

Behaviour of Prestressed Ultra-High Performance Concrete I-Beams Subjected to Shear and Flexure

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Abstract

Ultra-high performance concrete (UHPC) is a new type of concrete developed by selecting the particle sizes and gradation in the nano- and micro-scales targeting the highest possible packing. The resulting concrete with very high density is called UHPC. UHPC has very low permeability and hence it is very highly durable compared to traditional or high performance concrete (HPC). Micro reinforcement of UHPC by random distributed steel-synthetic fibers results in superior mechanical properties such as very high compressive and tensile strengths, high ductility, and high fatigue resistance. The material selection and early age curing processes, use of fiber reinforcement, and very high quality in production resulted in a very high initial cost of UHPC structures. In order to enable the mass production and cost effective use of the material, performance based design and optimization of UHPC structural members are required. This study is part of an NRC Canada research project to develop innovative, cost effective, and sustainable bridge structural systems using UHPC and other innovative materials. In this study, the estimation of shear and flexural capacities using the available approaches of international design guidelines of UHPC structures are comprehensively compared to a proposed truss models, linear and nonlinear finite element models. Several design trials intended to allow for an optimized use of the materials and a maximum load capacity was conducted for simply supported beams with one or two external loads, and having rectangular or I cross sections. Linear and non-linear finite element models are developed and their results were compared to the available international design recommendations. Truss models are proposed to simplify the stress analysis in the shear zone of the prestressed UHPC beams.

It is found that prestressed UHPC I-beam section gives the highest possible load capacity with minimum use of materials. The study shows that for the case of no stirrups, massive flexure and shear cracks initiate and propagate suddenly where a diagonal shear crack is fully developed and sudden collapse may expected. The proposed truss model gives very good match to nonlinear finite element analysis results for almost all the truss members. The results are significantly improved when additional struts are considered for both cases of beams with or without shear reinforcement. The study shows the importance of future experimental investigations to calibrate the proposed models.

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List of symbols

- A : the cross sectional area
- A_b : the area of the stirrup leg
- A_{bc} : the area of the bottom flange
- A_{tc} : the cross sectional area of top flange
- A_{st} : the area of the longitudinal reinforcement
- A_{fs-lc} : the steel area equivalent to the fibers contributing to the resistance of the bottom cord
- A_{fs-hz} : the steel area equivalent to the fibers oriented horizontally
- A_{fs-in} : the steel area equivalent to the inclined fibers
- $A_{cross-hz}$: the cross sectional area for the calculation of the steel fibers contributing to the bottom cord
- $A_{cross-in}$: the cross sectional area for the calculation of the steel fibers in the direction of inclined strut
- $A_{bot-cord}$: the area of the total steel area in the bottom cord
- A_{fs-vs} : the steel area equivalent to the fibers contributing to the resistance of the vertical strut
- A_{fs-vr} : the steel area equivalent to the fibers oriented vertically
- A_{fs-in} : the steel area equivalent to the inclined fibers
- $A_{cross-vr}$: the cross sectional area for the calculation of the steel fibers contributing to the vertical strut
- A_{vs} : area of the non-cracked tributary section of the vertical struts
- a_{ci} : the location of force component C_{ci} from the top compression fiber of the composite section
- b_w : width of web
- b_{fb} : the width of the bottom cords
- b_{ft} : the width of the top cords
- d_{fb} : the depth of the bottom cords
- d_{ft} : the depth of the top cords

$d_{bot-cord}$: the equivalent diameter of imaginary “steel bar” presenting the total steel area in the bottom cord

- C : the depth of the neutral axis
- C' : the distance from the extreme bottom fiber of the section to the neutral axis location
- C_{c1} : the resulting compressive force component in r_{fu}
- C_{c2} : the resulting compressive force component in r_{fo}
- C_c : the resultant compressive force
- e : the distance from the center of the bottom tendon to the neutral axis
- e' : the distance from the center of the top tendons to the neutral axis
- E_{ps} : modulus of elasticity of prestressing tendons
- E_{UHPC} : the modulus of elasticity of the UHPC.
- E_s : modulus of elasticity of the steel stirrup
- f'_c : design compressive strength of UHPC, (MPa)
- P_{sh} : the ultimate shear capacity of the section
- P_u : the ultimate moment capacity
- ε_1 : the effective strain in the prestressing steel
- ε_2 : the decompression strain
- ε_3 : the ultimate strain in the tendons
- ρ_{ft} : the ratio of the total tensile flexural reinforcement
- ρ_s : the volumetric ratio of the total transverse reinforcement
- M_u : the ultimate bending moment for the beam section

JESC 2006 symbols

- f_{cj} : the concrete compressive strength at age j days
- f_{pu} : the ultimate prestressing stress in the tendons
- f_{pe} : the effective prestressing stress at service load

F_{st} : the stress transfer factor
 I : the moment of inertia of the beam section
 M_C : the first cracking or ultimate applied moment based on the loading stage
 n : the number of the stirrups
 P_{ebot} : the bottom prestressed force in the bottom tendons
 P_{etop} : the top prestressed force in the top tendons
 P_{ed} : effective tensile force of tendons from the JESC, 2006
 PO_r : the ratio of the fibers across any crack that are pulled-out
 V_{rpcd} : the contribution of UHPC excluding the contribution of the fibers
 V_{fd} : contribution of fiber reinforcement to the shear capacity
 V_{ped} : the contribution of prestressing reinforcement to the shear resistance
 $V_{s,max}$: the transverse reinforcement capacity
 w_u : the ultimate crack width
 Z : distance from the location of compressive stress resultant to the centroid of tension steel, which can generally be set to $\frac{d}{1.15}$
 γ_b : member factor that may generally be set to 1.3
 f_{vd} : design average tensile strength perpendicular to diagonal cracks of UHPC
 β_u : an angle between member axis and a diagonal crack, where β_u is to be greater than 30°
 τ : average shear stress calculated from design shear force
 σ'_{xu} and σ'_{yu} : average compressive stress, along member axis and perpendicular to the member axis, respectively
 β_0 : angle formed by a diagonal crack and a line at 45° from the member axis, when it is not subjected to axial force
 γ_b : member factor that can be set to 1.3
 α_p : angle formed by tendons and member axis
 α : the non-dimensional optimization factor of the section
 γ_b : member factor that can be set to 1.3
 ε_u : the ultimate strain of 0.0035
 $\varepsilon_{cu} = \varepsilon_u$: the ultimate compressive strain
 ε_{bc} : the yielding strain

$R_{f_{hz}}$: the ratio of the fibers that are oriented in the longitudinal direction of the beam
 $R_{f_{in}}$: the ratio of the fibers that are oriented in an inclined direction of ($\pm 45^\circ$) to the horizontal direction of the beam
 $R_{f_{vr}}$: the ratio of the fibers that are oriented in the vertical direction of the beam
 r_{fu} : the effective depth of the component C_{c1}
 r_{fo} : the effective depth of the component C_{c2}
 $\sigma(w)$: the experimental characteristic post-cracking stress corresponding to a crack width w
 β_u : the angle of the compression struts and lower-bounded to 30° (AFG-IR Cl 7.3,3, 2002)

AFGC, 2002 symbols

b_0 and z : the effective web shear width and depth, respectively
 d_p : effective depth
 h : is the total height of the section
 I_C : the moment of inertia of the beam section
 K : the fiber orientation coefficient for general effect
 m : the number of compressive force components
 S : the area of the fiber effect
 V_c : the composite contribution of UHPC matrix and the fiber
 V_f : the shear resistance provided by the average fiber tensile resistance (before fiber pull out) acting along the diagonal cracks
 V_p : the contribution of the prestressing steel reinforcement to the shear resistance
 V_s : the contribution of the shear reinforcement, or the stirrups, to the shear resistance (V_s)

ε_1 : the effective strain in the prestressing steel
 ε_2 : the decompression strain
 ε_3 : the ultimate strain in the tendons

Chapter 1

Introduction

1.1 General

Ultra-high performance concrete (UHPC) is a new type of concrete developed by selecting the particle sizes and gradation of the constituent materials in nano- and micro-scales, targeting the highest possible packing. The resulting concrete with very high density is called UHPC. UHPC has very low permeability and hence it is highly durable compared to the traditional or high performance concretes (HPC). The material shows very high compressive strength but with very brittle failure mode. However, the use of fibers as micro reinforcement changes the UHPC failure mode to a ductile one. Significant test programs have been performed around the world to determine the optimum fiber properties, such as; volumetric content, fiber type, length and distribution homogeneity. It has been found that the micro reinforcement of UHPC, in the form of randomly distributed steel synthetic fibers, results in superior mechanical properties, such as; very high compressive and tensile strengths, high ductility, and high fatigue resistance. In order to control early age shrinkage, and to have higher mechanical properties, heat and steam treatment are typically used at precast plants for standard production of precast UHPC.

The material selection and early age curing processes, use of fiber reinforcement, and the high quality production contribute to the high initial cost of UHPC when used in structural applications. In order to enable mass production and cost effective use of the material, performance based design and optimization of UHPC structural members become important. The trials involved for design/optimization of UHPC structures lead to several uncertainties that call for further research and developments. Among these research problems are:

- (i) The optimized sections are slender, which may results in possible fiber orientation that reduce UHPC strength and can develop premature failure;

- (ii) in order to achieve the maximum flexural capacity of a UHPC member, it is required to add traditional shear reinforcement (stirrups) and this can result in dispersion and further orientation of fibers;
- (iii) the applicability of existing cracking and post-cracking models of traditional concrete for the modeling of UHPC has not been verified;
- (iv) the applicability and/or accuracy of existing methods for estimating shear resistance of concrete structures to UHPC structures has not been experimentally confirmed.

This study is part of a NRC Canada research project to develop innovative, cost effective, and sustainable bridge structural systems using UHPC, advanced composites and high performance steel. In this study, the estimation of shear and flexural capacities using the available approaches of international design guidelines for UHPC structures are compared with a proposed truss model, as well as with linear and nonlinear finite element models. Several design trials were used to minimize the use of materials while maximizing the load capacity for simply supported beams having rectangular and I-sections, subjected to one or two external loads. The study shows the importance of future experimental investigation to calibrate the proposed models and to develop a simplified and accurate shear design approach.

1.2 Objectives

The main objectives of the current research project are to: (i) study the effect of traditional shear reinforcement on shear and flexure capacity of hybrid (traditional and fibre reinforcement) reinforced prestressed UHPC beams; (ii) investigate the accuracy of existing simplified shear and flexural capacity estimation approaches relative to the proposed truss and finite element models; and (iii) evaluate cracking patterns in relation to nonlinear behaviour of UHPC and the modes of failure. This study is part of a research program to be undertaken by NRC Canada with a long term objective of developing a reliable, accurate and simple methodology for shear and flexure capacity predictions of hybrid reinforced prestressed UHPC beams. The current phase of the investigation has been carried out in collaboration with the University of Ottawa.

1.3 Scope

The scope includes:

- Literature review;

- Design of non-prestressed UHPC simply supported beams with rectangular sections that are critical for flexure or shear;
- Design prestressed UHPC simply supported beams with rectangular and I-sections that are critical for flexure or shear;
- Develop a truss model to simplify the procedure involved in estimating the behavior of prestressed UHPC I-beams;
- Develop linear and non-linear finite element models (using DIANA TNO software) for the above;
- Compare the results of different approaches for estimating flexure and shear capacities;
- Studying the crack pattern, non-linear behavior and failure mechanisms for different prestressed UHPC I-beams.

1.4 Thesis structure

This thesis has seven chapters, as follows:

- Chapter 1: Introduction, objectives and scope.
- Chapter 2: Literature review that summarizes the theoretical, experimental, technical, and design backgrounds for UHPC development, incorporating mechanical behavior, durability, shrinkage and early age curing, quality control, truss modeling, finite element modeling, and design of UHPC structures according to the available international guidelines.
- Chapter 3: Development of a design procedure for UHPC beams, including sectional optimization for rectangular and I-section, reinforcement and prestressing requirements, designed for flexure and shear.
- Chapter 4: Linear and nonlinear finite element modeling of prestressed UHPC I-beams.
- Chapter 5: Development of a proposed truss model for prestressed UHPC beams.
- Chapter 6: Conclusions and future work.
- Appendices.

Chapter 2

Literature Review

2.1 Introduction

Ultra high performance concrete (UHPC) is a material which has been developed to respond to the needs for high resistance to the penetration of corrosion agents like chloride. UHPC mix includes a water-to-cementations material ratio of less than 0.25, internal fiber reinforcement to ensure non brittle behavior, and high binder content with special aggregate. Moreover, UHPC can reach proper rheological properties, in spite of its very low water content, having enough granular packing and high-range water-reducing admixtures (Graybeal, 2007a). However, the high density of UHPC and the use of micro-reinforcement result in advanced mechanical properties in addition to its superior durability characteristics.

Significant testing programs were performed worldwide to establish the optimum fiber properties; such as the volumetric ratio, fiber type and length, as well as their distribution and homogeneity. It was found that the micro reinforcement in UHPC, through random distribution of steel synthetic fibers, results in superior mechanical properties such as very high compressive and tensile strengths, high ductility, and high fatigue resistance. Heat and steam curing of UHPC mixes are typically used as part of the standard production of precast members. This controls early age shrinkage while promoting higher mechanical properties. UHPC offers compressive strengths of 150-400 MPa, and flexural tensile strengths of 30-40 MPa.

2.1.1 Description of UHPC

A new generation of concrete, named Reactive Powder Concrete (RPC), was developed in France in the 1990s, which showed exceptional durability and strength due to the optimized selection, proportioning and mixing of constituent materials. The particle gradation in RPC mixes is optimized to produce minimum void ratio. The largest granular material in the RPC mix consists of very fine sand with a particle size ranging between 15 μm to 600 μm . The cement particles in the mix have a size ranging in the order of 15 μm .

UHPC mixes also contain small particles such as quartz flour with a particle size of 10 μm and silica fume with a particle size of 1 μm . The silica fume has the smallest particle size in the mix. In UHPC mixes, silica fume and quartz flour are added to the mix in suitable percentages to increase the performance of concrete. Silica fume is a pozzolanic material that reacts with calcium hydroxide generated by the hydration of Portland cement. This reaction produces additional binder material called calcium silica hydrate, which increases the properties of hardened concrete. Silica fume improves the cohesion properties of fresh concrete, and decreases segregation and bleeding. The very fine size of silica fume particles reduces the voids in hardened concrete, promoting low permeability and enhanced mechanical properties (Association Française de Genie Civil, AFGC, 2002).

Ultra-high performance fiber-reinforced concrete (UHPC) is defined as concrete having characteristic compressive strength in excess of 20 ksi (150 MPa), with steel fibers that result in ductile behavior. Because of the low water-cement ratio of UHPC, a good portion of Portland cement particles remain un-hydrated. The un-hydrated cement particles remain un-reacted within the mix, and function as fine aggregate (NDOR, 2008). In a related study, Ma and Schneider (2002) gradually replaced part of the cement in the mix with quartz flour of equivalent volume. The strength remained the same when the replacement of cement remained below 30% by weight.

According to the American Concrete Institute (ACI), UHPC is defined as a concrete that has requirements for a combination of special characteristics and uniformity. This cannot be reached by using conventional constituent materials, mixing methods, and curing procedures. The characteristics and requirements for UHPC include; i) ease in placement (good ability to fill and pass), ii) high early strength, iii) long-term mechanical properties, iv) low permeability, and v) long life in harsh environments (NDOR, 2008).

A comprehensive investigation was conducted by the United States Federal Highway Administration (FHWA) in (1987) on the use of different types of concrete. The major objective was to improve the standards, maintenance, and rehabilitation of highways and bridges. The research program included UHPC as a new innovative concrete material. In this study, UHPC

was categorized as a concrete material with maximum water-cement ratio of 0.2 and minimum compressive strength of 150 at 28 days (NDOR, 2008).

2.1.2 Challenges for the structural use of UHPC

The early age curing processes, use of fiber reinforcement, and very high quality in production, as well as the use of special constituent materials, result in high initial cost for UHPC structures. In order to enable mass production and cost effective use of the material, performance based design and optimization of UHPC structural members become essential. The design/optimization of UHPC structures calls for further research and developments. Included among research needs are; (i) optimization of sections and associated fiber orientation that may have effects on UHPC strength; (ii) use of shear reinforcement to attain maximum flexural capacity without adversely affecting fibre orientation; (iii) applicability of existing traditional concrete models for cracking and post-cracking behavior of UHPC; and (iv) applicability and/or accuracy of existing methods of estimating shear resistance of UHPC.

2.1.3 Literature review

The literature review in this chapter is intended to provide background information on different aspects of UHPC as follows:

- i. Introduction of general physical and chemical characteristics of UHPC;
- ii. Available research and practice on mechanical properties of UHPC for structural modeling and design;
- iii. The effects of fibers on UHPC;
- iv. Use of non-prestressed and prestressed reinforcement in UHPC;
- v. Analysis techniques (simplified and advanced) for UHPC structures.

2.2 General Properties of UHPC

2.2.1 Composition of UHPC

Depending on application, UHPC may have different material compositions. A typical composition is shown in Table 2.1. UHPC has a highly homogeneous and a finely graded concrete matrix consisting of fine sand with particle sizes ranges between 150 μm and 600 μm , cement with particles having an approximate average diameter of 15 μm , crushed quartz with an

average diameter of 10 μm , and silica fume. The sand provides the largest granular material, whereas the silica fume provides the smallest particles to fill up the voids between the cement and the crushed quartz particles (Akhnoukh, 2008 and Ma and Schneider, 2002). The mix also includes steel fibres and admixtures to help with the process of mixing (typically super-plasticizers and accelerators). Table 2.2 provides the chemical composition of steel fibres used in UHPCs.

Table 2.1 Typical UHPC composition (Akhnoukh, 2008 and Ma and Schneider, 2002)

Material	Amount (kg/m^3)	Percent by weight (%)
Portland cement	712	28.5
Fine sand	1020	40.8
Silica fume	231	9.3
Ground quartz	211	8.4
Super-plasticizer	30.7	1.2
Accelerator	30	1.2
Steel fiber	156	6.2
Water	109	4.4

Table 2.2 Chemical composition of steel fibers (Akhnoukh, 2008 and Ma and Schneider, 2002)

Element	Composition Percent by weight (%)
Carbon	69-76
Silicon	15-30
Manganese	40-60
Phosphorus	≤ 2.5
Sulfur	≤ 2.5
Chromium	≤ 8
Aluminum	≤ 0.3

2.2.2 Shrinkage of UHPC

Limited number of studies focused on the shrinkage of UHPC. The focus was placed on the effects of curing and heat treatment as it relates to two components of shrinkage; autogenous shrinkage and drying shrinkage. Different studies, as well as the French and Japanese design recommendation, indicated that most of the shrinkage in UHPC takes place during steam/heat curing. The long term shrinkage of precast UHPC elements, produced under high quality plant condition, is in the range of the shrinkage that occurs in ordinary concrete elements.

Koh et al. (2011) tested three different UHPC mixes with water cement ratios of less than 0.2, having compressive strength of 180 MPa and flexure strength of 35 MPa, subjected to a heat treatment of up to 90°. The mixes included 5% expansive additives and 1% shrinkage reducing agents. Heat treatment was used to accelerate hydration, which increased the autogenous shrinkage, but did not affect the drying shrinkage. Two types of specimens were investigated; one with steel fibers and the other without steel.

Autogenous Shrinkage: The use of steel fibers resulted in a reduction in the autogenous shrinkage by around 20%. Koh, et al. (2011) showed that UHPC without the steel fiber had as large autogenous shrinkage strains as 1000×10^{-6} while the shrinkage strain of UHPC mixed with 2% steel fibers was limited to reached 800×10^{-6} . The autogenous shrinkage increases with acceleration of hydration during curing when high-temperature curing is set at 90° C. However, the ultimate autogenous shrinkage is still similar to that takes place at 20° C. The purpose of using heat treatment is to accelerate strength gain (see Figure 2.1), but it also accelerates autogenous shrinkage. Koh, et al. (2011) observed that ultimate autogenous shrinkage can be reduced by applying variable temperature (see Figure 2.2). The researchers also investigated the effect of adding an expansive additive and a shrinkage reducing agent on UHPC shrinkage properties (Koh et al. 2011). It was found that these admixtures decreased the autogenous shrinkage by up to 43% as illustrated in Figure 2.3.

Drying shrinkage: Due to the very dense packing of cementitious materials and fine aggregate particles in UHPC mixes, they often develop little drying shrinkage. This highly packed density obstructs evaporation, resulting in very small quantities of evaporated water (Koh et al. 2011). In general UHPC has very small drying shrinkage of approximately 100×10^{-6} as compared with conventional concrete, which may have shrinkage strains of about 150×10^{-6} .

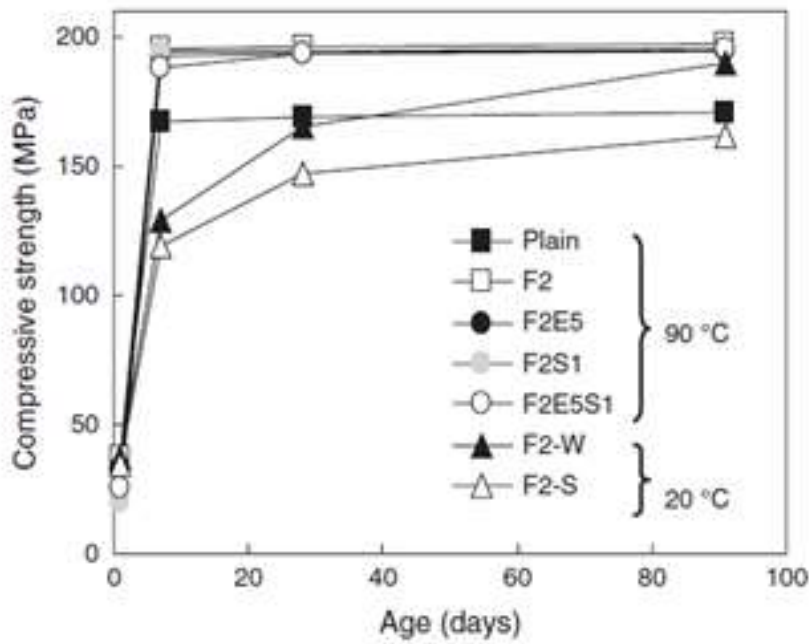


Figure 2.1 Compressive Strength gain in UHPC (Koh et al. 2011)

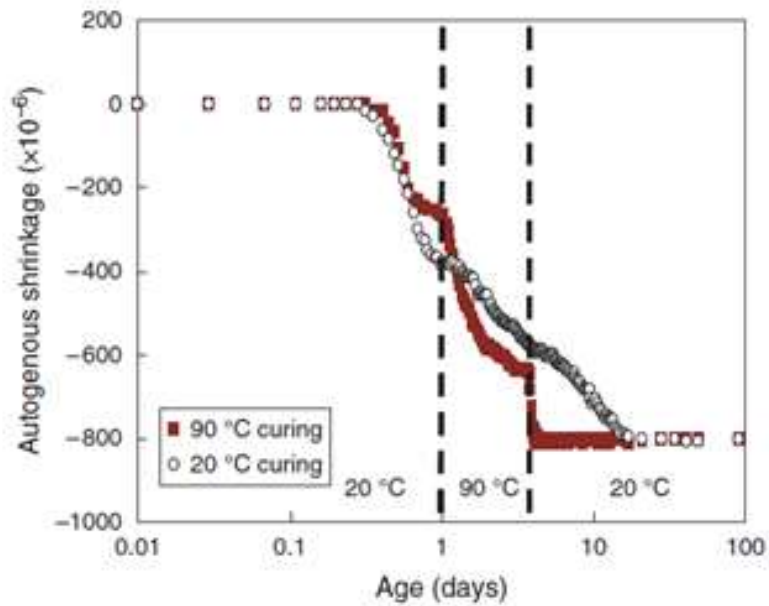


Figure 2.2 Effect of curing temperature on autogenous shrinkage (Koh et al. 2011)

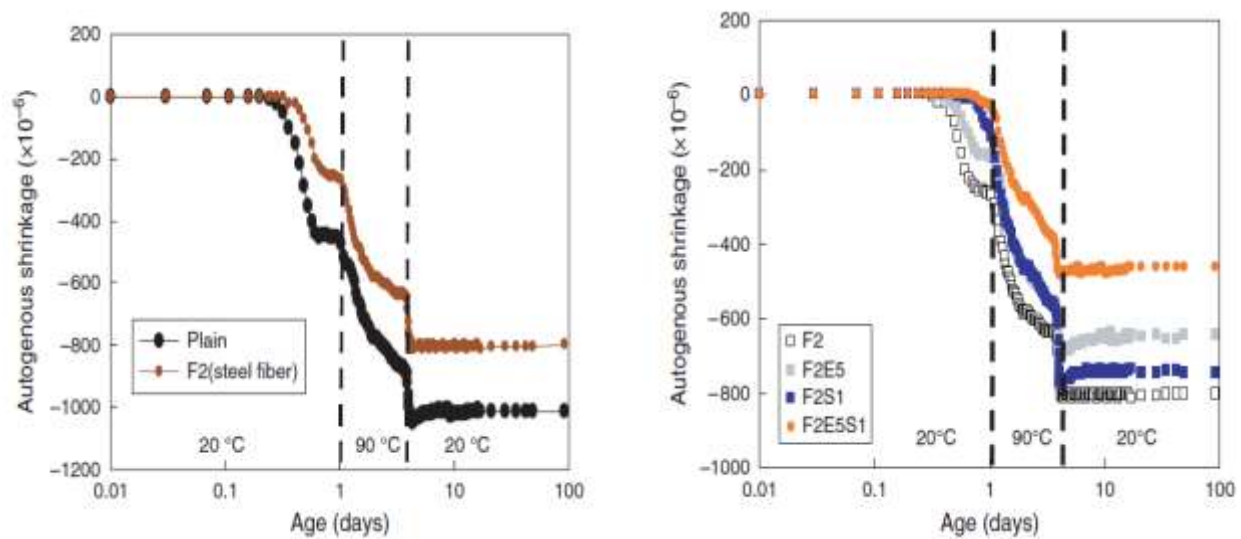


Figure 2.3 Effect of the steel fibers (F2), expansive additive (E5), and shrinkage reducing agent (S1) on autogenous shrinkage of UHPC (Koh et al, 2011)

In general, the autogenous shrinkage accounts for more than 85% of total shrinkage. Hence, the effects of autogenous shrinkage should be taken into consideration during design and construction of structures. Total shrinkage of UHPC can be reduced up to 50% compared to HPC when expansive additives and shrinkage reducing agents are included in the mix (Koh et al. 2011).

2.2.3 Durability of UHPC

Durability of concrete is considered to be the most important factor in performance and service life of concrete structures. Many experimental and analytical studies were conducted recently to improve the durability of concrete structures. Some of these studies focused on controlling the penetration of corrosive agents. Others concentrated on the optimization of fiber content. These studies led to the development of UHPC as a highly durable material for use in high performance concrete structures. In 2006, Graybeal and Tanesi investigated the durability of a specific type of UHPC called Ductal. Table 2.3 gives typical composition of UHPC (Ductal).

Table 2.3 Typical Constituent materials for UHPC (Graybeal and Tanesi, 2007)

Constituent	Amount (kg/m^3)	Percent by weight
Portland cement	712	28.5
Fine sand	1,020	40.8
Silica fume	231	9.3
Ground quartz	211	8.4
Superplasticizer	30.7	1.2
Accelerator	30	1.2
Steel fibers	156	6.2
Water	109	4.4

The researchers conducted three sets of tests with; (i) steam treated up to $90^{\circ}C$; (ii) non-steam treated; and (iii) delayed steam treated up to $90^{\circ}C$ starting 15 days after casting. The durability of these specimens was investigated in terms chloride penetration, scaling resistance, abrasion resistance, and freeze-thaw degradation resistance. The results are discussed below.

Chloride Penetration: Graybeal and Tanesi (2007) prepared specimens with a diameter of 102 mm and a height of 76 mm, and then applied a sodium chloride solution on the surface after 28 days of curing. The tests indicated that for different curing regimes of UHPC, the average chloride content at a particular depth was less than 0.05 kg/m^3 . In most cases, the average penetration was less than 0.02 kg/m^3 . The volume of chlorides penetrated into the UHPC was proved to be very low. A contributing factor was the discontinuous capillary porosity of UHPC which decreased permeability significantly.

Scaling Resistance: Graybeal and Tanesi (2007) performed scaling resistance tests on 8 square 76 mm thick slabs of 356 mm by 356 mm. A solution of calcium chloride is applied on each slab surface in room temperature. The slabs were then placed in a freezer at $-18^{\circ}C$. This procedure was repeated every day up to 50 or 70 cycles to simulate a harsh environment (freeze-thaw cycles). No scaling was noticed on any of the eight slabs, which were subjected to different curing regimes. The high scaling resistance of UHPC observed in these tests could be attributed

to the low water-cement ratio and high silica fume content used in the mixes. Similar results were observed by Gagne et al. (1990), Cantin and Pigeon (1996), and Toutanji et al. (1998).

Abrasion Resistance: Graybeal and Tanesi (2007) examined abrasion resistance of UHPC by casting 152 mm standard cylinders for each curing regime. The abrasion resistance is generally measured by taking the amount of abraded concrete from the surface, which can be abraded off using a rotating cutter within a time period of 2 min. Since UHPC normally has different finished surfaces, three different concrete surfaces were tested for each curing system. Figure 2.4 presents the results of abrasion weight loss tests based on ASTM C-944.

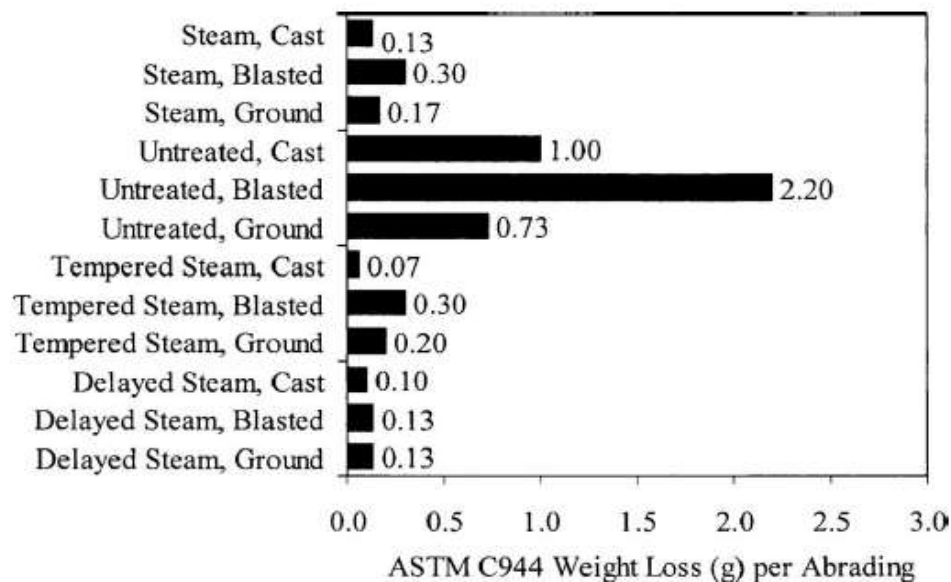


Figure 2.4 ASTM C 944 weight loss test results (Graybeal and Tanesi 2007)

Freeze-Thaw Degradation Resistance: Freeze-thaw degradation resistance is often defined as resistance to water penetration through a well-built voided microstructure. Graybeal and Tanesi (2007) reported on prisms having dimensions of 76 by 102 by 406 mm that were subjected to different curing regimes to examine the freeze-thaw resistance of UHPC. Cyclic thermo-loading, with temperatures ranging between **-18 and 4.4°C**, was applied 5 or 6 weeks after curing. These specimens were exposed to 690 thermo-load cycles, which resulted in light pitting of cured specimens on the surface. In contrast, more pitting was observed in uncured specimens. Furthermore, the difference in relative dynamic modulus (RDM) was measured as required by ASTM C 666 test method. Accordingly, the percentage difference in squares of the resonant frequency at any loading cycle was compared with the initial frequency before the application of

thermo-load cycles. RDM is expected to decrease as the deterioration of concrete is increased. The resonant frequency decreases when internal micro-cracking is formed and the cracks start propagating. The tests revealed that RDM decreased in uncured specimens while there was no obvious change in steam-cured specimens.

Graybeal and Tanesi (2007) explained that the treated UHPC specimens showed significant resistance to freeze-thaw cycles due to the use of pozzolanic materials such as silica fume and fly ash. These materials result in a very dense concrete matrix. On the other hand, uncured UHPC specimens have less resistance to freeze-thaw cycles due to incomplete hydration, which results in more porous concrete matrix and increased permeability.

Other researchers performed several tests on steam-cured and uncured specimens to investigate the freeze-thaw resistance of different UHPC mixes. They observed that UHPC exhibits high freeze-thaw resistance for both curing types. Lee et al. (2005) performed standard ASTM C 666 tests on UHPC specimens. They found that steam cured specimens retained 90% of their RDM after 1,000 thermo-loading cycles. They also observed an increase in RDM of some specimens, and attributed this behavior to possible complete hydration of cementitious materials. Bonneau et al. (1997) also performed similar tests and observed that after 300 cycles, 100% of RDM had been maintained. Figure 2.5 shows the resonant frequency of freeze/thaw prisms.

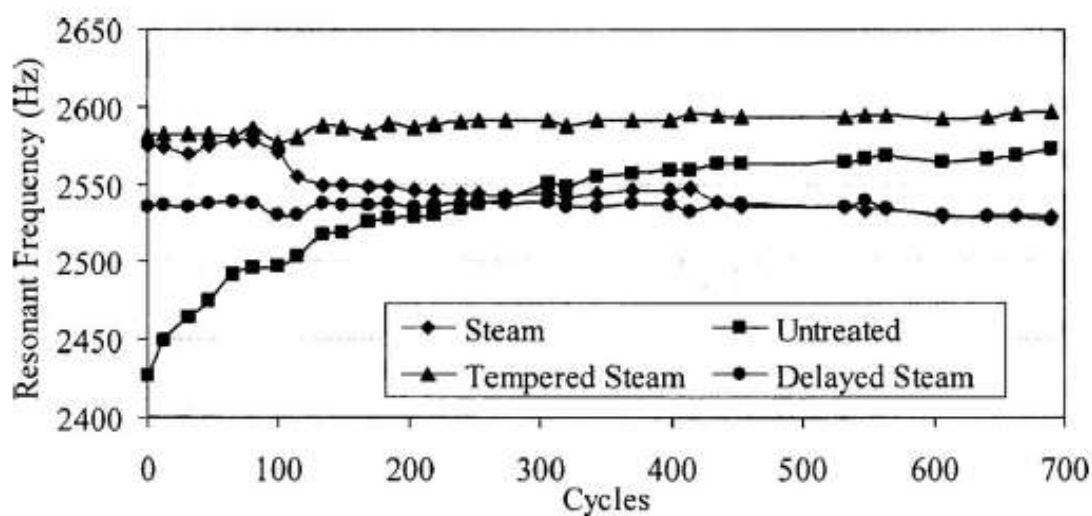


Figure 2.5 Resonant frequency of freeze/thaw prisms (Graybeal and Tanesi 2007)

2.3 Mechanical characteristics of UHPC

As was indicated earlier, UHPC develops ultra-high durability because of densely packed mixes and very low water cement ratios (< 0.2). High percentages of cement and silica fume, and fine aggregate (sand) without the use of coarse aggregate also contribute to improved mechanical characteristics (Almansour and Lounis, 2008). Uniformly distributed fibers ranging between 2 to 12% (by volume) are used to reinforce UHPC at the micro-level. Fiber length could be constant or variable, ranging between 1 to 25 mm. UHPC is classified into three types based on the fiber length and their volumetric ratio; i) UHPC with high proportions of fibers; ii) UHPC with intermediate proportion of long fibers (proposed in France in 1995); and iii) UHPC with very high percentage of fibers of different length (Rossi 2008).

It was also shown earlier that heat/steam treatment, as well as the use of very fine aggregates, reduce autogenous shrinkage. However, the same parameters affect negatively the superior macro-level mechanical properties. Compared to high performance concrete (HPC), commercially available UHPC has superior properties such as very high compressive strength, high tensile strength, and superior durability. It was found that when the hardened state of UHPC is reached with high quality control, the concrete age has a minor effect on strength, as shown in Figure 2.6 (JSCE, 2006).

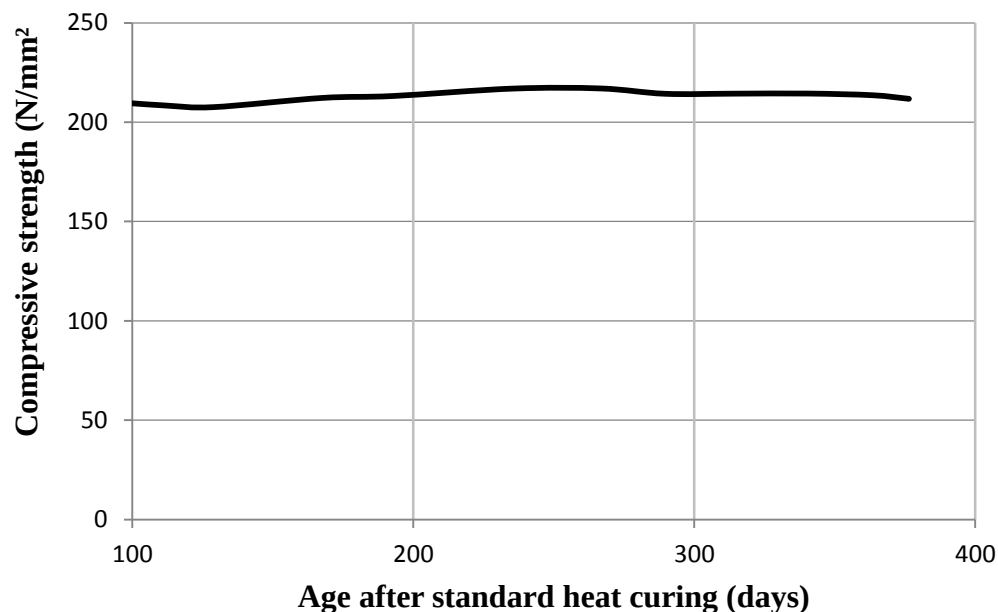


Figure 2.6 Long-term strength of UHPC following heat curing (JSCE, 2006)

2.3.1 Compressive strength

Several studies have been conducted to investigate the compressive strength of UHPC. Some of these studies focused on the development of UHPC stress-strain relationships using standard four inch cylinder tests (4" by 8" or 100 mm diameter by 200 mm height). These results are summarized in Figure 2.7. Other studies focused on the effect of fiber reinforcement. Comparison of failure modes of UHPC and HPC shows that UHPC has a more ductile failure mode in compression due to the presence of fibers, while HPC experiences an explosive (very brittle) failure mode (FWHA report 2007).

According to JSCE (2006) recommendations, when commercially available UHPC's constituent materials shown in Table 2.1 are used, the average compressive strength can be estimated to be 194 MPa, with a standard deviation of 7.7 MPa. This is illustrated in Figure 2.7. Considering the possible variation of compressive strength, the characteristic value can be conservatively set to a lower bound of 175 MPa (JSCE 2006).

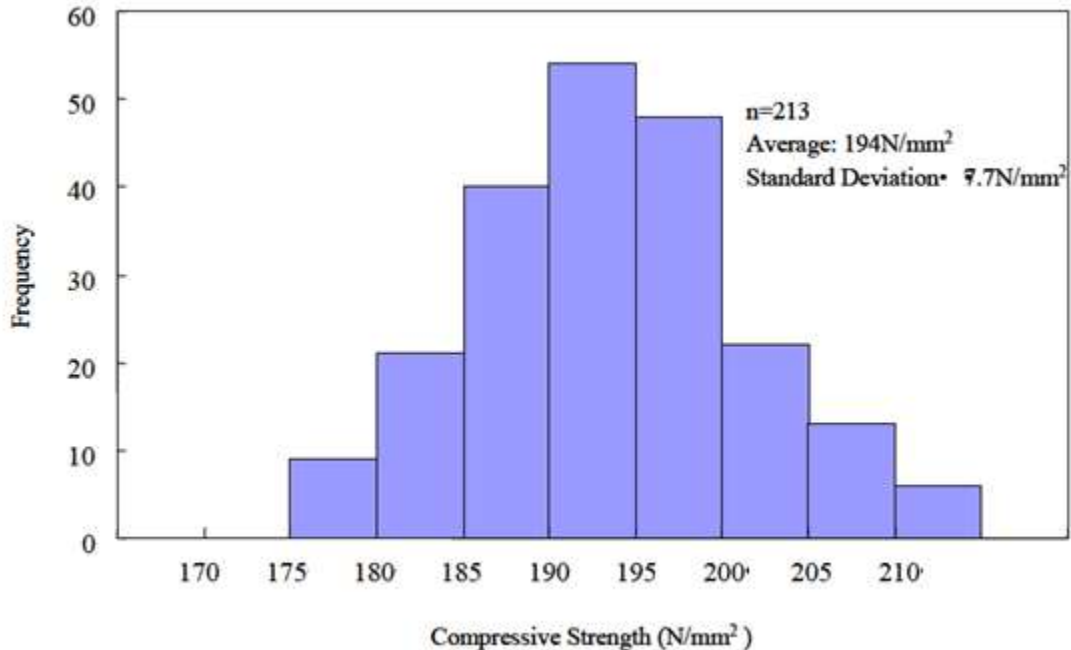


Figure 2.7 Frequency distribution of compression test results using 100 mm cylinders (JSCE, 2006)

2.3.1 Tensile strength and tension softening

UHPC exhibits significantly higher tensile strength prior to cracking than its tensile strength after cracking (JSCE, 2006). Upon first cracking, the crack widths increase with increasing load. During this stage of loading, tensile stresses transfer across the crack through fibers. Under increasing load, successive pull-out of the fibers and/or fracture takes place. Average behavior of UHPC specimen shows tension softening in direct tension (JSCE, 2006). The average tensile strength of commercially available UHPC is found to vary between 10 to 15 MPa. The tensile flexural strength ranges between 24 MPa to 29 MPa as shown in Figure 2.8. Figure 2.9 illustrates the tension softening diagram (JSCE, 2006). For design purposes, the first cracking strength is defined as the tensile stress beyond which the stress-strain relationship is no longer linear, and the rate of change in strain due to the change in stress increases successively. The Japanese recommendation suggests using the splitting-cylinder test to quantify UHPC tensile strength. For example, UHPC with constituent materials as specified in Table 2.1 has an average splitting tensile strength of 12 MPa with a standard deviation of 1.3 MPa as shown in Figure 2.8 (JSCE, 2006).

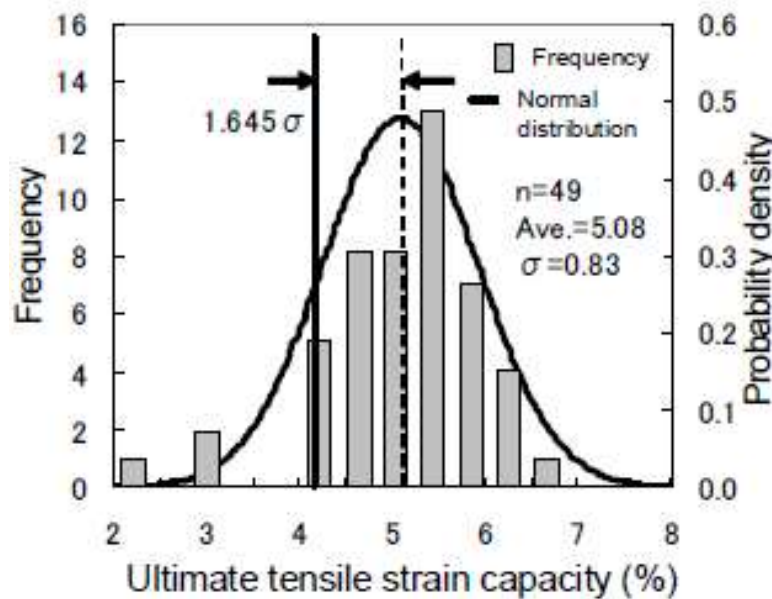


Figure 2.8 Distribution of splitting-cylinder test using 100 mm cylinders (JSCE 2006)

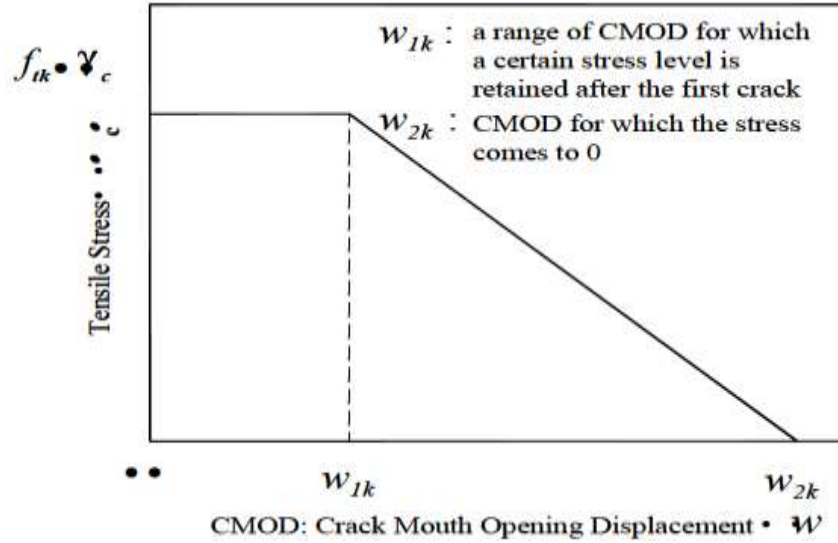


Figure 2.9 Tension softening diagram (JSCE, 2006)

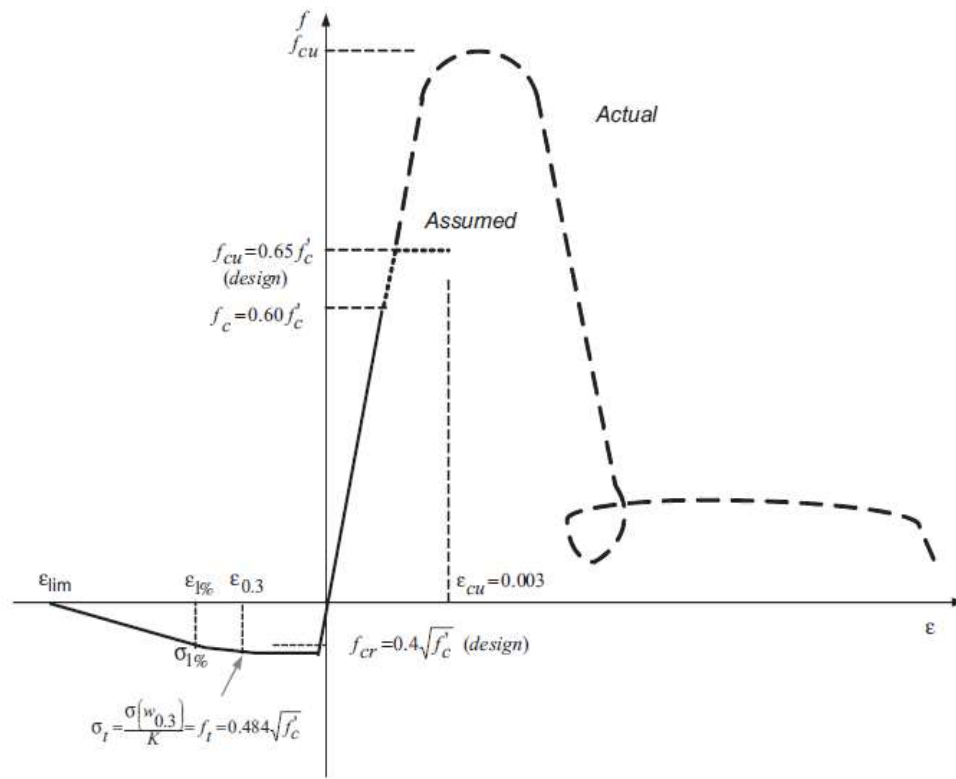
2.3.2 Stress-strain relationship of UHPC

Many studies showed that UHPC has improved ductility in the post peak region of stress-strain relationship, which is explained by the fiber micro-reinforcement. The compressive strength of UHPC is greatly affected by the type, shape, and content ratio of fiber reinforcement. Tests show that UHPC behaves linearly up to very high compressive stress close to the ultimate compressive strength, and then the compressive resistance declines gradually over very wide strain range, as shown in Figure 2.10. To account for the uncertainty associated with the behavior of UHPC as a new material, and the variability of test results, the stress-strain relationship of UHPC in compression is limited to a maximum compressive strength equal to only 65% of UHPC cylinder strength. The ultimate strain of UHPC in compression is also limited between 0.003 and 0.0035 as shown in Figure 2.10(a) and 2.10(b). On the other hand, the tensile stress-strain relationship is linear up to the first crack, then approximately remains constant over a wide range of strains as shown in Figure 2.10 (a).

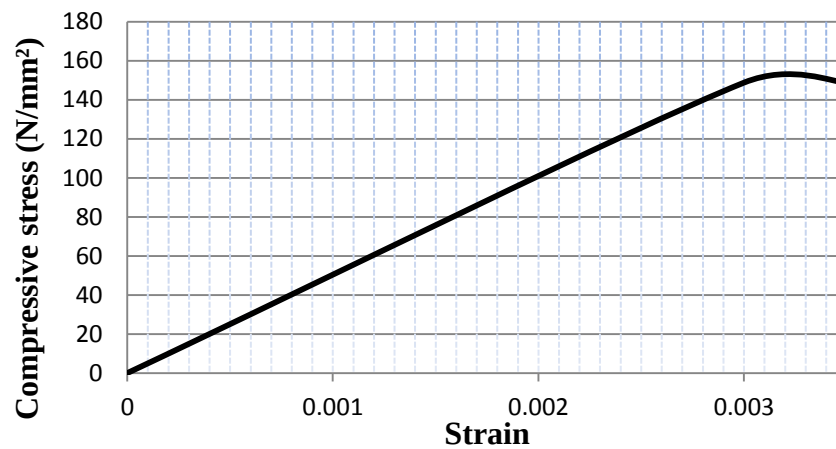
2.3.3 Shear capacity of UHPC

AFGC (2002), JSCE (2006) and some researchers proposed simplified approaches to estimate the contributions of UHPC to shear resistance. Many research programs involved the investigation of UHPC shear capacity in beams with only fibers in concrete as reinforcement,

without the use of traditional stirrup reinforcement. Voo et al. (2005) investigated shear capacity of I-beams with fiber reinforced UHPC, without the traditional shear reinforcement.



