470	0.451	0.436	0.422	0.410	0.400	0.390
0.02	0.445	0.419	0.400	0.384	0.371	0.361	0.351	0.343	0.336
0.03	0.405	0.382	0.364	0.351	0.340	0.331	0.323	0.317	0.311
0.04	0.378	0.357	0.342	0.330	0.320	0.312	0.306	0.300	0.295
0.05	0.358	0.339	0.325	0.315	0.306	0.299	0.294	0.289	0.284
0.06	0.342	0.325	0.313	0.304	0.296	0.290	0.285	0.280	0.276
0.07	0.330	0.314	0.303	0.295	0.288	0.282	0.277	0.273	0.270
0.08	0.319	0.305	0.295	0.287	0.281	0.276	0.272	0.268	0.265
0.09	0.310	0.298	0.288	0.281	0.275	0.271	0.267	0.263	0.260
0.10	0.303	0.291	0.282	0.276	0.271	0.266	0.263	0.260	0.257
0.12	0.290	0.280	0.273	0.267	0.263	0.259	0.256	0.253	0.251
0.14	0.280	0.272	0.266	0.261	0.257	0.254	0.251	0.249	0.247
0.16	0.272	0.265	0.260	0.255	0.252	0.249	0.247	0.245	0.243
0.18	0.266	0.259	0.255	0.251	0.248	0.245	0.243	0.241	0.240
0.20	0.260	0.255	0.251	0.247	0.245	0.242	0.240	0.239	0.237

Table 14(c) Deflection Coefficients $\frac{Pa^4}{abD_x} \times 10^{-1}$									
Torsion Parameter μ									
$\frac{D_y}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN)= 0.6									
0.01	0.452	0.427	0.407	0.391	0.378	0.367	0.357	0.349	0.342
0.02	0.385	0.364	0.348	0.336	0.326	0.318	0.311	0.305	0.300
0.03	0.349	0.332	0.319	0.309	0.301	0.294	0.289	0.284	0.280
0.04	0.326	0.311	0.301	0.292	0.286	0.280	0.276	0.272	0.268
0.05	0.309	0.297	0.288	0.281	0.275	0.270	0.266	0.263	0.260
0.06	0.297	0.286	0.278	0.272	0.267	0.263	0.260	0.257	0.254
0.07	0.287	0.278	0.271	0.265	0.261	0.257	0.254	0.252	0.250
0.08	0.279	0.271	0.265	0.260	0.256	0.253	0.250	0.248	0.246
0.09	0.272	0.265	0.260	0.255	0.252	0.249	0.247	0.245	0.243
0.10	0.266	0.260	0.255	0.252	0.248	0.246	0.244	0.242	0.240
0.12	0.258	0.253	0.249	0.246	0.243	0.241	0.239	0.237	0.236
0.14	0.251	0.247	0.243	0.241	0.239	0.237	0.235	0.234	0.233
0.16	0.246	0.242	0.239	0.237	0.235	0.234	0.232	0.231	0.230
0.18	0.241	0.239	0.236	0.234	0.233	0.231	0.230	0.229	0.228
0.20	0.238	0.236	0.234	0.232	0.230	0.229	0.228	0.227	0.226
ASPECT RATIO (WIDTH/SPAN)= 0.5									
0.01	0.381	0.361	0.345	0.333	0.324	0.316	0.309	0.303	0.298
0.02	0.324	0.310	0.299	0.291	0.285	0.279	0.275	0.271	0.267
0.03	0.296	0.285	0.278	0.272	0.267	0.263	0.259	0.256	0.254
0.04	0.278	0.270	0.264	0.260	0.256	0.253	0.250	0.248	0.246
0.05	0.267	0.260	0.256	0.252	0.249	0.246	0.244	0.242	0.240
0.06	0.258	0.253	0.249	0.246	0.243	0.241	0.239	0.237	0.236
0.07	0.252	0.248	0.244	0.241	0.239	0.237	0.236	0.234	0.233
0.08	0.247	0.243	0.240	0.238	0.236	0.234	0.233	0.232	0.230
0.09	0.243	0.240	0.237	0.235	0.233	0.232	0.231	0.229	0.228
0.10	0.239	0.237	0.234	0.233	0.231	0.230	0.229	0.228	0.227
0.12	0.234	0.232	0.230	0.229	0.228	0.227	0.226	0.225	0.224
0.14	0.231	0.229	0.227	0.226	0.225	0.224	0.223	0.223	0.222
0.16	0.228	0.226	0.225	0.224	0.223	0.222	0.222	0.221	0.220
0.18	0.226	0.225	0.223	0.222	0.222	0.221	0.220	0.220	0.219
0.20	0.225	0.223	0.222	0.221	0.220	0.220	0.219	0.219	0.218

Table 15(a) Moment Coefficients $M_{ab} = \frac{Pa^2}{x^2}$									
Torsion Parameter μ									
$\frac{D_v}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN) = 1.0									
0.01	1.174	1.123	1.079	1.039	1.004	0.972	0.943	0.916	0.892
0.02	1.005	0.959	0.920	0.886	0.855	0.828	0.803	0.781	0.761
0.03	0.922	0.878	0.841	0.809	0.781	0.756	0.734	0.714	0.696
0.04	0.869	0.826	0.790	0.760	0.734	0.710	0.690	0.671	0.655
0.05	0.830	0.788	0.754	0.725	0.699	0.678	0.658	0.641	0.625
0.06	0.801	0.759	0.726	0.697	0.673	0.652	0.634	0.617	0.603
0.07	0.777	0.736	0.703	0.676	0.653	0.632	0.615	0.599	0.585
0.08	0.757	0.716	0.684	0.658	0.635	0.616	0.599	0.584	0.570
0.09	0.740	0.700	0.668	0.642	0.621	0.602	0.585	0.571	0.558
0.10	0.725	0.685	0.655	0.629	0.608	0.590	0.574	0.560	0.547
0.12	0.700	0.661	0.632	0.608	0.587	0.570	0.555	0.542	0.530
0.14	0.680	0.642	0.613	0.590	0.571	0.554	0.540	0.527	0.516
0.16	0.662	0.626	0.598	0.576	0.557	0.542	0.528	0.516	0.505
0.18	0.648	0.612	0.585	0.564	0.546	0.531	0.518	0.506	0.495
0.20	0.635	0.600	0.574	0.553	0.536	0.522	0.509	0.498	0.487
ASPECT RATIO (WIDTH/SPAN) = 0.9									
0.01	1.059	1.013	0.972	0.936	0.904	0.875	0.849	0.825	0.804
0.02	0.910	0.867	0.830	0.799	0.771	0.746	0.725	0.705	0.687
0.03	0.835	0.794	0.759	0.730	0.705	0.683	0.663	0.645	0.630
0.04	0.787	0.747	0.714	0.686	0.663	0.642	0.624	0.608	0.593
0.05	0.753	0.713	0.681	0.655	0.632	0.613	0.596	0.581	0.567
0.06	0.725	0.686	0.656	0.631	0.609	0.591	0.575	0.561	0.548
0.07	0.703	0.665	0.636	0.611	0.591	0.573	0.558	0.545	0.532
0.08	0.685	0.647	0.619	0.595	0.576	0.559	0.544	0.531	0.520
0.09	0.669	0.632	0.604	0.582	0.563	0.547	0.533	0.520	0.509
0.10	0.655	0.619	0.592	0.570	0.552	0.536	0.523	0.511	0.500
0.12	0.631	0.597	0.572	0.551	0.534	0.519	0.506	0.495	0.485
0.14	0.612	0.580	0.555	0.536	0.519	0.506	0.493	0.483	0.473
0.16	0.596	0.565	0.542	0.523	0.508	0.494	0.483	0.473	0.463
0.18	0.582	0.553	0.530	0.513	0.498	0.485	0.474	0.464	0.455
0.20	0.570	0.542	0.521	0.503	0.489	0.477	0.466	0.457	0.448

Table 15(b) Moment Coefficients $M_x = \frac{Pa^2}{ab}$									
Torsion Parameter μ									
$\frac{D_y}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN)= 0.8									
0.01	0.947	0.903	0.866	0.833	0.804	0.779	0.756	0.735	0.716
0.02	0.815	0.774	0.741	0.712	0.688	0.666	0.647	0.630	0.615
0.03	0.748	0.709	0.678	0.652	0.630	0.610	0.594	0.578	0.565
0.04	0.705	0.667	0.638	0.613	0.593	0.575	0.560	0.546	0.534
0.05	0.673	0.637	0.609	0.586	0.567	0.550	0.536	0.523	0.512
0.06	0.648	0.613	0.587	0.565	0.547	0.531	0.518	0.506	0.495
0.07	0.627	0.594	0.569	0.548	0.531	0.516	0.504	0.492	0.482
0.08	0.610	0.578	0.554	0.534	0.518	0.504	0.492	0.481	0.471
0.09	0.595	0.565	0.542	0.523	0.507	0.494	0.482	0.471	0.462
0.10	0.582	0.553	0.531	0.513	0.498	0.485	0.473	0.463	0.454
0.12	0.561	0.534	0.513	0.496	0.482	0.470	0.460	0.450	0.442
0.14	0.543	0.518	0.499	0.483	0.470	0.459	0.449	0.440	0.432
0.16	0.529	0.505	0.487	0.472	0.460	0.449	0.440	0.431	0.424
0.18	0.517	0.495	0.477	0.463	0.451	0.441	0.432	0.424	0.417
0.20	0.506	0.485	0.469	0.456	0.444	0.434	0.426	0.418	0.411
ASPECT RATIO (WIDTH/SPAN)= 0.7									
0.01	0.835	0.794	0.760	0.731	0.706	0.684	0.664	0.646	0.630
0.02	0.719	0.682	0.652	0.627	0.606	0.587	0.571	0.557	0.544
0.03	0.659	0.624	0.597	0.575	0.556	0.540	0.526	0.514	0.503
0.04	0.619	0.587	0.563	0.542	0.525	0.511	0.498	0.487	0.477
0.05	0.590	0.561	0.538	0.519	0.503	0.490	0.478	0.468	0.459
0.06	0.568	0.540	0.519	0.501	0.487	0.474	0.463	0.454	0.445
0.07	0.550	0.524	0.504	0.487	0.473	0.462	0.451	0.442	0.434
0.08	0.534	0.510	0.491	0.476	0.463	0.451	0.442	0.433	0.425
0.09	0.521	0.498	0.481	0.466	0.454	0.443	0.434	0.425	0.418
0.10	0.510	0.489	0.471	0.458	0.446	0.436	0.427	0.419	0.411
0.12	0.492	0.472	0.457	0.444	0.433	0.424	0.415	0.408	0.401
0.14	0.477	0.459	0.445	0.433	0.423	0.414	0.406	0.399	0.393
0.16	0.465	0.449	0.436	0.424	0.415	0.406	0.399	0.392	0.386
0.18	0.456	0.440	0.428	0.417	0.408	0.400	0.393	0.386	0.381
0.20	0.447	0.433	0.421	0.411	0.402	0.395	0.388	0.381	0.376

Table 15(c) Moment Coefficients $M_x = \frac{Pa^2}{ab}$										
Torsion Parameter μ										
$\frac{D_y}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
ASPECT RATIO (WIDTH/SPAN)= 0.6										
0.01	0.723	0.686	0.656	0.631	0.609	0.591	0.575	0.560	0.547	
0.02	0.621	0.589	0.564	0.543	0.526	0.512	0.499	0.487	0.477	
0.03	0.567	0.540	0.518	0.501	0.486	0.473	0.462	0.453	0.444	
0.04	0.533	0.509	0.490	0.474	0.461	0.450	0.440	0.431	0.423	
0.05	0.508	0.486	0.469	0.455	0.443	0.433	0.424	0.416	0.409	
0.06	0.489	0.469	0.454	0.441	0.430	0.420	0.412	0.404	0.398	
0.07	0.474	0.456	0.442	0.430	0.419	0.410	0.403	0.395	0.389	
0.08	0.462	0.445	0.432	0.420	0.411	0.402	0.395	0.388	0.382	
0.09	0.451	0.436	0.423	0.413	0.404	0.396	0.388	0.382	0.376	
0.10	0.443	0.428	0.416	0.406	0.397	0.390	0.383	0.377	0.371	
0.12	0.429	0.416	0.405	0.396	0.388	0.380	0.374	0.368	0.363	
0.14	0.418	0.406	0.396	0.387	0.380	0.373	0.367	0.361	0.356	
0.16	0.410	0.399	0.389	0.381	0.374	0.367	0.361	0.356	0.351	
0.18	0.403	0.393	0.383	0.375	0.368	0.362	0.356	0.351	0.347	
0.20	0.398	0.387	0.379	0.371	0.364	0.358	0.352	0.347	0.343	
ASPECT RATIO (WIDTH/SPAN)= 0.5										
0.01	0.608	0.577	0.553	0.533	0.516	0.502	0.490	0.479	0.469	
0.02	0.520	0.497	0.479	0.464	0.451	0.440	0.431	0.422	0.415	
0.03	0.477	0.458	0.443	0.431	0.420	0.411	0.403	0.396	0.389	
0.04	0.450	0.434	0.421	0.410	0.401	0.393	0.386	0.379	0.373	
0.05	0.431	0.417	0.406	0.396	0.388	0.380	0.374	0.368	0.362	
0.06	0.417	0.405	0.395	0.386	0.378	0.371	0.365	0.359	0.354	
0.07	0.407	0.395	0.386	0.377	0.370	0.363	0.358	0.352	0.347	
0.08	0.398	0.388	0.379	0.371	0.364	0.357	0.352	0.347	0.342	
0.09	0.392	0.382	0.373	0.365	0.358	0.352	0.347	0.342	0.338	
0.10	0.386	0.376	0.368	0.361	0.354	0.348	0.343	0.338	0.334	
0.12	0.378	0.368	0.360	0.353	0.347	0.341	0.336	0.332	0.327	
0.14	0.372	0.363	0.354	0.347	0.341	0.336	0.331	0.327	0.323	
0.16	0.368	0.358	0.350	0.343	0.337	0.332	0.327	0.322	0.319	
0.18	0.366	0.355	0.347	0.339	0.333	0.328	0.323	0.319	0.315	
0.20	0.364	0.353	0.344	0.337	0.330	0.325	0.320	0.316	0.312	

Table 16(a) Moment Coefficients $M_y = \frac{Pa^2}{ab}$

Torsion Parameter μ									
$\frac{D}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN)= 1.0									
0.01	0.117	0.112	0.108	0.104	0.100	0.097	0.094	0.092	0.089
0.02	0.141	0.135	0.130	0.125	0.121	0.117	0.113	0.110	0.107
0.03	0.158	0.151	0.145	0.140	0.135	0.130	0.126	0.123	0.119
0.04	0.171	0.164	0.157	0.151	0.146	0.141	0.137	0.133	0.129
0.05	0.183	0.175	0.167	0.161	0.155	0.150	0.145	0.141	0.137
0.06	0.194	0.184	0.176	0.169	0.163	0.158	0.153	0.148	0.144
0.07	0.203	0.193	0.184	0.177	0.170	0.165	0.159	0.155	0.150
0.08	0.212	0.201	0.192	0.184	0.177	0.171	0.166	0.161	0.156
0.09	0.220	0.208	0.198	0.190	0.183	0.177	0.171	0.166	0.161
0.10	0.228	0.215	0.205	0.196	0.189	0.182	0.176	0.171	0.166
0.12	0.241	0.227	0.216	0.207	0.199	0.192	0.186	0.180	0.175
0.14	0.254	0.238	0.226	0.216	0.208	0.201	0.194	0.188	0.183
0.16	0.265	0.249	0.236	0.225	0.216	0.208	0.202	0.196	0.190
0.18	0.275	0.258	0.244	0.233	0.224	0.216	0.209	0.203	0.197
0.20	0.285	0.266	0.252	0.240	0.231	0.223	0.215	0.209	0.203
ASPECT RATIO (WIDTH/SPAN)= 0.9									
0.01	0.105	0.101	0.097	0.093	0.090	0.087	0.085	0.082	0.080
0.02	0.127	0.122	0.117	0.112	0.109	0.105	0.102	0.099	0.096
0.03	0.143	0.136	0.131	0.126	0.121	0.117	0.113	0.110	0.107
0.04	0.155	0.148	0.141	0.136	0.131	0.127	0.123	0.119	0.116
0.05	0.166	0.158	0.151	0.145	0.139	0.135	0.130	0.126	0.123
0.06	0.176	0.167	0.159	0.152	0.146	0.141	0.137	0.133	0.129
0.07	0.185	0.174	0.166	0.159	0.153	0.148	0.143	0.139	0.135
0.08	0.193	0.181	0.173	0.165	0.159	0.153	0.148	0.144	0.140
0.09	0.200	0.188	0.179	0.171	0.164	0.158	0.153	0.149	0.144
0.10	0.206	0.194	0.184	0.176	0.169	0.163	0.158	0.153	0.149
0.12	0.219	0.205	0.194	0.185	0.178	0.172	0.166	0.161	0.157
0.14	0.229	0.214	0.203	0.194	0.186	0.179	0.173	0.168	0.164
0.16	0.239	0.223	0.211	0.201	0.193	0.186	0.180	0.175	0.170
0.18	0.248	0.231	0.218	0.208	0.200	0.193	0.186	0.181	0.176
0.20	0.255	0.238	0.225	0.215	0.206	0.199	0.192	0.186	0.181

Table 16(b) Moment Coefficients $M_y - \frac{Pa^2}{ab}$									
Torsion Parameter μ									
$\frac{D_y}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN)= 0.8									
0.01	0.094	0.090	0.086	0.083	0.080	0.078	0.075	0.073	0.071
0.02	0.114	0.108	0.104	0.100	0.096	0.093	0.090	0.088	0.085
0.03	0.128	0.122	0.116	0.111	0.107	0.104	0.100	0.097	0.095
0.04	0.140	0.132	0.126	0.121	0.116	0.112	0.108	0.105	0.102
0.05	0.149	0.141	0.134	0.128	0.123	0.119	0.115	0.112	0.109
0.06	0.158	0.148	0.141	0.135	0.130	0.125	0.121	0.117	0.114
0.07	0.165	0.155	0.147	0.141	0.135	0.130	0.126	0.122	0.119
0.08	0.172	0.161	0.153	0.146	0.140	0.135	0.131	0.127	0.123
0.09	0.178	0.167	0.158	0.151	0.145	0.140	0.135	0.131	0.127
0.10	0.184	0.172	0.163	0.155	0.149	0.144	0.139	0.135	0.131
0.12	0.194	0.181	0.171	0.163	0.157	0.151	0.146	0.142	0.138
0.14	0.202	0.189	0.179	0.171	0.164	0.158	0.153	0.148	0.144
0.16	0.210	0.196	0.185	0.177	0.170	0.164	0.158	0.154	0.149
0.18	0.217	0.202	0.192	0.183	0.175	0.169	0.164	0.159	0.155
0.20	0.223	0.208	0.197	0.188	0.181	0.174	0.169	0.164	0.159
ASPECT RATIO (WIDTH/SPAN)= 0.7									
0.01	0.082	0.079	0.075	0.073	0.070	0.068	0.066	0.064	0.062
0.02	0.100	0.095	0.091	0.087	0.084	0.081	0.079	0.076	0.074
0.03	0.113	0.107	0.102	0.097	0.094	0.090	0.087	0.085	0.082
0.04	0.123	0.116	0.110	0.105	0.101	0.097	0.094	0.091	0.089
0.05	0.131	0.123	0.117	0.112	0.107	0.103	0.100	0.097	0.094
0.06	0.138	0.130	0.123	0.117	0.112	0.108	0.105	0.102	0.099
0.07	0.145	0.135	0.128	0.122	0.117	0.113	0.109	0.106	0.103
0.08	0.150	0.140	0.133	0.126	0.121	0.117	0.113	0.110	0.107
0.09	0.155	0.145	0.137	0.130	0.125	0.121	0.117	0.113	0.110
0.10	0.159	0.149	0.141	0.134	0.129	0.124	0.120	0.116	0.113
0.12	0.166	0.156	0.147	0.141	0.135	0.130	0.126	0.122	0.119
0.14	0.173	0.162	0.153	0.146	0.141	0.136	0.131	0.128	0.124
0.16	0.178	0.167	0.159	0.152	0.146	0.141	0.136	0.132	0.129
0.18	0.183	0.172	0.163	0.156	0.150	0.145	0.141	0.137	0.133
0.20	0.188	0.176	0.168	0.161	0.155	0.149	0.145	0.141	0.137

Table 16(c) <u>Moment Coefficients</u> $M_y = \frac{Pa^2}{ab}$ <u>Torsion Parameter μ</u>									
$\frac{D_y}{D_x}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ASPECT RATIO (WIDTH/SPAN)= 0.6									
0.01	0.071	0.068	0.065	0.062	0.060	0.058	0.056	0.054	0.053
0.02	0.087	0.082	0.078	0.074	0.072	0.069	0.067	0.065	0.063
0.03	0.098	0.091	0.087	0.083	0.079	0.077	0.074	0.072	0.070
0.04	0.105	0.099	0.093	0.089	0.085	0.082	0.080	0.077	0.075
0.05	0.112	0.104	0.099	0.094	0.090	0.087	0.084	0.082	0.079
0.06	0.117	0.109	0.104	0.099	0.095	0.091	0.088	0.086	0.083
0.07	0.121	0.114	0.108	0.103	0.099	0.095	0.092	0.089	0.087
0.08	0.125	0.117	0.111	0.106	0.102	0.098	0.095	0.092	0.090
0.09	0.128	0.121	0.114	0.109	0.105	0.101	0.098	0.095	0.093
0.10	0.131	0.124	0.117	0.112	0.108	0.104	0.101	0.098	0.095
0.12	0.137	0.129	0.123	0.117	0.113	0.109	0.106	0.103	0.100
0.14	0.141	0.133	0.127	0.122	0.117	0.113	0.110	0.107	0.104
0.16	0.145	0.137	0.131	0.126	0.121	0.117	0.114	0.111	0.108
0.18	0.148	0.141	0.135	0.129	0.125	0.121	0.118	0.114	0.112
0.20	0.151	0.144	0.138	0.133	0.128	0.124	0.121	0.118	0.115
ASPECT RATIO (WIDTH/SPAN)= 0.5									
0.01	0.060	0.057	0.054	0.051	0.049	0.048	0.046	0.045	0.043
0.02	0.072	0.068	0.064	0.061	0.059	0.057	0.055	0.053	0.052
0.03	0.080	0.075	0.071	0.068	0.065	0.063	0.060	0.059	0.057
0.04	0.085	0.080	0.076	0.072	0.070	0.067	0.065	0.063	0.061
0.05	0.090	0.084	0.080	0.076	0.073	0.071	0.069	0.067	0.065
0.06	0.093	0.088	0.083	0.080	0.077	0.074	0.072	0.070	0.068
0.07	0.096	0.091	0.086	0.083	0.080	0.077	0.074	0.072	0.070
0.08	0.098	0.093	0.089	0.085	0.082	0.079	0.077	0.075	0.073
0.09	0.101	0.095	0.091	0.088	0.084	0.082	0.079	0.077	0.075
0.10	0.103	0.098	0.093	0.090	0.087	0.084	0.081	0.079	0.077
0.12	0.106	0.101	0.097	0.094	0.090	0.088	0.085	0.083	0.081
0.14	0.109	0.105	0.101	0.097	0.094	0.091	0.089	0.086	0.084
0.16	0.112	0.108	0.104	0.100	0.097	0.094	0.092	0.089	0.087
0.18	0.115	0.110	0.106	0.103	0.100	0.097	0.095	0.092	0.090
0.20	0.118	0.113	0.109	0.106	0.102	0.100	0.097	0.095	0.093

Table 17 Frequency Ratio - First Antisymmetric Mode $n = 2$

$c = \frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4}$ (multiply by $(m^2 \pi^2 / a^2) \sqrt{\frac{D_x}{D_0}}$, to get frequencies in rps)

Torsion Parameter μ

mc	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(D_x/D_y)/\sqrt{D_x/D_y} = 0$									
0.25	2.213	2.965	3.559	4.069	4.520	4.925	5.302	5.654	5.987
0.50	1.404	1.713	1.974	2.205	2.411	2.600	2.776	2.941	3.097
0.75	1.196	1.362	1.509	1.642	1.764	1.877	1.984	2.084	2.179
1.00	1.114	1.216	1.308	1.394	1.474	1.549	1.619	1.687	1.750
1.25	1.074	1.142	1.204	1.263	1.318	1.371	1.421	1.469	1.515
1.50	1.052	1.100	1.144	1.187	1.227	1.265	1.302	1.338	1.373
1.75	1.038	1.073	1.107	1.138	1.169	1.198	1.226	1.254	1.281
2.00	1.029	1.056	1.082	1.106	1.130	1.153	1.175	1.197	1.220
2.50	1.018	1.036	1.052	1.068	1.083	1.098	1.103	1.128	1.172
3.00	1.013	1.024	1.036	1.048	1.058	1.068	1.078	1.099	1.125
3.50	1.009	1.018	1.026	1.034	1.043	1.050	1.058	1.075	1.091
4.00	1.007	1.013	1.020	1.026	1.033	1.039	1.044	1.050	1.058
4.50	1.005	1.010	1.017	1.022	1.025	1.031	1.037	1.042	1.047
5.00	1.004	1.008	1.014	1.019	1.021	1.025	1.028	1.032	1.039
6.00	1.003	1.006	1.009	1.011	1.014	1.017	1.020	1.024	1.027
7.00	1.003	1.005	1.007	1.009	1.011	1.013	1.015	1.017	1.019
8.00	1.002	1.003	1.005	1.007	1.007	1.008	1.010	1.010	1.011
9.00	1.002	1.003	1.004	1.005	1.007	1.008	1.009	1.010	1.011
10.00	1.001	1.002	1.003	1.004	1.005	1.006	1.007	1.008	1.009
$(D_x/D_y)/\sqrt{D_x/D_y} = 0.1$									
0.25	1.281	2.214	2.967	3.561	4.068	4.513	4.930	5.311	5.652
0.50	1.048	1.402	1.714	1.980	2.209	2.416	2.606	2.786	2.949
0.75	1.012	1.195	1.364	1.514	1.647	1.772	1.886	1.994	2.095
1.00	1.009	1.113	1.218	1.313	1.401	1.481	1.557	1.629	1.698
1.25	1.005	1.090	1.143	1.209	1.270	1.326	1.380	1.431	1.480
1.50	1.002	1.051	1.102	1.149	1.192	1.234	1.274	1.312	1.349
1.75	1.001	1.037	1.075	1.111	1.144	1.176	1.206	1.235	1.263
2.00	1.001	1.028	1.058	1.085	1.111	1.136	1.160	1.183	1.205
2.50	1.000	1.017	1.037	1.055	1.072	1.088	1.104	1.119	1.134
3.00	1.000	1.012	1.025	1.038	1.050	1.061	1.073	1.084	1.094
3.50	1.000	1.008	1.019	1.028	1.037	1.045	1.057	1.062	1.067
4.00	1.000	1.006	1.014	1.021	1.028	1.035	1.041	1.047	1.053
4.50	1.000	1.004	1.011	1.017	1.023	1.027	1.032	1.037	1.042
5.00	1.000	1.003	1.009	1.013	1.018	1.022	1.026	1.030	1.034
6.00	1.000	1.001	1.006	1.009	1.012	1.015	1.018	1.021	1.024
7.00	1.000	1.001	1.004	1.006	1.009	1.011	1.013	1.015	1.017
8.00	1.000	1.000	1.003	1.003	1.007	1.008	1.009	1.012	1.013
9.00	1.000	1.000	1.003	1.004	1.005	1.007	1.008	1.009	1.010
10.00	1.000	1.000	1.002	1.003	1.004	1.005	1.006	1.007	1.008

Table 18 Frequency Ratio-Second Symmetric Mode, $n=3$

$c = \frac{b}{a} \left(\frac{x}{D_y} \right)^{1/4}$ (multiply by $(m^2 \pi^2 / a^2) \frac{D_x}{D_0}$ to get frequencies in rps)

Torsion Parameter μ

mc	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(D_1/D_y) / \sqrt{D_x/D_y} = 0$									
0.25	36.485	36.729	36.925	37.170	37.365	37.561	37.805	38.001	38.196
0.50	9.343	9.549	9.756	9.962	10.156	10.349	10.542	10.723	10.903
0.75	4.359	4.560	4.749	4.928	5.106	5.273	5.433	5.589	5.743
1.00	2.672	2.850	3.017	3.175	3.323	3.465	3.599	3.731	3.859
1.25	1.934	2.092	2.235	2.369	2.493	2.612	2.723	2.830	2.932
1.50	1.567	1.700	1.821	1.933	2.038	2.136	2.230	2.318	2.403
1.75	1.368	1.479	1.580	1.674	1.761	1.844	1.921	1.995	2.066
2.00	1.252	1.344	1.429	1.507	1.580	1.649	1.715	1.777	1.837
2.50	1.135	1.199	1.259	1.314	1.366	1.416	1.464	1.510	1.554
3.00	1.082	1.129	1.172	1.212	1.251	1.288	1.323	1.358	1.391
3.50	1.055	1.089	1.121	1.152	1.181	1.210	1.237	1.272	1.291
4.00	1.039	1.065	1.090	1.114	1.137	1.161	1.182	1.203	1.235
4.50	1.029	1.050	1.070	1.089	1.107	1.125	1.143	1.160	1.179
5.00	1.023	1.039	1.055	1.071	1.086	1.101	1.113	1.130	1.144
6.00	1.015	1.026	1.037	1.048	1.059	1.071	1.080	1.090	1.101
7.00	1.010	1.019	1.027	1.035	1.043	1.051	1.062	1.067	1.082
8.00	1.008	1.014	1.020	1.026	1.032	1.038	1.044	1.051	1.057
9.00	1.006	1.011	1.016	1.021	1.027	1.032	1.036	1.041	1.047
10.00	1.005	1.009	1.005	1.017	1.021	1.025	1.029	1.033	1.036
$(D_1/D_y) / \sqrt{D_x/D_y} = 0.1$									
0.25	36.361	36.536	36.780	36.976	37.220	37.415	37.659	37.855	38.050
0.50	9.175	9.392	9.610	9.815	10.008	10.213	10.405	10.585	10.777
0.75	4.207	4.411	4.609	4.797	4.979	5.149	5.320	5.479	5.633
1.00	2.527	2.720	2.897	3.064	3.219	3.369	3.508	3.644	3.770
1.25	1.808	1.977	2.132	2.274	2.407	2.533	2.649	2.761	2.868
1.50	1.458	1.604	1.736	1.857	1.968	2.073	2.171	2.265	2.353
1.75	1.277	1.400	1.510	1.612	1.705	1.793	1.874	1.953	2.026
2.00	1.176	1.279	1.371	1.456	1.535	1.608	1.677	1.743	1.806
2.50	1.082	1.154	1.220	1.281	1.334	1.390	1.440	1.488	1.534
3.00	1.045	1.097	1.145	1.189	1.230	1.270	1.307	1.343	1.378
3.50	1.027	1.066	1.102	1.136	1.167	1.197	1.226	1.254	1.281
4.00	1.018	1.048	1.076	1.102	1.127	1.150	1.173	1.195	1.217
4.50	1.012	1.036	1.059	1.079	1.099	1.118	1.137	1.155	1.172
5.00	1.009	1.028	1.047	1.064	1.080	1.095	1.111	1.125	1.140
6.00	1.005	1.002	1.031	1.043	1.055	1.066	1.077	1.087	1.098
7.00	1.004	1.013	1.023	1.031	1.040	1.048	1.056	1.064	1.072
8.00	1.003	1.010	1.017	1.024	1.030	1.037	1.043	1.049	1.055
9.00	1.002	1.007	1.013	1.019	1.024	1.029	1.034	1.039	1.043
10.00	1.001	1.006	1.011	1.015	1.019	1.023	1.027	1.031	1.035

Table 19 Frequency Ratio- Second Antisymmetric Mode $n = 4$

$$c = \frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4} \quad (\text{multiply by } (m^2 \pi^2 / a^2) \frac{D_x}{D_0} \text{ to get}$$

frequencies in rps)

