Integrated Modelling for Supply Chain Planning and Multi-Echelon Safety Stock Optimization in Manufacturing Systems

Thesis submitted to the
School of Graduate Studies and Research in
partial fulfillment of the requirements for the Degree of

Master of Applied Science
in
Mechanical Engineering

by
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Abstract

Optimizing supply chain is the most successful key for manufacturing systems to be competitive. Supply chain (SC) has gotten intensive research works at all levels: strategic, tactical, and operational levels. These levels, in some researches, have integrated with each other or integrated with other planning issues such as inventory. Optimizing inventory location and level of safety stock at all supply chain partners is essential in high competitive markets to manage uncertain demand and service level. Many works have been developed to optimize the location of safety stock along supply chain, which is important for fast response to fluctuation in demand. However, most of these studies focus on the design stage of a supply chain. Because demand at different horizon times may vary according to different reasons such as the entry of different competitors on market or seasonal demand, safety stock should be optimized accordingly. At the planning (tactical) level, safety stock can be controlled according to each planning horizon to satisfy customer demand at lower cost instead of being fixed by a decision taken at the strategic level. On the other hand, most studies that consider safety stock optimization are tied to a specific system structure such as serial, assembly, or distribution structure.

This research focuses on formulating two different models. First, a multi-echelon safety stock optimization (MESSO) model for general supply chain topology is formulated. Then, it is converted into a robust form (RMESSO) which considers all possible fluctuation in demand and gives a solution that is valid under any circumstances. Second, the safety stock optimization model is integrated with tactical supply chain planning (SCP) for manufacturing systems. The integrated model is a multi-objective mixed integer non-linear programming (MINLP) model. This model aims to minimize the total cost and total time. A case study for each model is provided and the numerical results are analyzed.

Acknowledgements

First and foremost, I would like to thank my supervisor, Dr. Ming Liang, for providing me with the opportunity to work with him, for his constructive insightful suggestions, and his careful and patient examination of my thesis. His support has allowed me to achieve my goals successfully. He has not only professionally influenced me, but working alongside him also brought me an invaluable source of personal growth.

I am thankful for the financial support of my government, Kingdom of Saudi Arabia, which has giving me the opportunity to study and live in Canada.

In addition, I want to express my appreciation to my parents who always give me their instrumental support and have demonstrated patience during my absence from home. Their love will always be with me.

I also would like to thank my beloved wife for her faithful and spiritual support on my study and for the sacrifice she has made for me.

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Nomenclature

Indices

c	Component families ($c = 1, 2, ..., C$)
H	Tactical planning horizon
i	Product families ($i = 1, 2, ..., I$)
j	Manufacturing sites ($j = 1, 2, ..., J$)
k	Distribution centers ($k = 1, 2, ..., K$)
s	Suppliers ($s = 1, 2, ..., S$)
t	Tactical periods ($t = 1, 2, ..., T$)
z	Scenarios ($z = 1, 2, ..., Z$)

Sets

C_s	Set of component families that can be produced by supplier s
C_i	Set of component families used to produce product family i
I_j	Set of product families that can be manufactured in manufacturing site j
I_k	Set of product families that can be sold in distribution center k

J_s Set of manufacturing sites that can be supplied by supplier *s*

K_j Set of distribution centers that can be supplied by site *j*

S_j Set of suppliers that can supply site *j*

Parameters

$CMOT_j$ Over-time cost in manufacturing site *j*

CP_{cs} Production cost of component family *c* of supplier *s*

$CSOT_s$ Over-time cost of supplier *s*

D_{ikt} Demand of product family *i* in distribution center *k* in time period *t*

HD_{ik} Holding cost of one unit of product family *i* in distribution center *k* per standard time unit

HJ_{ij} Holding cost of one unit of product family *i* in manufacturing site *j* per standard time unit

HS_{cs} Holding cost of one unit of component family *c* in supplier *s* per standard time unit

JSA_j Storage space available in site *j*

KSA_k Storage space available in distribution center *k*

LB_{ij} Minimum production volume of product family *i* in manufacturing site *j*

MOT_j Over-time available in manufacturing site *j* per standard time period

N_t Number of standard time units in tactical period *t*

PA_j Production capacity of site *j* per standard time unit

PI_{ij} Production cost of product family *i* in manufacturing site *j*

SA_s Production capacity of supplier *s* per standard time unit

SOT_s Over-time available from supplier *s* per standard time period

SSA_s Storage space available at the site of supplier *s*

TIS_{cs} Guaranteed service time of component family *c* to be received by supplier *s*

TJ_{ij} Manufacturing time of product family *i* in manufacturing site *j*

TJK_{jk} Time required for transportation from manufacturing site *j* to distribution center *k*

TK_{ik} Preparation time of product family *i* in distribution center *k*

TOK_{ik} Guaranteed service time of product family *i* to be ready to send out of distribution center *k*

TS_{cs} Manufacturing time of component family *c* in supplier *s*

TSD_{ijk} Transportation cost of product family i between manufacturing site j and distribution center k

TSJ_{sj} Time required for transportation from supplier s to manufacturing site j

TSS_{csj} Transportation cost of component family c between supplier s and manufacturing site j

UB_{ij} An arbitrarily chosen large scalar number which represents maximum production volume of product family i in manufacturing site j

UC_{ik} Unfulfilled demand cost of one unit of product family i in distribution center k

UT_{ik} Time required for re-planning unfulfilled demand of one unit of product family i in distribution center k

α_i Storage space required to store one unit of product family i in manufacturing site j and distribution center k

β_c Storage space required to store one unit of component family c at the site of supplier s

λJ_{ij} Safety factor of demand of product family i at manufacturing site j

λK_{ik} Safety factor of product family i at distribution center k

λS_{cs} Safety factor of demand of component family c at the site of supplier s

μK_{ik} Mean value of demand of product family i at distribution center k

σK_{ik} Standard deviation value of demand of product family *i* at distribution center *k*

ρ_{ci} Amount of component family *c* used to produce one unit of product family *i*

Variables

FJ_{ij} Fraction of amount of product family *i* that should be held at manufacturing site *j*

FS_{cs} Fraction of amount of component family *c* that should be held at the site of supplier *s*

ICS_{cst} Inventory of component family *c* of supplier *s* at the end of tactical period *t*

IPD_{ikt} Inventory of product family *i* in distribution center *k* at the end of tactical period *t*

IPS_{ijt} Inventory of product family *i* in manufacturing site *j* at the end of tactical period *t*

$NLTJ_{ij}$ Net lead time of manufacturing site *j* to deliver product family *i*

$NLTK_{ik}$ Net lead time of distribution center *k* to deliver product family *i*

$NLTS_{cs}$ Net lead time of supplier *s* to deliver component family *c*

SSJ_{ij} Amount of safety stock of product family i at manufacturing site j

SSK_{ik} Amount of safety stock of product family i at distribution center k

SSS_{cs} Amount of safety stock of component family c at the site of supplier s

TIJ_{ij} Guaranteed service time of product family i to be received by manufacturing site j

TIK_{ik} Guaranteed service time of product family i to be received by distribution center k

TOJ_{ij} Guaranteed service time of product family i to be ready to send out of manufacturing site j

TOS_{cs} Guaranteed service time of component family c to be ready to send out of supplier s

UD_{ikt} Unfulfilled demand of product family i in distribution center k at the end of tactical period t

XCP_{cst} Amount of component family c produced by supplier s in time period t

$XMOT_{jt}$ Over-time capacity utilized by manufacturing site j in time period t

XP_{ijt} Amount of product family i produced in manufacturing site j in time period t

XSM_{ijkt} Amount of product family i shipped from manufacturing site j to distribution center k in time period t

$XSOT_{st}$ Over-time capacity utilized by supplier *s* in time period *t*

XSS_{csjt} Amount of component family *c* transferred from supplier *s* to manufacturing site *j* in time period *t*

μJ_{ij} Mean value of product family *i* at manufacturing site *j*

μS_{cs} Mean value of component family *c* at the site of supplier *s*

σJ_{ij} Standard deviation value of product family *i* at manufacturing site *j*

σS_{cs} Standard deviation value of component family *c* at the site of supplier *s*

Binary variables

X_{ijt} Equal to 1 if product family *i* is manufactured in site *j* in time period *t*, 0 otherwise

Chapter 1. Introduction

1.1 Supply chain planning and safety stock optimization

Supply chain (SC) is a system which has entities, processes, and outputs. This system illustrates the requirements, the sequences and the flows of procuring raw material, converting raw material into a product, and then distributing these products. It might include other details such as people, machines, inventory, routes, locations, capacity, etc. These details differ according to the aim(s) of modelling supply chain. There are three different levels of representing a supply chain, i.e., the strategic level, the tactical level, and the operational level. The strategic level deals with top view and long time setup of a supply chain. The design of a supply chain, the selection of suppliers, as well as the locations of manufacturing sites and distribution centers are examples of some issues that should be dealt with at the strategic level. The specifications of manufacturing resources, production policies and quantities, lot sizes, and inventory levels are mid-term tactical level concerns. At the operational level, scheduling, part and tool selection, machine loading, job shop organizing, and processes sequencing are the issues that should be considered daily or weekly.

The main purpose of supply chain planning is to minimize the cost and delivery time which affects customer satisfaction and helps companies to be competitive. Supply chain planning is the key to success in the global market. As such, numerous studies have been done in this field using optimization tools. Much work has been carried out for each level of supply chain planning often separately from the planning of other levels. Very few studies are reported in dealing with the integration of the multi-level supply chain planning problems. Many of these studies focus on inventory optimization.

In manufacturing systems, supply chain optimization and its responsiveness are greatly influenced by considering inventory. Inventory and amount of safety stock are important issues in a supply chain which should be integrated with supply chain optimization to manage demand uncertainties and to maintain customer service level.

1.2 Outline of the work

Some studies have been carried out to optimize the inventory location and the amount of safety stock along the supply chain. However, most, if not all, of previous studies of safety stock optimization focus on design stage (i.e. strategic level) issues of supply chain, which is correct from a location design perspective but inappropriate for safety stock management at the planning stage. The reason is that it is impossible to accurately predict the customer demand at the design stage due to the uncertainties involved in the long time span between the design stage and the production. At the planning (tactical) level, safety stock can be more accurately controlled according to each planning horizon to satisfy customer demand at lower cost instead of being fixed by a decision taken at the strategic level. Demand at different horizon times may vary according to different reasons such as the entry of different competitors into the market, seasonal demand, and short product life cycle. As such, the safety stock should and can be optimized.

In this work, two models have been developed: one deals with safety stock in multi-echelon systems in a robust form, and the other is an integrated model of tactical level of supply chain planning (SCP) with a multi-echelon safety stock optimization (MESSO) format.

In the first model, i.e. the model for robust multi-echelon safety stock optimization, the demand follows normal distribution. An event that follows normal distribution may occur at any point under the distribution graph. It may be within one standard deviation, two standard deviations, three standard deviations, or outside of three standard deviations but with very small probability 0.4%. This fact means that the regular formula of calculating the amount of safety stock could lead to shortage in the presence of demand fluctuations. Therefore, different standard deviations of demand are considered to be the scenarios of the robust model. After solving the model, the result of the robust model is compared with that of the typical safety stock optimization model.

A multi-objective integrated model for supply chain planning and multi-echelon safety stock optimization has also been developed in this work. The minimization of total time and of total cost are the objectives of the model. The output of the model yields optimal safety stock in each tactical horizon.

1.3 Organization of the thesis

Hereafter, the thesis is structured as follows. Chapter 2 presents a literature review of available studies on supply chain planning, safety stock optimization, and integration of supply chain with safety stock optimization. The limitations of previous studies have been highlighted, and the motivation of this study is then presented at the end of the chapter.

Chapter 3 introduces a robust safety stock optimization model for multi-echelon general structure systems. A brief review of robust modelling is provided at the beginning of the chapter. The guaranteed-service approach is used for optimizing safety stock. The nonlinear model is presented with the objective of minimizing total holding cost at all levels subject to service levels for internal and external customers.

In Chapter 4, a case study is presented to illustrate the application of the multi-echelon safety stock optimization model and its efficiency in regular and robust forms. The results are also compared.

Chapter 5 presents an integrated model for supply chain planning and safety stock optimization with two objective functions: the minimizations of the total cost and total time. The modified fuzzy-Chebyshev programming method is used to optimize the two conflicting objectives.

A case study to illustrate the application of the integrated model is presented in Chapter 6. The results obtained from different optimization methods and considerations are investigated and compared. The efficiency of the model is presented by showing the flexibility of the manufacturing operation due to different objectives.

Finally, Chapter 7 summarizes the work that has been carried out through this study and outlines a few potential opportunities for future research.

Chapter 2. Literature Review

The goal of this thesis is to provide an integrated multi-objective model for supply chain planning and multi-echelon safety stock optimization. The minimization of total costs and lead time are the objectives of the model. The safety stock optimization model for multi-echelon general network systems that can be valid at any demand variations is studied in the first part. Then, the integrated multi-objective model is developed and solved. The following sections in this chapter provide a review for the main topics of this work, i.e., supply chain planning, multi-echelon safety stock optimization, and integration of supply chain planning with MESS optimization.

2.1 Supply chain planning

Different models and approaches have been developed to address supply chain issues from various perspectives. Modelling of a supply chain can range from simple linear models to stochastic mixed integer nonlinear models. The complexity of modelling supply chain depends on the stages to be covered, time periods, and other concerns that should be taken into account such as inventory, service level, uncertainty of demand, sale price, and reliability. It is difficult, if not impossible, to find one supply chain model that fits all the supply chains in different organizations.

Supply chain in manufacturing systems is complex. It involves raw materials, manufacturing processes, and the distribution of finished products. Each of these main components is a system which has inputs, processes, and outputs. Supply chain can range from a very simple system such as a serial system that has one supplier, one manufacturer, and one distribution center, to a very complex one that has multiple suppliers, multiple manufacturing sites with similar or different operations, multiple distribution centers, multiple products, and multiple time periods.

2.1.1 Supply chain configuration and modelling

Supply chain has different topologies. Five of them are mentioned in the work of Willems (1999): serial system, assembly system, distribution system, spanning tree system, and general system. Briefly, a serial system is the system in which each node has at most one predecessor (upstream node) and at most one successor (downstream node). An assembly system is the system where any node is connected to at most one successor. A system in which any node has only one predecessor is called a distribution system. A spanning tree system has no restriction on the number of predecessors and successors that are connected to a node, yet all nodes must be connected. However, there is one restriction: no cycles are allowed. Finally, a general system has no restrictions. Figure 2.1 illustrates the configurations of these topologies. The first four system types can be considered as special cases of the general system.

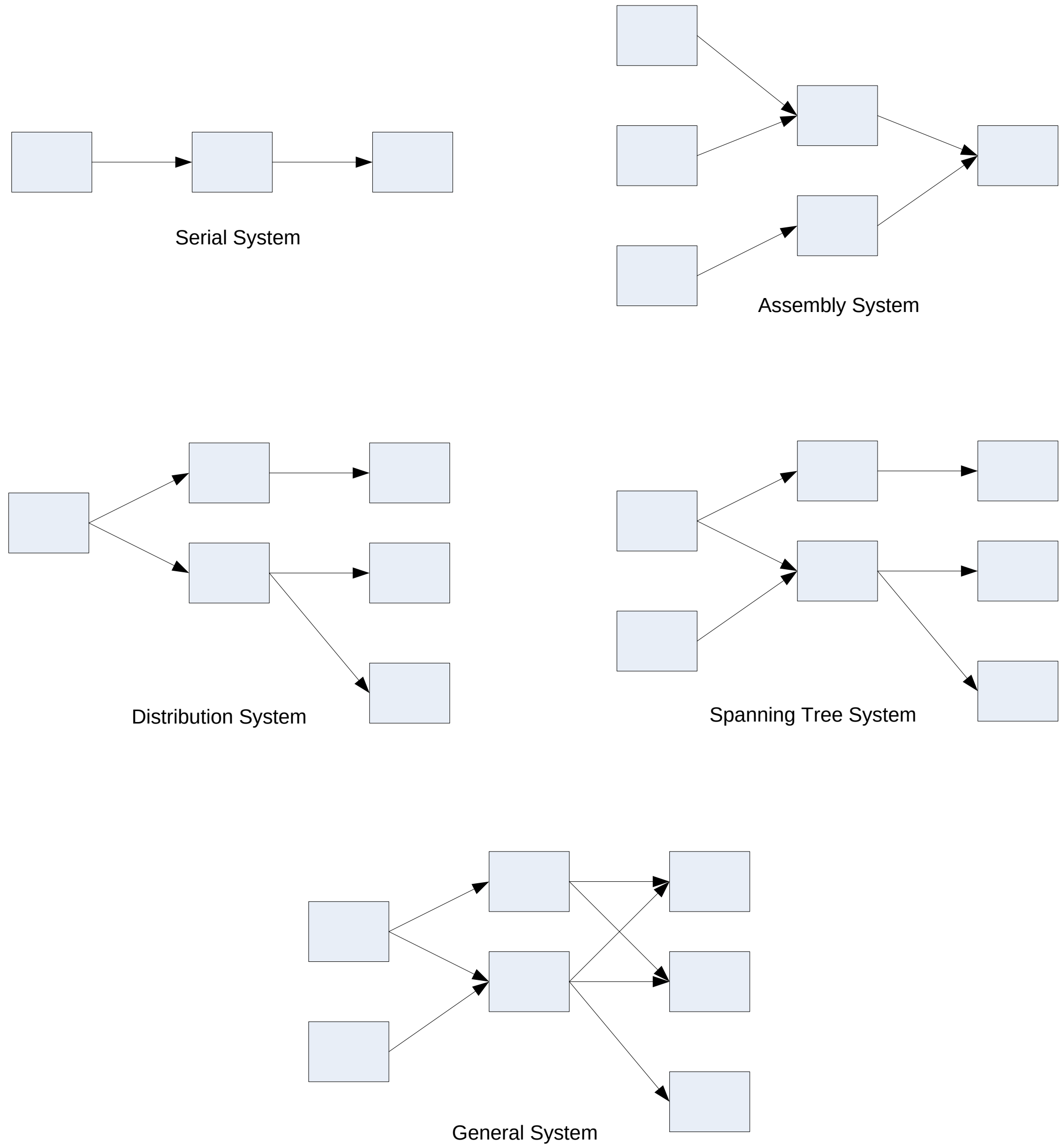

Figure 2.1: Supply chain topologies

Supply chain problems can be divided into three main problems: strategic, tactical, and operational problems (Schmidt and Wilhelm 2000; Peidro et al. 2009). Strategic problems deal with design and configuration issues of supply chains with a time frame ranging from half a decade to one decade. The tactical supply chain problems last from one to two years and deal with utilization of resources and cover suppliers, production facilities, transportation systems, inventory control, distribution centers, etc. Operational problems concern with daily to weekly scheduling issues such as scheduling, sequencing, lot sizing, movement routing, etc. Similarly, based on the frequency of taking decisions and the time frame that is impacted by these decisions, Chopra and Meindl (2008) mentioned that supply chain decisions can be divided into three associated groups: strategic, tactical, and operational, which cover long-term, mid-term, and short-term planning decisions respectively.

