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Switching Adaptive Control
of
Robot Manipulators

by

Mansour Kabganian

A Thesis Presented to the University of Ottawa
in partial fulfilment of the requirements for the degree of

Doctor of Philosophy
in
Mechanical Engineering

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In the name of God
the most Gracious, the most Merciful

For their patience and prayers
dedicated to my parents
Abdolkarim Kabganian and Hekmat Shoveiz

and

for her sacrifice and support
dedicated to my wife
Parastoo Badei

Abstract

A switching adaptive-PD controller for the trajectory tracking problem of robotic manipulators subjected to large and abrupt changes of the system parameters is developed. The manipulator arm and the payload are considered, in the most general form, to have a highly nonlinear dynamical model with unknown or partially known parameters. A switching controller is proposed to give the system the ability to deal with abrupt and large changes of parameters. The proposed control system is comprised of two different schemes called, in this thesis, the low-level and the high-level controllers. The high-level controller is an adaptive version of the computed torque control scheme and the low-level controller is a simple PD regulator with an on-line parameter estimator.

The system switches from the adaptive to the PD controller for a limited period when abrupt and large changes in parameters are observed. An on-line parameter estimator identifies the new parameters of the manipulator during the course of low-level regulation. This identification eases smooth switching from PD to adaptive control in the next stage of the process. A least-squares parameter estimator modified by a type of moving window, also known as exponential bounded gain forgetting, is used for this purpose. The robot dynamic response

is filtered to make the estimator independent of joint acceleration measurements.

For designing the adaptive part of the system, the Lyapunov stability criterion is utilized. Typically, the choice of the Lyapunov function to be used in the stable design of highly nonlinear systems is not easy and requires insight into the problem. A new methodology based on the direct exploitation of the generalized Krasovskii theorem is presented. This straightforward utilization of the theorem provides an easy means for the choice of the Lyapunov function for robot manipulators. A parameter-adaptive controller with a new adaptation law is developed based on a new Lyapunov function. The derived adaptive scheme is adopted for a computed torque control system. The boundedness of the vectors of the system states and parameter errors are proven. This guarantees the global stability of the high-level controller and the convergence of its tracking error. A priori knowledge of the system is used to avoid estimating all the parameters and to accelerate the performance of the control system.

The stability and robustness of the switching mechanism are studied by using the Lyapunov method. In the case of the switch from adaptive to PD mode, the robustness is justified intuitively and theoretically. The more critical part of the switching mechanism is that of switching from PD mode to adaptive mode. This is due to the robustness limitations of the high level controller. The Lyapunov study of the switching mechanism resulted in a criterion for finding the states under which the upper bound of the parameter mismatch and the disturbance torque vector does not cause instability. The results of this criterion leads the monitoring function to the choice of the suitable states for switching back to the adaptive controller.

Numerical simulations, using the proposed control scheme for a 3DOF articulated robot

manipulator, are presented for testing the performance of the control system. The software and codes were developed using C-language and Matlab. In a typical test for the PD mode, the identification of a 100% change in parameters took 2.67 seconds. The results are reliable for the switching back action. In the adaptive mode, a complete identification with no a priori knowledge took 6.5 seconds. This may seem slow in some applications. The identification scheme with a priori knowledge takes 1.3 seconds. In the cases with the priority for tracking rather than identification, the tracking mode is more appropriate. A combination of the identification mode followed by the tracking mode is recommended for switching from the PD to the adaptive controller.

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Above all, I thank Almighty God for making possible the seemingly impossible.

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Nomenclature

$\hat{}$	'hat sign'	Estimated value in adaptive mode or predicted value in PD estimator.
$\sim$	'tilde sign'	Error value.
$(\ )_d$	'd subscript'	Desired value.
$(\ )_f$	'f subscript'	Filtered quantity.

Roman Letters

A	Homogeneous transformation matrix for the links.
a_i	Length of the ith link of the manipulator arm.
c_i	$\cos(\theta_i)$.
c_{ij}	$\cos(\theta_i + \theta_j)$.
$c_{2(ij)}$	$\cos[2(\theta_i + \theta_j)]$.
$C(\theta,\dot{\theta})$	Damping matrix of the manipulator dynamics.

$\mathbb{C}(\theta)$	'n × n × n' matrix of the centrifugal and Coriolis effects in the manipulator dynamics.
d_i	Offset distance of the *i*th joint of the manipulator arm.
$\tilde{D}$	Disturbance torque vector.
E	Gripper transform matrix.
e_i	Location of the center of mass of the *i*th link with respect to the *i*th joint.
e	New location of the center of mass of the last link with the payload.
$e(t)$	Prediction error vector in the PD estimator.
$f(\dot{\theta})$	Joints friction vector.
$G(\theta)$	Gravity vector of the manipulator dynamics.
$h(\theta,\dot{\theta})$	Position and velocity dependent terms vector of the manipulator dynamics.
$H(\theta)$	Inertia matrix of the manipulator dynamics.
I	Identity matrix.
k	Gain matrix magnitude in the PD estimator mode.
k_0	Maximum gain matrix magnitude.
K	Kinetic energy of the manipulator.
K_{ij}	Elements of the regressor matrix in adaptive mode.
K_P	Position servo gain matrix.
K_V	Velocity servo gain matrix.
$K(\theta)$	Kinetic energy matrix of the manipulator.
$K(\theta,\dot{\theta},\ddot{\theta})$	Regressor matrix in the adaptive mode.
L	Lagrangian.

m	Total mass of the last link and the payload.
m_i	Mass of the ith link of the manipulator arm.
P	Potential energy of the manipulator.
$\mathbf{P}$	Unknown parameter functions vector in the adaptive mode.
$\wp$	Unknown parameter functions vector in the PD estimator mode.
$\mathcal{P}$	Symmetric positive definite matrix in Krasovskii theorem.
$\mathbf{q}$	Generalized joint coordinates vector.
$\mathbf{Q}$	Symmetric positive definite matrix in Krasovskii theorem.
s_i	$\sin(\theta_i)$.
s_{ij}	$\sin(\theta_i + \theta_j)$.
$s_{2(ij)}$	$\sin[2\,(\theta_i + \theta_j)\,]$.
$\mathbf{T}$	Actuators torque vector.
V	Lyapunov function.
$V(\theta, \dot{\theta})$	Centrifugal and Coriolis torque vector of the manipulator dynamics.
$W(t)$	Regressor matrix in the PD estimator mode.
$\mathbf{x}$	State vector of the system.
$y(t)$	Output vector of the system in the PD estimator mode.
$\mathbf{Z}$	Base transform matrix.

Greek Letters

α	$\boldsymbol{Ax}$
α_i	Offset angle (twist) of the ith link of the manipulator arm.
β	$\boldsymbol{B\hat{H}^{-1}K\tilde{P}}$
γ	$\boldsymbol{A^T \mathcal{P} + \mathcal{P}A}$
$\boldsymbol{\Gamma}$	Adaptation gain matrix.
$\boldsymbol{\Gamma}(t)$	PD estimator gain matrix.
δ	Error in the last link rotation angle θ_3, due to the payload misalignment.
$\boldsymbol{\theta}$	Joints displacement (rotation) vector.
θ_i	Displacement (rotation) of the ith joint of the manipulator arm.
λ	Forgetting factor in the modified PD estimator.
λ_i	Natural frequencies of the critically damped system.
λ_0	Maximum forgetting rate.
ν	$\boldsymbol{B\hat{H}^{-1}\tilde{D}}$
ω	Frequency of the desired trajectory.
$\boldsymbol{\Phi}$, $\boldsymbol{\Psi}$	Switching parameters.
$\boldsymbol{\omega}$	Natural frequencies vector.

Chapter 1

Introduction

1.1 General

Due to the general configuration of robot manipulators, their open loop linkage structure, and the independent and concurrent actuators that can be exercised on individual links, they can be commanded to carry out highly dextrous material handling tasks and to track complicated trajectories. However, this advantage makes them very sophisticated, and sometimes quite expensive. Not all current industrial robots are efficient. Thus, in order to take full advantage of these machines, they must be improved to their utmost capacity. The use of advanced controllers have a significant role to play in making a more efficient use of robot manipulators. One of the most currently researched topics related to the control of mechanical manipulators is the use of advanced controllers to enhance robot performance and to cause better functioning by the manipulator arm.

Most present day industrial robots use simple, conventional control laws that are completely error driven. These types of controllers limit the performance capability of robots. Since linearization and decoupling of the system equations are not considered, changes in configuration dependent dynamic parameters and interaction between the joints cause errors. A perfect design would select fixed gains in the control law for critically damped response, but this is not possible for all configurations.

A practical solution for this problem is to design for the worse case that will, unfortunately, introduce over-damped response and will slow the robot performance. Another design is to choose some average gains to approximate the critical damping response at a point that generally is chosen to be the center of the robot's work-space. However, for the different configurations, the response becomes either over-damped or under-damped. This design will not only introduce slow performance but will also cause some overshoots.

In most of the industrial robot applications, the above-mentioned designs are acceptable. However, in some cases where perfect tracking is required, advanced controllers should be implemented. For example, many applications in the category (Groover et. al., 1988) of material-handling and machine-loading and/or machine-unloading are carried out with simple positional controllers. However, some processing applications, such as continuous arc welding and spray painting, may require more advanced controllers. Other categories that require advanced controllers are assembly and inspection, in which the industry has shown great interest because of their economic potential.

Besides the aforementioned manufacturing applications, there exists a rapidly growing interest in other applications of robotics such as medical care, military and space robots in which the task complexity and demands require the use of more advanced controllers.

Most of the advanced control schemes require, as part of the control law, a dynamic model of the physical system to be controlled. The main hindrances to the use of such controllers for robot manipulators are: uncertainties of some parameters of the system; slow changes in parameters during the operation of the robot; and abrupt changes in parameters due to sudden changes in operating conditions.

During the last decade, a great deal of research work on controller design of robotic manipulators has focussed on the use of dynamic terms of the plant models in the control laws. These efforts have been hampered by the large number of mathematical operations required to incorporate such terms and the limit in the performance-price ratio of computers. As the number of dynamic terms of the model increases, so does the computational time, thus rendering this approach unsuitable as a practical real-time controller. The deficiencies of these controllers become more prevalent when the update rate of the controller signal becomes the limiting factor preventing high robot operating speeds. The limitations of the performance-price ratio of computing hardware have usually forced controller designers to use simple controllers as much as possible. Thus, for a long time, the use of the dynamic model of the manipulator did not seem practical.

Parameter uncertainties, time and configuration variation in system parameters, along with imprecise knowledge of the model, require the use of identification and adaptation techniques to render such advanced controllers theoretically viable. Such techniques, however, add to the computational load, and further, reduce the practicality of model based or computed torque techniques.

The aforementioned problems are the major hindrances in using the manipulator capabilities to their full advantage. The dynamical behaviors of these advanced machines have been simplified to an extent that the designed controllers have not been optimum. Industry has suffered from the lack of a mechanical approach to the problem for years, while one of the most advanced open loop mechanisms has been controlled with some inefficient controllers.

Recent advances in computer technology, and the decrease of computer hardware cost, along with improvements and innovations in the schemes to compute robot manipulator dynamic models, have refocused attention on the problem of practical implementation of advanced techniques for manipulator controls. The development of these techniques may be achieved through an intensive analysis and the use of the mathematical and the dynamical aspects of manipulator structure to design more powerful controllers.

In the main body of this research, continuous time formulations are used. This choice is based on the fact that "nonlinear physical systems are continuous in nature and are hard to meaningfully discretize, while digital control systems may be treated as continuous time systems in analysis and design if high sampling rates are used," (Slotine and Li, 1991). The availability of inexpensive computation generally allows high sampling rates. On the other hand, some simulations and numerical implementations contain discrete time sampling.

1.2 Objectives of the Thesis

Robot manipulators are generally used as aids in production operations. This usage requires that they operate at high speeds and results in high link accelerations and decelerations. The dynamic equations of the system are coupled, contain time and configuration varying parameters, and are highly nonlinear. When the robot manipulator has to perform an exact tracking of a specified trajectory for executing its task, the use of a nonlinear model-based controller is a necessity. The terminologies 'exact tracking' and 'perfect tracking' in this thesis stand for the designs with critically damped responses.

In all model-based compensators a major problem is parameter uncertainty, i.e., imprecise knowledge or change in parameters during the work of the manipulator. This problem results in a parameter mismatch between the model and the actual system. The parameter mismatch may be in the form of either small and slow, or large and abrupt, changes. The model of the robot manipulator and the controller will be unreliable due to this parameter uncertainty.

These difficulties in practical applications have been a major factor for the preference to use simple regulators that are robust to parameter uncertainties. The current research work focuses on resolving the difficulties that have so far precluded the use of advanced controllers in practical applications. The proposed solution consists of implementing a controller that utilizes a simple but robust regulator during a parameter estimation, and an advanced controller when the accuracy of the system model falls within acceptable limits. A switching mechanism oversees a smooth transition between the two control schemes.

1.2.1 Trajectory Tracking Problem

In many applications of robotic manipulators there is a need for some kind of trajectory tracking and obstacle avoidance control design. The term trajectory tracking stands for the process of following a specified trajectory through space with zero or small tracking error. The applications of robotic manipulators cover a wide range from processing to assembly and from medical care to use in military and space programs. These applications may include a variety of tasks such as arc welding, spray painting, etc. Sometimes there are restrictions on the maximum time to complete a task, or to avoid overshooting and bumping into obstacles.

Because of high nonlinearities and couplings in the dynamical structure of robot manipulators, achieving critically damped response and perfect trajectory tracking is not possible with traditional proportional derivative controllers.

A suitable approach for this problem, ignoring the controllers' limitation in dealing with parameter uncertainty, is the Model-Based Nonlinear Control Scheme. In the broad area of this nonlinear control scheme, the Computed Torque Method is the one utilized in the model-based portion of the controller used in this work. This control scheme, with the assumption of a perfect knowledge of the parameters, has a good capability for disturbance suppression and tracking the desired trajectories in different configurations of the manipulator. This is elaborated in more detail in Chapter 4.

1.2.2 Parameter Uncertainty

There are very few robotic applications where one can use the assumption of constant system parameters and the assumption of perfect knowledge. In robot manipulators, perturbation and imprecise knowledge of the parameters are common and frequent problems. There will be some slow changes of the parameters during the manipulator's operation, even if the perfect knowledge of the model is available. The source of these changes may be inherent to the system or result from the operating environment.

Model Reference Adaptive Control (MRAC) was chosen to solve the problem of small changes in parameters. A parameter adaptive controller with a new adaptation law was developed by a straightforward use of the generalized Krasovskii theorem, which results in a new Lyapunov function.

The choice of the Lyapunov function (Khalil, 1992; Slotine and Li, 1991) to be used in the design of highly nonlinear stable systems is not easy and requires insight into the problem. A large number of classical and advanced design methods have been developed to deal with linear control systems or nonlinear systems that can be linearized. For highly nonlinear systems, the choice of the design method is limited. The Lyapunov-based approaches are generally used for dealing with such problems. An important limitation of this method lies in the fact that finding a Lyapunov function for the system is difficult. In robot manipulators, due to the large number of parameters, it is even more complicated. An intuitive understanding of the physics of the system and often a trial and error search for the function is required. The approach utilized here is based on a direct exploitation of the generalized Krasovskii theorem. A new methodology for generating a Lyapunov function for robot manipulators is presented. This straightforward approach to the theorem, provides an easy means for the choice of the Lyapunov function.

The derived adaptive scheme was adopted for a computed torque control system. The system was proven to be bounded in variable vectors, hence globally stable and the convergence of the tracking error was guaranteed. The Lyapunov stability method was used for this purpose.

Due to lubrication in the joints, a major part of the joint friction forces can be assumed as viscous. Howevere, some part of the friction forces may be considered as an abrupt change and will be discussed later. This assumption, which is realistic in a well-lubricated condition for manipulator joints, eliminates the discontinuity problem caused by dry friction. The resulting viscous friction in joints can be considered as a kind of parameter uncertainty, (Craig, 1986). Thus, the structure of the model of the joint friction is known, although some parameters in the model, like friction coefficients, may be unknown.

In this part of the research it is assumed that the parameters of the system change slowly. This is a natural phenomenon in manipulators resulting from joint wear, parameter drift, change in quantity or some properties of the joint lubricant, etc. These issues will be considered in more detail in Chapter Four.

1.2.3 A Priori Knowledge

Based on the discussions presented so far, it is clear that the dynamical model of the system is not precisely known. Thus, while some parts or some parameters of the manipulator are unknown, a priori knowledge from other parts or parameters is available. The system, in this case, is considered to be partially known. This approach results in the major advantage of avoiding estimation of all the system parameters, and accelerates the performance of the control system.

The system is parameterized in a manner that a vector of error variables linear in the parameters is obtained. This approach limits the computations and avoids slowing the calculations and hence results in an efficient controller. Only the partially known part of the system is updated in every iteration. For more detail the reader is referred to Chapter 4.

1.2.4 Abrupt and Large Parameter Change and External Disturbances

In many applications, the system may have fast and large changes of parameters and disturbances. These abrupt changes can occur when an object is grasped or dropped, when an element having a major role in the dynamics of the system fails, or when some important or unknown phenomenon is not modeled or is poorly modeled. The abrupt and large change in the model-based control systems is a critical problem and is not easily solved through classical adaptive control approaches. For systems that experience such changes, the general solution is to use the highly robust PD controllers irrespective of their non-optimality.

Conventional adaptive controllers and parameter identification techniques cannot cope with abrupt and large changes in system parameters. Consequently it is proposed to implement a control system capable of switching between two different schemes of control. In normal conditions where the system parameter variations and disturbances are small, the system switches to the adaptive controller. When abrupt or large changes occur, the system switches to a simple, but more robust, controller for a limited time. While operating under this conventional controller, the system operation is non-optimal. However, stability is guaranteed and productivity is not lost. The system is switched back to the high-level controller when the extraneous and severe conditions have been identified.

A monitoring function is chosen to follow the events in the system and to organize for the optimum way of switching. When the system is switched to the low-level controller, the manipulator is steered back to the desired trajectory after the deviation caused by the abrupt change. An on-line parameter estimator identifies the new parameters of the system in the course of low-level regulation. When switching back to the high-level controller and after identifying the system, a smooth transition between the two control schemes is necessary to avoid inducing disturbances due to switching. The criterion for the time of switching is directly related to the robustness capabilities of the higher level controller.

The switching approach to deal with abrupt changes in parameters and disturbances will be presented in more detail in Chapter 3. This will not be in the form of a design, but more in a schematic presentation of the idea. The complete design and implementation of the control system, with the results of the simulations, will be presented in Chapter 5.

1.3 Thesis Outline

An outline of the remaining chapters follows:

A comprehensive literature survey is presented in Chapter Two and is divided into different sections according to the different trends of the research related to this work.

The general concept of the switching controller is given in Chapter Three. The concept is presented in a general format whereby the overall controller switches between a high fidelity but limited robustness controller and a low fidelity but more robust controller.

Kinematics and dynamics of the manipulator are presented in the beginning of Chapter Four. The Lagrangian formulation is chosen for dynamic analysis. Some mathematical manipulations related to dynamical terms, kinetic energy, and quadratic forms used throughout the thesis, are given in the Appendix A.

The design of the model-reference adaptive control system for the manipulator is the major subject of Chapter Four. The computed-torque controller is presented in Section 4.3. A parameter adaptive computed-torque controller is developed in the remaining portion of chapter. The results of the simulations of the adaptive control mode are presented in the last section of Chapter 4.

The complete design and implementation of the switching Adaptive-PD control system, with the results of simulations, are presented in Chapter 5. A critical study in the robustness of different schemes and of the switching action is also presented in this chapter.

The summary, conclusions, and future work are presented in Chapter 6. A list of references is presented at the end of the thesis.

Chapter 2

Literature Survey

2.1 Introduction

Robotic manipulators have enjoyed extensive attention in the field of automation during the last two decades. This chapter starts with a brief review of the historical development of robotics. A comprehensive literature survey of the related topics is then presented. The literature considered here mainly concerns the control of robotic manipulators.

2.2 A Brief History

Industrial robots have their origins in the field of automation. In this section, some inventions in the field of automation are reviewed, although not all of them deal directly with robotics. In the mid-1700s, Jacques de Vaucanson constructed some human-sized mechanical dolls that played music. In 1801, J.Jacquard invented the Jacquard loom, a programmable machine for weaving threads or yarn into cloth. In 1805, Henri Mailardet built a mechanical doll, which could draw pictures. This invention is on display in the Franklin Institute in Philadelphia, Pennsylvania. In the field of textile production, there were many mechanical inventions such as Hargreaves' spinning jenny (1770), Crompton's mule spinner (1779), Cartwright's power loom (1785) and the Jacquard loom (1801). For more detail on the chronology of developments related to robotics technology, the reader is referred to Groover et al. (1988).

Industrial robots have a major part of their roots in numerically controlled (NC) machine tools and the master-slave teleoperators. The latter, sometimes referred to as telecherics, are the remote manipulators mechanically controlled by a human operator at a remote location. Sometimes the arm can be electrically powered. Master-slave teleoperators were invented during the second world war to manipulate and move radioactive materials

In 1946, George C. Devol invented a controller device that could record electrical signals and play them back to operate a mechanical machine. John Parsons proposed the idea of using punched cards storing position data to control the axes of a machine tool. At Massachusetts Institute of Technology a research and development project supported by the U.S. Air Force was initiated to implement his idea and invention. In 1952, a three-axis milling machine was used to demonstrate the NC prototype. Subsequent work at MIT led to the development of APT (Automatically Programmed Tooling).

Currently, the common use of the master-slave teleoperators is in the handling of dangerous substances in atomic energy laboratories. In 1947, research started at Argonne National Laboratory on master-slave systems.

The combination of NC and master-slave machines led to the formation of the basis of robotic manipulators. In March 1954, the British inventor Cyril Walter Kenward applied for a patent for a robotic device design. The patent was issued in 1957. The first commercial robot, controlled by limit switches and cams, was introduced by Planet Corporation in 1959. George C. Devol designed the "programmed article transfer" in 1954. Based on his idea, Unimation Inc. developed the first "Unimate" industrial robot in 1959. This was a hydraulically driven robot using numerical control principles.

Research work in robotics took a major leap forward when the kinematic equations of manipulators and their solution were introduced by Denavit and Hartenberg (1955) and robot's spatial information was represented mathematically by Roberts (1965) and Peiper (1968). Researchers focused later on generating trajectories and controlling the manipulator to move along them. Some of these works are attributed to: Paul (1972, 1978), Whitney (1969), Kahn and Roth (1971), and Taylor (1979).

The application of dynamic analysis to robotic manipulators seems to have been started by Kahn and Roth (1971). They studied a multi degree of freedom mechanical manipulator based on the work of Uicker (1965).

The use of a nonlinear dynamic model of manipulators in control schemes seems to have been initiated by the work of Paul (1972), and Freund (1975, 1977). Paul investigated the 'Inverse Problem Technique', including on-line computation of a complete model of manipulator dynamics. The main drawback of this method is that it contains a large number of computations. Freund used tools from Lie Algebra to discuss the decoupling and linearizing of nonlinear systems. Some other contributions in this control method are attributed to Lewis

(1974), and Markiewicz (1973).

A great deal of research has been focused on various approaches to the control of robotic manipulators. Of these contributions, the literature survey is confined only to the works more related to the present research.

2.3 Inverse Dynamics Methods

Paul (1972) was the first to investigate the Inverse Problem Technique that was later named the Computed Torque Technique in the robotics literature by Bejczy (1974). Pavlov and Timofeyev (1976) took a similar approach. The technique includes on-line computation of the complete model of manipulator dynamics (i.e., computation of driving torques by solving the equation of motion of the manipulator using the measured values of coordinates and velocities of the robot and the computed values of accelerations). The method requires a large number of mathematical operations and computations that is a major drawback for on-line control of manipulators. Paul, however, suggested that some terms in the dynamic model can be precalculated off-line and stored so as to speed up the technique. Even with such enhancements, the method was found to be too slow and useful only for off-line computations.

Although many enhancements were done to speed up the computations, for a long time these sophisticated control algorithms found use only in theoretical research. Furthermore, due to parameter uncertainty, computational and round off errors, singularities, near singularities and other similar problems, these techniques were never accepted as viable for practical applications. Some of the most recent research on the inverse dynamics method that do not employ adaptation are presented here. The adaptive control schemes are investigated in the next section.

Stability robustness of robot controllers using the computed torque scheme was investigated by Yaz and Zohdy (1990). They assumed that the only unknowns are the exact values of the equivalent masses of the links. Unknown masses were treated as deterministic structured perturbations initially, and bounds were found on the degree of uncertainty that could be tolerated by the design using Lyapunov's second method. Masses were modeled as random structured perturbations, which resulted in a nonlinear stochastic equation for modeling the manipulator. In this case, bounds on uncertainty variances that guarantee asymptotic stability of the manipulator were found. The theoretical bounds obtained were examined using the solution of the Lyapunov equation associated with the globally linearized computed torque model.

For the trajectory following problem of a robot manipulator, Qu et al. (1991) showed the computed torque control scheme to be robust with respect to unknown dynamics by choosing the feedback gains and the estimates of the nonlinear dynamics. However, the problem of unknown or imprecisely known parameters are not addressed. The choices for the constant gains depend only on the coefficients of a polynomial bound of the unknown dynamics. The asymptotic stability of the position and velocity tracking errors was investigated.

Erlic and Lu (1992) presented a reduced order manipulator velocity observer. The observer is used to provide a smooth velocity estimate for a computed torque controller. By using a Lyapunov approach, it was shown that the modified computed torque controller is locally asymptotically stable for trajectory tracking. Experimental verifications of the results were performed on the second and third links of a PUMA-560.

In the above-mentioned research as in any other investigation in robotics, the problem of parameter uncertainty should not have been bypassed. More realistic authors have considered the problem, employing either an indirect approach or some kind of estimation. The author believes these approaches are no longer in the category of the classical computed torque control method.

2.4 Parameter Uncertainty and Adaptive Control

2.4.1 General

In recent years, there has been a fast decline in the price of computer hardware, and large improvements in the performance-price ratio. Therefore, the model-based methods have been reconsidered as a practical means for robot control.

The main problem to be solved is that of imprecise knowledge of the changing parameters. The parameters, need to be estimated or identified on-line through adaptation techniques. The present survey focuses on the Model Reference Adaptive Control scheme (MRAC). Many methods have been proposed for system identification and parameter estimation, and many different approaches have been used for developing adaptation algorithms.

Design of the model reference adaptive control systems typically employs one of the following approaches: 1. Local parametric optimization theory, 2. Lyapunov functions, or 3. Hyperstability and positivity concepts.

The stability problem has a significant role in Model Reference Adaptive Control (MRAC) of robot manipulators. Due to the high nonlinearity of the manipulator's dynamical equations, the proof of asymptotic stability seems impossible with numerical optimization techniques. The second method is the approach utilized in the present work. So, the survey will be more focused on the Lyapunov-based model reference adaptive control schemes. The third approach is beyond the scope of this research.

The local parametric optimization method seems to be used first by Dubowsky and DesForges (1979) to develop a model reference adaptive control system based on steepest decent method to minimize a cost function of the error between the model and the system. The major problem, which is not considered in their work is the guarantee for asymptotic stability of the control system. The discrete time version of the method and the experimental test on an industrial robot was presented by Dubowsky (1981, 1984).

None of the proposals for using the optimization technique addressed the global asymptotic stability of the control system. An effort for using the optimization technique was also made at the beginning of the current research and achieved good results in identifications. However, in designing a stable control and an adaptation law, the method did not work. As mentioned earlier, the proof of the global asymptotic stability requires the analytical solution of the differential equations governing the system. Due to the complexity and nonlinearity of the manipulator's dynamical equations, solving the problem with numerical optimization techniques seems impossible.

2.4.2 Lyapunov-Based Model Reference Adaptive Control

Takegaki and Arimoto (1981) seem to be the first who considered the applicability of model reference adaptive control in robotics. A linear centralized control law with feed-forward compensation of acceleration and gravitational effects is studied. The Lyapunov function approach is applied to change the coefficients within the control law in such a way as to drive the output error to zero. It is not quite clear how the gravity compensation affects the tracking quality, especially when weights of payloads vary over a wide range.

Most of the research in the Lyapunov-based model reference adaptive control of manipulators have been focused on the design of the control system and the adaptation law separately, and looking for the proof for global asymptotic stability, at the end. This approach needs an explicit estimation of the parameters and does not always guarantee stability. The Lyapunov theory is used for analyzing the stability of the system. The works of Han et al. (1987), Tamura and Nagahama (1988), Cat and Janos (1988), Dawson and Lewis (1989), Cherchas and Voss (1989), Wen (1990a, 1990b), Nicoletti (1991), Whitcomb et al. (1991), Stepanenko and Yuan (1991a) and Arnautovic and Koivo (1990) are mainly in this category.

Another group of researchers have used the Lyapunov direct method not only as a criterion to check the stability, but also for adaptive control design. In this approach the direct adaptation mechanisms are derived from the implementation of the Lyapunov theorem. The designed adaptive controllers do not require explicit parameter estimations. The method used in the present research for designing the adaptive control part belongs to this category.

Landau and Horowitz (1988, 1989) proposed an approach for the analysis and synthesis of fixed and adaptive controllers for robot manipulators. The approach provides a basis for new adaptation algorithms that use Lyapunov functions. Choi et al. (1985) proposed a control law consisting of two components: a nominal control and a variational control. The variational control part was derived based on the Lyapunov direct method. Yi et al. (1987) proposed an adaptive control method and a robust adaptive control method, using modified Lyapunov functions to derive the adaptation laws. Nguyen and Pooran (1989), developed an adaptive control scheme for position control of a 6-degree-of-freedom robot end-effector to study an autonomous assembly. The design controller is based on the concept of model reference adaptive control and the Lyapunov direct method. The assumption of slowly time varying robot motion is a limitation of the proposed methods.

Gu and Tongue (1990) used the Lyapunov function approach to derive a new algorithm for model based adaptive motion control for robot manipulators. The technique does not require a knowledge of state accelerations, and high feedback gains are not needed to ensure stability. The parameter ranges allowed for stability are required to be explicitly expressed. This is to ensure that the estimations remain bounded, and is a limitation for the proposed technique.

Oh and Jamshidi (1987) proposed a decentralized adaptive control scheme for the trajectory tracking of a general n degree of freedom robot manipulator. The controller consists of feedforward from the desired trajectory based on the inverse system of the model, PID feedback from the actual trajectory, and auxiliary input for the compensation of the neglected terms in modeling in each subsystem. The gains are derived and adjusted by the model reference adaptive control theory using the Lyapunov direct method. This is a major difference between this work and the present thesis. The gain adjustment, lowers the robustness of the bounded for the acceleration. In a reply, the authors acknowledge the error in their paper.

The works of Oh et al. (1988), Choi and Bien (1988), Menon and Garg (1988), Gu and Tongue (1988), Lewis et al. (1988), Wen and Bayard (1988), Bayard and Wen (1988), Johansson (1990), Shi and Singh (1990), Flashner and Skowronski (1991), Leahy (1990), Bolourchi and Hess (1990) and Feng and Palaniswami (1991) are other examples of the direct derivation of the model reference adaptive controller through the Lyapunov theorem approach.

Another group of work, which typically may be considered as robust model reference adaptive control, is aimed at modifying the control laws for robustness enhancement. Janos and Cat (1988) and Somlo and Cat (1989) proposed robust model reference adaptive control structures and schemes for the control of robot manipulators without knowing the parameters of the system. Wen et al. (1988b, 1992) proposed a sliding-mode-like modification to their joint-level control laws for robustness-enhancing when coulomb and viscous friction and model parameter errors are present. Qu et al. (1992) added a robust control part to their proposed controller for robustification due to possible unknown functional dynamics, estimation error, and disturbances. Fu (1992) proposed a robust adaptive decentralized control algorithm, which did not require explicit system parameter estimation. A useful evaluation and comparison between some forms of robust model-based control schemes was presented by Leahy et al. (1990). Modifications to model-based control structures and the development and evaluation of adaptive primary and robust secondary controllers is pursued by Leahy et al. (1991). Kang et al. (1989), introduced a robust adaptive control scheme for robot manipulators. For achieving the optimal adaptive controller, a performance index is chosen to be minimized.

