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Effects of High Construction Loads on the Short Term and
Long Term Deflections of Slabs

by

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A thesis
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ABSTRACT

When shore and reshore operations are used to construct multifloor flat slab type buildings, high transient construction loads are imposed on the supporting slabs in the system. These temporary live loads during construction due to shoring procedures have been shown in many cases to be in excess of design working loads and can exceed the calculated ultimate loads, leading to overstressing of the slabs, especially when the concrete has not attained its specified 28-day strength. When these construction loads are removed, some of the initial elastic deformations will not be recovered, and the higher than expected construction loading will result in a permanent increase of slab deflections.

Concrete creeps under sustained loads, which leads to a gradual increase of slab deflection under sustained load. The magnitude of creep depends largely on the age of concrete at loading and the dependence of creep on concrete age is very strongly marked at early ages; that is, the earlier the load is applied to concrete, the larger will be the magnitude of creep. Shrinkage of concrete will have a similar effect on the deformation as creep in that the differential shrinkage of reinforced concrete member

produces warping of the slab that can contribute as much as 30 per cent of the total long term deflection.

To evaluate the effects of both the intensity of construction loads and the age of concrete at loading on the short-term and long-term deflections of flat slab structures, a comprehensive laboratory study was undertaken to obtain the instantaneous and time-dependent deformations of three identical reinforced concrete flat plate structures and five one-way slabs. These eight slabs, each 63.5 mm thick, were made with normal weight concrete with no entrained air. Research results indicate that concrete loaded at an early age results in large creep deflection in addition to large immediate deflection because of the low modulus of elasticity of the premature concrete. The longterm deflection of the flat plates were as high as 5 times the initial elastic deflection after 700 days under sustained load, and 7 times for one-way slabs after 10 months.

The longterm deflections calculated by the ACI multiplier are shown to be inadequate as the multiplier does not take into account the effects of heavy construction loads applied on slabs soon after the placement of concrete. Two design methods are given to calculate the long term deflection of slabs in terms of the peak construction load relative to the slab strength.

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NOMENCLATURE

A_s	tension reinforcement
A_s'	compression reinforcement
b	section width
C	column width
C_u	ultimate creep coefficient
C_t	creep coefficient at time t
d	effective depth of section
D	flexural rigidity (EI) per unit width
e_g	eccentricity of equivalent tensile force with respect to gross uncracked section (Appendix M)
E	modulus of elasticity
E_c	modulus of elasticity of concrete
E_s	modulus of elasticity of steel
f_c'	specified 28-day concrete strength
f_{cbt}	tensile strength of concrete cube (Appendix O-1)
h	thickness
I	second moment of area
I_e	effective second moment of area
I_g	second moment of area of gross section
K	deflection coefficient
K'	reduction factor
K_w	boundary-condition coefficient

L span length
 L_n clear span
 M_{cr} cracking moment
 M_{max} maximum moment applied at a section
 M_1 equivalent end moment
 M_2 equivalent end moment
 M_3 equivalent mid-span moment
 Q uniform load
 t time in days
 W_s equivalent strip load
 X, Y orthogonal coordinates in plane of plate
 δ deflection normal to plane of plate
 σ stress
 ϵ strain
 ϵ_{shu} ultimate shrinkage strain
 ρ tensile reinforcement ratio (A_s/bd)
 ρ' compressive reinforcement ratio (A_s'/bd)
 ϕ curvature
 μ Poisson's ratio
 ξ creep factor (Eq. 4.1)

subscripts : c - due to creep
 cc - of concrete
 i - initial or elastic
 s - of steel
 sh - due to shrinkage
 t - value at time 't'

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To my parents

and to Elise, my better half

Chapter I

INTRODUCTION

1.1 FLAT PLATE SLABS

In the construction of multistorey concrete buildings, it is almost a universal practice to place concrete on forms which are supported from the next highest slab which in turn is reshored from lower slabs in the system. As a result of this construction procedure, the supporting slabs are subjected to considerable load during the period of construction, and it is possible that these construction loads on a slab system may be higher than the full service load for which the slabs were designed.

The imposition of load on young concrete can lead to undesirable levels of stress and deformation in the slabs. A short-term overload of such magnitude at an early age may cause extensive cracking that will be detrimental to the structure's subsequent performance. It is important that, the performance of a concrete floor be considered with reference to not only the service or design loads but also the load history during the construction of the building.

Concrete creeps under load, having the characteristic of deforming under steady external load. Creep of the concrete increases with time and magnitude of loading, and in many cases, creep continues until fracture occurs. Creep failure is characterized by excessive plastic deformation under sustained load. For structures that are to be subjected to early construction loads and relatively high service load over a long period of time, the long term structural response can be the principal concern.

1.2 CONSTRUCTION LOADS IN FLAT SLAB STRUCTURES

Design of reinforced concrete floor slabs must take into consideration both the design and the construction loads. The design loads are specified, but the construction loads are usually not well defined, and it is desirable to determine the variations in magnitude of these loads which occur during the construction period. Any analysis of the construction process must take into account the number of levels of formwork/shores and the number of levels of reshores. Figure 1.1 shows the loads carried by the floor slabs and shores for a typical construction cycle of three levels of shores and one floor poured per week [Grundy & Kabaila, 54]. The loads, to which the floor slabs and the shores are subjected, are expressed in multiples, called the floor loading ratio, of the floor self weight plus the formwork deadload.

In the design of reinforced concrete slabs, it is quite often that the slab self-weight or dead load is a large proportion of the total service or design load. If the design live load to dead load ratio of these particular building slabs is 0.5, the total design ultimate load, which is equal to 1.7 times the live load plus 1.4 times the dead load, becomes 2.25 times the dead load. There is a risk during the construction process that a concrete floor slab may be required to carry a design dead load of 1.4 times the slab weight, at a slab age of 5 days, when the available strength of the slab is only 60 per cent of its specified 28-day strength (Figure 1.2, compressive strength curve). That is, the available design ultimate strength of (0.6×2.25) or 1.35 times the dead load would be slightly less than 1.4 times dead load required to support one floor slab.

Overloading can also occur to the lowest slab in the supporting system during the erection of a slab building. For a flat slab building constructed with 3 levels of shores and a cycle time of 7 days, the applied load during construction reaches a maximum value of 2.36 times the dead load for floor level 3 (Figure 1.1), whereas the total design load capacity of the slab is only 2.25 times the slab dead load for slabs with a design live load to dead load ratio of 0.5.

Referring to Appendix A and for a design live/dead load ratio of 0.5, it can be deduced from Figure 1.2 (strength vs time) when concrete compressive strengths at 7, 14 and 21 days are 0.70, 0.88 and 0.96 of the 28-day design strength, that the sum of the available strengths of these three supporting slabs is 5.72 times the dead load, which is only slightly larger than the total applied load of (4.0×1.4) or 5.6 times the dead load,

1.3 IMMEDIATE AND LONG-TERM DEFLECTIONS OF FLAT SLABS

In the past, the deflection of reinforced concrete flat plate slabs has been of little interest to practicing engineers because low service loads produced relatively small deflections. The situation would be entirely different if during the construction process the slabs are subjected to temporary construction loads which are greater than the permanent design loads.

As the concrete has attained only low strength when this transient construction load is applied, the ratio of the stress applied to the strength available is high, resulting in large immediate deflections because of the low modulus of elasticity of the concrete. This temporary overload can also result in cracks which in turn reduce the stiffness of the slabs. The deflection caused by these early loads are probably irrecoverable and thus, the

deflection of slabs can be excessive in the longterm due to the effects of creep of concrete. These excessive deflections create the following common problems.

- noticeable sagging of floors
- cracking of partitions and walls built on the slabs
- improperly fitting architectural components and services

A survey by Mayer and Rush [77] indicated that the most frequent cause of damage in reinforced concrete structures was excessive deflection, which affected the structural serviceability and structural integrity.

1.4 SCOPE OF STUDY

The main object of this research was to determine the effects of high early age loads on the short- and long-term deflections of reinforced concrete flat slabs. The effects of the modulus of elasticity of concrete at various ages for various stress histories, creep and shrinkage of concrete on the slab deformation were also examined.

Different shoring schedules can change the load history to which a slab is subjected. To determine the effect of loading histories on the deflections of flat slabs, a total of eight one-third scale slab models were subjected to

different load histories corresponding to different construction methods. In the course of this test program, the time-dependent behaviour of three reinforced concrete flat-plate slabs and five one-way slabs has been followed, from construction, through the removal of the formwork and the application of temporary and sustained loads, over a continuous period of 10 to 23 months.

A summary of the literature review and experimental study on the effects of construction loads on the deflections of flat slabs is presented in this report. After an introductory chapter, chapter 2 treats the analysis of construction process and compares the construction loads imposed on slab buildings due to different construction sequences. In chapter 3, the relevant analytic approaches and the finite element method for calculating the elastic deflections of reinforced concrete two-way slab systems are presented. Chapter 4 describes the general development of the analysis of creep and shrinkage characteristics of concrete and goes on to deal with their contributions to the longterm deflections of reinforced concrete slabs. Chapter 5 is devoted to the literature review, both analytical and experimental, of longterm deflections of reinforced concrete slab. Chapter 6 gives details of the present experimental study on one-third scale model slabs. Chapter 7 presents and discusses the test results. Based on these model studies together with the test results from the literature,

conclusions and recommendations are given in chapter 8 on the effects of early loading on the deflections of reinforced concrete slabs.

Chapter II

CONSTRUCTION LOADS ON FLAT SLAB STRUCTURES

The basic object of the design of a reinforced concrete structure is to ensure that the loads imposed on it are carried safely not only during service but also during the erection of the structure. Uncertainty about the magnitude of the loads imposed on floor slabs during the period of construction has always concerned designers, especially with recent trends towards shallower members and longer spans.

Mayer and Rush in 1967 [77] noted that, more buildings collapsed during construction than under service loads. { As an example, on March 2, 1972, portions of the Skyline Plaza Apartment building under construction in Fairfax county, Va., collapsed, resulting in 14 deaths and 34 injured. In a recently published review of this catastrophe, Carino et al [27] concluded that the collapse was due to:

- premature removal of shores,
- low strength of the unshored concrete floors, and
- colder than anticipated ambient temperature, which reduced the rate of strength gain of the concrete.

Attention has been given in the last three decades to estimating the load history during construction. Several methods for calculating the loads to which concrete floor slabs are submitted in the course of construction will be briefly reviewed in this chapter.

2.1 NIELSEN, 1952

In 1952, Nielsen [88] presented a detailed analysis for determining the loads applied to concrete floors during construction. Nielsen's investigation consisted of both theoretical and field studies dealing with the load distribution between the floor slabs and the formwork. The construction loads carried by the slabs and shores were expressed as load ratios, which were factors by which the self weight of slab and the formwork dead load had to be multiplied.

This analysis considered the compatibility of deformations between the elastic shores and the slabs by means of influence surfaces. The major assumptions made included,

1. shores could be represented by a uniform elastic support,
2. compression in the shores were taken as the reactions at the elastic supports,
3. effects of shrinkage and creep of concrete and shores were neglected, and

4. the modulus of elasticity of concrete increased with time.

This detailed analysis provided a basis for determining the imposed loads and deformations in connected slabs and the supporting shores. This early work led to a completely different approach for the design and construction of multistory concrete buildings. Due to the unreasonably lengthy calculations involved in the analysis, its application in practice is limited.

2.2 GRUNDY AND KABAILA, 1963

With a few simplifying assumptions, Grundy and Kabaila [54] developed a simple analytical method for determining the loads imposed on floor slabs and shores through the construction sequence of a flat slab concrete building. The three major assumptions were :

1. shores are infinitely rigid by comparison with the slabs
2. loads carried by the shores are assumed to be uniformly distributed to the slab floor supporting these shores and,
3. load applied to the shoring system is distributed between the slabs in proportion to their relative flexural stiffnesses.

The first assumption enables the loads imposed on the system, either by placing of a new slab or by the removal of props, to be distributed between the supporting slabs, in direct proportion to their flexural stiffnesses. Analytical results of this approach indicated that the errors by introducing constant flexural stiffness for all connecting slabs or taking into account the increasing slab flexural stiffness with age were not appreciable. The flexural stiffness of an uncracked section is only slightly affected by the amount of reinforcement and may be taken to be directly proportional to the elastic modulus of concrete, E_c .

Calculated by the Grundy and Kabaila method, the loads carried by the shores and the slabs with three levels of shores, constant elastic modulus for all floors and a construction cycle of 7 day are shown in Figure 2.1. The absolute maximum load ratio calculated is 2.36 for a slab at level 3 at 3 cycles, and a converged load ratio of 2 for a typical floor.

In the discussion on the effects of the number of shoring floors on construction loads, Grundy and Kalaiba showed that increasing the number of shoring levels increased the absolute maximum load ratio as well as the age of the slab at which this maximum load is applied. For 2, 3 and 4 levels of shores, the absolute maximum load ratios

were 2.25 at 2 cycles, 2.36 at 3 cycles and 2.43 at 4 cycles respectively. While the age of application of the maximum load on a particular floor slab may vary, there is virtually no change in the converged construction loads on any typical floor slab whether 2, 3 or 4 floor levels were shored.

Grundy and Kabaila's simplified method of analysis provides a simple and straight forward approach to the calculation of construction loads for shored construction.

2.3 TAYLOR, 1967

The formwork stripping program recommended by Taylor [114] in 1967 employed the technique of slackening and tightening adjustable shores under a floor slab before placing concrete to the ~~top~~ level slab. Application of this technique reduces the shoring loads on the bottom slabs but increases the loads on the upper slabs. However, it has the disadvantage that continuous and cautious supervision is needed.

2.4 MAROSSZEKY 1972

Marosszeky [73] presented an analytical concept of reshoring of floors during construction. A similar analysis was also developed independently by Agarwal and Gardner [9] at the same time. The method consists of completely releasing a floor and immediately reshoring it. A floor is

allowed to carry its self weight at time T, where T is the construction time per floor. The two basic operations involved are replacing the lowest-level shores with reshores and removing the bottom level of reshores before the top level floor is poured. This method of reshoring is commonly used in the construction of multistory flat slab buildings.

Constructors using such a system have found to be considerably less severe than those obtained using shores and no reshores. Figure 2.2 gives the loads in a slab structure for a construction sequence using one level of shore and two levels of reshores. It is clear that the imposed loads on the slabs during the construction procedure do not exceed 1.34 D.L. but are applied to the floor slab at a slab age of one cycle.

2.5 AGARWAL AND GARDNER, 1974

Field measurements of loads imposed on shores and reshores were reported by Agarwal and Gardner [9] from two highrise flat slab buildings in 1974, to determine the actual load ratios to which the slabs and shores/reshores were subjected during the various construction stages. The field measurements agreed closely with the calculated maximum shore loads and confirmed that, the lower supporting slabs in a shore-reshore construction schedule can be subjected to loads greater than the design load of the structure.

Design tables [9,48] were given to determine the permissible shoring schedules for various live to dead load ratios, rates of construction, types of cement and ambient temperatures. As reshoring is effective in reducing the maximum loads experienced by the supporting slabs, and that the greater the number of shores, the greater the construction loads applied to the slabs, one level of forms plus reshores was recommended by Agarwal and Gardner.

2.6 CANTOR AND RIZZI, 1982

An alternative approach, the reshore-preshore procedures, was proposed by Cantor and Rizzi in 1982 [26]. Basically, preshoring employs the technique of installing a shore, which is not removed in the stripping process so that, the maximum span of unsupported concrete in both directions is never greater than 2.5 to 3 m. (8 to 10 ft.) According to Cantor and Rizzi, no form shall be stripped unless the concrete strength is greater than 65 % of the specified 28-day strength and that the permanent shore or the preshore can be removed when all reshores are placed. Figure 2.3 shows a construction schedule using wood shoring with permanent shore or preshore for a particular floor slab.

2.7 CHAN, 1984

Chan [32] investigated the load histories due to preshore construction for various one-way slab systems. Incorporating simplifying assumptions and using the STRUDL computer program, Chan carried out a two-dimensional analysis to estimate the load distribution pattern in the slab-form-reshore system during construction, in which beams of unit width were used to model concrete floors. The shore and form system considered in Chan's analysis was a 'flying form system' as shown in Figure 2.4B. Preshoring practice was adopted in the analysis, and the computing procedures included ;

1. determination of reactions of the shores (X and Z) and deflections of the floors in operation A (Figure 2.4A-ii)
2. determination of reactions of the shores and deflections of the floors in operation B (Figure 2.4A-iii), with an additional point load of the same magnitude but opposite direction as the reaction of shore Y in operation A, and
3. determination of moments and deflections at the end of the preshoring operation (Figure 2.4A-iv) as a combination of results from 1 and 2.

Preshore analysis demands knowledge of the slab flexural stiffness, form, shore and reshore stiffness and

their locations and the reshoring sequence. The load distributions of the third-floor slab by means of Grundy and Kabaila's shoring system, Marosszeky's reshoring system and Chan's preshoring system are shown in Figure 2.5. It can be noted from Figure 2.5 that Chan's results are very close to those of shore and reshore system. Chan's calculations of the preshore technique lie between those results by shoring-only system and shore-reshore system. Nevertheless, due to the high technical complexity in the preshoring system and the high loads applied to flat plate slabs in Grundy's shoring system, it is thus easy enough to understand why shore-reshore system is the most widely accepted practice in multistory flat plate building construction.

2.8 SUMMARY

The floor loading ratios computed by various authors and those measured by Agarwal & Gardner [9] and Lasisi & Ng [70] using shores instrumented with electrical resistance strain gages are summarized in Table 2.1. Both the computed and measured values verify that the floor loading ratios during construction can exceed two. None of the literature in Table 2.1 has been published in the last few years and the greater use of the reshoring technique results in smaller slab loads during construction.

9

The loads on the floor slabs and supporting shores and reshores for various shore/reshore combinations can be calculated by the accepted methods of mechanics provided the calculations include load history effects. The field measured shore loads show good agreement with those calculated. Most of the published field measured shore loads used shoring only and date from the early 70's and indicated slab construction loads approximately twice slab dead load. The recent use of reshoring has reduced construction loads but little confirmatory field experimental evidence is available.

Chapter III

DEFLECTION OF REINFORCED CONCRETE TWO-WAY SLAB SYSTEMS

3.1 GENERAL

Calculating the elastic deflections of a two-way slab system necessitates solution of the biharmonic equation,

$$\frac{\partial^4 W}{\partial^4 X} + \frac{2 \partial^4 W}{\partial X^2 \partial Y^2} + \frac{\partial^4 W}{\partial^4 Y} = \frac{Q(X,Y)}{D} \quad [3.1]$$

and the general solution to Equation 3.1 is of the form;

$$W = \frac{K Q L^4}{D} \quad [3.2]$$

W = deflection normal to plane of plate

X, Y = orthogonal coordinates in plane of plate

$Q(X, Y)$ = load at any point (X, Y) on plane of plate

D = flexural rigidity (EI) per unit width

$$= E h^3 / (12 (1 - \mu^2))$$

E = Young's modulus

h = plate thickness

L = panel span

μ = Poisson's ratio

K = deflection coefficient which varies with different boundary conditions

It is extremely difficult to develop a mathematical expression which satisfies Equation 3.1 at every point (X,Y), and which also satisfies the actual boundary conditions, especially when the stiffening effect of columns, column capitals, cracking of the concrete, inhomogeneity of concrete, anisotropy of local reinforcement, continuity of the slab systems and the aspect ratios must be taken into consideration in the derivation of such an expression. Instead of solving Equation 3.1 for the exact solutions, a number of simple approximate methods based on idealizations and simplifications of the true problem, have been suggested by many investigators.

3.2 THEORY OF PLATES

Timoshenko and Woinowsky-Krieger[122] presented in tabular form the elastic solutions of the deflection coefficients for ideal boundary conditions and full panel loading cases. For a plate of isotropic material under uniform loading, K in Eq.3.2 changes from 0.0263 for a single isolated panel to 0.00581 for an interior panel without checkerboard loading. Since concrete is not isotropic, and that the torsional moments in the slab were neglected in the analysis, Timoshenko's solution may be used only with an arbitrary reduction on the value of modulus of elasticity for the material.

3.3 DIAGONAL BEAM STRIP ANALOGY

Based on the assumption of a continuous strip of uniform width extending between diagonal columns, Sneddon and Hill [111] presented a simple method for the calculation of deflections for two-way slabs. Values of the deflection coefficients were found to be between 0.00531 for square panels and 0.0028 for rectangular panels with an aspect ratio of 1.5. This procedure did not take into account the torsional moments in the slab and was similar to the diagonal beam analogy adopted later by Portland Cement Association [PCA, 97] and used by Concrete Reinforcing Steel Institute [CRSI, 36].

In 1965, based on the diagonal beam strip analogy, the Portland Cement Association [97] suggested certain values of the deflection coefficients, for interior flat plates, ranging from 0.00528 for square panels to 0.0266 for rectangular panels with aspect ratio of 2.0. A value of 0.00528 for K is obviously a lower bound value, simply because of the use of ideal end restraint conditions in the analysis. These values of K were also used by CRSI in the preparation of the CRSI handbook [36].

A variation of the diagonal beam analogy has been used in this study for calculating the deflection of an isolated plate. In this method of analysis, the deflection at the centre of an uniformly loaded plate is estimated by consid-

ering a fixed-end beam of unit width with a span equal to the diagonal length of the plate. The intensities of the uniform distributed loads on the slabs and on the diagonal beam of unit width are equal. The deflection at the centre of the plate can then be easily calculated.

3.4 FINITE DIFFERENCE METHOD

Finite difference equations were used by Vanderbilt et al [124,125] to determine the values of K for linearly elastic, homogeneous, continuous plates with specified loadings and boundary conditions. Solutions were obtained on nine square panels, arranged three-by-three, for uniform load as well as pattern loading. The deflection coefficients for an uniformly loaded interior square flat plate was estimated to be 0.00441 for $C/L=0.1$ (C/L = ratio of column width to span length). To compare with the value of $K=0.00581$ given by Timoshenko and Woinowsky-Kriegers, i.e., for $C/L=0.0$, K was found to be 0.00581 in this case.

3.5 EQUIVALENT FRAME METHOD AND ADOSS

Nilson and Walters [89] and Kripanarayanan and Branson [68] presented an approximate method for the calculation of deflection coefficients of two-way slabs, using the concepts and terminology of the equivalent frame analysis given in the 1971 ACI code [3].

In this approach, the mid-span deflection of an equivalent frame was considered as the sum of that of the panel assumed to be fixed at both ends, and that due to the known rotation at each of the two support lines (Appendix B). This method also considers the deformation of a typical panel in one direction at a time, and the total deformation at any point being equal to the sum of the contributions in each direction.

This equivalent frame method of analysis was also used by the Canadian Portland Cement Association [25] in the preparation of the computer program ADOSS (Analysis and Design of Slab Systems). ADOSS is a standard computer package used to analyse and design slabs and can handle five different slab structures.

3.6 MODIFIED FRAME ANALYSIS OR THE S-METHOD

This is a simplified two-dimensional approximate method presented by Vanderbilt, Sozen and Seiss[124,125], for continuous and discontinuous plates on columns. By considering the lines of contraflexure in the slab at approximately one fifth span from the centerlines of the supporting columns as shown in Appendix C. The real structure is assumed to consist of column strips ($0.4 L_2$ wide and L_1 long slab beams) and plate elements ($0.6L_1 \times 0.6 L_2$), and the deflection at the centre of a typical span is made up of three parts,

1. the deflection of the centre of the uniformly loaded longer column strip ($0.4 L_2$ and L_1 long) with respect to its supporting columns (fixed-end beam problem). The loaded area includes the column strip and the additional triangular or trapezoidal portion contributed by the adjacent slab panels.
2. the deflection at the edge of the above strip with respect to its centre. This is a cantilever unit-width beam case. The beam is loaded with both an uniform distributed load over its length ($0.2L_2$) and a concentrated load of $0.3 \cdot L_2 \cdot Q$ at the free end of the cantilever beam where Q is the load per unit length, and
3. the deflection of the centre of the uniformly loaded simply supported (assumed) interior panel which is $0.6L_1$ by $0.6L_2$ in size.

L_1 and L_2 represent the long and short spans respectively. This method is very flexible in the way that it permits loss of stiffness due to the cracking of concrete, discontinuity at slab boundaries and irregular column grids. Comparison between calculated and measured deflections of the test structures in a subsequent chapter clearly indicates that the S-method gives the best approximation to the measured initial deflection of the several methods considered.

3.7 BEAM DEFLECTION FORMULAS

Using the design moments obtained from the equivalent frame method or the direct design method of the ACI code, Scanlon and Murray [106] proposed a hand-calculation procedure which is based on standard beam deflection formulas (Appendix D). By considering each strip as a beam under uniform load, W_s , and end moments (M_1 and M_2), the deflection at midspan of a column or middle strip of a two-way slab can be computed from the expression,

$$\delta = \frac{5 W_s L_n^4}{384 E I_e} - \frac{(M_1 + M_2) L_n^2}{16 E I_e} \quad [3.3]$$

W_s = equivalent strip load from

$$W_s L_n^2 / 8 = (M_1 + M_2) / 2 + M_3 = M_o \text{ (strip)}$$

L_n = clear span of member

The first term simply represents the deflection for a uniformly loaded simply supported beam, whilst the second term takes into consideration the fixed end constraints.

The effective modulus of elasticity approach as outlined in ACI Building Code [4], i.e. Equation 3.4 in section 3.9, was employed to include the effect of flexural cracks. To account for the tensile stress induced in a slab due to restrained shrinkage, a reduced value for the modulus of rupture of $4\sqrt{f_c'}$ was suggested to replace $7.5\sqrt{f_c'}$ as recommended by ACI, where f_c' is in p.s.i.

3.8 FINITE ELEMENT METHOD

The finite element method provides a powerful method for computation of plate deflections. The effect of time and cracking of concrete can be included in the analysis to determine the time-dependent deflections of two-way slabs.

ICES STRUDL-II[113], one of the most widely accepted computer programs in the field of structural engineering, was used in the present study to evaluate the initial deflections of the three flat plates. Plate bending square elements as shown in Figure 3.1A were used.

A finite element tangent stiffness formulation, coupled with a time step integration, was used by Van Greunen [126] to analyze reinforced and prestressed slab and panel structures, including the time-dependent effects due to load history, temperature history, creep, shrinkage and aging of concrete and the relaxation of stress in the prestressing steel. A computer program (NOPARC -Nonlinear Analysis of Prestressed and Reinforced Concrete Slabs and Panels) was developed to carry out this analysis. NOPARC can be used to trace the load-deformation response of thin reinforced and prestressed concrete structures under short term or sustained loads, through the elastic, inelastic and up to the ultimate ranges.

Concrete was represented by nonlinear constitutive relationship which incorporates tensile cracking at a limiting stress, tensile stiffening between cracks and the strain-softening phenomenon beyond the maximum compressive strength. The steel reinforcement was represented by a bilinear strain hardening model approximating the Bauschinger effect, and the reinforced concrete section was modelled as a layered system of concrete and "equivalent smeared" steel layers. Shear failure, bond slip, dowel action of reinforcement and aggregate interlock at cracks were not considered in the computer program.

Multi-layered triangular shell elements with fifteen degrees of freedom are used in the computer program with the finite elements discretization for quarter of the flat plate model shown in Figure 3.1B. A summary of the shell element and the time dependent analysis is given in Appendix E.

3.9 CRACKING OF CONCRETE

Prediction of the deflection of reinforced concrete two-way slabs is complicated due to the effect of cracking of concrete. The effect of concrete cracking is generally included in the calculations of second moment of area by using I_e (effective second moment of area) instead of I_g (second moment of area of gross uncracked section), and ACI Building Code [3,4,5] has proposed the following expression to determine an applicable value of I_e ,

$$I_e = \left(\frac{M_{cr}}{M_{max}} \right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right) I_{cr} \quad [3.4]$$

$\leq I_g$

M_{cr} = cracking moment

M_{max} = maximum bending moment in the span

I_g = second moment of area of the gross uncracked concrete section (neglecting reinforcement)

I_{cr} = second moment of area of the cracked transformed section

In the determination of slab deflections, the effective second moment of area can be taken as the average value of the positive and negative moment region values in the slab. Equation 3.4 was derived on the basis of test results of simple span beams, and was permitted for two-way slabs under the 1977 and 1983 versions of ACI Building Code, ACI-318[4,5].

In a state-of-the-art paper on deflection by ACI Committee 435 in 1974 [7], it was stated that application of any of the above mentioned methods would in general underestimate the deflection of two-way slabs.

Chapter IV

TIME DEPENDENT CONCRETE PROPERTIES

4.1 TIME-DEPENDENT DEFLECTIONS

The deflection of a reinforced concrete beam or two-way slab can be considered to consist of three components, namely, the initial or elastic deflection, the shrinkage deflection, and the creep deflection, with the last two components usually being referred to as the additional longterm or plastic deflection. Using the literature reviewed in the previous chapter, the instantaneous deflection can be predicted by using the modulus of elasticity corresponding to the age of concrete.

It has been the practice to estimate the additional deflections using empirically determined multiplying factors. ACI Committee 318 [4,5] recommended the use of Equation 4.1 to determine the multiplier which predicts the additional longterm deflection for both beams and two-way slab systems.

$$2 - 1.2 \frac{A_s'}{A_s} > 0.6 \quad \text{ACI-318-77 (4)}$$

[4.1]

$$\frac{\xi}{1 + 50 \rho'} > 0.6 \quad \text{ACI-318-83 (5)}$$

where ρ' , the compression steel ratio, shall be the value at midspan for simple and continuous span, and the time dependent factor ξ for sustained load be taken equal to 2, 1.4, 1.2 or 1.0 for 5 years or more, 1 year, 6 months or 3 months.

The final deflection would then be the sum of the initial deflection plus the additional deflection due to creep and shrinkage which is now equal to the initial deflection times the multiplying factor. The multiplier given in Equation 4.1 reflects the influence of compression reinforcement in reducing the additional longterm deflections:

Deflection multiplying factors provide a simple and convenient means of estimating additional deflections, however, these multipliers are different from country to country although similar materials are used. As compression steel is seldom provided in slabs, Equation 4.1 gives a maximum value of 2.0 for $\rho' = 0$. It has been shown in many cases [40,63,103,118,119] that, the final deflection could be as high as 6 to 12 times the initial deflection, resulting in unsatisfactory service load performance. Scanlan and Murray [105] suggested that calculating creep and shrinkage effects separately using the ACI 209 recommended procedures [2] would provide a more accurate prediction than the single multiplier method of the ACI code [4,5].

4.2 CREEP AND SHRINKAGE

Creep is dependent on the magnitude of the applied stress whilst shrinkage is independent of the stress in the concrete. Nevertheless, creep and shrinkage are interrelated in that both are affected by the same principal variables. These variables are [84] :

- age of concrete at initial loading
- environmental conditions. : curing conditions, relative humidity, and ambient temperature; for example, creep is about twice as large for 50% relative humidity as for 100% relative humidity.
- thickness and size of member, lower effective thickness (equal to twice the volume divided by the exposed surface area) gives higher shrinkage and creep.
- slump of concrete, a mix with lower water/cement ratio results in lower shrinkage and creep.
- cement content, a concrete mix with lower cement content at the same strength exhibits lower shrinkage and creep.
- % fines, larger aggregates permits the use of a leaner mix and thus results in lower shrinkage and creep.
- air content, creep is larger for higher air content.

4.2.1 Creep

It is well known that creep is the property of concrete by which it continues to deform with time under sustained loads, but the reason why concrete creeps has not yet been fully determined. In 1958, Freudenthal and Roll [46] divided the strains in concrete under sustained compression into three components. These were the instantaneous recoverable, time-dependent recoverable and the time-dependent irrecoverable strains, defined as the elastic, delayed elastic and flow respectively. Freudenthal and Roll proposed a rheological model for concrete with five elements in series representing five strain mechanisms,

1. elastic deformation
2. delayed elastic deformation due to the interaction between solid and fluid phases
3. irrecoverable viscous creep
4. consolidation due to irrecoverable seepage, and
5. deformation due to microcracking (local fracture)

The last four items were referred to as creep deformation. As is illustrated in Figure 4.1 that after the stress is removed, a portion of the creep strain or deformation will remain even after a long period of time.

Discussions on a number of hypotheses, based on certain possible mechanisms of deformation, were given by Kesler and

Ali in 1964 [67] and later in more detail by Neville in 1970 [85]. As Neville pointed out, on a phenomenological basis, at least six broad mechanisms of creep could be distinguished as follows,

- Mechanical Deformation theory : Reoverable delayed elastic deformation due to the wetness of the concrete under pressure and the irrecoverable deformation due to the closing of voids in the hydrated cement paste.

- Viscous Flow : hydrated cement paste is treated as a highly viscous fluid, and there is a sliding between gel particles which is largely irrecoverable.

- Plastic Flow : plastic deformation as a result of slip between the planes of maximum shear in the crystal lattice. Volume changes were involved in viscous flow but not in plastic flow.

- Seepage of Gel Water : a viscous flow of the dispersion medium (internal seepage). External applied stress changes the internal vapour pressure equilibrium and hence gel water content. The amount of decrease in volume depends on the relative amount of unfilled space.

- Delayed Elasticity : superimposition of stresses does not produce a superimposition of strains in that a non-linear stress-strain relationship applies to concrete.

- microcracking : part of the deformation due to creep is due to local fracture which is irrecoverable.

Kesler and Ali and Neville suggested, none of these available mechanisms is capable of explaining the major aspects of observed creep behavior. Incorporating the seepage mechanism and the visco-elastic behavior of the cement gel, Kesler and Ali[67] proposed a hypothesis on the basis that the time dependent deformations under load are mainly due to the imperfectly crystalline colloidal components of hydrated cement and associated absorbed moisture. In a discussion of this paper, Powers [98] stressed that shrinkage and creep are not fundamentally different; shrinkage is caused by triaxial tension in the water which pulls absorbed water from load-bearing areas, while creep is caused by external force, pushing absorbed water out of these areas. When these two forces act together, the resultant strains can be complicated by the triaxial forces due to drying and uniaxial force due to load. In addition, Powers concluded that it would be unrealistic to consider shrinkage and creep as independent phenomena simply because the resultant strains are caused by loading and drying at the same time.

Neville [84] stated that it is possible that the actual creep of concrete involves more than one mechanism and likely that its behaviour is a combination of seepage and

viscous shear theories. A complete description of other 'integrated' creep theories, which were based on more than one possible mechanism of deformation, can be referred to in the papers presented by Neville [84], Neville and Meyers [87], Cinlar, Bazant and Osman [34], and Glucklich and Ishai [52].

In the application of rate theory to the time-dependent deformations of concrete, Wittman and Lukas in 1974[134] concluded that the total deformation due to sustained load is dependent only upon the internal stress field which is caused by inhomogeneous moisture distribution and the stress field induced by external load.

In view of the conclusions made in various publications [34,46,52,67,84,87,98], it is apparent that creep is proportional to stress under sustained stresses less than $0.5f_c'$, and that creep is due to the combination of the following:

- closing of internal voids
- viscous flow of the cement-water paste
- crystalline flow in the aggregates
- flow of water out of the cement gel due to drying and external load

4.2.1.1 Creep Strain

The stress-strain relation for concrete is a function of time. As illustrated in Figure 4.1, strain increases with time under load from an initial strain ϵ_i gradually increasing to $(\epsilon_i + \epsilon_c)$. Under a sustained stress, the net strain could be many times as large as the initial strain and thus the creep induced deflection should be of considerable importance in the design of flat slab structure. Creep of concrete does not affect strength but increases the strain to approach possibly the limiting strain at which failure can occur.

4.2.1.2 Creep Coefficient

In an extensive study by Muller and Hilsdorf[82], involving the comparison of six prediction methods developed by national and international organizations for creep coefficients of concrete with experimental data, 102 tests were carried out and evaluated. Table 4.1 summarizes the parameters taken into account in various methods.

The experimental parameters included: age at loading of 1 to 3300 days, relative humidity of 45 to 100 per cent, specimen size of 70 to 610 mm, concrete strength of 17 MPa to 55 MPa, slow, normal and rapid hardening portland cements etc. Muller and Hilsdorf concluded that since the errors in the prediction of creep functions were large, no matter

which method was applied, lower and upper bound creep functions should be considered and evaluated for creep sensitive structures.

Different models for the prediction of creep coefficients were suggested by the Concrete Society [37,95] and CEB-FIP-70 [29] and are reproduced in Appendixes F and G. According to ACI committee 209 [2,19], the creep coefficient C_t at any time t can be approximated by, (Appendix H)

$$C_t = \frac{t^{0.6}}{10 + t^{0.6}} C_u \quad [4.2]$$

Equation 4.2 is for the standard conditions only, i.e., a relative humidity of 40 percent, a member thickness of at least 6 in., a slump of 3 in. and concrete loaded at 7 days etc. Correction factors are given in Appendix H for non-standard conditions. C_u is the ultimate creep coefficient and is defined as the ratio of creep to initial strain. In the absence of specific creep data for local conditions, C_u , varying from 1.3 to 4.25, can be taken as 2.35.

4.2.2 Shrinkage

The shrinkage of concrete is a phenomenon in which concrete members decrease in dimension due to the withdrawal of water. Shrinkage strain is defined as the ratio of the

total shortening of the member to its original length, and the free shrinkage strain of plain concrete has maximum values in the range between 400 and 1100 millionths for type I cements and commonly used aggregates and proportions depending on ambient temperatures and relative humidities.

ACI committee 209 [2,19], CEB-FIP-70 [29] and the Concrete Society [59] have proposed empirical methods to predict shrinkage behaviour. The models given by CEB-FIP-70 and the Concrete Society are given in Appendixes I and J respectively. According to ACI committee 209 [2], shrinkage after age 7 days for moist cured concrete can be expressed by (Appendix K),

$$\epsilon_{sh} = \frac{t}{t + 35} \epsilon_{shu} \quad [4.3]$$

Equation 4.3 again is valid only for the standard conditions mentioned earlier for the standard creep equation. ϵ_{shu} is the ultimate shrinkage and ranges from 0.000415 to 0.001070, with an average of 0.000780 in/in.

4.2.3 Deflection due to Creep and Shrinkage

An increase in the deflection of a reinforced concrete slab under longterm load can be attributed primarily to :

- (a) creep of concrete,

- (b) shrinkage of concrete,
- (c) a reduction in the modulus of elasticity of concrete and,
- (d) cracking of concrete.

How much should the longterm modulus of elasticity of concrete be reduced remains unknown but its effect is usually included in determination of the creep coefficient. To include the effect due to cracking of concrete, an effective second moment of area from Equation 3.4 can be used. The longterm deflections due to creep and shrinkage will be discussed in the following sections.

4.2.3.1 Deflection Due to Creep

Deflection due to creep depends upon the creep characteristics of the concrete used, the amount of compressive reinforcement in the concrete section and the amount of cracking that has taken place. For a fully cracked section as shown in Figure 4.2 and based on test results by Vanderbilt et al [124], the ratio of the increase in strain due to creep (ϵ_c) to the initial strain at first loading (ϵ_i) was determined to be equal to $(3-\rho'/\rho)$ for rectangular sections. The insignificant increase in steel stress with time was neglected in this analysis. For an uncracked section, no simple relationship between creep and initial

strains is available; however, it was concluded [124] that the increase in deflection due to creep of a uncracked beam is more than that of a fully cracked beam.

ACI Committee 209 [2] introduced the following empirical expression for calculating the additional deflection due to creep only, δ_c ,

$$\delta_c = K' C_t \delta_i \quad [4.4]$$

where K' is the reduction factor due to compressive reinforcement and is equal to $1.0 - 0.6(A_s'/A_s) > 0.40$, C_t is the creep coefficient from equation 4.2, and δ_i is the initial elastic deflection.

4.2.3.2 Deflection Due To Shrinkage

Shrinkage results in shortening but no curvature in plain concrete. For singly reinforced sections or sections containing unequal amounts of tensile and compressive reinforcement, the top fibres shorten due to shrinkage (for a positive moment region) and the shortening of the bottom fibres are resisted though not prevented, by the reinforcement. These unsymmetrical strains cause warping of the section. For thin slabs, because compression reinforcement is not usually provided, these additional deflections due to shrinkage induced warping can be considerable and will result in warping deflection.

In general, the effects of shrinkage on deflection are dependent on the boundary conditions of the member. The shrinkage curvature (φ_{sh}) was developed by Vanderbilt et al [124] to be

$$\varphi_{sh} = 0.0035 \frac{(\varphi - \varphi')}{d} \quad [4.5]$$

For a simply supported beam as shown in Figure 4.3a, the mid-span deflection can be determined from the following expression,

$$\delta_{sh} = \frac{\varphi_{sh} L^2}{8} \quad [4.6]$$

whereas for a fixed-end beam (Figure 4.3b), the expression becomes

$$\delta_{sh} = \frac{\varphi_{sh} L^2}{16} \quad [4.7]$$

It should be noted that shrinkage has a direct effect on deflections due to the shrinkage curvature in a simply supported beam, whereas in a fixed-end beam, the shrinkage has an indirect effect on deflections due to the cracking resulting from shrinkage causing a loss of stiffness.

Warping, as a result of differential shrinkage strains or rate of drying across a section, is a function of time and is strictly a geometrical phenomenon due to dimensional changes and is not a function of the strength or the modulus of elasticity of concrete. Among the several methods for computing shrinkage warping are those of Pickett, the equivalent force method, Miller, and the A.C.I.'s empirical method.

Pickett's [96] theoretical expressions for the deformations of concrete beams and slabs due to shrinkage warping were based on the assumption that the laws governing the development of shrinkage stresses in concrete during drying were analogous to those governing the development of thermal stresses in an ideal body during cooling. Theoretical equations (Appendix L) as well as tables and curves were given for the shrinkage warping and stresses in slabs and beams. The equations were derived assuming,

1. shrinkage is linearly related to change in moisture content,
2. the flow of moisture in concrete follows the law of diffusion and
3. concrete is homogeneous, isotropic and obeys Hooke's law.

In the equivalent force method [44] or the so-called fictitious force method, each element in the concrete cross

section is first assumed to be subjected to a uniform shrinkage strain, which is possible only if the steel is compressed by a fictitious force T_o , or $A_s E_s \epsilon_{sh}$ as shown in Figure A of Appendix M). An equivalent eccentric tension force of $A_s E_s \epsilon_{sh}$, or $(A_s - A_s') E_s \epsilon_{sh}$ should compression steel be provided, would be applied at steel level to the entire transformed area of the member due to the release of the steel compression force (Figure B of Appendix M). The resulting deflection of the beam or slab due to shrinkage could then be computed by considering the effect on the transformed cross section of this externally applied force, equal and opposite to the fictitious force. The beam is subjected to a uniform moment of $(M = -T_o e)$ and the shrinkage induced curvature can be evaluated.

To include the effect of cracking as well as the reduced value of the longtime modulus of elasticity of concrete, Branson and Christiason [19] suggested that for reinforced concrete drying symmetrically, better results could be obtained by using $E_c/2$ and I_g (for gross section) instead of E_{ct} (reduced elastic modulus of concrete for longterm loading) and I_{ct} (for cracked transformed section) such that, (Appendix M)

$$\phi_{sh} = \frac{L_o e_g}{E_c I_g / 2} \quad [4.8]$$

Miller's method [80] assumed that the extreme fibre of the beam furthest from the tension steel (method refers to singly reinforced concrete members only) shrinks the same amount as the free shrinkage of concrete and the relationship between curvature and strains can be depicted as shown in Appendix N. The amount of curvature can be calculated by means of the following formula,

$$\phi_{sh} = \frac{\epsilon_{sh} - \epsilon_s}{d} = \frac{\epsilon_{sh}}{d} \left(1 - \frac{\epsilon_s}{\epsilon_{sh}} \right) \quad [4.9]$$

Empirical values of 0.1 and 0.3 for ϵ_s/ϵ_{sh} were suggested by Miller for heavily and moderately singly-reinforced members respectively, i.e., Equation 4.9 can be reduced to

$$\phi_{sh} = \begin{cases} 0.7 \frac{\epsilon_{sh}}{d} & \text{MODERATELY REINFORCED BEAM} \\ 0.9 \frac{\epsilon_{sh}}{d} & \text{HEAVILY REINFORCED BEAM} \end{cases} \quad [4.10]$$

Another empirical equation, or otherwise known as a modification of Miller's formula, was proposed by ACI 209 [2] and Branson and Christiason [19] in which, the steel ratios were expressed in percentages. Equations 4.11 and 4.12 are applicable to both singly and doubly reinforced

members. The use of thickness, rather than the conventional effective thickness, was shown to be convenient and provide adequate agreement with available experimental data.

According to Branson and Christiason [19] and ACI committee 209 [2], the longterm deflection due to shrinkage warping alone can be determined by the following expression,

$$\delta_{sh} = K_w \varphi_{sh} L^2 \quad [4.11]$$

K_w = shrinkage coefficient depending on end constraints

= 1/16 for both ends continuous

= 11/128 for one end continuous

= 1/8 for simply supported

= 1/2 for cantilever

ϵ_{sh} = shrinkage strain

h = member thickness

ρ = tensile reinforcement percentage

ρ' = compression reinforcement percentage

φ_{sh} = shrinkage curvature in radian/unit length

$$\varphi_{sh} = 0.7 \frac{\epsilon_{sh}}{h} (\rho - \rho')^{\frac{1}{3}} \sqrt{\frac{(\rho - \rho')}{\rho}} \quad \text{FOR } (\rho - \rho') \leq 3 \% \quad [4.12]$$

$$= \frac{\epsilon_{sh}}{h} \quad \text{FOR } (\rho - \rho') > 3 \%$$

Chapter V

LITERATURE REVIEW ON LONGTERM DEFLECTIONS OF REINFORCED CONCRETE SLAB SYSTEMS

5.1 ANALYTICAL STUDIES

A literature review of the time-dependent deflection of reinforced concrete slabs will be given in this section.

5.1.1 Taylor, 1970

The two calculation methods proposed by Taylor to determine the longterm deflection of flat plates were based on an initial elastic deflection by means of the S-method as presented in Section 3.6, together with

1. a procedure recommended by Branson [17,18] or
2. a simple multiplier according to Taylor[117].

In the calculation of initial deflection, the column strip was assumed to be fully cracked, and both the cantilever element and the simply supported plate element being 50% cracked. The first method considered creep and shrinkage effects separately in that the deflection due to shrinkage alone was determined using Branson's procedure (Equations 4.11 and 4.12) and the additional deflection due to creep only was equal to the initial deflection multiplied

by a creep factor. Values of the creep factor were dependent upon the ratio of compressive to tensile reinforcement and estimated by,

$$C_t = 1.7 - \left(\frac{A_s'}{A_s} \right) \quad [5.1]$$

The alternative procedure proposed by Taylor [117] considered shrinkage and creep effects together and the longterm deflection was equal to the initial deflection multiplied by two factors. The first factor took into account the reduction of longterm deflection due to the application of compressive reinforcement and was equal to $(1 - 0.6(A_s'/A_s))$. The second factor was the combined shrinkage and creep multiplier, depending solely on the average relative humidity and age of the concrete when loaded. When loaded at 7, 14 or 28 days at an average humidity of 50%, factors of 4.2, 3.0 and 1.8 were recommended for the determination of the total longterm deflection of a flat plate without compressive reinforcement.

5.1.2 Scanlon and Murray, 1974

Scanlon and Murray [105] presented a finite element model coupled with a time integration procedure to simulate the time-dependent behaviour of reinforced concrete slabs

subjected to sustained load. The method of analysis included the effects of compression steel, concrete cracking, shrinkage and creep of concrete and was based on the following three assumptions,

1. concrete is a linear viscoelastic material,
2. the time dependent response of concrete is identical in tension or compression,
3. temperature effects were excluded.

For a 8-in. (213 mm) thick interior flat plate slab designed in accordance with the direct method of ACI 318-71 Building Code[3], the relationship between the deflection to span ratio and the cube of the span to depth ratio for both short- and long-term deflections was developed. For a C/L ratio of 0.1 and a sustained load equal to the design dead load plus 20 per cent of the design live load, the ratios of final deflections after 800 days to the initial deflections at 7 days were estimated to be 3.5 for cracked section or 4.4 for uncracked section.

5.1.3 Rangan, 1976

By considering the deflection at the centre of a rectangular flat plate as the sum of the deflections of a column-beam strip in the long span and a middle-beam strip in the short span, Rangan [99] proposed a simple approximate method of analysis for the calculation of longterm

deflection of flat plates and flat slabs. Basically, the method applied the technique of overestimating the initial deflection by assuming a fully cracked column-strip and a partially cracked middle-beam strip. With the multiplying factor of 2.35 as recommended by ACI Committee 209 [2], the longterm deflection of a flat plate was computed and compared to Taylor's experimental result [116,119] of a flat plate structure. The measured initial and longterm deflections after 850 days of Taylor's flat plate slab were 0.11 in. (2.8 mm) and 0.72 in. (18.3 mm), giving a final to initial deflection ratio of 6.5. Using Rangan's simple method, the initial deflection was computed as 0.2 in. (5.1 mm), giving a computed final deflection of 0.67 in. (17.0 mm) and a final to initial deflection ratio of only 3.35.

5.1.4 Wilson, 1982

Based on small scale one-way slab tests and simplified assumptions, Wilson [131] presented a simplified method for calculation of deflection-time relations of reinforced concrete beams and slabs. The time-dependent deflections were separated into the instantaneous deflection δ_i , the deflection caused by creep effects $\delta_c(t)$ and the deflection caused by shrinkage in compressive zone $\delta_{sh}(t)$.

Equations (Appendix O-1) were developed from test results (Appendix O-2) on one-way slabs, which were 0.16 m

thick and had a simply supported span length of 4.2 m with non-cut-off bars. A total of nine slab tests were carried out in which six were loaded to failure at different ages with the other three being loaded by their own weight for four weeks, after removal of formwork at 1, 2 and 7 days respectively.

A straight-line relation between the square root of loading time and the measured deflections was assumed. The method of analysis suggested measurements of deflection be taken at 2, 4, 9, 16, 25 and 28 days after casting and that the intersection between this straight line and the deflection axis approximated the initial deflection δ_i . The time-deflection curve as well as the ultimate deflection based on the measurements made on the working stage could then be estimated by means of the proposed method of calculation. Ultimate shrinkage strain was taken equal to 0.0005 for an average relative humidity of 40 %.

5.1.5 Sbarounis, 1984

A simplified method for the calculation of longterm deflections of multishored slabs was proposed by Sbarounis[102], in which the total deflection is equal to a simple multiplier times the immediate secant service dead load deflection, δ_{sd} (Appendix P)

$$\text{MULTIPLIER} = 1 + 2.8 \left(\frac{C_u}{2.35} \right) + 1.2 \left(\frac{\epsilon_{shu}}{800} \right) \quad [5.2]$$

Secant service load deflection, defined as the sum of the immediate deflection (δ_{ss}) due to slab weight (W_{ss}) and that due to service dead load other than slab weight (δ_{osd}), can be computed from the maximum construction load deflection (δ'_{max}) by the formula,

$$\delta_{sd} = \delta'_{max} \left(\frac{W_{sd}}{W_{max}} \right) \quad [5.3]$$

where δ'_{max} is the deflection due to the maximum construction load (W_{max}) when applied as a single load increment and W_{sd} is the total service dead load or the total sustained load. On the basis of an ultimate creep coefficient of 2.35 and an ultimate shrinkage strain of 800×10^{-6} as recommended by ACI Committee 209 [2], the multiplier in Equation 5.2 becomes 5.

From Equation 5.2 and excluding the deflection due to shrinkage and with an ultimate creep coefficient of 2.35, the ratio of the initial plus creep deflection (δ_{ec}) to immediate service dead load deflection (δ_{sd}) is 3.8. This factor of 3.8 for the total of immediate and creep deflections was derived for average conditions of δ_n/δ_{ss} of 2.3 and δ_{osd}/δ_{ss} of 0.3 where δ_n is the successive deflection corresponding to construction loads (W_n) at nth construction cycle of stripping and casting. It was calculated as a

function of the number of floors in the supporting assembly, for uncracked and cracked slabs and for a rate of construction being two floors per week. The ratio was found to be independent on the number of hardened slabs-(N) in the supporting system simply because as the value of N increases, both δ_{ec} and δ_{sd} increase resulting in a relatively unchanged ratio.

Table 5.1 summarizes the long term deflection ratios using the above mentioned analytical methods.

5.2 CASE STUDIES

A statistical collection of 181 cases of building damage due to excessive deflections of reinforced concrete building components in Europe from 1960 to 1965 was evaluated by Mayer and Rush [77]. In order of their importance, the following types of damage were reported,

partition wall cracks	106 times
excessive slab deflection	60 times
cracks in r.c. components	39 times
cracks around slab supports	29 times
damage to plaster and finishes	13 times
isolated deflection damage	12 times

A total of 259 instances of damage were recorded in 181 cases. It was also established that the problems were attributable to design errors 137 times (45%), size deficiencies 76 times (25%), errors in workmanship 56 times (19%) and calculation errors 33 times (11%). Damage was most frequently observed in two-way reinforced plates and that frequently, more than one possibility of damage and cause contributed to the same damage.

5.2.1 Washa, 1947

In 1947, Washa [127] presented the test results on eight oneway slabs, 12X3 in. (305X76 mm) in cross section, subjected to sustained loads for 5 years. The main variables in test specimens included three concrete slumps, 0.5", 4" and 8" (13, 102, and 203 mm), two water/cement ratios, 0.6 and 0.84, three span/depth ratios, 30, 50 and 70 corresponding to reinforcement ratios of 0.79, 0.79 and 1.58 % respectively, and two curing methods, dry and sealed specimens. It was found that the longterm deflections increased as the square of the slab span/depth ratio. Figure 5.1 represents the relation of plastic flow deflection ($\delta_{total} - \delta_{initial} - \delta_{shrinkage\ warping}$) to span/depth ratio.

Of the eight different slabs tested by Washa, slab G with span length of 7.5 ft. (2286 mm) and a slump of 4

in.(102 mm), similar to the oneway slab models tested in the present program, had a reinforcement ratio of 0.79 % and an estimated ultimate design load of 240 lb/ft (3.5 kN/m). The sustained load, weight of slab plus superimposed load, was 107 lb/ft (1.63 kN/m), which would be equivalent to the design dead load should the design live load to dead load ratio be taken as 0.5. The initial deflection at 28 days was measured as 0.24 in.(6.1 mm) and a longterm deflection of 0.93 in.(23.6 mm) was recorded at 5 years., i.e. a ratio of final deflection to initial deflection of 3.875.

5.2.2 Washa and Fluck, 1952

To determine the influence of compression reinforcement on the plastic flow of reinforced concrete beams, Washa and Fluck [128] conducted tests on 34 simply supported concrete beams under sustained loading for 2.5 years. The term plastic flow included the deflections due to cracking of concrete, concrete shrinkage, concrete creep-etc., in addition to the immediate elastic deflection during loading. There were five different test beam cross sections and the span-to-depth ratios ranged from 20 to 70. The beams, with compressive to tensile reinforcement ratios of 0, 0.5 and 1.0, were uniformly loaded by their own weight and an superimposed uniform load after 14 days of curing. The total applied loads produced bending moments approximately equal to the design resisting moments and hence all the beams exhibited cracking after loading.

The test results showed clearly the beneficial effect of compressive reinforcement in reducing excessive plastic flow. The plastic flow deflection for beams with equal tensile and compressive reinforcement was only one-half of that for beams with tensile reinforcement only. The average longterm deflection was, about three times the immediate elastic deflection for singly reinforced beams. It was also found that the additional longterm deflection for beams with an L/d ratio of 70 was 4 to 6 times that for beams with a L/d ratio of 20.

5.2.3 C.S.I.R.O., Sydney, 1961

In 1961, The Mechanics and Physics section of the Division of Building Research [C.S.I.R.O.,40] conducted tests on a lightweight flat plate structure during and after the erection of the structure. The flat plate slab was 3.5 in.(89 mm) thick and spanned three bays of 9 ft.(2953 mm) in one direction and three bays of 12 ft.(3937 mm) in the other with cantilevers of 6 ft.(1969 mm) at each end of the longer span. No edge beams or torsional reinforcement near the edge columns were provided.

