APPLICATION OF INPUT-OUTPUT ANALYSIS TO ENERGY STUDIES

by

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ABSTRACT

Many energy models have been built, differing in complexity, size, methodology and purpose of application to improve the decision making concerning energy policy.

The present methods based on econometric approach represent the economy in an aggregated fashion. Previous utilisation of the input-output statistical data in energy analysis proved that they represent a valuble source of detailed knowledge of the economy. Also most of the present models do not include technological constraints, capacity constraints and their utilisation.

In this thesis input-output statistical data are used in the rectangular format for the development of mathematical models for energy modeling and analysis with a particular stress on representation of technology, industrial capacity constraints. The following models were developed.

Two methods to compute the energy intensity coefficients based on rectangular input-output analysis are developed. Energy intensity coefficient gives the total energy required (direct plus indirect) to satisfy a final demand worth a dollar by an industry. Numerical examples for the coefficients are computed for the industries classified in the small aggregation canadian input-output data, for 1971.

An input-output linear programming model is developed for assisting the decisions regarding production and import of energy commodities when their supplies are limited by their production capacities. The model is applied using 1965 small aggretation input-output data. A parametric study is made to study the variation of shadow prices of energy commodities with increasing energy availability.

An input-output linear programming model incorporating an energy supply network is developed to forecast the demand for electric energy for medium term future. This model links the demand for electric energy to the final demand of goods and services produced in the economic system.

A dynamic model is developed in order to study the impact of uncertainity in energy availability in the long run on the growth of the economic system. This model maximises the gross domestic product, keeping the capacities and inventory levels at optimal levels. A monte-carlo simulation study is performed, using illustrative data for representing uncertain energy availability.

Optimal control theory is applied to formulate a dynamic input-output optimising model in continuous time. The objective is to minimize the utility of imports for a given time period. The advantage of continuous time optimising models is that they can be used to compute the best possible solution.

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Chapter I

STATE OF THE ART IN ENERGY MODELING

1.1 INTRODUCTION

The beginning of increases of oil prices in 1973, led the policy makers, scientists and economists all over the world to recognize the importance of modeling approach to help decisions regarding energy policy. Energy is required for the production and transportation of goods and services and is an extremely important item for maintaining the standard of living in a society. Studies have linked the consumption of energy to the Gross National Product of nations [54]. If Gross National Product is considered to be a measure of performance of a nation, then the importance of energy as an essential ingredient for the sustenance of a nation's health cannot be ignored. Numerous models have been built to this date to help make decisions concerning energy policy, differing in size, complexity, methodology, purpose, geographical scope and time frame (short term and long term). The variety of purposes for which energy models are built are listed below.

1.2 PURPOSES OF ENERGY MODELING

Some specific purposes for energy modeling are

1. To evaluate the effect of alternative public policies (such as energy pricing, pollution regulation etc.) on the energy consumption. The effect could be studied for a specific commodity or for the overall energy consumption.
2. To evaluate the effect of energy policies such as pricing and import regulations on energy demand, unemployment, Gross national product (GNP), environmental pollution etc. Griffin analysed in detail these types of effects [20].
3. To forecast the demand for energy consumption.
4. To forecast the prices of energy commodities (for example 'price of crude oil in 1990')
5. To evaluate the short term and long term effects of uncertainity in energy supply (from domestic and foreign sources) on the economic system. Usually the impact on prices, unemployment and GNP are studied.
6. To evaluate the feasibility of introducing new technologies in the future. The feasibility studies are based on the availability of technology (depends partly on the rate of expenditure on R&D), economic viability and demand for energy. Examples are [15,18].

7. To formulate optimal plans for meeting specified goals, such as the demand for goods and services at a minimum energy consumption or energy cost.
8. To specify the total energy flows in an economic system, starting from primary energy sources to the final demand with inter-fuel conversions.

1.3 CLASSIFICATION OF ENERGY MODELS

Energy models differ widely in terms of methodology, geographical scope, time frame and complexity. They can be classified in various ways. Classifications with examples are presented below.

1.3.1 Classification Based on Time Frame

Energy models can be classified based on the time domain for which they can be used. They are

1.3.1.1 Short Term Models

These models are used to forecast or compute an optimal plan, in a short term, usually upto one year. Such models provide a detailed forecast. For example if we consider an electricity demand forecasting model, the demand by the time of the day and by the season is provided. Short term models do not take into consideration explicitly technological changes and investments. For example the model in [50] is useful to forecast the demand for electricity in the short term.

1.3.1.2 Medium Term Models

In these models, the time domain is usually from one to five years. These models also do not take into account technological changes and investments. But they include energy production capacity constraints. The output of such models give only aggregate values (for example : Gigajoules of energy per year)

1.3.1.3 Long Term Models

These models are used for long range plans and projections. In these models technological changes and investments are introduced. Their range is usually from six years to several decades. They are usually dynamic models, though static models also exist. With such models the effects of changes in energy producing technologies and the uncertainities in energy resources availability on the economic system can be evaluated.

1.3.2 Classification Based on Topical Scope

The categories that fall into topical scope of classification are [25]

1.3.2.1 Sectorial Models

They are built for specific energy commodities (Crude oil for example [14]) and are either supply or demand models. Supply models are usually the models of technological systems involving extraction, conversion and transportation activities represented in the form of a network. The solution methods employed are simulation and linear programming. They are used to find the energy supply from different sources (electricity for example) for a given set of energy production costs and energy producing capacities. Econometric techniques have also been used to predict supply levels, relating supply to prices of energy commodities. There are also demand models which compute the demand for energy commodities based on factors such as energy prices, population growth,income level etc. [49]. The demand models are based on econometric techniques. They relate the demand for energy commodities to the factors mentioned above. Both time-series and cross sectional data are employed.

1.3.2.2 Industry Market Models

The supply and demand models if used separately, cannot be applied to compute equilibrium prices and quantities of energy commodities. When they are combined together they can be used to compute equilibrium points. Industry market models incorporate both supply and demand models to form a supply-demand model. This type of models exist for specific fuels and energy forms (such as electricity) [30]. Such models can be used to study the effect of price control of energy commodities on supply-demand balance. To achieve supply-demand balance in addition to econometric techniques, linear programming and simulation have also been used [32].

1.3.2.3 Energy System Models

These models include interlinked supply and demand submodels for all the energy commodities. Models of this type are useful for evaluating comperhensive energy policy. The supply submodel contains the technological features of the energy system represented in the form of a network, which can be formulated into linear programming format. The demand submodel is based on econometric techniques, [23].

1.3.2.4 Energy-Economy Models

These models couple the energy system to the rest of the economic system so that the effect of economic system variables on energy demand and supply and vice-versa can be studied. These are usually large-scale models coupling supply, demand and macro-economic models together. Examples of these types of models are PIES [26],CANDIDE energy sector extension [31] and the model developed by Fuller and Ziemba [19].

1.3.3 Classification Based on Methodology

Energy models can be classified based on the methodology employed in building them. The models involve either a basic technique or a combination of the basic techniques.

1.3.3.1 Mathematical Programming

This technique is used to represent the supply sub-models as well as the macroeconomic submodels of a large-scale model.

The technological features of the energy supply system can be incorporated in mathematical programming. Important technological features are capacity constraints, inter-fuel conversions transportation and transmission losses. In mathemetical programming models the variables represent the activity levels (such as electricity produced from coal) using specific processes. The mathematical programming techniques provide an optimal bound of the objectives desired and as such they can be interpreted as the results which can be achieved when right decisions are made. Models based on optimisation techniques are more useful for planning purposes. Of all the mathematical techniques, linear programming is the most widely used. Some examples are [2,23,24,42]. Linear programming technique is discussed in chapter IV. The limitation of linear programming is the assumption of linear relationships. Non-linear programming techniques have also been used [18], but not as frequently due to the difficulties in obtaining solution. Mathematical programming methods are suitable for equilibrium models of energy systems. The equilibrium models are in a sense simulation models represented in their dual form.

1.3.3.2 Network Analysis

This technique is used to represent a system rather than to solve numerically a problem. The energy system is represented in the form of a graph connecting the various activities. The relationships among the variables need not be assumed linear. Once the energy system is represented in the form of a network any solution technique may be employed. Linear programming is the most common solution technique though simulation technique has also been used [11].

1.3.3.3 Econometric Techniques

Econometric techniques are mainly employed for demand models where network models are not feasible because of diversified sources of demand. In econometric techniques, the demand for energy commodity is linked to other factors such as price, income etc. For example in mathematical form they can be represented as

$$Z_t = a\, X_{1,t} + b\, X_{2,t} + \epsilon$$

where Z is the dependent variable and

X_1, X_2 are the independent variables

a and b are the constants to be determined

Subscript refers to time and

ϵ is a random variable characterising the error.

Using time series data equations of best fit are estimated based on least squares technique. Econometric techniques are used to represent the aggregate behaviour. They are best suited for short-term and medium-term future since the relationships between the dependent and independent variables expressed in the form of equations are not valid in the long term due to price and technological changes.

1.3.3.4 Systems Dynamics

This is a modeling technique which takes into consideration, the cause and effect relationship. It is an application of feedback principle to socio-economic problems. The system to be modeled is divided into subsystems and the cause and effect relationships are established. Based on the cause-effect links mathematical models are built. The parameters of the model are estimated based on the actual data. The structure of the model depends upon the model builder. There is much flexibility in this technique and hence they can encompass very large problems. These models employ simulation techniques to obtain solution. An example is in [32].

1.3.3.5 Input-Output Analysis

This method of modeling is useful for macro-economic modeling and allows the modeler to build a large and dis-aggregated model.Input-output models have been coupled with energy supply models [27]. Input-output analysis is explained in Chapter II in detail.

1.3.4 Classification Based on Other Characteristics of An Energy Model

In addition to the above types of classification, energy models can also be categorized based on whether they are static or dynamic models. Static models do not take time into consideration and can be applied to one time period. Dynamic models incorporate time as an independent variable. An example is in [38]. In dynamic models investments are also treated as variables. When dynamic models are formulated as optimising problems, their solution assumes a perfect foresight of the decision maker.

Another classification separates deterministic models from stochastic models. In order to evaluate the effects of uncertainity of energy supply on energy prices and rest of the economic system stochastic models have been developed [29]. Post optimal sensitivity analysis is another viable alternative method for investigating the effects and significance of uncertainities.

1.4 SOME COMMENTS

From the brief review of energy modeling, it could be seen that the models differ widely in terms of geographical scope, application, methodology, complexity, time frame and the time characteristics of a model. In spite of such large differences some general comments can be made.

1. There is no single model which is the best for all purposes. Some models are better suited than others for a particular application. This is because each methodology, model scope in terms of time and geography, level of aggregation have weaknesses and strengths. The model evaluation depends upon the purpose for which it is intended.

2. Some models are more demanding in terms of computation time and data. Data requirement can be a hinderance in applying a model. Computation time and data requirement affect the cost of model application.
3. Large integrated energy models consisting of submodels for demand, supply and price-quantity equilibrium, can work properly only when the sub-models are compatible.
4. Because of the large number of factors affecting supply, demand and prices of energy commodities, all models represent approximately the reality. Demand originates from many sources and hence cannot be forecasted exactly. Though supply models in general represent technological systems, still the representation is not perfect. For example the model using linear programming (LP) does not consider the changes in efficiency of power plants at different loads. Price-quantity equilibrium for energy commodities is very complex; since energy prices are controlled by governments and also only a few organizations are involved in their supply creating an imperfect market situation. The outputs of energy models, hence, do not provide final answers but only guide lines. The decision maker has the decision prerogative of choosing a policy and models are helpful to discriminate among alternative policies.

Chapter II

INPUT-OUTPUT ANALYSIS

2.1 INTRODUCTION

Input-output analysis is concerned with the quantitative analysis of interdependence between producing and consuming sectors which constitute an economic system. The theory is useful to study the input-output relationships amongst the sectors.

2.2 INPUT-OUTPUT TABLES

An input-output table is the representation of an economic system, which is divided into various sectors. The sectors produce goods and services (commodities). It is an accounting frame-work for an economic system and gives a detailed picture of the activity levels (flow of goods and services amongst the sectors) for a particular time period. The time period for which data is collected and compiled is usually one year.

The sectors that comprise the economy are known as industries. The term 'industry' means a group of establishments producing the same or similar outputs. The above definition of an industry is general and includes 'house-hold' sector whose output is labour. The purity of a sector depends upon the level of detail of the economic system into sectors. For a highly detailed classification, some sectors would become single commodity sectors, producing one homogenous commodity.

A commodity can be defined as a good or service which has economic value. Because of the existence of production processes which give rise to joint products and by-products and also because of feasibility of producing a commodity by more than one process, very detailed classification of sectors does not lead to more accuracy in the analysis. As the classification is more detailed, the cost of data collection and compilation increases.

In the table, a detailed transactions of goods and services which the industries buy and sell are given (see Table 2.1). Such tables form a part of the 'System of National Accounts' as recommended by the United Nations Statistical Commission [52].

The industries that comprise the economic system are inter-dependent upon one another to produce goods and services, required for intermediate inputs and final demand. Final demand consists of goods and services required for consumption (house-hold consumption, government consumption and capital consumption), replenishing inventory and export.. The interdependence among the industries is due to the fact that the outputs of industries serve partly as inputs to other industries and also to the same industry. Such inputs are known as 'intermediate inputs' to distinguish from primary inputs. Primary inputs are those inputs which the industries consume, but which are not produced within the economic system. Thus depending upon the system classification one could consider labour, profits, taxes and non-competing imports as primary inputs. Non-competing imports are import of those commodities which are not produced within the economic system.

The flow of commodities among industries and to final demand is shown in Figure 2.1. The total output of an industry is given by the basic balancing equation as

amount of output of industry j consumed by itself +

amount of output of industry j sold to other industries +

amount of output of industry j sold to final demand =

total output of industry j

The general accounting system of an input-output table is shown in Table 2.1 [12].

Referring to the same table, the following accounting identities have to be satisfied.

1. For each commodity the total supply (including imports) should equal total demand (including intermediate inputs).

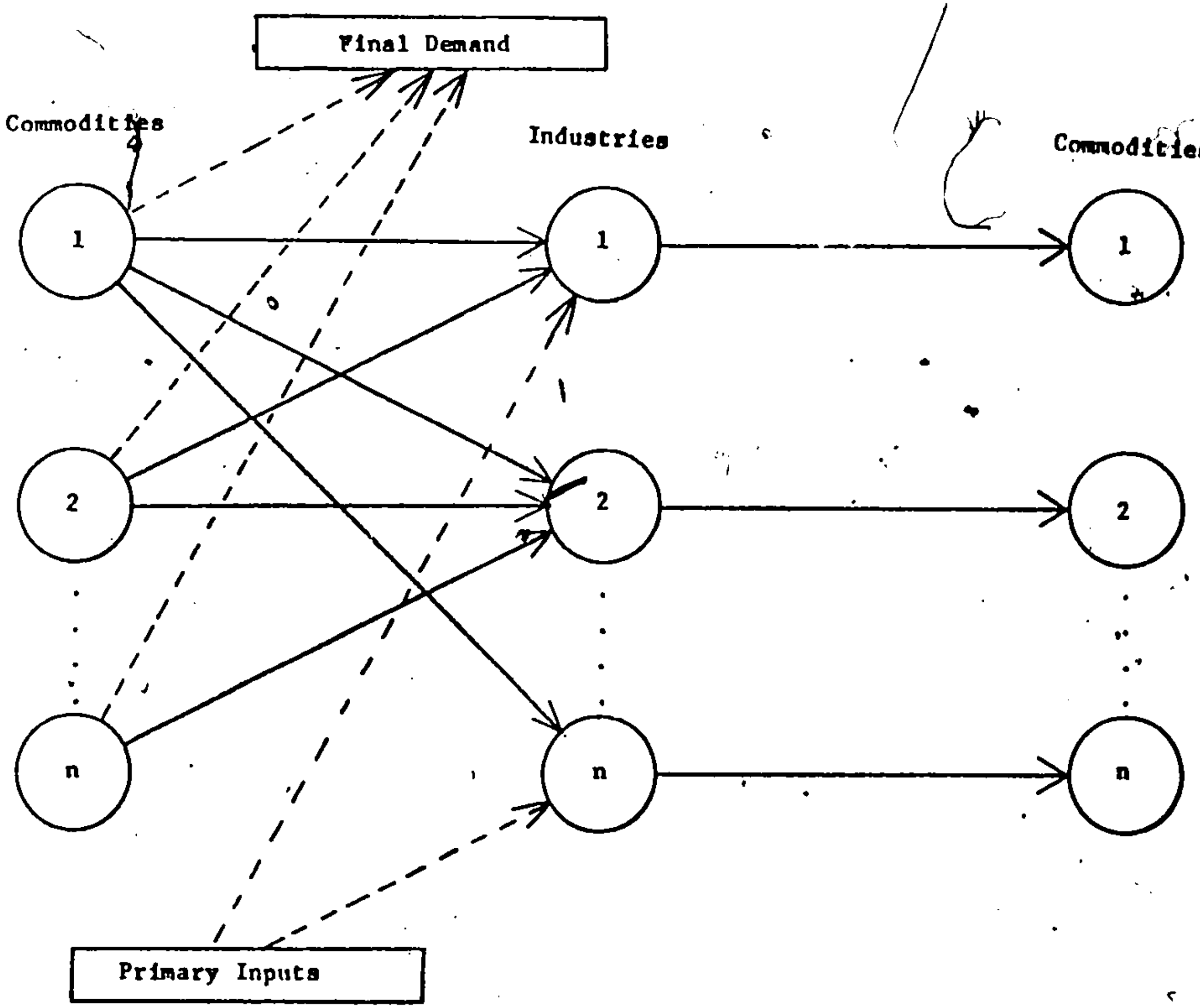

Fig. 2.1 Flow of commodities and primary inputs in Square input-output model.

		PURCHASING SECTORS								
		INTERMEDIATE USE SECTOR					FINAL USE	TOT. SUPP.		
		1	2	3	n				Imp.	Prod.
PURCHASING SECTOR	1	X_{11}	X_{12}	X_{13}	$\dots X_{1n}$	W_1	Y_1	Z_1	M_1	X_1
	2	X_{21}	X_{22}	X_{23}	$\dots X_{2n}$	W_2	Y_2	Z_2	M_2	X_2
	3	X_{31}	X_{32}	X_{33}	$\dots X_{3n}$	W_3	Y_3	Z_3	M_3	X_3
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	n	X_{n1}	X_{n2}	X_{n3}	$\dots X_{nn}$	W_n	Y_n	Z_n	M_n	X_n
TOT. PROD. INPUTS		U_1	U_2	U_3	$\dots U_n$					
PRIMARY INPUTS		V_1	V_2	V_3	$\dots V_n$					
TOTAL PROD.		X_1	X_2	X_3	$\dots, X_n$		Y	Z	M	X

Y_j is the final use of commodity j

X_i is the total production of commodity i.

Z_i is the total supply of commodity i.

M_i is the import of commodity i.

X_{ij} is the amount of commodity i used by sector j.

U_j is the total use by sector j of produced inputs purchased from other sectors and itself.

V_j is the total of primary inputs to sector j.

W_j is the total intermediate use of commodity j.

Table 2.1 Input-output accounting system.

$$Z_i = M_i + \hat{X}_i = \sum_{j=1}^{n} X_{ij} + Y_i = W_i + Y_i \qquad (2.1)$$

2: The total production in each sector should be equal to the value of inputs purchased from other sectors plus the sum of primary inputs in that sector, i.e.

$$X_j = \sum_{i=1}^{n} X_{ij} + V_j = U_j + V_j \qquad (2.2)$$

2.3 UNITS

All the entries in the tables i.e. production, consumption, primary inputs, imports and final demands are expressed in monetary units. The entries could also be expressed in physical units such as tons of steel. But if mathematical analysis has to be performed it would not be possible, when the units are different for different commodities. Thus the second accounting identity (eq.2.2) would not be satisfied since physical quantities of different inputs consumed by each sector cannot be meaningfully added. Each column in the transactions table gives the inputs that go into column industry from the corresponding row industry. Transaction table is industry by industry sales and purchases table which is in the first quadrant of the table in Table 2.1.

2.4 THE MODEL OF ECONOMIC SYSTEM

In an input-output model which divides the economic system into sectors, the output of each sector depends upon the output level of other sectors as a result of interdependence amongs various sectors. Consider for example the production of automobiles. The inputs to an automobile industry include among others steel, glass, plastics and copper. The industries which supply the automobile industry with these inputs, need at their turn inputs from other industries. If it is assumed that each sector produces only one homogenous commodity and if the relationships between the level of inputs and the level of outputs are linear for an 'n' sector economy, a set of 'n' equations in 'n' unknown production values, i.e. one for each sector, can be derived. This type of models are known as static models since the analysis can be applied to only one time period.

2.5 CLOSED AND OPEN MODELS

Depending upon the way in which final demand is treated the models can be characterized into closed and open models.

