

Plastic Interaction Relations for Elliptical and Semi-Elliptical Hollow Sections

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Abstract

The advancement of the structural steel manufacturing industry has led to the recent emergence of steel members with Elliptical Hollow Sections (EHS) and Semi Elliptical Hollow Sections (SEHS). Although these sections are gaining popularity among architects, the lack of design guidelines specifically tailored towards these sections inhibits their efficient structural use. Within this context, this thesis provides several steps towards the development of such guidelines.

A review of the manufacturing process of hot-rolled steel sections is conducted with emphasis on hollow structural sections. The main factors affecting the formation of residual stresses during cooling of the sections are discussed.

Lower bound plastic interaction relations for EHS subjected to combinations of axial force, bi-axial bending moments and torsion are then derived. The formulation is based on the lower bound theorem of plasticity and the maximum distortional energy density yield criterion. Its applicability for conducting the cross-sectional interaction check in structural steel design problems is illustrated through a practical example. A simplified and conservative interaction equation is then proposed based on curve fitting of the results of the lower bound solution.

Upper bound interaction relations are next developed for EHS subjected to combinations of axial force, bi-axial bending moments, torsion and bimoments. The formulation is based on kinematically admissible strain fields within the context of the upper bound theorem of plasticity. The interaction relations derived successfully capture the effect of confining radial strains present at welded end sections, as well as sections that are free to deform in the radial direction away from end welded sections. An iterative solution technique is developed to solve the resulting highly non-linear system of interaction relations.

The effects of residual stresses and initial imperfections on axial compressive resistance of hot-rolled EHS are then incorporated into the lower bound interaction relations. Towards that goal, the thermo-mechanical properties of steel were extracted from the literature. A thermo-mechanical finite element model was developed for prediction of residual stresses

in rolled sections. The validity of the model was assessed by comparison against residual stress measurements available in the literature. The model is then applied to predict the residual stresses in hot-rolled EHS.

A series of geometric and material nonlinear finite element analyses is conducted on columns of EHS sections. The analyses include predicted residual stresses and initial out-of-straightness imperfections in order to determine the inelastic buckling capacity of EHS members and generate column curves for EHS sections. The column curves are subsequently compared to those based on Canadian, American and European design codes. Two column curve equations are proposed in a format similar to that of the Canadian Standards for buckling about major and minor axes. The column curves were subsequently combined with the interaction relations developed to provide design rules for EHS members under combined loads.

The last contribution of the thesis provides a formulation of lower bound interaction relations for SEHS subject to combinations of axial force, bi-axial bending moments and torsion. An iterative scheme for solving the parametric form of the interaction relations is developed and a grid of admissible stress resultant combinations is generated. A series of trial functions are fitted to the grid of internal force combinations and two simplified and conservative interaction equations are proposed.

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Glossary

FEA:	Finite Element Analysis
EHS:	Elliptical Hollow Section
SEHS:	Semi-Elliptical Hollow Section
HSS:	Hollow Square Section
LPF:	Load Proportionality Factor
RS:	Residual Stress
GNL:	Geometric nonlinearity
ELPL:	Elasto-Plastic
CSA:	Canadian Standards Association
AISC:	American Society of Civil Engineers
AISI:	American Iron and Steel Institute
ASTM:	American Society for Testing and Materials

CHAPTER 1

Introduction

1.1. Significance

The advancement of the structural steel manufacturing industry has led to the recent emergence of steel members with Elliptical Hollow Sections (EHS) and their companion, semi-elliptical hollow sections (SEHS) (Fig. 1.1).

Members of oval cross-section have an original shape and are aesthetically pleasing. Thus, they are starting to gain popularity among architects. These sections have been used in major structures in Europe such as Heathrow, London, UK airport (Fig. 1.2) and Madrid, Spain airport (Fig. 1.3). In Canada, EHS members have been used in two award winning buildings: the skylight of the Electronic Arts stair in Vancouver, BC and the Legends Centre in Oshawa, ON (Packer 2008b). They also used structurally in Telus atrium Vancouver, BC (Packer 2009), society bridge, Braemar, Scotland (Packer 2008a)

From a structural viewpoint, elliptical sections offer the structural advantages of closed sections (i.e., high resistance to torsion and lateral torsional buckling). Their differing major and minor axis properties provide more efficient use of material when the bending moments about the major axis are larger than those about the minor axis. As compression members, EHS can be more economic than circular or square hollow sections for members with an effective length about the minor axis that is smaller than that of the major axis.

Despite the recent interest in adopting EHS in structural designs, the lack of design procedures specifically geared towards EHS design forces engineers either to (a) conduct their design in an overly conservative manner (Gardner and Ministro 2005) or (b) resort to complex and time consuming finite element solutions.

The objective of this thesis is to provide a precise but simple guideline for design of EHS and SEHS. This guideline must be simple enough to be practically utilized in design offices while it is specifically tailored for elliptical hollow sections.



Fig. 1.1: Elliptical and semi-elliptical hollow sections
(Photos courtesy of Ancofer (2008))

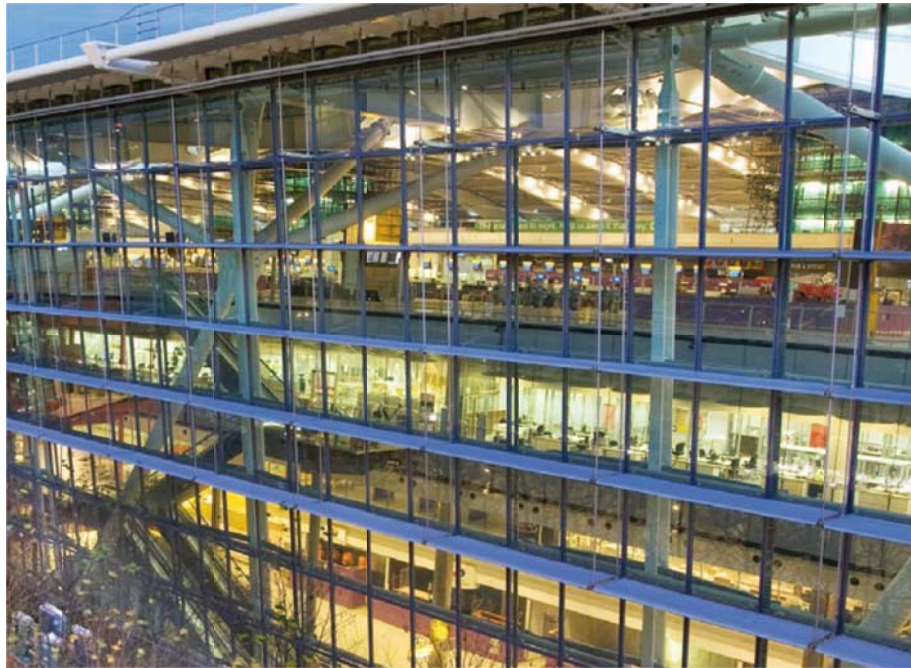


Fig. 1.2: Heathrow, London, UK airport, Terminal 5
Photos from McKechnie et al. (2009)



Fig. 1.3: Madrid, Spain airport, Terminal 4
Photos from “spanish-airport-guide.com/madrid-airport.html”

1.2. Manufacturing of Hot-Rolled Elliptical Hollow Sections

Elliptical hollow sections are produced either as hot finished welded section or seamless hot-rolled section. In this section the manufacturing process of the seamless hot-rolled EHS is described. A clear understanding of the manufacturing process of EHS is essential to reliably predict the residual stresses induced within these sections.

The semi-elliptical sections are generally cold-formed. The main mechanism in formation of residual stresses in cold formed sections is the plastification of the plate under forming forces. This is while the residual stresses in hot-rolled section are created because of thermo-machanical behaviour of the section during cooling period after rolling. Therefore the residual stresses within these sections are completely different and will not be investigated in this study.

1.2.1. The Hot Rolling Process

The hot-rolling process is widely used in the steel industry to manufacture structural steel sections. Once the section passes through the last roll, its temperature is about 900–1000°C (Richardson 1996; Baskiyar 2001). At this temperature, it is commonly assumed the state of stress is completely released in the section (Krauss 1990). Thermal residual stresses occur throughout the cooling process while their pattern and magnitude depend on the thermo-mechanical history of the product. At such initial temperature, it normally takes several hours for the product to reach room temperature. Therefore, it is common to speed up the cooling rate through controlled water spraying for a short period at the beginning of cooling process prior to air cooling. Although water spraying reduces the cooling time, it induces considerably more residual stresses in the section compared to air alone cooling.

From a structural point of view, the presence of residual stresses can reduce the buckling resistance for columns and beams, particularly those with intermediate slenderness ratios. Thus, various steel design standards (e.g., AISC-360 2005; EN 1993-1-1 2005; CAN/CSA-S16 2009) incorporate residual stress effects on the capacity of members typically through empirical equations (such as column curve equations).

1.2.2. Manufacturing of Hot-Rolled Sections

The production of long steel members involves a series of common processes. These are a) heating of the ingot, b) rolling to proper contour and dimensions, c) cooling to atmospheric temperature, d) straightening and e) storage (Fig. 1.4). During the heating process, steel ingots are heated near the melting point. At this temperature, steel colour turns to orange yellow (Totten 2006). Then, the soft steel passes through a series of rollers to gradually form into its desired shape. After passing through the final rolls, the cooling process starts. Since the cooling process tends to cause extreme residual stresses at the end of the member, its two ends are cut and returned to the ingot making factory for reuse. At the cutting stage, the member is cut into its final length. Depending on the cross section and cooling regime, the member may need to pass through a few straightening rollers. The straightening process is meant to reduce the deformations caused by non uniform cooling of the member but could introduce more residual stresses into the member. This process is an important step in unsymmetrical sections such as rails (Fig. 1.5) but is generally unnecessary for doubly symmetric sections (Fig. 1.6 and Fig. 1.7).

After passing through the last roll, the final product has a temperature of about 1000°C . The cooling process starts at this temperature. Different cooling techniques are used in the steel manufacturing industry including furnace cooling, air cooling, forced air cooling, water misting, water spraying, impinging jet water cooling, oil quenching and water quenching, which are sorted from the slowest to the fastest (Totten 2006). Many combinations of the above methods are used in industry depending on the final product characteristics desired. For structural sections, cooling is predominantly done by air. A short period of controlled water spray cooling may precede air cooling (Richardson 1996; Reggio et al. 2002).

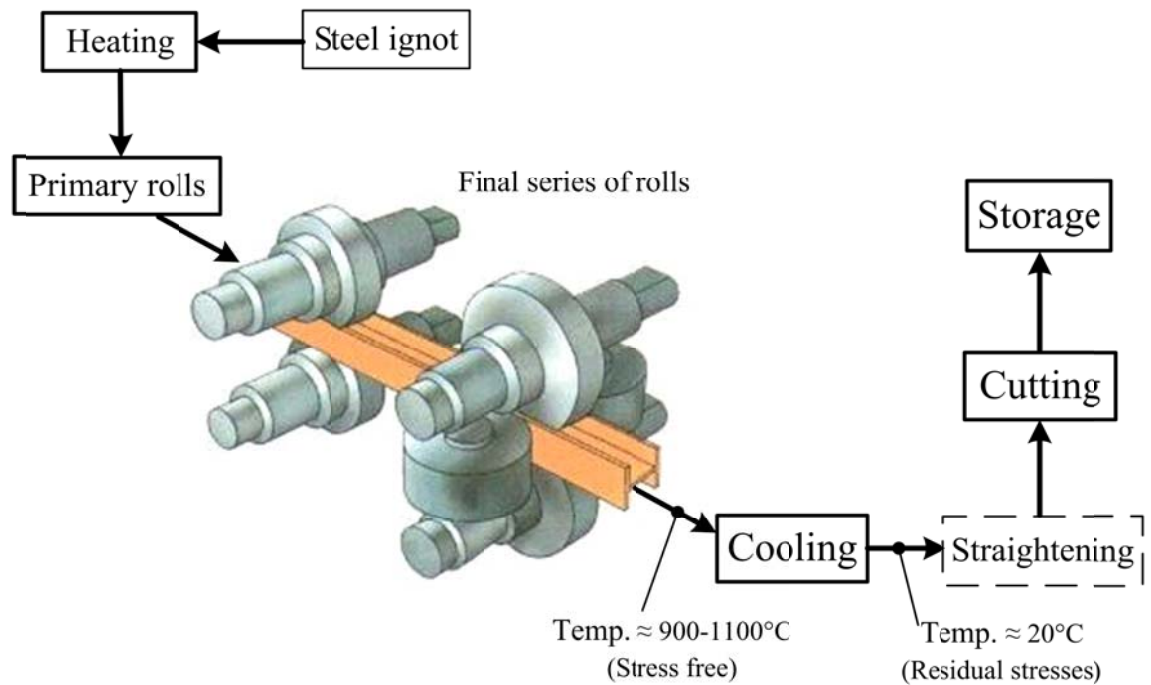


Fig. 1.4: Hot-rolling manufacturing processes

1.2.3. Hot-Rolling of Tubular Sections

The manufacturing of seamless pipes and tubes including elliptical hollow sections (EHS) begins with a round steel ingot heated to about 1200°C (Reggio et al. 2002). Two series of conical-shaped rolls draw the red-hot billet over a piercing mandrel (Fig. 1.8 and Fig. 1.9). The rolling action causes the metal flow over and around the mandrel to create a hollow pipe shell (McGannon 1964). The pipe shell then passes through a series of rolls to obtain its final shape, diameter and wall thickness (Fig. 1.10). The remaining processes are identical to those of other hot-rolled sections (i.e, Fig. 1.4).



Fig. 1.5: Cooling bed and straightener machines for rail sections
(Photo from Espinoza 2007)



Fig. 1.6: A cooling bed for wide flange sections
(Photo from Espinoza 2007)



Fig. 1.7: A cooling bed for pipes
(Photo from VMTubes 2009)

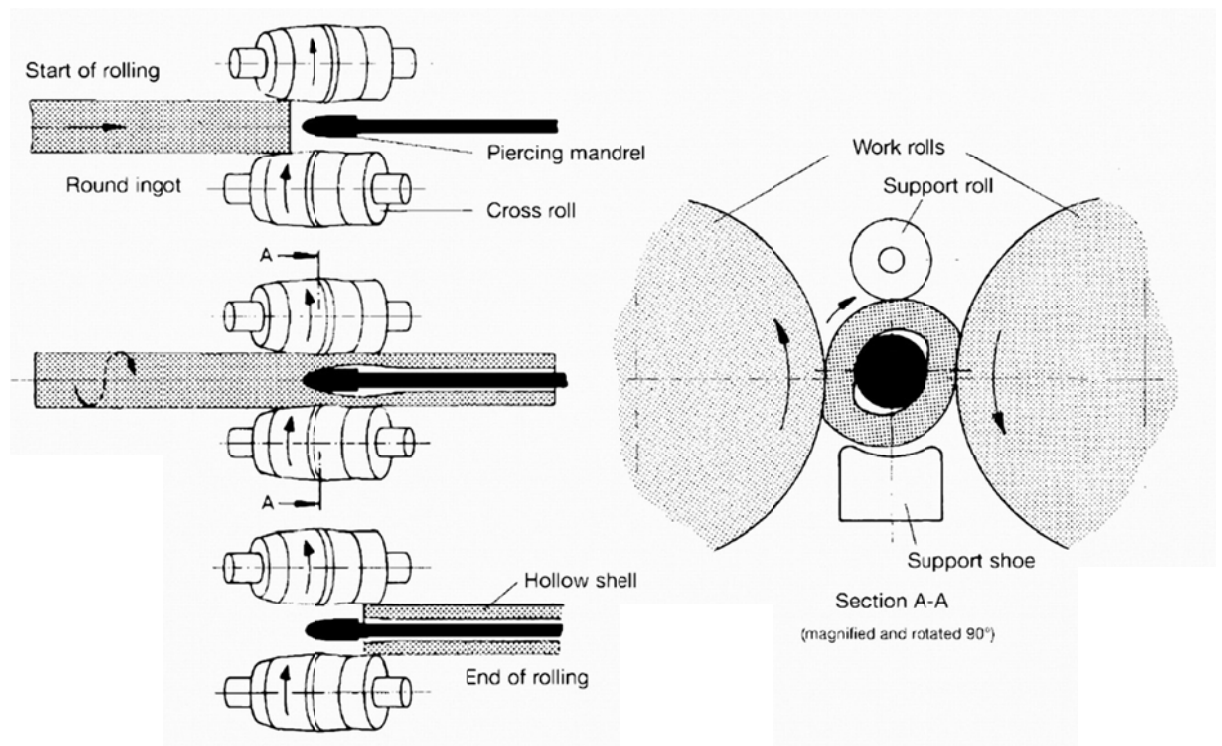


Fig. 1.8: Schematic of piercing a round ingot
(Figure from Brensing and Sommer 2008)



Fig. 1.9: Piercing mandrel (Photo from VMTubes 2008)



Fig. 1.10: HSS passing through the shaping rollers
(Photo from VMTubes 2008)

1.2.4. Formation of Residual Stresses

The interaction history among temperature, deformation and microstructure of steel body creates residual stresses. This interaction is schematically shown in Fig. 1.11, where each arrow shows in which way a process can affect the other processes. As time passes the temperature in the steel body drops in different rates causing a temperature field. The temperature field creates stress and strains in the body, changes thermo-mechanical properties of the steel material and triggers the latent heat. On the other hand body strains

create deformation heat and changes boundary surfaces thus affecting the temperature field. Besides, stresses can trigger phase transformation in the steel. The steel micro-structure dictates the thermo-mechanical properties of steel, releases latent heat and induces additional strains and consequently stresses. As will be discussed later, although in general there are six ways that the three processes can interact, not all of these ways are of practical importance in the cooling process of a hot-rolled section. For instance, the stress induced phase transformation is not likely to occur during cooling because it will exist if sudden high stresses are applied to a steel body, similar to what happens in an impact (Totten et al. 2002). The exact same argument holds true for deformation released heat.

