CAHIER DE RECHERCHE #2307E
Département de science économique
Faculté des sciences sociales
Université d'Ottawa

WORKING PAPER #2307E
Department of Economics
Faculty of Social Sciences
University of Ottawa

# Risk Sharing in a Dual Labor Market[*]

Jonathan Créchet[†]

October 2023

---

[*] I am grateful to Baris Kaymak, Ronald Wolthoff, Immo Schott, Pierre Brochu, and Michel Poitevin for their extensive advice. I also thank Louphou Coulibaly, Gabriela Galassi, Jason Garred, Adam Lavecchia, Vikram Manjunath, Myra Mohnen, Louis-Phillipe Morin, and participants at seminars and conferences including the Carleton Macro-Finance Workshop, the Annual Conference of the Canadian Macroeconomic Study Group, and the European Search-and-Matching Conference. Finally, I thank the Social Sciences and Humanities Research Council of Canada for funding this research and the CIREQ for its financial support during the writing of my Ph.D. thesis. All errors are mine.

[†] Department of Economics, University of Ottawa, 120 University Private, Ottawa, Ontario, Canada, K1N 6N5; e-mail: jcrechet@uottawa.ca.

# 1 Introduction

Labor-market duality, defined as a coexistence of permanent, open-ended jobs with strict firing restrictions and temporary, fixed-term jobs with a large incidence in worker flows, is a prominent feature of Continental European economies.[1] Labor-market reforms of employment protection legislation have been high on the political agenda of European countries in the past decades, fueling a continued interest of academics. A critical question is to understand the trade-offs underlying the sorting of firms and workers into permanent and temporary contracts and the economic mechanisms leading to the coexistence of the two types of jobs. This question is key for evaluating the substitutability between these jobs and, in turn, for studying the large variations in the employment share of temporary jobs across countries and over time.

Yet, the answer to this question is not fully understood. The macroeconomic literature on dual labor markets has built on the search-and-matching model of Mortensen and Pissarides (1994) (the MP model). A common approach to introducing a distinction between permanent and temporary jobs in this framework is to assume contracts with different firing costs that capture regulatory constraints on layoffs. However, in the standard framework with risk neutrality and no contractual friction, firing costs incur efficiency losses for agents starting an employment relationship, and workers and firms always jointly prefer to sign a temporary contract rather than a permanent one. Hence, this framework has no rationale for the coexistence of permanent and temporary jobs unless one imposes ad hoc restrictions on structural (policy-invariant) parameters or the agents' contract space. Such an approach has been largely dominant in the macroeconomic literature on dual labor markets.

This paper's contribution is to study a search-and-matching model of a dual labor market with risk aversion and long-term employment contracts. In this framework, risk-averse workers and risk-neutral employers agree on contracts subject to limited commitment (Thomas and Worrall (1988), Thomas and Worrall (2007)) and featuring endogenous separations (Spear and Wang (2005), Wang (2011)). The contract terms include a choice between a permanent and a temporary contract, defined as contract *types* with different firing costs. Employers want to promise wage and employment stability in exchange for a lower wage, but their commitment ability is limited. In this context, firing costs serve as a commitment device allowing for the provision of additional insurance, as shown by Karabay and McLaren (2011). However, this additional insurance comes with lower expected revenues due to labor retention and costly

---

[1] In France and Spain, two countries widely considered as epitomes of labor-market duality in Europe, temporary contracts account for the vast majority of employment inflows (Cahuc, Charlot, and Malherbet (2016)) and for a significant share of unemployment cyclical fluctuations (Silva and Vázquez-Grenno (2013), Hairault, Le Barbanchon, and Sopraseuth (2015)). In the European Union, the prevalence of temporary employment has dramatically increased among the youths over the long run, representing as much as half of the total employment of individuals aged 15-24 in the late 2010s (versus 24% in 1985, OECD (2023)).

separations. When choosing between offering a permanent or a temporary contract, the employer trades off the gains from commitment versus flexibility.

The contracting problem is embedded into a frictional labor market with free entry of vacancies, bargaining over contracts, and heterogeneous match quality. In a steady-state equilibrium, permanent and temporary jobs potentially coexist as a result of optimal contracting combined with heterogeneity in match quality. The insurance considerations dominate over the need for flexibility as the match quality is high: a permanent contract is optimal for high-productivity matches, whereas a temporary contract is optimal elsewhere. It follows that with a sufficiently dispersed match-quality distribution, employment inflows are divided between permanent and temporary jobs, and so are the equilibrium employment stocks—even in the absence of regulation on temporary contracts. The combination of limited commitment and endogenous separations that characterize the contractual environment yields this result. Under full commitment, there are no efficiency gains associated with firing costs, and all contracts should be temporary. However, the limited-commitment assumption cannot explain labor-market duality alone. Without endogenous separations, i.e., contracts with incentive-compatibility constraints that can be satisfied in any history of shocks as in the standard model of Thomas and Worrall (1988), firing costs are a "pure" commitment device coming with no efficiency losses: all contracts should be permanent.

A quantitative evaluation of the model examines its implications for the effect of employment protection legislation—firing costs and the regulation of temporary contracts—on steady-state aggregate labor-market outcomes. The model is calibrated to the U.S. labor market, taken as a benchmark laissez-faire economy. Plausible and parsimonious variations in the institutional environment of the model can replicate the employment share of temporary jobs in France and account for most of the difference in this employment share between France and Spain. The model generates plausible (untargeted) equilibrium flows, validating the main mechanism. Temporary contracts account for the substantial majority of employment inflows and outflows, as seen in the two latter countries (Cahuc et al. (2016)).

The model implies that temporary contracts crowd out permanent jobs without employment gains, in line with the most recent empirical evidence (García-Pérez, Marinescu, and Vall Castello (2019), Daruich, Di Addario, and Saggio (2023), and Cahuc, Carry, Malherbet, and Martins (2022)). I propose a decomposition of comparative statics quantifying job creation and job substitution effects (of permanent jobs) associated with temporary contracts. Promoting these contracts induces large substitution flows and mild job creation effects. The decomposition also shows that temporary jobs imply substantial reallocation of labor and composition changes resulting in higher employment outflows. The net result is an decrease in the prevalence of permanent employment but no decline in unemployment, consistent with European trends of past decades.

**Literature.** This work is primarily related to the literature examining the combined effect of temporary contracts and firing costs in labor markets with search frictions. The MP model—the dominant framework for analyzing dual labor markets—has no intrinsic rationale for employment protection.[2] As such, a common approach in this literature has been to impose exogenous restrictions on the agents' choice set, which is sensible for questions related to the spread of temporary jobs in Europe (e.g., Cahuc and Postel-Vinay (2002), Blanchard and Landier (2002), Bentolila, Cahuc, Dolado, and Le Barbanchon (2012), Faccini (2014)). However, by construction, this approach leaves open the questions related to the substitutability of permanent and temporary contracts or the sorting into the two types of jobs.

This limitation has motivated models with equilibrium mechanisms leading to the coexistence of permanent and temporary jobs. Some papers assume exogenous differences in structural parameters across contract types (e.g., Alonso-Borrego, Fernández-Villaverde, and Galdón-Sánchez (2005) and Caggese and Cuñat (2008)).[3] In addition, Berton and Garibaldi (2012) build a framework with segmented search between submarkets for permanent and temporary jobs and constant wages à la Hall (2005). In this setting, (risk-neutral) workers value a stable, permanent contract more than a temporary one since wages are constant.[4] On the other hand, employers make more profits with a temporary job. An indifference vacancy-posting condition implies positive job creation in both submarkets: employers trade off higher profits of an (occupied) temporary position with the higher job-filling rate of a (vacant) permanent position.

Moreover, Cahuc, Charlot, and Malherbet (2016) introduce a model with heterogeneity in the exogenous duration of production opportunities across jobs. Both the contract type *and* the duration of temporary contracts are choice variables. Importantly, Cahuc et al. (2016) assume that temporary contracts cannot be destroyed before the agreed term, reflecting legislation in several European countries, where temporary contracts have penalties for premature termination.[5] Then, when choosing the contract type, the agents trade off the firing costs of permanent contracts and the costs from inefficient retention of a match in a temporary contract.[6]

---

[2] Since the agents bargain over the total surplus of matches, both the employer and the worker are worse off upon hiring in the presence of non-transferable firing costs. Firing costs induce anticipations of future separation costs passing through profits *and* wages. Moreover, as Lazear (1990) shows, severance payments are neutral when agents are risk-neutral and wages are flexible.

[3] In addition to these, Cao, Shao, and Silos (2011) and Franceschin (2020) propose models with on-the-job search and non-contractible search intensity. Permanent workers exert relatively low search effort in equilibrium, implying endogenous sorting between the two types of jobs.

[4] Berton and Garibaldi (2012) assume that permanent contracts cannot be endogenously terminated. An employer must wait for an exogenous separation before terminating an unproductive match, and the wage must remain constant.

[5] Cahuc et al. (2016) implicitly assume that temporary contracts have infinite premature termination costs.

[6] It follows that in the presence of small transaction costs, permanent contracts are optimal in matches with a stable enough productivity level (and temporary contracts are optimal otherwise). In the absence of transaction

The main innovation of the present paper to the literature on dual labor markets is the introduction of risk aversion and history-contingent employment contracts. In Berton and Garibaldi (2012), wage rigidity is the key assumption leading to the coexistence of heterogeneous contracts. In Cahuc et al. (2016), it is the assumption that temporary contracts are subject to high enough premature termination penalties. It is clear that these assumptions are relevant in regard to existing legislation. Yet, the current literature still lacks explanations of labor-market duality that encompass a variety of institutional settings. The main objective of my paper is to fill this gap. I consider contracts with a large action space, which allows for assuming flexible wages and for considering a parsimonious, generic approach to modeling the policy environment.

The idea that a full understanding of the effect of employment protection in a search-and-matching model requires a departure from linear utility goes back to Pissarides (2001). The literature has been mostly interested in the implications of market incompleteness for the effect of severance payments (Alvarez and Veracierto (2001), Fella and Tyson (2013), Cozzi and Fella (2016), Lalé (2019), Dolado, Lalé, and Siassi (2021)). The main similarity of my paper with this literature is the introduction of market incompleteness in the MP framework. Nonetheless, none of these papers examines the coexistence of permanent and temporary contracts, nor provides rationales for the presence of non-transferable firing costs, an important component of employment protection legislation (and a potential source of efficiency gains in the present framework).

In addition, I borrow the idea from Karabay and McLaren (2011) of firing costs as a commitment device. Similar to the latter, I consider a labor market with search frictions and limited commitment. However, Karabay and McLaren (2011) assume an exogenous population of firms, no heterogeneity, full bargaining power to the employer, and do not distinguish between permanent and temporary contracts. I build a model of a dual labor market with free entry of firms, heterogeneity in match quality, and positive bargaining power for workers and firms. Also, I consider contracts with history-contingent plans for separation as in Spear and Wang (2005) and Wang (2011), showing that this is the combined effect of limited commitment and endogenous separations that is key to the trade off between different contract types. Moreover, I propose a novel quantitative analysis of a labor market with limited commitment and firing costs studying cross-country outcomes and the effect of employment protection reforms. This framework is used to quantify the general equilibrium welfare effects of firing costs, whereas the results in Karabay and McLaren (2011) are of analytical nature.

Lastly, this paper is related to the literature studying dynamic employment contracts under limited commitment (e.g., Thomas and Worrall (1988), Thomas and Worrall (2007), Rudanko

---

costs, it would be optimal, at the time of hiring, to agree on a sequence of arbitrarily short temporary contracts until a negative productivity shock occurs, thereby avoiding the need for a permanent contract.

(2009)) and dynamic contracts with endogenous employment separations (e.g., Spear and Wang (2005), Wang (2011), Wang and Yang (2015a), Wang and Yang (2015b), Wang and Yang (2019)). In terms of modeling, it is closely related to Wang and Yang (2019), analyzing contracts under limited commitment and on-the-job search. In the latter, separations are triggered by stochastic job offers instead of idiosyncratic productivity shocks, as in the present paper and which is at the source of the commitment-flexibility trade off explaining the coexistence of contracts. More generally, and to the best of my knowledge, the latter body of work does not study labor-market duality and only one paper examines firing costs, Wang (2013).[7] In this paper, the relevant friction is moral hazard, and firing costs are associated with lower workers' welfare in equilibrium. In contrast, my model with limited commitment implies that firing costs potentially create welfare gains for workers.

## 2 Model

### 2.1 Environment

**Preferences and technology.** Time is discrete with an infinite horizon and indexed by $t \geq 0$. The economy is populated with infinitely lived identical workers and identical employers. Workers are risk averse and employers risk neutral. The population of workers is constant and normalized to one, and the population of employers is endogenous. The expected lifetime utility of a worker in period $t \geq 0$ is given by

$$E_t\left[\sum_{\tau=t}^{\infty} \beta^{(\tau-t)} u(c_\tau)\right], \tag{1}$$

where $E_t$ denotes the expectation conditional on the information at time $t$, $\beta \in (0,1)$ is a discount factor, $u(.)$ is a period utility function and $c_t \geq 0$ is consumption at time. The period utility function is continuously differentiable, has $u'(c) > 0$ and $u''(c) < 0$ for all $c \geq 0$. Workers cannot save. In each period, they consume their entire income, and they are endowed with one indivisible unit of time that can be allocated to labor. Employers have access to a linear production technology using labor as the only input and with the consumption good as output.

The amount of output produced by a worker and a firm in a match at time $t$ is $y_t = z_t \times x$. The component $x \geq 0$ represents the match permanent productivity component, and is referred to as the *match quality*. The value of this component is randomly drawn at the beginning of the match (see below) and remains constant throughout. The variable $z_t \geq 0$ is called the match-output *stochastic component*, assumed to follow a simple first-order Markov process with support $Z = [0, \bar{z}]$, with $\bar{z} > 0$. As in Mortensen and Pissarides (1994), assume that any new

---

[7] In addition, Wang and Yang (2015a) examines the effect of unemployment insurance.

match starts with $z_t = \overline{z}$; in subsequent periods, a productivity shocks occurs with probability $\lambda$ and a new value for $z$ is drawn from a distribution with cdf. $G$, and pdf. $g$. In what follows, denote by $G_z(.|z)$ the cumulative distribution of the next-period shock $z'$ given current value $z$, when this allows for greater clarity: $G_z(z'|z) \equiv (1-\lambda)\mathcal{I}(z' \leq z) + \lambda G(z')$ for all $z \in Z$, where $\mathcal{I}$ is the indicator function taking the value of one when the associated expression is true (zero otherwise).

**Search and matching.** The labor market features search frictions. Workers are either unemployed or employed. Employers can operate production units ("jobs") that are either vacant or occupied by a worker. An unemployed worker receives income $b > 0$ in each period, and the firm with a vacant job (a vacancy) pays posting costs $\kappa > 0$ per period. I denote by $U$ the expected lifetime utility of an unemployed worker and by $\mathcal{J}_0 \geq 0$ the expected discounted profits of an employer with a vacant job, both determined in equilibrium.

Search is random, and matching is stochastic (e.g., Pissarides (2000)): unemployed workers and employers with a vacancy enter in contact randomly and then observe their potential match quality. Specifically, the number of contacts between unemployed workers and vacancies in period $t \geq 0$ is equal to $m(u_t, v_t)$, where $u_t \geq 0$ is the number of unemployed workers, and $v_t \geq 0$ is the number of vacancies. The function $m$ represents a standard matching function with constant returns to scale and is continuous, strictly increasing and strictly concave in each argument. The contact rate of a worker is $p(\theta_t) = m(1, \theta_t)$, where $\theta_t = v_t/u_t$ denotes the labor-market tightness at $t \geq 0$. The contact rate of the a firm is $q(\theta_t) = m(1/\theta_1, 1)$. Upon contact, the worker and the firm draw a potential match quality $x$ from a distribution with cdf. $G_x$, pdf. $g_x$, and support $X = [0, \overline{x}]$; assume $g_x$ positive and continuous over its support and let $b < \overline{x} \times \overline{z}$, ensuring the formation of a positive mass of jobs in equilibrium.

Upon contact, the worker and the employer agree on a possible employment contract, with the terms of this contract being set by bargaining (see below). The agents are allowed to set the contract terms *prior to* the revelation of the match quality $x$, that is, they have access to contracts specifying actions contingent on the possible realizations of $x$. In addition, a contract can specify a contingency plan of actions for all possible histories of the match stochastic component $z_t$. Finally, assume that exogenous match separations occur with probability $\delta$ in each period. Next, I describe in more detail the contracting environment.

**Contracting.** A contract specifies a contingency plan of non-negative wage payments (i.e., consumption for the worker) and separation probabilities for each possible history of the match-specific output. In addition and importantly, the contract design involves making choices over the contract *type*. A contract can be of type *permanent* or *temporary*. A permanent contract (PC) is subject to firing costs $F \geq 0$, paid by the employer upon an endogenous separation from a match. Exogenous separations are interpreted as quits and do not incur the payment of firing costs. A temporary contract (TC) is exempt from firing costs.[8] There are no severance payments: contracts do not include plans for payments after a separation.[9] After separation, the worker and the firm simply terminate their contract and do not further exchange.

Consistent with existing legislation, temporary contracts are subject to *hiring* and *duration* restrictions: in most countries, hiring on temporary contracts is only allowed in specific circumstances defined by law, and these contracts have a mandated maximum duration and a limited number of renewals. The hiring restrictions are captured by a parameter $\phi_0 \in [0,1]$, which represents the exogenous probability that the agents are *not* allowed to sign a TC at the start of a potential employment relationship. In other words, when an unemployed worker and a vacancy enter into contact in the labor market, they are exogenously allowed to choose between a PC and a TC with probability $1-\phi_0$; otherwise (with probability $\phi_0$), they are required to sign a PC if they wish to form an employment relationship. The particular value taken by $\phi_0$ is insignificant for the main results in the paper (in fact, the benchmark quantitative analysis assumes $\phi_0 = 0$, i.e., an absence of hiring restrictions). Considering hiring regulations is meant to conduct quantitative experiments.[10]

The duration restrictions are materialized by a parameter $\phi$, which is the exogenous probability that a TC is terminated and that the agents are required to sign a permanent contract if they wish to continue the employment relationship. In other words, when the duration restrictions apply (with probability $\phi$), the agents matched in a TC must either convert their current contract into a PC or separate. In such an environment, a TC specifies a contingency plan for separation probabilities (or, equivalently, for probabilities of conversion of the TC into a PC) in cases where the duration restrictions binds.[11]

---

[8] Assuming no firing costs for temporary contracts is a simplification that is common in the literature on dual labor markets. However, as explained in Cahuc et al. (2016), in several countries, the legislation makes it more costly to terminate a temporary contract before its term than a permanent contract. For instance, according to the French legislation, an early termination of a TC (i.e., before the term initially specified by the contract) initiated by the employer is in principle not allowed (except in special cases such as serious misconduct or force majeure). Nevertheless, when employers have information about the distribution of future states, they can in theory adjust the duration of a temporary job to minimize the costs incurred by negative shocks. The key distinction that this paper intends to capture is that a temporary contract presumably has relatively more flexibility and a higher risk of unemployment. A difference in firing costs can parsimoniously capture this distinction.

[9] This assumption is for tractability and is innocuous for the qualitative results, which hold as long as the employer has no access to contract with a complete set of history-contingent severance payments. This would be at odds with the assumption of limited commitment and would contradict empirical evidence suggesting that a minority of U.S. workers receive severance payments (Chetty (2008)). With a friction limiting the provision of severance plans, firing costs improve the employer's ability to provide insurance against income fluctuations in a context with limited commitment.

[10] In particular, this allows for conducting counterfactuals where the hiring regulation is tightened to explore the effect of labor-market reforms having eased the restrictions on temporary contracts in European countries over past decades, following e.g., Faccini (2014)

[11] This is in addition to wage payments and the plan for separation probabilities in cases where the regulation

It is assumed that the employer and the worker have limited ability to commit to the terms of their employment contract (Thomas and Worrall (1988)). The agents are allowed to renege on a contract and have incentives to do so when they can credibly threaten the other party to leave their match. In this context, a feasible contract must satisfy, in all histories such that the continuation of the contract is a possible outcome, a participation constraint for the worker and the employer, stating that both agents are better off in the contract than by taking their respective outside options. In this setting, firing costs associated with the PC constitute a commitment device that relaxes the employer's participation constraint in some states (Karabay and McLaren (2011)), an aspect that is key to the choice between a PC and a TC in this model.

The contract design can be decomposed into two steps. The first step specifies actions contingent on the match-quality realization, including deciding whether to form an employment relationship and choosing the contract type. The second step consists of setting history-contingent actions given the contract-type choice made in the first step (and given the match quality drawn upon hiring). I present below the contract definition backward, starting with the definition of a permanent and a temporary contract, leaving to the end of this subsection the presentation of the first-step problem.

A permanent contract consists of a list of history-contingent wage payments and separation rules, as per models of dynamic contracting with endogenous separations (e.g., Spear and Wang (2005), Wang (2011)). The contracting problem is analyzed in a recursive form which has a predetermined promised lifetime worker's utility value as a state variable summarizing the contract's past outcome (Green (1987), Spear and Srivastava (1987)). For a given match quality $x$, a permanent contract is a list

$$\sigma_P = \left\{ w_P(V,z), s_P(V,z,z'), \nu_P(V,z,z') : z \in Z_P,\ V \in \mathcal{V}_P(z),\ z' \in Z \right\}, \tag{2}$$

where $Z_P \subset Z$ is the set of values of the match-output stochastic component $z$ such that there exists a feasible PC, $V$ is the worker's current promised lifetime utility value, and $\mathcal{V}_P(z)$ is the set of expected lifetime utility values that can be delivered to the worker by a feasible PC given state $z \in Z_P$. The variables $Z_P$ and $\mathcal{V}_P$ are endogenous and determined in equilibrium.

Hence, in each attainable state, the contract specifies a wage payment $w_P(V,z) \geq 0$ and a set of actions contingent on the next-period's shock $z'$. These actions are described by: (i) a separation probability $s_p(V,z,z') \in [\delta,1]$ (recall that $\delta$ is the exogenous separation probability); (ii) expected continuation lifetime utility values delivered to the worker, such that $\nu_P(V,z,z') \in \mathcal{V}_P(z')$ for $z' \in Z_P$ and $s_P(V,z,z') < 1$; and $\nu_P(V,z,z') = U$ for $z' \in Z \setminus Z_P$ or $s_P(V,z,z') = 1$.[12]

---

does *not* bind, i.e., when there is no requirement to terminate the TC.

[12] When $z' \in Z \setminus Z_P$ (i.e., no continuation is feasible), $s_P(V,z,z') = 1$ and $\nu_P(V,z,z') = U$: the contract must end with probability one and the worker goes back to unemployment, obtaining the corresponding lifetime utility value.

Similarly to permanent contracts, a temporary contract specifies plans for wage payments and separation probabilities. However, in the case of a TC, these plans include actions contingent on situations where the exogenous duration restrictions bind (with probability $\phi$). Specifically, for a given match quality, the temporary contract is defined by

$$\sigma_T = \left\{ w_T(V,z),\, s_j(V,z,z'),\, \nu_j(V,z,z') : z \in Z_T,\, V \in \mathcal{V}_T(z),\, z' \in Z;\, j \in \{T,c\} \right\}, \tag{3}$$

where $Z_T$ and $\mathcal{V}_T(z)$ are defined analogously to their counterparts for the permanent contract. In each attainable state, the temporary contract specifies a wage payment $w_T(V,z)$ and a list of actions for the next period. The eventuality of an exogenous mandated termination of the TC is reflected upon: $s_T(V,z,z')$ and $\nu_T(V,z,z')$ describe the separation probability and continuation value in cases where the duration restrictions do not bind, whereas $s_c(V,z,z')$ and $\nu_c(V,z,z')$ are for actions contingent on cases where the TC is exogenously required to be converted into a PC (or terminated). The index $j \in \{T,c\}$ captures this distinction introduced by the exogenous duration restrictions.

The first step of the design of a contract sets a plan that is specified right after a worker and a firm enters into contact and just before observing the potential match quality $x$. This plan is called a *hiring rule*, defined as a list

$$\xi = \left\{ h_P(V_0,x), h_T(V_0,x), \nu_P(V_0,x), \nu_T(V_0,x) : V_0 \in \mathcal{V}_0,\, x \in X \right\}, \tag{4}$$

where $V_0$ represents the worker's initial expected lifetime utility in the potential match (or initial matching utility value) and $\mathcal{V}_0$ is the set of such values $V_0$ that can be delivered by a feasible contract, prior to observing the match quality. In equilibrium, I will let $V_0$ be determined by Nash bargaining given $\mathcal{V}_0$, the agents' outside options, and given the employer's expected matching profits generated by optimal contracting (see below).

Moreover, $h_P(x) \in [0,1]$ and $h_T(x) \in [0, 1-\phi_0]$ represent the probability of hiring in a permanent and temporary contract, respectively: hiring takes place with probability $h_P(x) + h_T(x) \leq 1$ given match quality $x$. For clarity but with abuse of notation, I denote the hiring worker's continuation values by $\nu_P$ and $\nu_T$ but these are to be distinguished from these appearing in definitions (2) and (3), which belong to the case of an ongoing match. These continuation values determine the worker's expected lifetime utility conditional on the match quality and the contract type, in cases where the match carries on beyond the first step (i.e., in cases where hiring takes place).

## 2.2 Contracting problem

This section presents the contracting problem prevailing in a labor market equilibrium. In equilibrium, the employer designs a Pareto efficient contract maximizing the expected lifetime discounted profits, subject to delivering to the worker an initial value $V_0 \in \mathcal{V}_0$ and to feasibility constraints. The analysis in this subsection and the following treats the agents' outside options (the unemployment utility value $U$ and the profits of a vacant job $\mathcal{J}_0$) as exogenous. Recall that these are determined in equilibrium (see subsection (2.4)), but treated as given from the perspective of the agents in a match agreeing on a contract. As before, the analysis proceeds backward decomposing the problem into two steps: I first analyze the design of permanent and temporary contracts conditional on the match quality and then an efficient hiring rule.

**Permanent contract.** Consider an employer with a permanent contract in a match with quality $x$. For clarity, let the dependence of the model's variable on $x$ be understood for now. This employer's expected lifetime discounted profit value is a solution to

$$\begin{aligned} J_P(V,z) = \max_{w_P,(s_P(z'),\nu_P(z'))_{z'\in Z}} & zx - w_P \\ & + \beta \int_{\underline{z}}^{\overline{z}} \Big[(1-s_P(z'))(J_P(\nu_P(z'),z') - \mathcal{J}_0)\Big] dG_z(z'|z) + \beta\mathcal{J}_0 + \chi(F,z) \end{aligned} \tag{5}$$

subject to

$$w_P \geq 0 \tag{6}$$

$$\chi(F,z) = \beta\Big[\int_{\underline{z}}^{\overline{z}} (s_P(z') - \delta) dG_z(z'|z)\Big](-F) \tag{7}$$

$$u(w_P) + \beta \int_{\underline{z}}^{\overline{z}} \Big[(1-s_P(z'))(\nu_P(z') - U)\Big] dG_z(z'|z) + \beta U = V \tag{8}$$

$$s_P(z') \in [\delta, 1], \text{ for } z' \in Z_P \text{ and } s_P(z') = 1, \text{ for } z' \in Z \setminus Z_P \tag{9}$$

$$\nu_P(z') \geq U, \text{ for } z' \in Z \text{ such that } s_P(z') < 1 \tag{10}$$

$$J_P(\nu_P(z'),z') \geq \mathcal{J}_0 - F, \text{ for } z' \in Z \text{ such that } s_P(z') < 1. \tag{11}$$

for all $z \in Z_P$ and all $V \in \mathcal{V}_P(z)$. The problem consists of setting actions to maximize the employer's profit function (5) under constraints (6)-(11). Constraint (6) comes from the non-negativity requirement for consumption and the absence of savings. In (7), $\chi(F,z)$ represents the discounted expected costs incurred by firing costs $F$ given the choice for separation probabilities. This reflects that exogenous separations do not have firing costs. Condition (8) represents the promise-keeping constraint, such that the contract continuation terms are consistent with the worker receiving predetermined promised lifetime utility $V$. The constraint (9) puts restrictions on the probabilities of separation. Equations (10) and (11) represent the worker's and the firm's sets of participation constraints, respectively. These require that in states in which continuation has a positive probability ($s_P < 1$), the worker and the firm are better off in the match than by taking their outside options. Note that firing costs enter not only in the profit equation (5) through the term $\chi(F,z)$ but also in the participation constraint of the employer.