(a) design stress-strain relationship suggested by Almansour & Lounis (2008)



(b) Design stress-strain relationship suggested by Japanese Recommendations (JSCE 2006)

Figure 2.10 Idealized strain-stress relationship of UHPC

Seven prestressed I-beams having UHPC with different volumetric fiber reinforcement ratios were tested the beams had a clear span of 4.0 m and a cross sectional depth of 650 mm. The cross section of the I-beams is illustrated in Figure 2.11. The fiber content varied between 1.25 to 2.5 %, consisting of two types; (i) normal (straight) fibers of 13 mm long and 0.2 mm diameter (type-I); and (ii) hooked-end fibers, 30mm long and 0.5 mm diameter (type-II). Figure 2.11 and Table 2.4 show the test details. The researchers showed that the fiber type and the fiber reinforcement percentage ratio had no influence on the first cracking load. UHPC specimens with end hooked fibers showed lower shear strength compared to UHPC specimens with straight fibers. Furthermore, high level of prestressing resulted in an increase in the overall stiffness of specimens. They also found that the shear crack patterns of specimens were the same. The cracks started in the web, and then propagated towards the top flange. It was further observed that higher the fiber ratio was, the slower the crack propagation was.

Table 2.4 Prestressing and fiber details of Voo et al. (2005) specimens

Beam No.	Prestressing Force (kN)		σ_{top}	σ_{bot}	σ_{ave}	Fiber Volume (%)
	Top Flange	Bottom Flange				
SB1	0	0	0	-27.2	0	Type I- 2.5
SB2	450	900	-4.72	-13.6	-14.3	Type I- 2.5
SB3	225	450	-2.36	-13.6	-7.15	Type I- 2.5
SB4	225	450	-2.36	-13.6	-7.15	Type I- 1.25
SB5	225	450	-2.36	-13.6	-7.15	Type I- 1.5 Type II- 1.0
SB6	225	450	-2.36	-13.6	-7.15	Type II- 2.5
SB7	225	450	-2.36	-13.6	-7.15	Type I- 1.88 Type II- 0.62

In most available design guidelines and previous research, the estimation of ultimate shear strength V_u of UHPC was expressed as the sum of the contribution of three components; (i) UHPC, (V_c); (ii) shear reinforcement, (V_s); and (iii) prestressing. The contribution of UHPC is the sum of the contributions of the matrix and the fibers, assuming random distribution of fiber. However; there were limited test results in support of the generalization of available simplified approaches. The applicability of these approaches to different structural members, sectional dimensions, reinforcement arrangements, and casting methods, has not yet been

verified. There is lack of comprehensive studies on the effect of fiber distribution (and possible fiber orientation), hybrid shear reinforcement (fibres in combination with transverse shear reinforcement) and high prestressing forces.

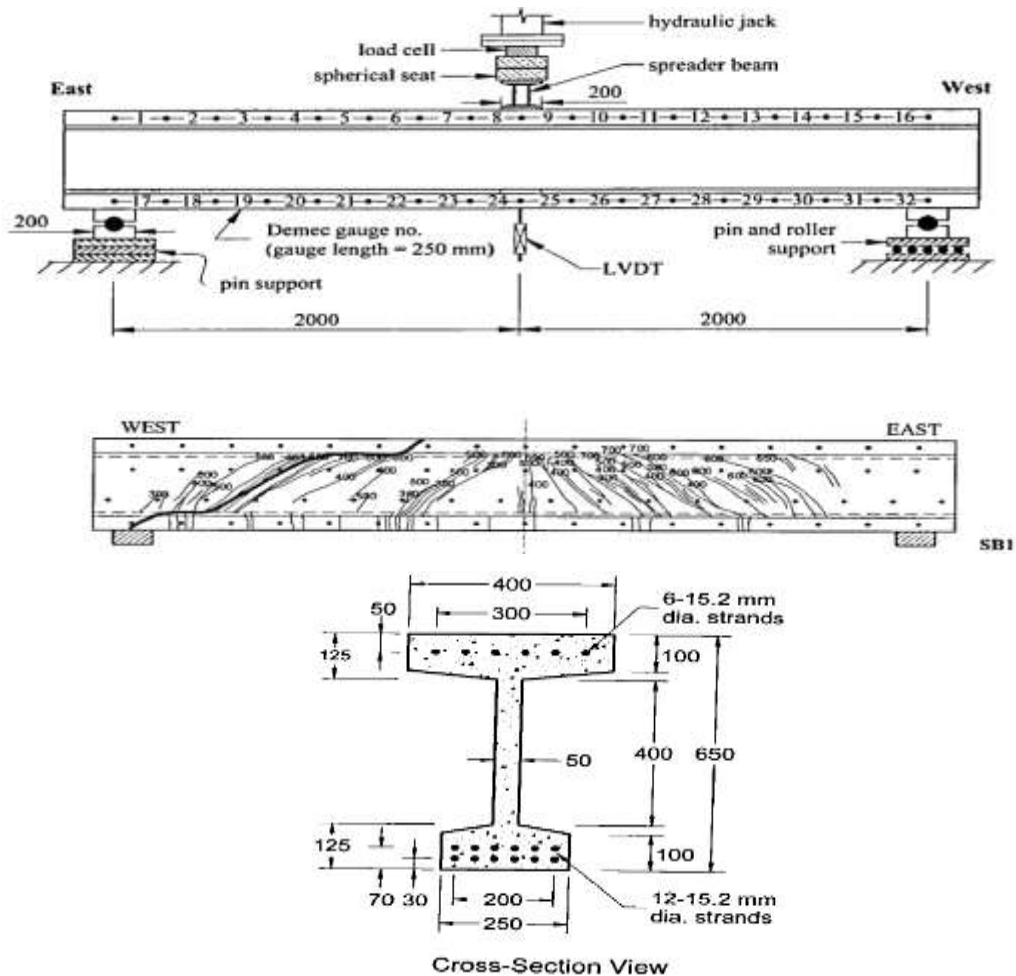


Figure 2.11 Test setup, crack pattern and cross section of specimens by Voo et al. (2005)

2.4 Fiber reinforcement and its effects on mechanical properties of UHPC

The use of fiber reinforcement is as old as the use of conventional reinforced concrete (Rossi 2008). Fibers are used as internal reinforcement in UHPC to enhance its tensile strength where they help in bridging micro cracks and delay the development of large cracks. Several studies investigated the structural performance of fiber reinforced concrete in flexure. The focus of these

studies was on the behavior of steel fibers and concrete during different stages of structural loading and related response at different stages, such as the linear elastic stage, cracking, fibre pull-out and/or fracture, and structural collapse. Figure 2.12 shows different types of fibers used in fiber-reinforced concrete (Martine et al. 2009).



Figure 2.12 Different fiber types used for fiber reinforced concrete (Martine et al. 2009)

It was found that fiber reinforcement significantly affects UHPC mechanical properties in the hardened state. This includes compressive strength, tensile strength, shear strength, and ductility (Martinie et al. 2009). Researchers found that UHPC mechanical properties are influenced by the aspect ratio of fibers, which is defined as the ratio of length l to diameter d of the fiber (Martinie et al. 2009). However, it was found that further addition of fibers could reduce the workability of concrete mix. A measure of fibre content may be the product $r \times \phi_f$, where r is the aspect ratio, and ϕ_f is the fiber volume fraction (Martinie et al. 2009).

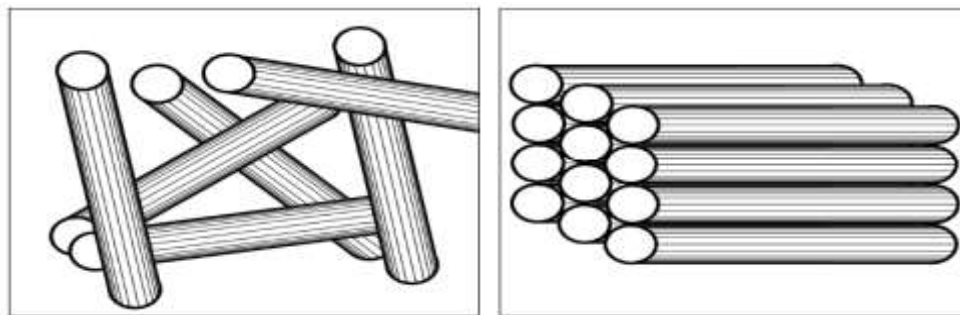


Figure 2.13 Apparent volumes occupied by fibers in random packing for various fiber aspect ratios (Martinie et al. 2009)

When the fiber volume fraction is very high and exceeds a critical limit, the fiber reinforced concrete cannot flow. As this critical fiber volume fraction decreases, fibers tend to form clumps or balls (Martinie et al. 2009). Even though the mechanical properties in hardened state can be increased with increasing fiber volume fraction, this fraction should not exceed a specific value, which presents a compromise between the workability of the fresh state and the mechanical behavior in the hardened state. On the other hand, it is also observed that the critical fiber volume fraction decreases with increasing volume of coarse particles (or aggregate), as well as with increasing aspect ratio of fibers. However, there is no optimum proportion of fibers that can be identified for the production of high-strength UHPC with a proper fluidity of fresh concrete. It is important to note that fiber volume fraction effects fiber distribution in hardened UHPC. This may result in reduction of strength. The effect of fibre orientation on overall UHPC strengths and modes of failure have not been studied in a comprehensive manner. Figure 2.14 shows two examples of high (but acceptable) fiber volumetric fractions on the consistency of UHPC mixes.



Figure 2.14 Consistency of cement paste mixed with fibers having various aspect ratios and water-cement ratio of 0.4 (Martinie et al. 2009)

Kang and Kim (2011) investigated the effect of fiber orientation on tensile strength of UHPC using direct tension tests. This was done by applying tensile forces on two sided notched specimens, as shown in Figure 2.15. UHPC was cast in two direction; (i) by placing concrete parallel (PL) to the tensile stress direction; and (ii) by placing concrete in the transverse (TL) direction (or perpendicular to the tensile stress direction). The results showed that the specimens with PL casting developed 10% higher strength than those cast in the transverse direction (TL). Figure 2.16 shows the two concrete placement directions.

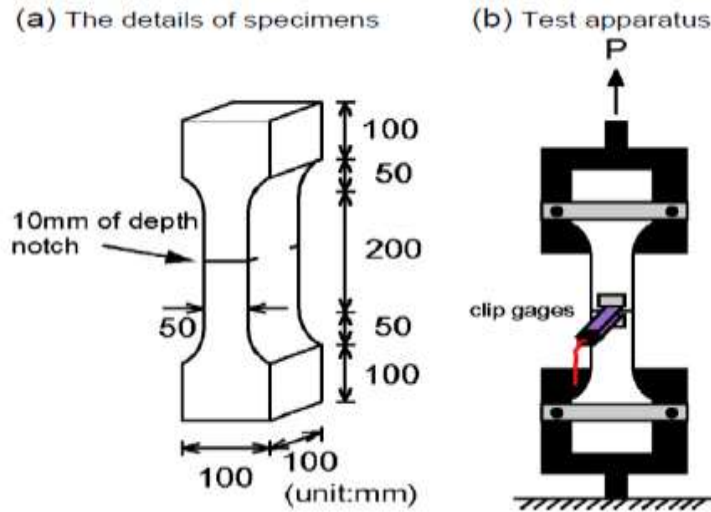


Figure 2.15 Direct tensile test specimen and apparatus (Kang and Kim 2011)

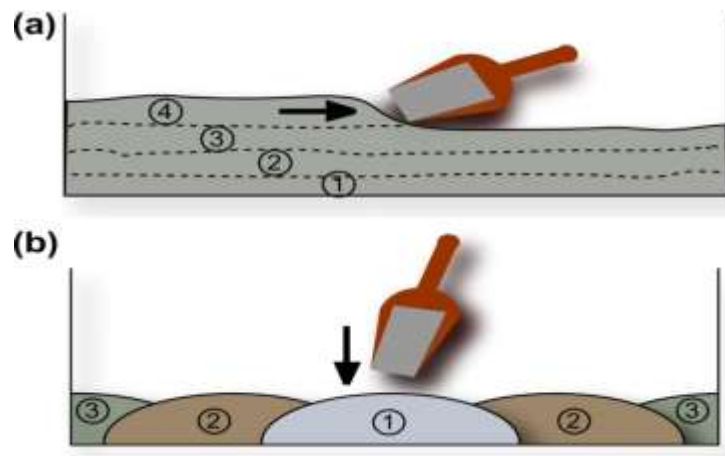


Figure 2.16 Specimen preparation by (a) placing concrete parallel to the longitudinal direction; and (b) placing concrete in the transverse direction (Lee et al. 2010)

The researchers indicated that the tensile strength of UHPC has two main components; (i) the pre-cracking tensile component, which relies on the mechanism of elastic shear transfer between the matrix and the fibers; and (ii) the post-cracking component that depends on pull-out and tensile strength of fibers (Kang and Kim 2011). It was observed that the second component, or the post-cracking strength, improved for specimens cast in the PL direction. Figure 2.17 shows the crack initiation strength, while Figure 2.18 shows the overall tensile behavior in both types of specimens.

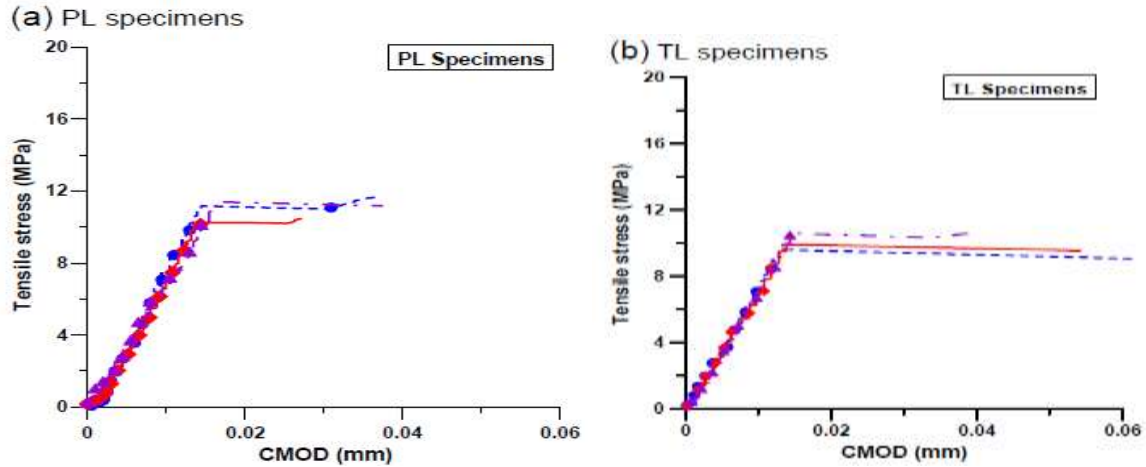


Figure 2.17 Initial tensile behaviors including the first cracking (Kang and Kim, 2011)

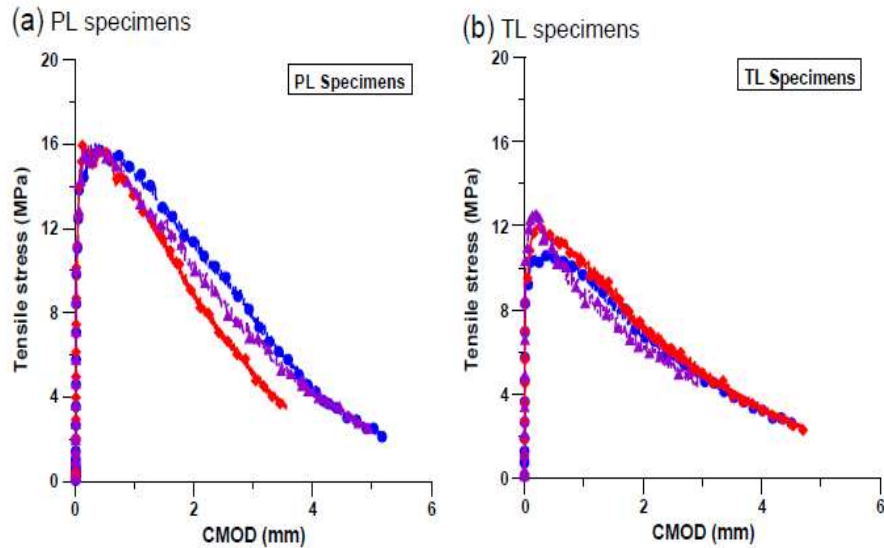


Figure 2.18 Overall tensile behaviors (Kang and Kim 2011)

Kang et al. (2010) conducted an experimental study to evaluate the effect of fiber orientation and dispersion on flexure behavior of UHPC. In order to evaluate the effect of two different concrete placement directions, the specimens with dimensions of $100 \times 100 \times 400$ mm were tested using three-point loads, as shown in Figure 2.19. The specimens were cut transversely (TC), horizontally (HC), and vertically (VC) to investigate the fiber dispersion and orientation. It was found that the specimens cast parallel to the longitudinal direction had more uniformly dispersed fibres in the transverse cross section (PL-TC) when compared with other specimens. Fiber reinforcement consisting of steel fibers of 13 mm length and 0.2 mm diameter were used in all the reinforced specimens, while non-reinforced specimens were used for control tests. They

observed that fluidity and placement direction of wet UHPC affected the distribution of steel fibers, including both the orientation and dispersion. They also observed that the distribution of steel fibers can be influenced by the flow direction of UHPC mix and the shape of the structural element.

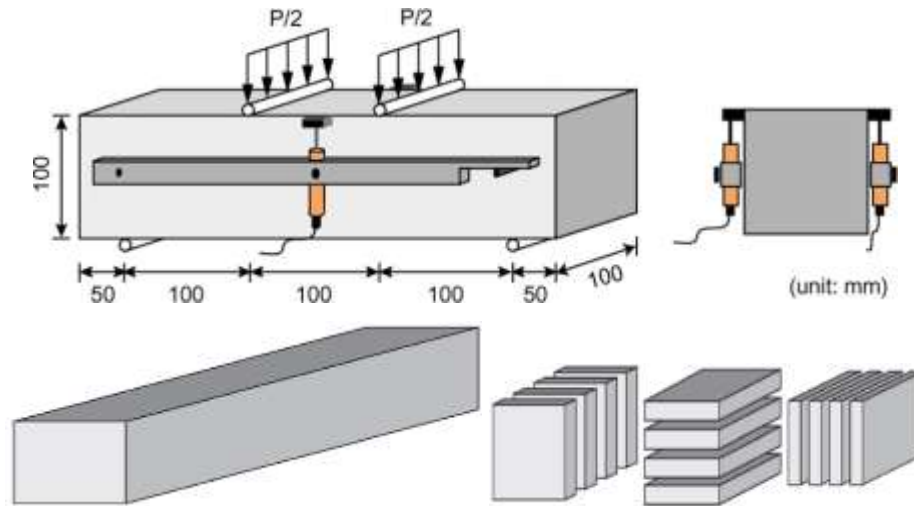


Figure 2.19 Flexural tensile test apparatus (Lee et al. 2010)

During the above experimental research it was observed that the PL specimens had only 5% larger cracking strength than the TL specimens. This is because the crack initiation is influenced mainly by the concrete matrix strength. However, the flexural strength is improved when the placement direction is parallel to the element direction, and the fibers are longitudinally oriented. This orientation in the direction of tensile stresses results in an increase in the tensile strength. Kang et al. (2010) found that PL specimens gave average flexure strength of 60% higher than the average flexural strength of TL specimens.

2.5 Optimum fiber volume ratio in hybrid reinforced UHPC elements

2.5.1 Effects of traditional reinforcement on fiber reinforced UHPC

The performance of reinforced UHPC structural elements was investigated by a number of researchers. Fehling et al. (2010) conducted experimental investigation of structural elements that had both the fibres and re-bars (hybrid reinforcement). They used steel fibres with variable volumetric ratios and variable fiber dimensions (length and diameter). The investigation involved

tests for bond, direct tension, flexural bending and biaxial bending. The volumetric fiber ratio varied between 0.9 % and 2.5 %. It was found that the specimens with volumetric fiber ratios of 0.9 % resulted in the highest bond strength of fibres with a relatively small longitudinal displacement (0.1 – 0.2 mm). Also, the fiber pull-out strength increased as the diameter and length of the steel fiber increased. The flexural bending tests indicated that the specimens with 0.9 % volumetric fiber ratio did not show strain hardening behavior after developing visual cracks. The cracks propagated suddenly upon formation. However, specimens with combined traditional reinforcement (steel re-bars) and steel fibers developed strain hardening behavior with well distributed cracks (crack spacing of about 5 times smaller than those in fibre reinforced specimens). The test results showed that the tension stiffening effect became more apparent when the fiber content increased. However, the crack spacing and crack width did not reduce proportionally. Instead, the addition of re-bars was able to limit crack widths significantly. The researchers observed that even with a low fiber content, the use of re-bars resulted in small spacing of cracks and reduced crack widths without developing the strain hardening behavior. Figure 2.20 shows the test setup and sample crack pattern from one of the tests conducted by Fehling et al. (2010).



Figure 2.20 Four-point-bending test; test setup and crack pattern (Fehling et al. 2010)

2.5.2 Effects of fiber reinforcement ratio on flexural behavior of UHPC

Meade and Graybeal (2010) conducted an experimental investigation on ultra-high performance concrete (UHPC) beams with different fiber content and light traditional flexural reinforcement. Sixteen beams with 4.9 m span and 150 mm by 380 mm cross section were tested. The beams

were simply supported. The two point loads were applied on the beam at 760 mm from the centerline. Figure 2.21 shows the details of the beams dimensions, reinforcement and test set-up.

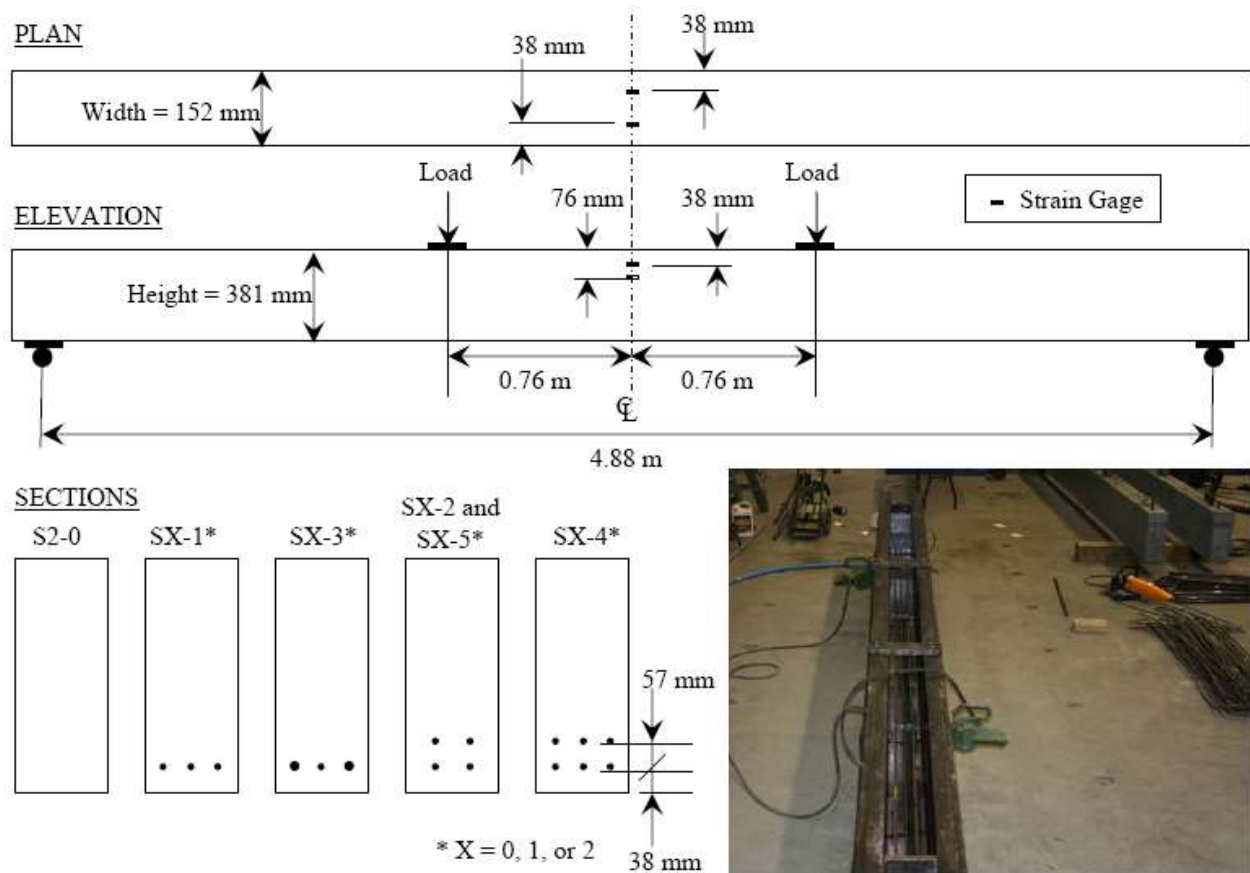


Figure 2.21 Beam dimensions, reinforcement and test setup (Meade and Graybeal 2010)

Figure 2.22 depicts views of the test setup. The beams were reinforced with less than 1% of re-bars and fiber volume ratios that ranged between 0%, 1%, and 2%. There were beams that were only reinforced with 2% fibres for comparison. Table 2.5 shows the details of test specimens for each of the 16 beams tested. Meade and Graybeal (2010) tested UHPC cylinders, both in compression and in bending. The cylinders had steel fiber ratios of 0%, 1%, and 2%. The compression tests involved cylinders with 76 mm diameter. The specimens that had 0 % fibers showed lower compressive strength than others, with severe crushing and fragmentation, while the specimens with 1% to 2% fibres showed a ductile failure mode without fragmentation but limited improvement in compressive strength. The results are tabulated in Table 2.7.



Figure 2.22 The flexural test set-up (Meade and Graybeal 2010)

Table 2.5 Summary of beams and reinforcement (Meade and Graybeal 2010)

Beam Designation	Fiber Reinforcing Content (%)	Mild Steel Reinforcement	Mild Steel Reinforcement Area (mm²)	Mild Steel Reinforcement Ratio (%)
S0-1	0	3-#10	213	0.41
S0-2	0	4-#10	284	0.55
S0-3	0	1-#10 + 2-#13	329	0.64
S0-4	0	6-#10	426	0.83
S0-5	0	4-#13	516	1.00
S1-1	1	3-#10	213	0.41
S1-2	1	4-#10	284	0.55
S1-3	1	1-#10 + 2-#13	329	0.64
S1-4	1	6-#10	426	0.83
S1-5	1	4-#13	516	1.00
S2-0	2	N/A	N/A	N/A
S2-1	2	3-#10	213	0.41
S2-2	2	4-#10	284	0.55
S2-3	2	1-#10 + 2-#13	329	0.64
S2-4	2	6-#10	426	0.83
S2-5	2	4-#13	516	1.00

The splitting cylinder tests involved 102 mm diameter cylinders. They indicated that the specimens without fiber reinforcement had relatively low strength at first crack initiation. The cylinders with 1% and 2% fiber volume ratios resulted in 20% and 23% higher strengths at first cracking. These results are shown in Table 2.6. In contrast, the specimens with 0% fiber volume

ratio could not maintain their peak splitting strength, and failed immediately after cracking. This indicates that a significant improvement was attained in fibre reinforced UHPC. The increase in cracking capacity with increasing fibre content is shown in Figure 2.23. The increases obtained were 2 and 2.6 times the cracking strength of plain concrete when 1% and 2% fibres were used, respectively.

Table 2.6 Compression and split cylinder test results (Meade and Graybeal 2010)

Beam Series	Compressive Strength, MPa		Split Cylinder Cracking Strength, MPa		Split Cylinder Peak Strength, MPa	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
S0	170	27.6	10.0	0.85	N/A	N/A
S1	194	15.6	12.0	1.72	18.8	0.87
S2	203	13.6	12.3	1.13	24.4	2.20

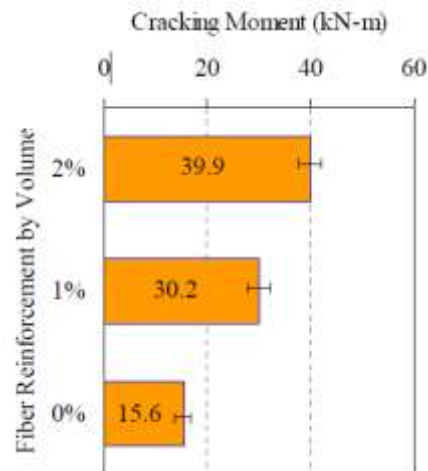


Figure 2.23 Average moment at first cracking as a function of fiber reinforcement (Meade and Graybeal 2010)

Figure 2.24 shows the load-deflection relationships for the three beam groups for different fiber volumetric ratios. For the case of no fiber reinforcement, the ultimate load is increased with increasing reinforcement ratio. However, the ductility is reduced significantly with use of high reinforcement ratios, resulting in brittle collapses at ultimate, without post-ultimate softening. Fibre-reinforced UHPC shows an increased strength with increasing fibre content and a wide range of post failure softening. It is concluded that ductile behavior with high strength occurs when the fiber volume ratio is around 1%. It is not yet clear whether this ratio is the optimum

ratio for all cases of design, when hybrid reinforcement is used. So far the available studies confirm that this ratio is preferred in hybrid reinforced UHPC. Hence, this ratio is used in the current study when combined with traditional re-bars.

2.5.3 Effects of hybrid reinforcement on shear strength of UHPC

Tellen et al (2010) conducted an experimental study on two UHPC I-beams to investigate the shear resistance of UHPC beams with fiber and traditional reinforcement. One of the beams was reinforced using traditional reinforcement for flexure and shear, while the other beam was only reinforced for flexure. The fibers used in this test were 13 mm long, 0.16 mm in diameter and had volumetric ratio of 3 %. Figure 2.25 shows the test setup and the reinforcement for the two beams tested. The researchers observed the tendency of the fibers to be oriented in a direction parallel to the longitudinal beam direction. They also did not justify whether they had optimized the fiber volumetric ratio, or they had an optimization study to identify the traditional shear reinforcement ratio. The study showed that diagonal shear crack occurred, originating from micro cracks, with an inclination of 45° relative to the shear reinforcement in the beam with transverse bars. Diagonal shear cracks occurred with a flatter angle of 35° when the beam did not have the vertical reinforcement (see Figure 2.26). The beam with flexural and transverse bars failed due to the rupturing of shear reinforcement. It is clear that the researchers used small area of transverse shear reinforcement. In order to have more significant change in shear and flexural behavior of the beam, higher shear reinforcement is required. On the other hand, the beam without the transverse bars failed due to shear at a lower load, as soon as the fibers pulled out. The researchers also observed that the post-peak residual flexural strength of both beams was due to the flanges acting independently in double curvature, spanning over the failed portion of the web. This mechanism cannot be ensured in every beam section or in prestressed concrete. Tellen et al. (2010) proposed a simple truss model with 45° struts for both beams with and without transverse bars, in presence of steel fibres. The area and yield strength of reinforcing bars were replaced with those of the UHPFRC web, and the fibers were assumed to act in a direction perpendicular to the cracks. This truss analogy treats UHPFRC fibers as reinforcement with a variable angle of inclination, bridging shear cracks at an angle perpendicular to the length of the crack. The researchers observed that providing shear reinforcing bars in the web was a viable approach for providing sufficient shear capacity.

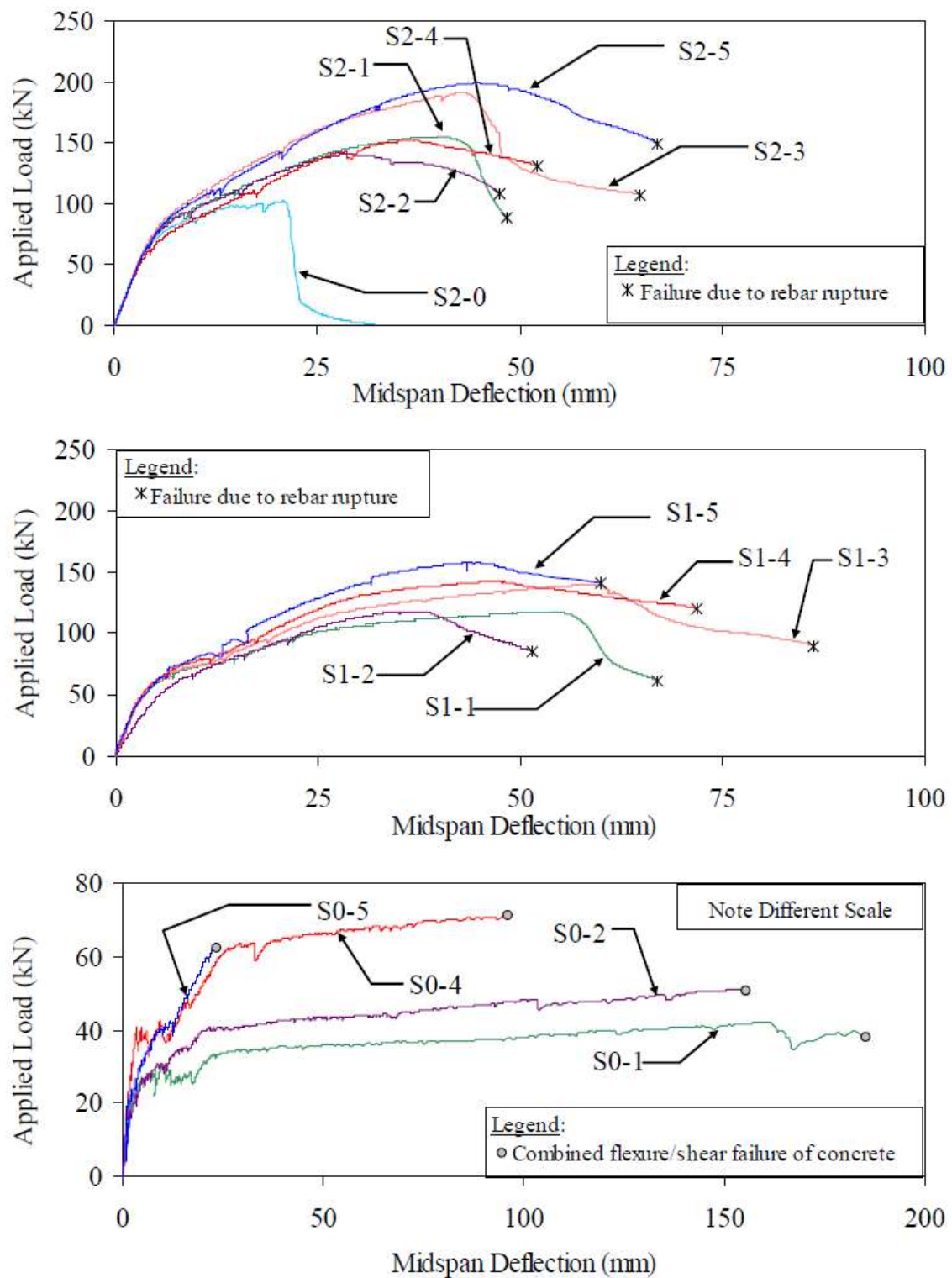


Figure 2.24. Load-deflection responses for beams (Meade and Graybeal 2010)

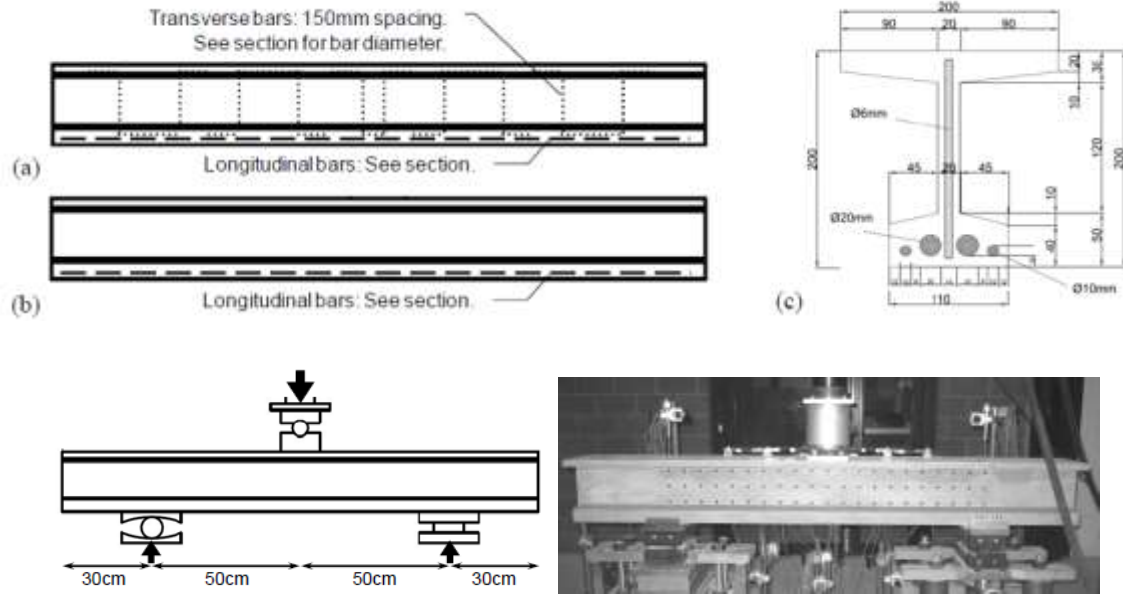


Figure 2.25 The details and test set-up for beams (Tellen et al. 2010)

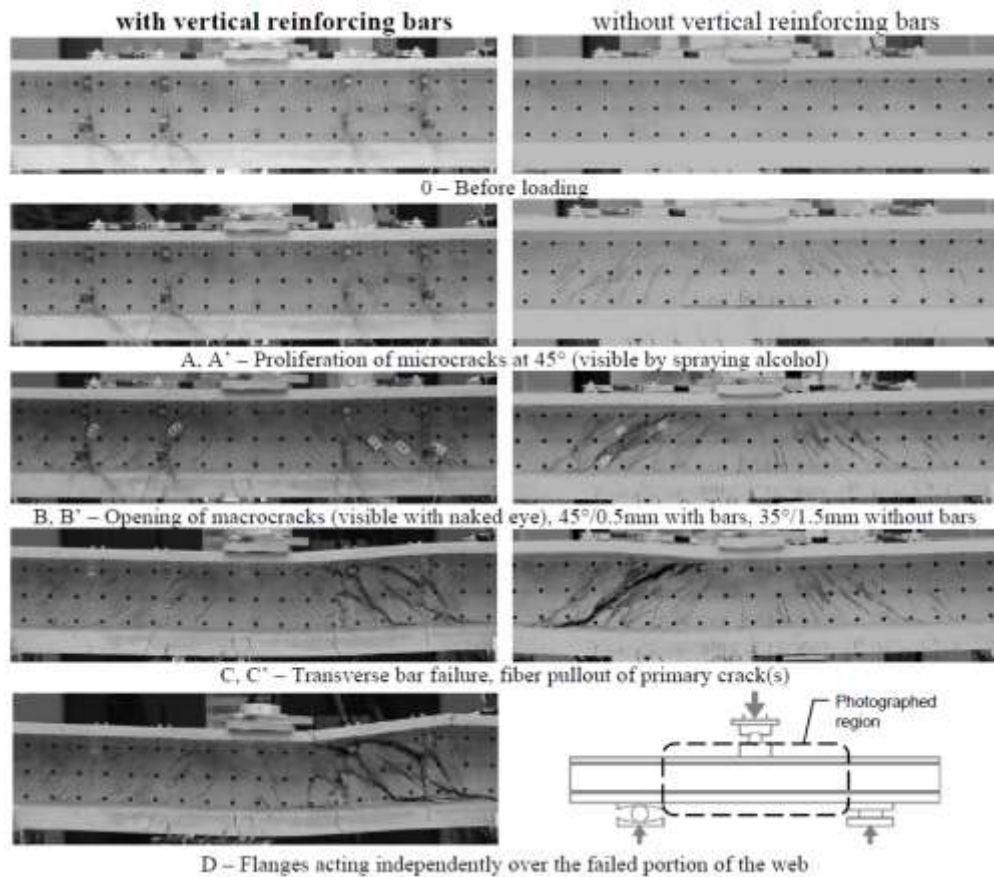


Figure 2.26 The failure modes and the crack pattern (Tellen, et al, 2010)

They claimed that the truss model resulted in good agreement with experimental results. Generalization of their approach may not be possible without conducting a parametric study on variable fiber and traditional reinforcement ratios, and different sectional shape and size, especially when the beam is prestressed. On the other hand assuming that the fibers are all perpendicular to the inclined crack presents a contradiction to the researchers' observation of the tendency of the fibers to orient horizontally.

2.6 Truss modeling

In the following section, a very brief historical and conceptual background of truss analogy for reinforced concrete behavior is presented. The application of truss modeling to prestressed concrete beams with tendons only is also reviewed.

2.6.1 Truss analogy

With the early evolution of reinforced concrete as a construction material, Ritter (1899) and Morsch (1908) proposed the concept of truss analogy to explain the flow of forces in cracked reinforced concrete. The concept was widely accepted and adopted by many researchers, design guidelines and codes. Dilger (1966) conducted a comprehensive analytical investigation to analyze the cracked shear zone using a constant angle continuum truss model. Paulay (1971) conducted an experimental and analytical study to evaluate the effectiveness of truss analogy using variable angle. Later in 1975, Park and Paulay proposed a constant truss angle to investigate the cracked elastic stiffness for structural elements subjected to shear loadings. Vecchio and Collins (1986) developed the Modified Compression Field Theory (MCFT) incorporating concrete tensile resistance to study column shear resistance. MacGregor (1992), proposed a shear analysis approach for deep beams using Strut-and-Tie models.

Top region (or flange in the case of an I-beam) of a simply supported beam is normally under compression (C) and the bottom region (bottom flange in the case of an I-beam) is in tension (T). The middle part of a beam (or the web for the case of an I-beam) is under variation of principle compressive and tensile stresses, the trajectories of which are orthogonal to each other at any point of intersection. These two sets of principle stress isolines are simulated in a truss model as struts under compression (parallel to the cracks) and under tension (normal to the cracks). As the

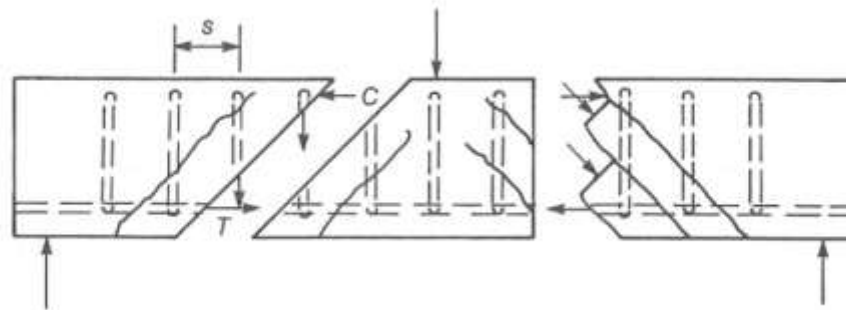
simple truss includes vertical and inclined struts, the vertical members resist tensile stresses forming ties, representing transverse reinforcement and the inclined members resist compressive stresses forming concrete struts.

Figure 2.27(a) shows the forces in a cracked beam where the top part is under compression, the bottom flexural steel and the vertical reinforcement are under tension, while the concrete in-between the cracks is under compression. In Figure 2.27(b), the beam is represented by a truss where the top cord and bottom cord are in compression and tension, respectively. The vertical truss vertical members represent the vertical (shear) reinforcement, effects of which are summed in one equivalent steel area under tension and the inclined struts are under compression (MacGregor and Wight, 2005).

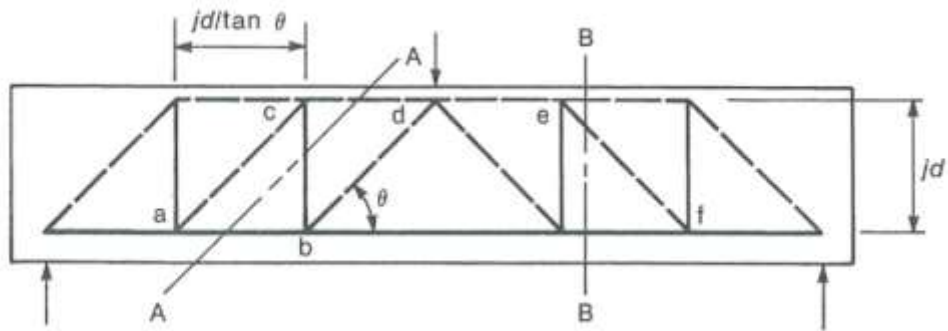
The truss model can be statically indeterminate. Often, the truss models are used to represent a load condition that corresponds to the yielding of shear reinforcement. This implies that the stirrups are replaced by vertical truss ties, each having a tensile force of $A_v f_y$ where A_v is the cross sectional area of the stirrups represented by a truss element, and f_y is the steel yield strength. This often results in a statically determinate truss (MacGregor and Wight, 2005).

2.6.1.1 Constant angle truss

The constant angle truss model is the first and the most popular simplified model originally developed by Ritter (1899) and Morsch (1908). This truss model is usually used for long beams as is shown in Figure 2.28, or beam-columns, and requires that the shear span of the beam-column must be longer than the length that is defined by the crack angle ($L = jd \cot \theta$, in Figure 2.29). It is assumed that the transverse steel is uniformly distributed over the beam length, and differential shear forces act in the vertical direction. The member forces in the truss can be calculated using static equilibrium. Figure 2.29 shows a typical constant angle truss for a simply supported beam under single external force.



(a) Internal forces in a cracked beam.



(b) Pin-jointed truss.

Figure 2.27 Constant angle truss for a cracked beam (MacGregor and Wight 2005)

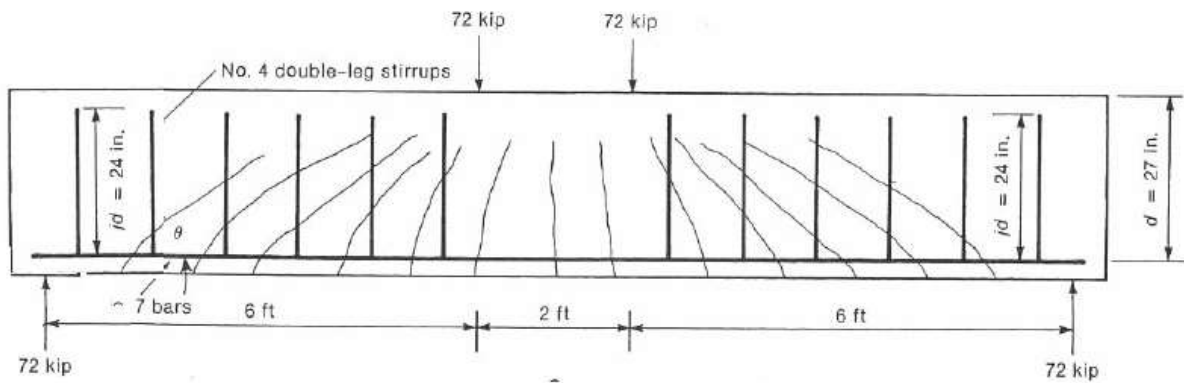


Figure 2.28 Example of a cracked reinforced concrete beam (MacGregor and Wight 2005)

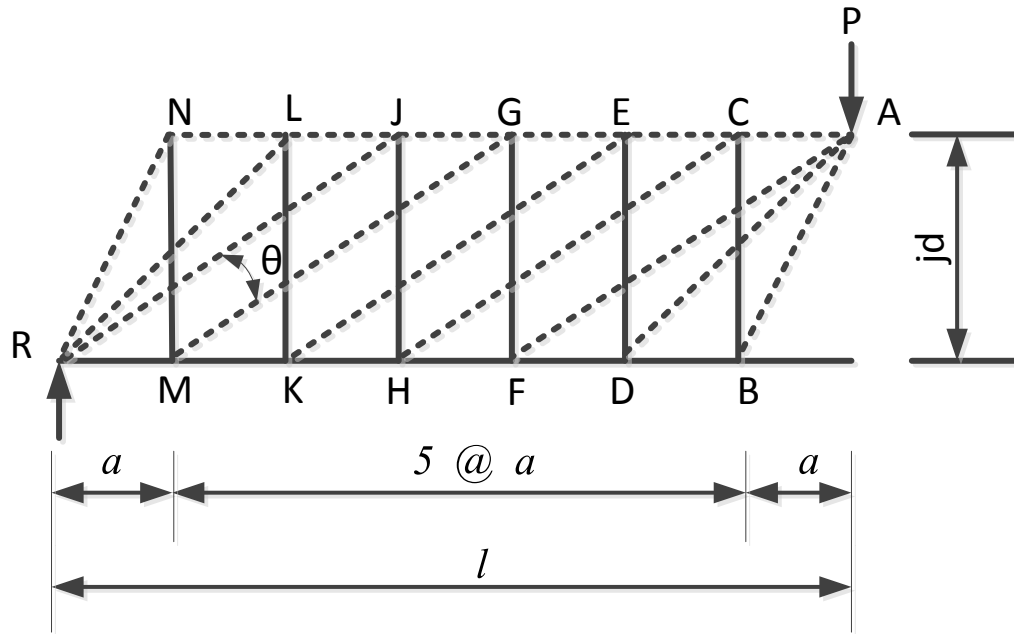


Figure 2.29 Truss analogy for a constant angle truss model (MacGregor and Wight 2005)

2.6.1.2 Variable angle truss

Variable angle truss model is based on the assumption that the stresses in a beam are distributed from concentrated loads (or reactions) in a D-shaped region. The stresses distribution may be non-linear, disturbed by cracks. The D-regions can also develop in the corners, bends, openings, and other regions of discontinuity. In the variable truss model, a single differential truss element is subjected to a differential shear force that is not uniformly distributed over the longitudinal axis of the beam, forming variable angles of inclined struts. Figure 2.30 shows a sample variable angle truss in a simply supported beam, subjected to a single force applied at the mid-span.

2.6.2 Truss Models for Prestressed Concrete Beams

Lertsamattiyakul et al. (2004) proposed a simplified truss model to evaluate shear capacity and failure patterns in prestressed concrete (PC) slender beams without transverse reinforcement. The researches established their model as an improvement to the truss model adopted by JSCE (2002), which neglects the influence of prestressing forces generated by straight tendons on shear capacity of prestressed concrete beams. The proposed truss model was validated using nonlinear finite element model (FEM) and available experimental results. The FEM was conducted using DIANA software. Figure 2.31 shows the details of the FEM model discretization.