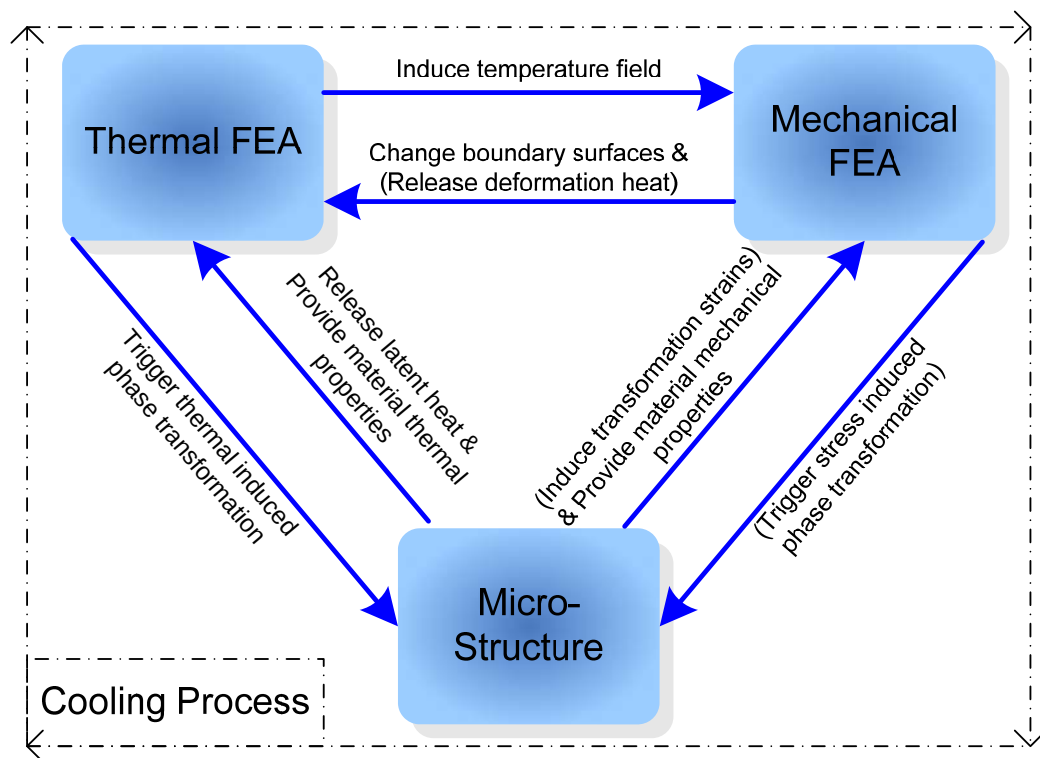


Fig. 1.11: The interaction amongst temperature, stress and micro-structure

The cooling process starts after the hollow section member passes through the final series of rollers. During this process, various parts of the section cool at different rates which induce residual stresses. The outside surface of the member has a much higher cooling rate compared to the inside surface because

1. The sprayed water affects primarily the outside surface;

2. The air inside a hollow section is mainly trapped and thus dissipates energy at a slower rate than the free moving air on the outside surface;
3. Each segment of the inside wall of the section has direct view to another segments of the wall with essentially the same temperature, which considerably reduces the heat loss by radiation of the inside surface;
4. The outside surface has a larger area compared to the inside.

The large difference of heat loss between the outside and inside surface of hollow sections creates different rate of contraction in the section which results in compressive residual stresses on the outside.

1.3. Thesis Outline

This thesis is organized in a paper format. Studies related to the mechanical capacity (i.e., interaction relations) of the elliptical and semi-elliptical hollow sections in absence of residual stresses (i.e., chapters 2, 3 and 6) were published. These studies are applicable to cross-sectional strength and are based on sections attaining their full plastic resistance. A study (in press) was conducted to predict the residual stresses induced in EHS during the cooling process and develop column curves for columns with EHS sections (chapter 5). Both studies are subsequently merged in Appendix II to develop plastic interaction relations for members under combined loads which incorporate residual stresses.

In Chapter 1, the manufacturing process of the EHS is reviewed. An understanding of the manufacturing of EHS sections is key to reliably predict the residual stresses and incorporate them into the interaction equations developed. In Chapter 2, interaction equations for cross-section capacity design of EHS under combined loading (i.e., tensile force, bi-axial bending moments and torsion) are developed. Then a single simplified and conservative interaction equation more appropriate for design offices is proposed. This single interaction equation has been shown that is less stringent than the current Canadian Standards equation for hollow sections. In Chapter 3, upper bound interaction equations are developed for EHS. In addition to axial force, bending and twisting moments, these relationships capture the effect of bimoments, and confining radial strains present at welded end sections. Chapter 4 provides background material for thermo-mechanical analysis

required for predicting residual stresses including the fundamentals of the formulations involved, solution techniques, assumptions, steel properties, and finite element model specifics. The material provided in this chapter is a prerequisite for the study conducted in Chapter 5 which provides a comprehensive study for the prediction of the residual stresses in EHS members through a thermo-mechanical finite element analysis. The study also uses the predicted residual stresses to develop column curves for EHS members and incorporates them into the interaction relations developed in Chapter 2 to design beam columns. Chapter 6 develops lower bound interaction relations for semi elliptical hollow sections and Chapter 7 provides a summary of the contributions of the study and recommendations for future research.

CHAPTER 2

Plastic Interaction Relations for Elliptical Hollow Sections¹

2.1. Abstract

Interaction expressions are developed for steel elliptical hollow sections (EHS) subjected to combination of axial force, bi-axial bending moments and torsion. The formulation is based on the lower bound theorem of plasticity and the maximum distortional energy density yield criterion. The interaction expressions developed consist of a set of elliptical integrals which are cast in a dimensionless form and numerically evaluated to obtain a grid of internal force combinations lying on the yield surface sought. A series of trial functions are fitted to the grid of internal force combinations and a simplified and conservative interaction equation is selected. The interaction relation relates axial force, biaxial bending and twisting moment combinations for EHS sections of common geometries. Its applicability for conducting the cross-sectional interaction check in structural steel design problems is illustrated through a design example.

Key words: Steel, Elliptical Hollow Section, Plasticity, Yield Surface, Interaction Relations

2.2. Introduction and Scope

The advancement of the structural steel manufacturing industry has led to the recent emergence of steel members with Elliptical Hollow Sections (EHS). Members of oval

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cross-sections have an original shape and are aesthetically pleasing. Thus, they are starting to gain popularity among architects. From a structural viewpoint, oval sections offer the structural advantages of closed sections (i.e., high resistance to torsion and lateral torsional buckling). Their differing major and minor axes properties provide more efficient use of material when the bending moments about the major axis are larger than those about the minor axis. As compression members, EHS can be more economic than circular or square hollow sections for members with an effective length about the minor axis that is smaller than that of the major axis. Despite the recent interest in adopting EHS in structural designs, the lack of design procedures specifically geared towards EHS design has forced engineers to conduct their design in an overly conservative manner (Gardner and Ministro 2005). Within this context, the present paper aims at developing refined interaction relations specific to EHS subject to general loading combination involving axial force, biaxial bending, and twisting moments. The interaction relations are applicable to compact sections (i.e., sections able to attain their plastic resistance prior undergoing local buckling).

2.3. Literature Review

2.3.1. Research Related to Elliptical Hollow Sections

The buckling and post buckling behaviour of thin-walled oval cylindrical shells as structural elements has attracted the interest of engineers in the 1960s to 1970s. In one of the earliest attempts (Kempner 1962), the buckling resistance of elliptical cylinders under pure axial compression was found to be in good agreement with that of a thin-walled circular cylinders, when the radius is replaced by the maximum radius of curvature of the ellipse in the classical buckling load formula. The buckling behaviour of thin walled elliptical sections has been investigated both analytically and experimentally under pure axial compression (Hutchinson 1968; Feinstein et al. 1971) and under combinations of axial force and bending moments (Kempner and Chen 1974; Chen and Kempner 1976). These studies were conducted on very thin walled sections made from polyester films and were unable to reach their section plastic resistance.

Steel Elliptical Hollow Sections (EHS) emerged as a structural section only recently. Thus, only a few relevant studies were published on EHS to date. This includes the work of Grigorenko and Rozhok (2002) who investigated the effects of variation of load eccentricity and thickness on the stress fields of cylinders with elliptical hollow sections. A series of compression and bending tests were conducted on elliptical hollow sections cylinders (Gardner and Ministro 2004; Gardner 2005). The study was followed by numerical analysis which has suggested the validity of finite element models in predicting the buckling behaviour of EHS under such loading. The behaviour of gusset plate connections to elliptical section under pure axial forces was recently investigated (Willibald et al. 2006). A finite element study aimed at investigating the local buckling behaviour of EHS was conducted by Zhu and Wilkinson (Zhu and Wilkinson 2007). Gardner and Chan (2007) proposed limits for classification of EHS under axial force, and bending about the major and minor axes. They developed analytical expressions for the yield compressive strength and plastic bending resistances of hot-rolled EHS and verified them against experimental results (Chan and Gardner 2008a; b). Recently, the global elastic buckling of EHS has been investigated (Ruiz-Teran and Gardner 2008) under compressive loads.

2.3.2. Interaction Relations for Hollow Structural Sections of Other Geometries

The general principles for the analytical development of lower bound and upper bound plastic interaction relations are presented in Chen and Atsuta (1972) and Hodge (1981). Morris and Fenves (1969) established lower bound plastic interaction relations for hollow structural rectangular sections subjected to combinations of biaxial bending, torsion, and axial force. Chen and Atsuta (1972) formulated lower and upper bound interaction relations for box sections under biaxial bending and axial forces. Pillai and Ellis (1971) conducted an experimental study on hollow structural sections subjected to combined axial forces and uniaxial bending moments and proposed a simplified interaction relationship based on their work. Interaction relationships for square hollow structural beam columns subjected to biaxial moments and axial forces were proposed by Pillai (1974) and later verified by an experimental investigation (Pillai and Kurian 1977). Pillai's interaction relations were

adopted in the Canadian steel Standards (CAN/CSA-S16 1974) and subsequent versions for the strength design of compact sections (Classes 1 and 2). The effect of residual stresses on the plastic resistance of welded box sections was investigated by Zhou and Chen (1986). The effect of biaxial shear was incorporated into the formulation of interaction relations for square hollow members (Mohareb and Ozkan 2004) subjected to biaxial bending, biaxial shear, twisting moments and axial forces.

For pipe sections, a number of interaction relations for pipes under various combinations of internal forces were provided in a summary by Gerald and Becker (1957). The solutions provided are applicable for pipes with large diameter to thickness ratio and include local buckling effects. For stockier pipes, the fully plastic resistance of the section can be attained or nearly attained as recognized in structural steel standards, e.g., (CAN/CSA-S16 2001) and steel pipeline standards (CAN/CSA-Z662 2007), before the occurrence of local buckling (e.g., Hu et al. 1993; Mohareb and Murray 1999; Mohareb 2002; 2003; Ozkan and Mohareb 2003). The interaction relations formulated by Mohareb (2001) were utilized in developing a pipe finite element which efficiently modelled the elasto-plastic behaviour of pipelines (Nowzartash and Mohareb 2004). Given that EHS have emerged only recently, similar interaction relations for EHS have not been developed, to the best of the knowledge of the authors. Within this context, a lower bound plastic interaction relation is developed for EHS subject to axial load, biaxial bending moments and twisting moments.

2.4. Statement of Problem

Plastic interaction relations for EHS subject to the combined action of biaxial bending, torsion and axial force are sought. It is assumed that the fully plastic resistance of the cross section, (commonly referred to as the cross-sectional capacity at limit state in design codes) will be attained. The formulation is based on the lower bound theorem of plasticity (e.g., Hodge 1981) in which stress distributions consistent with the material constitutive law are postulated and a lower bound interaction relation is recovered. Such a lower bound solution underestimates the capacity of the cross-section and thus is suitable for cross-sectional checks conducted as part of the design process. A steel elliptical hollow section (Fig. 2.1) subject to axial force N (positive when tensile), bending moments M_x , M_y acting about x and y axes respectively and twisting moment T is considered. The positive directions of

M_x, M_y and T is shown in Fig. 2.1. It is required to determine whether the elliptical section is able to withstand the action of the applied forces while assuming the fully plastic resistance of the section is reached. For this purpose, it is required to find an interaction relationship in the form $f(N, M_x, M_y, T) = 0$ such that the condition $f < 0$ is met for any physically possible combination of internal forces, $f = 0$ corresponds to a fully plastic state of the cross section and the condition of $f > 0$ is unattainable under the assumptions of the formulation.

2.5. Assumptions

2.5.1. Cross-Sectional Distortion

The cross section is assumed to remain undistorted under the action of internal forces induced. This assumption is justifiable for EHS with small diameter to thickness ratios, where the cross-sectional ovalization/distortion is negligible. In such sections, the fully plastic resistance can normally be attained, or nearly attained, before the occurrence of local buckling. Such section is classified as class 2 in the Eurocode 3 (EN 1993-1-1 2005) and Canadian steel standard (CAN/CSA-S16 2001). According to Eurocode 3 (EN 1993-1-1 2005) a circular section is class 2 if $D_0/t \leq 70\varepsilon^2$ where $\varepsilon^2 = 235/F_y$, D_0 is the outside diameter of the pipe and the yield strength F_y is expressed in MPa. A similar but less stringent relation exists in the Canadian standard (CAN/CSA-S16 2001) as $D_0/t \leq 18,000/F_y$.

By adopting the approximation in Kempner (1962), an EHS may be conservatively treated, for classification purpose, as a circular hollow section of radius R equal to the maximum radius of curvature of the ellipse, i.e., $R = D_1^2/2D_2$, D_1 and D_2 , being the major and minor diameters of the ellipse, respectively. Thus an EHS under bending is conservatively considered class 2 if the inequality $D_1^2/(tD_2) \leq 18,000/F_y$ holds. A finite element study (Gardner and Chan 2007) corroborated by laboratory tests indicated that EHS under bending about major axis axial force and minor axis bending or a combination thereof can

safely reach the plastic limit when $D_1^2/(tD_2) \leq 21,100/F_y$. For bending about major axis when $D_1/D_2 > 1.115$ that happens if $D_1^2/(tD_2) \leq 41,000/F_y$.

2.5.2. Idealized Stress-Strain Relationship

Because EHS steel exhibits a distinct yield plateau (Zhu and Wilkinson 2007), a bilinear elastic-perfectly plastic stress versus strain representation is adopted. The first line, representing the linear range of deformation, passes through the origin and has a slope identical to the initial slope of a tension coupon tests. The second line is assumed to have zero slope and aligned with the yield plateau of the stress-strain relationship for a tension coupon test. The additional capacity of the steel material due to strain hardening is neglected, leading to a lower bound approximation for the section plastic resistance.

2.5.3. Yield Criterion

Elliptical Hollow Section steel is assumed to yield in accordance with the maximum distortional energy density yield criterion (e.g., Boreisi and Sidebottom 1985), i.e., a given point on an EHS (Fig. 2.2) will attain yielding when the following condition is met

$$\frac{1}{2} \left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{11} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2) \right] = F_y^2 \quad (2.1)$$

where $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}$ and σ_{23} are stress tensor components at the point considered and F_y is the yield strength of the EHS material.

