

**Peter Schat's Tone Clock: The Steering Function and Pitch-
Class Set Transformation in *Genen***

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“The clock simply elucidates what our chromatic language is [and demonstrates] that there is a natural chromatic order, existing of itself, by virtue of the almighty power of number. [The tone clock] belongs to all, and it can work for anyone.” —Jenny McLeod

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Abstract

Dutch composer Peter Schat's (1935-2003) pursuit of a compositional system that could generate and preserve intervallic relationships, while allowing the composer as much flexibility as possible to manipulate musical material, led him to develop the tone-clock system. Fundamentally comprised of the twelve possible trichords, the tone clock permits each to generate a complete twelve-tone series through the "steering" principle, a concept traced to Boulez's technique of pitch-class set multiplication. This study serves as an overview of Schat's tone-clock system and focuses primarily on the effects of the steering function in "Genen" (2000). Furthermore, I expand on the tone-clock system by combining transformational theory with Julian Hook's uniform triadic transformations and my proposed STEER and STEERS functions, which express the procedures of the steering principle as a mathematical formula. Using a series of transformational networks, I illustrate the unifying effect steering has on different structural levels in "Genen," a post-tonal composition.

Keywords: Peter Schat, tone clock, steering, Genen, intervallic prime form, chromatic tonality, Pierre Boulez, pitch-class set multiplication, transformational theory, Julian Hook, Jenny McLeod, David Lewin, uniform triadic transformations

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Chapter One

Peter Schat's "despair at the lack of harmonic differentiation in dodecaphony and serialism"¹ was likely a significant factor that led to the eventual development of his tone-clock system. Though this latter system is primarily centered on twelve-tone composition, it is much more flexible than twelve-tone serialism since the pitch classes contained in the sets derived may be more freely manipulated. Furthermore, composers can increase the overall compositional unity of a work through an essential component of the tone-clock system: a technique called "steering." The primary objective of my thesis is to present the tone-clock system and, more importantly, to examine the role the steering principle plays in unifying a twelve-tone composition. I argue that, by generating and preserving coherent pitch-class set relationships, steering positively affects and thereby increases the overall unity of a musical work. In addition, my thesis also develops a means through which one may analyze a tone-clock composition; currently, no such analytical model exists. This analytical approach encompasses elements of transformational theory to illustrate the transformations influenced by the steering principle and how steering can produce a musical work that consists of a network of interrelated pitch-class sets.

Schat first introduced the concept of the tone clock in a series of articles published in the Dutch journal *Key Notes*. Several articles printed between 1963 and 1992 were later translated and compiled by Jenny McLeod into a book that was simply titled "The Tone Clock." This indispensable resource, as well as other relevant sources for my thesis,

¹ Peter Schat, *The Tone Clock. With an Introduction and Translation by Jenny McLeod* (Amsterdam: Harwood Academic Publishers, 1993), Preface.

is discussed in the literature review. This chapter also includes Schat's important biographical information, as well as a brief discussion regarding his compositional development leading up to the conception of his tone-clock system.

1.1: Biography of Peter Schat

Peter Schat was born in 1935 to a non-musical, protestant family in Utrecht, the Netherlands. Surprisingly, he aspired to become a composer from an early age, confessing that, while in the third grade, he filled in a questionnaire on his expected profession with "composer."² Schat's musical output may be divided into three phases in which he varied his compositional methods and ideologies. The first phase, spanning roughly through the 1950's and 1960's, is marked by Schat's concentration on and adherence to twelve-tone serialism; following in the footsteps of Arnold Schoenberg and Pierre Boulez, Schat focused on the single tone as the primary source of material for the compositional process. From the mid-1960's to 70's, he turned his attention towards the relationship of tones, more specifically intervallic content and interval permutations rather than note permutations. The third phase began in the 1980's when Schat elected to focus on larger collections, more precisely trichords. It was during this period that he developed the tone clock, a system through which musical material could be generated and be readily applied to composition.³

While studying composition with Henk Stam in Utrecht, Schat made his debut as a composer with his piece "Passacaglia and Fugue for organ," which was premiered at the Dom cathedral in 1954. The year 1954 proved to be an eventful one as Schat was

² Peter Schat, "VII: Curriculum" *Peter Schat's Personal Website*. <http://www.peterschat.nl/curriculum.html>

³ Erik Voermans, "Peter Schat's Search for Chromatic Order," *Key Notes* 28 (1994): 16-17.

invited to become a pupil of Kees van Baaren, who had recently acquired the position of director at the Utrecht Conservatory. The decision to study with van Baaren would alter the path of Schat's career as a composer since Van Baaren introduced him to dodecaphony and serialism. In the twentieth-century dodecaphonic and serial music was, in general, not well received by Dutch composers. The Dutch composer Willem Pijper, who rejected dodecaphony, likely influenced this negative attitude. One unnamed critic wrote of Alban Berg's "Der Wein" that such music "had meaning only as an intellectual construction."⁴ Van Baaren later modified this attitude toward serialism and dodecaphony when he began teaching composition at the Royal Conservatory in The Hague in 1958, since, unlike Pijper, he was an advocate of serial composition.⁵ For several years, van Baaren would play a critical role in Schat's development as he instructed the young composer in the techniques of classical counterpoint and harmony, as well as Viennese dodecaphony. This period marks the start of Schat's first phase of compositional development. In 1957, Schat pursued piano studies at the Conservatory, but according to his teacher, Jaap Callenbach, spent too much time composing rather than practicing his keyboard techniques. This resulted in "sufficient, but not brilliant" piano skills; however, Schat gained a valuable understanding of the demands put on musical performers.⁶ Although his parents emigrated to America to escape from the Soviets that same year, he remained in Europe as he believed that it would be more beneficial for his musical development. Van Baaren would have such a profound impact on Schat that when he was hired as the director of the Royal Conservatory in The Hague in 1958, Schat followed

⁴ Beatrix Baas, "Dutch 20th Century Piano Music Part II: Catching Up with International Trends," *Key Notes* 14 (1981): 1.

⁵ Ibid.

⁶ Schat, "VII: Curriculum."

him to pursue his formal training. Schat's studies with van Baaren were supplemented by regular attendance at the *Gaudeamus Music Weeks* in the Netherlands where he met influential composers such as Stockhausen, Kagel, and Ligeti. Schat gained recognition with avant-garde music enthusiasts at the International Society for Contemporary Music, the Donaueschinger Musiktage, and the Gaudeamus Foundation through his compositions "Mosaics" (1959), "Entelechy" (1961), and "Signalement" (1962).⁷ After concluding his studies with van Baaren, he began enrolling in the Darmstadt summer courses where he observed and was fascinated by Pierre Boulez at work. When, in 1960, Boulez announced the launch of a master class in Basel, Schat departed "with trembling knees" to study with the influential composer. Schat recalls living in a "monastic village" for two years while pursuing his studies with Boulez during which time he composed "Entelegies I & II" (1960,1961); Schat considered the latter his "thesis on Boulezian thinking."⁸

The 1960's were a tumultuous decade as they were a time of revolution and protest. Schat, along with Louis Andriessen, Reinbert de Leeuw, Misha Mengelberg, and Jan van Vlijmen, all pupils of van Baaren's at The Hague, took part in political activism and protested against musical censorship by the Dutch government. The group of five composers, later referred to as "The Five," began a campaign asking the Dutch Concertgebouw Orchestra to program more new music, as they believed the orchestra's repertoire was too dependent on German late-Romanticism. What began as a simple musical protest soon became a political one after the group discovered that the Dutch government had been censoring musical life in the Netherlands thereby controlling its musical society. In 1969, The Five launched what later became known as the "Nutcracker

⁷ Jos Wouters, "Dutch Music in the 20th-century," *The Musical Quarterly* 51/1 (1965): 109.

⁸ Schat, "VII: Curriculum."

Campaign,” a further protest against the Concertgebouw that would ultimately influence Dutch musical life with the establishment of groups that supported modern music and promoted Dutch composers.⁹ The mid-1960’s marked the beginning of Schat’s second phase of compositional thought as his focus shifted from note permutations to interval permutations. Experimental works from this period include “Improvisations from the Labyrinth” (1964), written graphically in a clock form, as well as “Clockwise and Anti-Clockwise” (1967) which expanded on the clock form concept. It was after composing “On Escalation” in 1967 that he decided he had reached the limits of serialism, claiming that the organization of all parameters in music was a compositional “dead end” and refocused his efforts towards an interval-based system.¹⁰

Schat’s enthusiasm for interval-based permutation is evident in his essay titled “Circular Fragment of a Theory” (1966) in which he introduces and discusses his concept of a symmetrical “all-interval series” in relation to symmetrical twelve-tone rows. Schat proposes that “tonal law,” which classifies a musical idea as tonal, consists of a series with a fixed center, much like a tonic in conventional tonal music. This includes Schoenberg’s conception of the tone row, as the center essentially shifts over all the possible tones of the equal-tempered system and can be manipulated through four transformations: transposition, inversion, retrograde, and retrograde inversion.¹¹ While a symmetrical series normally only has two forms, the prime/retrograde form and the inversion/retrograde-inversion form, Schat conceived a method of obtaining four related series. This function involved a set of permutations requiring a double rotation of the

⁹ Rudy Koopman, “On Music and Politics-Activism of Five Dutch Composers,” *Key Notes* 4 (1976): 20, 21, 22, 32.

¹⁰ Richard Dufallo, *Trackings: Composers Speak with Richard Dufallo* (New York: Oxford University Press, 1989), 371.

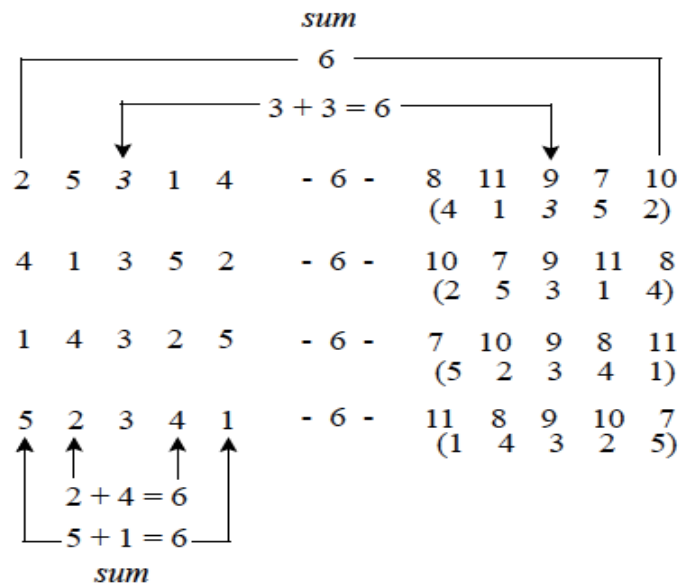
¹¹ Schat, *The Tone Clock*, 3.

intervals within a symmetrical interval-based series.¹² Schat proposed six conditions for the series: (1) “the halves of the series mirror each other; (2) all the intervals smaller than 6 are to the left, the intervals greater than 6 to the right; (3) both halves of the row are interchangeable; (4) the series halves can each in themselves be mirrored around the interval 3; (5) starting from the mirror points [the integer] 3, the sum of the intervals [namely 6] is constant; (6) other permutations of the intervals 1 to 5 (included) do not meet the conditions”¹³ (see example 1-1(a)). The arrows indicate how the sum of the interval classes around, and including, the mirror points is consistently 6. The same is true of the interval classes in identical order positions around the central mirror point, interval class 6. For example, in the first series (left-hand portion), the sum of the interval classes in the first and last positions, 2 and 4 respectively, is 6. Similarly, the sum of the interval classes in the first order position of each half of the series where the mirror point is interval class 6 (i.e., 2 and 4) is also 6. The larger integers of mod 12 on the right-hand side of the schema are reduced to the corresponding interval class in parentheses below to better convey the addition process. In other words, this means that the interval-class distance in semitones from a pitch class on the left side to its mirror point on the right is always 6.

¹² Ibid., xi.

¹³ Ibid., 4-5.

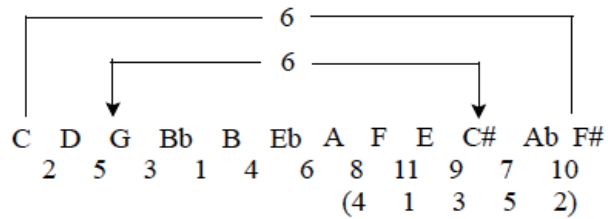
Example 1-1(a): Schema of Schat's "all interval series"¹⁴



When this concept is applied to generate a collection of pitches, a series like the one in example 1-1(b) is obtained. It features the first interval configuration in example 1-1(a) and illustrates how one would construct a series using this method. In essence, one selects a pitch class as the point of departure and constructs a series by following the all-interval series. This process can be repeated using all four permutations to generate four series related via interval permutation. Notice that the distance in semitones between the pitch-class integers around the mirror point (interval class 6) is consistently 6 (C – F # = 6, D – A b = 6, and so forth). Further exploration of interval permutation techniques will follow shortly because they show the progression that led to the tone-clock system. The techniques can be found in the works “Canto General” (1974), “Houdini” (1974-6), and “First Symphony” (1978).

¹⁴ Ibid., 5.

Example 1-1(b): Series generated through the all interval series



Schat taught composition in The Hague in 1974 for nine years before deciding to abandon his position and focus on composing. He credits his teaching experience with helping him develop his concept of “chromatic tonalities” that would eventually become the basis for the tone clock.¹⁵ In the early 1980s Schat decided to focus on larger pitch collections, especially trichords, as he maintained that trichords were the “minimum definition of tonality” because the “notes of a [trichord] remain individually audible in a chord, making it possible to investigate their mutual relationships.”¹⁶ A shift in focus from interval to trichord marked the start of the third and final phase of Schat’s compositional career, and his musical process now centered on the trichord and interval permutations. It was during this third period that Schat finalized the tone clock, as well as the principles that accompanied it, including the concept of “steering,” and began composing exclusively with the system. (The tone-clock system will be discussed in greater detail in the methodology section.) Some works from this period include the opera “Monkey Subdues the White-Bone Demon” (1980), “Symphony No.2” (1983), and “De Hemel” (1990).

¹⁵ Schat, “VII: Curriculum.”

¹⁶ Schat, *The Tone Clock*, 57.

1.2: Development of Schat's Compositional Techniques

Since Schat's conception of the tone clock would not have been possible were it not for the musical developments of the late nineteenth and early twentieth centuries, a brief summary of this period and how it relates to Schat's system is in order. For centuries the most indispensable tool for a composer was, and continues to be, a functional tone system. Schat contends: "I cannot create ideas, musical ideas, if I do not have something at hand, and that something is always a tone-system."¹⁷ Rokus de Groot describes a tone system as an "assemblage of intervals" consisting of, and established by, four parameters: (1) the entire collection of pitches; (2) the temperament that determines the interrelationships; (3) the designs that are basic to the tone relationships within a scale; and (4) the structures that are common to melodies and harmonies, such as modal patterns and cadential formulae. De Groot applies the four parameters to sixteenth-century Western European art music and determines that the pitch collection is derived from the chromatic scale, the temperament is the diatonic scale, the basic design is the hexachord, and the common structures include cadential formulae and "the functional differentiation of pitches into fundamental, reciting tone, etc."¹⁸ The incorporation of all four parameters is also well illustrated in the cohesive relationships found in the framework of the tonal system. The potential of the tonal system is undeniable, as it has allowed for the creation of an abundance of musical material for hundreds of years and whose influence persists to the present day. However, in the latter part of the nineteenth century, the tonal system fell out of favour with composers and was thereafter rarely used in Western European art music. As composers began to explore and experiment with new

¹⁷ Rokus de Groot, "Peter Schat's Tone clock: A Proposal for a New Tonality Considered," *Key Notes* 19 (1984): 5.

¹⁸ *Ibid.*

collections of sounds that were unsustainable by the tonal system, many of them sought a new tone system for composition; they wanted a system that would reproduce the long-standing, coherent relationships established in the diatonic tonal system without referring back to traditional harmony and tonality. In other words, composers searched for a modern tone system that would rival the diatonic tonal system in its capacity to generate coherent pieces of music. The new system, however, would serve as a framework for creating atonal works. No system was more significant, enduring, or influential than that of Arnold Schoenberg's twelve-tone row serialism. Schoenberg's method rejected the concept of a tonal center by promoting the equality of all twelve tones, as well as by eliminating the traditional concepts of dissonance and consonance, an attitude best represented by the phrase "the emancipation of the dissonance."¹⁹ Since the order of the twelve tones in a tone row was fixed and no tone could be repeated until the row had been completed, Schoenberg's tone-row system was fundamentally concerned with the affiliation of one tone to the next.

The rigidity of Schoenberg's system was a source of strength as it allowed for musical coherence and consistency, though it could also be construed as a great limitation. Once the order of the tone row had been established, the prime row could only be manipulated by means of transposition, inversion, retrograde, and retrograde inversion. While this feature allowed for the creation of a coherent piece of music, simultaneously facilitating the analytical process, it came at the cost of limiting the composer to a fixed ordered pitch-class set, as well as to a restricted number of manipulations. This limiting characteristic of the serial twelve-tone row technique was

¹⁹ Arnold Schoenberg "Composition with Twelve Tones," *Style and Idea: Selected Writings of Arnold Schoenberg*, ed. Leonard Stein (Berkeley: 1975): 218.

problematic for some composers, especially Schat, who desired a more flexible means of composing atonal twelve-tone music. Schat believed that manipulating the series theoretically “[undermined] the original principle of the series [which was] to maintain a specific order of notes.”²⁰

Schat was sensitive to the issue of pitch relationships and flexibility, and became highly critical of Schoenberg’s serial technique. As a practising composer under the tutelage of Boulez, he was familiar with and held the beliefs of the traditional serialist composer: the requirement for pitch collections to refrain from establishing a tonic, the forbidden octave repetition of a pitch class, and, in adherence to the ethos of the time, “composition of music at a desk, never at the piano.”²¹ However, Schat was unable to reconcile the individualization of pitch classes often involved in twelve-tone serial composition. He believed that pitch was of the utmost importance and argued that the pitch classes in Schoenberg’s system were not sufficiently “capable [of] defining their place from within [the composition]” or the composer capable “of justifying a choice of tone,” consequently causing the harmony to remain “impotent and out of control.”²² Schat concluded that serialism had reached its limitations with Boulez, Stockhausen, and other composers applying integral serialism to their works, as all parameters of music could be manipulated without the composer’s musical intuition being a factor. This dissatisfaction with tone-row serialism as a manipulation of numbers, rather than of the relationship of musical pitches, caused him to abandon the technique and seek an alternative; this alternative needed to focus on intervallic pitch relationships in order to

²⁰ Schat, *The Tone Clock*, 29.

²¹ *Ibid.*, xi.

²² *Ibid.*, 15.

develop a coherent chromatic harmony and allow the composer greater flexibility in the manipulation of the twelve tones.

Schat began experimenting with an interval-driven method in his composition “Canto General,” (1974) and developed a technique he referred to as “interval steering.” Rather than focusing on pitches, the procedure functioned as a cyclic permutation of intervals within a chord that excluded octave doublings in order to generate harmonic movement. He referred to the permutations as inversions of the initial chord (see example 1-2). The integers in example 1-2 represent the intervallic distance in semitones between pitch classes contained in the chord, with the note D₄ as a pedal tone. The directed-interval classes are then cycled using the interval steering technique. This process involves an upward vertical shift of all the directed-interval classes with the uppermost directed-interval class relocating to the bottom of the subsequent chord resulting in a harmonic progression generated by chords with no octave doublings.

Example 1-2: Interval steering technique in “Canto General” m.8 ²³

8	8	7	4	3
8	7	4	3	8
7	4	3	8	8
4	3	8	8	7
3	8	8	7	4

²³ Ibid., 18.

Schat also extended the technique of interval steering to melodies. Melodic permutation involved a series of intervals that proceeded through a circular rotation to develop melodic material (see example 1-3).²⁴ The melodic interval-steering technique was used extensively in the circus-opera “Houdini” (1974-6), wherein the initial interval-class series, represented by the integers between pitch classes showing pairs of ascending and descending intervals, was rotated to the left to generate five variations. The initial series, beginning on C-natural, ascends five-semitones and is followed by a two-semitone descent. It then proceeds with a six-semitone ascent and a five-semitone descent, and so on. Shifting the second ascending/descending interval group (six up, five down), while maintaining the C-natural starting point and a transfer of the first interval group (five up, two down) to the end of the series, generates the first variation.

Example 1-3: Melodic interval steering²⁵

Initial interval class series

Variation obtained through "interval steering"

The interval-steering technique was further developed in Schat’s “First Symphony” (1978), in which he began organizing various twelve-note series by their intervallic content rather than by individual pitch classes. This was done in order to

²⁴ Ibid., 17.