**Temporary contract.** The employer's expected lifetime discounted profit value with a temporary contract is

$$J_T(V,z) = \max_{\tilde{\sigma}_T} zx - w_T + \beta \int_{\underline{z}}^{\overline{z}} \Big[ (1-\phi)(1-s_T(z'))(J_T(\nu_T(z'),z') - \mathcal{J}_0) \\ + \phi(1-s_c(z'))(J_P(\nu_c(z'),z') - \mathcal{J}_0) \Big] dG_z(z'|z) + \beta \mathcal{J}_0 \tag{12}$$

subject to

$$w_T \geq 0 \tag{13}$$

$$u(w_T) + \beta \int_{\underline{z}}^{\overline{z}} \Big[ (1-\phi)(1-s_T(z'))\big(\nu_T(z') - U\big) \\ + \phi(1-s_c(z'))\big(\nu_c(z') - U\big) \Big] dG_z(z'|z) + \beta U = V \tag{14}$$

$$s_T(z') \in [\delta,1], \text{ for } z' \in Z_T, \text{ and } s_T(z') = 1 \text{ for } z' \in Z \setminus Z_T \\ s_c(z') \in [\delta,1], \text{ for } z' \in Z_P, \text{ and } s_c(z') = 1 \text{ for } z' \in Z \setminus Z_P \tag{15}$$

and subject to the set of participation constraints

$$\nu_T(z') \geq U, \text{ for } z' \in Z \text{ such that } s_T(z') < 1 \\ \nu_c(z') \geq U, \text{ for } z' \in Z \text{ such that } s_c(z') < 1 \tag{16}$$

for the worker and

$$J_T(\nu_T(z'),z') \geq \mathcal{J}_0, \text{ for } z' \in Z \text{ such that } s_T(z') < 1 \\ J_P(\nu_c(z'),z') \geq \mathcal{J}_0, \text{ for } z' \in Z \text{ such that } s_c(z') < 1 \tag{17}$$

for the employer, for all $z \in Z_T$ and $V \in \mathcal{V}_T(z)$. The choice variables

$$\tilde{\sigma}_T \equiv \Big\{ w_T, s_j(z'), \nu_j(z') : j \in \{T,c\}, z' \in Z \Big\}$$

are set to maximize the profit (12) under constraints (13)-(17). Note, again, the dependence of the choice variables on $j \in \{T,c\}$, which reflects the distinct states induced by the exogenous (random) requirement to convert the TC into a PC.

**Hiring rule.** The first step of the equilibrium contracting problem designs the hiring rule given the maximized profit functions $J_P$ and $J_T$. It is set to maximize the employer's expected discounted profits prior to drawing the match quality, and subject to delivering initial lifetime utility value $V_0$ to the worker according to

$$\mathcal{J}(V_0) = \max_{\tilde{\xi}} \\ \int_{\underline{x}}^{\overline{x}} \Big[ h_P(x')(J_P(\nu_P(x'),\overline{z},x') - \mathcal{J}_0) + h_T(x')(J_T(\nu_T(x'),\overline{z},x') - \mathcal{J}_0) \Big] dG_x(x') + \beta \mathcal{J}_0, \tag{18}$$

subject to

$$\int_{\underline{x}}^{\overline{x}} \Big[ h_P(x')(\nu_P(x') - U) + h_T(x')(\nu_T(x') - U) \Big] dG_x(x') + \beta U = V_0 \tag{19}$$

$$h_P(x) \in [0,1] \text{ for } x \in X_P, \text{ and } h_p(x) = 0 \text{ for } x \in X \setminus X_P \tag{20}$$

$$h_T(x) \in [0, 1 - \phi_0] \text{ for } x \in X_T, \text{ and } h_T(x) = 0 \text{ for } x \in X \setminus X_T \tag{21}$$

$$h_P(x) + h_T(x) \le 1 \ \text{ for all } x \in X \tag{22}$$

$$\nu_j(x) \ge U \text{ for all } x \text{ s.t. } h_j(x) > 0, \ j \in \{P, T\} \tag{23}$$

$$J_j(\nu_j(x), \overline{z}, x) \ge \mathcal{J}_0 \text{ for all } x \text{ s.t. } h_j(x) > 0, \ j \in \{P, T\}, \tag{24}$$

for $V_0 \in \mathcal{V}_0$; $\tilde{\xi}$ is a list of choice variables for the hiring-step actions, in accordance with definition (4). Moreover, I denote by $X_P$ and $X_T$ the set of match quality realizations such that there exists a feasible permanent and temporary contract, respectively. Notice that the formulation of the problem makes explicit the dependence of the profit functions $J_P$ and $J_T$ on the match quality.

Hence, the hiring rule is set to maximize the profits (18) under the constraint that the contract delivers initial expected lifetime utility $V_0$ to the worker according to (19) and under the participation constraints (23) and (24). Finally, (20) to (22) specifies the admissible probabilities, reflecting the hiring regulation represented by the parameter $\phi_0$. The solution to this step of the contracting problem describes the choice between a permanent and temporary contract as taking place in equilibrium.

## 2.3 Optimal contract

This section analyzes the design of an optimal contract. The two following propositions describe key features of the permanent and temporary contract that are preliminary to the contract-choice analysis. The following restricts the analysis to a match quality $x$ high enough so that both a permanent and a temporary contract are feasible.

**Proposition 1.** *Consider a match in any state $z \in Z_P$, $V \in \mathcal{V}_P(z)$. An optimal **permanent contract** features (i) a separation threshold $\underline{z}_P \in Z$ and (ii) wage-updating thresholds $\{\overline{w}_P(z') : z' \ge \underline{z}_P\}$ such that:*

*(i) the separation probability is*

$$\delta + (1 - \delta)\lambda G(\underline{z}_P);$$

*(ii) conditional on continuation, the next-period wage is*

$$\min\Big(w_P(V), \overline{w}_P(z')\Big), \tag{25}$$

*where $w_P(V)$ is the current-state wage (independent of $z$), and $\overline{w}_P(z')$ is the wage such that the employer's participation constraint (11) binds in next-period state $z'$.*

The following proposition is for the optimal temporary contract.

**Proposition 2.** *In any state $z \in Z_T$ and $V \in \mathcal{V}_T(z)$, an optimal* ***temporary contract*** *features (i) separation thresholds $\underline{z}_T$ and $\underline{z}_c$ and (ii) wage-updating thresholds $\{\overline{w}_T(z') : z' \geq \underline{z}_T\}$ and $\{\overline{w}_c(z') : z' \geq \underline{z}_c\}$ such that:*

*(i) a separation occurs with probability*

$$\delta + (1-\delta)\Big\{(1-\phi)\lambda G(\underline{z}_T) + \phi\Big[(1-\lambda)\mathcal{I}(z < \underline{z}_c) + \lambda G(\underline{z}_c)\Big]\Big\}, \tag{26}$$

*and the probability of a conversion into a permanent contract is*

$$(1-\delta)\phi\Big[(1-\lambda)\mathcal{I}(z \geq \underline{z}_c) + \lambda(1 - G(\underline{z}_c))\Big]; \tag{27}$$

*(ii) conditional on continuation in a temporary contract, the next-period wage is*

$$\min\Big(w_T(V,z), \overline{w}_T(z')\Big),$$

*whereas conditional on a conversion into a permanent contract, it is*

$$\min\Big(w_T(V,z), \overline{w}_c(z')\Big),$$

*where $w_T(V,z)$ is the current-state wage. Moreover $\overline{w}_T(z')$ is the wage such that the constraint (17) binds conditional on continuation in a temporary contract and $\overline{w}_c(z')$ is the wage such that (17) binds conditional on a conversion into a permanent contract in next-period state $z'$.*

The proofs of the propositions and lemmas in the main text are presented in appendix A. The optimal PC and TC are characterized in terms of a list of threshold functions that determine efficient wage dynamics and separation rules. The wage follows simple updating rules determined by the maximum feasible wages $\overline{w}_P(z')$, $\overline{w}_T(z')$, and $\overline{w}_c(z')$ in continuation states.[13] Efficient risk sharing commands that the wage remains constant across histories unless it needs to be adjusted downward to allow for the match continuation (i.e., to avoid an inefficient separation). In addition, a separation occurs with probability one in states such that there is no feasible contract providing non-negative continuation surplus to the worker and employer simultaneously, which is the case when productivity falls below the thresholds $\underline{z}_P$, $\underline{z}_T$, or $\underline{z}_c$.[14]

Hence, the optimal contract design brings together features of wage dynamics in models of the labor market with limited commitment (Thomas and Worrall (1988), Thomas and Worrall

---

[13] Note that in this setting, the worker's reservation wage is constant across states and contract types: the worker's outside option is independent of the shocks to the match productivity.

[14] Notice that this characterization has special cases where an endogenous separation is never optimal: in such cases, $\underline{z}_P = \underline{z}$ (or $\underline{z}_T, \underline{z}_c = \underline{z}$).

(2007)) and features of efficient separation decisions from models with search-and-matching frictions and endogenous separations (Mortensen and Pissarides (1994)). Note also that in a TC, the mandated conversion requirements affect the wage dynamics through the wage-updating threshold $\overline{w}_c$: with probability $\phi$ the relevant participation constraint for the separation and wage-dynamics rules is that of the employer facing the requirement to convert the TC into a PC. Hence, in the case of the TC, the worker's income dynamics is shaped by productivity shocks and by "conversion shocks" induced by the duration restrictions.

The two following lemmas analyze the distinguishing features of the optimal permanent and temporary contract that are key to the contract-type choice.

**Lemma 1.** *If $F > 0$, we have that $\underline{z}_P \leq \underline{z}_T \leq \underline{z}_c$, with strict inequalities for match quality $x$ low enough. Moreover, $\overline{w}_P(z) > \overline{w}_T(z)$ for all $z \geq \underline{z}_T$ and $\overline{w}_P(z) > \overline{w}_c(z)$ for all $z \geq \underline{z}_c$.*

This lemma implies that the probability of separation is higher in a temporary than a permanent contract, and that conditional on continuation, a wage cut is more likely in the temporary contract conditional on a given wage. This comes from the fact that (i) as expected, firing costs relax the participation constraint in the PC and (ii) that mandated conversion requirements impose additional restrictions on the set of feasible wage and separation rules in the TC. This implies the following key result for the choice between a PC and a TC.

**Lemma 2.** *Consider $z > \underline{z}_T$. (i) For $V = U$, $w_P(V) = w_T(V,z)$, implying that $J_T(V,z) \geq J_P(V,z)$. (ii) For all $V \in \mathcal{V}_P(z) \cap \mathcal{V}_T(z)$, $1/u'(w_T(V,z)) \geq 1/u'(w_P(V))$ implying that $\partial J_P(V,z)/\partial V \geq \partial J_T(V,z)/\partial V$. Moreover, the latter hold with strict inequalities if $V > U$.*

Part (i) says that the period wage costs of delivering to the worker a lifetime utility equal to the value of unemployment are independent of the contract type. This implies that lifetime profits are higher in a temporary contract delivering a lifetime value equal to that of unemployment. Part (ii) says that the marginal cost of delivering higher utility to the worker is greater in a temporary contract due to risk aversion, and due to the difference in income risk across the two contract types as implied by lemma 1. The permanent contract is less risky from the perspective of the worker because firing costs expand the continuation constraint set in a PC. In contrast, the duration restrictions reduce the same set in a TC. It follows that a PC has higher room for providing insurance against idiosyncratic shocks.[15] In other words, the temporary contract has a risk-compensating wage premium. It is important to note that this does *not* imply that temporary workers are paid a higher wage in equilibrium, which is counterfactual to empirical evidence (e.g., Bonhomme and Jolivet (2009)). Essentially, this result describes the trade-off that an employer faces when designing a contract: it is optimal to choose a PC because the losses

---

[15] Note that most of the results in this subsection are stated under the condition that $F > 0$: this is because it is assumed throughout that $F \geq 0$, as the benchmark quantitative model of section 3 will have $F = 0$.

from low flexibility are offset by the extra costs *that would have been required* to compensate the worker for the additional risk in a temporary contract (given lifetime utility).

**Contract choice.** The following proposition characterizes efficient hiring and contract-choice rules in terms of the realized match quality.

**Proposition 3.** *(i) The set of initial expected lifetime utility that can be delivered to the worker by a feasible contract is a non-empty, closed and bounded interval* $\mathcal{V}_0 = [U, \overline{V}_0]$. *(ii) Let* $F > 0$. *For any* $V_0 \in \mathcal{V}_0$*, an optimal hiring rule has thresholds* $\underline{x}_T$, $\underline{x}_P$*, and* $\hat{x}(V_0)$ *with* $0 < \underline{x}_T < \underline{x}_P < \hat{x}(V_0)$*, such that, when an unemployed worker and an employer with a vacant job enter into contact:*

- *a permanent contract is formed with probability*

$$\phi_0\Big[1 - G_x(\underline{x}_P)\Big] + (1-\phi_0)\Big[1 - G_x(\hat{x}(V_0))\Big], \tag{28}$$

- *a temporary contract is formed with probability*

$$(1-\phi_0)\Big[G_x(\hat{x}(V_0)) - G_x(\underline{x}_T)\Big], \tag{29}$$

  *and no match is formed with remaining probability.*

*In addition, an optimal hiring rule has a "target" wage* $w_0(V_0)$ *such that the match initial wage conditional on match quality* $x$ *is*

$$\min\Big(w_0(V_0), \overline{w}_c(\overline{z}, x)\Big), \tag{30}$$

*if a permanent contract is formed, where* $\overline{w}_c(\overline{z}, x)$ *is the wage such that the employer's participation constraint binds given* $x$ *and*

$$\min\Big(w_0(V_0), \overline{w}_T(\overline{z}, x)\Big) \tag{31}$$

*if a temporary contract is formed, with* $\overline{w}_T(\overline{z}, x)$ *analogously defined.*

High-quality matches end up sorting into permanent contracts and the other into temporary contracts as a result of the trade-off between flexibility and commitment underlying the contract choice. In low-quality matches with $x \in [\underline{x}_T, \hat{x}(V))$, the gains from greater flexibility dominate in the sense that it is optimal to make the temporary contract formation contingent on such realization. In high quality matches with $x \geq \hat{x}(V_0)$, the gains from better commitment dominate and a permanent contract is optimal. This trade-off is relevant when the hiring regulation is not binding, that is when both the PC and TC are available (with probability $1 - \phi_0$). When only the PC is available (with probability $\phi_0$), hiring occurs when $x \geq \underline{x}_P$. Finally, notice that $\underline{x}_T < \underline{x}_P$: the hiring reservation match-quality level is higher in a PC due to firing costs.

The proposition also describes an efficient initial wage plan. Due to the worker's preference for smooth consumption, the employer would like to offer a constant wage across states, independently of the contract type. However, the wage plan is constrained by limited-commitment frictions and must be set so that participation constraints are satisfied in all states with hiring as a possible outcome.

The next proposition characterizes properties of the hiring-stage profit function $\mathcal{J}$, a prerequisite for the labor-market equilibrium analysis that follows.

**Lemma 3.** *The maximized profit function* $\mathcal{J} : \mathcal{V}_0 \to \mathbb{R}_+$ *defined by* (18) *is strictly decreasing, strictly concave, continuously differentiable, and satisfies* $\mathcal{J}(\overline{V}_0) = \mathcal{J}_0$. *Further,* $\mathcal{J}'(V_0) = -1/u'(w_0(V_0))$.

Due to the assumption that the contract is designed before observing the match quality $x$, $\mathcal{J}$ is continuously differentiable and strictly concave.[16] This facilitates embedding the contracting problem in an environment with Nash bargaining, as it is done in the following labor-market market equilibrium section. Moreover, there is a unique $\overline{V}_0$ solving $\mathcal{J}(\overline{V}_0) = \mathcal{J}_0$ with the implicit assumption that the value of a vacant job $\mathcal{J}_0$ is low enough to ensure that matching potentially yields a positive surplus to both agents; this will be the case in equilibrium as vacancy free-entry will drive $\mathcal{J}_0$ to zero. This $\overline{V}_0$ represents the willingness to pay of an employer in the worker's utility terms, and any $V_0 \in [U, \overline{V}_0]$ yields a positive surplus to both agents. In addition, the employer's marginal cost of increasing the worker's initial lifetime matching utility is equal to the inverse of the marginal utility of the initial (unconstrained) consumption of that worker (itself equal to the target wage $w_0$ defined in the previous proposition).[17] The latter statement, which will prove useful in the equilibrium characterization that follows, comes from an envelope condition. It remains to study the distribution of the surplus in the context of an equilibrium of the labor market with bargaining, search frictions, and free entry of vacancies as in the following subsection.

## 2.4 Labor-market equilibrium

The section provides an analysis of a steady-state equilibrium of the labor market, in which the value of unemployment $U$, the profits of a vacancy $\mathcal{J}_0$, and the worker's initial matching lifetime utility value $V_0$ are endogenous and determined according to the following conditions.

---

[16] Allowing the agents to bargain over the contract choice given the realization of $x$ potentially yields a nonconvex problem as the Pareto-frontier of the two contracts potentially intersect in the employer's profit- worker's utility value space.

[17] More specifically, the marginal cost is equal to the inverse of the marginal utility of unconstrained consumption, i.e., the consumption (i.e., worker's income) level contingent on match quality such that hiring occurs and no participation constraints bind.

First, the worker's expected lifetime utility of unemployment is given by

$$U = u(b) + \beta\Big[p(\theta)V_0 + (1 - p(\theta))U\Big]. \tag{32}$$

With probability $p(\theta)$, the worker enters into contact with an employer, obtaining value $V_0$ and staying unemployed otherwise. Moreover, the worker's initial matching value is determined through Nash bargaining following

$$\max_{v_0 \in [U, \overline{V}_0]} (v_0 - U)^{\gamma} (\mathcal{J}(v_0) - \mathcal{J}_0)^{1-\gamma}, \tag{33}$$

where $\gamma \in (0,1)$ represents the relative bargaining power of the worker, where $\overline{V}_0$ is the employer's willingness to pay to form a match as discussed in lemma 3. Given properties of $\mathcal{J}$ (lemma 3) the objective is strictly concave and the problem has necessary and sufficient conditions for an optimum

$$V_0 - U = -\frac{\gamma}{1-\gamma}\Big[\mathcal{J}'(V_0)\Big]^{-1} (\mathcal{J}(V_0) - \mathcal{J}_0), \tag{34}$$

which implicitly determines the worker's initial matching value $V_0$ given $U$ and $\mathcal{J}$: the worker's marginal benefit of increasing the employment contract expected lifetime utility is equal to the (appropriately weighted) employer's marginal cost.

Finally, free entry of vacancies implies the zero-profit condition $\mathcal{J}_0 = 0$, so the labor-market tightness satisfies the indifference condition

$$q(\theta) = \Big[\beta\mathcal{J}(V_0)/\kappa\Big]^{-1} : \tag{35}$$

the expected cost of meeting a worker equals the employer's expected profits of such a meeting.

The steady-state equilibrium labor-market flows and stocks can be expressed in terms of the optimal contract characterization and the equilibrium variables $\theta$, $V_0$, and $U$. The steady-state unemployment-to-employment (UE) transition probability is

$$\Lambda_{UE} = p(\theta)\Big[(1 - \phi_0)(1 - G_x(\underline{x}_T)) + \phi_0(1 - G_x(\underline{x}_P))\Big], \tag{36}$$

whereas the transition probability from unemployment to a permanent job (UP) and to a temporary job (UT) can be written as

$$\Lambda_{UP} = p(\theta)\Big[(1 - \phi_0)(1 - G_x(\hat{x}(V_0))) + \phi_0(1 - G_x(\underline{x}_P))\Big] \tag{37}$$

$$\Lambda_{UT} = p(\theta)(1 - \phi_0)\Big[G_x(\hat{x}(V_0)) - G_x(\underline{x}_T)\Big], \tag{38}$$

respectively. Recall that $\underline{x}_T, \underline{x}_P$, and $\hat{x}(V_0)$ are the hiring cutoffs of proposition 3, and that $1-\phi_0$ materializes the hiring restrictions on TC. The transition probabilities from a temporary to a permanent job (TP) satisfies

$$\Lambda_{TP} = (1-\delta)\phi \int_{\underline{x}_T}^{\overline{x}} \Big\{ (1-\lambda)\Big(1 - H_T(\underline{z}_c(x')|x\Big) + \lambda\Big[1 - G(\underline{z}_c(x'))\Big] h_{T,x}(x') \Big\} dx'. \tag{39}$$

whereas the transition probabilities to unemployment from a permanent (PU) and a temporary job (TU) are

$$\Lambda_{PU} = \int_{\underline{x}_P}^{\overline{x}} \overline{s}_P(x') h_{P,x}(x') dx'; \quad \Lambda_{TU} = \int_{\underline{x}_P}^{\overline{x}} \overline{s}_T(x') h_{T,x}(x') dx', \tag{40}$$

with

$$\begin{aligned}
\overline{s}_P(x) &\equiv \delta + (1-\delta)\lambda G_z(\underline{z}_P(x')) \\
\overline{s}_T(x) &\equiv \delta + (1-\delta)\Big\{ \lambda\Big[(1-\phi)G_z(\underline{z}_T(x')) + \phi G_z(\underline{z}_c(x'))\Big] + \phi(1-\lambda)H_T(\underline{z}_c(x)|x) \Big\},
\end{aligned}$$

representing the probability of separation in a permanent and temporary contract conditional on match quality $x$, as per lemmas 1 and 2 (with the dependence of relevant thresholds on $x$ being made explicit). These flows depend on the equilibrium density functions of match quality for permanent and temporary jobs, $h_{P,x}$ and $h_{T,x}$; moreover, $H_T(.|x)$ is the equilibrium c.d.f. of stochastic productivity conditional on match quality $x \geq \underline{x}_T$ in the pool of temporary jobs. These distributions are described in the supplementary appendix. Finally, labor-market stocks can be described in terms of these transition probabilities as in

$$u = \frac{\Lambda_{EU}}{\Lambda_{UE} + \Lambda_{EU}}; \quad n_T = \frac{\Lambda_{UT}}{\Lambda_{TU} + \Lambda_{TP}} \times \frac{u}{1-u}; \quad n_P = 1 - n_T, \tag{41}$$

for the unemployment rate and the employment share of temporary and permanent jobs, respectively, and where

$$\Lambda_{EU} = \frac{n_P}{1-u}\Lambda_{PU} + \frac{n_T}{1-u}\Lambda_{TU} \tag{42}$$

is the unconditional transition probability from employment to unemployment. An equilibrium definition is as follows.

**Definition 1.** *A steady-state equilibrium is a list of value functions $J_P, J_T, \mathcal{J}$, $U$ and policy functions $\sigma_P^*$, $\sigma_T^*$, and $\xi^*$; an initial matching utility value $V_0$; a labor-market tightness $\theta$; and labor-market stocks $u$, $n_T$, $n_P$, such that: (i) the value functions $J_P$ and $J_T$ and the policy functions $\sigma_P^*$, $\sigma_T^*$ solve (5) and (12) given $U$ and given $\mathcal{J}_0 = 0$; (ii) $\mathcal{J}$ and $\xi^*$ solve (18) given $J_P$, $J_T$, and $U$; (iii) $U$, $V_0$, and $\theta$ satisfy (32), (34), and (35) given $\mathcal{J}$; (iv) $u$, $n_T$, $n_P$ satisfy (41) given $\theta$, $\sigma_P^*$, $\sigma_T^*$, and $\xi^*$.*

The following proposition presents a key equilibrium feature, which is the coexistence of permanent and temporary jobs—the main result of the paper. This proposition relates this coexistence to a simple condition on the upper bound of the match quality distribution $\overline{x}$ sufficient under the assumption that the unconditional match quality distribution is strictly positive, $g_x > 0$.

**Proposition 4.** *Assume that $F > 0$ and $\phi_0 < 1$: firing costs are positive and temporary contracts are not forbidden. In equilibrium and for $\overline{x}$ high enough, $n_T > 0$ and $n_P > 0$ (for any $\phi \in [0,1]$, $\phi_0 \in [0,1)$): permanent and temporary jobs coexist, regardless of the regulation on temporary jobs.*

An equilibrium solution features a coexistence of permanent and temporary contracts when the unconditional quality of matches—a primitive of the model—is sufficient dispersed in terms of its range (unless there are no firing costs or temporary contracts are forbidden). This follows from proposition 3 and from the stock-flow conditions (36) to (41). This is *regardless* of the parameter values for the hiring and duration/renewal restrictions on temporary jobs: this holds even when $\phi_0 = \phi = 0$, i.e., in the total absence of regulation on temporary jobs.

Specifically, this result follows from the optimal-contract (hiring rule) characterization in proposition 3: recalling that $g_x$ is positive over its support $X = [0,\overline{x}]$, $\overline{x}$ high enough implies values for the flows $\Lambda_{UP} > 0$ and $\Lambda_{UT} > 0$ from (37) and (38), implying the equilibrium stock values $n_P > 0$ and $n_T > 0$. In contrast, the standard search-and-matching model with endogenous separations (Mortensen and Pissarides (1994)) requires $\phi_0 > 0$ or $\phi > 0$ to have an equilibrium with permanent jobs. Key to this result are the assumptions of long-term contracting under limited commitment *and* endogenous separation plans as a component of the contract.

On the one hand, with the employers having full commitment ability, it is clear that temporary jobs would be always weakly preferred to the permanent ones. In such a case, firing costs would not provide any gain in terms of risk sharing and would only distort separation decisions and impose additional costs. On the other hand, in the absence of eventuality of separations (i.e., with feasible contracts lasting forever as in e.g., Thomas and Worrall (1988)), firing costs would serve as a "pure" commitment device that generates efficiency gains to any new match. It is then clear that a permanent contract is weakly preferred in all cases. The combination of these two features, together with a sufficiently high degree of dispersion for the match quality distribution implies the formation of an equilibrium mass of jobs in which opting for more flexibility is optimal and a mass for which more insurance coming from improved commitment instead is optimal.

## 3 Quantitative analysis

This section proposes a quantitative analysis of the model. First, the model is calibrated to the U.S. labor market for 1990-2010, a benchmark laissez-faire economy. Second, a validation exercise changes the model's parameters to mimic an introduction of salient European labor-market institutions into the benchmark. It shows that the model reproduces stylized facts of the long-run dynamics of temporary jobs in France and Spain. Finally, the section studies the model's implications for the following two questions: (i) the employment effect of labor market reforms relaxing hiring constraints on temporary contracts interacted with European-type institutions and (ii) the welfare effect of firing costs.

## 3.1 Calibration

The model is calibrated to the U.S. labor market for 1990-2010, the benchmark. The time unit is a month. The matching function has Cobb-Douglas form $m(u,v) = Au^{\eta}v^{1-\eta}$ ($A > 0$, $\eta \in (0,1)$). Upon hiring, the quality of matches is drawn from a log-normal distribution with mean normalized to one, parameters $\mu_x > 0$ and $\sigma_x^2 > 0$. The idiosyncratic-shock component of output is uniformly distributed with support $Z = [0,1]$ (Mortensen and Pissarides (1999)). The discount factor is equal to $\beta = 0.9957$, consistent with a 5% annual discount rate. The elasticity of matching is $\eta = 0.5$, a conventional value, and the worker's relative bargaining power parameter is set to $\alpha = \eta$, in line with the Hosios condition (Hosios, 1990). Following Jung and Kuhn (2019), I set the probability of exogenous separation $\delta$ to 0.24%. Finally, $F = 0$ to mimic the lax employment protection legislation in the U.S.[18]

The following parameters are internally calibrated: the non-work income level $b$, the matching efficiency $A$, the vacancy posting cost $\kappa$, the probability of a match-specific shock $\lambda$, and the variance of the log match quality $\sigma_x^2$. Following Shimer (2005), the labor-market tightness $\theta$ is normalized to one, and the vacancy-posting cost $\kappa$ is deduced from the free-entry condition (35). In addition and to save on computation, I calibrate the model in partial equilibrium, in the sense that I treat the worker's utility value of unemployment $U$ as given and then compute the non-work income $b$ that is consistent, in equilibrium, with $U$, the other parameters, and the normalization $\theta = 1$. I target $b$ to be equal 0.4 unit of output (Shimer (2005)). In equilibrium of the calibrated model, this value implies an average replacement ratio equal to 0.39. Moreover, the matching efficiency $A$ is set to target an unemployment rate equal to 5.6%, the U.S. average over 1990-2010; $\lambda$ targets a monthly employment-to-unemployment monthly transition probability equal to 2% (Jung and Kuhn (2014) and Shimer (2012)). The parameter $\sigma_x^2$, driving heterogeneity in match quality and, therefore, in separation rates, is calibrated using information from the Job-Tenure supplement of the CPS for 1996-2010. An individual with less than one year of tenure is 3.13 times more likely to experience a monthly separation (into non-employment or to another employer) than those with one to 15 years of tenure. In the model, the counterpart to this statistic is the ratio of separation probabilities

---

[18] With $F = 0$, the distinction between permanent and temporary contracts is meaningless and the TC restriction parameters $\phi_0$ and $\phi$ are irrelevant.