For an infinitesimal element of volume, the relevant stress components are the normal stress $\sigma = \sigma_{11}$ and the tangential shearing stress $\tau = \sigma_{12}$. Longitudinal stress σ is induced by the axial forces and/or bending moments. Shearing stress τ is caused by the Saint Venant torsion twisting moments. Normal stresses due to warping are neglected in the present formulation. As a simplification, the through thickness shear stresses σ_{13} and σ_{23} are assumed negligible. Also, the normal to the mid-surface stress σ_{33} and the circumferential stress σ_{22} are assumed negligible. Based on these simplifications, the maximum distortional energy density yield criterion simplifies to $\sigma^2 + 3\tau^2 = F_y^2$ or

$$\sigma = \pm \sigma^* = \pm \sqrt{F_y^2 - 3\tau^2} \quad (2.2)$$

where σ^* is a maximum longitudinal stress that can be attained in the presence of a shear stress τ .

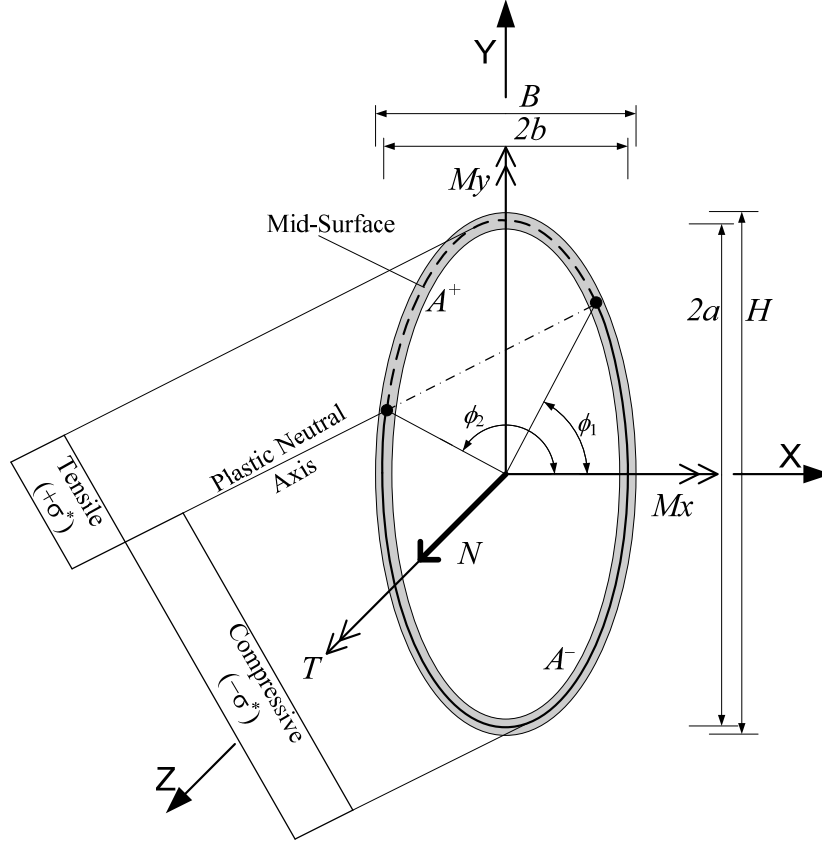


Fig. 2.1: EHS under combined axial force, torsion and bi-axial bending moments

2.5.4. Other Assumptions

No distinction is made between the true stress, based on the deformed infinitesimal element of area, and the engineering stress, based on the undeformed infinitesimal element of area. Also, the logarithmic strain is assumed to be nearly equal to the engineering strain as the formulation is restricted to small strains.

According to the previous studies (Gardner and Ministro 2004; Chan and Gardner 2008b), the effects of residual stress on hot rolled EHS resistance are believed to be low since: a) negligible deformations take place when material tensile coupons are machined from the EHS wall, and b) EHS steel is observed to exhibit a distinct yield point in tensile coupon

results and stub column tests, which is characteristic of low residual stress values. Therefore, residual stresses are not incorporated into the present study.

The formulation does not include the global buckling effects and thus is not intended to predict the resistance of members governed by the overall buckling mode of failure.

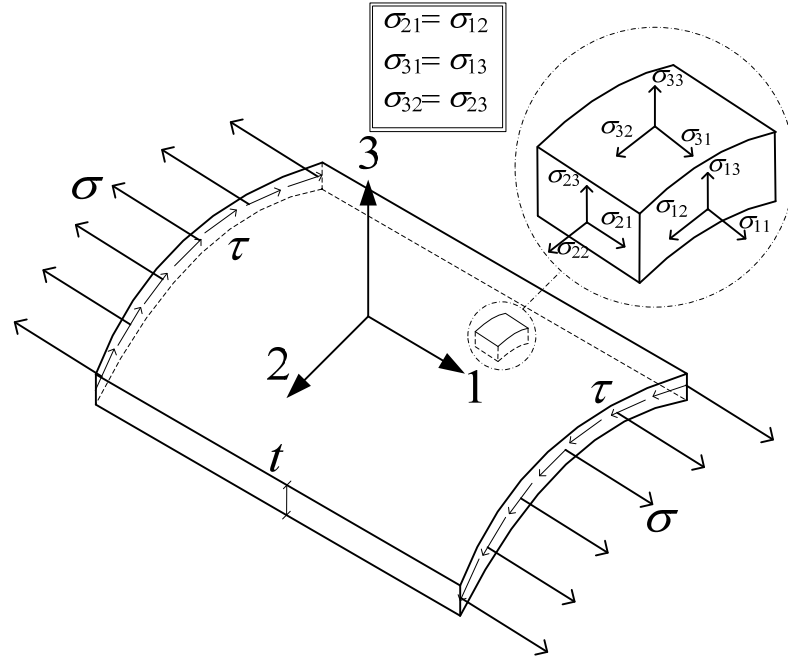


Fig. 2.2: Stress components acting on an EHS element

2.6. Formulation

Consider an EHS with a mid-surface major diameter $2a$, minor diameter $2b$ and thickness t . For a thin-walled closed section, the shearing stress τ can be assumed constant through the wall thickness t (Boresi and Sidebottom 1985), and is related to the torsion T through $T = 2A_s t \tau$, in which A_s is the area enclosed by the section mid-surface. In the case of an EHS, $A_s = \pi ab$. Thus, for an EHS, the relationship between shearing stress τ and the applied twisting moment T , is

$$T = 2\pi ab t \tau \quad (2.3)$$

For an elliptical section to fully plastify under bending moments, a portion of the cross-section, A^+ has to attain the maximum tensile stress σ^* while the remaining portion of the cross-section, A^- has to attain the maximum compressive stress $-\sigma^*$ (Fig. 2.1). In this figure, the dashed line along the mid-surface denotes the tensile region and the solid line depicts the compressive region of the section.

A generic point P (Fig. 2.3) on the section mid-surface subtends an angle ϕ at the ellipse centroid, relative to the positive X axis direction. On the figure, two concentric circles of radii a and b are overlaid. Point Q lying on the circumference of the larger circle has the same y coordinate as point P. Also, point T lying on the circumference of the smaller circle has the same x coordinate as that of point P. The coordinates of point Q are $(a \cos \beta, a \sin \beta)$, those of point T are $(b \cos \beta, b \sin \beta)$, while those of point P are $(b \cos \beta, a \sin \beta)$ where angle β is depicted in Fig. 2.3. An element of area of the ellipse cross-section is defined as $dA = tds = t\sqrt{dx^2 + dy^2}$ or $dA = ta\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta$.

The position of the plastic neutral axis (PNA) is defined by two angles ϕ_1 and ϕ_2 where $2\pi + \phi_1 \geq \phi_2 \geq \phi_1$ or $2\pi + \beta_1 \geq \beta_2 \geq \beta_1$. It is assumed that $\sigma = \sigma^+ = \sigma^*$ in the domain $\beta_2 \geq \beta \geq \beta_1$ and $\sigma = \sigma^- = -\sigma^*$ in the domain $2\pi + \beta_1 \geq \beta \geq \beta_2$.

For a section under the combined action of axial force N (tension is positive), biaxial bending moments M_x, M_y and twisting moments T , one has:

$$N = \int_A \sigma dA = \int_{A^+} \sigma^+ dA + \int_{A^-} \sigma^- dA = \int_{A^+} \sigma^* dA - \int_{A^-} \sigma^* dA = \int_{2A^+} \sigma^* dA - \int_A \sigma^* dA = 2 \int_{\beta_1}^{\beta_2} \sigma^* ta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta - \int_0^{2\pi} \sigma^* ta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.4)$$

The bending moment about major axis, M_x is obtained by

$$M_x = \int_A \sigma y dA = \int_{A^+} \sigma^* y dA - \int_{A^-} \sigma^* y dA = 2 \int_{A^+} \sigma^* y dA - \int_A \sigma^* y dA = 2 \int_{\beta_1}^{\beta_2} \sigma^* ta^2 \sin \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.5)$$

In Eq. 2.5, the identity $\int_A \sigma^* y dA = 0$ was used. Similarly,

$$M_y = \int_A \sigma x dA = 2 \int_{\beta_1}^{\beta_2} \sigma^* t \frac{b}{a} a^2 \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.6)$$

In Eqs. 2.4 through 2.6, angles β_1 and β_2 can be respectively bound by $-\pi/2 \leq \beta_1 \leq \pi/2$ and $\beta_1 < \beta_2 \leq \beta_1 + 2\pi$ without losing the generality of the formulation.

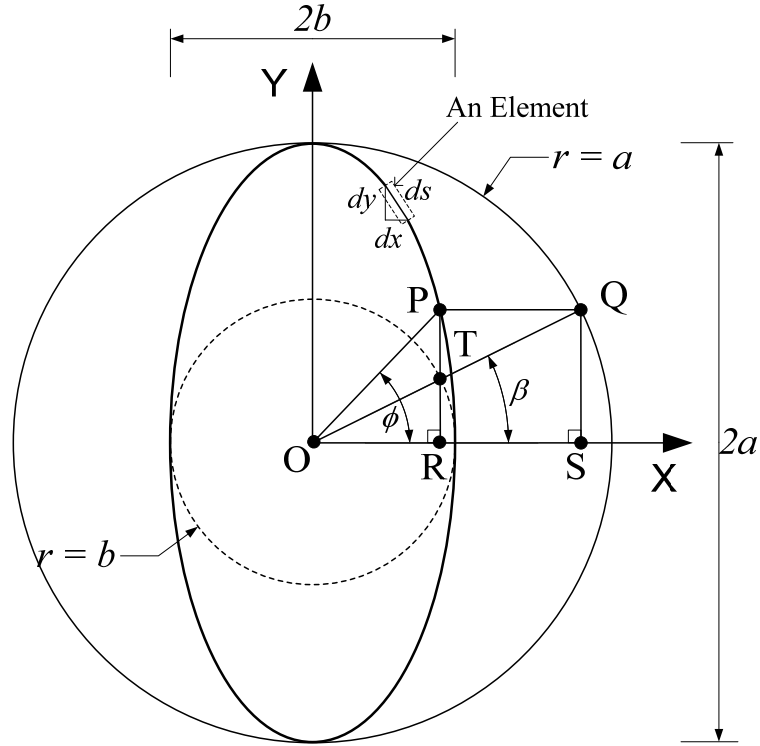


Fig. 2.3: Parametric relations of an Ellipse

2.6.1. Expressions for Plastic Resistances

It is desirable to normalize the plastic relations sought to make them universally applicable to any EHS geometry. Towards this goal, one needs to identify the limiting resistance value for each internal force in the absence of all other internal forces. These limiting resistances are special cases of Eqs. 2.3 through 2.6. From Eq. 2.3, knowing that the maximum torsional stress occurs when the longitudinal stress vanishes, one can express the plastic torsional capacity of the cross section, T_p as

$$T_p = 2\pi abt \frac{F_y}{\sqrt{3}} \quad (2.7)$$

The limiting tensile axial force, N_y is attained when the applied torsion is zero (i.e., $\sigma^* = F_y$) and when no bending moments are acting on the cross section, i.e., $\beta_2 = 2\pi + \beta_1$, therefore from Eq. 2.4 by arbitrarily taking $\beta_1 = 0$, one has

$$N_y = F_y t a \int_0^{2\pi} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.8)$$

The plastic resisting moment about major axis, M_{xp} occurs in the absence of torsion, minor axis bending and axial force. Under this scenario, the conditions $\beta_1 = 0$ and $\beta_2 = \pi$ hold, and Eq. 2.5 gives

$$M_{xp} = 2F_y t a^2 \int_0^\pi \sin \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.9)$$

Similarly, for the plastic resisting moment about the minor axis M_{yp} , $\beta_1 = -\pi/2$ and $\beta_2 = \pi/2$, Eq. 2.6 gives

$$M_{yp} = 2F_y t \frac{b}{a} a^2 \int_{-\pi/2}^{\pi/2} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.10)$$

It is noted that Eq. 2.8 is essentially identical to Eq. 3 of Chan and Gardner (Chan and Gardner 2008b). Also, Eqs. 2.9 and 2.10 are identical to Eqs. 5 and 6 of Chan and Gardner (Chan and Gardner 2008a). Equations 2.3 through 2.6 can be divided respectively by Eqs. 2.7 through 2.10 resulting in a set of non-dimensional equations. The non-dimensional equations recovered are universally valid for all elliptical hollow sections irrespective of their geometries.

$$T_r = \frac{T}{T_p} = \frac{\sqrt{3}\tau}{F_y} = \tau_r \quad (2.11)$$

in which τ_r is the ratio of shear stress τ to the yield shear stress $F_y/\sqrt{3}$

$$N_r = \frac{N}{N_y} = \sqrt{1 - \tau_r^2} \left(\frac{2 \int_{\beta_1}^{\beta_2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta}{\int_0^{2\pi} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta} - 1 \right) \quad (2.12)$$

$$M_{xr} = \frac{M_x}{M_{xp}} = \sqrt{1 - \tau_r^2} \frac{\int_{\beta_1}^{\beta_2} \sin \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta}{\int_0^\pi \sin \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta} \quad (2.13)$$

$$M_{yr} = \frac{M_y}{M_{yp}} = \sqrt{1 - \tau_r^2} \frac{\int_{\beta_1}^{\beta_2} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta}{\int_{-\pi/2}^{\pi/2} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta} \quad (2.14)$$

In Eqs. 2.11 through 2.14, T_r , N_r , M_{xr} and M_{yr} respectively are the ratios of torsion, axial force, bending moment about x axis and bending moment about the y axis relative to their respective plastic resistances (in the absence of other internal forces). These are the parametric equations of the interaction relation sought.

Conceptually, in order to recover an explicit form of the interaction relation, one needs to eliminate constants τ_r , β_1 and β_2 from Eqs. 2.11 through 2.14. It is noted however, that the integrals appearing in these equations cannot be explicitly evaluated. Thus, an exact closed form expression for the interaction equations for elliptical cross-sections of general dimensions is unattainable. For a given section, knowing values of any three of the four internal forces applied N , M_x and M_y , and T , Eqs. 2.11 to 2.14 will provide τ_r , β_1 and β_2 as well as the missing internal force.

2.6.2. Admissible Internal Force Combinations

It is clear that each of the internal forces (N, M_x, M_y, T) must meet the conditions $-1 \leq N_r \leq 1$, $-1 \leq M_{xr} \leq 1$, $-1 \leq M_{yr} \leq 1$, and $-1 \leq T_r \leq 1$, respectively. These conditions are necessary but not sufficient in ensuring that a given load combination $(N_r, M_{xr}, M_{yr}, T_r)$ is admissible. For that, one could multiply the load combination $(N_r, M_{xr}, M_{yr}, T_r)$ by an unknown scalar α , and solve Eqs. 2.11 through 2.14 for the four constants α , τ_r , β_1 and β_2 . This is equivalent to solving the dimensionless interaction equation $F(\alpha N_r, \alpha M_{xr}, \alpha M_{yr}, \alpha T_r) = 0$. Given the non-linearity of Eqs. 2.11 through 2.14, the solution is iterative. The scheme developed for the solution is presented in Appendix A. The value of α is an indicator of the proximity of the applied load combination

$(N_r, M_{xr}, M_{yr}, T_r)$ to the yield surface and is called load proportionality factor. A value of unity denotes a load combination lying on the interaction surface. When $\alpha < 1$, the combination needs to be scaled down in order to lie on the yield surface, i.e., is unattainable. In contrast, $\alpha > 1$ denotes that the load combination needs to be magnified in order to lie on the yield surface (i.e., the load combination does not cause the section to fully plastify).