²⁵ Ibid., 19-20.

reduce the rigidity of the traditional Schoenbergian series with the aim of “[keeping] the basic compositional material as flexible as possible, until such a time as it crystallizes into a melody, a chord, [or] a rhythm.”²⁶ Twelve-note series were divided into four trichords, and organized based on their intervallic content. Furthermore, the four trichords could be divided into two trichords consisting of different interval classes. Example 1-4 presents four trichords from the twelve-note series labeled “1st movement;” two trichords are comprised of one semitone and four semitones, and two trichords of two semitones and four semitones. The integers represent the number of semitones between pitch classes to demonstrate how the trichords are constructed. The interval steering occurs when the intervallic configuration of the first two trichords is mirrored by the subsequent trichords. “Variation I” is derived by the alteration of one of the interval classes; in this case, the four-semitone interval is replaced by a three-semitone interval. Schat explains that the relationships between the trichords generated by the interval steering in each variation of his “First Symphony” are carefully maintained. The composer preserves the relationship between the larger interval classes (4 or 3) and the smaller interval classes (1 or 2), as well as the mirror forms, for the trichords. The pitch classes that comprise each trichord, as well as the trichords themselves, may occur in any order and are therefore capable of producing numerous variations in the compositional process.²⁷

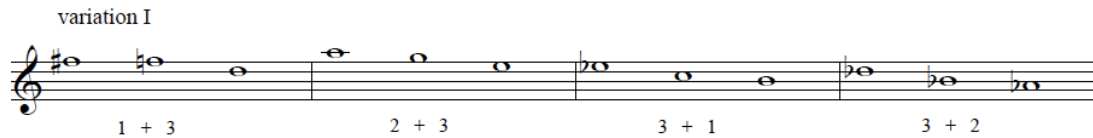
Example 1-4: Twelve-note series divided into trichords in “First Symphony”.

1st movement

1 + 4 2 + 4 4 + 1 4 + 2

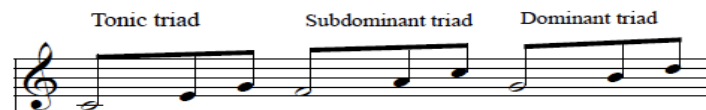
²⁶ Ibid., 28.

²⁷ Ibid., 28 and 30.



The interval-steering techniques used by Schat in his “First symphony” would eventually lead to the development of the more extensive tone-clock system as a compositional tool. Schat believed that the trichord was the minimal structure necessary to determine tonality and establish the tonal identity of a note.²⁸ This belief was founded on the principles previously recognized in the tonal system, in which the intervals of a major triad, transposed to the appropriate diatonic scale degree—tonic, subdominant, and dominant—outline the pitches of a major tonality (see example 1-5).

Example 1-5: Tonic, subdominant, and dominant triads in C-major.



It was this fundamental principle that Schat wanted to integrate into a twelve-tone based system, proposing that “the regime of the triad be extended to the chromatic scale, with the understanding that the starting point [was] no longer the ‘natural triad’, but every possible combination of three notes.”²⁹ Once Schat had isolated, identified, and named the twelve possible chromatic trichords, he applied the principles of interval steering to generate twelve-tone “harmonic fields,” as opposed to twelve-tone rows which were conceived melodically and in which the order of the tones were fixed. He referred to

²⁸ Ibid., 57.

²⁹ Ibid., 77. As the term triad is commonly associated with the major and minor triads of traditional tonality, Schat refers to these as the “natural triads.”

these collections as the “twelve tonalities of the chromatic scale.”³⁰ Schat argued that intervallic relationships could determine the “key” of a composition within an atonal framework the same way that a tonality was established by the relationships of the diatonic tonal system.³¹ It is important to note that Schat’s notion of tonality greatly differed from the conventional understanding of the term. He maintained that so long as the word “tonal” was used to describe music based on the “natural triad,” confusion regarding terms like tonal and atonal would continue. That is why he proposed to refer to pieces as being either “tonical” or “atonical.” The term “tonical” referred to pieces revolving around a central tone, while atonical to pieces with no central tone.³² Schat’s tone-clock system could then be interpreted as a type of atonality with reference to a chromatic pitch collection and temperament.³³

The interval-steering technique, which gradually evolved to become a crucial element of the tone clock, was eventually referred to as the “steering” principle. Schat’s conception of steering was subconsciously influenced by Boulez’s frequency multiplication technique, though he was not aware of this fact until McLeod revealed it to him, as will be discussed in more detail in chapter two. Despite Boulez’s influence, the two techniques produced significantly different outcomes. Though Schat had great admiration for Boulez and was heavily influenced by Boulezian serial techniques, he could not bring himself to compose using such a complex and unintuitive method as frequency multiplication. McLeod argues that “it is impossible to deduce the principle

³⁰ Ibid., 78.

³¹ Schat related his conception of “chromatic tonalities” to the work of Belgian musicologist François-Joseph Fétis who defined tonality as “the necessary connection between the notes of a tone-system” in his *Traité complet de la théorie et de la pratique de l’harmonie*. A full discussion of Fétis’s theories falls outside the scope of this study and will not be addressed here.

³² Schat, “On Harmony and Tonality,” *Key Notes* 19 (1984): 3.

³³ Rokus de Groot, “Peter Schat’s Tone Clock,” 2.

from [Boulez's] scores because he treats the 12-note series as a hidden generator of the actual notes."³⁴ While Boulez preferred to compose using "consciously structured complexity that [resisted] easy comprehension," Schat favoured an approach that was more tangible and "out in the open."³⁵ Schat could not reconcile that the frequency multiplication technique could not function without the use of octave doublings, which are permitted in the initial multiplication of pitch-class sets, but excluded thereafter in compositional practice. Moreover, Boulez's method of dividing a twelve-note series into smaller pitch-class sets and multiplying them by each other could not yield what Schat referred to as "chromatic tonalities."³⁶ Further explanation and discussion on the function of pitch-class set multiplication and its relation to steering will follow in chapter two.

1.3: Literature Review

The literature used for this study can be organized into two categories: (1) literature pertaining to Peter Schat and the tone clock, and (2) literature related to the transformation of pitch-class sets. Unfortunately, the scholarly literature on Schat and his tone-clock system is limited. Conversely, there is an abundance of literature relating to pitch-class set transformation, therefore requiring one to be more selective with relevant sources. Since a theoretical framework for the analysis of a tone-clock work does not exist, I draw from select sources in the field of transformational analysis to develop an appropriate analytical method. In the following paragraphs, I discuss *The Tone Clock* (1993) - a book containing a collection of articles written by Schat that have been

³⁴ Schat, "VIII: Clockwise."

³⁵ Schat, *The Tone Clock*, xvii.

³⁶ Jenny McLeod, e-mail correspondence with author, March 10, 2014.

translated from Dutch to English by McLeod - the content of Schat's personal website, and articles written by others on Schat's system. I then survey select literature on Boulez since this composer influenced Schat's "steering" principle, wherein pitch-class sets are generated through transformations. I also review Lewin's writings in relation to transformational networks in his book *Generalized Musical Intervals and Transformations* as a tool for the analysis of a tone-clock composition. In addition, Hook's UTT formula is adapted and used as part of transformational networks to more accurately depict the steering function.

1.3.1: Tone-Clock Literature

In *The Tone Clock*, Schat describes how he came to develop the tone clock, how the system works, and how the "steering" principle is applied. *The Tone Clock* remains the quintessential primary source for understanding the fundamentals of Schat's compositional system and the reasons for which it was developed. The book consists of a collection of essays published between 1963 and 1992 and provides a chronology of Schat's earlier experimentation with compositional techniques that led him to develop the tone-clock system. It includes actual musical examples and analyses of some early pieces that use the processes of the tone-clock system, as well as a full analysis of his piece "Symposium" (1989), detailing the application of tone-clock principles. The book also contains some of Schat's philosophies about composition and post-tonal music.

Schat's personal website, the content of which he himself maintained until his death in 2003, provides useful autobiographical information, information about obtaining manuscripts and scores, as well as explanations on the development of the tone clock. The website, located at www.peterschat.com, also contains Schat's more recent writings

on the tone clock, anecdotes about his life, and personal opinions on the compositional process. Moreover, it includes a section entitled “Artefacts,” which provides a catalogue of all his works, encompassing diverse mediums such as discs, books, scores, and videos. The two sources discussed thus far, *The Tone Clock* and Schat’s personal website, are crucial to my study as they were written and/or monitored by Schat himself, thereby preventing any misrepresentation of his compositional techniques and ideologies.

Rokus de Groot examines and discusses the tone clock in great detail in a series of articles published in the Dutch composers’ journal *Key Notes* (Donemus). These articles support the tone-clock system as a viable method for post-tonal composition. De Groot presents the fundamentals of the system to readers unfamiliar with it by exploring and clarifying certain aspects of it, as well as comparing it to other tone systems, such as the diatonic system. He also examines the tone clock from a historical perspective, elaborates on some concepts and metaphors associated with the system, and gives examples of musical application. These articles include “Peter Schat’s Tone Clock: A Proposal for a New Tonality Considered” (1984), “The Clockmaker as Musician: The Tone Clock in Motion” (1984), and “The Wheels of the Tone Clock: the Musician as Clockmaker” (1984). De Groot’s articles are relevant because they present a perspective other than Schat’s on the tone-clock system and assist one to better understand the theory behind it. His analyses concerning the characteristics of a tone system, as well as his labeling of a “composite tonality” are of particular interest to this study.

Finally, Jenny McLeod’s *Tone Clock Theory Expanded: Chromatic Maps I & II* (1994) proposes useful terminology for the analysis of tone-clock based music since she defines concepts associated with various elements of the system. I adopt her concept of

Intervallic Prime Forms (IPF) to refer to the tone-clock trichords and tetrachords. Similar to the prime form classification used in set theory, the IPFs label pitch-class sets according to their intervallic content and permit the reader to quickly identify and make connections between specific pitch-class sets in an analysis.

1.3.2: Pitch-class Set Theory and Transformational Theory Literature

Once the fundamentals of the tone clock have been outlined, a methodology must be devised to accurately convey the procedures of the system. To explain Schat's "steering" principle, I draw on Boulez's pitch-class set multiplication technique since these two methods overlap in significant ways to derive pitch-class material. I also borrow and expand on some of the analytical methods presented by Stephen Heinemann in "Pitch-Class Set Multiplication in Theory and in Practice" to show how pitch-class set multiplication can be expressed as a mathematical function that reflects the process. Understanding that steering is a product of a mathematical function that evolved from Boulez's techniques assists in developing a practical method to represent the steering procedure, as well as an approach to analyze such a process with actual musical events. These concepts are explored in greater detail in chapter two.

David Lewin introduced the notion of the transformational network as a means of illustrating the transformation of one musical object to a second musical object by focusing on the process rather than simply the end result. His book *Generalized Musical Intervals and Transformations (GMIT)* contains a detailed description of this analytical approach, of which the principal elements consist of nodes and arrows that depict the transformational process. Chapter two of my study will show that transformational networks are an effective means of portraying the process of steering because steering is,

in essence, a series of transformations that are applied to a pitch-class set. I therefore borrow the analytical approaches presented as transformational networks in *GMIT*, though they will be adapted so as to appropriately depict the steering principle at various hierarchical levels.

The transformational networks are modified through the substitution of conventional transformations with an adaptation of Julian Hook's uniform triadic transformations (UTT) formula, first proposed in his article "Uniform Triadic Transformations." I borrow Hook's UTT formula to better represent the steering function when it operates as part of a transformational network.

As previously mentioned, the scholarly literature on the tone clock is limited. Since an analytical method has not yet been designed for tone-clock works, my focus throughout this thesis, along with exploring the inner-workings of the tone clock, is to propose such an analytical approach by combining different aspects surveyed into one methodology. Most essential are Schat's *The Tone Clock*, Heinemann's "Pitch-Class Set Multiplication in Theory and in Practice," and Lewin's *GMIT* as they will provide useful tools to demonstrate the way in which the tone-clock system and steering principle work in theory and in compositional practice. All of these are explored in greater detail in the following chapter.

1.4: Thesis Outline

This thesis is divided into four chapters. The theoretical and technical aspects of the tone-clock system are dealt with in chapter two through a discussion of the system's fundamental principles and practical applications for composition with a primary focus

on the steering principle. I also propose a methodology for the analysis of a musical work that uses the tone-clock system, referencing relevant writings in relation to transformational theory and pitch-class set multiplication techniques. These methods facilitate the eventual analysis of a tone-clock composition in chapter three. An analysis of Schat's composition "Genen" follows in chapter three, in which the main focus is the transformation of pitch-class sets through the steering principle and how this serves as a unifying factor in a post-tonal composition. This is illustrated using numerous transformational networks coupled with my proposed STEER and STEERS functions. The analysis culminates with an all-encompassing transformational network that integrates all the pitch-class sets discussed throughout chapter three. The thesis ends with a concluding chapter that synthesizes the ideas presented throughout.

Chapter Two: The Tone Clock

McLeod describes Schat as “a musical mind that never stopped hunting for a more systematic, more differentiated solution to the problem of chromatic harmony” and that no solution was acceptable to him unless it involved all twelve pitch classes and the decentralization of a pitch or pitch class as tonic.³⁷ Schat’s search for such a solution resulted in the eventual development of the tone clock, which extends Schoenberg’s conception of twelve-tone composition and also utilizes all twelve pitch-classes of the equal-tempered system. Schat, however, sought a more flexible method of composing using the twelve pitch classes than was possible in the conventional twelve-tone row system of the Second Viennese School. Though composers such as Webern had previously segmented their twelve-tone rows into four similarly configured pitch-class sets (or trichords), none had developed a compositional system that could easily manipulate unordered trichords to derive other materials. Schat sought a system that would allow for the flexibility of unordered trichordal subsets within a twelve-tone composition while maintaining coherent pitch-class relationships. This interest lay in the compositional process, rather than the classification of subsets. He proposed that his tone clock be used as a generator of “chromatic tonalities” and not as a “description of all possible harmonies and tone movements.”³⁸ The system provides the means of deriving material rich in pitch-class relationships in a twelve-tone context, while generating

³⁷ Schat, *The Tone Clock. With an Introduction and Translation by Jenny McLeod* (Amsterdam: Harwood Academic Publishers, 1993), xi.

³⁸ *Ibid.*, 83.

twelve-tone “harmonic fields.”³⁹ While the system was originally intended to generate compositional material that would “forbid the automatic responses of diatonic tonality,”⁴⁰ the system, with the trichord at its core, doubles as a comprehensive tool for the classification of trichords and larger pitch-class sets.

This chapter introduces the fundamental theoretical and technical aspects of Peter Schat’s tone-clock system, including its application to compositional practice, and proposes an analytical approach for representing the transformations influenced by the tone clock as a transformational network. The chapter consists of six sections and begins by introducing key terminology associated with the tone clock, more specifically McLeod’s IPF classification system. Section 2.1 also presents the primary components of the tone-clock system, the twelve chromatic trichords. This leads to an exploration of the significance of a graphical representation of the tone clock in the form of a clock module referred to as the “Zodiac of the Twelve Tonalties” in section 2.2. A detailed discussion concerning an essential component of the system referred to by Schat as the “steering” principle follows in section 2.3. The primary application of steering as a method of trichordal transposition used to generate pitch-class sets of modulo 12 is examined first and is then supported by examples of how it may be applied in a variety of ways. A discussion on the existence of various levels of steering used to generate interrelated material in a composition ensues in section 2.4. Section 2.5 seeks to establish an important link between Schat’s steering principle and Boulez’s pitch-class set multiplication technique. By building on Heinemann’s work with pitch-class set

³⁹ Jenny McLeod, e-mail correspondence with author, March 10, 2014. This term differs from twelve-tone rows, which are conceived melodically and in which the order of the pitch classes are fixed as a series. Twelve-tone harmonic fields, rather, consist of four transpositions of a trichord, each of which may come in any order. Furthermore, the constituent pitch classes of each trichord may also come in any order.

⁴⁰ Schat, *The Tone Clock*, 83.

multiplication, the steering principle can be formalized into a mathematical function, STEER, which accurately depicts the process of steering. The significance of the function becomes evident when it is applied for analytical purposes in chapter three. Select aspects of Lewin's transformational theory are then discussed and incorporated into the methodology to assist in the analysis of tone-clock based compositions. Finally, section 2.6 focuses on the concept of transformational networks and how they may serve to illustrate the process of steering. Transformational networks are coupled with an adaptation of Hook's algebraic formula for uniform triadic transformations to more accurately depict the steering function.

2.1: Tone Clock “Hours” and Intervallic Prime Forms

Schat developed the tone-clock system as a compositional tool in which the focus was not on the individual pitch or the manipulation of individual parameters, but rather on coherent and logical relationships between trichordal subsets. For Schat, the trichord was the “minimal definition of tonality” because a single note cannot establish tonality, nor can two notes (an interval or dyad).⁴¹ One can draw parallels to major and minor triads in conventional tonal music to better grasp this concept. It is only once a major or minor third is added to a perfect-fifth interval that a triad can be classified as major or minor. The principle is further applied to generate diminished and augmented triads to produce four distinct triads within the tonal system. It is from this foundation that Schat proceeded with his twelve-tone based system. The tone clock functions on the premise

⁴¹ Ibid., 57.

that only twelve trichords exist within pitch-class space⁴², resulting in what Schat referred to as “the twelve tonalities.”⁴³ The composer isolated and named these twelve trichords, generated through interval classes, and organized them in a diagram resembling a clock face. The clock face is divided into twelve modules, referred to as “hours,” each representing one trichord. Before discussing the tone clock, some terminology specific to Schat must be presented.

To maintain consistency with Schat’s terminology, individual tone-clock trichords will be referred to by their intervallic prime form (IPF); this concept, comparable to Forte’s prime form in set theory, was introduced by Jenny McLeod to describe a pitch-class set’s intervallic content. McLeod used four criteria to determine a pitch-class set’s IPF: (1) the set must be presented in its most compact form; (2) the smallest interval comes first; (3) symmetrical forms are preferred over any asymmetrical form; and (4) sets with fewest interval classes are preferred.⁴⁴ The IPF associates the prime form and its inversion to a single clock hour, but also includes labels to differentiate the two. The term inversion, however, is not interpreted in the traditional sense where pitch or pitch classes are inverted around an axis. Rather, for Schat, inversion implies a rotation of directed-interval classes, so a configuration of <1+3> semitones would invert to <3+1> semitones regardless of the pitch-classes involved (see example 2-1). For this example, the trichord containing C-C # -E <1+3> could invert to C-D # -E <3+1> and be classified as a member of the same hour group as it simply

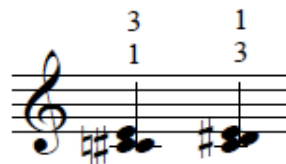
⁴² Forte’s trichord prime forms support the conclusion that only twelve trichords exist; therefore, Schat was not alone in recognizing this property.

⁴³ Schat, *The Tone Clock*, 57.

⁴⁴ Jenny McLeod’s *Tone Clock Theory Expanded: Chromatic Maps I & II* (Wellington: Victoria University of Wellington School of Music, 1994), 15-16.

involves a rotation of the upper and lower intervals, indicated as integers above the staff. This overlaps with the inversion of set classes in set theory as both trichords in example 2-1 share the same prime form (014) and would therefore be considered part of the same set class. Furthermore, Schat associated each IPF to one clock hour and represented each with a Roman numeral. The first hour (I) consists of a trichord containing one semitone plus one semitone, IPF <1+1> or the prime form (0,1,2), the second hour (II) containing one semitone plus two semitones <1+2> or (0,1,3), and so forth (see example 2-2).

Example 2-1: Inversion of directed-interval classes



Example 2-2: Schat’s tone-clock hours (Intervallic Prime Forms)⁴⁵



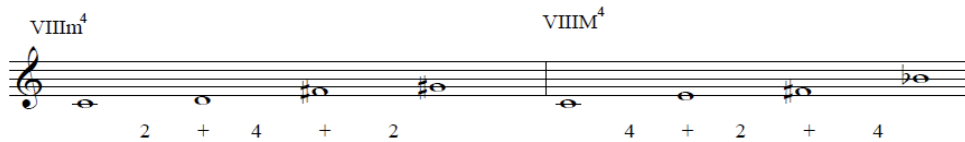
⁴⁵ It is important to note that the trichords need not begin on C-natural, but rather, and more importantly, need to preserve the intervallic relationships of each “hour.” Each clock position represents all of the transpositions and inversions (as derived by Schat through the rotation of directed-interval classes) of a trichord.