Formwork was removed 10 days after placement of concrete and the slab was submitted to its own weight, which was 32 psf (1.532 KPa). Variation of deflection with time was recorded for 201 days before an additional load of 31.2

psf(1.494 KPa) was applied to the slab. Progressive deflection of the slab under the uniformly distributed load plus the slab dead load continued for another 52 days.

The immediate deflections under slab dead load were small but the longterm deflections after 8 months were of the order of twelve times the initial deflections. The initial deflection and the increase in deflection at the centre of one interior panel relative to the supporting columns whose longterm settlement was estimated to be 0.13", were found to be 0.05 in. (1.27 mm) and 0.5 in. (12.7 mm) respectively, a ten-folded increase after 8 months. It was suggested that the portions of additional deflections because of creep of concrete and progressive cracking of the slab due to the thermal, shrinkage and settlement (of supports) stresses were roughly equal. In other words, additional deflection due to creep alone was 0.25 in. (6.35 mm) which was five times the initial elastic deflection.

The large longterm deflections under slab dead load and test load were attributed to :

- the expanded shale aggregate used in the structure, which was later found to have abnormally high creep and shrinkage characteristics, and

- the high and fluctuating outdoor temperatures during the course of the study, producing higher creep and shrinkage strains than for normal and constant temperatures.

5.2.4 Jenkins, Plowman and Haseltine, 1965

Results of an investigation by Jenkins, Plowman and Haseltine [64] into the causes of deflection of heated concrete floors on a number of Scottish flats were described. The reinforced concrete test floor spanned 12'-5.5" (3797 mm) and 15'-4" (4674 mm), 5" (127 mm) thick and was supported on three sides by load-bearing walls but was completely free on the fourth in the short direction.

At the start of the test, the slab was 4.5 years old and a measured deflection of 0.8 in. (20.3 mm) was recorded at midspan of the slab free edge. The mid-depth temperatures of concrete slab ranged from 60 F to 95 F for the next two years, in which period the total loading on the slab was 140 psf (6.7 KPa), which was larger than the load of 110 psf (5.27 KPa) on the slab for the first 4.5 years. An additional deflection of 0.41 in. (10.4 mm) was measured giving a total deflection of 1.21 in. (30.7 mm) over 6.5 years. The deflection to short span ratio approximated 1/125. which was certainly considered to be very high as a deflection to span ratio of about 1/240 is usually allowed by practice.

Full-scale laboratory tests were also carried out on simply supported slabs similar to the Scottish flats but only 2' (610 mm) wide. The slabs were loaded to their full design value of 40 psf (1.92 KPa) in addition to self weight. The

instantaneous dead load deflection was assumed to be equal to 0.05" (1.27 mm). For the two different aggregate concrete used, deflections of the slabs at 60 F concrete temperatures were 0.45" (11.4 mm) and 0.51" (13 mm) at 740 days, with the equivalent values at 140 F being 0.53" (13.5 mm) and 0.62" (15.7 mm) respectively, corresponding to deflection/span ratio of 1/270 and 1/220.

Results from slabs with equal tensile and compressive reinforcement also showed that the relative longterm deflections of these slabs were about 70% of the equivalent singly reinforced specimens.

5.2.5 Taylor and Heiman, 1970

Investigation into the initial and longterm deflection of a reinforced concrete flat plate was conducted by Taylor [119] in 1970.. The test slab, a 8-in. (203 mm) thick concrete slab with a longer span-to-depth for an interior panel of 31, was added as a roof to an existing 3.5 years old single story building. The total design load of the roof slab was 185 psf (8.86 KPa) which included the slab dead load of 110 psf (5.27 KPa).

The initial deflection due to slab dead load, taken when formwork for the roof slab was stripped at 14 days, was measured to be 0.11 in. (2.8 mm) at the centre of an interior panel. The measured longterm deflections of the roof slab

after 2.5 years in service were about 6 to 8 times the initial deflections.

In a follow-up paper by Taylor and Heiman [120], the longterm deflections of the same slab were measured after 9 years. The total deflection of the panel centre point of an interior slab was found to be 0.96 in. (24.4 mm), which was 8.7 times the initial deflection of 0.11 in. (2.8 mm). The total deflection of the slab was considered to consist of:

1. initial deflection = 15 %, including the deflections due to the slab dead load, superimposed loads and finishes, and initial cracking,
2. longterm deflection due to cracking = 19 %,
3. longterm deflection due to creep of concrete = 40 %, and
4. longterm deflection due to shrinkage of concrete = 26 %.

5.2.6 Taylor, 1971

The test area was part of the second level of a reinforced concrete flat slab structure with 9.5" (241 mm) thick slab and a longer span-to-depth ratio of 36. A typical interior panel had the dimensions of 28'-3" x 26'-0" (8.61 M.x7.92 M.), column size of 20 " (508 mm) and drop panels of 96"x96"x6" (2.44x2.44x0.15 M). The design dead load was 120 psf (5.75 KPa) which was the slab dead load, and

the design live load was 60 psf(2.87 KPa). From the available data in the paper [118], the formwork was apparently stripped 56 days after the placing of concrete.

Measurements on the longterm deflection of the test slab under a sustained load of 120 psf (slab self weight, 5.75 KPa) plus the time-varying car parking live load were taken up to 8 years. The measured initial deflection at the time of formwork stripping and the longterm deflections after 1, 3.5 and 8 years were 0.14"(3.56 mm.), 0.55"(13.94 mm.), 0.88"(22.35 mm.) and 0.94"(23.88 mm.) accordingly. The ratios of the respective longterm to initial deflections were therefore 3.93 after 1 year, 6.28 after 3.5 years, and 6.71 after 8 years.

5.2.7 Heiman and Taylor, 1972

Part of a second level of a car park was chosen as a test slab in this investigation [58]. It was a flat plate structure with a typical interior panel size of 24'-9"x23'-9" (7.54x7.24 M), and a slab thickness of 9.5"(241 mm), giving a longer span-to-depth ratio of 31. The design loadings included the slab dead load of 115 psf(5.51 KPa), and car park live load of 60 psf(2.87 KPa). Normal weight concrete with a cement to flyash ratio of 4.25 by weight was used.

A sustained load of 115 psf (5.51 KPa), i.e. the slab self weight, was applied 17 days after placing of concrete, and longterm deflections of panel centre point of the slab was measured from the time of casting up to 3.5 years. The long term deflections for a typical interior panel after 610 days and 3.5 years were measured to be 0.74" (18.8 mm) and 0.86" (21.8 mm), which were 2.18 and 2.53 times the elastic deflection of 0.34" (8.6 mm) due to slab dead load.

The relatively high initial deflection was attributed to the settlement of the props and the unexpected high construction load applied to the slab at early ages. Extensive cracks were discovered on both the under sides and top of the test slab. Despite the cracks not being discovered until two months after casting, it was believed that some of the cracks did appear soon after the placing, likely due to the high construction load estimated to be approximately 170 psf (8.14 KPa) which was actually equal to the full design load (dead plus live load) of the slab.

5.2.8 Heiman, 1973

A comparison, by Heiman [57], was made between calculated and measured deflections of eleven lightweight concrete beams on four different floors of the Tower Building in Australia Square, Sydney, Australia under service and test loads for a period of five years. The

50-story circular tower was 600 ft.(183 M.) high, 134.5 ft.(41 m) in diameter, and had a centre service core of 162 ft.(19 m.) in diameter. The beams with an overall depth of 20.5 in.(521 mm), spanned 36 ft. (11 m) between the core and the peripheral columns. The slab was 4.65 in.(118 mm) thick. and the props under the beams were removed 25 to 32 days after casting.

Although the design live load of the slab was 100 psf(4.79 KPa), a construction load of 121 psf(5.8 KPa) was imposed on the 38th floor when it was supporting three upper floors. The deflection due to this overload was 0.33"(33.8 mm) which was much higher than the elastic deflection of 0.13"(3.3 mm) due to a test load 100 psf(4.79 KPa) when the slab was 3.5 months old. The recovery was approximately equal to the elastic deflection of 0.13"(3.3mm) when the test load of 100 psf(4.79 KPa). was removed at 280 days.

On the contrary, when the construction load was removed, a recovery of only 0.01"(0.25 mm) was obtained. The primary reason was because overload applied to slabs at early ages produced permanent deflection, due to high stress/strength ratio.

The long term deflections of the test slab after 2 and 5 years were identical, i.e. 0.65"(16.5 mm). This total deflection included the unrecoverable deflection due to construction load, shrinkage deflection and the creep deflection due to service load.

5.2.9 Jenkins, 1974

Field tests were carried by Jenkins [62] in 1970 on the fourth level of a 5-story flat plate building at the University of Adelaide, Australia, to determine the behaviour of reinforced concrete slabs. The test slab spanned 21'-3"x18'-10" (6.48x5.74 m) with a column width to span ratio of 0.1. The slab depth was designed to be 9 in. (229 mm) but later measurements indicated an average depth of 9.57 in. (243 mm), resulting in a maximum span-to-depth ratio of 26.7. The total design load of the slab included a live load of 120 psf (5.75 KPa) plus a design dead load equal to the self weight of the slab which was also 120 psf (5.75 KPa).

Formwork was stripped after ten days and the initial elastic deflection at the centre of the panel was 2 mm. due to slab dead load. However, during construction of the building, a maximum deflection of 5.33 mm. was recorded when the upper floor was poured. When all the props were removed at about 28 days, the deflection dropped to 4.6 mm. Unfortunately, the magnitude of the construction load was not known.

The mid-point deflection of the test slab was 7.9 mm. after one year under self weight, which was four times the initial deflection of 2 mm. The total deflection was 11.7 mm. when the slab was 27 months old and under permanent

service load of less than 60 psf(2.88 KPa). No cracks were noticed throughout the experiment.

5.2.10 Yamamoto, 1982

Results of laboratory testing on seven one-way slabs were reported by Yamamoto in 1982[135]. Of the seven fixed-end slabs (0.4 m wide) tested, slabs 1 to 5 had span/depth ratios of 30 (3.6 m long and 0.12 m thick) and slabs 6 and 7 had span/depth ratios of 38.5 (5.0 m long and 0.13 m thick). The formworks were removed at 1,2,3,7,28,3,28 days for slabs 1 to 7 accordingly.

All seven slabs were subjected to a maximum load, for a period of up to two weeks, of twice the slab dead load but at different ages of the concrete (at 7,7,7,14,28,7 and 28 days respectively) and that the sustained load (at 21,21,21,28,42,28 and 42 days) was 1.5 times the slab dead load for all seven slabs. The reinforcement was so arranged that the maximum load of twice the slab dead was about 70% of the yield point.

Immediate and longterm deflections for up to 112 days after casting of the slabs were measured. The ratios between the longterm deflections and the initial deflections for slabs 1 to 5 varied from 4.43 to 6.74, and from 3.65 to 4.73 for slabs 6 and 7.

5.2.11 Sbarounis, 1984

Deflections with respect to the supporting columns were recorded one year after casting, by Sbarounis [103] at 175 bays on 13 floors of the upper part of a multistory lightweight concrete flat plate building. The flat plate had a center-to-center span of 21.6'x22.4' (6.58x6.83 m), with an average calculated thickness of 7.25" (184 mm). The design loads for the slab included 20 psf (0.96 KPa) for partitions, 40 psf (1.92 KPa) and 67 psf (3.21 KPa) for slab dead load.

The calculated slab deflection for a service load of 87 psf or 4.17 KPa, (slab dead load plus partition loading) and a 28-day concrete strength of 4000 psi (27.8 MPa) was 0.21" (5.33 mm). The measured one-year deflections ranged from 0.53" (13.46 mm) to 2.16" (54.86 mm), with 90% of them exceeding 1" (25.4 mm) and 10% exceeding 1.72" (43.7 mm). The average measured deflection was computed as 1.35" (34.29 mm) with a standard deviation of 0.29" (7.37 mm) and a coefficient of variation of 21.2%. The resulting ratio of measured longterm deflection to initial 28-day deflection averaged 6.4 and ranged from 2.5 to 10.3 after one year.

5.3 CONTROL OF DEFLECTION

Dealing with the control of deflections, ACI 318-83 Building Code [5] stated, 'reinforced concrete members subject to flexure shall be designed to have adequate stiffness to limit deflections or any deformations that may adversely affect strength or serviceability of a structure at service loads'.

In practice, methods of deflection control include (i) limiting span-depth ratio and (ii) limiting deflections. They are known as the indirect control method and the direct control method accordingly. For the first approach, the geometric span-to-depth ratios are limited to qualitatively acceptable values. For the second approach, the slab has to be designed such that the predicted deflection falls within acceptable limits.

The accepted span/depth ratios and limiting deflections vary from country to country. Neville et al [86] examined the code requirements of approximate 90 countries and found that, of the 28 that had specific clauses dealing with deflection control, the deflection limits averaged about $L/360$. In ACI Building Code [5], deflection limitations of $L/360$ for immediate deflection due to live load and $L/240$ for total deflection are specified when no damage to nonstructural elements is likely. For continuous two-way slabs without beams or drop panels, a minimum thickness of 5 in. (127 mm) is recommended by ACI.

The deflection limit specified by ACI 318-83 for longterm deflection is $L/240$, which, if a multiplier of 3 is used, requires a short-time deflection of $L/720$, which is very severe and sometimes impossible to satisfy in design.

5.4 SUMMARY

Table 5.2 presents a summary of the published measurements of long term deflections of reinforced concrete slab structures. The long term deflections are multiples of the initial deflection and cannot be ignored in designing for serviceability. The ratios of total deflection to initial deflection range from 2.2 after 20 months to 8.7 after 9 years. However, except for Yamamoto and Sbarounis, the construction loads and resulting deflections were not given.

Chapter VI

DESCRIPTION OF TEST AND INSTRUMENTATION

2 An essential part of this investigation consists of laboratory studies dealing with the short- and long-term deflections of floor slabs under various temporary and sustained loadings. A total of eight specimens in two series were tested at the Structures Laboratory in the University of Ottawa.

The first series of tests was performed on three single bay reinforced concrete flat plate slabs of approximately one-third scale. Parallel to the deformation study, tests were also carried out to determine the instantaneous and time-dependent concrete properties of the three normal weight concretes used for the slabs. Compressive strength and the modulus of elasticity of concrete together with the creep and shrinkage characteristics of the concrete were determined.

The second series of tests consisted of five reinforced concrete one-way slabs identical in all details but subjected to different load histories. Time-dependent concrete properties, namely compressive strength, modulus of elasticity and shrinkage of concrete were also recorded.

6.1 SERIES 1: FLAT PLATES

THE TEST STRUCTURES

Three loading tests on small scale concrete flat slabs were carried out in the Structures Laboratory of the University of Ottawa. All three slabs were square, 63.5 mm thick, supported by four 203x203 mm columns, with clear spans of 1.96 m. (Figure 6.1). Consequently, the ratio of the minimum span length to the slab thickness was 31.0. Table 6.1 gives details of the three flat slab models, which were designed according to the empirical method of analysis given in ACI 318-77 Building Code [4].

Concrete for the three slabs were supplied by a local premixed concrete plant. Retarder was used in all the concrete mixes, providing good workability during the placement of the concrete, and interior vibration was used in making all the slabs, cylinders and prisms. Table 6.2 gives details of the concrete proportions and properties used in the casting of the three slabs. Although the 28-day strength was specified to be 28.0 MPa. for all slabs, the actual concrete strengths obtained for the three slabs ranged from 28.0 MPa to 45.5 MPa at 28 days as illustrated in Figure 6.2 and Table 6.2.

Reinforcement for all three slabs comprised plain welded wire mesh and the typical stress-strain curve of wire

reinforcement is shown in Figure 6.3. These figures are mean values obtained from three test bars subjected to tension tests.

The wire had no definite yield point as illustrated in Figure 6.3. The yield strength by the 0.2 per cent offset was found to be about 430 MPa.. ACI 318-77 Building Code [4] specifies that the yield strength be taken equal to the strength corresponding to a strain of 0.35 per cent or 417 MPa for welded wire reinforced slabs. In this case, the strength corresponding to a strain of 0.35 per cent was 417 MPa, which would be used as the specified yield strength of reinforcing bars.

The negative moment steel ratio was 0.00253 and the steel extended over the outer thirds of the slabs. The positive moment steel ratio varied from 0.00507 in the column strips to 0.00253 in the middle strip and extended to the edges of the slabs. Details of the reinforcement layout can be found in Figure 6.1B.

The four corner columns, pinned at both ends as illustrated in Figure 6.1 to represent the points of inflection in the lower and upper columns, were reinforced with 4 No.15 deformed bars (total area = 800 mm.sq.) with an average yield strength of 417 MPa. Ties for the columns, spaced at 152 mm were fabricated from 64 mm plain bars with a yield strength 417 MPa.

Torsional reinforcement was provided in the slabs, and consisted of galvanized square mesh with 1.6 mm diameter bars per 25.4 mm. A total of four layers of reinforcing mesh were used (two near the top surface and two near the bottom surface of the slab), and extended 610 mm from the column faces as shown in Figure 6.4.

Slab 1 was cast on 1983-02-18, slab 2 on 1983-06-10 and slab 3 on 1983-09-23 as indicated in Figure 6.5, which also shows curves representing the temperature and relative humidity of the air in the interior of the room.

For slabs 1 and 2, forms were stripped after 4 days, and the slabs were loaded at 7 days to simulate the early construction loads, while forms were stripped after 10 days and the slab was loaded at 28 days for slab 3 to simulate the assumed standard loading procedure. The intensities of the applied loads varied for each of the three slabs.

A total of 40 standard cylinders, 152 mm diameter and 304 mm long, were taken from each slab mix, cured under the same conditions as the test slabs and tested at different ages of concrete. Figure 6.2 gives the time-dependent concrete properties for the three slabs.

LOADING

Direct analysis of structures by scale model tests is concerned with obtaining strains or deformations due to

specified loading. In order to reproduce the load-deformation or load-strain characteristics of a full-scale loaded slab, the model slabs should experience the same distributions and intensity of stress as the prototype structure.

Small scale models have relatively less self weight than required to give the desired stress levels. For this study, the live load to dead load ratio was chosen, which specified the dead load contribution to the ultimate flexural capacity. The ultimate flexural moment capacity was calculated from the section properties and the uniformly distributed load required to obtain this moment could be calculated from the live load to dead load ratio.

In the present study, the design live load to dead load ratios were 1.0 for slab 1 and 0.5 for slabs 2 and 3, resulting in design dead loads of 1.820 KPa, 2.523 KPa and 2.523 KPa accordingly for the three slabs. Details of load calculations for the three models can be found in Appendix Q. In view of the thin concrete slab and its relatively light self weight (1.485 KPa), additional dead load had to be applied to each slab to obtain the desired dead-load stress level. For simplicity, it was decided that the construction load be expressed as a simple multiple of twice the calculated dead load and that the sustained load as one-and-a-half times the calculated dead load. This allowed the slab models to be subjected to the same stresses as a full size slab.

The formwork was left in place until the removal of the props. No attempt was made to cure the concrete afterwards. For slabs 1 and 2, additional loads of 2.16 KPa and 3.55 KPa corresponding a total load of twice the calculated dead load were applied at 7 days over the whole span of the flat plates and subsequently sustained loads of 2.73 KPa and 3.79 KPa at 28 days. For slab 3, total sustained load of 3.79 KPa was applied at 28 days. Details of the loading histories on the three slab models were given in Figures 6.6A and 6.6B.

The load was applied by stacking concrete blocks on the slab. Two different sizes or weights of concrete blocks were used, namely 152 mm blocks (14.4 Kg each) and 203 mm blocks (16.6 Kg each).

DEFLECTION

The measurement of deflections was confined to 36 gauge points on the slab surface, and was made by 18 dial gauges with a 0.01 mm accuracy for one half of the slab and by 18 steel scales graduated to 0.25 mm. for the other half of the slab. Dial readings were taken at the centres of the slabs and at one-third points both on the principal axes and the diagonals. Measurements against the steel scales were made by high precision survey's level. The locations and numbers of the points at which deflection measurements were made are shown in Figure 6.7.

STRAIN

The reinforcement was instrumented with electric resistance gauges at critical locations (Figure 6.8) to measure the strains in the reinforcement. A total of 24 gauges mounted on the reinforcing bars were used for each slab. The gauges were checked before and after placing the concrete, and were fully waterproofed. In spite of numerous precautions, a majority of these strain gauges did not function properly during the tests and these sets of readings were later abandoned. The malfunction of the strain gauges is attributed to the chemical reaction of the concrete moisture and the damage by the vibration during the placement of concrete.

SHRINKAGE

To measure the unrestrained linear shrinkage of the concrete, the length change of four concrete prisms for each slab model was measured by means of a micrometer (0.001 mm accuracy), and according to the requirements of ASTM C157-64T [11]. The specimens were 76x102x381 mm prisms for slabs 1 and 2 and 76x63x381 mm prisms for slab 3.

CREEP

A test of concrete under sustained constant compression to determine the creep behaviour of the concrete, was carried out on 152x305 mm concrete cylinders in a loading

frame in accordance with the requirements of ASTM C512-74 [1] with one modification. As it was not practical to fabricate 10 spring loaded creep frames, a load frame of bearing plates and tie rods was employed as illustrated in Figure 6.9. This load frame was capable of maintaining load to within 3 per cent of the predetermined load level, by adjusting the nuts to apply the required loads to the three concrete cylinders through the three one-inch diameter threaded rods.

As shown in Figure 6.9, the three threaded rods were strain gauged, calibrated by a tension test and used as a load cell. Loads were applied by means of hydraulic jacks with reference to the strain readings of the steel rods. In order to ensure linear relationship between the creep strains and applied stress, stress levels were kept below 40 per cent of the concrete strength at the day of testing. The sustained load produced stresses equal to 20 to 40 per cent of the compressive strength of the test concrete at the time of testing.

To measure the time-dependent strains of concrete under sustained load, machined steel pins were installed (152 mm apart) in the steel moulds in which the concrete cylinders were cast. Strains were measured with a specially made portable gauge, designated as the creep gauge, between the two steel bars. In taking readings, the creep gauge was

first set to a fixed reading over a calibrated gauge length. This creep gauge has a gauge length of 152 mm and an accuracy of 0.0025 mm. All concrete cylinders were stored in steel moulds during the first few days after casting under identical conditions as the slabs, and were demoulded at 96 hours. Subsequently, up to the time of the beginning of the creep test and during the test, they were placed in the same test room as the three slabs.

The measurement of deformations of specimens under load and a check of the stress applied were made regularly, daily during the first week, weekly during the first month and then monthly. At 28 days and again at 6 months, one set of three cylinders under compression were removed from the frames and were tested for concrete strength and modulus of elasticity.

Creep tests were conducted on concrete under both compression and flexure. In addition to the flat plate models, six beam control specimens were manufactured for each of the slab models (Figure 6.10). Two plain, two singly reinforced and two doubly reinforced concrete beams control specimens were subjected to sustained loads of 35 and 40 percent of the beam capacity.

6.2 SERIES 2 : ONE-WAY SLABS

A second series of five reinforced concrete one-way slabs, subjected to various levels of loads at different ages of concrete, were tested in order to investigate the effects of loading history on the short- and long-term deformation behavior of the slabs.

THE TEST STRUCTURES

In this experimental phase, five identical one-way slabs were tested. All slabs, as shown in Figure 6.11 and Table 6.1, were 2160x203x63.5 mm in size, reinforced with 5 mm diameter wire in one direction, and cast on the same day (1984-03-16) from the same mix. Again, ready mixed concrete from a local plant was used, with a target 28-day strength of 28.0 MPa. The concrete mix proportion is given in Table 6.2. The principal reinforcement comprised identical plain bars as used in the flat plate tests (Figure 6.3). For a live/dead load ratio of 0.5, the calculated dead load of the one way slab models was found to be 1.00 kN/m (Appendix R).

LOADING

During the first 28 days after the placing of concrete, the uniform loading on the slabs varied not only between each slab but also from time to time for each of them. There were two control specimens among the five test slabs. Of the two control specimens, A and B, B was subjected to

the same loading history as flat slab models 1 and 2 in the first series of tests, i.e. slab self weight from day 4 to 7, twice the calculated dead load during the period from day 7 until day 28, followed by a sustained load totalled 1.5 times the calculated dead load. After the formwork was stripped at day 5, control specimen A was subjected to a superimposed dead load in addition to its self weight, resulting in a total load equal to the calculated dead load, and not until 28 days that an additional load of half the calculated dead load was put on to the slab.

For the first 28 days, slabs 3, 4 and 5 were subjected to different loading histories according to the various construction sequences of shore plus reshore, preshore and shoring only (Figure 2.5). The sustained load after day 28 was identical for all five slabs, i.e. 1.5 times the calculated dead load. The magnitudes and schedule of the loading applied to each of the slabs individually are given in Figure 6.12.

INSTRUMENTATION

Only the mid-span deformations of the five one-way slabs were measured under load. After the formwork was stripped, measurements were made every day for the first month and then weekly afterwards. In addition, shrinkage of four concrete prisms (76x102x381 mm), compressive strength, and modulus of elasticity of concrete were also measured at different ages of concrete.

Chapter VII

PRESENTATION AND DISCUSSION OF RESULTS

7.1 INTRODUCTORY REMARKS

This research involves an experimental investigation into the effects of early age construction loads on the short term and long term deflections of reinforced concrete slabs, including the effects of creep and shrinkage of concrete. Two series of experimental slabs were constructed and subjected to various construction load histories, followed by a constant sustained load over a long period of time. The first series of tests consisted of three identical flat plate models, with model 1 having a design live load to dead load ratio of 1.0 and a ratio of 0.5 for models 2 and 3. The second series of tests consisted of eight identical one-way slab models, all having a design live load to dead load ratio of 0.5. Each of these eight slab models was subjected to different load histories.

For each slab, cylinders were taken to measure the strength and the elastic modulus of concrete at various stages, prisms to measure the shrinkage of concrete and cylinders to measure the combined effect of shrinkage and creep of concrete. Deflections of slabs subjected to load

were recorded for a period of 10 months for one-way slab models and up to 700 days for flat plate models.

7.2 CONCRETE PROPERTIES

The compressive strength and the modulus of elasticity of concrete for the slab models have been summarized in Table 6.2 and again in Figure 6.1A (compressive strength) and Figure 6.2B (modulus of elasticity). All results are average of three cylinders. Although the 28-day concrete strength had been specified as 28.0 MPa for the two series of slab models, scatter of results was obtained, with 28-day compressive strengths of concrete varying between 28.0 MPa for flat plate model 1 to 45.5 MPa for flat plate model 2. A comparison between the measured compressive strength and the modulus of elasticity of concrete and those computed by ACI 209 [2] is satisfactory as shown in Figures 6.2.

7.3 SHRINKAGE

Four concrete prisms were made for each slab model to determine the longitudinal shrinkage of concrete due to drying. All specimens were cured for the first three days after casting, under the same environmental conditions as for the slab models to ensure the same effects from the ambient temperature and relative humidity. Initial readings (original lengths of specimens) were taken on day 3 when

the specimens were demoulded. The first sets of readings on the length changes of concrete specimens were recorded on the day of form removal, day 4 for flat plates 1 and 2 and for one-way slabs and day 10 for flat plate 3. The resulting shrinkage strains are plotted against time in Figures 7.1 to 7.4, each point is the average of results from four concrete prisms.

It is interesting to note that shrinkage strains increased very rapidly during the early portion of the observation period and then remained constant or decreased slightly afterwards, depending on the environmental conditions. The longterm shrinkage of concrete due to drying of concrete during the period from 10 months to 700 days varied from 645 to 786 microstrains (Table 7.1A). Published results of linear shrinkage tests indicated that the ultimate shrinkage strains ranged from 0.0006 under mild conditions to about 0.003 under severe drying conditions.

Under identical environmental conditions, large concrete members dry more slowly than small ones and accordingly, shrinkage goes down with increasing member size. Owing to the small size of the concrete prisms for shrinkage measurements for flat plate model 3, higher shrinkage strains were thus anticipated for flat plate model 3 than for the other two flat plate models and the one way slab models. However, test results (Table 7.1A) showed that

concrete for flat plate model 2 exhibited the least shrinkage strains with the highest 28-day concrete strength of 45.5 MPa and that concrete for flat plate model 1 had the largest shrinkage strains with the lowest 28-day concrete strength of 28.0 MPa. Comparing the shrinkage strains of the four different concretes used (Table 7.1A), it is apparent that the ultimate shrinkage is a function of the quality of the concrete.

Values of measured shrinkage strains of normal weight concretes and those calculated by various empirical methods and the experimental data from this study are summarized in Table 7.1B. It has not been possible to provide a single shrinkage model which fitted extremely well to the results (Figs. 7.1 to 7.4) nevertheless, the experimental data generally lay within the predicted curves of the various models, and that the agreement between the measured concrete shrinkage with the prediction method of the ACI 290 [2] is very good for all the slabs.

7.4 CREEP OF CONCRETE IN COMPRESSION

Strains in concrete under constant load can be measured in terms of three main components, elastic, creep and shrinkage. Strains developed at the time of loading are regarded as elastic strains, and the time-dependent strain remaining after deduction of shrinkage strains is creep.

Parallel to the linear shrinkage tests, investigation into the creep behaviour of concrete was carried out on concrete cylinders which were 304 mm long, 152 mm in diameter and were loaded in test rigs as shown in Figure 6.9. Creep tests on concrete cylinders were not carried out for the one-way slab models.

All of the concrete cylinders, a total of 12 (4 sets of 3) cylinders at the start of the creep test for each flat plate model, were subjected to a constant load of up to 40 % of the strength at testing to determine the creep for each of the concretes used for the three flat plate models. At this level of stress under consideration, the amount of creep can be taken to be directly proportional to the stresses in the concrete. Constant compressive loads were applied to the concrete cylinders at different ages; i.e. day 7 for slabs 1 and 2 and day 28 for slab 3. The environment of the laboratory was uncontrolled and both temperature and humidity varied widely as shown in Figure 6.5. All the loaded cylinders were accompanied by unstressed control concrete prisms (Section 7.3) to establish the shrinkage under identical atmospheric conditions.

For the concrete used in slab 1, the sustained load (on the 12 concrete cylinders) produced a stress equal to approximately 40% of the nominal concrete strength at the day of testing. Two series of creep tests were carried out

for slab 2 with stress/ strength ratios of 0.348 (for 6 of the 12 concrete cylinders) and 0.255 (for the other 6 cylinders) to determine the effects stress/strength ratio to the creep strains of concrete. For slab 3, the stress/strength ratio was 0.265 for all 12 concrete cylinders. The corresponding stress to 28-day concrete strength ratios were 0.325 for slab 1, 0.273 and 0.200 for slab 2 and 0.265 for slab 3. The measured strains are given as a function of the loading time in Figures 7.5 to 7.7, with each curve representing the mean of 6 to 12 cylinders or 12 to 24 sets of readings with 2 sets of readings from each cylinder. The measured strains include the initial strain and the total creep plus shrinkage strains measured on concrete cylinders under a compressive stress. The results of concrete cylinder tests are also given in Table 7.2, with Tables 7.3 and 7.4 summarizing the measured and calculated values of creep strain and creep coefficients by various authors. Results available from the cylinder tests indicated that strains of concrete under sustained load were in the order of 7.0 to 8.5 times those measured on initial loading (row C of Table 7.2).

For concrete used in slab 2, the concrete cylinders were loaded at 7 days and the ratio of measured creep strain to initial strain was found to be 4.55 for a stress/strength ratio of 0.255 and 4.53 for a stress/strength ratio of 0.348 (row J of Table 7.2). For the concrete used in slab 1, the

ratio was found to be 4.02. For slab 3, the creep ratio was 5.05 where the concrete cylinders were not loaded until 28 days. A review of test results (row J of Table 7.2) with those predicted by Bazant [12] (row L of Table 7.2) shows fairly good agreement.

It can be recognized from the creep tests that creep was influenced significantly by the stress-strength ratio of the concrete. For the two stress/strength ratios of concrete used in slab 2 a comparison of creep curves (Figures 7.6A and 7.6B) showed that the final deformation of the concrete under a heavier load (12.3 MPa) was approximately 1.25 ($=2745/2188$ in row B of Table 7.2) times that of the less-loaded concrete (9.0 MPa) when the stress ratio between the two concretes was 1.36. Creep strain is proportional to the stress/strength ratio of the concrete at the time of loading and that creep under constant stress from an early age is likely to be more than that allowed by ACI 209 [2] (compare row J to row M of Table 7.2).

Test results available from the literature (Table 7.3) indicate that the creep strains are in the order of 0.4 to 1.2 microstrain/psi. The results of the present tests produce basic creep of 109 to 131 microstrain/MPa or 0.77 to 0.91 microstrain/psi for the three types of concrete used in the three slab models (row H of Table 7.2).

At first glance at Figures 7.5 to 7.7, it seems rather unrealistic that the family of creep curves corresponding to ACI model will not be approached closely. However, the inclusion of shrinkage in combination with the creep effect indicate much better agreement between the experimental and analytical values.

For the purpose of the present analysis, the time-dependent shrinkage strain was assumed to follow the ACI model. Nevertheless, no account has been taken of the influence of temperature on shrinkage and creep. It can be found from row L of Table 7.2 and Figures 7.5 to 7.7 that Bazant's creep model provides a close agreement with the experimental results; It is mainly because Bazant's models (Appendix S) included not only shrinkage and specific creep, but also a third variable, the drying creep. As Gamble and Parrott [47] pointed out that, in addition to specific creep and shrinkage effects, the drying creep alone could account as much as another 40 or 60 % of the shrinkage strains (Appendix T). It is worth mentioning that for none of the specimens was the measured creep strain smaller than computed.

Of the 4 sets of creep tests on a total of 12 concrete cylinders, 2 sets for slab 1 and 1 set for slab 3 were designed to examine the effect of sustained load on the modulus of elasticity of concrete; i.e. a period of

constant stress followed by complete removal of load and a compression test on the then-unloaded concrete cylinders. Test results (asterisked figures in Table 6.2) indicated little difference of both the compressive strength and the elastic modulus between the creep loaded concrete and the control concrete.

For slab 3, one set of three creep concrete cylinders was unloaded at 68 days, i.e. 40 days after the application of the sustained load. These cylinders were reloaded and unloaded immediately after the release of the sustained load to determine the elastic modulus of the creep concrete. After two cycles of loading and unloading, the concrete cylinders were again loaded to the same sustained load before the load release. The time-varying strains of this set of creep concrete cylinders are given in Figure 7-7B. Comparing Figures 7-7 A and B, it appears that the unloading-reloading process did not affect the long-term creep strains of the concrete.

7.5 CREEP OF CONCRETE IN FLEXURE

To determine the otherwise different responses of concrete due to sustained load in flexure and to examine the significance of the compression reinforcement, two plain, two singly reinforced and two doubly reinforced concrete beams were subjected to constant load up to 40% of the beam

capacity. Details of the loading geometry and of the beam properties are given in Figure 6.10.

The deflections recorded for the beam control specimens are given in Figures 7.8 to 7.10 and Table 7.5. The sustained dead load deflections after a period of 10 months were found to be the order of 3 to 5 times the initial values. The evidence of test results from the beam specimens and from elsewhere [128] show that compressive reinforcement is very effective in reducing the longterm deflection of reinforced concrete beams. Under identical sustained loads and environmental conditions, the longterm deflections of singly reinforced beams were found to be 18 to 37 percent larger than those with tension and compression reinforcement.

In every case, the specimens with compression steel showed the least long term deflections and compression reinforcement did reduce the creep deflection. Deflections due to shrinkage were reduced by the use of compression reinforcement simply because without compression steel, appreciable warping might occur.

It should be mentioned that due to the relatively low reinforcement ratio (tension and compression) in the reinforced concrete beams, the use of equal amount of compression reinforcement provided only a slight advantage on the reduction of time dependent deflections. Moreover,

the test results shown in Table 7.5 illustrate the strength dependence of longterm deflection. Under equal magnitude of sustained loads, beams with higher concrete strength exhibited the least final deflection for plain concrete, single reinforced or doubly reinforced concrete beams.

From the test results of concrete beams (Table 7.5) and of concrete cylinders (Table 7.2), it is apparently that concrete creeps less in flexure than in compression. Take flat plate model 1 for an example, the ratio of final deformation to initial deformation after 700 days was found to be 7.13 for concrete under axial compression and only 3.02 to 4.88 for the control beams.

7.6 INITIAL ELASTIC DEFLECTIONS

The measured and calculated elastic deflections of the three flat plate models by each of the methods described in Chapter 3 are given in Table 7.6. The measured elastic deflections due to unloading for slabs 1 and 2 or loading for slab 3 at 28 days varied from 0.73 mm to 1.10 mm (corresponding to an equivalent load of 2.3 KPa and an elastic modulus of concrete of 28 MPa), exceeding those calculated by the given methods which ranged from 0.50 to 0.67 mm. The measured deflections were found to exceed those calculated using any of the given methods. The discrepancies can be attributed to the following causes,

- although the slabs were examined for cracks before and after loading and no cracks were detected, nonvisible hair-lined tension cracks might have occurred, resulting in a loss of stiffness.

- during the removal of formwork and the application of concrete blocks on to the slabs, vibrations of the slabs due to the movements seemed probable, thus affecting the accuracy of the dial gages above or under the slabs.

- analytic calculations were based on fixed-ends conditions for the columns. This condition might not hold in actual case, where rotation of columns seemed probable.

Before analysis of the test results, it is necessary to examine the results for internal consistency. Comparing directly the deflections of the slab models is not possible due to the different load histories and concrete properties for the slab models. However, the elastic deflections can be normalized with respect to the load and the concrete modulus of elasticity by the following formula,

$$\delta = \frac{KWL^4}{EI} \text{ where } K = \frac{1}{384} \text{ for fully fixed beams [7.1]}$$

$$\text{i.e. } \frac{\delta E}{W} = \text{constant}$$

$\delta E/w$ values for the three flat plate models are listed in Table 7.7, varying from 9020000 mm for slab 1 on day 28 to 22990000 mm for slab 3 on day 10. It is obvious that the three slabs cannot be considered to be identical, even though the effect of time on the concrete properties has been taken into consideration. Consequently, the experimental results will have to be analysed separately for each slab and compared with theoretical predictions. This conclusion is not unexpected as deflections are very sensitive to boundary conditions.

Measured and calculated elastic deflection for the five one-way slabs are given in Table 7.8. Prediction of the elastic deflections were computed by the Finite Element computer program, NOPARC [126]. It was found that NOPARC did give reasonably accurate elastic deflections but did not give consistent results for long term analysis.

Comparison of $\delta E/w$ values for all five one-way slab models at various ages of concrete are summarized in Table 7.9 and again in Figure 7.11. This constant parameter ($\delta E/w$) has a mean value of 10760 and a standard deviation of 1851. With the benefit of linear regression analysis, a regression line was obtained as indicated in Figure 7.11, having a conditional standard deviation of 1815. It is clear that this linear regression line approaches closely to the constant mean value of 10760. Indeed, values of $\delta E/w$ at

ages of 4 days and 28 days by the regression line were found to be 10390 and 11230 respectively, resulting in an average value of 10870, almost identical to the mean value of 10760.

For a $\delta E/W$ value of 10760 and by means of Equation 7.1, a K value of 1.22/384 was obtained, which is very close to a fully-fixed beam case having a K value of 1/384. In conclusion, the five one-way slab models can be analysed as fully fixed beams and that the internal consistency of test results is excellent.

7.7 LONG TERM DEFLECTIONS

7.7.1 Flat Plate Models

The immediate deflections are seldom of any practical importance, but the long term deflections can reach annoying proportions. Creep under load and warping due to shrinkage of concrete are important and can cause large increase in deflection with a lapse of time. Plots of deflection-time curves for the three flat plate models and of the profiles of the loaded slabs are shown in Figures 7.12 to 7.14 with Tables 7.10 to 7.12 summarizing the test results for each of the three flat plate models.

For flat plates 1 and 2 of the first series of tests, the formwork was stripped after the concrete had cured for four days, and the slab was subjected to its own weight. At 7 days, additional dead load was applied over the whole span

of the slab to give a total dead load equal to twice the design dead load, simulating the construction load at an early age of concrete. At 28 days, portion of the construction load was removed, and the total remaining load (sustained constant load) was reduced to one-and-a-half times the design dead load. For flat plate model 3, the concrete was allowed to cure for 10 days before the formwork was removed and the slab was subjected to its own weight. At 28 days, superimposed dead load was applied to the slab, giving a total sustained load of one-and-a-half times the design dead load. The load histories for the three models have been given as shown in Figures 6.6A and 6.6B.

Although the absolute value of initial deflection differs, each deflection curve shows approximately the same gradient. Comparing slab 1 with slab 2, it was found that (Figs. 7.12 and 7.13) slab 2 showed the larger initial deflection due to higher load during the initial stage of the concrete age, and subsequently a larger longterm deflection at the end of the observation period. For slab 1, final deflection of 6.37 mm after 700 days for an initial deflection of 1.29 mm were obtained, and a final value of 6.94 mm (589 days) for an initial deflection of 1.67 mm for slab 2. Final deflections were of the order of 4.94 and 4.16 times the initial deflections. For slab 3 with an initial deflection of 2.12 mm, long term deflection totalled 8.48 mm after 483 days, resulting in a deflection ratio of

8.48/2.12 or 4.00. Table 7.13 presents a summary of the test results of the three flat plate models.

Due to the higher magnitude of applied loads as a result of a design live load to dead load ratio of 0.5 for slab 2 instead of 1.0 as for slab 1, it was expected that the increase in longterm deflection for slab 2 would be much more than that for slab 1. However, this has not been the case. The smaller than anticipated increase in mid-span deflection of slab 2 (5.27 mm over 589 days) relative to that of slab 1 (5.08 mm over 700 days) were due primarily to the quality of concrete. Though the design strength of concretes were specified to be 28.0 MPa for all the slabs, the actual strengths of concrete from ready-mixed concrete plants were found to be 45.5 MPa for slab 2 and 28.0 MPa for slab 1 at 28 days. If it is assumed that "the relationship between deflection of slab and the elastic modulus of concrete is linear", the net mid-span deflection after 28 days and the longterm deflection for slab 2 would have been 2.39 mm and 9.94 mm respectively (the bracketed figures in rows D and F of Table 7.11). Indeed, the increase in mid-span deflection due to creep and shrinkage would be 7.55 mm which was 43 percent more than that for slab 1, i.e., the same magnitude in the increase of deflection (43%) as for the applied loads (40% for construction load and 38% for sustained load).

The magnitude of the long-term deflection was influenced significantly by the deflections or loading histories that occur during construction period. As Table 7.13 (row K) indicates, over 40 per cent of the final deflection occurred during the first 28 days after the casting of the slabs, and that the creep deflection (row H of Table 7.13) was about 2.5 to 3.0 times the initial deflection.

7.7.2, One-way Slab Models

This second series of the laboratory investigation was carried out to examine the effects of different loading histories (Figure 6.12) corresponding to various construction methods, compared to the arbitrarily chosen loading procedure applied to the three flat slab models in series 1. It was hoped that this continued research would yield additional information on the longterm structural behaviour of floor slabs subjected to early construction loads of different magnitudes.

Figures 7.15 to 7.21 show the deflection-time curves for the five slab models which were subjected to different loading histories within the first 28 days after the placement of concrete. In view of Figures 7.15 to 7.21 and Tables 7.14 to 7.18 which represent the test results on the one-way slab models, and of Table 7.19 which summarizes the test results of all five slabs, the following observations can be made.

Measured 28-day deflections of the five slabs indicated that deflections were highly influenced by the magnitude of the incremental loads. An examination of the measured deflections during the construction process suggested that if heavy load were applied at this time, appreciable differences in deflections would occur between the most heavily loaded slab (model 5) and the least loaded slab (model 1). The 28-day final deflections were 2.74 mm for slab model 5 and only 1.66 mm for slab model 1 (row B of Table 7.19), a factor of 1.65. The differences carried through when a comparison of the long term deflections was made between these two slab models, i.e., 4.07 mm for slab model 5 and 3.10 mm for slab model 1 (row E of Table 7.19).

An examination of the deflection curves (Figure 7.16) reveals that the slopes of all these curves were virtually the same after day 28; subsequent deflections after removal of the construction loads seemed to be independent of the prior loading. In other words, the additional deflections due to creep and shrinkage of concrete during the first 28 days of loading could not be recovered despite removal of the load and the identical sustained load on the slabs afterwards.

It is equally interesting to note that the increases in deflections from day 28 to the end of the observation period (i.e. 10 months) for all the five slabs were very close

indeed, 1.44, 1.24, 1.35, 1.34 and 1.33 mm for slabs 1 to 5 accordingly. However, due to the distinct loading histories in the initial stages, final deflections after 10 months for the individual slabs were found to range from 3.10 mm (slab 1) to 4.07 mm (slab 5) for span/deflection ratios of 631 and 481 respectively.

The net deflections after 28 days, taken as the sum of deflections due to loading (plus) and unloading (minus) alone and excluding deflections due to creep and shrinkage, were measured to be 0.60, 0.63, 0.62, 0.60 and 0.58 mm for slabs 1 to 5 respectively. The corresponding final deflections after 10 months were 3.10 mm, 3.96 mm, 3.18 mm, 3.35 mm and 4.07 mm, giving ratios of final deflection to net deflection at 28 days of 5.17, 6.29, 5.12, 5.58 and 7.02 (row J of Table 7.19). The corresponding creep ratios (creep deflection/initial deflection in row H of Table 7.19), varied from 2.98 for the least loaded slabs (#1 and #3) to 4.79 for the most heavily loaded slab (#5), which were larger than the usually assumed value of 2.35 [ACI, 2).

7.8 CONTROL OF DEFLECTION

As far as the deflection limits of $L/240$ as specified by ACI 318-83 Building Code [5] is concerned, it is disturbing that for flat plate models 1, 2 and 3, the measured values of $L/307$, $L/282$ and $L/231$ (Tables 7.10 to

7.12) were very close to or exceeded the specified deflection limit.

Unlike the three flat slab models in which the deflection/span ratios were very close to or higher than allowable ($L/240$), the resulting deflection/span ratios for all the five one-way slabs varied from $1/481$ for slab model 5 (most heavily loaded) to $1/631$ for slab model 1 (least loaded), which were below the specified deflection limit of $1/240$ by ACI 318-83 Building Code[5]. In short, because the simple multiplier for additional longterm deflection as recommended by the ACI 318-83 Building Code was based on experimental data on one way slabs and though was stated to be equally applicable to two-way slabs, its applications to flat slabs may result in serious errors in predicting the longterm or total deflection.

7.9 ANALYSIS AND COMPARISON OF TEST RESULTS

To analyse the long term behaviour of reinforced concrete slabs, initial elastic deflection, deflection due to shrinkage of concrete and deflection due to creep of concrete are the main concerns. As stated in Section 3.9, that errors in estimating the elastic deflection of two-way reinforced concrete slabs seem inevitable, and since the long term deflection is usually several times the elastic deflection, poor correlation between measured and calculated

long term deflections is foreseen should the calculated elastic deflections be used. For a meaningful comparison between results and with the benefit of experimental data, the measured elastic deflections will be used herein as the initial elastic deflections.

7.9.1 A.C.I. Models

According to clause 9.5.2.5 of the ACI 318-83 Building Code [5], the absolute maximum ratio of additional deflection due to creep and shrinkage to initial deflection for slabs can be taken as 2 after five years or 1.2 after one year (Eq.4.1). If the initial deflection of a slab was taken to be equal to the net deflection after 28 days (1.29, 1.67 and 2.12 mm for models 1 to 3), the final mid-span deflections of the three flat plate models would then be three times the net deflection, i.e. 3.87 mm, 4.91 mm and 6.36 mm accordingly which were much lower than the measured values of 6.37 mm, 6.94 mm and 8.48 mm respectively (Tables 7.10 to 7.12).

For the five one way slab models, three times the initial deflections would give long term deflections of 1.80, 1.89, 1.86, 1.80 and 1.74 mm when the measured long term deflections were 3.10, 3.96, 3.18, 3.35 and 4.07 mm accordingly. The ACI 318-83 figures do not seem to be acceptable simply because no distinctions were made for the

design live load to dead load ratio and the heavy construction load applied to the slabs soon after the placement of concrete.

Comparison between calculated long term deflections by ACI-209 model (Eqs. 4.3 and 4.11) and the measured deflections from this study (Figs. 7.12 to 7.14 for flat plate models and Figs. 7.17 to 7.21 for one-way slab models) and from other available test results and field measurements are illustrated in Fig. 7.22 and also in Table 7.20. In every case, the measured long term deflections were found to exceed those calculated by ACI-209 model with a mean of 0.74 and a standard deviation of 0.12 (correlation coefficient=0.9447). Use of a creep factor which is time dependent only and not stress or loading history dependent needs to be reconsidered.

7.9.2 Sbarounis's Model

Comparison of measured deflections and those calculated by Sbarounis's model (Eqs. 5.2 and 5.3) is given in Fig. 7.23 and Table 7.21. Sbarounis's model is the only model to have included the effects of construction load on the long term deflections of slabs. The construction load effects are included implicitly in calculating the secant service load deflection (δ_{sd} in Eq. 5.3).

A relatively better agreement between the measured and calculated deflections (than the ACI 209 model) was obtained as shown in Fig.7.23, with a mean of 0.83 and a standard deviation of 0.19 (correlation coefficient=0.9725). However, Sbarounis's model does not have any means of taking account of the creep occurring during the construction period due to the construction loads, which can be seen to be considerable from Figs.7.12 to 7.14, and that the possibly significant contribution of shrinkage deflection is included simply as a multiplier of the secant deflection.

7.9.3 Proposed Models

MODEL A :

Any method proposed for use in a design office must be reasonably simple to use while including the most significant parameters. For the longterm deflection problem, the most important parameters are elastic deflection due to construction loads, creep deflections due to construction loads, elastic deflection due to sustained load and creep deflection due to sustained load. To include these major construction parameters, the longterm deflection of flat slab structures can be approximated by the following equation;

$$\delta_{total} = \delta_{sust} + \delta_{shrink} + C_u (\delta_{cons} \frac{C.L.}{ULT.} + \delta_{sust}) \quad [7.2]$$

where

δ_{total} = longterm deflection.

δ_{cons} = deflection due to construction load with the elastic modulus taken at the appropriate age

δ_{sust} = deflection due to service (sustained) load

δ_{shrink} = deflection due to shrinkage by ACI-209 (Equations 4.11 and 4.12)

C_u = ACI-209 creep coefficient (Equation 4.2)

C.L. = maximum construction load

Ult. = load capacity of slab = 1.4 D.L.+1.7 L.L.

Application of this equation in comparison with measured deflections from this study (Figs. 7.12 to 7.14 for flat plate models and Figs. 7.17 to 7.21 for one-way slab models) and other test results is given in Fig. 7.24 and also in Table 7.22. It is obvious from Fig. 7.24 that the correlation between measured and calculated deflections is good (mean=0.95, standard deviation=0.11, correlation coefficient=0.9904), and the proposed method is simple to use.

MODEL B :

From the test results of the five one-way slab models and taking into account of both the peak load factor and the

average load factor during the first 28 days after a new slab is cast, test results (Table 7.23) give clear indication that the final deflection of a slab depends to a greater extent upon the peak load factor rather than upon the average load factor. The longterm deflection of slab 5 with the highest peak load factor of 2.36 was found to be 4.07 mm which was greater than that of slab 2 (3.96 mm) with the highest average load factor of 1.80. Probable reasons for this phenomenon are the possibility of cracks developed due to this peak load and that the relatively high immediate deflection with subsequent higher creep deflection might not be recoverable upon removal of this peak load.

Since longterm deflection due to shrinkage warping must be stress or load independent, creep deflection will depend upon two major parameters; (i) peak load/ultimate load ratio, to include the effect due to the magnitude of the construction load and (ii) the ratio between the concrete strength at first loading or at the time when this peak (construction) load was applied and the specified 28-day concrete strength, to include the effect of concrete age at loading. The results of the tests (rows H and M of Table 7.19) in terms of the creep coefficients (creep deflection/elastic or 28-day net deflection) for the five slab models subjected to various peak loads at different ages and of the peak load factors are given in Figure 7.25.

It was found that for a period of up to 10 months under loading, a linear relationship exists between the peak load factor and the creep coefficient and can be described by the following equation,

$$C.C. = 2.50 + (P.L.F. - 0.55) 4.18 \quad [7.3]$$

where

C.C. = CREEP COEFFICIENT @ 306 DAYS

P.L.F. = PEAK LOAD FACTOR = $\frac{\text{PEAK LOAD}}{\text{ULT. LOAD}} \sqrt{\frac{f_c}{f_c'}}$

PEAK LOAD = MAXIMUM APPLIED LOAD

ULT. LOAD = LOAD CAPACITY OF SLAB

f_c = CONCRETE STRENGTH WHEN PEAK LOAD IS APPLIED

f_c' = 28-DAY CONCRETE STRENGTH

From A.C.I. 209's model for time-dependent creep coefficient (Equation 4.2), the creep coefficient at 306 days can be estimated to be about 0.756 of the ultimate creep coefficient. Equation 7.3 can therefore be replaced by the following expression for the ultimate creep coefficient;

$$\begin{aligned} \text{ULT.C.C.} &= 3.3 + (P.L.F. - 0.55) 5.5 \quad [7.4A] \\ &> 2.35 \end{aligned}$$

A minimum value of 2.35 for ultimate creep coefficient is suggested as is recommended according to A.C.I. 209's creep model (Equation 4.2). Long term deflection of a slab can therefore be calculated by the following formula;

$$\delta_{total} = \delta_{initial} (1 + C_t) + \delta_{shrink} \quad [7.4B]$$

where $C_t = \text{ULT. CC} \frac{t^{0.6}}{10 + t^{0.6}}$

δ_{shrink} = SHRINKAGE DEFLECTION (EQ. 4.11)

Figures 7.12 to 7.14 illustrate the application and much improved accuracy by Equation 7.4 compared to the measured long term deflections of the three flat plate models tested in this study. Between measured and calculated deflections by equation and 7.4, it can be seen from Table 7.24 and Fig. 7.26 that the correlation is excellent with a mean of 1.01 and a standard deviation of 0.10 (correlation coefficient=0.9969).

7.10 CONCLUDING REMARKS

This research involved an experimental investigation into the effect of early age construction loads on the short term and long term deflections of reinforced concrete slabs. Including construction effects, creep and shrinkage of concrete, total long term deflections of up to 5 times the measured or 8 times the calculated 28-day service load deflection have been measured. Test results show that long term deflection is a problem and cannot be ignored in the design process if reinforced concrete slab structures are to behave adequately during construction and service.

Two approximate methods are given to calculate long term deflection which include the major construction param-

eters and give reasonable results for the eight slabs tested as well as for some other test results and field measurements. Since in practice, designers continue to rely upon experimental information and empirical expressions about longterm behaviour of concrete, the two design methods (Equations 7.2 and 7.4) taking into account the effects of early age construction loads are satisfactory to calculate the long term deflection of slabs.

Chapter VIII

CONCLUSION

During incremental construction a ~~multi~~story flat slab building is subjected to the combined effects of dead loads and construction loads. This report describes in brief both the analytical methods for the computation of two-way slab deflections and laboratory tests to determine the instantaneous and longterm deformations of reinforced concrete flat plate and one-way slab structures. Several loading histories were considered to represent various construction methods commonly adopted in practice.

8.1 CONCLUSION

On the basis of the results from the present study and from the literature, the following conclusions can be made.

(1) The errors in the prediction of elastic deflections are large no matter which method is used. Discrepancies could be attributed to the torsional effects in the regions between the columns and the slab elements. The classical theories of plates, based on the assumption of ideally elastic concrete plate, do not necessarily reflect the true state of stress in a slab.

(2) Of the several methods employed in computing the initial deflections of flat plates, only the modified frame method or the S-method and the beam deflection formulas gave reasonably comparable values. As the computation of slab deflection has become a routine design office procedure, Scanlon and Murray's modified beam deflection formulas give a simple and convenient design means for predicting the initial deflections of flat plate slabs.

(3) Time dependent creep and shrinkage have significant effects on the total or long term deflections of the flat plates. Total deflections of up to 4.94, 4.16 and 4.00 times the initial deflections have been reached for flat plate 1 after 700 days, flat plate 2 after 589 days and flat plate 3 after 483 days respectively.