2.6.3. Special Case: Thin-Walled Circular Sections

A thin walled circular section may be considered as a special case of an EHS with equal minor and major radii. In this case, the expression $(b/a)^2 \sin^2 \theta + \cos^2 \theta = 1$ holds true. By setting $a = b = r$, Eqs. 2.12 to 2.14 simplify into

$$N_r = \sqrt{1 - T_r^2} \left(\frac{\beta_2 - \beta_1}{\pi} - 1 \right) \quad (2.15)$$

$$M_{rx} = \frac{1}{2} \sqrt{1 - T_r^2} (-\cos \beta_2 + \cos \beta_1) \quad (2.16)$$

$$M_{yr} = \frac{1}{2} \sqrt{1 - T_r^2} (\sin \beta_2 - \sin \beta_1) \quad (2.17)$$

By eliminating β_1 and β_2 from all three equations, the relations simplify to

$$M_{xr}^2 + M_{yr}^2 = (1 - T_r^2) \cos^2 \left(\frac{\pi N_r}{2 \sqrt{1 - T_r^2}} \right) \quad (2.18)$$

Equation 2.18 is identical to Eq. 15 in Mohareb (2002) when the internal pressure and shear terms are assumed to vanish. Also, by setting $M_{yr} = T_r = 0$, Eq. 2.18 simplifies to $M_{xr}^2 = \cos^2(\pi N_r/2)$ which is identical to Eq. 3 in Hu. et al. (1993), where it was shown to agree well with experimental results for pipes subjected to bending and axial force.

2.6.4. Yield Surface for EHS of Common Geometries

According to Corus (2005), all elliptical hollow sections currently produced have an aspect ratio of two. The dimensions are measured from outer surface of the profile. The smallest available section is EHS-150×75×4 and the largest one is EHS-500×250×16, where the

first two numbers indicate the height and width of the section and the third number is the thickness, all in millimetres. For this range of available profiles, the value of $b/a = (B - t)/(H - t)$ varies from 0.467 to 0.490, where B and H are respectively the cross section width and height measured from the outside fibres of the section and t is the wall thickness. Except for a single profile designation (i.e., EHS-200×100×12.5), b/a always exceeds 0.470. Thus, the results reported in the present studies are conservatively based on $b/a = 0.470$. By setting $b/a = 0.470$ in Eqs. 2.8 to 2.10, and using Simpson's rule for numerical integration, one obtains the following equations

$$N_y = F_y t a \int_0^{2\pi} \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta = 4.770 F_y t a \quad (2.19)$$

$$M_{xp} = 2 F_y t a^2 \int_0^\pi \sin \beta \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta = 2.695 F_y t a^2 \quad (2.20)$$

$$M_{yp} = 2 F_y t (0.47) a^2 \int_{-\pi/2}^{\pi/2} \cos \beta \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta = 1.594 F_y t a^2 \quad (2.21)$$

By setting $b/a = 0.470$ in equations 2.12 to 2.14, and noting that $\tau_r = T_r$, the following equations are obtained

$$N_r^* = \frac{N_r}{\sqrt{1 - T_r^2}} = 0.4193 \int_{\beta_1}^{\beta_2} \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta - 1 \quad (2.22)$$

$$M_{xr}^* = \frac{M_{rx}}{\sqrt{1 - T_r^2}} = 0.7422 \int_{\beta_1}^{\beta_2} \sin \beta \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.23)$$

$$M_{yr}^* = \frac{M_{ry}}{\sqrt{1 - T_r^2}} = 0.5898 \int_{\beta_1}^{\beta_2} \cos \beta \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (2.24)$$

Table 2.1 provides the axial force ratio N_r^* (modified for torsion) for any set of given moment ratios M_{xr}^* and M_{yr}^* (also modified for torsion). The table was generated for a 0.05×0.05 grid. For any set of specified modified moment ratios M_{xr}^* and M_{yr}^* , the nonlinear system of equations based on Eqs. 2.23 and 2.24 is solved for values of β_1 and β_2 . Equations 2.23 and 2.24 are re-written as

$$G_1(\beta_1, \beta_2) = M_{xr}^* - 0.7422 \int_{\beta_1}^{\beta_2} \sin \beta \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta = 0 \text{ and}$$

$G_2(\beta_1, \beta_2) = M_{yr}^* - 0.5898 \int_{\beta_1}^{\beta_2} \cos \beta \sqrt{(0.47)^2 \sin^2 \beta + \cos^2 \beta} d\beta = 0$ and Newton method is adopted to express an improved solution guess vector $\langle \beta_1 \ \beta_2 \rangle_{n+1}$ in terms of the previous iteration vector $\langle \beta_1 \ \beta_2 \rangle_n$

$$\begin{Bmatrix} \beta_1 \\ \beta_2 \end{Bmatrix}_{n+1} = \begin{Bmatrix} \beta_1 \\ \beta_2 \end{Bmatrix}_n - \left[\begin{array}{cc} \frac{\partial G_1}{\partial \beta_1} & \frac{\partial G_1}{\partial \beta_2} \\ \frac{\partial G_2}{\partial \beta_1} & \frac{\partial G_2}{\partial \beta_2} \end{array} \right]_n^{-1} \begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix}_n \quad (2.25)$$

The iterative process continues until the square root of the sum of squares (SRSS) of functions G_1 and G_2 become smaller than a specified tolerance (i.e, 10^{-6}). In Eq. 2.25, it is noted that while G_1 and G_2 are implicit functions of β_1 and β_2 , their gradients $\partial G_i / \partial \beta_j$ have an explicit form, i.e.,

$$\begin{aligned} \partial G_1 / \partial \beta_1 &= 0.7422 \sin \beta_1 \sqrt{(0.47)^2 \sin^2 \beta_1 + \cos^2 \beta_1} \\ \partial G_2 / \partial \beta_1 &= 0.5898 \cos \beta_1 \sqrt{(0.47)^2 \sin^2 \beta_1 + \cos^2 \beta_1} \\ \partial G_1 / \partial \beta_2 &= -0.7422 \sin \beta_2 \sqrt{(0.47)^2 \sin^2 \beta_2 + \cos^2 \beta_2} \\ \partial G_2 / \partial \beta_2 &= -0.5898 \cos \beta_2 \sqrt{(0.47)^2 \sin^2 \beta_2 + \cos^2 \beta_2} \end{aligned} \quad (2.26a-d)$$

Given β_1 and β_2 , the corresponding modified axial force ratio $N_r^* = N_r / \sqrt{1 - T_r^2}$ is numerically computed by evaluating the right hand side of Eq. 2.22 using Simpson's rule.

The definition of the modified ratios N_r^* , M_{xr}^* , and M_{yr}^* reduces the number of parameters in the interaction from four to three arguments while still implicitly accounting for the effect of torsion. The interaction surface for the positive octant of the $N_r^* - M_{xr}^* - M_{yr}^*$ space is depicted in Fig. 2.4. Also, Fig. 2.5 depicts the relations between the modified moment ratios M_{xr}^* and M_{yr}^* for various values of modified axial force ratios.

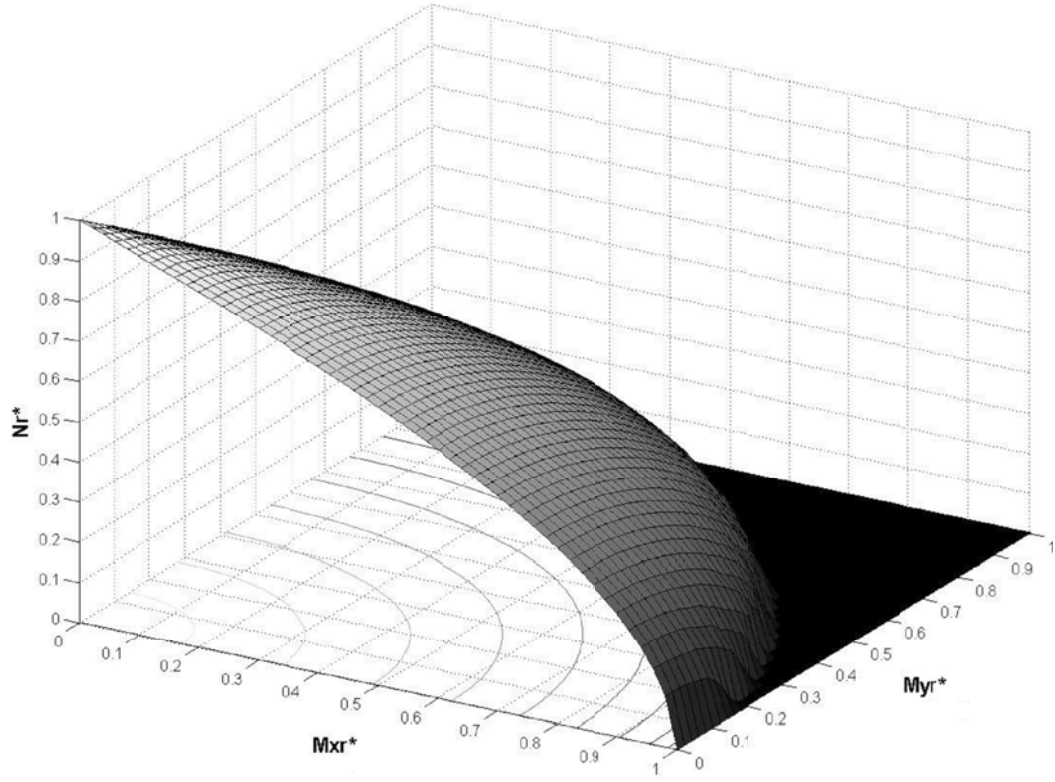


Fig. 2.4: Interaction surface for EHS with $b/a = 0.47$

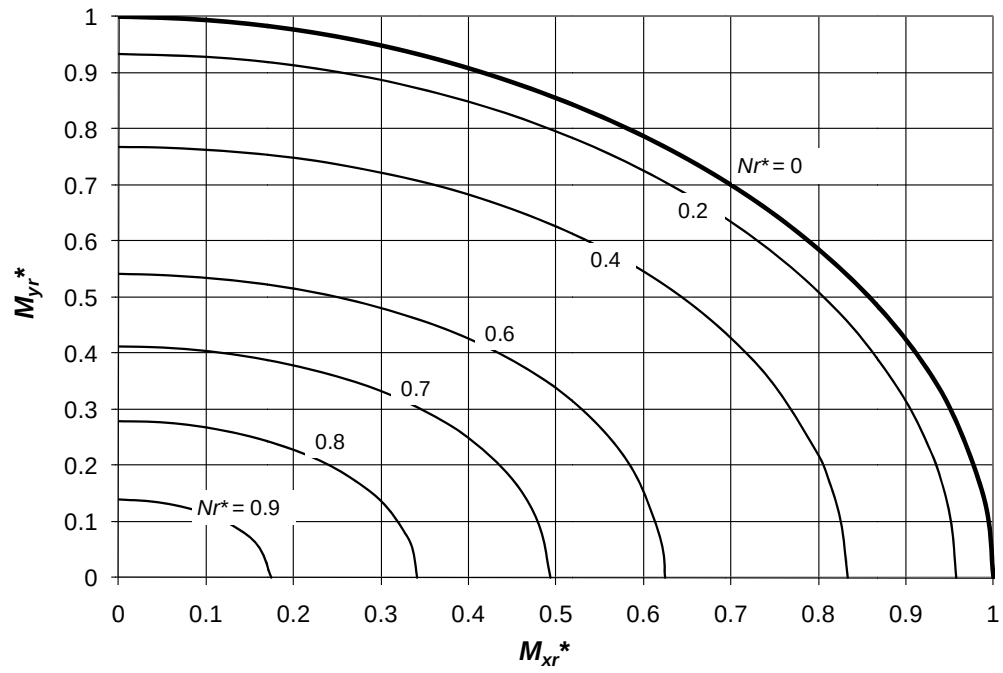


Fig. 2.5: EHS Interaction surface for constant N_r^* ($b/a = 0.47$)

Table 2.1: Exact interaction surface for EHS with $b/a = 0.47$

$-N_r^*$		M_{xr}^*																					
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00	
M_{yr}^*	0.00	1.00	0.97	0.94	0.92	0.89	0.86	0.83	0.79	0.76	0.73	0.69	0.66	0.62	0.58	0.54	0.49	0.44	0.38	0.31	0.22	0.00	
	0.05	0.96	0.96	0.93	0.91	0.88	0.85	0.82	0.79	0.76	0.73	0.69	0.66	0.62	0.58	0.53	0.49	0.44	0.38	0.31	0.22	-	
	0.10	0.93	0.92	0.91	0.89	0.87	0.84	0.81	0.78	0.75	0.72	0.69	0.65	0.61	0.57	0.53	0.48	0.43	0.37	0.30	0.21	-	
	0.15	0.89	0.89	0.88	0.86	0.84	0.82	0.80	0.77	0.74	0.71	0.67	0.64	0.60	0.56	0.52	0.47	0.42	0.36	0.29	0.19	-	
	0.20	0.86	0.85	0.85	0.83	0.82	0.80	0.77	0.75	0.72	0.69	0.66	0.63	0.59	0.55	0.51	0.46	0.41	0.34	0.27	0.16	-	
	0.25	0.82	0.82	0.81	0.80	0.79	0.77	0.75	0.73	0.70	0.67	0.64	0.61	0.57	0.53	0.49	0.44	0.39	0.32	0.25	0.12	-	
	0.30	0.78	0.78	0.78	0.77	0.75	0.74	0.72	0.70	0.68	0.65	0.62	0.59	0.55	0.51	0.47	0.42	0.37	0.30	0.21	0.03	-	
	0.35	0.75	0.75	0.74	0.73	0.72	0.71	0.69	0.67	0.65	0.62	0.59	0.56	0.53	0.49	0.45	0.40	0.34	0.27	0.17	-	-	
	0.40	0.71	0.71	0.70	0.70	0.69	0.67	0.66	0.64	0.62	0.59	0.57	0.53	0.50	0.46	0.42	0.37	0.31	0.23	0.10	-	-	
	0.45	0.67	0.67	0.67	0.66	0.65	0.64	0.62	0.60	0.58	0.56	0.53	0.50	0.47	0.43	0.38	0.33	0.26	0.17	-	-	-	
	0.50	0.63	0.63	0.63	0.62	0.61	0.60	0.59	0.57	0.55	0.53	0.50	0.47	0.44	0.39	0.35	0.29	0.21	0.08	-	-	-	
	0.55	0.59	0.59	0.59	0.58	0.57	0.56	0.55	0.53	0.51	0.49	0.46	0.43	0.40	0.35	0.30	0.24	0.14	-	-	-	-	
	0.60	0.55	0.55	0.55	0.54	0.53	0.52	0.51	0.49	0.47	0.45	0.42	0.39	0.35	0.31	0.25	0.17	-	-	-	-	-	
	0.65	0.51	0.51	0.50	0.50	0.49	0.48	0.47	0.45	0.43	0.41	0.38	0.34	0.30	0.25	0.18	-	-	-	-	-	-	
	0.70	0.46	0.46	0.46	0.45	0.45	0.43	0.42	0.40	0.38	0.36	0.33	0.29	0.24	0.17	-	-	-	-	-	-	-	
	0.75	0.42	0.42	0.41	0.41	0.40	0.39	0.37	0.35	0.33	0.30	0.27	0.22	0.16	-	-	-	-	-	-	-	-	
	0.80	0.37	0.37	0.36	0.36	0.35	0.33	0.32	0.30	0.27	0.24	0.19	0.13	-	-	-	-	-	-	-	-	-	
	0.85	0.31	0.31	0.31	0.30	0.29	0.28	0.26	0.23	0.20	0.15	0.06	-	-	-	-	-	-	-	-	-	-	
	0.90	0.25	0.25	0.24	0.23	0.22	0.20	0.18	0.14	0.08	-	-	-	-	-	-	-	-	-	-	-	-	
	0.95	0.17	0.17	0.16	0.15	0.13	0.10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	1.00	0.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	

2.6.5. Approximate Interaction Relations

This section aims at finding a simple function describing the interaction between the modified normalized modified internal forces $N_r^*, M_{xr}^*, M_{yr}^*$. Towards this goal, the numeric values in Table 2.1 were used to find a best fit surface for the points. Five approximate candidate functions $g_j(N_r^*, M_{xr}^*, M_{yr}^*)$ ($j = 1...5$) were examined for their ability to fit the numerical values. These are:

$$\begin{aligned}
 g_1(N_r^*, M_{xr}^*, M_{yr}^*) &= M_{xr}^{*a_1^1} + M_{yr}^{*a_2^1} + N_r^{*a_3^1} - 1 = 0 \\
 g_2(N_r^*, M_{xr}^*, M_{yr}^*) &= a_5^2 M_{xr}^{*a_1^2} + a_6^2 M_{yr}^{*a_2^2} + N_r^{*a_3^2} \\
 &\quad + (1 - a_4^2) M_{xr}^* + (1 - a_5^2) M_{yr}^* - 1 = 0 \\
 g_3(N_r^*, M_{xr}^*, M_{yr}^*) &= a_4^3 M_{xr}^{*a_1^3} + a_5^3 M_{yr}^{*a_2^3} + a_6^3 N_r^{*a_3^3} \\
 &\quad + (1 - a_4^3) M_{xr}^* + (1 - a_5^3) M_{yr}^* + (1 - a_6^3) N_r^* - 1 = 0 \\
 g_4(N_r^*, M_{xr}^*, M_{yr}^*) &= \sqrt{M_{xr}^{*a_1^4} + M_{yr}^{*a_2^4}} + N_r^{*a_3^4} - 1 = 0 \\
 g_5(N_r^*, M_{xr}^*, M_{yr}^*) &= \left(M_{xr}^{*a_1^5} + M_{yr}^{*a_2^5} \right)^{a_4^5} + N_r^{*a_3^5} - 1 = 0
 \end{aligned} \tag{2.27a-e}$$

where a_k^j are fitting parameters for the candidate function j which are to be determined from nonlinear regression analysis. A common feature between the candidate functions $g_j(N_r^*, M_{xr}^*, M_{yr}^*)$ is the fact that they all meet the conditions $g_j(1, 0, 0) = g_j(0, 1, 0) = g_j(0, 0, 1) = 0$.