Torsion Parameter μ

mc	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(D/D_y) / \sqrt{D_x/D_y} = 0$									
0.25	100.163	100.343	100.523	100.702	100.882	101.061	101.241	101.420	101.599
0.50	25.194	25.374	25.553	25.712	25.890	26.048	26.227	26.384	26.542
0.75	11.333	11.502	11.679	11.832	11.998	12.156	12.314	12.472	12.628
1.00	6.500	6.667	6.827	6.987	7.141	7.294	7.441	7.582	7.723
1.25	4.290	4.450	4.603	4.752	4.896	5.036	5.169	5.300	5.424
1.50	3.112	3.265	3.410	3.547	3.679	3.805	3.926	4.043	4.157
1.75	2.425	2.567	2.701	2.828	2.947	3.062	3.170	3.274	3.375
2.00	1.994	2.129	2.250	2.265	2.473	2.576	2.674	2.767	2.857
2.50	1.533	1.641	1.740	1.833	1.920	2.003	2.080	2.155	2.227
3.00	1.313	1.400	1.480	1.554	1.624	1.690	1.753	1.813	1.871
3.50	1.199	1.268	1.332	1.392	1.448	1.501	1.552	1.602	1.649
4.00	1.135	1.190	1.241	1.299	1.336	1.379	1.422	1.462	1.501
4.50	1.096	1.141	1.183	1.222	1.260	1.299	1.332	1.365	1.398
5.00	1.071	1.108	1.143	1.176	1.207	1.238	1.267	1.296	1.324
6.00	1.043	1.069	1.094	1.117	1.140	1.162	1.184	1.205	1.226
7.00	1.029	1.048	1.066	1.083	1.101	1.117	1.134	1.150	1.166
8.00	1.021	1.035	1.049	1.062	1.076	1.089	1.107	1.115	1.138
9.00	1.015	1.027	1.038	1.048	1.060	1.070	1.080	1.085	1.110
10.00	1.012	1.021	1.030	1.039	1.047	1.056	1.065	1.073	1.081
$(D/D_y) / \sqrt{D_x/D_y} = 0.1$									
0.25	100.083	100.263	100.442	100.622	100.723	100.901	101.081	101.260	101.439
0.50	25.094	25.274	25.432	25.611	25.790	25.948	26.126	26.283	26.461
0.75	11.226	11.404	11.573	11.741	11.908	12.066	12.224	12.381	12.537
1.00	6.402	6.572	6.737	6.896	7.054	7.207	7.359	7.505	7.645
1.25	4.193	4.369	4.517	4.671	4.818	4.957	5.096	5.230	5.357
1.50	3.021	3.178	3.329	3.472	3.607	3.737	3.861	3.982	4.098
1.75	2.338	2.487	2.627	2.758	2.883	3.001	3.114	3.222	3.324
2.00	1.917	2.055	2.183	2.303	2.417	2.524	2.625	2.722	2.814
2.50	1.466	1.581	1.687	1.786	1.877	1.963	2.045	2.123	2.197
3.00	1.259	1.353	1.438	1.517	1.591	1.661	1.726	1.789	1.849
3.50	1.156	1.231	1.300	1.364	1.423	1.479	1.533	1.584	1.633
4.00	1.100	1.160	1.217	1.268	1.317	1.363	1.407	1.449	1.490
4.50	1.068	1.117	1.163	1.206	1.246	1.284	1.320	1.355	1.389
5.00	1.049	1.089	1.127	1.162	1.196	1.228	1.258	1.288	1.317
6.00	1.028	1.056	1.083	1.109	1.133	1.156	1.178	1.200	1.221
7.00	1.017	1.039	1.059	1.078	1.096	1.113	1.130	1.147	1.163
8.00	1.012	1.028	1.044	1.058	1.072	1.086	1.099	1.112	1.125
9.00	1.009	1.023	1.034	1.045	1.056	1.067	1.079	1.088	1.099
10.00	1.007	1.017	1.027	1.036	1.045	1.054	1.063	1.071	1.080

Table 20 Frequency Ratio - Third Symmetric mode, $n = 5$

$$c = \frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4} \text{ (multiply by } (m^2 \pi^2 / a^2) \frac{D_x}{\rho_0} \text{, to get}$$

frequencies in rps)

Torsion Parameter μ

mc	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$(D_1/D_y) / \sqrt{D_x/D_y} = 0$									
0.25	196.184	196.283	196.495	196.595	196.807	196.910	197.198	197.218	197.430
0.50	49.158	49.314	49.470	49.625	49.780	49.935	50.090	50.245	50.371
0.75	21.959	22.108	22.257	22.405	22.553	22.713	22.860	23.007	23.141
1.00	12.444	12.598	12.745	12.891	13.037	13.182	13.320	13.463	13.600
1.25	8.054	8.205	8.351	8.496	8.635	8.774	8.907	9.039	9.171
1.50	5.684	5.830	5.974	6.110	6.246	6.377	6.505	6.631	6.753
1.75	4.270	4.413	4.548	4.682	4.810	4.934	5.055	5.172	5.286
2.00	3.365	3.502	3.633	3.760	3.881	3.998	4.110	4.220	4.327
2.50	2.333	2.459	2.577	2.689	2.796	2.898	2.995	3.089	3.180
3.00	1.809	1.920	2.024	2.122	2.214	2.302	2.385	2.466	2.543
3.50	1.519	1.615	1.705	1.789	1.868	1.943	2.015	2.084	2.149
4.00	1.350	1.432	1.509	1.580	1.648	1.712	1.773	1.833	1.889
4.50	1.245	1.316	1.381	1.442	1.500	1.555	1.608	1.659	1.708
5.00	1.179	1.239	1.294	1.347	1.396	1.444	1.489	1.533	1.576
6.00	1.104	1.148	1.189	1.228	1.265	1.301	1.336	1.369	1.402
7.00	1.067	1.100	1.131	1.160	1.189	1.217	1.244	1.270	1.296
8.00	1.046	1.071	1.095	1.119	1.141	1.163	1.185	1.206	1.227
9.00	1.033	1.053	1.073	1.091	1.109	1.127	1.145	1.162	1.179
10.00	1.025	1.041	1.057	1.072	1.087	1.102	1.117	1.131	1.145
$(D_1/D_y) / \sqrt{D_x/D_y} = 0.1$									
0.25	196.072	196.283	196.383	196.595	196.695	196.907	197.006	197.218	197.317
0.50	49.102	49.258	49.414	49.541	49.695	49.851	50.006	50.160	50.315
0.75	21.885	22.033	22.182	22.343	22.490	22.638	22.785	22.932	23.078
1.00	12.374	12.522	12.675	12.821	12.967	13.112	13.256	13.400	13.536
1.25	7.983	8.134	8.284	8.428	8.567	8.710	8.843	8.982	9.112
1.50	5.617	5.762	5.909	6.048	6.183	6.317	6.448	6.574	6.699
1.75	4.201	4.346	4.487	4.620	4.752	4.878	5.001	5.121	5.237
2.00	3.298	3.440	3.573	3.703	3.826	3.947	4.062	4.173	4.282
2.50	2.272	2.401	2.524	2.640	2.749	2.855	2.955	3.052	3.144
3.00	1.754	1.870	1.978	2.079	2.175	2.266	2.352	2.436	2.514
3.50	1.471	1.572	1.666	1.754	1.837	1.915	1.989	2.060	2.128
4.00	1.308	1.396	1.476	1.552	1.623	1.690	1.753	1.814	1.874
4.50	1.210	1.285	1.354	1.419	1.480	1.537	1.590	1.644	1.684
5.00	1.149	1.213	1.272	1.328	1.380	1.429	1.476	1.529	1.565
6.00	1.082	1.129	1.174	1.215	1.254	1.299	1.327	1.361	1.395
7.00	1.050	1.086	1.120	1.151	1.181	1.210	1.238	1.265	1.291
8.00	1.033	1.061	1.087	1.112	1.136	1.158	1.181	1.202	1.223
9.00	1.023	1.046	1.067	1.086	1.106	1.124	1.142	1.159	1.176
10.00	1.017	1.035	1.052	1.069	1.084	1.099	1.104	1.129	1.143

Table 21 Frequency Ratio - Third Antisymmetric Mode, $n = 6$

$$c = \frac{b}{a} \left(\frac{D_x}{D_y} \right)^{1/4} \quad (\text{multiply by } (m^2 \pi^2 / a^2) \frac{D_x}{D_y}, \text{ to get frequencies in rps})$$

Torsion Parameter U

mc	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	$(D_x/D_y) / \sqrt{D_x/D_y} = 0$								
0.25	324.205	324.305	324.405	324.505	324.749	324.849	324.948	325.192	325.292
0.50	81.168	81.304	81.440	81.575	81.711	81.846	81.981	82.152	82.287
0.75	36.157	36.305	36.436	36.583	36.714	36.861	36.991	37.137	37.267
1.00	20.417	20.561	20.696	20.839	20.973	21.106	21.248	21.381	21.513
1.25	13.137	13.281	13.419	13.557	13.688	13.825	13.955	14.084	14.219
1.50	9.167	9.335	9.471	9.603	9.734	9.865	9.994	10.119	10.247
1.75	6.828	6.963	7.097	7.226	7.355	7.482	7.606	7.726	7.845
2.00	5.299	5.432	5.562	5.690	5.815	5.937	6.055	6.169	6.283
2.50	3.525	3.653	3.775	3.894	4.009	4.121	4.229	4.334	4.434
3.00	2.589	2.709	2.824	2.933	3.038	3.138	3.235	3.328	3.419
3.50	2.050	2.161	2.265	2.364	2.458	2.549	2.635	2.718	2.798
4.00	1.721	1.821	1.916	2.004	2.089	2.168	2.245	2.319	2.389
4.50	1.511	1.601	1.685	1.764	1.839	1.910	1.978	2.043	2.106
5.00	1.373	1.453	1.527	1.597	1.664	1.726	1.787	1.849	1.900
6.00	1.205	1.277	1.335	1.389	1.441	1.491	1.539	1.585	1.630
7.00	1.134	1.183	1.228	1.272	1.313	1.352	1.391	1.428	1.464
8.00	1.090	1.129	1.165	1.199	1.232	1.264	1.296	1.326	1.355
9.00	1.064	1.095	1.124	1.152	1.179	1.205	1.231	1.260	1.280
10.00	1.047	1.072	1.096	1.120	1.142	1.164	1.186	1.207	1.227

$$(D_x/D_y) / \sqrt{D_x/D_y} = 0.1$$

0.25	324.062	324.161	324.405	324.505	324.605	324.849	324.948	325.048	325.292
0.50	81.096	81.232	81.368	81.503	81.675	81.810	81.945	82.080	82.215
0.75	36.093	36.241	36.388	36.519	36.666	36.796	36.943	37.073	37.219
1.00	20.363	20.498	20.642	20.785	20.919	21.052	21.194	21.327	21.459
1.25	13.085	13.224	13.362	13.499	13.636	13.772	13.902	14.037	14.166
1.50	9.141	9.279	9.415	9.551	9.686	9.816	9.946	10.070	10.198
1.75	6.773	6.911	7.044	7.176	7.305	7.432	7.558	7.681	7.800
2.00	5.244	5.379	5.513	5.641	5.765	5.889	6.009	6.126	6.241
2.50	3.472	3.603	3.727	3.849	3.966	4.079	4.189	4.289	4.399
3.00	2.540	2.663	2.780	2.892	2.999	3.102	3.200	3.297	3.389
3.50	2.004	2.118	2.226	2.328	2.425	2.517	2.606	2.691	2.773
4.00	1.679	1.783	1.881	1.972	2.060	2.142	2.221	2.296	2.369
4.50	1.474	1.567	1.655	1.737	1.814	1.888	1.957	2.024	2.089
5.00	1.340	1.423	1.501	1.574	1.643	1.708	1.770	1.829	1.886
6.00	1.189	1.254	1.315	1.373	1.427	1.478	1.527	1.574	1.620
7.00	1.114	1.166	1.214	1.259	1.302	1.343	1.382	1.420	1.457
8.00	1.074	1.115	1.154	1.190	1.224	1.258	1.289	1.320	1.350
9.00	1.051	1.084	1.116	1.145	1.173	1.200	1.227	1.252	1.277
10.00	1.037	1.064	1.097	1.114	1.138	1.160	1.182	1.204	1.225

Table 22 Correction tables for Variation of Frequency Ratio R with Poisson's

$\frac{D_1}{D_2}$		Ratio Parameter $(D_1/D_2)/\sqrt{b_x/b_y}$						$\frac{mc}{b_x} = \frac{b_y}{a} \cdot \left(\frac{D_1}{D_2}\right)^{1/4}$					
		$\mu = 1, mc =$						$\mu = .9, mc =$					
$\sqrt{b_x/b_y}$		.25	.5	1.0	1.5	2.0	5.0	.25	.5	1.0	1.5	2.0	5.0
2	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.025	0.8950	0.9358	0.9796	0.9883	0.9935	0.9991	0.9871	0.9878	0.9931	0.9962	0.9960	0.9964
	0.050	0.7759	0.8678	0.9500	0.9757	0.9859	0.9976	0.9732	0.9767	0.9858	0.9919	0.9930	0.9965
	0.075	0.6333	0.7921	0.9226	0.9617	0.9771	0.9956	0.9586	0.9648	0.9782	0.9871	0.9986	0.9959
	0.100	0.5550	0.7250	0.9000	0.9500	0.9650	0.9900	0.9440	0.9520	0.9707	0.9826	0.9880	0.9972
3	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.025	1.0000	0.9949	0.9868	0.9836	0.9861	0.9974	0.9999	0.9977	0.9937	0.9948	0.9954	0.9993
	0.050	0.9986	0.9909	0.9735	0.9664	0.9712	0.9940	0.9987	0.9942	0.9884	0.9895	0.9910	0.9984
	0.075	0.9973	0.9871	0.9599	0.9489	0.9556	0.9903	0.9974	0.9919	0.9832	0.9842	0.9890	0.9976
	0.100	0.9966	0.9820	0.9455	0.9303	0.9393	0.9857	0.9962	0.9884	0.9771	0.9791	0.9822	0.9964
4	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.025	1.0000	0.9992	0.9962	0.9932	0.9915	0.9953	0.9992	0.9992	0.9973	0.9966	0.9966	0.9989
	0.050	0.9992	0.9976	0.9923	0.9857	0.9817	0.9901	0.9992	0.9984	0.9952	0.9931	0.9928	0.9975
	0.075	0.9994	0.9968	0.9885	0.9783	0.9715	0.9845	0.9992	0.9977	0.9926	0.9892	0.9890	0.9960
	0.100	0.9992	0.9948	0.9848	0.9709	0.9613	0.9788	0.9984	0.9970	0.9899	0.9857	0.9852	0.9945
5	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.025	1.0000	1.0000	0.9983	0.9973	0.9950	0.9941	0.9994	1.0000	0.9990	0.9981	0.9975	0.9984
	0.050	0.9994	0.9994	0.9972	0.9940	0.9990	0.9878	0.9994	0.9994	0.9979	0.9962	0.9949	0.9963
	0.075	0.9994	0.9994	0.9955	0.9908	0.9851	0.9813	0.9994	0.9988	0.9963	0.9943	0.9919	0.9950
	0.100	0.9994	0.9994	0.9944	0.9881	0.9800	0.9748	0.9994	0.9988	0.9950	0.9919	0.9894	0.9932
6	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.025	0.9996	0.9996	0.9991	0.9994	0.9975	0.9941	0.9996	0.9996	0.9996	0.9988	0.9985	0.9982
	0.050	0.9996	0.9996	0.9987	0.9991	0.9950	0.9882	0.9996	0.9996	0.9987	0.9976	0.9967	0.9964
	0.075	0.9996	0.9991	0.9978	0.9985	0.9921	0.9819	0.9996	0.9996	0.9983	0.9964	0.9949	0.9946
	0.100	0.9996	0.9991	0.9973	0.9972	0.9895	0.9758	0.9996	0.9996	0.9975	0.9952	0.9934	0.9926

TABLE 23A
AMPLIFICATION FACTORS A_F - STAGE I
INITIAL BRIDGE CONDITIONS - BRIDGE AT REST
Crossing Frequency $f_C = 0.5\pi$

f_g	f_R								A_F
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3π	1.1734	1.7379	2.2485	1.3034	1.4243	1.4052	1.2970	1.1953	0.01
6π	1.1334	1.6454	3.2018	1.4200	1.0862	1.1991	1.3497	1.3578	
9π	1.1041	1.4681	3.8230	1.3931	1.3515	1.2193	1.1067	1.2184	
12π	1.0980	1.5652	4.0055	1.3642	1.2580	1.3333	1.2223	1.0389	
15π	1.0930	1.5257	4.0652	1.3929	1.3464	1.2528	1.3281	1.2798	
18π	1.0971	1.5459	4.0295	1.4025	1.3055	1.3069	1.2863	1.3146	
3π	-1.1591	1.6824	2.1685	1.2999	1.4245	1.4048	1.2967	1.1951	0.03
6π	1.1280	1.5181	2.6640	1.4146	1.0858	1.1995	1.3496	1.3575	
9π	1.0921	1.4552	2.7183	1.3878	1.3517	1.2187	1.1065	1.1344	
12π	1.0923	1.4977	2.6837	1.3567	1.2580	1.3326	1.2225	1.0388	
15π	1.0967	1.4453	2.6777	1.3896	1.3464	1.2522	1.3281	1.2796	
18π	1.0782	1.4984	2.6014	1.3965	1.3054	1.1349	1.2862	1.3144	
3π	1.1444	1.6325	2.0918	1.2963	1.4246	1.4042	1.2963	1.1950	0.05
6π	1.1239	1.4531	2.2530	1.4097	1.0853	1.1999	1.3494	1.3572	
9π	1.0790	1.4294	2.3081	1.3822	1.3518	1.2180	1.1063	1.1342	
12π	1.0788	1.4408	2.2568	1.3514	1.2579	1.3317	1.2226	1.0387	
15π	1.0777	1.4305	2.2037	1.3859	1.3464	1.2516	1.3281	1.2792	
18π	1.0937	1.4249	2.3121	1.3899	1.3052	1.3047	1.2861	1.3141	
3π	1.1219	1.5626	1.9870	1.2909	1.4247	1.4035	1.2953	1.1941	0.08
6π	1.1125	1.3722	2.0497	1.4025	1.0845	1.1489	1.3493	1.3567	
9π	1.0896	1.4242	1.9700	1.3735	1.3519	1.2169	1.1060	1.1338	
12π	1.0874	1.3793	2.0090	1.3427	1.2578	1.3306	1.2230	1.0386	
15π	1.0857	1.4111	2.0459	1.3798	1.3462	1.2508	1.3282	1.2788	
18π	1.0875	1.3856	2.0092	1.3849	1.3022	1.1341	1.2860	1.3136	
3π	1.1305	1.5208	1.9208	1.2874	1.4249	1.4030	1.2947	1.1936	0.10
6π	1.1029	1.3226	1.9327	1.3977	1.0840	1.1483	1.3491	1.3564	
9π	1.0728	1.4119	1.7712	1.3675	1.3520	1.2162	1.1058	1.2185	
12π	1.0944	1.3537	1.8582	1.3388	1.2577	1.3298	1.2229	1.0386	
15π	1.0767	1.3820	1.7701	1.3753	1.3461	1.2502	1.3282	1.2785	
18π	1.0901	1.3794	1.9258	1.3819	1.3021	1.3018	1.2859	1.3132	
3π	1.1603	1.3534	1.6460	1.2700	1.4255	1.4006	1.2925	1.1917	0.20
6π	1.1221	1.2700	1.4587	1.3750	1.0816	1.2030	1.3484	1.3548	
9π	1.0745	1.2724	1.6143	1.3410	1.3521	1.2126	1.1048	1.2187	
12π	1.0874	1.3155	1.5911	1.3245	1.2571	1.3258	1.2236	1.0385	
15π	1.0703	1.2985	1.6235	1.3588	1.3460	1.2474	1.3281	1.2770	
18π	1.0873	1.2711	1.6734	1.3670	1.3016	1.2960	1.2854	1.3116	

TABLE 23B
AMPLIFICATION FACTORS A_r - STAGE I
INITIAL BRIDGE CONDITIONS - BRIDGE AT REST
Crossing Frequency $f_C = \pi$
 $f_R =$

f_B	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	M_R
3π	1.5277	1.7624	1.9280	1.6906	1.3712	1.2301	1.1639	1.1283	0.01
6π	1.1734	1.7379	2.2485	1.3035	1.4243	1.4052	1.2970	1.1953	
9π	1.1071	1.1548	2.8242	1.3066	1.2972	1.3362	1.3325	1.3207	
12π	1.1334	1.6454	3.3056	1.4199	1.0862	1.1991	1.3497	1.3577	
15π	1.0989	1.5757	3.6092	1.3410	1.2804	1.0826	1.2352	1.2787	
18π	1.1041	1.4681	3.8229	1.3931	1.3515	1.2193	1.1067	1.2184	
3π	1.5321	1.7445	1.9101	1.6828	1.3674	1.2289	1.1633	1.1279	0.03
6π	1.1591	1.6824	2.1685	1.2998	1.4244	1.4047	1.2967	1.1952	
9π	1.1069	1.6386	2.5000	1.2993	1.2966	1.3354	1.3328	1.3207	
12π	1.1280	1.5181	2.6640	1.4146	1.0858	1.1994	1.3496	1.3575	
15π	1.1103	1.4534	2.6924	1.3375	1.2806	1.0827	1.2350	1.2789	
18π	1.0921	1.4553	2.2827	1.3878	1.3517	1.2187	1.1065	1.2184	
3π	1.5359	1.7268	1.8927	1.6749	1.3642	1.2266	1.1626	1.1276	0.05
6π	1.1442	1.1632	2.0918	1.2962	1.4246	1.4042	1.2963	1.1950	
9π	1.1025	1.5367	2.2819	1.2921	1.2959	1.3345	1.3332	1.3207	
12π	1.1239	1.4531	2.2530	1.4097	1.0853	1.2000	1.3494	1.3572	
15π	1.1165	1.3822	2.3367	1.3340	1.2809	1.0828	1.2348	1.2790	
18π	1.0790	1.4295	2.3082	1.3822	1.3518	1.2180	1.1063	1.1342	
3π	1.5402	1.7010	1.8669	1.6633	1.3593	1.2240	1.1603	1.1258	0.08
6π	1.1219	1.5626	1.9870	1.2909	1.4248	1.4035	1.2952	1.1941	
9π	1.1049	1.4133	1.9929	1.2819	1.2950	1.3332	1.3336	1.3207	
12π	1.1125	1.3722	2.0497	1.4024	1.0845	1.2005	1.3492	1.3567	
15π	1.1193	1.3878	1.9280	1.3289	1.2813	1.0830	1.2395	1.2793	
18π	1.0896	1.4241	1.9700	1.3735	1.3520	1.2169	1.1060	1.1338	
3π	1.5423	1.6840	1.8500	1.6556	1.3562	1.2225	1.1593	1.1254	0.10
6π	1.1305	1.5208	1.9208	1.2874	1.4249	1.4030	1.2948	1.1935	
9π	1.1103	1.3470	1.8252	1.2753	1.2943	1.3323	1.3334	1.3207	
12π	1.1029	1.3226	1.9326	1.3977	1.0840	1.1484	1.3491	1.3564	
15π	1.1148	1.3586	1.8498	1.3255	1.2815	1.0832	1.2343	1.2795	
18π	1.0973	1.4118	1.7712	1.3676	1.3520	1.2162	1.1058	1.2186	
3π	1.5450	1.6045	1.7092	1.6169	1.3402	1.2144	1.1545	1.1221	0.20
6π	1.1603	1.3534	1.6460	1.2700	1.4255	1.4007	1.2925	1.1917	
9π	1.1230	1.2488	1.6029	1.2439	1.2912	1.3280	1.3355	1.3207	
12π	1.1221	1.2700	1.4587	1.3750	1.0815	1.1453	1.3483	1.3548	
15π	1.0960	1.3344	1.5195	1.1311	1.2824	1.0837	1.2331	1.2804	
18π	1.685	1.2724	1.6143	1.2000	1.3522	1.2126	1.1048	1.2187	

TABLE 23C
AMPLIFICATION FACTORS A_T - STAGE I
INITIAL BRIDGE CONDITIONS - BRIDGE AT REST
Crossing Frequency $f_C = 1.4\pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3π	1.6590	2.0724	2.1540	1.7073	1.4410	1.3399	1.2832	1.2572	0.01
6π	1.2456	1.6578	1.9625	1.5349	1.2350	1.0777	-	-	
9π	1.1704	1.7005	2.3074	1.2507	1.4289	1.4214	1.3145	1.2110	
12π	1.0953	1.7619	2.7324	1.2546	1.3276	1.3609	1.2721	1.2851	
15π	1.1523	1.6733	3.1390	1.4116	1.1737	1.2307	1.3725	1.3734	
18π	1.1084	1.6366	3.4372	1.4069	1.1061	1.1693	1.3221	1.3347	
3π	1.6747	2.0473	2.1316	1.8997	1.4371	1.3330	1.2828	1.2555	0.03
6π	1.2446	1.5736	1.8580	1.5288	1.2318	1.0787	-	-	
9π	1.1570	1.6520	2.1532	1.2475	1.4289	1.4205	1.3141	1.2108	
12π	1.1058	1.6357	2.4800	1.2471	1.3271	1.3600	1.3009	1.2853	
15π	1.1480	1.5863	2.5570	1.4055	1.1731	1.2312	1.3724	1.3732	
18π	1.1039	1.4834	2.6913	1.4021	1.1034	1.1695	1.3219	1.3343	
3π	1.6899	2.0226	2.1097	1.6921	1.4339	1.3300	1.2822	1.2550	0.05
6π	1.2435	1.4977	1.7602	1.5227	1.2289	1.0791	-	-	
9π	1.1273	1.6063	2.0849	1.2444	1.4287	1.4198	1.3135	1.2104	
12π	1.1144	1.5273	2.2521	1.2398	1.3264	1.3591	1.3013	1.2855	
15π	1.1439	1.5098	2.2992	1.3994	1.1725	1.2320	1.3724	1.3730	
18π	1.0996	1.3689	2.2682	1.3978	1.1008	1.1699	1.3217	1.3340	
3π	1.7114	1.9867	2.0775	1.6808	1.4293	1.3277	1.2794	1.2540	0.08
6π	1.2416	1.4002	1.6258	1.5137	1.2246	1.0805	-	-	
9π	1.1486	1.5444	1.9872	1.2392	1.4284	1.4188	1.3123	1.2096	
12π	1.1250	1.3990	1.9519	1.2295	1.3255	1.3578	1.3020	1.1689	
15π	1.1304	1.4098	2.0639	1.3905	1.1716	1.2329	1.3725	1.3726	
18π	1.0886	1.3391	2.0248	1.3914	1.0993	1.1704	1.3215	1.3336	
3π	1.7246	1.9634	2.0565	1.6733	1.4262	1.3263	1.2785	1.2524	0.1
6π	1.2401	1.3447	1.5447	1.5078	1.2217	1.0813	-	-	
9π	1.1597	1.5055	1.9275	1.2366	1.4283	1.4182	1.3118	1.2088	
12π	1.1313	1.3472	1.7979	1.2227	1.3243	1.3569	1.3025	1.2856	
15π	1.1215	1.3521	1.9240	1.3847	1.1711	1.2335	1.3725	1.3723	
18π	1.0799	1.3374	1.9198	1.3872	1.0990	1.7706	1.3213	1.3332	
3π	1.7745	1.8545	1.9565	1.6355	1.4107	1.3179	1.2733	1.2490	0.2
6π	1.2307	1.2783	1.4502	1.4784	1.2076	1.0852	-	-	
9π	1.1869	1.3498	1.6705	1.2214	1.4275	1.4150	1.3089	1.2060	
12π	1.1458	1.2470	1.6280	1.1908	1.3217	1.3525	1.2652	1.2860	
15π	1.1210	1.2817	1.4876	1.8583	1.1682	1.2367	1.3723	1.3712	
18π	1.1063	1.3118	1.4853	1.3669	1.0987	1.1720	1.3203	1.3316	

TABLE 23D
AMPLIFICATION FACTORS A_F - STAGE I
INITIAL BRIDGE CONDITIONS - BRIDGE AT REST
Crossing Frequency $f_c = 1.8 \pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3π	1.6708	2.1553	2.1344	1.6250	1.4211	1.3418	1.3042	1.2835	0.01
6π	1.4536	1.6444	1.8208	1.6576	1.3334	1.1825	1.1120	1.0745	
9π	1.1115	1.7625	2.1900	1.4359	1.3373	1.2845	1.1774	1.0854	
12π	1.1024	1.6762	1.4030	1.2224	1.4259	1.4239	1.3186	1.2144	
15π	1.1031	1.7610	2.7213	1.2938	1.3439	1.3737	1.2774	1.2598	
18π	1.1469	1.7159	3.0498	1.3843	1.2246	1.2746	1.3693	1.3651	
3π	1.6803	2.1303	2.1141	1.6189	1.4175	1.3412	1.3040	1.2833	0.03
6π	1.4555	1.6293	1.8054	1.6501	1.3297	1.1813	1.1114	1.0742	
9π	1.1102	1.6903	2.0929	1.4309	1.3382	1.2838	1.1776	1.0857	
12π	1.1509	1.6304	2.1376	1.2194	1.4257	1.4233	1.3181	1.2142	
15π	1.1134	1.6293	2.4629	1.2123	1.3433	1.3728	1.2818	1.2600	
18π	1.1448	1.3640	2.5018	1.3778	1.2240	1.2738	1.3693	1.3649	
3π	1.6925	2.1058	2.0938	1.6129	1.4150	1.3384	1.3037	1.2836	0.05
6π	1.4570	1.6145	1.7903	1.6226	1.3266	1.1791	1.1107	1.0738	
9π	1.1088	1.6256	2.0020	1.4260	1.3390	1.2838	1.1776	1.0859	
12π	1.1374	1.5889	2.0729	1.2166	1.4254	1.4225	1.3174	1.2139	
15π	1.1236	1.5182	2.2272	1.2049	1.3427	1.3718	1.2810	1.2603	
18π	1.1393	1.5304	2.3084	1.3713	1.2233	1.2378	1.3694	1.3649	
3π	1.7143	2.0699	2.0642	1.6036	1.4112	1.3363	1.3007	1.2823	0.08
6π	1.4582	1.5927	1.7678	1.6314	1.3218	1.1768	1.1086	1.0729	
9π	1.1066	1.5385	1.8766	1.4187	1.3401	1.2841	1.1772	1.0854	
12π	1.1578	1.5292	1.9814	1.2122	1.4250	1.4214	1.3162	1.2130	
15π	1.1355	1.3923	1.9231	1.1945	1.3418	1.3705	1.2798	1.2604	
18π	1.1268	1.4202	2.0483	1.3619	1.2224	1.2718	1.3696	1.3645	
3π	1.7305	2.0465	2.0447	1.5980	1.4085	1.3348	1.2999	1.2806	0.10
6π	1.4584	1.5785	1.7530	1.6240	1.3188	1.1753	1.1078	1.0717	
9π	1.1050	1.4877	1.7994	1.4138	1.3410	1.2843	1.1772	1.0851	
12π	1.1685	1.4937	1.9230	1.2094	1.4248	1.4207	1.3156	1.2122	
15π	1.1411	1.3196	1.8178	1.1877	1.3411	1.3696	1.2796	1.2604	
18π	1.1163	1.3586	1.8978	1.3557	1.2218	1.2397	1.3697	1.3644	
3π	1.8124	1.9374	1.9522	1.5661	1.3961	1.3279	1.2955	1.2778	0.20
6π	1.4544	1.5106	1.6824	1.5872	1.3034	1.1676	1.1033	1.0687	
9π	1.0963	1.3046	1.4910	1.3897	1.3495	1.2853	1.1771	1.0848	
12π	1.1936	1.3440	1.6780	1.1954	1.4234	1.4171	1.3125	1.2099	
15π	1.2571	1.2614	1.6396	1.1557	1.3380	1.3652	1.2820	1.2611	
18π	1.1020	1.2828	1.5381	1.3267	1.2188	1.2670	1.3701	1.3636	

