

**Does Production Function Specification Matter for
Macroeconomic Effects of Energy Shocks?**

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Abstract

Following Dhawan, Jeske and Silos (2010), this paper firstly examines whether the explanatory power of a “spillover” effect to the Great Moderation will change if we change the form of production function in Dhawan, Jeske and Silos (2010). Then it compares impulse responses of key macroeconomic variables to energy price shocks given different production function specifications.

This paper finds that production function specification doesn’t seem to play an important role in explaining of the explanatory power of a “spillover” effect to the Great Moderation, but it may have an impact on the responses of some of the key macroeconomic variables to energy price shocks. One noteworthy result emerges with the two-level CES production function that combines labour and energy inputs in the first nesting, and then combines this aggregate with capital, i.e. the (LE)K case. Here, the responses of consumption, working hours and wage may be sensitive to the slightly positive “spillover” effect in the late period estimated by Dhawan, Jeske and Silos (2010).

1. Introduction

The macroeconomic effects of energy shocks have been getting attention for decades. Many researchers have examined different channels through which energy shocks may influence key macroeconomic variables. For example, Mork and Hall (1980) assumed short-run wage and price rigidity in their model, and found that an increase in energy price increased inflation and unemployment and decreased real output, investment and consumption. Bernanke (1983) suggested that uncertainty about the future energy prices would delay current investment and create investment fluctuations if the investment decisions were irreversible. Loungani (1986) empirically showed that oil shocks had reallocative effects on employment across industries, could explain fluctuations in unemployment.

There are also researchers trying to figure out a relationship between energy price shocks and some specific economic phenomena. For instance, Dhawan, Jeske and Silos (2010) estimated the “spillover” effects from energy price shocks to the productivity process and introduced it into a dynamic stochastic general equilibrium (DSGE) model similar to Kim and Loungani (1992). They found that the disappearance of this spillover effect is an important reason of the big drop in macroeconomic volatility in USA since the middle 1980s (the “Great Moderation”).

Even though there is a large number of works on energy shocks, few of them consider the role of different specific production function forms on the macroeconomic effects of energy shocks in a DSGE model. Intuitively, an increase in the energy price will decrease the energy use, and the direct effect on output of

reduced energy using depends on the way through which energy is introduced into a production function. So the form of the production function could matter for macroeconomic effects of energy shocks.

There have been many studies on the form of a production function that includes more than two inputs (capital and labour, typically). One of the earliest papers in this area is Sato (1967), which presents a two-level constant-elasticity-of-substitution (CES) production function with n inputs. Then more specifically, with three factors (capital K , labour L , energy E), Kemfert (1998) showed that a two-level CES production function had three cases: (KE)L, (KL)E, and (LE)K. For example, (KE)L means that capital and energy are combined in a CES form first, which then is combined with labour in a CES form (Kemfert (1998), Page 251):

$$Y = A[a(bK^{-\alpha} + (1-b)E^{-\alpha})^{-\beta/\alpha} + (1-a)L^{-\beta}]^{-1/\beta} \quad (1),$$

where $A > 0$ is a constant, a, b are share parameters ($0 < a < 1$, $0 < b < 1$), $\alpha = (1 - \sigma_{K,E}) / \sigma_{K,E}$, $\beta = (1 - \sigma_{KE,L}) / \sigma_{KE,L}$, $\sigma_{K,E}$ is the substitution elasticity between capital and energy, $\sigma_{KE,L}$ is the substitution elasticity between (KE) and labour. The explanations are similar for the (KL)E and (LE)K cases.

Many researchers have even estimated elasticities of substitution between capital, labour and energy in a two-level CES production function in each of the three cases above. For example, Kemfert (1998) estimated elasticities of substitution for the aggregate German industry and seven industrial sectors. Van der Werf (2008) did the same estimation for seven industries and twelve OECD countries. Dissou, Karnizova and Sun (2012) estimated elasticities of substitution for ten industries in Canada.

This paper analyzes the role of production function specifications on macroeconomic effects of energy shocks. More specifically, it follows Dhawan, Jeske and Silos (2010) to examine two aspects. The first one is to examine whether the explanatory power of a “spillover” effect to the Great Moderation will change if we change the form of production function in Dhawan, Jeske and Silos (2010). The second aspect is to compare impulse responses of key macroeconomic variables to energy shocks, given different production specifications.

This paper is organized as follows. Section 2 describes the DSGE model from Dhawan, Jeske and Silos (2010), which I will use for simulation. Section 3 lists the production function forms to be included into the model. Section 4 calibrates the parameters in each production function form. Section 5 simulates the model and shows the results. Finally, section 6 concludes.

2. Model Description

Dhawan, Jeske and Silos (2010) examined the link between energy price shocks and the productivity process, and investigated whether this link can explain the “Great Moderation”. They empirically found that energy price shock had a significantly negative effect on the productivity from 1970 to 1982 (the early period), but from 1982 to 2005 (the late period) this effect was positive and not significant. So they concluded that the “spillover” effects from energy price shocks to the productivity process occurred in the early period and then disappeared in the late period. Finally they showed that the lower “spillover” effect in the late period can explain the “Great

Moderation” since the middle 1980s.

I use the DSGE model from Dhawan, Jeske and Silos (2010) with different forms of a production function, so I can examine whether the explanatory power of the lower “spillover” effect to the “Great Moderation” will change if we change the form of the production function.

The DSGE model from Dhawan, Jeske and Silos (2010) is as follows.

Households’ lifetime utility is a function of consumption c and leisure $1-h$ in each period t .

$$U = E_0 \sum_{t=0}^{\infty} \beta^t [\varphi \log c_t + (1-\varphi) \log(1-h_t)] \quad (2),$$

where β is the discount factor and φ is consumer’s preference weight on consumption ($0 < \beta < 1$, $0 < \varphi < 1$).

The representative firm’s production function includes three inputs (capital k , labour h and energy e) and an aggregate technology shock z , which I will talk about further in section “Production Function Selection”.

$$y_t = z_t (\eta k_{t-1}^{\nu} + (1-\eta) e_t^{\nu})^{\alpha/\nu} h_t^{1-\alpha} \quad (3),$$

where η and α are share parameters ($0 < \eta < 1$, $0 < \alpha < 1$), and $\nu = (\sigma_{K,E} - 1) / \sigma_{K,E}$.

The entire income is spent in three ways: consumption for non-energy goods c , investment i and energy input e .

$$c_t + i_t + p_t e_t = y_t \quad (4),$$

where p is the relative price of energy.

The price of energy is assumed to be exogenous, and they choose an ARMA(1,1) process for the energy price movements based on the Akaike Information Criterion

(AIC):

$$\log p_t = \rho^p \log p_{t-1} + \varepsilon_t^p + \xi \varepsilon_{t-1}^p \quad (5),$$

$$\varepsilon_t^p \sim N(0, \sigma_p^2)$$

where ρ^p is the persistence parameter for the logarithm of energy price and ξ is the coefficient for the lag innovation to the energy price shock.

The formula for investment is

$$i_t = k_t - (1 - \delta)k_{t-1} \quad (6),$$

where δ is the depreciation rate of capital ($0 < \delta < 1$).

One thing special about this DSGE model from Dhawan, Jeske and Silos (2010) is the stochastic process for technology shock z , which is different in early and late periods because of the possible “spillover” effect from the oil price shock. The empirical results of Dhawan, Jeske and Silos (2010) showed that the spillover effect occurred in the early period and then disappeared in the late period. In the early period, $t \leq t^*$ (t^* = the third quarter of 1982),

$$\log z_t = \rho^z \log z_{t-1} + \varepsilon_t^z + \gamma_1^1 \varepsilon_{t-1}^p + \gamma_1^2 \varepsilon_{t-2}^p + \gamma_1^3 \varepsilon_{t-3}^p + \gamma_1^4 \varepsilon_{t-4}^p \quad (7),$$

$$\varepsilon_t^z \sim N(0, \sigma_{z,1}^2).$$

Then in the late period, $t > t^*$,

$$\log z_t = \rho^z \log z_{t-1} + \varepsilon_t^z + \gamma_2^1 \varepsilon_{t-1}^p + \gamma_2^2 \varepsilon_{t-2}^p + \gamma_2^3 \varepsilon_{t-3}^p + \gamma_2^4 \varepsilon_{t-4}^p \quad (8),$$

$$\varepsilon_t^z \sim N(0, \sigma_{z,2}^2).$$

The estimated parameters (vectors γ_1 and γ_2) and the variance of the shocks differ between the early and the late samples. ρ^z is the persistence parameter for technology process, the parameters γ s represent the “spillover” effects from energy price shocks

to the productivity process. According to Dhawan, Jeske and Silos (2010)'s estimation, $\gamma_1 = [-0.092, -0.026, -0.067, -0.046]$ in the early period. All γ_1 s are negative and significant. In contrast, $\gamma_2 = [0.015, 0.021, 0.027, 0.011]$ in the late period. Now the elements of γ_2 s are not significant, which represents disappearance of the “spillover” effect.

The first order necessary optimal conditions for the equilibrium of the model are as follows (Dhawan, Jeske and Silos (2008)).

Households' choice between consumption and leisure at period t:

$$(1 - \varphi) \frac{1}{1 - h_t} = \varphi \frac{1}{c_t} w_t \quad (9).$$

The wage equals the marginal product of labour:

$$w_t = (1 - \alpha) \frac{y_t}{h_t} \quad (10).$$

The real interest rate equals the marginal product of capital:

$$r_t = y_t \alpha \frac{\eta k_{t-1}^{\nu-1}}{\eta k_{t-1}^{\nu} + (1 - \eta) e_t^{\nu}} \quad (11).$$

The energy price equals the marginal product of energy:

$$p_t = y_t \alpha \frac{(1 - \eta) e_t^{\nu-1}}{\eta k_{t-1}^{\nu} + (1 - \eta) e_t^{\nu}} \quad (12).$$

Households' consumption choice between periods t and t+1 (the Euler equation):

$$1 = \beta E \left\{ \frac{c_t}{c_{t+1}} (1 + r_{t+1} - \delta) \right\} \quad (13).$$

3. Production Function Selection

Equation (3) shows the production function in Dhawan, Jeske and Silos (2010), which firstly combines capital and energy in a CES form and then combines (KE) and labour in a Cobb-Douglas form. This function implies a constant substitution elasticity between capital and energy, and an unitary substitution elasticity between (KE) and labour.

But we may expect different simulation results if we change the form of the production function. Intuitively, an increase in the energy price will decrease the energy use, and the direct effect on output of the reduced energy use depends on the way through which energy is introduced into the production function. Furthermore, Kim and Loungani (1992) showed that no matter when technology shock and energy price shock were both allowed or when only energy price shock was allowed, the volatilities of key macroeconomic variables generated by simulation were lower in CES case than in Cobb-Douglas case.

Since the two-level constant-elasticity-of-substitution production function is the general case of the Cobb-Douglas form, I choose a two-level CES function in most cases. Following the study from Kemfert (1998), van der Werf (2008) and Dissou, Karnizova and Sun (2012), I use a production function with three factors (capital, labour and energy) in three cases: (KE)L, (KL)E and (LE)K, in which, for example, (KE)L means capital and energy are combined in a CES form, and then this aggregate and labour are combined in a CES form.

An important thing about the CES production function is to determine the

substitution elasticities between inputs. I will use the estimation results for USA by van der Werf (2008) (page 2972, Table 3) in each of the three cases. Note that in the (KE)L case in van der Werf (2008), the elasticity of substitution between capital and energy is 0.9999, and the elasticity of substitution between (KE) and labour is 0.9793. That is, the results do not reject the unitary value of each of the two elasticities, further, they do not reject a common elasticity between them (van der Werf (2008), page 2972-2974). So I will use the Cobb-Douglas function in the (KE)L case.

In the (KL)E case, the results reject the unitary value of each of the two elasticities ($\sigma_{K,L}$ and $\sigma_{KL,E}$), and reject a common elasticity between them (van der Werf (2008), page 2972-2974). So I will use the two-level CES function in the (KL)E case. The test results are the same for the two elasticities ($\sigma_{L,E}$ and $\sigma_{LE,K}$) in the (LE)K case (van der Werf (2008), page 2972-2974), so I will also use the two-level CES function in the (LE)K case.