(4) Although general design is covered by the ACI building code, the current building code provisions on deflection consider only a few of the many factors affecting long-term deflections. Creep multipliers based on beam test data are not suitable for flat slabs and the thickness and thickness-to-span limitations prescribed by the Code cannot be expected to prevent excessive deflections in every case.

(5) Sustained loads affected the deformation, but its effect on the modulus of elasticity of concrete was less than expected. For flat plates 1 and 3, the modulus of elasticity of loaded and unloaded (after 21, 40 or 178 days;

Table 6.2) concrete cylinders were identical to that of concrete free of any stresses.

(6) It was found experimentally that the creep deformation of concrete can exceed elastic deformation by a factor of 5 and additional deformation due to drying of concrete alone can account to as much as another 2 times the elastic deformation.

(7) The experimental data showed the difference in creep coefficients for the three different concretes and the variations of creep coefficient with age of loading. The creep coefficient increased as the sustained stress level increased or as the stress/strength ratio of concrete at loading increased.

(8) Effects of member size on shrinkage strains were less obvious. However, the dependence of shrinkage upon quality of concrete is tremendous. The recorded concrete shrinkage strains were much less for flat plate model 2 (645 microstrains after 589 days) with a 28-day concrete strength of 45.5 MPa compared to that for the other slab models with lower concrete strengths, 786 microstrains for flat plate model 1 after 700 days (concrete strength = 28.0 MPa), 698 microstrains for flat plate model 3 after 483 days (concrete strength = 34.1 MPa), and 742 microstrains for one way slab models after 306 days (concrete strength = 33.6 MPa).

(9) Since shrinkage due to drying of concrete is often assumed to be stress independent, the ratios of final strains to initial strains vary, depending on the magnitudes of initial strains or stresses.

(10) ACI-209's [2] shrinkage equation (Eq.4.3) is easy to use and appeared to provide a good estimation of the shrinkage strains.

(11) ACI-209's [2] creep equations (Eqs.4.2 and 4.4) tend to underestimate the creep deformation for reinforced concrete slabs.

(12) Compression reinforcement is very effective in reducing the time dependent longterm deflection of reinforced concrete beams and logically should be equally effective for two-way slabs.

(13) The sequence of stripping or reshoring has a significant influence on longterm deflections. Different construction processes produced total deflections as high as seven times the initial deflections with the use of Grundy's shoring system (one way slab model 5), or as low as five times the initial deflections with the use of shoring/reshoring system (one way slab model 3).

(14) Stress is not a reliable way of controlling deflections. For structures that are to be subjected to moderately high stress at service load over a long period of

time, creep should become a design consideration, because the greatest risk would appear to be the longterm deflection.

(15) Creep deflection is peak load and not average load dependent, i.e., high transient load would result in a larger creep deflection than for a sustained load of moderate magnitude during construction process.

(16) Two empirical formulas are presented and have been proved to be satisfactory in computing the longterm deflection of a slab, taking into consideration the effects of construction loads.

8.2 RECOMMENDATIONS

(1) The discrepancy in the measured and computed deflections emphasizes the need for better method for computing immediate deflections of two-way slabs.

(2) Future full-scaled tests on flat slabs are recommended to take into the consideration of the following parameters : (a) variation in construction and sustained loads; (b) rate of strength development of the concrete; (c) curing methods; (d) span/depth ratios; and (e) interior or exterior slab panels.

(3) Compression steel is recommended for flat slabs so as to reduce or eliminate the possibly appreciable warping, especially in thin slabs.

(4) In order to avoid extensive cracking and thus high immediate and long term deflections of flat slabs, it is desirable to determine the exact variations in magnitudes of the dead and live loads which may occur during the construction period.

(5) Though designers and engineers prefer a shortened stripping time for floor slabs, it is recommended that specifications be made of the concrete strength required before the removal of form so as to prevent damage or even failure to slabs due to premature removal of forms.

(6) Usual practice calls for the following precautions, which should be followed to reduce excessive deflections;

(a) control construction loads to prevent cracking of concrete during construction; (b) ensure top reinforcement locations to achieve maximum stiffness at supports and (c) proper curing.

(7) The key to reduce the long term deflection is to keep any transient load during construction period to a minimum, simply because the long term deflection is sensitive more to the peak load than to the average load to which a slab is subjected.

(8) Long term deflection of a slab consists of an initial deflection due to service and/or early construction loads, plus the deflections due to creep and shrinkage, and

equations 7.2 (model A) and 7.4 (model B) can be used to approximate the long term deflection of a slab.

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(133) Winter G. and Nilson A.H., 'Design of Concrete Structures', 9th Edition, McGraw Hill Book Co., New York, 1979

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(135) Yamamoto T., 'Longterm Deflections of Reinforced Concrete Slabs Subjected to Overloading at An Early Age', International Conference on "Concrete at Early Ages", RILEM, Ecole Nationale des Ponts et Chausses, Paris, April 6-8, 1982, Vol.1, pp.103-108

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TABLES-

AUTHOR	MAXIMUM VALUE, CONVERGED VALUES IN BRACKETS				COMMENT
	M = 2	M = 3	M = 4	M = 5	
NIELSEN [88]	2.17 ^{xx} 2.00 ⁼	2.28 ^x 2.56 ⁺			VALUES FOR FLOOR LEVEL 2 ONLY x x TIMBER PROPS (N = 5) x TIMBER PROPS (N = 1) + STEEL PROPS (N = 1) = OBSERVED
GRUNDY AND KABAILA [54]	2.25 (2.00)	2.36 (2.00)	2.43 (2.00)		(N = 5)
BERESFORD [14]	2.25 (2.00)	2.36 * (2.00) 2.35 * (2.06) 2.44 * (2.32)	2.45 (2.00)	2.50 (2.00)	* OBTAINED FOR RAPID HARDENING, NORMAL AND SLOW MATURING CONCRETES RESPECTIVELY (N = 5)
BLAKEY AND BERESFORD [15]	2.25 2.25**	2.30			** STEPWISE CONSTRUCTION
BERESFORD [14]		2.2- 1.5			(N = 4) - OBSERVED
MAROSSZEKY [73]		1.33			RESHORING TECHNIQUE - 1 LEVEL OF SHORING AND 2 LEVELS OF RESHORING
CHAN [32]		1.62			STRUDL ANALYSIS USING PRESHORING TECHNIQUE (N = 5)
AGARWAL AND GARDNER [9]		2.02 (A) 2.11 (B)			MEASURED VALUES (A) 3 LEVELS OF SHORES, 4 LEVELS OF RESHORES AND A CONSTRUCTION RATE OF TWO FLOORS A WEEK (B) 3 LEVELS OF SHORES, NO RESHORES AND A CONSTRUCTION RATE OF ABOUT ONE FLOOR A WEEK
LASISI AND NG [70]		1.47 (N=5) 1.56 (1)			(1) OBSERVED - CONSTRUCTION LIVE LOAD INCLUDED AND A CONSTRUCTION CYCLE OF ONE FLOOR PER 10 DAYS

M = NUMBER OF LEVELS OF SHORING USED

N = TIME IN DAYS FOR REMOVAL OF LOWEST LEVELS OF SHORES AFTER CONCRETING TOP FLOOR

TABLE 2.1 FLOOR LOADING RATIOS

PREDICTION METHOD						
REQUIRED INPUT DATA	ACI	BaP	C78	DIN	BCS	C70
SIZE AND SHAPE OF MEMBER	X	X	X	X	X	X
AGE AT LOADING	X	X	X	X	X	X
HUMIDITY OF ENVIRONMENT	X	X	X	X	X	X
COMPRESSIVE STRENGTH OF CONCRETE	X	X	X	X	X	X
CONSISTENCY OF FRESH CONCRETE	X		X	X		
TEMPERATURE		X	X	X		X*
TYPE OF CEMENT	X	X	X	X	X	X
CEMENT CONTENT		X				X
UNIT WEIGHT OF CONCRETE	X	X				
WATER-CEMENT RATIO		X				X
SAND-AGGREGATE RATIO	X	X				
AGGREGATE-CEMENT RATIO		X				
AIR CONTENT OF CONCRETE	X					
CURING CONDITIONS **		X				

* DURING THE CURING PERIOD PRIOR TO LOADING

** HUMIDITY AND BEGINNING OF DRYING

ACI - ACI COMMITTEE 209

[2]

BaP - BAZANT AND PANULA

[12]

C78 - CEB/FIP MODEL CODE 1978

[30]

DIN - GERMAN PRESTRESSED CONCRETE CODE

[41]

BCS - BRITISH CONCRETE SOCIETY

[20]

C70 - CEB/FIP RECOMMENDATIONS 1970

[29]

TABLE 4.1 REQUIRED INPUT DATA FOR CREEP
OF CONCRETE

	RATIO = $\frac{\text{LONG TERM DEFLECTION}}{\text{INITIAL DEFLECTION}}$	
TAYLOR [117]	4.2 - LOADED AT 7 DAYS 3.0 - LOADED AT 14 DAYS 1.8 - LOADED AT 28 DAYS	
SCANLON & MURRAY [105]	3.5 - CRACKED SECTION 4.5 - UNCRACKED SECTION	
RANGANE[99]	3.35 - CRACKED SECTION	
WILSON [131]	CONCRETE STRENGTH AT TIME OF LOADING	
	5.10 - 0.6 f_c' 4.16 - 0.8 f_c' 3.78 - 1.0 f_c'	
SBAROUNIS [102]	5.00 X δ_{sd} * (EQUATIONS 5.2 AND 5.3)	

* δ_{sd} = SUM OF THE IMMEDIATE DEFLECTION DUE TO SLAB WEIGHT AND THAT DUE TO SERVICE LOAD OTHER THAN SLAB WEIGHT
= $\delta'_{max} (W_{sd}/W_{max})$

δ'_{max} = DEFLECTION DUE TO CONSTRUCTION LOD. W_{max}

W_{max} = MAXIMUM CONSTRUCTION LOAD WHEN APPLIED AS A SINGLE INCREMENT

W_{sd} = TOTAL SERVICE DEAD LOAD OR THE TOTAL SUSTAINED LOAD

TABLE 5.1 ANALYTICAL LONG TERM DEFLECTION RATIOS

AUTHOR	YEAR	LONGER SPAN-MM	AGE OF INIT.-DEF. (DAYS)	INIT.-DEF. (MM)	AGE OF FINAL DEFLECTION	FINAL DEF-MM	RATIO*	DEF/SPAN**
C.S.I.R.O.	1961	3937	10	1.27	8 MONTHS	12.7	10.0	1/310
TAYLOR & HEIMAN	1970	6293	14	2.80	9 YEARS	24.4	8.7	1/258
TAYLOR	1971	8676	56	3.56	1 YEAR 3.5 YEARS 8.0 YEARS	13.97 22.35 23.40	3.9 6.3 6.7	1/621 1/388 1/370
HEIMAN & TAYLOR	1972	7544	17	8.60	610 DAYS 3.5 YEARS	18.87 21.80	2.2 2.5	1/401 1/346
JENKINS	1974	6477	10	2.00	1 YEAR 27 MONTHS	7.90 11.70	4.0 5.9	1/820 1/554
YAMAMOTO	1982	3600	1	1.29	112 DAYS	5.71	4.4	1/630
		3600	2	0.95	112 DAYS	5.35	5.6	1/673
		3600	3	0.81	112 DAYS	5.46	6.7	1/659
		3600	7	0.77	112 DAYS	4.92	6.4	1/732
		3600	28	0.72	112 DAYS	3.92	5.4	1/918
		5000	3	4.15	112 DAYS	19.62	4.7	1/255
		5000	28	4.05	112 DAYS	14.78	3.6	1/338
SBAROUNIS	1984	6828	28	5.33	1 YEAR	34.29*	6.4	1/199

* FINAL DEFLECTION/INITIAL DEFLECTION

** FINAL DEFLECTION/LONGER SPAN LENGTH

+ AVERAGE DEFLECTION OF 175 BAYS ON 13 FLOORS OF A FLAT PLATE STRUCTURE

TABLE 5.2 SUMMARY OF MEASURED LONG TERM DEFLECTIONS

	FLAT PLATES			ONE-WAY SLABS				
SLAB DESIGNATION	1	2	3	1	2	3	4	5
DATE OF CASTING	18-2 1983	10-6 1983	23-9 1983	16-3 1984	16-3 1984	16-3 1984	16-3 1984	16-3 1984
CLEAR SPAN (M)	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
SQUARE COLUMN SIZE (MM)	203	203	203	203	203	203	203	203
SLAB THICKNESS (MM)	63.5	63.5	63.5	63.5	63.5	63.5	63.5	63.5
SLUMP (MM)	76	76	76	76	76	76	76	76
TIME AT FORM STRIPPING (DAYS)	4	4	10	5	4	5	5	5
28-D CONCRETE STRENGTH (MPA)	28.0	45.5	34.1	33.6	33.6	33.6	33.6	33.6
STEEL YIELD STRENGTH, MPA	417	417	417	417	417	417	417	417
STEEL YOUNG'S MODULUS GPA	200	200	200	200	200	200	200	200
MAIN STEEL DIAMETER, MM	5	5	5	5	5	5	5	5
ρ_{min} FOR WWF REINF. SLABS (%)	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180
ρ_{min} PROVIDED (%)	0.253	0.253	0.253	0.380	0.380	0.380	0.380	0.380
ρ_{max} PROVIDED (%)	0.507	0.507	0.507	0.380	0.380	0.380	0.380	0.380
TORSIONAL REINFORCEMENT	YES •	YES •	YES •	--	--	--	--	--
YIELD STRENGTH -TOR. REINF. MPA	400	400	400	--	--	--	--	--

• FIGURE 6.4

TABLE 6.1 PROPERTIES OF SLAB MODELS

	SLAB 1	SLAB 2	SLAB 3	ONE-WAY SLABS
	CONCRETE MIX PROPORTIONS (KG/CU.M)			
CEMENT	305	360	335	330
WATER	172	160	165	179
AGGREGATE	1009	1050	1040	1009
SAND	863	790	820	864
	f _c IN MPA		E _c IN GPa	
	f _c	E _c	f _c	E _c
DAY 3	17.8	----	----	----
DAY 4	19.9	21.7	30.9	29.4
DAY 5	----	----	34.7	31.5
DAY 7	22.6	24.5	35.8	32.9
DAY 10	24.8	25.5	----	----
DAY 11	----	----	----	----
DAY 12	----	----	38.0	34.9
DAY 14	26.2	25.9	38.6	----
DAY 21	----	----	----	----
DAY 28	28.0 25.8 •	26.9 26.9 •	45.5	38.5
DAY 68	----	----	----	34.8 33.4 31.3 •
DAY 185	30.0 29.5 •	27.9 27.9 •	46.9	39.5
1 YEAR	30.0	28.6	47.6	39.9
2 YEARS	30.2	----	----	----

* - CREEP CYLINDERS INITIALLY LOADED AT
7 DAYS FOR SLAB 1 OR
28 DAYS FOR SLAB 3

TABLE 6.2 CONCRETE PROPERTIES

	FLAT PLATE MODEL 1	FLAT PLATE MODEL 2	FLAT PLATE MODEL 3	ONE-WAY SLAB MODEL
CONCRETE STRENGTH (MPA)	28.0	45.5	34.1	33.6
SHRINKAGE STRAINS	786 (700 DAYS)	645 (589 DAYS)	698 (483 DAYS)	742 (306 DAYS)

(SHRINKAGE IN MICRO-STRAINS)

TABLE 7.1A MEASURED CONCRETE SHRINKAGE STRAINS

SHRINKAGE STRAINS (MICOR IN/IN)																							
A.C.I. [2]	800 FOR STANDARD CONDITIONS : SLUMP = 3 IN; AIR CONTENT = 0 TO 6 %, R.H.=40% , MIN.THICKNESS = 6 IN. AND CEMENT = 470 - 752 LB./CU.YD																						
ZELTIN, THORNTON AND LEW [136]	<table><tr><th></th><th>HIGH</th><th>AVERAGE</th><th>LOW</th></tr><tr><td>LIGHTWEIGHT STANDARD</td><td>1010</td><td>850</td><td>790</td></tr><tr><td>FINELY GROUND</td><td>770</td><td>660</td><td>600</td></tr><tr><td>HIGH EARLY STRENGTH</td><td>810</td><td>680</td><td>630</td></tr><tr><td></td><td>820</td><td>700</td><td>640</td></tr></table>				HIGH	AVERAGE	LOW	LIGHTWEIGHT STANDARD	1010	850	790	FINELY GROUND	770	660	600	HIGH EARLY STRENGTH	810	680	630		820	700	640
	HIGH	AVERAGE	LOW																				
LIGHTWEIGHT STANDARD	1010	850	790																				
FINELY GROUND	770	660	600																				
HIGH EARLY STRENGTH	810	680	630																				
	820	700	640																				
FINTEL [45]	150 EXPANSION 400 - 800 CONTRACTION AT 50% R.H.																						
LEA [71]	<table><tr><th>WATER/CEMENT RATIO</th><th>AGGREGATE/CEMENT RATIO</th></tr><tr><td>0.4</td><td>0.5</td></tr><tr><td>800</td><td>1200</td></tr><tr><td>550</td><td>850</td></tr><tr><td>400</td><td>600</td></tr><tr><td>300</td><td>400</td></tr><tr><td>200</td><td>300</td></tr></table> - EXPERIMENTAL DATA OBTAINED FROM 5 IN. SQUARE SECTIONS FOR A R.H. OF 50 % AND AT A TEMPERATURE OF 71°F (21°C)			WATER/CEMENT RATIO	AGGREGATE/CEMENT RATIO	0.4	0.5	800	1200	550	850	400	600	300	400	200	300						
WATER/CEMENT RATIO	AGGREGATE/CEMENT RATIO																						
0.4	0.5																						
800	1200																						
550	850																						
400	600																						
300	400																						
200	300																						
TROXELL, RAPHAEL AND DAVIS [123]	900 1 YEAR - R.H. = 50 , AND 66 TO 85 % OF THE 1100 20 YEARS 20-YEAR SHRINKAGE OCCURRED IN ONE YEAR																						
CONCRETE SOCIETY (HOBBS AND PARROTT, REF.59)	900 - 6 MONTHS AT 50 % (INDOOR) 950 - 30 YEARS																						
DRYSDALE, MIRZA AND McCUTCHEON [42]	677 (ONE YEAR) $\epsilon_{SH} = 0.000111 + 0.00024 \cdot \log(\text{AGE}-28)$ WHERE $\epsilon_{SH}(\text{AGE}=28 \text{ DAYS}) = 0.000111 = \text{SHRINKAGE STRAIN AT 28 DAYS}$																						
BAZANT/PANULA [12]	450 - 1200 FORM COLLECTED EXPERIMENTAL DATA SLAB 1 : 1150 SLAB 2 : 510 SLAB 3 : 853 ONE-WAY SLABS : 794 (ESTIMATED FROM APPENDIX S)																						

TABLE 7.1B SUMMARY OF MEASURED SHRINKAGE OF CONCRETE

	SLAB 1	SLAB 2	SLAB 3
(A) INITIAL STRAIN (MICRO.MM/MM)	372	278 380 •	278
(B) FINAL STRAIN	2652	2188 2745 •	2380
(C) STRAIN RATIO = B/A	7.13	7.87 7.22 •	8.56
(D) STRAIN INCREASE (= A-B)	2280	1910 2365 •	2102
(E) MEASURE SHRINKAGE STRAIN**	786	645	698
(F) DRYING CREEP ***	311	255 365 •	276
(G) PLASTIC FLOW (= D-E-F)	1183	1010 1355 •	1128
(H) BASIC CREEP = (G) /STRESS (MICRO-STRAIN PER MPA)	131	111 109 •	124
(I) INCREASE IN STRAIN DUE TO CREEP ONLY (=D-E)	1494	1265 1720 •	1404
(J) CREEP COEFFICIENT (= I/A)	4.02	4.55 4.53 •	5.05
(K) TOTAL CREEP STRAINS (BAZANT)	1384	1294 1553 •	1113
(L) CREEP COEFFICIENT (= K/A)	3.72	4.65 4.09 •	4.00
(M) CREEP COEFFICIENT (ACI)	1.97	1.93 1.93 •	1.89

(STRAINS IN 0.000001 MM/MM)

- * APPLIED STRESS = 12.3 MPA
APPLIED STRESS = 9.0 MPA FOR ALL OTHER CYLINDERS

- ** 700, 589 AND 483 DAYS FOR SLABS 1, 2 AND 3

- *** BY GAMBLE AND PAROTT'S EQUATIONS (APPENDIX T)
DRYING CREEP = $K\sigma\epsilon_{sh}$

$K = 0.044 / \text{MPa}$; ϵ_{sh} = SHRINKAGE

σ = APPLIED STRESS IN MPA

$K = 0.396$ FOR STRESS OF 9.00 MPA

$K = 0.566$ FOR STRESS OF 12.3 MPA

TABLE 7.2 TEST RESULTS OF CONCRETE CYLINDERS
UNDER SUSTAINED COMPRESSION

	SPECIFIC CREEP (ULTIMATE CREEP PER UNIT OF SUSTAINED STRESS)
FINTEL [45]	0.2 TO 2 MICRO. IN/IN/PSI
NEVILLE [84]	1.4 2000 PSI 0.8 4000 PSI .55 6000 PSI 0.4 8000 PSI FOR CONCRETE LOADED AT 7 DAYS WITH 28-DAY STRENGTH OF MICRO IN/IN/PSI $Ct = C(28) \cdot (-6.19 + 2.15 \cdot \log_e(t))^{0.38}$ I.e. AT 365 DAYS, $Ct = 2.0 \cdot C(28)$ $Ct =$ LONGTERM SPECIFIC CREEP IN MICRO IN/IN/MPA $C(28) =$ SPECIFIC CREEP AFTER 28 DAYS
DRYSDALE, MIRZA AND McUTCHEON [42]	CREEP STRAIN = $-A + B \log(t)$ (LEAST SQUARE FIT OF EXPERIMENTAL DATA) $A = 1030400 \cdot Z^3 + 971 \cdot Z^2 + 0.57 \cdot Z + 0.000177$ $B = 1858300 \cdot Z^3 + 1775 \cdot Z^2 + 1.90 \cdot Z + 0.000732$ $Z = \epsilon_{EL} - 0.000500$ $\epsilon_{EL} =$ ELASTIC STRAIN I.e. APPLIED STRESS 1000 PSI, CREEP STRAIN = 0.5 MICRO-IN/IN/PSI 1500 PSI, (AFTER 100 = 0.67 2000 PSI, DAYS) = 0.9
BAZANT AND PANULA [12]	DOUBLE POWER LAW (APPENDIX-S) : $C_0(t, t') = \frac{\phi}{E_0} (t'^{-m} + \alpha) (t - t')^n$ EXPERIMENTAL DATA : 0.2 TO 1.2 MICRO-IN/IN/PSI FOR CONCRETE LOADED AT OR AFTER 7 DAYS
H. C. FU (1985) (EXPERIMENTAL DATA)	0.76 TO 0.91 MICRO-STRAIN/PSI (109 TO 131 MICRO-STRAIN/MPA) - ROW H OF TABLE 7-2

TABLE 7.3 SUMMARY OF SPECIFIC CREEP OF CONCRETE

	CREEP COEFFICIENT (ULTIMATE CREEP FACTOR)
A.C.I. [2]	2.35 FOR STANDARD CONDITIONS : CEMENT CONTENT = 470 TO 720 LB/CU.YD MOIST CURED, RELATIVE HUMIDITY = 40% SLUMP = 3 IN. MIN. THICKNESS = 6 IN. AGE AT LOADING = 7 DAYS ETC.
CEB-FIP-70 [29]	3.25 FOR R.H. OF 30 % - AGE AT LOADING = 28 DAYS 3.10 FOR R.H. OF 40 % - EFFECTIVE THICKNESS < 50 CM • - WATER/CEMENT = 0.5 - CEMENT = 330 KG/CU.M. CORRECTION FACTOR = 1.4 - FOR CONCRETE LOADED AT 7 DAYS CREEP COEFFICIENT = $1.4 \times 3.10 = 4.34$
CONCRETE SOCIETY [37.95]	3.75 AGE AT LOADING = 7 DAYS - R.H. = 40 % - EFFECTIVE THICKNESS < 200 MM
BAZANT/PANULA [12]	3.72 •• ESTIMATED FOR SLAB 1 4.65 •• ESTIMATED FOR SLAB 2 4.09 •• ESTIMATED FOR SLAB 2 4.00 •• ESTIMATED FOR SLAB 3 (REFER TO ROW L OF TABLE 7.2)
H. C. FU (1985) (EXPERIMENTAL DATA)	4.02 SLAB 1 4.55 SLAB 2 4.53 SLAB 2 5.05 SLAB 3 (REFER TO ROW J OF TABLE 7.2)

• EFFECTIVE THICKNESS = 2 X VOLUME/EXPOSED SURFACE AREA

TABLE 7.4 SUMMARY OF CREEP COEFFICIENT OF CONCRETE

	INITIAL (A) DEFLECTION	FINAL (B) DEFLECTION	DEF. RATIO (=B/A)
SLAB 1 (700 DAYS)			
PLAIN CONCRETE BEAM	8.0	24.2	3.02
SINGLE R. C. BEAM	21.0	102.5	4.88
DOUBLY R. C. BEAM	20.0	75.0	3.75
SLAB 2 (589 DAYS)			
PLAIN CONCRETE BEAM	6.0	16.1	3.02
SINGLE R. C. BEAM	16.0	47.0	2.94
DOUBLY R. C. BEAM	15.0	40.7	2.71
SLAB 3 (483 DAYS)			
PLAIN CONCRETE BEAM	7.0	22.4	3.20
SINGLE R. C. BEAM	17.0	85.3	5.01
DOUBLY R. C. BEAM	16.0	62.1	3.88

(DEFLECTIONS IN 0.01 MM)

CONCRETE STRENGTHS : 28.0 MPA FOR SLAB 1
45.5 MPA FOR SLAB 2
34.1 MPA FOR SLAB 3

LOADS ON CONTROL BEAMS :
445 N ON PLAIN CONCRETE BEAMS
1335 N ON SINGLY OR DOUBLY R.C. BEAMS

TABLE 7.5 TEST RESULTS OF CONTROL BEAMS
(CREEP OF CONCRETE IN FLEXURE)

	UDL KPA	E (MPA)	POISSON RATIO- ν	DEFLECTION AT MID-SPAN .01MM	REMARKS
H. C. FU (1985)	2.3	28	---	78 (SLAB 1) 73 (SLAB 2) 1-10 (SLAB 3)	EXPERIMENTAL DATA ¹
ADOSS - CPCA [25]	2.3	28	0.0	58	EQUIVALENT FRAME METHOD [10.14]
STRUDL II [113]	2.3	28	0.2	55	FINITE ELEMENT METHOD
STRUDL II [113]	2.3	28	0.0	60	FINITE ELEMENT METHOD
TIMOSHENKO [122]	2.3	28	0.0	50	ELASTIC SOLUTION (K=0.00581) •
SNEDDON & HILL [111]	2.3	28	0.0	46 60 ••	DIAGONAL BEAM STRIP ANALOGY (K=0.00531) •
VANDERBILT ET AL [124]	2.3	28	0.0	50	FINITE DIFFERENCE & MOMENT DISTRIBUTION
VANDERBILT ET AL [124]	2.3	28	0.0	67	S-METHOD
SCANLON & MURRAY [106]	2.3	28	0.0	64	BEAM DEFLECTION FORMULAS
NOPARC [126]	2.3	28	0.0	61	F.E.M. WITH TIME STEP INTEGRATION SCHEME

• DEFLECTION = $K Q L^4 / D$ (EQ. 3.2)

•• USING DEFLECTION FORMULA FOR A FIXED-ENDS BEAM

- DEFLECTION = $Q L^3 / (384 \cdot E \cdot I)$

- $L = \sqrt{2} \cdot 1.96 \text{ M} = 2.77 \text{ M}$

- $I = I_G$

- $Q = \text{LOAD PER UNIT LENGTH}$

1 - ELASTIC DEFLECTIONS AT 28 DAYS

TABLE 7.6 ELASTIC DEFLECTIONS OF FLAT PLATES

	SLAB 1	SLAB 2	SLAB 3
ON DAY 4	922	1463	---
ON DAY 7	1075	1107	---
ON DAY 10	---	---	2299
ON DAY 28	902	883	1423

($\delta E/W$ IN 10000 MM)

TABLE 7.7 SUMMARY OF $\delta E/W$ VALUES
(FLAT PLATE MODELS)

SLAB NO	1		2		3		4		5	
DAY NO.	M	C	M	C	M	C	M	C	M	C
4 *	----	----	14.0	10.0	----	----	----	----	----	----
4 **	----	----	11.0	10.6	----	----	----	----	----	----
5 *	13.5	9.75	----	----	14.5	9.75	13.0	9.75	15.0	9.75
5 **	28.5	25.0	----	----	26.5	25.0	26.0	25.0	29.5	25.0
7	----	----	60.0	47.0	10.5	11.2	11.2	11.2	8.00	11.1
12	----	----	----	----	-10.	-107	-6.0	-6.8	5.00	3.50
14	----	----	----	----	11.0	10.6	11.5	10.3	16.5	10.3
19	----	----	----	----	-11.5	-104	-5.0	-5.0	8.00	7.70
21	----	----	----	----	11.5	10.4	12.5	10.1	10.5	9.75
26	----	----	----	----	-11.5	-103	-225	-188	-58.	-41.
28	18.0	15.0	-22.	-15.	21.0	15.0	19.0	15.0	23.5	15.0
*** NET DEF AT 28-D	60.0	49.8	63.0	52.6	62.0	50.5	60.0	50.8	58.0	51.0
TOTAL DEF. (A)	310.	313.	396.	351.	318.	378.	335.	377.	407.	355.

- * DUE TO SLAB DEAD LOAD
- ** DUE TO ADDITIONAL DEAD LOAD
- *** DUE TO LOADING/UNLOADING ONLY
(+ FOR LOADING AND - FOR UNLOADING)

(A) AFTER 306 DAYS

M - MEASURED DEFLECTIONS (0.01 MM)
C - COMPUTED DEFLECTIONS BY NOPARC [126]

TABLE 7.8 ELASTIC DEFLECTIONS OF ONE WAY
SLAB MODELS

SLAB NO	1	2	3	4	5
DAY NO.	$\delta E/W$	$\delta E/W$	$\delta E/W$	$\delta E/W$	$\delta E/W$
4 *	---	12000	---	---	---
4 **	---	9430	---	---	---
5 *	12040	---	12930	11600	13380
5 **	10890	---	10130	9940	11280
7	---	11700	8430	9000	6430
12	---	---	8400	7790	12980
14	---	---	9330	10050	14410
19	---	---	9890	9140	9040
21	---	---	9940	11130	9350
26	---	---	10030	10760	12650
28	10730	13110	12510	13200	14000

* DUE TO SLAB DEAD LOAD

** DUE TO ADDITIONAL DEAD LOAD

$\delta E/W$ (MEAN) = 10760

$\delta E/W$ (STANDARD DEVIATION) = 1851

LINEAR REGRESSION ANALYSIS :

LEAST SQUARE REGRESSION LINE :

$$\delta E/W = 10255 + 34.78 (T)$$

T = AGE OF SLAB IN DAYS

CONDITIONAL STANDARD DEVIATION = 1815

(REFER TO FIGURE 7-11)

TABLE 7.9 SUMMARY OF $\delta E/W$ VALUES
(ONE WAY SLAB MODELS)

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 4 (FORM REMOVAL)	+64	DUE TO SLAB DEAD LOAD
(B) AT DAY 7 (ADDITIONAL DEAD LOAD)	+96	TOTAL LOAD = A + B
(C) AT DAY 28 PARTIAL LOAD REMOVED	-31	25 % OF TOTAL LOAD REMOVED
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	129	FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	322 ** 291	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE PARTIAL LOAD REMOVAL
(F) TOTAL DEFLECTION AFTER 700 DAYS	637	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	307	= SPAN/(F) ; SPAN = 1956 MM
(H) TIME DEPENDENT DEFLECTION RATIO	4.94	= F/D

TABLE 7.10 MEASURED DEFLECTIONS OF FLAT PLATE MODEL 1

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 4 (FORM REMOVAL)	+75	DUE TO SLAB DEAD LOAD
(B) AT DAY 7 (ADDITIONAL DEAD LOAD)	+121	TOTAL LOAD = A + B
(C) AT DAY 28 PARTIAL LOAD REMOVED	-29	25 % OF TOTAL LOAD REMOVED
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	167 (239)	+ FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	331 ** 302	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE PARTIAL LOAD REMOVAL
(F) TOTAL DEFLECTION AFTER 589 DAYS	694 (994)	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	282 (197)	= SPAN/(F) ; SPAN = 1956 MM
(H) TIME DEPENDENT DEFLECTION RATIO	4.16 (4.16)	= F/D

() - DEFLECTIONS READJUSTED WITH RESPECT TO MODULAR RATIO OF SLAB 2 AND SLAB 1
 E_c (SLAB 2)/ E_c (SLAB 1) = 1.432 ; I.E. 167 X 1.432 = 239 FOR (D) & ETC.,

TABLE 7.11 MEASURED DEFLECTIONS OF FLAT PLATE MODEL 2

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 10 (FORM REMOVAL)	+110	DUE TO SLAB DEAD LOAD
(B) AT DAY 28 (ADDITIONAL DEAD LOAD)	+102	TOTAL LOAD = A + B
(C) AT DAY 28 PARTIAL LOAD REMOVED	---	25 % OF TOTAL LOAD REMOVED
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	212	+ FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	345 237 **	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE ADDITIONAL LOAD ON
(F) TOTAL DEFLECTION AFTER 483 DAYS	848	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	231	= SPAN/(F) ; SPAN = 1956 MM
(H) TIME DEPENDENT DEFLECTION RATIO	4.00	= F/D

TABLE 7.12 MEASURED DEFLECTIONS OF FLAT PLATE MODEL 3

	SLAB 1	SLAB 2	SLAB 3
(A) 28-DAY NET DEFL.	129	167	212
(B) TOTAL DEFLECTION AT 28 DAYS	291	302	345
(C) 28-DAY SHRINK. (1)	347	347	398
SHRINKAGE DEF. (2)	57	57	66
(D) CREEP DEFL. AT 28 DAYS (=B-C-A)	105	78	67
(E) FINAL DEFL.	637 (700 DAYS)	694 (589 DAYS)	848 (483 DAYS)
(F) TOTAL SHRINK. (1)	786	645	698
SHRINKAGE DEF. (2)	130	107	116
(G) TOTAL CREEP DEFL. (=E-F-A)	378	420	520
(H) G/A	2.930	2.515	2.453
(J) E/A	4.94	4.16	4.00
(K) B/E	0.46	0.44	0.41
(L) F/E	0.204	0.154	0.137

(DEFLECTIONS IN 0.01 MM)

- (1) MEASURED SHRINKAGE IN MICRO-STRAIN
(2) A.C.I. MODEL - EQUATIONS 4.11 AND 4.12

CLEAR SPAN OF SLAB = 1.96 M. HOWEVER, DUE TO NEGATIVE REINFORCEMENT AT COLUMN CORNERS AND THAT THEORETICALLY, SHRINKAGE DEFLECTION IS ZERO FOR EQUAL AMOUNT OF +VE AND -VE REINFORCEMENT. I.E. SPAN LENGTH SUBJECTED TO SHRINKAGE WARPING = $1.96/2 = 0.98\text{M}$ THEN ITS DIAGONAL LENGTH = $\sqrt{2}(980\text{ MM}) = 1386\text{ MM}$, WHICH CAN BE ASSUMED TO BE SIMPLY SUPPORTED WITH A REINFORCEMENT RATIO OF 0.253 %

- G = CREEP DEFLECTION (PLASTIC FLOW)
H = CREEP COEFFICIENT = CREEP DEFL./ELASTIC DEFL.
J = FINAL DEFL./ELASTIC (28-DAY NET) DEFLECTION
K = 28-DAY DEFLECTION/TOTAL DEFLECTION
L = SHRINKAGE DEFLECTION/TOTAL DEFLECTION

TABLE 7.13 SUMMARY OF TEST RESULTS OF FLAT PLATE MODELS

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 5 (FORM REMOVAL)	13.5 28.5	DUE TO SLAB DEAD LOAD DUE TO ADDITIONAL LOAD
(B) AT DAY 7 (ADDITIONAL DEAD LOAD)	----	TOTAL LOAD = A + B
(C) AT DAY 28	+18	ADDITIONAL DEAD LOAD ON
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	60	+ FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	148 ** 166	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE ADDITIONAL LOAD (C) ON
(F) TOTAL DEFLECTION AFTER 306 DAYS	310	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	631	= SPAN/(F) ; SPAN = 1956 MM
(H) TIME DEPENDENT DEFLECTION RATIO	5.17	= F/D

TABLE 7.14 MEASURED DEFLECTIONS OF ONE WAY SLAB 1

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 4 (FORM REMOVAL)	14.0 11.0	DUE TO SLAB DEAD LOAD DUE TO ADDITIONAL LOAD
(B) AT DAY 7 (ADDITIONAL DEAD LOAD)	60.0	TOTAL LOAD = A + B
(C) AT DAY 28	-22	25 % OF TOTAL LOAD REMOVED
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	63	+ FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	294 ** 272	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE PARTIAL LOAD (C) REMOVAL
(F) TOTAL DEFLECTION AFTER 306 DAYS	396	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	494	= SPAN / (F) ; SPAN = 1956 MM
(H) TIME DEPENDENT DEFLECTION RATIO	6.29	= F / D

TABLE 7.15 MEASURED DEFLECTIONS OF ONE WAY SLAB 2

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 5 (FORM REMOVAL)	14.5 26.5	DUE TO SLAB DEAD LOAD DUE TO ADDITIONAL LOAD LOAD
(B) AT DAY 7 (ADDITIONAL DEAD LOAD)	-----	TOTAL LOAD = A + B
(C) AT DAY 28	+21	ADDITIONAL DEAD LOAD ON
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	62	+ FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	162 ** 183	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE ADDITIONAL LOAD (C) ON
(F) TOTAL DEFLECTION AFTER 306 DAYS	318	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	615	= SPAN/(F) : SPAN = 1956 MM
(H) TIME DEPENDENT DEFLECTION RATIO	5.12	= F/D

TABLE 7.16 MEASURED DEFLECTIONS OF ONE WAY SLAB MODEL 3

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 5 (FORM REMOVAL)	13.0 26.0	DUE TO SLAB DEAD LOAD DUE TO ADDITIONAL LOAD
(B) AT DAY 7 (ADDITIONAL DEAD LOAD)	-----	TOTAL LOAD = A + B
(C) AT DAY 28	+19	ADDITIONAL DEAD LOAD ON
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	60	+ FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	182 ** 201	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE ADDITIONAL LOAD ON
(F) TOTAL DEFLECTION AFTER 306 DAYS	335	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	584	= SPAN/(F) : SPAN = 1956 MM
(H) TIME DEPENDENT DEFLECTION RATIO	5.58	= F/D

TABLE 7.17 MEASURED DEFLECTIONS OF ONE WAY SLAB MODEL 4

	DEFLECTION (0.01 MM)	REMARKS
(A) AT DAY 5 (FORM REMOVAL)	15.0 29.5	DUE TO SLAB DEAD LOAD DUE TO ADDITIONAL LOAD
(B) AT DAY 7 (ADDITIONAL DEAD LOAD)	----	TOTAL LOAD = A + B
(C) AT DAY 28	+24	ADDITIONAL DEAD LOAD ON
(D) NET DEFLECTION AT 28 DAY (DUE TO LOADING/UNLOADING ONLY)	58	+ FOR LOADING AND - FOR UNLOADING
(E) TOTAL DEFLECTION AT 28 DAY	240 ** 274	INCLUDING CREEP/SHRINKAGE EFFECTS ** BEFORE ADDITIONAL LOAD ON
(F) TOTAL DEFLECTION AFTER 306 DAYS	407	INCLUDING CREEP/SHRINKAGE EFFECTS
(G) SPAN TO DEFLECTION RATIO	481	= SPAN/(F) • SPAN = 1956 MMK
(H) TIME DEPENDENT DEFLECTION RATIO	7.02	= F/D

TABLE 7.18 MEASURED DEFLECTIONS OF ONE WAY SLAB MODEL 5

	SLAB 1	SLAB 2	SLAB 3	SLAB 4	SLAB 5
(A) 28-DAY NET DEFL.	60	63	62	60	58
(B) TOTAL DEFLECTION AT 28 DAYS	166	272	183	201	274
(C) 28-DAY SHRINK. (1)	380	380	380	380	380
SHRINKAGE DEF. (2)	36	36	36	36	36
(D) CREEP DEFL. AT 28 DAYS (=B-C-A)	70	173	85	105	180
(E) FINAL DEFLECTION (10 MONTHS)	310	396	318	335	407
(F) TOTAL SHRINK. (1)	742	742	742	742	742
SHRINKAGE DEF. (2)	71	71	71	71	71
(G) TOTAL CREEP DEFL. (=E-F-A)	179	262	185	204	278
(H) G/A (FIG. 7.25)	2.98	4.16	2.98	3.40	4.79
(J) E/A	5.17	6.29	5.12	5.58	7.02
(K) F/E	0.23	0.18	0.22	0.21	0.17
(L) PEAK LOAD f_c/f_c'	1.00	2.00	1.34	1.62	2.35
	0.757	0.814	0.814	0.960	0.960
(M) PEAK LOAD FACTOR	0.510	0.985	0.660	0.736	1.066

(DEFLECTIONS IN 0.01 MM)

(1) MEASURED SHRINKAGE IN MICRO-STRAIN

(2) A.C.I. MODEL - EQUATIONS 4.11 AND 4.12

CLEAR SPAN OF SLAB = 1.96 M. HOWEVER, DUE TO NEGATIVE REINFORCEMENT AT FIXED ENDS AND THAT THEORETICALLY, SHRINKAGE DEFLECTION IS ZERO FOR EQUAL AMOUNT OF +VE AND -VE REINFORCEMENT.

i.e., SPAN LENGTH SUBJECTED TO SHRINKAGE WARPING = $1.96/2$ OR 0.98 M WHICH CAN BE ASSUMED AS SIMPLY SUPPORTED WITH A REINFORCEMENT RATIO OF 0.380 %

G/ = CREEP DEFLECTION (PLASTIC FLOW)

H/ = CREEP COEFFICIENT = CREEP DEFL./ELASTIC DEFL.

J = FINAL DEFL./ELASTIC (28-DAY NET) DEFLECTION

K = SHRINKAGE DEFLECTION / TOTAL DEFLECTION

L = PEAK LOAD = MAXIMUM CONSTRUCTION LOAD (FIGURE 6.12)

$L = f_c/f_c' = \frac{\text{CONCRETE STRENGTH AT LOADING}}{\text{SPECIFIED 28-DAY CONCRETE STRENGTH}}$

M = (PEAK LOAD/ULTIMATE LOAD) / $\sqrt{f_c/f_c'}$

TABLE 7.19 TEST RESULTS OF ONE-WAY SLAB MODELS

ACI-209 MODEL : EQUATIONS 4.4 AND 4.11 FOR CREEP AND SHRINKAGE

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(J)	(K)	(L)
	SPAN/ DEPTH RATIO	STRIP FORM AT 7 DAYS	INIT. DEFL. (MM)	ULT. CREEP COEFF.	CREEP COEFF. C1	CREEP DEFL. = E * C	SHRINK DEFL. (MM)	TOTAL DEFL. = C + F + G	LOADING PERIOD (DAYS)	MEAS. DEFL. (MM)	CAL. MEAS. (J/L)
H.C.FU (1985) FLAT SLAB 1	30.8	4	1.29	2.50	2.09	2.70	1.30 ¹	5.29	700	6.37	0.83
H.C.FU (1985) FLAT SLAB 2	30.8	4	1.87	2.50	2.05	3.42	1.07 ¹	6.16	589	6.94	0.89
H.C.FU (1985) FLAT SLAB 3	30.8	10	2.12	2.35	1.89	4.00	1.16 ¹	7.28	483	8.48	0.86
YAMAMOTO (1982) ONEWAY SLAB 1	30.0	1	1.29	2.94	1.851	2.37	1.16 ²	4.82	112	5.71	0.84
YAMAMOTO (1982) ONEWAY SLAB 2	30.0	2	0.95	2.71	1.705	1.82	1.16 ²	3.73	112	5.35	0.70
YAMAMOTO (1982) ONEWAY SLAB 3	30.0	3	0.81	2.58	1.626	1.32	1.16 ²	3.29	112	5.46	0.60
YAMAMOTO (1982) ONEWAY SLAB 4	30.0	7	0.77	2.35	1.481	1.14	1.16 ²	3.07	112	4.92	0.62
YAMAMOTO (1982) ONEWAY SLAB 5	30.0	28	0.72	2.35	1.481	1.07	1.16 ²	2.85	112	3.92	0.75
YAMAMOTO (1982) ONEWAY SLAB 6	38.5	3	4.15	2.58	1.626	6.75	2.38 ²	13.28	112	19.62	0.68
YAMAMOTO (1982) ONEWAY SLAB 7	38.5	28	4.05	2.35	1.481	6.00	2.38 ²	12.43	112	14.78	0.84
WASHA (1947) BEAM C	30.0	28	6.10	2.35	2.117	12.51	4.32	23.33	1825	23.68	0.99
JENKINS (1974) FLAT PLATE	26.7	10	2.00	2.35	1.99	3.98	1.50 ³	7.48	810	11.70	0.64
SBAROUNIS (1984) - 175 BAYS ON 13 FLOORS (FLAT PLATE)	35.0	28	5.33	2.35	1.82	9.70	3.76 ³	18.79	365	34.3 ⁴	0.55
TAYLOR AND HEIMAN (1970) - FLAT PLATE	31.0	14	4.70 ⁵	2.35	2.18	10.25	3.74 ³	18.68	3285	24.4	0.77
H.C.FU (1985) ONEWAY SLAB 1	30.8	5	0.60	2.43	1.84	1.10	0.71	2.41	306	3.10	0.78
H.C.FU (1985) ONEWAY SLAB 2	30.8	4	0.63	2.50	1.89	1.19	0.71	2.53	306	3.96	0.64
H.C.FU (1985) ONEWAY SLAB 3	30.8	5	0.62	2.43	1.84	1.14	0.71	2.47	306	3.18	0.78
H.C.FU (1985) ONEWAY SLAB 4	30.8	5	0.60	2.43	1.84	1.10	0.71	2.41	306	3.35	0.72
H.C.FU (1985) ONEWAY SLAB 5	30.8	5	0.58	2.43	1.84	1.07	0.71	2.36	306	4.07	0.58

TABLE 7.20 COMPARISON OF ACI-209 MODEL
WITH TEST RESULTS

1 • TABLE 7.13 (ROW F)

2 • MEASURED SHRINKAGE STRAIN = 0.000670 AT 112 DAYS
DUE TO EQUAL AMOUNT OF POSITIVE AND NEGATIVE REINFORCEMENT AT FIXED ENDS.
EFFECTIVE SPAN LENGTH SUBJECTED TO SHRINKAGE WARPING = 0.5 X SPAN LENGTH

3 • DUE TO INSUFFICIENT INFORMATION,
SHRINKAGE DEFLECTION = 20% FINAL DEFLECTION
= $((C) + (F)) / 4.0$

4 • AVERAGE OF 175 BAYS ON 13 FLOORS OF A FLAT-PLATE STRUCTURE ONE YEAR
AFTER CASTING WITH FINAL DEFLECTIONS RANGING FROM 13.46 TO 54.86 MM

5 • ULTIMATE LOAD = $1.4(110) + 1.7(75) = 281$ PSF
MEASURED DEFLECTION DUE TO SLAB DEAD LOAD (110 PSF) = 2.8 MM AT 14 DAYS
I.E. DEFLECTION DUE TO SERVICE LOAD = $(110+75) \cdot 2.8 / 110$ MM = 4.7 MM

TABLE 7.20 (CONTINUED)

$$\delta_{total} = \delta_{sd} \times \left(2.8 \frac{C_u}{2.35} + 1.2 \frac{\epsilon_{shu}}{800} + 1 \right) \quad [5.2]$$

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(J)	(K)	(L)	(M)
	SPAN/ DEPTH RATIO	STRIP FORM AT 7 DAYS	δ_{s1} (MM)	C.L. AND SER.-L. δ_{s2} (MM)	CREEP COEFF Ct	SHRINK COEFF	MULTI- PLIER. =1+F+G	TOTAL DEFL. =H X E	LOADING PERIOD (DAYS)	MEAS. DEFL. (MM)	CAL. MEAS. (J/L)	
H.C.FU (1985) FLAT SLAB 1	30.8	4	1.60 ¹	3.64 2.73 KPA	1.20	2.48	1.14	4.62	5.54	700	6.37	0.87
H.C.FU (1985) FLAT SLAB 2	30.8	4	1.96 ¹	5.03 3.785 KPA	1.47	2.44	1.13	4.57	6.72	589	6.94	0.97
H.C.FU (1985) FLAT SLAB 3	30.8	10	2.12 ¹	3.785 3.785 KPA	2.12	2.38	1.12	4.50	9.54	483	8.48	1.13
YAMAMOTO (1982) ONEWAY SLAB 1	30.0	1	1.53	2.0 DL 1.5 DL	1.1475	2.20	1.00	4.20	4.82	112	5.71	0.84
YAMAMOTO (1982) ONEWAY SLAB 2	30.0	2	1.20	2.0 DL 1.5 DL	0.9000	2.03	1.00	4.03	3.63	112	5.35	0.68
YAMAMOTO (1982) ONEWAY SLAB 3	30.0	3	1.06	2.0 DL 1.5 DL	0.7950	1.94	1.00	3.94	3.13	112	5.46	0.57
YAMAMOTO (1982) ONEWAY SLAB 4	30.0	7	1.02	2.0 DL 1.5 DL	0.7650	1.76	1.00	3.76	2.88	112	4.92	0.53
YAMAMOTO (1982) ONEWAY SLAB 5	30.0	28	0.96	2.0 DL 1.5 DL	0.7200	1.76	1.00	3.76	2.71	112	3.92	0.69
YAMAMOTO (1982) ONEWAY SLAB 6	38.5	3	5.24	2.0 DL 1.5 DL	3.9300	1.94	1.00	3.94	15.48	112	19.62	0.79
YAMAMOTO (1982) ONEWAY SLAB 7	38.5	28	5.08	2.0 DL 1.5 DL	3.8100	1.76	1.00	3.76	14.31	112	14.78	0.97
WASHA (1947) BEAM C	30.0	28	6.10	1.5 DL 1.5 DL	6.10	2.52	1.18	4.70	28.67	1825	23.68	1.21
JENKINS (1974) FLAT PLATE	26.7	10	3.27 ³	198PSF 180PSF	2.82	2.38	1.15	4.53	12.77	810	11.70	1.09
TAYLOR AND HELMAN (1970) - FLAT PLATE	31.0	14	4.70 ⁴	110PSF 110PSF	4.70	2.60	1.19	4.79	22.50	3285	24.4	0.92
H.C.FU (1985) ONEWAY SLAB 1	30.8	5	0.60 ⁵	1.5 DL 1.5 DL	0.6000	2.19	1.08	4.27	2.56	306	3.10	0.83
H.C.FU (1985) ONEWAY SLAB 2	30.8	4	0.85	2.0 DL 1.5 DL	0.6375	2.25	1.08	4.33	2.76	306	3.96	0.70
H.C.FU (1985) ONEWAY SLAB 3	30.8	5	0.62	1.5 DL 1.5 DL	0.6200	2.19	1.08	4.27	2.65	306	3.18	0.83
H.C.FU (1985) ONEWAY SLAB 4	30.8	5	0.635	1.62DL 1.5 DL	0.5852	2.19	1.08	4.27	2.50	306	3.35	0.75
H.C.FU (1985) ONEWAY SLAB 5	30.8	5	0.925	2.36DL 1.5 DL	0.5904	2.19	1.08	4.27	2.52	306	4.07	0.62

TABLE 7.21 COMPARISON OF SBAROUNIS'S MODEL
WITH TEST RESULTS

- δ_{total} : TOTAL DEFLECTION OF SLAB UNDER SUSTAINED LOAD
 δ_{ed} : SECANT DEFLECTION = $\delta_{cl} \times \frac{SER.L.}{C.L.}$ (EQ. 5.3)
 δ_{cl} : DEFLECTION DUE TO CONSTRUCTION LOAD
 C_t : CREEP COEFFICIENT = $2.0 C_u / 2.35$ (C_u - A.C.I. 209 MODEL, EQ. 4.2)
C.L. : CONSTRUCTION LOAD
SER.L. : SERVICE OR SUSTAINED LOAD
- 1 : SUMMATION OF DEFLECTIONS DURING THE CONSTRUCTION LOAD (FIRST 28 DAYS)
 - (SLAB 1) = 0.64 + 0.96 = 1.60 MM
 - (SLAB 2) = 0.75 + 1.21 = 1.96 MM
 - (SLAB 3) = 1.10 + 1.02 = 2.12 MM
 - 2 : CONSTRUCTION LOAD = 2 X SLAB DEAD LOAD
 SUSTAINED LOAD = 1.5 X SLAB DEAD LOAD
 - 3 : FROM GIVEN DEFLECTION-TIME CURVE, IMMEDIATE DEFLECTION DUE TO CONSTRUCTION LOAD WHEN THE FLOOR ABOVE WAS POURED = 0.05 INCH = 1.27 MM.
 I.E. DEFLECTION DUE TO SLAB DEAD LOAD AND CONST. LOAD = 2 + 1.27 = 3.27 MM
 WHICH CORRESPONDS TO AN EQUIVALENT LOAD OF (3.27/2.0) X 120 PSF OR 196 PSF
 FOR AN ULT. LOAD = 1.4(120) + 1.7(60) = 270 PSF.
 PEAK LOAD/ULTIMATE LOAD = 196/270 = 0.726
 SERVICE LOAD = 120 + 60 = 180 PSF
 - 4 : ULTIMATE LOAD = 1.4(110) + 1.7(75) = 281 PSF
 MEASURED DEFLECTION DUE TO SLAB DEAD LOAD (110 PSF) = 2.8 MM AT 14 DAYS
 I.E. DEFLECTION DUE TO SERVICE LOAD = (110+75)/110 X 2.8 = 4.7 MM
 - 5 : TABLE 7-19 - DEFLECTION DUE TO CONSTRUCTION LOAD

(SLAB 1) = 0.60 MM	= 0.600 MM (AT 28 DAYS)
(SLAB 2) = 0.63 + 0.22	= 0.850 MM (AT 7 DAYS)
(SLAB 3) = 0.62	= 0.620 MM (AT 28 DAYS)
(SLAB 4) = 0.60 - 0.19 + 0.225	= 0.635 MM (AT 21 DAYS)
(SLAB 5) = 0.58 - 0.235 + 0.58	= 0.925 MM (AT 21 DAYS)
 - 6 : FIRST VALUE :
 - (SLAB 1) MAXIMUM LOAD = SUSTAINED LOAD = 1.50 X CALCULATED DEAD LOAD
 - (SLAB 2) MAXIMUM LOAD = LOAD AT 7 DAYS = 2.00 X CALCULATED DEAD LOAD
 - (SLAB 3) MAXIMUM LOAD = SUSTAINED LOAD = 1.50 X CALCULATED DEAD LOAD
 - (SLAB 4) MAXIMUM LOAD = LOAD AT 21 DAYS = 1.62 X CALCULATED DEAD LOAD
 - (SLAB 5) MAXIMUM LOAD = LOAD AT 21 DAYS = 2.36 X CALCULATED DEAD LOAD
 SECOND VALUE :
 FOR ALL FIVE SLABS, SUST. LOAD (28 DAYS) = 1.50 X CALCULATED DEAD LOAD

TABLE 7.21 (CONTINUED)

$$\delta_{total} = \delta_{sust} + c_u \left(\delta_{cl} \frac{C.L.}{ULT.} + \delta_{sust} \right) + \delta_{sh} \quad [7.2]$$