According to the type of the supply chain problem and topics that should be covered in the planning of a supply chain, modelling approaches and the complexity of the modelling are different. Two comprehensive reviews about modelling supply chains should be mentioned here. The first one is done by Mina and Zhou (2002) who provided a review on modelling supply chain, main problems and important components of supply chains, main variables and constraints, integration, and modelling classifications. The second one is conducted by Sarimveis et al. (2008) and concerns the methods used in modelling supply chains. Four different methods have been used: deterministic, stochastic, game theoretic, and simulation methods. These methods have been mentioned before by Beamon (1998). A model is described as a deterministic model when all its parameters are known. If there is at least one parameter that is unknown yet follows a probabilistic distribution, then the model can be described as a stochastic model. The supply chain economic game-theoretic models and models based on simulation are mainly used to evaluate the performance of different supply chain strategies.

Capturing the reality of a supply chain increases the complexity of the model (Simchi-Levi et al. 2000). For instance, considering multi-conflict objectives makes the model more complex and difficult to solve. However, including multi-objective functions in planning supply chains is often essential to capture some of the reality of the system. An example is the integration of material procurement, production, and inventory control to study the tread-off of different objectives such as total cost, lead time, service level, etc.

Even though integration is defined as a source of the complexity of modelling by Simchi-Levi et al. (2000), sometimes, different levels of supply chain issues should be integrated for more accurate and comprehensive plans. Hurtubise et al. (2004) introduced a method to solve combination models of planning and scheduling (tactical and operational level) in a supply chain network. A two-phase solution method is used to reduce the complexity of planning and scheduling modelling for production and transportation of a product in a supply chain. The method simply uses the output results of aggregate planning solutions as input data to the detailed operational scheduling. The objectives are to minimize the travel and production cost. Similarly, Fakharzadeh-Naeini (2011) developed a model for supply chain tactical and operational levels. He used an iterative algorithm to optimize the integrated model which works back and forth between two separated models, tactical and operational. Another example of supply chain integration is the work of Patel et al. (2009). They developed an integrated mix-integer programming model for strategic, tactical, and operational levels of a supply chain. With high production rate required to satisfy customers' demands, the model aims to minimize the total costs of distribution, storage, inventory and operations.

2.1.2 Optimize multi-objective supply chain model

In manufacturing systems, minimizing total costs of a supply chain plan is not the only objective, especially in the competitive global market. In addition to minimizing costs, some other objectives, such as service level, delivery time (responsive time), and shortages, might be equally or more important than cost and should be optimized. Different methods have been used to optimize multi-objective models.

There are several methods to optimize multiple objectives simultaneously. Here, only two are reviewed. The first one is to convert the multi-objective problem into a single-objective using a weighted sum or weighted goal programming method where each objective is multiplied by a factor that represents the weight of importance of that objective (Corner and Buchanan 1995). The other method is known as the ε-constraint method, and is used to solve all objectives and find a set of solutions called Pareto optimal solutions (Chankong and Haimes 1983). One objective is optimized while the others are used as un-equality constraints. The solution set gives the decision makers trade-off solutions among all objectives so that they can choose a solution that fits best in their situations. Moreover, the joint use of the Pareto concept or weighted sum method and artificial intelligence methods has been employed to develop multi-objective solution methods. For instance, Moncayo-Martínez and Zhang (2011) used ant colony optimization as a meta-heuristic approach for designing a supply chain for assembly networks. Their meta-heuristic is called Pareto-Ant Colony method and has been used to solve multi-objective models.

Liu and Liang (2008) modified the fuzzy-Chebyshev programming method to solve multi-objectives models. Their method has several advantages. First, unlike the conventional weighted sum method that subjectively assigns weight factors to each objective, the weight factors in their method are optimized using a fuzzy membership function. Secondly, the conventional fuzzy-Chebyshev programming method treats all objectives equally. The modified fuzzy-Chebyshev programming (MFCP) method developed by Liu and Liang (2008) can treat objectives with different weights according to the importance order of the objectives. As such, the MFCP method is used in this thesis, and more details about this method will be given in Chapter 5.

2.1.3 Uncertainty

A main issue in supply chain planning that has attracted many researchers is uncertainty. Peidro et al. (2009) provided a great review about dealing with uncertainty in supply chain planning. There are three main uncertainties in a supply chain, i.e., demand uncertainty, supply uncertainty, and process uncertainty. Details are also provided for each source of uncertainty and modelling approach. They emphasized that the most important uncertainty among the three is demand uncertainty. Then, they indicated the common approaches used for modelling supply chains under uncertainty are the analytical approach, the artificial intelligence approach, the simulation approach, and the hybrid approach. Their review covers all three types of supply chain problems: strategic, tactical, and operational.

The uncertainty in a supply chain can be addressed by treating certain related parameters as stochastic or by reducing the effects of uncertainty of parameters. An example of treating uncertainty is using a fuzzy membership function for uncertain parameters. Petrovic et al. (1999) studied a serial single-product supply chain in an uncertainty environment. By using fuzzy sets, they addressed the uncertainty issues related to demand and supplier reliability. Inventory is available at each member of a supply chain and is often controlled based on periodic review (order-up-to level). Simulation is the tool used to study the behavior of the supply chain in uncertain environments when the inventory is fully decentralized and partially coordinated. Similarly, Liang (2008) and Liang and Cheng (2009) solved the production-distribution planning decision problem with two conflict functions: total cost and delivery time. They used fuzzy sets to represent the uncertainties of three constraints, i.e., demand, machine capacity, and labor level while the other constraints are assumed to be certain. With a given minimum acceptable membership level, they converted the fuzzy parameters into crisp numbers using the weights of the most pessimistic, most likely, and most optimistic values of each fuzzy parameter. Then, they proposed the solution procedure. However, obtaining a satisfactory decision from the model depends on the experience of the decision maker because the values of minimum acceptable membership levels and the probabilities (weights) of the most pessimistic, most likely, and most optimistic are subjectively adjusted. Also, Peidro et al. (2009) and Peidro et al. (2010) proposed a mathematical programming model for supply chain planning where demand, supply and process are uncertain parameters. These uncertainties are presented as fuzzy sets. The objective function of the model is to minimize the costs of idleness, raw material, inventory, backlog and transport for different customer satisfaction levels. Another way of handling uncertainties is using scenarios to represent different values for uncertain parameters with known probabilities. Chen and Lee (2004) considered demand and price uncertainties in their scheduling model. Demand uncertainty is represented by different scenarios with known probability and the uncertainty of price represented by fuzzy sets which describe the preference prices to the seller and to the buyer. Their mixed-integer nonlinear programming model is a multi-objective formulation for multi-product, multi-stage, and multi-period applications. Four objective functions are considered in their model, which are maximizations of fair profit among all partners, safe inventory levels, customer service levels, and robustness of selected objectives.

The effects of uncertainty on supply chain can be mitigated by controlling inventory or by optimizing safety stock. More-detailed reviews are presented in section 2.2, which is about optimizing safety stock (SS). Then, section 2.3 will review the issues about integrating supply chain planning and SS optimization.

2.2 Multi-echelon safety stock optimization (MESSO)

All competitors in the global market are striving to meet customers' demands and to improve theie level of customer satisfaction. As mentioned in the previous section, uncertainty is one of the major issues in supply chain planning, and inventory is a tool to deal with this issue. Some studies have been carried out to analyze the whole inventory system at different levels (e.g., Clark and Scarf 1960 and Clark and Scarf 1962). They are among the first to study the multi-echelon inventory systems starting with a two-echelon inventory model. They proved the optimality of a base-stock policy for the pure-serial inventory system and developed an efficient decomposition method for the optimal base-stock ordering policy. The importance of considering a multi-echelon inventory system is to keep the stock level balanced from the bottom to the top of inventory systems (Chiang and Monahan 2005).

Though safety stock is often considered as the first line of defence against uncertainties in manufacturing systems such as uncertainties of demand, processing time, and supply, optimizing safety stock is a very challenging task. Lesnaia (2004) showed that the safety stock placement problem is a concave NP-hard problem which has the same complexity explained before by Sahni (1974). Even in a simplified supply chain such as a serial supply chain, optimizing safety stock is still NP-hard (Sitompul et al. 2006).

There are two main approaches to optimize SS in multi-echelon systems (Graves and Willems 2003; Klosterhalfen and Minner 2006). The first one is the stochastic-service approach (SSA) where the safety stock is used as the only procedure to take care of supply chain uncertainties. The randomness of this approach comes from the probability of availability at upstream nodes in a supply chain, so that lead time is considered as a stochastic variable. The second approach is called guaranteed-service approach (GSA). In this approach, other procedures (e.g., increasing production rate by applying overtime or speeding up production) can be used besides safety stock (SS) to deal with uncertainties. The principle of GSA is that the quoting time should not be exceeded to provide the services to the downstream nodes from their predecessors. More details and implementations of these approaches are available in Simpson (1958), Clark and Scarf (1960), Chapter 3 written by Graves and Willems (2003) appearing in the handbook edited by de Kok and Graves (2003), and Klosterhalfen and Minner (2006). A new approach is developed by Klosterhalfen et al. (2013). They introduced an integrated safety stock optimization method called hybrid-service approach (HSA). It is a combination of the stochastic-service approach and the guaranteed-service approach, which ensures the overall cost-optimality for each stage and computes the required inventory.

Due to the complexity of optimizing SS in a multi-echelon system, researchers have attempted to simplify the system. All research papers accessed for this thesis work, starting from the work of Simpson (1958) are limited to serial systems and at most to two-stage systems. The first milestone extension in this area was the work of Inderfurth (1991) in which a dynamic programming algorithm is developed to optimize the safety stock in a multi-echelon inventory system. His work was based on the early study of Simpson (1958). His result was suitable for serial systems and divergent systems. As mentioned before, a serial system is one in which any node at any stage can only receive material from one upstream node and can only send the material to one downstream node. On the other hand, a divergent system (also called distribution system), as Inderfurth (1991) illustrated in his paper, is one where any node in a production/distribution system receives its required material from only one upstream node, but it can send materials to more than one node in the downstream level. The objective function aims to minimize the cost of holding inventory with restricted customer service level. For the same system structure, Inderfurth (1994) investigated practical safety stock problems for multi-stage inventory systems when using both the pull and push production strategies. Besides the serial and divergent systems, Inderfurth and Minner (1998) made further investigations to include another structure called convergent system (assembly system). In a convergent system, every node has at most one direct successor, but it can have more than a node in the upstream level. Starting from a general system structure they simplified the system according to a specific consideration of service measures and classified complexity according to three simpler structures, i.e., serial, divergent, and convergent systems. Even in some recent works such as that of Klosterhalfen et al. (2013), serial systems are still the main focus. Also, Badurina et al. (2013) solved the problem using genetic algorithm and simulated annealing. Their work includes both serial and divergent systems.

Moreover, safety stock has been studied from different perspectives in manufacturing systems. For instance, Hung and Chang (1999) studied how to determine the safety stock to be used against the uncertainty of the manufacturing processes to ensure on-time delivery and provide better customer service. Another example is integrating SS with production (Teimoury et al. 2010). In their study, two models are developed. The first one is an inventory control model developed using the queuing theory aiming to find the level of safety stock and reorder quantity to satisfy the minimum cost. The second is a production planning model which calculates production quantity. They considered inventory only for the final product based on the continuous review (s,Q) policy. Because the inventory is specified for the final product, there is no optimization for safety stock throughout the whole supply chain stages. Another example is integrating SS with demand forecasting (Beutel and Minner 2012). They presented two different frameworks for integrating demand forecasting with safety stock planning when demand is affected by different external factors such as sale price, price change, and weather. Firstly, they set required safety stocks according to the estimated errors of demand forecasting by using a regression model. Secondly, by using linear programming, they optimized inventory function for different objectives according to different service level constraints. The next section provides a review of the integration of safety stock optimization with supply chain planning.

2.3 Supply chain and safety stock integration

Integrating inventory control and safety stock optimization in supply chain planning is an attractive topic for many researchers and practitioners. Thanha et al. (2008) conducted a comprehensive review of supply chain modelling and methods that are used in the last decade. However, all the models that they reviewed were for the strategic (design) level. Even after 2008, the studies of integrating supply chain planning with inventory were still limited to the design stage. For instance, a mixed integer model was developed by Gebennini et al. (2009) to determine the number of facilities required and their locations along a production-distribution system and to incorporate inventory control decision, production rate, and service level to optimize safety stock. The structure of the system is a distribution network of supply chain.

For more supply chain structures, Simchi-Levi and Zhao (2005) studied the impacts of safety stock in three different supply chain systems: serial system, assembly system, and distribution system. The stochastic service approach was used and inventory in each stage was controlled using continuous review based stock policy. The demand follows independent Poisson processes while lead time is exogenously determined.

As mentioned before, optimizing safety stock in a multi-echelon system is NP-hard. As such, an integrated model for supply chain planning and safety stock optimization is also NP-hard. Consequently, researchers have attempted to simplify the model structures. The simplest supply chain network is proposed by Jha and Shanker (2009). They integrated a vendor-buyer model with inventory for a serial supply chain system which contains only two-echelon (vendor and buyer) and one item. Diabat and Richard (2009) developed a model that considers location and decision of a multi-echelon inventory system. They simplified the model so that it contains only one product, one manufacturing site, and multiple retailers. In the same year, Wan and Cong (2009) presented a simulation model of a multi-echelon inventory system for a simple supply chain structure with one manufacturer, one distribution center, and three retailers. Due to the simplicity in modelling and solution procedure, they chose to use simulation by the ARENA software instead of the traditional mathematical methods.

In the inventory control area, the first study for the integration of inventory with supply chain in a multi-echelon system based on mathematical programming is reported by You and Grossmann (2010). The guaranteed service approach is used in their modelling process. The mixed-integer non-linear programming (MINLP) model can be used to determine the network structure of the multi-echelon supply chain, and to make inventory and transportation decisions.

For safety sock optimization, Jung et al. (2004) and Jung et al. (2008) used discrete event simulation and linear programming to evaluate and control safety stock and service level at each stage. The linear programming model uses safety stock and service level as decision variables. In Jung et al. (2004), they considered demand uncertainty, and in Jung et al. (2008), they focused on production capacity. However, in both cases, the internal service time between stages is not optimized. Also, by using simulation, Rawata and Altiok (2008) studied the effect of using different safety stock policies on supply chain performance. In the same year, Li et al. (2008) integrated discrete event simulation with genetic algorithm to solve the problem for two-echelon inventory systems.