My production function selection is as follows:

Case1: (KE)L

$$y_t = z_t A k_{t-1}^{\alpha_1} e_t^{\alpha_2} h_t^{1-\alpha_1-\alpha_2} \quad (3.1)$$

This Cobb-Douglas function implies the elasticities of substitution between any two inputs are equal to one.

Case2: (KL)E:

$$y_t = z_t [\alpha_1 (\alpha_2 (A_1 k_{t-1})^a + (1-\alpha_2) (A_2 h_t)^a)^{b/a} + (1-\alpha_1) e_t^b]^{1/b} \quad (3.2)$$

This two-level CES function implies a constant substitution elasticity between capital and labour, and a constant substitution elasticity between (KL) and energy. The

substitution elasticity between capital and energy and the substitution elasticity between labour and energy vary overtime.

Case3: (LE)K:

$$y_t = z_t [\alpha_1 (A_1 (\alpha_2 h_t^a + (1 - \alpha_2) e_t^a)^{1/a})^b + (1 - \alpha_1) (A_2 k_{t-1})^b]^{1/b} \quad (3.3)$$

This form implies a constant substitution elasticity between labour and energy, and a constant substitution elasticity between (LE) and capital. The elasticity of substitution between capital and labour and the elasticity of substitution between capital and energy vary overtime.

A, A_1, A_2 are constants in each of the three cases, and I will talk about them further in section “Calibration”.

Different forms of the production function modify the first order conditions for wage, interest rate and energy price in each of the three cases above.

Case 1: (KE)L

$$w_t = (1 - \alpha_1 - \alpha_2) \frac{y_t}{h_t} \quad (10.1)$$

$$r_t = \alpha_1 \frac{y_t}{k_{t-1}} \quad (11.1)$$

$$p_t = \alpha_2 \frac{y_t}{e_t} \quad (12.1)$$

Case 2: (KL)E

$$w_t = \frac{\alpha_1 (1 - \alpha_2) A_2^a y_t h_t^{a-1} (\alpha_2 A_1^a k_{t-1}^a + (1 - \alpha_2) A_2^a h_t^a)^{b/a-1}}{\alpha_1 (\alpha_2 A_1^a k_{t-1}^a + (1 - \alpha_2) A_2^a h_t^a)^{b/a} + (1 - \alpha_1) e_t^b} \quad (10.2)$$

$$r_t = \frac{\alpha_1 \alpha_2 A_1^a y_t k_{t-1}^{a-1} (\alpha_2 A_1^a k_{t-1}^a + (1 - \alpha_2) A_2^a h_t^a)^{b/a-1}}{\alpha_1 (\alpha_2 A_1^a k_{t-1}^a + (1 - \alpha_2) A_2^a h_t^a)^{b/a} + (1 - \alpha_1) e_t^b} \quad (11.2)$$

$$p_t = \frac{(1-\alpha_1)y_t e_t^{b-1}}{\alpha_1(\alpha_2 A_1^a k_{t-1}^a + (1-\alpha_2)A_2^a h_t^a)^{b/a} + (1-\alpha_1)e_t^b} \quad (12.2)$$

Case 3: (LE)K

$$w_t = \frac{\alpha_1 A_1^b \alpha_2 y_t h_t^{a-1} (\alpha_2 h_t^a + (1-\alpha_2)e_t^a)^{b/a-1}}{\alpha_1 A_1^b (\alpha_2 h_t^a + (1-\alpha_2)e_t^a)^{b/a} + (1-\alpha_1)A_2^b k_{t-1}^b} \quad (10.3)$$

$$r_t = \frac{(1-\alpha_1)A_2^b y_t k_{t-1}^{b-1}}{\alpha_1 A_1^b (\alpha_2 h_t^a + (1-\alpha_2)e_t^a)^{b/a} + (1-\alpha_1)A_2^b k_{t-1}^b} \quad (11.3)$$

$$p_t = \frac{\alpha_1 A_1^b (1-\alpha_2)y_t e_t^{a-1} (\alpha_2 h_t^a + (1-\alpha_2)e_t^a)^{b/a-1}}{\alpha_1 A_1^b (\alpha_2 h_t^a + (1-\alpha_2)e_t^a)^{b/a} + (1-\alpha_1)A_2^b k_{t-1}^b} \quad (12.3)$$

4. Calibration and Solution

I solve the model numerically, using a log-linearized version of the model around the steady state (the same method as in Dhawan, Jeske and Silos (2010)). The computer code is included in the Appendix C.

According to Dhawan, Jeske and Silos (2010)'s estimation for the equations (5), (7) and (8), $\rho^p=0.921$ and $\rho^z=0.912$, which suggest high persistence for energy price and technology processes. $\sigma_{z,1}^2=0.0000545$ and $\sigma_{z,2}^2=0.0000208$ suggest the volatility of technology shock is lower in the late period. $\sigma_p^2=0.001$ suggests the volatility of energy price shock is much larger than technology shock. Finally, the estimate of ξ in (5) is $\xi=0.375$. (Dhawan, Jeske and Silos (2010), page 719, Table 2). Dhawan, Jeske and Silos (2010) also calibrated $\beta=0.99$, $\varphi=0.3376$ and $\delta=0.0154$ to match $k/y=12$, $e/y=0.0544$ and $h=0.3$ (page 721, Table 4).

Even though I can use their results for the parameters above, I still need to determine the parameter values for the production function in three cases. So I

calibrate each parameter based on the results of the steady state values from Dhawan, Jeske and Silos (2010).

4.1. (KE)L

In the deterministic steady state, the equations (10.1) and (12.1) give us

$$wh/y = 1 - \alpha_1 - \alpha_2,$$

$$e/y = \alpha_2/p,$$

and the equation (3.1) gives

$$1 = zA(k/y)^{\alpha_1} (e/y)^{\alpha_2} (h/y)^{1-\alpha_1-\alpha_2}.$$

Further, in the deterministic steady state, $z=1$ and $p=1$ ($\sigma_z^2=0, \sigma_p^2=0$) (Dhawan, Jeske and Silos (2010)).

In this case, the unknown parameters are α_1, α_2 and A . According to Dhawan, Jeske and Silos (2010), $k/y=12$, $e/y=0.0544$, $h/y=0.281$, $w^*h/y=0.64$. So we have:

$$\alpha_2 = e/y = 0.0544,$$

$$\alpha_1 = 1 - \alpha_2 - wh/y = 0.3056,$$

$$A = (k/y)^{-\alpha_1} (e/y)^{-\alpha_2} (h/y)^{\alpha_1+\alpha_2-1} = 1.2354.$$

Adding a constant $A=1.2354$ in the production function can help matching the steady state value of every single variable from Dhawan, Jeske and Silos (2010).

4.2. (KL)E

The unknown parameters are $a, b, \alpha_1, \alpha_2, A_1, A_2$. I can calculate the values for a, b given the estimation results for the elasticity of substitution between capital and labour and the substitution elasticity between (KL) and energy from van der Werf (2008), for USA, $\sigma_{K,L} = 0.3191$, $\sigma_{KL,E} = 0.547$ (Page 2972, Table 3). So we have

$$a = \frac{\sigma_{K,L} - 1}{\sigma_{K,L}} = -2.1338, b = \frac{\sigma_{KL,E} - 1}{\sigma_{KL,E}} = -0.828$$

The reason for adding constants A_1, A_2 in this case is that if I follow the same way above to match $k/y=12$, $e/y=0.0544$ and $w^*h/y=0.64$ without A_1, A_2 , then the resulting value for α_2 will be out of the reasonable range $(0,1)$.

In the deterministic steady state, the equations (3.2) and (12.2) give us

$$1/(1 - \alpha_1) = (e/y)^{b-1} \quad (16)$$

$$1 = \alpha_1(\alpha_2 A_1^a (k/y)^a + (1 - \alpha_2) A_2^a (h/y)^a)^{b/a} + (1 - \alpha_1)(e/y)^b \quad (17)$$

Let $B = \alpha_2 A_1^a (k/y)^a + (1 - \alpha_2) A_2^a (h/y)^a$, the equations (3.2) and (11.2) give us

$$r = \alpha_1 \alpha_2 A_1^a (k/y)^{a-1} B^{b/a-1} \quad (18)$$

The equations (3.2) and (10.2) give us

$$w = \alpha_1 (1 - \alpha_2) A_2^a (h/y)^{a-1} B^{b/a-1} \quad (19)$$

To match the values $k/y=12$, $e/y=0.0544$, $h/y=0.281$, $r=0.0255$, $w^*h/y=0.64$ from Dhawan, Jeske and Silos (2010), we have

$$\alpha_1 = 0.995, x_1 = \alpha_2 A_1^a = 56.9543, x_2 = (1 - \alpha_2) A_2^a = 0.0395.$$

4.3. (LE)K

In this case, we have the parameters $a, b, \alpha_1, \alpha_2, A_1, A_2$. Again, I calculate the values of a, b given the estimation results from van der Werf (2008), for USA, $\sigma_{L,E} = 0.8584$, $\sigma_{LE,K} = 0.2852$ (page 2972, Table 3). So we have

$$a = \frac{\sigma_{L,E} - 1}{\sigma_{L,E}} = -0.165, b = \frac{\sigma_{LE,K} - 1}{\sigma_{LE,K}} = -2.5$$

I add constants A_1, A_2 for the same reason as in (KL)E case, to ensure that the share parameters are between zero and one.

In the deterministic steady state, the equations (3.3) and (11.3) give us

$$r = (1 - \alpha_1)A_2^b(k/y)^{b-1} \quad (20)$$

Divide (10.3) by (12.3), we have

$$w = \frac{\alpha_2}{1 - \alpha_2} (h/e)^{a-1} \quad (21)$$

The equations (12.3) and (3.3) give us

$$1 = \alpha_1 A_1^b (1 - \alpha_2) (e/y)^{a-1} (\alpha_2 (h/y)^a + (1 - \alpha_2) (e/y)^a)^{b/a-1} \quad (22)$$

To match the values $k/y=12$, $e/y=0.0544$, $h/y=0.281$, $r=0.0255$, $w*h/y=0.64$ from Dhawan, Jeske and Silos (2010), we have

$$x_1 = \alpha_1 A_1^b = 0.0218, x_2 = (1 - \alpha_1) A_2^b = 152.64, \alpha_2 = 0.939 \quad .$$

5. Simulation

5.1. Comparing “Spillover” Effects to The Great Moderation

Dhawan, Jeske and Silos (2010) showed that the simulated volatility of output would drop by 53% from the early to the late period if the “spillover” effect and volatility of technology shock were both lower in the late period. The drops in the simulated volatilities of consumption, investment and working hours are 56.83%, 52.3% and 51.57%, respectively. When they set the volatility of technology shock at the average level in both periods and examine only the influence of the lower “spillover” effect in the late period, the volatility of output would drop by 36.96%, and the drops in volatilities of consumption, investment and working hours are 38.88%, 38.18% and 34.83%, respectively. (Dhawan, Jeske and Silos (2010), page 722, Table 5)

To make the simulation results comparable with Dhawan, Jeske and Silos (2010), I follow the same steps in each of the three cases. I firstly run the model with different parameter values for spillover effects ($\gamma^1, \gamma^2, \gamma^3, \gamma^4$) and volatility of innovation to technology shock (σ_z) between the early and late periods. Then I also set the volatility of technology shock at the average level in two periods to examine only the spillover effect. (see Dhawan, Jeske and Silos (2010), page 721-722)

The results for the three specifications of the production function are reported in Table 1 to 3. In each case, column “B” reports the simulated volatilities of output, consumption, investment and working hours when both ($\gamma^1, \gamma^2, \gamma^3, \gamma^4$) and σ_z are allowed to change between the two periods. Column “S” reports the simulated volatilities of the four macroeconomic variables when only ($\gamma^1, \gamma^2, \gamma^3, \gamma^4$) are allowed to change between the two periods. The last two columns represent the percentage drops in the volatilities of the four variables from the early to the late period.

5.1.1 Case 1: (KE)L

Table 1 shows the simulated volatilities of output, consumption, investment and working hours in both the early and the late periods in the (KE)L case. The lower spillover effect and the decrease in the volatility of TFP together can generate a drop in the volatility of output by about 53%, which is the same as Dhawan, Jeske and Silos (2010). The drops in the volatilities of consumption, investment, working hours are about 58%, 52%, 52%, respectively, also close to their results.