For the Drucker's convexity criterion (Boresi and Sidebottom 1985) to be met, the inequality

$$N_r^* \frac{\partial g_j}{\partial N_r^*} + M_{rx}^* \frac{\partial g_j}{\partial M_{xr}^*} + M_{ry}^* \frac{\partial g_j}{\partial M_{yr}^*} > 1.0 \tag{2.28}$$

must be satisfied for the chosen fitting parameters.

For each candidate function, the sum of the squares of errors

$$E_j(a_k^j) = \sum_{p=1}^{p_{\max}} \left[g_j^2(a_k^j) \right]_p \tag{2.29}$$

was minimized in the $\{a_k^j\}$ space. In Eq. 2.29, $p_{\max}$ is the number of data points for the nonlinear regression (i.e., 340 for Table 2.1). A conceptual representation is depicted in Fig. 2.6 where the black dot represents a loading combination as determined in Table 2.1 and the white dot represents the corresponding approximation on the candidate interaction function g_j .

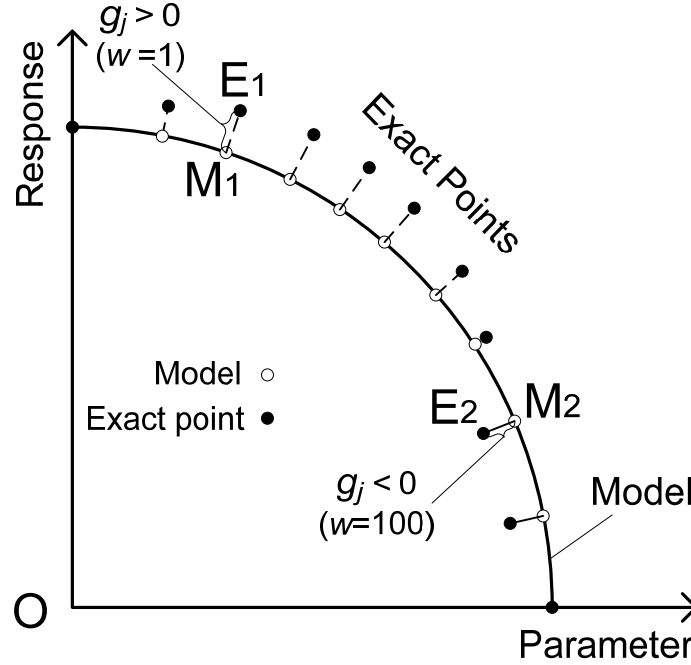


Fig. 2.6: Error in curve-fitting procedure

The sum of the squares of errors E_j for each candidate function g_j is minimized by enforcing the conditions

$$\left\{ \frac{\partial E_j(a_k^j)}{\partial a_l^j} \right\} = \left\{ \sum_{p=1}^{p_{\max}} 2 \left[g_j(a_k^j) \frac{\partial g_j(a_k^j)}{\partial a_l^j} \right]_p \right\} = \{0\} \quad (2.30)$$

in which, index p denotes a given point on the yield surface and the summation is performed for all points $p_{\max}$ in Table 2.1. The explicit expressions for the derivatives $\{\partial g_j(a_k^j)/\partial a_l^j\}$ of the five candidate functions are presented in Appendix B. By using Newton's method, and given a solution vector for the fitting parameters $\{a_k^j\}_n$ based on the n^{th} iteration, an improved guess solution vector $\{a_k^j\}_{n+1}$ is given by

$$\{a_k^j\}_{n+1} = \{a_k^j\}_n - \left[\frac{\partial}{\partial a_l^j} \left(g_j(a_k^j) \frac{\partial g_j(a_k^j)}{\partial a_k^j} \right) \right]^{-1} \left\{ g_j(a_k^j) \frac{\partial g_j(a_k^j)}{\partial a_k^j} \right\}_n \quad (2.31)$$

In Eq. 2.31, the terms of matrix $\left[\partial(g_j \partial g_j / \partial a_k^j) / \partial a_l^j \right]_n$ are numerically computed through the central finite difference technique and the matrix obtained is then inverted. The value of the best fit parameters and the corresponding error for each candidate function is tabulated in Table 2.2. It is noted that each set of parameters was found to satisfy the convexity condition (Eq. 2.28). Equation 2.27e provides the best fit to the numerical value in Table 2.1 while Eq. 2.27d provides a close second best fit. Equation 2.27d is simpler and allows isolating the effect of torsion after some simple manipulation, and was thus adopted in the following steps.

Table 2.2: Parameters of the fitted models

Candidate Function j	a_1^j	a_2^j	a_3^j	a_4^j	a_5^j	a_6^j	Sum of Squares of Errors E_j^*
1	2.380	1.608	1.058	—	—	—	6.718
2	5.882	5.292	1.087	0.537	0.396	—	1.078
3	4.657	3.639	9.917	0.650	0.558	-0.066	1.072
4	2.262	1.750	1.757	—	—	—	0.090
5	2.317	1.808	1.826	0.457	—	—	0.054

*) Based on Table 2.1.

2.6.6. Conservative Interaction Relation

Whilst Eq. 2.27d provides an excellent representation of the yield surface, it leads to slightly un-conservative results half of the time. In order to obtain a conservative curve fit suitable for design, a weighted regression analysis based on the same form of the function (i.e., Eq. 2.27d) is performed. The error weight for un-conservative prediction (such as point M_2 in Fig. 2.6) was taken as 100 time larger than that of the error weight for a conservative prediction (such as point M_1). , i.e.,

$$\begin{aligned}
E_j &= \sum_{p=1}^{p_{\max}} \left[w_p \left(g_j(a_k^j) \right)_p^2 \right] \\
w_p &= 1 \quad g_j \geq 0 \\
w_p &= 100 \quad g_j < 0
\end{aligned} \tag{2.32}$$

The obtained expression was found to be

$$\sqrt{M_{xr}^{*2.0} + M_{yr}^{*1.70}} + N_r^{*1.75} = 1.0 \tag{2.33}$$

Equation 2.33 provides an interaction surface that is conservative for 99.5% of the points. Nowhere was it found to overestimate the table values by more than half a percent. In Fig. 2.7 the ratio $N_r^{*1.75}$ on the vertical axis is plotted versus $\sqrt{M_{xr}^{*2.0} + M_{yr}^{*1.70}}$ on horizontal axis for the points of Table 2.1. On the same figure, Eq. 2.33 is overlaid as a solid line. It is apparent that the proposed equation provides a conservative fit for the numerical values.

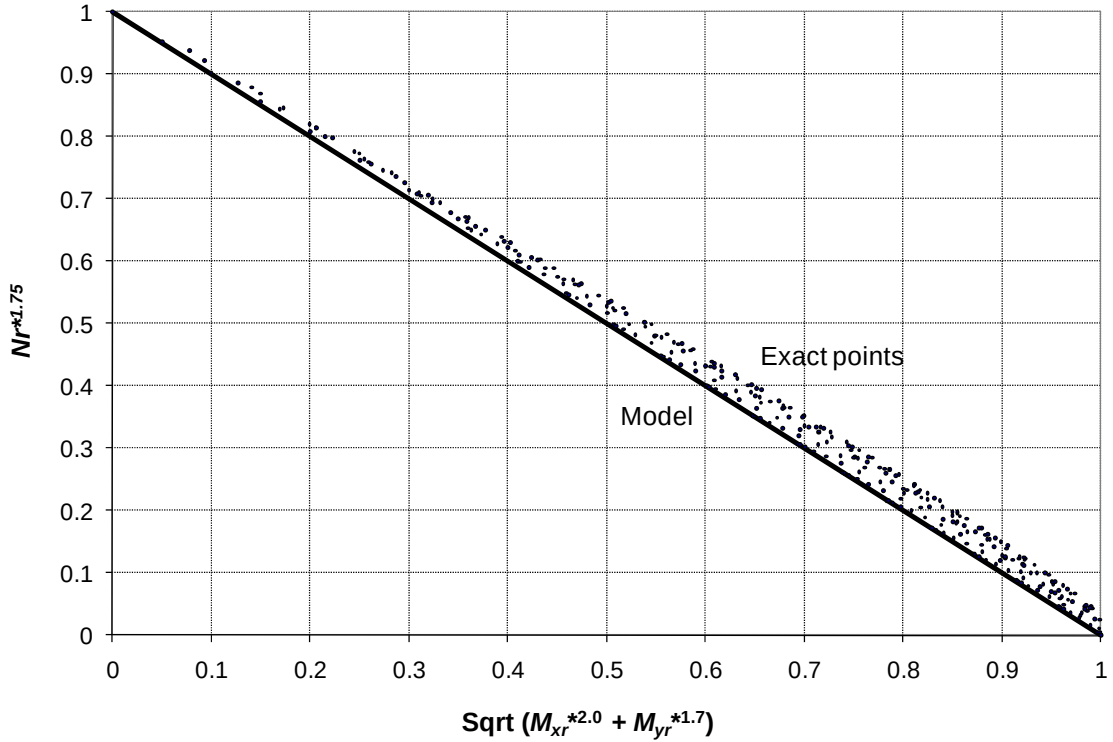


Fig. 2.7: Conservative interaction equation ($b/a = 0.47$)

For example, when $M_{xr}^* = 0.2$ and $M_{yr}^* = 0.3$, Eq. 2.33 predicts a modified axial force ratio $N_r^* = 0.74$. The corresponding value for N_r^* in Table 2.1 is 0.75. The proposed interaction equation provides an excellent approximation while erring slightly on the conservative side.

When $M_{xr}^* = 0.4$ and $N_r^* = 0.5$, Eq. 2.33 predicts a value for M_{yr}^* of 0.52 which is less than the value of 0.56 obtained from Table 2.1.

As depicted in Fig. 2.7, the proposed equation yields nearly exact values for high modified axial force ratios. It tends to slightly underestimate the section capacity as the modified axial force ratio decreases. Equation 2.33 can be expressed in terms of M_{xr} , M_{yr} , N_r and T_r by substituting M_{xr}^* , M_{yr}^* and N_r^* with their definitions provided in Eqs. 2.22 to 2.24 as

$$\left(\frac{M_{xr}}{\sqrt{1-T_r^2}} \right)^{2.0} + \left(\frac{M_{yr}}{\sqrt{1-T_r^2}} \right)^{1.7} + 2 \left(\frac{N_r}{\sqrt{1-T_r^2}} \right)^{1.75} - \left(\frac{N_r}{\sqrt{1-T_r^2}} \right)^{3.5} = 1 \quad (2.34)$$

2.6.7. Design Example

A 3 m span steel EHS-250×125×8 member (Fig. 2.8) has an yield strength of 350 MPa .

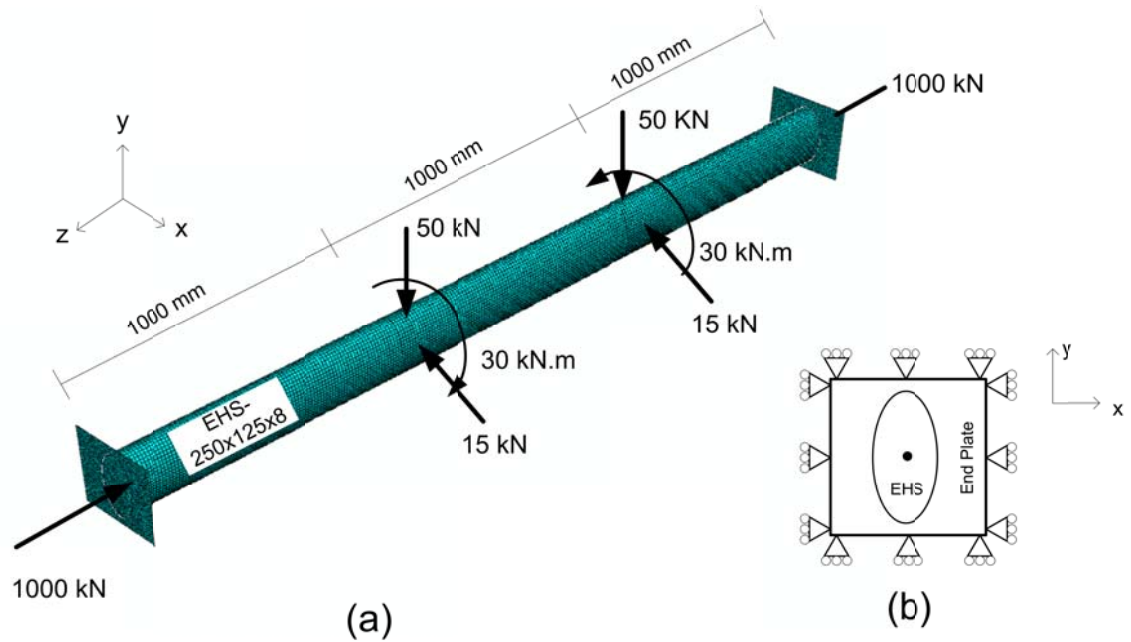


Fig. 2.8: (a) FEA model, (b) End plates and boundary conditions

The member is subject to a 1000 kN compressive force as shown, two 50 kN loads each acting at the one-third span points inducing a major bending moment in the middle section

of 50 kNm. Also, it is subject to 15 kN loads acting at the same point inducing a minor bending moment of 15 kNm. Two 30 kNm torsional moments are applied to the member as shown. Under the given twisting moments, two reaction torsional moments of 10 kNm each are generated at both ends. Thus, the twisting moment in the middle section is 20 kNm. It is thus required to determine, based on a cross-sectional resistance, whether the member withstands the above combination of internal forces (i.e. $N = -1,000$ kN . $M_x = 50$ kNm $M_y = 15$ kNm and $T = 20$ kNm)

For the given cross-section $D_1 = 250$ mm , $D_2 = 125$ mm , $a = 121$ mm , $b = 58.5$ mm , $t = 8$ mm and $b/a = 0.483$. Other limit states involving the presence of residual stresses are outside the scope of the present problem.

In order to use the formulation in this study the section must meet class 2 requirements. For a section under pure axial force, the value of the equivalent diameter to thickness ratio for an EHS $D_1^2/(tD_2)$ is 62.5, which is essentially at the borderline $21,100/F_y = 60.3$ between Classes 2 and 3 (2007) . For major axis bending moments, this value is much smaller than the limiting value of $41,000/F_y = 117.1$. These limits are taken from experimental study by Gardner and Chan (2007). For combined loading involving axial force and bending, one may expect a classification borderline between Class 2 and Class 3 for $D_1^2/(tD_2)$ ranging between 60.3 and 117.1 and the section can be assumed as meeting Class 2 requirements.

The minimum radius of gyration for the section is $r_{\min} = r_y = 45$ mm . Assuming pin-end conditions, the Euler buckling resistance of the member is $A \pi^2 E r_{\min}^2 / KL^2 = 1998$ kN which is much higher than the applied axial force. This suggests that the axial resistance is governed by yield rather than global buckling considerations.

Solution 1:

The exact lower bound interaction equations are used. Using the procedure described in Appendix A, one obtains $\alpha = 1.026$ which indicates that the capacity of the section is 2.6% higher than the applied loads. In other words, the section is attaining 97.5% of its plastic resistance. The values of β_1 and β_2 calculated from Eqs. 2.5 and 2.6 are substituted into

Eq. 2.4 to obtain the maximum axial force $N = -1,045 \text{ kN}$ that can be tolerated by the section.

Solution 2:

In order to verify the solution, a finite element model based on ABAQUS 6.7-1 (SIMULIA 2007a) was developed (Fig. 2.8). The model consisted of one middle and two end sections. Longitudinally, each member had a uniform mesh consisting of 100 elements, each having a length of 10 mm. Circumferentially, the model had 56 elements. The size of the 13 elements surrounding the major axis were half those surrounding the minor axis. The S4R element, a four-noded shell element with reduced integration, was used to model the EHS wall.

Steel material was assumed as linearly elastic-perfectly plastic. A modulus of elasticity $E = 200 \text{ GPa}$, a Poisson's ratio $\nu = 0.3$ and a yield strength $F_y = 350 \text{ MPa}$ were taken.

