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AN ADAPTIVE CONTROLLER USING IMPULSE RESPONSE IDENTIFICATION

by

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ABSTRACT

This thesis considers a method of designing an adaptive control system that uses a sampled pulse response identification of the plant.

A second order system model is considered and various figures of merit are computed from the pulse response. The ratio of the largest absolute value of the samples of the pulse response to the sum of all the absolute values of the samples is chosen as the best figure of merit. The adjustment procedure uses a relator figure which is a monotone function of the system gain in order to find the direction of the maximum of the figure of merit.

The system was simulated on a PDP-8 /TR-48 computer configuration. The adjustment procedure was shown to converge to the desired control.

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CHAPTER I

INTRODUCTION

There is a certain class of systems that needs to be controlled very well. For example, a large distillation column in an oil refinary has a very valuable output. A slight change in such parameters as temperature, pressure, or reaction time may cause the end products to be worth much less than they would be under proper control With the advent of atomic power, atomic reactors could also be in this class of systems. A slight change in ion density or fuel impurity could effect a significant drop in power output. Airplanes and spacecraft could also be in this class of systems. The rapid variations in such parameters as altitude, heat, gravitational force, air pressure, velocity, and weight could cause serious problems when the craft is to be kept on an exact course.

For such systems, the controller, an adaptive controller, would measure the system variations to calculate its own strategy to keep the system operating within specifications. When an adaptive controller is chosen over an ordinary time-variant controller to control the system, the added expense must be justified. An ordinary feedback controller compensates for unexpected inputs. The system parameter variations are considered to be insignificant. An adaptive controller also compensates for parameter variations or variations in the operating point of non-linear systems. (See Fig. 1) There are two schemes that the adaptive controller can employ. The first, which is considered only in passing, is a hill climbing process of a cost functional on a cost surface. The cost functional is an algorithm relating operating cost to the value of the output.

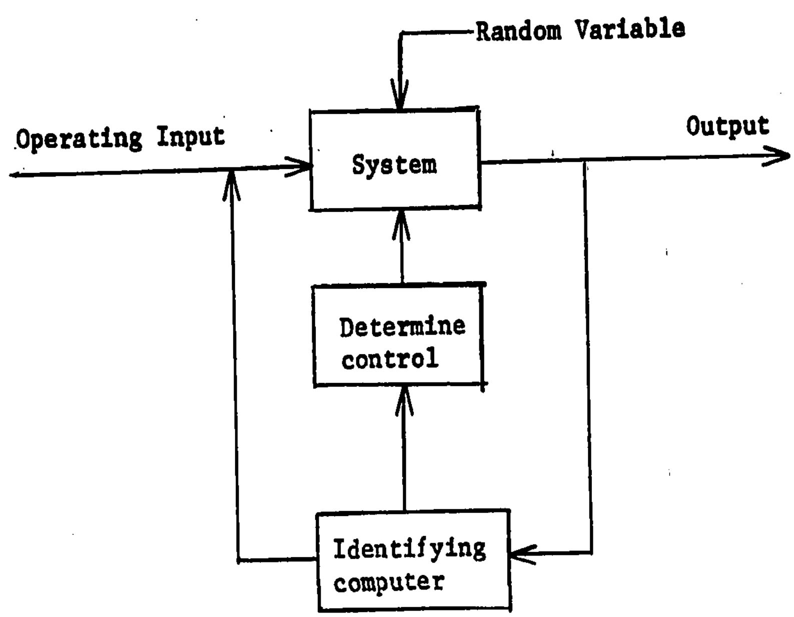


Fig. An Adaptive System

In effect, the cost per unit output is computed as a function of the various states and inputs in state space. This has the advantage that only the present time need be considered for control. The disadvantage is that the search is difficult, being on a multidimensional surface. It can also lead to local rather than global optimum.

The second scheme obtains an input-output record of the system while it is operating. This record is used to calculate some property of the system. A control is then adjusted to improve that property. This process takes place repeatedly during the system operation so that the system will tend to operate in some optimal state. Also, the adaptive process should be such that the normal operation of the system is disturbed as little as possible. The mathematics in an adaptive controller are usually done by a computer to speed up the process and make it truly automatic. A digital computer is usually chosen over an analog computer because of its versatility.

There are two methods for manipulating the input-output record; in the time domain and in the frequency domain. Both these methods first estimate a mathematical model and then calculate some performance index using the model. The controller then searches for the control which yields the optimum performance index. The models can be time functions such as impulse or step respon s, frequency responses, or state space models.

Although they are more difficult to estimate, the most useful form of system models, from the point of view of calculating the adaptive control, are the state space and frequency response models. A state space model, for example, can be used as a basis for computing optimal control strategies, while a frequency response model may be used in an algorithm for choosing the control parameters. No satisfactory general method for design starting from an impulse response model is known.

On the other hand, the easiest and least costly of these models to estimate is the impulse response model. The trouble with the frequency response model is that it is difficult to estimate in the presence of noise. It has a longer identification time (9) and more expense (3). It is possible to derive the frequency response from the correlation function by Wiener (15) filter theory, but this too is very difficult. State space models may be obtained by model reference methods (14), for example. These models again require some type of multivariable optimization. As a result, they are difficult to program.

The easiest method of computing a system model with a digital computer is by correlation techniques. The data records, which are functions of time, are more amenable to correlation functions than to functions of frequency. Even with correlation functions, computing the performance index is still a complex problem. Dynamic programming, Pontryagin functions, and variational calculus are three common methods used to get the performance index. (7)

In summation, the correlation functions are easy to calculate but difficult to use. On the other hand, the frequency response and state space models are difficult to calculate, but, as a system model, are relatively easy to use. This thesis first discusses the sampled correlation function as a system model in chapter II. A method of determining a performance index will be developed in chapter III, which compares the estimate of the actual impulse response with an ideal impulse response. This is roughly the same idea that is common in frequency response analysis. Chapter IV introduces a method which facilitates the optimization process by indicating in which direction the control must be changed. Chapter V presents the experimental work, wherein a real time second order system is simulated on an analog computer, while the system gain is controlled from a digital computer. Chapter VI concludes the thesis with suggestions for further study.

CHAPTER II

IDENTIFICATION

A) A Correlation Review

A review of correlation theory is now in order. A system with an impulse response h(t), an input x(t), and an output y(t) (See Fig. 2) will have the following relationship between input and output.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \qquad \text{eqn. 2-1}$$

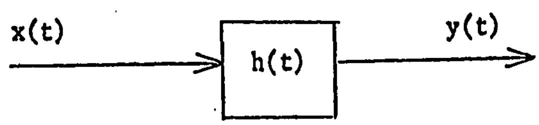


Fig. 2 A System

From 2.1

$$x(t+\lambda)\ y(t) = x(t+\lambda) \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\ x(t+\lambda)\ x(t-\tau)\ d\tau . \qquad (2.2a)$$

$$E\{x(t+\lambda)\,y(t)\} = \int_{-\infty}^{\infty} h(\tau)\; E\{x(t+\lambda)\; x(t-\tau)\}\,d\tau$$

(where E is expectation)

or

$$\phi_{xy}(\lambda) = \int_{-\infty}^{\infty} h(\tau)\;\; \phi_{xx}(\lambda+\tau)\,d\tau . \tag{2.2b}$$

where ϕ is the correlation function.