A more interesting work in robust adaptive control was done by Slotine and Li (1987).

An adaptive robot control algorithm was derived, which consisted of a PD feedback part and a full dynamics feedforward compensation part. The unknown manipulator and payload parameters were estimated on-line. The algorithm requires neither feedback of joint accelerations nor inversion of the estimated inertia matrix. A drawback of this controller is that the estimated parameters of the system do not converge to their true values. However, the primary concern in adaptive control design is the tracking error. Another point to be considered is the necessity for some kind of sufficient richness condition to insure convergence of the estimated parameters.

Slotine and Li (1988) demonstrated their work on a high-speed two degree of freedom semi-direct-drive robot. Spong et al. (1990) commented on Slotine and Li (1988) work, and established the stability in the sense of Lyapunov of the adaptive motion controller for the robot manipulator reported by them.

Repperger (1989) made a comparison between Lyapunov-based adaptive control algorithms and majorizing function approach, which is more of a dominant eigenvalue method. In the second method a linear function bounds the modeling error, which implicitly involves different types of nonlinearity. A gravity vector nonlinearity is studied as an example. This latter approach is more of a dominant eigenvalue method and is compared with the time constant approach using the Lyapunov functions.

At the end of the section, two new enhancements that have been done in the adaptive control scheme, are briefly mentioned. The first is the adaptive algorithm for learning control of a class of nonlinear systems, such as robotic manipulators, proposed by Pourboghrat (1988). The design is based on the combination of direct adaptive control and the learning process. The algorithm is applicable only in cases where the motion is repeated in several cycles. The second is a detailed analysis of the 'composite' adaptive controllers provided by Slotine and Li (1989). In these controllers, the parameter adaptation algorithm is driven by both the tracking error and the prediction error.

2.5 Abrupt Changes in Parameters

Most of the model reference adaptive control schemes for manipulators are established with the 'slowly-varying parameters' assumption for the trajectory tracking problem. This is true in many practical cases such as parameter drifts, joint lubrication warming up during the work, etc. In practical robot applications slow variation in parameters is a small subset of the more general parameter variation problem. Abrupt changes in parameters have a more dominant effect on robot performance than slow changes. The 'slow-varying' assumption will be violated when a sudden change in friction coefficient occurs or when an unknown object is grasped, etc. When strong external disturbances and sudden and large changes of the parameters are accounted for in the robot manipulator model, the problem of designing an asymptotically stable controller becomes very difficult. This is one of the important topics dealt within this research, and needs to be considered in the literature survey.

Maliotis et al. (1990), proposed some robust adaptive control schemes for a class of partially known robot systems. Using the Lyapunov approach, under a slowly time-varying assumption a magnitude bound and a maximum achievable convergence rate for the error was derived. The authors believe that the gains in the adaptive portion of the algorithm may be made as high as desired in order to approach the maximum achievable convergence rate. The effect of these adaptive gains does depend on the unknown dynamics. Next, the slowly time-varying assumption is relaxed and the abrupt change condition is imposed by selecting high gains. However, a major drawback for choosing high gains is the degradation of the system performance due to the sensitivity of the adaptive systems.

Mills (1990) proposed a nonlinear controller for the trajectory control of a rigid link robotic manipulator. The local asymptotic stability of the proposed controller is guaranteed by the Lyapunov approach. The closed-loop system is shown to be robust to parameter perturbations caused by unknown payloads, viscous joint friction, and parameter uncertainty in the dynamic model of the manipulator. The latter causes error in the control law parameters. To satisfy the sufficiency conditions for local asymptotic stability in the presence of large dynamic parameter perturbations, which principally is caused by manipulation of unknown payloads, the proposal is a particular high gain nonlinear control term. High gain feedback is used in the presence of kinematic and dynamic controller modeling errors and large dynamic parameter uncertainty. The negative effect of the high gains on the system performance is again a major disadvantage of the proposed method.

Yuan and Stepanenko (1992) presented an adaptive controller for rigid-body robotic manipulators. The controller was shown to be stable and robust with respect to a limited class of external disturbances. The authors claim that the robustness of the adaptive controller is established without the 'slow-varying' assumption. A Lyapunov-type stability analysis indicates that the tracking error and compensation error will eventually converge into a closed region. The authors believe that the size of this region is adjustable by the controller parameters. However, in the case of abrupt and large changes, this will end up to selecting some unrealistic controller parameters that will degrade the system performance.

In all the above-mentioned research, the authors claim to bypass the 'slowly varying parameters' assumption, although, different techniques have been reported. The general approach can be summarized as follows. Due to the sensitivity of the adaptive control systems, the robustness to parameter perturbations and external disturbances is not unlimited. The region of the convergence in these systems is highly state dependent. For adjusting the size of this region, all the research has been focused on the manipulation of the control system parameters. This will limit the performance of the controller, especially when some more realistic phenomena, such as link flexibility and vibrations, are considered.

2.6 Switching Type Controllers

The switching control of highly nonlinear systems is a new and important area of research. Unfortunately, no successful work is reported in the switching adaptive control of robotic manipulators.

The general problem of switching in adaptive systems is dealt with by Narendra (1991). In this paper it is distinguished between different situations in adaptive control theory in which switching control has been used. In addition, the specific problem of switching adaptive control when prior information concerning the plant transfer function is limited, is discussed. An important point to be noted is that only the abstract of this article is presented in a conference. The complete paper with rigorous mathematical analysis and the detail of the necessary results, seems not to have been published.

Chapter 3

Switching Control of Manipulators

3.1 General

In this chapter the concept of dealing with abrupt change in parameters is introduced. The basic idea behind the proposed switching adaptive controller and its hierarchical structure is clarified. Throughout this chapter, the major concepts and the basic idea are presented physically and elaborated intuitively, rather than using mathematical and theoretical tools. Mathematical designs will be presented in later chapters. To achieve this readability, a conceptual and schematic presentation of the idea of the switching controller is employed.

The complete design and implementation of the switching control system will be presented in Chapter 5. The development of the adaptive part of the control system and its stability analysis will be presented in the following chapter.

3.2 Abrupt and Large Parameter Changes and Disturbances

In the modeling of robotic manipulators, a large number of practical assumptions and simplifications are introduced to reduce the order and complexity of the model and render it solvable. The most notable examples of unmodelled phenomena and operational uncertainties are the backlash in the drive train and in the joints, dry, or near dry friction due to less than perfect lubrication, unknown payloads, etc. The effect of these phenomenons is not a slow and gradual variation in the parameters of the model, but rather a sudden and large change.

The presence of abrupt change in the model-based control system is a major problem that is not easily solved through the classical adaptive control approach. An exaggerated schematic diagram for the displacement of a joint with a typical, trajectory tracking, adaptive model-based controller is shown in Figure 3.1.

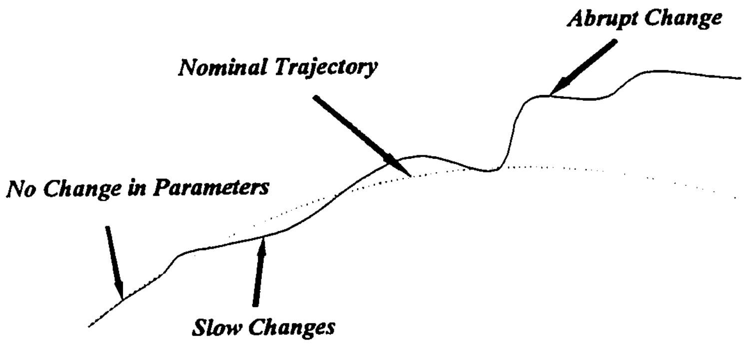

Figure 3.1 Exaggerated joint displacement with abrupt changes.

The abrupt change in parameters causes, in the best case scenario, a deviation from the desired trajectory and, in the worse case, may cause system instability. In cases where system parameters are unknown, or when they undergo abrupt and large changes, control system designers have typically resorted to using traditional PD controllers.

In a trajectory tracking problem with nonlinearities, couplings and parameter uncertainties, the number of the computations required for conventional adaptive controllers is so enormous that the parameter estimation process cannot cope with fast parameter change in real-time. The desired control system should have two different performances under two different circumstances. It should be able to react fast enough to the abrupt change in parameters, and then use its normal high-level controller when the abrupt change or strong disturbance has passed. This switching however, should not compromise stability and robustness.

3.3 Switching Controller

The proposed solution to the problem of abrupt change in parameters is to design a control system capable of switching between different schemes of controllers. A switching occurs when an abrupt change in parameters happens during an ordinary performance of the manipulator. Under normal operating conditions of constant or slowly varying parameters, the system would be controlled by the adaptive controller. When large parameter changes occur, the system is switched to a lower-level but more robust controller for a limited period. By losing, temporarily, the advantages of the high level controller, the control scheme guarantees stability, robustness, feasibility and productivity. The system is switched back to the original controller when it gets close to relatively steady and normal conditions. During this process, the system has had enough time for estimation and adaptation of the parameters.

A monitoring function is chosen to follow the events in the system and to organize for the optimum way of switching. When the system is switched to the conventional controller, an on-line parameter estimator starts identifying the new parameters of the system. A criterion that is derived from the Lyapunov stability analysis indicates for the monitoring function the states under which the upper bound of the disturbance torque vector and the abrupt changes in parameters would not cause instability. Implementing the results of this criterion gives the monitoring function the ability to decide the best time to switch to the advanced controller. The system is forced toward the desired controller, with some reasonable transitions in control signals, in order to obtain a smooth connection between the trajectories of the two different schemes. This smooth connection is necessary to avoid the generation of new disturbances in the system due to the switching action.

3.4 The Hierarchical Procedure

The general procedure and steps that the control system follows to complete a given task are considered in this section. This presentation is intended to give the reader a heuristical motivation and a physical feeling of the entire structure of the controller from a global point of view.

The control system incorporates a kind of hierarchical control structure that supervises switching between different control schemes. In this thesis, the two different control schemes utilized in the proposed control system are called conventional and advanced controllers. The conventional controller is a simple PD regulator and the advanced scheme is an adaptive computed-torque controller. The switching procedure is clarified in a time-historical sequence, with the aid of some explanatory figures. The proposed control system will be shown in block-diagram whenever necessary. The blocks used in this kind of presentation may be entirely unknown, namely black boxes, or partially known, which sometime are called grey boxes, (Bekey, 1970). The comprehensive development and design of the complete Switching Adaptive-PD control system, will be presented in Chapter 5.

A schematic and exaggerated joint displacement for a manipulator with the proposed switching controller is shown in Figure 3.2. The desired trajectory in joint space is indicated by a dashed line. Starting and ending points are A_1 and A_n. The manipulator starts from the initial point A_1 at time zero, with the system switched into the conventional controller, Figure 3.3. The reason for starting with the conventional regulator is the imperfect knowledge of parameters at this point. The system starts the parameter estimation process immediately.

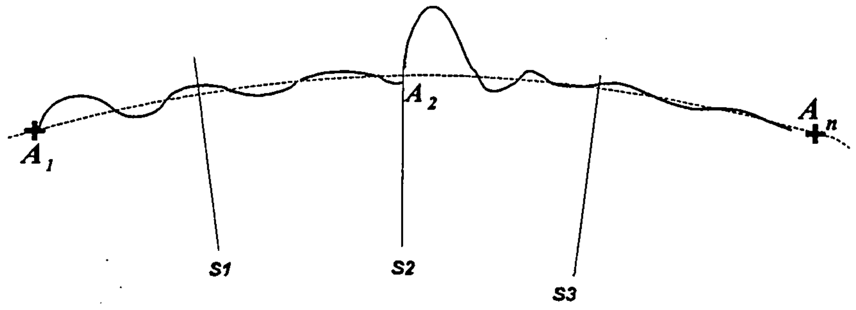

Figure 3.2 Exaggerated Joint Displacement with Switching Controller.

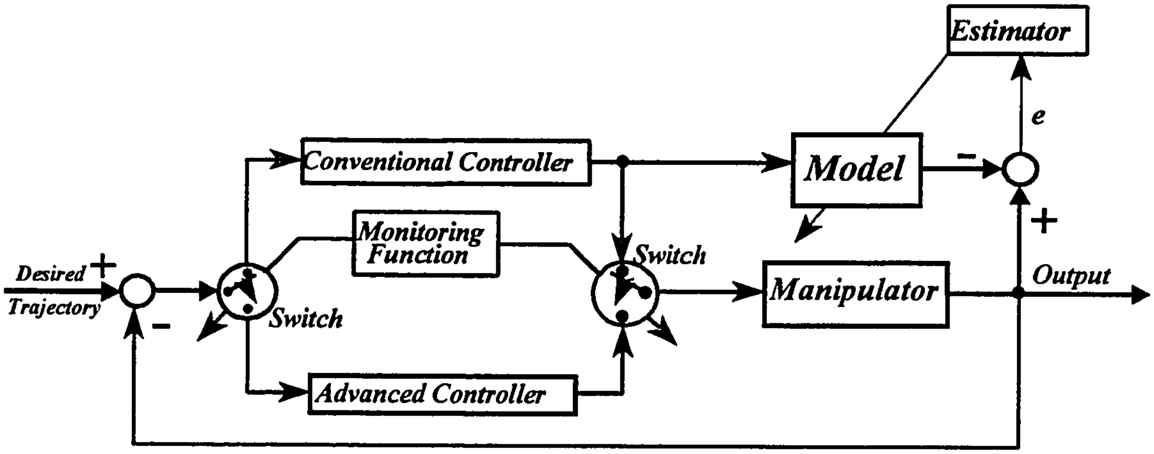

Figure 3.3 Switching Controller.

Note that all the points on the trajectory can be any set of the time history of the trajectory: position, velocity or acceleration. The desired position trajectory, however, should be smooth to avoid generating unboundedness angular velocities and accelerations.

The low-level regulator should be a completely error-driven, positional controller. The main specifications and capabilities of this controller are as follow:

- It should be a simple and therefore, a fast regulator.
- It should a have highly exponential stability designed for the worse cases possible in the system, due to the abrupt parameter changes.

As mentioned earlier, a feedback PD compensator is chosen in this work as the low-level controller.

At point A_1, where the manipulator has started to move, the on-line parameter estimator initiates the parameter identification process. The monitoring function monitors the system's performance and checks the states with the results of the robustness criterion and then decides when to switch to the advanced controller, Figure 3.3. The robustness criterion to be used by the monitoring function was derived from the Lyapunov stability analysis of the system. The main consideration for choocing this criterion was the robustness capabily of the advanced controller. Detailed formulations are presented in Chapter 5.

By instantaneously monitoring the output information of the system, a condition suitable for switching may be recognized by the monitoring function. At this instant shown by the point S_1 in Figure 3.2, it is decided to switch the system to a high-level controller. This controller is the dominant scheme in the system. The manipulator is supposed to stay in this situation, unless some abrupt change in the system happens. The main capabilities and requirements of this advanced controller are:

- It should be able to track the desired trajectory.
- It should be capable of coping with the slow and small parameter perturbations.

This controller is usually chosen from the category of model-based controllers. In the present research, the Adaptive Computed Torque Control was utilized.

The robot controller stays in the adaptive scheme unless some abrupt change happens in the system, as indicated by point A_2 in Figure 3.2. By recognizing this sudden change at point S_2, the monitoring function decides to switch back again into the conventional controller. Again, by realizing the normal conditions at point S_3, the monitoring function decides to switch back to the advanced controller.

The last two steps, at points S_2 and S_3, may be repeated in every circumstance that an abrupt change is identified, and the process will be continued untill the task is completed.

3.5 Switching Adaptive-PD Control of Manipulators

In this section, a general explanation for the entire control system is presented. This is carried out with the aid of the block diagram in Figure 3.2. As mentioned earlier, a comprehensive development and design of the switching adaptive control system will be presented later in Chapter 5. The main structure of the adaptive controller is adopted from a computed-torque control scheme. This adaptive computed-torque controller will be discussed in detail and designed in Chapter Four. It is designed for achieving exponential stability and parameter estimation convergence. This section, focuses more on the switching function.

The control signal in the adaptive mode is constructed from two parts, servo and model-based portions. The inertia matrix of the manipulator is present in the servo portion, and other dynamical terms are compensated for in the model-based portion. The general dynamic equation of the robot manipulator in joint-space (Asada and Slotine, 1986) is given by

$$T = H(\theta)\,\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) \tag{3.1}$$

The detail of the formulations will be presented in Chapter 4. The major concern here, is only to clarify the switching controller. By choosing the servo portion control law as

$$u = \ddot{\theta}_d + K_v(\dot{\theta} - \dot{\theta}_d) + K_p(\theta - \theta_d) \tag{3.2}$$

and showing the estimated values by the (^) sign, the adaptive computed-torque control law becomes:

$$T = \hat{H}(\theta)u + \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta) \tag{3.3}$$

By introducing

$$\hat{h}(\theta,\dot{\theta}) = \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta) \tag{3.4}$$

the control-law, in a more compact form, is given by:

$$T = \hat{H}(\theta)\,u + \hat{h}(\theta,\dot{\theta}) \tag{3.5}$$

Introducing the switching parameters Ψ and Φ, Equation 3.5 can be restated as:

$$T = \Psi u + \Phi \tag{3.6}$$

The operation of the monitoring function and the switch can be seen by choosing Ψ in Figure 3.4 equal to the unit matrix 'I' and disconnecting the model-based portion of the compensator (i.e., setting Φ equal to the zero vector). The resulting system will be a PD controller. In other words, for achieving the low-level controller, it is enough to choose ($\Psi = I,\ \Phi = \mathbf{0}$) and in order to switch back to the high-level controller, Ψ and Φ are set to

$$\Psi = \hat{H}(\theta)$$

$$\Phi = \hat{h}(\theta, \dot{\theta})$$

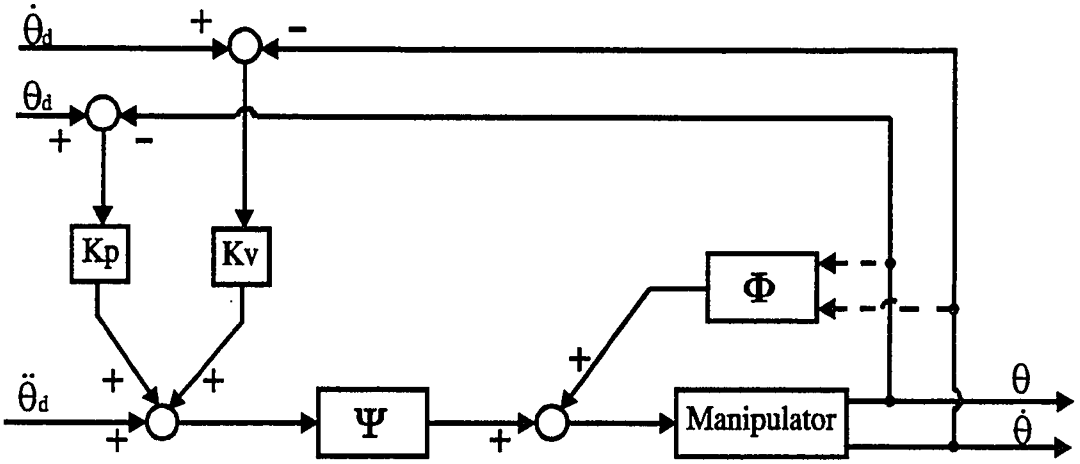

Figure 3.4 Switching Parameters

Chapter 4

Dynamics and Adaptive Control of Mechanical Manipulators

4.1 Introduction

The dynamic model of the robot manipulator is derived using Lagrange equations of motion. The model, in the most general form, is considered as a set of n jointed rigid links forming an open kinematic chain. The joints are all assumed to be revolute. However, with the general formulations derived here, it is easy to modify the results for non-revolute joints. The gravity effect is considered in the dynamic model. The friction effect, as was discussed earlier, can be considered either as a slow varying viscosity in the well-lubricated case, or as an abrupt parameter change in the dry friction case. An imprecise knowledge of the model is assumed. Hence, some or all of the parameter values may be unknown or imprecisely known.

In order to illustrate the results and evaluate the performance of the control system, a

3DOF revolute manipulator arm is chosen. The schematic diagram of the manipulator is shown in Figure 4.1.

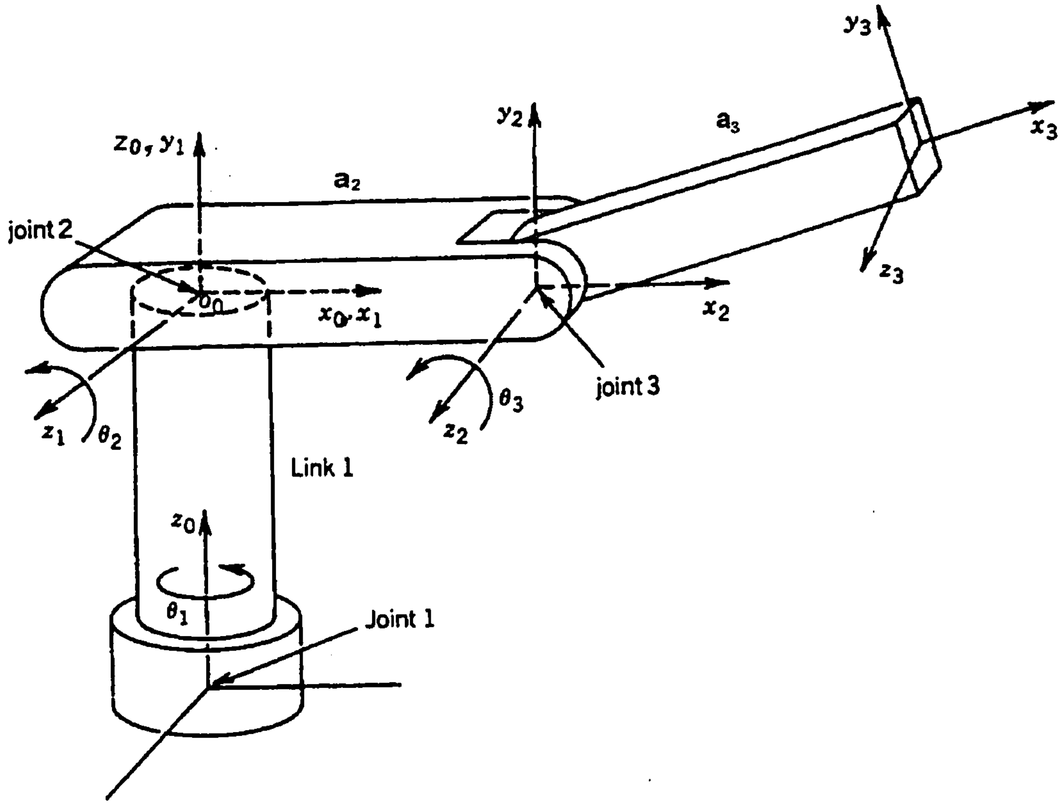

Figure 4.1 Test Manipulator.

The reason for choosing a three-dimensional spatial manipulator as an example, rather than the common and simple structured two-link planar arm, is to achieve more realistic results for studying the effects of different phenomena.

By deriving the dynamic model of the system, a computed-torque control system is designed. This scheme is upgraded later to an adaptive controller which will be exploited not only as a base for the final design of the switching controller, but also for comparing the advantages of the proposed control law.

Some mathematical lemma related to dynamical terms, kinetic energy, quadratic forms, etc., as well as the necessary manipulations, are given in Appendix A.

4.2 Dynamic Structure of the Manipulator

4.2.1 Introduction

In this section, the dynamic model of the robot manipulator is derived for both the general form and the particular form of the test manipulator used for the simulations. This approach is employed throughout the research work

4.2.2 Kinematics

The manipulator is kinematically modeled as revolute jointed rigid links forming an open kinematic chain. A schematic diagram of the manipulator is shown in Figure 4.1. The references for joint angles are given in Figure 4.2. Using the Denavit-Hartenberg (D-H) convention, the link parameters are listed in Table 4.1. For a thorough description of the notations, refer to Denavit and Hartenberg (1955), Asada and Slotine (1986) and Craig (1986).

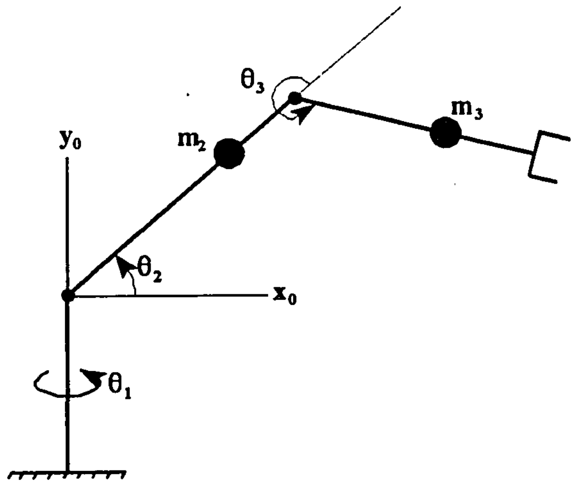

Figure 4.2 Joint Angles of the Manipulator.

Joint Var.	**Link**	**Frame**	θ_i	d_i	a_i	α_i
θ_1	1	0-1	θ_1	0	0	90
θ_2	2	1-2	θ_2	0	a_2	0
θ_3	3	2-3	θ_3	0	a_3	0

Table 4.1: The Link Parameter Table

The homogeneous transformation matrix **A** for the links is

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(4.1)}$$

Z and **E**, the base and the gripper transform matrices are

$$Z = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(4.2)}$$

where a, b and c are the base coordinates in the global reference coordinates. For brevity the notations $s_i = \sin(\theta_i), \ldots$ and $c_{ij} = \cos(\theta_i + \theta_j)$, will be used for the remainder of the work.

The forward and inverse kinematic equations of the manipulator are given by Equations4.3 and 4.4 respectively as follows.

$$\begin{aligned} x &= a + a_2 c_1 c_2 + a_3 c_1 c_{23} \\ y &= a_2 s_1 c_2 + a_3 s_1 c_{23} + b \\ z &= c + a_2 s_2 + a_3 s_{23} \end{aligned}$$

$$\begin{aligned} \dot{x} &= -(a_2 s_1 c_2 + a_3 s_1 c_{23})\,\dot{\theta}_1 - (a_2 c_1 s_2 + a_3 c_1 s_{23})\dot{\theta}_2 - (a_3 c_1 s_{23})\dot{\theta}_3 \\ \dot{y} &= -(a_2 s_1 c_2 + a_3 s_1 c_{23})\,\dot{\theta}_1 - (a_2 c_1 s_2 + a_3 c_1 s_{23})\dot{\theta}_2 - (a_3 c_1 s_{23})\dot{\theta}_3 \\ \dot{z} &= (a_2 c_2 + a_3 c_{23})\dot{\theta}_2 + a_3 c_{23}\dot{\theta}_3 \end{aligned} \qquad \textbf{(4.3)}$$

$$\begin{aligned} \ddot{x} &= -a_4 s_1 \ddot{\theta}_1 - a_5 c_1 \ddot{\theta}_2 - a_3 c_1 s_{23} \ddot{\theta}_3 + a_6 \\ \ddot{y} &= a_4 c_1 \ddot{\theta}_1 - a_5 s_1 \ddot{\theta}_2 - a_3 s_1 s_{23} \ddot{\theta}_3 + a_7 \\ \ddot{z} &= a_4 \ddot{\theta}_2 + a_3 c_{23} \ddot{\theta}_3 + a_8 \end{aligned}$$

$$
\begin{aligned}
\theta_1 &= \arctan\left(\frac{y-b}{x-a}\right) \\
\theta_2 &= \arccos\left(\frac{AC \pm \sqrt{A^2C^2 + (A^2+B^2)(B^2-C^2)}}{A^2+B^2}\right) \\
\theta_3 &= \arcsin\left(\frac{F\cot\theta_2 \pm \sqrt{F^2\cot^2\theta_2 + (1+\cot^2\theta_2)(1-F^2)}}{\cot^2\theta_2 + 1}\right) \\
\dot{\theta}_3 &= \frac{a_4(s_1\dot{y} + \dot{x}c_1)}{a_3(-a_4 s_{23} + a_5 c_{23})} \\
\dot{\theta}_2 &= \frac{\dot{z}}{a_4} - \left(\frac{a_3 c_{23}}{a_2 c_2 + a_3 c_{23}}\right)\dot{\theta}_3 \\
\dot{\theta}_1 &= \frac{\dot{x} + a_5 c_1 \dot{\theta}_2 + a_3 c_1 s_{23} \dot{\theta}_3}{-a_4 s_1} \\
\ddot{\theta}_3 &= -\frac{a_4 c_1}{a_2 a_3 s_3}\ddot{x} - \frac{a_4 s_1}{a_2 a_3 s_3}\ddot{y} - \frac{a_5}{a_2 a_3 s_3}\ddot{z} + \frac{a_4 a_7 s_1 + a_4 a_6 c_1 + a_5 a_8}{a_2 a_3 s_3} \\
\ddot{\theta}_2 &= -\frac{a_3}{a_4} c_{23} \ddot{\theta}_3 + \frac{1}{a_4}(\ddot{z} - a_8) \\
\ddot{\theta}_1 &= -\frac{1}{a_4 s_1}(c_1 a_5 \ddot{\theta}_2) + a_3 c_1 s_{23} \ddot{\theta}_3 - \ddot{x} - a_6)
\end{aligned}
\qquad \textbf{(4.4)}
$$

where

$$
\begin{aligned}
A &= a_2 c_1 (x-a) \\
B &= a_2 c_1^2 (z-c) \\
C &= -\frac{1}{2}\left[(x-a)^2 + c_1^2 (z-c)^2 + c_1^2 (a_2^2 - a_3^3)\right]
\end{aligned}
\qquad \textbf{(4.5)}
$$

and

$$\begin{aligned}
a_4 &= a_2 c_2 + a_3 c_{23} \\
a_5 &= a_2 s_2 + a_3 s_{23} \\
a_6 &= -a_5 c_1 \dot{\theta}_1^2 + 2a_5 s_1 \dot{\theta}_1 \dot{\theta}_2 + 2a_3 s_1 s_{23} \dot{\theta}_1 \dot{\theta}_3 - a_4 c_1 \dot{\theta}_2^2 - 2a_3 c_1 c_{23} \dot{\theta}_2 \dot{\theta}_3 - a_3 c_1 c_{23} \dot{\theta}_3^2 \qquad (4.6) \\
a_7 &= -a_4 s_1 \dot{\theta}_1^2 - a_4 s_1 \dot{\theta}_2^2 - a_3 s_1 c_{23} \dot{\theta}_3^2 - 2 a_5 c_1 \dot{\theta}_1 \dot{\theta}_2 - 2a_3 c_1 c_{23} \dot{\theta}_2 \dot{\theta}_3 - a_3 c_1 c_{23} \dot{\theta}_3^2 \\
a_8 &= -a_5 \dot{\theta}_2^2 - a_3 s_{23} \dot{\theta}_3^2 - 2a_3 s_{23} \dot{\theta}_2 \dot{\theta}_3
\end{aligned}$$

4.2.3 Dynamics

A schematic diagram of the manipulator's kinetic model is given in Figure 4.3. The Lagrangian is given by

$$L = K - P \qquad (4.7)$$

where

$$\begin{aligned}
K = {} & \frac{1}{2} [m_2 e_2^2 c_2^2 + m_3 (a_2 c_2 + e_3 c_{23})^2] \dot{\theta}_1^2 + \frac{1}{2} [m_2 e_2^2 + m_3 (a_2^2 + e_3^2 + 2 a_2 e_3 c_3)] \dot{\theta}_2^2 \\
& + \frac{1}{2} m_3 e_3^2 \dot{\theta}_3^2 + m_3 (e_3^2 + a_2 e_3 c_3) \dot{\theta}_2 \dot{\theta}_3 \qquad (4.8)
\end{aligned}$$

$$P = m_2 g e_2 s_2 + m_3 g (a_2 s_2 + e_3 s_{23})$$

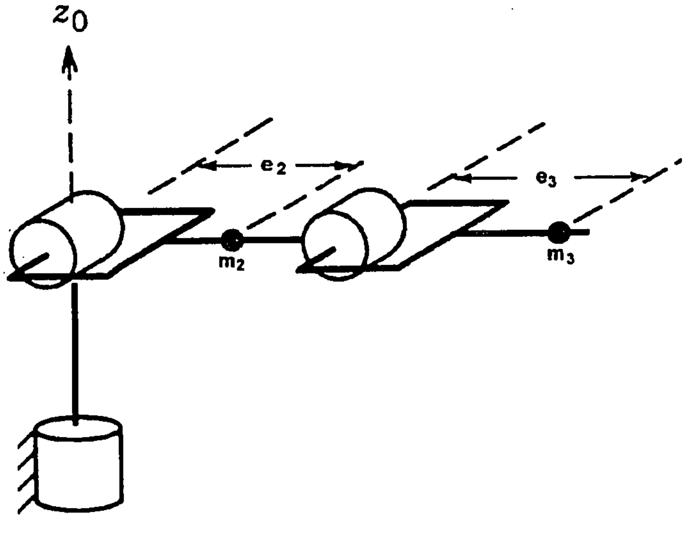

Figure 4.3 Kinetic Model of the Manipulator.