All transverse loads are applied as a ring load at the interfaces between the end section and the middle section. Only the resultants of these transverse loads are depicted in Fig. 2.8a. Torsion was applied as edge loads along the tangential direction. The axial force is applied at the end-sections as traction loads acting in the longitudinal direction. All loads were simultaneously applied to the model and the analysis progressed incrementally until full plastification of the middle cross-section was achieved. Analysis does not include second order effects due to geometric nonlinearity.

Two thick plates are modeled at both ends of the member such that they simulate a pin-pin condition about the major and minor axes (Fig. 2.8b). The joints on the horizontal edges of these plates are restrained against vertical movement and joints on the vertical edges are restrained against horizontal movement (i.e., they restrain twisting deformation). The central joint of one of the plates is restrained from longitudinal movement.

The maximum axial force attained by the model was 1,046 kN which is in excellent agreement with that predicted by the interaction coefficients presented in Solution 1.

Solution 3:

The exact aspect ratio b/a of the cross-section section is 0.483. This is slightly different from the value $b/a \approx 0.47$ adopted in the present study. The use of the interaction relations developed will introduce a slight approximation on the conservative side. From Eqs. 2.7, 2.19, 2.20 and 2.21 one can determine the section capacities in the absence of other internal forces as $T_p = 72 \text{ kNm}$, $N_y = 1,615 \text{ kN}$, $M_{xp} = 110 \text{ kNm}$, $M_{yp} = 65 \text{ kNm}$. The corresponding dimensionless internal force ratios are $T_r = 0.278$, $M_{xr}^* = 0.473$ and $M_{yr}^* = 0.248$. Linear interpolation on the grid in Table 2.1 provides the prediction $N_r^* = -0.658$ or $N = -1,021 \text{ kN}$, which is 2.3% less than the first and second solutions.

Solution 4:

A last solution is provided by applying the approximate conservative interaction relation (Eq. 2.34). By substituting the load ratios into Eq. 2.34 and solving for axial force, the axial force ratio obtained is $N_r^* = -0.599$ corresponding to $N = -967 \text{ kN}$, which is 7.5% less than the exact value. As expected, the last solution is the most conservative. In contrast to Solutions 1 to 3, Solution 4 conservatively predicts that the section is unable to withstand the applied load combination.

2.7. Summary and Conclusion

Interaction relations for elliptical hollow sections under combination of axial force, bi-axial bending moments and twisting moment were developed. The relations provide cross-sectional strength design criteria whether a section can withstand a combination of given internal forces. The relations are based on the fully plastic condition and will govern the design in cases where localized strains do not occur before the fully plastic resistance of the section is mobilized. The solution has been verified by using powerful finite element software, ABAQUS. The interaction equations simplified for the sections available in the market and appropriate design parameters, table and diagrams were provided. These relations were more simplified to one explicit equation as a conservative interaction relationship suitable for design purposes.

Previously established interaction relations for pipes are recovered as a special case of the obtained relations.

2.8. Notations

A	Cross section area
a, b	EHS center-line major and minor radius
D_0	Diameter of a pipe
D_1, D_2	EHS center-line major and minor diameter
E	Steel modulus of elasticity
F_y	Steel yield stress
I	Section minimum moment of inertia
K	Effective length factor
L	Member length
M_x	Resultant bending moments about X (major) axis
M_{xp}	Plastic capacity of section for bending moments about X axis
M_{rx}	Bending moment resistance about X axis
M_y	Resultant bending moments about Y (minor) axis
M_{yp}	Plastic capacity of section for bending moments about Y axis
M_{ry}	Bending moment resistance about Y axis
N	Resultant axial force
N_p	Plastic capacity of section for axial force
N_r	Axial force resistance excl. global buckling and residual stresses
r	Section radius of gyration
T	Resultant twisting moments
T_p	Plastic capacity for twisting moments
T_r	Torsional resistance
t	EHS or pipe thickness
β	Parameter of ellipse [i.e., $x = b \cos(\beta)$, $y = a \sin(\beta)$]
σ	Longitudinal stress
σ^*	Maximum longitudinal stress in the presence of τ
τ	Tangential shear stress

2.9. Appendix A: Formulation for Iterative Procedure

We rewrite equations 2.11 through 2.14 after scaling the given load combination (N, M_x, M_y, T) by a factor α as

$$\begin{aligned}
 H_1(\alpha, \tau_r, \beta_1, \beta_2) &= \alpha T_r - \tau_r = 0 \\
 H_2(\alpha, \tau_r, \beta_1, \beta_2) &= \alpha N_r - \sqrt{1 - \tau_r^2} \left(\frac{2}{I_1} \int_{\beta_1}^{\beta_2} S(\beta) d\beta - 1 \right) = 0 \\
 H_3(\alpha, \tau_r, \beta_1, \beta_2) &= \alpha M_{xr} - \frac{\sqrt{1 - \tau_r^2}}{I_2} \int_{\beta_1}^{\beta_2} \sin \beta S(\beta) d\beta = 0 \\
 H_4(\alpha, \tau_r, \beta_1, \beta_2) &= \alpha M_{yr} - \frac{\sqrt{1 - \tau_r^2}}{I_3} \int_{\beta_1}^{\beta_2} \cos \beta S(\beta) d\beta = 0
 \end{aligned} \tag{2.35a-d}$$

in which $S(\beta) = \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}$ and the integrals $I_1 = \int_0^{2\pi} S(\beta) d\beta$, $I_2 = \int_0^\pi \sin \beta S(\beta) d\beta$, and $I_3 = \int_{-\pi/2}^{\pi/2} \cos \beta S(\beta) d\beta$ are numerically evaluated using Simpson's rule.

It is now required to solve the system of non-linear functions H_1 through H_4 in arguments $\alpha, \tau_r, \beta_1, \beta_2$. Using Newton's method, and given a guess solution vector $\langle \alpha \ \tau_r \ \beta_1 \ \beta_2 \rangle_n^T$ based on the n th iteration, an improved solution vector $\langle \alpha \ \tau_r \ \beta_1 \ \beta_2 \rangle_{n+1}^T$ for the $(n+1)$ th iteration would be determined from

$$\begin{Bmatrix} \alpha \\ \tau_r \\ \beta_1 \\ \beta_2 \end{Bmatrix}_{n+1} = \begin{Bmatrix} \alpha \\ \tau_r \\ \beta_1 \\ \beta_2 \end{Bmatrix}_n - \nabla_n^{-1} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{Bmatrix}_n \tag{2.36}$$

in which

$$\nabla_n = \begin{bmatrix} \frac{\partial H_1}{\partial \alpha} & \frac{\partial H_1}{\partial \tau_r} & \frac{\partial H_1}{\partial \beta_1} & \frac{\partial H_1}{\partial \beta_2} \\ \frac{\partial H_2}{\partial \alpha} & \frac{\partial H_2}{\partial \tau_r} & \frac{\partial H_2}{\partial \beta_1} & \frac{\partial H_2}{\partial \beta_2} \\ \frac{\partial H_3}{\partial \alpha} & \frac{\partial H_3}{\partial \tau_r} & \frac{\partial H_3}{\partial \beta_1} & \frac{\partial H_3}{\partial \beta_2} \\ \frac{\partial H_4}{\partial \alpha} & \frac{\partial H_4}{\partial \tau_r} & \frac{\partial H_4}{\partial \beta_1} & \frac{\partial H_4}{\partial \beta_2} \end{bmatrix}_n =$$

$$\begin{bmatrix} T_r & -1 & 0 & 0 \\ N_r & \frac{\tau_r}{\sqrt{1-\tau_r^2}} \left(\frac{2 \int_{\beta_1}^{\beta_2} S(\beta) d\beta}{I_1} - 1 \right) & \frac{2\sqrt{1-\tau_r^2}}{I_1} S(\beta_1) & -\frac{2\sqrt{1-\tau_r^2}}{I_1} S(\beta_2) \\ M_{rx} & \frac{\tau_r \int_{\beta_1}^{\beta_2} \sin \beta S(\beta) d\beta}{I_2 \sqrt{1-\tau_r^2}} & \frac{\sqrt{1-\tau_r^2}}{I_2} \sin \beta_1 S(\beta_1) & -\frac{\sqrt{1-\tau_r^2}}{I_2} \sin \beta_2 S(\beta_2) \\ M_{ry} & \frac{\tau_r \int_{\beta_1}^{\beta_2} \cos \beta S(\beta) d\beta}{I_3 \sqrt{1-\tau_r^2}} & \frac{\sqrt{1-\tau_r^2}}{I_3} \cos \beta_1 S(\beta_1) & -\frac{\sqrt{1-\tau_r^2}}{I_3} \cos \beta_2 S(\beta_2) \end{bmatrix}_n$$

2.10. Appendix B: Formulation for Regression Analysis

The following formulas are forming the gradient of functions, needed for regression.

$$g_1 = M_{xr}^{*a_1^1} + M_{yr}^{*a_2^1} + N_r^{*a_3^1} - 1 = 0$$

$$\frac{\partial g_1}{\partial a_k^1} = \begin{Bmatrix} \partial g_1 / \partial a_1^1 \\ \partial g_1 / \partial a_2^1 \\ \partial g_1 / \partial a_3^1 \end{Bmatrix} = \begin{Bmatrix} M_{xr}^{*a_1^1} \ln(M_{xr}^*) \\ M_{yr}^{*a_2^1} \ln(M_{yr}^*) \\ N_r^{*a_3^1} \ln(N_r^*) \end{Bmatrix} \quad (2.37)$$

Function g_2 is a special case of g_3 when $a_7^3 = 0$, thus the equation of $\partial g_3 / \partial a_k^3$ may be reduced accordingly to be utilized for $\partial g_2 / \partial a_k^2$.

$$g_3 = a_5^3 M_{xr}^{*a_1^3} + a_6^3 M_{yr}^{*a_2^3} + a_7^3 N_r^{*a_3^3} + (1 - a_5^3) M_{xr}^* + (1 - a_6^3) M_{yr}^* + (1 - a_7^3) N_r^* - 1 = 0 \quad (2.38)$$

$$\frac{\partial g_3}{\partial a_k^3} = \begin{Bmatrix} \partial g_3 / \partial a_1^3 \\ \partial g_3 / \partial a_2^3 \\ \partial g_3 / \partial a_3^3 \\ \partial g_3 / \partial a_5^3 \\ \partial g_3 / \partial a_6^3 \\ \partial g_3 / \partial a_7^3 \end{Bmatrix} = \begin{Bmatrix} a_5^3 M_{xr}^{*a_1^3} \ln(M_{xr}^*) \\ a_6^3 M_{yr}^{*a_2^3} \ln(M_{yr}^*) \\ a_7^3 N_r^{*a_3^3} \ln(N_r^*) \\ M_{xr}^{*a_1^3} - M_{xr}^* \\ M_{yr}^{*a_2^3} - M_{yr}^* \\ N_r^{*a_3^3} - N_r^* \end{Bmatrix} \quad (2.39)$$

Function g_4 is a special case of g_5 when $a_4^5 = 0.5$ thus the equation for $\partial g_5 / \partial a_k^5$ may be reduced accordingly to be utilized for $\partial g_4 / \partial a_k^4$.

$$g_5 = \left(M_{xr}^{*a_1^5} + M_{yr}^{*a_2^5} \right)^{a_4^5} + N_r^{*a_3^5} - 1 \quad (2.40)$$

$$\frac{\partial g_5}{\partial a_k^5} = \begin{Bmatrix} \partial g_5 / \partial a_1^5 \\ \partial g_5 / \partial a_2^5 \\ \partial g_5 / \partial a_3^5 \\ \partial g_5 / \partial a_4^5 \end{Bmatrix} = \begin{Bmatrix} a_4^5 \left(M_{xr}^{*a_1^5} + M_{yr}^{*a_2^5} \right)^{a_4^5 - 1} M_{xr}^{*a_1^5} \ln(M_{xr}^*) \\ a_4^5 \left(M_{xr}^{*a_1^5} + M_{yr}^{*a_2^5} \right)^{a_4^5 - 1} M_{yr}^{*a_2^5} \ln(M_{yr}^*) \\ \left(M_{xr}^{*a_1^5} + M_{yr}^{*a_2^5} \right)^{a_4^5} \ln \left(M_{xr}^{*a_1^5} + M_{yr}^{*a_2^5} \right) \\ N_r^{*a_3^5} \ln(N_r^*) \end{Bmatrix} \quad (2.41)$$

CHAPTER 3

Upper Bound Plastic Interaction Relations for Elliptical Hollow Sections²

3.1. Abstract

A family of interaction expressions are developed for steel elliptical hollow sections (EHS) subjected to combinations of axial force, bi-axial bending moments, torsion and bimoments. The formulation is based on kinematically admissible strain fields within the context of the upper bound theorem of plasticity.

The interaction relations derived successfully capture the effect of confining radial strains present at welded end sections, as well as sections that are free to deform in the radial direction away from end welded sections.

The interaction expressions developed consist of a set of elliptical integrals which are cast in a dimensionless form applicable for EHS of general geometries. An iterative solution technique is developed to solve the resulting non-linear relations. The applicability of the resulting interaction relations for conducting the cross-sectional check is illustrated through examples. A comparison with shell finite element analysis results illustrates the validity of the interaction relations derived.

Key words: Steel, Elliptical Hollow Section, Plasticity, Yield Surface, Bimoments

² Nowzartash, F., and Mohareb, M. (2010). "Upper Bound Plastic Interaction Relations for Elliptical Hollow Sections." *J. of Eng. Mech., ASCE*, 136, 1015-1027

3.2. Introduction and Scope

Steel members with elliptical hollow sections (EHS) have recently emerged into the family of structural sections. Since members of such cross-sections are aesthetically pleasing, their popularity is increasing among architects. From a structural engineering viewpoint, elliptical sections have desirable properties as well. They have a high resistance to torsion and lateral torsional buckling. As compression members, their differing major and minor axis properties provide efficient use of material when the effective length about the minor axis is smaller than that of the major axis. Also, they become more economical than circular or square hollow sections when the bending moments about the major axis are larger than those about the minor axis. The lack of procedures specifically geared towards EHS design has forced engineers to either conduct their design in an overly conservative manner or to resort to time consuming finite element analysis. Within this context, the present study aims at developing refined interaction relations specific to EHS subject to general combinations of internal forces involving axial force, biaxial bending, twisting moments and bimoments induced by torsion. The interaction relations are applicable to sections capable of attaining their plastic resistance prior to undergoing local buckling.

3.3. Literature Review

The literature review focuses on two aspects relevant to the present study. These are a) research related to EHS, and b) research related to upper bound interaction relations.

3.3.1. Research Related to Elliptical Hollow Sections

Interest in the buckling and post buckling behaviour of thin-walled oval cylindrical shells dates back to the years 1960s and 1970s. Kempner (1962) showed that the buckling resistance of an elliptical cylinder under pure axial compression is in good agreement with that of a thin-walled circular cylinder with a radius equal to the maximum radius of curvature of the elliptical section. The buckling behaviour of thin walled elliptical sections has been investigated for elements subject to pure compressive axial force (Hutchinson 1968), combined axial compression and bending moments (Kempner and Chen 1974) and combined axial compression and asymmetric bending (Chen and Kempner 1976). The effects of end fixity conditions on members under compression were experimentally

investigated (Feinstein et al. 1971). These studies focused on very thin walled sections made from polyester films that are unable to reach their section plastic resistance prior experiencing local buckling.

Only recently has the steel manufacturing industry introduced steel EHS as a structural section. Thus, only a few relevant studies were published to date on EHS. This includes a study on effects of load eccentricity on the stress fields of cylinders with elliptical hollow sections (Grigorenko and Rozhok 2002), and a series of compression and bending tests conducted on elliptical hollow sections cylinders (Gardner and Ministro 2004; Gardner 2005). These studies were followed by finite element analyses which were shown to successfully predict the behaviour of EHS under such loading. Willibald et al. (2006) studied the behaviour of gusset plate connections to elliptical section under pure axial forces. Zhu and Wilkinson (2007) conducted a finite element study to investigate the local buckling behaviour of EHS. A classification guideline for EHS under axial force, and bending about the major and minor axes was proposed by Gardner and Chan (2007). They developed analytical expressions for the yield compressive strength, plastic bending and shear resistances of hot-rolled EHS and verified them against experimental results (CAN/CSA-Z662 2007; Chan and Gardner 2008a; b). More recently, their work has been extended to stainless steel EHS (Silvestre 2008). The local elastic buckling of EHS including the shear deformation was investigated by Ruiz-Teran and Gardner (2008). Silvestre (2008) formulated a buckling solution for EHS and studied the buckling failure modes of EHS under axial compression in a thorough parametric finite element analysis. Recently, Nowzartash and Mohareb (2009) developed a lower bound interaction relation for EHS under combined action of axial force, bi-axial bending moments and twisting moments based on statistically admissible stress field. They also extended their formulation to Semi-Elliptical Hollow sections (Nocedal and Wright 1999). The present paper complements the previous study by developing an upper bound interaction relation for EHS based on kinematically admissible strain fields. In addition to the internal forces in Nowzartash and Mohareb (2009), the present formulation captures the bimoment effect taking place in EHS under twisting moments, as well as confining effects which take place at welded end sections

3.3.2. Research Related to Upper Bound Interaction Relations

Upper bound interaction relations were developed for various sections subject to axial force and bending moments (Chen and Atsuta 1972), under bending and twisting moments and for pipes under general loading (Mohareb 2003). In these solutions, upper bound interaction relations were obtained by equating the dissipation in internal strain energy to the change in external work done through associated generalized incremental strains. Internal strain energy is induced by assumed kinematically admissible strain increments. For problems involving a single non-zero stress component, kinematically admissible strain increment fields can be intuitively postulated (e.g., Hodge 1981). For problems involving more complex strain/stress states, general expressions for strain increment fields can be obtained by taking the appropriate derivatives of continuous displacement fields. This approach was used in early works (Hill and Siebel 1953; Gaydon and Nuttall 1957). This approach was adopted in the present study under the simplifying assumption that the EHS material is incompressible. The obtained incremental strain fields are kinematically admissible and are used to derive an upper bound plastic interaction relation for EHS subject to axial load, biaxial bending moments, twisting moments and bi-moments.