On the other hand, from (2. 1)

$$y(t)\,x(t-\lambda) = \int_{-\infty}^{\infty} h(\tau)\; x(t-\tau)\;\; x(t-\tau)\,x(t-\tau)\,d\tau . \tag{2.3a}$$

From (2.31)

$$E\{y(t)\,x(t-\lambda)\} = \int_{-\infty}^{\infty} h(\tau)\;\; E\{x(t-\tau)\;\; x(t-\lambda)\}\,d\tau$$

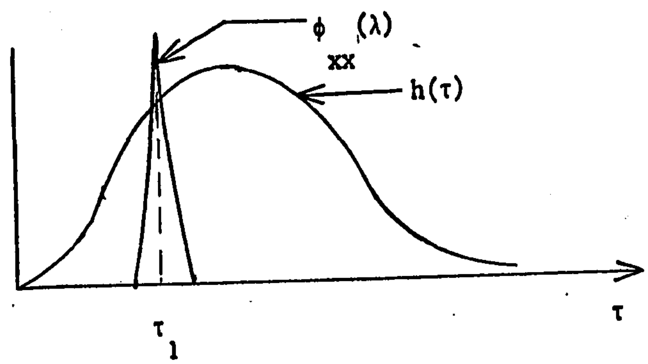


Fig. 3 A Graphical Derivation Of a Crosscorrelation Function

or

$$\phi_{yx}(\lambda) = \int_{-\infty}^{\infty} h(\tau)\;\; \phi_{xx}(\lambda-\tau)\,d\tau . \tag{2.3b}$$

If, in (2.2b) and (2.3b), we have

$\phi_{xx}(\tau) = \delta(\tau),$

$$\phi_{yx}(\lambda) = h(\lambda), \tag{2.4}$$

$$\phi_{xy}(\lambda) = h(-\lambda). \tag{2.5}$$

B) <u>The Sampled Correlation Functions</u>

As this theory is to be applied using a digital computer, the sampled version of eqn. 2-1 must be derived. (10) If the system is stable with zero initial conditions, the sampled version of eqn. 2-1 for one point of time is:

$$y(K) = \sum_{n=0}^{K} x(K-n)h(n)T \qquad \text{eqn. 2-6}$$

In the following discussion, the first output sample that is considered is y(KT), so that the effects of initial conditions will have been eliminated.

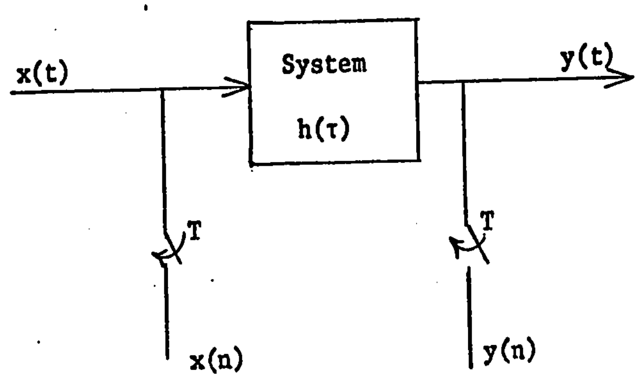


Fig. 4 The Sampled System

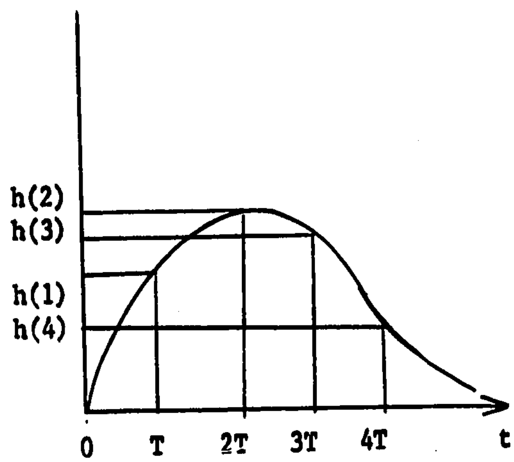


Fig. 5 The Impulse Response And Its Digital Estimate

If the samples of y(t) are expressed as a vector, the sampled version of eqn. 2-6 is:

$$\underline{Y} = \underline{X}\,\underline{H}\,T \qquad \text{eqn. 2-7}$$

Where:

$$\underline{Y} = \begin{bmatrix} y(K) \\ y(K+1) \\ \cdot \\ \cdot \\ y(K+N) \end{bmatrix} \qquad \text{eqn. 2-8}$$

$$\underline{X} = \begin{bmatrix} x(K) & x(K-1) & . & . & . & x(0) \\ x(K+1) & x(k) & . & . & . & x(1) \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ x(K+N) & x(K+N-1) & & . & . & x(N) \end{bmatrix} \quad \text{eqn. 2-9}$$

$$\underline{H} = \begin{bmatrix} h(0) \\ h(1) \\ . \\ . \\ . \\ h(K) \end{bmatrix} \quad \text{eqn. 2-10}$$

In these equations, K is chosen such that the time (K+1)T is longer than the significant part of the impulse response h(t). T is the sampling period.

C) <u>The Identifying Input</u>

At this point, the psuedo - autocorrelation and crosscorrelation functions will be introduced.

$$\hat{\phi}_{xx}(i) = \frac{1}{N+1} \sum_{n=0}^{N} x(n)\ x(n+i) \qquad \text{eqn. 2-11}$$

$$\hat{\phi}_{xy}(i) = \frac{1}{N+1} \sum_{n=0}^{N} x(n)\ y(n+i) \qquad \text{eqn. 2-12}$$

These are used as elements in the psuedo correlation matrices, $\hat{\Omega}_{xx}$ and $\hat{\Omega}_{xy}$ defined by the following equations.

$$\hat{\Omega}_{xx} = \underline{X}^T \underline{X} \qquad \text{eqn. 2-13}$$

$$\hat{\Omega}_{xy} = \underline{X}^T \underline{Y} \qquad \text{eqn. 2-14}$$

We can see from eqn. 2-7 that these matrices can be used to estimate the impulse response. If both sides of eqn. 2-7 are premultiplied by the transpose of the $\underline{X}$ matrix, X^T, the impulse response can be solved for.

$$\underline{H} = \frac{1}{T} \hat{\Omega}_{xx}^{-1} \hat{\Omega}_{xy} \qquad \text{eqn. 2-15}$$

The hat ($\wedge$) notation is used to show that these functions are estimates computed from the actual input - output samples.