When examining trichord XI, one realizes that this trichord is a conventional minor triad, and through the inversion of its interval classes yields a major triad. The existence of this phenomenon in the tone clock allows for the extension of the tonal system's major-minor principle into the chromatic realm. It was McLeod who observed the potential of Schat's system to subsume this aspect of the tonal system and thus proposed that a trichord with the smaller interval first was minor ($IIm = 1 + 2$), and its inversion ($IIM = 2 + 1$) was major. Since the minor form represents the most compact form of a set, it serves as the referential set for the tone-clock hour. The four symmetrical trichords, I $\langle 1+1 \rangle$, VI $\langle 2+2 \rangle$, X $\langle 3+3 \rangle$, and XIII $\langle 4+4 \rangle$, each containing a single interval class, are neither major nor minor. Trichord IX $\langle 2+5 \rangle$, however, is an anomaly, as it has the distinct quality of being expressed in one of three ways. It may be presented in its minor form $\langle 2+5 \rangle$, its major form $\langle 5+2 \rangle$, or, more uncommonly, in a symmetrical form $\langle 5+5 \rangle$.

In combination with the trichordal hours, Schat's system requires the use of select symmetrical tetrachords to derive his "harmonic fields." Tetrachords, indicated by a superscript 4, may also be referred to as major or minor based on the configuration of their intervals. Symmetrical tetrachords are derived through the alternation of interval classes contained within trichords, so that a tetrachord derived from trichord $IIm \langle 1+2 \rangle$ would consist of the following interval classes: $IIm^4 = 1+2+1$. The resulting tetrachord produces an intervallic palindrome that categorizes the tetrachord as symmetrical. Tetrachordal inversions are derived in the same way as trichordal inversions and are labeled based on the first interval class that appears. For example, symmetrical tetrachord

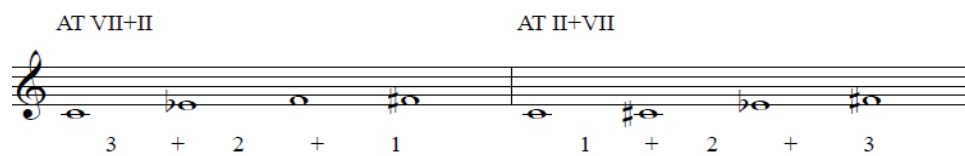
VIII⁴ may be expressed in its minor form as VIII^m⁴ <2+4+2> or VIIM⁴ <4+2+4> in its major form (see example 2-3).

Example 2-3: Symmetrical tetrachords VIII^m⁴ and VIIM⁴



Asymmetrical tetrachords are also possible and exist in one of two forms: (1) as a result of a combination of trichords, or (2) as a subset of a larger pitch-class collection. An asymmetrical tetrachord (AT) will be produced when trichords that share an interval class are juxtaposed. In example 2-4, trichords VII <2+3> and II <1+2> can be combined because they share interval-class 2. Depending on which of the two juxtaposed trichords appears first, the resulting tetrachord will emerge in one of two forms: as either ATVII+II or ATII+VII. Note that asymmetrical tetrachords, unlike their symmetrical counterparts, have interval-class mirror inversion forms rather than major and minor forms, so that ATVII+II <3+2+1> would invert to II+VII <1+2+3>.

Example 2-4: Asymmetrical tetrachords ATVII+II and II+VII



The second configuration of asymmetrical tetrachords emerges as a subset of a symmetrical pentachord, indicated by the letters SP and a superscript 5. I have labeled

these incomplete symmetrical pentachords (ISP). Like symmetrical tetrachords, symmetrical pentachords consist of an intervallic palindrome wherein a trichord's interval classes are inverted and juxtaposed and can be expressed in major or minor form (see example 2-5(a)). Before proceeding, it is important to make a distinction between symmetrical pentachords and what McLeod refers to as oedipus pentachords. Oedipus pentachords, indicated simply by a superscript 5, consist of alternating interval classes dependent on the trichord from which they are derived and also have a major and minor form (see example 2-5(b)).⁴⁶ Omitting either the first or last pitch class of a major or minor SP produces an asymmetrical tetrachord. For example, to obtain a tetrachord from $SPIIM^5 \langle 2+1+1+2 \rangle$, one may omit the last pitch class, resulting in $ISPIIM^5 \langle 2+1+1 \rangle$, or the first pitch class, yielding $ISPIIM^5 \langle 1+1+2 \rangle$. The same process may be applied to the minor form of SPII to produce additional asymmetrical tetrachords (see example 2-6). The applicability of asymmetrical tetrachords will be examined further in relation to the steering principle, following a discussion of the clock module.

Example 2-5(a): Symmetrical pentachords $SPIIm^5$ and $SPIIM^5$

$SPIIm^5$ $SPIIM^5$
 1 + 2 + 2 + 1 2 + 1 + 1 + 2

Example 2-5(b): Asymmetrical pentachords IIm^5 and IIM^5

IIm^5 IIM^5
 1 + 2 + 1 + 2 2 + 1 + 2 + 1

⁴⁶ McLeod, *Tone Clock Theory Expanded*, 12.

Example 2-6: Two configurations of ISPIIM⁵ and ISPIIm⁵

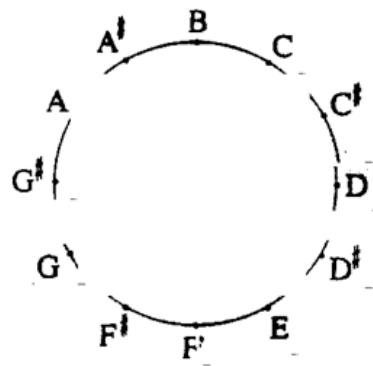
The image displays four musical staves in treble clef, arranged in two rows. The top row shows two configurations of ISPIIM⁵. The first staff has notes G4, A4, Bb4, and C5 with interval numbers 2, +, 1, +, 1 below. The second staff has notes G4, Ab4, A4, and B4 with interval numbers 1, +, 1, +, 2 below. The bottom row shows two configurations of ISPIIm⁵. The first staff has notes G4, Ab4, Bb4, and C5 with interval numbers 1, +, 2, +, 2 below. The second staff has notes G4, A4, B4, and C5 with interval numbers 2, +, 2, +, 1 below.

2.2: The Clock Module

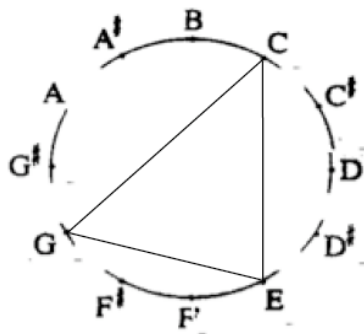
Analogous with the clock metaphor, trichords appear clockwise around a clock face and are arranged in ascending order based on the interval classes they contain. Schat identifies these with Roman-numeral labels. Moreover, he includes diagrams with smaller clock modules representing the tone-clock trichords when explaining how these trichords are derived. The smaller modules consisting of the twelve pitch classes are arranged clockwise, like hour points on a clock face, with B-natural at the 12 o'clock position (see example 2-7). Graphic representations of each trichord are then obtained by connecting the required points on the clock face. For example, one can derive a C-major triad by connecting the hour points C-E-G (see example 2-8(a)). This principle can then be extended to include the twelve trichords isolated by Schat. Not only is a trichord's prime form included, but the four transpositions necessary to generate all twelve pitch classes without pitch-class repetition as well (see example 2-8(b) with trichord I). This process is applied to all twelve trichords to produce twelve unique representations of each hour. The resulting modules are then arranged clockwise as the tone clock, also known as the "Zodiac of the Twelve Tonalities" (see example 2-9). Schat describes the zodiac-clock

diagram as “the visual image of [...] the regime of the triad over the chromatic scale,”⁴⁷ leading him to label his system as the Tone Clock. It is important to discuss how Schat arrived at the trichordal transpositions used in the twelve modules, as these were not arbitrarily chosen. They are, in fact, derived through a technique fundamental to the tone-clock system known as “steering.”

Example 2-7: Twelve-note module in the form of a clock face⁴⁸



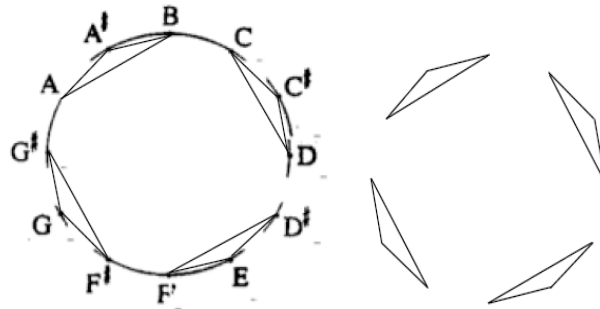
Example 2-8(a): C-major triad on the clock face



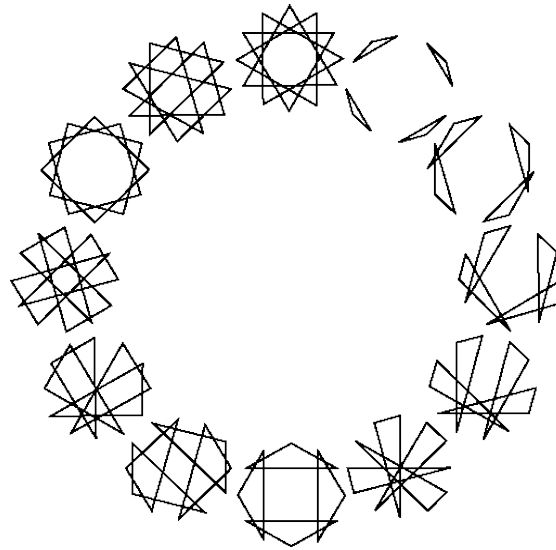
⁴⁷ Schat, *The Tone Clock*, 82.

⁴⁸ *Ibid.*, 75.

Example 2-8(b): Trichord I, prime form and three transpositions⁴⁹



Example 2-9: The Zodiac of the Twelve Tonalities⁵⁰



2.3: The “Steering” Principle

Along with the twelve trichords, an essential⁵⁰ component for generating “chromatic tonalities” consists of the operation identified by Schat as “steering.” This

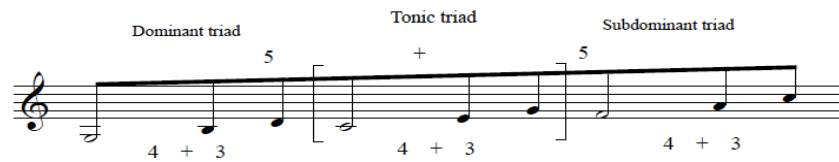
⁴⁹ This graphic representation of trichord I (1+1 semitones) includes all possible transpositions using all twelve pitch-classes without repeated pitch classes. Beginning on C-natural, the four trichords include (1) C-C#-D, (2) D#-E-F, (3) F#-G-G#, and (4) A-A#-B. The four-triangle diagram is then used as a representation of the first hour of the tone clock.

⁵⁰ Peter Schat, “VIII: Clockwise” *Peter Schat’s Personal Website*. <http://www.peterschat.nl/clockwise.html>

operation essentially directs, or “steers,” a tone-clock trichord through its transpositions to produce an unordered pitch-class set of modulo 12. McLeod argues that the steering principle is in fact a function that promotes growth and reproduction in music since “any given note has the power to generate [...] a group of notes,” and asserts that the concept of steering had been a musical practice since the medieval era.⁵¹ The steering principle can be applied to a tonal context and expressed using Schat’s tone-clock trichords. In traditional tonal music, musical organization revolves around a central pitch class, or tonic, from which more material is generated. For example, the central pitch class in C-major is, of course, the pitch-class “C,” which, in order to denote a major tonality, generates a C-major triad, since the triad establishes tonality. One can then introduce the other two fundamental triads to establish tonality: the subdominant (F-major) and dominant (G-major) triads. Once analyzed using Schat’s tone-clock hours, it becomes apparent that a C-major tonality is determined by three trichords from hour XI <3+4> in their major form <4+3>. The steering principle comes into effect as the subdominant and dominant triads are positioned five semitones (or a perfect fourth) from, and symmetrically around, the central tonic triad. This yields a steering of trichord XIM <4+3> by the symmetrical form of trichord IX <5+5> (see example 2-10). The interval content of the tonic, subdominant, and dominant trichords is identified by the integers below the staff, while the interval content of the steering trichord is notated above the staff. Open noteheads indicate each trichord’s root to more clearly demonstrate the steering effect of hour IX in operation.

⁵¹ Ibid.

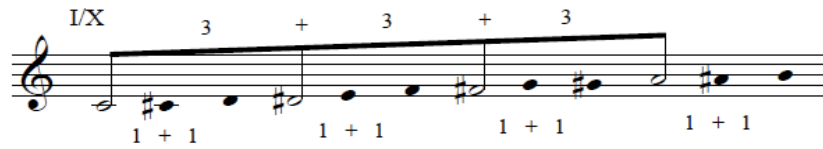
Example 2-10: Trichord steering in C-major



Schat's conception of "steering," as it relates to the tone clock, functions in a similar manner as the triads in example 2-10, though he did not allow for the formation of a central pitch, nor did he restrict himself to using only conventional major and minor triads. Schat realized that by transposing a select trichord three times he could generate all twelve pitch classes without pitch-class repetition. As his interest lay exclusively in the realm of twelve-tone composition, he found this property very useful for the compositional process. The steering principle essentially transposes a tone-clock trichord three times to generate a twelve-tone "harmonic field:" a pitch-class set of modulo 12, from which a composer may freely manipulate not only the order of the "sub-fields," or constituent trichords, but also the order of the pitch classes contained within each trichordal "sub-field." A symmetrical tetrachord steering determines the manner in which a trichord is transposed. A trichord steering by a tetrachord, the content of which is derived from a different tone-clock trichord's intervals, generates an unordered pitch-class set with a unique configuration of pitch classes. In example 2-11, trichord I, represented by the pitches C, C #, and D <1+1>, is effectively steered by tetrachord X⁴ <3+3+3> to produce all twelve pitch classes.⁵²

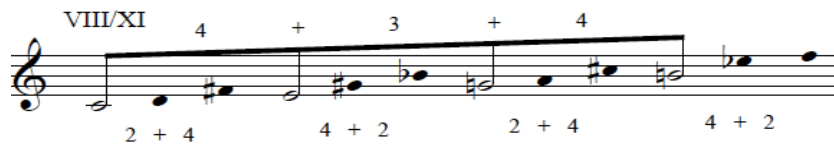
⁵² In set-theory terminology, pitch-class set 3-1 (0,1,2) undergoes a T₃ transformation each time it is transposed to generate a twelve-tone unordered pitch-class set. With reference to specific pitches, C maps on to D#, C# maps on to E, D maps on to F, and then the process is repeated for the next transposition.

Example 2-11: Trichord I <1+1> steering by tetrachord X⁴ <3+3+3>

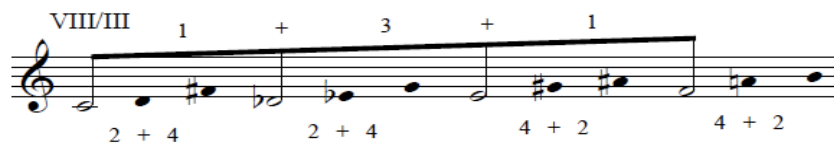


The trichord and steering combination in example 2-11 yields an ascending chromatic scale; the open noteheads represent the trichord X steering of trichord I (closed noteheads). This technique of trichordal transposition is applicable to all twelve trichords and each trichord is individually steered by a different tetrachord. Furthermore, some trichords possess the property of having more than one steering using different tetrachords to produce distinctive twelve-tone unordered pitch-class sets. For example, trichord VIII <2+4> may be steering by tetrachord XIM⁴ <4+3+4> (see example 2-12(a)) or by tetrachord IIIIm⁴ <1+3+1> (see example 2-12(b)).

Example 2-12(a): Trichord VIII <2+4> steering by tetrachord XIM⁴ <4+3+4>



Example 2-12(b): Trichord VIII <2+4> steering by tetrachord IIIIm⁴ <1+3+1>



Asymmetrical hours, such as VIII, will yield a pitch-class set that consists of two transpositions of the intervallic prime form ($VIII_m=2+4$) and two of its inversion ($VIII_M=4+2$) to generate all twelve pitch classes without repeated pitch classes. The major or minor form does not affect the trichords' affiliation with a tone-clock hour so long as the interval classes remain constant. Therefore, whether trichord VIII appears in its major or minor form is irrelevant. Notice how both examples 2-12(a) and 2-12(b) contain two instances of the major and minor forms of trichord VIII. The order of their appearance is irrelevant so long as they are present in the final unordered pitch-class set.

The steering function is expressed using Roman numerals separated by a backslash with the first Roman numeral representing the tone-clock hour followed by the backslash and finally the "steering hour." For example, trichord VIII steering by tetrachord IX⁴, in major or minor form, would be expressed as VIII/IX. Following Schat's notation, the "steering hour" is assumed to be a tetrachord unless it is steering another tetrachord, in which case the "steering hour" is a trichord.

An exception to the trichord steering principle applies solely to trichord X, which cannot be steering by a tetrachord to produce all twelve pitch classes. This is due to the intervallic content of trichord X $\langle 3+3 \rangle$ which yields a diminished triad. To obtain all twelve pitch classes without repetition, trichord X must be in tetrachord form and steering by a trichord (see example 2-13). Schat believed that this anomaly in the system strengthened his argument for it, and compared it to the Pythagorean comma or the problem of squaring the circle.⁵³ McLeod, instead, interprets the X hour as a "gateway"

⁵³ Peter Schat, "The Tone Clock, or the Zodiac of the Twelve Tonalities," *Key Notes* 17 (1983): 12.

between the tone-clock trichords and the generation of pitch-class sets using tetrachords.⁵⁴

Example 2-13: Tetrachord $X^4 \langle 3+3+3 \rangle$ steering by trichord I $\langle 1+1 \rangle$



Not all tetrachords can steer trichords and yield a pitch-class set of modulo 12 without repeated pitch-classes. Schat identifies a total of twenty possible steerings for the full tone clock, as shown in Table 2-1. Much like the tone clock trichords, Schat’s system requires that the steerings appear in their most compact form; therefore, even though a steering of the tenth hour by the fourth hour (X/IV) is possible, it is simply a variant of a steering by the first hour (X/I) as it produces no new tetrachords and is therefore excluded from the table (see example 2-14(a)). The steering function may also be used to “modulate” between trichords that share a common “steering hour,” which serves to establish a logical and continuous relationship between them (see example 2-14(b)). Since hour X steers trichords I and IV, both of these two trichords may be generated from the pitch classes used to express hour X. So the pitch-class set containing C, D#, F#, and A, representing tetrachord $X^4 \langle 3+3+3 \rangle$, becomes the progenitor of “chromatic tonalities” based on trichords I or IV, and permits for the seamless transition between them.

⁵⁴ Jenny McLeod, e-mail correspondence with author, August 18, 2014.

Table 2-1: Tone clock hours and their steerings

Tone clock hour	Number of steerings	Steering hour(s)
I	1	X
II	1	VIII
III	2	V, VII
IV	3	VI, VIII, X
V	1	II
VI	2	V, X
VII	1	VIII
VIII	3	III, IV, XI
IX	2	II, X
X	1	I
XI	3	VI, VIII
XII	1	I

Example 2-14(a): Tetrachord X⁴ steerings comparing hours I and IV

Example 2-14(a) shows two musical tetrachords. The first is labeled X/I and consists of three groups of three notes each, with fingerings 1, +, 1. The second is labeled X/XII and consists of three groups of three notes each, with fingerings 4, +, 4.

Example 2-14(b): “Modulation” of hours I and IV by X steering

Example 2-14(b) shows a modulation from I/X to IV/X. The I/X section consists of four groups of two notes each, with fingerings 1 + 1. The IV/X section consists of four groups of two notes each, with fingerings 4 + 1, and a final group of three notes with fingerings 3 + 3.

Additionally, Schat proposes that a combination of trichords may also produce all twelve pitch classes. These particular pitch-class sets are generated in one of two ways: (1) as a juxtaposition of trichords, each steered by its own respective hour, or (2) by allowing a single trichord to be steering by two different hours. The first method essentially juxtaposes two contrasting hours, such as II <1+2> and VII <2+3>, which coexist within the same twelve-tone “harmonic field” creating what de Groot refers to as a “composite tonality”⁵⁵ (see example 2-15(a)). Within the generated pitch-class set, each hour individually follows its conventional steering. In this example, both II and VII are steered by hour VIII, though this is not necessarily a requirement as shown in example (2-15(b)), where trichord I <1+1> is steered by X <3+3>, and III <1+3> by V <1+5>. When combining hours, both the IPF and the inversion are present to generate a pitch-class set with twelve different elements.

Example 2-15(a): “Composite tonality” of trichords II+VII

The musical notation for Example 2-15(a) shows a sequence of notes on a staff. Above the staff, the label "II + VII" is centered. Below the staff, the label "II/VIII" is positioned above the first note, and "VII/VIII" is positioned below the first two notes. The notes are: C4 (quarter), D4 (quarter), E4 (quarter), F4 (quarter), G4 (quarter), A4 (quarter), B4 (quarter), C5 (quarter), D5 (quarter), E5 (quarter), F5 (quarter), G5 (quarter). Below the staff, four steering hour labels are placed: "1 + 2" under the first two notes, "3 + 2" under the third and fourth notes, "2 + 3" under the fifth and sixth notes, and "2 + 1" under the seventh and eighth notes.