2.5.1 Closed input-output model

If the final demand sector is treated as an industry, it is implied that the levels of final demand depend upon the output level of other industries. This type of model can explain consumption. Final demand is treated as an endogenous variable (see Figure 2.2). The group of household consumers is treated as an industry, whose inputs are commodities and whose output is labour. Labour is consumed by other industries. A closed static model cannot be truly closed because capital investment (part of final demand) cannot be explained by current production levels [28].

2.5.2 Open input-output model

Another form of input-output model is the open model. Here final demand is assumed to be created independent of the activity levels of other industries. The final demand consists of household consumption, government expenditure, changes in stocks, export and capital expenditure. When the final demands are known, gross output of commodities can be calculated using linearity assumption (see section 2.4). A schematic diagram of open input-output model is given in Figure 2.3.

2.6 METHODS FOR SOLVING THE MODEL

For the open input-output model when a set of expenditures on final demand is given, it is possible to find the total (gross) production requirements of commodities. The gross production of a commodity is equal to the sum of the amount sold to final demand and the amount sold to other industries (intermediate inputs).

Considering the equation (2.1)

$$M_i + X_i = \sum_{j=1}^{n} X_{ij} + Y_i \qquad (2.3)$$

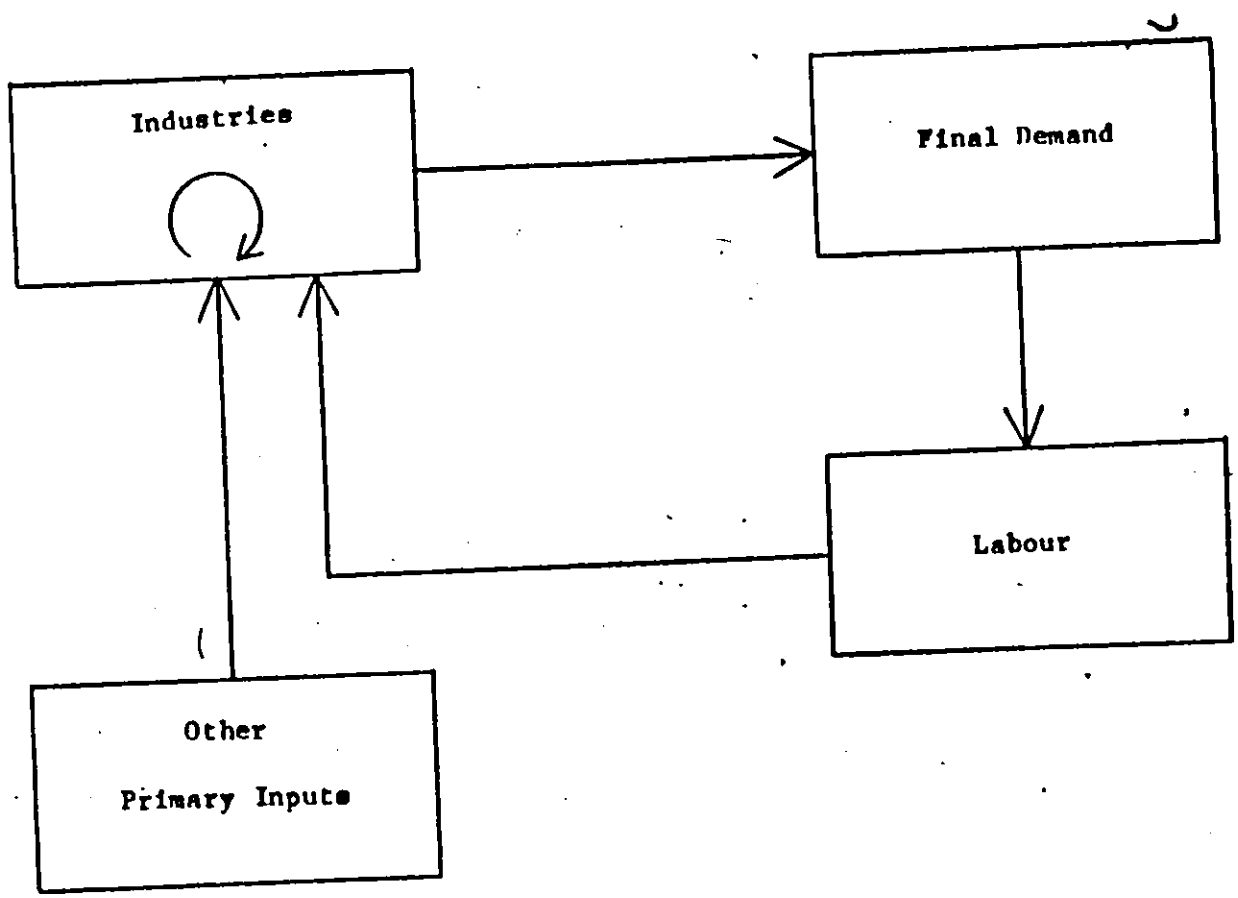

Fig 2.2 Closed input-output model (the arrow in loop represents inter-industry transactions).

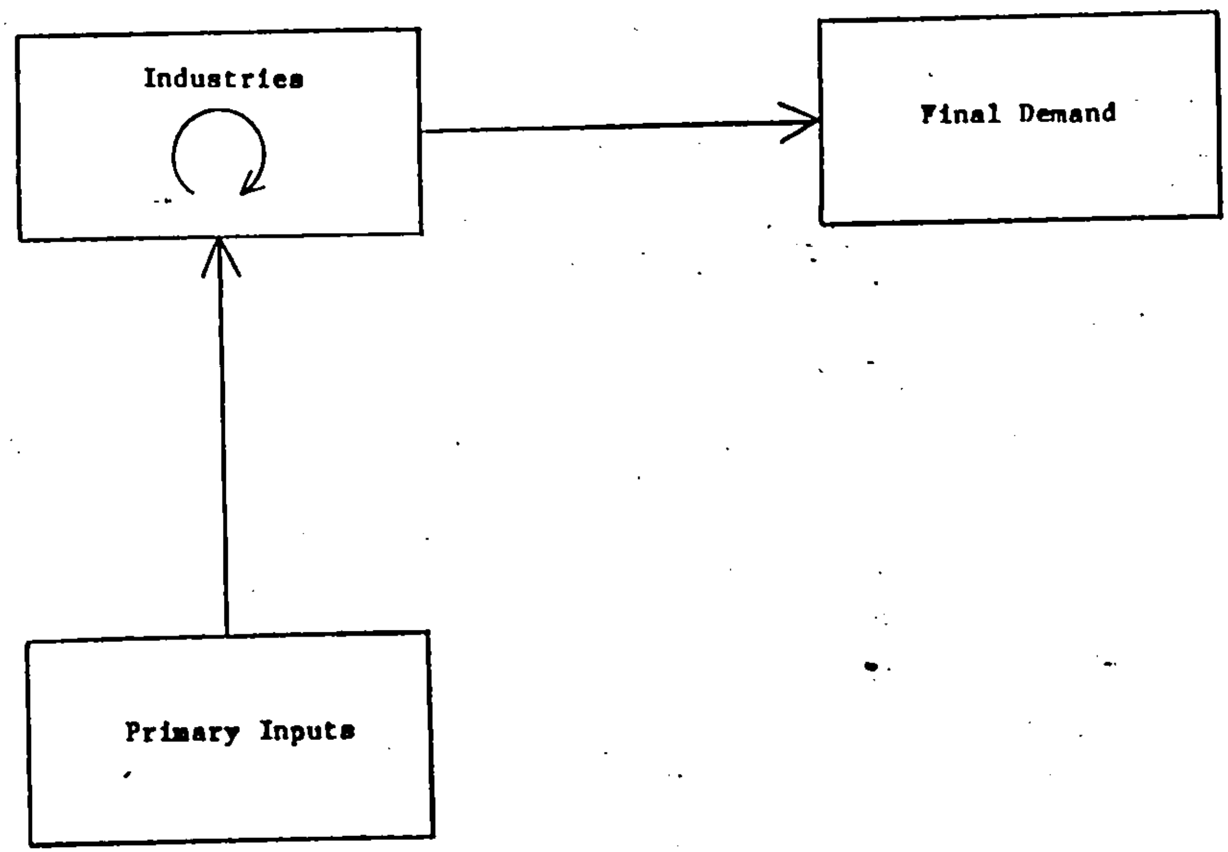

Fig. 2.3 Open input-output model (the arrow in loop represents inter-industry transactions).

If the imports are determined outside the system, i.e., if the imports are known, the balance equation can be written as

$$X_i = \sum_{j=1}^{n} X_{ij} + Y_i - M_i \qquad (2.4)$$

For 'n' commodities this system forms a set of 'n' equations in 'n' unknown gross production values, one for each commodity. With the assumption of linear relationship between the outputs and inputs for an industry an input coefficient a_{ij} is defined as

$$a_{ij} = X_{ij} / X_j \quad (\$/\$)$$

a_{ij} is the output of sector i consumed by sector j, per unit of i th sector's output. Now equation (2.4) can be written as

$$X_i = \sum_{j=1}^{n} a_{ij} X_j + Y_i - M_i$$

Denoting $Y_i - M_i$ as F_i

$$X_i = \sum_{j=1}^{n} a_{ij} X_j + F_i \qquad (2.5)$$

When expressed in matrix form F will be equal to

$$F = (I - A)X \qquad (2.6)$$

where F is the column vector of final demands less imports.

X is the column vector of total commodity outputs.

I is an identity matrix of order n by n.

If (I-A) is a non-singular matrix, equation (2.6) can be written as

$$X = (I - A)^{-1} F \qquad (2.7)$$

Equation (2.7) is the expression for gross production of commodities for square input-output analysis. It is known as square input-output analysis because of the one to one mapping between industries and commodities.

Because of the interdependence among industries, the output of an industry in order to satisfy a given final demand, requires direct inputs (to produce its output) and also indirect inputs. Indirect inputs are needed to produce direct inputs. Again to produce indirect inputs , other inputs are needed. This process of inputs requirements continues until the inputs become very small and can be represented by a converging series. This can be seen from equation (2.7) when the matrix $(I - A)^{-1}$ is expanded in a series as shown.

$$X = (I + A + A^2 + A^3 + A^4 + \ldots) F \qquad (2.8)$$

2.7 CONDITIONS FOR THE SOLUTION TO EXIST

For any solution to equation (2.7), to be meaningful, the values of production of all the commodities have to be equal or greater than zero. Negative production values do not have any meaning. The non-negativity of outputs (production) is guarenteed by the Hawkins-Simon Conditions, which are

1. The input coefficients a_{ij} for all commodities must be less than unity. This requirement is essential for any viable economy, otherwise it would mean that one or more industries require more than one unit of direct input of it's own product to produce one unit of it's own product.
2. The value of the determinant of the matrix (I - A) must be greater than zero.

2.8 ASSUMPTIONS UNDERLYING INPUT-OUTPUT MODEL

The square open input-output model rests on the following assumptions [37].

1. Each industry produces one commodity and no other industry produces the same commodity. So far while discussing the model the terms 'industry' and 'commodity' were used synonymously, because of this assumption. This assumption rules out the possibility of joint products and by-products.
2. The commodity produced by an industry cannot be substituted by another commodity. Validity of this assumption depends upon the aggregation level of industries. However even if the classification of industries were made such that each industry produced only one output, the assumption is not truly valid.
3. Input-Output model is a linear economic model. It is assumed that the inputs to an industry are strictly

proportional to the output of that industry. This implies that the model ignores economies of scale.

4. The model also assumes that the economic system was in equilibrium when the data was collected, meaning that the supplies of commodities were equal to demand.

5. When the model is used to project the gross production requirement of commodities it is assumed that the input coefficients remain constant during the projected period of time. The value of input coefficients do change over time, reflecting changes in relative prices and changes in technologies. Methods have been developed to forecast the changes in the magnitude of input coefficients [1].

2.9 USES OF INPUT-OUTPUT MODELS

1. Policy Evaluation: The model can be used to study the effects of alternative policies on the economic system, measured by such indices such as Gross National Product (GNP), employment level etc.

2. The model is also used for forecasting the economic activity when the final demands for commodities are known. The difference between policy evaluation and forecasting is that policy evaluation may be thought as conditional forecasting. Input-output models when used for forecasting purposes are less valuble be-

cause they ignore price effects. They can be used for commodities whose demand is inelastic.

3. The model can also be used to study the changes in the structure of an economic system. These changes are due to the combined effect of price and technological changes which can be separared [40].

2.10 RECTANGULAR INPUT-OUTPUT MODEL

In order to reduce the limitations due to aggregation of commodities in the square input-output model, the inter-industry transactions can be represented in two tables, the input and the output tables, the input table giving the commodities consumed by industries and the output table giving the production of commodities by industries. This type of tabulation has the advantage of being able to accomodate multiple product industries. In rectangular input-output model, the economic system consists of industries producing commodities. An industry can produce more than one commodity. Commodities are distinguished from industries, in that they are more in number. This format was recommended by the U.N [52] and adopted by Canada [45]. The flow to industries consist of commodities produced by industries and primary inputs. These inputs are used by industries to produce their own commodities. The commodities are used to meet the inter-industry requirements and final demand. The flow of commodities among industries and to final demand is shown in Figure 2.4.

The inter-industry transactions are represented in two tables:
Use (input) matrix and Make (output) matrix. The Use matrix gives the detail of intermediate inputs (commodities) consumed by industries and the Make matrix gives the detail of outputs of industries, classified by commodities.

The accounting framework for this type of model is shown in Table 2.2 [45]. The transactions in the format given in Table 2.2 are recorded in \$.

The model determines the vector of total commodities output, given the vector of final demands for commodities.
By definition the elements of the vector q are

$$q_j \ (\$) = \sum_{i=1}^{N} V_{ij} \ (\$) \ ; \ j = 1 \text{ to } n \qquad (2.9)$$

and the elements of the vector G are

$$G_i \ (\$) = \sum_{k=1}^{n} V_{ik} \ (\$) \ ; \ k = 1 \text{ to } N \qquad (2.10)$$

Using accounting balance

$$q_j \ (\$) = \sum_{i=1}^{N} U_{ij} \ (\$) + e_j \ (\$) \qquad (2.11)$$

In order to define the model two matrices of coefficients are required in the Canadian model.

The market-share matrix D is defined as

$$D_{ik} = V_{ik} \ / \ q_k \ (\$/\$) \qquad (2.12)$$

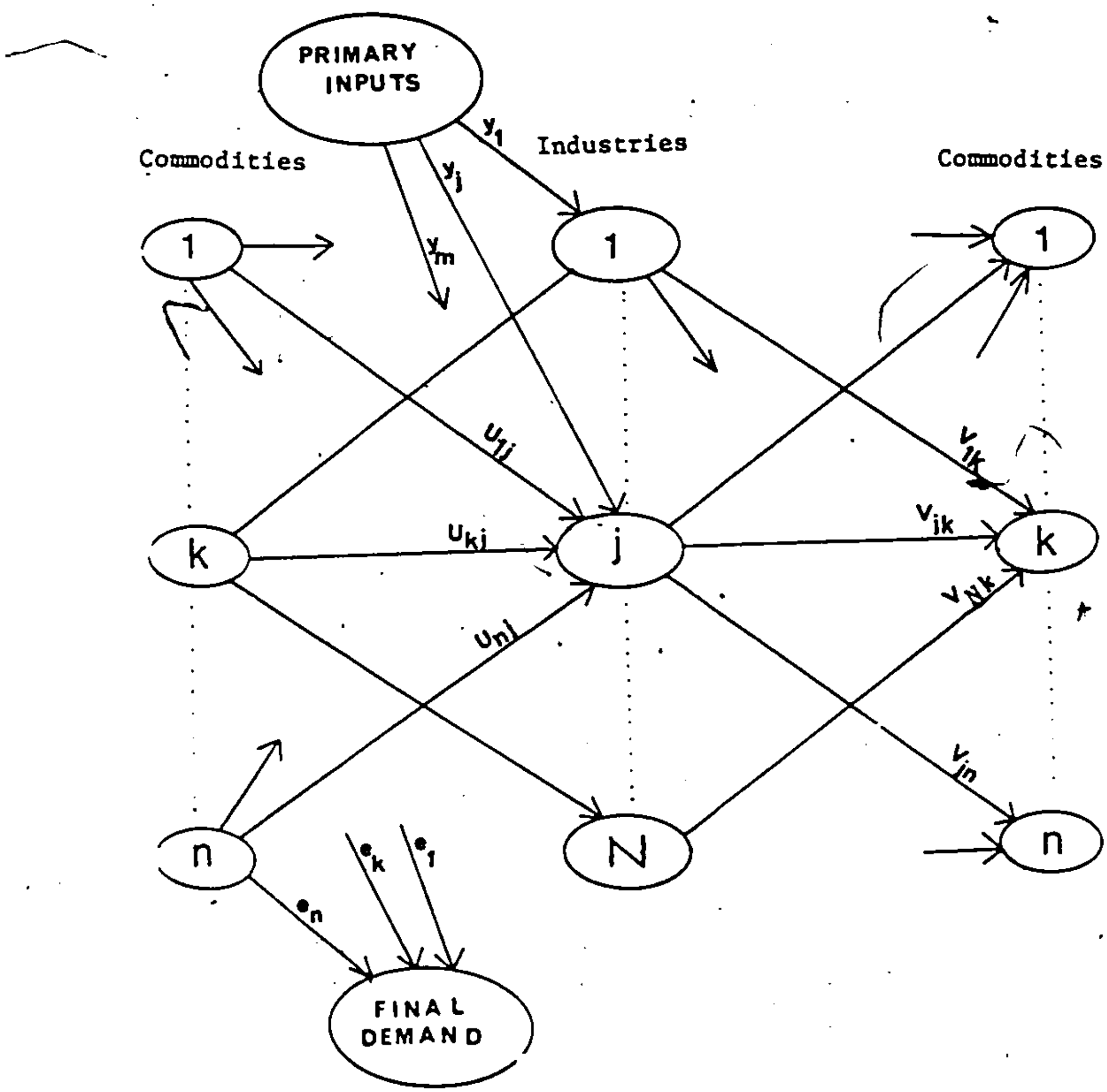

Fig. 2.4 Flow of commodities and primary inputs in Rectangular input-output model.

	Competitive Commodities	Industries	Final Demand Less Imports	Total
Competitive Commodities		U	e	q
Industries	V			g
Primary Inputs		y'	y_e	
Total	q'	g'		

Notation

Capital letters are used for matrices, lower case letters for vectors and scalars. Column vectors are unprimed; row vectors are primed.

U is a matrix of the values of intermediate inputs.

V is a matrix of the values of outputs.

q is a vector of the values of total commodity outputs.

g is a vector of the values of total industry outputs.

e is a vector of the values of final demand less imports for competitive commodities.

y' is a vector of the values of the primary inputs of industries.

y_e is a scalar representing the value of the primary inputs associated with final demand less imports.

Table 2.2 Accounting frame-work for Rectangular input-output analysis.

All the elements in each column of the 'make matrix V' divided by the element of the corresponding column of the vector q gives the vector D.

Input coefficients matrix B is defined as

$$B_{ij} = U_{ij} \,/\, G_j \quad (\$/\$) \tag{2.13}$$

substituting the relations obtained in equations (2.12) and (2.13) in equations (2.10) and (2.11)

$$G_i \ (\$) = \sum_{k=1}^{n} D_{ik} \ q_k \ (\$) \tag{2.14}$$

$$q_j \ (\$) = \sum_{i=1}^{N} B_{ji} \ G_i \ (\$) + e_j (\$) \tag{2.15}$$

Eliminating G in the previous equation, by using equation (2.14)

$$q_j = \sum_{i=1}^{N} \sum_{k=1}^{n} B_{ji} \ D_{ik} \ q_k + e_j$$

or in matrix notation

$$q = B.D\, q + e \tag{2.16}$$

from which one can get

$$q \ (\$) = (I - BD)^{-1} e \ (\$) \tag{2.17}$$

where I is an identity matrix of order n by n.

Eliminating q in equation (2.14) yields

$$G \ (\$) = (I - DB)^{-1} D \, e \ (\$) \tag{2.18}$$

In (2.18) I is an identity matrix of order N by N.

Similar to the matrix 'A' of input coefficients in the square input-output model, the matrix (BD) is a matrix of direct input coefficients, but taking into account the constant market shares of industries for each commodity.

2.11 ASSUMPTIONS

In addition to the last three assumptions in the square model the following assumptions are made in rectangular input-output model.

1. 'Each commodity is produced by industries in fixed proportions relative to other industries'. This assumption justifies the linear equation (2.14) of the market -share of commodities. It means that each industry maintains its market-share in the total market of a commodity irrespective of the value of total production of a commodity.
2. 'The inputs of to an industry are proportional to its output'. This assumption means that the inputs required to produce a dollar's worth of output of an industry are in strict proportion to the total output of that industry, regardless of product mix. This assumption implies a linear production function (eq.2.15) and industry based technology.

2.12 DISCUSSION

In the models discussed so far, nothing has been mentioned about imports, exports and stock changes of commodities. These transactions are adjusted in the final demand. So in effect, imports, exports and stock changes have to be predetermined before the models can be applied.

As mentioned earlier, rectangular input-output model can accomodate multiple product industries. But the model has inherent weakness, due to the assumption of constant market shares. Industries cannot be expected to have constant market shares for a long period of time. Thus large number of commodities, taking into account the possibility of joint products could be represented at the expense of market-share assumption.