Table 1: Benchmark parameter values

| | | |
|---|---|---|
| *Preset* | | |
| $\beta$ | Discount factor | 0.996 |
| $\sigma$ | Relative risk aversion | 2.5 |
| $\eta$ | Elasticity of matching | 0.5 |
| $\gamma$ | Worker's bargaining power | 0.5 |
| $\delta$ | Exogenous separation probability | 0.0024 |
| $F$ | Firing costs | 0 |
| *Internal* | | |
| $b$ | Non-work income | 0.383 |
| $A$ | Matching efficiency | 0.600 |
| $\lambda$ | Probability of idiosyncratic shock | 0.068 |
| $\sigma_x^2$ | Log match-quality variance | 0.204 |
| $\mu_x$ | Log match-quality mean | -0.102 |
| $\kappa$ | Vacancy posting costs | 2.096 |

Notes: parameter values in the benchmark U.S. calibrated economy. The exogenous probability of separation $\delta$ is set following Jung and Kuhn (2019). The other preset parameters are standard. The targets for the internally calibrated parameters are an unemployment rate equal to 5.6%, an aggregate EU probability of 2%, non-work income $b = 0.4$ (Shimer (2005)), and relative separation rates by tenure estimated from the Job Tenure Supplement of the CPS (less than one year of tenure to 1-15 years). Moreover, $\mu_x = -\sigma_x^2/2$. The value of $\kappa$ is set to be consistent with the normalization that the labor-market tightness $\theta = 1$ (Shimer (2005)). In addition, the model is calibrated in partial equilibrium with the value of unemployment $U$ exogenous, and the parameter $b$ is computed accordingly (see the text for details). The model fit to the calibration targets is shown in Table 2, Panel A.

to unemployment.[19] I then let $\mu_x = -\sigma_x^2/2$, a normalization. The resulting parameters are reported in Table 1 and the model fit to the targets is illustrated in Panel a) of Table 2.

## 3.2 Institutions and cross-country outcomes

This subsection presents an experiment introducing counterfactual labor-market policies into the U.S. benchmark economy. The primary goal is to assess the model's ability to account for salient cross-country variation in labor market outcomes in response to parsimonious parameter changes. I consider policies consistent with the cases of France and Spain. Not only these countries are considered as epitomes of dual labor markets attracting great attention in the

---

[19] These ratios are computed using unweighted averages of separation rates conditional on yearly tenure, to control for composition differences between the data and the model.

Table 2: Data and model targeted statistics

| | *Target* | *Model* |
|---|---|---|
| *a) U.S. benchmark* | | |
| Unem. rate (%) | 5.61 | 5.64 |
| EU rate (%) | 2.00 | 2.00 |
| Sep. rate tenure ratio | 3.13 | 3.17 |
| Non-work income, $b$ | 0.40 | 0.38 |
| *b) "French' counterfactual* | | |
| Unem. rate | 9.20 | 9.21 |
| TP rate (%) | 3.43 | 3.44 |
| $b/E(w)$ | 0.55 | 0.55 |
| $F/E(w)$ | 1.50 | 1.49 |
| *c) "Spanish' counterfactual* | | |
| Unem. rate | 13.28 | 13.54 |
| TP rate | 2.16 | 2.16 |
| $b/E(w)$ | 0.58 | 0.58 |
| $F/E(w)$ | 1.80 | 1.80 |

Notes: targeted and model statistics for the U.S. benchmark economy (Panel a)) and the counterfactual model economies with "French-type" (b) and "Spanish-type" institutions (c). The U.S. employment separation ratio by tenure (less than one year to 1-15 years) is estimated from the Job Tenure Supplement of the CPS (1996-2010). The U.S. value of non-work income of $b = 0.4$ is from Shimer (2005); replacement ratios ($b/E(w)$) are from Bentolila et al. (2012), and firing costs (relative to the average equilibrium wage $F/E(w)$) are from Cahuc et al. (2016). Transition probabilities are estimates from Jung and Kuhn (2014) (U.S.), Givord and Wilner (2015) (France), and Silva and Vázquez-Grenno (2013) (Spain).

literature, but they also display sharp differences in the prevalence of temporary jobs. Note that this exercise is not meant to mimic the French and Spanish economies. Rather, it examines the extent to which the model captures non-targeted empirical outcomes in response to plausible policy differences—a validation exercise.

In this experiment, preference and production technology primitives remain the same across economies ($\beta$, $\sigma$, $\sigma_x^2$, $\mu_x$, $\lambda$). I also keep constant the matching technology and bargaining parameters $\kappa$, $\eta$, and $\gamma$. The efficiency of matching $A$ is set following Jung and Kuhn (2014), studying the business cycle dynamics of the German and U.S. labor markets. Their calibration suggests a value for $A$ in the U.S. that is twice as large as for Germany.[20] The unemployment benefit level $b$ is calibrated to match a wage replacement ratio of 0.55 in France and 0.58 in Spain (Bentolila et al., 2012). As suggested by Cahuc et al. (2016), firing costs $F$ are set to be equal to 1.5 of the average wage in France and 1.8 of that in Spain.

In addition, the hiring restriction parameter is $\phi_0 = 0$ in the two economies, consistent with the fact that temporary contracts represent the vast majority of hiring flows in France and Spain (Cahuc et al., 2016), which suggests lax regulation (or, at least, low enforcement of this regulation).[21] To calibrate the duration restriction on temporary contracts $\phi$, I target empirical transition probabilities from temporary to permanent jobs (TP). For Spain, Silva and Vázquez-Grenno (2013) estimate a monthly TP rate equal to 2.16% for 1993-2010. For France, I take quarterly transition probabilities estimated by Givord and Wilner (2015) for 2002-2010 and infer monthly probabilities following Shimer (2012) and Silva and Vázquez-Grenno (2013).[22] This yields a monthly TP rate equal to 3.43%. Finally, using transition matrices from Givord and Wilner (2015) and Silva and Vázquez-Grenno (2013), I compute the following steady-state unemployment rates: 9.2 % for France and 13.3 % in Spain. The exogenous separation probability $\delta$ is adjusted to match these statistics. Table 3 shows the parameter values for the counterfactual economies along with the benchmark. The targeted and model moments are shown in Panels b) and c) of Table 2.

Table 3: Parameter values for the counterfactual model economies

| | | *Counterfactual* | | *Benchmark* |
|---|---|---|---|---|
| | | *French-type* | *Spanish-type* | |
| $A$ | Matching efficiency | 0.300 | 0.300 | 0.600 |
| $b$ | Non-work income | 0.500 | 0.530 | 0.383 |
| $F$ | Firing costs | 1.37 | 1.65 | 0 |
| $\phi_0$ | TC hiring regulation | 0 | 0 | - |
| $\phi$ | TC duration regulation | 0.045 | 0.030 | - |
| $\delta$ | Exogenous separation | 0.0008 | 0.0019 | 0.0024 |

Notes: parameter values for the counterfactual economies with “French-type” and “Spanish-type” institutions (and their benchmark counterparts). The matching efficiency $A$ and TC hiring regulation strictness $\phi_0$ are preset (see the text for details). The parameters $b$, $F$, $\phi$, and $\delta$ are set to match the statistics in Table 2.

Table 4 reports untargeted outcomes of the counterfactual economies. To put these in

---

[20] According to Elsby et al. (2013), France, Germany, and Spain have unemployment outflow rates ranging from 6 to 8% in 1990-2010, suggesting an efficiency of matching of a similar magnitude in these three countries.

[21] The parameter $\phi_0$ will be increased later on when analyzing the effect of temporary contracts (subsection 3.3).

[22] Silva and Vázquez-Grenno (2013) and Givord and Wilner (2015) use longitudinal data from labor-force surveys.

empirical perspective, it also reports estimates from Givord and Wilner (2015) and Silva and Vázquez-Grenno (2013).[23] First, the table reports monthly transition probabilities in and out of employment and of permanent jobs. The French-type institutions imply worker flows in the model that are close to their data counterparts. The latter is also true, albeit to a lesser extent, for the model with Spanish-type institutions. Second and most importantly, the model captures most of the large difference in the prevalence of temporary employment between France and Spain. The table displays employment shares of temporary jobs implied by the empirical transition probabilities. The economy with French-type institutions has an employment share of 12.7%, close to the data (11.9%). The Spanish-type model has a temporary employment share of 20%, versus 24% in the data. Moreover, the counterfactual model predicts that temporary jobs account for one-half to two-thirds of the employment inflows and for the large majority of the employment inflows. The same is observed in data for France and Spain.[24]

Figure 1 represents policy and distribution functions to interpret the model's outcomes. For conciseness, the figure focuses on the counterfactual economy with French-type institutions. Panel a) plots the difference in the hiring surplus in a permanent contract and a temporary one, across values of the realized match quality and the worker's initial matching value. Specifically, it represents

$$(1-\beta)\Delta\mathcal{S}(\nu_0,x) = (1-\beta)\left[J_P(\nu_P(\nu_0,x),\bar{z},x) - J_T(\nu_T(\nu_0,x),\bar{z},x) + \frac{\nu_P(\nu_0,x)-\nu_T(\nu_0,x)}{u'(w_0(\nu_0))}\right] \quad (43)$$

over some values for the match quality $x$ and feasible matching worker's lifetime utility values $\nu_0$. In (43), $\nu_P(\nu_0,x)$ and $\nu_T(\nu_0,x)$ respectively denote the worker's lifetime employment utility value in a permanent and a temporary contract, conditional on drawing match quality $x$ and after prior agreement on a hiring rule delivering matching value $\nu_0$ to the worker (determined according to (34) in equilibrium). The value of (43) measures the surplus difference between the two contract types in equivalence terms of lifetime consumption annuities.

The figure shows that only matches in higher percentiles of the sampling distribution generate more surplus in a permanent contract. This is consistent with the fact that employment inflows are largely dominated by temporary jobs. Panel b) shows simulated equilibrium distributions of the match quality conditional on the type of contract. The figure shows large heterogeneity between permanent and temporary jobs, implying stark differences in separation rates. The latter explains the large magnitude of the share of temporary jobs in employment

---

[23] One caveat of this data-model comparison is that estimates from Givord and Wilner (2015) and Silva and Vázquez-Grenno (2013) are based on frameworks distinguishing between unemployment and non-participation, whereas the model ignores participation flows. Still, these estimates illustrate facts characterizing the composition of employment stocks and flows and allowing for gauging the empirical relevance of the model's sorting mechanisms.

[24] Cahuc et al. (2016), using spell administrative data find that temporary jobs account for 90% of employment inflows in France and Spain in the 2000s.

Table 4: Counterfactual labor-market stock and flows (non-targeted)

| | *France* | | *Spain* | |
|---|---|---|---|---|
| | *Data* | *Model* | *Data* | *Model* |
| *Transition rates (%)* | | | | |
| Unem. to Emp. (UE) | 8.43 | 11.75 | 5.32 | 10.79 |
| Emp. to Unem. (EU) | 1.09 | 1.19 | 0.76 | 1.69 |
| Unem. to Perm. (UP) | 1.83 | 1.71 | 0.41 | 1.48 |
| Perm. to Unem. (PU) | 0.33 | 0.62 | 0.15 | 0.83 |
| Employment stock, temp. contracts (TC) share (%) | 11.88 | 12.66 | 24.26 | 20.17 |
| Unemployment to employment flows, TC share | 73.18 | 85.44 | 92.29 | 86.24 |
| Employment exit flows, TC share | 53.93 | 54.31 | 67.36 | 60.58 |

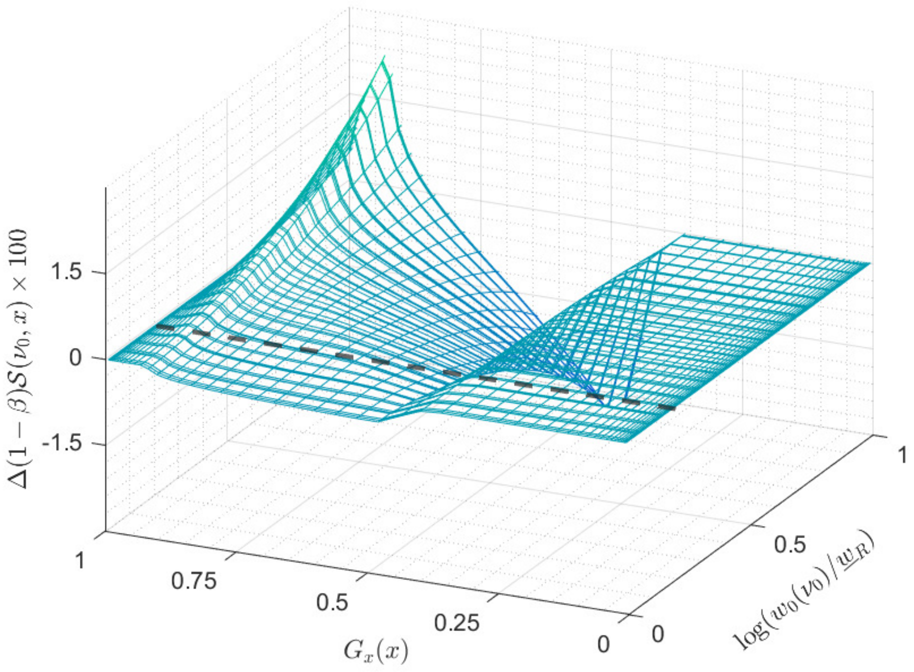
Notes: select statistics for the counterfactual "French-type" and "Spanish-type" model economies (versus their empirical counterparts). The transition probabilities are taken from Givord and Wilner (2015) for France (private sector excluding self-employment) and Silva and Vázquez-Grenno (2013) for Spain. Monthly transition probabilities for France are computed by applying the time-aggregation adjustment in Shimer (2012) to quarterly transition probabilities in Givord and Wilner (2015). The empirical employment share of temporary jobs is computed from steady-state stocks implied by empirical transition probabilities. Empirical employment exit flows are defined as flows into unemployment and non-participation.

outflows. The model's optimal sorting behavior implies that temporary jobs are low-surplus "marginal" jobs accounting for the majority of employment flows, as seen in the data and the model.

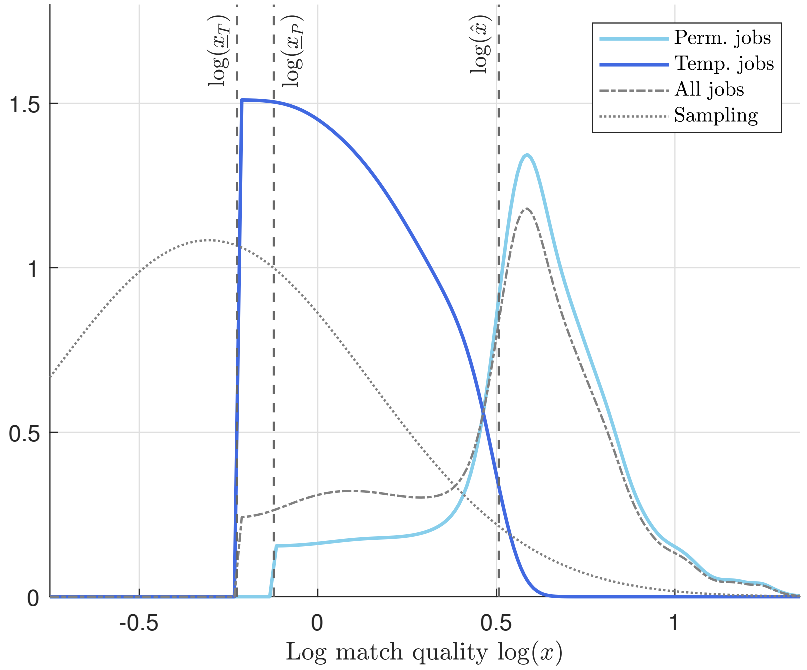
Lastly, the supplementary appendix presents exercises that decompose the differences in outcomes between the U.S. baseline and the French and Spanish-type counterfactuals (Tables A.1 and A.2). Matching efficiency is the main contributor to U.S.-France unemployment differences and employment protection ($F$) plays no role. In addition, the differences between France and Spain in the share of temporary jobs are attributable to (i) the exogenous separation probability $\delta$ and (ii) the regulation on TC duration and renewal, $\phi$ (in order of importance).

## 3.3 Policy experiments

**The employment effect of temporary contracts.** The following revisits the widely studied question of the consequences of labor-market reforms relaxing hiring restrictions on temporary contracts in Europe in the post-oil-shock era. The experiment is an increase in the parameter capturing the strictness of hiring restrictions on temporary contracts, $\phi_0 \in [0,1]$ in the model



(a) Matching surplus: PC vs. TC



(b) Equilibrium simulated distributions of match quality

Figure 1: Equilibrium outcomes in the "French" counterfactual economy

Notes: equilibrium policy and distribution functions in the French-type calibrated economy. Panel a): hiring surplus difference (equation (43)) vs. match-quality quantiles and the worker's initial matching values, in terms of the "target" hiring wage $w_0$ (proposition 3). The dashed line represents the equilibrium level of the hiring wage determined by condition (33). Panel b): Gaussian kernel density estimates (bandwidth 0.1) of simulated equilibrium distributions and the sampling distribution log match quality $g_x$. The vertical lines show hiring cutoffs $\underline{x}_T$, $\underline{x}_P$, and $\hat{x}$ of proposition 3.

economies with counterfactual, French and Spanish-type, institutions.

In equilibrium, a change in $\phi_0$ shifts employment inflows (the unemployment to employment transition probability, or UE rate) through two main channels: a *tightness* (firms' expected profits and vacancy creation incentives changes) and a *selection* channel (firms adjust their hiring decisions contingent on match-quality realizations). The latter selection channel depends on the number of temporary jobs created in response to the reform (on top of the labor-market tightness response) net of the number of temporary jobs substituted for permanent jobs, which would have been formed regardless. These selection effects are inherent in the presence of heterogeneity in match quality and, foremost, inherent in the model's optimal sorting into permanent and temporary jobs.

More formally, denote by $\mathcal{A}(\phi_0) \equiv (1-\phi_0)\left[1 - G_x(\underline{x}_T)\right] + \phi_0 G_x(\underline{x}_P)$ the probability of hiring conditional on a worker-firm contact, or match acceptance probability: $\Lambda_{UE} = p(\theta)\mathcal{A}$. Consider two alternative regimes for the hiring regulation, indexed by $\phi_0$ and $\phi_0'$, in $[0,1)$. Then, from (36), the associated difference in the UE rate can be approximated by (making explicit the dependence on policy regimes)

$$\Lambda_{UE}(\phi_0') - \Lambda_{UE}(\phi_0) \approx \underbrace{\frac{\sum_{\tilde{\phi}_0} \Lambda_{UE}(\tilde{\phi}_0)}{2} \frac{p(\theta(\phi_0')) - p(\theta(\phi_0))}{p(\theta(\tilde{\phi}_0))}}_{\text{(i) tightness}}$$

$$+ \underbrace{(\phi_0' - \phi_0)\frac{\sum_{\tilde{\phi}_0} \Lambda_{UE}(\tilde{\phi}_0)}{2} \frac{G_x(\hat{x}(\phi_0')) - G_x(\underline{x}_T(\phi_0))}{\mathcal{A}(\tilde{\phi}_0)}}_{\text{(ii-a) temp. job creation}} - \underbrace{(\phi_0' - \phi_0)\frac{\sum_{\tilde{\phi}_0} \Lambda_{UE}(\tilde{\phi}_0)}{2} \frac{G_x(\hat{x}(\phi_0')) - G_x(\underline{x}_P(\phi_0))}{\mathcal{A}(\tilde{\phi}_0)}}_{\text{(ii-b) perm. job substitution}}. \tag{44}$$

The details are in the supplementary appendix. The difference in the UE rate can be decomposed as follows. The first term (i) measures the contribution of the change in the labor-market tightness and the associated contact probability $p(\theta)$. The second and third terms give the contribution of the above-mentioned selection channel. It consists of (ii-a) the change in the proportion of matches forming a temporary contract *net of* (ii-b) the change in the proportion of matches that would have been viable in a permanent contract and that are substituted, as a result, by a temporary contract. In expression (44), is understood that summations are over the two alternative policy regimes ($\tilde{\phi}_0 \in \{\phi_0, \phi_0'\}$).[25]

---

[25] Further, the approximation comes from $(x_T(\phi_0), x_P(\phi_0), \hat{x}(\phi_0)) \approx (x_T(\phi_0'), x_P(\phi_0'), \hat{x}(\phi_0'))$, the hiring cutoffs of proposition 3 are approximately equal across policy regimes. A change in $\phi_0$ has no direct equilibrium effect on optimal hiring decisions conditional on match quality realizations. In contrast, such a change directly affects the sorting of matches into permanent and temporary contracts, which is the result of policy decisions *and* the probability $\phi_0$ (see (36)). A change in $\phi_0$ shifts the reservation wage through the labor market tightness, a second-order, indirect equilibrium effect on the optimal hiring thresholds. A similar discussion applies to thresholds for separation decisions conditional on the contract type (lemmas 1 and 2).

In parallel, the magnitude of employment outflows responds to two main effects. First, there is a shift in the equilibrium proportion of jobs subject to firing restrictions (i.e., permanent jobs), which changes the probability of labor *retention* in response to idiosyncratic shocks. This is a standard channel, operating in the canonical search-and-matching model. Second, and due to endogenous sorting, there is a change in the employment-pool *composition*, as labor and jobs optimally reallocate as a result of the reform. Formally, one can approximate the change in the EU rate across policy regimes by

$$
\begin{aligned}
\Lambda_{EU}(\phi_0') - \Lambda_{EU}(\phi_0) \approx\; & \overbrace{\Big(n_P(\phi_0') - n_P(\phi_0)\Big) \sum_{\tilde{\phi_0}} \int_{x' \in \tilde{X}(\tilde{\phi_0})} \Big(\bar{s}_P(x';\tilde{\phi_0}) - \bar{s}_T(x';\tilde{\phi_0})\Big) \tilde{h}(x')dx'}^{\text{(i) retention policy change}} \\
& + \underbrace{\Big(n_P(\phi_0') - n_P(\phi_0)\Big) \sum_{\tilde{\phi_0}} \int_{x' \in X} \Big(h_P(x';\tilde{\phi_0}) - h_T(x';\tilde{\phi_0})\Big) \tilde{s}(x')dx'}_{\text{(ii-a) between-job reallocation}} \\
& + \underbrace{\frac{\sum_{\tilde{\phi_0}} n_P(\tilde{\phi_0})}{2} \int_{x' \in X} (h_P(x';\phi_0') - h_P(x';\phi_0)) \tilde{s}_P(x')dx'}_{\text{(ii-b) permanent-job reallocation}} \\
& + \underbrace{\frac{1 - \sum_{\tilde{\phi_0}} n_P(\tilde{\phi_0})}{2} \int_{x' \in X} (h_T(x';\phi_0') - h_T(x';\phi_0)) \tilde{s}_T(x')dx'}_{\text{(ii-c) temporary-job reallocation}}. \qquad (45)
\end{aligned}
$$

Tildes indicate auxiliary objects derived from equilibrium variables (and their averages) across policy regimes. See the supplementary appendix for the details. Component (i) measures the contribution of the standard retention channel. The three other terms (ii-a to ii-c) measure the role of reallocation. Component (ii-a) reflects the contribution of match reallocation between permanent and temporary jobs, whereas (ii-b) and (ii-c) measures the contribution of the match reallocation within these two distinct pools of employment.[26] The total reallocation effect gives the contribution of the shift in the composition of the entire pool of employment and jobs.

The numerical implementation is as follows. I consider the model economies with French and Spanish-type institutions. In these two economies, the hiring regulation parameter $\phi_0 = 0$. There are no hiring restrictions on the type of contract, implying an equilibrium employment share of temporary jobs equal to 12% and 20%. I then impose a counterfactual increase in hiring restrictions to make these employment shares 5.5% as prevailing in Europe in the early 1980s, prior to the wave of reforms promoting temporary contracts (Faccini (2014)). This

---

[26] In the case of the decomposition based (45) as well, the approximation comes from assuming that policy decisions (optimal separation decisions conditional on state variables) are approximately constant across policy regimes.

requires imposing $\phi_0 = 0.55$ in the French-type economy and $\phi_0 = 0.72$ in the Spanish. In these economies, 45 and 28% of potential new matches, respectively, are allowed to choose the contract type. I interpret the results as uncovering the causal effect of promoting temporary contracts and present variations in outcomes associated with a shift from the more to the less strict policy regime (from $\phi_0 > 0$ to $\phi_0 = 0$).

Table 5 shows the results for employment stocks and flows. A reform promoting temporary contracts does *not* generate employment gains. The effect, if anything, is positive in the Spanish-type economy, where the unemployment rate increases by 0.4 percentage points. Both employment inflows and outflows are higher. However, the magnitude of the changes in the flows in and out of the permanent employment pool is disproportionate to the changes in the aggregate, indicating substantial shifts in the composition of jobs.

Table 5: The employment effect of temporary contracts

| | *French-type institutions* | | *Spanish-type institutions* | |
|---|---|---|---|---|
| | *No restrictions* $\phi_0 = 0$ | *Pre-reform* $\phi_0 = 0.55$ | *No restrictions* $\phi_0 = 0$ | *Pre-reform* $\phi_0 = 0.72$ |
| Unemp. rate (%) | 9.21 | 9.22 | 13.54 | 13.16 |
| TC emp. share | 12.66 | 5.56 | 20.17 | 5.65 |
| UE rate (%) | 11.75 | 10.76 | 10.79 | 9.36 |
| UP rate | 1.71 | 6.27 | 1.48 | 6.79 |
| EU rate (%) | 1.19 | 1.09 | 1.69 | 1.42 |
| PU rate | 0.62 | 0.85 | 0.83 | 1.20 |

Notes: evaluation of the effect of temporary contracts on employment stocks and flows in the model economies with French and Spanish-type institutions. The columns "No restrictions" present equilibrium outcomes (in percentage) in economies without hiring restrictions on temporary contracts, i.e., $\phi_0 = 0$."Pre-reforms" present outcomes in economies with $\phi_0 > 0$, set to generate an employment share of temporary jobs equal to 5.5%, the average in Continental Europe in the early 1980s (prior to reforms promoting temporary contracts, Faccini (2014)).

The decomposition based on (44) and (45) explores further these composition changes and their link to the aggregate flows. The results are reported in Table 6. The table shows the absolute values of components of (44) and (45), as well as their contribution to percentage-point changes in the unemployment rate in response to the reform (see the supplementary appendix for details). The following conclusions can be drawn. First, the channels inherent in the presence of heterogeneity and endogenous sorting across job types represent the main quantitative effects. The *selection* channel represents almost 90% of the total UE rate change in the two economies. Similarly, the EU rate change is almost entirely explained by job reallocation effects. In contrast, the standard tightness and retention channels have low quantitative significance.

Table 6: Decomposition of the effect of temporary contracts on employment flows

| | *French-type* | | *Spanish-type* | |
|---|---|---|---|---|
| | *Absolute* | *Unem. change* | *Absolute* | *Unem. change* |
| *UE rate change (p.p.)* | 0.99 | -0.74 | 1.43 | -1.65 |
| (i) tightness | 0.11 | -0.08 | 0.21 | -0.24 |
| (ii) selection | 0.89 | -0.66 | 1.22 | -1.41 |
| (ii-a) temp. job creation | 5.50 | -4.10 | 6.65 | -7.66 |
| (ii-b) perm. job substitution | -4.55 | 3.39 | -5.31 | 6.11 |
| *EU rate change (p.p.)* | 0.10 | 0.74 | 0.27 | 2.02 |
| (i) retention | 0.01 | 0.11 | 0.03 | 0.25 |
| (ii) reallocation, total | 0.08 | 0.62 | 0.22 | 1.67 |
| (ii-a) between perm. and temp. jobs | 0.30 | 2.22 | 0.55 | 4.10 |
| (ii-b) within perm. jobs | -0.21 | -1.51 | -0.33 | -2.50 |
| (ii-c) within temp. jobs | -0.01 | -0.10 | 0.01 | 0.07 |

Notes: decomposition of the effect of lowering the temporary-contract hiring restriction parameter $\phi_0$ on steady-state equilibrium unemployment flows in counterfactual economies with "French" and "Spanish-type" institutions. The decomposition is based on equations (44) and (45). The table reports changes in equilibrium outcomes implied by a decline in the parameter $\phi_0$ from 0.54 (French) and 0.71 (Spanish) to 0. The second and third columns show changes in the percentage-point value of components of (44) and (45). The third and fourth columns show the percentage-point contribution of these components to the change in the unemployment rate.