TABLE 236
AMPLIFICATION FACTORS A_p - STAGE I
INITIAL BRIDGE CONDITIONS - BRIDGE AT REST
Crossing frequency $f_c = 2.2\pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	1.6913	2.0787	1.9826	1.5054	1.3539	1.2962	1.2685	1.2535	0.01
6 π	1.5830	1.8635	2.0137	1.7097	1.4001	1.2680	1.2058	1.1724	
9 π	1.2860	1.5964	1.8546	1.5617	1.2519	1.0804	1.0016	-	
12 π	1.1535	1.7667	2.2414	1.3743	1.3933	1.3569	1.2190	1.1514	
15 π	1.1588	1.6700	2.4554	1.2050	1.4223	1.4240	1.3195	1.2151	
18 π	1.1068	1.7580	2.7103	1.1953	1.2770	1.3813	1.2884	1.2413	
3 π	1.6866	1.0572	1.9666	1.5009	1.3510	1.2960	1.2683	1.2533	0.03
6 π	1.5904	1.8433	1.9939	1.7018	1.3962	1.2670	1.2454	1.1720	
9 π	1.2853	1.5087	1.7996	1.5553	1.2486	1.0792	1.0011	-	
12 π	1.1317	1.7024	2.1525	1.3700	1.3937	1.3570	1.2589	1.1514	
15 π	1.1456	1.6163	2.1821	1.2022	1.4220	1.4232	1.3190	1.2148	
18 π	1.1189	1.6234	2.4478	1.1878	1.3533	1.3805	1.2879	1.2416	
3 π	1.6897	2.0362	1.9500	1.4965	1.3492	1.2932	1.2685	1.2536	0.05
6 π	1.5972	1.8232	1.9745	1.6937	1.3929	1.2643	1.2046	1.1716	
9 π	1.2844	1.4305	1.6519	1.5489	1.2457	1.0774	1.0006	-	
12 π	1.1217	1.6446	2.0678	1.6251	1.3942	1.3566	1.2487	1.1514	
15 π	1.1419	1.5764	2.0641	1.1994	1.4217	1.4223	1.3183	1.2144	
18 π	1.1290	1.5110	2.2123	1.1804	1.3526	1.3795	1.2871	1.2418	
3 π	1.6872	2.0006	1.9261	1.4898	1.3463	1.2917	1.2658	1.2532	0.08
6 π	1.6056	1.7939	1.9458	1.6818	1.3881	1.2617	1.2021	1.1707	
9 π	1.2826	1.3775	1.5556	1.5394	1.2413	1.0759	1.6056	-	
12 π	1.0975	1.5656	1.9511	1.3594	1.3950	1.3564	1.2480	1.1509	
15 π	1.1162	1.5190	1.9744	1.1953	1.4211	1.4212	1.3170	1.2136	
18 π	1.1410	1.4063	1.9031	1.1698	1.3517	1.3782	1.2859	1.2420	
3 π	1.6920	1.9854	1.9103	1.4852	1.3444	1.2905	1.2651	1.2511	0.10
6 π	1.6101	1.7750	1.9272	1.6738	1.3848	1.2601	1.2011	1.1692	
9 π	1.2813	1.3680	1.5450	1.5332	1.2385	1.0740	-	-	
12 π	1.0819	1.5180	1.8782	1.3551	1.3954	1.3563	1.2478	1.1502	
15 π	1.1712	1.4845	1.9198	1.1926	1.4208	1.4204	1.3169	1.2128	
18 π	1.1481	1.3811	1.8318	1.1630	1.3511	1.3773	1.2851	1.2421	
3 π	1.7399	1.8909	1.8344	1.9610	1.3348	-	1.2615	1.2487	0.20
6 π	1.6220	1.6850	1.1839	1.6345	1.3687	1.2520	1.1961	1.1659	
9 π	1.2720	1.3214	1.4998	1.5022	1.2240	1.0066	1.2251	-	
12 π	1.0884	1.3399	1.5832	1.3344	1.3982	1.3554	1.2464	1.1491	
15 π	1.1962	1.3387	1.6800	1.1792	1.4192	1.4167	1.3132	1.2104	
18 π	1.1645	1.2715	1.6984	1.1311	1.3480	1.3729	1.2815	1.2430	

TABLE 23F
AMPLIFICATION FACTORS A_c - STAGE I
INITIAL BRIDGE CONDITIONS - BRIDGE AT REST
Crossing frequency $f_c = 2.5\pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3π	1.6993	1.9602	1.8341	1.9090	1.2887	1.2431	1.2210	1.2043	0.01
6π	1.6352	1.9853	2.1043	1.7175	1.4284	1.3090	1.2529	1.2224	
9π	1.3932	1.5570	1.7375	1.6261	1.3031	1.1451	1.0716	1.0328	
12π	1.1468	1.7482	2.1475	1.4635	1.3023	1.2384	1.1338	1.0450	
15π	-	1.7380	2.2485	1.3035	1.4243	1.4052	1.2970	1.1953	
18π	1.1413	1.6956	2.5622	1.1622	1.4087	1.4185	1.3167	1.2124	
3π	1.6888	1.9419	1.8210	1.4056	1.2862	1.2432	1.2216	1.2096	0.03
6π	1.6474	1.9620	2.0826	1.7097	1.4243	1.3080	1.2523	1.2221	
9π	1.3940	1.3438	1.7237	1.6190	1.2996	1.1440	1.0710	1.0324	
12π	1.1455	1.6725	2.0480	1.4583	1.3031	1.2388	1.1343	1.0453	
15π	1.1591	1.6824	2.1686	1.2998	1.4245	1.4047	1.2967	1.1951	
18π	1.1326	1.5785	2.2868	1.1597	1.4083	1.6177	1.3162	1.2120	
3π	1.6805	1.9245	1.8073	1.4020	1.2847	1.2407	1.2217	1.2099	0.05
6π	1.6588	1.9393	2.0614	1.7019	1.4210	1.3053	1.2517	1.2218	
9π	1.3972	1.5308	1.7102	1.6118	1.2965	1.1419	1.0704	1.0321	
12π	1.1441	1.6046	1.9543	1.4530	1.3039	1.2390	1.1343	1.0456	
15π	1.1443	1.6325	2.0918	1.2963	1.4246	1.4042	1.2962	1.1950	
18π	1.1466	1.5414	2.0404	1.1573	1.4078	1.4168	1.3154	1.2116	
3π	1.6219	1.8980	1.7877	1.3969	1.2826	1.2394	1.2190	1.2079	0.08
6π	1.6740	1.9060	2.0302	1.6903	1.4162	1.3029	1.2491	1.2206	
9π	1.3930	1.5119	1.6902	1.6013	1.2919	1.1397	1.0684	1.0313	
12π	1.1418	1.2192	1.8256	1.4453	1.3053	1.2397	1.1341	1.0453	
15π	1.1219	1.5625	1.9870	1.2910	1.4248	1.4035	1.2912	1.1942	
18π	1.1637	1.9910	1.9525	1.1537	1.4072	1.4156	1.3142	1.2105	
3π	1.6685	1.8809	1.7747	1.3932	1.2808	1.2384	1.2182	1.2073	0.10
6π	1.6829	1.8842	2.0097	1.6822	1.4130	1.3012	1.2681	1.2191	
9π	1.3939	1.4995	1.6772	1.5942	1.2890	1.1381	1.0676	1.0302	
12π	1.1402	1.4616	1.7472	1.4400	1.3061	1.2400	1.1342	1.0450	
15π	1.1305	1.5208	1.9208	1.2879	1.4249	1.4030	1.2998	1.1936	
18π	1.1732	1.4587	1.9020	1.1513	1.4067	1.4148	1.3135	1.2098	
3π	1.6753	1.7990	1.7117	1.3734	1.2310	1.2341	1.2157	1.2059	0.20
6π	1.7128	1.7824	1.9132	1.6930	1.3973	1.2928	1.2429	1.2157	
9π	1.3869	1.4401	1.6140	1.5593	1.2739	1.1309	1.0635	1.0275	
12π	1.1310	1.2799	1.4347	1.4146	1.3112	1.2419	1.1348	1.0452	
15π	1.1603	1.3534	1.6460	1.2700	1.4255	1.4006	1.2425	1.1917	
18π	1.1948	1.3242	1.6791	1.1395	1.4045	1.4106	1.3100	1.2073	

TABLE 24A
AMPLIFICATION FACTORS A_n - STAGE II
INITIAL BRIDGE CONDITIONS: CORRESPONDING TO LAST OF I.
Crossing Frequency $f_c = 0.5 \pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3π	1.1563	2.2857	2.2534	1.3302	1.4165	1.4013	1.2925	1.1946	0.01
6π	1.1329	1.6919	3.1411	1.4198	1.1004	1.1916	1.3581	1.3645	
9π	1.0798	1.4853	3.4936	1.4005	1.3468	1.2175	1.1167	1.2231	
12π	1.0865	1.4665	3.5955	1.3461	1.2609	1.3300	1.2197	1.0881	
15π	1.1315	1.4632	3.4324	1.3844	1.3415	1.2488	1.3249	1.2712	
18π	1.1533	1.4137	3.3391	1.3695	1.2962	1.2965	1.2797	1.2982	
3π	1.1306	2.0053	2.1654	1.3300	1.4179	1.4010	1.2922	1.1944	0.03
6π	1.1279	1.6873	2.5971	1.4111	1.0992	1.1922	1.3579	1.3642	
9π	1.0936	1.4661	2.5860	1.2370	1.3476	1.2166	1.1160	1.2230	
12π	1.0846	1.3959	2.5126	1.3403	1.2609	1.3286	1.2184	1.0479	
15π	1.1167	1.3363	2.3744	1.3783	1.3418	1.2482	1.3248	1.2708	
18π	1.1808	1.3631	2.2101	1.3640	1.2959	1.2953	1.2794	1.2977	
3π	1.1226	1.7945	2.0849	1.3296	1.4192	1.4005	1.2916	1.1942	0.05
6π	1.1169	1.5404	2.2145	1.4027	1.0979	1.1756	1.3577	1.3637	
9π	1.0193	1.5662	2.2254	1.3882	1.3483	1.2155	1.1155	1.2230	
12π	1.1107	1.4069	2.0515	1.3347	1.2609	1.3271	1.2172	1.0477	
15π	1.1375	1.3870	2.0173	1.3742	1.3419	1.2475	1.3247	1.2703	
18π	1.2051	1.3396	2.1918	1.3590	1.2955	1.2941	1.2793	1.2972	
3π	1.1507	1.5757	1.9743	1.3282	1.4211	1.3999	1.2905	1.1934	0.08
6π	1.1231	1.3881	2.0192	1.3912	1.0961	1.1944	1.3573	1.3629	
9π	1.1118	1.4782	1.8734	1.3787	1.3493	1.2140	1.1147	1.2229	
12π	1.1164	1.3044	1.8353	1.3767	1.2607	1.3248	1.2153	1.0474	
15π	1.1387	1.2969	1.8814	1.3676	1.3419	1.2466	1.3246	1.2695	
18π	1.1985	1.2916	2.0204	1.3591	1.2946	1.2923	1.2790	1.2938	
3π	1.1640	1.4789	1.9080	1.3272	1.4222	1.3995	1.2900	1.1927	0.10
6π	1.1401	1.3204	1.9042	1.3836	1.0949	1.1952	1.3571	1.3623	
9π	1.1000	1.3636	1.6916	1.3721	1.3499	1.2130	1.1141	1.2229	
12π	1.1128	1.2870	1.7955	1.3236	1.2606	1.3233	1.2140	1.0472	
15π	1.1122	1.3117	1.7847	1.3628	1.3417	1.2460	1.3247	1.2690	
18π	1.1861	1.3370	1.7443	1.3584	1.2940	1.2911	1.2788	1.2960	
3π	1.1673	1.3097	1.6282	1.3181	1.4281	1.3975	1.2875	1.1906	0.20
6π	1.1574	1.2504	1.4572	1.3511	1.0896	1.1992	1.3558	1.3597	
9π	1.0716	1.2791	1.5640	1.3387	1.3520	1.2075	1.1113	1.1528	
12π	1.0743	1.3045	1.6882	1.3162	1.2592	1.3158	1.2091	1.0463	
15π	1.0989	1.2379	1.6022	1.3357	1.3392	1.2450	1.3248	1.2665	
18π	1.1533	1.2277	1.5251	1.3494	1.2902	1.2856	1.2778	1.2958	

TABLE 24B
AMPLIFICATION FACTORS A_F - STAGE II
INITIAL BRIDGE AND VEHICLE CONDITIONS $\gamma = 0$, $f_c = \pi$

f_B	f_R							M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0
3 π	1.5189	1.8395	1.9298	1.5555	1.3732	1.2540	1.1810	1.1388
6 π	1.1564	2.2854	2.2534	1.3301	1.4166	1.4013	1.2925	1.1946
9 π	1.0950	1.8567	2.7644	1.3467	1.3029	1.3468	1.3365	1.3185
12 π	1.1329	1.6919	2.9653	1.4698	1.1003	1.1916	1.3581	1.3646
15 π	1.0783	1.5486	3.4490	1.3129	1.2651	1.0951	1.2437	1.2764
18 π	1.0799	1.4852	3.4933	1.4003	1.3468	1.2175	1.1166	1.2231
3 π	1.5231	1.7887	1.9124	1.5489	1.3726	1.2540	1.1810	1.1387
6 π	1.1307	2.0054	2.1654	1.3299	1.4180	1.4010	1.2921	1.1944
9 π	1.0944	1.6454	2.4759	1.3427	1.3027	1.3465	1.3365	1.3184
12 π	1.1279	1.6874	2.5971	1.4111	1.0991	1.1924	1.3579	1.3641
15 π	1.0994	1.4015	2.6172	1.3122	1.2645	1.0948	1.2438	1.2765
18 π	1.0935	1.4662	2.5861	1.3941	1.3476	1.2165	1.1160	1.1573
3 π	1.5300	1.7475	1.8951	1.5429	1.3725	1.2529	1.1811	1.1388
6 π	1.1225	1.7946	2.0849	1.3294	1.4193	1.4005	1.2916	1.1942
9 π	1.1056	1.5123	2.2582	1.3386	1.3025	1.3460	1.3366	1.3184
12 π	1.1170	1.5604	2.2145	1.4027	1.0980	1.1932	1.3576	1.3657
15 π	1.0846	1.3997	2.2585	1.3114	1.2639	1.0946	1.2439	1.2766
18 π	1.1056	1.5663	2.2255	1.3880	1.3484	1.2155	1.1154	1.2229
3 π	1.5441	1.6991	1.8697	1.5349	1.3724	1.2524	1.1794	1.1328
6 π	1.1507	1.5757	1.9746	1.3281	1.4211	1.3999	1.2906	1.1933
9 π	1.1168	1.4231	1.9671	1.3321	1.3022	1.3455	1.3368	1.3187
12 π	1.1230	1.3611	2.0197	1.3911	1.0961	1.1944	1.3533	1.3629
15 π	1.0957	1.3645	1.6740	1.3102	1.2630	1.0941	1.2441	1.2768
18 π	1.1118	1.4782	1.7787	1.3786	1.3494	1.2140	1.1146	1.2229
3 π	1.5555	1.6742	1.8530	1.5304	1.3722	1.2521	1.1790	1.1371
6 π	1.1640	1.4790	1.9081	1.3271	1.4223	1.3994	1.2900	1.1926
9 π	1.1237	1.4051	1.7965	1.3277	1.3020	1.3451	1.3368	1.3179
12 π	1.1401	1.3205	1.9042	1.3835	1.0949	1.1952	1.3570	1.3623
15 π	1.1023	1.3121	1.7943	1.3095	1.2625	1.0940	1.2442	1.2771
18 π	1.1000	1.3636	1.7142	1.3719	1.3499	1.2130	1.1146	1.2229
3 π	1.6213	1.6030	1.7718	1.5154	1.3722	1.2504	1.1773	1.1355
6 π	1.1673	1.1391	1.6282	1.3181	1.4282	1.3975	1.2874	1.1606
9 π	1.0956	1.2694	1.5851	1.3037	1.3012	1.3432	1.3372	1.3174
12 π	1.1573	1.2504	1.4572	1.3511	1.0896	1.1993	1.3557	1.3597
15 π	1.0727	1.3027	1.5569	1.3045	1.2614	1.0934	1.2448	1.2778
18 π	1.0716	1.2791	1.5640	1.3386	1.3520	1.2075	1.1112	1.2229

TABLE 24C
AMPLIFICATION FACTORS A_R - STAGE II
INITIAL BRIDGE AND VEHICLE CONDITIONS $\mu = 0, f_c = 1.4\pi$

f_B	f_R								A_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	1.7385	2.5415	2.3230	1.7485	1.4315	1.3233	1.2893	1.2774	0.01
6 π	1.2089	2.2256	2.3578	1.5634	1.3222	1.1757	1.0499	-	
9 π	1.2563	1.3152	3.0526	1.2387	1.4681	1.4436	1.3159	1.2075	
12 π	1.1258	1.5567	3.3866	1.2538	1.3357	1.3275	1.3681	1.3319	
15 π	1.1324	1.7092	3.2891	1.3471	1.1787	1.2576	1.3875	1.3737	
18 π	1.0922	1.7036	3.2611	1.4365	1.0950	1.1911	1.3222	1.3229	
3 π	1.7430	2.4465	2.2735	1.7435	1.4276	1.3224	1.2887	1.2770	0.03
6 π	1.2130	1.9650	2.1393	1.5448	1.3244	1.1766	1.0507	-	
9 π	1.2419	1.3717	2.6711	1.2365	1.4682	1.4428	1.3137	1.2073	
12 π	1.1198	1.6021	2.6073	1.2468	1.3348	1.3266	1.3682	1.3320	
15 π	1.1446	1.5662	2.4889	1.3417	1.1776	1.2577	1.3872	1.3734	
18 π	1.0921	1.7002	2.5980	1.4323	1.0939	1.1908	1.3221	1.3227	
3 π	1.7486	2.3617	2.2267	1.7365	1.4245	1.3198	1.2883	1.2766	0.05
6 π	1.2159	1.7539	1.9478	1.5265	1.3268	1.1770	1.0508	-	
9 π	1.2247	1.4209	2.3719	1.2345	1.4681	1.4418	1.3147	1.2069	
12 π	1.1108	1.5362	2.1791	1.2398	1.3339	1.3255	1.3686	1.3319	
15 π	1.1529	1.4709	2.0166	1.3365	1.1767	1.2580	1.3870	1.3731	
18 π	1.0964	1.5825	2.2040	1.4282	1.0928	1.1905	1.3220	1.3225	
3 π	1.7560	2.2431	2.1620	1.7275	1.4199	1.3174	1.2860	1.2758	0.08
6 π	1.2218	1.5138	1.7474	1.5000	1.3302	1.1784	1.0514	-	
9 π	1.1955	1.4693	2.0631	1.2314	1.4680	1.4406	1.3134	1.2059	
12 π	1.0996	1.5914	1.8929	1.2296	1.3325	1.3240	1.3689	1.3318	
15 π	1.1587	1.4003	2.0331	1.3300	1.1751	1.2584	1.3867	1.3725	
18 π	1.0999	1.2875	2.0010	1.4220	1.0917	1.1900	1.3218	1.3221	
3 π	1.7592	2.1713	2.1221	1.7218	1.4169	1.3157	1.2850	1.2746	0.10
6 π	1.2243	1.3965	1.6859	1.4827	1.3357	1.1793	1.0518	-	
9 π	1.1744	1.4828	1.9350	1.2293	1.4680	1.4397	1.3127	1.2052	
12 π	1.0910	1.5707	1.7756	1.2226	1.3315	1.3230	1.3692	1.3317	
15 π	1.1554	1.3708	1.8993	1.3261	1.1741	1.2586	1.3865	1.3721	
18 π	1.1005	1.3953	1.8928	1.4179	1.0900	1.1897	1.3217	1.3218	
3 π	1.7537	1.8986	1.9639	1.6917	1.4024	1.3078	1.2803	1.2715	0.20
6 π	1.2319	1.2876	1.4654	1.4020	1.3439	1.1834	1.0542	-	
9 π	1.1797	1.1328	1.6583	1.2193	1.4676	1.4355	1.3094	1.2028	
12 π	1.1355	1.2835	1.6620	1.1880	1.3268	1.3179	1.3703	1.3314	
15 π	1.1054	1.3138	1.5301	1.3136	1.1691	1.2596	1.3885	1.3703	
18 π	1.0857	1.3034	1.4875	1.3969	1.0852	1.1884	1.3211	1.3207	

TABLE 24d
AMPLIFICATION FACTORS A_0 = STAGE II
INITIAL BRIDGE AND VEHICLE CONDITIONS $\neq 0$, $f_c = 1.8\pi$

f_B	fR								$\frac{A_0}{A_1}$
	0.40	0.60	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	1.3604	1.7853	1.9759	1.6060	1.2931	1.1642	1.1042	1.0719	0.01
6 π	1.5589	1.8450	2.4567	1.5115	1.4396	1.3454	1.2705	1.2199	
9 π	1.1128	1.6149	2.1808	1.5621	1.3837	1.2902	1.1675	1.0727	
12 π	1.2452	1.3316	2.9381	1.2267	1.4561	1.4339	1.3099	1.2024	
15 π	1.1209	1.5245	3.1809	1.2104	1.3491	1.3458	1.3366	1.2992	
18 π	1.1410	1.5941	3.0016	1.3961	1.2302	1.3111	1.3650	1.3620	
3 π	1.3537	1.7824	1.9750	1.6045	1.2903	1.1653	1.1048	1.0723	0.03
6 π	1.5443	1.7592	2.3632	1.5120	1.4397	1.3458	1.2706	1.2201	
9 π	1.1042	1.4612	2.0836	1.5512	1.3843	1.2900	1.1677	1.0729	
12 π	1.2439	1.3709	2.5564	1.2250	1.4516	1.4329	1.3093	1.2021	
15 π	1.1083	1.3290	2.5130	1.2021	1.3485	1.3447	1.3371	1.2995	
18 π	1.1374	1.6170	2.4725	1.3897	1.2294	1.3097	1.3653	1.3619	
3 π	1.3476	1.7790	1.9744	1.6028	1.2894	1.1619	1.1055	1.0733	0.05
6 π	1.5302	1.5302	2.2778	1.5107	1.4401	1.3450	1.2708	1.2202	
9 π	1.1043	1.3471	1.9925	1.5405	1.3849	1.2894	1.1673	1.0728	
12 π	1.2407	1.4041	2.2751	1.2234	1.4509	1.4317	1.3088	1.2016	
15 π	1.0936	1.4968	2.1563	1.1939	1.3479	1.3434	1.3378	1.2998	
18 π	1.1263	1.5480	2.2743	1.3836	1.2285	1.3081	1.3655	1.3617	
3 π	1.3397	1.7739	1.9728	1.6004	1.2879	1.1611	1.1021	1.0724	0.08
6 π	1.5099	1.6209	2.1637	1.5105	1.4405	1.3449	1.2697	1.2199	
9 π	1.1057	1.3659	1.8655	1.5251	1.3859	1.2890	1.1667	1.0724	
12 π	1.2332	1.5254	2.0137	1.2210	1.4499	1.4301	1.3070	1.2007	
15 π	1.1030	1.5639	1.8615	1.1816	1.3469	1.3416	1.3388	1.2995	
18 π	1.1113	1.4632	2.0165	1.3747	1.2271	1.3058	1.3659	1.3614	
3 π	1.3360	1.7703	-	1.5985	1.2865	1.1605	1.1018	1.0704	0.10
6 π	1.4980	1.5880	2.0957	1.5107	1.4409	1.3448	1.2696	1.2189	
9 π	1.1063	1.3950	1.7897	1.5151	1.3864	1.2887	1.1665	1.0720	
12 π	1.2234	1.5586	1.9126	1.2193	1.4492	1.4290	1.3062	1.1998	
15 π	1.1222	1.5751	1.7952	1.1734	1.3461	1.3404	1.3394	1.3000	
18 π	1.1224	1.4325	1.8670	1.3687	1.2263	1.3043	1.3662	1.3612	
3 π	1.3309	1.7477	1.9635	1.5866	1.2813	1.1571	1.0996	1.0688	0.20
6 π	1.4584	1.5212	1.8347	1.5122	1.4420	1.3442	1.2687	1.2182	
9 π	1.0904	1.2887	1.4866	1.4689	1.3892	1.2876	1.1657	1.0715	
12 π	1.1881	1.4324	1.6727	1.2108	1.4459	1.4236	1.3022	1.1970	
15 π	1.1794	1.3541	1.6719	1.1325	1.3426	1.3346	1.3425	1.3009	
18 π	1.1350	1.2497	1.5617	1.3412	1.2217	1.2966	1.3673	1.3603	

TABLE 24E
AMPLIFICATION FACTORS A_r = Stage II
INITIAL BRIDGE AND VEHICLE CONDITIONS $\neq 0$, $f_c = 2.2\pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	1.3588	1.8461	1.8249	1.3682	1.1931	1.1220	1.0869	1.0677	0.01
6 π	1.6161	1.4196	2.3524	1.6276	1.3703	1.2579	1.2119	1.1922	
9 π	1.2798	2.3454	1.9069	1.5722	1.1910	1.0594	-	-	
12 π	1.1427	1.9677	2.9950	1.5168	1.3372	1.3432	1.2630	1.1798	
15 π	1.2180	1.3389	2.5926	1.2133	1.4308	1.4195	1.3014	1.1966	
18 π	1.1016	1.5577	2.8181	1.1845	1.3573	1.3649	1.2946	1.2592	
3 π	-	-	-	-	-	-	-	-	0.03
6 π	1.6399	1.8944	2.2902	1.6187	1.3671	1.2568	1.2111	1.1916	
9 π	1.2869	2.0906	1.7881	1.5559	1.1919	1.0582	-	-	
12 π	1.1364	1.8747	2.6260	1.5138	1.3395	1.3437	1.2633	1.1800	
15 π	1.2212	1.4024	2.2600	1.2120	1.4301	1.4185	1.3008	1.1962	
18 π	1.0994	1.5552	2.4224	1.1759	1.3568	1.3637	1.2954	1.2597	
3 π	-	-	-	-	-	-	-	-	0.05
6 π	1.6639	1.8702	2.2314	1.6100	1.3646	1.2542	1.2102	1.1909	
9 π	1.2930	1.8390	1.6755	1.5400	1.1932	1.0562	-	-	
12 π	1.1294	1.7822	2.3547	1.5107	1.3415	1.3440	1.2634	1.1802	
15 π	1.2222	1.4699	2.1030	1.2106	1.4295	1.4173	1.3000	1.1958	
18 π	1.1229	1.7874	2.1492	1.1673	1.3562	1.3622	1.2963	1.2600	
3 π	1.-	-	-	-	-	-	-	-	0.08
6 π	1.6991	1.8358	2.1498	1.5980	1.3607	1.2517	1.2076	1.1897	
9 π	1.3004	1.1552	1.5445	1.5166	1.1954	1.0538	-	-	
12 π	1.1181	1.6523	2.0779	1.5058	1.3445	1.3447	1.2634	1.1801	
15 π	1.2208	1.5686	1.9621	1.2085	1.4285	1.4157	1.2987	1.1949	
18 π	1.1520	1.4792	1.8541	1.1546	1.3553	1.3602	1.2976	1.2604	
3 π	1.-	-	-	-	-	-	-	-	0.10
6 π	1.7219	1.8139	2.0996	1.5905	1.3582	1.2500	1.2062	1.1882	
9 π	1.3039	1.4887	1.5320	1.5013	1.1967	1.0523	-	-	
12 π	1.1100	1.5746	1.9511	1.5024	1.3465	1.3451	1.2635	1.1798	
15 π	1.2162	1.5856	1.9000	1.2071	1.4279	1.4147	1.2979	1.1941	
18 π	1.1672	1.4938	1.8091	1.1460	1.3547	1.3589	1.2984	1.2607	
3 π	1.-	-	-	-	-	-	-	-	0.20
6 π	1.8182	1.7152	1.8948	1.5584	1.3466	1.2418	1.2003	1.1839	
9 π	1.3107	1.3344	1.4736	1.4283	1.2054	1.0443	-	-	
12 π	1.0740	1.3261	1.5958	1.4831	1.3561	1.3470	1.2642	1.2762	
15 π	1.1889	1.4151	1.6733	1.1994	1.4250	1.4094	1.2943	1.1916	
18 π	1.2000	1.3577	1.6658	1.1033	1.3516	1.3522	1.3027	1.2622	

TABLE 24F
AMPLIFICATION FACTORS A_F = STAGE II
INITIAL BRIDGE AND VEHICLE CONDITIONS $\neq 0$, $f_c = 2.5\pi$

f_B	f_R								M_R
	0.40	0.60	1.0	2.0	3.0	4.0	5.0	6.0	
3π	-	-	-	-	-	-	-	-	0.01
6π	1.7760	2.5591	2.6320	1.5907	1.4215	1.3338	1.3107	1.3051	
9π	1.5180	1.5609	2.5673	1.5445	1.4129	1.3249	1.2541	1.2049	
12π	1.1634	1.5777	2.4034	1.5508	1.4257	1.3088	1.1748	1.0780	
15π	1.1563	2.2860	2.2536	1.3302	1.4165	1.4014	1.2925	1.1947	
18π	1.1679	1.7957	2.8354	1.1649	1.4138	1.4347	1.3398	1.2466	
3π	-	-	-	-	-	-	-	-	0.03
6π	1.7554	2.4631	2.5527	1.6829	1.4176	1.3325	1.3094	1.3043	
9π	1.5105	1.5481	2.4393	1.5413	1.4116	1.3243	1.2537	1.2047	
12π	1.1091	1.4718	2.1982	1.5440	1.4265	1.3086	1.1748	1.0779	
15π	1.1307	2.0056	2.1654	1.3300	1.4180	1.4010	1.2922	1.1944	
18π	1.1689	1.7190	2.4508	1.1636	1.4145	1.4343	1.3395	1.2469	
3π	-	-	-	-	-	-	-	-	0.05
6π	1.7959	2.3734	2.4769	1.6759	1.4143	1.3299	1.3086	1.3035	
9π	1.5023	1.5375	2.3245	1.5377	1.4105	1.3231	1.2535	1.2045	
12π	1.1593	1.3873	2.0314	1.5298	1.4274	1.3078	1.1744	1.0779	
15π	1.1226	1.7947	2.0849	1.3295	1.4193	1.4005	1.2915	1.1942	
18π	1.1683	1.6430	2.2566	1.1623	1.4151	1.4337	1.3391	1.2462	
3π	-	-	-	-	-	-	-	-	0.08
6π	1.8119	2.2508	2.3700	1.6657	1.4092	1.3270	1.3064	1.3025	
9π	1.4885	1.5243	2.1737	1.5324	1.4087	1.3216	1.2518	1.2038	
12π	1.1496	1.3105	1.8373	1.5092	1.4285	1.3073	1.1734	1.0773	
15π	1.1507	1.5757	1.9745	1.3283	1.9211	1.3998	1.2905	1.1934	
18π	1.1660	1.5317	2.0499	1.1604	1.4161	1.4330	1.3383	1.2457	
3π	-	-	-	-	-	-	-	-	0.10
6π	1.8212	2.1764	2.3030	1.6590	1.4060	1.3251	1.3051	1.3013	
9π	1.4787	1.5164	2.0865	1.5288	1.4077	1.3208	1.2511	1.2028	
12π	1.1405	1.3359	1.7354	1.4959	1.4293	1.3069	1.1731	1.0767	
15π	1.1640	1.4790	1.9080	1.3671	1.4223	1.3994	1.2900	1.1927	
18π	1.1627	1.4649	1.9527	1.1590	1.4167	1.4325	1.3380	1.2453	
3π	-	-	-	-	-	-	-	-	0.20
6π	1.8430	1.8869	2.0245	1.6274	1.3897	1.3155	1.2991	1.2972	
9π	1.4295	1.4762	1.7692	1.5092	1.4017	1.3165	1.2481	1.2005	
12π	1.1152	1.2598	1.3964	1.4352	1.4326	1.3048	1.1714	1.0753	
15π	1.1673	1.3097	1.6283	1.3181	1.4282	1.3975	1.2875	1.1906	
18π	1.1795	1.2682	1.6599	1.1521	1.4199	1.4301	1.3361	1.2439	

