

On the Optimization of Open Truss Interlaced Composite Structures Featuring Continuous Yarns

by

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Abstract

Topology optimization (TO) is the process of maximizing the structural efficiency of parts by removing material from zones within the part envelope that do not carry significant loads. Designs produced with TO typically include free forms and intricate shapes that lend well to additive manufacturing (AM) techniques when traditional materials are used. However, conventional slicing used in AM techniques is not suited for manufacturing structural, continuous yarn polymer-matrix fibre reinforced composites (PMFRCs) as it cannot lead to carbon yarns extending along load transmission paths defined in 3D. Due to this limitation, it is not currently possible to perform true 3D TO on PMFRCs with continuous yarns as already possible with metal and polymer parts. As a result, PMFRCs may outperform metal alloys in terms of tabulated specific properties but in the current state of industrial technology, additively manufactured metal structures can outperform structures made from PMFRCs. Previous work showed that TO continuous yarn PMFRC parts can be made by interlacing dry continuous yarn in 3D. The configuration of the resulting dry yarn truss is then optimized for maximizing specific structural stiffness of the resulting 3D TO PMFRC part through iterative yarn tension adjustments. The objective of the present work is to further optimize the geometries of PMFRC trusses by means of a seven-step sequential process. The process begins with a static finite element analysis (FEA) to extract principal stress fields and proceeds through a series of algorithmic steps including stress path generation, main stress path selection, triangulated structure construction via k -neighbours triangulation, optimization of yarn alignment with the principal stress field, graph-based construction of individual yarn paths and graph-based structural thickening to properly model yarn cross-sections. Together, these steps enable the design of truss-like structures that follow mechanically optimized stress trajectories.

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Chapter 1

Introduction

1.1 Motivation

Polymer-matrix fibre-reinforced composites (PMFRC) have become structural materials of choice for demanding structural and load-bearing applications because of their exceptional specific mechanical properties and relative ease of manufacturing, especially in smaller series. These important advantages have made the materials prevalent in the aerospace sector, where more than 50% of the structure of civilian transport aircraft are routinely made of PMFRCs.

The last decades saw the parallel emergence of additive manufacturing processes for parts made of polymers and, of more relevance here, metal alloys. Additive manufacturing (AM) processes enable the production of structural metal parts featuring geometry that cannot be made using subtractive manufacturing processes such as computer numerical control (CNC) machining. Critically, additive manufacturing enables the economic production of trellis/lattice parts with topological optimization, where material is deposited only along paths of high stresses under select load cases. These technologies have led to structures that are highly efficient on a mass basis [1].

PMFRC manufacturing processes such as liquid moulding, advanced tape placement lay up (ATP/AFP), prepreg autoclaving and most others are inherently additive in nature. Despite this, true topology optimization (TO) has not happened with PMFRC, for two reasons [2]. Firstly, the usage of continuous yarns in structural PMFRC restricts flexibility in yarn orientations. Current printed reinforced plastic parts typically feature short fibres or, sometimes, layered continuous yarns [3–6] that can be oriented optimally in 2D but not in 3D along paths of higher stresses [8]. Secondly, industrial PMFRC processes generally do not enable routine production of structures optimized as trellis/lattices. While some examples of manufacturing were reported, the resulting truss-like structures are typically simple and feature a repeating basic structural unit [8–12] such as isogrids [13, 14]. Few examples of non-repeating 3D structures were reported [15].

As a result, PMFRCs may outperform metal alloys in terms of tabulated specific material properties but in the current state of technology, additively manufactured metal structures can outperform structures made from PMFRCs. PMFRC parts remain limited to beam and shell geometries because of limitations to PMFRC manufacturing processes. This is the main challenge that this work addresses: bringing the increased performance of true TO to structural PMFRC parts featuring continuous carbon fibre yarns.

1.2 Topology Optimization in Composite Parts

TO is a mathematical method which optimizes the spatial distribution of material within a defined domain for a given set of loads, boundary conditions and constraints with the goal of maximizing the structural performance of the system [16].

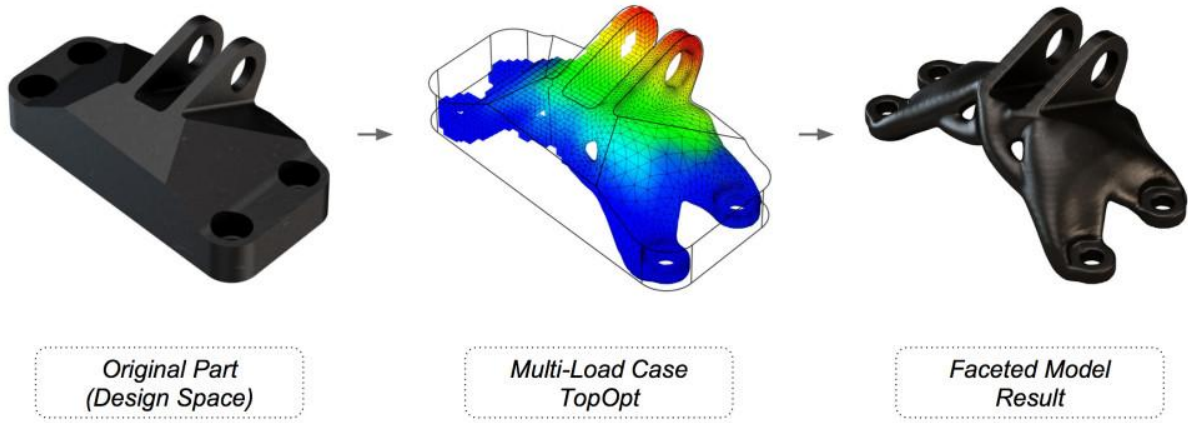


Figure 1.2.1: Overview of TO workflow [17]

TO maximizes the structural performance and efficiency of parts by removing redundant material from areas that do not need to carry significant loads, ultimately reducing weight or solving design challenges such as reducing the effects of resonance or thermal stress. Designs produced with TO typically include free forms and intricate shapes that lend well to AM techniques. AM typically utilizes a 3D computer-aided design (CAD) model of the part, slicing it along a single direction into many very thin cross-section profiles. Cross-section perimeters are then traced either by a laser, electron beam, extrusion nozzle or jetting nozzle and the area contained within the perimeters is filled with a hatching pattern. Once a layer has been deposited/molten/cured, the next layer is added until the whole part has been generated [18].

However, the conventional slicing process used for manufacturing parts using AM is not fit to manufacturing PMFRCs due to constraints imposed by required orientations and continuity of yarns for optimal part performance. In fact, one of the greatest challenges in printing continuous carbon fibre reinforced composites with complex shapes and high mechanical performance are limitations of printing path design, whereas carbon fibres cannot be printed or deposited along preferred load transmission paths of composite parts, beyond simple, trivial cases [20].

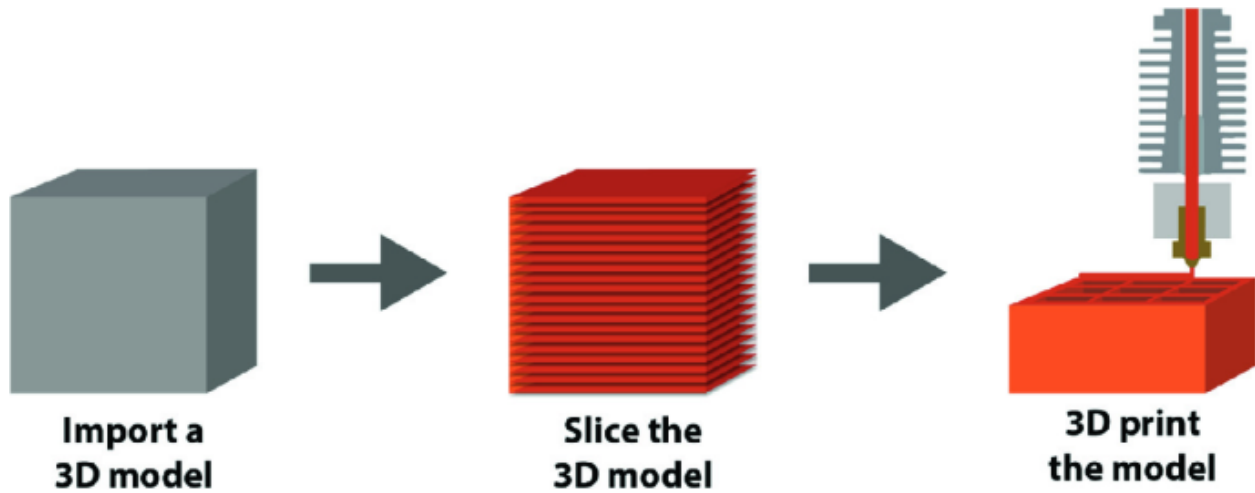
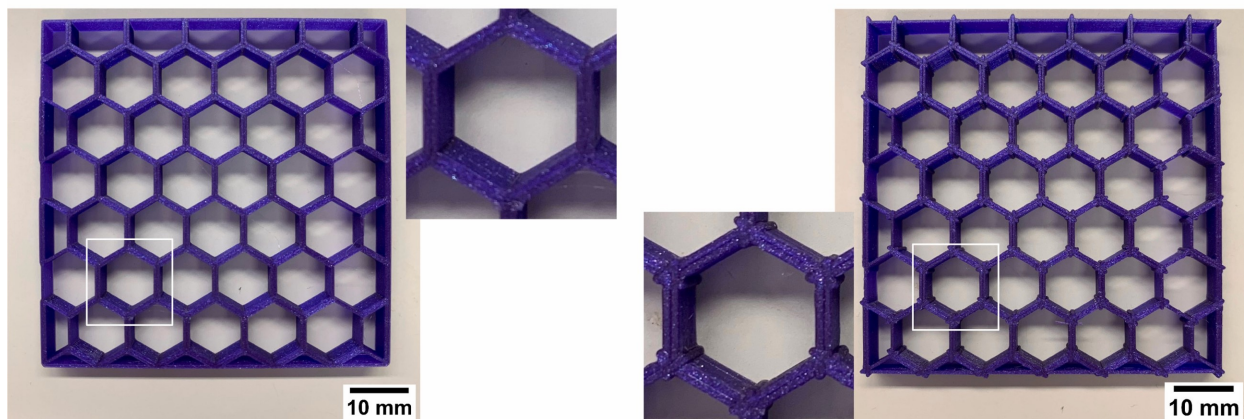


Figure 1.2.2: Overview of conventional slicing process for AM [19]

Under current methods the geometries of 3D-printed PMRFCs with continuous yarns are limited to shapes with relatively simple, and constant cross-sectional geometry, or parts that are quite thin such as shells and plates where yarns only extend in-plane [20, 24–26]. Due to these geometric limitations, it is not currently possible to perform true TO in 3D on PMRFCs with continuous yarns, the likes of which are already seen with metal and polymer parts.



(a) Honeycomb grid [24]



(b) Suspension plate [20]

Figure 1.2.3: Composite parts featuring continuous yarns produced with 3D printing

1.3 Project Overview and Objectives

The main objective of this thesis is to develop and validate a sequential process for the design of 3D TO continuous yarn PMFRC parts. First, an initial isotropic part volume subjected to specified loading conditions is analyzed using the finite element method (FEM) to determine a stress field and paths of high principal stress. Points along these paths of high principal stress as well as points defining surfaces on which boundary conditions and loads are applied are used in a k -neighbours triangulation, producing an initial interlaced PMFRC structure. This structure is then optimized to maximize alignment of its members along orientations of principal stress paths. Lastly, by modelling the structure using graph theory, paths for individual interlaced dry yarns are extracted and thickened to properly model yarn cross-sections. These steps enable true 3D TO in manufactured PMFRC parts featuring continuous fibre yarns.

This chapter introduces a sequential process for the design of 3D TO continuous yarn PMFRC parts subjected to specific load cases. Taking up from the work of Liao [27], the goal is to distribute PMFRC material within a part envelope in a way that reduces both mass and strains under specified load cases maximizing specific structural stiffness. In Liao's work, interlacing patterns and initial structure geometry were generated arbitrarily. The process introduced in this chapter generates non-arbitrary, optimized interlacing patterns and PMFRC part geometries through a number of sequential steps. A general overview of the process is shown in Figure 1.3.1.

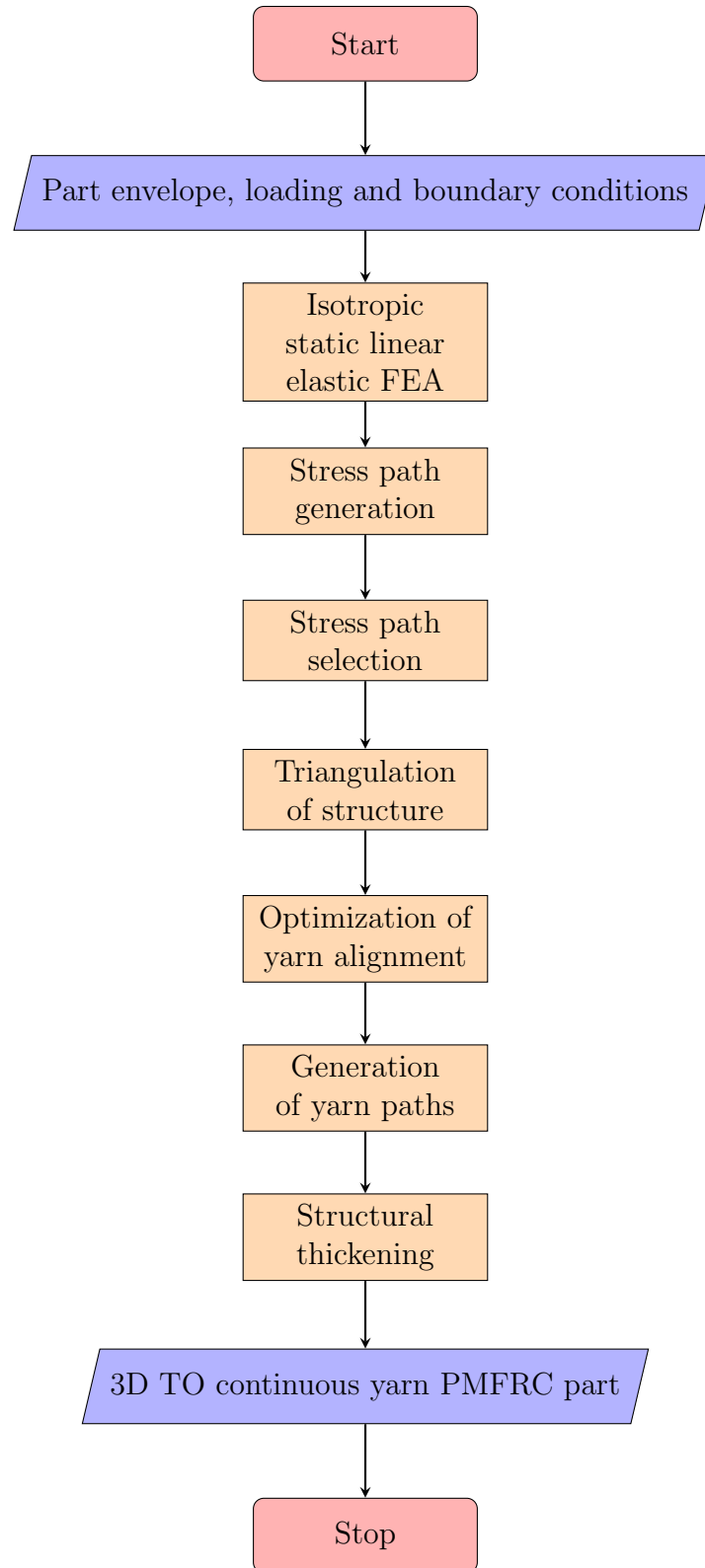


Figure 1.3.1: General overview of the process for the design of 3D TO continuous yarn PMFRC parts

In step 1, static linear elastic structural FEA is performed on isotropic material contained within the part envelope, for specified loading and boundary conditions. The principal stress vector field is extracted. In step 2, a large number of path lines tangent to the resulting principal stress vector field at every point are generated, labelled as initial principal stress paths. Seed points for initial principal stress path generation are positioned uniformly throughout the part volume to ensure that paths are distributed throughout the part in a generally uniform manner, hence the entire volume of the part is populated with initial principal stress paths. In step 3, a small number of the most significant principal stress paths are selected, labelled as main principal stress paths. A prioritization index is associated with each initial stress path based on stress levels, length and the current distribution of selected stress paths within the part volume. Principal stress paths are foundational elements of triangulated interlaced dry yarn trusses. In step 4, points along the stress paths as well as points where loads and boundary conditions are applied are used for generating the triangulated interlaced dry yarn truss, using k -neighbours triangulation. Triangulated structures are generally sought for their stiffness performance [28]. In step 5, an optimization loop is performed where the interlacing points of the structure are displaced to maximize yarn alignment with the directions of the principal stress vector field. In step 6, paths for individual interlaced dry yarns are generated using a distance prioritization scheme along with Dijkstra’s algorithm. Finally in step 7, the yarns are thickened by attributing them circular cross-sections; in steps 1-6, only 1D cross-sections are considered. An algorithm to detect and solve interferences between thickened yarns is developed using the vertex contraction operation for graphs.

1.4 Thesis Contents

This thesis is divided into 10 chapters. Chapter 1 is an introduction to the thesis topic and project motivation. Chapter 2 highlights relevant literature. Chapter 3 presents the 13 design

cases that were analyzed in this work. Chapter 4 presents the generation of stress paths from principal stress fields. Chapters 5 and 6 highlight and compare the use of Delaunay and k -neighbours triangulation to generate triangulated, dry interlaced structures. Chapter 7 showcases the optimization problem that was posed to maximize the alignment of yarns with directions of principal stress. Chapter 8 presents the generation of interlacing sequences for the structures using graph theory. Chapter 9 shows the preliminary implementation of a structural thickening algorithm that augments the dimensions of the interlaced structures' cross-sections from 1D to 2D. Finally, Chapter 10 concludes on the project and highlights areas of future work.

1.5 Thesis Contributions

By creating a novel process for the design of 3D TO continuous yarn PMFRC parts, the algorithms were designed to achieve the following:

- Stress path generation from principal stress vector fields
- Stress path importance indexing
- k -neighbours triangulation to generate dry interlaced structures
- Maximization of yarn alignment with directions of principal stress fields
- Interlacing yarn path pattern generation
- Extending the dimension of graph-based representations of structures by introducing 2D cross-sections and managing interference

These algorithms form a framework that systematically generates optimized dry interlaced structures featuring continuous yarns for a given an initial part volume, loads and boundary conditions. This enables optimal continuous yarn placement in eventual 3D printed PMFRC parts, a key component of true TO in 3D.

Chapter 2

Literature Review

2.1 Topology Optimization

TO is a mathematical method used for determining the optimal material distribution within a given design space, subject to constraints and objectives such as minimizing compliance or maximizing stiffness under set loads and conditions. The optimization procedure determines which points of the design domain should be full of material and which void [29]. The general TO problem is formulated as:

$$\min_{\rho} f(\rho) \tag{2.1}$$

subject to:

$$g_i(\rho) \leq 0, \quad i = 1, 2, \dots, m \tag{2.2}$$

$$h_j(\rho) = 0, \quad j = 1, 2, \dots, n \tag{2.3}$$

$$0 < \rho_{\min} \leq \rho(x) \leq 1, \quad x \in \Omega \tag{2.4}$$

where $\rho(x)$ is the material density function, where $\rho = 1$ represents solid material and $\rho = 0$ represents void; $f(\rho)$ is the objective function, which may represent compliance, natural frequency, or other performance measures; $g_i(\rho)$ are inequality constraints, such as volume; $h_j(\rho)$ are equality constraints, such as equilibrium conditions, and Ω is the design domain in which topology optimization is performed. A common objective in TO is to minimize structural compliance, which is equivalent to maximizing stiffness. The compliance-based TO problem is formulated as:

$$\min_{\rho} C(\rho) = \int_{\Omega} \sigma(\rho) : \varepsilon(\rho) d\Omega \quad (2.5)$$

subject to:

$$K(\rho)u = F \quad (2.6)$$

$$\int_{\Omega} \rho(x) d\Omega \leq V_{\max} \quad (2.7)$$

where $C(\rho)$ is the compliance, representing the total strain energy stored in the structure; σ and ε are the stress and strain fields, respectively; $K(\rho)$ is the global stiffness matrix; u is the displacement field; F is the external force vector, and $V_{\max}$ is the allowable material volume fraction. To solve the TO problem as formulated, several methods have been developed, such as homogenization [30], density-based approaches [31], level set methods [32], moving morphable component [33], and bi-directional evolutionary structural optimization [34]. These methods typically utilize the finite element method, where the continuous design domain is discretized into a number of finite elements. The result is an optimized structure that distributes material efficiently for load-bearing performance, Figure 2.1.1.

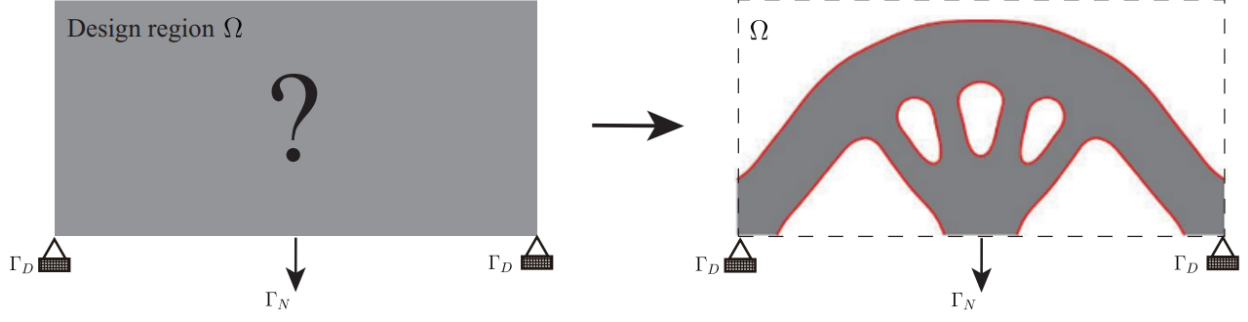


Figure 2.1.1: Design space before and after applying TO [29]

2.2 Topology Optimization of Composite Parts

In the case of PMFRCs, TO involves additional design variables that control fibre orientations. The problem is extended to optimize both material distribution and fibre orientations:

$$\min_{\rho, \theta} C(\rho, \theta) = \int_{\Omega} \sigma(\rho, \theta) : \varepsilon(\rho, \theta) d\Omega \quad (2.8)$$

subject to:

$$K(\rho, \theta)u = F \quad (2.9)$$

$$\int_{\Omega} \rho(x) d\Omega \leq V_{\max} \quad (2.10)$$

$$\theta_{\min} \leq \theta(x) \leq \theta_{\max}, \quad x \in \Omega \quad (2.11)$$

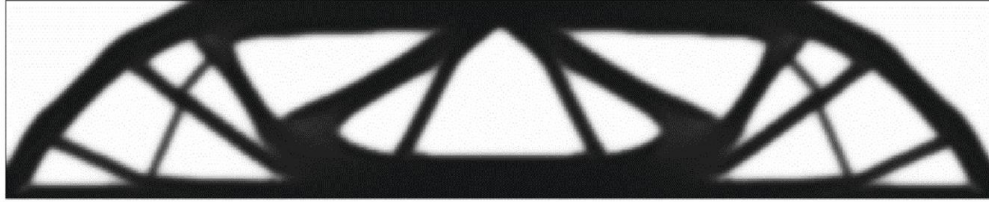
where $\theta(x)$ represents the fibre orientation at each point in the structure and $K(\rho, \theta)$ is the anisotropic stiffness matrix. TO methods for composite parts can be categorized broadly into two types: sequential optimization and concurrent optimization [35]. The former technique relies on determining the structural topology first and arranging the fibre orientations at the second stage, or vice versa. This strategy decouples the optimization procedures for density and anisotropy, simplifying problem formulation and preventing numerical in-

stabilities that can arise from mixing the two different types of design variables. Dividing optimization into two smaller sub-problems reduces overall computational load. Standard TO algorithms can also be applied to matrix optimization without immediately considering fibre anisotropy. Additionally, since fibre optimization occurs after material distribution is finalized, constraints from manufacturing techniques can be better incorporated.

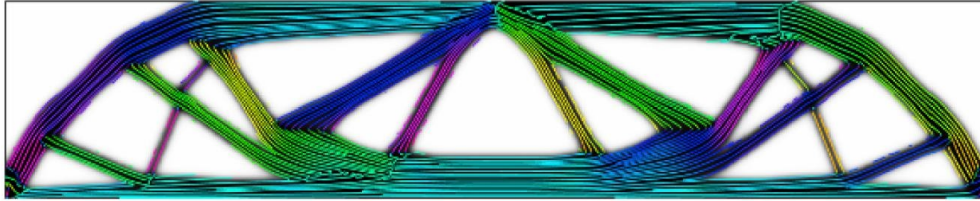
Motivated by these factors, many researchers have explored different variations of the sequential design method. For example, Lee et al. [36] proposed a three-step sequential method for the optimization of 3D printed PMFRCs through compliance minimization. The first step provides the matrix distribution by performing an isotropic TO using a density-based method, while the second and third steps provide an optimized continuous fibre orientation field and discrete fibre paths respectively, Figure 2.2.1. Similar three-step approaches were also proposed by Nikoleta [37] and Peeters et al. [38] that leverage lamination parameters to produce optimized variable stiffness laminates, Figure 2.2.2.

Sequential TO has also been addressed utilizing principal stress fields to guide fibre path generation. Zhang et al. [39] and Li et al. [20] both proposed two-step sequential processes for PMFRCs produced using 3D printing. The first step involves an isotropic TO using a density-based method to obtain a matrix distribution, while the second step consists of using a streamline approach to generate fibre paths. That is to say, the principal stress field that is obtained during initial TO is used for generating lines that are tangent to the maximum principal stress orientations, to produce fibre paths that maximize stiffness in the final part. The work of Pedersen [40,41] showed that maximizing stiffness in PMFRCs requires aligning fibre orientations with maximum principal stress directions. Zhang et al's process is illustrated in Figure 2.2.3.

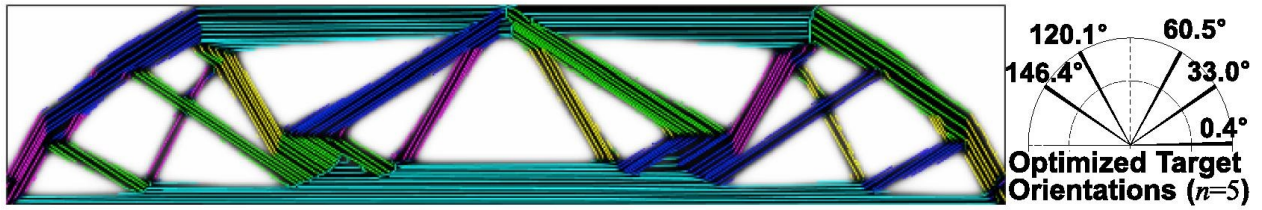
Contrarily to sequential optimization, concurrent TO optimizes the material distribution



(a)



(b)



(c)

Figure 2.2.1: (a) Isotropic design, (b) continuous fibre orientation field and (c) discrete fibre paths of the Messerschmitt–Bølkow–Blohm (MBB) beam produced by the optimization process proposed by Lee et al. [36]

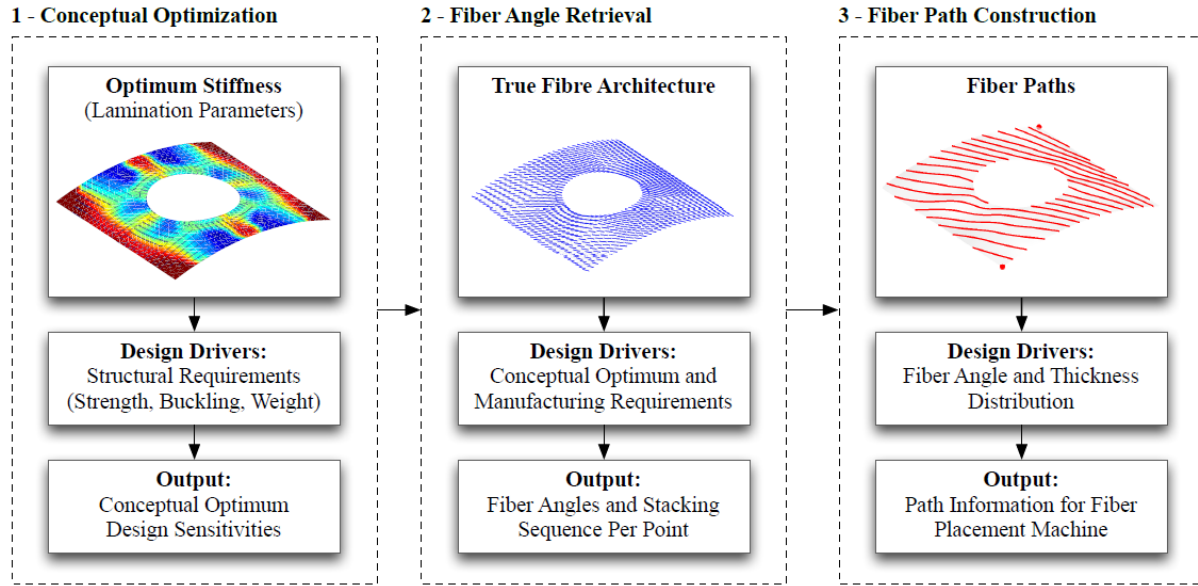


Figure 2.2.2: Three-step optimization process for variable stiffness laminates [37]

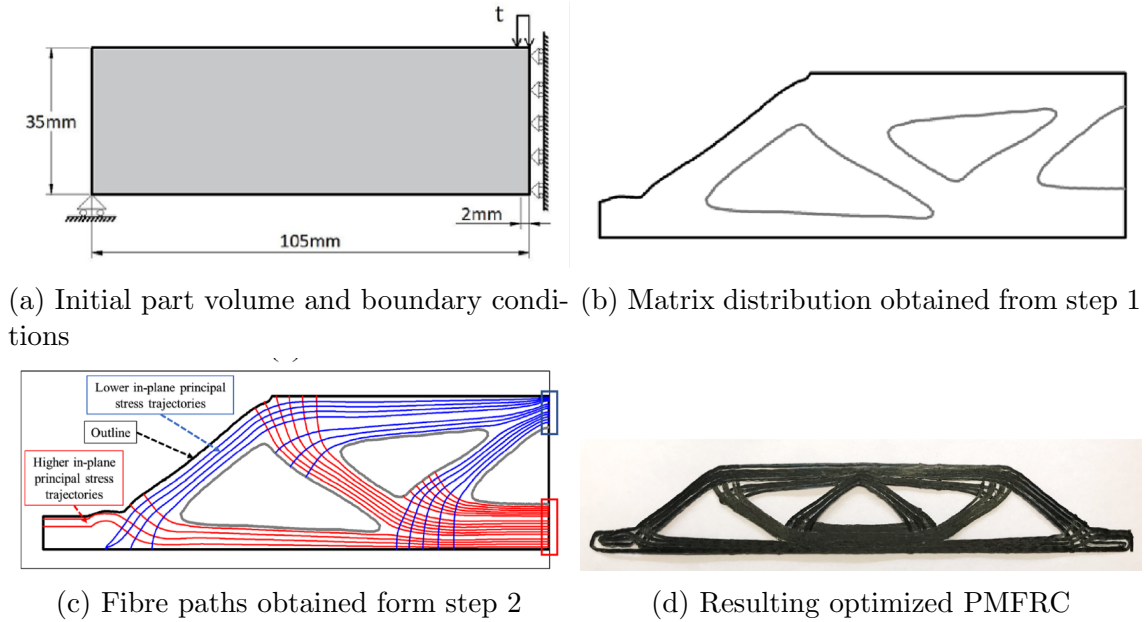


Figure 2.2.3: Zhang et al's streamline-based TO process for 3D printing [39]

and fibre orientation simultaneously. With concurrent TO, the optimization problem becomes significantly more complex, requiring a multi-field optimization approach often utilizing gradient-based methods [42–44], level-set methods with fibre orientation tracking [45–47] or genetic algorithms [48] to solve coupled design variables. This being said, there has still been a growing inclination toward the concurrent approach since it may provide higher structural performance in certain problems such as those involving highly anisotropic materials, multi-scale structures, or stress-driven designs where both the topology and material orientation must be simultaneously optimized to achieve optimal performance [35].

Kim et al. [49] proposed a homogenization and density-based hybrid approach for concurrent TO. The density based method determines whether or not material is void or composite within the part topology, while the homogenization method specifies the microstructure of unit cells within the topology. Size and orientation parameters of the microstructures are then treated as the design variables to be optimized, Figure 2.2.4. The authors considered three different types of cells, all of which were proven effective on multiple geometries including the Messerschmitt–Bölkow–Blohm (MBB) beam, Figure 2.2.5.

The majority of studies in the literature only address fibre path generation for 2D or 2.5D designs [35]. The TO methods proposed by Schmidt et al. [50], Jung et al. [51] and Luo et al. [52] are among rare works that consider the derivation of 3D continuous fibre trajectories in the context of TO. Schmidt et al. hybridized a gradient-based algorithm with simulated annealing to tackle the highly non-convex nature of concurrent TO. The versatility of this approach was shown via different design examples including large-scale industrial products under multi-axial loads, force inverters, and porous structures, Figure 2.2.6.

Similarly to Kim et al. [49], Jung et al. utilized a multiscale approach combined with a homogenization-based method to produce optimized PMFRCs geometries. Jung et al. ad-

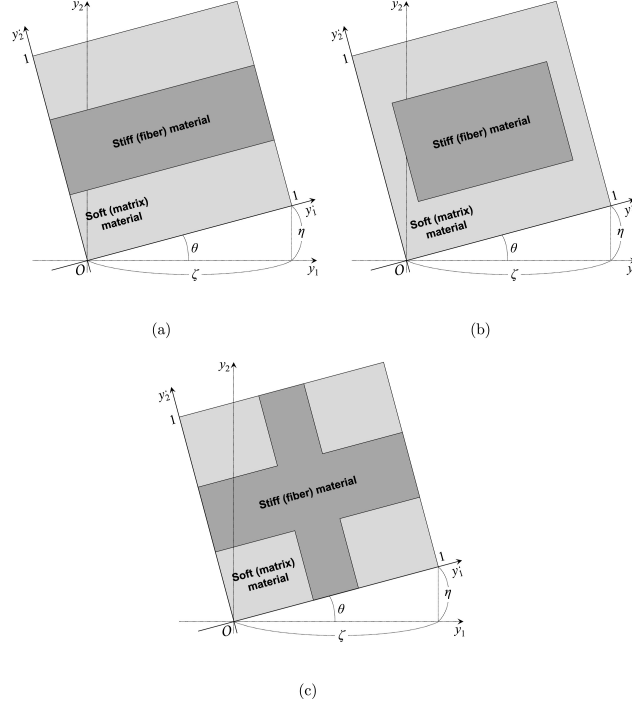
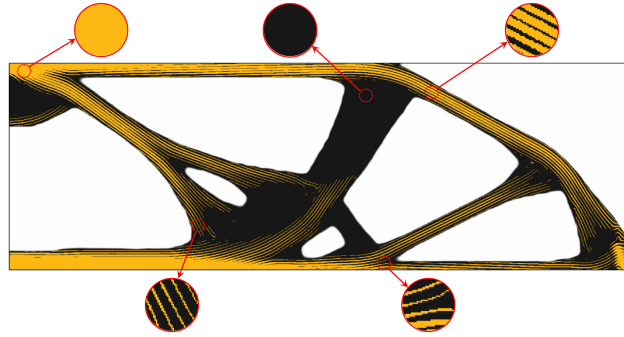


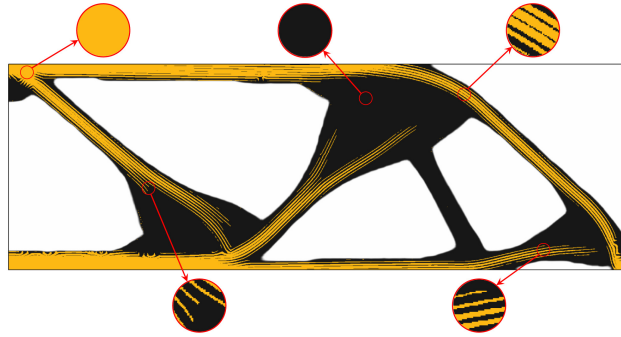
Figure 2.2.4: (a) Unidirectional fibre layout unit cell (b) Rectangular shape fibre layout unit cell (c) Cross shape fibre layout unit cell from Kim et al.'s framework [49]

ditionally used a de-homogenization scheme to obtain continuous 3D fibre paths from the optimized fibre orientation field, Figure 2.2.8. It is of note that the design approach proposed in the paper is restricted to composites with circular fibres arranged in square arrays. Luo et al. utilized density based methods to optimize the initial part topology, while continuous fibre paths were generated using a method inspired by field-based toolpath generation methods combined with a continuity constraint, Figure 2.2.8.

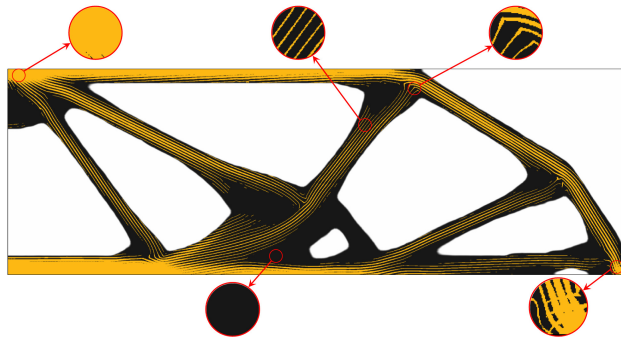
When considering 3D fibre path generation for TO, the resulting PMFRCs typically feature complex geometry and fibre path morphology [35]. This issue becomes prominent for 3D composite designs with fibre paths perpendicular to the printing plane. Such structures cannot be created through planar layer-by-layer printing since they require multi-axis AM [53, 54]. Consequently, further studies on 3D printing approaches are being conducted, focusing on both software [55, 56] and hardware/experiments [53, 57, 58].



(a)



(b)



(c)

Figure 2.2.5: (a) Unidirectional unit cell TO (b) Rectangular unit cell TO (c) Cross unit cell TO for the MBB beam using Kim et al.'s framework [49]

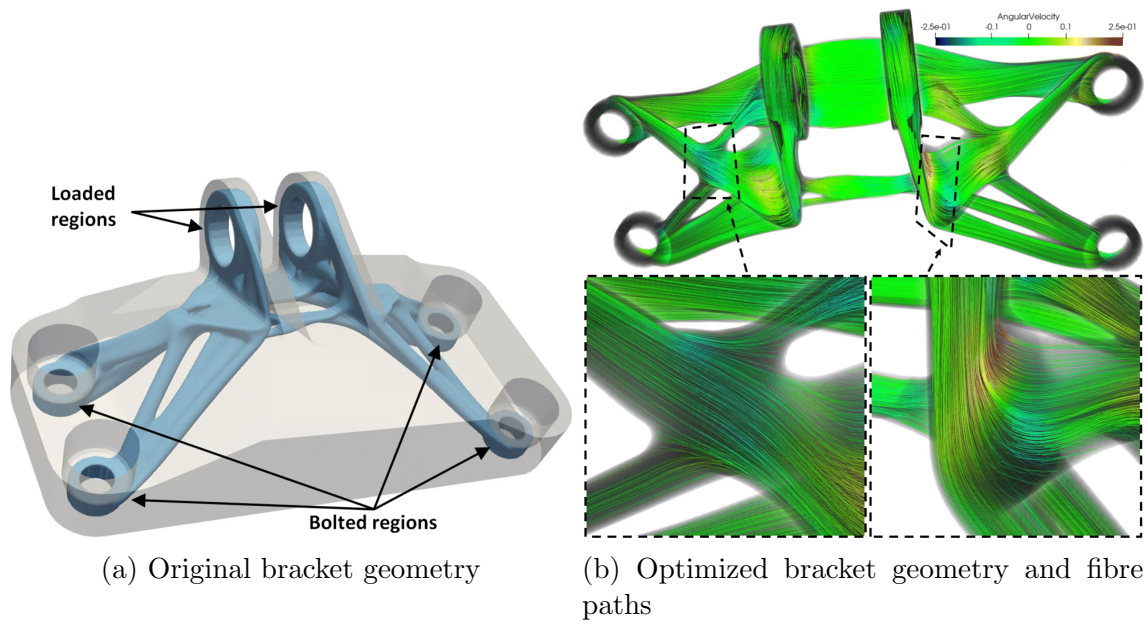


Figure 2.2.6: Optimized bracket geometry obtained from Schmidt et al's gradient-based algorithm [50]

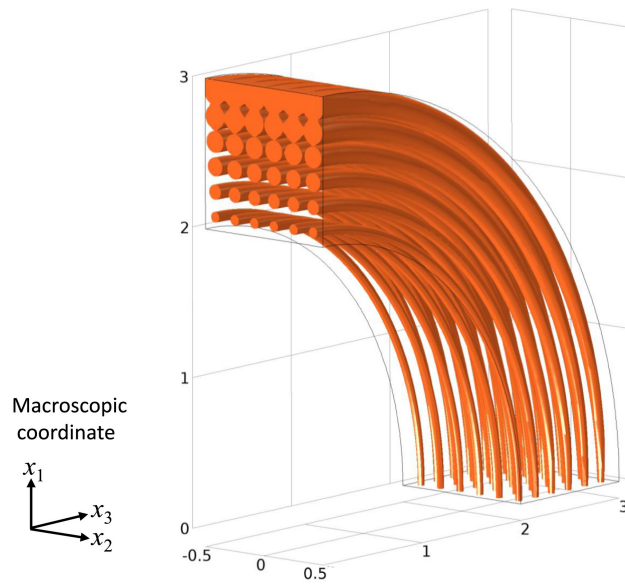


Figure 2.2.7: 3D fibre paths generated by Jung et al. [51]

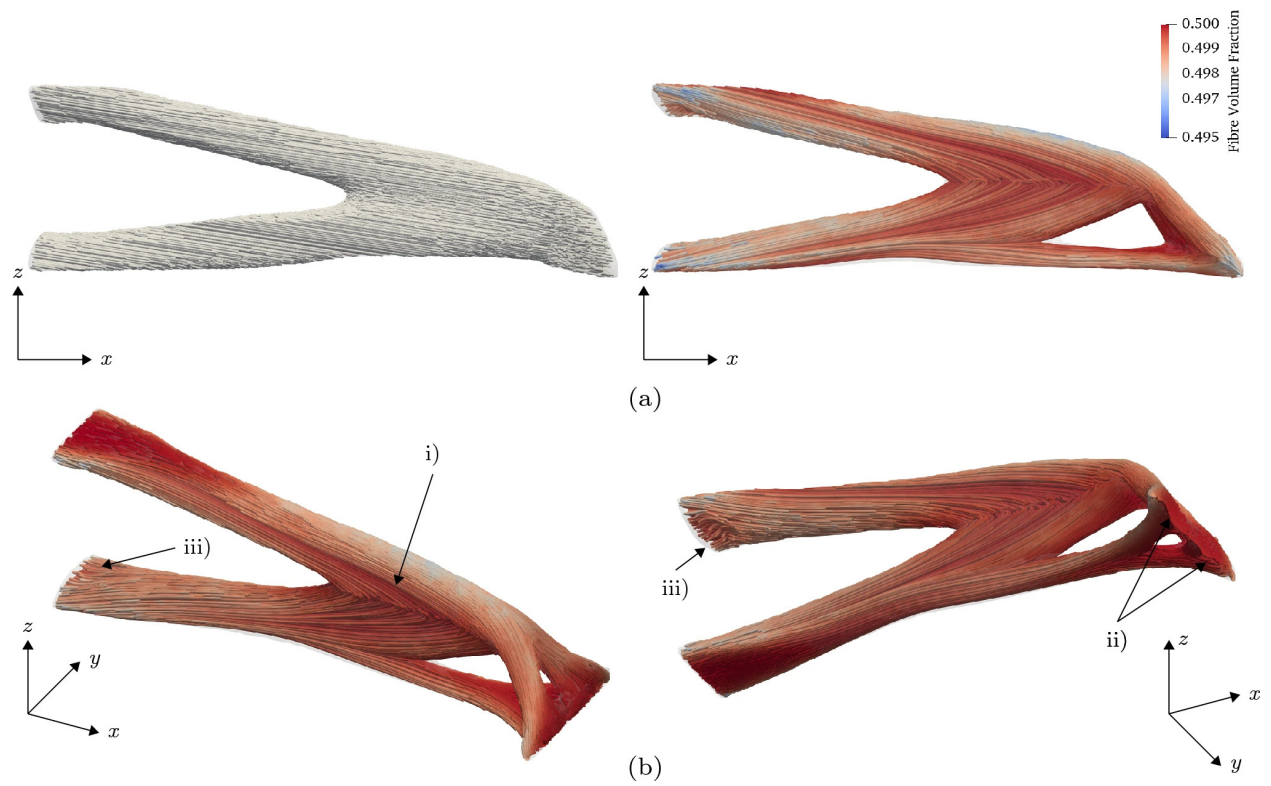


Figure 2.2.8: Topology and fibre path optimization results from Luo et al. [52] (a) Designs for constant (left) and spatially varying (right) fibre volume fractions (b) Regions highlighting (i) sharp fibre turns occurring due to abrupt change in principal stress directions (ii) discontinuous fibres and (iii) variations of spacing between fibres

2.3 3D Printing of Composite Parts

While the reinforcement of PMRCs by short fibres seems possible for nearly all AM methods, reinforcement by continuous fibres is possible with only some of them. The adjustment of fibre orientation is only possible for processes that utilize a moving print head [59]. Fused Deposition Modelling (FDM) or Fused Filament Fabrication (FFF, analogous to FDM) can be used for producing PMFRCs [60].

FDM typically utilizes a continuous polymer filament as a feedstock material. The polymer filament feeds into an extruder, which is then heated to a quasi-liquid state, enabling it to pass through a heated extruding orifice where it is fused in-place on a print surface Figure 2.3.1. This extruding apparatus is typically mounted onto an X-Y computer numerical control (CNC) gantry allowing the formation of complex geometries. Once a layer pattern is complete, the print platform drops down or the extruding orifice rises to deposit a subsequent layer of material. Through the deposition of successive layers, 3D parts are fabricated.

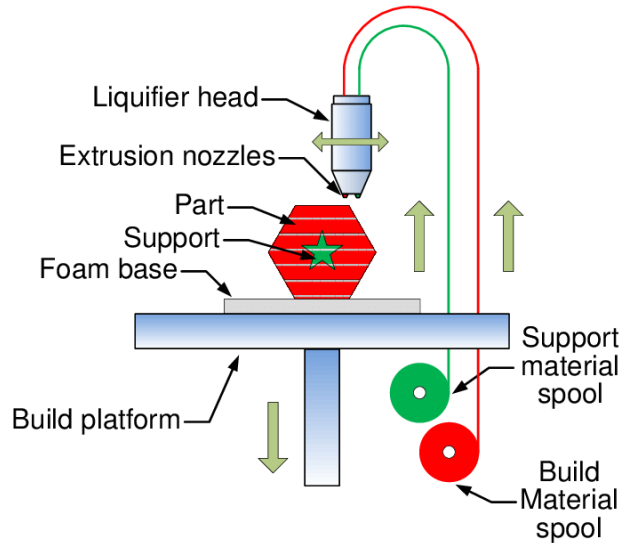


Figure 2.3.1: Traditional FDM process [60]

One method to produce PMFRCs by FDM is to simply change the feedstock material to a prefabricated composite filament, such as textile mixtures of thermoplastic and reinforcing fibres or pre-impregnated rovings. This leads to a constant fibre-volume ratio in the fabricated part. An additional modification that can be made to the traditional FDM process is to utilize two print stages, with separate heads to first print the matrix, then the continuous fibre reinforcements, Figure 2.3.2 [61]. Additionally, fibres can be impregnated in the heating block of the nozzle itself, Figure 2.3.3 [20]. Crucially, in all cases—including commercial systems such as those from Anisoprint [62] and Markforged [63]—the layer-by-layer stacking principle is used for producing PMFRC parts. This strongly limits orientations that continuous fibres can take in the 3D domain, drastically limiting the PMFRC geometries and fibre orientations, Figure 2.3.4.

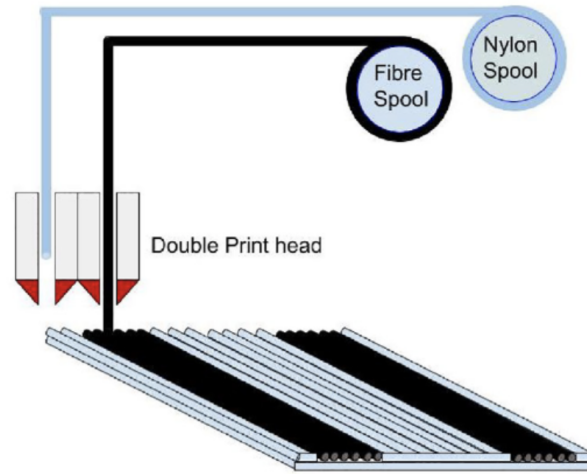


Figure 2.3.2: FDM process using a double print head for PMFRCs [61]

The freehanging 3D printing method proposed Liu et al. [64] by was developed in order to break geometrical limitations imposed by traditional 3D printing of PMFRCs. As opposed to layer-by-layer stacking that can only produce relatively simple geometries with relatively constant cross-sections, or thin parts such as shells and plates, the freehanging method can produce overhanging and undercut structures with the guidance of the nozzle's spatial movement. Impregnation of the carbon fibre takes place in the nozzle.

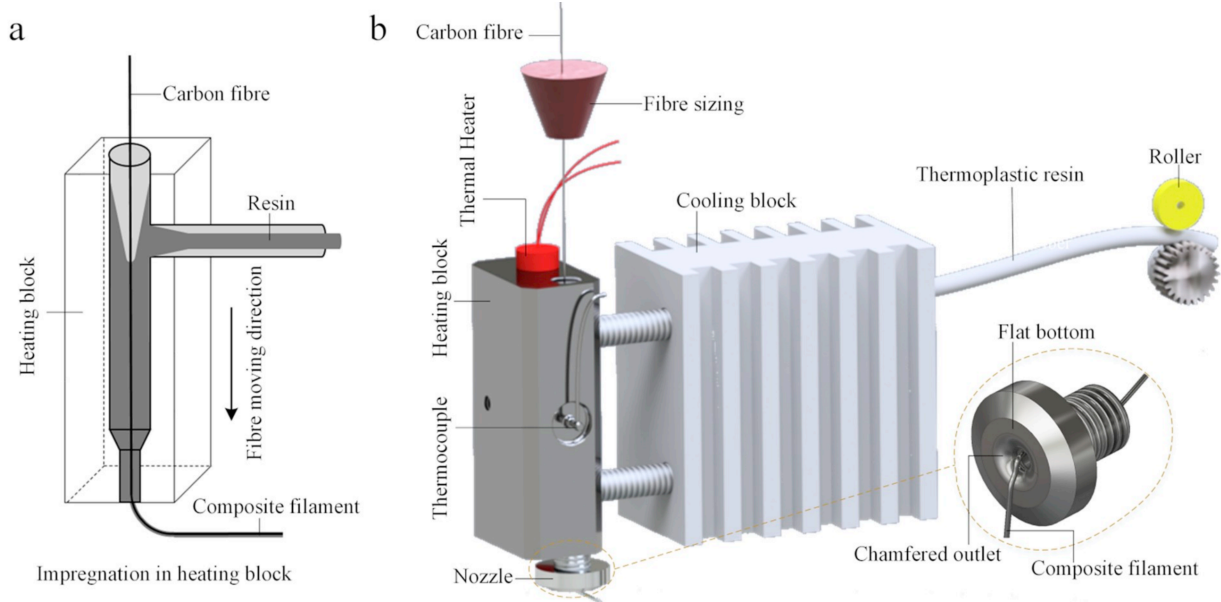
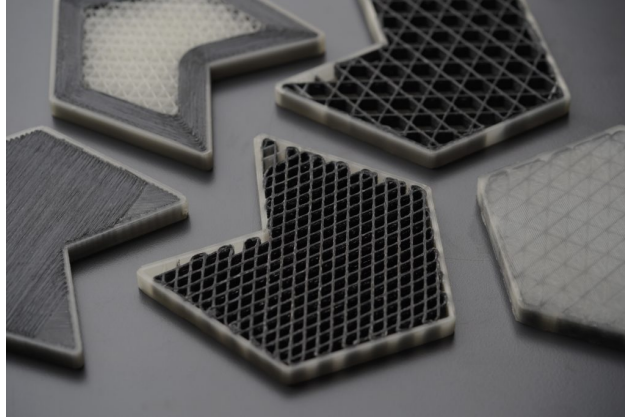


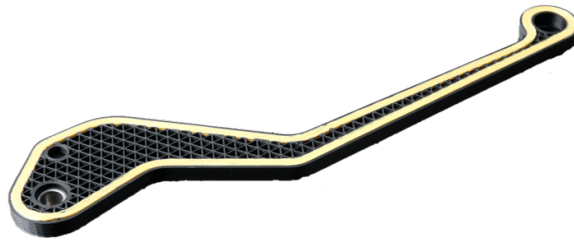
Figure 2.3.3: Impregnation of carbon fibre and resin matrix in printing head [20]

As a result, continuous fibres can be better aligned with load transmission paths in the part. There are four typical printing stages to produce overhangs/undercuts: the horizontal stage, upward stage, dangling stage and downward stage as shown by Figure 2.3.6. However, one key challenge when producing these structures is the generation of printing paths that avoid fibre breakage, debonding and structural deformations while printing. Thus, the feeding speed, extrusion capacity, cooling air flows and additional printing path modifications are used as control parameters and tuned throughout the four phases of the typical printing process.

At the horizontal stage, the distance between nozzle and print bed is reduced, increasing the bonding strength between panel and part. For the upward stage, the nozzle is raised by an angle α that is dependent on part geometry. To avoid debonding between the part and print bed, the matrix extrusion capacity is increased before the nozzle is lifted, and the cooling air flow is intensified. Structural deformations are unavoidable during the dangling stage, due to movement of the nozzle while the part is curing. This being said, modifications



(a) Various parts produced by Anisoprint [65]



(b) Motorcycle handle produced by Markforged [66]

Figure 2.3.4: Commercially 3D printed PMFRCs featuring continuous yarns

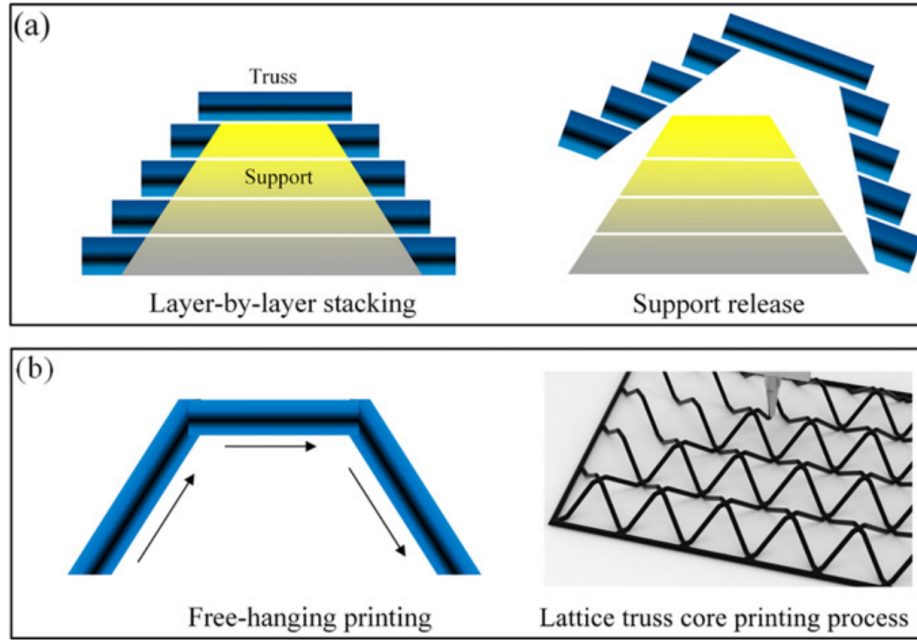


Figure 2.3.5: Comparison between traditional and freehanging 3D printing methods [64]

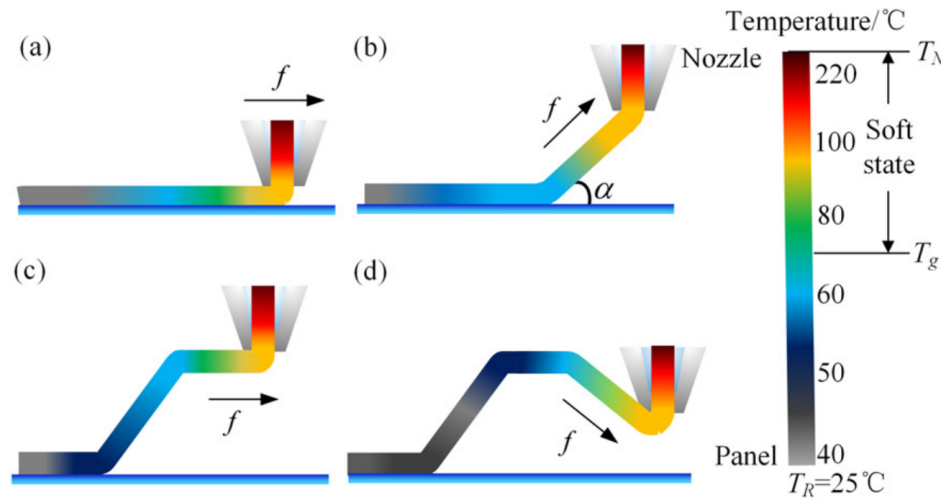


Figure 2.3.6: (a) Horizontal stage (b) upward stage (c) dangling stage and (d) downward stage of freehanging [64]

are brought to the printing path for the upward stage to introduce additional structural deformation such that the printing path is in its target position after thermal deformation. Different types of geometries produced by freehanging 3D printing are shown in Figure 2.3.7.

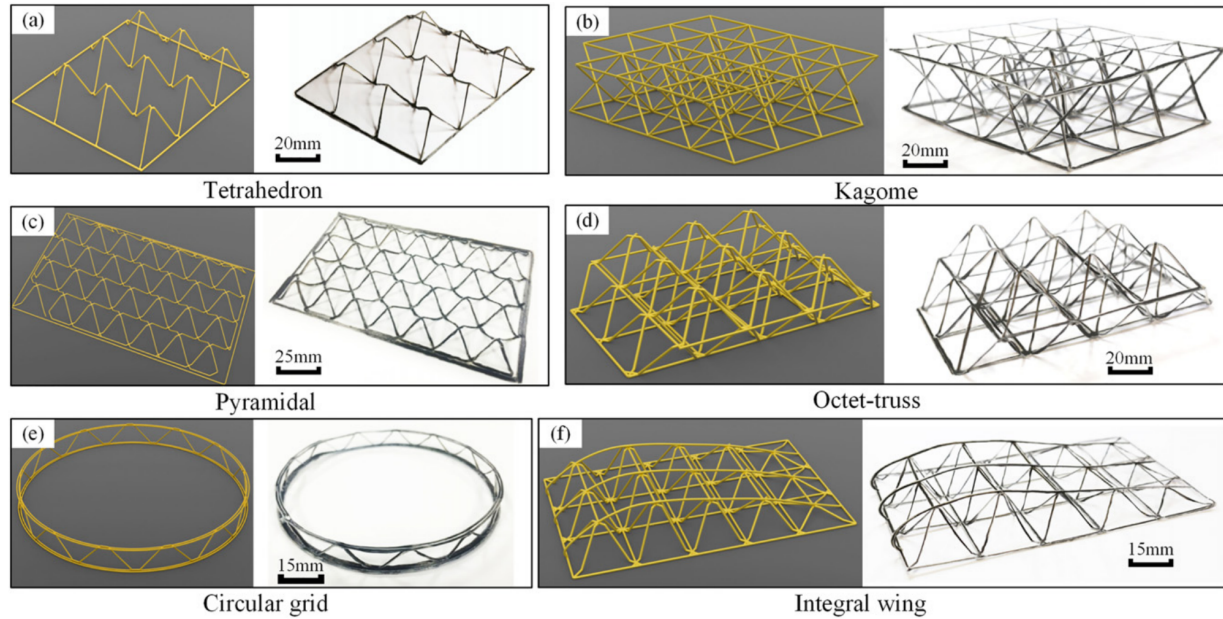


Figure 2.3.7: Truss shapes produced with freehanging 3D printing methods [64]

To bypass the shape limitations inherent to conventional 3D printing techniques, Liao [27] produced non-periodic open structure continuous-fibre PMFRC parts by interlacing continuous dry yarns subjected to individually controlled tensions, bringing yarns to an optimal geometric configuration. Combining with resin led to proof-of-concept PMFRC trusses, Figure 2.3.8. In Liao's work, interlacing patterns and initial structure geometry were generated arbitrarily.

Industrial adoption of continuous fibre 3D printing has grown in recent years, enabling the production of anisotropic, mechanically efficient components through advanced additive manufacturing techniques. Continuous Composites Inc. has pioneered a novel additive manufacturing technology known as Continuous Fibre 3D Printing (CF3D), which enables the deposition of continuous fibres impregnated with a thermosetting resin in freeform space

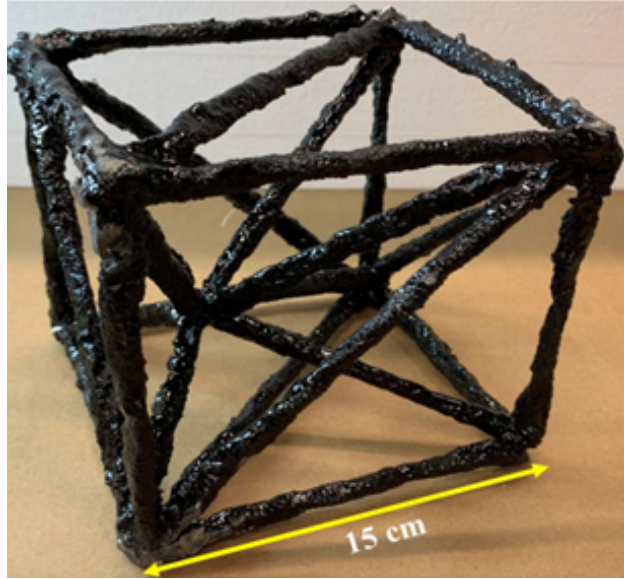


Figure 2.3.8: Open truss composite structure/part produced by Liao [27]

without the need for support structures [67]. Unlike traditional layer-by-layer composite fabrication, CF3D integrates the curing and placement processes, allowing for complex geometries and optimized fibre alignment in three dimensions, Figure 2.3.9. This approach enables structurally efficient, lightweight designs with tailored anisotropic properties and has positioned CF3D as a promising solution for high-performance applications in aerospace, automotive, and defense sectors.

Reinforce3D has developed the Continuous Fibre Injection Process (CFIP), an innovative post-processing technology that reinforces pre-manufactured 3D printed or molded parts by injecting continuous fibres directly into their internal channels Figure 2.3.10 [69]. Unlike conventional continuous fibre deposition methods, CFIP separates part shaping from fibre reinforcement, allowing fibres to be inserted after the structure is formed. This decoupling enables more flexible fibre path optimization and allows reinforcement in complex geometries, such as brake pedals Figure 2.3.11, that are difficult to access during the initial fabrication. The process significantly enhances mechanical performance while remaining compatible with existing additive manufacturing and molding techniques.



Figure 2.3.9: Continuous Composites Inc's CF3D process [68]

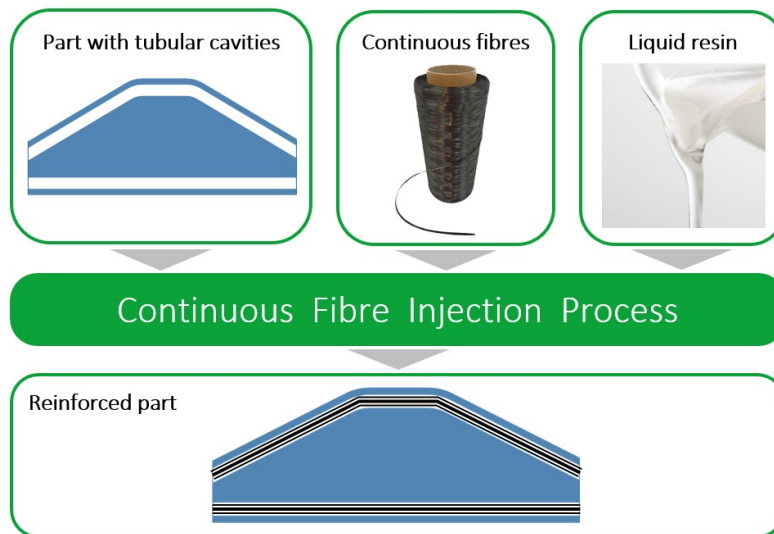


Figure 2.3.10: Illustration of CFIP [69]



Figure 2.3.11: Brake pedal produced with CFIP [69]

Chapter 3

Design Cases

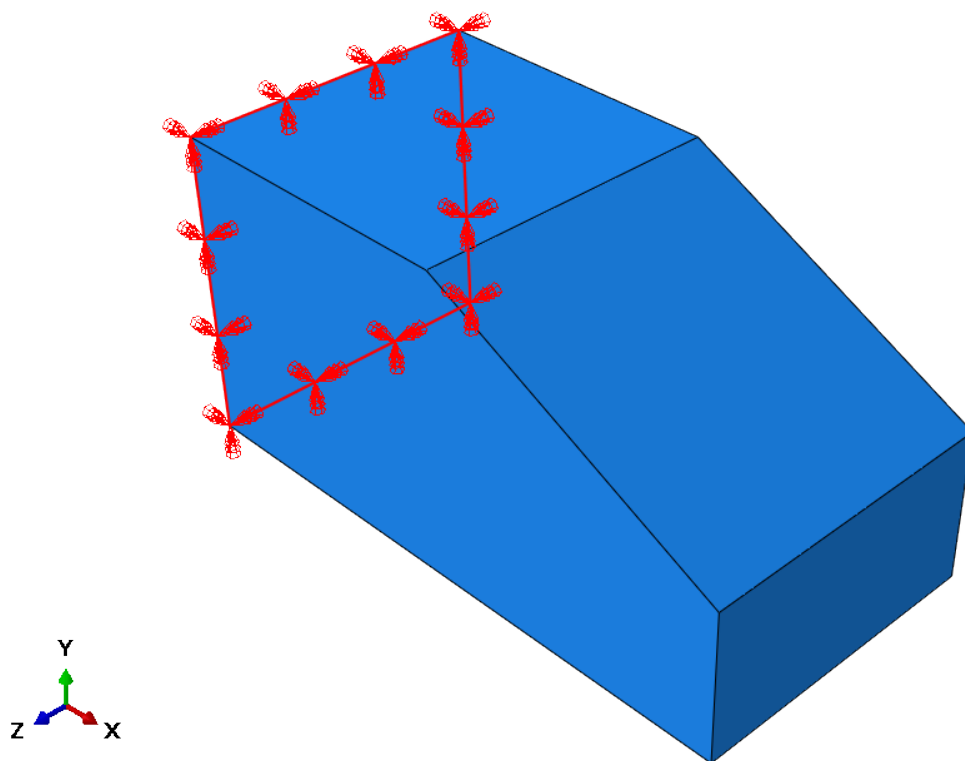
This chapter describes the geometries, applied loads and boundary conditions that will be optimized. In total, 13 different design cases will be considered, including 7 different geometries and 13 unique load cases. This was done to consider a wide variety of test cases toward validating the proposed topology optimization framework. Each design case is labelled using the format GxLCy, where G denotes the geometry variant and LC represents the load case. For example, G1LC1 refers to Geometry 1 under Load Case 1. The full set of design cases is summarized in Table 3.1.

Table 3.1: Summary of design cases

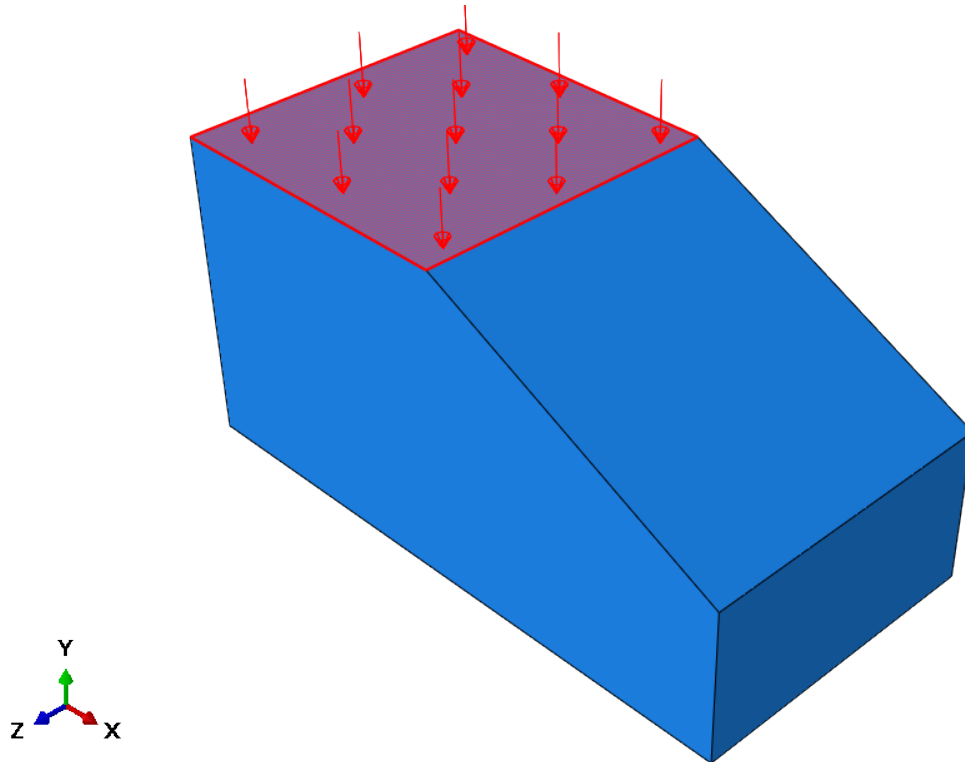
Design case	Description	Demonstrator type
G1LC1	Trapezoidal brick - compression	Academic
G2LC1	Holed plate - tension	Academic
G3LC1	Rectangular plate - bending	Academic
G4LC1	Round bar - torsion	Academic
G5LC1	GE bracket - vertical tension	Industrial
G5LC2	GE bracket - horizontal tension	Industrial
G5LC3	GE bracket - bending	Industrial
G6LC1	Connecting rod - compression	Industrial
G6LC2	Connecting rod - tension	Industrial
G6LC3	Connecting rod - bending	Industrial
G7LC1	Running blade - vertical load	Industrial
G7LC2	Running blade - vertical and 40% body weight lateral load	Industrial
G7LC3	Running blade - vertical and 80% body weight lateral load	Industrial

3.1 Academic Demonstrators

Four simple geometries, not representative of real mechanical parts, were considered initially to validate algorithm outputs during the initial stage of development. These were labelled academic demonstrators, the first of which corresponds to a trapezoidal brick. The first design case, labelled geometry 1 load case 1 (G1LC1) corresponds to the trapezoidal brick in compression, Figure 3.1.1. This design case was proposed to validate algorithm output when applied to geometries featuring sharp corners. The second proposed academic demonstrator is a holed plate. The second design case, labelled geometry 2 load case 1 (G2LC1) corresponds to the holed plate in tension, Figure 3.1.2. This load case was proposed to validate algorithm when applied to geometries featuring circular features such as holes.