In fig. 3, one point of the crosscorrelation curve is shown graphically to be the area under the product of the autocorrelation curve and the system impulse response. For the crosscorrelation function to be proportional to the impulse response, the input autocorrelation function should be as close to an impulse response as possible.

A family of signals that satify this restriction is the binary psuedo-noise sequences. These also have the advantages of being easily generated by shift registers, and having a fixed computable inverse. A binary psuedo-noise sequen e (6) has the property that:

$$\phi_{xx}(0) = A^2 \qquad \text{eqn. 2-16}$$

$$\phi_{xx}(\gamma) = \frac{-A^2}{N} \quad \gamma \neq 0 \qquad \text{eqn. 2-17}$$

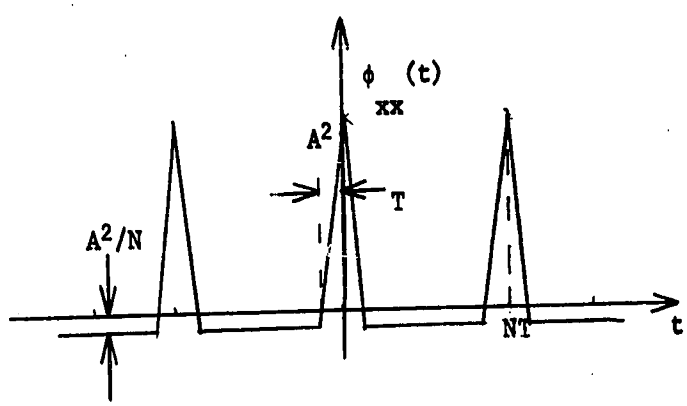


Fig. 6 Autocorrelation Function Of a Binary Psuedo-Noise Sequence

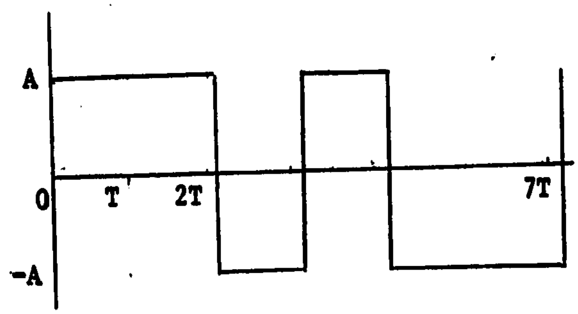


Fig. 7 A Binary Psuedo-Noise Sequence Of Length Seven

Where A is the amplitude of the binary signal, and N is the length of the sequence or the number of bauds in a period. Such a signal has an autocorrelation matrix:

$$\Omega_{xx} = \begin{bmatrix} A^2 & \frac{A^2}{N} & \cdot & \cdot & \cdot & \cdot & \frac{A^2}{N} \\ \frac{A^2}{N} & A^2 & \frac{A^2}{N} & \cdot & \cdot & \cdot & \frac{A^2}{N} \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ \cdot & & & & & \cdot & \frac{A^2}{N} \\ \frac{A^2}{N} & \cdot & \cdot & \cdot & \cdot & \frac{A^2}{N} & A^2 \end{bmatrix} \qquad \text{eqn. 2-18}$$

Levin shows that the inverse of this matrix is (11):

$$\Omega^{1}_{xx} = C \begin{bmatrix} 2 & 1 & \cdot & \cdot & \cdot & \cdot & 1 \\ 1 & 2 & 1 & \cdot & \cdot & \cdot & 1 \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ \cdot & & & & & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & & 1 & 2 \end{bmatrix} \qquad \text{eqn. 2-19}$$

Where C is a constant;

If eqn. 2-19 is applied to eqn. 2-5, the sampled version of the impulse response is estimated.

$$\underline{H} = \frac{C}{T} \begin{bmatrix} 2 & 1 & \cdot & \cdot & \cdot & 1 \\ 1 & 2 & \cdot & \cdot & \cdot & 1 \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 1 & \cdot & \cdot & \cdot & 1 & 2 \end{bmatrix} \begin{bmatrix} x(1)y(1) & \cdot & \cdot & \cdot & x(N)y(N) \\ x(1)y(2) & \cdot & \cdot & \cdot & x(N)y(N+1) \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ x(1)y(K) & \cdot & \cdot & \cdot & x(N)y(N+K-1) \end{bmatrix}$$

eqn. 2-20

The relationship between the system impulse response and the input-output data record is expressed in eqn. 2-20.

D) Errors Of Operation

The point could now be made that the identifying input would disrupt the system from its normal operation. Certainly it does, but recent papers discuss ways of minimizing the disturbance at the expense of identification time and accuracy. (8) These tradeoffs can be computed with the use of probability theory. Maximum likelihood and Bayesian estimates (11) are tools that have already been studied. The advantage with this theory is that it can be applied in such a way that operating inputs and identifying inputs can be applied simultaneously.

This chapter will be concluded with a few remarks about sampling periods and sequence lengths. The duration of the estimated impulse response, KT, should theoretically be longer than the significant duration of the impulse response of the real system. Also, the sampling period, T, should obey Shannon's sampling theorem. That is, T should be one half the period or less of the highest frequency component desired. Hazleman (8) has shown that the error due to initial conditions and/or periodicity of the test signal can be significantly reduced by applying the test signal at least one signal period prior to observation, and by making this period at least five times the dominant time constant of the system.

From this, it is tempting to make the input sequence as long as possible. While it is true that the error due to bias effects are inversely proportional to the sequence length, mean square error due to wide band noise is proportional to sequence length. Thus a tradeoff must be made. The above mentioned errors are caused by the assumption that the input autocorrelation function is a delta function. It is this assumption that makes eqn. 2-5 valid.

The cause of these errors can be clearly seen if we consider the continuous relationship and compare it with the estimated relationship.

$$y(T) = y(0) + \int_0^T h(\tau)\, x(t-\tau)\, d\tau \qquad \text{eqn. 2-21}$$

If only one pulse is considered, the input x(t) is constant and so can be taken outside the integral.

$$y(T) = y(0) + x(0) \int_0^T h(\tau)\, d\tau \qquad \text{eqn. 2-22}$$

Or more generally:

$$y(NT) = y(\{N-1\}T) + x(\{N-1\}T) \int_{\{N-1\}T}^{NT} h(\tau)\, d\tau$$

eqn. 2-23

From eqn. 2-7, the corresponding sampled data version of this equation is:

$$y(NT) = y(\{N-1\}T) + x(\{N-1\}T)\ h(T)\ T \qquad \text{eqn. 2-24}$$

For equations 2-23 and 2-24 to be equal, the integral of eqn. 2-23 must be equal to T h(T). That is, if there is no instrumentation error and no noise, the estimate at time NT of the impulse response is:

$$h(NT) = \frac{1}{T} \int_{\{N-1\}T}^{NT} h(\tau)\ d\tau \qquad \text{eqn. 2-25}$$

It is apparent from eqn 2-25 that the estimate h(NT), will not be exactly on the impulse response, resulting in the previously mentioned errors. The points which make this estimate good are that the sum of the estimated points will be close to the integral of the impulse response, and that the estimated points will be close to the actual points of the impulse response. The estimate will have no errors if the sampling period T is zero, because the integral of eqn. 2-25 will merely be the H(NT) sample.