From the practical point of view, input-output models have been for policy analysis and forecasting. But their application is limited by the availability of upto date input-output data for computing the current values of input coefficients due to the difficulty in data collection and compilation. The tables published lag behind about five years. The latest tables published for Canada are for 1978. So when the models are used, the input coefficients have to be modified, to consider the changes in technology and price changes. Methods of updating the coefficients have been developed [1].

The difference between square and rectangular input-output models can be seen from figures (2.1) and (2.4). The relationship between the two models, is derived in appendix A.

2.13 INPUT-OUTPUT TABLES OF THE CANADIAN ECONOMY

The input-output tables of the Canadian economy are published by 'Statistics Canada'. The first set of tables in rectangular format for the year 1961 appeared in 1969 [45]. Since then tables are being published for each successive year. The latest set of tables available is for 1978.

The tables are published at three levels of aggregation: small, medium and large. The number of commodities, industries and categories of final demand for each level of aggregation are given below.

	No. of Ind.	No.of Comm.	No. of F.D Categories
Small	16	43	14
Medium	43	92	14
Large	191	586	136

The tables published are

1. The make (output) matrix
2. The Use (input) matrix
3. The Final Demand matrix
4. The Impact (inverse) matrix

All the transactions in the above tables (except for the last one) are in nominal dollars. The Make and the Use matrices are the output and the input matrices as defined earlier. The final Demand matrix gives the expenditure on final demand by the following categories:

-Consumer expenditure durable and non-durable.

-construction.

-Machinery and equipment.

-Exports and re-exports.

-Government expenditure.

-Inventory changes and imports.

Import of commodities is treated as negative demand.

Using the same notation as in the previous section, the impact (inverse) matrix is $(I - BD)^{-1} D$; it gives the variation of direct plus indirect industrial output for one dollar variation of final demand for each commodity.

For the sake of uniformity of valuation of inputs, outputs and the expenditure on final demand the transactions are recorded in producers' prices and do not include trade and tranportation margins. All the trade margins are treated as separate commodities.

The tables are also published in 1961 and 1971 constant dollars, for the sake of comparison between different years.

α_j is the energy intensity for commodity j (J/$)

ε_k is the energy intensity for industry k (J/$)

V is the matrix of values of outputs ($)

q is a vector of values of total commodity outputs ($)

G is a vector of values of total industry outputs ($)

B is the matrix of industry inputs coefficients ($/$)

$\hat{B}$ is the modified matrix of industry inputs coefficients (method II)

D is the market-share matrix ($/$)

$\hat{D}$ is the modified market-share matrix (method II)

E is the energy i sold to industry j (J)

E is the energy i sold to final demand (J)

e is a vector of values of final demand less imports ($)

$\hat{q}$ is a modified vector of total commodity outputs (method II)

$\hat{e}$ is a modified vector of final demand less imports

U is a matrix of values of intermediate inputs ($)

T is the total energy requirement for a given final demand of commodities (J)

NOMENCLATURE FOR CHAPTER III

Chapter III

ENERGY INTENSITY COEFFICIENTS CALCULATION USING RECTANGULAR INPUT-OUTPUT TABLES

3.1 ENERGY INTENSITY COEFFICIENTS: DEFINITION AND USES

Whenever goods and services (commodities) are consumed, energy is also consumed. This is because energy is required to produce commodities. Commodities require different amount of energy to be produced. Information about the energy required to produce commodities is useful as an aid to energy policy decisions. Energy intensity coefficients are indices to study the energy requirement for production of commodities. The coefficients give the value of energy required (in energy or monetary units), for producing a commodity. In the production of any commodity energy is consumed directly as process energy requirement and indirectly as the energy required in the previous processes to produce the inputs in the given commodity production. The energy requirement to produce commodities varies widely. For example a ton of Aluminium requires different amount of energy than a ton of cement. The coefficients could be computed at micro (company) and macro (national) level. At the micro level this involves in computing the energy reqirement either direct or direct plus indirect (total) for a particular production process, in order to produce unit output.

At the macro level instead of studying the process, the energy requirement direct or total is computed for commodities or industries in an economic system [9,21,56]. Different uses exist for the coefficients as outlined below.

At th micro level the coefficients can be used to study the production processes, to improve the efficiency of operation in terms of energy consumption. They can also be used to compare different industrial processes, to produce the same commodity and to study the feasibility of new and modified processes. These studies are made at the company level and hence the system boundaries (for the production process) are chosen depending upon the purpose of the study.

At the macro level, the coefficients are used as an aid to policy decision making. The direct coefficients are useful to study the time trends in energy requirement for producing commodities. They can also be used for planning regional energy balances, by locating industries such that regions are self sufficient in terms of energy.

Total energy coefficients are useful for the following purposes.

1. To forecast total energy requirement of fuels, in the short term, when the final demands for goods and services are known.
2. To determine the impact of a policy on the overall energy consumption (for example additional taxes on steel production).

3. To check whether there is any disparity between energy costs and monetary costs [56].
4. To study the relationship between physical production and demand for energy commodities [56].

3.2 METHODS OF CALCULATION

3.2.1 Energy Analysis of Technological Processes

In the case of energy intensity coefficients for a given technological process, study is made on the actual system. The process is represented by a sequence of operations linked by flow of materials [6]. When the total energy intensity coefficients are computed at micro level, the analysis is known as vertical analysis [10]. In vertical analysis, after taking into account the direct energy of the process, indirect energy needs have to be traced. This is done by studying the production processes of the raw materials. The number of processes to be studied theoretically can be very large. In practice the study is stopped after certain number of processes. An example of micro analysis for producing polymers is in [5]. The major disadvantages of micro analysis are

1. It is tedious and time consuming.
2. Because of the truncation of tracing indirect energy needs, the error is difficult evaluate [8].

3.2.2 Input-Output Approach

At the macro level when the energy intensity coefficients are required for all the commodities produced within the economic system the above method can be used but would be very tedious. Moreover the coefficients computed thus could not be used to forecast the gross energy requirement because of the uncertainity in the error. While using the micro approach care has to be taken to see that double counting does not occur. For example once the electric energy requirement for a furnace is taken into account, the coal required to produce the same electricity should not be added. When there are joint products the allocation of energy requirement to each product is difficult.

The macro level method is based upon input-output analysis. Using input-output analysis direct and total energy intensity coefficients can be computed for commodities [9,21,40,56]. The calculation of direct energy coefficients using input-output analysis is straightforward. Direct energy intensity coefficients from square and rectangular input-output tables have been calculated [17,40]. The total energy intensity coefficients can also be calculated using input-output tables. Application of input-output analysis to calculate the total energy coefficients has been developed by D.Wright [56] and R.A.Herendeen [21]. Any nation which compiles data in square input-output format, can use the methods developed by Wright, Herendeen, Bullard and Herendeen

[9]. In Canada the Input-output accounts are based on rectangular input-output model [45]. The methods developed to for square input-output tables cannot be applied to rectangular input-output tables. In this chapter two methods to compute the total energy intensity coefficients using rectangular input-output tables are developed [35].

3.3 UNITS

The coefficients can be computed in three types of units as [49]

1. x dollars of energy per dollar of commodity. For a commodity k, if the total energy coefficient is α_k then

$$x = \frac{\text{Energy consumed (\$)}}{\text{Dollar's worth of commodity k (\$)}} = \alpha_k$$

2. y joules of energy per dollar of commodity. Using the same notation

$$y = \frac{\text{Energy consumed (joules)}}{\text{Dollar's worth of commodity k}} = \alpha_k$$

3. z joules of energy per physical unit (eg. ton) of commodity

$$z = \frac{\text{Energy consumed (joules)}}{\text{Physical unit of commodity k}} = \alpha_k$$

Coefficients of the third type are the most useful and can be calculated, only when the commodities are homogenous. For example 'Chemical and chemical products' (commodity No.27 in the small aggregation input-output tables) is not a homogenous commodity. In the absence of data for homogenous commodities, coefficients expressed in energy units per dollar are desirable as energy units are not affected by inflation and price changes.

Using input-output tables, three techniques can be adopted to obtain the coefficients in energy units per dollar's worth of a commodity.

In the first method monetary input-output data and average price of energy commodity are used [56]. This method does not consider the various grades of fuels and price discrimination among the customers by the energy suppliers. Analysis based on the second and third methods involve the use actual energy data and monetary input-output data [9,21]. The analyses in this chapter is of the second and third types.

3.4 METHODOLOGY

For the square input-output analysis, the total energy intensity coefficients can be computed using the following methods.

1. Multiply the row corresponding to the energy commodity of the matrix $(I - A)^{-1}$ by the average price of the energy commodity [56]. Matrix 'A' is the matrix of input coefficients in the square analysis.

   If α_k is the total energy intensity coefficient for a commodity k (J/\$) and if p_j is the price of the energy commodity j in (\$/J), α_k can be written as

$$\alpha_k = p_j \; (I - A)^{-1}_{jk} \qquad (3.1)$$

2. Multiply the direct energy use coefficients obtained using the actual energy consumption data by the matrix $(I - A)^{-1}$.

   If e_k and α_k are the direct and total energy use coefficients in (J/\$) respectively, for a non-energy commodity k, then they are related by

$$\alpha_k = \sum_{\ell=1}^{n} e_\ell \; (I - A)^{-1}_{\ell k} \qquad (3.2)$$

   where n is the number of commodities.

3. Invert a matrix (I - A) after replacing the row corresponding to the energy commodity in the transactions matrix by the actual energy data.

   If α_k is the total energy coefficient for a commodity k in (J/\$) and if H is the matrix of input coefficients, computed after replacing the monetary data by the actual energy data in the transactions matrix, for the energy commodity then

$$\alpha_k = (I - H)^{-1}_{jk} \qquad (3.3)$$

where j is the energy commodity type.

For computing the total energy coefficients Herendeen proved that the second and third methods are superior [22].

In the analysis presented in this chapter, in addition to the market share assumption (see section 2.11), the energy input to an industry expresses in energy units is assumed to be proportional to the gross output of an industry, expressed in dollars.

3.5 METHOD 1

This method is based on the idea developed by Herendeen [19] for square input-output tables. For rectangular tables, it is possible to compute energy intensity coefficients for commodities as well as for industries. The relationship between the two for this method is given below.

$$\sum_{j=1}^{N} \varepsilon_j V_{jk} = \alpha_k q_k \qquad (3.4)$$

where ε_j (J/\$) and α_k (J/\$) are the total energy intensity coefficients for industry j and commodity k respectively. The notation of rectangular input-output analysis is the same as in the previous chapter (section 2.10).

According to the market share assumption (eq.2.13)

$$V_{jk} = D_{jk}\ q_k \qquad (3.5)$$

substituting (3.5) in (3.4) one gets

$$\sum_{j=1}^{N} \varepsilon_j\ D_{jk} = \alpha_k \qquad (3.6)$$

The total of the energy commodity i is the sum of the energy commodity sold to industries and the energy sold to final demand, which is

$$E_i = \sum_{j=1}^{N} E_{ij} + E_{if} \qquad (3.7)$$

where

E_{ij} (J) is the energy sold to industry j

E_{if} (J) is the energy sold to final demand.

equation (3.7) can be written as

$$E_i = \sum_{j=1}^{N} (E_{ij} / G_j)\ G_j + E_{if} \qquad (3.8)$$

(G_j is the total output of industry j in dollars)
Defining (E_{ij} / G_j) as R_{ij} (J/\$), one can write (3.8) as

$$E_i = \sum_{j=1}^{N} R_{ij}\ G_j + E_{if} \qquad (3.9)$$

The total output of industry j is given by (see section 2.10)

$$G_j = \sum_{k=1}^{N} \sum_{\ell=1}^{n} (I - DB)^{-1}_{jk}\ D_{k\ell}\ e_\ell \qquad (3.10)$$

substituting (3.10) in (3.9)

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{n} R_{ij} (I - DB)^{-1}_{jk} D_{kl} e_l + E_{if} \qquad (3.11)$$

The final demand for commodities can be translated into final demand for industries using the market share assumption as

$$F_k = \sum_{l=1}^{n} D_{kl} \cdot e_l \qquad (3.12)$$

where F_k (\$) is the final demand for the output of industry k.

Similarly for the industry p producing the energy commodity i

$$F_p(\$) = \sum_{k=1}^{n} \breve{D}_{pk} \quad e_k \qquad (3.13)$$

Rewriting equation (3.11) as

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{n} R_{ij} (I - DB)^{-1}_{jk} D_{kl} e_l + \left(E_{if} / F_p\right) F_p \qquad (3.14)$$

Defining

$$W_{ip} \text{ (J/\$)} = E_{if}/F_p \quad \text{for the industry p producing the energy commodity}$$
$$= 0 \quad \text{otherwise} \qquad (3.15)$$

Equation (3.14) can be written as

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{l=1}^{n} R_{ij} (I - DB)^{-1}_{jk} D_{kl} e_l + W_{ip} F_p \qquad (3.16)$$

when there is more than one energy commodity, equation (3.16) can be written in matrix notation as

$$E\ (J) = [\ R(I - DB)^{-1} + W\ ]\ F$$

or using (3.12)

$$E\ (J) = [\ R\ (I - DB)^{-1} + W\ ]\ D\ e \qquad (3.17)$$

Equation (3.17) can be used to compute the gross energy requirement when the final demand for commodities are known.

It is shown in appendix A that the rectangular input-output analysis can be converted to square input-output analysis using the relationships given by equations (A.11),(A.12) and (A.13). The same equations are given below.
when the number of industries equals the number of commodities

$$V_{jk} \neq 0 \text{ for } j=k=1,2,\ldots N \qquad (3.18)$$

$$V_{jk} = 0 \text{ otherwise}$$

and $n = N$

$$q_j = G_j \text{ for } j=1,2,\ldots N \qquad (3.19)$$

$$D_{jk} = 1 \text{ for } j=k=1,2,\ldots,N$$

$$D_{jk} = 0 \text{ for } j=k \qquad (3.20)$$

The matrix D becomes an identity matrix. Hence the equation (3.17) for square input-output tables becomes

$$E = [\ R\ (I - B)^{-1} + W\ ]\ e \qquad (3.21)$$

which is identical to the equation obtained by Herendeen [21]. The energy intensity coefficients for industries are also energy intensity coefficients for commodities.

3.6 METHOD 2

This method is based on the idea of 'conservation of embodied energy' developed by Bullard and Herendeen [9] which is 'The energy consumed at a particular stage of production is considered to be passed over to the next stage embodied in the product'. For example the energy used in converting Aluminium ore into finished Aluminium is assumed to be contained within the metal. The losses of energy in the production processes are also assumed to be a necessity in this method. This is done in order to account for all the energy consumed in the economic system. The concept is illustrated in Figure 3.1, for a system with two industries producing three commodities. Referring to the same figure, energy enters the economic system as primary energy and is used to produce secondary energy commodities by industries. Secondary energy commodities are used by other industries to produce non-energy commodities. The commodities are used as raw materials by industries (inter-industry requirement). The energy leaves the economic system embodied in the final demand of commodities. The energy losses within the system are also included in the embodied energy.

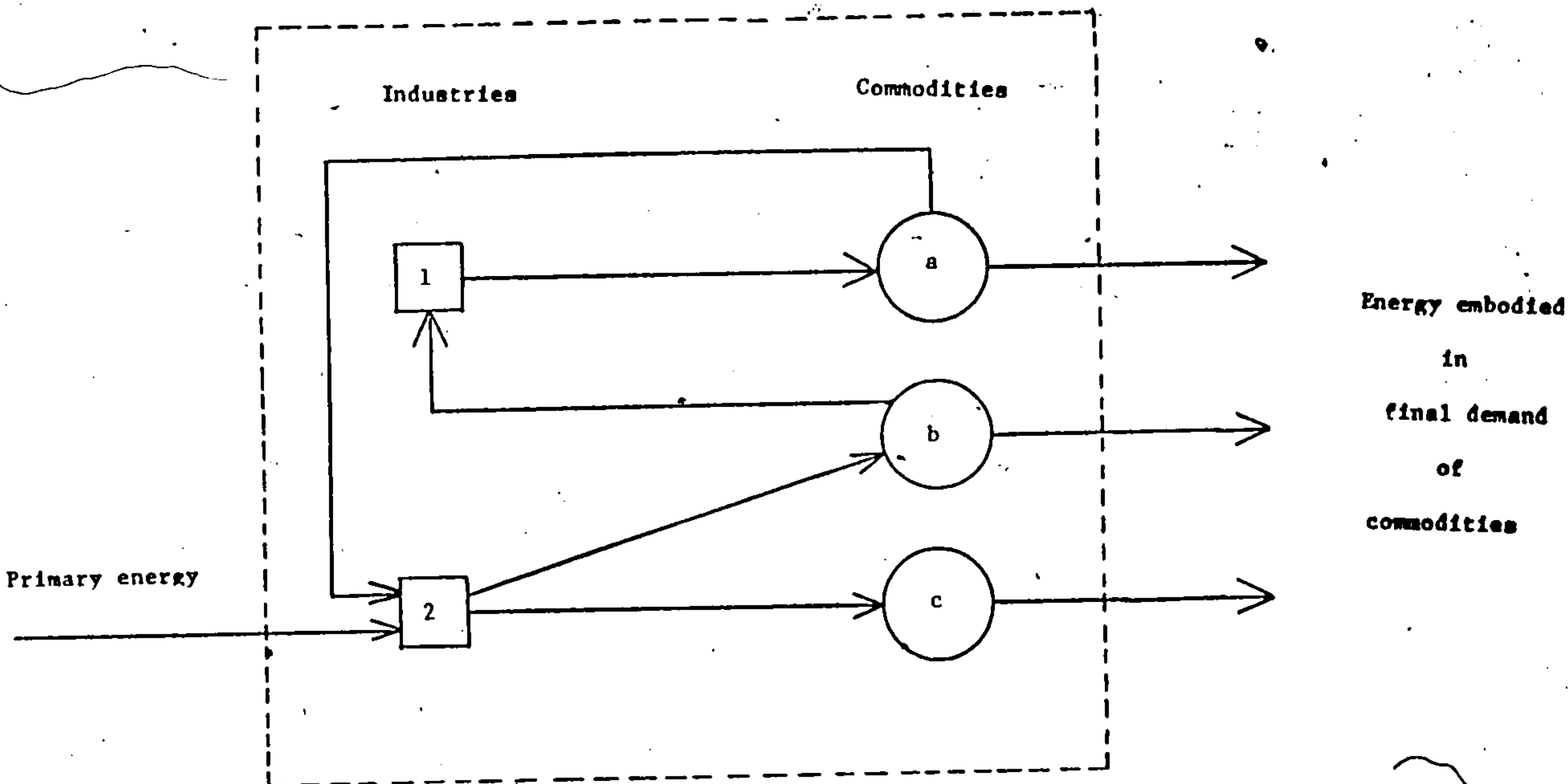

Fig. 3.1 Energy flow in an economic system.

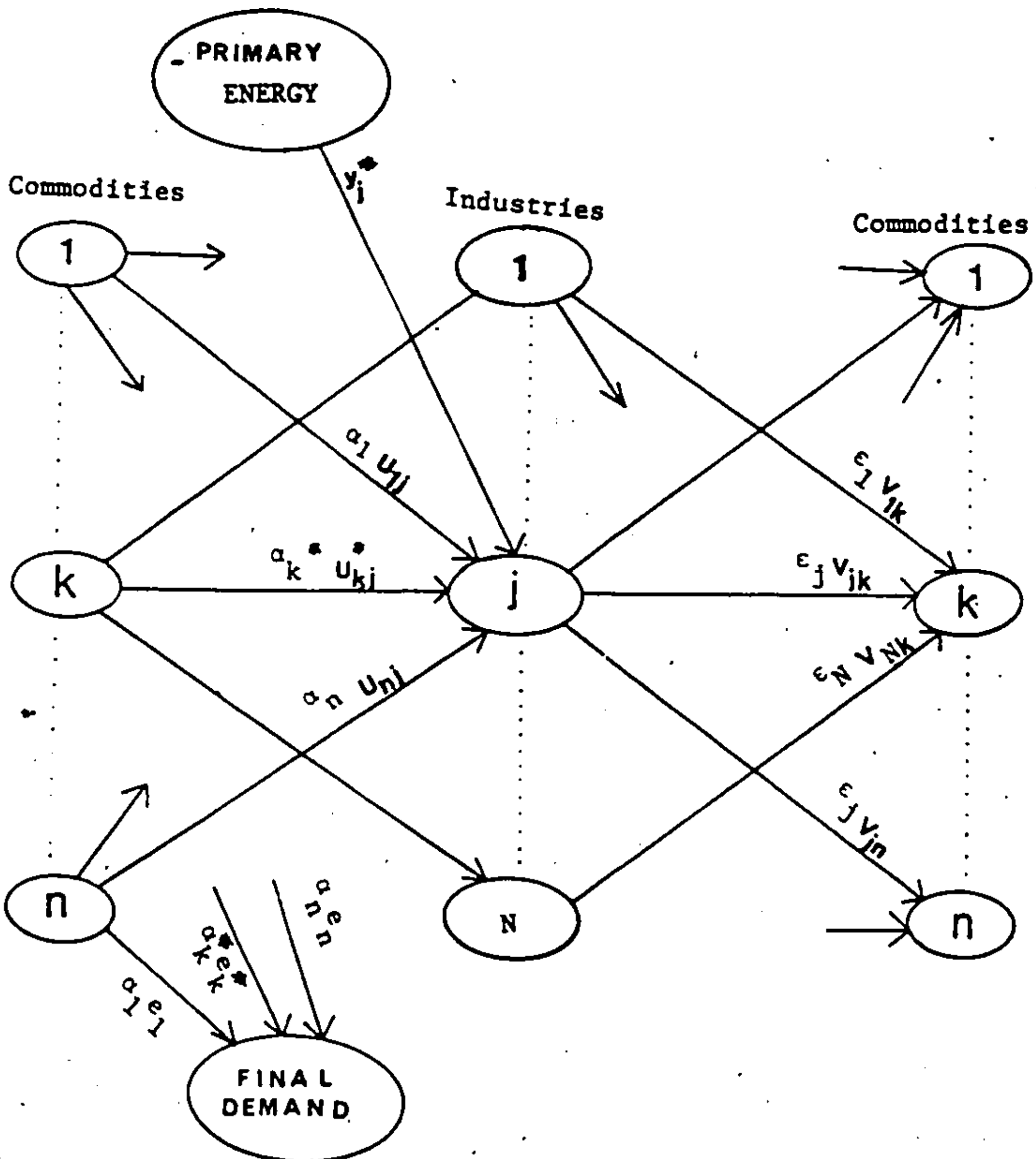

Fig. 3.2 Energy flow in rectangular input-output model.