For stockier circular pipe sections, the upper bound solution in (Mohareb 2003) was shown to coincide with the lower bound solution (Mohareb 2001) since the postulated strain increment fields in (Mohareb 2003) and the postulated stress fields (Mohareb 2001) both were “exact” within the limitations of formulations. In contrast, the interaction relations developed in the present upper bound solution are found to be distinct from those based on the recently developed lower bound solution (Nowzartash and Mohareb 2009).

3.4. Statement of Problem

An upper bound interaction relation for steel elliptical hollow sections (EHS) subject to the combined action of biaxial bending, torsion, bimoments and axial force is sought. It is assumed that the fully plastic resistance of the cross section will be attained. An EHS (Fig. 2.2) subject to axial force N (positive when in tension), bending moment components M_x and M_y about the principal axes X and Y in the plane of the cross section, twisting moment T and associated bimoment B is considered. It is required to determine whether or not the

EHS can withstand the action of the combined stress resultants under the assumption that the fully plastic resistance of the section is reached.

Mathematically, it is required to find an interaction relation of the form $f(N, M_x, M_y, T_w, T_{sv}, B) = 0$ such that the condition $f < 0$ is met for a physically possible combination of internal forces, $f = 0$ corresponds to a fully plastic state of the cross section, and the condition $f > 0$ is unattainable under the assumptions of the formulation.

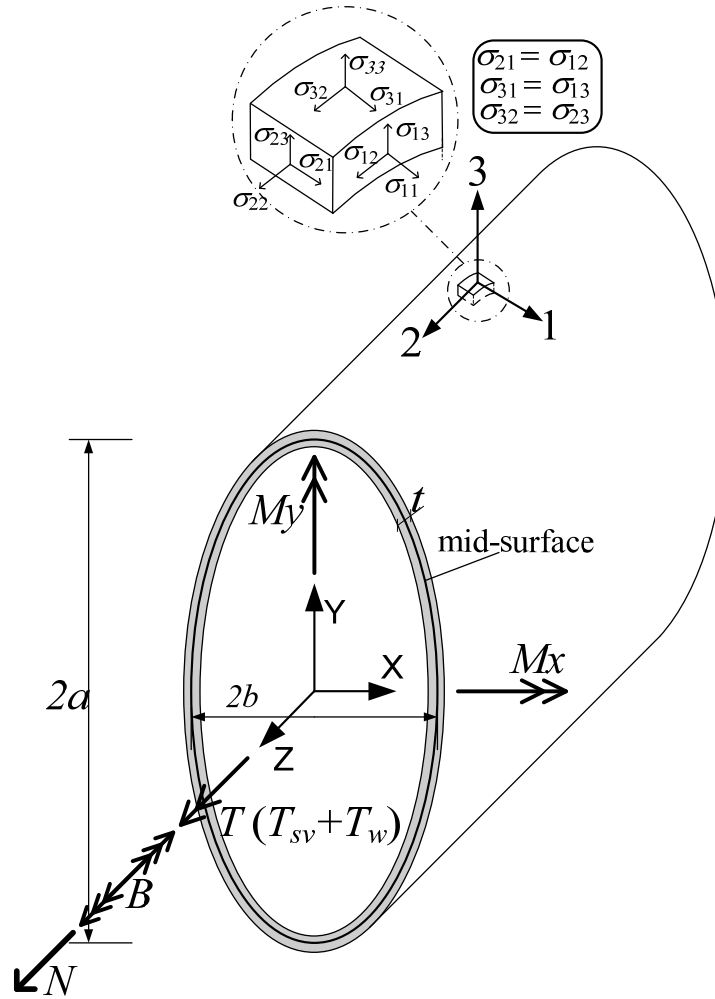


Fig. 3.1: Loads and stresses acting on an EHS member

3.5. Assumptions

3.5.1. Kinematics

The kinematic assumptions made are:

1. The interaction relations obtained in this paper for EHS subjected to axial force, bi-axial bending moments, torsion and bimoments do not consider the simultaneous combination with shear forces. Therefore, these interaction relations are applicable when shear forces are negligible.
2. In line with the Euler-Bernoulli beam theory, for a section under bending and axial deformation (no twisting), it is assumed that a section originally plane and normal to the member axis remains plane and perpendicular to the member axis after deformation. This assumption implies that shear deformation is neglected in the formulation. The shear deformations can be certainly neglected since the shear stresses have been assumed to be negligible according to the previous assumption.
3. The Bencosker (1954) warping function for closed section is adopted to describe the warping deformation induced by torsion.
4. Strain gradients across the section wall are assumed to vanish. This assumption is approached for relatively thin-walled sections such as EHS currently available in the steel construction industry.
5. Hoop strains are assumed proportional to the longitudinal strains. As subsequently discussed, this assumption was observed to be valid in a wide range of shell finite element analyses.

Under the assumptions above, an infinitesimal element of volume within the section wall undergoes two types of strains, a shear strain tangent to the section mid-surface and a longitudinal strain. All other strain components are assumed negligible within the present development.

3.5.2. Constitutive Modeling

Tension coupon tests show that EHS steel exhibits a distinct yield plateau (Zhu and Wilkinson 2007). Therefore, a bilinear elastic-perfectly plastic stress versus strain representation is adopted. The first line, representing the linear range of deformation, passes through the origin and has a slope identical to the initial slope of a tension coupon tests. The second line has zero slope (i.e., strain hardening is neglected) and passes through the yield point. The additional capacity of the section due to strain hardening is neglected.

Steel is assumed to yield in accordance with the maximum distortional energy density yield criterion (Boresi and Sidebottom 1985) which means a point on the EHS section (Fig. 2.2) yields when the following condition is met

$$(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{11} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2) = 2F_y^2 \quad (3.1)$$

where $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}$ and σ_{23} are stress tensor components at the point considered and F_y is the yield strength of the EHS material.

For an infinitesimal element of volume taken from the section wall, the relevant stress components are the longitudinal normal stress $\sigma = \sigma_{22}$, the hoop stress $\sigma_H = \sigma_{11}$ and the tangential shearing stress $\tau = \sigma_{12}$. Longitudinal stress σ is induced by the axial forces, bending moments and/or warping. Shearing stress τ is caused by twisting moments. As a simplification, shear stresses σ_{13} and σ_{23} acting through the thickness and the radial stress σ_{33} are assumed negligible. This assumption is realistic for thin walled sections. Based on these assumptions, the maximum distortional energy density yield criterion simplifies to

$$\sigma^2 - \sigma\sigma_H + \sigma_H^2 + 3\tau^2 - F_y^2 = 0 \quad (3.2)$$

In metal plasticity, an associative plasticity rule is commonly adopted. Thus, the yield function in Eq. 3.2 is used as a plastic potential to quantify the incremental plastic strains.

3.5.3. Other assumptions

Other assumptions made in the formulation are: a) no distinction is made between true and engineering stresses, b) the cross-section attains its plastic resistance prior undergoing local buckling, c) residual stress effects are neglected, in line with the results in (Gardner and

Ministro 2004; Chan and Gardner 2008b), and d) global buckling is not a governing mode of failure.

3.6. Postulated Displacement Fields

3.6.1. Displacements due to bending and extension

A generic point P on the section mid-surface subtends an angle θ at the ellipse centroid relative to the positive X-axis (Fig. 3.2a). The section centroid undergoes a displacement w_0 in the longitudinal direction z as a result of axial force (Fig. 3.2b). Also, the section undergoes angles of rotation $\phi_x = \phi_x(z)$ about the X-axis (Fig. 3.2c) and $\phi_y = \phi_y(z)$ about the Y-axis (Fig. 3.2d) caused by bending. The location of point P in the deformed configuration is point P' .

The coordinates of a point P on the section are $(r \cos \theta, r \sin \theta)$, where r is the distance of point P from the section centroid O . When the section undergoes longitudinal displacement w_0 and angles of rotation $\phi_x(z)$ and $\phi_y(z)$, the longitudinal displacement of point P is

$$w(\theta, z) = w_0(z) + r\phi_x(z) \sin \theta - r\phi_y(z) \cos \theta \quad (3.3)$$

The coordinates of point P can be expressed as $(b \cos \beta, a \sin \beta)$ where angle β is depicted in Fig. 3.3. Based on the geometric relations $r \sin \theta = a \sin \beta$ and $r \cos \theta = b \cos \beta$ (Fig. 3.3), Eq. 3.3 is expressed as

$$w(\beta, z) = w_0(z) + a\phi_x(z) \sin \beta - b\phi_y(z) \cos \beta \quad (3.4)$$

Also, as a result of angles of rotation $\phi_x = \phi_x(z)$, $\phi_y = \phi_y(z)$, the centroidal axis O of an EHS of length z undergoes in plane displacements u_0, v_0 (Fig. 3.2e) given by

$$u_0(z) = \int_0^z \phi_y(z) dz \quad (3.5)$$

$$v_0(z) = - \int_0^z \phi_x(z) dz \quad (3.6)$$

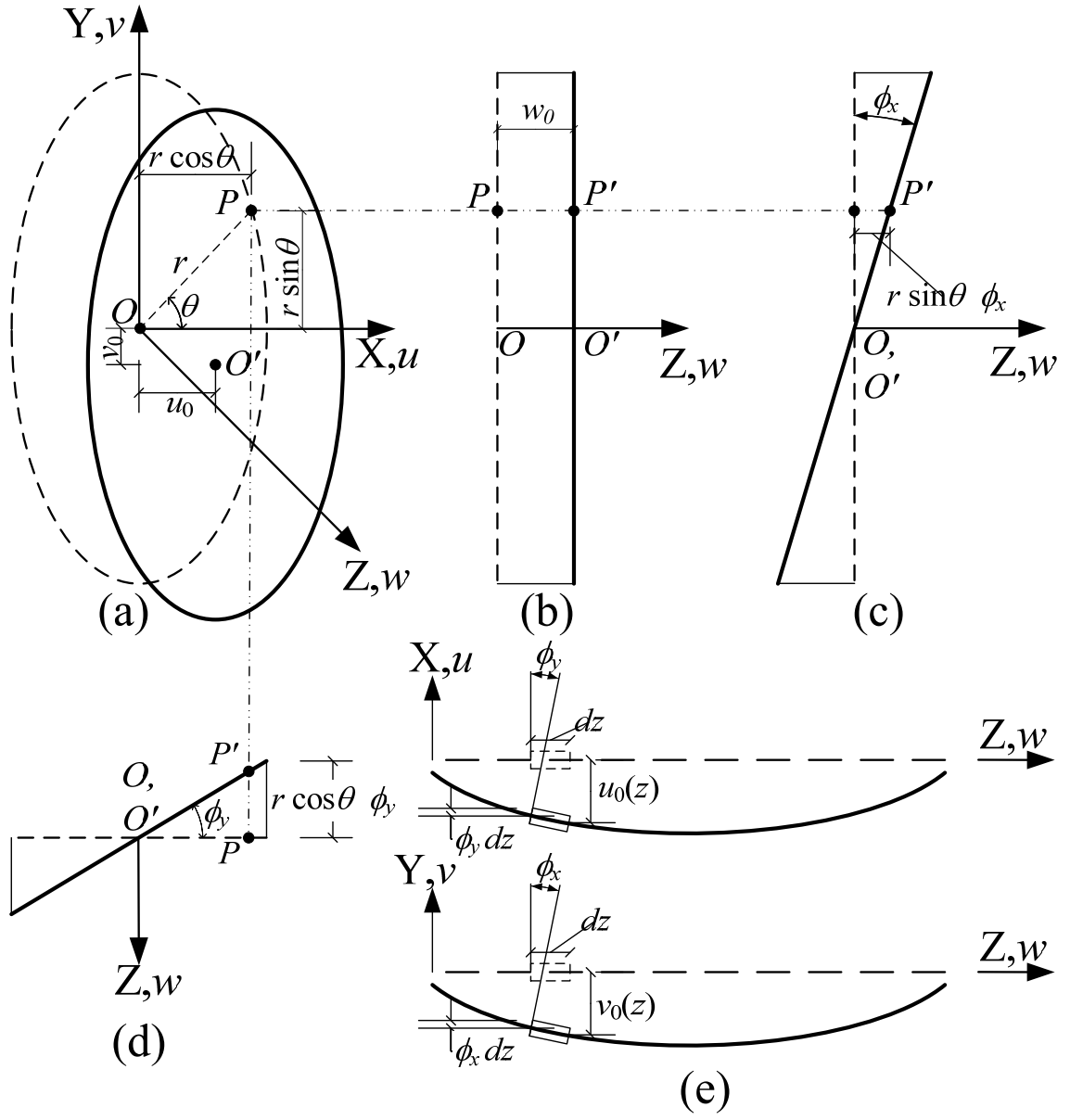


Fig. 3.2: Displacements of EHS under extension and rotation about X and Y axes

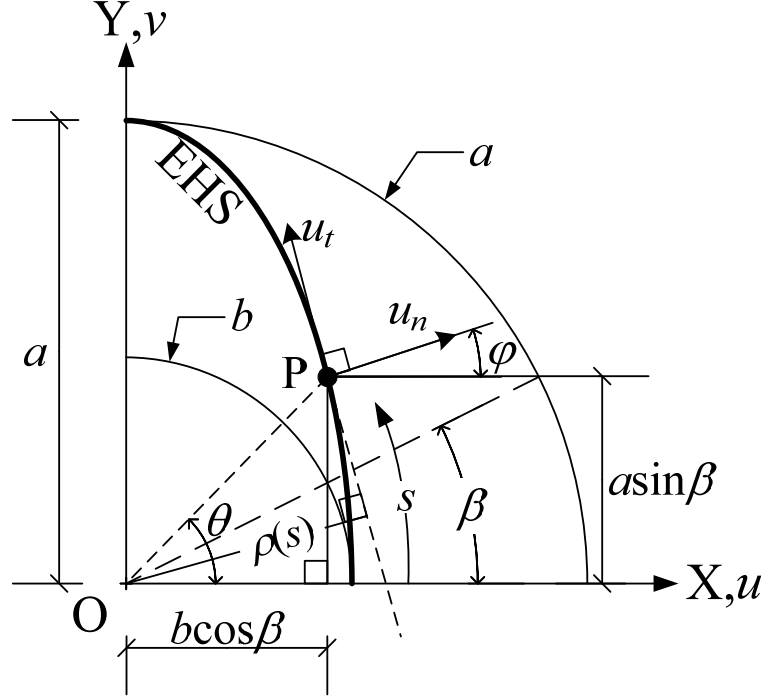


Fig. 3.3: Geometry and displacement components of an EHS

3.6.2. Displacements due to Torsion and Warping

A generic point P on the section mid-surface subtends an angle θ at the centroid of the ellipse (Fig. 3.4), relative to the positive X axis. The section undergoes an angle of twist $\psi(z)$ about the section centroid O . Under the assumption of no cross-sectional distortion, the location of point P in the deformed configuration is point P'' .