CHAPTER III

OPTIMIZATION

A) General Technique

The trouble with this identification method is that there are very few ways of using it to determine the optimal control. One method found in the literature was developed by Aeronutronics Inc. (5) This method uses the ratio of positive area to negative area of the impulse response as a performance index. Anderson et. al. (1) also discuss the possibility of using the model to control relative stability, rise time, or gain.

The purpose of this work is to find a more general method by which the impulse response can be used in an adaptive control system. The system that is to be controlled should have a specific ideal relationship between the system input and output. That is, an ideal but often unrealisable crosscorrelation function can be derived from the system specifications. The optimum system is a realisable system that is as close to the ideal as possible. A figure of merit calculated from the impulse response estimate of the system should have a maximum (or minimum) that coincides with the optimum system.

B) The Problem

As a specific example, an adaptive communication channel will be studied. The communication channel is a linear time-varying system, wherein the ideal output is identical to the input, but delayed by some unspecified time, τ. The operating input and the random variables are unavailable for either observation or manipulation. Because the ideal output is identical to the input, the ideal crosscorrelation function should be the same shape as the input autocorrelation function. Therefore, the ideal crosscorrelation function is a maximum at time τ, where τ is the delay of the system. This can be stated mathematically as:

$$\phi_{xy}(\tau) = A^2 \qquad \text{eqn. 3-1}$$

$$\phi_{xy}(\mu) = \frac{-A^2}{N} \qquad \mu \neq \tau \qquad \text{eqn. 3-2}$$

Stated in matrix form, this is:

$$\Omega_{xy_{ideal}} = \begin{bmatrix} -A^2/N \\ \cdot \\ \cdot \\ \cdot \\ -A^2/N \\ A^2 \\ -A^2/N \\ \cdot \\ \cdot \\ \cdot \\ -A^2/N \end{bmatrix} \qquad \text{eqn. 3-3}$$

The closer the Ω_{xy} matrix is to the ideal crosscorrelation matrix, the better is the system. This is equivalent to saying that the larger one sample $\phi_{xy}(\tau)$ is compared to all other samples $\phi_{xy}(\mu)$, the better is the system.

C) <u>The Figure Of Merit</u>

From this point, the figure of merit must be chosen. In determining which figure of merit is best, the three following qualities must be considered.

1) The maximum of the figure of merit should occur at the control yielding the optimum system.

2) The figure of merit should be such that the problem of local optimum points is avoided. This would be of considerable help in the adjustment procedure for the optimum point.

3) The figure of merit should be easily computable.

From eqn. 3-3 and the paragraph following it, we can say that the figure of merit may be such that is proportional to a function of the largest crosscorrelation function sample $\phi_{xy}(\tau)$, and inversely proportional to a function of the other crosscorrelation samples $\phi_{xy}(\mu)$. It should have the three qualities discussed above. Such a figure of merit is:

$$\text{F.M.} = \frac{|\hat{\phi}_{xy}(\tau)|}{\sum_{\mu=0}^{K} |\hat{\phi}_{xy}(\mu)|} \qquad \text{eqn. 3-4}$$

This particular figure of merit was found to be the best of several figures of merit in a series of experiments. A second order system was chosen for these experiments because the optimal damping factor is generally agreed to be 0.7.

The purpose of the experiments was to find the damping factor which yielded the maximum figure of merit for different sampling periods. The results of the experiments are given in their respective graphs. Perhaps the most obvious choice of figure of merit is:

$$\text{F.M.} = \frac{\hat{\phi}_{xy}(\tau)}{\sum_{\mu=0}^{K} \hat{\phi}_{xy}(\mu)} \qquad \text{eqn. 3-5}$$

However in a highly oscillatory system, the denominator tends toward zero because the sum of the positive terms is about equal to the sum of the negative terms. This gives a large figure of merit to an undesirable underdamped system. This figure of merit does not meet the first requirement of the above list at all.

To avoid this problem, the samples could be squared. This would yield the following figure of merit.

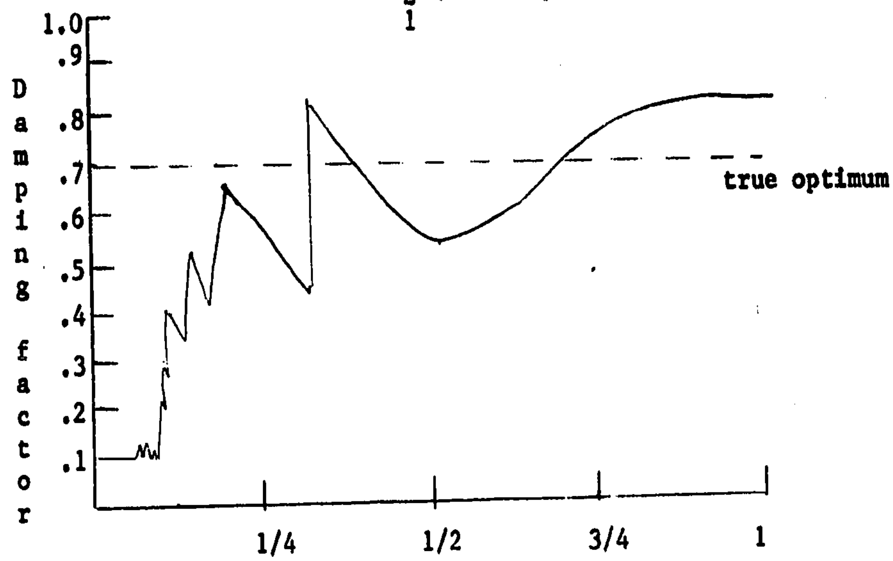
$$\text{F.M.} = \frac{\hat{\phi}^2_{xy}(\tau)}{\sum_{\mu=0}^{K} \hat{\phi}^2_{xy}(\mu)} \qquad \text{eqn. 3-6}$$

This figure of merit does give better results than eqn. 3-5 although it is still somewhat unsatisfactory. (see Figs. 8 and 9). The largest figure of merit does not occur sufficiently close to the true system optimum. The problem of the local maximum also occurs. These problems are apparently caused by the nature of the squaring process. Large numbers are made larger, and small numbers are made smaller, relative to each other. Thus, an underdamped system may have a greater figure of merit than the true optimum due to the distortion of the squaring. Raising the samples to higher powers would only increase the distortion. It is interesting to note that a damping factor of 0.5 seems to be the optimum setting. This would be the case if an integral square error function is used. (12)

From here, the next step would be to use the absolute value in computing the figure of merit. If this is done, the figure of merit of eqn. 3-4 is derived. The experimental results for this figure of merit are shown in Fig. 10 and Fig. 11. Fig. 11 is the results of an experiment wherein the system shown in Fig. 12 was simulated and the gain was varied. Aside from giving experimental results in favor of the chosen figure of merit, these tests point to the importance of the sampling period.