Vanteddu et al. (2007) studied placement of safety stock with consideration of cost and response time at the design stage. Using the GSA, Funaki (2012) integrated safety stock placement with supply chain network design. The objective is to minimize all costs related to safety stock placement. Li and Jiang (2012) solved the safety stock optimization problem by integrating constraint programming technique with genetic algorithm for general acyclic supply chain networks. They converted a supply chain network problem into a scheduling network problem so that they were able to use algorithms that are developed in the area of project scheduling to solve the safety stock placement problem.

Gupta (2008) studied inventory management and safety stock placement costs as part of the work in the design of a multi-echelon supply chain. Like many other works, his multi-echelon supply chain is a serial system. Two separate models for inventory and supply chain have been developed. The work of Egri (2012) is one of the most recent works that only considers serial supply chains for simplicity. He investigated the safety stock placement problem in serial supply chains.

2.4 Motivation

The above review suggests that the integration of safety stock optimization with supply chain planning has not been studied adequately and the limited integration studies are confined to the design stage of simple supply chains. Obviously, the "optimal" results obtained in such a fashion are not applicable for complex systems with a general supply chain topology and are infeasible even for simple supply chains. The reasons are simple: a) such results are obtained based on simple structures which cannot be directly applicable to complex systems, and b) more uncertainties will develop during the large time gap between the design and planning stages and hence the previously obtained "optimal" results become out-dated even for a simple supply chain. In light of the above observations, the objectives of this thesis are set to:

1. Develop a multi-echelon safety stock optimization (MESSO) model for supply chain systems with a general configuration,
2. Convert the MESSO model into a robust-MESSO model to mitigate the uncertainty issue,
3. Develop a multi-objective formulation to integrate the MESSO model with a supply chain planning model,
4. Apply the modified fuzzy-Chebyshev programming approach to solve the multi-objective problem, and
5. Provide numerical examples for these models to illustrate their applications.

Chapter 3. Robust Multi-Echelon Safety Stock Model

Inventory is the main defence line that companies use against fluctuations in demand. However, inventory control itself is a challenging topic, particularly in a supply chain environment that faces many uncertainties. More specifically, it is highly desirable to determine the right amount of safety stock to minimize the impact of shortage, whereas the inventory cost can also be minimized at the same time. This is especially important due to the demand uncertainty. This gives rise to the need to develop a robust model that is valid at any given situation and that yields the best solutions despite uncertainty. To this end, a robust model is formulated and compared with the traditional optimization model of the problem.

3.1 Introduction to robust modelling

The robust optimization method was introduced by Mulvey et al (1995). It is a method for formulating problems that have uncertain parameters based on known scenarios. The authors defined two types of robustness of the solution of a model, i.e., *solution robustness* if the solution remains *nearly optimal* for all scenarios, and *model robustness* if the solution is *almost feasible* for all scenarios.

Consider the following programming model (Mulvey et al 1995):

$$\min \quad c^T x + d^T y \tag{3.1}$$

Subject to

$$Ax = b \tag{3.2}$$

$$Bx + Cy = e \tag{3.3}$$

$$x, y \geq 0 \tag{3.4}$$

where *x* and *y* are respectively decision and control variables. If we have a set of scenarios $z = \{1, 2, ..., Z\}$, and a subset of uncertain parameters $\{d_z; B_z; C_z; e_z\}$ associated with probability of scenario ρ_z , $\sum_z \rho_z = 1$, then the robust format for the model will be as follows:

$$\min \quad \sigma\left(x, y_1, y_2, \ldots, y_z\right) + \omega\rho\left(\delta_1, \delta_2, \ldots, \delta_z\right) \tag{3.5}$$

The first term in equation (3.5) represents solution robustness, and the second term represents model robustness. While the solution robustness term is responsible for meeting the minimum objective value, the model robustness term is responsible for panelizing violation solutions. ω is the trade-off weight between solution robustness and model robustness. Constraints (3.2) - (3.4) can be written as constraints (3.6) - (3.8) below.

$$Ax = b \tag{3.6}$$

$$B_z x + C_z y_z + \delta_z = e \qquad \forall\ z \in Z \tag{3.7}$$

$$x \geq 0, y_z \geq 0,\ \delta_z \geq 0 \qquad \forall\ z \in Z \tag{3.8}$$

where δ_z is the deviation of the violation of the control constraint for scenario *z*.

If we consider $\xi = c^T x + d^T y$ as a cost function of the model, the uncertain cost function can be written as $\xi_z = c^T x + d_z^T y_z$ with probability ρ_z. Then, the traditional objective function is converted to include two terms: mean and standard deviation of new stochastic scenario-based objective function, i.e.,

$$\sigma(o) = \sum_{z\in Z} \rho_z \xi_z + \lambda \sum_{z\in Z} \rho_z \left(\xi_z - \sum_{z'\in Z} \rho_{z'} \xi_{z'}\right)^2 \tag{3.9}$$

where λ is solution variance weight factor, which controls the sensitivity of the solution. The higher the value of λ, the less sensitive the solution. Based on the study of Yu and Li (2000) of the computational effort of the robust model, Al-e-hashem et al. (2011) rewrite the equation (3.9) as

$$\sigma(o) = \sum_{z\in Z} \rho_z \xi_z + \lambda \sum_{z\in Z} \rho_z \left[\left(\xi_z - \sum_{z'\in Z} \rho_{z'} \xi_{z'}\right) + 2\theta_z\right] + \omega \sum_{z\in Z} \rho_z \delta_z \tag{3.10}$$

and added the following equations to the constraints:

$$\xi_z - \sum_{z\in Z} \rho_z \xi_z + \theta_z \geq 0 \qquad \forall\ z \in Z \tag{3.11}$$

$$\theta_z \geq 0 \qquad \forall\ z \in Z \tag{3.12}$$

As θ_z shows the deviation of violation, they interpreted constraint (3.11) as: if ξ_z is greater than $\sum_{z\in Z} \rho_z \xi_z$, then $\theta_z = 0$, but if $\sum_{z\in Z} \rho_z \xi_z$ is greater than ξ_z, then $\theta_z = \sum_{z\in Z} \rho_z \xi_z - \xi_z$.

3.2 Problem description

Planning and optimizing safety stock in multi-echelon systems is not a trivial issue due to the uncertainty of demand and the complexity of the model. There are two main approaches to plan safety stock: the stochastic-service approach (SSA) and the guaranteed-service approach (GSA). While safety stock in SSA is regarded as the only means to deal with supply and demand uncertainty, operating flexibility is used beside safety stock to encounter the demand uncertainty in GSA. The difficulty of implementing the first approach (SSA) is due to the fact that the service time depends on the availability of material in upstream points and on the probability of stock-out in downstream points.

In this work, the GSA is used to formulate the safety stock planning model because it is easier to implement theoretically and practically. Each partner in a supply chain quotes a service time to its successors that can be met. After this quoted time, the successor should receive the material requested from its predecessors. Decentralized inventory policy is often used, which means that each partner in the system has inventory and their safety stock should be optimized in coordination with other partners in the same echelon or in other echelons. The demand follows an independent normal distribution. The objective of the model is to minimize the cost of holding safety stock with consideration of customer service level. The system to be considered is general in nature, meaning that any partner (node) at any level can receive material/product from any predecessors and can send material/product to any following successors as shown in

Figure 3.1. Also, a node can receive the same material from different predecessor at the same time.

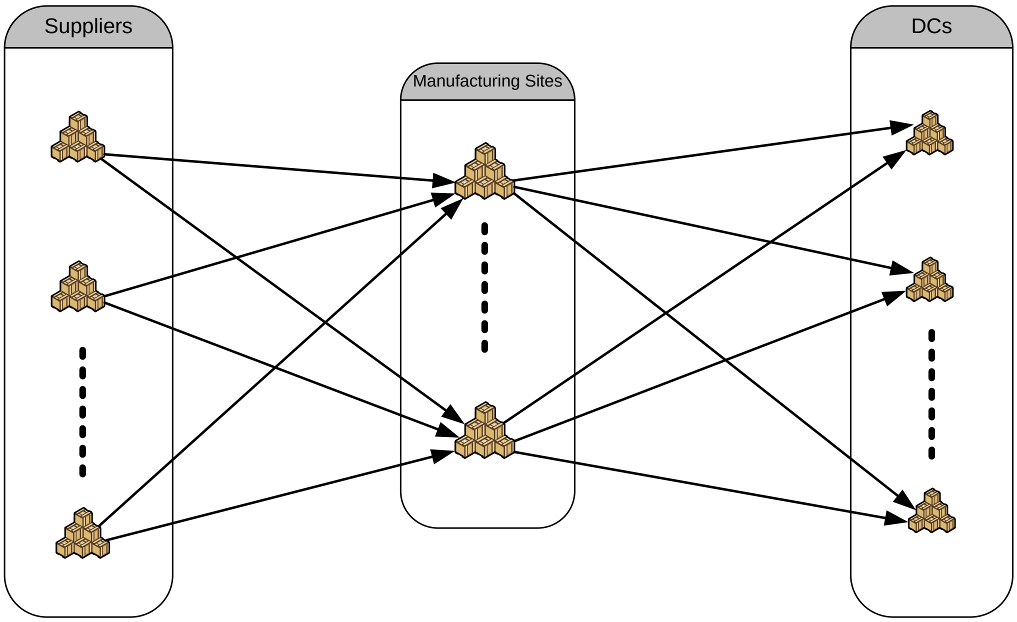

Figure 3.1: The network expression of the problem under consideration

Three different levels are considered, i.e., distribution centers, manufacturing sites, and suppliers. Each member at any level has to consider the safety stock amount to meet the quoted delivery time and to maintain service level.

3.3 The MESSO mathematical model

The multi-echelon safety stock optimization (MESSO) model aims to minimize the total holding costs of all supply chain partners with respect to the time and service level relationships among them.

As shown in Figure 3.1, the first level facing customer demand is distribution centers. A distribution center should hold safety stock that can absorb the normally-distributed demand with respect to a certain service level which is usually determined by the top management according to the organization policy.

Safety stock at a distribution center k for product i can be calculated by

$$SSK_{ik} = \lambda K_{ik} \, \sigma K_{ik} \sqrt{NLTK_{ik}} \tag{3.13}$$

where SSK_{ik} is the amount of safety stock required to cover demand uncertainty bounded by mean and $\pm\sigma K_{ik}$, according to distribution center service level factor λK_{ik} and replenishment lead time $NLTK_{ik}$ specified by

$$NLTK_{ik} = TIK_{ik} + TK_{ik} - TOK_{ik} \tag{3.14}$$

Equation (3.14) is used to calculate the net replenishment lead time $NLTK_{ik}$, which is affected by the time quoted by the previous level (manufacturing site), transportation time, processing time, and the time quoted by the distribution center for delivery, as illustrated by Figure 3.2.

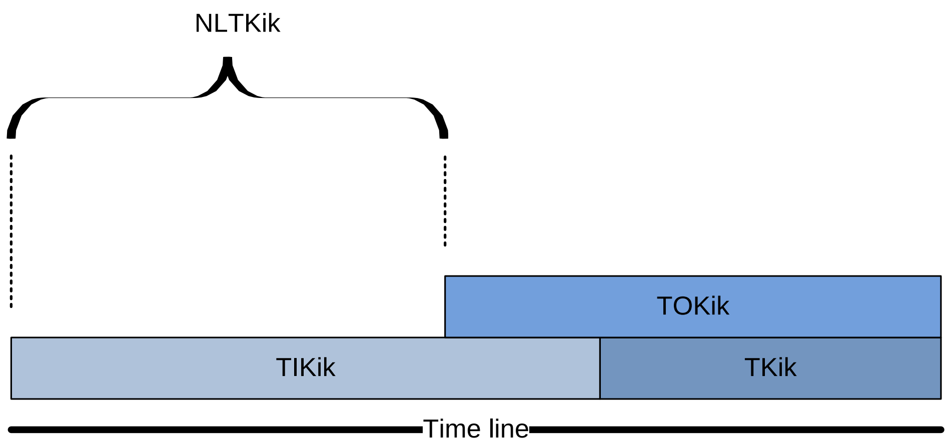

Figure 3.2: Net replenishment lead time.

The minimum quoted time for delivery TOK_{ik} can be zero which means immediate delivery, while the maximum value is the replenishment time from the previous level plus processing time in a distribution center, i.e.,

$$0 \leq TOK_{ik} \leq TIK_{ik} + TK_{ik} \tag{3.15}$$

The guaranteed time for a product to be in a distribution center is equal to the maximum of the summation of time quoted from a manufacturing site TOJ_{ij} plus transportation time TJK_{jk} from manufacturing site j to distribution center k, as expressed below.

$$TIK_{ik} = \max\left\{TOJ_{ij} + TJK_{jk}\right\} \tag{3.16}$$

Similarly, safety stocks in manufacturing sites and suppliers can be calculated respectively using equations (3.17) to (3.20), and equations (3.21) to (3.23), as follows:

$$SSJ_{ij} = \lambda J_{ij}\, \sigma J_{ij} \sqrt{NLTJ_{ij}} \tag{3.17}$$

$$NLTJ_{ij} = TIJ_{ij} + TJ_{ij} - TOJ_{ij} \tag{3.18}$$

$$0 \le TOJ_{ij} \le TIJ_{ij} + TJ_{ij} \tag{3.19}$$

$$TIJ_{ij} = \max\left\{TOS_{cs} + TSJ_{sj}\right\} \tag{3.20}$$

$$SSS_{cs} = \lambda S_{cs}\, \sigma S_{cs} \sqrt{NLTS_{cs}} \tag{3.21}$$

$$NLTS_{cs} = TIS_{cs} + TS_{cs} - TOS_{cs} \tag{3.22}$$

$$0 \le TOS_{cs} \le TIS_{cs} + TS_{cs} \tag{3.23}$$

$$all\,variables\,are\,non\,negitivite \tag{3.24}$$

Service level factors λK_{ik}, λJ_{ij}, and λS_{cs} may be common among all supply chain levels or independent for each level.

The standard deviation of the demand that is used to calculate safety stock at manufacturing sites or suppliers is dependent on the standard deviation of the demand at distribution centers. Because any manufacturing site can manufacture and deliver some or all products to one or more distribution centers, the standard deviation at a manufacturing site can be calculated as the square root of the sum of the standard deviation of a product at distribution centers, $\sigma J_{ij} = \sqrt{\sum_k \left(\sigma K_{ik}\right)^2}$. However, a distribution center can receive a product from more than one manufacturing site with different percentages. This percentage can be represented by variable FJ_{ij} in a manufacturing site and by FS_{cs} at a supplier, as in expressions (3.25) and (3.26) respectively. These two variables take value between 0 and 1, and the summation of each one at all manufacturing sites and suppliers sites is equal to 1, as illustrated by expressions (3.27) and (3.28). For suppliers, standard deviations are not for the product but for components that are required by manufacturing sites to produce products, so that the correlation between product and components is used to calculate the equivalent standard deviations for suppliers as in expression (3.26).

$$\sigma J_{ij} = \sqrt{\sum_k \left(\sigma K_{ik}\, FJ_{ij}\right)^2} \tag{3.25}$$

$$\sigma S_{cs} = \sqrt{\sum_i \sum_j \left(\sigma J_{ij}\, FS_{cs}\, \rho_{ci}\right)^2} \tag{3.26}$$

$$\sum FJ_{ij} = 1 \tag{3.27}$$

$$\sum FS_{cs} = 1 \tag{3.28}$$

where ρ_{ci} represents the amount of a component used in a product. Then, the objective function can be expressed as

$$\min \quad Cost \tag{3.29}$$

where

$$Cost = \sum_s \sum_{c \in C_s} HS_{CS}\, SSS_{cs} + \sum_j \sum_{i \in I_j} HJ_{ij} SSJ_{ij} + \sum_k \sum_{i \in I_k} HD_{ik}\, SSK_{ik} \tag{3.30}$$

The above objective function represents all the costs of holding safety stocks at all partners of the supply chain. The first term is the holding cost of the safety stock at all suppliers for all components. The cost of holding the safety stock of all products in manufacturing sites and distribution centers are presented by the second and third terms respectively.

3.4 Robust MESSO model (RMESSO)

The uncertainty of demand in the safety stock model may affect the value of standard deviation of the demand at a distribution center, σK_{ik}. Assuming that the demand follows normal distribution, we can consider the demand within different control limits: $(\mu \pm \sigma)$ with a probability of 68.2%, $(\mu \pm 2\sigma)$ with a probability of 95.4%, or $(\mu \pm 3\sigma)$ with a probability of 99.7%, (see Figure 3.3).

