PARAMETER ESTIMATION IN ELECTRIC POWER SYSTEMS USING STOCHASTIC MODELS

by

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ABSTRACT

The scope of this study aims to encompass in detail some of the most prominent features involved in the parameter and state estimation of electric power systems.

The main items presented in this study include a review of state estimation methods, a detailed development and formulation of the AEP algorithm and its programmed implementation. In addition, a method is proposed for the improved pricing of power production based on a stochastic model. Attention is also focused on the error detecting capabilities of the algorithm.

Two numerical examples are presented which demonstrate the successful application of the method developed to obtain statistically optimal parameters for the incremental cost curve in the presence of uncertainty in the fuel and maintenance costs.

TABLE OF CONTENTS

		PAGE
Acknowledgements		i
Abstract		ii
Nomenclature		v
List of Figures		ix
List of Tables		x
CHAPTER		
I.	Introduction	1
II.	Estimation Techniques: A Review	4
	IIa. General	4
	IIb. Least Squares Model	7
	IIc. Bayesian Model	8
	IId. Fisher Model	9
III.	AEP Method for State Estimation	13
	IIIa. Problem Description	13
	IIIb. Problem Formulation and Mathematical Development	13
	IIIc. Solution Procedure	21
	IIId. A Numerical Example	23
	IIIe. Results and Observations	27
IV.	Incremental Cost Curve Parameter Estimation	34
	IVa. General	34
	IVb. Mathematical Development	34
	IVc. Numerical Examples	38
	(i) 5-Bus Power System	38
	(ii) 25-Bus Power System	43

CHAPTER		PAGE
V.	Summary and Conclusions	50
	Bibliography	52
	Appendix A - Economic Dispatch	59
	Appendix B - Listing of the Fortran Program for Economic Dispatch Solution.	63
	Appendix C - Listing of the Fortran Program for AEP Method of State Estimation.	78
	Curriculum Vitae	84

NOMENCLATURE

SYMBOL	DESCRIPTION
*	denotes conjugation.
^	a caret over a symbol denotes the estimated value.
| |	denotes the magnitude of the quantity inside the vertical lines.
( )	numbers in these brackets designate equation numbers.
[ ]	numbers in these brackets refer to the bibliography.
$(\ )^t$	denotes transpose of the matrix.
$(\ )^{-1}$	denotes inverse of the matrix.
A	bus incidence matrix relating network nodes to lines.
A_F	a fixed known cost.
A_{gp}; A_{gq}	appropriate submatrices of A for p-type and q-type buses.
B_p ; B_q	appropriate submatrices of A for p-type and q-type buses.
C	a suitable matrix relating $\hat{x}$ to z .
C_p ; C_q	vectors, as defined in (3.25) and (3.26)

SYMBOL	DESCRIPTION
C_n	slope of the linearized incremental cost curve.
D_n	intercept of linearized incremental cost curve on the vertical axis.
D_p ; D_q	diagonal matrices, as defined in (3.12) and (3.13).
$E\{\ \}$	expected value.
F	total production cost.
F_L	augmented cost function.
F_n	production cost of unit n.
G_i	Generator at bus i.
H	a known matrix describing a linear relationship between z and x.
H_p ; H_q	diagonal matrices with elements V_p / z_j^* and V_q / z_j^* respectively.
I	identity matrix.
$(IC)_i$	incremental cost of unit i.
$(ITL)_i$	incremental transmission loss associated with unit i.
J; J_p; J_q	objective functions to be minimized.
K_p ; K_q	vectors with elements $V_p^2 \, y_{j,p}^*$ and $V_q^2 \, y_{j,q}^*$ respectively.

SYMBOL	DESCRIPTION
L_i	load at bus i .
MVA	megavolt amperes.
MW	megawatts.
MX	megavars.
P_{G_n}	real generated power of unit n.
P_D	total demand in the system.
P_L	total losses in the system.
P_j, P_k	real power at j and k .
Q_j, Q_k	reactive power at j and k .
R	error covariance matrix ; $R = E\{v v^t\}$.
R_{jk}	real part of the complex impedance obtained from the bus impedance matrix with a composite load bus as reference.
S_i	complex bus power i.e. $S_i = P_i + jQ_i$.
$S_{j,pq}$	complex power flow from bus p to bus q on line j.
$S_{j,qp}$	complex power flow from bus q to bus p on line j.
$S_{pq}^m ; S_{pq}^c$	complex measured and calculated line flow from bus p to bus q.
$S_{qp}^m ; S_{qp}^c$	complex measured and calculated line flow from bus q to bus p.

SYMBOL	DESCRIPTION
tr	trace of a matrix. (sum of diagonal elements)
v	measurement error or disturbance
v_p ; v_q	line voltage vectors.
V_p ; V_q	bus voltages at buses p and q .
W_p ; W_q	$\ell \times \ell$ diagonal matrices of weighting factors.
W, W_o	suitables matrices for estimated state variables ($\hat{x} = Wz + W_o$) .
x	state vector.
x_g	known voltage vector.
$y_{j,p}$; $y_{j,q}$	line shunt admittances at buses p and q.
z	known measurement vector.
z_j	series impedance on line j .
α_{jk} ; β_{jk}	suitable coefficients as defined by (A10) and (A11).
δ_j ; δ_k	bus voltage phase angles.
ϵ_n	random variable representing uncertainty in incremental cost.
Λ	Lagrange multiplier matrix.
λ_L	Lagrange multiplier.
μ	mean, or expected value of x .
ν	random variable describing error in operating costs.
Σ	minimum error covariance matrix for x ; $E\{(x - \hat{x})(x - \hat{x})^t\}$.
Σ_z	minimum error covariance matrix for z ; $E\{(z - \hat{z})(z - \hat{z})^t\}$.
ψ	covariance matrix : $E\{(x - \mu)(x - \mu)^t\}$.

LIST OF FIGURES

FIGURE		PAGE
1.	Schematic diagram for information flow during state estimation.	6
2.	Typical line between buses p and q.	14
3.	Typical facility to obtain line power flows.	15
4.	Three bus power system.	24
5.	Equivalent circuit of transmission lines.	24
6.	Typical load variation.	25
7.	Estimated voltage magnitude for bus 2.	30
8.	Estimated voltage phase angle for bus 2.	31
9.	Estimated voltage magnitude for bus 3.	32
10.	Estimated voltage phase angle for bus 3.	33
11.	Incremental cost curve for a generating unit.	36
12.	The system model.	37
13.	5-Bus power system.	38
14.	Cost convergence characteristic for 5-Bus power system.	42
15.	25-Bus power system.	44

LIST OF TABLES

TABLE		PAGE
I.	MW and MX schedules for load L_2.	25
II.	Line power flows for varying load.	28
III.	5-Bus system impedance and line charging data.	39
IV.	5-Bus system loads and initial generation.	39
V.	Estimated parameters for the 5-Bus system.	41
VI.	New generation schedule for 5-Bus system.	41
VII.	25-Bus system impedance and line charging data.	43
VIII.	25-Bus system loads and initial generation.	45
IX.	Estimated parameters for the 25-Bus system.	48
X.	New generation schedule for 25-Bus system.	49

CHAPTER I

INTRODUCTION

During the past several years, tremendous advances in digital computer technology have revolutionized the field of state estimation in electric power systems. This development is marked by a dramatic improvement in system monitoring and control of power systems which has been brought about largely by the wide-spread usage of high speed multiprocessor real-time computers.

Data acquisition systems are being incorporated as an integral part of several major electrical utilities to monitor actual conditions in order to support the system operators and production supervisors in the operation of their system. Moreover, they enable the system operators to have readily available up-to-date information about the system variables and system structure. These include the status of the circuit breakers, generating equipment, transformers line-end open indicators, selected bus voltages as well as the real and reactive line power flows. This, coupled with existing and planned computer facilities, provides the system operator with greatly expanded telemetering facilities and the means of improving the security of the system. The implementation of state estimation techniques furthermore, provides the system operator with a reliable and complete data base that allows for tighter security margins which are essential in providing the customer with a highly reliable power supply.

As has been frequently pointed out [1], state estimation not only

1. Reference may be made, in particular, to an invited paper by F.C. Schweppe and E.J. Handschin [39], detailing this point.

provides a reliable data base comprising the bus voltage magnitudes and difficult-to-measure voltage phase angles, but may also help to detect bad data points by means of error checking routines. Moreover, it may help to give a better understanding of the dynamic behaviour of the system and to suggest an improved metering configuration.

The general theory underlying state estimation techniques is reviewed in chapter two, which focuses on understanding the basic estimation problem by examining the information that is available prior to the measurements or obtained during measurements. On the basis of the information that is available one can say that the Least squares technique is well suited for on-line data processing in electric power systems. This decision is reached keeping in mind that the estimation algorithm is simple enough to be used on a process computer with limited storage capacity and computing speed.

Chapter Three describes in detail the so-called AEP[1] method which is used to obtain an estimate of the state of the system from measurements of the power flows in the lines. If the lines are measured at both ends a redundant set of measurements will be obtained. A simple but illustrative example is given on the application of this method.

Chapter Four applies the method of Least squares (which is basic to the AEP method) to obtain statistically optimal parameters describing a linear incremental cost curve for a generating unit in the

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1. The abbreviation AEP stands for the American Electric Power Service Corporation, New York.

presence of uncertainty in the fuel and maintenance costs. The estimation of the parameters is carried out from the knowledge of allocated real generated powers i.e. inputs to the estimator are the incremental and operating costs of each generating unit.

The concluding chapter attempts to interpret the results obtained in the context of theoretical predictions set out in the earlier chapters. This is followed by a bibliography and appendices that complete the thesis.

All the programming has been done using Fortran programming language on the IBM 360 System Model 65.

CHAPTER II.

ESTIMATION TECHNIQUES: A REVIEW

IIa. General : Although the concept of estimation is quite old[1], it has found widespread applications in the past few decades, particularly in the field of engineering sciences. In the area of power systems, Schweppe [40,41,42] first suggested the idea of developing static state estimators for control functions associated with the system operating under steady-state conditions. He outlined some modelling concepts and computational problems associated with the implementation of these estimators. Since then, the recognized need for on-line solutions to problems in the electric power industry has stimulated the development of new estimation techniques [9,10,25,26,27,31,44].

The basic estimation problem consists in obtaining an estimate of some variables (usually the state variables) from measurements or a set of observations made on the system. The problem of developing a mathematical model describing how the observations are related to the state variables is of considerable importance. If the model is wrong, the subsequent estimation procedures and computational efforts are of no use. A reasonably precise model could no doubt be formulated, but it may be so complex that no existing method could be applied to solve it.

For simplicity, therefore, assuming that there is a linear relationship between the set of measurements and the state variables, one may write :

$$z = Hx + v \qquad (2.1)$$

1. Legendre (1805) and Gauss (1809) laid the foundations of estimation theory.

in which

z	=	known measurement set, m vector
x	=	state variables to be estimated, n vector
H	=	known m x n matrix
v	=	measurement error or disturbance, m vector

A typical estimate for (2.1) can be expressed as

$$\hat{x} = Wz + Wo \qquad (2.2)$$

in which W and W o are suitable matrices.

Figure 1 schematically depicts the flow of information during power system state estimation.[1] In many existing and proposed control centers, telemetered data is received at periodic intervals of the order of few seconds. This data (represented by data acquisition system in Figure 1) is fed into the state estimator which provides the operator with reliable estimates $\hat{x}$ of the state variables. If bad data is present in the information being fed in, the estimator has the capability of detecting such erroneous data.

There are three important models for the estimation process [38] that depend on the information which is available on x or v prior to the measurements or obtained during measurements. These models are:

Least Squares Model: nothing is said about x or v.

Bayesian Model : x and v are random variables with a known distribution.

Fisher Model : x is completely unknown but v is a random variable.

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1. The output of the state estimator provides a reliable data base from which control and dispatching decisions are made and various display, monitoring, alarm, logging and interrogation functions are fed.

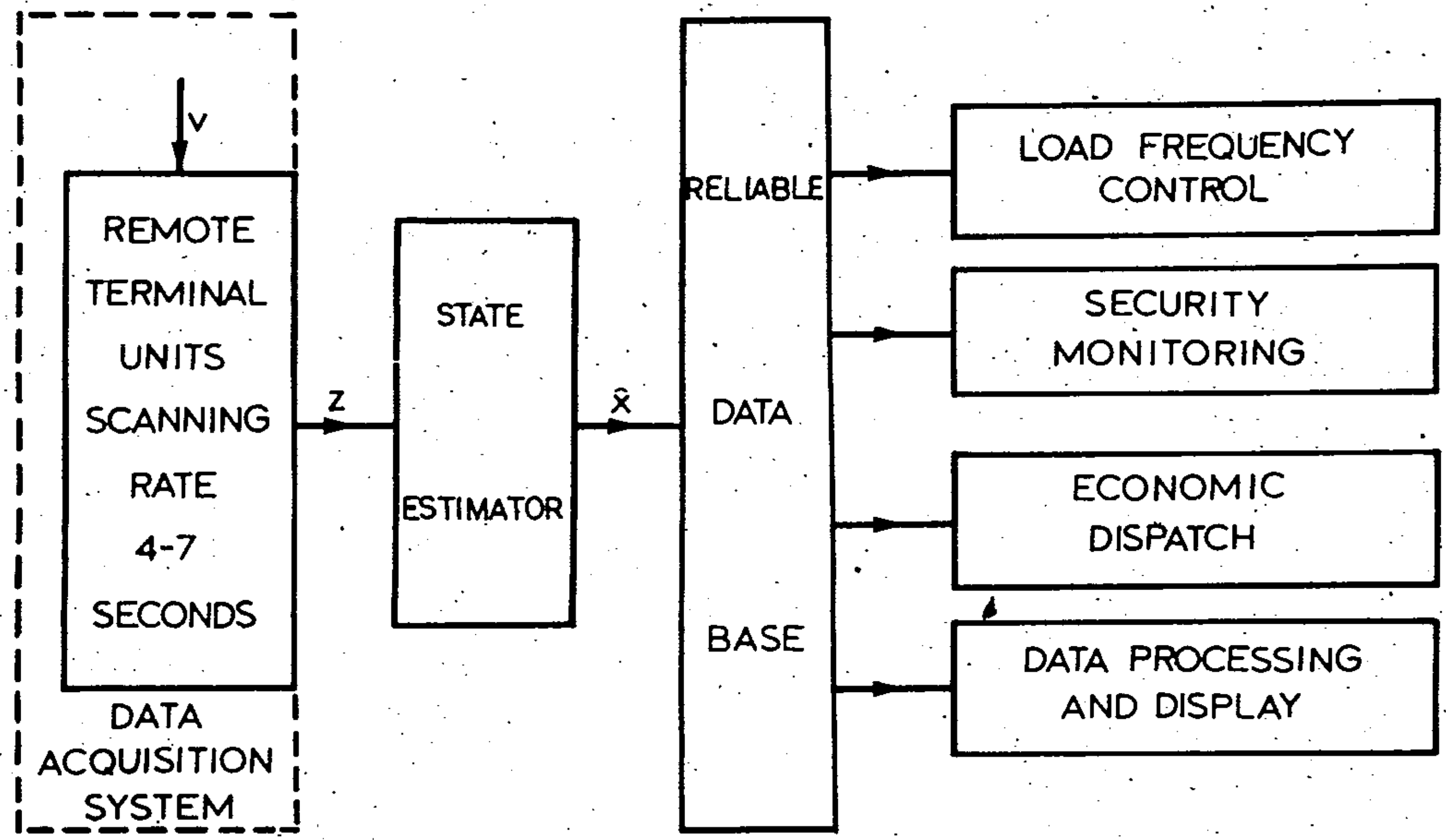

Fig. 1. Schematic diagram for information flow during state estimation

Next, the models for these estimators are discussed in detail. Although only the Least Squares model has been applied in the following chapters, the other models are given for the sake of completeness. The estimators are defined in matrix and vector notations which makes their implementation a little easier on a digital computer using standard matrix subroutines.

IIb. Least Squares Model : The Least squares method is the oldest and most widely used estimation procedure. Some of its popularity is due to the fact that nothing is assumed about the probability distribution concerning the state variables x or measurement errors v. The Least squares technique has a very simple interpretation in that it provides the state estimate which minimizes the sum of the squared residuals. A residual is defined as the difference between the actual observation and the observation predicted by the state estimate through the mathematical model. Given a set of observations z, the estimates $\hat{x}$ are obtained by minimizing the objective function

$$J(x) = (z - Hx)^t R^{-1} (z - Hx) \qquad (2.3)$$

in which R^{-1} may be identified as a positive definite weighting matrix. Normally, it is quite reasonable to use weights which are elements of the inverse of the error covariance matrix. The weighted Least squares method provides statistically optimal estimates which are well defined.

Taking the derivative of (2.3) one obtains

$$\frac{\partial J(x)}{\partial x} = -2H^t R^{-1} (z - Hx) \qquad (2.4)$$

Setting (2.4) to zero one obtains the estimate

$$\hat{x} = (H^t R^{-1} H)^{-1} H^t R^{-1} z \qquad (2.5)$$

In order to evaluate the performance of the estimator, x and v may be assumed as random variables with

$$E\{x\} = \mu \; ; \quad E\{v\} = 0 \quad ; \quad E\{xv^t\} = 0$$

Then, it can be shown [29] that $\hat{x}$ is unbiased since

$$E\{\hat{x}\} = E\{x\}$$

<u>IIc. Bayesian Model</u>: Estimates obtained with this method are based on the Bayes rule. Bayesian estimates could be obtained provided one makes use of the prior information about the probability distribution of the state variables x and the measurement errors v. Let x and v be random variables with mean and covariance matrix given by

$$E\{x\} = \mu \; ; \quad E\{(x-\mu)(x-\mu)^t\} = \psi$$

$$E\{v\} = 0 \; ; \quad E\{vv^t\} = R \; ; \quad E\{xv^t\} = 0$$

The estimate $\hat{x}$ is given by

$$\hat{x} = Wz + Wo \tag{2.7}$$

which also gives a minimum error covariance matrix.