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(J)	(K)	(L)	(M)
	SPAN/ DEPTH RATIO	STRIP FORM AT 7 DAYS	δ_{cl} (IN)	C.L. ULT.-L. RATIO	δ_{sust}	δ_{sh}	ULT. CREEP COEFF.	CREEP COEFF. C1	TOTAL DEFL. [7.2]	LOADING PERIOD (DAYS)	MEAS. DEFL. (IN)	CAL. MEAS. (J/L)
H.C.FU (1985) FLAT SLAB 1	30.8	4	1.80 ¹	0.645	0.930 ²	1.30 ³	2.50 ⁴	2.09 ⁵	6.35	700	6.37	1.00
H.C.FU (1985) FLAT SLAB 2	30.8	4	1.96 ¹	0.890	0.890 ²	1.07 ³	2.50	2.05	7.36	589	6.94	1.06
H.C.FU (1985) FLAT SLAB 3	30.8	10	1.10 ¹	0.260	1.680 ²	1.16 ³	2.35	1.89	6.56	483	8.48	0.77
YAMAMOTO (1982) ONEWAY SLAB 1	30.0	1	1.53	0.70	0.720 ⁶	1.16 ⁷	2.938	1.851	5.20	112	5.71	0.91
YAMAMOTO (1982) ONEWAY SLAB 2	30.0	2	1.20	0.70	0.750 ⁶	1.16 ⁷	2.707	1.705	4.62	112	5.35	0.86
YAMAMOTO (1982) ONEWAY SLAB 3	30.0	3	1.06	0.70	0.750 ⁶	1.16 ⁷	2.560	1.626	4.34	112	5.46	0.79
YAMAMOTO (1982) ONEWAY SLAB 4	30.0	7	1.02	0.70	0.750 ⁶	1.16 ⁷	2.350	1.481	4.08	112	4.92	0.83
YAMAMOTO (1982) ONEWAY SLAB 5	30.0	28	0.96	0.70	0.720 ⁶	1.16 ⁷	2.350	1.481	3.94	112	3.92	1.00
YAMAMOTO (1982) ONEWAY SLAB 6	38.5	3	5.24	0.70	3.270 ⁶	2.38 ⁷	2.580	1.626	16.93	112	19.62	0.86
YAMAMOTO (1982) ONEWAY SLAB 7	38.5	28	5.08	0.70	3.090 ⁶	2.38 ⁷	2.350	1.481	15.31	112	14.78	1.04
WASHA (1947) BEAM 0	30.0	28	2.14 ⁸	0.156	6.10 ⁹	4.32	2.35	2.117	24.04	1825	23.68	1.02
JENKINS (1974) FLAT PLATE	26.7	10	3.27 ¹⁰	0.726	2.00	2.68 ¹¹	2.35	1.99	13.38	810	11.70	1.14
SBAROUNIS (1984) - 175 DAYS ON 13 FLOORS (FLAT PLATE)	35.0	28	9.62 ¹²	0.826	5.33	7.37 ¹¹	2.35	1.82	36.85	365	34.3 ¹³	1.07
TAYLOR AND HEIMAN (1970) - FLAT PLATE	31.0	14	2.80 ¹⁴	0.391	4.70 ¹⁵	4.33 ¹¹	2.35	2.18	21.66	3285	24.4	0.89
H.C.FU (1985) ONEWAY SLAB 1	30.8	5	0.42 ¹⁶	0.44 ¹⁷	0.54 ¹⁸	0.71 ¹⁹	2.43	1.84	2.59	306	3.10	0.84
H.C.FU (1985) ONEWAY SLAB 2	30.8	4	0.85	0.889	0.66	0.71	2.50	1.89	4.06	306	3.96	1.03
H.C.FU (1985) ONEWAY SLAB 3	30.8	5	0.525	0.578	0.630	0.71	2.43	1.84	3.06	306	3.18	0.96
H.C.FU (1985) ONEWAY SLAB 4	30.8	5	0.635	0.720	0.570	0.71	2.43	1.84	3.17	306	3.35	0.95
H.C.FU (1985) ONEWAY SLAB 5	30.8	5	0.925	1.040	0.705	0.71	2.43	1.84	4.48	306	4.07	1.10

TABLE 7.22 COMPARISON OF TEST RESULTS WITH MODEL A

- δ_{total} : TOTAL DEFLECTION OF SLAB UNDER SUSTAINED LOAD
- $\delta_{sust.}$: DEFLECTION DUE TO SUSTAINED (SERVICE) LOAD
- $\delta_{ol.}$: DEFLECTION DUE TO CONSTRUCTION LOAD (BEFORE SUSTAINED LOAD ON)
- C_t : CREEP COEFFICIENT (A.C.I.'S CREEP MODEL)
- C.L. : CONSTRUCTION LOAD
- ULT.L. : ULTIMATE LOAD CAPACITY OF SLAB
- δ_{sh} : SHRINKAGE DEFLECTION BY A.C.I. MODEL (EQUATIONS 4.11 AND 4.12)
- 1 : SUMMATION OF DEFLECTIONS DURING CONSTRUCTION PERIOD (FIRST 28 DAYS)
 (SLAB 1) = $0.64 + 0.96 = 1.60$ MM
 (SLAB 2) = $0.75 + 1.21 = 1.96$ MM
 (SLAB 3) = $1.10 + 0.00 = 1.10$ MM
 - 2 : $\delta(SUST)$ = DEFLECTION AT 28 DAYS X (SUSTAINED LOAD/LOAD AT 28 DAYS)
 (SLAB 1) = $0.31 \times 2.730/0.910 = 0.93$ MM
 (SLAB 2) = $0.29 \times 3.785/1.245 = 0.89$ MM
 (SLAB 3) = $1.02 \times 3.785/2.300 = 1.68$ MM
 - 3 : SHRINKAGE DEFLECTION FROM TABLE 7-13
 - 4 : ULTIMATE CREEP COEFFICIENT ACCORDING TO A.C.I.'S CREEP MODEL
 - 5 : TIME DEPENDENT CREEP COEFFICIENT (EQUATION 4.2)
 - 6 : = DEFLECTION DUE TO REMOVAL OF HALF SLAB DEAD LOAD AFTER THE CONSTRUCTION PERIOD MULTIPLIED BY 3.0 BECAUSE SUSTAINED LOAD = $1.5 \times$ SLAB DEAD LOAD
 - 7 : TABLE 7-20 (COLUMN G)
 - 8 : DEFLECTION DUE TO BEAM DEAD LOAD OF 37.5 PSF (ULT. LOAD = 240 PSF)
 - 9 : DEFLECTION DUE TO SUSTAINED LOAD OF 107 PSF
 - 10 : FROM NOTE 3 OF TABLE 7-21, CONS.LOAD/ULT.LOAD = $196/270 = 0.726$
 DEFLECTION DUE TO CONSTRUCTION LOAD = 3.27 MM
 - 11 : FROM THE RESULTS OF FLAT SLAB MODELS (TABLE 7-13).
 SHRINKAGE DEFLECTION = $0.2 \times$ TOTAL DEFLECTION
 - 12 : DEFLECTION DUE TO SERVICE LOAD OF 87 PSF = 5.33 MM
 DEFLECTION DUE TO CONSTRUCTION LOAD (157 PSF) = $5.33 \times 157/87 = 9.62$ MM
 - 13 : AVERAGE DEFLECTION OF 175 BAYS ON 13 FLOORS OF A FLAT PLATE STRUCTURE ONE YEAR AFTER CASTING WITH ONE-YEAR DEFLECTION RANGING FROM 13.46 MM TO 54.86 MM
 - 14 : DEFLECTION DUE TO SLAB DEAD LOAD (NOTE 4 OF TABLE 7.21)
 - 15 : DEFLECTION TO SERVICE LOAD (NOTE 4 OF TABLE 7.21)
 - 16 : TABLE 7-8 (SLAB 1) = $0.60 - 0.18 = 0.42$ M
 (SLAB 2) = $0.63 + 0.22 = 0.85$ MM (BEFORE
 (SLAB 3) = $0.62 - 0.21 + 0.115 = 0.525$ MM SUSTAINED
 (SLAB 4) = $0.60 - 0.19 + 0.225 = 0.635$ MM LOAD ON)
 (SLAB 5) = $0.58 - 0.235 + 0.58 = 0.925$ MM
 - 17 : CONST. LOADS FOR SLABS 1 TO 5 WERE 1.0, 2.0, 1.34, 1.62 AND 2.36 TIMES THE CAL. DEAD LOAD, AND THE ULTIMATE LOAD = $2.25 \times$ CAL. DEAD LOAD
 - 18 : = DEFLECTION AT 28 DAYS X (SUSTAINED LOAD/LOAD AT 28 DAYS)
 - 19 : TABLE 7-19 (ROW F)

TABLE 7.22 (CONTINUED)

DAYS	4	5	7	12	14	19	21	26	28	H*	A*	FINAL DEFL. (MM)
1	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	3.10
2	0.40	0.40	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	1.80	3.96
3	0.00	1.00	1.34	1.00	1.34	1.00	1.34	1.00	1.00	1.34	1.22	3.18
4	0.00	1.00	1.34	1.12	1.45	1.29	1.62	1.00	1.00	1.62	1.34	3.35
5	0.00	1.00	1.34	1.45	1.78	2.03	2.35	1.00	1.00	2.35	1.66	4.10

SLAB NUMBER

LOAD FACTORS

LOAD ON SLAB = LOAD FACTOR X DESIGN DEAD LOAD OF SLAB
= LOAD FACTOR X 68.5 LB/FT

- H = HIGHEST (PEAK) LOAD DURING THE FIRST 28 DAYS
- A = AVERAGE LOAD DURING THE FIRST 28 DAYS
(BEFORE SUSTAINED LOAD OF 1.5 BEING PUT ON AT 28 DAYS)

SAMPLE CALCULATIONS IN COMPUTING THE AVERAGE LOAD, A.

$$\text{SLAB 2 : } A = (0.4(3) + 2.0(21)) / (3+21) = 43.2/24 = 1.80$$

$$\begin{aligned} \text{SLAB 5 : } A &= \frac{(1.0(2) + 1.34(5) + 1.45(2) + 1.78(5) + 2.03(2) + 2.35(5) + 1.0(2))}{(2+5+2+5+2+2)} \\ &= 38.31/23 = 1.66 \end{aligned}$$

TABLE 7.23 LOAD FACTORS ON ONE-WAY SLAB MODELS

$$F = 3.3 + (E-0.55)5.5 > 2.35 \quad [7.4]$$

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(J)	(K)	(L)	(M)
	SPAN/ DEPTH RATIO	STRIP FORM AT 7 DAYS	INIT. DEFL. (MM)	PEAK/ ULT. RATIO	$\frac{(D)}{\sqrt{f_c}}$	CREEP COEFF. EQU. 24	CREEP DEFL. = (C) X (F)	SHRINK DEFL. = (H)	TOTAL DEFL. = C+G+H	LOADING PERIOD (DAYS)	MEAS. DEFL. (MM)	CAL. MEAS. (J/L)
H.C.FU (1985) FLAT SLAB 1	30.8	4	1.11 ¹	0.645	0.718	4.224 3.533 ²	3.92	1.30 ³	6.24	700	6.37	0.99
H.C.FU (1985) FLAT SLAB 2	30.8	4	1.06 ¹	0.889	1.003	5.782 4.758	3.04	1.07 ³	7.17	589	6.94	1.03
H.C.FU (1985) FLAT SLAB 3	30.8	10	1.97 ¹	0.667	0.667	3.944 3.167	6.24	1.16 ³	9.37	483	8.48	1.10
YAMAMOTO (1982) ONEWAY SLAB 1	30.0	1	1.29	0.70 ⁴	0.881 ^A	5.010 3.155	4.07	1.16 ⁵	6.52	112	5.71	1.14
YAMAMOTO (1982) ONEWAY SLAB 2	30.0	2	0.95	0.70	0.851	4.956 3.122	2.97	1.16	5.08	112	5.35	0.95
YAMAMOTO (1982) ONEWAY SLAB 3	30.0	3	0.81	0.70	0.851	4.956 3.122	2.53	1.16	4.46	112	5.50	0.82
YAMAMOTO (1982) ONEWAY SLAB 4	30.0	7	0.77	0.70	0.740	4.345 2.737	2.11	1.16	4.04	112	4.92	0.82
YAMAMOTO (1982) ONEWAY SLAB 5	30.0	28	0.72	0.70	0.700	4.125 2.599	1.87	1.16	3.75	112	3.92	0.96
YAMAMOTO (1982) ONEWAY SLAB 6	38.5	3	4.15	0.70	0.851	4.956 3.122	12.96	2.38	19.49	112	19.62	0.99
YAMAMOTO (1982) ONEWAY SLAB 7	38.5	28	4.05	0.70	0.700	4.125 2.599	10.53	2.38	16.96	112	14.78	1.15
WASHA (1947) BEAM 9	30.0	28	6.10	0.446	0.446	2.728 2.460	15.00	4.32	25.42	1825	23.68	1.07
JENKINS (1974) FLAT PLATE	26.7	10	2.00	0.726 ⁶	0.726	4.268 3.623	7.25	2.31 ⁷	11.56	810	11.70	0.99
SBAROUNIS (1984) - 175 BAYS ON 13 FLOORS (FLAT PLATE)	35.0	28	5.33	0.826 ⁸	0.997	5.704 4.420	23.56	7.22 ⁷	36.11	365	34.3 ⁹	1.05
TAYLOR AND HEIMAN (1970) - FLAT PLATE	31.0	14	4.70 ¹⁰	0.391 ^B 0.658 ^C	0.417 ^B 0.658 ^C	3.894 ^C 3.614	16.98	5.42 ⁷	27.10	3265	24.4	1.10

(D) = PEAK LOAD / ULTIMATE LOAD

(E) = PEAK LOAD / (ULTIMATE LOAD X $\sqrt{f_c/f'_c}$) WHERE

f_c IS THE COMPRESSIVE STRENGTH OF CONCRETE AT FIRST OR PEAK LOADING

(H) = SHRINKAGE DEFLECTION BY A.C.I. MODEL (EQUATIONS 4.11 AND 4.12)

1 = DUE TO THE DISCREPANCIES IN THE CALCULATED AND MEASURED ELASTIC DEFL., IT WAS DECIDED FOR A MEANINGFUL COMPARISON THAT ELASTIC (INITIAL) DEF. = DEF. AT 28 DAYS DUE TO UNLOADING (SLABS 1 & 2) OR LOADING (SLAB 3) TIMES (SUSTAINED LOAD/LOAD AT 28 DAYS), MULTIPLIED BY A CORRECTION FACTOR (C.F.) FOR THE AGE OF CONCRETE AT FIRST LOADING
C.F. = $1/(\sqrt{f_c/f'_c})$ WHERE f_c IS THE CONCRETE STRENGTH AT FIRST LOADING

TABLE 7.24 COMPARISON OF MODEL B WITH TEST RESULT

1. FOR SLAB 1, DEFLECTION AT 28 DAYS DUE TO UNLOADING = 0.31 MM
 SUSTAINED LOAD/LOAD AT 28 DAYS = $2.730/0.910 = 3.0$
 $f_c/f_c' = 0.7118$ (FIRST LOADING AT 4 DAYS)
 1. INITIAL DEFLECTION = $0.31 \times 3.0 / \sqrt{0.7118} = 1.11$ MM
 FOR SLAB 2, FIRST LOADING AT 4 DAYS:
 INITIAL DEFLECTION = $0.29 \times (3.785/1.245) (1/\sqrt{0.68}) = 1.06$ MM
 FOR SLAB 3, FIRST LOADING AT 10 DAYS:
 INITIAL DEFLECTION = $1.02 \times (3.785/2.300) (1/\sqrt{0.80}) = 1.97$ MM
2. USING A.C.I.'S CREEP MODEL (EQ.4.2) TO CALCULATE THE CREEP COEFFICIENT (2ND VALUE IN COL.F) CORRESPONDING TO THE LOADING PERIOD OF EACH SLAB WITH ULT.CREEP COEFF. (1ST VALUE IN COL.F) FOUND FROM MODEL B (EQ.7.4)
3. TABLE 7.13 (ROW F)
4. AS GIVEN IN THE LITERATURE,
 "70% OF THE YIELD POINT AT THE MAXIMUM LOADING DURING CONSTRUCTION PERIOD"
5. MEASURED SHRINKAGE STRAIN = 0.000670 AT 112 DAYS
 DUE TO EQUAL AMOUNT OF POSITIVE AND NEGATIVE REINFORCEMENT AT FIXED ENDS,
 EFFECTIVE SPAN LENGTH SUBJECTED TO SHRINKAGE WARPING = $0.5 \times$ SPAN LENGTH
6. FROM GIVEN DEFLECTION-TIME CURVE, IMMEDIATE DEFLECTION DUE TO CONSTRUCTION LOAD WHEN THE FLOOR ABOVE WAS POURED = 0.05 INCH = 1.27 MM.
 1. DEFLECTION DUE TO SLAB DEAD LOAD AND CONST. LOAD = $2 \times 1.27 = 3.27$ MM
 WHICH CORRESPONDS TO AN EQUIVALENT LOAD OF $(3.27/2.0) \times 120$ PSF OR 196 PSF
 FOR AN ULT. LOAD = $1.4(120) + 1.7(60) = 270$ PSF,
 PEAK LOAD/ULTIMATE LOAD = $196/270 = 0.726$
7. FROM THE RESULTS OF FLAT SLAB MODELS (ROW K OF TABLE 7.19),
 SHRINKAGE DEFLECTION = $0.2 \times$ TOTAL DEFLECTION
 1. SHRINKAGE DEFL. = $((C) + (G)) / 4.0$
8. FROM SBAROUNIS'S MODEL STRUCTURE, CONSTRUCTION LOAD = 157 PSF (CASE 2)
 ULTIMATE LOAD = $1.4(60+27) + 1.7(40) = 190$ PSF
 PEAK LOAD/ULTIMATE LOAD = $157/190 = 0.826$
 ESTIMATE $f_c/f_c' = 0.7$ (AGE OF 7 DAYS AT LOADING)
9. AVERAGE DEFLECTION OF 175 BAYS ON 13 FLOORS OF A FLAT PLATE STRUCTURE ONE YEAR AFTER CASTING WITH ONE-YEAR DEFLECTION RANGING FROM 13.46 TO 54.86 MM
10. ULTIMATE LOAD = $1.4(110) + 1.7(75) = 281$ PSF
 MEASURED DEFLECTION DUE TO SLAB DEAD LOAD (110 PSF) = 2.8 MM AT 14 DAYS
 1. DEFLECTION DUE TO SERVICE LOAD = $(110+75)/110 \times 2.8 = 4.7$ MM
- A. TWO LOADING CONDITIONS WERE CONSIDERED FOR ALL 7 SLABS, 1. SLAB DEAD LOAD AFTER FORM REMOVAL AND TWICE THE SLAB D.L. DURING CONSTRUCTION PERIOD. IN EVERY BUT ONE SLAB (#1) THAT THE CONSTRUCTION LOADING RESULTED IN A LARGER PEAK LOAD FACTOR; FOR SLAB #1, FORMWORK WAS STRIPPED AFTER ONE DAY
 $f_c/f_c' = 0.165$ AND PEAK LOAD/ULT. LOAD = $0.7/2 = 0.35$
 HENCE, PEAK LOAD FACTOR = $0.35/\sqrt{0.165} = 0.861$
 FOR A CONSTRUCTION LOAD OF TWICE THE SLAB D.L. AND APPLIED AT 7 DAYS,
 PEAK LOAD FACTOR = $0.7/\sqrt{0.677} = 0.851 < 0.861$ THUS, 0.861 WAS USED
- B. CASE 1 : SLAB DEAD LOAD APPLIED AT 14 DAYS;
 PEAK LOAD/ULTIMATE LOAD = $110/281 = 0.391$
 $f_c/f_c' = 0.88$
 1. PEAK/ULT. $(1/\sqrt{(f_c/f_c')}) = 0.417$
- C. CASE 2 : SERVICE LOAD APPLIED AT 28 DAYS;
 PEAK LOAD/ULTIMATE LOAD = $185/281 = 0.658$
 $f_c/f_c' = 1.00$
 1. PEAK/ULT $(1/\sqrt{(f_c/f_c')}) = 0.658 > 0.417$
 HENCE, CREEP COEFF. = $3.3 + (0.658-0.55)5.5 = 3.894$

TABLE 7.24 (CONTINUED)

FIGURES

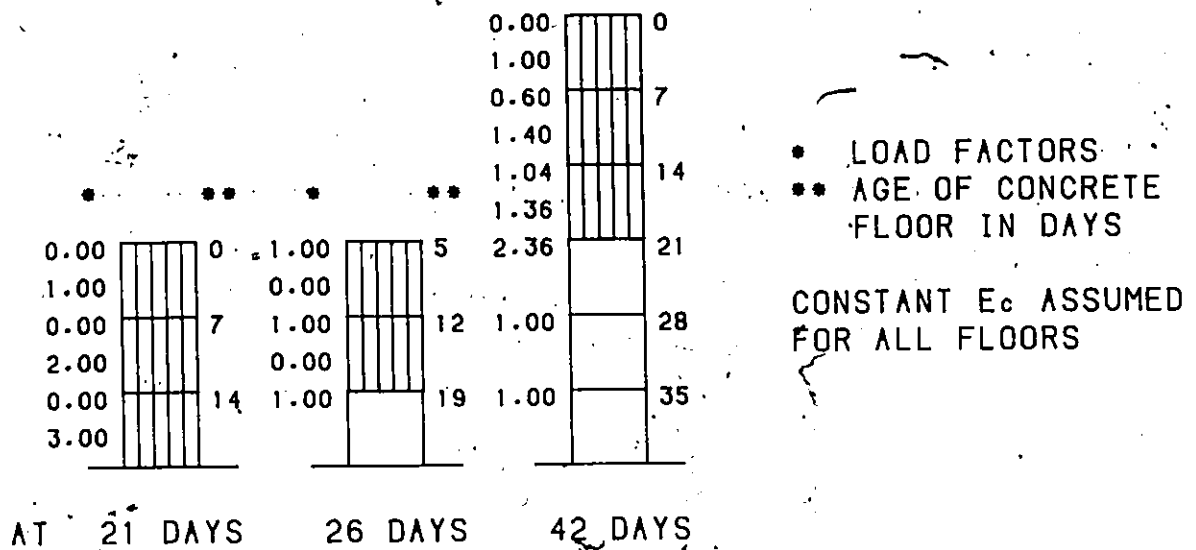


FIG.1.1 TYPICAL LOAD DISTRIBUTIONS ON CONCRETE FLOORS [54]

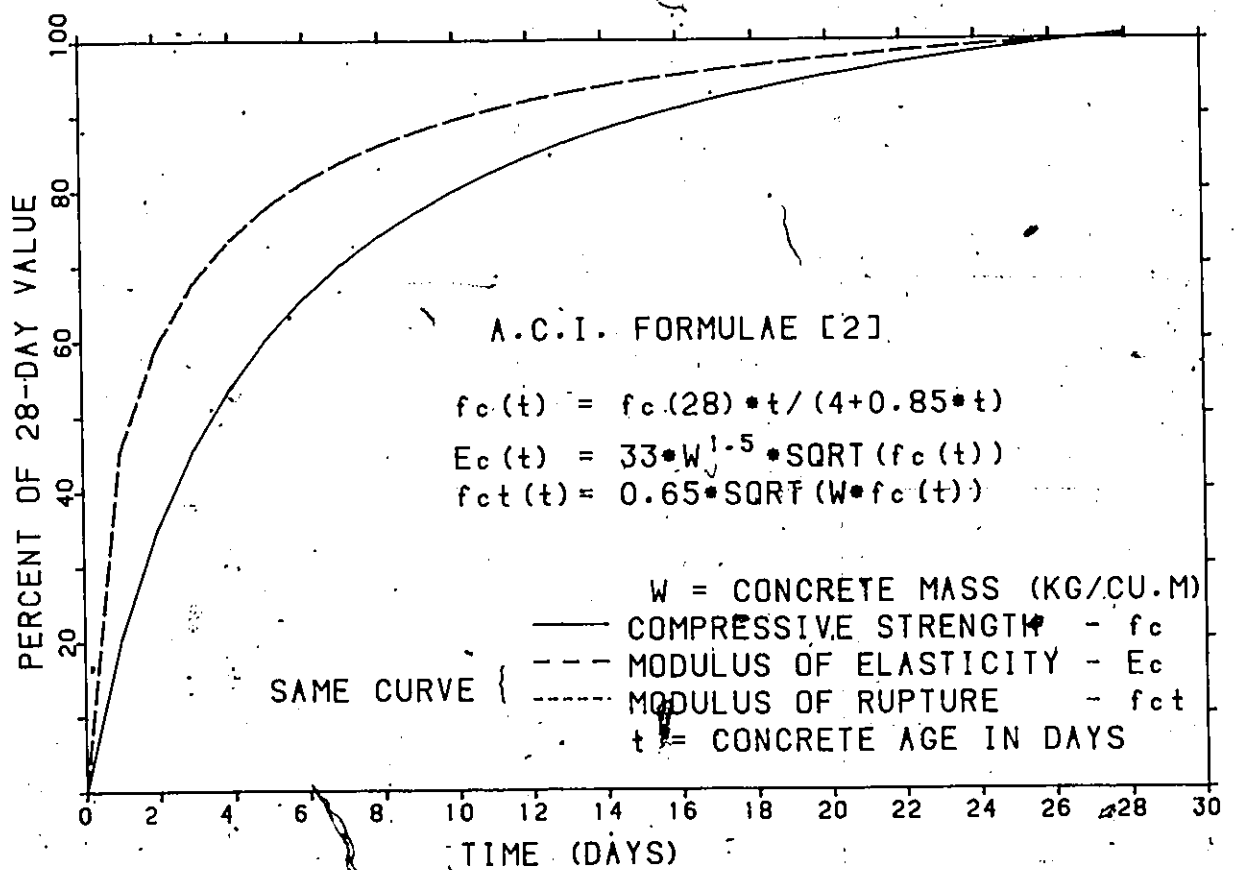


FIG.1.2 TIME DEPENDENT PROPERTIES OF CONCRETE

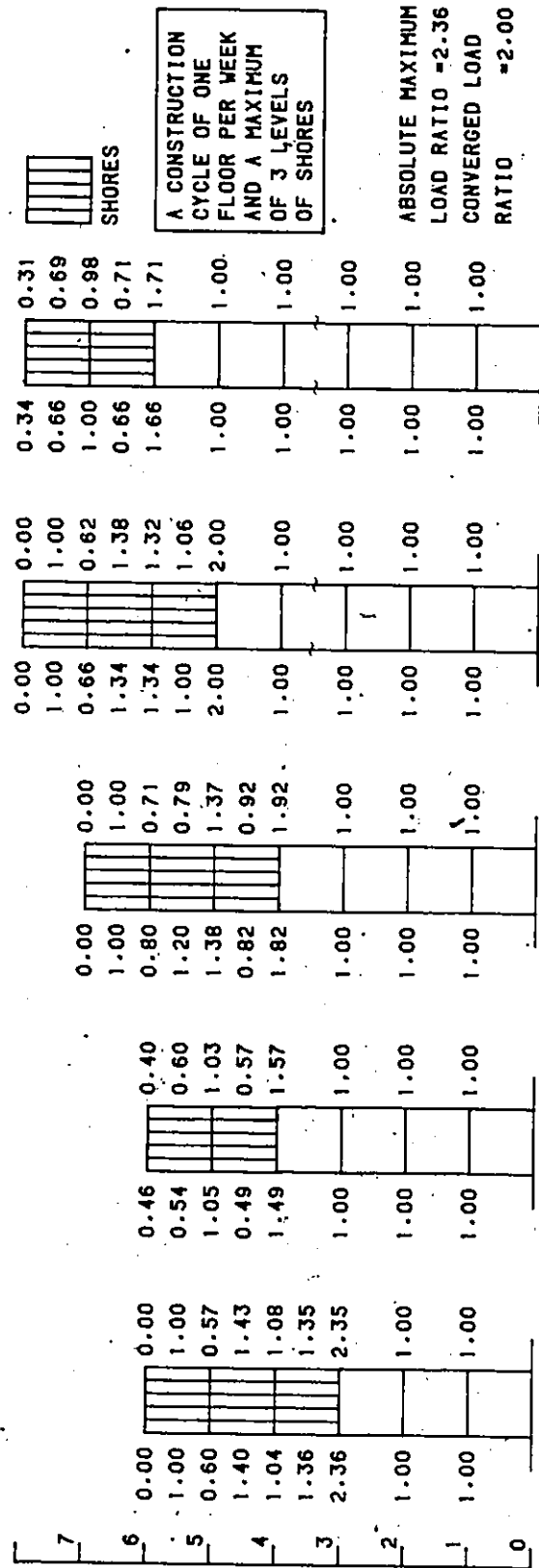
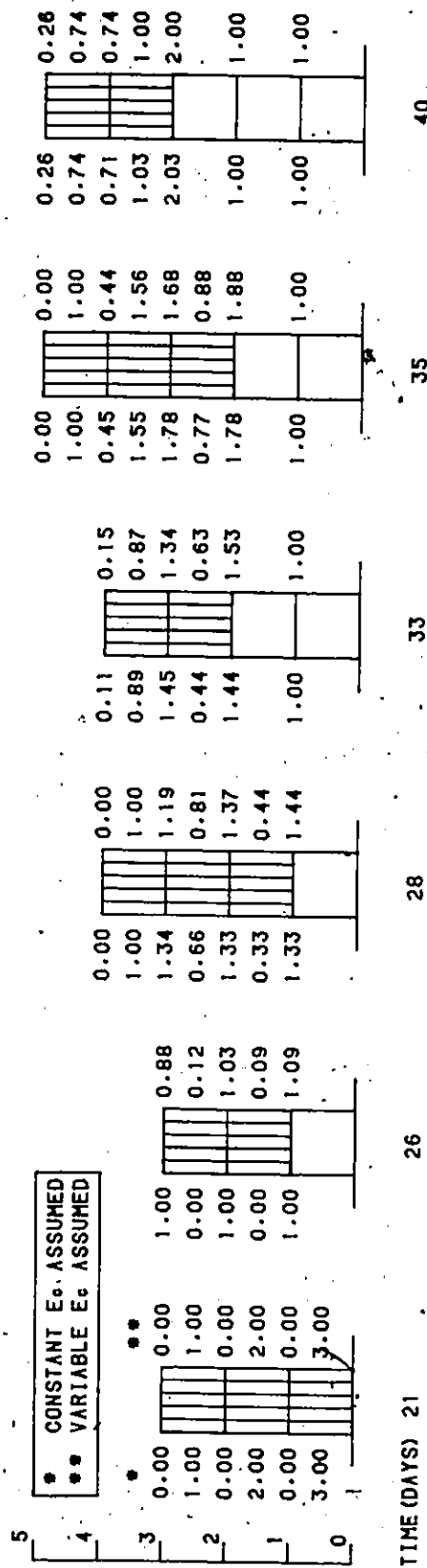


FIG.2.1 CONSTRUCTION LOADS WITH 3 LEVELS OF SHORES [54]

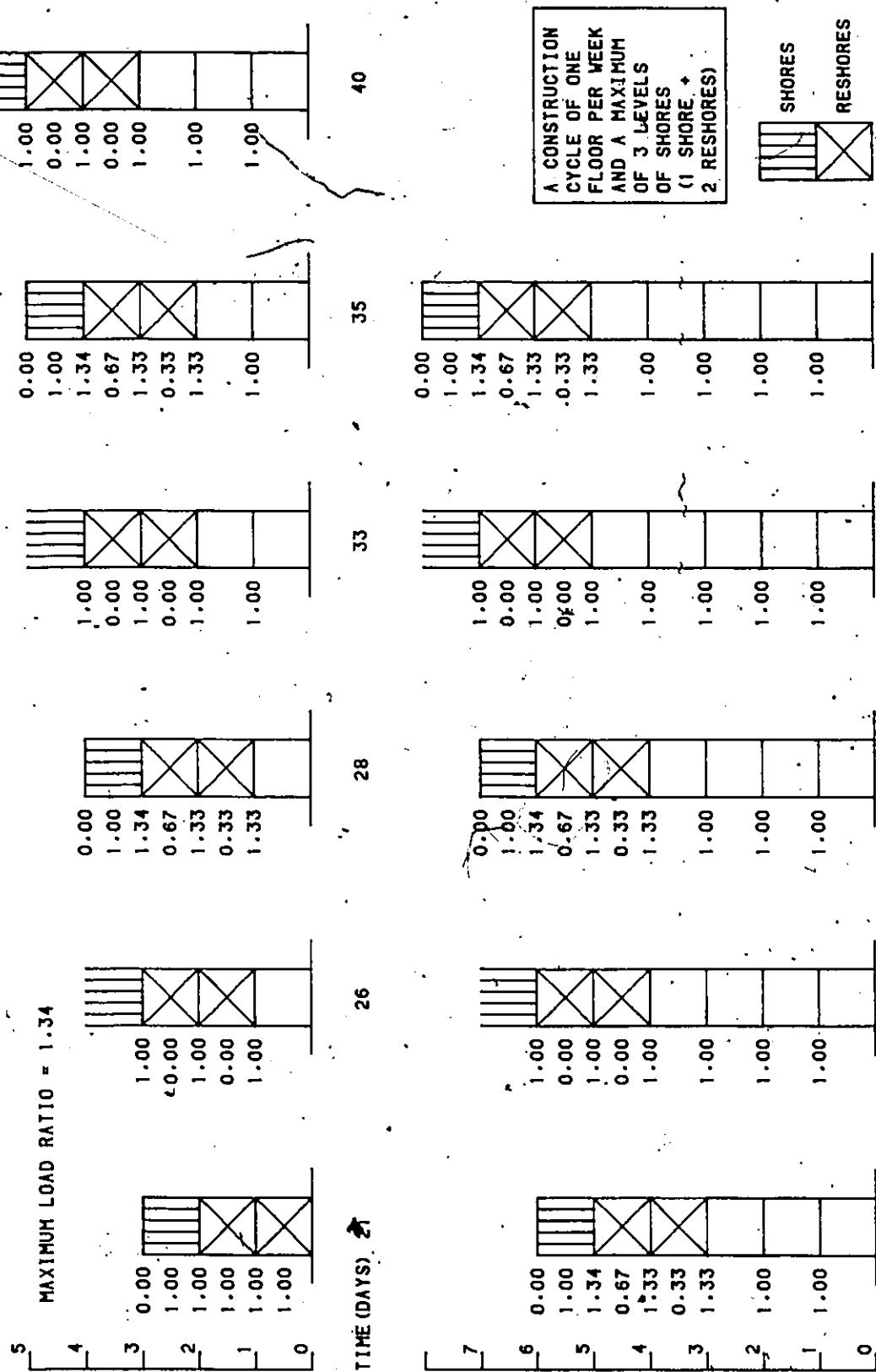
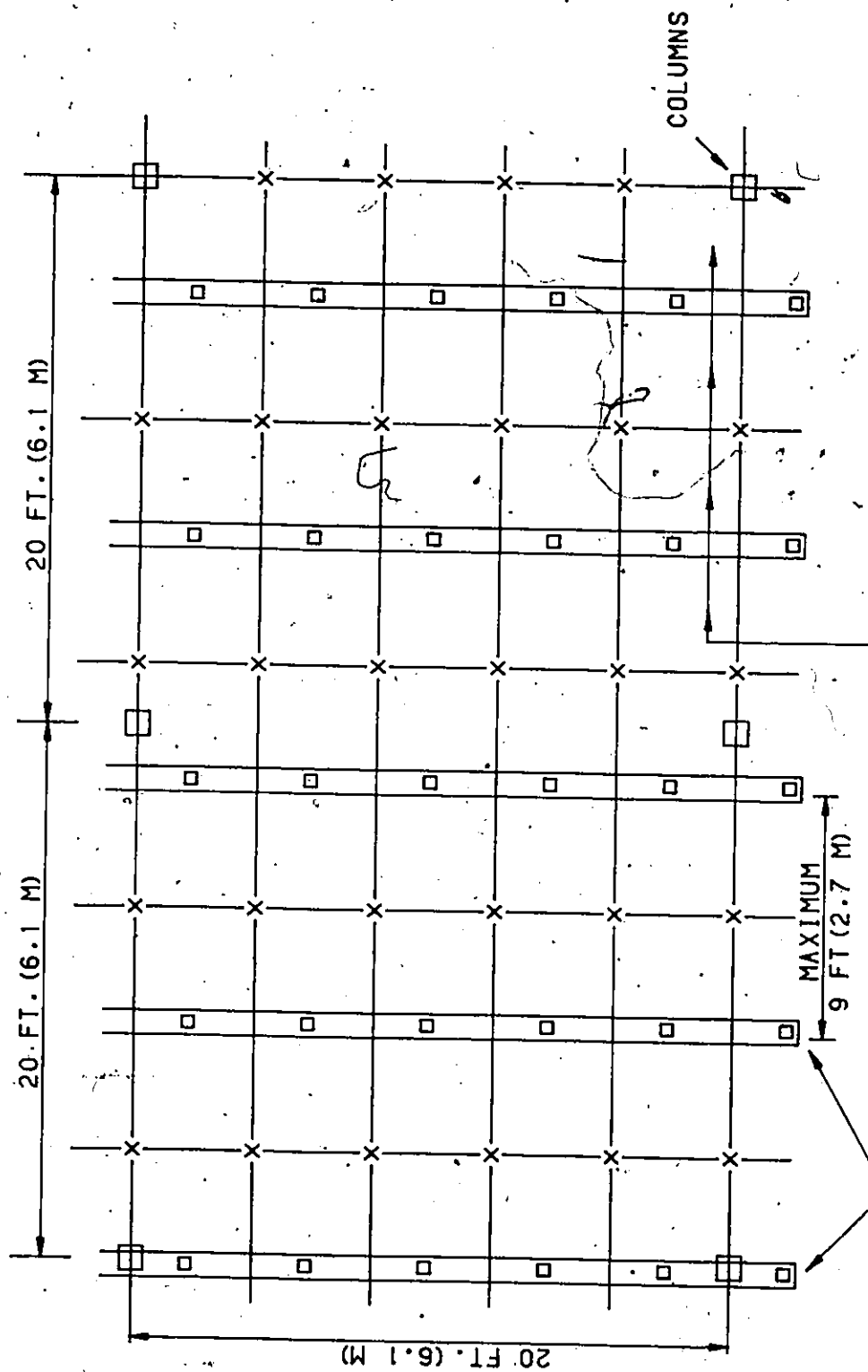


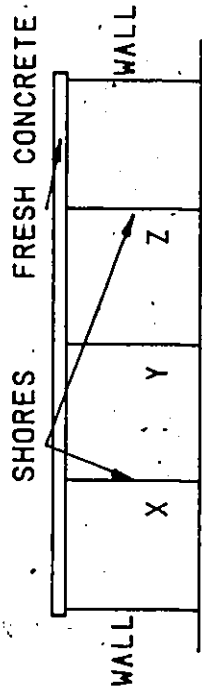
FIG.2.2.2 CONSTRUCTION LOADS WITH 1 LEVEL OF SHORES AND 2 LEVELS OF RESHORES [73]



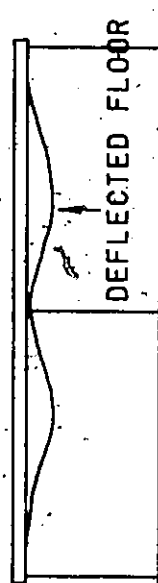
MIN. 5/8 IN. PLYWOOD SHEETS
(4 FT. X 8 FT.)
- TO BE REPLACED BY RESHORES
DURING STRIPPING PROCESS

PERMANENT SHORE STRIP
(1 FT. X 8 FT.)
- TO BE REMOVED AFTER ALL
RESHORES ARE INSTALLED

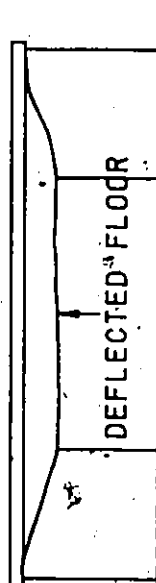
FIG.2.3 SHORING WITH PRESORE OR PERMANENT SHORE [26]



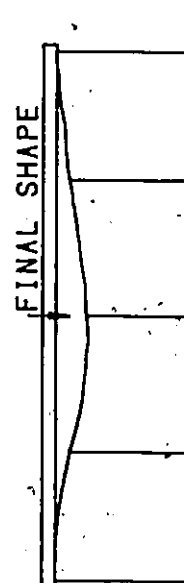
I) PLACING OF CONCRETE



II) OPERATION A - RESHORING X AND Z WITH Y IN PLACE

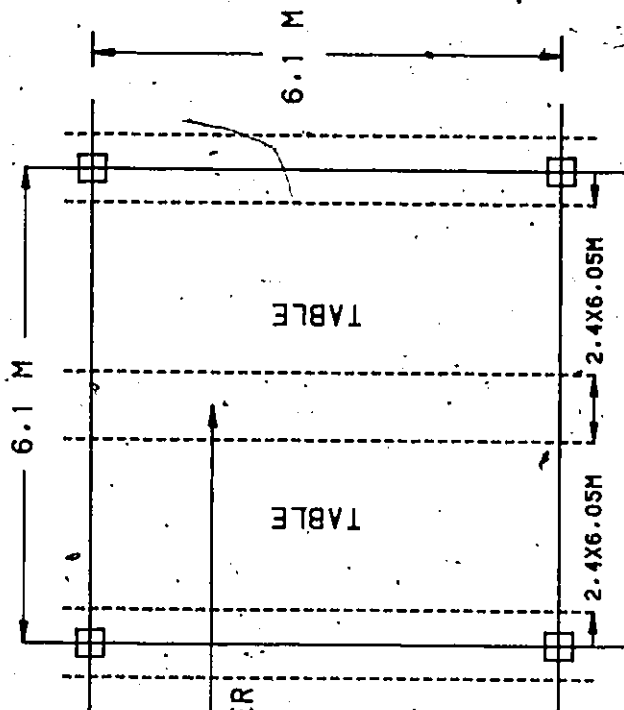


III) OPERATION B - X & Z RESHORED THEN SHORE Y REMOVED



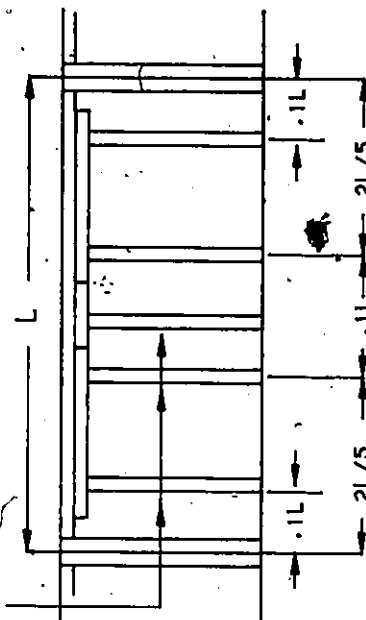
IV) RESHORING OPERATION COMPLETED

(A) PROCEDURES



PLAN

TABLE SUPPORT TRUSSES
(TO LOWER AND RAISE THE TABLE)



ELEVATION

(B) FLYING FORM AND SHORING

FIG.2.4 SCHEMATIC REPRESENTATION OF THE PRESORING TECHNIQUE [32]

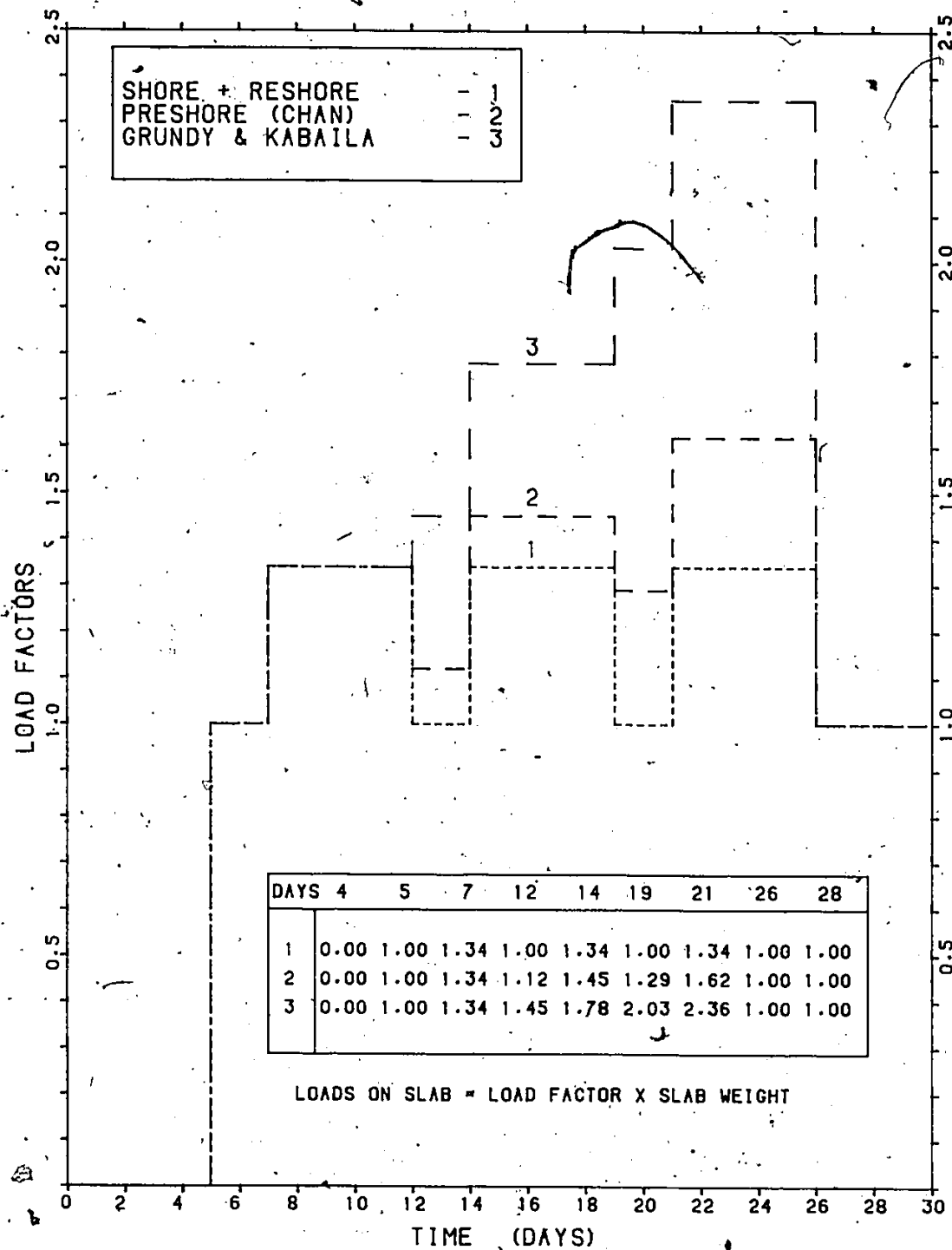
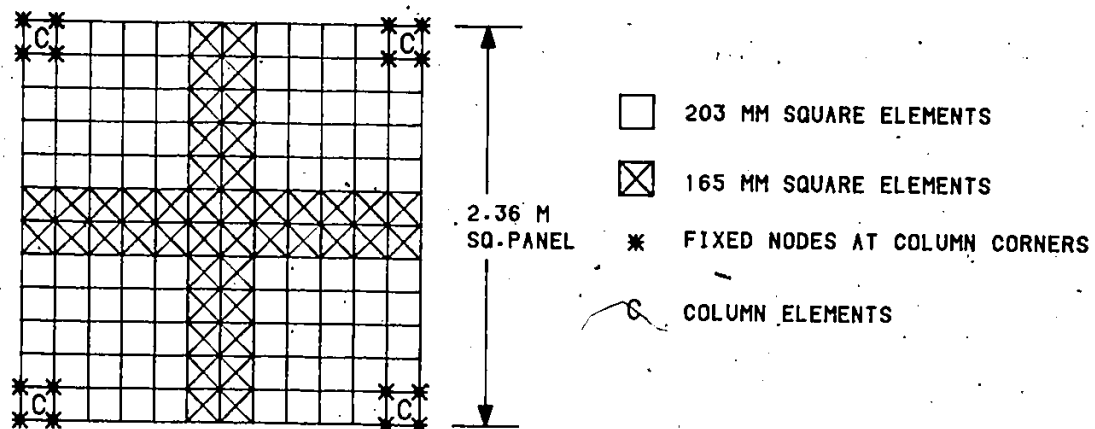
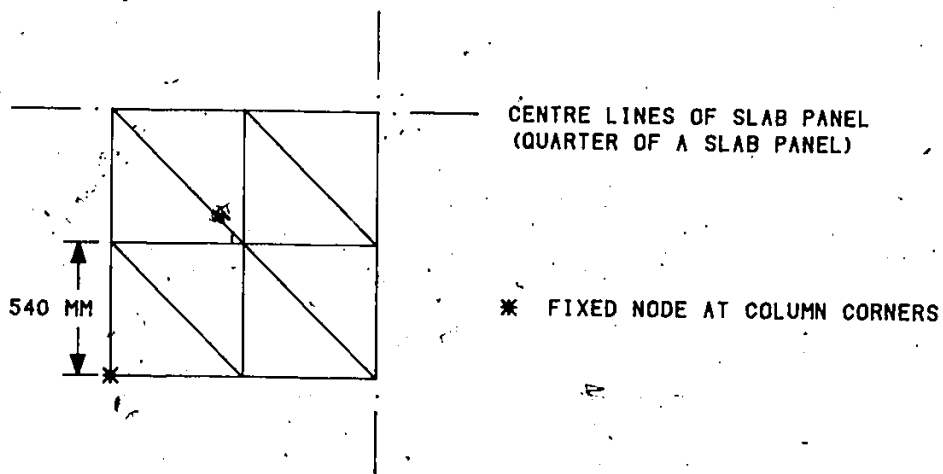


FIG.2.5 LOADING HISTORIES ON FLAT SLAB BUILDINGS

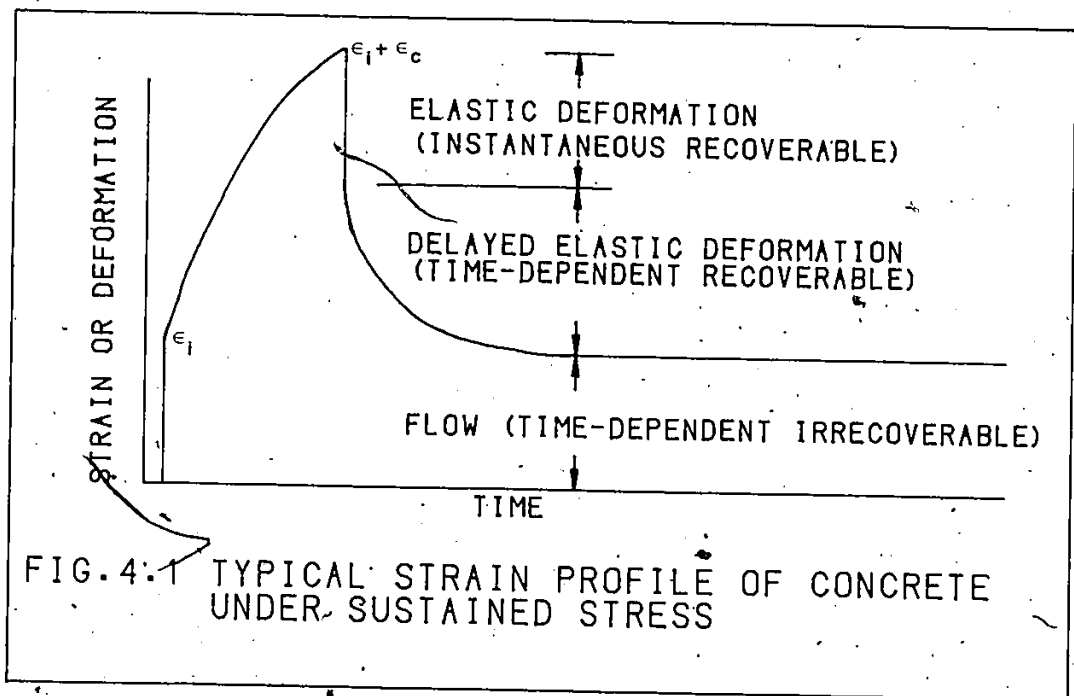


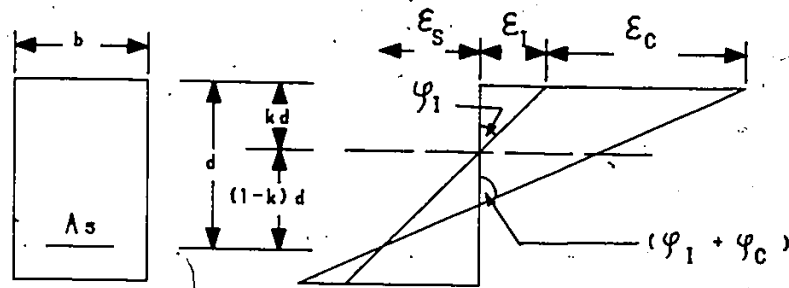
(A) STRUDL [113]



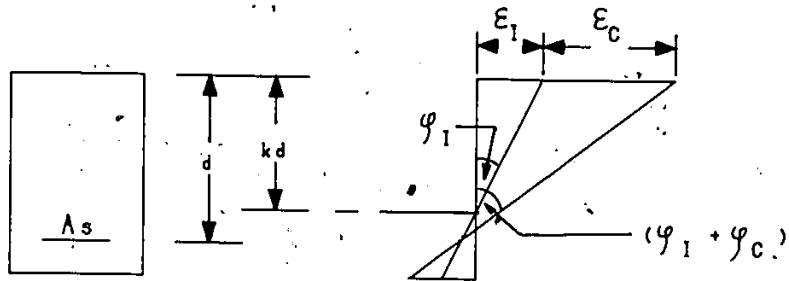
(B) NOPARC [126]

FIG.3.1 FINITE ELEMENTS DISCRETIZATION





FULLY CRACKED SECTION

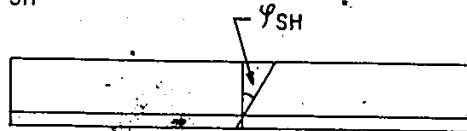


UNCRACKED SECTION

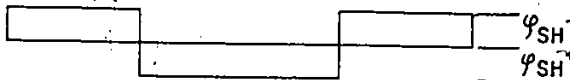
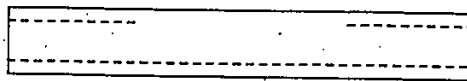
φ = CURVATURE AND ϵ = STRAIN
 SUBSCRIPTS : I - INITIAL OR ELASTIC
 C - DUE TO CREEP OF CONCRETE
 S - STRAIN IN STEEL