For an economic system with N industries producing n commodities, the energy flows are shown in Figure 3.2.

α_k and ε_j are the total energy intensity coefficients of commodity k and industry j respectively.
The notation for rectangular input-output analysis is the same as in the previous chapter (section 2.10). If there are 'r' energy commodities of the total number of commodities, energy balance for an industry j can be written as

Total energy embodied in energy commodity i consumed by industry j

+

Total energy embodied in non-energy commodity i consumed by industry j

+

Primary energy consumed by industry j

=

Total energy embodied in the output of industry j

Using the notation of rectangular input-output analysis, equation would be

$$\sum_{i=1}^{r} \alpha_i^{*} U_{ij}^{*} + \sum_{i=r+1}^{n} \alpha_i U_{ij} + Y_j^{*} = \varepsilon_j G_j \qquad (3.22)$$

where

Y_j^{*} is the primary energy consumed by industry j (J). It is termed as 'energy extracted from the earth'

α_i is the total energy intensity coefficient for a non-energy commodity i (J/$)

α_k^* is the energy intensity coefficient for an energy commodity k (J/J)

ε_l is the energy intensity coefficient for an industry l (J/$)

U_{ij}^* is the intermedite input of energy commodity i to industry j (J)

Dividing equation (3.22) by G_j throught gives

$$\sum_{i=1}^{r} (\alpha_i^* U_{ij}^* / G_j) + \sum_{i=r+1}^{n} (\alpha_i U_{ij} / G_j) + Y_j^*/G_j = \varepsilon_j \qquad (3.23)$$

for j = 1 to N

A new industry inputs coefficient matrix $\hat{B}$ is defined as given below

$$\hat{B}_{ij} = U_{ij}^* / G_j \quad (J/\$) \quad i = 1 \text{ to } r, \; j=1 \text{ to } N$$

$$\hat{B}_{ij} = U_{ij} / G_j \quad (\$/\$) \quad i=r+1 \text{ to } n, \; j=1 \text{ to } N \qquad (3.24)$$

The following vectors are also defined.

Row vector $\hat{\alpha}$

$$\hat{\alpha}_i = \alpha_i^* \quad (J/J) \quad \text{for } i = 1 \text{ to } r$$
$$= \alpha_i \quad (J/\$) \quad \text{for } i=r+1 \text{ to } n$$

Row vector ε whose elements are

$$\varepsilon_j \ (J/\$) \quad \text{for } j=1 \text{ to } N$$

Row vector S

$$S_j = Y_j^*/G_j \ (J/\$) \quad \text{for } j=1 \text{ to } N$$

Equation (3.23) can be written in matrix form as

$$\hat{\alpha}\,\hat{B} + S = \varepsilon \tag{3.25}$$

For a commodity produced by more than one industry, an energy balance equation can be written relating the embodied energy in the commodities to the energy in the output of industries.

(Total energy intensity coefficient of industry j).(total production of commodity k by industry j)

=

(Total energy intensity coefficient for commodity k).(total production of commodity k)

Referring to figure 3.2 the relationship is

$$\sum_{j=1}^{N} \varepsilon_j V_{jk} = \alpha_k^* q_k^* \quad \text{for } k=1 \text{ to } r \tag{3.26}$$

$$\sum_{j=1}^{N} \varepsilon_j V_{jk} = \alpha_k q_k \quad \text{for } k=r+1 \text{ to } n \tag{3.27}$$

q_k^* is the output of energy commodity k (J)

Now a new market-share matrix $\hat{D}$ is defined as

$$\widehat{D}_{ji} = V_{ji}/q_i^{*}(\$/J) \quad i=1 \text{ to } r$$
$$\quad j=1 \text{ to } N$$
$$= V_{ji}/q_i\,(\$/\$) \quad i=r+1 \text{ to } n \qquad (3.28)$$
$$\quad j=1 \text{ to } N$$

A diagonal matrix $\widehat{q}$ is also defined such that

$$\widehat{q}_{ii} = q_i^{*}\ (J) \qquad i=1 \text{ to } r$$
$$= q_i\ (\$) \qquad i=r+1 \text{ to } n$$

Equations (3.26) and (3.27) can be combined and written in matrix form

$$\varepsilon V = \widehat{\alpha}\,\widehat{q} \qquad (3.29)$$

Equation (3.28) can be written as below

$$\widehat{D}\,\widehat{q} = V \qquad (3.30)$$

Substituting equation (3.30) in equation (3.29)

$$\varepsilon\,\widehat{D} = \widehat{\alpha} \qquad (3.31)$$

Using equation (3.25)

$$\varepsilon\,\widehat{D}\widehat{B} + S = \varepsilon$$

or

$$\varepsilon = S(I - \widehat{D}\widehat{B})^{-1} \qquad (3.32)$$

From equation (3.31) one gets

$$\widehat{\alpha} = S(I - \widehat{D}\widehat{B})^{-1}\widehat{D} \qquad (3.33)$$

3.7 DISCUSSION

The total energy required 'T', when the final demand of commodities is given, can be computed as

$$T = \hat{\alpha}\,\hat{e} \tag{3.34}$$

using the coefficients calculated from the second method. $\hat{e}$ is the column vector of final demands, which are expressed in joules for energy commodities.

The conservation of embodied energy leads to the fact that energy extracted from the earth is used to satisfy the final demand of goods and services. Energy extracted from the Earth includes imported energy. Appendix B proves this for an illustrative case, with two industries producing three commodities.

When computing the total energy requirement of secondary energy (electricity for example), in the second method the inputs extracted from the earth should be substituted by the inputs going to the production of secondary energy. The total energy obtained thus would be the requirement of primary energy (coal for example) for the production of secondary energy.

When the energy commodity is produced by more than one industry, (say two) ,then two sets of coefficients are computed, one set for the energy commodity produced by each industry. This is done by substituting the appropriate values in the vector S , in the second method. The energy intensity coefficient for (that type of energy commodity) would be the sum of the corresponding elements of the sets of coefficients obtained. Since the outputs of a commodity are valued at the same rate, the output of an energy commodity in joules is proportional to its value in dollars (see appendix C). In the first method, the elements of the vector W are taken non-zero for the industries producing the energy commodity.

The coefficients can be computed for primary and secondary energy commodities, for which actual energy data exist. There are also energy inputs for which data does not exist and are not part of the input-output commodity classification. For example solar energy input for the production of food grains is not accounted for, in the analysis.

The computed coefficients would be the average values and do not represent the energy required by specific processes. Errors in the input-output data are transmitted into the coefficients. In addition, because of the linearity assumption of input-output models, changes in the efficiencies of production of energy commodity are not considered. For example the coal input in joules, to produce a dollar's worth of electricity would remain the same, irrespective of the level of electricity production.

In the static input-output model, in the inputs matrix only the current inputs (raw materials) are included. All the capital expenditure is a component of the final demand of goods and services. Hence the energy required for capital purchases is not included in the coefficients.

The coefficients could be used only for short-term energy demand forecasting. The values of the coefficients depend upon the mix of processes of producing secondary energy from primary energy. For example the coefficient representing the consumption of coal (directly and indirectly) for the production of electricity is dependent upon the actual values of the production of electric energy using coal. If nuclear fuel is used to produce electric energy at a higher level, the value of the coefficient goes down.

When the coefficients are computed using rectangular input-output tables, market-shares constancy is assumed. When the market-shares change, the values of energy intensity coefficients also change.

The coefficients give the total requirement of energy in joules per dollar of final demand. It is not possible to distinguish the use of energy as a fuel and as a feedstock.

The energy intensity coefficients for industries, computed using the two methods are not comparable because of the differences in market-share assumptions (equations 2.12 and 3.28). However it has been shown that the results obtained by the second method are superior [22].

3.8 ENERGY INTENSITY COEFFICIENTS FOR CANADA

As an illustration, energy intensity coefficients for industries have been computed using small aggregation Canadian input-output data [47]. The coefficients have been calculated separately for different energy commodities, i.e., for coal, natural gas and electricity. Computation is done using 1971 data in nominal dollars.. The energy data in gigajoules is taken from Statistics Canada working paper [46]. The values of the coefficients for the methods are shown in Tables 3.1 and 3.2.

INDUSTRY	COAL	NATURAL GAS	ELECTRIC ENERGY
AGRICULTURE	0.006	0.007	0.006
FORESTRY	0.003	0.006	0.003
FISHING & HUNTING	0.003	0.005	0.003
MINES & OIL WELLS	0.034	0.572	0.012
MANUFACTURING	0.011	0.017	0.012
CONSTRUCTION	0.005	0.008	0.005
TRANSPORTATION	0.004	0.023	0.003
COMMUNICATION	0.002	0.003	0.002
UTILITIES	0.165	0.034	0.221
WHOLESALE TRADE	0.003	0.006	0.003
RETAIL TRADE	0.005	0.016	0.005
FINANCING	0.002	0.006	0.002
BUSINESS SERVICES	0.003	0.006	0.004
TRANSP. MARGINS	0.004	0.023	0.004
OFFICE SUPPLIES	0.008	0.013	0.008
TRAVEL & ADVERT.	0.006	0.011	0.006

10^8 J/\$

Table 3.1 Energy intensity coefficients for Canada for 1971 based on the first method.

INDUSTRY	COAL	NATURAL GAS	ELECTRIC ENERGY
AGRICULTURE	0.0044	0.0102	0.0053
FORESTRY	0.0028	0.0065	0.0028
FISHING & HUNTING	0.0038	0.0086	0.0030
MINES & OIL WELLS	0.1429	0.3125	0.0108
MANUFACTURING	0.0142	0.0325	0.0114
CONSTRUCTION	0.0081	0.0183	0.0050
TRANSPORTATION	0.0029	0.0072	0.0034
COMMUNICATION	0.0012	0.0029	0.0019
UTILITIES	0.0055	0.0199	0.3332
WHOLESALE TRADE	0.0018	0.0041	0.0027
RETAIL TRADE	0.0016	0.0045	0.0048
FINANCING	0.0012	0.0030	0.0021
BUSINESS SERVICES	0.0027	0.0062	0.0037
TRANSP. MARGINS	0.0029	0.0073	0.0040
OFFICE SUPPLIES	0.0095	0.0217	0.0080
TRAVEL & ADVERT.	0.0059	0.0136	0.0058

10^{8} J / $

Table 3.2 Energy intensity coefficients for Canada for 1971 based on the second method.

e_j is the share of Gross Domestic Product of commodity j at factor cost (\$/\$)

D is the market-share matrix (\$/\$)

$L_{1,i}$ is the limit on production of i th commodity (\$)

$L_{2,i}$ is the limit on export of i th commodity (\$)

$L_{3,i}$ is the limit on import of i th commodity (\$)

N is the number of industries

n is the number of commodities

B is a matrix of industry inputs coefficients (\$/\$)

M_j is the import of commodity j (\$)

EX_j is the export of commodity j (\$)

FN_j is the final inventory of commodity j (\$)

IN_j is the initial inventory of commodity j (\$)

e is a vector of values of final demands of commodities (\$)

l_k is the coefficient characterizing the variability of energy resource k

q is a vector of values of total commodity outputs (\$)

NOMENCLATURE FOR CHAPTER IV

Chapter IV

EVALUATION OF UNCERTAINITY EFFECTS ON ENERGY PLANNING: A STOCHASTIC INPUT-OUTPUT LINEAR PROGRAMMING MODEL

4.1 INTRODUCTION

Static rectangular input-output analysis is described in the form of a set of linear equations (section 2.10). Using such analysis, the gross production requirements of commodities can be computed when the final demand for commodities is known. Since the model is expressed as a set of linear equations, a unique solution of gross production requirements is obtained. The static rectangular analysis can also be formulated as a linear programming model [16,37] with certain advantages which are mentioned later. Static input-output analysis can be formulated into linear programming models in various ways [13,16,33,37]. In this chapter, the static input-output analysis in the form of a linear programming problem, incorporating production, exports, imports and inventories is presented. The model is applied in the form of parametric programming to study the effect of production capacity limits of energy commodities, in particular mineral fuels on the economic system. The shadow prices of energy commodity at different production levels can be used when making decision as to import of energy commodity or increase in capacity of energy commodity production. Small aggregation input-output data has been used for numerical applications.

4.2 LINEAR PROGRAMMING

Linear programming falls into the class of optimisation problems. In optimising problems there exists a function of some variables (unknown) to be extremized (maximized or minimized) subject to certain constraints on the variables which are also expressed as functions. In linear programming all the functions are linear in the unknown variables. Formally linear programming is defined as [16] 'The analysis of problems in which a linear function of a number of variables is to be maximised (or minimised) when those variables are subject to a number of restraints in the form of linear inequalities'.

4.2.1 The Primal Problem

Mathematically a linear programming problem can be represented as

$$\text{Maximise } J = C\,X \tag{4.1}$$

subject to

$$A\,X \leqslant B \tag{4.2}$$

$$X \geqslant 0 \tag{4.3}$$

where

C is a row vector of order (1 by n) of the coefficents of the objective function.

X is a column vector of order (n by 1) of the unknown variables.

A is a matrix of coefficients of order (m by n)

B is a column vector of order (m by n) of the right hand side of the inequality (4.2).

n is the number of unknowns

m is the number of constraints.

The inequalities (4.1) and (4.2) are the objective function and the constraints respectively. In addition to the m constraints represented by the inequality (4.2), additional n constraints are imposed by the inequalities (4.3) such that the unknown variables do not take negative values.

It is clear from the inequality (4.2) that when m=n and when the symbol ' $\leq$ ' is replaced by '=', the inequalities are converted into a system of linear equations. For such a system a unique solution exists, which may not satisfy the conditions imposed by (4.3).

Given the inequalities (4.2) and (4.3), it can be seen that there exists no more a unique solution, but a set of infinite solutions. For such a system a desirable solution can be obtained if an objective function can be chosen to suit the needs of the individual who formulates the problem. The solution is obtained in the form of values of the variables, maximising (or-minimising) the linear objective function. In a general case

$$m \gtreqless n$$

When the values of the vectors C and B and the matrix A are known, methods of finding solution exist [7]. This problem is known as the primal problem.

4.2.2 The dual Problem

For every linear programming problem represented (4.1)-(4.3), the dual linear programming problem can be formulated as a set of unknown variables (known as dual variables)

$$\text{Minimise } k = B^T Y \tag{4.4}$$

subject to

$$A^T Y \geq C \tag{4.5}$$

$$Y \geq 0 \tag{4.6}$$

where

Y is a column vector (m by n) of the unknown dual variables.

Superscript T indicates transposition.

For both the formulations { (4.1)-(4.3) and (4.4)-(4.6) } the value of the function as the solution will be the same [7].

The dual variables can be interpreted as shadow prices. In the case of optimal resource allocation problem, if the constraints in inequality (4.2) represent the availability of scarce resources and if the objective function represents the Gross Domestic Product (GDP), the solution to the dual problem will be the marginal values in terms of GDP. The shadow prices can be considered as the valuations of limited resources. If a solution to the primal problem shows excess availability of a resource say b_j, the corresponding dual varible y_j will have zero value. In general the shadow prices represent the marginal contribution to the objective function.

4.3 ADVANTAGES OF FORMULATING INPUT-OUTPUT ANALYSIS IN LINEAR PROGRAMMING FORMAT

Static input-output analysis can be formulated in linear programming form. Some advantages are listed below.

1. Computation of optimal solution: Unlike static input-output analysis, the solution obtained from a linear programming problem is an optimal one. A model using 'minimising the labour costs' as the objective function gives a production program for producing commodities meeting all the final demands and minimising total labour cost.
2. Choice of criterion for optimality: In the linear programming formulation the objective can be chosen depending upon the needs of the policy analyst. The following criteria could be used [53].

a) Maximise GDP

b) minimise imports

c) Maximise consumption

d) Minimise energy consumption

e) Minimise total costs

f) Minimise labour costs

Each of the above criteria has its own usefulness. For scarce energy resources, the fifth criterion could be the most suitable.

3. Shadow Price Determination: The solution to the dual LP problem is obtained using Simplex algoritm as a by-product. The dual solution, represents the shadow prices and gives an indication of the resources which can improve the value of the objective function.

4. Capacity Constraints: In the linear programming formulation, the capacity limits for producing commodities can be represented as constraints. In this way, it can be found, if the plans are feasible or not.

4.4 LINEAR PROGRAMMING MODEL USING INPUT-OUTPUT STATISTICS

The formulation of the linear programming model is

$$\text{Maximise} \quad \left(\sum_{j=1}^{n} c_j \quad q_j\right) \tag{4.7}$$

subject to

$$\sum_{j=1}^{n} (I-BD)_{ij} \; q_j - EX_i + M_i - FN_i \quad = e_i - IN_i \tag{4.8}$$

$$i = 1 \text{ to } n$$

and

$$q_i \le L_{1,i} \qquad i = 1 \text{ to } n \qquad (4.9)$$

$$EX_i \le L_{2,i} \qquad i = 1 \text{ to } n \qquad (4.10)$$

$$M_i \le L_{3,i} \qquad i = 1 \text{ to } n \qquad (4.11)$$

$$FN_i = IN_i + \text{change in inventory level} \qquad (4.12)$$

For the energy commodity k

$q_k \le l_k$. actual energy production in \$ in a particular year.

where

c_j is the share of gross domestic product (GDP) of commodity j at factor cost in (\$/\$).

$$c_j = \sum_{k=1}^{N} d_k \; D_{kj} \qquad (4.13)$$

where

d_k is the share of GDP at factor cost of industry k per unit of output of k which includes labour, profits and taxes.

D_{kj} is the general element of the market-share coefficients matrix.

$L_{1,i}$ is the limit on the production of i th commodity.

$L_{2,i}$ is the limit on the export if i th commodity.

$L_{3,i}$ is the limit on the import of i th commodity.

N is the number of industries.

B_{ij} and D_{ij} are the elements of the industry inputs coefficients matrix and market-share coefficients matrix respectively in (\$/\$)

q_j is the gross output of commodity j in (\$)

EX_j is the export of commodity j in (\$)

M is the import of commodity in (\$)

FN_j is the final inventory of commodity j in (\$)

IN is the initial inventory of commodity in (\$). This is an input to the programming problem.

e_i is the final demand less imports for commodity i in (\$)

l_k is the coefficient characterising the variability of energy resource.

The linear programming model is formulated such that the effects on GDP, commodity outputs and shadow prices of commodities could be computed when the coefficient l_k is treated as a random variable. The effects of randomness of final demands and coefficients B_{ij} and D_{ij} can also be computed.

4.5 MODEL APPLICATION

For the purpose of illustration, the model has been applied using the small aggregation input-output data [47]. The energy sector is represented by mineral fuels aggregating coal, natural gas and crude mineral oils. The limit on the availability of energy commodity was varied by giving different values to the coefficient l_k . A parametric programming is made, by increasing the value of l_k from l_k =1.0 till the shadow price corresponding to the energy commodity became equal to zero. The data used was for 1965. The study is carried out in the following sequence.

1. Calculation of the optimal solution with the capacities for producing energy commodities equivalent to their actual values in 1965 in ($).
2. If the shadow price corresponding to the energy commodity has a non-zero shadow price, parametric study is performed by increasing the capacity for producing the energy commodity. This is done by increasing the value of the coefficient l_k (k=energy commodity) and computing the optimal solution. If the shadow price for the energy commodity is again greater than zero., the value of the coefficient l_k is increased and the optimal solution is found. The steps are repeated, until the shadow price for the energy commodity becomes zero.