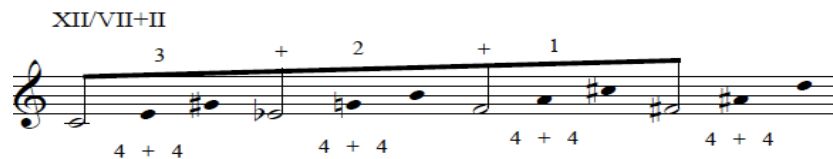
Example 2-15(b): “Composite tonality” of trichords I+III

The musical notation for Example 2-15(b) shows a sequence of notes on a staff. Above the staff, the label "I + III" is centered. Below the staff, the label "I/X" is positioned above the first note, and "III/V" is positioned below the first two notes. The notes are: C4 (quarter), D4 (quarter), E4 (quarter), F4 (quarter), G4 (quarter), A4 (quarter), B4 (quarter), C5 (quarter), D5 (quarter), E5 (quarter), F5 (quarter), G5 (quarter). Below the staff, four steering hour labels are placed: "1 + 1" under the first two notes, "3 + 1" under the third and fourth notes, "1 + 3" under the fifth and sixth notes, and "1 + 1" under the seventh and eighth notes.

⁵⁵ Rokus de Groot, “The Wheels of the Tone Clock: The Musician as Clockmaker,” *Key Notes 19* (1984): 17.

The second method of hour combination occurs when a select trichord is steered by more than one hour. This method may be referred to as “compound steering” and involves an AT. In other words, the interval classes from two trichords will be incorporated into the tetrachordal steering function (see example 2-16). In this example, trichord XII <4+4> is steered by a combination of hours VII_m <2+3> and IIM <2+1> to produce twelve pitch classes. As a result, the trichord is no longer steered in the conventional manner by means of a symmetrical tetrachord, but instead by the interval classes of two trichords, which result in an AT. In more precise terms, trichord XII is steered by ATVII+II.

Example 2-16: Trichord XII/VII+II



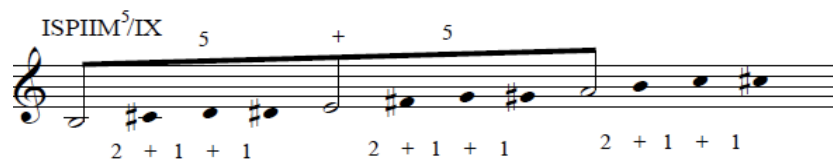
In practice, Schat applied the steering principle more liberally, often in ways that deviated from his original conception of the technique. He frequently used steering to generate pitch-class sets larger or smaller than twelve pitch classes and allowed pitch-class repetition when deriving pitch-class sets. This was likely done because of the technique’s capacity for generating an infinite quantity of coherent material while maintaining a relationship with the tone-clock trichords. However, Schat advised against the combination of too many hours claiming that it could lead to “hard, bare octave collisions and many ‘uncontrolled’ chords [arising].”⁵⁶ In essence, a pitch-class set of

⁵⁶ Schat, *The Tone Clock*, 103.

any size may be steering by any other pitch-class set to generate material. Moreover, any pitch may become a generator, or “steering-note,” of new material resulting in various levels of steering. A discussion on the various structural levels of steering will follow shortly.

The flexibility of Schat’s model presents a composer with the opportunity to use also an asymmetrical tetrachord as a starting point for generating a pitch-class set. For example, $ISPIIM^5 \langle 2+1+1 \rangle$ may be steering by the symmetrical form of trichord IX $\langle 5+5 \rangle$ to generate material related to trichord II (see example 2-17). This steering arrangement produces a pitch-class set consisting of twelve pitches with only eleven distinct pitch classes present, as the C-sharp is repeated due to the transposition of the initial tetrachord. Schat permitted pitch-class duplication such as this; likely because he did not want to limit his system to generating only sets of modulo 12. The steering principle does not only occur at the surface level, but may also be applied to derive larger structures.

Example 2-17: Tetrachord $ISPIIM^5$ steering by IX



2.4: Deep-Level Steering versus Surface-Level Steering

The discussion and examples thus far have focused on surface-level steering, as this is the easiest level to analyze. In compositional practice, however, there exist

multiple levels of steering because any pitch class or pitch-class set may generate new material and therefore become a steering note. It is this process that preserves the relationships of the various steering levels to each other and contributes to the overall cohesion of the music produced using the tone-clock system. In other words, the steering principle does not simply derive twelve distinct pitch classes using the tone-clock hours: it is also used to generate a network of interrelated material from which an entire work may be composed. For example, one may begin composing a work by selecting trichord VIII <2+4> steering by symmetrical tetrachord XIM⁴ <4+3+4>, representing the surface-level steering (see example 2-18). The pitch-class set begins on E and not C in this case. The next level of steering develops when the pitch-class set is itself steering on a second, deeper level by the tetrachord X⁴ <3+3+3>. The whole set undergoes a T₃ transformation three consecutive times resulting in four transpositions of VIII/XI steering by X⁴.

Example 2-18: Deeper level steering of VIII/IX by X⁴

VIII/XI

VIII / XI / X

2 + 4 4 + 2 2 + 4 4 + 2

4 3 4

3

The T_3 transformation may be expressed more clearly as the second-level steering of symmetrical tetrachord XI^4M , which is the tetrachord steering of trichord VIII, both of which are steering at the second level by tetrachord X^4 . The potential for generating new material is essentially limitless and may continue until a composer has derived enough to create a work. The sets generated in example 2-18 can be divided into three levels of “steering:” (1) the surface level, consisting of VIII/XI, (2) the second level, with XI^4/X , and (3) the third, or deepest, level, showing the “steering hour” tetrachord X^4 as the generator of the whole excerpt, which could itself be a subset of some larger set (see examples 2-19(a)(b)(c)). Though example 2-19 is generated from top to bottom beginning with VIII/XI, it is also possible to produce the same result by starting with tetrachord X^4 and deriving the material from the deep level to the surface level.

Example 2-19(a): Surface-level steering: VIII/XI



(6,9,e,0,2). However, the process of multiplication does not align directly with traditional mathematics; rather the process of addition is used to derive the new pitch-class sets. In other words, the multiplication function, represented by the circled multiplication symbol \otimes , involves the addition of integers (see example 2-20, right-hand matrix). Since pitch-class set (0,5,3) is “multiplied” by (6,9), $6 + 0 = 6$, $6 + 3 = 9$, $6 + 5 = 11(e)$, and so forth. Schat rejected the use of this procedure because Boulez permitted pitch-class duplication in the initial calculations, which were subsequently eliminated during the compositional process. In example 2-20, the doubling of pitch-class 9 is removed from the final “product” but the other integers generated by the multiplication of pitch class 6 are not.

Example 2-20: Boulez’s Pitch Multiplication Technique⁵⁹

The multiplication technique is adapted for Schat’s tone-clock system in the form of the steering principle and is expressed in terms of trichords multiplied by tetrachords or vice versa (see example 2-21). Before multiplication can occur, trichords must be reduced to their IPF and begin with pitch-class integer (0). This is essentially the same procedure as the normalization process used to produce the ois in Boulez. In example 2-21, the IPF of trichord I $\langle 1+1 \rangle$ beginning on pitch-class integer (0), or the prime form

initial pitch-class integer 4 $\langle 4\{7,9,1\} \rangle$ is $\{0,3,5,9\}$. The ois transposes a set to begin on 0 and lists the integers in ascending order, yielding a set in its most compact form.

⁵⁹ Ibid. Note the repetition of pitch-class (9) as a result of the multiplication.

(0,1,2), is multiplied by tetrachord $X^4 \langle 3+3+3 \rangle$, which also begins with pitch-class integer (0), resulting in the pitch-class set (0,3,6,9). Each pitch class of tetrachord X^4 generates a trichord that is reproduced vertically above any given pitch class (see example 2-21, right-hand matrix); therefore, pitch-class (0) generates trichord (0,1,2), pitch-class (3) generates trichord (3,4,5), and so forth.

Example 2-21: Trichord I (0,1,2) multiplied by tetrachord X^4 (0,3,6,9)

Trichord (IPF)

2	2	5	8	e
1	1	4	7	t
0	0	3	6	9
⊗	0	3	6	9

(Tetrachord)

Though Boulez’s pitch-class multiplication technique successfully produces pitch-class sets of modulo 12 when multiplying a symmetrical trichord, as in example 2-21, the multiplication procedure becomes problematic when multiplying asymmetrical trichords. This is true for two reasons. Firstly, when an asymmetrical trichord steers to generate twelve distinct pitch classes it typically requires that two major and two minor forms of that trichord be present. This cannot be achieved with a single multiplication of pitch classes, as it will multiply either only the major form or minor form by a select tetrachord. Secondly, because only one form of an asymmetrical trichord can be multiplied at once, pitch-class duplication is inevitable (see example 2-22). The result of multiplying the minor form of trichord II $\langle 1+2 \rangle$ by $VIII m^4 \langle 2+4+2 \rangle$, expressed in integer notation as $(013) \otimes (0268)$, produces a pitch-class set with only ten distinct pitch

classes. The pitch-classes (3), in the second trichord, and (9), in the fourth trichord, are duplicated as a result of the multiplication procedure.

Example 2-22: Trichord IIm multiplied by tetrachord VIIIm⁴ (013) ⊗ (0268)

Trichord (IPF)

3	3 5 9 e
1	1(3)7(9)
0	0 2 6 8
⊗	0 2 6 8

(Tetrachord)

The solution to this problem involves performing two separate multiplications: one multiplication of a trichord's major form and another of its minor form. The results must then be combined to yield the distinct trichord and steering combination that generates four distinct trichords without pitch-class repetition (see example 2-23(a)). The operation $IIm \otimes VIIIm^4$ produces the first and third trichords, indicated by the circled integers 1 and 3, while the operation $IIM \otimes VIIIm^4$ completes the pitch-class set by producing the second and fourth trichords, circled integers 2 and 4. Trichords containing pitch-class repetition, as in the first trichord (023) of the second matrix, are marked by an X and are excluded from the final juxtaposition of trichords. The resulting pitch-class set, originally labeled II/VIII by Schat, may then be more accurately labeled $II(mMmM)/VIII(m)^4$, where (m/M) designate the minor form (m) and major form (M) respectively (see example 2-23(b)). This new labeling more clearly reflects the trichordal content, in terms of its minor/major configuration, necessary for multiplication, as well as the form of the tetrachord, major or minor, that is used. If the initial trichord or tetrachord is symmetrical, there is no indication of major or minor form.

Example 2-23(a): Multiplication matrices of $\text{IIIm} \otimes \text{VIIIm}^4$ and $\text{IIM} \otimes \text{VIIIm}^4$

The image shows two musical staves. The first staff is labeled $\text{IIIm} (0 1 3)$ and contains the notes C, D, and E. The second staff is labeled $\text{VIIIm}^4 (0 2 6 8)$ and contains the notes C, D, F, and G. Below these staves are two multiplication matrices. The first matrix is for $\text{IIIm} \otimes \text{VIIIm}^4$ and the second is for $\text{IIM} \otimes \text{VIIIm}^4$. Both matrices have a circled X in the bottom-left corner and a circled 0 in the bottom-right corner. The first matrix has circled 1 and 3 above the first two columns, and circled 2 and 4 above the last two columns. The second matrix has circled 2 and 4 above the last two columns.

	①	③		②	④
3	3	9	X	3	5
1	1	7	X	2	4
0	0	6	X	0	2
⊗	0	2	6	8	⊗

Example 2-23(b): $\text{II}(\text{mMmM})/\text{VIIIm}^4$

The image shows a musical staff labeled $\text{II}(\text{mMmM})/\text{VIIIm}^4$. The staff contains a sequence of notes: C, D, E, F, G, A, B, C. Below the staff are four circled numbers: ① (0 1 3), ② (2 4 5), ③ (6 7 9), and ④ (8 t e).

The steering principle can be formalized as the function STEER, which can be applied to trichordal and tetrachordal pitch-class sets to generate additional musical material and expressed as $(x)\text{STEER}(y)$. Theoretically, the x and y variables may be substituted by a pitch-class set of any size. For instance, let the formula $(\text{TRI}(x))\text{STEER}(\text{TET}(y)) = \text{TRI}(\text{m}/\text{M}) \otimes \text{TET}((\text{ipc})\text{m}/\text{M})^4$, where TRI is a tone-clock trichord's Roman numeral classification, TET is the Roman numeral of a tetrachord derived from a tone-clock trichord, and where m/M designates the configuration of major and minor forms. The initial pitch-class integer from which TET is derived (ipc) can be interpreted as the tetrachord's root. This is necessary because not all tetrachordal pitch-class sets will begin on (0) or C. The formula $\text{TRI}(\text{m}/\text{M}) \otimes \text{TET}((\text{ipc})\text{m}/\text{M})^4$ produces a set of directed-interval classes consisting of twelve distinct pitch classes. The

configuration of minor/major forms in TRI(m/M) determines which of the pitch classes from TET(m/M)⁴ are used for the purpose of multiplying. In other words, the (m/M) designating the minor/major configuration corresponds with the pitch classes of the select tetrachord TET(m/M)⁴. For example, VIII(mmMM) ⊗ III((0)m)⁴ indicates that TRI(m), the minor form of the initial trichord VIII, will be multiplied by the first two pitch classes of III((0)m)⁴. Therefore, one half of the formula can be expressed as TRI(026) ⊗ TET(01xx). The other half of the formula is TRI(M), the major form of the trichord VIII, multiplied by the last two pitch-classes of TET(m)⁴, expressed as TRI(046) ⊗ TET(xx45) (see example 2-24). A simplified formula of the one shown in example 2-24 can be expressed as VIII ⊗ 0(m)1(m)4(M)5(M), wherein III((0)m)⁴ has been calculated so as to show its constituent pitch-class integers and the manner in which they steer the trichord VIII(m/M). In other words, the (m/M) configuration and the pitch classes from which the trichords are derived are more clearly represented. This formula can then be used to determine the resulting pitch-class set (see example 2-25).

Example 2-24: VIII STEER III = VIII(mmMM) ⊗ III(m)⁴

TRI(mm	MM)	
① ②	③ ④	
6	6 7	6 t e
2	2 3	4 8 9
0	0 1	0 4 5
⊗	0 1	⊗ 4 5
TET(m)		

VIII(mmMM)/III(m)⁴

Example 2-25: Solving the VIII STEER III formula

$$\begin{aligned}\text{VIII STEER III} &= \text{VIII}(\text{mmMM}) \otimes \text{III}((0)\text{m})^4 \\ &= \text{VIII}(\text{mmMM}) \otimes 0145 \\ &= \text{VIII} \otimes 0(\text{m})1(\text{m})4(\text{M})5(\text{M}) \\ &= 0(026)1(026)4(046)5(046) \\ &= (026)(137)(48\text{t})(59\text{e}) \\ &= (\text{C},\text{D},\text{F}\#)(\text{D}\flat,\text{E}\flat,\text{G})(\text{E},\text{G}\#, \text{A}\#)(\text{F},\text{A},\text{B})\end{aligned}$$

The STEER function can be applied to the multiplication of any TRI(m/M) consisting of any configuration of major and minor forms. Furthermore, the STEER function may be performed using pitch-class sets of any size so that a tetrachord could be multiplied, or steered, by a trichord to generate tetrachordal sets, or a tetrachord by a tetrachord to generate sets with more than twelve notes. For example, STEER could be applied to show how $X^4 \otimes \text{XII}(8)$, or tetrachord X multiplied by trichord XII with the initial pitch-class 8, produces a pitch-class set with subsets segmented as tetrachords rather than trichords. The same process may even be used to generate pitch-class sets smaller than twelve notes so that a trichord may be multiplied by a trichord to yield a set consisting of only nine pitch classes.

Though both steering and pitch-class set multiplication are conceptually similar, they produce significantly different outcomes. Steering, when used conservatively, is used to derive pitch-class sets of modulo 12 without pitch-class repetition. In contrast, pitch-class multiplication techniques used by Boulez serve as a means to generate an infinite quantity of material through pitch-class set multiplication and permits pitch-class repetition that is subsequently eliminated during the compositional process. Although one could not arrive at Schat's "chromatic tonalities" by using Boulez's method for the

reasons discussed above, namely the multiplication of two forms of a trichord required to generate all twelve pitch-classes, it is clear that pitch-class set multiplication is an integral component of Schat's tone-clock system in the form of steering. When mapping musical transformations in Schat's compositions, the STEER function becomes an essential component of transformational networks, as it is capable of clearly encompassing pitch-class set relationships that unify the work.

2.6: Application of Transformational Theory

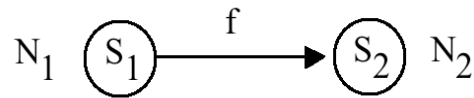
David Lewin presents the concept of transformational networks in his *Generalized Musical Intervals and Transformations* (1987) as a tool to shift the focus of the analyst from musical results to musical processes.⁶⁰ In essence, a transformational network graphically illustrates how musical objects are transformed into one another through a series of nodes and arrows. Lewin defines a transformation network as “an ordered sextuple [consisting of] S, NODES, ARROW, SGP, TRANSIT, and CONTENTS.”⁶¹ S represents a set of musical objects transformed by SGP, a semigroup of transformations on S. NODES and ARROW provide the visual elements of the graph, in which CONTENTS assigns elements of S to the nodes and TRANSIT assigns elements of SGP to the arrows. Lewin illustrates this with a generalized diagram that depicts nodes N_1 and N_2 whose CONTENTS consist of S_1 and S_2 respectively, and are related via TRANSIT f , which is a transformation on S, also referred to as SGP (see example 2-26).

⁶⁰ David Lewin, *Generalized Musical Intervals and Transformations* (New Haven and London: Yale University Press, 1987), xxxi.

⁶¹ *Ibid.*, 196.

A transformational network, such as the one in example 2-26, can be used as a visual representation of the STEER function.

Example 2-26: Generalized Transformational Network⁶²



$$S_1 = \text{CONTENTS}(N_1); S_2 = \text{CONTENTS}(N_2)$$

$$f = \text{TRANSIT}(N_1, N_2); f(S_1) = S_2$$

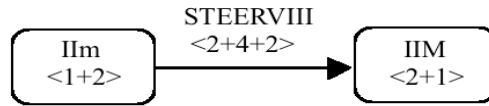
If this concept is applied to a tone-clock trichord, such as trichord II <1+2> steered by VIII^{m4} <2+4+2>, then the CONTENTS of $N_1 = \text{IIm} = S_1$ and the CONTENTS of $N_2 = \text{IIM} = S_2$. The trichords are related via $f = I_5$ (inversion around the axis D^{\sharp}/E^{\flat}) or steering by VIII (see example 2-27(a)). Though both networks yield the same result, the methods used to transform the initial trichord are significantly different. The first network (i) generalizes the process of steering a trichord, which in this instance reverses the trichord's interval-class content and transposes it by an interval determined by the steering hour. The second network (ii) involves pitch-class inversion and transposition around an axis. From this initial example, a transformational network illustrating the steering operations in a pitch-class set derived from II/VIII^{m4} can be constructed (see example 2-27(b)). The transformational network is circular because it conveys Schat's notion that trichordal subsets may come in any order and are therefore all related when applied in the compositional process. The network clearly depicts the manner in which trichordal subsets relate to each other through inversion and transposition; however, the

⁶² Ibid.

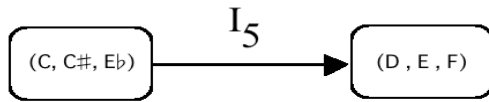
process of the steering hour is concealed. The I_5 operation does not express the transposition by two semitones directed by the interval classes of the steering hour, VIII.

Example 2-27(a): Transformational network illustrating $II_m \rightarrow IIM$ via I_5 or steering by VIII

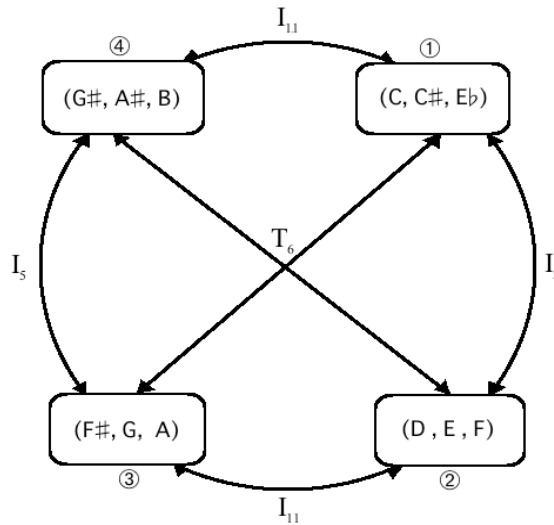
(i)



(ii)



Example 2-27(b): Transformational network of $II/VIII_m^4$



Though accurate in its depiction of trichordal relationships, the transformation network in 2-27(b) fails to capture the influence of steering as Schat conceived it. In essence, Schat's steering consisted of a pitch-class set directed by the interval classes of another. When Schat depicts a pitch-class set derived from the tone clock, he ensures that

the vertices (open noteheads) of each trichord clearly illustrate the effect that the interval configuration of the steering hour has on the initial set. The problem with representing the process of steering through a transformational network is that it does not overlap with simple inversion and transposition.