The estimators W and Wo are given by

$$W = (H^t R^{-1} H + \psi^{-1})^{-1} H^t R^{-1}$$

$$Wo = (H^t R^{-1} H + \psi^{-1}) \cdot \psi^{-1} \; E\{x\}$$

The minimum error covariance matrix is given by[1]

$$\Sigma = E\{(x - \hat{x})(x - \hat{x})^t\} \tag{2.8}$$

or

$$\Sigma = \psi - \psi H^t (H \psi H^t + R)^{-1} H \psi \tag{2.9}$$

1. The covariance matrix predicts the magnitude of the estimation error and hence provides a measure of confidence for the estimated state.

Writing (2.7) in the following form

$$\hat{x} = E\{x\} + \psi H^t (H \psi H^t + R)^{-1} (z - H E\{x\}) \qquad (2.10)$$

it is clear that $\hat{x}$ is unbiased since

$$E\{\hat{x}\} = E\{x\} = \mu$$

A Bayesian model is one which is completely defined in the probabilistic sense. It uses all the prior information about the probability distribution of the random variables. But in engineering practice it seldom occurs that such information is available. Thus, Bayesian methods have a limited applicability.

IId. <u>Fisher Model</u> : This model is characterized by the assumption that the state variables are completely unknown and the measurement error is a random variable.

Assume that

$$E\{v\} = 0 \quad ; \quad E\{vv^t\} = R$$

The estimate

$$\hat{x} = Wz \qquad (2.11)$$

gives the minimum error covariance matrix.

The estimator W is given by

$$W = (H^t R^{-1} H)^{-1} H^t R^{-1} \qquad (2.12)$$

The minimum error covariance matrix is given by

$$\Sigma = (H^t R^{-1} H)^{-1} \qquad (2.13)$$

As can be seen by comparing (2.5) and (2.11), the Least squares and Fisher estimates are equivalent. It follows that the only weighting matrix that produces statistically optimal estimates is the inverse of the error covariance matrix R.

To prove (2.12), let

$$\hat{x} = Cz \tag{2.14}$$

be the minimum variance linear unbiased estimate such that

$$E\{\hat{x}\} = E\{x\}$$

and $E\{|x - \hat{x}|^2\}$ be a minimum. But

$$\begin{aligned} E\{|x - \hat{x}|^2\} &= \text{tr } E[(x - \hat{x})(x - \hat{x})^t] \\ &= \text{tr } E[(x - CHx - Cv)(x - CHx - Cv)^t] \\ &= \text{tr } CRC^t \end{aligned} \tag{2.15}$$

where for an unbiased estimate one requires $CH = I$ (Since $E\{\hat{x}\} = CHx$)

Thus, one must minimize $\text{tr } CRC^t$ subject to $CH = I$. Using a Lagrange multiplier matrix Λ, the problem reduces to minimizing

$$\text{tr } (CRC^t - CH\Lambda^t)$$

Taking the derivative with respect to C and equating to zero one obtains

$$RC^t - H\Lambda^t = 0 \tag{2.16}$$

Thus

$$C = \Lambda H^t R^{-1} \tag{2.17}$$

Multiplying both sides of (2.17) by H and noting that $CH = I$ one obtains

$$\Lambda = (H^t R^{-1} H)^{-1} \tag{2.18}$$

Substituting (2.18) into (2.17), one gets

$$C = (H^t R^{-1} H)^{-1} H^t R^{-1} \tag{2.19}$$

This completes the proof.

The least squares technique is perhaps the best suited for power system state estimation. It is very simple to use and yields good results, particularly when the noise disturbance is small. With the proper choice of the weighting matrix, this method provides statistically optimal estimates. On the basis of the kind of information available and the suitability for on-line data processing, the Least squares method is used in the following chapters to estimate the state variables in a power system.

Within the scope of state estimation one may consider two processes: a static process giving rise to static state estimation [9, 10] and a dynamic process giving rise to dynamic state estimation [8, 31, 38, 44, 48]. In the static state estimation process, such as the AEP method of chapter III, a snapshot scan of the system state is considered to obtain the measurements. A snapshot scan implies that all the meters are read at the same instant of time and fed into the computer as one vector. These snapshot scans are received at periodic intervals of the order of a few seconds. The steady-state model of a power system could be interpreted as one scan. However, it is not satisfying to employ a static-state logic on a time-varying system. In dynamic state estimation, the dynamic behaviour of the power system, including the data acquisition system, must be modelled. Considering the time behaviour of the system state, a dynamic model [8] may be defined by a relation of the form

$$x(t + \Delta t) = x(t) + w(t) \tag{2.20}$$

i.e. the state at time $(t + \Delta t)$ will be the same as that at time t, except for some uncertainty represented by a white-noise process $w(t)$.

If the observation equation (2.1) is non-linear, it is possible to obtain a solution by either using iterative techniques or by a linearization process. In any case, an understanding of static state estimation provides the proper framework for analysis of dynamic systems.

Although a power system is subjected to random disturbances as well as time-varying load demands, the minute-by-minute changes in the state variables and line flows are small. Hence, under normal operating conditions, a quasi-steady-state mode may be assumed.

CHAPTER III

AEP METHOD FOR STATE ESTIMATION

This method utilizes line power flow measurements to provide an estimate of the state variables. It is a very simple and effective method and is under various stages of implementation at several power system control centers [35]. This method requires modest computing and storage requirements resulting in less programming effort. It also has excellent convergence characteristics which are an essential prerequisite for on-line implementation.

IIIa. Problem Description : The 'state' of a power system may be represented by a vector, the state vector, made up of bus voltage magnitudes and phase angles. In other words, with the knowledge of complex bus voltages, one knows the total distribution of line flows in a power system.

If real and reactive power flows in all transmission lines are considered to be a set of measurements, one can use the method of Least squares to obtain the state estimates. If line flows at both ends of a transmission line are measured, a redundant set of measurements are available which acts as a safeguard against possible loss of metered data.

IIIb. Problem Formulation and Mathematical Development :

Consider a typical line between two buses as shown in Fig. 2. The line has been replaced by the usual π - equivalent circuit. The important parameters are :

V_p, V_q	-	bus voltages at buses p and q
$S_{j,pq}$	-	power flow from bus p to bus q on line j
$S_{j,qp}$	-	power flow from bus q to bus p on line j
z_j	-	series impedance on line j
$y_{j,p}$; $y_{j,q}$	-	line shunt admittance at buses p and q respectively including all elements to ground.

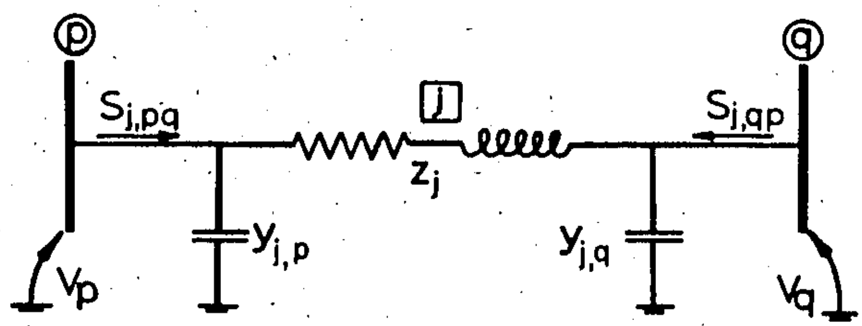

Fig. 2. Typical line between buses p and q

Typical instrumentation needed to provide one of these line power flows is shown in Fig. 3.

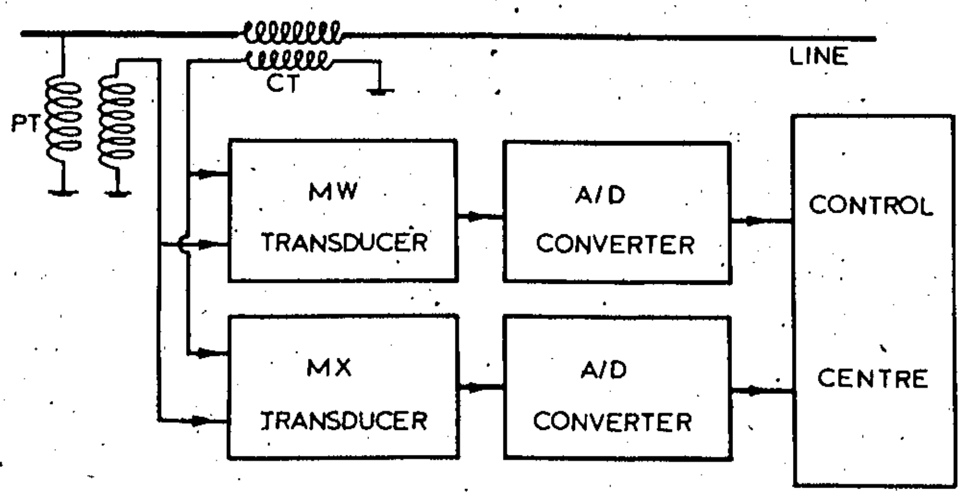

Fig. 3. Typical facility to obtain line power flows

According to Dopazo [10], the errors in a digital gathering station, similar to the one shown in Fig. 3, are typically :

Analog to digital conversion:	0.10% of full scale
MW and MX transducers :	0.25 - 0.5% of full scale
Instrument transformers :	2% of MVA measured.

From Fig. 2 the line power flows are given by

$$S_{j,pq} = V_p \left[\frac{(V_p - V_q)}{z_j} + V_p\, y_{j,p} \right]^* \qquad (3.1)$$

and

$$S_{j,qp} = V_q \left[\frac{(V_q - V_p)}{z_j} + V_q \, y_{j,q} \right]^* \qquad (3.2)$$

in which '*' signifies conjugation.

Letting

$$v_{j,pq} = V_p - V_q \qquad (3.3)$$

$$v_{j,qp} = V_q - V_p \qquad (3.4)$$

one obtains

$$S_{j,pq} = V_p \left[\frac{v_{j,pq}}{z_j} + V_p \, y_{j,p} \right]^* \qquad (3.5)$$

$$S_{j,qp} = V_q \left[\frac{v_{j,qp}}{z_j} + V_q \, y_{j,q} \right]^* \qquad (3.6)$$

Define the ℓ - vector for the line flows as :

$$S_p = \begin{bmatrix} S_{1,pq} \\ \vdots \\ S_{j,pq} \\ \vdots \\ S_{\ell,pq} \end{bmatrix} \; ; \; S_q = \begin{bmatrix} S_{1,qp} \\ \vdots \\ S_{j,qp} \\ \vdots \\ S_{\ell,qp} \end{bmatrix}$$

and the state vector of not more than n - 1 variables is defined as :

$$X = \begin{bmatrix} V_1 \\ \vdots \\ V_p \\ \vdots \\ V_q \\ \vdots \\ V_n \end{bmatrix}$$

The Least squares estimation problem consists in minimizing the function

$$J_p(x) = (S_{pq}^m - S_{pq}^c)^{*t} \; W_p(S_{pq}^m - S_{pq}^c) \qquad (3.7)$$

in which S_{pq}^m are the measured line flows and S_{pq}^c are the calculated line flows from (3.1) from the knowledge of the state variables to be estimated. The term W_p is an $\ell \times \ell$ diagonal matrix of weighting factors.

Similarly for the line flows at bus q one obtains

$$J_q(x) = (S_{qp}^m - S_{qp}^c)^{*t} \; W_q(S_{qp}^m - S_{qp}^c) \qquad (3.8)$$

Defining the line voltages in vector form as

$$v_p = \begin{bmatrix} v_{1,pq} \\ \vdots \\ v_{j,pq} \\ \vdots \\ v_{\ell,pq} \end{bmatrix} \quad ; \quad v_q = \begin{bmatrix} v_{1,qp} \\ \vdots \\ v_{j,qp} \\ \vdots \\ v_{\ell,qp} \end{bmatrix}$$

it is possible to express (3.5) and (3.6) as

$$S_{pq} = H_p \, v_p^{*} + K_p \tag{3.9}$$

$$S_{qp} = H_q \, v_q^{*} + K_q \tag{3.10}$$

in which

H_p = diagonal matrix with elements V_p / z_j^{*}

H_q = diagonal matrix with elements V_q / z_j^{*}

K_p = vector with elements $|V_p|^2 \, y_{j,p}^{*}$

K_q = vector with elements $|V_q|^2 \, y_{j,q}^{*}$

The matrices so defined are valid for all buses p and q.

Substituting (3.9) and (3.10) into (3.7) and (3.8) one obtains

$$J_p(x) = (H_p v_p^{*m} - H_p v_p^{*c})^{*t} \, W_p (H_p v_p^{*m} - H_p v_p^{*c})$$

or

$$J_p(x) = (v_p^{m} - v_p^{c})^{t} \, D_p (v_p^{m} - v_p^{c})^{*} \tag{3.11}$$

in which

$$D_p = H_p^* W_p H_p \tag{3.12}$$

Similarly

$$J_q(x) = (v_q^m - v_q^c)^t D_q (v_q^m - v_q^c)^* \tag{3.13}$$

in which

$$D_q = H_q^* W_q H_q \tag{3.14}$$

The matrices D_p and D_q are diagonal matrices and they are a function of the state variables.

The line voltages are related to the bus voltages through a relation of the form

$$v = AV \tag{3.15}$$

in which A is a suitable bus incidence matrix with elements +1, -1, or 0. The following notation is used:

A_{jk} = +1 if the j^{th} line is connected to k^{th} bus and directed away from the k^{th} bus.

A_{jk} = -1 if the j^{th} line is connected to k^{th} bus and directed toward the k^{th} bus.

A_{jk} = 0 if the j^{th} line is not connected to k^{th} bus.

To obtain a meaningful solution, however, the voltage at one bus (or more) must be specified. In practice, the voltage at two or three buses are measured to ensure against loss of this important piece of data without which the state estimator algorithm would be disabled.

Partitioning (3.15) into known and unknown state variables one obtains for bus p :

$$v_p = A_{gp} x_g + B_p x \qquad (3.16)$$

in which x_g is a known voltage vector and x are the unknown state variables. The matrices A_{gp} and B_p are appropriate sub-matrices of the bus incidence matrix.

Similarly for bus q one obtains :

$$v_q = A_{gq} x_g + B_q x \qquad (3.17)$$

Substituting (3.16) and (3.17) into (3.11) and (3.13) one gets:

$$J_p(x) = (v_p^m - A_{gp} x_g - B_p x)^t \; D_p (v_p^m - A_{gp} x_g - B_p x)^* \qquad (3.18)$$

$$J_q(x) = (v_q^m - A_{gq} x_g - B_q x)^t \; D_q (v_q^m - A_{gq} x_g - B_q x)^* \qquad (3.19)$$

In order to proceed with the static state estimation process one must minimize (3.18) and (3.19) with respect to the state variables. Before this is done, however, one must make the following assumptions:

(a) The dependence of D_p and D_q on x will be introduced iteratively.[1]

(b) Although the line voltages are not measured directly, it will greatly simplify the problem if these are assumed independent of x. This, in essence, means that

$$\frac{dv_p^m}{dx} = 0 \; ; \quad \frac{dv_q^m}{dx} = 0$$

---

1. This artifice linearizes the set of equations at the current iteration. Also, D_p and D_q are new weighting matrices.

Taking the derivative of (3.18) and (3.19) with respect to x and remembering that D_p and D_q are diagonal matrices one obtains:

$$\frac{dJ_p(x)}{dx} = -2\,B_p^t\,D_p(v_p^m - A_{gp}x_g - B_p x) \qquad (3.20)$$

$$\frac{dJ_q(x)}{dx} = -2B_q^t\,D_q(v_q^m - A_{gq}x_g - B_q x) \qquad (3.21)$$

Setting (3.20) and (3.21) to zero one gets:

$$(B_p^t\,D_p\,B_p)\,\hat{x} = B_p^t\,D_p(v_p^m - A_{gp}x_g) \qquad (3.22)$$

$$(B_q^t\,D_q\,B_q)\,\hat{x} = B_q^t\,D_q(v_q^m - A_{gq}x_g) \qquad (3.23)$$

Recognizing that $B_q = -B_p$ and $A_{gq} = -A_{gp}$ and summing (3.22) and (3.23) one finally gets :

$$[\,B_p^t(D_p + D_q)\,B_p]\,\hat{x} = B_p^t(D_p v_p^m - D_q v_q^m) + C_p + C_q \qquad (3.24)$$

in which

$$C_p = -B_p^t\,D_p\,A_{gp}\,x_g \qquad (3.25)$$

$$C_q = -B_p^t\,D_q\,A_{gp}\,x_g \qquad (3.26)$$

Equation (3.24) is the basic iterative formula for the solution of the state variables.

IIIc. Solution Procedure :

Step 1: Measure the line powers S_{pq}^m, S_{qp}^m.

Step 2 : Make an initial estimate for the state vector.