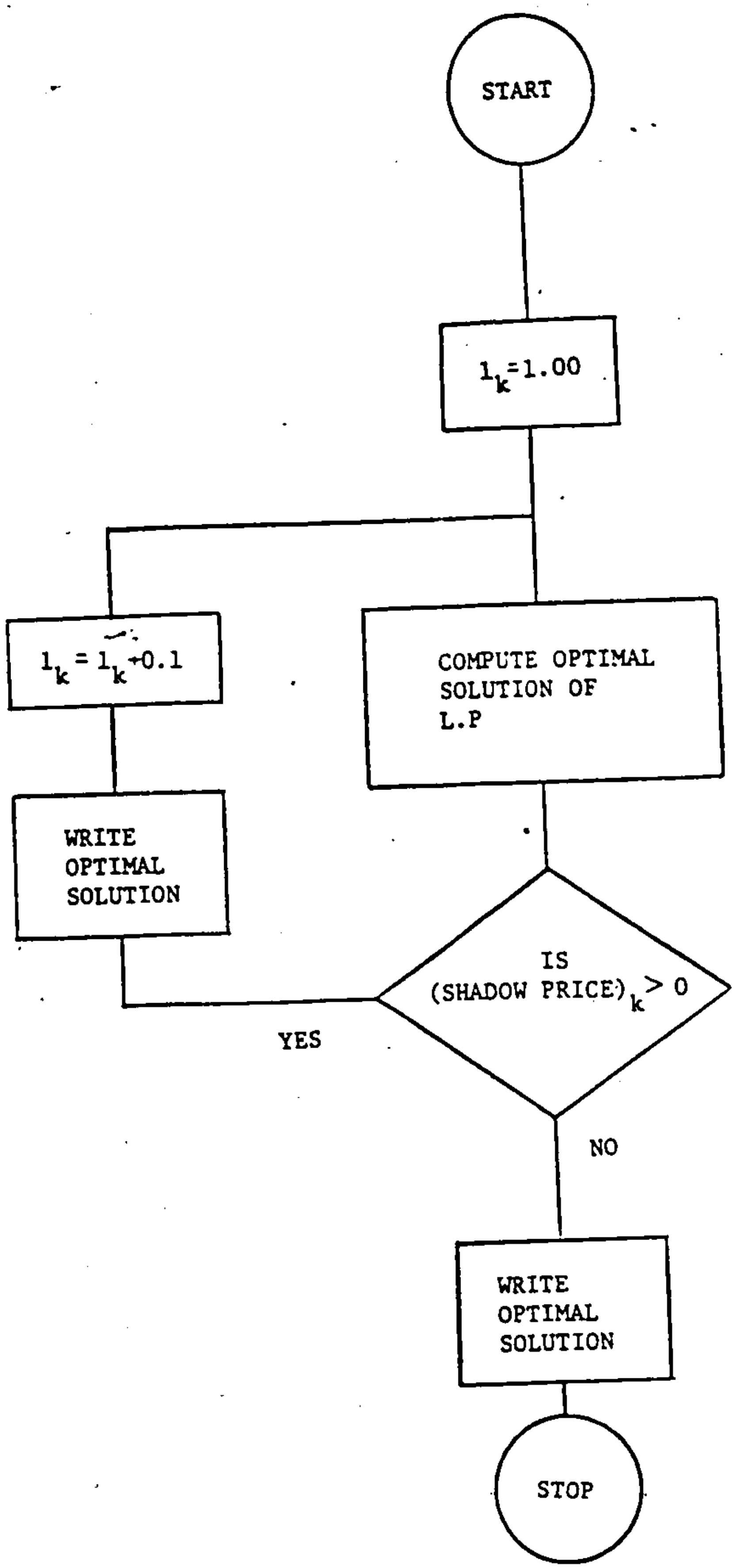

Fig. 4.1 Flow chart for parametric study

The process of parametric study is shown in the form of a flow chart in Figure 4.1. The variation of GDP and shadow price for energy commodity for an increasing capacity of energy commodity production is shown in Figure 4.2. The values of shabow prices, for one particular case of l_k are shown in Table 4.1.

4.6 DISCUSSION

The shadow prices obtained have units (\$/\$). They represent the marginal contribution to the objective function. When the shadow price for a commodity is very large, it would be worthwhile to increase the capacity for producing that commodity, or importing that commodity. The rules for producing and importing a commodity are

Produce a commodity j if

(shadow price) (\$/\$) $\geq$ Investment in \$ to increase production of commodity j by one \$ +1

Import a commodity j if

(shadow price) (\$/\$) $\geq$ 1 (\$) but less than the above value.

The study showed that the shadow price for the energy commodity is very sensitive to the value of the coefficient l_k, when its value is near to 1.0.

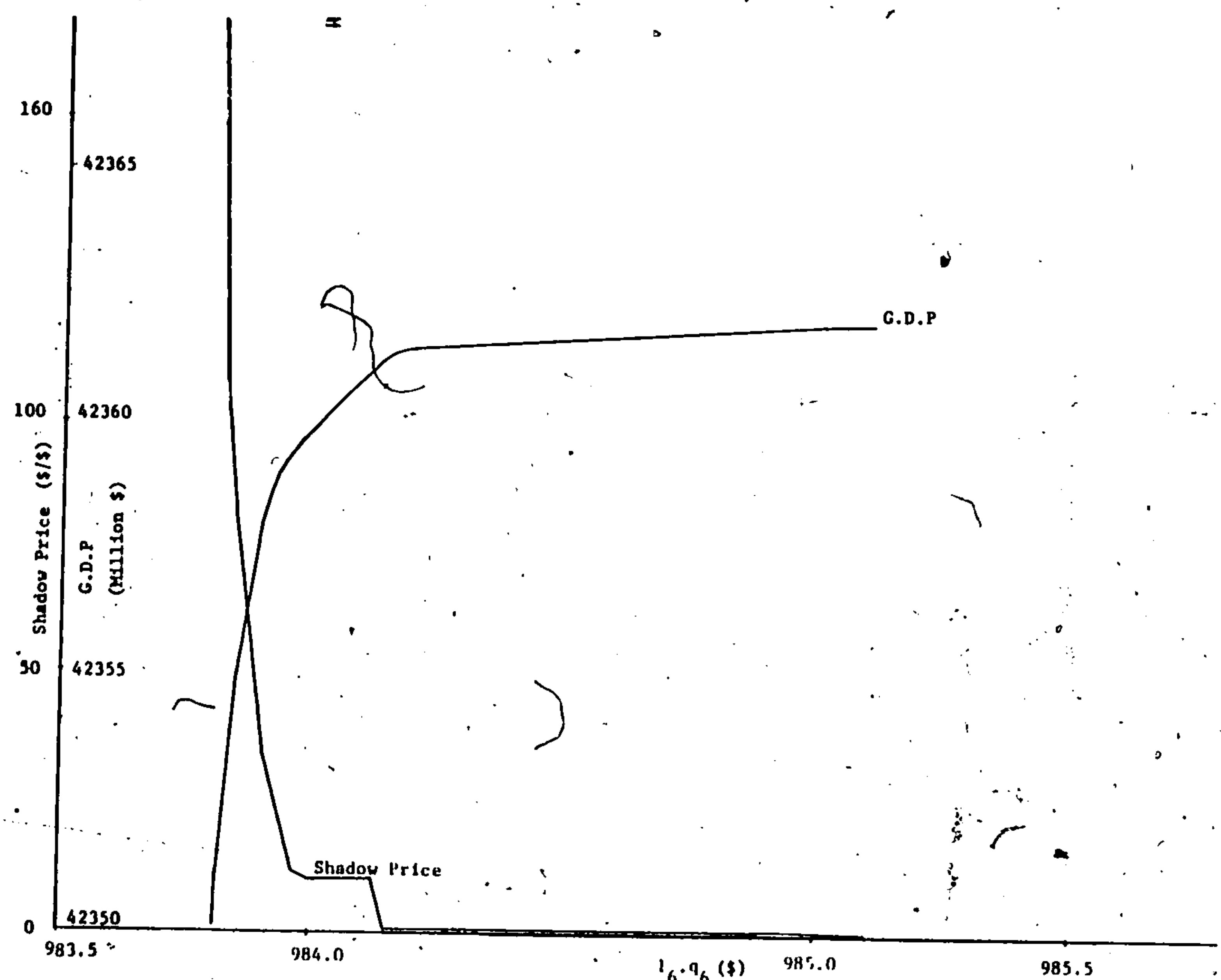

Fig. 4.2 Variation of G.D.P and shadow price of energy commodity for an increasing energy commodity availability.

GROSS DOMESTIC PRODUCT (M$) 42359.88

COMMODITY	PRODUCTION(M$)	SHADOW PRICE($/$)
GRAINS	1113.83	0.58
OTHER AGRICULTURAL PRODUCTS	2819.04	0.57
FORESTRY PRODUCTS	1016.60	0.48
FISHING AND TRAPPING PROD.	173.45	0.66
METALLIC ORES & CONCENTRATES	1570.67	0.58
MINERAL FUELS	983.99	11.18
NON-METALLIC MINERALS	366.81	0.60
SERVICES INCIDENTAL TO MINING	255.37	0.62
MEAT AND DAIRY PRODUCTS	3311.89	0.41
FRUIT,VEG.,MISC. FOOD PRODUCTS	2658.72	0.0
BEVERAGES	770.51	0.0
TOBACCO & TOBACCO PRODUCTS	376.61	0.0
RUBBER,LEATHER & PLASTICS	948.42	0.0
TEXTILE PRODUCTS	1294.72	0.0
CLOTHING	1348.52	0.0
LUMBER & OTHER WOOD PRODUCTS	1516.57	0.0
FURNITURE & FIXTURES	490.60	0.0
PAPER & PAPER PRODUCTS	2943.98	0.0
PRINTING & PUBLISHING	1080.49	0.0
PRIMARY METAL PRODUCTS	3581.24	0.0
METAL FABRICATED PRODUCTS	2295.66	0.0
MACHINERY & EQUIPMENT	1428.20	0.0
AUTOS ,TRUCKS & OTHER EQPT.	3900.73	0.0
ELEC. AND COMM. PRODUCTS	1972.39	0.010
NON-METALLIC MINERAL PROD.	1036.91	5.72
PETROLEUM AND COAL PROD.	1507.43	0.0
CHEMICALSANND CHEM., PROD.	2079.92	0.004
MISC., MAN'FD PRODUCTS	752.65	0.0
RESIDENTIAL CONSTRUCTION	2218.30	0.0
NON-RESIDENTIAL CONSTN.	5481.08	0.0
REPAIR CONSTRUCTION	1798.67	0.0
TRANSP. AND STORAGE	4621.48	0.60
COMMUNICATION SERVICES	1445.33	0.79
OTHER UTILITIES	1356.70	0.45
WHOLESALE MARGINS	4014.00	0.54
RETAIL MARGINS	4429.92	0.62
IMPUTED RENT OWNER OCPD. DWEL.	3112.73	0.59
OTHER FINANCE'INS.,REAL ESTATE	6071.11	0.58
BUSINESS SERVICES	1353.50	0.58
PERSONAL & OTHER MISC. SERVICES	5955.88	0.60
TRANSPORTATION MARGINS	2086.82	0.0
OPERATING,OFFICE,LAB, & FOOD	4016.98	0.0
TRAVEL,ADVTG. AND PROMOTION	2013.40	0.0

Table 4.1 Sample result of parametric study.

X_j is the total production of commodity j ($)

c_j is the share of Gross Domestic Product of commodity j at factor cost ($/$)

B is the matrix of industry inputs coefficients matrix ($/$)

D is the market-share matrix ($/$)

n is the number of commodities

G is a vector of values of total industry outputs ($)

N is the number of industries

Z_1 is the electric energy produced from coal (J)

Z_2 is the electric energy produced from gas (J)

Z_3 is the electric energy produced from oil (J)

Z_4 is the electric energy produced from hydro source (J)

Z_5 is the electric energy produced fron nuclear energy source (J)

Z_6 is the gas obtained from gas well (J)

Z_7 is the gas produced from coal (J)

Z_8 is the gas produced from oil (J)

p_j is the price of energy commodity j ($/J)

η_{ij} is the efficiency of conversion from fuel i to energy commodity j

K_j is the capacity for producing energy using process Z_j

NOMENCLATURE FOR CHAPTER V

Chapter V

ELECTRIC ENERGY DEMAND FORECASTING USING INPUT-OUTPUT ANALYSIS

5.1 INTRODUCTION

Forecasting is needed to formulate plans. The demand for electric energy is required for determining

(a) Production capacities

(b) Transmission capacities

(c) Supply of fuels needed to produce electricity

Forecasts can be made at regional or national levels and at various levels of detail.

5.2 ELECTRIC ENERGY DEMAND MODELS

There are various models which can be used to forecast the demand for electric energy. Depending upon the relationship between the energy and the economic system, the models can be classified into three categories.

5.2.1 Extrapolation Methods

These methods involve fitting trend curves to historical data. When the curve is projected into the future the value of the forecast is obtained. The trend of the past electric energy consumption is assumed to continue into the future. The analytical functions used in curve fitting include [48]

(a) Straight line $Y = a + b X$

(b) Parabola $Y = a + b X + C X^2$

(c) S curve $Y = a + b X + c X^2 + d X^3$

(d) Exponential $Y = c e^{dX}$

(e) Gompertz $Y = \ln^{-1}(a + c e^{dX})$

The coefficients are determined using the least-squares technique.

The other techniques falling into this category are

(i) Moving average method

(ii) Exponential smoothing

Methods (i) and (ii) are similar to the trend extrapolation techniques. But the weight given to more recent data can be varied in (i) and (ii). The electric energy data is de-seasonalised for these techniques and the forecast obtained is re-seasonalised. This is done because electric energy is a commodity with seasonal demand. With these techniques, only data pertaining to the consumption of electric energy is required. The relationship between electric energy and the activity in the economic system is not taken into account.

5.2.2 Causal Methods

Techniques falling into this category forecast the demand for electric energy by linking the activity levels in the economic system to the energy demand. They are

5.2.2.1 Correlation and regression techniques

The demand for electric energy is linked to other activity levels and demographic factors. As an example, demand for electricity can be linked to the production of commodities and population growth. The relationships can be linear or non-linear. With the available forecasts of the other activity levels, the demand for electricity is forecasted. The demand is usually forecasted for three classes of customers: residential, commercial and industrial. Two examples, one for for industrial and one for residential electric energy demand are given.

1. Example for Industrial Electric Energy Demand [3].

   The equation determines the demand for electricity by primary metals industry.

   $$\ln(E)=0.35-0.461\ln(PC)+0.22\ln(PK)-0.32\ln(PO)-1.94\ln(PE)-1.07\ln(W)$$

   where

   E is the kwh electricity purchased by primary metals industry.
   PC is the price of coal
   PO is the price of oil
   PK is the price of coke
   PE is the price of electricity

W is the average wage rate of production workers in primary metals industry

2. Example for Residential Electric Energy Demand [55]. In this example the demand for electric energy is given by

$$Q = k + b_1 P + b_2 G + b_3 Y + b_4 R + b_5 C + \epsilon$$

where

Q is the average electricity consumption per household (kwh per year)

P is the average price of electricity per kwh

G is the average price of natural gas (cents per therm)

Y is the median family income ($)

R is the average number of rooms per household

C is the number of degree days

ϵ is the random error term

5.2.2.2 Network Models

Network models are useful to determine the production of electric energy from different fuels, when the demands for electric energy for various sectors are available. In addition to electric energy, network models of overall energy system also determine the production of other fuels (gas, oil etc.) by the available processes. This is accomplished by the use of optimisation techniques

5.2.3 Energy-Economy interaction models

Using energy-economy integrated models, the effect of energy scarcity on the economic system and also the effect of production of other commodities on the energy system could be studied.

5.3 ADVANTAGES AND DISADVANTAGES

1. The methods in the first category are not based on causal relationship. They rely upon the past trend, which may not continue into the future. However, they are simple to use and need little data.
2. Techniques in the second category fail to consider the energy-economy link to the full extent. They recognize only that electric energy demand is linked to the demand of other sectors. They take into account only one way relationship between the economic system (some variables) and the demand for electric energy. For example if energy production is related to transportation sector, these methods fail to predict what would happen to energy sector when there is a change in the activity level of transportation sector.

In the network models of energy systems, optimisation is usually done to meet the demand for energy at a minimum cost of the energy system. In the economic sense, such goals are too narrow. The overall objective of the economic system may not be minimising energy cost but some other criterion which affects the whole society such as unemployment. Such objectives cannot be incorporated in the stand alone network models.

3. The third category models are integrated models. The energy system is linked to the rest of the economic system. These models could be formulated as optimising problems with the optimising criterion reflecting the good of the whole society. The disadvantage of such models is that they require extensive data.

5.4 APPLICATION OF INPUT-OUTPUT ANALYSIS

Input-output analysis could be applied to forecast the total demand for electric energy. Equation (2.17) could be directly used to find the total production requirement of electric energy in monetary units when the final demands for goods and services are known. Alternatively energy intensity coefficients developed in chapter III could be used to compute the total demand for electric energy in energy units.

The above methods are useful for specific purposes. The first method using equation (2.17) can be used for overall economic planning. The second technique gives the forecast in energy units. But both the methods do not provide the quantities of electricity to be produced from different fuels.

The input-output linear programming model in chapter IV could be used for planning purposes. Again this method gives the total electricity requirement in monetary units.

Moreover these models are not useful when a more detailed forecast is required. For example, it is not possible to forecast electric energy generation using different fuels. Only the average technology representing the mix of the available technologies is reflected. When the final demand for commodities is very high more electric energy has to be produced. This may result in the necessity of using fuel oil to produce electric energy, resulting in higher requirement of fuel oil. Effect of variation in final demand of goods and services, on the production mix of electric energy is not reflected in these models.

In this chapter a model based on rectangular input-output analysis, in a linear programming format combined with a network representation of the energy system is presented [34]. This model can be used to forecast the total demand for electric energy when the final demand for for commodities is known.

5.5 ENERGY SYSTEM NETWORK

The energy system represented in a network form is shown in Figure 5.1. The nodes on the left hand side of the network are the sources of primary energy and the nodes in the middle are the total output of primary and secondary energy commodities. The nodes on the right hand side are the demand for energy commodities. The demands are the sum of inter-industry requirements and the final demands for all the commodities (except the inter-industry requirement within the energy sector).

The network representation of the technological system shown in Figure 5.1 has the following technologies for producing electric energy incorporated in it.

1. Directly from coal using thermal plants.
2. From hydro power.
3. From nuclear fuel.
4. From gas.

In the network, the mining of coal, gas and conversion of coal to gas and fuel oil to gas are also presented. In this example transportation and transmission losses are neglected, but the model can incorporate them. All the flows in the network are in energy units. This network is incorporated into the linear programming model of the economic system.

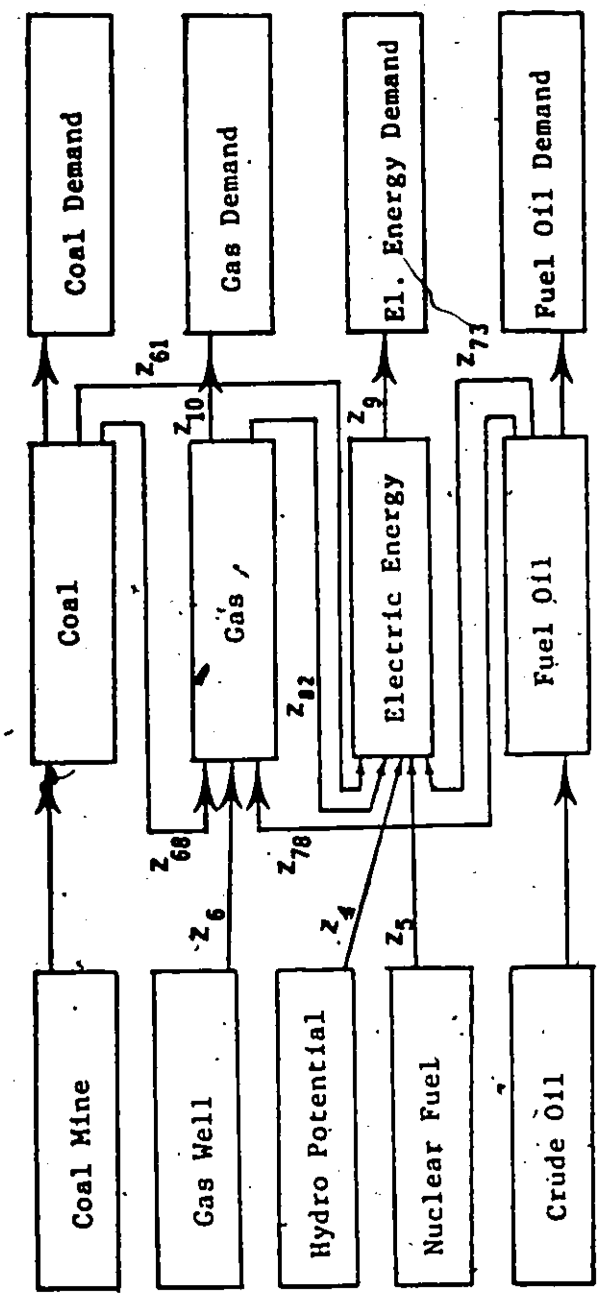

Fig. 5.1 Energy system network

5.6 LINEAR PROGRAMMING MODEL

In this linear programming model, the objective is to maximise the Gross Domestic Product (GDP), subject to the constraints on the total commodity production to satisfy final demands of commodities and the availability of capacities of industries.