The Lagrange equations of motion for the manipulator joint torques are given by:

$$T_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} \tag{4.9}$$

Solving Equations 4.7 to 4.9 for the three joint torques gives:

$$\begin{aligned} T_1 = {} & [(m_2 e_2^2 + m_3 a_2^2) c_2^2 + m_3 e_3^2 c_{23}^2 + 2 m_3 a_2 e_3 c_2 c_{23}] \ddot{\theta}_1 \\ & -2[(m_2 e_2^2 + m_3 a_2^2) c_2 s_2 + m_3 e_3^2 c_{23} s_{23} + m_3 a_2 e_3 s_2 c_{23} \\ & + m_3 a_2 e_3 c_2 s_{23}] \dot{\theta}_1 \dot{\theta}_2 - 2[m_3 e_3^2 c_{23} s_{23} + m_3 a_2 e_3 c_2 s_{23}] \dot{\theta}_1 \dot{\theta}_3 \end{aligned} \tag{4.10}$$

$$\begin{aligned} T_2 = {} & (m_2e_2^2 + m_3a_2^2 + m_3e_3^2 + 2m_3a_2e_3c_3)\ddot{\theta}_2 + m_3e_3(e_3 + a_2c_3)\ddot{\theta}_3 \\ & + [m_2e_3^2 + m_3a_2^2)c_2s_2 + m_3e_3^2c_{23}s_{23} + m_3a_2e_3s_2c_{23} + m_3a_2e_3c_2s_{23}]\dot{\theta}_1^2 \\ & - m_3e_3a_2s_3\dot{\theta}_3^2 - 2m_3a_2e_3s_3\dot{\theta}_2\dot{\theta}_3 + (m_2e_2 + m_3a_2)g\,c_2 + m_3ge_3c_{23} \end{aligned} \tag{4.11}$$

$$\begin{aligned} T_3 = {} & m_3e_3(e_3 + a_2c_3)\ddot{\theta}_2 + m_3e_3^2\ddot{\theta}_3 + m_3e_3(e_3s_{23}c_{23} + a_2s_{23}c_2)\dot{\theta}_1^2 \\ & + m_3a_2e_3s_3\dot{\theta}_2^2 + m_3ge_3c_{23} \end{aligned} \tag{4.12}$$

These joint torque equations can be cast in vector-matrix form as follows:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} H_{11} & 0 & 0 \\ 0 & H_{22} & H_{23} \\ 0 & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 & h_{112} & h_{113} \\ h_{211} & \dfrac{\dot{\theta}_3^2}{\dot{\theta}_1\dot{\theta}_2}h_{233} & \dfrac{\dot{\theta}_2}{\dot{\theta}_1}h_{223} \\ h_{311} & \dfrac{\dot{\theta}_2}{\dot{\theta}_1}h_{322} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_1\dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ G_2 \\ G_3 \end{bmatrix} \tag{4.13}$$

where the terms are given by

$$\begin{aligned} H_{11} &= (m_2e_2^2 + m_3a_2^2)c_2^2 + m_3e_3^2c_{23}^2 + 2m_3a_2e_3c_2c_{23} \\ H_{22} &= m_2e_2^2 + m_3a_2^2 + m_3e_3^2 + 2m_3a_2e_3c_3 \\ H_{23} &= H_{32} = m_3(e_3^2 + a_2e_3c_3) \\ H_{33} &= m_3e_3^2 \end{aligned} \tag{4.14}$$

$$
\begin{aligned}
h_{112} &= -2[(m_2e_2^2+m_3a_2^2)c_2s_2+m_3e_3^2c_{23}s_{23}+m_3a_2e_3s_2c_{23}+m_3a_2e_3c_2s_{23}] \\
h_{211} &= -\frac{1}{2}h_{112} \\
h_{113} &= -2m_3e_3s_{23}(e_3c_{23}+a_2c_2) \\
h_{311} &= -\frac{1}{2}h_{113} \\
h_{322} &= -h_{233} = -\frac{1}{2}h_{223} = m_3a_2e_3s_3
\end{aligned}
\qquad \textbf{(4.15)}
$$

$$
\begin{aligned}
G_2 &= (m_2e_2+m_3a_2)gc_2+m_3ge_3c_{23} \\
G_3 &= m_3ge_3c_{23}
\end{aligned}
\qquad \textbf{(4.16)}
$$

The triple indexing scheme of h_{ijk} in Equation 4.15 is due to the structure of the n × n × n matrix of centrifugal and Coriolis effects $\mathbb{C}(\theta)$ (Vukobratovic et al., 1985). The full form is given in Appendix A. In compact form, Equation 4.13 is given by

$$T = H(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) \qquad \textbf{(4.17)}$$

where T is the vector of actuators torques, $H(\theta)$ is the inertia matrix of the manipulator[1] that is configuration dependent, $C(\theta,\dot{\theta})$ and $G(\theta)$ are the damping matrix and gravity vector of the manipulator.

The second term in the above equation is the torque vector arising from centrifugal and Coriolis forces. The entire term will be represented by $V(\theta,\dot{\theta})$, and called 'centrifugal and Coriolis torque vector' or 'velocity dependent vector' for brevity. Thus

---

[1] Sometimes called 'mass matrix' of the manipulator.

$$T = H(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) \tag{4.18}$$

In a more general form with the presence of joint frictions $f(\dot{\theta})$, and external disturbance and unmodelled elements dynamics **D**, the complete form of Equation 4.18 follows:

$$T = H(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + f(\dot{\theta}) + G(\theta) + D \tag{4.19}$$

4.3 Model-Based Nonlinear Control

4.3.1 Trajectory Tracking Problem

In most of the applications of robotic manipulators, some kind of trajectory tracking and obstacle avoidance control design is required. The term 'Perfect Trajectory Tracking' or 'Perfect Tracking,' stands for following a desired trajectory through space with zero, or close to zero, tracking error.

The applications of robotic manipulators are wide, and range from processing to assembly, and from medical care to military and space applications. Industrial applications include a variety of tasks such as arc welding, spray painting, etc. Sometimes there are requirements on the maximum time to complete a task, on the extent of overshoot, or on avoidance of obstacles. In many of these applications a perfect trajectory tracking is required.

Due to the high nonlinearities and couplings in the dynamical structure of robot manipulators, a conventional PID controller cannot be adjusted to provide critical damping throughout the work volume. As mentioned in the first chapter, a practical solution to this problem is to adjust the PID gains for the worse case. This, however, results in an over-damped response for the remainder of the configuration space, and, consequently, imperfect tracking. Over-damping also slows down the robot operation, which in some applications becomes a major concern. Another approach is to choose some average gains to approximate the critical damping response at a point that generally is chosen to be the center of the robot's work-space. This would result in the response being either over-damped or under-damped, depending on the robot configuration. In most of the industrial robot applications, such a controller design is unacceptable due to the presence of overshoots.

One solution to this problem is to use a Model-Based Nonlinear Control Scheme. This control scheme, with the assumption of perfect knowledge of the parameters, has a good capability for disturbance suppression and for tracking the desired trajectories irrespective of the configuration of the manipulator.

4.3.2 Computed Torque Control

Model-based nonlinear control scheme is a generic term and refers to several methods. Among the numerous control schemes belonging to this category, the feedback linearization method is utilized as a basis for the high-level controller in the proposed control system. This method is called Computed Torque Control Technique in robotic manipulator application (Paul, 1972; Lee and Chung, 1982). The technique, is also sometimes referred to as the nonlinearity cancellation method (An et al., 1988).

Because of the high nonlinearities involved in robotic applications, the so-called static or quasi-static linearization methods are not sufficient. These methods try to approximately linearize the dynamics around an equilibrium point, whether the point is fixed or is moving.

By contrast, the feedback linearization method uses measured output parameters to algebraically transform a nonlinear system dynamics into a fully, or partly, linear one.

Although this method has been used in many nonlinear applications and has attracted the attention of many researchers in robotics, it has some major drawbacks. The most important weaknesses that are of more concern here are the full state measurement requirement, and the fact that it does not guarantee robustness to parameter uncertainties or disturbances. It should be emphasized at this point that in, the proposed high-level controller, the computed torque method is used only as a basis for the adaptive scheme.

Recalling the manipulator equation of motion, Equation 4.18, a control law is chosen with a structure similar to the manipulator's dynamics as follows:

$$\boldsymbol{T} = \boldsymbol{H}(\theta)\,\boldsymbol{u} + \boldsymbol{V}(\theta,\dot{\theta}) + \boldsymbol{G}(\theta) \qquad \textbf{(4.20)}$$

Solving Equations 4.18 and 4.20 gives:

$$\ddot{\theta} = \boldsymbol{u} \qquad \textbf{(4.21)}$$

Since the inertia matrix of the manipulator is always positive definite, hence invertible, the control problem is reduced to that of controlling the system given by Equation 4.21. This system represents a set of n decoupled double-integrators. By introducing the tracking error as

$$\tilde{\theta} = \theta_d - \theta \qquad \textbf{(4.22)}$$

in which the tilde sign '~' implies the error, and subscript d stands for the desired value. The servo law is chosen as

$$\boldsymbol{u} = \ddot{\theta}_d + \boldsymbol{K}_V \dot{\tilde{\theta}} + \boldsymbol{K}_P \tilde{\theta} \tag{4.23}$$

where $\boldsymbol{K}_P$ and $\boldsymbol{K}_V$ are servo gain matrices. For a 3DOF system, these can be written as

$$\boldsymbol{K}_P = \begin{bmatrix} k_{P1} & 0 & 0 \\ 0 & k_{P2} & 0 \\ 0 & 0 & k_{P3} \end{bmatrix}, \quad \boldsymbol{K}_V = \begin{bmatrix} k_{V1} & 0 & 0 \\ 0 & k_{V2} & 0 \\ 0 & 0 & k_{V3} \end{bmatrix} \tag{4.24}$$

Substituting Equation 4.23 into Equation 4.20 and equating the result to Equation 4.18 gives the error equation as follows

$$\ddot{\tilde{\theta}} + \boldsymbol{K}_V \dot{\tilde{\theta}} + \boldsymbol{K}_P \tilde{\theta} = \boldsymbol{0} \tag{4.25}$$

For exponential stability, $\boldsymbol{K}_P$ and $\boldsymbol{K}_V$ are chosen as

$$\begin{aligned} K_P &= diag[\lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2] \\ K_V &= diag[2\lambda_1, 2\lambda_2, \ldots, 2\lambda_n] \end{aligned} \tag{4.26}$$

where λ_i are the natural frequencies of the critically damped system or the rate of decay of the tracking error (Spong and Vidyasagar, 1989). Now the controller is decomposed into two parts as shown in Figure 4.4. The first part is the model-based portion. It uses the manipulator dynamics to linearize and decouple the arm to a set of unit masses to be controlled. The second part is the servo portion, and acts as a simple trajectory following controller for n independent unit masses.

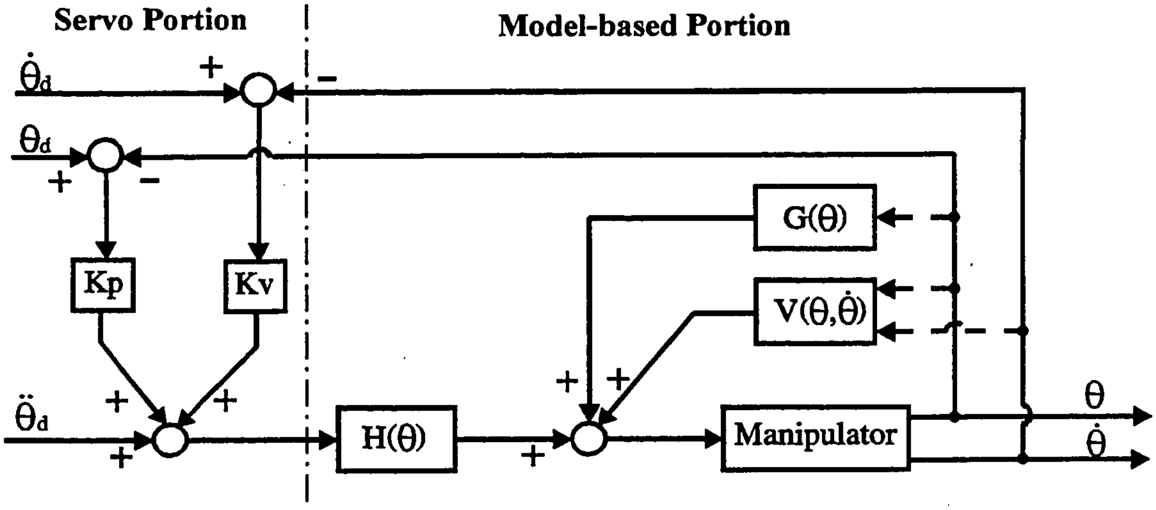

Figure 4.4 Computed Torque Method.

In practice, the major limitation of this method is the presence of parameter uncertainty. In these formulations, a perfect knowledge of the dynamical model and the absence of disturbance was assumed. This is the motivation for upgrading the controller to an adaptive scheme, which is discussed in the following section.

4.4 Model Reference Adaptive Control (MRAC)

4.4.1 Parameter Uncertainty

For mechanical manipulators, parameter perturbations and imprecise knowledge of parameters are a reality. Though a good knowledge of the model and the parameters may be obtained for a given point, change of the parameters during the manipulator's operation is inevitable. The source of these changes may be inside the system, such as friction and wear, and/or from the environment, such as collision with obstacles and viscous friction for robots submerged in fluids. Therefore, for practical robot operations, the assumptions of 'constant parameters' and 'perfect knowledge' of the system are much removed from reality. For a geometrical description of the trajectory tracking problem with the presence of small parameter uncertainties, an exaggerated joint motion is represented in Figure 4.5.

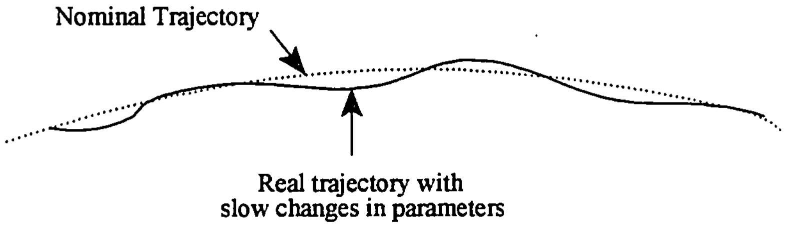

Figure 4.5 Slow Changing Parameters.

The model reference adaptive control (MRAC) scheme is frequently employed to solve the problem of slow changes in parameters. In this work, a parameter-adaptive control with a new adaptation law is developed from a new Lyapunov function obtained by a direct approach to the generalized Krasovskii theorem. The suggestion of this approach is for the specific reason that follows.

A large number of classical and advanced design methods have been developed to deal with linear control systems, or nonlinear systems, that can be linearized. For highly nonlinear systems, the choice of design methods is limited. The Lyapunov-based approaches are generally used for dealing with such problems. A major limitation of this method lies in the fact that finding an appropriate Lyapunov function for the system is difficult. It requires insight into the problem and often a trial and error search for a suitable function. In robot manipulators, due to the large number of the parameters, this is very difficult. The approach utilized here, is based on a direct exploitation of the generalized Krasovskii theorem. This straightforward approach to the theorem, provides a clear method for developing a Lyapunov function.

4.4.2 Adaptive Computed Torque Control

Parameter uncertainties and variation in system parameters, coupled with imprecise knowledge of the model, necessitate the use of identification, parameter estimation, and adaptation techniques to render the model-based controllers feasible. The main idea is to infer the values of the parameters, or its error values, from the measurements of input and output signals of the system. Usually, in real time control, and in circumstances where there is not enough time for precomputations, efficient on-line parameter adjustment mechanisms are required. This becomes more important when the guarantee for stability is required.

In this section, the previously designed computed-torque control system will be upgraded to an adaptive scheme. The Lyapunov stability design method is used for designing the controller. The system is considered to be partially known, and a vector of the error variables is found that is linear in the parameters.

The adaptive version of the computed-torque controller is chosen to have the same structure as Equation 4.20, i.e.

$$\boldsymbol{T} = \hat{\boldsymbol{H}}(\theta)\,\boldsymbol{u} + \hat{\boldsymbol{V}}(\theta,\dot{\theta}) + \hat{\boldsymbol{G}}(\theta) \qquad \textbf{(4.27)}$$

or

$$\boldsymbol{T} = \hat{\boldsymbol{H}}(\theta)\boldsymbol{u} + \hat{\boldsymbol{B}}(\theta)[\dot{\theta}\,\dot{\theta}] + \hat{C}(\theta)[\dot{\theta}^2] + \hat{\boldsymbol{G}}(\theta) \qquad \textbf{(4.28)}$$

where the hat (^) sign implies the estimated value. The servo portion of the control law, for exponential stability, is chosen as

$$\boldsymbol{u} = \ddot{\theta}_d + \boldsymbol{K}_V\dot{\tilde{\theta}} + \boldsymbol{K}_P\tilde{\theta} \qquad \textbf{(4.29)}$$

$\boldsymbol{K}_P$ and $\boldsymbol{K}_V$ are the same as in Equation 4.26. A schematic diagram showing the structure of the controller is shown in Figure 4.6. The condensed form of the manipulator dynamic equation used in Figure 4.6, follows:

$$\boldsymbol{T} = \boldsymbol{H}(\theta)\ddot{\theta} + \boldsymbol{h}(\theta,\dot{\theta}) \qquad \textbf{(4.30)}$$

where

$$h(\theta,\dot{\theta}) = V(\theta,\dot{\theta}) + G(\theta) \qquad \textbf{(4.31)}$$

However, in the adaptive case, the hat signs for estimated values are necessary.

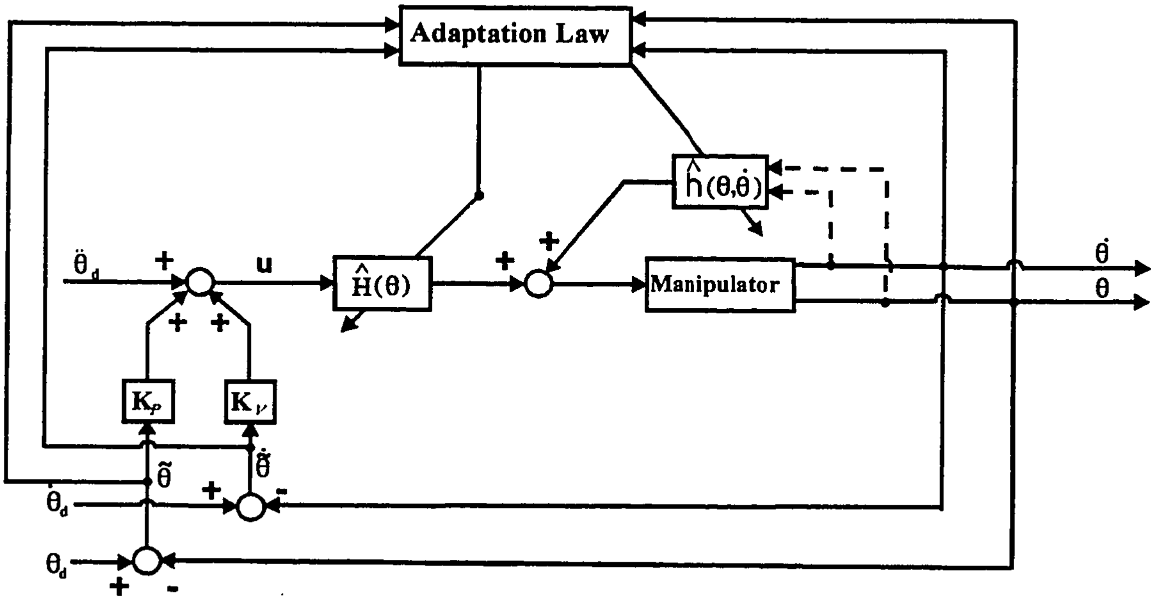

Figure 4.6 Adaptive Computed Torque Controller

4.4.3 Error Equation

The error Equation 4.25 can be modified, due to parameter uncertainty, by equating the dynamics of the manipulator to Equation 4.27, i.e.

$$\hat{\boldsymbol{H}}(\theta)\boldsymbol{u} + \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta) = \boldsymbol{H}(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) \qquad \textbf{(4.32)}$$

Substituting for $\boldsymbol{u}$ from Equation 4.29 into Equation 4.32 gives

$$\hat{\boldsymbol{H}}(\theta)[\ddot{\theta}_d + \boldsymbol{K}_V\dot{\tilde{\theta}} + \boldsymbol{K}_P\tilde{\theta}] + \hat{V}(\theta,\dot{\theta}) + \hat{G}(\theta) = \boldsymbol{H}(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) \qquad \textbf{(4.33)}$$

Using the error term $\ddot{\tilde{\theta}} = \ddot{\theta}_d - \ddot{\theta}$, and rearranging gives

$$\hat{\boldsymbol{H}}(\theta)[\ddot{\tilde{\theta}} + \boldsymbol{K}_V\dot{\tilde{\theta}} + \boldsymbol{K}_P\tilde{\theta}] = [\boldsymbol{H}(\theta) - \hat{\boldsymbol{H}}(\theta)]\ddot{\theta} + V(\theta,\dot{\theta}) - \hat{V}(\theta,\dot{\theta}) + G(\theta) - \hat{G}(\theta) \qquad \textbf{(4.34)}$$

The Dynamic Model Errors (DME) are the errors in the dynamical model used in the controller arising from errors in the parameters of the model. By introducing the following notations for the DMEs

$$\begin{aligned} \tilde{\boldsymbol{H}}(\theta) &= \boldsymbol{H}(\theta) - \hat{\boldsymbol{H}}(\theta) \\ \tilde{V}(\theta,\dot{\theta}) &= V(\theta,\dot{\theta}) - \hat{V}(\theta,\dot{\theta}) \qquad \textbf{(4.35)} \\ \tilde{G}(\theta) &= G(\theta) - \hat{G}(\theta) \end{aligned}$$

the error equation simplifies to

$$\ddot{\tilde{\theta}} + K_V \dot{\tilde{\theta}} + K_P \tilde{\theta} = \hat{H}^{-1}(\theta)[\tilde{H}(\theta)\ddot{\theta} + \tilde{V}(\theta, \dot{\theta}) + \tilde{G}(\theta)] \qquad \textbf{(4.36)}$$

Introducing the following notation for torque signal error

$$\tilde{T} = \tilde{H}(\theta)\ddot{\theta} + \tilde{V}(\theta, \dot{\theta}) + \tilde{G}(\theta) \qquad \textbf{(4.37)}$$

and substituting that in Equation 4.36 gives

$$\ddot{\tilde{\theta}} + K_V \dot{\tilde{\theta}} + K_P \tilde{\theta} = \hat{H}^{-1}(\theta)\,\tilde{T} \qquad \textbf{(4.38)}$$

4.4.4 A Priori Knowledge and Linear Parameterization

In some adaptive control designs, all the parameters and sometimes the entire process is assumed to be unknown. In these cases, the identification of the entire system is necessary and much computation needs to be executed in every time-step for the identification process. Except for some specific circumstances, this is not a recommended engineering approach. In the majority of applications, having some initial information, even though imprecise, is more beneficial and would reduce the computational effort.

Based on the discussions presented so far, in the current research, the dynamical model of the system is imprecisely known. Thus, while some parts or some parameters of the manipulator are unknown, a priori knowledge from other parts or parameters is available. This a priori knowledge is employed and the system is considered partially known.

The system is parameterized in a way that a vector of error variables is derived. This vector is linear in the parameters. By not considering the entire system as a complete unknown process, more efficiency is maintained, the number of computations is decreased and, therefore, the speed is increased. The only part of the system which is updated in every iteration is the changing, or partially known, part.

In the estimation and adaptation part of the proposed controller, the parameter values need to be inferred from available data. Linear parameterization is considered for this purpose. A vector of the parameter function errors caused by uncertainties is found and it is determined to be linear in the parameters. This a priori knowledge incorporated into the MRAC, together with the Lyapunov function candidate, will guide the choice of the adaptation law and the design of the controller. Let the k × 1 vector of parameter functions errors be

$$\tilde{\boldsymbol{P}} = [\,\tilde{P}_1 \; \tilde{P}_2 \; \tilde{P}_3 \; \dots \; \tilde{P}_k\,]^T \tag{4.39}$$

where k is the minimum number of the combinations of the system parameters which should be estimated and updated in each iteration. The inertia torque terms of any multiple link robot can be written in the linear form as follows

$$\tilde{\boldsymbol{T}} = \tilde{\boldsymbol{H}}(\theta)\,\ddot{\theta} + \tilde{\boldsymbol{V}}(\theta, \dot{\theta}) + \tilde{\boldsymbol{G}}(\theta) = \boldsymbol{K}(\theta, \dot{\theta}, \ddot{\theta})\,\tilde{\boldsymbol{P}} \tag{4.40}$$

where $K(\theta,\dot{\theta},\ddot{\theta})$ is an $n \times k$ regressor matrix of the trajectory functions, whose components will be derived later. Note that, for brevity, regressor matrix arguments are sometimes ignored. It should be noted that this $\boldsymbol{K}$ is different from kinetic energy matrix $\boldsymbol{K}(\theta)$ presented in the appendix A.

In the test manipulator, the parameters m, e, and δ are estimated and adapted in each iteration after a change in the system occurs. As an example, when grasping an unknown payload, the total mass of the last link and the payload, combined, is shown as m. The new location of the center of mass of the last link is given by e. The error in the last link rotation angle, θ_3, due to the payload misalignment, is shown by δ. By introducing the new parameters, the dynamic equation of the manipulator in the Equation 4.13 can be rewritten as

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} H'_{11} & 0 & 0 \\ 0 & H'_{22} & H'_{23} \\ 0 & H'_{32} & H'_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 & h'_{112} & h'_{113} \\ h'_{211} & \dfrac{\dot{\theta}_3^2}{\dot{\theta}_1 \dot{\theta}_2} h'_{233} & \dfrac{\dot{\theta}_2}{\dot{\theta}_1} h'_{223} \\ h'_{311} & \dfrac{\dot{\theta}_2}{\dot{\theta}_1} h'_{322} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ G'_2 \\ G'_3 \end{bmatrix} \tag{4.41}$$

where

$$\begin{aligned} H'_{11} &= (m_2 e_2^2 + m a_2^2) c_2^2 + m e^2 c_{23\delta}^2 + 2 m a_2 e c_2 c_{23\delta} \\ H'_{22} &= m_2 e_2^2 + m a_2^2 + m e^2 + 2 m a_2 e c_{3\delta} \\ H'_{23} &= H'_{32} = m e (e + a_2 c_{3\delta}) \\ H'_{33} &= m e^2 \end{aligned} \tag{4.42}$$

and

$$h'_{112} = -2\,[(m_2e_2^2+m\,a_2^2)c_2s_2+m\,e^2c_{23\delta}s_{23\delta}+m\,a_2es_2c_{23\delta}+ma_2ec_2s_{23\delta}]$$

$$h'_{211} = -\frac{1}{2}\,h'_{112}$$

$$h'_{113} = -2\,[m\,e^2c_{23\delta}s_{23\delta}+ma_2ec_2s_{23\delta}] \tag{4.43}$$

$$h'_{311} = m\,e^2c_{23\delta}\,s_{23\delta}+ma_2es_2c_{23\delta}+ma_2e\,c_2\,s_{23\delta}$$

$$h'_{322} = -h'_{233} = -\frac{1}{2}\,h'_{223} = m\,a_2e\,s_{3\delta}$$

and

$$G'_2 = (m_2e_2+m\,a_2)gc_2+m\,gec_{23\delta}$$

$$G'_3 = mge\,c_{23\delta} \tag{4.44}$$

Let the elements of the vector $\tilde{P}$ in Equation 4.39 be

$$\tilde{P}_1 = m-\hat{m}$$

$$\tilde{P}_2 = m\,e^2c_\delta^2-(\widehat{me^2c_\delta^2})$$

$$\tilde{P}_3 = m\,e^2s_\delta^2-(\widehat{me^2s_\delta^2})$$

$$\tilde{P}_4 = m\,e^2s_{2(\delta)}-(\widehat{m\,e^2\,s_{2(\delta)}})$$

$$\tilde{P}_5 = me\,c_\delta-(\widehat{m\,e\,c_\delta}) \tag{4.45}$$

$$\tilde{P}_6 = mes_\delta-(\widehat{m\,e\,s_\delta})$$

$$\tilde{P}_7 = m\,e^2-(\widehat{me^2})$$

$$\tilde{P}_8 = me^2c_{2(\delta)}-(\widehat{m\,e^2c_{2(\delta)}})$$

The large hat sign implies the estimated value for all the terms inside the brackets. Equation 4.13 gives

$$\begin{bmatrix}\tilde{T}_1\\ \tilde{T}_2\\ \tilde{T}_3\end{bmatrix}=\begin{bmatrix}\tilde{H}_{11} & 0 & 0\\ 0 & \tilde{H}_{22} & \tilde{H}_{23}\\ 0 & \tilde{H}_{32} & \tilde{H}_{33}\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{bmatrix}+\begin{bmatrix}0 & \tilde{h}_{112} & \tilde{h}_{113}\\ \tilde{h}_{211} & \dfrac{\dot{\theta}_3^2}{\dot{\theta}_1\dot{\theta}_2}\tilde{h}_{233} & \dfrac{\dot{\theta}_2}{\dot{\theta}_1}\tilde{h}_{223}\\ \tilde{h}_{311} & \dfrac{\dot{\theta}_2}{\dot{\theta}_1}\tilde{h}_{322} & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1^2\\ \dot{\theta}_1\dot{\theta}_2\\ \dot{\theta}_1\dot{\theta}_3\end{bmatrix}+\begin{bmatrix}0\\ \tilde{G}_2\\ \tilde{G}_3\end{bmatrix} \qquad \textbf{(4.46)}$$

where the components are derived as

$$\tilde{H}_{11}=a_2^2c_2^2\tilde{P}_1+c_{23}^2\tilde{P}_2+s_{23}^2\tilde{P}_3-\frac{1}{2}s_{2(23)}\tilde{P}_4+2a_2c_2c_{23}\tilde{P}_5-2a_2c_2s_{23}\tilde{P}_6$$
$$\tilde{H}_{22}=a_2^2\tilde{P}_1+\tilde{P}_7+2a_2c_3\tilde{P}_5+2a_2s_3\tilde{P}_6$$

$$\tilde{H}_{23}=\tilde{H}_{32}=a_2(c_3\tilde{P}_5-s_3\tilde{P}_6)$$

$$\tilde{H}_{33}=\tilde{P}_7$$

$$\tilde{h}_{112}=-2c_2s_2a_2^2\tilde{P}_1-c_{2(23)}\tilde{P}_4-s_{2(23)}\tilde{P}_8-2a_2s_{(2\theta_2+\theta_3)}\tilde{P}_5-2a_2c_{(2\theta_2+\theta_3)}\tilde{P}_6$$
$$\tilde{h}_{113}=-c_{2(23)}\tilde{P}_4-2a_2c_2s_{23}\tilde{P}_5-2a_2c_2c_{23}\tilde{P}_6-s_{2(23)}\tilde{P}_8$$
$$\tilde{h}_{211}=-\frac{1}{2}\tilde{h}_{112}$$
$$\tilde{h}_{223}=-2a_2(s_3\tilde{P}_5+c_3\tilde{P}_6)$$
$$\tilde{h}_{233}=2\,(a_2s_3\tilde{P}_5+a_2c_3\tilde{P}_6)$$
$$\tilde{h}_{311}=-\frac{1}{2}\tilde{h}_{113}$$
$$\tilde{h}_{322}=-\tilde{h}_{233}=-\frac{1}{2}\tilde{h}_{223}$$

$$\tilde{G}_2=a_2gc_2\tilde{P}_1+gc_{23}\tilde{P}_5-gs_{23}\tilde{P}_6$$
$$\tilde{G}_3=gc_{23}\tilde{P}_5-gs_{23}\tilde{P}_6$$