Step 3 : Calculate H_p, H_q, K_p, K_q, D_p and D_q . Decide on the weighting factors W_p, W_q .

Step 4 : Calculate the line voltages v_p^m and v_q^m from the knowledge of S_{pq}^m and S_{qp}^m . (Use equations (3.9) and (3.10)) .

Step 5 : Calculate the state vector using (3.24)

Step 6 : Repeat steps 3 through 5 until the change in x between two successive iterations is less than a predetermined tolerance.

Error detection is a problem that is often encountered when a set of measurements is fed into a digital computer for processing. Completely erroneous results may be obtained if some of the bad data lies outside the limits given by probability theory. These bad data could be caused by many things, such as failure of A/D converters, sensor failure, noise outbursts in communication lines etc. . Such bad data must be eliminated in order to obtain meaningful results. Error detection and correction has been the topic of much recent interest[12, 19, 23, 30, 35, 39] . One way to detect and correct bad data is to compute the magnitude of the variance between the computed and measured power flows and eliminating or correcting those measurements that exceed a variance larger than the specified value. This error correction procedure could be easily incorporated in the AEP algorithm :

If $z = Hx + v$ and $E\{vv^t\} = R$; $E\{v\} = 0$

then,

$$\hat{x} = (H^t R^{-1} H)^{-1} H^t R^{-1} z \tag{3.27}$$

$$\Sigma = E\{(x - \hat{x})(x - \hat{x})^t\}$$

or

$$\Sigma = (H^t R^{-1} H)^{-1} \tag{3.28}$$

Similarly for z :

$$\hat{z} = H\hat{x} \tag{3.29}$$

$$(z - \hat{z}) = H(x - \hat{x}) + v \tag{3.30}$$

The error covariance matrix for z is

$$\Sigma_z = E\{z - \hat{z})(z - \hat{z})^t\}$$

or

$$\Sigma_z = E\{[H(x - \hat{x}) + v][H(x - \hat{x}) + v]^t\}$$

or

$$\Sigma_z = -H(H^t R^{-1} H)^{-1} H^t + R \tag{3.31}$$

If σ_{z_j} is an element of this matrix, then a measurement is in error if $\sigma_{z_j} > \sigma_{sp}$, in which $\sqrt{\sigma_{sp}}$ is a suitably chosen specified standard deviation.

III d. A Numerical Example :

Consider the simple system shown in Fig. 4.

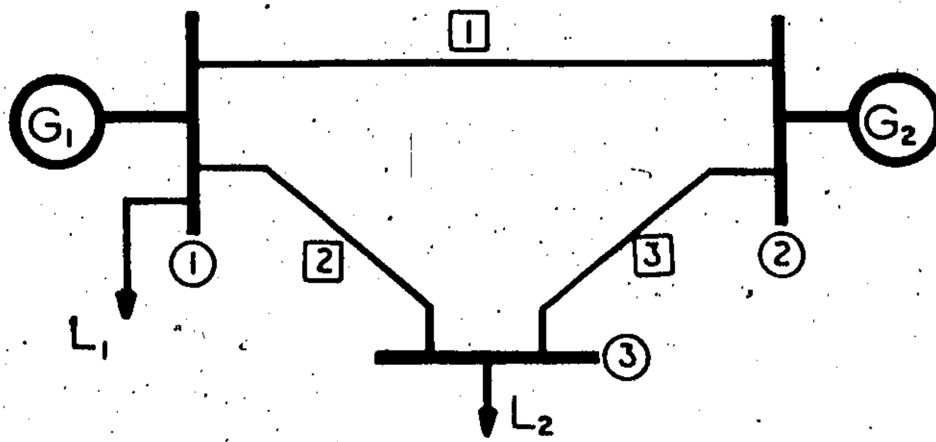

Fig. 4. Three bus power system

The system consists of two generating units, two loads and three transmission lines. The voltage at bus 1 is fixed and measured at one per unit and a phase angle of zero degrees. The transmission lines are all identical and may be represented by an equivalent circuit as shown in Fig. 5, on a 50 MVA, 120 kV base.

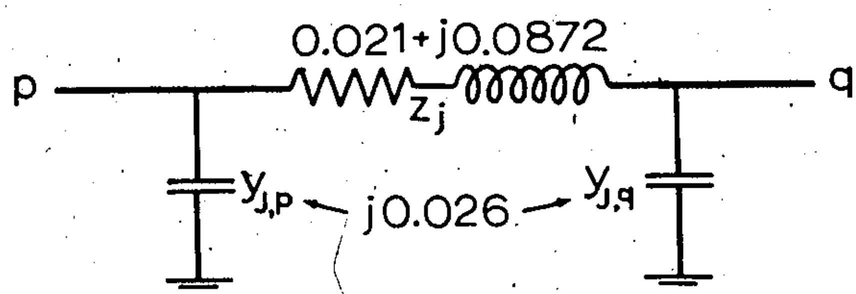

Fig. 5. Equivalent circuit of transmission lines

Load L_1 is assumed to be constant at $S_{L1} = 0.6 + j0.4$. Load L_2 is a typical industrial load as shown in Fig. 6 and tabulated in Table I[1]. Generator G_2 supplies a constant power $S_{G2} = 0.6 + j1.0$.

1: In practice, the state estimation process is carried out every 4-7 seconds. However, there is no loss of generality, as far as the objective of this numerical example is concerned, to consider an hourly scanning rate for the remote terminal units.

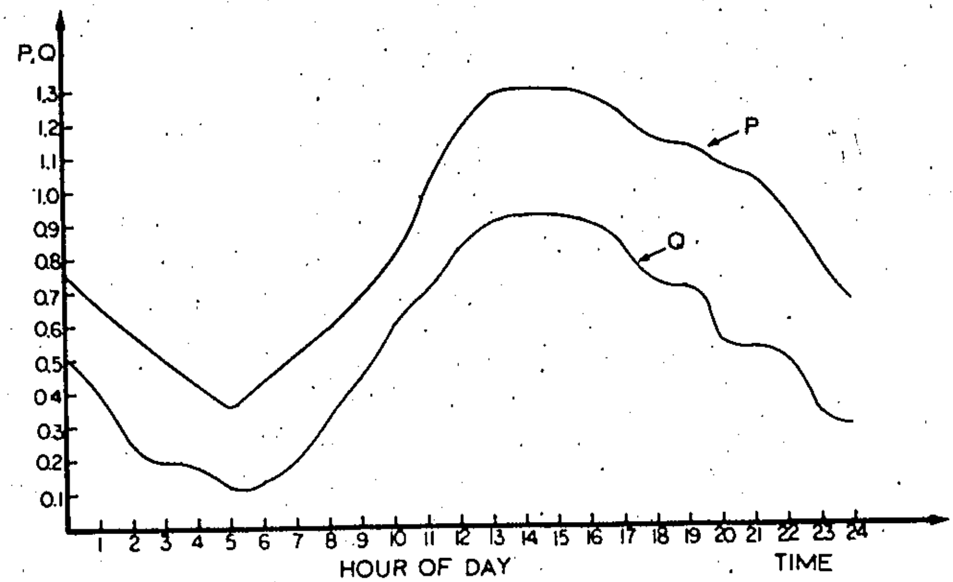

Fig. 6. Typical load variation

Table I. MW and MX schedules for load L_2

Time (h)	MW (pu)	MX (pu)
0	0.75	0.50
1	0.66	0.40
2	0.58	0.24
3	0.50	0.20
4	0.43	0.18
5	0.36	0.12
6	0.44	0.14
7	0.52	0.20
8	0.59	0.34
9	0.68	0.44
10	0.80	0.60
11	1.00	0.68
12	1.16	0.82
13	1.29	0.90
14	1.30	0.92
15	1.30	0.92
16	1.29	0.90
17	1.23	0.84
18	1.15	0.72
19	1.14	0.70

Table I (Cont'd...)

Time (h)	MW (pu)	MX (pu)
20	1.07	0.54
21	1.04	0.52
22	0.93	0.50
23	0.79	0.32
24	0.66	0.28

The line voltages may be expressed in terms of the bus voltages by a relation of the form (see equation (3.16)) :

$$\begin{bmatrix} v_{1,1} \\ v_{2,1} \\ v_{3,2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} V_1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$$

for the p - type buses.

Similarly for the q - type buses one obtains (see equation (3.17)):

$$\begin{bmatrix} v_{1,2} \\ v_{2,3} \\ v_{3,3} \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} V_1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$$

The line flows for the p-type buses are given by (see equation (3.9)):

$$\begin{bmatrix} S_{1,1} \\ S_{2,1} \\ S_{3,2} \end{bmatrix} = \begin{bmatrix} \frac{V_1}{z_1^*} & 0 & 0 \\ 0 & \frac{V_1}{z_2^*} & 0 \\ 0 & 0 & \frac{V_2}{z_3^*} \end{bmatrix} \begin{bmatrix} v_{1,1}^* \\ v_{2,1}^* \\ v_{3,2}^* \end{bmatrix} + \begin{bmatrix} V_1^2 \cdot y_{1,1}^* \\ V_1^2 \, y_{2,1}^* \\ V_2^2 \, y_{3,2}^* \end{bmatrix}$$

A similar relation holds for the q-type buses as given by (3.10).

The 'exact' power flow in the lines was obtained from the load flow solution. These values are tabulated in Table II.

IIIe. Results and Observations :

The estimated bus voltage magnitudes and phase angles are obtained by the AEP method for 0%, ± 10% and ± 20 % measurement errors in the line power flows. The results are shown in Figs. 7 through 10. As can be clearly seen from an examination of these graphs, the estimated state variables are all within reasonable limits and consistent with the measurement error. The solution for no error in the measurements coincides with the solution obtained from the load flow.

A listing of the Fortran program for the AEP method appears in Appendix C. The convergence of the iterative procedure was quite rapid. The inverse of the matrices was obtained using Gaussian elimination. Sparse matrix techniques were not required since in this simple example the matrices to be inverted are all full.

A redundant set of measurements were used to obtain the state estimates. As pointed out by Poretta [35], the AEP state estimation algorithm allows weighting of the individual line power flow measurements through the weighting matrices W_p and W_q. For this example, W_p and W_q were assumed to be identity matrices.

Table II. Line Power Flows for Varying Load [MW/MX]

Hours	Line 1 From bus 1 to bus 2 From bus 2 to bus 1		Line 2 From bus 1 to bus 3 From bus 3 to bus 1		Line 3 From bus 2 to bus 3 From bus 3 to bus 2	
	MW	MX	MW	MX	MW	MX
0	-0.13575	-0.53124	0.30261	-0.03821	0.45848	0.49203
	0.14157	0.50802	-0.30068	0.00122	-0.44934	-0.50125
1	-0.16598	-0.56475	0.24197	-0.10941	0.42726	0.45476
	0.17273	0.54523	-0.24058	0.06985	-0.41941	-0.46985
2	-0.19205	-0.61672	0.18817	-0.21835	0.39975	0.39705
	0.20024	0.60292	-0.18662	0.17897	-0.39338	-0.41900
3	-0.21912	-0.62964	0.13504	-0.24677	0.37211	0.38186
	0.22787	0.61807	-0.13361	0.20678	-0.36640	-0.40678
4	-0.24296	-0.63597	0.08868	-0.26135	0.34796	0.37400
	0.25211	0.62601	-0.08732	0.22094	-0.342711	-0.40097
5	-0.26623	-0.65469	0.04242	-0.30151	0.32387	0.35226
	0.27611	0.64768	-0.04075	0.26217	-0.31928	-0.38218
6	-0.23902	-0.64859	0.09527	-0.28720	0.35144	0.36013
	0.24845	0.63977	-0.09361	0.24793	-0.34635	-0.38794
7	-0.21227	-0.62962	0.14829	-0.24627	0.37905	0.38216
	0.22095	0.61782	-0.14678	0.20659	-0.37323	-0.40663
8	-0.18956	-0.58458	0.19515	-0.15187	0.40305	0.43240
	0.19695	0.56760	-0.19400	0.11111	-0.39601	-0.45116
9	-0.15942	-0.55153	0.25553	-0.08175	0.43416	0.46929
	0.16583	0.53065	-0.25408	0.04254	-0.42591	-0.48253
10	-0.11914	-0.49746	0.33688	0.032549	0.47583	0.52884
	0.12417	0.47113	-0.33494	-0.06707	-0.46556	-0.53293
11	-0.05018	-0.46719	0.47184	0.09964	0.54562	0.56238
	0.05438	0.43761	-0.46685	-0.12323	-0.53315	-0.55675
12	0.00502	-0.41538	0.58167	0.21031	0.60178	0.61788
	-0.00178	0.38206	-0.57343	-0.21987	-0.58657	-0.60012
13	0.05044	-0.38345	0.67113	0.27931	0.64751	0.65148
	-0.04764	0.34843	-0.65976	-0.27556	-0.63019	-0.62445

Table II. (Cont'd) ...

Hours	Line 1 From bus 1 to bus 2 From bus 2 to bus 1		Line 2 From bus 1 to bus 3 From bus 3 to bus 1		Line 3 From bus 2 to bus 3 From bus 3 to bus 2	
	MW	MX	MW	MX	MW	MX
14	0.05383	-0.37613	0.67828	0.29471	0.65113	0.65923
	-0.05114	0.34072	-0.66651	-0.28922	-0.63349	-0.63075
15	0.053825	-0.37615	0.67828	0.29471	0.65113	0.65925
	-0.051139	0.34073	-0.66651	-0.28922	-0.63349	-0.63078
16	0.05036	-0.38349	0.67111	0.27928	0.64756	0.65151
	-0.04757	0.34847	-0.65974	-0.27553	-0.63025	-0.62447
17	0.02945	-0.40641	0.62936	0.23059	0.62638	0.62744
	-0.02634	0.37263	-0.61969	-0.23415	-0.61031	-0.60587
18	0.00197	-0.44993	0.57345	0.13874	0.59813	0.58104
	0.00185	0.41894	-0.56600	-0.15187	-0.58399	-0.56814
19	-0.00142	-0.45701	0.56643	0.12389	0.59461	0.57346
	0.00538	0.42652	-0.55924	-0.13819	-0.58075	-0.56182
20	-0.02490	-0.51248	0.51746	0.00713	0.57004	0.51368
	0.02995	0.48626	-0.51182	-0.02892	-0.55818	-0.51107
21	-0.03523	-0.51982	0.49708	-0.00809	0.55956	0.50573
	0.04045	0.49425	-0.49189	-0.01506	-0.54812	-0.50494
22	-0.07342	-0.52866	0.42301	-0.02881	0.52122	0.49587
	0.07891	0.50415	-0.41925	-0.00044	-0.51079	-0.49956
23	-0.12038	-0.58941	0.32824	-0.15735	0.47254	0.42887
	0.12744	0.57108	-0.32560	0.12289	-0.46440	-0.44289