TABLE 25A
AMPLIFICATION FACTORS A_F = Stage III
INITIAL BRIDGE AND VEHICLE CONDITIONS $F_0 f_C = 0.5\pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3π	2.1057	2.7376	3.4196	2.6000	2.4659	2.5910	2.4226	2.2367	0.01
6π	2.0935	2.9066	4.2344	2.8282	2.1325	2.3434	2.6238	2.6832	
9π	2.0820	2.8437	4.4708	2.7805	2.5202	2.3696	2.2140	2.3746	
12π	2.0982	2.7112	4.4408	2.6919	2.3600	2.6620	2.4219	2.0589	
15π	2.1787	2.7242	4.1881	2.7690	2.5741	2.5174	2.6173	2.5347	
18π	2.2724	2.6259	4.0465	2.6793	2.5408	2.6343	2.5744	2.6484	
3π	2.0670	2.5108	3.0242	2.5916	2.4721	2.5913	2.4224	2.2364	0.03
6π	2.1017	2.8569	3.3284	2.8114	2.1305	2.3452	2.6234	2.6824	
9π	2.1061	2.5846	3.0143	2.6067	2.5221	2.0979	2.2130	2.3743	
12π	2.0987	2.5175	3.3070	2.6829	2.4275	2.6583	2.4217	2.0582	
15π	2.1809	2.3678	3.2197	2.6950	2.5763	2.5142	2.5815	2.5356	
18π	2.3590	2.4602	3.0884	2.6646	2.5426	2.6322	2.4627	2.6481	
3π	2.0421	2.2645	2.8903	2.5832	2.4776	2.5913	2.4219	2.2363	0.05
6π	2.1072	2.7547	2.8522	2.7937	2.1285	2.3276	2.6230	2.6816	
9π	1.9983	2.5801	2.9529	2.7433	2.5231	2.3678	2.2120	2.3741	
12π	2.1343	2.3176	2.9678	2.6685	2.4287	2.6543	2.4212	2.0576	
15π	2.1939	2.4954	3.0505	2.7103	2.5767	2.5108	2.5798	2.5364	
18π	2.2242	2.4484	3.1185	2.6964	2.5433	2.6293	2.5752	2.6478	
3π	2.0380	2.0485	2.7312	2.5704	2.4858	2.5914	2.4221	2.2351	0.08
6π	2.1276	2.5071	2.6397	2.7660	2.1253	2.3448	2.6226	2.6803	
9π	2.0487	2.4853	2.7612	2.7155	2.5233	2.3657	2.2105	2.3739	
12π	2.0597	2.2641	2.7681	2.6456	2.4299	2.6480	2.4202	2.0568	
15π	2.0389	2.3851	2.7852	2.6793	2.5741	2.5056	2.6182	2.5373	
18π	2.3467	2.3140	2.9415	2.6256	2.5421	2.6237	2.5755	2.6458	
3π	2.0235	2.0948	2.6384	2.5622	2.4908	2.5914	2.4199	2.2340	0.10
6π	2.1473	2.3402	2.6007	2.7471	2.1238	2.3447	2.6221	2.6794	
9π	2.0543	2.4440	2.5869	2.7038	2.5250	2.3639	2.2095	2.3739	
12π	2.0305	2.2827	2.7156	2.6298	2.4301	2.6435	2.4194	2.0563	
15π	2.0626	2.1875	2.7204	2.5855	2.5702	2.5020	2.6181	2.5378	
18π	2.3069	2.3302	2.5900	2.6137	2.5400	2.6191	2.5756	2.6464	
3π	1.9777	2.0816	2.2579	2.5182	2.5143	2.5911	2.4168	2.2311	0.20
6π	2.0620	2.1052	2.3889	2.6752	2.1180	2.3438	2.6196	2.6750	
9π	2.0000	2.3023	2.3511	2.5973	2.5510	2.3505	2.2041	2.2925	
12π	1.9653	2.3034	2.5590	2.4778	2.4261	2.6182	2.4147	2.0533	
15π	2.0607	2.1127	2.5770	2.5544	2.5929	2.4840	2.6150	2.5383	
18π	2.1694	2.2047	2.3545	2.5023	2.5516	2.5856	2.5742	2.6441	

TABLE 25B
AMPLIFICATION FACTORS A_F - STAGE III
INITIAL BRIDGE AND VEHICLE CONDITIONS $\neq 0$
 $f_C = \pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	2.5447	2.5629	3.1313	3.1097	2.6000	2.2952	2.1231	2.0235	0.01
6 π	2.1057	2.8378	3.4196	2.6000	2.4660	2.5910	2.4226	2.2336	
9 π	2.1380	2.7350	3.9284	2.5817	2.4790	2.6633	2.5823	2.4689	
12 π	2.0935	2.9066	4.0955	2.8282	2.1325	2.3455	2.6213	2.6832	
15 π	2.0908	2.9264	4.4362	2.6473	2.4299	2.0835	2.4480	2.5288	
18 π	2.0822	2.8436	4.4707	2.7804	2.5202	2.3696	2.2140	2.3745	
3 π	2.5341	2.4384	3.0544	3.0907	2.5941	2.2937	2.1227	2.0230	0.03
6 π	2.0670	2.5109	3.0242	2.5915	2.4722	2.5737	2.4225	2.2365	
9 π	2.1346	2.6250	3.3080	2.5726	2.4795	2.6619	2.5814	2.4690	
12 π	2.1017	2.8570	3.3284	2.8113	2.1305	2.2078	2.6234	2.6823	
15 π	2.0544	2.5346	3.2016	2.6403	2.4242	2.0824	2.4478	2.5286	
18 π	2.1060	2.5847	3.1667	2.7639	2.5220	2.3689	2.2128	2.3086	
3 π	2.5263	2.3302	2.9815	3.0721	2.5896	2.2893	2.1282	2.0226	0.05
6 π	2.0420	2.2645	2.8903	2.5831	2.4776	2.5912	2.4219	2.2362	
9 π	2.1280	2.5065	2.9149	2.5643	2.4798	2.6602	2.5806	2.4692	
12 π	2.1072	2.7547	2.8522	2.7936	2.1284	2.2095	2.6230	2.6815	
15 π	2.0319	2.5436	2.8523	2.6327	2.4187	2.0814	2.4477	2.5284	
18 π	2.0891	2.5577	2.9875	2.5890	2.5231	2.3678	2.2118	2.3740	
3 π	2.5171	2.1915	2.8774	3.0453	2.5829	2.2855	2.1169	2.0211	0.08
6 π	2.0379	2.0486	2.7305	2.5701	2.4858	2.5914	2.4205	2.2350	
9 π	2.1155	2.2704	2.5393	2.5535	2.4802	2.6578	2.5792	2.4692	
12 π	2.1275	2.5023	2.6397	2.7660	2.1251	2.3450	2.6226	2.6803	
15 π	2.0503	2.4243	2.6951	2.6200	2.4098	2.0796	2.4476	2.5281	
18 π	2.0487	2.4734	2.5826	2.7155	2.5233	1.9557	2.2104	2.3647	
3 π	2.5118	2.1111	2.8119	3.0281	2.5784	2.2830	2.1153	2.0179	0.10
6 π	2.0235	2.0948	2.6384	2.5622	2.4909	2.5914	2.4199	2.2339	
9 π	2.1070	2.2216	2.2530	2.5438	2.4803	2.6562	2.5783	2.4641	
12 π	2.1473	2.3404	2.6007	2.7470	2.1238	2.3447	2.6221	2.6794	
15 π	2.0631	2.3818	2.2735	2.6108	2.4035	2.0787	2.4475	2.5280	
18 π	2.0543	2.4440	2.6013	2.5527	2.5252	2.3638	2.2094	2.3642	
3 π	2.5182	1.8010	2.5271	2.9524	2.5566	2.2709	2.1073	2.0123	0.20
6 π	1.9773	1.9193	2.2578	2.5181	2.5143	2.5910	2.4168	2.2310	
9 π	1.9929	2.3323	2.2317	2.4873	2.4805	2.6479	2.5739	2.4692	
12 π	2.0619	2.1052	2.3838	2.6751	2.0730	2.2230	2.6195	2.6750	
15 π	1.9811	2.3056	2.4786	2.5602	2.3789	2.0741	2.4465	2.5270	
18 π	1.9999	2.3021	2.3511	2.5973	2.5511	2.3504	2.2040	2.3732	

TABLE 25C
AMPLIFICATION FACTORS A_p - STAGE III
INITIAL BRIDGE AND VEHICLE CONDITIONS $\neq 0$
 $f_c = 1.4\pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	2.3602	2.8500	3.1544	2.7961	2.2271	1.9838	1.8759	1.8206	0.01
6 π	2.2196	2.8836	3.4373	3.0877	2.5469	2.2183	1.9931	1.6521	
9 π	2.2617	1.8760	4.0829	2.4482	2.5878	2.7042	2.5130	2.3106	
12 π	2.1835	2.3828	4.3873	2.4121	2.5265	2.6571	2.6253	2.4894	
15 π	2.0994	2.8106	3.3260	2.7353	2.2774	2.4866	2.6310	2.6708	
18 π	2.0550	2.9952	4.3164	2.8283	2.1835	2.2917	2.5782	2.6279	
3 π	2.3463	2.6816	3.0684	2.7863	2.2212	1.9834	1.8764	1.8209	0.03
6 π	2.2163	2.5595	3.1690	3.0565	2.5442	2.2175	1.9930	1.6515	
9 π	2.2365	1.8976	3.4471	2.4398	2.5912	2.7036	2.5122	2.3102	
12 π	2.1657	2.4994	3.3953	2.4049	2.5258	2.6553	2.6244	2.4887	
15 π	2.1053	2.7335	3.2643	2.7219	2.2765	2.4857	2.6331	2.6703	
18 π	2.0612	2.8582	3.3046	2.8187	2.1769	2.2903	2.5781	2.6276	
3 π	2.3338	2.5340	2.9897	2.7762	2.2172	1.9788	1.8757	1.8208	0.05
6 π	2.2131	2.3080	2.9394	3.0262	2.5420	2.2154	1.9922	1.6509	
9 π	2.2006	1.9098	3.1186	2.4315	2.5940	2.7029	2.5111	2.3095	
12 π	2.1433	2.4493	2.8526	2.3957	2.5248	2.6532	2.6237	2.4878	
15 π	2.1217	2.6355	2.7340	2.7072	2.2755	2.4849	2.6305	2.6697	
18 π	2.0829	2.7689	2.8292	2.8080	2.1700	2.2889	2.5782	2.6572	
3 π	2.3137	2.3470	2.8821	2.7613	2.2111	1.9757	1.8712	1.8194	0.08
6 π	2.2085	2.0337	2.6941	2.9818	2.5385	2.2136	1.9902	1.6487	
9 π	2.1370	2.0890	2.7737	2.4191	2.5980	2.7021	2.5086	2.3076	
12 π	2.1018	2.3705	2.4310	2.3786	2.5232	2.6504	2.6228	2.4863	
15 π	2.1443	2.4566	2.5123	2.6884	2.2740	2.4835	2.6302	2.6687	
18 π	2.0982	2.4266	2.7147	2.7902	2.1636	2.2868	2.5783	2.6267	
3 π	2.2966	2.2425	2.8173	2.7516	2.2070	1.9736	1.8700	1.8167	0.10
6 π	2.2060	1.8997	2.6018	2.9527	2.5364	2.2125	1.9893	1.6464	
9 π	2.0899	2.2103	2.6257	2.4108	2.6003	2.7016	2.5075	2.3065	
12 π	2.0829	2.3625	2.2401	2.3704	2.5221	2.6484	2.6223	2.4853	
15 π	2.1423	2.2987	2.4723	2.6786	2.2729	2.4826	2.6300	2.6680	
18 π	2.1048	2.4439	2.6821	2.7770	2.1601	2.2854	2.5783	2.6263	
3 π	2.1699	1.8892	2.5583	2.7008	2.1873	1.9633	1.8641	1.8129	0.20
6 π	2.2136	1.6383	2.2587	2.8158	2.5205	2.2063	1.9858	1.6404	
9 π	1.9808	2.2634	2.2580	2.3699	2.6144	2.6970	2.5022	2.3015	
12 π	2.0347	2.3493	2.2903	2.3131	2.5189	2.6384	2.6193	2.4820	
15 π	1.9711	2.2757	2.3732	2.6202	2.2671	2.4776	2.6282	2.6647	
18 π	2.0028	2.1338	2.4468	2.7075	2.1378	2.2075	2.5779	2.6243	

TABLE 25D
AMPLIFICATION FACTORS A_F - STAGE VII
INITIAL BRIDGE AND VEHICLE CONDITIONS $\neq 0$
 $f_C = 1.8\pi$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	1.4665	1.4029	-	-	-	-	1.2289	1.1842	0.01
6 π	2.6052	2.5730	3.6094	3.1016	2.7032	2.4183	2.2412	2.1309	
9 π	2.0548	2.2636	3.1833	2.9820	2.5291	2.2650	2.0500	1.9010	
12 π	2.2741	2.8797	4.0017	2.4047	2.5981	2.7238	2.5322	2.3284	
15 π	2.1823	2.2840	4.2196	2.3183	2.5431	2.6828	2.6021	2.4642	
18 π	2.1449	2.6591	4.1234	2.7429	2.3666	2.5791	2.5692	2.6193	
3 π	1.4377	1.8895	-	-	-	-	1.2304	1.1849	0.03
6 π	2.5782	2.4190	3.4440	3.0873	2.6977	2.4167	2.2406	2.1307	
9 π	2.9407	2.0742	3.0318	2.4599	2.5256	2.2632	2.0496	1.9007	
12 π	2.2595	1.8860	3.7176	2.3971	2.6000	2.7224	2.5314	2.3280	
15 π	2.1609	2.1996	3.3288	2.3101	2.5435	2.6806	2.6019	2.4639	
18 π	2.1325	2.7272	3.2806	2.2784	2.3653	2.5766	2.5694	2.6191	
3 π	1.4136	1.8758	-	-	-	-	1.2315	1.1868	0.05
6 π	2.5518	2.2932	3.2957	3.0731	2.6931	2.4129	2.2397	2.1303	
9 π	2.0348	1.9331	2.9053	2.9383	2.5224	2.2605	2.0484	1.9002	
12 π	2.2394	1.8827	3.0087	2.3695	2.6014	2.7215	2.5302	2.3273	
15 π	2.1317	2.3561	2.8419	2.2999	2.5437	2.6781	2.6016	2.4636	
18 π	2.1088	2.6637	2.9187	2.7147	2.3638	2.5740	2.5699	2.6188	
3 π	1.3850	1.8563	-	-	-	-	1.2278	1.1869	0.08
6 π	2.5137	2.1424	3.1007	3.0522	2.6862	2.4094	2.2355	2.1386	
9 π	2.0301	1.8851	2.7282	2.9080	2.5175	2.2376	2.0458	1.8986	
12 π	2.1970	2.1733	2.7112	2.3781	2.6041	2.7196	2.5280	2.3256	
15 π	2.1156	2.2985	2.4656	2.2849	2.5439	2.6747	2.6010	2.4626	
18 π	2.0755	2.4552	2.4984	2.6983	2.3615	2.5701	2.5707	2.6183	
3 π	-	1.8440	2.0115	-	-	-	1.2277	1.1034	0.10
6 π	2.4909	2.0603	2.9861	3.0388	2.6816	2.4070	2.2340	2.1259	
9 π	2.0276	1.8652	2.6227	2.8880	2.5142	2.2556	2.0446	1.8973	
12 π	2.1600	2.3133	2.5907	2.3705	2.6069	2.7184	2.5267	2.3242	
15 π	2.1152	2.3789	2.3863	2.2755	2.5439	2.6723	2.6007	2.4619	
18 π	2.0742	2.2803	2.3945	2.6854	2.3599	2.5675	2.5709	2.6178	
3 π	-	1.7871	2.2252	-	-	-	1.2273	1.1829	0.20
6 π	2.4326	1.7638	2.5543	2.9709	2.6582	2.3950	2.2262	2.1209	
9 π	2.0138	1.6472	2.2205	2.7931	2.4973	2.2458	2.0388	1.8932	
12 π	2.0169	2.3476	2.2567	2.3325	2.6159	2.7116	2.5206	2.3195	
15 π	2.0763	2.4141	2.3003	2.2161	2.5428	2.6603	2.5989	2.4592	
18 π	2.0103	2.2709	2.3229	2.6170	2.3539	2.5544	2.5760	2.6158	

TABLE 25 E
AMPLIFICATION FACTORS A_F - STAGE III
INITIAL BRIDGE AND VEHICLE CONDITIONS $\neq 0$

$$f_C = 2.2\pi$$

f_B	f_R								M_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	1.1618	1.6124	-	-	-	-	-	-	0.01
6 π	2.5774	2.5983	3.4628	3.1047	2.5281	2.2386	2.0976	2.0227	
9 π	2.3061	3.0527	3.0426	3.1212	2.4336	2.1166	1.8519	1.7284	
12 π	2.0367	2.5656	3.9368	2.8668	2.2427	2.3842	2.2554	2.0562	
15 π	2.2566	1.8750	3.7430	2.3725	2.5909	2.7229	2.5377	2.3354	
18 π	2.1638	2.2692	3.4074	2.2556	2.5550	2.7036	2.5630	2.4268	
3 π	1.1936	1.5866	-	-	-	-	-	-	0.03
6 π	2.5839	2.4907	3.3351	2.0843	2.5204	2.2361	2.0965	2.0218	
9 π	2.3042	2.6779	2.8803	3.0913	2.4297	2.1136	1.8500	1.7273	
12 π	2.0254	2.4173	3.85035	2.8545	2.4259	2.3864	2.1027	2.0968	
15 π	2.2486	1.9070	3.1323	2.3642	2.5929	2.7213	2.5366	2.3347	
18 π	2.1532	2.3559	3.0779	2.2456	2.5554	2.7012	2.5629	2.4267	
3 π	1.2267	1.5653	-	-	-	-	-	-	0.05
6 π	2.5898	2.3907	3.2183	3.0646	2.5146	2.2306	2.0450	2.0209	
9 π	2.3020	2.3945	2.7324	3.0623	2.4263	2.1090	1.8479	1.7262	
12 π	2.0138	2.2768	3.1974	2.8421	2.4247	2.3882	2.2567	2.0971	
15 π	2.2306	1.9402	2.8326	2.3569	2.5946	2.7196	2.5352	2.3339	
18 π	2.1624	2.3104	2.8413	2.2387	2.5555	2.6984	2.5628	2.4265	
3 π	1.2782	1.5414	-	-	-	-	-	-	0.08
6 π	2.5952	2.2544	3.0609	3.0358	2.5048	2.2253	2.0894	2.0187	
9 π	2.2976	2.0670	2.5504	3.0204	2.4214	2.1036	1.8429	1.7234	
12 π	1.9970	2.0911	2.8763	2.8232	2.4230	2.3912	2.2568	2.0969	
15 π	2.1967	2.2255	2.6567	2.3459	2.5969	2.7170	2.5326	2.3319	
18 π	2.1641	2.2110	2.4703	2.2252	2.5555	2.6945	2.5624	2.4255	
3 π	1.3.39	1.5301	-	-	-	-	-	-	0.10
6 π	2.5963	2.1714	2.9662	3.0166	2.4986	2.2219	2.0871	2.0153	
9 π	2.2945	1.9702	2.4998	2.9929	2.4178	2.1000	1.8405	1.7207	
12 π	1.9855	1.9829	2.7260	2.8106	2.4218	2.3931	2.2573	2.0963	
15 π	2.1633	2.3552	2.5727	2.3385	2.5983	2.7153	2.5311	2.3305	
18 π	2.1600	2.3271	2.4088	2.2139	2.5553	2.6919	2.5622	2.4249	
3 π	1.5030	-	-	-	-	-	-	-	0.20
6 π	2.5907	1.8385	2.5932	2.9244	2.4678	2.2051	2.0759	2.0077	
9 π	2.2991	1.6659	2.2801	2.8606	2.4015	2.0821	1.8285	1.7126	
12 π	1.9488	1.6244	2.2697	2.7462	2.4154	2.4019	2.2605	2.0975	
15 π	2.0315	2.3564	2.2521	2.3033	2.6042	2.7094	2.5239	2.3251	
18 π	2.0949	2.4140	2.2958	2.1543	2.5533	2.6796	2.5611	2.4223	

TABLE 25F
AMPLIFICATION FACTORS A_F - STAGE III
INITIAL BRIDGE AND VEHICLE CONDITIONS $\phi = 0$
 $f_c = 2.5 \pi$

f_B	f_R								K_R
	0.40	0.80	1.0	2.0	3.0	4.0	5.0	6.0	
3 π	1.4958	1.9087	1.6574	1.2108	1.1391	1.1216	1.1111	1.1050	0.01
6 π	2.5825	3.0135	3.5865	2.9847	2.4215	2.1730	2.0629	2.0064	
9 π	2.5652	2.3220	3.7017	3.1338	2.6809	2.4028	2.2295	2.1205	
12 π	2.1252	2.2456	3.4166	3.0100	2.5943	2.3042	2.0752	1.9234	
15 π	2.1057	2.8380	3.4197	2.5996	2.4658	2.5910	2.4226	2.2366	
18 π	2.2237	2.2972	3.9612	2.2756	2.5981	2.7620	2.5982	2.4055	
3 π	1.4738	1.8511	1.6223	1.2043	1.1354	1.1213	1.1111	1.1050	0.03
6 π	2.5743	2.8185	3.4458	2.9694	2.4142	2.1716	2.0619	2.0059	
9 π	2.5453	2.2441	3.5011	3.1166	2.6742	2.4002	2.2282	2.1196	
12 π	2.1182	2.0978	3.1630	2.9837	2.5910	2.3023	2.0744	1.4296	
15 π	2.0670	2.5110	3.0243	2.5916	2.4721	2.5912	2.4224	2.2365	
18 π	2.2146	2.1830	3.2877	2.2688	2.6007	2.7608	2.5973	2.4049	
3 π	1.4565	1.7984	1.5894	1.1976	1.1330	1.1180	1.1117	1.1054	0.05
6 π	2.5669	2.6471	3.7174	2.9546	2.4087	2.1661	2.0608	2.0053	
9 π	2.5255	2.1718	3.3242	3.0991	2.6682	2.3958	2.2269	2.1189	
12 π	2.1089	1.9803	2.9582	2.9583	2.5878	2.2991	2.0731	1.9223	
15 π	2.0420	2.2627	2.8903	2.5832	2.4777	2.5913	2.4217	2.2352	
18 π	2.1970	2.1116	2.9590	2.2620	2.6029	2.7593	2.5961	2.4043	
3 π	1.4363	1.7262	1.5432	1.1880	1.1290	1.1158	1.1083	1.1031	0.08
6 π	2.5526	2.4287	3.1445	2.9330	2.4000	2.1616	2.0561	2.0032	
9 π	2.4960	2.0712	3.0980	3.0729	2.6591	2.3908	2.2223	2.1167	
12 π	2.0921	1.8553	2.7192	2.9008	2.5829	2.2957	2.0700	1.9207	
15 π	2.0378	2.0485	2.7312	2.5704	2.4858	2.5914	2.4204	2.2351	
18 π	2.1648	2.2356	2.7190	2.2518	2.6061	2.7572	2.5938	2.4426	
3 π	1.4265	1.6834	1.5148	1.1817	1.1259	1.1142	1.1073	1.1019	0.10
6 π	2.5395	2.3055	3.0411	2.9183	2.3943	2.1586	2.0541	2.0003	
9 π	2.4769	2.0087	2.9696	3.0563	2.6531	2.3878	2.2202	2.1141	
12 π	2.0799	1.8335	2.5925	2.8966	2.5797	2.2933	2.0688	1.9189	
15 π	2.0235	2.0948	2.6384	2.5621	2.4909	2.5913	2.4199	2.2339	
18 π	2.1383	2.2945	2.6021	2.2450	2.6081	2.7557	2.5926	2.4012	
3 π	1.4086	1.5243	-	1.1486	1.1126	1.1068	1.1025	1.0986	0.20
6 π	2.4288	1.8842	2.6392	2.8471	2.3660	2.1437	2.0449	1.9941	
9 π	2.4204	1.7383	2.5034	2.9719	2.6227	2.3720	2.2104	2.1073	
12 π	2.0573	1.6024	2.1709	2.7862	2.5628	2.2817	2.0619	1.9142	
15 π	1.9778	2.0817	2.2579	2.5171	2.5142	2.5906	2.4168	2.2310	
18 π	2.0506	2.2674	2.2288	2.2136	2.6162	2.7511	2.5864	2.3966	

Table 26 -

Expressions for D_x and ρ_o in terms of dimensionless parameters α_s, β_s and ξ_s .

Line	Term.	Equation
1	D_x of orthotropic plate.	$D_x \big _{\text{orthotropic plate}} = D_x \big _{\text{plate}} + D_x \big _{\text{stiffeners.}}$
2	e_x - depth of c.G. from top of slab.	$e_x = \frac{H}{2} \left[\frac{\alpha_s^2 + \beta_s (1 - \alpha_s)^2}{\alpha_s + \beta_s (1 - \alpha_s)} \right]$
3	$D_x \big _{\text{plate}}$	$D_x \big _{\text{plate}} = \frac{EH^3}{12(1-\nu)} \cdot \alpha_s^3 \cdot \left[1 + 12 \left(\frac{e_x}{h} - \frac{1}{2} \right)^2 \right].$ in which e_x is given in line 2.
4	$D_x \big _{\text{stiffeners}}$	$D_x \big _{\text{stiffeners}} = E \cdot \beta_s \frac{H^3}{12} \left[(1 - \alpha_s)^3 + 12(1 - \alpha_s) \left(\frac{1 - \alpha_s}{2} - \frac{e_x}{H} \right)^2 \right]$ e_x - given in line 2.
5	ρ_o - mass density of plate per unit area.	$\rho_o = \frac{\rho H}{g} \left[\alpha_s + \beta_s (1 - \alpha_s) \right]$

Table 27. Effect of α_s and β_s on maximization of frequency coefficients

α_s	β_s	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	β_s	Maximum Coefficients
		$K_F (H. \sqrt{\frac{E_g}{\rho}} \cdot m \frac{\pi}{2})$										
	0.10	0.3110	0.3218	0.3181	0.3127	0.3076	0.3032	0.2994	0.2962	0.2937	0.19	0.321830
	0.15	0.2866	0.3121	0.3153	0.3130	0.3092	0.3052	0.3014	0.2979	0.2947	0.29	0.315313
	0.20	0.2641	0.2987	0.3081	0.3095	0.3078	0.3050	0.3018	0.2986	0.2954	0.40	0.309495
	0.25	0.2453	0.2848	0.2990	0.3039	0.3045	0.3033	0.3011	0.2984	0.2957	0.47	0.304612
	0.30	0.2303	0.2717	0.2894	0.2971	0.3000	0.3005	0.2995	0.2977	0.2955	0.57	0.300531
	0.35	0.2192	0.2603	0.2800	0.2900	0.2950	0.2970	0.2973	0.2965	0.2951	0.67	0.297345
	0.40	0.2119	0.2508	0.2715	0.2882	0.2898	0.2932	0.2948	0.2957	0.2945	0.78	0.295090
	0.45	0.2079	0.2437	0.2644	0.2770	0.2847	0.2895	0.2922	0.2935	0.2937	0.89	0.293789
	0.50	0.2111	0.2390	0.2588	0.2717	0.2803	0.2859	0.2896	0.2918	0.2930	0.98	0.293412

Table 28. Effect of α_B , β_B and ξ_B on Maximisation of Mass

ratio coefficients K_M .

$$\left(\frac{\delta_c}{a^4 \rho} \right)$$

Maximum

α_B	β_B	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	β_B	K_M
						$\xi_B = 1.0$						
0.10		2.9296	2.8392	2.6398	2.4681	2.3302	2.2192	2.1286	2.0534	1.9897	0.12	2.945110
0.15		1.5193	1.6219	1.5770	1.5092	1.4434	1.3880	1.3343	1.2903	1.2520	0.19	1.622330
0.20		0.9145	1.0435	1.0537	1.0309	0.9992	0.9668	0.9367	0.9080	0.8825	0.26	1.056000
0.25		0.6072	0.7229	0.7517	0.7500	0.7365	0.7188	0.7000	0.6815	0.6639	0.33	0.753215
0.30		0.4319	0.5285	0.5621	0.5701	0.5665	0.5577	0.5464	0.5343	0.5221	0.41	0.570110
0.35		0.3231	0.4027	0.4358	0.4482	0.4502	0.4468	0.4405	0.4328	0.4245	0.48	0.450318
0.40		0.2521	0.3170	0.3477	0.3619	0.3671	0.3672	0.3644	0.3599	0.3544	0.55	0.367629
0.45		0.2044	0.2564	0.2840	0.2986	0.3056	0.3081	0.3077	0.3055	0.3022	0.63	0.308231
0.50		0.1717	0.2127	0.2367	0.2508	0.2588	0.2628	0.2641	0.2638	0.2622	0.72	0.264231
						$\xi_B = 3.0$						
0.10		3.5245	3.6924	3.6267	3.5353	3.4513	3.3791	3.3179	3.2658	~~3.2273~~	0.19	3.693530
0.15		1.7283	1.9470	1.9708	1.9492	1.9215	1.8786	1.8449	1.8142	1.7867	0.28	1.971650
0.20		0.9971	1.1810	1.2263	1.2275	1.2149	1.1975	1.1779	1.1583	1.1396	0.35	1.229970
0.25		0.6403	0.7841	0.8320	0.8438	0.8403	0.8302	0.8173	0.8112	0.7943	0.42	0.843973
0.30		0.4472	0.5612	0.6475	0.6250	0.6286	0.6252	0.6184	0.6098	0.6005	0.49	0.628595
0.35		0.3296	0.4185	0.4594	0.4768	0.4849	0.4855	0.4824	0.4773	0.4711	0.56	0.485825
0.40		0.2517	0.3240	0.3592	0.3721	0.3855	0.3883	0.3877	0.3851	0.3811	0.63	0.388404
0.45		0.2053	0.2594	0.2892	0.3059	0.3148	0.3190	0.3200	0.3191	0.3169	0.70	0.320026
0.50		0.1721	0.2139	0.2390	0.2541	0.2631	0.2681	0.2703	0.2707	0.2698	0.78	0.270742

Table 29. Comparison of Optimum Coefficients of frequency and mass ratios by theoretical (table 27 and 28) and linear relationship between α_s , β_s and ξ_s (Equation 102).