The disappeared “spillover” effect solely can generate a 37% decrease in the volatility of output and 43%, 36%, 36% drops in the volatilities of consumption,

investment and working hours, respectively. Only the percentage drop in the volatility of consumption is a little higher, other results are similar to Dhawan, Jeske and Silos (2010).

5.1.2 Case 2: (KL)E

Table 2 gives the simulated volatilities of output, consumption, investment and working hours in both the early and the late periods in the (KL)E case. The lower spillover effect and the decrease in the volatility of TFP together can generate drops in the volatility of output, consumption, investment and working hours by about 53%, 58%, 51%, 52%, respectively. These are also very close to the results in Dhawan, Jeske and Silos (2010).

The drops in the volatilities of output, consumption, investment and working hours generated only by the disappeared “spillover” effect are about 35%, 41%, 36% and 36% respectively, there are at most only 2 basis points difference to Dhawan, Jeske and Silos (2010).

5.1.3 Case 3: (LE)K

Table 3 shows the simulated volatilities of output, consumption, investment and working hours in both the early and the late periods in the (LE)K case. The drops in the volatilities of output, consumption, investment and working hours generated by the lower spillover effect and the reduced volatility of TFP together are 53%, 55%, 51% and 50%, respectively, which are still similar to the results from Dhawan, Jeske and Silos (2010).

The drops in the volatilities of output, consumption, investment and working hours

generated only by the disappeared “spillover” effect are all about 35%, in which the drop in the volatility of consumption is about 4 basis points less than the result in Dhawan, Jeske and Silos (2010).

5.2. Comparing Impulse Responses

To compare the impulse responses of key macroeconomic variables to the energy price shock in three cases, I firstly examine the models with higher spillover effect and volatility of technology shock in the early period, then the models with lower spillover effect and volatility of technology shock in the late period. Figure 1 to Figure 9 give the impulse responses of the logarithms of key variables to a one-standard-deviation energy price shock. The standard deviation is the same in all simulations, and is equal to the square root of 0.001. In each figure impulse responses represent percentage deviations from the values in the steady state.

5.2.1 Early Period

Figure 1 to Figure 3 give the results in the early period with higher spillover effect and volatility of technology shock (setting $\gamma_1 = [-0.092, -0.026, -0.067, -0.046]$ and $\sigma_{z,1}^2 = 0.0000545$ estimated by Dhawan, Jeske and Silos (2010)). In all three cases, a one-standard-deviation energy price shock will increase the logarithm of the relative price of energy by 4.31 basis points and decrease the logarithm of technology by 0.66 basis points because of the significantly negative spillover effect in the early period. The logarithm of energy use will drop by 5.16, 2.76 and 3.98 basis points in the (KE)L, the (KL)E and the (LE)K case respectively because using energy is more

costly. Because the unitary substitution elasticity makes it easier to replace energy by other inputs in the Cobb-Douglas form, but the substitution elasticity between (KL) and energy is only 0.547 in the (KL)E case for USA (van der Werf (2008)), the energy use drops most in the (KE)L case and drops least in the (KL)E case.

5.2.1.1 Case 1: (KE)L

In the (KE)L case, a one-standard-deviation energy price shock can decrease the logarithm of consumption by 0.47 basis points and decrease the logarithm of working hours by 0.8 basis points. It will also decrease the logarithm of investment by 6.2 basis points and decrease the logarithm of capital stock by 0.71 basis points. It will also decrease the logarithm of interest rate by about 1.3 basis points and lower the logarithm of wage by 0.69 basis points. Finally, it will decrease the logarithm of output by 1.49 basis points.

The results above are as expected. An increase in the energy price will increase the cost of production, and decrease production. The reduced technology from the negative spillover effect can amplify this result and enlarge the “scale” effect of drop in production. If the “scale” effect is bigger than the substitution effect, the demands for capital and labour will decrease, so the capital use, the labour use, the interest rate and the wage will all decrease. Consumption and investment will drop because of the reduced income.

5.2.1.2 Case 2: (KL)E

In the (KL)E case, a one-standard-deviation energy price shock can decrease the logarithm of consumption by 0.57 basis points which is more than that in the (KE)L

case and decrease the logarithm of working hours by 0.37 basis points which is much less than the (KE)L case. It will decrease the logarithm of investment by 4.16 basis points and decrease the logarithm of capital stock by 0.43 basis points, both are less than the (KE)L case. It will decrease the logarithm of interest rate by 1.47 basis points and lower the logarithm of wage by 0.66 basis points. And it will decrease the logarithm of output by 1.08 basis points, which is less than the (KE)L case.

5.2.1.3 Case 3: (LE)K

In the (LE)K case, after a one-standard-deviation energy price shock, the logarithm of consumption will drop by 0.53 basis points. The logarithm of working hours will drop by 0.27 basis points, this is even more less. The logarithm of investment will decrease by 4.04 basis points, this is close to the (KL)E case but less than the (KE)L case. The logarithm of capital stock will decrease by 0.42 basis points, which is very close to the (KL)E case. The logarithm of interest rate will drop by 1.69 basis points, this is larger than previous cases. The logarithm of wage will decrease by 0.6 basis points, which is lower than previous cases. The logarithm of output will drop by 1.06 basis points, which is close to the (KL)E case.

5.2.1.4 Comparison across Three Cases

After a one-standard-deviation energy price shock, the energy use will drop most in the Cobb-Douglas case, and drop least in the (KL)E case. The drops in output, working hours, investment and capital stock are largest in the Cobb-Douglas case, and the drop in consumption is smallest in the Cobb-Douglas case. The drops in working hours and wage are smallest in the (LE)K case, the drop in interest rate is largest in

the (LE)K case.

Consequently, the assumption regarding the form of the production function and the values of the elasticities of substitution between inputs matter for the magnitude of the predicted theoretical impulse response.

5.2.2 Late Period

Figure 4 to Figure 6 show the impulse responses in the late period with the lower spillover effect and volatility of technology shock (setting $\gamma_2 = [0.015, 0.021, 0.027, 0.011]$ and $\sigma_{z,2}^2 = 0.0000208$ estimated by Dhawan, Jeske and Silos (2010)). In all three cases, a one-standard-deviation energy price shock will increase the logarithm of the relative price of energy by 4.31 basis points, this is the same as the early period. But the logarithm of technology will increase by 0.21 basis points because of the slightly positive spillover effect from energy price to technology process estimated by Dhawan, Jeske and Silos (2010). The logarithm of the energy use will drop by 4.68, 2.56 and 3.7 basis points in the (KE)L, the (KL)E and the (LE)K case respectively.

5.2.2.1 Case 1: (KE)L

In the (KE)L case, a one-standard-deviation energy price shock can decrease the logarithm of consumption by about 0.03 basis points and decrease the logarithm of working hours by 0.23 basis points, as shown in Figure 4. The logarithm of investment will decrease by 1.74 basis points and the logarithm of capital stock will drop by about 0.07 basis points. The logarithm of interest rate will drop by 0.35 basis

points and the logarithm of wage will decrease by 0.13 basis points. The logarithm of output will drop by 0.36 basis points.

5.2.2.2 Case 2: (KL)E

In the (KL)E case, a one-standard-deviation energy price shock can decrease the logarithm of consumption by about 0.045 basis points, as shown in Figure 5. The logarithm of working hours will drop by 0.12 basis points, which is less than the (KE)L case. The logarithm of investment will drop by 1.31 basis points and the logarithm of capital stock will decrease by about 0.05 basis points, both of which are less than the (KE)L case. The logarithm of interest rate will decrease by 0.44 basis points, which is larger than the (KE)L case, and the logarithm of wage will drop by about 0.09 basis points which is less than the (KE)L case. And the logarithm of output will drop by 0.18 basis points, which is less than the (KE)L case.

5.2.2.3 Case 3: (LE)K

In the (LE)K case, as reported in Figure 6, a one-standard-deviation energy price shock can decrease the logarithm of investment by 1.34 basis points, and the logarithm of capital stock will drop by about 0.05 basis points, these are very close to the (KL)E case. The logarithm of interest rate will decrease by 0.65 basis points, which is larger than two other cases. And the logarithm of output will drop by 0.19 basis points, this is very close to the (KL)E case.

Unexpectedly, in this case, consumption, working hours and wage respond differently, compared to the previous cases. After a one-standard-deviation energy price shock, the logarithm of consumption will increase by about 0.047 basis points,

the logarithm of working hours will decrease by about 0.046 basis points in the first period but increase by about 0.08 basis points in the fifth period, and the logarithm of wage will increase by about 0.05 basis points.

One possible reason for the unexpected responses of consumption, working hours and the wage might be the slightly positive “spillover” effect from energy price shock to the technology process in the late period estimated by Dhawan, Jeske and Silos (2010). The increased technology can weaken the drop in output and lower the “scale” effect of reduced output. The demand for labour may increase if the “scale” effect is less than the substitution effect between energy and labour ($\sigma_{L,E} = 0.8584$ for USA estimated by van der Werf (2008)). So working hours and wage increase and consumption may also increase because of the higher wage.

In next section, I set the parameters of “spillover” effect ($\gamma^1, \gamma^2, \gamma^3, \gamma^4$) in the late period to be zero to see whether this will influence the results.

5.2.3 Late Period Modified (No Spillover Effects)

Figure 7 to Figure 9 show the impulse responses in the late period without spillover effect ($\gamma_2 = [0, 0, 0, 0]$). After a one-standard-deviation energy price shock, the logarithm of energy price will increase by 4.31 basis points in all three cases, and the logarithm of energy use will drop by 4.72, 2.57 and 3.74 basis points in the (KE)L, the (KL)E and the (LE)K case, respectively. One thing noteworthy is that the responses of consumption, working hours and wage now turn to be negative after shutting down the spillover effect entirely.

The logarithm of consumption will drop by 0.12 basis points in the (KE)L and (LE)K cases and decrease by 0.15 basis points in the (KL)E case, which are close to each other. The logarithm of working hours will drop by 0.23 and 0.1 basis points in the (KE)L and the (KL)E case, respectively, and drop by about 0.005 basis points in the (LE)K case. Although shutting down the spillover effect change the way working hours responses in the (LE)K case, this response in Figure 9 is still close to zero, probably because the substitution effect between labour and energy is still large in the (LE)K case.

The logarithm of investment will drop by 1.74, 1.18 and 1.12 basis points in the three cases, respectively. The logarithm of capital stock will decrease by 0.19, 0.12 and 0.11 basis points in the three cases, respectively. And the logarithm of interest rate will drop by 0.39, 0.44 and 0.68 basis points in the (KE)L, the (KL)E and the (LE)K cases respectively.

The logarithm of wage will decrease by 0.18 basis points in the (KE)L and (KL)E cases and drop by 0.12 basis points in the (LE)K case. Finally, the logarithm of output will drop by 0.41 basis points in the (KE)L case and decrease by 0.21 basis points in the (KL)E and (LE)K cases.

The comparison results across the production function forms are very qualitatively similar to the results in the early period. For example, the energy use will drop most in the Cobb-Douglas case, and drop least in the (KL)E case. The drops in output, working hours, investment and capital stock are largest in the Cobb-Douglas case. The drops in working hours and wage are smallest in the (LE)K case, the drop in

interest rate is largest in the (LE)K case. The quantitative magnitudes of the impulse responses of key macroeconomic variables are less than the results in the early period.

6. Conclusion

Following Dhawan, Jeske and Silos (2010), this paper firstly examines whether the explanatory power of a “spillover” effect to the Great Moderation will change if we change the form of the production function in Dhawan, Jeske and Silos (2010). Then it compares the impulse responses of key macroeconomic variables to energy price shocks, given different production function specifications.

This paper chooses three production function forms to do the comparison: a Cobb-Douglas form with three inputs (capital K, labour L, energy E), a two-level CES function in the (KL)E case, and a two-level CES function in the (LE)K case.

This paper finds that production function specification doesn't seem to play an important role on the explanatory power of a “spillover” effect to the Great Moderation. The only noticeable difference is that the drop in volatility of consumption generated only by “spillover” effect is larger in the Cobb-Douglas form and lower in the (LE)K case of the two-level CES form, compared to the result from Dhawan, Jeske and Silos (2010).