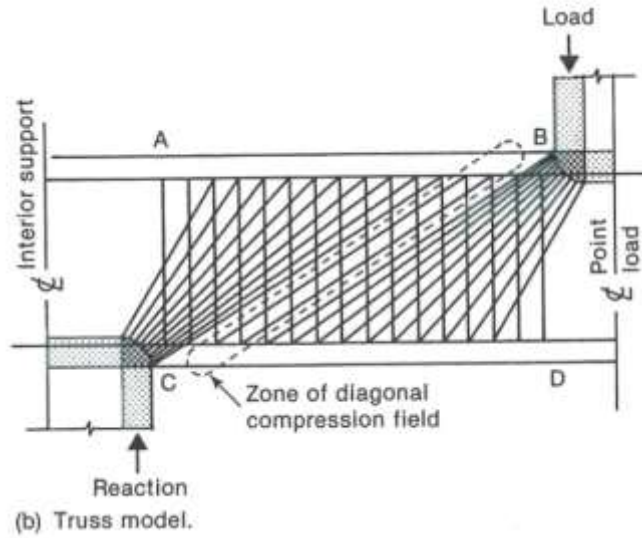


Figure 2.30 Truss analogy for constant angle truss model (MacGregor and Wight 2005)

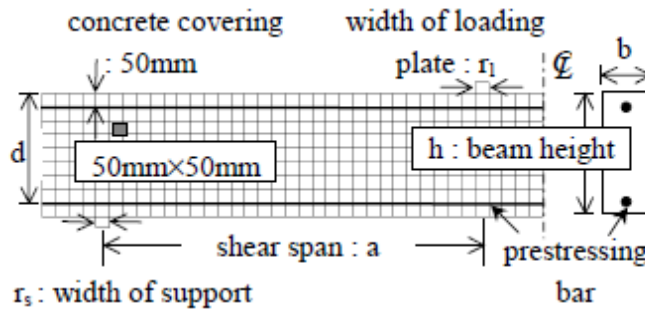


Figure 2.31 The details of FEM model discretization (Lertsamattiyakul et al. 2004)

Using the simple beam theory, the researchers defined stresses in critical sections of the beam. They were interested in studying the effect of the variation of stresses at the extreme bottom fiber (σ_l) and at the extreme top fiber (σ_u) on the critical shear zone. The stress distribution in the critical shear zone was found using a nonlinear finite element model. In the first parametric study, the researchers studied the case of zero stress resultant at the extreme top fiber ($\sigma_u = 0$) and variable stress resultants at the bottom fiber ($\sigma_l = 3$ to 15 MPa). Figure 2.32 shows that with the increase in the bottom fiber stresses the concentrated inclined stresses became steeper, and eventually vanish as a compression arch develop between the applied force and the reaction. In the second parametric study, the researchers studied the case of zero stress resultant at the extreme bottom fiber ($\sigma_l = 7$ MPa) and variable stress resultants at the top fiber ($\sigma_u = -2$, to $+$

7 MPa). Figure 2.33 shows that, this condition, once again resulted in steeper and narrower concentrated inclined stresses, while a compression arch developed between the applied force and the support reaction.

The researchers realized that the traditional truss models do not account for the observed arching action, which provides further shear resistance to PC beams even after the formation and propagation of diagonal shear cracks. These observations indicate that the arch action that develops in PC beams can maintain shear resistance until the compression failure occurs. The stress flow observed in FEM analyses indicated two compression stress flow streams, one along the inclined concentrated stress flow, and the other along the compression arch (see Figures 2.32 and 2.33). Accordingly, the researchers suggested a variable angle truss model with two inclined struts as shown in Figure 2.33. The angle of inclination of struts (θ) was found from the average inclination of compressive stress streams (the arch and the concentrated stress fields). The spacing of truss panels could be established from the beam height and the angle of inclination ($m = h \times \cot \theta$). This is illustrated in Figure 2.34. It is important to indicate that the prestressing forces in the truss model were applied as external forces.

2.7 Summary of Previous Research and Research Needs

Ultra high performance concrete (UHPC) was developed for higher resistance to corrosion agents, like chlorides, that tend to penetrate in concrete and reduce long term durability and performance of the material. UHPC mixes have water-cement ratios of less than 0.25. They contain micro fiber reinforcement to improve ductility. They also have high binder content with special aggregates. The resulting high density fibre reinforced UHPC material results in superior mechanical properties and greater durability. UHPC offers compressive strength of 150-400 MPa, flexural tensile strength of 30-40 MPa , and post-cracking tensile strength in excess of 5-15 MPa.

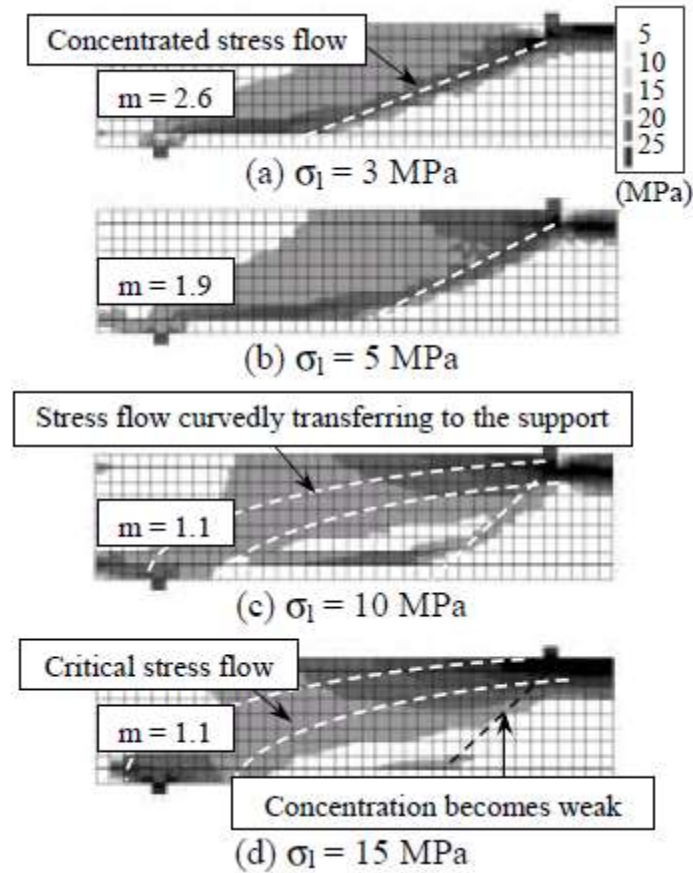


Figure 2.32 Stress distributions in the critical shear zone when ($\sigma_u = 0$) and ($\sigma_1=3, 5, 10$ and 15 MPa) (Lertsamattiyakul et al 2004)

The use of special materials, early age curing processes, fiber reinforcement, and high quality production requirements are factors that contribute to the high initial cost of UHPC structures. Cost effective production and use of the material in the construction industry require optimization of structural members. Previous efforts towards design and optimization of UHPC structures led to several challenges that call for further research and development. The optimized sections result in slender members. The slenderness affects fiber orientation, which may reduce UHPC strength, causing premature failure. The flexural capacity of UHPC members cannot be developed unless traditional shear reinforcement (stirrups) is used. However the presence of stirrups may generate further disturbance in fibre orientation. These point need further research before an optimum design for UHPC elements can be implemented. Furthermore, the applicability of existing cracking and post-cracking models developed for traditional concrete to UHPC requires validation. Experimental data, required for such validation is badly missing in the literature (Almansour and Lounis, 2009).

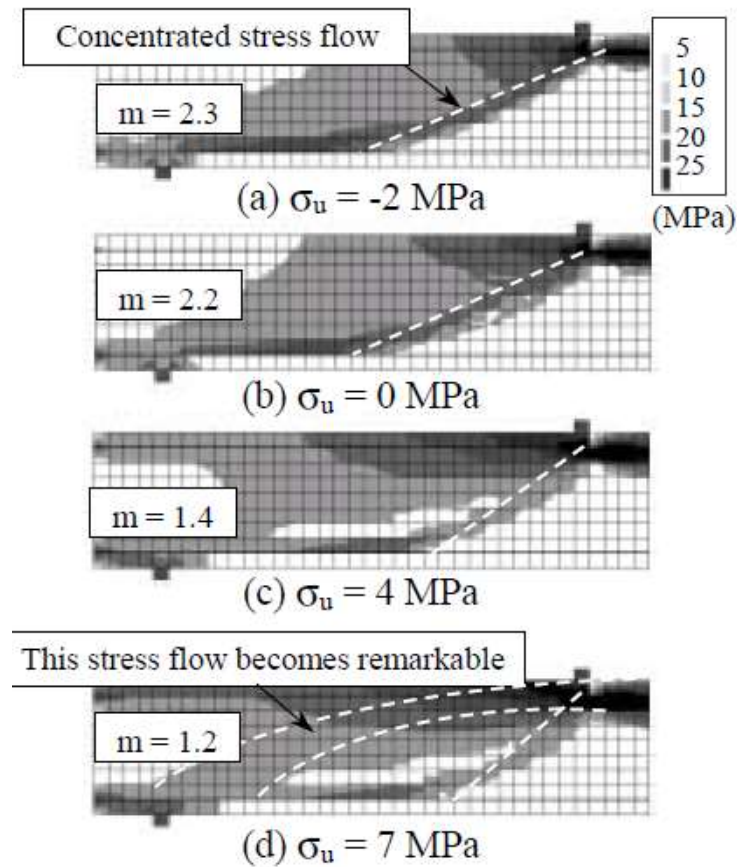
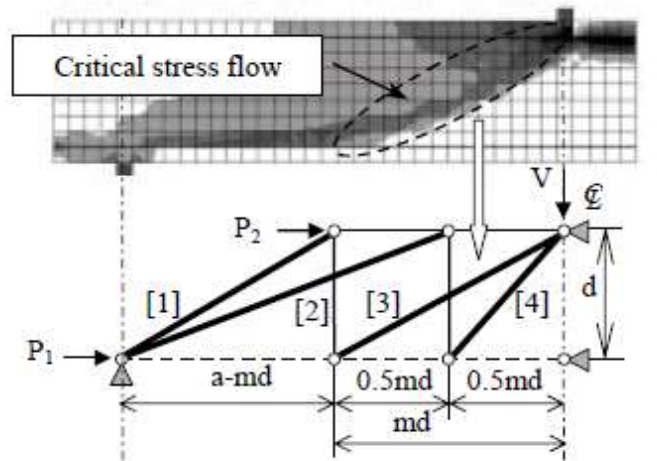


Figure 2.33 Stress distributions in the critical shear zone when ($\sigma_l = 7$ MPa) and ($\sigma_u = -2$ to 7 MPa) (Lertsamattiyakul et al, 2004)

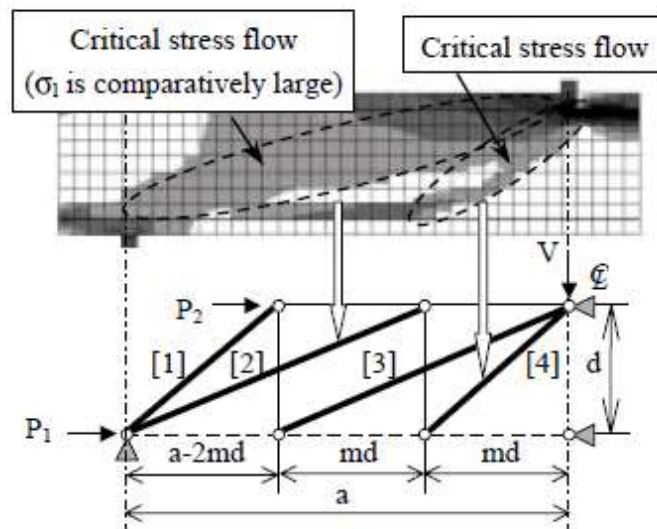
Since the major factor for developing ultra-high strength in UHPC is the presence of fiber reinforcement, many studies have focused on the effects of fiber type, fibre size, and fiber orientation. Research is still lacking in the area of fiber orientation as affected by the pouring of fresh concrete and its relation to the structural element shape, size and density of reinforcement. Some researchers investigated the effects of pouring direction of fibre orientation (Lee et al. 2010, Kang and Kim 2011). They concluded that pouring parallel to the longitudinal direction gave much higher flexural strength than pouring transverse to the longitudinal direction. However, the effect of the casting process on shear capacity of UHPC has not been investigated.

Simplified shear design procedure recommended for UHPC structural elements are based on simple truss models. These models are based on the assumption that the randomly distributed

fibers in UHPC could enable a force flow in UHPC beams that is analogous to reinforced concrete. There is no experimental evidence for such hypothesis. It is believed that the orientation of fibers would play an important role in shear capacity.



(a) Model 1: $m > 1.0$



(b) Model 2: $m \leq 1.0$

Figure 2.34 Proposed simplified truss models (Lertsamattiyakul et al. 2004)

For optimum use of UHPC, hybrid reinforcement (fiber and stirrups) offers potentials for increased shear capacity. Further experimental and analytical research is needed in combined use of fibres and stirrups. Furthermore, more research is needed for the development of new and improved analytical techniques for shear strength assessment. It has been found that traditional

truss models do not account for arching action in slender prestressed concrete (PC) beams that was observed in FE analyses. The arching action provides further shear resistance to PC beams even after the formation and propagation of diagonal shear cracks. The analytical observations indicate that, for prestressed concrete beams, the arching action could maintain shear resistance until the failure occurs due to the compression crushing of concrete.

Chapter 3

Design Procedure and Selection of Beams

3.1 Introduction

A number of prestressed UHPC beams with hybrid reinforcement (also including non-prestressed re-bars) were selected and designed for performance assessment. The beams were intended to form test specimens in the subsequent experimental phase of the research program, which is outside the scope of the current phase of investigation. Therefore, the beams selected are scaled models of those that may be designed for use in practice. However, they are sufficiently large so that the scale effects do not become a concern, and the ordinary size constituent materials can be used.

The experimental program is planned to be carried out at the NRC-Construction Structural Laboratory with a test setup that allows testing of 3.0 m long beams. Therefore the span length was selected accordingly. The beams are reinforced with different amounts of stirrups to investigate their shear behavior. An important consideration for the beam design is the optimization of the use of UHPC as a material. This implies that the amount of UHPC material is minimized while shear and/or flexural strengths are maximized.

The beam selection was done in stages. First, the significance of prestressing was investigated. This was done first designing non-prestressed rectangular UHPC sections. Different designs were performed, using non-prestressed re-bars and stirrups. The results indicate that the beams with a rectangular section require excessive amounts of steel reinforcement because of the high compressive strength of UHPC. Next, prestressed concrete design was considered. Different levels of prestressing were considered while maintaining the rectangular shape for the cross-section. The designs, once again, indicated need for a large number of prestressing strands and

large cross-sectional sizes. Therefore, the use of I-sections was investigated next. It was found that prestressed I-beams gave the highest capacity with minimum materials. Furthermore, the relatively low percentage of prestressing reinforcement needed resulted in fewer obstructions during concrete casting, promoting proper fiber orientation.

This chapter presents the highlights of the different beam designs considered. The focus is placed on: (i) The procedures followed in design of both non-prestressed and prestressed UHPC; (ii) advantages and disadvantages of the beam sections considered; and (iii) characteristics of final beam designs.

3.2 Mechanical Properties of Different Materials

The mechanical properties of UHPC, prestressing tendons (high strength steel), and non-prestressed reinforcing steel (black steel) used in design are given in Table 3.1. The stress-strain relationships, showing nonlinear material characteristics, are illustrated in Chapter 4. The prestressing tendons are specified to fail at 95% of their ultimate strength, and the tensile strength of UHPC is taken based on the JSCE, 2006 design equation ($0.4\sqrt{f'_c}$). The steel fibre geometric properties are 13mm length and 0.2mm diameter.

Table 3.1 Material properties used for analysis (JSCE, 2006)

Material	Property	Design Value
UHPC	Compressive Strength (f'_c , MPa)	175
	Tensile Strength (f_{cr} , MPa)	5.3
	Maximum Design Strain in Compression	0.0035
	Elastic Modulus MPa	50,000
	Poisson's Ratio	0.2
	Mass Density kg/m ³	2,500
Tendons (High Strength Steel)	Nominal Strength (MPa)	1860
	Ultimate design strength (MPa)	1767
	Pretension Stress (MPa)	1376
	Elastic Modulus MPa	200,000
	Poisson's Ratio	0.3
	Mass Density kg/m ³	7,800
Re-bars-Stirrups (400R Carbon Steel)	Nominal Strength (MPa)	670
	Yield Stress (MPa)	430
	Elastic Modulus MPa	200,000
	Poisson's Ratio	0.3
	Mass Density kg/m ³	7,800

3.3 Design procedures

Analytical tools used for the design of beams include available empirical formulae, truss models, and non-linear finite element methods. Because design guidelines do not exist for UHPC in Canada and the US, the design procedures presented in this chapter are based on French and Japanese recommendations. In addition, proposed design approaches for Canada and the US, presented recently by researchers, are also considered when applicable. It has to be noted that load factors and material resistance factors are ignored in this study, and nominal capacities are used in design, as representative values for estimating beam capacities in the subsequent experimental program.

3.3.1 Shear design procedures

3.3.1.1 Japanese shear design recommendations

The UHPC shear design procedure presented in this section is based on the Japanese recommendations (JSCE 2006). Accordingly, the shear design capacity, V_{yd} , is the sum of three components; those due to i) UHPC, ii) the fibres, and iii) the prestressing reinforcement. The relevant expressions are presented below with units of MPa, mm, and Newton.

$$V_{yd} = V_{rped} + V_{fd} + V_{ped} + V_s \quad (3.1)$$

Where V_{rped} is the contribution of UHPC without the fibers;

$$V_{rped} = 0.18 \sqrt{f'_c} \cdot b_w \frac{d}{\gamma_b} \quad (3.2)$$

V_{fd} is the contribution of fiber reinforcement.

$$V_{fd} = \left(\frac{f_{vd}}{\tan \beta_u} \right) * b_w * \frac{z}{\gamma_b} \quad (3.3)$$

Where;

$$\beta_u = \frac{1}{2} \tan^{-1} \left(\frac{2\tau}{\sigma'_{xu} - \sigma'_{yu}} \right) - \beta_0 \quad (3.4)$$

The transverse reinforcement contribution can be generally obtained by employing the 45-degree truss analogy:

$$V_{s,max} = \frac{A_v f_y d}{s} \quad (3.5)$$

V_{ped} is the contribution of prestressing to shear resistance:

$$V_{ped} = P_{ed} * \sin \frac{\alpha_p}{\gamma_b} \quad (3.6)$$

3.3.1.2 AFGC shear design recommendations

The shear resistance of UHPFRC is the sum of three components: (i) the composite contribution of UHPC matrix and the fiber (V_c) and the shear resistance provided by the average fiber tensile resistance (before fiber pull out) acting along the diagonal cracks (V_f) and; (ii) the contribution of the prestressing steel reinforcement to the shear resistance (V_p) (iii) the contribution of the shear reinforcement, or the stirrups, to the shear resistance (V_s). Then the shear resistance is:

$$V_r = (V_c + V_f) + V_p + V_s \quad (3.7)$$

$$V_c = \frac{0.24\phi_c}{\gamma_E} \sqrt{f_{cj}} b_0 z \quad (3.8)$$

Where the coefficient γ_E characterizes the current uncertainty regarding the possibility of extrapolating the design equations established for HPC for which $f'_c \leq 85$ MPa to UHPFRC where, $f'_c \geq 150$ MPa.

The UHPC contribution to shear strength is simplified to be $V_c = 0.16\sqrt{f'_c} b_0 z$ (Almansour & Lounis, 2009). The fiber contribution V_f is represented by:

$$V_f = \frac{S\sigma_p}{\gamma_{bf} \tan \beta_u} \quad (3.9)$$

Where σ_p is the residual tensile strength that is represented by:

$$\sigma_p = \frac{1}{K} \frac{1}{w_{lim}} \int_0^{w_{lim}} \sigma(w) dw \quad (3.10)$$

where, $w_{lim} = \max(w_u, 0.3 \text{ mm})$, $w_u = l_c \varepsilon_u$, $l_c = \frac{2}{3} h$; $S = 0.9 b_0 d_p$ or $S = b_0 z$; $K = 1.25$; $\gamma_{bf} = 1.3$ and.

It has been found that using the shear resistance calculation method presented by (AFGC, 2002) recommendations are complicated as it needs multiple tests data and many trials. On the other hand, limited experimental results are available to support the French recommendations empirical formulae for shear resistance (FHWA-HRT-06-115, 2006).

3.3.2 Flexure Analysis and Design

Flexural design for reinforced or prestressed UHPC is done in as much the same manner as conventional ultimate strength design for reinforced and prestressed concrete. The principles of strain compatibility are employed with the assumption of plane strains before bending remaining plane after bending. The application of the procedure to UHPC is illustrated in the following sections, with appropriate limits for materials.

3.3.2.1 Flexure analysis and design of non-prestressed beams

Currently there are no design guidelines for UHPC in North America. The Canadian Highway Bridge Design Code CHBDC (CSA-S6 2006) limits the ultimate strength of concrete to 85 MPa, whereas the ultimate strength of UHPC ranges between 150 MPa and 400 MPa.

The procedure described by Almansour and Lounis (2009) for flexural design of non-prestressed UHPC beams is presented in the following section. The procedure begins by assuming reinforcement ratio (ρ), and sectional dimensions. The depth of the fully stressed compressive zone, or the zone under constant compressive stress of (f_{cgu}), shown in Fig. 3.1, is given by:

$$r_{fu} = \left(\frac{\varepsilon_{cu} - \varepsilon_{bc}}{\varepsilon_{cu}} \right) \times c \quad (3.11)$$

$$r_{fo} = \left(\frac{\epsilon_{bc}}{\epsilon_{cu}} \right) \times c \quad (3.12)$$

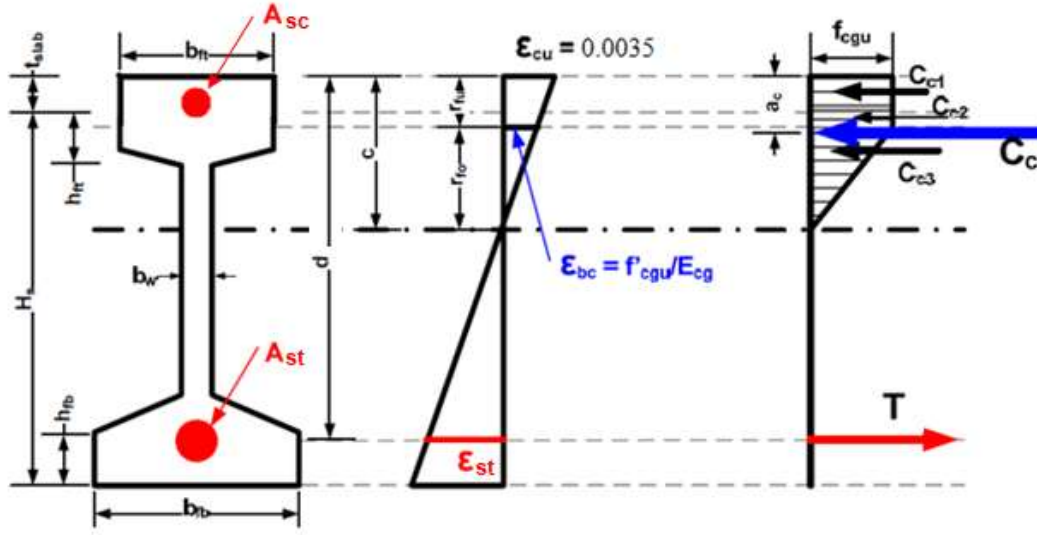


Figure 3.1 Flexural strain and stress distributions at ultimate for UHPC beams (Almansour and Lounis, 2009)

The compressive force component in the top flange can be computed as follows:

$$C_{c1} = b_e * f_{cgu} * r_{fu} \quad (3.13)$$

The compressive force component in r_{fo} is given by:

$$C_{c2} = 0.5 * b_{ft} * f_{cgu} * r_{fo} \quad (3.14)$$

The resultant compressive force is given by;

$$C_c = \sum_{i=1}^m C_{ci} \quad (3.15)$$

The point of application of the resultant C_c from the top compression fiber of the composite section is given by:

$$a_c = \frac{\sum_{i=1}^m C_{ci} \times a_{ci}}{\sum_{i=1}^m C_{ci}} \quad (3.16)$$

The centroid of the steel rebar is calculated through trial-and-error until the equilibrium of internal forces is satisfied ($C_c = T$). Hence, the flexural resistance of UHPFRC is obtained by:

$$M_u = C_c (d_p - a_c) \quad (3.17)$$

3.3.2.2 Flexural analysis and design of prestressed UHPC beams

Equations (3.11) through (3.17) can be used to analyze and design UHPC sections in flexure when they are reinforced by non-prestressed reinforcement (re-bars). This section provides an overview of sectional analysis of UHPC beams when they are prestressed, following the steps outlined by Nawy (2008). Accordingly, once the sectional properties are determined, the area of prestressing steel can be calculated as follows:

$$A_{ps} = \frac{M_n}{0.74 * f_{pu} * h} \quad (3.18)$$

The effective strain ε_1 in prestressing tendons can be written in terms of the effective prestressing stress f_{pe} , and the elastic modulus of steel E_{ps} prior to yielding.

$$\varepsilon_1 = \varepsilon_{pe} = \frac{f_{pe}}{E_{ps}} \quad (3.19)$$

When the compressive stress in the surrounding concrete at the level of steel is neutralized by the tensile stress in steel, a decompression strain $\varepsilon_{decomp} = \varepsilon_2$ is developed.

$$\varepsilon_2 = \varepsilon_{decomp} = \frac{P_e}{A_c E_c} \left(1 + \frac{e^2}{r^2} \right) \quad (3.20)$$

Where A_c : area of UHPC section

E_c : modulus of elasticity of UHPC

The maximum compressive strain at the extreme compression fiber at ultimate is assumed to be $\varepsilon_c = 0.0035 \text{ mm/mm}$ (JSCE, 2006). At this stage of loading, the strain at the level of prestressing steel becomes;

$$\varepsilon_3 = \varepsilon_c \left(\frac{d_p - c}{c} \right) \quad (3.21)$$

Where, c is the depth of the neutral axis. Hence, the total tensile strain in the prestressing steel is as follows:

$$\varepsilon_s = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad (3.22)$$

The extreme compression fiber stress can be written as:

$$f_{top} = -\frac{P_{ebot} e}{I_c} - \frac{P_{ebot}}{A_c} + \frac{P_{etop} e' c'}{I_c} - \frac{P_{etop}}{A_c} - \frac{M_c \times c}{I_c} \quad (3.23)$$

The extreme tensile fiber stresses can be expressed as:

$$f_{bot} = \frac{P_{ebot} e C}{I_C} - \frac{P_{ebot}}{A_C} - \frac{P_{etop} e' C'}{I_C} - \frac{P_{etop}}{A_C} + \frac{M_C \times C}{I_C} \quad (3.24)$$

3.4 Beam designs

A number of different beams were designed with simple supports, loaded with either two point loads at 1/3 points for flexure-dominant behavior, or a single point load at distance $a = 1.05 m$ from the right support to generate shear-dominant behavior. The shear-span-to-effective depth ratio (a/d) was equal or greater than 2.1 to avoid deep beam action. Different beam sections were designed to fulfill the following requirements for simply supported beams.

- Ultimate capacity not exceeding 5000 kN during testing;
- Over-strength and under-strength sections in flexure so that the failure is either initiated by shear or by flexure.
- Sections reinforced with non-prestressed or prestressed reinforcement for flexure, and with stirrups for shear, with randomly distributed steel fibers in concrete.
- Sufficiently wide spacing of stirrups to avoid fiber segregation and non-uniform fiber distribution during concrete casting.

3.4.1 Rectangular UHPC beam designs

Four beams with rectangular sections were designed; two as reinforced UHPC beams (reinforced with non-prestressed steel bars) and the other two as prestressed beams. Each pair either had a flexure critical or shear critical beam. The shear capacity of the beams was calculated according to the Japanese recommendations. The beam dimensions and flexural reinforcement were proportioned such that the flexural capacity in one beam was approximately 50% higher than that associated with shear failure, while the other was designed to fail in flexure.

Figure 3.2 shows the details of reinforced concrete beams designed, having 24 #25 bars. This was an excessive amount of steel, needed to balance high internal compressive forces generated by UHPC. The US size #3 reinforcement (with 3/8 in diameter) was used as shear reinforcement to obtain close spacing of ties for the investigation of the effect of stirrup spacing (50mm) on the

distribution of fibers. Figure 3.3 shows the design of prestressed concrete beams. The prestressing steel consisted of 17 tendons at the bottom for service load stresses and 9 tendons at the top to resist tensile stresses associated with prestress transfer. Each tendon was assumed to have 150 mm² nominal steel area. This was once again very high quantity of steel. Table 3.2 shows the details of these four UHPC beams. In order to compare the efficiency of different beams, a non-dimensional factor was introduced as follows:

$$\alpha = \frac{A_s}{\sqrt{\frac{I}{A}}} \quad (3.25)$$

Where A_s is the total (top and bottom) area of steel (non-prestressed or prestressed). The ratio reflects the total steel area needed for a given moment resistance, represented by the radius of gyration of a section. As the parameter (α) is reduced, the section becomes more efficient, implying higher moment resistance with reduced steel area, and reduced UHPC material. Table 3.2 shows that (α) for prestressed rectangular sections are less than half the values for non-prestressed beams, demonstrating the efficiency of prestressed beams.

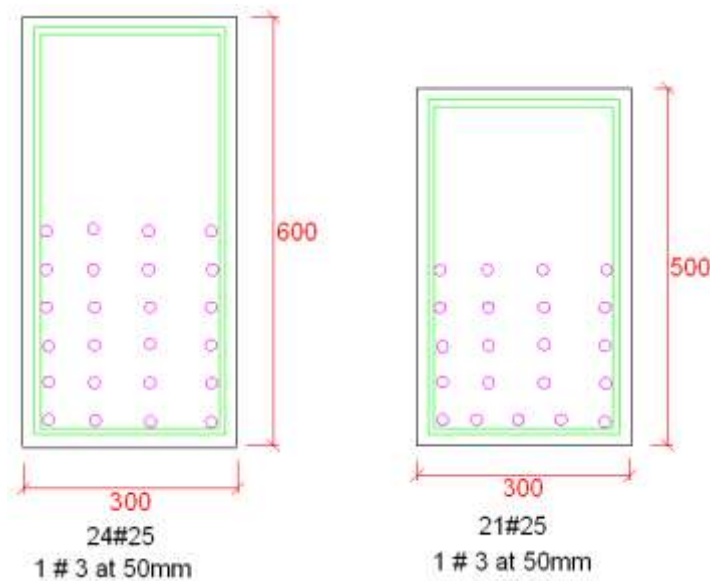


Figure 3.2 Cross-sectional properties of non-prestressed beams (flexure and shear dominant)

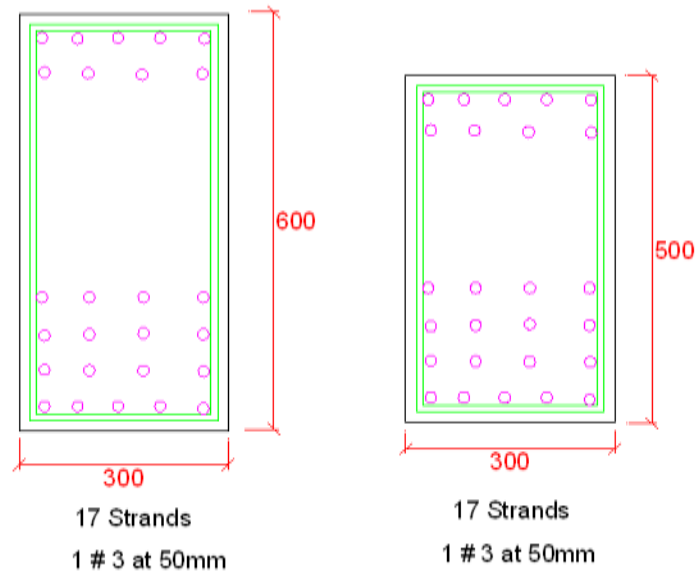


**Figure 3.3 Cross-sectional properties of prestressed beams
(flexure and shear dominant)**

Table 3.2 Design quantities for non-prestressed and prestressed UHPC rectangular beams

Beam Type	ρ_f	ρ_s	A_s mm^2	$\frac{A_s}{\sqrt{I}\sqrt{A}}$	P_{sh} KN	P_u KN	M_u $KN.m$
Non-prestressed (shear failure)	0.068	0.0084	12216	71	2597	3408	2277
Non-prestressed (flexural failure)	0.071	0.0056	10689	62	2597	2X1616	1616
Prestressed (shear failure)	0.022	0.0101	3900	27	1949	3028	2068
Prestressed (flexural failure)	0.026	0.0067	3900	27	1949	2x1637	1637

3.4.2 Design of UHPC beams with I-Sections

As demonstrated in the previous section, the efficiency of rectangular UHPC sections is low, because; (i) the ratio of the moment of inertia to the area of a rectangular section is low; (ii) the lever arm for internal moment resistance is small; and (iii) any increase in beam depth results in a corresponding increase in shear resistance with a marginal increase in moment capacity because the moment capacity is basically affected by the effective depth (d) of a designed beam. The sectional efficiency was improved when prestressing was used. Further improvement is evident when rectangular section is replaced by I section. Therefore, additional 7 UHPC beams were designed. They include; (i) five shear critical prestressed UHPC beams; and (ii) two flexure critical prestressed UHPC beams.

The shear-critical beams had stirrup spacing as the design variable. Accordingly, a beam without any transverse shear reinforcement, and 4 others with different stirrup spacing were designed. The beams all had simple supports with a 3.0 m span length and a single concentrated load applied at $a = 1.05$ m away from the left support. Flexure-critical beams either had closely spaced stirrups in the support region or no stirrups. The prestressing strands in these beams were designed to produce predominantly flexural behavior.

BS1 with 50mm stirrup spacing

This beam has 50 mm stirrup spacing, which makes it the most heavily reinforced beam in the transverse direction. This stirrup spacing results in a large shear capacity that demanded a large number of strands. Three layers of bottom strands and a single layer of top strands are used. The cross-sectional details are illustrated in Fig. 3.4.

BS2 with 100mm stirrup spacing

Beam BS2 has 100mm stirrup spacing. Due to the increase in the stirrup spacing, the beam capacity is less than that of BS1, which results in less number of strands. Hence, the strands are arranged in two layers in the bottom flange. Also, the top longitudinal strands are organized in one layer. Figure 3.5 shows the design details.

BS3 with 150mm stirrup spacing

This beam has a stirrup spacing of 150 mm, as shown in Fig. 3.6. The increased stirrup spacing and resulting reduction in shear resistance results in 12 strands as longitudinal prestressing steel. Only 4 strands are required in the top flange because of the reduced tension in the top fiber.

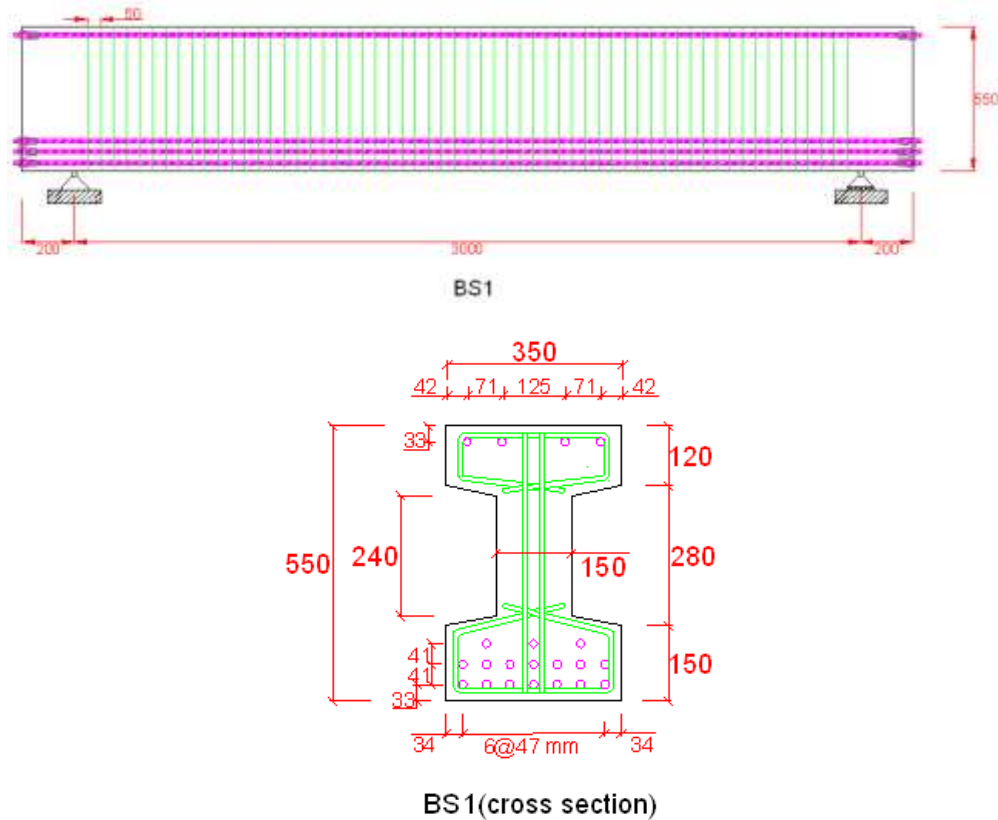
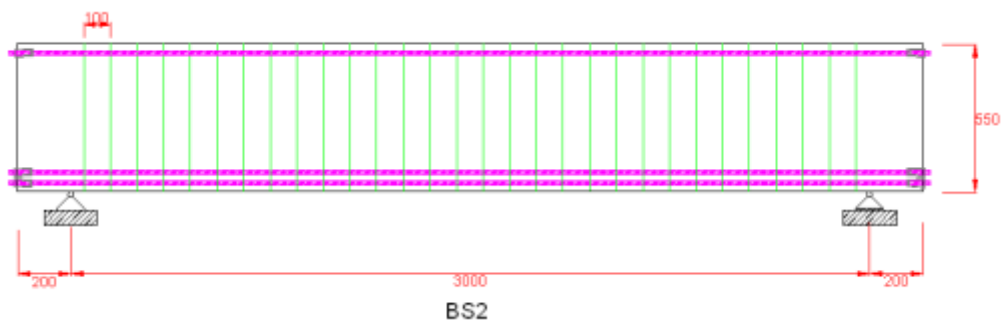
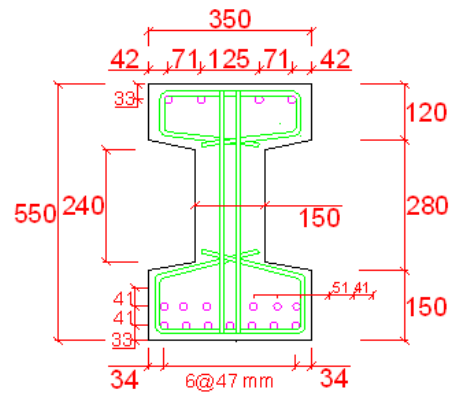


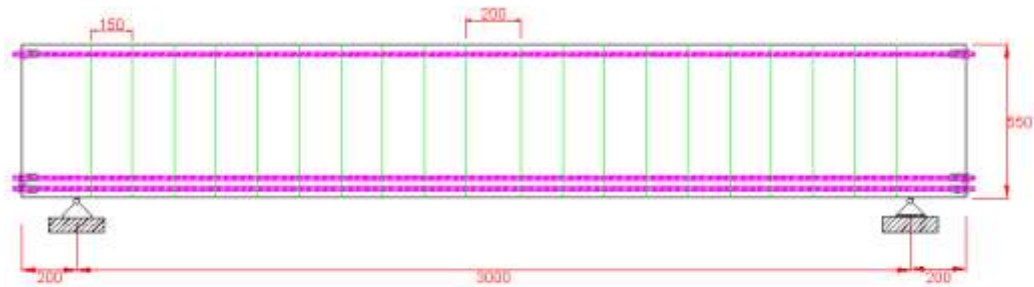
Figure 3.4 Design details of BS1



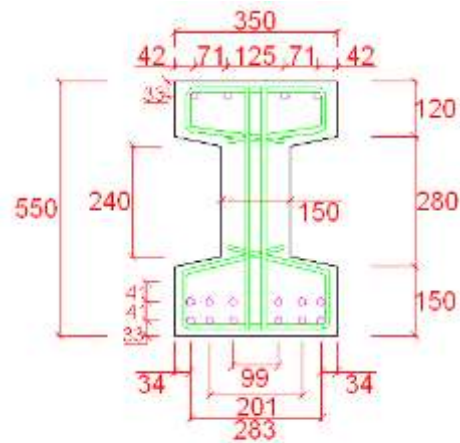


BS2 (cross section)

Figure 3.5 Design details of BS2



BS3



BS3 (cross section)

Figure 3.6 Design details of BS3

BS4 with 200mm stirrup spacing

The details of Beam BS4 are illustrated in Figure 3.7. This beam has the widest stirrup spacing of 200mm. There is no shear reinforcement in the middle 300mm of the specimen. The number of bottom and top strands is 11 and 4, respectively.

BS5 without stirrups

BS5, without any stirrup steel is illustrated in Figure 3.8. This beam is intended to demonstrate the contribution of fibers in concrete to shear resistance. Its performance will be compared with the Japanese recommendations for design.

FL1 and FL2

In addition to the shear-critical beams discussed above, two flexure-dominant beams were designed. The first beam (FL1) has a stirrup spacing of 50mm in the shear failure zone, and the second beam (FL2) has no transverse reinforcement. They both have 13 strands at the bottom, and 3 strands at the top. The details are illustrated in Figs. 3.9 and 3.10.

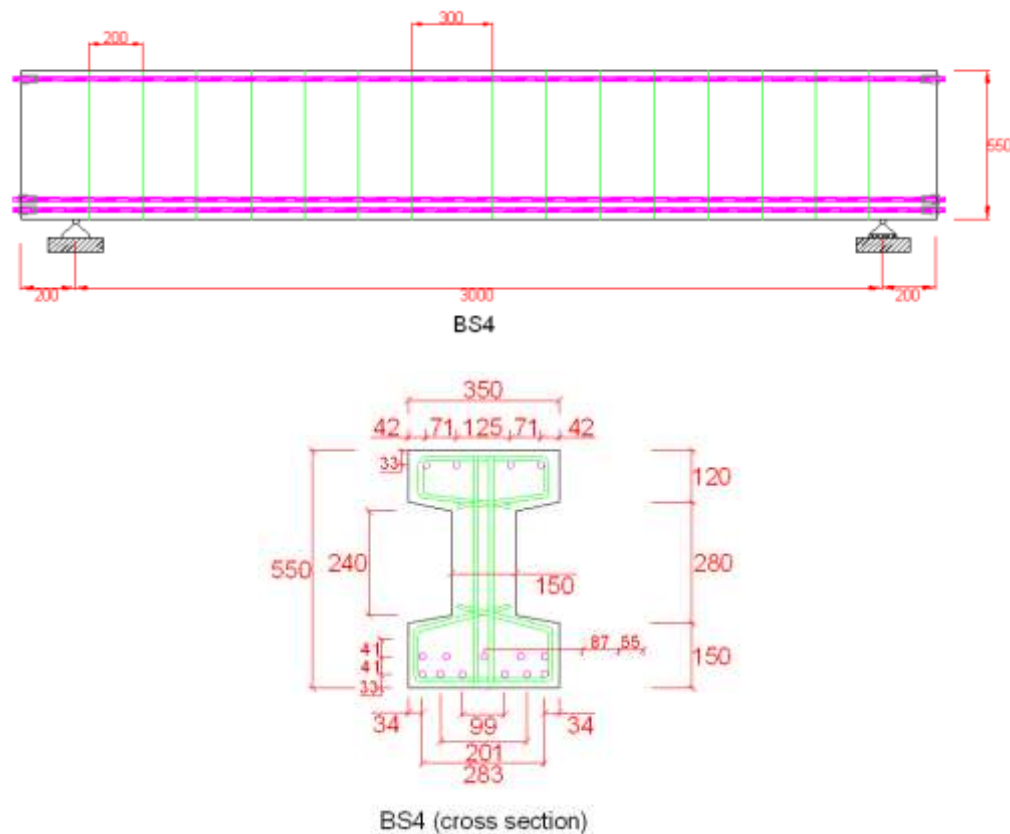
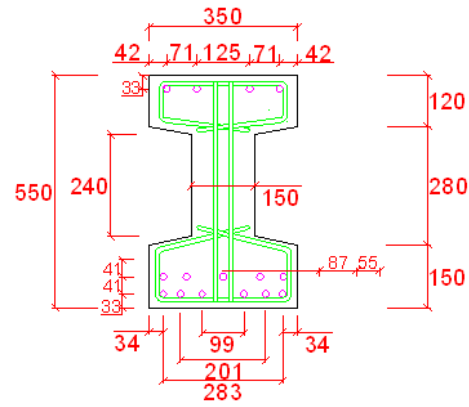


Figure 3.7 Design details of BS4

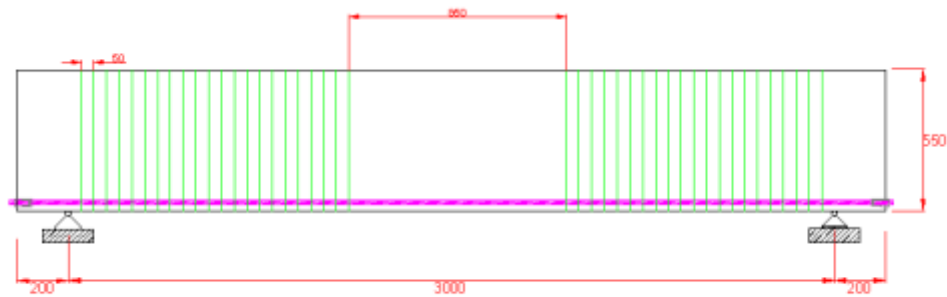


BS5

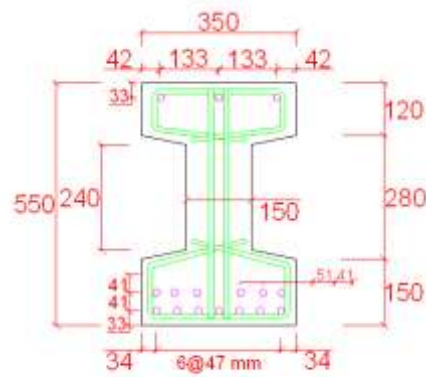


BS5 (cross section)

Figure 3.8 Design details of BS5



FL1



FL1 (cross section)

Figure 3.9 Design details of FL1

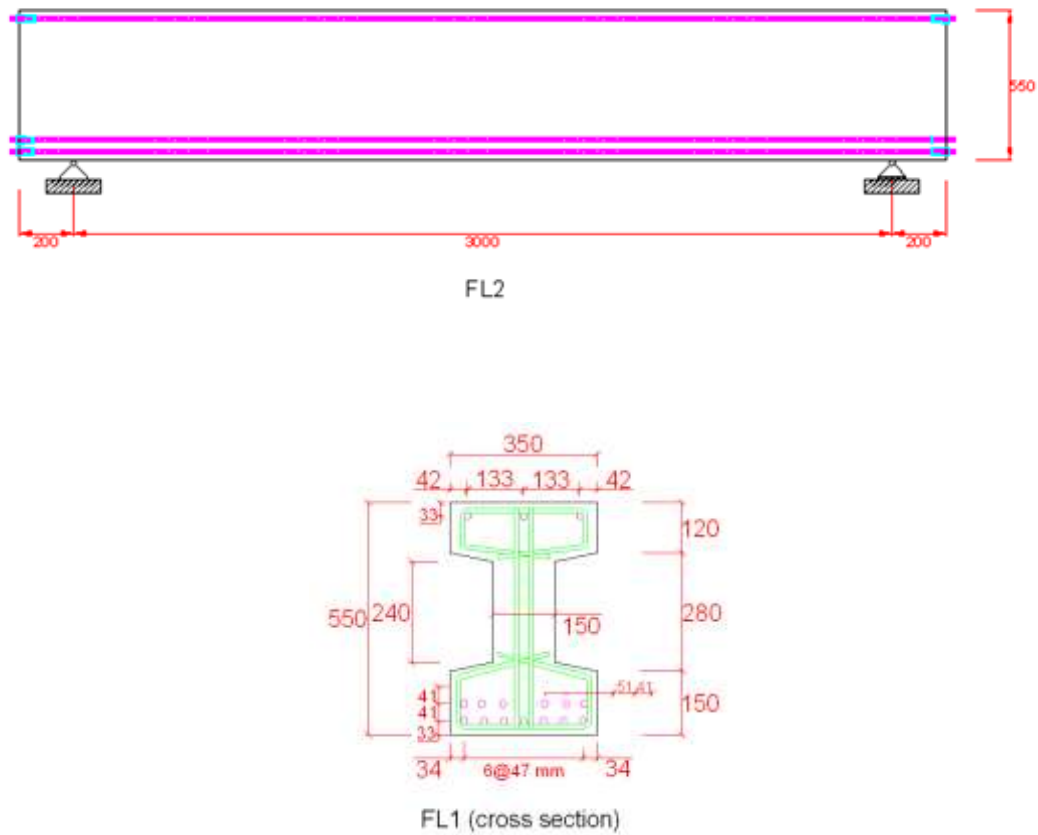


Figure 3.10 Design details of FL2

Table 3.6 summarizes the design details for all I-beams. The table indicates that for the same moment and shear capacity, lower amounts of reinforcement and UHPC are needed for prestressed UHPC I-beams in comparison with rectangular sections for which the values are tabulated in Table 3.3. The non-dimensional optimization factor (α) has much lower values, indicating as low as 15% of the corresponding values for rectangular non-prestressed sections.

Tables 3.4 and 3.5 show extreme fiber stresses in critical sections and tendon stresses for all I-beams at ultimate and prestress transfer load stages.