The formulation is as follows:

$$\text{Max} \sum_{j=1}^{n} c_j \quad X_j \qquad (5.1)$$

subject to

$$\sum_{j=1}^{n} (I-BD)_{ij} \, X_j \geq e_i \qquad (5.2)$$

for i = 1 to n

and

$$\sum_{j=1}^{n} D_{kj} \, X_j \leq G_k \quad \text{for } k = 1 \text{ to } N \qquad (5.3)$$

where

c_j is the share of commodity j at factor cost of G.D.P

X_j is the total production of commodity j ($)

I is an identity matrix (n by n)

n is the number of commodities (for small aggregation Canadian input-output data n= 43)

In the equations (5.1)-(5.3) the relationships are given in ($). In the input-output model, electric energy is treated as a single commodity irrespective of the source from which it is produced.

In order to introduce the energy system network in the linear programming model, additional unknowns are introduced. In this model all the variables denoted by X are in ($) and the variables denoted by Z are in (J).

Let $Z_1, Z_2, \ldots Z_5$ be the electric energy produced from coal, gas, oil, hydro and nuclear fuels.
Z_6, Z_7, Z_8 be the gas obtained from well, coal and oil respectively.

The summation of production by different processes equals the total production of the specific energy commodity.

$$\sum_{i=1}^{5} Z_i = Z_9 \tag{5.4}$$

where Z_9 is the total electric energy produced and

$$\sum_{i=6}^{8} Z_i = Z_{10} \tag{5.5}$$

Z_{10} is the total gas production.

If p_1, p_2, p_3 and p_4 are the prices ($/J) of coal, oil, gas and electric energy respectively, the relationship between dollar quantities and joule quantities for energy commodities are given by

$$p_3 \; Z_{10} = X_8 \tag{5.6}$$

$$p_4 \; Z_9 = X_{38} \tag{5.7}$$

where X_8 and X_{36} are the production of gas and electric energy in dollars as they appear in equations (5.1)-(5.3)

The inputs to produce energy commodities are obtained by using conversion efficiencies from fuel i to energy commodity j, $\eta_{i\text{-}j}$. For coal fired power plants, one can write

$$Z_{6\text{-}1}\ \eta_{6\text{-}1} = Z_1 \qquad (5.8)$$

where $Z_{6\text{-}1}$ is the coal input to produce electric energy. It is constrained by the availability of coal as in the following inequality.

$$\sum_{\substack{j=1 \\ \neq 36}}^{n} (I-BD)_{6j}\ X_j - (p_1 / \eta_{6\text{-}1})\ Z_1 \geq e_6 \qquad (5.9)$$

$\eta_{6\text{-}1}$ is the efficiency of producing electricity from coal. The inequality (5.9) replaces in the system (5.2) the relationship corresponding to coal. If one also considers the production of gas from coal, the inequality (5.9) would change to

$$\sum_{\substack{j=1 \\ \neq 8,36}}^{n} (I-BD)_{6j}\ X_j - (p_1/\eta_{6\text{-}1})\ Z_1 + (p_1/\eta_{6\text{-}8})\ Z_7 \geq e_6 \qquad (5.10)$$

where

$\eta_{6\text{-}8}$ is the efficiency of conversion of coal to gas.
In a similar manner, one can write equations for electric energy produced from gas.

$$Z_{8\text{-}2}\eta_{8\text{-}2} = Z_2 \qquad (5.11)$$

and

$$\sum_{\substack{j=1 \\ \neq 36}}^{n} (I-BD)_{8j} X_j - (p_3 / \eta_{8.2}) Z_2 \geq e_8 \qquad (5.12)$$

For electric energy produced from oil

$$Z_{7.3} \; \eta_{7.3} = Z_3 \qquad (5.13)$$

and

$$\sum_{\substack{j=1 \\ \neq 36}}^{n} (I-BD)_{7j} X_j - (p_2 / \eta_{7.3}) Z_3 \geq e_7 \qquad (5.14)$$

Commodity 7 corresponds to fuel oil. When the production of gas from fuel oil is also included equation (5.14) becomes

$$\sum_{\substack{j=1 \\ \neq 8,36}}^{n} (I-BD)_{7j} X_j - (p_2 / \eta_{7.3}) Z_3 - (p_2 / \eta_{7.8}) Z_8 \geq e_7 \qquad (5.15)$$

In the equations (5.11)-(5.15)

$\eta_{8.2}$ is the efficiency of producing electricity from gas.

$\eta_{7.3}$ is the efficiency of electricity from fuel oil.

$\eta_{7.8}$ is the efficiency of gas from fuel oil.

In a similar manner (5.12) and (5.18) replace in the system (5.2) for gas and fuel oil.

Since nuclear fuel and hydro potential are not in the input-output commodity classification, the production of electricity by these inputs is given fixed values. This is also justified by the fact that electricity from these inputs meets the base load.

Constraints on the production of electricity and gas by each process have also been introduced to represent the capacity of each type of process as given below.

$$Z_i \leq K_i \quad \text{for} \quad i = 1 \text{ to } 3 \text{ and } 6 \text{ to } 8$$

$$Z_i = K_i \quad \text{for} \quad i = 4 \text{ and } 5 \qquad (5.16)$$

5.7 DISCUSSION

1. The advantage of this model is, it converts the final demand for various commodities, into gross demand for electric energy. It links the demand for electric energy to the rest of the economic system in a disaggregated manner.
2. When the capacities of plants for producing gas and electricity in joules and the final demand of commodities in dollars are given, the total electricity from different fuels can be calculated.
3. Similarly gas production from different sources is computed.
4. In addition GDP and the gross production of non-energy commodities (in $) are computed.
5. The model needs large amount of data.

5.8 NUMERICAL RESULTS

The model has been run using small aggregation Canadian Input-output data in rectangular format for 1971 [47]. The efficiencies of conversion of various energy forms into electric energy and gas have been taken within the domain of practical values.

Figures (5.2) and (5.3) show the demand for electric energy and gas for an increasing final demand of commodities. They also show the quantites of electric energy and gas production from different sources. To obtain feasible solutions, the capacities of industries have been increased correspondingly.

The following average values of the efficiencies of conversion have been considered for Canadian utilities in 1971.

$$\eta_{6.1} = 33.00\ \%$$

$$\eta_{8.2} = 28.57\ \%$$

$$\eta_{7.3} = 28.00\ \%$$

$$\eta_{6.8} = 86.95\ \%$$

$$\eta_{7.8} = 86.95\ \%$$

The following prices of energy commodities have been assumed.

$$p_1 = 0.2 \quad (\$/GJ)$$

$$p_2 = 0.7 \quad (\$/GJ)$$

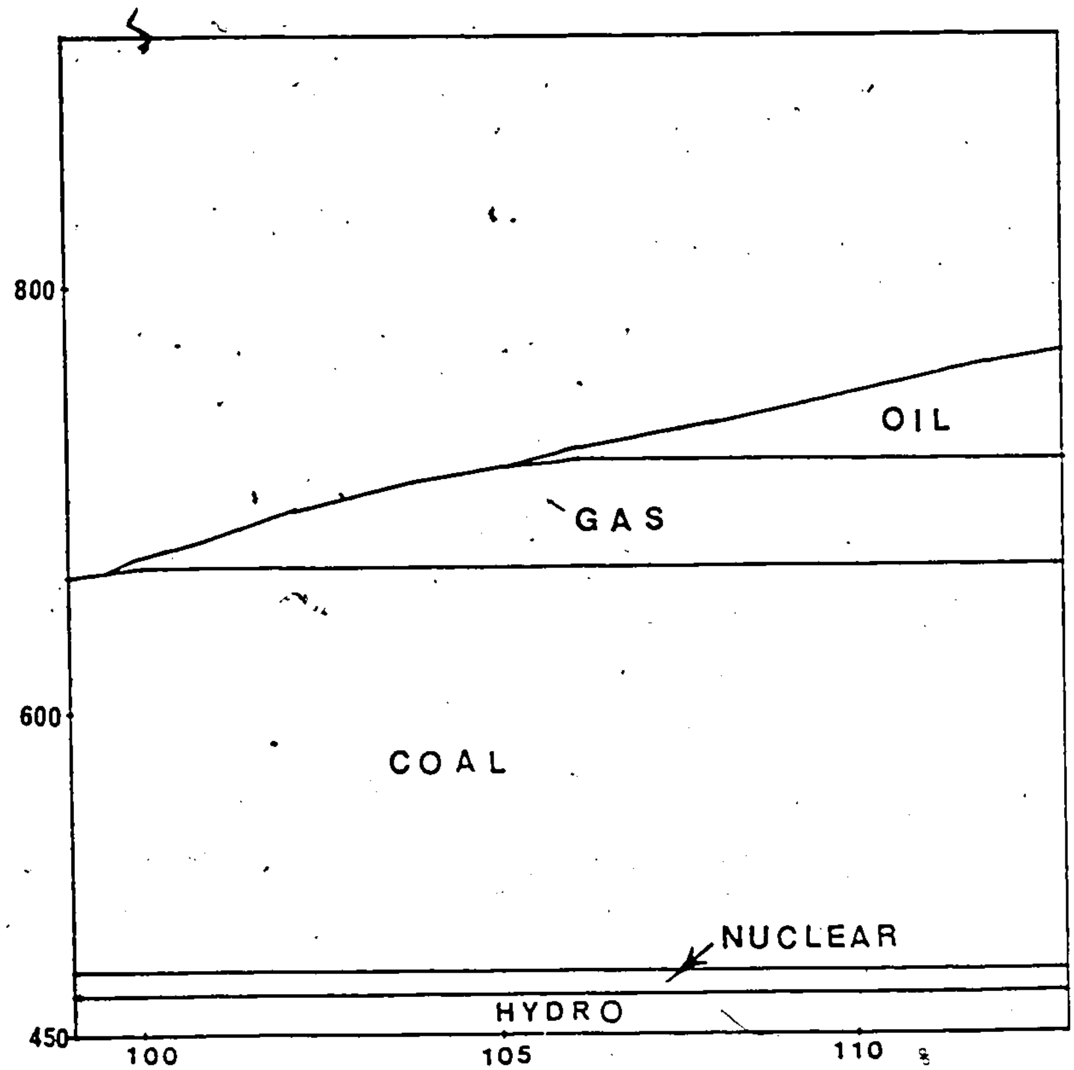

Fig. 5.2 Electric energy production from different fuels for a variable final demand. (Energy units in 10^{15} Joules). (For 1971 data).

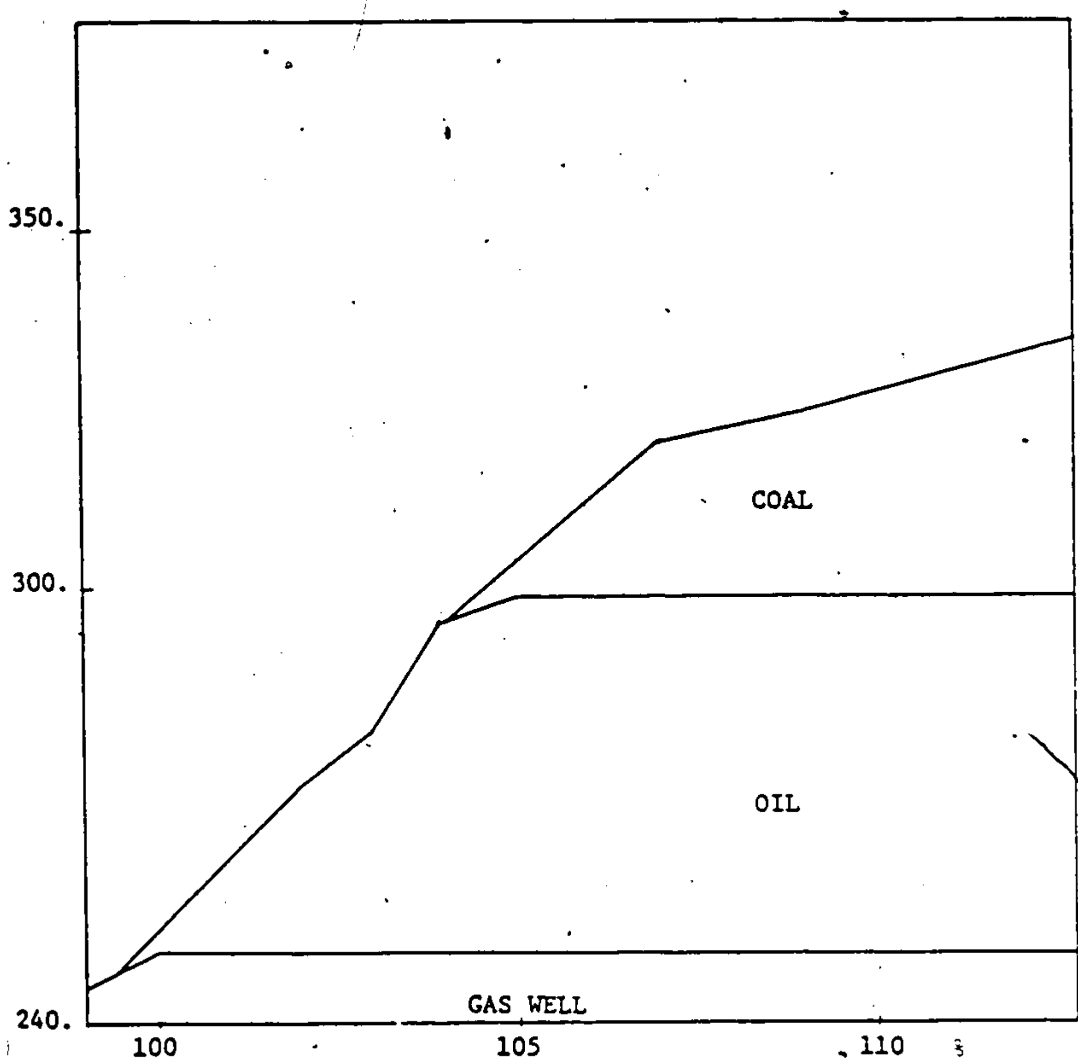

Fig. 5.3 Gas production from different sources for a variable final demand. (Energy units in 10^{15} Joules). (For 1971 data).

$p_3 = 0.4$ (\$/GJ)

$p_4 = 3.0$ (\$/GJ)

The values of electric energy produced from hydro and nuclear processes have been taken approximately equal to the production values in 1971 [46].

Discrete Time Model

$q(t)$ is a vector of values of total commodity outputs ($) in time period 't' ($)

$X(t)$ is a vector of capacities for producing commodities in time period 't' ($)

$M_j(t)$ is the import of commodity j in time period 't' ($)

$EX_j(t)$ is the export of commodity j in time period 't' ($)

$IN_i(t)$ is the initial inventory of commodity i in time period 't' ($)

$FN_j(t)$ is the final inventory of commodity j in time period 't' ($)

$L_{1,j}$ is the limit on final inventory of commodity j ($)

$L_{2,j}$ is the limit on the export of commodity j ($)

p_j is the cost of importing commodity j ($/$)

k_j is the share of GDP of commodity j ($/$)

f is the average opportunity cost of capital ($/$)

Continuous Time Model

$x_j(t)$ is the production of commodity j. ($)

$\dot{x}_j(t)$ is the rate of change of $x_j(t)$.

$m_j(t)$ is the import of commodity j ($)

$f_j(t)$ is the final demand of commodity j ($)

R is a matrix which is equal to $(CD)^{-1}(I-BD)$

V is a matrix which is equal to $(CD)^{-1}$

a, b, s are known constants

T is the time horizon for the dynamic model

NOMENCLATURE FOR CHAPTER VI

Chapter VI

OPTIMAL DYNAMIC INPUT-OUTPUT MODELS FOR ENERGY PLANNING

6.1 INTRODUCTION

The models included in chapters IV and V are based on the theory of static input-output analysis. Static analysis does not take into account the dynamic nature of the economic systems and the results obtained are applicable only to the particular time period, for which the model has been run. For example, using the model in chapter V, one can find the optimal values of electric energy and gas production starting from various fuels for a particular year. Such models can be applied to short-term and medium-term future.

The model included in this chapter is based on dynamic input-output analysis, which recognizes the dynamic character of economic systems wherein production, investment and demand for commodities are functions of time. Using this model, the optimal growth pattern of an economic system can be computed.

6.2 DYNAMIC INPUT-OUTPUT THEORY

For square input-output tables, this theory was developed by Leontief [28]. Here inter-sectorial dependence for intermediate inputs as well as for investment requirements are taken into consideration.

Dynamic input-output analysis is similar to static analysis, except that a new set of coefficients, to take into account the capital requirements are defined. Capital coefficients describe the amount of commodities a sector must hold in order to maintain its capacity at full utilisation per unit of its capacity.

The element of capital coefficient matrix C (n by n for square input-output analysis with n sectors) is defined as [28]

> c_{ji} is the amount of commodity j which sector i must hold in order to produce unit output of commodity i ($/$) when the sector i is operating at full capacity.

For a sector i, the equation for dynamic input-output analysis in a differential equation form can be written as [28]

$$x_i(t) - \sum_{j=1}^{n} a_{ij} \; x_j(t) - \sum_{j=1}^{n} c_{ij} \; \dot{x}_j(t) = f_i(t) \qquad (6.1)$$

where

$x_i(t)$ is the production of commodity i at time 't'

$\dot{x}_i(t)$ is the rate of change of production of commodity i at time 't'. Here time lag for invesment is taken zero.

$f_i(t)$ is the final demand of commodity i at time 't' and does not include capital expenditure.

a_{ij} is the element of input-output coefficients of matrix A

c_{ij} is the element of capital coefficients matrix C

There would be one equation for each sector and hence a system of n differential equations. If the time path of final demand $f_i(t)$ is knowm for all commodities the set of equations can be solved for the gross outputs of commodities at any point of time, provided the levels of output at the initial time are given.

The same system could be represented in difference equation form [28].

$$x_i(t)-\sum_{j=1}^{n} a_{ij} x_j(t)-\sum_{j=1}^{n} c_{ij}\{x_j(t+1)-x_j(t)\}= f_i(t) \quad (6.2)$$

for i = 1 to n

assuming the invesment lag to be one time period. In a manner similar to the differential equation form (6.1) when f (t) and initial values of $x_i(t)$ are given for all commodities, the set of interconnected equations (6.2) can be solved for $x_i(t+1)$.

The introduction of capital coefficients matrix 'C' in the analysis signifies the need for both current inputs (as raw materials) and capital inputs for a sector to produce it's output.

6.3 DYNAMIC RECTANGULAR INPUT-OUTPUT ANALYSIS

In dynamic rectangular input-output analysis, similar to the dynamic square input-output analysis, a matrix C (n by N) is defined. An element of matrix C is

> c_{ji} is the amount of commodity j, which industry i must hold to produce unit output, operating at full capacity.

Using the same notation as in static rectangular input-output analysis (section 2.10) equations in differential form would be

$$q_i(t)-\sum_{j=1}^{n}(BD)_{ij}\,q_j(t)-\sum_{j=1}^{n}(CD)_{ij}\,\dot{q}_j(t)=e_i(t) \qquad (6.3)$$

for i = 1 to n

and in difference equation form as

$$q_i(t)-\sum_{j=1}^{n}(BD)_{ij}\,q_j(t)-\sum_{j=1}^{n}(CD)_{ij}\{q_j(t+1)-q_j(t)\}=e_i(t)$$

for i = 1 to n (6.4)

The solutions to the above systems of equations can be obtained as in dynamic square input-output analysis.

6.4 DISCUSSION

1. Dynamic input-output analysis takes into account the capital requirements and generates a time path of gross production for a given final demand of commodities.
2. The analyses can be extended to cases, where investment is more than one time period in discrete time form (6.2 and 6.4).
3. The analysis can be transformed into optimisation problems, minimising or maximising certain variables over a given time period.
4. The above analyses are open models of economic systems. The final demands are independently determined. These types of models cannot explain recessions and trade cycles.
5. The analyses assume that all industries operate at full capacities. This assumption is an over simplification and justifies the linear programming approach to incorporate capacity constraints and unused capacities.

6.5 DYNAMIC DISCRETE LINEAR PROGRAMMING MODEL

In this section an optimising dynamic model in linear programming format [36] is developed, incorporating production, capacities, imports and inventories as variables. In the model, constraints on dis-investment are also introduced, which could be used to find the unused capacities as well.

Using the model, the effect of limited availability of primary energy resources or any other scarce natural resource on the growth of the economy could be studied.