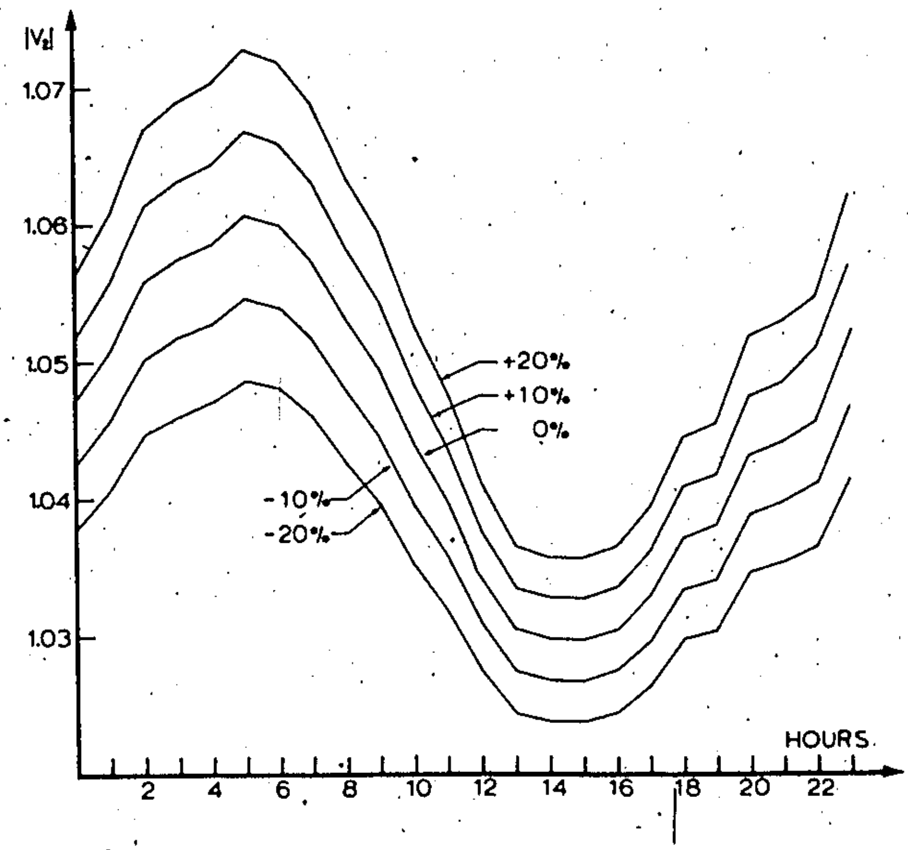

Fig. 7 Estimated voltage magnitude for bus. 2

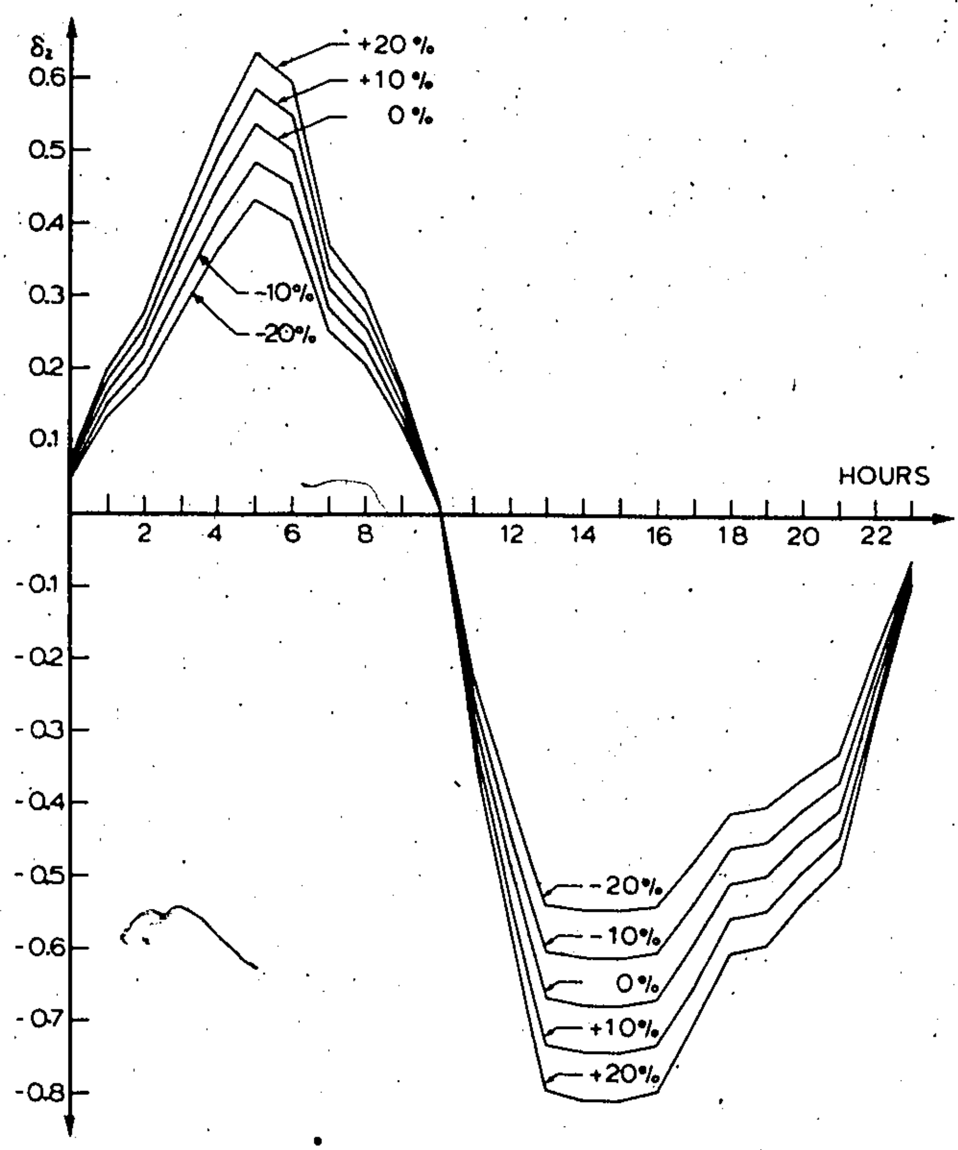

Fig. 8 Estimated voltage phase angle for bus 2

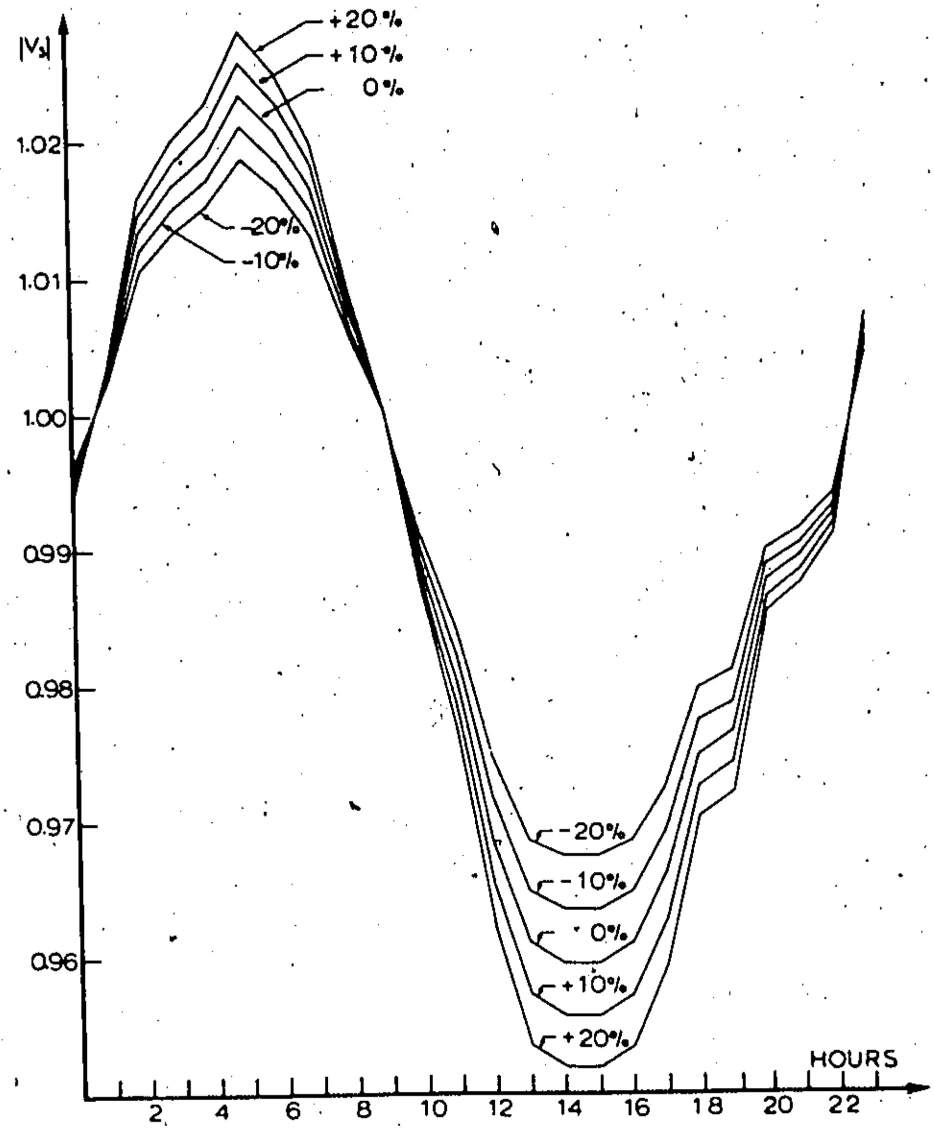

Fig. 9. Estimated voltage magnitude for bus 3

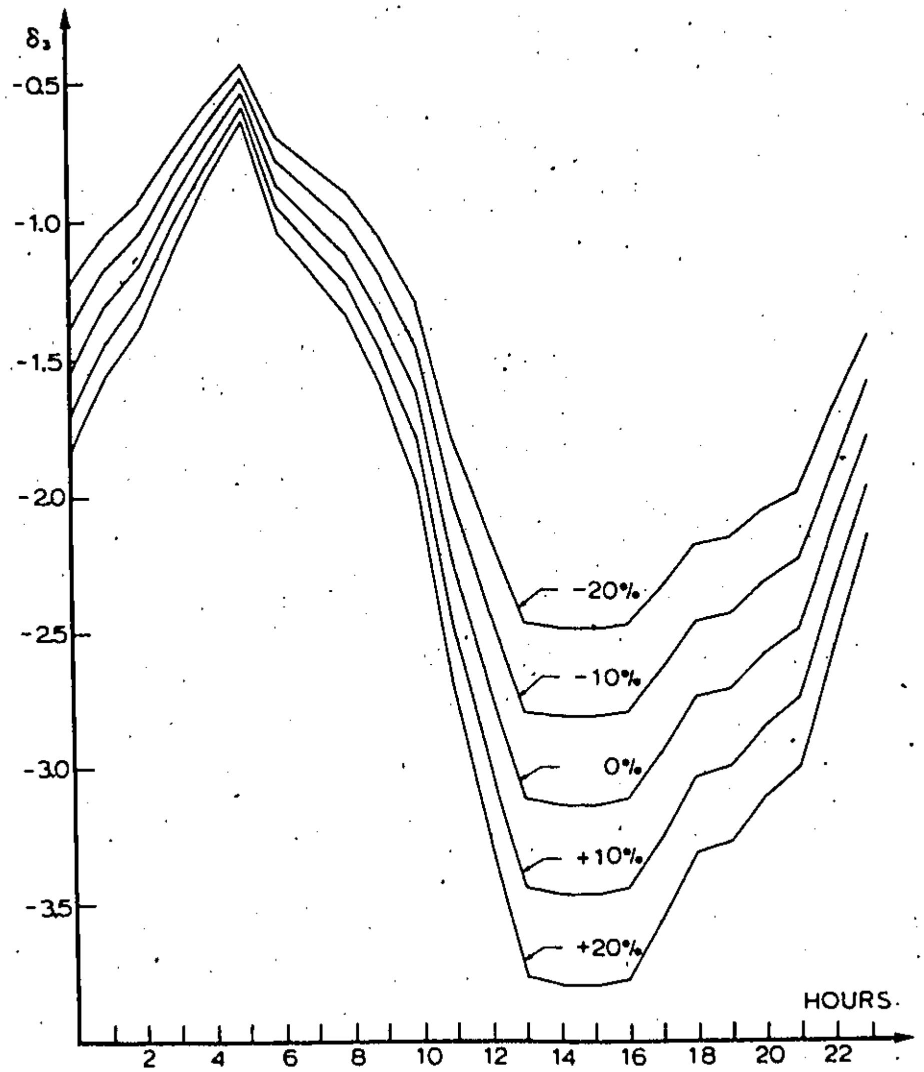

Fig. 10. Estimated voltage phase angle for bus 3

CHAPTER IV

INCREMENTAL COST CURVE PARAMETER ESTIMATION

IVa. General :

To dispatch a power system economically the operating costs are minimized as the load changes in time. To achieve this objective an accurate knowledge of the parameters of the incremental cost curve is of importance. In this chapter, the method of Least squares is applied to obtain statistically optimal parameters in the presence of error in the incremental cost.

IVb. Mathematical Development :

The incremental cost of operating a generating unit is a measure of the change in cost input (Δ F) with respect to power output (Δ P). The incremental cost curve describes a plot of the slope ($\Delta F / \Delta P$) in dollars per megawatt-hour versus the power input in megawatts.

In most cases, the incremental cost of a generating unit may be represented by a piecewise linear characteristic consisting of one or more segments of the form

$$\frac{dF_n}{dP_{G_n}} = C_n P_{G_n} + D_n \qquad (4.1)$$

in which F_n and P_{G_n} are the operating cost and generated real powers of unit n, respectively, and C_n and D_n are the parameters to be estimated.

If one assumes that there is some uncertainty in the operating cost of a unit caused by uncertain pricing of energy resulting from environmental constraints, fluctuating fuel costs, etc., one may wish to estimate the parameters C_n and D_n assuming a relation of the form

$$\frac{dF_n}{dP_{G_n}} = C_n P_{G_n} + D_n + \epsilon_n \tag{4.2}$$

in which ϵ_n is a random variable representing the uncertainty in the operating cost, or equivalently, in the incremental operating cost.

Clearly, small changes in the parameters will affect the economic allocation of real generated powers.

Letting

$$\lambda_n = \frac{dF_n}{dP_{G_n}} \tag{4.3}$$

and substituting (4.3) into (4.2) one obtains

$$\lambda_n = C_n P_{G_n} + D_n + \epsilon_n \tag{4.4}$$

Fig. 11 depicts a typical linearized incremental cost curve. The dotted lines merely represent the changes in the slope caused by small changes in C_n.

Expressing (4.4) in matrix form one obtains

$$\lambda = P_G C + D + \epsilon \tag{4.5}$$

in which P_G is a square diagonal matrix.

Assuming that D is known, it is possible to define the function

$$J(C) = (\lambda - D - P_G C)^t R^{-1} (\lambda - D - P_G C) \tag{4.6}$$

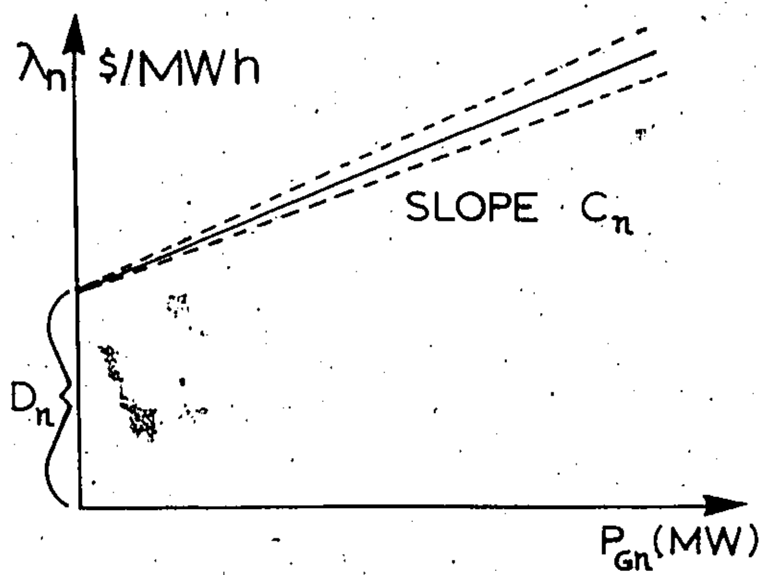

Fig. 11. Incremental cost curve for a generating unit.

in which R is the error covariance matrix given by

$$R = E(\epsilon\epsilon^t) \qquad (4.7)$$

An estimate of C may be obtained by minimizing (4.6) with respect to C. The result for this estimate is given by

$$\hat{C} = (P_G^t R^{-1} P_G)^{-1} P_G^t R^{-1} (\lambda - D) \qquad (4.8)$$

Clearly, since P_G is a square diagonal matrix, (4.8) reduces to

$$\hat{C} = P_G^{-1} (\lambda - D) \qquad (4.9)$$

which is the result of the state estimation process when the number of measurements is equal to the number of estimated variables.

An estimate of D may be obtained from the cost equations which may be written as a 2nd order polynomial in P_G [22] :

$$F = P_G D + \frac{1}{2} P_G^{\,t} P_G \hat{C} + A_F + \nu \qquad (4.10)$$

in which A_F is a fixed known cost and ν is a random variable describing the error in the operating cost. Its distribution is similar to that of ϵ .

The best estimate of D is given by

$$\hat{D} = P_G^{-1} (F - A_F) - \frac{1}{2} P_G \hat{C} \qquad (4.11)$$

Equations (4.8) and (4.11) are solved simultaneously for $\hat{C}$ and $\hat{D}$.

Conceptually, it is possible to think of the estimation process as a device in which the input variables (measurements) are given by the operating cost of the generating units as well as their incremental costs. From these measurements the parameters C_n and D_n are estimated. This is shown schematically in Fig. 12.

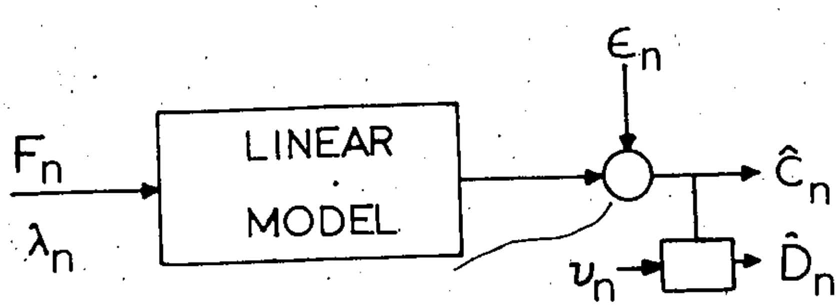

Fig. 12. The System Model.

IVc. Numerical Examples:

The method developed has been successfully applied to 5-bus and 25-bus power systems which have been described in a paper by Gungor et al. [17] .

(i) 5-Bus Power System : This system, which consists of three generating units, four loads and seven branches, is shown in Fig. 13.

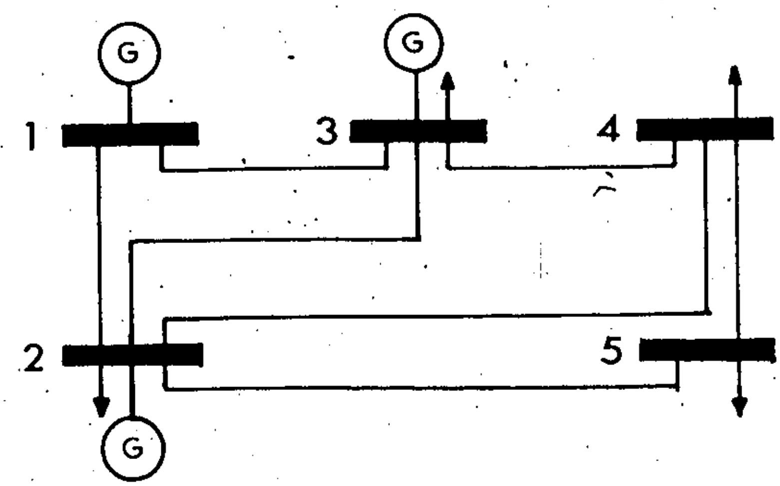

Fig. 13. The 5-Bus power system

Per unit impedances and line charging susceptance on a 100 MVA base are given in Table III.