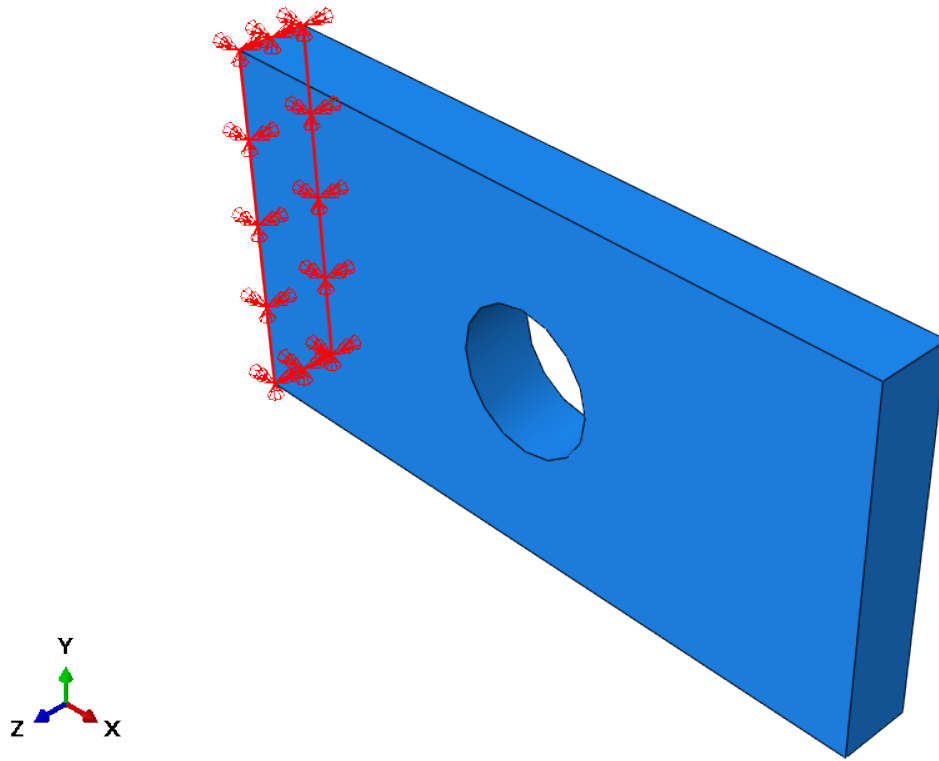


(a) Built-in left face

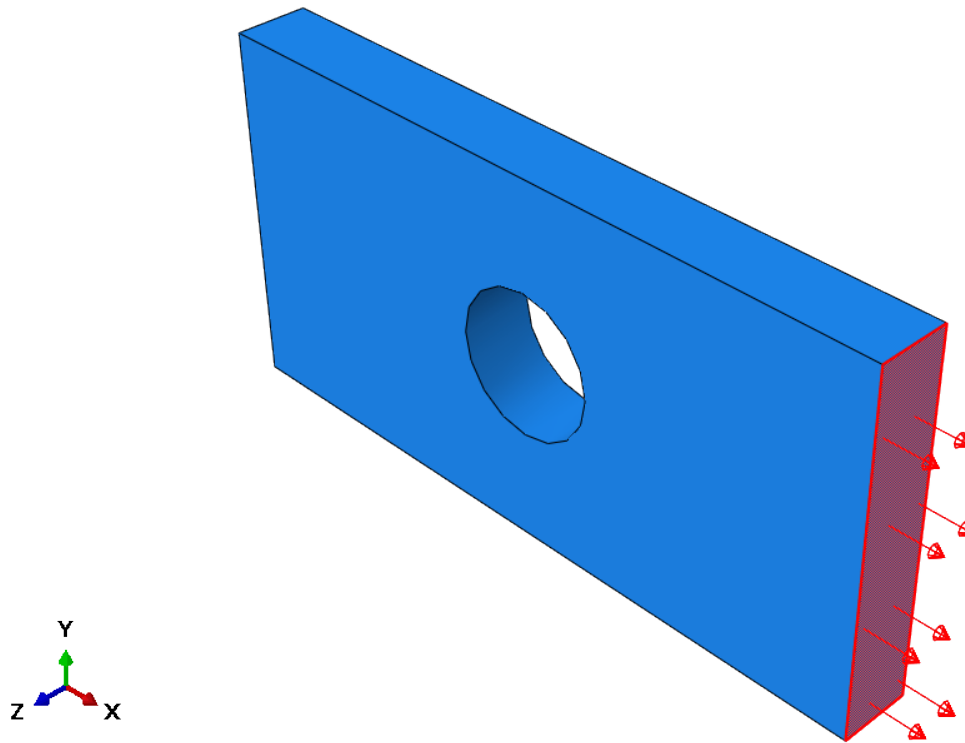


(b) -50 MPa axial stress on top face

Figure 3.1.1: Depiction of design case G1LC1



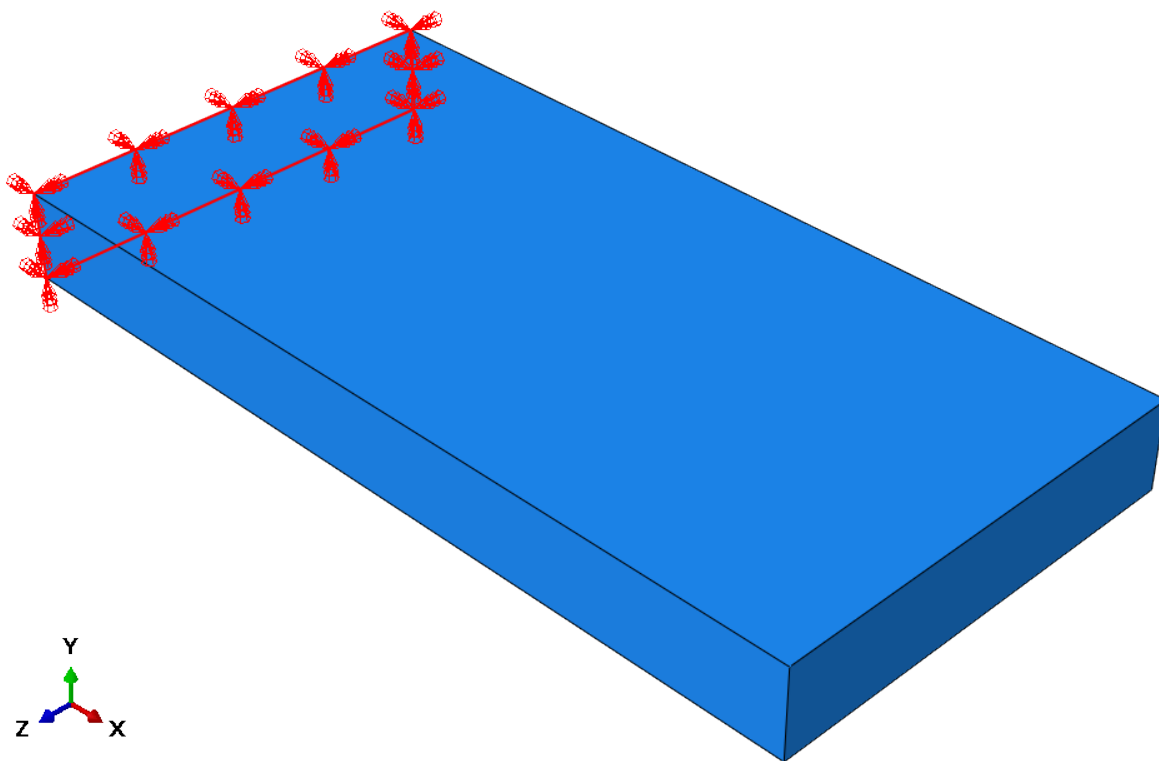
(a) Built-in left face



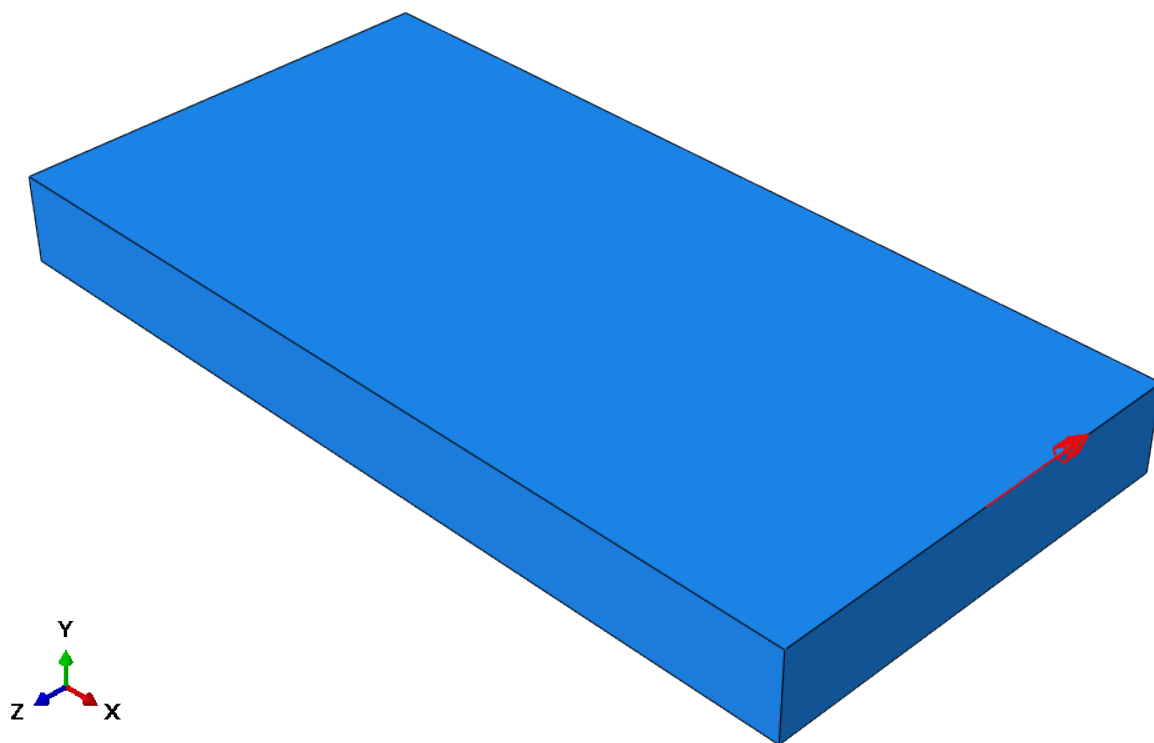
(b) 100 MPa axial stress on right face

Figure 3.1.2: Depiction of design case G2LC1

The final two academic demonstrators correspond to a plate and a round bar. The third design case, labelled geometry 3 load case 1 (G3LC1), corresponds to the plate being loaded in bending in the xy -plane (Figure 3.1.3) while the fourth design case, geometry 4 load case 1 (G4LC1), corresponds to the round bar in torsion (Figure 3.1.4), were proposed to validate algorithm output when applied to out-of-plane loads, rather than purely in-plane loads.

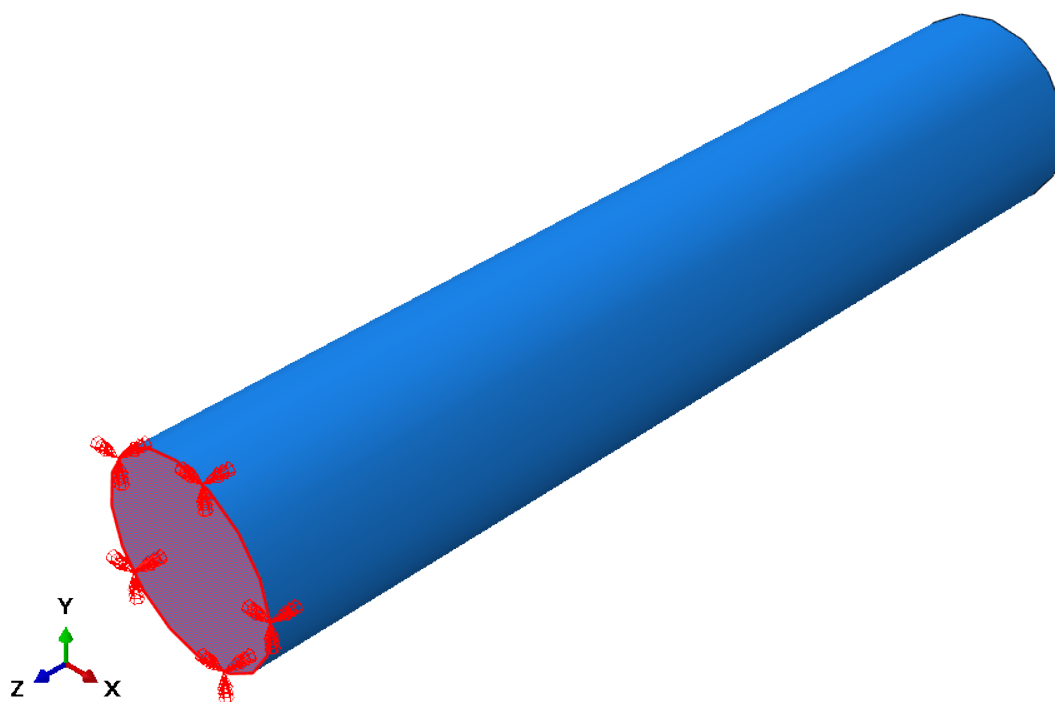


(a) Built-in left face

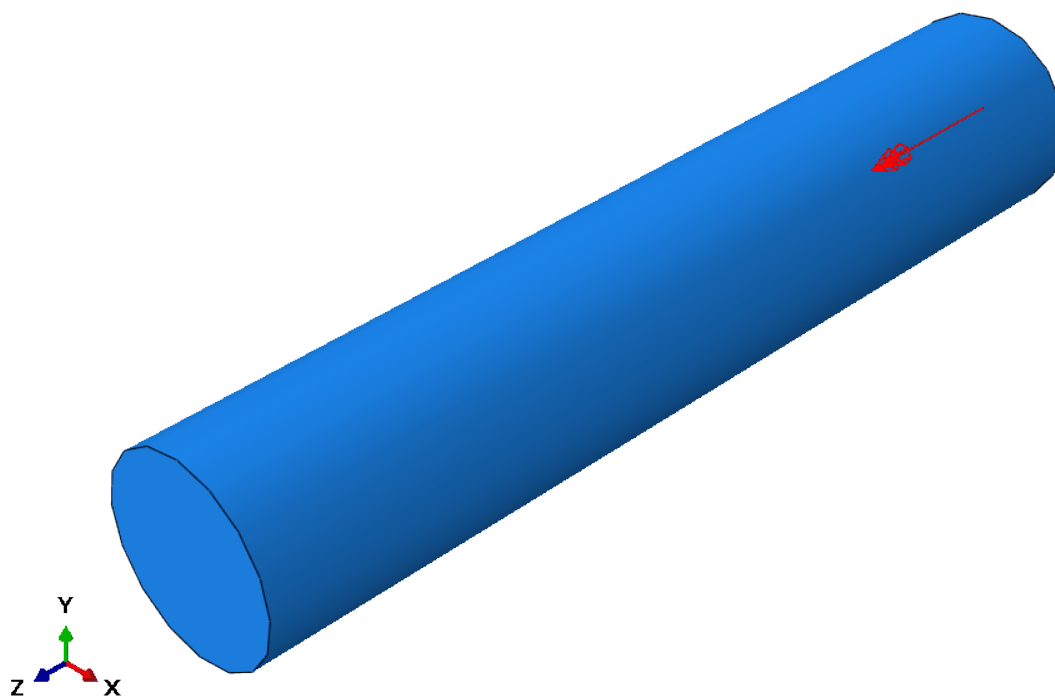


(b) $-10000 \text{ N} \cdot \text{mm} \hat{k}$ moment on right face

Figure 3.1.3: Depiction of design case G3LC1



(a) Built-in left face



(b) $10000 \text{ N} \cdot \text{mm} \hat{k}$ moment on right face

Figure 3.1.4: Depiction of design case G4LC1

3.2 Industrial Demonstrators

Three geometries labelled industrial demonstrators and inspired from real mechanical parts were also considered in addition to the four academic demonstrators. For each of these industrial demonstrators, three different load cases were considered, totalling 9 different design cases.

3.2.1 GE Bracket

General Electric (GE) launched a competition in 2013 challenging engineers to optimize one of their jet engine brackets for four different load cases [70]. The GE bracket geometry corresponds to the first industrial demonstrator. Load cases 3 and 4 (Figure 3.2.1) correspond to the first and third industrial design cases (G5LC1 and G5LC3 respectively) considered for this work. The second industrial design case (G5LC2) corresponds to load condition 3 in Figure 3.2.1, but the force is applied at an angle of -42° from the vertical. Boundary conditions for design cases G5LC1-G5LC3 are depicted in Figure 3.2.2, while the applied loads are shown in Figures 3.2.3-3.2.5. The STL file for the bracket was procured from *GrabCad* [70].

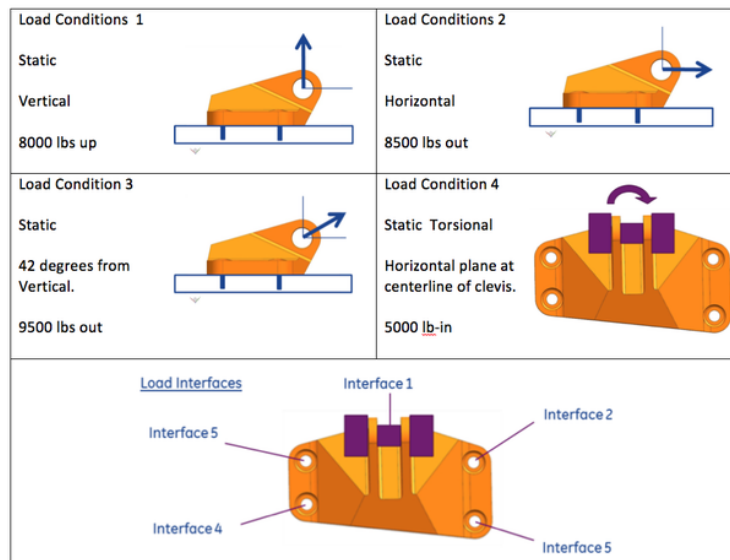


Figure 3.2.1: GE bracket challenge load cases (load cases 3 and 4 are used for this work) [70]

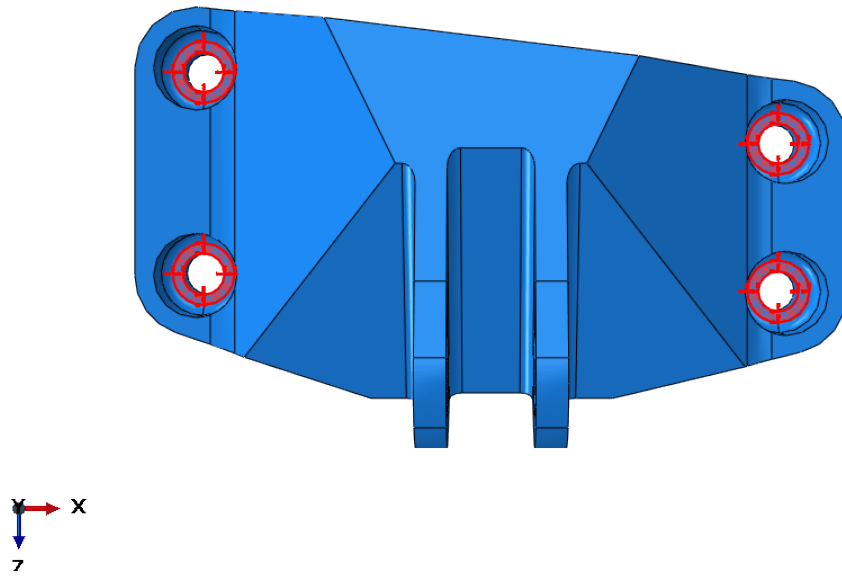


Figure 3.2.2: Built-in bottom holes - design cases G5LC1-G5LC3

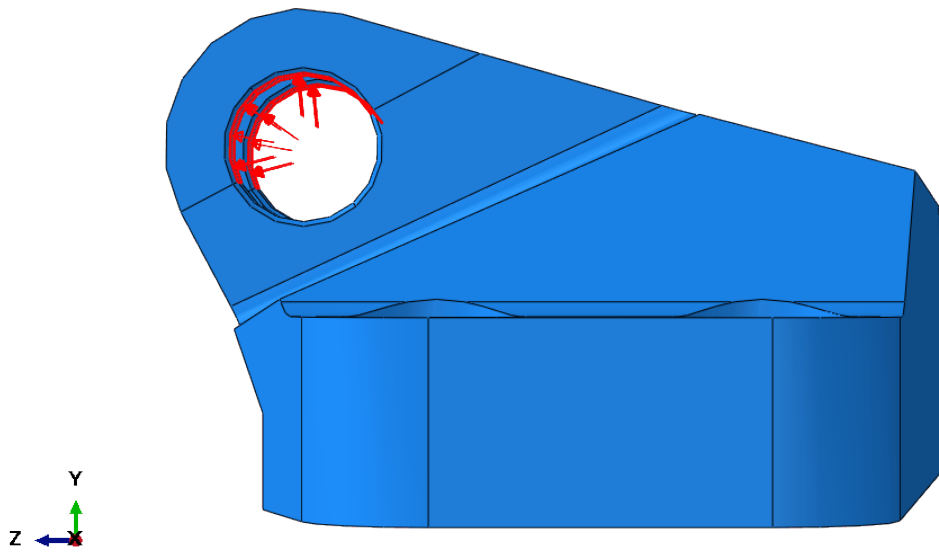


Figure 3.2.3: -200 MPa axial stress on top holes - design case G5LC1

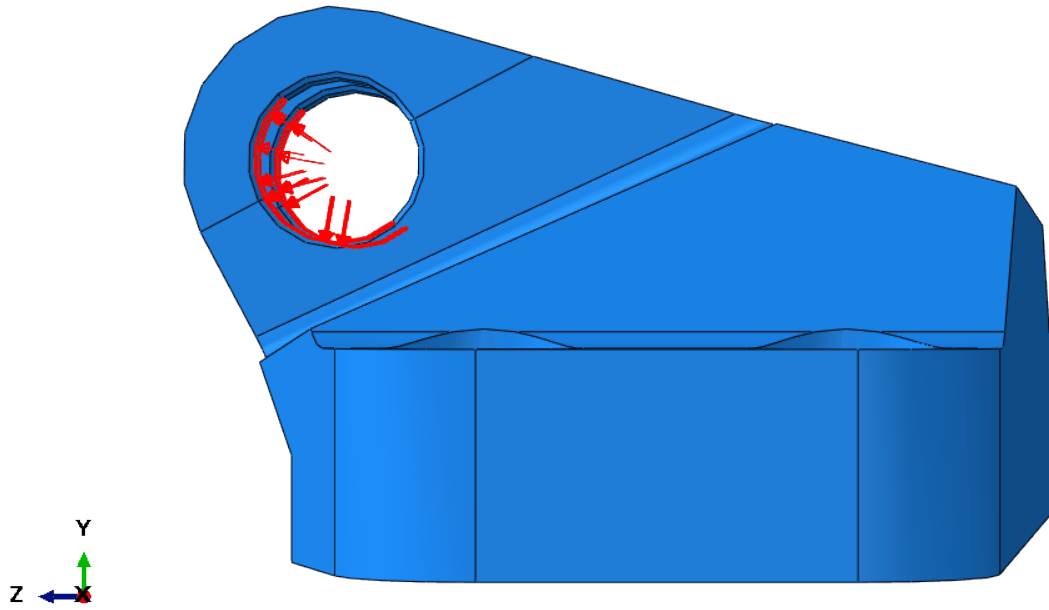


Figure 3.2.4: -200 MPa axial stress on top holes - design case G5LC2

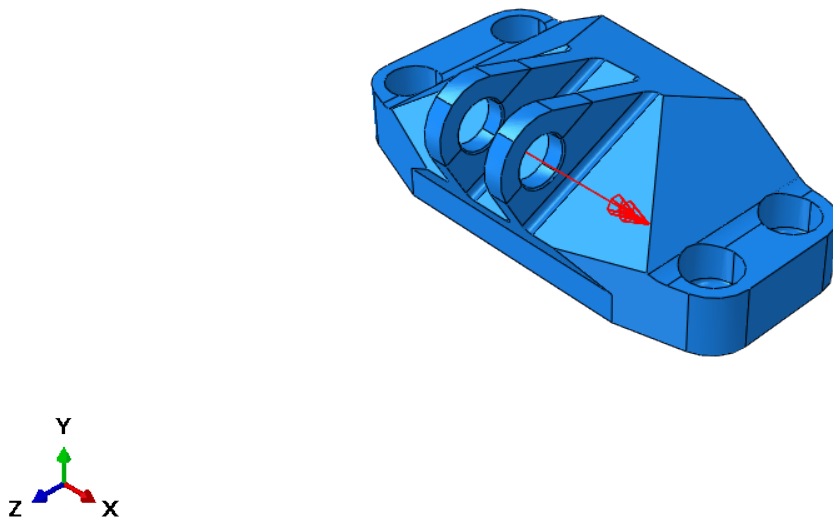


Figure 3.2.5: 565 000 N · mm $\hat{i}$ moment on top holes - design case G5LC3

3.2.2 Connecting Rod

The second industrial demonstrator is that of a connecting rod which was also obtained from *GrabCad* [71]. Boundary conditions applied to each connecting rod design case are identical, Figure 3.2.6. Design cases 1 and 2 (G6LC1 and G6LC2 respectively), Figures 3.2.7 and 3.2.8, correspond to the peak compressive and tensile forces that are exerted on the rod during a four stroke engine cycle combustion cycle. These loads occur at crank angles of 0° and 360° respectively [72]. Design case 3 (G6LC3), Figure 3.2.9, consists of a bending moment applied about the length of the rod. While this load case may not represent a typical loading scenario for connecting rods in real-world applications, it was included to ensure a broader range of load conditions for comprehensive analysis.

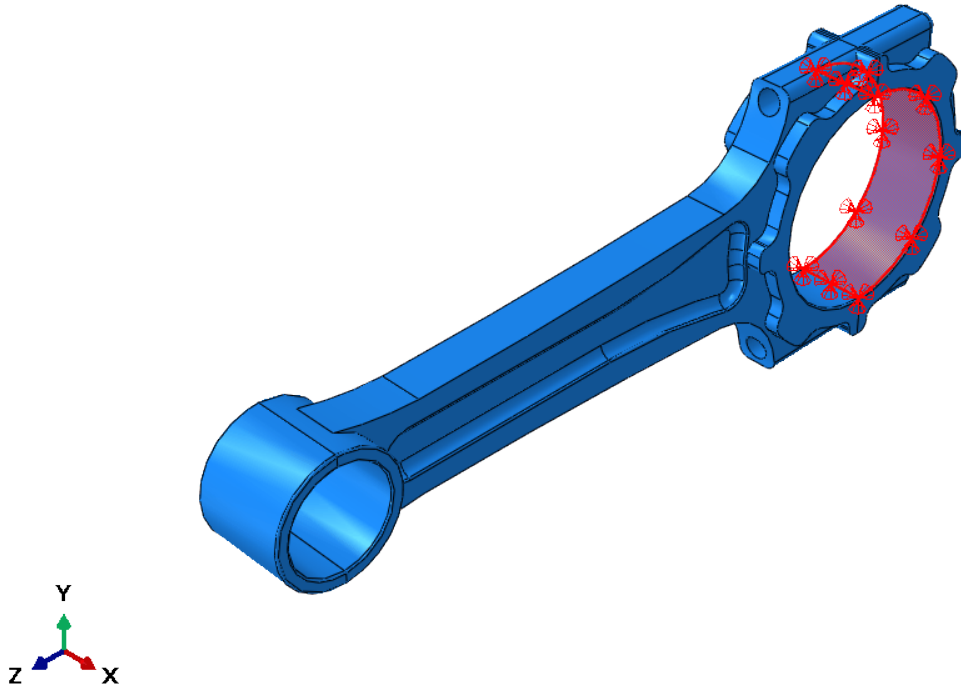


Figure 3.2.6: Pinned right half of right hole face - design cases G6LC1-G6LC3

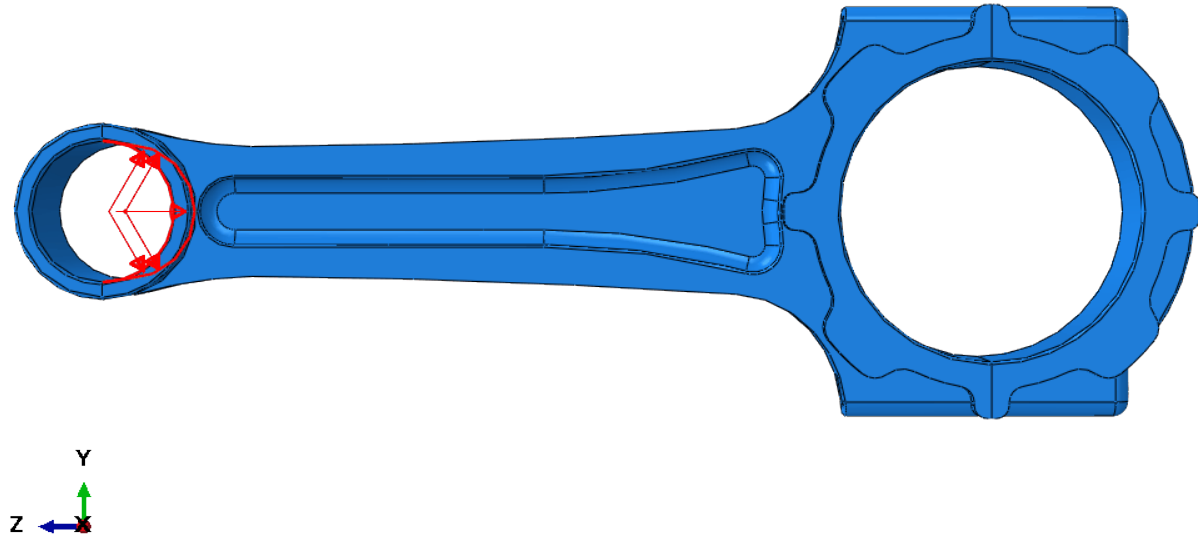


Figure 3.2.7: -80 MPa axial stress on right half of left hole face - design case G6LC1

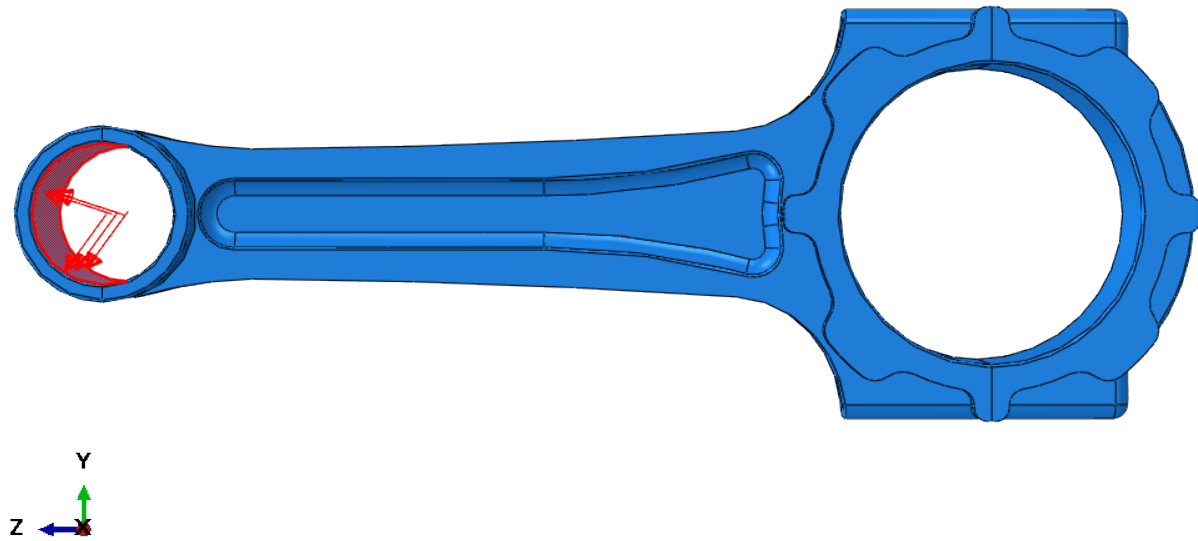


Figure 3.2.8: 15 MPa axial stress on left half of left hole face - design case G6LC2

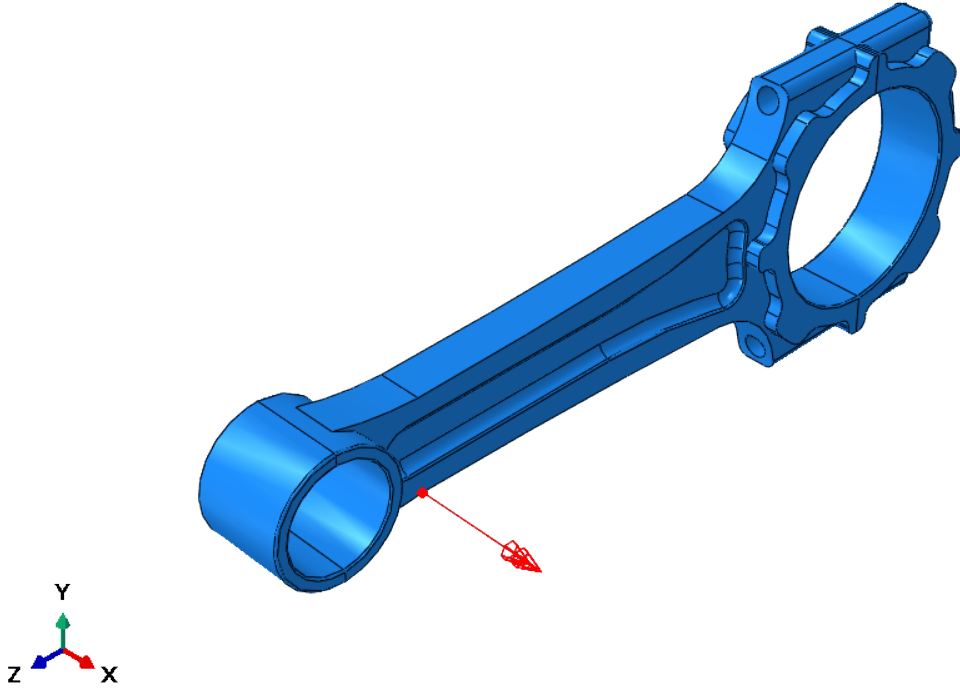


Figure 3.2.9: $100\,000\text{ N} \cdot \text{mm} \hat{i}$ moment on face - design case G6LC3

3.2.3 Running Blade

The final industrial part envelope corresponds to a running blade, a prosthetic limb used by athletes with lower-limb amputations. Boundary conditions applied to the running blade design cases are shown in Figure 3.2.10. The first design case (G7LC1) corresponds to an athlete weighing 85 kg loading their full body weight on the blade vertically when they take a step, Figure 3.2.11. Design cases 2 and 3 (G7LC2 and G7LC3 respectively) correspond to an athlete taking a turn while running with different turning radii, Figure 3.2.13. This being said, a lateral load is applied to the bottom of the blade, in addition to the load that is applied in design case G7LC1. The lateral loads applied for cases G7LC2 and G7LC3 are equivalent to 40% bodyweight and 80% bodyweight respectively. Figure 3.2.12 serves to better illustrate the point of application of the lateral load, but is not representative of an analyzed design case. Table 3.2.1 provides a summary of all design cases analyzed in the scope of this work.

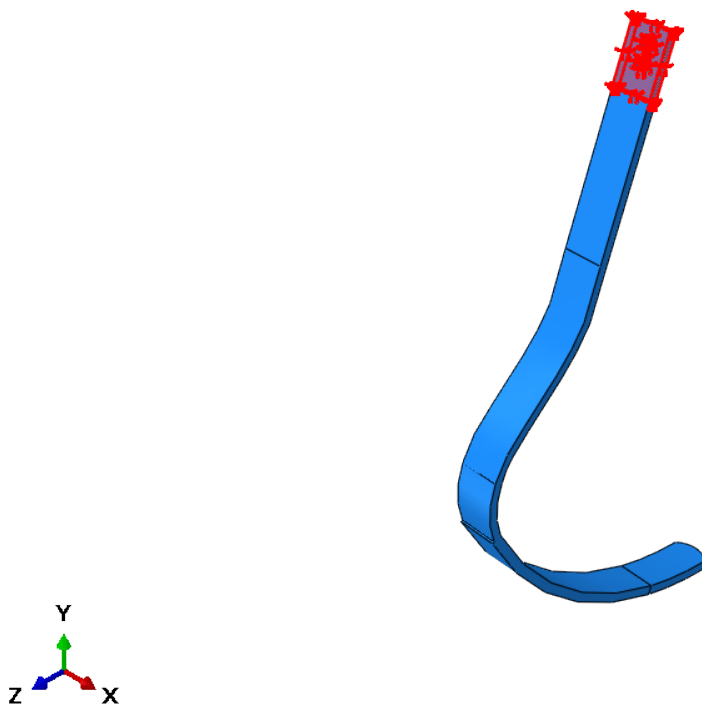


Figure 3.2.10: Encastred upper attachment - design cases G7LC1-G7LC3

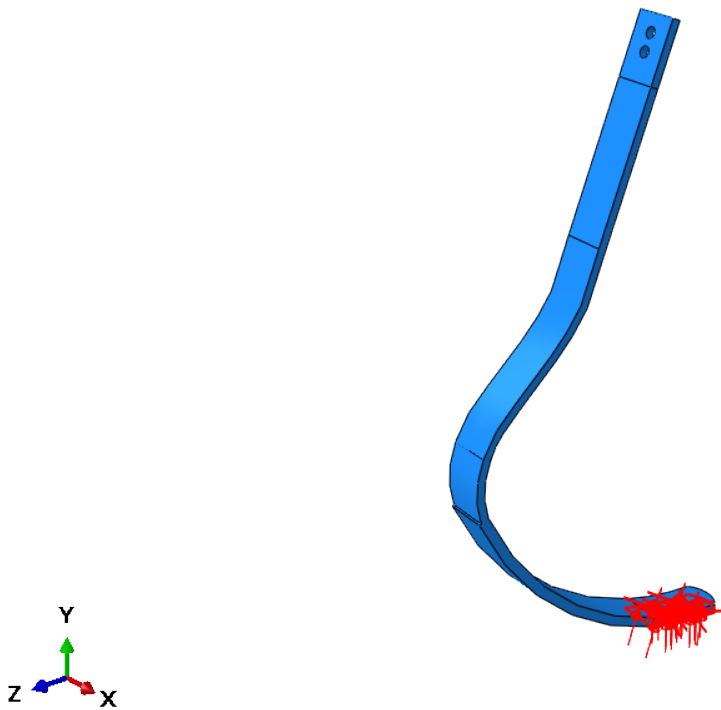


Figure 3.2.11: 0.05 MPa axial stress applied on lower surface of foot - design case G7LC1

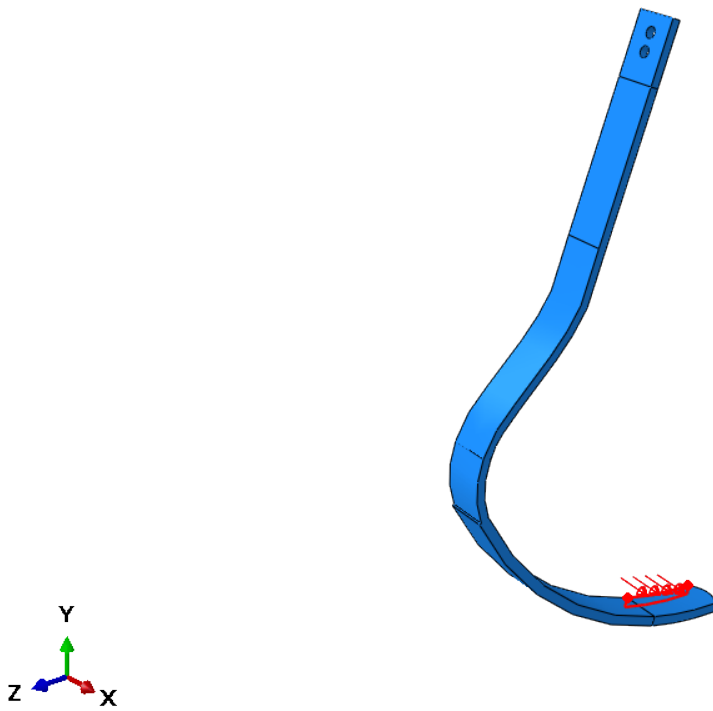


Figure 3.2.12: Depiction of lateral load application point - design cases G7LC2 and G7LC3

Chapter 4

Stress Path Generation and Selection

This chapter presents methodology for steps 1-3 of the sequential 3D TO PMFRC part design process. Methodology for these steps is illustrated through results obtained for 13 different design cases. In step 1, static linear elastic structural FEA is performed on isotropic material contained within the part envelope, for specified loading and boundary conditions. The principal stress vector field is extracted. In step 2, a large number of paths locally tangent to the principal stress vectors are generated; these paths are labelled initial stress paths. Seed points for initial stress path generation are positioned throughout the part envelope to ensure that initial stress paths are distributed in a generally uniform manner, and that the entire volume of the part is populated with initial stress paths. In step 3, a small number of the most significant stress paths are selected, labelled as main stress paths. A prioritization index is associated with each initial stress path based on stress levels, length and distance from other selected stress paths within the part envelope. Stress paths are foundational elements of triangulated interlaced dry yarn trusses generated in further steps, ensuring that PMFRC material is located in highly loaded zones within the part envelope, along directions of higher stress.

4.1 Step 1: Principal Stress Vector Field

Stiffness optimization of 3D TO PMFRC parts require that a large proportion of yarns be aligned longitudinally with principal normal stresses in the part. In step 1, linear elastic FEA is performed on a continuous isotropic solid corresponding to the part envelope. Principal stress vectors are determined for each element by calculating eigenvectors of the Cauchy stress tensor [73] for specified load case and boundary conditions. A 3D vector field is produced by selecting the direction vectors of corresponding to highest principal stress amplitudes. FEA simulations were performed using ABAQUS 2019 with material and mesh properties found in Tables 4.1.1 and 4.1.2 respectively. Material properties are representative of a typical epoxy resin. Mesh sizes were chosen considering resolution of the vectors fields and contained computing time. Principal stress vector fields appear in Figures 4.1.1-4.1.4, Section 4.1.1 for academic demonstrators and in Figures 4.1.5-4.1.10, Section 4.1.2 for industrial demonstrators.

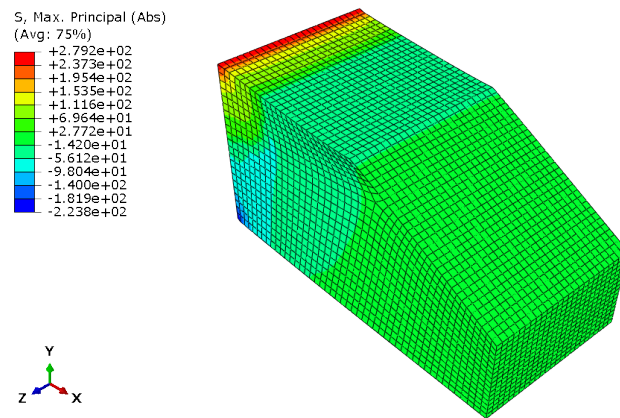
Table 4.1.1: Material properties

Young's modulus (GPa)	Density (g/cm ³)	Poisson's ratio
3.0	1.2	0.3

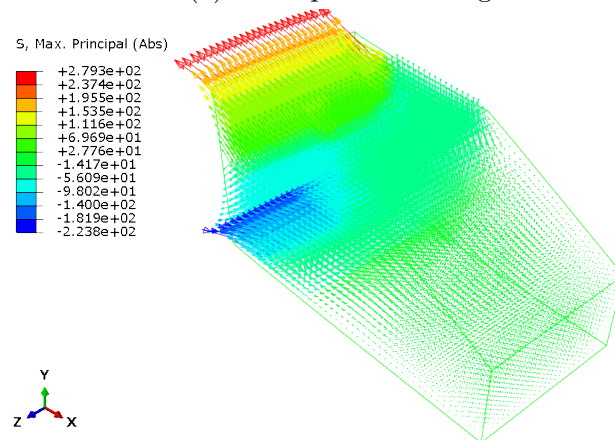
Table 4.1.2: FEA mesh properties

Design case	Number of nodes	Number of elements	Element type
G1LC1	25 324	22 575	C3D8R
G2LC1	9 000	7 045	C3D8R
G3LC1	7 957	6 250	C3D8R
G4LC1	6 529	5 550	C3D8R
G5LC1-G5LC3	83 000	56 201	C3D10
G6LC1-G6LC3	38 786	22 943	C3D10
G7LC1-G7LC3	5 426	2 767	C3D10

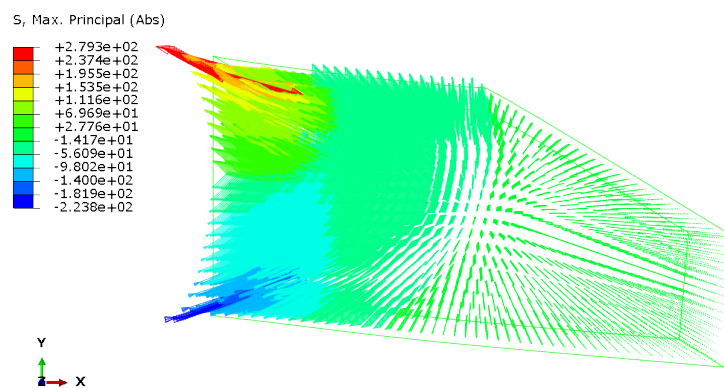
4.1.1 Academic Demonstrators - Principal Stress Fields



(a) Principal stress magnitudes

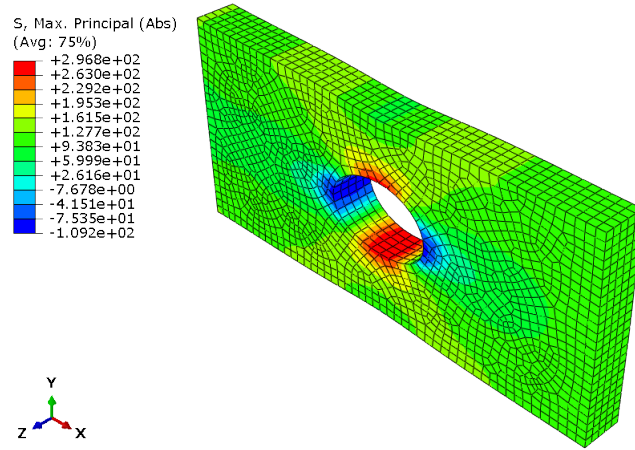


(b) Principal stress vectors (isometric view)

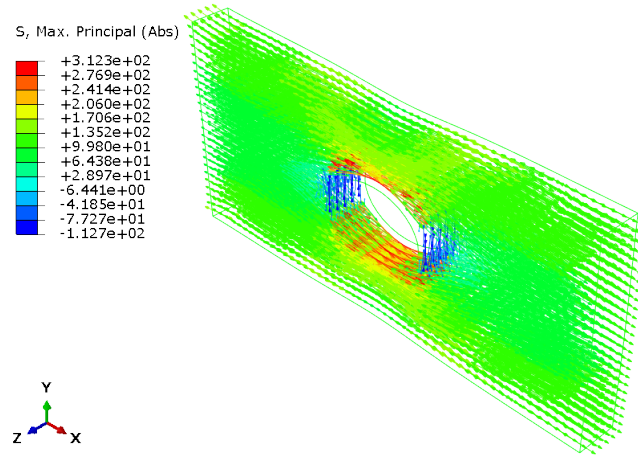


(c) Principal stress vectors (side view)

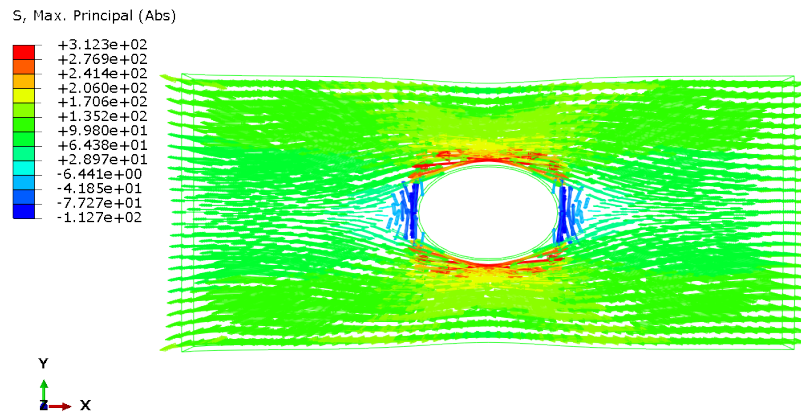
Figure 4.1.1: Principal stress field - design case G1LC1 (MPa)



(a) Principal stress magnitudes

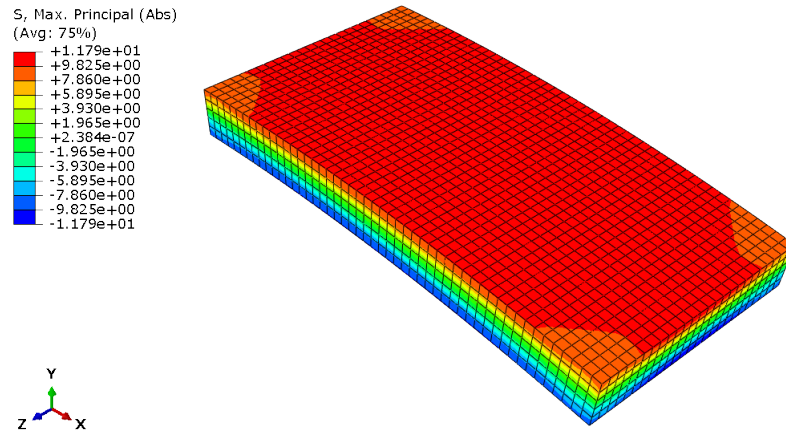


(b) Principal stress vectors (isometric view)

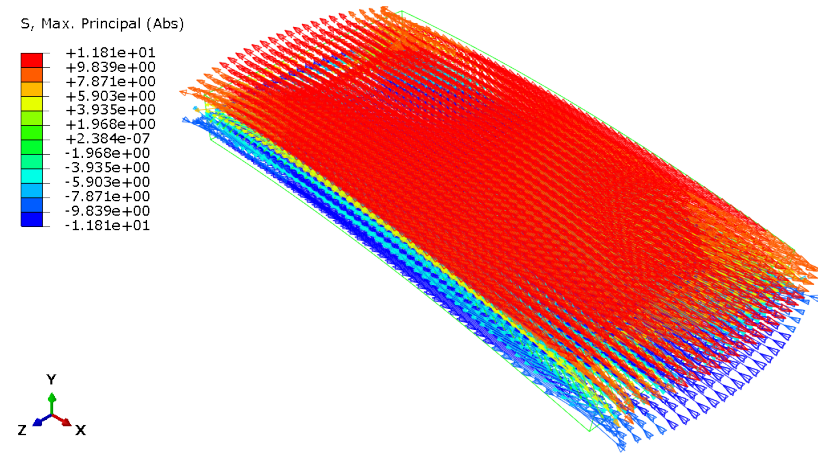


(c) Principal stress vectors (side view)

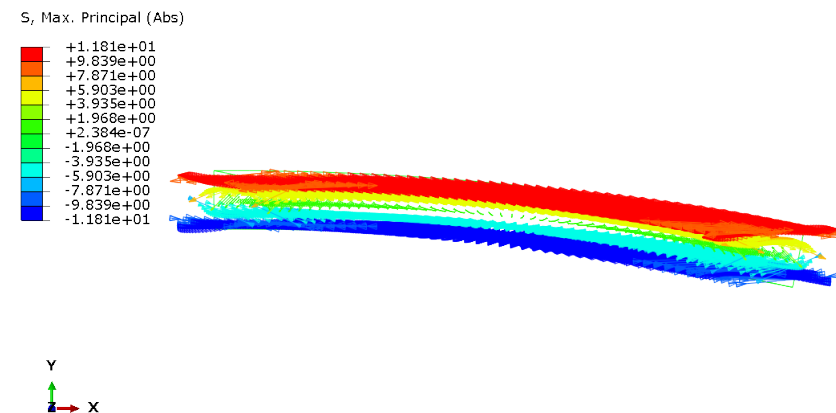
Figure 4.1.2: Principal stress field - design case G2LC1 (MPa)



(a) Principal stress magnitudes

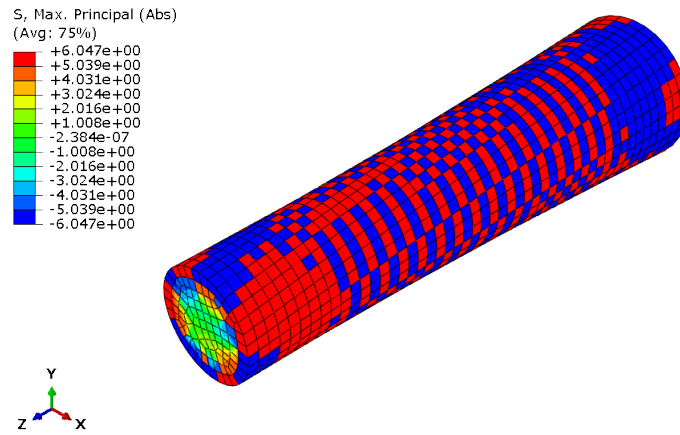


(b) Principal stress vectors (isometric view)

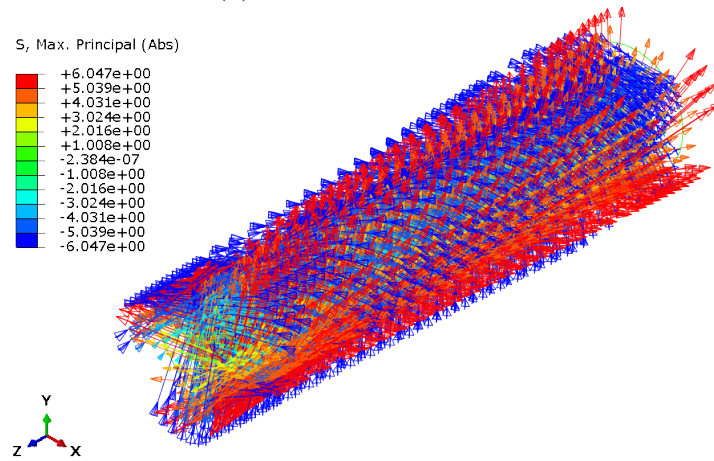


(c) Principal stress vectors (side view)

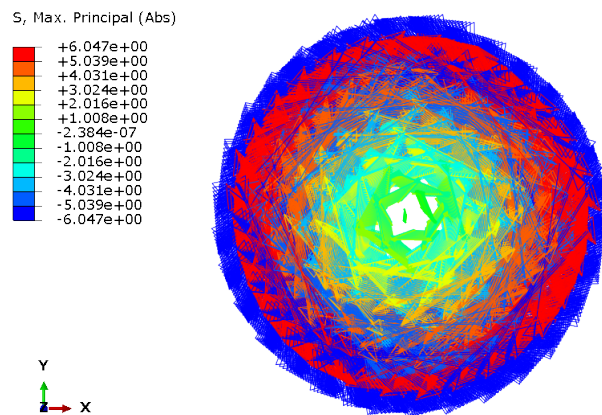
Figure 4.1.3: Principal stress field - design case G3LC1 (MPa)



(a) Principal stress magnitudes



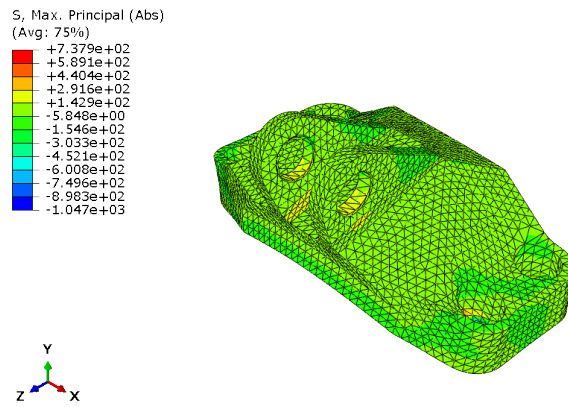
(b) Principal stress vectors (isometric view)



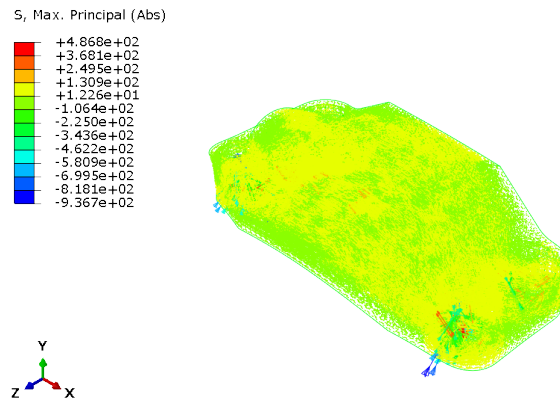
(c) Principal stress vectors (front view)

Figure 4.1.4: Principal stress field - design case G4LC1 (MPa)

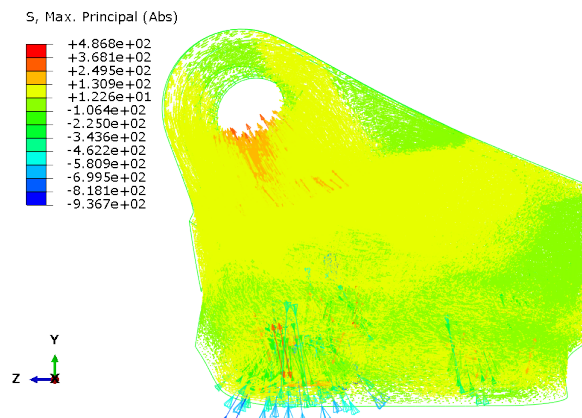
4.1.2 Industrial Demonstrators - Principal Stress Fields



(a) Principal stress magnitudes

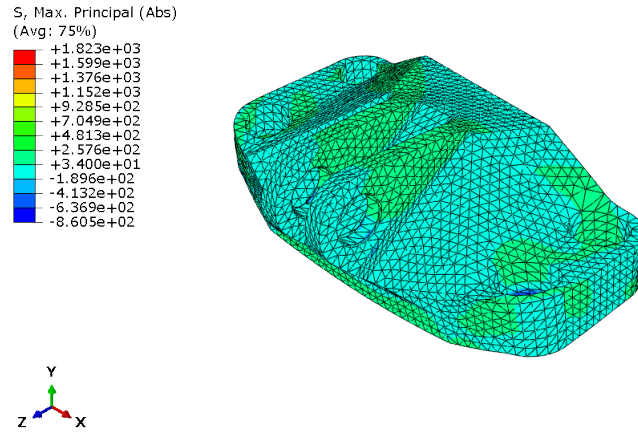


(b) Principal stress vectors (isometric view)

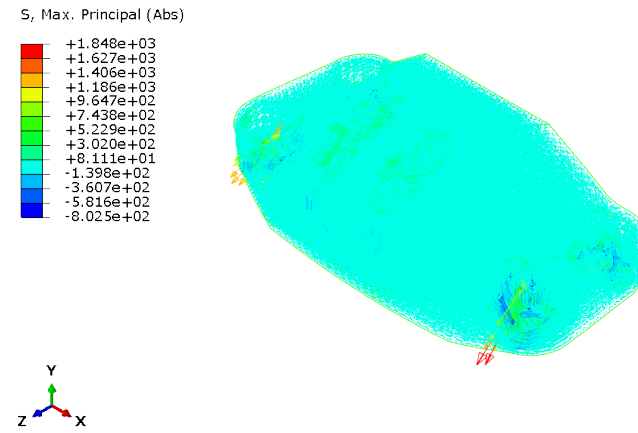


(c) Principal stress vectors (side view)

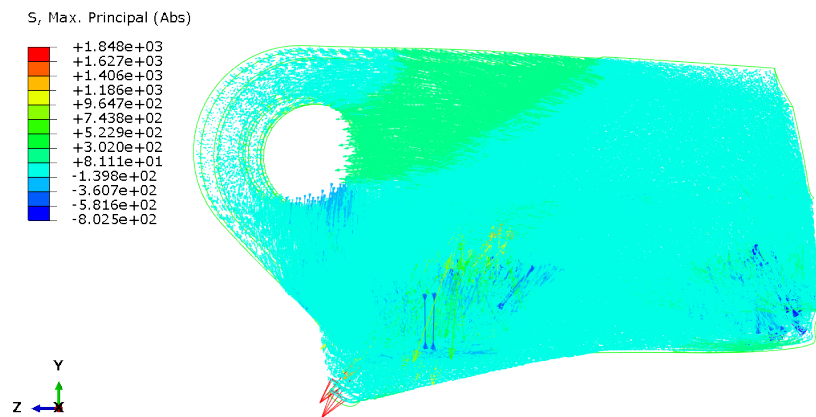
Figure 4.1.5: Principal stress field - design case G5LC1 (MPa)



(a) Principal stress magnitudes

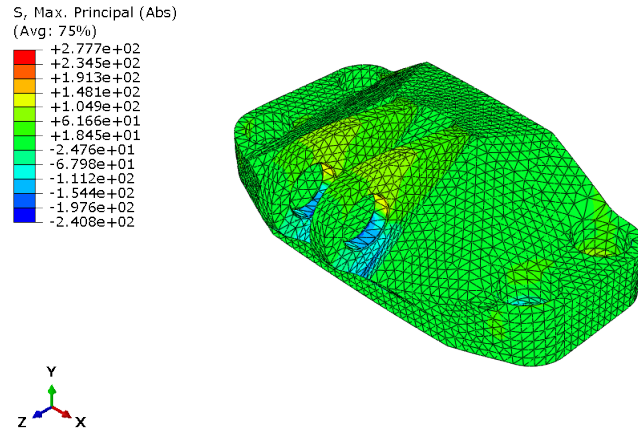


(b) Principal stress vectors (isometric view)

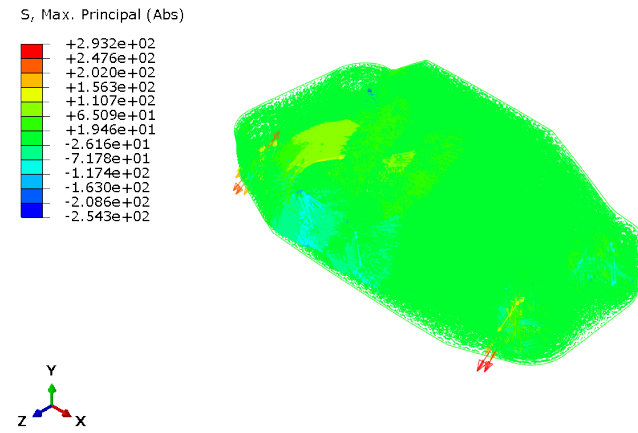


(c) Principal stress vectors (side view)

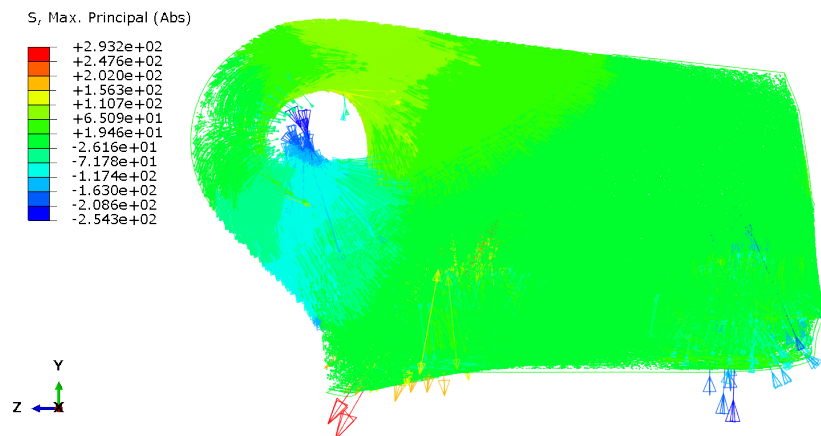
Figure 4.1.6: Principal stress field - design case G5LC2 (MPa)



(a) Principal stress magnitudes

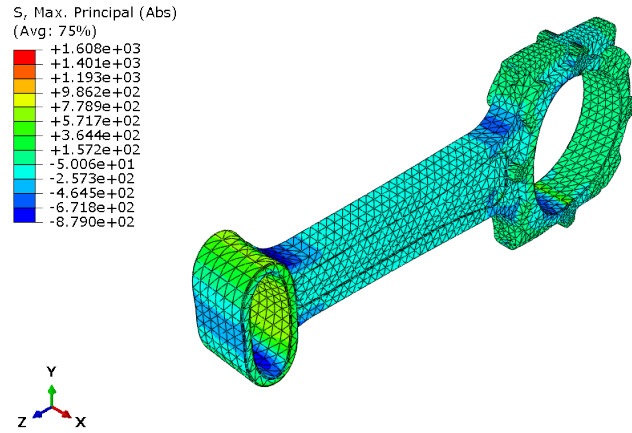


(b) Principal stress vectors (isometric view)

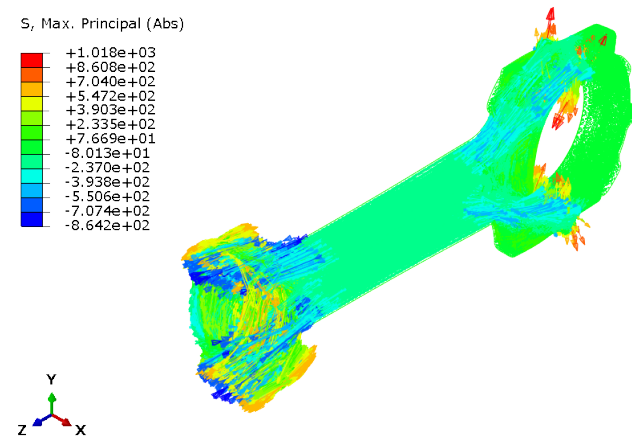


(c) Principal stress vectors (side view)

Figure 4.1.7: Principal stress field - design case G5LC3 (MPa)



(a) Principal stress magnitudes

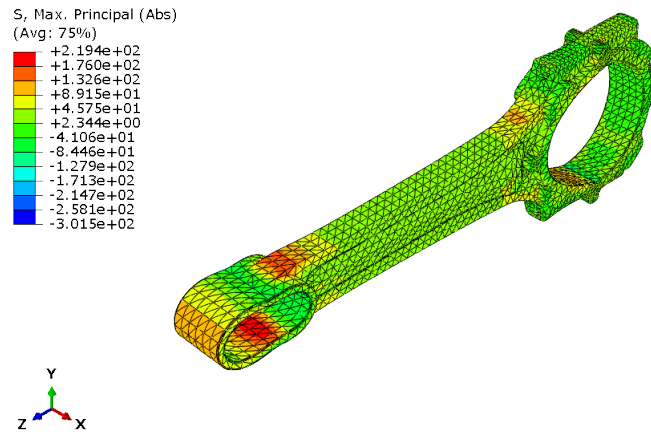


(b) Principal stress vectors (isometric view)

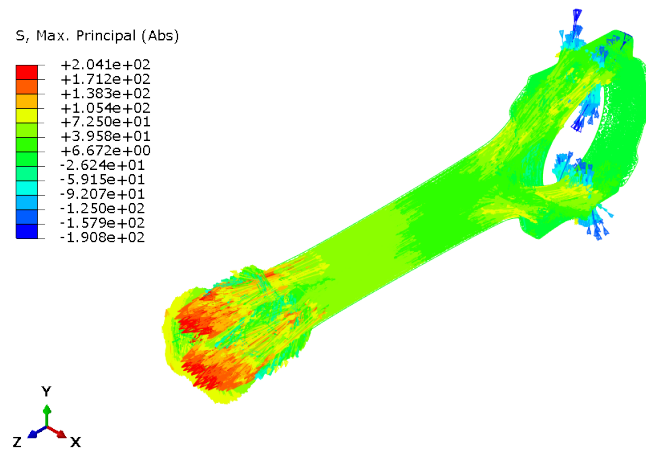


(c) Principal stress vectors (side view)

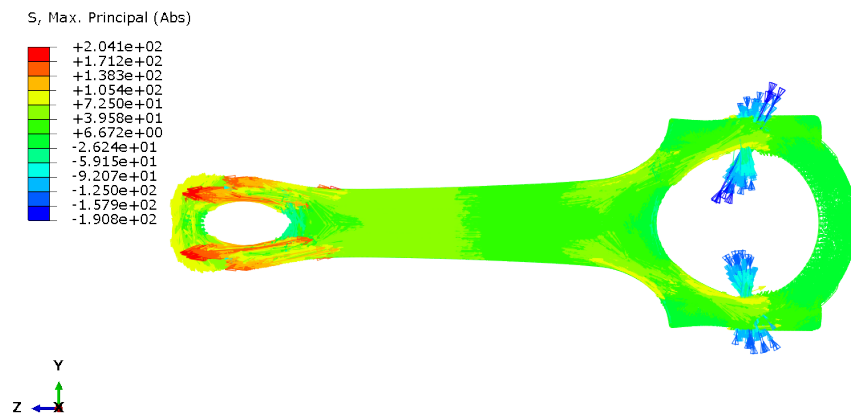
Figure 4.1.8: Principal stress field - design case G6LC1 (MPa)



(a) Principal stress magnitudes

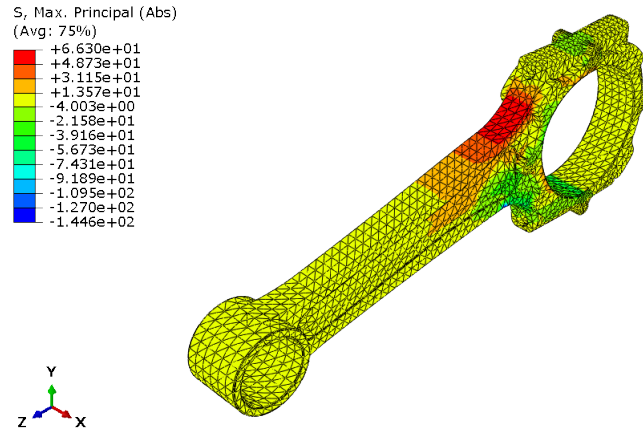


(b) Principal stress vectors (isometric view)

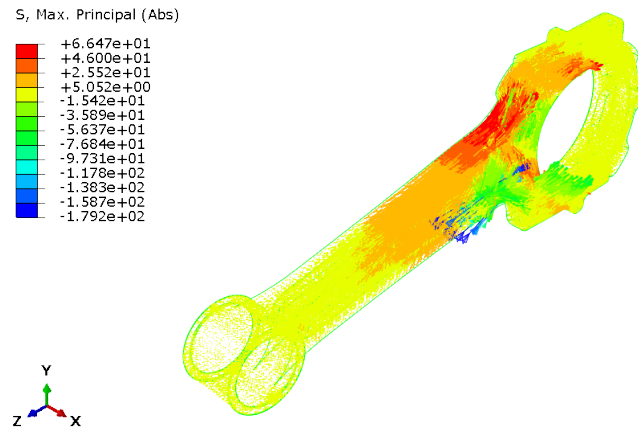


(c) Principal stress vectors (side view)

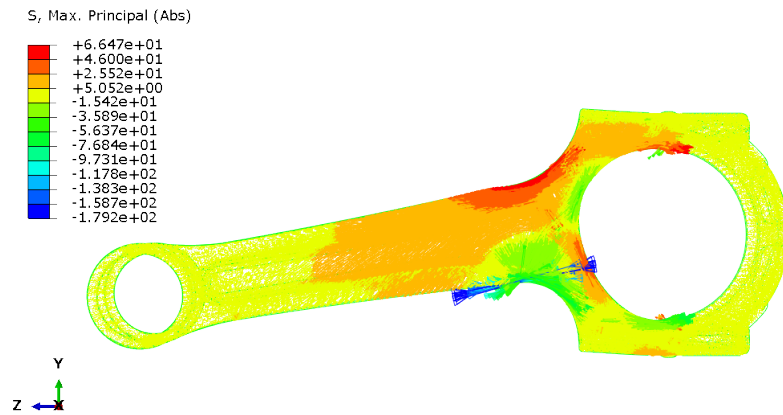
Figure 4.1.9: Principal stress field - design case G6LC2 (MPa)



(a) Principal stress magnitudes

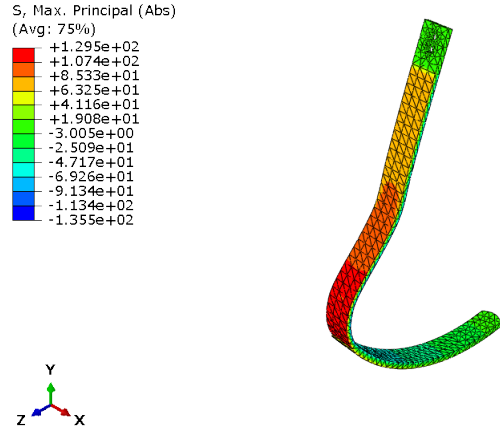


(b) Principal stress vectors (isometric view)

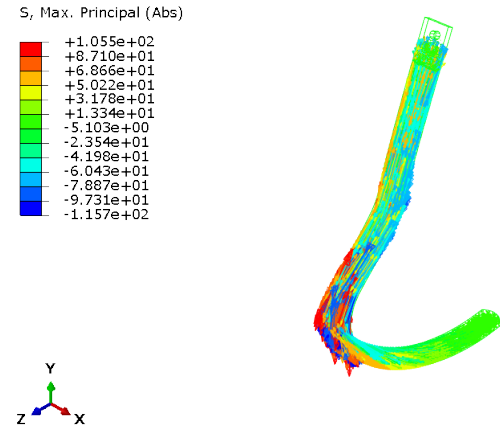


(c) Principal stress vectors (side view)

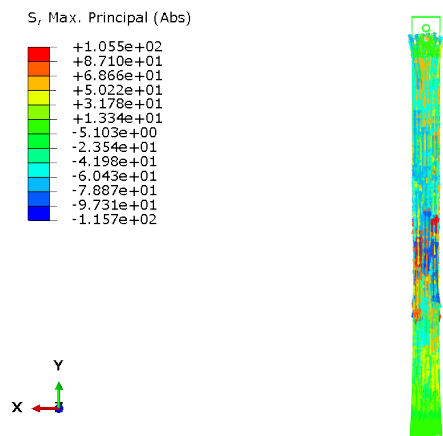
Figure 4.1.10: Principal stress field - design case G6LC3 (MPa)



(a) Principal stress magnitudes

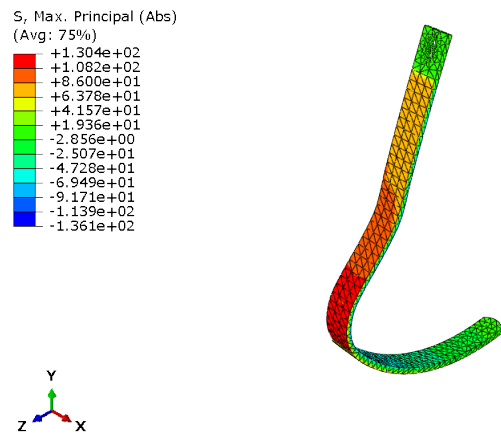


(b) Principal stress vectors (isometric view)

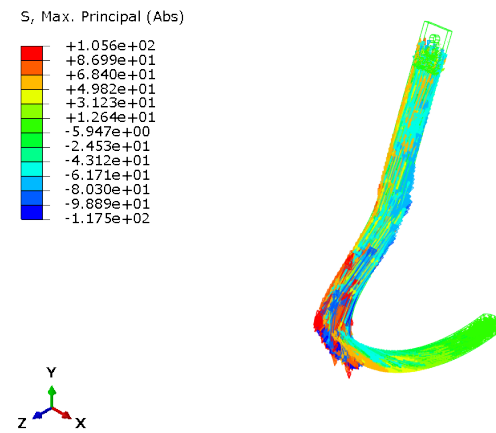


(c) Principal stress vectors (front view)

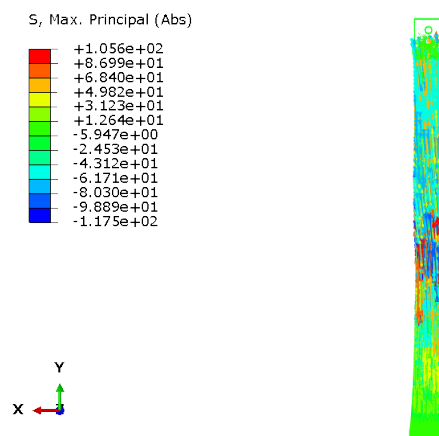
Figure 4.1.11: Principal stress field - design case G7LC1 (MPa)



(a) Principal stress magnitudes

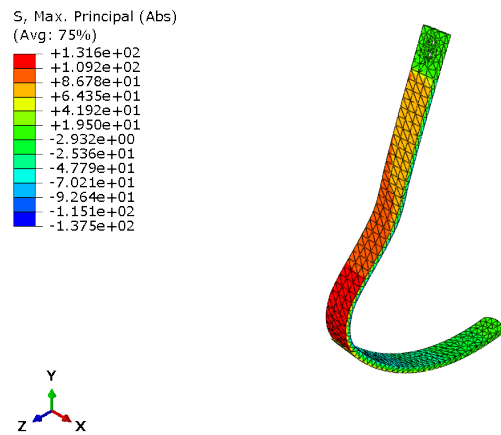


(b) Principal stress vectors (isometric view)

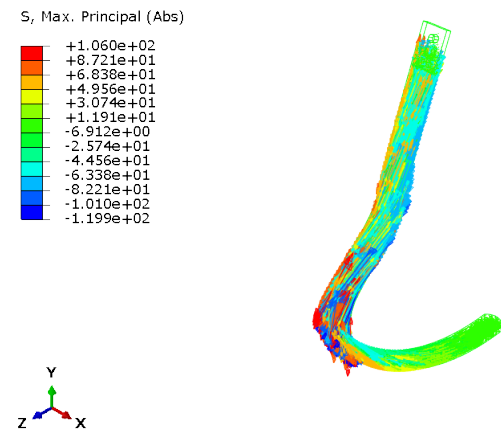


(c) Principal stress vectors (front view)

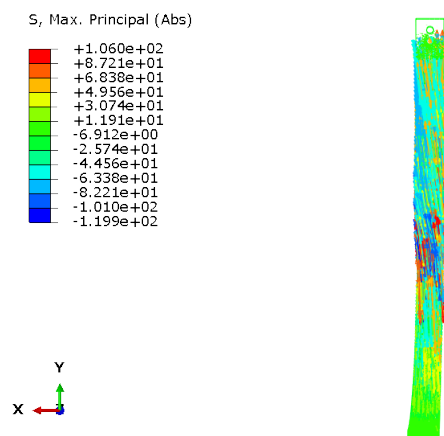
Figure 4.1.12: Principal stress field - design case G7LC2 (MPa)



(a) Principal stress magnitudes



(b) Principal stress vectors (isometric view)



(c) Principal stress vectors (front view)

Figure 4.1.13: Principal stress field - design case G7LC3 (MPa)

4.2 Step 2: Initial Stress Paths

Stress paths S tangent to the principal stress field at every point are defined as follows:

$$\nabla S(x, y, z) = l(x, y, z)\hat{\mathbf{i}} + m(x, y, z)\hat{\mathbf{j}} + n(x, y, z)\hat{\mathbf{k}} \quad (4.1)$$

where l , m and n are direction cosines in the x , y and z directions respectively. Initially, evenly spaced seed points are distributed as a uniform grid extending throughout the part envelope to ensure that initial stress paths are present everywhere in the part. In some symmetric design cases, such as G3LC1, a single plane of points sufficed for generating a suitable amount of stress paths Figure 4.2.4. Individual initial stress paths originate at each seed point, and may change direction at each adjacent element in the finite element mesh as they align with local principal stress vectors, Figure 4.2.1. Bidirectional stress path growth terminates when the envelope is reached or when transition between elements produce directional discontinuities beyond a set threshold. A large number of initial stress paths are generated in step 2, which are then filtered using a prioritization index in step 3. Initial stress paths appear in Figures 4.2.2-4.2.5, Section 4.2.1 for academic demonstrators and in Figures 4.2.6-4.2.11, Section 4.2.2 for industrial demonstrators.