Second, job-substitution flows are large, in the sense that these account for the large majority of temporary-job creation associated with the reform (around 90% in both economies). In other words, job substitution largely offsets job creation effects, mitigating the positive response of the aggregate UE flows. Third, there are quantitatively large reallocation movements in response to the reform. On the one hand, the distribution of permanent jobs shifts towards high-quality matches due to the substitution effect, as reflected in the sharp relative drop in PU rates (Table 5). On the other hand, the entire employment pool shifts towards low-quality matches as labor reallocates into temporary employment, resulting in higher employment outflows. As the table shows, the associated unemployment variation is substantial: the labor reallocation effect increases the unemployment rate by 0.6 points in the French case and 1.7 in the Spanish.

In conclusion, the model corroborates the view that the liberalization of temporary contracts has been associated with crowding out of permanent employment without significant employment gains—as well as by the observation that unemployment has been persistently high in past decades in Europe despite numerous labor-market reforms spreading temporary jobs.

This model with optimal sorting allows for quantifying the importance of job substitution and job creation flows, as opposed to standard models based on the MP framework. The model's endogenous substitution effect is key to rationalizing the empirical employment trends in Europe: the large majority of flows into temporary jobs reflect the substitution of permanent jobs rather than job creation. Moreover, the model's endogenous composition changes due to temporary contracts implies, as seen above, substantial increases in unemployment.[27]

**Welfare implications.** I now turn to analyzing the implications of the model for the effect of firing costs on welfare. The assumptions of risk aversion and limited commitment potentially open an efficiency role for firing costs at the micro-matching level. How do these gains and costs balance out in equilibrium and at the aggregate level? The following examines outcomes in the benchmark (U.S.) calibrated model with different firing cost values $F$, assuming no regulation on temporary jobs ($\phi_0 = \phi = 0$).

Let $\pi(V,z,x)$ represent the lifetime annuity income with a present value equal to the employer's lifetime profits in state $(V,x,z)$. In addition, let $c_e(V)$ be the certainty equivalent lifetime consumption of an employment contract providing utility $V$ to a worker. Define $c_u$ in the same way but for an unemployed worker (with equilibrium lifetime utility $U$). Let the equilibrium averages of $\pi$ and $c_e$ be respectively denoted by $\overline{\pi}(F)$ and $\overline{c}_e(F)$ when firing costs are set to $F \geq 0$. These averages are taken over the relevant equilibrium distributions of match states. In addition, consider the following social welfare function:

$$\mathcal{W}(F) = (1-u)(\overline{\pi}(F) + \overline{c}_e(F)) + uc_u(F) - \theta u \kappa - (1-u) n_P \frac{\Lambda_{UP} - \delta}{1-\delta} F, \tag{46}$$

for $F > 0$. This is equal to the total consumption value of profits, employment contracts, and the unemployment status net of firms' hiring and firing costs (per time unit). This statistic comes with the important caveat that it makes strong assumptions regarding the distribution of profits and firing costs.[28] As such, it should be complemented with an examination of the components of this function in isolation.

The results are shown in Table 7, for firing costs equivalent to values from one to 12 months of the benchmark equilibrium wage. Firing costs affect total output through (i) the productivity of labor and (ii) employment; high values of $F$ have a significantly negative impact on productivity, inducing substantial output losses. As an illustration, firing costs equal to

---

[27] It is important to note that these results regarding employment effects are positive in nature. There is no efficiency ground in the model for regulating temporary jobs, as this would introduce extra constraints on the agents' contracting problem. The welfare analysis that follows addresses the model's normative implications. It shows that welfare gains to workers can be generated by firing costs precisely in the case where there is no regulation on temporary contracts.

[28] It implicitly assumes that firing costs are non-distributed, a source of efficiency loss. Moreover, it assumes that workers have no claim to firm ownership.

Table 7: Baseline model outcomes under alternative firing-cost regimes (relative to $F = 0$, in %)

| *Firing costs/benchmark wage* | *one month* | *three months* | *six months* | *12 months* |
|---|---|---|---|---|
| Total output (% change) | -0.01 | 0.01 | -0.39 | -1.51 |
| Employment (% change) | -0.14 | -0.07 | 0.00 | 0.10 |
| Labor productivity (% change) | 0.14 | 0.08 | -0.39 | -1.61 |
| *Profits and labor costs (p.p. change)* | | | | |
| Average profits $\overline{\pi}$ | -0.76 | -0.26 | -1.04 | -2.06 |
| Hiring costs/output | 0.21 | -0.02 | -0.21 | -0.43 |
| Average log wage | 0.33 | 0.16 | 0.84 | 0.76 |
| *Welfare (% change)* | | | | |
| Unemployed workers, $c_u$ | 0.04 | 0.09 | 0.15 | 0.22 |
| All workers, $\overline{c}_e$ | 0.05 | 0.17 | 0.78 | 1.08 |
| Social welfare, $\mathcal{W}(F)$ | -0.93 | -0.11 | -0.02 | -0.34 |

Notes: equilibrium outcomes associated with an increase in firing costs $F$ in the benchmark, U.S., calibrated economy, assuming no regulation on temporary jobs: $\phi_0 = \phi = 0$. Firing costs are expressed in terms of the average equilibrium wage in the benchmark economy. Outcomes are reported in difference to the benchmark. Profits are measured in lifetime annuity income terms and workers' welfare is measured in annuity certainty equivalent consumption. The aggregate social welfare statistics $\mathcal{W}(F)$ is given by (46).

12 months of the benchmark average wage reduce output by 1.5% and productivity by 1.6%. The lower productivity comes from a higher propensity of match retention in response to idiosyncratic shocks. Yet, the average equilibrium wage is higher with $F$, reflecting better insurance for workers from a lower propensity of wage cut. This insurance comes at the expense of the average equilibrium profits, lower with firing costs.

In terms of welfare, both the employed *and* the unemployed workers are better off with stricter employment protection. Firing costs equal six months of the benchmark equilibrium wage increase the average expected lifetime utility value (in lifetime consumption equivalent) of employed workers by 0.8% and of unemployed workers by 0.1%. Twelve-month firing costs induce an increase of 1.1 and 0.2% respectively for these two groups. The social welfare function (46) suggests aggregate welfare losses in the presence of firing costs: the negative effect on profits outweighs the positive effects on worker's welfare (with the caveats mentioned above).

The novel result is that the welfare of unemployed workers increases with firing costs. These results are in contrast with standard models with complete markets, where firing costs unambiguously reduce the welfare of unemployed workers. The quantitative analysis shows that firing costs are associated with potential welfare gains for both employed and unemployed workers in the presence of risk aversion and limited commitment.

## 4 Conclusion

This paper builds an equilibrium model of a labor market with search frictions, idiosyncratic productivity shocks, and dynamic employment contracts signed under limited commitment. The agents in this model have access to a menu of contract types with different separation costs, temporary and permanent contracts. In equilibrium, permanent and temporary jobs coexist when the match-quality distribution is sufficiently dispersed in terms of its range. This coexistence arises from the presence of limited commitment as friction impeding the provision of insurance by employers to workers. Firing costs associated with permanent jobs mitigate the commitment friction, but this comes at the cost of lower flexibility and costly separations. This trade-off between commitment and flexibility explains the coexistence of contracts with heterogeneous explicit job security rules, as seen in the so-called dual labor markets with a strong divide between permanent and temporary jobs and, more generally, in OECD countries.

The model implies that firing costs are potentially associated with significant welfare gains in favor of both employed *and* unemployed workers, in contrast to models with complete markets, where employment protection makes the unemployed worse off. The following implications for policy can be drawn. First, since profit losses due to firing costs are substantial, the social desirability of employment protection in the presence of limited commitment depends on the distribution of firm ownership across agents. Second, the welfare gains materialize when firing costs effectively apply to high-quality matches. The welfare experiment assumes no restriction on temporary contracts: this is the optimal sorting of matches into the different contract types that generate welfare gains for workers. A policy design potentially implying a differentiated treatment of matches based on their quality is one where firing costs increase with tenure since, in equilibrium, high-quality matches have a higher survival probability. Consistent with this conjecture, such tenure-based employment protection schemes prevail in many OECD countries. To rationalize this pervasiveness, future work should analyze the optimal design of tenure-dependent employment protection in the presence of limited commitment and heterogeneity.

# Bibliography

Alonso-Borrego, C., J. Fernández-Villaverde, and J. E. Galdón-Sánchez (2005). Evaluating Labor Market Reforms: A General Equilibrium Approach. Working Paper 11519, National Bureau of Economic Research.

Alvarez, F. and M. Veracierto (2001). Severance Payments in an Economy with Frictions. *Journal of Monetary Economics 47*(3), 477–498.

Bentolila, S., P. Cahuc, J. J. Dolado, and T. Le Barbanchon (2012). Two-Tier Labour Markets in the Great Recession: France versus Spain. *Economic Journal 122*(562), 155–187.

Benveniste, L. M. and J. A. Scheinkman (1979). On the Differentiability of the Value Function in Dynamic Models of Economics. *Econometrica*, 727–732.

Berton, F. and P. Garibaldi (2012). Workers and Firms Sorting into Temporary Jobs. *Economic Journal 122*(562), 125–154.

Blanchard, O. and A. Landier (2002). The Perverse Effects of Partial Labour Market Reform: Fixed-Term Contracts in France. *Economic Journal 112*(480), 214–244.

Bonhomme, S. and G. Jolivet (2009). The Pervasive Absence of Compensating Differentials. *Journal of Applied Econometrics 24*(5), 763–795.

Caggese, A. and V. Cuñat (2008). Financing Constraints and Fixed-Term Employment Contracts. *Economic Journal 118*(533), 2013–2046.

Cahuc, P., P. Carry, F. Malherbet, and P. S. Martins (2022). Employment Effects of Restricting Fixed-Term Contracts: Theory and Evidence. CEPR Discussion Paper No. DP16875.

Cahuc, P., O. Charlot, and F. Malherbet (2016). Explaining the Spread of Temporary Jobs and its Impact on Labor Turnover. *International Economic Review 57*(2), 533–572.

Cahuc, P., F. Malherbet, and J. Prat (2019). The Detrimental Effect of Job Protection on Employment: Evidence from France. CEPR Discussion Paper No. DP13767.

Cahuc, P. and F. Postel-Vinay (2002). Temporary Jobs, Employment Protection and Labor Market Performance. *Labour Economics 9*(1), 63–91.

Cao, S., E. Shao, and P. Silos (2011). Fixed-Term and Permanent Employment Contracts: Theory and Evidence. Bank of Canada Working Paper No. 2011-21.

Chetty, R. (2008). Moral Hazard versus Liquidity and Optimal Unemployment Insurance. *Journal of Political Economy 116*(2), 173–234.

Cozzi, M. and G. Fella (2016). Job Displacement Risk and Severance Pay. *Journal of Monetary Economics 84*, 166–181.

Daruich, D., S. Di Addario, and R. Saggio (2023, 02). The Effects of Partial Employment Protection Reforms: Evidence from Italy. *The Review of Economic Studies*, rdad012.

Dolado, J. J., E. Lalé, and N. Siassi (2021). From Dual to Unified Employment Protection:

Transition and Steady State. *Quantitative Economics 12*(2), 547–585.

Elsby, M. W., B. Hobijn, and A. Şahin (2013). Unemployment Dynamics in the OECD. *Review of Economics and Statistics 95*(2), 530–548.

Faccini, R. (2014). Reassessing Labour Market Reforms: Temporary Contracts as a Screening Device. *Economic Journal 124*(575), 167–200.

Fella, G. and C. J. Tyson (2013). Privately Optimal Severance Pay. *The BE Journal of Macroeconomics 13*(1), 415–453.

Franceschin, R. (2020). Choosing Employment Protection: the Role of on-the-Job Search and Ability Learning. Available at https://drive.google.com/file/d/1h5u2nfrot3V-n18k1AU9Y144e95AL90_/view.

García-Pérez, J. I., I. Marinescu, and J. Vall Castello (2019). Can Fixed-Term Contracts put Low Skilled Youth on a Better Career Path? Evidence from Spain. *The Economic Journal 129*(620), 1693–1730.

Garibaldi, P. and G. L. Violante (2005). The Employment Effects of Severance Payments with Wage Rigidities. *The Economic Journal 115*(506), 799–832.

Givord, P. and L. Wilner (2015). When Does the Stepping-Stone Work? Fixed-Term Contracts versus Temporary Agency Work in Changing Economic Conditions. *Journal of Applied Econometrics 30*(5), 787–805.

Green, E. J. (1987). Lending and the Smoothing of Uninsurable Income. In *Contractual Arrangements for Intertemporal Trade*, pp. 3—25. Minnesota Studies in Macroeconomics series.

Hairault, J.-O., T. Le Barbanchon, and T. Sopraseuth (2015). The Cyclicality of the Separation and Job Finding Rates in France. *European Economic Review 76*, 60–84.

Hall, R. E. (2005). Employment Fluctuations with Equilibrium Wage Stickiness. *American Economic Review 95*(1), 50–65.

Hosios, A. J. (1990). On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies 57*(2), 279–298.

Jung, P. and M. Kuhn (2014). Labour Market Institutions and Worker Flows: Comparing Germany and the U.S. *Economic Journal 124*(581), 1317–1342.

Jung, P. and M. Kuhn (2019). Earnings Losses and Labor Mobility over the Life Cycle. *Journal of the European Economic Association 17*(3), 678–724.

Karabay, B. and J. McLaren (2011). Pareto-Improving Firing Costs? *European Economic Review 55*(8), 1083–1093.

Lalé, E. (2019). Labor-Market Frictions, Incomplete Insurance and Severance Payments. *Review of Economic Dynamics 31*, 411–435.

Lazear, E. P. (1990, 08). Job Security Provisions and Employment. *The Quarterly Journal of Economics 105*(3), 699–726.

Mortensen, D. T. and C. A. Pissarides (1994). Job Creation and Job Destruction in the Theory of Unemployment. *Review of Economic Studies 61*(3), 397–415.

Mortensen, D. T. and C. A. Pissarides (1999). Job Reallocation, Employment Fluctuations and Unemployment. *Handbook of Macroeconomics 1*, 1171–1228.

OECD (2023). Organisation for Economic Co-operation and Development online Data (OECD Data). Url: https://data.oecd.org/. Consulted Feb 1, 2023.

Pissarides, C. A. (2000). *Equilibrium unemployment theory, 2nd edition*. MIT press.

Pissarides, C. A. (2001). Employment Protection. *Labour Economics 8*(2), 131–159.

Rudanko, L. (2009). Labor Market Dynamics under Long-Term Wage Contracting. *Journal of Monetary Economics 56*(2), 170–183.

Shimer, R. (2005). The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review 95*(1), 25–49.

Shimer, R. (2012). Reassessing the Ins and Outs of Unemployment. *Review of Economic Dynamics 15*(2), 127–148.

Silva, J. I. and J. Vázquez-Grenno (2013). The Ins and Outs of Unemployment in a Two-Tier Labor Market. *Labour Economics 24*, 161–169.

Spear, S. E. and S. Srivastava (1987). On Repeated Moral Hazard with Discounting. *Review of Economic Studies 54*(4), 599–617.

Spear, S. E. and C. Wang (2005). When to Fire a CEO: Optimal Termination in Dynamic Contracts. *Journal of Economic Theory 120*(2), 239–256.

Thomas, J. and T. Worrall (1988). Self-Enforcing Wage Contracts. *Review of Economic Studies 55*(4), 541–554.

Thomas, J. P. and T. Worrall (2007). Limited Commitment Models of the Labour Market. *Scottish Journal of Political Economy 54*(5), 750–773.

Wang, C. (2011). Termination of Dynamic Contracts in an Equilibrium Labor Market Model. *Journal of Economic Theory 146*(1), 74–110.

Wang, C. (2013). Incentives and the Cost of Firing in an Equilibrium Labor Market Model with Endogenous Layoffs. *International Economic Review 54*(2), 443–472.

Wang, C. and Y. Yang (2015a). Equilibrium Matching and Termination. *Journal of Monetary Economics 76*, 208–229.

Wang, C. and Y. Yang (2015b). Outside Opportunities and Termination. *Games and Economic Behavior 91*, 207–228.

Wang, C. and Y. Yang (2019). Optimal Self-Enforcement and Termination. *Journal of Economic Dynamics and Control 101*, 161–186.

# Supplementary Appendix

## A Proofs

### A.1 Proof of proposition 1

The following two lemmas present results that are required to characterizing a solution to problem (5) as in proposition (1). Additional details on the contractual environment are useful. Consider a worker-firm match starting at time $t_0 \geq 0$. For clarity, let $t_0 = 0$ and denote by $\sigma_{P,0}$ a permanent contract (PC) starting at time 0. Recalling that by assumption, all matches start with fixed productivity $z_0 = \underline{z}$, denote by $z^t \equiv (z_0, z_1, ..., z_t)$ an element of $\{z_0\} \times Z^t$, where $Z^t$ denotes the $t$-fold product of the support $Z$ of the stochastic output component $z_t$, for all $t > 0$. Each $z^t$ is a (potential) history of the match-output stochastic component (or productivity shocks) from time 0 to $t \geq 0$. Moreover, denote by $(z^{t-1}, z_t) \in \{z_0\} \times Z^t$ the history from time 0 to $t$ that is composed of the history $z^{t-1}$ followed by productivity $z_t$ at time $t$.

A permanent contract specifies a stopping time $0 < T \leq +\infty$ and a contingency plan for separation probabilities, which is a sequence of functions $\{s_t\}_{t=1}^{T} = \{s_1, ..., s_T\}$ defined by

$$s_t : \tilde{Z}^{t-1}(\{s_\tau\}_{\tau=1}^{t-1}) \times Z \to [\delta, 1], \tag{A.1}$$

with

$$\tilde{Z}^t(\{s_\tau\}_{\tau=1}^{t}) \equiv \Big\{(z^{t-1}, z_t) \in \{z_0\} \times Z^t \, : \, z^{t-1} \in \tilde{Z}^{t-1}(\{s_\tau\}_{\tau=1}^{t-1}) \text{ and } s_t(z^t) < 1\Big\}, \tag{A.2}$$

for all $1 \leq t \leq T$, and with initial value

$$s_1 : \{z_0\} \times Z \to [\delta, 1], \tag{A.3}$$

and

$$\tilde{Z}^1(s_1) = \Big\{(z_0, z_1) \in \{z_0\} \times Z \, : \, s_1(z_0, z_1) < 1\Big\}. \tag{A.4}$$

Hence, $\tilde{Z}^t(\{s_\tau\}_{\tau=1}^{t}) \subset \{z_0\} \times Z^t$ is the set of histories of the shock up to time $t$ such that the probability of continuation up to this time is positive given the separation plan specified by the contract. In other words, it is the set of possible histories conditional on the continuation of the match up to time $t \geq 0$ given the previous actions specified by the separation plan of the contract. A PC also sets a wage contingency plan

$$w_t : \tilde{Z}^t(\{s_\tau\}_{\tau=1}^{t}) \to \mathbb{R}_+, \tag{A.5}$$

for $0 \leq t \leq T$.

The employer's expected lifetime discounted profits associated with a PC $\sigma_{P,0}$ satisfies, at time $t$ and history $z^t$ (treating as understood the functional dependence on match quality $x$)

$$\pi_{P,t}(z^t;\sigma_{P,0}) = xz_t - w_t(z^t) + \beta E_t\Bigg\{ \sum_{\tau=t+1}^{\tilde{T}} \beta^{\tau-(t+1)}\Big(1-\mathcal{S}_{\tau-1|t}(z^{\tau-1})\Big)\Big[(1-s_\tau(z^{\tau-1},z_\tau))(xz_\tau - w_\tau(z^{\tau-1},z_\tau)) + s_\tau(z^{\tau-1},z_\tau)\mathcal{J}_0\Big] + \beta^{\tilde{T}}\mathcal{J}_0 - \tilde{\chi}(F;\sigma_{P,0})\Bigg\} \tag{A.6}$$

for all $t$, $0 \leq t \leq T$ and $z^t \in \tilde{Z}^t$. The term $\tilde{\chi}(F;\sigma_{P,0})$ denote the expected costs implied by the separation rule of the contract that are due to firing costs. Moreover, $E_t$ denotes the expectation conditional on history $z^t$ and $t \leq \tilde{T} \leq T$ represents the random separation date implied by the stopping time $T$ and the separation plan that is part of the contract. In addition,

$$1 - \mathcal{S}_{\tau|t}(z^\tau) = \Pi_{\tau'=t}^{\tau}(1 - s_{\tau'}(z^{\tau'})) \tag{A.7}$$

is the match continuation probability up to time $\tau$ in history $z^\tau$, conditional on continuation up to time $t$, in history $z^t$. In the above expressions, it is understood that $z^\tau$ is a history with first $t$ elements equal to $z^t$, that is $z^\tau = (z^t, z_{t+1}, \ldots z_\tau)$.

The worker's expected lifetime discounted utility in a PC $\sigma_{P,0}$ at time $t$ and history $z^t$ is

$$v_{P,t}(z^t;\sigma_{P,0}) = u(w(z^t)) + \beta E_t\Bigg\{ \sum_{\tau=t+1}^{\tilde{T}} \beta^{\tau-(t+1)}\Big(1-\mathcal{S}_{\tau-1|t}(z^{\tau-1})\Big)\Big[(1-s_\tau(z^{\tau-1},z_\tau))u(w_\tau(z^{\tau-1},z_\tau)) + s_\tau(z^{\tau-1},z_\tau)U\Big] + \beta^{\tilde{T}}U\Bigg\}, \tag{A.8}$$

for all $t \in [0,T]$, and $z^t \in \tilde{Z}^t$.

To be feasible, a PC $\sigma_{P,0}$ (and the associated separation plan) must be such that the following participation constraints are satisfied in all histories that can attained with non-zero probabilities:

$$\pi_{P,t}(z^t;\sigma_{P,0}) \geq \mathcal{J}_0 - F \tag{A.9}$$

$$v_{P,t}(z^t;\sigma_{P,0}) \geq U, \tag{A.10}$$

for all $t \in [0,T]$ and all $z^t \in \tilde{Z}^t$.

The following lemma characterizes a separation plan in an optimal contract. It characterizes the set of shocks such that continuation is feasible and an optimal separation decision. It states that optimal separation decisions follow a simple cutoff rule, such that a voluntary separation occurs at time $t \geq 0$ if and only if $z_t < \underline{z}_P$ for some $\underline{z}_P \in Z$.

**Lemma 4.** *(i) For all $t \geq 0$, the set of match-output stochastic components $z_t$ such that there exists a feasible permanent contract is an interval $Z_P = [\underline{z}_P, \overline{z}] \subset Z$; (ii) in an optimal permanent contract, the separation probability is*

$$s_t(z^{t-1}, z_t) = \delta + (1-\delta)\mathcal{I}(z_t < \underline{z}_P), \tag{A.11}$$

*for all $t > 0$, $z^{t-1} \in \{z_0\} \times Z_P^{t-1}$, and all $z_t \in Z$.*

*Proof:* Part (i). Consider a contract $\underline{\sigma}_P$ paying wage $w_t(z^t) = w(U) \equiv u^{-1}\Big[(1-\beta)U\Big]$ in all $t$, $z^t$ with positive probability of continuation, and with separation rule specified by

$$s_t(z^t) = \begin{cases} \delta \text{ if } \pi_{P,t}(z^t; \underline{\sigma}_P) \geq \mathcal{J}_0 - F \\ 1 \text{ if } \pi_{P,t}(z^t; \underline{\sigma}_P) < \mathcal{J}_0 - F, \end{cases} \tag{A.12}$$

for all $t$, $z^t$. Moreover, notice that $w(U) \geq b > 0$ since in equilibrium $U \geq u(b)/(1-\beta)$. Therefore, the contract $\underline{\sigma}_P$ is feasible: it has positive wage payments and delivers non-negative surplus to the worker and the employer in all histories with continuation of the match; specifically, it delivers to the worker lifetime utility equal to the unemployment value $U$ in all histories.

The employer's lifetime profits associated with contract $\underline{\sigma}_P$ can be recursively expressed as

$$\begin{aligned} &\pi_{P,t}((z^{t-1}, z_t); \underline{\sigma}_P) = \\ &xz_t - w(U) + \beta(1-\delta)\int_{\underline{z}}^{\overline{z}} \max\Big\{\pi_{P,t+1}((z^{t-1}, z_t, z_{t+1}); \underline{\sigma}_P), \mathcal{J}_0 - F\Big\} dG_z(z_{t+1}|z_t) + \beta\delta\mathcal{J}_0, \end{aligned} \tag{A.13}$$

for all $t$, $z^{t-1}$, and all $z_t \in Z$, where $(z^t, z_t, z_{t+1})$ denotes history $z^t$ followed by shocks $z_t$ and $z_{t+1}$. Observing that $\pi_{P,t}((z^{t-1}, z_t); \underline{\sigma}_P)$ is independent of time $t$ and of the particular realized history $z^{t-1}$, the lifetime profits associated with contract $\underline{\sigma}_P$ when the current productivity is $z$ can be written as

$$\pi_P(z; \underline{\sigma}_P) = xz - w(U) + \beta(1-\delta)\int_{\underline{z}}^{\overline{z}} \max\Big\{\pi_P(z'; \underline{\sigma}_P), \mathcal{J}_0 - F\Big\} dG_z(z'|z) + \beta\delta\mathcal{J}_0 \tag{A.14}$$

for all $z \in Z$.

Notice that to be feasible in a given state $z \in Z$, the contract $\underline{\sigma}_P$ must satisfy $\pi_P(z; \underline{\sigma}_P) \geq \mathcal{J}_0 - F$. Using standard dynamic-programming arguments, it can be shown the functional equation (A.14) has a unique solution, and that $\pi_P(.; \underline{\sigma}_P) : Z \to \mathbb{R}$ is continuous and strictly increasing in $z$.

As such, if $\pi_P(z; \underline{\sigma}_P) \geq \mathcal{J}_0 - F$ for $z = \underline{z}$, the same holds for any $z \in Z$. In that case, $\underline{z}_P = \underline{z}$ and $Z_P = Z$: there exists a feasible PC for all $z \in Z$. In the alternative case where $\pi_P(z; \underline{\sigma}_P) < \mathcal{J}_0 - F$ for $z = \underline{z}$, there is a unique cutoff $\underline{z}_P > \underline{z}$ solving $\pi_P(\underline{z}_P; \underline{\sigma}_P) = \mathcal{J}_0 - F$, such that the profits satisfy $\pi_P(z; \underline{\sigma}_P) \geq \mathcal{J}_0 - F$ if and only if $z \geq \underline{z}_P$. In this case, $Z_P = [\underline{z}_P, \overline{z}]$ is a strict subset of $Z$ (with $\underline{z}_P > \underline{z}$).

I now show that in the case where $\underline{z}_P > \underline{z}$, there is no feasible PC in any state $z$ such that $z < \underline{z}_P$. Let $\underline{z}_P > \underline{z}$, and consider a contract $\underline{\sigma}'_P$ identical to $\underline{\sigma}_P$, except that it specifies $s_t(z^{t-1}, z_t) < 1$ for some arbitrary $t \geq 0$, $z^{t-1} \in \{z_0\} \times Z_P^t$, and $z_t \in [\underline{z}_P - \varepsilon, \underline{z}_P) \cap Z$, and for some arbitrary number $\varepsilon > 0$. Such a contract is *not* feasible, since it has

$$\pi_{P,t}((z^{t-1}, z_t); \underline{\sigma}'_P) < \mathcal{J}_0 - F,$$

for these particular arbitrary $t$, $z^{t-1}$, $z_t$, and for any number $\varepsilon > 0$.