The model of the economic system is shown in Figure 6.1. Referring to the same figure

V_{ij} is the output of commodity i by industry j (\$)

U_{ji} is the current input of commodity j to industry i (\$)

W_{ji} is the capital input of commodity j to industry i (\$)

M_k is the import of commodity k (\$)

Y_j is the primary input to industry j (\$)

N is the number of industries

n is the number of commodities

The model is formulated in linear programming format here

$$\text{Max} \sum_{j=1}^{n} \sum_{t=1}^{T} \{k_j q_j(t) - p_j M_j(t) - f Z_j(t) - f[FN_j(t) + IN_j(t)]/2\} \tag{6.5}$$

subject to

$$\sum_{j=1}^{n} (I-BD)_{\ell j}\, q_j(t) - \sum_{j=1}^{n} (CD)_{\ell j}\, [X_j(t+1) - X_j(t)] - \sum_{j=1}^{n} \alpha_j (CD)_{\ell j}\, X_j(t) + M_\ell(t) - EX_\ell(t) - FN_\ell(t) + IN_\ell(t) = e_\ell(t)$$

$$\text{for } \ell = 1 \text{ to } n \text{ and } t = 1 \text{ to } T \tag{6.6}$$

$$FN_j(t) = IN_j(t+1) \tag{6.7}$$

$$Z_j(t)=X_j(t)-q_j(t) \qquad \text{(unused capacity)} \qquad (6.8)$$

$$X_j(t+1)-X_j(t) \geq 0 \qquad \text{(disinvestment)} \qquad (6.9)$$

$$\sum_{t=1}^{T} \sum_{j=1}^{n} EX_j(t) \geq \sum_{t=1}^{T} \sum_{j=1}^{n} M_j(t) \qquad \text{(Trade balance)} \qquad (6.10)$$

$$FN_j(t) \leq L_{1,j} \qquad (6.11)$$

$$EX_j(t) \leq L_{2,j} \qquad (6.12)$$

where

$q_j(t)$ is the production of commodity j in time period t ($)

$X_i(t)$ is the capacity for producing commodity i in time period t ($)

$M_j(t)$ is the import of commodity j in time period t ($)

$EX_j(t)$ is the export of commodity j in time period t ($)

$IN_i(t)$ is the beginning inventory of commodity i in time period t ($)

$FN_j(t)$ is the final inventory of commodity j in time period t ($)

$L_{i,j}$ is the limit on the final inventory of commodity j

$L_{2,j}$ is the limit on the export of commodity j.

p_j is the cost of importing commodity j ($/$)

k_j is the share of GDP of commodity j ($/$)

α_j is a factor for real depreciation ($/$) per year

f is the average opportunity cost of capital ($/$)

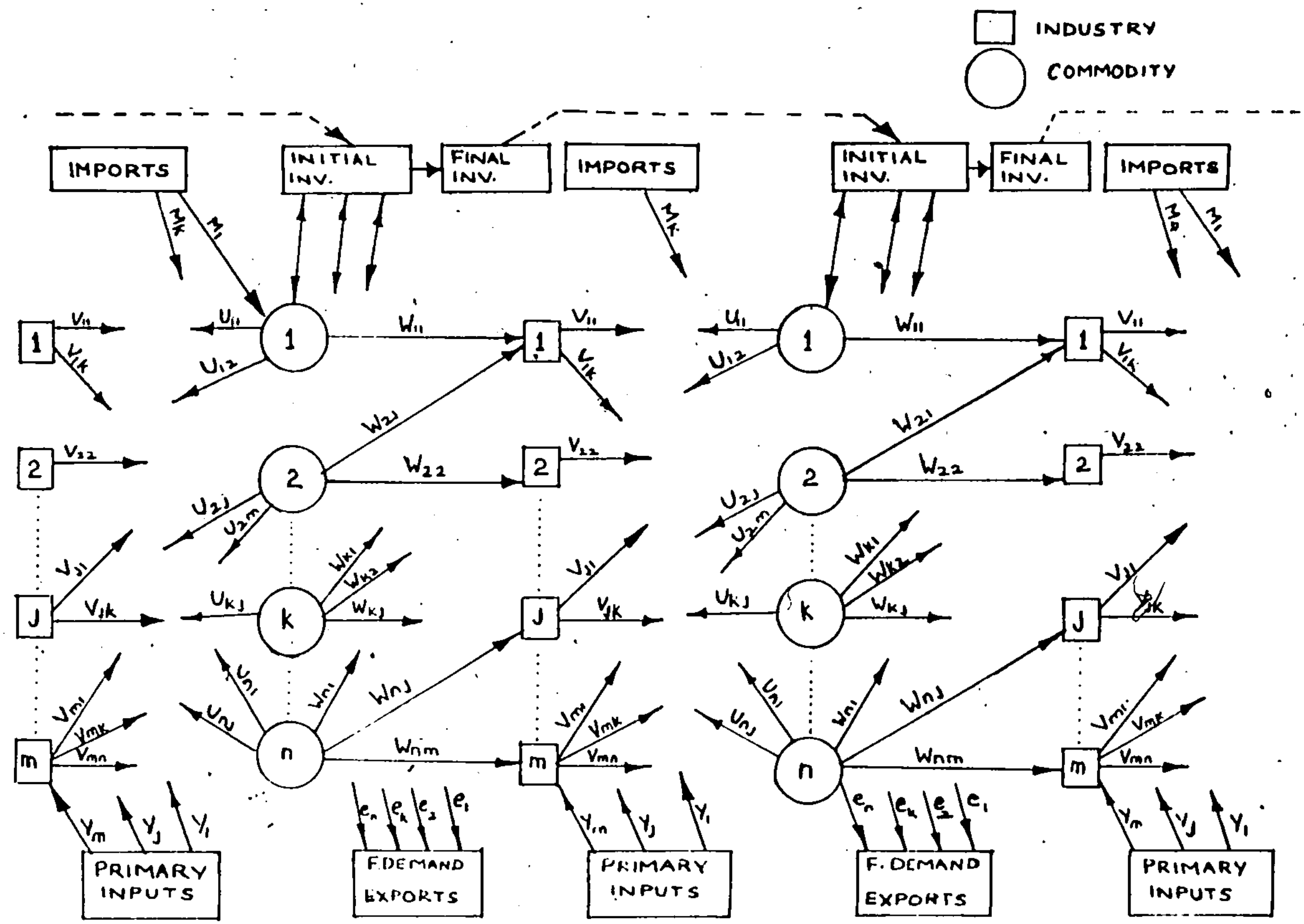

Fig. 6.1 Dynamic model of economic system

6.6 MODEL CHARACTERISTICS

The inputs to the model are final demand, for all time periods, limits on exports, initial and final inventory levels and capacities for the first time period. The final values of capacities in T th period can also be specified. The model determines production, capacities, imports, exports and inventories maximising GDP. The industry capacities G and their utilisation H can be related to the commodity capacities and utilisation.

$$G = D X \quad (6.13)$$

$$H = D q \quad (6.14)$$

In this model one time period has been taken as the investment lag. The inputs and outputs are expressed in constant dollars.

The growth of a sector in this model is determined by the availability of natural resources. The availability could be limited domestically or from foreign sources. The effect of limited availability of one commodity were studied by introducing limits on the production and import of that commodity.

Changes in technology can be introduced in the model by the way of time dependent current and capital coefficients.

The advantages of this model are

1. This model distinguishes between production, capacity and unused capacity.
2. It attaches a penalty to unused capacities which assures the optimal pattern of investment over time.
3. Inventories are also given penalties so as to take into consideration the inventory carrying costs.

In this formulation of the model, no discounting factor is used for monetary values of production and other variables. Hence the data should be in constant dollars. Inventories are given limits in the model to represent storage capacities.

6.7 MONTE-CARLO MODELLING APPROACH

The model is run with illustrative data, with two industries producing three commodities for five time periods as the actual capital coefficients data is not available for Canada. Taking the first commodity to represent the energy resource, the model is run to simulate the uncertainity of energy resource availability, both within the country and from foreign sources. The uncertainity is assumed to be normally distributed with known means and variances for all the time periods. In the model run the values of the variances are taken higher for energy uncertainity for more distant future. The results of a sample run are shown in Figures (6.2)-(6.4).

The following data has been assumed.

$$U = \begin{bmatrix} 10 & 15 \\ 20 & 10 \\ 10 & 25 \end{bmatrix} \qquad e = \begin{bmatrix} 30 \\ 20 \\ 25 \end{bmatrix} \qquad q = \begin{bmatrix} 55 \\ 50 \\ 60 \end{bmatrix}$$

$$V = \begin{bmatrix} 35 & 25 & 20 \\ 20 & 25 & 40 \end{bmatrix} \qquad g = \begin{bmatrix} 80 \\ 85 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.08 & 0.16 \\ 0.16 & 0.08 \\ 0.08 & 0.24 \end{bmatrix} \qquad B = \begin{bmatrix} 0.125 & 0.176 \\ 0.125 & 0.118 \\ 0.125 & 0.294 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.636 & 0.50 & 0.333 \\ 0.364 & 0.50 & 0.667 \end{bmatrix}$$

$$BD = \begin{bmatrix} 0.144 & 0.151 & 0.159 \\ 0.202 & 0.184 & 0.162 \\ 0.187 & 0.210 & 0.238 \end{bmatrix} \qquad CD = \begin{bmatrix} 0.109 & 0.120 & 0.134 \\ 0.131 & 0.120 & 0.106 \\ 0.139 & 0.160 & 0.187 \end{bmatrix}$$

$$\alpha = \begin{bmatrix} 0.25 & 0.25 & 0.25 \end{bmatrix}.$$

$$e(1) = \begin{bmatrix} 40 \\ 40 \\ 40 \end{bmatrix} \quad e(2) = \begin{bmatrix} 35 \\ 35 \\ 77 \end{bmatrix} \quad e(3) = \begin{bmatrix} 60 \\ 60 \\ 60 \end{bmatrix} \quad e(4) = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} \quad e(5) = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix}$$

$$EX(1), EX(2), EX(3), EX(4), EX(5) \leqslant \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

$f = 0.5$ $\qquad$ $X_1(6) \leqslant 160.$

$$X(1) = \begin{bmatrix} 110 \\ 60 \\ 40 \end{bmatrix}, \quad FN(1), FN(2), FN(3), FN(4), FN(5) \leqslant \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

$$k_j = \begin{bmatrix} 1.0 & 1.0 & 1.0 \end{bmatrix} \quad \text{for all 't'}$$

If mean and standard deviation are denoted by QN and QD respectively for production and MN and MD for import respectively, their assumed values are

$QN_1(1)=110.00$	$QD_1(1)=6.00$
$QN_1(2)=116.00$	$QD_1(2)=9.50$
$QN_1(3)=122.00$	$QD_1(3)=14.50$
$QN_1(4)=129.00$	$QD_1(4)=19.00$
$QN_1(5)=150.00$	$QD_1(5)=27.00$

$MN_1(1)=10.00$	$MD_1(1)=0.50$
$MN_1(2)=10.00$	$MD_1(2)=1.50$
$MN_1(3)=10.00$	$MD_1(3)=2.00$
$MN_1(4)=10.00$	$MD_1(4)=2.50$
$MN_1(5)=10.00$	$MD_1(5)=3.00$

For the purpose of simulation, normalized random variables (0,1) were generated. If Z is the outcome of the random variable corresponding to the energy commodity availability within the country, in the first year, constraints were imposed for its production as

$$q_1(1) \leq QN_1(1) + Z \cdot QD_1(1)$$

For each run of the model there were ten values (five for the production and five for import of energy commodity) which depended on the outcome of the corresponding random variables.

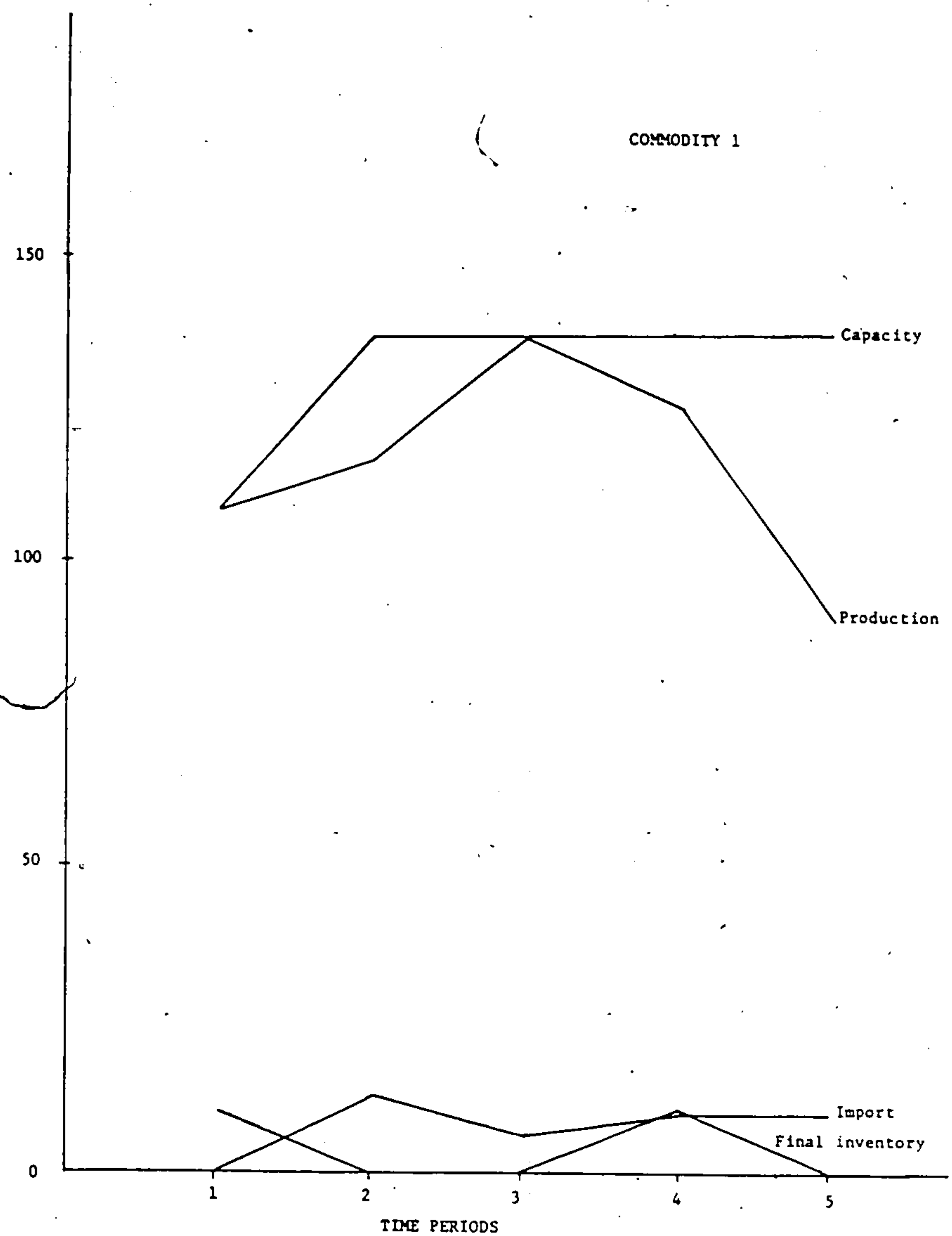

Fig. 6.2 Time path of variables for the first commodity.

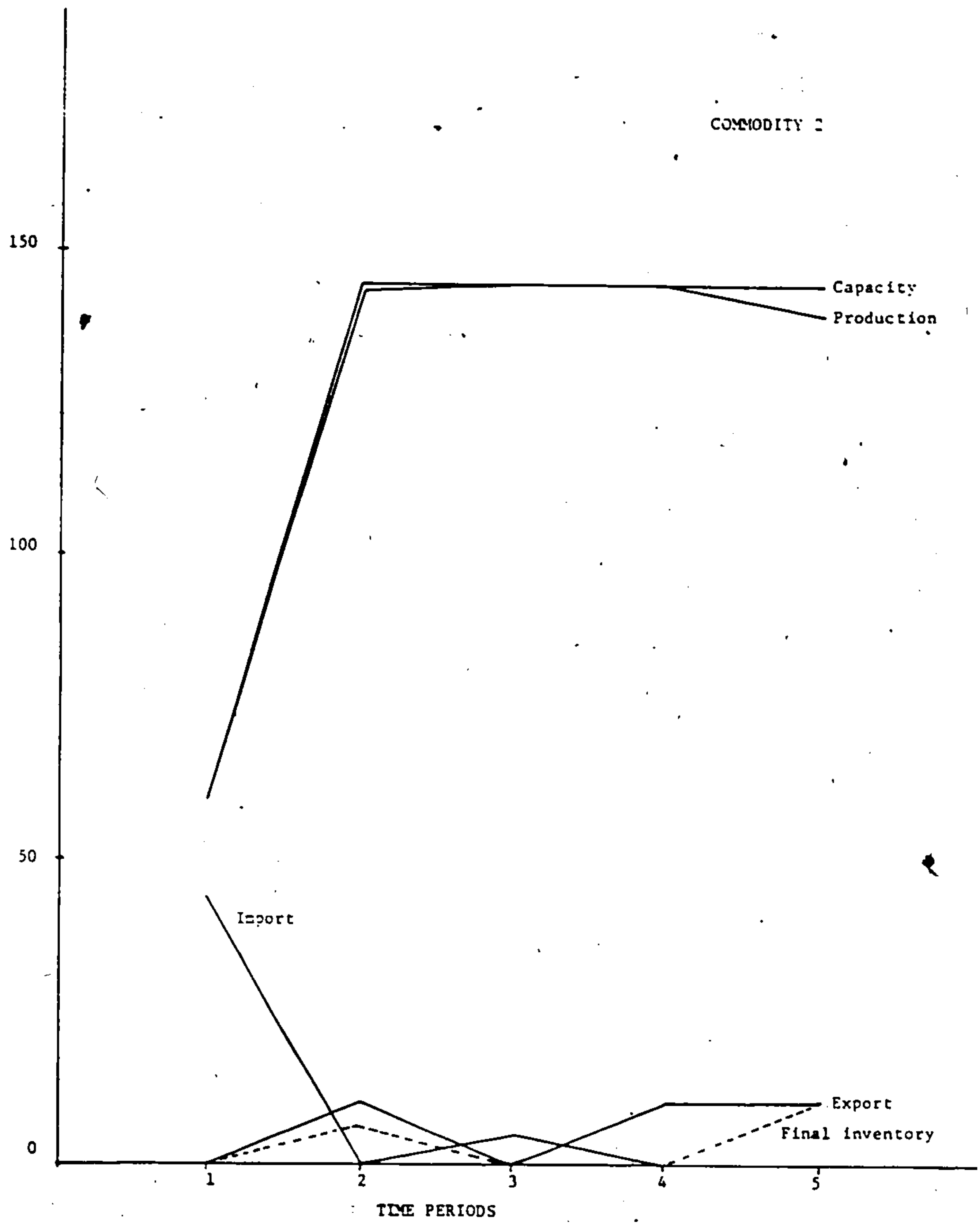

Fig. 6.3 Time path of variables for the second commodity.

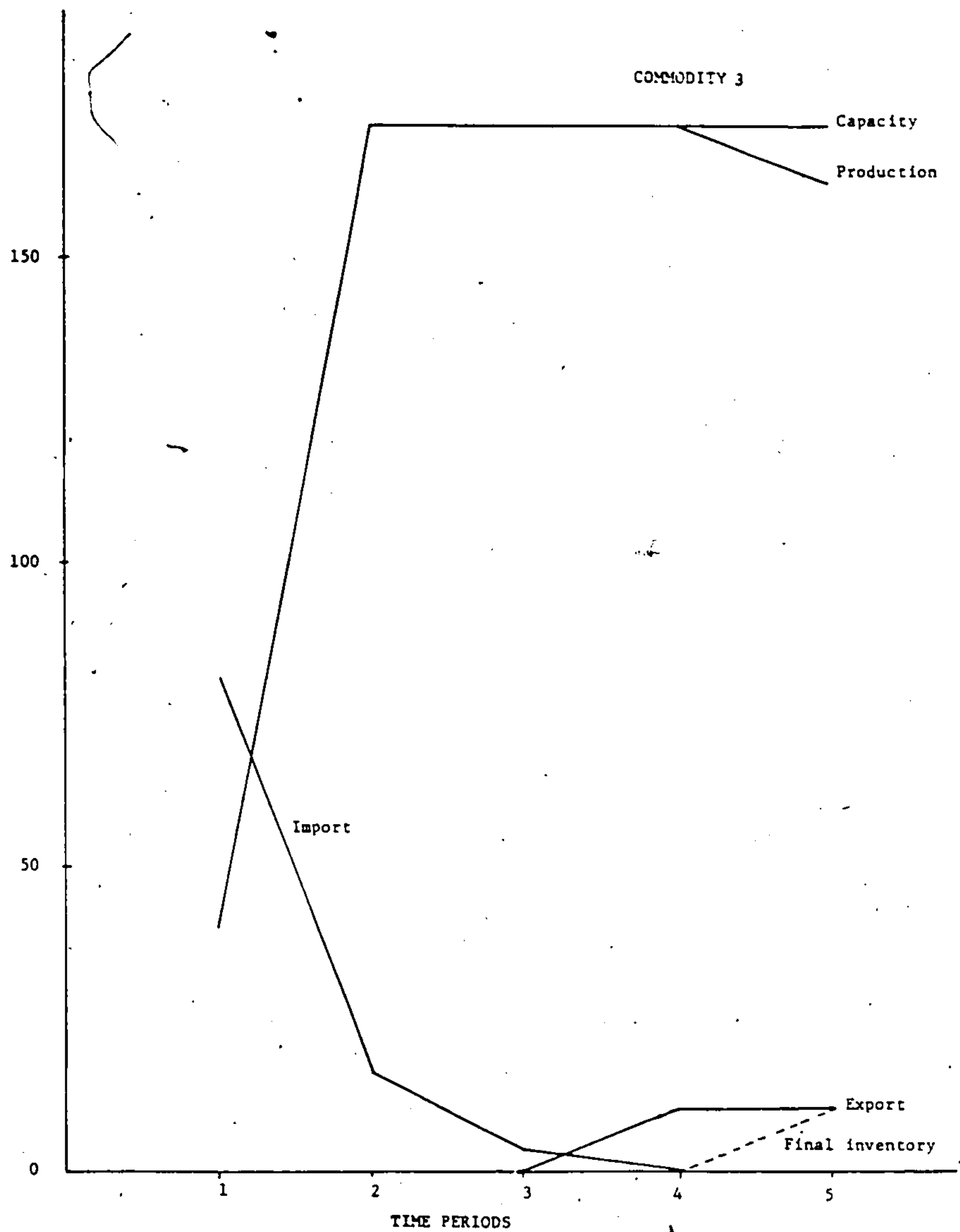

Fig. 6.4 Time path of variables for the third commodity.