FIG.4.2 STRAIN DISTRIBUTION IN R.C.BEAM

φ_{SH} - CURVATURE DUE TO SHRINKAGE



(a) SIMPLY SUPPORTED BEAM



(b) FIXED-END BEAM

FIG.4.3 SHRINKAGE CURVATURES IN R.C. BEAMS

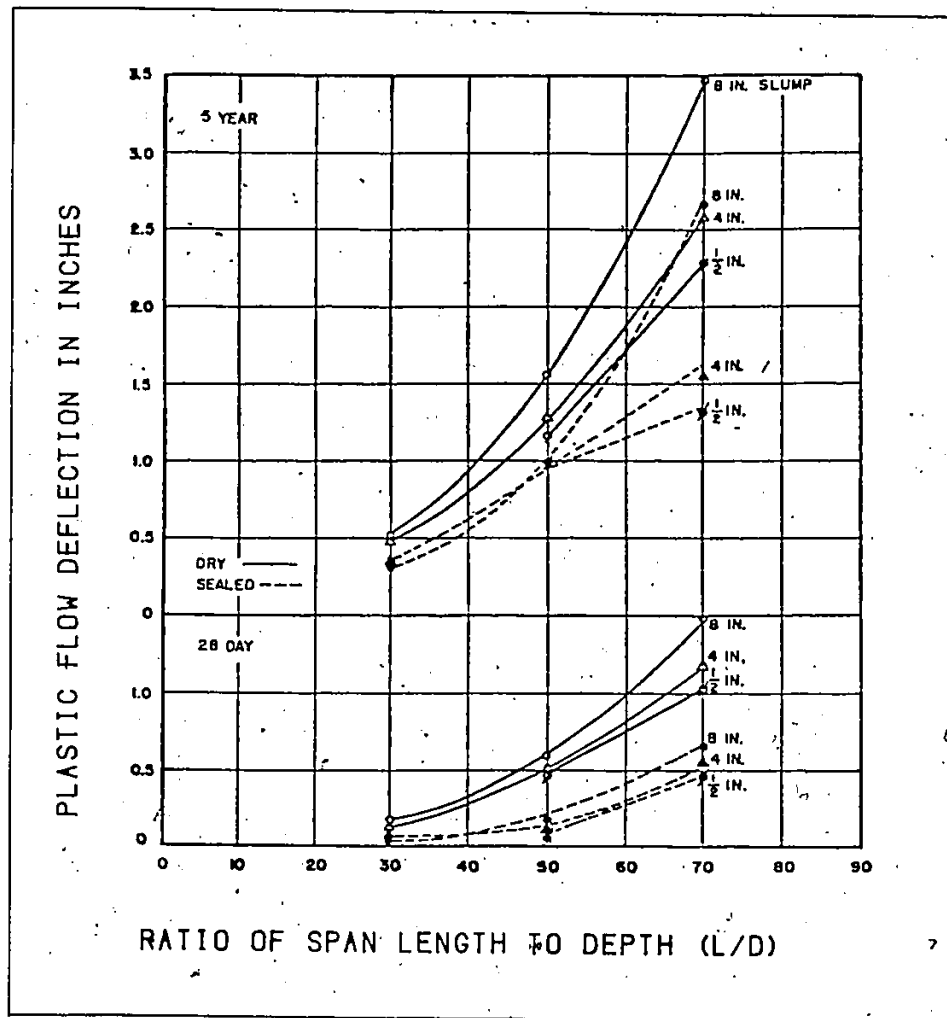
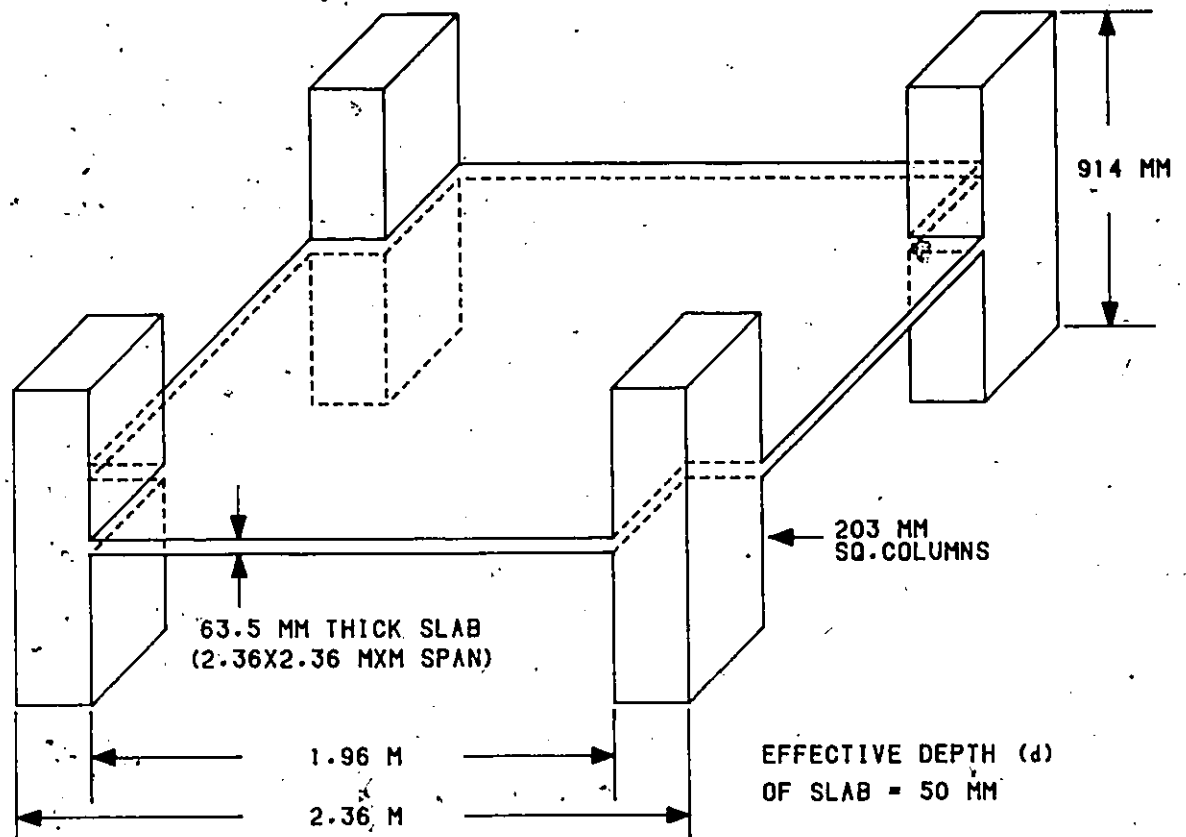
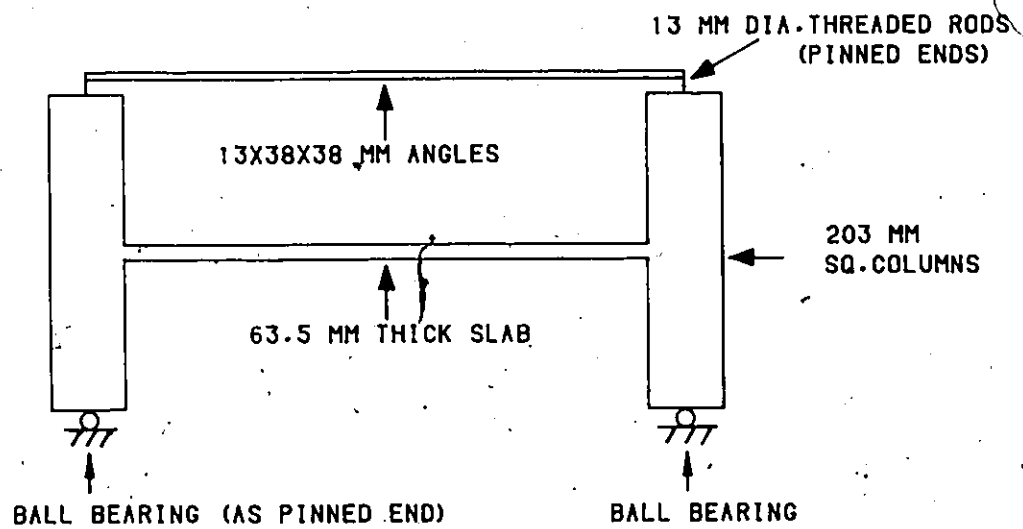


FIG.5.1 RELATIONSHIP BETWEEN PLASTIC FLOW DEFLECTION AND SPAN/DEPTH RATIO [127]



FLAT PLATE STRUCTURES



SUPPORT CONDITIONS

FIG.6.1A EXPERIMENTAL FLAT PLATE STRUCTURES

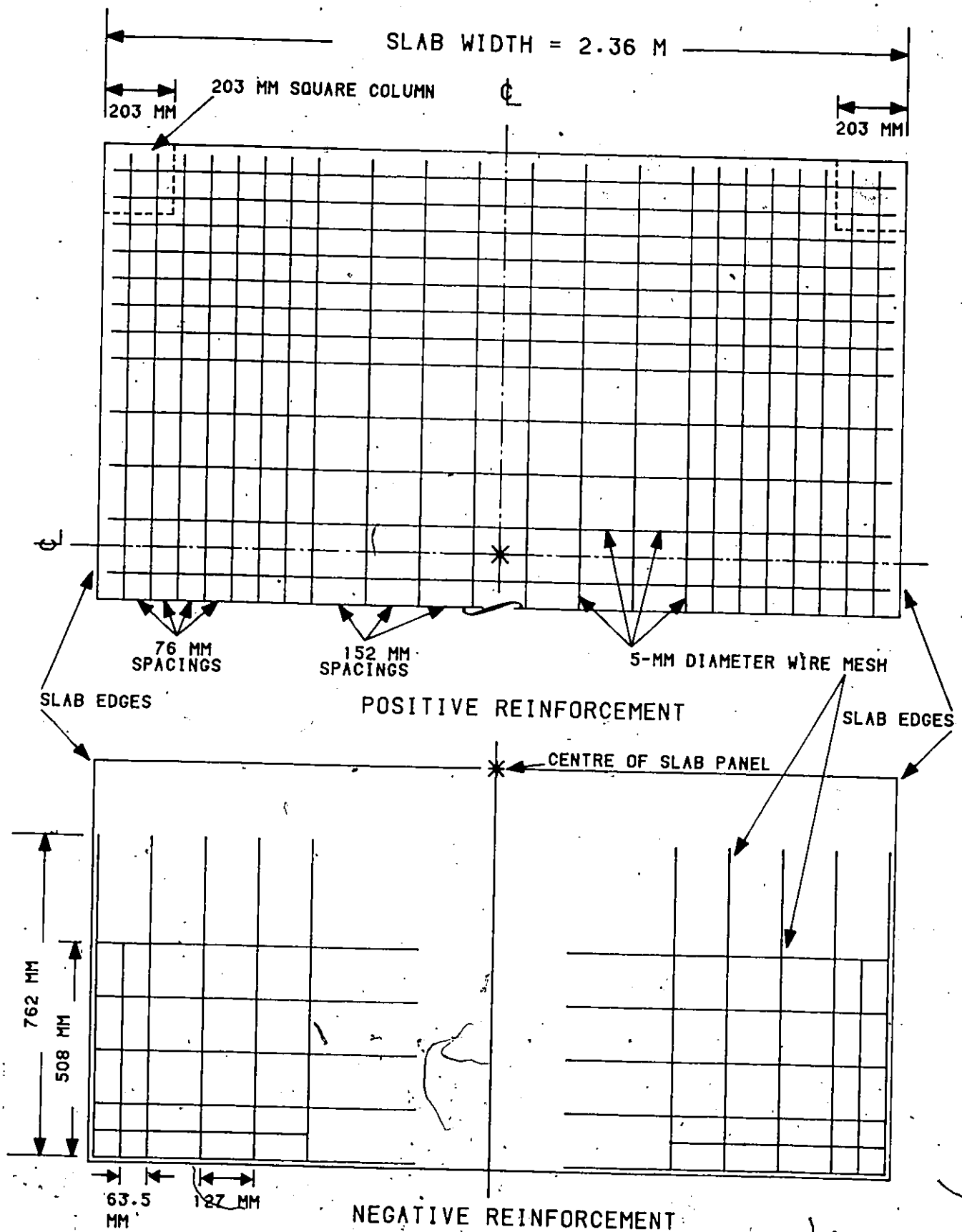
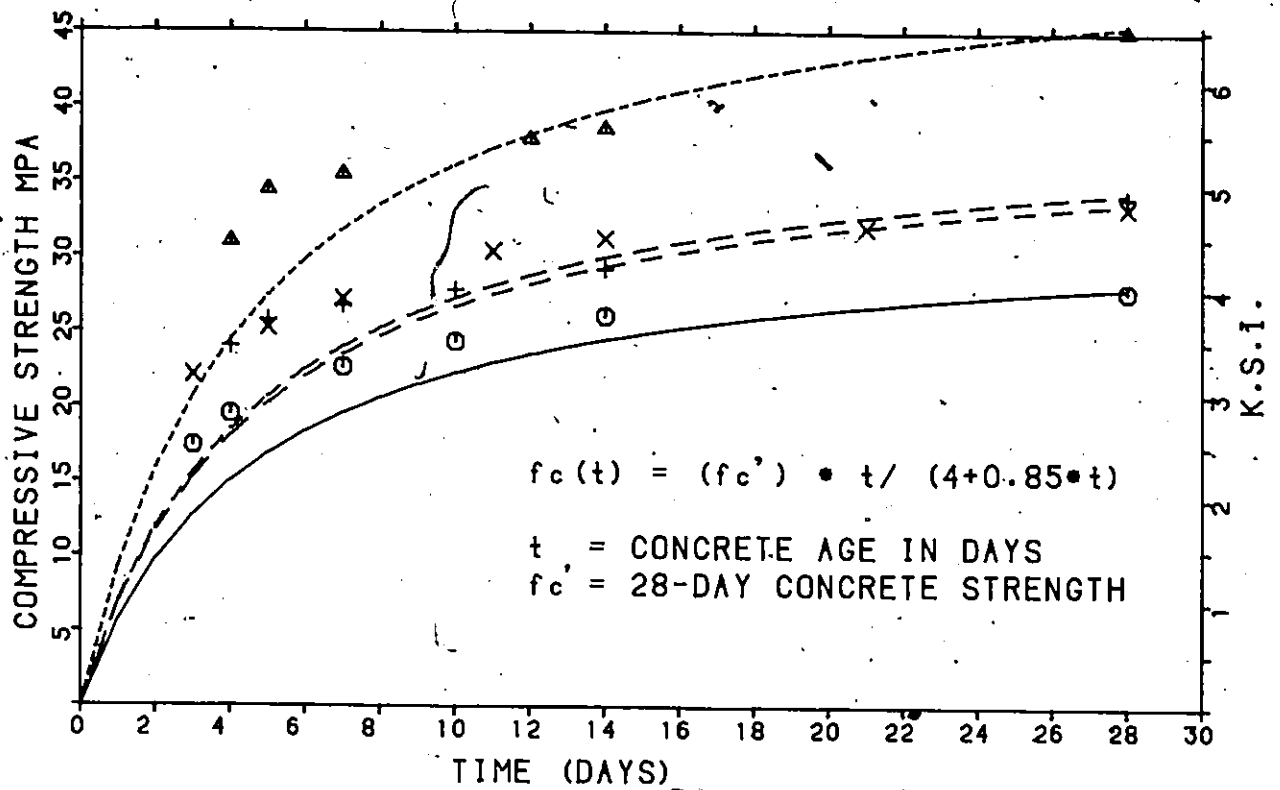
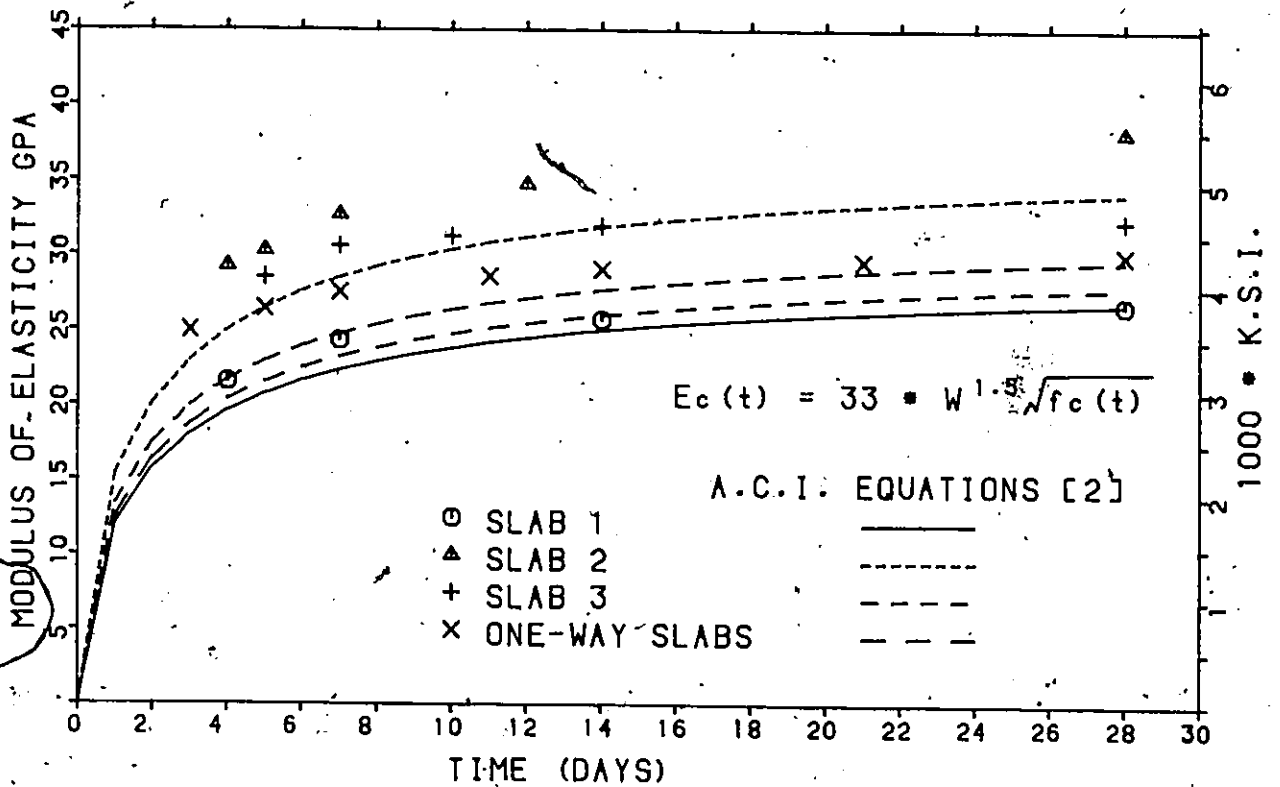


FIG.6.1B REINFORCEMENT LAYOUT



(A) COMPRESSIVE STRENGTH .VS. TIME



(B) MODULUS ELASTICITY .VS. TIME

FIG.6.2 MEASURED CONCRETE PROPERTIES

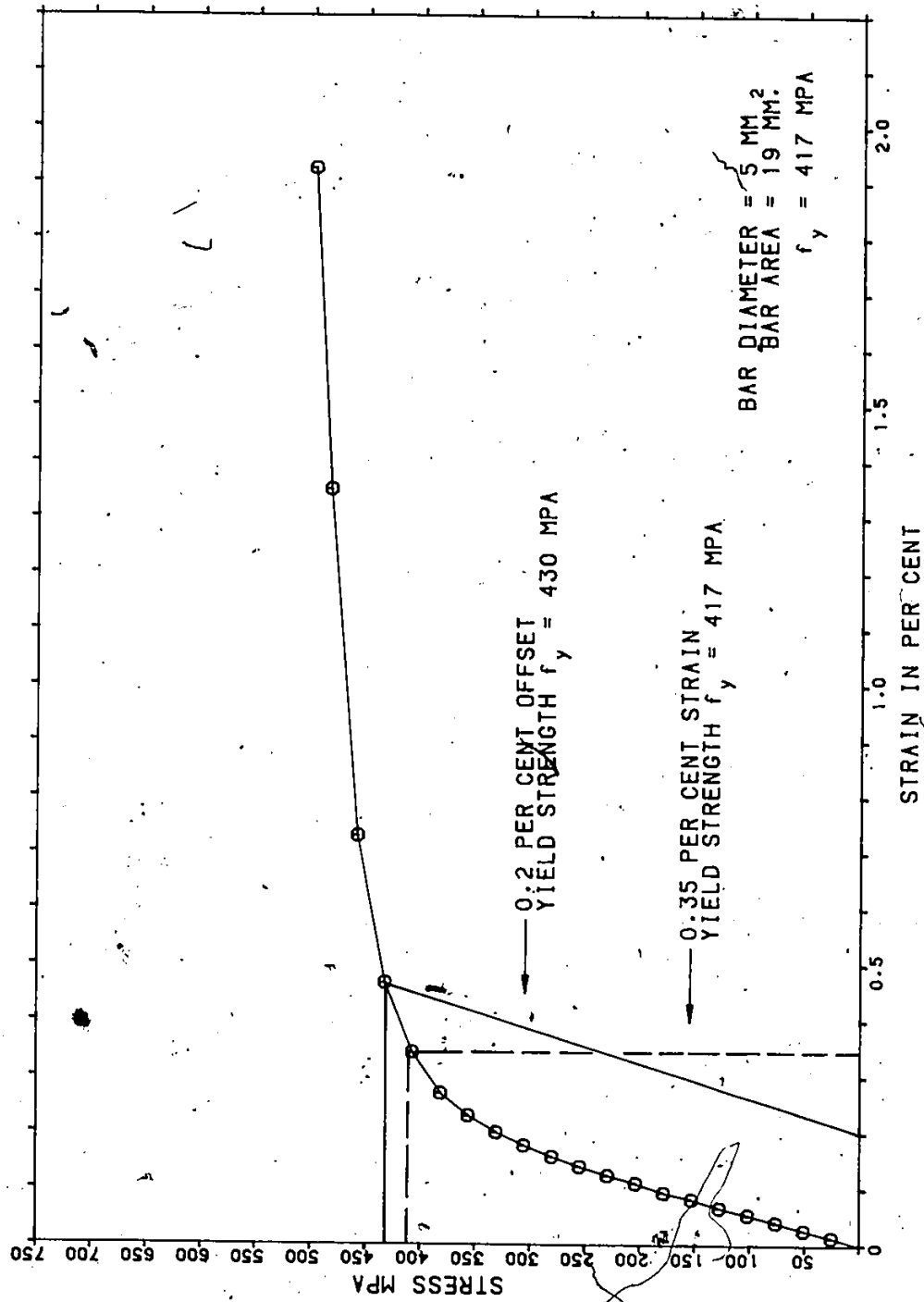
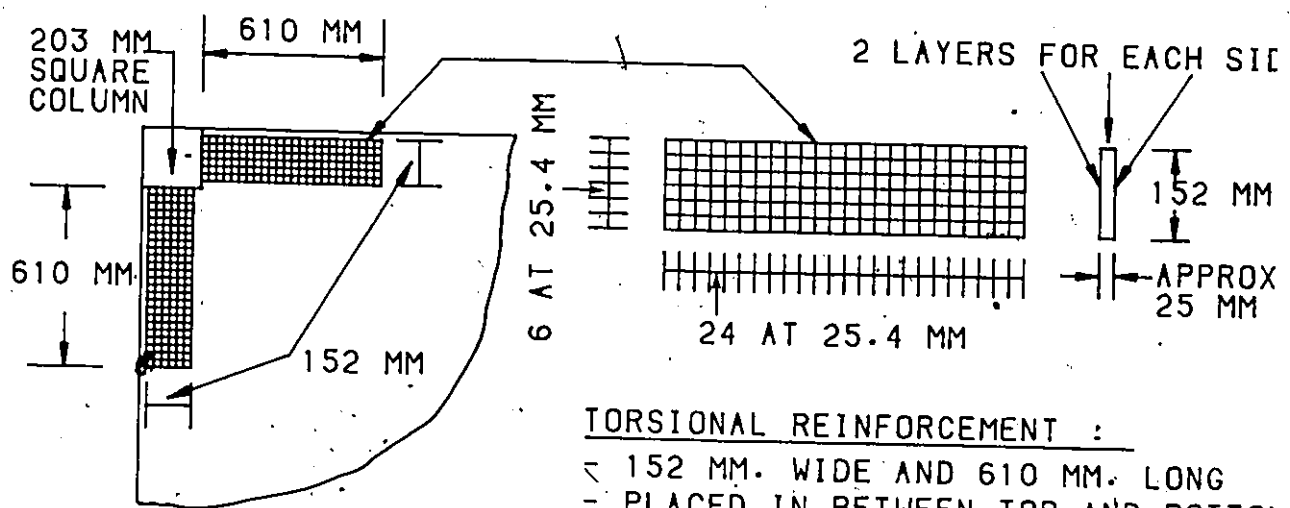


FIG.6.3 STRESS-STRAIN CURVE OF REINFORCING BAR



TORSIONAL REINFORCEMENT :

≤ 152 MM. WIDE AND 610 MM. LONG
 - PLACED IN BETWEEN TOP AND BOTTOM
 MAIN REINFORCEMENT

DIAMETER = 1.6 MM

ULT. STRENGTH = 660 MPa

ESTIMATED f_y = 400 MPa

FIG.6.4 TORSIONAL REINFORCEMENT IN TEST SLABS

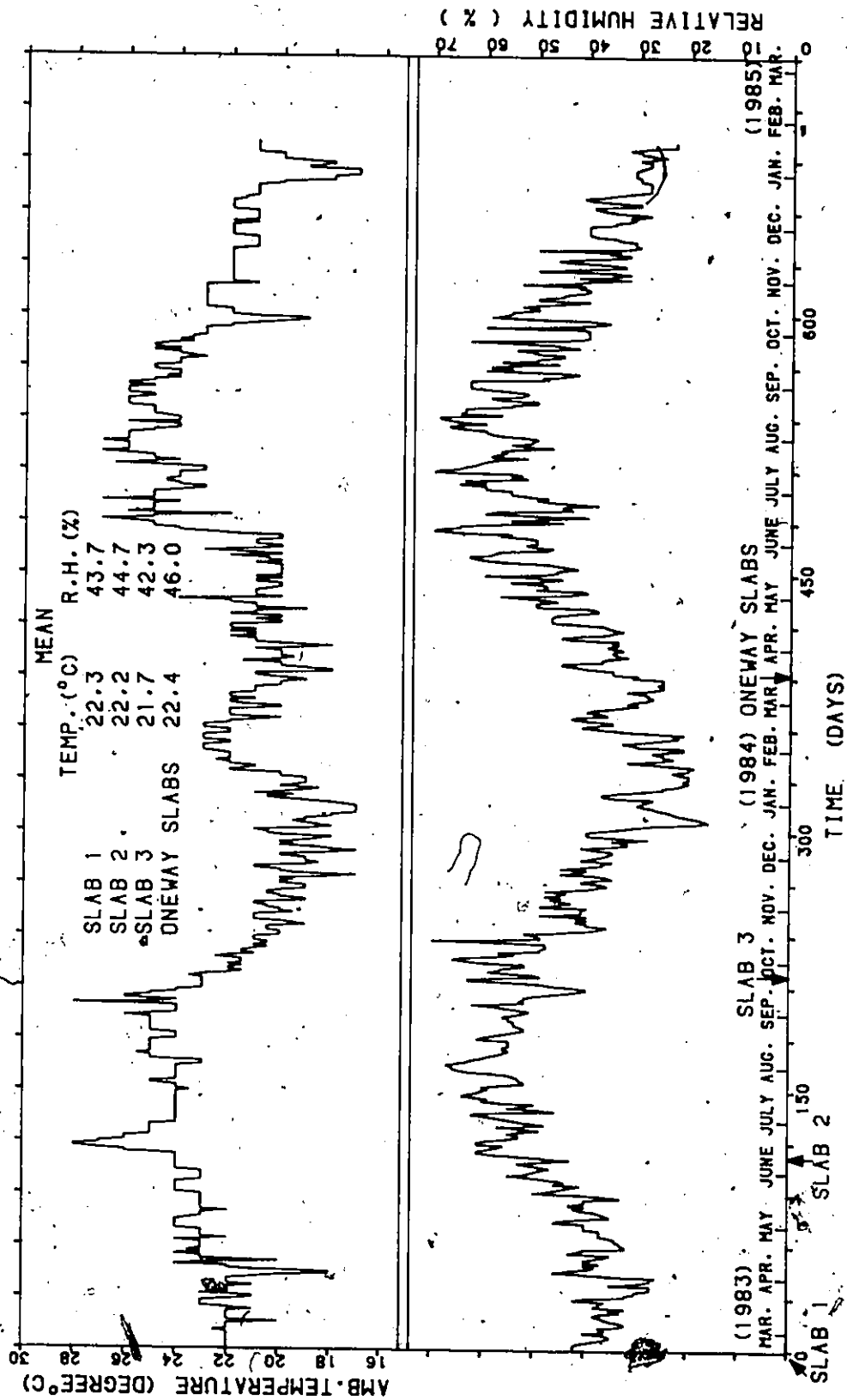
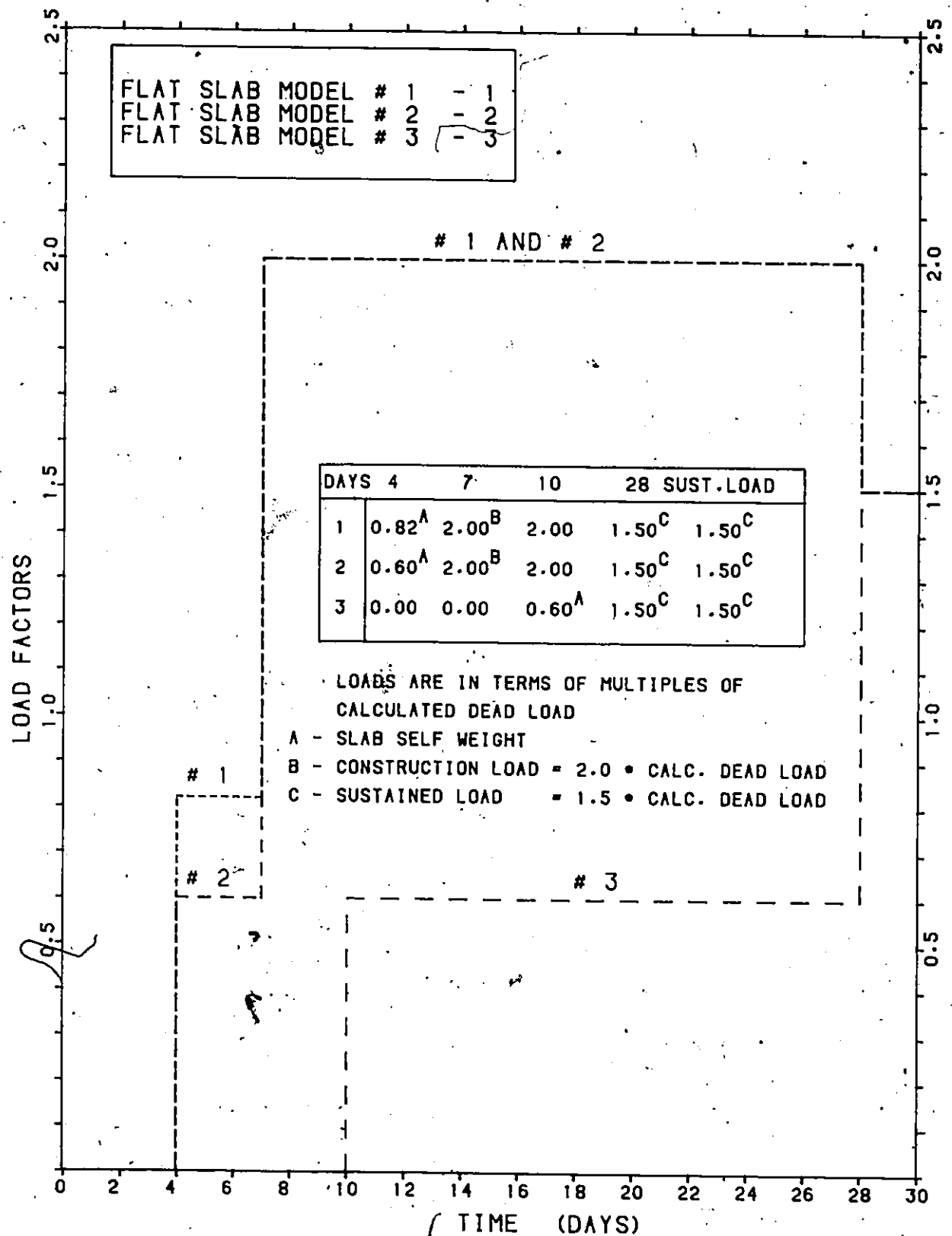
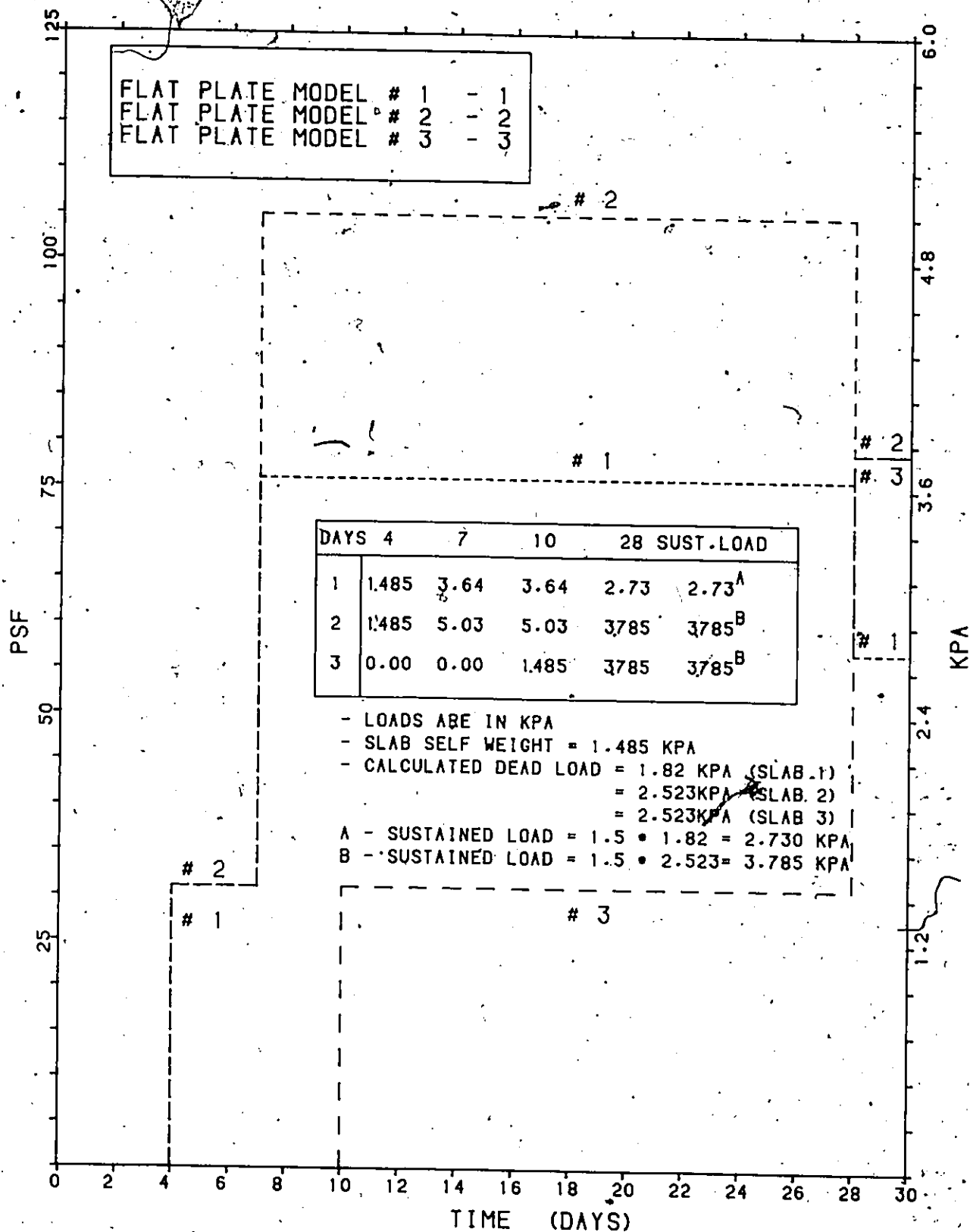


FIG. 6.5 AMBIENT TEMPERATURE AND RELATIVE HUMIDITY VERSUS TIME



(A) LOADS IN TERMS OF MULTIPLES OF CALCULATED DEAD LOAD
FIG. 6.6A LOADING HISTORIES ON FLAT PLATE MODELS



(B) LOADS IN TERMS OF KPA AND PSF
FIG. 6.6B LOADING HISTORIES ON FLAT PLATE MODELS

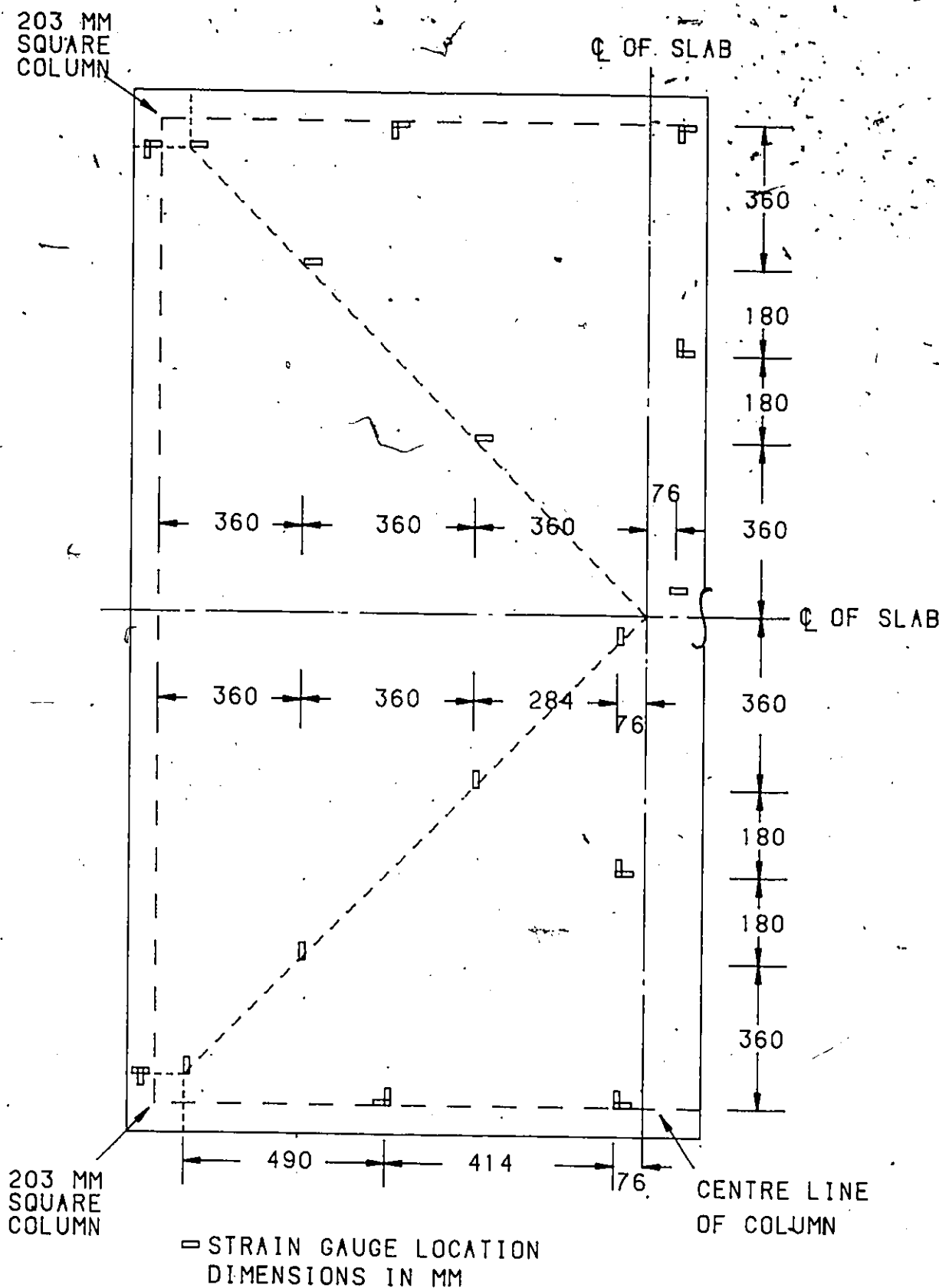


FIG.6.8 LOCATIONS OF RENIFORCEMENT STRAIN MEASUREMENT

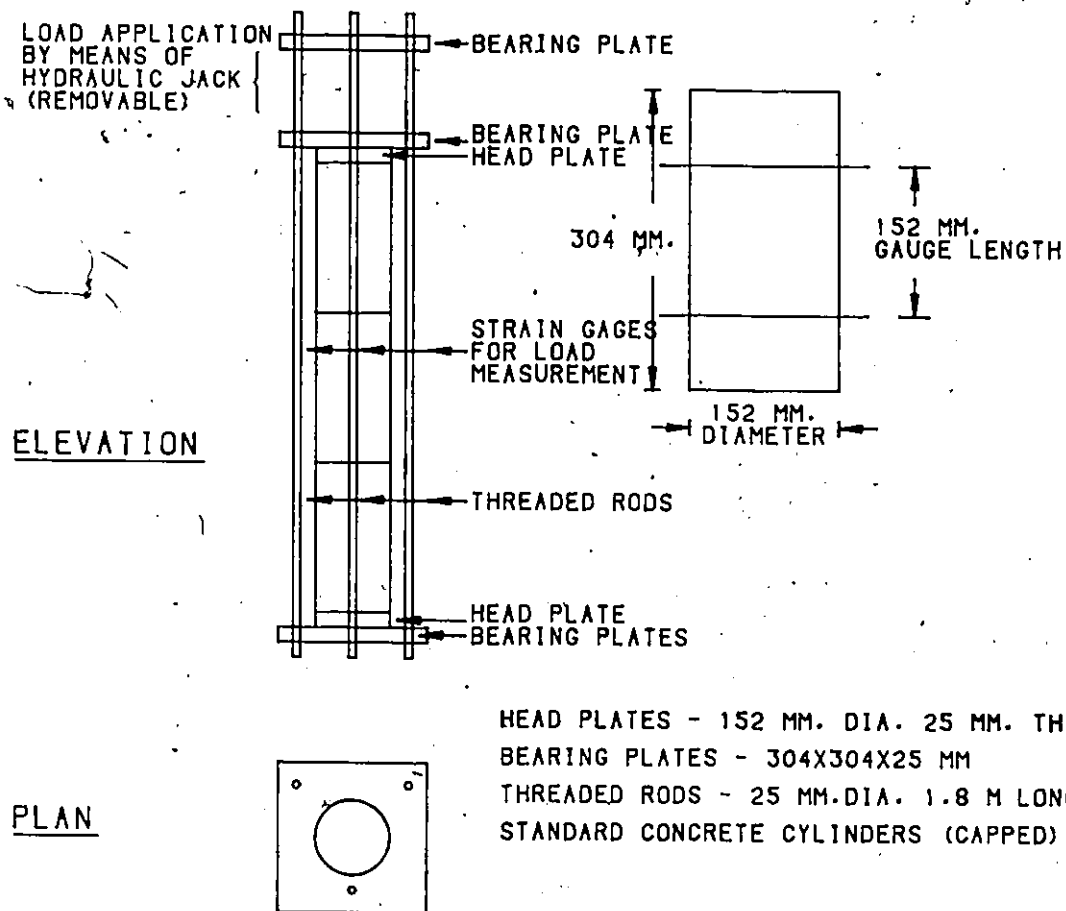
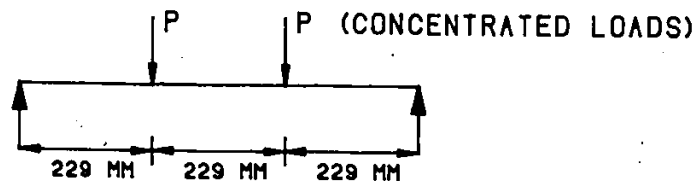


FIG.6.9 APPARATUS SET-UP FOR CREEP TEST



RECTANGULAR SECTION - WIDTH = 76.0 MM
 - DEPTH = 63.5 MM
 - REINFORCING BAR DIA. = 5 MM

	REINFORCEMENT RATIO (%)	SUSTAINED LOAD (2•P IN N)
PLAIN CONCRETE	-----	445
SINGLY REINFORCED	0.507	1335
DOUBLY REINFORCED	0.507 ($\rho = \rho'$)	1335

FIG.6.10 CONTROL BEAM SPECIMENS

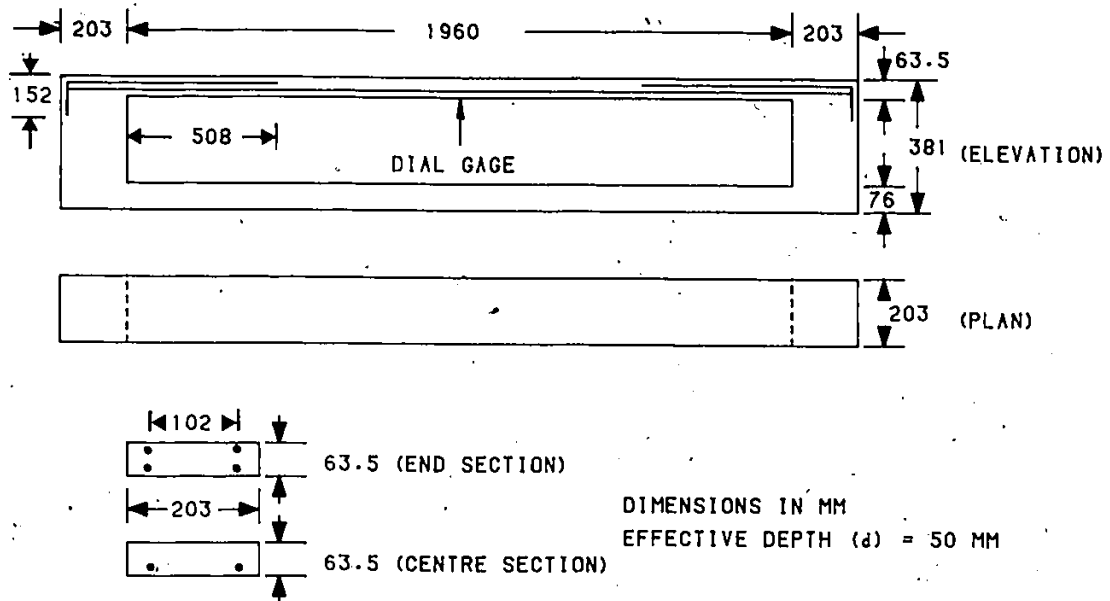


FIG.6.11 DETAILS OF ONEWAY SLAB MODELS

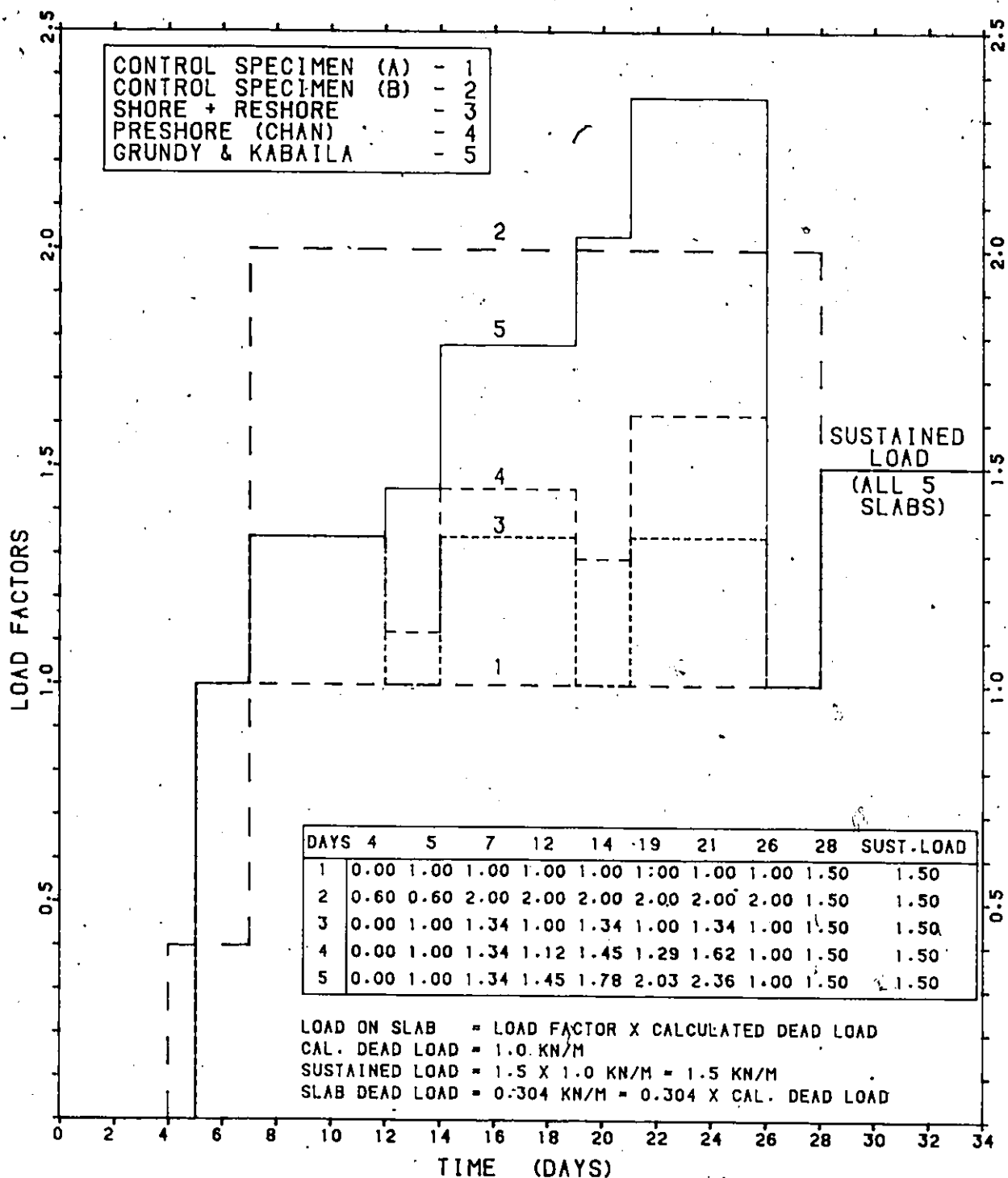


FIG.6.12 LOADING HISTORIES ON ONE-WAY SLAB MODELS

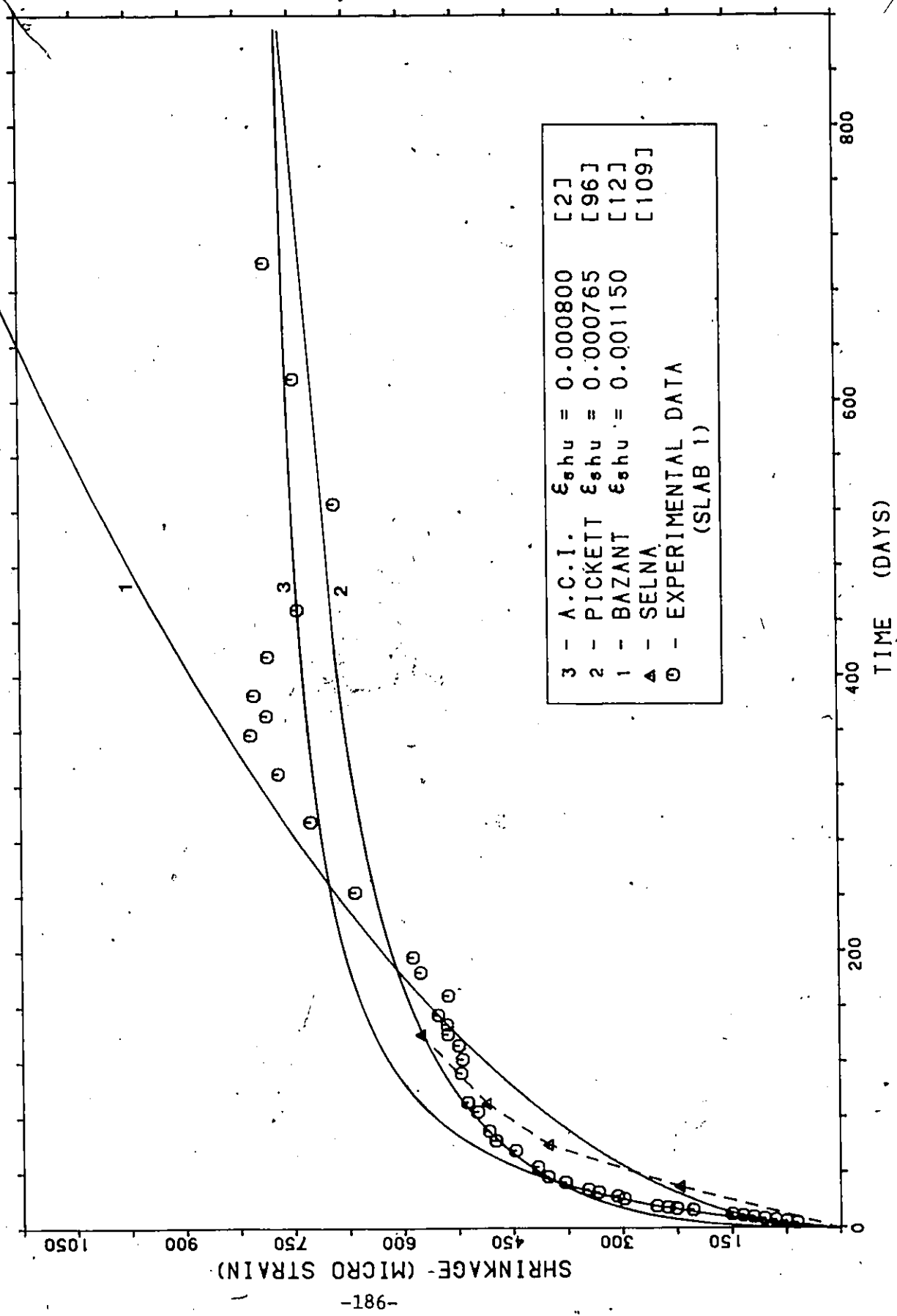


FIG.7.1 LONGITUDINAL SHRINKAGE OF CONCRETE (SLAB 1)

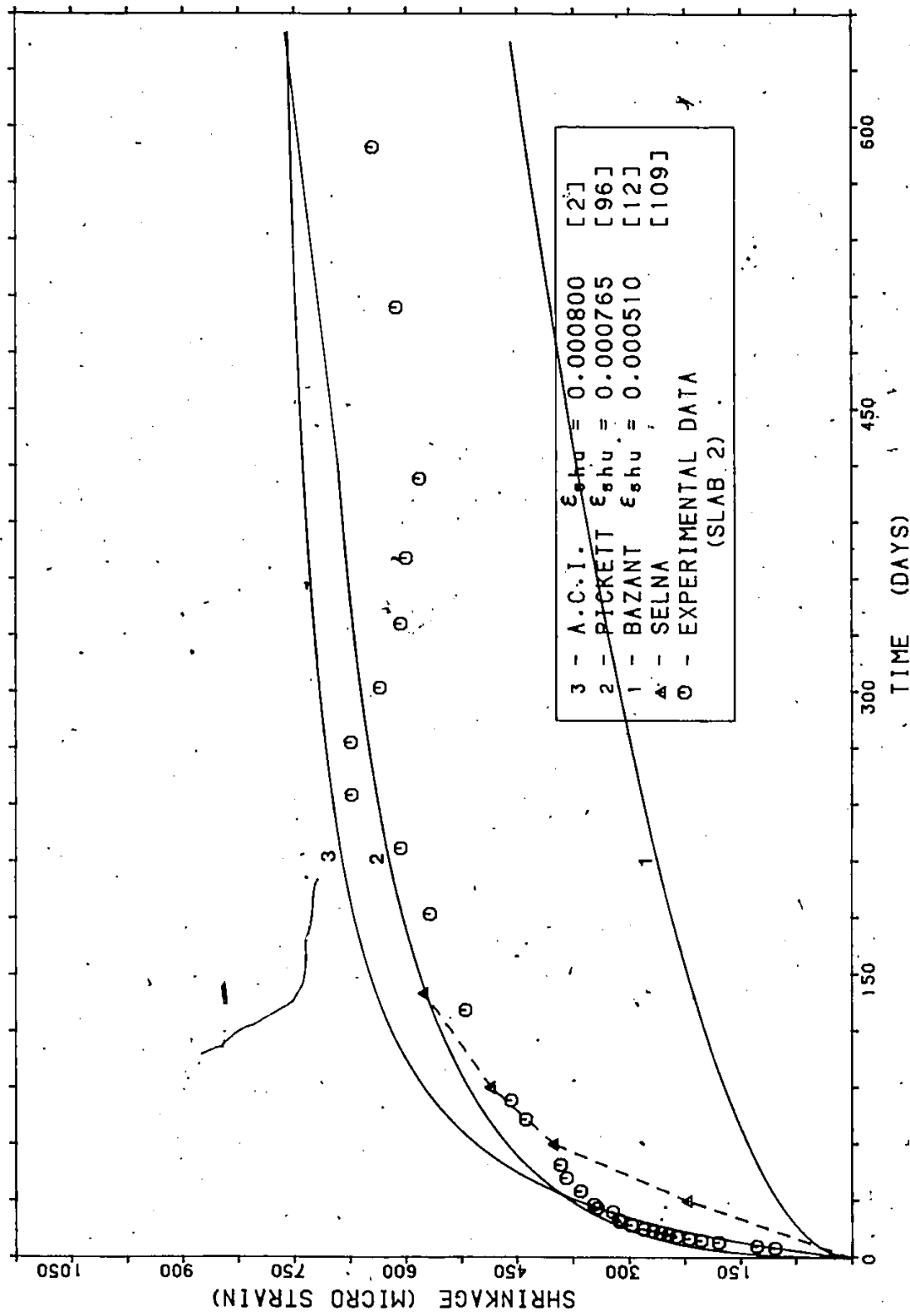


FIG.7.2 LONGITUDINAL SHRINKAGE OF CONCRETE (SLAB 2)