The specification of the production function may play a role in explaining the responses of some of the key macroeconomic variables to energy price shocks. For example, the responses of working hours, investment and capital stock are larger in the Cobb-Douglas form than in the two-level CES (KL)E and (LE)K forms. In the

two-level CES (LE)K form, the response of interest rate is larger than in the other two cases, while the response of the real wage is smaller than in the other two cases. The response of output is larger in the Cobb-Douglas form, and it is very close in the other two cases. The energy use decreases mostly in the Cobb-Douglas form and drops the least in the (KL)E case.

One thing noteworthy is that with the two-level CES production function in the (LE)K case, the responses of consumption, working hours and the real wage to energy price shock turns to be positive because of the slightly positive “spillover” effect in the late period estimated by Dhawan, Jeske and Silos (2010). Shutting down the “spillover” effect entirely can change them back to negative, although the response of working hours remains close to zero even after that.

Appendix A: Tables

Table 1. Simulated Volatilities of Key Macroeconomic Variables in Case 1: (KE)L

	Early Period		Late Period		% Drop (\approx)	
	B	S	B	S	B	S
y	2.42	2.19	1.14	1.39	-53	-37
c	0.52	0.47	0.22	0.27	-58	-43
i	10.82	9.79	5.19	6.27	-52	-36
h	1.43	1.29	0.69	0.83	-52	-36

Note: B = “Baseline” defined as the same as in Dhawan, Jeske and Silos (2010), page 721, which includes different parameter values for spillover effects ($\gamma^1, \gamma^2, \gamma^3, \gamma^4$) and volatility of innovation to technology shock (σ_z) between early and late periods. S = “Spillover Only” defined as the same as in Dhawan, Jeske and Silos (2010), page 721-722, which means σ_z doesn’t change in two periods. y=“output”, c=“consumption”, i=“investment”, h=“working hours”. Volatilities are the standard deviations of log-deviations from HP-filtered series, same as Dhawan, Jeske and Silos (2010), page 722, table 5. The explanations are the same for Table 2 and Table 3.

Table 2. Simulated Volatilities of Key Macroeconomic Variables in Case 2: (KL)E

	Early Period		Late Period		% Drop (\approx)	
	B	S	B	S	B	S
y	1.79	1.60	0.85	1.04	-53	-35
c	0.71	0.64	0.30	0.38	-58	-41
i	7.45	6.76	3.62	4.33	-51	-36
h	0.64	0.58	0.31	0.37	-52	-36

Table 3. Simulated Volatilities of Key Macroeconomic Variables in Case 3: (LE)K

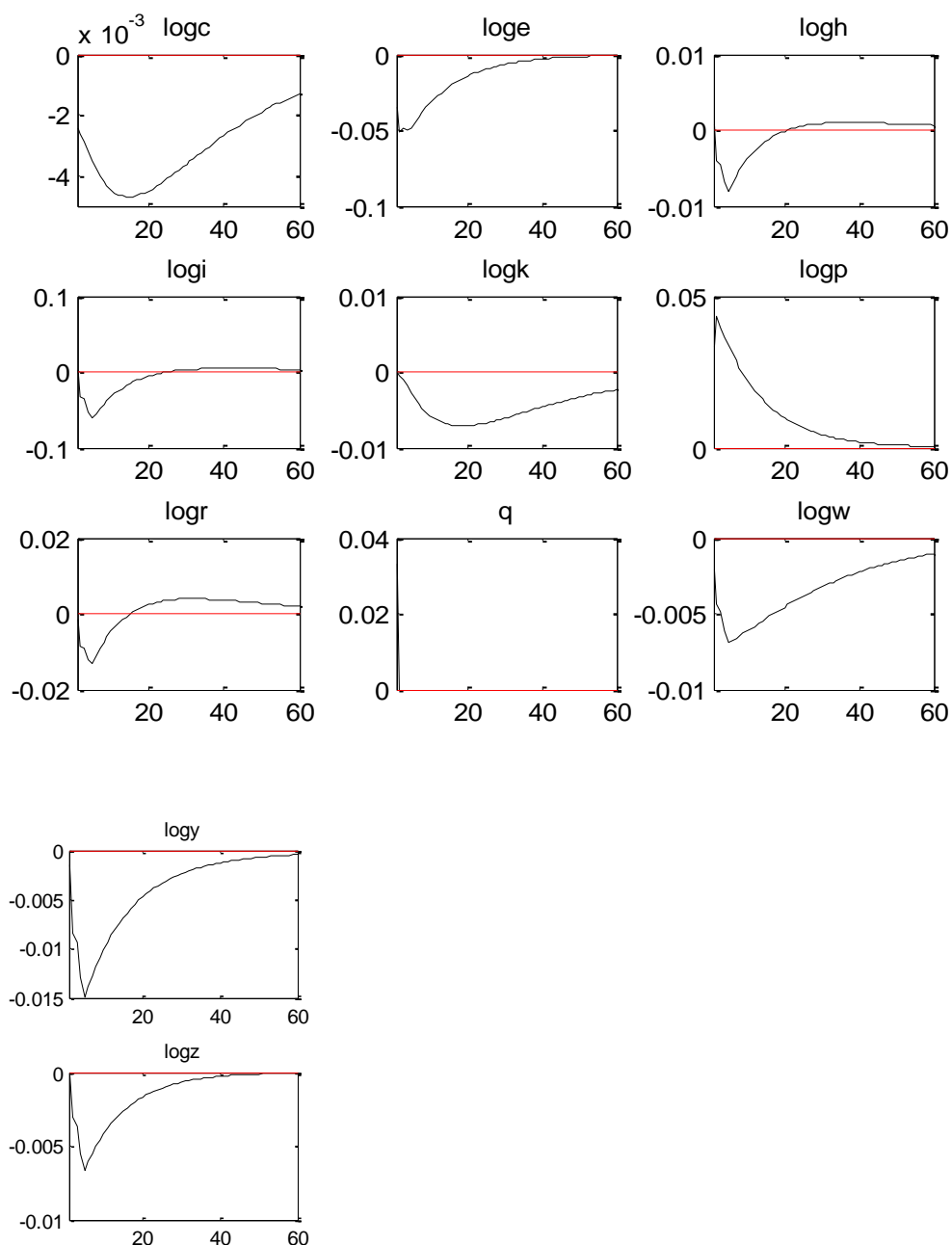
	Early Period		Late Period		% Drop (\approx)	
	B	S	B	S	B	S
y	1.75	1.57	0.83	1.02	-53	-35
c	0.64	0.57	0.29	0.37	-55	-35
i	7.22	6.54	3.56	4.26	-51	-35
h	0.54	0.49	0.27	0.32	-50	-35

Appendix B: Figures

Figure 1

Impulse Responses to Energy Price Shock with Negative Spillover in the Early

Period for Case 1: (KE)L



Note: q is an auxiliary variable in Dynare codes to help adding the lagged energy price shocks into the productivity and the energy price processes (Dhawan, Jeske and Silos (2009)). The explanations are the same for Figure 2 to Figure 9.

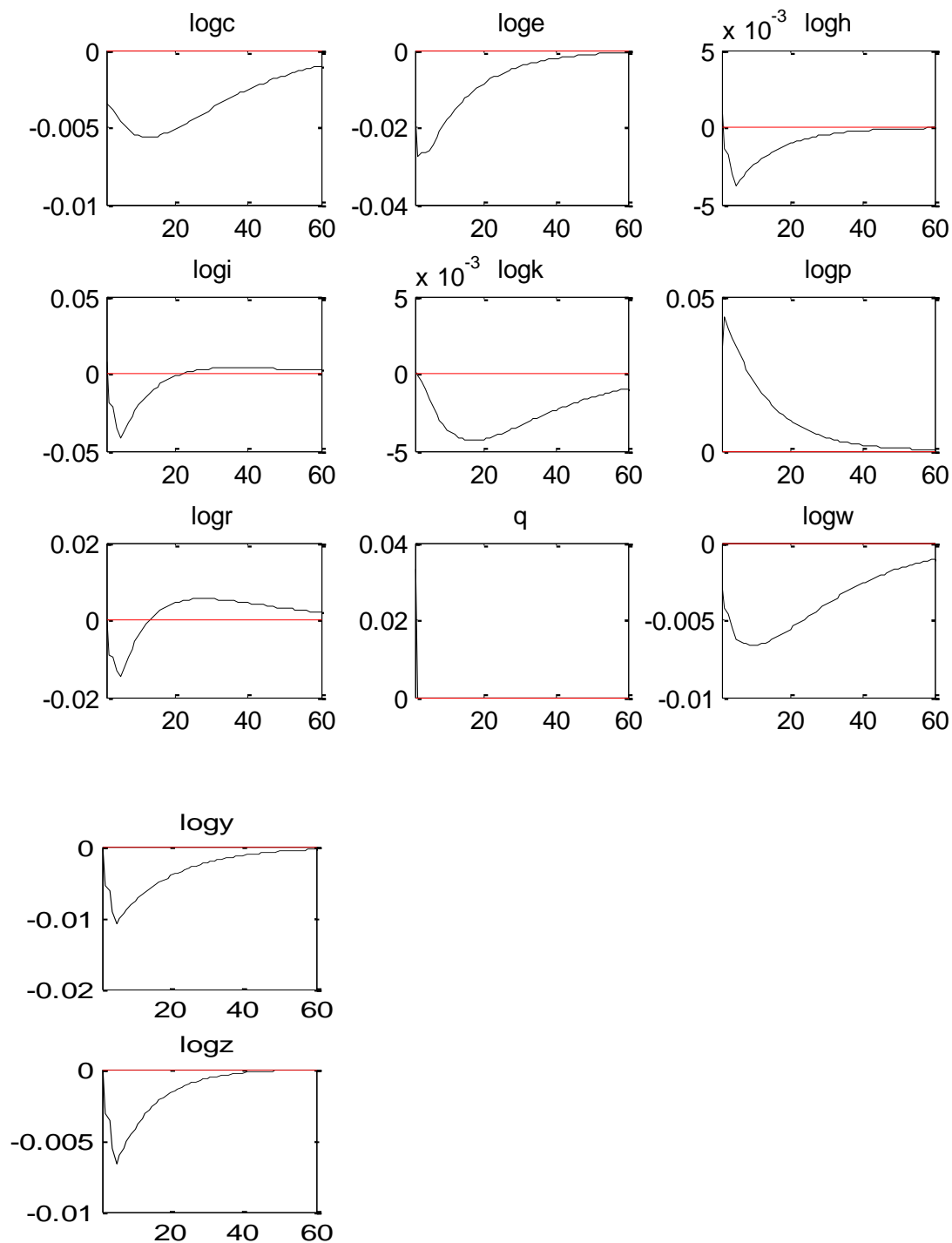
Figure 2**Impulse Responses to Energy Price Shock with Negative Spillover in the Early****Period for Case 2: (KL)E**

Figure 3

Impulse Responses to Energy Price Shock with Negative Spillover in the Early Period for Case 3: (LE)K