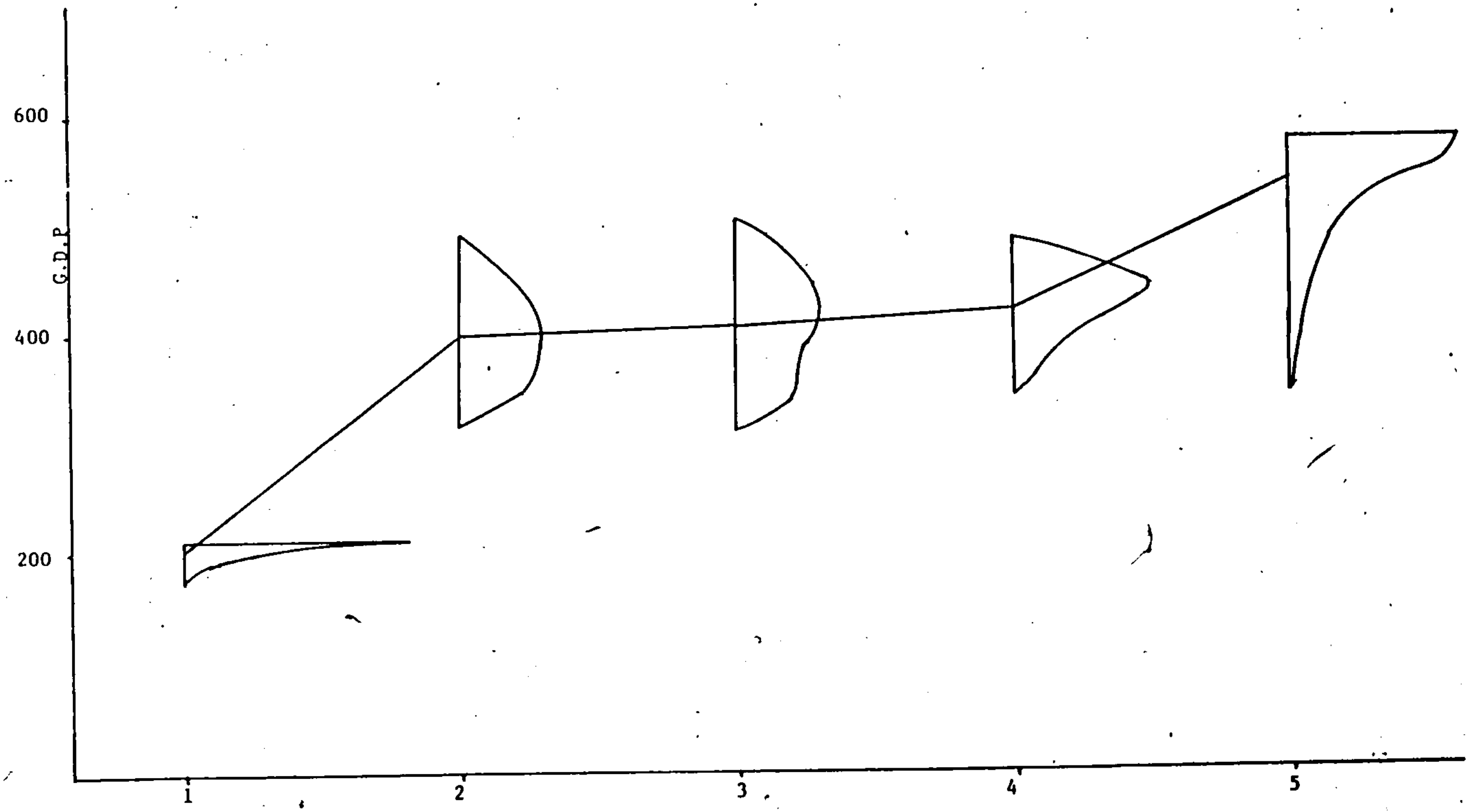

Fig. 6.5 Results of simulation study (the mean G.D.P are joined by straight lines).

The model was run for fifty times. The resulting mean G.D.P for each time period and the distribution of G.D.P are shown in Fig. 6.5. It was found from the study that the distributions of G.D.P depend upon the availability of capacities for commodities production.

6.8 DYNAMIC MODEL IN CONTINUOUS TIME

The model included in section 6.5 is a discrete time model. Optimising models can be formulated in continuous time [4,8,44,51]. The advantage of continuous time models is that they can be used to find the best possible (ideal) solutions. A solution to a discrete time model will converge to the solution of the continuous time model of the same system, as the time intervals are made smaller. The optimal solution for continuous time models will be in the form of continuous functions (time paths) of time and it would be a better solution than the solution of a discrete model in terms of optimality.

Solutions to continuous time models can be obtained by applying the theory of optimal controls. Optimal control theory is concerned with the search for the control which attains the desired objective while minimizing (or maximizing) a defined system criterion [43]. The problem of optimization theory may be subdivided into four interrelated parts [43].

(i) Definition of goal.

(ii) Knowledge of the current position with respect to the goal.

(iii) Knowledge of all environmental factors influencing the past, present and future.

(iv) Determination of the best policy from the goal definition (i) and knowledge of the current state (ii) and environment (iii).

The formulation of the model for a two commodity economic system is given below.

$$\text{Min } J = \int_0^T (a_1 e^{b_1 m_1} + a_2 e^{b_2 m_2}) \, dt \qquad (6.15)$$

subject to

$$\dot{x}_1 = R_{11} x_1 + R_{12} x_2 + V_{11} m_1 + V_{12} m_2 - V_{11} f_1 - V_{12} f_2 \qquad (6.16)$$

$$\dot{x}_2 = R_{21} x_1 + R_{22} x_2 + V_{21} m_1 + V_{22} m_2 - V_{21} f_1 - V_{22} f_2 \qquad (6.17)$$

$$x_1 \geq 0 \ ; \ x_2 \geq 0 \qquad (6.18)$$

$$0 \leq m_1 \leq s_1 \ ; \ 0 \leq m_2 \leq s_2 \qquad (6.19)$$

with $f_1(t)$ and $f_2(t)$ being the known functions, with the given initial conditions $x_1(0)$ and $x_2(0)$.

In the above formulation

$x_j(t)$ is the production of commodity j. Its rate of change is denoted by $\dot{x}_j(t)$.

$m_j(t)$ is the import of commodity j.

$f_j(t)$ is the final demand of commodity j.

R_{ij} is the general element of the matrix $(CD)^{-1}[I-BD]$, where C, B and D are the matrices as defined in the previous model.

V_{ij} is the general element of the matrix $(CD)^{-1}$.

a_1, a_2, b_1, b_2, s_1 and s_2 are known constants.

T is the time horizon for the optimization problem.

In the model the objective is to minimize the utility (in this case penalty) of imports, for the time period. Various forms of utility functions exist and the function chosen is an acceptable one [39]. The constants in the utility function are chosen such that the penalties inrease exponentially with import levels.

The problem is to find an optimal level of imports at every instant of time in order to minimize the objective function. Since imports are the decision variables at the discretion of the decision maker, they are the control variables to guide the system to the desired objective.

For the above problem to be solved, it has to be put in in a more convient form to include the constraints (6.16)-(6.19). They can be included by introducing a new variable defined as [43]

$$\dot{x}_3 = [(s_1 - m_1)^2 m_1^2 H_1 + (s_2 - m_2)^2 m_2^2 H_2 + x_1^2 H_3 + x_2^2 H_4] \quad (6.20)$$

H_1, H_2, H_3 and H_4 are step functions with the following conditions.

$$H_1 = 0 \quad \text{if } (s_1 - m_1)m_1 \geq 0$$
$$= k \quad (s_1 - m_1)m_1 < 0$$

$$H_2 = 0 \quad \text{if } (s_2 - m_2)m_2 \geq 0$$
$$= k \quad (s_2 - m_2)m_2 < 0$$

$$H_3 = 0 \quad \text{if } x_1 \geq 0$$
$$= k \quad x_1 < 0$$

$$H_4 = 0 \quad \text{if } x_2 \geq 0$$
$$= k \quad x_2 < 0$$

and

$x_3(0) = 0$; the terminal time requirement is $x_3(T) = 0$.

k is a positive penalty constant.

The above additional unknown function $x_3(t)$ is added to the objective function, Then the problem can be formulated as

$$\text{Min } J = \int_0^T (a_1 e^{b_1 m_1} + a_1 e^{b_2 m_2})dt + x_3(T) \qquad (6.21)$$

subject to

$\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$ as defined previously with the given conditions

$x_1(0)$, $x_2(0)$, $x_3(0)$ and $x_3(T)$.

At this stage the maximum principle (also described in [43]) for continuous time can be applied.

The Hamiltonian for the reformulated problem is

$$\begin{aligned} H = {} & a_1 e^{b_1 m_1} + a_2 e^{b_2 m_2} + \lambda_1 (R_{11} x_1 + R_{12} x_2 + V_{11} m_1 + V_{12} m_2 \\ & - V_{11} f_1 - V_{12} f_2) + \lambda_2 (R_{21} x_1 + R_{22} x_2 + V_{21} m_1 + V_{22} m_2 \\ & - V_{21} f_1 - V_{22} f_2) + \lambda_3 [H_1 (s_1 - m_1)^2 m_1^2 + H_2 (s_2 - m_2)^2 m_2^2 + H_3 x_1^2 \\ & + H_4 x_2^2] \end{aligned} \qquad (6.22)$$

where $\lambda(t)$ are the cannonical variables.

Using the maximum principle one obtains (H with a subscript variable denotes partial differentiation of Hamiltonian with respect to that variable).

$$H_{x_1} = \lambda_1 R_{11} + \lambda_2 R_{21} + 2\lambda_3 x_1 H_3 = -\dot{\lambda}_1 \qquad (6.23)$$

$$H_{x_2} = \lambda_1 R_{12} + \lambda_2 R_{22} + 2\lambda_3 x_2 H_4 = -\dot{\lambda}_2 \qquad (6.24)$$

$$H_{x_3} = 0 = -\dot{\lambda}_3 \qquad (6.25)$$

With the given conditions for λ's as $\lambda_1(\tau)=0$ and $\lambda_2(\tau)=0$ for free end point conditions of $x_1(t)$ and $x_2(t)$.

Differentiating the Hamiltonian partially with respect to m_1 and m_2

$$H_{m_1} = a_1 b_1 e^{b_1 m_1} + v_{11}\lambda_1 + V_{21}\lambda_2 + \lambda_3 H_1 (2m_1 s_1^2 + 4m_1^3 - 6m_1^2 s_1) = 0 \qquad (6.26)$$

and

$$H_{m_2} = a_2 b_2 e^{b_2 m_2}\ v_{12}\lambda_1 + V_{22}\lambda_2 + \lambda_3 H_2 (2m_2 s_2^2 + 4m_2^3 - 6m_2^2 s_2) = 0 \qquad (6.27)$$

The expressions for optimal values of imports m_1 and m_2 can be found from (6.26) and (6.27), in terms of a_1, a_2, b_1, b_2, s_1, s_2, H_1 and H_2 & λ. The expressions for m_1 and m_2 have to be substituted in (6.16), (6.17) and (6.20).

At this stage one is left with six differential equations (6.16), (6.17) & (6.20) and (6.23)-(6.25), having six boundary conditions. The boundary conditions are

$$x_1(0)=x_{10} \quad ; \quad x_2(0)=x_{20}$$

$$x_3(0)=0 \quad ; \quad x_3(T)=0$$

$$\lambda_1(T)=0 \quad ; \quad \lambda_2(T)=0$$

These equations have to be solved simultaneously. Observing the boundary conditions, it may be seen that they are complex. The system of differential equations cannot be solved in a straightforward way. Equation (6.25) does not have any conditions at all and equations (6.16)-(6.17) have the initial conditions, whereas (6.23)-(6.24) have terminal conditions.

The system can be solved numerically using an iterative technique known as 'the method of adjoints' [41]. Adjoint equations are explained as [41] 'with every set of linear ordinary differential equations is associated a companion set of equations called the adjoint equations, whose matrix of coefficients is the negative transpose of the matrix of the original set of differential equations'. The method attempts to convert all the terminal conditions into initial conditions using iterative technique. Once the initial conditions for all the equations are known, the system can be solved using any available numerical technique for solving ordinary differential equations. However the solution is not assured and depends upon the convergence properties.

Some comments about continuous time models.

In general it is difficult to obtain solutions to continuous time models whenever there are many constraints. When the model is complex it is difficult to obtain even numerical solutions.

Continuous time models give an insight into the nature of economic systems to understand the complex interactions. It may be noted that in the model the investment lag (the time between investment and the beginning of production) is nil which is unrealistic.

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APPENDIX A

Linear algebraic input-output model in dollar relationships.

Consider an economic system having N industries and n commodities.

A set of balance equations are given for the input-output dollar transactions as

$$q_i = \sum_{j=1}^{N} U_{ij} + e_i \quad \text{for } i = 1 \text{ to } n \tag{A.1}$$

$$\sum_{i=1}^{n} U_{ij} + Y_j = G_j \quad \text{for } j = 1 \text{ to } N \tag{A.2}$$

$$\sum_{j=i}^{N} V_{jk} = q_k \tag{A.3}$$

$$\sum_{k=1}^{n} V_{jk} = G_j \tag{A.4}$$

or in matrix notation

$$q = U L + e \tag{A.5}$$

$$J U + Y = G \tag{A.6}$$

$$V L + Y = G \tag{A.7}$$

$$J V = q \tag{A.8}$$

where

L and J are unit column and row vectors respectively. Other notation is the same as earlier (Table 2.2)

Input-output models have in common two general assumptions.

(i) Linearity

(ii) Time invariance

Besides this, the rectangular model has market-share and industry technology assumptions. With the above assumptions one can derive

$$q = (I-BD)^{-1} e \qquad (A.9)$$

$$G = (I-DB)^{-1} D e \qquad (A.10)$$

In the case of square input-output model, there is one to one mapping between industries and commodities, i.e, each industry produces only one commodity and that each commodity is produced by only one industry. This results in

$$V_{jk} \neq 0 \quad \text{for } j=k=1,2,\ldots..N$$

$$(A.11)$$

$$V_{jk} = 0 \quad \text{otherwise}$$

and $N=m$

$$q_j = G_j \quad \text{for} \quad j=1,2,\ldots..,N \qquad (A.12)$$

$$D_{jk} = 1 \quad \text{for } j=k=1,2,\ldots.,N$$

$$\bar{D}_{jk} = 0 \quad \text{for } j=k \tag{A.13}$$

The matrix D becomes a non-informative identity matrix so that equations (A.9) and (A.10) are reduced to a single equation.

$$q = (I-B)^{-1}e \tag{A.14}$$

resulting in the original Leontief input-output model. Matrix B is the industry input coefficients matrix, which was denoted as A in (2.7). Vectors q and e are the vectors of gross output and final demand of commodities.

APPENDIX B

A numerical example is given for an economy with two industries producing three commodities. Let the first commodity be the energy commodity. The numerical values are

$$U = \begin{bmatrix} \begin{matrix} 2 \\ (5) \end{matrix} & \begin{matrix} 1 \\ (20) \end{matrix} \\ 3 & 2 \\ 1 & 5 \end{bmatrix} ; \quad e = \begin{bmatrix} \begin{matrix} 3 \\ (15) \end{matrix} \\ 4 \\ 2 \end{bmatrix} ; \quad q = \begin{bmatrix} \begin{matrix} 6 \\ (40) \end{matrix} \\ 9 \\ 8 \end{bmatrix}$$

$$V = \begin{bmatrix} 2 & 6 & 1 \\ 4 & 3 & 7 \end{bmatrix} ; \quad G = \begin{bmatrix} 9 \\ 14 \end{bmatrix}$$

All the values are in dollars except for the figures in brackets, which are in joules. Using (3.24) and (3.28) one can obtain

$$(I-\widehat{D}\widehat{B})^{-1} = \begin{bmatrix} 1.6025 & 0.6810 \\ 0.8506 & 2.3732 \end{bmatrix}$$

Y_1^* and Y_2^* are assumed to be proportional to their dollar values so that

$Y_1^* = 13.333$

$Y_2^* = 26.666$

$$S\,(I-\widehat{D}\widehat{B})^{-1} = \begin{bmatrix} 2.3741 & 1.009 \\ 1.6201 & 4.5203 \end{bmatrix}$$

The column sum of this matrix gives the energy intensity coeficients.

$\mathcal{E} = \{3.9942 \quad 5.5293\}$

The coefficients for commodities are

$\mathcal{E}\,\widehat{D} = \widehat{\alpha} = \{0.7526 \quad 4.5054 \quad 5.5293\}$

For the final demand

$$\hat{e} = \begin{bmatrix} (15) \\ 4 \\ 2 \end{bmatrix}$$

Total energy would be

$$T = \hat{a}\ \hat{e} = 40 \text{ joules}$$

which is equal to $Y_1^* + Y_2^*$

APPENDIX C

Consider an economy with two industries producing three commodities. Let the first commodity be the energy commodity. Energy balance equation can be written for each industry as

$$\alpha_1^* U_{11}^* + \alpha_2 U_{21} + \alpha_3 U_{31} + Y_1^* = \varepsilon_1 G_1 \qquad \text{(C1)}$$

$$\alpha_1^* U_{12}^* + \alpha_2 U_{22} + \alpha_3 U_{32} + Y_2^* = \varepsilon_2 G_2 \qquad \text{(C2)}$$

Summing (C1) and (C2) yields

$$\alpha_1^* (U_{11}^* + U_{12}^*) + \alpha_2 (U_{21} + U_{22}) + \alpha_3 (U_{31} + U_{32}) + Y_1^* + Y_2^* = \varepsilon_1 G_1 + \varepsilon_2 G_2 \qquad \text{(C3)}$$

Since $\sum_{j=1}^{N} U_{ij} + e_i = q_i$ we can write

$$U_{11}^* + U_{12}^* = q_1^* - e_1^*$$

$$U_{21} + U_{22} = q_2 - e_2 \qquad \text{(C4)}$$

$$U_{31} + U_{32} = q_3 - e_3$$

Equation (C3) can be written as

$$\alpha_1^* (q_1^* - e_1^*) + \alpha_2 (q_2 - e_2) + \alpha_3 (q_3 - e_3) + Y_1^* + Y_2^* = \varepsilon_1 G_1 + \varepsilon_2 G_2 \qquad \text{(C5)}$$

$$\varepsilon_1 V_{11} + \varepsilon_2 V_{21} = \alpha_1^* q_1^* \qquad \text{(C6)}$$

$$\varepsilon_1 V_{12} + \varepsilon_2 V_{22} = \alpha_2 q_2 \qquad \text{(C7)}$$

$$\varepsilon_1 V_{13} + \varepsilon_2 V_{23} = \alpha_3 q_3 \qquad \text{(C8)}$$

Summing (C6), (C7) and (C8)

$$\varepsilon_1(V_{11}+V_{12}+V_{13}) + \varepsilon_2(V_{21}+V_{22}+V_{23}) = \alpha_1^* q_1^* + \alpha_2 q_2 + \alpha_3 q_3 \quad \text{(C9)}$$

We know that $\sum_{i=1}^{n} V_{ji} = G_j$ so that equation (A9) will be

$$\varepsilon_1 G_1 + \varepsilon_2 G_2 = \alpha_1^* q_1^* + \alpha_2 q_2 + \alpha_3 q_3 \quad \text{(C10)}$$

Substituting (C10) in (C5) we get $-\alpha_1^* e_1^* - \alpha_2 e_2 - \alpha_3 e_3 + Y_1^* + Y_2^* = 0$ which shows that the direct and indirect energy used for satisfying final demand of goods and services is equal to the primary energy extracted from the earth.

This analysis can be easily generalised to n commodities and N industries.