A solution to this problem involves the application of concepts introduced by Julian Hook to depict *uniform triadic transformations*, hereafter UTT. Hook uses an algebraic framework to illustrate triadic transformations, wherein each UTT consists of a sign (+ or –) and two transposition levels. The sign is used to indicate whether the transformation preserves (+) or reverses (–) a triad’s mode. The transposition level consists of two integers of mod 12, the first for a major triad and the second for minor, and serves to indicate the interval by which the root of a select triad is transposed.

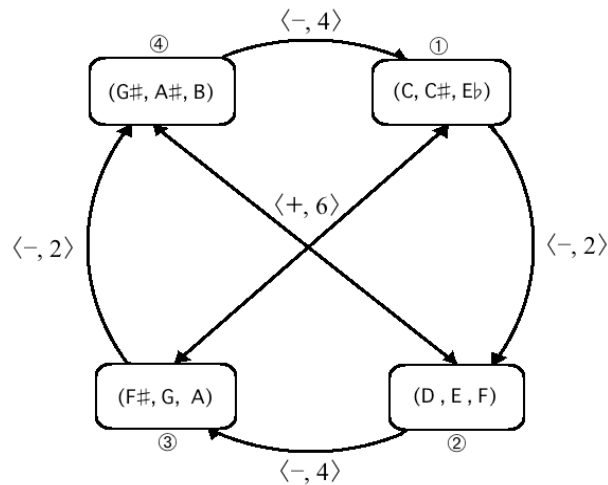
Therefore, the mapping of C major to E minor is expressed by the formula $\langle -, 4, 8 \rangle$, wherein the major form is reversed to minor and the root, C, is transposed by 4 semitones to E.⁶³ The inverse would involve mapping E minor to C major through a mode reversal and transposition by 8 semitones. Hook’s UTT formula can be adapted to better articulate the steering function as part of a transformational network. Though Hook’s model consists of two transpositional levels, one for major triads and a second for minor, I use only one transpositional level to represent the interval between two roots. Therefore, each altered UTT consists of a sign and a single transpositional level. Like Hook’s UTT, the sign serves as an indicator of whether a tone-clock trichord’s mode is preserved or reversed, and the transpositional level specifies the transposition interval of a trichord’s root. For example, performing the operation $\langle -, 2 \rangle$ on trichord II_m $\langle 1+2 \rangle$ with the root

⁶³ Julian Hook, “Uniform Triadic Transformations,” *Journal of Music Theory*, 46/2 (2002): 59.

note C (C-C#-Eb) reverses the trichord's mode from minor to major and transposes its root upward by two semitones, yielding IIM <2+1> with the root note D (D-E-F). The transformation can then be applied to create the network in example 2-26(b) to more accurately depict the effect of the steering hour VIIIIm⁴ <2+4+2> (see example 2-28). The UTT in example 2-28 serve to appropriately express the transformations undergone from one trichord to the next, that is the change of mode (if one occurs) and transposition intervals implicit in the steering hour. The circled integers denote the order in which the trichords are derived according to the steering hour.

Example 2-28: Application of UTT to transformational network of II/VIIIIm⁴

II(mM mM)	
(m / M) ① ② ③ ④	
3 / 3 3 5 9 e	
1 / 2 1 4 7 t	
0 / 0 0 2 6 8	
⊗ 0 2 6 8	=
└──┬──┘	II(mMmM)/VIII(m) ⁴
VIII(m)	



In summary, Schat conceived of the tone clock not as a descriptor of all the possible harmonies and pitch-class relationships, but as a generator of “chromatic

tonalities,” or pitch-class sets of modulo 12, with the objective of using it as a tool to establish and preserve coherent pitch-class relationships throughout a twelve-tone piece. At the core of the tone-clock system are the twelve trichords in pitch-class space that are arranged on a clock-face diagram referred to as the “Zodiac of the Twelve Tonalities,” wherein each trichord represents an hour or a tonality. Each chromatic tonality results from a succession of transformations influenced by the steering principle, which is applied to each of the twelve tone-clock trichords individually. Steering is a process involving select tetrachords, which are used to determine the manner by which an initial trichord is transposed in order to derive a pitch-class set with twelve distinct pitch elements. The resulting pitch-class sets can then be used as material in the compositional process or can be further transformed through steering to yield additional interrelated material. The origins of Schat’s steering can be traced back to Boulez’s technique of pitch-class set multiplication. In essence, steering transforms a trichord into a twelve-note series by multiplying an initial pitch-class set with a second pitch-class set, much like Boulez’s pitch-class set multiplication technique. In the case of the tone clock, the process entails multiplying a trichord by a tetrachord ($3 \times 4 = 12$) or a tetrachord by a trichord ($4 \times 3 = 12$) to generate “chromatic tonalities.” The procedure can be expanded to derive pitch-class sets larger and smaller than twelve notes. For example, a tetrachord may steer a tetrachord ($4 \times 4 = 16$) or a trichord may steer a trichord ($3 \times 3 = 9$). The steering principle can be formalized as the STEER function, which presents a more accurate representation of the process involved. Transformational networks, useful tools for showing the conventional operations of transposition and inversion in post-tonal works, also have the potential to show the process involved with the steering principle.

By adapting Hook's UTT formula as transformations for the steering function, transformational networks can more accurately represent the steering process. One can undertake an analysis of Schat's compositions by using transformational networks in combination with Hook's UTT formula and the STEER function, and demonstrate how steering can generate coherent pitch-class set relationships to unify a post-tonal composition.

Chapter Three: Analytical Applications of the Tone Clock in “Genen”

This chapter focuses on the importance of the steering principle and the role that it plays in generating and preserving coherent pitch-class set relationships that contribute to the overall unity of a twelve-tone composition. In the ensuing analysis, I explore the significance of steering in Schat’s composition “Genen” (2000). “Genen” is an ideal piece for analysis due to its length (spanning a mere 204 measures), its minimal instrumentation consisting of only violin and piano, and because one can surmise that Schat’s methods for composing with the tone clock were better developed and more refined after nearly two decades of experimentation with the system. However, embarking on such an analysis requires careful consideration as to the optimal manner in which to illustrate pitch-class set relationships that are generated through steering at various levels. This is because an analytical system with the purpose of illustrating the steering function does not exist, or has not yet been formalized, making an analysis of a tone-clock based composition more challenging. Though Schat’s compositions are comprised of primarily twelve-tone writing, analytical methods using set theory do not capture the essence of the steering principle; therefore, a different analytical approach is required. As discussed in chapter two, transformational networks coupled with an adaptation of Hook’s UTT formula are a practical and effective way of depicting the transformations that constitute the “chromatic tonalities” as Schat conceived them. This is because steering generates material through a series of transformations to a pitch-class set. Furthermore, transformational networks can illustrate deeper level steering that produces large-scale pitch-class set transformations that are not always evident. The

STEER function outlined in chapter two also becomes indispensable when undertaking such an analysis because it produces a more accurate representation of the procedures required for the successful steering of a pitch-class set, trichordal or otherwise. The STEER function is an integral component of transformational networks that depict the deeper level steerings in “Genen.”

This chapter is divided into three subsections. Section 3.1 discusses the manner in which transformational networks are applied to illustrate the transformation of sets, followed by the analysis in Section 3.2. I limit my analysis to a large portion of the opening and closing sections of the piece, since this material is representative of the work. More specifically, I examine the first major section of the piece, which spans the first sixty-six measures, as well as an excerpt from the closing section (mm.179-204). Lastly, the data presented throughout the analysis is summarized and synthesized as concluding remarks in section 3.3.

3.1: Application of Transformational Networks

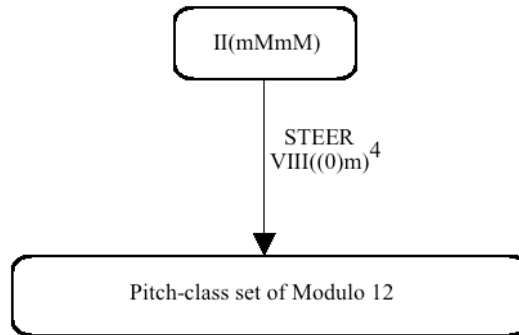
While the transformational network presented in example 2-28 at the end of chapter two is effective for showing steering on the surface level, a different model is necessary to demonstrate steering on a larger scale. The STEER function is most applicable in this situation. All three networks in example 3-1 represent an identical procedure that once applied to trichord II generates a twelve-tone series segmented into trichordal subsets. The three networks shown are necessary to establish the equivalence of the final network in 3-1(c) to the previous two, as well as to demonstrate the process by which it is arrived. For example, if one wishes to present a transformational network

which shows how trichord II is transformed into a twelve-note series through steering by tetrachord VIII, a generalized transformational network like the one in example 3-1(a) could be used. This network is a literal graphic representation of the formula $II\text{STEER}VIII = II(mMmM) \otimes VIII((0)m)^4$. The contents of the upper node represent the first portion of the STEER formula that depicts trichord II accompanied by the minor/major configuration that is applied when multiplying by the tetrachord $VIII m^4$, indicated by the arrow labeled $STEER VIII((0)m)^4$. The diagram in example 3-1(b) expands the formula to show the pitch classes implied by the $VIII((0)m)^4$ portion of the STEER function. The contents of the lowest node present the trichordal subsets derived from the multiplication of trichord II by tetrachord VIII, yielding a twelve-note series or a “chromatic tonality.” The diagram also pairs the forms of trichord II, minor or major, with the appropriate pitch class of tetrachord VIII so that pitch-class 0 yields a II minor trichord, pitch-class 2 yields a II major trichord, and so forth. The upper node in example 3-1(c) contains pitch classes that constitute the minor form of trichord II, as it is representative of that class of trichord on the tone-clock zodiac discussed in chapter two, and presents a more concrete musical example of the STEER function. The major form of trichord II would also be possible since steering usually involves both forms of a select trichord. Example 3-1(c) replaces the pitch-class integers in 3-1(b) and substitutes them with specific pitch classes. The first pitch class of the steering notes is identical to that of the trichord. This will always be the case in transformational networks that express concrete musical events because the representation is more precise, thereby creating a stronger association with the musical event in question. It is important to recognize that example 3-1(a) implies the same transformation as the more context-specific 3-1(c). My

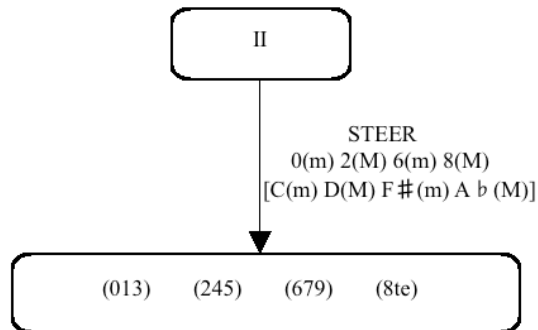
analysis will employ networks like the one shown in example 3-1(c) because they most clearly demonstrate how a select trichord is transformed into a twelve-note series through the process of steering.

Example 3-1: Transformational networks depicting STEER = II(mMmM) ⊗ VIII((0)m)⁴

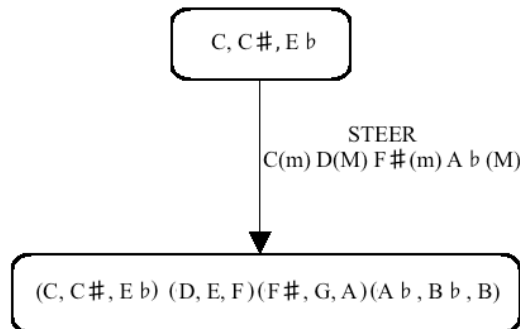
(a)



(b)



(c)

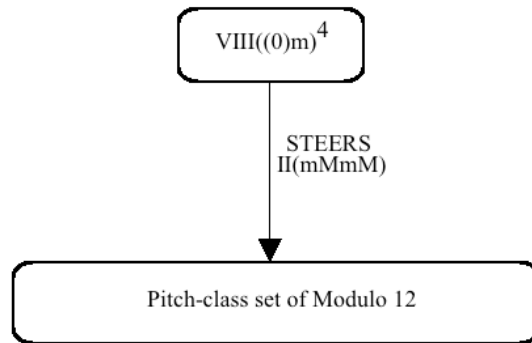


A transformational network may also be produced to show the process by which a twelve-note series is derived from steering notes rather than from a trichord as the starting point. This additional function is useful to demonstrate how sets may be inversely derived from a deeper-level structure, thereby illustrating greater structural unity. The derivation of a pitch-class set in this manner simultaneously generates the four trichordal transpositions that constitute the set. This requires a slight modification of the STEER function, which is achieved by adding an “S” to the end, so as to appropriately depict the process by which a pitch-class set is derived using the steering notes as a point of departure rather than the trichord. The process can be illustrated using an arrow that carries the function STEERS, in which an initial pitch-class set *steers* a second. For example, the transformational networks in example 3-1 can be reproduced so that the upper node contains the steering notes and the arrow expresses the notion that it STEERS some trichord. The modified formula would then read $VIII(0)m^4 \otimes II(mMmM)$, though it would produce the same results as the previous example. Example 3-2 shows three equivalent networks that proceed from the most generalized (3-2(a) and (b)) to the most precise and depict the STEERS function as it is applied to tetrachord $VIII(0)m^4$ and trichord $II(mMmM)$. One can observe that the lower node of example 3-2(c) contains the same twelve-note series as example 3-1(c); therefore, one can conclude that both processes will generate the same pitch-class set with the order of operations reversed. Like the previous example, the minor/major configuration of trichord II must be paired with the appropriate pitch class of the steering tetrachord. Therefore, pitch-class C generates a II minor trichord, D a II major trichord, F# a II minor trichord, and Ab a II major trichord. The first pitch class of both the trichord and tetrachord in example 3-2(c)

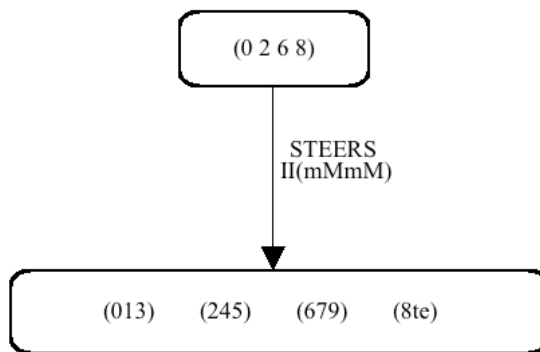
is identical, as was the case in example 3-1(c). This is a practical way of expressing how a real musical object is transformed through steering. In other words, though they are much more specific than the generalized STEER/(S) functions call for, the networks in example 3-2(c) and example 3-1(c) illustrate how a pitch-class set, trichord or tetrachord, beginning with the pitch-class C, is transformed throughout a musical passage.

Example 3-2: Transformational networks depicting STEERS = VIII((0)m)⁴ ⊗ II(mMmM)

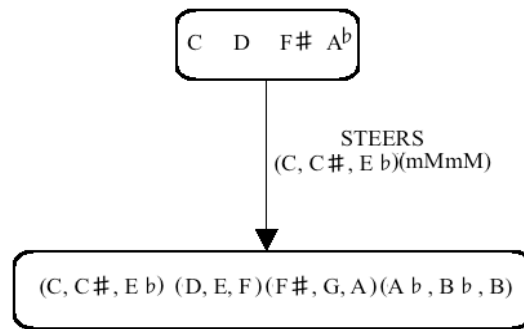
(a)



(b)



(c)



3.2: Analysis of “Genen”

An analysis of “Genen” demonstrates how the practical application of the tone clock and its principles can generate a coherent and unified musical work. My interpretation focuses primarily on the role of the steering principle as a generator of musical material at multiple levels, as well as its function in maintaining coherent intervallic relationships throughout the piece. This is accomplished by employing concepts and terminology, such as IPF and the multiple levels of steering, introduced in chapter two, as well as the analytical approach using transformational networks discussed above. My main argument posits that, though “Genen” consists of several seemingly unrelated “chromatic tonalities,” the musical material may be traced back to a single source. In other words, I contend that one pitch-class set acts as a generator of musical material thereby unifying the piece in a way that is not necessarily apparent.

My analysis begins by exploring the different surface-level steerings found in the piano accompaniment, as they are most quickly and easily identified due to the trichordal organization of each part. Schat clearly explores the harmonic potential of each trichordal subset to produce two simultaneous and contrasting harmonic layers. That is, two tonalities occur simultaneously as a series of chords. The contrast is emphasized by the

use of two different time signatures in each hand: a $\frac{5}{8}$ compound meter in the right-hand part, and a $\frac{2}{4}$ simple meter in the left-hand part. The right-hand part, consisting of the pitch-classes (C, E, A \flat) with the intervallic configuration of <4+4>, clearly implies tone-clock hour XII. The left hand performs the pitch-classes (F, B \flat , C) and can therefore be interpreted in one of three ways: (1) the minor form of hour IX <2+5> = (B \flat , C, F), (2) the major form of IX <5+2> = (F, B \flat , C), or (3) as the symmetrical form of IX <5+5> = (C, F, B \flat). I opt for the symmetrical form of hour IX to align with McLeod’s third (symmetrical forms are preferred over any asymmetrical form) and fourth (sets with fewest interval classes are preferred) criteria for selecting a trichord’s prime form. This interpretation may seem counterintuitive because of the blatant <2+5> configuration exhibited throughout “Genen;” however, it makes more sense analytically to label the set using the symmetrical form for reasons that will be discussed later in the analysis.

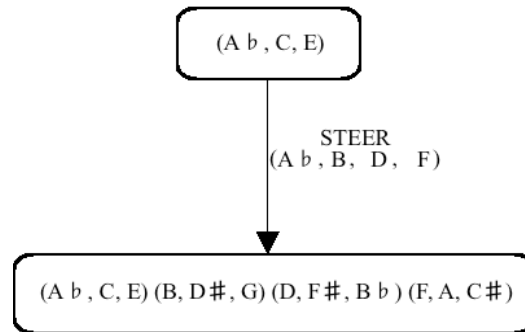
Further investigation of the right-hand sonorities reveals that it performs three more iterations of trichord XII through mm. 2–11, which when arranged appropriately result in the “chromatic tonality” XII/X. The material in the right-hand part may then be expressed as the following formula:

$$\text{XIISTEERX} = \text{XII} \otimes \text{X}^4(8) = (\text{A}\flat, \text{C}, \text{E}) (\text{B}, \text{D}\sharp, \text{G}) (\text{D}, \text{F}\sharp, \text{B}\flat) (\text{F}, \text{A}, \text{C}\sharp)^{64}$$

Furthermore, a transformational network like the one depicted in example 3-1 can be generated to visually represent the STEER function shown above (see example 3-3). Note that, like example 3-1, the trichord and steering tetrachord share the same initial pitch class (A \flat).

⁶⁴ There is no need to indicate the minor and major configurations in this case because both the initial trichord (XII: 4+4) and the steering tetrachord (X⁴: 3+3+3) consist of only one interval class and are therefore symmetrical.

Example 3-3: Transformational Network depicting XIISTEERX = XII ⊗ X⁴ (8)

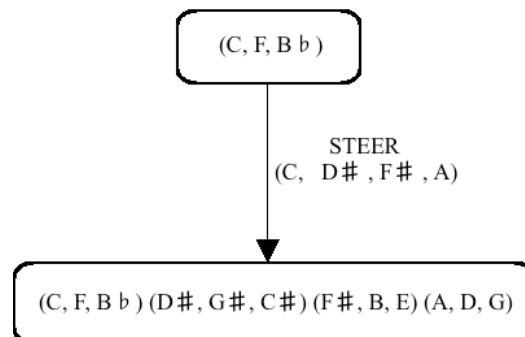


By extracting the trichordal sets in mm. 2-11 of the left-hand part, one can conclude that it unfolds a steering of trichord IX by X, or the “chromatic tonality” IX/X. This arrangement is expressed by the formula:

$$\text{IXSTEERX} = \text{IX} \otimes \text{X}^4(0) = (\text{C, F, B } b) (\text{D}\#, \text{G}\#, \text{C}\#) (\text{F}\#, \text{B, E}) (\text{A, D, G})$$

A transformational network that conveys the above formula is shown in example 3-4 below.

Example 3-4: Transformational Network depicting IXSTEERX = IX ⊗ X⁴ (0)



The analyzed excerpt (mm. 1–11) is reproduced in example 3-5 with the trichordal subsets of each pitch-class set circled and labeled with circled integers. The circled integers indicate the order in which the trichords appear in each chromatic tonality. For example, the first trichord of the XII/X tonality, (A \flat , C, E) in the right hand, is indicated by the adjacent ① in m. 2, as is the first trichord of IX/X in the left-hand part.