Table III 5 - Bus System Impedance and Line Charging Data

Bus Code	Impedance	Charging
1 - 2	0.02+j0.06	j0.030
1 - 3	0.08+j0.24	j0.025
2 - 3	0.06+j0.18	j0.020
2 - 4	0.06+j0.18	j0.020
2 - 5	0.04+j0.12	j0.015
3 - 4	0.01+j0.03	j0.010
4 - 5	0.08+j0.24	j0.025

The initial bus voltages, generation schedule and system load are listed in Table IV.

Table IV 5-Bus System Loads and Initial Generation

Bus	Voltage		Generation		Load	
	Magnitude	Angle	MW	MX	MW	MX
1	1.060	0.0	0.984	0.0	0.00	0.00
2	1.056	0.0	0.400	0.3	0.20	0.10
3	1.044	0.0	0.300	0.1	0.45	0.15
4	1.041	0.0	0.0	0.0	0.40	0.05
5	1.030	0.0	0.0	0.0	0.60	0.10

The cost functions in \$/h are given by the expressions

$$F_1 = 140 + 200P_{G_1} + 60P_{G_1}^2$$

$$F_2 = 120 + 150P_{G_2} + 75P_{G_2}^2$$

$$F_3 = 80 + 180P_{G_3} + 70P_{G_3}^2$$

and the incremental costs are given by the equations

$$\lambda_1 = 2.0 + 1.2P_{G_1}$$

$$\lambda_2 = 1.5 + 1.5P_{G_2}$$

$$\lambda_3 = 1.8 + 1.4P_{G_3}$$

in which the generated powers must be expressed in per unit.

The economic dispatch solution (see appendices A and B) is given by

$P_{G_1} = 0.426039$; $\lambda_1 = 2.51125$; $F_1 = 236.09835$

$P_{G_2} = 0.690571$; $\lambda_2 = 2.53586$; $F_2 = 259.35227$

$P_{G_3} = 0.550231$; $\lambda_3 = 2.57032$; $F_3 = 200.23436$

The parameters C_n and D_n are estimated using (4.8) and (4.11) with a constant $\frac{1}{2}$ % error in the operating costs of each generating unit and a variable error of $\frac{1}{2}$ %, 1%, 2% and 3% in the incremental costs. The error covariance matrices are assumed to be identity matrices. The results are given in Table V.

Table V Estimated parameters $\hat{C}_n$ and $\hat{D}_n$ for the 5-Bus system.

Error %	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{D}_1$	$\hat{D}_2$	$\hat{D}_3$
0	1.2	1.5	1.4	2.0	1.5	1.8
$\frac{1}{2}$	1.128881	1.482339	1.380583	2.042852	1.524872	1.823532
1	1.187829	1.519062	1.427297	2.030295	1.512192	1.810680
2	1.305720	1.592506	1.520728	2.005182	1.486833	1.784976
3	1.423611	1.665951	1.614156	1.980069	1.461473	1.759273

As can be seen from Table V, the estimated parameters are within reasonable limits and consistent with the errors considered. Dispatching the system (see Appendices A and B) with the estimated parameters will produce a different allocation of generated powers. This new economic allocation should, however, be close to the original generation schedule. This can be seen in Table VI.

Table VI The new generation schedule for 5-Bus system.

Error %	P_{G_1}	P_{G_2}	P_{G_3}
0	0.426039	0.690571	0.550231
$\frac{1}{2}$	0.425970	0.690528	0.550182
1	0.426021	0.690561	0.550219
2	0.425955	0.690508	0.550167
3	0.426109	0.690630	0.550291

The cost convergence characteristic for the 5-bus power system in the case of 0% error is shown in Fig. 14. This characteristic has been plotted from the results of the computer program for economic dispatch solution (See appendix B). The convergence of the iterative procedure was quite rapid. In all cases, convergence to a tolerance of 0.00001 was achieved in eight iterations.

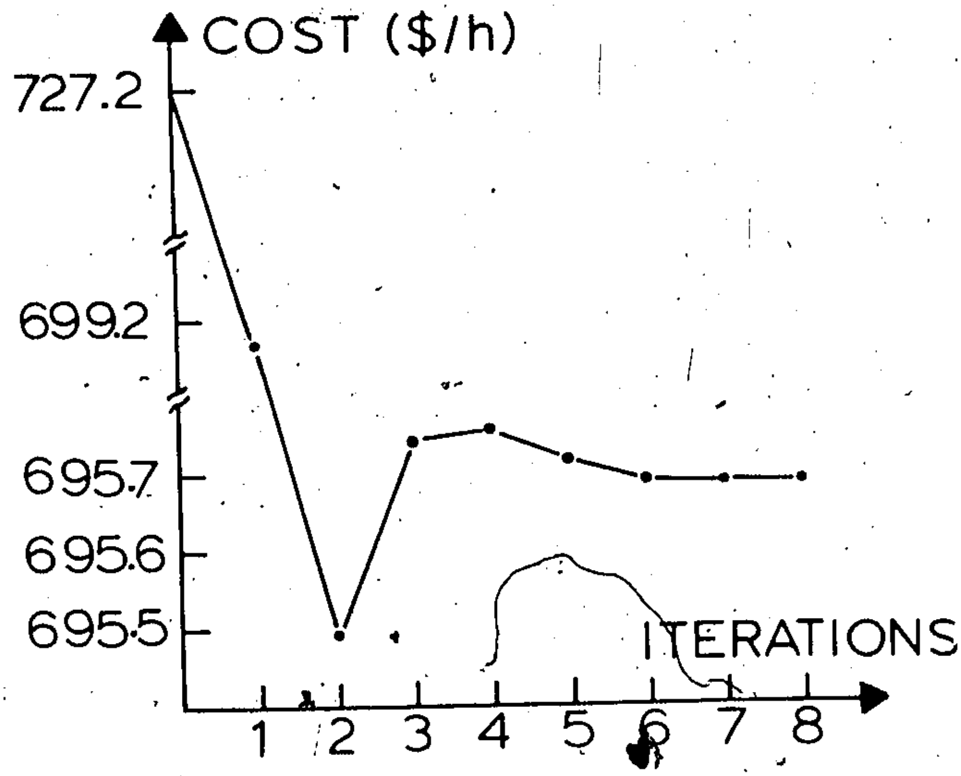

Fig. 14 Cost convergence characteristic for the 5-bus power system in the case of 0% error.

(ii) 25-Bus Power System :

In order to see the results on a large power system, a 25-bus system of Fig. 15 is considered next. This system consists of five generating units, twenty four loads and thirty five branches. Table VII gives the impedance and line charging data in per unit on a 100 MVA base.

Table VII 25-Bus System

Impedance and Line Charging Data

Bus Code	Impedance	Line-charging Susceptance
1-3	0.0720 + j0.2876	j0.0179
1-16	0.0290 + j0.1379	j0.0337
1-17	0.1012 + j0.2799	j0.0148
1-19	0.1487 + j0.3897	j0.0224
1-23	0.1085 + j0.2245	j0.0573
1-25	0.0753 + j0.3593	j0.0873
2-6	0.0617 + j0.2935	j0.0186
2-7	0.0511 + j0.2442	j0.0155
2-8	0.0579 + j0.2763	j0.0175
3-13	0.0564 + j0.1478	j0.0085
3-14	0.1183 + j0.3573	j0.0185
4-19	0.0196 + j0.0514	j0.0113
4-20	0.0382 + j0.1007	j0.0220
4-21	0.0970 + j0.2547	j0.0558
5-10	0.0497 + j0.2372	j0.0577
5-17	0.0144 + j0.1269	j0.1335
5-19	0.0929 + j0.2442	j0.0140
6-13	0.0263 + j0.0691	j0.0040
7-8	0.0529 + j0.1465	j0.0078
7-12	0.0364 + j0.1736	j0.0110
8-9	0.0387 + j0.1847	j0.0118
8-17	0.0497 + j0.2372	j0.0572
9-10	0.0973 + j0.2691	j0.0085
10-11	0.0898 + j0.2359	j0.0135
11-17	0.1068 + j0.2807	j0.0161
12-17	0.0460 + j0.2196	j0.0139
14-15	0.0281 + j0.0764	j0.0044
15-16	0.0256 + j0.0673	j0.0148
17-18	0.0806 + j0.2119	j0.0122
18-19	0.0872 + j0.2294	j0.0132
20-21	0.0615 + j0.1613	j0.0354
21-22	0.0414 + j0.1087	j0.0238
22-23	0.2250 + j0.3559	j0.0169
22-24	0.0970 + j0.2595	j0.0567
24-25	0.0472 + j0.1458	j0.0317

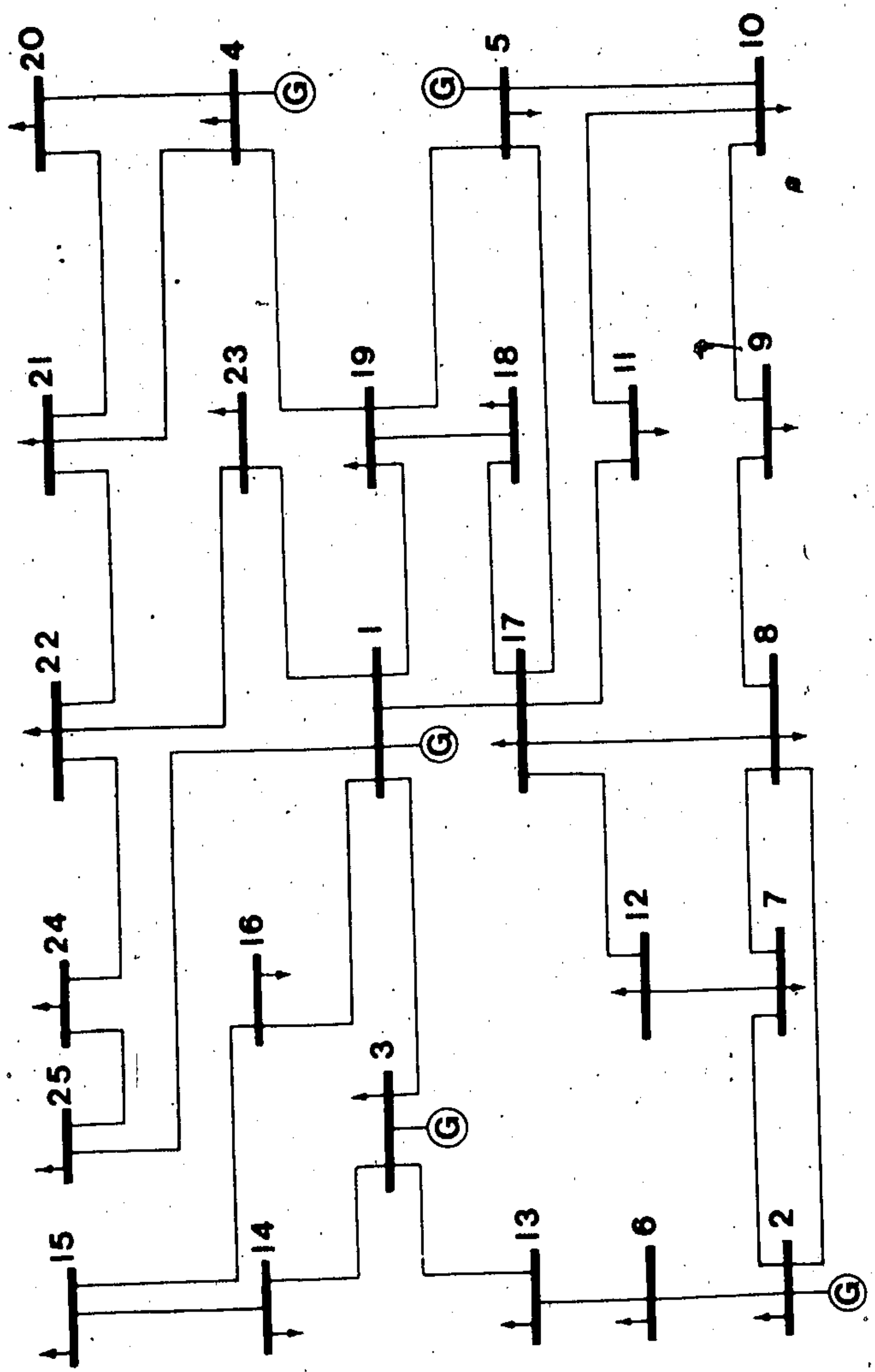

Fig. 15. 25-Bus power system

The initial bus voltages, generation schedule and system load are given in Table VIII.

Table VIII 25-Bus System Loads and Initial Generation

Bus	Voltage Magnitude pu	Voltage Angle Degrees	Generation MW	Generation MVAR	Load MW	Load MVAR
1	1.020	0.0	54	0	00	0
2	0.889	0.0	100	-17	10	3
3	0.959	0.0	150	4	50	17
4	0.891	0.0	50	-4	30	10
5	0.885	0.0	200	-47	25	8
6	0.905	0.0	0	0	15	5
7	0.884	0.0	0	0	15	5
8	0.886	0.0	0	0	25	0
9	0.875	0.0	0	0	15	5
10	0.883	0.0	0	0	15	5
11	0.892	0.0	0	0	5	0
12	0.892	0.0	0	0	10	0
13	0.914	0.0	0	0	25	8
14	0.935	0.0	0	0	20	7
15	0.942	0.0	0	0	30	10
16	0.963	0.0	0	0	30	10
17	0.902	0.0	0	0	60	20
18	0.888	0.0	0	0	15	5
19	0.896	0.0	0	0	15	5
20	0.882	0.0	0	0	25	8
21	0.894	0.0	0	0	20	7
22	0.912	0.0	0	0	20	7
23	0.972	0.0	0	0	15	5
24	0.940	0.0	0	0	15	5
25	0.959	0.0	0	0	25	8

The cost functions in \$ / h are given by the expressions :

$$F_1 = 40 + 180 \, P_{G_1} + 15 \, P_{G_1}^2$$

$$F_2 = 60 + 170 \, P_{G_2} + 30 \, P_{G_2}^2$$

$$F_3 = 100 + 210 \, P_{G_3} + 12 \, P_{G_3}^2$$

$$F_4 = 25 + 200 \, P_{G_4} + 80 \, P_{G_4}^2$$

$$F_5 = 120 + 190 \, P_{G_5} + 10 \, P_{G_5}^2$$

and the incremental costs are given by the equations

$$\lambda_1 = 1.8 + 0.30 \, P_{G_1}$$

$$\lambda_2 = 1.7 + 0.60 \, P_{G_2}$$

$$\lambda_3 = 2.1 + 0.24 \, P_{G_3}$$

$$\lambda_4 = 2.0 + 1.60 \, P_{G_4}$$

$$\lambda_5 = 1.9 + 0.20 \, P_{G_5}$$

in which $P_{G_1}, P_{G_2}, \ldots, P_{G_5}$ must be expressed in per unit.

The economic dispatch (See appendix A) solution is given by

$P_{G_1} = 1.426540$; $\lambda_1 = 2.22796$; $F_1 = 327.30244$

$P_{G_2} = 0.756377$; $\lambda_2 = 2.15383$; $F_2 = 205.74727$

$P_{G_3} = 0.784106$; $\lambda_3 = 2.28819$; $F_3 = 272.04012$

$P_{G_4} = 0.259369$; $\lambda_4 = 2.41499$; $F_4 = 82.25558$

$P_{G_5} = 2.409550$; $\lambda_5 = 2.38191$; $F_5 = 635.87371$

Proceeding in a similar manner to that of the 5-bus system, the estimated parameters $\hat{C}_n$ and $\hat{D}_n$ are obtained for the 25-bus system and shown in Table IX .

Table IX. Estimated parameters $\hat{C}_n$ and $\hat{D}_n$ for 25-Bus system.

Error %	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$	$\hat{D}_1$	$\hat{D}_2$	$\hat{D}_3$	$\hat{D}_4$	$\hat{D}_5$
0	0.3	0.6	0.24	1.60	0.20	1.80	1.70	2.10	2.00	1.90
1/2	0.299531	0.592509	0.224930	1.570830	0.198931	1.811805	1.716433	2.123255	2.019688	1.914483
1	0.315151	0.620986	0.254113	1.663944	0.208817	1.800664	1.705663	2.111814	2.007563	1.902573
2	0.346387	0.677939	0.312478	1.850164	0.228588	1.778384	1.684124	2.088931	1.983413	1.878753
3	0.377623	0.734891	0.370844	2.036392	0.248359	1.756104	1.662585	2.066049	1.959262	1.854934

Again, it can be seen from Table IX that the estimated parameters are within reasonable limits and consistent with the errors considered. Dispatching the 25-bus power system (See appendix A) with the estimated parameters will produce a new economic allocation of generated powers. This new generation schedule is close to the original schedule, as can be seen in Table X.

Table X. The new generation schedule for 25-bus system.