By introducing the following abbreviations

$$s_{2(23)} = \sin[2(\theta_2 + \theta_3)]$$
$$c_{2(23)} = \cos[2(\theta_2 + \theta_3)] \quad \textbf{(4.47)}$$

and factoring out the $\tilde{p}_i$'s, the Equation 4.46 can be cast in the matrix form of Equation 4.40 as follows

$$\begin{bmatrix} \tilde{T}_1 \\ \tilde{T}_2 \\ \tilde{T}_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} & 0 & K_{18} \\ K_{21} & 0 & 0 & K_{24} & K_{25} & K_{26} & K_{27} & K_{28} \\ 0 & 0 & 0 & K_{34} & K_{35} & K_{36} & K_{37} & K_{38} \end{bmatrix} \begin{bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \tilde{p}_3 \\ \tilde{p}_4 \\ \tilde{p}_5 \\ \tilde{p}_6 \\ \tilde{p}_7 \\ \tilde{p}_8 \end{bmatrix} \quad \textbf{(4.48)}$$

where the K_{ij} are grouping parameters given in the following equation.

$$
\begin{aligned}
K_{11} &= a_2^2 c_2^2 \ddot{\theta}_1 - 2 a_2^2 c_2 \dot{\theta}_1 \dot{\theta}_2 \\
K_{12} &= c_{23}^2 \ddot{\theta}_1 \\
K_{13} &= s_{23}^2 \ddot{\theta}_1 \\
K_{14} &= -\frac{1}{2} s_{2(23)} \ddot{\theta}_1 - c_{2(23)} \dot{\theta}_1 (\dot{\theta}_2 + \dot{\theta}_3) \\
K_{15} &= 2 a_2 [c_2 c_{23} \ddot{\theta}_1 - s_{(2\theta_2+\theta_3)} \dot{\theta}_1 \dot{\theta}_2 - c_2 s_{23} \dot{\theta}_1 \dot{\theta}_3] \\
K_{16} &= -2 a_2 [c_2 s_{23} \ddot{\theta}_1 + c_{(2\theta_2+\theta_3)} \dot{\theta}_1 \dot{\theta}_2 + c_2 c_{23} \dot{\theta}_1 \dot{\theta}_3] \\
K_{18} &= -s_{2(23)} \dot{\theta}_1 (\dot{\theta}_2 + \dot{\theta}_3) \\
K_{21} &= a_2^2 \ddot{\theta}_2 + a_2 c_2 s_2 \dot{\theta}_1 + a_2 g c_2 \\
K_{24} &= \frac{1}{2} c_{2(23)} \dot{\theta}_1^2 \\
K_{25} &= 2 a_2 c_3 \ddot{\theta}_2 + a_2 c_3 \ddot{\theta}_3 + a_2 s_{(2\theta_2+\theta_3)} \dot{\theta}_1^2 - a_2 s_3 \dot{\theta}_3^2 - 2 a_2 s_3 \dot{\theta}_2 \dot{\theta}_3 + g c_{23} \\
K_{26} &= -2 a_2 s_3 \ddot{\theta}_2 - a_2 s_3 \ddot{\theta}_3 + a_2 c_{(2\theta_2+\theta_3)} \dot{\theta}_1^2 - a_2 c_3 \dot{\theta}_3^2 - 2 a_2 c_3 \dot{\theta}_2 \dot{\theta}_3 - g s_{23} \\
K_{27} &= \ddot{\theta}_2 \\
K_{28} &= \frac{1}{2} s_{2(23)} \dot{\theta}_1^2 \\
K_{34} &= K_{24} \\
K_{35} &= a_2 c_3 \ddot{\theta}_2 + a_2 c_2 s_{23} \dot{\theta}_1^2 + a_2 s_3 \dot{\theta}_2^2 + g c_{23} \\
K_{36} &= -a_2 s_3 \ddot{\theta}_2 + a_2 c_{23} \dot{\theta}_1^2 + a_2 c_3 \dot{\theta}_2^2 - g s_{23} \\
K_{37} &= \ddot{\theta}_3 \\
K_{38} &= K_{28}
\end{aligned}
\qquad \textbf{(4.49)}
$$

The closed loop error dynamics can be derived by substituting for $\tilde{T}$ from Equation 4.40 into Equation 4.38 to give

$$\ddot{\tilde{\theta}} + \boldsymbol{K}_V \dot{\tilde{\theta}} + \boldsymbol{K}_P \tilde{\theta} = \hat{\boldsymbol{H}}^{-1}(\theta) \boldsymbol{K}(\theta, \dot{\theta}, \ddot{\theta}) \tilde{\boldsymbol{P}} \qquad \textbf{(4.50)}$$

For the particular manipulator under study, the components of $\boldsymbol{K}$ and $\tilde{\boldsymbol{P}}$ are given by Equations 4.49 and 4.45 respectively.

4.4.5 State Space Representation

The states of the system are chosen to be

$$x = [\ \tilde{\theta} \quad \dot{\tilde{\theta}}\ ]^T \tag{4.51}$$

For an n degree of freedom manipulator, the state vector has a dimension of 2n. The closed loop error dynamics in Equation 4.50 can be written in the form

$$\dot{x} = Ax + B\,(\hat{H}^{-1} K \tilde{P}) \tag{4.52}$$

where **A** and **B** are block-partitioned matrices as follows

$$A = \begin{bmatrix} 0 & I \\ -K_P & -K_V \end{bmatrix}, \quad B = \begin{Bmatrix} 0 \\ I \end{Bmatrix} \tag{4.53}$$

I and **0** are n × n identity and zero matrices respectively. For exponential stability, K_P and K_V are chosen the same as Equation 4.26. Let the output be

$$Y = \Omega x \tag{4.54}$$

where Ω is an $n \times 2n$ matrix given by

$$\Omega = [4\omega^3 \mathcal{P}_B \quad \mathcal{P}_A + 7\omega^2 \mathcal{P}_B] \tag{4.55}$$

where

$$\omega = diag[\lambda_1, \lambda_2, \ldots, \lambda_n] \tag{4.56}$$

and $\mathcal{P}_A$ and $\mathcal{P}_B$ are n × n submatrices of the block-partitioned symmetric positive definite matrix

$$\mathcal{P} = diag[\mathcal{P}_A \quad \mathcal{P}_B] \tag{4.57}$$

or in the more detail

$$\begin{aligned} \mathcal{P}_A &= diag[\mathcal{P}_1 \quad \mathcal{P}_2 \ldots \mathcal{P}_n] \\ \mathcal{P}_B &= diag[\mathcal{P}_{n+1} \quad \mathcal{P}_{n+2} \ldots \mathcal{P}_{2n}] \end{aligned} \tag{4.58}$$

Thus, the state space representation of the system is given by

$$\begin{cases} \dot{x} = A\,x + B\,(\hat{H}^{-1} K \tilde{P}) \\ Y = \Omega x \end{cases} \tag{4.59}$$

4.4.6 Adaptation Law

For the autonomous and controllable system of Equation 4.59, let A be the Jacobian matrix of the nonlinear system

$$\frac{\partial f}{\partial x} = A \tag{4.60}$$

By the generalized Krasovskii theorem (Khalil, 1992; Slotine and Li, 1991), the sufficient condition for asymptotic stability, with the origin as the equilibrium point, is that there exist symmetric positive definite matrices $\mathcal{P}$ and Q such that for all non-zero states, the matrix

$$F(x) = A^T \mathcal{P} + \mathcal{P} A + Q \qquad \textbf{(4.61)}$$

is negative-semi-definite in some neighborhood of the origin. In this case, the positive definite function $V(x) = f^T \mathcal{P} f$ is a Lyapunov function for the system.

It is to be noted that an important assumption in the theorem for introducing the Lyapunov function is the negligence of parameter uncertainty. In the case under study, the parameter uncertainty cannot be neglected. Thus, another quadratic term is augmented to the above-mentioned function, due to the uncertainty errors. The following Lyapunov function candidate is proposed:

$$V = f^T \mathcal{P} f + \tilde{P}^T \Gamma^{-1} \tilde{P} \qquad \textbf{(4.62)}$$

where $\mathcal{P}$ and Γ are positive definite symmetric matrices. By defining f to be the nonlinear equation governing the entire system, a unique and original Lyapunov function candidate will be obtained. The Lyapunov function and its time derivative are investigated for the stability of the system and the boundedness of the parameters. Both quadratic terms in the function are positive, because $\mathcal{P}$ and Γ are positive definite symmetric matrices. Thus, the Lyapunov function candidate is positive for all non-zero states

$$V(x) > 0 \qquad \textbf{(4.63)}$$

The nonlinear system represented in state-space form, recalled from Equation 4.59 is

$$f = \dot{x} = A\,x + B\,(\hat{H}^{-1}K\ \tilde{P}) \tag{4.64}$$

The time derivative of the Lyapunov function candidate, by considering that the vector $\tilde{P}$ is linear in the parameter functions, is given by

$$\dot{V} = \dot{f}^{T}\mathcal{P}\,f + f^{T}\mathcal{P}\,\dot{f} + 2\,\tilde{P}^{T}\Gamma^{-1}\dot{\tilde{P}} \tag{4.65}$$

From the chain-rule it follows

$$\dot{f} = \frac{\partial f}{\partial x}\,\dot{x} = A\,f \tag{4.66}$$

Substituting for $\dot{f}$ in Equation 4.65 gives

$$\begin{aligned} \dot{V} &= f^{T}A^{T}\mathcal{P}f + f^{T}\ \mathcal{P}A\,f + 2\,\tilde{P}^{T}\,\Gamma^{-1}\,\dot{\tilde{P}} \\ &= f^{T}(A^{T}\mathcal{P} + \mathcal{P}A)f + 2\,\tilde{P}^{T}\Gamma^{-1}\,\dot{\tilde{P}} \end{aligned} \tag{4.67}$$

Substituting for f from Equation 4.64 gives

$$\dot{V} = (A\,x + B\hat{H}^{-1}K\tilde{P})^{T}(A^{T}\mathcal{P} + \mathcal{P}A)(\,Ax + B\,\hat{H}^{-1}\,K\tilde{P}) + 2\,\tilde{P}^{T}\Gamma^{-1}\,\dot{\tilde{P}} \tag{4.68}$$

Using the following notations for brevity

$$\begin{aligned} Ax &= \alpha \\ B\hat{H}^{-1}K\tilde{P} &= \beta \\ A^{T}\mathcal{P} + \mathcal{P}A &= \gamma \end{aligned} \tag{4.69}$$

Equation 4.68 can be written as follows

$$\dot{V} = (\alpha^T+\beta^T)(\gamma)(\alpha+\beta) + 2\tilde{P}^T\Gamma^{-1}\dot{\tilde{P}}$$

$$= \alpha^T\gamma\alpha + \beta^T\gamma\beta + 2\beta^T\gamma\alpha + 2\tilde{P}^T\Gamma^{-1}\dot{\tilde{P}} \tag{4.70}$$

From Equation 4.61 and the fact that Q is positive definite, it can be concluded that the matrix

$$\gamma = A^T\mathcal{P} + \mathcal{P}A = F(x) - Q \tag{4.71}$$

is negative definite. This proves that the first and second term of Equation 4.70 are negative. Expanding the remainder two terms and rearranging gives:

$$\begin{aligned}\dot{V} &= \alpha^T\gamma\alpha + \beta^T\gamma\beta + 2\tilde{P}^T(B\hat{H}^{-1}K)^T(A^T\mathcal{P} + \mathcal{P}A)(Ax) + 2\tilde{P}^T\Gamma^{-1}\dot{\tilde{P}} \\ &= \alpha^T\gamma\alpha + \beta^T\gamma\beta + 2\tilde{P}^T[(B\hat{H}^{-1}K)^T(A^T\mathcal{P} + \mathcal{P}A)(Ax) + \Gamma^{-1}\dot{\tilde{P}}]\end{aligned} \tag{4.72}$$

To guarantee the stability, it is enough to equate the last term to zero

$$\dot{\hat{P}} = -\Gamma(B\hat{H}^{-1}K)^T(A^T\mathcal{P} + \mathcal{P}A)(Ax) \tag{4.73}$$

From the relationship

$$P - \hat{P} = \tilde{P} \tag{4.74}$$

since **P** is constant, it follows that

$$\dot{\hat{P}} = -\dot{\tilde{P}} \qquad (4.75)$$

Therefore, the parameter adaptation mechanism may be derived as follows

$$\dot{\hat{P}} = \Gamma\, K^T \hat{H}^{-1} B^T (A^T\, \mathcal{P} + \mathcal{P} A)(A\, x) \qquad (4.76)$$

or in a more compact form

$$\dot{\hat{P}} = \Gamma\, K^T \hat{H}^{-1} \Omega\, x \qquad (4.77)$$

This adaptation law is derived following the proposed methodology. It has a structure that, similar to other adaptation laws reported in the literature, contains the terms of gain, regressor, and inertia matrices. However, the main difference lies in the fact that the proposed one uses the matrix Ω that contains the natural frequencies of the system. For this adaptation law, the time derivative of the Lyapunov function candidate in Equation 4.70 is negative for all non-zero states

$$\dot{V}(x) < 0 \qquad (4.78)$$

Therefore, the Lyapunov function candidate in Equation 4.62 is proven to be a Lyapunov function and the values of x and $\tilde{P}$ are bounded. This guarantees the global stability of the adaptive computed-torque controller proposed in the present chapter. The boundedness of the states implies the convergence of the tracking error.

4.5 Implementation Remarks

The inverse of the estimated inertia matrix exists in the parameter update law. This may result in a singular matrix during the course of estimations. The matrix will remain positive definite and invertible if all the estimated parameters are confined to stay in a region close to the real parameter values. A priori upper and lower bounds for the parameter function estimates are imposed. The parameters are reset in cases where the estimates move outside the limits. The resetting action guarantees the boundedness of the estimates that is essential for perfect tracking. However, it generates a step change in the vector of the parameter error functions. Thus, care should be taken in choosing reasonable values for the bounds and resettings.

Since the joint acceleration terms in the adaptation law are difficult to measure, the application of a first order filter reduces these terms to lower order terms, i.e., velocities that can be easily measured. The application of the first order filter in the PD estimator, is presented in detail in the next chapter. The software implementation of the filter in the adaptive controller is similar to the case of the PD estimator.

4.6 Simulation Results

4.6.1 General

A three link spatial revolute manipulator arm was chosen for simulation and for assessing the performance of the control system as shown in Figure 4.1. The schematic diagram of the manipulator's kinetic model is given in Figure 4.3. The manipulator is studied when it picks up or drops a payload with unknown mass. The manipulator parameters were selected to be

$$\begin{aligned} m_2 &= 10kg \quad , \quad m_3 = 5kg \\ a_2 &= 1m \quad , \quad a_3 = 0.6m \\ e_2 &= 0.6m \quad , \quad e_3 = 0.25m \end{aligned} \tag{4.79}$$

The grasping of an unknown payload or dropping some part of the payload, was studied as the change in the system. In the case of dropping some part of the payload, the total mass of the last link and the payload was initially chosen to be equal to 5kg. The parameters m, e, and δ are considered to be estimated and adapted in each iteration after a change in the system occurs.

Simulations were performed with two different goals, tracking and identification, both with the desired trajectories in the harmonic form

$$\begin{aligned} \theta_{d1} &= A_1 + B_1[\sin(10\omega t + \phi_1) + \sin(8\omega t + \phi_1)] \\ \theta_{d2} &= A_2 + B_2[\sin(6\omega t + \phi_2) + \sin(4\omega t + \phi_2)] \\ \theta_{d3} &= A_3 + B_3[\sin(2\omega t + \phi_3) + \sin(\omega t + \phi_3)] \end{aligned} \tag{4.80}$$

A random disturbance vector opposing the joint motions is considered as the dry friction. All the results are shown after a switch from the PD regulator to the adaptive mode, at time $t = 0$.

4.6.2 Tracking

In this test, the first priority was given to the achievement of a fast and accurate tracking, rather than the identification. Therefore, there was no need for persistent excitation. The test was performed with $\omega=0.5$rad/sec and different step changes in the payload. A random disturbance vector, as unmodelled dynamics and dry friction, was added in the joints. The disturbance was considered to have an amplitude equal to 5% of the typical maximum control signal. The effect of abrupt parameter change on the tracking, was investigated by dropping different parts of the payload.

The results of the tracking test for a medium step change in the payload are considered here. Medium change was considered to be 50% change in the payload mass. The total mass of the third link and the payload was changed from 10kg to 5kg at time t=1sec. The tip patherror and the servo errors are given in Figures 4.7 and 4.8. The figures show that accurate tracking is achieved in less than one second. The maximum deviation from the steady state condition in the tip path error due to the parameter change is 2.31mm. To avoid redundancy in this thesis, future reference to 'a maximum deviation' will indicate the maximum deviation from steady state condition caused by parameter change.

Figure 4.8 shows that the maximum deviations in the first, second and last joint tracking errors are 0.00035rd, -0.00160rd and 0.00316rd respectively. Steady state errors equal to -0.0019rd and -0.00568rd remain in the second and third joints.

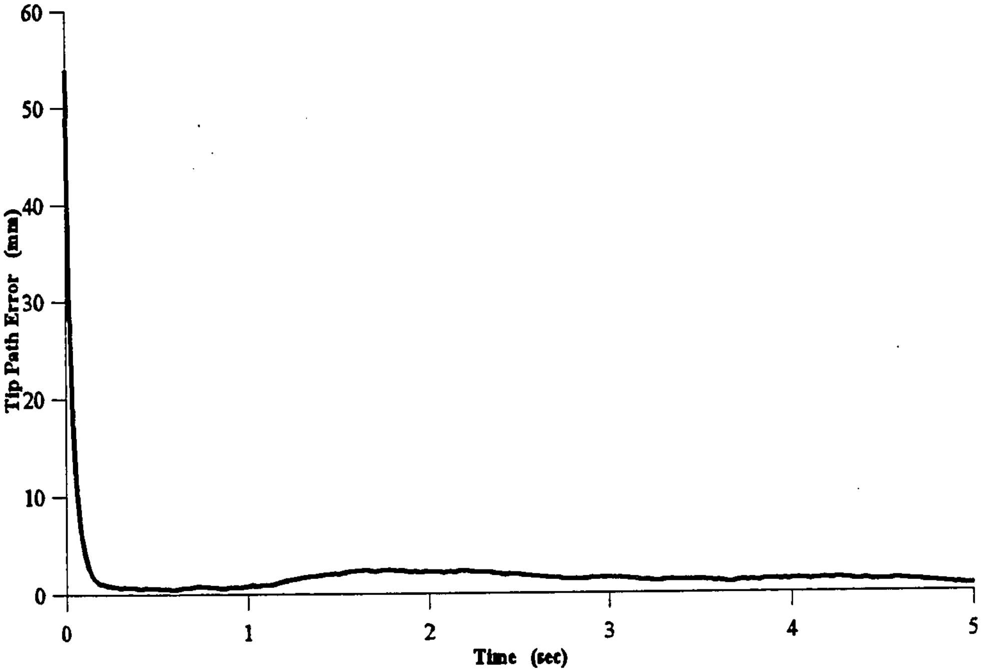

Figure 4.7 Tip Path Error for Tracking with Medium Step Change.

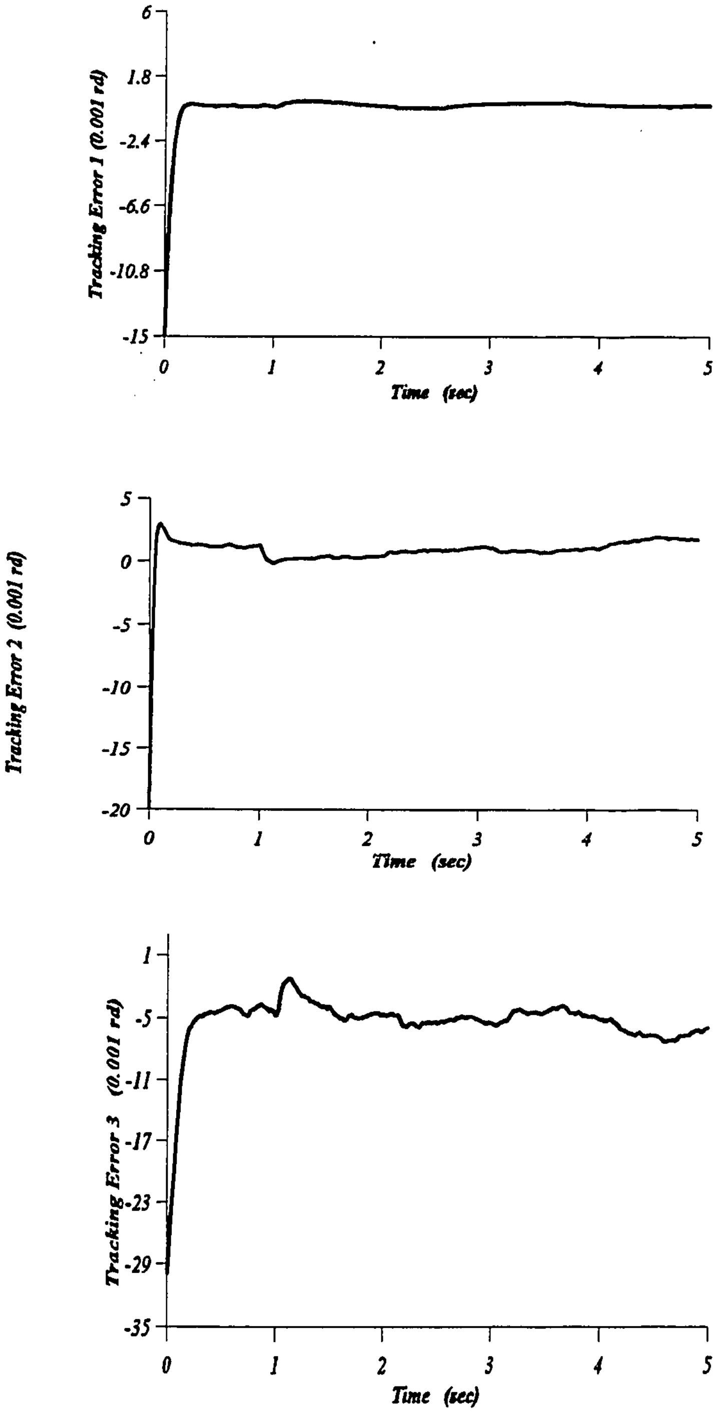

Figure 4.8 Servo Errors in Tracking Mode with Medium Step Change.

The results of the tracking test for small step change in the payload are given here. Small change was considered to be 25% change in the payload mass. The total mass of the third link and the payload was changed from 10kg to 7.5kg at time t=1sec. The tip path error and the servo errors are given in Figures 4.9 and 4.10 respectively. Figure 4.9 shows that perfect tracking is achieved in 0.37sec. The maximum deviation in the tip path error is 0.58mm.

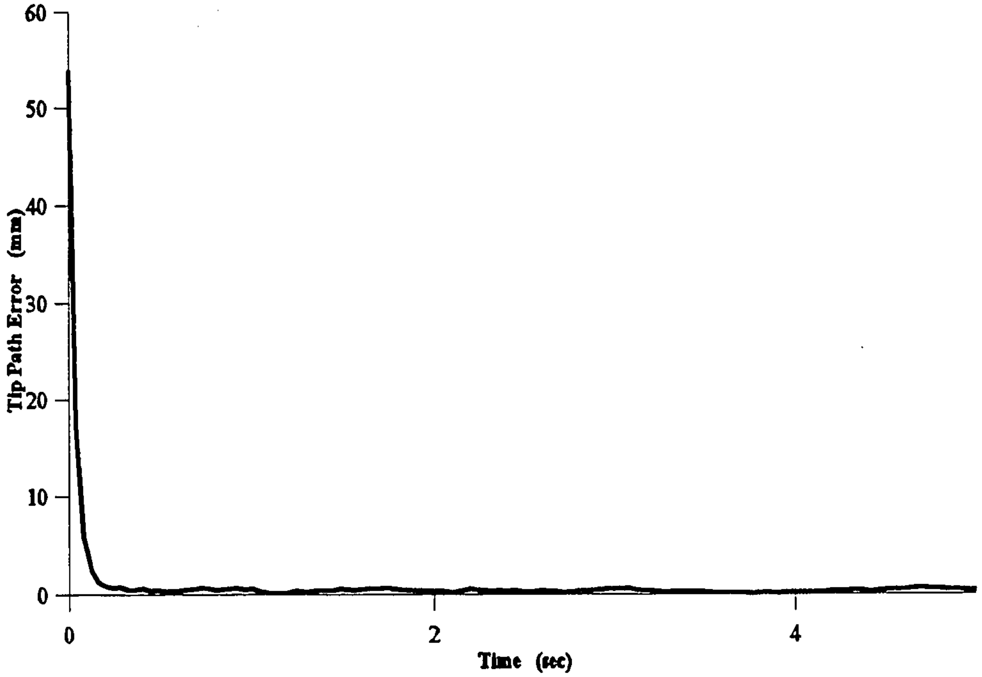

Figure 4.9 Tip Path Error for Tracking with Small Step Change.

Figure 4.10 shows that the deviation in the first joint is negligible and the maximum deviation in the second and third joint tracking errors are -0.00089rd and 0.003rd respectively. A steady state error equal to -0.00375rd remains in the third joint.

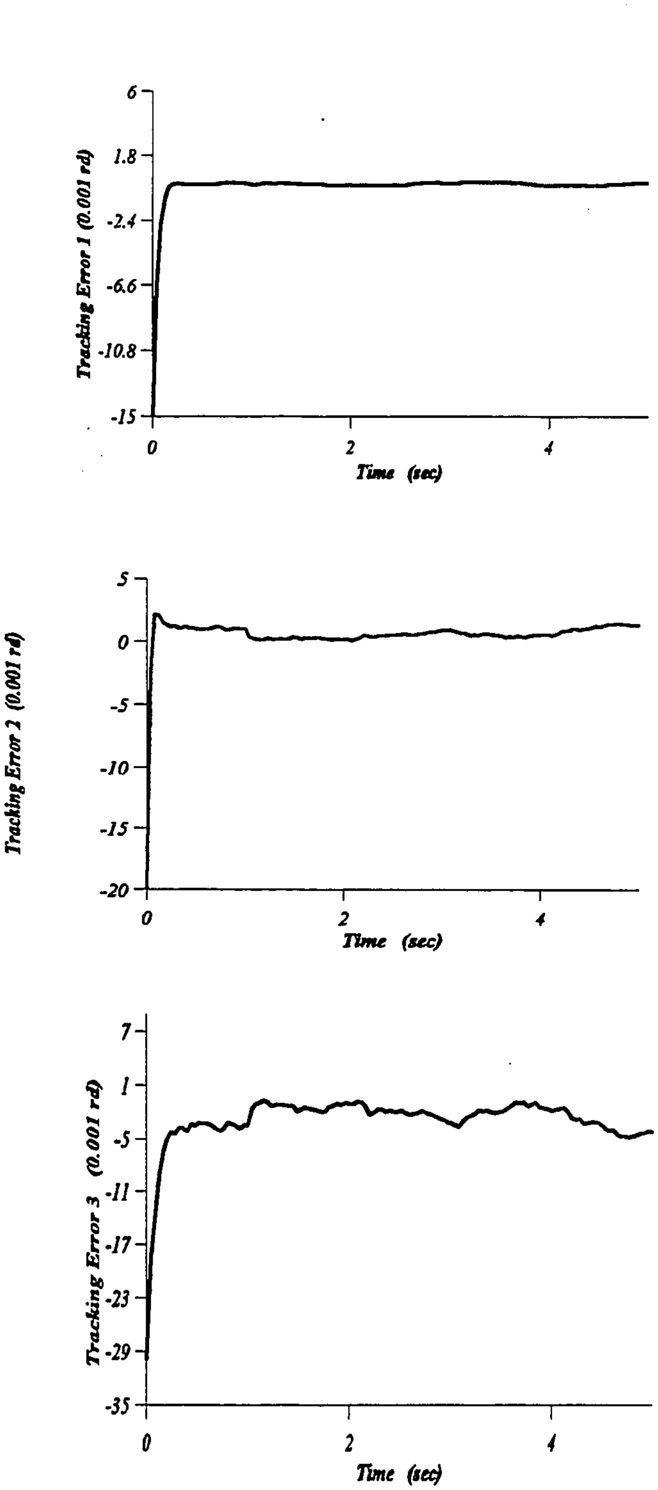

Figure 4.10 Servo Errors in Tracking Mode with Small Step Change.

The results of the tracking test for large step change in the payload are considered here. Large change in this mode was considered to be 75% change in the payload mass. The total mass of the third link and the payload was changed from 10kg to 2.5kg at time t=1sec. The tip path error and the servo errors are given in Figures 4.11 and 4.12. The figures show that acceptable tracking is achieved in 2.96sec. The maximum deviation in the tip path error is 3.73mm.

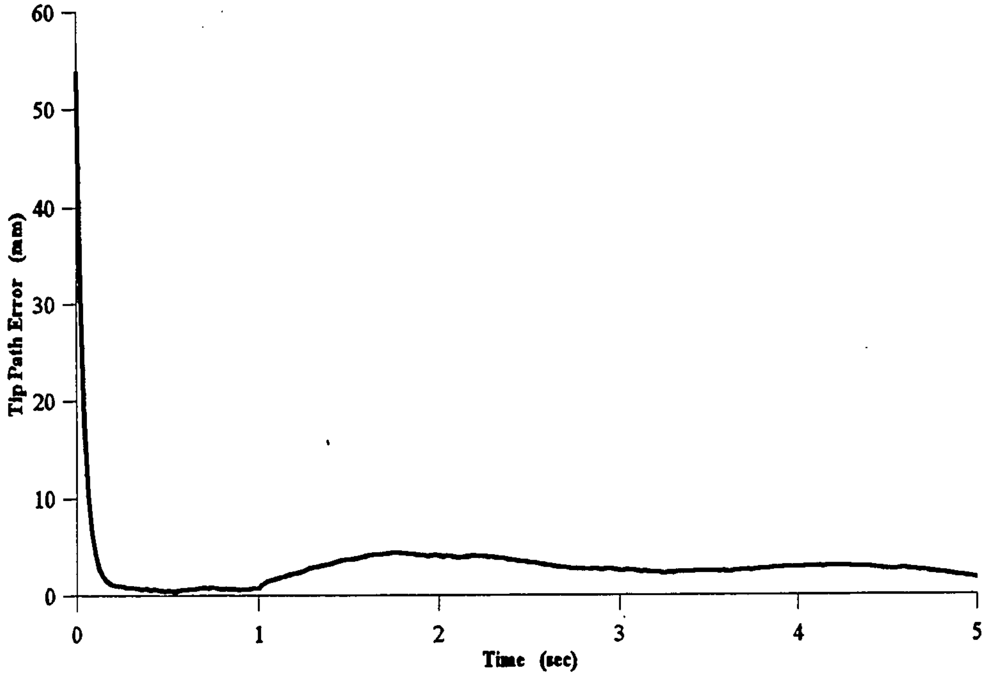

Figure 4.11 Tip Path Error for Tracking with Large Step Change.