Table 3.3 Design quantities for prestressed UHPC I beams

Type	ρ_{ft}	ρ_s	A_{st} mm^2	$\frac{A_{st}}{\sqrt{\frac{I}{A}}}$	P_{sh} KN	P_u KN	M_u $KN.m$
BS1	0.0182	0.0098	2850	16	1818	3018	2060
BS2	0.0139	0.0049	2550	14	1380	2423	1654
BS3	0.0128	0.0033	2100	12	1224	2256	1540
BS4	0.0117	0.0025	1950	11	1136	2133	1456
BS5	0.0117	—	1950	11	548	2095	1430
FL1	0.0139	0.0065	2250	13	1818	2×1656	1656
FL2	0.0139	—	2250	13	548	2×1656	1656

Table 3.4 Stresses in critical design sections and tendons at ultimate load stage

Stress (MPa)	Beam type						
	BS1	BS2	BS3	BS4	BS5	FL1	FL2
Stress at extreme tensile fiber	47.8	22.7	17.5	13.6	14.3	26.1	26.1
Stress at extreme compressive fiber	-42.0	-20.8	-16.6	-13.5	-12.3	-22.2	-22.2
Stress in tendons	1785	1788	1790	1820	1820	1785	1785

Table 3.5 Stresses in critical design sections and tendons at transfer load stage

Stress (MPa)	Beam type						
	BS1	BS2	BS3	BS4	BS5	FL1	FL2
Stress at extreme tensile fiber	-7.4	-11.1	-11.6	-12.0	-9.4	-8.3	-8.3
Stress at extreme compressive fiber	-54.3	-38.9	-35.4	-32.1	-31.7	-38.7	-37.2
Stress in tendons	1376	1376	1376	1376	1376	1376	1376

Table 3.6 Summary of beam design for I-Beams

Beam Name	Strands (Type Pr EN 10138-3)			Stirrups - #3 US (Bar area:71 mm ²)		Concrete Strength (MPa)	Fibre Ratio (%)
	Area (mm ²)	Number		Spacing (mm)	Number		
		Bottom	Top				
BS1	150	17	4	50	59	175	0.9
BS2	150	13	4	100	29	175	0.9
BS3	150	12	4	150	20	175	0.9
BS4	150	11	4	200	15	175	0.9
BS5	150	11	3	N/A	N/A	175	2
FL1	150	13	3	N/A	N/A	175	0.9
FL2	150	13	3	N/A	N/A	175	2

Chapter 4

Finite element Analysis of Prestressed UHPC beams

4.1 Introduction

Prestressed UHPC beams with I-sections give the best load capacity-to-cross-sectional area ratio, as well as the best load capacity-to-moment of inertia ratio. This is demonstrated in Chapter 3. More refined analysis than those conducted in Chapter 3 is required to study the effect of shear reinforcement (stirrups) on shear and flexural load capacities of prestressed UHPC beams with random distribution of fibers. Finite element method (FEM) is expected to give more accurate deformation and stress distributions, initiation and development of cracks, and stress concentration zones and their effects. FEM also helps in defining the possible failure modes. This chapter presents the finite element modeling (FEM) of prestressed UHPC beams, convergence study, stress checks and failure loads for different beams designed in Chapter 3.

The primary purpose of the FEM method employed as part of this study is to evaluate the safety and accuracy of existing UHPC shear design approaches recommended (mainly JSCE 2006). FEM is also used to investigate the effectiveness of the simplified truss model suggested in JSCE 2006 for estimating the shear capacity of prestressed UHPC beams with hybrid reinforcement (combined stirrups and internal fibres). The JSCE recommendations do not include guidance on using hybrid shear reinforcement. Furthermore, they do not provide a solid method for calculating the contribution of shear reinforcement. As shown in the previous chapter, it is necessary to use significant amount of stirrup reinforcement to attain load carrying capacities consistent with the high strength of UHPC, which cannot be reached with the use of fiber reinforcement alone. Hence, it is important to investigate stress distributions in critical flexural and shear regions of beams, and the resulting failure mechanisms. This involves stress distributions in beams at different stages of loading, with different shear reinforcement ratios. FEM analysis also enables preliminary assessment of stress concentration zones and possibility

of establishing criteria for material optimization. An important outcome of the FEM analysis is expected to be the establishment of the basis for a truss model that can better represent the flow of forces at ultimate stage of loading.

UHPC is a fiber reinforced concrete material where very fine aggregate is used (no traditional coarse or fine aggregate are used, as discussed in Section 2.2). It is cast and heat/steam cured with high level of quality control. The resulting material has different stress-strain characteristics and fracture behavior when compared with traditional concrete. The applicability of available FEM software to UHPC, including the assumptions and mechanical properties of materials, has not yet been proven. In addition, the role of steel fibers and their contribution to mechanical properties of UHPC after cracking is typically not considered in FEM modelling. Furthermore, the use of shear reinforcement, high percentage of prestressing steel, and slender cross sections in UHPC elements result in significant disturbance of fiber distribution (and possibly orientation). The high fiber orientation can further violate the assumption of isotropic properties of UHPC, changing its behavior to that of an orthotropic or even fully anisotropic material. The coupled action of orthotropic behavior due to fiber orientation, and high stress concentration zones in the flange, the web and the flange-web connection can result in serious cracks and/or premature crushing that the FEM model is not capable of capturing.

The software (Diana TNO) is used in this study, which is an advanced computer program developed specifically for modeling the structural behavior of reinforced concrete members. The selection of this software is based on its availability and it's well recognized capabilities in modeling the behavior of reinforced concrete structures. The FEM is anticipated to predict the failure mode when the concrete reach extreme compression strains or the prestressing steel reach its ultimate strength. During the modeling process, extreme care was directed to ensure the convergence of the model while identifying possible similarities and differences in material properties and structural behavior between UHPC and traditional concrete elements.

4.2 General Characteristics: Geometry, Boundary Conditions, and Loading

Two major external loading sets were modeled in this study: (i) three-point loading; and (ii) four-point loading. The first loading set was to study the shear performance of prestressed beams

where a single load was applied at a distance (a) from the support such that the shear-span-to-effective depth ratio (a/d) exceeded 2 (see Figure 4.1). The second loading set was to study the flexural performance of prestressed beams where two point loads were applied at one third and two thirds of the span. Simple support condition was assumed with a hinge support on one side, and a roller support on the other side. The center-to-center span length was taken as 3.0 m.

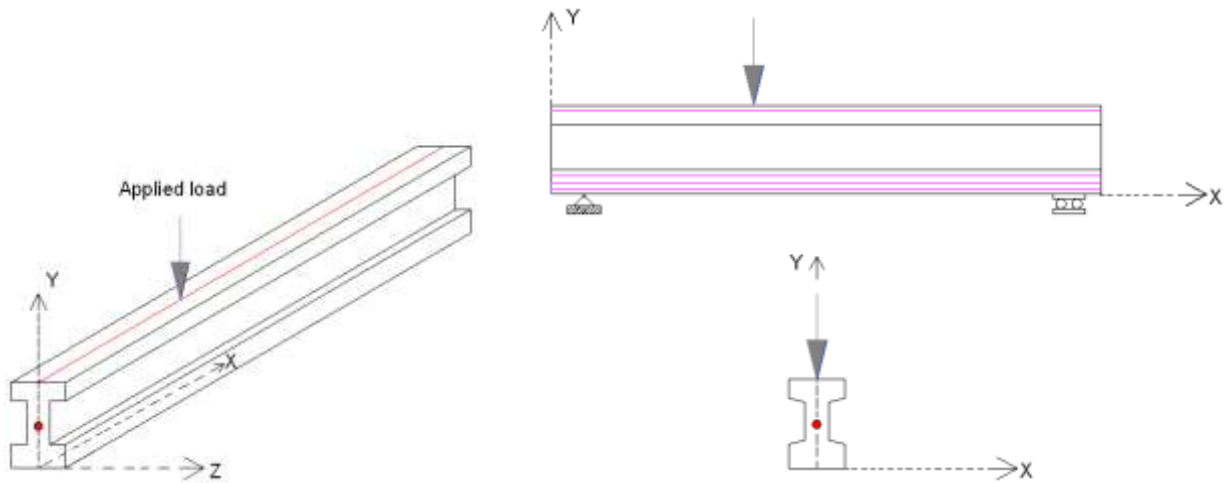


Figure 4.1 Overview of the prestressed UHPC beam, shear loading case

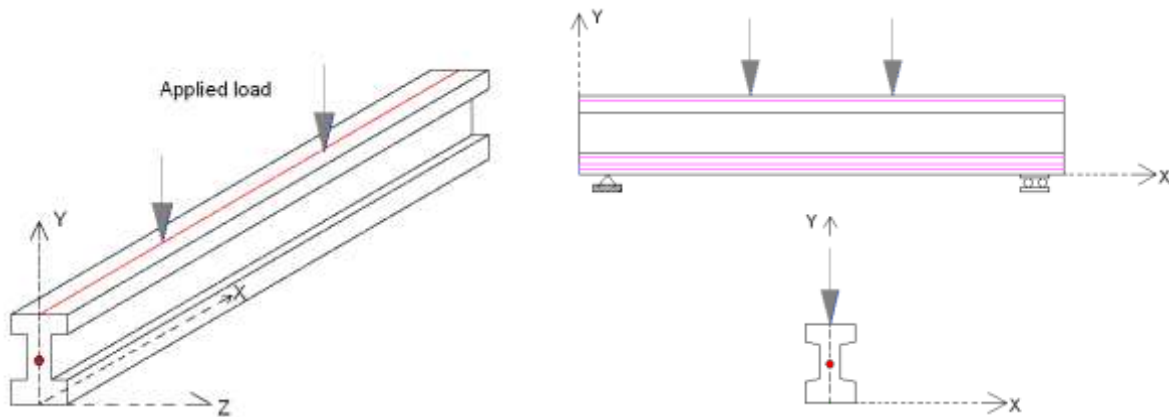


Figure 4.2 Overview of the prestressed UHPC beam, flexural loading case

4.3 Mechanical Properties of Materials

The stress strain relationship of UHPC used in the FEM model was the relationship given in the Japanese recommendations (JSCE, 2006). This relationship is shown in Figure 4.3 (a & b). It

provides a conservative simplification of typical test results of UHPC. More precise models have not yet been developed for structural analysis. Two types of steel were used in the FEM modeling of beams; i) ASTM A416 Grade 270 (1860 MPa) tendons for prestressing and ii) #3 (9.5 mm diameter) rebars for stirrup reinforcement. Figures 4.4 and 4.5 show the stress strain relationships for the stirrup steel and the prestressing tendons, respectively.

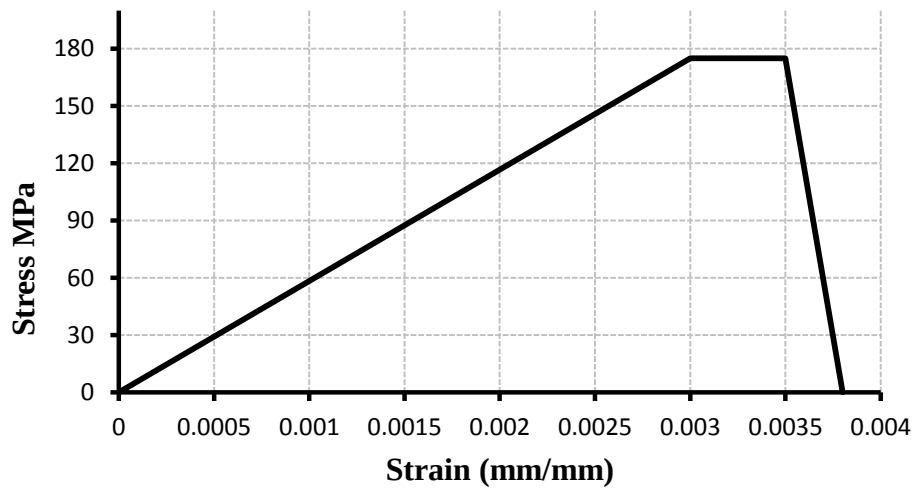
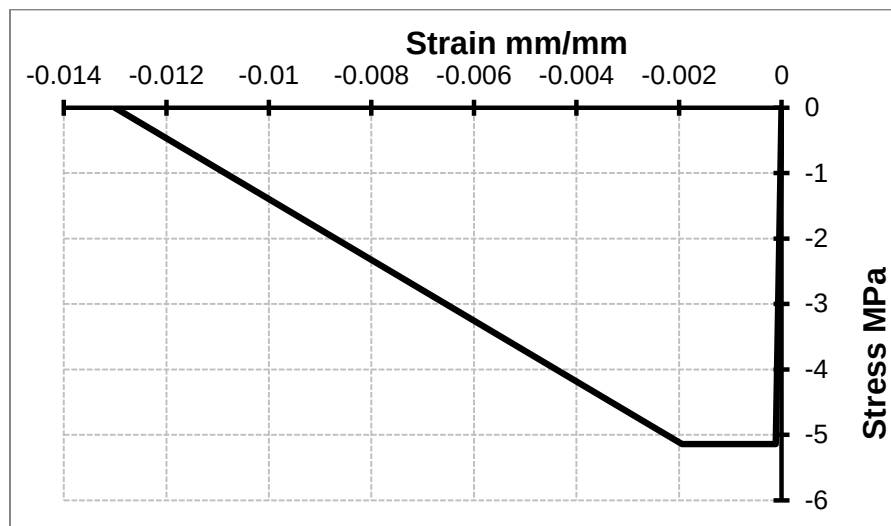


Figure 4.3 (a) Idealized stress-strain relationship of UHPC (Compression region)



(b) Idealized stress-strain relationship of UHPC (Tension region)

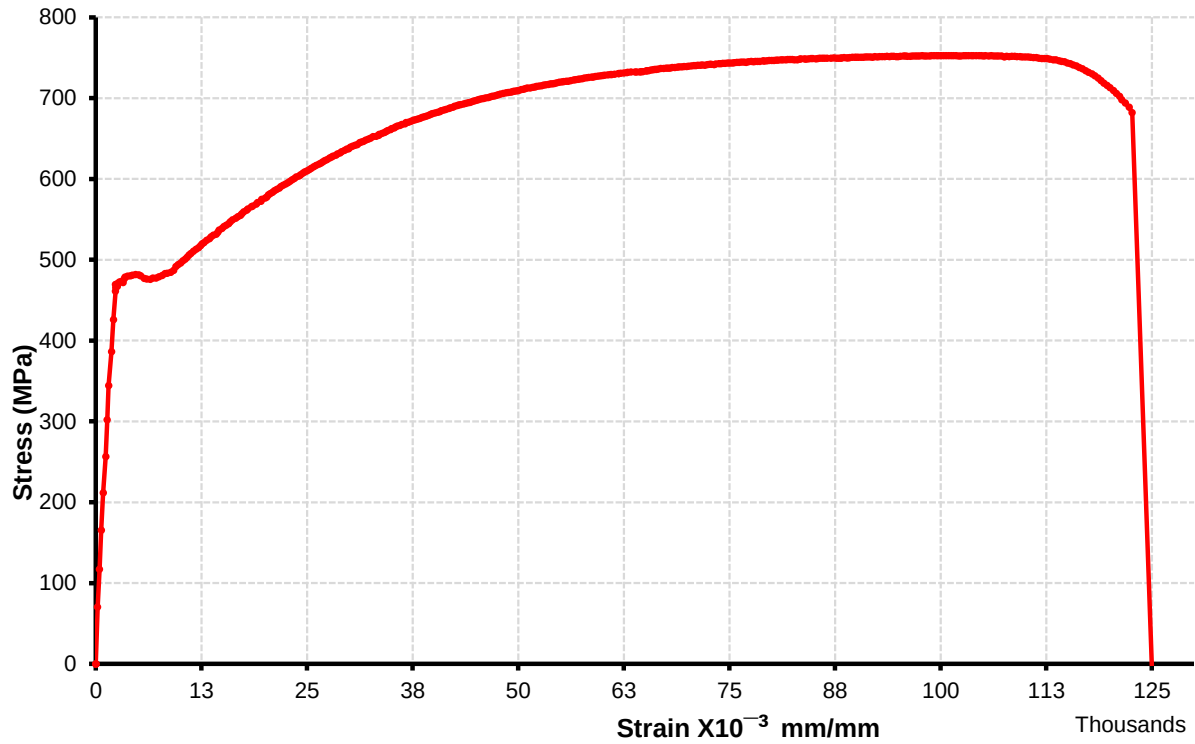


Figure 4.4 Typical stress-strain relationship for stirrup steel obtained through coupon tests

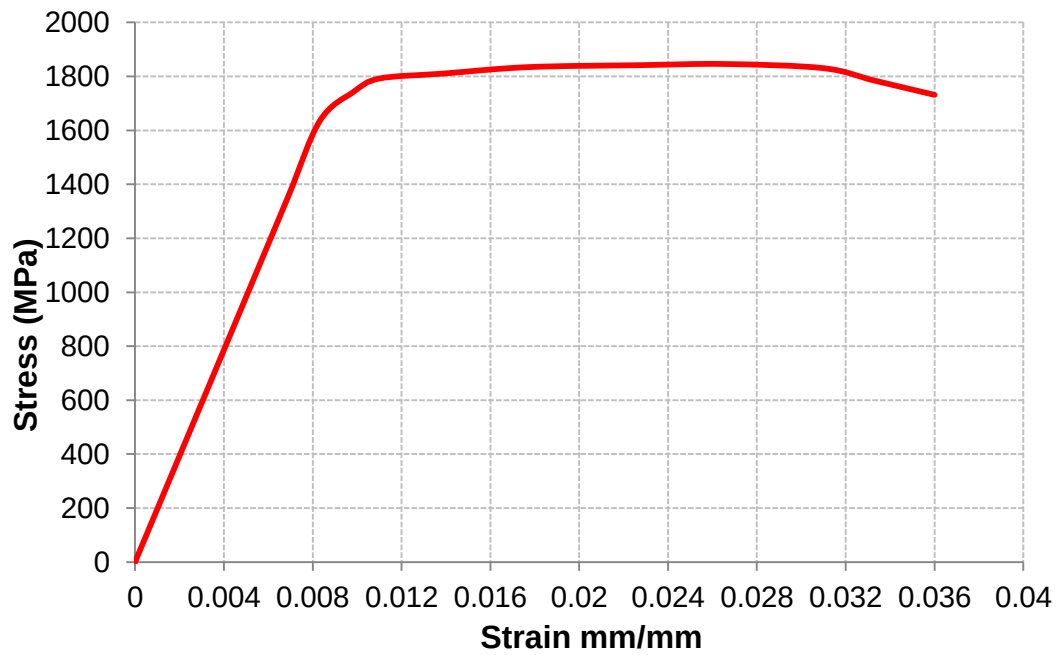


Figure 4.5 Stress-strain relationship of prestressing steel, ASTM A416 Grade 270

Linear elastic parameters were used for modeling UHPC in the linear FEM model, as summarized in Table 4.1. The non-linear FEM model for UHPC consists of a total strain crack model. Multi-linear softening was assumed in the tensile regime, and multi-linear conservative approximation was assumed for the compressive behavior of UHPC up to ultimate strength (which is defined as the failure of UHPC through crushing). The values used are given in Table 4.2. In Diana, the total strain-crack model consists of three sub-models; i) orthogonal fixed crack model, ii) orthogonal rotating crack model, and iii) switching from rotating to fixed crack model. The crack model used in the software is referred to as “the total strain based rotating crack model.” For the non-prestressed steel reinforcement, von Mises plasticity model with strain hardening was selected with the yield strength specified in Table 4.3.

Table 4.1 Linear elastic parameters for the models

Linear elastic parameters	Syntax	UHPC	Steel
Elastic Modulus MPa	YOUNG	50,000	200,000
Poisson's Ratio	POISON	0.2	0.3
Mass Density kg/m ³	DENSIT	2,500	7,800

Table 4.2 Nonlinear parameters for UHPC-models (for FEM modeling in Diana)

Nonlinear parameters	Syntax	UHPC
Compressive strength (MPa)	COMSTR	175
Tensile strength (MPa)	TENSTR	5.29
Fracture energy(N.m/m ²)	GF1	100
Crack model	TOTCRK	Rotate
Tensile curve	TENCRV	Multi-linear
Compression curve	COMCRV	Multi-linear

Table 4.3 Nonlinear parameters for the embedded steel in the FEM model

Nonlinear parameters	Steel
Plasticity model	Von Mises
Yield strength MPa	440

4.4 Modeling Approach and Discretization

One of the important steps in FEM modeling is the selection of appropriate element modeling approach, which includes the element selection, and the generation and optimization of the mesh. The details of the development of finite element modeling of concrete structures are outside of the scope of this thesis. The focus in this chapter is on general engineering concepts that help interpret the analysis results and the comparisons made with simplified modeling approaches.

The prestressed UHPC beam analyzed is symmetrical in the vertical plane, about the axis that passes through the center of gravity of the cross section. All the external and self-loads, as well as the center-lines of supports are assumed to lie in the plane of symmetry (see Figures 4.1 and 4.2). Because of the symmetry of the section, loading and boundary conditions, the beam is assumed to be under no torsional moment. The out of plane-of-symmetry deformations/strains are assumed to be zero. Hence, the I-beam can be simulated using shell elements where all the loads and reactions are acting in the shell plane. The deformations, strains and stresses acting in the beam are simulated by the in-plane deformations, strains and stresses of the model. The major difference between the uses of plane stress/plane strain elements and the shell element is that the shell element can directly simulate all the in-plane and out-of-plane stresses and forces. Also, shell element are typically used for structure where the thickness is not important compared to its length and width of the member. The cross section of the beam consists of three parts: (i) bottom flange: 150 mm height by 350 mm width; (ii) web: 280 mm height by 150 mm width; and (iii) top flange: 120 mm height by 350 mm width. Therefore, the resulting two dimensional model involved three types of elements: (i) shell elements to simulate the UHPC with two thicknesses: 350 mm for the flange and 150 mm for the web; (ii) tendon/cable elements to simulate the prestressing tendons; and (iii) frame elements to simulate the shear reinforcement. The model uses the software built-in models for bond between the reinforcement and the UHPC. These models were originally developed for traditional reinforced concrete. The discretization followed the concept of uniform element dimensions throughout the beam, while sizes of the elements were optimized through a convergence study, as discussed below.

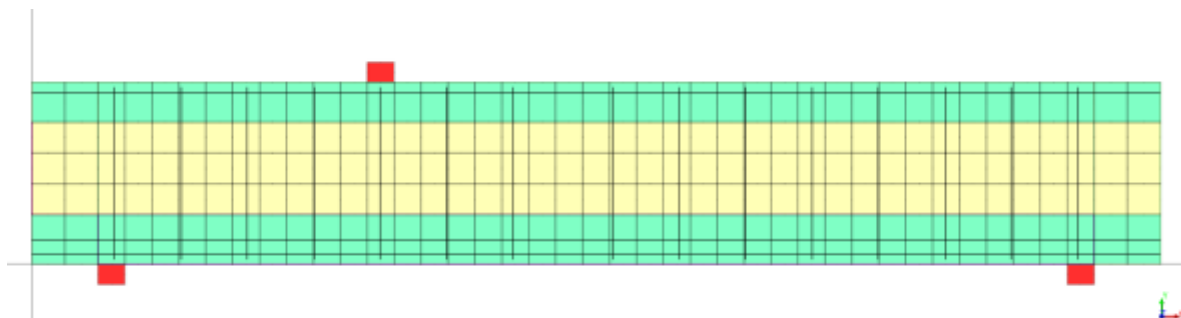
The accuracy of the FEM analysis and the computational time and stability are highly depending on the model configuration and FEM mesh. In general, the number of elements, number of joints

per element, degree of numerical integration, and hence the total number of joints in the finite element model are critical for numerical stability. For nonlinear models, the running time and the accuracy of the model are also dependent on the load step, which has to be optimized.

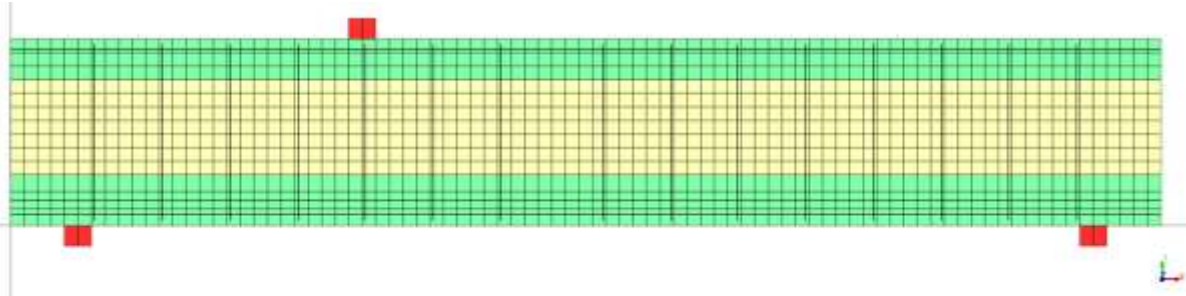
4.5 Convergence study and Optimum mesh

In order to ensure the accuracy and numerical efficiency of the finite element model a converging study was conducted based on the maximum deflection in the beam. The approach is simplified by using constant element dimensions (or $L_{\text{element}} \times B_{\text{element}}$, where L_{element} is the length of the element and B_{element} is the breadth of the element). Selecting B_{element} equal to L_{element} further simplifies the meshing process and the convergence study. The convergence study is then conducted by starting with a reasonable element side dimension (B_{element} or L_{element} , which are equal in this case), then reducing this dimension successively and comparing the maximum deformation for different mesh sizes.

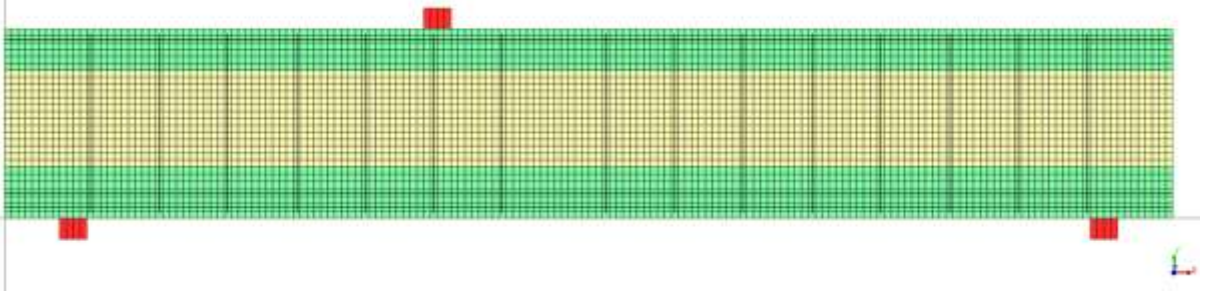
The first element size in this convergence study was 80mm that resulted in 292 elements and maximum displacement of 0.94679 mm (see Figure 4.6-a). The second trial involved an element size of 80 mm, which resulted in a maximum displacement of 1.03231 mm with 1169 elements (see Figure 4.6-b). The third trial was conducted with finer square meshes with a 20 mm mesh size, which gave a displacement of 1.04146mm with 4675 elements (see Figure 4.6-c). The finest size of element was used in the fourth trial with a mesh size of 10mm. This resulted in a maximum displacement of 1.04319 mm with 18700 elements (see Figure 4.6-d). In all these cases, a concentrated static load of 500 kN was applied at 850 mm from the left support. Figure 4.7 shows the numerical convergence of maximum displacement where no significant change is observed when the element size is reduced beyond 20 mm (4675 elements).



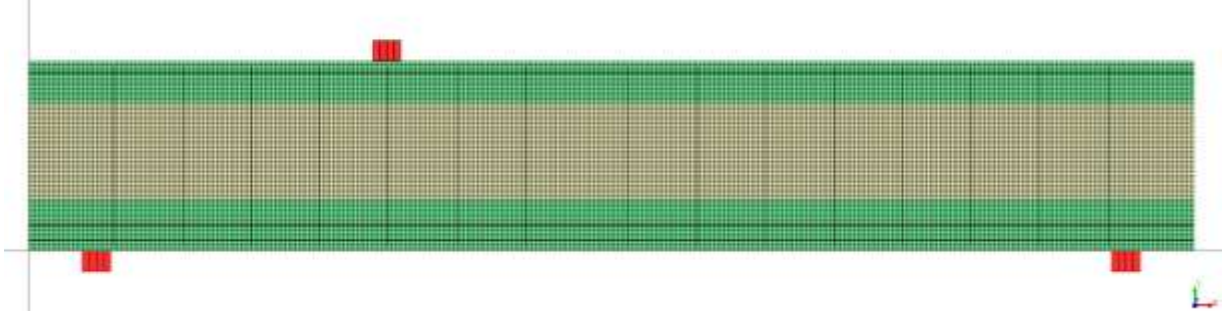
(a) Element size is 80 X 80 mm



(b) Element size is 40 X 40 mm



(c) Element size is 20 X 20 mm



(d) Element size is 10 X 10 mm

Figure 4.6 Different mesh arrangements for the convergence study

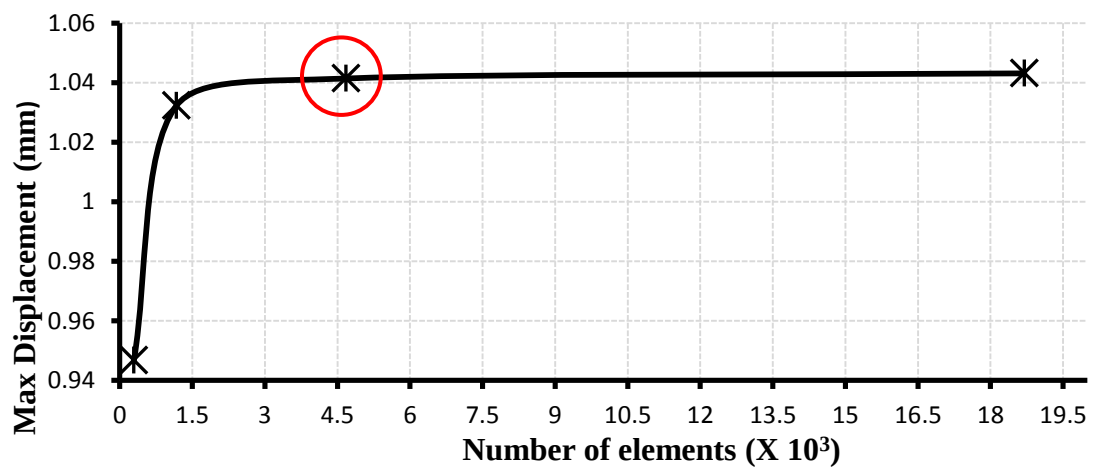


Figure 4.7 Maximum displacement versus number of elements in linear elastic

4.6 Load Cases

The loading sequences in both linear and nonlinear finite element analysis approaches were selected to represent the actual construction practice. The prestressing stress was assumed to be pretension, where the strands were pre-tensioned prior to the casting of UHPC. In practice, the tendons and the shear reinforcement are usually placed in their final locations prior to applying the pre-tensioning stress to the tendons (FHWA-HRT-11-020, 2010). The UHPC is then cast in the form, and heat/steam cured. The stress transfer to concrete is done by either releasing the prestressing jacks, or by cutting the tendons. The initial stress level is equal to 74% of the tendons ultimate strength (1860 MPa). It has been reported that the UHPC reaches almost its full compressive and tensile strengths after heat/steam curing, and prestress losses are less than those observed in conventional normal or high performance concretes.

4.6.1 Load Cases for Linear static FEM Model

Three loading cases were considered in the linear FEM model; (i) self-weight (which is called body force in Diana), (ii) prestressing forces (in the transfer stage); and (iii) external load(s), selected to be either one or two point loads in the current study.

4.6.1.1 Load Case 1: Self Weight

Figure 4.8 shows the deflected shape of Beam FL1 under self-weight, as a sample beam deflection profile. In the finite element model this load was simulated as a body force, which was defined as gravity load.

4.6.1.2 Load Case 2: Prestressing Forces

The prestressing tendons are mainly used in the tension flange of I-sections (the bottom flange in all the beams considered). Since cracks are not allowed due to prestressings, tendons are also used in the top flange to stop any possible crack initiation due to prestress transfer stresses. Tensile stresses in the top fibre can exceed crack limits under two conditions: (i) immediately after prestressing release; and (ii) in the support region (see Figure 4.9) and this mainly due to the very high level of prestressing in the bottom flange and the simple support. The stresses in the top fibre are kept within allowable limits by adding prestressing tendons in the top flange.

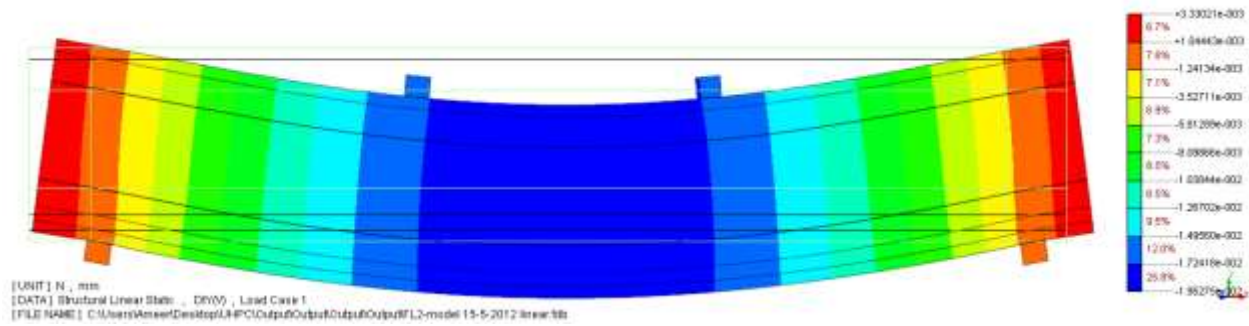


Figure 4.8 Load case 1 or self-load (Beam FL1)

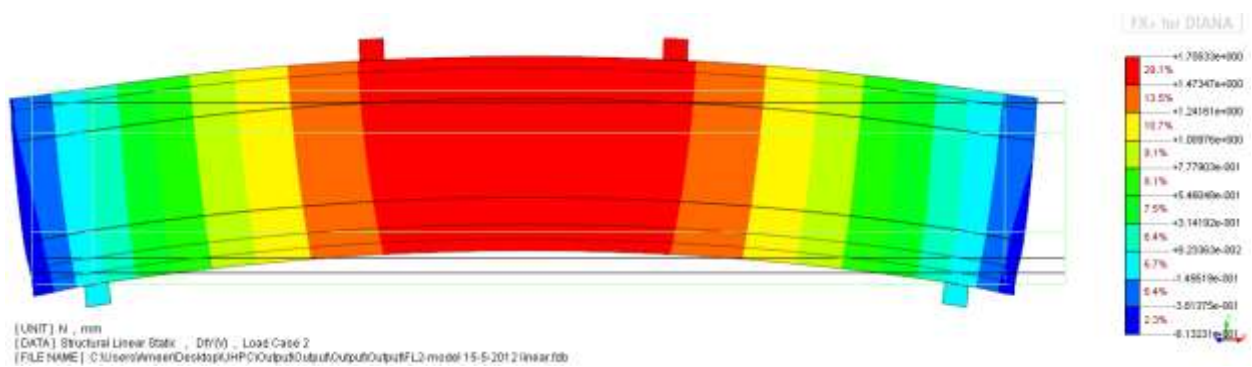


Figure 4.9 Load case or prestress forces (Beam FL1)

4.6.1.3 Load Case 3: External Loads

Two types of external loads are considered in the current investigation; (i) single external load applied to shear-critical beams (see Figure 4.10); (ii) two point loads applied to flexure-critical beams (see Figure 4.11).

Beams, for which linear analysis was conducted, were subjected to all three load cases discussed above (Load Cases 1, 2 and 3). This implies that the beams were subjected to prestressing forces; gravity self-weights, as well as externally applied point loads.

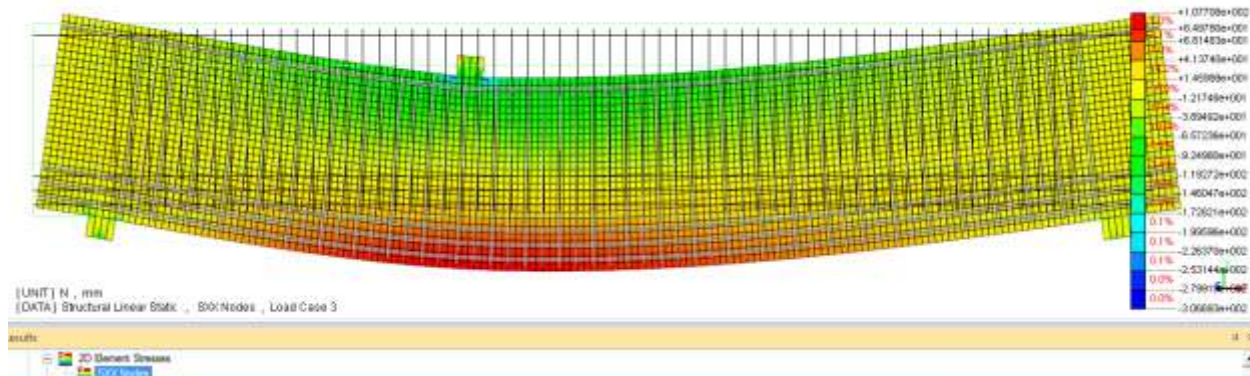


Figure 4.10 Load case 3 or external load (Beam BS1)

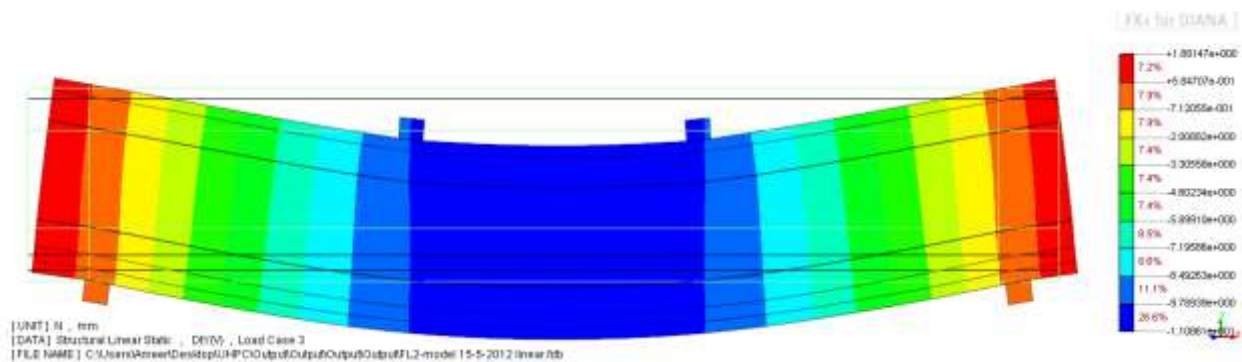


Figure 4.11 Load case 3 or external load (Beam FL1)

4.6.2 Load Cases for Nonlinear analysis

Beams analyzed for nonlinear behavior were subjected to monotonically increasing external loads. The analysis approach employed is capable of investigating gradual development of stresses; and initiation, development, and propagation of cracks. The self-weight and prestress forces remained constant, and the external loads were imposed over 10 steps, with intervals of 10% of the total load at each step. Each step of the load was further divided to sub-steps, the magnitude of which was controlled by convergence criteria that were based on the required level of accuracy.

4.7 Linear Finite Element Analysis

The results of linear analyses are presented in this section. As discussed earlier, two types of beams with two different failure modes were considered:

- (i) Prestressed UHPC beams designed to fail in flexure (FL): These UHPC-beams were over-designed in shear to promote flexural failure. Four points loading is selected for this

case (two external point loads and two support reactions), as shown in Figure 4.2. Two beams of this type were analyzed; one beam with “heavy” shear reinforcement and the second beam without any shear reinforcement.

(ii) Prestressed UHPC beams designed to fail in shear (BS): The UHPC-beams in this category were over-designed in flexure to develop shear failures. Three points loading was selected for this case (one external point load and two support reactions) as shown in Figure 4.1. Five beams of this type were investigated; one beam had no shear reinforcement while the other beams had stirrups uniformly spaced at 50 mm, 100 mm, 150 mm and 200 mm.

In the linear model all materials were assumed to be linear elastic. The stress-strain relationships of UHPC, prestressing steel and stirrup steel were all assumed to be linear up to failure. The beam was modeled using shell elements. Special frame elements were used for tendons and stirrups. The crack initiation and propagation were ignored and the strains of UHPC and steel reinforcement were assumed to satisfy the strain compatibility.

4.7.1 Flexure Critical UHPC Beams

Figure 4.12 illustrates the FEM model for the first flexure-dominant beam, FL1. The beam was subjected to two point loads through steel plates, which were used to avoid stress concentration. The same steel plates were also used at the supports. The optimum mesh size, as established by the convergence study was used. The properties of prestressing strands and shear reinforcement were specified as indicated in Table 3.5 of Chapter 3.



Figure 4.12 The FEM model for Beam FL1

The stirrups in FL1 were placed at 50 mm spacing on both sides of the beam in the support regions, extending two stirrups beyond the application of support reactions. The ultimate load

was defined either by the compression crushing of concrete at $f'_c = -175 \text{ MPa}$ and the corresponding strain of $\epsilon_{ult} = 0.0035$, or the tension failure of the strands in the lower flange at $0.95f_u = 1767 \text{ MPa}$. Figure 4.13 shows that the maximum compression in the top flange is less than the ultimate stress of UHPC, while Figure 4.14 shows that the tendon steel in the mid-span region has reached its ultimate stress of 1760 MPa. The local effects of the loads are shown in Figure 4.13 where very high compressive stress is localized under the loading plate.

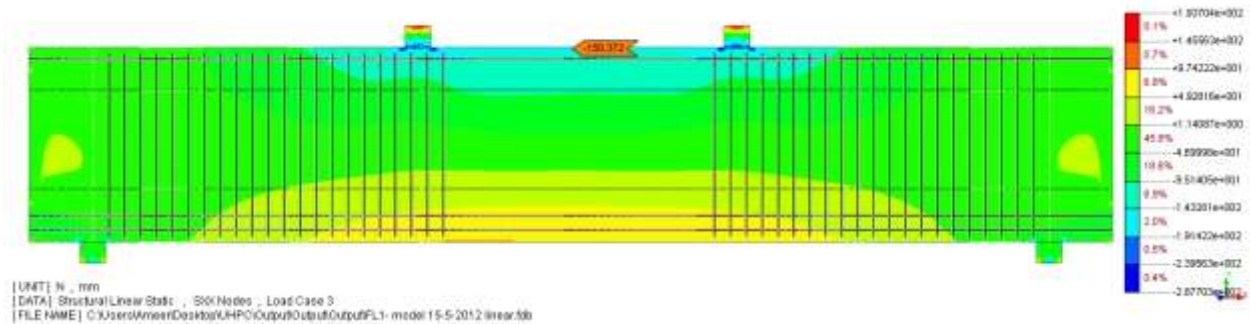


Figure 4.13 Distribution of longitudinal stresses in UHPC (S_{xx}) at ultimate load (FL1)

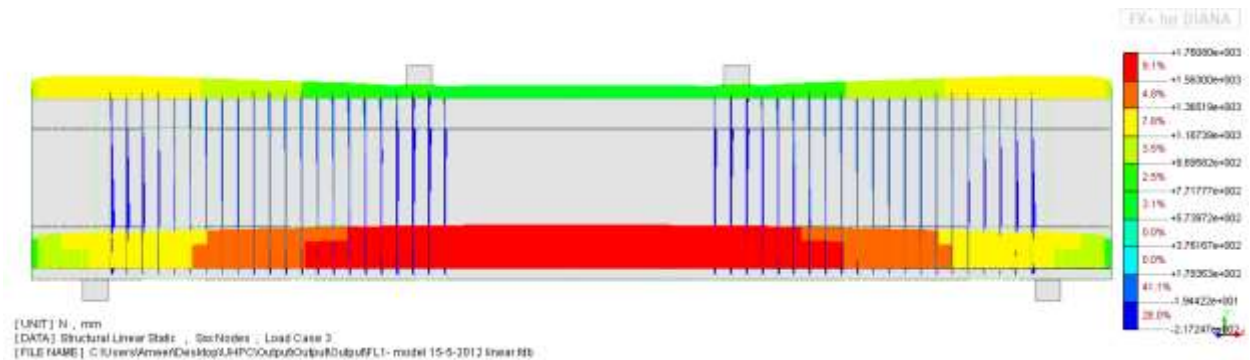


Figure 4.14 Distribution of tendon stresses at ultimate (FL1)

The FEM model for beam FL2 was the same as that for beam FL1 shown in Figure 4.12, except it did not contain any shear reinforcement. Figure 4.15 shows concrete compressive stresses at ultimate load condition, indicating that compression in the top flange was less than that at ultimate. Figure 4.16 shows that the tendon stresses in the mid-span region reached the ultimate tensile stress of 1760 MPa. Figure 4.15 further indicates low to high tensile stresses were developed in all regions of the beam except for the top flange around the two external loading points.

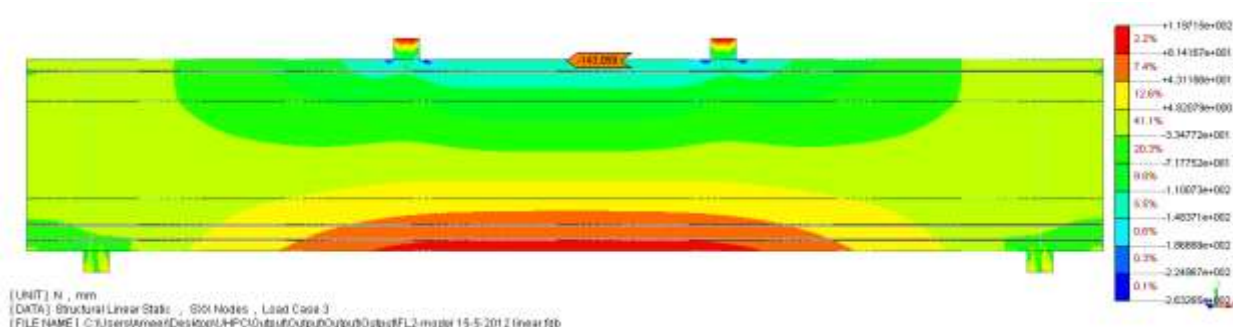


Figure 4.15 Distribution of longitudinal stresses (S_{xx}) at ultimate (FL2)

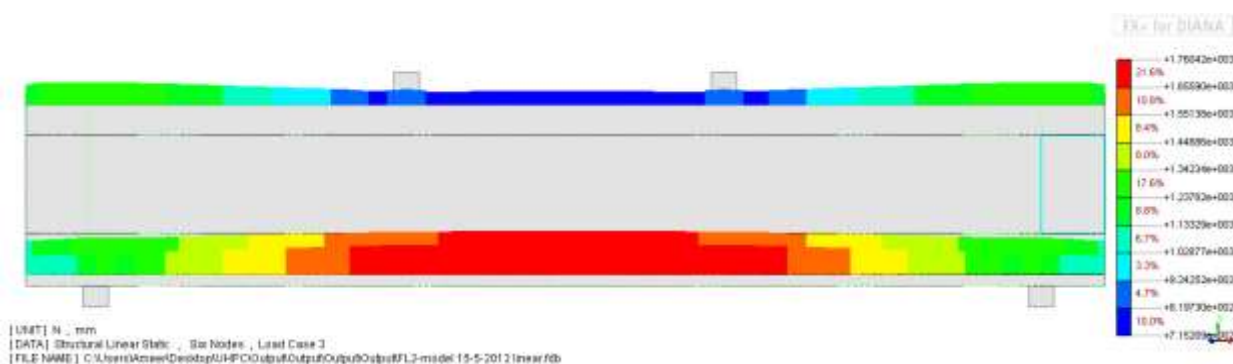


Figure 4.16 Distribution of tendon stresses at ultimate (FL2)

4.7.2 Shear-Critical UHPC Beams

Five shear-critical prestressed UHPC beams were modeled and analyzed. The stirrups in the beams were distributed uniformly over the length having four different centre-to-centre spacing of 50 mm, 100 mm, 150 mm, 200 mm, with an additional beam without any stirrups. The beams were designed, as described in Chapter 3, Sec. 3.4.2, to fail in shear prior to reaching their flexural design capacities. The elastic FEM analysis conducted in this section was not able to capture shear failures, since tensile cracking of UHPC was not modeled (hence the subsequent set of analyses in Sec. 4.8 involving non-linear FEM analysis). However, transverse stresses were investigated, showing the significance of shear stresses in the beams. The Analyses were allowed to continue up to the flexural failure point, overestimating beam failure loads.

A typical beam model is illustrated in Figure 4.17, with a single point load applied at one-third the span length. The load was applied through a narrow but thick high strength steel plate (ASTM A992 - Grade 50). The same steel plates were placed at the supports as shown in Figure 4.19. The optimum mesh size established earlier by the convergence study was used in all beam models. The design of each beam is as indicated in Chapter 3 (Table 3.5).

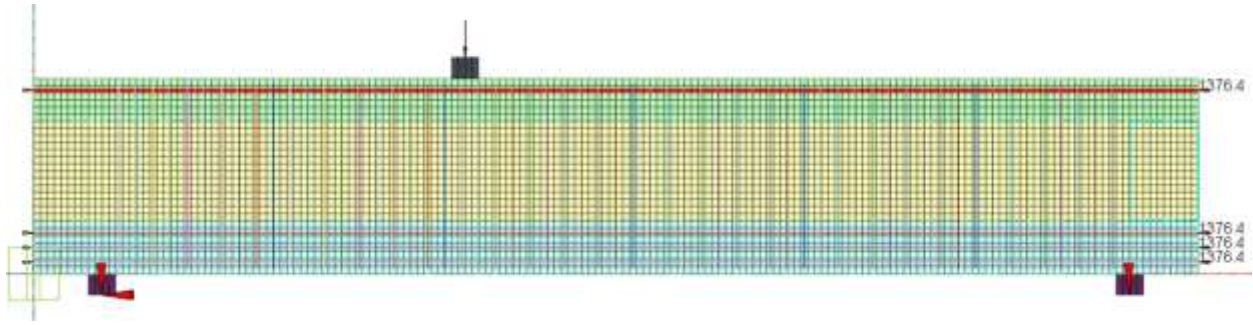


Figure 4.17 Typical shear-dominant beam model (Beam BS1)

The boundary conditions for all the beams were identical, and consisted of a pin support at the left support and a roller support at the right support. The bottom flange tendons were distributed in three layers, while the top tendons were distributed in one layer as detailed in Sec. 3.4.2. Figure 4.17 shows the FEM model for beam BS1 as an example of beams used in this category, with 50 mm stirrup spacing.

Beam BS1 had the highest amount of shear reinforcement. Figure 4.18 indicates that the maximum compressive stress developed in the compression zone of the top flange was less than the ultimate stress of UHPC. The figure further indicates the distribution of low to high compressive stresses everywhere in the beam. A large compression arch has developed between the external load and the supports. On the other hand the distribution of transverse stresses shows high compressive stresses forming “bulbs” at or around the loading plate and the two supporting plates as shown in Figure 4.19, with transverse tensile stresses elsewhere in the beam reaching approximately 40 MPa in tension. Figure 4.20 demonstrates tensile stresses in tendons, with tension in the bottom flange reaching the ultimate stress of 1723 MPa.