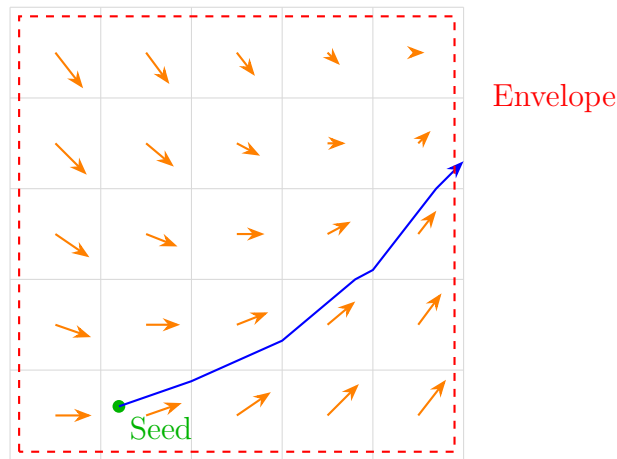
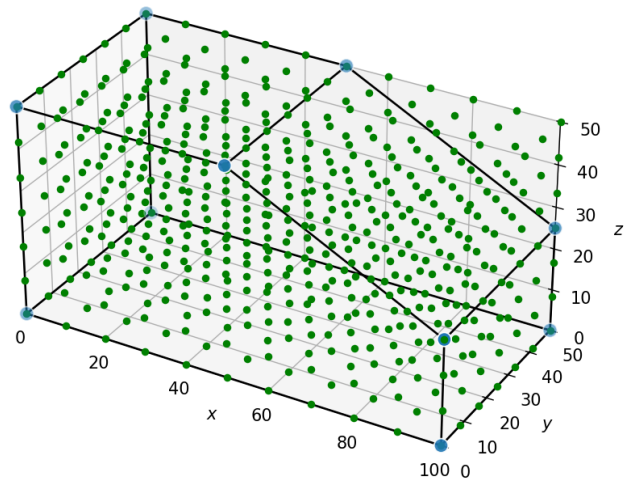
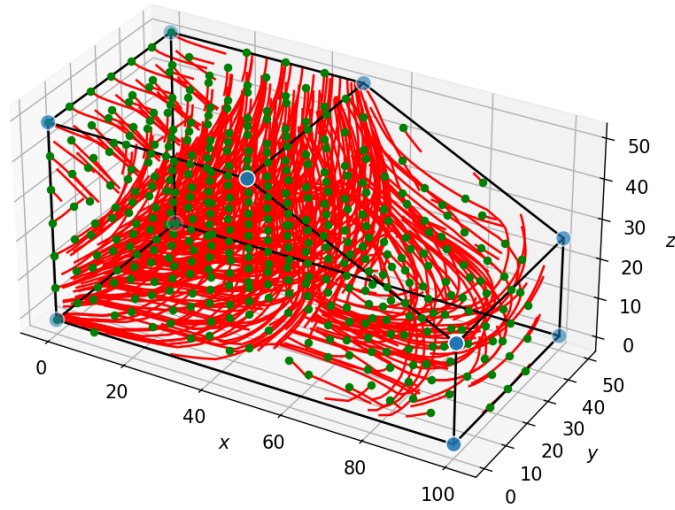


Figure 4.2.1: Stress path propagation through a finite element mesh, principal stress vectors in orange

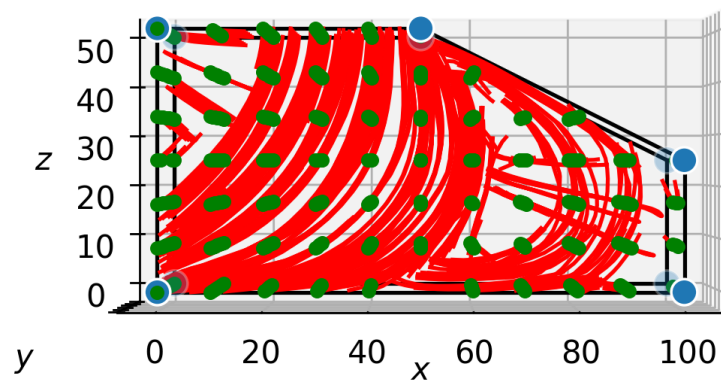
4.2.1 Academic Demonstrators - Initial Stress Paths



(a) 11 x 7 x 7 grid of seed points distributed along the x -axis

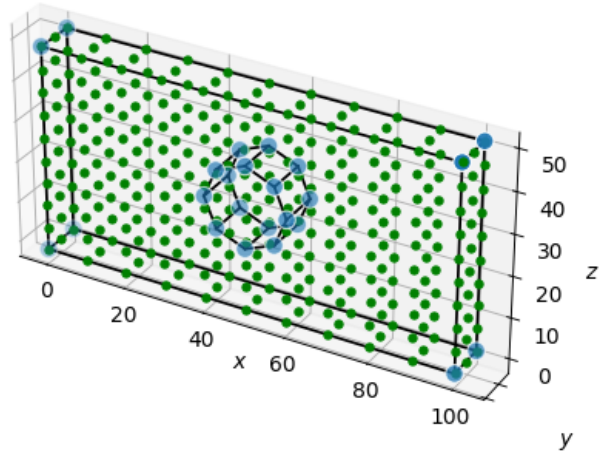


(b) Stress paths (isometric view)

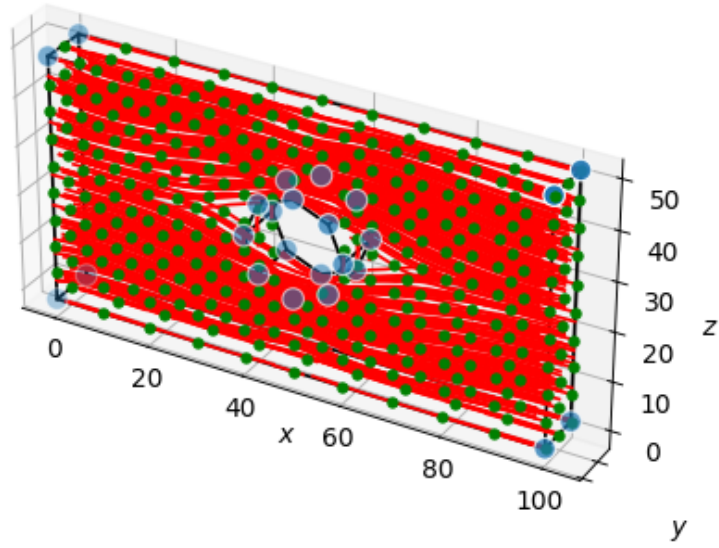


(c) Stress paths (side view)

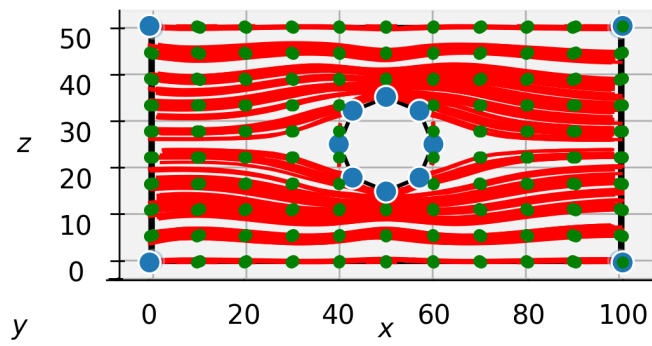
Figure 4.2.2: Initial stress paths - design case G1LC1



(a) 11 x 3 x 10 grid of seed points

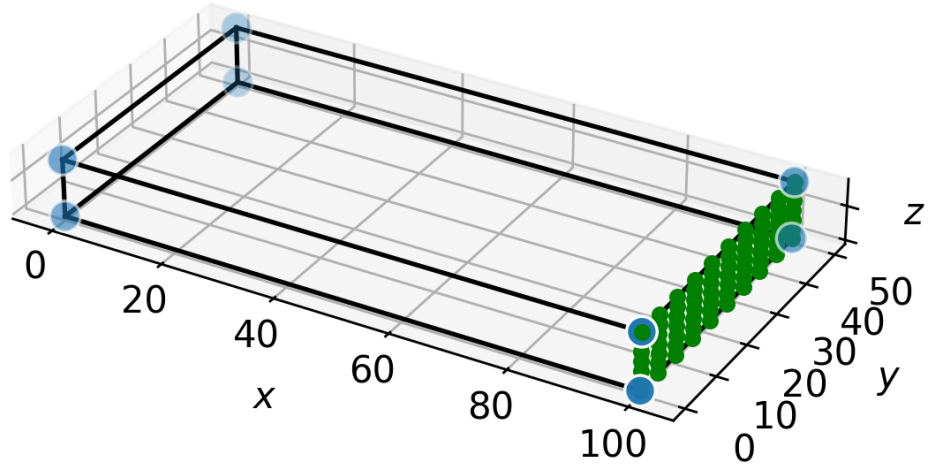


(b) Stress paths (isometric view)

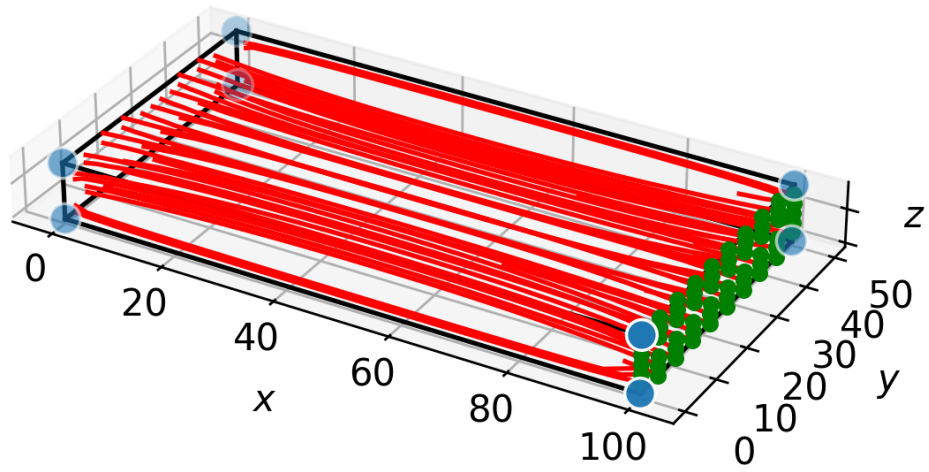


(c) Stress paths (side view)

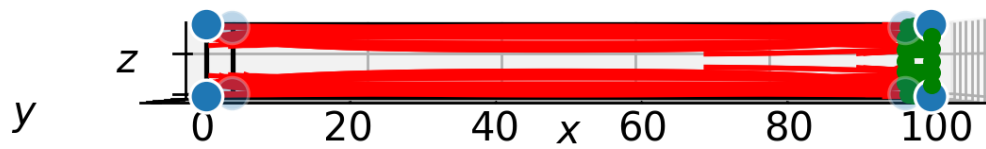
Figure 4.2.3: Initial stress paths - design case G2LC1



(a) 1 x 10 x 7 grid of seed points

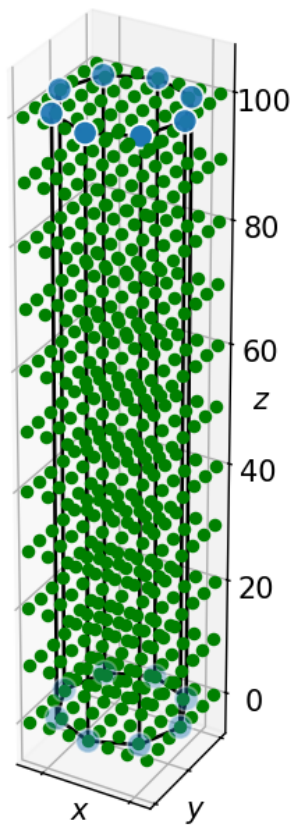


(b) Stress paths (isometric view)

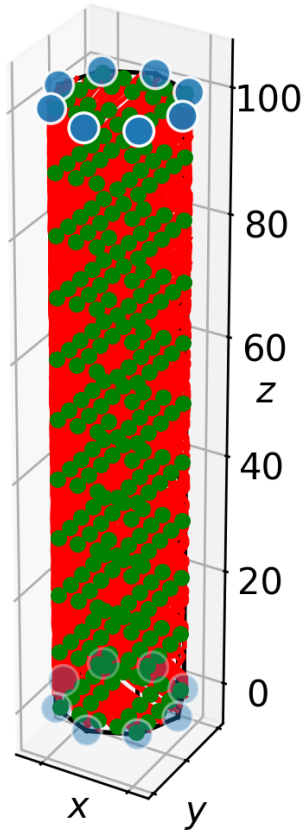


(c) Stress paths (side view)

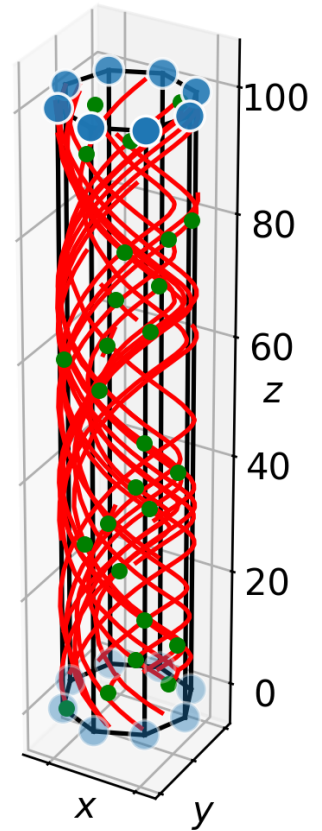
Figure 4.2.4: Initial stress paths - design case G3LC1



(a) 10 x 7 x 7 grid of seed points



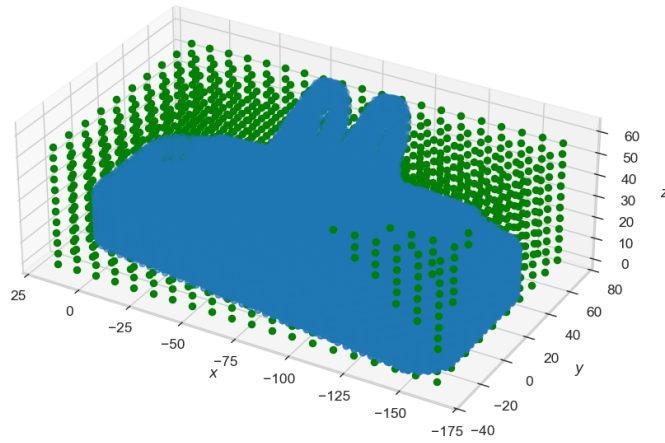
(b) All generated stress paths (isometric view)



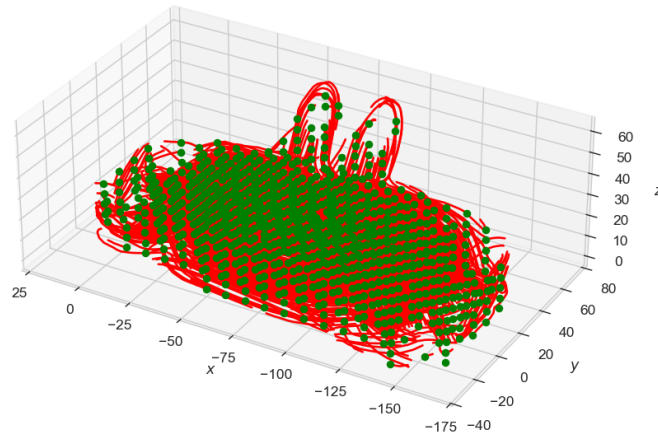
(c) 10% of the generated stress paths (isometric view)

Figure 4.2.5: Initial stress paths - design case G4LC1

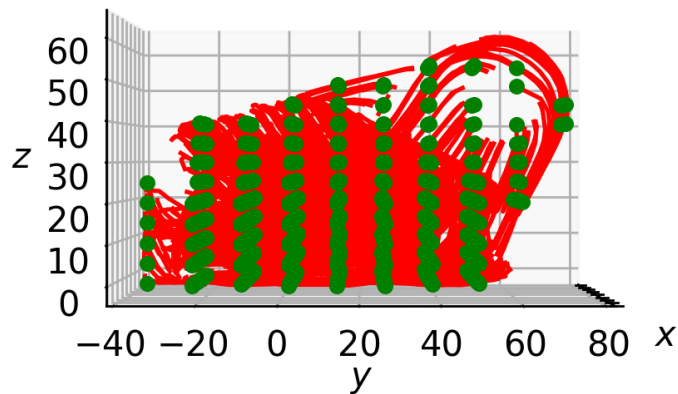
4.2.2 Industrial Demonstrators - Initial Stress Paths



(a) 10 x 12 x 20 grid of seed points

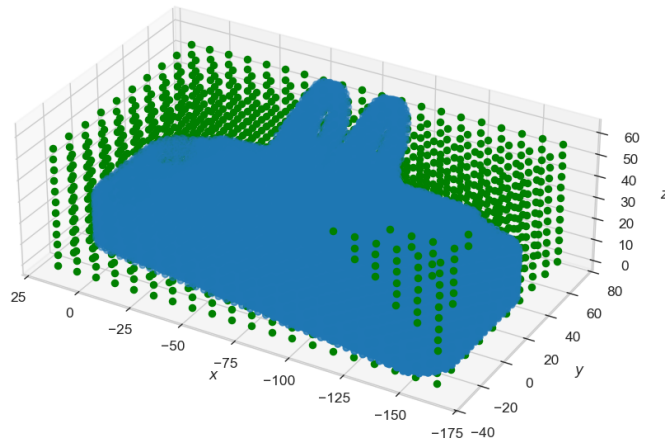


(b) Stress paths (isometric view)

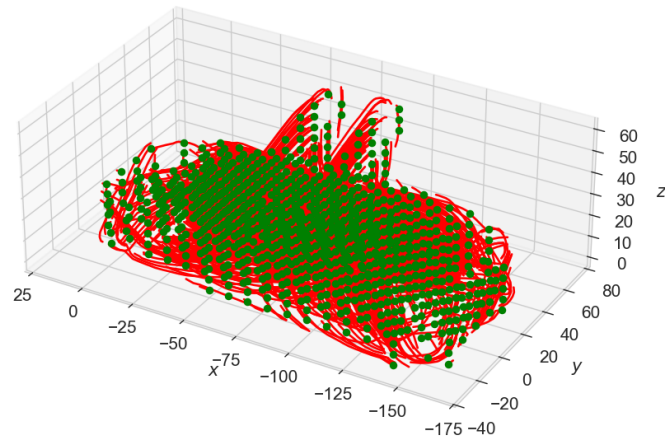


(c) Stress paths (side view)

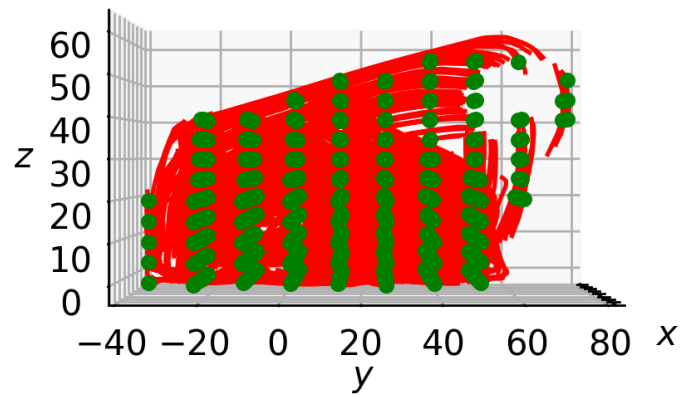
Figure 4.2.6: Initial stress paths - design case G5LC1



(a) 10 x 12 x 20 grid of seed points

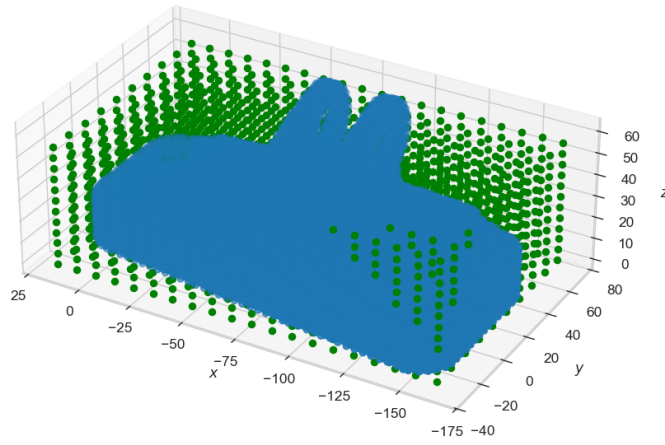


(b) Stress paths (isometric view)

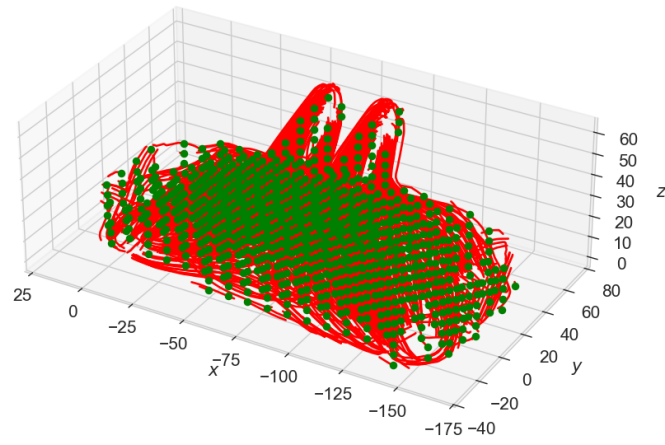


(c) Stress paths (side view)

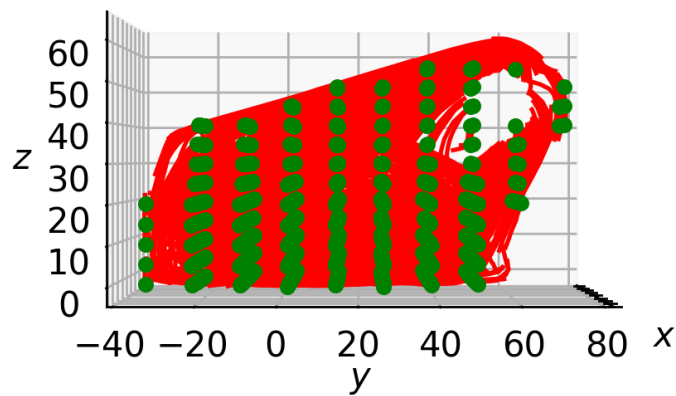
Figure 4.2.7: Initial stress paths - design case G5LC2



(a) 10 x 12 x 20 grid of seed points



(b) Stress paths (isometric view)



(c) Stress paths (side view)

Figure 4.2.8: Initial stress paths - design case G5LC3

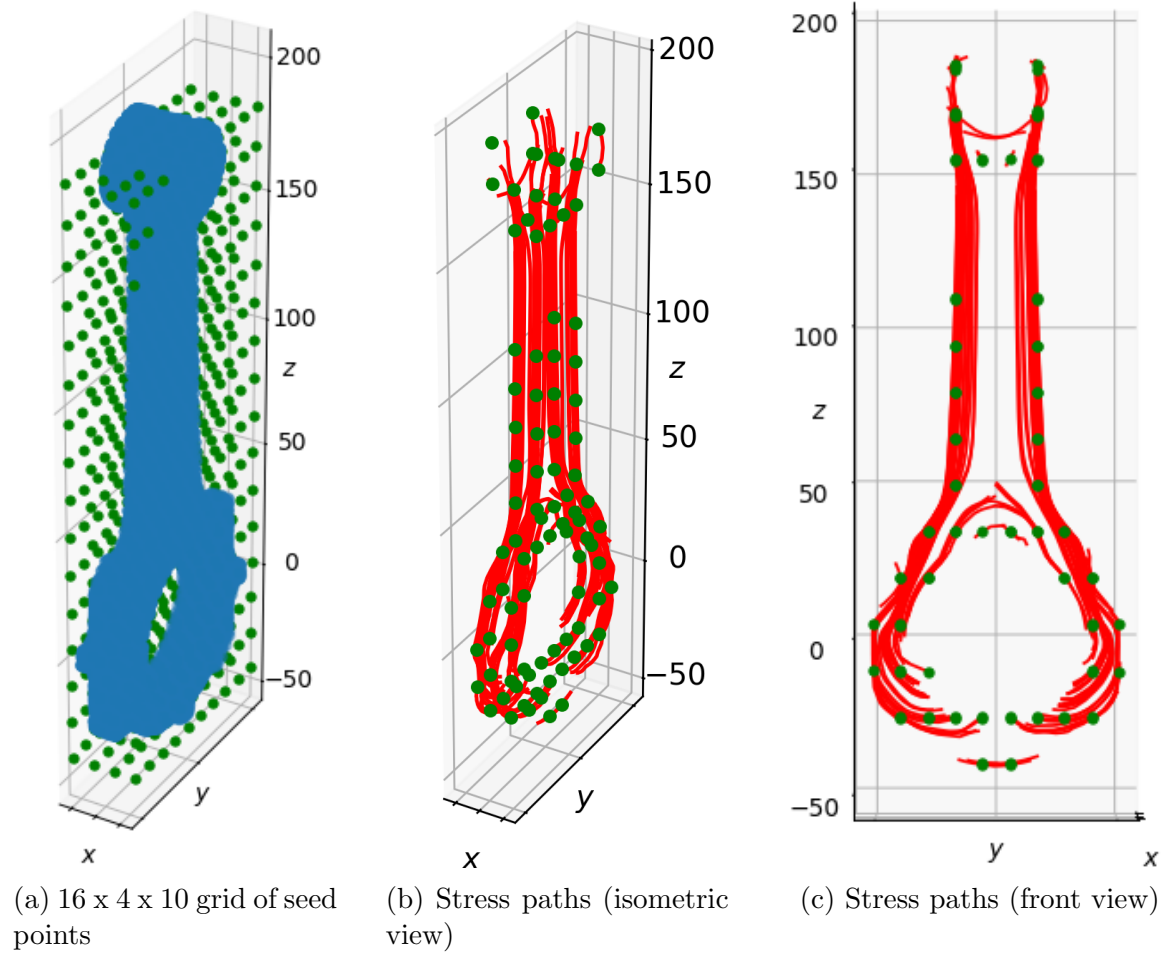


Figure 4.2.9: Initial stress paths - design case G6LC1

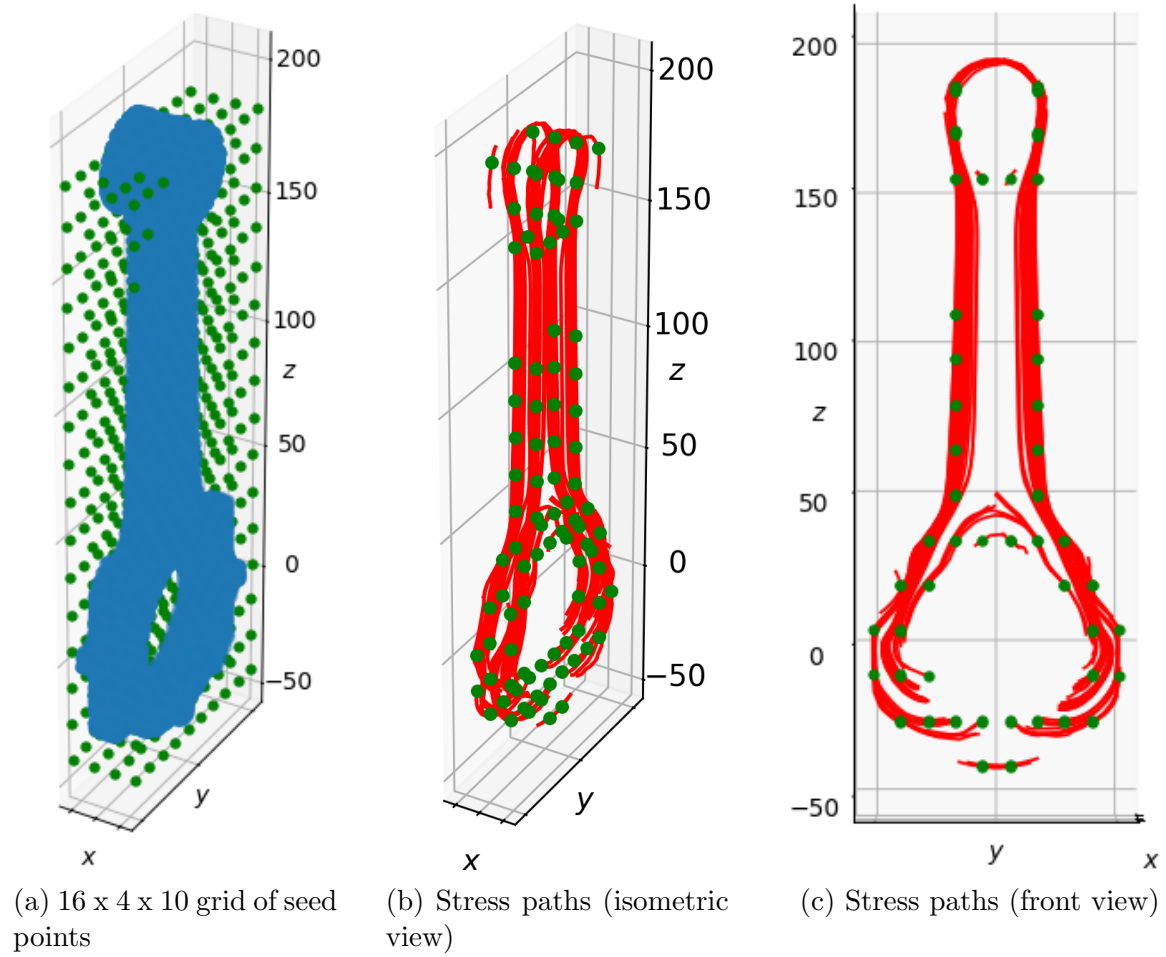


Figure 4.2.10: Initial stress paths - design case G6LC2

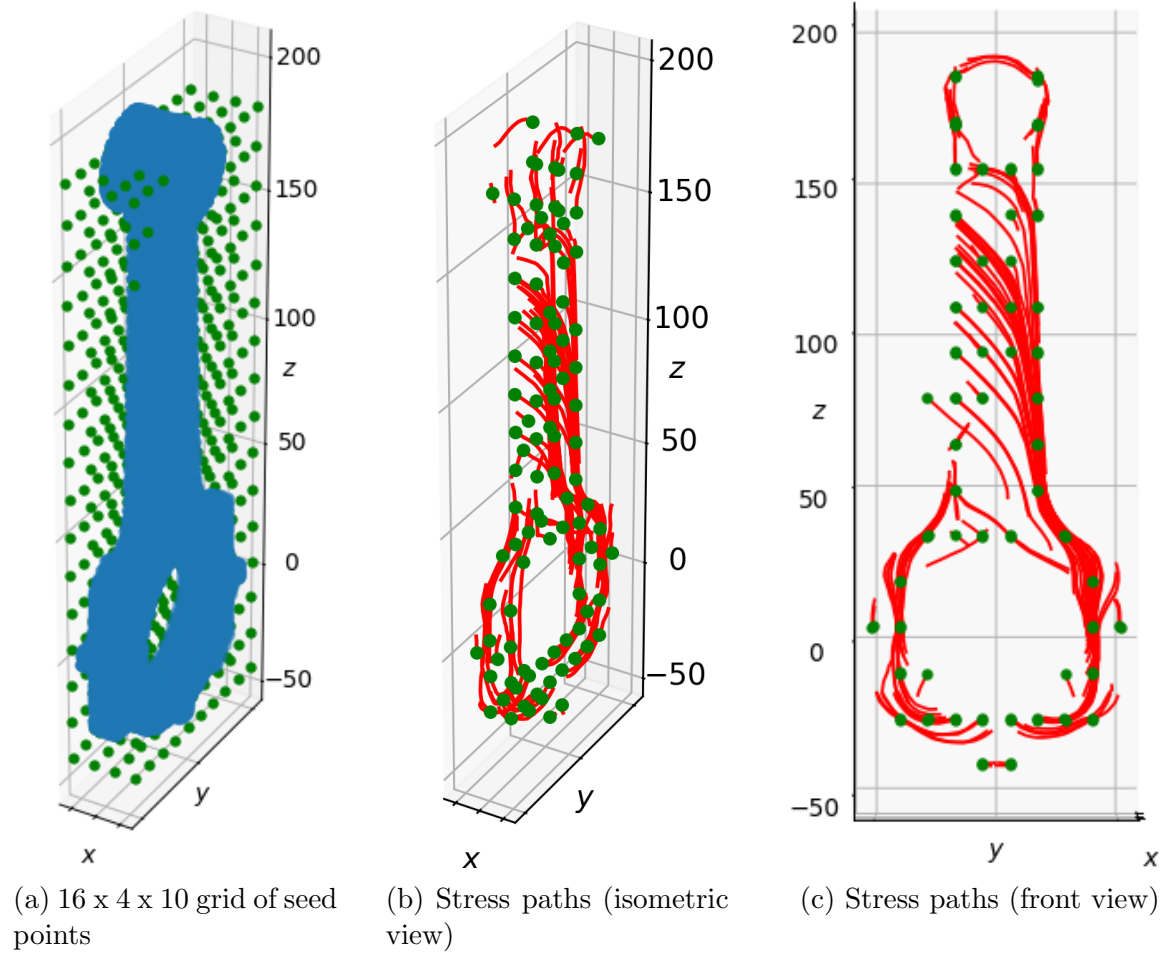


Figure 4.2.11: Initial stress paths - design case G6LC3

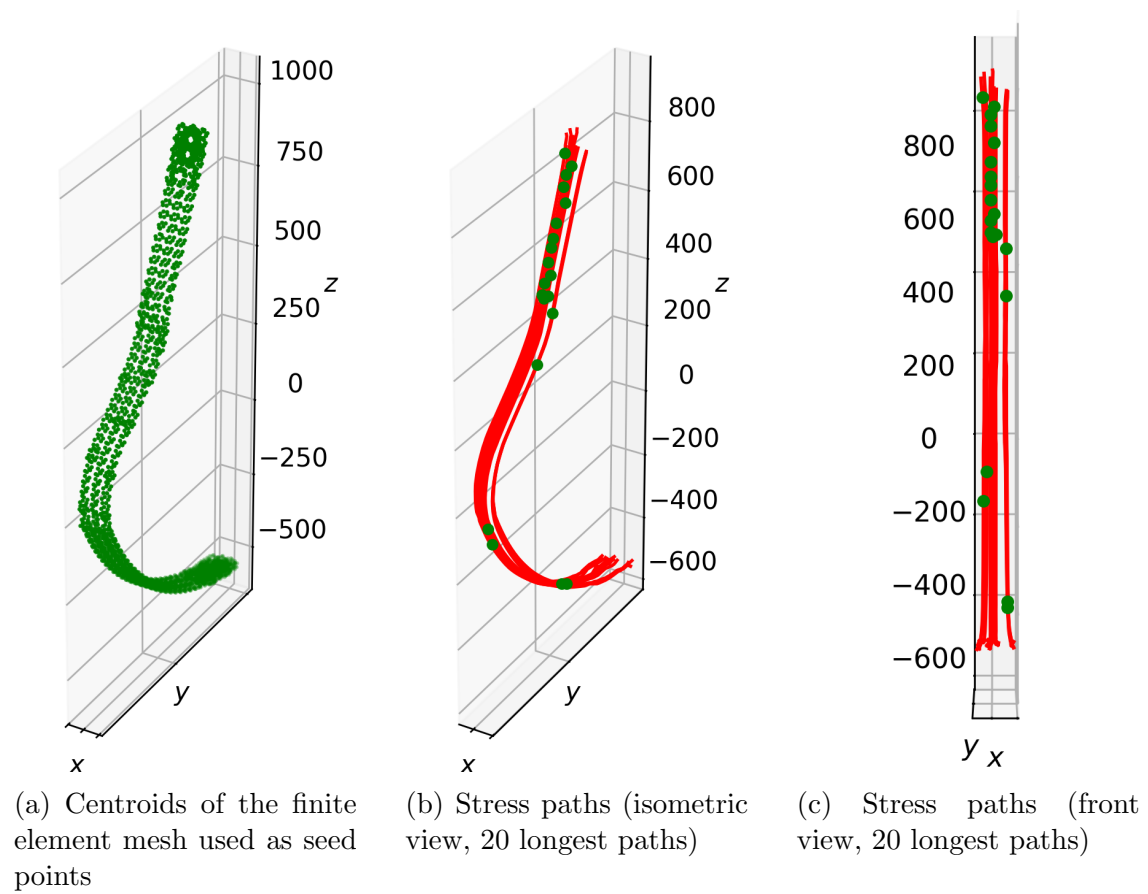


Figure 4.2.12: Initial stress paths - design case G7LC1

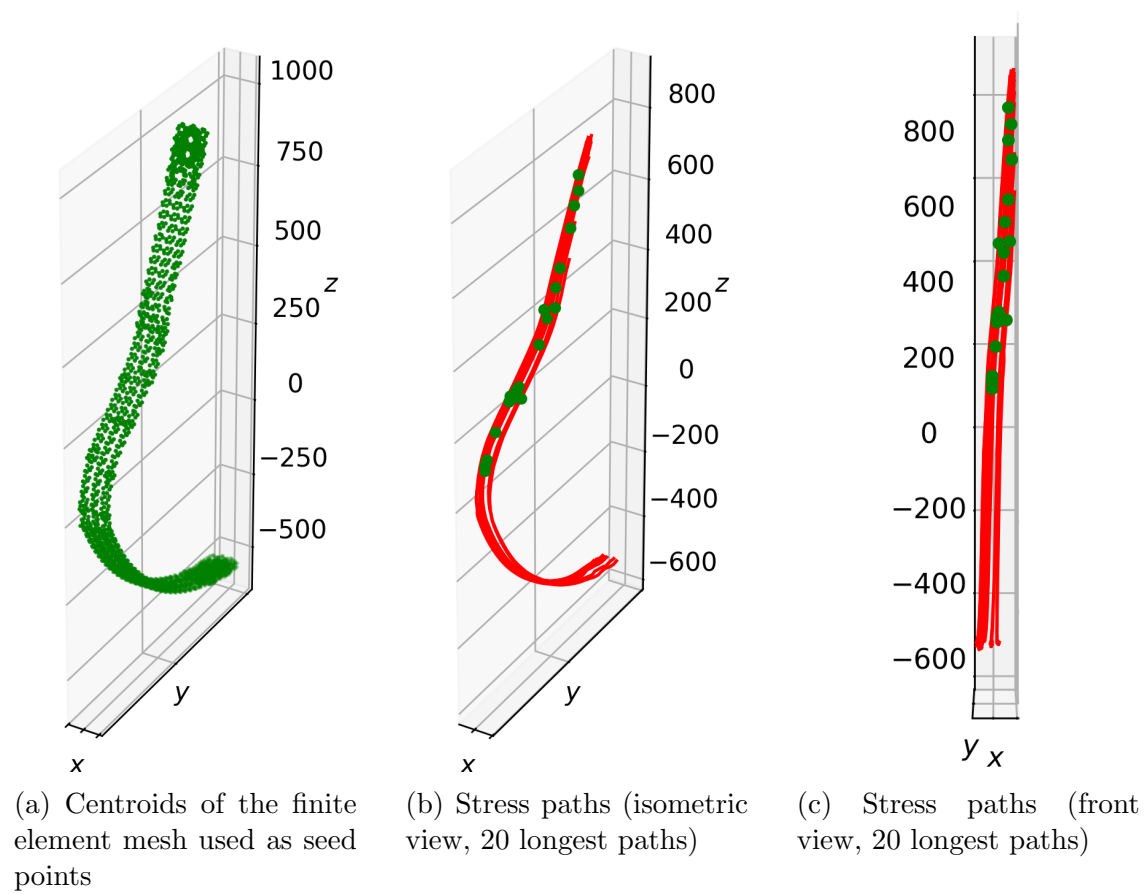


Figure 4.2.13: Initial stress paths - design case G7LC2

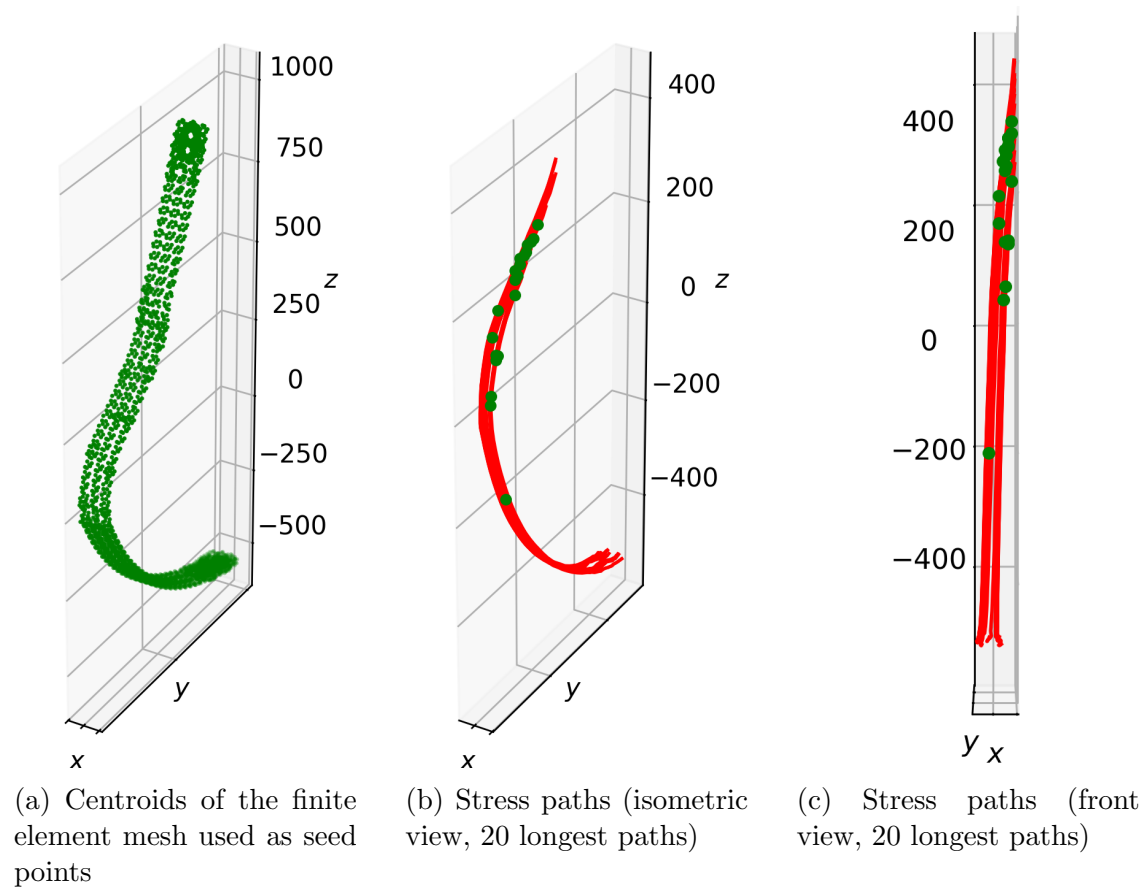


Figure 4.2.14: Initial stress paths - design case G7LC3

4.3 Step 3: Selection of Main Stress Paths

Step 3 consists in selecting the most relevant stress paths along all stress paths generated in step 2. The purpose of selection is that in step 4, points defined on stress paths are used along with points located where loads and boundary conditions are applied, for generating triangulated interlaced dry yarn trusses. Physically, points on stress paths correspond to yarn interlacing points. Including larger numbers of interlacing points produces trusses of increasing complexity, reducing eventual manufacturability. Therefore, initial stress paths produced in step 2 must be filtered to lead to a contained number of main stress paths. Seed points that define either surfaces where boundary conditions and loads are applied, or geometric features such as holes, are imposed in the stress path selection along with the stress paths generated from these points. This ensures that these surfaces are defined properly during the triangulation process in step 4.

The part designer sets a small integer number n of main principal stress paths to be considered. A prioritization index P is then calculated for each initial stress path as follows:

$$P_i = c_1 S_i^* + c_2 D_i^* + c_3 M_i^* \quad (4.2)$$

where

$$\begin{cases} S_i = \sum_{j=1}^{k-1} \sigma_j ||\mathbf{p}_j - \mathbf{p}_{j+1}|| \\ D_i = \text{Var}(d), \quad d = \{||\mathbf{c}_i - \mathbf{s}_\ell||^2, \quad 1 \leq \ell \leq \# \text{ of selected points} \} \\ M_i = ||\mathbf{c}_i - \mathbf{GMs}|| \end{cases} \quad (4.3)$$

S, D and M correspond to the stress-length product, variance penalty and clustering terms respectively. The $*$ superscript denotes that these terms are normalized between 0

and 1 for all candidate stress lines considered. Index i corresponds to the i^{th} candidate initial stress path. c_1 , c_2 and c_3 are adjustable weighing factors for the strength-length product, variance penalty and clustering term respectively.

The stress length term promotes paths that carry higher stresses and extend longer in the part envelope. Here, $\mathbf{p}_j$ and $\mathbf{p}_{j+1}$ correspond to the coordinates of the j^{th} point and the next consecutive point on a stress path, while σ_j denotes the principal stress at the j^{th} point. k corresponds to the number of points generated along the stress path.

The variance penalty promotes a uniform distribution of selected stress paths in the volume of the part. Here, Var is the variance function, $\mathbf{c}_i$ corresponds to the centre point of the i^{th} candidate stress path, and $\mathbf{s}_\ell$ corresponds to the coordinate of the ℓ^{th} selected point. Selected points are either the centroids of stress paths that were already selected or points imposed for the selection process. In choosing negative values for weight c_2 , lower variances of the squared distance terms are favoured, ensuring that selected stress paths are evenly spaced relative to each other, leading to uniform stress line distributions.

The clustering term prevents local clustering of selected stress paths in zones of large point density. This term is of particular importance for design cases G5LC1-G5LC3 due to the large amount of points defining the bracket's holes. Here, $\mathbf{GM}$ corresponds to the coordinates of the geometric median of the selected points. The geometric median corresponds to the point minimizing the sum of distances to the selected points.

Main stress paths are selected iteratively, calculating prioritization indices for every generated stress path at each iteration. Figures 4.3.1-4.3.4 and Figures 4.3.5-4.3.13 illustrate the selection process for academic and industrial demonstrators respectively. It is of note that due to the thinness of the running blade, it was difficult to achieve stress path selections

that minimized clustering, which is particularly highlighted in Figures 4.3.12 and 4.3.13.

4.3.1 Academic Demonstrators - Main Stress Paths

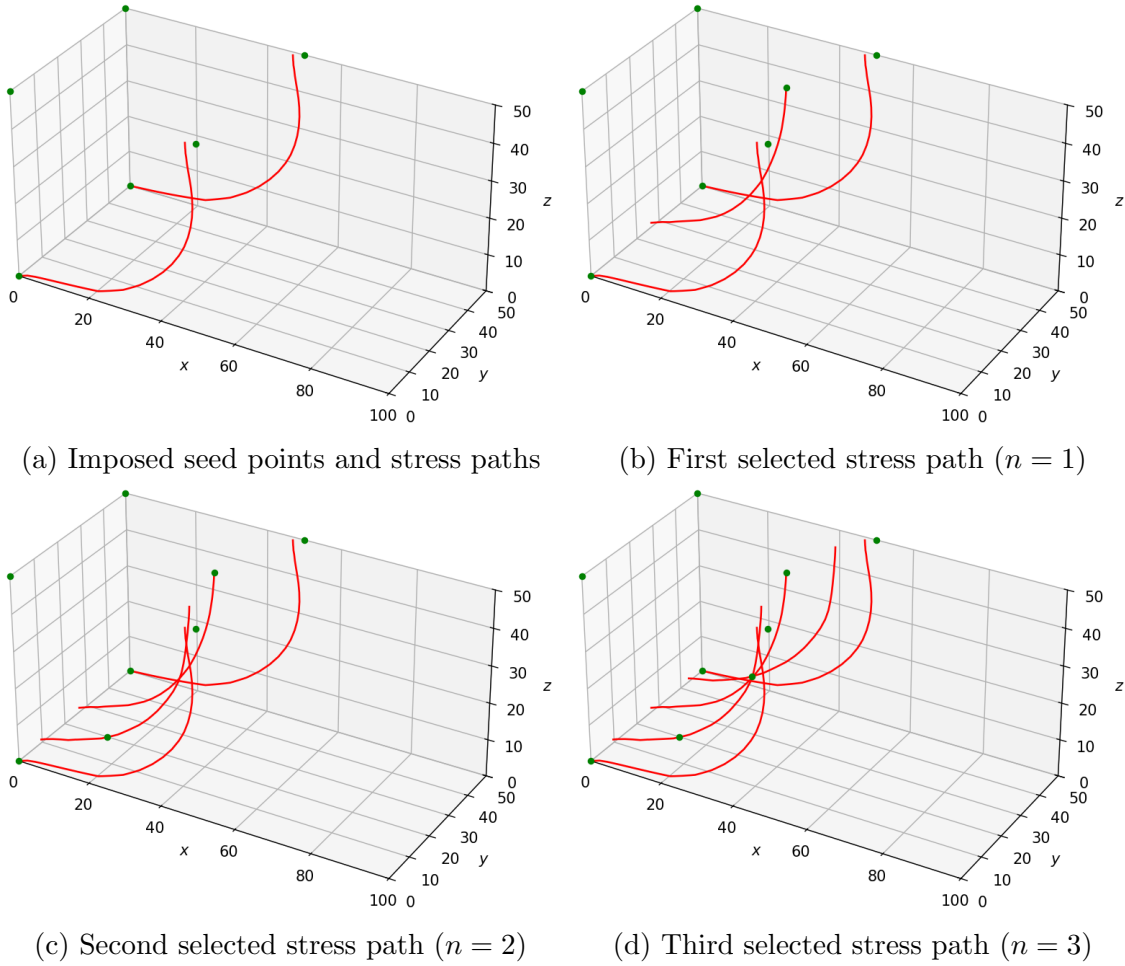
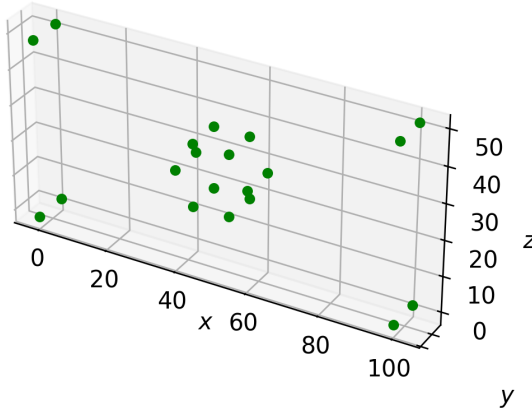
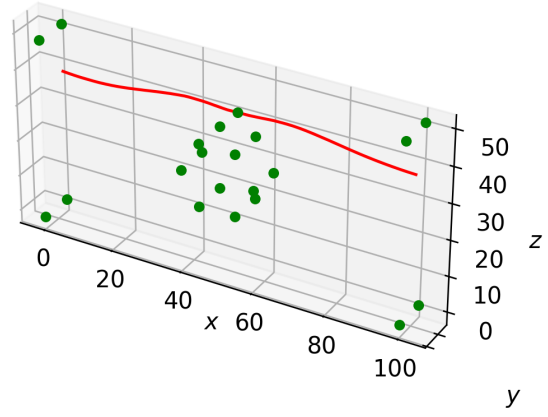


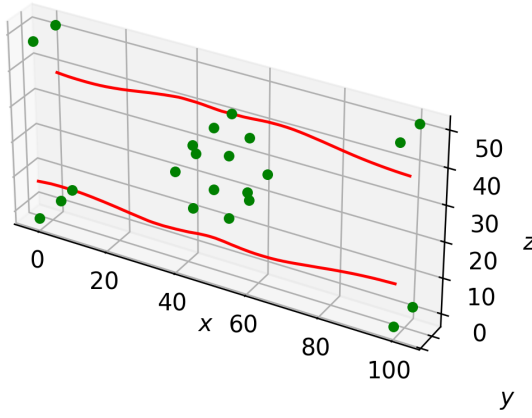
Figure 4.3.1: Selection of three main stress paths - design case G1LC1 ($c_1 = 1$, $c_2 = -3$, $c_3 = 1$)



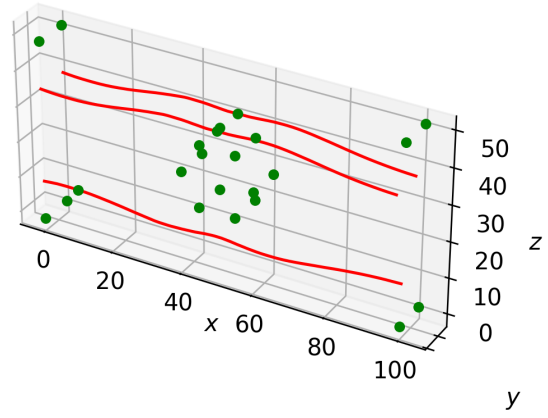
(a) Imposed seed points



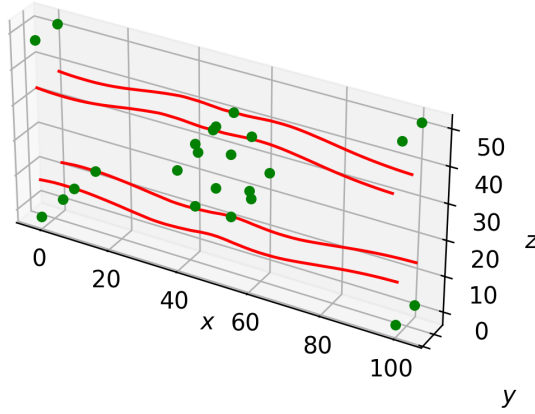
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

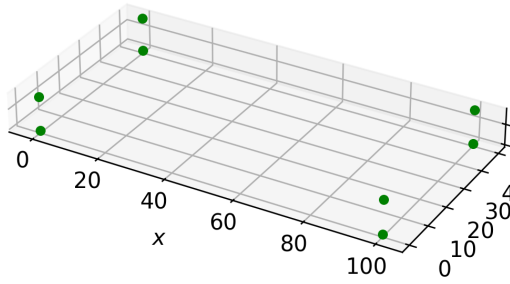


(d) Third selected stress path ($n = 3$)

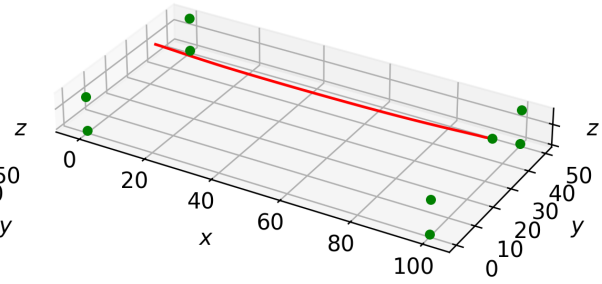


(e) Fourth selected stress path ($n = 4$)

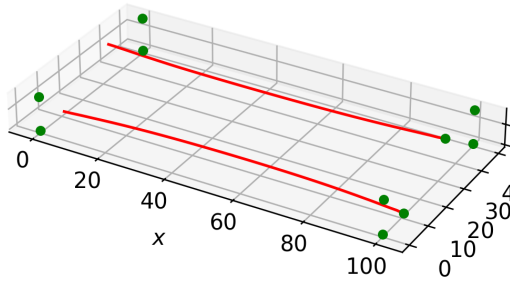
Figure 4.3.2: Selection of four main stress paths - design case G2LC1 ($c_1 = 1$, $c_2 = -10$, $c_3 = 1$)



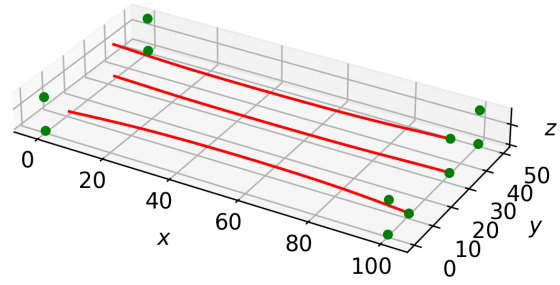
(a) Imposed seed points



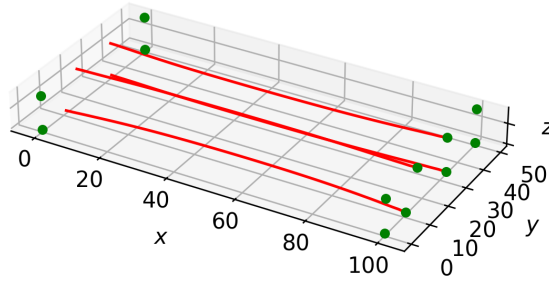
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

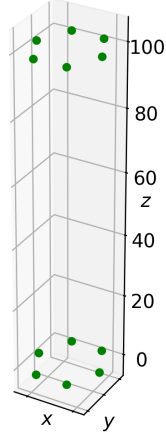


(d) Third selected stress path ($n = 3$)

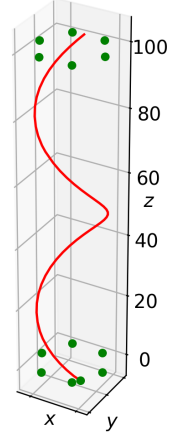


(e) Fourth selected stress path ($n = 4$)

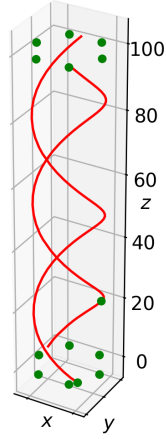
Figure 4.3.3: Selection of four main stress paths - design case G3LC1 ($c_1 = 1$, $c_2 = -10$, $c_3 = 1$)



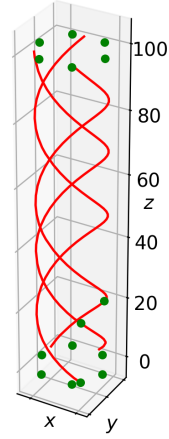
(a) Imposed seed points



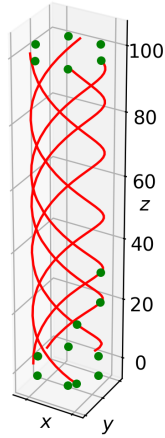
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)



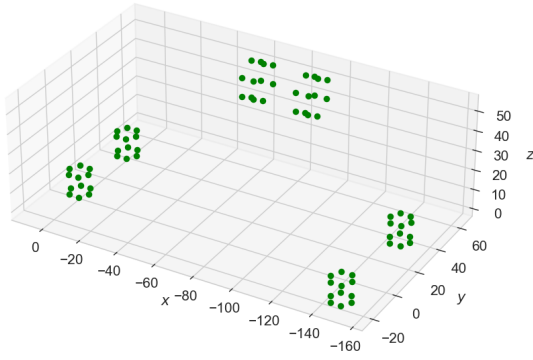
(d) Third selected stress path ($n = 3$)



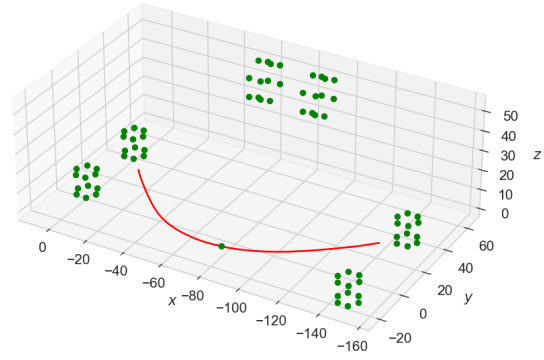
(e) Fourth selected stress path ($n = 4$)

Figure 4.3.4: Selection of four main stress paths - design case G4LC1 ($c_1 = 1$, $c_2 = -10$, $c_3 = 1$)

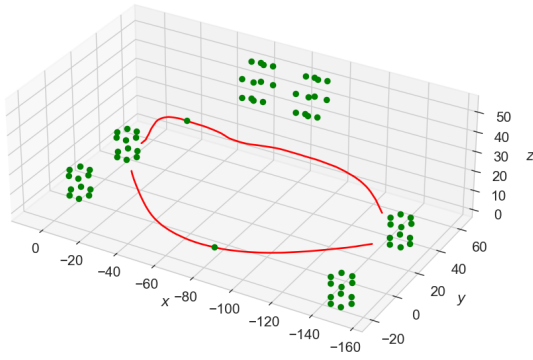
4.3.2 Industrial Demonstrators - Main Stress Paths



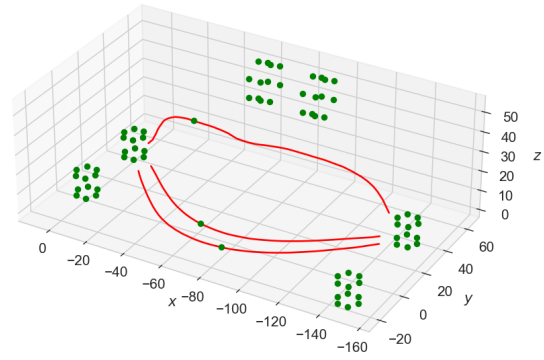
(a) Imposed seed points



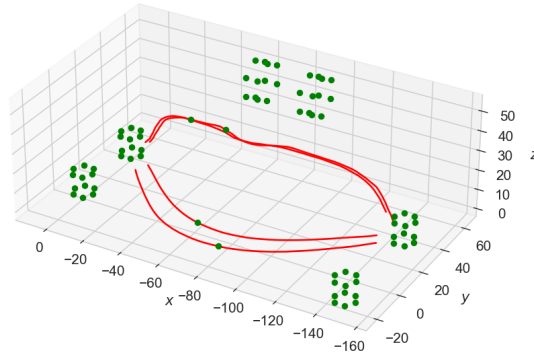
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

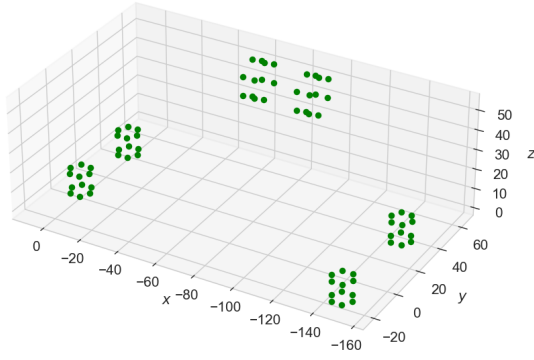


(d) Third selected stress path ($n = 3$)

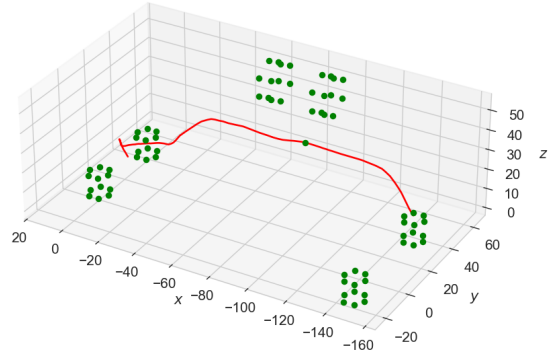


(e) Fourth selected stress path ($n = 4$)

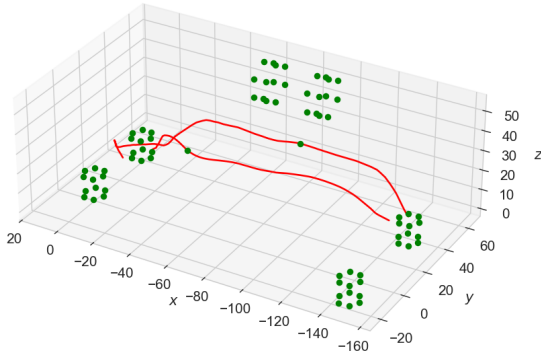
Figure 4.3.5: Selection of four main stress paths - design case G5LC1 ($c_1 = 15$, $c_2 = -100$, $c_3 = 8.5$)



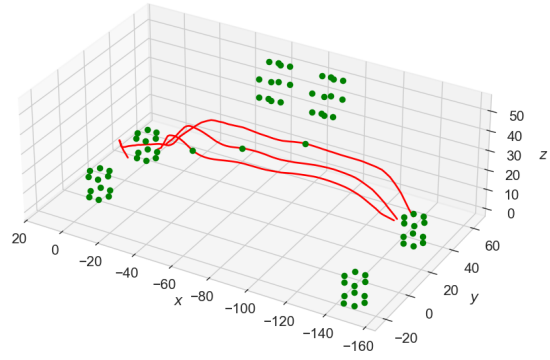
(a) Imposed seed points



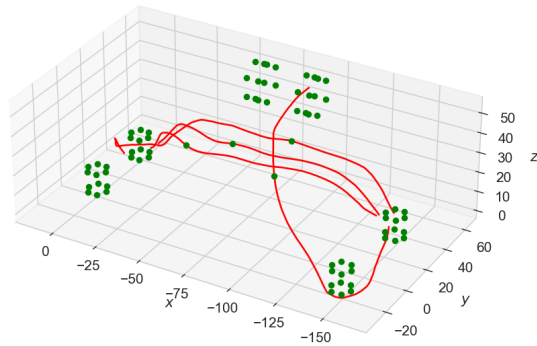
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

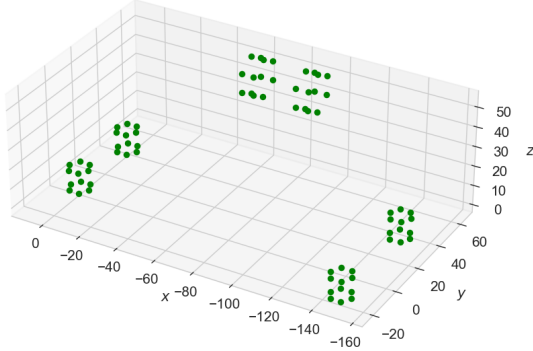


(d) Third selected stress path ($n = 3$)

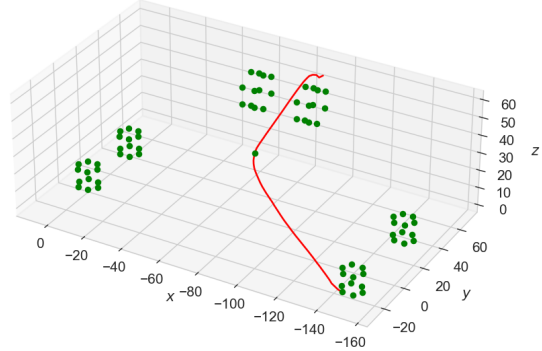


(e) Fourth selected stress path ($n = 4$)

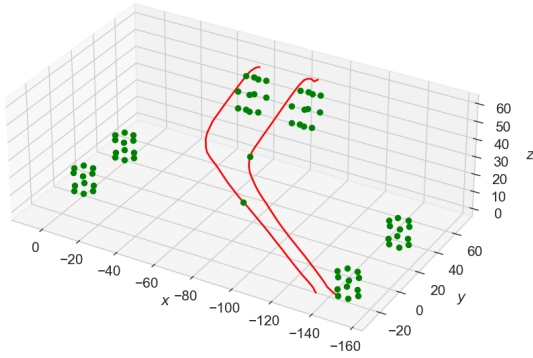
Figure 4.3.6: Selection of four main stress paths - design case G5LC2 ($c_1 = 10$, $c_2 = -5$, $c_3 = 2.5$)