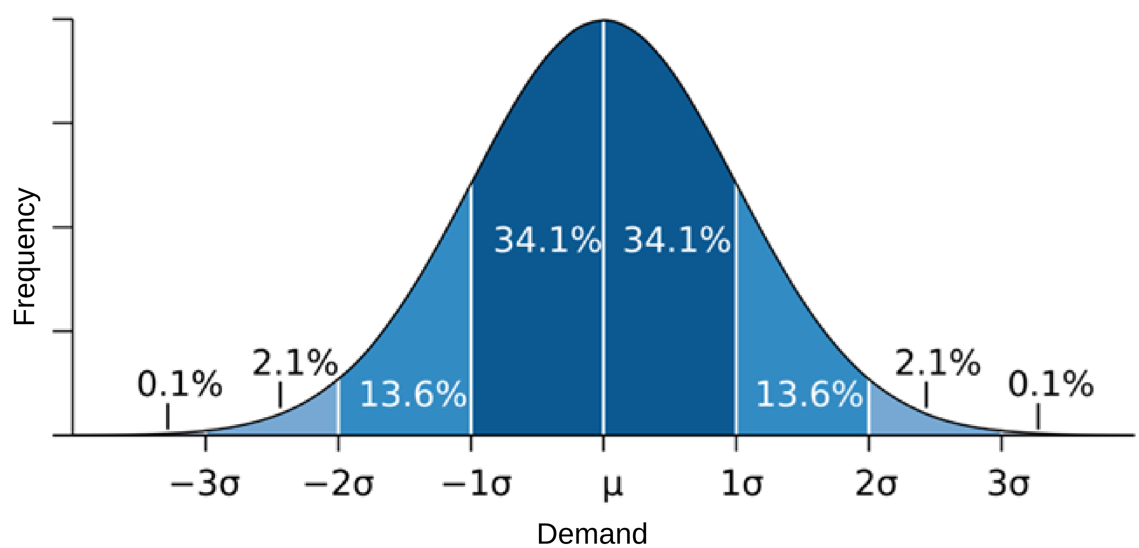

Figure 3.3: Normal distribution (adapted from Wikipedia)

According to these different limits, three scenarios can be created based on the value of standard deviation of the demand that should be covered by safety stock:

Scenario 1: when safety stock should cover σ of demand with a probability of 68%. This scenario is used when the demand is within limits $(\mu \pm \sigma)$.

Scenario 2: when safety stock should cover 2σ of demand with a probability of 27%. This scenario is used when the demand is out of limits $(\mu \pm \sigma)$ but within limits $(\mu \pm 2\sigma)$.

Scenario 3: when safety stock should cover 3σ of demand with a probability of 5%. This scenario is used when the demand is out of limits $(\mu \pm 2\sigma)$ but within limits $(\mu \pm 3\sigma)$.

Now, the robust model can be formulated as follows:

Objective function

$$\min \quad \sum_z P_z Cost_z + \lambda \sum_z P_z \left[\left(Cost_z - \sum_{z'} P_{z'} Cost_{z'} + 2\theta_z \right) \right] + \omega \sum_z \sum_k \sum_{i \in I_k} P_z \delta_{ikz} \quad (3.31)$$

The first and second terms are the mean and variance of the objective function while the last term is the model robustness measure. P_z is the probability of occurrence of scenario z, and λ denotes the weight placed on solution variance. The higher λ is, the less sensitive the data under all scenarios will be. ω is a parameter used for the trade-off between solution robustness and model robustness.

Parameters and variables that are affected by scenarios will still have the same indications and meanings with respect to scenario z. For example, while σS_{cs} is the standard deviation of the demand of component family c at supplier s, σS_{csz} is the standard deviation of the demand of component family c at supplier s for scenario z. Constraints (3.13), (3.17), and (3.21) in the above will be replaced by constraints (3.32), (3.33), and (3.34) as follows:

$$SSS_{csz} = \lambda S_{cs}\, \sigma S_{csz} \sqrt{NLTS_{cs}} \quad (3.32)$$

$$SSJ_{ijz} = \lambda J_{ij}\, \sigma J_{ijz} \sqrt{NLTJ_{ij}} \quad (3.33)$$

$$SSK_{ikz} = \lambda K_{ik}\, \sigma K_{ikz} \sqrt{NLTK_{ik}} + \delta_{ikz} \quad (3.34)$$

$$Cost_z - \sum_{z'} P_{z'} Cost_{z'} + \theta_z \geq 0 \quad (3.35)$$

$$\delta_{ikz} \geq 0, and\, \theta_z \geq 0 \quad (3.36)$$

Constraint (3.34) is a control constraint used to calculate the amount of safety stock required in each distribution center, and δ_{ikz} is the deviation of the violation of the control constraint according to scenario z for product family i at distribution center k. Constraint (3.35) comes from linearization of the original robust objective function consisting of only mean and standard deviation of the objective function as follows:

$$\sum_z P_z Cost_z + \lambda \sum_z P_z \left(Cost_z - \sum_{z'} P_{z'} Cost_{z'} \right)^2 \quad (3.37)$$

The standard deviation values at manufacturing sites and suppliers are modified to consider different scenarios. The modified versions of the two are

$$\sigma J_{ijz} = \sqrt{\sum_k \left(\sigma K_{ikz}\, FJ_{ij}\right)^2} \tag{3.38}$$

$$\sigma S_{csz} = \sqrt{\sum_i \sum_j \left(\sigma J_{ijz}\, FS_{cs}\, \rho_{ci}\right)^2} \tag{3.39}$$

$$all\, variables\, are\, non\, negitivite \tag{3.40}$$

Chapter 4. MESSO Case Study

In this chapter, a three-level supply chain with three suppliers, two manufacturing sites, and three distribution centers is studied. Three different products can be produced in manufacturing sites using six different components. The non-linear problem is implemented using the AIMMS software (CONOPT solver). The standard deviation of the demand is assumed to be 10. Tables 4.1 to 4.5 list the values of the parameters.

Table 4.1: Holding cost and processing time at distribution centers

Product family (i)	Holding cost (HD_{ik}) ($)			Processing time (TK_{ik}) (min)		
	Distribution centers (k)			Distribution centers (k)		
	1	2	3	1	2	3
1	10	11	11	2	3	4
2	17	15	16	3	3	3
3	24	23	19	2	3	2

Table 4.2: Holding cost and processing time at manufacturing sites

Product family (i)	Holding cost (HJ_{ij}) ($)		Production time (TJ_{ij}) (min)	
	Manufacturing site (j)		Manufacturing site (j)	
	1	2	1	2
1	10	13	3	6
2	18	17	4.5	8
3	23	19	3.5	4

Table 4.3: Holding cost and production time of suppliers

Component family (c)	Holding cost (HS_{cs}) ($)			Production time (TS_{cs}) (min)		
	Supplier (s)			Supplier (s)		
	1	2	3	1	2	3
1	1.6	1	1.3	2	4	2.5
2	1.5	1.2	1	2	3	3.5
3	1	1.6	1.4	4	2	4
4	1.3	1.7	1.5	4.5	2	3.5
5	1.4	1	1.2	2	4.5	3
6	1	1.5	1.3	5	3	3

Table 4.4: Transportation time between levels

Supplier (s)	Transportation time (TSJ_{sj}) – Manufacturing site (j)		Manufacturing Site (j)	Transportation time (TJK_{jk}) – Distribution centers (k)		
	1	2		1	2	3
1	2	4	1	2	3.5	5
2	3	3.5	22	5	4	3
3	5	2				

Table 4.5: Number of components required for each product

Component family (c)	No. of components required for each product – ρ_{ci} – Product family (i)		
	1	2	3
1	1	1	1
2	2	2	2
3	1	1	1
4	4	4	4
5	2	2	2
6	1	1	1

The time quoted by suppliers to start preparing components, TIS_{cs}, is assumed to be 0.1 time units for all suppliers. Similarly, the time quoted at distribution centers for products to be ready for delivery to customers, TOK_{ik}, is set to be 1 time unit. The standard deviation for demand normal distribution is 10 for all products, and the service levels at all supply chain partners should be 97.5%, which means that the values of service level factors are 1.96. λ and ω are chosen to be 0.001 and 1 respectively.

The model is solved four times with different values of the demand standard deviation, and one last time as a robust model with considerations of all scenarios. In the first time, the safety stock model is solved for three standard deviations, which means the maximum amount of safety stock to cover all possible variations in demand. Secondly, the model is solved to cover only one standard deviation of demand. In the third and fourth times, the model is solved to cover the average of the values of standard deviation. While in the third time, the average is calculated with equal chance (i.e. $\left(\sigma + 2\sigma + 3\sigma\right)/3$), in the fourth time, the probability is considered when the average standard deviation is calculated (i.e. $\left(0.68\right)\sigma + \left(0.27\right)2\sigma + \left(0.05\right)3\sigma$). All these four solutions are for the regular model, not the robust one. Finally, the robust model is solved considering all scenarios.

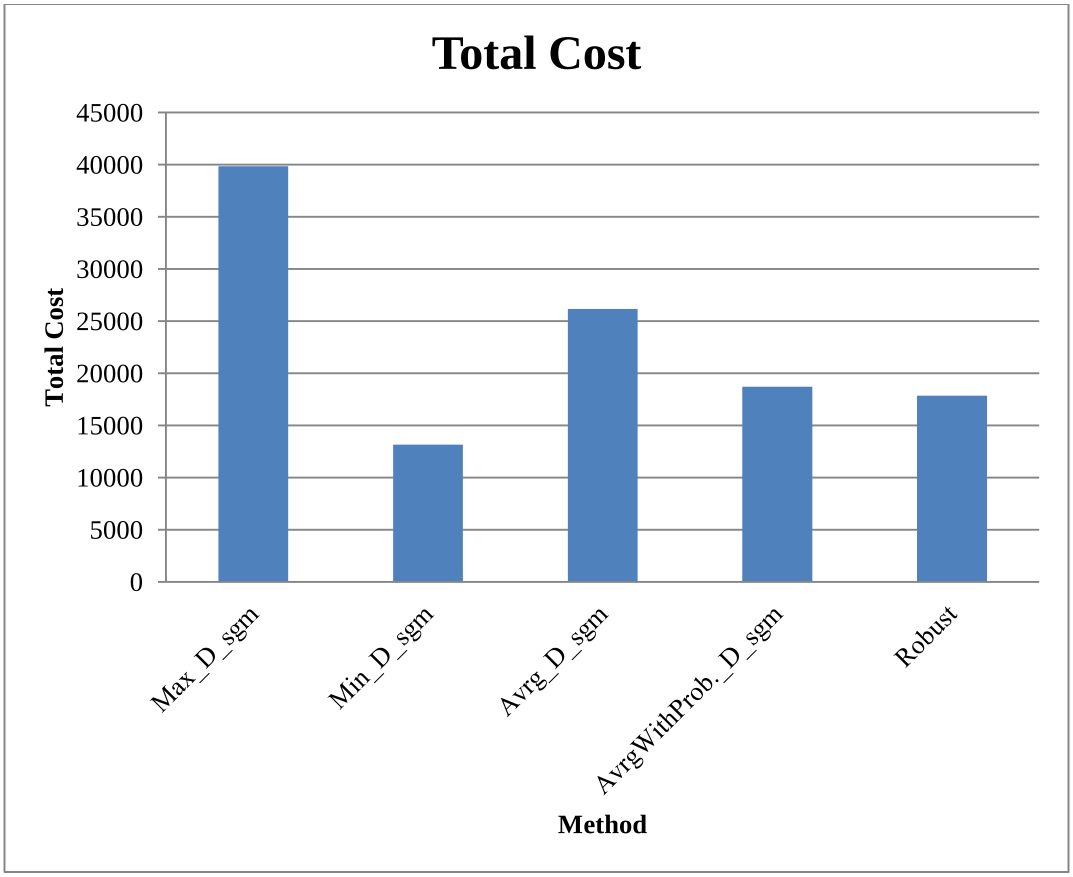

Figure 4.1: The comparison of costs obtained by different solution methods

Figure 4.1 shows the cost of each solution. The first and second column (Max_D_sgm and Min_D_sgm) represent the cost when safety stock should cover 3σ and σ respectively. The cost in the first case is very high and this case is not practical. The second case is the most popular way to calculate the safety stock, but it causes shortages for some demands when they are out of $\mu \pm \sigma$ with a probability of about 32%. The closest cost to the robust result occurs when we consider the average standard deviation with respect to their probabilities.

In Figure 4.2, the amounts of safety stock are compared. In the case study, it is interesting to note that in the robust solution, the model enforces the replenishment lead time at the supplier and the manufacturing sites to be 0 and holds the safety stock at distribution centers only when the fluctuation of demand occurs.

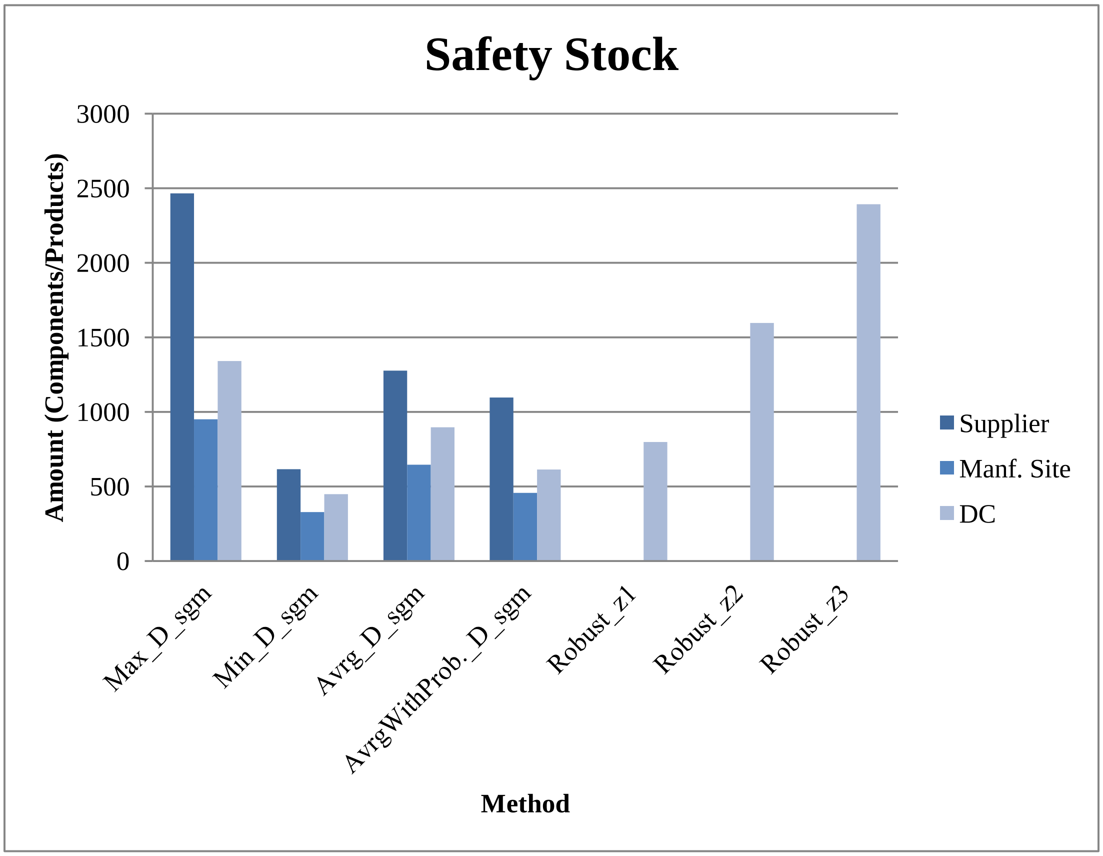

Figure 4.2: Safety stocks obtained by different solution methods

One of the main advantages of the proposed MESSO model is that it optimizes the amount of safety stock that each supplier, manufacturing site, and distribution center should hold in a supply chain network. The two variables introduced in the model, FJ_{ij} and FS_{cs}, work well in the optimization of the amount of safety stock. For instance, Table 4.6 shows the amounts of safety stock at each manufacturing site and Table 4.7 indicates the optimized percentage obtained by the model. Similarly, Table 4.8 and 4.9 are for suppliers.

Table 4.6: Amounts of safety stocks of each product at the two manufacturing sites

Product family (i)	Safety stock (SSJ_{ij}) (product units)	
	Manufacturing site (j)	
	1	2
1	100.43	48.84
2	91.43	71.05
3	42.61	103.23

Table 4.7: Optimized percentages of holding safety stock at manufacturing sites

Product family (i)	FJ_{ij} (%)	
	Manufacturing site (j)	
	1	2
1	69	31
2	59	41
3	29	71

Table 4.8: Amounts of safety stocks of each component at supplier sites

Component family (c)	Safety stock (SSS_{cs}) (component units)		
	Supplier (s)		
	1	2	3
1	23.35	74.91	7.87
2	53.30	33.62	154.85
3	102.59	15.66	0.00
4	87.35	130.45	106.08
5	46.85	101.32	58.03
6	34.44	25.07	40.17

Table 4.9: Optimized percentages of safety stocks at supplier sites

Component family (c)	FS_{cs} (%)		
	Supplier (s)		
	1	2	3
1	26	67	7
2	26	14	61
3	79	21	0
4	19	55	26

5	26	44	30
6	27	31	42

It should be pointed out that the model is valid for different supply chain topologies by using variables FJ_{ij} and FS_{cs} as parameters and by setting their values to 0 or 1. Moreover, the model can be used for different assumptions. For example, in our numerical example we assume that the components can be offered from all suppliers to all manufacturing sites. We can also assume that only one supplier can provide a certain component, or a manufacturing site cannot produce a certain product. The validity of our model comes from considering the general system so that the other system structures are special cases of the general system.