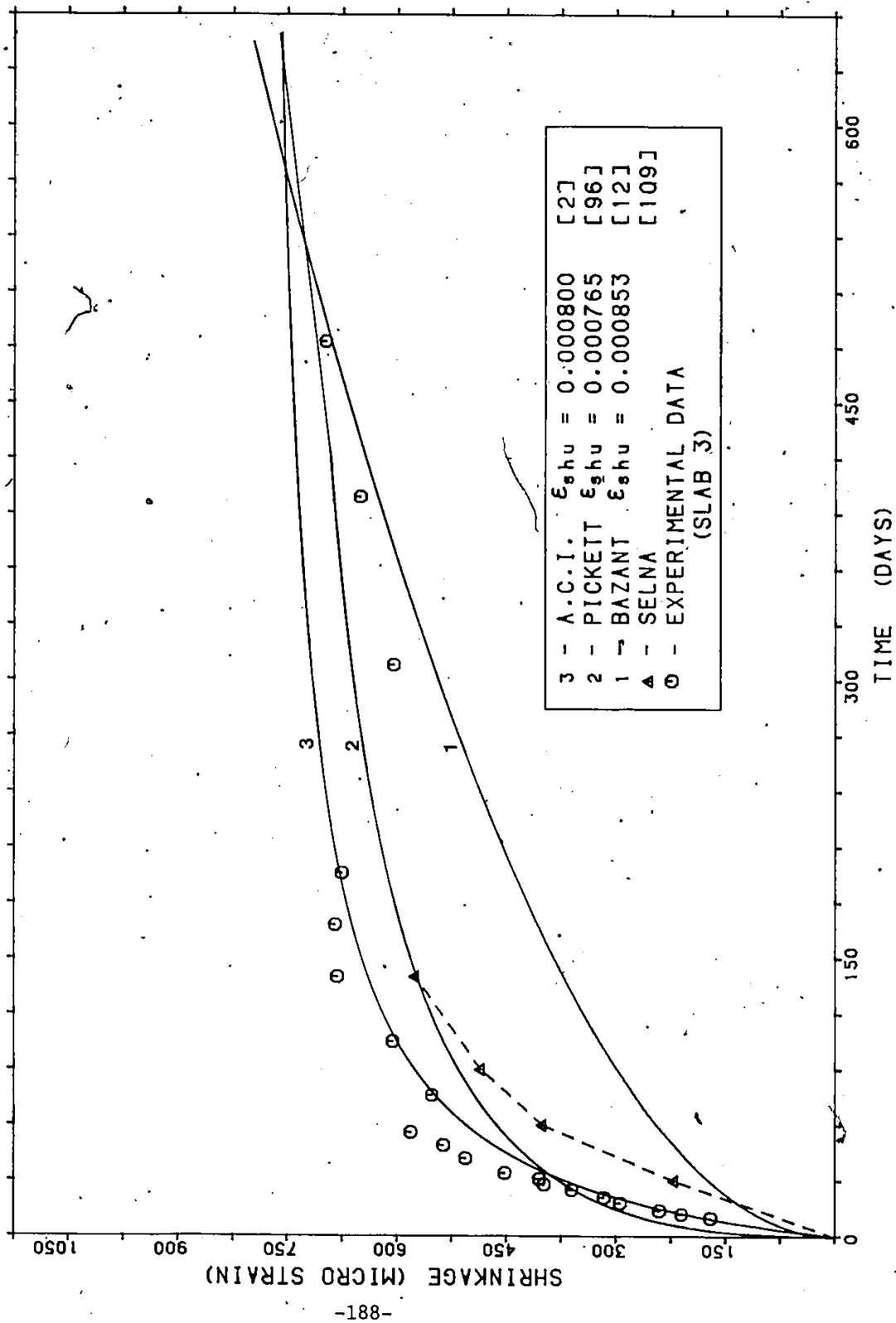


FIG.7.3 LONGITUDINAL SHRINKAGE OF CONCRETE (SLAB 3)

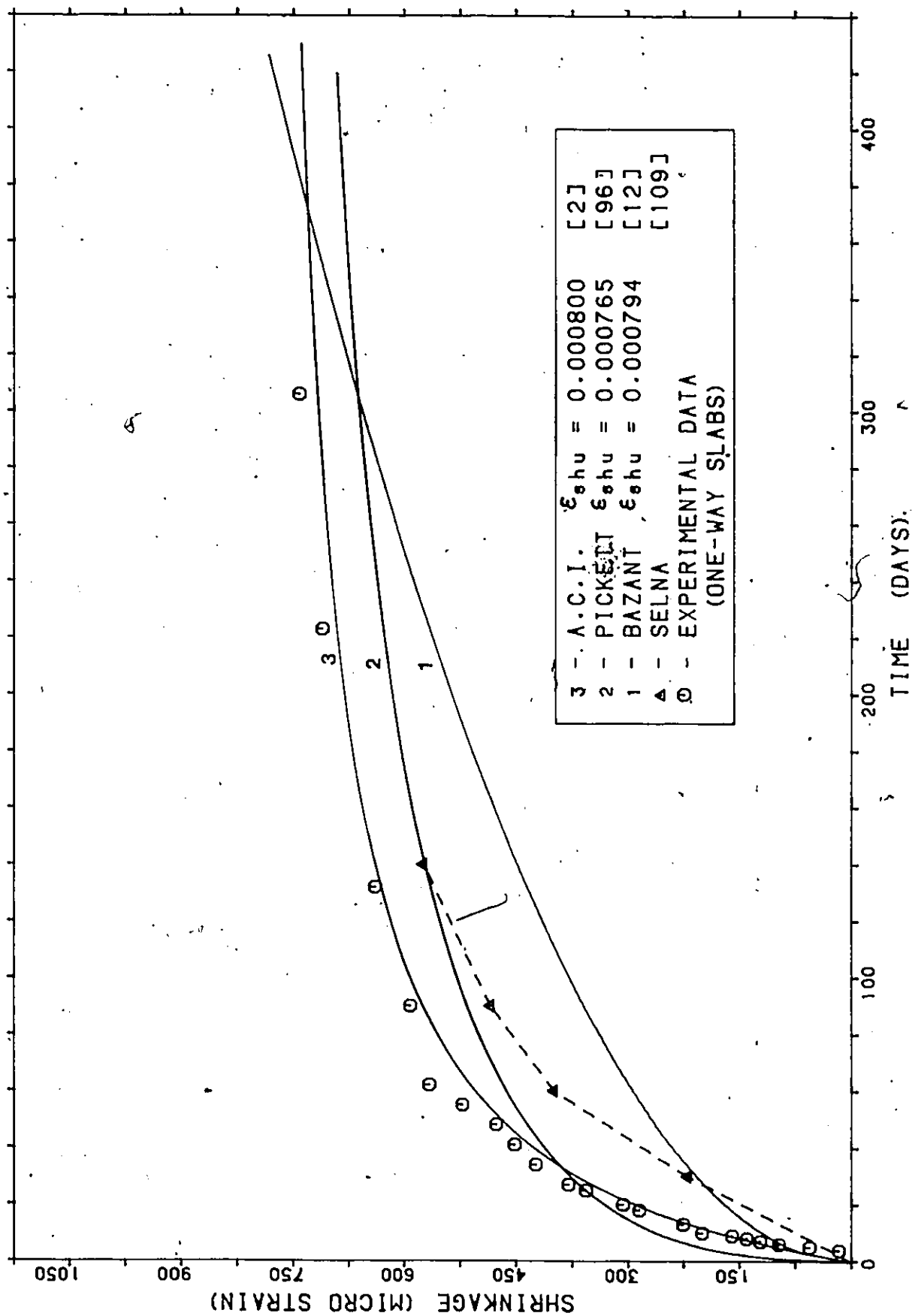


FIG.7.4 LONGITUDINAL SHRINKAGE OF CONCRETE (ONE-WAY SLABS)

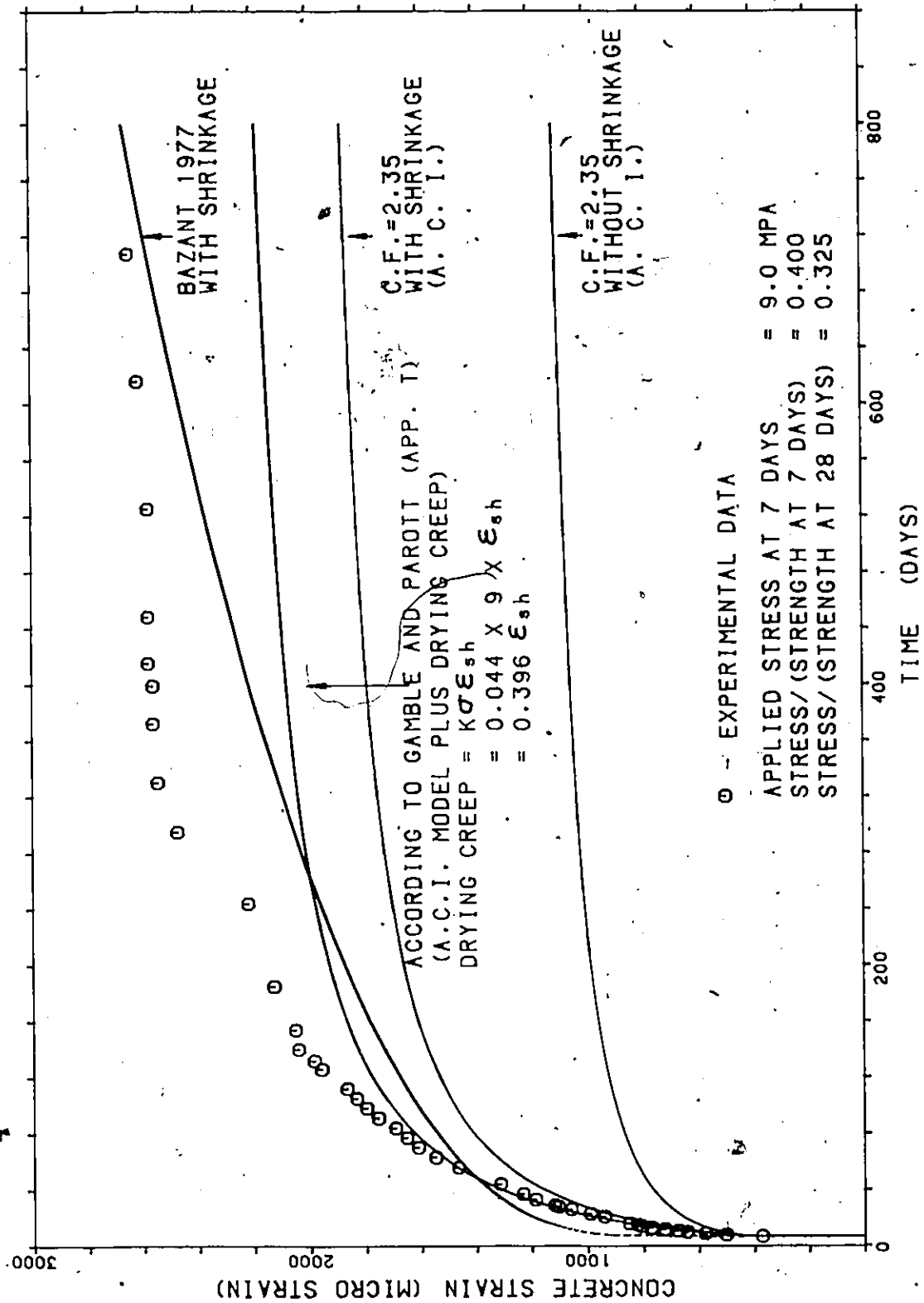


FIG.7.5 TIME-DEPENDENT CONCRETE STRAIN (SLAB 1)

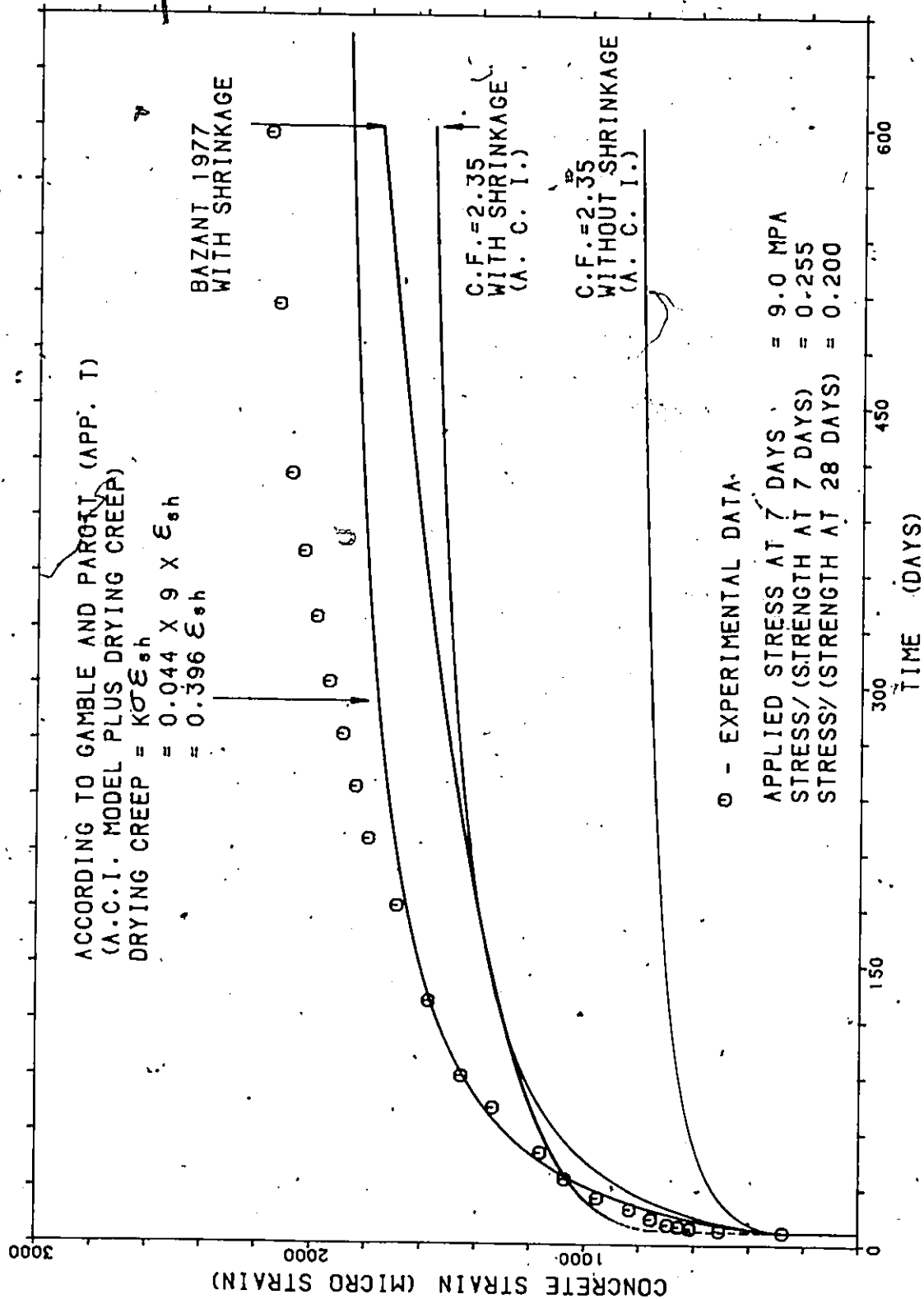


FIG. 7.6A TIME-DEPENDENT CONCRETE STRAIN (SLAB 2)
(SUSTAINED STRESS = 9.0 MPa)

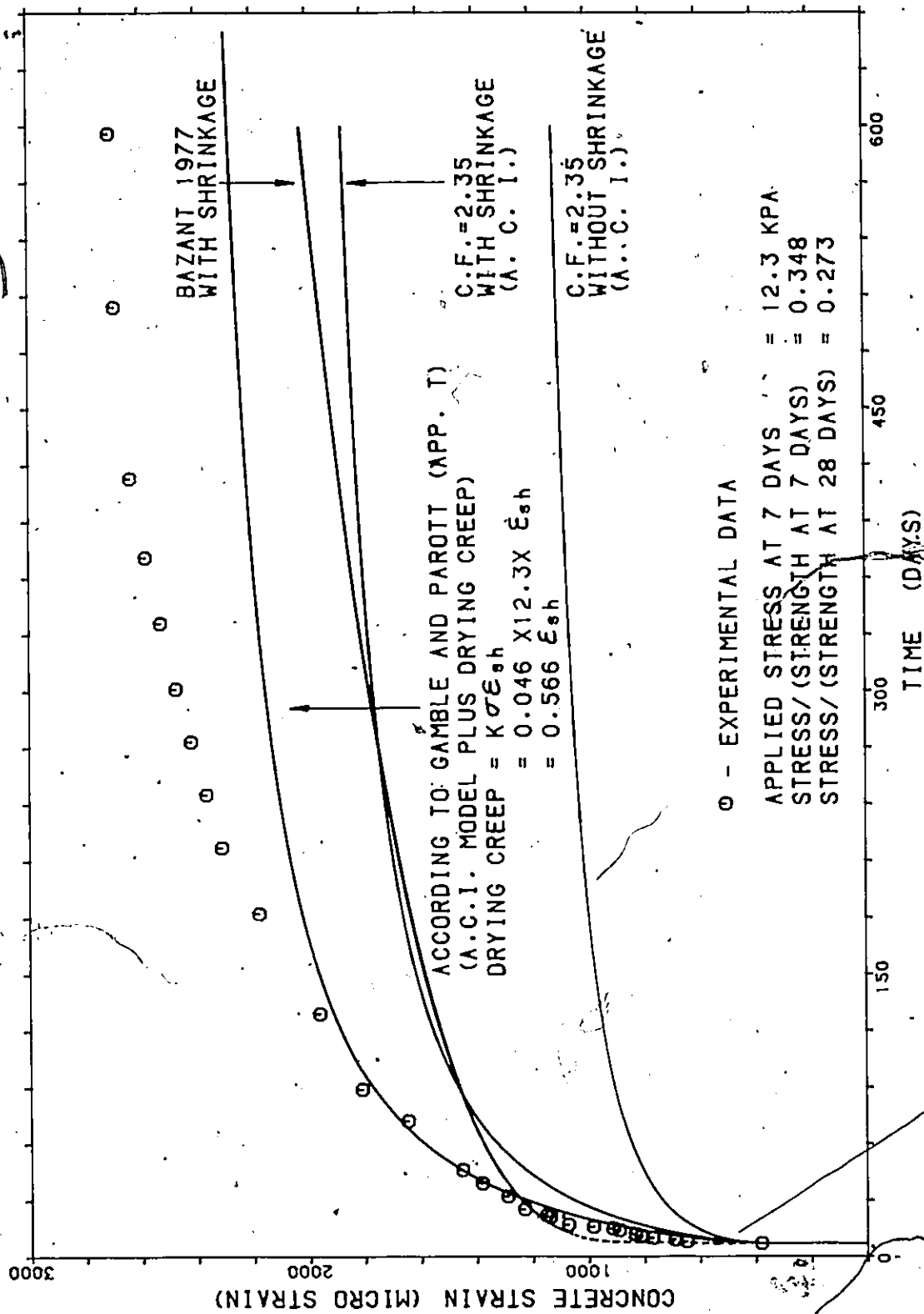


FIG.7.6B TIME-DEPENDENT CONCRETE STRAIN (SLAB 2)
(SUSTAINED STRESS = 12.3 KPA)

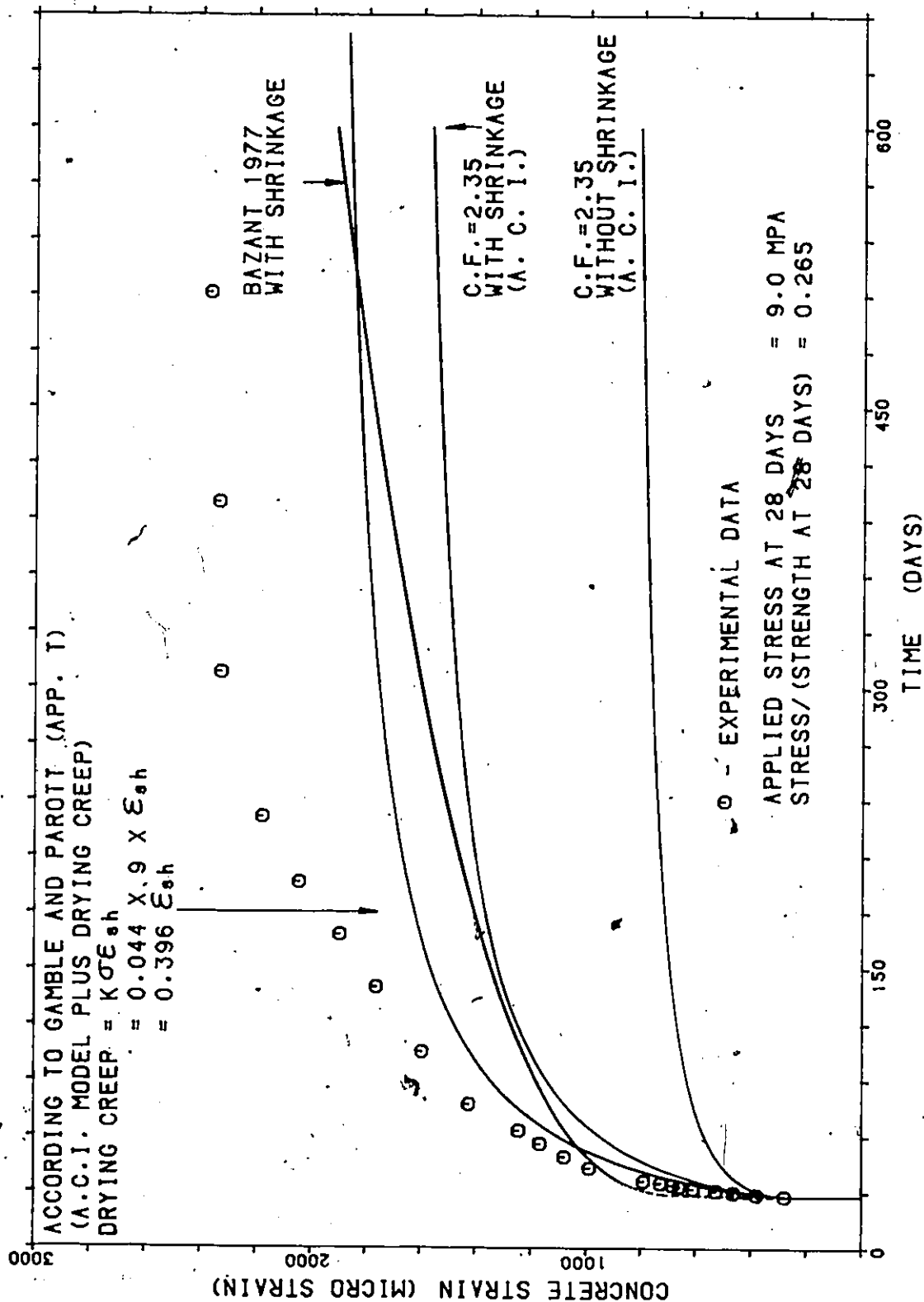


FIG. 7.7A TIME-DEPENDENT CONCRETE STRAIN (SLAB 3)
(CONSTANT LOAD FROM 28 DAYS TO 483 DAYS)

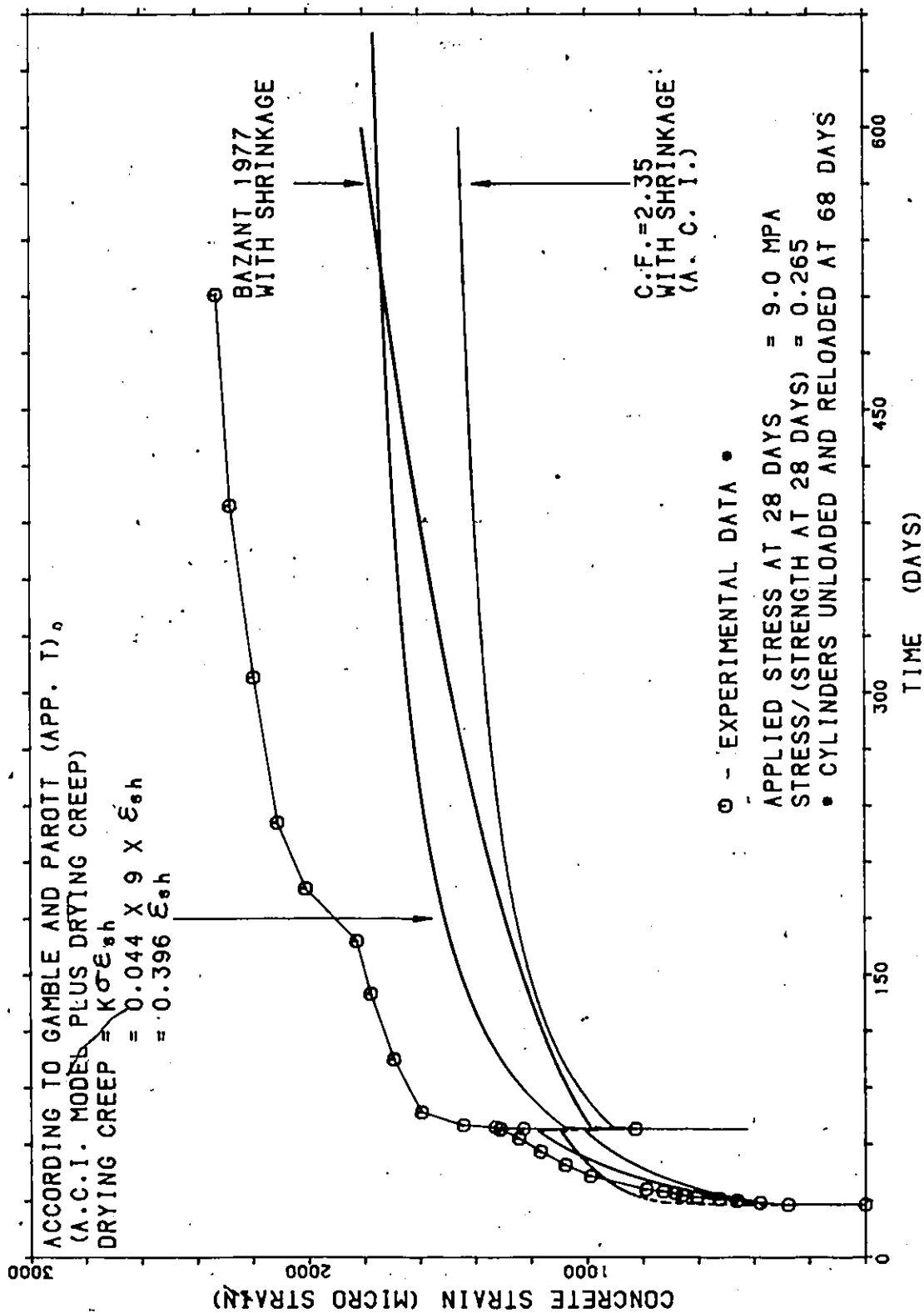


FIG.7.7B TIME-DEPENDENT CONCRETE STRAIN (SLAB 3)
(CYLINDERS WERE UNLOADED/RELOADED AT 68 DAYS)

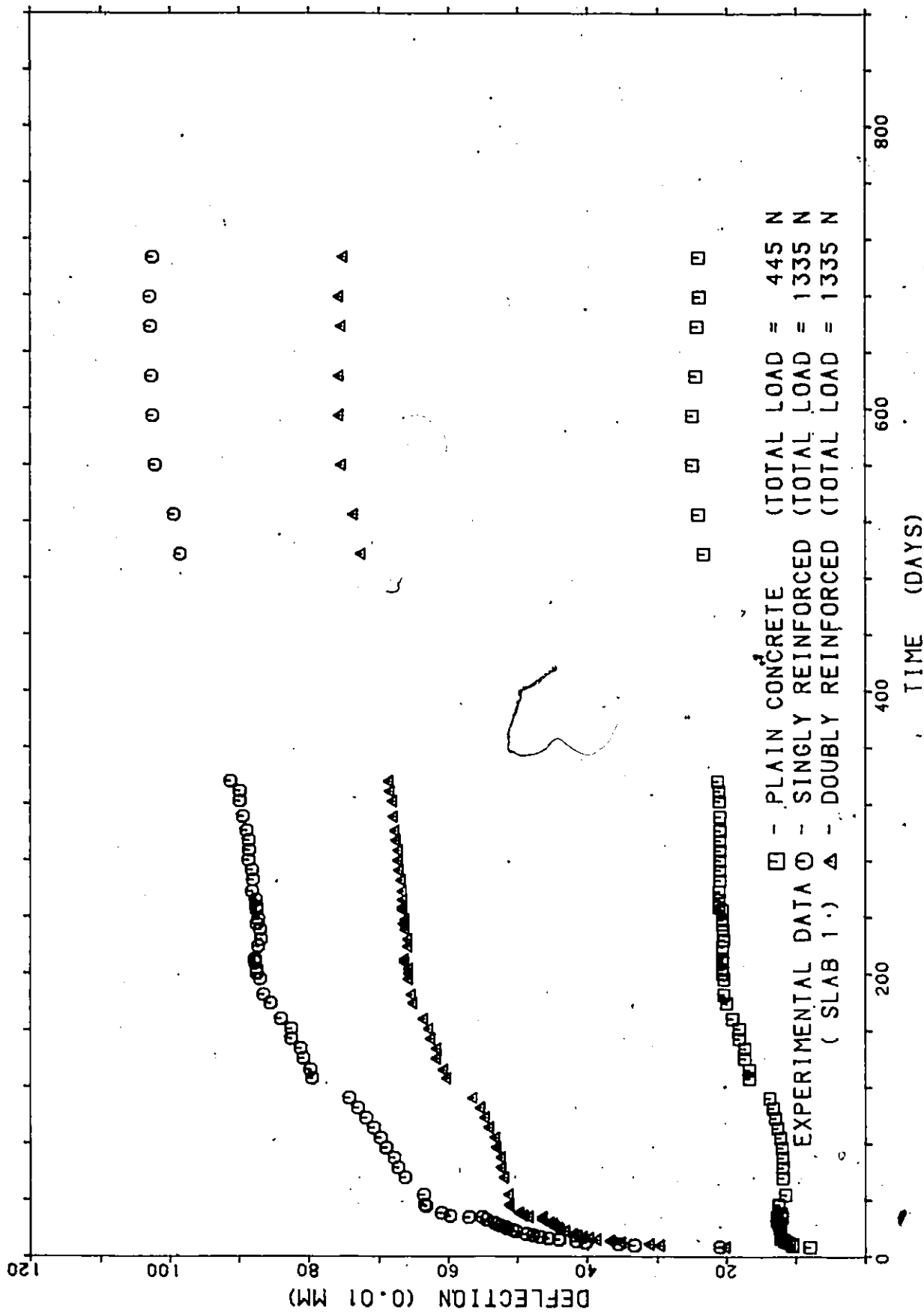


FIG.7.8 TIME-DEPENDENT BEAM DEFLECTIONS (SLAB 1)

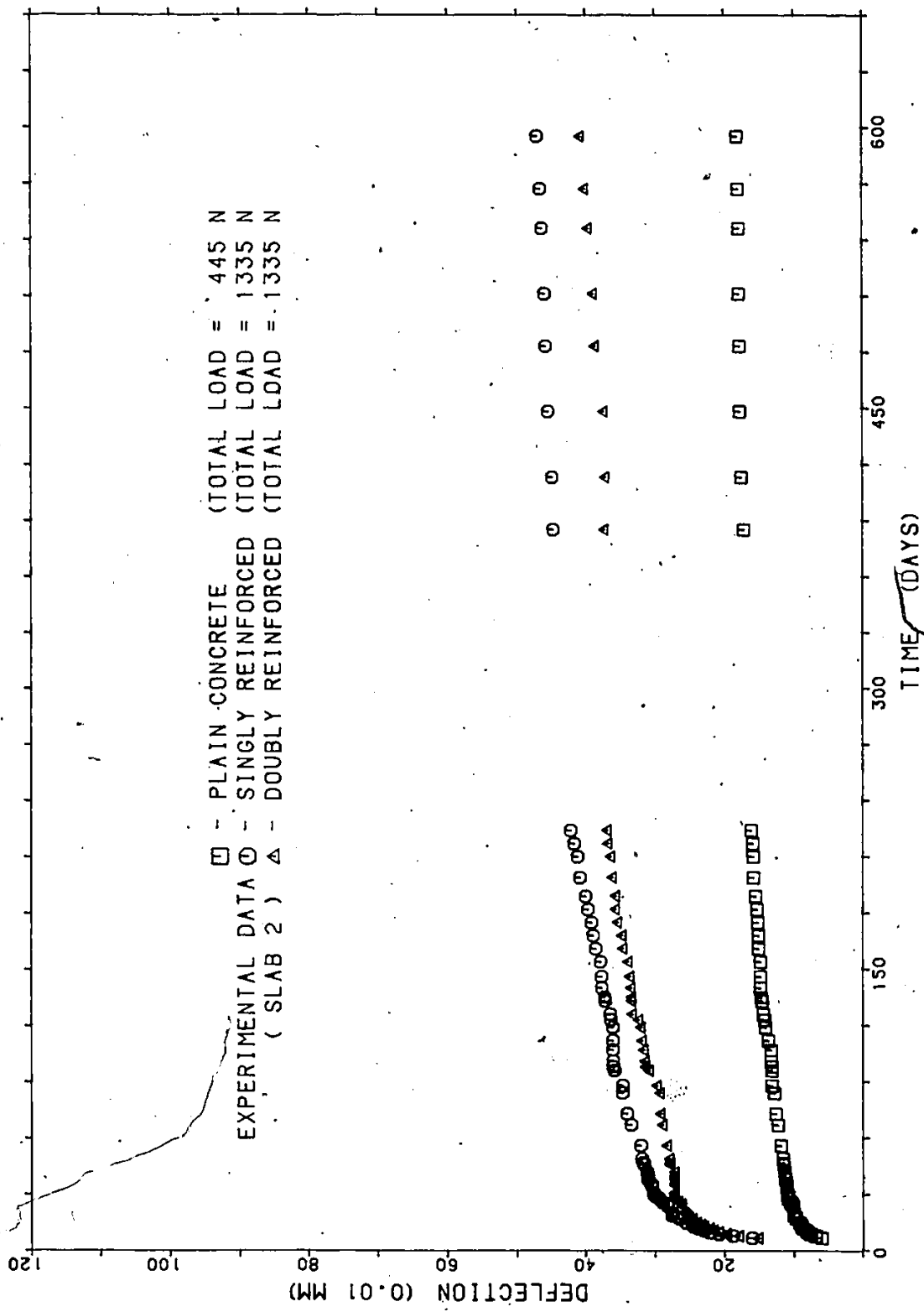


FIG.7.9 TIME-DEPENDENT BEAM DEFLECTIONS (SLAB 2)

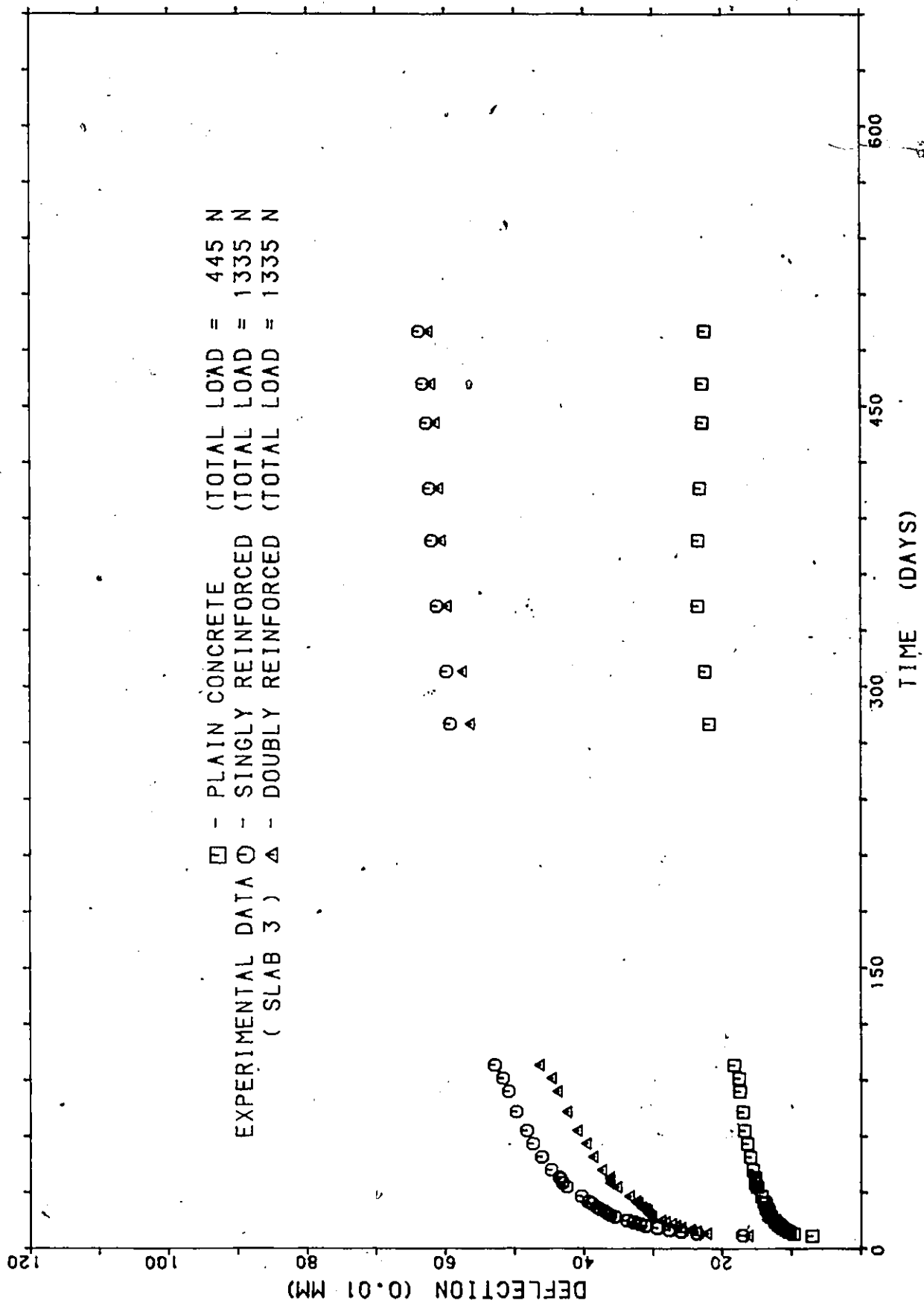


FIG.7.10 TIME-DEPENDENT BEAM DEFLECTIONS (SLAB 3)

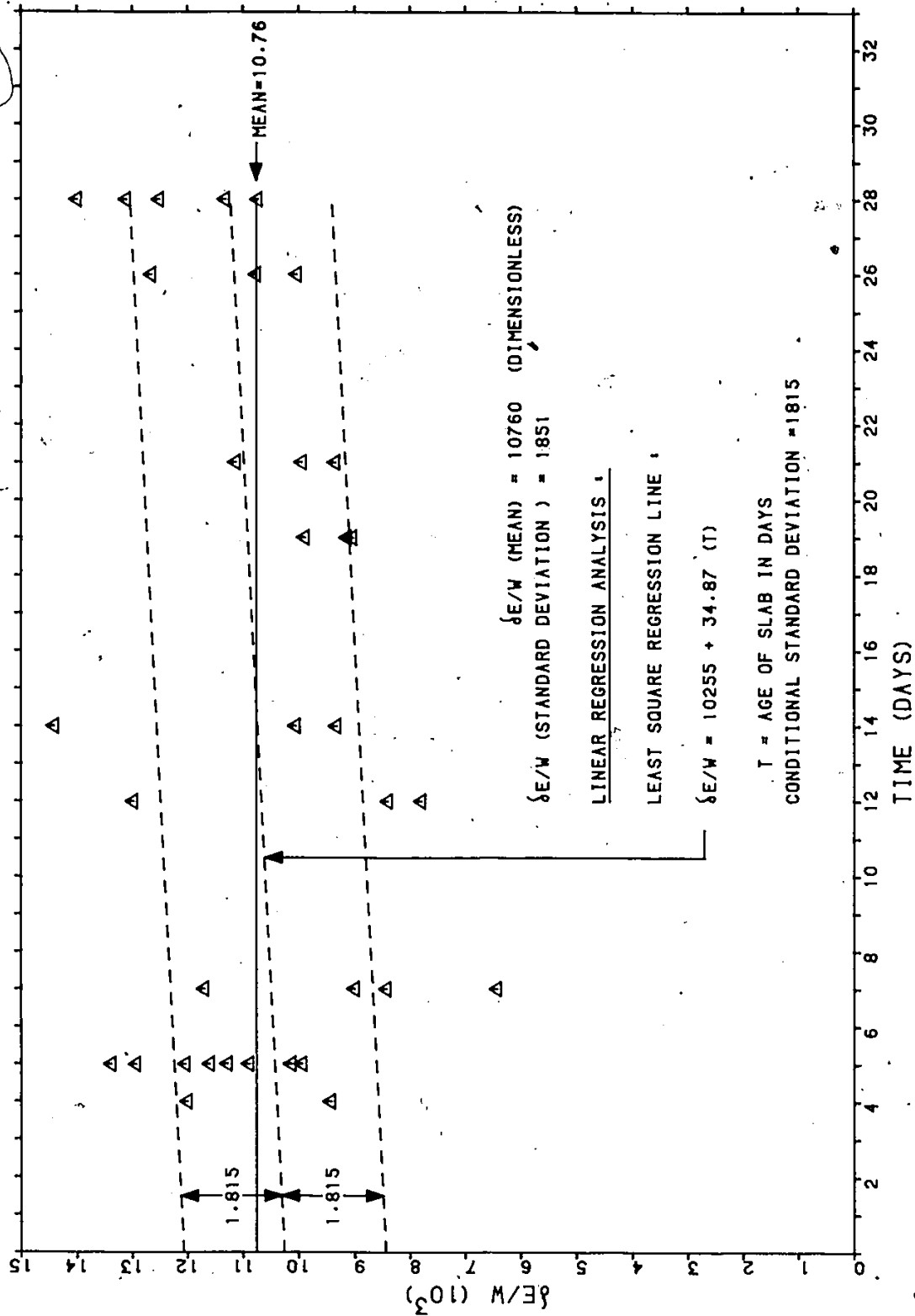


FIG.7.11 INTERNAL CONSISTENCY OF TEST RESULTS
(ONE WAY SLAB MODELS)

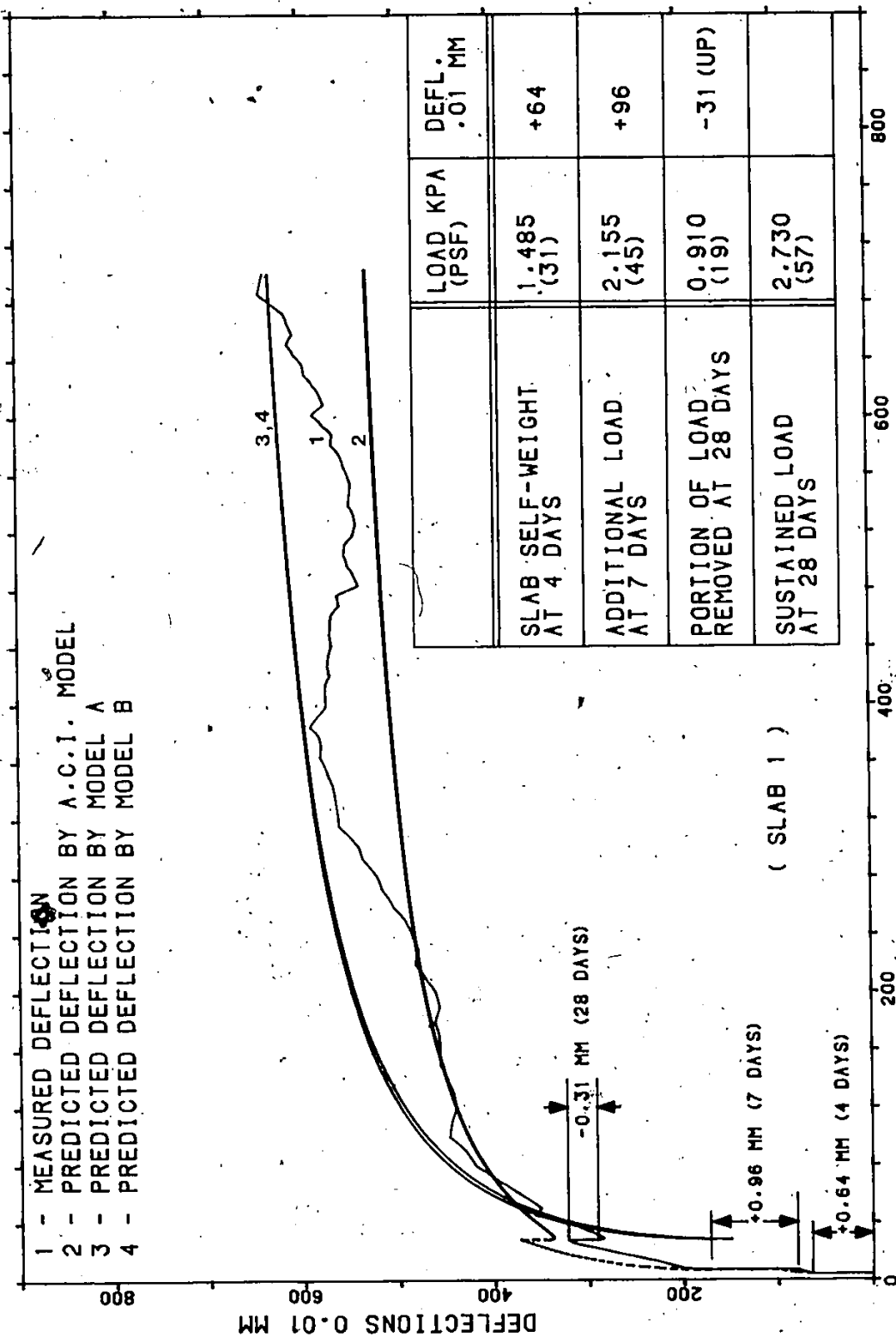


FIG.12A MEASURED DEFLECTION-TIME CURVE (FLAT PLATE MODEL 1)

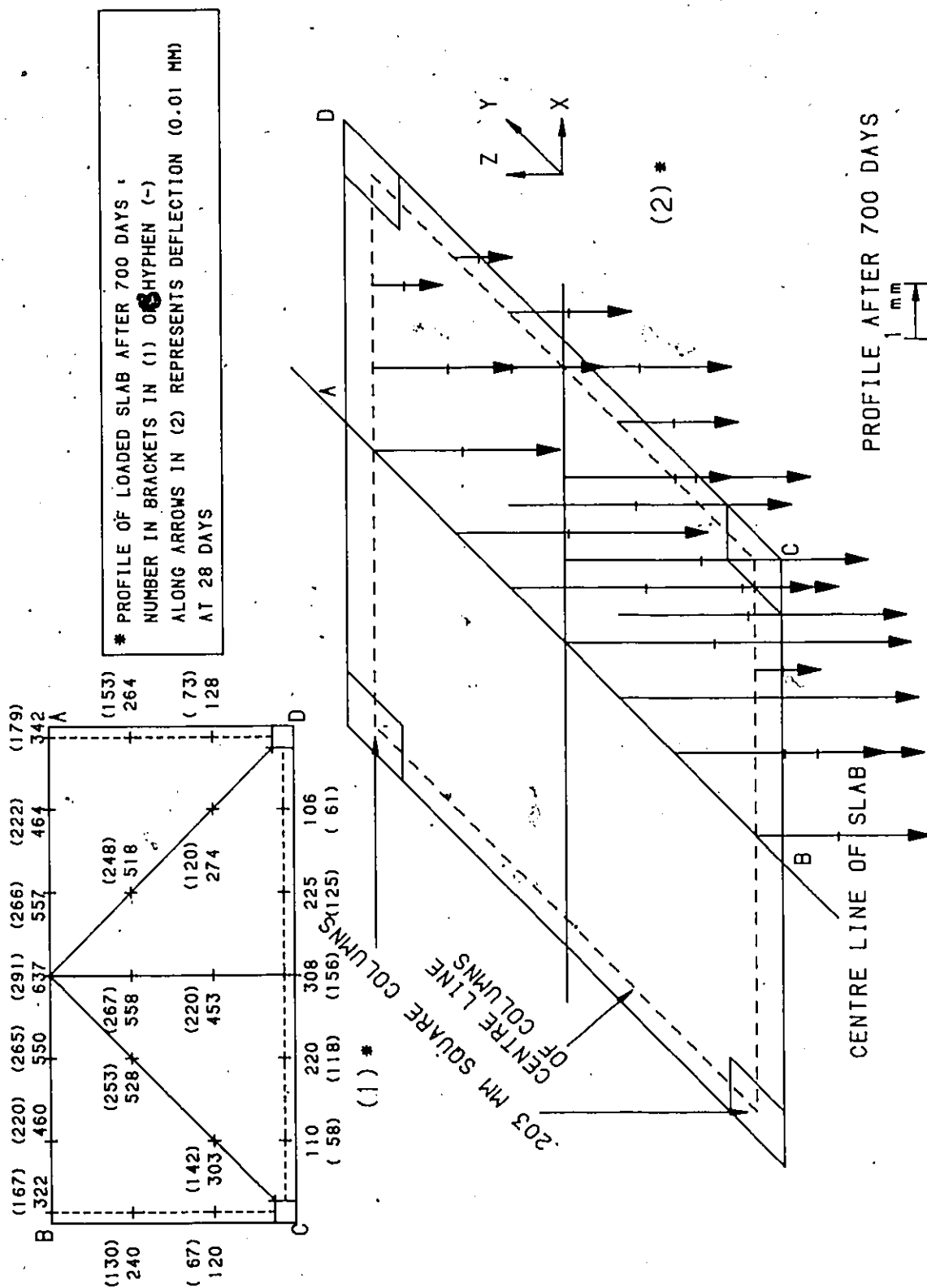


FIG.7.12B PROFILE OF DEFLECTED SLAB (FLAT PLATE MODEL 1)

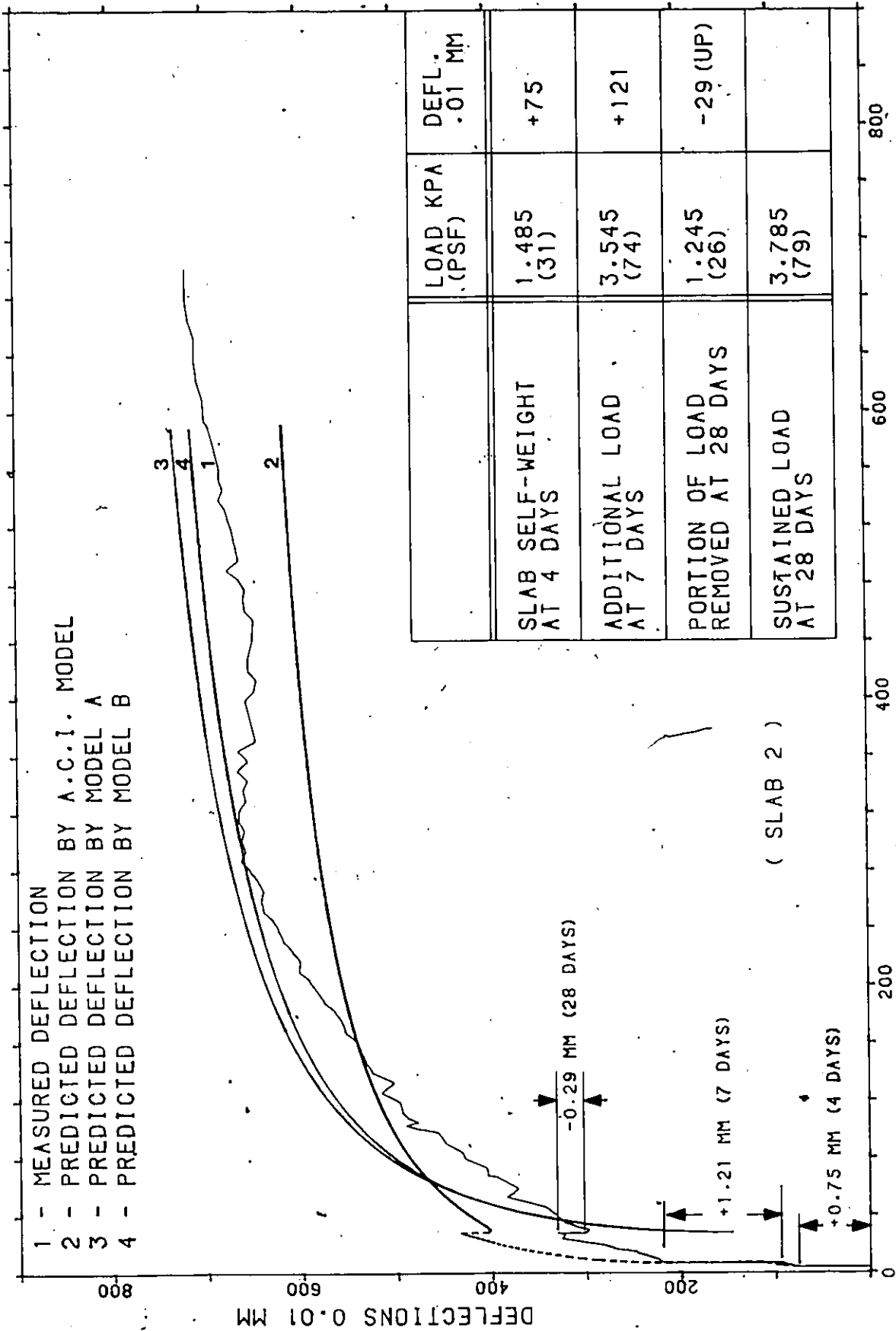


FIG.13A MEASURED DEFLECTION-TIME CURVE (FLAT PLATE MODEL '2)