Figure 4.12 shows that the maximum deviations in the first, second and last joint tracking errors are 0.00064rd, -0.00144rd and 0.00228rd respectively. A steady state error equal to -0.0129rd remains in the last joint.

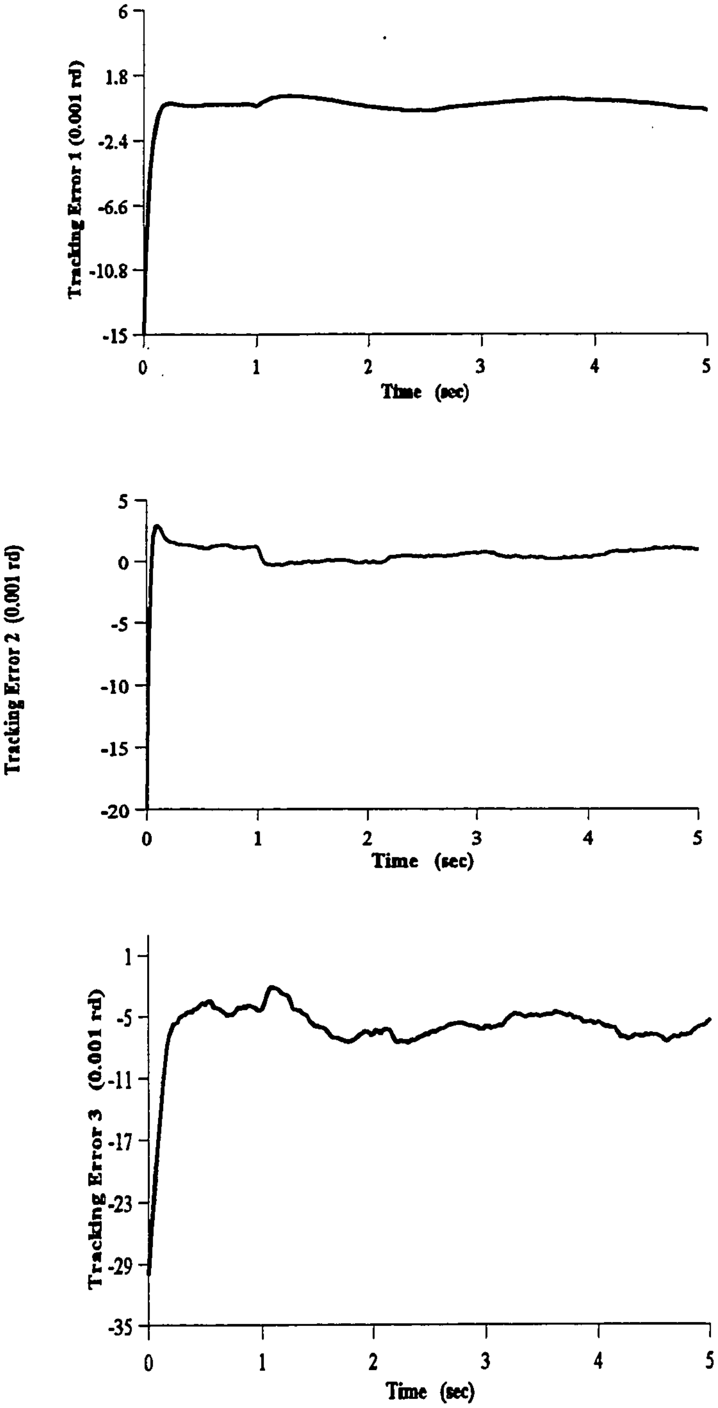

Figure 4.12 Servo Errors in Tracking Mode with Large Step Change.

4.6.3 Identification of the Step Change in Parameters

In this test, the first priority was given to the complete identification of the true values of the parameters, rather than fast tracking. Thus, to satisfy the condition for persistent excitation, the frequency in the desired trajectory was chosen from 5 rad/sec to 8 rad/sec. However, the choice of lower frequency is the condition for tracking action, which was considered in the previous subsection.

In this test, all parameters were tuned to the true values at the beginning and after two seconds a step change was imposed by dropping part of the payload. This affected the location of the center of mass of the last link and the new magnitude of the payload. To achieve a better evaluation of the results, two different sets of simulations were run: the identification of different step-size changes in parameters and the robustness to the external disturbance.

4.6.3.1 Identification of Different Step Changes in Parameters

The effect of step change in parameters on the identification was studied by dropping different parts of the payload. A reasonable value of the random disturbance amplitude, equal to 5% of the typical maximum control signal, was considered in all of the simulations.

The results of the identification test for medium step changes in the payload and the location of the center of mass are shown in Figure 4.13. A medium change was considered to be 50% change in the payload mass. The total mass of the last link and the payload was changed from 10kg to 5kg at time t = 2sec. In this test, the frequency of the desired trajectory was 8 rad/sec. By choosing realistic and reasonable gains for the controller and the adaptation mechanism, the complete identification of the step change took approximately 0.3 second. The tracking errors given in Figure 4.14 are disturbed for less than 0.4 second. The maximum deviation in the first, second and last joint servo errors are 0.045rd, -0.011rd and -0.02rd respectively.

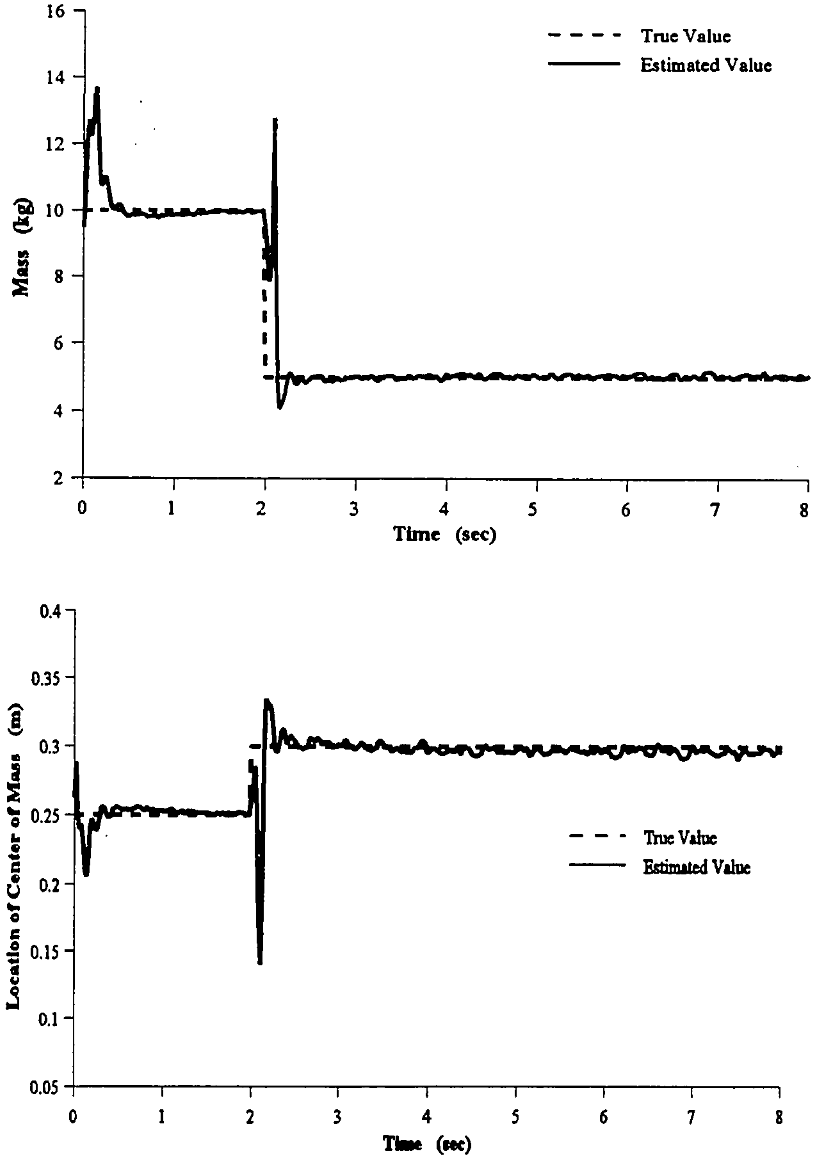

Figure 4.13 Identification with Medium Step Change and Disturbances.

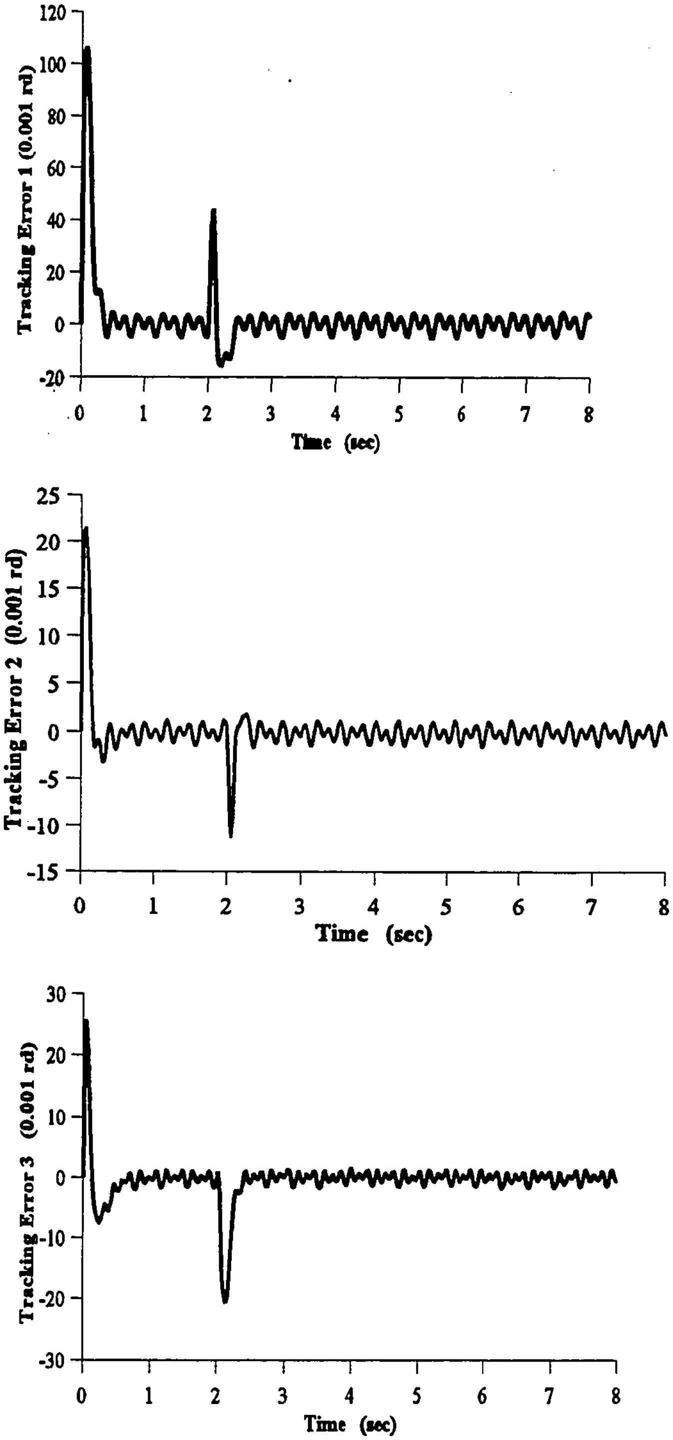

Figure 4.14 Tracking Errors in Identification with Medium Step Change.

The result of the identification of a small step change in the mass is shown in Figure 4.15. The small change is considered to be 25% change in the payload mass. The total mass of the last link and the payload was changed from 5kg to 3.75kg at time t = 2sec. Figure 4.15 shows the reversal overshoot after the step change is vanished or very much reduced. This is due to the small tracking errors. The tracking errors are given in Figure 4.16. The complete identification of the step change took between 0.2 and 0.3 second.

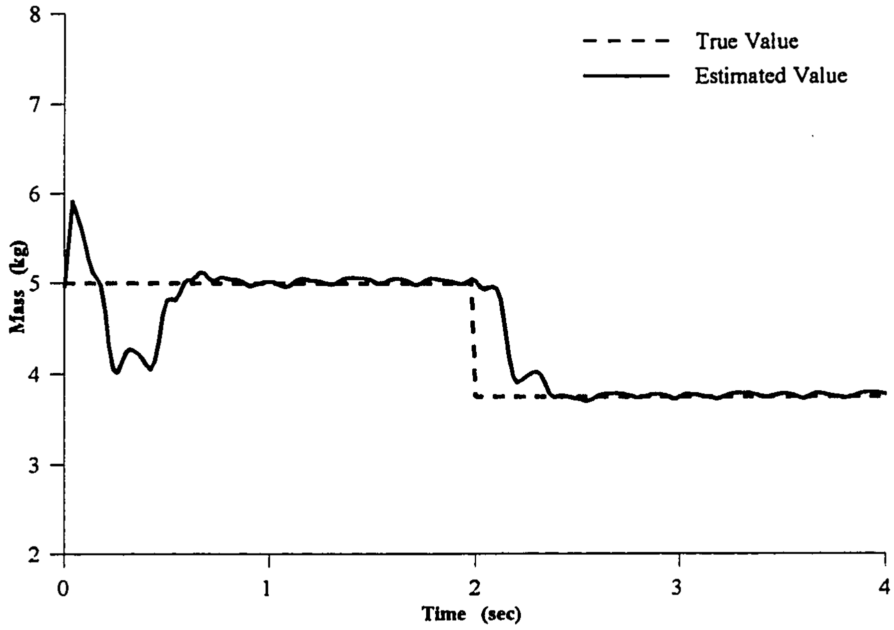

Figure 4.15 Identification with Small Step Change.

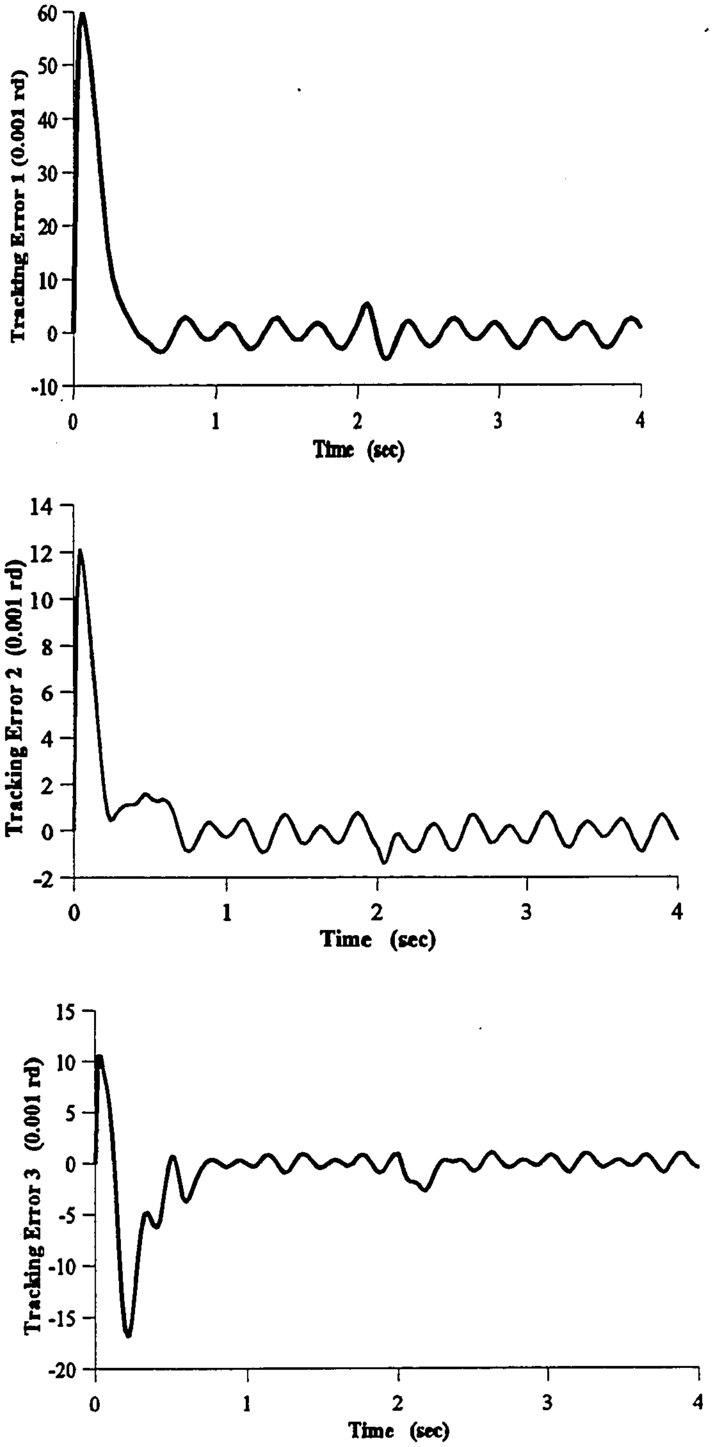

Figure 4.16 Tracking Errors in Identification with Small Step Change.

The results of the identification test for a large parameter step change are shown in Figure 4.17 and 4.18. The large change was considered to have a 100% change in the payload mass. The total mass of the last link and the payload was changed from 5kg to 10kg at time t = 2sec. Identification of the step change was achieved in one second as seen in Figure 4.17. The tracking errors given in Figure 4.18 diminish to acceptable levels in 1.3sec. The maximum deviations in the first, second and last joint servo errors are -0.05rd, 0.01rd and 0.039rd respectively.

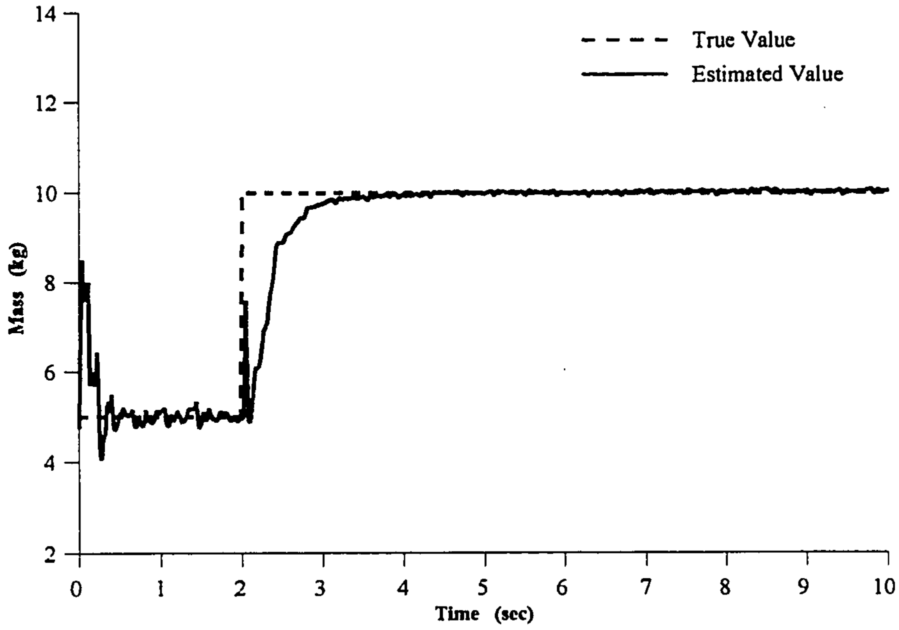

Figure 4.17 Identification with Large Parameter Step Change.

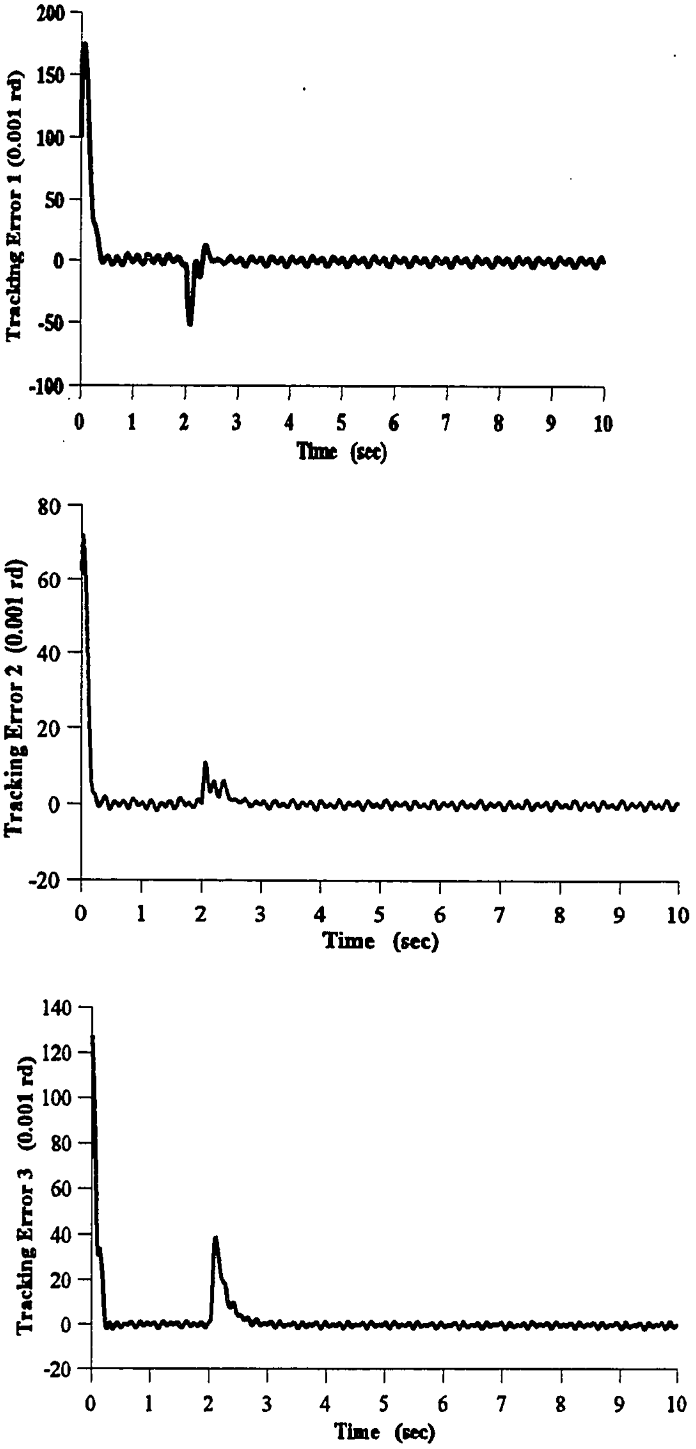

Figure 4.18 Tracking Errors in Identification with Large Parameter Step Change.

4.6.3.2 Robustness Test to External Disturbances

In this set of simulations, the magnitude of the random disturbance was changed in order to study the robustness of the control system to external noise acting on each joint. The test was performed with constant step parameter changes.

The results of the identification test and the related tracking errors for a typical step change with a medium magnitude disturbance are given in Figures 4.13 and 4.14. For this type of disturbance, the maximum magnitude of the noise vector was set to be approximately 5% of the typical maximum control signal vector. Perturbations in parameter identification varied between zero and 0.23kg for the actual value of 5kg. The steady state error in the mass identification was close to 0.1kg. The maximum deviations in the first, second and last joint servo errors are 0.045rd, -0.011rd and -0.02rd respectively.

Tests for small and large disturbance magnitude were also conducted to study the effect of noise on the system. Results of the simulations for the same step change with a small magnitude disturbance are given in Figure 4.19. The related tracking errors are shown in Figure 4.20. For a small disturbance, the maximum amplitude vector of the noise was chosen to be less than 1% of the typical maximum control signal vector. Steady state error in the parameter identification is very much close to zero.

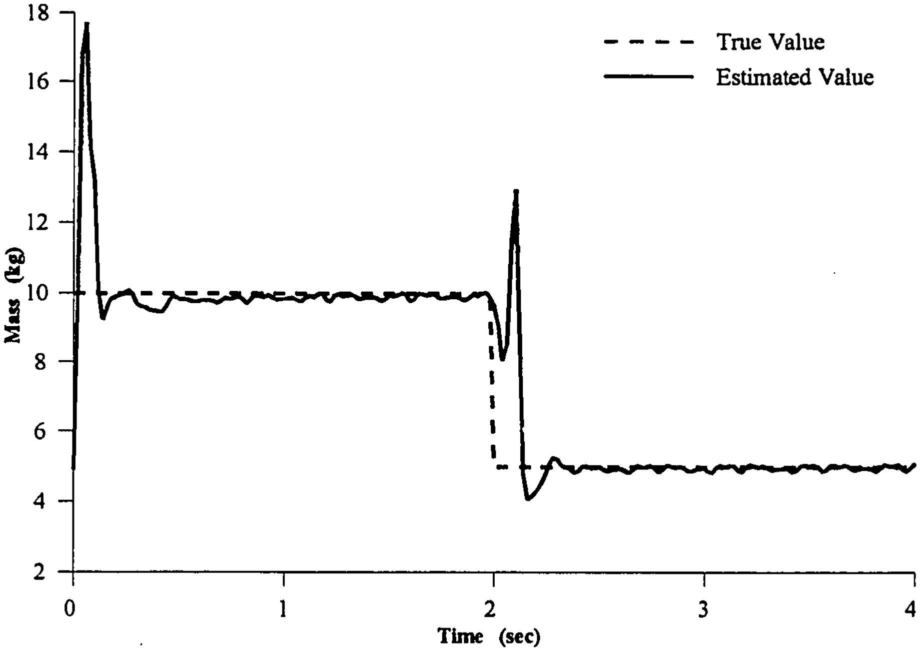

Figure 4.19 Identification with Small Amount of Disturbance.

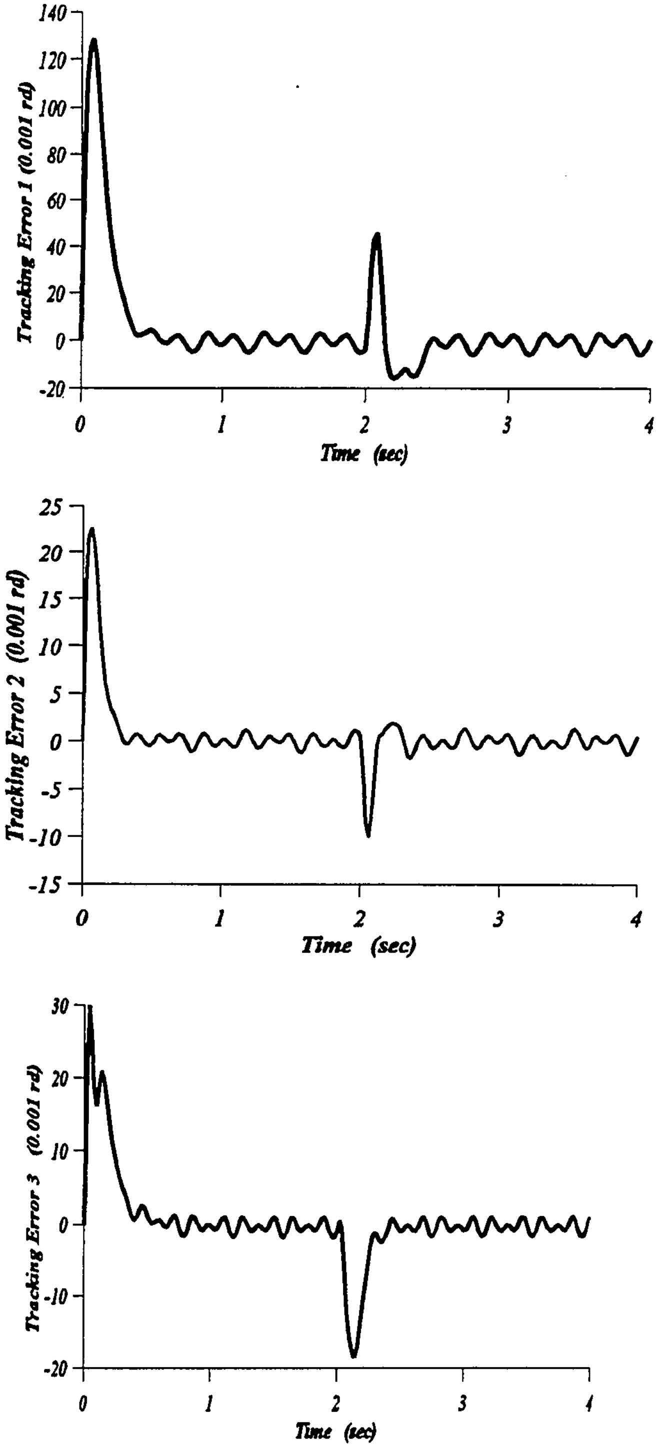

Figure 4.20 Tracking Errors in Identification with Small Disturbances.

Results of the simulations for the same step change with a large amount of disturbance are shown in Figure 4.21. A large disturbance in this set of experiments was chosen to have a maximum amplitude vector close to 50% of the typical maximum control signal vector. Perturbations in the mass identification varies between +16kg to -2.8kg for the true value of 5kg. The related tracking errors are shown in Figure 4.22.

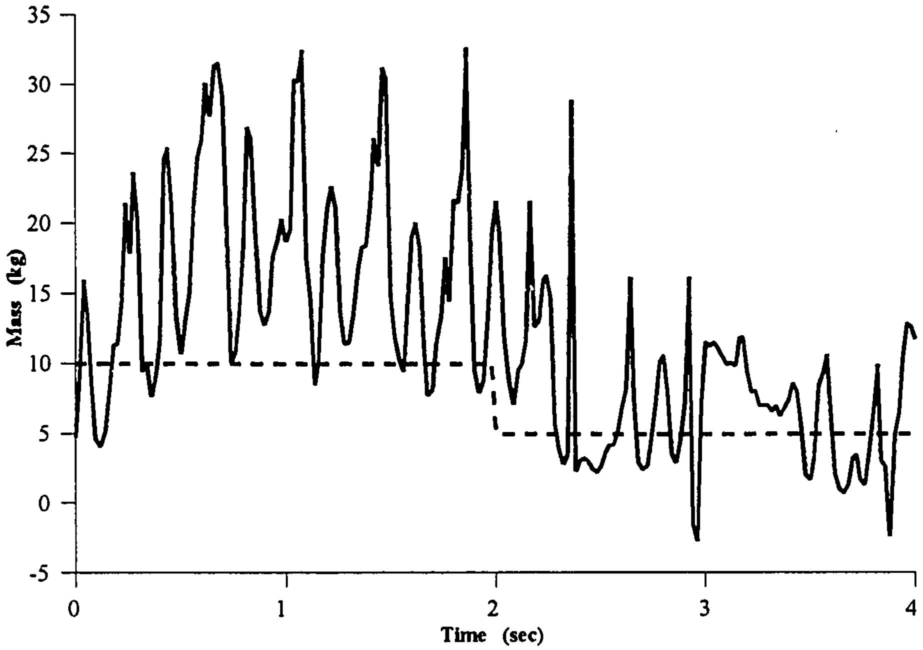

Figure 4.21 Identification with Large Amount of Disturbance.