α_s	Frequency Coefficients		Mass Coefficients $\xi_s = 1.0$		Mass Coefficients $\xi_s = 3.0$	
	Table 27	Equation 102	Table 28	Equation 102	Table 28	Equation 102
0.10	0.321837	0.321820	2.945110	2.935030	3.693530	3.689110
0.15	0.315313	0.315271	1.622330	1.620320	1.971650	1.970660
0.20	0.309495	0.309495	1.056600	1.055670	1.229970	1.229830
0.25	0.304612	0.304544	0.753215	0.753098	0.843973	0.843973
0.30	0.300531	0.300466	0.570110	0.570033	0.628595	0.628484
0.35	0.297345	0.297298	0.450318	0.450279	0.485825	0.485492
0.40	0.295090	0.295069	0.367729	0.367616	0.388404	0.388022
0.45	0.293789	0.293738	0.308231	0.308231	0.320026	0.320026
0.50	-	-	0.264231	0.264186	0.270742	0.270403

TABLE 30

Comparison of frequency coefficients of optimum sections by
 methods of frequency and mass-ratio maximizations

αS	Frequency by Maximum Fre- quency Approach	Frequencies by Maximum Mass Ratio Approach					
		$\xi_B = 0-1$	1.5	2	2.5	3	3.5
.1	.3218	.3194	.3201	.3207	.3211	.3214	.3216
.15	.3153	.3129	.3137	.3142	.3147	.3150	.3152
.20	.3095	.3072	.3079	.3085	.3089	.3092	.3094
.25	.3045	.3022	.3030	.3035	.3040	.3043	.3044
.30	.3005	.2980	.2988	.2994	.2999	.3002	.3004
.35	.2973	.2946	.2955	.2961	.2966	.2970	.2972

Table 31(a) Rigidities of Optimum Sections in Units, $\alpha_B = 0.10, 0.15$ and 0.20

$$\frac{D_x}{E} \times 10^5$$

$$\frac{D_y}{E} \times 10^5$$

ξ_B	$\frac{D_x}{E} \times 10^5$						$\frac{D_y}{E} \times 10^5$					
	6	7	8	9	10	11	6	7	8	9	10	11
	$\alpha_B = 0.10$											
1.0	0.2593	0.0498	0.0791	0.1180	0.1680	0.2305	0.3068	0.3092	0.4910	0.7329	1.0435	1.4314
1.5	0.2683	0.0515	0.0818	0.1221	0.1739	0.2385	0.3175	0.3145	0.4994	0.7455	1.0615	1.4560
2.0	0.2771	0.0532	0.0845	0.1261	0.1796	0.2463	0.3279	0.3196	0.5075	0.7576	1.0787	1.4797
2.5	0.2857	0.0549	0.0871	0.1300	0.1851	0.2540	0.3380	0.3245	0.5154	0.7693	1.0953	1.5025
3.0	0.2941	0.0565	0.0897	0.1339	0.1906	0.2614	0.3480	0.3293	0.5229	0.7805	1.1113	1.5245
3.5	0.3024	0.0581	0.0922	0.1376	0.1959	0.2688	0.3577	0.3339	0.5302	0.7914	1.1268	1.5457
	$\alpha_B = 0.15$											
1.0	0.1072	0.0206	0.0327	0.0488	0.0695	0.0953	0.1268	0.1508	0.3157	0.4712	0.6709	0.9204
1.5	0.1109	0.0213	0.0338	0.0505	0.0718	0.0985	0.1312	0.2022	0.3210	0.4792	0.6823	0.9359
2.0	0.1144	0.0220	0.0349	0.0521	0.0741	0.1017	0.1354	0.2054	0.3261	0.4868	0.6931	0.9508
2.5	0.1179	0.0226	0.0359	0.0537	0.0764	0.1048	0.1395	0.2085	0.3310	0.4942	0.7036	0.9651
3.0	0.1213	0.0233	0.0370	0.0552	0.0786	0.1078	0.1435	0.2114	0.3358	0.5012	0.7136	0.9789
3.5	0.1246	0.0239	0.0380	0.0567	0.0807	0.1108	0.1474	0.2143	0.3403	0.5080	0.7233	0.9922
	$\alpha_B = 0.20$											
1.0	0.0563	0.0108	0.0172	0.0256	0.0365	0.0500	0.0666	0.1440	0.2287	0.3413	0.4860	0.6667
1.5	0.0581	0.0112	0.0177	0.0265	0.0377	0.0517	0.0688	0.1464	0.2325	0.3470	0.4941	0.6777
2.0	0.0600	0.0115	0.0183	0.0273	0.0389	0.0533	0.0709	0.1487	0.2361	0.3524	0.5018	0.6883
2.5	0.0617	0.0119	0.0188	0.0281	0.0400	0.0549	0.0730	0.1509	0.2396	0.3576	0.5092	0.6984
3.0	0.0635	0.0122	0.0194	0.0289	0.0411	0.0564	0.0751	0.1530	0.2429	0.3626	0.5163	0.7082
3.5	0.0652	0.0125	0.0199	0.0297	0.0422	0.0579	0.0771	0.1550	0.2461	0.3674	0.5231	0.7176

Table 31(b) Rigidities of Optimum Sections in Units, $\alpha_B = 0.25, 0.30$ and 0.35

α_B	$\frac{D_x}{E} \times 10^5$						$\frac{\sqrt{D_x D_y}}{E} \times 10^3$					
	6	7	8	9	10	11	6	7	8	9	10	11
	$\alpha_B = 0.25$											
$\frac{D_x}{D_y}$												
1.0 0.0337	0.0065	0.0103	0.0153	0.0218	0.0300	0.0399	0.1115	0.1770	0.2642	0.3761	0.5160	0.6868
1.5 0.0348	0.0067	0.0106	0.0158	0.0225	0.0309	0.0412	0.1133	0.1798	0.2685	0.3822	0.5243	0.6979
2.0 0.0359	0.0069	0.0109	0.0163	0.0232	0.0319	0.0424	0.1150	0.1826	0.2726	0.3881	0.5323	0.7085
2.5 0.0369	0.0071	0.0113	0.0168	0.0239	0.0328	0.0437	0.1166	0.1852	0.2765	0.3937	0.5400	0.7188
3.0 0.0379	0.0073	0.0116	0.0173	0.0246	0.0337	0.0449	0.1182	0.1877	0.2803	0.3990	0.5474	0.7285
3.5 0.0389	0.0075	0.0119	0.0177	0.0252	0.0346	0.0460	0.1198	0.1902	0.2839	0.4042	0.5545	0.7380
	$\alpha_B = 0.30$											
1.0 0.0220	0.0042	0.0067	0.0100	0.0142	0.0195	0.0260	0.0900	0.1429	0.2134	0.3038	0.4167	0.5547
1.5 0.0227	0.0044	0.0069	0.0103	0.0147	0.0202	0.0268	0.0914	0.1452	0.2168	0.3086	0.4233	0.5635
2.0 0.0234	0.0045	0.0071	0.0106	0.0151	0.0208	0.0276	0.0928	0.1474	0.2200	0.3132	0.4297	0.5719
2.5 0.0240	0.0046	0.0073	0.0109	0.0156	0.0214	0.0284	0.0941	0.1494	0.2231	0.3176	0.4357	0.5799
3.0 0.0247	0.0047	0.0075	0.0112	0.0160	0.0219	0.0292	0.0954	0.1514	0.2260	0.3219	0.4415	0.5876
3.5 0.0253	0.0049	0.0077	0.0115	0.0164	0.0225	0.0299	0.0966	0.1533	0.2289	0.3259	0.4471	0.5950
	$\alpha_B = 0.35$											
1.0 0.0152	0.0029	0.0046	0.0069	0.0099	0.0135	0.0180	0.0749	0.1190	0.1776	0.2529	0.3469	0.4617
1.5 0.0157	0.0030	0.0048	0.0071	0.0102	0.0140	0.0186	0.0761	0.1208	0.1804	0.2568	0.3523	0.4689
2.0 0.0162	0.0031	0.0049	0.0074	0.0105	0.0144	0.0191	0.0772	0.1226	0.1830	0.2605	0.3574	0.4757
2.5 0.0166	0.0032	0.0051	0.0076	0.0108	0.0148	0.0197	0.0783	0.1243	0.1855	0.2641	0.3623	0.4822
3.0 0.0170	0.0033	0.0052	0.0078	0.0110	0.0152	0.0202	0.0793	0.1259	0.1879	0.2675	0.3670	0.4885
3.5 0.0175	0.0034	0.0053	0.0080	0.0113	0.0155	0.0207	0.0802	0.1274	0.1902	0.2708	0.3715	0.4945

Table 32. Rigidities of Optimum Sections giving maximum frequencies.

$h = 6 \text{ in. to } 11 \text{ in.}$

$$\frac{D_x D_y}{E} \times 10^3, h =$$

$$\frac{D_x}{E} \times 10^5, h =$$

α_s	$\frac{D_x}{D_y} \times 10^3$	6	7	8	9	10	11	6	7	8	9	10	11
0.10	0.3262	0.0626	0.0995	0.1485	0.2114	0.2900	0.3860	0.3468	0.5507	0.8220	1.1704	1.6055	2.1369
0.15	0.1342	0.0258	0.0409	0.0611	0.0870	0.1193	0.1588	0.2224	0.3532	0.5272	0.7506	1.0297	1.3708
0.20	0.0700	0.0134	0.0214	0.0319	0.0454	0.0623	0.0829	0.1607	0.2552	0.3809	0.5423	0.7439	0.9902
0.25	0.0417	0.0080	0.0127	0.0190	0.0270	0.0371	0.0494	0.1240	0.1970	0.2940	0.4186	0.5743	0.7643
0.30	0.0271	0.0052	0.0083	0.0123	0.0176	0.0241	0.0320	0.0999	0.1587	0.2368	0.3372	0.4626	0.6157
0.35	0.0187	0.0036	0.0057	0.0085	0.0121	0.0166	0.0221	0.0830	0.1317	0.1966	0.2800	0.3841	0.5112

Table 33 -

Details of design charts for optimum sections.

Unit No.	Design Parameters	Variable	Constants		Fig. No.	Remarks
			Individuals	Group		
1	Mass Coefficient K_M	ξ_s	1.0	Six values	31(a)	<u>Symbols:-</u> ξ_s = Width of stiffeners (t) Thickness of slab (b) a_s = Thickness of slab (b) Total depth of section (H) A_R = Width of plate (b) Span " " (a) Coefficients K_M, K_F, K_D . These are Mass ratios, Frequencies and Deflections of optimum sections. Multipliers: $K_M = \frac{w_c \cdot D_x D_y}{\rho \cdot a^4 \cdot H} \times 10^3$ $K_F = \frac{H \cdot E_s}{\rho} \cdot \frac{\pi^2}{a^2}$ CPS. $K_D = \frac{P a^4}{a b D_x D_y} \times 10^{-2}$ D_x and D_y of opti. sections having slab 6" to 11" are obtained from table 31 and 32 <u>Note:</u> Coefficients are obtained using both criterias of optimization and are given by solid lines for max mass & dotted lines for max freq. in each of these cases, except Fig. 37.
2		"	0.9	$a_s = 0.1$ to	31(b)	
3		"	0.8	$a_s = 0.35$	31(c)	
4		"	0.7	covered in	31(d)	
5		"	0.6	each case	31(e)	
6		"	0.5	of A_R	31(f)	
7		A_R	1.0		32(a)	
8		"	1.5		32(b)	
9		"	2.0		32(c)	
10		"	2.5	— Do —	32(d)	
11		"	3.0		32(e)	
12		"	3.5		32(f)	
13		a_s	1.0	Two values	33(a)	
14		"	0.9	of $\xi_s = 1.0$ &	33(b)	
15		"	0.8	ξ_s	33(c)	
16		"	0.7	$\xi_s = 3.0$	33(d)	
17		"	0.6	were considered for	33(e)	
18		"	0.5	each A_R	33(f)	
19	Frequency Coefficient	a_s		$\xi_s = 1.0$ & 3.0	34(a)	
20	K_F	ξ_s		$a_s = 0.1$ to 0.35	34(b)	
21	Deflection Coefficient	ξ_s	1.0	Six values	35(a)	
22	K_D	"	0.9	of in	35(b)	
23		"	0.8	$a_s = 0.1$ to	35(c)	
24		"	0.7	$a_s = 0.35$	35(d)	
25		"	0.6	considered	35(e)	
26		"	0.5	in each case of A_R	35(f)	
27			A_R	1.0		36(a)
28		"	1.5		36(b)	
29		"	2.0	— Do —	36(c)	
30		"	2.5		36(d)	
31		"	3.0		36(e)	
32		"	3.5		36(f)	
33		a_s	1.0	Six values	37(a)	
34		"	1.5	A_R in	37(b)	
35		"	2.0	$A_R = 1.0$ to	37(c)	
36		"	2.5	$A_R = 0.5$	37(d)	
37		"	3.0	were taken	37(e)	
38		"	3.5	for each ξ_s	37(f)	

Table 34(a) -

Details - Influence surfaces for deflections
of optimum sections.

<u>No.</u>	<u>S_s</u>	<u>A_R</u>	<u>a_s</u>	<u>Fig. No.</u>	<u>Remarks</u>
1	1.0	1.0	0.25	38(a)	
2	"	"	0.30	38(b)	
3	"	"	0.35	38(c)	
4	"	"	0.40	38(d)	
5	"	"	0.45	38(e)	
6	"	"	0.50	38(f)	
7	1.0	0.50	0.25	39(a)	
8	"	"	0.30	39(b)	
9	"	"	0.35	39(c)	
10	"	"	0.40	39(d)	
11	"	"	0.45	39(e)	
12	"	"	0.50	39(f)	
13	1.50	1.0	0.25	40(a)	
14	"	"	0.30	40(b)	
15	"	"	0.35	40(c)	
16	"	"	0.40	40(d)	
17	"	"	0.45	40(e)	
18	"	"	0.50	40(f)	
19	1.50	0.50	0.25	41(a)	
20	"	"	0.30	41(b)	
21	"	"	0.35	41(c)	
22	"	"	0.40	41(d)	
23	"	"	0.45	41(e)	
24	"	"	0.50	41(f)	
25	2.0	1.0	0.25	42(a)	
26	"	"	0.30	42(b)	
27	"	"	0.35	42(c)	
28	"	"	0.40	42(d)	
29	"	"	0.45	42(e)	
30	"	"	0.50	42(f)	
31	2.0	0.50	0.25	43(a)	
32	"	"	0.30	43(b)	
33	"	"	0.35	43(c)	
34	"	"	0.40	43(d)	
35	"	"	0.45	43(e)	
36	"	"	0.50	43(f)	
37	2.50	1.0	0.25	44(a)	
38	"	"	0.30	44(b)	
39	"	"	0.35	44(c)	
40	"	"	0.40	44(d)	
41	"	"	0.45	44(e)	
42	"	"	0.50	44(f)	

Table 34(b) -

Details Influence Surfaces for deflections
of optimum sections.

<u>No.</u>	<u>g_s</u>	<u>A_R</u>	<u>a_s</u>	<u>Fig. No.</u>	<u>Remarks</u>
43	2.50	0.50	0.25	45(a)	
44	"	"	0.30	45(b)	
45	"	"	0.35	45(c)	
46	"	"	0.40	45(d)	
47	"	"	0.45	45(e)	
48	"	"	0.50	45(f)	
49	3.0	1.0	0.25	46(a)	
50	"	"	0.30	46(b)	
51	"	"	0.35	46(c)	
52	"	"	0.40	46(d)	
53	"	"	0.45	46(e)	
54	"	"	0.50	46(f)	
55	3.0	0.50	0.25	47(a)	
56	"	"	0.30	47(b)	
57	"	"	0.35	47(c)	
58	"	"	0.40	47(d)	
59	"	"	0.45	47(e)	
60	"	"	0.50	47(f)	

Table 35(a). Dimensions (inch units) and details of models in series A, B, C and D.

b - width: t - width of stiffener: H-total height of
a - span: s - c/c spacing of stiffener: h - thick-
ness of plate.

Series	Model	b	a	t	H-h	h	s	b/a	H
A	A1	20	20	3/4	1/4	1/16	4.0	1.00	5/16
	A2	20	25	3/4	1/4	1/16	4.0	0.80	5/16
	A3	20	30	3/4	1/4	1/16	4.0	0.67	5/16
	A4	20	35	3/4	1/4	1/16	4.0	0.57	5/16
	A5	20	40	3/4	1/4	1/16	4.0	0.50	5/16
B	B1	20	20	3/4	1/4	1/8	4.0	1.00	3/8
	B2	20	25	3/4	1/4	1/8	4.0	0.80	3/8
	B3	20	30	3/4	1/4	1/8	4.0	0.67	3/8
	B4	20	35	3/4	1/4	1/8	4.0	0.57	3/8
	B5	20	40	3/4	1/4	1/8	4.0	0.50	3/8
C	C1	12	12	1/4	1/2	1/16	2.4	1.00	9/16
	C2	12	15	1/4	1/2	1/16	2.4	0.80	9/16
	C3	12	18	1/4	1/2	1/16	2.4	0.67	9/16
	C4	12	21	1/4	1/2	1/16	2.4	0.57	9/16
	C5	12	24	1/4	1/2	1/16	2.4	0.50	9/16
D	D1	12	12	1/4	1/4	1/16	2.4	1.00	5/16
	D5	12	24	1/4	1/4	1/16	2.4	0.50	5/16

Table 35(b). Rigidities of Models

Series

Rigidity	A	B	C	D
D _x	9481.63	16538.30	34556.50	6198.33
D _y	223.57	1788.58	223.57	223.57
D _x /D _y	42.40	9.25	154.56	27.72
D _{xy}	876.85	1370.30	1112.30	352.15

Table 36. Calibration of Vehicle stiffeners and
calibration beam.

Typical test data.

<u>Load</u> <u>(Lbs)</u>	<u>Deflection x 10⁻³ in.</u>	<u>Strain x 10⁻⁶</u>
0	0	0
2	35	286
4	74	606
6	110	908
8	150	1240
10	185	1600
12	225	1900
14	263	2200
16	300	2500
18	335	2850
20	375	3150
22	413	

Table 38(a), (b)
Fundamental and Higher Modes - Experimental

Series A Model		A_1						A_2						A_3						A_4						A_5						Total
Series	n	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	
A	1	/	/	/	-	/	/	/	/	/	/	/	/	-	-	-	-	-	-	/	/	/	/	/	/	/	/	/	-	-	-	$A_1 - 9$
	2	/	/	-	/	-	/	/	-	/	-	/	-	/	-	/	-	/	-	/	-	/	-	/	-	/	-	/	-	/	-	$A_2 - 11$
	3	-	-	/	-	-	/	-	/	-	-	/	-	-	/	-	-	/	-	/	-	/	-	/	-	-	/	-	-	/	-	$A_3 - 2$
	4	-	-	-	-	-	/	-	-	-	-	/	-	-	-	-	-	/	-	-	-	-	-	/	-	-	/	-	-	/	-	$A_4 - 13$
	5	-	-	-	-	-	/	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	$A_5 - 12$
	6	-	-	-	-	-	/	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	

Series	B Model	B ₁						B ₂						B ₃						B ₄						B ₅						Total
		1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	
B	1	✓	-	✓	-	✓	-	✓	✓	✓	-	✓	-	✓	✓	✓	✓	✓	-	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	B ₁ - 6
	2	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	B ₂ - 10
	3	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	✓	-	B ₃ - 15
	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	B ₄ - 15
	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	B ₅ - 13
	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	

Table 39

Programme of Testing - Influence line and Forced Vibration tests

Influence line tests

Model	Load on	deflection and strain output												Remarks
		CC2		CL2		CLL2		CC3		CL3		CLL3		
		Def	Str	Def	Str	Def	Str	Def	Str	Def	Str	Def	Str	
A1	1-1,2-2,3-3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	1) Load placed on 3 pts in turn and deflection strains obtained 2) Vehicle placed at s/4, s/2, 3s/4 and only strains obtained for both series A and B See Fig. 60 for symbols
A2	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
A3	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
A4	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
A5	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
B1	1-1,2-2,3-3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
B2	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
B3	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
B4	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
B5	"	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	

Table 40

Forced Vibration Tests (numerical printed output by DAS)

Model	B	$\phi = .5\pi$				$\phi = 1.5\pi$				Strain		Remarks
		CC2		CL2		CC2		CL2		CLL2		
A1	0.5	✓		—		✓		✓		✓		1) For each vehicle running on 1-1, and 2-2 strain output were obtained. 2) Speed was 3.3'/sec. 3) Vehicle frequency - 5 cps 4) Strain gauges CC2, CL2, CLL2 at centre section and on stiffeners See Fig. 60 for symbols
A2	const.	✓		—		✓		✓		✓		
A3		✓		—		✓		✓		✓		
A4		✓		—		✓		✓		✓		
A5		✓		—		✓		✓		✓		
B1	.5	✓		✓		✓		✓		✓		
B2	const.	✓		—		✓		✓		✓		
B3		✓		✓		✓		✓		✓		
B4		✓		✓		✓		✓		✓		
B5		✓		✓		✓		✓		✓		

Table 41 -

Details of static tests data given in table A1 to table A7.
Tests - as per programme in columns 1, 2, 3, 4 and 5 of
table 37. See fig 60 for symbols of locations.

<u>Model</u>	<u>Table</u>	<u>No of Cycles.</u>	<u>Details.</u>
A1	A1 (a)	2	Deflection at 9 points and strains at
A2	" (b)	2	centre for A1, A2, A3 and only
A3	" (c)	1	deflections for A4, A5
A4	" (d)	2	Load at centre for all models.
A5	" (e)	2	
B1	A2 (a)	2	Deflection at 9 points and strains
B2	" (b)	2	at centre for all models.
B3	" (c)	2	Load at centre for all models.
B4	" (d)	2	
B5	" (e)	2	
C1	A3 (a)	1	Deflection at 9
C1	" (b)	1	strain at 8 points
C2	A4 (a)	1	Load deflection under point
C3	" (b)	1	Load at centre.
C4	" (c)	1	
C5	A5 (a)	2	Load deflection at 15 points and
C5	" (b)	1	strains at 9 points.
D1	A6 (a)	2	Load deflection at 15 points
D5	A7 (a)	2	Load - strain at 15 points
D5	"	1	Load - strain at 9 points.

Table 42 -

Details of Influencelines travelling static load and vehicle
test data given in table A8 to A11. Tests as per
programme of testing in columns 8 and 9 of table 37 and 39
see fig 60 for symbols of locations.

<u>Model</u>	<u>Table No.</u>	<u>Details</u>
B1	A8 (a)	Influence lines - travelling point loads
B2	" (b)	on 1-1, 2-2, 3-3.
B3	" (c)	
B4	" (d)	
B5	" (e)	
A2	A9 (a)	---- Do ----
A4	" (b)	
A5	" (c)	
B1 to B5	A10	Influence lines travelling vehicle - static load on 1-1, 2-2
A1 to A5	A11	---- Do ----

Table 43 -

Details of tables for data in forced vibration tests -
Tests as per schedule in table 40. Symbols in Fig. 60.

Model	Load on	ϕ	Table	No:	Tests
			cc2	cL2	
B1	1-1	1.5 π	A12(a)	A12(b)	3
	"	0.5 π			
	2-2	1.5 π			
	"	0.5 π			
B2	1-1	1.5 π	A13(a)	A13(b)	3
	"	0.5 π			
	2-2	1.5 π			
	"	1.5 π			
B3	1-1	1.5 π	A14(a)	A14(b)	3
	"	0.5 π			
	2-2	1.5 π			
	"	1.5 π			
B4	1-1	1.5 π	A15(a)	A15(b)	3
	"	1.5 π			
	2-2	1.5 π			
	"	1.5 π			
B5	1-1	1.5 π	A16(a)	A16(b)	3
	"	0.5 π			
	2-2	1.5 π			
	"	0.5 π			
A1	1-1	1.5 π	A17(a)	-	2
	"	0.5 π			
	2-2	1.5 π			
	"	0.5 π			
A2	1-1	1.5 π	A18(a)	-	2
	"	0.5 π			
	2-2	1.5 π			
	"	0.5 π			
A4	1-1	1.5 π	A19(a)	-	2
	"	0.5 π			
	2-2	1.5 π			
	"	0.5 π			
A5	1-1	1.5 π	A20(a)	-	2
	"	0.5 π			
	2-2	1.5 π			
	"	0.5 π			

Table 45. Theoretical deflections $\times 10^{-3}$, Unit load at centres.
Models in series A, B, C and D.

$$\frac{x}{a} = 0.1 \text{ to } 0.5, \quad \frac{y}{b} = 0 \text{ to } 0.4.$$

Model	Deflections at -								
	$\frac{x}{a}$	0.1	0.2	0.3	0.4	0.5	0.5	0.5	
	$\frac{y}{b}$	0	0	0	0	0	0.2	0.4	0.5
A1	0.4676	0.9120	1.2951	1.5945	1.7205	0.7446	0.5100	0.3400	
A2	0.7402	1.4433	2.0488	2.5210	2.7294	1.5267	1.0520	0.8500	
A3	1.0964	1.1363	3.0300	3.7240	4.0145	2.6226	2.2000	1.8500	
A4	1.5576	3.0319	4.2949	5.2693	5.6752	4.1817	3.0800	2.8600	
A5	2.1467	4.1734	5.9030	7.2269	7.7745	6.1719	4.6884	4.5000	
B1	0.2231	0.4350	0.6170	0.7586	0.8180	0.5563	0.2913	0.2030	
B2	0.3700	0.7202	1.0200	1.2509	1.3470	1.0533	0.7126	0.5857	
B3	0.5784	1.1236	1.5877	1.9410	2.0865	1.7741	1.3858	1.2328	
B4	0.8652	1.6773	2.3644	2.8808	3.0910	2.7685	2.3531	2.1836	
B5	1.2469	2.4130	3.3939	4.1218	4.4147	4.0865	3.6561	3.4751	
C1	0.0706	0.1378	0.1957	0.2409	0.2600	0.0828	0.0133	0.0047	
C2	0.1100	0.2158	0.3065	0.3774	0.4072	0.1797	0.0499	0.0257	
C3	0.1607	0.3134	0.4449	0.5476	0.5908	0.3208	0.1258	0.0790	
C4	0.2225	0.4338	0.6155	0.7570	0.8163	0.5112	0.2527	0.1785	
C5	0.2984	0.5814	0.8292	1.0125	1.0913	0.7570	0.4400	0.3394	
D1	0.2784	0.5430	0.7709	0.9490	1.0239	0.5444	0.1686	0.0665	
D5	1.3252	2.5742	3.6372	4.4446	4.7789	4.0678	3.1502	2.7684	

Table 46. Theoretical strains $\times 10^{-6}$, under unit
load at centre.

Models in series A, B, C and D.

<u>Model</u>	<u>Strain $\times 10^{-6}$</u>
A1	7.6830
A2	8.8255
A3	10.4931
A4	12.8368
A5	16.5084
B1	4.5657
B2	5.3619
B3	6.5700
B4	8.2324
B5	10.8714
C1	5.8880
C5	12.1267
D5	30.5021

Table 47. Frequency ratio R and frequencies in cycles per second for models in Series A. Empirical method.

Frequencies in CPS.

Frequency ratio

m n	Frequency ratio						Frequencies in CPS.					
	1	2	3	4	5	6	1	2	3	4	5	6
	MODEL A1						ASPECT RATIO (WIDTH/SPAN)= 1.00					
1	1.00	1.00	1.00	1.00	1.00	1.00	70.40	281.60	633.59	1126.38	1759.97	2534.36
2	1.43	1.18	1.10	1.07	1.05	1.04	100.74	332.80	699.70	1204.18	1847.44	2630.13
3	1.72	1.23	1.12	1.07	1.05	1.04	121.16	346.85	707.73	1206.92	1845.54	2624.10
4	2.54	1.47	1.23	1.14	1.09	1.07	179.05	414.56	781.21	1284.13	1925.41	2705.99
5	3.44	1.70	1.34	1.20	1.13	1.09	242.32	479.73	847.45	1350.51	1991.46	2771.51
6	4.80	2.07	1.51	1.30	1.20	1.14	337.90	584.25	959.25	1467.38	2111.84	2894.43
	MODEL A2						ASPECT RATIO (WIDTH/SPAN)= 0.80					
1	1.00	1.00	1.00	1.00	1.00	1.00	45.06	180.22	405.50	720.98	1126.38	1621.99
2	1.55	1.24	1.14	1.10	1.07	1.05	70.02	224.05	463.17	789.53	1204.18	1707.65
3	2.03	1.34	1.17	1.10	1.07	1.05	91.54	240.68	474.73	796.45	1206.92	1706.64
4	3.26	1.69	1.34	1.21	1.14	1.10	146.79	305.09	545.09	870.80	1284.13	1786.03
5	4.66	2.05	1.51	1.30	1.20	1.14	209.91	369.50	610.83	937.12	1350.51	1852.17
6	6.74	2.61	1.77	1.46	1.30	1.22	303.76	470.50	718.55	1050.10	1467.38	1971.94

Continued

4

Table 47 - Continued, Models A3, A4 and A5

MODEL		A3		ASPECT RATIO (WIDTH/SPAN)= 0.67		A4		ASPECT RATIO (WIDTH/SPAN)= 0.57		A5		ASPECT RATIO (WIDTH/SPAN)= 0.50	
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.67	1.31	1.12	1.09	1.07	1.09	1.07	1.09	1.07	1.09	1.07	1.09	1.07
3	2.38	1.45	1.23	1.14	1.10	1.14	1.10	1.14	1.10	1.14	1.10	1.14	1.10
4	4.09	1.95	1.47	1.29	1.19	1.14	1.14	1.27	1.07	1.14	1.14	1.27	1.07
5	6.11	2.46	1.70	1.42	1.28	1.20	1.20	1.91	0.93	1.20	1.20	1.91	0.93
6	9.07	3.24	2.07	1.64	1.42	1.30	1.30	2.83	0.74	1.30	1.30	2.83	0.74
MODEL		A4		ASPECT RATIO (WIDTH/SPAN)= 0.57		A5		ASPECT RATIO (WIDTH/SPAN)= 0.50		A5		ASPECT RATIO (WIDTH/SPAN)= 0.50	
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.79	1.37	1.22	1.15	1.11	1.09	1.09	1.12	1.09	1.12	1.09	1.12	1.09
3	2.77	1.58	1.30	1.19	1.13	1.09	1.09	1.15	1.09	1.15	1.09	1.15	1.09
4	5.03	2.23	1.62	1.37	1.25	1.18	1.18	1.15	1.09	1.15	1.09	1.15	1.09
5	7.78	2.92	1.93	1.55	1.37	1.26	1.26	1.78	0.86	1.26	1.26	1.78	0.86
6	11.78	3.97	2.42	1.84	1.56	1.40	1.40	2.70	0.74	1.40	1.40	2.70	0.74
MODEL		A5		ASPECT RATIO (WIDTH/SPAN)= 0.50		A5		ASPECT RATIO (WIDTH/SPAN)= 0.50		A5		ASPECT RATIO (WIDTH/SPAN)= 0.50	
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.90	1.43	1.26	1.18	1.13	1.10	1.10	1.12	1.09	1.12	1.09	1.12	1.09
3	3.19	1.72	1.37	1.23	1.16	1.12	1.12	1.15	1.09	1.15	1.09	1.15	1.09
4	6.08	2.54	1.77	1.47	1.32	1.23	1.23	1.07	0.93	1.23	1.23	1.07	0.93
5	9.68	3.44	2.18	1.70	1.47	1.34	1.34	1.70	0.86	1.34	1.34	1.70	0.86
6	14.07	4.60	2.81	2.07	1.72	1.51	1.51	2.61	0.68	1.51	1.51	2.61	0.68

Table 48. Frequency ratio R and frequencies in cycles per second for models in series B. Empirical Method.

		Frequency ratio R						Frequencies in CPS.					
m \ n		1	2	3	4	5	6	1	2	3	4	5	6
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	74.72	298.89	672.50	1195.55	1868.05	2690.00
	1.30	1.13	1.08	1.05	1.04	1.03	1.03	96.80	337.22	723.04	1255.99	1936.89	2766.18
	1.87	1.23	1.11	1.07	1.04	1.03	1.03	139.68	368.98	747.42	1274.89	1951.33	2776.76
	3.31	1.58	1.26	1.15	1.09	1.07	1.07	247.30	470.92	844.31	1369.39	2044.91	2870.17
	5.20	1.99	1.42	1.23	1.15	1.10	1.10	388.32	596.20	954.98	1470.27	2139.41	2960.35
	7.94	2.67	1.70	1.38	1.23	1.16	1.16	593.47	796.81	1142.68	1646.20	2306.95	3122.25
2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	47.82	191.29	430.40	765.16	1195.56	1721.60
	1.38	1.17	1.10	1.07	1.05	1.04	1.04	65.95	223.75	473.98	817.83	1255.99	1788.85
	2.33	1.36	1.17	1.10	1.07	1.05	1.05	111.45	259.39	502.46	841.00	1274.89	1804.12
	4.57	1.90	1.40	1.22	1.15	1.10	1.10	218.77	364.09	602.00	937.24	1369.39	1897.81
	7.57	2.59	1.67	1.37	1.23	1.16	1.16	362.02	496.13	720.86	1045.59	1470.27	1993.39
	11.84	3.66	2.13	1.61	1.38	1.26	1.26	566.09	700.29	916.59	1230.69	1646.20	2162.36

MODEL B1 ASPECT RATIO (WIDTH/SPAN)= 1.00

MODEL B2 ASPECT RATIO (WIDTH/SPAN)= 0.80

Continued

Table 48 Continued. Models B3, B4, B5.