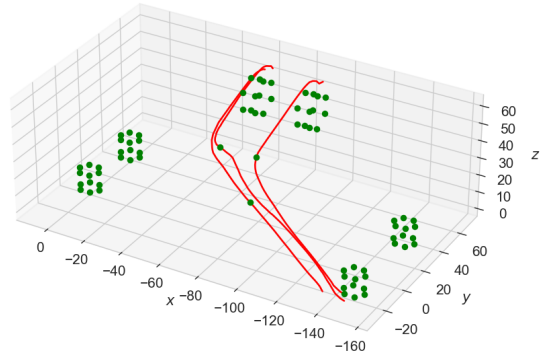
(a) Imposed seed points



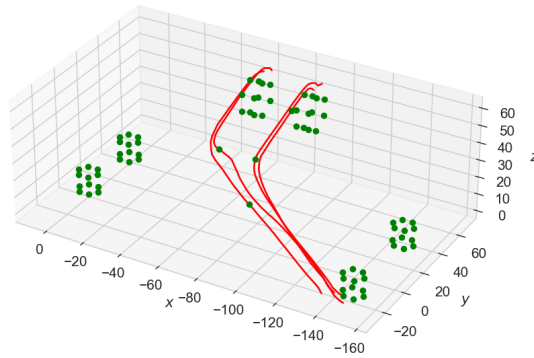
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

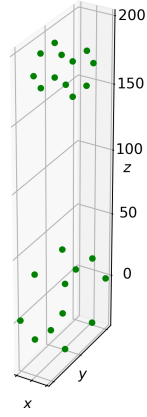


(d) Third selected stress path ($n = 3$)

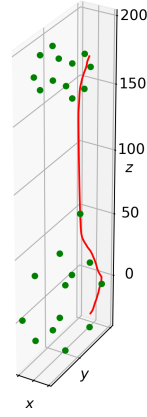


(e) Fourth selected stress path ($n = 4$)

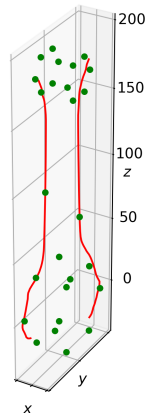
Figure 4.3.7: Selection of four main stress paths - design case G5LC3 ($c_1 = 5$, $c_2 = -10$, $c_3 = 5$)



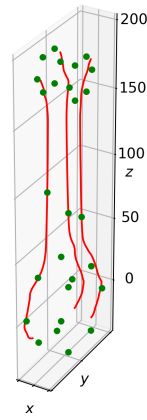
(a) Imposed seed points



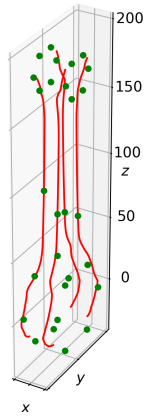
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

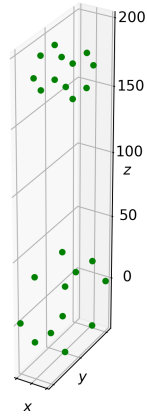


(d) Third selected stress path ($n = 3$)

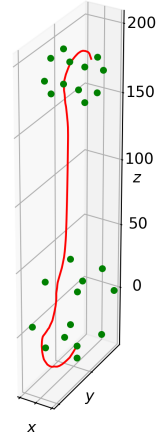


(e) Fourth selected stress path ($n = 4$)

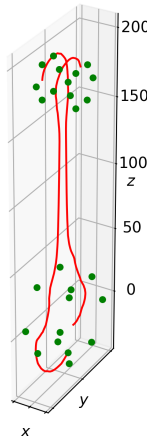
Figure 4.3.8: Selection of four main stress paths - design case G6LC1 ($c_1 = 7.5$, $c_2 = -100$, $c_3 = 25$)



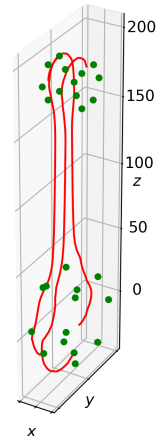
(a) Imposed seed points



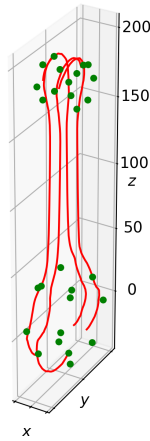
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

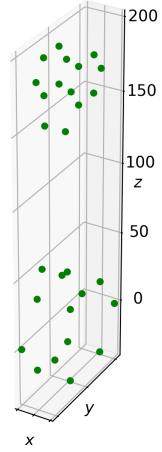


(d) Third selected stress path ($n = 3$)

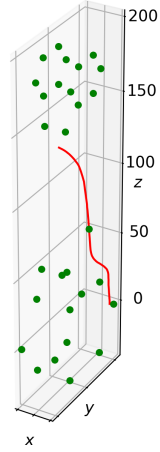


(e) Fourth selected stress path ($n = 4$)

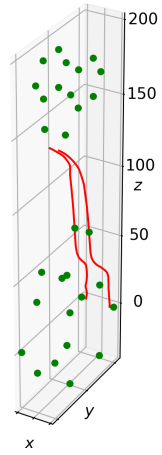
Figure 4.3.9: Selection of four main stress paths - design case G6LC2 ($c_1 = 7.5$, $c_2 = -100$, $c_3 = 5$)



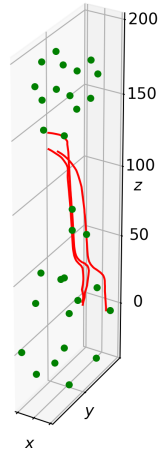
(a) Imposed seed points



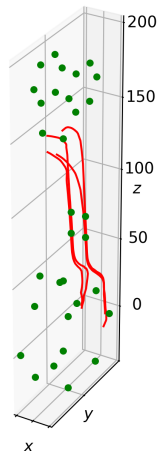
(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)



(d) Third selected stress path ($n = 3$)



(e) Fourth selected stress path ($n = 4$)

Figure 4.3.10: Selection of four main stress paths - design case G6LC3 ($c_1 = 7.5$, $c_2 = -100$, $c_3 = 25$)

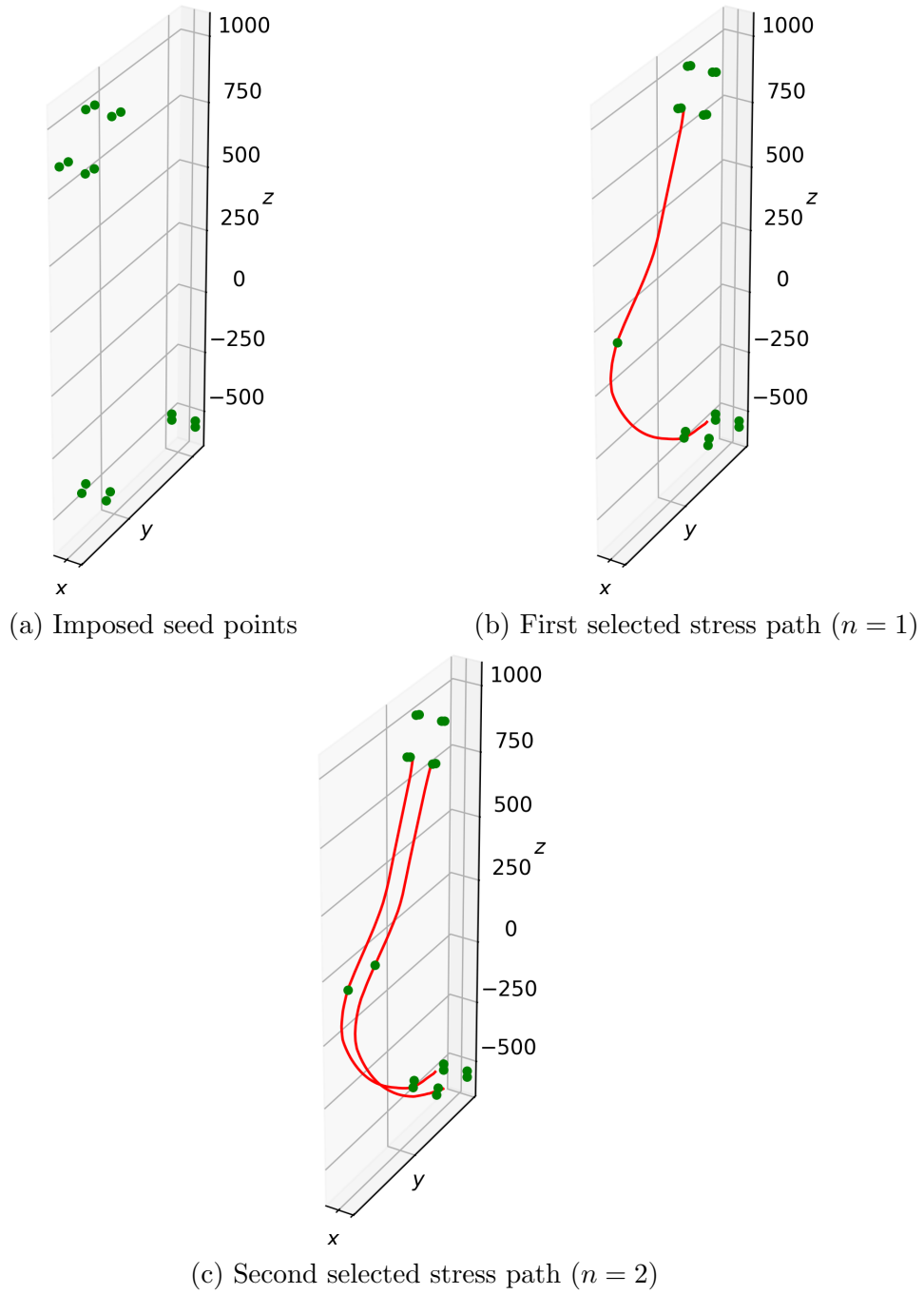
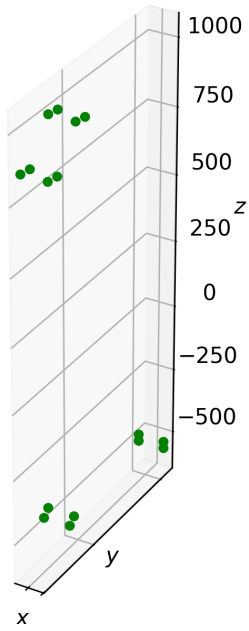
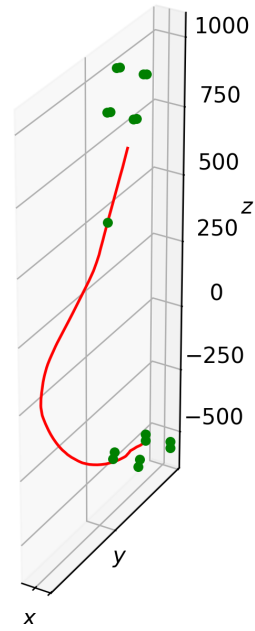


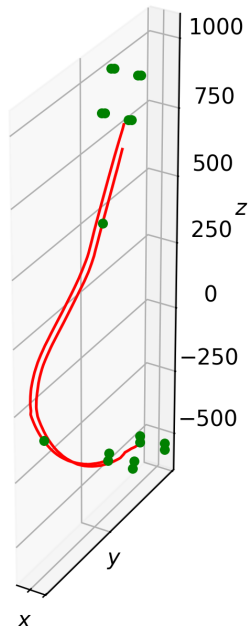
Figure 4.3.11: Selection of two main stress paths - design case G7LC1 ($c_1 = 1$, $c_2 = -10$, $c_3 = 2.5$)



(a) Imposed seed points



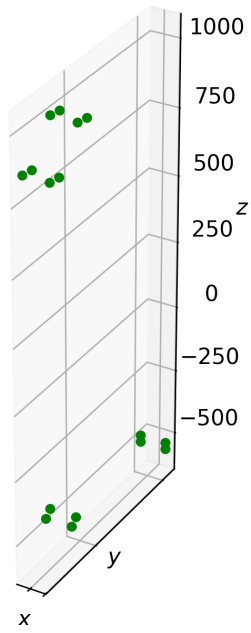
(b) First selected stress path ($n = 1$)



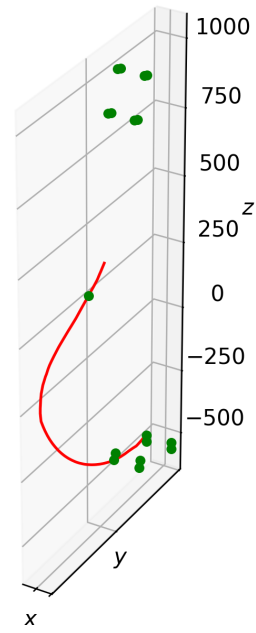
(c) Second selected stress path ($n = 2$)

Story:

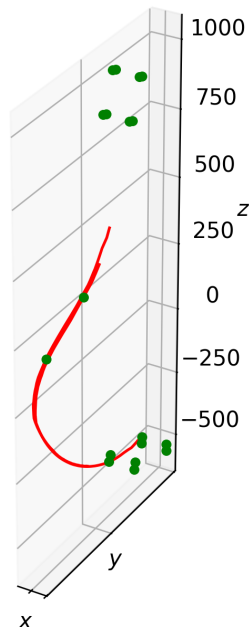
Figure 4.3.12: Selection of two main stress paths - design case G7LC2 ($c_1 = 0.1$, $c_2 = -10$, $c_3 = 10$)



(a) Imposed seed points



(b) First selected stress path ($n = 1$)



(c) Second selected stress path ($n = 2$)

Figure 4.3.13: Selection of two main stress paths - design case G7LC3 ($c_1 = 10$, $c_2 = -10$, $c_3 = 25$)

Chapter 5

Initial Structure Generation Using Delaunay Triangulation

In step 4 of the sequential TO process, points along the stress paths as well as points where loads and boundary conditions are applied are used for generating triangulated interlaced dry yarn trusses. This chapter presents a first attempt at developing the methodology for step 4 along with results for the 4 academic demonstrators. This first attempt was based on Delaunay triangulation [74] and generated initial structures that featured a very large amount of edges leading to undesired manufacturing complexity. Graph sparsification techniques were applied to these initial structures in an attempt to reduce the amount of edges. However, this led to structures that lacked connectivity, ultimately requiring the development of a new triangulation algorithm presented in Chapter 6.

5.1 Step 4 a): Delaunay Triangulation of Stress Paths

5.1.1 Delaunay Triangulation

The Delaunay triangulation of a discrete set of points $\mathbf{P}$ produces triangles between points such that no point belonging to $\mathbf{P}$ is contained within any circumcircle of the triangulation, Figure 5.1.1. This fundamental property of the Delaunay triangulation is known as the empty circle property [74]. The empty circle property can be extended to 3D points, forming a set of tetrahedra such that the circumsphere of each tetrahedron does not contain any other point belonging to $\mathbf{P}$ in its interior. Delaunay triangulation is widely used in mesh generation due to the empty circle property maximizing the minimum dihedral angle [75–77]. This leads to well-shaped elements that improve numerical stability in simulations.

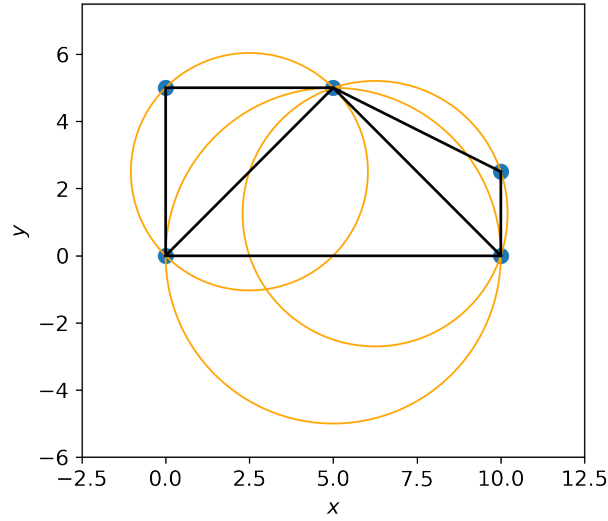


Figure 5.1.1: Depiction of Delaunay triangulation and empty circle property in 2D

5.1.2 Graph Theory

Graph theory is the study of graphs, which are mathematical structures used for modelling pairwise relations between objects. These objects correspond to mathematical abstractions labelled vertices, and pairs of vertices labelled edges. Let $G = (V, E)$ denote graph G

composed of the set of vertices V and set of edges E that connect vertices [78]. Here, E is defined as a multiset of pairs of connected vertices as follows:

$$E \subseteq \{\{x, y\} \mid x, y \in V\} \quad (5.1)$$

The vertices x, y of an edge are called the endpoints of an edge; the edge is said to be incident to x and y . Graphs can be further classified as weighted or unweighted [78]. In unweighted graphs edges have no associated numerical values, Figure 5.1.2. These graphs are useful in applications where only the structure of connections matters, such as social networks or connectivity in a network topology. Contrarily, in weighted graphs each edge $(u, v) \in E$ has an associated weight $w(u, v)$, Figure 5.1.3. These weights can represent distances, costs, capacities, or probabilities. Weighted graphs are commonly used in shortest path problems, such as in transportation networks and routing algorithms. Triangulated interlaced dry yarn trusses are modelled as weighted graphs, with vertices, edges and weights corresponding to interlacing points, yarns segments and the distances between interlacing points respectively.

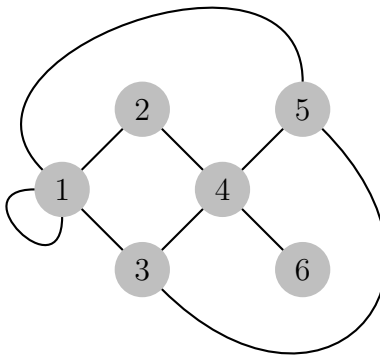


Figure 5.1.2: Example of unweighted graph with 6 vertices and 9 edges

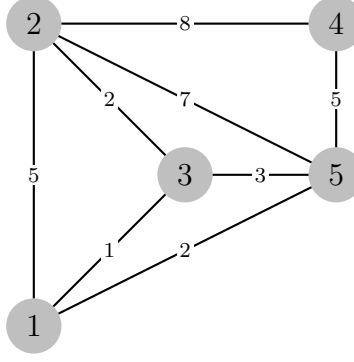


Figure 5.1.3: Example of weighted graph with 5 vertices and 8 edges

The average clustering coefficient measures the overall tendency of vertices in a graph to cluster together. For each vertex v , the local clustering coefficient c_v is the fraction of pairs of v 's neighbours that are connected by an edge. The average clustering coefficient is the mean of all vertices' local clustering coefficients [79]:

$$C = \frac{1}{n} \sum_{v \in G} c_v \quad (5.2)$$

The average clustering coefficient is used for quantifying the complexity of triangulated interlaced dry yarn trusses.

5.1.3 Initial Triangulation

For each stress path selected in step 3, a small number k of equidistant points are sampled along the stress path. This set of points is then used for performing 3D Delaunay triangulation based on Python's SciPy library, generating an initial triangulated interlaced truss. It is of note that certain geometric features such as holes and fillets are lost during typical Delaunay triangulation. A first post-processing step where edges are removed from these regions was used for re-establishing these features. This triangulation process was performed on all academic design cases, ranging from $k = 2$ to $k = 6$ points per stress path. Triangulation results can be found in Appendix A. For each triangulation result, the number of edges in the structure was documented, allowing to determine the complexity of the structures pro-

duced as a function of the number of points, Figure 5.1.4. Delaunay triangulation results for academic demonstrators appear in Figures 5.1.5-5.1.19

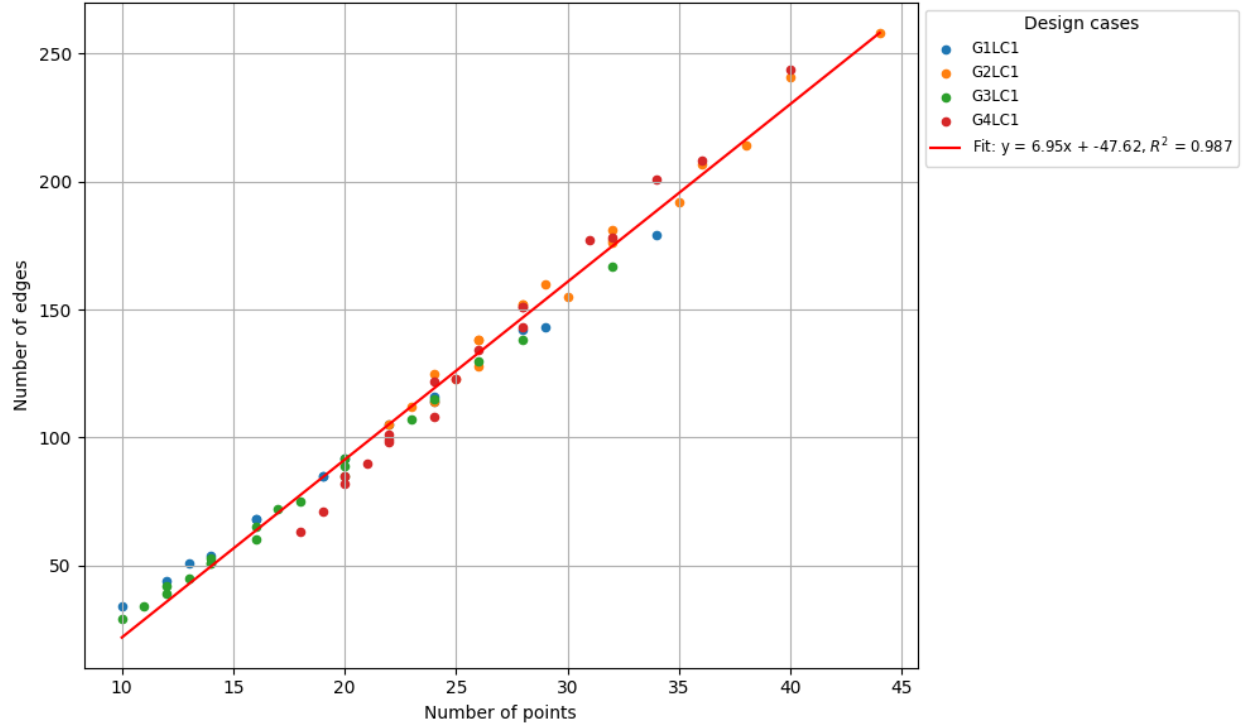
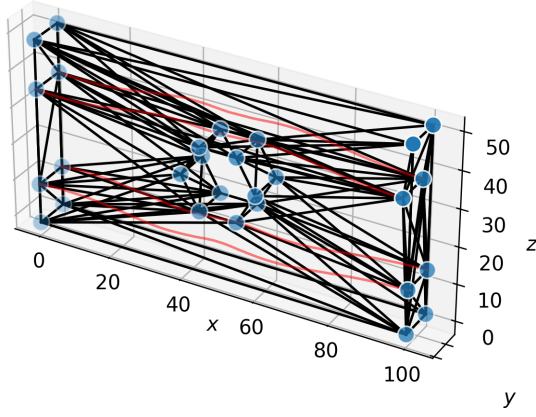
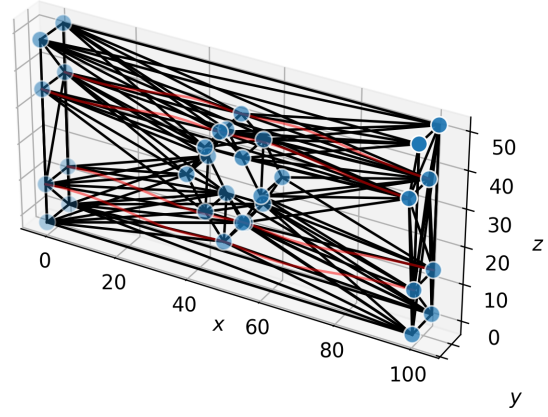


Figure 5.1.4: Number of edges in Delaunay triangulated trusses as a function of number of points for all academic demonstrators

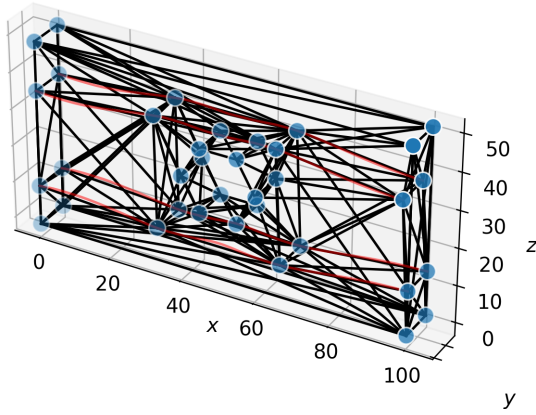
Structure complexity increases linearly with the number of points. Delaunay triangulation produces highly dense structures with clusters. This is particularly problematic for structures that require many points for defining load and boundary surfaces, such as hole surroundings in Figure 5.1.5: most edges in the structure appear to emanate from this zone due to the points defining the hole. In an attempt to mitigate this, graph sparsification techniques were employed to remove redundant edges of the structure; this is discussed in the next section.



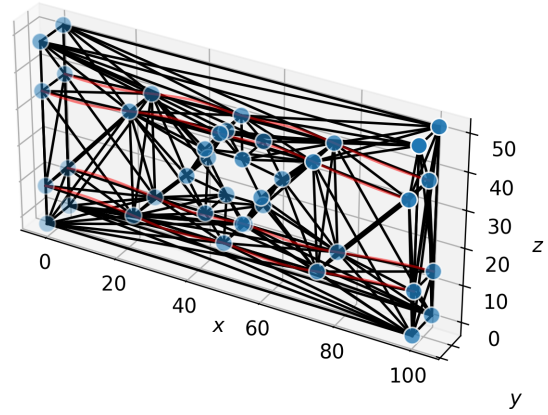
(a) 2 points per main stress path ($k = 2$)



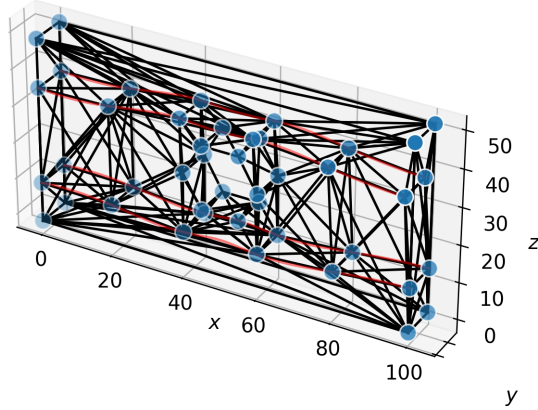
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)



(d) 5 points per main stress line ($k = 5$)



(e) 6 points per main stress path ($k = 6$)

Figure 5.1.5: Delaunay triangulation - design case G2LC1 with varying amounts of points per main stress path, $n = 4$ main stress paths, main stress paths in red

5.1.4 Graph Sparsification

Graph sparsification is a technique that approximates a given graph by a sparse graph using a subset of its vertices and/or edges. The goal of sparsification is to maintain specific graph properties relevant to the downstream task while minimizing the graph's size [80]. In graph sparsification, the prune rate ρ is defined as the proportion of edges removed from the original graph. Four different sparsification algorithms were considered in this work: Local Degree [81], Local Similarity [82], SCAN Similarity [82] and Simmelian Backbones [83] as they are deterministic in nature [80,83], meaning that if you run the same algorithm on the same input multiple times, you always get the exact same result.

Results obtained on the holed plate produced with $n = 4$ stress paths and $k = 6$ points per path are shown in Figures 5.1.6-5.1.9. In all cases the sparsified structures are not fully connected, making them undesirable from a structural perspective. This demonstrates the need for a triangulation method less sensitive to the number of points used, eliminating the need for sparsification. A new triangulation algorithm was developed to remedy this problem, which is presented in Chapter 6.

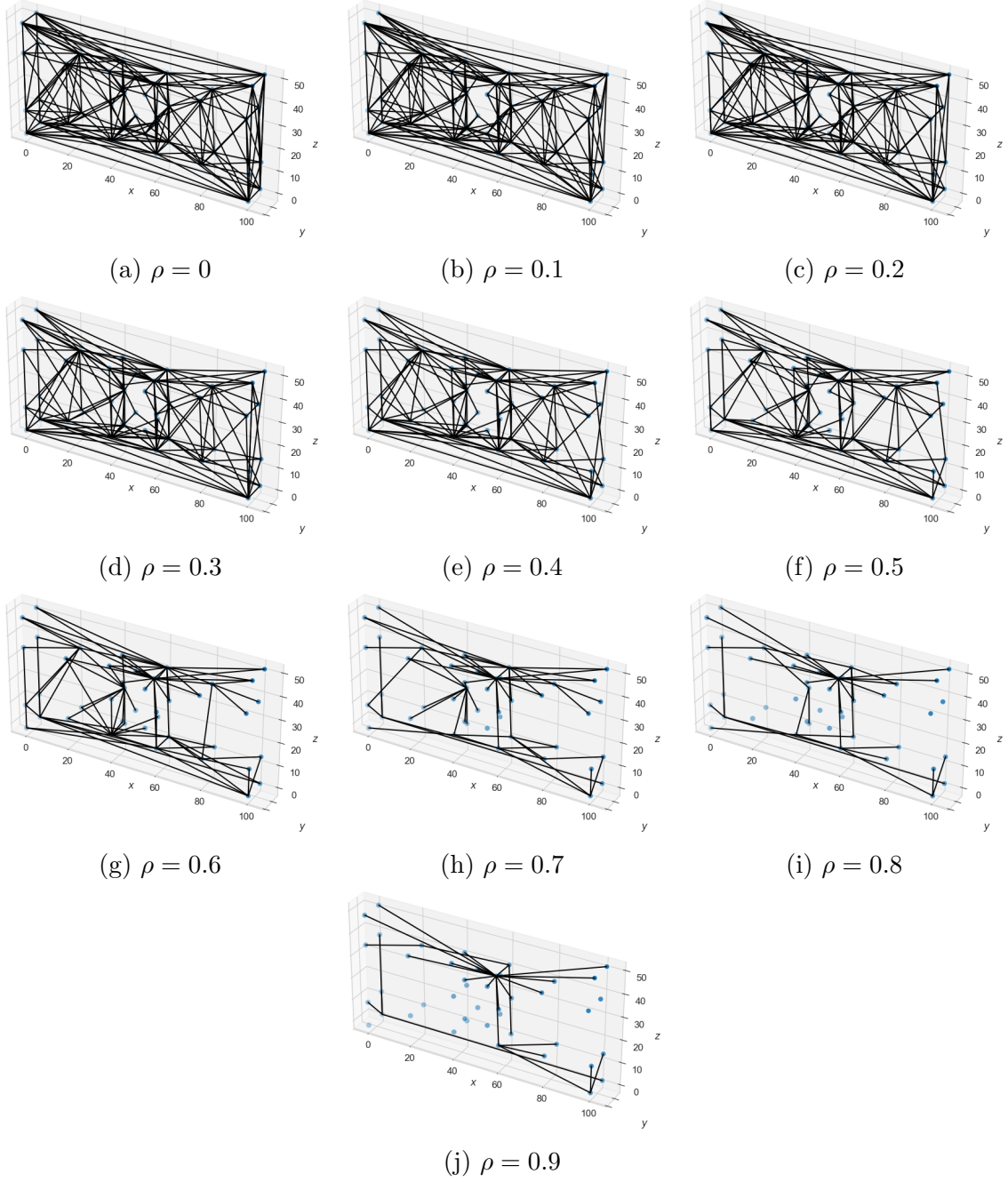


Figure 5.1.6: Sparsification results - design case G2LC1 using a Local Degree Sparsifier with varying prune rate ρ

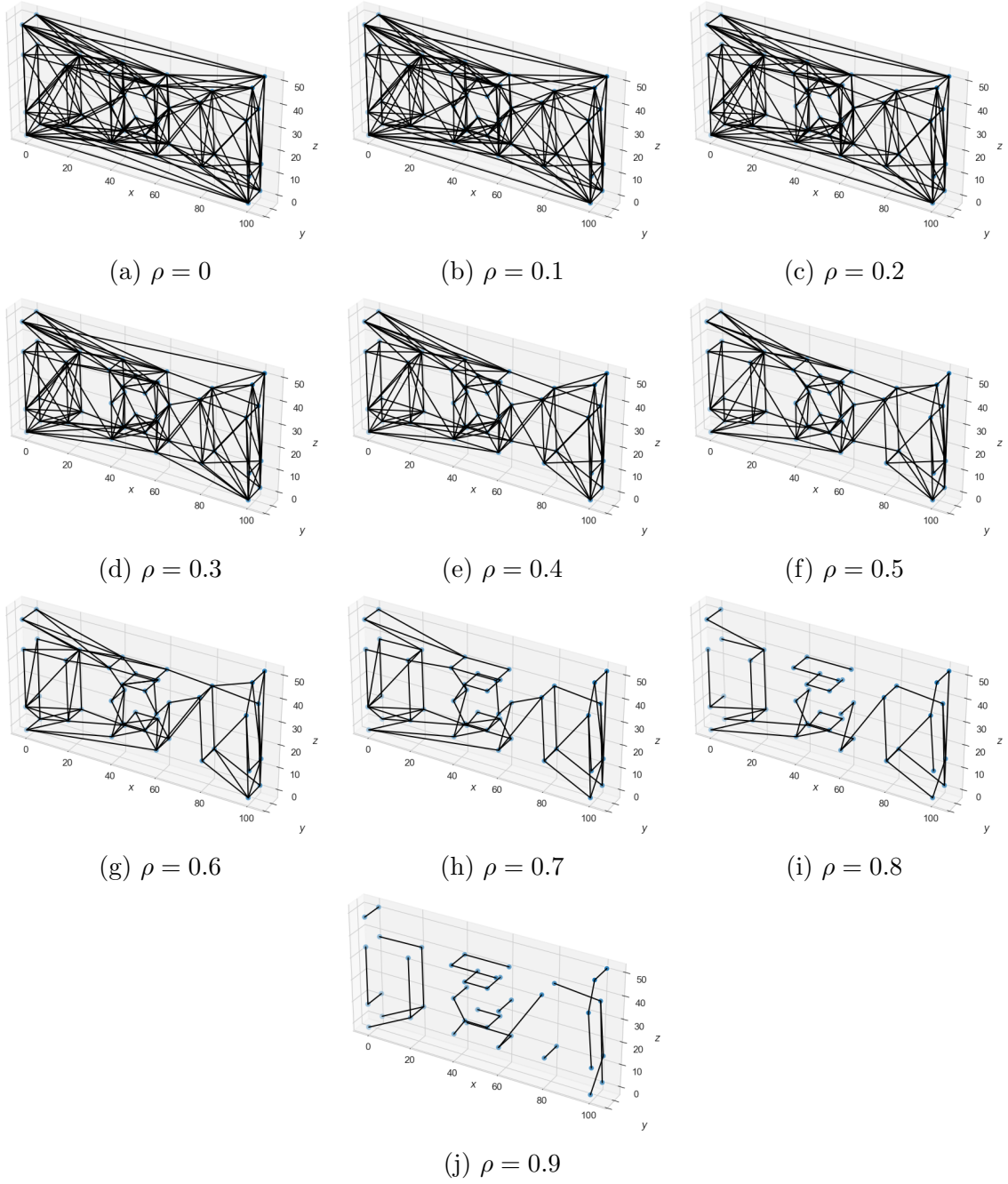


Figure 5.1.7: Sparsification results - design case G2LC1 using a Local Similarity Sparsifier with varying prune rate ρ

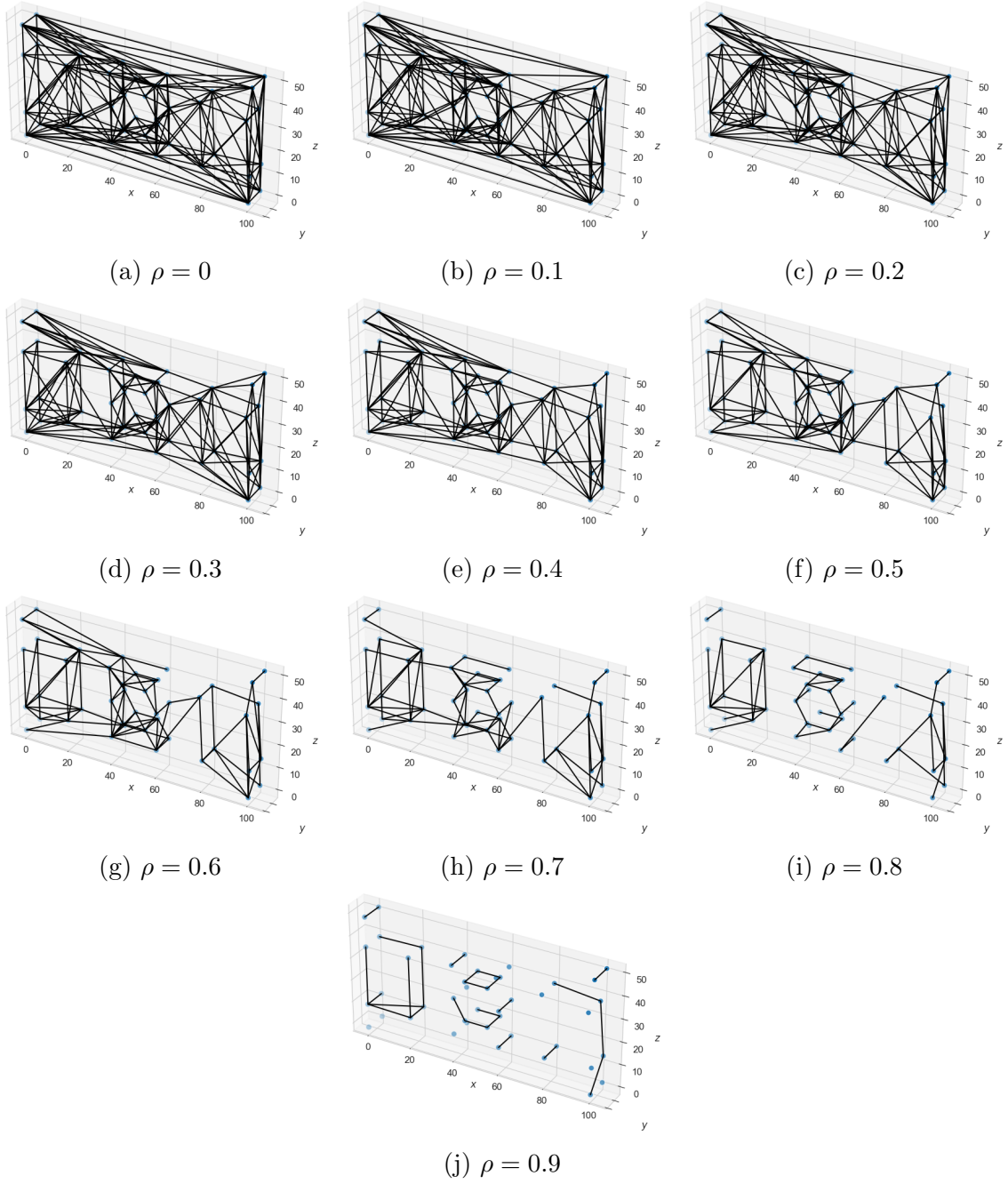


Figure 5.1.8: Sparsification results - design case G2LC1 using a Scan Similarity Sparsifier with varying prune rate ρ

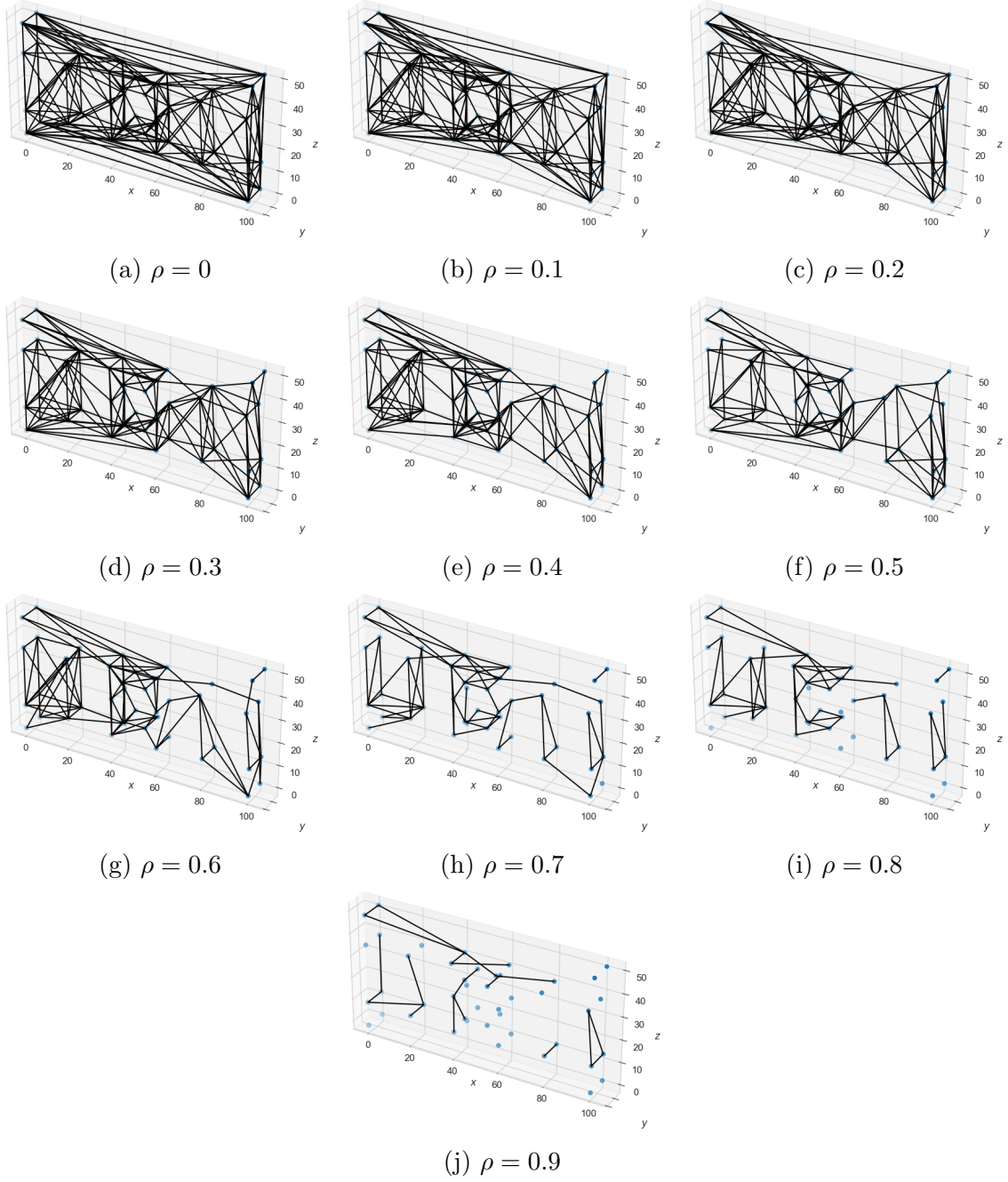


Figure 5.1.9: Sparsification results - design case G2LC1 using a Simmelian Backbones Spar-sifier with varying prune rate ρ

Chapter 6

Initial Structure Generation with k -neighbours Triangulation

In this chapter, the final methodology for step 4 of the sequential TO process is presented, along with results for all 13 design cases. In step 4, points along the stress paths as well as points where loads and boundary conditions are applied are used for generating triangulated interlaced dry yarn trusses. To this end, a k -neighbours triangulation algorithm was developed as described in this chapter. Here, edges are formed between a point and its k nearest neighbours, where number k is defined by the user.

6.1 Step 4 b): k -neighbours Triangulation of Stress Paths

In a step similar to the first one described for the generation of trusses with Delaunay triangulation, a first set of edges are constructed for defining surfaces where boundary conditions and loads are applied, and a second set of edges are constructed between sequential points on stress paths. Then for every point in the structure, k edges extending from the point to its k closest neighbours by Euclidean distance are created; in the process, no duplicate edges are produced. The number e of edges in a structure generated by k -neighbours triangulation can be determined using

$$e(k) = n_f(k + 1) + i(j(k + 1) - 1) \quad (6.1)$$

where n_f corresponds to the number of frozen vertices, k is the number of neighbours, i is the number of main stress paths and j is the number of points samples per stress path. Frozen vertices refer to vertices that define either surfaces where boundary conditions and loads are applied or geometric features such as holes. Their positions remain fixed during further steps of the TO framework. This is done to ensure that these features are preserved during the remainder of the optimization framework. Similarly to Delaunay triangulation, for a set number of main stress paths and points sampled per path, structure complexity increases linearly with the number of neighbours. In 3D Delaunay triangulation, the smallest internal solid angle in each tetrahedron created is maximized by the algorithm [75–77]. Due to this condition, edges in Delaunay triangulated structures tend to extend across large regions of the part envelope, resulting in edges occupying a larger fraction of the part envelope. This results in structures with very dense reinforcement clusters as illustrated in Chapter 5. This behaviour is not observed with k -neighbours triangulation as distance between edge end points is minimized. Figures 6.1.1-6.1.4 demonstrate that for each academic demonstrator, the average edge length in a structure generated using Delaunay triangulation is nearly

always greater than its equivalent for structures generated by k -neighbours triangulation, regardless of the k value or number of triangulation points used.

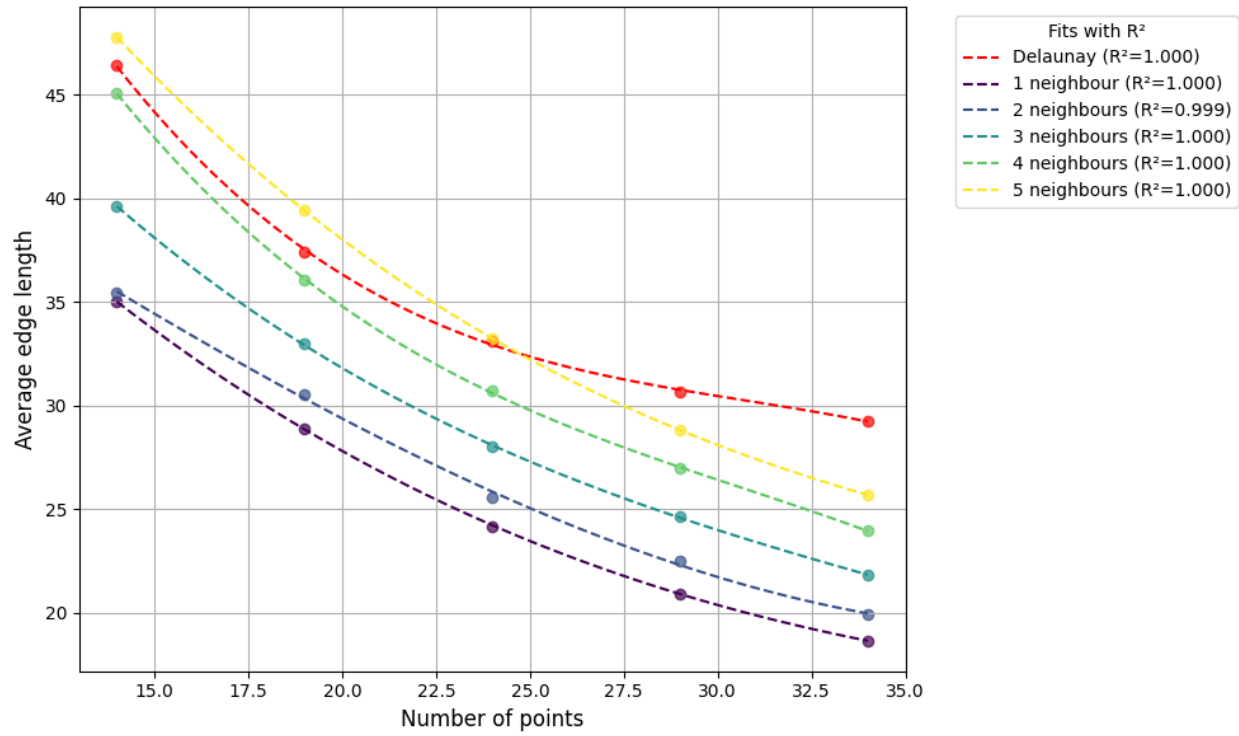


Figure 6.1.1: Average edge length as a function of number of triangulation points for Delaunay and k -neighbours triangulation - design case G1LC1

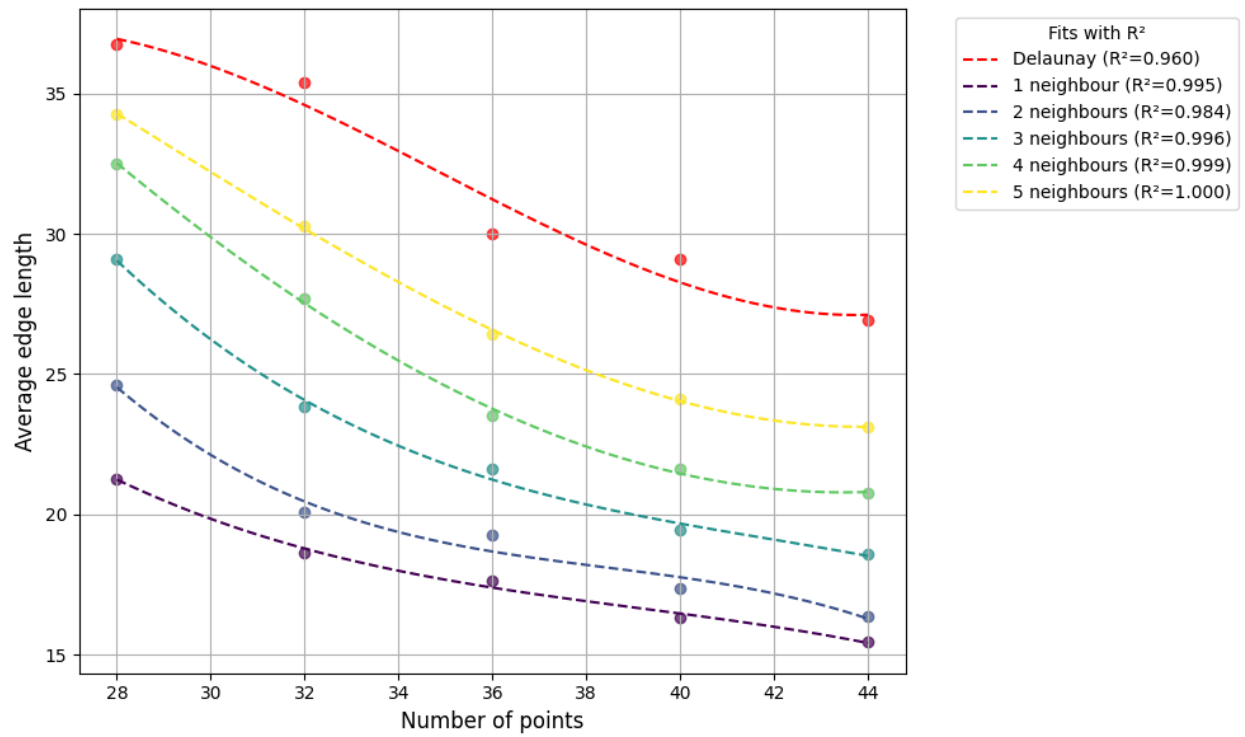


Figure 6.1.2: Average edge length as a function of number of triangulation points for Delaunay and k -neighbours triangulation - design case G2LC1

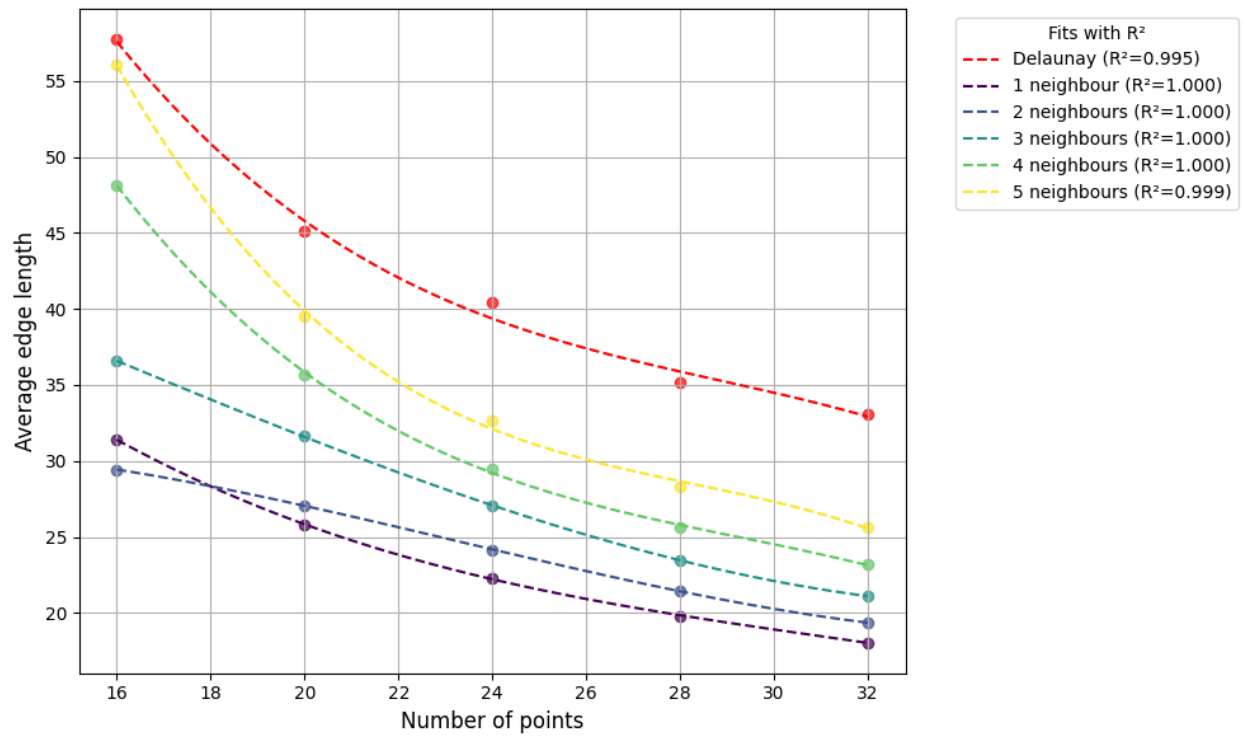


Figure 6.1.3: Average edge length as a function of number of triangulation points for Delaunay and k -neighbours triangulation - design case G3LC1

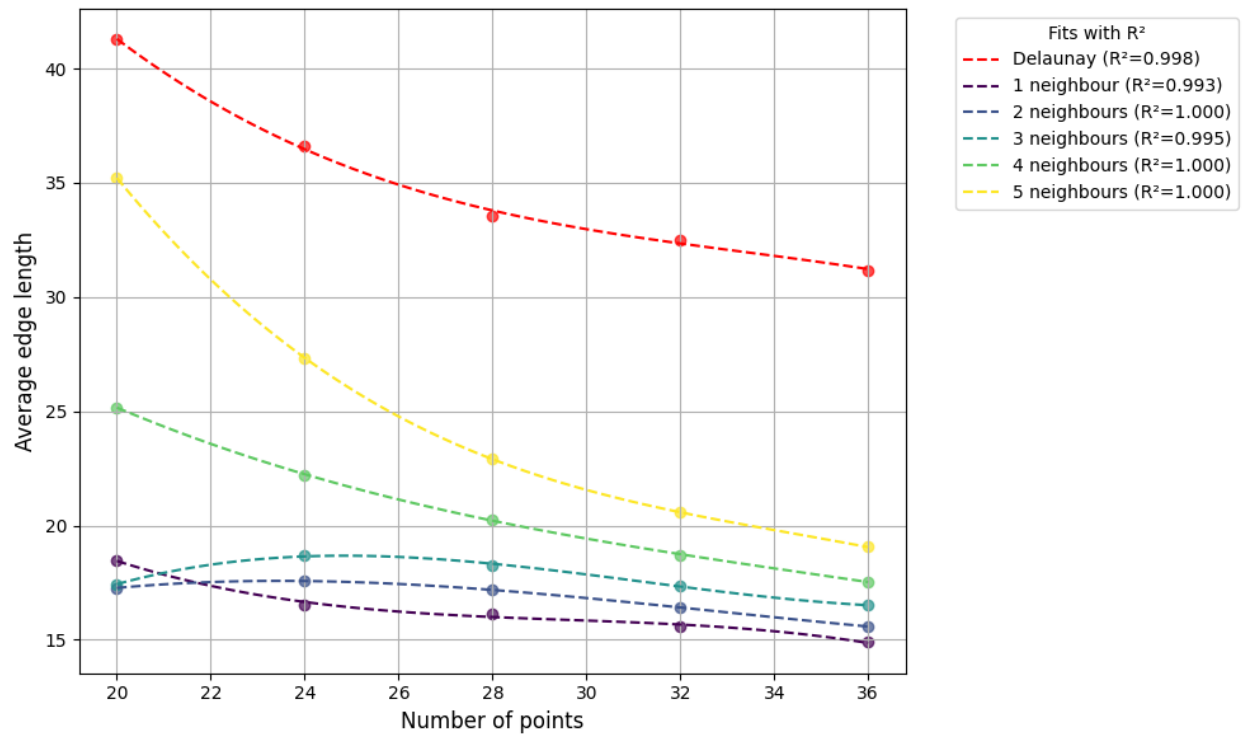


Figure 6.1.4: Average edge length as a function of number of triangulation points for Delaunay and k -neighbours triangulation - design case G4LC1

An additional metric was developed for providing more insight into the relationship between k values and edge clustering, with the aim of minimizing the latter. In order to guide the part designer in choosing more favourable values of k , a modified clustering coefficient was defined as follows:

$$C_{\text{modified}} = C \cdot \frac{|E|}{|V|} \quad (6.2)$$

where C is the clustering coefficient of the graph (Equation (5.2)), $|E|$ is the number of edges in the graph and $|V|$ is the number of vertices in the graph. The modified clustering coefficient scales clustering coefficient C by the ratio of the number of edges to the number of vertices in the graph. This scaling accounts for variations in graph size and density, providing a metric that reflects the structural properties of the graph in relation to its edge-to-vertex ratio. Plots of modified clustering coefficient as a function of total edge length were generated for each academic demonstrator, for Delaunay triangulation and k -neighbours triangulation with varying k values, Figures 6.1.5-6.1.8. Each point on a curve corresponds to a different number of points sampled per stress path, ranging from 2 to 6 and in all cases, the maximum number of main stress paths selected in Section 4.3 was used. For each design case, using a value of $k = 3$ and sampling 6 points per stress path for the triangulation produces structures that are less complex (lower clustering coefficient) than those obtained from Delaunay triangulation, all the while maximizing the cumulative edge distance. In other words, the total amount of reinforcement in the structure is maximized, while controlling structural complexity. In some cases such as G1LC1 or G2LC2, $k = 4$ neighbours also produces structures that are less complex than Delaunay triangulation. But in general $k = 3$ neighbours with 6 points per stress path are the parameters that shall be used for the generation of triangulated structures in following steps. These parameters were selected as a compromise between maximizing reinforcement coverage and maintaining manageable structural complexity, based on preliminary testing across the academic demonstrators. While

this compromise limits the total amount of reinforcement that can be applied, it significantly improves manufacturability by reducing geometric complexity and avoiding features that would be difficult or impractical to fabricate. The k -neighbours triangulation results for G2LC1 are shown in Figure 6.1.9, while results for all design cases are found in Appendix B.

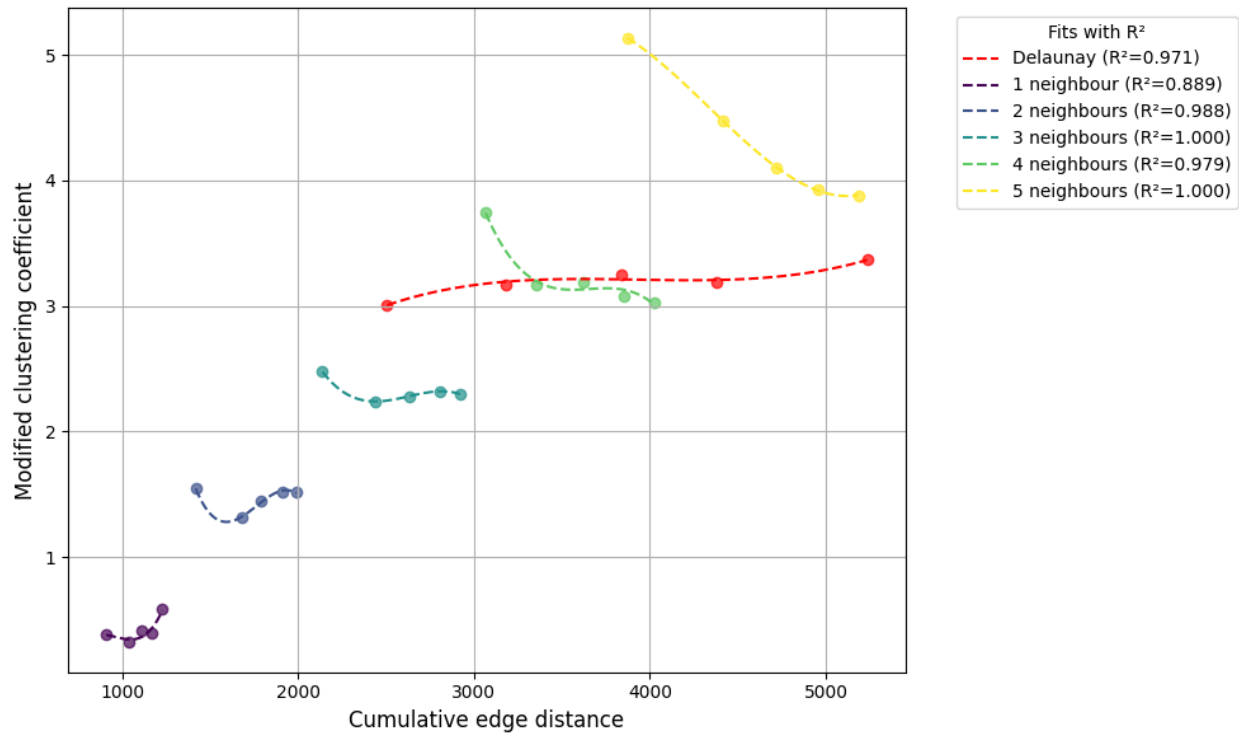


Figure 6.1.5: Modified clustering coefficient as a function of cumulative edge distance for Delaunay and k -neighbours triangulation - design case G1LC1

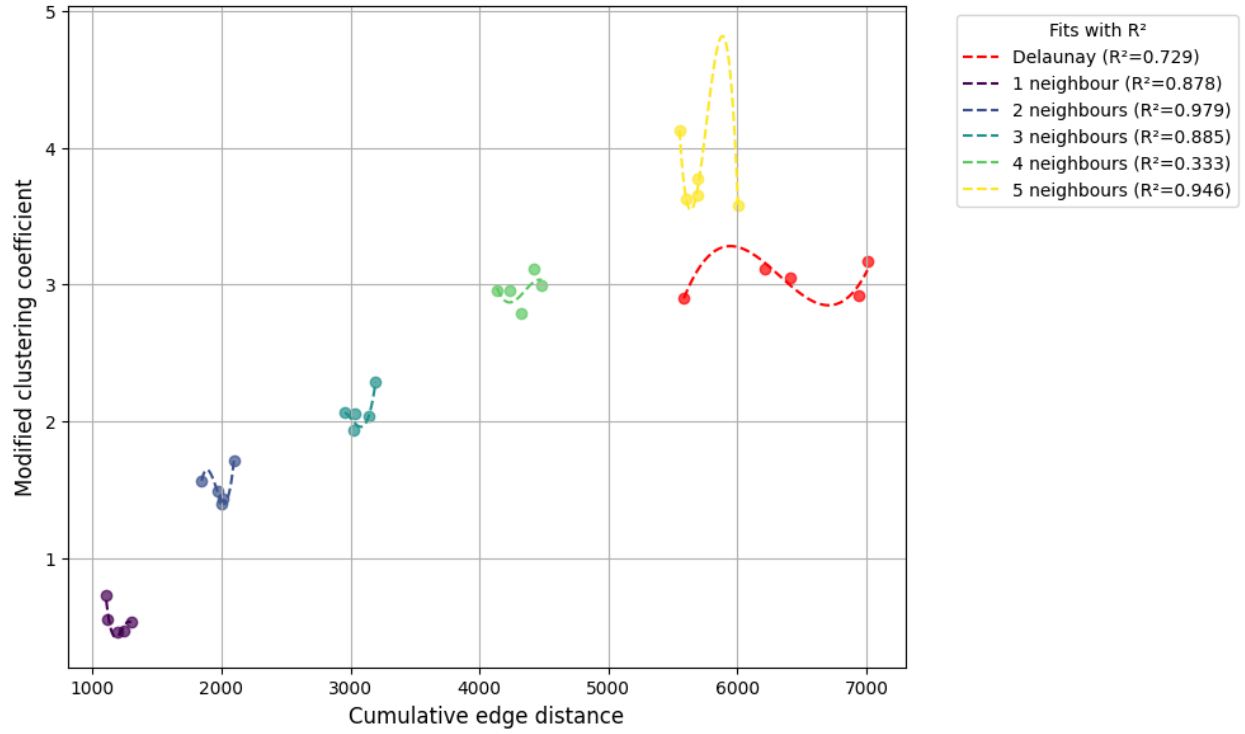


Figure 6.1.6: Modified clustering coefficient as a function of cumulative edge distance for Delaunay and k -neighbours triangulation - design case G2LC1

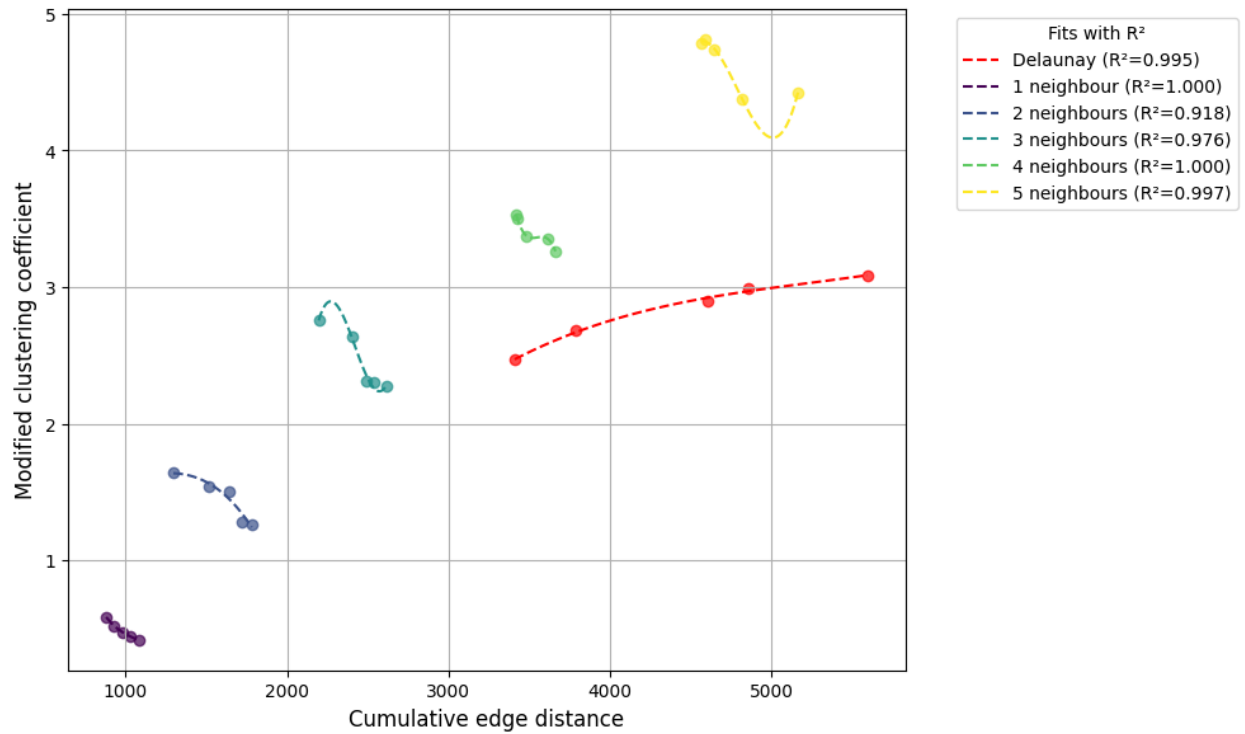


Figure 6.1.7: Modified clustering coefficient as a function of cumulative edge distance for Delaunay and k -neighbours triangulation - design case G3LC1

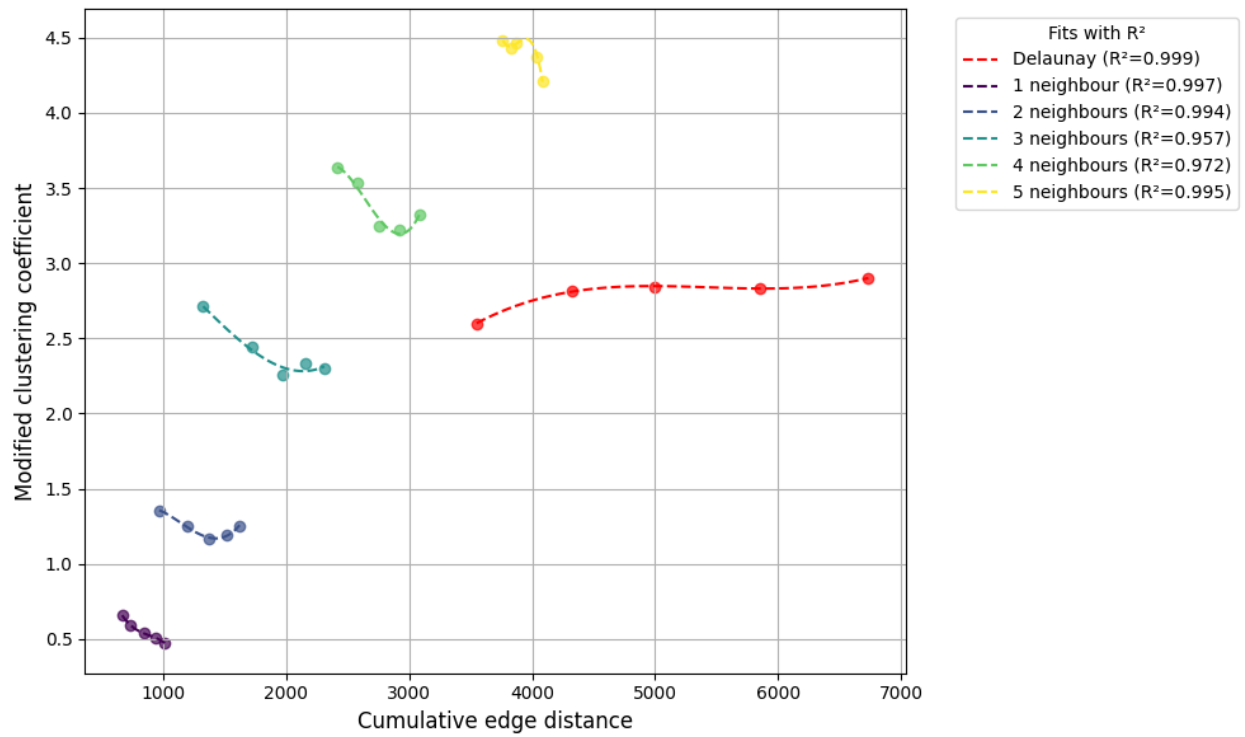
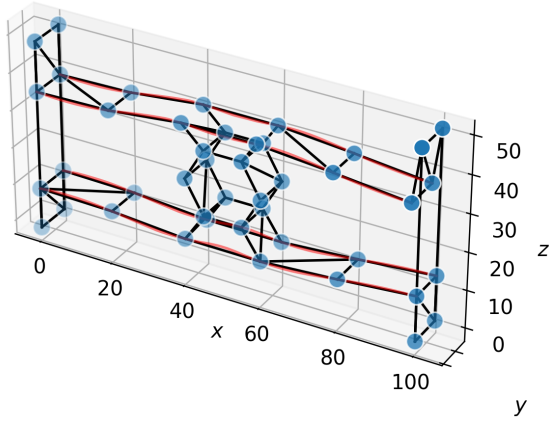
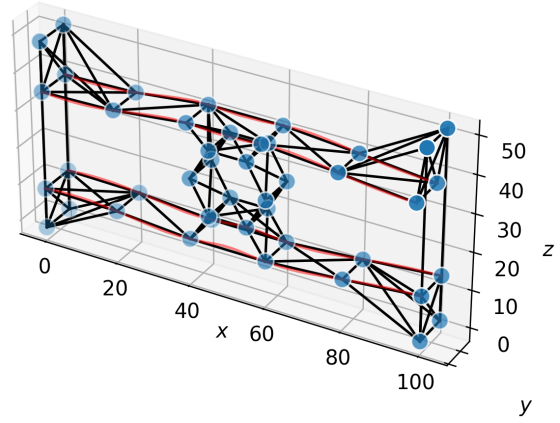


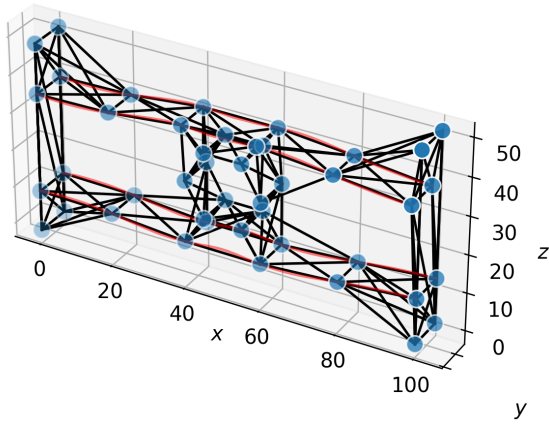
Figure 6.1.8: Modified clustering coefficient as a function of cumulative edge distance for Delaunay and k -neighbours triangulation - design case G4LC1



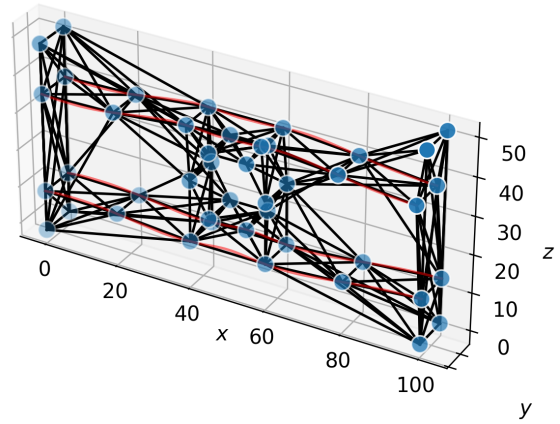
(a) $k = 1$ neighbour



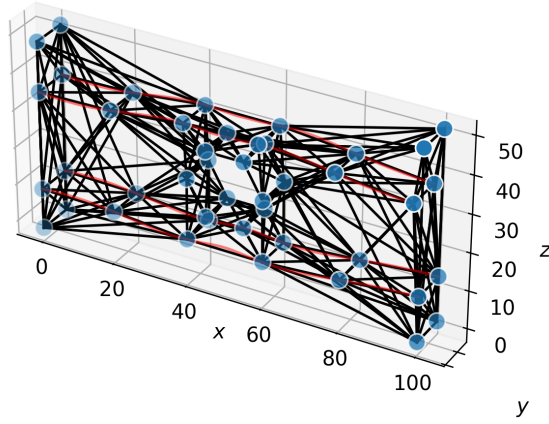
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours



(d) $k = 4$ neighbours



(e) $k = 5$ neighbours

Figure 6.1.9: k -neighbours triangulation - design case G2LC1 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red

Chapter 7

Triangulated Structure Optimization

In this chapter, the methodology for step 5 of the sequential TO process is presented along with results for all 13 design cases. In step 5, an optimization problem is set, where interlacing points in the triangulated structure are displaced in an effort to maximize alignment of yarns with the principal stress vector field. Symmetry penalties and hole penalties are introduced to increase structure symmetry where design cases are symmetrical, and to ensure that holes and other geometric entities are not obstructed by interlacing points as optimization proceeds.