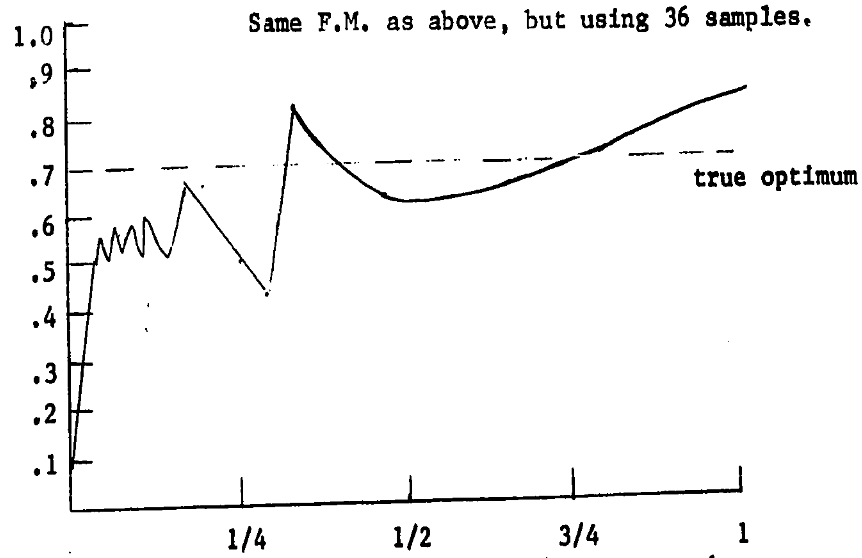
Fig. 8

$$\text{F.M.} = \frac{(\text{max. sample})^2}{\sum_{1}^{16} (\text{all samples})^2}$$



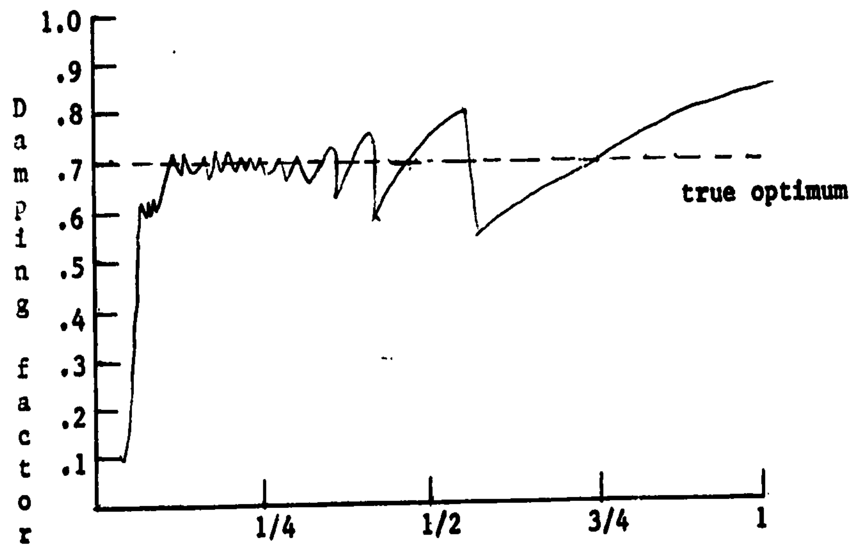
Fract. of max. sampling period allowed by Shannon's sampling theorem.

Same F.M. as above, but using 36 samples.



Vert and horiz. scales are the same as above.

Fig. 9



Fract of max. sampling period allowed by Shannon's sampling theorem.

Fig. 10 F.M. $= \frac{|\text{Max. Sample}|}{\sum_{1}^{16} |\text{samples}|}$

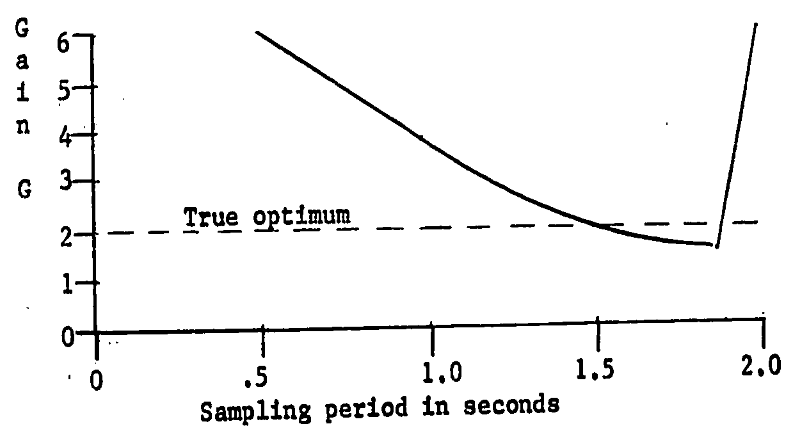


Fig. 11 F.M. $= \frac{|\text{Max. Sample}|}{\sum_{1}^{16} |\text{samples}|}$

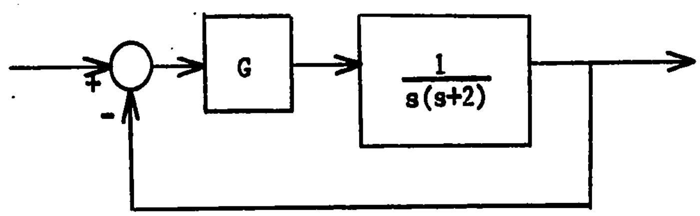


Fig. 12

At first glance, Fig. 11 would seem to disprove the claims made for this figure of merit. This is not so. We must remember that in this system, the natural frequency is equal to the gain G. When the sampling is fast, the figures of merit for the systems with low natural frequencies are small because the time used to calculate the impulse response is too short. When the sampling is slow, the systems with high natural frequencies are sampled improperly, according to Shannon's sampling theorem. The useful range of sampling lies somewhere between these two extremes. The best sampling period would be less than half the maximum allowed by Shannon's sampling theorem. The useful range of sampling can be extended if the number of samples in the estimate is increased.