MODEL		B3		ASPECT RATIO (WIDTH/SPAN)= 0.67		B4		ASPECT RATIO (WIDTH/SPAN)= 0.57		B5		ASPECT RATIO (WIDTH/SPAN)= 0.50	
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.46	1.21	1.13	1.09	1.07	1.05	1.06	1.08	1.06	1.13	1.10	1.08	1.08
3	2.88	1.50	1.23	1.14	1.09	1.07	1.09	1.12	1.09	1.23	1.16	1.11	1.11
4	6.09	2.30	1.58	1.32	1.21	1.15	1.28	1.44	1.20	1.40	1.37	1.26	1.26
5	10.44	3.33	1.99	1.54	1.34	1.23	1.47	1.75	1.32	1.79	1.62	1.42	1.42
6	16.55	4.88	2.67	1.90	1.56	1.38	1.78	2.25	1.52	2.30	2.03	1.70	1.70
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.55	1.25	1.16	1.11	1.08	1.06	1.08	1.11	1.06	1.18	1.10	1.08	1.08
3	3.52	1.67	1.31	1.18	1.12	1.09	1.12	1.18	1.09	1.31	1.16	1.11	1.11
4	7.05	2.77	1.79	1.44	1.28	1.20	1.28	1.44	1.20	1.79	1.62	1.42	1.42
5	13.80	4.20	2.38	1.75	1.47	1.32	1.47	1.75	1.32	2.38	2.03	1.70	1.70
6	22.08	6.30	3.30	2.25	1.78	1.52	1.78	2.25	1.52	3.30	2.67	2.03	2.03
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.64	1.30	1.18	1.13	1.10	1.08	1.10	1.13	1.08	1.30	1.16	1.11	1.11
3	4.25	1.87	1.40	1.23	1.16	1.11	1.16	1.23	1.11	1.40	1.37	1.26	1.26
4	9.86	3.31	2.03	1.58	1.37	1.26	1.37	1.58	1.26	2.03	2.03	1.70	1.70
5	17.66	5.20	2.82	1.99	1.62	1.42	1.62	1.99	1.42	2.82	2.67	2.03	2.03
6	20.43	7.94	4.04	2.67	2.03	1.70	2.03	2.67	1.70	4.04	2.67	2.03	2.03

Table 49. Frequency ratio R and frequencies in cycles per second for models in series A. Theoretical approach.

m n	Frequency ratio R						Frequencies in CPS.					
	1	2	3	4	5	6	1	2	3	4	5	6
MODEL A1												
1	1.00	1.00	1.00	1.00	1.00	1.00	70.41	281.64	633.69	1126.56	1760.25	2534.76
2	1.35	1.09	1.05	1.01	1.01	1.01	95.05	306.99	665.37	1142.33	1790.00	2560.00
3	1.69	1.20	1.09	1.05	1.04	1.04	119.27	337.97	690.72	1186.27	1850.00	2630.00
4	2.48	1.41	1.21	1.12	1.09	1.05	179.04	397.11	766.76	1257.24	1910.00	2660.00
5	3.38	1.63	1.32	1.87	1.13	1.09	237.97	457.95	833.30	1336.10	1986.00	2730.00
6	4.63	1.92	1.48	1.30	1.19	1.14	325.98	540.75	939.75	1461.15	2100.00	2880.00
MODEL A2												
1	1.00	1.00	1.00	1.00	1.00	1.00	45.05	180.20	405.45	720.80	1126.25	1621.80
2	1.49	1.12	1.04	1.03	1.01	1.01	66.88	201.82	422.48	742.42	1140.33	1640.00
3	2.02	1.32	1.14	1.08	1.05	1.04	91.00	237.86	463.02	748.46	1182.56	1690.00
4	3.12	1.64	1.29	1.20	1.10	1.09	140.56	295.54	524.65	865.68	1242.25	1770.00
5	4.52	1.93	1.50	1.30	1.19	1.14	203.63	347.79	608.58	934.15	1341.36	1830.00
6	6.63	2.51	1.75	1.43	1.30	1.21	298.68	452.30	709.13	1031.47	1460.75	1958.00

Continued

Table 49 Continued. Models A3, A4 and A5.

MODEL		A3	A S P E C T		R A T I O		(WIDTH/SPAN) = 0.67				
1	1.00	1.00	1.00	1.00	1.00	31.29	125.16	281.61	500.64	782.25	1126.44
2	1.57	1.25	1.08	1.08	1.06	49.19	156.45	302.73	540.69	829.19	1176.00
3	2.32	1.43	1.22	1.13	1.09	72.47	178.98	343.00	565.72	851.09	1197.41
4	3.85	1.89	1.42	1.22	1.18	116.68	236.55	401.01	611.78	923.06	1261.61
5	5.90	2.41	1.69	1.40	1.27	184.61	301.89	476.77	698.89	991.89	1343.84
6	8.94	3.15	2.07	1.59	1.41	279.73	393.75	582.93	797.53	1102.97	1460.99
MODEL		A4	A S P E C T		R A T I O		(WIDTH/SPAN) = 0.57				
1	1.00	1.00	1.00	1.00	1.00	22.98	91.92	206.82	367.68	574.50	827.28
2	1.68	1.32	1.18	1.08	1.10	38.63	121.43	244.05	397.09	631.95	852.10
3	2.73	1.54	1.28	1.14	1.11	62.76	141.74	264.52	418.42	637.12	892.64
4	4.99	2.12	1.60	1.33	1.22	114.58	194.87	330.50	487.54	702.04	971.23
5	7.69	2.87	1.91	1.55	1.36	176.92	263.81	394.82	569.17	783.62	1020.86
6	11.56	3.80	2.39	1.80	1.56	265.72	349.30	493.47	661.82	896.22	1147.44
MODEL		A5	A S P E C T		R A T I O		(WIDTH/SPAN) = 0.50				
1	1.00	1.00	1.00	1.00	1.00	17.60	70.40	158.40	281.60	440.00	636.60
2	1.69	1.34	1.18	1.08	1.04	29.66	94.27	186.91	303.57	456.72	654.43
3	3.18	1.69	1.36	1.21	1.14	55.99	119.26	215.42	339.61	500.28	687.53
4	5.88	2.51	1.72	1.42	1.26	103.49	176.42	272.61	400.99	554.40	763.28
5	9.55	3.41	2.12	1.69	1.46	168.08	240.28	335.97	475.60	642.40	842.22
6	14.67	4.78	2.79	2.02	1.68	258.02	336.51	441.62	568.83	739.20	931.35

Table 50. Frequency ratio and frequencies in cycles per second for Models in series B. Theoretical method.

Frequency Ratio R

Frequency in cps.

m n	1	2	3	4	5	6	1	2	3	4	5	6
A S P E C T R A T I O (W I D T H / S P A N) = 1.0												
1	1.00	1.00	1.00	1.00	1.00	1.00	74.71	298.84	672.50	1195.55	1868.05	2690.00
2	1.20	1.05	1.02	1.01	1.01	1.01	90.07	314.45	687.96	1210.95	1890.00	2724.00
3	1.87	1.22	1.09	1.05	1.03	1.02	139.47	363.94	736.21	1258.41	1930.39	2752.02
4	3.10	1.52	1.22	1.12	1.08	1.05	231.64	453.24	821.85	1341.27	2011.44	2831.90
5	4.99	1.97	1.41	1.22	1.14	1.10	372.83	587.71	949.88	1463.83	2130.31	2947.71
6	7.56	2.58	1.67	1.36	1.23	1.15	564.62	770.98	1124.39	1630.23	2289.89	3102.74
A S P E C T R A T I O (W I D T H / S P A N) = 0.80												
1	1.00	1.00	1.00	1.00	1.00	1.00	47.80	191.20	430.20	764.80	1195.00	1720.80
2	1.31	1.08	1.05	1.02	1.01	1.01	62.78	206.88	462.00	800.00	1210.95	1752.00
3	2.33	1.34	1.15	1.08	1.05	1.04	111.38	256.79	494.88	828.67	1258.41	1784.02
4	4.27	1.82	1.35	1.19	1.12	1.08	204.21	347.45	582.66	913.65	1341.27	1865.39
5	7.26	2.53	1.66	1.36	1.22	1.15	347.88	484.48	714.55	1040.48	1463.85	1984.64
6	11.30	3.51	2.08	1.59	1.36	1.25	540.34	671.22	894.31	1213.27	1630.24	2145.72

Table 50. Continued. Models B3, B4 and B5.

$\frac{m}{n}$		Frequency Ratio R						Frequency in cps.					
		1	2	3	4	5	6	1	2	3	4	5	6
MODEL B3		A S P E C T						R A T I O (WIDTH/SPAN) = 0.67					
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	33.20	132.80	298.80	563.20	830.00	1195.20
2	1.44	1.12	1.05	1.04	1.03	1.02	1.02	47.64	148.41	314.45	575.00	862.00	1210.95
3	2.88	1.49	1.22	1.12	1.08	1.05	1.05	95.57	198.36	363.95	595.48	893.62	1258.41
4	5.69	2.18	1.52	1.38	1.18	1.12	1.12	189.06	289.73	453.24	682.27	978.22	1341.27
5	10.03	3.22	1.97	1.52	1.33	1.22	1.22	332.96	428.51	587.73	812.44	1104.26	1463.83
6	15.88	4.70	2.58	1.86	1.54	1.36	1.36	527.19	617.51	770.98	989.84	1275.93	1630.16
MODEL B4		A S P E C T						R A T I O (WIDTH/SPAN) = 0.57					
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	24.39	97.56	219.51	390.24	609.75	878.04
2	1.47	1.16	1.07	1.04	1.03	1.02	1.02	38.25	113.09	235.18	405.91	641.00	893.79
3	3.51	1.67	1.30	1.17	1.10	1.07	1.07	85.71	162.83	285.00	455.03	673.86	941.66
4	7.37	2.61	1.71	1.39	1.25	1.17	1.17	178.80	254.64	375.90	543.23	759.98	1025.96
5	13.30	4.04	2.33	1.73	1.46	1.31	1.31	324.50	394.74	511.62	675.89	888.89	1151.48
6	21.29	5.99	3.16	2.19	1.75	1.51	1.51	519.33	583.38	697.36	856.67	1064.67	1332.35
MODEL B5		A S P E C T						R A T I O (WIDTH/SPAN) = 0.50					
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	18.67	74.68	168.03	298.72	466.75	672.12
2	1.63	1.21	1.09	1.05	1.04	1.02	1.02	31.92	90.09	183.71	314.45	498.00	687.96
3	4.23	1.87	1.39	1.22	1.14	1.09	1.09	79.12	139.47	233.67	363.94	531.35	736.22
4	9.30	3.10	1.93	1.52	1.32	1.22	1.22	173.74	231.64	324.61	453.24	618.76	821.85
5	17.08	4.99	2.75	1.97	1.60	1.41	1.41	318.99	372.83	462.34	587.73	750.00	949.88
6	27.53	7.56	3.87	2.58	1.99	1.67	1.67	514.22	593.48	649.84	771.00	928.90	1124.39

Table 51. Comparison of experimental frequencies with those by theoretical and empirical method. Models A
TH-Theory, EM-Empirical, EX- Experimental.

m	n	TH	EM	EX	m	n	TH	EM	EX
<u>Model A1</u>					<u>Model A2</u>				
1	1	70.41	70.41	67.00	1	1	45.05	45.05	43.00
1	2	95.05	100.75	96.00	1	2	66.88	70.02	71.50
1	3	119.27	121.16	120.00	1	3	91.00	91.55	96.50
1	5	237.97	242.33	250.00	1	4	140.56	146.80	150.00
1	6	325.98	337.91	340.00	1	5	203.63	209.91	220.00
2	1	281.64	281.64	282.00	2	1	180.20	180.20	175.00
2	3	337.97	346.86	330.00	2	5	347.50	369.51	330.00
2	5	457.95	479.34	440.00	3	1	405.45	405.45	390.00
3	3	690.72	707.74	698.00	3	5	608.58	510.84	620.00
<u>Model A4</u>					4	1	720.80	720.80	730.00
1	1	22.98	22.98	24.00	4	5	934.15	937.13	920.00
1	2	38.63	41.12	40.00	5	1	1126.25	1126.25	1200.00
1	3	62.76	63.65	66.00	<u>Model A5</u>				
1	4	115.58	115.59	118.00	1	1	17.60	17.60	19.00
1	5	176.96	178.86	188.00	1	2	29.66	31.70	31.00
2	1	91.92	91.92	86.00	1	3	55.99	56.20	60.00
2	3	141.74	145.40	140.00	1	5	168.08	170.44	180.00
3	1	206.82	206.82	208.00	2	1	70.40	70.40	70.00
3	3	264.52	268.83	236.00	2	3	119.26	121.16	118.00
3	4	330.50	334.26	342.00	2	5	240.28	242.33	240.00
4	4	487.74	505.63	455.00	3	1	158.40	158.40	150.00
5	4	702.04	720.57	715.00	3	3	215.42	217.49	208.00
6	4	971.23	979.91	980.00	4	3	339.61	346.86	360.00
					4	4	400.99	414.57	400.00
					4	6	568.83	584.26	600.00

Table 52. Comparison of Frequencies.

TH. - Theory; EM. - Empirical, EX. - Experimental.

m	n	TH. Model B1	EM.	EX.	m	n	TH. Model B3	EM.	EX.
1	1	74.81	74.81	79.50	1	1	33.20	33.20	36.00
1	3	139.47	139.68	145.00	1	2	47.64	48.61	52.00
1	5	372.83	388.33	370.00	1	3	95.57	96.75	102.50
2	1	298.84	298.84	300.00	1	4	189.06	202.23	195.00
2	3	363.94	368.98	390.00	1	5	332.96	346.60	380.00
3	1	672.39	672.39	660.00	2	1	132.80	132.80	129.00
Model B2					2	3	198.36	199.61	220.00
1	1	47.80	47.80	45.00	2	5	428.51	442.38	488.00
1	2	62.78	65.95	62.50	3	1	298.80	298.80	308.00
1	3	111.38	111.46	120.50	3	2	314.45	337.23	335.00
1	5	347.88	363.03	375.00	3	5	587.73	596.21	630.00
2	1	191.20	191.20	205.00	3	6	770.98	796.82	800.00
2	3	256.79	259.40	265.00	4	3	595.48	604.47	600.00
2	5	484.48	496.34	500.00	4	4	682.27	702.94	735.00
3	1	430.20	430.20	455.00	5	3	893.62	906.70	960.00
3	3	494.88	502.46	525.00	Model B4				
3	5	714.55	720.87	725.00	1	1	24.39	24.39	26.00
Model B5					1	2	38.25	37.83	38.00
1	1	18.67	18.67	19.50	1	3	85.71	85.99	100.00
1	2	31.92	30.66	32.00	1	4	179.80	191.53	190.00
1	3	79.12	79.47	86.00	2	1	97.56	97.56	96.00
1	4	173.74	184.10	168.00	2	3	162.83	163.37	165.00
1	5	318.99	329.81	348.00	2	5	394.74	409.93	450.00
2	1	74.68	74.68	76.00	3	1	219.51	219.51	215.00
2	3	139.47	139.68	140.00	3	2	235.18	253.82	255.00
2	5	372.83	388.33	429.00	3	3	285.00	288.27	310.00
3	2	183.71	199.01	193.00	3	5	511.00	522.29	548.00
3	3	233.67	235.75	260.00	3	6	697.36	725.63	770.00
3	5	462.34	474.81	520.00	4	1	390.24	390.24	410.00
4	3	363.94	370.93	408.00	4	3	455.03	461.89	485.00
4	5	587.73	596.21	660.00	4	5	675.89	682.70	720.00

Table 53. Percent errors in frequencies by Empirical method and experimental.
EM - Empirical, EX - Experimental.

m	n	EM	EX	m	n	EM	EX
<u>Model A1</u>				<u>Model A2</u>			
1	1	-	4.84	1	1	-	4.55
1	2	6.00	1.00	1	2	4.69	6.90
1	3	1.58	0.60	1	3	0.60	6.04
1	5	1.83	5.05	1	4	4.44	6.72
1	6	3.66	4.30	1	5	3.08	8.04
2	1	-	0.12	2	1	-	2.89
2	3	2.63	2.36	2	5	6.33	5.03
2	5	4.76	3.91	3	1	-	3.80
3	3	2.46	1.05	3	5	16.06	1.88
				4	1	-	1.27
				4	5	0.32	1.51
				5	1	-	6.55
<u>Model A4</u>				<u>Model A5</u>			
1	1	-	4.43	1	1	-	7.95
1	2	6.45	3.55	1	2	7.00	4.51
1	3	1.42	5.16	1	3	0.38	7.16
1	4	0.80	2.98	1	5	1.40	7.09
1	5	1.07	6.24	2	1	-	0.57
2	1	-	6.44	2	3	1.59	1.06
2	3	2.58	1.22	2	5	0.85	0.12
3	1	-	0.57	3	1	-	5.30
3	3	1.62	10.78	3	3	1.00	3.30
3	4	1.14	3.48	4	3	2.13	6.00
4	4	3.67	6.71	4	4	3.39	0.25
5	4	2.64	1.84	4	6	2.71	5.47
6	4	0.89	0.90				

Table 54. Percent Errors in Frequencies

TH. - Theory; EM. - Empirical, Ex. - Experimental

m	n	EM.	EX.	m	n	EM.	EX.
Model B1				Model B3			
1	1	-	5.66	1	1	~~2.03~~	8.43
1	3	0.15	3.96	1	2	~~2.03~~	9.14
1	5	~~4.15~~	0.76	1	3	~~2.23~~	7.25
2	1	-	0.39	1	4	6.97	3.14
2	3	1.38	7.16	1	5	4.10	14.12
3	1	-	1.84	2	1	-	2.86
Model B2				2	3	0.63	10.90
1	1	-	5.85	2	5	3.24	13.88
1	2	5.05	0.45	3	1	-	3.08
1	3	0.02	8.19	3	2	7.24	6.53
1	5	4.06	7.80	3	5	1.44	7.19
2	1	-	7.20	3	6	3.35	3.76
2	3	1.02	3.20	4	3	1.51	0.75
2	5	2.44	3.20	4	4	3.02	7.73
3	1	-	5.76	5	3	1.46	7.42
3	3	1.53	6.08	Model B4			
3	5	0.88	1.46	1	1	-	6.60
Model B5				1	2	1.09	0.65
1	1	-	4.45	1	3	0.32	16.67
1	2	3.95	0.25	1	4	6.52	5.67
1	3	0.44	8.69	2	1	-	1.60
1	4	5.96	3.30	2	3	0.33	1.33
1	5	3.45	9.09	2	5	3.85	13.99
2	1	-	1.76	3	1	-	2.05
2	3	0.15	0.38	3	2	7.92	8.43
2	5	4.16	15.06	3	3	11.47	10.87
3	2	8.33	5.05	3	5	2.21	7.24
3	3	0.89	11.26	3	6	3.96	10.47
3	5	2.70	12.47	4	1	-	5.06
4	3	1.92	12.11	4	3	15.08	6.59
4	5	1.44	12.30	4	5	10.00	6.52

Table 55(a). Peak Dynamic deflection $\times 10^{-3}$ in.

Models in series A. $B = 0.5$, $\phi = 1.5\pi$

Model	Peaks.	Vehicle on 1-1		Vehicle on 2-2	
		CC2	CLL2	CC2	CLL2
A1	1	23.724	6.325	10.280	15.025
	2	13.751	3.668	5.959	8.709
	3	49.155	13.108	21.300	31.132
	4	5.243	1.398	2.272	3.320
	5	2.658	0.708	1.152	1.683
A2	1	30.955	10.318	20.637	25.223
	2	18.833	6.278	12.555	15.345
	3	98.155	32.719	65.437	79.979
	4	20.887	6.962	13.924	17.019
	5	37.913	12.638	25.276	30.892
A4	1	39.920	24.566	33.779	39.920
	2	12.189	7.500	10.314	12.189
	3	154.170	54.876	130.451	154.170
	4	41.622	25.613	35.218	41.621
	5	172.682	106.266	146.115	172.682
	6	19.916	12.254	16.852	19.916
	7	56.954	35.030	48.167	56.924
A5	1	49.514	34.385	42.637	52.265
	2	2.923	2.030	2.517	3.086
	3	238.975	165.955	205.784	252.252
	4	53.071	36.855	45.700	56.020
	5	280.080	195.000	241.800	296.400
	6	2.614	1.815	2.291	2.758
	7	169.776	117.900	146.196	179.208
	8	9.907	6.880	8.531	10.457
	9	9.842	6.835	8.475	10.389

Table 55(b). Peak Dynamic deflections $\times 10^{-3}$ in
Models in series A: B = 0.5, $\alpha = 0.5\pi$

Model	Peak	Vehicle on 1-1		Vehicle on 2-2	
		CC2	CLL2	CC2	CLL2
A1	1	3.601	0.960	1.561	2.281
	2	2.824	0.753	1.224	1.789
	3	50.168	13.378	21.739	31.773
	4	12.795	3.412	5.544	8.104
	5	23.662	6.310	10.254	14.986
A2	1	7.293	2.441	4.862	5.942
	2	1.849	0.619	1.233	1.507
	3	82.204	27.401	54.802	66.981
	4	28.912	9.667	19.274	23.558
	5	85.744	28.581	57.163	69.665
	6	5.694	1.898	3.796	4.640
	7	10.841	3.614	7.227	8.833
A4	1	10.244	6.304	8.868	10.244
	2	3.817	2.349	3.230	3.817
	3	118.435	72.883	100.214	118.435
	4	44.247	27.229	37.440	44.247
	5	212.960	131.053	180.198	212.960
	6	53.843	33.011	45.390	53.843
	7	139.511	85.853	118.048	139.511
	8	5.335	3.283	4.514	5.335
	9	19.271	11.859	16.306	19.271
A5	1	11.124	7.725	9.579	11.742
	2	12.780	8.875	11.005	13.490
	3	138.038	95.860	118.866	145.707
	4	13.738	9.540	11.829	14.501
	5	275.450	191.290	237.193	290.753
	6	47.995	33.330	41.329	50.662
	7	285.550	198.300	245.892	301.420
	8	20.412	14.175	17.577	21.546
	9	71.244	49.475	61.349	75.202

Table 56(a) Peak Dynamic deflections $\times 10^{-3}$ in.						
Models in series B. $B = 0.5$, $d = 1.5\pi$						
Model	Peak	Vehicle on 1-1			Vehicle on 2-2	
		CC2	CL2	CLL2	CC2	CLL2
B1	1	9.914	7.435	4.956	7.667	9.294
	2	6.029	4.522	3.014	4.712	5.712
	3	20.211	15.158	10.106	15.632	18.948
	4	2.269	1.702	1.134	1.755	2.127
	5	1.158	0.869	0.579	0.896	1.086
B2	1	13.461	11.234	8.426	11.796	12.357
	2	8.627	7.189	5.392	7.549	7.908
	3	42.860	35.717	26.788	37.502	39.289
	4	10.159	8.466	8.350	8.889	9.313
	5	17.134	14.278	10.709	14.991	15.705
B3	1	14.701	12.167	10.139	12.674	15.208
	2	9.946	8.231	6.859	8.574	10.288
	3	58.089	48.074	40.062	50.078	60.093
	4	17.019	14.085	11.737	14.671	17.606
	5	47.107	38.985	32.488	40.610	48.731
	6	4.769	3.976	3.289	4.116	5.494
	7	5.311	4.395	3.662	4.579	5.494
B4	1	18.006	16.676	14.005	16.339	18.740
	2	10.525	9.745	8.186	9.550	10.914
	3	81.700	75.650	63.545	74.140	84.720
	4	26.079	24.148	20.284	23.665	27.045
	5	90.736	84.016	70.573	82.345	94.096
	6	15.795	14.625	12.285	13.333	16.360
	7	29.346	27.173	22.825	26.629	30.433
B5	1	22.620	20.010	19.140	20.010	22.620
	2	7.940	7.024	6.718	7.024	7.940
	3	101.158	89.486	85.595	89.486	101.158
	4	29.207	25.837	24.714	25.837	29.207
	5	142.561	126.111	120.628	126.111	142.561
	6	26.516	23.456	22.436	23.456	26.516
	7	86.642	76.745	73.313	76.745	86.642
	8	5.760	5.095	4.874	5.095	5.760
	9	7.714	6.824	6.527	6.824	7.714

Table 56 (b). Peak dynamic deflections $\times 10^{-3}$ in models in series B.

$B = 0.40, \phi = 0.5\pi$

Model	Peak	Vehicle on 1-1			Vehicle on 2-2		
		cc2	cL2	cLL2	cc2	cL2	cLL2
B1	1	1.317	0.988	0.659	1.019	1.445	1.335
	2	1.631	1.223	0.815	1.261	1.784	1.529
	3	21.166	15.875	10.583	16.371	23.151	19.849
	4	5.376	4.034	2.689	4.159	5.882	5.012
	5	10.699	8.024	5.349	8.275	11.702	10.030
B2	1	2.575	2.146	1.609	2.253	2.575	2.361
	2	1.636	1.363	1.022	1.431	1.636	1.499
	3	32.978	27.482	20.612	28.856	32.978	30.230
	4	12.869	10.724	8.043	11.260	12.869	11.796
	5	38.778	37.315	24.236	33.930	38.778	35.547
	6	2.806	2.338	1.754	2.455	2.806	2.572
	7	5.053	4.211	3.158	4.422	5.053	4.632
B3	1	3.572	2.956	2.464	3.080	3.695	3.695
	2	1.452	1.202	1.002	1.253	1.503	1.503
	3	41.247	34.135	28.445	35.558	42.670	42.670
	4	16.249	13.448	11.207	14.009	16.810	16.810
	5	65.441	54.158	45.132	56.415	67.898	67.898
	6	8.602	7.119	5.932	7.416	8.899	8.899
	7	17.532	14.509	12.091	15.114	18.139	18.139
	8	0.498	0.412	0.343	0.429	0.515	0.515
B4	1	5.022	4.650	3.906	4.557	5.208	5.227
	2	1.323	1.225	1.029	1.200	1.372	1.377
	3	48.443	44.855	37.678	43.957	50.238	50.417
	4	14.931	13.825	11.613	13.549	15.484	15.539
	5	101.515	93.995	78.955	92.115	105.274	105.650
	6	29.470	27.288	22.921	26.742	30.562	30.671
	7	70.930	65.677	55.169	64.364	73.559	73.822
	8	5.000	4.630	3.889	4.537	5.186	5.204
	9	12.801	11.853	9.556	11.615	13.275	13.322
B5	1	5.995	5.268	5.039	5.268	5.995	6.184
	2	2.850	2.522	2.412	2.522	2.850	2.960
	3	62.938	55.676	53.255	55.676	62.938	65.359
	4	28.567	25.271	24.172	25.271	28.567	29.666
	5	147.779	130.727	125.044	130.727	147.719	163.463
	6	45.587	40.327	38.574	40.327	45.587	47.350
	7	136.465	120.719	115.470	120.719	136.445	141.714
	8	24.878	22.007	21.051	22.007	24.878	25.834
	9	47.275	43.821	40.003	43.821	47.275	49.094

Table 57 - Comparison of Dynamic Strains. 1 - Theory ; 2 - Experiment ; 3 - % Error.

Model Vehicle	Vehicle on 1-1									Vehicle on 2-2								
	cc2			cL2			cLL2			cc2			cL2			cLL2		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
B1	125	130	4.00	90	100	11.11	65	70	7.69	125	115	8.00	130	138	6.10	115	120	4.34
B2	190	170	10.53	-	-	-	100	110	10.00	175	160	8.57	-	-	-	175	150	14.29
B3 1.5 "	200	185	7.50	160	130	18.75	120	121	0.83	150	160	6.66	130	160	23.00	150	165	10.00
B4	180	190	5.55	190	170	10.52	145	130	10.34	150	160	6.66	210	180	14.29	170	190	11.76
B5	250	220	12.00	220	190	13.63	170	160	5.88	250	230	8.00	280	260	7.15	290	260	10.34
B1	150	130	13.33	95	90	5.26	75	60	20.00	125	110	12.00	140	125	10.71	120	110	8.33
B2	145	120	17.24	-	-	-	90	98	8.88	110	140	27.27	-	-	-	120	140	16.66
B3 0.5 "	210	205	2.38	175	160	8.57	120	125	4.16	190	180	5.26	190	170	10.53	185	175	5.40
B4	230	200	13.03	200	175	12.50	160	130	12.75	225	180	20.00	270	220	18.51	250	220	12.00
B5	220	190	13.63	210	160	23.80	190	140	26.31	220	200	9.09	-	-	-	260	225	13.46
A1	230	250	8.69	-	-	-	70	85	21.42	140	160	14.28	-	-	-	220	260	18.18
A2 1.5 "	145	110	4.61	-	-	-	160	150	6.25	240	215	10.42	-	-	-	325	290	10.77
A4	350	310	11.43	-	-	-	210	220	4.76	270	310	14.81	-	-	-	320	330	3.33
A5	450	410	8.88	-	-	-	290	260	10.34	420	370	11.90	-	-	-	470	420	10.64
A1	240	215	10.41	-	-	-	70	80	14.28	130	132	1.54	-	-	-	215	200	6.98
A2 0.5 "	200	215	7.50	-	-	-	190	85	15.00	145	150	3.45	-	-	-	210	230	9.52
A4	430	380	11.63	-	-	-	280	260	7.12	355	330	7.04	-	-	-	430	390	9.30
A5	430	360	16.27	-	-	-	295	285	3.39	400	360	10.00	-	-	-	450	380	15.55

Table 58

Distribution of errors in experimental results

Accuracy in	Total Tests	Distribution of Error							
		0-5%	5%-10%	10%-15%	15%-20%	20%-25%	25%-30%	30%-35%	35%-40%
Static deflections	17	5	7	3	2	-	-	-	-
Frequencies Empirical Method	77	66	11	-	-	-	-	-	-
Frequencies Experimental	94	43	37	14	-	-	-	-	-
Dynamic strains under vehicle	28	4	8	11	4	1	-	-	-
Dynamic strains away from vehicle run	56	9	20	20	5	2	-	-	343

APPENDIX V

TEST DATA

TABLE A1 TO A20

Table A1(a). Load-- deflection and strain at centre.
Model A1.

LOAD	STRN	CRE3	CC3	CLE3	CRE1	CC1	CLE1	CRE2	CC2	CLE2
0	0	0	0	0	0	0	0	0	0	0
5	44	1	4	1	1	5	1	1	7	1
10	90	3	9	2	3	10	2	3	15	3
15	138	4	14	4	4	15	4	5	22	5
20	184	6	20	5	6	21	5	7	31	7
25	233	7	25	6	7	26	7	9	39	9
30	282	9	31	8	9	32	8	10	47	10
25	236	8	26	7	7	27	7	9	40	9
20	191	6	21	5	6	22	7	7	32	7
15	144	5	16	4	5	17	4	5	24	5
10	96	4	11	3	3	12	3	3	16	3
5	51	2	6	1	1	7	2	2	9	1
0	5	1	1	0	0	1	0	0	1	0

0	0	0	0	0	0	0	0	0	0	0
5	44	1	4	1	1	5	1	1	7	2
10	89	3	10	3	3	10	3	3	14	3
15	135	4	15	4	4	15	4	5	22	5
20	183	6	20	5	6	20	6	7	30	7
25	230	7	25	7	7	25	7	9	38	9
30	280	9	31	9	9	31	9	11	46	10
25	234	8	26	7	8	26	7	9	38	9
20	187	6	21	5	6	21	6	9	31	7
15	140	5	16	4	5	16	5	6	23	5
10	94	3	10	3	3	10	3	4	15	4
5	48	2	5	2	2	5	2	2	8	2
0	4	1	1	0	0	0	1	0	0	0

Model A2 .

0	0	0	0	0	0	0	0	0	0	0
5	59	3	8	2	4	8	2	5	12	6
10	121	6	16	5	6	16	5	8	25	8
15	184	9	24	8	10	23	8	13	38	12
20	248	12	32	11	13	32	11	17	50	17
25	311	16	41	15	16	40	15	22	63	22
20	255	13	33	12	13	32	12	17	51	18
15	195	10	25	9	10	24	9	13	39	14
10	131	7	17	6	7	17	6	9	26	9
5	66	3	8	3	3	8	3	5	13	5
0	3	0	0	0	0	0	0	0	0	0

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Table A1(c). Load - deflection.
Model A3.

LOAD	CLE1	CC1	GRE1	CLE2	CC2	GRE2	CLE3	CC3	CRE3
0	0	0	0	0	0	0	0	0	0
5	6	13	6	8	20	9	6	13	6
10	13	28	13	18	41	22	13	27	13
15	20	42	20	28	58	31	20	41	20
20	27	57	33	38	84	39	28	56	29
25	34	28	34	48	106	49	35	30	34
20	27	58	27	39	86	38	28	57	27
15	20	44	20	28	65	29	21	44	20
10	13	20	13	19	44	19	14	30	13
5	7	15	7	9	23	10	7	14	7
0	0	1	0	0	2	1	1	2	0

Table A1(d). Load deflection.
Model A4.