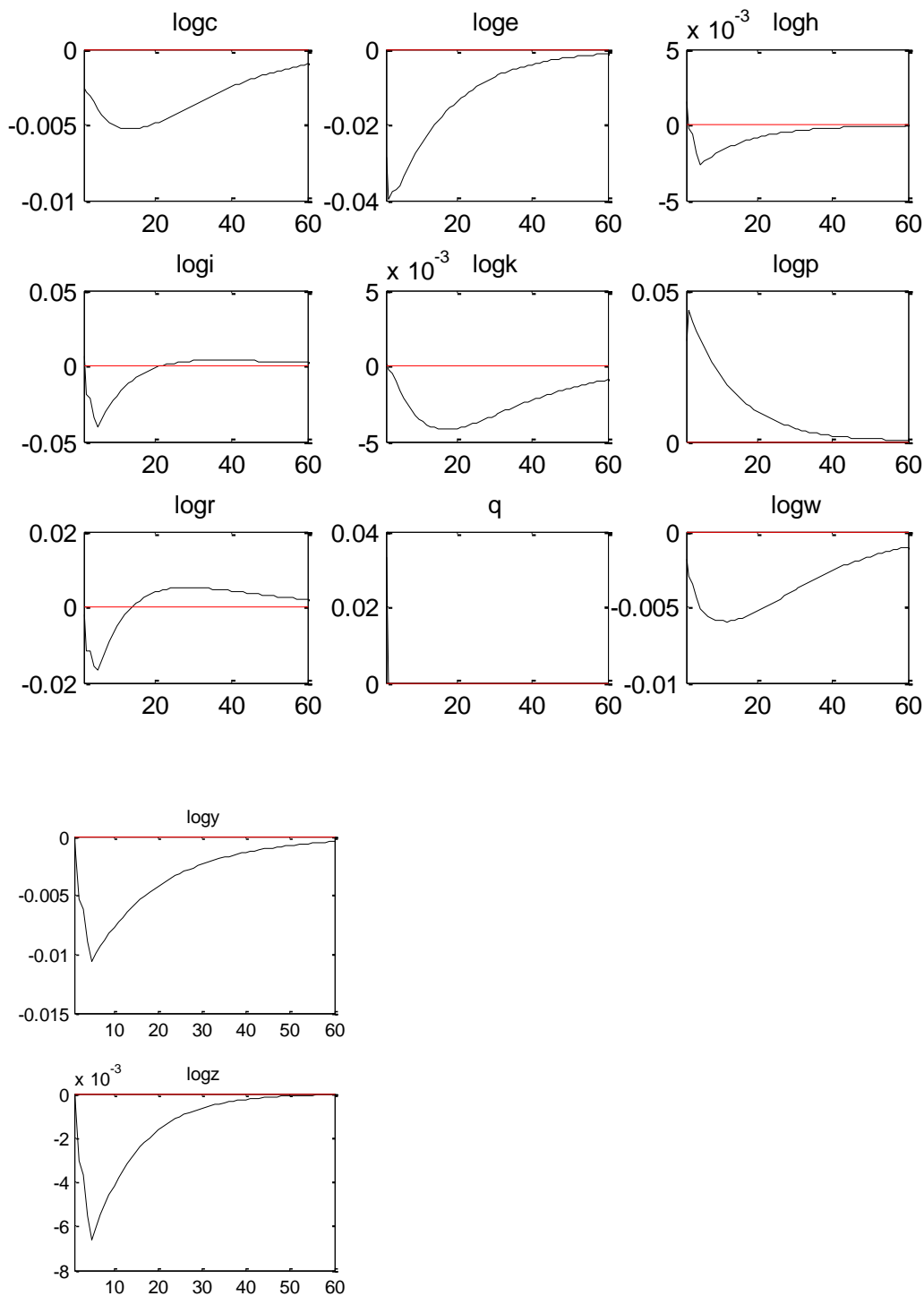


Figure 4

Impulse Responses to Energy Price Shock with Positive Spillover in the Late Period for Case 1: (KE)L

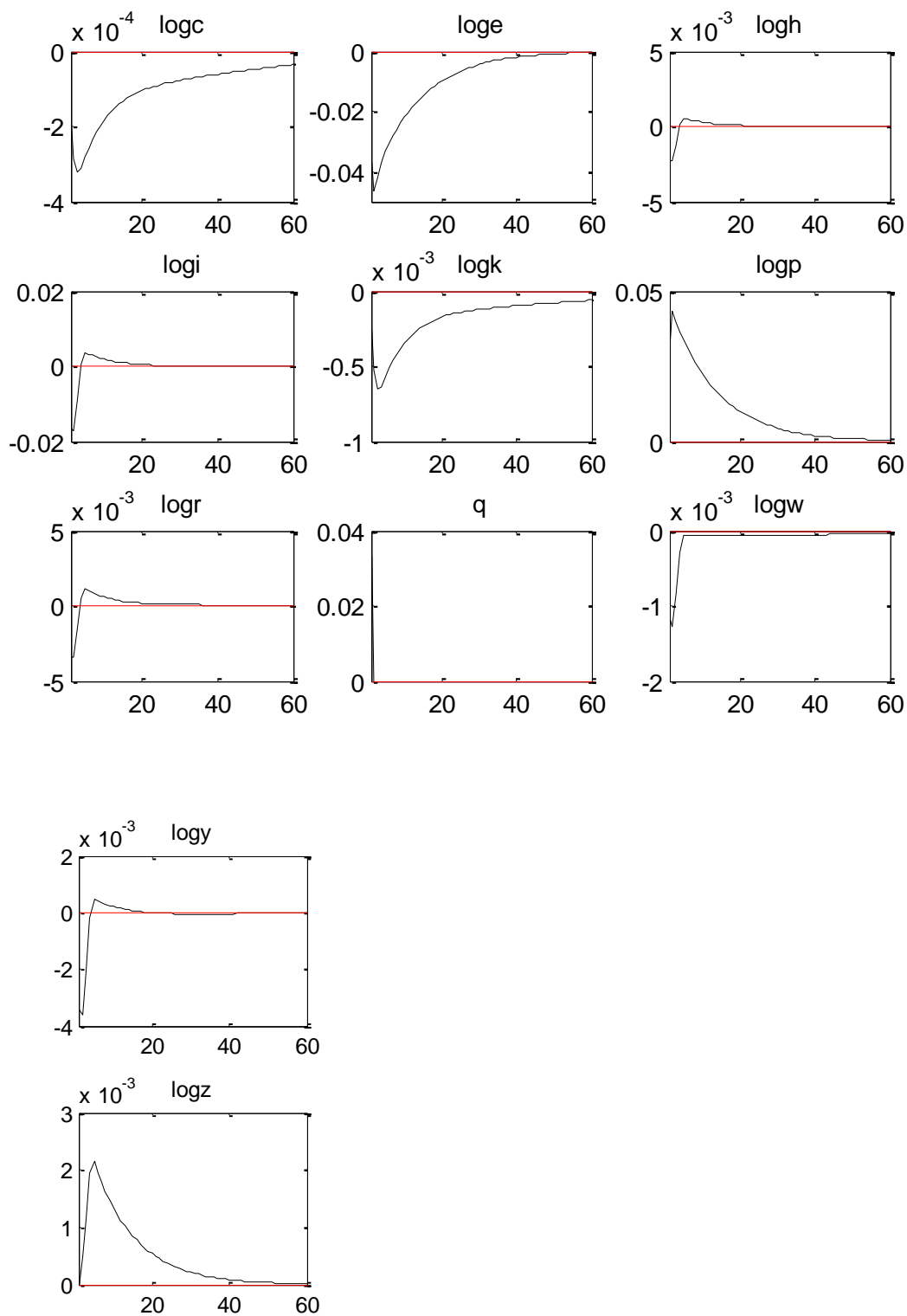


Figure 5

Impulse Responses to Energy Price Shock with Positive Spillover in the Late Period for Case 2: (KL)E