CHAPTER IV

THE ADJUSTMENT PROCEDURE

Now that a suitable figure of merit has been chosen, the question of how to get the optimal control arises. That is, in which direction will the control be varied? To aid in the adjustment, a relator figure determined from the digital impulse response estimate is used. A relator figure is another type of performance index. Its only requirement is that it is monotonic with respect to the total variation of the system. In other words, its derivative never changes sign for the range of variations of the system. The relator figure is used to determine the direction in which the control must be changed. Keeping in mind requirement no. 2 for the figure of merit, the adjustment would take place on a relator figure vs. a figure of merit graph. This graph would have one universal maximum (minimum) without any other local maxima. When the system varies, the relator figure, figure of merit, and system performance also vary. If a decrease in the relator figure and the figure of merit occurs (see Fig. 13) then the control must be such that it increases the relator figure until the figure of merit starts to decrease just past the optimal control. It is

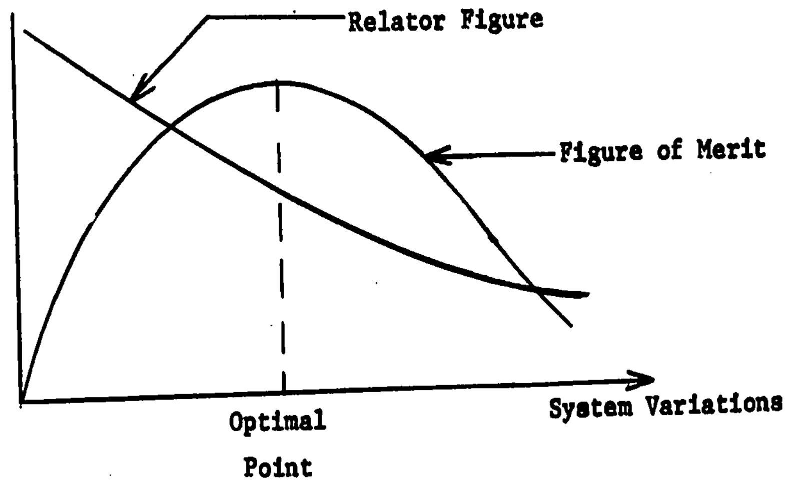


Fig. 13 Control Performance

obvious that the choice of relator figure is important, and must be chosen from some apriori knowledge of the system variations. In viewing the system in the Laplace plane, the time varying parameters may be a pair of complex conjugate poles moving along a control locus. A control locus is a path along which the control forces the variable parameters to move. If, as in Fig. 14, the poles are always moving closer to the imaginary axis, the system is becoming more and more oscillatory. Therefore, a good relator figure would be a measure of the system oscillation. Such a measure could be the sum of the negative samples of the impulse response, or simply the sum of all the samples.

In like manner, the random variations of the system

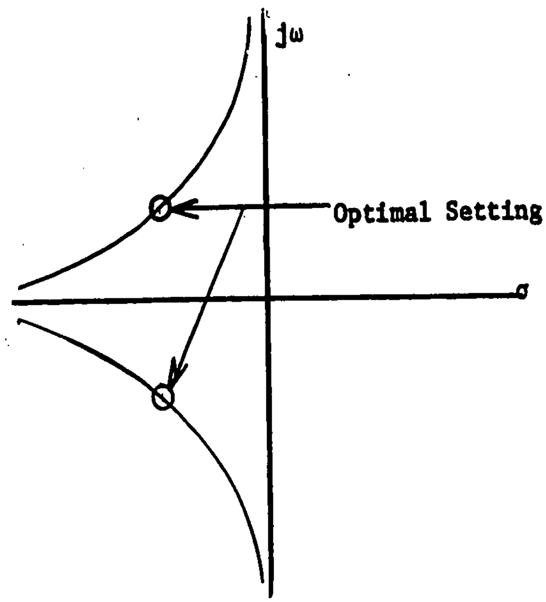


Fig. 14 A Control Locus

could also generate a locus. If the random locus were the same as the control locus, the optimal value of the relator figure could be stored, and the adjustment procedure would merely keep the relator figure at the value for optimum performance without actually computing the figure of merit. In other words, the control policy would merely be to keep the poles at the optimum point. The relator figure and the figure of merit would have constant optimal values.

If this were not true, the random variable would force the poles off one control locus onto another, and the optimal

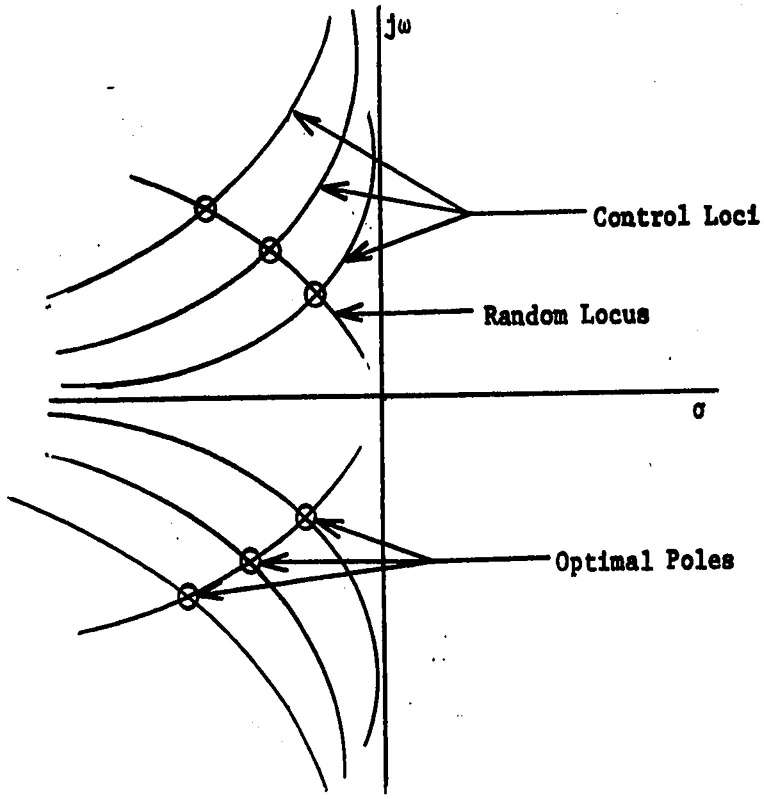


Fig. 15 A Family Of Control Loci

control will have changed. Under such conditions, we would have a family of control loci which are dependant on the random variable.