The same labeling system will be used for the remainder of the analysis to indicate the order of the constituent trichordal subsets of pitch-class sets being discussed.

Example 3-5: Tonality XII/X and IX/X in mm. 1–11

On the second page of the score, the XIII/X set introduced on the previous page is transferred to the left hand and then back to the right hand in mm. 12 and 14, as indicated by the arrows between barlines and across the left and right-hand parts in example 3-6.

(Arrows are used in the score to signal a change of voice for a given pitch-class set.)

Example 3-6 shows that the VIII/X set undergoes several changes of voice. The left hand simultaneously proceeds to cycle through three transformations of the IX/X set beginning in m. 14, which persists through mm. 20 and 25.

Example 3-6: “Genen” piano mm.12–25

Schat introduces a new “tonality” in mm. 12–13 in the piano’s right-hand part (see example 3-6). The steering of this new tonality could be interpreted in one of two ways because both steerings yield identical trichordal subsets, though in distinctive configurations. The set could be identified as either VIII/IV or as VIII/XI, and the pitch-class sets are expressed as the formulae below:

$$\text{VIII/STEERIV} = \text{VIII}(\text{mMmM}) \otimes \text{IV}((\text{e})\text{m}^4) = (\text{B}, \text{C}\#, \text{F}) (\text{E}\flat, \text{G}, \text{A}) (\text{E}, \text{F}\#, \text{B}\flat) (\text{A}\flat, \text{C}, \text{D})$$

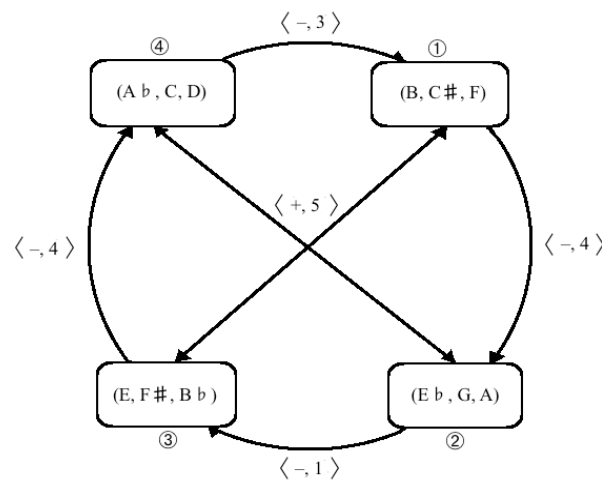
$$\text{VIII/STEERXI} = \text{VIII}(\text{mMmM}) \otimes \text{XI}((\text{4})\text{m}^4) = (\text{E}, \text{F}\#, \text{B}\flat) (\text{A}\flat, \text{C}, \text{D}) (\text{B}, \text{C}\#, \text{F}) (\text{E}\flat, \text{G}, \text{A})$$

The VIII/IV and VIII/XI sets, along with their respective steerings, may be more closely examined in the transformational networks provided in example 3-7. Clearly both sets consist of identical trichords, however the steering hour changes how they are derived. In example 3-7(a) they follow an interval-steering pattern of <4+1+4>, whereas example 3-

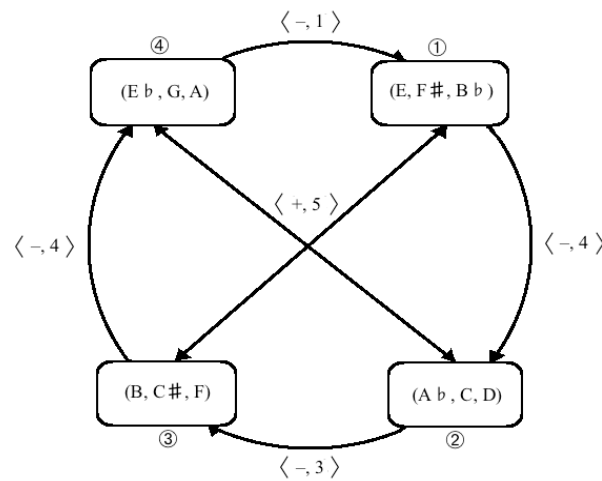
7(b) is generated through a $\langle 4+3+4 \rangle$ configuration, therefore permitting two legitimate interpretations. This raises an important question: Why would the analyst prefer one interpretation over another? This leads one to consider how the composer originally conceived of it and if the composer's intentions should influence the analytical process.

Example 3-7: Transformational networks of VIII/IV and VIII/XI

(a) VIII/IV



(b) VIII/XI



Through an examination of Schat's sketches for "Genen," it becomes evident that he considered this pitch-class set as VIII/IV (see example 3-8). My preliminary analysis, however, was conducted without Schat's sketches and because the first trichordal subset that emerges in the music is the first of a VIII/XI steering, I concluded that this was the most convincing interpretation. I, therefore, proceeded under this assumption and it allowed me to produce some substantial analytical observations. It was interesting, then, to find that Schat had conceived of this set differently. However, there is nothing in his sketches that suggest the VIII/IV steering is any more valid or significant than a VIII/XI steering. That is, he did not exploit its potential to generate further material or to unify the piece in any way. This opens the possibility to have more than one interpretation of the set, since both steerings exist within the same set. In essence, the two steerings complement one another and permit the analyst to exploit the dual steering nature to highlight different aspects of a composition. I have chosen to preserve my original interpretation of the set as VIII/XI because it can better illustrate the compositional unity found in "Genen." Further discussion regarding compositional unity and the significance of the VIII/XI set will come later in the analysis. Furthermore, the first trichord of the right-hand part in m. 12 is the first trichordal subset of the VIII/XI steering, which strengthens my position. It is also worth noting that all of the pitch-class sets discussed thus far have started with the first trichordal subset from which the set is derived. This property shared by all three pitch-class sets strengthens my argument for preferring the VIII/XI interpretation to that of VIII/IV, as the VIII/XI interpretation follows the same trend. My analysis will therefore identify these sets as VIII/XI.

Example 3-8: Schat's sketches of VIII/IV steering⁶⁵



The same VIII/XI set transpires in the violin part in mm. 18–20, though the trichordal subsets are used in a different configuration, as indicated by the circled integers (see example 3-9). A transposition of the VIII/XI set occurs in mm. 22 and 24 of the right-hand piano part following an identical rhythmic configuration as the initial set in mm. 12–13. The transposed set is also present in mm. 23–24 of the violin melody (see example 3-10). The trichordal subsets are derived using the formula:

$$\text{VIII STEER XI} = \text{VIII}(\text{mMmM}) \otimes \text{XI}((7)\text{m}^4) = (\text{G}, \text{A}, \text{C}\#) (\text{B}, \text{D}\#, \text{F}) (\text{D}, \text{E}, \text{G}\#) (\text{F}\#, \text{B}\flat, \text{C})$$

It then becomes evident that the VIII/XI set in m. 22 has undergone a T_3 transformation in relation to the initial VIII/XI set in mm.12–13 (see example 3-11). This transposed set is referred to and labeled as VIII/XI (B), whereas the initial set is VIII/XI (A).

Example 3-9: VIII/XI set – violin mm.18–20



⁶⁵ Sketches are courtesy of Nederlands Muziek Instituut, The Hague, The Netherlands.

Example 3-10: VIII/XI set – violin and piano mm.22–24

The image shows a musical score for violin and piano. The violin part (top staff) has a melodic line with circled trichordal subsets and circled integers 1, 2, 3, and 4. The piano part (bottom two staves) has a rhythmic pattern of chords with circled trichordal subsets and circled integers 1, 2, 3, and 4. Dynamics include *mf* and *p*. The score is labeled VIII/XI (B) in three places.

Example 3-11: T_3 transformation of set VIII/XI (A) to VIII/XI (B)

$$(E, F \#, B \flat) (A \flat, C, D) (B, C \#, F) (E \flat, G, A) \xrightarrow{T_3} (G, A, C \#) (B, D \#, F) (D, E, G \#) (F \#, B \flat, C)$$

Two more iterations of VIII/XI sets occur on the third page of the score in mm. 25–28 in the violin part (see example 3-12). The sets are labeled VIII/XI (C) and (D) respectively and the trichordal subsets are again circled and marked by circled integers for easy identification. A second manifestation of sets (C) and (D) occurs in the right-hand piano part in mm. 28–30 following the same rhythmic pattern as sets (A) and (B), thereby strengthening the association between all four sets. The sets VIII/XI (C) and (D) are represented by the following formulae:

$$(C) \text{ VIII STEER XI} = \text{VIII}(mMmM) \otimes \text{XI}((t)m^4) = (B \flat, C, E) (D, F \#, G \#) (F, G, B) \\ (A, C \#, D \#)$$

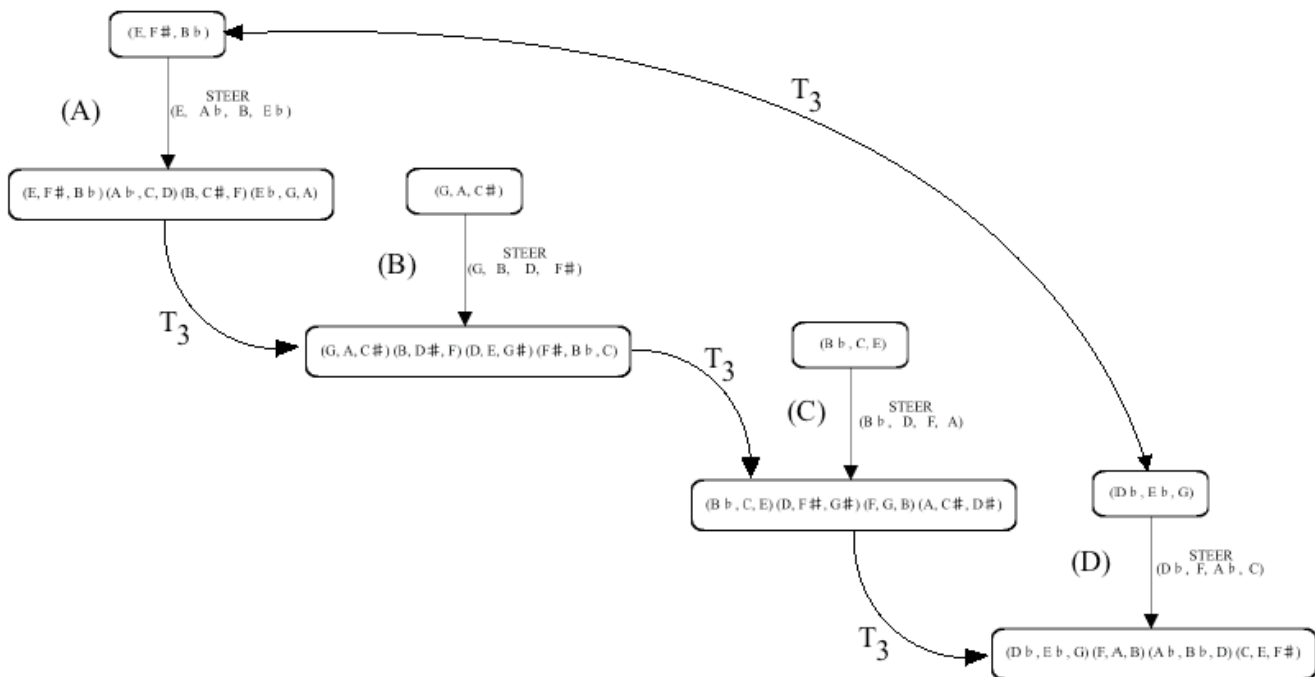
$$(D) \text{ VIII STEER XI} = \text{VIII}(mMmM) \otimes \text{XI}((1)m^4) = (D \flat, E \flat, G) (F, A, B) (A \flat, B \flat, D) \\ (C, E, F \#)$$

Each VIII/XI set is consistently transposed by three semitones, suggesting that they are steered on a second level by hour X <3+3+3>. A transformational network showing four different iterations of the VIII/XI set transformed by T_3 is provided in example 3-13, wherein each musical object is a VIII/XI set represented by a transformational network like the ones discussed earlier in the chapter.

Example 3-12: Two VIII/XI sets played by the violin in mm. 25–28



Example 3-13: Transformational network showing T_3 transformations of VIII/XI sets



Example 3-13 shows that the initial VIII/XI set (A), which begins with the trichord (E, F #, B b), is transformed through three transpositions throughout mm. 12–30. This phenomenon clearly implies a second-level steering by hour X that is expressed as VIII/XI/X. The second-level steering and surface-level steering must be realized as two formulae, which are shown and expanded in example 3-14. Firstly, the XI⁴ tetrachord is steered by the X⁴ tetrachord, XISTEERX, resulting in a set with sixteen pitch classes that can be partitioned as four tetrachordal subsets. The tetrachordal subsets can then in turn act as steering notes for trichord VIII to generate the VIII/XI sets discussed earlier. For example, using the first tetrachordal subset in example 3-14, (48e3), generates the set VIII/XI (A) (the full set can be viewed in example 3-7(b)).⁶⁶ So as to not become completely detached from actual musical depictions of these pitch-class sets, example 3-15 shows the transformational network in example 3-13 and the formulae discussed above as conventional musical notation. Open noteheads with stems beamed and pointing down depict the hour X steering, while the smaller beamed open noteheads pointing up represent the tetrachordal subsets of the XI/X set; the latter therefore become the steering notes of the VIII/XI sets. The filled-in noteheads represent the VIII trichord and its various transformations.

⁶⁶ The pitch-classes (48e3) serve as the starting pitch classes for the trichordal subset.

Example 3-14: XISTEERX and VIIISTEERXI expanded

$$\begin{aligned}
 \text{XISTEERX} &= \text{XIM}^4 \otimes \text{X}(4)^4 \\
 &= (047e) \otimes 47t1 \\
 &= 4(047e) 7(047e) t(047e) 1(047e) \\
 &= (48e3) (7e26) (t259) (1580) \\
 &= (E, A \flat, B, E \flat) (G, B, D, F \sharp) (B \flat, D, F, A) (D \flat, F, A \flat, C)
 \end{aligned}$$

$$\begin{aligned}
 \text{VIIISTEERXI} &= \text{VIII}(mMmM) \otimes \text{XI}((4)m^4) \\
 &= \text{VIII}(mMmM) \otimes 48e3 \\
 &= \text{VIII} \otimes 4(m) 8(M) e(m) 3(M) \\
 &= 4(026) 8(046) e(026) 3(046) \\
 &= (46t) (802) (e15) (379) \\
 &= (E, F \sharp, B \flat) (A \flat, C, D) (B, C \sharp, F) (E \flat, G, A)
 \end{aligned}$$

Example 3-15: Musical notation depicting VIII/XI/X steering

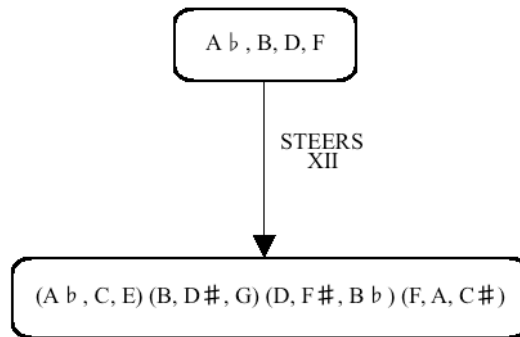


Three distinctive pitch-class sets representing three chromatic tonalities have been identified thus far: (1) XII/X, (2) IX/X, and (3) VIII/XI/X. The significance of hour X can not be overlooked, since it serves as the steering hour for three different sets, though not necessarily on the same structural level. It steers trichords IX and XII on the surface level and steers VIII/XI sets on a second level. Furthermore, the three different steerings that generate three different sets originate from three distinct tetrachords. This can be observed in examples 3-16 (a) through (c), which consist of three transformational networks that depict how the hour X tetrachords generate all the pitch-class set material

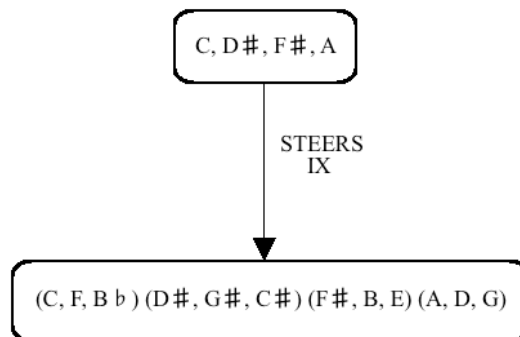
using the STEERS function. The first tetrachord in example 3-16(a), consisting of the pitch-classes $A\flat$, B, D, and F, STEERS trichord XII and generates the XII/X tonality. The IX/X set in example 3-16(b) is produced through the steering of trichord IX by the pitch-classes C, $D\sharp$, $F\sharp$, and A. Finally, the tetrachord E, G, $B\flat$, and $D\flat$ in example 3-16(c) steers the VIII/XI sets on a second level thereby generating four distinct, yet still interrelated, sets.

Example 3-16: X tetrachords generating sets XII/X, IX/X, and VIII/XI/X

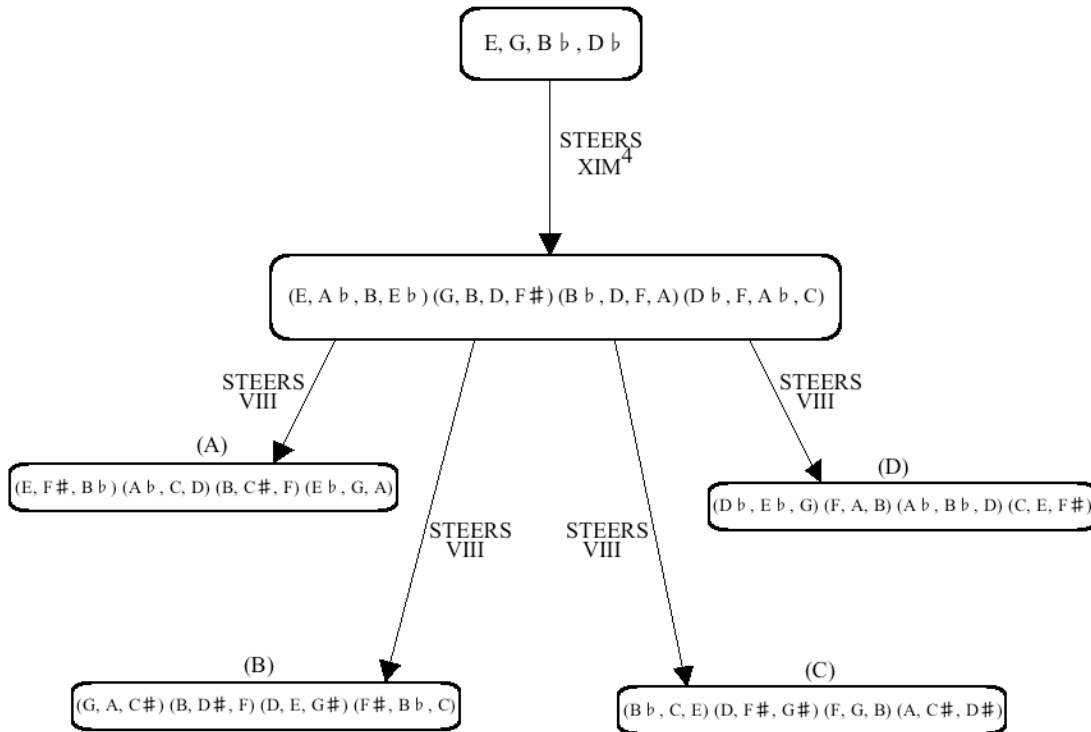
(a)



(b)



(c)



The majority of the material in the first forty-five measures of “Genen” can be traced back to the tetrachordal sets discussed above. It is appropriate then to question whether these three tetrachordal sets are used to generate additional material throughout the composition. Table 3-1 summarizes the tone-clock hours and their steerings from mm. 34–45 in all three voices prior to the introduction of new or varied material. It is important to note that only measures in which significant changes occur are included. Also, the configuration of the trichordal subsets can be traced using the circled integers provided below the steering label. The majority of the violin part material is generated from the set VIII/XI (A) introduced in example 3-7(b), though occasionally the material will mirror the IX/X set being played simultaneously by the piano. When examining the score one finds that the right-hand part juxtaposes the IX/X tonality with the XII/X

tonality that is simultaneously played by the left hand. For the sake of clarity only the IX/X tonality is included in the right-hand piano analysis. The simile marks indicate that the tonality in the previous column is still in effect.