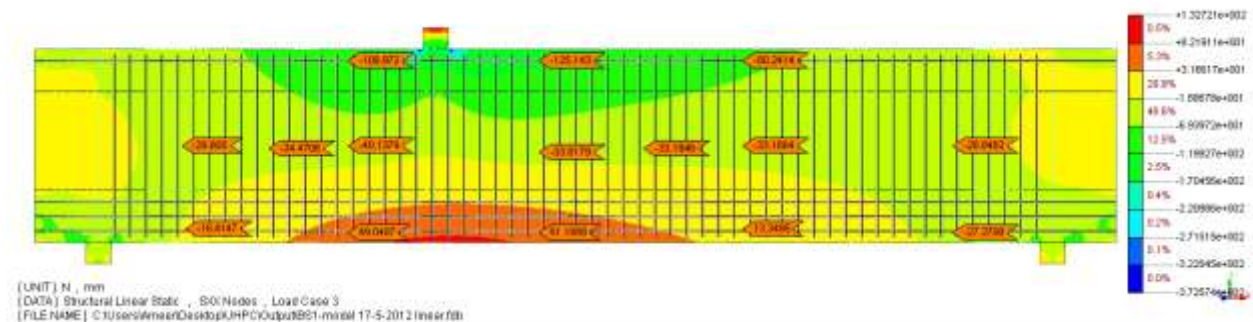


Figure 4.18 Distribution of longitudinal stresses (S_{xx}) at ultimate load (BS1)

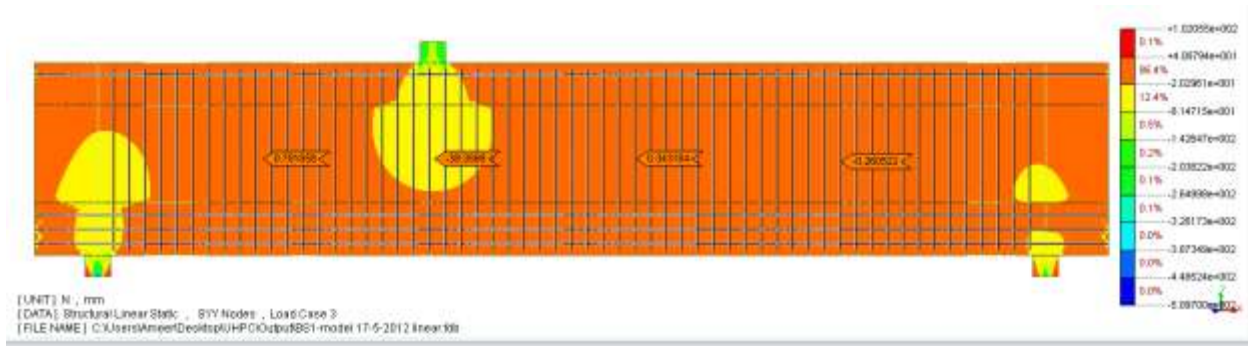


Figure 4.19 Distribution of transverse stresses (S_{yy}) at ultimate load (BS1)

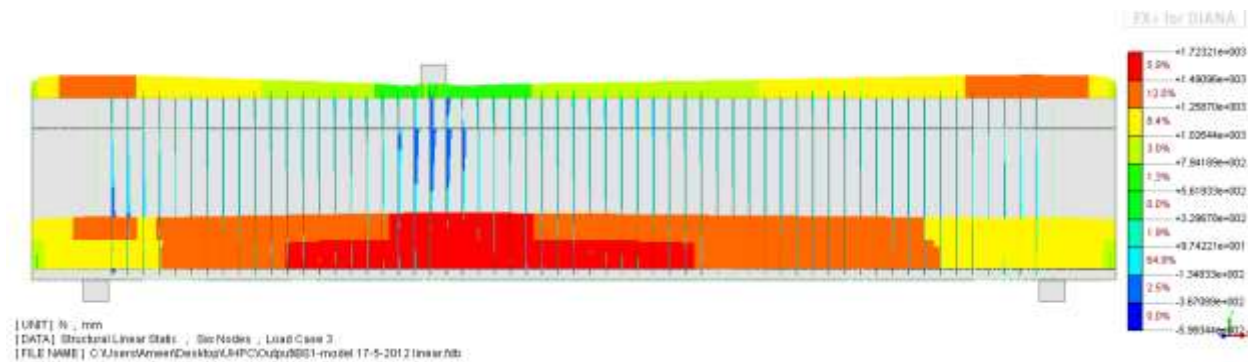


Figure 4.20 Distribution of tendon stresses at ultimate load (BS1)

Similar behavior was observed in beam BS2 with the second highest amount of shear reinforcement. Figure 4.21 shows longitudinal stress distribution in concrete. It is indicated that the maximum compressive fibre stress is much less than the crushing strength of UHPC. The figure also shows distribution of low to high compressive stresses everywhere in the beam with lower intensities than those observed in beam BS1. A large arch of compressive stresses linking the top compression zone to the supports is shown in Figure 4.21. On the other hand the distribution of the transverse stresses, shown in Figure 4.22, indicates high compressive stress “bulbs” forming at the loading point and the two supports. Figure 4.23 shows that the tendons in the bottom flange have reached their ultimate strength of 1775 MPa.

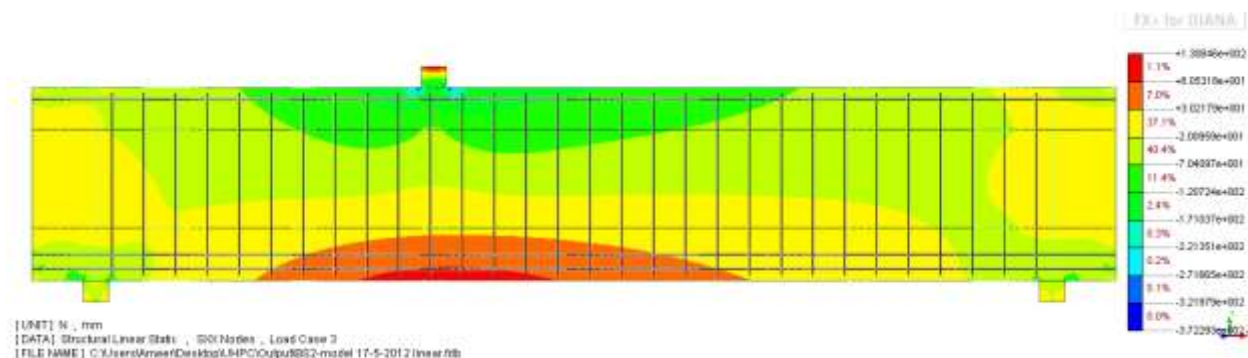


Figure 4.21 Distribution of longitudinal stresses (S_{xx}) at ultimate load (BS2)

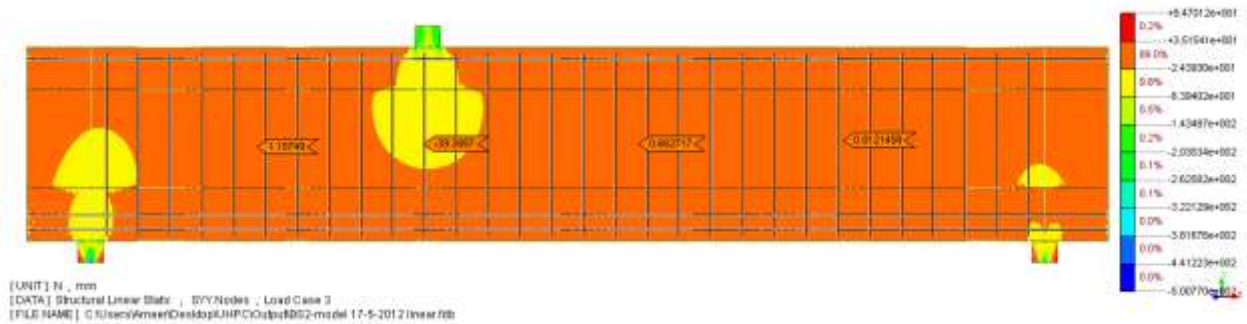


Figure 4.22 Distribution of transverse stresses (S_{yy}) at ultimate load (BS2)

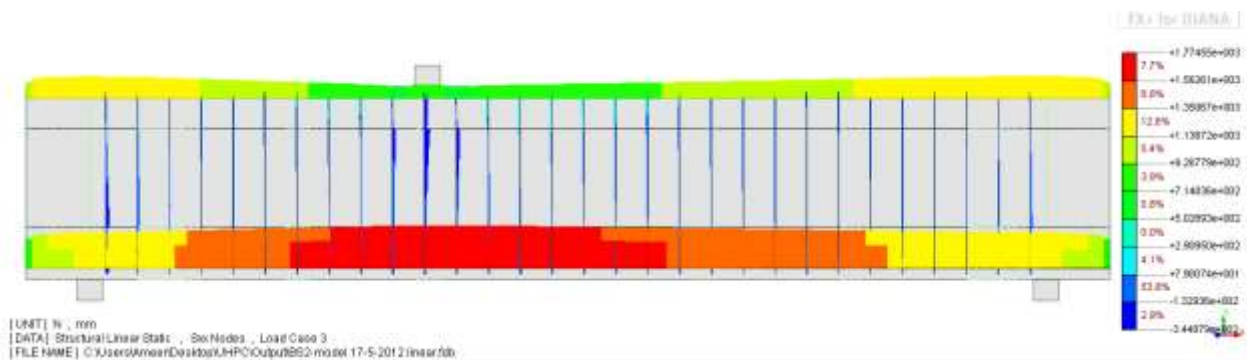


Figure 4.23 Distribution of tendon stresses at ultimate load (BS2)

With the wider stirrup spacing of 150 mm in beam BS3, the compression zones were reduced and the tension zones became wider, especially at the beam ends. Figure 4.24 shows the maximum compression zone in the top flange is less than the ultimate stress of UHPC. The figure also indicates a wide distribution of low to high tensile stresses in all regions of the beam at the bottom and at beam ends. These tensile stresses are higher than those observed in beams BS1 and BS2. Unlike beams BS1 and BS2, the compression arch between the applied load and the supports is not continuous. The distribution of transverse stresses shown in Figure 4.25 indicate the same compressive stress “bulbs” around the loading plate and the two supporting plates, with similar intensity and distribution as those observed in beams BS1 and BS2. Figure 4.26 shows the prestressing tendons in the bottom flange has reached their ultimate stress of 1771 MPa.

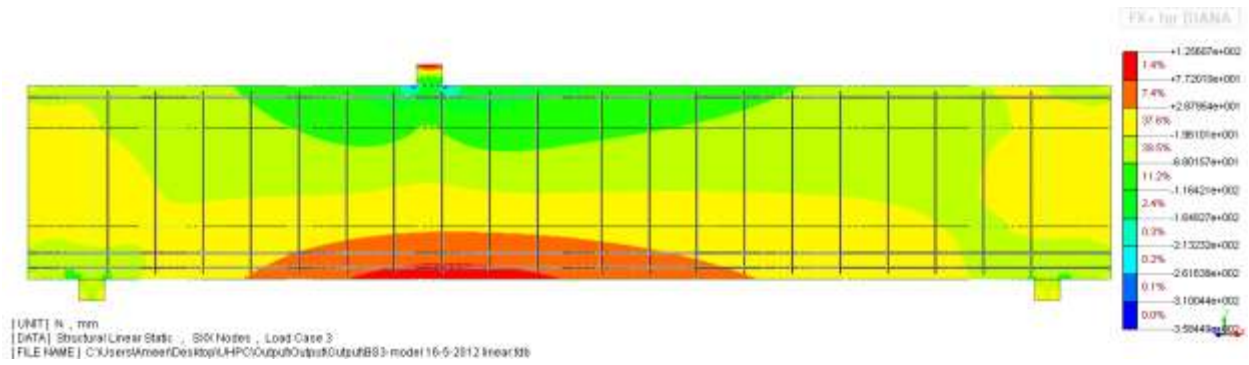


Figure 4.24 Distribution of longitudinal stresses (S_{xx}) at ultimate load (BS3)

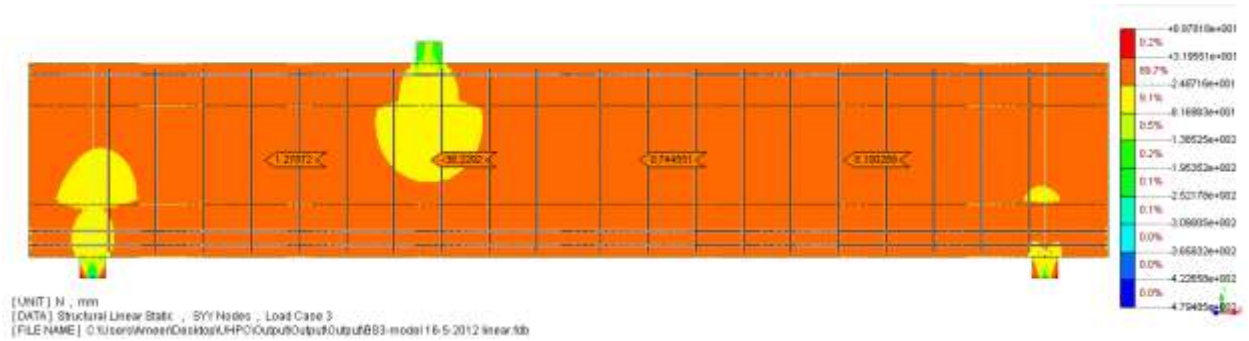


Figure 4.25 Distribution of transverse stresses (S_{yy}) at ultimate load (BS3)

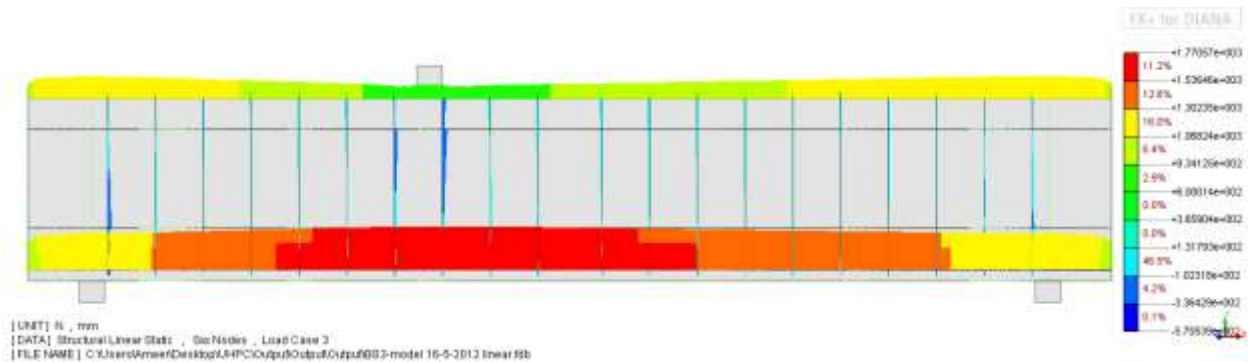


Figure 4.26 Distribution of tendon stresses at ultimate load (BS3)

Increasing the spacing of stirrups to 200 mm in Beam BS4 resulted in further reductions in compression regions and increased tension zones. Figure 4.27 shows that the maximum compressive stress in the top flange is below the ultimate strength of UHPC. The figure indicates a wide distribution of low to high tensile stresses in the bottom and at both ends of the beam as compared to BS2 and BS3. The compression arch in the beam is further reduced and the link between the point of application of the load and the supports has become weaker. The distributions of transverse stresses show the same high compressive stress “bulbs” observed in

previous beams at and around the loading plate and the two supporting plates. This is shown in Figure 4.28. The tendon stresses are illustrated in Figure 4.29 and shows that the maximum tendon stress has reached its ultimate value of 1781 MPa.

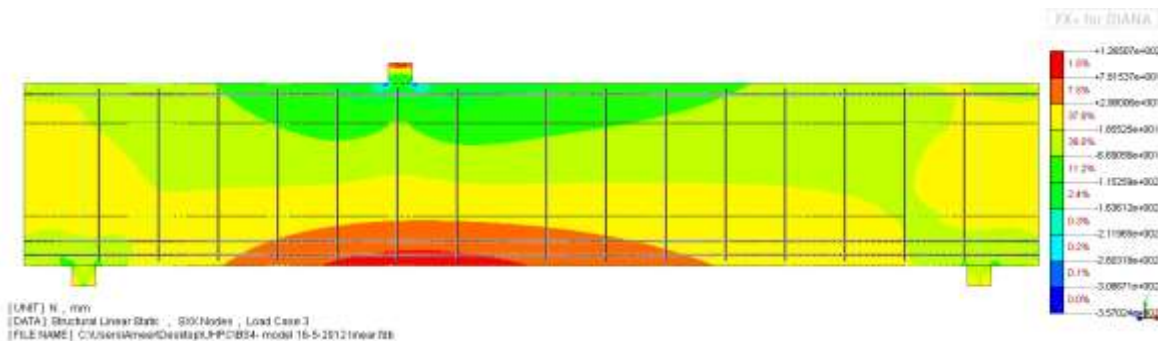


Figure 4.27 Distribution of longitudinal stresses (S_{xx}) at ultimate load (BS4)

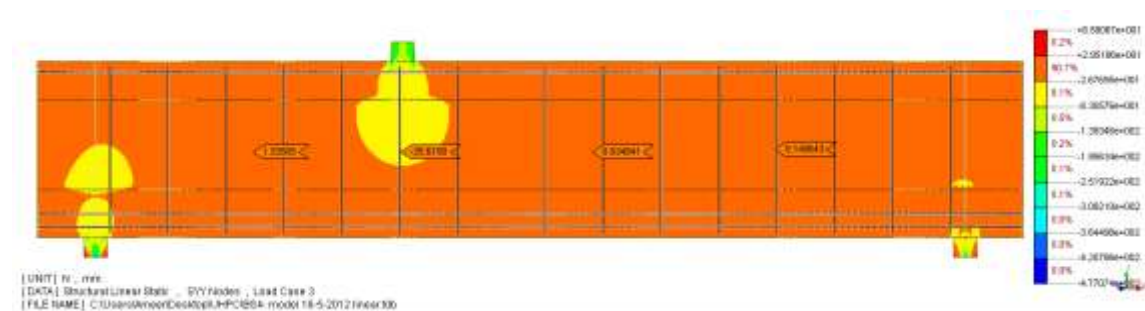


Figure 4.28 Distribution of transverse stresses (S_{yy}) at ultimate load (BS4)

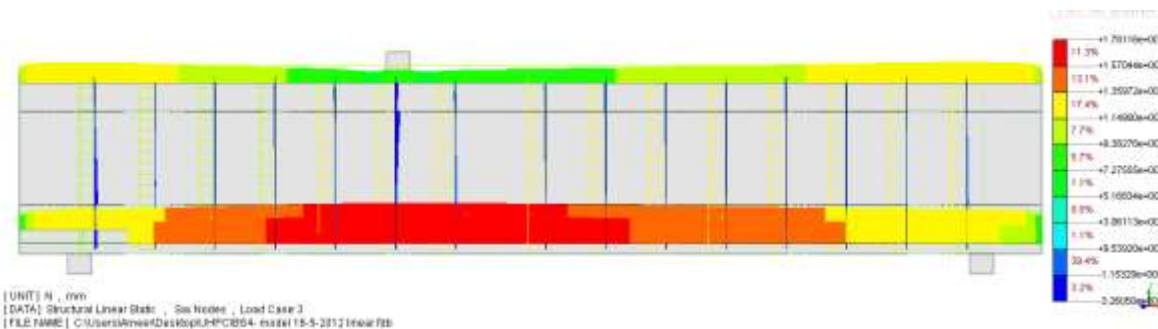


Figure 4.29 Distribution of tendon stresses at ultimate load (BS4)

The last beam analyzed using elastic FEM analysis was Beam BS5 with no stirrups. The compression zones in the beam were minimized while the tension zones covered most of the beam, especially in the high shear zone (left side of the beam). Figure 4.30 indicates that the maximum compressive stress in the top flange was less than the ultimate strength of UHPC. The figure also shows a wide distribution of high tensile stresses in the bottom and end regions of the beam. The increase in tensile stresses was the highest of all shear-critical beams. The

compression arch in the beam between the applied load and the supports was further reduced. The distributions of transverse stresses show the same high compressive stress “bulbs” observed in the earlier beams around the loading plate and the two supporting plates, as illustrated in Figure 4.31. The tendon steel in the critical mid-span region has reached its ultimate stress of 1778 MPa as indicated in Figure 4.32.

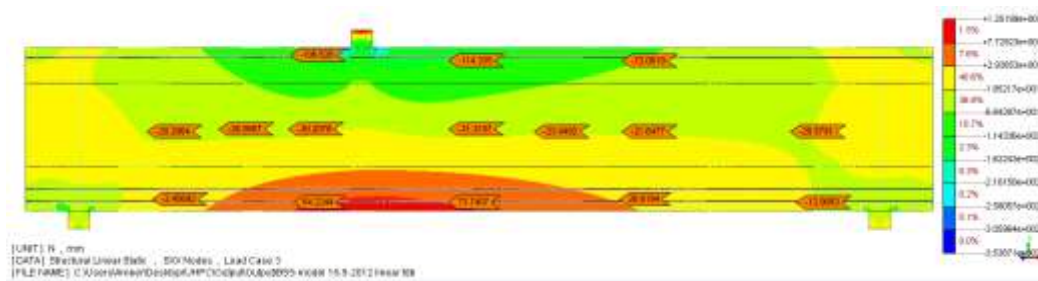


Figure 4.30 Distribution of longitudinal stresses (S_{xx}) at ultimate load (BS5)



Figure 4.31 Distribution of transverse stresses (S_{yy}) at ultimate load (BS5)

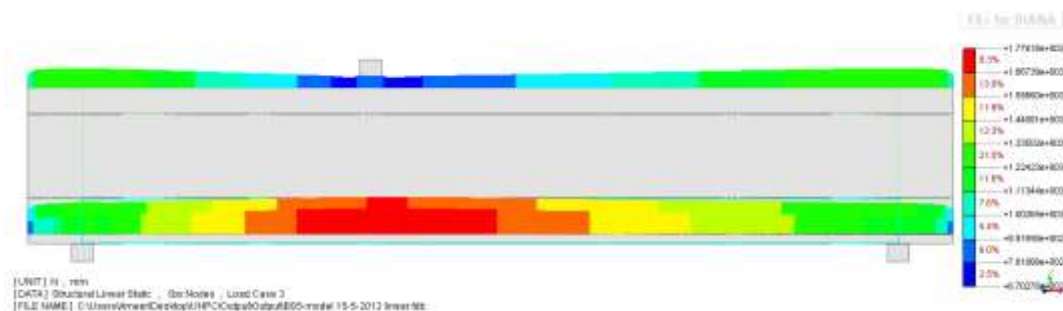


Figure 4.32 Distribution of tendon stresses at ultimate load (BS4)

4.8 Non-Linear Finite Element Analysis

Nonlinear FEM analysis was performed for only four key beams. These consisted of; (i) flexure-critical beam FL1, (ii) shear-critical beam BS1 with maximum shear reinforcement; BS3 with 150 spaced stirrups; and (iii) shear-critical beam BS5 without shear reinforcement. The objective of the non-linear FEM analysis was to investigate damage states in beams and define the failure

modes. It also helped identify the flow of forces at ultimate load stage, providing guidance for the subsequent task in research, i.e., the development of truss models. The results of the FEM analysis are briefly presented in the following sections.

The beams investigated in the current study were all assumed to be pretensioned. The tendons were subjected to prestressing force prior to the casting of UHPC. The prestressing stress was transferred after the heat/steam curing of UHPC. At this stage, it was assumed that 95% of the UHPC strength was developed. The external loads were assumed to be applied after the UHPC has reached its full strength.

DIANA finite element program offers a model for post tensioned concrete elements. Hence the following approach was used to simulate the pretension force and the monotonic loading up to collapse as follows:

1. The prestressing stress at the service load stage was assumed to be equal to the initial stress minus the prestress losses. The resulting prestress effect was superimposed on the effects of externally applied load at the center of each layer of tendons.
2. The equivalent prestressing force was assumed to be applied at the same stage of loading as the beam self-weight.
3. In the nonlinear range, the stress strain relationship of prestressing steel was initialized at the state of stress defined in (1) as shown in Figure 4.33.
4. The distribution of stresses along the beam consisted of the sum of the initial prestress (under constant eccentricity) and the flexural stress. This stress distribution is symmetrical about the mid-span for flexure-dominant beams where the external load was applied at mid-span. This is shown in Figure 4.34. The distribution is asymmetrical about the center of the beam for shear-dominant beams as in this case the peak stress occurs at the point of application of the external load at $1/3$ the span (see Figure 4.35).

Two important stages of loading are highlighted in the following sections; i) the formation of first flexural cracking; and ii) the ultimate load. The distribution of stresses and the crack patterns are discussed relative to each beam analyzed.

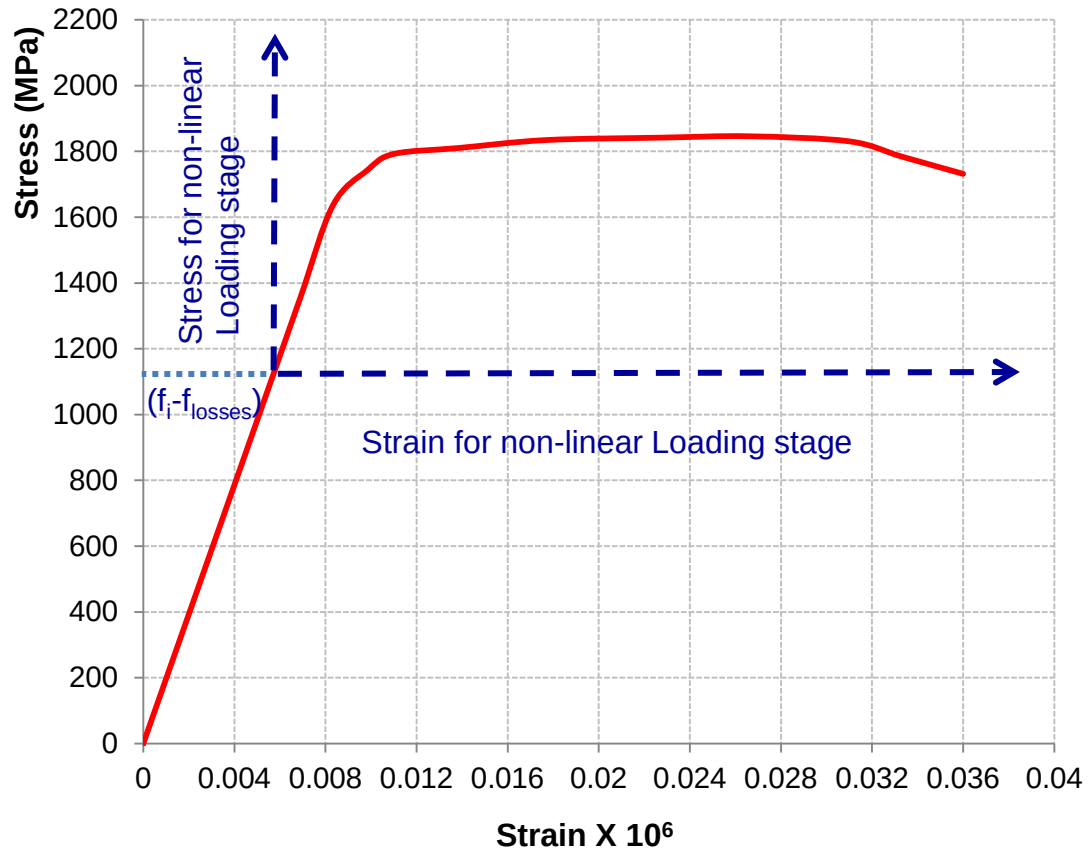


Figure 4.33 Stress-strain relationship of prestressing steel, ASTM A416 Grade 270

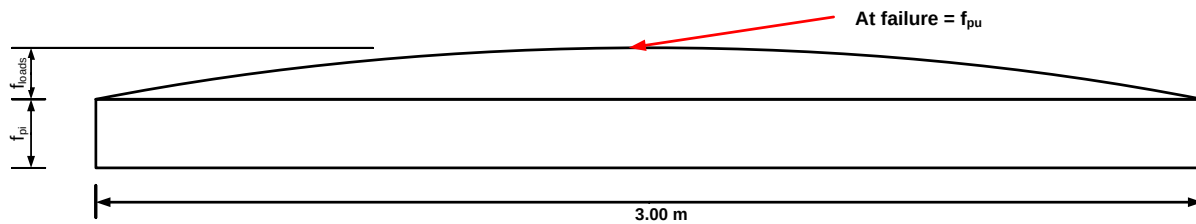


Figure 4.34 Stresses in prestressing tendons for FL1 (nonlinear model - ultimate load)

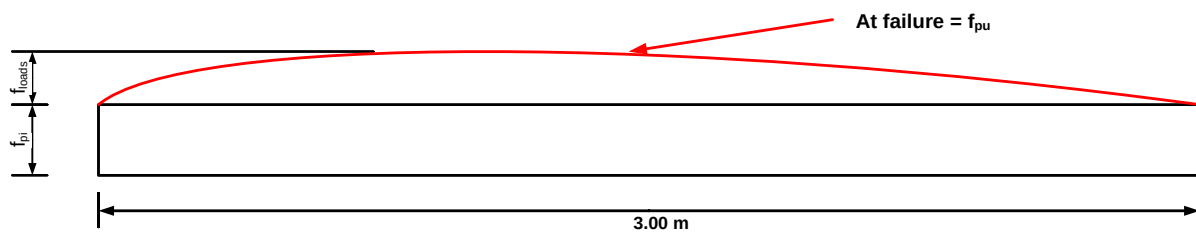


Figure 4.35 Stresses in prestressing tendons for BS1 (nonlinear model - ultimate load)

4.8.1 Flexure-Dominant UHPC Beam

The only flexure-dominant beam that was analyzed using a non-linear model was beam FL1. The reinforcement arrangement for the beam is shown in Figure 4.36, where the stirrups are placed at 50 mm c/c in two shear zones near the supports. Two point loads were applied at 1/3 locations as illustrated in Figure 4.13 with the same steel plates (ASTM A992 Grade 50) as for the beam analyzed elastically, having cross section dimensions of (60 mm X 60 mm) and a length equal to the flange width. Similar steel plates were used at the supports. The nonlinear analysis was conducted in 10 monotonically-increasing load steps. However, the results, in the form of stress distributions along the beam length are presented only at first cracking and at ultimate. The ultimate load was attained either when the maximum stress in the critical compression region reached the compressive strength of UHPC, or the tensile stress in the extreme tensile tendons reached their ultimate strength.

The model showed that the first flexural crack emerged when the applied load reached 1470 kN. The compressive stresses at this stage of loading were dominant, covering 99.6 % of the beam. The maximum compressive and tensile stresses were distributed around the mid span. The maximum compressive stress in this case was 106 MPa or 60% of the UHPC compressive strength. The maximum tensile stress in the tendons was 1529 MPa or 82% of the ultimate strength of steel. At the formation of the first flexural crack, the compressive stress in the beam formed an arch between the two supports and the two external loads, as illustrated in Figure 4.36. The support zones also developed high compressive stresses. The transverse stresses at the formation of first flexural crack show uniform but very low compressive stresses across the beam, as depicted in Figure 4.37. Stresses in the loading and support plates showed fluctuations between tension and compression. For assessing stresses in prestressing strands, the initial stress in tendons should be added to those computed from the FEM analysis. This is done in Figure 4.38, which shows the distribution of tendon stresses, as well as stresses in stirrups (for all steel in the beam). The tendons in the compression zone show compressive stresses, which means that the tendon initial tensile stress is reduced. Figure 4.39 shows the initiation of flexural cracks in the tension zone, at beam mid span.

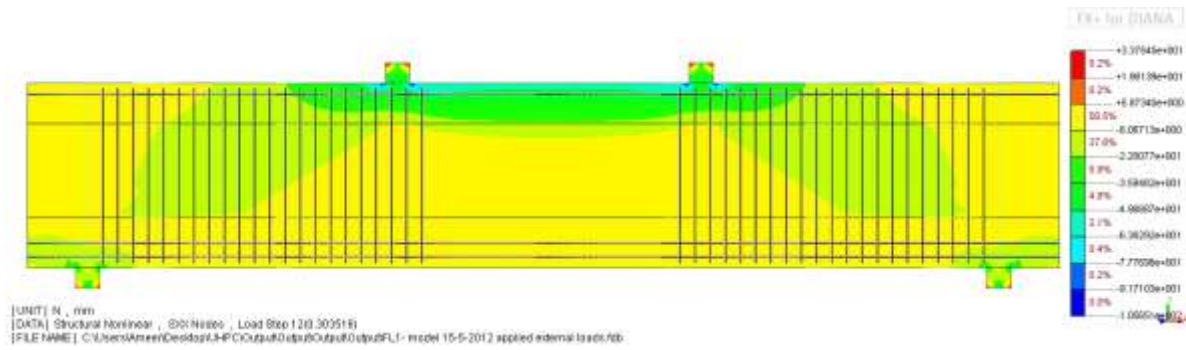


Figure 4.36 Longitudinal stresses (S_{xx}) in FL1 at flexural crack initiation

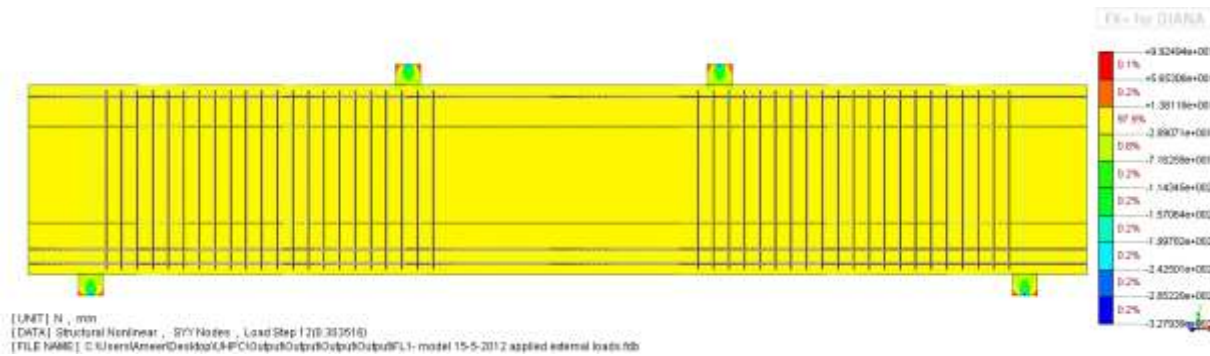


Figure 4.37 Transverse stresses (S_{yy}) in FL1 at flexural crack initiation

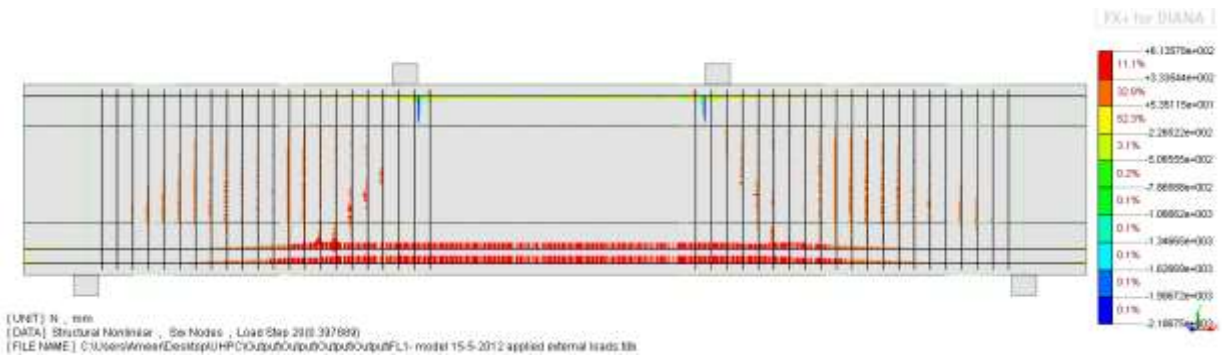


Figure 4.38 Tendon and stirrup stresses in beam FL1 at flexural crack initiation

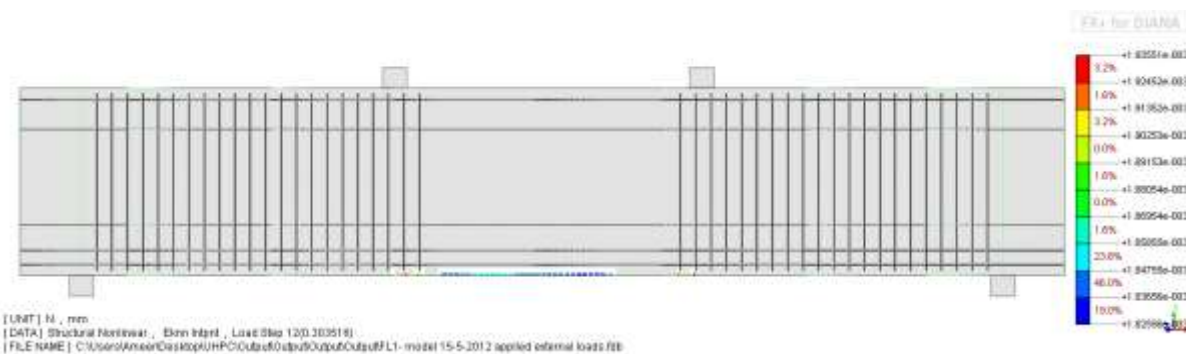


Figure 4.39 Distribution of flexural cracks in FL1 at cracking load

The ultimate load capacity of beam FL1 was reached at 2×2450 kN of total load. Compressive and tensile stresses are the highest in the mid span region. However, the flexural and shear cracks that occurred across the beam disturbed the typical distribution of compressive stresses. The distribution of longitudinal stresses can be seen in Figure 4.40. The maximum compressive stress in this case is 136 MPa or 77.5 % of UHPC ultimate compressive strength. The maximum tensile stress in the tendons is 1784 MPa or 96 % of the ultimate strength of steel. High compressive stresses are observed in areas of stress concentration, i.e., the locations of external forces and support reactions, as well as the prestress force transfer points at beam ends. The tension zone has increased significantly at this stage of loading, covering 37.4 % of the beam. The compression zone, on the other hand, decreased from 99.6 % at first cracking to 62.6 % at ultimate. Transverse stresses increased from a maximum value of 14 MPa at first flexural cracking to 28 MPa at ultimate, with very small regions of compression below the loads as demonstrated in Figure 4.41. Figure 4.42 shows the distribution of stresses in tendons and stirrups. It indicates a significant increase in tendon tensile stresses within the bottom tension zone of mid-span regions, and the development of small tensile stresses in stirrups. Figure 4.43 shows extensive flexural cracking in the bottom flange, while major shear cracks are distributed in the web on both sides of the beam between external loads and supports. Shear cracks started to form in the web, near the supports, and joined the flange cracks, forming inclined shear-flexure cracking. It should be mentioned that the nonlinear FE model employed was for predicting crack patterns based on fracture mechanics, and developed for traditional concrete where the effects of fibers and the absence of aggregate matrix in the mix were not considered.

Figure 4.40 Longitudinal stresses (S_{xx}) in FL1 at ultimate

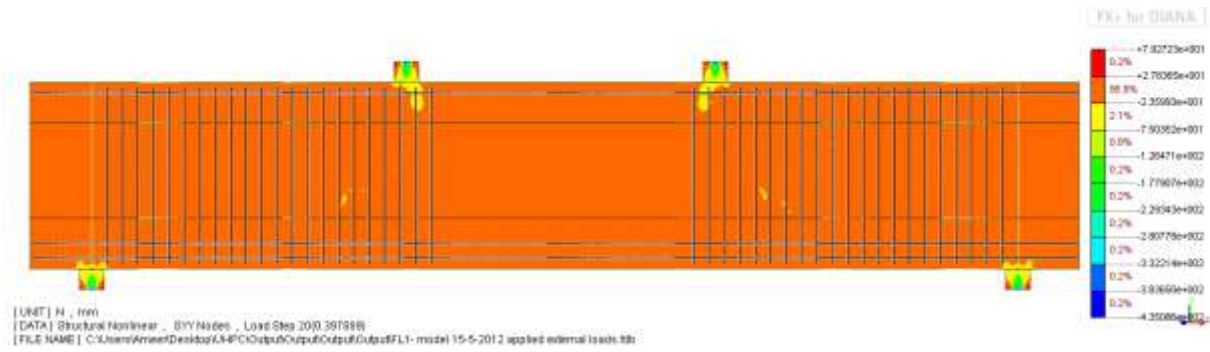


Figure 4.41 Transverse stresses (S_{yy}) in FL1 at ultimate

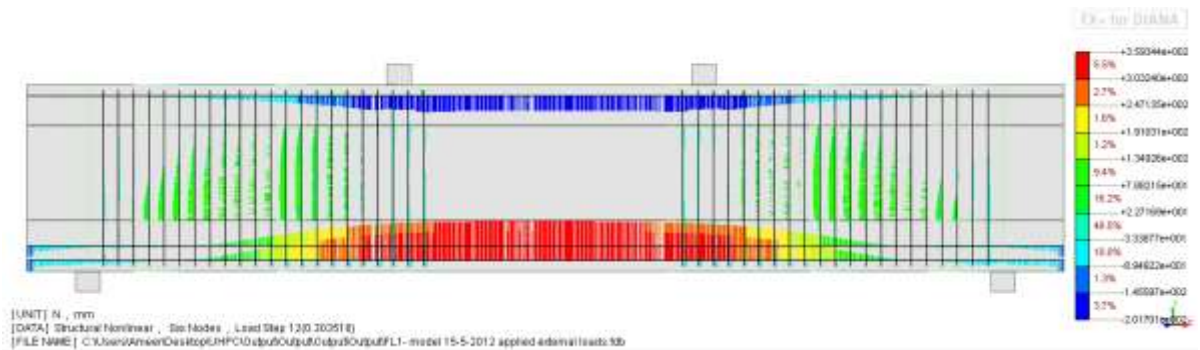


Figure 4.42 Tendon and stirrup stresses in beam FL1 at ultimate

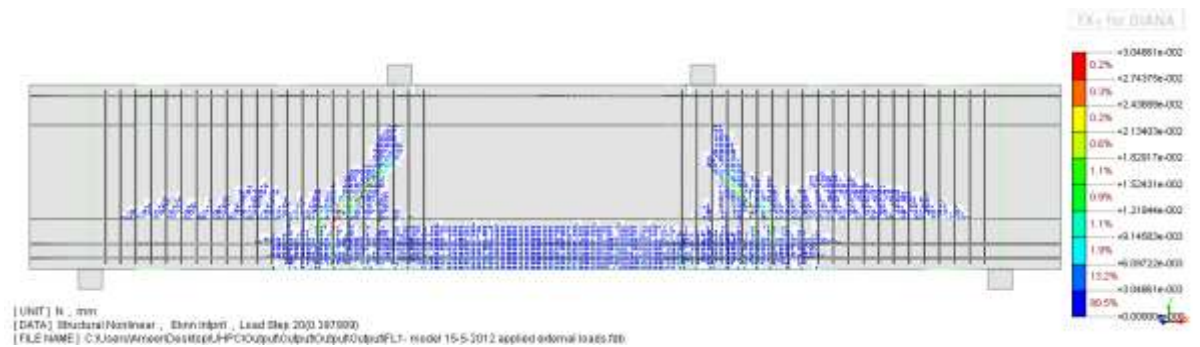


Figure 4.43 Distribution of cracks in beam FL1 at ultimate

4.8.2 Shear-Dominant UHPC Beams

Three shear critical prestressed UHPC beams were analyzed using nonlinear models. The beams had either 50 mm, 150 mm stirrup spacing (BS1, BS3) or did not contain any transverse reinforcement (BS5). The objective of the analyses was to investigate stress distributions at different stages of loading. Furthermore, the initiation, formation and progression of cracks at critical stages of loading were studied. The beams were subjected to a single concentrated load as shown in Figure 4.17. The same figure also shows the reinforcement arrangement, as well as the loading and the support setup. Similar to the beam FL1 discussed in the preceding section, a

single external load was applied on a thick steel plate. Two similar thick steel plates were used for the beam supports.

Nonlinear finite element analysis was conducted for three beams in 10 monotonically increasing load steps. However, two critical stages are presented here: (i) the first cracking load; and (ii) the ultimate load. The ultimate load is reached when the diagonal shear crack is fully formed and the beam starts experiencing its final failure/collapse when either the tensile stress in the extreme tension strand reaches its rupturing capacity or the compressive stress in the critical compression zone reaches its limit

The model for beam BS1 shows that the first shear crack initiated at the intersection of the bottom flange and the web after the formation of hairline flexural cracks in the bottom flange near the load application region. The longitudinal compressive stresses have formed an unsymmetrical arch between the applied load and the supports as shown in Figure 4.44. High tensile stresses were recorded in the bottom flange, which also extended towards the top flange under the applied load. The maximum compressive stress at the formation of initial shear cracks was 144 MPa or 82 % of UHPC ultimate strength. The maximum tensile stress in tendons, at the same stage of loading was 1547 MPa or 83% of the ultimate rupturing strength of steel. The dominant longitudinal stresses were tensile stresses, which covered 69 % of the beam, while the compression arch zone covered 32 % of the beam. Figure 4.45 shows the distribution of transverse stresses, indicating low and uniform tension in almost everywhere in the beam, but a small concentrated compressive stress zone of about 66 MPa that appear below the applied load, at the top flange and at the flange-web connection area.

Figure 4.46 shows the distribution of stresses in tendons and stirrups. High tensile stresses are observed in the tendons below the applied load, and in the shear reinforcement in the high shear zone between the applied load and the nearest support. The final stress distribution in tendons can be found by superimposing initial tendon stresses on stresses obtained from the FEM analysis. Shear cracks initiated when the applied load approached 2590 kN, as shown in Figure 4.47.

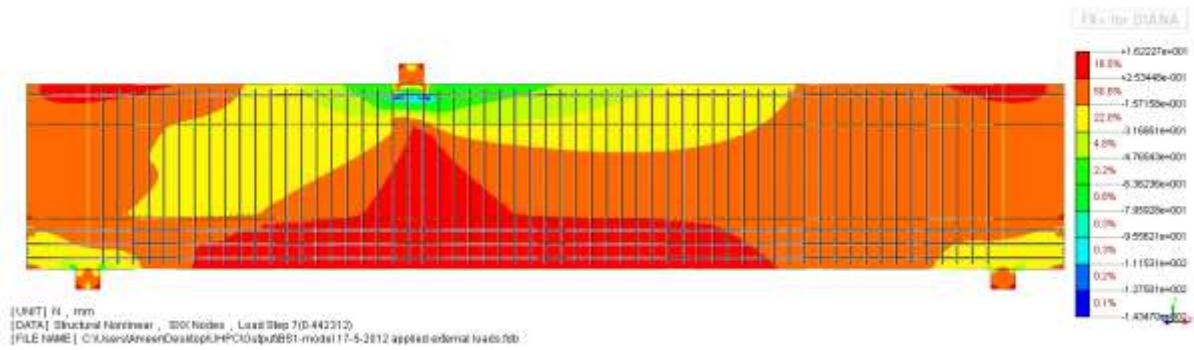


Figure 4.44 Longitudinal stresses (S_{xx}) in BS1 at the initiation of first flexural crack, obtained from nonlinear analysis

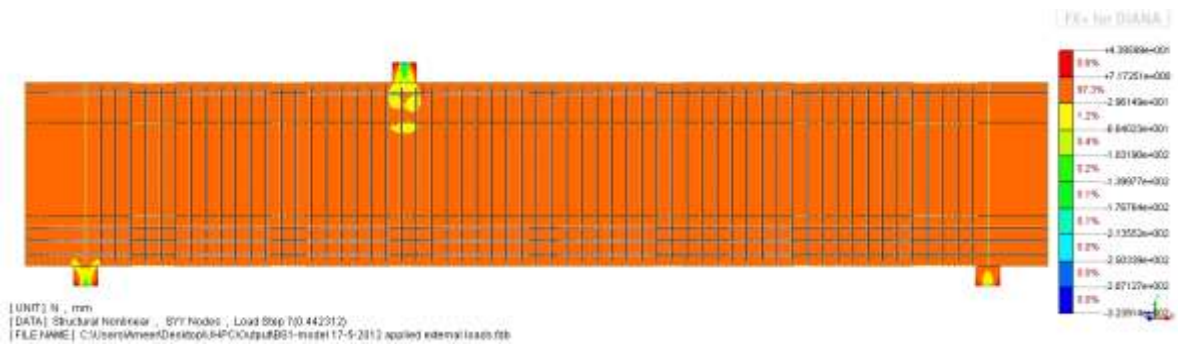


Figure 4.45 Transverse stresses (S_{yy}) in BS1 at the initiation of first flexural crack, obtained from nonlinear analysis

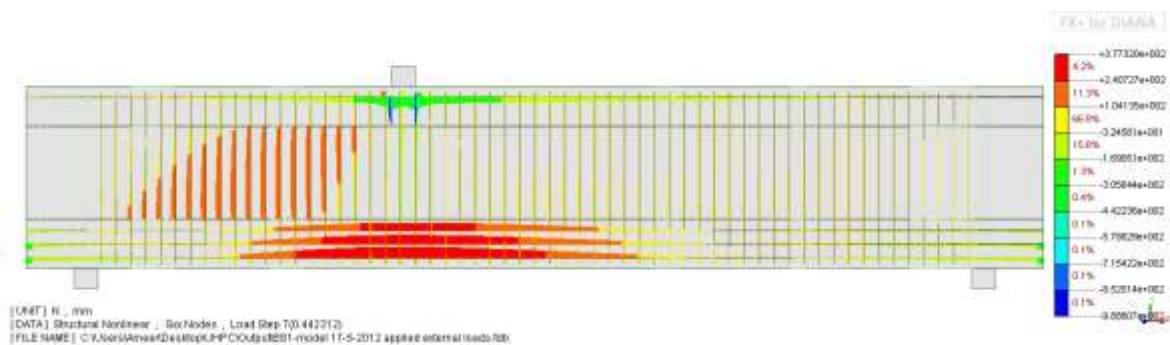


Figure 4.46 Stresses in tendons and stirrups for beam BS1 at the initiation of first flexural crack, obtained from nonlinear analysis

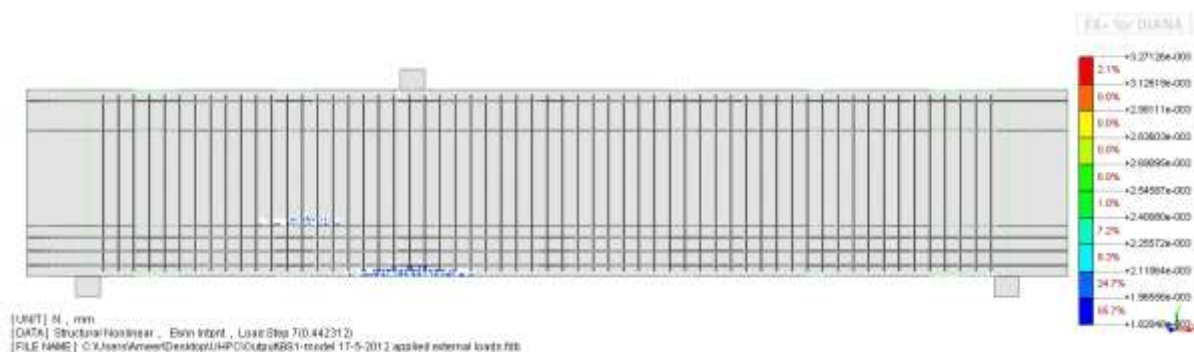


Figure 4.47 Distribution of cracks in beam BS1 at the initiation of first flexural crack, obtained from nonlinear analysis

The analysis indicated that the ultimate load was 3700 kN. At this stage of loading, maximum compressive and tensile stresses were distributed around the applied load. Wide flexural cracks were observed, localized in a narrow region just below the applied load. The maximum compressive stress in this case was reduced to 119 MPa or 68 % of the UHPC ultimate strength. The maximum tensile stress in the tendons was 1744 MPa or 94 % of the rupturing strength of steel. The dominant longitudinal stress was compression, which covered 60 % of the beam, while the tensile stress zone covered the remaining 40 %. High compressive stresses occurred along the diagonal shear struts, as well as the external load and support regions. This is shown in Figure 4.48. The longitudinal stresses were asymmetrical and showed high tensile stresses under the external load and at support regions. Figure 4.49 shows the distribution of transverse stresses, where tensile stresses are around 11 MPa and are uniformly distributed everywhere except for the small stress concentration zone that appears at the location of the external load, where compression reached to 110 MPa.