Chapter 5. Integration of MESSO with Supply Chain Planning

5.1 Introduction

As mentioned before, optimizing supply chain planning is important for an organization to be competitive. As such, supply chain planning has attracted intensive research activities at all levels (i.e., strategic, tactical, and operational). Inventory is one of the most important issues in supply chain planning. The proper selection of inventory location and level of safety stock at all supply chain partners is essential in highly competitive markets to manage uncertainty demand and service level. In the current literature, some studies (e.g., Osman and Demirli 2012) have been optimized the location of safety stock along supply chain for fast response to fluctuation in demand, and different safety stock policies (centralized and decentralized). However, most of these studies focus on the issues of the design stage of a supply chain. The outcome may not be applicable because of the many unexpected events that may occur during the long time span between the design and production stages. Hence, the postponement of safety stock optimization to the planning stage is more accurate due to the reduced uncertainties. At the planning (tactical) level, the safety stock can be controlled according to each planning horizon to satisfy customer demand at lower cost instead of being fixed by a decision taken at the strategic level. Demand at different horizons may vary for different reasons such as the entry of different competitors into the market, or seasonal demand, and hence safety stock should be optimized accordingly. Furthermore, to ensure the feasibility and the guidance of the higher level planning, the optimization processes should not be done in isolation from other stages of supply chain planning. Hence, an extended model from the supply chain tactical model of Fakharzadeh-Naeini (2011) is integrated with safety stock in a multi-echelon inventory system in this chapter.

5.2 Problem description and formulation

Supply chain planners have a common goal, which is to meet customer expectations at the lowest possible total cost. The costs to be considered in this chapter include raw material costs, manufacturing costs, transportation costs, holding costs, backlog costs, and overtime costs. The demand is stochastic with a known normal distribution $N(\mu, \sigma^2)$. The problem is formulated as a multi-objective mixed-integer non-linear model for a multi-product multi-level multi-period system. Suppliers, manufacturing sites, and distribution centers are the levels in the supply chain to be studied. Suppliers can offer some or all components to any manufacturing site, and the latter can produce any product and send it to distribution centers. However, the suppliers are different in terms of cost and lead time. The same can be said for manufacturing and distribution centers. Different tactical horizons are studied which may have similar or different tactical period lengths. A tactical horizon can be divided into a number of tactical periods (see Figure 5.1).

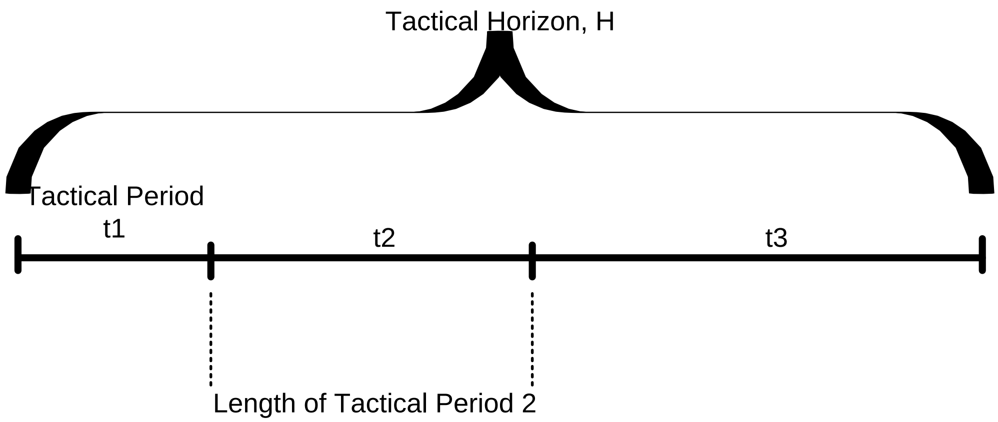

Figure 5.1: Tactical horizon and tactical periods

The inventory of each supply chain partner is illustrated in Figure 5.2. Optimizing safety stock in a multi-echelon inventory system in this work follows a guaranteed service approach.

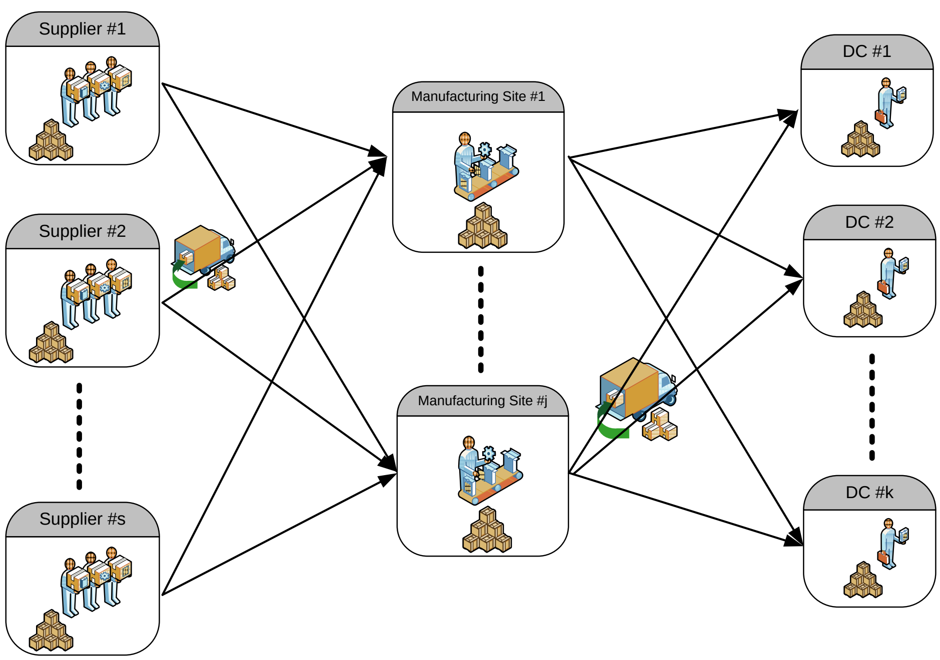

Figure 5.2: Supply chain structure

The model aims to optimize two conflicting objective functions: cost and lead time. The former includes five different costs: production costs, transportation costs, over time costs, inventory costs, and backlog costs. The latter, i.e., the lead time objective function, includes production time and transportation time at suppliers and manufacturing sites, as well as preparation time at distribution centers. Transportations between levels are incapacitated and materials are shipped out as soon as the required amount of materials becomes available. In other words, the transportation time is used once the components or products are shipped to a downstream point. The model and other constraints are presented in more detail in the following sections.

5.3 Mathematical model

In this model, the safety stock optimization in a multi-echelon inventory system (MEIS) is integrated with a supply chain planning model subject to several sets of constraints. The objective of the model is to minimize the total lead time and total costs. The model covers three types of supply chain partners: manufacturers, suppliers, and distributers. The model constraints and objective function are as follows.

Production capacities:

Usually, a production system has limits on its production capacity. The capacity can be formulated as constrain (5.1). If product family i is assigned to be produced in manufacturing site j, then X_{ijt} will equal 1, and 0 otherwise. The amount of production in this time period XP_{ijt} should not exceed the production facility capacity UB_{ij} and not be any lower than LB_{ij}, i.e.,

$$LB_{ij} X_{ijt} \leq XP_{ijt} \leq UB_{ij} X_{ijt} \qquad \forall i \in I_j, j, t \tag{4.1}$$

The amount of components provided by suppliers is not restricted so that manufacturing sites can have any amount of components needed.

Material balance:

Material balance constraints link different supply chain levels. Also, they are the links between time periods. For instance, constraint (5.2) expresses that the inventory of product family *i* at the end of a tactical period *t* at distribution center *k* is equal to the amount of inventory from a previous time period, plus the sum of all products of family *i* shipped from manufacturing site *j* in the current time period and the unfulfilled demand in current time *t*, minus the demand required in *t* and the unfulfilled demand from the previous time *t*-1. Because backlogging is allowed, the unfulfilled demand term is added to the material balance equation at distribution centers. Hence one has

$$IPD_{ikt} - UD_{ikt} = IPD_{ikt-1} - UD_{ikt-1} + \sum_{j \in j_k} XSM_{ijkt} - D_{ikt} \qquad \forall i \in I_j, j, k \in k_j, t \tag{4.2}$$

In the following, constraints (5.3) and (5.4) represent the material balance at manufacturing sites and suppliers respectively. Constraint (5.3) shows that the amount of inventory at the end of a time period is equal to the amount of inventory carried from the previous time period plus the amount of product family *i* produced in time *t* at manufacturing site *j* minus the amount of product sent to all distribution centers *k*'s. Similarly, constraint (5.4) shows that the amount of inventory of component family *c* at supplier *s* at time *t* is equal to the quantity of components carried from previous time *t*-1 plus the amount of components produced in time *t* minus the total amount of components sent to all manufacturing sites *j*'s.

$$IPS_{ijt} = IPS_{ijt-1} + XP_{ijt} - \sum_{k \in k_j} XSM_{ijkt} \qquad \forall i \in I_j, j, k \in k_j, t \tag{4.3}$$

$$ICS_{cst} = ICS_{cs(t-1)} + XCP_{cst} - \sum_{j \in J_s} XSS_{csjt} \qquad \forall c \in C_s, s, j \in J_s, t \tag{4.4}$$

Inventory boundaries:

The amount of inventory of component *c* at supplier *s* and product *i* at site *j* or distribution center *k* at any time is restricted. The lower bound is equal to the amount of safety stock that should be held at any time while the upper bound is equal to the mean demand multiplied by the total lead time plus safety stock. Constraints (5.5) and (5.6), constraints (5.7) and (5.8), and constraints (5.9) and (5.10) specify the lower and upper bounds of the amount of inventory at distribution centers, manufacturing sites, and supplier respectively.

$$IPD_{ikt} \geq SSK_{ik} \qquad \forall i \in I_k, k, t \tag{4.5}$$

$$IPD_{ikt} \leq \mu K_{ik} NLTK_{ik} + SSK_{ik} \qquad \forall i \in I_k, k, t \tag{4.6}$$

$$IPS_{ijt} \geq SSJ_{ij} \qquad \forall i \in I_j, j, t \tag{4.7}$$

$$IPS_{ijt} \leq \mu J_{ij}\, NLTJ_{ij} + SSJ_{ij} \qquad \forall i \in I_j, j, t \tag{4.8}$$

$$ICS_{cst} \geq SSS_{cs} \qquad \forall c \in C_s, s, t \tag{4.9}$$

$$ICS_{cst} \leq \mu S_{cs}\, NLTS_{cs} + SSS_{cs} \qquad \forall c \in C_s, s, t \tag{4.10}$$

Safety stock optimization:

Safety stock is calculated according to safety factors set by management, standard deviation of the demand at a location, and net lead time. It is optimized using the guaranteed-service approach. Constraints (5.11) – (5.21) are used to optimize the safety stock at all supply chain members simultaneously. Here, the guaranteed time of products or components to be in distribution centers or manufacturing sites, TIK_{ik} and TIJ_{ij}, is equal to the maximum of time quoted by the upstream members plus transportation time. For example, if product *i* requires three components *c*1, *c*2, and *c*3, then manufacturing site *j* cannot start producing product *i* until the last component arrives. These constraints are written as

$$SSK_{ik} = \lambda K_{ik}\, \sigma K_{ik} \sqrt{NLTK_{ik}} \tag{4.11}$$

$$NLTK_{ik} = TIK_{ik} + TK_{ik} - TOK_{ik} \tag{4.12}$$

$$0 \leq TOK_{ik} \leq TIK_{ik} + TK_{ik} \tag{4.13}$$

$$TIK_{ik} = \max\left\{TOJ_{ijk} + TJK_{jk}\right\} \tag{4.14}$$

$$SSJ_{ij} = \lambda J_{ij} \, \sigma J_{ij} \sqrt{NLTJ_{ij}} \quad (4.15)$$

$$NLTJ_{ij} = TIJ_{ij} + TJ_{ij} - TOJ_{ijk} \quad (4.16)$$

$$0 \le TOJ_{ijk} \le TIJ_{ij} + TJ_{ij} \quad (4.17)$$

$$TIJ_{ij} = \max\left\{TOS_{csij} + TSJ_{sj}\right\} \quad (4.18)$$

$$SSS_{cs} = \lambda S_{cs} \, \sigma S_{cs} \sqrt{NLTS_{cs}} \quad (4.19)$$

$$NLTS_{cs} = TIS_{cs} + TS_{cs} - TOS_{csij} \quad (4.20)$$

$$0 \le TOS_{csij} \le TIS_{cs} + TS_{cs} \quad (4.21)$$

However, the mean internal demand between distribution centers and manufacturing sites, and that between manufacturing sites and suppliers, are calculated according to external demand placed by customers at distribution centers. If the demand for product *i* at distribution center *k* has a mean equal to μK_{ik} and a variance of σK_{ik}^2 , then the means and standard deviations at a manufacturing site and a supplier can be calculated as follows:

$$\mu J_{ij} = \sum_k \mu K_{ik} \quad (4.22)$$

$$\sigma J_{ij} = \sqrt{\sum_k \left(\sigma K_{ik}\right)^2} \quad (4.23)$$

$$\mu S_{cs} = \sum_i \sum_j \mu J_{ij} \, \rho_{ci} \quad (4.24)$$

$$\sigma S_{cs} = \sqrt{\sum_i \sum_j \left(\sigma J_{ij} \, \rho_{ci} \right)^2} \tag{4.25}$$

where ρ_{ci} is the number of components c required for product i.

Because, if required, more than one supplier can provide materials to the same or different manufacturing sites simultaneously, and multiple manufacturing sites can serve multiple distribution centers at the same time, the model becomes more complex and duplication could happen. To avoid duplication in the calculation of inventory limits and safety stock at suppliers, manufacturing sites, and distribution centers, the related fractions are introduced to optimize the amount of materials that each supply chain partner should hold. These fractions can be used as parameters when they are known to evaluate a supply chain plan, or they can be used as variables for optimization. According to these fraction values (i.e. FJ_{ij} and FS_{cs}), mean and standard deviation values at each level of supply chain must be recalculated. In the following, equations (5.26) and (5.27) are used to calculate mean and standard deviation of the demand at distribution centers respectively, equations (5.28) to (5.31) are provided to calculate the mean and standard deviation values at manufacturing sites and suppliers according to the mean and standard deviation values at distribution centers.

$$\mu K_{ik} = \frac{\sum_t D_{ikt}}{T} \text{, } where \, T \, is \, the \, number \, of \, time \, periods \tag{4.26}$$

$$\sigma K_{ik} = \sqrt{\frac{\sum_t \left(D_{ikt} - \mu K_{ik} \right)^2}{T}} \tag{4.27}$$

$$\mu J_{ij} = \sum_k \mu K_{ik} \, FJ_{ij} \tag{4.28}$$

$$\sigma J_{ij} = \sqrt{\sum_k \left(\sigma K_{ik} \, FJ_{ij}\right)^2} \tag{4.29}$$

$$\mu S_{cs} = \sum_i \sum_j \mu J_{ij} \, FS_{cs} \rho_{ci} \tag{4.30}$$

$$\sigma S_{cs} = \sqrt{\sum_i \sum_j \left(\sigma J_{ij} \, FS_{cs} \, \rho_{ci}\right)^2} \tag{4.31}$$

The following constraints, (5.32) and (5.33), ensure that the fraction values of potential suppliers and manufacturing sites be equal to 1, which means the relative amount of components or product at a supplier or a manufacturing site is exactly the amount required by a distribution center.

$$\sum FJ_{ij} = 1 \tag{4.32}$$

$$\sum FS_{cs} = 1 \tag{4.33}$$

Time availability (production capacity):

Production capacities and availabilities at manufacturing sites and suppliers are controlled by constraints (5.34) - (5.37), which state that the amount of production at any time period should not exceed the available time at that time period. The number of time units available at each tactical period is represented by parameter N_t. N_t can be the same or different from one tactical period to another and can be considered as the length of tactical period *t*. In addition, overtime is another capacity for production at time period *t*, which can be used when necessary, but it should not exceed a certain amount of time, as specified by constraints (5.36) to (5.37)

$$\sum_{i \in I_j} TJ_{ij}\, XP_{ijt} \leq PA_j\, N_t + XMOT_{jt} \qquad \forall i \in I_j, j, t \tag{4.34}$$

$$\sum_{c \in C_s} TS_{cs}\, XCP_{cst} \leq SA_s\, N_t + XSOT_{st} \qquad \forall c \in C_s, S, t \tag{4.35}$$

$$XMOT_{jt} \leq MOT_j\, N_t \qquad \forall j, t \tag{4.36}$$

$$XSOT_{st} \leq SOT_s\, N_t \qquad \forall s, t \tag{4.37}$$

Storage capacity:

Each supply chain member has a limited space for storage. Constraints (5.38) to (5.40) represent the maximum limits of storage facilities at distribution centers, manufacturing sites, and suppliers respectively.

$$\sum_{i \in I_k} \propto_i IPD_{ikt} \leq KSA_k \qquad \forall k, t \tag{4.38}$$

$$\sum_{i \in I_j} \propto_i IPS_{ijt} \leq JSA_j \qquad \forall i \in I_j, j, t \tag{4.39}$$

$$\sum_{c \in C_s} \beta_c\, ICS_{cst} \leq SSA_s \qquad \forall s, t \tag{4.40}$$

Other constraints:

The conversion of components into products is expressed by

$$\sum_{j \in J_s} XSS_{csjt} = \sum_{i \in I_j} \rho_{ci} XP_{ijt} \qquad \forall c \in C_s, s, j \in J_s, t \tag{4.41}$$

According to this relationship between products and components, planners should inform the suppliers of the amount of components required at each time period.