Table 3-1: Tone-clock trichords and steerings mm. 34–45

Measure(s)	34	35	37	40	42	43	44–45
Violin	N.A.	VIII/XI (A) ④	IX/X ④	VIII/XI (A) ②	VIII/XI (A) ③	IX/X ③	VIII/XI (A) ①
Piano R.H.	IX/X ②	/	IX/X ④	IX/X ①	/	IX/X ③	/
Piano L.H.	XII/X ④	/	XII/X ②	XII/X ③	/	XII/X ①	/

A variation of the IX/X tonality occurs in mm. 46–55 in the right-hand part. Rather than continue using a set identical to the previous forty-five measures, Schat varies the manner in which the IX/X tonality is derived in m. 46. Whereas the initial IX/X set is generated through the (C, D #, F #, A) tetrachord⁶⁷, this variation uses the (A b, B, D, F) tetrachord⁶⁸ and must therefore be labeled IX/X (B) (see example 3-17). In other words, the entire set undergoes a T₄ transformation. The XII/X sets present in the left hand are interspersed with the IX/X sets in the right hand, like in mm. 34–45. Each manifestation of the IX/X set is circled and the first iteration of each subset is labeled with a circled integer to denote its configuration in the set.

⁶⁷ This set is the diminished-seventh chord and therefore only has three distinct forms.

⁶⁸ This set uses the enharmonically equivalent pitch-class G# rather than A b.

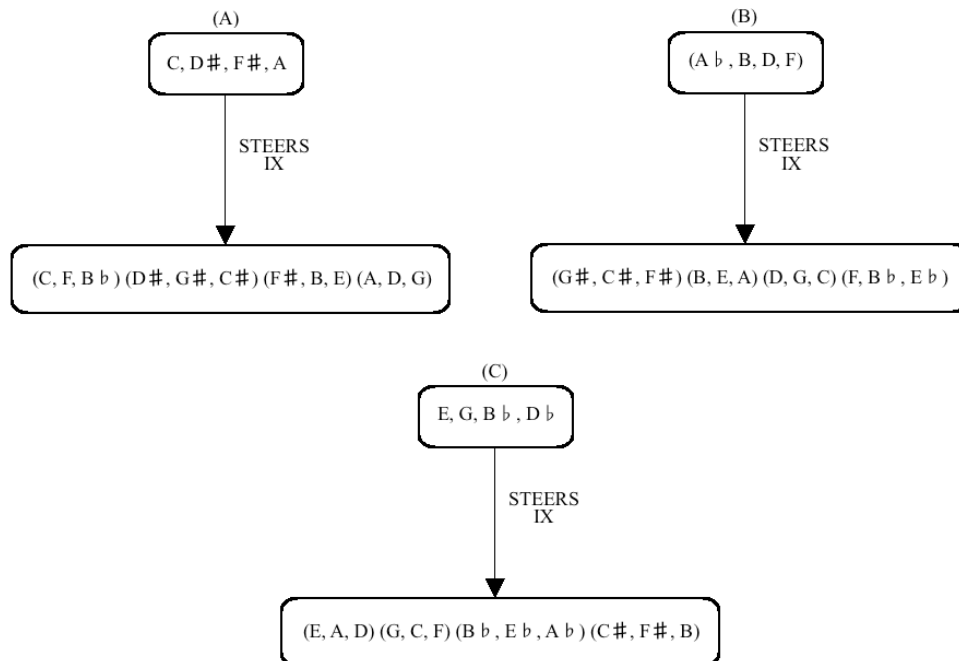
Example 3-17: IX/X (B) in mm. 46–52 derived from (G #, B, D, F)

A second variation of the IX/X tonality, hereafter IX/X (C), occurs in the right-hand piano part in mm. 60–66 wherein the steering notes are again transformed via T_4 (see example 3-18). As a result, the steering notes are transformed to E, G, B \flat , and C \sharp : the same pitch-class set as the third tetrachord discussed above that generates the tonality VIII/XI/X. In other words, the three steering tetrachords shown in example 3-16 are used to derive three IX/X sets (see example 3-19). This brings the number of total sets derived from these tetrachords to eight: one XII/X set, four VIII/XI/X sets, and three XI/X sets. Furthermore, the IX/X sets are steered on a second level by hour XII <4+4>, the implications of which will be discussed in section 3.3.

Example 3-18: IX/X set in mm. 60–66 derived from (E, G, B \flat , C \sharp)⁶⁹

The musical score for Example 3-18 consists of two systems. The first system is labeled 'IX/X(C)' and 'XII/X'. It features a piano part (p) and a harpsichord part (mp *espr.*). Circled chords in the piano part are numbered 2, 4, and 1. The second system starts at measure 64 and contains circled chords numbered 3.

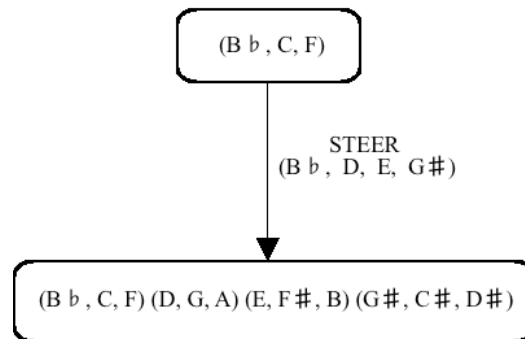
Example 3-19: IX/X sets derived from X⁴ tetrachords



⁶⁹ This set uses the enharmonically equivalent pitch-class C \sharp rather than D \flat .

The connection between the three tetrachordal sets listed above and the role they play in generating the IX/X sets would not have been possible had the hour IX trichords not been initially categorized as the symmetrical form <5+5>. If the asymmetrical forms of trichord IX – <2+5> and <5+2> – had been used to derive the tonality in m.1, the resulting set would have consisted of a trichord IX steering by hour VIII <4+2+4> with the steering notes B♭, D, E, and G♯ (see example 3-19). Consequently, the steering notes of the ensuing variations would also be modified and therefore no longer be consistent with the tetrachords identified in example 3-16. This inconsistency ultimately disrupts the potential for greater compositional unity throughout the piece. Though the content of the constituent trichords in example 3-20 is identical to that of example 3-4, the steering notes no longer coincide with the other X⁴ tetrachords identified as steering hours. The symmetrical form of trichord IX is therefore preferred to the asymmetrical form because it corresponds with the X⁴ steering of the other sets. This phenomenon thereby permits a higher degree of compositional unity because the majority of the pitch-class set material can be traced back to three progenitor sets.

Example 3-20: Transformational network depicting IX/VIII set



The tetrachordal sets as generators of pitch-class material remain significant in the final twenty-five measures of the score. For instance, a second steering of trichord VIII <2+4> by III⁴ <1+3+1> occurs on page fourteen and fifteen in mm. 179–182 (see example 3-21). This set is expressed by the formula:

$$\text{VIII STEER III} = \text{VIII}(\text{mmMM}) \otimes \text{III}((\text{e})\text{m}^4) = (\text{B}, \text{C}\#, \text{F}) (\text{C}, \text{D}, \text{F}\#) (\text{E}\flat, \text{G}, \text{A}) (\text{E}, \text{A}\flat, \text{B}\flat)$$

A contrapuntal texture is created by the right and left-hand parts, which consist of the same VIII/III subsets as the violin reproduced as chords and by a XII/X set, respectively. Initially, it seems that the VIII/III set cannot be derived from the X⁴ tetrachords, however, further examination of the score demonstrates that this is not the case. In fact, the set can relate to the XII/X set that accompanies it in the left hand, as will be demonstrated shortly.

Example 3-21: VIII/III and XII/X sets in mm. 179–182

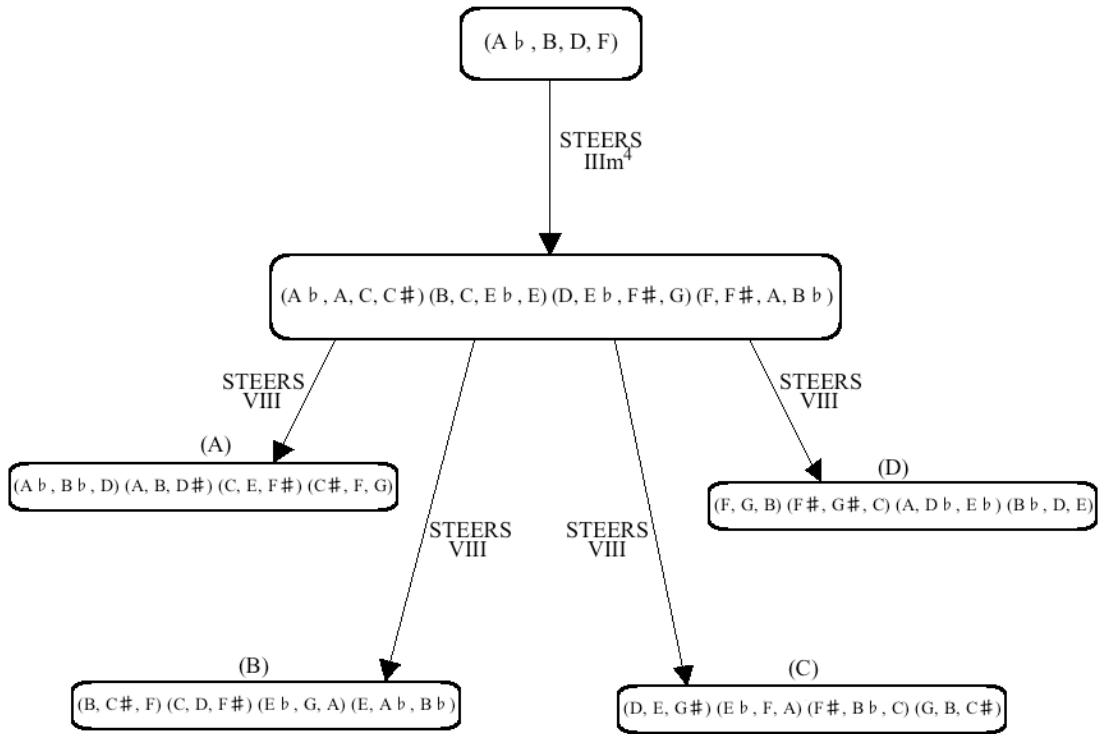
An excerpt from mm. 189–194 is provided in example 3-22, which shows the piano now divided into three voices. The two upper staves consist of a series of VIII/III sets, while the lower staff continues to unfold a XII/X set interspersed with dyads of interval-class 4, which could be classified as hour XII dyads. Once more, trichordal subsets are circled and labeled with circled integers. Brackets above the staves serve to

distinguish between the various VIII/III sets, which are also identified alphabetically from A to D. The content of the first bracketed section in m.189 is identical to the VIII/III set in example 3-21, hereafter VIII/III (A). The next bracket contains the set VIII/III (B) which has undergone a T_3 transformation. Applying successive T_3 transformations derives the content of subsequent bracketed sections consisting of sets C and D. In other words, the VIII/III set is steered on a second level by X^4 . A transformational network depicting the two levels of steering is provided in example 3-23, wherein the tetrachord responsible for the second-level steering (A \flat , B, D, F) is identical to the tetrachord steering the XII/X set that accompanies the VIII/III sets. In other words, the sets have identical underlying steering notes. Though seemingly an isolated set, the VIII/III set in mm. 179–182, in fact, results from a second-level steering by X^4 , as demonstrated by the music in mm. 189–194 showing that it is derived from the steering notes of the XII/X set at the start of the piece.

Example 3-22: VIII/III and XII/X sets in the piano mm.189–194

The image shows a musical score for piano, measures 189-194. The score is divided into four sections labeled (A), (B), (C), and (D). The top staff is labeled VIII/III and the bottom staff is labeled XII/X. Arrows labeled T_3 indicate transformations between sections. Circled numbers 1-4 highlight specific notes in the VIII/III set.

Example 3-23: Transformational network depicting VIII/III/X



As previously discussed, three IX/X sets in example 3-19 are individually steered by the X⁴ tetrachords. I have proposed that these tetrachords are steered on a second level by hour XII, which has further implications for this analysis. By observing the pitch classes of the three tetrachordal sets steered by XII, it becomes evident that they produce a X/XII tonality that consists of twelve distinct pitch classes from which all the material discussed thus far is derived (see example 3-24). Therefore, this set can be identified as the primary generator of pitch-class sets for the majority of “Genen” and, consequently, the main underlying structure. The network in example 3-24 depicts the three tetrachordal sets steered by XII <4+4>, while table 3.2 shows the X/XII tetrachordal subsets and the pitch-class sets they generate at the surface level and second level. The bracketed (4)

indicates that the second-level steering generates four sets at the surface level. The total number of sets derived by the X⁴ tetrachords has now reached twelve.

Example 3-24: Transformational network showing tonality X⁴/XII⁷⁰

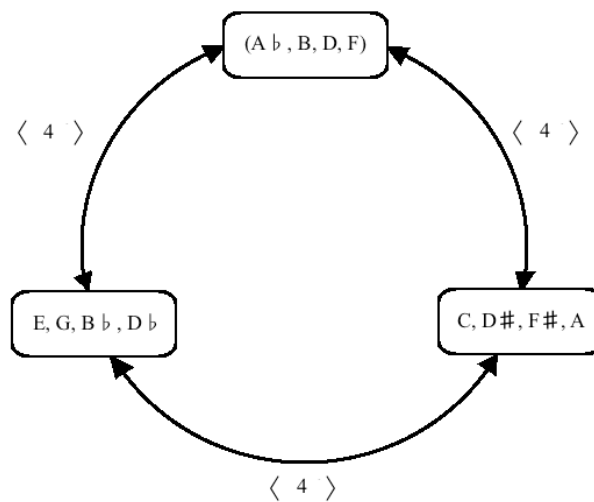


Table 3-2: Tetrachordal subsets and the pitch-class sets they generate

X/XII Tetrachords	Surface-level Steering		Second-level Steering
(A♭, B, D, F)	XII/X	IX/X (B)	VIII/III/X (4)
(C, D♯, F♯, A)	IX/X (A)		
(E, G, B♭, D♭)		IX/X (C)	VIII/XI/X (4)

Though it may seem that the violin melody in mm. 1–11 has been overlooked thus far, it is simply because it was not yet possible to present a meaningful analysis in terms of its relationship to other pitch-class sets and the overall unity of “Genen.” Being aware of the existence of a deep-level steering by hour XII now gives an analysis of the main

⁷⁰ No plus or minus is necessary to denote the mode of the UTTs because hour X appears as three symmetrical tetrachords and therefore does not carry a major or minor classification.

theme significance that would not have been evident otherwise. The material in the opening measures is more ambiguous than that which has been analyzed up to this point primarily because the subsets are not as easily identified since they are not organized as simply as in previous examples.

Schat's sketches of the pitch-class sets and motivic material that most closely resemble the main theme are included in example 3-25, but do not provide an easy solution to establish a strong relationship between the violin melody and the remainder of the work. It is clear that the set in example 3-25 denotes a II/XII tonality which is then used to produce the melody in the lower part of the sketch. Notice that the melody emphasizes the first pitch class of the preceding subset with longer note values. In other words, the tied half-note E \flat is the first pitch class of the second subset, the G is part of the third subset, and the final-pitch class B is a return to the first subset. However, in practice Schat does not restrict himself to the simplicity of the sketch and opts for a more developed theme. This presents some difficulties for the analyst because the material does not adhere to the II/XII set shown in the sketch. A strong argument can be made for a slightly more eloquent interpretation of the main theme that is also more analytically persuasive and logical in terms of the set from which it is derived.

Example 3-25: Schat's sketches of the main theme: tonality and melody⁷¹



By altering the sketch, Schat produces a theme with greater complexity, resulting in material that makes a straightforward analysis more difficult (see example 3-26). The subsets of the II/XII set, shown in the sketch, are circled in example 3-26 so that they may be located more easily. When compared to the original sketch of the theme, it is obvious that Schat added a considerable amount of material. For example, an added C \sharp occurs in m. 1, as well as an E \sharp in mm. 5 and 8; however, one could argue that the C \sharp is implied in the original sketch because Schat rarely uses naturals to cancel a previous accidental, even within the same measure. Similarly, the II/XII set in the sketch does not account for the appearance of A \flat /G \sharp in mm. 8–10, a feature that I still cannot explain other than to label them, as well as the C and E \sharp , as passing tones. This will be discussed in greater detail shortly. Furthermore, the sketch does not account for the overlap of certain subsets that occur in mm. 2–5 and mm. 6–10. I would like to propose that a different interpretation can better account for the material in the main theme.

⁷¹ Sketches are courtesy of Nederlands Muziek Instituut, The Hague, The Netherlands.

Example 3-26: “Genen” main theme mm. 1–11



The most convincing point for the argument that a different set plays a key role here involves the notes of the opening motive, which span mm. 1–2. Rather than identify the tied half-note $E\flat$ as a constituent of the ensuing subset, it would seem more logical to attribute it to the same subset as the pitch classes that precede it, especially since the same pitch classes persist in mm. 3 and 5. The resurfacing of the unaltered motive several times throughout “Genen” provides further support for a different interpretation. Another four manifestations of the opening motive are presented in example 3-27, none of which are followed by the second subset ($E\flat$, F , $F\sharp$). This phenomenon leads me to reconsider the $E\flat$ as a member of the first subset, thereby changing it from the trichord (B , $C\sharp$, D) to the tetrachord (B , $C\sharp$, D , $E\flat$). As a result the intervallic configuration changes from $IIM\langle 2+1\rangle$ to $ISPIIM^5\langle 2+1+1\rangle$.⁷² Consequently, this modification alters the configuration of the other subsets and ultimately the entire set, though the steering notes remain unchanged (see example 3-28). The parenthesized notes represent the aforementioned passing tones that are present in the opening theme. This interpretation is supported by the other tied half notes, G in m.7 and B in m.11, of the main theme, which are constituents of the second and third subsets in example 3-28, respectively.

⁷² ISP is the acronym used for incomplete symmetrical pentachord. Therefore, the $\langle 2+1+1\rangle$ tetrachord is derived from the first three interval classes of symmetrical pentachord $IIM^5\langle 2+1+1+2\rangle$.

Furthermore, the tied half notes can be interpreted as a gateway between subsets because the final note of any given subset is the same as the first of the subsequent subset.

My interpretation can also account for the consistent subset overlap that occurs in mm. 8–10, an element that is not possible if we analyze this material based on the II/XII set. One could argue that because subsets and the pitch classes contained therein may come in any order that the material of the theme could be attributed to the II/XII set. This would violate, however, the essence of whatever tonality is in use. If the subsets may be segmented and used in any way imaginable, then the resulting material becomes too chaotic and cannot be easily traced back to the set from which it is derived, essentially defeating the purpose of generating sets using the tone-clock system.

Example 3-27: Four manifestations of the opening motive throughout “Genen”

(a) mm. 13–15



(b) mm. 30–33



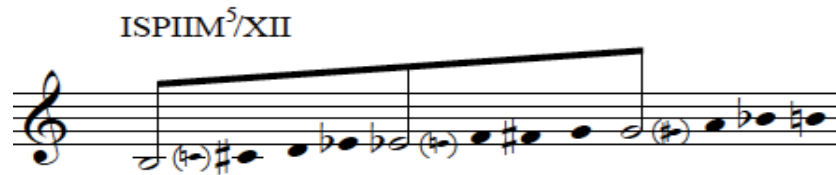
(c) mm. 56–57



(d) mm. 130–132

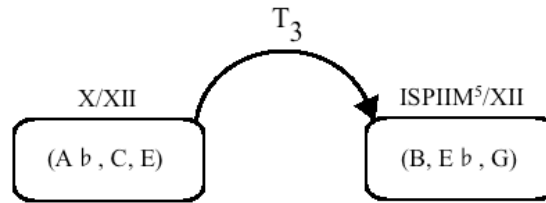


Example 3-28: ISPIIM⁵/XII



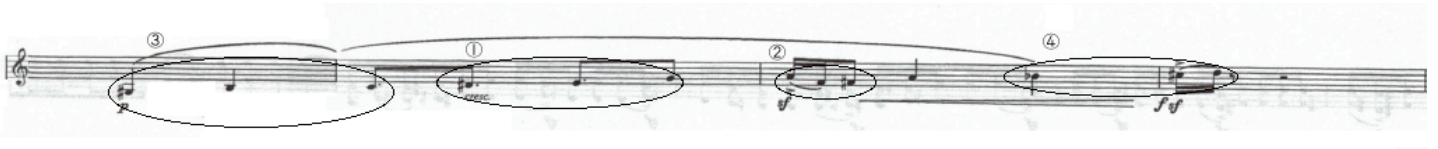
How, then, do the steering notes of the ISPIIM⁵ set relate to what has been identified as the progenitor set, X/XII? I have already established that hour X plays a significant role in generating material throughout “Genen,” identified in my previous analyses as the steering hour for twelve sets. This means that all sets contain T₃ transformations at some structural level. Not only does the ISPIIM⁵ set share the same steering hour as the progenitor set, that is, hour XII, but the steering notes result from a T₃ transformation of the X/XII steering notes (see example 3-29). The steering notes of the X/XII set (A^b, C, E) are derived from the tetrachordal sets discussed previously (refer to table 3-2). They are arranged according to the order in which they appear in “Genen” and are then transposed by three semitones to produce the ISPIIM⁵ steering notes. This process again emphasizes the significance of T₃ transformations for the piece.