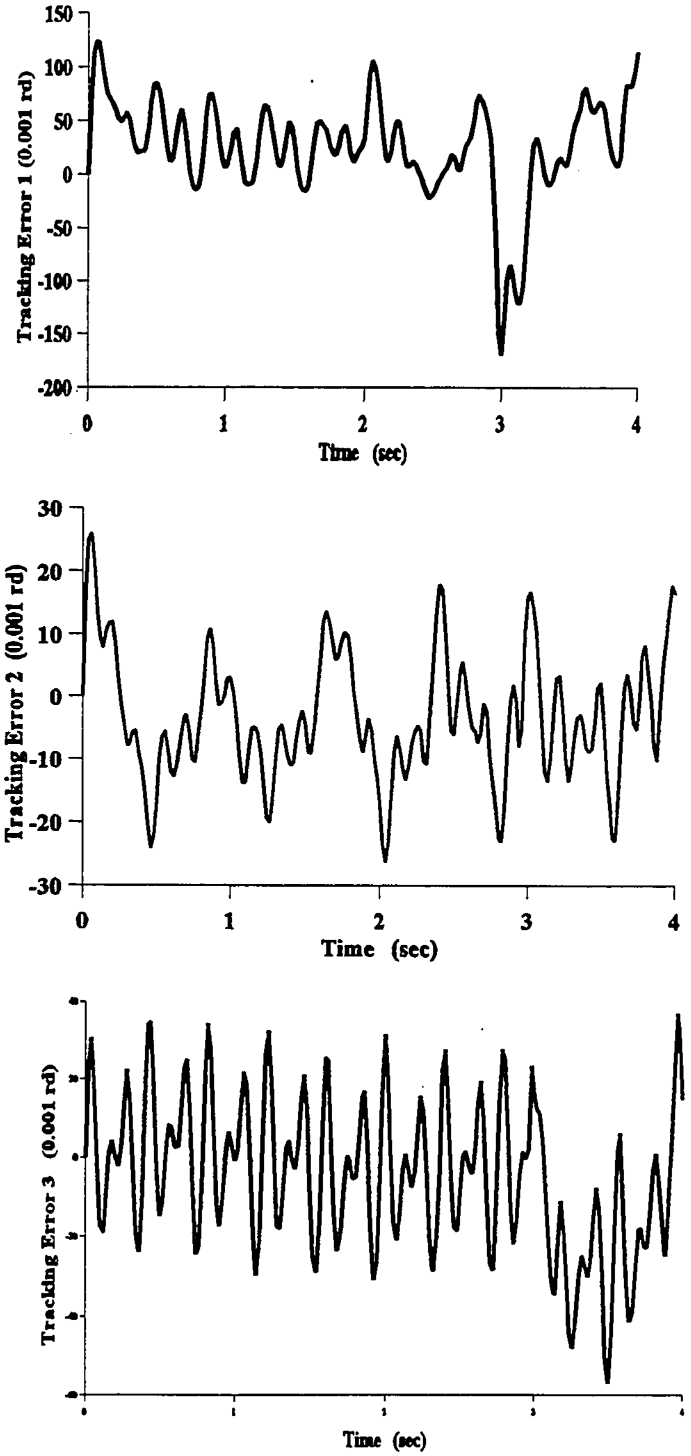

Figure 4.22 Tracking Errors in Identification with Large Disturbances.

Some tests were conducted with very large disturbance for the same step change. In these experiments the maximum amplitude vector of the disturbance was chosen more than 200% of the typical maximum control signal vector. The control system failed in achieving stable tracking of the parameters.

4.6.4 Identification with No A Priori Knowledge

In this test, the first priority was given to the complete identification of the true values of the parameters, rather than fast tracking. Thus, to satisfy the condition for persistent excitation, the frequency in the desired trajectory was chosen to be high. Some results of the identification test, with $\omega = 5$ are given in Figures 4.23 and 4.24. The system was assumed with no a priori knowledge of parameter functions, to simulate a situation with complete unknown parameters. By choosing realistic and reasonable controller and adaptation gains, the complete identification took approximately 6.5 seconds. This may seem slow in some applications, and will be improved with some a priori knowledge. The small and acceptable steady state errors in identifications, which were between one and 3%, show the effectiveness of the persistency in excitations. Small disturbances in the tracking errors, are due to the high frequency in the desired trajectory. There is no steady state error in joint trackings.

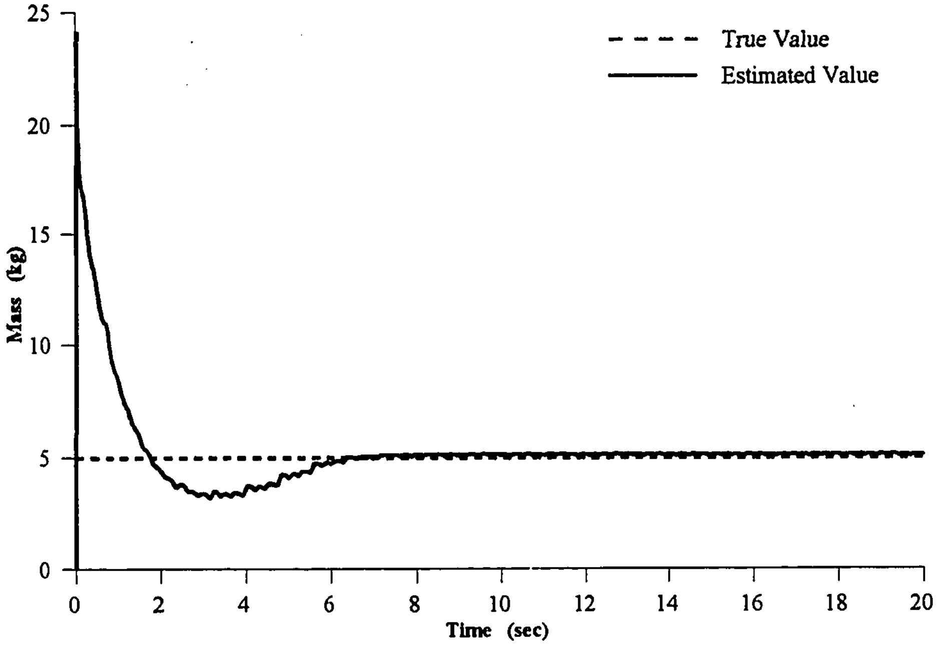

Figure 4.23 Identification with No A Priori Knowledge

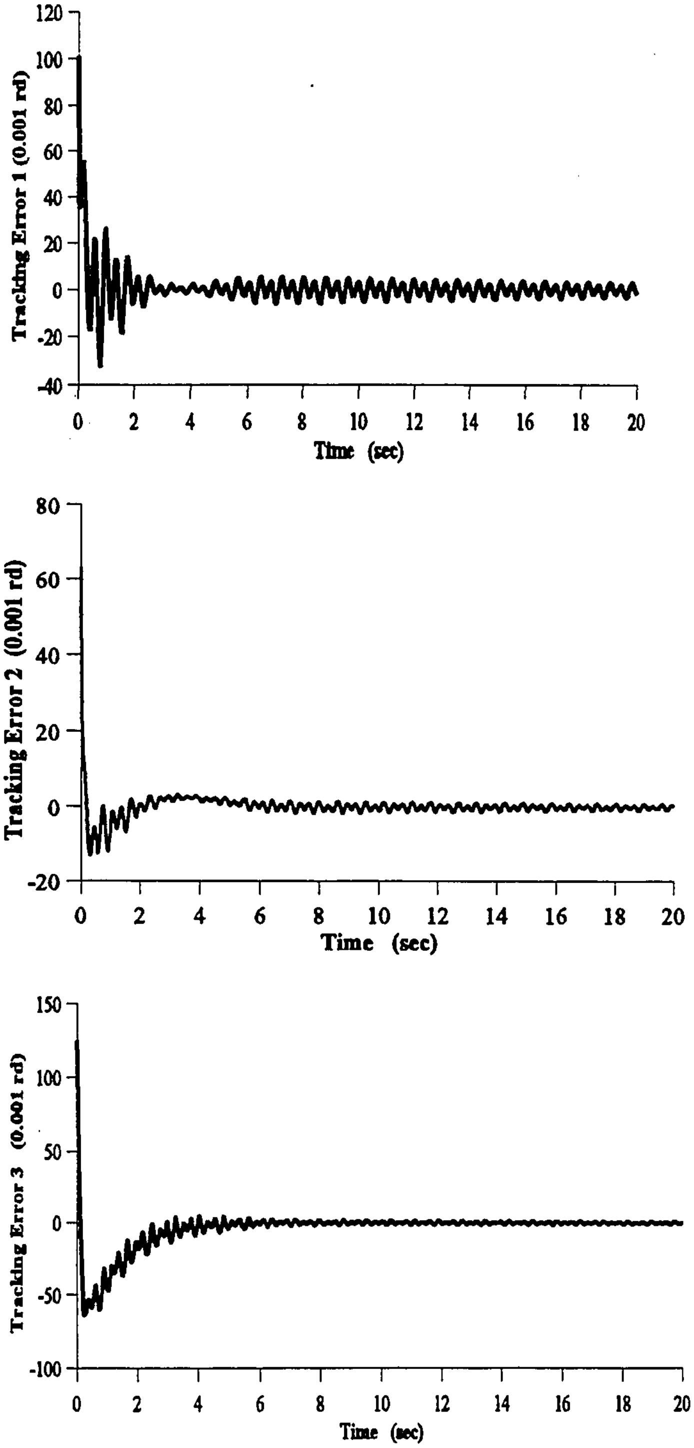

Figure 4.24 Tracking Errors in Identification with No A Priori Knowledge

The same test was conducted with random disturbances to simulate dry friction in the joints and other unmodelled dynamics. The results are shown in Figures 4.25 and 4.26. Disturbances on every joint were considered to have a maximum amplitude equal to 50% of the typical maximum control signal.

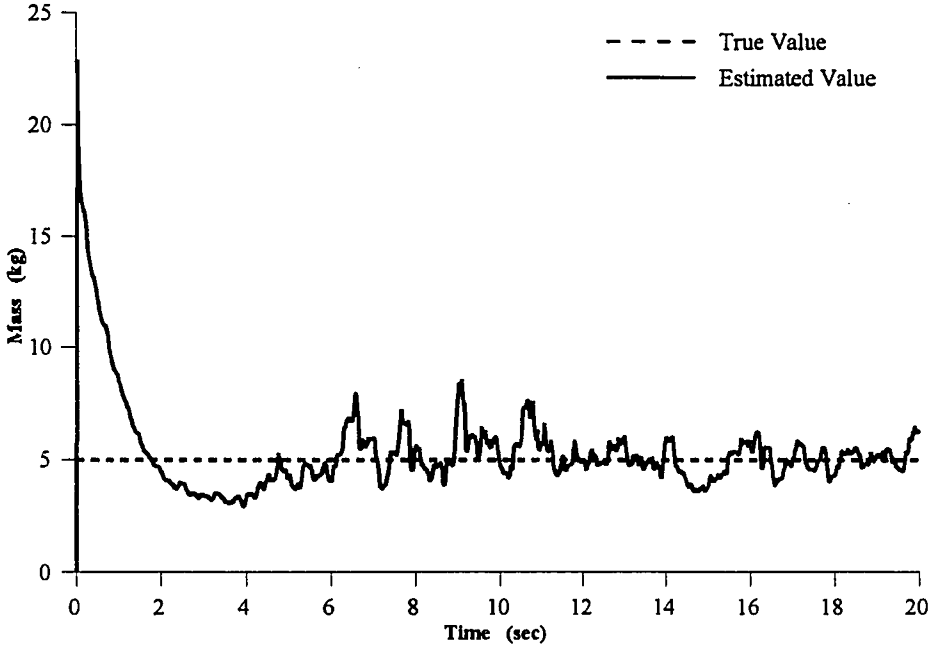

Figure 4.25 Identification with No A Priori Knowledge and Disturbance.

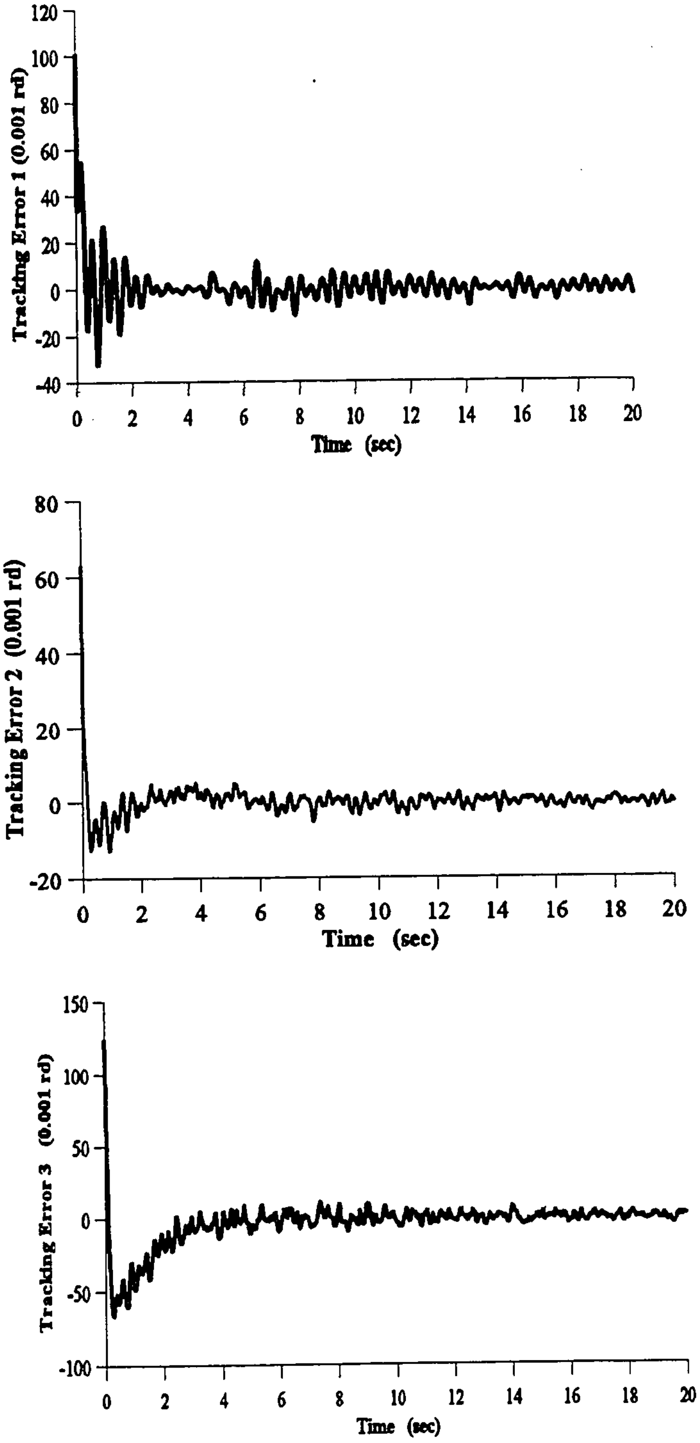

Figure 4.26 Tracking Errors in Identification with No A Priori Knowledge and Disturbance.

4.6.5 The Effect of Manipulator Parameters

The analytical formulations of the dynamics and control of the robot manipulator presented in this thesis are general. However, a few assumptions, such as link rigidity, were made. The general design of the control system gives good flexibility in the choice of manipulator parameters for the simulations. The only modifications required are in the controller gains and the adaptation mechanism gains, which should be adjusted.

The results of the simulations for a different set of manipulator parameters are given in Figures 4.27 and 4.28. The new parameters of the manipulator were selected to be

$$\begin{array}{ll} m_2 = 20kg & , \; m_3 = 15kg \\ a_2 = 1.5m & , \; a_3 = 1m \\ e_2 = 0.7m & , \; e_3 = 0.4m \end{array} \qquad \textbf{(4.81)}$$

The total mass of the last link and the payload was changed from 15kg to 7kg at time $t = 2$sec. The system performance appears quite similar to the manipulator under the same conditions, i.e., the size of the parameter step change and the amount of noise. This may be concluded from a comparison of the results with Figures 4.13 and 4.14.

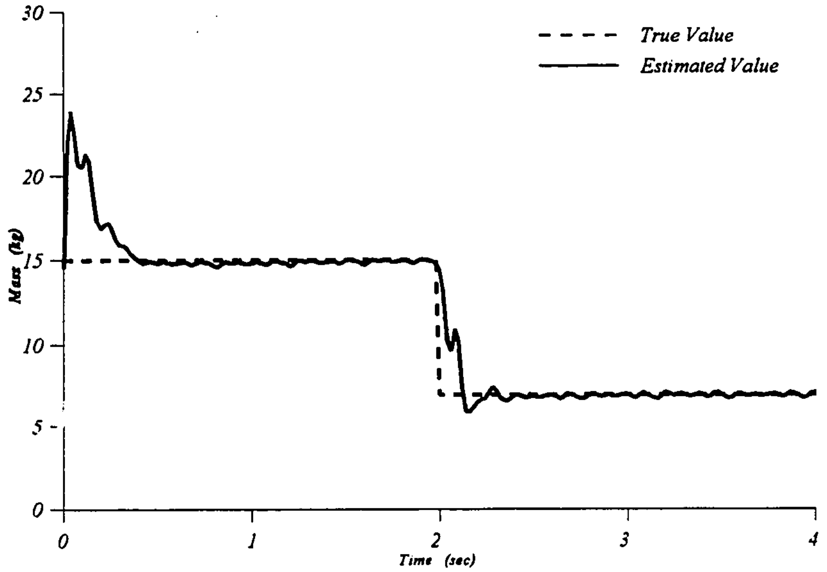

Figure 4.27 Identification with Different Manipulator Parameters.

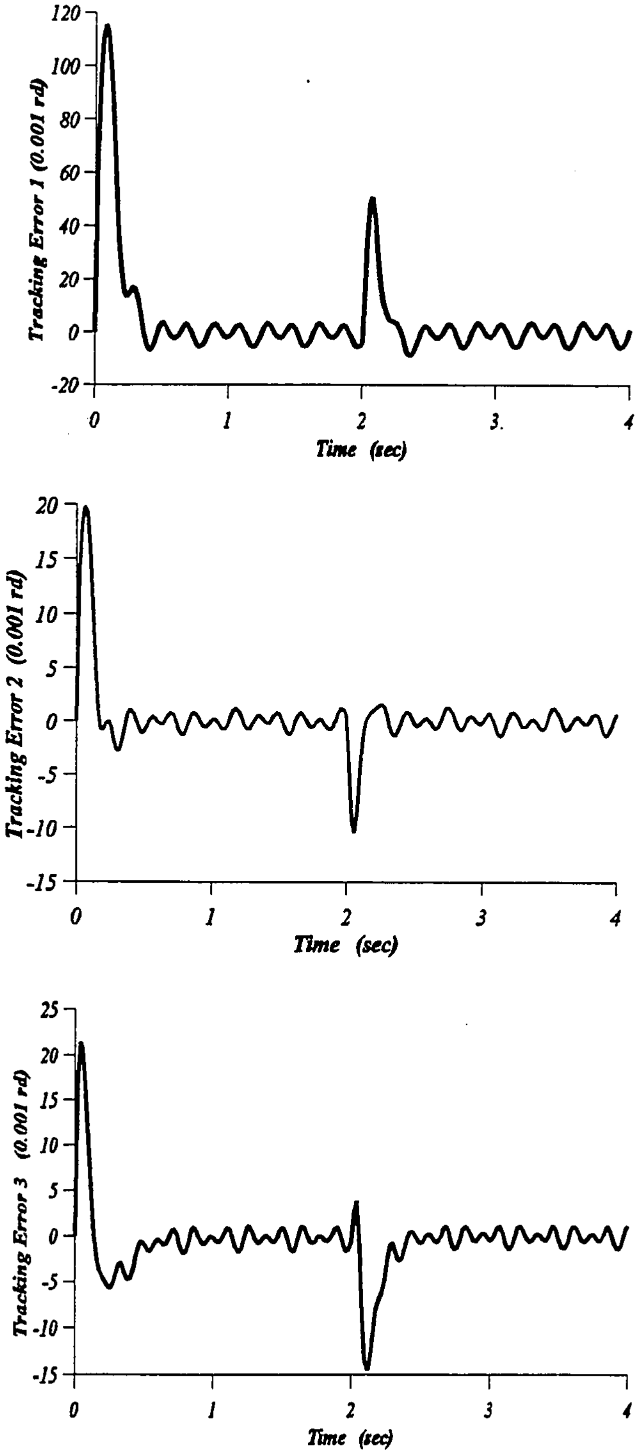

Figure 4.28 Tracking Errors for Different Manipulator Parameters.

4.6.6 Summary for Simulations

The results of the simulations for different step change in parameters show that the system is stable if the magnitude of disturbance is not large. However, in the presence of large magnitude disturbance, the size of the step change will have a critical effect on the stability of the system.

In the test with a large step parameter change (i.e., 100% change in the mass and 5% disturbance magnitude,) the identification of a 5kg mass change required 1.3sec. However, this is a limitation for the case where the system examines a sequence of parameter changes, and a fast processor for running the adaptation algorithm is not available.

The results of the robustness test show that, for the small and medium disturbance magnitude, the adaptive scheme is robust and capable of coping with the large change in parameters. By increasing the disturbance magnitude to 50%, the identification process loses its accuracy and the estimates of the 5kg mass fluctuate in a large and unacceptable range between -2.8kg to +16kg. The tests with stronger disturbance failed due to the instability of the tracking errors and the loss of parameter convergence. In such situations, a switch to the low-level scheme is again imposed by the monitoring function

The main reason for such correlation between the large parameter change and external disturbance may be concluded from the Lyapunov stability and robustness analysis of the system. The negative effect of the disturbance should be compensated with the other terms that are parameter dependent. For small change in parameters, this negative effect is well compensated. However, in the case of large and abrupt change in parameters, the system will become unstable. The theoretical justifications for this phenomenon will be presented in the robustness study in Section 5.4.

Chapter 5

Switching Adaptive-PD Control of Mechanical Manipulators

5.1 Introduction

The design of the overall control system is presented in this chapter. The general concept for the proposed switching controller was discussed in Chapter 3. Three major parts should be considered in the design: the adaptive control scheme with on-line parameter estimation and adaptation, the low-level controller that is a simple PD regulator with an on-line estimator, and the switching mechanism.

The major components of the system are shown in Figure 5.1, and are discussed in detail in the following sections. Stability is considered in all three parts. The adaptive control scheme was designed and illustrated in Chapter 4. The low level controller with the on-line estimator and switching mechanism are discussed in this chapter.

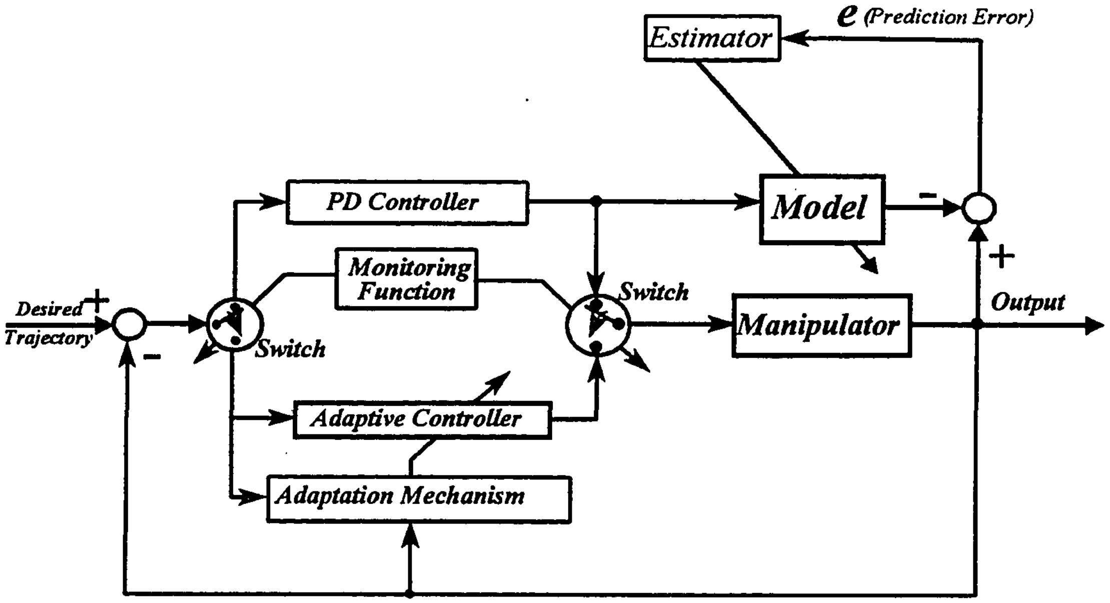

Figure 5.1 Switching Adaptive-PD Controller.

5.2 On-Line Parameter Estimator

As discussed in Chapter 3, the system remains in the adaptive mode while working under normal conditions that contain continuous and moderate changes of parameters. The system switches to the PD controller for a limited period when abrupt and/or large changes in parameters, that may impede the operation of the high-level controller, occur. On-line parameter estimation is required while the system is controlled by such a robust controller. This is to identify the change in the parameters of the manipulator during the course of operation of the low-level regulator. The identification process is necessary for smooth switching from PD to adaptive controller in the next stage of the process.

The required characteristics for the estimator are: high speed in identifying large stepwise changes in parameters and good robustness to disturbances and noise. Modification for tracking the time varying parameters is presented in Section 5.5. The least squares parameter estimator (Astrom and Wittenmark, 1989; Li and Slotine, 1987) was selected as the basis for designing the on-line estimator. The scheme has the capability of averaging out disturbances and noise (Slotine and Li 1991).

A linearized estimation model is necessary to implement the least-squares estimator. This model establishes the following linear relationship between the vector of unknown parameter functions and the available data from the system:

$$y(t) = W(t)\,\rho \tag{5.1}$$

Where $y(t)$ is the n dimensional output vector of the system, $W(t)$ is the $n \times m$ regressor matrix that is a trajectory dependent signal, and ρ is the unknown parameter functions vector. The unknown parameter functions are defined differently from those in the previous chapter, although they are functions of the same parameters. The choice of $y(t)$ as the first order filter of the rate of change of mechanical energy is most appropriate for the manipulator, and is given by

$$y(t) = [\frac{d}{dt}E(\theta,\dot{\theta})]_f \tag{5.2}$$

By the principle of energy conservation, the rate of change of mechanical energy is equal to the power input to the actuators, i.e.

$$\frac{d}{dt}E(\theta,\dot{\theta}) = T^{T}\dot{\theta} \tag{5.3}$$

The total mechanical energy in the manipulator under study is expressed as

$$\begin{aligned} E &= K + P \\ &= \frac{1}{2}[m_2 e_2^2 c_2^2 + m_3(a_2 c_2 + e_3 c_{23})^2]\dot{\theta}_1^2 + \frac{1}{2}[m_2 e_2^2 + m_3(a_2^2 + e_3^2 + 2a_2 e_3 c_3)]\dot{\theta}_2^2 \\ &+ \frac{1}{2}m_3 e_3^2\dot{\theta}_3^2 + m_3(e_3^2 + a_2 e_3 c_3)\dot{\theta}_2\dot{\theta}_3 + m_2 g e_2 s_2 + m_3 g(a_2 s_2 + e_3 s_{23}) \end{aligned} \tag{5.4}$$

where ***K*** and ***P*** are given in Equation 4.8. A linear parameterization results in the following equation:

$$E(\theta,\dot{\theta}) = V(\theta,\dot{\theta})\wp = [V_1\ V_2\ V_3\ V_4]\begin{bmatrix} \wp_1 \\ \wp_2 \\ \wp_3 \\ \wp_4 \end{bmatrix} \tag{5.5}$$

where the elements of $V(\theta,\dot{\theta})$ are given by

$$\begin{aligned} V_1 &= \frac{1}{2}[c_2^2\dot{\theta}_1^2 + \dot{\theta}_2^2] \\ V_2 &= \frac{1}{2}c_{23}\dot{\theta}_1^2 + \frac{1}{2}(\dot{\theta}_2^2 + \dot{\theta}_3^2) \\ V_3 &= a_2 c_2 c_{23}\dot{\theta}_1^2 + a_2 c_3(\dot{\theta}_2^2 + \dot{\theta}_2\dot{\theta}_3) + g s_{23} \\ V_4 &= g s_2 \end{aligned} \tag{5.6}$$

and the elements of parameter functions vector $\wp$ are given by

$$\begin{aligned}\wp_1 &= m_2 e_2^2 + m_3 a_2^2 \\ \wp_2 &= m_3 e_3^2 \\ \wp_3 &= m_3 e_3 \\ \wp_4 &= m_2 e_2 + m_3 a_2\end{aligned} \tag{5.7}$$

Solving Equations 5.3 and 5.5 gives

$$\frac{dV(\theta,\dot{\theta})}{dt}\,\wp = T^T\dot{\theta} \tag{5.8}$$

Since the joint acceleration terms in Equation 5.8 are difficult to measure, the application of a first order filter $\frac{1}{s+\lambda}$ to both sides of the equation reduces these terms to lower order terms, i.e., velocities that can be easily measured. In this filter, s is the laplace operator, and, λ is a constant positive gain. The left side of the Equation 5.8, after the application of the filter, may be written, in the s domain, as follows:

$$\begin{aligned}\frac{sV(s)}{s+\lambda}\,\wp &= (1-\frac{\lambda}{s+\lambda})\,V(s)\,\wp \\ &= [\,V(s)-\lambda\,(\frac{1}{s+\lambda})V(s)\,]\,\wp\end{aligned}$$

Thus, the Equation 5.8, in the time domain, is given as

$$(V-\lambda V_f)\,\wp = [T^T\dot{\theta}]_f \tag{5.9}$$

where the index f stands for the filtered quantities. The filtering process introduces a gain of $1/\lambda$. Therefore, the filtered signals will be smaller than the real values. This will decrease the estimation speed by a factor of λ. In the discrete implementation of the filter, the magnitude of λ was chosen equal to 0.99, which does not have a significant effect on the speed of estimation process. For smaller λ, one may multiply both sides of the equation by a constant. By choosing the regressor matrix in Equation 5.1 as

$$W = V - \lambda V_f \tag{5.10}$$

The estimation model assumes a form of energy, leading to a significant decrease in the computation time as compared with traditional models.

5.2.1 Parameter Update Law

While the parameter update law utilized the tracking error in the MRAC derived in the previous chapter, here, the parameter update law is driven by the prediction error as defined in the following equation

$$e(t) = \hat{y}(t) - y(t) \tag{5.11}$$

where $\hat{y}(t)$ is the predicted output derived from the estimation model, and based on the estimated parameter functions vector $\hat{\rho}$. $\hat{y}(t)$ is given by

$$\hat{y}(t) = W(t)\,\hat{\rho} \tag{5.12}$$

The estimated parameters at time t are generated by minimizing j, the total function of the prediction error given in the following relationship, with respect to $\hat{\rho}$:

$$J = \int_0^t \| y(s) - W(s)\,\hat{\rho}(t) \|^2 ds \tag{5.13}$$

Substituting for norm of a function as $\|F\| = F^T . F$ in the minimization procedure, results in

$$\int_0^t [-\,2\,W^T(s)\,y(s) + 2W^T(s)W(s)\,\hat{\rho}(t)]\,ds = 0 \tag{5.14}$$

Rearranging to isolate $\hat{\rho}$ gives

$$\hat{\rho}(t) = [\int_0^t W^T(s)W(s)\,ds]^{-1} \int_0^t W^T(s)\,y(s)\,ds \tag{5.15}$$

Let

$$\Gamma(t) = [\int_0^t W^T(s)W(s)\,ds]^{-1} \tag{5.16}$$

the parameter update law in the integral form is derived as follows

$$\hat{\rho}(t) = \Gamma(t) \int_0^t W^T(s)\,y(s)\,ds \tag{5.17}$$

To achieve a recursive algorithm rather than computing the integral in every time step, a differential equation form of the update law is more desirable. The time derivative of Equation 5.17 and the definition 5.16 leads to

$$\Gamma^{-1}\dot{\hat{\rho}} = W^T y - W^T W \hat{\rho} \tag{5.18}$$

Using the definition of the prediction error, results in the differential equation form of the parameter update law

$$\dot{\hat{\rho}}(t) = -\Gamma(t)\, W^T(s)\, e(t) \tag{5.19}$$

where $\Gamma(t)$ being the estimator gain matrix, needs to be updated on-line through a gain update law. Using the fact that $\frac{d}{dt}(\Gamma\Gamma^{-1}) = 0$ gives:

$$\dot{\Gamma}\Gamma^{-1} + \Gamma \frac{d}{dt}(\Gamma^{-1}) = 0 \tag{5.20}$$

The time derivative of the inverse of the gain matrix is obtained from the Equation 5.16 as follows

$$\frac{d}{dt}[\Gamma^{-1}] = W^T(t) W(t) \tag{5.21}$$

Solving Equations 5.20 and 5.21, gives the estimator gain update law as follows:

$$\dot{\Gamma} = -\Gamma W^T(t) W(t) \Gamma \tag{5.22}$$

5.3 Stability and Robustness of the System When Switching From MRAC to PD Control

5.3.1 General

The control system under study switches back and forth between the high level (adaptive) controller and the low level (PD) regulator. In this section the stability and robustness of the first switching action are studied. The low level controller is a local scheme consisting of independent linear controllers at each joint, in the following form

$$T = K_V\dot{\tilde{\theta}} + K_P\tilde{\theta} \qquad \textbf{(5.23)}$$

where $\tilde{\theta} = \theta_d - \theta$, and θ_d is the fixed vector of desired joint displacements.