Error %	P_{G_1}	P_{G_2}	P_{G_3}	P_{G_4}	P_{G_5}
0	1.42654	0.756377	0.784106	0.259369	2.40955
1/2	1.42657	0.756369	0.784048	0.259385	2.40958
1	1.42650	0.756402	0.784158	0.259358	2.40952
2	1.42650	0.756415	0.784201	0.259340	2.40949
3	1.42649	0.756428	0.784207	0.259336	2.40947

CHAPTER V

SUMMARY AND CONCLUSIONS

This thesis has presented three basic state estimation methods: the Least squares model, the Bayesian model and the Fisher model. These mathematical models have been presented in vector and matrix notations which makes their implementation easier on a digital computer using standard matrix subroutines. The AEP method for static state estimation is based on the Least squares model and one obtains the estimate of the state from measured line flows. The method is easy to conceive, simple to implement and very efficeint. Measurements at both ends of a line are very effective in detecting and identifying bad data points. It is apparent that state estimation gives more weight to lines of lower impedance and allows, therefore, the separation of power system into areas. The AEP algorithm was chosen for this study on the basis of the desirability of the data required and on the effectiveness and simplicity of the mathematical formulation.

Since the matrices D_p and D_q depend only on bus voltage magnitudes, which have small variations with system conditions, the assumption that D_p and D_q are independent of $\hat{x}$ in the application of equations (3.18) and (3.19) is quite reasonable and justified. With this assumption the state estimation algorithm is greatly simplified. This approximation in a way, is equivalent to keeping matrices D_p and D_q constant at its final value during the iterative procedure since the bus voltage magnitudes can be predicted quite accurately at the instant matrices D_p and D_q are computed.

This study also includes a method of estimating the parameters describing the linear incremental cost curve from actual values corresponding to an economic dispatch solution. The inputs to the estimator are the incremental cost of each generating unit and the actual operating costs. Since these may not be known accurately, the estimated parameters may differ from the actual parameters from which the economic solution was obtained. In this way a method has been provided for the improved pricing of power production.

Several major electrical utilities, including Ontario Hydro, are in the process of carrying out feasibility studies aimed at determining the goodness and practicality of state estimation algorithms. When one considers all the factors and circumstances involved in developing a concept of this nature, one feels that state estimation algorithms for on-line implementation will soon be the rule rather than the exception. There should be little doubt that today's interest in this field will become tomorrow's demand for such efficient procedures. Every day new ideas are suggested which expand, better and supersede the existing ones. More work is needed in describing the dynamic behaviour of the system and at devising on-line contingency methods during abnormal conditions.

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APPENDIX A

ECONOMIC DISPATCH

One of the primary objectives in the economic operation of power systems is the scheduling of the real power outputs of generators to minimize total fuel consumption while supplying all load requirements. This scheduling process i.e. economic allocation of generating capacity, is known as Economic Dispatch.

The total production cost F (in dollars per hour) can be expressed as a sum of production costs F_i of each generating unit i. The problem amounts to minimizing the cost function

$$F = \sum_{i=1}^{n} F_i (P_{G_i}) \tag{A1}$$

subject to the constraints

$$\sum_{i=1}^{n} P_{G_i} = P_D + P_L \quad ; \quad i = 1, 2, \ldots, n \tag{A2}$$

and[1]

$$P_{G_{i,min}} \leq P_{G_i} \leq P_{G_{i,max}} \tag{A3}$$

where P_{G_i} = real generated power of unit i

P_D = total load demand, $P_D = \sum_{i=1}^{n} P_{D_i}$

P_L = the real power losses.

Using the Lagrangian multiplier method, one obtains the augmented cost function :

$$F_L = \sum_{i=1}^{n} F_i - \lambda_L \left(\sum_{i=1}^{n} P_{G_i} - P_D - P_L \right) \tag{A4}$$

---

1. Additional constraints may be included as discussed in [20, 34].

in which λ_L is the Lagrangian multiplier. Taking the partial derivative of (A4) with respect to P_{G_i} and equating to zero, one obtains

$$\frac{\partial F_L}{\partial P_{G_i}} = \frac{\partial F_i}{\partial P_{G_i}} - \lambda_L \left(1 - \frac{\partial P_L}{\partial P_{G_i}}\right) = 0 \qquad \text{(A5)}$$

Note that P_D is considered to be a constant while taking the derivative.

Letting,

Incremental cost $(IC)_i = \dfrac{\partial F_i}{\partial P_{G_i}}$

and Incremental transmission loss $(ITL)_i = \dfrac{\partial P_L}{\partial P_{G_i}}$

(A5) can be written as

$$(IC)_i = \lambda_L \left[1 - (ITL)_i \right] ; \quad i = 1, 2, \ldots, n \qquad \text{(A6)}$$

It should be noted that there are n optimum dispatch equations (A6) plus the $(n+1)^{th}$ equation (A2) to solve for (n+1) unknowns $P_{G_1}, P_{G_2}, \ldots, P_{G_n}, \lambda_L$.

The incremental transmission losses $(ITL)_i$ are obtained from the loss equation. There are two methods which are commonly used in arriving at the loss equation.

(i) B-Coefficients Method: This method [1,20] expresses the loss equation in terms of coefficients for the network and the real powers. The loss equation is given by

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \qquad \text{(A7)}$$

and incremental losses are given by

$$\frac{\partial P_L}{\partial P_m} = 2 \sum_n B_{mn} P_n \quad \text{(A8)}$$

in which B_{mn} matrix represent a set of coefficients reflecting the actual system voltages and the network configuration.

(ii) Bus Impedance Method : This method [14, 33] utilizes the bus impedance matrix with composite load bus as reference in obtaining the loss equation :

$$P_L = \sum_{j=1}^{n} \sum_{k=1}^{n} [\alpha_{jk}(P_j P_k + Q_j Q_k) + \beta_{jk}(Q_j P_k - P_j Q_k)] \quad \text{(A9)}$$

where $$\alpha_{jk} = \frac{R_{jk}}{|V_j| \, |V_k|} \cos (\delta_j - \delta_k) \quad \text{(A10)}$$

$$\beta_{jk} = \frac{R_{jk}}{|V_j| \, |V_k|} \sin (\delta_j - \delta_k) \quad \text{(A11)}$$

in which R_{jk}'s are obtained from the bus impedance matrix with composite load bus as reference. The bus voltage magnitudes as well as the phase angles $|V_j|$, $|V_k|$, δ_j, δ_k are obtained from the load flow solution. In this case

$$(ITL)_j \approx 2 \sum_{k=1}^{n} (\alpha_{jk} P_k - \beta_{jk} Q_k) \quad \text{(A12)}$$

Solution Procedure for Economic Dispatch

STEP 1. Obtain the bus impedance matrix with composite load bus as reference for the network.

STEP 2. Assume initial values for $P_{G_1}, P_{G_2}, \ldots, P_{G_n}$.

STEP 3. Compute all bus powers $S_i = P_i + j\,Q_i$ and all bus voltages using the load flow.

STEP 4. Calculate power losses $P_L = \sum_{i=1}^{n} P_{G_i} - P_D$

STEP 5. Calculate $(ITL)_i$ using (A8) or (A12).

STEP 6. Choose an appropriate value for the Lagrange multiplier λ_L and compute $(IC)_i$ using (A6)

STEP 7. Calculate P_{G_i}'s using a known relation of the form

$$P_{G_i} = \alpha_i + \beta_i\,(IC)_i + \gamma_i\,(IC)_i^2 + \ldots$$

STEP 8. Check for convergence i.e. whether

$\left| \sum_{i=1}^{n} P_{G_i} - P_D - P_L \right|$ is less than a predetermined tolerance. If not, repeat steps 6 through 8.

STEP 9. Repeat steps 3 through 9 until the change in P_{G_i} i.e. $\left| P_{G_i}^{(new)} - P_{G_i}^{(old)} \right|$ between two successive iterations is less than a predetermined tolerance.

A computer program for the economic dispatch solution appears in Appendix B. The bus impedance method for arriving at the loss equation is used in this computer program. The load flow as mentioned in step 3 is based on the Gauss-Seidel iterative technique.