Suppose now that there exists a feasible contract $\underline{\sigma}''_P$ with the same separation rule as $\underline{\sigma}'_P$. Since this contract is assumed feasible, it must have

$$\pi_{P,t}((\underline{z}^{t-1}, z_t); \underline{\sigma}''_P) \geq \mathcal{J}_0 - F > \pi_{P,t}((z^{t-1}, z_t); \underline{\sigma}'_P), \tag{A.15}$$

for the same $t$, $z^{t-1}$, $z_t$. Moreover, $\underline{\sigma}''_P$ must deliver to the worker a lifetime utility value at least equal to that of unemployment, $U$.

The first inequality of the latter condition (A.15) means that $\underline{\sigma}''_P$ must have a different wage plan and, therefore, must deliver a different expected consumption profile to the worker than the non-feasible contract $\underline{\sigma}'_P$. However, since the worker is risk averse, it would be weakly be more costly for the employer to deliver lifetime utility $U$ with such a contract $\underline{\sigma}''_P$ than with $\underline{\sigma}'_P$, which has a constant worker's consumption level $w(U)$ across all continuation histories. As such, we must have

$$\pi_{P,t}((\underline{z}^{t-1}, z_t); \underline{\sigma}''_P) \leq \pi_{P,t}((z^{t-1}, z_t); \underline{\sigma}'_P),$$

which contradicts (A.15). Since $t$, $z_t$, and $\varepsilon > 0$ where chosen arbitrarily, the same holds for any deviation from the cutoff separation rule associated with contract $\underline{\sigma}_P$. Therefore, for all $t \geq 0$, there is no feasible PC for $z_t < \underline{z}_P$.

Part (ii). Consider a feasible PC $\sigma^*_{P,0}$ with the cutoff separation rule as in lemma 4. Consider an alternative PC $\tilde{\sigma}_{P,0}$, similar to $\sigma^*_{P,0}$ but with $s_t(z^t) > \delta$ in some $t \geq 0$ and some $z^t \in \{z_0\} \times Z_P^t$. The employer's change in profits at $t = 0$ from deviating from $\sigma^*_{P,0}$ to $\tilde{\sigma}_{P,0}$ satisfies

$$\begin{aligned}
&\pi_{P,0}(z_0; \tilde{\sigma}_{P,0}) - \pi_{P,0}(z_0; \sigma^*_{P,0}) \\
&\propto (1 - s_t(z^t))\pi_{P,t}(z^t; \tilde{\sigma}_{P,0}) + s_t(z^t)\mathcal{J}_0 + (s_t(z^t) - \delta)(-F) - (1 - \delta)\pi_{P,t}(z^t; \sigma^*_{P,0}) - \delta \mathcal{J}_0 \\
&= -(s_t(z^t) - \delta)(\pi_{P,t}(z^t; \sigma^*_{P,0}) - (\mathcal{J}_0 - F)) - (s_t(z^t) - \delta)F \leq 0,
\end{aligned}$$

where the equality of the third line follows from $\pi_{P,t}(z^t; \sigma^*_{P,0}) = \pi_{P,t}(z^t; \tilde{\sigma}_{P,0})$ for $z^t$ where the deviation in the separation rule occurs (since by assumption, the contracts are identical otherwise). The inequality follows from $s_t(z^t) > \delta$ and from the fact that $\sigma^*_{P,0}$ is feasible and, therefore, satisfies (A.9). Moreover, the worker's change in lifetime utility from the deviation from $\sigma^*_{P,0}$ to $\tilde{\sigma}_{P,0}$ is

$$v_{P,0}(z_0; \tilde{\sigma}_{P,0}) - v_{P,0}(z_0; \sigma^*_{P,0}) \propto -(s_t(z^t) - \delta)(v_{P,t}(z^t; \sigma^*_{P,0}) - U) \leq 0, \tag{A.16}$$

where the inequality follows from the fact that $\sigma^*_{P,0}$ is feasible and, therefore, satisfies (A.10). Therefore, for any given feasible wage plan, any feasible separation plan that differs from the one specified by $\sigma^*_{P,0}$ leaves the employer *and* the worker worse off. The separation rule stated in lemma 4 is optimal.

The next lemma characterizes the set of lifetime utility values that can be delivered to a worker by a feasible PC when continuation is feasible.

**Lemma 5.** *For all $t \geq 0$ and all $z_t \geq \underline{z}_P$, the set of expected lifetime utility values that can be delivered to the worker by a feasible PC at time $t$, in state $z_t$ is a closed and bounded interval $\mathcal{V}_P = [U, \overline{v}_P(z_t)]$, $\overline{v}_P(z_t) \geq U$ (independent of the history leading to $z_t$). Moreover, $\overline{v}_P(z_t) = U$ for $z_t = \underline{z}_P$, and $\overline{v}_P(z_t)$ is strictly increasing with $z_t$ for $z_t \geq \underline{z}_P$, for all $t \geq 0$*

*Proof.* Consider a permanent contract $\overline{\sigma}_P(z_t)$ with an efficient cutoff separation rule as specified in lemma 4. Let this contract pay wage $\overline{w}_P(z_t)$ at time $t$ and state $z_t \geq \underline{z}_P$ independently of the history leading to $z_t$, where $\overline{w}_P : Z_P \to \mathbb{R}_+$ is defined by

$$\begin{aligned}
&\pi_{P,t}((z^{t-1}, z); \overline{\sigma}_P(z)) = \\
&\qquad xz - \overline{w}_P(z) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \max\Big(\pi_{P,t+1}((z^{t-1}, z, z'); \overline{\sigma}_P(z)), \mathcal{J}_0 - F\Big) dG_z(z'|z) \\
&\qquad + \beta(1-\delta)G_z(\underline{z}_P|z)(\mathcal{J}_0 - F) + \beta\delta(-F),
\end{aligned} \tag{A.17}$$

with

$$\pi_{P,t}((z^{t-1}, z); \overline{\sigma}_P(z)) = \mathcal{J}_0 - F, \tag{A.18}$$

for any $t \geq 0$, $z^{t-1} \in \{z_0\} \times Z_P^{t-1}$, and $z \geq \underline{z}_P$. Finally, let $\overline{\sigma}_P(z_t)$ have a wage plan satisfying the wage updating rule

$$w_\tau(z^{\tau-1}, z_\tau) = \min\Big\{w_{\tau-1}(z^{\tau-1}), \overline{w}_P(z_\tau)\Big\}, \tag{A.19}$$

for all $\tau > t$, $z^{\tau-1}$, and all $z_\tau \geq \underline{z}_P$.

In words: a voluntary separation occurs if and only if $z < \underline{z}_P$; the contract has a constant wage across time until the employer's participation constraint binds, in which case it is adjusted downward; the wage payment in state $z_t$, given by (A.17), is such that the firm's lifetime profits are equal to its outside option. Note that this wage dynamics is consistent with the efficient wage updating rule in long-term contracts under limited commitment as characterized by Thomas and Worrall (1988) and Thomas and Worrall (2007).

Such a PC $\overline{\sigma}_P(z_t)$ leaves the worker with lifetime utility

$$\begin{aligned}
&v_{P,t}((z^{t-1}, z_t); \overline{\sigma}_P(z_t)) = u(\overline{w}_P(z_t)) \\
&\qquad + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} v_{P,t}((z^{t-1}, z_t, z_{t+1}); \overline{\sigma}_P(z_t)) dG_z(z_{t+1}|z_t) + \beta\Big[\delta + (1-\delta)G_z(\underline{z}_P|z_t)\Big]U.
\end{aligned} \tag{A.20}$$

for all $t$ and $(z^{t-1}, z_t)$. Using the wage updating rule (A.19), we have that the lifetime utility value (A.20) is independent of $t$ and $z^{t-1}$. Specifically, we can write

$$v_{P,t}((z^{t-1}, z_t); \overline{\sigma}_P(z_t)) = \overline{v}_P(z_t), \tag{A.21}$$

where $\overline{v}_P : Z_P \to \mathbb{R}$ is defined by

$$\overline{v}_P(z) = u(\overline{w}_P(z)) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \min\Big\{\overline{v}_P(z), \overline{v}_P(z')\Big\} dG_z(z'|z) + \beta\Big[\delta + (1-\delta)G_z(\underline{z}_P|z)\Big]U, \tag{A.22}$$

for all $z \geq \underline{z}_P$, with $w_P$ defined by (A.17). It can be shown using standard arguments that the functional equation (A.22) has a unique solution.

Moreover, since the employer's profits are strictly increasing in $z_t$, we have that $\overline{w}_P(z_t) \geq w(U) \equiv u^{-1}[(1-\beta)U]$ for all $z_t \geq \underline{z}_P$, due to the fact that $w(U)$ is the constant wage such that the employer's surplus is null in state $z_t$. This implies that $\overline{v}_P(z_t) \geq U$ because the period utility function $u$ is strictly increasing in consumption. Also, a contract $\overline{\sigma}_P(z_t)$ leaves, by construction, the employer better off in any history with continuation of the match. It follows that the PC $\overline{\sigma}_P(z_t)$, delivering lifetime utility $\overline{v}_P(z_t)$ in any state $z_t \geq \underline{z}_P$, is feasible. Moreover, since the period utility function $u$ is continuous and strictly increasing in consumption, it is clear that any lifetime utility level $v \in [U, \overline{v}_P(z_t)]$ can be delivered by a feasible contract.

It remains to show that there is no feasible PC that can deliver $v > \overline{v}_P(z_t)$ in any state $z_t \geq \underline{z}_P$. Consider a PC $\overline{\sigma}'_P(z_t)$ identical to $\overline{\sigma}_P(z_t)$ except that it has an additional wage payment of an amount $\varepsilon > 0$ in an arbitrary history $z^\tau \in \{z_0\} \times Z_P^\tau$ with $\tau \geq t$. Since, by construction of contract $\overline{\sigma}(z_t)$, we have that the profits satisfy $\pi_{P,t}((z^{t-1}, z_t); \overline{\sigma}(z_t)) = \mathcal{J}_0 - F$ in this specific $z_t$, this additional payment, must be compensated by a negative variation in the wage payment in another history $z^{\tau'} \in \{z_0\} \times Z_P^{\tau'}$, $\tau' \geq t$. Denote by $\varepsilon' > 0$ the absolute value of this variation.

The employer's change in expected lifetime profits associated with deviating from $\overline{\sigma}_P(z_t)$ to $\overline{\sigma}'_P(z_t)$ at time $t \geq 0$, state $z_t$ is

$$[\beta(1-\delta)(1-\lambda G(\underline{z}_P))]^{\tau-t}\Pi_\tau(z^\tau|z_t)(-\varepsilon) + [\beta(1-\delta)(1-\lambda G(\underline{z}_P))]^{\tau'-t}\Pi_{\tau'}(z^{\tau'}|z_t)\varepsilon', \tag{A.23}$$

where $\Pi_\tau(z^\tau|z_t)$ denotes the probability of reaching history $z^\tau$ at time $\tau$ conditional on state $z_t$ at $t$. To be feasible, this change in profits must be non-negative, i.e., it must satisfy

$$\varepsilon' \geq \Gamma(z^\tau, z^{\tau'})\varepsilon, \tag{A.24}$$

where

$$\Gamma(z^\tau, z^{\tau'}) \equiv [\beta(1-\delta)(1-\lambda G(\underline{z}_P))]^{\tau-\tau'}\frac{\Pi_\tau(z^\tau|z)}{\Pi_{\tau'}(z^{\tau'}|z)}. \tag{A.25}$$

The worker's change in lifetime utility from the same deviation is

$$\begin{aligned}
&[\beta(1-\delta)(1-\lambda G(\underline{z}_P))]^{\tau-t}\pi_\tau(z^\tau|z_t)\Big[u(w_\tau(z^\tau)+\varepsilon)-u(w_\tau(z^\tau))\Big] \\
&\qquad\qquad -[\beta(1-\delta)(1-\lambda G(\underline{z}_P))]^{\tau'-t}\pi_{\tau'}(z^{\tau'}|z_t)\Big[u(w_{\tau'}(z^{\tau'})-\varepsilon')-u(w_{\tau'}(z^{\tau'})\Big] \\
&\le [\beta(1-\delta)(1-\lambda G(\underline{z}_P))]^{\tau-t}\pi_\tau(z^\tau|z_t)\Big[u(w_\tau(z^\tau)+\varepsilon)-u(w_\tau(z^\tau))\Big] \\
&\qquad\qquad -[\beta(1-\delta)(1-\lambda G(\underline{z}_P))]^{\tau'-t}\pi_{\tau'}(z^{\tau'}|z_t)\Big[u(w_{\tau'}(z^{\tau'})-\Gamma(z^\tau,z^{\tau'})\varepsilon)-u(w_{\tau'}(z^{\tau'})\Big] \\
&\equiv h(\varepsilon),
\end{aligned} \tag{A.26}$$

where the inequality follows from the feasibility condition (A.24). Note that $h(0)=0$. Moreover,

$$\frac{dh(\varepsilon)}{d\varepsilon} \propto u'(w_\tau(z^\tau)+\varepsilon)-u'(w_\tau(z^\tau)-\Gamma(z^\tau,z^{\tau'})\varepsilon). \tag{A.27}$$

Two cases can be distinguished, depending on the value of $\tau'$.

Case 1: Suppose that $\tau' \ge \tau$. Then, we must have $w_\tau(z^\tau) \ge w_{\tau'}(z^{\tau'})$, since the wage is decreasing over time in contract $\overline{\sigma}_P(z_t)$, due to updating rule (A.19). Then, the right-hand side of equation (A.27) is negative due to $\Gamma(z^\tau,z^{\tau'})\varepsilon > 0$ and the strict concavity of $u$. In this case, $h(\varepsilon) < 0$ for all $\varepsilon > 0$: any $\varepsilon > 0$ leaves the worker worse-off.

Case 2: Now, suppose $\tau' < \tau$, implying that $w_\tau(z^\tau) \le w_{\tau'}(z^{\tau'})$. There are two possible sub-cases, depending on the nature of the latter inequality. Case 2.1: we have $w_\tau(z^\tau) = w_{\tau'}(z^{\tau'})$. The RHS of (A.27) is negative, due to $u'' < 0$. Then, $h(\varepsilon) < 0$ for all $\varepsilon > 0$: as in case 1, any $\varepsilon > 0$ leaves the worker worse-off.

Case 2.2: $w_\tau(z^\tau) < w_{\tau'}(z^{\tau'})$. However, note that a such wage profile implies that the participation constraints of the employer must be binding between time $\tau'$ and $\tau$ in the *initial* contract $\underline{\sigma}_P(z_t)$, consistent with the wage updating rule (A.19). That is, it must bind in some history $z^{\tau''}$ with $\tau' < \tau'' \le \tau$. However, with $\tau' < \tau'' \le \tau$ and $\varepsilon > 0$, the contract $\overline{\sigma}'(z_t)$ is strictly more costly to the employer in any history $z^{\tau''}$ than the initial contract $\overline{\sigma}(z_t)$, when considering the forward-looking profits expected at time $\tau''$. Therefore, we should have

$$\pi_{P,\tau''}(z^{\tau''};\overline{\sigma}(z_t)) = \mathcal{J}_0 - F > \pi_{P,\tau''}(z^{\tau''};\overline{\sigma}'(z_t)).$$

This implies that $\overline{\sigma}'(z_t)$ with deviation occurring at time $\tau' < \tau$ and in history $z^{\tau''}$ does not satisfy the employer's participation constraint in that history and, therefore, is not feasible.

Therefore, in state $z_t \ge \underline{z}_P$, there is no feasible deviation from contract $\overline{\sigma}(z_t)$ yielding to the worker an expected lifetime utility value strictly higher than $\overline{v}_P(z)$. This implies that the set of feasible utility values in state $z_t \ge \underline{z}_P$ is an interval $[U, \overline{v}_P(z)]$, for all $t \ge 0$.

Finally, the fact that $\overline{v}_P(\underline{z}_P) = U$ follows from $\pi_P(\underline{z}_P;\underline{\sigma}_P) = \mathcal{J}_0 - F$ (recall that $\underline{\sigma}_P$ is the constant-wage contract paying $w(U)$, which delivers lifetime utility $U$ to the worker in all histories such that the match continues). The fact that $\overline{v}_P$ is strictly increasing with $z$ is due to the fact that the profit function $\pi_{P,t}$ is strictly increasing with $z_t$, and so is the constrained wage function $\overline{w}_P$, defined by (A.17), and on which depends the function $\overline{\nu}_P$.

**Lemma 6.** *(i) There exists a unique solution $J_P$ to the functional equation (5); (ii) for any $z \in Z_P$, $J_P(.,z)$ is strictly decreasing, strictly concave, and continuously differentiable.*

Part (i). Let

$$\Omega_P = \Big\{(V,z) \in \mathcal{V} \times Z_P \,:\, V \in [U, \overline{\nu}_P(z)]\Big\}, \tag{A.28}$$

and consider the function $T_P J : \Omega_P \to \mathbb{R}_+$ defined by

$$\begin{aligned} T_P J(V,z) = \max_{\{w_P,(\nu_P(z'))_{z' \geq \underline{z}_P}\} \in \Gamma_P(V,z)} & \; xz - w_P + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \Big(J(\nu_P(z'),z') - (\mathcal{J}_0 - F)\Big) dG_z(z'|z) \\ & + \beta[\mathcal{J}_0 - (1-\delta)F] \end{aligned} \tag{A.29}$$

for $z \in Z_P$ and $V \in \mathcal{V}_P(z) = [U, \overline{\nu}_P(z)]$, where

$$\begin{aligned} \Gamma_P(V,z) = \Bigg\{ w_P, (\nu_P(z'))_{z' \geq \underline{z}_P} \; & : w_P \geq 0 \\ & u(w_P) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} (\nu_P(z') - U) dG_z(z'|z) + \beta U = V \\ & \nu_P(z') \in [U, \overline{\nu}_P(z')],\; z' \geq \underline{z}_P \Bigg\}, \end{aligned} \tag{A.30}$$

for all $z \geq Z_P$, $V \in \mathcal{V}_P(z)$ and for a function $J : \Omega_P \to \mathbb{R}_+$. From lemmas 4 and 5, an optimal contract yields profits that are the value of a function $J_P : \Omega_P \to \mathbb{R}_+$ satisfying $T_P J_P = J_P$, that is $J_P$ is a fixed point of the operator $T_P$ defined by (A.29).

Let $B(\Omega_P)$ denotes the set of bounded real-valued functions with domain $\Omega_P$, equipped with the sup-norm metric. Observe that $J \in B(\Omega_P)$ implies that $T_P J \in B(\Omega_P)$. Indeed, we must have that $w_P \geq 0$ according to conditions associated with (A.30). Moreover, using constraint (8), we have that

$$\begin{aligned} w_P(V,z) &= u^{-1}\Big\{V - \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} (\nu_P(z') - U) dG_z(z'|z) - \beta U\Big\} & \text{(A.31)} \\ &\leq u^{-1}\Big\{V - \beta U\Big\}, & \text{(A.32)} \end{aligned}$$

for all $z \geq \underline{z}_P$, $V \in \mathcal{V}_P(z)$, since we must have $\nu_P(z') \geq U$ for all $z' \geq \underline{z}_P$ from (A.30). Hence, $T_P J$ is bounded and $T_P$ defines an operator mapping the space $B(\Omega_P)$ into itself.

Moreover, the operator $T_P$ satisfies the sufficient Blackwell's conditions for being a contraction in $B(\Omega_P)$. First, the monotonicity condition is satisfied. Take $J, J' \in B(\Omega_P)$, with $J < J'$ for all $(V,z) \in \Omega_P$. Let $\tilde{w}$ and $\tilde{v}$ be maximizers to the RHS of (A.29) evaluated at $J$, and $\tilde{w}'$ and $\tilde{v}'$ be maximizers for $J'$. We have:

$$\begin{aligned} T_P J'(V,z) &= xz - \tilde{w}'(V,z) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \Big(J'(\tilde{v}'(z'),z') - (\mathcal{J}_0 - F)\Big) dG_z(z'|z) + \beta[\mathcal{J}_0 - (1-\delta)F] \\ &\geq xz - \tilde{w}(V,z) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \Big(J'(\tilde{v}(z'),z') - (\mathcal{J}_0 - F)\Big) dG_z(z'|z) + \beta[\mathcal{J}_0 - (1-\delta)F] \\ &> xz - \tilde{w}(V,z) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \Big(J(\tilde{v}(z'),z') - (\mathcal{J}_0 - F)\Big) dG_z(z'|z) + \beta[\mathcal{J}_0 - (1-\delta)F] \\ &= T_P J(V,z), \end{aligned}$$

for all $z \geq \underline{z}_P$, $V \in \mathcal{V}_P(z)$, where the weak inequality of the second line comes from that $\tilde{w}'$ and $\tilde{v}'$ are optimal for $J'$ and from that $\tilde{w}$ and $\tilde{v}$ are feasible since these are maximizers for $J$. The strict inequality of the third line comes from $J' > J$.

Moreover, $T_P$ satisfies the discounting condition. Recall that $\beta \in (0,1)$. Taking any $a > 0$, we have

$$\begin{aligned} T_P(J(V,z)+a) = &\max_{\{w_P,(v_P(z'))_{z'\geq \underline{z}_P}\}\in\Gamma_P(V,z)} xz - w_P + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \Big[J(v_P(z'),z') - (\mathcal{J}_0 - F)\Big] dG_z(z'|z) \\ &\qquad + \beta[\mathcal{J}_0 - (1-\delta)F] + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} a\, g_z(z'|z)dz' \\ &= T_P J(V,z) + \beta(1-\delta)\Big[1 - G_z(\underline{z}_P|z)\Big]a \\ &< T_P J(V,z) + \beta a \end{aligned} \tag{A.33}$$

for all $z, V$ and all $J \in B(\Omega_P)$. It follows that the operator $T_P$ has a unique fixed point $J_P = T_P J_P$ solution to the Bellman equation (5) in the complete metric space $B(\Omega_P)$.

Part (ii). The fact that $J_P$ is strictly decreasing in $V$ follows from the fact that the period utility function $u$ is strictly increasing combined with the equality constraint (8). Moreover, $J_P$ strictly concave follows from the strict concavity of $u$.

I now show that $J_P$ is continuously differentiable. Take any $z \geq \underline{z}_P$ and let the function $W : \Omega_P \to \mathbb{R}$ be defined by

$$\begin{aligned} W(V',z) = xz - \hat{w}(V',V,z) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \Big[J_P(\tilde{v}_P(V,z,z'),z') - (\mathcal{J}_0 - F)\Big] dG_z(z'|z) \\ + \beta[\mathcal{J}_0 - (1-\delta)F] \end{aligned} \tag{A.34}$$

with

$$\hat{w}(V',V,z) = u^{-1}\Big\{V' - \beta\int_{\underline{z}_P}^{\overline{z}} (\tilde{v}_P(V,z,z') - U) dG_z(z'|z) - \beta U\Big\}, \tag{A.35}$$

for $z \geq \underline{z}_P$, $V \in [U, \overline{v}_P(z)]$, where $\tilde{v}_P$ denotes maximizers of the RHS of (5). For $z > \underline{z}_P$ and $V \in (U, \overline{v}_P(z))$, there is a neighborhood of $V$ in $(U, \overline{v}_P(z))$ denoted by $\mathcal{N}(V)$ such that for any $V' \in \mathcal{N}(V)$, the actions $\hat{w}(V', V, z)$ and $\tilde{v}_P(V, z, .)$ are feasible but not necessarily optimal. It follows that

$$W(V', z) \leq J(V', z), \tag{A.36}$$

for $z > \underline{z}_P$, $V' \in \mathcal{N}(V)$, with $W(V', z) = J(V', z)$ if $V' = V$. From Benveniste and Scheinkman (1979), since the function $J_P(., z)$ is concave, it is differentiable at $V \in (U, \overline{v}(z))$ with

$$\frac{\partial J_P(V,z)}{\partial V} = -\left\{u'\left[V - \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} (v_P^*(z') - U)\, dG(z'|z) - \beta U\right]\right\}^{-1}, \tag{A.37}$$

which is continuous due to the fact that the period utility function $u$ is continuously differentiable.

*Optimality conditions.* Using lemmas 4 and 5, problem (5) can be written as

$$\begin{aligned} J_P(V,z) = \max_{\{w_P, (v_P(z'))_{z' \geq \underline{z}_P}\}} \; & xz - w_P + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} \left[J_P(v_P(z'), z') - (\mathcal{J}_0 + F)\right] dG_z(z'|z) \\ & + \beta[\mathcal{J}_0 - (1-\delta)F] \end{aligned} \tag{A.38}$$

such that

$$w_P \geq 0 \tag{A.39}$$

$$u(w_P) + \beta(1-\delta)\int_{\underline{z}_P}^{\overline{z}} (v_P(z') - U)\, dG_z(z'|z) + \beta U = V \tag{A.40}$$

$$v_P(z') - U \geq 0, \text{ for } z' \geq \underline{z}_P \tag{A.41}$$

$$\overline{v}_P(z') - v_P(z') \geq 0, \text{ for } z' \geq \underline{z}_P, \tag{A.42}$$

for $z \geq \underline{z}_P$ and $V \in \mathcal{V}_P(z)$, where $\overline{v}_P(z')$ solves

$$J_P(\overline{v}_P(z'), z') = \mathcal{J}_0 - F,$$

for $z' \geq \underline{z}_P$. In addition, define $\overline{w}_P(z') \equiv w_P(\overline{v}_P(z'), z')$. Notice that these variables $\overline{v}_P$ and $\overline{w}_P$ are equivalent to those defined by (A.17) and (A.22). Recall that $\overline{w}_P$ is strictly increasing in $z$ over $[\underline{z}_P, \overline{z}]$.

Recall from the above discussion (proof of lemma 4 that the contract $\underline{\sigma}_P$ paying constant wage $w(U)$ whenever continuation is feasible is a contract delivering efficiently lifetime utility equal to the unemployment value $U$ to the worker. Notice from this discussion that the wage $w(U)$ is the lowest wage that is paid in an optimal contract, since the worker's utility is strictly increasing in consumption: any PC delivering lifetime utility $V > U$ must have wage payments that are weakly greater than $w(U)$ in all histories. Moreover, remember that this wage has $w(U) = u^{-1}((1-\beta)U) > b > 0$. As such, the following focuses on a solution with $w_p > 0$ ((A.39) slack).

Denote by $\gamma, \mu_w(z'), \mu_e(z')$ the Lagrange multipliers associated with constraints (A.40) to (A.42). The necessary conditions for a solution to (5) are

$$1 - \gamma u'(w_P) = 0 \tag{A.43}$$

$$\frac{\partial J_P(\nu_P(z'), z')}{\partial V} + \gamma + \mu_w(z') - \mu_e(z') = 0 \tag{A.44}$$

$$\mu_w(z')\Big(\nu_P(z') - U\Big) = 0 \tag{A.45}$$

$$\mu_e(z')\Big(\overline{\nu}_P(z') - \nu_P(z')\Big) = 0 \tag{A.46}$$

$$\gamma,\ \mu_w(z'),\ \mu_e(z') \geq 0, \tag{A.47}$$

for $z \geq \underline{z}_P$, $V \in \mathcal{V}_p(z)$, and $z' \geq \underline{z}_P$. Moreover, by the envelope theorem, we have:

$$\frac{\partial J_P(V,z)}{\partial V} = -\gamma, \tag{A.48}$$

for $z \geq \underline{z}_P$, $V \in \mathcal{V}_p(z)$. The FOC's (A.43) and (A.44) and the envelope condition (A.48) imply

$$\frac{1}{u'(w_P(\nu_P(z'), z'))} = \frac{1}{u'(w_P)} + \mu_w(z') - \mu_e(z'), \tag{A.49}$$

for all $z' \geq \underline{z}_P$, where $w_P(\nu_P(z'), z')$ denotes the optimal next-period wage associated with next-period state $z'$ and with the worker's continuation lifetime utility value $\nu_P(z')$.

First, notice that $\mu_w(z') > 0$ and $\mu_e(z') > 0$ imply that $\nu_P(z') = \overline{\nu}_P(z') = U$, which holds if and only if $z' = \underline{z}_P$ (see lemma 5). Therefore, $w_P(\nu_P(z'), z') = w(U)$ for $z' = \underline{z}_P$. Moreover, for $z' > \underline{z}_P$, $\mu_w(z') > 0$ and $\mu_e(z') = 0$ implies, from (A.45), that $\nu_P(z') = U$, and therefore, $w_P(\nu_P(z'), z') = w(U)$. However, from (A.49), and from the strict concavity of $u$, this also implies that $w_P < w(U)$, contradicting that $w(U)$ is the lowest wage in an optimal PC.