5.3.2 Stability

The structure of the PD robot regulator shown in Equation 5.23 may be intuitively viewed as an appendage of artificial springs and dampers attached to each joint of a mechanical manipulator. These passive mechanical elements will force the system toward a set-point. Therefore, the stability and robustness are intuitively acceptable, however, theoretical justifications are considered in this section.

Simple PD controllers have extensively been used by engineers in industrial robots. However, due to high nonlinearity in dynamics of manipulators, the stability was not justified theoretically for a considerable period of time. Arimoto and Miyazaki (1983) have proven the global stability of such schemes by using the Lyapunov stability theory. The Lyapunov function candidate

$$V = \frac{1}{2}\left[\dot{\theta}^T H \dot{\theta} + \theta^T K_p \theta\right] \tag{5.24}$$

is suggested for the study of the global stability of the system. The function is positive except in $\theta = \theta_d$, because both quadratic terms are positive as a result of symmetric and positive definiteness of the matrices K_P and H. The time derivative of the function is

$$\dot{V} = \frac{1}{2}\dot{\theta}^T \dot{H} \dot{\theta} + \dot{\theta}^T H \ddot{\theta} + \tilde{\theta}^T K_P \dot{\tilde{\theta}} \tag{5.25}$$

It is to be noted that friction promotes the stability. In the absence of friction and gravity in Equation 4.17 and considering $\dot{\tilde{\theta}} = -\dot{\theta}$, the above equation results in

$$\dot{V} = \frac{1}{2}\dot{\theta}^T \dot{H} \dot{\theta} + \dot{\theta}^T [T - C(\theta,\dot{\theta})\dot{q}] - \tilde{\theta}^T K_P \dot{\theta} \tag{5.26}$$

Applying the control law in Equation 5.23 yields

$$\dot{V} = \frac{1}{2}\dot{\theta}^T \dot{H} \dot{\theta} + \dot{\theta}^T K_P \tilde{\theta} - \dot{\theta}^T K_V \dot{\theta} - \dot{\theta}^T C(\theta,\dot{\theta})\dot{\theta} - \tilde{\theta}^T K_P \dot{\theta} \tag{5.27}$$

Using $\dot{H} = 2C$, from Equation A.9 in Appendix A, it can be concluded that

$$\dot{V} = -\dot{\theta}^T K_V \dot{\theta} \leq 0 \tag{5.28}$$

Furthermore, the manipulator arm cannot remain at any position where $\tilde{\theta} \neq \mathbf{0}$. This may be concluded from the proof of non-zero acceleration in such configurations (Asada and Slotine, 1986). Therefore, $\dot{\boldsymbol{\theta}}$ and $\dot{V}$ cannot remain zero in these situations and the manipulator must settle down only at positions where $\tilde{\boldsymbol{\theta}} = \mathbf{0}$ and $\dot{\boldsymbol{\theta}} = \mathbf{0}$. Thus, V is a Lyapunov function and the system is globally asymptotically stable.

In the presence of disturbance, another term, non-correlated with the states, will be added to the Equation 5.28. Negativeness of the new $\dot{V}$, therefore the global stability of the system, would be guaranteed, by a suitable choice of $\boldsymbol{K}_V$ for the a bounded disturbance.

5.3.3 Robustness to Parameter Uncertainties and Disturbances

Referring to the discussions presented in the earlier section, the robustness of the resulting mechanical system is also intuitively verified. As mentioned earlier, theoretical justifications are also required.

In the stability study of PD robot controllers, the time derivative of the Lyapunov function in Equation 5.28 was shown to be independent of the parameters of the manipulator, i.e., $\dot{V}$ depends only on $\boldsymbol{K}_V$. This implies that the rate of convergence is independent of the mass parameters. Therefore, the control law is robust to parameter uncertainties. In practice, upper bounds are imposed on $\boldsymbol{K}_P$ and $\boldsymbol{K}_V$ due to the presence of high frequency unmodeled dynamics, and an integral control action with the gain $\boldsymbol{K}_i$ is added to the control law. This will constantly attenuate the errors caused by dry friction and gravity.

Two major problems occur in the switching action from the high-level controller to the PD regulator: parameter uncertainty and disturbances. Any change in the states and the value of the control signal, will act as an initial condition for this linear local scheme. Thus, the convergence of the states will be guaranteed, as long as the upper bounds for the gains tolerate the closed loop error dynamics.

The stability and robustness study in Section 5.3, confirms the stability and robustness of the low level controller to the parameter uncertainties and external disturbances. A critical study is required for the higher level controller, due to the parameter dependency of the time derivative of its Lyapunov function. This subject is discussed in the following section.

5.4 Robustness of the System in Switching from PD to MRAC

5.4.1 Introduction

The stability of the adaptive scheme was studied in Chapter 4 while designing the control system and the adaptation mechanism. Using the proper adaptation law, global stability was guaranteed and convergence of the parameter estimates was achieved. This was investigated only with parameter uncertainty, whereas the effect of disturbances was neglected. The presence of disturbances is a critical issue, especially in the switching action under study. The switching action from PD to MRAC is a potential source of instability in the control system. Thus the present section focuses on the robustness issue of the proposed MRAC.

In the robustness study of the adaptive system, two major sources for modeling errors are considered. The first is the parameter mismatch $\tilde{\boldsymbol{P}}$, which was called the vector of parameter functions errors in Chapter 4. The second is the disturbance that was not considered in the stability study and design of the adaptive system, although both the parameter mismatch and the disturbances were imposed in the simulations to achieve more realistic results. It is easy to show from the structure of the Lyapunov function and its time derivative that the unbounded disturbances will make the system unstable. In this section the robustness of the system to bounded disturbances is studied.

5.4.2 Robustness - Switching Criterion for Monitoring Function

Recalling Equation 4.40 and adding a disturbance torque vector $\tilde{D}$ leads to

$$\tilde{T} = \tilde{H}(\theta)\ddot{\theta} + \tilde{V}(\theta,\dot{\theta}) + \tilde{G}(\theta) + \tilde{D} = K(\theta,\dot{\theta},\ddot{\theta})\tilde{P} + \tilde{D} \tag{5.29}$$

This will alter the nonlinear Equation 4.64 governing the system to

$$f = \dot{x} = Ax + B\hat{H}^{-1}K\,\tilde{P} + \nu \tag{5.30}$$

where $\nu = B\hat{H}^{-1}\tilde{D}$ is a $2n \times 1$ vector. Substituting the above equation in the Lyapunov function candidate in Equation 4.62, and calculating the time derivative gives

$$\begin{aligned}\dot{V} &= (\alpha^T+\beta^T+\nu^T)(\gamma)(\alpha+\beta+\nu) + 2\tilde{P}^T\Gamma^{-1}\dot{\tilde{P}} \\ &= \alpha^T\gamma\alpha + \beta^T\gamma\beta + \nu^T\gamma\nu + 2\beta^T\gamma\alpha + 2\alpha^T\gamma\nu + 2\beta^T\gamma\nu + 2\tilde{P}^T\Gamma^{-1}\dot{\tilde{P}}\end{aligned} \tag{5.31}$$

All the variables in the above equation have been introduced in Chapter 4. Using a similar procedure to that carried out in Section 4.4.6, yields

$$\begin{aligned}\dot{V} = {} & \alpha^T\gamma\alpha + \beta^T\gamma\beta + \nu^T\gamma\nu + 2\alpha^T\gamma\nu + 2\beta^T\gamma\nu \\ & + 2\tilde{\boldsymbol{P}}^T[(\boldsymbol{B}\hat{\boldsymbol{H}}^{-1}\boldsymbol{K})^T(\boldsymbol{A}^T\mathcal{P} + \mathcal{P}\boldsymbol{A})(\boldsymbol{A}\boldsymbol{x}) + \dot{\tilde{\boldsymbol{P}}}]\end{aligned} \tag{5.32}$$

Rearranging for more brevity gives

$$\begin{aligned}\dot{V} = {} & \alpha^T\gamma\alpha + \beta^T\gamma\beta + \nu^T\gamma\nu + 2f^T\gamma\nu \\ & + 2\tilde{\boldsymbol{P}}^T[(\boldsymbol{B}\hat{\boldsymbol{H}}^{-1}\boldsymbol{K})^T(\boldsymbol{A}^T\mathcal{P} + \mathcal{P}\boldsymbol{A})(\boldsymbol{A}\boldsymbol{x}) + \dot{\tilde{\boldsymbol{P}}}]\end{aligned} \tag{5.33}$$

Choosing the same adaptation law as before, results in

$$\dot{V} = \alpha^T\gamma\alpha + \beta^T\gamma\beta + \nu^T\gamma\nu + 2f^T\gamma\nu \tag{5.34}$$

In the limits of validity for this result, it should be noted, as mentioned before, that the above equation refers only to the robustness study of manipulators with rigid links. Hence, it does not cover the disturbances exciting the modes of the flexible arms.

The first three quadratic terms in Equation 5.34 are negative for all states. The last term is generated by the disturbance and its sign depends on both the magnitude of the disturbance and upon the size of the abrupt parameter change. Thus, the sign of the time derivative of the Lyapunov function should be studied under different conditions. To achieve a switching criterion for the monitoring function, some mathematical analysis of the sign of $\dot{V}$ is necessary. The Equation 5.34 may be rewritten in the following manner:

$$\dot{V} = -|f_1(x^2)| - |f_2(\tilde{\boldsymbol{P}}^2)| - |f_3(\tilde{\boldsymbol{D}}^2)| + f_4(x, \tilde{\boldsymbol{D}}^2, \tilde{\boldsymbol{D}}, \tilde{\boldsymbol{P}}) \tag{5.35}$$

The first term of this equation contains the square of the state (**x**), while the last term contains only (**x**). In the presence of disturbance, the sign of $\dot{V}$ depends on both parameter change and disturbance, while, in the absence of disturbance, the sign is negative for all configurations. The criterion for the monitoring function should provide the states under which the upper bound of the disturbance torque vector and the abrupt changes in the system would not cause instability. By choosing a reasonable negative value for $\dot{V}$, and, by considering the upper bounds of the disturbance torque vector as well as the parameter mismatch, Equation 5.34 gives the states for switching. Implementing the results of this criterion gives the monitoring function the ability to decide upon a suitable time to switch back to the adaptive controller.

A conclusion from the robustness study in this section is the fact that the most critical situation for the switching adaptive-PD controller is the switch back action from the PD mode to the adaptive scheme. This is due to the limitations of the robustness of the high level controller. The system stays in PD mode and continues the identification since the estimation of the abrupt change in parameters, plus the other disturbances, reach a level that is tolerable by the adaptive controller. In a more specific expression, the total value of the terms in Equation 5.34 should reach to an acceptable negative value to assure the stability and robustness. This is accomplished through an on-line checking of the states in the region of the stability and robustness.

5.5 Time Varying Parameters and Moving Window

The standard least-squares estimator has a poor tracking capability for time varying parameters. An intuitive reason assumed for this deficiency is the nature of the least-squares method in fitting all available data, including those with less errors, before and after the instance of switching. This characteristic makes the estimation process very slow and sometimes impossible in the presence of time varying parameters. To improve the least-squares estimator, a type of moving window over the data is used. An "exponential data forgetting" (Li and Slotine, 1987), modified by a "Bounded Gain Forgetting (BGF)" technique (Slotine and Li, 1991), is chosen for this purpose.

The moving window is implemented here through a first order filter of the prediction error square in the Equation 5.13. Thus, the cost function to be minimized will change to

$$J = \int_0^t e^{-\lambda(t-s)} \, \| y(s) - W(s)\hat{\rho}(t) \|^2 ds \qquad \textbf{(5.36)}$$

where λ is the forgetting factor. With a time varying forgetting factor, the term λ(t-s) would be replaced by $\int_s^t \lambda(r)dr$. Similar to Section 5.2, one can conclude that the parameter update law is still the same as Equation 5.19. The gain update law is therefore modified to

$$\dot{\Gamma} = \lambda(t)\Gamma - \Gamma W^T(t) W(t)\Gamma \qquad \textbf{(5.37)}$$

To avoid the gain windup problem in the absence of persistent excitation, the 'Bounded Gain Forgetting (BGF)' was implemented. This technique changes the width of the window depending on the persistency of excitation (Slotine and Li, 1991):

$$\lambda(t) = \lambda_0 \left(1 - \frac{\|\Gamma\|}{k_0}\right) - \Gamma\, W^T(t) W(t)\, \Gamma \qquad (5.38)$$

where λ_0 is the maximum forgetting rate and k_0 is the maximum gain matrix magnitude.

5.6 Implementation Remarks

The major precautions necessary in design and implementation in order to maintain acceptable robustness and to minimize the problems of both large change in parameters and switching action are as follows:

In the case of large and abrupt change in parameters, the control system must switch to the PD regulator. This is due to the robustness limitations of the high level controller. The last estimated and adapted parameters in high-level controller are used as the initial values for the PD mode estimator.

The on-line estimator starts right after the abrupt and large change in parameters. The system remains in PD mode and the identification continues, until an acceptable model and minimum parameter mismatch suitable for the switching back action is obtained.

The monitoring function switches back to the high-level controller when some reasonable parameter variations and bounded disturbances in the region of the robustness of the adaptive mode are realized. The monitoring function switching criterion derived and elaborated in the previous section is related to the negativeness of the total value of the terms in Equation 5.34. By monitoring the system's performance, and checking the states with the results of this robustness criterion, the monitoring function decides when to switch to the higher-level controller. The most updated parameters computed by the on-line estimator in the PD mode are transferred to the high-level controller. These estimated parameters will be used as the initial estimations in the adaptive scheme.

In the limits of validity for the robustness results in Section 5.4 it should be noted , as mentioned before, that it refers only to rigid link manipulators. Hence, it does not cover the disturbances exciting the modes of the flexible arms.

In the off-line calculations of the switching states from the Equation 5.34, care should be taken in the choice of the reasonable negative value for $\dot{V}$ and the upper bounds for disturbance torque vector and parameter change. Large negative value for $\dot{V}$, and conservative upper bounds for disturbance and parameter mismatch, will result to a small region of stability. This may cause the system to stay in the low-level mode for the entire working period.

Upper and lower bounds are imposed on the estimated parameter functions as described in Chapter 4. These bounds are checked in every iteration of the estimations and parameter functions are reset if the estimated values are higher or lower than the bounds.

5.7 Simulation Results

5.7.1 General

The three link articulated manipulator arm described in Chapter 4 is chosen for simulations and for testing the performance of the switching control system. The manipulator and its kinetic model are shown in Figures 4.1 and 4.3. The parameters of the manipulator are given below

$$\begin{aligned} m_2 &= 10\,kg \ , \quad m_3 = 15\,kg \\ a_2 &= 1\,m \ , \quad a_3 = 0.6\,m \\ e_2 &= 0.6\,m \ , \quad e_3 = 0.25\,m \end{aligned} \tag{5.40}$$

For simulating a large and abrupt change in the switching controller, as an example, the manipulator is studied again when it picks up or drops a payload with unknown mass. The new parameters 'm' and 'e' are estimated in each iteration after a change in the system occurs. 'm' is the total mass of the last link and the payload, which will change when the unknown payload is grasped or dropped. 'e' is the new location of the center of mass of the last link. Changes in 'm' and 'e' are represented by 'dm' and 'de' respectively.

Simulations were performed for different levels of change in parameters, random disturbances, and persistency of excitations. Only some typical results out of numerous tests are presented in this section. First, the effects of different sizes of the step change in the payload are investigated and then the robustness of the estimator and the system to the different levels of disturbance is studied. These issues have been theoretically analyzed in the robustness study of the previous sections.

The desired trajectories were chosen in the same harmonic form of Equation 4.80. A random disturbance vector opposing the joint motions is considered in the model. An example of that is dry friction. All the results are shown after a switch from the adaptive mode to the PD regulator, at time t =0.

5.7.2 Switch to PD with Large Change in Parameters

In this test, the system is switched to PD mode with a large step change in parameters. A large change in the payload and the mass of the last link is defined as a change from 60 to 70%. The last estimated parameters in the adaptive mode, m_e and e_e , are used as the starting values for the identification in the PD mode. These two initial values and the change in parameters are as follow

$$\begin{aligned} m_e &= 15kg \\ e_e &= 0.25m \\ dm &= 10\,kg \\ de &= -\,0.05m \end{aligned} \qquad \textbf{(5.40)}$$

As was the case in some tests in Chapter 4, a small amount of random noise was applied. The persistent excitation condition was satisfied for the trajectories with the frequencies close to $\omega = 0.9$ rad/sec. The tip path error, is given in Figure 5.2. The figure shows that, after an overshoot ending at $t = 0.12$sec, a tip path steady state error of approximately 0.015 mm remains.

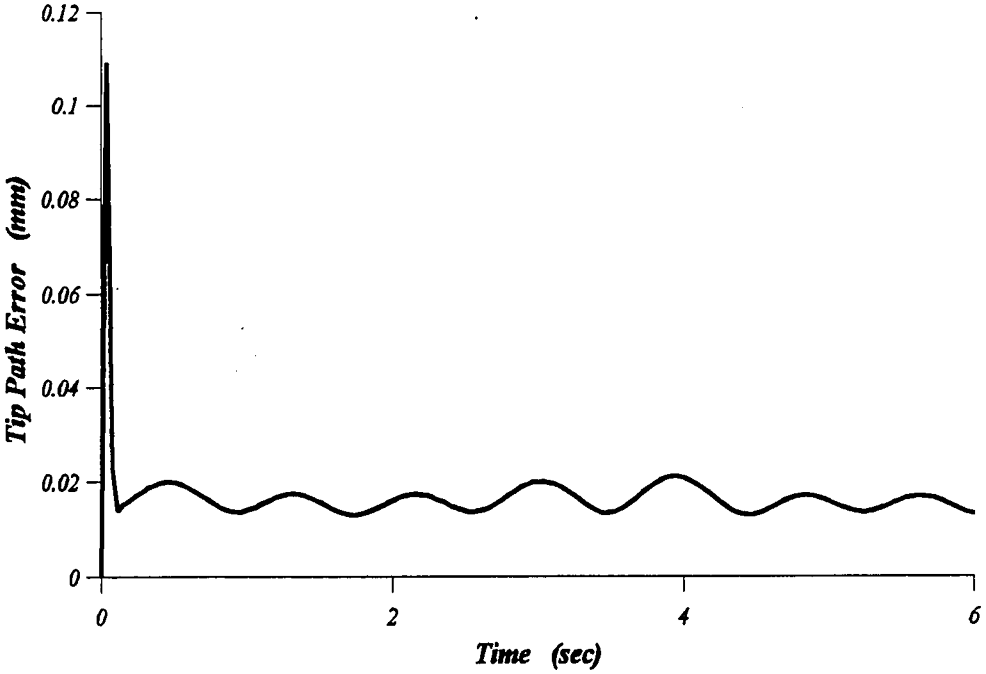

Figure 5.2 Tip Path Error in PD Mode with Large Parameter Change.

The mass identification is presented in Figure 5.3. By choosing realistic and reasonable gains for the controller and the estimator, an acceptable identification of the abrupt change was accomplished within 2.54sec. The estimated mass fluctuates between 26.2kg and 24.3kg. This is reliable for switching back action, but obviously not perfect. A steady state estimation error of 0.68kg remains.

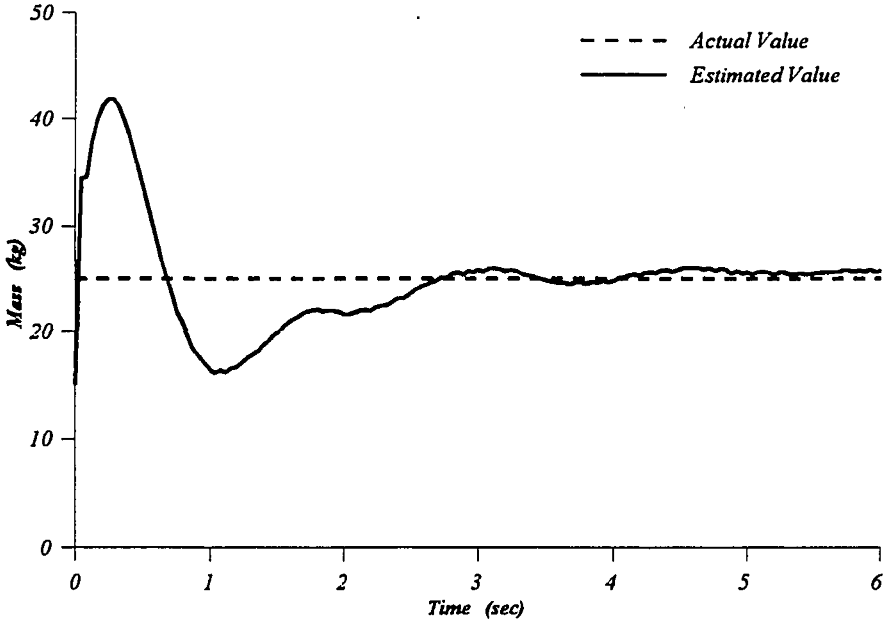

Figure 5.3 Identification with Large Step Change in Parameters.

The tracking errors for this test, are given in Figure 5.4. The effect of disturbance in the order of high frequencies is more obvious in the last joint servo error (tracking error of the third joint). This is due to the small mass moment of inertia of the third link with respect to the last joint. Furthermore, the noise in the first joint does not affect the tracking error. The maximum deviations in the first, second, and last joint tracking errors are 6.7e-5rd, 7e-6rd and 3.5e-6rd respectively. Steady state error in the second and third joint servos are 7e-6 and 3.5e-6 respectively.

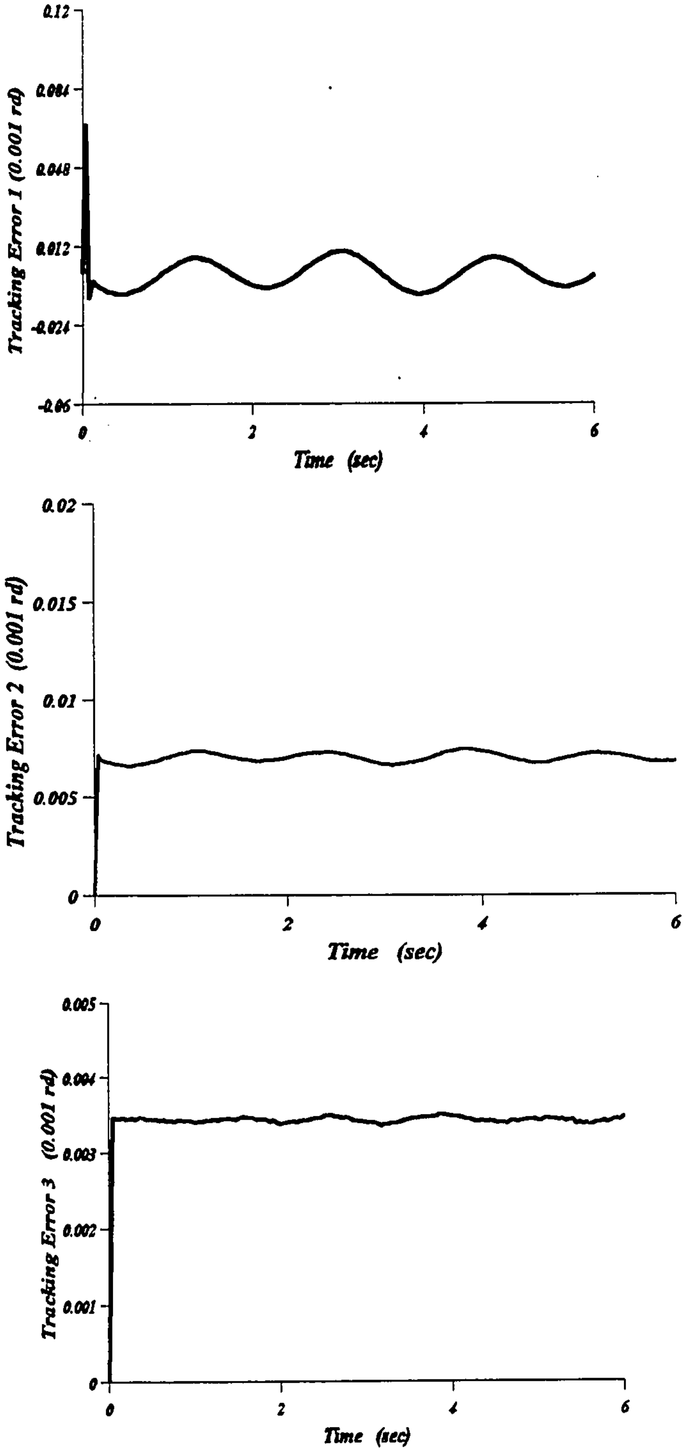

Figure 5.4 Tracking Errors in PD Mode with Large Parameter Step Change

5.7.3 Switch to PD with Very Large Change in Parameters

In this test, the system was switched to PD mode with a very large step change in parameters. A change in the payload approximately equal to the mass of the last link was considered a very large change. The initial estimation values and the step change in parameters were chosen as

$$\begin{aligned} m_e &= 18kg \\ e_e &= 0.25m \\ dm &= 15kg \\ de &= -0.05m \end{aligned} \tag{5.41}$$

A small amount of random noise, the same as in the earlier test, was applied. The persistent excitation condition was chosen equal to the previous test. The tip path error, is given in Figure 5.5. The figure shows that a steady state error of approximately 0.017mm remains in the tip path error after an overshoot of 0.97mm.

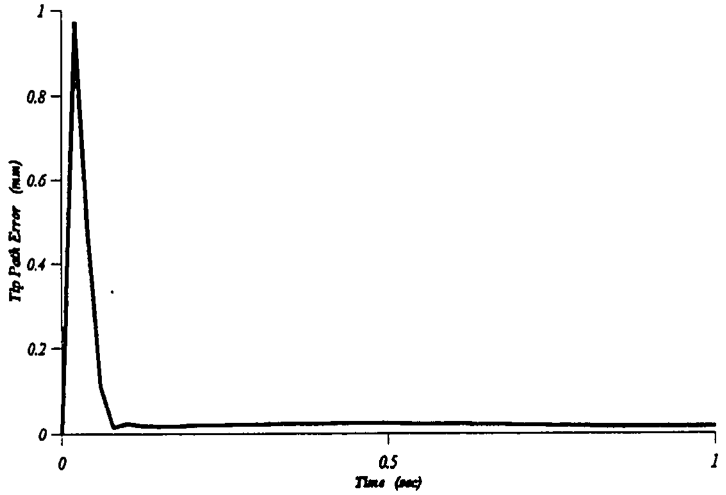

Figure 5.5 Tip Path Error in PD Mode with Very Large Parameter Change.

The mass identification is presented in Figure 5.6. The identification of the step change was achieved in 2.67 seconds. The large deviations in this period are due to the nature of the PD controllers and their inability to deal effectively with nonlinear systems. The mass estimate, after the overshoots, fluctuates between 28.8kg to 31.8kg and a steady state error of approximately 1.2kg remains.

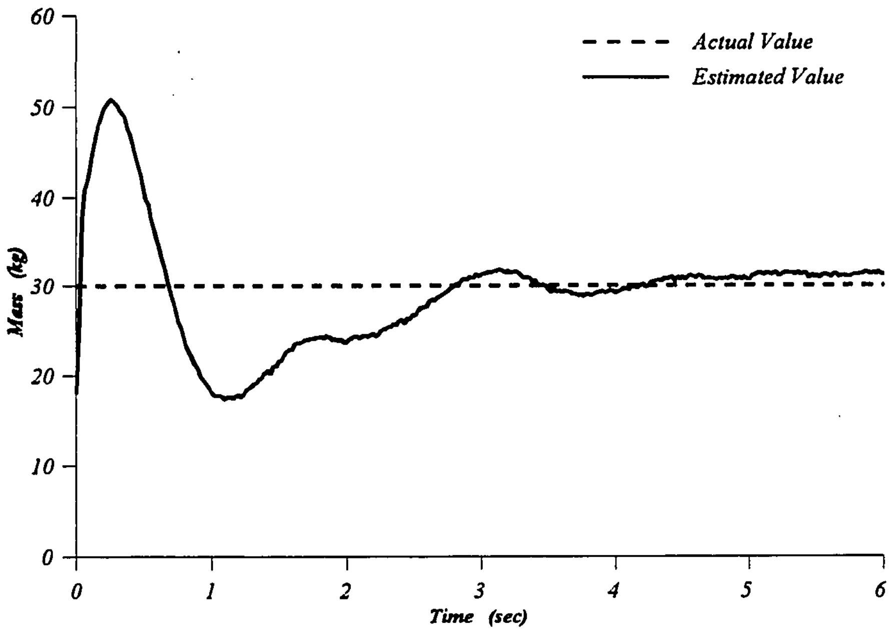

Figure 5.6 Identification with Very Large Step Change in Parameters.

The tracking errors are shown in Figure 5.7. The figure shows again that the effect of high frequency disturbance is more obvious, in the form of high frequency perturbations, in the third joint servo error. The maximum deviations in the first, second and last joint tracking errors are 6.1e-4rd, 1.2e-5rd and 3.5e-6rd respectively. Steady state errors in the joint servos are 5.3e-5rd, 8.3e-6rd and 3.5e-6rd respectively.