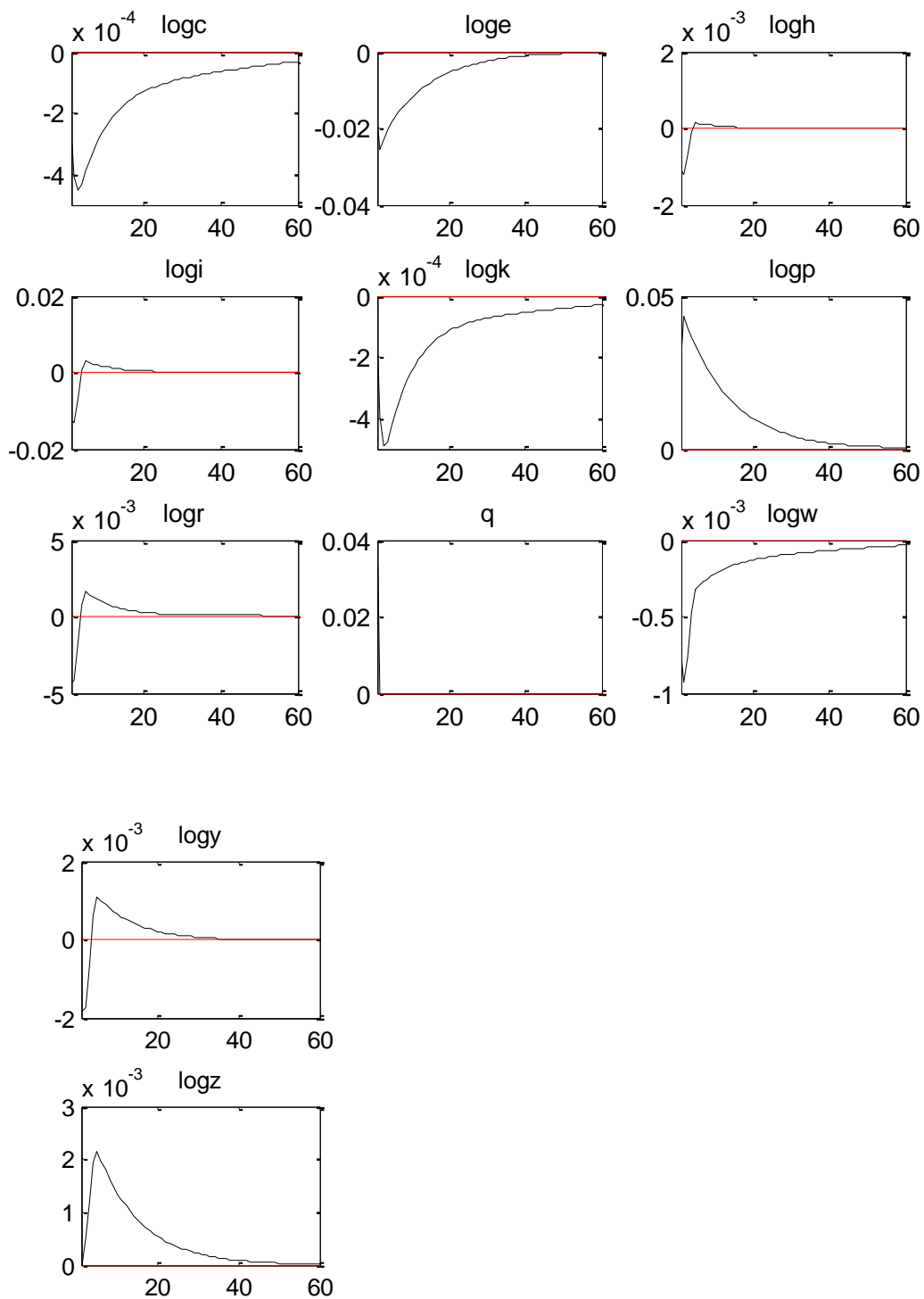


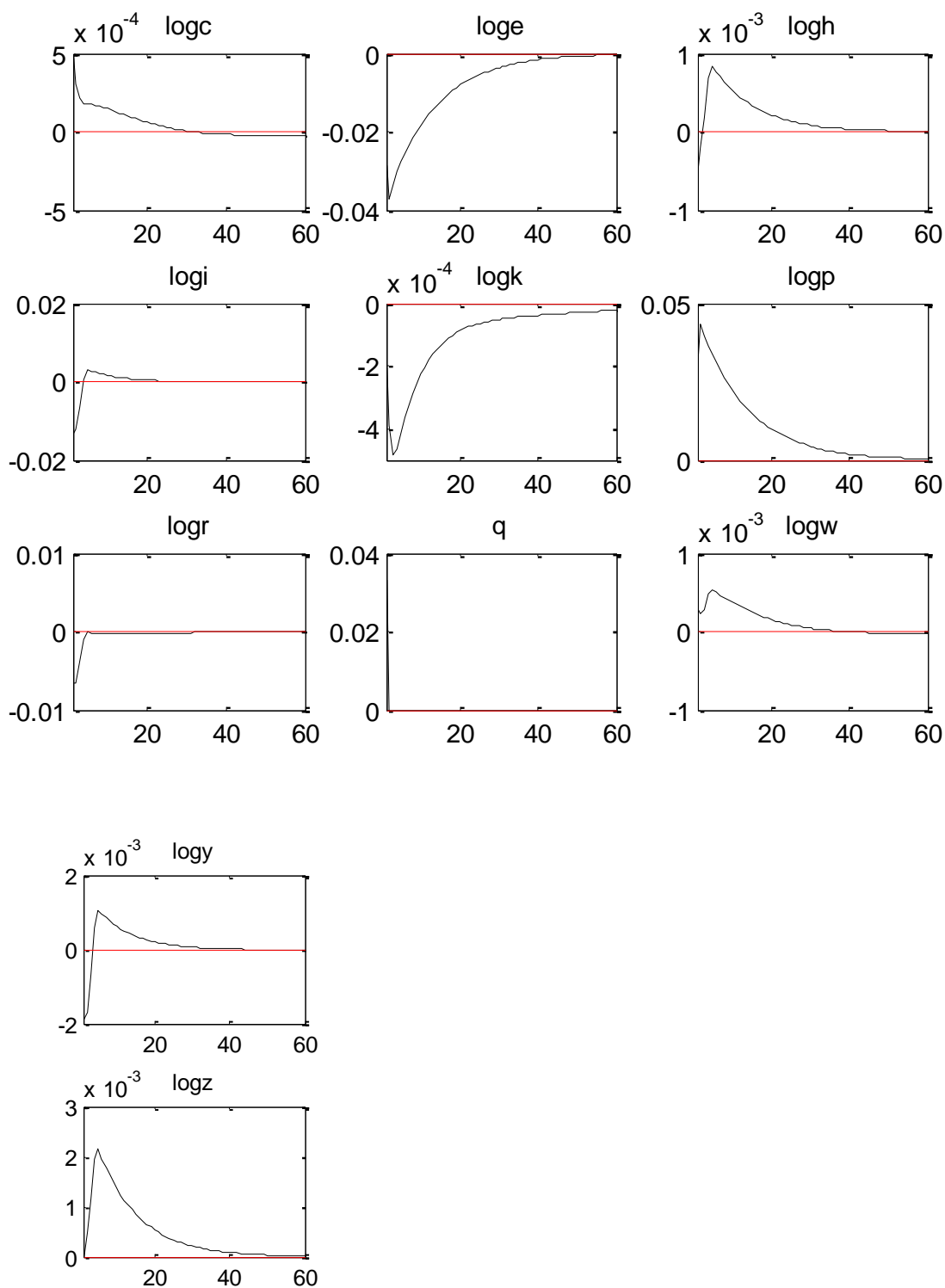
Figure 6**Impulse Responses to Energy Price Shock with Positive Spillover in the Late****Period for Case 3: (LE)K**

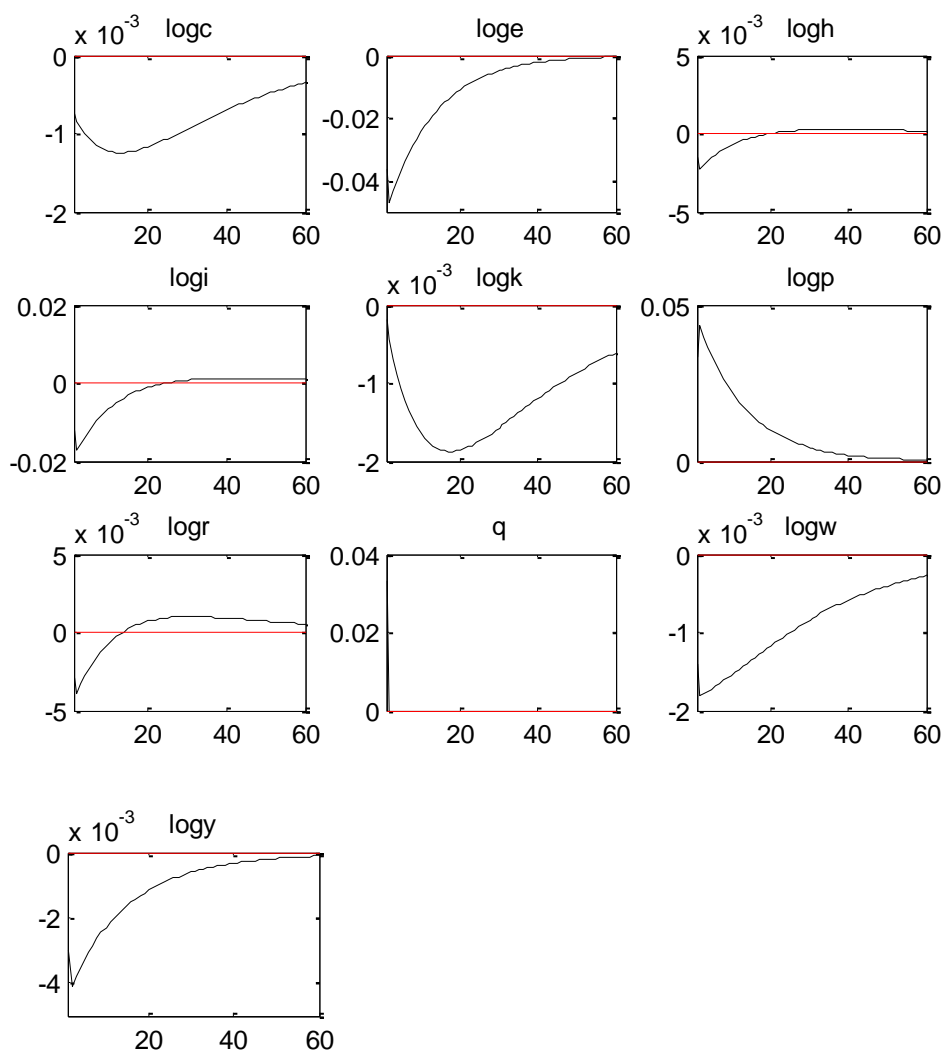
Figure 7**Impulse Responses to Energy Price Shock with No Spillover in the Late Period****(Modified) for Case 1: (KE)L**Note: Setting $\gamma_2 = [0, 0, 0, 0]$ in the late period.

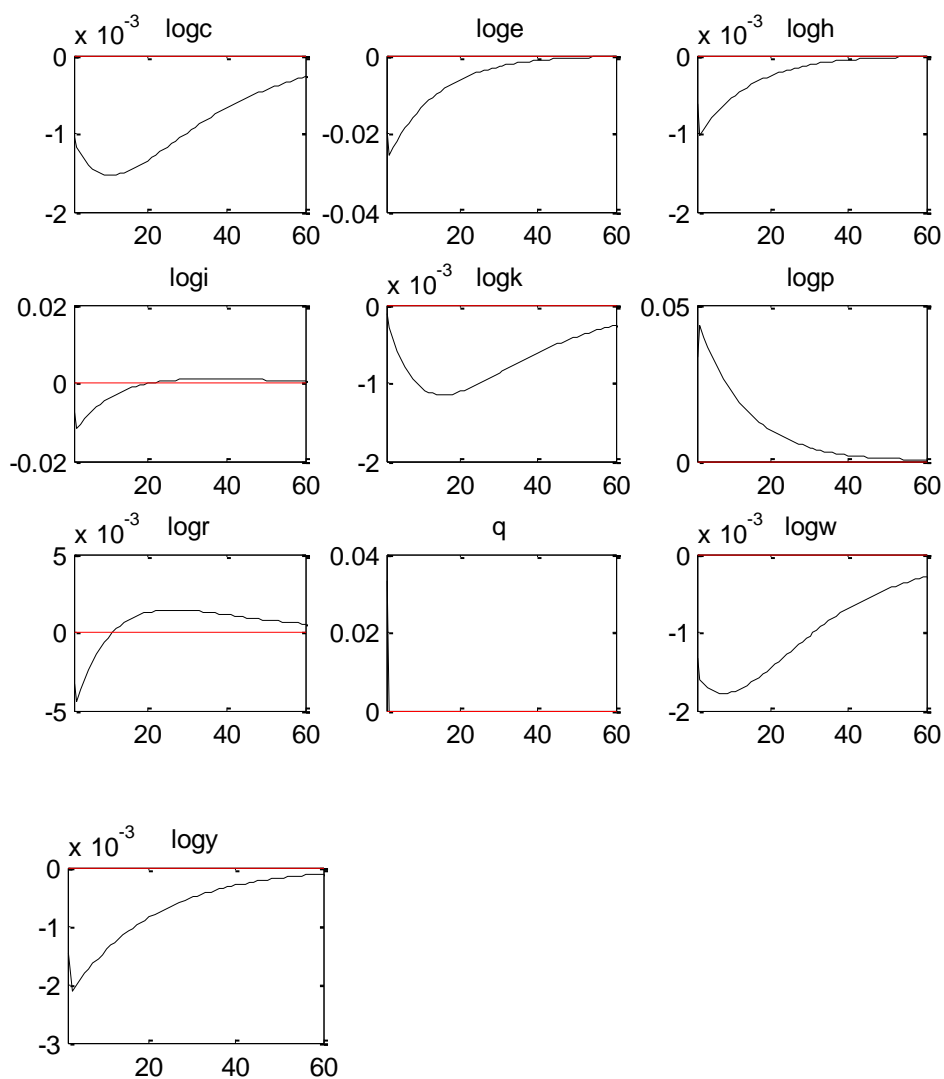
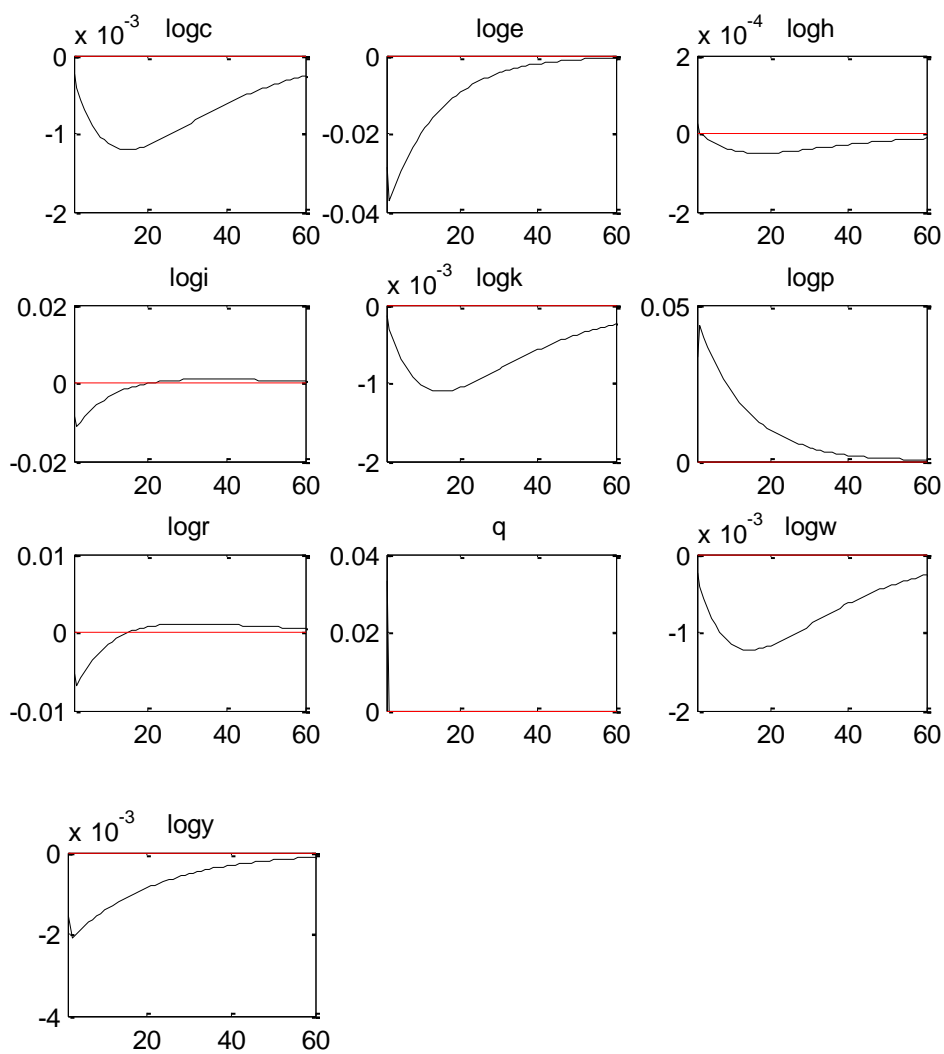
Figure 8**Impulse Responses to Energy Price Shock with No Spillover in the Late Period****(Modified) for Case 2: (KL)E**Note: Setting $\gamma_2 = [0, 0, 0, 0]$ in the late period.