Finally, all variables are non-negative, i.e.,

$$all\,variables\,are\,non\,negitivite \tag{4.42}$$

Objective Functions:

The model has two objective functions: minimization of total cost (5.43) and minimization of lead time (5.49). The total cost includes production costs at suppliers and manufacturing sites (*PC*), inventory costs at all levels (*IC*), backlog costs (*BC*), transportation costs between levels (*TC*), and overtime costs (*OC*). These costs are presented by equations (5.44) to (5.48) respectively.

$$Min\ Cost = PC + IC + BC + TC + OC \tag{4.43}$$

where:

$$PC = \sum_t \sum_j \sum_{i \in I_j} PI_{ij}\, XP_{ijt} + \sum_t \sum_s \sum_{c \in C_s} CP_{cs}\, XCP_{cst} \tag{4.44}$$

$$IC = \sum_t \sum_j \sum_{i \in I_j} HJ_{ij} N_t\, IPS_{ijt} + \sum_t \sum_k \sum_{i \in I_k} HD_{ik} N_t IPD_{ikt} + \sum_t \sum_s \sum_{c \in C_s} HS_{cs} N_t\, ICS_{cst}$$

(4.45)

$$BC = \sum_t \sum_k \sum_{i \in I_k} UC_{ik}\, UD_{ikt} \quad (4.46)$$

$$TC = \sum_t \sum_s \sum_{j \in J_s} \sum_{c \in C_s} TSS_{csj}\; XSS_{csjt} + \sum_t \sum_j \sum_{k \in K_j} \sum_{i \in I_j} TSD_{ijk}\, XSM_{ijkt} \quad (4.47)$$

$$OC = \sum_t \sum_s CSOT_c\; XSOT_{st} + \sum_t \sum_j CMOT_j\; XMOT_{jt} \quad (4.48)$$

The second objective function, (5.49), deals with times required at each level until the products become available at distribution centers. Four terms are in the time objective function: *TCS*, which is the total time required for all components at all suppliers until they are shipped to manufacturing sites; *TIM*, the total time needed for all product families at all manufacturing sites until they are delivered to distribution centers; *TID*, the total time required for all products at all distribution centers until they are ready for customers; and *BT*, the total time required for the unfulfilled demand. These four cost components are expressed by equations (5.50) to (5.53) respectively.

$$\min Time = TCS + TIM + TID + BT \quad (4.49)$$

where:

$$TCS = \sum_t \sum_s \sum_{c \in C_s} (NLTS_{cs} + TS_{cs}(XCP_{cst} - 1)) \quad (4.50)$$

$$TIM = \sum_t \sum_j \sum_{i \in I_j} (NLTJ_{ij} + TJ_{ij}(XP_{ijt} - 1)) \quad (4.51)$$

$$TID = \sum_t \sum_k \sum_{i \in I_k} (NLTK_{ik} + TJ_{ij}(D_{ikt} - UD_{ikt})) \quad (4.52)$$

$$BT = \sum_t \sum_k \sum_{i \in I_k} UT_{ik} UD_{ikt} \quad (4.53)$$

5.4 Method used to solve the multi-objective model

As mentioned in the literature review, optimizing a multi-objective problem can be done using different methods. In the present research, the method used is Liu and Liang (2008)'s modified fuzzy-Chebyshev programming method (MFCP).

Suppose we have a maximization problem with F objective functions, Z_f (f=1,2,…,F). The method can be written in steps as follows:

1) Find the maximum and minimum values of the objective functions, Z_f^U and Z_f^L, by solving the problem as a single objective problem.

2) Define the importance of each objective and sort them according to their importance increasingly.

3) Assign a dummy variable to each objective function which represents the deviation level of the objective called satisfaction level, δ_f, where $\delta_f \leq \dfrac{Z_f - Z_f^L}{Z_f^U - Z_f^L}$.

4) Arranging these satisfaction level indicators in ascending order, $\delta_1 > \delta_2 > \ldots > \delta_f > \delta_{f+1} > \ldots > \delta_F$, where δ_1 is the most important and δ_F is the least important.

5) Use weight, w_f, to define the relationship between two immediate neighbouring satisfaction level indicators, i.e., $\delta_{f+1} = w_f \delta_f$, or

$$\delta_2 = w_1\delta_1; \ldots; \delta_F = w_{F-1}\delta_{F-1}$$

where w_f is a self-adjusted weight generated according to fuzzy function of the objective with respect to the maximum and minimum values of objective functions obtained in step 1, and it represents the best deviation level between each two consecutive objectives. It is calculated as $w_f = \frac{Z_f - Z_f^L}{Z_f^U - Z_f^L}$

6) Finally, re-write the model as follows:

$$\max \delta_1 \tag{4.54}$$

$S.T.$

$$\delta_f \le \frac{Z_f - Z_f^L}{Z_f^U - Z_f^L} \qquad \forall f = 1,2,\ldots,F \tag{4.55}$$

$$\delta_{f+1} = w_f \delta_f \qquad \forall f = 1,2,\ldots,F \tag{4.56}$$

$$w_f = \frac{Z_f - Z_f^L}{Z_f^U - Z_f^L} \qquad \forall f = 1,2,\ldots,F \tag{4.57}$$

All original constraints

In this thesis, the problem is formulated as a minimization problem with two objective functions, cost and time, and thus two dummy variables are assigned to each objective function (i.e. δ_{Cost} for cost objective, and δ_{Time} for time objective). If we assume that minimizing cost is more important than minimizing time, then the order is

$\delta_{Cost} > \delta_{Time}$. If the opposite is the case, then the order should be written as $\delta_{Time} > \delta_{Cost}$. It is assumed here that minimizing the total cost is more important than minimizing the total lead time, i.e., $\delta_{Cost} > \delta_{Time}$. In step 5, because this is a minimization problem, the relationship between satisfaction level indicators is calculated as $\delta_f = w_{f+1}\delta_{f+1}$, or $\delta_{Cost} = w_{Time}\delta_{Time}$ in the context of our model, where w_{Time} is a fuzzy function that represents the relative importance relationship between cost and time.

According to this discussion, our model can be rewritten using MFCP as follows:

$$\min \ \delta_{Cost} \tag{4.58}$$

$s.t.$

$$\delta_{Cost} = w_{Time}\delta_{Time} \tag{4.59}$$

$$w_{Time} = \frac{Time - Time_L^*}{Time_U^* - Time_L^*} \tag{4.60}$$

$$\delta_{Time} \geq \frac{Time - Time_L^*}{Time_U^* - Time_L^*} \tag{4.61}$$

$$\delta_{Cost} \geq \frac{Cost - Cost_L^*}{Cost_U^* - Cost_L^*} \tag{4.62}$$

All other constraints, (4.1) – (4.21) and (4.26) – (4.42)

Chapter 6. Case Study for the Integrated Model of Supply Chain Planning and MESSO

The mixed-integer non-linear problem (MINLP) model proposed in the previous chapter is solved using AIMMS software (**A**IMMS **O**uter **A**pproximation AOA solver). For the case study, the model is solved 11 times using the weighted sum method for deterministic weights ranging from 0 to 1 with an increment of 0.1. It is done as such to obtain a detailed trade-off between cost and time. Then, it is solved twice when the weight is systematically generated and self-adjusted using the MFCP method: one when minimizing total costs is more important than minimizing total time and the other when minimizing total time is more important than minimizing total costs.

In this case study, six components are produced by three suppliers, and they are moved to two manufacturing sites to convert them into three products. Then the products are shipped to three distribution centers to satisfy customers' demand in three tactical time periods. The time units for each tactical period, N_t, are different. The first tactical period has one standard unit, the second one has two standard units, and the third has four. We assume that the service factor for safety stock calculation at all supply chain members is 1.96. Tables 6.1 to 6.16 show the values of the parameters required for this case study.

Table 6.1: Production and holding costs at supplier sites

Supplier (s)	Component family (c), ($)	Production cost (CC_{cs}), ($)	Holding cost (HS_{cs}), ($)
1	1	3.5	1.6
	2	4	1.5
	3	1	1
	4	1	1.3
	5	3.8	1.4
	6	1	1
2	1	1	1
	2	2.4	1.2
	3	4	1.6
	4	3.4	1.7
	5	1	1
	6	4	1.5
3	1	1.5	1.3
	2	1	1
	3	2	1.4
	4	2	1.5

	5	2	1.2
	6	2.5	1.3

Table 6.2: Production and holding costs at manufacturing sites

Product family (i)	Production cost (PC_{ij}), ($)		Holding cost (HJ_{ij}), ($)	
	Manufacturing site (j)		Manufacturing site (j)	
	1	2	1	2
1	20	37	10	13
2	42	26	18	17
3	48	65	23	19

Table 6.3: Holding costs at distribution centers

Product family (i)	Holding cost (HD_{ik}), ($)		
	Distribution center (k)		
	1	2	3
1	10	11	11
2	17	15	16
3	24	23	19

Table 6.4: Required time for components at supplier sites

Component family (c)	Production time (TS_{cs}), (min)		
	Supplier (s)		
	1	2	3
1	1	2	1.1
2	1	1.5	2
3	2	1	1.5
4	2.2	1	1.6
5	1	2.2	1.5
6	3	1	1.5

Table 6.5: Required time for production at manufacturing sites

Product family (i)	Production time (TJ_{ij}), (min)	
	Manufacturing site (j)	
	1	2
1	2	1
2	1.8	3
3	3.5	2

Table 6.6: Processing time required at distribution centers

Product family (i)	Processing time (TK_{ij}), (min)		
	Distribution center (j)		
	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1

Table 6.7: Costs of transporting components from suppliers to manufacturing sites

Component family (c)	Supplier (s)	Transportation cost of components (TSS_{csj}), ($)	
		Manufacturing site (j)	
		1	2
1	1	2.5	1
	2	3.1	1.3
	3	1	3
2	1	2.6	1
	2	2	1.1
	3	1	2.8

	1	3	1.3
3	2	2.1	1.2
	3	1.2	2.9
	1	2	1
4	2	1.5	1
	3	1	2
	1	2.8	1
5	2	2	1.1
	3	1.1	3.5
	1	3.4	1.2
6	2	1.1	1.8
	3	1	3

Table 6.8: Costs of transporting products from manufacturing sites to distribution centers

Product family (i)	Manufacturing site (j)	Transportation cost of products (TSD_{ijk}), ($) — Distribution center (k)		
		1	2	3
1	1	8	5.1	3
	2	3.5	5	7
2	1	9	7	5
	2	4	6	9
3	1	11.1	8	6.5
	2	7	9.5	12

Table 6.9: Transportation time from suppliers to manufacturing sites

Supplier(s)	Transportation time (TSJ_{sj}) — Manufacturing site (j)	
	1	2
1	1	3
2	1.5	1.7
3	3	1

Table 6.10: Transportation time from manufacturing sites to distribution centers

Manufacturing site (j)	Transportation time (TJK_{jk}) Distribution center (k)		
	1	2	3
1	1	1.5	3
2	3	1.7	1

Table 6.11: Available regular time, available overtime, and overtime cost at supplier sites

Supplier (s)	Available time (min) (SA_s)	Available overtime (min) (SOT_s)	Overtime cost ($) ($CSOT_s$)
1	5000	250	4.4
2	6000	300	5
3	7000	350	6.4

Table 6.12: Available regular time, available overtime, overtime cost, and production capacity at manufacturing sites

Manufacturing site (j)	Available Time (PA_j) (min)	Available overtime (MOT_j) (min)	Overtime cost ($CMOT_j$) ($)	Production capacity (units) (UB_{ij}) Product family (i) 1	2	3
1	2400	240	5.6	10000	8000	12000
2	3200	320	6.4	8000	11000	12000

Table 6.13: Storage capacity for each time period at all supply chain members

Time (t)	Available storage capacity (SSA_s) Supplier (s) 1	2	3	(JSA_j) Manufacturing site (j) 1	2	(KSA_k) Distribution center (k) 1	2	3
1	500	500	500	2400	3200	600	540	480
2	500	500	500	2400	3200	600	540	480
3	500	500	500	2400	3200	600	540	480

Table 6.14: Unit size of components and products

Unit's size			
Product family (i)	α_i	Component family (c)	β_c
1	1	1	1
2	1.3	2	1
3	2.2	3	1
		4	1
		5	1
		6	1

Table 6.15: Number of components required for each product

Component family (c)	Component needed per product (ρ_{ci})		
	Product family (i)		
	1	2	3
1	2	1	2
2	2	2	2
3	0	2	1
4	3	4	4

5	2	2	0
6	1	0	1

Table 6.16: Backlog cost and re-planning time for each product family at distribution centers

Product family (i)	Distribution center (k)	Backlog cost (UC_{ik})	Backlog re-planning time (UT_{ik})
1	1	1000	500
	2	1000	500
	3	1000	500
2	1	1000	500
	2	1000	500
	3	1000	500
3	1	1000	500
	2	1000	500
	3	1000	500

The model is solved for different values of importance weight, *w*, to study the behaviour of the model and give a set of solutions that illustrates the trade-off between cost and time objectives using equation (6.1) below as the objective function. In this case, the value of *w* is deterministic and subjectively selected from 0 to 1 with an increment of 0.1 at each time.

$$\min \quad w\left[\frac{Cost - Cost_L^*}{Cost_L^*}\right] + (1-w)\left[\frac{Time - Time_L^*}{Time_L^*}\right] \tag{4.63}$$

The 11 solutions (associated with the 11 *w* values) of this problem are presented in Figure 6.1. From these cost and time values, the decision makers are able to select the best combination according to their experiences.

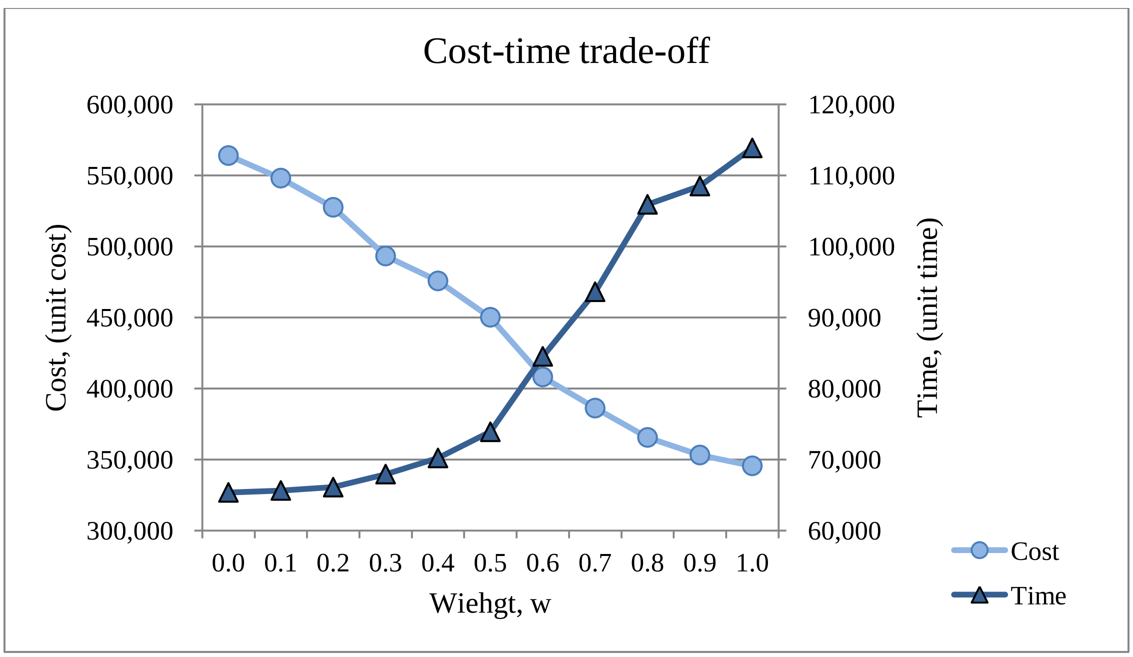

Figure 6.1: The cost and time values associated with different weights (*w*)

The lowest cost in this example is 345,466.2 cost units achieved when w = 1. At the same value of w, the time objective has the worst value which is 113,849.97 time units. When w = 0, the cost is the highest at 563,872.82, which is about 63% higher than the lowest cost. However, time objective is improved by about 43% and value amounts to 65,351.77 time units.