7.1 Step 5: Structural Optimization

Let

$$\mathbf{x} = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n) \quad (7.1)$$

denote the vector of movable points within the dry interlaced structure, where each element $\mathbf{p}_i$ of the vector denotes the x, y, z coordinates of the i -th point. The coordinates of frozen vertices as defined in Chapter 6 are not included in $\mathbf{x}$ to ensure that they are not displaced during optimization; this is done to preserve critical geometric characteristics such as holes

and fillets or surfaces where boundary conditions and loads are applied. The following optimization problem can then be set:

$$\mathbf{min} f(\mathbf{x}) = -c_1 A + c_2 SP + c_3 HP \quad (7.2)$$

where

$$\begin{cases} A = \sum_{e \in E} \sigma(\mathbf{m}) |S_C(\mathbf{v}_e, \boldsymbol{\theta}(\mathbf{m}))| \\ SP = \sum_{\mathbf{p} \in \mathbf{x}} \sum_{\mathbf{s} \in \mathbf{S}} \|\text{Ref}_{\mathbf{p}}(\mathbf{s}) - \mathbf{q}\|^2 \\ HP = \sum_{e \in E} \sum_{\mathbf{h} \in \mathbf{H}} \frac{1}{\|\mathbf{m} - \mathbf{h}\|^2} \end{cases} \quad (7.3)$$

such that

$$\begin{cases} x_{\min} \leq \mathbf{p}_{\mathbf{n},1} \leq x_{\max} \\ y_{\min} \leq \mathbf{p}_{\mathbf{n},2} \leq y_{\max} \\ z_{\min} \leq \mathbf{p}_{\mathbf{n},3} \leq z_{\max} \end{cases} \quad (7.4)$$

where A is the alignment coefficient, SP is the symmetry coefficient, and HP is the hole penalty. $x_{\min}$, $x_{\max}$, $y_{\min}$, $y_{\max}$, $z_{\min}$ and $z_{\max}$ correspond to the bounds of the initial part volume. Alignment coefficient A aligns the edges of the graph representation of the structure with principal stress directions. Here, E is the set of all edges forming the graph representation of the structure; σ is the magnitude of the principal stress; $\mathbf{m}$ is the midpoint of an edge; S_C is the cosine similarity; $\mathbf{v}_e$ is the slope of edge e , and $\boldsymbol{\theta}$ is the local principal stress orientation vector. A compares the alignment of each edge in the graph with the local principal stress vector evaluated at the midpoint of the edge, scaling by the magnitude of the principal stress. This scaling ensures that the optimization favours edges that are aligned with regions of higher principal stress magnitude. The alignment coefficient is

the main metric used to measure the alignment of the structure with the principal stress field.

Symmetry coefficient SP promotes symmetry within the structure as points are being displaced during the optimization process. Here, $\mathbf{S}$ is the set of planes of symmetry within the part. $\text{Ref}_{\mathbf{p}}(\mathbf{s})$ is the reflection of point p about plane of symmetry $\mathbf{s}$, and $\mathbf{q}$ is the movable point in the structure that is closest to $\text{Ref}_{\mathbf{p}}(\mathbf{s})$. For each movable point within the structure, this term minimizes the distance between the reflection of the point being displaced and existing points that are near that reflection, promoting displacements that create or preserve existing symmetries within the structure.

Hole penalty term HP is included in the cost function to prevent movable points from obstructing holes or other geometric features that must be preserved. Here, $\mathbf{h}$ is the center of a hole and $\mathbf{H}$ is the set of hole centers for a part volume. Using an inverse square law, this term increases greatly when points are placed within holes, ensuring that holes are not obstructed during the optimization process.

Constants c_1 , c_2 , and c_3 are included to ensure dimensional consistency within the formulation. Generally, c_1 has units of L^2/F , c_2 has units of $1/\text{L}^2$, and c_3 has units of L^2 , where L and F denote units of length and force, respectively. For the optimization process, values of $c_1 = 1 \text{ mm}^2/\text{N}$, $c_2 = 1/\text{mm}^2$, and $c_3 = 1 \text{ mm}^2$ were assigned, consistent with the units used in the finite element simulations.

7.2 Optimization Results

The structural optimization problem was solved using the Broyden-Fletcher-Goldfarb-Shanno algorithm [84] for all 13 design cases. Optimized structures can be found in Appendix C. A summary of improvements in alignment coefficients post-optimization can be found in Table

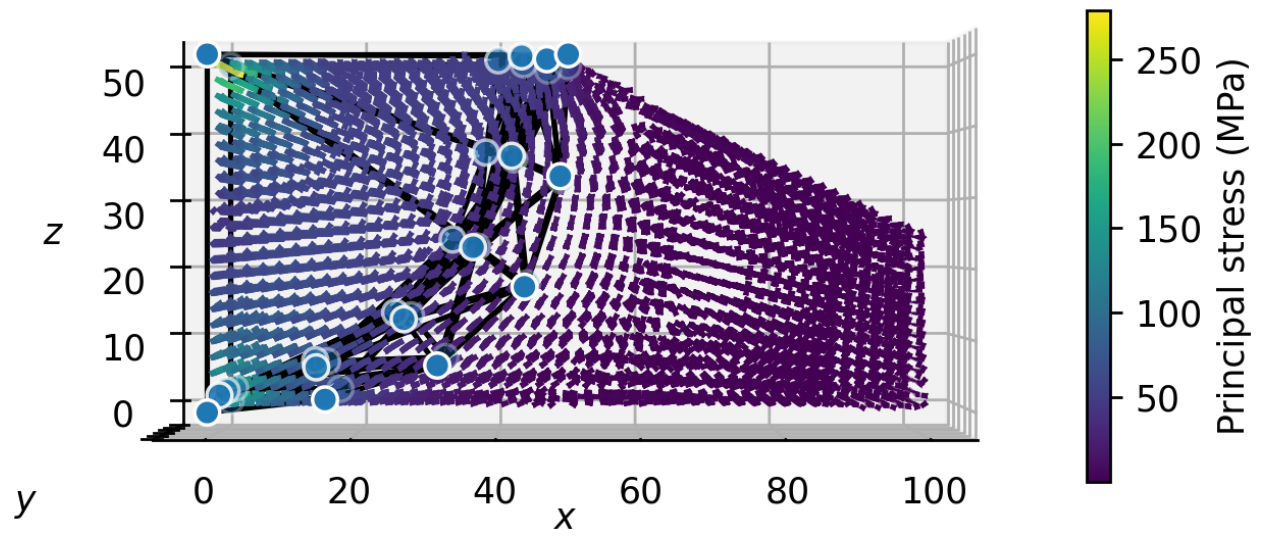
7.2.1. Results are mixed, ranging from 0.55 to 23.54% improvement in structural alignment with an average improvement of 11.1%.

Table 7.2.1: Summary of structural optimization results

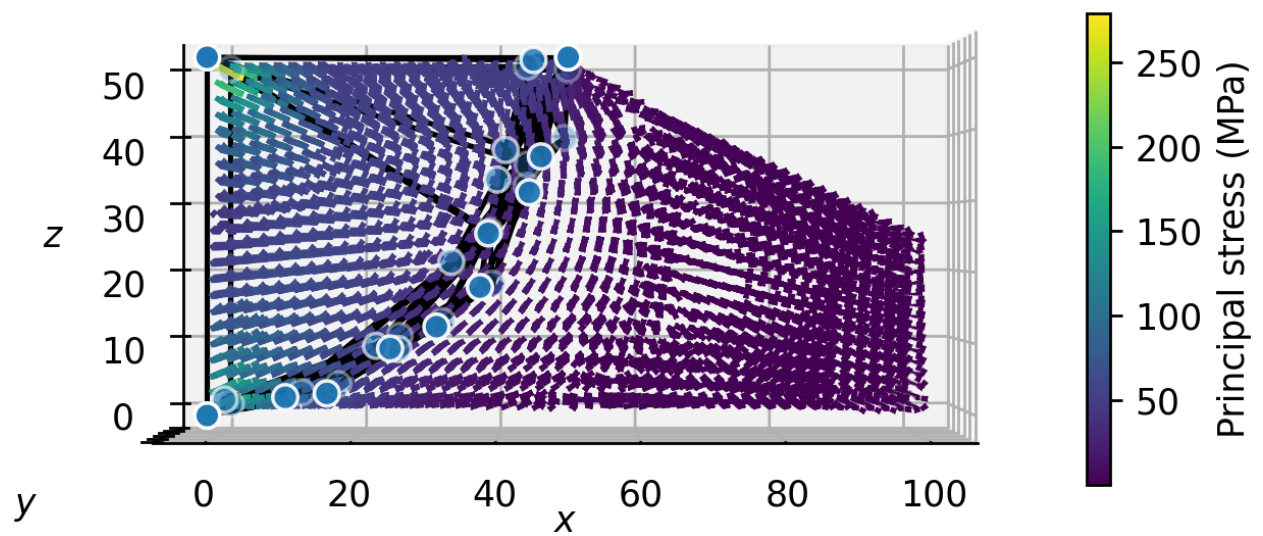
Design case	Alignment coefficient (MPa)		Percentage increase (%)
	Unoptimized	Optimized	
G1LC1	4 250	4 779	12.45
G2LC1	11 488	13 852	20.58
G3LC1	388	405	4.38
G4LC1	361	363	0.55
G5LC1	13 632	14 714	7.94
G5LC2	21 076	23 773	12.80
G5LC3	7 000	7 516	7.37
G6LC1	25 789	30 907	19.85
G6LC2	6 087	6 792	11.58
G6LC3	1 309	1 497	14.36
G7LC1	1 253	1 548	23.54
G7LC2	1 630	1 712	5.03
G7LC3	1 701	1 759	3.41

Optimization results for G1LC1 and G2LC1, Figures 7.2.1 and 7.2.2, illustrate that point displacement predominantly occurs toward regions of higher principal stress, leading to increases in principal stress alignment of 12.45% and 20,58% for design cases G1LC1 and G2LC1 respectively. This alignment with the stress field suggests that the optimized designs are mechanically efficient, as material is guided to areas where it is most needed to resist loading. Furthermore, both structures mostly maintain symmetry about their respective planes, demonstrating the effectiveness of the symmetry coefficient. Contrarily, certain design cases such as G3LC1 exhibit limited point displacement, leading to optimization results that are nearly identical to their pre-optimized structures, Figure 7.2.3. In this instance, the structure failed to reorient itself meaningfully toward regions of higher principal stress, resulting in a marginal improvement in principal alignment of 4.38%. Certain design cases such as G6LC1 exhibited significant improvements in principal stress alignment but did not produce geometries that appear structurally sound, Figure 7.2.4. Although G6LC1 achieved

a 19.85% increase in principal stress alignment, the resulting structure does not intuitively resemble an optimized part, with many edges extending outside the initial volume. This outcome highlights the potential need for stronger or more effective volume constraints within the optimization process to ensure that structural integrity and manufacturability are preserved. To properly assess the variability in optimization performance, a sensitivity analysis was performed that is presented in the following section.

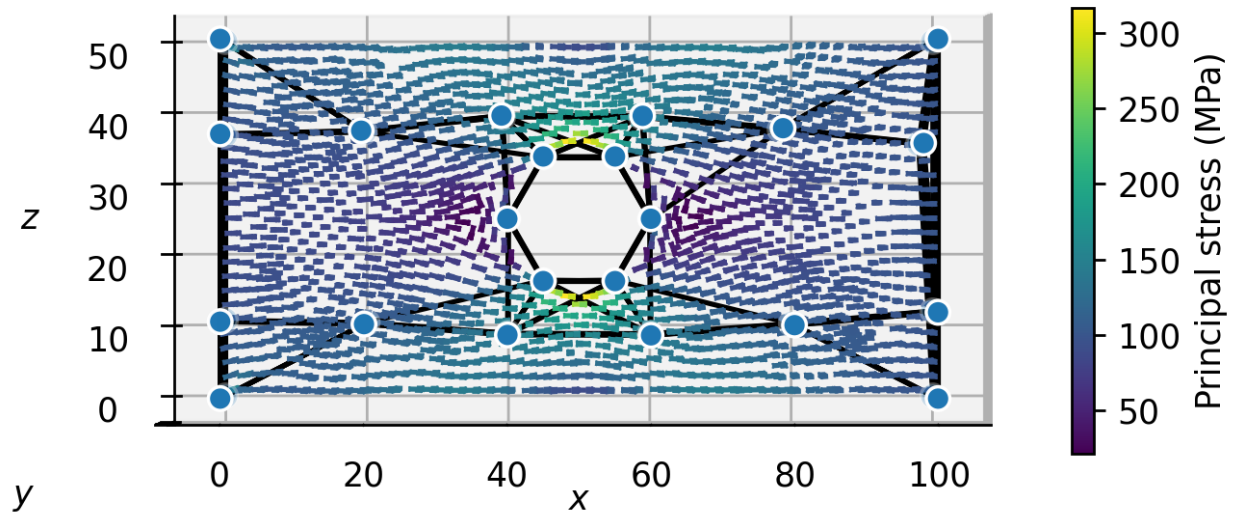


(a) Unoptimized side view

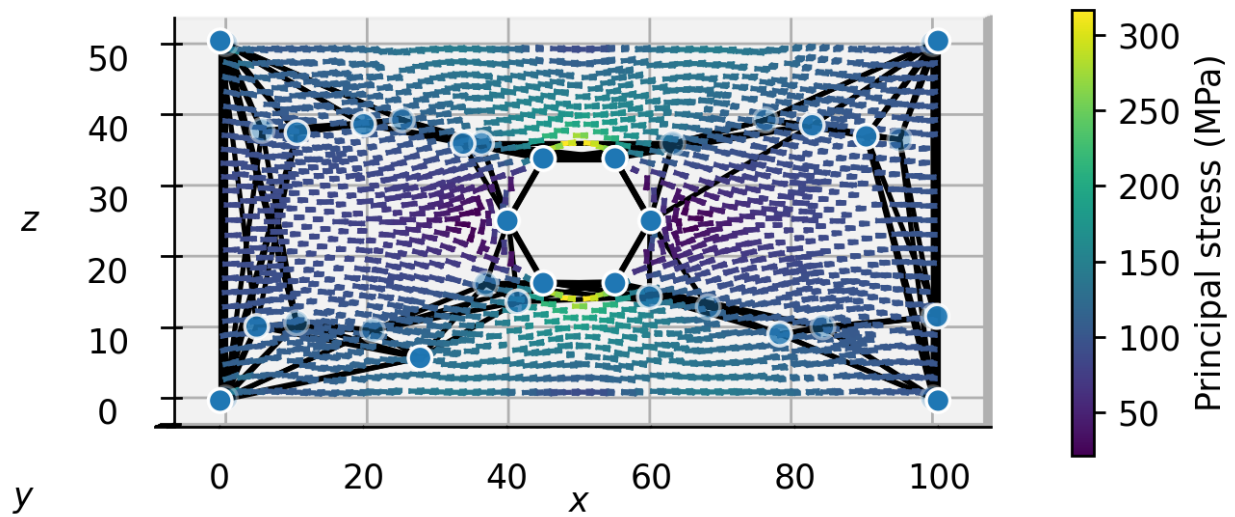


(b) Optimized side view

Figure 7.2.1: Optimized structure alignment compared to principal stress field - design case G1LC1

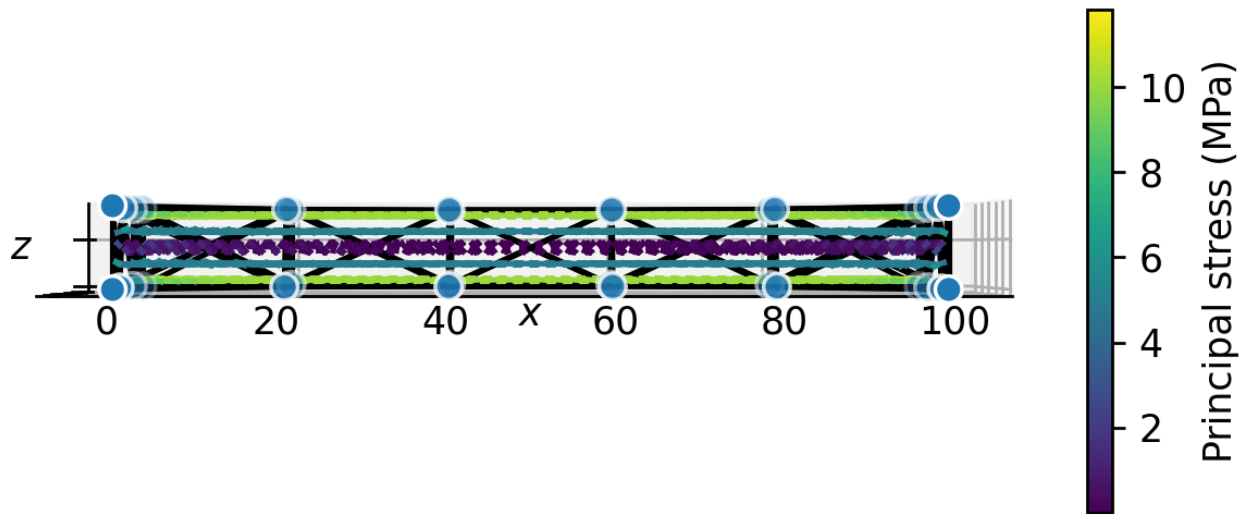


(a) Unoptimized side view

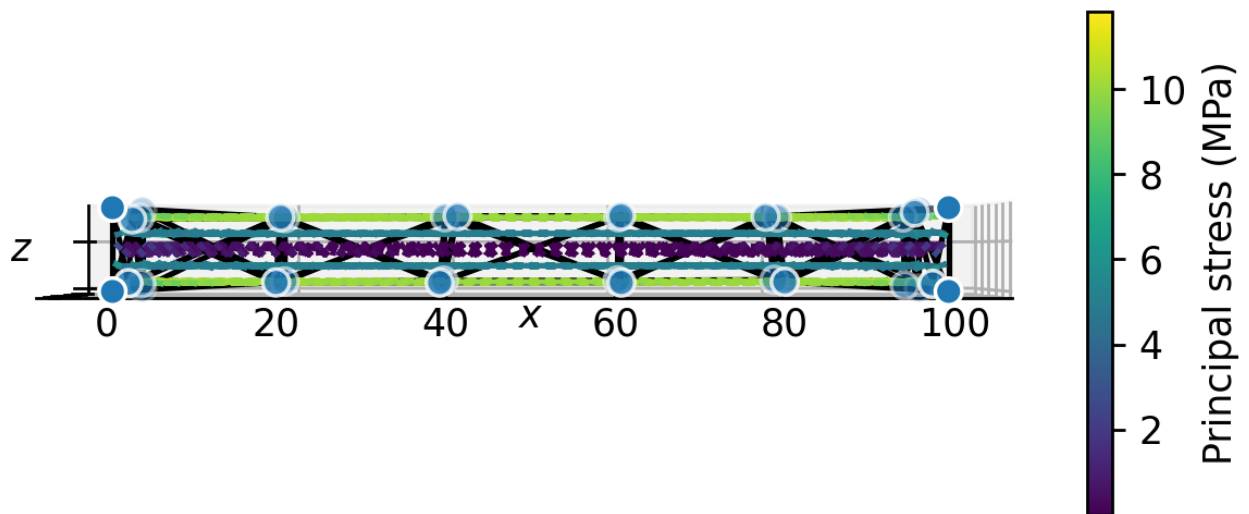


(b) Optimized side view

Figure 7.2.2: Optimized structure alignment compared to principal stress field - design case G2LC1

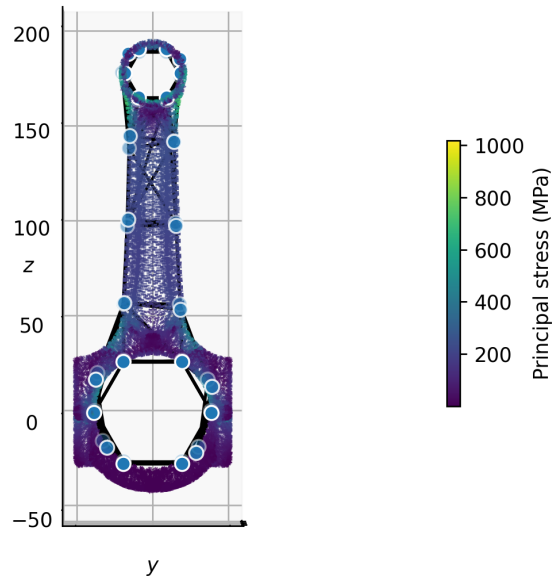


(a) Unoptimized side view

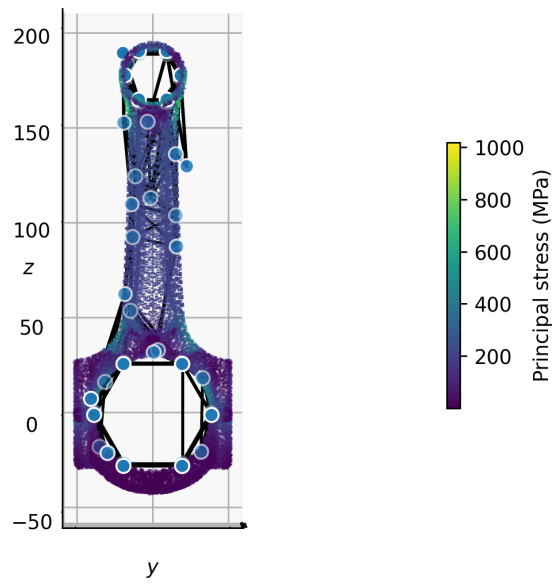


(b) Optimized side view

Figure 7.2.3: Optimized structure alignment compared to principal stress field - design case G3LC1



(a) Unoptimized side view



(b) Optimized side view

Figure 7.2.4: Optimized structure alignment compared to principal stress field - design case G6LC1

7.3 Sensitivity Analysis

Variability in optimization performance can be explained by considering the effects of alignment and symmetry coefficients on the cost function. In fact, evaluating alignment and symmetry coefficients of initial structures gives insights into whether the optimization process is primarily driven by structural alignment or symmetry, Table 7.3.1. If the alignment coefficient is of higher magnitude than the symmetry coefficient, the minimization of the cost function will be significantly more sensitive to variations in alignment coefficient, and vice-versa for the symmetry coefficient. Table 7.3.1 shows that only 5 design cases are driven primarily by the alignment coefficient: G1LC1, G2LC1, as well as cases G5LC1-G5LC3, since they do not possess any symmetry. Contrarily, G4LC1 and G6LC1-G7LC3 are primarily driven by the symmetry coefficient of the cost function.

Table 7.3.1: Initial alignment and symmetry coefficients

Design case	Alignment coefficient (MPa)	Symmetry coefficient (mm ²)	Ratio
G1LC1	4 250	306	13.89
G2LC1	11 488	1 076	10.67
G3LC1	388	787	0.493
G4LC1	361	1 530 237	0.000236
G5LC1	13 632	0	N/A
G5LC2	21 076	0	N/A
G5LC3	7 000	0	N/A
G6LC1	25 789	176 767	0.146
G6LC2	6 087	176 820	0.0344
G6LC3	1 309	113 042	0.0116
G7LC1	1 253	276 597	0.00453
G7LC2	1 630	277 171	0.00588
G7LC3	1 701	277 422	0.00621

To determine the sensitivity of the optimization process to the position of each vertex in the structure, the gradients of the alignment and symmetry coefficients were calculated as follows:

$$\begin{cases} \nabla A = \frac{\partial A}{\partial x} \hat{\mathbf{i}} + \frac{\partial A}{\partial y} \hat{\mathbf{j}} + \frac{\partial A}{\partial z} \hat{\mathbf{k}} \\ \nabla SP = \frac{\partial SP}{\partial x} \hat{\mathbf{i}} + \frac{\partial SP}{\partial y} \hat{\mathbf{j}} + \frac{\partial SP}{\partial z} \hat{\mathbf{k}} \end{cases} \quad (7.5)$$

Magnitudes of the gradients at each vertex were approximated by finite differences then normalized between 0 and 1 to construct sensitivity maps for the alignment and symmetry coefficients, Figures 7.4.1-7.4.13. Sensitivity maps for all design cases can be found in Appendix D. These maps indicate which vertices are important drivers in the optimization process. For a given vertex, a gradient of higher magnitude corresponds to the optimization process being more sensitive to the displacement of that vertex, and vice-versa. A vertex that has a normalized gradient higher than the mean normalized gradient is labelled a driver vertex, meaning it drives the optimization process in a significant way.

Observing design cases where optimization processes are primarily driven by the alignment coefficient, such as G1LC1 and G2LC1, it can be seen that having low alignment driving vertex densities leads to higher increases in alignment post-optimization. This behaviour occurs because the structure is generally less sensitive to individual vertex displacements. As a result, the algorithm can perform local displacements to improve alignment without significantly disturbing the orientation of neighbouring edges, allowing the structure to adapt more easily in a clear, coordinated direction. In contrast, when the structure is more sensitive to individual vertex displacements, vertex movements become constrained since even small displacements cause large fluctuations in overall alignment of the structure. This leads to a phenomenon referred to as "structural gridlock," where points in the structure are unable to undergo displacement, leading to marginal gains in principal stress alignment. Structural gridlock is most clearly observed in Figure 7.3.3, which feature the highest density of driving

vertices among the design cases analyzed, which also produced one of the worst optimization results with a 4.38% alignment improvement.

For design cases where the optimization process is primarily driven by the symmetry coefficient, a second condition must be satisfied to achieve significant improvements in alignment: vertices that contribute to symmetry optimization must also influence alignment. In Figure 7.3.4 a), two vertices have normalized alignment coefficient gradient magnitudes of 1, meaning that they drive the alignment optimization significantly. In Figure 7.3.4 b), the same vertices show normalized symmetry coefficient gradient magnitudes around 0.5, meaning that they are still moderately sensitive to symmetry optimization. Under normal conditions for design cases that are driven by the symmetry coefficient, alignment driving vertices would not be displaced significantly, since their influence on the dominant symmetry coefficient is minimal. However, because these vertices also contribute meaningfully to the symmetry optimization, they are displaced during the process resulting in a substantial 19.85% increase in alignment, even though optimization is driven by the symmetry coefficient. This example highlights the need to scale the alignment and symmetry terms of the cost function for each design case based on the initial alignment-to-symmetry coefficient ratios, in order to achieve more consistent optimization results.

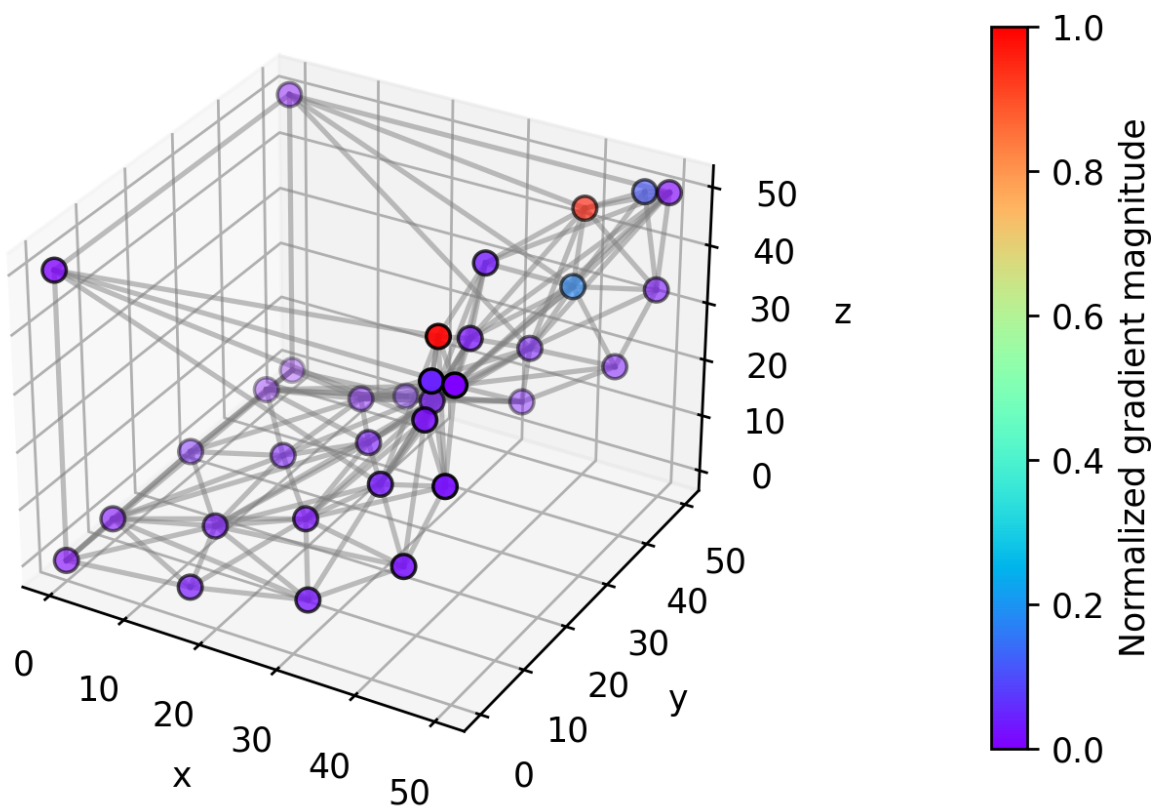


Figure 7.3.1: Alignment coefficient sensitivity map - design case G1LC1

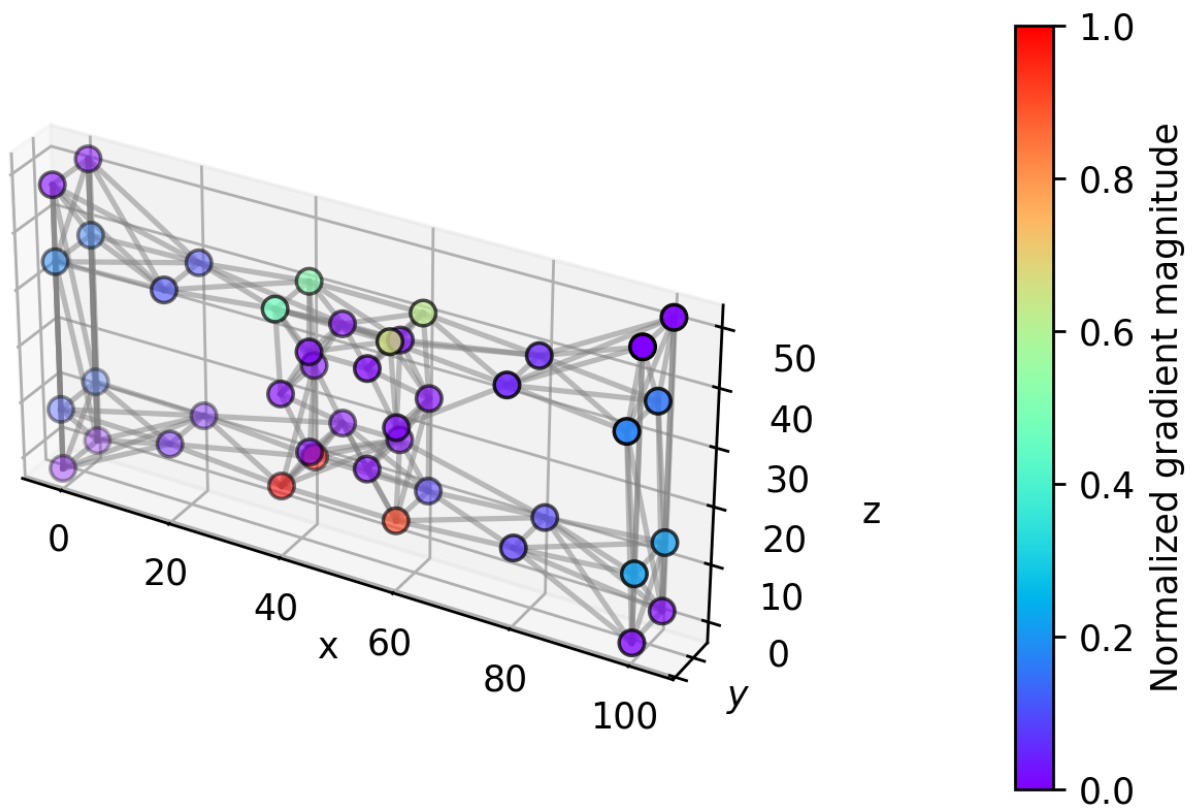


Figure 7.3.2: Alignment coefficient sensitivity map - design case G2LC1

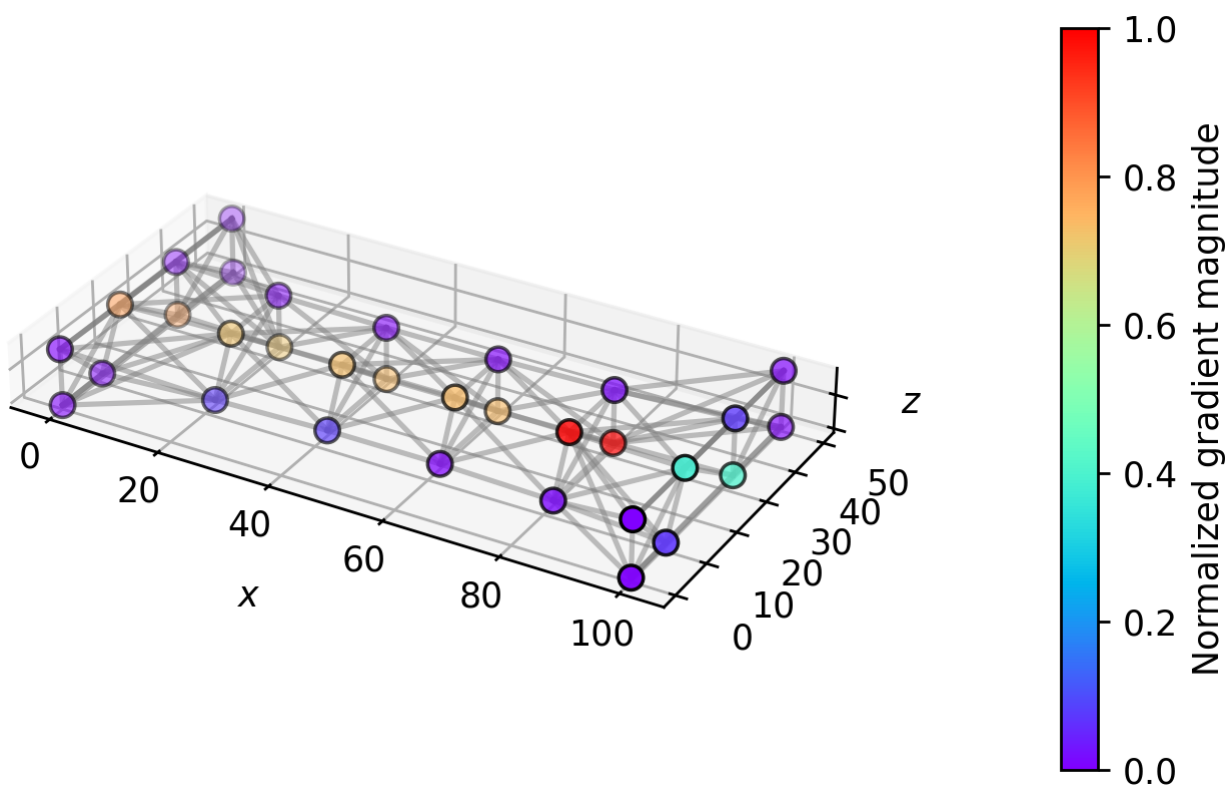
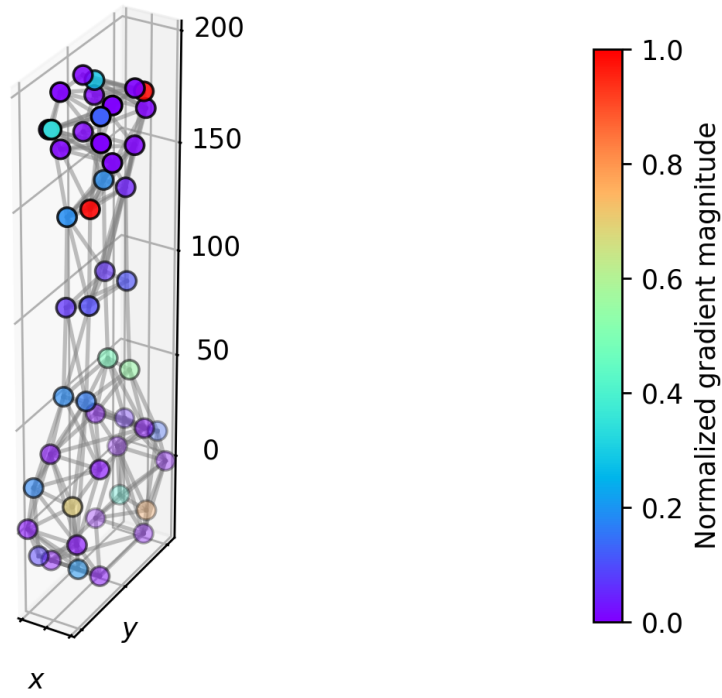
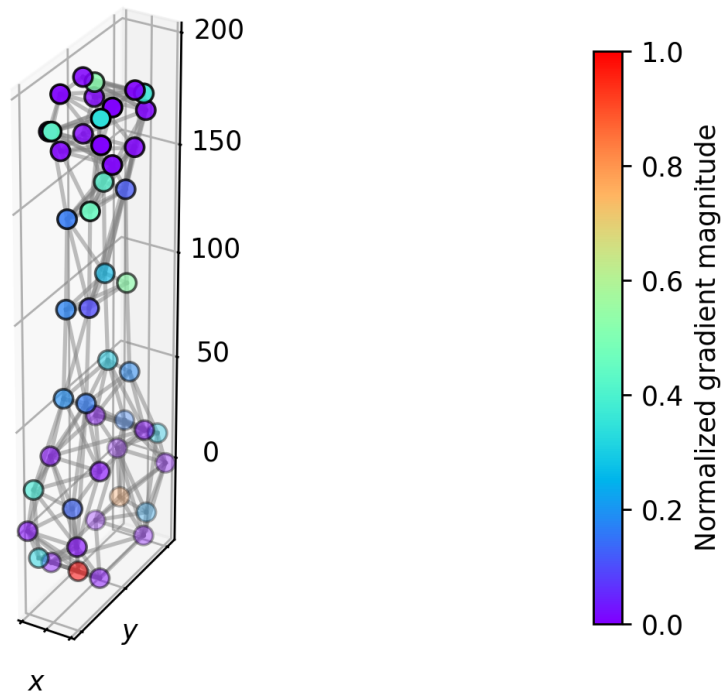


Figure 7.3.3: Alignment coefficient sensitivity map - design case G3LC1



(a) Alignment coefficient sensitivity map



(b) Symmetry coefficient sensitivity map

Figure 7.3.4: Sensitivity maps - design case G6LC1

Chapter 8

Yarn Path Generation

In this chapter, the methodology for the sixth and penultimate step of the sequential TO process is presented, along with results for all 13 design cases. In step 6, yarn paths that cover all edges of the dry interlaced structure are generated in a controlled, efficient manner. Using a distance prioritization scheme along with Dijkstra’s algorithm [85], it is possible to generate sets of yarns that allow the formation of the dry structure once interlaced, while minimizing the amount of yarns used.

8.1 Step 6: Yarn Path Generation

Yarn paths correspond to sequences of positions that individual yarns go through, that enable the formation of the dry structure when they are interlaced. The biggest challenge in generating sets of paths that cover all yarns in the structure is the very large number of possible combinations, which increases tremendously with the number of edges in a graph. The primary goal of step 6 is to generate a set of yarn paths that cover all edges of the graph in a controlled, efficient manner. Two axioms were established for generating allowable yarn paths. Firstly, yarns must originate and end at points on the part envelope, to ensure that the positions of these points are fixed and the yarns can be tensioned adequately. Secondly,

different yarns can overlap over a given edge within the structure.

The edges of the graph are first sorted in decreasing order of distance measured from the edge midpoint to the closest envelope vertex. For a given edge, the closest envelope node to each of both edge endpoints is identified, Figure 8.1.1 b). For the two endpoints of each edge, Dijkstra’s algorithm [85] is executed with the envelope vertex to which it is closest, determining the shortest path from each end point to reach an envelope vertex, Figure 8.1.1 c). At this stage, the construction of yarn paths begins. Each path is progressively formed by merging the original edge with the two shortest paths reaching the envelope, creating an allowable yarn path, Figure 8.1.1 d). The following distance function is used for Dijkstra’s algorithm:

$$D(u, v, n) = d(u, v) + 10^5 H(n - 1) \quad (8.1)$$

where d is the Euclidean distance, u is the candidate edge midpoint, v is the target envelope vertex, H is the Heaviside function, and n is the number of times the candidate edge is already featured in the set of constructed yarn paths. The Euclidean distance term serves to minimize the distance between the path being formed and the envelope vertex that is being targeted, while the Heaviside function serves as an overlap penalty. The penalty magnitude of 10^5 units was chosen to ensure that reused edges are strongly penalized relative to typical Euclidean distances (1–100 units), effectively discouraging overlap unless absolutely necessary. Functionally, this minimizes the mass of the structure as it favours the generation of yarn paths that feature fewer edges, as well as reducing the number of overlaps between yarn paths. Once a yarn path is formed, each edge it contains is marked as ‘used’, meaning that penalties are applied during the execution of Dijkstra’s algorithm by means of the Heaviside function if it appears in subsequent paths, promoting the use of edges that are not currently covered by yarns. This process is repeated until every edge in the structure is covered.

The example in Figure 8.1.1 features exaggerated levels of triangulation to better illustrate the yarn path generation process. An edge count map demonstrating the number of yarn paths that cross an edge in a graph, or the edge usage count, can be found in Figure 8.1.2 for G1LC1. The edge count map shows that each edge in the structure is covered by a yarn, and that edge overlap between yarns is minimized since most edges feature a single yarn path. Edge count maps for each design case are found in Appendix E.

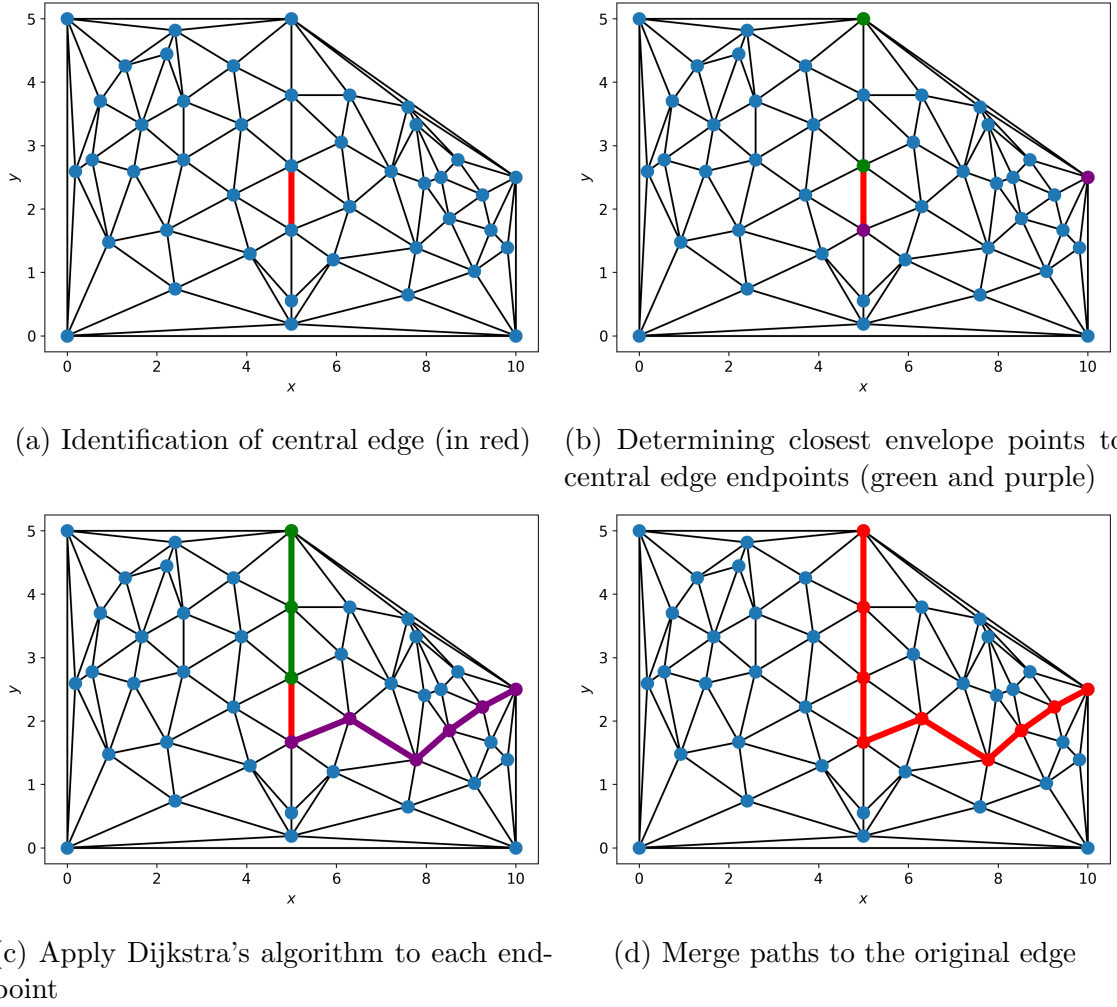


Figure 8.1.1: Sequential merging process used to generate an allowable yarn path

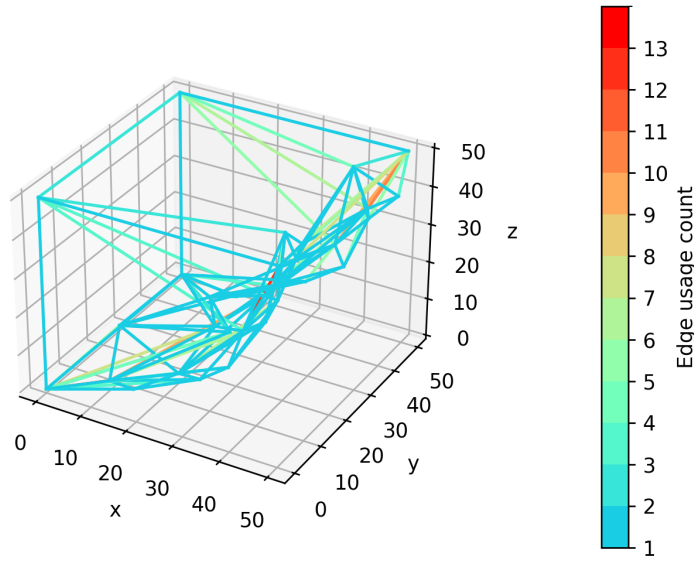


Figure 8.1.2: Edge count map - design case G1LC1

8.2 Yarn Path Analysis

As shown by the edge count maps in Appendix E, it is apparent that in all cases most edges feature little to no overlap between yarn paths. This is confirmed by analyzing edge counts for each design case, Figure 8.2.1. In fact, an edge count of one features a staggering 86.38% of the time, demonstrating the effectiveness of the algorithm in preventing edge overlap between yarn paths.

This results in yarn paths that are relatively short in terms of edge count, as shown in Figure 8.2.2. In fact, 38.31% of the generated yarn paths are "trivial paths" consisting of only a single edge, 24.87% consist of two edges, and 20.86% consist of three edges. This could lead to an unnecessarily high number of yarn paths for a given design case, which in turn complicates the interlacing sequence required to produce the structure. This highlights the potential benefit of an overlap penalty that scales with the number of overlaps between yarn paths, as opposed to a flat penalty. The flat penalty causes most edges to become

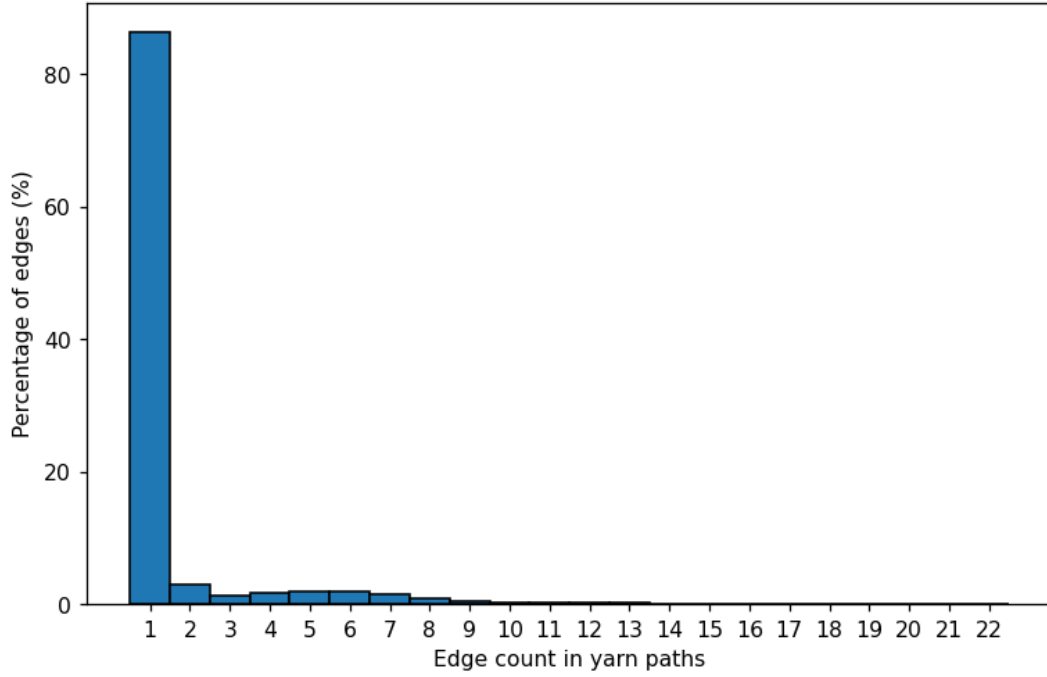


Figure 8.2.1: Histogram of edge usage count for all 13 design cases

effectively unusable by the algorithm once a critical number of yarn paths are generated, since each edge is used only once. As a result, the overlap penalty is applied to most edges in the structure, preventing many envelope edges from being incorporated into the yarn paths. Consequently, paths with only a single edge are generated. This is best illustrated by Figure 8.2.3, which shows that the average number of edges per yarn path decreases as more yarn paths are generated across all 13 design cases. While the algorithm initially produces longer yarn paths, the average path length decreases as more paths are created to minimize overlaps. Figures 8.2.4 and 8.2.5 display the first four and last four yarn paths generated for G4LC1, highlighting the change in path lengths as the algorithm progresses. Implementing a scaling overlap penalty would prevent this issue, encouraging the growth of longer yarn paths and enhancing the efficiency of the interlacing patterns.

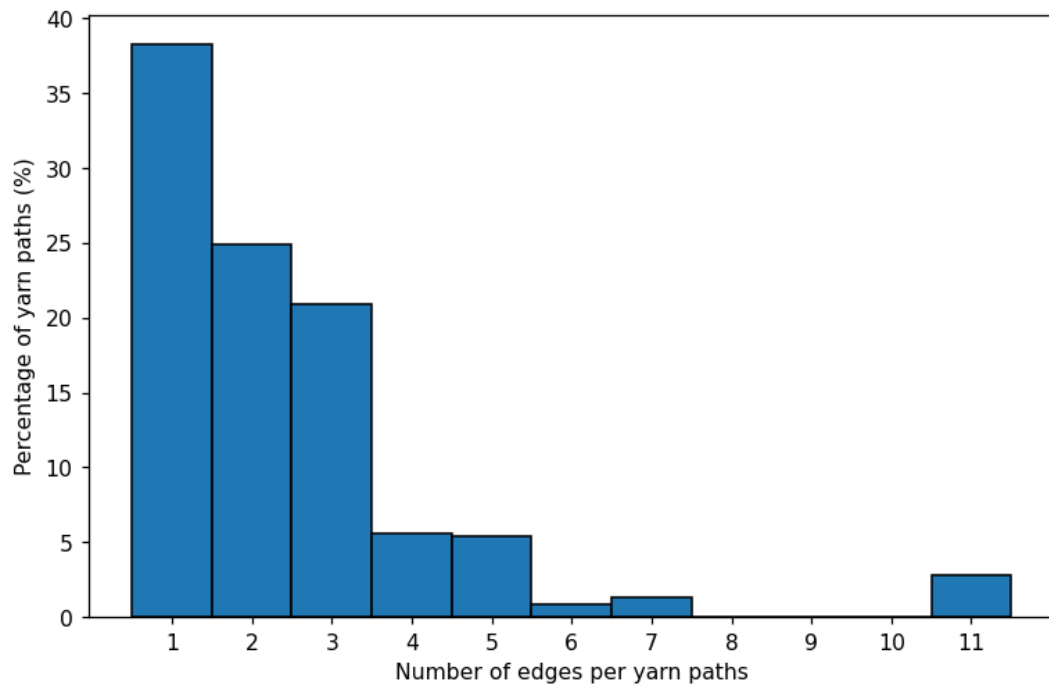


Figure 8.2.2: Histogram of number of edges per yarn path for all 13 design cases

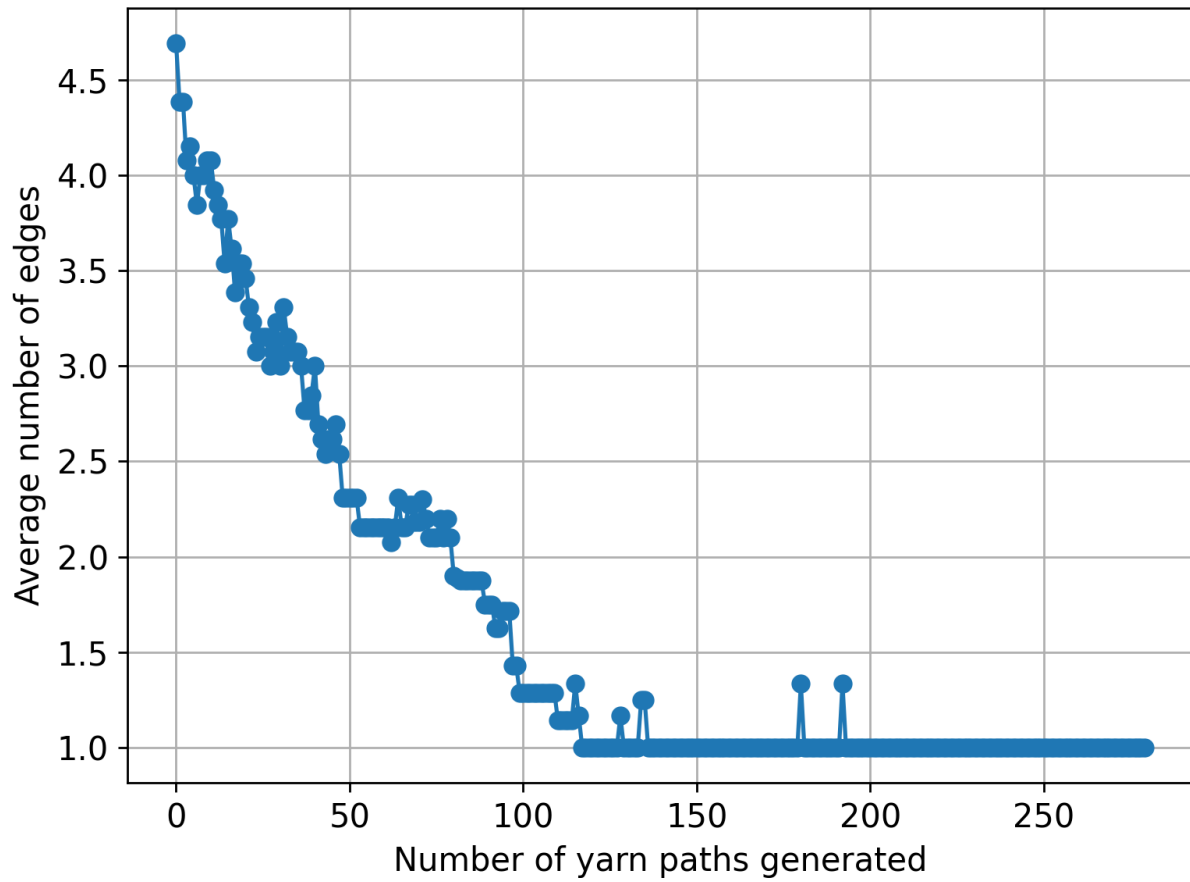


Figure 8.2.3: Average number of edges per path as a function of the number of yarn paths generated - all 13 design cases

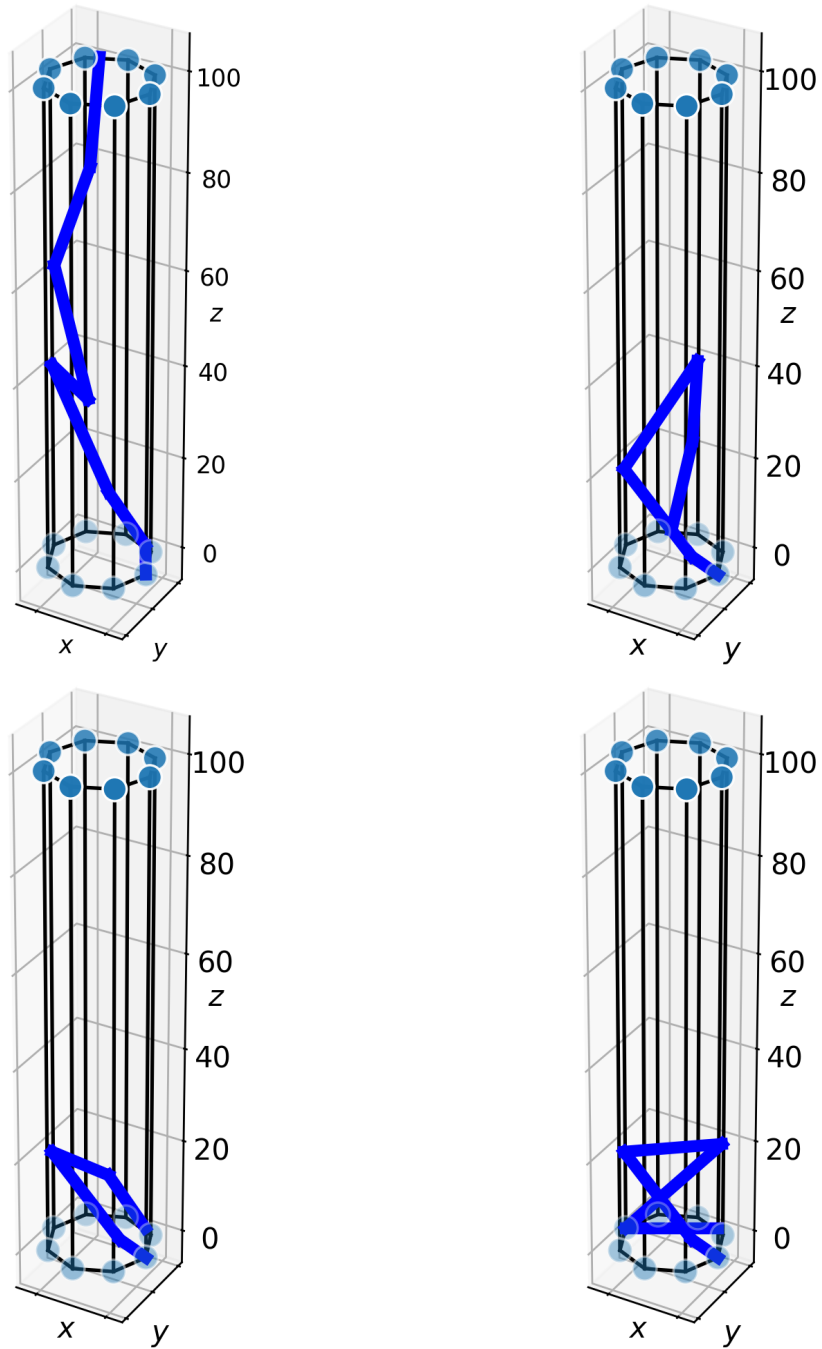


Figure 8.2.4: First four yarn paths generated - design case G4LC1

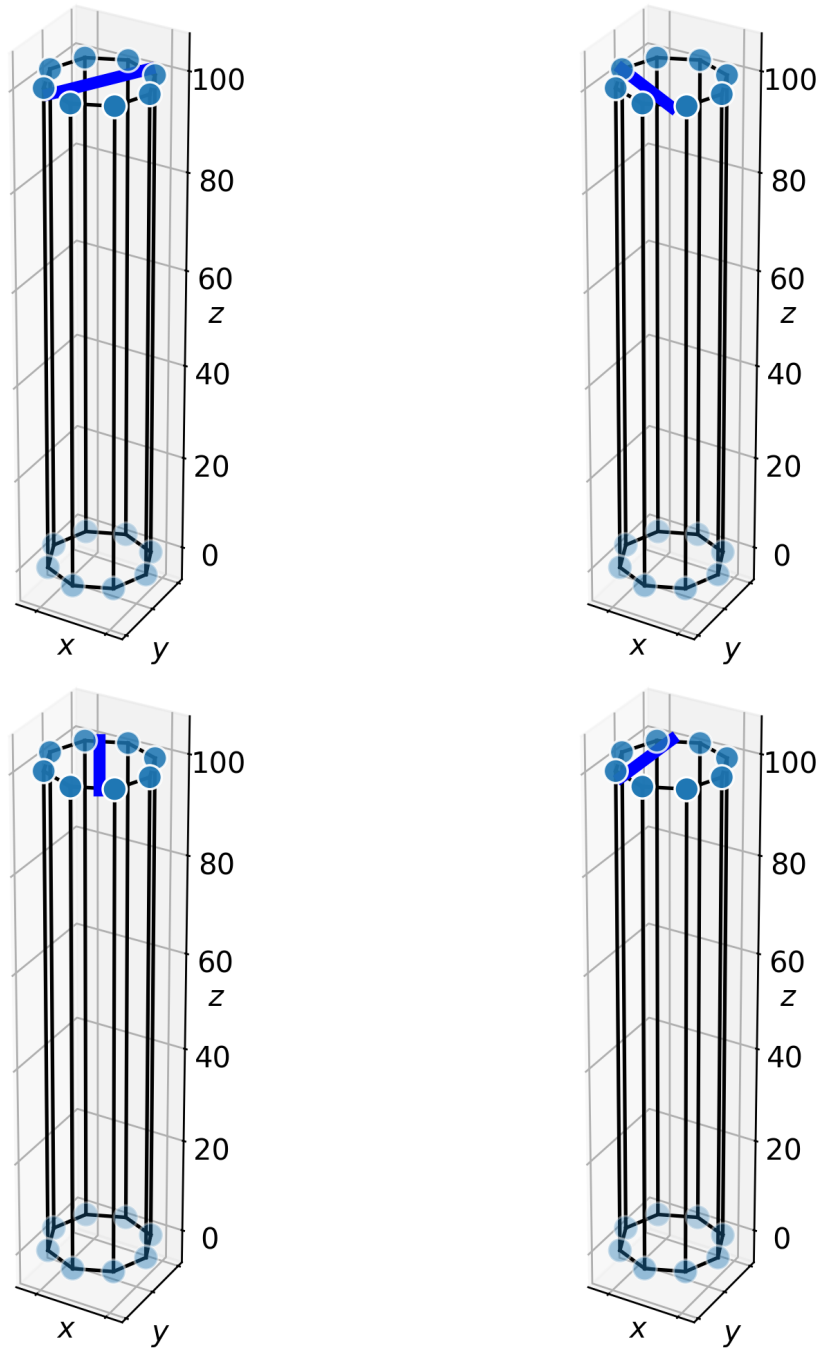


Figure 8.2.5: Last four yarn paths generated - design case G4LC1

Chapter 9

Structural Thickening

In this chapter, the methodology for the seventh and final step of the sequential TO process is presented. In step 7, the yarns are thickened by attributing them circular cross-sections. Up to this point, only 1D cross-sections have been considered for the triangulated structures, even though in practice the manufactured structures will indeed have cross-sections. In this chapter, an algorithm to detect and solve interferences between thickened yarns is developed using the vertex contraction operation for graphs. The algorithm presented in this chapter is a preliminary implementation as time constraints limited further refinement; however, it provides a foundation for future optimization and validation.

Before presenting the methodology for thickening the yarns, it is important to recall a few practical considerations regarding the deposition process and yarn paths. First, for a given path, the process deposits a constant amount of yarn. Second, slight fluctuations in yarn tension, and fibre volume fraction may occur but they are expected to be minor, meaning that the cross-sectional area associated with a given path remains effectively constant along the path. Third, the quantity of yarns may vary from one path to another to accommodate structural or design requirements. Fourth, while the cross-sectional area is constant along a path, the shape of the section may vary depending on the geometry of the surfaces around

which the yarns are guided and on the behaviour of yarns at interlacing points — an effect that has been preliminarily explored but is not currently controlled [86]. Therefore, at this stage of development, assuming circular cross-sections provides a sound initial basis for the thickening of yarns.

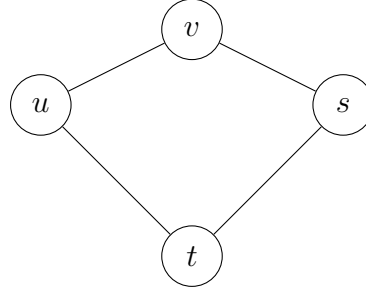
Building on these considerations, the proposed structural thickening algorithm extends the 1D graph-based representation of dry interlaced yarn structures developed in steps 1–6. In this approach, 2D circular cross-sections are attributed to the yarns, and intersecting cylinders are merged to improve structural continuity and resolve. This method enhances manufacturability while maintaining alignment with key design constraints such as structural symmetry and connectivity.

9.1 Vertex Contraction

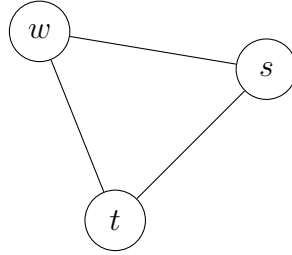
Vertex contraction is an operation in graph theory that merges two adjacent vertices u and v into a single vertex, effectively removing edge (u, v) and reassigning all edges incident to either u or v to the newly formed vertex. This transformation depicted in Figure 9.1.1 simplifies the graph while preserving its essential connectivity properties [87]. The cylinder merging process that is described in the next section is primarily driven by vertex contraction, which enables the combination of geometrically close yarns in dry interlaced structures. Formally, given a graph $G = (V, E)$, contracting an edge $(u, v) \in E$ results in a new graph $G' = (V', E')$ where:

- Vertices u and v are replaced by new vertex w ;
- Any edge previously connected to u or v is now connected to w ;
- If an edge existed between u and v , it is removed;

- If both u and v shared common neighbours, parallel edges may be introduced which can optionally be merged into a single edge with appropriate weight adjustments.



(a) Before vertex contraction



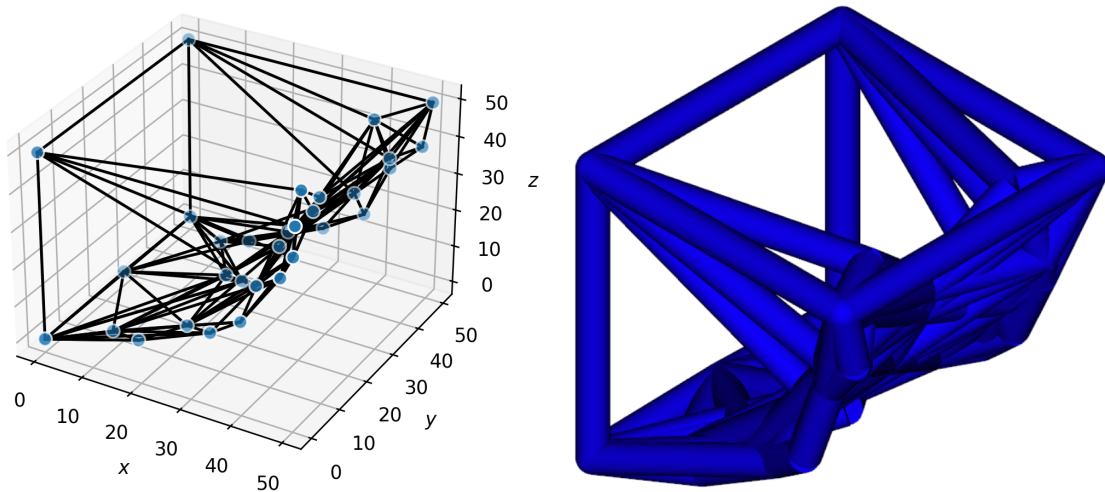
(b) After contracting vertices u and v

Figure 9.1.1: Depiction of vertex contraction

9.2 Cylinder Merging

The part designer begins by setting a target radius r for yarn cross-sections. The 1D edges of the initial graph are then converted to cylinders of radius r , Figure 9.2.1. The start and end coordinates of each segment are extracted, and their respective direction vectors and lengths are computed. These properties form the basis for determining intersections between cylinders.

To identify potential intersections, the algorithm computes the average axis distance between all pairs of cylinders, Figure 9.2.2. Instead of evaluating distances along the full cylinder lengths, 10 equidistant points are sampled along the length of the shorter cylinder to improve computational efficiency. This also ensures that intersections between cylinders



(a) Initial graph with 1D cross-sections (b) Graph with 2D circular cross-sections

Figure 9.2.1: Design case G1LC1 with $r = 2.5$ mm

of markedly unequal lengths can be detected adequately.

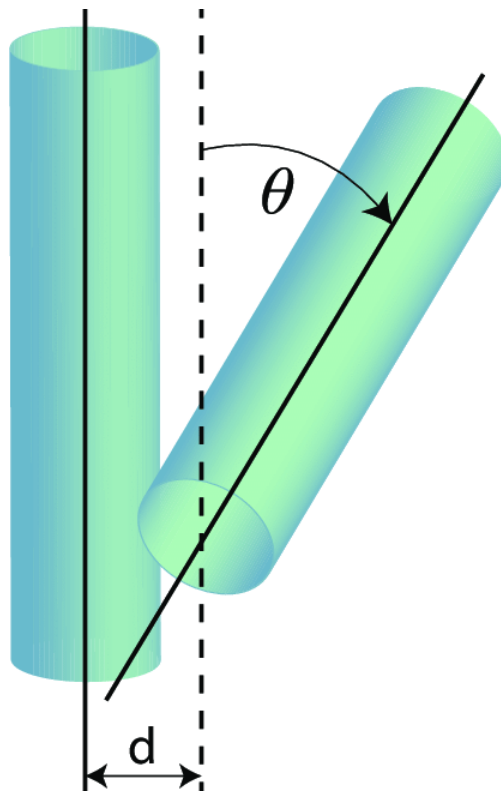


Figure 9.2.2: Axis distance definition [88]

A threshold-based criterion determines whether two cylinders are considered close enough to be merged:

$$\bar{d}_{\text{axis}} \leq \alpha r \quad (9.1)$$

where $\bar{d}_{\text{axis}}$ is the average distance between the axes of the two cylinders, r is the radius of the cylinder, and α is a user-defined intersection multiplier, set to 1.25, that controls the maximum allowable distance for merging. Once the pairs of intersecting cylinders are identified, the algorithm ensures that merging the two edges will not disconnect the graph. Before merging any intersecting cylinders, the algorithm verifies that the structural integrity of the graph is preserved: each candidate merge operation is tested by temporarily removing the intersecting edges and simulating a merged connection. If this operation results in a disconnected graph, the merge is discarded. This ensures that merging operations do not introduce unintended discontinuities in the structure.