Hence, for $z' > \underline{z}_P$, we must have $\mu_e(z') \geq 0$ and $\mu_w(z') = 0$. This implies, from (A.49) and from $u'' < 0$, $w_P(V,z) \leq w_P(\nu_P(z'), z')$ for all $z' > \underline{z}_P$. If $w_P(\nu_P(z'), z') < w_P(V,z)$, then we must have, from (A.49) that $\mu_e(z') > 0$. From (A.46), this means that $\nu_e(z') = \overline{\nu}_e(z')$. Therefore, by definition, $w_P(\nu_P(z'), z') = \overline{w}_P(z') = w_P(\overline{\nu}_P(z'), z')$. In addition, if $w_P(V,z) = w_P(\nu_P(z'), z')$, $\mu_e(z') = 0$, and $\nu_P(z') \leq \overline{\nu}_P(z')$, implying that $w_P(\nu_P(z'), z') \leq \overline{w}_P(z')$.

Hence, given any current state $(V, z)$, the next-period wage is

$$w_P(\nu_P(z'), z') = \begin{cases} w_P(V,z) & \text{if } w_P(V,z) \leq \overline{w}_P(z') \\ \overline{w}_P(z') & \text{otherwise,} \end{cases} \tag{A.50}$$

for $z' \geq \underline{z}_P$. This is consistent with the wage updating rule in proposition 1. Finally, observe that, since productivity shocks are i.i.d., the continuation value satisfies, given current state $(V, z)$

$$\begin{aligned} \nu_P(z') &= \mathcal{I}(w_P(V,z) \leq \overline{w}_P(z'))V + \mathcal{I}(w_P(V,z) > \overline{w}_P(z'))\overline{\nu}_P(z') \\ &= \mathcal{I}(V \leq \overline{\nu}_P(z'))V + \mathcal{I}(V > \overline{\nu}_P(z'))\overline{\nu}_P(z') \\ &= \min(V, \overline{\nu}_P(z')) \end{aligned}$$

for $z' \geq \underline{z}_P$: due to i.i.d. shocks, the worker's continuation is equal to the worker's promised value if the wage stays constant, and is independent of $z'$. Therefore, the wage function solves

$$\begin{aligned} w_P(V,z) &= u^{-1}\left\{V - \beta(1-\delta)\left[\int_{\underline{z}_P}^{\overline{z}} \min(V, \overline{\nu}_P(z'))dG_z(z'|z) + G_z(\underline{z}_P|z)U\right] - \beta\delta U\right\} \\ &= u^{-1}\left\{V - \beta(1-\delta)\left[(1-\lambda)V + \lambda\int_{\underline{z}_P}^{\overline{z}} \min(V, \overline{\nu}_P(z'))dG(z') + \lambda G(\underline{z}_P)U\right] - \beta\delta U\right\}, \\ &= w_P(V), \end{aligned} \tag{A.51}$$

for all $z \geq \underline{z}_P$, and $V \in \mathcal{V}_P(z)$, where the second line follows from the definition of $G_z(.|z)$. The wage function is independent of $z$, as stated in proposition 1. □

## A.2 Proof of proposition 2

Denote by $\sigma_{T,0}$ a temporary contract (TC) signed at time $t = 0$. This specifies: a stopping time $0 < T \leq +\infty$, separation probabilities $s_{T,t}(z^t)$ and $s_{c,t}(z^t)$ (contingent on whether the exogenous duration restrictions are slack or binding, see subsection 2.1), a wage payment, $w(z^t)$, and an expected lifetime utility value that is delivered to the worker conditional on a conversion of the TC into a PC, $\nu_c(z^t)$, for all $t \geq 0$ and all histories $z^t$ such that the probability of continuation up to $t$ is positive. Denote by $\pi_{T,t}(z^t;\sigma_{T,0})$ the expected lifetime profits at time $t$, history $z^t$ of TC $\sigma_{T,0}$ and by $v_{T,t}(z^t;\sigma_{T,0})$ the worker's expected lifetime utility value. A feasible TC (and the associated separation plan) satisfies

$$\begin{aligned} \pi_{T,t}(z^t;\sigma_{T,0}) &\geq \mathcal{J}_0 \\ v_{T,t}(z^t;\sigma_{T,0}) &\geq U, \end{aligned} \tag{A.52}$$

conditional on continuation into the TC, and

$$\begin{aligned} \nu_c(z^t) &\geq U \\ J_P(\nu_c(z^{t-1}, z_t), z_t) &\geq \mathcal{J}_0 \end{aligned} \tag{A.53}$$

conditional on conversion into a PC, for all time $t \geq 0$, and all histories $z^t$ where continuation up to time $t$ has positive probability given the separation plan in that contract. Notice that the latter feasibility constraint is expressed in terms of the stationary recursive PC profit function $J_P$ (depending on current $z_t$ only).

The following lemmas characterize the state space of a feasible TC (and an optimal separation plan).

**Lemma 7.** *(i) For all $t \geq 0$, the set of values of $z_t$ such that a conversion of a TC into a permanent contract is feasible is an interval $Z_c = [\underline{z}_c, \overline{z}] \subset Z$. In an optimal TC, the probability of separation satisfies, in cases where the exogenous duration restriction binds:*

$$s_{c,t}(z^{t-1}, z_t) = \delta + (1-\delta)\mathcal{I}(z_t < \underline{z}_c), \tag{A.54}$$

*for all $t$, $z^{t-1}$ such that the probability of continuation is positive up to time $t$ and for all $z_t \in Z$. (ii) The set of continuation lifetime utility values that can be delivered to the worker conditional on a conversion of the TC into a PC at time $t$, in state $z_t$ is an interval $\mathcal{V}_c(z) = [U, \overline{v}_c(z)]$, for all $t \geq 0$ and all $z_t \geq \underline{z}_c$.*

*Proof:* Part (i). To be feasible, conversion of a TC into a PC in history $(z^{t-1}, z_t)$ must be such that the participation constraints (A.53) are satisfied. Since $J_P$ is strictly decreasing in the worker's lifetime utility value (lemma 6) and strictly increasing the current productivity, there exists a value $v_{c,t}(z^{t-1}, z_t)$ satisfying these two constraints in state $z_t$ if and only if $J_P(U, z_t) \geq \mathcal{J}_0$ for this $z_t$. Therefore, by the continuity of $J_P$, there is a cutoff $\underline{z}_c \geq \underline{z}$ such that conversion is feasible if and only if $z \geq \underline{z}_c$, and $Z_c = [\underline{z}_c, \overline{z}]$. Specifically, this cutoff is $\underline{z}_c = \underline{z}$ if $J_P(U, \underline{z}) \geq \mathcal{J}_0$, and it solves $J_P(U, \underline{z}_c) = \mathcal{J}_0$ otherwise.

Note that since $\underline{z}_P$ (the separation cutoff in $z_t$ for the permanent contract) satisfies $J_P(U, \underline{z}_P) = \mathcal{J}_0 - F$ for $J_P(U, \underline{z}) \geq \mathcal{J}_0 - F$, we must have $\underline{z}_P \leq \underline{z}_c$. This implies that $\underline{z}_c \in Z_P$, so that $\underline{z}_c$, defined as the unique solution to $J_P(U, z) = \mathcal{J}_0$ is well defined (i.e., such a solution exists).

Lastly, observe that setting $s_{c,t}(z^{t-1}, z_t) > \delta$ for any $z_t \geq \underline{z}_c$ could not be optimal since there exists $V \geq U$ such that $J_P(V, z_t) \geq \mathcal{J}_0$ for such $z_t$, that is, there exists a value $V$ such that the surplus of the worker and the employer after conversion into a PC is non-negative.

Part (ii). From the property of the profit function $J_P$, the maximum lifetime utility value that can be delivered to the worker conditional on a conversion of the TC into a PC in state $z \geq \underline{z}_c$, denoted by $\overline{v}_c(z)$, solves $J_P(\overline{v}_c(z), z) = \mathcal{J}_0$. Note that the maximum feasible value conditional on continuation in a PC solves $J_P(\overline{v}_P(z), z) = \mathcal{J}_0 - F$, which implies that $\overline{v}_P(z) \geq \overline{v}_c(z)$ and $\overline{v}_c(z) \in \mathcal{V}_P(z)$ for all $z \geq \underline{z}_c$: a solution $J_P(\overline{v}_c(z)) = \mathcal{J}_0$ exists and is unique.

**Lemma 8.** *(i) For all $t \geq 0$, the set of values of $z_t$ such that a continuation of the TC is feasible is an interval $Z_T = [\underline{z}_T, \overline{z}] \subset Z$. (ii) We have $z_T \leq z_c$. (iii) In an optimal TC, the optimal separation is*

$$s_{T,t}(z^{t-1}, z_t) = \delta + (1-\delta)\mathcal{I}(z_t < \underline{z}_T), \tag{A.55}$$

*for all $t$, $z^{t-1}$ such that the continuation probability up to time $t$ is positive, and for all $z_t$.*

*Proof.* Part (i). As in lemma 4, consider a constant-wage contract $\underline{\sigma}_T$ paying $w(U)$ whenever continuation into a TC or conversion into a PC is feasible, and delivering lifetime utility $U$ to the worker in all histories. The associated employer's lifetime profits are

$$\begin{aligned}
\pi_{T,t}((z^{t-1},z_t);\underline{\sigma}_T) &= \pi_T(z_t;\underline{\sigma}_T) \\
&= xz_t - w(U) \\
&\quad + \beta(1-\delta)(1-\phi)\int_{\underline{z}}^{\overline{z}} \max\Big\{\pi_T(z_{t+1};\underline{\sigma}_T),\mathcal{J}_0\Big\} dG_z(z_{t+1}|z_t) \\
&\quad + \beta(1-\delta)\phi\int_{\underline{z}_c}^{\overline{z}} J_P(U,z_{t+1})\,dG_z(z_{t+1}|z_t) + \beta[\delta + (1-\delta)\phi G_z(\underline{z}_c|z_t)]\mathcal{J}_0,
\end{aligned} \tag{A.56}$$

for all $z_t$, and which is independent of time $t$ and of the history leading to state $z_t$. Since $\underline{\sigma}_T$ delivers lifetime utility $U$ to the worker, it requires $\pi_T(z_t;\underline{\sigma}_T) \geq \mathcal{J}_0$ to be feasible in a given state $z_t$. Once again (similarly to the proof of lemma 4), by the worker's risk aversion, there exists a feasible contract in state $z_t$ if only if the constant-wage contract $\underline{\sigma}_T$ is feasible in that state. Therefore, observing that $\pi_{T,t}(.;\underline{\sigma}_T)$ is continuous and strictly increasing in $z_t$, there exists a cutoff $\underline{z}_T \in Z$ such that $\underline{\sigma}_T$ is feasible in state $z_t$ if and only if $z_t \geq \underline{z}_T$, and $Z_T = [\underline{z}_T, \overline{z}]$.

Part (ii). Due to $F \geq 0$ and since the constant-wage contracts $\underline{\sigma}_T$ and $\underline{\sigma}_P$ pay the same wage equal to $u^{-1}[(1-\beta)U]$, we have that $\pi_T(z;\underline{\sigma}_T) \geq \pi_P(z;\underline{\sigma}_P)$ for all $z$. This implies that $\underline{z}_T \leq \underline{z}_c$ since these satisfy $\pi_T(\underline{z}_T;\underline{\sigma}_T) = \pi_P(\underline{z}_c;\underline{\sigma}_P) = \mathcal{J}_0$.

Part (iii). The fact that the stated TC separation rule is optimal follows from the same reasoning as in the proof of lemma 4: separations are inefficient whenever continuation leaves the worker and the employer with a non-negative surplus.

**Lemma 9.** *Assume that the set of values such that a temporary contract exists for $z \geq \underline{z}_T$, is a closed and bounded interval $\mathcal{V}_T(z) = [U, \overline{v}_T(z)]$. (i) There exists a unique value function $J_T$ solution to the Bellman equation (12). (ii) For any $z \geq \underline{z}_T$, the function $J_T(.,z)$ is strictly decreasing, strictly concave, and continuously differentiable.*

*Proof.* Part (i). Let $\Omega_T = \Big\{(V,z) \in \mathcal{V} \times Z_T : V \in [U, \overline{v}_T(z)]\Big\}$, and let

$$\begin{aligned}
T_T J(V,z) = \max_{\tilde{\sigma}_T \in \Gamma_T(V,z)} & \; xz - w_T \\
&+ \beta(1-\delta)\int_{\underline{z}_c}^{\overline{z}} \Big[(1-\phi)J(\nu_T(z'),z') + \phi J_P(\nu_c(z'),z') - \mathcal{J}_0\Big] dG_z(z'|z) \\
&+ \beta(1-\delta)(1-\phi)\int_{\underline{z}_T}^{\underline{z}_c} (J(\nu_T(z'),z') - \mathcal{J}_0)\,dG_z(z'|z) + \beta\mathcal{J}_0,
\end{aligned} \tag{A.57}$$

where

$$\tilde{\sigma}_T = \Big\{ w_T, (\nu_T(z'))_{z' \geq \underline{z}_T}, (\nu_c(z'))_{z' \geq \underline{z}_c} \Big\}, \tag{A.58}$$

and

$$\begin{aligned}
\Gamma_T(V,z) = \Bigg\{ \tilde{\sigma}_T \, : \, & w_T \geq 0 \\
& u(w_T) + \beta(1-\delta) \int_{\underline{z}_c}^{\bar{z}} \Big[ (1-\phi)\nu_T(z') + \phi \nu_c(z') - U \Big] dG_z(z'|z) \\
& \qquad + \beta(1-\delta)(1-\phi) \int_{\underline{z}_T}^{\underline{z}_c} (\nu_T(z') - U) dG_z(z'|z) + \beta U = V && \text{(A.59)} \\
& \nu_T(z') \in [U, \bar{\nu}_T(z')], \text{ for all } z' \geq \underline{z}_T && \text{(A.60)} \\
& \nu_c(z') \in [U, \bar{\nu}_c(z')], \text{ for all } z' \geq \underline{z}_c \Bigg\} && \text{(A.61)}
\end{aligned}$$

for all $(V,z) \in \Omega_T$ and for $J : \Omega_T \to \mathbb{R}$. Let $B(\Omega_T)$ denote the space of bounded real-valued function defined on $\Omega_T$, endowed with the sup-norm metric. Observe that $J \in B(\Omega_T)$ implies that $T_T J \in B(\Omega_T)$ since $J_P$ is bounded. Therefore, $T_T$ is an operator mapping $B(\Omega_T)$ into itself. A solution to (12) must be a fixed point to this operator.

Observe that $T_T : B(\Omega_T) \to B(\Omega_T)$ satisfies the sufficient Blackwell conditions. Once again, letting $J' > J$ implies that $T_T J' > T_T J$, so that the monotonicity condition is satisfied. Moreover, taking any $a > 0$, we have

$$\begin{aligned}
T_T(J+a) &= T_T J(V,z) + \beta(1-\delta) \int_{\underline{z}}^{\bar{z}} (1-\phi) \mathcal{I}(z' \geq \underline{z}_T) a g_z(z'|z) dz' \\
&< T_T J(V,z) + \beta a.
\end{aligned} \tag{A.62}$$

Hence, the discounting condition is satisfied. Therefore, $T_T$ has a unique fixed point in $B(\Omega_T)$, equal to $J_T$ defined by (12).

Part (ii). $J_T$ is strictly decreasing and strictly concave from the properties of the period utility function $u$, the properties of the PC profit function $J_P$, and from the equality constraint (14). $J_T$ is continuously differentiable from the fact that the period utility function $u$ is continuously differentiable. □

*Optimality conditions.* From lemmas 7 and 8, problem (12) can be expressed as

$$\begin{aligned}
J_T(V,z) = \max_{\tilde{\sigma}_T \in \Gamma_T(V,z)} \; & xz - w_T \\
& + \beta(1-\delta) \int_{\underline{z}_c}^{\bar{z}} \Big[ (1-\phi) J_T(\nu_T(z'),z') + \phi J_P(\nu_c(z'),z') - \mathcal{J}_0 \Big] dG_z(z'|z) \\
& + \beta(1-\delta)(1-\phi) \int_{\underline{z}_T}^{\underline{z}_c} (J_T(\nu_T(z'),z') - \mathcal{J}_0) dG_z(z'|z) + \beta \mathcal{J}_0
\end{aligned} \tag{A.63}$$

such that

$$\begin{aligned} u(w_T) + \beta(1-\delta)\int_{\underline{z}_c}^{\overline{z}} \Big[(1-\phi)\nu_T(z') + \phi\nu_c(z') - U\Big] dG_z(z'|z) & \\ + \beta(1-\delta)(1-\phi)\int_{\underline{z}_T}^{\underline{z}_c} (\nu_T(z') - U) dG_z(z'|z) + \beta U = V & \qquad \text{(A.64)} \end{aligned}$$

$$\nu_c(z') - U \geq 0, \text{ for } z' \geq \underline{z}_c \qquad \text{(A.65)}$$

$$\overline{\nu}_c(z') - \nu_c(z') \geq 0, \text{ for } z' \geq \underline{z}_c \qquad \text{(A.66)}$$

$$\nu_T(z') - U \geq 0, \text{ for } z' \geq \underline{z}_T \qquad \text{(A.67)}$$

$$\overline{\nu}_T(z') - \nu_T(z') \geq 0, \text{ for } z' \geq \underline{z}_T \qquad \text{(A.68)}$$

for $z \geq \underline{z}_T$, $V \in \mathcal{V}_T(z)$. Attach multiplier $\gamma \geq 0$ to constraint (A.64); attach $\mu_{w,c}(z') \geq 0$ and $\mu_{e,c}(z') \geq 0$ to (A.65) and (A.66); finally, attach $\mu_{w,T}(z') \geq$ and $\mu_{e,T}(z') \geq 0$ to (A.67) and (A.68). We have the first-order conditions:

$$1 - \gamma u'(w_T) = 0 \qquad \text{(A.69)}$$

$$\frac{\partial J_P(\nu_c(z'), z')}{\partial V} + \gamma + \mu_{w,c}(z') - \mu_{e,c}(z') = 0, \text{ for } z' \geq \underline{z}_c \qquad \text{(A.70)}$$

$$\frac{\partial J_T(\nu_T(z'), z')}{\partial V} + \gamma + \mu_{w,T}(z') - \mu_{e,T}(z') = 0, \text{ for } z' \geq \underline{z}_T \qquad \text{(A.71)}$$

with the complementary slackness conditions

$$\mu_{w,c}(z')(\nu_c(z') - U) = 0 \qquad \text{(A.72)}$$

$$\mu_{e,c}(z')(\overline{\nu}_c(z') - \nu_c(z')) = 0, \qquad \text{(A.73)}$$

for $z' \geq \underline{z}_c$ and

$$\mu_{w,T}(z')(\nu_T(z') - U) = 0 \qquad \text{(A.74)}$$

$$\mu_{e,T}(z')(\overline{\nu}_T(z') - \nu_T(z')) = 0, \qquad \text{(A.75)}$$

for $z' \geq \underline{z}_T$. The envelope theorem implies

$$\frac{\partial J_T(V, z)}{\partial V} = -\gamma, \qquad \text{(A.76)}$$

for all $z \in Z_T$, $V \in \mathcal{V}_T(z)$. Using FOC (A.70) and (A.71), the complementary slackness conditions (A.72) to (A.75), and the envelope theorem, and following the same lines of argument as in the proof of proposition A.1, we have the wage updating rule:

$$w_T(\nu_P(z')), z') = \min(\overline{w}_T(z'), w_T(V, z)) \qquad \text{(A.77)}$$

for $z' \geq \underline{z}_T$, which specifies the wage conditional on continuation in a TC, and that

$$w_P(\nu_P(z'), z) = \min(\overline{w}_c(z'), w_T(V, z)) \qquad \text{(A.78)}$$

for $z' \geq \underline{z}_c$, which specifies the wage conditional on continuation after conversion into a PC, with the constrained wage satisfying

$$\begin{aligned} w_T(z') &= w_T(\overline{v}_T(z'), z'), \text{ for } z' \geq \underline{z}_T \\ w_c(z') &= w_P(\overline{v}_c(z')), \text{ for } z' \geq \underline{z}_c, \end{aligned} \tag{A.79}$$

and where

$$\begin{aligned} J_T(\overline{v}_T(z'), z') &= \mathcal{J}_0, \text{ for } z' \geq \underline{z}_T \\ J_P(\overline{v}_c(z'), z') &= \mathcal{J}_0, \text{ for } z' \geq \underline{z}_c. \end{aligned} \tag{A.80}$$

□

## A.3 Proof of lemma 1

The following uses the properties of the optimal permanent and temporary contracts to recast the initial profit functions (5) and (12) in terms of auxiliary functions depending on the current wage payment instead of the promised lifetime utility. Specifically, consider a match in an optimal PC paying current wage $\omega$, with given current productivity $z \geq \underline{z}_P$. Following the optimal separation and wage updating rules in proposition 1, the employer in this match receives expected lifetime discounted profits given by

$$\tilde{J}_P(\omega, z) = xz - \omega + \beta(1-\delta)\Big[(1-\lambda)\tilde{J}_P(\omega, z) + \lambda \int_{\underline{z}}^{\overline{z}} \max(\tilde{J}_P(\omega, z'), \mathcal{J}_0 - F)\, dG(z')\Big] + \beta\delta\mathcal{J}_0, \tag{A.81}$$

for all $\omega \geq w(U) \equiv u^{-1}((1-\beta)U)$ and $z \geq \underline{z}_P$. Similarly, the employer's profits in an optimal TC is, following proposition 2,

$$\begin{aligned} \tilde{J}_T(\omega, z) = xz - \omega &+ \beta(1-\delta)(1-\phi)\Big[(1-\lambda)\tilde{J}_T(\omega, z) + \lambda \int_{\underline{z}}^{\overline{z}} \max(\tilde{J}_T(\omega, z'), \mathcal{J}_0)\, dG(z')\Big] \\ &+ \beta(1-\delta)\phi\Big[(1-\lambda)\max(\tilde{J}_P(\omega, z), \mathcal{J}_0) + \lambda \int_{\underline{z}}^{\overline{z}} \max(\tilde{J}_P(\omega, z'), \mathcal{J}_0)\, dG(z')\Big] \\ &+ \beta\delta\mathcal{J}_0, \end{aligned} \tag{A.82}$$

for all $\omega \geq w(U)$ and all $z \geq \underline{z}_T$.

The policy functions for optimal separations and wage updating in a PC and a TC can be characterized using these auxiliary profit functions. Consider match quality $x$ such that $\tilde{J}_P(w(U), \underline{z}) < \mathcal{J}_0 - F$ and $\tilde{J}_T(w(U), \underline{z}) < \mathcal{J}_0$ (recall that $\underline{z}$ is the lower bound of the stochastic productivity support $Z$). The separation cutoffs $\underline{z}_P, \underline{z}_T$, and $\underline{z}_c$ satisfy, in terms of $\tilde{J}_P$ and $\tilde{J}_T$

$$\begin{aligned} \tilde{J}_P(w(U), \underline{z}_P) + F &= \mathcal{J}_0 \\ \tilde{J}_T(w(U), \underline{z}_T) &= \mathcal{J}_0 \\ \tilde{J}_P(w(U), \underline{z}_c) &= \mathcal{J}_0 : \end{aligned} \tag{A.83}$$

separation occurs after productivity shocks low enough so that a constant-wage contract delivering to the worker lifetime value equal to that of unemployment $U$ yields negative surplus to the employer (see subsection A.1 and A.2). In the case where $x$ is high in the sense that $\tilde{J}_P(w(U),\underline{z}) > \mathcal{J}_0$, $\underline{z}_P = \underline{z}$; similarly, we could have, for some high $x$, that $\underline{z}_T = \underline{z}$ or $\underline{z}_c = \underline{z}$.

Similarly, the cutoffs for optimal wage updating, $\{\overline{w}_P(z) : z \geq \underline{z}_P\}$, $\{\overline{w}_T(z) : z \geq \underline{z}_T\}$, and $\{\overline{w}_c(z) : z \geq \underline{z}_c\}$ solve, in terms of the auxiliary profit functions $\tilde{J}_P$ and $\tilde{J}_T$:

$$\begin{aligned}
&\tilde{J}_P(\overline{w}_P(z),z) + F = \mathcal{J}_0, \text{ for } z \geq \underline{z}_P \\
&\tilde{J}_T(\overline{w}_T(z),z) = \mathcal{J}_0, \text{ for } z \geq \underline{z}_T \\
&\tilde{J}_P(\overline{w}_c(z),z) = \mathcal{J}_0, \text{ for } z \geq \underline{z}_c.
\end{aligned} \tag{A.84}$$

In words, these wage cutoffs are such that the employer's participation constraint binds in the relevant state.

Consider now the following remark related to the effect of firing costs $F$ on the auxiliary profit functions.

**Remark 1.** *(i) If $F = 0$, $\tilde{J}_P(\omega,z) = \tilde{J}_T(\omega,z)$ for all $\omega,z$. (ii) Moreover:*

$$\frac{\partial(\tilde{J}_P(\omega,z)+F)}{\partial F} > 0 \tag{A.85}$$

*and*

$$0 > \frac{\partial \tilde{J}_T(\omega,z)}{\partial F} > \frac{\partial \tilde{J}_P(\omega,z)}{\partial F}, \tag{A.86}$$

*for all $\omega \geq w(U)$, and all $z \in Z$.*

Point (i) follows from the fact that a PC and TC are identical when firing costs are zero, and point (ii) follows from standard dynamic programming arguments. Notice that the auxiliary functions $\tilde{J}_P$ and $\tilde{J}_T$ as defined by (A.81) and (A.82) can be computed over the entire support of the stochastic component, $Z$ (hence the fact that the remark is expressed for all $z \in Z$). As a result of point (i) and (ii), when $F > 0$, we have

$$\tilde{J}_P(\omega,z) + F > \tilde{J}_T(\omega,z) > \tilde{J}_P(\omega,z), \tag{A.87}$$

for all $\omega, z$. This, combined with conditions (A.83), implies that $\underline{z}_P < \underline{z}_T < \underline{z}_c$ for match quality $x$ low enough (which holds with weak inequalities for all values of $x$). Combined with conditions (A.84), this implies that $\overline{w}_P(z) > \overline{w}_T(z)$ for $z \geq \underline{z}_T$ and $\overline{w}_P(z) > \overline{w}_c(z)$ for $z \geq \underline{z}_c$. □

## A.4 Proof of lemma 2

Consider the wage as a function of the promised expected lifetime value of the worker, $w_P$ (proposition 1) defined by (A.51). Consider the auxiliary function $\tilde{V}_P(.,z) : [w(U), \overline{w}_P(z)] \to \mathbb{R}$

be defined by $\tilde{V}_P(\omega,z) = w_P^{-1}(\omega)$, for $z \geq \underline{z}_P$. This represents the expected lifetime utility value received by a worker in an optimal PC with current wage $\omega$ implied by the associated wage updating and separation rule (as a function of this $\omega$). Let $\tilde{V}_T(.,z) \equiv w_{T,z}^{-1}$, with $w_{T,z}^{-1}$ the inverse of the TC wage function $w_T(.,z)$ (proposition 2), represents the same value function, but for a worker in an optimal TC.

We have, in the case of a PC, and following proposition 1:

$$\begin{aligned}\tilde{V}_P(\omega,z) = u(\omega) + \beta(1-\delta)\Bigg[(1-\lambda)\tilde{V}_P(\omega,z) + \lambda\int_{\underline{z}_P}^{\overline{z}} \min\Big(\tilde{V}_P(\omega,z'),\overline{v}_P(z')\Big)dG(z')\Bigg]\\ + \beta\Bigg[\delta + (1-\delta)\lambda G(\underline{z}_P)\Bigg]U,\end{aligned} \tag{A.88}$$

for all $\omega \geq w(U)$ and all $z \geq \underline{z}_P$, where

$$\overline{v}_P(z) = \tilde{V}_P(\overline{w}_P(z),z), \tag{A.89}$$

for $z \geq \underline{z}_P$, is the worker's lifetime utility in an optimal PC paying the (constrained) wage $\overline{w}_P(z)$ associated with the employer's participation constraint in state $z \geq \underline{z}_P$.