Then, the model is solved twice with weight, w, as a variable, which is an output of the MFCP model. In the first run, it is assumed that the cost objective is more important than the time objective. In the second, the opposite is assumed, i.e., the time objective is more important than the cost objective. The results are plotted in Figure 6.2 and listed in Table 6.17.

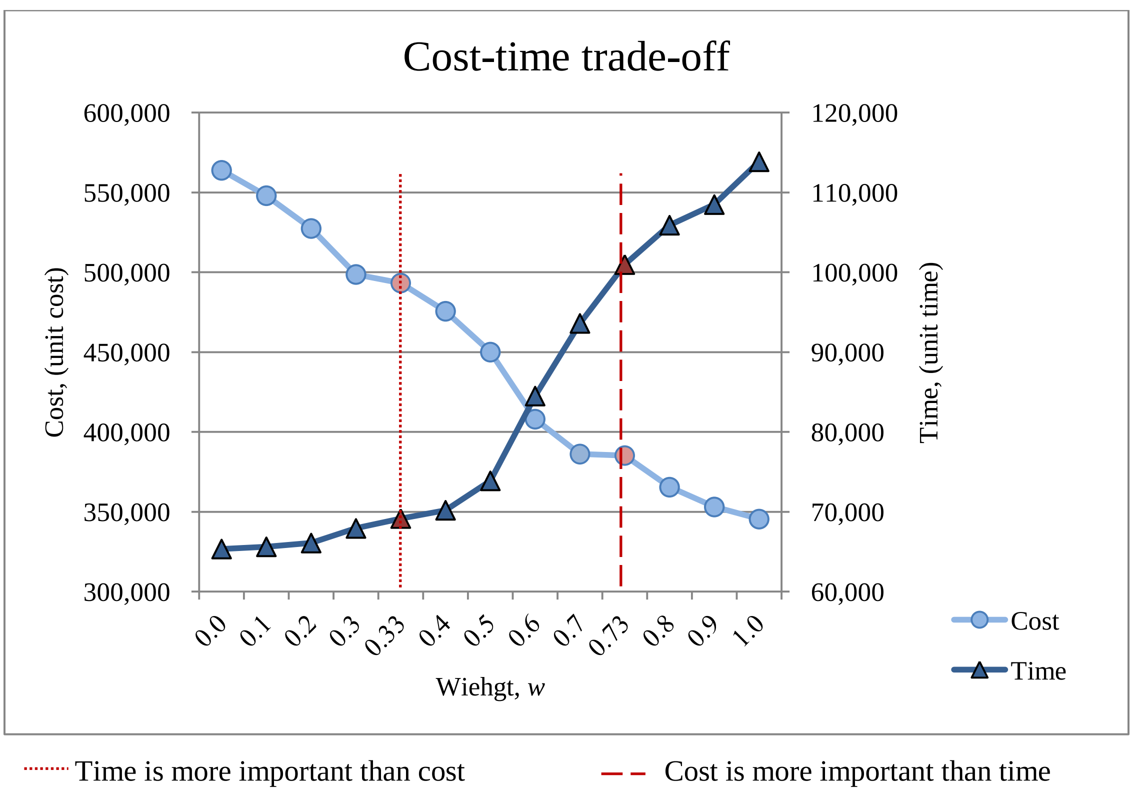

Figure 6.2: Comparison of the objectives values resulted from the MFCP model and from pre-determined value of (w)

When cost is more important than time, the optimal weight is 0.7322, the total cost is 385,123.84, and the total time is 100,945.00, which represents a 11.48% and 54.46% deviation from their best values respectively. If time is more important than cost, the optimal weight is 0.3301, the cost and time become 498,661.49 and 69,158.70 with deviations of 44.34% and 5.82% from their best values (obtained using the single objective models) respectively.

Finally, Table 6.17 shows the operation flexibility of the manufacturing system when the objective changes.

Table 6.17: Comparison of cost, time, and production rate when cost is more important than time and vice versa

Cost is more important than time					Time is more important than cost				
Total cost	385,123.84				Total cost	498,661.49			
Total time	100,945.00				Total time	69,158.70			
w	0.7322				*w*	0.3301			
Amount of production XP_{ijt}					Amount of production XP_{ijt}				
Manf. site (*j*)	Tactical time (*t*)	Product family (*i*) 1	2	3	Manf. site (*j*)	Tactical time (*t*)	Product family (*i*) 1	2	3
1	1	407.3	18.1	263.2	1	1	386.1	1	1
	2	599	170	0		2	394.8	342.1	0
	3	1050	1	0		3	688.1	2.5	340
2	1	1	280.4	81.6	2	1	0	464.6	319.2
	2	1	340	240		2	205.2	0	240
	3	0	1079	510		3	361.9	1077.5	170

Chapter 7. Summary and Directions for Future Studies

The objectives of this thesis that are outlined in Chapter 1 have been achieved. The following two sections summarize the work carried out through this research and present some points as potential opportunities for future work.

7.1 Summary

This study has provided a methodology to model safety stock in multi-echelon inventory systems and to integrate it with supply chain planning. Unlike most related studies, general network topology structure of supply chain is considered. In summary, the following has been accomplished:

1) Safety stock model has been formulated for a multi-echelon supply chain network structure. Then, it is converted into a robust multi-echelon safety stock optimization model to cover a set of scenarios. The results obtained from the robust model are compared with those obtained using a regular multi-echelon safety stock model.
2) An integrated model for supply chain planning and safety stock optimization in multi-echelon manufacturing systems has been developed. Two objectives, i.e., total cost and total time, are considered. Due to the conflict between the two objectives, the model has been solved using subjectively selected weights and the MFCP approach. Both results are investigated and compared.
3) Two case studies have been conducted using the AIMMS. The first one is an implementation of robust multi-echelon safety stock optimization. The result was compared with a multi-echelon safety stock optimization model. Both models are

for a general system structure. The second case study is an integrated model using pre-determined weight and the MFCP method. The results were compared, and they clearly illustrated the flexibility of operations according to different objectives to obtain a plan that is responsive enough to the fluctuating customer demand. The MFCP method can accommodate the decision maker's preferences in a less subjective manner.

7.2 Future work

To further improve the integrated SC planning and MESS optimization, the above study could be extended in the following directions:

1) *Incorporation of stochastic production and transportation times into both RMESSO model and integrated model of MESSO*

   In this study, production and transportation times are assumed to be deterministic. Only the customer demand is treated as a stochastic parameter in this work. Due to the unexpected failure in the production system and the potential unreliability of the transportation system, considering uncertainty or randomness in production and transportation times would be an interesting topic for future research.

2) *Incorporation of shipping restrictions into the proposed integrated model*

   In this study, there is no restriction on shipping capacity, which means that no matter the amount of products or components required to be shipped, they are shipped once so that the transportation time is the same for any amount. However, this is not always the case. Therefore, incorporating shipping capacity and different shipping options into the integrated model would be a potential topic for future work.

3) *Extending the proposed integrated model by including the operational level*

This study considers supply chain planning and safety stock optimization at the tactical level. Operational scheduling with a detailed inventory system optimization (e.g. reorder point and quantity) in a multi-echelon form in a tactical plan would result in more reliable and robust plans. This would be a very attractive topic for future research.

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Appendices

Appendix A – AIMMS model for robust multi-echelon safety stock optimization model

```
MAIN MODEL Main_RMESSO

 DECLARATION SECTION
  comment   :  "NODES:
              identifier      : FN
              index domain    : f
              definition      : NetOutflow <=S(f)
              text            : factory supply node for f;

              identifier      : CN
              index domain    : c
              definition      : NetInflow >= D(c)
              text            : customer demand node for c ;

          ARC:
              identifier      : Flow
              index domain    : (f,c) | T(f,c)
              range           : nonnegative
              from            : FN(f)
              to              : CN(c)
              cost            : T(f,c) ;"
```

```
SET:
  identifier   :  Product
  index        :  i
  definition   :  ElementRange(from:1,to:3,prefix:'Product - ') ;

SET:
  identifier   :  Component
  index        :  c
  definition   :  ElementRange(from:1,to:6,prefix:'Component - ') ;

SET:
  identifier   :  ManufacturingSite
  index        :  j
  definition   :  ElementRange(from:1,to:2,prefix:'Manufacturing Site - ') ;

SET:
  identifier   :  DistributionCenter
  index        :  k
  definition   :  ElementRange(from:1,to:3,prefix:'Distribution Center - ') ;

SET:
  identifier   :  Supplier
  index        :  s
  definition   :  ElementRange(from:1,to:3,prefix:'Supplier - ') ;
```

SET:

identifier : Scenario

indices : z, zs

definition : ElementRange(from:1,to:3,prefix:'Scenario - ') ;

PARAMETER:

identifier : HJ

index domain : (i,j) ;

PARAMETER:

identifier : HD

index domain : (i,k) ;

PARAMETER:

identifier : HS

index domain : (c,s) ;

PARAMETER:

identifier : RO

index domain : (c,i) ;

PARAMETER:

identifier : IRO

index domain : (c,i)

definition : if (RO(c,i)>0) then

```
                    1
                  else
                   0
                  endif  ;

PARAMETER:
  identifier        :  LMDK
  index domain      :  (i,k) ;

PARAMETER:
  identifier        :  LMDJ
  index domain      :  (i,j) ;

PARAMETER:
  identifier        :  LMDS
  index domain      :  (c,s) ;

PARAMETER:
  identifier        :  TK
  index domain      :  (i,k) ;

PARAMETER:
  identifier        :  TJ
  index domain      :  (i,j) ;

PARAMETER:
```

identifier : TS

index domain : (c,s) ;

PARAMETER:

identifier : TOK

index domain : (i,k)

range : nonnegative ;

VARIABLE:

identifier : TOJ

index domain : (i,j)

range : nonnegative ;

VARIABLE:

identifier : TOS

index domain : (c,s)

range : nonnegative ;

PARAMETER:

identifier : TIS

index domain : (c,s) ;

PARAMETER:

identifier : TSJ

index domain : (s,j) ;

PARAMETER:

identifier : TJK

index domain : (j,k) ;

PARAMETER:

identifier : Sigma

index domain : (i,k,z)

range : nonnegative ;

PARAMETER:

identifier : STDK

index domain : (i,k,z)

range : nonnegative

definition : Sigma(i,k,z) ;

PARAMETER:

identifier : P

index domain : (z) ;

PARAMETER:

identifier : Omiga ;

PARAMETER:

identifier : Lamda ;

VARIABLE:

identifier : STDJ

index domain : (i,j,z)

range : nonnegative

definition : sqrt(sum[k,((STDK(i,k,z)*FJ(i,j))^2)]) ;

VARIABLE:

identifier : STDS

index domain : (c,s,z)

range : nonnegative

definition : sqrt(sum[(i,j),((STDJ(i,j,z)*FS(c,s)*RO(c,i))^2)]) ;

VARIABLE:

identifier : SSK

index domain : (i,k,z)

range : nonnegative ;

VARIABLE:

identifier : SSJ

index domain : (i,j,z)

range : nonnegative ;

VARIABLE:

identifier : SSS

index domain : (c,s,z)

range : nonnegative ;

VARIABLE:

identifier : NLTK

index domain : (i,k)

range : nonnegative ;

VARIABLE:

identifier : NLTJ

index domain : (i,j)

range : nonnegative ;

VARIABLE:

identifier : NLTS

index domain : (c,s)

range : nonnegative ;

VARIABLE:

identifier : TIK

index domain : (i,k)

range : nonnegative ;

VARIABLE:

identifier : TIJ

index domain : (i,j)

range : nonnegative ;

VARIABLE:

identifier : FJ

index domain : (i,j)

range : nonnegative ;

CONSTRAINT:

identifier : SiteToDCFaraction

index domain : (i)

definition : sum[j,FJ(i,j)]=1 ;

VARIABLE:

identifier : FS

index domain : (c,s)

range : nonnegative ;

CONSTRAINT:

identifier : SupplierToSiteFaraction

index domain : (c)

definition : sum[s,FS(c,s)]=1 ;

VARIABLE:

identifier : Thita

index domain : (z)

range : nonnegative ;

VARIABLE:

identifier : Dlt

index domain : (i,k,z)

range : nonnegative ;

VARIABLE:

identifier : SSKC

index domain : (z)

range : nonnegative

definition : sum[k,sum[i,HD(i,k)*SSK(i,k,z)]] ;

VARIABLE:

identifier : SSJC

index domain : (z)

range : nonnegative

definition : sum[j,sum[i,HJ(i,j)*SSJ(i,j,z)]] ;

VARIABLE:

identifier : SSSC

index domain : (z)

range : nonnegative

definition : sum[s,sum[c,HS(c,s)*SSS(c,s,z)]] ;

VARIABLE:

identifier : TotalCost

range : free

definition : sum[z,P(z)* (SSJC(z)+ SSKC(z)+ SSSC(z))]+

Lamda*(sum[z,P(z)*((SSJC(z)+ SSKC(z)+ SSSC(z))-
(sum[zs,P(zs)*(SSJC(zs)+SSKC(zs)+SSSC(zs))])+2*Thita(z))])+

Omiga*(sum[z,sum[k,sum[i,P(z)*Dlt(i,k,z)]]]) ;

CONSTRAINT:

identifier : SafetyStockAtSite

index domain : (i,j,z)

definition : SSJ(i,j,z)=LMDJ(i,j)*STDJ(i,j,z)*sqrt(NLTJ(i,j)) ;

CONSTRAINT:

identifier : NetLeadTimeAtSite

index domain : (i,j)

definition : NLTJ(i,j)=TIJ(i,j)+TJ(i,j)-TOJ(i,j) ;

CONSTRAINT:

identifier : TOJupper

index domain : (i,j)

definition : TOJ(i,j)<=TIJ(i,j)+TJ(i,j) ;

CONSTRAINT:

identifier : GTimeToSite

index domain : (i,j)

definition : TIJ(i,j)=max((c,s),((TOS(c,s)+TSJ(s,j))*IRO(c,i))) ;

CONSTRAINT:

identifier : SafetyStockAtDC

index domain : (i,k,z)

definition : SSK(i,k,z)=STDK(i,k,z)*LMDK(i,k)*sqrt(NLTK(i,k))+Dlt(i,k,z) ;

CONSTRAINT:

identifier : NetLeadTimeAtDC

index domain : (i,k)

definition : NLTK(i,k)=TIK(i,k)+TK(i,k)-TOK(i,k) ;

CONSTRAINT:

identifier : TOKupper

index domain : (i,k)

definition : TOK(i,k)<=TIK(i,k)+TK(i,k) ;

CONSTRAINT:

identifier : GTimeToDC

index domain : (i,k)

definition : TIK(i,k)=max(j,(TOJ(i,j)+TJK(j,k))) ;

CONSTRAINT:

identifier : SafetyStockAtSupplier

index domain : (c,s,z)

definition : SSS(c,s,z)=LMDS(c,s)*STDS(c,s,z)*sqrt(NLTS(c,s)) ;

CONSTRAINT:

identifier : NetLeadTimeAtSupplier

index domain : (c,s)

definition : NLTS(c,s)=TIS(c,s)+TS(c,s)-TOS(c,s) ;

```
CONSTRAINT:
   identifier          :  TOSupper
   index domain        :  (c,s)
   definition          :  TOS(c,s)<=TIS(c,s)+TS(c,s)  ;

CONSTRAINT:
   identifier          :  Feasiblity
   index domain        :  (z)
   definition          :  (SSJC(z)+SSKC(z)+SSSC(z))-
                       sum[zs,P(zs)*(SSJC(zs)+SSKC(zs)+SSSC(zs))]+Thita(z)>=0  ;

MATHEMATICAL PROGRAM:
   identifier          :  RMESSO
   objective           :  TotalCost
   direction           :  minimize
   constraints         :  AllConstraints
   variables           :  AllVariables
   type                :  Automatic ;

ENDSECTION  ;

PROCEDURE
  identifier           :  MainInitialization

ENDPROCEDURE  ;

PROCEDURE
```

```
  identifier          :  MainExecution
  body    :
    Solve RMESSO;

 ENDPROCEDURE  ;

 PROCEDURE
  identifier          :  MainTermination
  body    :
    return DataManagementExit();

 ENDPROCEDURE  ;

ENDMODEL Main_RMESSO ;
```

Appendix B – AIMMS model for integrated model of supply chain planning and safety stock optimization