CHAPTER V

SIMULATION AND CONTROL

A) The Experiment

After the proper figure of merit was chosen, the question remained whether or not this type of control would work. To find out, a closed loop second order system was simulated in real time on a TR-48 analog computer. The open loop feedback attenuation, considered to be the random variable, was set at a value of 0.3. The open loop system gain, considered to

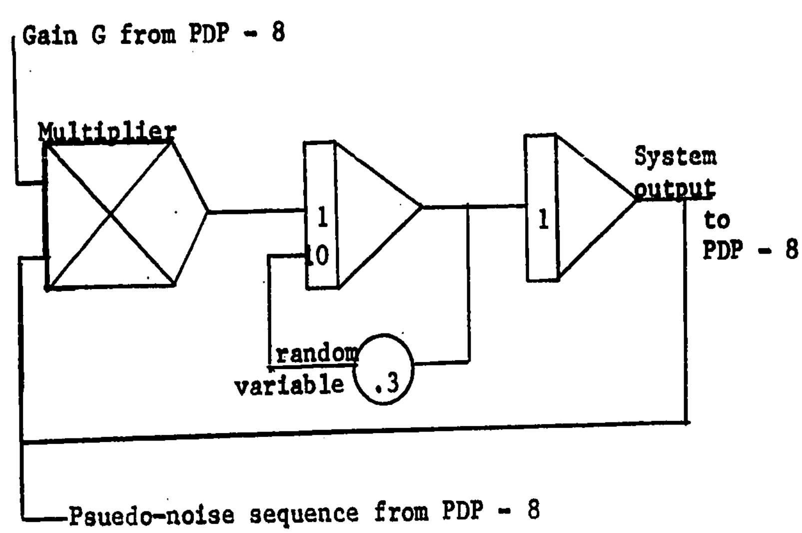


Fig. 16 The Analog Circuit

be the control variable, was initially set at a value of 8.0 (see Fig. 16). The system was then controlled by the PDP - 8 digital computer.

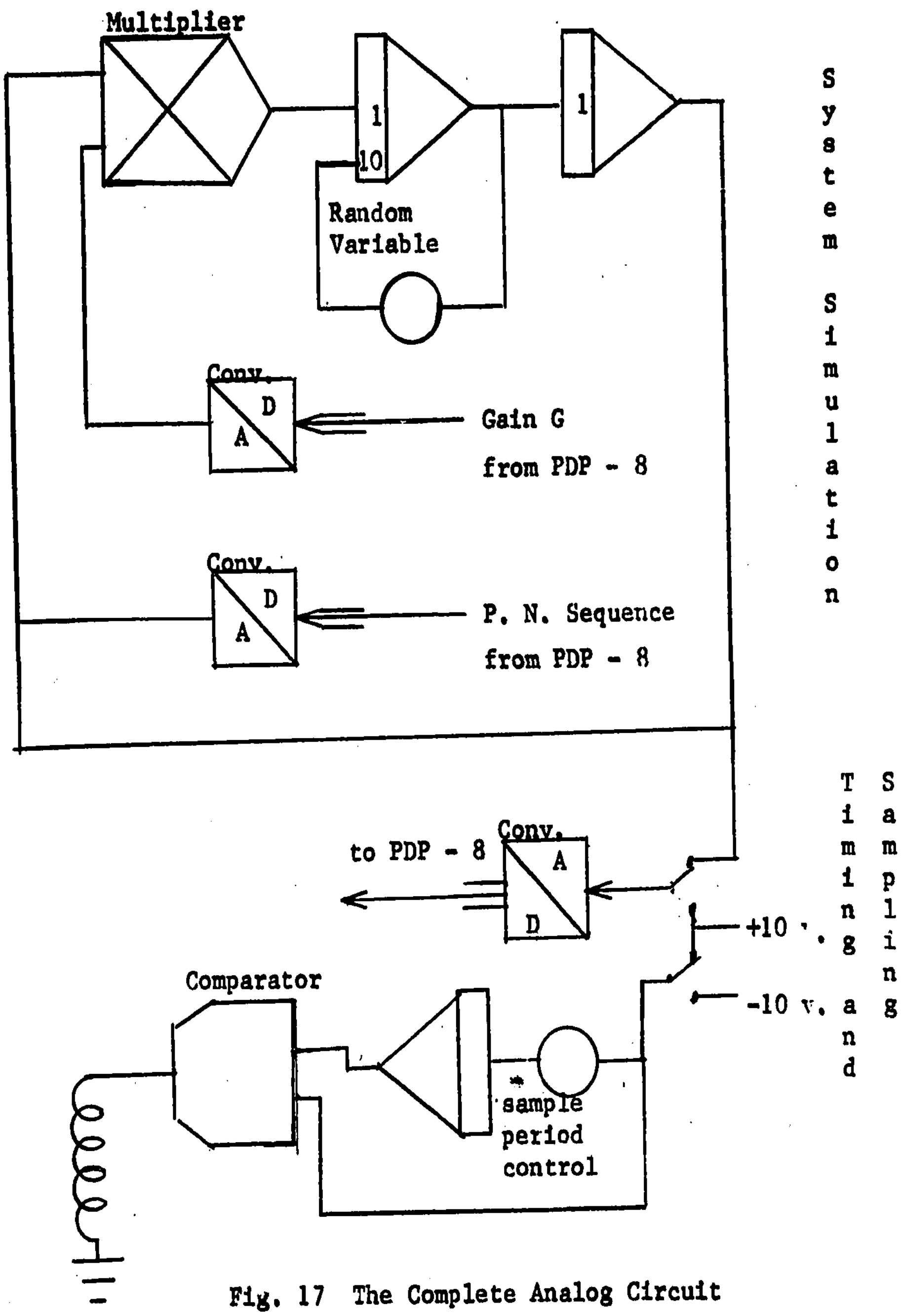


Fig. 17 The Complete Analog Circuit

The psuedo - noise sequence amplitude was approximately one volt and the sampling period was about one second. The psuedo - noise sequence was 31 bauds long, and the estimated crosscorrelation function had ten samples. The test signal amplitude and the sampling rate were controlled by analog potentiometers. The complete analog circuit showing sampling control, test signal amplitude control, and converters is shown in Fig. 17.

B) The Digital Program

The digital program must accept the correct data from the A/D converters and output the psuedo - noise sequence via a D/A converter from the memory. The sampled data from the input and output are then used to estimate the crosscorrelation function. The figure of merit and the relator figure are then computed and compared with their previous values. The gain is then changed in accordance with the changes in the two performance indices. The flow chart for the digital program is given in Fig. 18. To get an estimate, there must be 31 digital outputs for the psuedo - noise sequence, and 41 digital inputs to compute the crosscorrelation matrix as indicated in eqn. 2-20. Therefore, to avoid rearranging a memory after each identification, the sequence is outputted until x(31) is reached so that the identification will again start at x(0). Because the program only compares respective values, the constant C in eqn. 2-20 is ignored.