Example 3-29: T_3 transformation of X/XII steering notes



The question is, then, do the steering notes of the ISPIIM⁵ set have greater significance in “Genen?” In other words, do they recur elsewhere as members of a different set or are they perhaps a generator that produces numerous sets? In either case, there is an increased level of overall unity for the piece. In fact, an identical transformation has already occurred in an earlier set. The XII/X set explored at the outset of this analysis contains this very transformation as the first trichordal subset (A \flat , C, E) is transformed into (B, D \sharp , G). Therefore, the second trichord of the XII/X set is responsible for generating the ISPIIM⁵ set. Further investigation of Schat’s sketches leads to the conclusion that the same pitch classes that steer the ISPIIM⁵ set operate as generators of additional sets. The (B, D \sharp , G) set is used again to steer a III/VII set on a third level (see example 3-30). Example 3-30(a) depicts set III/VII/XII(A), wherein the pitch-class B begins the (B, E \flat , G) steering of III/VII. The trichords are themselves responsible for steering a series of dyads with the configuration <2-4-2>. This intervallic configuration could be identified as representing hour VIII dyads, henceforth labeled as VIII², wherein the superscript 2 denotes a dyad associated with a select tone clock trichord. Example 3-30(b) features sketches of the three VIII²/III/VII/XII sets derived through the B, E \flat , and G trichord, which can be found by isolating the first pitch classes of the trichords that occur in the third position labeled by the circled integer ① below set

Example 3-31: Violin melody consisting of set VIII²/III/VII/XII (A) mm.183–186



3.3: Concluding Remarks

Numerous pitch-class sets have been identified and examined throughout my analysis of “Genen;” however, my main objective was not to simply to highlight such sets. Rather, I have focused primarily on the manner in which the sets are derived through the steering principle and ultimately how steering contributes to the overall unity of the piece. Table 3.3 includes pitch-class sets with the appropriate steering level in relation to the progenitor set (X/XII); this is accompanied by a graphic representation of the data in the form of a transformational network that applies the STEERS function in example 3-32. Table 3-3 classifies the steering levels according to the top-down orientation of the network presented in example 3-32, essentially providing a summary of all the sets explored in the analysis and showing how they are derived. The first column consists of a set, the second column provides the set used to derive the set in the first column, and the third column describes the hierarchical relationship of the set in the first column to the progenitor set in terms of steering. The levels here differ from the steering levels discussed earlier, since the levels in the table do not necessarily relate to how the sets appear in the music. In other words, sets that are on different levels could both be classified as surface-level steering depending on how they are used in the score. For instance, in the music, set XII/X appears as a surface-level steering of the progenitor set, as does set VIII/XI (A). However, they exist on different levels in relation to the

progenitor set according to example 3-32. That is, VIII/XI (A) is in fact the surface-level steering of XIM^4 , which is steered on a second level by the progenitor set.

Table 3-3: “Genen” sets and their associated steering levels

Set	Steering Set	Steering-level relation to progenitor set
XII/X	X/XII	First level
IX/X (A)	X/XII	First level
IX/X (B)	X/XII	First level
IX/X (C)	X/XII	First level
XIM^4/X	X/XII	First level
VIII/XI (A)	XIM^4/X	Second level
VIII/XI (B)	XIM^4/X	Second level
VIII/XI (C)	XIM^4/X	Second level
VIII/XI (D)	XIM^4/X	Second level
$VIIIm^4/XII$	XII/X	Second level
III/VII (A)	$VIIIm^4/XII$	Third level
III/VII (B)	$VIIIm^4/XII$	Third level
III/VII (C)	$VIIIm^4/XII$	Third level
ISPIIM ⁵ /XII	$VIIIm^4/XII$	Third level

The top node of the transformational network in example 3-32 contains the progenitor set, from which of all other sets discussed in the analysis may be traced through the STEERS function; numerous arrows mark these relationships. The subsets found in the upper nodes become steering notes for the subsequent sets found below. The example also shows that the progenitor set may be referred to as the underlying principal

tonality of the piece. Therefore, the X⁴/XII set is responsible for steering at the deepest level and generates a variety of other distinct, yet interrelated, sets.

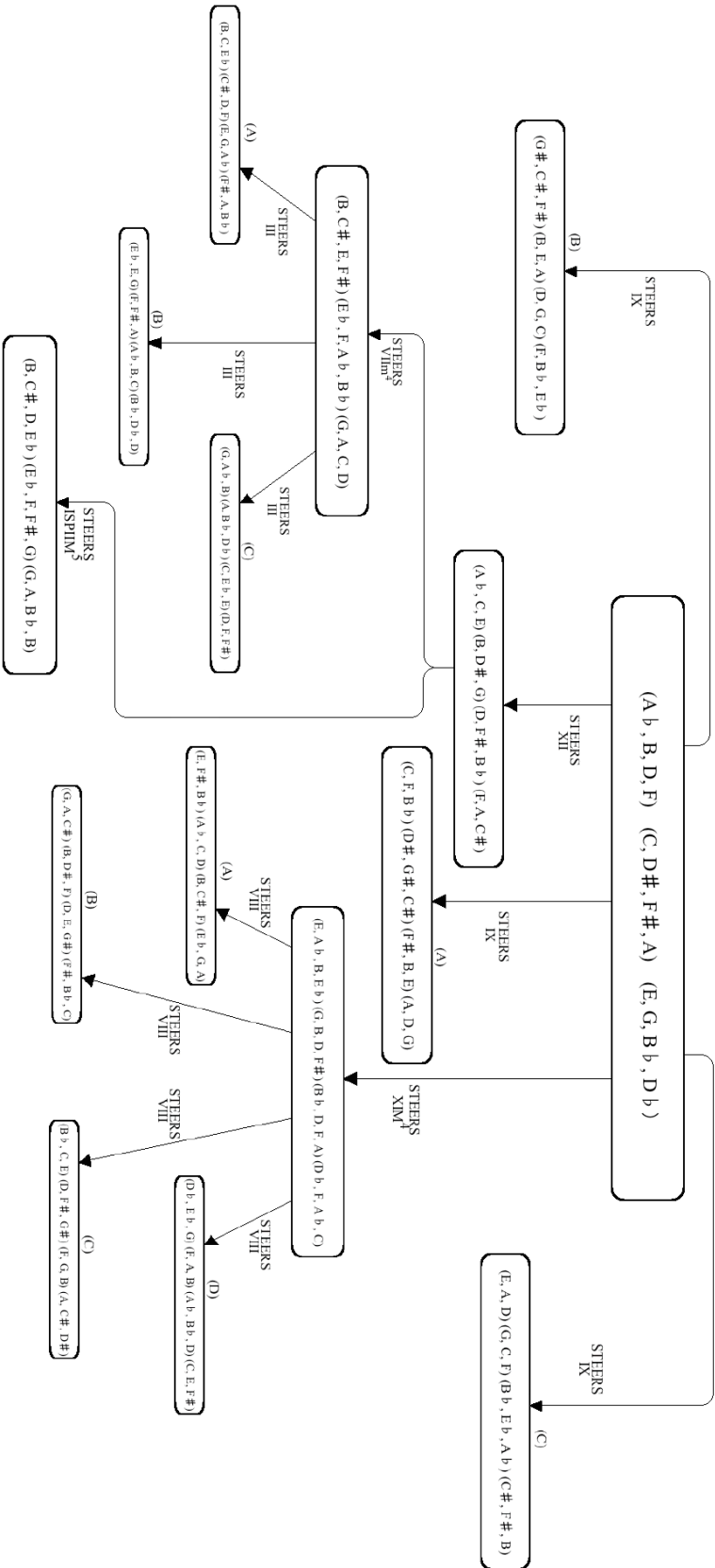
The network in example 3-32 aligns well with McLeod's observations regarding the properties of the steering principle:

It is the combined musical principles of growth and reproduction: i.e. the idea that any given note has the power to generate, or give birth to, a group of notes (an interval, triad, tetrad, etc.) - coupled with the idea of transposition, the power of a group of notes to reproduce replicas of itself elsewhere in the system.⁷⁴

The composite network demonstrates how a subset of a “tonality” can reproduce through steering to generate larger sets and whose subsequent subsets can in turn reproduce to generate further material. As McLeod proposes, any note has the potential to generate a collection of notes. In the case of this analysis, any set can be transformed into the steering notes of a larger set. It is this property of the steering function that allows it to have tremendous influence over the unity of a composition. If we interpret the network in example 3-32 as an organic organism such as a tree, the underlying X/XII set can be understood as the roots and trunk, and the sets it generates as the branches and leaves that sprout from the trunk. The potential to sprout new branches, or sets, is virtually limitless and, despite how isolated the new sets may initially appear, they can all be traced back to the same root.

⁷⁴ Peter Schat, “VIII: Clockwise” *Peter Schat's Personal Website*. <http://www.peterschat.nl/clockwise.html>

Example 3-32: Composite transformational network of "Genen" sets



The model presented throughout the course of this study is not representative of all of Schat's compositional output using the tone clock. My primary objective was to demonstrate the steering principle's undeniable capacity for producing a highly unified post-tonal composition. In "Genen," this becomes evident when one traces the origins of several sets appearing at the surface level to one of three tetrachordal subsets that are constituents of the progenitor set. In other words, the tetrachordal subsets can be identified as steering notes at some hierarchical level. Once aware of the possibility that sets may be somehow linked at a deeper level through steering, analysts exploring other tone-clock compositions can attempt to discover similar connections. Such connections may not necessarily exist to the same degree as they appear in "Genen," but I am convinced that similar relationships influenced by the steering principle will likely emerge. Nevertheless, Schat's concept of steering as a method of pitch-class transformation and reproduction remains an innovative and creative way of unifying a composition in post-tonal music. Schat's tone-clock works merit further investigation because his system is a viable and effective method of post-tonal composition.

Chapter 4: Conclusion

4.1: Synthesis

The main goal of this thesis was to explore the theoretical aspects of the tone clock as a compositional tool to ultimately illustrate how one of its fundamental principles, steering, could be used to produce a post-tonal work with a high degree of unity. Furthermore, I aimed to develop an analytical method by which one could discuss efficiently the role that steering has on various pitch-class sets throughout a musical work by adopting McLeod's intervallic prime forms (IPF). The method needed to clearly display each set and make the procedures used to derive them apparent. I included transformational theory as a part of the model because transformational networks serve as the optimal medium to present small-and large-scale connections throughout my analysis. I proposed mathematical formulae to convey more efficiently the steering procedure in the form of the STEER and STEERS functions, in combination with an adaptation of Hook's formula for uniform triadic transformations.

In addition to its potential as a compositional tool, the tone clock serves as a method for classifying and organizing the twelve possible trichordal pitch-class sets using McLeod's IPFs, and therefore provides an alternative to set theory taxonomy. In contrast to the prime forms in set theory, which organize trichordal sets using unordered pitch-class sets, the IPF system focuses on a trichord's intervallic structure, similar to a successive-interval array. McLeod notes that "Schat's numbering and order of the triads are exactly the same as Forte's, [though Schat] had no knowledge of or interest in set theory."⁷⁵ For Schat, the intervallic structure was crucial for the identification of a

⁷⁵ Jenny McLeod, *Tone Clock Theory Expanded*, 8.

trichord. The IPF system is also necessary to facilitate the analysis of tone-clock compositions because set theory cannot be used to explain the effects of the steering principle. That is, set theory could be used to identify a passage consisting of trichordal sets that share the same unordered intervallic structure; however, it would easily overlook the steering of those trichords by another set, which could have large-scale implications. McLeod also argues that set theory is far more abstract than Schat's theory by including integer notations that "make the groups too hard to distinguish from one another and thus on the whole [make them] impossible to remember,"⁷⁶ moreover, she considers the use of Forte's set numbers which cause one to "[lose] touch with their actual structure,"⁷⁷ another weakness of pitch-class set system, which her IPFs resolve. In her dissertation, Petrella further promotes the use of IPFs by outlining some performance implications for pianists playing a tone-clock piece. She proposes that "as [a] work becomes more familiar, the harmonies begin to be recognized by the ear, the eyes and the hand, and [that] Schat's system becomes a predictable series of harmonic progressions and combinations" in the same way that one recognizes commonly used musical gestures, such as cadences and progressions, in a Mozart Sonata.⁷⁸ A composer or analyst can achieve the same level of familiarity with the structure of pitch-class sets derived from the tone clock when they become accustomed to identifying them using IPFs.

Though Schat was not the first to segment twelve-tone rows into four trichords, he was the first to incorporate the notion of steering – a principle that allowed a second set

⁷⁶ Ibid., 4.

⁷⁷ Ibid., 5.

⁷⁸ Diana Helfers Petrella, "The Tone Clock: Peter Schat's System and an Application to his Etudes for Piano and Orchestra, Opus 39, a Lecture Recital, and Three Recitals of Selected Repertoire by Bartók, Clementi, Copland, Debussy, Griffes, Mozart, Liszt, Rachmaninoff, Ravel, and Schumann" (PhD diss., University of North Texas, 2002), 54.

to direct the transposition of the first. My analyses primarily focused on showing the processes involved in pitch-class set steering, thereby illustrating the manner in which sets are transformed. This was accomplished by comparing and contrasting Schat's steering technique with Boulez's pitch-class set multiplication technique, as the procedures of the latter influenced those of the former. Transformational networks were used to illustrate the various pitch-class set transformations that generated material throughout the analyzed excerpts, as were the newly proposed STEER and STEERS functions – mathematical representations of the steering process – and Hook's UTT formula. Using these analytical tools, I proceeded to show that numerous sets present throughout "Genen" could be traced to a single progenitor set as a result of various levels of steering. This analytical interpretation explained how Schat's work consists of several interrelated pitch-class sets. In other words, seemingly isolated sets appearing at the surface level could be linked to deeper-level structures and were therefore connected on a larger scale. This demonstrates the steering principle's, and by association the tone clock's, capacity for producing a unified post-tonal composition.

4.2: Future Explorations

Since flexibility is built into the tone-clock system, it could also be applied to repertoire other than that of the contemporary Western European art tradition. For example, Theo Hoogstins borrowed the tone clock to derive idiomatic jazz harmonies, typically seventh chords or chords with various extensions, to compose works in a jazz context. Example 4-1 shows four sets used to derive the harmonies in mm. 21–28 of his piece "La rivière souterraine." Each set consists of three tetrachordal subsets with the

intervallic configuration <2+3+2>, or VII^m⁴. The tetrachords are then steered by hour XII, and then on a second level by hour X. This steering combination yields a series of minor-eleventh tetrachords, of which the first three are interpreted as (using jazz chord symbols) D^m11/C, B^bm11/A^b, and F[#]m11/E.⁷⁹ The flexibility of Schat’s model allows it to be applied to this jazz work, leading to the conclusion that its potential to derive and analyze works of different genres seems limitless.

Example 4-1: VII/XII/X sets used in mm. 21–28 of “La rivière souterraine”⁸⁰



4.3: Concluding Remarks

The statement “art is no science – there is nothing to prove”⁸¹ sums up well Schat’s philosophy about composition. Schat insisted that the tone clock was not a compositional theory, nor was it an answer as to how one might compose great music, but was merely a tool or system one could use during the compositional process. The

⁷⁹ Tracking the root movement B^b, D, and F[#], or that of the lowest chord member, reveals the hour XII steering. Note that the minor-eleventh chords appear as third-inversion chords to accommodate the intervallic configuration of hour VII.

⁸⁰ Theo Hoogstins, “Chapter 4: Harmonies” *Peter Schat’s Tone Clock in jazz and improvised music*. <http://www.hoogstins.com/toonklok/artikel/harmo-e.htm>.

⁸¹ Peter Schat, *The Tone Clock*, 58.

tone-clock system's appeal is its inherent ability to generate coherent and consistent pitch-class sets, which Schat termed "chromatic tonalities," as well as its potential for producing deep structural connections through the steering principle. It also permits much more flexibility, in comparison to the rigidity of Schoenberg's twelve-tone serialism, in how one might use the constituents (trichords and pitch-classes) of the twelve-tone sets it produces. Most importantly, the tone clock provides composers with a means to express their creativity and individuality through a coherent system that can generate significant pitch-class set relationships that serve to greatly unify a composition.

“Art is no science is no politics – and vice versa. And I’m not a politician, or a scientist.

I’m just a composer.” —Peter Schat

Glossary

Asymmetrical tetrachord (AT): A set consisting of four pitch classes wherein the constituent interval classes do not form an intervallic palindrome. They occur in one of two forms: (1) when two hours that share an interval class are juxtaposed, as in $\langle 1+2+3 \rangle$ where interval-class 2 is shared by hours II $\langle 1+2 \rangle$ and VII $\langle 2+3 \rangle$, and (2) as a subset of a symmetrical pentachord (see also ISP), as in $\langle 1+4+4 \rangle$, the first four interval classes of the pentachord $IVm^5 \langle 1+4+4+1 \rangle$.

Atonical/tonical: Terms used by Schat in place of atonal and tonal to describe what he believes are two different approaches to post-tonal composition. An atonical composition contains no semblance of a central tone or tonic, while a composition that does reference a central pitch (tonic) is labeled as tonical. A composition may then be based on any pitch-class set and fall under the category of either atonical or tonical.

Chromatic tonality: The derivation of a twelve-note series through the transformation of a select tone-clock trichord. For Schat, a twelve-note series consisting of four transformations of a trichord constituted a tonality because he believed that the trichord was the smallest structure that could be used to define a tonality.

Composite tonality: A method of hour combination that juxtaposes two contrasting hours, each steered by its own respective hour, within the same pitch-class set to produce a new tonality.

Compound Steering: A method for hour combination wherein a select trichord is steered by more than one hour in the form of an asymmetrical tetrachord (see also form (1) of **Asymmetrical tetrachord**).

Deeper-level steering: Classified according to the number of levels traversed. Steering on a second level is labeled as second-level steering, and so forth.

Harmonic field: A pitch-class set with twelve distinct elements generated using the tone clock. A twelve-note “harmonic field” differs from a twelve-tone row, which is conceived melodically and in which the order of the pitch classes are fixed as a series. Twelve-tone “harmonic fields,” rather, consist of four transpositions of a trichord, each of which may come in any order. Furthermore, the constituent pitch classes of each trichord may also come in any order (see also **Harmonic subfield**) thereby presenting the composer with a much more flexible harmonic and melodic palate.

Harmonic subfield: Refers to a pitch-class set’s constituent trichordal subsets. The pitch classes in each subset may be used in any order.

Hour: A term analogous with the clock metaphor used to label the tone clock’s constituent trichords. Each tone-clock trichord can be referred to by a corresponding hour on the clock face (see also **Zodiac of the Twelve Tonalities**).

Incomplete symmetrical pentachord (ISP): A subset of a symmetrical pentachord, which as a result generates an asymmetrical tetrachord (see also form (2) of **Asymmetrical tetrachord**).

Initially ordered pitch-class set (io set): An element of the pitch-class set multiplication process. It is a partially-ordered set that must be reduced to its normal form before being multiplied by an unordered set (see **Normal form**).

Initial pitch class (ipc): Used as part of the STEER function and refers to the pitch class that substitutes as the root of pitch-class set.

Intervallic Prime Form (IPF): Comparable to Forte's concept of prime forms in set theory in that they can both be used to describe the intervallic content of a given set. Whereas set theory would label the prime form of the set 3-1 as (0,1,2), the IPF label would be a Roman numeral I and denote the interval configuration <1+1>.

Normal Form/ordered pitch-class intervallic structure (ois): The normal form of a partially ordered set. For example, the ois of the io set with the initial pitch-class integer 4 <4{7,9,1}> is {0,3,5,9}.

Steering: An operation used to transform a pitch-class set through multiplication. The manner through which the set is transformed is determined by a series of directed-interval classes from a second pitch-class set.

STEER(S): A function consisting of a formula that illustrates how pitch-class sets are derived through the steering principle. This is achieved through a process of multiplying a set that has been designated the io set by a second set, whose ipc has been determined (see also **Initially ordered pitch-class set** and **initial pitch class**). The product of the multiplication is a larger third set.

Surface-level Steering: Steering that occurs locally and is most apparent.

Tonical: See atonical/tonical

Tone-clock Hour: A tone-clock trichord analogous to the clock metaphor. In essence, it is a labeling system that uses Roman numerals so that one may quickly and easily distinguish between the twelve trichords of the tone clock.

Uniform Triadic Transformation (UTT): Introduced by Hook and uses an algebraic framework to illustrate triadic transformations. Each UTT consists of a sign (+ or -) and two transposition levels. The sign is used to indicate whether the transformation preserves (+) or reverses (-) a triad's mode. The transposition level consists of two integers of mod 12, the first for a major triad and the second for minor, and serves to indicate the interval by which the root of a select triad is transposed.

Zodiac of the Twelve Tonalities: A composite representation of the individual hour modules arranged on a clock face (see also **Hour**).

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