APPENDIX B

LISTING OF THE FORTRAN PROGRAM FOR ECONOMIC DISPATCH SOLUTION

```
      REAL IC,ITL,LMBDA,LOSSES
      INTEGER BRANCH,TYPE1,TYPE2,T1,T2,GENS,LOADS,GN,GL,FROM,TO,GL1,
     1TP1,TP2
      LOGICAL TYP1,TYP2,OT1,OT2
      COMPLEX ZPRIM,YPRIM,ZBUS,YBUS,V,S,YSHUNT,YC,CONJG,A,AT,C2,CT2,
     1C3,CT3,R0,R1,R2,CMPLX,YBUS0,SFLOW
      COMPLEX ZBUS0,ZBUS1,ZBUS2,ZBUS3,YBUS0,YBUS1
      DIMENSION ZPRIM(7,7),YPRIM(7,7),YBUS(5,5),V(5),S(5),
     1YSHUNT(5,5),YC(5),VMAG(5),QMIN(5),P(5),Q(5),FROM(7),TO(7),
     2QMAX(5),PG(5),QG(5),PD(5),QD(5),T1(4),T2(1),A(7,5),AT(5,7),
     3YBUS0(5,5),R0(5,7),ALFA(3),BETA(3),GAMMA(3),DELTA(3),IC(3)
      DIMENSION SFLOW(5,5),PMAX(7),PMIN(7),C(3,3),D(3,3),PGOLD(7),
     1VMAG0(5),ZBUS1(7,7),ZBUS2(4,4),C2(7,4),
     2CT2(4,7),R1(4,7),ZBUS3(3,3),C3(4,3),CT3(3,4),R2(3,4),
     3A1(3),B1(3),C1(3),ITL(3),E(3,3)
C
C
C     READ NUMBER OF NODES AND BRANCHES,THE NUMBER OF GENERATORS AND LOADS
C
C
      READ 1,NODE,BRANCH,GENS,LOADS
    1 FORMAT(4I5)
      BASE=100.0
      ND1=NODE-1
      GN=GENS+1
      GL=GENS+LOADS
      GL1=GL-1
      LB=2*BRANCH
      TYP2=.TRUE.
      OT1=.TRUE.
      OT2=.FALSE.
C
C
C     READ THE BUS INCIDENCE MATRIX
C
C
      DO 40 I=1,BRANCH
      DO 40 J=1,NODE
      READ 51,A(I,J)
      AT(J,I)=A(I,J)
   40 CONTINUE
C
C
C     READ THE PRIMITIVE IMPEDANCE MATRIX
C     AND OBTAIN THE PRIMITIVE ADMITTANCE MATRIX
C
C
      DO 50 I=1,BRANCH
      READ 51,(ZPRIM(I,J),J=1,BRANCH)
      READ 1,FROM(I),TO(I)
   51 FORMAT(2F10.0)
   50 CONTINUE
      CALL MATINV(YPRIM,ZPRIM,BRANCH,LB)
C
C
C     READ THE SHUNT ADMITTANCE AT EACH NODE
C
C
```

```
      DO 4446 I=1,NODE
      DO 4446 J=1,NODE
      YSHUNT(I,J)=(0.0,0.0)
 4446 CONTINUE
      YSHUNT(1,2)=(0.0,0.030)
      YSHUNT(1,3)=(0.0,0.025)
      YSHUNT(2,3)=(0.0,0.020)
      YSHUNT(2,4)=(0.0,0.020)
      YSHUNT(2,5)=(0.0,0.015)
      YSHUNT(3,4)=(0.0,0.010)
      YSHUNT(4,5)=(0.0,0.025)
      YSHUNT(2,1)=(0.0,0.030)
      YSHUNT(3,1)=(0.0,0.025)
      YSHUNT(3,2)=(0.0,0.020)
      YSHUNT(4,3)=(0.0,0.010)
      YSHUNT(4,2)=(0.0,0.020)
      YSHUNT(5,2)=(0.0,0.015)
      YSHUNT(5,4)=(0.0,0.025)
C
C
C     READ THE CHARGING SHUNT CAPACITORS
C
C
      YC(1)=(0.0,0.0)
      YC(2)=(0.0,0.0)
      YC(3)=(0.0,0.0)
      YC(4)=(0.0,0.0)
      YC(5)=(0.0,0.0)
C
C
C     TOLERANCES
C
C
      EPS=1.0E-5
      EPS1=5.0E-4
      EPS2=1.0E-5
C
C
C     FORM THE BUS ADMITTANCE MATRIX
C
C
      CALL ADMIT(A,AT,YBUS0,NODE,BRANCH,YPRIM,R0)
C
C
C     OBTAIN THE BUS IMPEDANCE MATRIX WITH A COMPOSITE LOAD AS REFERENCE
C     OBTAIN THE IMPEDANCE MATRIX FOR REFERENCE FRAME 1
C
C
      DO 10 I=1,7
      DO 10 J=1,7
      ZBUS1(I,J)=(0.0,0.0)
 10   CONTINUE
      ZBUS1(2,2)=(0.016857,0.0505715)
      ZBUS1(3,2)=(0.0125714,0.0377144)
      ZBUS1(4,2)=(0.016857,0.0505715)
      ZBUS1(5,2)=(0.0125714,0.0377144)
      ZBUS1(6,2)=(0.0134285,0.0402858)
      ZBUS1(7,2)=(0.0157142,0.047143)
```

```
      ZBUS1(2,3)=(0.0125714,0.0377144)
      ZBUS1(3,3)=(0.0297143,0.089143)
      ZBUS1(4,3)=(0.0125714,0.0377144)
      ZBUS1(5,3)=(0.0297143,0.089143)
      ZBUS1(6,3)=(0.0262857,0.0788572)
      ZBUS1(7,3)=(0.0171428,0.0514287)
      ZBUS1(2,4)=(0.016857,0.0505715)
      ZBUS1(3,4)=(0.0125714,0.0377144)
      ZBUS1(4,4)=(0.016857,0.0505715)
      ZBUS1(5,4)=(0.0125714,0.0377144)
      ZBUS1(6,4)=(0.0134285,0.0402858)
      ZBUS1(7,4)=(0.0157142,0.047143)
      ZBUS1(2,5)=(0.0125714,0.0377144)
      ZBUS1(3,5)=(0.0297143,0.089143)
      ZBUS1(4,5)=(0.0125714,0.0377144)
      ZBUS1(5,5)=(0.0297143,0.089143)
      ZBUS1(6,5)=(0.0262857,0.0788572)
      ZBUS1(7,5)=(0.0171428,0.0514287)
      ZBUS1(2,6)=(0.0134285,0.0402858)
      ZBUS1(3,6)=(0.0262857,0.0788572)
      ZBUS1(4,6)=(0.0134285,0.0402858)
      ZBUS1(5,6)=(0.0262857,0.0788572)
      ZBUS1(6,6)=(0.0317143,0.0951429)
      ZBUS1(7,6)=(0.0195237,0.0585715)
      ZBUS1(2,7)=(0.0157142,0.047143)
      ZBUS1(3,7)=(0.0171428,0.0514287)
      ZBUS1(4,7)=(0.0157142,0.047143)
      ZBUS1(5,7)=(0.0171428,0.0514287)
      ZBUS1(6,7)=(0.0195237,0.0585715)
      ZBUS1(7,7)=(0.0436507,0.130952)
C
C
C     OBTAIN THE IMPEDANCE MATRIX FOR REFENCE FRAME 2
C
C
      DO 65 I=1,GL
      DO 65 J=1,GN
      READ 51,C2(I,J)
      CT2(J,I)=CONJG(C2(I,J))
65    CONTINUE
      CALL Z2(ZBUS1,ZBUS2,C2,CT2,NODE,GENS,LOADS,R1,GN,GL)
C
C
C     OBTAIN THE IMPEDANCE MATRIX FOR REFENCE FRAME 3
C
C
      DO 66 I=1,GN
      DO 66 J=1,GENS
      READ 51,C3(I,J)
      CT3(J,I)=CONJG(C3(I,J))
   66 CONTINUE
      CALL Z3(ZBUS2,ZBUS3,C3,CT3,GENS,GN,R2)
C
C
C     OBTAIN THE BUS ADMITTANCE MATRIX WITH THE ADDITIONAL SHUNT REACTANCE
C
C
      CALL ADMIT1(YBUS,NODE,YSHUNT,YC,YBUS0)
```

```
C
C
C     SLACK BUS  VOLTAGE.  THE SLACK BUS IS NUMBER 1
C
C
      V(1)=(1.06,0.0)
      PG(1)=0.0
      QG(1)=0.0
      PD(1)=0.0
      QD(1)=0.0
      P(1)=PG(1)-PD(1)
      Q(1)=QG(1)-QD(1)
      S(1)=CMPLX(P(1),Q(1))
C
C
C     READ THE NUMBER OF TYPE1 AND TYPE2 BUSES
C
C
      READ 5,TYPE1,TYPE2
    5 FORMAT(2I2)
      READ 6,(T1(I),I=1,TYPE1)
      READ 6,(T2(I),I=1,TYPE2)
    6 FORMAT(16I5)
C
C
C     SET  THE ACCELERATING FACTOR
C
C
      ACCEL=1.4
C
C
C     THE STATEMENTS THAT FOLLOW ARE FOR TYPE2 BUSES ONLY
C
C
      IF ( .NOT. TYP2 )    GO TO 777
C
C
C     READ THE VOLTAGE MAGNITUDE FOR TYPE2 BUSES
C
C
      DO 7 I=1,TYPE2
      IF(TYPE2.EQ.0.0) GO TO 7
      M1=T2(I)
      READ 2,V(M1),PG(M1),PD(M1)
      PGOLD(M1)=PG(M1)
      QD(M1)=0.0
      QG(M1)=0.0
      Q(M1)=QG(M1)-QD(M1)
      P(M1)=PG(M1)-PD(M1)
      S(M1)=CMPLX(P(M1),Q(M1))
      VMAG(M1)=CABS(V(M1))
      VMAGO(M1)=VMAG(M1)
      READ 2,QMAX(M1),QMIN(M1)
    7 CONTINUE
    2 FORMAT(8F10.0)
  777 CONTINUE
C
C
```

```
C     READ INITIAL ESTIMATES FOR TYPE1 BUS VOLTAGES
C
C
      DEMAND=0.0
      DO 19 I=1,TYPE1
      M1=T1(I)
      READ 14,V(M1),PG(M1),QG(M1),PD(M1),QD(M1)
 14   FORMAT(6F10.6)
      PG(1)=0.984
      P(M1)=PG(M1)-PD(M1)
      Q(M1)=QG(M1)-QD(M1)
      DEMAND=DEMAND+PD(M1)
      S(M1)=CMPLX(P(M1),Q(M1))
   19 CONTINUE
C
C
C     READ THE COEFFICIENTS FOR THE COST CURVES
C
C
      READ 100,(ALFA(I),BETA(I),GAMMA(I),I=1,GENS)
      READ 100,(A1(I),B1(I),C1(I),I=1,GENS)
 100  FORMAT(3F10.0)
      PRINT 1000,NODE,BRANCH,GENS,LOADS,TYPE1,TYPE2
 1000 FORMAT(1H1,'NODE=',I2/1H,'BRANCHES=',I2/1H,'GENERATORS=',I2/
     11H ,'LOADS=',I2/1H,'TYPE1=',I2/1H,'TYPE2=',I2)
    3 FORMAT(2I5)
      PRINT 1001
 1001 FORMAT(1H0,'BUS INCIDENCE MATRIX')
      PRINT 1030,((A(I,J),J=1,NODE),I=1,BRANCH)
 1030 FORMAT(10(2X,F4.0))
 1031 FORMAT(4(2X,F4.0))
 1002 FORMAT(6(2X,1PE15.5))
      PRINT 1003
 1003 FORMAT(1H0,'PRIMITIVE IMPEDANCE MATRIX')
      PRINT 1004,((ZPRIM(I,J),J=1,BRANCH),I=1,BRANCH)
 1004 FORMAT(4(2X,1PE15.5))
      PRINT 1005
 1005 FORMAT(1H0,'PRIMITIVE ADMITTANCE MATRIX')
      PRINT 1004,((YPRIM(I,J),J=1,BRANCH),I=1,BRANCH)
      PRINT 1006
 1006 FORMAT(1H0,'BUS ADMITTANCE MATRIX WITHOUT SHUNT REACTANCE')
      PRINT 1002,((YBUS0(I,J),J=1,NODE),I=1,NODE)
      PRINT 1007
 1007 FORMAT(1H0,'BUS ADMITTANCE MATRIX WITH  SHUNT REACTANCE')
      PRINT 1002,((YBUS(I,J),J=1,NODE),I=1,NODE)
      PRINT 1009
 1009 FORMAT(1H0,'IMPEDANCE MATRIX FOR REFERENCE FRAME 1')
      PRINT 1002,((ZBUS1(I,J),J=1,GL),I=1,GL)
      PRINT 1010
 1010 FORMAT(1H0,'TRANSFORMATION MATRIX FROM FRAME 1 TO FRAME 2')
      PRINT 2030,((C2(I,J),J=1,GN),I=1,GL)
 2030 FORMAT(8(2X,F10.4))
      PRINT 1011
 1011 FORMAT(1H0,'IMPEDANCE MATRIX FOR REFERENCE FRAME 2')
      PRINT 1002,((ZBUS2(I,J),J=1,GN),I=1,GN)
      PRINT 1012
 1012 FORMAT(1H0,'TRANSFORMATIUON MATRIX FROM FRAME 2 TO FRAME 3')
      PRINT 3030,((C3(I,J),J=1,GENS),I=1,GN)
```

```
3030 FORMAT(6(2X,F10.4))
     PRINT 1013
1013 FORMAT(1H0,'IMPEDANCE MATRIX FOR REFERENCE FRAME 3')
     PRINT 1004,((ZBUS3(I,J),J=1,GENS),I=1,GENS)
     PRINT 1014
1014 FORMAT(1H0,'INITIAL VOLTAGE AND POWERS AT THE BUSES')
     DO 1015 I=1,NODE
     PRINT 1016,I,V(I),I,S(I)
1016 FORMAT(1H ,'V(',I2,') = ',1PE15.5,'+J',1PE15.5,2X,'S(',I2,') = ',
    11PE15.5,'+J',1PE15.5)
1015 CONTINUE
     PRINT 1017
1017 FORMAT(1H0,'GENERATED POWERS AND DEMAND')
     DO 1018 I=1,NODE
     PRINT 1019,I,PG(I),I,PD(I),I,QD(I)
1019 FORMAT(1H ,'PG(',I2,') = ',1PE15.5,2X,'PD(',I2,') = ',1PE15.5,
    12X,'QD(',I2,') = ',1PE15.5)
1018 CONTINUE
     DO 1121 I=1,TYPE2
     M1=T2(I)
1121 CONTINUE
     PRINT 1020
1020 FORMAT(1H0,'COST COEFFICIENTS')
     PRINT 1021,(A1(I),B1(I),C1(I),I=1,GENS)
     PRINT 1021,(ALFA(I),BETA(I),GAMMA(I),I=1,GENS)
1021 FORMAT(3(2X,1PE15.5))
     CALL CST(A1,B1,C1,PG,GENS,COST,BASE)
     PRINT 1022,COST
1022 FORMAT(1H0,'INITIAL COST = ',1PE15.5)
C
C
C    SET THE ITERATION COUNTER FOR THE LOAD FLOW
C    MAKE AN INITIAL ESTIMATE FOR THE INCREMENTAL COST OF RECEIVED POWER
C
C
     LMBDA=0.3
     MU=1.
     DO 1040 I=1,GENS
1040 PGOLD(I)=PG(I)
 999 CONTINUE
     DO 756 I=1,NODE
     VMAG(I)=VMAGO(I)
 756 CONTINUE
C
C
C    COMPUTE ALL THE BUS POWERS AND BUS VOLTAGES FROM A LOAD FLOW
C
C
     CALL LFLOW(NODE,V,VMAG,QMAX,QMIN,S,YBUS,P,Q,TYPE1,TYPE2,ACCEL,
    1EPS,T1,T2,TYP2,QT1,QT2)
     DO 601 J=1,NODE
     P(J)=REAL(S(J))
     Q(J)=AIMAG(S(J))
     PG(J)=P(J)+PD(J)
     QG(J)=Q(J)+QD(J)
 601 CONTINUE
C
C
```

```
C       COMPUTE THE LOSSES
C
C
        LOSSES=0.0
        DO 5000 I=1,NODE
 5000   LOSSES=LOSSES+REAL(S(I))
        PRINT 701, LOSSES,DEMAND
  701   FORMAT(1H0,'LOSSES =',1PE15.5,2X,'DEMAND = ',1PE15.5)
C
C
C       COMPUTE THE PARAMETERS FOR THE ITL'S
C
C
        CALL PARAM(C,D,ZBUS3,V,GENS,NODE,DELTA,B)
C
C
C       COMPUTE THE ITL'S
C
C
        DO 600 M1=1,GENS
        SUM=0.0
        DO 700 K=1,GENS
        SUM=SUM+PG(K)*C(M1,K)-QG(K)*D(M1,K)
  700   CONTINUE
        ITL(M1)=2.0*SUM
  600   CONTINUE
C
C
C       COMPUTE THE IC'S
C
C
        DLMDA=5.0E-3
        KT=0
 6666   CONTINUE
        DO 400 M1=1,GENS
        IC(M1)=LMBDA*(1.0-ITL(M1))
  400   CONTINUE
C
C
C       COMPUTE THE GENERATED POWERS AT TYPE2 BUSES AND TOTAL GENERATION
C
C
        GEN=0.0
        DO 77 M1=1,GENS
        PG(M1)=ALFA(M1)+BETA(M1)*IC(M1)+GAMMA(M1)*(IC(M1)**2)
        GEN=GEN+PG(M1)
   77   CONTINUE
        TP=GEN-(DEMAND+LOSSES)
        KT=KT+1
        IF((KT .EQ. 20) .OR. (KT. EQ.50)) DLMDA=DLMDA/10.0
        IF(TP) 78,780,79
   78   LMBDA=LMBDA+DLMDA
        GO TO 6666
   79   LMBDA=LMBDA-DLMDA
        IF(ABS(TP)  .GE.  EPS1)   GO TO 6666
  780   CONTINUE
        PRINT 965,TP
  965   FORMAT(2X,'TP = ',1PE11.4)
```

```
C
C
C     CHECK FOR CONVERGENCE IN THE GENERATED POWERS
C
C
      DO 72 M1=1,GENS
      PGDF=ABS(PG(M1)-PGOLD(M1))
      PGOLD(M1)=PG(M1)
      IF(PGDF-EPS2) 992,992,990
 992  CONTINUE
      IF (M1 .GE. GENS)  GO TO 991
 72   CONTINUE
 990  CONTINUE
      DO 888 M1=1,GENS
      P(M1)=PG(M1)-PD(M1)
      S(M1)=CMPLX(P(M1),Q(M1))
 888  CONTINUE
      CALL CST(A1,B1,C1,PG,GENS,COST,BASE)
      CALL MONIT(MU,PG,S,LMBDA,ITL,IC,GENS,NODE,COST)
      MU=MU+1
      IF (MU .GT. 10)  GO TO 991
      GO TO 999
 991  CONTINUE
      CALL FLOW(SFLOW,FROM,TO,BRANCH,NODE,YPRIM,V,YSHUNT,YC)
      STOP
      END
```

```
      SUBROUTINE MATINV(Z,Y,N,M2)
      COMPLEX Z,Y,B,D,E
      DIMENSION Z(N,N),Y(N,N),B(50,90)
      DO 10 I=1,N
      DO 10 J=1,N
10    B(I,J) =Y(I,J)
      M1=N+1
      DO 100 I=1,N
      DO 200 J=M1,M2
      M3=I+N
      IF(M3-J) 12,13,12
13    B(I,J)=(1.0,0.0)
      GO TO 200
12    B(I,J)=(0.0,0.0)
200   CONTINUE
100   CONTINUE
      DO 6 I=1,N
      D=B(I,I)
      DO 7 J=1,M2
7     B(I,J)=B(I,J)/D
      DO 8 L=1,N
      IF (L .EQ. I)  GO TO 8
      E=B(L,I)
      DO 9 J=1,M2
9     B(L,J)=B(L,J)-E*B(I,J)
8     CONTINUE
6     CONTINUE
      DO 17 I=1,N
      DO 17 J=M1,M2
17    Z(I,J-N)=B(I,J)
      RETURN
      END
```

```
      SUBROUTINE  MATPLY(R,A,B,N,M,L)
      DIMENSION R(N,L),A(N,M),B(M,L)
      COMPLEX R,A,B,SUM
      DO 1 K=1,N
      DO 2 I=1,L
      SUM=(0.0,0.0)
      DO 3 J=1,M
      SUM=SUM+A(K,J)*B(J,I)
3     CONTINUE
      R(K,I)=SUM
2     CONTINUE
1     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MONIT(MU,PG,S,LMBDA,ITL,IC,GENS,NODE,COST)
      COMPLEX S
      REAL LMBDA,ITL,IC
      INTEGER GENS
      DIMENSION PG(GENS),ITL(GENS),IC(GENS),S(NODE)
      PRINT 1,MU,LMBDA,COST
1     FORMAT(1H0,'ITERATION = ',I3,3X,'INCREMENTAL COST = ',1PE15.5,
     12X,'COST = ',1PE15.5)
      DO 2 I=1,GENS
      PRINT 3,I,PG(I),I,ITL(I),I,IC(I)
3     FORMAT(1H ,'PG(',I2,') = ',1PE15.5,3X,'ITL(',I2,') = ',1PE15.5,
     13X,'IC(',I2,') = ',1PE15.5)
2     CONTINUE
      DO 4 I=1,NODE
4     PRINT 5,I,S(I)
5     FORMAT(1H ,'S(',I2,') = ',1PE15.5,'+J',1PE15.5)
      RETURN
      END
```

```
      SUBROUTINE ADMIT(A,AT,YBUS,NODE,BRANCH,YPRIM,R)
      COMPLEX YBUS,YPRIM,A,AT,R
      INTEGER BRANCH
      DIMENSION YPRIM(BRANCH,BRANCH),YBUS(NODE,NODE),A(BRANCH,NODE),
     1AT(NODE,BRANCH),R(NODE,BRANCH)
      CALL MATPLY(R,AT,YPRIM,NODE,BRANCH,BRANCH)
      CALL MATPLY(YBUS,R,A,NODE,BRANCH,NODE)
      RETURN
      END
```

```
      SUBROUTINE ADMIT1(YBUS,NODE,YSHUNT,YC,YBUS0)
      COMPLEX YBUS,YSHUNT,YC,YBUS0,TEMP
      DIMENSION YBUS(NODE,NODE),YSHUNT(NODE,NODE),YC(NODE),
     1YBUS0(NODE,NODE)
      DO 1 I=1,NODE
      TEMP=(0.0,0.0)
      DO 2 J=1,NODE
      IF(I.EQ.J) GO TO 2
      TEMP=TEMP+YSHUNT(I,J)
      YBUS(I,J)=YBUS0(I,J)
2     CONTINUE
      YBUS(I,I)=YBUS0(I,I)+TEMP+YC(I)
1     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE Z2(ZBUS1,ZBUS2,C2,CT2,NODE,GENS,LOADS,R1,GN,GL)
      COMPLEX ZBUS1,ZBUS2,C2,CT2,R1
      INTEGER GENS,GN,GL
      DIMENSION ZBUS1(GL,GL),ZBUS2(GN,GN),C2(GL,GN),CT2(GN,GL),
     1R1(GN,GL)
      CALL MATPLY(R1,CT2,ZBUS1,GN,GL,GL)
      CALL MATPLY(ZBUS2,R1,C2,GN,GL,GN)
      RETURN
      END
```

```
      SUBROUTINE Z3(ZBUS2,ZBUS3,C3,CT3,GENS,GN,R2)
      COMPLEX ZBUS2,ZBUS3,C3,CT3,R2
      INTEGER GENS,GN
      DIMENSION ZBUS2(GN,GN),ZBUS3(GENS,GENS),C3(GN,GENS),CT3(GENS,GN),
     1R2(GENS,GN)
      CALL MATPLY(R2,CT3,ZBUS2,GENS,GN,GN)
      CALL MATPLY(ZBUS3,R2,C3,GENS,GN,GENS)
      RETURN
      END
```

```
      SUBROUTINE PARAM(C,D,ZBUS3,V,GENS,NODE,DELTA,R)
      COMPLEX ZBUS3,V
      INTEGER GENS
      DIMENSION C(GENS,GENS),D(GENS,GENS),ZBUS3(GENS,GENS),V(NODE),
     1DELTA(GENS),R(GENS,GENS)
      DO 1 J=1,GENS
      DO 2 K=1,GENS
2     R(J,K)=REAL(ZBUS3(J,K))
      DELTA(J)=ATAN(AIMAG(V(J))/REAL(V(J)))
1     CONTINUE
      DO 10 J=1,GENS
      DO 10 K=1,GENS
      T1=DELTA(J)-DELTA(K)
      T2=CABS(V(J))*CABS(V(K))
      C(J,K)=R(J,K)*COS(T1)/T2
      D(J,K)=R(J,K)*SIN(T1)/T2
10    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE   LFLOW(NODE,V,VMAG,QMAX,QMIN,S,YBUS,P,Q,TYPE1,TYPE2,
     1ACCEL,EPS,T1,T2,TYP2,OT1,OT2)
      COMPLEX YBUS,V,S,USE,TEMP,CONJG,CMPLX
      LOGICAL L,TYP2,OT1,OT2
      INTEGER TYPE2,TYPE1,T1,T2
      DIMENSION V(NODE),VMAG(NODE),QMAX(NODE),QMIN(NODE),S(NODE),
     1YBUS(NODE,NODE),P(NODE),Q(NODE),T1(TYPE1),T2(TYPE2)
      DIMENSION VP(10),VQ(10),ALFA(10),PSE(10),DEL(10),USE(10)
      ITER=1
      PI=4.0*ATAN(1.0)
      RAD=180.0/PI
   25 IBUS=1
      DO 180 I=1,NODE
  180 USE(I)=V(I)
      DVMAX=0.0
      IF (IBUS .EQ. 1) GO TO 401
   23 CONTINUE
      IF(OT1)  GO TO 2005
      IF(OT2)  GO TO 2000
      DO 240 I=1,TYPE2
      IF(IBUS .EQ. T2(I)) GO TO 2000
  240 CONTINUE
      DO 241 I=1,TYPE1
      IF(IBUS .EQ. T1(I)) GO TO 2005
  241 CONTINUE
 2000 CONTINUE
      L=.FALSE.
      CALL GEN(IBUS,V,NODE,QMAX,QMIN,S,YBUS,P,Q,L)
 2005 CONTINUE
C     A NEW VALUE OF VOLTAGE IS CALCULATED FOR ALL BUSES.
      TEMP=(0.0,0.0)
      DO 21 J=1,NODE
      IF (J - IBUS) 22,21,22
   22 TEMP=YBUS(IBUS,J)*USE(J)+TEMP
   21 CONTINUE
      V(IBUS)=(CONJG(S(IBUS))/CONJG(USE(IBUS))-TEMP)/YBUS(IBUS,IBUS)
      V(IBUS)=USE(IBUS)+ACCEL*(V(IBUS)-USE(IBUS))
      IF (L)  GO TO 232
      DO 231 I=1,TYPE2
      IF (IBUS  -   T2(I)) 232,233,232
  233 ALFA(IBUS)=ATAN(AIMAG(V(IBUS))/REAL(V(IBUS)))
      VP(IBUS)=VMAG(IBUS)*COS(ALFA(IBUS))
      VQ(IBUS)=VMAG(IBUS)*SIN(ALFA(IBUS))
      V(IBUS)=CMPLX(VP(IBUS),VQ(IBUS))
      GO TO 232
  231 CONTINUE
  232 CONTINUE
      DEL(IBUS)=CABS(V(IBUS)-USE(IBUS))
      USE(IBUS)=V(IBUS)
      IF (DEL(IBUS) - DVMAX)  401,400,400
  400 DVMAX=DEL(IBUS)
  401 IBUS=IBUS+1
      IF (IBUS - NODE) 500,500,501
  500 CONTINUE
      GO TO 23
  501 CONTINUE
      IF (DVMAX - EPS) 600,601,601
  601 CONTINUE
```

```
      ITER=ITER+1
      IF( ITER .GT. 5) GO TO 25
      PRINT 2,ITER,ACCEL
      PRINT 704,(V(I),I=1,NODE)
      PRINT 704,(S(I),I=1,NODE)
 704  FORMAT(6(2X,1PE11.4))
      GO TO 25
 600  CONTINUE
      DO 778 IBUS=1,NODE
      VMAG(IBUS)=CABS(V(IBUS))
      PSE(IBUS) =       RAD*ATAN(AIMAG(V(IBUS))/REAL(V(IBUS)))
 778  CONTINUE
C     COMPUTE THE SLACK GENERATOR POWER
      TEMP=(0.0,0.0)
      DO 26 J=1,NODE
      TEMP=YBUS(1,J)*V(J)+TEMP
 26   CONTINUE
      S(1)=V(1)*CONJG(TEMP)
      PRINT 1
 1    FORMAT(1H0,'LOAD FLOW RESULTS')
      PRINT 2,ITER,ACCEL
 2    FORMAT(1H0,2X,'ITERATION = ',I4,'   ACCELERATING FACTOR =',F5.2)
      DO 3 I=1,NODE
 3    PRINT 4,I,V(I),VMAG(I),PSE(I)
 4    FORMAT(1H ,2X,'V(',I2,')= ',1PE15.5,'+J',1PE15.5,'  = ',1PE15.5,
     1'EXP(',1PE11.4,')')
      PRINT 5,(I,S(I),I=1,NODE)
 5    FORMAT(2(2X,'S(',I2,') = ',1PE15.5,'+J',1PE15.5))
      RETURN
      END



      SUBROUTINE CST(A,B,C,PG,GENS,COST,BASE)
      INTEGER GENS
      DIMENSION A(GENS),B(GENS),C(GENS),PG(GENS)
      COST=0.0
      DO 1 I=1,GENS
 1    COST=COST+A(I)+B(I)*PG(I)*BASE+C(I)*PG(I)*PG(I)*BASE
      COST=COST+340.0
      RETURN
      END
```

```
      SUBROUTINE GEN(IBUS,V,NODE,QMAX,QMIN,S,YBUS,P,Q,L)
      COMPLEX TEMP,V,YBUS,S,CMPLX,CONJG
      LOGICAL L
      DIMENSION V(NODE),YBUS(NODE,NODE),S(NODE),P(NODE),Q(NODE),
     1QMAX(NODE),QMIN(NODE)
      TEMP=(0.0,0.0)
      DO 20 J=1,NODE
   20 TEMP=YBUS(IBUS,J)*V(J)+TEMP
      Q(IBUS)=AIMAG(V(IBUS)*CONJG(TEMP))
      IF(Q(IBUS) - QMAX(IBUS)) 3000,3000,3001
 3000 IF (Q(IBUS) - QMIN(IBUS)) 3003,3002,3002
 3001 Q(IBUS)=QMAX(IBUS)
      L=.TRUE.
      GO TO 3002
 3003 Q(IBUS)=QMIN(IBUS)
      L=.TRUE.
 3002 CONTINUE
      S(IBUS)=CMPLX(P(IBUS),Q(IBUS))
      RETURN
      END
```

```
      SUBROUTINE FLOW(SFLOW,FROM,TO,BRANCH,NODE,YPRIM,V,YSHUNT,YC)
      COMPLEX SFLOW,YPRIM,V,YSHUNT,YC,CONJG
      INTEGER FROM,TO,BRANCH
      DIMENSION SFLOW(NODE,NODE),FROM(BRANCH),TO(BRANCH),YPRIM(BRANCH,
     1BRANCH),YSHUNT(NODE,NODE),YC(NODE),V(NODE)
      PRINT 6005
 6005 FORMAT(1H0,'POWER FLOW BETWEEN BUSES'/)
      DO 1 I=1,BRANCH
      NR=FROM(I)
      NS=TO(I)
      SFLOW(NR,NS)=V(NR)*(CONJG(V(NR))-CONJG(V(NS)))*CONJG(YPRIM(I,I))
     1+V(NR)*CONJG(V(NR))*CONJG(YSHUNT(NR,NS))
      SFLOW(NS,NR)=V(NS)*(CONJG(V(NS))-CONJG(V(NR)))*CONJG(YPRIM(I,I))
     1+V(NS)*CONJG(V(NS))*CONJG(YSHUNT(NS,NR))
      PRINT 6007,NR,NS,SFLOW(NR,NS)
      PRINT 6007,NS,NR,SFLOW(NS,NR)
 6007 FORMAT(1H0,2X,'FROM NODE',I2,' TO NODE ',I2,2X,1PE15.5,'+J',
     11PE15.5)
    1 CONTINUE
      RETURN
      END
```