Figure 9**Impulse Responses to Energy Price Shock with No Spillover in the Late Period****(Modified) for Case 3: (LE)K**

Note: Setting $\gamma_2 = [0, 0, 0, 0]$ in the late period.

Appendix C: Codes

Note: The following Dynare codes are all modifications from Dhawan, Jeske and Silos (2009). Except the parameters calibrated from this paper, all other parameter values are from Dhawan, Jeske and Silos (2009). All initial values are from Dhawan, Jeske and Silos (2009).

Code 1: case1 early baseline

```
// This code is a modification from Dhawan&Jeske&Silos(2009)
// case1
// early period, spillover with higher volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
// alpha1, alpha2, a are calibrated from paper,
// other parameter values are from Dhawan&Jeske&Silos(2010)
parameters      beta alpha1 alpha2  a
                logpbar logzbar
                phi eta delta
                rhoz zq1 zq2 zq3 zq4 sigmaz1 pp2 rhop sigmap;
beta      = 0.9900000000000000 ;
alpha1    = 0.3056000000000000 ;
alpha2    = 0.0544 ;
a         = 1.2354 ;
logpbar   = 1.0000000000000000 ;
logzbar   = 1.0000000000000000 ;
phi       = 0.3376344086021510 ;
eta       = 0.9959458779155580 ;
delta     = 0.0153671949757884 ;
rhoz      = 0.9120000000000000 ;
zq1       = -0.0920000000000000 ; // early period
zq2       = -0.0260000000000000 ; // early period
zq3       = -0.0670000000000000 ; // early period
zq4       = -0.0460000000000000 ; // early period
sigmaz1   = sqrt(0.0000545) ; // higher volatility of TFP
rhop      = 0.9210000000000000 ;
pp2       = 0.3750000000000000 ;
sigmap    = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) = (1-alpha1-alpha2)*exp(logy-logh);
// (3) Definition of interest rate
```

```

exp(logr) = alpha1* exp(logy-logk(-1));
// (4) energy price = MPE
exp(logp) = alpha2* exp(logy-loge);
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*a*exp(alpha1*logk(-1))*exp(alpha2*loge)*exp((1-alpha1-alpha2)*logh);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsz+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz1*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
// initial values are from Dhawan&Jeske&Silos(2009)
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 2: case1 late baseline

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// late period, low spillover with lower volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
// alpha1, alpha2, a are calibrated from paper,
// other parameter values are from Dhawan&Jeske&Silos(2010)
parameters      beta alpha1 alpha2  a
                logpbar logzbar
                phi eta delta
                rhoz zq1 zq2 zq3 zq4 sigmaz2 pp2 rhop sigmap;
beta            = 0.9900000000000000 ;
alpha1          = 0.3056000000000000 ;
alpha2          = 0.0544 ;
a               = 1.2354 ;
logpbar         = 1.0000000000000000 ;
logzbar         = 1.0000000000000000 ;
phi             = 0.3376344086021510 ;
eta             = 0.9959458779155580 ;
delta           = 0.0153671949757884 ;
rhoz            = 0.9120000000000000 ;
zq1             = 0.0150000000000000 ; // late period
zq2             = 0.0210000000000000 ; // late period
zq3             = 0.0270000000000000 ; // late period
zq4             = 0.0110000000000000 ; // late period
sigmaz2         = sqrt(0.0000208) ; // lower volatility of TFP
rhop            = 0.9210000000000000 ;
pp2             = 0.3750000000000000 ;
sigmap          = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) = (1-alpha1-alpha2)*exp(logy-logh);
// (3) Definition of interest rate
exp(logr) = alpha1* exp(logy-logk(-1));
// (4) energy price = MPE
exp(logp) = alpha2* exp(logy-loge);
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =

```

```

exp(logz)*a*exp(alpha1*logk(-1))*exp(alpha2*loge)*exp((1-alpha1-alpha2)*logh);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz2*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 3 case1 early spillover only

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// early period, spillover with average volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters beta alpha1 alpha2 a

```

```

logpbar logzbar
phi eta delta
rhoz zq1 zq2 zq3 zq4 sigmaz pp2 rhop sigmap;
beta = 0.9900000000000000 ;
alpha1 = 0.3056000000000000 ;
alpha2 = 0.0544 ;
a = 1.2354 ;
logpbar = 1.0000000000000000 ;
logzbar = 1.0000000000000000 ;
phi = 0.3376344086021510 ;
eta = 0.9959458779155580 ;
delta = 0.0153671949757884 ;
rhoz = 0.9120000000000000 ;
zq1 = -0.0920000000000000 ; // early period
zq2 = -0.0260000000000000 ; // early period
zq3 = -0.0670000000000000 ; // early period
zq4 = -0.0460000000000000 ; // early period
sigmaz = sqrt(0.0000333) ;
rhop = 0.9210000000000000 ;
pp2 = 0.3750000000000000 ;
sigmap = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) = (1-alpha1-alpha2)*exp(logy-logh);
// (3) Definition of interest rate
exp(logr) = alpha1* exp(logy-logk(-1));
// (4) energy price = MPE
exp(logp) = alpha2* exp(logy-loge);
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*a*exp(alpha1*logk(-1))*exp(alpha2*loge)*exp((1-alpha1-alpha2)*logh);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);

```

```

// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 4 case1 late spillover only

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// late period, low spillover with average volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta alpha1 alpha2      a
                logpbar logzbar
                phi eta delta
                rhoz zq1 zq2 zq3 zq4 sigmaz pp2 rhop sigmap;
beta      = 0.9900000000000000 ;
alpha1    = 0.3056000000000000 ;
alpha2    = 0.0544 ;
a         = 1.2354 ;
logpbar   = 1.0000000000000000 ;
logzbar   = 1.0000000000000000 ;
phi       = 0.3376344086021510 ;

```

```

eta    = 0.9959458779155580 ;
delta  = 0.0153671949757884 ;
rhoz   = 0.9120000000000000 ;
zq1    = 0.0150000000000000 ; // late period
zq2    = 0.0210000000000000 ; // late period
zq3    = 0.0270000000000000 ; // late period
zq4    = 0.0110000000000000 ; // late period
sigmaz = sqrt(0.0000333) ; // average volatility of TFP
rhop   = 0.9210000000000000 ;
pp2    = 0.3750000000000000 ;
sigmap = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) = (1-alpha1-alpha2)*exp(logy-logh);
// (3) Definition of interest rate
exp(logr) = alpha1* exp(logy-logk(-1));
// (4) energy price = MPE
exp(logp) = alpha2* exp(logy-loge);
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*a*exp(alpha1*logk(-1))*exp(alpha2*loge)*exp((1-alpha1-alpha2)*logh);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*z_y(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc  = -0.20738977 ;
loge  = -2.84627709 ;
logh  = -1.20397280 ;
logi  = -1.62516014 ;
logk  = 2.55036010 ;
logp  = 0.00000000 ;

```

```

q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 5 case2 early baseline

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case2
// early period, spillover with higher volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta alpha1 x1 x2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz1 pp2 rhop sigmap;
beta = 0.9900000000000000 ;
alpha1 = 0.9950000000000000 ;
x1 =56.9543 ;
x2 = 0.0395 ;
a =-2.1338 ;
b =-0.828 ;
logpbar = 0.0000000000000000 ;
logzbar = 0.0000000000000000 ;
phi = 0.3376344086021510 ;
delta = 0.0153671949757884 ;
rhoz = 0.9120000000000000 ;
zq1 = -0.0920000000000000 ; // early period
zq2 = -0.0260000000000000 ; // early period
zq3 = -0.0670000000000000 ; // early period
zq4 = -0.0460000000000000 ; // early period
sigmaz1 = sqrt(0.0000545) ; // higher volatility of TFP

```

```

rhop    = 0.9210000000000000 ;
pp2     = 0.3750000000000000 ;
sigmap  = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x2*exp((a-1)*logh)/(alpha
1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x1*exp((a-1)*logk(-1))/(a
lpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (4) energy price = MPE
exp(logp) =
exp(logy)*(1-alpha1)*exp((b-1)*loge)/(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)
+(1-alpha1)*exp(b*loge));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*z y(-1)+szy*epsz+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz1*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc    = -0.20738977 ;
loge    = -2.84627709 ;
logh    = -1.20397280 ;
logi    = -1.62516014 ;
logk    = 2.55036010 ;
logp    = 0.00000000 ;
q       = 0.00000000 ;

```

```

logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul( periods=180000, drop=3200, irf=60, order=1, hp_filter=1600 );

```

Code 6 case2 late baseline

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case2
// late period, low spillover with lower volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta alpha1 x1 x2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz2 pp2 rhop sigmap;
beta           = 0.9900000000000000 ;
alpha1        = 0.9950000000000000 ;
x1            = 56.9543 ;
x2            = 0.0395 ;
a             = -2.1338 ;
b             = -0.828 ;
logpbar       = 0.0000000000000000 ;
logzbar       = 0.0000000000000000 ;
phi           = 0.3376344086021510 ;
delta         = 0.0153671949757884 ;
rhoz          = 0.9120000000000000 ;
zq1           = 0.0150000000000000 ; // late period
zq2           = 0.0210000000000000 ; // late period
zq3           = 0.0270000000000000 ; // late period
zq4           = 0.0110000000000000 ; // late period
sigmaz2       = sqrt(0.0000208) ; // lower volatility of TFP
rhop          = 0.9210000000000000 ;

```

```

pp2    = 0.3750000000000000 ;
sigmap = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x2*exp((a-1)*logh)/(alpha
1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*logc));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x1*exp((a-1)*logk(-1))/(a
lpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*logc));
// (4) energy price = MPE
exp(logp) =
exp(logy)*(1-alpha1)*exp((b-1)*logc)/(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)
+(1-alpha1)*exp(b*logc));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*logc))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+logc) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz2*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
logc = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;

```

```

logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 7 case2 early spillover only

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case2
// early period, spillover with average volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta alpha1 x1 x2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz pp2 rhop sigmap;
beta = 0.9900000000000000 ;
alpha1 = 0.9950000000000000 ;
x1 =56.9543 ;
x2 = 0.0395 ;
a =-2.1338 ;
b =-0.828 ;
logpbar = 0.0000000000000000 ;
logzbar = 0.0000000000000000 ;
phi = 0.3376344086021510 ;
delta = 0.0153671949757884 ;
rhoz = 0.9120000000000000 ;
zq1 = -0.0920000000000000 ; // early period
zq2 = -0.0260000000000000 ; // early period
zq3 = -0.0670000000000000 ; // early period
zq4 = -0.0460000000000000 ; // early period
sigmaz = sqrt(0.0000333) ; // average volatility of TFP
rhop = 0.9210000000000000 ;
pp2 = 0.3750000000000000 ;

```

```

sigmap      =  sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x2*exp((a-1)*logh)/(alpha
1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x1*exp((a-1)*logk(-1))/(a
lpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (4) energy price = MPE
exp(logp) =
exp(logy)*(1-alpha1)*exp((b-1)*loge)/(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)
+(1-alpha1)*exp(b*loge));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*z y(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;

```

```

logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 8 case2 late spillover only

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case2
// late period, low spillover with average volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta alpha1 x1 x2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz pp2 rhop sigmap;
beta           = 0.9900000000000000 ;
alpha1        = 0.9950000000000000 ;
x1            =56.9543 ;
x2           = 0.0395 ;
a             =-2.1338 ;
b            =-0.828 ;
logpbar       = 0.0000000000000000 ;
logzbar       = 0.0000000000000000 ;
phi          = 0.3376344086021510 ;
delta        = 0.0153671949757884 ;
rhoz         = 0.9120000000000000 ;
zq1          = 0.0150000000000000 ; // late period
zq2          = 0.0210000000000000 ; // late period
zq3          = 0.0270000000000000 ; // late period
zq4          = 0.0110000000000000 ; // late period
sigmaz       = sqrt(0.0000333) ; // average volatility of TFP
rhop         = 0.9210000000000000 ;
pp2          = 0.3750000000000000 ;
sigmap       = sqrt(0.001108) ;

```