DECLARATION SECTION

comment : "NODES:

identifier : FN

index domain : f

definition : NetOutflow <=S(f)

text : factory supply node for f;

identifier : CN

index domain : c

definition : NetInflow >= D(c)

text : customer demand node for c ;

ARC:

identifier : Flow

index domain : (f,c) | T(f,c)

range : nonnegative

from : FN(f)

to : CN(c)

cost : T(f,c) ;"

SET:

identifier : Product

index : i

definition : ElementRange(from:1,to:3,prefix:'Product - ') ;

SET:

identifier : Component

index : c

definition : ElementRange(from:1,to:6,prefix:'Component - ') ;

SET:

identifier : ManufacturingSite

index : j

definition : ElementRange(from:1,to:2,prefix:'Manufacturing Site - ') ;

SET:

identifier : DistributionCenter

index : k

definition : ElementRange(from:1,to:3,prefix:'Distribution Center - ') ;

SET:

identifier : Supplier

index : s

definition : ElementRange(from:1,to:3,prefix:'Supplier - ') ;

SET:

identifier : Time

index : t

definition : ElementRange(from:1,to:3,prefix:'Time-') ;

PARAMETER:

identifier : PC

index domain : (i,j) ;

PARAMETER:

identifier : CC

index domain : (c,s) ;

PARAMETER:

identifier : HJ

index domain : (i,j) ;

PARAMETER:

identifier : HD

index domain : (i,k) ;

PARAMETER:

identifier : HS

index domain : (c,s) ;

PARAMETER:

identifier : UC

index domain : (i,k) ;

PARAMETER:

identifier : UT

index domain : (i,k) ;

PARAMETER:

identifier : TSS

index domain : (c,s,j) ;

PARAMETER:

identifier : TSD

index domain : (i,j,k) ;

PARAMETER:

identifier : CSOT

index domain : (s) ;

PARAMETER:

identifier : CMOT

index domain : (j) ;

PARAMETER:

identifier : N

index domain : (t) ;

PARAMETER:

identifier : LB

index domain : (i,j) ;

PARAMETER:

identifier : UB

index domain : (i,j) ;

PARAMETER:

identifier : PA

index domain : (j) ;

PARAMETER:

identifier : SA

index domain : (s) ;

PARAMETER:

identifier : MOT

index domain : (j) ;

PARAMETER:

identifier : SOT

index domain : (s) ;

PARAMETER:

identifier : ALPHA

index domain : (i) ;

PARAMETER:

identifier : BETA

index domain : (c) ;

PARAMETER:

identifier : JSA

index domain : (j,t) ;

PARAMETER:

identifier : KSA

index domain : (k,t) ;

PARAMETER:

identifier : SSA

index domain : (s,t) ;

PARAMETER:

identifier : RO

index domain : (c,i) ;

PARAMETER:

identifier : TD

index domain : (i,k,t)

range : integer

definition : normal(90,10)*N(t) ;

PARAMETER:

identifier : D

index domain : (i,k,t)

range : integer

definition : TD(i,k,t) ;

PARAMETER:

identifier : LMDK

index domain : (i,k) ;

PARAMETER:

identifier : LMDJ

index domain : (i,j) ;

PARAMETER:

identifier : LMDS

index domain : (c,s) ;

PARAMETER:

identifier : TK

index domain : (i,k) ;

PARAMETER:

identifier : TJ

index domain : (i,j) ;

PARAMETER:

identifier : TS

index domain : (c,s) ;

PARAMETER:

identifier : TOK

index domain : (i,k) ;

VARIABLE:

identifier : TOJ

index domain : (i,j,k)

range : nonnegative ;

VARIABLE:

identifier : TOS

index domain : (c,s,i,j)

range : nonnegative ;

PARAMETER:

identifier : TIS

index domain : (c,s) ;

PARAMETER:

identifier : TSJ

index domain : (s,j) ;

PARAMETER:

identifier : TJK

index domain : (j,k) ;

VARIABLE:

identifier : X

index domain : (i,j,t)

range : binary ;

VARIABLE:

identifier : XP

index domain : (i,j,t)

range : nonnegative ;

VARIABLE:

identifier : XCP

index domain : (c,s,t)

range : nonnegative ;

VARIABLE:

identifier : IPS

index domain : (i,j,t)

range : nonnegative ;

VARIABLE:

identifier : IPD

index domain : (i,k,t)

range : nonnegative ;

VARIABLE:

identifier : ICS

index domain : (c,s,t)

range : nonnegative ;

VARIABLE:

identifier : UD

index domain : (i,k,t)

range : nonnegative ;

VARIABLE:

identifier : XSS

index domain : (c,s,j,t)

range : nonnegative ;

VARIABLE:

identifier : XSM

index domain : (i,j,k,t)

range : nonnegative ;

VARIABLE:

identifier : XSOT

index domain : (s,t)

range : nonnegative ;

VARIABLE:

identifier : XMOT

index domain : (j,t)

range : nonnegative ;

PARAMETER:

identifier : MEANK

index domain : (i,k)

range : nonnegative

definition : Mean(t,D(i,k,t)/N(t)) ;

VARIABLE:

identifier : MEANJ

index domain : (i,j)

range : nonnegative

definition : sum[k,(MEANK(i,k)*FJ(i,j))] ;

VARIABLE:

identifier : MEANS

index domain : (c,s)

range : nonnegative

definition : sum[(i,j),(MEANJ(i,j)*FS(c,s)*RO(c,i))] ;

PARAMETER:

identifier : STDK

index domain : (i,k)

range : nonnegative

definition : PopulationDeviation(t,D(i,k,t)/N(t)) ;

VARIABLE:

identifier : STDJ

index domain : (i,j)

range : nonnegative

definition : sqrt(sum[k,((STDK(i,k)*FJ(i,j))^2)]) ;

VARIABLE:

identifier : STDS

index domain : (c,s)

range : nonnegative

definition : sqrt(sum[(i,j),((STDJ(i,j)*FS(c,s)*RO(c,i))^2)]) ;

VARIABLE:

identifier : SSK

index domain : (i,k)

range : nonnegative ;

VARIABLE:

identifier : SSJ

index domain : (i,j)

range : nonnegative ;

VARIABLE:

identifier : SSS

index domain : (c,s)

range : nonnegative ;

VARIABLE:

identifier : NLTK

index domain : (i,k)

range : nonnegative ;

VARIABLE:

identifier : NLTJ

index domain : (i,j)

range : nonnegative ;

VARIABLE:

identifier : NLTS

index domain : (c,s)

range : nonnegative ;

VARIABLE:

identifier : TIK

index domain : (i,k)

range : nonnegative ;

VARIABLE:

identifier : TIJ

index domain : (i,j)

range : nonnegative ;

VARIABLE:

identifier : FJ

index domain : (i,j)

range : nonnegative ;

CONSTRAINT:

identifier : SiteToDCFaraction

index domain : (i)

definition : sum[j,FJ(i,j)]=1 ;

VARIABLE:

identifier : FS

index domain : (c,s)

range : nonnegative ;

CONSTRAINT:

identifier : SupplierToSiteFaraction

index domain : (c)

definition : sum[s,FS(c,s)]=1 ;

VARIABLE:

identifier : TotalCost

range : free

definition : sum[t,sum[j,sum[i,PC(i,j)*XP(i,j,t)]]]+
sum[t,sum[s,sum[c,CC(c,s)*XCP(c,s,t)]]]+
sum[t,sum[j,sum[i,HJ(i,j)*N(t)*IPS(i,j,t)]]]+
sum[t,sum[k,sum[i,HD(i,k)*N(t)*IPD(i,k,t)]]]+
sum[t,sum[s,sum[c,HS(c,s)*N(t)*ICS(c,s,t)]]]+
sum[t,sum[k,sum[i,UC(i,k)*UD(i,k,t)]]]+
sum[t,sum[s,sum[j,sum[c,TSS(c,s,j)*XSS(c,s,j,t)]]]]+
sum[t,sum[j,sum[k,sum[i,TSD(i,j,k)*XSM(i,j,k,t)]]]]+
sum[t,sum[s,CSOT(s)*XSOT(s,t)]]+sum[t,sum[j,CMOT(j)*XMOT(j,t)]]
;

VARIABLE:

identifier : TotalTime

range : free

definition : sum[t,sum[s,sum[c,NLTS(c,s)+(TS(c,s)*(XCP(c,s,t)-1))]]]+
sum[t,sum[j,sum[i,NLTJ(i,j)+(TJ(i,j)*(XP(i,j,t)-1))]]]+
sum[t,sum[k,sum[i,NLTK(i,k)+TK(i,k)*(D(i,k,t)-
UD(i,k,t))+UT(i,k)*UD(i,k,t)]]] ;

VARIABLE:

identifier : SgmT

range : nonnegative

definition : !W*SgmC;

comment : "For Cost Minimization (cost is more important than time):

Nothing (!)

For Time Minimization (time is more important than cost):

W*SgmC;" ;

VARIABLE:

identifier : SgmC

range : nonnegative

definition : W*SgmT

comment : "For Cost Minimization (cost is more important than time):

W*SgmT;

For Time Minimization (time is more important than cost):

Nothing (!)" ;

VARIABLE:

identifier : W

range : nonnegative

definition : !(TotalCost-246372)/(824741.91-246372);

(TotalTime-48965.7)/(155089.78-48965.7)

comment : "For Cost Minimization (cost is more important than time):

(TotalTime-48965.7)/(155089.78-48965.7);

For Time Minimization (time is more importance than cost);

(TotalCost-246372)/(824741.91-246372);" ;

CONSTRAINT:

identifier : CostRatio

definition : SgmC>=(TotalCost-246372)/(824741.91-246372) ;

CONSTRAINT:

```
  identifier          :  TimeRatio
  definition          :  SgmT>=(TotalTime-48965.7)/(155089.78-48965.7)  ;

VARIABLE:
  identifier          :  Sigma
  range               :  free
  definition          :  SgmC;
                        !SgmT;
  comment             :  "For Cost Minimization (cost is more important than time):
                              SgmC;
                              !SgmT;

                        For Time Minimization (time is more important than cost):
                              !SgmC;
                              SgmT;" ;

CONSTRAINT:
  identifier          :  UProductionLimits
  index domain        :  (i,j,t)
  definition          :  LB(i,j)*X(i,j,t)<=XP(i,j,t) ;

CONSTRAINT:
  identifier          :  LProductionLimits
  index domain        :  (i,j,t)
  definition          :  UB(i,j)*X(i,j,t)>=XP(i,j,t) ;
```

CONSTRAINT:

identifier : MaterialBalanceSite

index domain : (i,j,t)

definition : IPS(i,j,t)=IPS(i,j,t-1)+XP(i,j,t)-sum[k,XSM(i,j,k,t)] ;

CONSTRAINT:

identifier : MaterialBalanceDC

index domain : (i,k,t)

definition : IPD(i,k,t)-UD(i,k,t)=IPD(i,k,t-1)-UD(i,k,t-1)+
sum[j,XSM(i,j,k,t)]-D(i,k,t) ;

CONSTRAINT:

identifier : MaterialBalanceSupplier

index domain : (c,s,t)

definition : ICS(c,s,t)=ICS(c,s,t-1)+XCP(c,s,t)-sum[j,XSS(c,s,j,t)] ;

CONSTRAINT:

identifier : CapacitySite

index domain : (j,t)

definition : sum[i,TJ(i,j)*XP(i,j,t)]<=PA(j)*N(t)+XMOT(j,t) ;

CONSTRAINT:

identifier : CapacitySupplier

index domain : (s,t)

definition : sum[c,TS(c,s)*XCP(c,s,t)]<=SA(s)*N(t)+XSOT(s,t) ;

CONSTRAINT:

identifier : CapacitySiteOverTime

index domain : (j,t)

definition : XMOT(j,t)<=MOT(j)*N(t) ;

CONSTRAINT:

identifier : CapacitySupplierOverTime

index domain : (s,t)

definition : XSOT(s,t)<=SOT(s)*N(t) ;

CONSTRAINT:

identifier : StorageAtSite

index domain : (j,t)

definition : sum[i,ALPHA(i)*IPS(i,j,t)]<=JSA(j,t) ;

CONSTRAINT:

identifier : LStorageAtSite

index domain : (i,j,t)

definition : IPS(i,j,t)>=SSJ(i,j) ;

CONSTRAINT:

identifier : UStorageAtSite

index domain : (i,j,t)

definition : IPS(i,j,t)<=MEANJ(i,j)*NLTJ(i,j)+SSJ(i,j) ;

CONSTRAINT:

identifier : SafetyStockAtSite

index domain : (i,j)

definition : SSJ(i,j)=LMDJ(i,j)*STDJ(i,j)*sqrt(NLTJ(i,j)) ;

CONSTRAINT:

identifier : NetLeadTimeAtSite

index domain : (i,j,k)

definition : NLTJ(i,j)=TIJ(i,j)+TJ(i,j)-TOJ(i,j,k) ;

CONSTRAINT:

identifier : TOJupper

index domain : (i,j,k)

definition : TOJ(i,j,k)<=TIJ(i,j)+TJ(i,j) ;

CONSTRAINT:

identifier : TOJlower

index domain : (i,j,k)

definition : TOJ(i,j,k)>=0 ;

CONSTRAINT:

identifier : GTimeToSite

index domain : (i,j)

definition : TIJ(i,j)=max((c,s),(TOS(c,s,i,j)+TSJ(s,j))) ;

CONSTRAINT:

identifier : StorageAtDC

index domain : (k,t)

definition : sum[i,ALPHA(i)*IPD(i,k,t)]<=KSA(k,t) ;

CONSTRAINT:

identifier : LStorageAtDC

index domain : (i,k,t)

definition : IPD(i,k,t)>=SSK(i,k) ;

CONSTRAINT:

identifier : UStorageAtDC

index domain : (i,k,t)

definition : IPD(i,k,t)<=MEANK(i,k)*NLTK(i,k)+SSK(i,k) ;

CONSTRAINT:

identifier : SafetyStockAtDC

index domain : (i,k)

definition : SSK(i,k)=LMDK(i,k)*STDK(i,k)*sqrt(NLTK(i,k)) ;

CONSTRAINT:

identifier : NetLeadTimeAtDC

index domain : (i,k)

definition : NLTK(i,k)=TIK(i,k)+TK(i,k)-TOK(i,k) ;

CONSTRAINT:

identifier : TOKupper

index domain : (i,k)

definition : TOK(i,k)<=TIK(i,k)+TK(i,k) ;

CONSTRAINT:

identifier : TOKlower

index domain : (i,k)

definition : TOK(i,k)>=0 ;

CONSTRAINT:

identifier : GTimeToDC

index domain : (i,k)

definition : TIK(i,k)=max(j,(TOJ(i,j,k)+TJK(j,k))) ;

CONSTRAINT:

identifier : StorageAtSupplier

index domain : (s,t)

definition : sum[c,BETA(c)*ICS(c,s,t)]<=SSA(s,t) ;

CONSTRAINT:

identifier : LStorageAtSupplier

index domain : (c,s,t)

definition : ICS(c,s,t)>=SSS(c,s) ;

CONSTRAINT:

identifier : UStorageAtSuppier

index domain : (c,s,t)

definition : ICS(c,s,t)<=MEANS(c,s)*NLTS(c,s)+SSS(c,s) ;

CONSTRAINT:

identifier : SafetyStockAtSupplier

index domain : (c,s)

definition : SSS(c,s)=LMDS(c,s)*STDS(c,s)*sqrt(NLTS(c,s)) ;

CONSTRAINT:

identifier : NetLeadTimeAtSupplier

index domain : (c,s,i,j)

definition : NLTS(c,s)=TIS(c,s)+TS(c,s)-TOS(c,s,i,j) ;

CONSTRAINT:

identifier : TOSupper

index domain : (c,s,i,j)

definition : TOS(c,s,i,j)<=TIS(c,s)+TS(c,s) ;

CONSTRAINT:

identifier : TOSlower

index domain : (c,s,i,j)

definition : TOS(c,s,i,j)>=0 ;

CONSTRAINT:

identifier : RateComponentToProduct

index domain : (c,j,t)

definition : sum[s,XSS(c,s,j,t)]=sum[i,RO(c,i)*XP(i,j,t)] ;

```
  MATHEMATICAL PROGRAM:
    identifier          :  SCPandMESSO
    objective           :  Sigma
    direction           :  minimize
    constraints         :  AllConstraints
    variables           :  AllVariables
    type                :  Automatic ;
ENDSECTION  ;

PROCEDURE
  identifier            :  MainInitialization

ENDPROCEDURE  ;


PROCEDURE
  identifier            :  MainExecution
  body      :
   SCPandMESSO.CallbackAOA := 'OuterApprox::BasicAlgorithm';
    OuterApprox::IterationMax := 100;   ! Optional
      Solve SCPandMESSO;

ENDPROCEDURE  ;
```

```
PROCEDURE
  identifier            :  MainTermination
  body      :
    return DataManagementExit();
ENDPROCEDURE  ;


ENDMODEL Main_SCPandMESSO ;
```