Figure 4.50 shows the distribution of tendon and stirrup stresses. High tensile stresses occurred in tendons below the applied load, and in shear reinforcement in the high shear stress zone. A diagonal shear crack has fully formed and bounded by dense shear cracks within the high shear zone, as depicted in Figure 4.51. Smaller shear cracks have also formed on the right side of the external load.

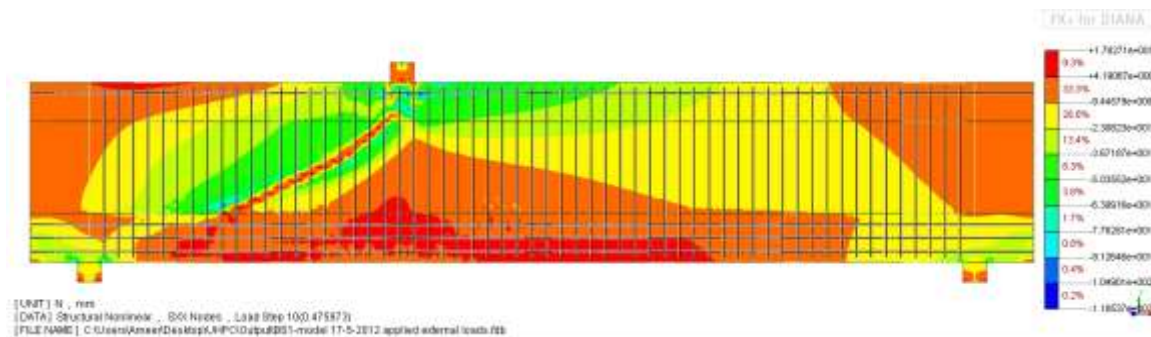


Figure 4.48 Longitudinal stresses (S_{xx}) in BS1 at ultimate, obtained from nonlinear analysis

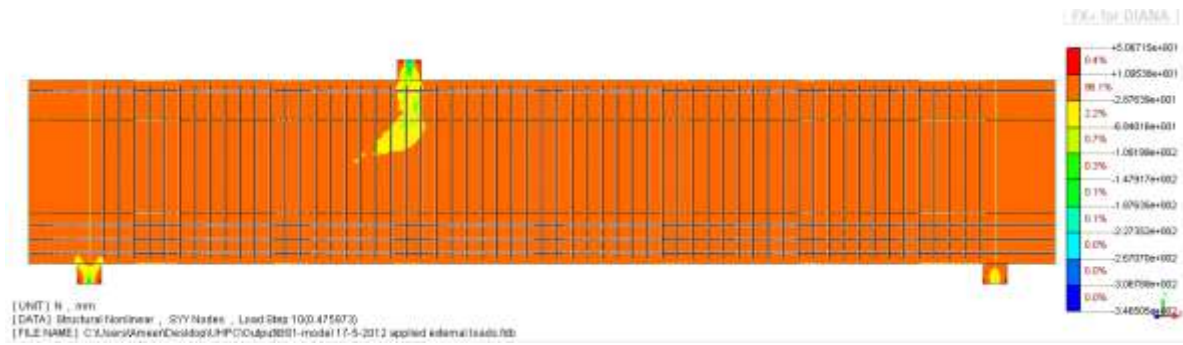


Figure 4.49 Transverse stresses (S_{yy}) in BS1 at ultimate, obtained from nonlinear analysis

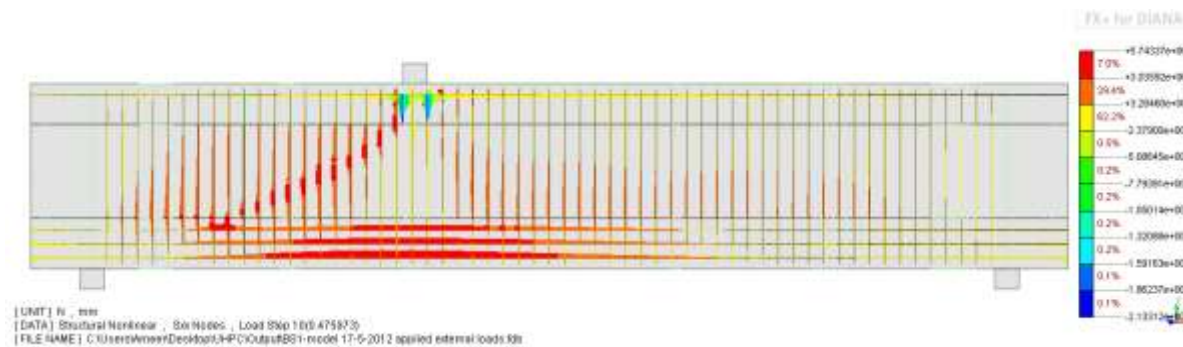


Figure 4.50 Tendon and stirrup stresses in BS1 at ultimate, obtained from nonlinear analysis

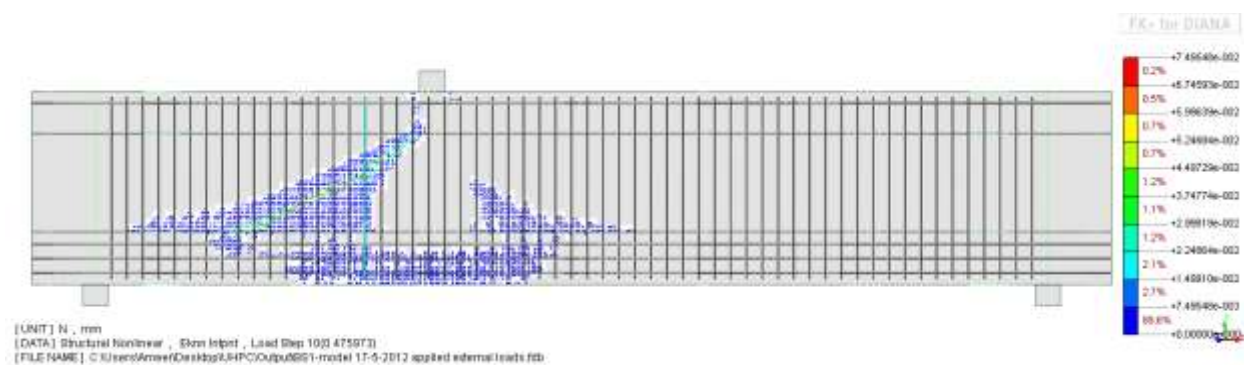


Figure 4.51 Distribution of cracks in BS1 at ultimate, obtained from nonlinear analysis

In a similar behavior to prestressed UHPC beam BS1, the nonlinear finite element model for beam BS3 shows that the first shear crack is initiated at the connection between the bottom

flange and the web in the high shear zone. This occurred after the formation of small flexural cracks in the bottom flange, below the applied load. However the cracked zones in this case are wider than those of beam BS1 (see Figure 4.47 and Figure 4.55). Shear cracks in this model initiated when the applied load approached 2440 kN. The compressive stresses formed asymmetrical arch, having a maximum stress zone below the applied load. High tensile stress zone (see Figure 4.52: the red zone) is distributed horizontally in the bottom flange, on the sides of the beam, and distributed as asymmetrical triangle below the applied load. The maximum compressive stress in this case is 144 MPa or 82 % of UHPC ultimate compressive stress. The maximum tensile stress in the tendons is 1600 MPa or 86% of the ultimate stress of the steel. The support regions and the beam ends where the tendon forces were applied show high compressive stresses.

The dominant longitudinal stress is the compressive stresses, which covers 70 % of the beam, while the tensile zones cover only 30 % of the beam. The compressive stresses show high values under the external load and at the top of supports as shown in Figure 4.52. Figure 4.54 shows the distribution of stresses in tendons and shear stirrups. High tendon stresses are observed below the applied load. As expected, high tensile stresses in shear reinforcement are formed in the high shear zone. Figure 4.53 shows the distribution of lateral stresses. The stresses are generally low, with uniform tension almost everywhere except for small concentrated compressive stress zones of up to 54 MPa that appears below the applied load. These compressive stresses occur in the flange and at the connection of top flange and the web.

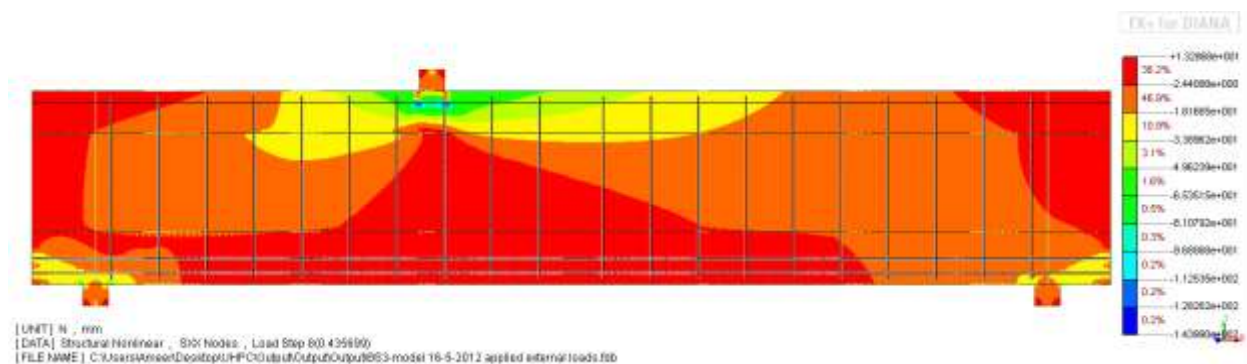


Figure 4.52 Longitudinal stresses (S_{xx}) in BS3 at the initiation of first flexural crack, obtained from nonlinear analysis

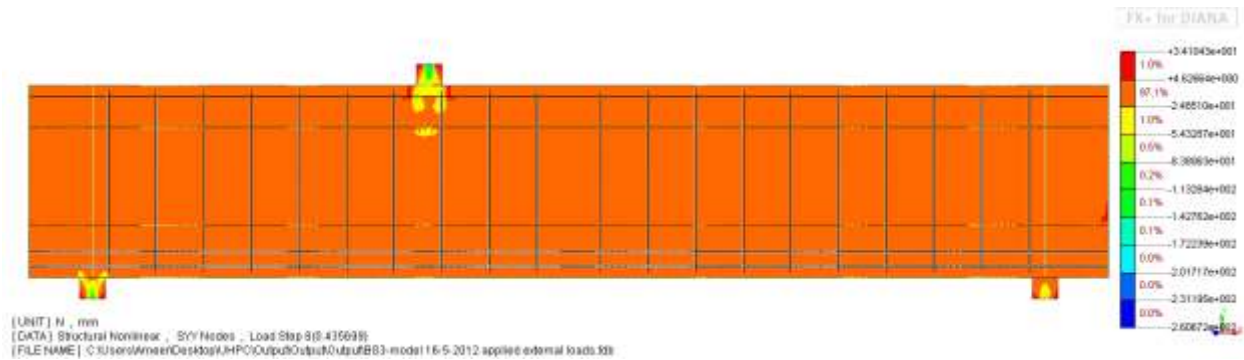


Figure 4.53 Transverse stresses (S_{yy}) in BS3 at the initiation of first flexural crack, obtained from nonlinear analysis

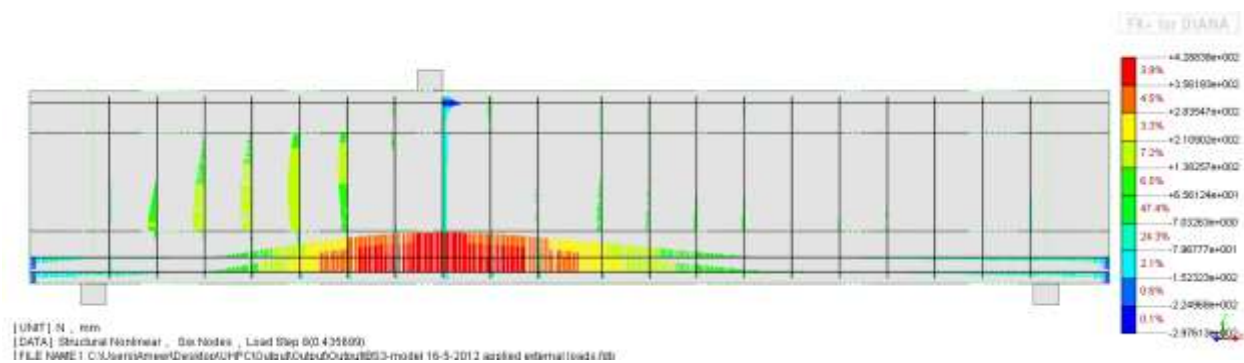


Figure 4.54 Tendon stresses in BS3 at the initiation of first flexural crack, obtained from nonlinear analysis

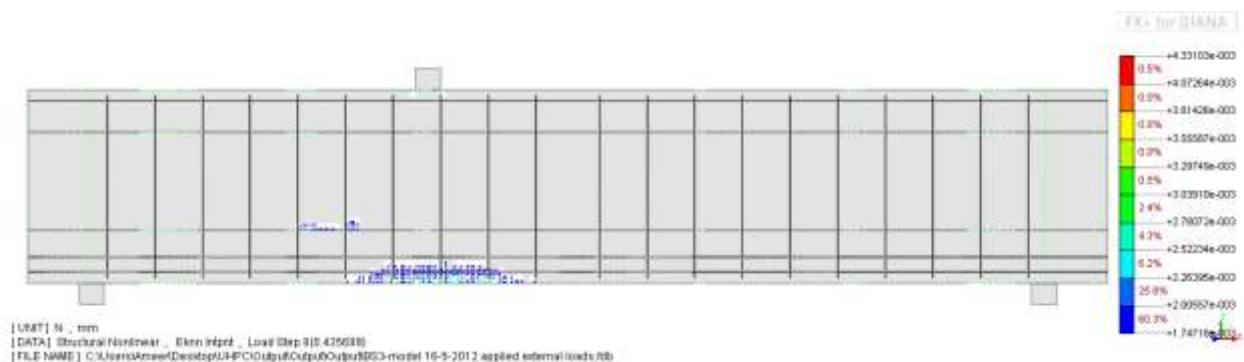


Figure 4.55 Distribution of cracks for beam BS3 at the initiation of first flexural crack, obtained from nonlinear analysis

The model shows that the ultimate load in this model is 3050 kN. The maximum compressive and tensile stresses are distributed at the applied load region. The diagonal shear crack is fully

formed in the high shear zone. This is depicted in Figure 4.56. A small shear crack zone is also formed on the right side of the external load. Heavy flexural cracks localized in a narrow zone below the applied load. The maximum compressive stress in this case is reduced to 129 MPa or 73 % of UHPC ultimate compressive stress. The maximum tensile stress in the tendons is 1723 MPa or 93 % of the ultimate steel strength.

The dominant longitudinal stresses are the compressive stresses, which covers 77 % of the beam. The tensile stress zones cover 23 % of the beam. Inclined compressive stresses become maximum in highly disturbed diagonal shear zones, external load zone, and the support zones. This is shown in Figure 4.56. The fluctuations of stresses in these zones may be explained by the formation of cracks and the stress transfer through the cracks. The longitudinal stresses show asymmetrical distribution, showing high tensile values at the load and support regions. Figure 4.58 shows the distribution of stresses in tendons and shear stirrups. Higher tendon stresses develop below the applied load. Similarly, higher tensile stresses occur in shear reinforcement in the high shear zone. Figure 4.57 shows the distribution of transverse stresses, with tensile stresses varying around 6.2 MPa. These stresses are uniformly distributed everywhere except for small stress concentration zones that occurred when the first crack has formed. The transverse stresses eventually increased to 91 MPa in compression.

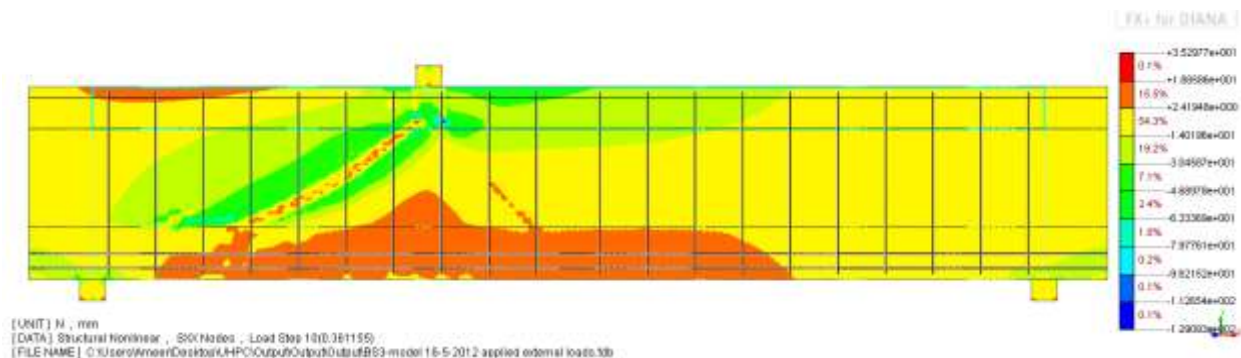


Figure 4.56 Longitudinal stresses (S_{xx}) in BS3 at ultimate, obtained from nonlinear analysis

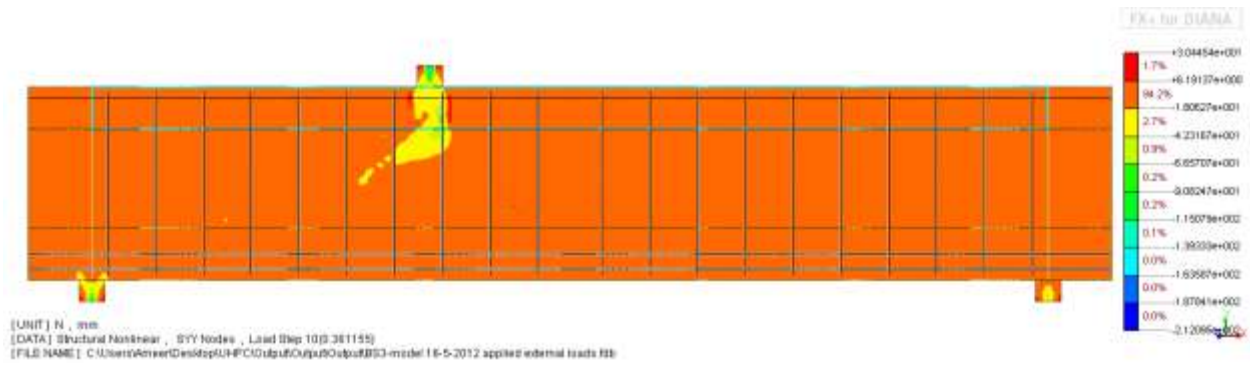


Figure 4.57 Transverse stresses (S_{yy}) in BS3 at ultimate, obtained from nonlinear analysis

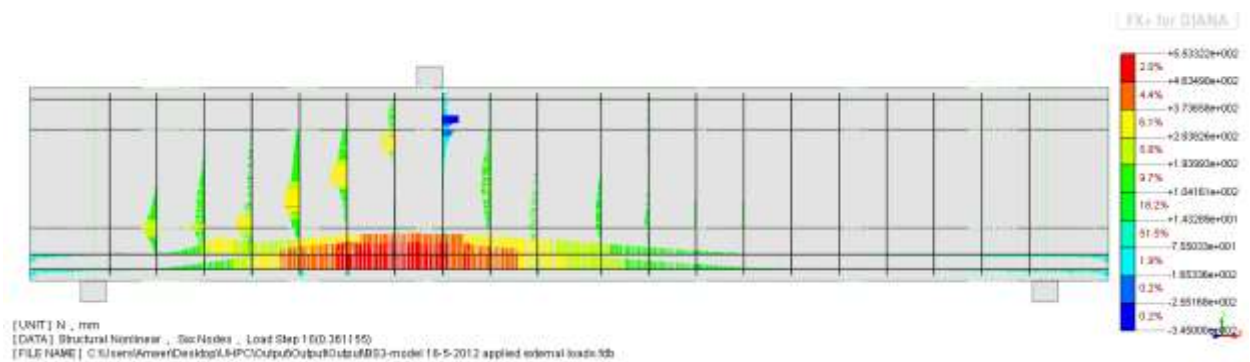


Figure 4.58 Tendons stresses in BS3 at ultimate, obtained from nonlinear analysis

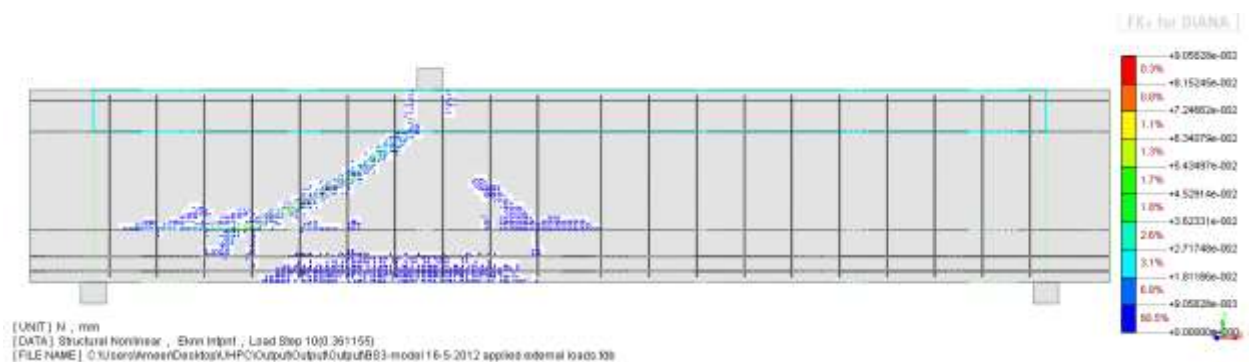


Figure 4.59 Distribution of cracks in BS3 at ultimate, obtained from nonlinear analysis

The Third shear-dominant beam that was investigated through non-linear FE analysis was beam BS5, which had no shear reinforcement. The distribution of longitudinal stresses in UHPC is shown in Figure 4.60 at the formation of first shear crack. The compressive stresses were

distributed in an asymmetrical manner in a narrow region under the applied load, covering about 23% of the beam. High tensile stress zone covered over 77 % of the beam. The maximum compressive stress in this case was 138 MPa or 79 % of the UHPC ultimate strength. The maximum tensile stress in the tendons reached 1666 MPa or 90 % of the ultimate rupturing strength of steel. The support regions, as well as the end regions of tendons, show high compressive stresses. Figure 5.61 shows the distribution of transverse stresses, where the tensile stresses are low and uniform covering 97 % of the beam. A small compressive stress zone, covering only 3 % of the beam, appeared below the applied load in the direction of the diagonal crack.

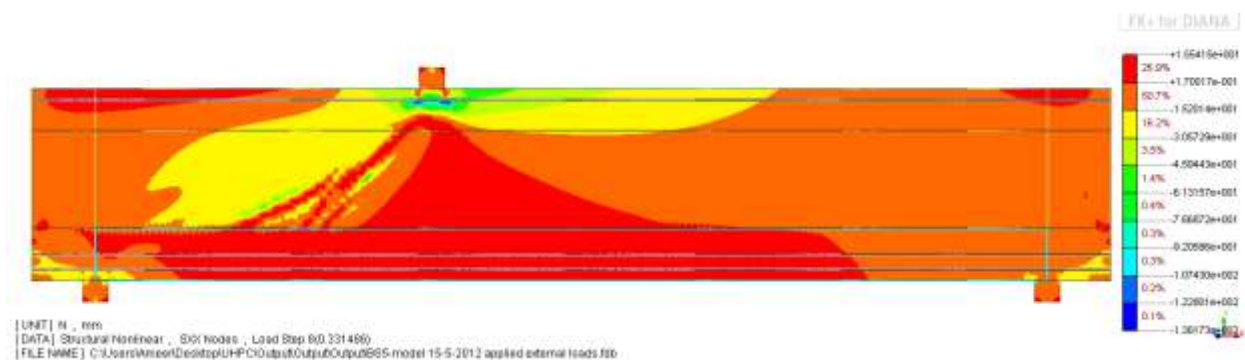


Figure 4.60 Longitudinal stresses (S_{xx}) in BS5 at the initiation of first flexural crack, obtained from nonlinear analysis

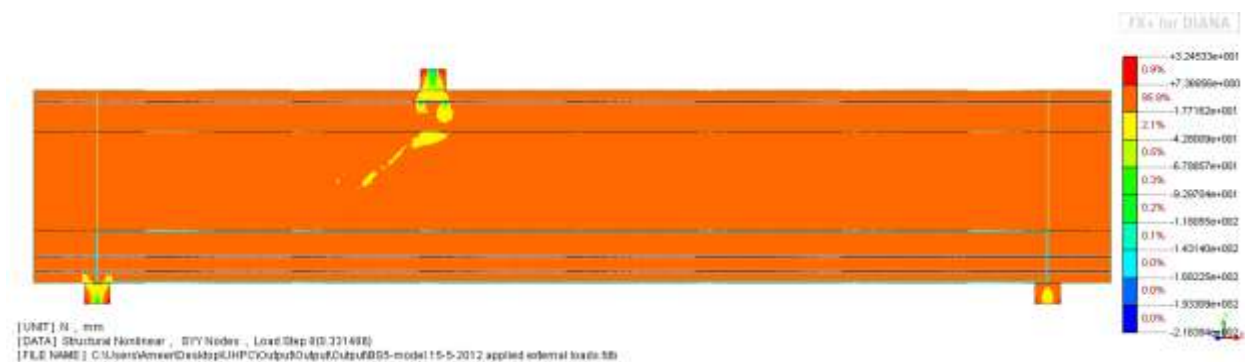


Figure 4.61 Transverse stresses (S_{yy}) in BS5 at the initiation of first flexural crack, obtained from nonlinear analysis

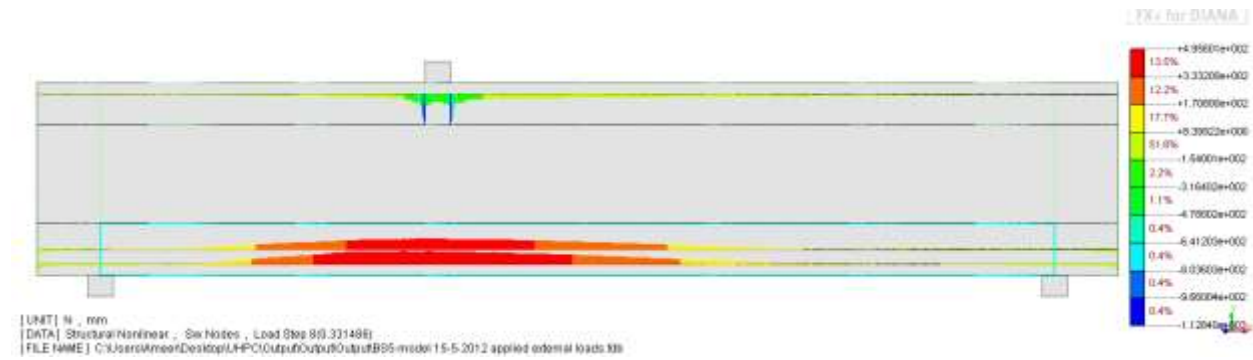


Figure 4.62 Tendon stresses in BS5 at the initiation of first flexural crack, obtained from nonlinear analysis

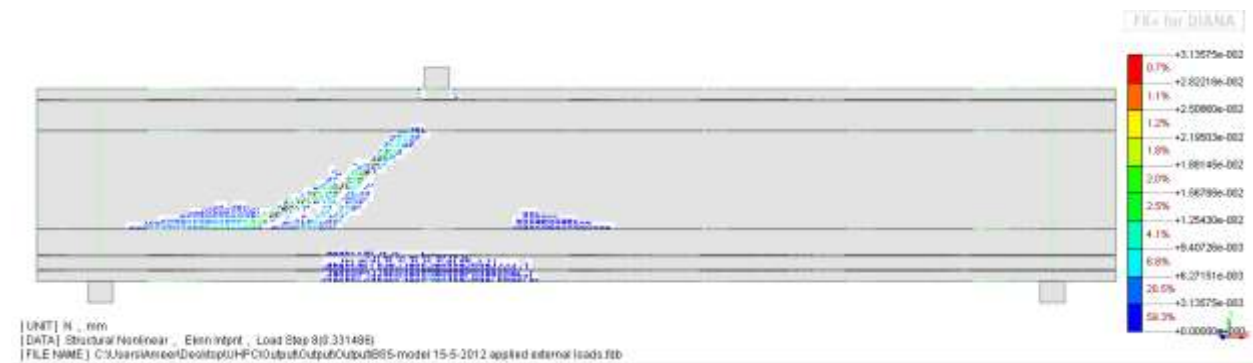


Figure 4.63 Distribution of cracks for beam BS5 at the initiation of first flexural crack, obtained from nonlinear analysis

The ultimate load for the non-linear model was 3300 kN. At this stage of loading the longitudinal compressive stresses showed asymmetrical distribution, forming a semi-arch zone, linking the two supports and the point of application of the external load. The compression zone covered about 47% of the beam with maximum stress taking place below the applied load. High tensile stresses occurred and covered over 53% of the beam as illustrated in Figure 4.64. The maximum compressive stress reduced to 123 MPa or 70 % of the UHPC compressive strength. The maximum tensile stress in the tendons reached 1763 MPa or 95% of the ultimate rupturing strength of steel. Figure 4.65 shows the distribution of transverse stresses, where the tensile stresses were low and uniform covering 90 % of the beam. A small compressive stress zone, covering 10 % of the beam appeared below the applied load and in the direction of diagonal crack. Figure 4.66 shows the distribution of stresses in tendons, where high stresses were generated below the applied load. The crack pattern at ultimate load condition is illustrated in Figure 4.67. It indicated that diagonal shear cracks have fully formed at this stage of loading, and were bounded by excessive shear cracks in the high shear zone. A small shear crack zone was

also formed on the right side of the external load. Wide flexural cracks were formed and were localized in a narrow band below the applied load.

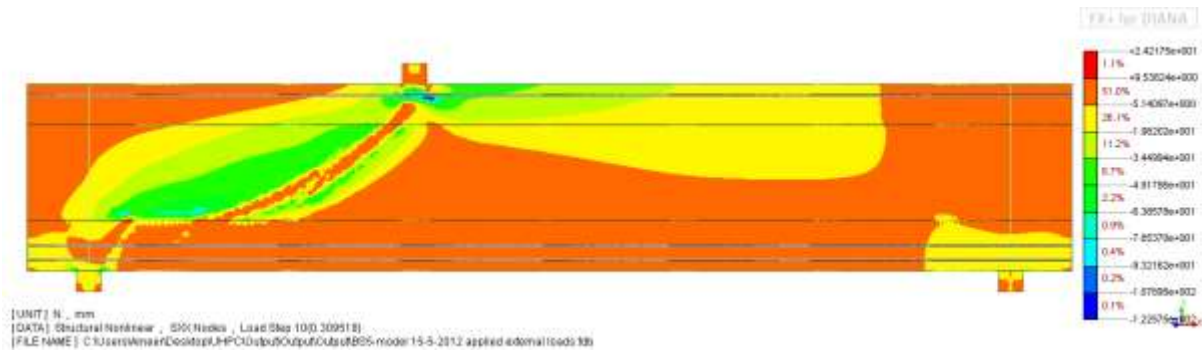


Figure 4.64 Longitudinal stresses (S_{xx}) in BS5 at ultimate, obtained from nonlinear analysis

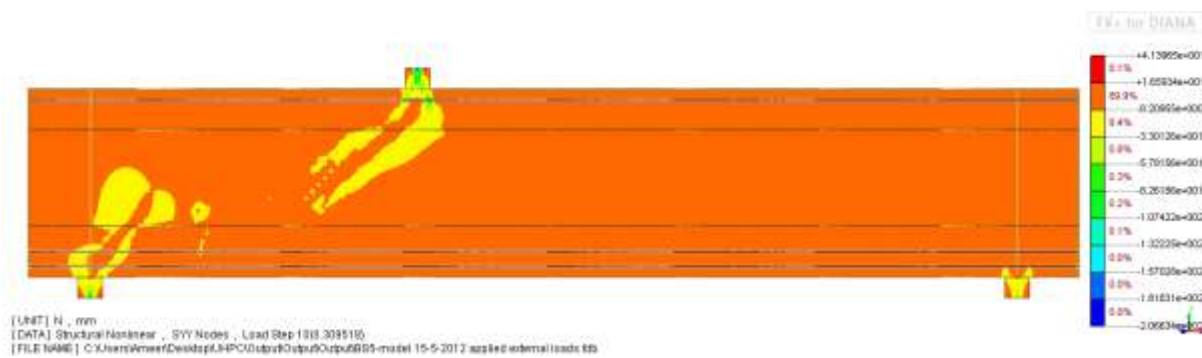


Figure 4.65 Transverse stresses (S_{yy}) in BS5 at ultimate, obtained from nonlinear analysis

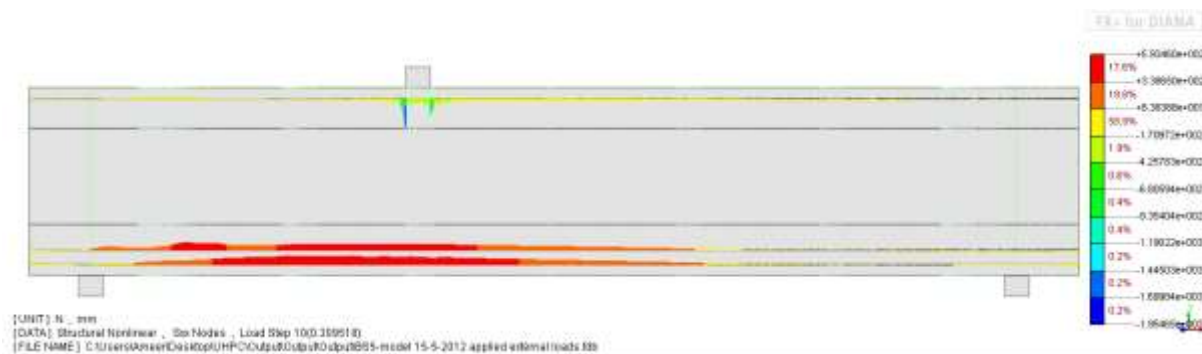


Figure 4.66 Tendons stresses in BS5 at ultimate, obtained from nonlinear analysis

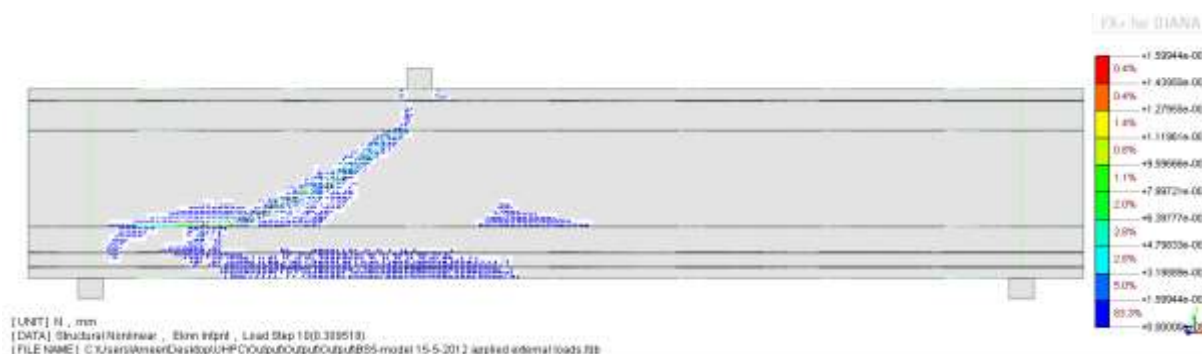


Figure 4.67 Distribution of cracks in BS5 at ultimate, obtained from nonlinear analysis

4.9 Deflection fields for Linear and Nonlinear Models

This section presents changes in vertical deflection fields under increasing external loads. An overview of displacements in different prestressed UHPC beams is provided for both linear and nonlinear models. The emphasis is placed on maximum displacements and their relationships to applied loads. Comparisons between finite element analysis results and computed deflections on the basis of JSCE recommendations are also presented for maximum deflections.

4.9.1 Linear Elastic Models

In linear analysis, all external, prestressing and self-weight forces were applied in the same load step. Hence, this analysis consisted of only one deflection field at ultimate when either the tensile or compressive stress limit was reached. The two beams designed to fail in flexure, FL1 and FL2, showed symmetrical deflection profiles about mid-span, as illustrated in Figures 4.68 and 4.69. The region of maximum deflection was also located at mid-span. Table 4.4 shows that the calculation of maximum deflection based on JSCE 2006 was more than three times the maximum deflection obtained from linear FE analysis.

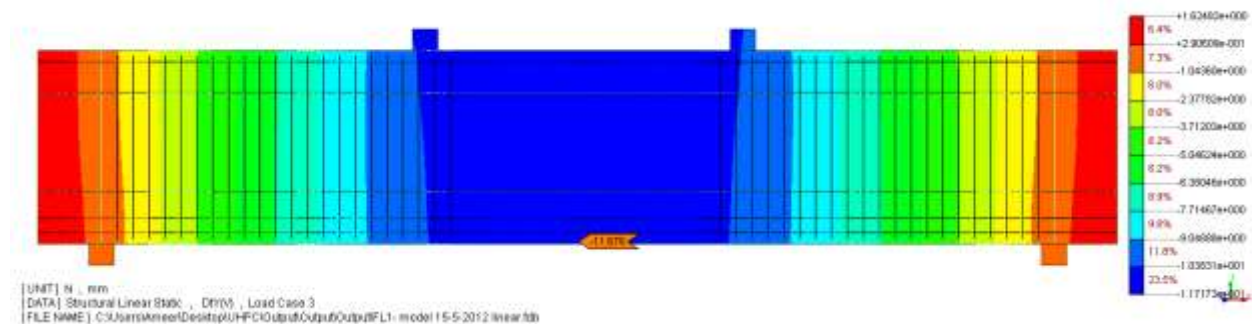


Figure 4.68 Distribution of vertical displacement (V) in FL1 obtained from linear FE analysis at ultimate

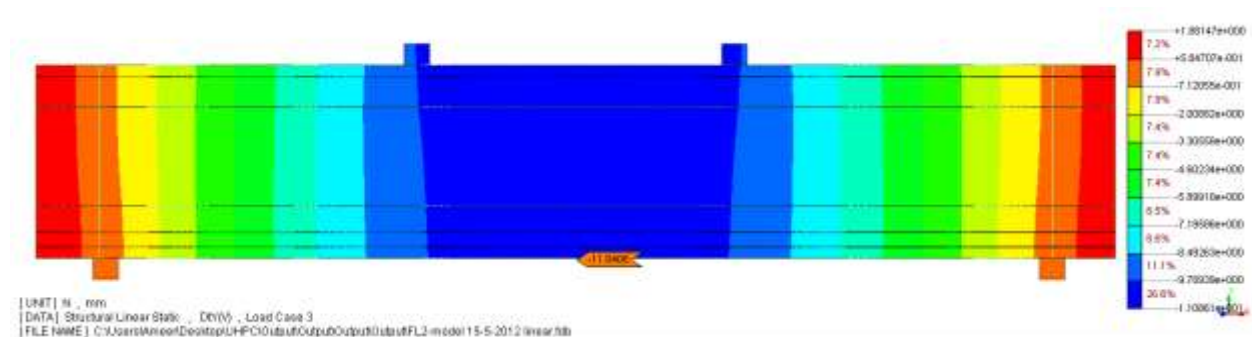


Figure 4.69 Distribution of vertical displacement (V) in FL2 obtained from linear FE analysis at ultimate

Figures 4.70 through 4.74 show vertical deflection fields for beams designed to failure in shear, i.e., BS1 through BS5. In all cases the deflections started increasing gradually from the support regions and reached their maximum values at a section to the right side of the point of application of external load. Table 4.4 shows that the computed maximum deflections based on JSCE 2006 provided reasonably close results to those computed by linear FE analysis.

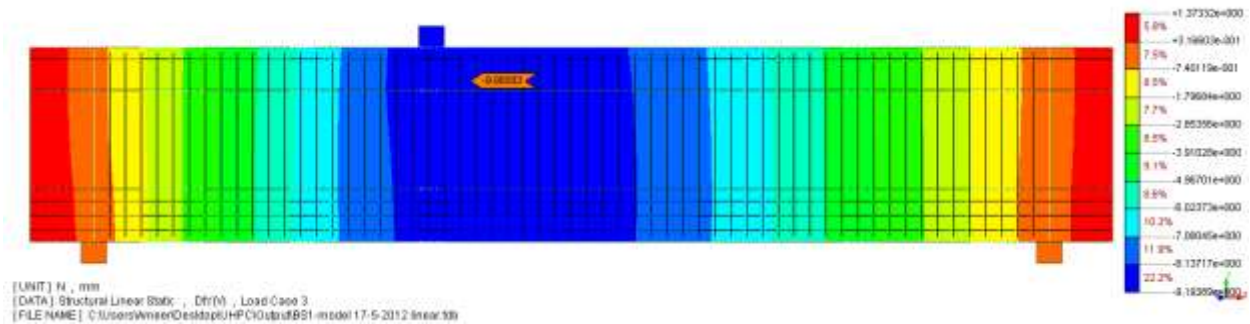


Figure 4.70 Distribution of vertical displacement (V) in BS1 obtained from linear FE analysis at ultimate

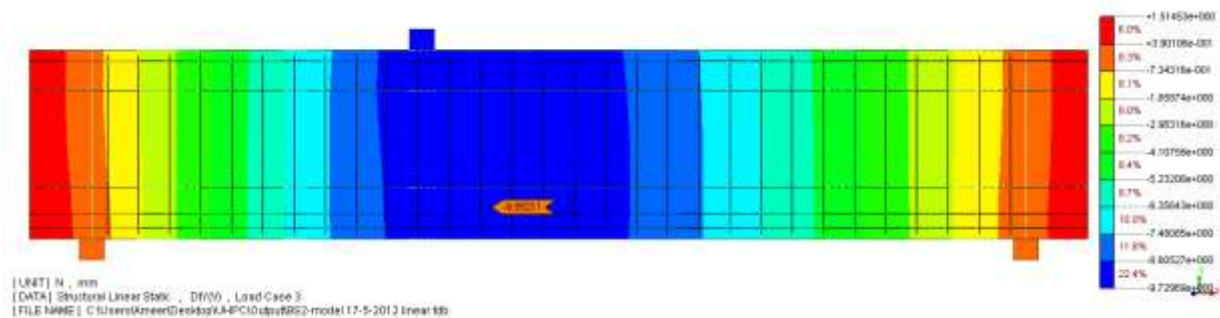


Figure 4.71 Distribution of vertical displacement (V) in BS2 obtained from linear FE analysis at ultimate

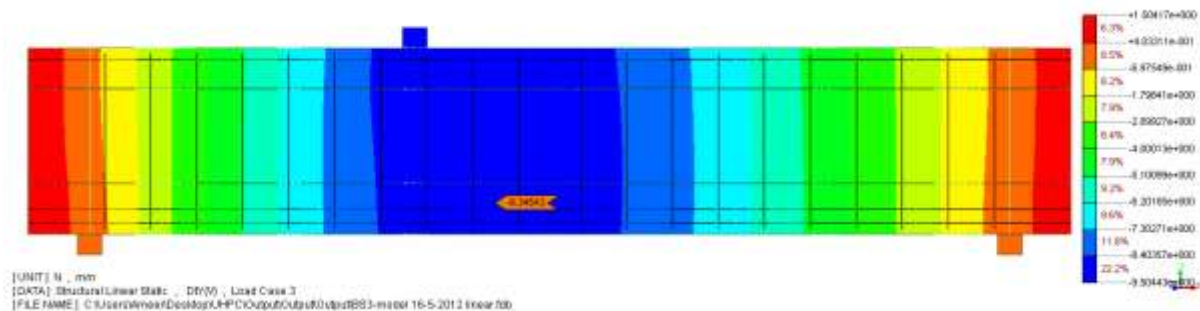


Figure 4.72 Distribution of vertical displacement (V) in BS3 obtained from linear FE analysis at ultimate

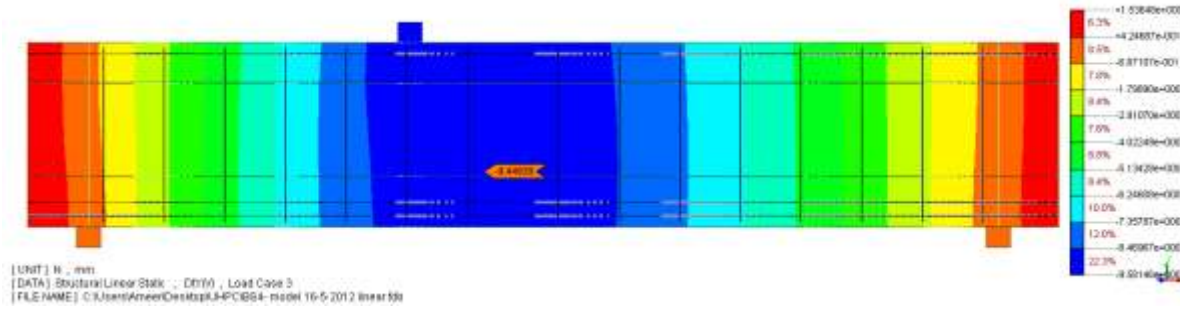


Figure 4.73 Distribution of vertical displacement (V) in BS4 obtained from linear FE analysis at ultimate

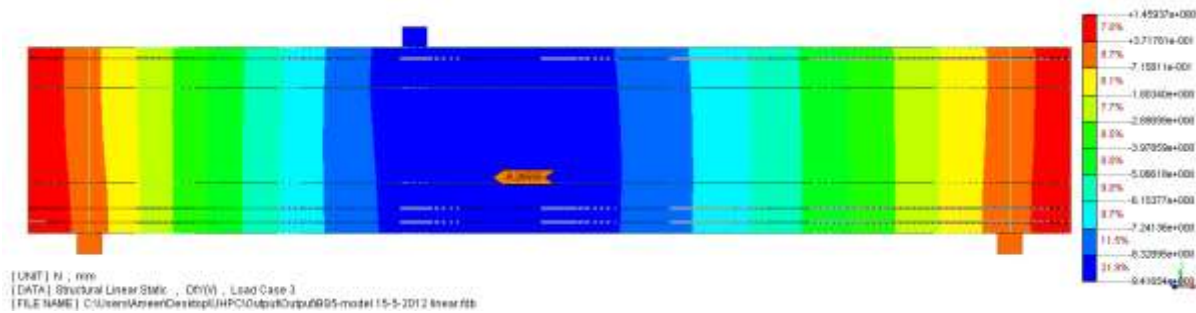


Figure 4.74 Distribution of vertical displacement (V) in BS5 obtained from linear FE analysis at ultimate

4.9.2 Deflection Fields of Nonlinear Elastic Models

During inelastic behavior of prestressed UHPC beams under vertical loads the initiation and propagation of cracks disrupt the continuity of deflection fields. In non-linear FE modeling this creates challenges. The distortion of cracked zones, together with high prestressing forces, result in inconsistent displacement fields, where the maximum deflection zone is hard to identify. The illustrated figures 4.75 and 4.76 show deflection fields for beam FL1 at the formation of first flexural cracks and at ultimate load. Comparisons of Figures 4.75 and Figure 4.76 show the effects of the development and propagation of flexural cracks on deflection fields.

Figures 4.77 and 4.78 show deflection fields for beam BS1 with maximum shear reinforcement at two stages of loading, i.e., at the formation of first shear cracking and at ultimate load.

The comparison between Figures 4.77 and 4.78 show the effects of the development and propagation of diagonal shear cracks on the deflection field in high shear zone. However, this effect is less than that observed in beam BS5 with no shear reinforcement, as presented below.