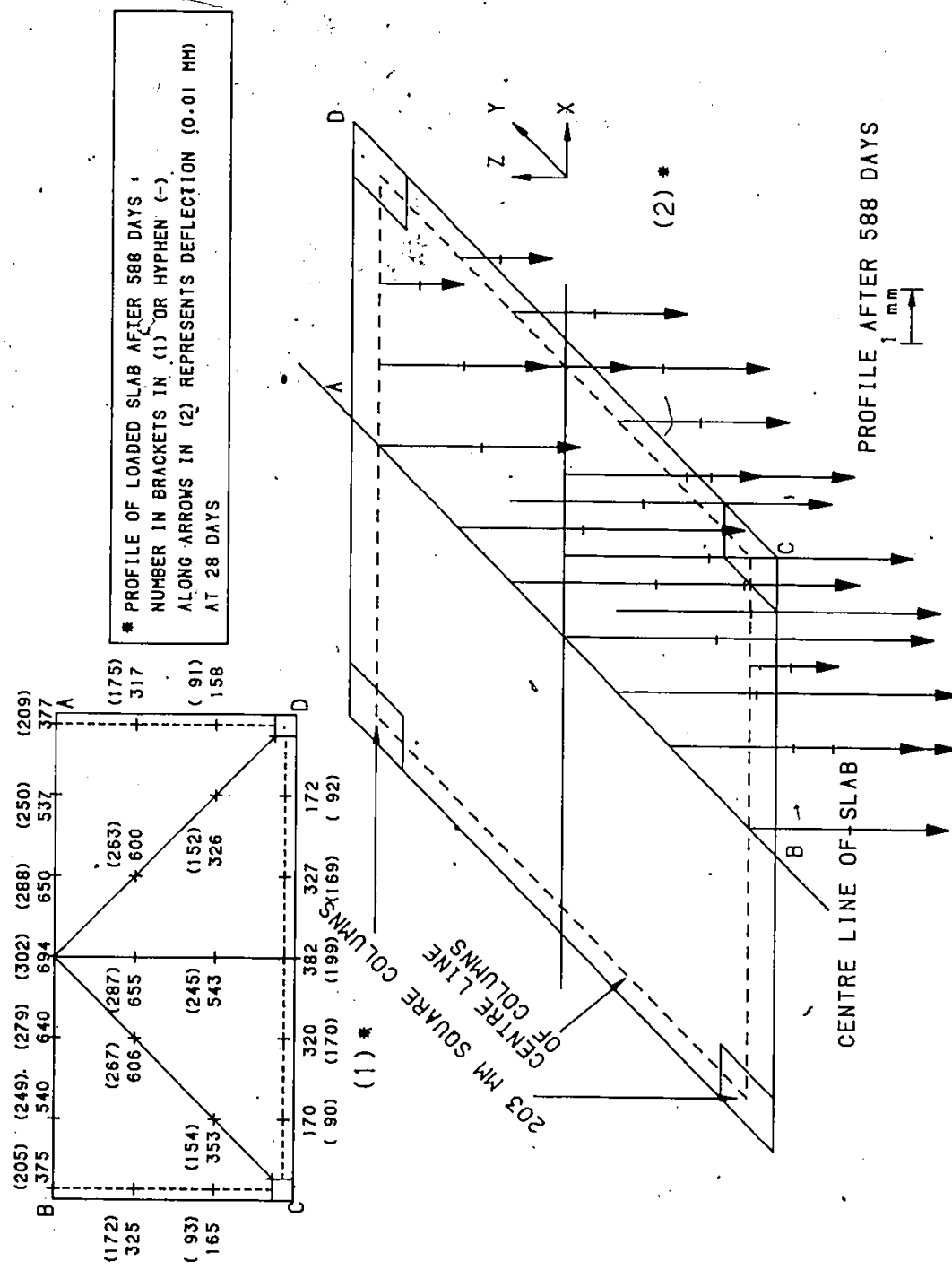


FIG.7.13B PROFILE OF DEFLECTED SLAB (FLAT PLATE MODEL 2)

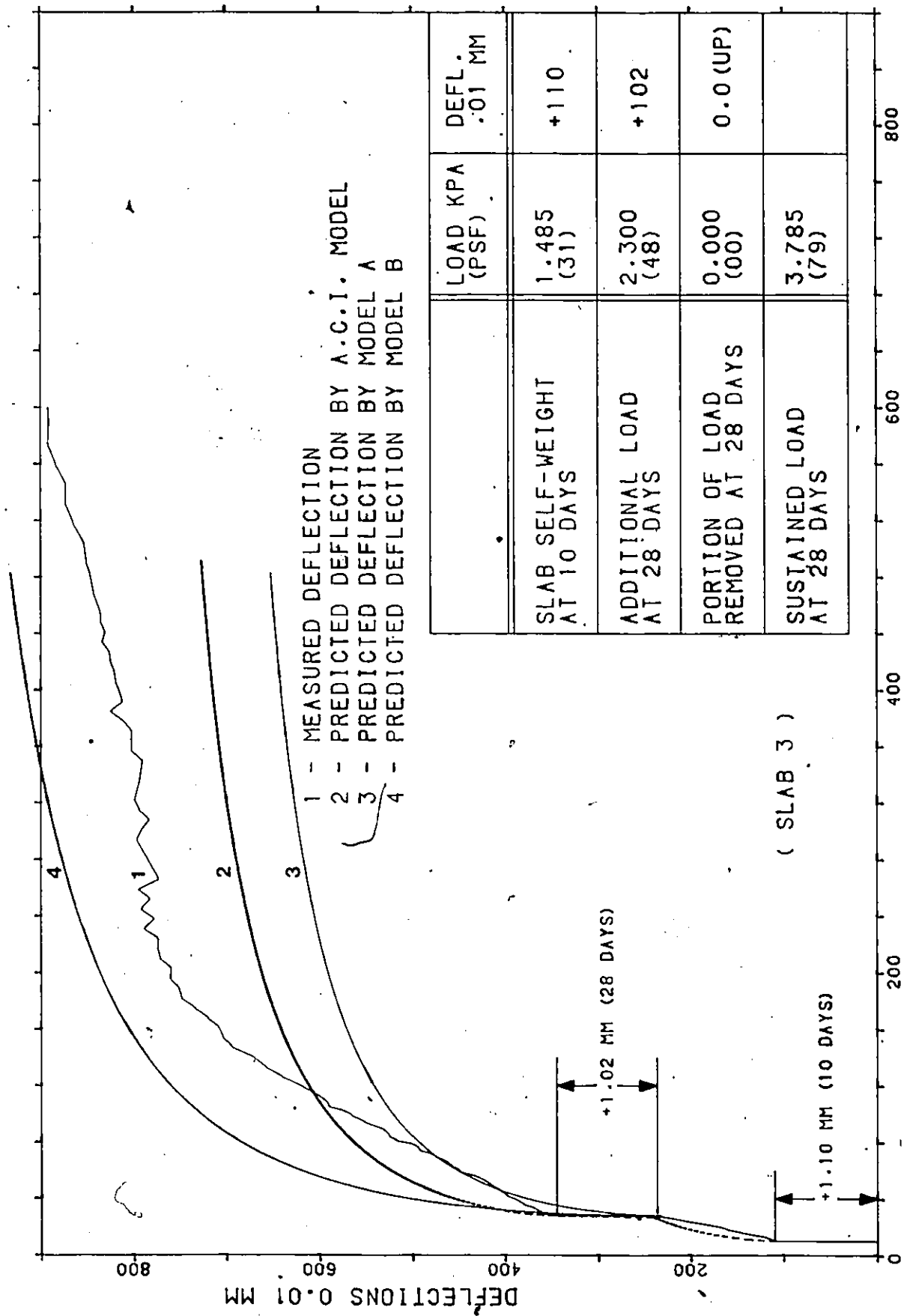


FIG.14A MEASURED DEFLECTION-TIME CURVE (FLAT PLATE MODEL 3)

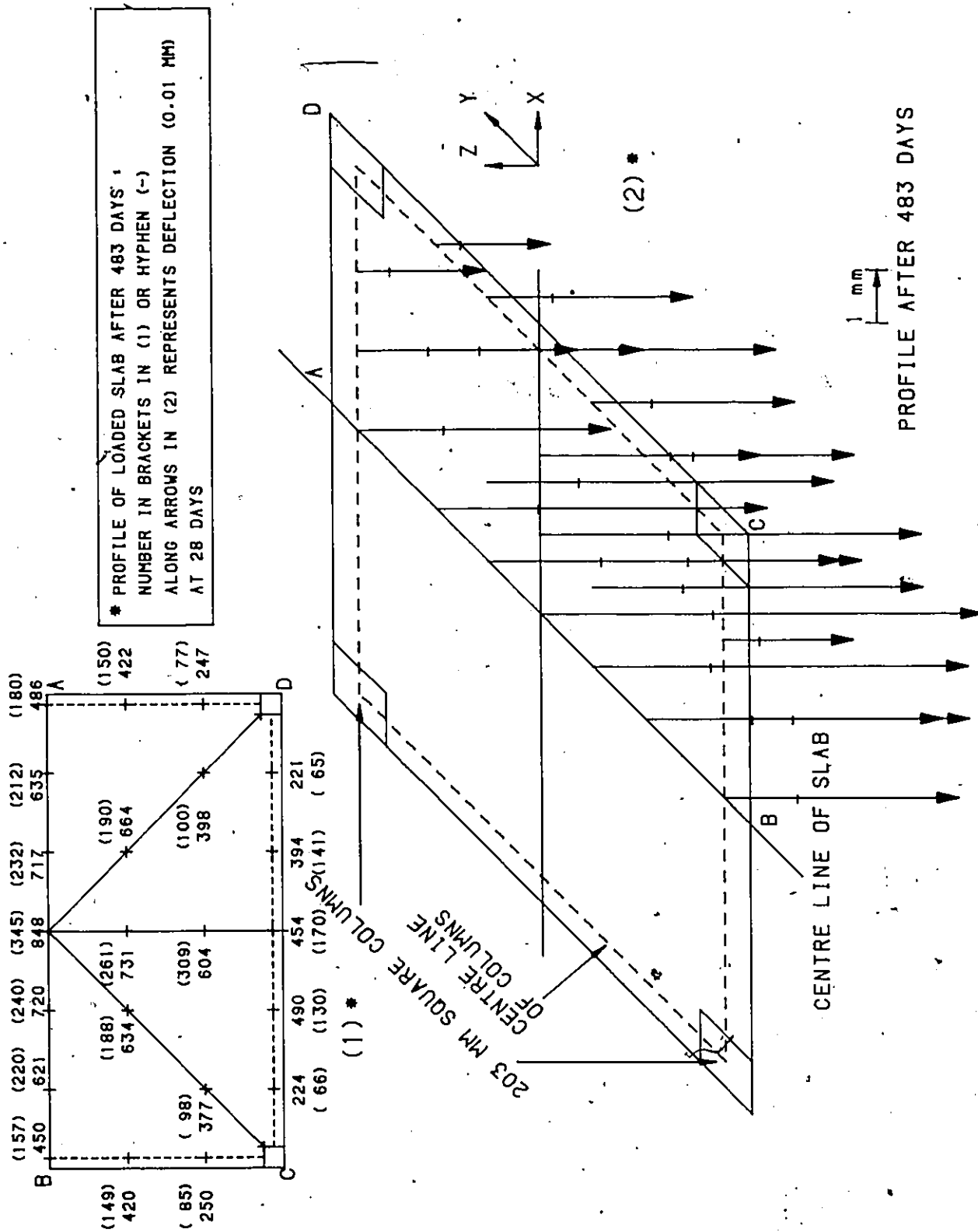


FIG.7.14B PROFILE OF DEFLECTED SLAB (FLAT PLATE MODEL 3)

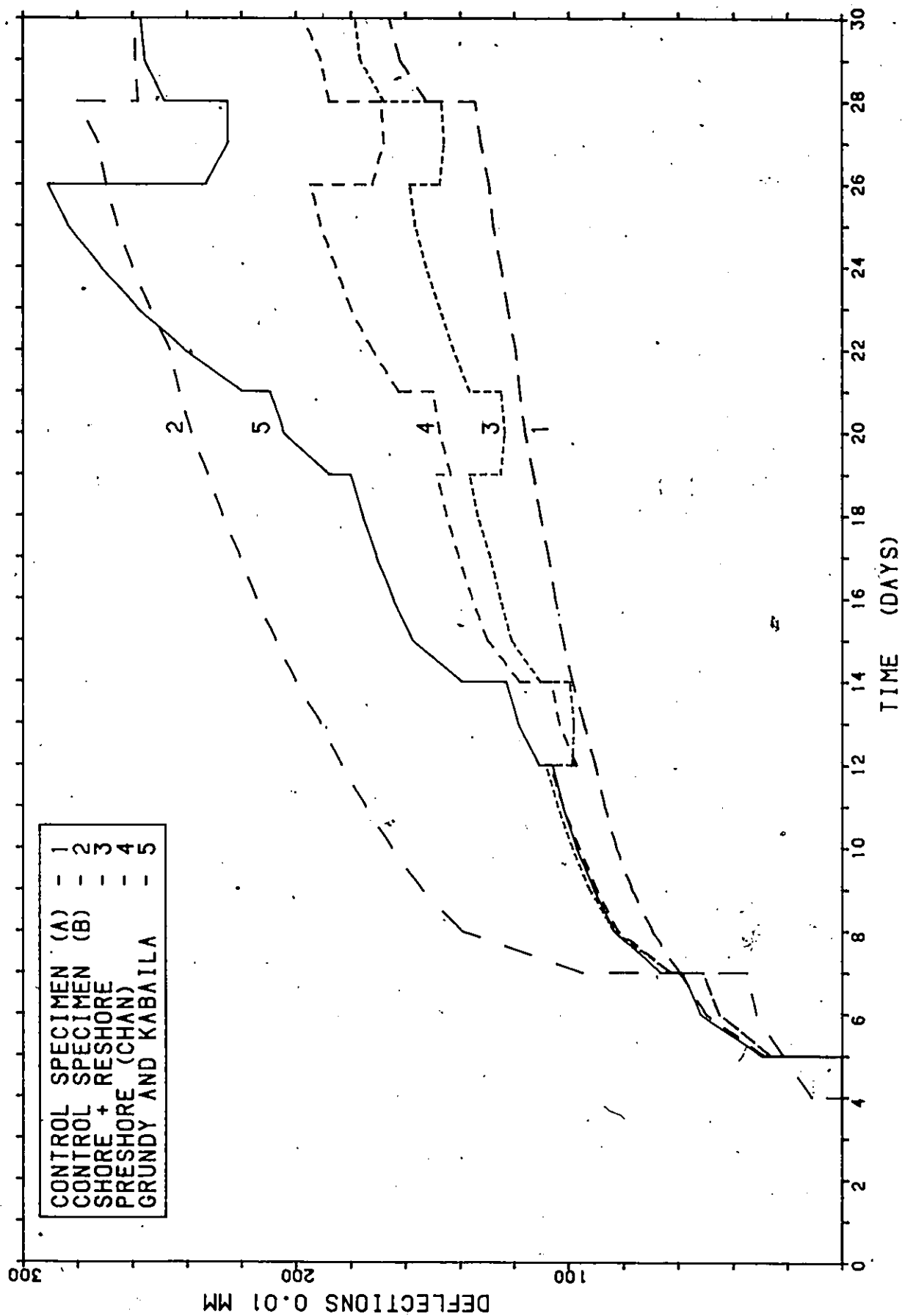


FIG.7.15 MEASURED DEFLECTIONS DUE TO CONSTRUCTION LOADS
(ONE WAY SLAB MODELS)

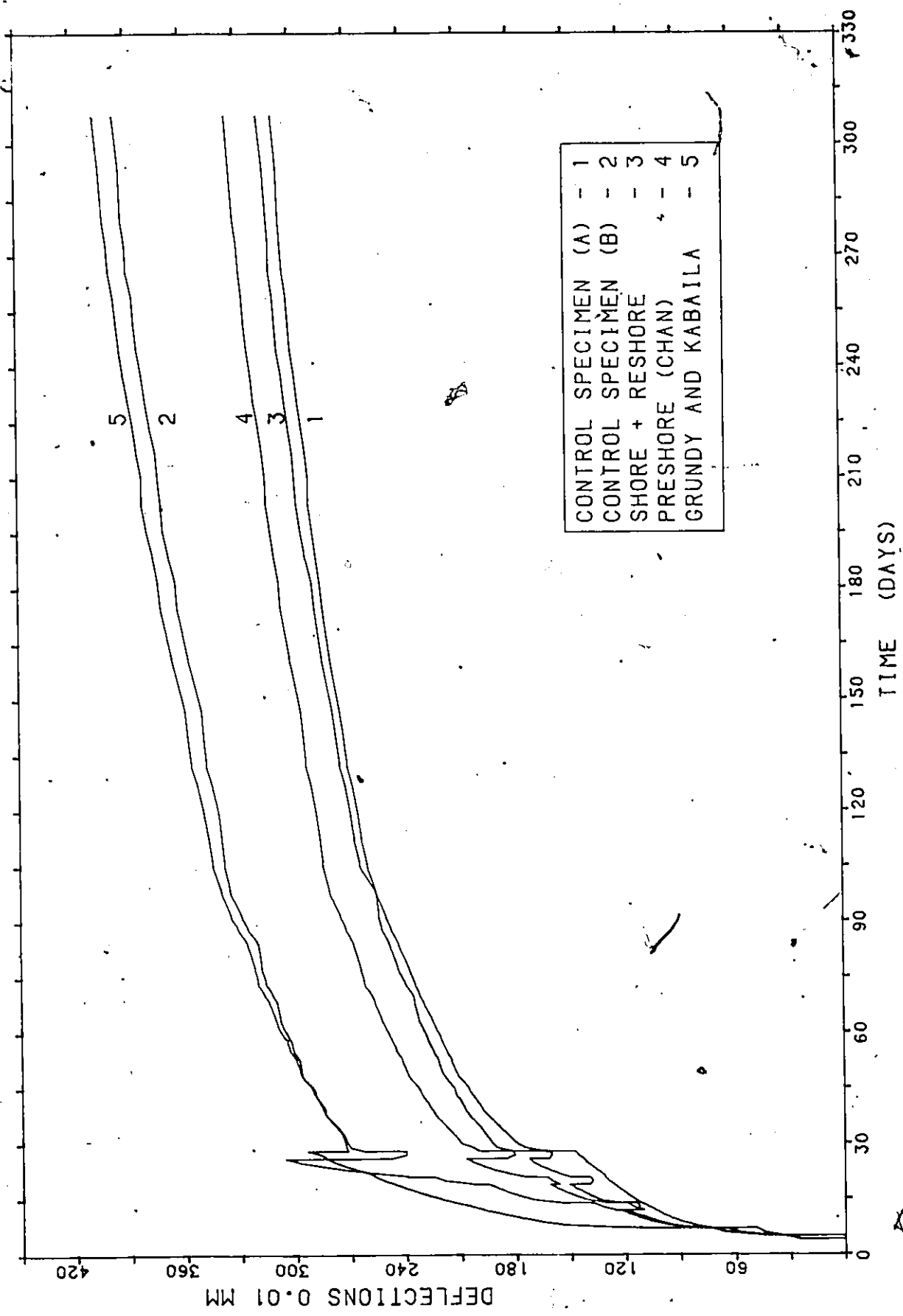


FIG.7.16 MEASURED DEFLECTION-TIME CURVES (ONE-WAY SLABS)

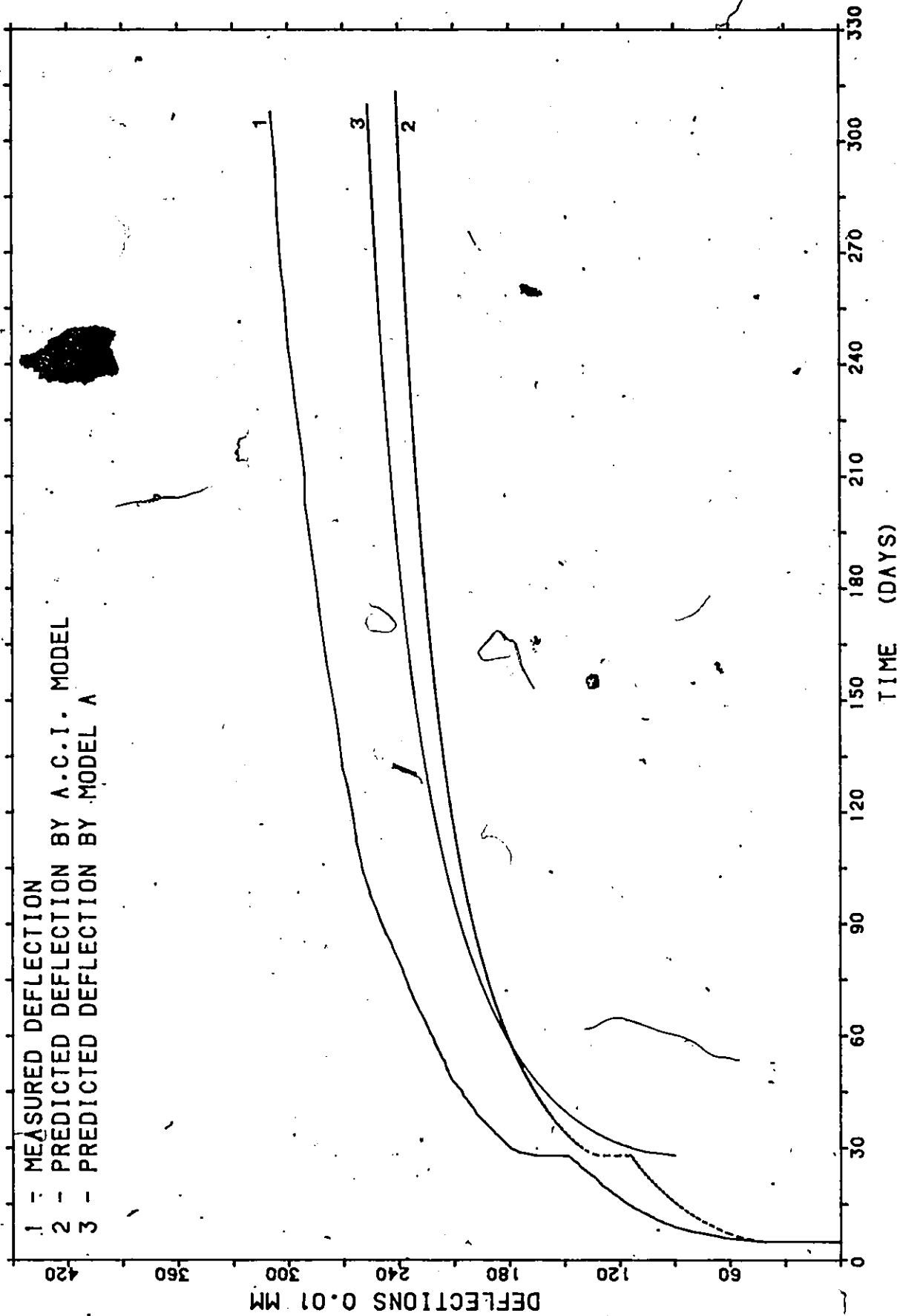


FIG. 7.17 DEFLECTION-TIME CURVE (ONEWAY SLAB #1)

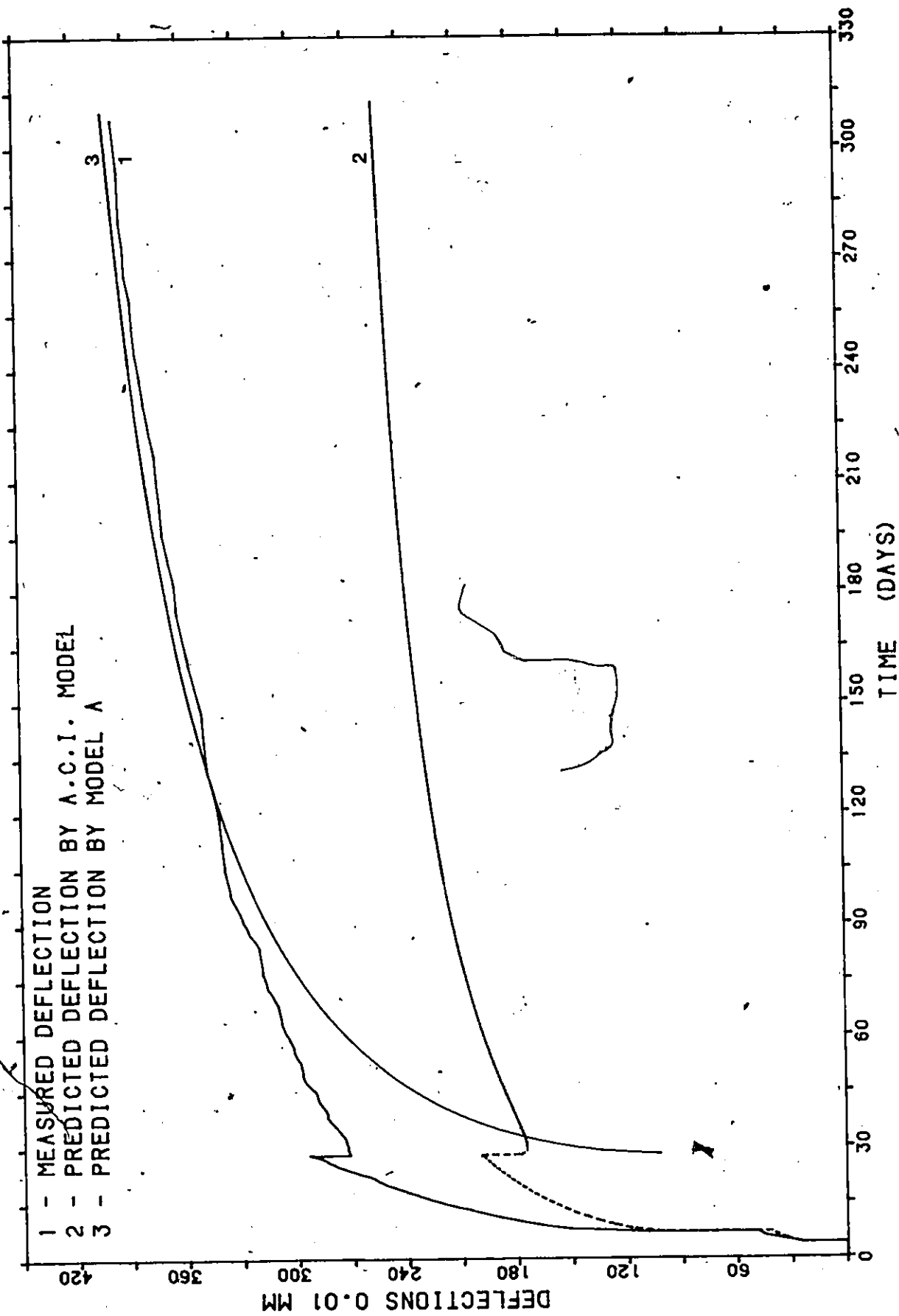


FIG.7.18 DEFLECTION-TIME CURVE (ONEWAY SLAB #2)

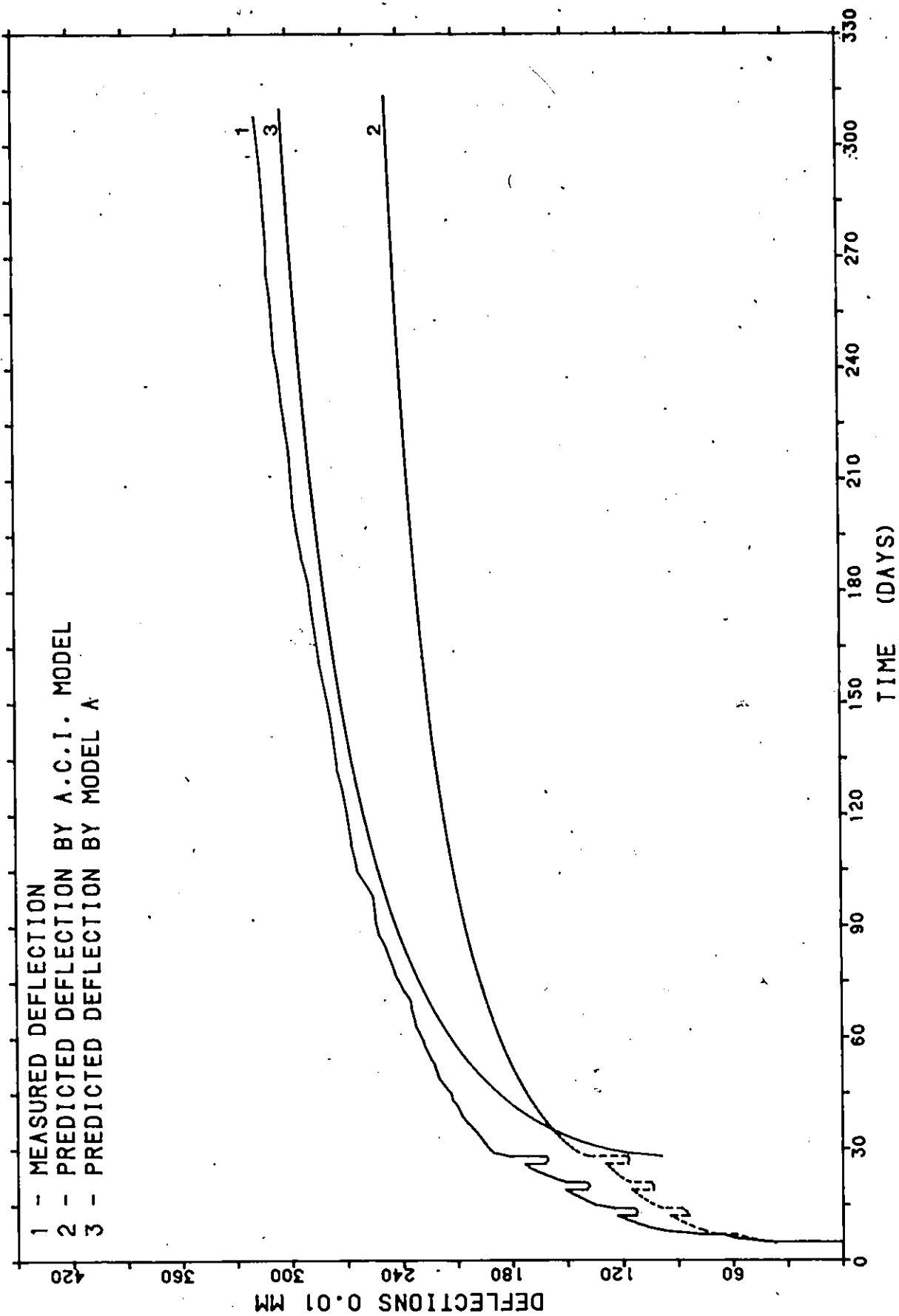


FIG.7.19 DEFLECTION-TIME CURVE (ONEWAY SLAB #3)

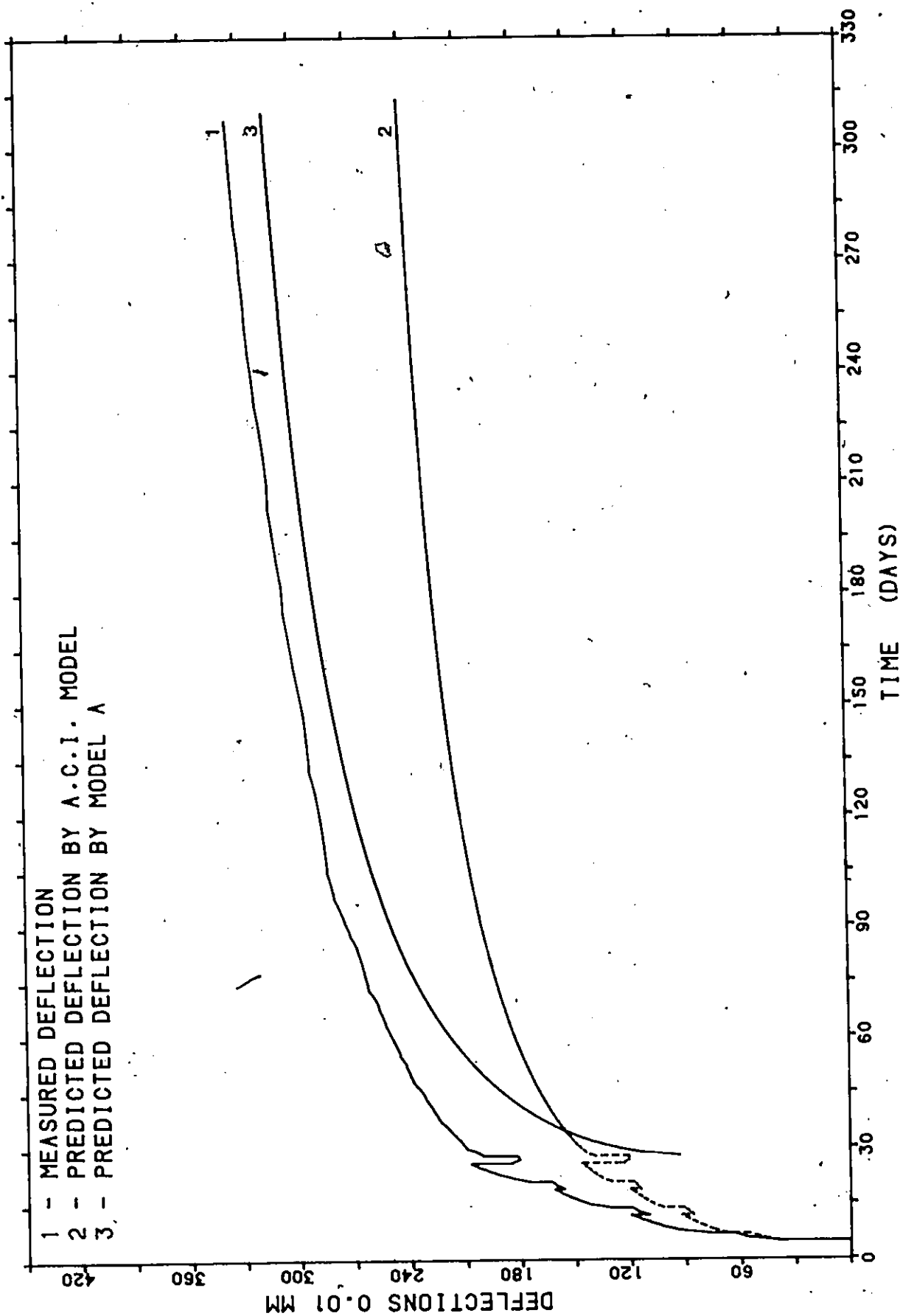


FIG.7.20 DEFLECTION-TIME CURVE (ONEWAY SLAB #4)

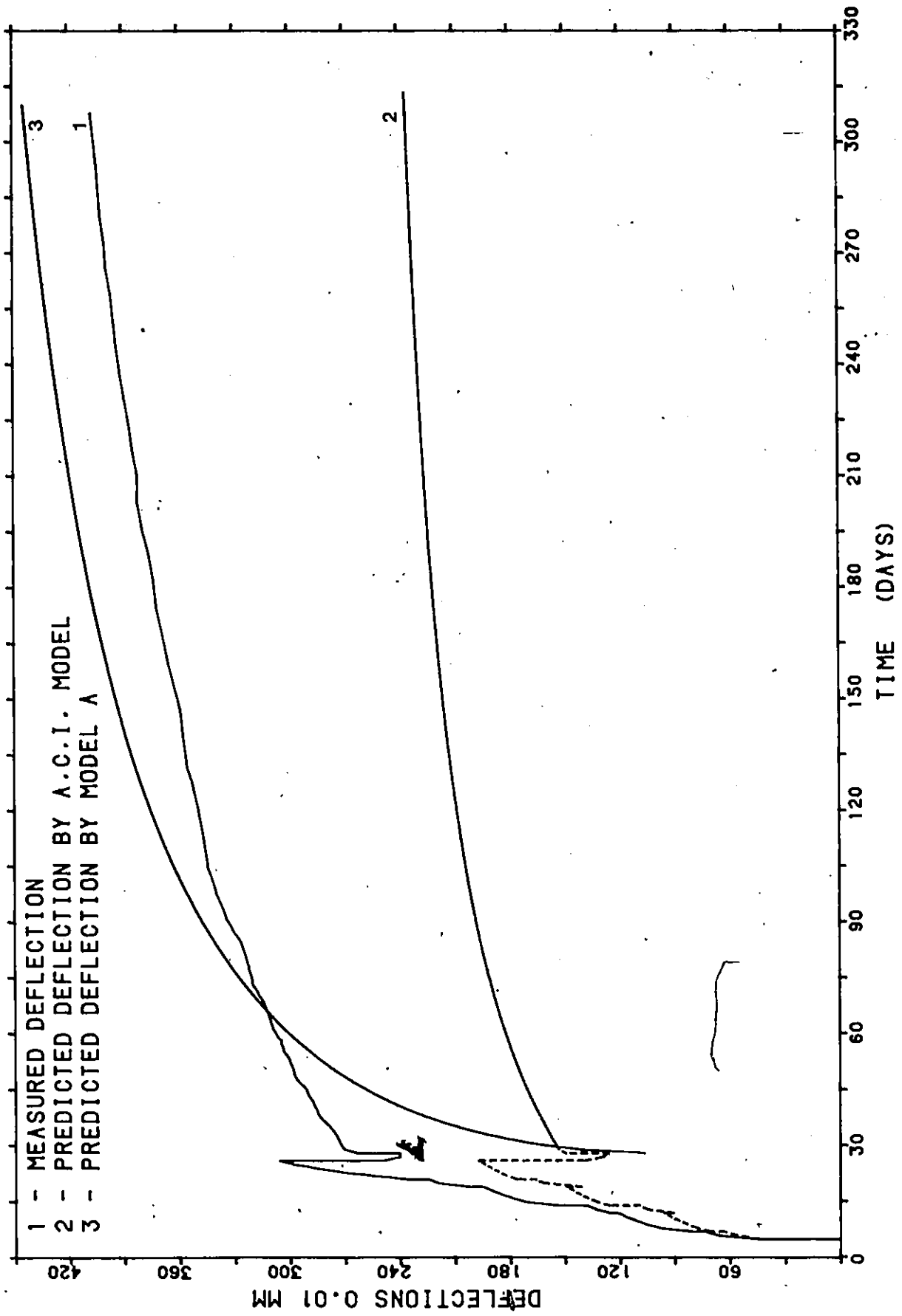


FIG.7.21 DEFLECTION-TIME CURVE (ONEWAY SLAB #5)

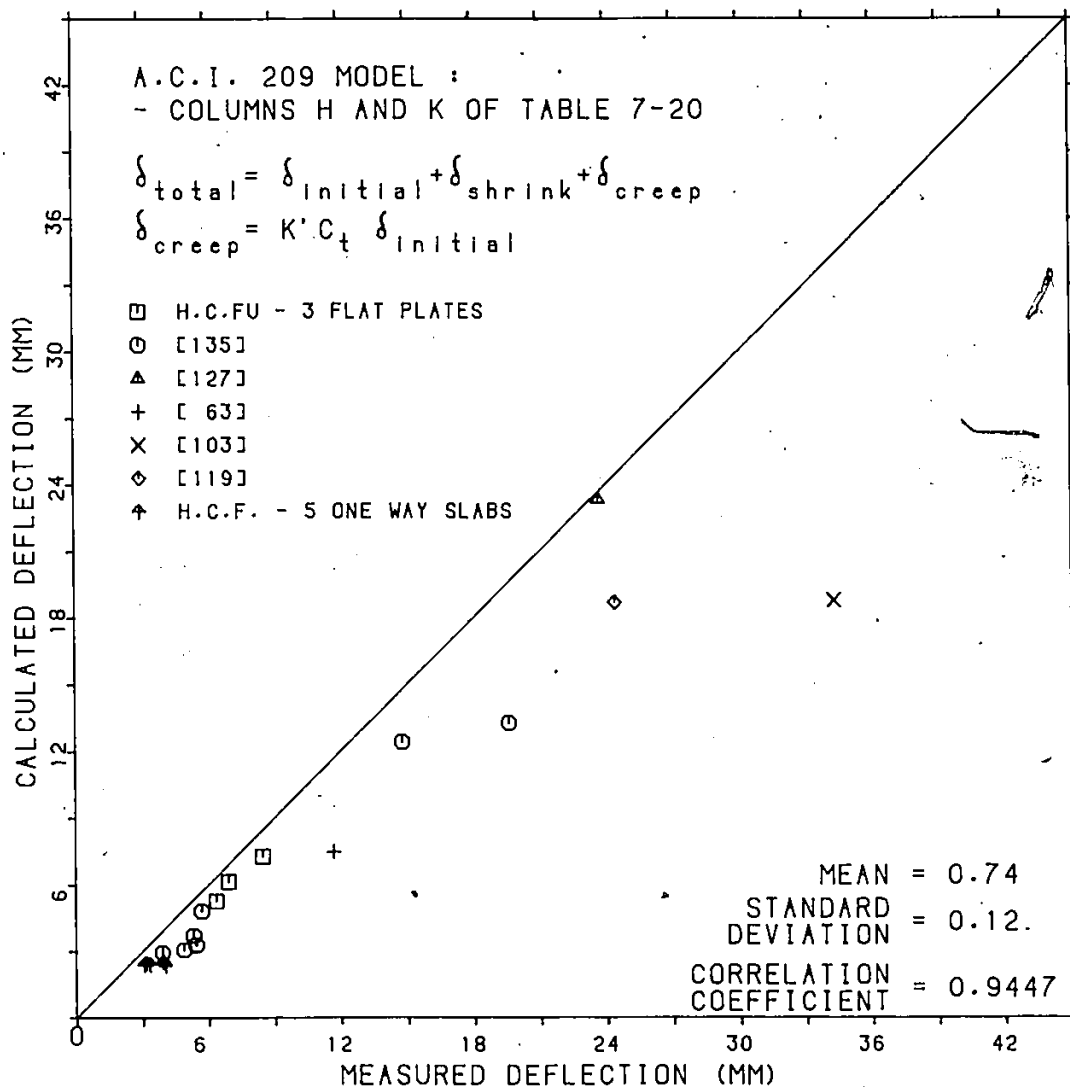


FIG.7.22 COMPARISON OF ACI-209 MODEL WITH TEST RESULTS

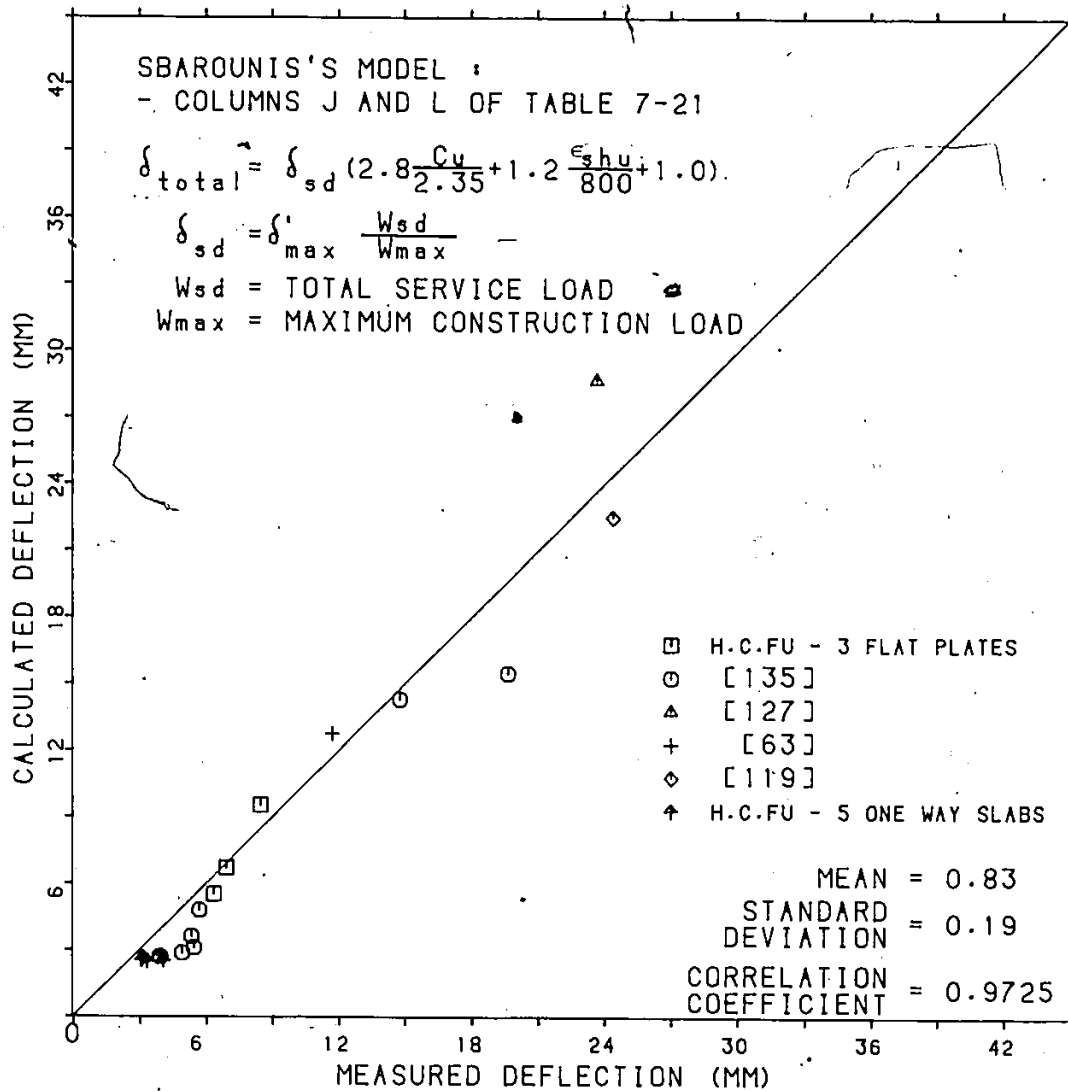


FIG.7.23 COMPARISON OF SBAROUNIS'S MODEL WITH WITH TEST RESULTS

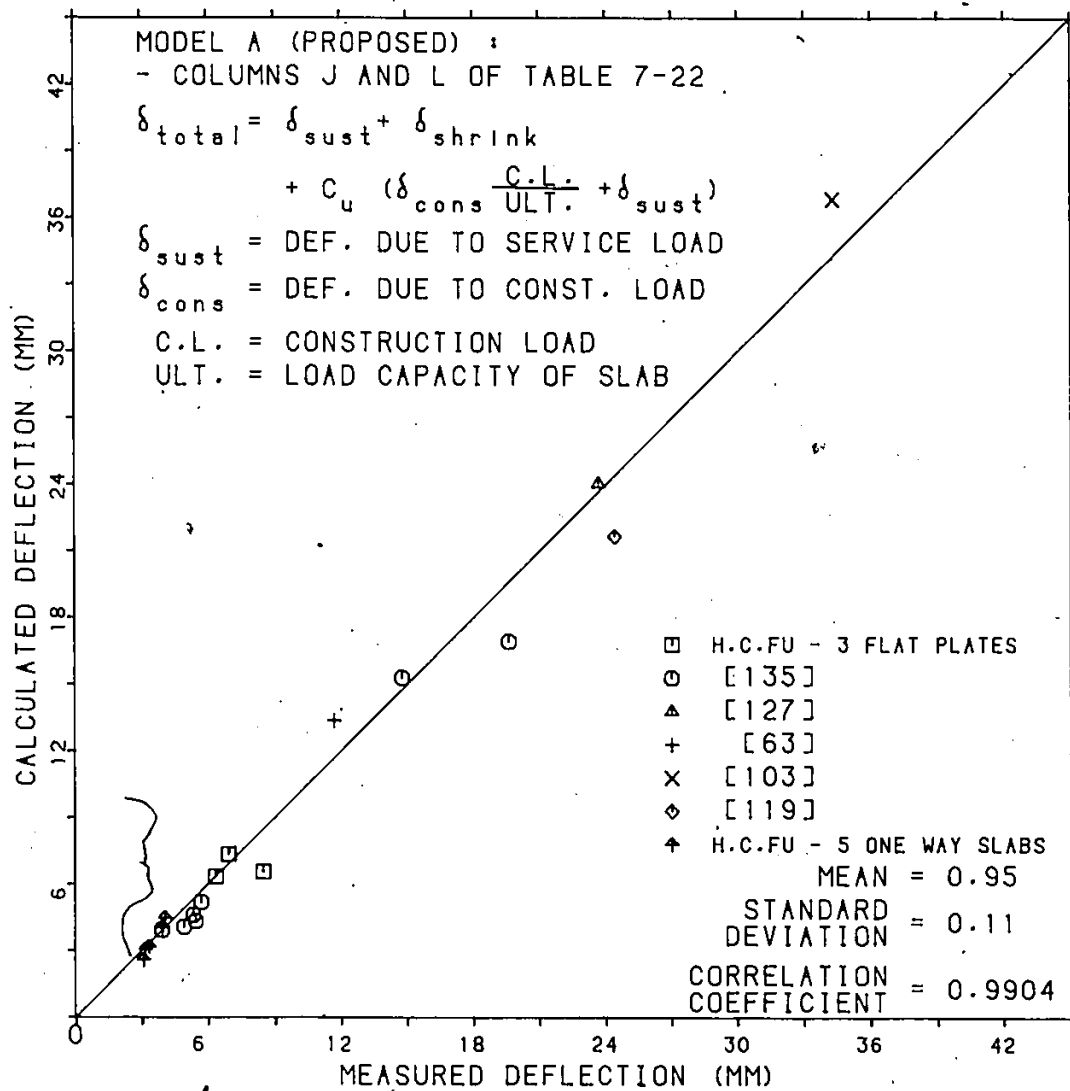


FIG.7.24 'COMPARISON OF MODEL A WITH TEST RESULTS

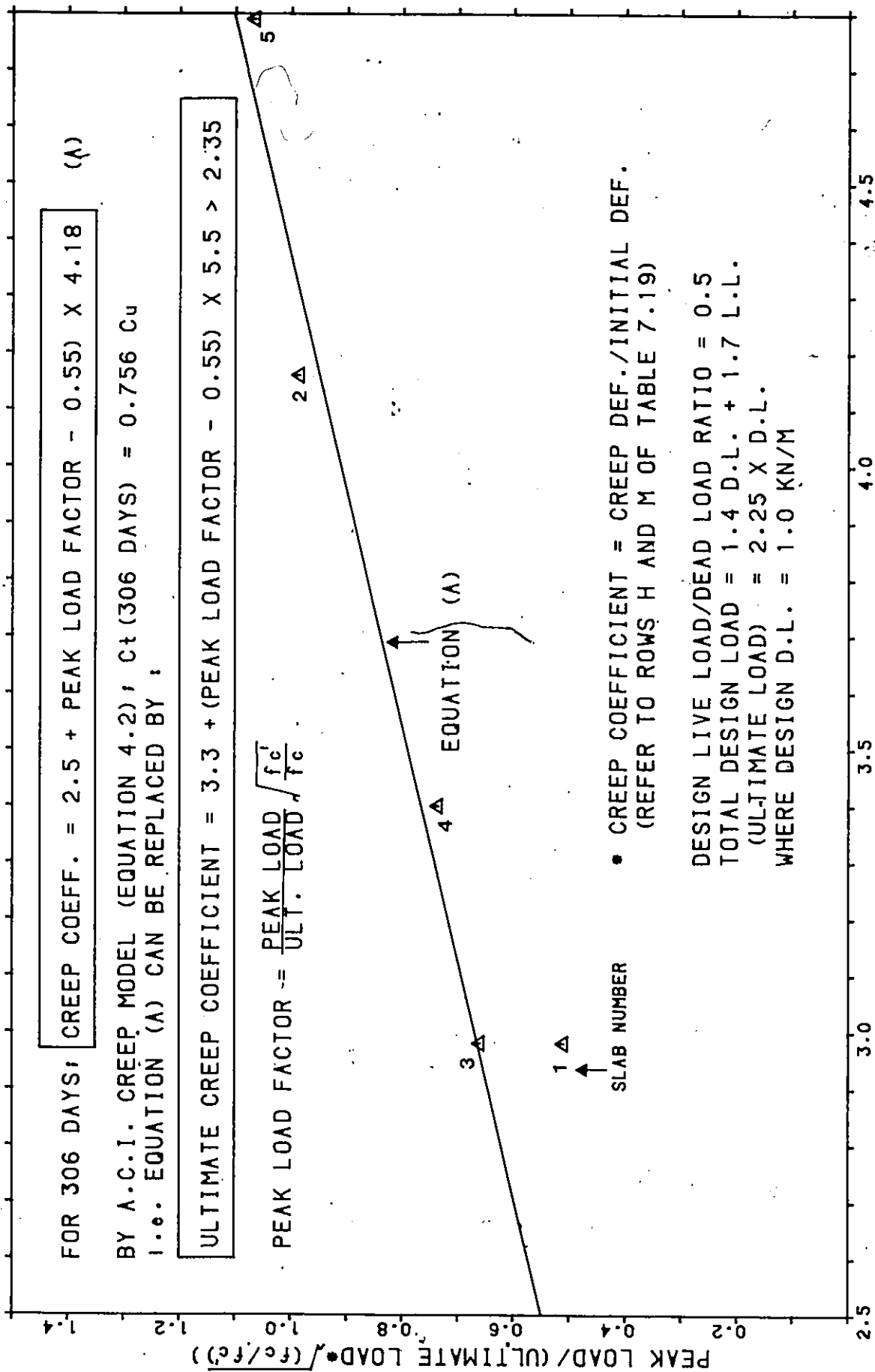


FIG. 7.25 CREEP COEFFICIENT .VS. PEAK LOAD FACTOR

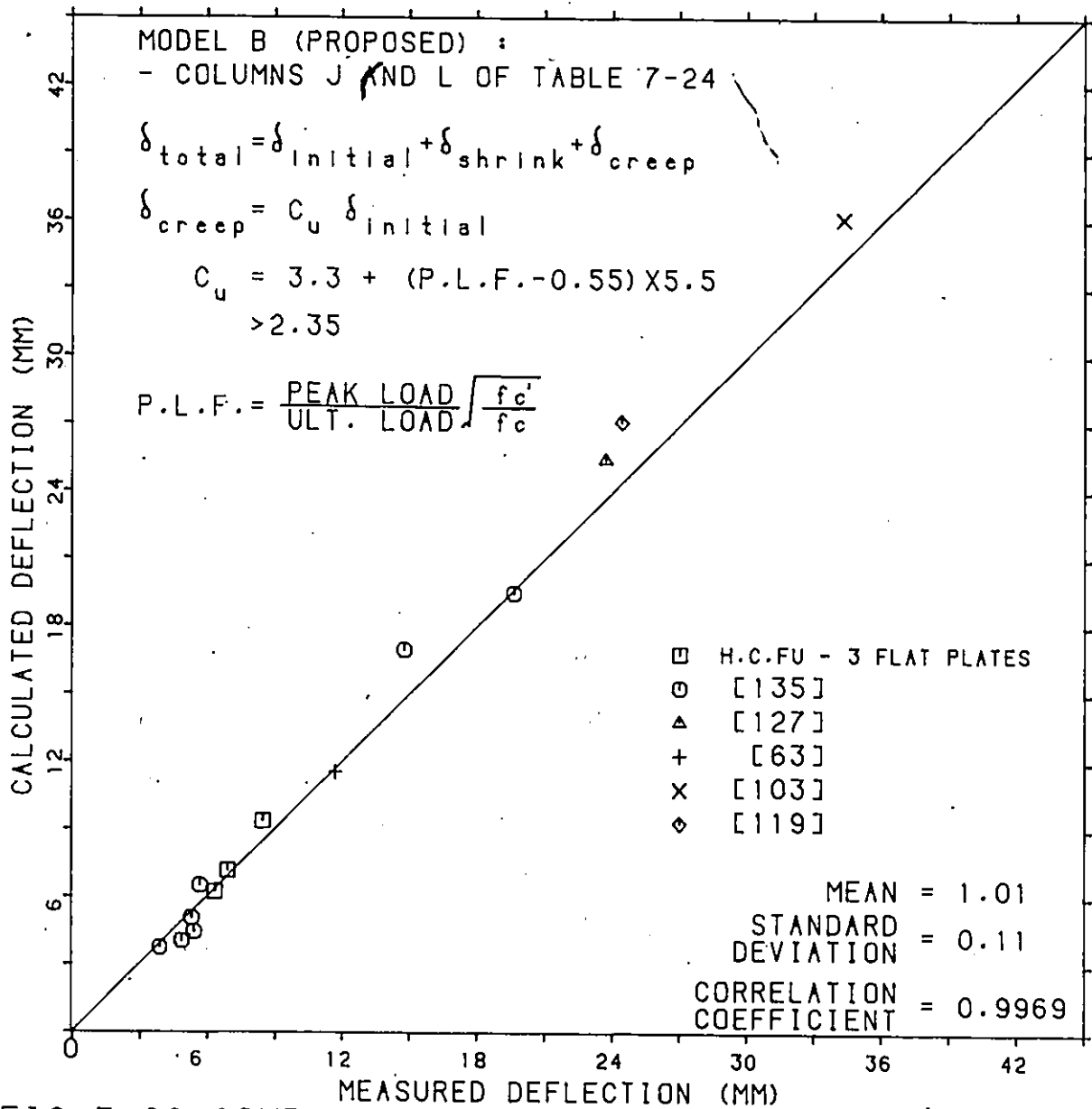


FIG.7.26 COMPARISON OF MODEL B WITH TEST RESULTS

APPENDIXES

$$\begin{array}{lcl} \text{LIVE LOAD/DEAD LOAD} & = & 0.5 \\ (\text{L.L.}) & (\text{D.L.}) & (1.0) \end{array}$$

$$\text{DESIGN LOAD} = 1.4 \text{ D.L.} + 1.7 \text{ L.L.}$$

$$= 2.25 \quad (3.1) \bullet \text{ D.L.}$$

$$\begin{array}{lcl} \text{DESIGN LOAD/DESIGN D.L.} & = & 2.25/1.4 \\ & = & 1.607 \end{array} \quad \begin{array}{l} (3.1/1.4) \\ (2.214) \end{array}$$

CONSIDER A CONSTRUCTION SEQUENCE USING
THREE LEVELS OF FORMS AND 7-DAY CYCLE (FIGURE 1.1)

CONCRETE STRENGTH AT (FIGURE 1.2)

$$7 \text{ DAYS} = 0.70 \times f_c'$$

$$14 \text{ DAYS} = 0.88 \times f_c'$$

$$21 \text{ DAYS} = 0.96 \times f_c'$$

$$\text{TOTAL} = 2.54 \times f_c'$$

$$\text{AVAILABE STRENGTH} = 2.54 \times 2.25 = 2.47 \times 3.10$$

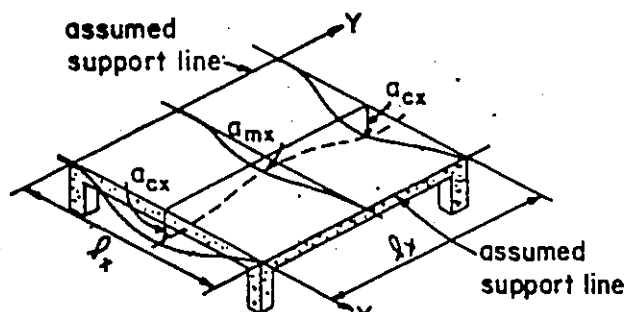
$$= 5.72 \quad 7.65$$

$$\approx 5.56 \bullet > 5.56 \bullet$$

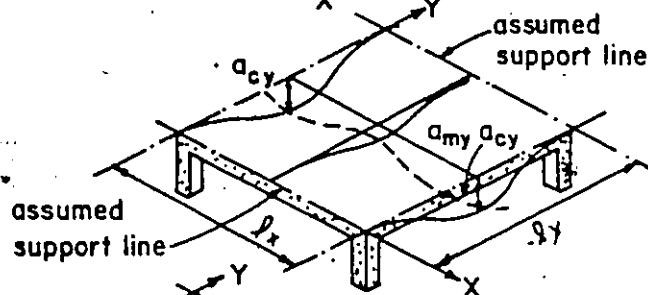
? O.K.