```

model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x2*exp((a-1)*logh)/(alpha
1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x1*exp((a-1)*logk(-1))/(a
lpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (4) energy price = MPE
exp(logp) =
exp(logy)*(1-alpha1)*exp((b-1)*loge)/(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)
+(1-alpha1)*exp(b*loge));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;

```

```

logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul( periods=180000, drop=3200, irf=60, order=1, hp_filter=1600 );

```

Code 9 case3 early baseline

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case3
// early period, spillover with higher volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta x1 x2 alpha2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz1 pp2 rhop sigmap;
beta            = 0.9900000000000000 ;
x1              = 0.0218 ;
x2              = 152.64 ;
alpha2         = 0.939 ;
a               = -0.165 ;
b               = -2.5 ;
logpbar        = 0.0000000000000000 ;
logzbar        = 0.0000000000000000 ;
phi            = 0.3376344086021510 ;
delta          = 0.0153671949757884 ;
rhoz           = 0.9120000000000000 ;
zq1            = -0.0920000000000000 ; // early period
zq2            = -0.0260000000000000 ; // early period
zq3            = -0.0670000000000000 ; // early period
zq4            = -0.0460000000000000 ; // early period
sigmaz1        = sqrt(0.0000545) ;
rhop           = 0.9210000000000000 ;
pp2            = 0.3750000000000000 ;
sigmap         = sqrt(0.001108) ;
model;

```

```

// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*alpha2*exp((a-1)*logh
)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*x2*exp((b-1)*logk(-1))/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)
+x2*exp(b*logk(-1)));
// (4) energy price = MPE
exp(logp) =
exp(logy)*x1*(1-alpha2)*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*exp((a-1)*
loge)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz1*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;

```

```

end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul( periods=180000, drop=3200, irf=60, order=1, hp_filter=1600 );

```

Code 10 case3 late baseline

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case3
// late period, low spillover with lower volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta x1 x2 alpha2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz2 pp2 rhop sigmap;
beta           = 0.9900000000000000 ;
x1             = 0.0218 ;
x2            = 152.64 ;
alpha2        = 0.939 ;
a             = -0.165 ;
b            = -2.5 ;
logpbar       = 0.0000000000000000 ;
logzbar       = 0.0000000000000000 ;
phi          = 0.3376344086021510 ;
delta        = 0.0153671949757884 ;
rhoz         = 0.9120000000000000 ;
zq1          = 0.0150000000000000 ; // late period
zq2          = 0.0210000000000000 ; // late period
zq3          = 0.0270000000000000 ; // late period
zq4          = 0.0110000000000000 ; // late period
sigmaz2      = sqrt(0.0000208) ; // low volatility of TFP
rhop         = 0.9210000000000000 ;
pp2          = 0.3750000000000000 ;
sigmap       = sqrt(0.001108) ;
model;
// (1) Labor vs consumption

```

```

(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*logc))^(b/a-1)*alpha2*exp((a-1)*logh
)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*logc))^(b/a)+x2*exp(b*logk(-1)));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*x2*exp((b-1)*logk(-1))/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*logc))^(b/a
)+x2*exp(b*logk(-1)));
// (4) energy price = MPE
exp(logp) =
exp(logy)*x1*(1-alpha2)*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*logc))^(b/a-1)*exp((a-1)*
logc)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*logc))^(b/a)+x2*exp(b*logk(-1)));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*logc))^(b/a)+x2*exp(b*logk(-1)))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+logc) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz2*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
logc = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;

```

```

shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 11 case3 early spillover only

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case3
// early period, high spillover with average volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta x1 x2 alpha2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz pp2 rhop sigmap;
beta            = 0.9900000000000000 ;
x1              = 0.0218 ;
x2              = 152.64 ;
alpha2         = 0.939 ;
a               = -0.165 ;
b               = -2.5 ;
logpbar        = 0.0000000000000000 ;
logzbar        = 0.0000000000000000 ;
phi            = 0.3376344086021510 ;
delta          = 0.0153671949757884 ;
rhoz           = 0.9120000000000000 ;
zq1            = -0.0920000000000000 ; // early period
zq2            = -0.0260000000000000 ; // early period
zq3            = -0.0670000000000000 ; // early period
zq4            = -0.0460000000000000 ; // early period
sigmaz        = sqrt(0.0000333) ;
rhop           = 0.9210000000000000 ;
pp2           = 0.3750000000000000 ;
sigmap        = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);

```

```

// (2) Definition of wage
exp(logw) =
exp(logy)*x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*alpha2*exp((a-1)*logh
)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*x2*exp((b-1)*logk(-1))/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)
+x2*exp(b*logk(-1)));
// (4) energy price = MPE
exp(logp) =
exp(logy)*x1*(1-alpha2)*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*exp((a-1)*
loge)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*z y(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;

```

```

var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 12 case3 late spillover only

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case3
// late period, low spillover with average volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta x1 x2 alpha2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz pp2 rhop sigmap;
beta            = 0.9900000000000000 ;
x1              = 0.0218 ;
x2              = 152.64 ;
alpha2         = 0.939 ;
a               = -0.165 ;
b               = -2.5 ;
logpbar        = 0.0000000000000000 ;
logzbar        = 0.0000000000000000 ;
phi            = 0.3376344086021510 ;
delta          = 0.0153671949757884 ;
rhoz           = 0.9120000000000000 ;
zq1            = 0.0150000000000000 ; // late period
zq2            = 0.0210000000000000 ; // late period
zq3            = 0.0270000000000000 ; // late period
zq4            = 0.0110000000000000 ; // late period
sigmaz        = sqrt(0.0000333) ; // average volatility of TFP
rhop           = 0.9210000000000000 ;
pp2            = 0.3750000000000000 ;
sigmap         = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage

```

```

exp(logw) =
exp(logy)*x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*alpha2*exp((a-1)*logh
)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*x2*exp((b-1)*logk(-1))/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)
+x2*exp(b*logk(-1)));
// (4) energy price = MPE
exp(logp) =
exp(logy)*x1*(1-alpha2)*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*exp((a-1)*
loge)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zzy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;

```

```

stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 13 case1 late modified

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// late period, no spillover with lower volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta alpha1 alpha2      a
                logpbar logzbar
                phi eta delta
                rhoz zq1 zq2 zq3 zq4 sigmaz2 pp2 rhop sigmap;
beta      = 0.9900000000000000 ;
alpha1    = 0.3056000000000000 ;
alpha2    = 0.0544 ;
a         = 1.2354 ;
logpbar   = 1.0000000000000000 ;
logzbar   = 1.0000000000000000 ;
phi       = 0.3376344086021510 ;
eta       = 0.9959458779155580 ;
delta     = 0.0153671949757884 ;
rhoz      = 0.9120000000000000 ;
zq1       = 0.0000000000000000 ; // late period
zq2       = 0.0000000000000000 ; // late period
zq3       = 0.0000000000000000 ; // late period
zq4       = 0.0000000000000000 ; // late period
sigmaz2   = sqrt(0.0000208) ; // lower volatility of TFP
rhop      = 0.9210000000000000 ;
pp2       = 0.3750000000000000 ;
sigmap    = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) = (1-alpha1-alpha2)*exp(logy-logh);
// (3) Definition of interest rate
exp(logr) = alpha1* exp(logy-logk(-1));

```

```

// (4) energy price = MPE
exp(logp) = alpha2* exp(logy-loge);
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*a*exp(alpha1*logk(-1))*exp(alpha2*loge)*exp((1-alpha1-alpha2)*logh);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz2*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 14 case2 late modified

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case2
// late period, no spillover with lower volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta alpha1 x1 x2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz2 pp2 rhop sigmap;
beta      = 0.9900000000000000 ;
alpha1    = 0.9950000000000000 ;
x1        =56.9543 ;
x2        = 0.0395 ;
a         =-2.1338 ;
b         =-0.828 ;
logpbar   = 0.0000000000000000 ;
logzbar   = 0.0000000000000000 ;
phi       = 0.3376344086021510 ;
delta     = 0.0153671949757884 ;
rhoz      = 0.9120000000000000 ;
zq1       = 0.0000000000000000 ; // late period
zq2       = 0.0000000000000000 ; // late period
zq3       = 0.0000000000000000 ; // late period
zq4       = 0.0000000000000000 ; // late period
sigmaz2   = sqrt(0.0000208) ; // lower volatility of TFP
rhop      = 0.9210000000000000 ;
pp2       = 0.3750000000000000 ;
sigmap    = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x2*exp((a-1)*logh)/(alpha
1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a-1)*x1*exp((a-1)*logk(-1))/(a
lpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge));
// (4) energy price = MPE
exp(logp) =

```

```

exp(logy)*(1-alpha1)*exp((b-1)*loge)/(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)
+(1-alpha1)*exp(b*loge));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(alpha1*(x1*exp(a*logk(-1))+x2*exp(a*logh))^(b/a)+(1-alpha1)*exp(b*loge))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz2*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

Code 15 case3 late modified

```

// This code is a modification from Dhawan&Jeske&Silos(2009)
// case3
// late period, no spillover with lower volatility of TFP
//periods 55000;
var logc loge logh logi logk logp logr q logw logy logz;
varexo epsz epsp;
parameters      beta x1 x2 alpha2  a b
                logpbar logzbar
                phi delta
                rhoz zq1 zq2 zq3 zq4 sigmaz2 pp2 rhop sigmap;
beta           = 0.9900000000000000 ;
x1             = 0.0218 ;
x2            = 152.64 ;
alpha2        = 0.939 ;
a             = -0.165 ;
b            = -2.5 ;
logpbar       = 0.0000000000000000 ;
logzbar       = 0.0000000000000000 ;
phi           = 0.3376344086021510 ;
delta         = 0.0153671949757884 ;
rhoz          = 0.9120000000000000 ;
zq1           = 0.0000000000000000 ; // late period
zq2           = 0.0000000000000000 ; // late period
zq3           = 0.0000000000000000 ; // late period
zq4           = 0.0000000000000000 ; // late period
sigmaz2       = sqrt(0.0000208) ; // low volatility of TFP
rhop          = 0.9210000000000000 ;
pp2           = 0.3750000000000000 ;
sigmap        = sqrt(0.001108) ;
model;
// (1) Labor vs consumption
(1-phi)/(1-exp(logh)) = phi*exp(logw-logc);
// (2) Definition of wage
exp(logw) =
exp(logy)*x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*alpha2*exp((a-1)*logh)
)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (3) Definition of interest rate
exp(logr) =
exp(logy)*x2*exp((b-1)*logk(-1))/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)
+x2*exp(b*logk(-1)));
// (4) energy price = MPE
exp(logp) =

```

```

exp(logy)*x1*(1-alpha2)*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a-1)*exp((a-1)*
loge)/(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)));
// (5) Euler equation
1 = beta*exp(logc-logc(+1))*(1+exp(logr(+1))-delta);
// (6) definition of output
exp(logy) =
exp(logz)*(x1*(alpha2*exp(a*logh)+(1-alpha2)*exp(a*loge))^(b/a)+x2*exp(b*logk(-1)))^(1
/b);
// (7) definition of investment
exp(logi) = exp(logk)-(1-delta)*exp(logk(-1));
// (8) resource constraint
exp(logc)+exp(logi)+exp(logp+loge) = exp(logy);
// (9) TFP process
// zy=perzy*zzy(-1)+szy*epsy+zqsum*(q(-1)+q(-2)+q(-3)+q(-4));
logz=rhoz*logz(-1)+sigmaz2*epsz+zq1*q(-1)+zq2*q(-2)+zq3*q(-3)+zq4*q(-4);
// (10) Price process
logp=rhop*logp(-1)+q+pp2*q(-1);
// (11) Need auxilliary variable q, because we can't use lagged exogenous shocks in Dynare
q=sigmap*epsp;
end;
initval;
logc = -0.20738977 ;
loge = -2.84627709 ;
logh = -1.20397280 ;
logi = -1.62516014 ;
logk = 2.55036010 ;
logp = 0.00000000 ;
q = 0.00000000 ;
logr = -3.67032446 ;
logw = 0.82313915 ;
logy = 0.06545345 ;
logz = 0.00000000 ;
end;
shocks;
var epsz;
stderr 1;
var epsp;
stderr 1;
end;
steady;
check;
stoch_simul(periods=180000,drop=3200,irf=60,order=1,hp_filter=1600);

```

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