Moreover, in the case of a TC, and following proposition 2:

$$\begin{aligned}\tilde{V}_T(\omega,z) &= u(\omega) + \beta(1-\delta)(1-\lambda)\Bigg[(1-\phi)\tilde{V}_T(\omega,z) + \phi\mathcal{I}(z \geq \underline{z}_c)\min\Big(\tilde{V}_P(\omega,z),\overline{v}_c(z)\Big)\Bigg]\\ &+ \beta(1-\delta)\lambda\Bigg[(1-\phi)\int_{\underline{z}_T}^{\overline{z}} \min(\tilde{V}_T(\omega,z'),\overline{v}_T(z'))dG(z') + \phi\int_{\underline{z}_c}^{\overline{z}} \min(\tilde{V}_P(\omega,z'),\overline{v}_c(z'))dG(z')\Bigg]\\ &+ \beta\Bigg\{\delta + (1-\delta)\Big[(1-\lambda)\phi\mathcal{I}(z < \underline{z}_c) + \lambda((1-\phi)G(\underline{z}_T) + \phi G(\underline{z}_c))\Big]\Bigg\}U,\end{aligned} \tag{A.90}$$

for all $\omega, z$, where

$$\overline{v}_T(z) = \tilde{V}_T(\overline{w}_T(z),z) \text{ for } z \geq \underline{z}_T \tag{A.91}$$

$$\overline{v}_c(z) = \tilde{V}_P(\overline{w}_c(z),z) \text{ for } z \geq \underline{z}_c, \tag{A.92}$$

which represents the worker's lifetime utility associated with constrained wages $\overline{w}_T(z)$ and $\overline{w}_c(z)$.

Consider the following remark related to the properties of the auxiliary value functions $\tilde{V}_P$ and $\tilde{V}_T$.

**Remark 2.** *Let $F > 0$. (i) For $\omega = w(U) \equiv u^{-1}((1-\beta)U)$, $\tilde{V}_P(\omega,z) = \tilde{V}_T(\omega,z) = U$ for all $z \geq \underline{z}_T$. (ii) Moreover:*

$$\frac{\partial\tilde{V}_P(\omega,z)}{\partial\omega} > \frac{\partial\tilde{V}_T(\omega,z)}{\partial\omega}, \tag{A.93}$$

*for all $\omega \geq w(U)$ and all $z \geq \underline{z}_T$.*

Point (i) says that the worker receives the lifetime value of unemployment $U$ in an optimal contract (PC or TC) paying constant wage $w(U) = u^{-1}((1-\beta)U)$, and so is indifferent between the two contract types. Point (ii) follows from:

$$\frac{\partial \tilde{V}_P(\omega,z)}{\partial \omega} = \frac{u'(\omega)}{1-\beta \mathcal{P}_P(\omega)} \tag{A.94}$$

$$\frac{\partial \tilde{V}_T(\omega,z)}{\partial \omega} = \frac{u'(\omega)}{1-\beta \mathcal{P}_T(\omega,z)}, \tag{A.95}$$

for all $z \geq \underline{z}_T$, where

$$\mathcal{P}_P(\omega) = (1-\delta)(1-\lambda) + \lambda(1 - G(\overline{w}_P^{-1}(\omega))), \tag{A.96}$$

where $\overline{w}_P^{-1}$ represents the inverse of the cutoff wage updating function $\overline{w}_P : [\underline{z}_P, \overline{z}] \to \mathbb{R}_+$ (proposition 1); as such, $\mathcal{P}_P(\omega)$ is the probability that the match continues *and* that the wage stays constant in the next period given a PC paying wage $\omega$ (independent of $z$ due to i.i.d. shocks). Moreover,

$$\begin{aligned}\mathcal{P}_T(\omega,z) = (1-\delta)\Big\{&(1-\lambda)\Big[(1-\phi) + \phi \mathcal{I}(z \geq w_c^{-1}(\omega))\Big] \\ &+ \lambda\Big[(1-\phi)(1-G(w_T^{-1}(\omega))) + \phi(1-G(w_c^{-1}(\omega)))\Big]\Big\},\end{aligned} \tag{A.97}$$

where $\overline{w}_T^{-1}$ and $\overline{w}_c^{-1}$ are the inverse of the wage updating functions $\overline{w}_T$ and $\overline{w}_c$ (proposition 2); this represents the same probability than the former equation, but in the case of a TC. Since, from lemma 1, with $F > 0$, $\overline{w}_T(z) < \overline{w}_P(z)$ for $z \geq \underline{z}_T$ (with both function being strictly increasing), it follows that $\overline{w}_T^{-1}(\omega) > \overline{w}_P^{-1}(\omega)$. Similarly, $\overline{w}_c^{-1}(\omega) > \overline{w}_P^{-1}(\omega)$. Hence, as a consequence of lemma 1,

$$\mathcal{P}_P(\omega) > \mathcal{P}_T(\omega,z), \tag{A.98}$$

and $\partial \tilde{V}_P(\omega,z)/\partial\omega > \partial \tilde{V}_T(\omega,z)/\partial\omega$ for all $\omega \geq w(U)$ and all $z \geq \underline{z}_T$.

Now, define $\rho(\omega,z)$ as

$$\tilde{V}_P(\omega,z) = \tilde{V}_T(\omega + \rho(\omega,z), z), \tag{A.99}$$

for all $\omega \geq w(U)$ and $z \geq \underline{z}_T$. This is the wage difference such that the worker is indifferent between an optimal PC paying $\omega$ and an optimal TC paying $\omega + \rho(\omega,z)$. We have, from the above remark that $\rho(w(U),z) = 0$ and that $\rho$ is strictly increasing in $\omega$. Finally, notice, from the definition of $\tilde{V}_P$ and $\tilde{V}_T$, that

$$w_T(V,z) - w_P(V) = \rho(w_P(V),z),$$

for $z \geq \underline{z}_T$ and $V \in \mathcal{V}_T(z) \cap \mathcal{V}_P(z)$. Hence, from the properties of the function $\rho$, $w_T(U,z) = w_P(U) = w(U)$. □

## A.5 Proof of proposition 3

This subsection treats the dependence of the model variables on match quality $x$ as explicit.

Part (i). Recall from subsections A.1 and A.2 that $\pi_P(z,x;\underline{\sigma}_P)$ and $\pi_T(z,x;\underline{\sigma}_T)$ denote the profits in a PC and a TC, respectively, associated with contracts paying constant wage $w(U) = u{-}1((1-\beta)U)$ and with efficient separation plans. These contracts deliver to the worker lifetime value equal to the value of unemployment $U$ (the worker's outside option). Such a contract is denoted $\underline{\sigma}_P$ in the case of a PC and $\underline{\sigma}_T$ in the case of a TC.

According to subsection A.1, there exists a feasible PC in a given state $(\overline{z},x)$ if $\pi_P(\overline{z},x;\underline{\sigma}_P) \geq \mathcal{J}_0$, that is if contract $\underline{\sigma}_P$ leaves the employer with positive surplus (recall that $\overline{z}$ is the value of the stochastic term $z$ for a new match, by assumption). Now, notice that $\pi_P(\overline{z},x;\underline{\sigma}_P) < 0$ for $x = 0$ (due to $b > 0$, implying that $u^{-1}((1-\beta)U) > 0$ since $U > b/(1-\beta)$ in equilibrium). Therefore, $\pi_P(\overline{z},0;\underline{\sigma}_P) < \mathcal{J}_0$ since $\mathcal{J}_0 \geq 0$.

Letting the upper bound of the support of match quality $X$, $\overline{x}$, be high enough, and since $\pi_P(.;\underline{\sigma}_P)$ is continuous and strictly increasing in $x$, there is a unique match quality value $\underline{x}_P$ solving $\pi_P(\underline{x}_P,\overline{z};\underline{\sigma}_P) = \mathcal{J}_0$. Hiring in a PC is feasible if and only if $x \geq \underline{x}_P$. Similarly, there is a unique match-quality value $\underline{x}_T$ solving $\pi_T(\overline{z},\underline{x}_T;\underline{\sigma}_T) = \mathcal{J}_0$, such that hiring in a TC is feasible if and only if $x \geq \underline{x}_T$. Moreover, with positive firing costs $F > 0$, we have that $\pi_P(\overline{z},x;\underline{\sigma}_P) < \pi_T(\overline{z},x;\underline{\sigma}_T)$ for all $x \geq 0$, which implies that $\underline{x}_P > \underline{x}_T$.

In addition, since for any $x \geq \underline{x}_T$ there exists a feasible (permanent or temporary) contract leaving the employer and the worker with positive surplus, an optimal hiring rule must be such that hiring occurs with probability one for any $x \geq \underline{x}_T$ (i.e., $h_P(x) + h_T(x) = 1$ for $x \geq \underline{x}_T$). As such, constraint (19) can be written as

$$\begin{aligned} V_0 =&(1-\phi_0)\int_{\underline{x}_T}^{\underline{x}_P} v_T(x')dG_x(x') + \int_{\underline{x}_P}^{\overline{x}} \Big[h_P(x')v_P(x') + (1-h_P(x'))v_T(x')\Big]dG_x(x') \\ &+\Big[(1-\phi_0)G_x(\underline{x}_T) + \phi_0 G_x(\underline{x}_P)\Big]U, \end{aligned} \tag{A.100}$$

for $V_0 \in \mathcal{V}_0$, with $v_P$ and $v_T$ such that (23) and (24) are satisfied and with $h_P(x) \in [\phi_0, 1]$ for $x \geq \underline{x}_P$. Therefore, the highest initial lifetime utility value $V_0$ that can be delivered to the worker by a feasible hiring rule, denoted by $\overline{V}_0$ is given by

$$\begin{aligned} \overline{V}_0 =&(1-\phi_0)\left[\int_{\underline{x}_T}^{\underline{x}_P} \overline{v}_T(x')dG_x(x') + \int_{\underline{x}_P}^{\overline{x}} \max(\overline{v}_P(x'),\overline{v}_T(x'))dG_x(x')\right] \\ &\phi_0 \int_{\underline{x}_P}^{\overline{x}} \overline{v}_P(x')dG_x(x') + \Big[(1-\phi_0)G_x(\underline{x}_T) + \phi_0 G_x(\underline{x}_P)\Big]U, \end{aligned} \tag{A.101}$$

where $\overline{v}_P(x) \geq U$ and $\overline{v}_T(x) \geq U$ denote the maximum feasible values upon hiring in a PC and a TC with match quality $x$, solving $J_P(\overline{v}_P(x),\overline{z},x) = \mathcal{J}_0$ (for $x \geq \underline{x}_P$) and $J_T(\overline{v}_T(x),\overline{z},x) = \mathcal{J}_0$ ($x \geq \underline{x}_T$). Since all participation constraints of the employer are binding for such a value

$\overline{V}_0$, and since the associated employer's profits functions $J_P$ and $J_T$ yields, by definition, the profits associated with optimal contracts, there exists no feasible hiring rule that can deliver $V_0 > \overline{V}_0$. Moreover, since $\overline{\nu}_P(x) \geq U$ or $\overline{\nu}_T(x) \geq U$ and since $J_P$ and $J_T$ are strictly decreasing and continuous in $V$, any $V \in [U, \overline{V}_0]$ is feasible.

Part (ii). From part (i), problem (18) can be expressed as

$$\begin{aligned}
\mathcal{J}(V_0) = \max_{\{(\nu_T(x'))_{x' \geq \underline{x}_T}, (\nu_P(x'), h_P(x'))_{x' \geq \underline{x}_P}\}} & (1-\phi_0) \int_{\underline{x}_T}^{\overline{x}_P} J_T(\nu_T(x'), \overline{z}, x') dG_x(x') \\
& + \int_{\underline{x}_P}^{\overline{x}} \Big[ h_P(x') J_P(\nu_P(x'), \overline{z}, x') + (1 - h_P(x')) J_T(\nu_T(x'), \overline{z}, x') \Big] dG_x(x') \\
& + \Big[ \phi_0 G_x(\underline{x}_P) + (1-\phi_0) G_x(\underline{x}_T) \Big] \mathcal{J}_0 \qquad \text{(A.102)}
\end{aligned}$$

such that (A.100) is satisfied and such that

$$\begin{aligned}
& (1 - h_P(x'))(h_P(x') - \phi_0), \text{ for } x' \geq \underline{x}_P && \text{(A.103)} \\
& h_P(x')(\nu_P(x') - U) \geq 0, \text{ for } x' \geq \underline{x}_P && \text{(A.104)} \\
& (1 - h_P(x'))(\nu_T(x') - U) \geq 0, \text{ for } x' \geq \underline{x}_T && \text{(A.105)} \\
& h_P(x')(\overline{\nu}_c(\overline{z}, x') - \nu_P(x')) \geq 0, \text{ for } x' \geq \underline{x}_P && \text{(A.106)} \\
& (1 - h_P(x'))(\overline{\nu}_T(\overline{z}, x') - \nu_T(x')) \geq 0, \text{ for } x' \geq \underline{x}_T, && \text{(A.107)}
\end{aligned}$$

for all $V_0 \in [U, \overline{V}_0]$, where $\overline{\nu}_c(\overline{z}, x')$ and $\overline{\nu}_T(\overline{z}, x')$ satisfy

$$\begin{aligned}
& J_P(\overline{\nu}_c(\overline{z}, x), \overline{z}, x) = \mathcal{J}_0 && \text{(A.108)} \\
& J_T(\overline{\nu}_T(\overline{z}, x), \overline{z}, x) = \mathcal{J}_0, && \text{(A.109)}
\end{aligned}$$

for $x \geq \underline{x}_P$ and $x \geq \underline{x}_T$, respectively. Attach non-negative multiplier $\gamma$ to (A.100) and attach non-negative $\xi(x')$, $\mu_{P,w}(x')$, $\mu_{T,w}(x')$, $\mu_{P,e}(x')$, and $\mu_{T,e}(x')$ to (A.103)-(A.107), respectively. The necessary conditions for an optimum are

$$\begin{aligned}
& \Delta_P(x') + \xi(x')(1 + \phi_0 - 2h_P(x')) = 0, \text{ for } x' \geq \underline{x}_P && \text{(A.110)} \\
& \frac{\partial J_P(\nu_P(x'), \overline{z}, x')}{\partial \nu_P(x')} + (\gamma + \mu_{P,w}(x') - \mu_{P,e}(x')) = 0, \text{ for } x' \geq \underline{x}_P \text{ and } h_P(x') > 0 && \text{(A.111)} \\
& \frac{\partial J_T(\nu_T(x'), \overline{z}, x')}{\partial \nu_T(x')} + (\gamma + \mu_{T,w}(x') - \mu_{T,e}(x')) = 0, \text{ for } x' \geq \underline{x}_T \text{ and } 1 - h_P(x') > 0, && \text{(A.112)}
\end{aligned}$$

where

$$\Delta_P(x) = J_P(\nu_P(x), \overline{z}, x) - J_T(\nu_T(x), \overline{z}, x) + \gamma\Big(\nu_P(x) - \nu_T(x)\Big) \qquad \text{(A.113)}$$

for $x \geq \underline{x}_P$, which represents the marginal value of increasing the probability of hiring into a PC. The complementary slackness conditions are:

$$\xi(x')(1-h_P(x'))(h_P(x')-\phi_0) = 0, \text{ for } x' \geq \underline{x}_P \tag{A.114}$$
$$\mu_{P,w}(x')h_P(x')(v_P(x')-U) = 0, \text{ for } x' \geq \underline{x}_P \tag{A.115}$$
$$\mu_{T,w}(x')(1-h_P(x'))(v_T(x')-U) = 0, \text{ for } x' \geq \underline{x}_T \tag{A.116}$$
$$\mu_{P,e}(x')h_P(x')(\overline{v}_P(x')-v_P(x')) = 0, \text{ for } x' \geq \underline{x}_P \tag{A.117}$$
$$\mu_{T,e}(x')(1-h_P(x'))(\overline{v}_T(x)-v_T(x')) = 0, \text{ for } x' \geq \underline{x}_T. \tag{A.118}$$

From the envelope theorem combined with conditions (A.111) and (A.112) (see subsections A.1 and A.2), we have that

$$\begin{aligned}
\frac{1}{u'(w_P(v_P(x'),x'))} &= \gamma + \mu_{P,w}(x') - \mu_{P,e}(x'), \text{ for } x' \geq \underline{x}_P \text{ and } h_P(x') > 0 \\
\frac{1}{u'(w_T(v_T(x'),\overline{z},x'))} &= \gamma + \mu_{T,w}(x') - \mu_{P,e}(x'), \text{ for } x' \geq \underline{x}_T \text{ and } 1-h_P(x') > 0,
\end{aligned} \tag{A.119}$$

where $w_P(v_P(x'),x')$ and $w_T(v_T(x'),\overline{z},x')$ denote the wage payment associated with hiring in a PC and in a TC, respectively. Following the same reasoning as in the proof of proposition 1 (see subsection A.1), we have the following rules for setting the hiring wage

$$\begin{aligned}
w_P(v_P(x'),x') &= \min\Big(w_0(V_0), \overline{w}_c(\overline{z},x')\Big), \text{ for } x' \geq \underline{x}_P \\
w_T(v_T(x'),\overline{z},x') &= \min\Big(w_0(V_0), \overline{w}_T(\overline{z},x')\Big), \text{ for } x' \geq \underline{x}_T,
\end{aligned} \tag{A.120}$$

where, again, $\overline{w}_c(\overline{z},x')$ and $\overline{w}_T(\overline{z},x')$ solve the relevant participation constraints for the employer; $w_0(V_0)$ is a "target" wage obtained using constraint (A.100) as seen below.

I now analyze an optimal hiring rule $h_P(x')$, $x' \geq \underline{x}_P$, which determines the choice between a PC and a TC. Consider the following lemma, relating the marginal value of hiring into a PC, $\Delta_P(x')$, to the choice of $h_P(x')$ at the optimum.

**Lemma 10.** *For any $x' \geq \underline{x}_P$, if $\Delta_P(x') \geq 0$, $h_P(x') = 1$ is optimal; if $\Delta_P(x') < 0$, $h_P(x') = \phi_0$ is optimal.*

*Proof:* If $\Delta_P(x') > 0$, then $\xi(x') > 0$, and $h_P(x') > (1+\phi_0)/2$ from FOC (A.110). However, the two latter strict inequalities require that $h_P(x') = 1$ to satisfy the complementary slackness condition (A.114). Following the same reasoning but reversing the inequalities, $\Delta_P(x') < 0$ requires that $h_P(x') = \phi_0$. Finally, for $\Delta_P(x') = 0$, any $h_P(x') \in [\phi_0, 1]$ is admissible regarding conditions (A.110) and (A.114). Therefore, $h_P(x') = 1$ for $\Delta_P(x') \geq 0$ is an admissible solution.

The next lemma characterizes the behavior of the marginal value of hiring in a PC as a function of the match quality, $\Delta_P$.

**Lemma 11.** *Let $F > 0$. We have that: (i) $\Delta_P(\underline{x}_P) < 0$; (ii) for $\overline{x}$ high enough, there exists a value of $x > \underline{x}_P$, denoted $\tilde{x}$, such that $\Delta_P(x) \geq 0$ for all $x \geq \tilde{x}$; (iii) If $\tilde{x}$ exists, $\Delta_P$ is continuous and strictly increasing in $x$ over $[\underline{x}_P, \tilde{x})$; otherwise, $\Delta_P$ is continuous and strictly increasing over $[\underline{x}_P, \overline{x}]$.*

*Proof:* As a preliminary step, notice that the marginal value of hiring in a PC can be written as

$$\Delta_P(x) = \mathcal{S}_P(x) - \mathcal{S}_T(x), \tag{A.121}$$

for all $x \geq \underline{x}_P$, where

$$\mathcal{S}_P(x) = J_P(\nu_P(x), \overline{z}, x) - \mathcal{J}_0 + \gamma(\nu_P(x) - U) \tag{A.122}$$

$$\mathcal{S}_T(x) = J_T(\nu_T(x), \overline{z}, x) - \mathcal{J}_0 + \gamma(\nu_T(x) - U), \tag{A.123}$$

represent the joint match surplus of hiring in a PC and a TC (in consumption units), respectively. Let the dependence of the problem's variables on the initial worker's lifetime utility $V_0$ be understood.

In addition, using the envelope theorem, we have that

$$\frac{d\mathcal{S}_P(x)}{dx} = \left.\frac{\partial J_P(V, \overline{z}, x)}{\partial x}\right|_{V=\nu_P(x)} + \mu_{e,P}(x)\frac{\partial \overline{v}_c(\overline{z}, x)}{\partial x}, \tag{A.124}$$

with $\overline{v}_c(\overline{z}, x)$ satisfying (A.108). Intuitively, a higher match quality increases the surplus through higher output (the first additive term), and through relaxing the constraint on hiring values (and the hiring wages) that can be feasibly delivered to the worker (the second term). Using FOC (A.119) and the wage rule (A.120), and from implicit differentiation of (A.108), we have, for the permanent-contract hiring surplus:

$$\frac{\partial \mathcal{S}_P(x)}{\partial x} = \frac{u'(w_P(\nu_P(x), x))}{u'(w_0)} \times \left.\frac{\partial J_P(V, \overline{z}, x)}{\partial x}\right|_{V=\nu_P(x)}, \text{ for } x \geq \underline{x}_P. \tag{A.125}$$

Similarly, for a temporary contract, the hiring surplus has:

$$\frac{\partial \mathcal{S}_T(x)}{\partial x} = \frac{u'(w_T(\nu_T(x), \overline{z}, x))}{u'(w_0)} \times \left.\frac{\partial J_T(V, \overline{z}, x)}{\partial x}\right|_{V=\nu_T(x)}, \textbf{ for } x \geq \underline{x}_T. \tag{A.126}$$

Part (i). By definition of the above defined hiring cutoffs $\underline{x}_P$ and $\underline{x}_T$, we have, in terms of the above surplus functions

$$\mathcal{S}_P(\underline{x}_P) = 0 \tag{A.127}$$

$$\mathcal{S}_T(\underline{x}_T) = 0. \tag{A.128}$$

Recalling that $\underline{x}_P > \underline{x}_T$ for $F > 0$, and due to $S_T$ strictly increasing with $x$ (see (A.126)), it follows that $\mathcal{S}_T(\underline{x}_P) > 0 = \mathcal{S}_P(\underline{x}_P)$, and $\Delta_P(x) < 0$ for $x = \underline{x}_P$.

Part (ii). For $\overline{x}$ high enough and since $J_P$ is strictly increasing in $x$, there exists a value $\tilde{x} \leq \overline{x}$ such that $J_P(U,z,x) \geq \mathcal{J}_0$ for all $z \in Z$ and for all $x \geq \tilde{x}$. For such values $x \geq \tilde{x}$, it is clear from (A.81) that there are no costs associated with $F$ in a permanent contract: separation are never optimal as the surplus remains positive in all states. In such a case, since firing costs increase the commitment ability of the employer, allowing the provision of extra wage stability to the worker, it is clear that a permanent contract is weakly preferred to a temporary one, providing, therefore, higher hiring surplus: $\Delta_P(x) \geq 0$ for $x \geq \tilde{x}$.

Part (iii). Let $\tilde{z}_P(x) \geq \underline{z}_P(x)$ represent the stochastic-productivity cutoff such that the employer's participation constraint binds in a continuing PC when $z' \leq \tilde{z}_P(x)$, given the hiring wage $w_P(\nu_P(x),x)$. Formally, $\tilde{z}_P(x) \equiv \overline{w}_{P,x}^{-1}(w_P(\nu_P(x),x))$, where $\overline{w}_{P,x}^{-1}$ represents the inverse of the cutoff updating wage function (see proposition 1 and subsection A.4), evaluated at match quality $x$, $\overline{w}_P(.,x) : [\underline{z}_P(x),\overline{z}] \to \mathbb{R}_+$. Define $\tilde{z}_T(x) \geq \underline{z}_T(x)$ in a similar way, but for a TC.

The derivative of the employer's lifetime profits in an optimal PC with respect to match quality $x$ satisfies

$$\left.\frac{\partial J_P(V,\overline{z},x)}{\partial x}\right|_{V=\nu_P(x)} = \Big[1-\beta(1-\delta)(1-\lambda)\Big]^{-1} \times \left\{\overline{z} + \frac{\beta(1-\delta)\lambda}{1-\beta(1-\delta)(1-\lambda G_z(\tilde{z}_P(x)))} \int_{\tilde{z}_P(x)} z' dG_z(z')\right\}. \tag{A.129}$$

On the other hand, in the case of an optimal TC, the derivative of the employer's lifetime profits w.r.t. match quality $x$ has upper bound given by

$$\left.\frac{\partial J_T(V,\overline{z},x)}{\partial x}\right|_{V=\nu_T(x)} \leq \Big[1-\beta(1-\delta)(1-\lambda)\Big]^{-1} \times \left\{\overline{z} + \frac{\beta(1-\delta)\lambda}{1-\beta(1-\delta)(1-\lambda G_z(\tilde{z}_T(x)))} \int_{\tilde{z}_T(x)} z' dG_z(z')\right\}, \tag{A.130}$$

for $x \geq \underline{x}_T$.

To compare (A.129) and (A.130), remember that $\overline{w}_P(z,x) > \overline{w}_T(z,x)$ for all $z \geq \underline{z}_T(x)$, and all $x \geq \underline{x}_P$ (lemma 1). Moreover, we must have, from (A.120) that $w_P(\nu_P(x),x) \leq w_T(\nu_T(x),x)$ since, from (A.86), $\overline{w}_c(\overline{z},x) < \overline{w}_T(\overline{z},x)$: the wage is lower in a PC due to tighter (hiring-stage) participation constraints.

Assume that $\tilde{x}$ exists. Then, $\tilde{z}_P(x) < \tilde{z}_T(x)$ for all $x \in [\underline{x}_P,\tilde{x})$: the probability of a wage cut or a separation is strictly lower in a PC than in a TC on the interval. The latter is true conditional on a given wage (lemma 1), implying it holds also in the case under study since, as discussed above, the hiring wage is lower in a PC.[29] In addition, if $\tilde{x}$ does not exist, then the strict inequality holds for all $x \in [\underline{x}_P,\overline{x}]$. It follows that

$$\frac{\partial J_P(V,\overline{z},x)}{\partial x} > \frac{\partial J_T(V,\overline{z},x)}{\partial x}, \tag{A.131}$$

---

[29] In contrast, if $x \geq \tilde{x}$, separations are never optimal, and there are wage values such that a wage cut never occurs in both contracts: $\tilde{z}_P(x) = \tilde{z}_T(x) = \underline{z}$ for some $x$.

which holds for $x \in [\underline{x}_P, \tilde{x})$ if $\tilde{x}$ exists and for $x \in [\underline{x}_P, \overline{x}]$ otherwise.

In addition, we have that the ratio of marginal utilities appearing in expression (A.125) and (A.126) satisfy, from the hiring-wage rule

$$\frac{u'(w_P(\nu_P(x),x))}{u'(w_0)} = \frac{u'\Big[\min(w_0(V_0), \overline{w}_c(\overline{z},x))\Big]}{u'(w_0)} \tag{A.132}$$

$$\frac{u'(w_T(\nu_T(x),\overline{z},x))}{u'(w_0)} = \frac{u'\Big[\min(w_0(V_0), \overline{w}_T(\overline{z},x))\Big]}{u'(w_0)}, \tag{A.133}$$

for $x \geq \underline{x}_P$. Therefore, since $\overline{w}_c(x,\overline{z}) < \overline{w}_T(x,\overline{z})$ (condition (A.86)), and from the strict concavity of the period utility function, the ratio of marginal utilities appearing in (A.125) is greater than the one in (A.126) over $[\underline{x}_P, \overline{x}]$. This, combined with (A.131), implies

$$\frac{\partial S_P(x)}{\partial x} > \frac{\partial S_T(x)}{\partial x}, \tag{A.134}$$

for $x \in [\underline{x}_P, \tilde{x})$ if $\tilde{x}$ exists and for $x \in [\underline{x}_P, \overline{x}]$ otherwise. Therefore, $\Delta_P$ is strictly increasing over the relevant interval, as stated in the lemma.

The following result follows from the properties of $\Delta_P(x)$. It provides a characterization of the choice between a PC and a TC upon hiring in terms of a cutoff rule associated with $x$.

**Corollary 1.** *(of lemma 11) For $\overline{x}$ high enough, there is a unique $\hat{x} \geq \underline{x}_P$ such that $h_P(x) = 1$ for $x \geq \hat{x}$ and $h_P(x) = \phi_0$ for $x \in [\underline{x}_P, \hat{x})$ is optimal: when the employer is exogenously allowed to choose between a permanent and a temporary contract, hiring in a PC is optimal for $x \geq \hat{x}$ and hiring in a TC is optimal for $x \in [\underline{x}_T, \hat{x})$.*

From the latter result, we have that hiring in a PC occurs in two distinct cases.