LOAD	GRE3	CC3	CLR3	GRE1	CC1	CLR1	GRE2	CC2	CLR2
0	0	0	0	0	0	0	0	0	0
5	10	18	11	9	17	9	14	28	15
10	19	36	23	19	36	21	28	57	31
15	30	55	35	30	55	33	44	86	48
20	41	73	48	41	73	45	60	115	64
25	53	91	60	53	91	57	77	143	82
20	43	75	49	44	75	46	63	118	66
15	33	57	38	34	58	36	49	90	50
10	23	41	26	24	39	24	33	62	34
5	12	21	15	13	21	13	18	33	18
0	3	3	3	3	3	2	3	2	2
0	0	0	0	0	0	0	0	0	0
5	8	17	10	8	17	10	12	28	12
10	18	35	22	18	35	21	26	56	29
15	28	54	34	28	54	32	42	84	45
20	39	72	46	39	73	44	58	113	61
25	50	89	58	49	90	55	74	140	79
20	40	73	47	41	74	75	60	115	63
15	30	56	36	30	56	33	44	87	47
10	20	38	24	21	38	23	30	60	31
5	10	19	13	10	20	12	15	31	15
0	0	0	0	0	2	1	0	2	0

Model A5

[illegible]

Table A2(a). Load - deflection and strain at centre.
Model B1

LOAD	STRN	GRE3	CC3	CLE3	GRE1	CC1	CLE1	GRE2	CC2	CLE2
0	0	0	0	0	0	0	0	0	0	0
5	20	0.5	2	0.5	0	2	0	1	3	1
10	40	1.5	4	1	1	4	1	2	6	2
15	60	2	6	2	1.5	6.5	1.5	3	10	3
20	82	3	9	3	3	9	3	4.5	13	4.5
25	104	4	11	4	4	11.5	4	6	16	6
30	131	5	14	5	5	14	5	7	20	7
35	DIS.	6	16	6	6	16	6	9	24	9
30	129	6	15	5.5	5.5	15.5	6	8	21	8
25	110	5	12.5	5	5	13.5	5	7	18	7
20	88	4	10.5	4	4	11	4.5	6	15	6
15	69	3.5	8.5	3.5	3	9.5	4	5	12	5
10	48	3	6	3	2	7	3.5	3.5	9	4
5	28	2	4	2	1	5	2.5	2	5.5	3
0	7	1	1.5	1	0	2.5	2	0	2	2
0	0	0	0	0	0	0	0	0	0	0
5	18	1	2	1	0.5	2	0.5	1	3	1
10	38	1.5	4	1	1	4	1	2	6	2
15	59	3	6	2	2	6	2	3	9	3
20	79	3.5	9	3	3	8	2.5	4	12	4
25	100	4	11	4	4	10	4	6	15	5
30	123	5	13	5	5	13	5	7	19	6
35	145	6	16	6	6	16	6	9	22	8
30	125	6	14	5	6	14	5	8	20	7
25	107	5	12	5	5	12	5	6	16	6
20	88	4	10	4	4	10	4	5	14	5
15	67	3	8	3	3	8	3	4	11	4
10	48	3	6	2	2	6	2	3	7	3
5	26	1	3	1	1	3	1	1	4	2
0	6	0	1	0	0	1	0	0	1	1

Table A2(b). Load - deflection and strain at centre.
Model B2

[illegible]

Table A2(c). Load - deflection and strain at centre.
Model B3

LOAD	STRN	CRB3	CC3	CLE3	CRE1	CC1	CLE1	CRE2	CC2	CLE2
0	0	0	0	0	0	0	0	0	0	0
5	40	4	6	5	5	5	5	6	10	6
10	80	9	12	9	9	12	9	12	19	13
15	120	13	19	14	14	19	14	19	29	19
20	160	18	25	18	19	26	19	26	39	26
25	200	22	31	23	23	33	23	33	48	33
30	241	27	38	27	29	39	28	40	58	40
35	281	33	44	33	34	46	33	46	68	46
30	241	28	39	29	30	40	29	41	60	40
25	202	24	32	24	24	36	24	34	50	34
20	162	19	27	19	20	28	20	28	41	27
15	122	15	21	15	16	22	16	22	31	21
10	84	11	15	11	11	16	11	15	22	15
5	44	7	9	6	7	9	7	9	13	9
0	4	2	3	2	2	3	2	3	4	3

0	0	0	0	0	0	0	0	0	0	0
5	40	4	6	4	4	6	4	6	9	6
10	78	8	9	8	8	12	9	11	18	12
15	118	12	17	13	12	18	13	18	27	18
20	157	17	23	17	17	25	18	24	36	24
25	197	21	30	22	22	31	22	31	46	31
30	238	26	36	26	27	37	27	38	56	38
35	278	30	42	31	31	42	32	45	65	45
30	240	27	37	27	28	39	28	39	57	39
25	202	23	31	23	24	33	24	33	48	33
20	161	18	25	18	19	27	19	27	39	26
15	122	14	19	14	14	21	14	20	30	20
10	83	10	13	10	10	14	10	14	20	14
5	44	5	7	5	5	8	5	8	11	8
0	5	1	1	1	1	2	1	2	2	2

Table A2(d). Load - deflection and strain at centre.
Model B4

LOAD	STRN	CRK3	CC3	GLE3	CRE1	CC1	GLE1	CRK2	CC2	GLE2
0	0	0	0	0	0	0	0	0	0	0
5	40	6	8	7	7	9	7	9	13	10
10	82	13	17	14	14	18	14	19	26	20
15	125	21	26	22	21	27	22	31	40	32
20	167	28	35	30	29	36	29	41	55	42
25	210	36	44	38	36	45	37	52	68	53
30	255	43	53	45	44	54	45	63	83	64
35	298	51	63	53	51	63	53	74	97	75
30	260	46	56	48	46	57	48	66	85	67
25	216	39	47	40	39	48	40	55	72	56
20	173	31	38	32	32	38	32	45	58	45
15	130	24	29	23	25	30	24	33	44	34
10	87	16	20	16	17	20	17	22	30	23
5	44	9	10	9	9	11	9	11	16	12
0	1	1	1	1	1	1	1	1	2	1
0	0	0	0	0	0	0	0	0	0	0
5	40	7	8	7	6	8	7	8	12	9
10	83	14	18	14	14	17	14	19	26	20
15	125	22	27	22	21	26	22	30	40	30
20	168	29	36	30	29	35	29	41	54	42
25	213	36	45	38	36	44	37	52	68	53
30	256	44	55	44	43	53	45	63	82	63
35	299	51	64	53	51	62	53	74	95	74
30	261	46	57	48	46	55	47	66	85	66
25	220	39	48	40	39	47	39	56	71	56
20	174	32	39	32	31	37	32	44	57	45
15	131	24	29	24	24	28	24	33	43	33
10	89	17	20	16	16	20	16	22	29	22
5	46	9	11	9	8	10	9	11	15	10
0	3	1	1	1	0	0	1	1	0	0

Table A3(a, b). Load deflection and strains. Model C1

(a)									
LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
10	2	2	1	1	1	0	0	0	2
20	3	4	2	5	1	3	5	1	6
30	5	7	4	13	2	6	12	2	10
40	8	10	6	30	3	12	16	2	16
50	10	12	8	37	3	16	23	3	19
60	11	14	9	42	4	19	27	4	23
70	14	17	12	60	5	23	38	4	33
60	12	15	10	51	4	19	32	3	27
50	10	13	8	45	4	16	25	3	23
40	9	10	7	39	3	10	18	3	19
30	7	8	5	30	2	6	10	2	6
20	4	5	4	13	2	4	3	1	3
0	1	1	1	5	1	1	0	0	0
(b)									
0	0	0	0	0	0	0	0	0	0
10	33	85	32	8	15	11	12	-	15
20	73	178	73	20	34	24	18	-	35
30	111	270	113	30	50	35	44	-	54
50	191	457	197	51	85	56	80	-	96
60	230	549	230	62	103	70	97	-	115
70	266	639	296	79	143	90	107	-	131
60	236	555	260	69	124	76	95	-	114
50	198	466	220	57	103	65	80	-	96
40	162	376	180	45	87	51	65	-	78
30	122	285	137	35	70	39	50	-	60
20	84	193	14	24	52	27	33	-	41
0	7	8	9	2	14	1	1	-	4

Table A4 (a, b, c): Load deflection $\times 10^{-3}$ at centre,
Models C2, C4, and C5.

Model C2			Model C3			Model C4		
Load	Deflection		Load	Deflection		Load	Deflection	
0	0	0	0	0	0	0	0	0
10	5	5	10	6	7	5	5	4
20	10	10	20	12	13	10	9	9
30	14	14	30	19	19	15	14	12
40	18	18	40	24	25	20	18	17
50	23	22	50	31	31	25	22	22
60	27	27	60	37	37	30	27	26
70	31	31	65	40	40	40	35	34
75	33	33	70	42	42	50	43	42
70	32	32	65	40	41	40	36	35
60	28	28	60	38	38	30	28	28
50	24	24	50	32	31	25	24	23
40	19	19	40	25	25	20	20	19
30	15	15	30	20	19	15	16	14
20	10	10	20	14	12	10	11	10
10	5	6	10	8	6	5	6	6
0	1	0	0	2	1	0	0	1

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Table A5(a). Load - deflection. Model C5
Deflection at quarter points and ends.

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
4	1	3	2	5	0	7	1	1	4
8	4	6	4	19	2	23	18	4	18
12	7	10	6	35	5	41	40	5	35
16	10	14	9	54	8	61	60	7	46
20	14	19	12	75	11	81	76	10	68
25	17	24	15	100	15	105	109	13	94
30	21	30	18	125	19	131	126	16	113
35	25	36	22	152	22	150	154	20	136
30	23	32	20	138	20	135	143	18	122
25	20	28	18	121	18	118	121	15	108
20	17	23	15	103	15	97	97	12	89
16	14	20	14	88	12	81	80	9	69
12	11	16	11	70	10	65	63	7	47
4	3	5	5	25	3	20	10	1	2

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
4	2	2	1	3	0	2	1	1	4
8	5	6	3	8	1	9	13	3	14
12	8	10	5	16	2	18	22	4	20
16	11	15	8	25	3	28	37	6	29
20	14	18	10	32	5	36	46	7	34
25	18	24	14	44	6	49	62	9	47
30	22	30	17	56	7	62	78	10	56
35	25	36	21	68	9	74	90	12	67
30	23	33	20	63	9	71	88	10	64
25	20	28	17	56	7	62	71	8	50
20	16	23	14	46	7	53	55	6	36
16	13	19	12	37	6	42	40	5	29
12	10	14	9	25	4	29	29	4	19
8	7	11	6	21	3	25	14	3	11

Table A5(b). Load - Strain. Model C5

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
4	-	51	9	18	23	12	13	21	15
8	-	101	18	22	47	24	26	40	30
12	-	153	20	31	65	33	37	58	43
16	-	205	31	46	88	46	51	80	60
20	-	259	42	58	110	58	65	102	77
25	-	325	54	72	135	72	81	127	98
30	-	395	67	83	159	87	100	155	120
35	-	459	79	99	185	101	116	181	139
30	-	400	68	85	157	87	101	158	123
25	-	340	57	72	131	73	87	133	106
20	-	278	46	56	101	58	71	109	88
16	-	228	35	43	79	45	59	89	71
12	-	177	25	31	57	33	46	58	55
8	-	125	16	18	36	22	35	47	40
4	-	71	4	6	13	8	19	25	22
0	-	17	1	1	6	3	3	3	5

Table A6. Load deflection. Model D1
Deflection at quarter points and ends.

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
5	0	4	2	10	2	7	6	2	11
10	1	8	5	22	4	22	24	4	23
15	2	12	8	37	5	38	39	5	36
20	4	16	11	51	6	52	49	6	50
25	7	20	13	65	8	66	60	8	63
30	10	24	16	80	11	81	71	11	76
20	6	18	12	60	7	62	59	7	63
10	0	9	6	33	3	36	3	3	35
0	0	1	2	7	1	9	3	1	12

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
5	4	4	2	0	-	2	8	-	6
10	6	8	5	4	-	5	10	-	12
15	10	12	8	7	-	10	13	-	16
20	13	16	10	13	-	15	17	-	23
25	17	21	13	19	-	21	22	-	28
30	20	24	15	24	-	27	30	-	32
20	15	17	11	18	-	22	24	-	21
10	7	9	6	8	-	11	12	-	7
0	1	1	0	0	-	1	4	-	0

Table A7(a) Load - deflection. Model D5

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
2	3	6	3	33	4	30	30	3	32
4	8	13	8	75	11	71	69	8	65
6	13	20	14	125	18	125	115	16	123
8	18	33	20	170	25	171	157	22	164
10	23	38	26	224	31	239	202	28	223
12	28	45	31	267	37	280	241	34	261
14	33	53	37	317	44	333	282	40	314
16	38	60	41	367	49	363	319	45	349
18	41	69	47	414	56	411	361	52	398
20	46	76	51	451	53	454	401	58	436
18	42	73	48	426	57	431	381	54	416
16	38	66	43	379	49	385	341	49	390
12	28	50	33	289	39	298	268	37	308
10	24	44	28	249	31	250	224	33	252
8	19	34	23	197	25	214	186	25	214
6	13	25	17	149	19	156	138	18	160
4	9	15	12	100	11	114	98	12	128
2	2	10	9	53	4	56	48	5	62
0	0	2	1	3	3	11	3	2	26

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
2	4	6	4	20	5	24	26	4	29
4	9	14	9	49	9	51	54	10	55
6	15	22	15	75	14	82	89	14	81
8	20	30	21	111	19	112	121	19	123
10	25	39	27	147	24	145	154	24	157
12	30	48	34	172	30	194	194	30	216
14	35	55	39	205	34	227	221	34	235
16	40	62	44	245	40	262	255	39	266
18	45	70	50	280	45	303	289	44	302
20	50	79	56	323	51	352	322	49	348
18	48	75	46	292	44	321	312	47	337
16	43	69	48	257	40	290	284	43	319
14	38	61	43	217	34	246	250	38	272
12	34	52	38	177	29	216	221	34	245
10	28	43	32	140	24	175	186	28	197
8	24	35	36	103	20	129	154	24	166
6	18	27	20	65	15	99	118	18	137
4	13	19	14	31	10	69	87	14	112
2	7	10	8	9	5	30	47	9	62
0	1	2	2	0	1	0	11	4	33

Table A7(b) Load - Strain. Model D5

LOAD	CC1	CC2	CC3	CR1	CR2	CR3	CL1	CL2	CL3
0	0	0	0	0	0	0	0	0	0
2	6	42	15	18	40	9	10	25	9
4	31	93	29	33	73	26	26	52	26
6	39	146	46	51	108	43	41	78	43
8	64	196	57	63	141	58	51	92	58
10	72	250	73	83	178	77	63	112	76
12	90	302	91	101	212	95	81	141	97
14	109	356	112	120	249	114	96	169	111
16	130	412	130	139	287	134	111	200	128
18	146	465	151	156	322	153	126	230	145
20	164	512	169	173	356	169	138	259	159
10	85	262	66	93	186	84	68	111	79
0	4	5	9	4	5	2	0	37	0

Table A8(a). Influence lines. Model B1
L-Load, D-deflection, s-strain.

L	cc2		cL2		cLL2		cc3		cL3		cLL3	
	D	S	D	S	D	S	D	S	D	S	D	S
LOAD ON 1 - 1 $x/a = 0.275$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	10	37	8	31	5	24	9	115	6	42	4	23
40	21	80	16	68	10	55	19	235	14	89	9	51
LOAD ON 2 - 1 $x/a = 0.500$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	12	74	8	49	5	30	8	43	6	31	4	22
40	26	173	18	106	11	67	18	98	14	69	9	52
LOAD ON 1 - 1 $x/a = 0.775$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	9	41	7	42	4	25	5	10	4	14	3	14
40	19	86	14	85	9	51	10	22	9	28	6	30
LOAD ON 2 - 2 $x/a = 0.350$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	8	43	11	46	10	61	6	43	10	74	8	53
40	17	92	23	94	20	128	14	93	20	154	15	111
LOAD ON 2 - 2 $x/a = 0.500$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	10	63	13	95	11	76	7	37	9	37	8	42
40	20	129	28	195	23	155	14	82	19	76	17	86
LOAD ON 2 - 2 $x/a = 0.650$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	9	54	11	85	9	62	6	20	7	16	7	24
40	18	112	23	176	19	130	12	48	14	38	13	54
LOAD ON 3 - 3 $x/a = 0.275$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	4	23	8	43	14	75	3	21	7	51	12	131
40	-9	51	17	90	29	157	7	46	14	109	26	172
LOAD ON 3 - 3 $x/a = 0.500$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	5	30	10	68	18	141	4	21	7	40	12	51
40	12	66	22	143	37	291	8	50	16	84	27	111
LOAD ON 3 - 3 $x/a = 0.775$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	4	21	8	50	14	84	3	13	5	19	9	16
40	9	47	16	104	29	173	5	30	10	41	17	37

Table AS(b). Influence lines. Model B2
L-Load, D-deflection, s-strain.

L	cc2		cL2		cLL2		cc3		cL3		cLL3	
	D	S	D	S	D	S	D	S	D	S	D	S
LOAD ON 1-1 $x/a = 0.240$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	15	33	13	-	10	35	14	100	12	48	8	-
40	29	70	25	-	19	73	28	203	22	99	16	-
LOAD ON 1-1 $x/a = 0.480$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	24	135	19	-	15	59	16	39	13	44	11	-
40	48	273	39	-	29	118	32	81	27	90	22	-
LOAD ON 1-1 $x/a = 0.720$												
0	0	0	0	0	0	0	0	0	0	0	0	-
20	18	52	15	-	12	45	10	16	10	19	8	-
40	35	106	30	-	24	92	20	33	19	41	17	-
LOAD ON 2-2 $x/a = 0.220$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	12	37	14	-	13	46	10	51	14	98	11	-
40	24	75	27	-	26	94	20	102	27	199	22	-
LOAD ON 2-2 $x/a = 0.460$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	20	84	24	-	21	98	14	47	16	44	15	-
40	39	170	47	-	43	197	27	95	33	90	31	-
LOAD ON 2-2 $x/a = 0.700$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	15	54	17	-	17	72	9	21	10	15	10	-
40	30	110	35	-	35	142	18	44	21	34	22	-
LOAD ON 3-3 $x/a = 0.240$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	10	35	15	-	20	66	8	32	12	65	19	-
40	20	62	29	-	42	134	16	64	24	130	39	-
LOAD ON 3-3 $x/a = 0.480$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	15	55	23	-	34	199	10	36	15	49	23	-
40	29	109	44	-	68	403	20	74	30	99	45	-
LOAD ON 3-3 $x/a = 0.720$												
0	0	0	0	0	0	0	0	0	0	0	0	0
20	12	42	17	-	25	89	7	21	10	19	14	-
40	24	86	34	-	50	184	15	46	21	43	29	-

Table A8(c). Influence lines. Model B3
L-Load, D-deflection, s-strain.

L	cc2		cL2		cLL2		cc3		cL3		cLL3	
	D	S	D	S	D	S	D	S	D	S	D	S
LOAD ON 1-1 $x/a = 0.279$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	17	34	14	40	12	34	16	40	8	44	8	31
30	36	74	30	80	26	76	32	184	21	94	19	67
LOAD ON 1-1 $x/a = 0.479$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	26	113	21	71	23	49	16	29	14	33	12	32
30	52	229	42	144	36	103	33	63	30	69	25	69
LOAD ON 1-1 $x/a = 0.679$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	21	44	17	50	15	41	12	13	10	15	9	19
30	42	91	36	103	30	85	24	31	22	35	20	42
LOAD ON 2-2 $x/a = 0.229$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	19	52	21	56	22	69	14	47	17	59	17	60
30	38	106	45	116	45	139	28	98	35	124	35	125
LOAD ON 2-2 $x/a = 0.429$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	22	66	26	115	27	90	14	28	17	28	17	36
30	45	157	55	234	55	184	29	62	34	61	35	77
LOAD ON 2-2 $x/a = 0.629$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	17	37	18	41	19	63	10	13	10	11	11	13
30	38	78	38	85	40	117	20	29	22	28	24	33
LOAD ON 3-3 $x/a = 0.279$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	14	35	19	56	27	69	10	31	15	59	24	117
30	28	74	40	115	45	141	21	66	30	124	47	242
LOAD ON 3-3 $x/a = 0.479$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	19	46	26	87	38	168	12	31	18	44	25	50
30	37	96	48	174	77	303	25	65	35	87	50	105
LOAD ON 3-3 $x/a = 0.679$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	16	40	21	64	30	51	10	19	13	21	17	19
30	32	83	45	130	62	135	20	43	27	46	35	44

Table A8(d). Influence lines, Model B4
L-Load, D-deflection, s-strain.

L	cc2		cL2		cLL2		cc3		cL3		cLL3	
	D	S	D	S	D	S	D	S	D	S	D	S
LOAD ON 1-1 $x/a = 0.250$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	27	42	25	48	22	46	25	103	22	66	18	45
30	54	82	52	98	46	92	51	209	45	133	37	90
LOAD ON 1-1 $x/a = 0.505$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	40	128	38	92	30	68	28	36	25	43	22	44
30	84	254	85	182	62	132	54	65	50	82	43	84
LOAD ON 1-1 $x/a = 0.765$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	24	31	23	39	18	38	14	11	14	14	12	16
30	50	68	40	82	41	80	29	26	28	32	25	36
LOAD ON 2-2 $x/a = 0.206$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	20	33	23	36	22	43	18	53	21	100	19	63
30	40	70	46	72	47	90	36	112	43	208	40	131
LOAD ON 2-2 $x/a = 0.465$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	35	83	40	137	40	140	24	37	27	38	27	45
30	72	170	84	278	83	281	60	81	57	82	57	98
LOAD ON 2-2 $x/a = 0.730$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	27	45	30	47	29	58	17	17	18	18	18	20
30	54	94	62	99	62	121	33	36	36	39	37	42
LOAD ON 3-3 $x/a = 0.250$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	21	43	29	61	37	67	17	40	23	70	34	140
30	43	85	58	121	76	135	36	78	48	143	68	285
LOAD ON 3-3 $x/a = 0.505$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	32	67	42	116	58	193	21	37	28	47	37	42
30	53	131	87	230	118	381	44	76	58	93	74	93
LOAD ON 3-3 $x/a = 0.765$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	22	40	28	54	36	59	14	18	16	17	21	19
30	44	81	57	109	74	120	28	37	33	37	42	36

Table A8(e). Influence lines. Model B5

L-Load, D-deflection, s-strain

L	cc2		cL2		cLL2		cc3		cL3		cLL3	
	D	S	D	S	D	S	D	S	D	S	D	S
LOAD ON 1 - 1 $x/a = 0.256$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	37	45	35	51	34	55	34	102	30	74	28	55
30	78	93	76	108	74	115	72	211	64	152	58	131
LOAD ON 1 - 1 $x/a = 0.481$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	52	139	50	98	45	80	35	33	33	41	32	44
30	112	289	106	198	98	167	76	76	72	95	69	104
LOAD ON 1 - 1 $x/a = 0.706$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	37	47	36	53	36	56	23	16	22	20	22	22
30	80	99	78	113	74	117	50	37	48	46	48	54
LOAD ON 2 - 2 $x/a = 0.269$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	37	56	42	58	45	73	31	60	36	79	37	81
30	80	118	90	119	96	153	66	127	75	168	76	74
LOAD ON 2 - 2 $x/a = 0.494$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	50	93	57	143	60	114	34	34	36	33	40	44
30	105	192	119	293	123	235	71	74	76	76	81	92
LOAD ON 2 - 2 $x/a = 0.719$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	34	46	38	54	40	61	21	16	22	13	25	12
30	72	97	80	131	86	128	45	35	48	33	52	33
LOAD ON 3 - 3 $x/a = 0.256$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	37	54	46	73	58	77	30	49	38	83	50	172
30	67	99	85	130	109	146	54	50	70	154	95	333
LOAD ON 3 - 3 $x/a = 0.481$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	45	73	60	128	78	195	32	38	40	45	50	59
30	95	153	124	250	160	402	68	83	83	98	103	128
LOAD ON 3 - 3 $x/a = 0.706$												
0	0	0	0	0	0	0	0	0	0	0	0	0
15	34	51	43	71	55	76	22	19	25	16	31	20
30	73	105	91	150	115	157	47	41	55	34	65	39

Table A9(a). Influence lines. Model A2
L-Load, D-Deflection, S-Strain.

L	CC2		CLL2		CC3		CLL3	
	D	S	D	S	D	S	D	S
LOAD ON 1-1								
0	0	0	0	0	0	0	0	0
15	31	64	11	39	34	207	7	31
30	62	131	23	79	60	415	15	60
LOAD ON 1-1								
0	0	0	0	0	0	0	0	0
15	39	233	12	43	23	62	8	22
30	79	463	26	92	49	128	18	56
LOAD ON 1-1								
0	0	0	0	0	0	0	0	0
15	22	51	8	28	11	29	4	9
30	46	105	17	58	25	61	11	26
LOAD ON 2-2								
0	0	0	0	0	0	0	0	0
15	16	43	19	67	14	58	15	72
30	34	90	38	135	29	118	29	140
LOAD ON 2-2								
0	0	0	0	0	0	0	0	0
15	25	79	28	112	18	63	20	69
30	51	162	60	225	38	128	40	144
LOAD ON 2-2								
0	0	0	0	0	0	0	0	0
15	18	61	23	82	11	31	14	33
30	40	123	47	169	25	72	29	75
LOAD ON 3-3								
0	0	0	0	0	0	0	0	0
15	7	23	37	119	6	17	31	200
30	17	51	77	245	14	38	64	412
LOAD ON 3-3								
0	0	0	0	0	0	0	0	0
15	10	35	51	262	8	21	32	73
30	23	70	104	529	18	46	64	161
LOAD ON 3-3								
0	0	0	0	0	0	0	0	0
15	6	23	31	88	4	10	17	24
30	13	45	65	179	9	24	35	54

Table A9(b). Influence lines. Model A4
L-Load, D-Deflection, S-Strain.

L	CC2		CLL2		CC3		CLL3	
	D	S	D	S	D	S	D	S
LOAD ON 1-1					$x/a = 0.236$			
0	0	0	0	0	0	0	0	0
10	36	45	20	35	39	155	14	28
20	71	93	43	76	77	311	32	61
LOAD ON 1-1					$x/a = 0.493$			
0	0	0	0	0	0	0	0	0
10	49	146	29	53	39	32	20	33
20	104	349	63	117	77	70	39	71
LOAD ON 1-1					$x/a = 0.750$			
0	0	0	0	0	0	0	0	0
10	33	25	19	40	20	12	5	17
20	69	73	42	81	43	26	20	35
LOAD ON 2-2					$x/a = 0.286$			
0	0	0	0	0	0	0	0	0
10	25	40	26	46	23	52	23	63
20	50	81	57	101	46	107	48	131
LOAD ON 2-2					$x/a = 0.545$			
0	0	0	0	0	0	0	0	0
10	44	92	47	104	34	40	30	58
20	89	206	97	225	67	85	64	120
LOAD ON 2-2					$x/a = 0.800$			
0	0	0	0	0	0	0	0	0
10	34	51	37	70	23	16	21	28
20	69	117	77	145	46	36	46	57
LOAD ON 3-3					$x/a = 0.236$			
0	0	0	0	0	0	0	0	0
10	17	28	45	62	14	17	43	177
20	36	56	92	130	28	35	86	333
LOAD ON 3-3					$x/a = 0.493$			
0	0	0	0	0	0	0	0	0
10	29	43	72	206	21	23	48	74
20	59	89	142	408	43	47	95	153
LOAD ON 3-3					$x/a = 0.750$			
0	0	0	0	0	0	0	0	0
10	20	29	43	66	13	10	25	25
20	41	61	89	134	29	23	52	55

Table A9(c). Influence lines. Model A5
L-Load, D-Deflection, S-Strain.

L	CC2		CLL2		CC3		CLL3	
	D	S	D	S	D	S	D	S
LOAD ON 1 - 1 $x/a = 0.288$								
0	0	0	0	0	0	0	0	0
10	50	43	33	45	50	124	25	38
20	104	89	74	97	100	253	54	83
LOAD ON 1 - 1 $x/a = 0.512$								
0	0	0	0	0	0	0	0	0
10	74	184	45	63	45	33	31	36
20	147	366	99	134	91	71	66	72
LOAD ON 1 - 1 $x/a = 0.738$								
0	0	0	0	0	0	0	0	0
10	46	44	31	38	26	16	17	15
20	94	92	66	83	55	33	38	33
LOAD ON 2 - 2 $x/a = 0.250$								
0	0	0	0	0	0	0	0	0
10	40	43	48	61	34	72	36	79
20	84	88	100	124	74	148	79	163
LOAD ON 2 - 2 $x/a = 0.475$								
0	0	0	0	0	0	0	0	0
10	61	96	72	117	43	41	46	55
20	125	197	151	243	88	87	100	117
LOAD ON 2 - 2 $x/a = 0.700$								
0	0	0	0	0	0	0	0	0
10	46	59	56	70	28	18	32	29
20	96	116	115	138	60	39	68	56
LOAD ON 3 - 3 $x/a = 0.288$								
0	0	0	0	0	0	0	0	0
10	34	40	76	84	26	32	69	187
20	74	80	155	172	57	72	141	375
LOAD ON 3 - 3 $x/a = 0.512$								
0	0	0	0	0	0	0	0	0
10	46	59	109	256	33	27	68	68
20	100	125	220	505	70	61	135	136
LOAD ON 3 - 3 $x/a = 0.738$								
0	0	0	0	0	0	0	0	0
10	34	40	70	75	23	15	39	28
20	72	86	143	156	47	31	80	61

Table A10.

Vehicle Static Crawl strains.