- A TOTAL OF 4 SLABS (4 X 1.4 D.L.) IN THE SYSTEM
(THREE OLD SLABS SUPPORTING ONE FRESH SLAB)

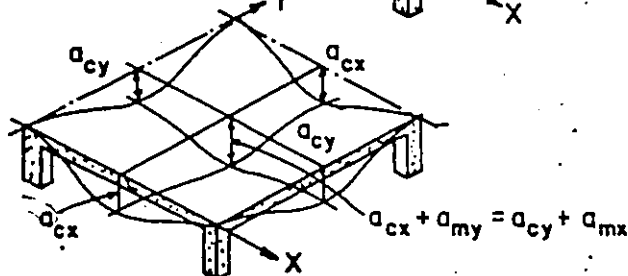
APPENDIX-A EFFECTS OF LIVE LOAD TO DEAD LOAD
RATIO IN FLAT SLAB STRUCTURES
DURING CONSTRUCTION PROCESS



(A) X-DIRECTION BENDING



(B) Y-DIRECTION BENDING



(C) COMBINED BENDING

DEFLECTION COMPUTATIONS :

- MID-SPAN DEFLECTION OF AN EQUIVALENT FRAME INCLUDES :
 - (A) THAT OF THE PANEL ASSUMED TO BE FIXED AT BOTH ENDS OF ITS SPANS
 - (B) PLUS THAT DUE TO THE KNOWN ROTATION AT EACH OF THE TWO SUPPORTING LINES

MID-SPAN REFERENCE DEFLECTION OF A UNIFORMLY LOADED EQUIVALENT FRAME WITH FIXED ENDS

$$\delta_{REF} = \frac{w l^4}{384 E_c I_{FRAME}}$$

W = UNIFORM LOAD

MID-SPAN DEFLECTION OF A COLUMN OR MIDDLE STRIP UNDER FIXED END CONDITIONS

$$\delta'_{STRIP} = \delta_{REF} \frac{M_{STRIP} E_c I_{FRAME}}{M_{FRAME} E_c I_{STRIP}} = \delta_{MX} \text{ OR } \delta_{MY}$$

ROTATION OF COLUMN (θ) = UNBALANCED MOMENT/STIFFNESS OF THE EQU. COLUMN
MID-SPAN DEFLECTION OF THE PANEL DUE TO END ROTATION = $\delta_{\theta} = \theta L / 8$

THE TOTAL MID-SPAN DEFLECTION FOR THE COLUMN OR MIDDLE STRIP EQUALS

$$\delta_{STRIP} = \delta'_{STRIP} + \delta_{\theta.L} + \delta_{\theta.R}$$

$\delta_{\theta.L}$ = CONTRIBUTION TO DEFLECTION DUE TO ROTATION OF LEFT SUPPORT

$\delta_{\theta.R}$ = CONTRIBUTION TO DEFLECTION DUE TO ROTATION OF RIGHT SUPPORT

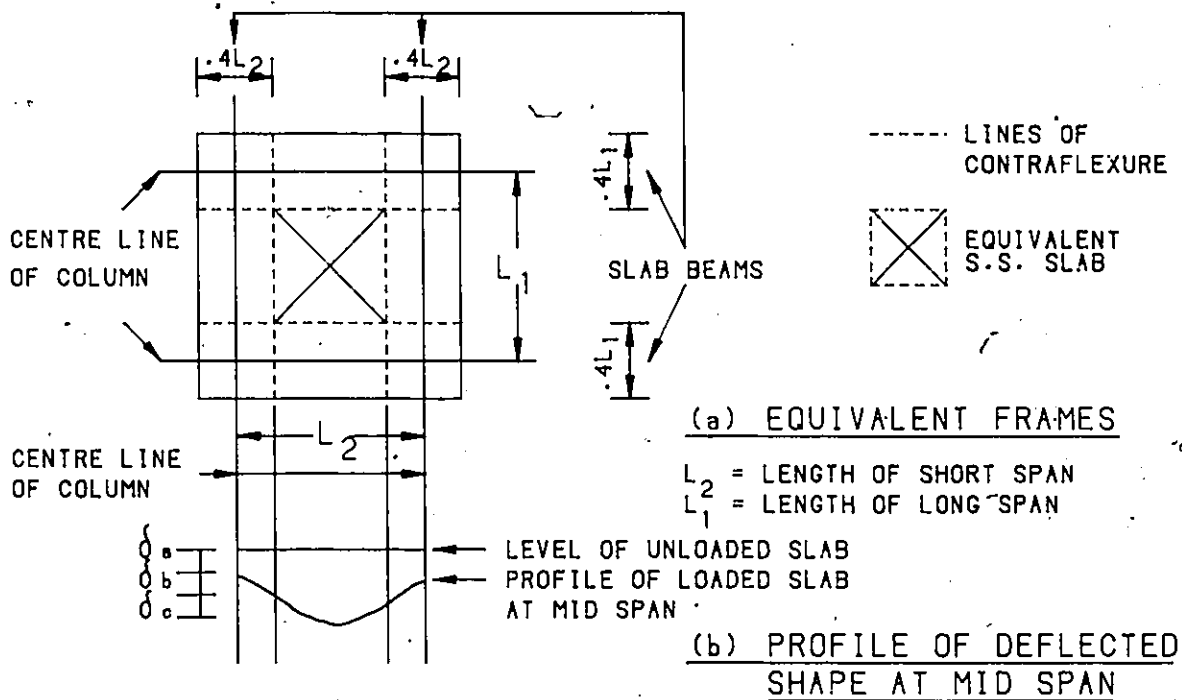
FOR SQUARE PANELS, THE MID-SPAN DEFLECTION IS

$$\delta = \delta_{CY} + \delta_{MX} = \delta_{CX} + \delta_{MY}$$

FOR RECTANGULAR PANELS, THE MID-SPAN DEFLECTION IS

$$\delta = (\delta_{CY} + \delta_{MX} + \delta_{CX} + \delta_{MY}) / 2$$

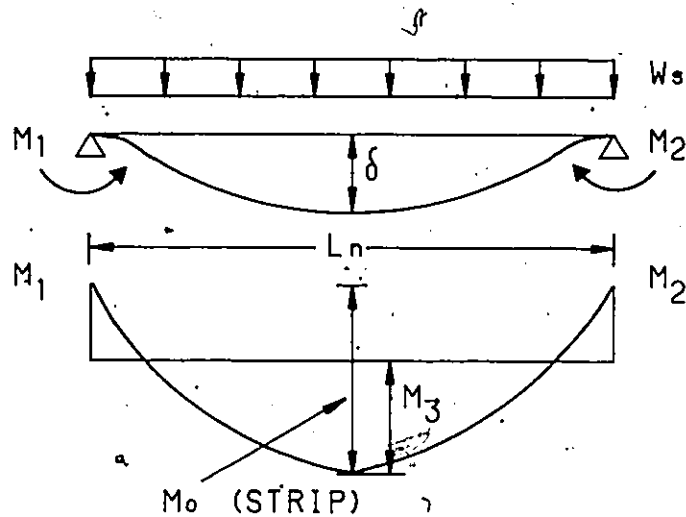
APPENDIX-B DEFLECTION COMPUTATIONS BY ADOSS [25]



- δ_a = DEFLECTION FOR FIXED-ENDS COLUMN STRIP ($0.4 L_2$ WIDE AND L_1 LONG)
 δ_b = DEFLECTION FOR CANTILEVER BEAM (UNIT WIDTH AND $0.2 L_2$ LONG)
 δ_c = DEFLECTION FOR SIMPLY SUPPORTED INTERIOR PLATE ($0.6 L_1 \times 0.6 L_2$)

APPENDIX-C S-METHOD FOR DEFLECTION ANALYSIS [124]

COLUMN OR MIDDLE STRIP WITH END MOMENTS



$$\delta = \frac{5 W_s L_n^4}{384 E I_o} - \frac{(M_1 + M_2) L_n^2}{16 E I_o}$$

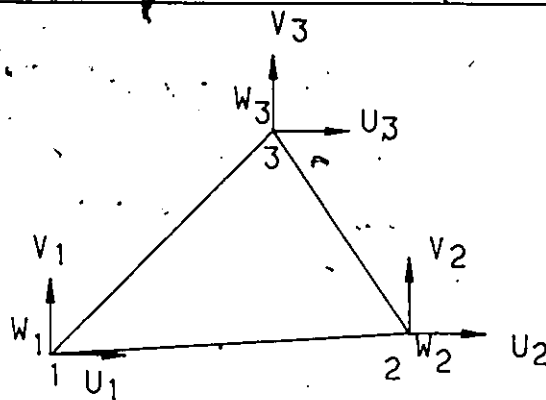
W_s = EQUIV. STRIP LOAD FROM

$$\begin{aligned} \frac{W_s L_n^2}{8} &= \frac{(M_1 + M_2)}{2} + M_3 \\ &= M_0 \text{ (STRIP)} \end{aligned}$$

I_o = I EFFECTIVE (EQ. 3.4)

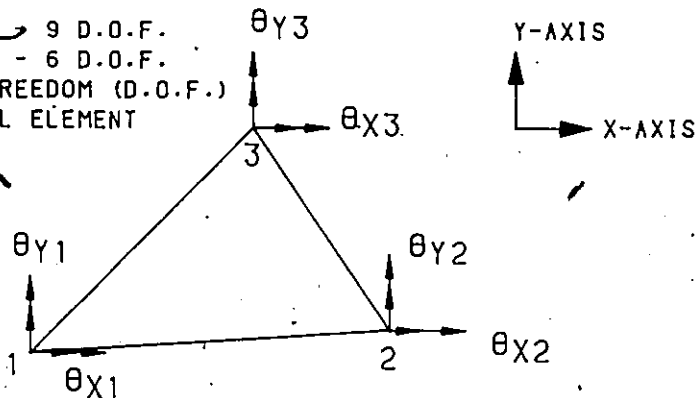
L_n = CLEAR SPAN

APPENDIX-D SIMPLIFIED BEAM APPROACH [106]



(i) CONSTANT STRAIN TRIANGLE

U_i, V_i, W_i DISPLACEMENTS - 9 D.O.F.
 θ_i ROTATIONS - 6 D.O.F.
 NUMBER OF DEGREES OF FREEDOM (D.O.F.)
 OF THE TRIANGULAR SHELL ELEMENT
 = 9 + 6 = 15



(ii) PLATE BENDING TRIANGLE

TRIANGULAR SHELL ELEMENT

TIME DEPENDENT ANALYSIS

WITH REFERENCE TO FIGURES 1 AND 2 ON NEXT PAGE :

- (1) CONSTANT LOAD ASSUMED FOR EACH TIME STEP $\Delta t_i = t_{i+1} - t_i$
- (2) FIND INCREMENTAL STRAIN (DUE TO CREEP, SHRINKAGE AND TEMPERATURE) FOR K-TH CONCRETE LAYER.

$$\{\Delta \epsilon_K\} = \{\Delta \epsilon_K^c\} + \{\Delta \epsilon_K^{sh}\} + \{\Delta \epsilon_K^T\}$$

T = DUE TO TEMPERATURE CHANGES

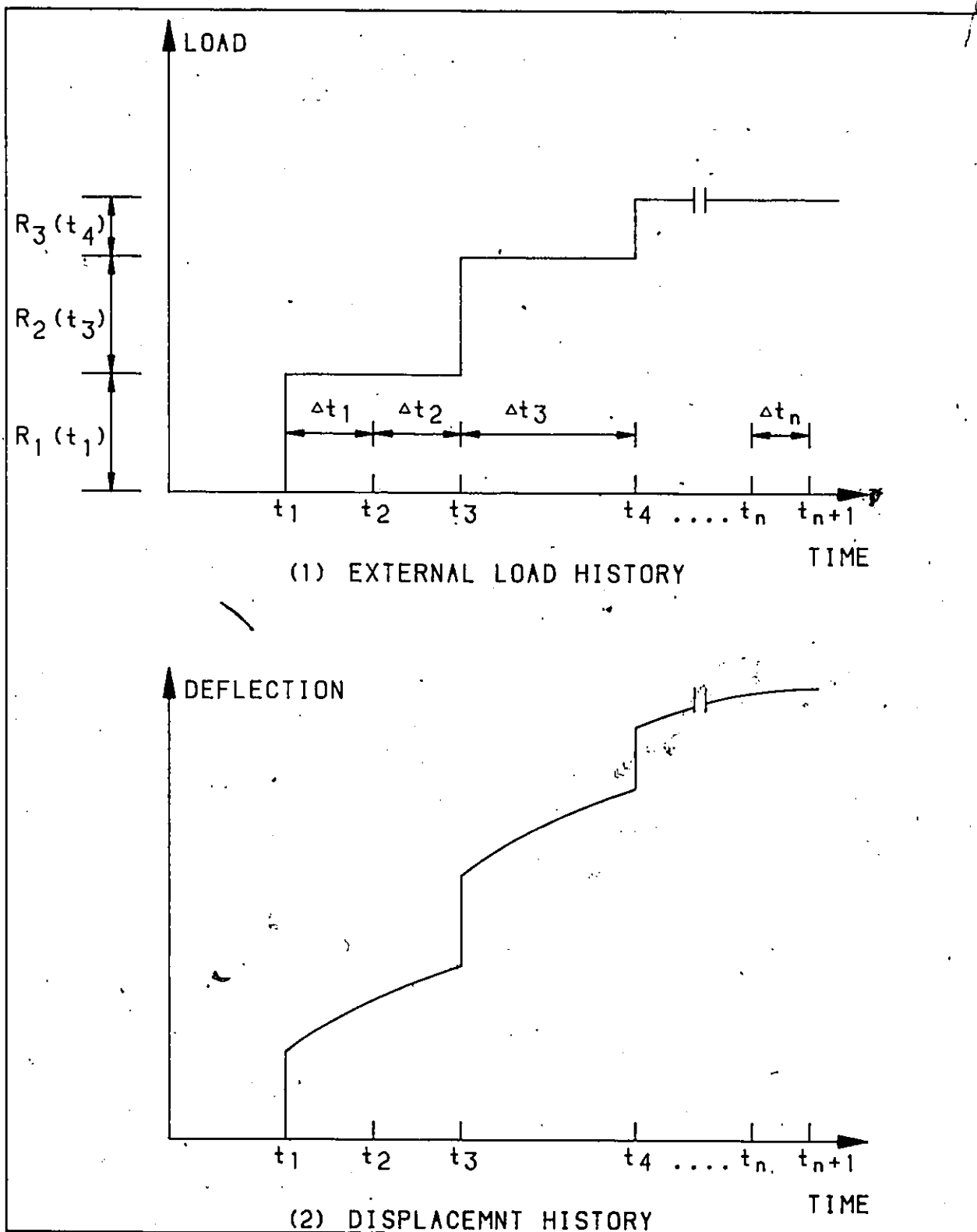
- (3) APPLY INTEGRATION EQUATIONS TO DETERMINE TIME LOAD [R].

$$[R] = \sum_{K=1}^N \int_A [B_K]^T [D_K] \{\Delta \epsilon_K\} dA$$

N = NUMBER OF CONCRETE LAYERS
 B = STRAIN-DISPLACEMENT MATRIX
 T = TRANSPOSE OF MATRIX
 D = MATERIAL MATRIX
 A = AREA OF ELEMENT

- (4) TRANSFER EQUIVALENT LOAD TO GLOBAL COORDINATES
- (5) COMPUTE ADDITIONAL DEFLECTION DUE TO THIS EQUIVALENT TIME LOAD WITHIN TIME STEP Δt_i

APPENDIX-E FINITE ELEMENT ANALYSIS BY NOPARC [126]



APPENDIX-E (CONTINUED)

THE FINAL (30 YEAR) LOAD-INDUCED STRAIN
= ELASTIC STRAIN + CREEP STRAIN

$$= \frac{\text{STRESS}}{E_t} + \frac{\text{STRESS} \times \text{CREEP FACTOR}}{E_t}$$

THE ELASTIC MODULUS AT THE TIME OF LOADING
(E_t) IS GIVEN BY :

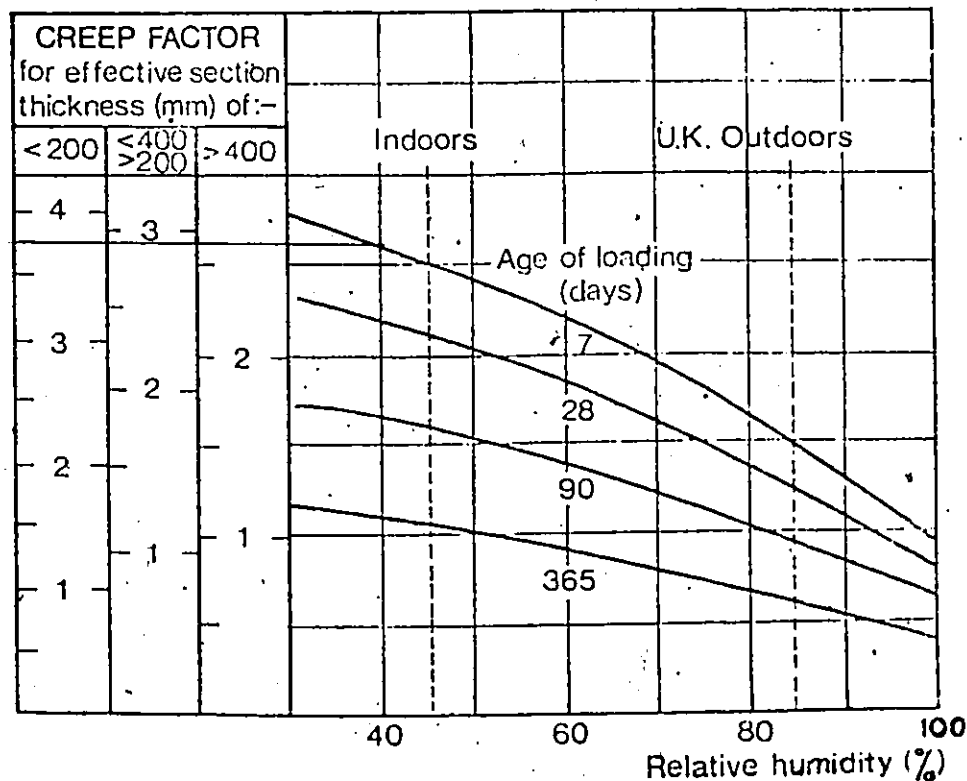
$$E_t = E_{28} (0.4 + 0.6 f_t / f_{28})$$

$$E_{28} = 20 + 0.2 f_{28} \text{ (KN/MM}^2\text{)}$$

STRENGTH RATIOS (f_t / f_{28}) :

AGE (DAYS)	7	28	90	365
OPC *	0.65	1.00	1.20	1.35
RHC **	0.75	1.00	1.15	1.20

- * ORDINARY PORTLAND CEMENT CONCRETE
- ** RAPID HARDENING CEMENT CONCRETE



APPENDIX-F MODEL OF CREEP FACTOR BY
THE CONCRETE SOCIETY [37.95]

$$C_{28}(t, t_0) = K_c^c K_d^c K_b^c K_e^c K_t^c$$

$C_{28}(t, t_0)$ IS THE CREEP COEFFICIENT AT ANY TIME, t ,
FOR CONCRETE LOADED AT $t_0 = 28$ DAYS

CORRECTION FACTORS :

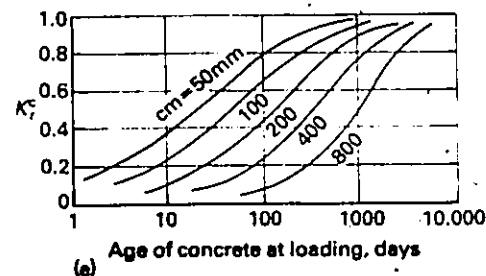
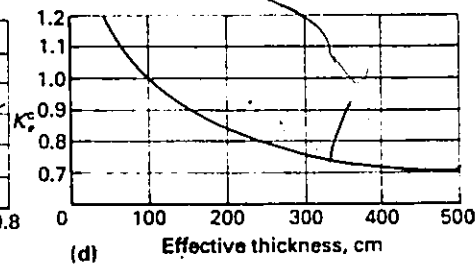
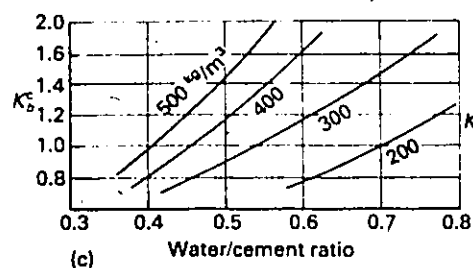
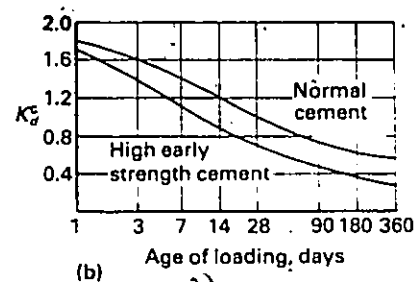
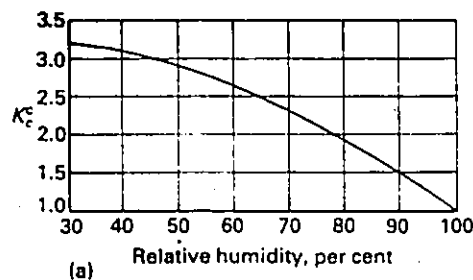
K_c^c = AMBIENT RELATIVE HUMIDITY

K_d^c = AGE OF CONCRETE AT LOADING

K_b^c = CONCRETE COMPOSITION

K_e^c = CROSS-SECTIONAL DIMENSION

K_t^c = THICKNESS COEFFICIENT



Note: Effective thickness, t , is the quotient of the area divided by half the perimeter in contact with the atmosphere. If one of the dimensions is large with respect to the others, the effective thickness is equal to the actual thickness.

APPENDIX-G CEB-FIP 70 MODEL OF CREEP PREDICTION [29]

$$C_t = K_s K_h K_H K_{t_o} \frac{(t - t_o)^{0.6}}{10 + (t - t_o)^{0.6}} C_u$$

C_t = CREEP COEFFICIENT
= CREEP STRAIN/INITIAL STRAIN

C_u = ULTIMATE CREEP COEFFICIENT TO BE DETERMINED
FROM EXPERIMENTAL DATA
= 2.35 FOR STANDARD CONDITIONS

K_s = CORRECTION FACTOR FOR SLUMP
= $0.81 + 0.07 S$; S = SLUMP IN. INCHES

K_h = CORRECTION FACTOR FOR SIZE OF MEMBER
= $1.0 - 0.0167 (SIZE - 6.0)$; $SIZE > 6$ IN.
= 1.0 SIZE < 6 IN.

K_H = CORRECTION FACTOR FOR RELATIVE HUMIDITY
= $1.27 - 0.0067 \cdot H$; $H > 40$

K_{t_o} = CORRECTION FACTOR FOR AGE AT LOADING
= $1.25 t_o^{-0.118}$ FOR MOIST CURED CONCRETE FOR 7 DAYS
(t_o = AGE AT LOADING IN DAYS.)

STANDARD CONDITIONS :

SLUMP = 2.7 INCHES.

REL. HUMIDITY = 40 %

MIN. THICKNESS = 6 INCHES OR LESS

LOADING AGE = 7 DAYS FOR MOIST CURED CONCRETE

* CORRECTION FACTORS FOR CEMENT CONTENT, AIR CONTENT
AND PERCENT FINES (BY WEIGHT) ARE NORMALLY NEGLECTED

APPENDIX-H A.C.I. MODEL FOR CREEP [2.19]

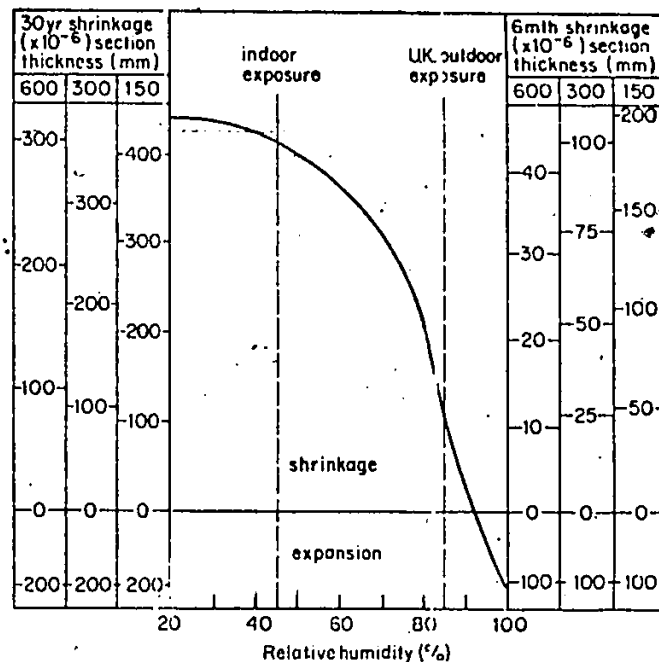
THE FOLLOWING INFORMATIONS ARE NEEDED FOR THE EVALUATION OF DRYING SHRINKAGE OF STRUCTURAL CONCRETE (NORMAL WEIGHT) WITH NORMAL WORKABILITY WITHOUT ADDITIVES :

- (1) EXPOSURE CONDITION
- (2) EFFECTIVE SECTION THICKNESS
(TWICE THE VOLUME DIVIDED BY EXPOSED SURFACE AREA)
- (3) RELATIVE AREA OF REINFORCING STEEL

SHRINKAGE OF SYMMETRICALLY REINFORCED CONCRETE :

$$= S (1 + K P)$$

WHERE S = POTENTIAL SHRINKAGE OF THE PLAIN CONCRETE
P = AREA OF STEEL RELATIVE TO THAT OF CONCRETE
K = 25 OR 15 FOR INTERNAL OR EXTERNAL EXPOSURE



APPENDIX-I MODEL OF DRYING SHRINKAGE BY THE CONCRETE SOCIETY [59]

$$\epsilon_{shu} = K_c^s K_d^s K_e^s K_p^s K_t^s$$

ϵ_{shu} IS ULTIMATE SHRINKAGE

CORRECTION FACTORS :

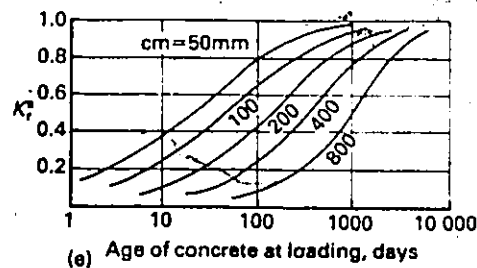
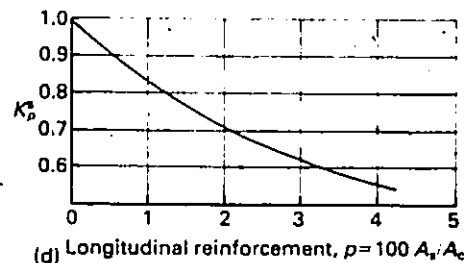
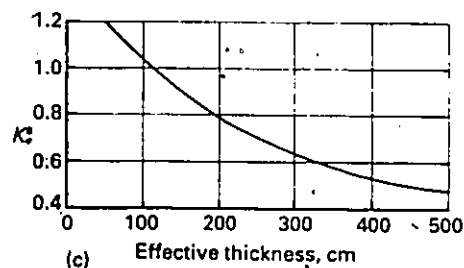
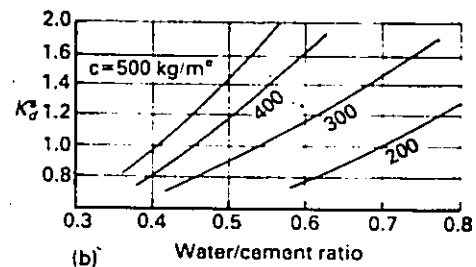
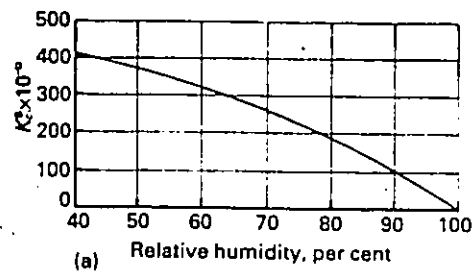
K_c^s = AMBIENT RELATIVE HUMIDITY

K_d^s = CONCRETE COMPOSITION

K_e^s = CROSS-SECTIONAL DIMENSIONS

K_p^s = LONGITUDINAL REINFORCEMENT

K_t^s = THICKNESS COEFFICIENT



Note:
For effective thickness see Fig. 2-4.

APPENDIX-J CEB-FIP 70 MODEL OF SHRINKAGE PREDICTION [29]

$$\epsilon_{sh}(t) = K_s K_h K_H \frac{(t-t_0)^B}{A + (t-t_0)^B} \epsilon_{shu}$$

$\epsilon_{sh}(t)$ = SHRINKAGE STRAIN AT TIME t

ϵ_{shu} = ULTIMATE SHRINKAGE STRAIN

t_0 = TIME FROM WHICH SHRINKAGE STARTS

A, B = CONSTANTS DETERMINED FROM EXPERIMENTS

$K_s K_h K_H$ = CORRECTION FACTORS FOR SLUMP OF CONCRETE MIX, SIZE OF CONCRETE MEMBER AND RELATIVE HUMIDITY

$$\epsilon_{sh}(t) = K_s K_h K_H \frac{(t-t_0)}{35 + (t-t_0)} \epsilon_{shu}$$

CORRECTION FACTORS :

$K_s = 0.89 + 0.041 \times S$; S = SLUMP IN INCHES

$K_h = 1.193 - 0.0322 d$; d = THICKNESS (IN.)

$K_H = 1.4 - 0.01 H$; $40 < H < 80$

$= 3.0 - 0.03 H$; $80 < H < 100$

(H = % RELATIVE HUMIDITY)

• CORRECTION FACTORS FOR CEMENT CONTENT, AIR CONTENT AND PERCENT FINES (BY WEIGHT) ARE NORMALLY NEGLECTED

APPENDIX-K A.C.I. MODEL FOR SHRINKAGE [2,19]

EQUATION OF DIFFUSION
(DIFFUSION OF HEAT ON THERMAL STRESSES)

$$K \left(\frac{\partial^2 S}{\partial X^2} + \frac{\partial^2 S}{\partial Y^2} + \frac{\partial^2 S}{\partial Z^2} \right) = \frac{\partial S}{\partial t}$$

S = FREE, UNRESTRAINED UNIT LINEAR SHRINKAGE STRAIN

K = DIFFUSION COEFFICIENT OF SHRINKAGE IN SQ.IN/DAY

ASSUMPTIONS :

- (1) FLOW OF WATER ENTIRELY BY VAPOR DIFFUSION
- (2) VAPOR PRESSURE OF THE WATER IN THE CONCRETE PROPORTIONAL TO THE MOISTURE CONTENT
- (3) PERMEABILITY INDEPENDENT OF THE MOISTURE CONTENT

$$T = 4 K_0 / b^2 \left(\sqrt{(2+t)/2} - 1 \right)$$

K_0 = INITIAL K = 0.1 SQ.IN/DAY

$$K = 0.10 \sqrt{(2/(2+t))} \text{ SQ.IN/DAY}$$

b = SECTIONAL WIDTH

S_{shu} = ULTIMATE SHRINKAGE (0.000765 IN/IN)

SIMPLIFIED EQUATIONS : -

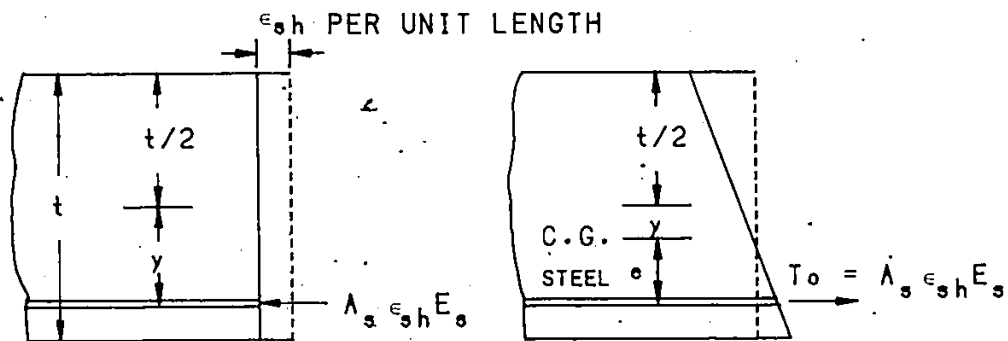
(A) IF $T < 0.05$

$$S/S_{shu} = 2 \sqrt{T/\pi}$$

(B) IF $T > 0.05$

$$\frac{S}{S_{shu}} = 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \left[e^{-(2n-1)^2 (\pi^2/4) T} \right]$$

APPENDIX-L PICKETTS SHRINKAGE EQUATIONS[96]



(A) UNIFORM SHORTENING

(B) DEFORMATION CAUSED BY STEEL

(A) CONCRETE FREE TO SHRINK WITH A FICTITIOUS COMPRESSIVE FORCE OF $A_s \epsilon_{sh} E_s$ AT CENTROID OF STEEL (FIGURE A)

(B) COMPRESSIVE FORCE OF STEEL RELEASED, RESULTING IN AN EQUIVALENT TENSILE FORCE, T_o , AT CENTROID OF STEEL
EQUIVALENT TENSILE FORCE. = T_o
= $A_s \epsilon_{sh} E_s$

APPLIED AT STEEL LEVEL TO THE GROSS TRANSFORMED AREA (FIGURE B)

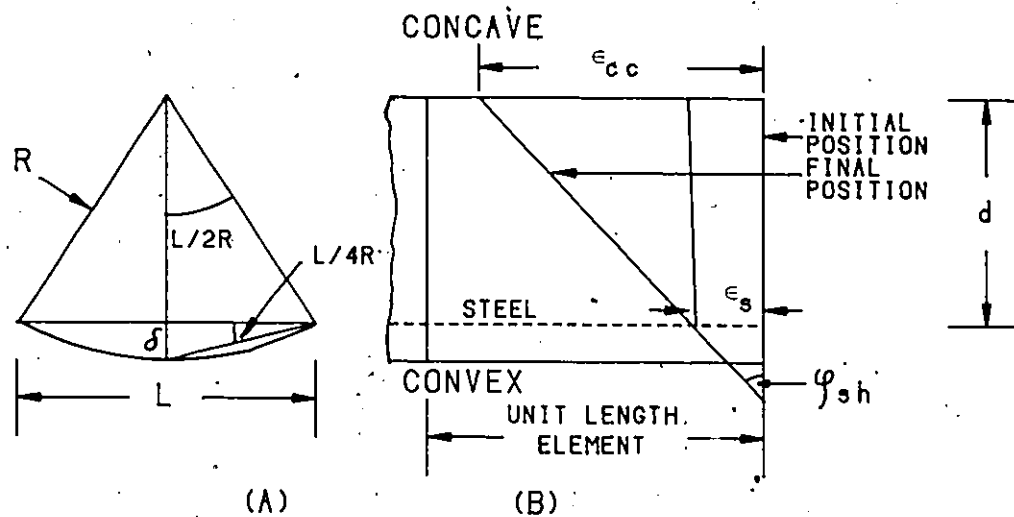
$$\text{CONCRETE STRESS} = \frac{T_o}{A} \pm \frac{T_o e y}{I}$$

$$\text{CURVATURE } \phi_{sh} = \frac{T_o e_g}{E_c I_g / 2}$$

$$\text{DEFLECTION } \delta_{sh} = \phi_{sh} K_w L^2$$

ϵ_{sh} = STRAIN IN STEEL, OR
= FREE SHRINKAGE STRAIN IN (A)
 K_w = COEFFICIENT (EQ.4.11)
 t = MEMBER THICKNESS
 e = ECCENTRICITY

APPENDIX-M EQUIVALENT FORCE METHOD [44]



$$\text{FROM (B): } \phi_{sh} = \frac{\epsilon_{cc} - \epsilon_s}{d} = \frac{\epsilon_{sh}}{d} \left(1 - \frac{\epsilon_s}{\epsilon_{sh}} \right)$$

$$\text{FROM (A): } \delta = \frac{L}{4R} \cdot \frac{L}{2} = \frac{L^2}{8R}$$

$$\text{i.e. } \phi_{sh} = \frac{1}{R} = \frac{8\delta}{L^2}$$

$$\delta = \frac{\phi_{sh} L^2}{8}$$

SIMPLY SUPPORTED BEAM

$$\delta = \frac{\phi_{sh} L^2}{16}$$

FIXED-END BEAM

$\epsilon_{sh} = \epsilon_{cc}$ = STRAIN IN CONCRETE DUE TO SHRINKAGE
 ϵ_s = STRAIN IN STEEL
 R = RADIUS OF CURVATURE
 d = EFFECTIVE DEPTH OF SECTION

APPENDIX-N GEOMETRY OF WARPING [80]

- (A) BASED ON THE STRAIGHT LINE ADOPTED TO THE MEASURED VALUES PLOTTED WITH RESPECT TO VARIABLE $\sqrt{t}$
- (B) AS A SUGGESTION, THE FOLLOWING POINTS OF TIME COUNTED FROM THE FORM REMOVAL ARE RECOMMENDED FOR THE MEASUREMENT, $t = 2, 4, 9, 16, 25$ DAYS

- (C) INTERSECTION BETWEEN STRAIGHT LINE. (POINTS FROM (B) .VS. $\sqrt{t}$) AND Y-AXIS IS $\delta(0)$

$$\delta(0) = \delta_1 = \text{INSTANTANEOUS DEFLECTION ; OR}$$

$$\delta_1 = \mu L^2 \frac{12}{bh^3} \left[\eta(0) \frac{M_{\max} - 1.25 M_{cr}(0)}{\mu E_s \left(\frac{d}{h} \right)^3} + \frac{1.25 M_{cr}(0)}{\phi(0) E_c(0)} \right]$$

$$= \delta(0) = \delta_1(0)$$

WHERE $\mu = (\delta EI / L^2)$ $M_{\max} = 5/48$ FOR S.S. BEAMS

$= 1/48$ FOR FIXED-ENDS BEAMS

$\eta(0) =$ FROM GIVEN FIGURES
(DEPENDING ON CURVATURE)

$\phi(0) =$ FROM GIVEN FIGURES
(DEPENDING ON SHAPE OF CROSS SECTION)

$(0) =$ INITIAL RESULTS AT TIME = 0

- (D) EFFECT OF CREEP

$$\delta_c(t) = \frac{\varphi(t)}{\varphi(\infty)} \left[\mu L^2 \frac{12}{bh^3} \left[\eta(0) \frac{M_{\max} - 1.25 M_{cr}(\infty)}{\mu E_s \left(\frac{d}{h} \right)^3} + \frac{1.25 M_{cr}(0)}{\phi(0) E_c(0)} \right] - \delta_1 \right]$$

OR $\delta_c(t) = \left[\varphi(t) / \varphi(\infty) \right] \times \delta_c(\infty)$

WHERE $\varphi(t) / \varphi(\infty) = \sqrt{t} / 14$; (FROM FIGURE)

APPENDIX-01 DEFLECTION EQUATIONS BY WILSON [131]

THEN $\delta_c(t) = \sqrt{t}/14 \times \delta_c(\infty)$
 $= \delta(t) - \delta(0) = \text{CREEP DEFORMATION}$

$\delta(t) = \delta_2(t) (1 + \phi(t)) = \text{TOTAL DEFORMATION}$

$\delta_2(t) = \text{CALCULATED DEFLECTION OF CRACKED BEAM}$

(E) EFFECT OF SHRINKAGE :

$\delta_{sh}(t) = (\delta_1 + \delta_c(t)) (1 + \phi(t))$

(F) TOTAL DEFLECTION, $\delta(t)$

$\delta(t) = \delta(0) + \delta_{sh}(t) + \delta_c(t)$

(G) - $\mu = \delta EI / L^2 M_{max}$

- $\phi(t) / \phi(\infty)$

Ec, $\eta, \phi, \gamma, \phi(t)$, FROM GIVEN FIGURES AND TABLES

- $M_{cr}(0) = f_{cbt}(0) \gamma(0) b h^2 / 6$

- $M_{cr}(\infty) = M_{cr}(0) / (1 + \phi(\infty))$

APPENDIX-01 (CONTINUED)

DEFLECTIONS IN MM	SLAB 1	SLAB 2	SLAB 3
FORMWOK STRIPPED AT	1 DAYS	2 DAYS	3 DAYS
MEASURED INITIAL DEFLECTION	11.79	3.07	1.54
COMPUTED INITIAL DEFLECTION	12.50	4.23	1.54
MEASURED 28-DAY DEFLECTION	20.36	8.57	6.00
COMPUTED 28-DAY DEFLECTION	18.21	8.93	5.71
INCREASE IN DEFLECTION (MEAS.)	8.57	5.50	4.46
INCREASE IN DEFLECTION (COMP.)	5.71	4.70	4.17

(A) ALL SLABS (ONE-WAY) HAD A THICKNESS OF 0.16 M AND WERE SIMPLY SUPPORTED WITH A SPAN LENGTH OF 4.2 M

(B) ALL SLABS WERE LOADED BY THEIR OWN WEIGHT DURING FOUR WEEKS AFTER REMOVAL OF FORMWORK

(C) DEFLECTION VALUES (MEASURED AND COMPUTED) WERE OBTAINED FROM GIVEN DEFLECTION-TIME CURVES

APPENDIX-02 TEST RESULTS BY WILSON [131]

TOTAL DEFLECTION = MULTIPLIER X δ_{sd}

$$\text{MULTIPLIER} = 1 + 2.8 C_u / 2.35 + 1.2 \epsilon_{shu} / 800$$

C_u = ULTIMATE CREEP COEFFICIENT

ϵ_{shu} = ULTIMATE SHRINKAGE STRAIN
(MICRO. IN/IN)

$$\begin{aligned} \delta_{sd} &= \delta_{ss} + \delta_{osd} \\ &= \delta'_{max} (W_{sd} / W_{max}) \end{aligned}$$

δ_{ss} = IMMEDIATE DEFLECTION DUE TO SLAB WEIGHT W_{ss}

δ_{osd} = IMMEDIATE DEFLECTION DUE TO SERVICE
DEAD LOAD OTHER THAN W_{ss}

δ'_{max} = IMMEDIATE DEFLECTION DUE TO W_{max}
APPLIED AS A SINGLE INCREMENT

W_{max} = MAXIMUM CONSTRUCTION LOAD
= DEAD LOAD + LIVE LOAD

W_{sd} = TOTAL SERVICE LOAD

APPENDIX-P DEFLECTION MULTIPLIER ACCORDING
TO SHAROUNIS [102]

SECTION PROPERTIES :

$$\begin{aligned}\rho &= 0.00507 \text{ (COLUMN STRIP)} \\ f_y &= 417 \text{ MPA} \\ f'_c &= 28 \text{ MPA} \\ d &= 50 \text{ MM}\end{aligned}$$

MOMENT CAPACITY OF SECTION

$$= 0.9 \rho b d^2 f_y \left(1 - \frac{0.59 \rho f_y}{f'_c} \right)$$

$$\begin{aligned}&= 4600 \text{ N-M/M} \\ &= 2470 \text{ N-M/(COLUMN STRIP)} \\ &\text{WHERE COLUMN STRIP} = 0.540 \text{ M}\end{aligned}$$

MAXIMUM MOMENT = 0.6 (0.7 M_o) FOR COLUMN STRIP

$$M_o = \frac{W L_1 L_n^2}{8}$$

$$L_1 = 2160 \text{ MM} ; \quad L_n = 1960 \text{ MM}$$

$$\text{I.E. } W = 5.677 \text{ KPA}$$

LOAD CAPACITY OF SLAB = 1.4 D + 1.7 L
WHERE D = DEAD LOAD AND L = LIVE LOAD

FOR MODEL 1 : L/D=1.0 LOAD CAPACITY OF SLAB = 3.1 D
HENCE, D=5.677/3.1 = 1.82 KPA

FOR MODEL 2 : L/D=0.5 LOAD CAPACITY OF SLAB = 2.25 D
HENCE, D=5.677/2.25 = 2.52 KPA

APPENDIX-Q LOAD CALCULATIONS FOR FLAT PLATE MODELS

SECTION PROPERTIES :

$$\begin{aligned}\rho &= 0.00380 \\ f_y &= 417 \text{ MPA} \\ f'_c &= 28 \text{ MPA} \\ d &= 50 \text{ MM} \\ b &= 203 \text{ MM} \\ \text{CLEAR SPAN} &= 1960 \text{ MM}\end{aligned}$$

MOMENT CAPACITY OF SECTION

$$\begin{aligned}&= 0.9 \rho b d^2 f_y \left(1 - \frac{0.59 \rho f_y}{f'_c} \right) \\ &= 700 \text{ N-M}\end{aligned}$$

$$\text{MAXIMUM MOMENT} = \frac{W L^2}{12}$$

$$\begin{aligned}\text{LOAD CAPACITY OF SLAB} &= 1.4 D + 1.7 L \\ \text{WHERE } D &= \text{DEAD LOAD AND } L = \text{LIVE LOAD}\end{aligned}$$

$$\begin{aligned}\text{FOR } L/D &= 0.5, \text{ LOAD CAPACITY OF SLAB} = 2.25 D = W \\ \text{HENCE, } D &= 1.00 \text{ KN/M}\end{aligned}$$

APPENDIX-R LOAD CALCULATIONS FOR
ONE WAY SLAB MODELS

(A) SQUARE-ROOT HYPERBOLIC LAW FOR SHRINKAGE

- FINAL SHRINKAGE

$$\epsilon_{\infty} = 1210 - 880 Y \quad \text{MICRO.IN/IN}$$

$$Y = (390 Z^{-4} + 1)^{-1}$$

$$Z = ((1.25 \sqrt{a/c} + 0.5 (g/s)^2) \times ((1+s/c)/(w/c))^{1/3} \sqrt{f_c'}) - 12$$

$$> 0.0$$

- ULTIMATE SHRINKAGE STRAIN $\epsilon_{shu} = \epsilon_{\infty} \cdot E(7+600)/E(t_0 + \tau_{sh})$

- SHRINKAGE STRAINS AT TIME t

$$\epsilon_{sh}(\hat{t}, t_0) = \epsilon_{shu} K_h S(\hat{t})$$

- c = CEMENT CONTENT KG/CU.M.
- w/c = WATER/CEMENT RATIO
- a/c = AGGREGATE/CEMENT RATIO
- g/s = GRAVEL/SAND RATIO
- s/c = SAND/CEMENT RATIO
- AGG. = SAND + GRAVEL
- f_c' = 28-DAY CYLINDER STRENGTH, KSI
- SAND < 4.7 MM (SIEVE #4)
- t = AGE OF CONCRETE
- t_0 = AGE WHEN DRYING BEGINS
- $\hat{t}$ = DURATION OF DRYING = $t - t_0$
- τ_{sh} = SHRINKAGE SQUARE HALF-TIME
- D = EFFECTIVE CROSS-SECTION THICKNESS, MM
- $= 2(V/S) = 2 \times (\text{VOLUME/EXPOSED SURFACE})$
- C_1 = DRYING DIFFUSIVITY OF NEARLY SATURATED CONCRETE AT REFERENCE TEMPERATURE: $T_0 = 23^\circ\text{C}$
- K_T = TEMPERATURE COEFFICIENT (IN ABSOLUTE TEMPERATURE)
- $= \text{CALCULATED FROM } T \text{ AND } T_0 \text{ IN KELVIN}$
- H = RELATIVE HUMIDITY

APPENDIX-S BAZANT'S MODELS FOR SHRINKAGE AND CREEP [12]

$$C_1^{ref} = 10 \text{ MM}^2/\text{DAY}$$

$$\begin{aligned} C_7 &= \text{ASSUMED VALUE OF } C_1 \text{ AT AGE OF 7 DAYS} \\ &= 0.125 \cdot c \cdot (w/c) - 12 \\ &= 7 \text{ IF } C_7 < 7 \\ &= 21 \text{ IF } C_7 > 21 \end{aligned}$$

$$\begin{aligned} \text{HUMIDITY DEPENDENCE: } K_h &= 1-H^3 \text{ for } H \leq 0.98 \\ &= -0.2 \text{ for } H = 1.00 \end{aligned}$$

SQUARE-ROOT HYPERBOLIC LAW IN TIME

$$S(\hat{t}) = \sqrt{\hat{t} / (\tau_{sh} + \hat{t})}$$

$$\tau_{sh} = 600 (K_s \cdot D / 150)^2 C_1^{ref} / C_1(t_0)$$

$$\begin{aligned} K_s = \text{SHAPE FACTOR} &= 1.0 && \text{INFINITE SLAB} \\ &= 1.15 && \text{INFINITE CYLINDER} \\ &= 1.25 && \text{INFINITE SQUARE PRISM} \\ &= 1.30 && \text{SPHERE} \\ &= 1.55 && \text{CUBE} \end{aligned}$$

AGE DEPENDENCE :

$$C_1(t) = C_7 K_T' (0.05 + \sqrt{(6.3/t)})$$

$$\epsilon_{shu} = \epsilon_{soo} \cdot E(7+600) / E(t_0 + \tau_{sh})$$

$$E(t') = E_{28} \cdot \sqrt{(t' / (4 + 0.85 t'))}$$

TEMPERATURE DEPENDENCE

$$K_T' = T/T_0 \cdot \text{EXP.}(5000/T_0 - 5000/T)$$

APPENDIX-S (CONTINUED)

(B) CREEP FUNCTION AT SIMULTANEOUS DRYING

CREEP FUNCTION :

$$J(t, t') = 1/E_0 + C_0(t, t') + C_d(t, t', t_0) - C_p(t, t', t_0)$$

E_0 = INITIAL (ELASTIC) MODULUS

$C_0(t, t')$ = BASIC CREEP (DOUBLE POWER LAW)

$C_d(t, t', t_0)$ = INCREASE OF CREEP DUE TO DRYING

$C_p(t, t', t_0)$ = DECREASE OF CREEP AFTER DRYING

t' = AGE AT LOAD APPLICATION

t_0 = AGE AT THE START OF DRYING

(1) BASIC CREEP : $C_0(t, t')$

$$C_0(t, t') = \varphi_1/E_0 * (t'^{-m} + \alpha) * (t-t')^n$$

WHERE $E_0, \varphi_1, \alpha, m, n$ ARE MATERIAL PARAMETERS:

$$1/E_0 = 0.09 + 1/(1.7 * Z_1^2) ; Z_1 = 0.00005 \rho^2 f_c'$$

$$1/E_0 = \text{IN } 0.000001/\text{PSI}$$

$$E_0 = 57000 \sqrt{f_c'} \quad (\text{A.C.I.})$$

ρ = UNIT MASS OF CONCRETE IN LB./CU.FT.

f_c' = 28-DAY CYLINDER STRENGTH IN KSI

$$\varphi_1 = \frac{10^3 n}{2(28^{-m} + \alpha)}$$

$$\alpha = 1/(40 * w/c) ; m = 0.28 + 1/(f_c')^2$$

$$X = \left[(2.1 (a/c) / (s/c))^{1.4} + 0.1 (f_c')^{1.5} (w/c)^{1/3} (a/g)^{2.2} \right]_{a_1} - 4$$

$$\text{IF } X > 4 \quad n = 0.12 + (0.07 X^6) / (5130 + X^6)$$

$$\text{IF } X < 4 \quad n = 0.12$$

a/g = 1.0 FOR CEMENT TYPES I, II

= 0.93

= 1.05

III
IV

APPENDIX-S (CONTINUED)

(2) $C_d(t, t', t_0)$:

$$C_d(t, t', t_0) = (\varphi'_d / E_0) t'^{-m/2} K_h' \epsilon_{shu} S_d(t, t')$$

$$\varphi'_d = (1 + (t' - t_0) / 10 \tau_{sh})^{-0.5} \varphi_d$$

$E_0, m, \epsilon_{shu}, \tau_{sh}$ FROM SHRINKAGE MODEL

$$S_d(t, t') = (1 + 10 \tau_{sh} / (t - t'))^{-C_d(n)}$$

= TIME SHAPE

$$C_d = 2.8 - 7.5 n$$

$$K_h' = |H_0^{1.5} - H^{1.5}|, H_0 = 0.98 \text{ to } 1.0$$

$$\varphi_d = 0.008 + 0.027(u) \quad \text{FOR } r > 0$$

$$= 0.008 \quad \text{FOR } r \leq 0$$

$$u = 1 / (1 + 0.7 r^{-1.4})$$

$$\text{AND } r = 56000 ((s/a) \cdot f_c')^{0.3} (g/s)^{1.3} ((w/c) / \epsilon_{s\infty})^{1.5} - 0.85$$

(3) $C_p(t, t', t_0)$

$$C_p(t, t', t_0) = C_p K_h'' S_p(t, t_0) C_o(t, t')$$

$$C_p = 0.83$$

$$K_h'' = |H_0^2 - H^2|, H_0 = 0.98 \text{ TO } 1.0$$

$$S_p(t, t_0) = \text{TIME SHAPE}$$

$$= (1 + 100 \tau_{sh} / (t - t_0))^{-n}$$

APPENDIX-S (CONTINUED)

THE TOTAL STRAIN IS

$$\epsilon_t = \epsilon_i + \epsilon_{bc} + \epsilon_{sh} (1 + K\sigma)$$

ϵ_t = TIME-DEPENDENT STRAIN

ϵ_i = INITIAL STRAIN

ϵ_{bc} = BASIC CREEP

ϵ_{sh} = SHRINKAGE STRAIN

$K = 0.043 / \text{MPa}$ AT 7.9 MPa

$K = 0.049 / \text{MPa}$ AT 15.9 MPa

σ = APPLIED STRESS IN MPa

(A) FOR $\sigma = 1300 \text{ PSI} = 9 \text{ MPa}$; $K=0.044$

$K = 0.396$

(B) FOR $\sigma = 1776 \text{ PSI} = 12.3 \text{ MPa}$; $K=0.046$

$K = 0.566$

APPENDIX-T DRYING CREEP ACCORDING TO
GAMBLE AND PARROTT [47]