Table 4.4 Load-displacement results from linear elastic analysis

Model	JSCE, 2006 load (kN)	FEM load (kN)	JSCE, 2006 deflection Δ_{JSCE} (mm)	FEM deflection Δ_{FEM} (mm)	Difference $\left(\frac{\Delta_{JSCE}-\Delta_{FEM}}{\Delta_{JSCE}}\right) \times 100$ (%)
FL1	2×1656	2×2650	39.5	11.7	70
FL2	2×1656	2×2500	39.5	11.0	72
BS1	3019	4230	7.5	9.1	-21
BS2	2423	4180	10.3	9.6	8
BS3	2256	4000	11.8	9.4	20
BS4	2133	3950	13.3	9.5	29
BS5	2095	3950	11.8	9.3	22

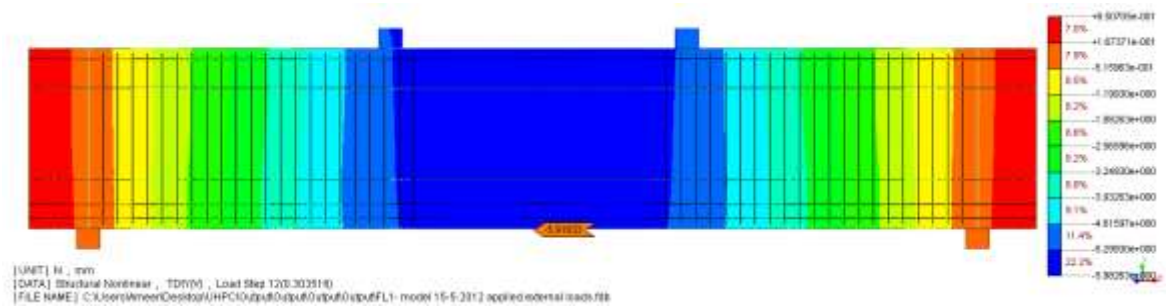


Figure 4.75 Distribution of vertical displacements (V) in FL1 at the formation of first flexural cracking, obtained from non-linear analysis

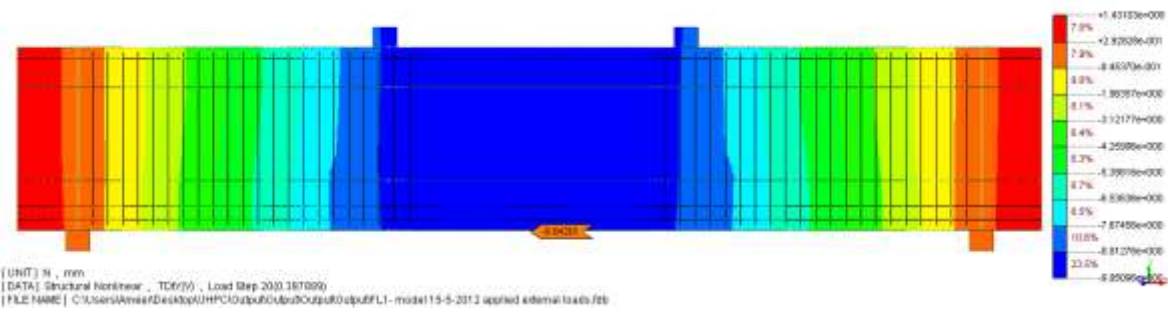


Figure 4.76 Distribution of vertical displacements (V) in FL1 at ultimate, obtained from non-linear analysis

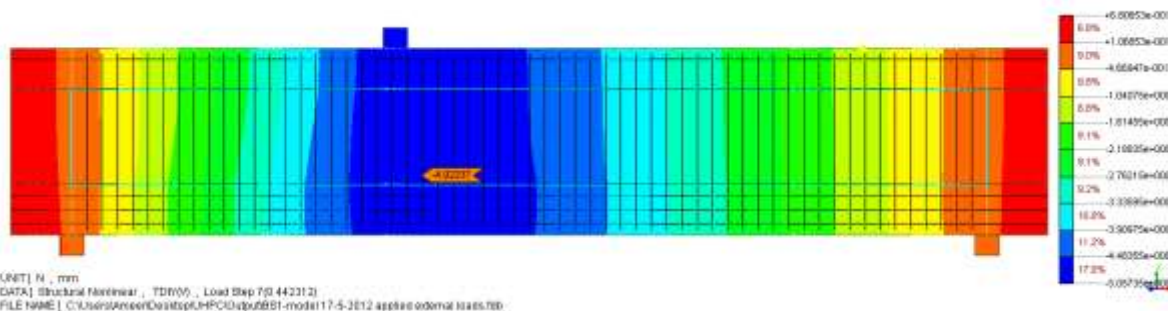
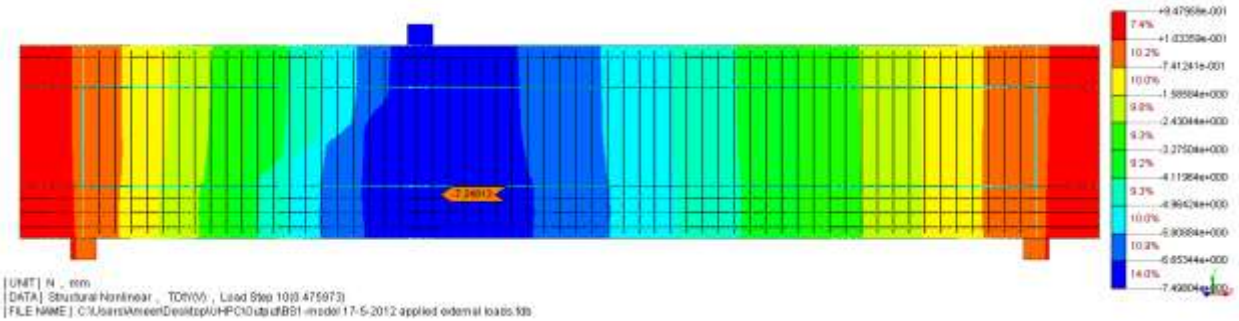


Figure 4.77 Distribution of vertical displacement (V) in BS1 at the formation of first shear crack, obtained from non-linear analysis



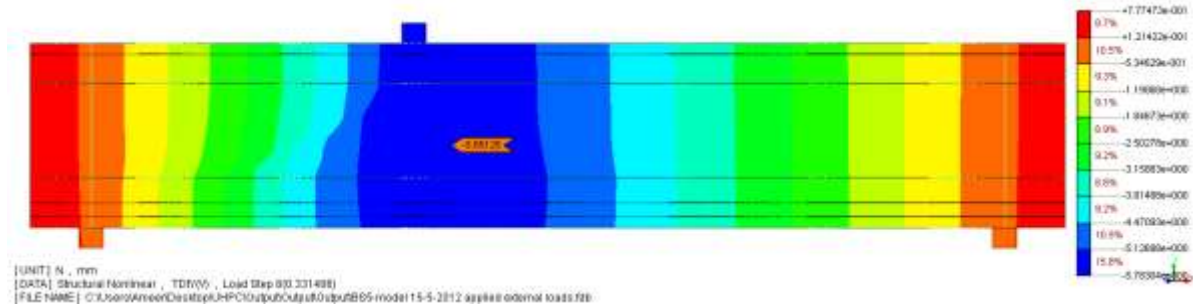


Figure 4.79 Distribution of vertical displacements (V) in BS5 at the formation of first shear crack, obtained from non-linear analysis

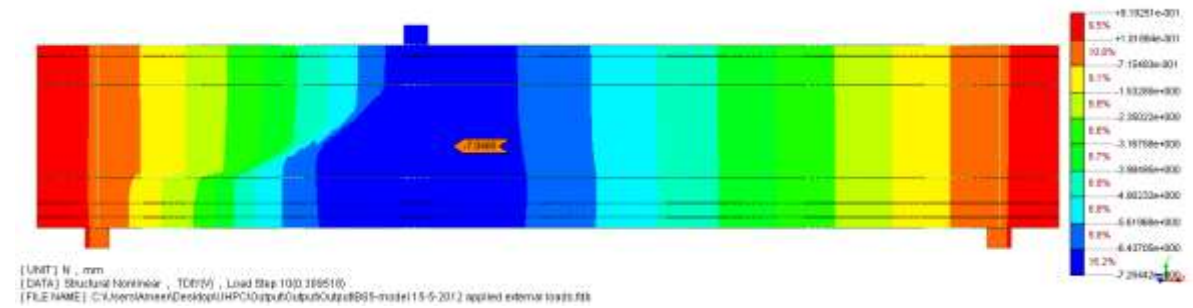


Figure 4.80 Distribution of vertical displacement (V) in BS5 at ultimate, obtained from non-linear analysis

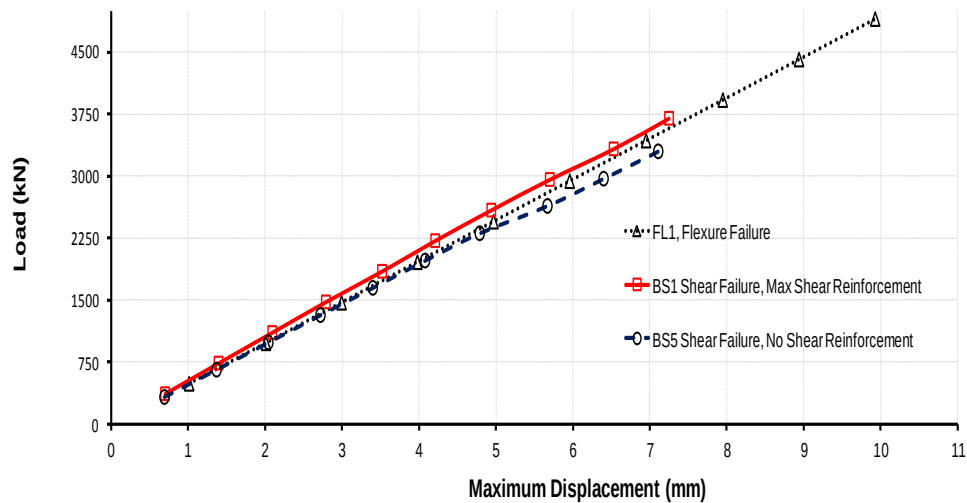


Figure 4.81 Load – maximum deflection relationship for Beams FL1, BS1 & BS5 (Nonlinear Model – 10 step loading)

Table 4.5 Load-displacement results from nonlinear analysis

Model	Load case	JSCE, 2006 load (kN)	FEM load	JSCE, 2006 deflection (mm)	FEM deflection (mm)	Difference $\left(\frac{\Delta_{JSCE}-\Delta_{FEM}}{\Delta_{JSCE}}\right) \times 100$ (%)
FL1	First Cracking	2 X 948	2 X 1470	0.5	5.9	-1054
	Ultimate	2 X 1656	2 X 2450	11.7	9.8	16
BS1	First Cracking	1709	2590	1.0	4.9	-407
	Ultimate	3019	3700	7.5	7.3	-3
BS5	First Cracking	1178	2640	0.7	5.7	-719
	Ultimate	2095	3300	11.8	7.1	41

Chapter 5

Truss Models for Prestressed UHPC Beams

5.1 Introduction

The shear design approach of JSCE 2006 is based on a traditional truss analogy with constant diagonal crack inclination of 45° . The applicability of this approach to fiber reinforced concrete, with or without prestressing and shear reinforcement (stirrups), has not yet been verified. Furthermore, the shear failure mechanism in fiber reinforced UHPC is not yet well understood. Therefore, the applicability of truss models to prestressed UHPC beams requires further research. This chapter provides simplified truss models that were developed as part of the current investigation for prestressed UHPC beams. The models are based on the results of linear and nonlinear FEM analyses described in Chapter 4. They include lateral compression struts associated with prestressing steel. The details of the truss models are described in the following section.

5.2 Development of Truss Models

Figure 5.1 shows the dimensions of a constant angle truss model for the shear-dominant beams designed in Chapter 3. The distance between the pin support (A) and the projection of the load application point on the bottom cord of the truss (or joint C in Figure 5.1) represents the shear span, which is divided into two “panels” of 525 mm length, each. The rest of the lower cord span from joint (C) to joint (F), with a total length of 1950 mm, is divided into three “panels” of 650 mm each. The height is assumed to be constant for all beam models, and is equal to the distance between the centroid of the bottom layer of prestressing tendons and the center of the top flange. This truss height is computed to be 484 mm. The truss is then formed by; i) a lower cord, consisting of members AB, BC, CD, and EF; ii) a top cord, consisting of members GH, HI, and IJ; iii) vertical ties, consisting of members BG, CH, DI, and EJ; and iv) inclined struts, consisting of members AG, BH, DH, EI, and FJ. The spacing of the truss panels on the left and right sides

are equal to a and b , respectively, i.e., $S_1 = a$ and $S_2 = b$. Since the truss height is constant, the inclination of diagonal struts on each side are also equal, i.e., $\theta_3 = \theta_1$ and $\theta_4 = \theta_2$.

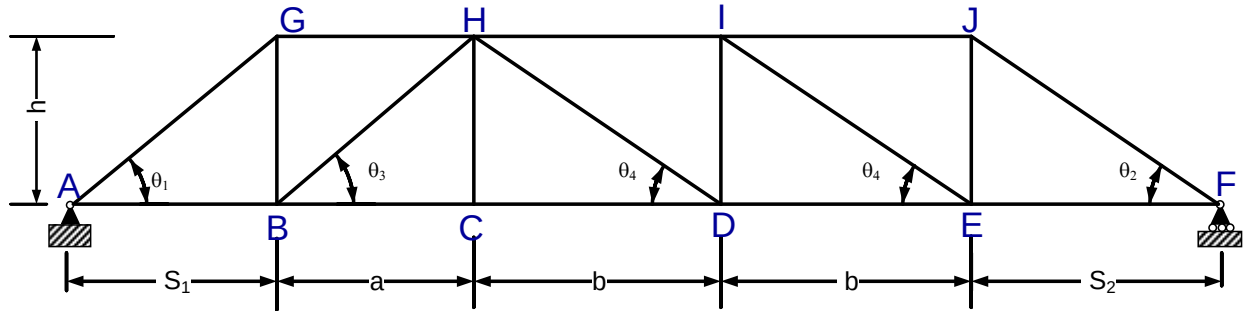


Figure 5.1 Proposed truss model without horizontal struts due to prestressing

5.2.1 Modeling the Prestressing Forces

In the proposed truss model, the prestressing forces are applied as external horizontal forces on the truss. The prestressing forces in lower flange tendons are represented by a horizontal force at the roller support (the right hand exterior joint of the lower cord in Figure 5.2) and a horizontal component of the pin support on the opposite side of the truss. The prestressing forces in top flange tendons are represented by two equal and opposite forces applied at two exterior joints of the upper cord. Figure 5.2 shows the two sets of external forces that simulate the effects of prestressing on the beams (F_{xt} in the “tension” cord at joints A and F, and F_{xc} in the “compression” cord at joints G and J).

5.2.2 Additional Inclined Struts

The effect of prestressing system on the shear strength of traditional prestressed concrete beams is usually accounted for, if there is a vertical component of the prestressing force generated by inclined or curved tendons. However researchers (Lertsamattiyakul, et al 2004) found that the distribution of prestressing stresses in the shear zone affects the flow of forces. It was shown that adding one or more lateral strut(s) results in a better match with finite element analysis. The need for additional inclined struts was investigated as part of the current investigation. This was done by adding one, two or three additional struts to the truss model. The stresses in the truss model

are then compared with those computed by linear and the nonlinear finite element analyses. The truss analysis was conducted using SAP2000 computer software. The following are the four truss patterns investigated:

- i. Basic statically determinate truss model, as illustrated in Figure 5.1, without additional inclined struts.
- ii. Basic truss plus one additional inclined strut linking the vertical load to the pin support as shown in Figure 5.2.
- iii. Basic Truss plus two additional inclined struts transferring stresses to the lower cord as illustrated in Figure 5.3.

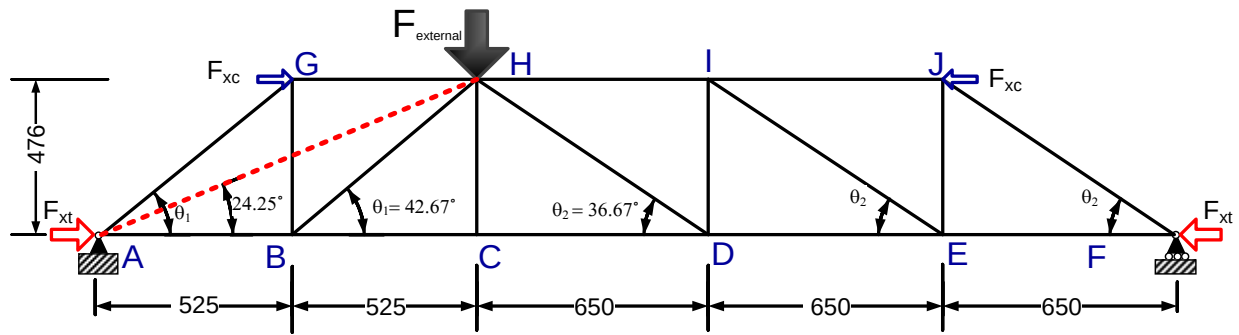


Figure 5.2 Proposed truss model with one additional inclined strut

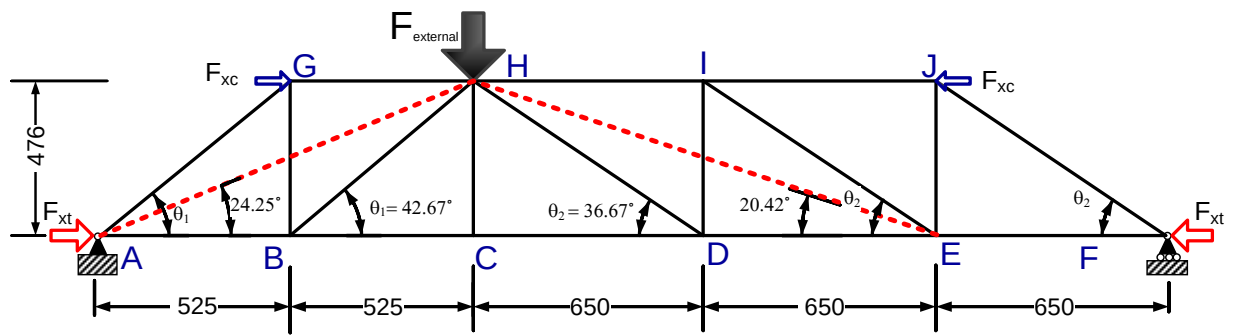


Figure 5.3 Proposed truss model with two additional inclined struts

- iv. Basic Truss plus three additional inclined struts three inclined struts linking the point of application of the load (H) to the pin support (A), joint (E), and the roller support (F) on the lower cord of the truss were added as illustrated in Figure 5.4.

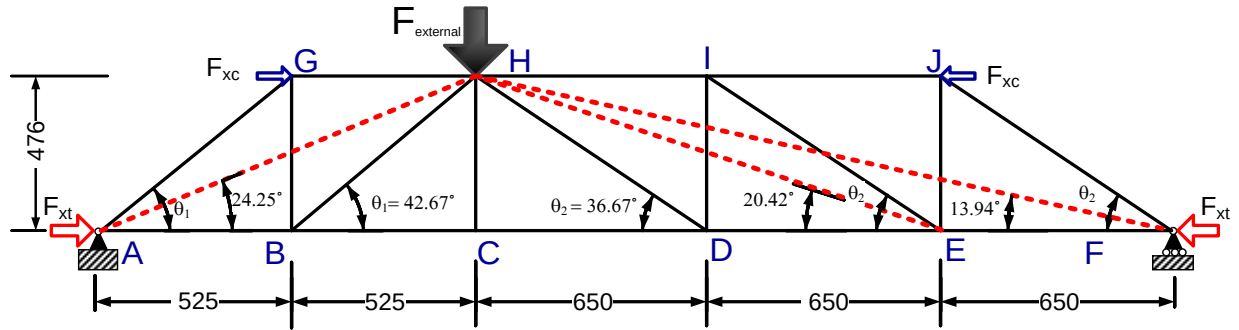


Figure 5.4 Proposed truss model with three additional inclined struts

5.2.3 Applied loads, Boundary Conditions, and Loading Cases

There are three types of forces applied on the truss models described in the preceding section: i) external gravity load applied on one of the top joints (joint H in Figure 5.2); ii) prestressing forces applied on the roller joint F (balanced by the reaction in the hinge support A); and iii) prestressing forces applied on the top cord at joints G and J. The boundary conditions (support conditions) of the truss are the same as those of the beam; consisting of a pin (left hand side) and a roller (right hand side). Figure 5.2 through Figure 5.4 show the truss models with external loads and support conditions. The focus in the truss analysis was on ultimate load case. However, for comparison with the results on nonlinear FE model in some cases, crack initiation load case was also investigated.

5.3 Truss Models for Comparison with Linear Finite Element Models

In this section, the results of structural analysis using truss models are presented for five shear critical prestressed UHPC beams designed in Chapter 3; i.e., Beams BS1 with , BS2, BS3, BS4, and BS5. Figure 5.5 shows the general state of stress in different members of trusses under a single external load. The top cord is in compression, the bottom cord is in tension, the inclined struts are in compression and the vertical ties are in tension, as expected. In the linear FEM, the beam deformed elastically without any cracks. Therefore, the stresses in stirrup steel or in steel fibers are equal to the stresses in the surrounding UHPC.

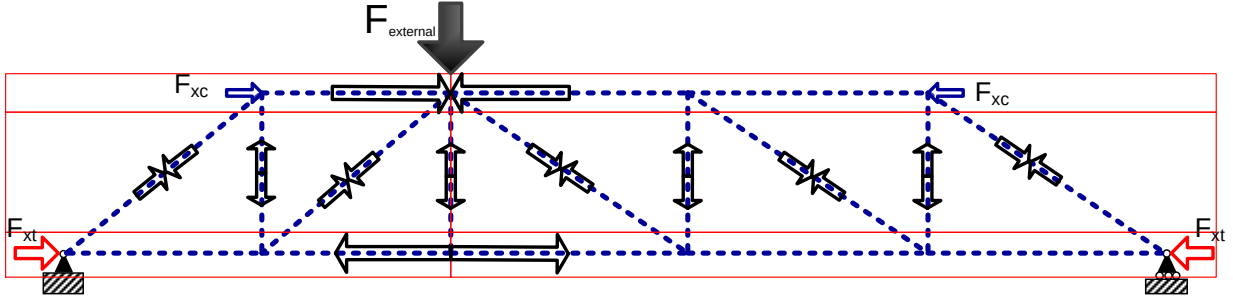


Figure 5.5 Concept of truss model simulating linear FEM

The truss members are modeled as rectangular UHPC elements with the following cross sectional areaa:

i. Bottom cords:

$$A_{bc} = b_{fb} \times d_{fb} \quad (5.1)$$

Where:

A_{bc} is the cross sectional area of bottom flange.

b_{fb} is the width of the bottom cord, equal to the width of the bottom flange (350mm).

d_{fb} is the depth of the bottom cord, equal to the depth of the bottom flange (150mm).

ii. Top cords:

$$A_{tc} = b_{ft} \times d_{ft} \quad (5.2)$$

Where:

A_{tc} is the cross sectional area of top flange.

b_{ft} is the width of the top cord, equal to the width of the top flange (350mm).

d_{ft} is the depth of the top cord, equal to the depth of the top flange (120mm).

iii. Vertical struts:

$A_{\text{vertical strut}} = \text{Vertical UHPC member area} + \text{UHPC area equivalent to the total area of stirrups}$

For left Vertical strut:

$$A_{vl} = \left(\frac{s_1 + a}{2} \right) \times b_w + \left[2n_{vl} \times A_b \times \left(\frac{E_s}{E_{UHPC}} - 1 \right) \right] \quad (5.3)$$

For Vertical strut under the applied load:

$$A_{vc} = \left(\frac{a+b}{2} \right) \times b_w + \left[2n_{vc} \times A_b \times \left(\frac{E_s}{E_{UHPC}} - 1 \right) \right] \quad (5.4)$$

For right vertical strut 1:

$$A_{vr1} = (b) \times b_w + \left[2n_{vr1} \times A_b \times \left(\frac{E_s}{E_{UHPC}} - 1 \right) \right] \quad (5.5)$$

For right vertical strut 2:

$$A_{vr2} = \left(\frac{b+s_2}{2} \right) \times b_w + \left[2n_{vr2} \times A_b \times \left(\frac{E_s}{E_{UHPC}} - 1 \right) \right] \quad (5.6)$$

Where: a and b are shown in Figure 1, b_w is the width of the web (150mm), n is the number of stirrups in the equivalent strut, A_b is the area of the stirrup leg, E_s is the modulus of elasticity of the steel stirrup, and E_{UHPC} is the modulus of elasticity of UHPC.

iv. Lateral struts:

$$A = b_w \times S' \quad (5.7)$$

where S' for different struts is as follows:

$$S'_{AG} = S_1 \sin \theta_1 \quad \text{for AG}$$

$$S'_{FJ} = S_2 \sin \theta_2 \quad \text{for FJ}$$

$$S'_{BH} = (2a) \sin \theta_3 \quad \text{for BH}$$

$$S'_{DH} = (2b) \sin \theta_4 \quad \text{for DH}$$

$$S'_{EI} = (2b) \sin \theta_4 \quad \text{for EI}$$

Where: a, b, S_1 , and S_2 are shown in Figure 1; b_w is the web width, S' is the width of the inclined strut, and $\theta_1, \theta_2, \theta_3$ and θ_4 are the inclination angles of inclined struts as shown in Figure 5.1. In the present case $S_1 = a$ and $S_2 = b$, hence $\theta_3 = \theta_1$ and $\theta_4 = \theta_2$; then:

$$S'_{AG} = a \sin \theta_1 \quad \text{for AG}$$

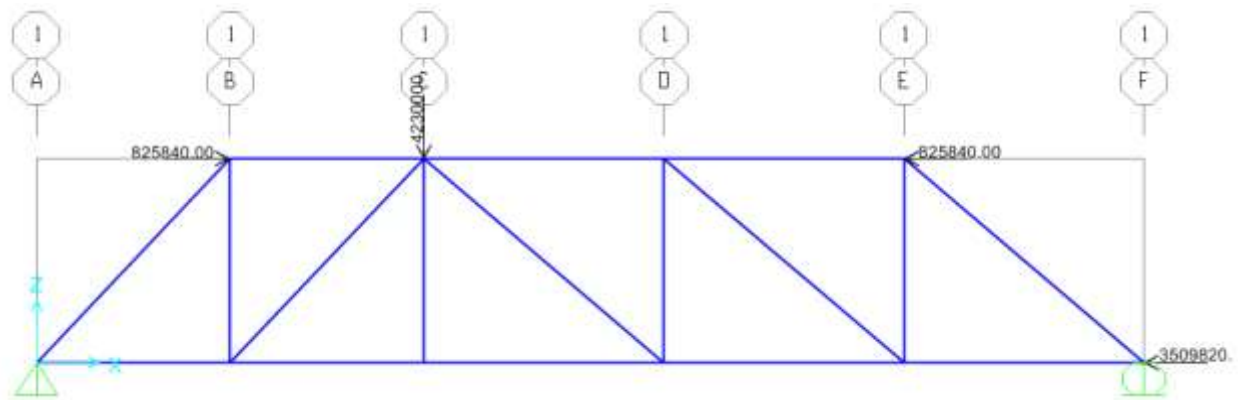
$$S'_{FJ} = b \sin \theta_2 \quad \text{for FJ}$$

$$S'_{BH} = (2a) \sin \theta_1 \quad \text{for BH}$$

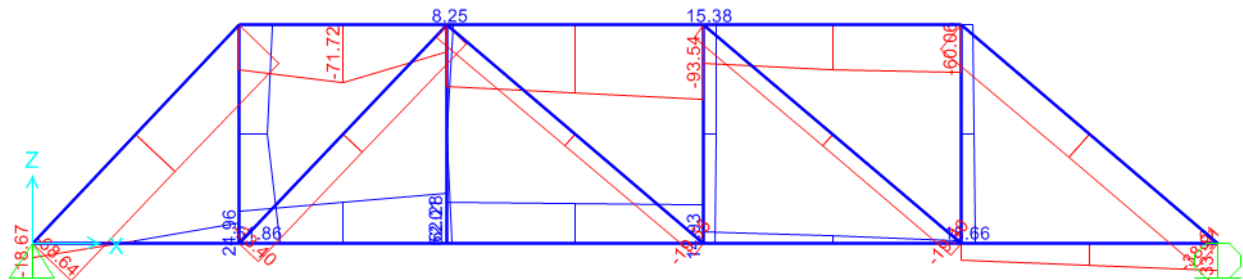
$$S'_{DH} = (2b) \sin \theta_2 \quad \text{for DH}$$

$$S'_{EI} = (2b) \sin \theta_2 \quad \text{for EI}$$

The stresses in truss members are the results of truss analysis conducted by SAP2000. Figures 5.6 through 5.10 show the analysis results for beams BS1, BS2, BS3, BS4 and BS5.

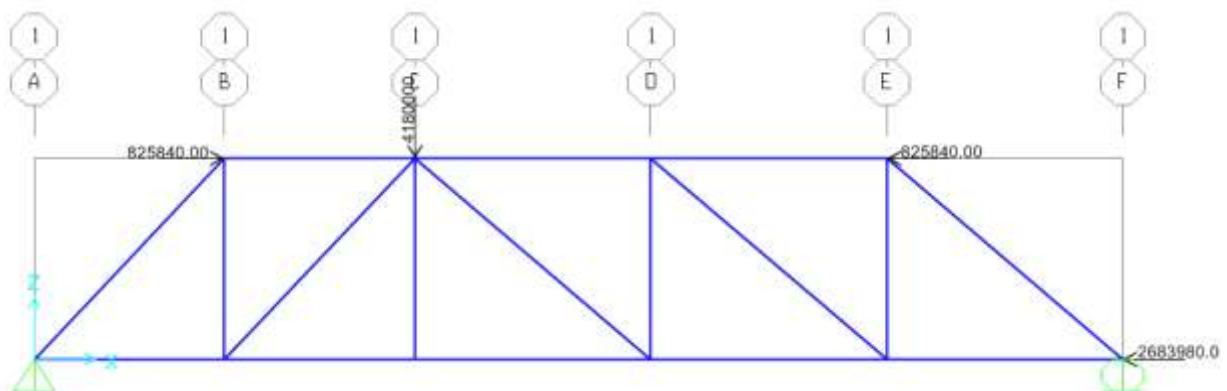


(a) External and prestressing forces



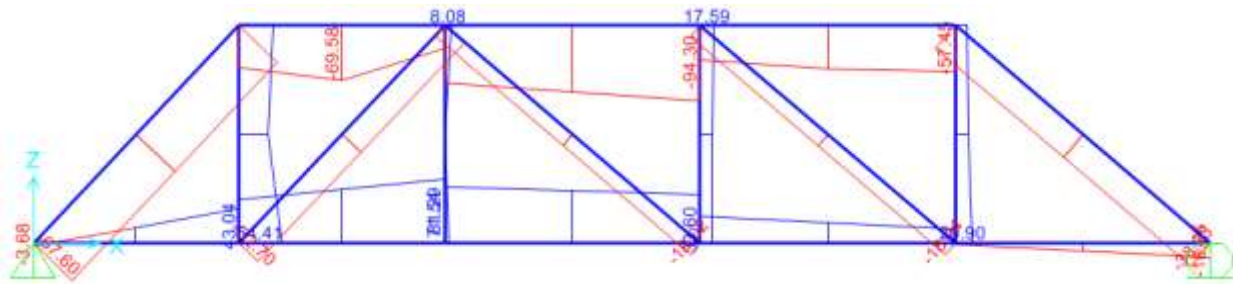
(b) Stresses of different Truss members

Figure 5.6 External forces and member stresses for beam BS1



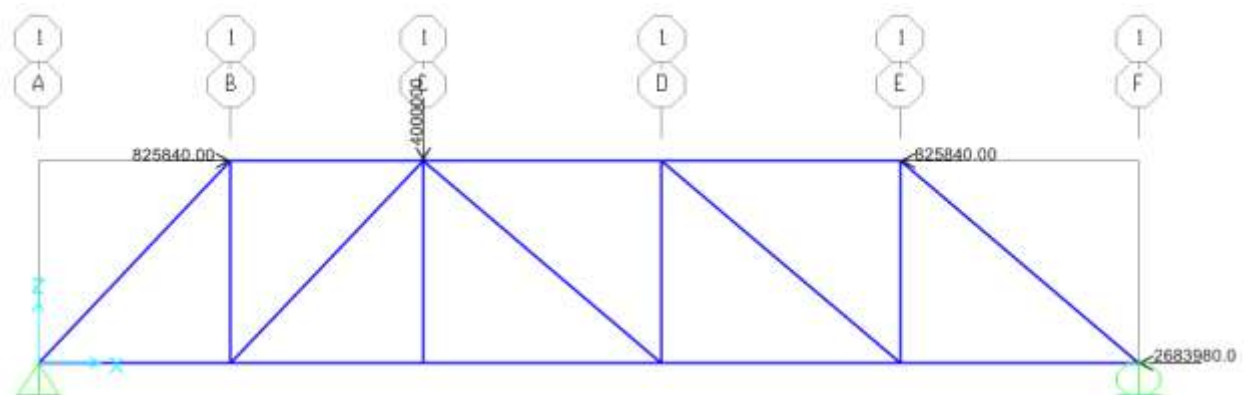
(a) External and prestressing forces

Figure 5.7 External forces and member stresses for beam BS2

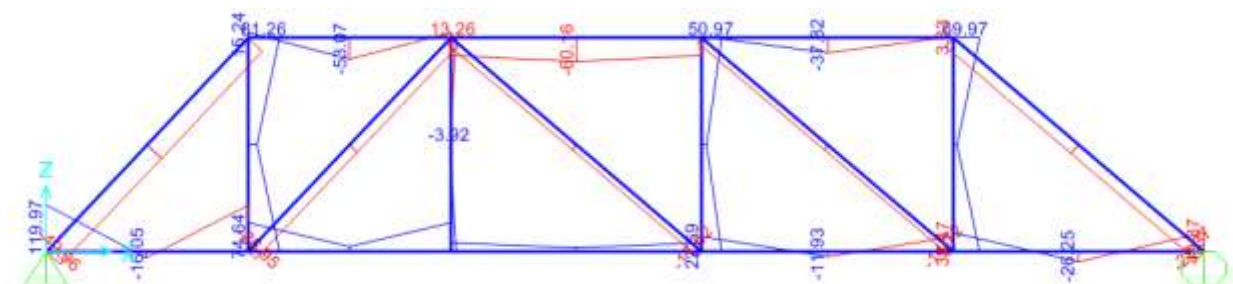


(b) Stresses of different Truss members

Figure 5.8(Con'd) External forces and member stresses for beam BS2

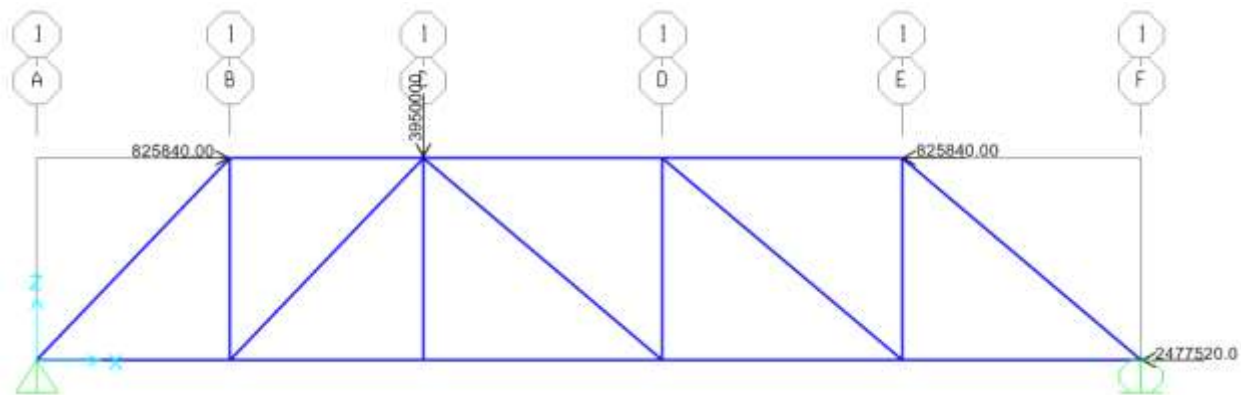


(a) External and prestressing forces

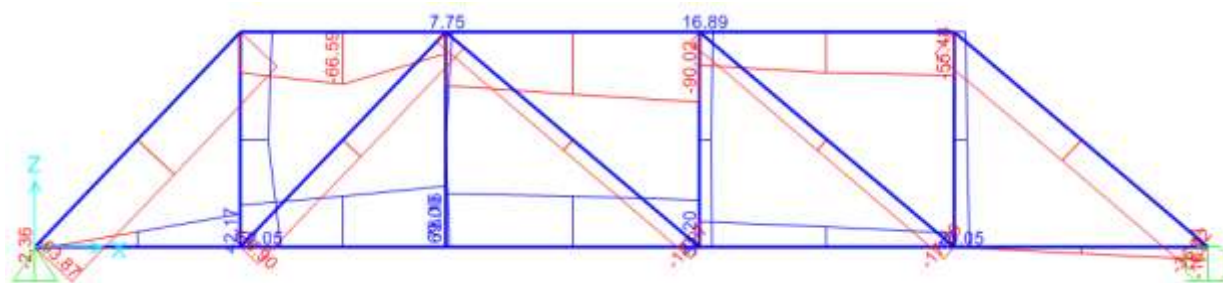


(b) Stresses of different Truss members

Figure 5.9 External forces and member stresses for beam BS3

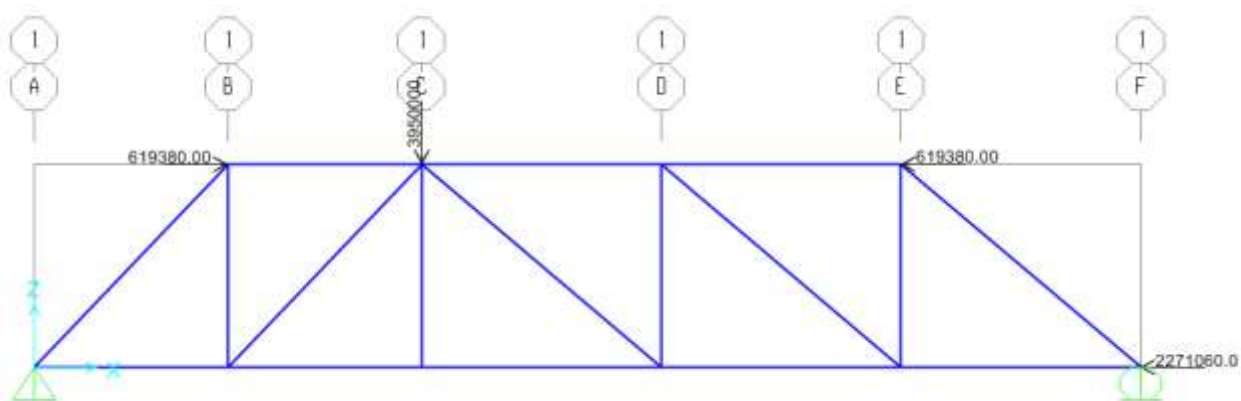


(a) External and prestressing forces



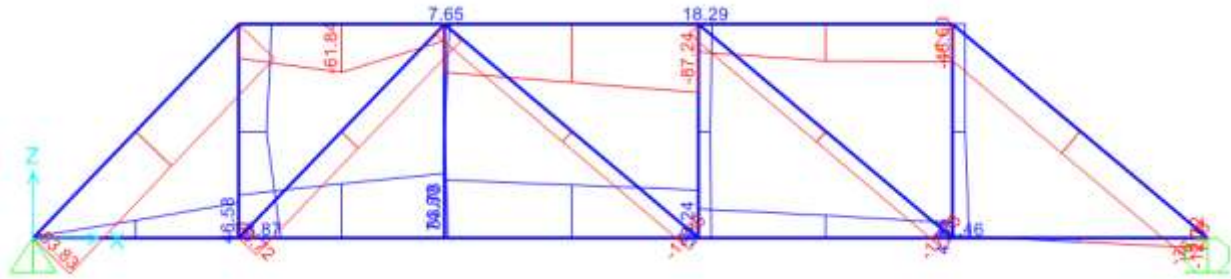
(b) Stresses of different Truss members

Figure 5.10 External forces and member stresses for beam BS4



(a) External and prestressing forces

Figure 5.11 External forces and member stresses for beam BS5



(b) Stresses of different Truss members

Figure 5.12 (Cont'd) External forces and member stresses for beam BS5

5.4 Comparisons of Truss Analysis with Linear FEM Analysis

The results of truss analysis are compared in this section with beam results obtained from linear FEM analysis. The emphasis is placed on stresses computed for each element. The influence of applying the effects of prestressing through external forces is first assessed. Then, the effect of adding one or more lateral struts on the distribution of stresses is examined. It is important to note that the truss elements consist of UHPC, with additional cross sectional areas assigned in proportion to the ratio of the modulus of elasticity of reinforcing steel in the element to that of UHPC.

5.4.1 Effect of Prestressing Force on Truss Model

Table 5.1 shows the effect of incorporating prestressing forces as external forces on the truss model for Beam BS1, which has the highest amount of shear reinforcement among the beams designed, with US #3 (71 mm² area) stirrups at 50 mm spacing. It provides the comparison of elastic FEM analysis results with those obtained from the truss analyses with and without the prestressing forces. In general, the difference between the two sets of results were very different for vertical ties due to stirrups, because the elastic FE model did not account for cracking, and the stirrups only become effective after the cracking of concrete. However, the application of prestressing forces improved the correlation in other elements, especially the tension members along the bottom cord.

A similar comparison is made in Table 5.2 for Beam BS5 with no shear reinforcement. A better match between the stresses in the truss members and the stresses of linear FEM is achieved in the lower and upper cords when the prestressing tendons were applied. However, in general the

correlation between the truss analyses and the linear FEM analysis was poor. This was attributed to the poor material models used in the FEM analysis, which did not account for concrete cracking.

Table 5.1 Effect of prestressing force on Beam BS1 when the basic truss model was used without additional inclined struts

	Member	FEM (MPa)	With Prestressing Forces		Without Prestress. Forces	
			Truss (MPa)	% Differ.	Truss (MPa)	% Differ.
Vertical Struts	BG	0.782	51.86	-6531	51.593	-6498
	CH	-39.36	8.248	121	6.387	116
	DI	0.34	15.39	-4425	26.45	-7679
	EJ	-0.26	18.66	7276	29.05	11273
Inclined Struts	AG	-40.67	-68.64	-69	-63.27	-56
	BH	-54.59	-33.4	39	-29.49	46
	DH	-42.16	-19.55	54	-16.69	60
	EI	-41.35	-19.40	53	-16.85	59
	FJ	-35.97	-38.91	-8	-35.51	1
Lower Cord	AB	-16.42	24.96	252	98.20	698
	BC	49.05	62.28	-27	142.78	-191
	CD	61.19	52.01	15	134.73	-120
	DE	13.35	14.93	-12	92.64	-594
	EF	-27.27	-33.92	-24	57.77	312
Upper Cord	GH	-110	-71.72	35	-44.49	60
	HI	-125.1	-93.54	25	-73.85	41
	IJ	-80.24	-60.06	25	-30.95	61

Table 5.2 Effect of prestressing force on Beam BS5 when the basic truss model was used without additional inclined struts

	Member	FEM (MPa)	With Prestressing Forces		Without Prestressing forces	
			Truss (MPa)	% Differ.	Truss (MPa)	% Differ.
Vertical Struts	BG	1.426	52.83	-3605	63.08	-4323
	CH	-36.1	7.81	122	6.45	118
	DI	0.974	17.25	-1671	27.02	-2674
	EJ	0.261	20.41	-7720	29.68	-11272
Inclined Struts	AG	-27.48	-63.95	-133	-67.73	-146
	BH	-41.09	-30.89	25	-31.54	23
	DH	-39.04	-17.95	54	-17.87	54
	EI	-26.99	-17.88	34	-18.03	33
	FJ	-25.66	-39.19	-53	-38	-48
Lower Cord	AB	-2.457	36.97	1605	105.23	4383
	BC	64.22	73.30	-14	153.27	-139
	CD	73.74	64.19	13	144.44	-96
	DE	26.62	28.39	-7	99.30	-273
	EF	-12.91	-20.48	-59	62.06	581
Upper Cord	GH	-106.6	-62.62	41	-47.73	55
	HI	-114.3	-86.43	24	-79.13	31
	IJ	-72.06	-49.69	31	-33.05	54

5.4.2 Effect of Number of Additional struts

The effect of adding additional inclined compression struts has been investigated by comparing truss analyses with the results of elastic FEM analysis in Table 5.3 and 5.4. For Beam BS1, where the stirrup reinforcement was maximum, an improved correlation with FEM results was observed in the vertical ties when the three additional inclined struts were used. However, the correlations were better in most of the inclined struts, as well as top and bottom cord stresses when the basic truss model was implemented. The comparison shows that adding three inclined

struts improved the correlation of stresses in vertical ties for Beam BS5 with no stirrup steel. Slight improvements in some of the inclined strut stresses were observed with increasing the number of inclined struts. In general it was observed that any improvement in correlation was limited because of the relatively poor material models used in the elastic FEM analysis.

Table 5.3 Effect of additional struts in Beam BS1

	Member	FEM (MPa)	No Additional Strut		Three Additional Struts	
			Truss (MPa)	% Diff	Truss (MPa)	% Diff
Vertical Struts	BG	0.782	51.86	-6531	22.61	-2791
	CH	-39.36	8.25	121	2.14	105
	DI	0.34	15.39	-4425	3.08	-806
	EJ	-0.26	18.66	7276	9.97	3935
Inclined Struts	AG	-40.67	-68.64	-69	-27.15	33
	AH	-37.96			-22.1	42
	BH	-54.59	-33.40	39	-13.61	75
	DH	-42.16	-19.55	54	-4.21	90
	EH	-35.42			-1.00	97
	EI	-41.35	-19.40	53	-4.07	90
	FH	-34.17			-14.66	57
	FJ	-35.97	-38.91	-8	-19.49	46
Lower Cord	AB	-16.42	24.96	252	35.99	319
	BC	49.05	62.28	-27	46.63	5
	CD	61.19	52.01	15	42.59	30
	DE	13.35	14.93	-12	33.93	-154
	EF	-27.27	-33.92	-24	15.86	158
Upper Cord	GH	-110	-71.72	35	-36.22	67
	HI	-125.1	-93.54	25	-40.75	67
	IJ	-80.24	-60.06	25	-35.36	56

Table 5.4 Effect of additional struts in Beam BS5

	Member	FEM (MPa)	No Additional Struts		One Additional Strut		Two Additional Struts		Three Additional Struts	
			Truss (MPa)	% Diff	Truss (MPa)	% Diff	Truss (MPa)	% Diff	Truss (MPa)	% Diff
Vertical Struts	BG	1.43	52.83	-3605	26.40	-1751	26.01	-1724	25.20	-1667
	CH	-36.1	7.81	122	11.80	133	8.34	123	1.28	104
	DI	0.97	17.25	-1671	17.55	-1702	11.51	-1082	6.09	-525
	EJ	0.26	20.41	-7720	20.48	-7746	16.22	-6115	7.74	-2866
Inclined Struts	AG	-27.48	-63.95	-133	-26.87	2	-26.86	2	-26.87	2
	AH	-29.72			-18.10	39	-18.51	38	-19.63	34
	BH	-41.09	-30.89	25	-13.44	67	-13.38	67	-13.16	68
	DH	-39.04	-17.95	54	-18.01	54	-11.98	69	-4.90	87
	EH	-25.02					-14.42	42	3.66	115
	EI	-26.99	-17.88	34	-17.79	34	-11.84	56	-4.80	82
	FH	-22.31							-11.83	47
Lower Cord	FJ	-25.66	-39.19	-53	-36.21	-41	-36.08	-41	-19.72	23
	AB	-2.46	36.97	1605	47.00	2013	46.80	2005	46.17	1979
	BC	64.22	73.30	-14	64.63	-1	62.36	3	58.40	9
	CD	73.74	64.19	13	60.22	18	57.31	22	55.44	25
	DE	26.62	28.39	-7	28.93	-9	36.29	-36	43.41	-63
Upper Cord	EF	-12.91	-20.48	-59	-20.47	-59	-20.58	-59	25.65	299
	GH	-106.6	-62.62	41	-34.79	67	-33.57	69	-30.39	72
	HI	-114.3	-86.43	24	-88.00	23	-71.19	38	-39.29	66
	IJ	-72.06	-49.69	31	-49.86	31	-53.08	26	-34.68	52

5.5 Comparisons of Truss Analysis with Nonlinear FEM Analysis

In this section, the comparison of the results obtained from nonlinear FEM analysis and truss analyses is presented. Three of the shear critical beams designed in Chapter 3, namely BS1, BS3 and BS5 are selected for the comparison. Beam BS1 has the highest shear reinforcement, consisting of #3 US (6.33 mm diameter) stirrups spaced at 50 mm and fiber reinforcement having 0.9% volumetric ratio. Beam BS3 has intermediate shear reinforcement, with the same stirrups spaced at 150 mm, and the same fibre content. Beam BS5, on the other hand has no

stirrups but increased fibre content of 2.0% volumetric ratio. The truss models are the same as those described in the preceding section.

5.5.1 Concepts and Assumptions for the Truss Models Employed

The truss models used in this section were developed with due considerations given to the contribution of fibres in UHPC. Upon the initiation of flexural or shear cracks, the fibers in UHPC provides crack control, bridging and transfer tensile stresses across the cracks. It has been observed by previous researchers (see Chapter 2, Section 2.6) that the fibers in UHPC have a tendency to orient themselves in the longitudinal direction. In contrast, both the French and the Japanese design recommendation assume uniform distribution of fibers. These recommendations suggest a reduction factor, accounting for the fiber orientation. The truss models used in the current investigation are based on the assumption that the ratio of the fibers that are oriented in the longitudinal direction constitute $R_{f_{hz}} = 40\%$, and the ratio of the fibers oriented in $\pm 45^\circ$ inclination form $R_{f_{in}} = 30\%$ and the rest of the fibers are oriented in the vertical direction ($R_{f_{vr}} = 30\%$) parallel to the stirrups. It is further assumed that the fibers in the inclined direction provide full tensile resistance in controlling diagonal cracks. Figure 5.11 shows three general cases, demonstrating the relationship between the fiber orientation and the crack orientation. In the lower flange flexural cracks are controlled with both the longitudinal fibres and the inclined fibers. The vertical fibers in this case are parallel to the crack direction. In the case of horizontal cracks at the lower flange-web connection, contributions are expected from the vertical and inclined fibers, while the horizontal fibers are not effective. Both horizontal and vertical fibres are effective in controlling the inclined cracks. As the cracks widths widen, however, some of the fibres pull out because of their short lengths (10 mm to 25 mm). It is assumed conservatively that 50% of the fibres are pulled-out ($PO_r = 50\%$) at ultimate.

The fibre area for each truss element is computed as explained below:

- i. **Bottom cords:** At ultimate load, the flexural cracks are assumed to be fully developed, and the fibers are assumed to contribute to crack control while providing tensile resistance together with the tendons.

Since the volumetric ratio of fibers is assumed to be $f_{vr} = 0.9\%$, the cross sectional area of fibres in the lower cord is:

$$A_{fs-lc} = (A_{fs-hz} + A_{fs-in} \times \cos 45^\circ) \quad (5.8)$$

Where: A_{fs-lc} is the steel area equivalent to the fibers in the bottom cord; A_{fs-hz} is the steel area equivalent to the fibers oriented horizontally; and A_{fs-in} is the steel area equivalent to the inclined fibers.