Models - A

Model	Position of Vehicle.	Vehicle on 1-1			CLL3	Vehicle on 2-2		
		CC2	CLL2	CC3		CC2	CLL2	CC3
A1	0	0	0	0	0	0	0	0
	a/4	113	33	139	7	89	75	63
	a/2	141	44	74	2	103	152	14
	3a/4	71	27	43	0	50	60	2
	a	0	0	0	0	0	0	0
A2	0	0	0	0	0	0	0	0
	a/4	120	64	185	55	94	114	182
	a/2	202	89	99	60	134	197	97
	3a/4	74	50	40	34	58	85	47
	a	0	0	0	0	0	0	0
A4	0	0	0	0	0	0	0	0
	a/4	153	103	222	91	102	144	170
	a/2	255	141	77	83	198	243	107
	3a/4	99	83	30	36	84	114	48
	a	0	0	0	0	0	0	0
A5	0	0	0	0	0	0	0	0
	a/4	125	135	242	111	120	170	206
	a/2	299	177	105	100	275	288	133
	3a/4	117	110	47	53	115	135	62
	a	0	0	0	0	0	0	0

Table A12(a) Dynamic - Strains. Model: B1 Location: CC2			
Vehicle on 1-1		Vehicle on 2-2	
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$
111 +00006 111 +00001 111 +00012 111 +00037 111 +00040 111 +00060 111 +00127 111 +00100 111 +00051 111 +00073 111 +00125 111 +00033 111 +00000 111 +00000 000 00000 000 00000	111 +00006 111 +00028 111 +00016 111 +00077 111 +00102 111 +00078 111 +00079 111 +00100 111 +00065 111 +00012 111 +00020 111 +00043 111 +00008 111 +00001 111 +00005 111 +00000 111 +00003 111 +00004 000 00000 000 00000	111 +00042 111 +00035 111 +00045 111 +00090 111 +00110 111 +00088 111 +00039 111 +00036 111 +00027 111 +00006 111 +00011 111 +00064 000 00000 000 00000	111 +00010 111 +00042 111 +00055 111 +00064 111 +00074 111 +00091 111 +00073 111 +00046 111 +00035 111 +00018 111 +00057 111 +00014 111 +00005 111 +00005 111 +00005 000 00000 000 00000

Table A12(c) - Dynamic - Strains. Model: B1 Location: CLL2

Vehicle on 1-1				Vehicle on 2-2	
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$
114 +00000 114 +00005 114 +00035 114 +00017 114 +00020 114 +00037 114 +00011 114 +00033 114 +00015 114 +00013 114 +00025 114 +00026 114 +00010 114 +00003 114 +00006 114 +00009 114 +00005 000 00000 000 00000	114 +00002 114 +00001 114 +00020 114 +00038 114 +00007 114 +00001 114 +00030 114 +00003 114 +00009 114 +00006 114 +00030 114 +00001 114 +00022 114 +00012 114 +00003 114 +00005 114 +00009 000 +00000 000 00000	114 +00025 114 +00031 114 +00050 114 +00009 114 +00020 114 +00091 114 +00071 114 +00047 114 +00091 114 +00028 114 +00007 114 +00002 000 00000 000 00000	114 +00000 114 +00000 114 +00012 114 +00058 114 +00068 114 +00017 114 +00065 114 +00101 114 +00118 114 +00088 114 +00032 114 +00012 114 +00012 000 00000 000 00000		

Table A13(b) - Dynamic - Strains. Model B2 Location: CLL2

[illegible]

Table A14(a) - Dynamic - Strains. Model B3 • Location: CC2

Vehicle on 1-1		Vehicle on 2-2	
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$
113 +0.000	113 +0.000	113 +0.000	113 +0.000
113 +0.002	113 +0.001	113 +0.002	113 +0.001
113 +0.003	113 +0.002	113 +0.003	113 +0.002
113 +0.004	113 +0.003	113 +0.004	113 +0.003
113 +0.005	113 +0.004	113 +0.005	113 +0.004
113 +0.006	113 +0.005	113 +0.006	113 +0.005
113 +0.007	113 +0.006	113 +0.007	113 +0.006
113 +0.008	113 +0.007	113 +0.008	113 +0.007
113 +0.009	113 +0.008	113 +0.009	113 +0.008
113 +0.010	113 +0.009	113 +0.010	113 +0.009
113 +0.011	113 +0.010	113 +0.011	113 +0.010
113 +0.012	113 +0.011	113 +0.012	113 +0.011
113 +0.013	113 +0.012	113 +0.013	113 +0.012
113 +0.014	113 +0.013	113 +0.014	113 +0.013
113 +0.015	113 +0.014	113 +0.015	113 +0.014
113 +0.016	113 +0.015	113 +0.016	113 +0.015
113 +0.017	113 +0.016	113 +0.017	113 +0.016
113 +0.018	113 +0.017	113 +0.018	113 +0.017
113 +0.019	113 +0.018	113 +0.019	113 +0.018
113 +0.020	113 +0.019	113 +0.020	113 +0.019
113 +0.021	113 +0.020	113 +0.021	113 +0.020
113 +0.022	113 +0.021	113 +0.022	113 +0.021
113 +0.023	113 +0.022	113 +0.023	113 +0.022
113 +0.024	113 +0.023	113 +0.024	113 +0.023
113 +0.025	113 +0.024	113 +0.025	113 +0.024
113 +0.026	113 +0.025	113 +0.026	113 +0.025
113 +0.027	113 +0.026	113 +0.027	113 +0.026
113 +0.028	113 +0.027	113 +0.028	113 +0.027
113 +0.029	113 +0.028	113 +0.029	113 +0.028
113 +0.030	113 +0.029	113 +0.030	113 +0.029
113 +0.031	113 +0.030	113 +0.031	113 +0.030
113 +0.032	113 +0.031	113 +0.032	113 +0.031
113 +0.033	113 +0.032	113 +0.033	113 +0.032
113 +0.034	113 +0.033	113 +0.034	113 +0.033
113 +0.035	113 +0.034	113 +0.035	113 +0.034
113 +0.036	113 +0.035	113 +0.036	113 +0.035
113 +0.037	113 +0.036	113 +0.037	113 +0.036
113 +0.038	113 +0.037	113 +0.038	113 +0.037
113 +0.039	113 +0.038	113 +0.039	113 +0.038
113 +0.040	113 +0.039	113 +0.040	113 +0.039
113 +0.041	113 +0.040	113 +0.041	113 +0.040
113 +0.042	113 +0.041	113 +0.042	113 +0.041
113 +0.043	113 +0.042	113 +0.043	113 +0.042
113 +0.044	113 +0.043	113 +0.044	113 +0.043
113 +0.045	113 +0.044	113 +0.045	113 +0.044
113 +0.046	113 +0.045	113 +0.046	113 +0.045
113 +0.047	113 +0.046	113 +0.047	113 +0.046
113 +0.048	113 +0.047	113 +0.048	113 +0.047
113 +0.049	113 +0.048	113 +0.049	113 +0.048
113 +0.050	113 +0.049	113 +0.050	113 +0.049
113 +0.051	113 +0.050	113 +0.051	113 +0.050
113 +0.052	113 +0.051	113 +0.052	113 +0.051
113 +0.053	113 +0.052	113 +0.053	113 +0.052
113 +0.054	113 +0.053	113 +0.054	113 +0.053
113 +0.055	113 +0.054	113 +0.055	113 +0.054
113 +0.056	113 +0.055	113 +0.056	113 +0.055
113 +0.057	113 +0.056	113 +0.057	113 +0.056
113 +0.058	113 +0.057	113 +0.058	113 +0.057
113 +0.059	113 +0.058	113 +0.059	113 +0.058
113 +0.060	113 +0.059	113 +0.060	113 +0.059
113 +0.061	113 +0.060	113 +0.061	113 +0.060
113 +0.062	113 +0.061	113 +0.062	113 +0.061
113 +0.063	113 +0.062	113 +0.063	113 +0.062
113 +0.064	113 +0.063	113 +0.064	113 +0.063
113 +0.065	113 +0.064	113 +0.065	113 +0.064
113 +0.066	113 +0.065	113 +0.066	113 +0.065
113 +0.067	113 +0.066	113 +0.067	113 +0.066
113 +0.068	113 +0.067	113 +0.068	113 +0.067
113 +0.069	113 +0.068	113 +0.069	113 +0.068
113 +0.070	113 +0.069	113 +0.070	113 +0.069
113 +0.071	113 +0.070	113 +0.071	113 +0.070
113 +0.072	113 +0.071	113 +0.072	113 +0.071
113 +0.073	113 +0.072	113 +0.073	113 +0.072
113 +0.074	113 +0.073	113 +0.074	113 +0.073
113 +0.075	113 +0.074	113 +0.075	113 +0.074
113 +0.076	113 +0.075	113 +0.076	113 +0.075
113 +0.077	113 +0.076	113 +0.077	113 +0.076
113 +0.078	113 +0.077	113 +0.078	113 +0.077
113 +0.079	113 +0.078	113 +0.079	113 +0.078
113 +0.080	113 +0.079	113 +0.080	113 +0.079
113 +0.081	113 +0.080	113 +0.081	113 +0.080
113 +0.082	113 +0.081	113 +0.082	113 +0.081
113 +0.083	113 +0.082	113 +0.083	113 +0.082
113 +0.084	113 +0.083	113 +0.084	113 +0.083
113 +0.085	113 +0.084	113 +0.085	113 +0.084
113 +0.086	113 +0.085	113 +0.086	113 +0.085
113 +0.087	113 +0.086	113 +0.087	113 +0.086
113 +0.088	113 +0.087	113 +0.088	113 +0.087
113 +0.089	113 +0.088	113 +0.089	113 +0.088
113 +0.090	113 +0.089	113 +0.090	113 +0.089
113 +0.091	113 +0.090	113 +0.091	113 +0.090
113 +0.092	113 +0.091	113 +0.092	113 +0.091
113 +0.093	113 +0.092	113 +0.093	113 +0.092
113 +0.094	113 +0.093	113 +0.094	113 +0.093
113 +0.095	113 +0.094	113 +0.095	113 +0.094
113 +0.096	113 +0.095	113 +0.096	113 +0.095
113 +0.097	113 +0.096	113 +0.097	113 +0.096
113 +0.098	113 +0.097	113 +0.098	113 +0.097
113 +0.099	113 +0.098	113 +0.099	113 +0.098
113 +0.100	113 +0.099	113 +0.100	113 +0.099

Table A14(b) - Dynamic - Strains. Model: B3 Location: CL2

Vehicle on 1-1			Vehicle on 2-2	
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	
114 +00312	14 +00600	14 +00307	114 +00307	114 +00307
114 +00301	14 +00029	114 +00312	114 +00312	114 +00312
114 +00312	114 +00035	114 +00331	114 +00331	114 +00331
114 +00329	114 +00077	114 +00321	114 +00321	114 +00321
114 +00340	114 +00066	114 +00322	114 +00322	114 +00322
114 +00368	114 +00086	114 +00308	114 +00308	114 +00308
114 +00403	114 +00099	114 +00377	114 +00377	114 +00377
114 +00372	114 +00140	114 +00347	114 +00347	114 +00347
114 +00367	114 +00112	114 +00321	114 +00321	114 +00321
114 +00393	114 +00085	114 +00308	114 +00308	114 +00308
114 +00447	114 +00095	114 +00308	114 +00308	114 +00308
114 +00374	114 +00097	114 +00308	114 +00308	114 +00308
114 +00340	114 +00097	114 +00308	114 +00308	114 +00308
114 +00303	114 +00069	114 +00308	114 +00308	114 +00308
114 +00305	114 +00066	114 +00308	114 +00308	114 +00308
114 +00320	114 +00001	114 +00308	114 +00308	114 +00308
114 +00317	114 +00017	114 +00308	114 +00308	114 +00308
114 +00291	114 +00012	114 +00308	114 +00308	114 +00308
114 +00299	114 +00067	114 +00308	114 +00308	114 +00308
114 +00297	114 +00067	114 +00308	114 +00308	114 +00308
100 +00000	100 +00069	100 +00000	100 +00000	100 +00000
100 +00000	100 +00000	100 +00000	100 +00000	100 +00000

Table A14(c) - Dynamic - Strains. Model: B3 Location: CLL2

Vehicle on 1-1			Vehicle on 2-2		
$\phi = 1.5\pi$	$\phi = 0.5\pi$		$\phi = 1.5\pi$	$\phi = 0.5\pi$	
111 -0.317	111 +0.019	111 -0.337	111 -0.337	111 -0.337	111 -0.337
111 -0.327	111 -0.008	111 -0.326	111 -0.326	111 -0.326	111 -0.326
111 +0.317	111 +0.001	111 -0.366	111 -0.366	111 -0.366	111 -0.366
111 -0.035	111 -0.005	111 -0.333	111 -0.333	111 -0.333	111 -0.333
111 -0.035	111 -0.006	111 -0.327	111 -0.327	111 -0.327	111 -0.327
111 -0.043	111 -0.003	111 -0.323	111 -0.323	111 -0.323	111 -0.323
111 -0.034	111 -0.013	111 -0.334	111 -0.334	111 -0.334	111 -0.334
111 -0.034	111 -0.008	111 -0.326	111 -0.326	111 -0.326	111 -0.326
111 -0.036	111 -0.006	111 -0.315	111 -0.315	111 -0.315	111 -0.315
111 -0.036	111 -0.007	111 -0.318	111 -0.318	111 -0.318	111 -0.318
111 -0.039	111 -0.002	111 -0.327	111 -0.327	111 -0.327	111 -0.327
111 -0.039	111 -0.012	111 -0.320	111 -0.320	111 -0.320	111 -0.320
111 -0.039	111 -0.007	111 -0.321	111 -0.321	111 -0.321	111 -0.321
111 -0.037	111 -0.001	111 -0.355	111 -0.355	111 -0.355	111 -0.355
111 -0.037	111 -0.005	111 -0.326	111 -0.326	111 -0.326	111 -0.326
111 -0.039	111 -0.001	111 -0.405	111 -0.405	111 -0.405	111 -0.405
111 -0.039	111 -0.008	111 -0.356	111 -0.356	111 -0.356	111 -0.356
111 -0.039	111 -0.011	111 -0.400	111 -0.400	111 -0.400	111 -0.400
111 -0.039	111 -0.005	111 -0.045	111 -0.045	111 -0.045	111 -0.045
111 -0.039	111 -0.005	111 -0.051	111 -0.051	111 -0.051	111 -0.051
111 -0.039	111 -0.005	111 -0.037	111 -0.037	111 -0.037	111 -0.037
111 -0.039	111 -0.005	111 -0.037	111 -0.037	111 -0.037	111 -0.037
111 -0.039	111 -0.005	111 -0.041	111 -0.041	111 -0.041	111 -0.041
111 -0.039	111 -0.005	111 -0.031	111 -0.031	111 -0.031	111 -0.031
111 -0.039	111 -0.005	111 -0.043	111 -0.043	111 -0.043	111 -0.043

Table A15(b) - Dynamic - Strains, Model: B4 Location: CL2

Vehicle on 1-1			Vehicle on 2-2	
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	
145 +00000	145 +00000	145 +00075	145 +00000	
145 +00051	145 +00050	145 +00098	145 +00029	
145 +00084	145 +00097	145 +00165	145 +00033	
145 +00096	145 +00066	145 +00162	145 +00052	
145 +00107	145 +00054	145 +00124	145 +00065	
145 +00075	145 +00165	145 +00140	145 +00090	
145 +00140	145 +00097	145 +00161	145 +00110	
145 +00164	145 +00120	145 +00171	145 +00092	
145 +00099	145 +00132	145 +00063	145 +00100	
145 +00098	145 +00092	145 +00070	145 +00142	
145 +00050	145 +00090	145 +00079	145 +00147	
145 +00136	145 +00076	145 +00070	145 +00121	
145 +00040	145 +00080	145 +00021	145 +00038	
145 +00038	145 +00058	145 +00001	145 +00142	
145 +00023	145 +00009	145 +00015	145 +00068	
000 000000	145 +00057	145 +00003	145 +00045	
000 000000	145 +00007	000 +00000	000 +00000	
	000 000000	000 +00000	000 +00000	
	000 000000	000 +00000	000 +00000	

Table A15(c) - Dynamic - Strains. Model: B4 Location: CLL2

Vehicle on 1-1				Vehicle on 2-2	
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$
146 +00000	146 +00000	146 +00000	146 +00000	146 +00000	146 +00000
146 +00052	146 +00052	146 +00052	146 +00052	146 +00052	146 +00052
146 +00071	146 +00071	146 +00071	146 +00071	146 +00071	146 +00071
146 +00094	146 +00094	146 +00094	146 +00094	146 +00094	146 +00094
146 +00073	146 +00073	146 +00073	146 +00073	146 +00073	146 +00073
146 +00060	146 +00060	146 +00060	146 +00060	146 +00060	146 +00060
146 +00120	146 +00120	146 +00120	146 +00120	146 +00120	146 +00120
146 +00120	146 +00120	146 +00120	146 +00120	146 +00120	146 +00120
146 +00096	146 +00096	146 +00096	146 +00096	146 +00096	146 +00096
146 +00069	146 +00069	146 +00069	146 +00069	146 +00069	146 +00069
146 +00068	146 +00068	146 +00068	146 +00068	146 +00068	146 +00068
146 +00132	146 +00132	146 +00132	146 +00132	146 +00132	146 +00132
146 +00024	146 +00024	146 +00024	146 +00024	146 +00024	146 +00024
146 +00051	146 +00051	146 +00051	146 +00051	146 +00051	146 +00051
146 +00063	146 +00063	146 +00063	146 +00063	146 +00063	146 +00063
146 +00037	146 +00037	146 +00037	146 +00037	146 +00037	146 +00037
146 +00001	146 +00001	146 +00001	146 +00001	146 +00001	146 +00001
146 +00004	146 +00004	146 +00004	146 +00004	146 +00004	146 +00004
000 000000	000 000000	000 000000	000 000000	000 000000	000 000000
000 000000	000 000000	000 000000	000 000000	000 000000	000 000000

Table. A16(a) - Dynamic - Strains. Model: B5 Location: CC2

Vehicle on J-1			Vehicle on 2-2	
$\phi \neq 1.5\pi$	$\delta = 0.5\pi$	$\delta = 1.5\pi$	$\delta = 0.5\pi$	$\delta = 1.5\pi$
140 +00082	140 +00000	140 +00021	140 +00046	140 +00046
140 +00079	140 +00023	140 +00067	140 +00013	140 +00013
140 +00119	140 +00032	140 +00014	140 +00017	140 +00017
140 +00101	140 +00042	140 +00056	140 +00025	140 +00025
140 +00117	140 +00057	140 +00065	140 +00070	140 +00070
140 +00116	140 +00060	140 +00114	140 +00097	140 +00097
140 +00195	140 +00066	140 +00115	140 +00092	140 +00092
140 +00176	140 +00097	140 +00096	140 +00113	140 +00113
140 +00172	140 +00095	140 +00086	140 +00102	140 +00102
140 +00160	140 +00097	140 +00076	140 +00158	140 +00158
140 +00235	140 +00126	140 +00229	140 +00163	140 +00163
140 +00220	140 +00166	140 +00137	140 +00175	140 +00175
140 +00173	140 +00166	140 +00064	140 +00170	140 +00170
140 +00087	140 +00120	140 +00122	140 +00297	140 +00297
140 +00145	140 +00132	140 +00137	140 +00142	140 +00142
140 +00135	140 +00142	140 +00137	140 +00158	140 +00158
140 +00113	140 +00067	140 +00105	140 +00122	140 +00122
140 +00048	140 +00059	140 +00071	140 +00137	140 +00137
140 +00070	140 +00056	140 +00067	140 +00131	140 +00131
140 +00077	140 +00071	140 +00072	140 +00096	140 +00096
140 +00048	140 +00099	140 +00097	140 +00071	140 +00071
140 +00055	140 +00013	140 +00013	140 +00065	140 +00065
140 +00048	140 +00007	140 +00007	140 +00075	140 +00075
140 +00052	140 +00003	140 +00003	140 +00030	140 +00030
000 +00000	140 +00000	000 +00000	140 +00050	140 +00050
000 +00000	140 +00000	000 +00000	140 +00043	140 +00043
	140 +00000	140 +00000	140 +00045	140 +00045

Table A16(b) - Dynamic - Strains. Model: B5 Location: CL2

Vehicle on 1-1			Vehicle on 2-2		
$\phi = 1.5\pi$	$\phi = 0.5\pi$		$d = 1.5\pi$	$d = 0.5\pi$	
141 +00115	141 +00000		141 +00000	141 +00000	
141 +00169	141 +00010		141 +00002	141 +00000	
141 +00107	141 +00003		141 +00037	141 +00105	
141 +00130	141 +00020		141 +00006	141 +00111	
141 +00163	141 +00023		141 +00023	141 +00127	
141 +00162	141 +00007		141 +00023	141 +00142	
141 +00140	141 +00006		141 +00010	141 +00139	
141 +00132	141 +00000		141 +00026	141 +00171	
141 +00207	141 +00077		141 +00009	141 +00190	
141 +00191	141 +00152		141 +00078	141 +00219	
141 +00157	141 +00089		141 +00120	141 +00210	
141 +00054	141 +00070		141 +00203	141 +00190	
141 +00152	141 +00060		141 +00173	141 +00150	
141 +00133	141 +00159		141 +00090	141 +00165	
141 +00112	141 +00093		141 +00021	141 +00137	
141 +00020	141 +00050		141 +00106	141 +00135	
141 +00021	141 +00045		141 +00106	141 +00123	
141 +00056	141 +00091		141 +00057	141 +00102	
141 +00021	141 +00031		141 +00002	141 +00085	
141 +00001	141 +00022		141 +00050	141 +00077	
141 +00050	141 +00014		141 +00023	141 +00032	
141 +00001	141 +00037		141 +00001	141 +00027	
141 +00001	141 +00008		141 +00012	141 +00020	
141 +00001	141 +00001		141 +00013	141 +00001	
141 +00001	141 +00003		141 +00000	141 +00001	

Table A16(c) - Dynamic - Strains. Model: B5 Location: CLL2

Vehicle on 1-1			Vehicle on 2-2		
$\phi = 1.5\pi$	$\phi = 0.5\pi$		$\phi = 1.5\pi$	$\phi = 0.5\pi$	
149 +00000	149 +00035 0	149 +0023 0	149 +00077 0	149 +00041	149 +00071
149 +00000	149 +00026 0	149 +00019 0	149 +00066 0	149 +00052	149 +00020
149 +00035	149 +00060 0	149 +00069 0	149 +00077 0	149 +00041	149 +00013
149 +00010	149 +00072 0	149 +00085 0	149 +00077 0	149 +00041	149 +00040
149 +00082	149 +00080 0	149 +00066 0	149 +00077 0	149 +00052	149 +00041
149 +00030	149 +00073 0	149 +00130 0	149 +00066 0	149 +00052	149 +00052
149 +00083	149 +00093 0	149 +00174 0	149 +00130 0	149 +00052	149 +00052
149 +00082	149 +00129 0	149 +00151 0	149 +00174 0	149 +00052	149 +00052
149 +00067	149 +00111 0	149 +00162 0	149 +00151 0	149 +00052	149 +00052
149 +00165	149 +00101 0	149 +00160 0	149 +00162 0	149 +00052	149 +00052
149 +00125	149 +00103 0	149 +00234 0	149 +00160 0	149 +00052	149 +00052
149 +00103	149 +00152 0	149 +00219 0	149 +00234 0	149 +00052	149 +00052
149 +00077	149 +00124 0	149 +00219 0	149 +00219 0	149 +00052	149 +00052
149 +00171	149 +00104 0	149 +00110 0	149 +00110 0	149 +00052	149 +00052
149 +00170	149 +00061 0	149 +00077 0	149 +00077 0	149 +00052	149 +00052
149 +00129	149 +00122 0	149 +00106 0	149 +00106 0	149 +00052	149 +00052
149 +00049	149 +00078 0	149 +00127 0	149 +00127 0	149 +00052	149 +00052
149 +00116	149 +00000 0	149 +00070 0	149 +00070 0	149 +00052	149 +00052
149 +00130	149 +00066 0	149 +00099 0	149 +00099 0	149 +00052	149 +00052
149 +00112	149 +00007 0	149 +00063 0	149 +00063 0	149 +00052	149 +00052
149 +00020	149 +00012 0	149 +00003 0	149 +00003 0	149 +00052	149 +00052
149 +00032	149 +00010 0	149 +00012 0	149 +00012 0	149 +00052	149 +00052
149 +00053	149 +00010 0	149 +00009 0	149 +00009 0	149 +00052	149 +00052
149 +00020	149 +00010 0	149 +00009 0	149 +00009 0	149 +00052	149 +00052
149 +00001	149 +00009 0	149 +00007 0	149 +00007 0	149 +00052	149 +00052
		149 +00009 0	149 +00009 0	149 +00052	149 +00052

Table A17(a) - Dynamic - Strains. Model : A1 Location: CC2

Vehicle on 1-1			Vehicle on 2-2		
$\phi = 1.5\pi$	$\phi = 0.5\pi$		$\phi = 1.5\pi$	$\phi = 0.5\pi$	
111 +00000 111 +00179 111 +00202 111 +00348 111 +00230 111 +00112 111 +00162 111 +00220 111 +00092 111 +00100 111 +00103 111 +00098 111 +00099 000 00000 000 00000	111 +00177 111 +00223 111 +00209 111 +00263 111 +00312 111 +00228 111 +00201 111 +00139 111 +00230 111 +00118 111 +00107 111 +00121 111 +00101 111 +00096 111 +00096 111 +00097 000 00000 000 10000		111 +00127 111 +00178 111 +00255 111 +00223 111 +00149 111 +00191 111 +00257 111 +00150 111 +00093 111 +00102 111 +00105 111 +00109 111 +00100 111 +00101 111 +00101 111 +00101 000 00000 000 00000	111 +00134 111 +00172 111 +00175 111 +00163 111 +00192 111 +00222 111 +00174 111 +00173 111 +00114 111 +00135 000 00000 000 00000	

Table A17(b) - Dynamic - Strains. Model: A1 Location: CL12

Vehicle on 1-1			Vehicle on 2-2	
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	
113 +00118	113 +00000	113 +00341	113 +00133	
113 +00154	113 +00116	113 +00254	113 +00203	
113 +00185	113 +00137	113 +00173	113 +00216	
113 +00118	113 +00171	113 +00170	113 +00256	
113 +00067	113 +00156	113 +00133	113 +00227	
113 +00114	113 +00095	113 +00136	113 +00187	
113 +00096	113 +00108	113 +00139	113 +00129	
113 +00094	113 +00134	113 +00102	113 +00119	
113 +00095	113 +00080	113 +00101	113 +00122	
113 +00096	113 +00100	113 +00099	113 +00103	
113 +00097	113 +00099	113 +00099	000 000000	
113 +00097	113 +00100	113 +00099	000 000000	
113 +00099	113 +00100	113 +00100	000 000000	
000 000000	113 +00100	000 000000	000 000000	
000 000000	000 000000	000 000000	000 000000	

Table A18(a) - Dynamic - Strains. Model : A2 Location : CC2

Vehicle on 1-1			Vehicle on 2-2		
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$
111 +00102 111 +00171 111 +00181 111 +00273 111 +00365 111 +00338 111 +00175 111 +00193 111 +00218 111 +00151 111 +00132 111 +00124 111 +00105 111 +00102 000 000000 000 000000	111 +00000 111 +00102 111 +00175 111 +00266 111 +00309 111 +00304 111 +00231 111 +00229 111 +00324 111 +00168 111 +00182 111 +00130 111 +00125 111 +00116 111 +00161 111 +00100 111 +00109 000 000000 000 000000	111 +00111 111 +00134 111 +00136 111 +00150 111 +00255 111 +00255 111 +00237 111 +00218 111 +00256 111 +00302 111 +00168 111 +00167 111 +00120 111 +00095 111 +00114 111 +00109 000 000000 000 000000	111 +00102 111 +00177 111 +00224 111 +00192 111 +00250 111 +00237 111 +00130 111 +00196 111 +00139 111 +00144 111 +00137 111 +00100 111 +00110 111 +00106 111 +00100 000 000000 000 000000		

Table 18(b) - Dynamic + Strains. Model : A2 Location: CLL2

Vehicle on 1-1			Vehicle on 2-2		
$\phi = 1.5\pi$	$\phi = 0.5\pi$		$\phi = 1.5\pi$	$\phi = 0.5\pi$	
113 +00155 113 +00149 113 +00189 113 +00188 113 +00092 113 +00161 113 +00229 113 +00126 113 +00091 113 +00097 113 +00103 113 +00105 113 +00099 113 +00099 113 +00100 113 +00100 000 000000 000 000000	113 +00121 113 +00139 113 +00194 113 +00177 113 +00162 113 +00169 113 +00139 113 +00214 113 +00150 113 +00167 113 +00137 113 +00089 113 +00113 113 +00111 000 000000 000 000000		113 +00172 113 +00209 113 +00313 113 +00269 113 +00218 113 +00196 113 +00285 113 +00261 113 +00092 113 +00124 113 +00091 113 +00164 000 000000 000 000000	113 +00000 113 +00234 113 +00241 113 +00314 113 +00225 113 +00231 113 +00313 113 +00169 113 +00200 113 +00167 113 +00126 113 +00197 113 +00095 113 +00095 113 +00090 113 +00099 000 000000 000 000000	

Table A19(a) - Dynamic - Strains. Model : A4 Location : CC2

Vehicle on 1-1			Vehicle on 2-2		
$\phi = 1.5\pi$	$\phi = 0.5\pi$		$\phi = 1.5\pi$	$\phi = 0.5\pi$	
111 -02707	111 -02500		111 -02555	111 -02636	
111 -02592	111 -02632		111 -02579	111 -02665	
111 -02650	111 -02566		111 -02585	111 -02635	
111 -02603	111 -02563		111 -02620	111 -02620	
111 -02616	111 -02557		111 -02620	111 -02547	
111 -02558	111 -02527		111 -02482	111 -02496	
111 -02396	111 -02458		111 -02396	111 -02514	
111 -02401	111 -02445		111 -02441	111 -02561	
111 -02512	111 -02427		111 -02578	111 -02512	
111 -02426	111 -02519		111 -02545	111 -02452	
111 -02402	111 -02516		111 -02525	111 -02456	
111 -02539	111 -02522		111 -02566	111 -02586	
111 -02627	111 -02526		111 -02584	111 -02613	
111 -02629	111 -02574		111 -02743	111 -02545	
111 -02558	111 -02647		111 -02614	111 -02553	
111 -02670	111 -02586		111 -02699	111 -02635	
111 -02607	111 -02608		111 -02692	111 -02694	
111 -02716	111 -02647		111 -02702	111 -02646	
111 -02693	111 -02691		111 -02703	111 -02711	
111 -02702	111 -02700		111 -02701	111 -02695	
111 -02430	111 -02699		000 -02600	111 -02703	
111 -02700	111 -02699		000 -02600	111 -02702	
111 -02700	000 -02600			111 -02702	
000 -02700	000 -02600			000 -02702	
000 -02700	000 -02600			000 -02702	
000 -02700	000 -02600			000 -02702	

Table A19(b) - Dynamic - Strains Model : A4 Location: CLL2

Vehicle on 1-1'				Vehicle on 2-2			
ϕ	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 0.5\pi$
113	-00600	113 -00600	113 -00777	113 -00600	113 -00777	113 -00000	113 -00000
113	-00615	113 -00660	113 -00577	113 -00660	113 -00577	113 -00635	113 -00635
113	-00696	113 -00691	113 -00577	113 -00691	113 -00577	113 -00542	113 -00542
113	-00602	113 -00640	113 -00635	113 -00640	113 -00635	113 -00519	113 -00519
113	-00501	113 -00608	113 -00579	113 -00608	113 -00579	113 -00638	113 -00638
113	-00616	113 -00634	113 -00620	113 -00634	113 -00620	113 -00563	113 -00563
113	-00604	113 -00634	113 -00497	113 -00634	113 -00497	113 -00476	113 -00476
113	-00602	113 -00566	113 -00430	113 -00566	113 -00430	113 -00390	113 -00390
113	-00461	113 -00525	113 -00301	113 -00525	113 -00301	113 -00447	113 -00447
113	-00552	113 -00558	113 -00301	113 -00558	113 -00301	113 -00307	113 -00307
113	-00545	113 -00619	113 -00545	113 -00619	113 -00545	113 -00567	113 -00567
113	-00650	113 -00579	113 -00389	113 -00579	113 -00389	113 -00508	113 -00508
113	-00489	113 -00495	113 -00530	113 -00495	113 -00530	113 -00542	113 -00542
113	-00562	113 -00508	113 -00530	113 -00508	113 -00530	113 -00520	113 -00520
113	-00547	113 -00622	113 -00499	113 -00622	113 -00499	113 -00660	113 -00660
113	-00667	113 -00656	113 -00699	113 -00656	113 -00699	113 -00633	113 -00633
113	-00644	113 -00597	113 -00578	113 -00597	113 -00578	113 -00622	113 -00622
113	-00552	113 -00559	113 -00597	113 -00559	113 -00597	113 -00680	113 -00680
113	-00692	113 -00638	113 -00671	113 -00638	113 -00671	113 -00714	113 -00714
113	-00701	113 -00707	113 -00589	113 -00707	113 -00589	113 -00680	113 -00680
113	-00717	113 -00637	113 -00690	113 -00637	113 -00690	113 -00700	113 -00700
113	-00692	113 -00705	113 -00707	113 -00705	113 -00707	113 -00703	113 -00703
113	-00702	113 -00698	113 -00703	113 -00698	113 -00703	113 -00702	113 -00702
113	-00703	113 -00699	000 -00650	113 -00699	000 -00650	000 -00600	000 -00600
000	-00000	113 -00701	000 -00650	000 -00701	000 -00650	000 -00000	000 -00000
000	-00000	113 -00700	000 -00650	113 -00700	000 -00650	000 -00000	000 -00000

Table A20(b) - Dynamic - Strains. Model: A5 Location: CLL2

Vehicle on 1-1				Vehicle on 2-2			
$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$	$\phi = 1.5\pi$	$\phi = 0.5\pi$
113 +00170	113 +00123	113 +00000	113 +00123	113 +00000	113 +00000	113 +00000	113 +00000
113 +00124	113 +00103	113 +00143	113 +00103	113 +00143	113 +00200	113 +00200	113 +00200
113 +00101	113 +00187	113 +00139	113 +00187	113 +00139	113 +00229	113 +00229	113 +00229
113 +00208	113 +00179	113 +00253	113 +00179	113 +00253	113 +00184	113 +00184	113 +00184
113 +00223	113 +00153	113 +00285	113 +00153	113 +00285	113 +00237	113 +00237	113 +00237
113 +00186	113 +00224	113 +00208	113 +00224	113 +00208	113 +00302	113 +00302	113 +00302
113 +00285	113 +00260	113 +00254	113 +00260	113 +00254	113 +00339	113 +00339	113 +00339
113 +00235	113 +00270	113 +00316	113 +00270	113 +00316	113 +00370	113 +00370	113 +00370
113 +00353	113 +00266	113 +00397	113 +00266	113 +00397	113 +00333	113 +00333	113 +00333
113 +00325	113 +00218	113 +00461	113 +00218	113 +00461	113 +00371	113 +00371	113 +00371
113 +00191	113 +00306	113 +00352	113 +00306	113 +00352	113 +00461	113 +00461	113 +00461
113 +00278	113 +00339	113 +00299	113 +00339	113 +00299	113 +00436	113 +00436	113 +00436
113 +00243	113 +00295	113 +00384	113 +00295	113 +00384	113 +00419	113 +00419	113 +00419
113 +00302	113 +00275	113 +00369	113 +00275	113 +00369	113 +00294	113 +00294	113 +00294
113 +00336	113 +00191	113 +00399	113 +00191	113 +00399	113 +00224	113 +00224	113 +00224
113 +00098	113 +00242	113 +00226	113 +00242	113 +00226	113 +00316	113 +00316	113 +00316
113 +00170	113 +00175	113 +00098	113 +00175	113 +00098	113 +00252	113 +00252	113 +00252
113 +00252	113 +00145	113 +00254	113 +00145	113 +00254	113 +00189	113 +00189	113 +00189
113 +00133	113 +00149	113 +00274	113 +00149	113 +00274	113 +00127	113 +00127	113 +00127
000 000000	113 +00117	113 +00104	113 +00117	113 +00104	113 +00090	113 +00090	113 +00090
000 000000	113 +00117	113 +00115	113 +00117	113 +00115	113 +00121	113 +00121	113 +00121
000 000000	113 +00104	113 +00117	113 +00104	113 +00117	113 +00099	113 +00099	113 +00099
					000 000000	000 000000	000 000000
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