Case (i): choosing a TC is *not* allowed (which is the case with probability $\phi_0$), and the surplus in a PC $\mathcal{S}_P(x)$ is non-negative. Since $\mathcal{S}_P(x) \geq 0$ for $x \geq \underline{x}_P$, hiring in a PC occurs if $x \geq \underline{x}_P$.

Case (ii): choosing a TC is allowed (which is the case with probability $1-\phi_0$), and the marginal value $\Delta_P(x)$ is non-negative. Assume that $\tilde{x}$ exists. Since $\Delta_P$ is continuous in $x$ and has $\Delta_P(x_P) < 0$, $\Delta_P(x) \geq 0$ for all $x \geq \tilde{x}$, and is strictly increasing over $[x_P, \tilde{x})$, there is a unique cutoff $\hat{x} \in (x_P, \tilde{x}]$ such that $\Delta_P(x) < 0$ for $x < \hat{x}$ and $\Delta_P(x) \geq 0$ otherwise. If $\tilde{x}$ does not exist but $\overline{x}$ is high enough, there is, by the properties of $\Delta_P$, a unique cutoff $\hat{x}$ similarly defined.

From the discussion for cases (i) and (ii), hiring in a PC occurs with probability $\phi_0[1 - G_x(\underline{x}_P)] + (1-\phi_0)[1 - G_x(\hat{x})]$, and hiring in a TC occurs with probability $(1-\phi_0)[G_x(\hat{x}) - G_x(x_T)]$, as stated in the proposition. In cases where $\overline{x}$ is such that $\Delta_P(x) < 0$ for all $x \geq \underline{x}_P$, let $\hat{x} = \overline{x}$, implying that the probability of hiring in a PC is zero. □

## A.6 Proof of lemma 3

Following proposition 3, the profit function $\mathcal{J}$ and the associated "target" hiring wage $w_0(V_0)$ and PC hiring cutoff $\hat{x}(V_0)$, solution to problem (18) satisfy:

$$\begin{aligned}
&\mathcal{J}(V_0) = \\
&\quad (1-\phi_0)\left[\int_{\underline{x}_T}^{\hat{x}(V_0)} (J_T(\nu_T(x'),\bar{z},x') - \mathcal{J}_0)dG_x(x') + \int_{\hat{x}(V_0)}^{\bar{x}} (J_P(\nu_P(x'),\bar{z},x') - \mathcal{J}_0)dG_x(x')\right] \\
&\quad + \phi_0 \int_{\underline{x}_P}^{\bar{x}} (J_P(\nu_P(x'),\bar{z},x') - \mathcal{J}_0)dG_x(x') + \mathcal{J}_0,
\end{aligned} \tag{A.135}$$

for all $V_0 \in \mathcal{V}_0$, where: (i) $\underline{x}_P$, and $\underline{x}_T$ represent the reservation cutoffs for hiring in a PC and a TC (proposition 3), solving $J_P(\underline{x}_P,\bar{z},U) = \mathcal{J}_0$ and $J_T(\underline{x}_T,\bar{z},U) = \mathcal{J}_0$; (ii) the hiring continuation values, $\nu_P(x)$, $x \geq \underline{x}_P$ and $\nu_T(x)$, $x \geq \underline{x}_T$ satisfy the set of conditions:

$$\begin{aligned}
V_0 = {} & (1-\phi_0)\left[\int_{\underline{x}_T}^{\hat{x}(V_0)} (\nu_T(x') - U)dG_x(x') + \int_{\hat{x}(V_0)}^{\bar{x}} (\nu_P(x') - U)dG(x')\right] \\
& + \phi_0 \int_{\underline{x}_P}^{\bar{x}} (\nu_P(x') - U)dG(x') + \beta U,
\end{aligned} \tag{A.136}$$

which follows from constraint (A.100), and

$$\begin{aligned}
\nu_P(x) &= \min\left(\nu_P^*(x), \overline{\nu}_P(x,\bar{z})\right), \text{ for } x \geq \underline{x}_P \\
\nu_T(x) &= \min\left(\nu_T^*(x), \overline{\nu}_T(x,\bar{z})\right), \text{ for } x \geq \underline{x}_T,
\end{aligned} \tag{A.137}$$

where $\nu_P^*(x)$ and $\nu_T^*(x)$ represent "unconstrained" hiring continuation lifetime utility values delivered to the worker, prevailing when the relevant participation constraints are slack:

$$\begin{aligned}
w_P(\nu_P^*(x), x) &= w_0(V_0), \text{ for } x \geq \underline{x}_P \\
w_T(\nu_T^*(x), \bar{z}, x) &= w_0(V_0), \text{ for } x \geq \underline{x}_T
\end{aligned} \tag{A.138}$$

with $w_P$ and $w_T$ the wage functions in a PC and a TC (see proposition 1 and subsection A.2). Moreover, $\overline{\nu}_P(x,\bar{z})$ and $\overline{\nu}_T(x,\bar{z})$ are the "constrained" continuation values, solving:

$$\begin{aligned}
J_P(\overline{\nu}_P(x,\bar{z}),\bar{z},x) &= \mathcal{J}_0, \text{ for } x \geq \underline{x}_P \\
J_T(\overline{\nu}_T(x,\bar{z}),\bar{z},x) &= \mathcal{J}_0, \text{ for } x \geq \underline{x}_T;
\end{aligned} \tag{A.139}$$

(iii) the cutoff for hiring into a PC when choosing a TC is allowed, $\hat{x}(V_0)$ solves

$$\Delta_P(\hat{x}(V_0)) = 0, \tag{A.140}$$

where $\Delta_P$ is defined by (A.113); using FOC's (A.110) and (A.119), this is equivalent to

$$J_P(\nu_P(\hat{x}(V_0)),\bar{z},\hat{x}(V_0)) - J_T(\nu_T(\hat{x}(V_0)),\bar{z},\hat{x}(V_0)) + \frac{\nu_P(\hat{x}(V_0)) - \nu_T(\hat{x}(V_0))}{u'(w_0(V_0))} = 0. \tag{A.141}$$

To see that $\mathcal{J}(\overline{V}_0) = \mathcal{J}_0$, recall that $\overline{V}_0$ is defined by (A.101), implying that $v_P(x) = \overline{v}_P(x)$ for $x \geq \underline{x}_P$ and $v_T(x) = \overline{v}_T(x)$ for $x \geq \underline{x}_T$ for $V_0 = \overline{V}_0$. This, together with (A.135) and (A.139), implies that $\mathcal{J}(V_0) = \mathcal{J}_0$.

To show that the profit function $\mathcal{J}$ is strictly decreasing, strictly concave, and continuously differentiable, apply the envelope theorem to its associated constrained maximization problem to compute the derivative of this function, satisfying:

$$\mathcal{J}'(V_0) = -\frac{1}{u'(w_0(V_0))} < 0, \tag{A.142}$$

for all $V_0 \in \mathcal{V}_0$, which follows from FOC's (A.111) and (A.119). Since $w_0$ is a continuous function of $V_0$ (following conditions (A.136) and (A.137)), this shows that the profit function $\mathcal{J}$ is strictly decreasing and continuously differentiable. Moreover, we have the second derivative

$$\mathcal{J}''(V_0) = \frac{dw_0(V_0)}{dV_0}\frac{u''(w_0(V_0))}{u'(w_0(V_0))} < 0, \tag{A.143}$$

for $V_0 \in \mathcal{V}_0$, where the strict inequality follows from the fact that $w_0$ is strictly increasing with $V_0$ and from the strict concavity of the period utility function $u$. □

# B Steady-state equilibrium distributions

The steady-state match quality density in the pool of permanent jobs, denoted $h_{P,x}$, satisfies in equilibrium

$$\begin{aligned} h_{P,x}(x) = {} & p(\theta)\frac{(1-\phi_0)\mathcal{I}(x \geq \hat{x}(V_0)) + \phi_0}{\delta + (1-\delta)\lambda G_z(\underline{z}_P(x))}\frac{u}{(1-u)n_P}g_x(x) \\ & + (1-\delta)\phi\frac{(1-\lambda)[1-H_T(\underline{z}_c(x)|x)] + \lambda[1-G_z(\underline{z}_c(x))]}{\delta + (1-\delta)\lambda G_z(\underline{z}_P(x))}\frac{n_T}{n_P}h_{T,x}(x), \end{aligned}$$

for $x \geq \underline{x}_P$ ($h_{P,x}(x) = 0$ otherwise), where $h_{T,x}$ is the match quality density in the pool of temporary jobs, satisfying

$$h_{T,x}(x) = \frac{p(\theta)(1-\phi_0)}{\delta + (1-\delta)\big[\phi + (1-\phi)\lambda G(\underline{z}_T(x))\big]}\frac{u}{(1-u)n_T}g_x(x). \tag{B.1}$$

for $\underline{x}_T \leq x < \hat{x}(V_0)$ ($h_{T,x}(x) = 0$ otherwise). In addition, $H_T(.|x)$ is the cumulative distribution function of $z$ conditional on match quality $x$, in the pool of temporary contracts, which satisfies:

$$H_T(z|x) = \begin{cases} 1, & \text{for } z = \overline{z} \\ \dfrac{(1-\delta)(1-\phi)\lambda[G_z(z) - G_z(\underline{z}_T(x))]}{\delta + (1-\delta)[(1-\phi)(1-G_z(z)) + \phi]}, & \text{for } z \in [\underline{z}_T(x), \overline{z}) \\ 0 & \text{otherwise,} \end{cases} \tag{B.2}$$

for all $x \geq \underline{x}_T$.

# C Decomposition of the employment effect of temporary contracts

Consider a value for the parameter for the strictness of the hiring regulation on temporary contracts $\phi_0 \in [0,1)$. In equilibrium, the transition rates between unemployment (U) and employment (E) satisfy:

$$\Lambda_{UE}(\phi_0) = p(\theta(\phi_0))(1-\phi_0)\Big[1 - G_x(\underline{x}_T(\phi_0))\Big] + \phi_0\Big[1 - G_x(\underline{x}_P(\phi_0))\Big] \tag{C.1}$$

$$\begin{aligned}\Lambda_{EU}(\phi_0) &= n_P(\phi_0)\int_{\underline{x}_P(\phi_0)}^{\overline{x}} \bar{s}_P(x';\phi_0)h_P(x';\phi_0)dx' + (1-n_P(\phi_0))\int_{\underline{x}_T(\phi_0)}^{\overline{x}} \bar{s}_T(x';\phi_0)h_T(x';\phi_0)dx' \\ &= n_P(\phi_0)\Lambda_{PU}(\phi_0) + (1-n_P(\phi_0))\Lambda_{TU}(\phi_0),\end{aligned} \tag{C.2}$$

where $\Lambda_{PU}$ and $\Lambda_{TU}$ represent employment separation rates (into U) from permanent and temporary jobs, respectively. For the discussion that follows, the dependence of equilibrium variables on the policy parameter $\phi_0 \in [0,1)$ is treated as explicit in the notation.

Consider an alternative policy regime indexed by $\phi_0' \in [0,1)$. The following identity is useful. Let some variables $x(\phi_0')$, $x(\phi_0)$ be such that $x(\tilde{\phi}_0) = y(\tilde{\phi}_0) \times z(\tilde{\phi}_0) > 0$, for $\tilde{\phi}_0 \in \{\phi_0, \phi_0'\}$. Then,

$$x(\phi_0') - x(\phi_0) = \Big(y(\phi_0') - y(\phi_0)\Big)\frac{z(\phi_0') + z(\phi_0)}{2} + \Big(z(\phi_0') - z(\phi_0)\Big)\frac{y(\phi_0') + y(\phi_0)}{2}. \tag{C.3}$$

Observe now that (C.1) can be written as:

$$\begin{aligned}\Lambda_{UE}(\phi_0) = p(\theta(\phi_0))\Big[&\Big(1 - G_x(\underline{x}_P(\phi_0))\Big) + (1-\phi_0)\Big(G_x(\hat{x}(\phi_0)) - G_x(\underline{x}_T(\phi_0))\Big) \\ &- (1-\phi_0)\Big(G_x(\hat{x}(\phi_0)) - G_x(\underline{x}_P(\phi_0))\Big)\Big]\end{aligned} \tag{C.4}$$

Using identity (C.3), the difference in (C.1) across policy regimes can be written as,

$$\begin{aligned}\Lambda_{UE}(\phi_0') - \Lambda_{UE}(\phi_0) = &\frac{\sum_{\tilde{\phi}_0 \in \{\phi_0, \phi_0'\}} \mathcal{A}(\tilde{\phi}_0)}{2}\Big(p(\theta(\phi_0')) - p(\theta(\phi_0))\Big) \\ &+ \frac{\sum_{\tilde{\phi}_0 \in \{\phi_0, \phi_0'\}} p(\theta(\tilde{\phi}_0))}{2}\Big(\mathcal{A}(\phi_0') - \mathcal{A}(\phi_0)\Big),\end{aligned} \tag{C.5}$$

where $\mathcal{A} \equiv \Big(1 - G_x(\underline{x}_P)\Big) + (1-\phi_0)\Big(G_x(\hat{x}) - G_x(\underline{x}_T)\Big) - (1-\phi_0)\Big(G_x(\hat{x}) - G_x(\underline{x}_P)\Big)$. In addition, with the approximation $\Big\{\underline{x}_T(\phi_0), \underline{x}_P(\phi_0), \hat{x}(\phi_0)\Big\} \approx \Big\{\underline{x}_T(\phi_0'), \underline{x}_P(\phi_0'), \hat{x}(\phi_0')\Big\}$ in hands, we have that

$$\mathcal{A}(\phi_0') - \mathcal{A}(\phi_0) \approx -\frac{\phi_0' - \phi_0}{2}\sum_{\tilde{\phi}_0}\Big[G_x(\hat{x}(\tilde{\phi}_0)) - G_x(x_T(\tilde{\phi}_0))\Big] + \frac{\phi_0' - \phi_0}{2}\sum_{\tilde{\phi}_0}\Big[G_x(\hat{x}(\tilde{\phi}_0)) - G_x(x_P(\tilde{\phi}_0))\Big], \tag{C.6}$$

where the summation is taken over $\{\phi_0, \phi_0'\}$. Combine with (C.5) and rearrange to get (44) in the main text.

Turning now to the EU rate difference across policy regimes, one can write, based on (C.2):

$$\begin{aligned}\Lambda_{EU}(\phi_0') - \Lambda_{EU}(\phi_0) = {} & (n_P(\phi_0') - n_P(\phi_0)) \frac{\sum_{\tilde{\phi}_0} \big(\Lambda_{PU}(\tilde{\phi}_0) - \Lambda_{TU}(\tilde{\phi}_0)\big)}{2} \\ & + \frac{\sum_{\tilde{\phi}_0} n_P(\tilde{\phi}_0)}{2} \big(\Lambda_{PU}(\phi_0') - \Lambda_{PU}(\phi_0)\big) \\ & + \frac{1 - \sum_{\tilde{\phi}_0} n_P(\tilde{\phi}_0)}{2} \big(\Lambda_{TU}(\phi_0') - \Lambda_{TU}(\phi_0)\big), \end{aligned} \tag{C.7}$$

for two policy regimes $\phi_0, \phi_0' \in [0,1)$. It is useful to define $X_P(\tilde{\phi}_0) \equiv \{x \in X | x \geq \underline{x}_P(\phi_0)\}$, i.e., $X_P(\tilde{\phi}_0)$ represents the support of the equilibrium match-quality distribution conditional on a permanent contract, in policy regime $\tilde{\phi}_0$. Define $X_T(\tilde{\phi}_0)$ similarly, but conditional on a temporary contract (and the associated hiring cutoff $\underline{x}_T(\phi)$). Finally, let $\tilde{X}(\tilde{\phi}_0) \equiv X_P(\tilde{\phi}_0) \cap X_T(\tilde{\phi}_0)$.

Based on (C.2), we have the following expression for the difference in the PU transition rate across policy regimes:

$$\begin{aligned}\Lambda_{PU}(\phi_0') - \Lambda_{PU}(\phi_0) = {} & \int_{x \in X_P(\phi_0) \cap X_P(\phi_0')} \big(h_P(x';\phi_0')\bar{s}_P(x';\phi_0') - h_P(x';\phi_0)\bar{s}_P(x';\phi_0)\big) dx' \\ & + \int_{x \in X_P(\phi_0') \backslash X_P(\phi_0)} h_P(x';\phi_0') s_P(x';\phi_0') dx' - \int_{x \in X_P(\phi_0) \backslash X_P(\phi_0')} h_P(x';\phi_0) s_P(x';\phi_0) dx'; \end{aligned}$$

use (C.3) and rearrange to get

$$\begin{aligned}\Lambda_{PU}(\phi_0') - \Lambda_{PU}(\phi_0) = {} & \int_{x \in X_P(\phi_0) \cap X_P(\phi_0')} \big(h_P(x';\phi_0') - h_P(x';\phi_0)\big) \frac{\bar{s}_P(x';\phi_0') + \bar{s}_P(x';\phi_0)}{2} dx' \\ & + \int_{x \in X_P(\phi_0) \cap X_P(\phi_0')} \big(\bar{s}_P(x';\phi_0') - \bar{s}_P(x';\phi_0)\big) \frac{h_P(x';\phi_0') + h_P(x';\phi_0)}{2} dx' \\ & + \int_{x \in X_P(\phi_0') \backslash X_P(\phi_0)} (h_P(x';\phi_0') - h_P(x';\phi_0)) s_P(x';\phi_0') dx' \\ & + \int_{x \in X_P(\phi_0) \backslash X_P(\phi_0')} (h_P(x';\phi_0') - h_P(x';\phi_0)) s_P(x';\phi_0) dx', \end{aligned} \tag{C.8}$$

which comes from (C.3) and from $h_P(.;\tilde{\phi}_0) = 0$ outside of its support. In the case where $s_P(x;\phi_0') \approx s_P(x;\phi_0)$ for $x \in X_P(\phi_0) \cap X_P(\phi_0')$, one can write

$$\Lambda_{PU}(\phi_0') - \Lambda_{PU}(\phi_0) \approx \int_{x \in X} (h_P(x';\phi_0') - h_P(x';\phi_0)) \tilde{s}_P(x') dx', \tag{C.9}$$

where

$$\tilde{s}_P(x) = \begin{cases} (\bar{s}_P(x;\phi_0') + \bar{s}_P(x;\phi_0))/2, & \text{for } x \in X_P(\phi_0) \cap X_P(\phi_0') \\ s_P(x;\phi_0), & \text{for } x \in X_P(\phi_0) \setminus X_P(\phi_0') \\ s_P(x;\phi_0'), & \text{for } x \in X_P(\phi_0') \setminus X_P(\phi_0) \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, one can write for the difference in the TU transition rate:

$$\Lambda_{TU}(\phi_0') - \Lambda_{TU}(\phi_0) \approx \int_{x\in X} (h_T(x';\phi_0') - h_T(x';\phi_0))\tilde{s}_T(x')dx', \tag{C.10}$$

where $\tilde{s}_T$ is similarly defined, but with respect to the pool of temporary employment (and the associated policy functions).

Moreover, we have that the difference between transition rates PU and TU satisfies

$$\begin{aligned} \Lambda_{PU}(\tilde{\phi}_0) - \Lambda_{TU}(\tilde{\phi}_0) = & \int_{x\in\tilde{X}(\phi_0)} \Big(s_P(x';\tilde{\phi}_0)h_P(x';\tilde{\phi}_0) - s_T(x';\tilde{\phi}_0)h_T(x';\tilde{\phi}_0)\Big)dx' \\ & + \int_{x\in X_P(\tilde{\phi}_0)\setminus X_T(\tilde{\phi}_0)} s_P(x';\tilde{\phi}_0)h_P(x';\tilde{\phi}_0)dx' \\ & - \int_{x\in X_T(\tilde{\phi}_0)\setminus X_P(\tilde{\phi}_0)} s_T(x';\tilde{\phi}_0)h_T(x';\tilde{\phi}_0)dx', \end{aligned}$$

for $\tilde{\phi}_0 \in \{\phi_0, \phi_0'\}$. Making use of (C.3) again, we have that

$$\begin{aligned} \Lambda_{PU}(\tilde{\phi}_0) - \Lambda_{TU}(\tilde{\phi}_0) = & \int_{x\in\tilde{X}(\phi_0)} \Big(s_P(x';\tilde{\phi}_0) - s_T(x';\tilde{\phi}_0)\Big)\frac{h_P(x';\tilde{\phi}_0) + h_T(x';\tilde{\phi}_0)}{2}dx' \\ & + \int_{x\in\tilde{X}(\phi_0)} \Big(h_P(x';\tilde{\phi}_0) - h_T(x';\tilde{\phi}_0)\Big)\frac{s_P(x';\tilde{\phi}_0) + s_T(x';\tilde{\phi}_0)}{2}dx' \\ & + \int_{x\in X_P(\tilde{\phi}_0)\setminus X_T(\tilde{\phi}_0)} \Big(h_P(x';\tilde{\phi}_0) - h_T(x';\tilde{\phi}_0)\Big)s_P(x';\tilde{\phi}_0)dx' \\ & + \int_{x\in X_T(\tilde{\phi}_0)\setminus X_P(\tilde{\phi}_0)} \Big(h_P(x';\tilde{\phi}_0) - h_T(x';\tilde{\phi}_0)\Big)s_T(x';\tilde{\phi}_0)dx' \end{aligned}$$

for $\tilde{\phi}_0 \in \{\phi_0, \phi_0'\}$. This can be more conveniently written as

$$\begin{aligned} \Lambda_{PU}(\tilde{\phi}_0) - \Lambda_{TU}(\tilde{\phi}_0) = & \int_{x\in\tilde{X}(\phi_0)} \Big(s_P(x';\tilde{\phi}_0) - s_T(x';\tilde{\phi}_0)\Big)\tilde{h}(x';\tilde{\phi}_0)dx' \\ & + \int_{x\in X} \Big(h_P(x';\tilde{\phi}_0) - h_T(x';\tilde{\phi}_0)\Big)\tilde{s}(x';\tilde{\phi}_0)dx', \end{aligned} \tag{C.11}$$

where

$$\tilde{s}(x;\tilde{\phi}_0) \equiv \begin{cases} (\bar{s}_P(x;\tilde{\phi}_0) + \bar{s}_T(x;\tilde{\phi}_0))/2, & \text{for } x \in \tilde{X}(\tilde{\phi}_0) \\ s_P(x;\tilde{\phi}_0), & \text{for } x \in X_P(\tilde{\phi}_0) \setminus X_T(\tilde{\phi}_0) \\ s_T(x;\tilde{\phi}_0), & \text{for } x \in X_T(\tilde{\phi}_0) \setminus X_P(\tilde{\phi}_0) \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\tilde{h}(x;\tilde{\phi}_0) \equiv \frac{h_P(x';\tilde{\phi}_0) + h_T(x';\tilde{\phi}_0)}{2},\ x \in \tilde{X}(\tilde{\phi}_0),$$

for $\tilde{\phi}_0 \in \{\phi_0, \phi_0'\}$. Insert (C.9), (C.10), and (C.11) into (C.7) to get (45) in the main text.

Finally, the steady-state equilibrium unemployment rate in policy regime $\phi_0$ satisfies

$$u(\phi_0) = \frac{\Lambda_{EU}(\phi_0)}{\Lambda_{EU}(\phi_0) + \Lambda_{UE}(\phi_0)}. \tag{C.12}$$

The difference in the unemployment rate across regimes can be written as

$$u(\phi_0') - u(\phi_0) = \tag{C.13}$$

$$\frac{1}{2}\left(\frac{\Lambda_{EU}(\phi_0')}{\Lambda_{EU}(\phi_0') + \Lambda_{UE}(\phi_0')} - \frac{\Lambda_{EU}(\phi_0')}{\Lambda_{EU}(\phi_0') + \Lambda_{UE}(\phi_0)} + \frac{\Lambda_{EU}(\phi_0)}{\Lambda_{EU}(\phi_0) + \Lambda_{UE}(\phi_0')} - \frac{\Lambda_{EU}(\phi_0)}{\Lambda_{EU}(\phi_0) + \Lambda_{UE}(\phi_0)}\right)$$
$$+ \frac{1}{2}\left(\frac{\Lambda_{EU}(\phi_0')}{\Lambda_{EU}(\phi_0') + \Lambda_{UE}(\phi_0')} - \frac{\Lambda_{EU}(\phi_0)}{\Lambda_{EU}(\phi_0) + \Lambda_{UE}(\phi_0')} + \frac{\Lambda_{EU}(\phi_0')}{\Lambda_{EU}(\phi_0') + \Lambda_{UE}(\phi_0)} - \frac{\Lambda_{EU}(\phi_0)}{\Lambda_{EU}(\phi_0) + \Lambda_{UE}(\phi_0)}\right). \tag{C.14}$$

The term of the first line of the RHS measures the contribution of the UE rate change to the change in the unemployment rate across regimes, in percentage points. The second line measures the contribution of the EU rate. Table 6 uses the latter to quantify the contribution of components in (44) and (45) to the unemployment rate change through the UE rate and the EU rate.

# D Additional results

## D.1 Cross-country outcomes: France vs. the U.S.

Table A.1: Decomposition of differences between the French-type and U.S. benchmark economies

| | Total | $A$ | $\delta$ | $b$ | $F$ |
|---|---|---|---|---|---|
| Reference param. value (U.S.) | | 0.60 | 0.0024 | 0.38 | 0 |
| Comparison param. (France) | | 0.30 | 0.0008 | 0.50 | 1.37 |
| Param. difference | | -0.30 | -0.0016 | 0.12 | 1.37 |
| Unem. rate (pp. difference) | 3.57 | -7.82 | 4.93 | 0.63 | 3.62 |
| *Transition rates (pp. difference)* | | | | | |
| UE | -21.77 | -4.55 | -21.25 | -12.51 | -21.63 |
| EU | -0.81 | -0.33 | -0.29 | -1.08 | -0.78 |

Notes: decomposition of differences in aggregate equilibrium outcomes between the "French-type" and U.S. baseline model economies. The first column reports the total difference in equilibrium outcomes between the French-type economy and the U.S. benchmark. The subsequent columns report the difference in outcomes after counterfactually imposing a given parameter in the U.S. baseline to be equal to its counterpart in the French economy, i.e., a measure of the total difference in equilibrium outcomes attributable to this parameter. The parameters under focus are the matching efficiency $A$ , the exogenous probability of separation $\delta$, the non-work income level $b$, and firing costs $F$. The first row indicates parameter values in the baseline (U.S.) economy, taken as reference point. The third row indicates the difference in parameter values between the baseline and French model economies.

## D.2 Cross-country outcomes: France vs. Spain

Table A.2: Decomposition of differences between the French and Spanish-type economies

| | Total | $\delta$ | $b$ | $F$ | $\phi$ |
|---|---|---|---|---|---|
| Reference param. value (French) | | 0.0008 | 0.50 | 1.37 | 0.045 |
| Comparison param. (Spanish) | | 0.0019 | 0.53 | 1.65 | 0.030 |
| Param. difference | | 0.0011 | 0.03 | 0.28 | -0.015 |
| *Stocks (pp. difference)* | | | | | |
| Unem. rate | 4.32 | 1.86 | 2.96 | 4.30 | 3.92 |
| TC emp. share | 7.52 | 4.48 | 6.62 | 7.47 | 4.46 |
| *Transition rates (p.p. difference)* | | | | | |
| UE | -0.97 | -0.92 | -0.06 | -0.94 | -0.98 |
| EU | 0.50 | 0.14 | 0.41 | 0.49 | 0.44 |

Notes: decomposition of differences in aggregate equilibrium outcomes between the "French-type" and "Spanish-type" model economies. The first column reports the total difference in equilibrium outcomes between the Spanish economy and the French-type economy. The subsequent columns report the difference in outcomes after counterfactually imposing a given parameter in the French model economy to be equal to its counterpart in the Spanish economy, i.e., a measure of the total difference in equilibrium outcomes attributable to this parameter. The parameters under focus are the exogenous probability of separation $\delta$, the non-work income level $b$, firing costs $F$, and the regulatory restriction on the duration of temporary contracts, $\phi$. The first row indicates parameter values in the French economy, taken as reference point. The third row indicates the difference in parameter values between the Spanish and French model economies.