To promote the formation of symmetric structural patterns for symmetric design cases, the algorithm optionally computes a symmetry score for each candidate pair of edges. This score is based on how well the pair corresponds to a mirrored counterpart across a defined symmetry plane. A score of 1.0 denotes perfect symmetry, while 0 indicates no symmetry. This score is calculated by reflecting the start points of the cylinders across a symmetry plane and searching for the closest matching pair among all candidate pairs. A normalized score is then returned indicating the quality of the match. Once all candidate pairs are evaluated and filtered based on connectivity, they are placed in a priority queue using their symmetry score and distance between their central axes. Let each cylinder pair have a score defined as the tuple:

$$\text{key} = (S, \bar{d}_{\text{axis}}) \quad (9.2)$$

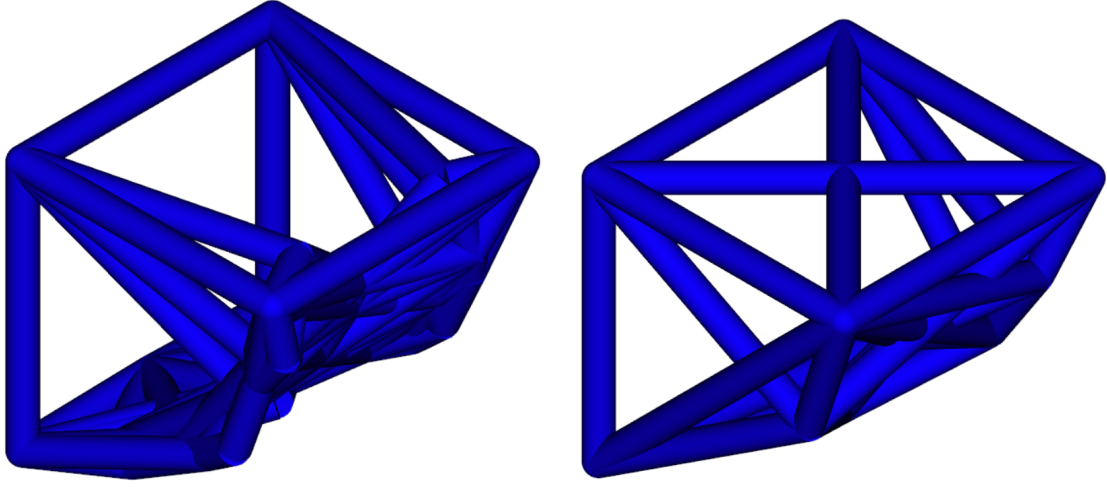
where S is the symmetry score. Pairs are inserted into the priority queue in ascending order of this key:

$$(S_1, \bar{d}_{\text{axis}_1}) < (S_2, \bar{d}_{\text{axis}_2}) \iff S_1 < S_2 \quad \text{or} \quad (S_1 = S_2 \text{ and } \bar{d}_{\text{axis}_2} < \bar{d}_{\text{axis}_1}) \quad (9.3)$$

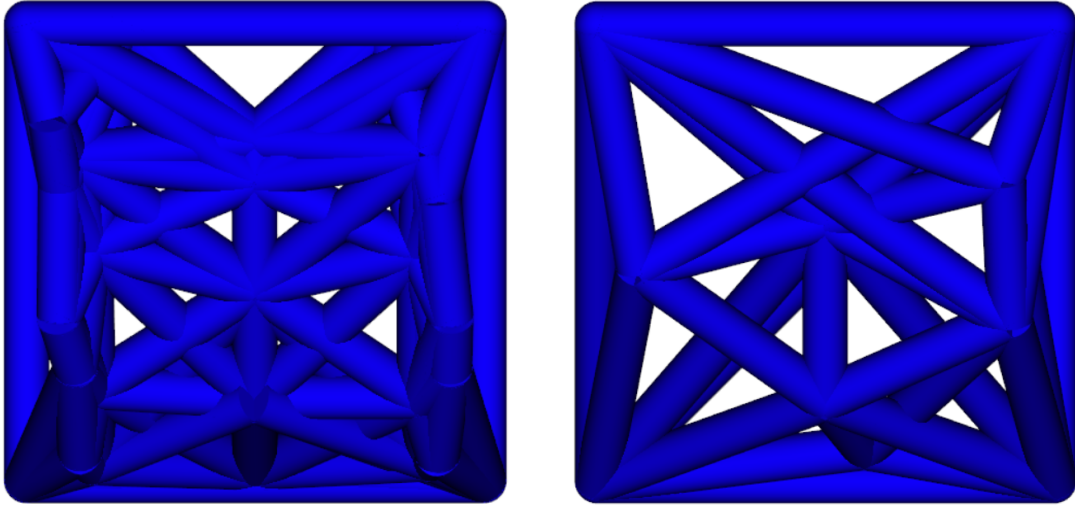
This means that pairs with larger symmetry scores come first, and among equal symmetry scores smaller $\bar{d}_{\text{axis}}$ are favoured. For design cases that feature no symmetry, pairs of cylinders are simply sorted in descending order of $\bar{d}_{\text{axis}}$.

For each valid pair of close cylinders, the merging process begins by aligning their start and end points. The merged cylinder's start and end positions are determined as weighted averages of the corresponding points from the two original cylinders, with weights proportional to their respective lengths. Vertex contraction is then performed on the pairs of start and endpoints of both cylinders. To preserve the overall structural layout, additional checks ensure that boundary nodes are not incorrectly merged. If one of the original start or end nodes corresponds to a boundary node, the merged position simply corresponds to the boundary node to ensure that it is not displaced.

The merging process is executed iteratively, with each iteration reducing the number of overlapping cylinders in the structure. After each merge, the graph is updated, and a new set of close cylinder pairs is identified. The process continues until no further merges are possible. If an iteration fails to reduce the number of intersecting cylinders, the algorithm terminates to prevent unnecessary computations. The result of this process is an optimized 3D structure where intersecting cylindrical beams have been merged while maintaining connectivity and symmetry, Figures 9.2.3. This final structure provides a more realistic representation of manufacturable reinforcement paths, reducing redundancy while preserving key design constraints. Performing the same series of vertex contractions on the yarn paths obtained in Step 7 generates the yarn paths for the thickened structure.



(a) Before cylinder merging, isometric view (b) After cylinder merging, isometric view



(c) Before cylinder merging, front view (d) After cylinder merging, front view

Figure 9.2.3: Design case G1LC1 with $r = 2.5$ mm before and after cylinder merging

The proposed algorithm effectively simplifies the structure, while maintaining structural connectivity. Figures 9.2.4-9.2.7 showcase the effect of increasing the target radius on the number of edges in the simplified graph. All curve fits were performed using fourth-degree polynomials. As the target radius increases, the number of edges in the graph decreases until attaining a lower limit, indicating that there is a critical radius where the structure cannot be further simplified without losing important geometric features such as boundary

surfaces, holes or fillets. This holds true for all design cases. For design cases G1LC1 through G6LC3, target radii were constrained between 0 and 10, reflecting characteristic lengths of the envelopes ranging from 100 to 234. For design cases G7LC1 to G7LC3, the target radii were expanded to between 0 and 50, corresponding to a characteristic length of the envelope of 1509 for these cases.

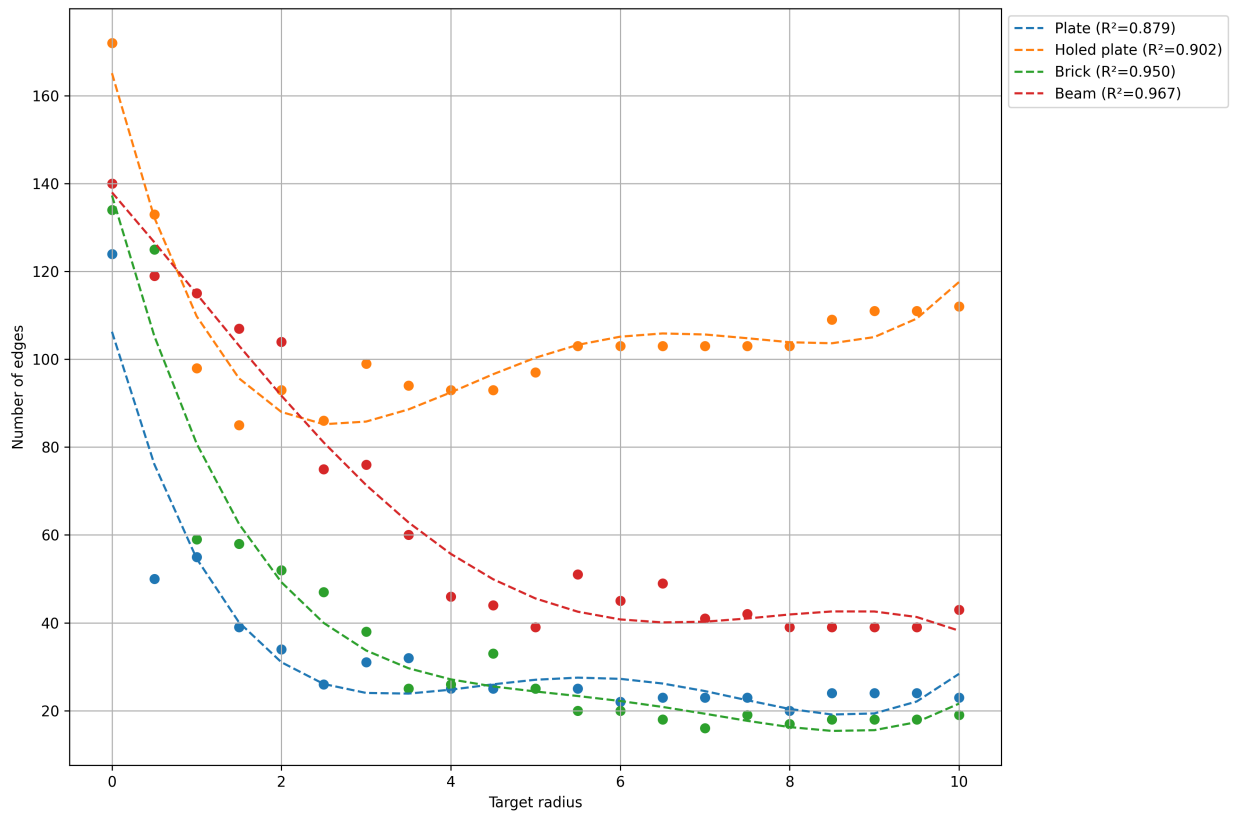


Figure 9.2.4: Number of edges as a function of target radius - design cases G1LC1-G4LC1

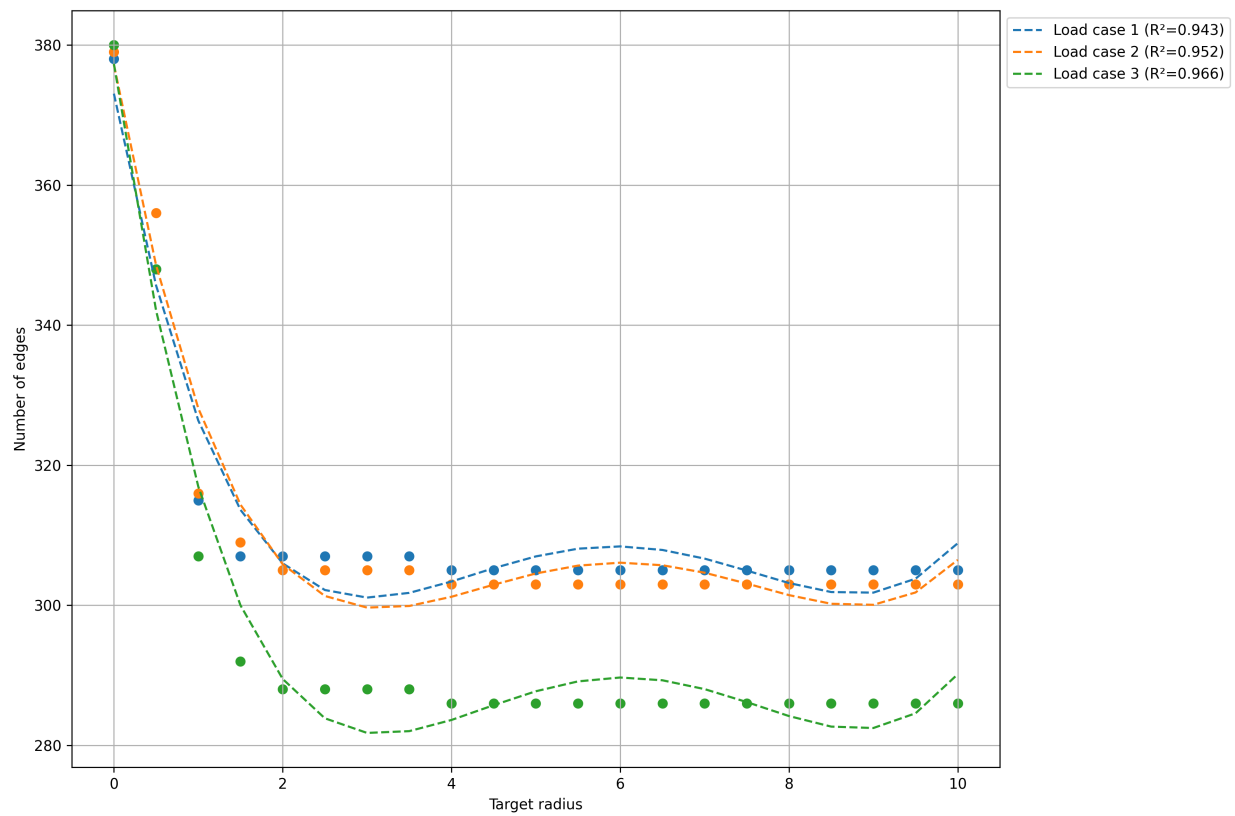


Figure 9.2.5: Number of edges as a function of target radius - design cases G5LC1-G5LC3

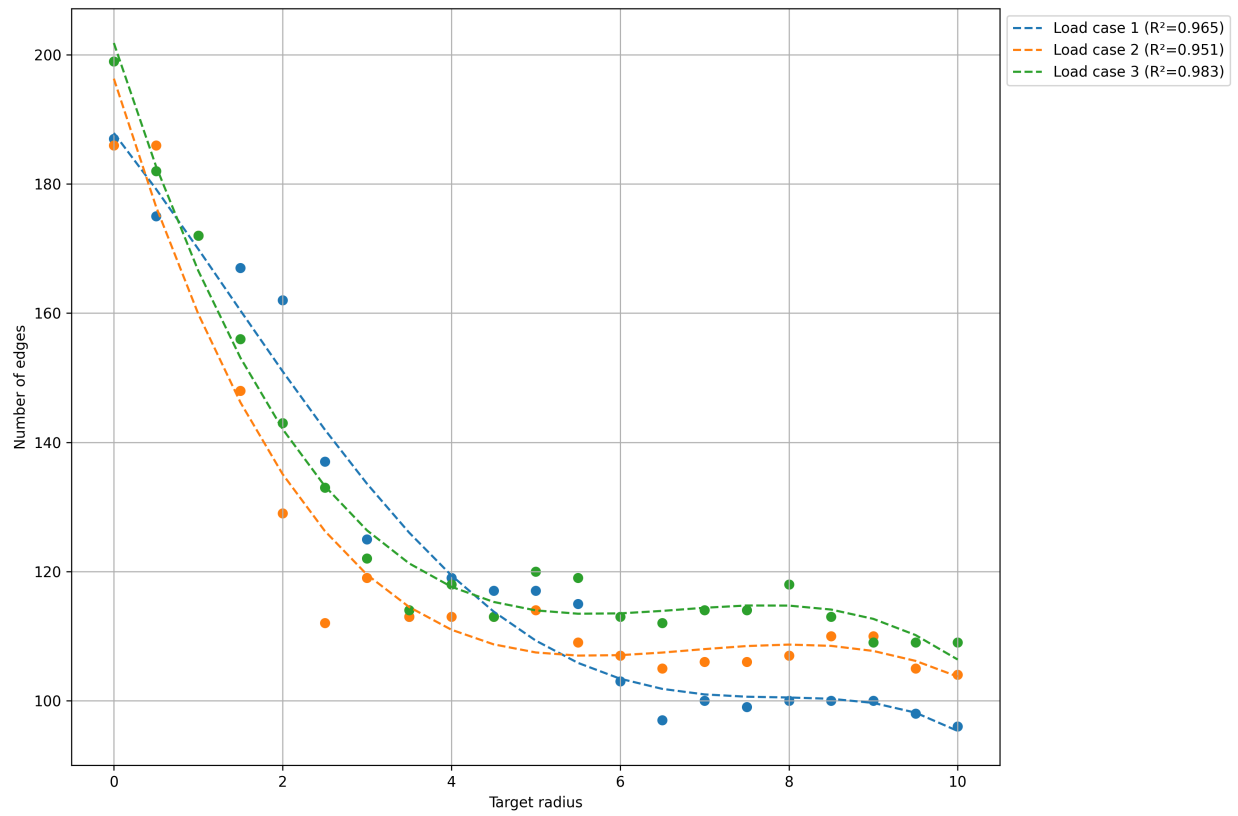


Figure 9.2.6: Number of edges as a function of target radius - design cases G6LC1-G6LC3

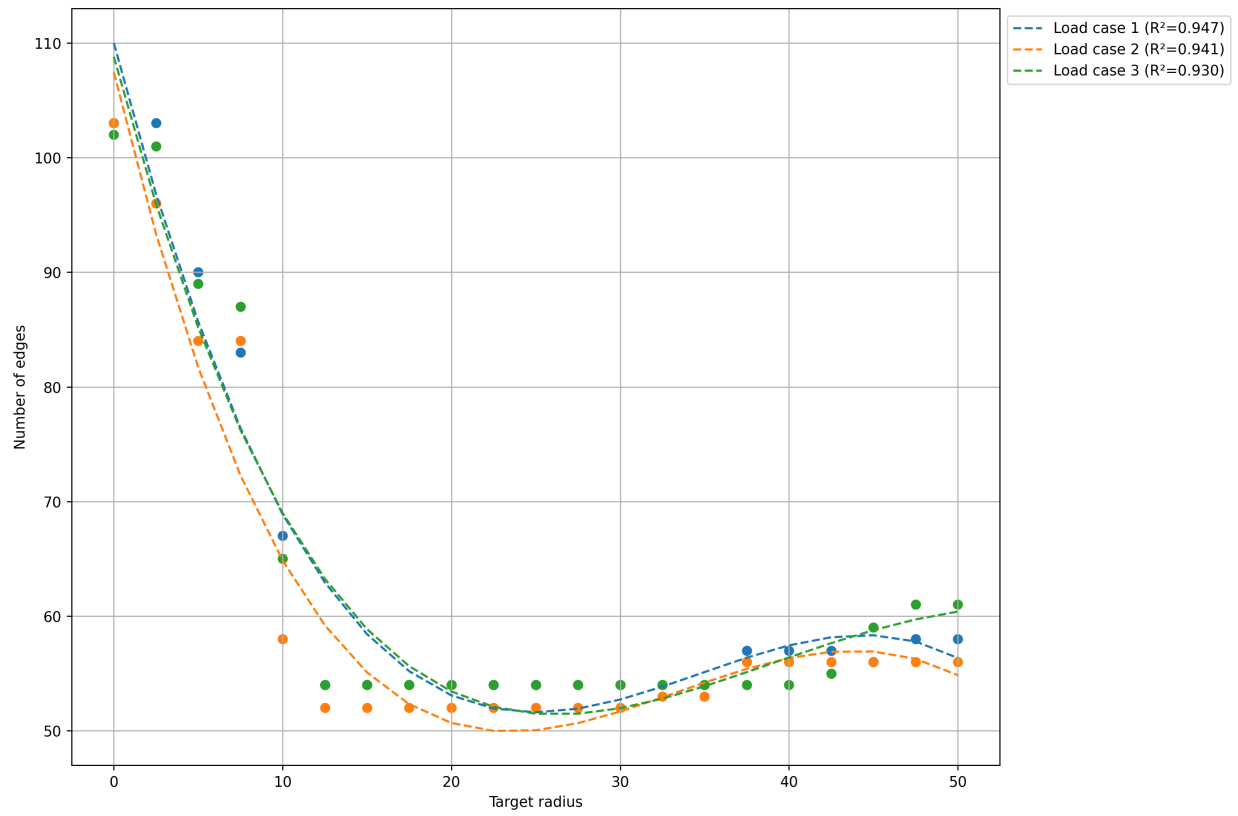
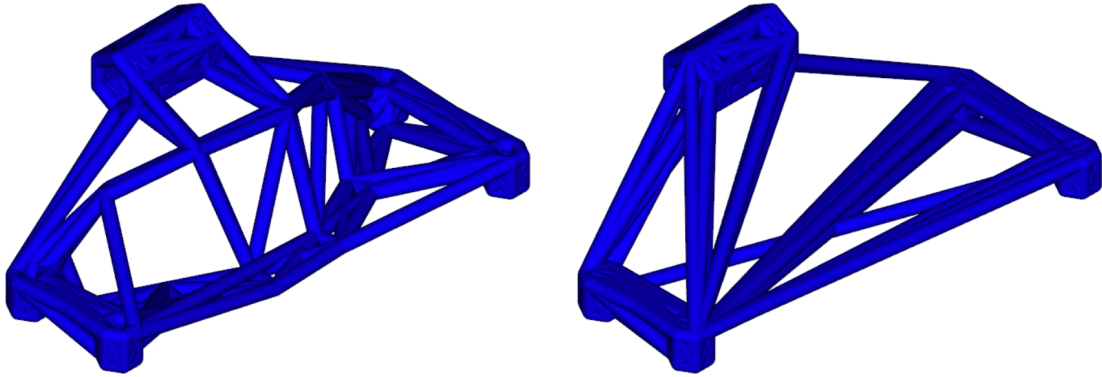


Figure 9.2.7: Number of edges as a function of target radius - design cases G7LC1-G7LC3

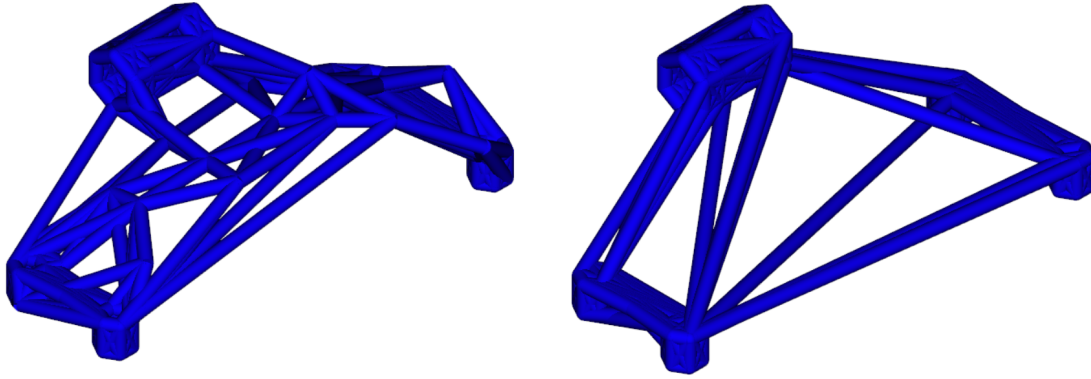
However, there are a few limitations to the current implementation of the algorithm. For example, in design cases featuring many envelope points that are not subject to displacement during the merging process, the amount of valid merge operations is limited. This results in simplified structures that vary quite strongly from their initial configurations, and that still feature a large number of intersecting cylinders. This is best exemplified by design cases G5LC1-G5LC3, where most edges in the final structure are between points that define its holes, Figures 9.2.8-9.2.10.

Additionally, symmetry is not guaranteed in the structure as shown in Figure 9.2.3. This is primarily due to the iterative nature of cylinder merging. After many merging iterations, any small asymmetry artefacts that have accumulated from previous steps in the TO framework compound, leading to asymmetrical graphs, even if the design case itself is symmetric. All in all, this version of the algorithm serves as a useful proof of concept for the generation of simplified, connected graphs with 2D cross-sections. However, it highlights significant limitations in the handling of symmetry and in the definition of surfaces and geometric features that need to be preserved throughout the TO framework.



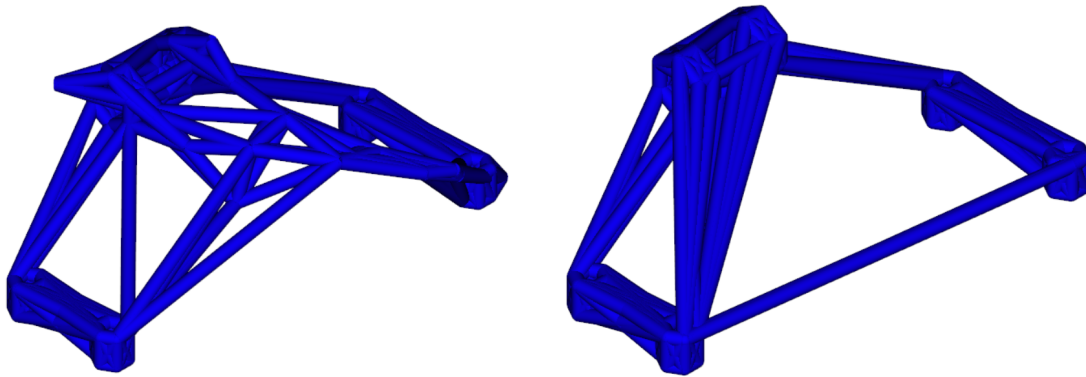
(a) Before cylinder merging, isometric view (b) After cylinder merging, isometric view

Figure 9.2.8: Design case G5LC1 with $r = 2.5$ mm before and after cylinder merging



(a) Before cylinder merging, isometric view (b) After cylinder merging, isometric view

Figure 9.2.9: Design case G5LC2 $r = 2.5$ mm before and after cylinder merging



(a) Before cylinder merging, isometric view (b) After cylinder merging, isometric view

Figure 9.2.10: Design case G5LC3 with $r = 2.5$ mm before and after cylinder merging

Chapter 10

Conclusion and Future Work

10.1 Conclusions

This thesis presents a novel framework for the generation and optimization of dry interlaced continuous yarn structures within the context of 3D TO for PMFRCs. The process begins with FEA performed in the continuous volume of the part envelope to extract principal stress fields and proceeds through a series of algorithmic steps including stress path generation, importance-based path selection, triangulated structure construction via k -neighbours, alignment optimization, yarn path generation and graph-based thickening. Together, these steps enable the design of truss-like structures that follow mechanically optimized stress trajectories, and that could prospectively be manufactured.

The core contribution lies in the development of a modular and extensible pipeline that integrates stress-informed geometry generation with discrete optimization and graph theory approaches to yarn placement. This pipeline bridges the gap between traditional TO and the manufacturing realities of 3D printed PMFRCs, where continuous yarn reinforcement is a requirement.

10.2 Future Work and Recommendations

One of the key limitations of the current approach is the lack of pure symmetry in the line selection process. While principal stress fields may exhibit inherent symmetry for certain design cases, the selection of stress paths does not always reflect this due to small numerical variations and the criteria used for selection. This effect is compounded at each step, particularly during the structural thickening, leading to asymmetric structures in symmetric design cases as shown in Chapter 9. To address this, a mirrored selection approach could be implemented for symmetrical design cases, ensuring that a chosen stress path enforces a corresponding counterpart across the symmetry plane.

Another challenge lies in tuning the optimization process to properly balance symmetry, alignment with the principal stress field, and constraints such as hole avoidance. Without carefully selected scaling factors, one objective may dominate at the expense of others, leading to suboptimal structures as observed in Chapter 7. An adaptive weighting scheme, where the relative importance of different terms is adjusted based on the structure’s current state, could offer a more robust approach. Additionally, normalizing each term based on its expected range may prevent imbalances in the optimization process.

The proposed framework also faces difficulties in handling holes and curved regions due to its discrete nature. Since the structure is generated from a finite set of points, representing curvature accurately requires a sufficiently high point density. However, the inclusion of a large amount of points leads to large levels of clustering during the triangulation process and applies mobility constraints to vertices during the optimization process. That is to say, during the yarn alignment optimization and structural thickening steps, fewer points can be displaced in the structure, reducing the effectiveness of these steps.

A more fundamental improvement to the method would involve optimizing the entire structure generation process rather than only applying yarn alignment optimization towards the end of the workflow. The current approach follows a sequential workflow, where FEA is performed first, followed by stress-aligned edge extraction, triangulation, and then a final optimization step. Instead, a more integrated approach could iteratively refine the structure throughout the FEA process, dynamically adding or removing edges in the structure as the stress field evolves. This would allow for a more adaptive and efficient generation of stress-aligned structures, potentially leading to better overall performance. This problem could lend itself well to machine learning methods, particularly physics informed neural networks or finite element neural networks [89], given the physical nature of the problem and the number of parameters that are to be optimized concurrently.

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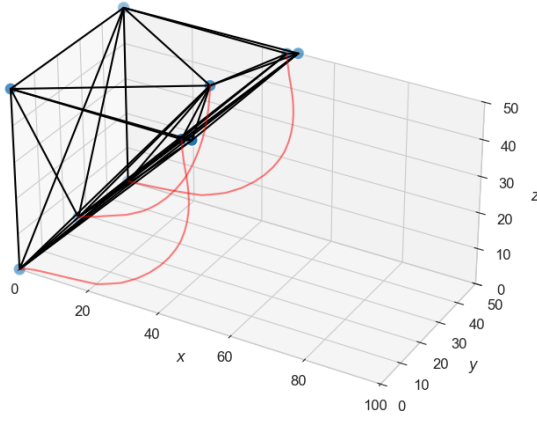
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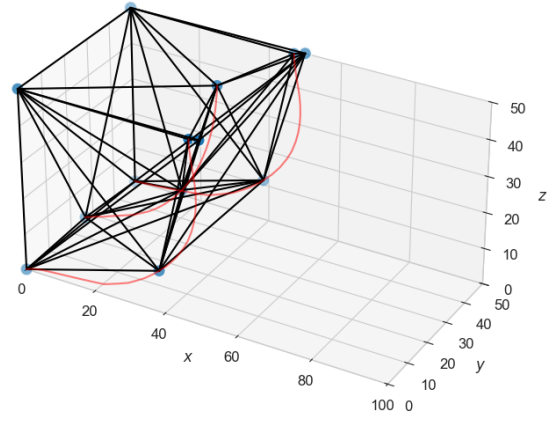
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Appendix A

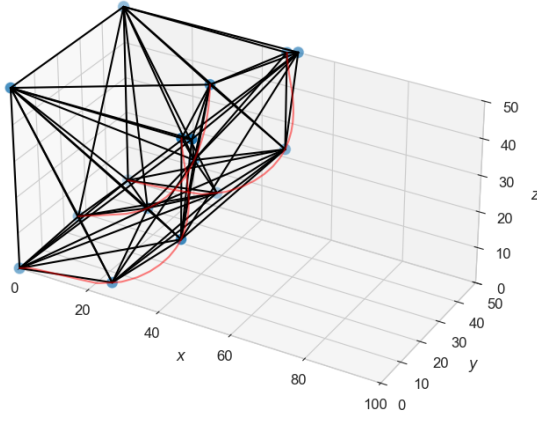
Delaunay Triangulation Results



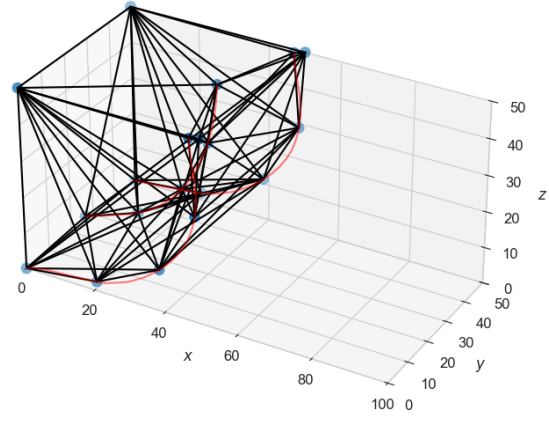
(a) 2 points per stress path ($k = 2$)



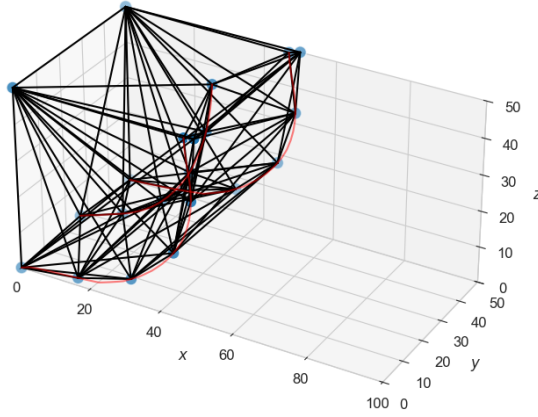
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

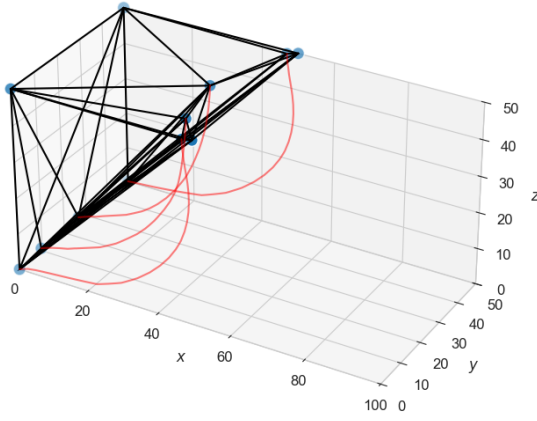


(d) 5 points per main stress path ($k = 5$)

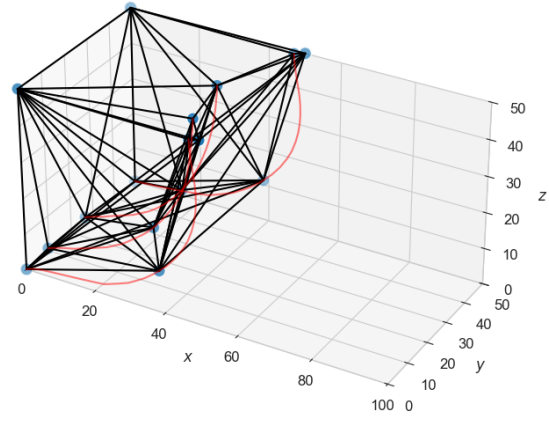


(e) 6 points per main stress path ($k = 6$)

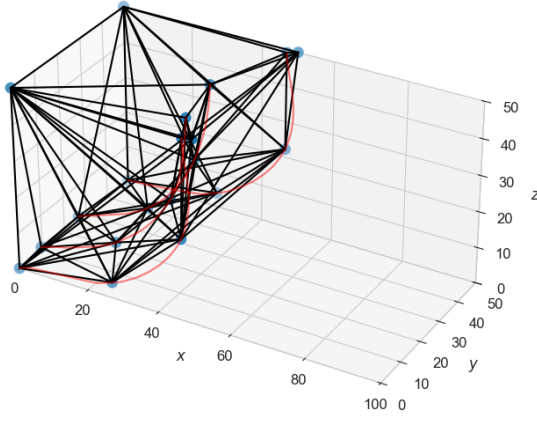
Figure A-1: Delaunay triangulation - design case G1LC1 with varying amounts of points per main stress path, $n = 1$ main stress path, main stress path in red



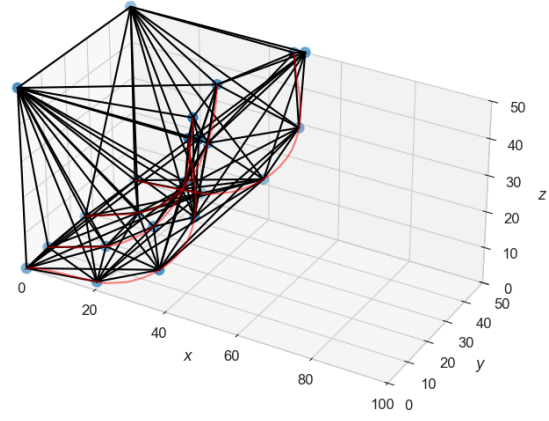
(a) 2 points per main stress path ($k = 2$)



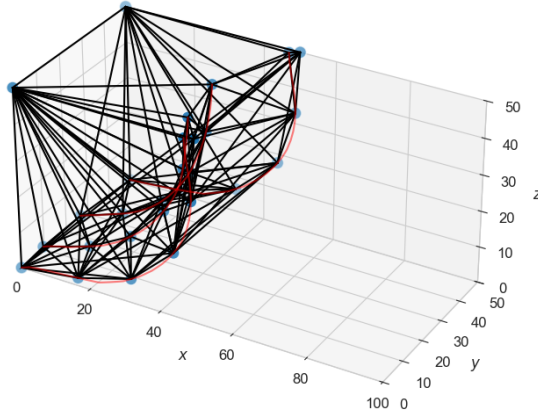
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

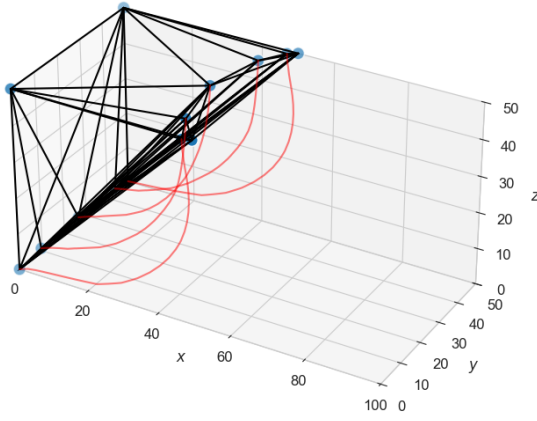


(d) 5 points per main stress path ($k = 5$)

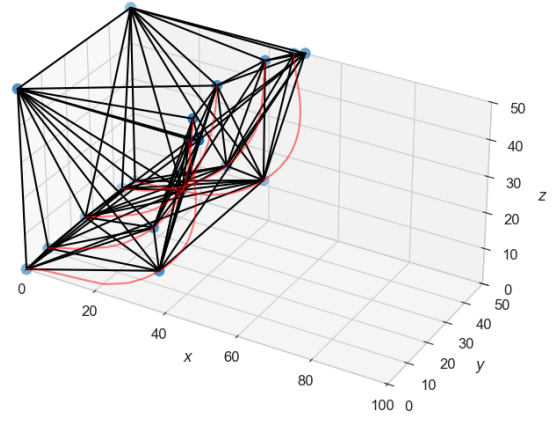


(e) 6 points per main stress path ($k = 6$)

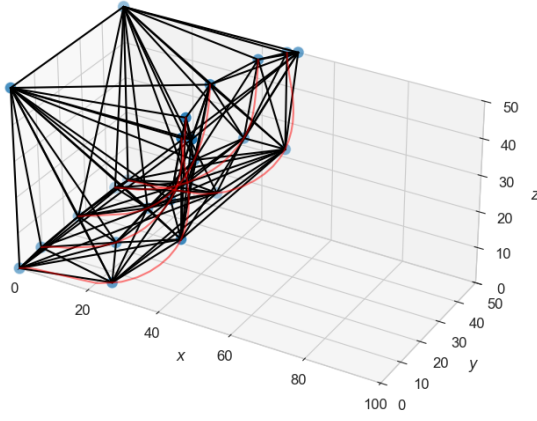
Figure A-2: Delaunay triangulation - design case G1LC1 with varying amounts of points per main stress path, $n = 2$ main stress paths, main stress paths in red



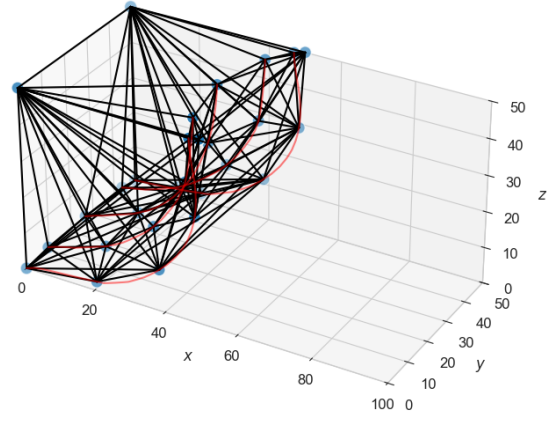
(a) 2 points per main stress path ($k = 2$)



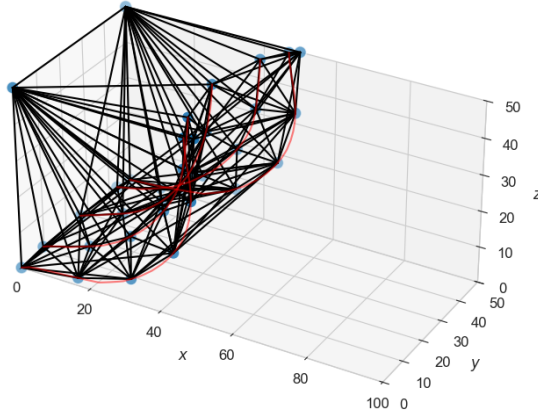
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

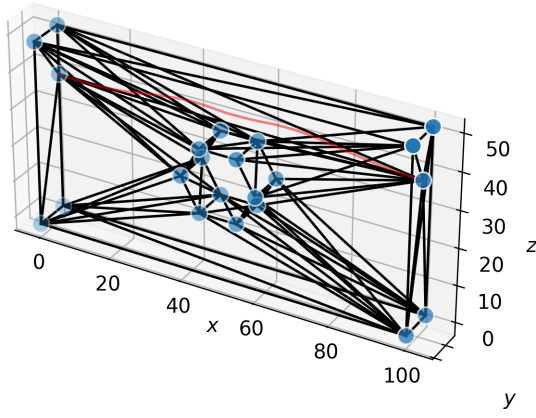


(d) 5 points per main stress path ($k = 5$)

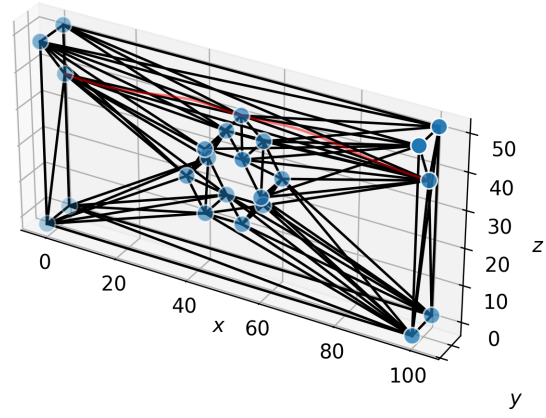


(e) 6 points per main stress path ($k = 6$)

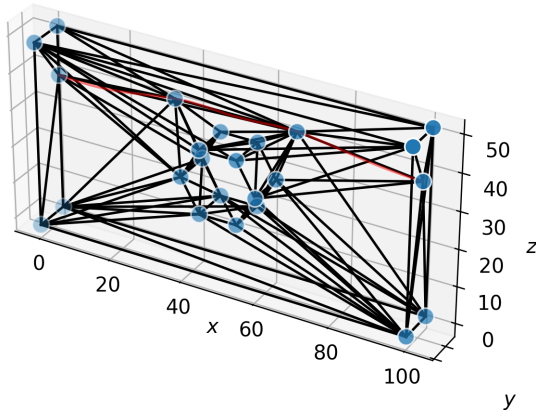
Figure A-3: Delaunay triangulation - design case G1LC1 with varying amounts of points per main stress path, $n = 3$ main stress paths, main stress paths in red



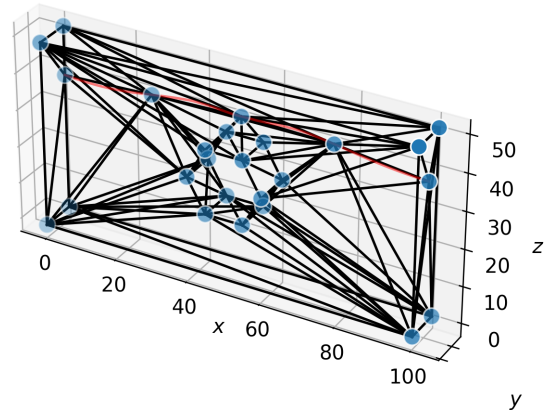
(a) 2 points per main stress path ($k = 2$)



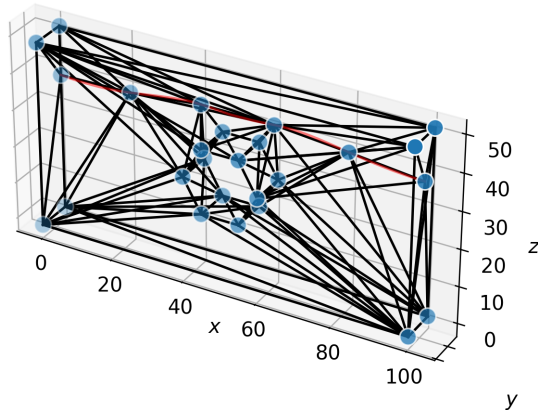
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

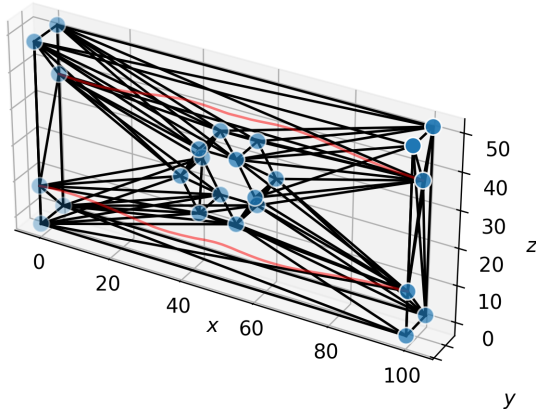


(d) 5 points per main stress path ($k = 5$)

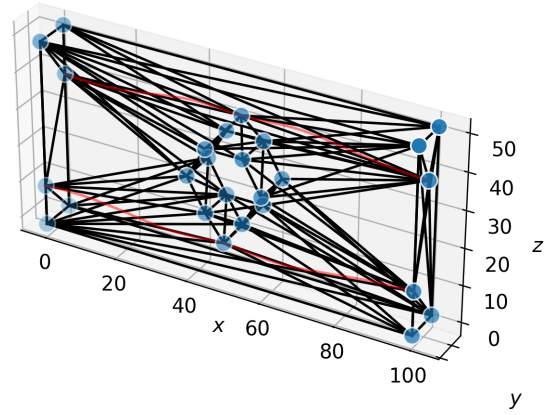


(e) 6 points per main stress path ($k = 6$)

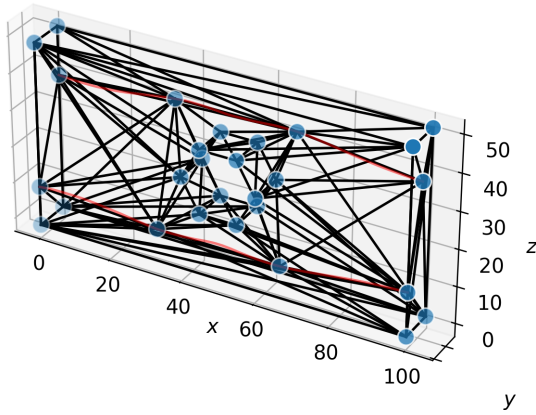
Figure A-4: Delaunay triangulation - design case G2LC1 with varying amounts of points per main stress path, $n = 1$ main stress path, main stress path in red



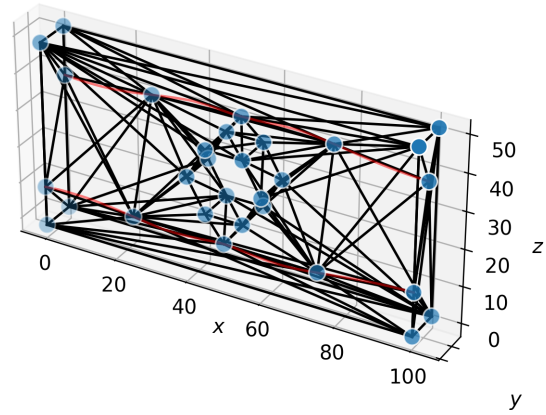
(a) 2 points per main stress path ($k = 2$)



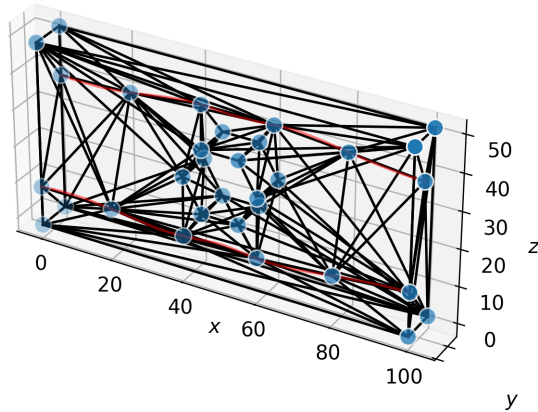
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

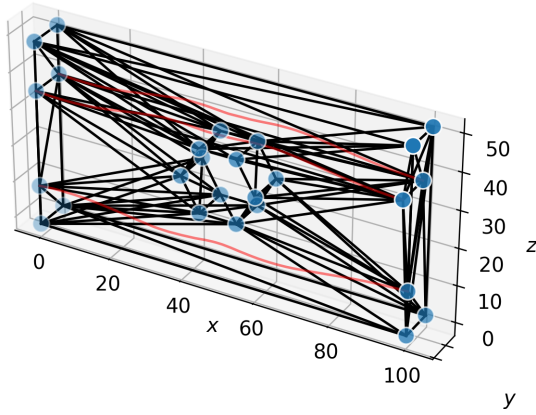


(d) 5 points per main stress path ($k = 5$)

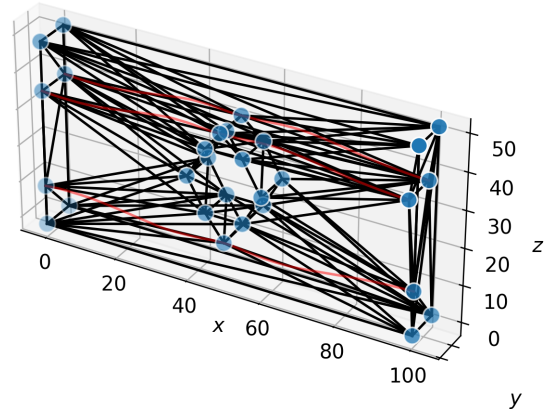


(e) 6 points per main stress path ($k = 6$)

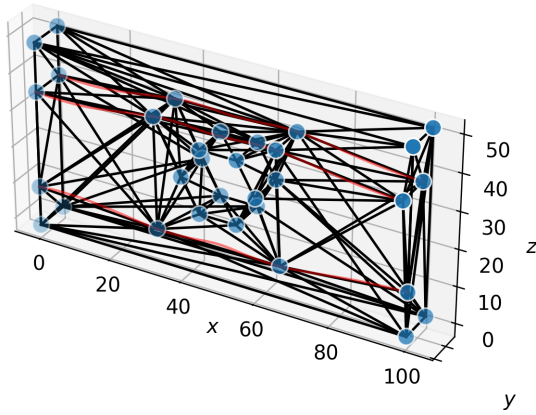
Figure A-5: Delaunay triangulation - design case G2LC1 with varying amounts of points per main stress path, $n = 2$ main stress paths, main stress paths in red



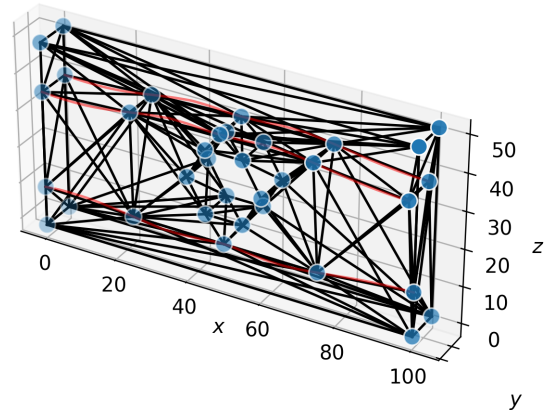
(a) 2 points per main stress path ($k = 2$)



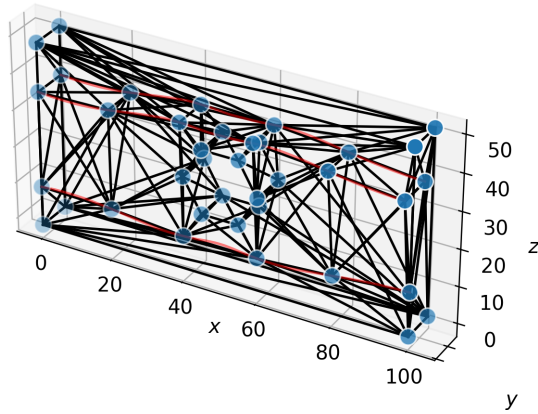
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

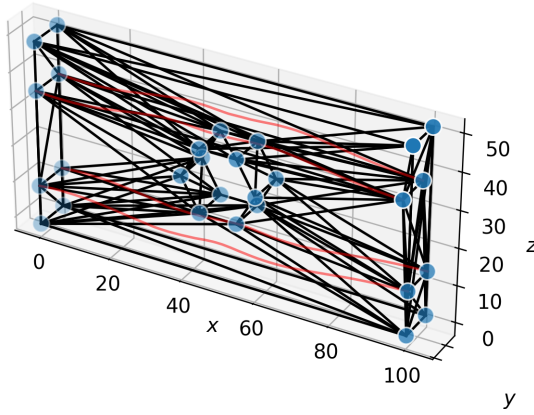


(d) 5 points per main stress path ($k = 5$)

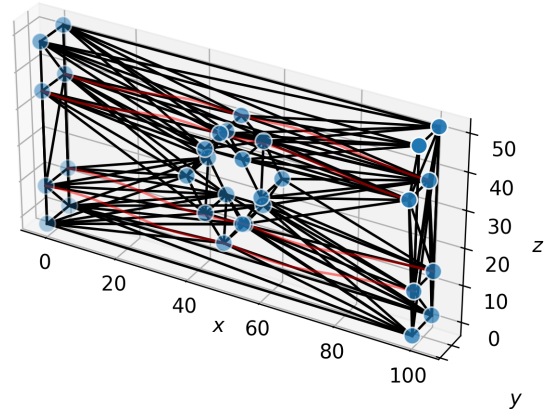


(e) 6 points per main stress path ($k = 6$)

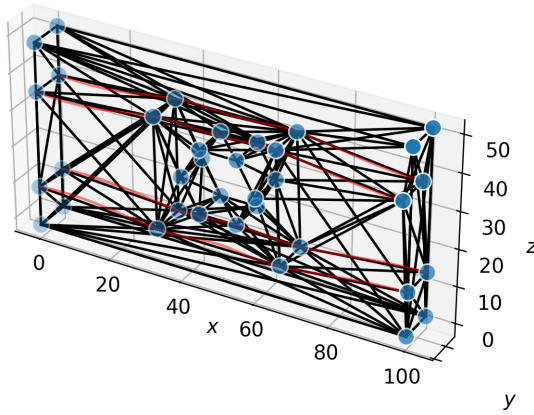
Figure A-6: Delaunay triangulation - design case G2LC1 with varying amounts of points per main stress path, $n = 3$ main stress paths, main stress paths in red



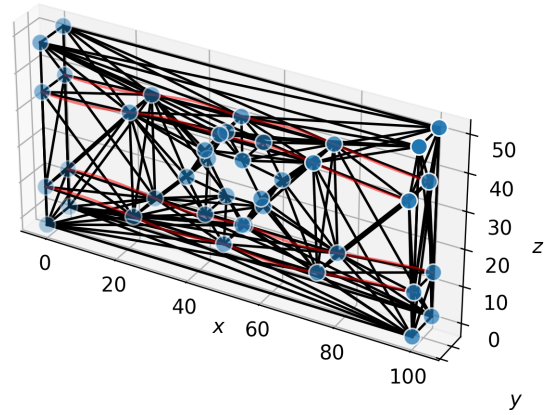
(a) 2 points per main stress path ($k = 2$)



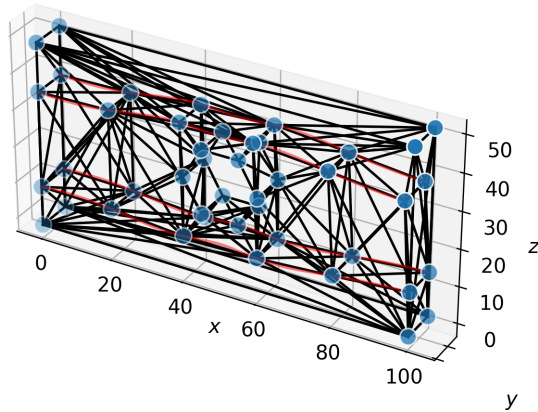
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

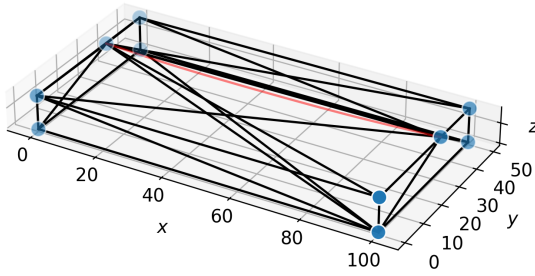


(d) 5 points per main stress line ($k = 5$)

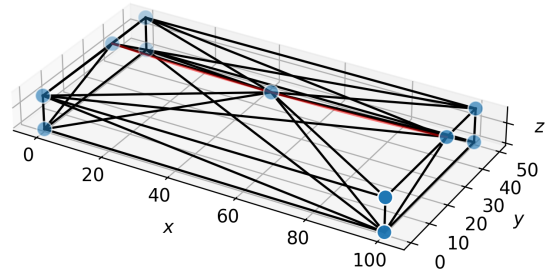


(e) 6 points per main stress path ($k = 6$)

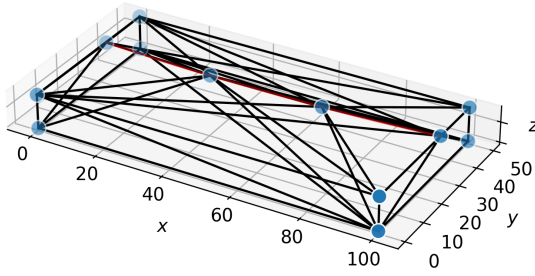
Figure A-7: Delaunay triangulation - design case G2LC1 with varying amounts of points per main stress path, $n = 4$ main stress paths, main stress paths in red



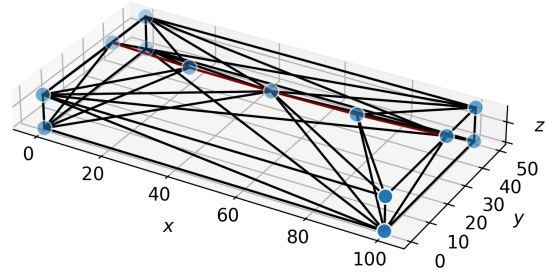
(a) 2 points per main stress path ($k = 2$)



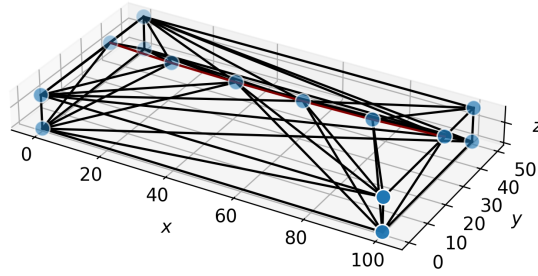
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

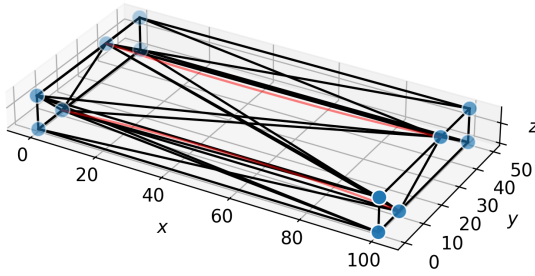


(d) 5 points per main stress path ($k = 5$)

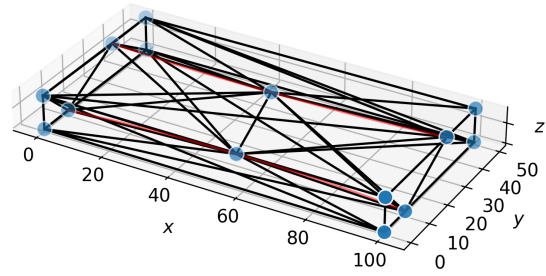


(e) 6 points per main stress path ($k = 6$)

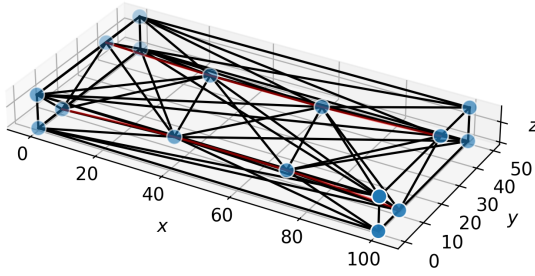
Figure A-8: Delaunay triangulation - design case G3LC1 with varying amounts of points per main stress path, $n = 1$ main stress path, main stress path in red



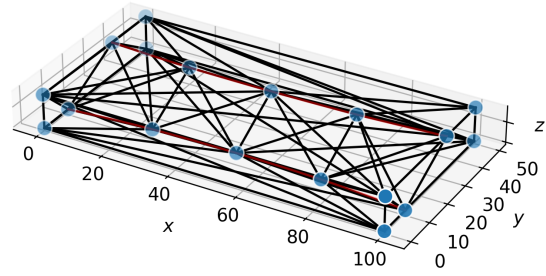
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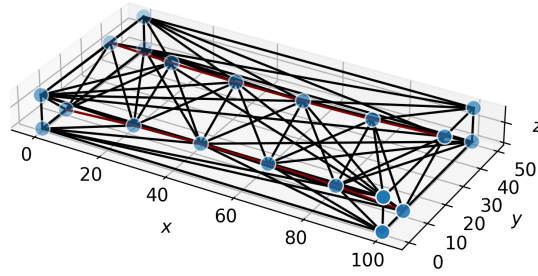
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)



(d) 5 points per main stress path ($k = 5$)



(e) 6 points per main stress path ($k = 6$)

Figure A-9: Delaunay triangulation - design case G3LC1 with varying amounts of points per main stress path, $n = 2$ main stress paths, main stress paths in red

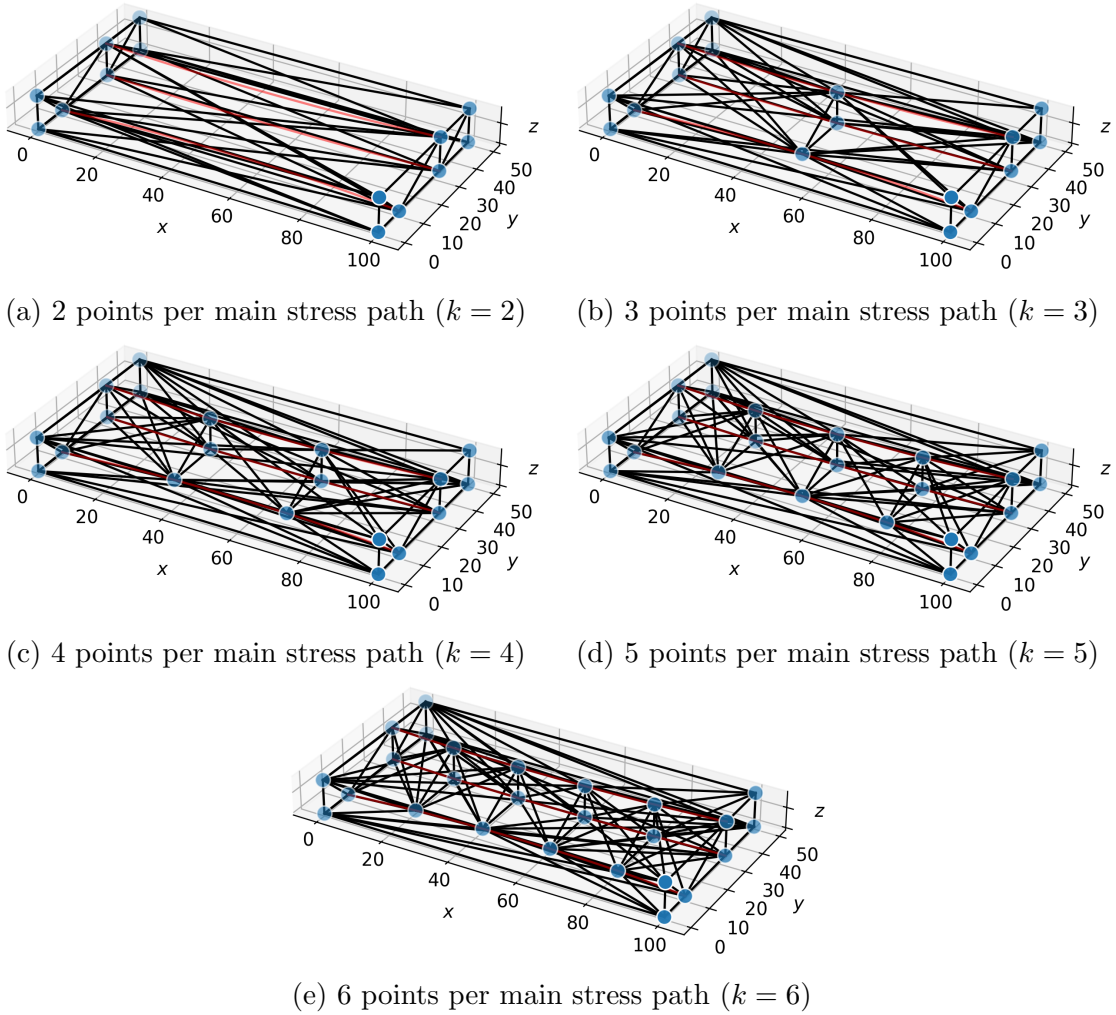
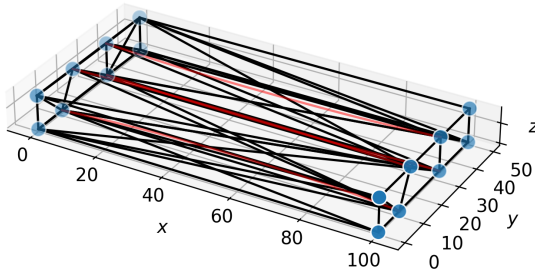
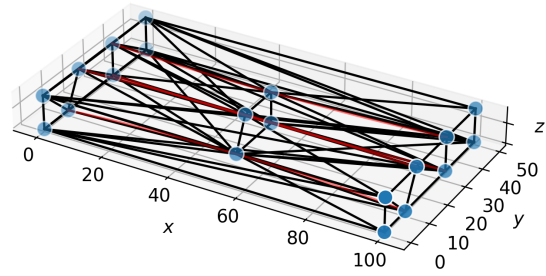


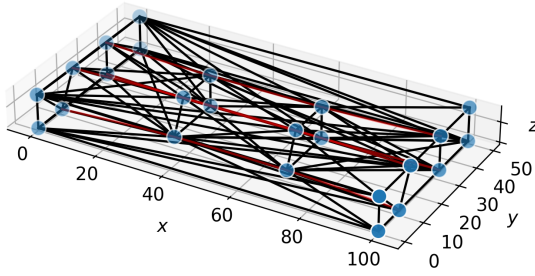
Figure A-10: Delaunay triangulation - design case G3LC1 with varying amounts of points per main stress path, $n = 3$ main stress paths, main stress paths in red



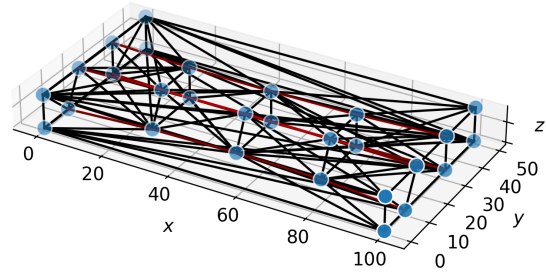
(a) 2 points per main stress path ($k = 2$)



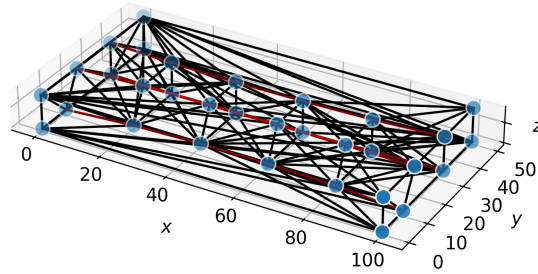
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

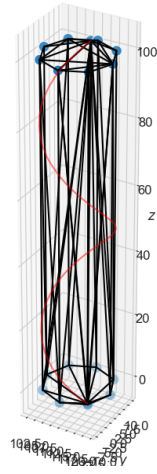


(d) 5 points per main stress path ($k = 5$)

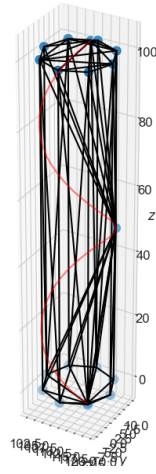


(e) 6 points per main stress path ($k = 6$)

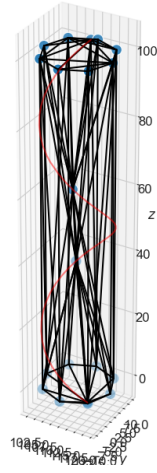
Figure A-11: Delaunay triangulation - design case G3LC1 with varying amounts of points per main stress path, $n = 4$ main stress paths, main stress paths in red



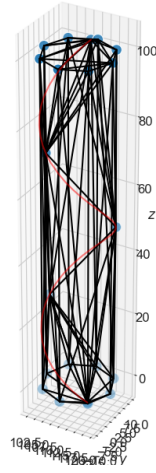
(a) 2 points per main stress path ($k = 2$)



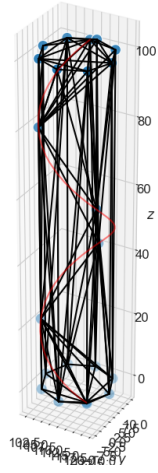
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

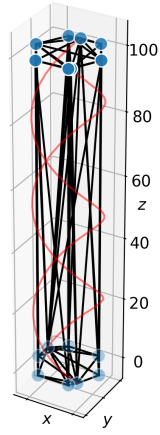


(d) 5 points per main stress path ($k = 5$)

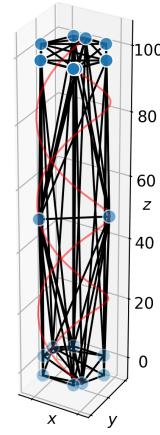


(e) 6 points per main stress path ($k = 6$)

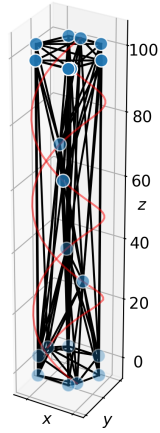
Figure A-12: Delaunay triangulation - design case G4LC1 with varying amounts of points per main stress path, $n = 1$ main stress path, main stress path in red



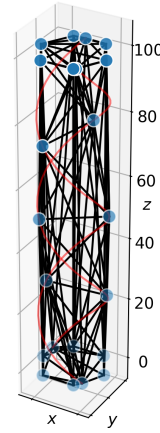
(a) 2 points per main stress path ($k = 2$)



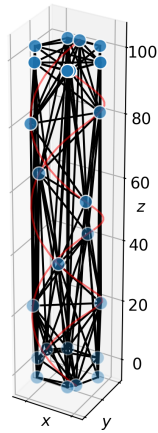
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

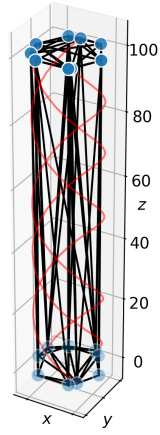


(d) 5 points per main stress path ($k = 5$)

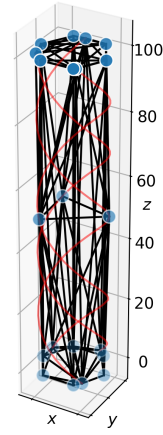


(e) 6 points per main stress path ($k = 6$)

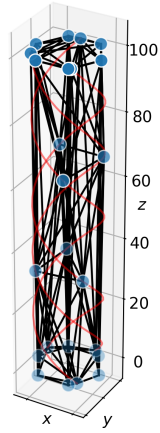
Figure A-13: Delaunay triangulation - design case G4LC1 with varying amounts of points per main stress path, $n = 2$ main stress paths, main stress paths in red



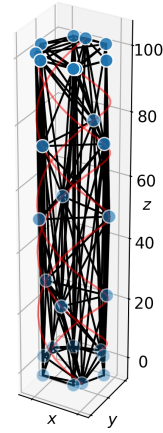
(a) 2 points per main stress path ($k = 2$)



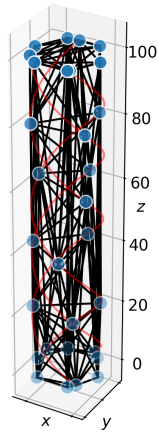
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)