APPENDIX C

LISTING OF THE FORTRAN PROGRAM FOR AEP METHOD OF STATE ESTIMATION

```
C
C
C     AEP METHOD FOR STATE ESTIMATION
C
C
      DIMENSION D(3,3),X(3,1),Z(3),B(3,2),BT(2,3),XT(3,1),W(3,3),XN(2,1)
      DIMENSION HP(3,3),HQ(3,3),SMP(3,1),SMQ(3,1),HPCONJ(3,3)
      DIMENSION HQCONJ(3,3),HPINV(3,3),HQINV(3,3),RP3(3,1),RQ3(3,1)
      DIMENSION HPW(3,3),HQW(3,3),DP(3,3),DQ(3,3),VMP(3,1),VMQ(3,1)
      DIMENSION AKP(3,1),AKQ(3,1),DPVMP(3,1),DQVMQ(3,1),DIF(3,1)
      DIMENSION DIF1(2,1),DIF2(2,1),CP(2,1),CQ(2,1),DPU(3,1),DQU(3,1)
      DIMENSION DB(3,2),R6(2,2),R7(2,2),U(3,1)
      DIMENSION STP(3,1),STQ(3,1)
      COMPLEX HP,HQ,SMP,SMQ,HPCONJ,HQCONJ,HPINV,HQINV,RP3,RQ3,HPW,HQW
      COMPLEX DP,DQ,VMP,VMQ,AKP,AKQ,DPVMP,DQVMQ,DIF,DIF1,DIF2,CP,CQ
      COMPLEX DPU,DQU,DB,R6,R7
      COMPLEX XN2,XN3,DEL2,DEL3
      COMPLEX D,XN,XT,X,W,Z,U,B,BT
      COMPLEX STP,STQ,YJP
C
C     READ BUS VOLTAGES FROM THE LOAD FLOW SOLUTION
C
      TIME=0.0
      DO 52 I=1,3
      READ 53,(XT(I,J),J=1,1)
 53   FORMAT(2F15.8)
 52   CONTINUE
C
C     READ LINE FLOWS FROM THE LOAD FLOW SOLUTION
C
      DO 22 I=1,3
      READ 23,(STP(I,J),J=1,1)
 23   FORMAT(2F15.8)
 22   CONTINUE
      DO 127 I=1,3
      READ 23,(STQ(I,J),J=1,1)
 127  CONTINUE
C
C     READ LINE IMPEDANCES
C
      Z(1)=(0.021,0.0872)
      Z(2)=Z(1)
      Z(3)=Z(1)
C
C     READ THE WEIGHTING FACTOR MATRIX & OTHER GIVEN DATA
C
      DO 60 I=1,3
      DO 60 J=1,3
      READ 61,W(I,J)
 61   FORMAT(2F5.0)
 60   CONTINUE
      DO 10 I=1,3
      DO 10 J=1,2
      READ 11,B(I,J)
      BT(J,I)=B(I,J)
 11   FORMAT(2F5.0)
 10   CONTINUE
C
```

```
C     INITIAL ESTIMATES OF X
C
      PI=4.0*ATAN(1.0)
      RAD=180.0/PI
      RESMAX=2.E-4
      KOUNT=0
      X(1,1)=(1.0,0.0)
      X(2,1)=(1.0,0.0)
      X(3,1)=(1.0,0.0)
 88   CONTINUE
      PRINT 89, TIME
 89   FORMAT(1H0,'TIME OF DAY IN HOURS=',F5.0)
      PRINT 97
 97   FORMAT(1H0//5X,'BUS VOLTAGES FROM LOAD FLOW')
      PRINT 53,((XT(I,J),J=1,1),I=1,3)
      PRINT 98
 98   FORMAT(1H0//5X,'LINE FLOWS FROM LOAD FLOW')
      PRINT 53,((STP(I,J),J=1,1),I=1,3)
      PRINT 53,((STQ(I,J),J=1,1),I=1,3)
      X(2,1)=XT(2,1)
      X(3,1)=XT(3,1)
      YJP=(0.0,0.0226)
      AKP(1,1)=Z(1)*YJP*X(1,1)
      AKP(2,1)=Z(2)*YJP*X(1,1)
      AKP(3,1)=Z(3)*YJP*X(2,1)
      AKQ(1,1)=Z(1)*YJP*X(2,1)
      AKQ(2,1)=Z(2)*YJP*X(3,1)
      AKQ(3,1)=Z(3)*YJP*X(3,1)
      X(2,1)=CONJG(X(2,1))
      X(3,1)=CONJG(X(3,1))
C
C     INTRODUCE ERROR TO GET MEASURED LINE FLOWS
C
      DO 24 I=1,3
      STP(I,1)=STP(I,1)+0.1*STP(I,1)
      STQ(I,1)=STQ(I,1)+0.1*STQ(I,1)
      SMP(I,1)=CONJG(STP(I,1))
      SMQ(I,1)=CONJG(STQ(I,1))
 24   CONTINUE
C
C     CALCULATION OF D-MATRICES
C
      DO 26 I=1,3
      DO 26 J=1,3
      HP(I,J)=(0.0,0.0)
      HQ(I,J)=(0.0,0.0)
 26   CONTINUE
      HP(1,1)=X(1,1)/Z(1)
      HP(2,2)=X(1,1)/Z(2)
      HP(3,3)=X(2,1)/Z(3)
      HQ(1,1)=X(2,1)/Z(1)
      HQ(2,2)=X(3,1)/Z(2)
      HQ(3,3)=X(3,1)/Z(3)
      DO 80 I=1,3
      DO 80 J=1,3
      HPCONJ(I,J)=CONJG(HP(I,J))
      HQCONJ(I,J)=CONJG(HQ(I,J))
 80   CONTINUE
```

```
      N1=3
      M1=3
      L1=3
      CALL MATPLY(HPW,HP,W,N1,M1,L1)
      CALL MATPLY(DP,HPW,HPCONJ,N1,M1,L1)
      CALL MATPLY(HQW,HQ,W,N1,M1,L1)
      CALL MATPLY(DQ,HQW,HQCONJ,N1,M1,L1)
      DO 133 I=1,3
      DO 133 J=1,3
      D(I,J)=(0.0,0.0)
  133 CONTINUE
      D(1,1)=DP(1,1)+DQ(1,1)
      D(2,2)=DP(2,2)+DQ(2,2)
      D(3,3)=DP(3,3)+DQ(3,3)
      PRINT 201,((D(I,J),J=1,3),I=1,3)
  201 FORMAT(6(2X,F10.4))
C
C     CALCULATION OF ( B'DB )
C
      N1=3
      M1=3
      L1=2
      CALL MATPLY(DB,D,B,N1,M1,L1)
      N1=2
      M1=3
      L1=2
      CALL MATPLY(R6,BT,DB,N1,M1,L1)
C
C     INVERSE OF ( B'DB )
C
      LB=2*N1
      CALL MATINV(R7,R6,N1,LB)
C
C     OTHER MATRIX CALCULATIONS
C
      N1=3
      M1=3
      LB=2*N1
      CALL MATINV(HPINV,HP,N1,LB)
      CALL MATINV(HQINV,HQ,N1,LB)
      L1=1
      CALL MATPLY(RP3,HPINV,SMP,N1,M1,L1)
      CALL MATPLY(RQ3,HQINV,SMQ,N1,M1,L1)
      DO 130 I=1,3
      VMP(I,1)=RP3(I,1)-AKP(I,1)
      VMQ(I,1)=RQ3(I,1)-AKQ(I,1)
  130 CONTINUE
      N1=3
      M1=3
      L1=1
      CALL MATPLY(DPVMP,DP,VMP,N1,M1,L1)
      CALL MATPLY(DQVMQ,DQ,VMQ,N1,M1,L1)
      DO 131 I=1,3
      DIF(I,1)=DPVMP(I,1)-DQVMQ(I,1)
  131 CONTINUE
      N1=2
      M1=3
      L1=1
```

```
      CALL MATPLY(DIF1,BT,DIF,N1,M1,L1)
      U(1,1)=(1.0,0.0)
      U(2,1)=(1.0,0.0)
      U(3,1)=(0.0,0.0)
      N1=3
      M1=3
      L1=1
      CALL MATPLY(DPU,D,U,N1,M1,L1)
      N1=2
      M1=3
      L1=1
      CALL MATPLY(CP,BT,DPU,N1,M1,L1)
      DO 132 I=1,2
      DIF2(I,1)=DIF1(I,1)-CP(I,1)
  132 CONTINUE
C
C     ESTIMATE  OF  THE  STATE  VECTOR  X
C
      N1=2
      M1=2
      L1=1
      CALL MATPLY(XN,P7,DIF2,N1,M1,L1)
      PRINT 18
   18 FORMAT(1H0//5X,'THE  STATE  ESTIMATE')
      PRINT 25,((XN(I,J),J=1,1),I=1,2)
   25 FORMAT(2(2X,F10.6))
      X2=REAL(XN(1,1))
      Y2=AIMAG(XN(1,1))
      S2=(X2**2)+(Y2**2)
      VMAG2=SQRT(S2)
      PHAS2=RAD*ATAN(Y2/X2)
      X3=REAL(XN(2,1))
      Y3=AIMAG(XN(2,1))
      S3=(X3**2)+(Y3**2)
      VMAG3=SQRT(S3)
      PHAS3=RAD*ATAN(Y3/X3)
      PRINT 121,VMAG2,PHAS2,VMAG3,PHAS3
  121 FORMAT(1H ,5X,'V(2)=',F9.6,'EXP(',F10.6,')',5X,'V(3)=',F9.6,'EXP('
     *,F10.6,')')
C
C     CALCULATION FOR 24 HOURS OF THE DAY
C
      IF(TIME.EQ.23.0) GO TO 99
      DO 86 I=2,3
      READ 53,(XT(I,J),J=1,1)
   86 CONTINUE
      DO 87 I=1,3
      READ 53,(STP(I,J),J=1,1)
   87 CONTINUE
      DO 128 I=1,3
      READ 53,(STQ(I,J),J=1,1)
  128 CONTINUE
      TIME=TIME+1.0
      GO TO 88
   99 STOP
      END
```

```
      SUBROUTINE MATINV(Z,Y,N,M2)
      COMPLEX Z,Y,B,D,E
      DIMENSION Z(N,N),Y(N,N),B(50,90)
      DO 10 I=1,N
      DO 10 J=1,N
10    B(I,J) =Y(I,J)
      M1=N+1
      DO 100 I=1,N
      DO 200 J=M1,M2
      M3=I+N
      IF(M3-J) 12,13,12
13    B(I,J)=(1.0,0.0)
      GO TO 200
12    B(I,J)=(0.0,0.0)
200   CONTINUE
100   CONTINUE
      DO 6 I=1,N
      D=B(I,I)
      DO 7 J=1,M2
7     B(I,J)=B(I,J)/D
      DO 8 L=1,N
      IF (L .EQ. I)  GO TO 8
      E=B(L,I)
      DO 9 J=1,M2
9     B(L,J)=B(L,J)-E*B(I,J)
8     CONTINUE
6     CONTINUE
      DO 17 I=1,N
      DO 17 J=M1,M2
17    Z(I,J-N)=B(I,J)
      RETURN
      END
```

```
      SUBROUTINE  MATPLY(R,A,B,N,M,L)
      DIMENSION R(N,L),A(N,M),B(M,L)
      COMPLEX R,A,B,SUM
      DO 1 K=1,N
      DO 2 I=1,L
      SUM=(0.0,0.0)
      DO 3 J=1,M
      SUM=SUM+A(K,J)*B(J,I)
3     CONTINUE
      R(K,I)=SUM
2     CONTINUE
1     CONTINUE
      RETURN
      END
```

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