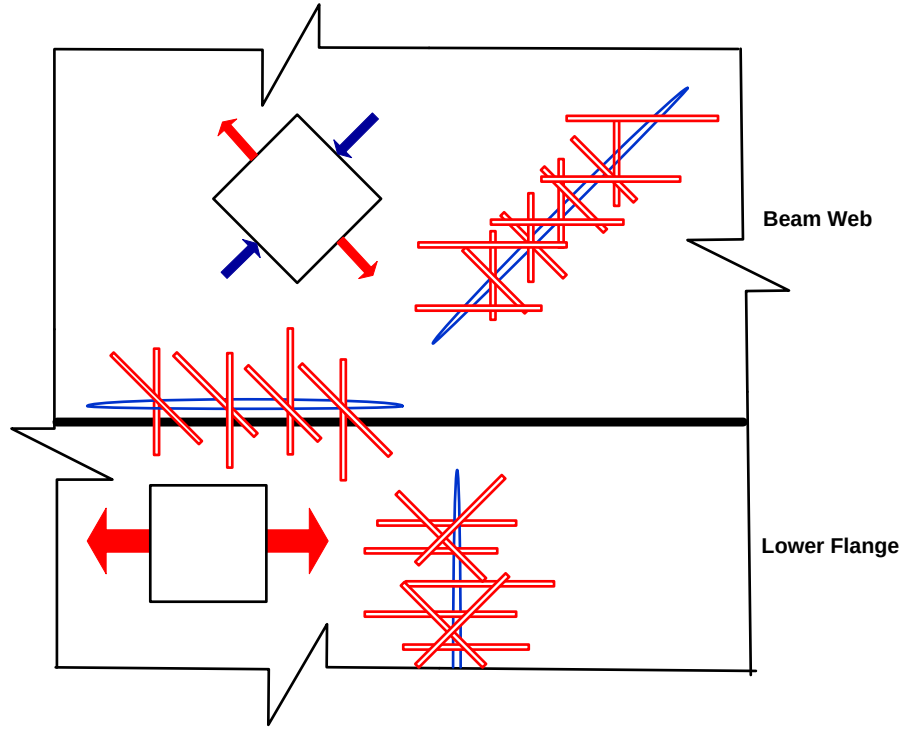


Figure 5.13 Fiber contributions to crack control

It is important to indicate that the truss is modeled as a linear elastic system. The fibre steel, as well as the tendons is assumed have the same modulus of elasticity and Poisson's ratio. Therefore, it is possible to combine all the steel in the bottom cord in one equivalent steel area. This equivalent steel area is calculated as follows:

$$A_{fs-hz} = f_{vr} \times A_{cross-hz} \times R_{fhz} \times PO_r \quad (5.9)$$

$$A_{fs-in} = f_{vr} \times A_{cross-in} \times R_{fin} \times PO_r \quad (5.10)$$

$$A_{bot-cord} = A_{fs} + A_{tendons} \quad (5.11)$$

$$d_{bot-cord} = \sqrt{\frac{A_{bot-cord} \times 4}{\pi}} \quad (5.12)$$

Where:

$A_{cross-hz}$ is the cross sectional area of steel fibers contributing to the bottom cord;

$A_{cross-in}$ is the cross sectional area of steel fibers in the direction of inclined strut;

$A_{bot-cord}$ is the total steel area in the bottom cord;

$d_{bot-cord}$ is the equivalent diameter of a steel element representing the total steel area in the bottom cord.

Note that the diameter of this equivalent “steel element”, or ($d_{bot-cord}$), is used as input for the sectional properties of bottom cord.

ii. Vertical ties:

The UHPC contribution to vertical ties is only through the fibers (until they pull-out). Additional contribution is provided by stirrups, when present. Hence, vertical truss ties consist of two components, i) contribution of fibres, and ii) contribution of stirrup steel. The contribution of fibres is computed as follows:

$$A_{fs-vs} = (A_{fs-vr} + A_{fs-in} \times \sin 45^\circ) \quad (5.13)$$

Where: A_{fs-vs} is the steel area equivalent to the fibers contributing to tensile resistance of vertical ties; A_{fs-vr} is the steel area equivalent to the fibers oriented vertically; and A_{fs-in} is the steel area equivalent to the inclined fibers. Hence the equivalent steel area that combines the contributions of stirrups and steel fibers in vertical ties can be calculated as follows:

$$A_{fs-vr} = f_{vr} \times A_{cross-vr} \times R_{fvr} \times PO_r \quad (5.14)$$

$$A_{ver-strut} = A_{fs-vs} + A_{stirrups} \quad (5.15)$$

$$A_{stirrups} = 2 n \times A_b \quad (5.16)$$

iii. Top cords:

The top cord is assumed to resist compression in the entire top flange.

$$A_{tc} = b_{ft} \times d_{ft} \quad (5.17)$$

Where:

A_{tc} is the cross sectional area of top flange.

b_{ft} is the width of top cord, which is equal to the width of the top flange (350mm).

d_{ft} is the depth of top cord, which equals to the depth of the top flange (120mm).

iv. Inclined struts:

The cross sectional area of each inclined strut is calculated as follows:

$$A = b_w \times S' \quad (5.18)$$

Where S' is defined below for different struts.

$$S'_{AG} = a \sin \theta_1, \quad \text{for AG}$$

$$S'_{FJ} = b \sin \theta_2 \quad \text{for FJ}$$

$$S'_{BH} = (2a) \sin \theta_1 \quad \text{for BH}$$

$$S'_{DH} = (2b) \sin \theta_2 \quad \text{for DH}$$

$$S'_{EI} = (2b) \sin \theta_2 \quad \text{for EI}$$

Where: a , b , S_1 , and S_2 are shown in Figure 5.1, b_w is the web width, S' is the width of inclined struts, and $\theta_1, \theta_2, \theta_3$ and θ_4 are the inclination angles of inclined struts (as shown in Figure 5.1). For the truss models analyzed in the current investigation; $S_1 = a$ and $S_2 = b$; hence $\theta_3 = \theta_1$ and $\theta_4 = \theta_2$.

Figure 5.12 shows forces acting on a prestressed UHPC beam when the diagonal shear crack is fully formed. The Figure shows that the top flange of the beam is in compression, the steel tendons and fibers in the bottom cord are in tension, the stirrups and fibers across diagonal shear crack are in tension. When stirrups are not used in the case of Beam BS5, the vertical ties consists of only the fibres in UHPC.

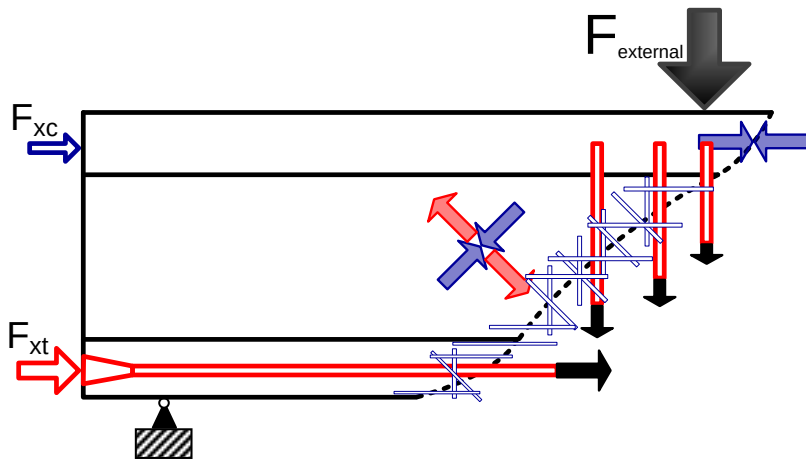


Figure 5.14 Forces in prestressed UHPC beams at diagonal crack formation

5.5.2 Stresses Transfer Factor for Steel Fibers

Beams with stirrups were modeled for nonlinear FEM analysis such that the vertical stresses would give the tensile stress in the stirrups. When stirrups do not exist, as in the case of Beam BS5; the FEM was not able to reflect the contribution of fibres to shear resistance. In this case, and to evaluate the stresses in the fibers, a vertical rebar is provided with a cross sectional area equivalent to the total cross sectional area of the fibers contributing to tensile resistance. This area is equal to the cross sectional area of the vertical tie element in the truss model. The stress in this equivalent rebar section represents the average stress in the fibers across the crack. Hence a stress transfer factor is proposed to convert uniformly distributed stresses in un-cracked tributary area of vertical tie element to stresses concentrated within the element that crosses inclined cracks. The stress transfer factor can be expressed as shown below:

$$F_{st} = \frac{A_{vs}}{A_{fs-vs}} \quad (5.19)$$

Where: F_{st} is the stress transfer factor, A_{vs} is area of the un-cracked tributary area of a vertical strut (i.e., in BG, CH, DI, or EJ), and A_{fs-vs} is the overall area of steel fibers resisting tensile stresses in the direction of vertical tie element.

$$A_{vsBG} = a \times b_w \quad (5.20)$$

$$A_{vsch} = \left(\frac{a+b}{2} \right) \times b_w \quad (5.21)$$

$$A_{vsDI} = b \times b_w \quad (5.22)$$

In the truss models adopted, $a = 525 \text{ mm}$ and $b = 650 \text{ mm}$, as shown in Figure 5.1 and the web width, $b_w = 150 \text{ mm}$. Furthermore;

$$A_{fs-vs} = (A_{fs-vr} + A_{fs-in} \times \sin 45^\circ) \quad (5.23)$$

$$A_{fs-vr} = f_{vr} \times A_{cross-vr} \times R_{fvr} \times PO_r \quad (5.24)$$

$$A_{fs-in} = f_{vr} \times A_{cross-in} \times R_{fin} \times PO_r \quad (5.25)$$

Following is a numerical example for calculating the stresses transfer factor:

For strut BG (see Figure 5.1):

$$A_{vsBG} = a \times b_w = 525 \times 150 = 78750 \text{ mm}^2$$

$$\begin{aligned} A_{fs-vr} &= f_{vr} \times A_{cross-vr} \times R_{fvr} \times PO_r = 0.02 \times \left(\frac{525 + 525}{2} \right) \times 150 \times 0.3 \times 1 \\ &= 472.5 \text{ mm}^2 \end{aligned}$$

$$A_{fs-in} = f_{vr} \times A_{cross-in} \times R_{fin} \times PO_r = 0.02 \times \left(\frac{525 + 525}{2} \right) \times 150 \times 0.3 \times 1 = 472.5 \text{ mm}^2$$

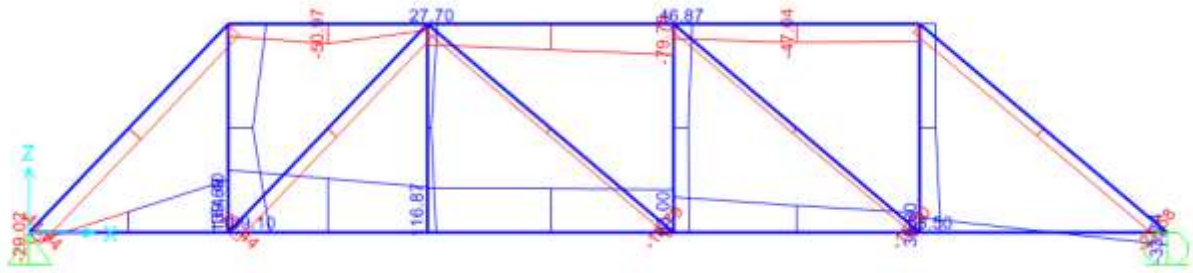
$$A_{fs-vs} = (A_{fs-vr} + A_{fs-in} \times \sin 45^\circ) = 472.5 + 472.5 \sin 45^\circ = 806.6222 \text{ mm}^2$$

$$F_{st} = \frac{A_{vs}}{A_{fs-vs}} = \frac{78750}{806.6222} = 97.63$$

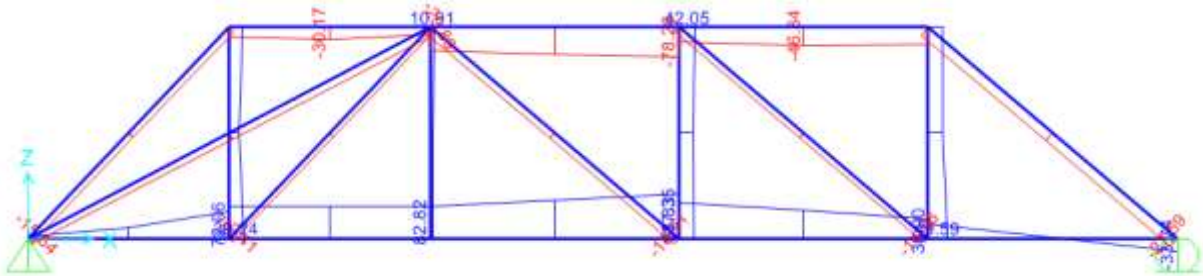
5.5.3 Truss Analysis Results for Beam BS1

Beam BS1 has the highest percentage of shear reinforcement (#3 US stirrups at 50 mm spacing). The nonlinear finite element analysis of this beam indicates hairline flexural cracks at the first crack initiation stage. These cracks formed in the bottom flange, below the applied load. The first shear crack initiated at the connection between the web and the bottom flange. The cracks widened gradually with increasing external load until the ultimate state was reached. The ultimate load stage was well above the first cracking load. The fibers resisted diagonal tension cracks until the cracks propagated and widened, at which stage a large percentage of the fibres pulled-out.

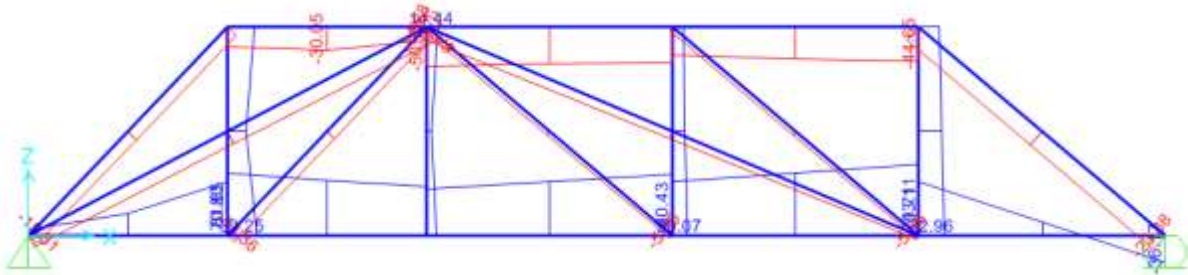
The nonlinear truss model presented in previous sections provides very good correlation with the results of nonlinear finite element analysis for almost all truss members, at ultimate load stage. The details of the first crack initiation load stage are not discussed in detail as this stage occurred early in loading, without significant stress development. The accuracy of results improved significantly when additional inclined struts are included in the truss model. Figures 5.13(a) to (d); show stresses in truss members at ultimate. Table 5.5 gives the comparison of truss analysis with different number of additional inclined struts relative to the inelastic FEM analysis.



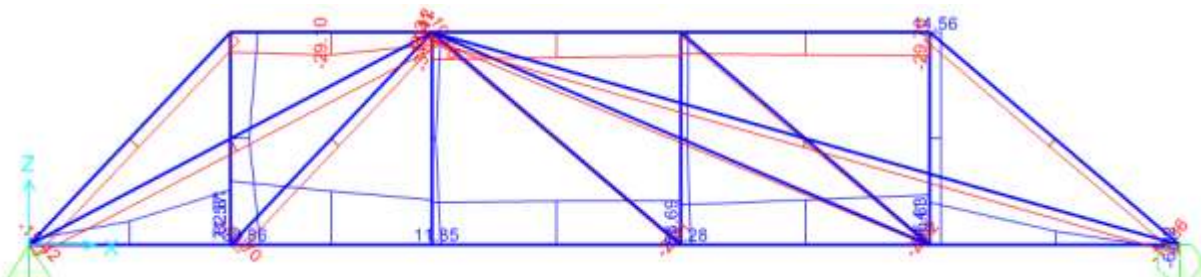
(a) BS1 Simple truss (No additional struts)



(b) BS1 one additional inclined strut



(c) BS1 two additional inclined struts



(d) BS1 three additional inclined struts

Figure 5.15 Stresses in truss members at ultimate (Beam BS1)

5.5.4 Truss Analysis Results for Beam BS3

Beam BS3 was similar to beam BS1, but had intermediate level of shear reinforcement with #3 US stirrups spaced at 150 mm. The nonlinear FEM analysis indicated the initiation of flexural cracks in the bottom flange, below the applied load. Diagonal tension cracks at the connection between the web and the bottom flange and propagated toward the web. The cracks became wider as the load increased at a rate which was slightly faster than that for Beam BS1. This continued until the ultimate state of stress was attained at a significantly higher load than the initial cracking load.

Table 5.5 Stresses in truss models compared to nonlinear FEM analysis for Beam BS1 at ultimate load stage

	Member	FEM	No additional struts		One additional strut		Two additional struts		Three additional struts	
			Truss (MPa)	%age Diff	Truss (MPa)	%age Diff	Truss (MPa)	%age Diff	Truss (MPa)	%age Diff
Vertical Struts	BG	439	1766.1	-302	449.18	-2	439.93	0	431.63	2
	CH	336.59	91.89	73	60.82	82	16.05	95	12.88	96
	DI	336.59	738.79	-119	743.88	-121	261.40	22	163.89	51
	EJ	33.00	740.13	-2143	741.21	-2146	677.48	-1953	258.74	-684
Inclined Struts	AG	-35.91	-61.94	-72	-16.46	54	-16.39	54	-16.34	54
	AH	-51.82			-27.80	46	-27.67	47	-27.57	47
	BH	-63.31	-28.92	54	-7.56	88	-7.19	89	-6.90	89
	DH	-8.79	-15.34	-75	-15.68	-78	-5.06	42	-2.99	66
	EH	-12.19					-16.27	-33	-25.31	-108
	EI	-15.96	-17.75	-11	-17.69	-11	-5.04	68	-2.903	82
	FH	-13.19							-19.91	-51
Lower Cord	FJ	-14.21	-34.00	-139	-34.04	-139	-33.02	-132	-13.41	6
	AB	1744.27	1353.0	22	1602.77	8	1577.64	10	1559.76	11
	BC	1744.27	2001.4	-15	1758.56	-1	1731.85	1	1704.46	2
	CD	1744.27	1818.4	-4	1809.01	-4	1729.78	1	1707.86	2
	DE	1744.27	1371.6	21	1371.21	21	1568.97	10	1614.08	7
Upper Cord	EF	1744.27	1338.3	23	1338.36	23	1337.50	23	1315.79	25
	GH	-22.17	-69.20	-212	-32.98	-49	-32.85	-48	-32.75	-48
	HI	-56.53	-82.14	-45	-85.56	-51	-46.09	18	-27.14	52
	IJ	-27.67	-54.12	-96	-54.10	-96	-53.24	-92	-33.47	-21

The truss model used for the beam provided a very good match with the results of the nonlinear finite element analysis for almost all truss elements, at the ultimate stage of loading. The results improved significantly with the addition of inclined struts. Figure 5.14 shows the stresses in truss elements. Table 5.6 shows the results of the truss model with three additional inclined struts at the ultimate load stage. At this stage of loading, once again the accuracy of results improved with the addition of inclined struts. This was especially true for stresses in the lower cord.

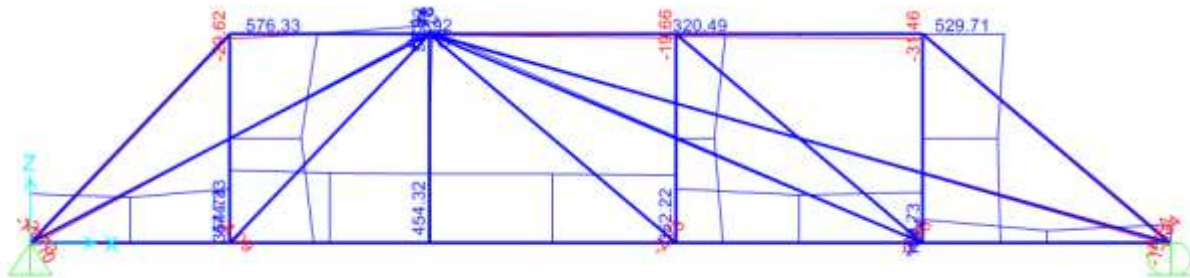


Figure 5.14 Stresses in truss members for Beam BS3 at ultimate load

5.5.5 Truss Analysis Results for Beam BS5

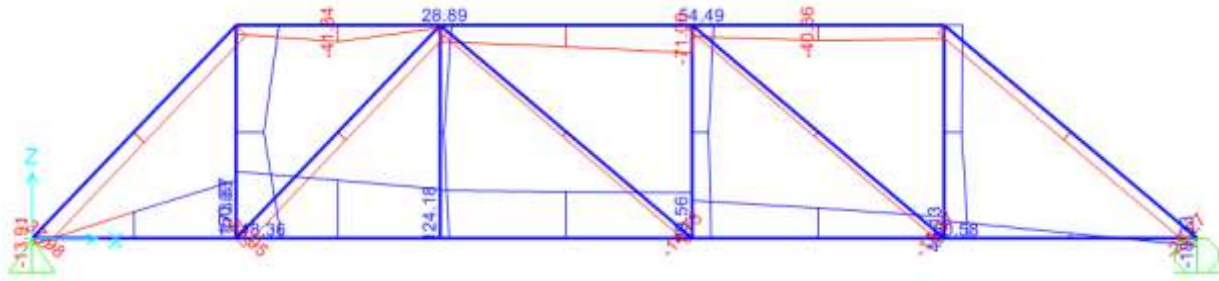
Beam BS5 only had the fiber in UHPC and the prestressing reinforcement, but no stirrup reinforcement. The nonlinear finite element analysis indicate that the cracks propagated quickly immediately after formation, resulting in massive distribution of cracks within the beam. The cracks were in the form of; (i) flexural cracks in the bottom flange; (ii) shear cracks at the connection between the web and the bottom flange; and (ii) fully developed diagonal shear cracks in the web. The ultimate state of stress was reached shortly after the shear crack initiation, as this beam did not have stirrup reinforcement.

The truss model presented in previous sections gives very good correlation with the results of nonlinear finite element analysis. The correlation improves significantly by adding additional inclined struts. Figures 5.15(a) to (d); show stresses in truss members for Beam BS5 at the crack initiation load stage. Table 5.7 shows the results of truss analyses at this stage of loading. The results indicate that adding more inclined struts improves the correlation with nonlinear FEM analysis.

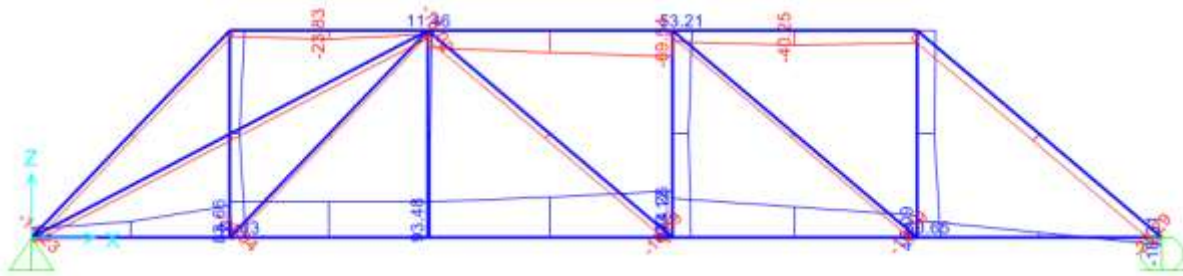
Table 5.6 Stresses in truss models for Beam (BS3) at ultimate load, compared with nonlinear FEM analysis

			Truss with three additional struts	
	Member	FEM	Truss	% Diff
Vertical Struts	BG	576.33	328.74	43
	CH	17.92	152.25	-750
	DI	320.49	193.99	39
	EJ	529.71	59.25	89
Inclined Struts	AG	-11.73	-7.93	32
	AH	-23.10	-29.17	-26
	BH	-4.09	-9.753	-139
	DH	-2.73	-3.03	-11
	EH	-2.12	-2.452	-15
	EI	-2.68	-1.169	56
	FH	-15.51	-11.32	27
Lower Cord	FJ	-11.34	-7.86	31
	AB	1731.1	1723.26	0
	BC	1850.7	1723.26	7
	CD	1830.7	1723.26	6
	DE	1728.6	1723.26	0
Upper Cord	EF	1527.1	1723.26	-13
	GH	-25.96	-21.802	16
	HI	-19.66	-24.37	-24
	IJ	-31.46	-20.30	35

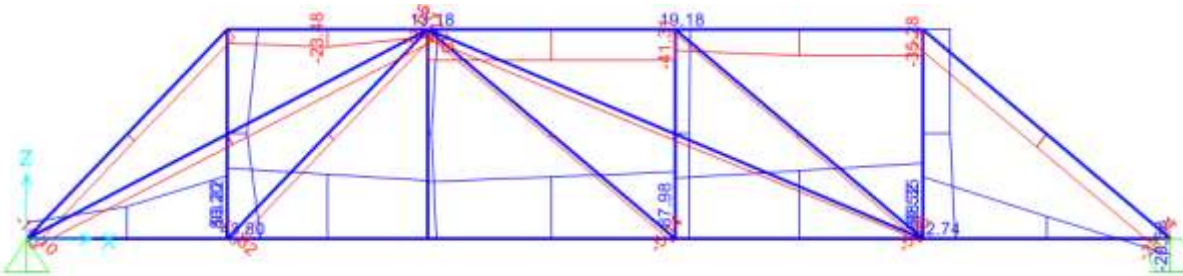
When the external load is further increased up to failure, which is initiated by the rupturing of tendons, the correlation of truss analyses with non-linear FEM analysis significantly decreases. Table 5.8 shows that adding three additional inclined struts once again improves the results.



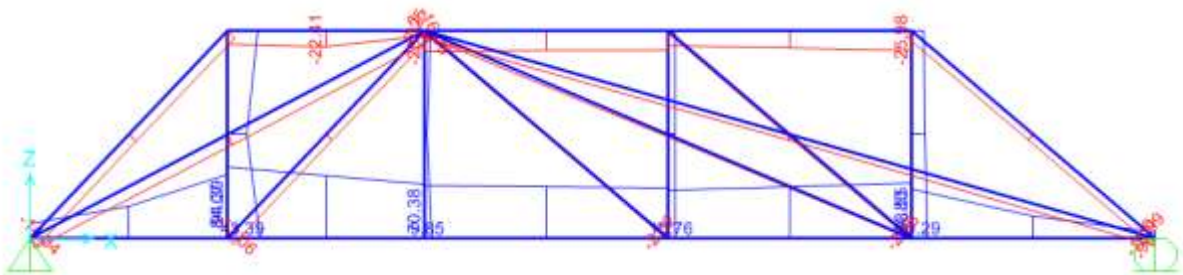
(a) Simple truss (No additional strut)



(b) One additional strut



(c) Two additional struts



(d) Three additional struts

Figure 5.16 Stresses in truss members for Beam BS5 at first crack formation

**Table 5.7 Stresses in truss elements and comparison with non-linear FEM analysis for
Beam BS5 at first cracking**

			No additional strut		One additional inclined strut		Two additional inclined struts		Three additional inclined struts	
	Member	FEM	Truss (MPa)	% Diff	Truss (MPa)	% Diff	Truss (MPa)	% Diff	Truss (MPa)	% Diff
Vertical Ties	BG	247.98	1806.34	-628	487.73	-97	464.18	-87	450.03	-81
	CH	291.91	95.70	67	38.83	87	8.05	97	12.68	96
	DI	117.16	885.79	-656	923.76	-688	337.05	-188	222.14	-90
	EJ	107.39	1058.46	-886	1057.2	-884	999.43	-831	383.66	-257
Inclined Struts	AG	-30.58	-44.43	-45	-9.89	68	-10.01	67	-10.08	67
	AH	-47.79			-20.75	57	-20.47	57	-20.31	57
	BH	-22.13	-19.53	12	-4.23	81	-3.94	82	-3.76	83
	DH	-11.25	-9.87	12	-10.48	7	-3.32	71	-2.20	80
	EH	-17.06					-10.7	37	-2.14	87
	EI	-12.82	-12.66	1	-12.58	2	-3.00	77	-2.19	83
	FH	-10.59							-13.79	-30
	FJ	-6.982	-24.72	-254	-24.74	-254	-24.77	-255	-9.66	-38
Lower Cord	AB	1665.5	1128.42	32	1667.1	0	1636.9 0	2	1618.54	3
	BC	1665.5	1830.96	-10	1774.3	-7	1736.4 0	-4	1713.36	-3
	CD	1665.5	1744.74	-5	1762.9	-6	1712.9 4	-3	1693.95	-2
	DE	1665.5	1429.46	14	1426.9	14	1613.7 2	3	1624.13	2
	EF	1665.5	917.66	45	918.01	45	918.54	45	1457.96	12
Upper Cord	GH	-39.06	-42.22	-8	-23.09	41	-20.63	47	-20.20	48
	HI	-44.68	-48.7	-9	-57.91	-30	-21.42	52	-16.27	64
	IJ	-20.47	-40.35	-97	-40.35	-97	-40.44	-98	-24.77	-21

**Table 5.8 Stresses in truss elements and comparison with non-linear FEM analysis for
Beam BS5 at ultimate**

	Member	FEM	No additional struts		One additional strut		Two additional struts		Three additional struts	
			Truss (MPa)	% Diff	Truss (MPa)	% Diff	Truss (MPa)	% Diff	Truss (MPa)	% Diff
Vertical Ties	BG	214.77	2230.12	-938	597.64	-178	691.72	-222	665.76	-210
	CH	386.62	117.26	70	73.66	81	8.46	98	24.66	94
	DI	426.648	1127.31	-164	1156.51	-171	545.9	-28	417.65	2
	EJ	146.45	1328.76	-807	1327.92	-807	1261.8	-762	600.06	-310
Inclined Struts	AG	-43.56	-55.49	-27	-14.82	66	-15.03	66	-15.15	65
	AH	-46.24			-23.98	48	-23.37	49	-19.32	58
	BH	4.37	-24.09	651	-6.38	246	-5.75	231	-5.42	224
	DH	-0.28	-12.16	-4198	-12.63	-4363	-5.35	-1789	-4.15	-1365
	EH	-2.78					-62.98	-2165	-38.62	-1289
	EI	-6.146	-15.72	-156	-15.66	-155	-5.38	13	-4.24	31
	FH	-5.08							-12.32	-143
	FJ	-5.7	-30.85	-441	-30.87	-442	-30.89	-442	-14.77	-159
Lower Cord	AB	1744.28	1525.29	13	1935.65	-11	1869.3	-7	1836.14	-5
	BC	1744.28	2161.58	-24	2095.47	-20	2012.4	-15	1970.78	-13
	CD	1744.28	2050.19	-18	2064.54	-18	1994.8	-14	1960.72	-12
	DE	1744.28	1669	4	1666.93	4	1819.4	-4	1817.59	-4
	EF	1744.28	1024.28	41	1024.62	41	1025.1	41	1598.61	8
Upper Cord	GH	-13.22	-53.32	-303	-24.73	-87	-23.2	-75	-22.50	-70
	HI	-20.58	-60.59	-194	-64.9	-215	-26.95	-31	-19.96	3
	IJ	-10.6	-46.63	-340	-46.63	-340	-46.73	-341	-30.03	-183

Chapter 6

Conclusions and Future Work

6.1 Introduction

Conclusions drawn from the current analytical investigation on performance of prestressed UHPC beams are presented in three major parts; (i) the optimization of UHPC beams; (ii) the application of finite element models; and (iii) the application of truss models. A summary of important conclusions and suggested future research are also presented in this chapter.

6.2 Optimization of UHPC Beams

The early part of the investigation involved the optimization of UHPC beams when used for combined flexure and shear. In search for efficient and cost effective UHPC beams for highest capacity with minimum use of UHPC, it was found that non-prestressed rectangular beams require excessive reinforcement and hence could not be economical. Prestressing UHPC rectangular beams also indicated the need for a large number of prestressing strands. In addition, as a large number of bottom strands was used to balance the inherently high compressive strength of UHPC, additional top flange prestressing became necessary to control tension associated with transfer stresses. The resulting beam would have too many layers of strands with reduced lever arm for internal moment resistance. This option was also found to be uneconomical.

It was concluded that prestressed UHPC I-beams gave the highest possible load capacity with minimum use of materials. The high moment of inertia to area ratio, the presence of top and bottom flanges (to accommodate prestressing tendons), and the high lever arm resulted in an economical section with very high moment resistance. Furthermore, the use of relatively low percentage of prestressing steel resulted in fewer obstructions for fiber orientation during casting.

6.3 Finite Element Models

The applicability of both linear and nonlinear FEM models for prestressed UHPC beams was investigated. It was found that the elastic FEM analysis gave only an approximate and preliminary view of stress distributions in UHPC beams. However, they did provide a reasonably good understanding of the general flow of stresses in UHPC, and the longitudinal and transverse reinforcement within the elastic range of deformations. It clearly showed the formation of compression arches linking the applied load to the ends of the bottom prestressing tendons and the supports. However, elastic FE models were unable to show the change in stress distributions at ultimate. The neutral axis at critical flexural zones remained at the mid-height level even at ultimate. It was concluded that the elastic model provided overestimation of load capacity and underestimation of stresses in tendons, while giving very low stresses in stirrups.

Nonlinear finite element models were capable of; (i) estimating the formation and propagation of cracks; (ii) predicting distributions of nonlinear stresses in UHPC as cracks develop and propagate; (iii) predicting nonlinear stresses in tendons and stirrups for use as damage indicators; and (iv) predicting the local effects of stress concentrations at points of applications of loads, prestressing forces and supports. The models were also capable of integrating the ductility of UHPC associated with fibres, for nonlinear behavior of beams. They were capable of capturing the effects of shear reinforcement, including the mechanism of failure. The nonlinear FEM analyses indicated two major failure mechanisms; (i) diagonal tension failure in the beam without stirrup reinforcement, which subsequently led to massive flexure and shear cracks, eventually resulting in a diagonal shear failure plane between the load and the nearest support; and (i) tendon rupturing in beams with sufficient stirrup reinforcement.

The proposed concept of applying prestressing as external forces and adjusting the stress-strain relationship of prestressing steel accordingly, showed acceptable nonlinear behavior. However, verification against experimental results is recommended.

The nonlinear FE models, on the other hand, were not capable of (i) estimating the effects of fiber stresses in cracked UHPC; (ii) integrating the effects of fiber orientation on beam properties and showing their effects on damage mechanism in extreme flexure and shear zones; and (iii)

estimating localized effect of high volume shear reinforcement and related disturbances in fiber distribution and resulting potential damage and loss of bond. Hence the models need calibrations of important parameters on the basis of a comprehensive test program.

6.4 Truss Models

Truss models were used for shear-dominant beams to study their performance in predicting beam behavior. A simple traditional truss model, connecting top and bottom cords with vertical ties and minimum number of 45° inclined struts was used first, and then improved by introducing additional inclined struts, fanning away from the external load. The accuracy of the truss analyses was verified against elastic and inelastic FEM analyses. When compared against elastic FEM analysis the correlation was not good, especially for stresses in vertical ties. This is explained by lack of crack simulation ability of the elastic FE model, as the cracks trigger vertical tensile stresses in truss tie elements. However, when nonlinear FEM analysis was employed for the verification of truss models, reasonably good agreements were obtained. The behaviour predictions improved by adding extra inclined struts that transfer the applied load to the supports.

6.5 Summary of Conclusions

The following conclusions summarize the major findings of the research project reported in this thesis:

- High compressive stresses in UHPC necessitate the use of high percentage of reinforcing steel to mobilize the available compressive strength of concrete. This results in unrealistically high percentage of non-prestressed reinforcement, congesting the beam cage, creating serious concrete placement problems.
- Use of UHPC in long span beams, such as bridge girders may be economical when prestressed. High strength prestressing strands provide sufficiently high tensile capacity to balance the high compressive strength of UHPC. The use of I-sections provides a more optimum solution instead of using rectangular sections.
- The effect of prestressing in FEM and truss analyses can be introduced by means of equivalent external forces, applied on concrete. For tendons having constant eccentricity, this translates into horizontal beam forces applied at the ends.

- Elastic FE models are not capable of important aspects of UHPC behavior. An important deficiency of such models is the inability to simulate concrete cracking. However, they do capture the flow of forces through compression struts which form the arching action.
- Non-linear FE models provide promising predictions of flexural and shear behavior for prestressed UHPC beams. Further research is needed to develop cracking models for fibre-reinforced UHPC for improved analytical predictions.
- Contrary to earlier recommendations, a minimum amount of stirrup steel is required to ensure the development of flexural strength in prestressed UHPC beams. Beyond the minimum stirrup reinforcement needed, the increase in transverse steel (or reduction in stirrup spacing) do not affect the beam behavior, as expected.
- Fibre performance in UHPC requires careful study and modeling techniques. Fibre orientation and performance across cracks require new models based on experimental evidence.
- The assumed fibre orientations of 40%, 30% and 30% in the longitudinal, inclined and vertical directions, respectively provide reasonable results for the type of prestressed UHPC beams considered.
- For the 10 mm fibres considered in the current investigation, assuming pull-out of 50% of fibres at the ultimate load condition when cracks become substantially wide, provides good predictions of beam behavior.
- Conventional 45° truss analogy cannot provide good estimates of structural behavior for shear-dominant prestressed UHPC beams subjected to an externally applied concentrated force. The use of additional inclined struts for the fanning region of the applied load, transferring stresses to supports and external prestressing force locations, improve behavior predictions significantly. The latter point becomes especially important in prestressed UHPC beams where the horizontal prestressing forces promote more arching action to form, which is best represented by inclined struts.
- The traditional truss models developed for ordinary concrete beams can be used for shear-dominant prestressed UHPC beams.

6.6 Future Work

The present study is part of an on-going research project at the NRC of Canada to develop innovative bridge structures using UHPC. The first phase of research, which is conducted in collaboration with the University of Ottawa, has resulted in observations and conclusions that suggest further research in the area, as indicated below:

- A comprehensive experimental study of prestressed UHPC beams, involving large-scale and full-scale test specimens;
- Investigation of the effect of transverse shear reinforcement on obstructing the distribution of fibres in UHPC and the resulting fibre orientations.
- Investigation and analytical modeling of crack formation, collapse mechanisms, and structural performance of prestressed UHPC beams.
- Study of the effect of fiber content in UHPC beams as it affects flexural and shear capacities of beams.
- Development and calibration of non-linear FE models that can capture all aspects of non-linear material behavior in cracked UHPC.
- Development of variable-angle truss models with additional inclined struts and verification against test results.
- Development of a new simplified shear design approach for prestressed UHPC beams using combined fibres and steel reinforcement (hybrid reinforcement) for flexure and shear.

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Appendix 1

A Details of the final design of UHPC-I Beam

A.1 Design of shear:

Shear capacity of UHPC can be obtained by

$$V_{yd} = V_{rpcd} + V_{fd} + V_{ped}$$

The recommendations give the design shear capacity V_{yd} of UHPC as the sum of the UHPC matrix capacity (V_{rpcd}), Fibre reinforcement contribution (V_{fd}), and the contribution of prestressing (V_{ped}).

Therefore,

The strength of UHPC matrix is given by

$$V_{rpcd} = 0.18\sqrt{f'_c} \cdot b_w \frac{d}{\gamma_b} = 135.4523 \text{ KN}$$

$$\beta_u = 30^\circ$$

The design average tensile strength is provided by

$$f_{vd} = 0.4\sqrt{f'_c} = 5.29 \text{ Mpa}$$

$$Z = \frac{d}{1.15} = 428.6957 \text{ mm}$$

The fiber strength in UHPC is given by

$$V_{fd} = \left(\frac{f_{vd}}{\tan\beta_u} \right) * b_w * \frac{Z}{\gamma_b} = 453.3534 \text{ KN}$$

The strength of shear reinforcement is provided by

$$V_{s,max} = \frac{A_v f_y d}{S} = 602.0516 \text{ KN}$$

$$V_{yd} = 1190.857 \text{ KN}$$

The shear force can be represented as following

$$P_{sh} = \left(\frac{V_{yd} * l}{l - 2.1d} \right) = 1818.380277 \text{ KN}$$

A.2 Design for flexure:

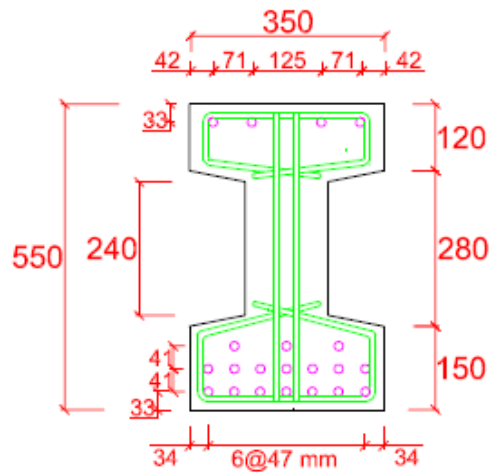


Figure A.1 Cross section of prestressed UHPC I-beam

$$b_w \times h = 150 \times 550 \text{ (See Figure A.3)}$$

The resulted shear load is;

$$P_{sh} = 1818.38 \text{ kN}$$

The applied load on the section can be calculated as follows;

$$P_m = 1.5 P_{sh} = 2668.874 \text{ kN}$$

The resulting moment from the applied load is given by;

$$M_n = \frac{P_m * (l - 2.1 * d)(2.1 * d)}{l} = 1809.544 \text{ KN.m}$$

Where: $l = 3m, d = 493mm$

The area of prestressing steel can generally be calculated as follows.

$$A_{sp} = \frac{M_n}{0.74 * f_{pu} * h} = 2390.352 \text{ mm}^2$$

Use 17 strands Ø 16 mm ($A_{sp} = 2550 \text{ mm}^2$)

The resulting tension force in the section is;

$$T_{sp} = A_{sp} * f_{pu} = 4564500 \text{ N}$$

From the trials the neutral axis is;

$$C = 91.5 \text{ mm}$$

$$r_{fu} = \left(\frac{\varepsilon_{cu} - \varepsilon_{bc}}{\varepsilon_{cu}} \right) \times C = 39.21 \text{ mm}$$

$$r_{fo} = \left(\frac{\varepsilon_{bc}}{\varepsilon_{cu}} \right) \times C = 52.29 \text{ mm}$$

The compression steel force is given by;

$$C'_s = \varepsilon'_s * E_s * A'_{sp} = 555020.3 \text{ N}$$

The compression force resulting from the compression zone is represented by;

$$C_{c1} = b_e * f_{cgu} * r_{fu} = 2401875 \text{ N}$$

$$C_{c2} = 0.5 * b_{ft} * f_{cgu} * r_{fo} = 1601250 \text{ N}$$

The total compression force is given by;

$$C_c = \sum_{i=1}^m C_{ci} = 4558145 \text{ N}$$

$$T_s = 4564500 \cong C_c = 4558145 \text{ N OK}$$

Therefore, the resulting moment from the resulting compressive force component ($C_c = 4558145 \text{ N}$) is equal to $M_u = 2060.111 \text{ kN.m}$.

From that the beam capacity can be obtained as follows:

$$P_u = \frac{(M_u * l)}{((l - a)a)} = 3018.478 \text{ kN}$$

This is the ultimate applied load that would cause the full failure for this beam.

A.3 Check for cracking in compression and tension fibers:

Stresses at transfer stage at the support:

$$f_{topi} = \frac{P_{ebot} e C}{I_c} - \frac{P_{ebot}}{A_c} - \frac{P_{etop} e' C'}{I_c} - \frac{P_{etop}}{A_c}$$

$$f_{boti} = -\frac{P_{ebot} e C}{I_c} - \frac{P_{ebot}}{A_c} + \frac{P_{etop} e' C'}{I_c} - \frac{P_{etop}}{A_c}$$

Therefore,

the top face stress is given by;

$$f_{top} = 2.176 \text{ MPa} < 0.85 \times 0.4\sqrt{175} = 4.5 \text{ MPa OK}$$

Also,

the bottom face stress is given by;

$$f_{bot} = -63.89 \text{ MPa} < f'_c = -175 \text{ MPa OK}$$

$$\text{Where } e = d_p - C = 493 - 135 = 358 \text{ mm}$$

$$e' = C - d' = 135 - 32.5 = 102.5 \text{ mm}$$

$$C' = 550 - 135 = 415 \text{ mm}$$

Stresses at transfer stage at mid-span:

$$f_{top} = -\frac{P_{ebot} e C}{I_C} - \frac{P_{ebot}}{A_C} + \frac{P_{etop} e' C'}{I_C} - \frac{P_{etop}}{A_C} - \frac{M_C \times C}{I_C}$$

$$f_{bot} = \frac{P_{ebot} e C}{I_C} - \frac{P_{ebot}}{A_C} - \frac{P_{etop} e' C'}{I_C} - \frac{P_{etop}}{A_C} + \frac{M_C \times C}{I_C}$$

The self-weight of the beam can be obtained by;

$$D_L = 3.446 \text{ kN/m}$$

The moment resulting from dead load is calculated as follows;

$$M = 3.876483 \text{ kN.m}$$

Therefore,

The top face stress is equal to:

$$f_{top} = 2.41 \text{ MPa} < 0.4\sqrt{175} = 5.291503 \text{ MPa OK}$$

Also,

The bottom face stress is given by;

$$f_{bot} = -64.123 \text{ MPa} < 0.85 f'_c = -175 \text{ MPa OK}$$

B Load-displacement for BS1

The displacement of UHPC-prestressing beams can be calorized as follows:

B.1 Load-displacement at first cracking

Before calculating the first cracking displacement, the cumbering displacement should be obtained as follows:

$$\Delta_i = 0.5 \left(\frac{l}{2} \right) \times \phi_i \times \frac{2}{3} \left(\frac{l}{2} \right) = 1.802 \text{ mm}$$

Where: Δ_i is the cumber displacement (uplift due to the prestressing forces).

Figure D.1 shows the simplified approach to obtain the cumbering uplift displacement (Δ_i).

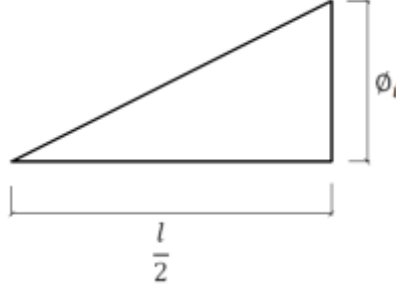


Figure B.1 The simplified approach to obtain the cumbering uplift displacement (Δ_i)

ϕ_i is the cumbering curvature and l is the length of the beam (3m).

$$\phi_i = \frac{\varepsilon_{topi} + \varepsilon_{boti}}{h} = 2.4025 \times 10^{-6}$$

Where: ε_{topi} is the top strain at transfer stage (cumber), ε_{boti} is the bottom strain at transfer stage (cumber), and h is the entire height of the beam.

$$\varepsilon_{topi} = \frac{f_{topi}}{E_{UHPC}} = 4.35 \times 10^{-5} \quad \text{and} \quad \varepsilon_{boti} = \frac{f_{boti}}{E_{UHPC}} = -0.00128$$

$$f_{topi} = \frac{P_{ebot} e C}{I_C} - \frac{P_{ebot}}{A_C} - \frac{P_{etop} e' C'}{I_C} - \frac{P_{etop}}{A_C} = 2.176 \text{ MPa}$$

$$f_{boti} = -\frac{P_{ebot} e C}{I_C} - \frac{P_{ebot}}{A_C} + \frac{P_{etop} e' C'}{I_C} - \frac{P_{etop}}{A_C} = -63.89 \text{ MPa}$$

Where: f_{topi} is the top stress at transfer stage (cumber), f_{boti} is the bottom strain at transfer stage (cumber), and E_{UHPC} is the modules of elasticity of UHPC.

$$\Delta_{cr} = 0.5 (a) \times \phi_{cr} \times \frac{2}{3} (a) = 1.85 \text{ mm}$$

Where: Δ_{cr} is the total displacement at first cracking

Figure D.2 shows the simplified approach to obtain the first cracking displacement (Δ_{cr}).

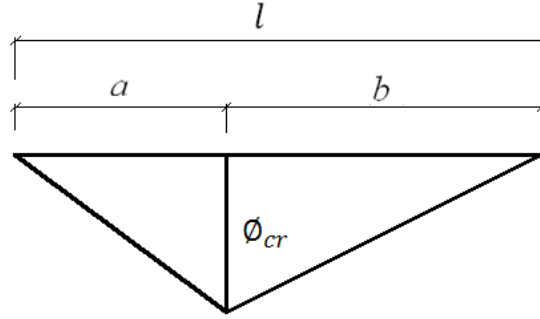


Figure B.2 The simplified approach to obtain the first cracking displacement (Δ_{cr})

$$\phi_{cr} = \left(\frac{\varepsilon_{botcr} + \varepsilon_{topcr}}{h} \right) = 5.032 \times 10^{-6}$$

Where: ε_{topcr} is the top strain at cracking, ε_{botcr} is the bottom strain at cracking, and h is the entire height of the beam.

$$\varepsilon_{botcr} = \frac{f_{botcr}}{E_c} = 0.000106 \quad \text{And} \quad \varepsilon_{topcr} = \frac{f_{topcr}}{E_c} = -0.00266$$

$$f_{topcr} = -\frac{P_{ebot} e C}{I_c} - \frac{P_{ebot}}{A_c} + \frac{P_{etop} e' C'}{I_c} - \frac{P_{etop}}{A_c} - \frac{M_{cr} \times C}{I_c} = -133.078 \text{ MPa}$$

$$f_{botcr} = \frac{P_{ebot} e C}{I_c} - \frac{P_{ebot}}{A_c} - \frac{P_{etop} e' C'}{I_c} - \frac{P_{etop}}{A_c} + \frac{M_{cr} \times C}{I_c} = 5.29 \text{ MPa}$$

Where: f_{topcr} is the top stress at cracking, f_{botcr} is the bottom strain at cracking, and E_{UHPC} is the modules of elasticity of UHPC.

B.2 Load-displacement at ultimate

The ultimate displacement is the result of

$$\Delta_u = A_1 \left(\frac{2}{3} a_1 \right) + A_2 \left(\frac{a_2}{2} + a_1 \right) + A_3 \left(\frac{2}{3} a_2 + a_1 \right) - \Delta_i = 7.47 \text{ mm}$$

$$A_1 = 0.5 \times a_1 \times \phi_{cr}, A_2 = a_2 \times \phi_{cr}, \text{ and } A_3 = 0.5 \times a_2 \times (\phi_u - \phi_{cr})$$

Where: ϕ_u is the ultimate curvature which is $\phi_u = \frac{\varepsilon_{cu}}{C}$

Figure D.3 shows the simplified approach for calculating the ultimate displacement.

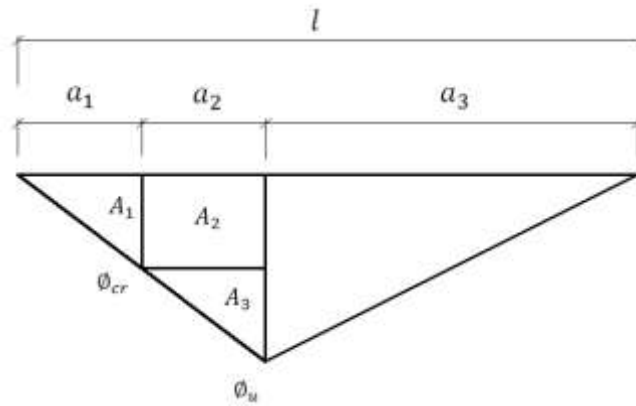


Figure B.3 The simplified approach for calculating the ultimate displacement (ϕ_u)

All the variables that are used to obtain the ultimate displacement (Δ_u) are described in Figure D.3. Note that, the ultimate displacement is obtained in terms of the first cracking and ultimate curvature (ϕ_{cr} and ϕ_u).