Under this scheme, each identification period took about one minute. After approximately two hours, the controller settled at its optimum. The main reasons it took so long were the small step size and the instrumentation errors. Both the digital to analog, and the analog to digital converters had about 5% error. These, together with the error in the analog circuit, gave an overall instrumentation error of over 10%. After each identification period, the gain was incremented by about 0.3 volts. The problem arises when the computer must differentiate between two gain settings so close together with an error of over 10% in each. Aside from these instrumentation errors, there are also the errors of approximation discussed at the end of Chapter 2. The point is that even under these conditions, the controller still drove the system to its optimal control.

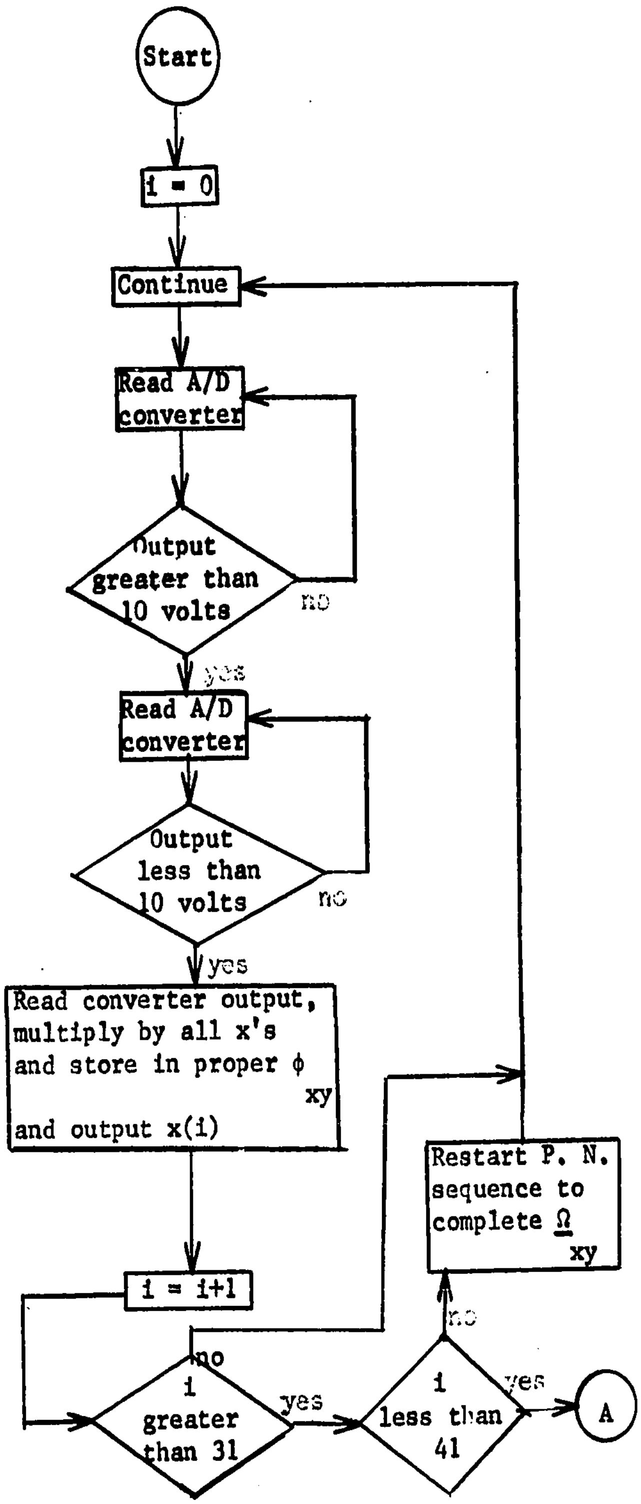


Fig. 18(a) The Digital Flow Chart

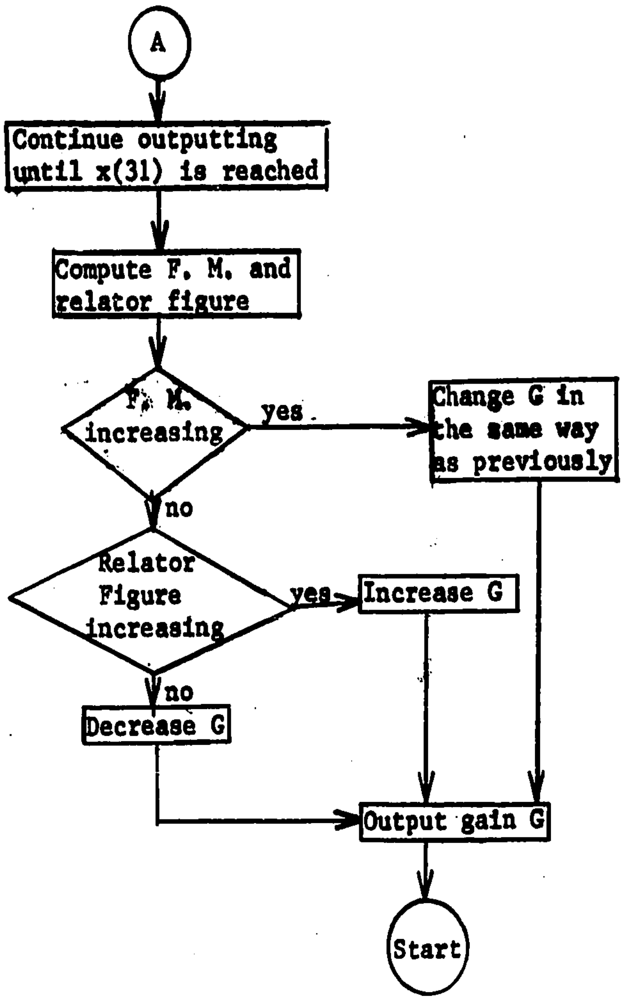


Fig. 18(b) The Digital Chart

CHAPTER VI

CONCLUSION

The purpose of this thesis was to find a useful format for using the impulse response estimate in an adaptive control. Once this was found, it was applied to a specific problem an adaptive communication channel. The theory was applied to a second order system, and worked in the presence of considerable error. Using this format, the theory of crosscorrelation estimation can now be applied to adaptive control systems.

The work in this area is far from complete. There are many areas that could investigated much further. A few of these are:

1) Controlling the amplitude of the psuedo - noise sequence used as an identifying input so that it is proportional to the error of the estimate. This would give a more accurate estimate when needed, while reducing the disturbance to the systems normal operation when not needed.

2) Making the step size of the control also variable. This would force the control closer to the optimum in a shorter time.

3) Using a Bayesian estimate with the identification. The work of Loo (11), which uses prior knowledge of the system if applied to this format, presents an interesting problem.

4) Developing this method of correlation comparison for some types of control systems other than communication channels.

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