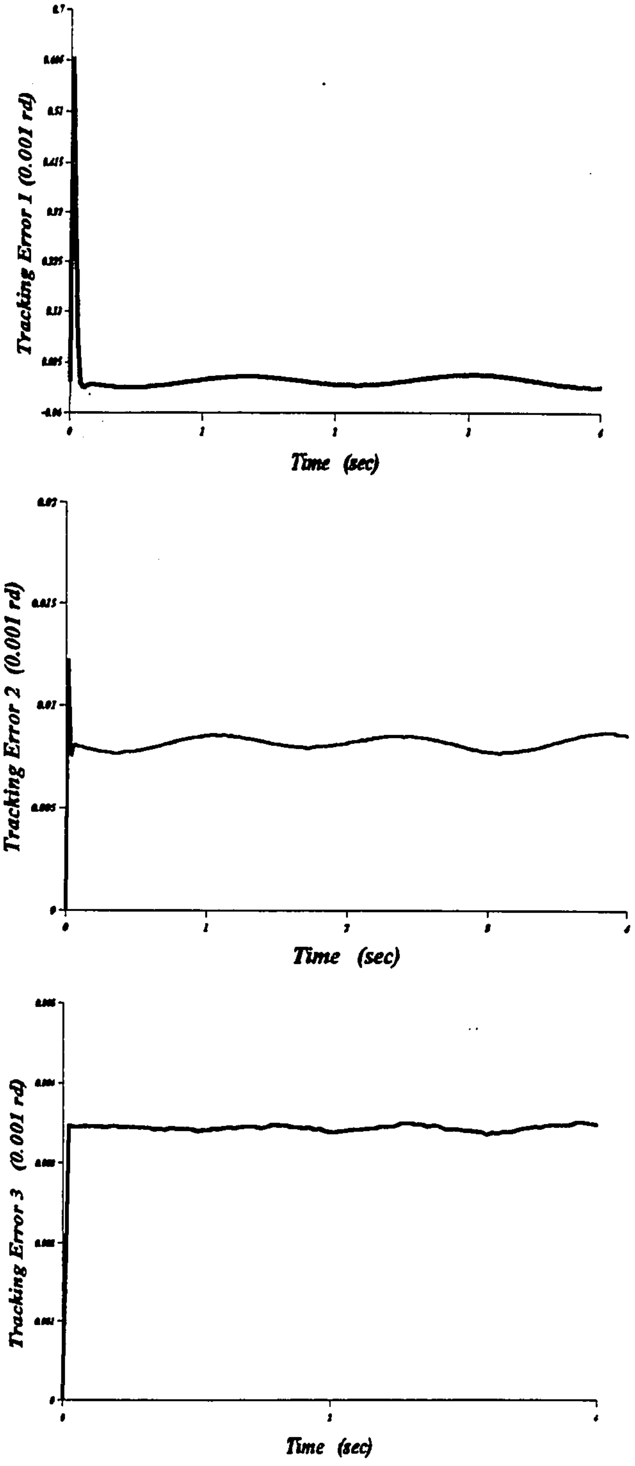

Figure 5.7 Tracking Errors in PD Mode with Very Large Parameter Step Change.

5.7.4 Robustness Test to External Disturbances

In this section, the sensitivity of the system is tested to evaluate its ability to reject external disturbances. The test was performed while the system was experiencing a large step parameter change. The imposed external disturbances acted on each joint.

5.7.4.1 Small Magnitude Disturbance

The results of the identification test and the related tracking errors for the large step change with a small magnitude random disturbance were given before in Figures 5.2, 5.3 and 5.4. The disturbance in this set of experiments had a maximum amplitude vector of approximately 5% of the typical maximum control signal vector. Comments on these results were provided in Section 5.7.2.

5.7.4.2 Medium Magnitude Disturbance

The results of the test for the same step change with a medium magnitude random disturbance are presented here. For this test, the maximum amplitude vector of the noise was chosen to be approximately 25% of the typical maximum control signal vector. The persistent excitation condition was satisfied for the trajectories with frequencies close to $\omega = 0.9$ rad/sec. The result of the mass identification is shown in Figure 5.8. The mass estimate, for the actual value of 25kg, fluctuates between 27.1kg to 25kg. The steady state error in the parameter identification is 1.4kg.

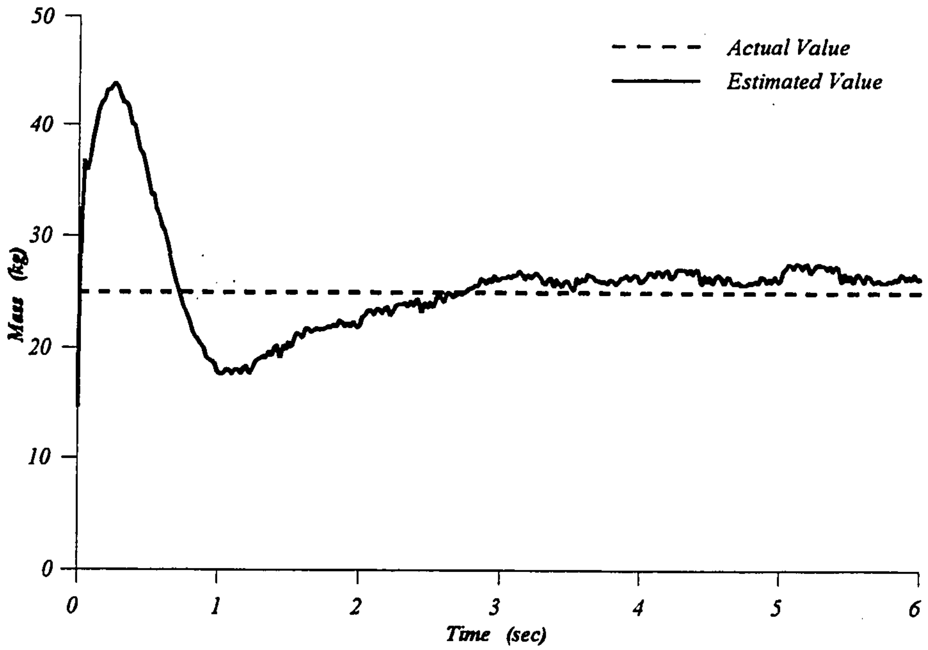

Figure 5.8 Identification with Medium Disturbance Magnitude.

The related tracking errors are shown in Figure 5.9. The effect of noise on the last joint causes perturbations in the order of 0.0005rd. The maximum deviations in the first, second and last joint tracking errors are -4.4e-5rd, 7.2e-6rd and 3.4e-6rd respectively. Steady state errors in the second and third joint servos are 7.1e-6rd and 3.5e-6rd respectively.

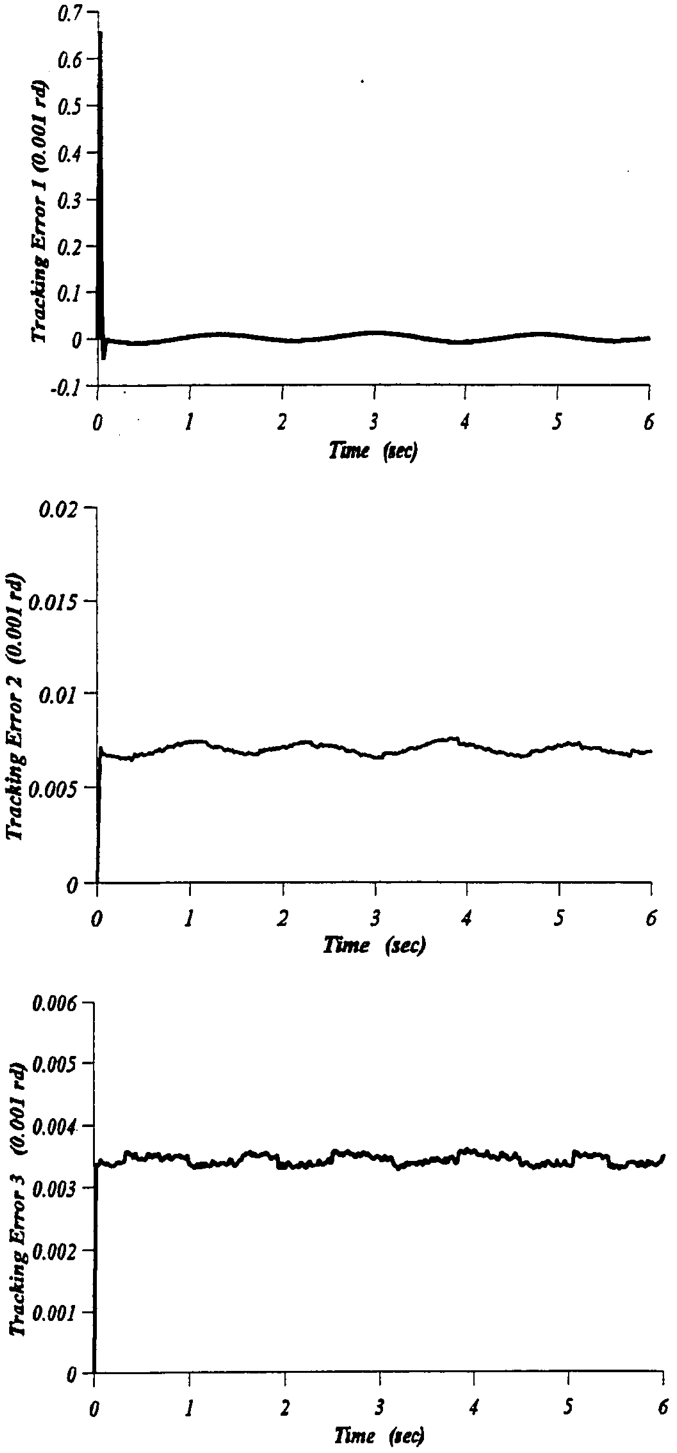

Figure 5.9 Tracking Errors in Identification with Medium Disturbance.

The tip path error is shown in Figure 5.10. The figure shows that after 0.1sec, a steady state tip path error of approximately 0.018mm remains. The overshoot is equal to 1.05mm.

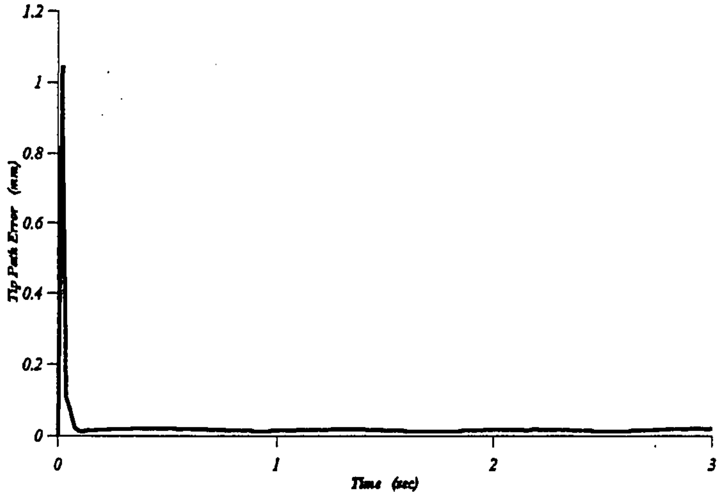

Figure 5.10 Tip Path Error with Medium Disturbance Magnitude.

5.7.4.3 Large Magnitude Disturbance

The identification test for the same step change with a large magnitude disturbance is presented in this section. A large disturbance in this set of experiments was chosen to have a maximum amplitude vector close to 50% of the typical maximum control signal vector. The persistent excitation condition was satisfied for the trajectories with $\omega = 0.9$ rad/sec. The result of the mass identification is presented in Figure 5.11. The mass estimate, for the actual value of 25kg, fluctuates between 30.3kg to 23.84kg. Steady state error in the parameter identification is approximately 2.7kg with a substantial amount of perturbations. An acceptable identification was achieved in 2.8sec.

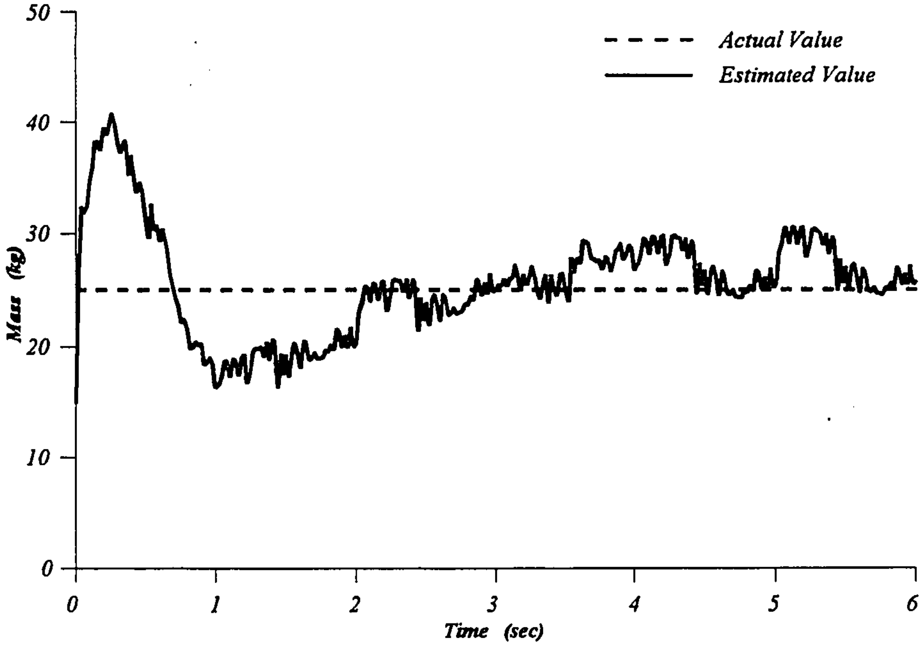

Figure 5.11 Identification with Large Disturbance Magnitude.

The related tracking errors are shown in Figure 5.12. The maximum deviations in the first, second and third joint tracking errors are -4.5e-5rd, 7e-6rd and 3.4e-6rd respectively. Steady state errors in the second and third joint servos are 7e-6rd and 3.5e-6rd respectively.

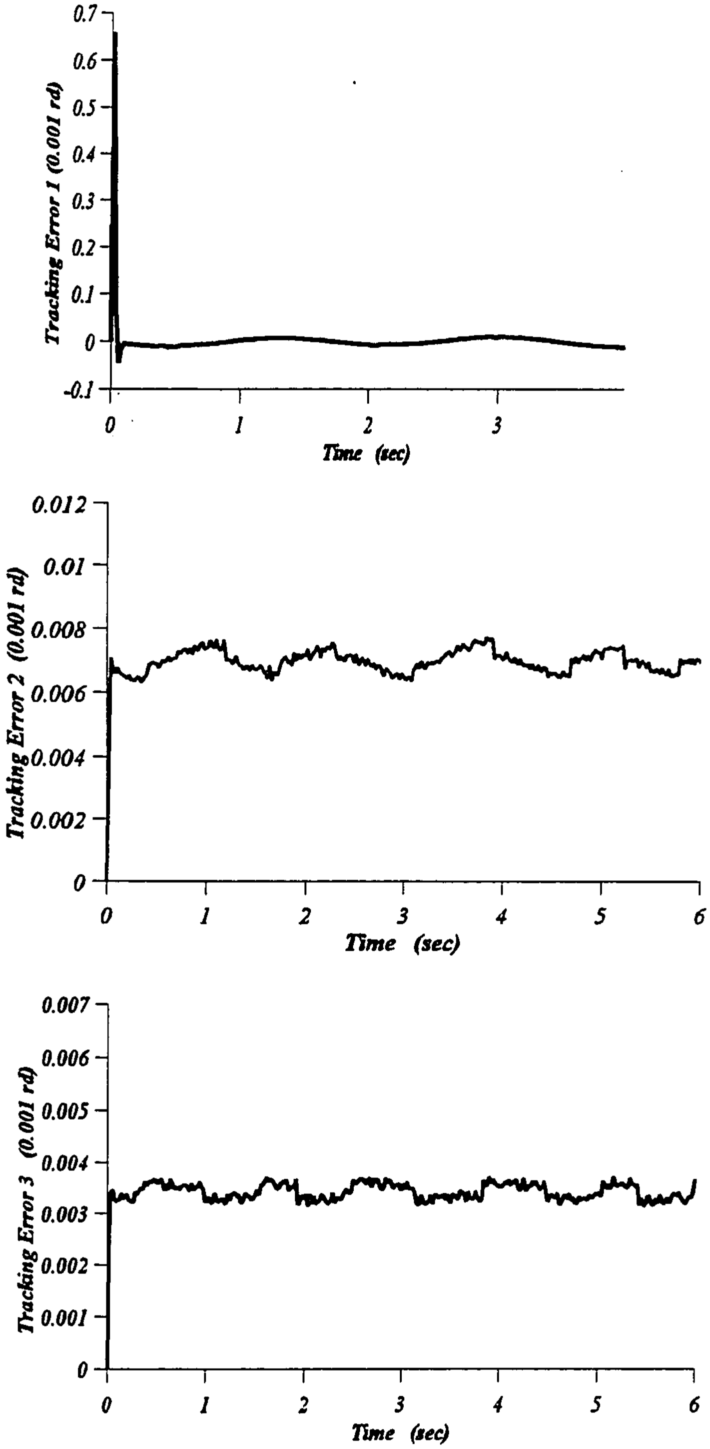

Figure 5.12 Tracking Errors in Identification with Large Disturbances.

The tip path error, is given in Figure 5.13. The figure shows that after 0.1sec, a steady state tip path error of approximately 0.02mm remains. The overshoot is 1.05mm.

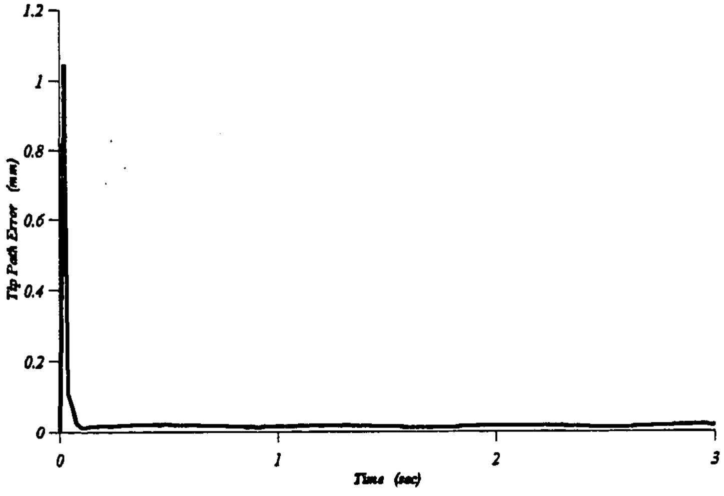

Figure 5.13 Tip Path Error with Large Disturbance Magnitude.

5.7.5 Summary for Simulations

The results of the abrupt change in parameters show that the tracking errors converge to a reasonable steady state situation rapidly. However, fluctuations in the tracking and parameter estimations remain. This is due to the nature of the PD regulators and their weakness in tracking the nonlinear systems. The on-line estimator has the same characteristics, because it is driven by predicted error.

In the test with abrupt mass change of 100%, the tip path error reaches to an acceptable range in less than 0.8sec. However, the system should stay in this mode for more than 2.67sec in order to achieve the estimations necessary for the switch mechanism. The estimation of an actual mass equal to 30kg fluctuates between 31.8kg to 28.8kg and a steady state error of 1.2kg remains, which is acceptable, comparing with the results of the adaptive mode.

The results of the robustness test show that the system is robust to large disturbance and large change in parameters. In the large magnitude disturbance test (i.e. 50% disturbance magnitude) the tip path error reaches 0.02mm in 0.1sec. However, the parameter estimation process needs 2.8sec for reasonable results. The estimation of the new 25kg mass fluctuates between 30.3kg to 23.8kg. A steady state error of 2.7kg in the mass estimation remains. This is acceptable, comparing with the results of the adaptive mode for the same conditions, which forces the monitoring function to switch the system to the low-level scheme.

Chapter 6

Summary, Conclusions and Future Work

6.1 Summary and Conclusions

The summary and conclusion for the various topics dealt with in the thesis are provided below.

6.1.1 Switching Control System

A switching control system was designed to deal with the abrupt and fast changes of the parameters. The scheme comprises two different controllers defined, in this work, as the 'conventional' and the 'advanced' controllers. The low-level controller is a simple PD regulator with an on-line parameter estimator. The high-level controller is an adaptive version of the computed-torque control scheme. The system also incorporates a hierarchical control structure through a monitoring function, which supervises switching between the different control schemes.

6.1.2 PD Mode with On-line Parameter Estimator

The switching controller was implemented to enable the system to tolerate abrupt and large changes of parameters. The system switches from the adaptive to the PD controller for a limited period of time whenever abrupt and large change in parameters occurs.

An on-line parameter estimator was used to identify new parameters of the manipulator during the course of conventional regulation. This identification eases smooth switching from the PD to the adaptive controller in the next stage of the process. A least-squares parameter estimator, modified by a type of moving window, i.e., 'exponential bounded gain forgetting,' was selected for this purpose. This estimator is fast in the identification of large stepwise changes. It has a good robustness to disturbances and noise, and, as well, benefits from old data forgetting and the ability to track the time varying parameters. In the design of the estimator, a first order filter was employed to eliminate the need to feedback joint acceleration terms.

The identification of a 100% change in parameters took approximately 2.67 seconds in the PD mode. The results are reliable for the switching back action, but are not perfect. A more accurate estimation and tracking, will be achieved in the adaptive mode.

6.1.3 Adaptive Control Mode

For designing the adaptive part of the system, the Lyapunov stability criterion was used. A parameter-adaptive controller with a new adaptation law was developed. A new methodology based on the generalized Krasovskii theorem for finding the Lyapunov function for robot controller design was proposed. The method is direct and systematic. The technique is general and implementable for all revolute jointed rigid link open kinematic chains and manipulators. The derived adaptive scheme was adopted for a computed-torque control system.

The manipulator arm and the payload were considered in the most general form, to have a partially known and highly nonlinear dynamical model. A priori knowledge for this partially known model was used to avoid estimating all the parameters and to accelerate the adaptation performance. A vector of the parameters error functions was derived such that it is linear in the parameters and the values of the parameters are updated through an adaptation law.

By introducing the associated adaptive control law and the adaptation mechanism, the boundedness of the vectors of the system states and parameters error functions were proven. This guaranteed the global stability of the higher-level controller and the convergence of its tracking error.

Five different cases were considered in simulations: (a) identification with emphasis on estimation of the true values of the parameters; (b) tracking with more focus on achieving fast and accurate trajectory tracking rather than the complete identification of the system; (c) identification with no a priori knowledge; (d) robustness study of the control system; and (e) the effect of different sets of parameters on the performance.

A priori upper and lower bounds for the parameter estimates were imposed. The parameters were reset in cases that the estimations moved outside these limits. The resetting process keeps the estimates in a range close to their actual values, and, therefore, guarantees positive definiteness and invertibility of the estimated inertia matrix and eliminates the possibility of instability. However, it generates a step change in the vector of the parameters error functions. Thus, care should be taken in choosing reasonable values for the bounds and resettings.

A persistent excitation condition was imposed to guarantee parameter error convergence and identification of the true values of the parameters. Insufficient persistent excitation may happen in some applications and will affect the convergence of the parameter estimates. Perfect tracking is maintained in such applications, but, the exact identification of the parameter values is impossible.

Tracking, for small and medium changes in parameters, was shown to be adequate and accurate. Disturbance rejection for both cases was also tested.. The results of Chapter 4 show that the adaptive computed torque scheme, as the advanced controller, is, by nature, not a fast controller. While the adaptation algorithm could identify an entirely unknown system, the speed of the estimation and adaptation of the parameter values was increased by utilizing a priori knowledge of the parameters. Another enhancement for this purpose was the choice of different speeds for two loops. The model based and adaptation loop may run slower than the servo feedback loop. An ideal controller strategy for fast and accurate tracking would use a combination of two modes: a short initial phase of parameter identification followed by a switch to the tracking mode. These remedies are effective only for the small or medium changes of parameters. For large and abrupt changes, the system is switched to the conventional regulator.

The complete identification, with no a priori knowledge, took approximately 6.5 seconds. This may seem slow in some applications. The identification process, with some a priori knowledge, for a step parameter change took 1.3sec. The small steady state error in identification shows the effectiveness of the persistency in excitations. In cases where the priority is for tracking, rather than identification, the tracking mode is more appropriate.

6.1.4 Switching Mechanism

The stability and robustness of the switching mechanism were studied in detail using the Lyapunov method. The switch from the adaptive to the PD mode was justified intuitively and theoretically. The more critical part of the switching process is that of switching from PD mode to adaptive mode, particularly considering the robustness limitations of the advanced controller. The sign of the time derivative of the Lyapunov function was proven to be state dependent. Thus, for sufficiently large states, the stability was guaranteed and the system was proven to be unstable for small states. The resulting Lyapunov study gave a criterion for finding the states under which the upper bound of the disturbance torque vector would not cause instability. Implementing the results of this criterion into the monitoring function facilitated the choice of the suitable time to switch back to the adaptive controller.

Two major criteria were considered in the implementation and control design part to solve the problems arising from the switch action: a) After an abrupt and large change in parameters or large disturbances, the switching mechanism will switch the controller to the PD mode. The controller will remain in this mode until the identification mechanism has determined an acceptable model for the system. The parameters of the model will not be fully identified at the time of switching to the high level controller. b) Some upper and lower bounds for all of the estimated parameter functions are imposed. The parameters are checked in every estimation iteration and are reset if they fall out of the bounds.

Numerical simulations, using the proposed control scheme for a 3DOF articulated robot manipulator, are presented under various practical conditions. The software and codes were developed using C-language and Matlab.

6.2 Contributions

The major contributions of this research can be summarized as:

A switching controller has been researched to give the system the ability of tolerating abrupt and large changes of parameters. The switching system utilizes two different control schemes: a simple PD regulator with an on-line estimator and a model reference adaptive control scheme. This results in a newly developed system that we call 'Switching Adaptive Control System'.

A new methodology based on the generalized Krasovskii theorem for finding the Lyapunov function for robot controller design is proposed. The method is direct and systematic. The technique is general and implementable for all revolute jointed rigid link open kinematic chains and manipulators.

A new Lyapunov function has been derived for the stability study of the control system. This was achieved by using the methodology developed during this research.

A parameter adaptive control system with a new adaptation mechanism was developed. The derived algorithm is different from other similar structures in a term that contains the natural frequencies of the system. The derived adaptive scheme has been adopted for a computed torque control system.

The robustness of the resulting switching adaptive-PD controller was investigated through the extension of the previously developed energy-like method. The Lyapunov analysis of the switching action from PD to adaptive mode, as the most critical part of the switching mechanism, resulted in the development of a criterion for finding the states under which the upper bound of the disturbance would not cause instability. Implementing the results of this criterion into the monitoring function facilitated the choice of the suitable time to switch back to the adaptive controller. The development of this criterion is a significant factor in the efficiency of the system.

6.3 Future Work

The proposed switching control method may prove useful for other applications where the system parameters experience severe changes. A trade off may be done in any incident where the abrupt and large changes in parameters occur during the usual performance of the system. By temporarily losing some advantages of an advanced controller, the guarantee for stability, robustness, effectiveness and productivity is maintained. The system may switch back to the original controller when it gets close to relatively steady and normal conditions. By the aid of the switching controller, the capabilities of two or more controllers may be united in one control system. Thus, the potential of the overall system to operate efficiently under different conditions will increase.

The proposed methodology for generating the Lyapunov function is general and not specific only to this application. It may, therefore, be studied for application to other types of robot manipulators and, perhaps, used by other adaptive control schemes. Among the most notable suggestions would be: the implementation of the method on flexible manipulator arms and the choice of a feedforward model reference adaptive scheme as the advanced controller.

Theoretical justifications to achieve the conditions that ensure the persistence of excitation have been accomplished only for linear systems. For nonlinear systems, such as robot manipulators, this assurance is problematic and not yet solved. Thus, intuition and experience, plus physical insight into the nonlinear system, is essential in the design and implementation. A study of the persistent excitation condition for the trajectories in robotic manipulators may be required to achieve a more justified and simplified design.

An experimental study of the proposed control system would be revealing and beneficial. The final verifications should be done within an experimental setup because many of the possible parameter variations and other phenomena, which would exist in real applications, may have been ignored in the simulations.

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Appendix A

Some Necessary Forms of Dynamical Terms

A.1 General

In this section some necessary forms of the dynamical terms in robotic manipulator structures are obtained. These mathematical forms are necessary in many cases, especially in controller design and stability analysis.

A.2 Quadratic forms

The Coriolis and centrifugal torque vector in Equation 4.18 can be written in a quadratic form of $\dot{\theta}^T \mathbb{C}(\theta)\dot{\theta}$:

$$V(\theta,\dot{\theta}) = \dot{\theta}^T \mathbb{C}(\theta)\dot{\theta} = \begin{bmatrix} \dot{\theta}^T \mathbb{C}_1(\theta)\dot{\theta} \\ \dot{\theta}^T \mathbb{C}_2(\theta)\dot{\theta} \\ \dot{\theta}^T \mathbb{C}_3(\theta)\dot{\theta} \end{bmatrix} \tag{A.1}$$

Where $\mathbb{C}(\theta)$ is called the $n \times n \times n$ matrix of centrifugal and Coriolis effects (Vukobratovic et al., 1985). For the manipulator under study, the components of $\mathbb{C}(\theta)$, may be derived as follows

$$
\begin{aligned}
\mathbb{C}_1(\theta) &= \begin{bmatrix} 0 & h_{112} & 0 \\ 0 & 0 & 0 \\ h_{113} & 0 & 0 \end{bmatrix} \\
\mathbb{C}_2(\theta) &= \begin{bmatrix} h_{211} & 0 & 0 \\ 0 & 0 & h_{223} \\ 0 & 0 & h_{233} \end{bmatrix} \qquad \textbf{(A.2)} \\
\mathbb{C}_3(\theta) &= \begin{bmatrix} h_{311} & 0 & 0 \\ 0 & h_{322} & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

where the values of h_{ijk} are given in Equation 4.15. To check this quadratic decomposition, by simply multiplying the matrices, one may have

$$
V(\theta,\dot{\theta}) = \begin{bmatrix} h_{112}\dot{\theta}_1\dot{\theta}_2 + h_{113}\dot{\theta}_1\dot{\theta}_3 \\ h_{211}\dot{\theta}_1^2 + h_{233}\dot{\theta}_3^2 + h_{223}\dot{\theta}_2\dot{\theta}_3 \\ h_{311}\dot{\theta}_1^2 + h_{322}\dot{\theta}_2^2 \end{bmatrix} \qquad \textbf{(A.3)}
$$

which could be derived from Equation 4.13.

A.3 Kinetic energy of the manipulator

The kinetic energy of the manipulator is

$$K = \frac{1}{2}\dot{\theta}^T H(\theta)\,\dot{\theta} = \frac{1}{2}[\dot{\theta}_1\ \dot{\theta}_2\ \dot{\theta}_3]\begin{bmatrix} H_{11} & 0 & 0 \\ 0 & H_{22} & 0 \\ 0 & 0 & H_{33} \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \qquad \textbf{(A.4)}$$

where the values of H_{ij} are given in (4.14).

A.3.1 Some Notes on Properties of the Kinetic Energy Matrix

1- Kinetic energy matrix of the manipulator is equal to its inertia matrix:

$$\boldsymbol{K}(\theta) = \boldsymbol{H}(\theta) \qquad \textbf{(A.5)}$$

Therefore, the scalar kinetic energy is as follows:

$$K = \frac{1}{2}\dot{\theta}^T \boldsymbol{K}(\theta)\,\dot{\theta} \qquad \textbf{(A.6)}$$

2- Time derivative of the kinetic energy is equal to the power input provided by the actuators and gravitational torques:

$$\frac{1}{2}\frac{d}{dt}(\dot{q}^T H \dot{q}) = \dot{q}^T H \ddot{q} + \dot{q}^T\ C(q,\dot{q})\dot{q} = \dot{q}^T(T-G) \qquad \textbf{(A.7)}$$

where q is the generalized joint coordinate vector.

3- A very useful lemma: $\dot{H} = 2C$, i.e., by differentiating the inertia matrix, one can derive the damping matrix.

Proof: Differentiating the left-hand side of Equation A.7 explicitly gives

$$\frac{1}{2}(\dot{q}^T \dot{H} \dot{q}) + \dot{q}^T H \ddot{q} = \dot{q}^T (T - G) \qquad \textbf{(A.8)}$$

By considering the general coordinate q instead of θ in Equation 4.17, replacing $H\ddot{q}$ from it and cancellation of the input terms $T - G$ from both sides, we will have

$$\dot{q}^T (T-G) = \frac{1}{2}(\dot{q}^T \dot{H} \dot{q}) + \dot{q}^T (T - C\dot{q} - G)$$

$$\dot{q}^T \dot{H} \dot{q} - 2\dot{q}^T C \dot{q} = 0 \qquad \textbf{(A.9)}$$

$$\dot{q}^T (\dot{H} - 2C) \dot{q} = 0$$

This result suggests that the matrix $\dot{H} - 2C$ be skew symmetric (Slotine and Li, 1991). An important and practical result is that by differentiating the inertia matrix one can derive the damping matrix.