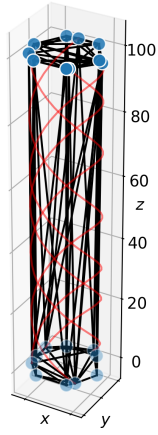


(d) 5 points per main stress path ($k = 5$)

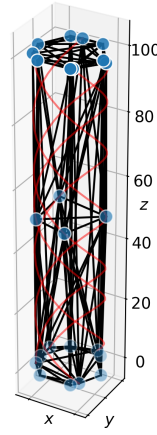


(e) 6 points per main stress path ($k = 6$)

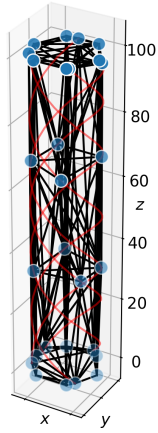
Figure A-14: Delaunay triangulation - design case G4LC1 with varying amounts of points per main stress path, $n = 3$ main stress paths, main stress paths in red



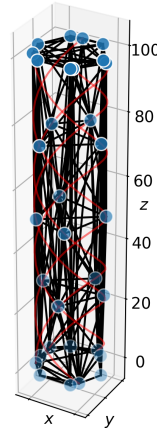
(a) 2 points per main stress path ($k = 2$)



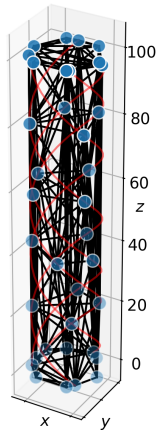
(b) 3 points per main stress path ($k = 3$)



(c) 4 points per main stress path ($k = 4$)



(d) 5 points per main stress path ($k = 5$)

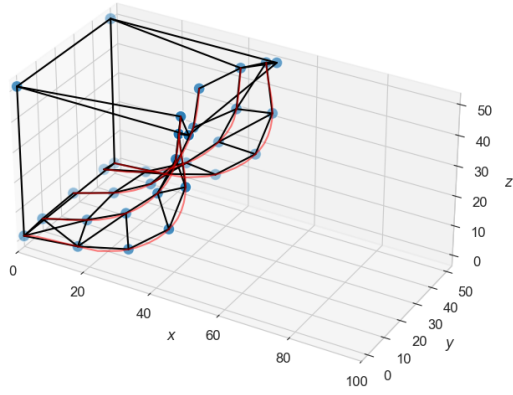


(e) 6 points per main stress path ($k = 6$)

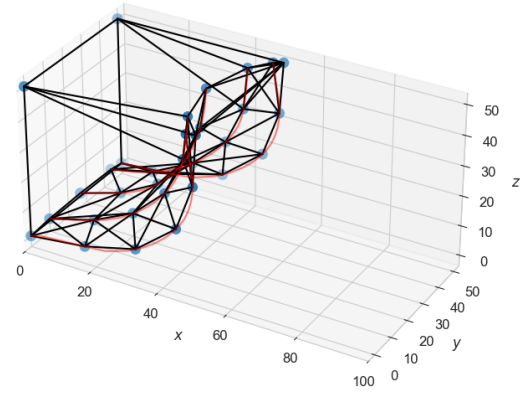
Figure A-15: Delaunay triangulation - design case G4LC1 with varying amounts of points per main stress path, $n = 4$ main stress paths, main stress paths in red

Appendix B

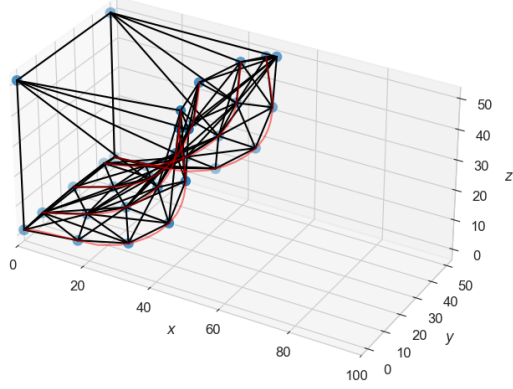
k -neighbours Triangulation Results



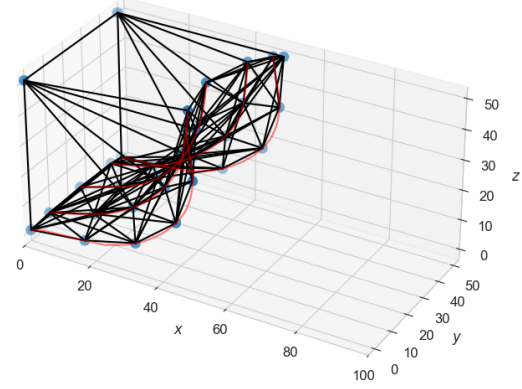
(a) $k = 1$ neighbour



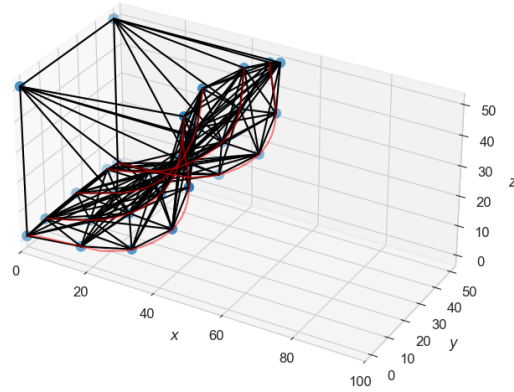
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

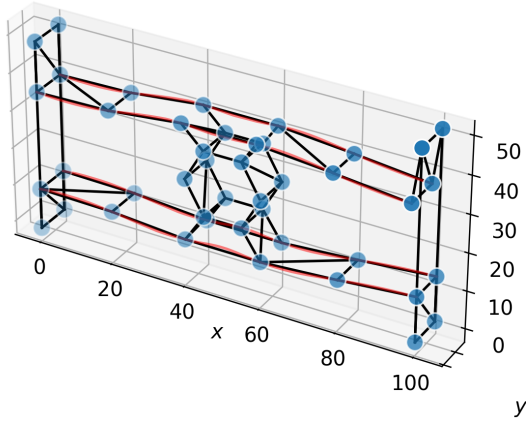


(d) $k = 4$ neighbours

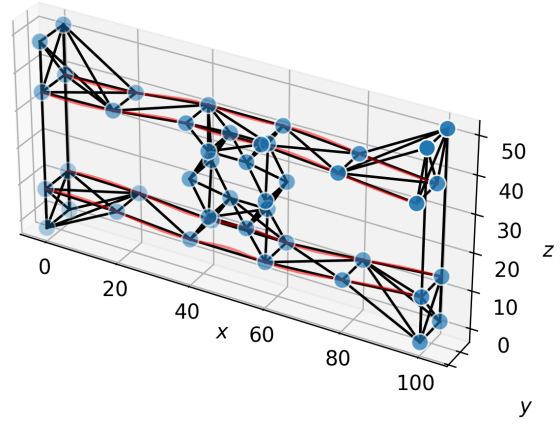


(e) $k = 5$ neighbours

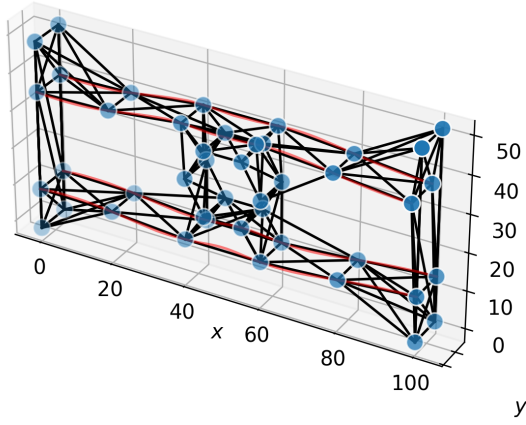
Figure B-1: k -neighbours triangulation - design case G1LC1 with different k values, 3 main stress paths with 6 points sampled per path, main stress paths in red



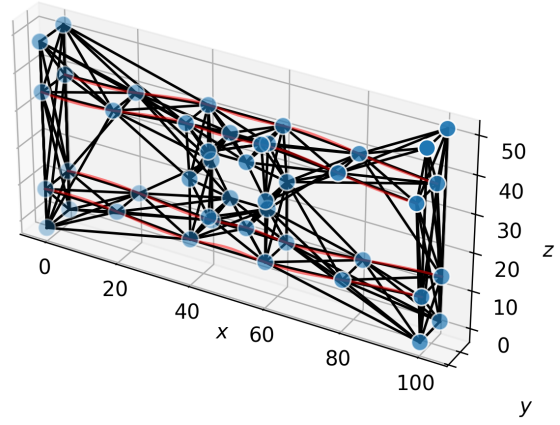
(a) $k = 1$ neighbour



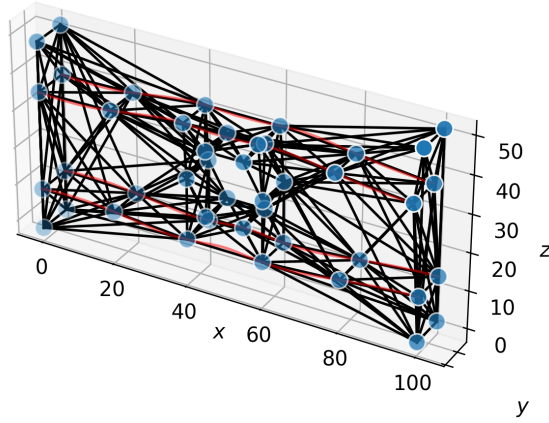
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours



(d) $k = 4$ neighbours



(e) $k = 5$ neighbours

Figure B-2: k -neighbours triangulation - design case G2LC1 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red

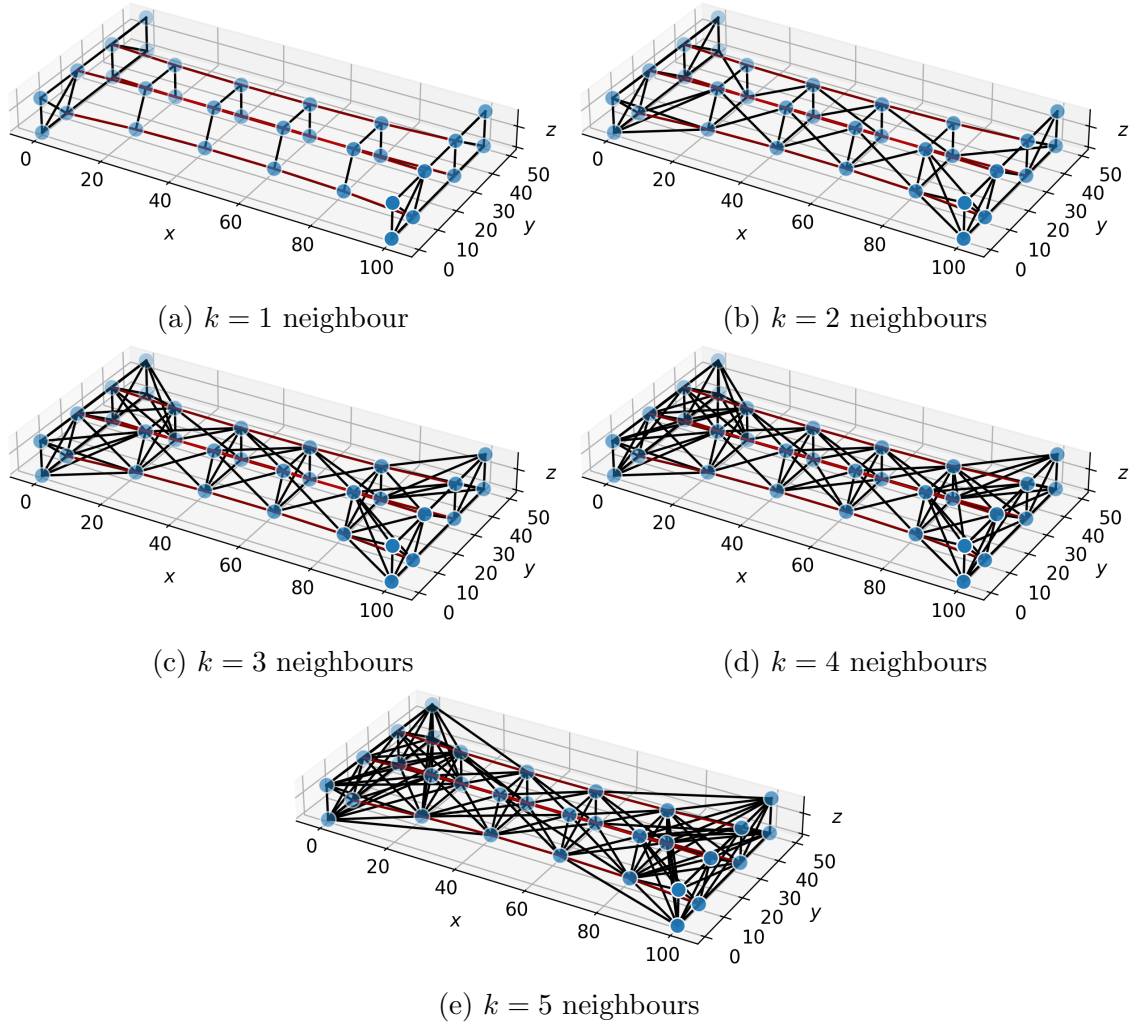
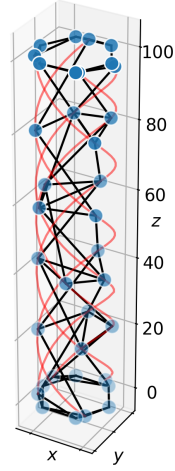
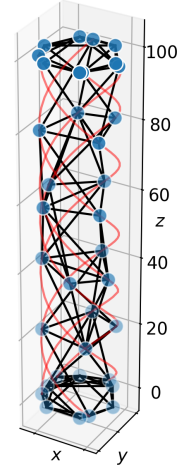


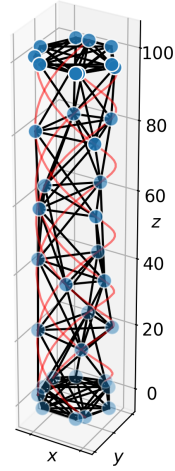
Figure B-3: k -neighbours triangulation - design case G3LC1 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



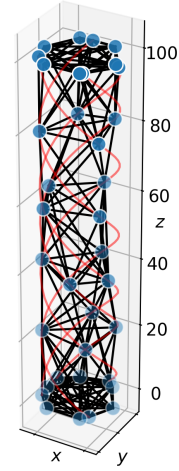
(a) $k = 1$ neighbour



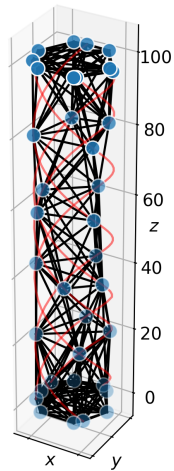
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

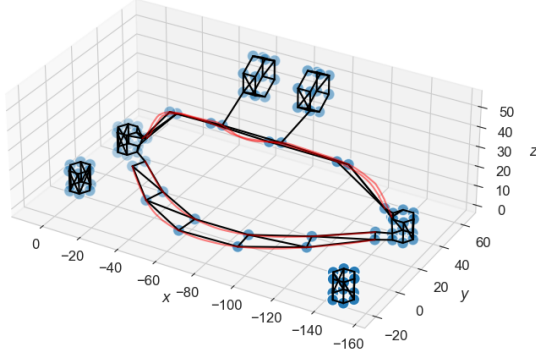


(d) $k = 4$ neighbours

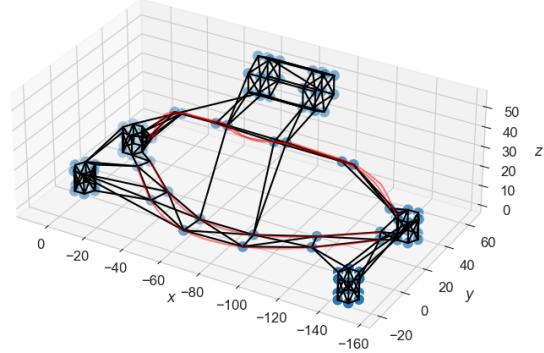


(e) $k = 5$ neighbours

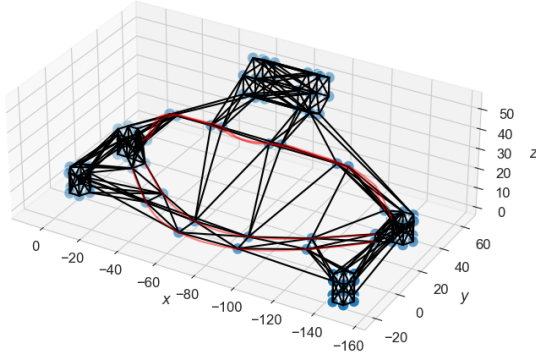
Figure B-4: k -neighbours triangulation - design case G4LC1 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



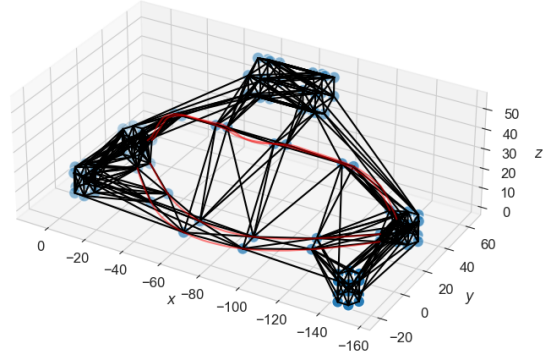
(a) $k = 1$ neighbour



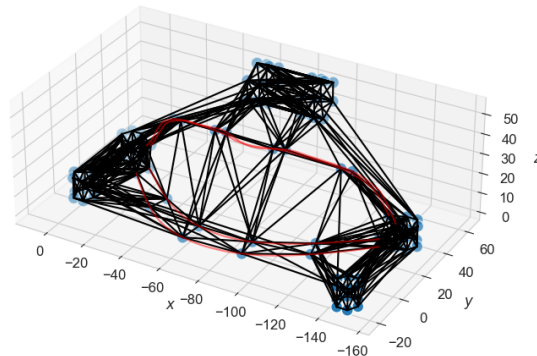
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

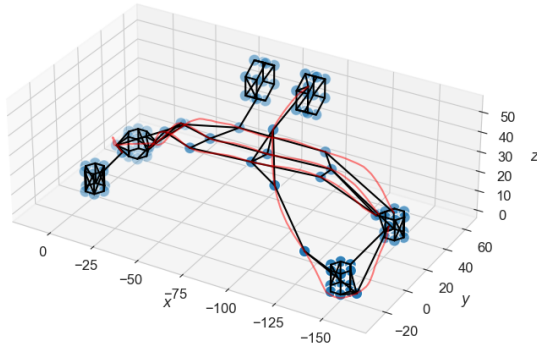


(d) $k = 4$ neighbours

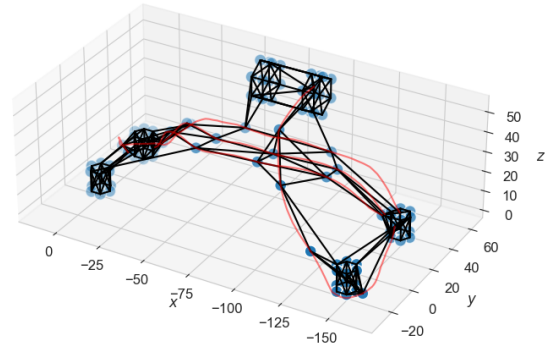


(e) $k = 5$ neighbours

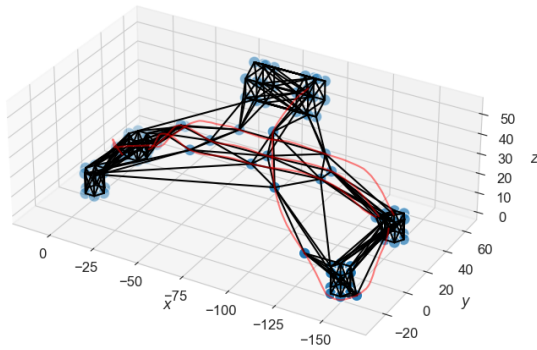
Figure B-5: k -neighbours triangulation - design case G5LC1 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



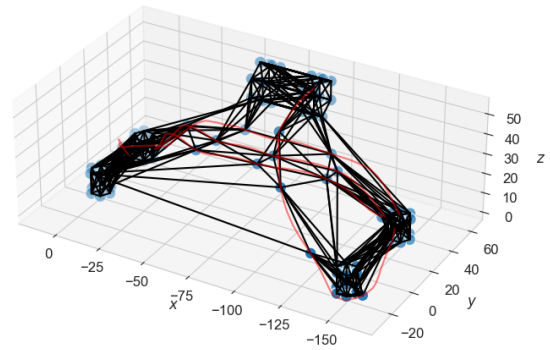
(a) $k = 1$ neighbour



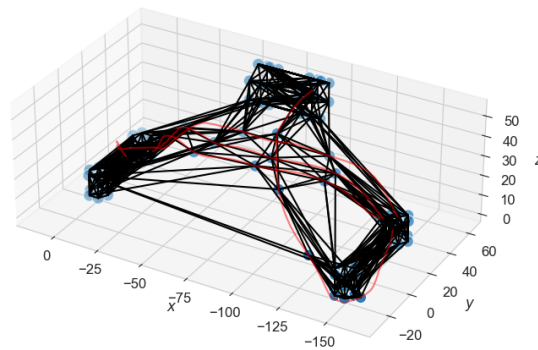
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

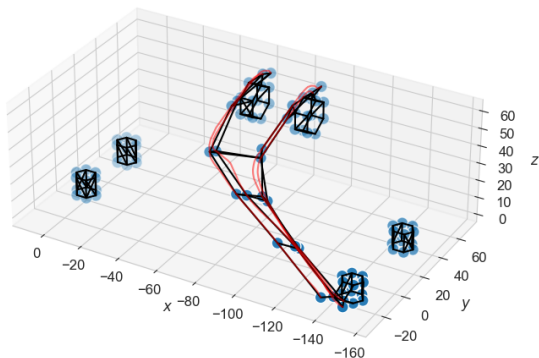


(d) $k = 4$ neighbours

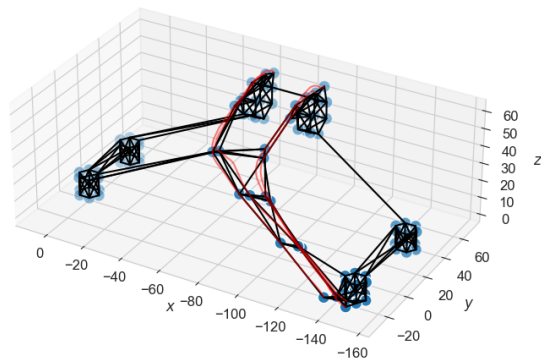


(e) $k = 5$ neighbours

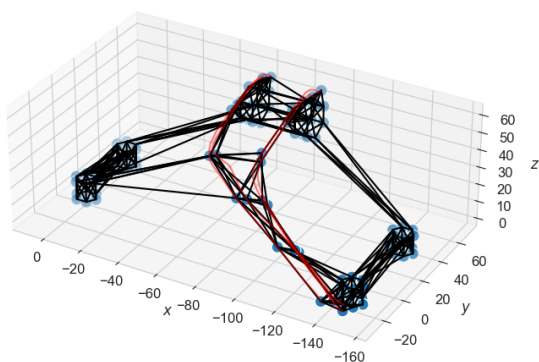
Figure B-6: k -neighbours triangulation - design case G5LC2 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



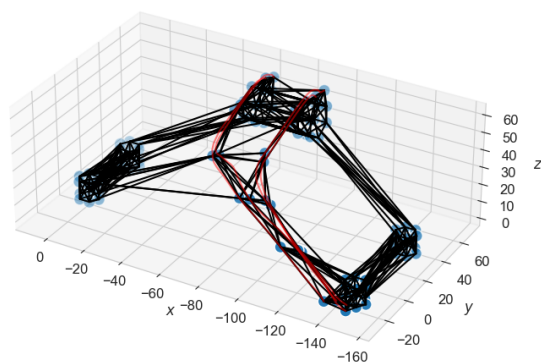
(a) $k = 1$ neighbour



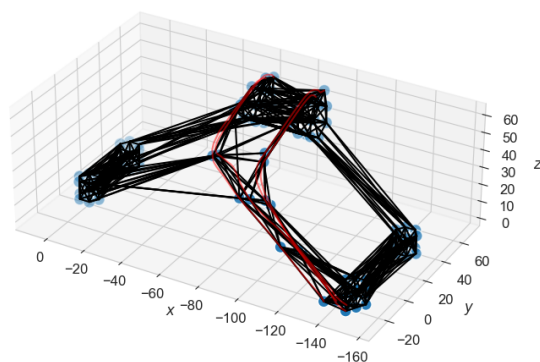
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

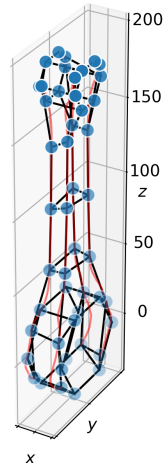


(d) $k = 4$ neighbours

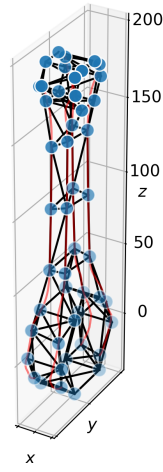


(e) $k = 5$ neighbours

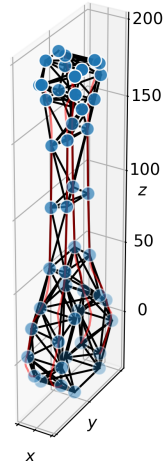
Figure B-7: k -neighbours triangulation - design case G5LC3 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



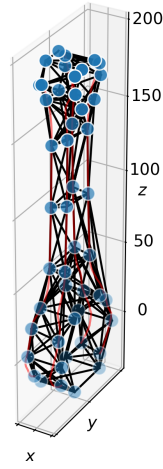
(a) $k = 1$ neighbour



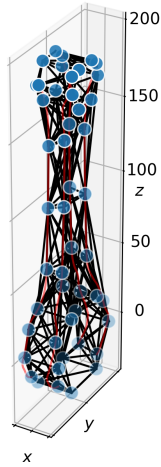
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

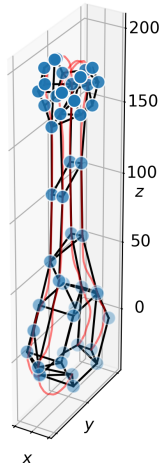


(d) $k = 4$ neighbours

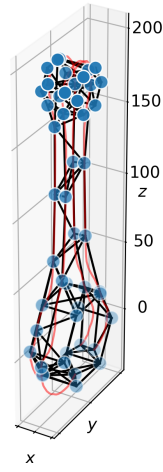


(e) $k = 5$ neighbours

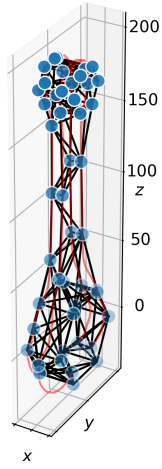
Figure B-8: k -neighbours triangulation - design case G6LC1 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



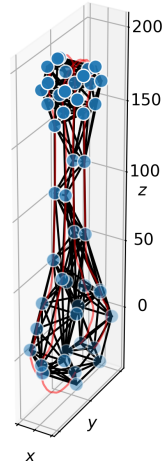
(a) $k = 1$ neighbour



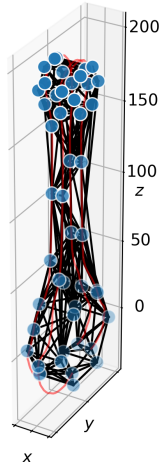
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

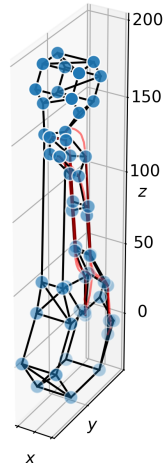


(d) $k = 4$ neighbours

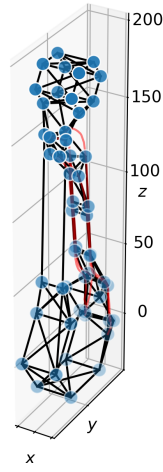


(e) $k = 5$ neighbours

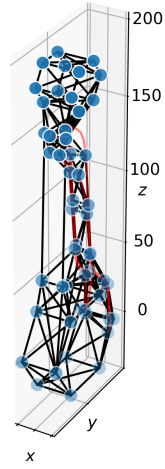
Figure B-9: k -neighbours triangulation - design case G6LC2 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



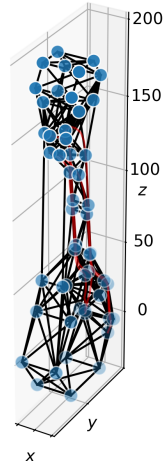
(a) $k = 1$ neighbour



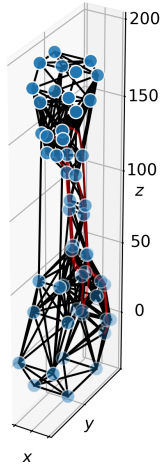
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

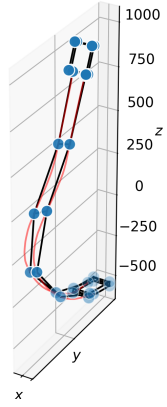


(d) $k = 4$ neighbours

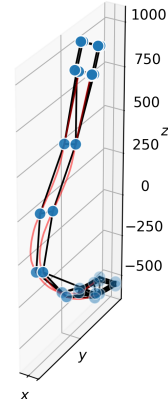


(e) $k = 5$ neighbours

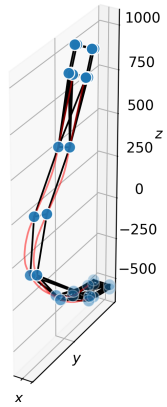
Figure B-10: k -neighbours triangulation - design case G6LC3 with different k values, 4 main stress paths with 6 points sampled per path, main stress paths in red



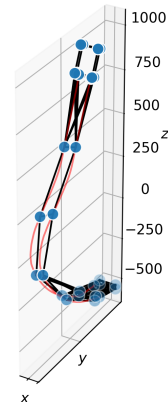
(a) $k = 1$ neighbour



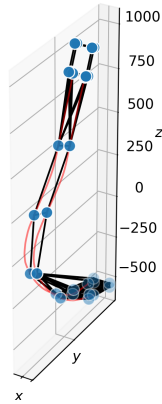
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

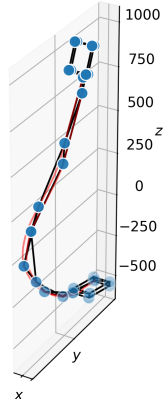


(d) $k = 4$ neighbours

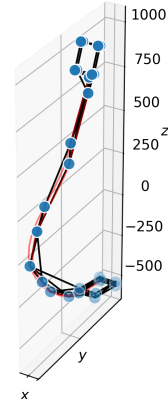


(e) $k = 5$ neighbours

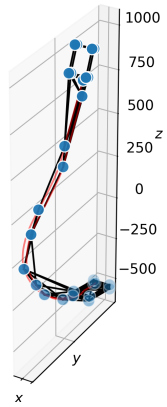
Figure B-11: k -neighbours triangulation - design case G7LC1 with different k values, 2 main stress paths with 6 points sampled per path, main stress paths in red



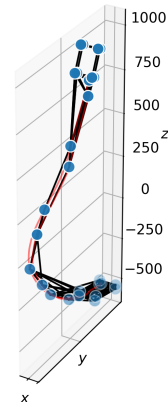
(a) $k = 1$ neighbour



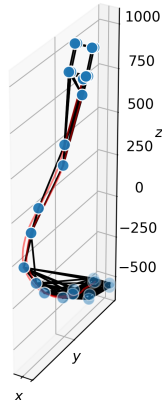
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours

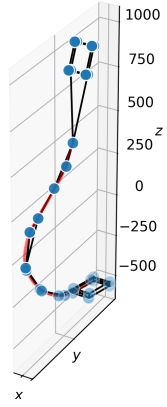


(d) $k = 4$ neighbours

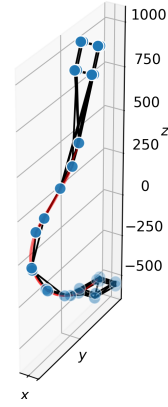


(e) $k = 5$ neighbours

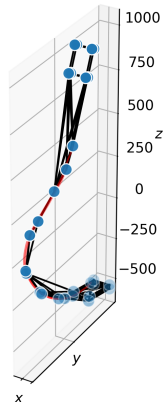
Figure B-12: k -neighbours triangulation - design case G7LC2 with different k values, 2 main stress paths with 6 points sampled per path, main stress paths in red



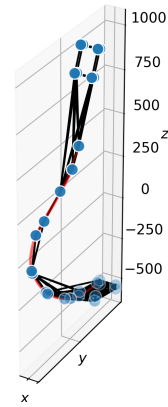
(a) $k = 1$ neighbour



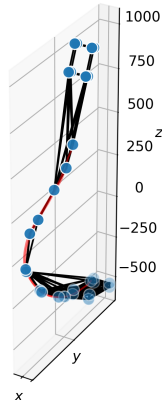
(b) $k = 2$ neighbours



(c) $k = 3$ neighbours



(d) $k = 4$ neighbours

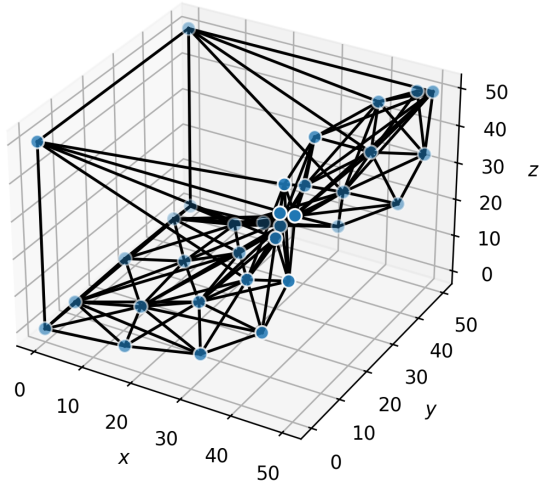


(e) $k = 5$ neighbours

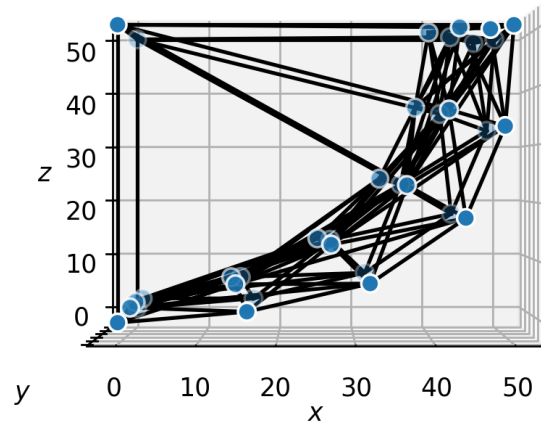
Figure B-13: k -neighbours triangulation - design case G7LC3 with different k values, 2 main stress paths with 6 points sampled per path, main stress paths in red

Appendix C

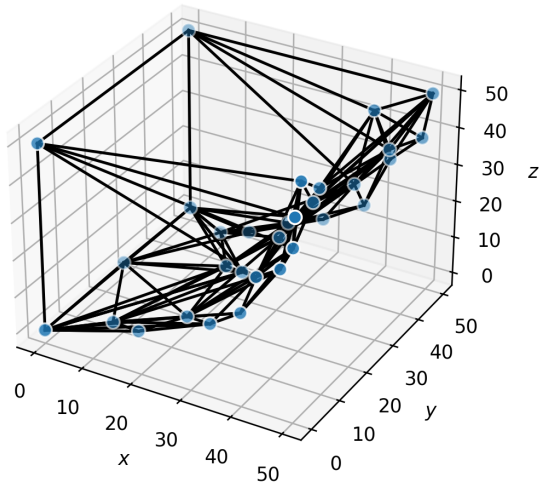
Structural Optimization Results



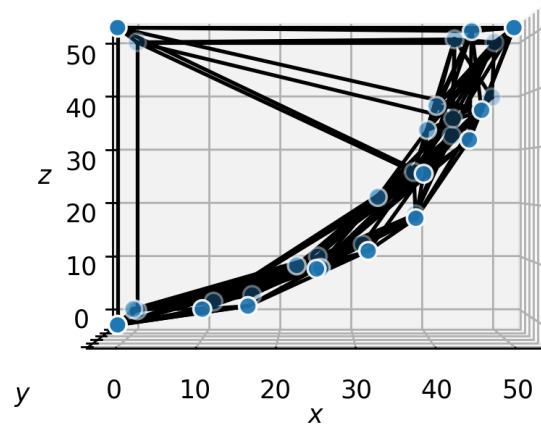
(a) Unoptimized isometric view



(b) Unoptimized side view

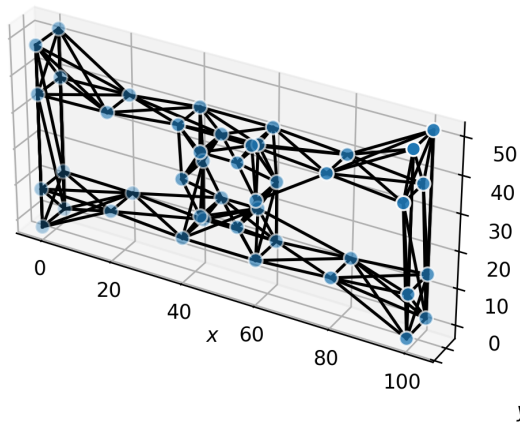


(c) Optimized isometric view

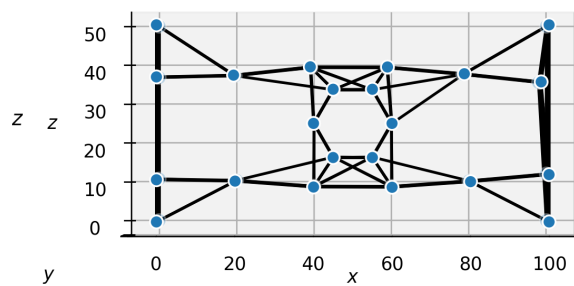


(d) Optimized side view

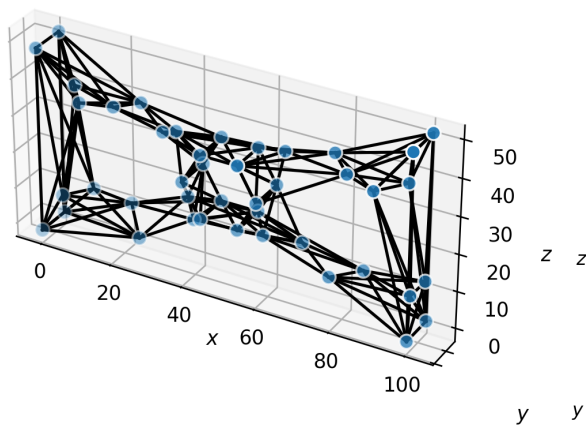
Figure C-1: Optimization results - design case G1LC1



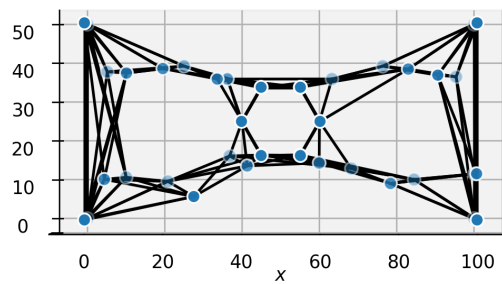
(a) Unoptimized isometric view



(b) Unoptimized side view

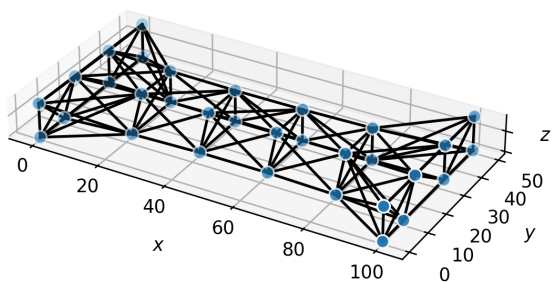


(c) Optimized isometric view

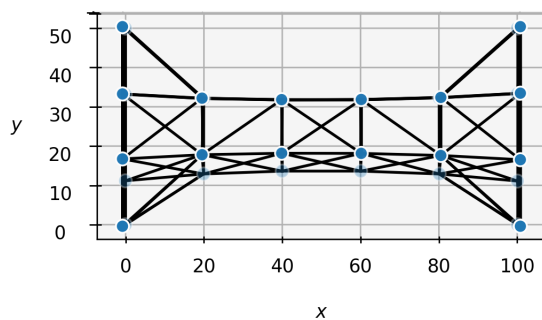


(d) Optimized side view

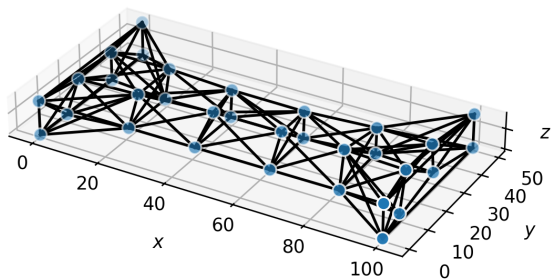
Figure C-2: Optimization results - design case G2LC1



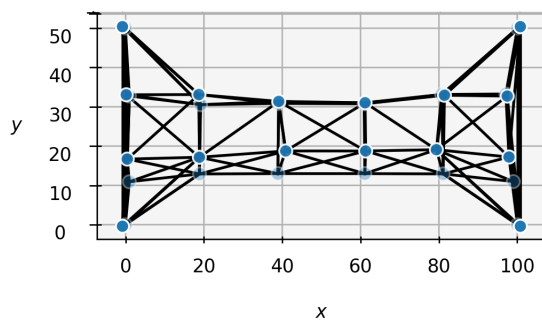
(a) Unoptimized isometric view



(b) Unoptimized top view

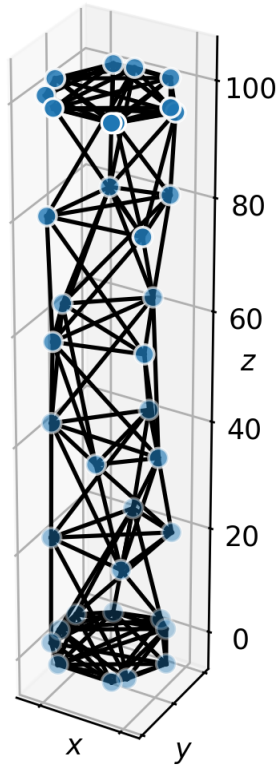


(c) Optimized isometric view

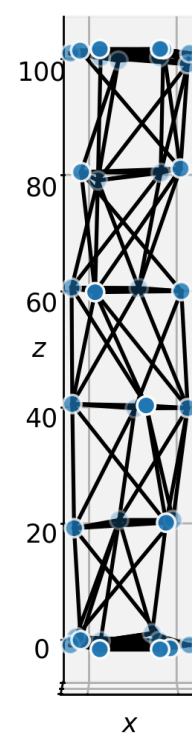


(d) Optimized top view

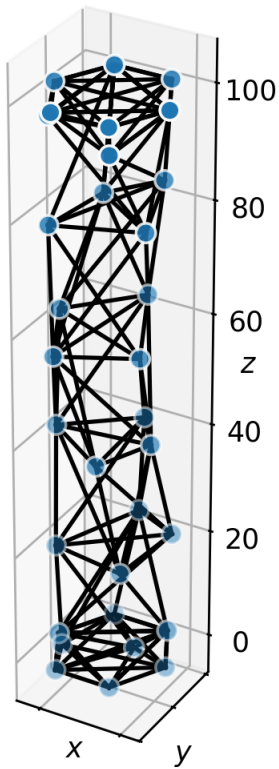
Figure C-3: Optimization results - design case G3LC1



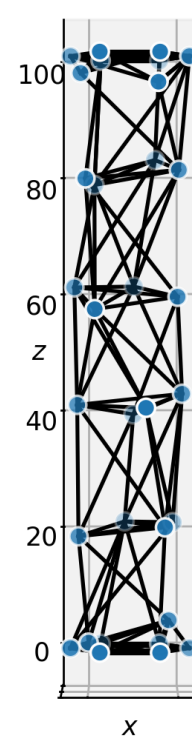
(a) Unoptimized isometric view



(b) Unoptimized side view

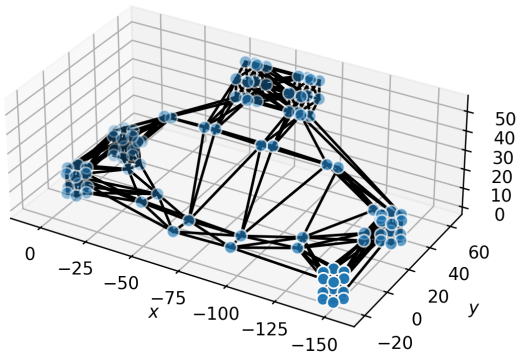


(c) Optimized isometric view

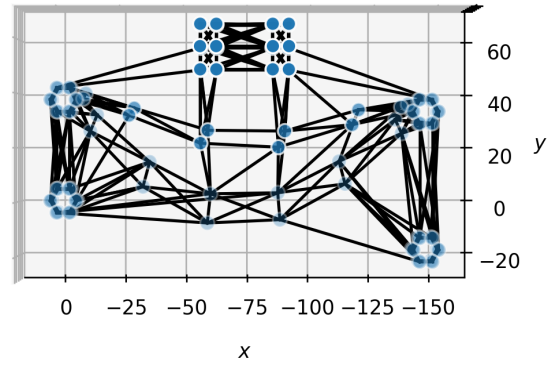


(d) Optimized side view

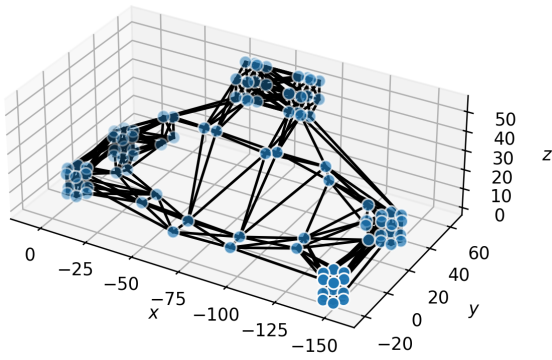
Figure C-4: Optimization results - design case G4LC1



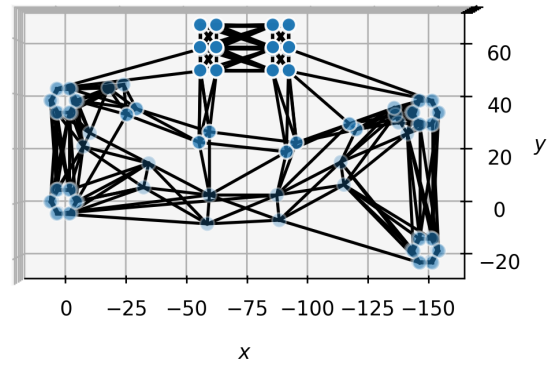
(a) Unoptimized isometric view



(b) Unoptimized top view

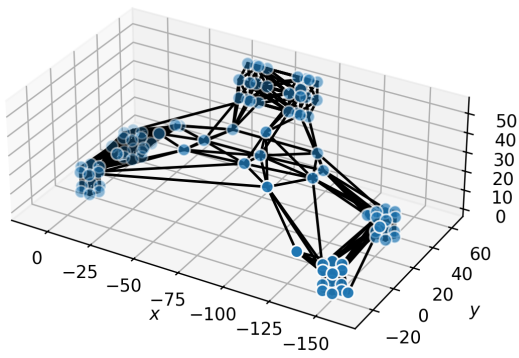


(c) Optimized isometric view

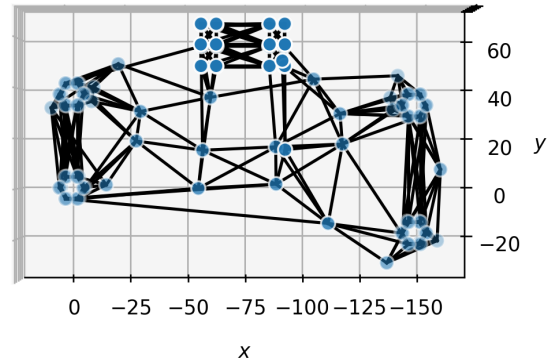


(d) Optimized top view

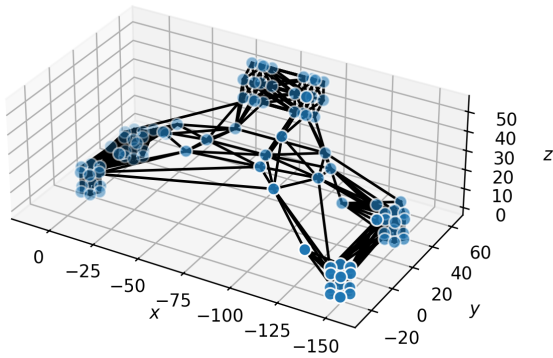
Figure C-5: Optimization results - design case G5LC1



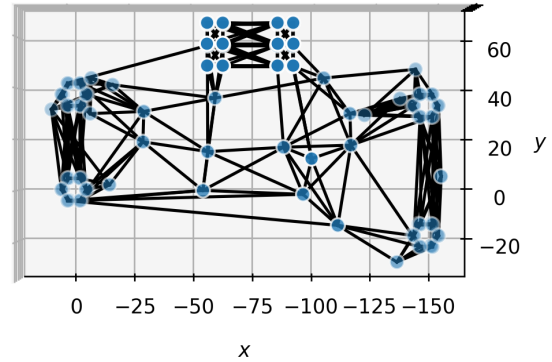
(a) Unoptimized isometric view



(b) Unoptimized top view

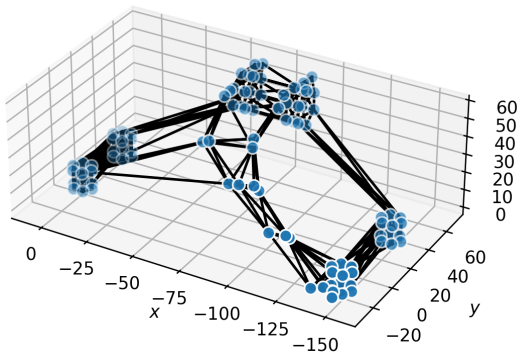


(c) Optimized isometric view

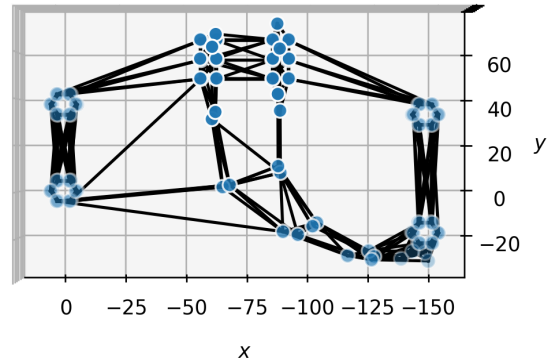


(d) Optimized top view

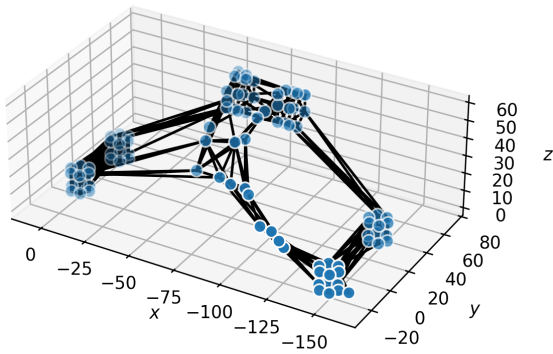
Figure C-6: Optimization results - design case G5LC2



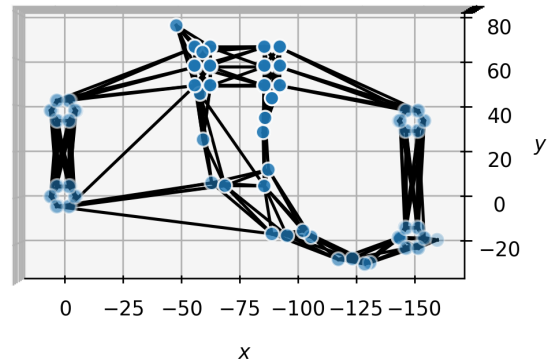
(a) Unoptimized isometric view



(b) Unoptimized top view

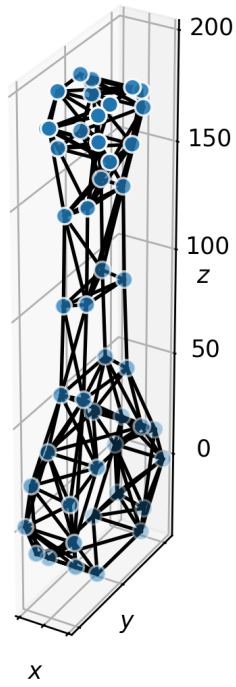


(c) Optimized isometric view

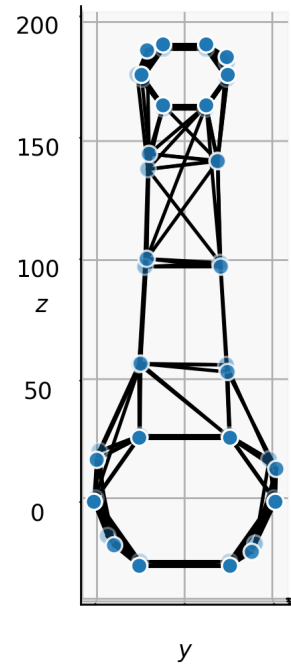


(d) Optimized top view

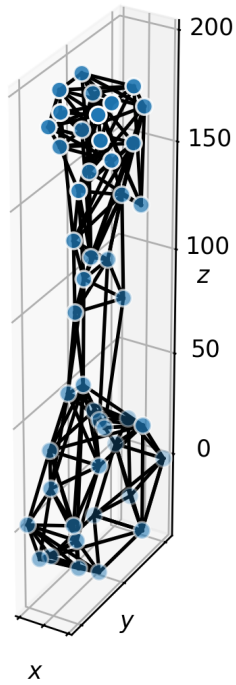
Figure C-7: Optimization results - design case G5LC3



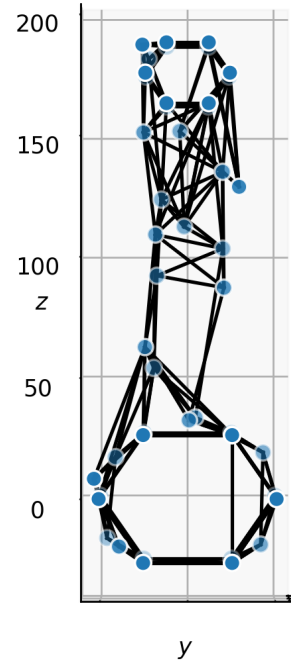
(a) Unoptimized isometric view



(b) Unoptimized front view

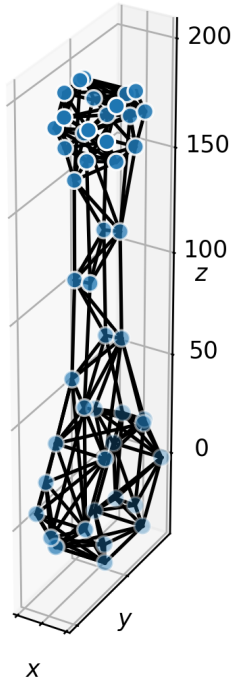


(c) Optimized isometric view

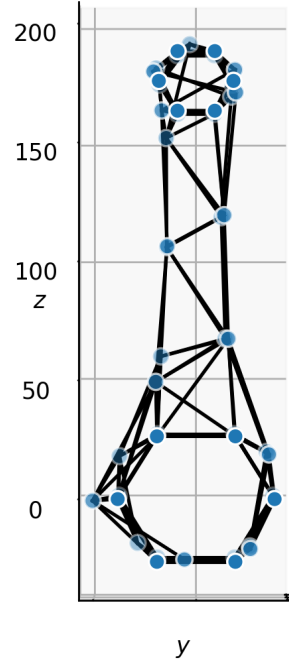


(d) Optimized front view

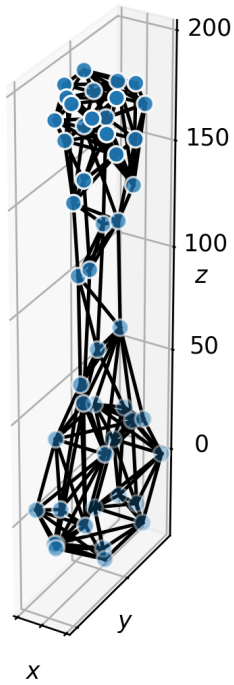
Figure C-8: Optimization results - design case G6LC1



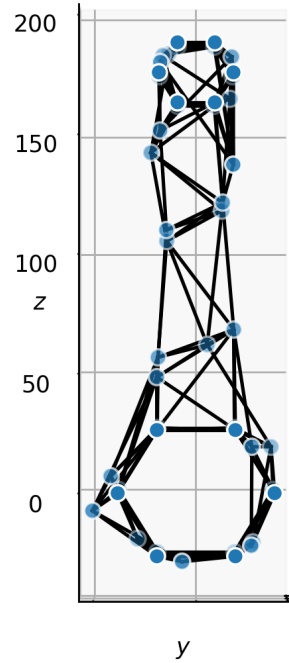
(a) Unoptimized isometric view



(b) Unoptimized front view

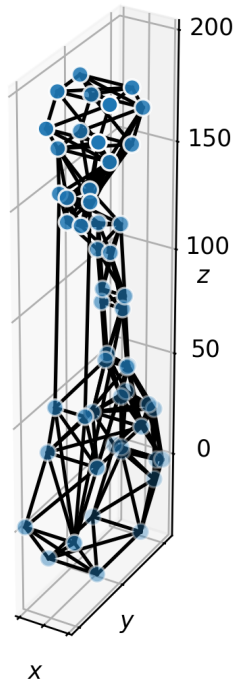


(c) Optimized isometric view

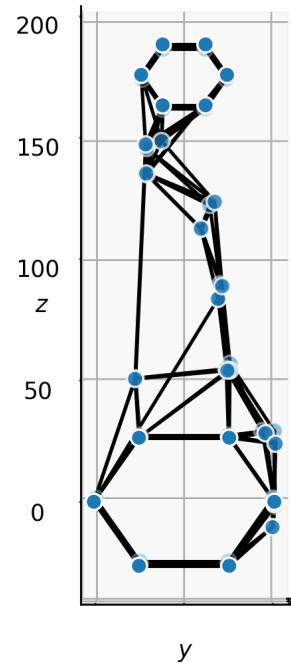


(d) Optimized front view

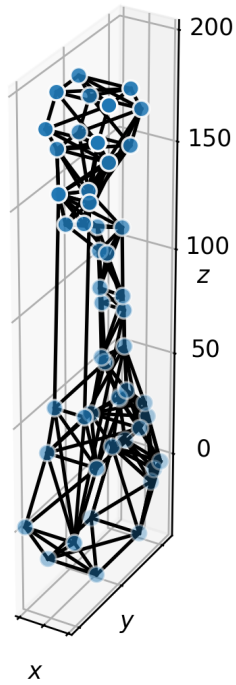
Figure C-9: Optimization results - design case G6LC2



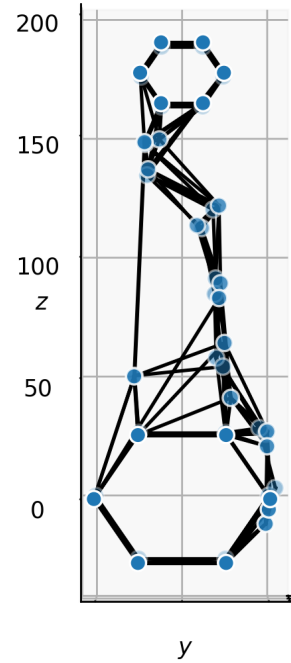
(a) Unoptimized isometric view



(b) Unoptimized front view

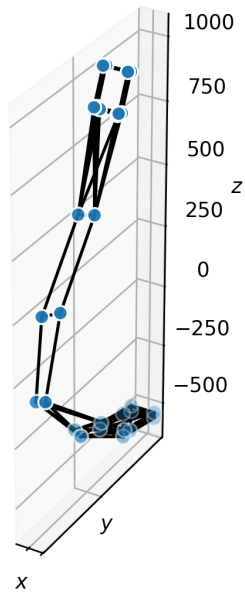


(c) Optimized isometric view

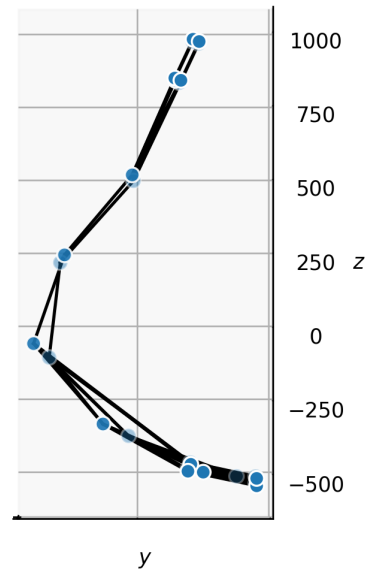


(d) Optimized front view

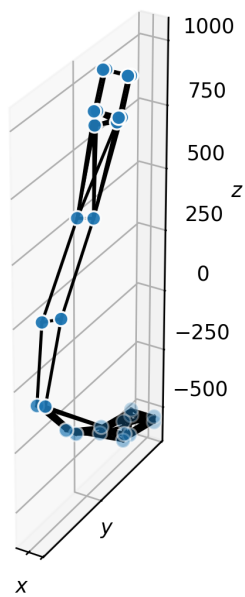
Figure C-10: Optimization results - design case G6LC3



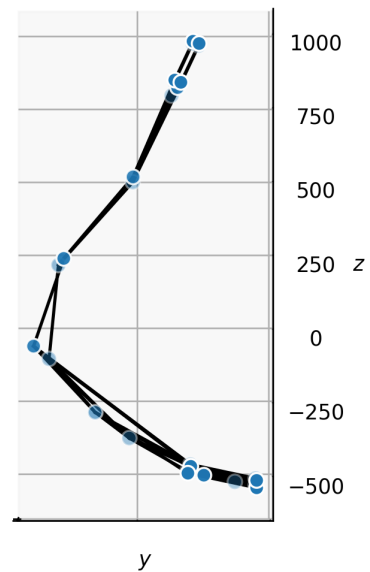
(a) Unoptimized isometric view



(b) Unoptimized side view

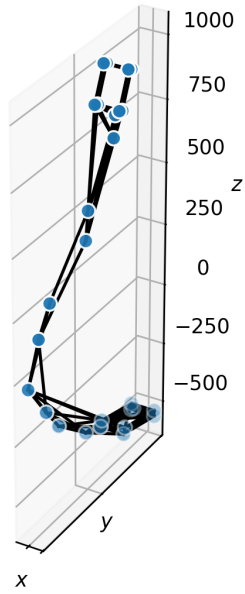


(c) Optimized isometric view

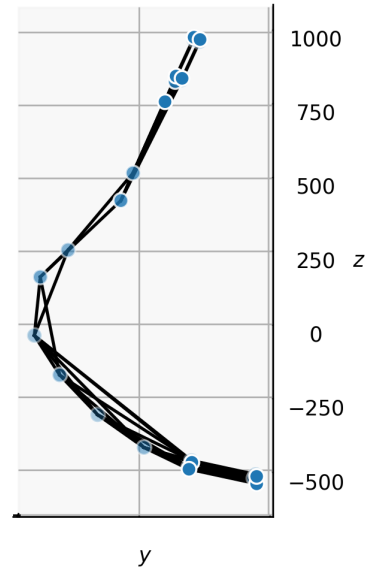


(d) Optimized side view

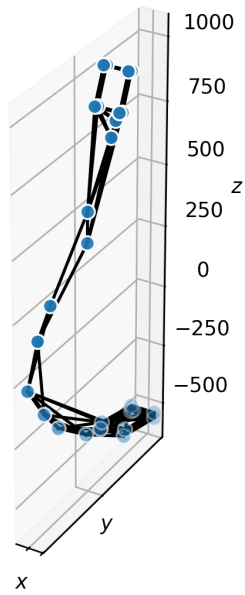
Figure C-11: Optimization results - design case G7LC1



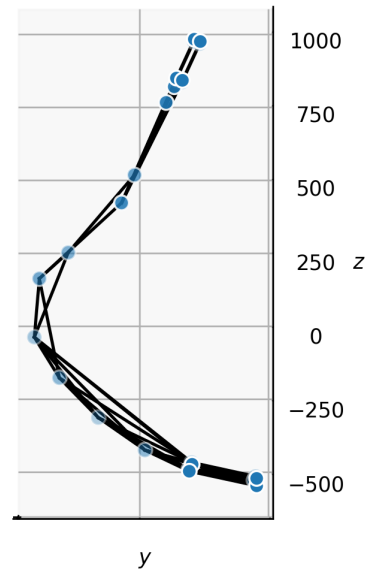
(a) Unoptimized isometric view



(b) Unoptimized side view

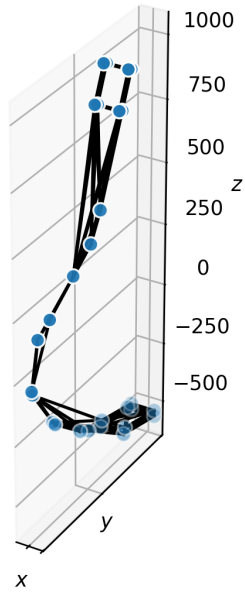


(c) Optimized isometric view

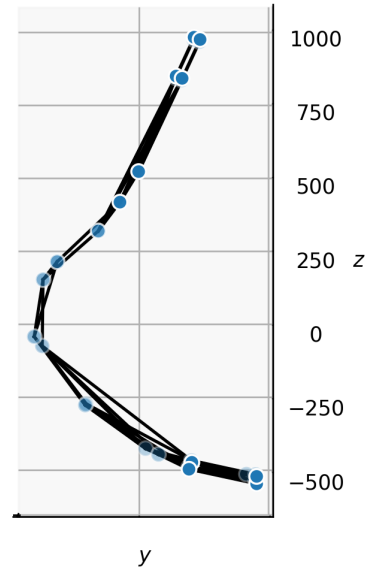


(d) Optimized side view

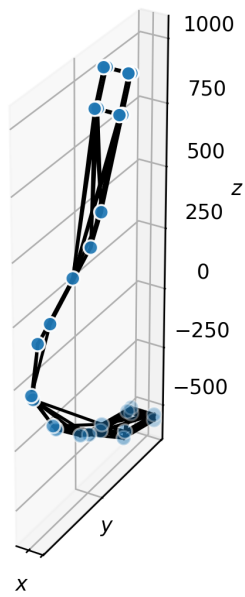
Figure C-12: Optimization results - design case G7LC2



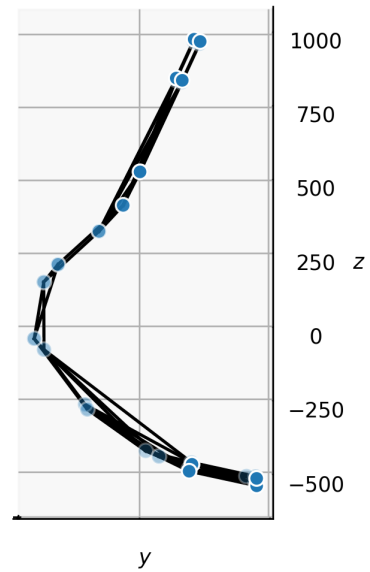
(a) Unoptimized isometric view



(b) Unoptimized side view



(c) Optimized isometric view



(d) Optimized side view

Figure C-13: Optimization results - design case G7LC3

Appendix D

Sensitivity Maps

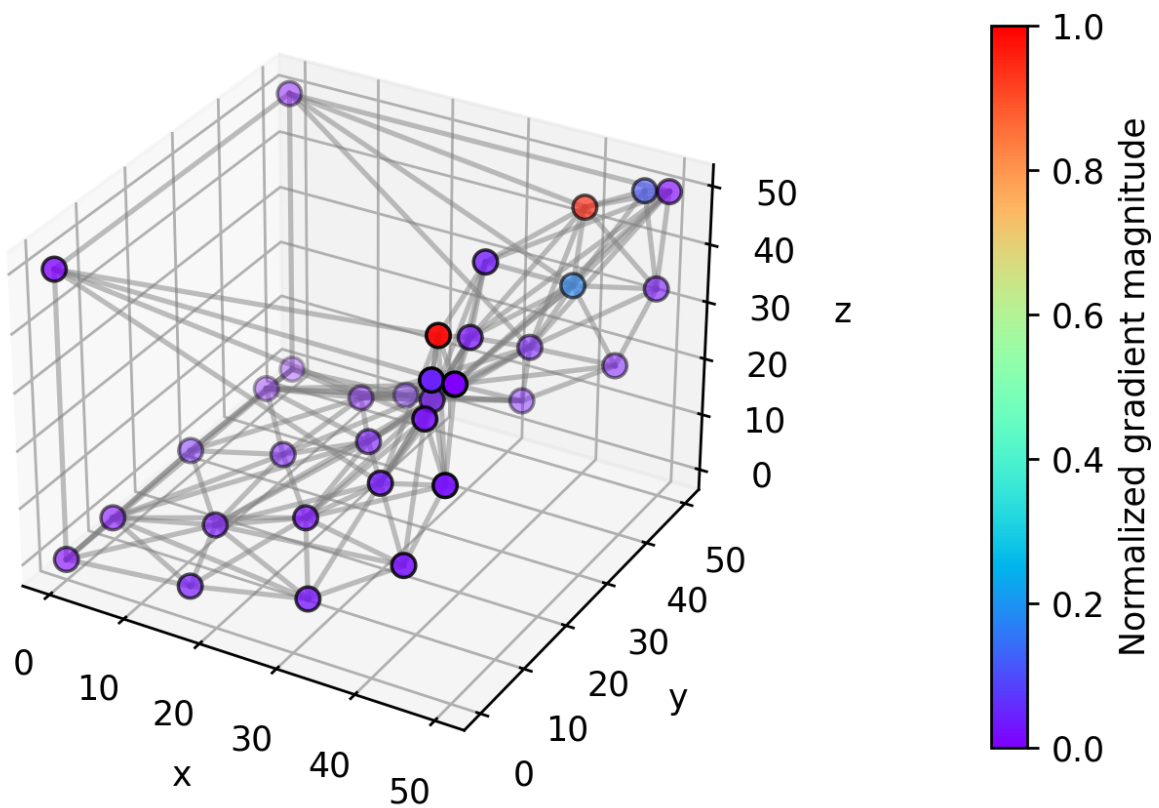


Figure D-1: Alignment coefficient sensitivity map - design case G1LC1

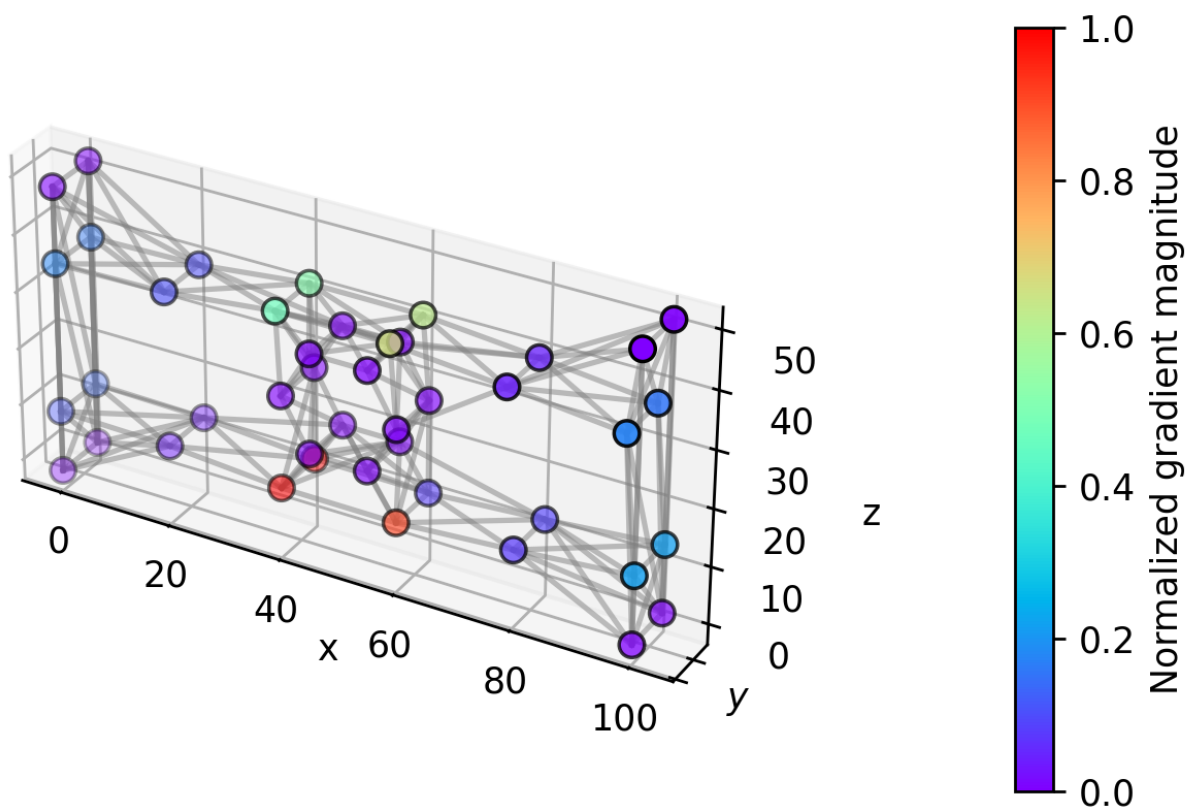
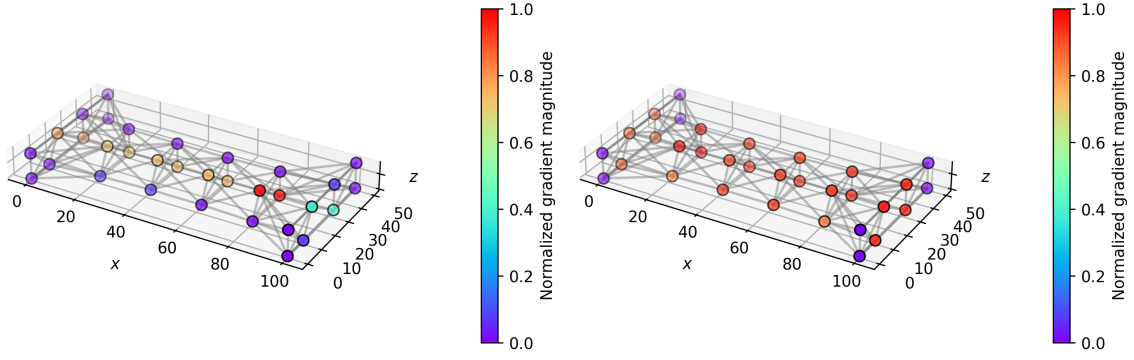
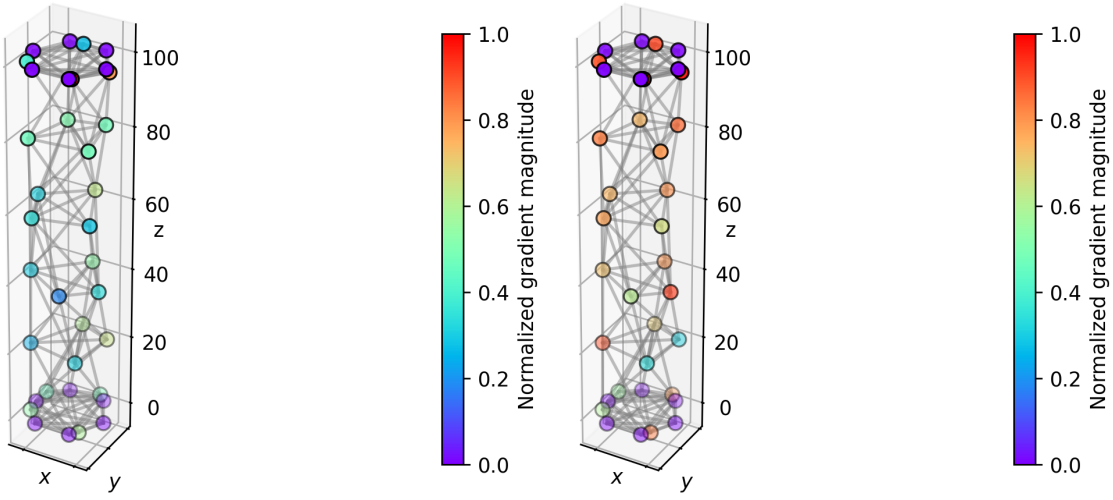


Figure D-2: Alignment coefficient sensitivity map - design case G2LC1



(a) Alignment coefficient sensitivity map (b) Symmetry coefficient sensitivity map

Figure D-3: Sensitivity maps - design case G3LC1



(a) Alignment coefficient sensitivity map (b) Symmetry coefficient sensitivity map

Figure D-4: Sensitivity maps - design case G4LC1

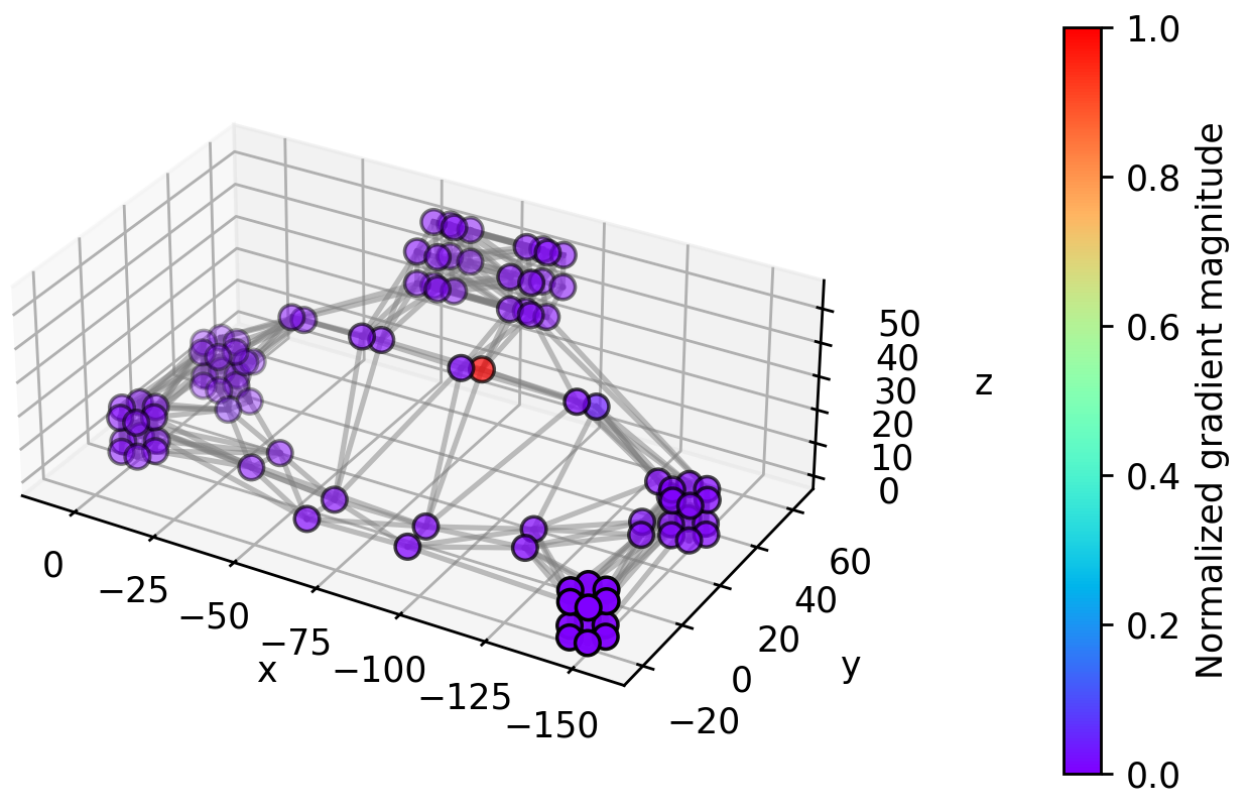


Figure D-5: Alignment coefficient sensitivity map - design case G5LC1

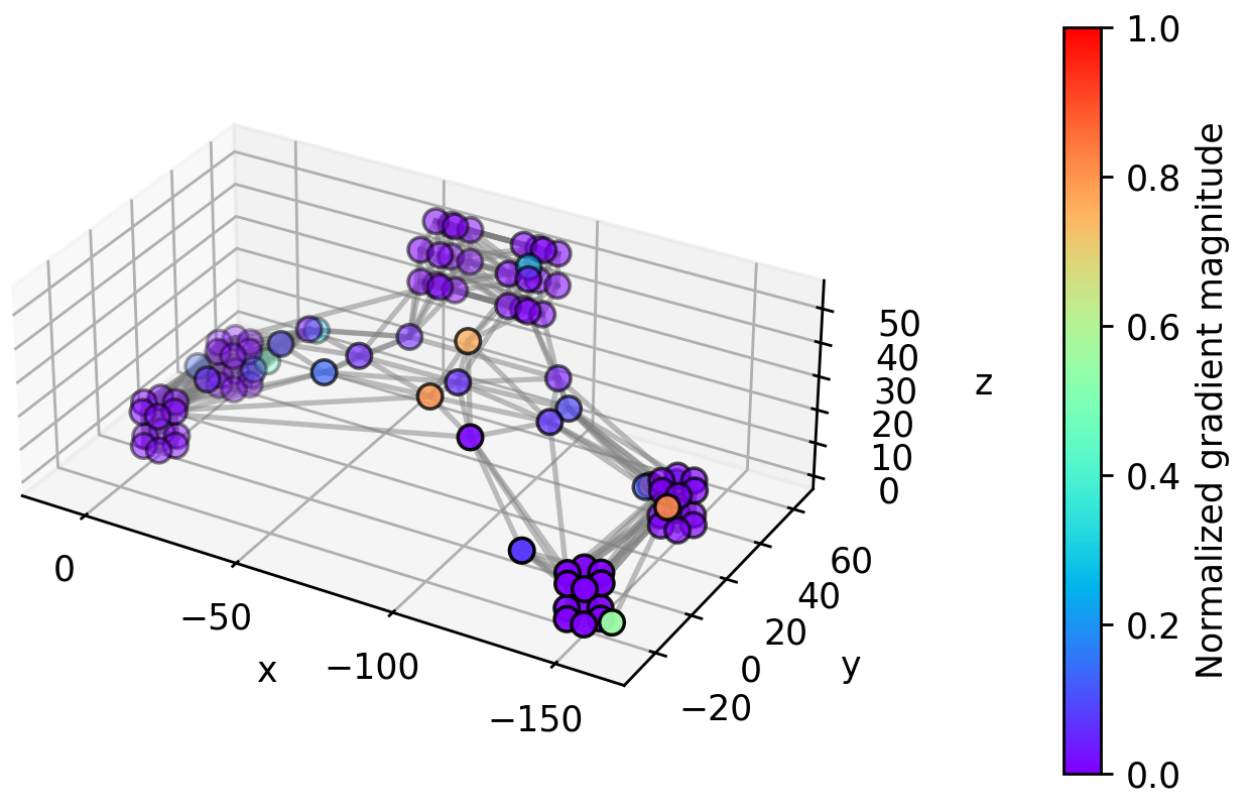


Figure D-6: Alignment coefficient sensitivity map - design case G5LC2

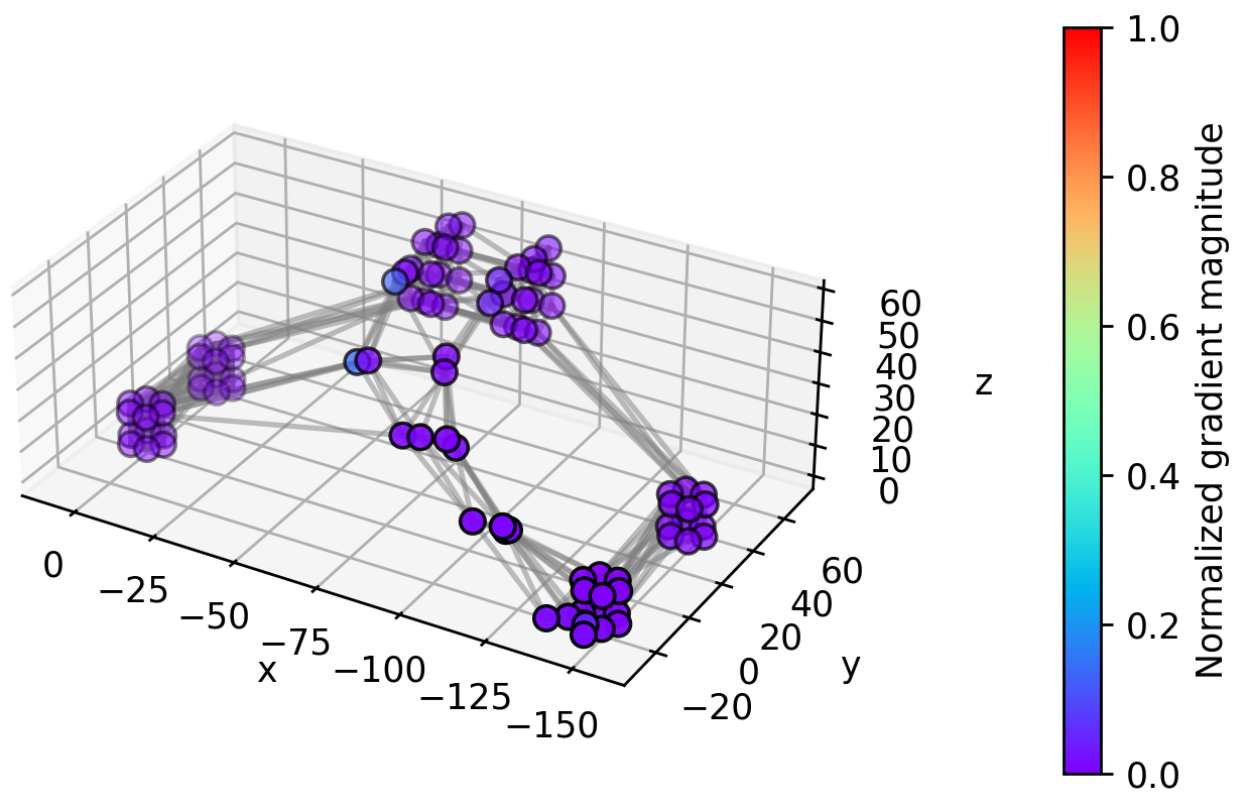


Figure D-7: Alignment coefficient sensitivity map - design case G5LC3

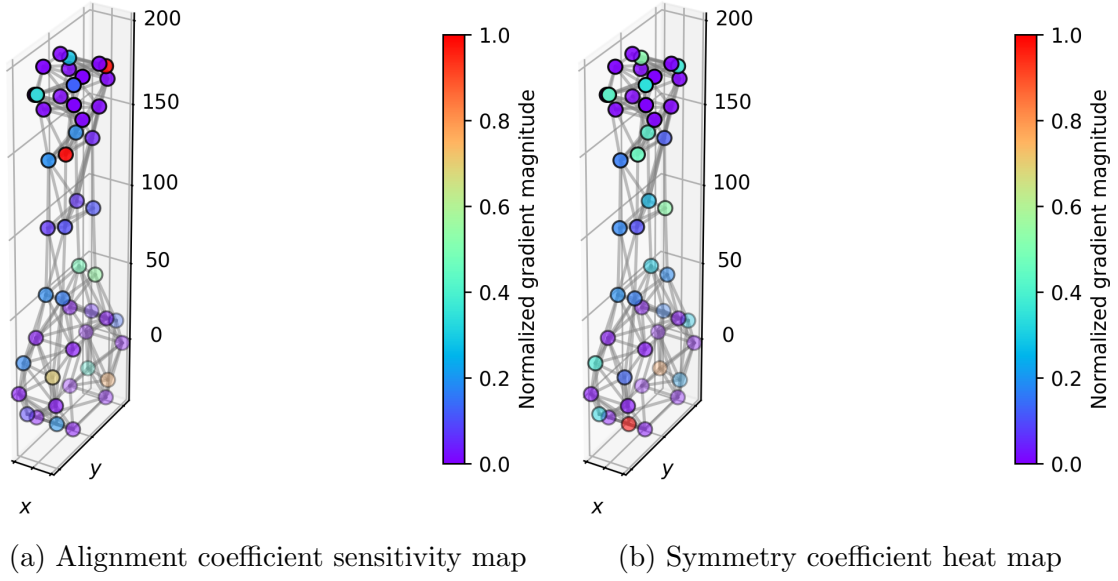


Figure D-8: Sensitivity maps - design case G6LC1

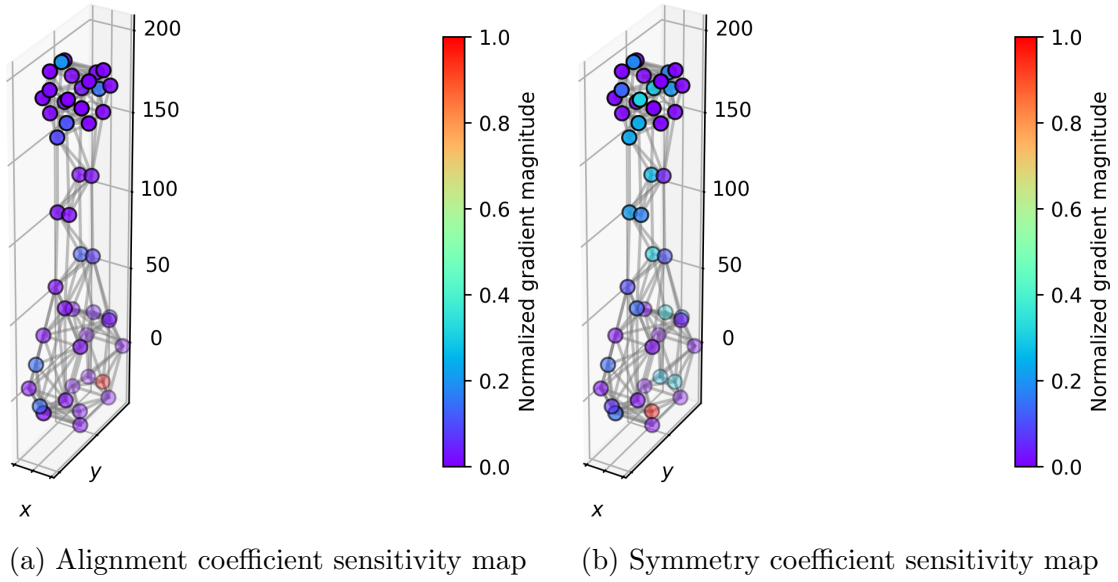


Figure D-9: Sensitivity maps - design case G6LC2

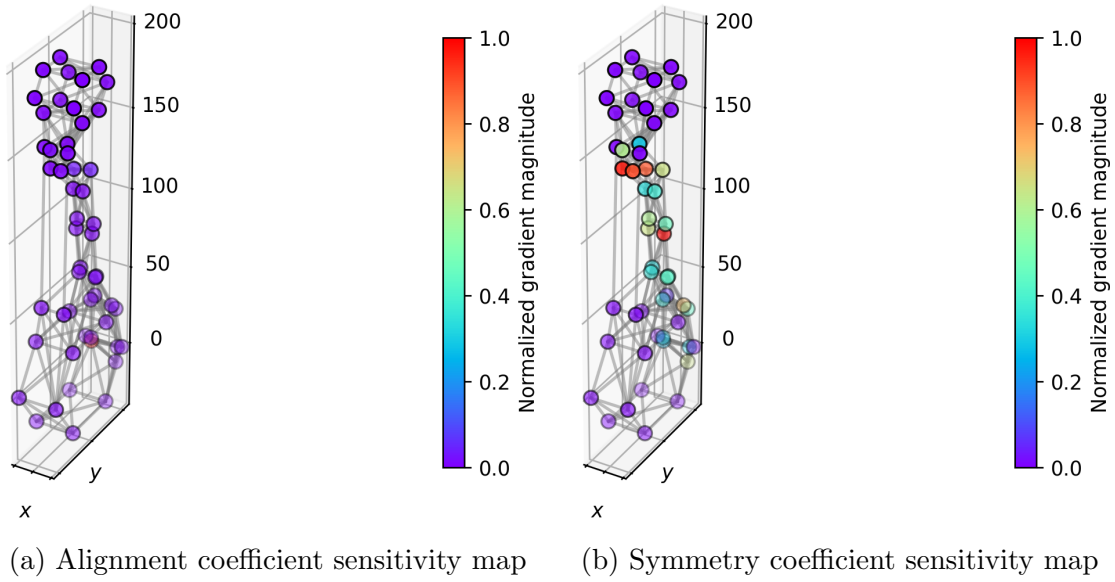


Figure D-10: Sensitivity maps - design case G6LC3

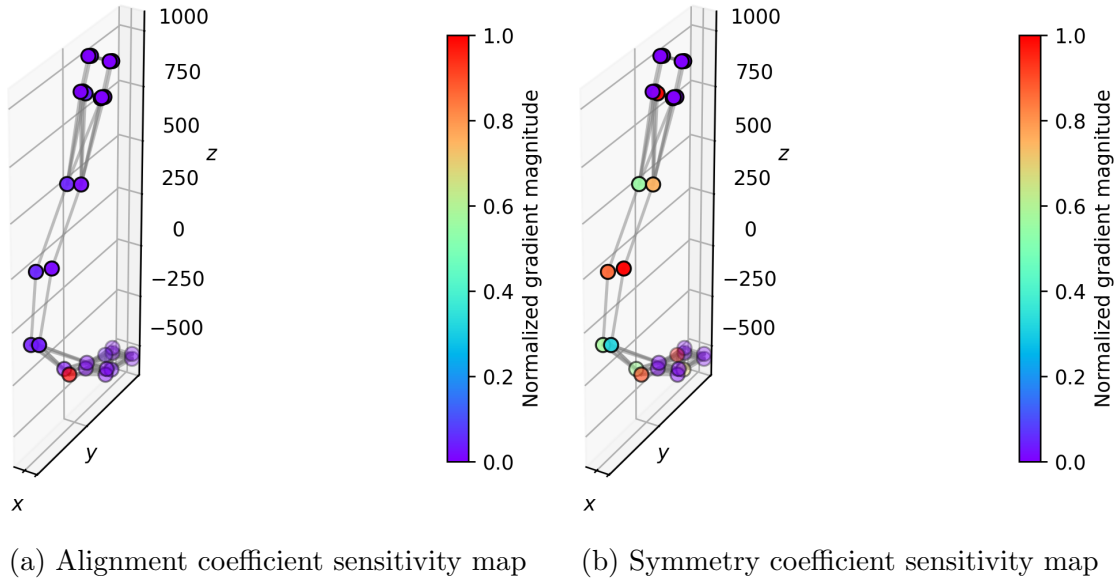
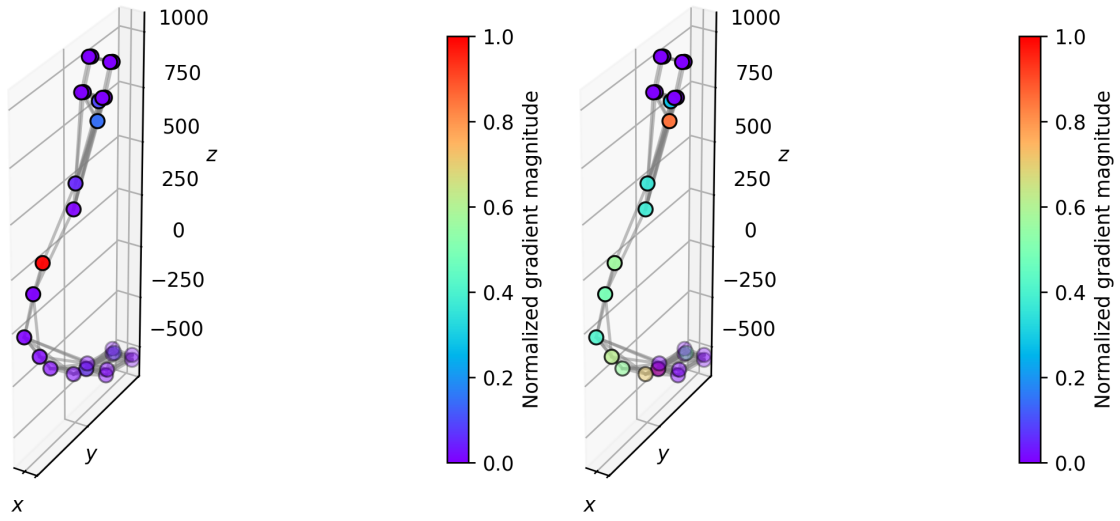
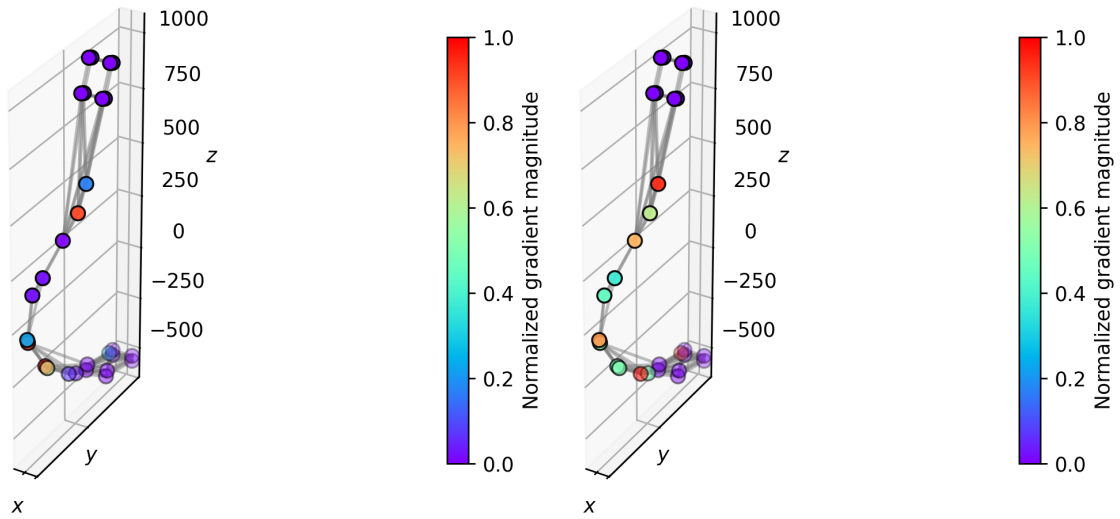


Figure D-11: Sensitivity maps - design case G7LC1



(a) Alignment coefficient sensitivity map (b) Symmetry coefficient sensitivity map

Figure D-12: Sensitivity maps - design case G7LC2



(a) Alignment coefficient sensitivity map (b) Symmetry coefficient sensitivity map

Figure D-13: Sensitivity maps - design case G7LC3

Appendix E

Edge Count Maps

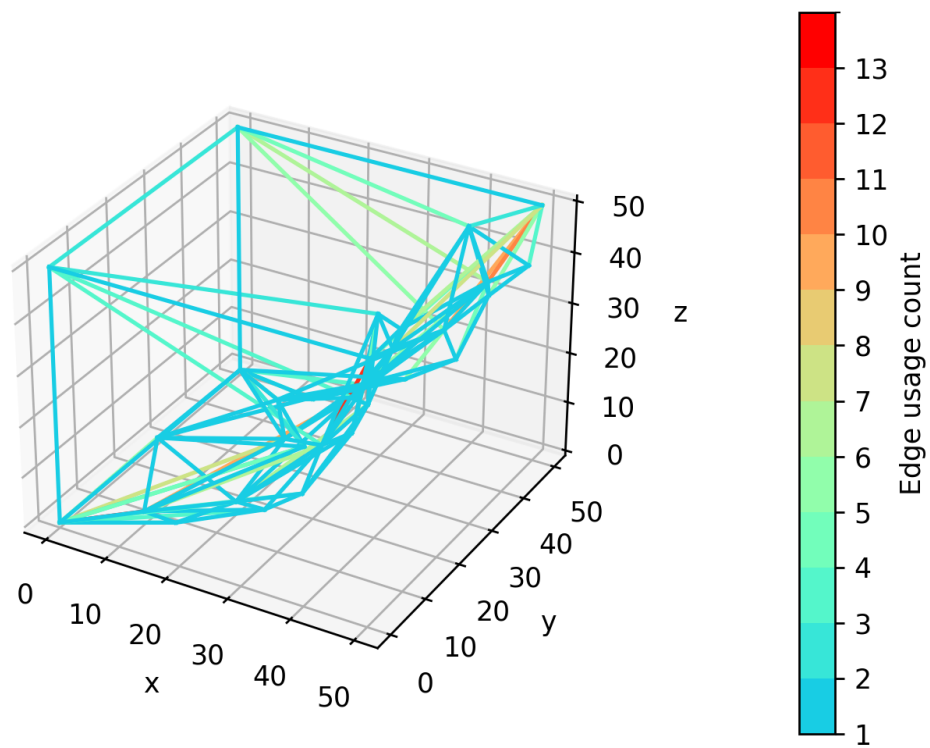


Figure E-1: Edge count map - design case G1LC1

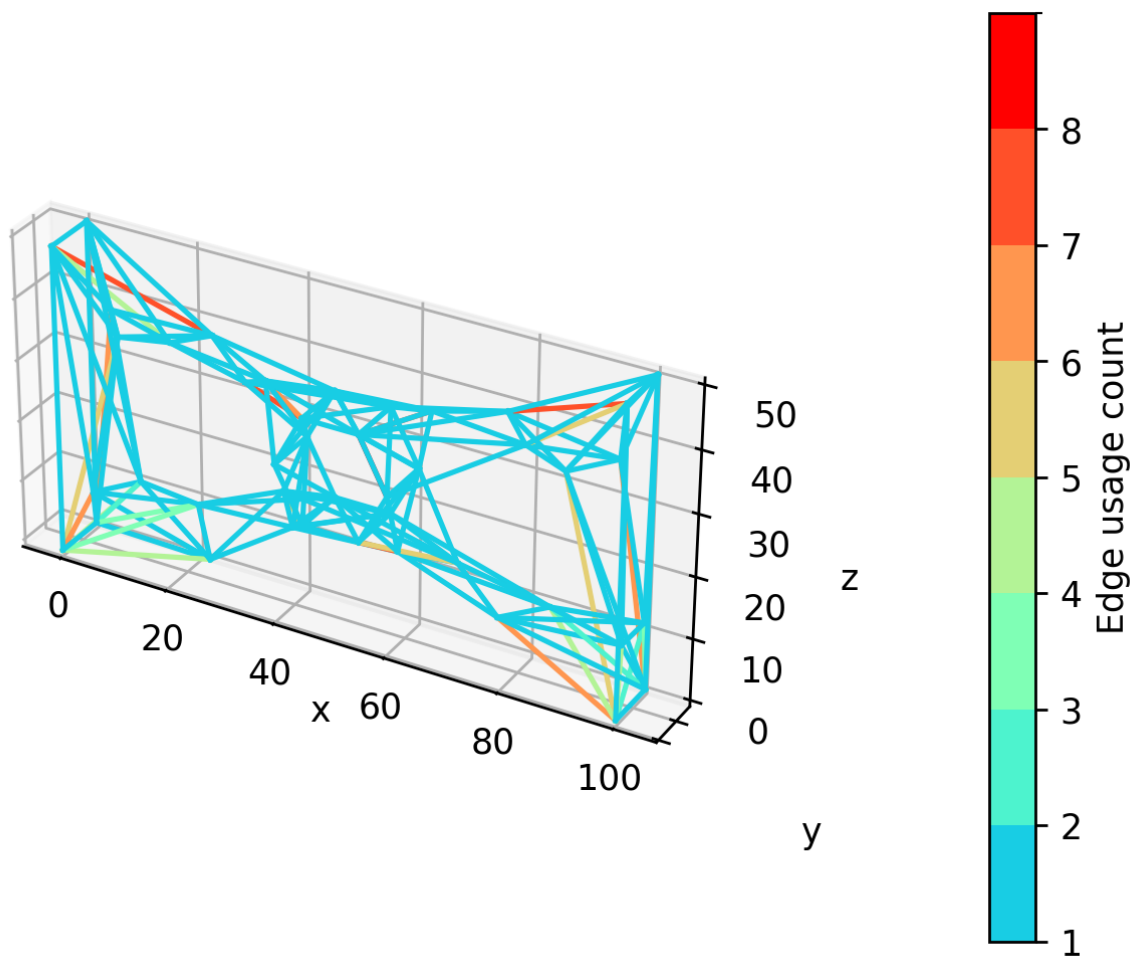


Figure E-2: Edge count map - design case G2LC1

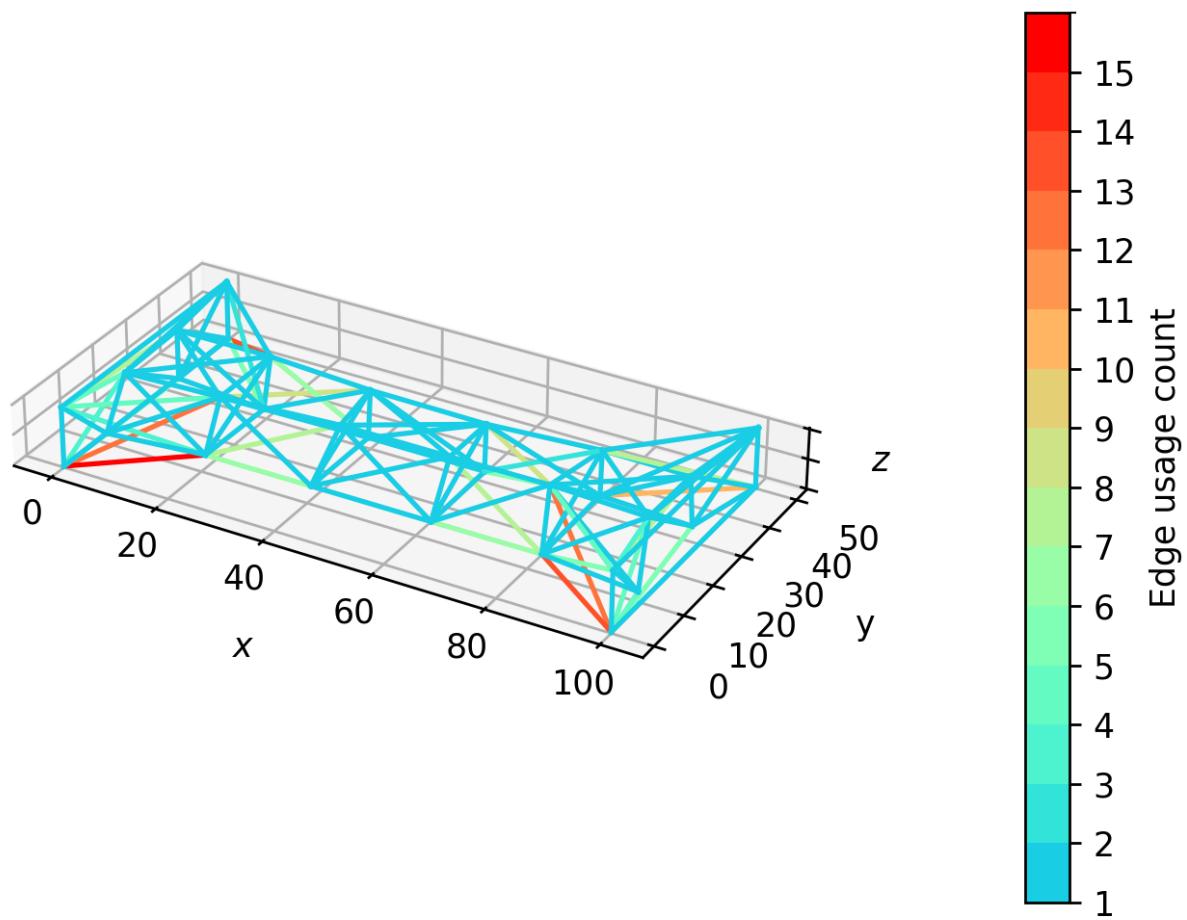


Figure E-3: Edge count map - design case G3LC1

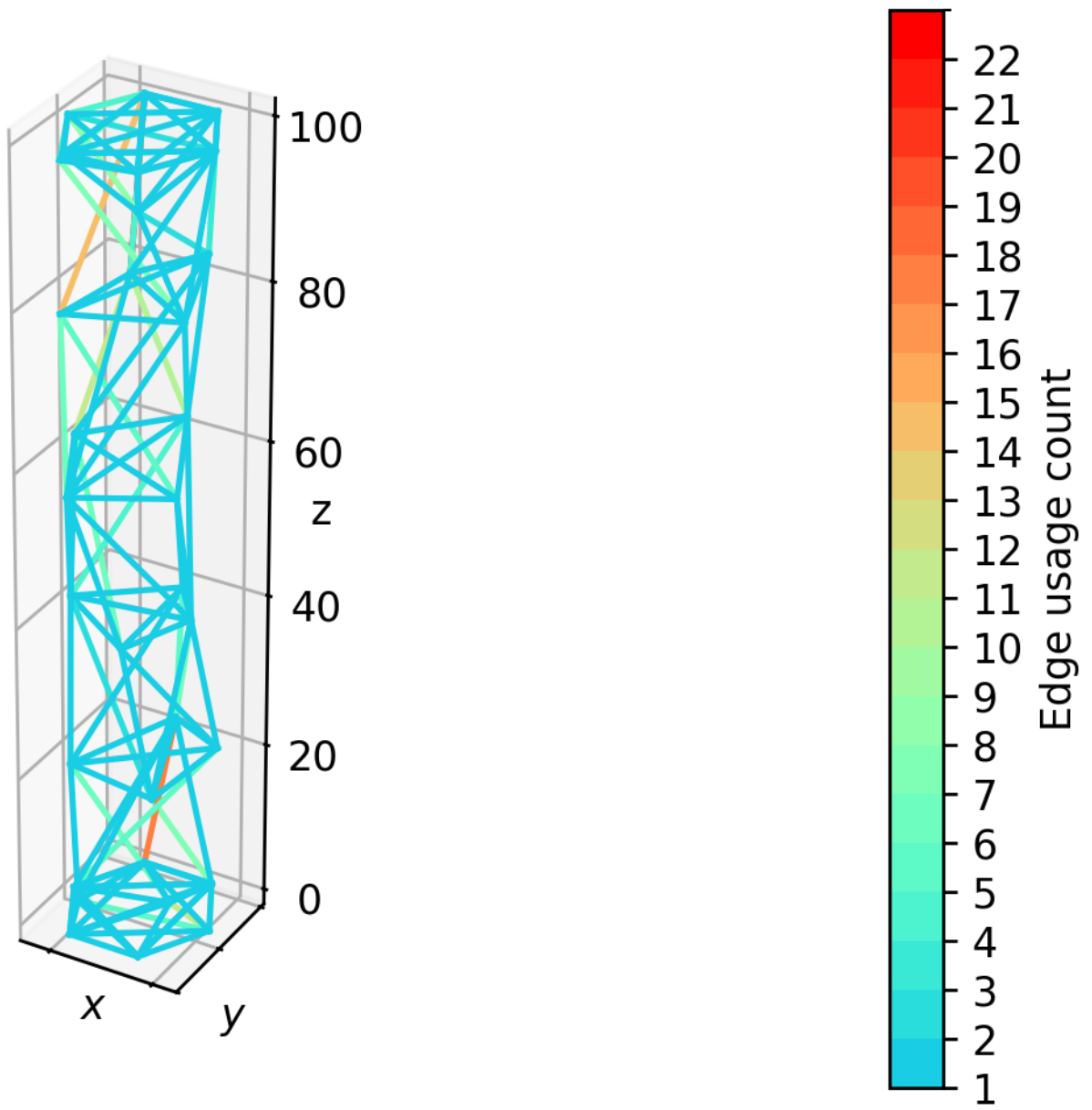


Figure E-4: Edge count map - design case G4LC1

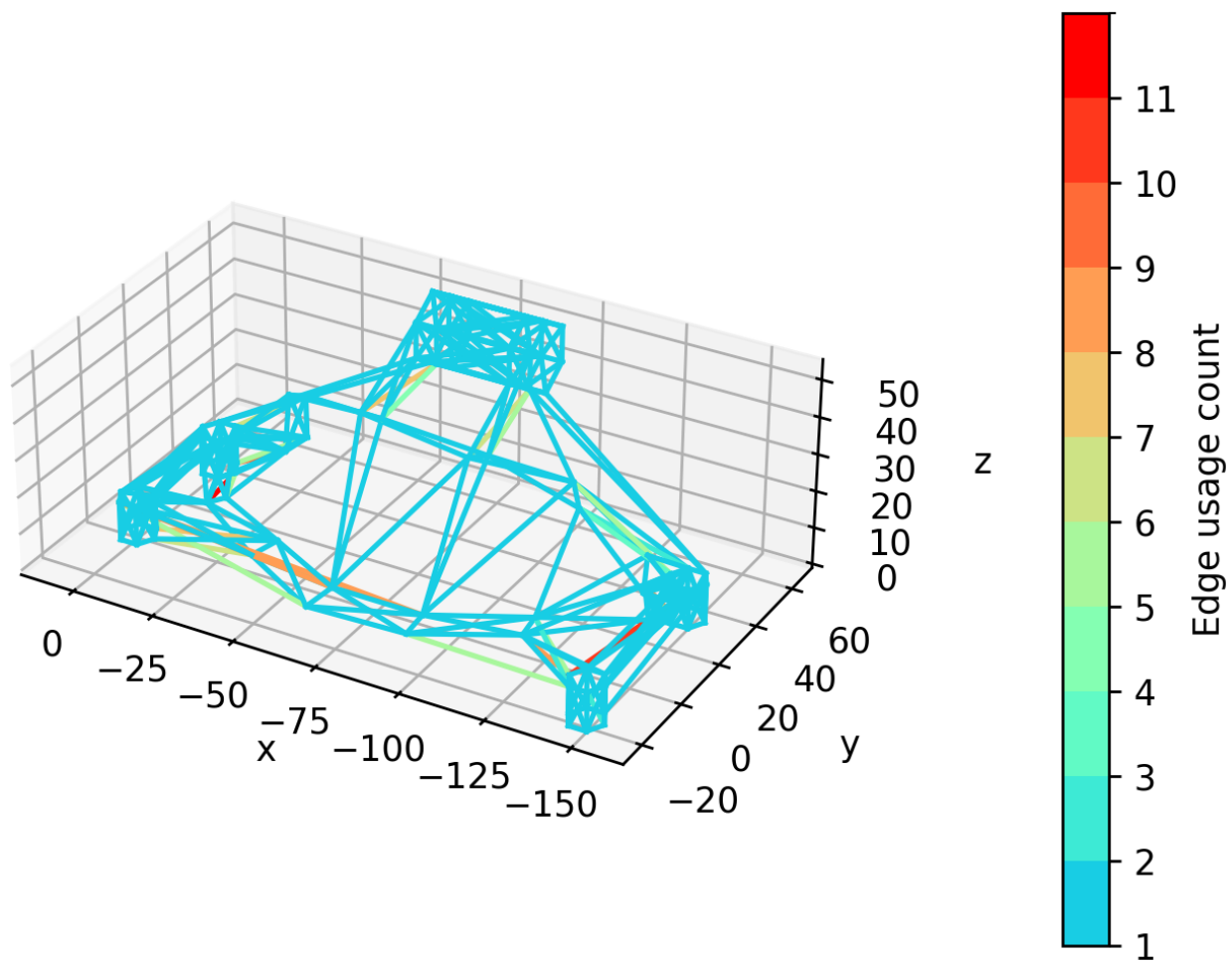


Figure E-5: Edge count map - design case G5LC1

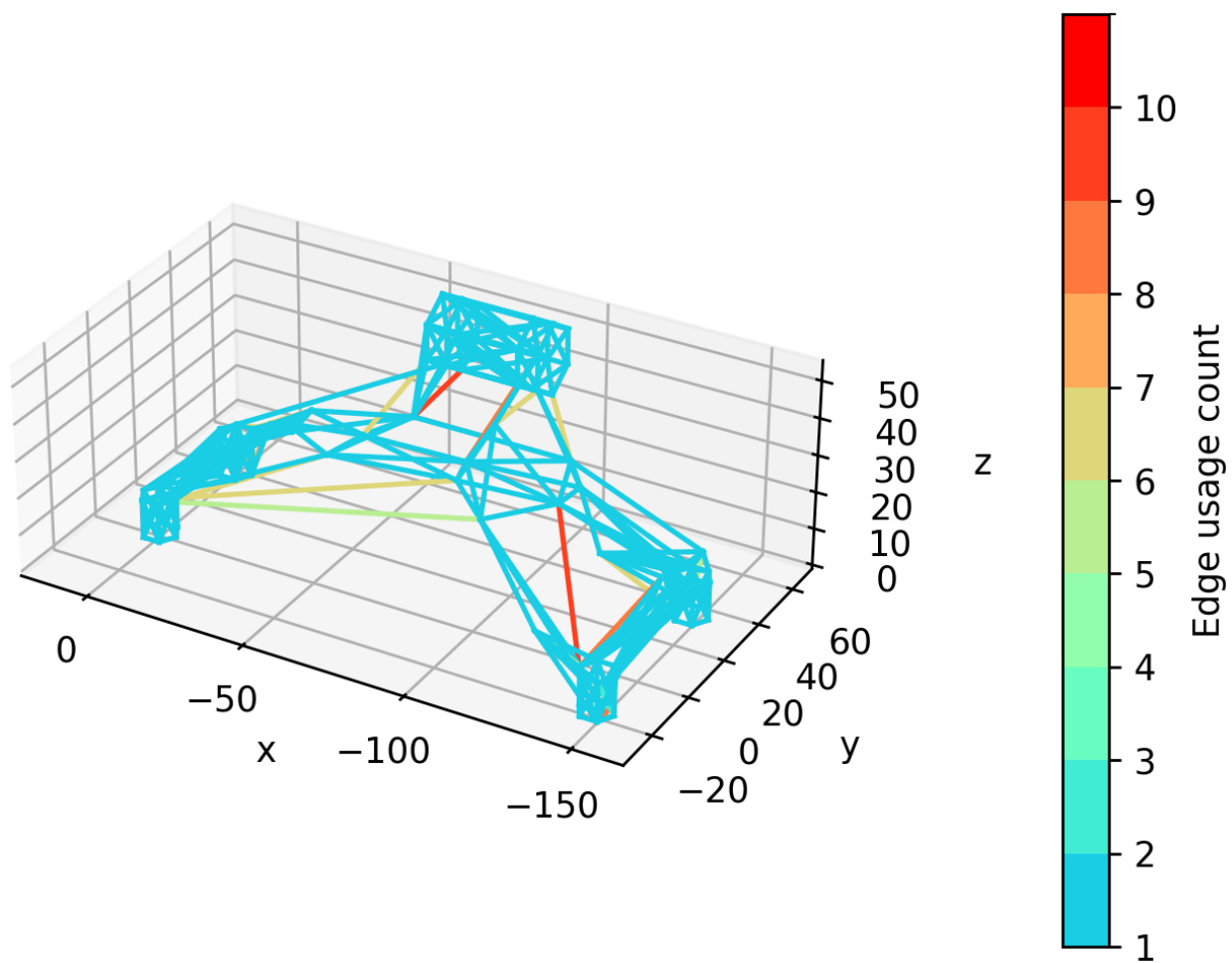


Figure E-6: Edge count map - design case G5LC2

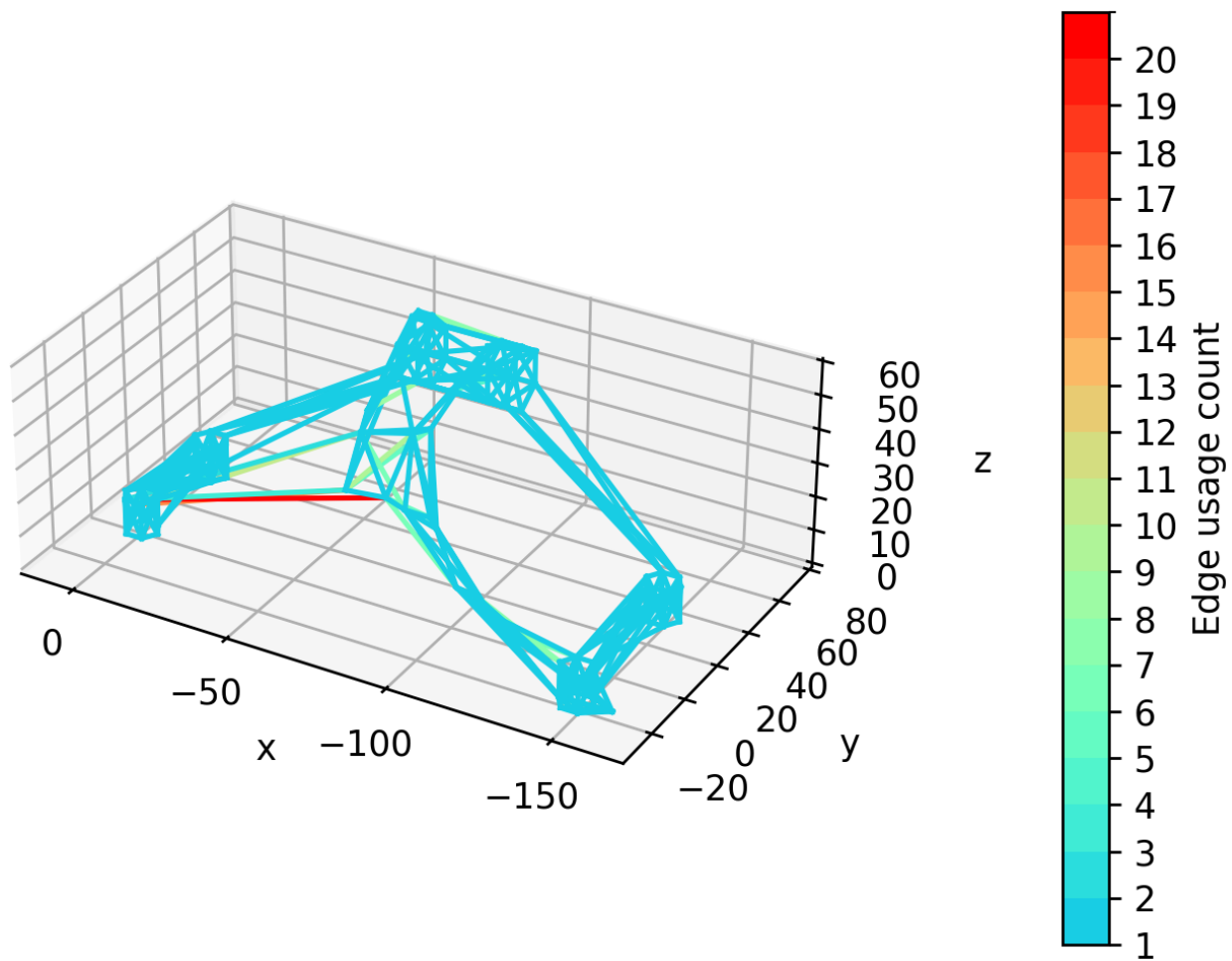


Figure E-7: Edge count map - design case G5LC3

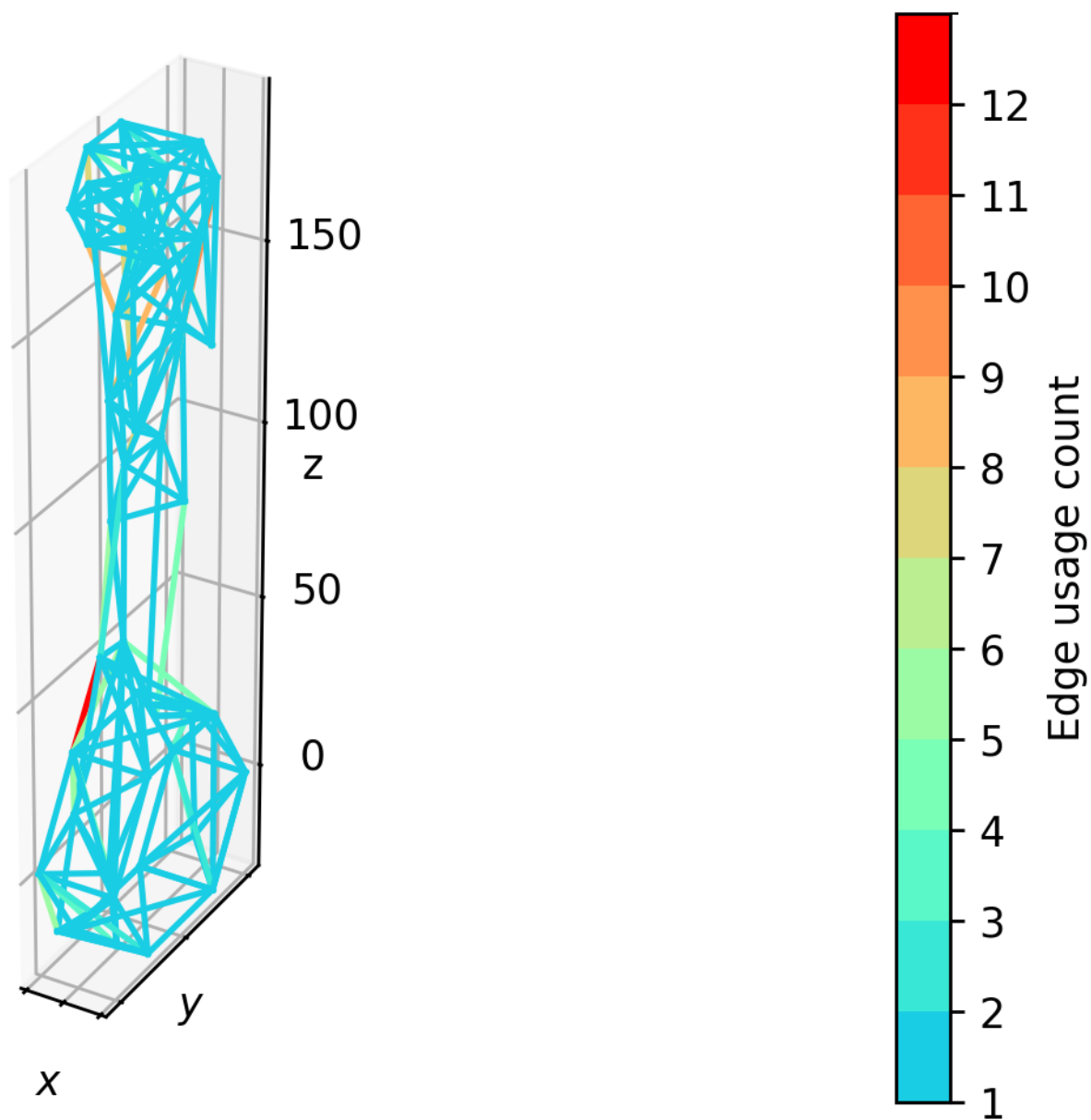


Figure E-8: Edge count map - design case G6LC1

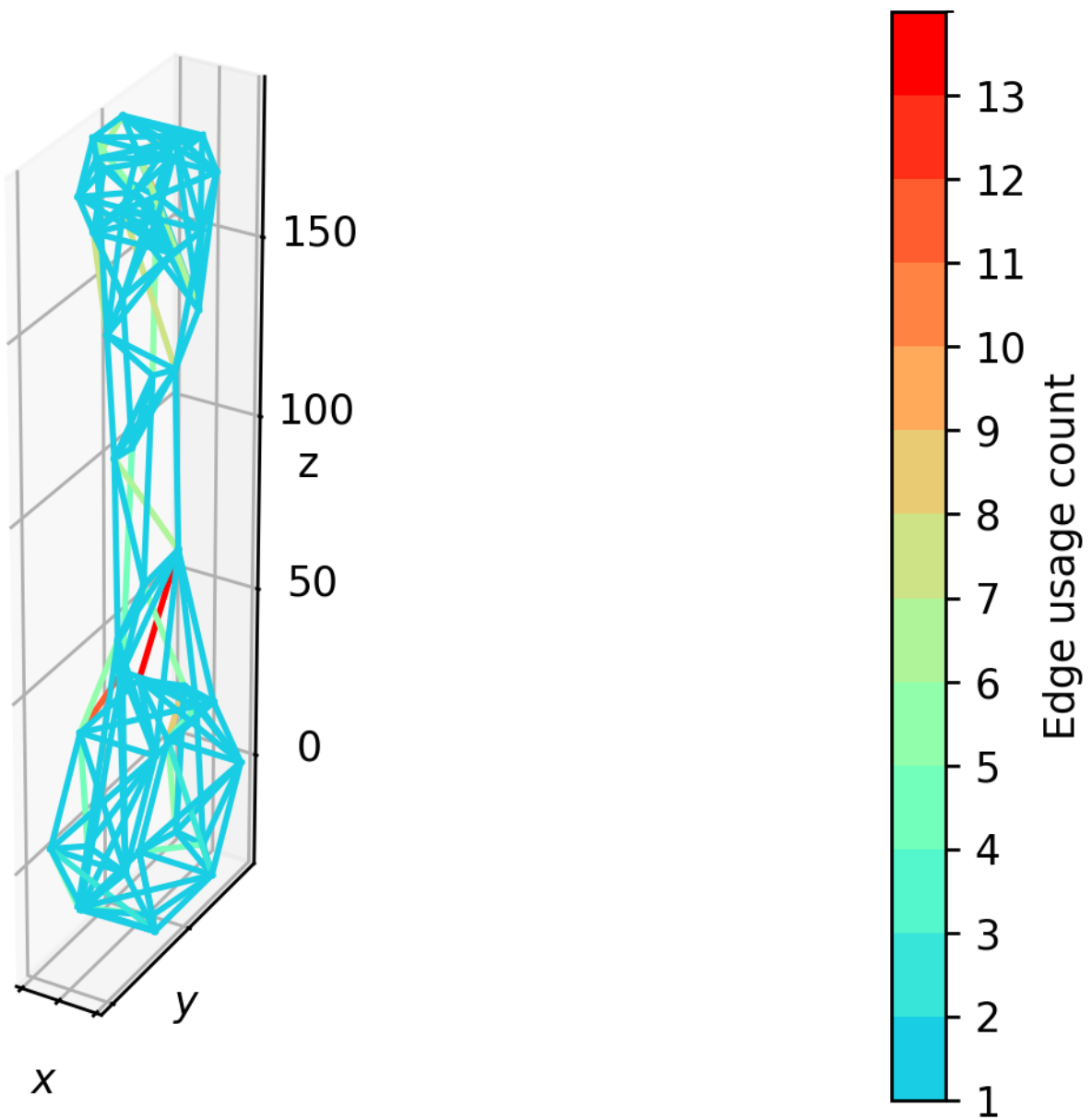


Figure E-9: Edge count map - design case G6LC2

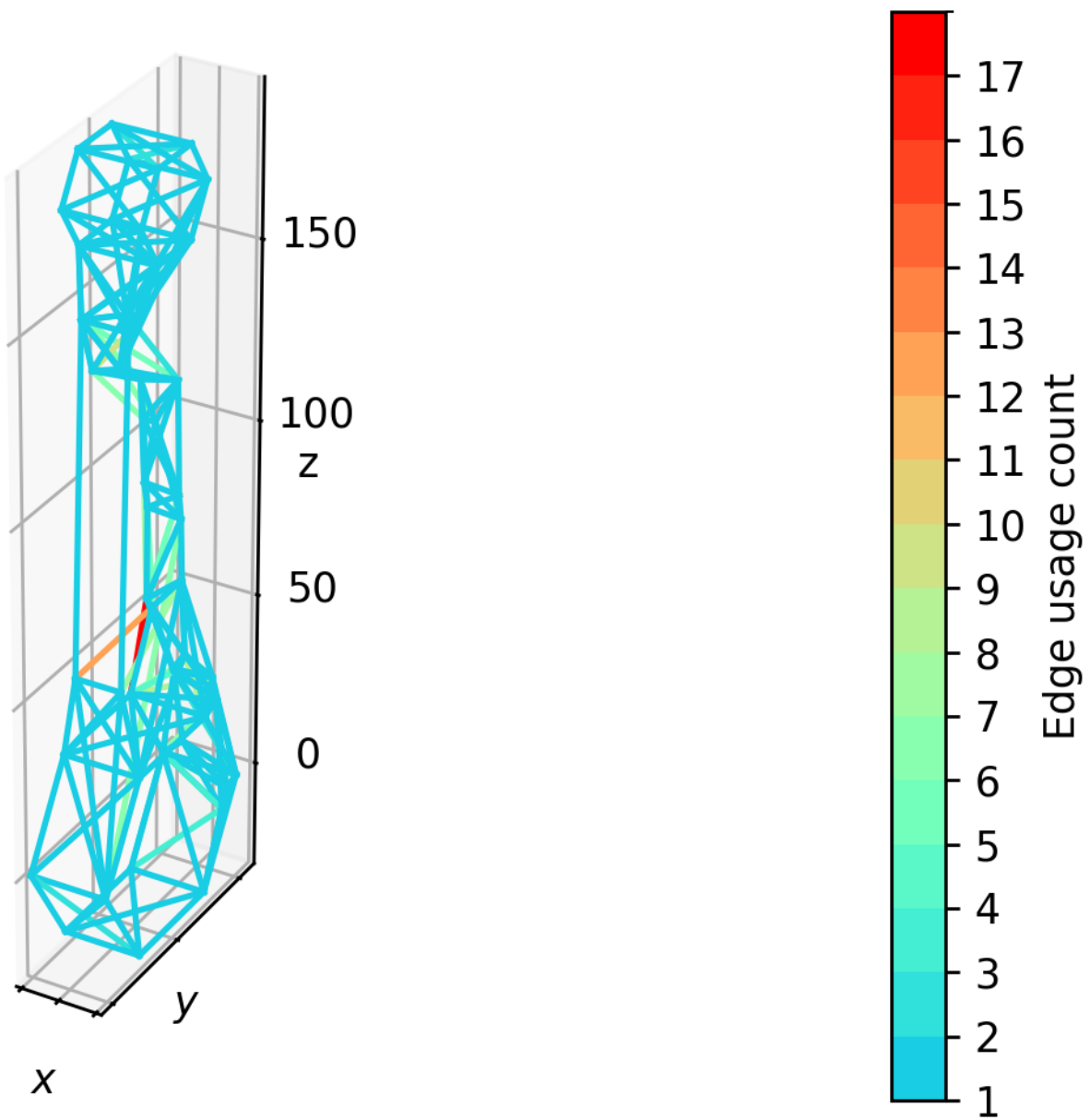


Figure E-10: Edge count map - design case G6LC3

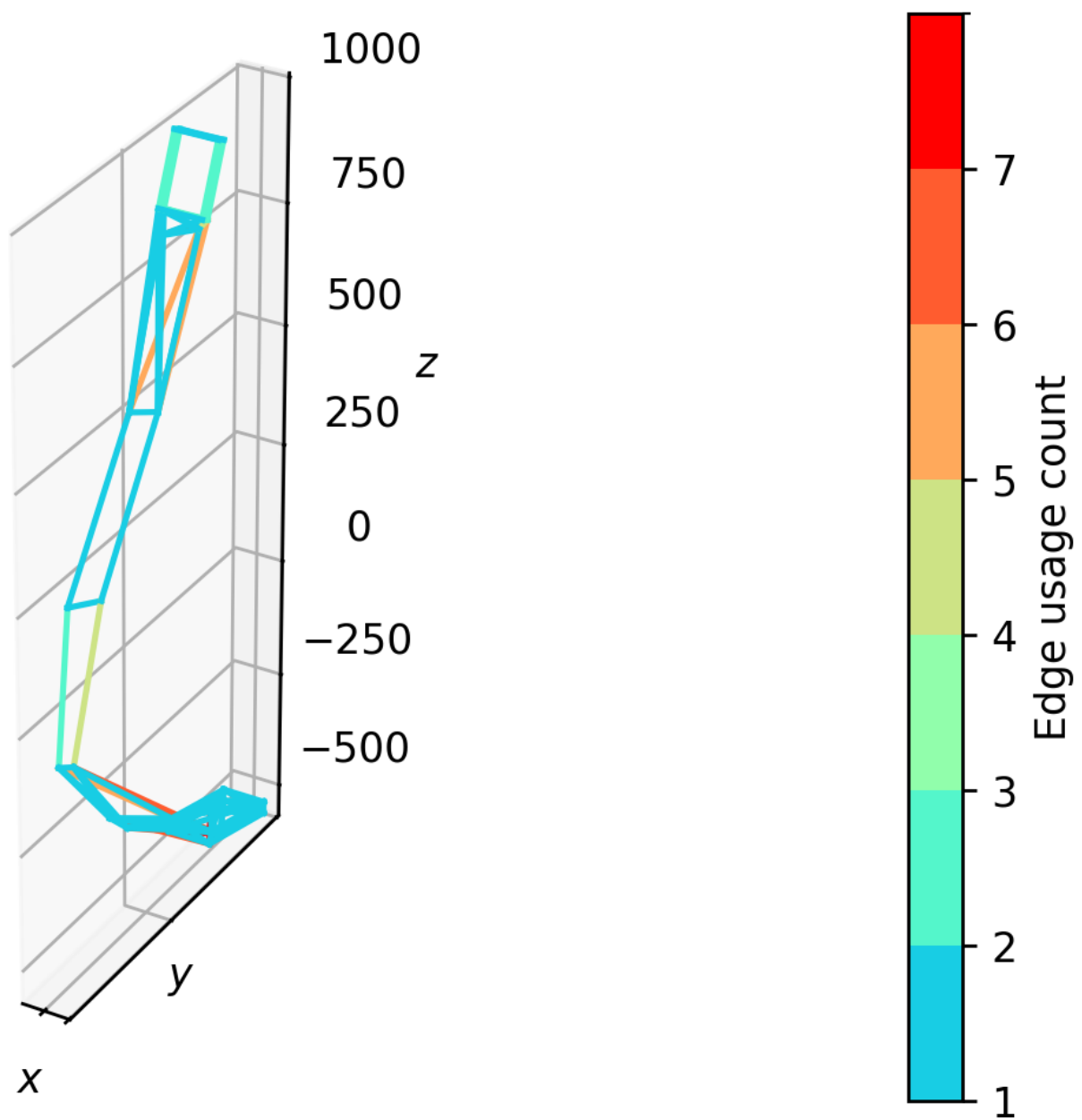


Figure E-11: Edge count map - design case G7LC1

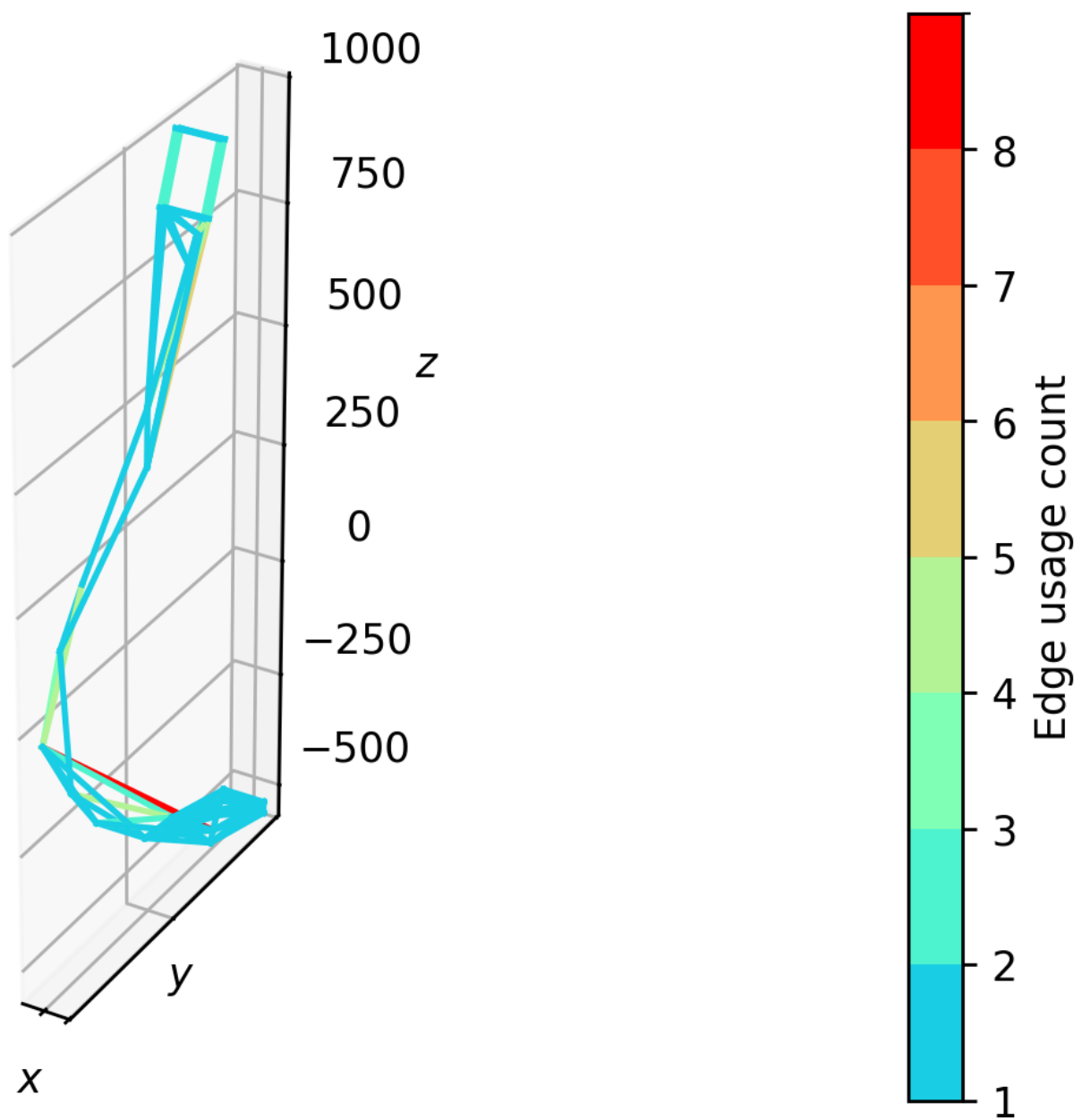


Figure E-12: Edge count map - design case G7LC2

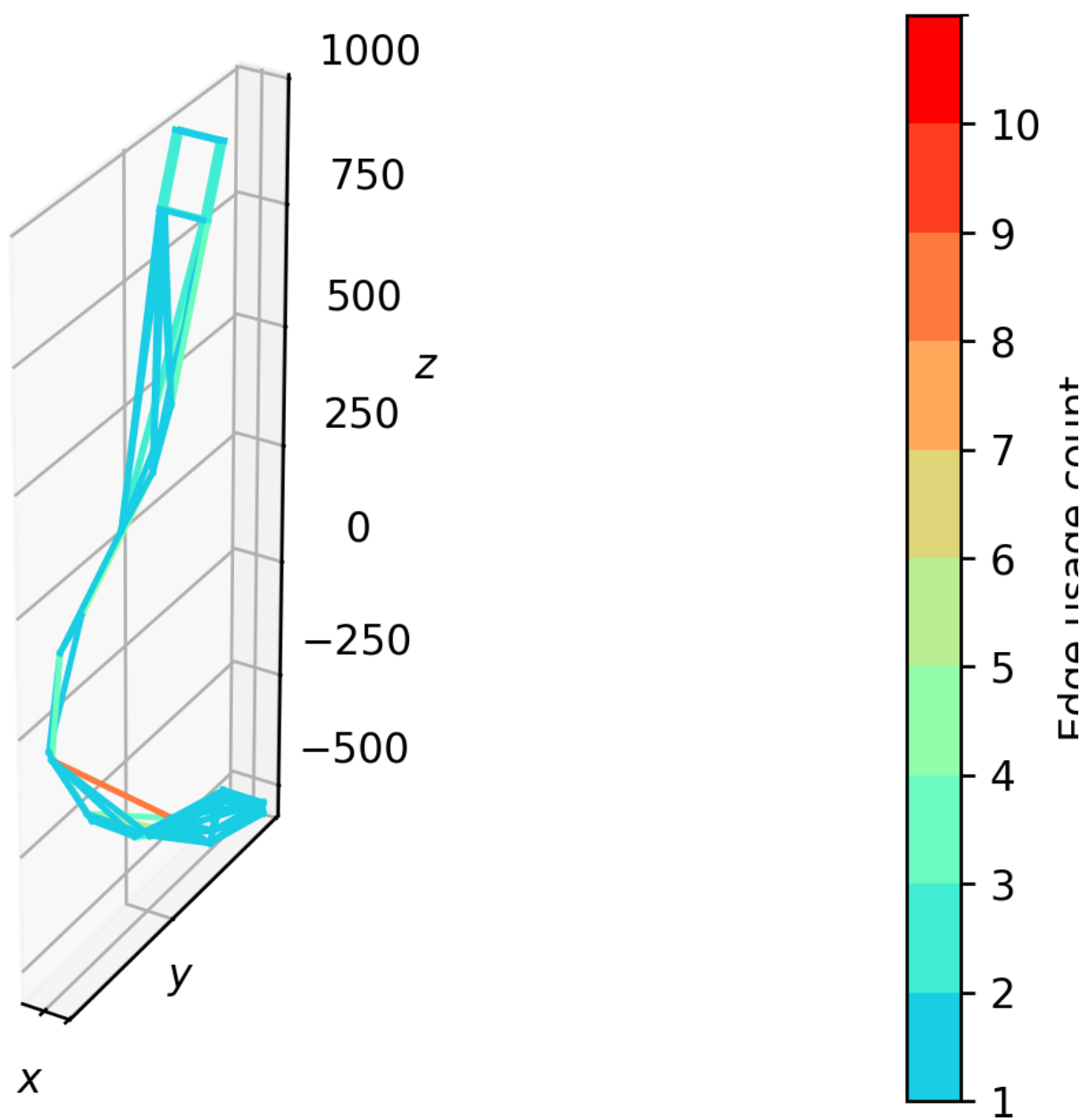


Figure E-13: Edge count map - design case G7LC3