The position vector of point OP'' relative to the displaced origin O is $(r \cos(\theta + \psi(z)), r \sin(\theta + \psi(z)))$. Thus, the in-plane displacement components due to twist $u_T(\theta, z)$ and $v_T(\theta, z)$, both defined along the X and Y axes respectively, can be expressed as

$$u_T(\theta, z) = r(\cos(\theta + \psi(z)) - \cos \theta) \cong -r\psi(z) \sin \theta \quad (3.7)$$

$$v_T(\theta, z) = r(\sin(\theta + \psi(z)) - \sin \theta) \cong r\psi(z) \cos \theta \quad (3.8)$$

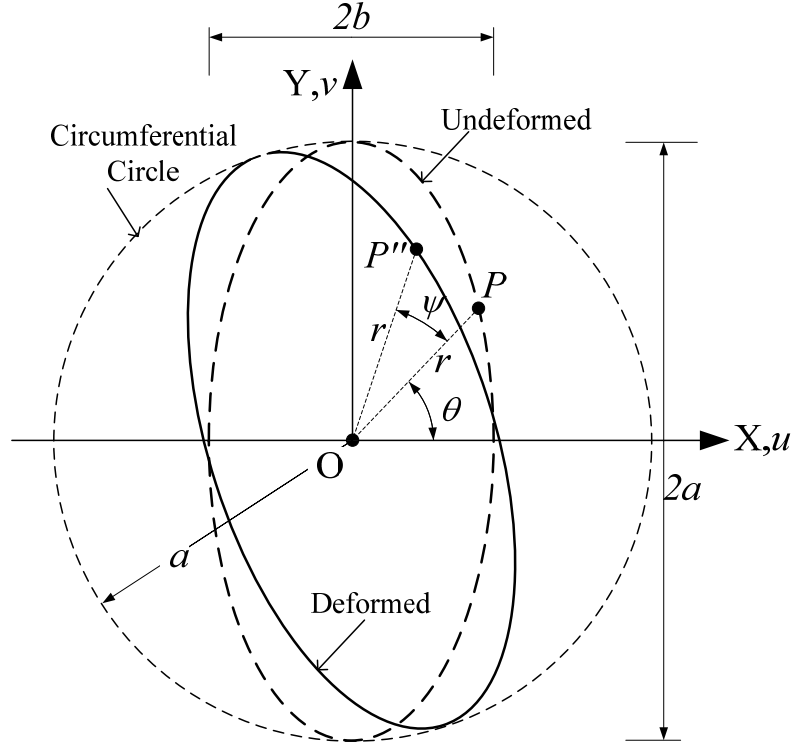


Fig. 3.4: Displacements of an EHS under twisting

The approximation in the right hand sides of Eqs. 3.7 and 3.8 holds for a small angle of twist ψ . Equations 3.7 and 3.8 can be written as a function of β as

$$u_T(\beta, z) = -a\psi(z) \sin \beta \quad (3.9)$$

$$v_T(\beta, z) = b\psi(z) \cos \beta \quad (3.10)$$

In addition to the in-plane displacements u_T and v_T , an elliptical section under torsion undergoes warping (i.e., out-of-plane deformation). The out-of-plane displacement of the cross-section is assumed to be proportional to the warping function $\omega(s)$ (Benscoter 1954), i.e.,

$$w_T(s, z) = -\Omega(z)\omega(s) \quad (3.11)$$

in which $\Omega(z)$ determines the magnitude of the longitudinal displacement due to warping effects as a function of the longitudinal coordinate z .

For a single cell cross-section, the warping function $\omega(s)$ as given by Bencoter (1954) is

$$\omega(s) = \int_0^s \left[\frac{2A}{t(s) \oint \frac{ds}{t(s)}} - \rho(s) \right] ds \quad (3.12)$$

where s is a coordinate along the ellipse arc length originating from positive X-axis and increasing in the counter-clockwise direction, A is the cross-sectional area enclosed by the mid-surface of the ellipse and $\rho(s)$ is the normal distance between the shear center (here coinciding with the centroid O), and the tangent to the middle surface of the cross-section at point of interest (Fig. 3.3). The sign convention for $\omega(s)$ in the present work is opposite to that of Bencoter (1954) in order to yield a positive longitudinal displacement in the first XY quadrant adopted in the present study.

For an EHS with constant thickness $t(s) = t$, Eq. 3.12 takes the form

$$\omega(s) = \int_0^s \left[\frac{2\pi ab}{p_r} - \rho(s) \right] ds \quad (3.13)$$

where p_r is the perimeter of the cross-section along the mid-surface.

Referring to Fig. 3.3, the distance between the displaced shear center (coinciding with the displaced centroid O' in Fig. 3.2) and the tangent to the point P is

$$\rho = OP \cos(\theta - \varphi) = b \cos \beta \cos \alpha + a \sin \beta \sin \alpha = \frac{ab}{\sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta}} \quad (3.14)$$

In Eq. 3.14, the relation $\tan \varphi = (b/a) \tan \beta$ is used. An infinitesimal arc element along the ellipse mid-surface is

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta \quad (3.15)$$

From Eqs. 3.14 and 3.15 by substituting in Eq. 3.13, the warping function ω can be expressed in terms of β as

$$\omega(\beta) = ab \left(\frac{2\pi}{p_r} \int_0^\beta \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta - \beta \right) \cong \frac{ab}{2} \frac{a-b}{a+b} \sin(2\beta) \quad (3.16)$$

The right hand side of Eq. 3.16 is based on an accurate approximation of the exact expression for $\omega(\beta)$. The dimensionless form of the exact and approximate forms of the warping function $\omega(\beta)$ are depicted in Fig. 3.5 for the special case $b/a = 0.5$.

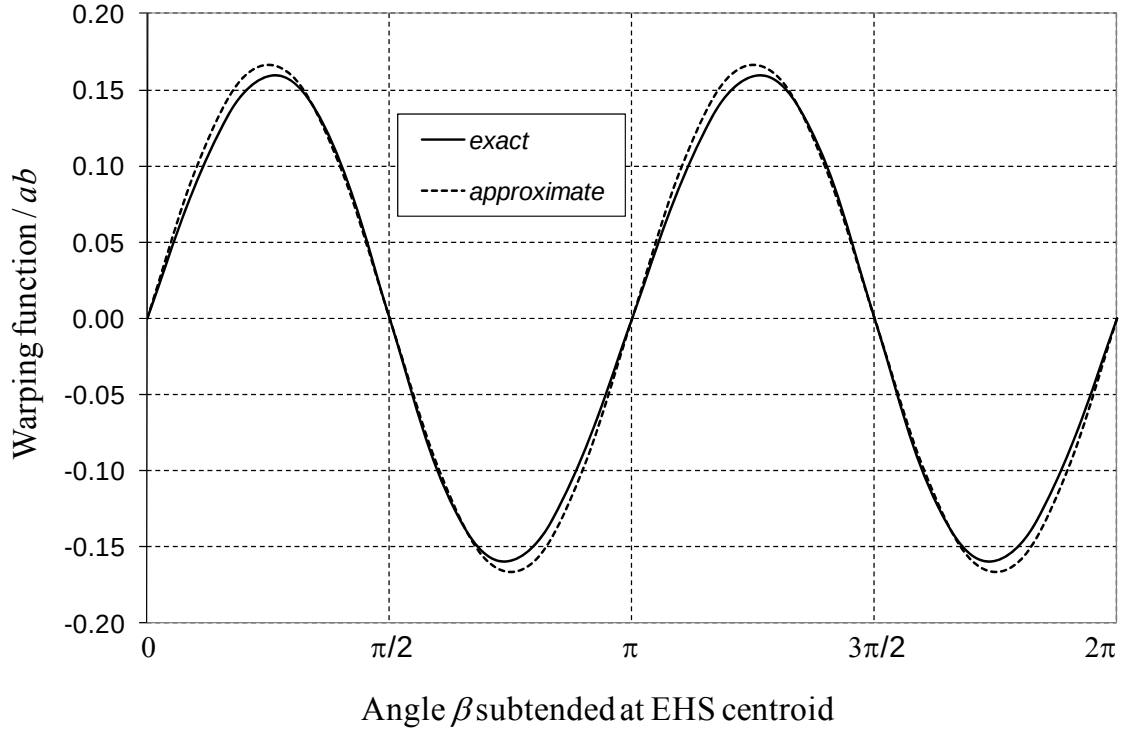


Fig. 3.5: Exact and approximate warping functions ($b/a = 0.5$)

3.6.3. Total Displacement Fields

By summing the displacement contributions in Eqs. 3.4 to 3.6 and 3.9 to 3.11, the total displacement fields for a generic point on the cross section of an EHS may be expressed as:

$$u(\beta, z) = -a\psi(z)\sin\beta + \int_0^z \phi_y(z)dz \quad (3.17)$$

$$v(\beta, z) = b\psi(z)\cos\beta - \int_0^z \phi_x(z)dz \quad (3.18)$$

$$w(\beta, z) = w_0(z) + a\phi_x(z)\sin\beta - b\phi_y(z)\cos\beta - \Omega(z)\omega(\beta) \quad (3.19)$$

The displacement fields, u and v can be resolved along the tangent to the surface (Fig. 3.3) as

$$u_t = -u \sin \alpha + v \cos \alpha \quad (3.20)$$

Upon substitution of Eqs. 3.17 and 3.18 into Eq. 3.20, the displacement tangent to the surface, u_t is found as

$$u_t = \frac{ab}{\sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta}} \psi' + \frac{-b \sin \beta \int_z \phi_y dz - a \cos \beta \int_z \phi_x dz}{\sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta}} \quad (3.21)$$

3.6.4. Expressions for strains

By taking the appropriate derivatives of Eqs. 3.19 and 3.21, one recovers the shear strain $\gamma = \gamma_{tz}(z)$ as

$$\gamma = \frac{\partial u_t}{\partial z} + \frac{\partial w}{\partial s} = \frac{ab}{\sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta}} \psi' + \left(\frac{ab}{\sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta}} - \frac{2ab\pi}{p_r} \right) \Omega \quad (3.22)$$

where the prime represents the first derivative with respect to z .

From Eq. 3.19, the longitudinal strain $\varepsilon = \varepsilon_{zz}(z)$ is obtained by differentiation with respect to z as

$$\varepsilon = \frac{\partial w}{\partial z} = a\phi'_x \sin \beta + b\phi'_y \cos \beta + w'_0 - \Omega' \frac{ab(a-b)}{2(a+b)} \sin(2\beta) \quad (3.23)$$

Eqs. 3.22 and 3.23 can be used to express the incremental plastic strains $\dot{\varepsilon}$ and $\dot{\gamma}$ for a cross-section in a fully plastic state. One obtains

$$\dot{\varepsilon} = a\dot{\phi}'_x \sin \beta + b\dot{\phi}'_y \cos \beta + \dot{w}'_0 - \dot{\Omega}' \frac{ab(a-b)}{2(a+b)} \sin(2\beta) \quad (3.24)$$

$$\dot{\gamma} = \frac{ab}{\sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta}} \dot{\psi}' + \left(\frac{ab}{\sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta}} - \frac{2ab\pi}{p_r} \right) \dot{\Omega} \quad (3.25)$$

3.6.5. Expressions for Incremental Plastic Strains

With reference to Eq.3.2, the function $f = \sigma^2 - \sigma\sigma_H + \sigma_H^2 + 3\tau^2 - F_y^2$ is adopted as a plastic potential. According to the normality rule (Nocedal and Wright 1999), the incremental plastic strain components $\dot{\varepsilon}_{ij}$ at a point on the wall of the members having coordinates (β, z) are given by $\dot{\varepsilon}_{ij} = d\lambda \partial f / \partial \sigma_{ij}$ in which $d\lambda = d\lambda(\beta, z)$ is a positive scalar. The incremental plastic strains are

$$\dot{\varepsilon} = d\lambda \frac{\partial f}{\partial \sigma} = d\lambda(2\sigma - \sigma_H) \quad (3.26)$$

$$\dot{\gamma} = d\lambda \frac{\partial f}{\partial \tau} = d\lambda(6\tau) \quad (3.27)$$

$$\dot{\varepsilon}_H = d\lambda \frac{\partial f}{\partial \sigma_H} = d\lambda(2\sigma_H - \sigma) \quad (3.28)$$

In line with Assumption 4, it is assumed that $\dot{\varepsilon}_H = \eta \dot{\varepsilon}$, where η is a proportionality factor relating hoop and longitudinal strains. The bounds of η are -0.5 and 0 , respectively corresponding to the limiting cases of zero hoop stress $\sigma_H = 0$ and zero incremental plastic hoop strain $\dot{\varepsilon}_H = 0$.

Experimentation with several elasto-plastic shell finite element models subjected to various combinations of loads under ABAQUS (SIMULIA 2007a) have shown that the assumption $\eta = 0$ is nearly approached at member ends where longitudinal stresses due to warping and hoop stresses do not vanish. Also, the assumption $\eta = -0.5$ is nearly approached away from the ends.

Equations 3.26 to 3.28 are solved for the components of the stress tensor to yield

$$\sigma = \left(\frac{2}{3} + \frac{\eta}{3} \right) \frac{\dot{\varepsilon}}{d\lambda} \quad (3.29)$$

$$\tau = \frac{1}{6} \frac{\dot{\gamma}}{d\lambda} \quad (3.30)$$

$$\sigma_H = \left(\frac{1}{3} + \frac{2\eta}{3} \right) \frac{\dot{\varepsilon}}{d\lambda} \quad (3.31)$$

The positive scalar $d\lambda$ may be derived by substituting Eqs. 3.29 to 3.31 in the plastic potential function, yielding

$$d\lambda = \frac{\sqrt{4(1+\eta+\eta^2)\dot{\epsilon}^2 + \dot{\gamma}^2}}{2\sqrt{3}F_y} = \frac{\sqrt{(1+\eta+\eta^2)\dot{\epsilon}^2 + \dot{\gamma}^2/4}}{\sqrt{3}F_y} \quad (3.32)$$

Alternative expressions for the positive scalar $d\lambda$ from Eqs. 3.29 to 3.31 are

$$d\lambda = \frac{\sqrt{(1+\eta+\eta^2)\dot{\epsilon}^2 + \dot{\gamma}^2/4}}{\sqrt{3}F_y} = \left(\frac{2}{3} + \frac{\eta}{3}\right) \frac{\dot{\epsilon}}{\sigma} = \frac{1}{6} \frac{\dot{\gamma}}{\tau} = \left(\frac{1}{3} + \frac{2\eta}{3}\right) \frac{\dot{\epsilon}}{\sigma_H} > 0 \quad (3.33)$$

3.6.6. Energy Expression

Based on the upper bound theorem of plasticity, the dissipation in internal energy U per unit length of a fully plastified cross-section of an EHS member due to assumed incremental kinematically admissible strain fields is equal to the change in external work W_e done by the generalized external stress resultant through the associated generalized incremental strains, or $U = W_e$, in which the change in the external work is given by

$$W_e = N\dot{w}_0' + M_x\dot{\phi}_x' + M_y\dot{\phi}_y' + T_w\dot{\Omega} + T_{sv}\dot{\psi}' + B\dot{\Omega}' \quad (3.34)$$

where the total applied torsion T has been divided into two components T_w and T_{sv} , representing torsion due to warping and saint-Venant's torsion, respectively.

The kinematically admissible incremental strain fields induce an internal energy dissipation U equal to

$$U = \int_A (\sigma\dot{\epsilon} + \tau\dot{\gamma} + \sigma_H\dot{\epsilon}_H) dA \quad (3.35)$$

From Eqs. 3.29 to 3.31, by substituting into Eq. 3.35, one obtains

$$U = \int_A \frac{2(1+\eta+\eta^2)\dot{\epsilon}^2 + \dot{\gamma}^2/2}{3d\lambda} dA \quad (3.36)$$

The dissipation in the internal strain energy U is derived by substituting Eq. 3.32 into Eq. 3.36 yielding

$$U = \frac{F_y}{\sqrt{3}} \int_A \sqrt{4(1+\eta+\eta^2)\dot{\epsilon}^2 + \dot{\gamma}^2} dA \quad (3.37)$$

3.6.7. Internal force resultants

The internal energy U is equated to the external work W_e , i.e., $U = W_e$. Using Eqs. 3.34 and 3.37 the stress resultants N, M_x, M_y, T_w, T_{sv} and B are obtained by taking the partial derivatives of the identity $U = W_e$ with respect to the corresponding generalized differential displacements, yielding $N = \partial U / \partial \dot{w}'_0$, $M_x = \partial U / \partial \dot{\phi}'_x$, $M_y = \partial U / \partial \dot{\phi}'_y$, $T_w = \partial U / \partial \dot{\Omega}$, $T_{sv} = \partial U / \partial \dot{\psi}'$ and $B = \partial U / \partial \dot{\Omega}'$.

The axial force is derived from the expression

$$N = \frac{\partial U}{\partial \dot{\epsilon}} \frac{\partial \dot{\epsilon}}{\partial \dot{w}'_0} = \frac{4(1+\eta+\eta^2)}{\sqrt{3}} a F_y t \int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} d\beta \quad (3.38)$$

where $\text{sgn}(\dot{\epsilon}) = \dot{\epsilon}/|\dot{\epsilon}|$. The bending moment about major axis is

$$M_x = \frac{\partial U}{\partial \dot{\epsilon}} \frac{\partial \dot{\epsilon}}{\partial \dot{\phi}'_x} = \frac{4(1+\eta+\eta^2)}{\sqrt{3}} a^2 F_y t \int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} \sin \beta d\beta \quad (3.39)$$

The bending moment about minor axis is

$$M_y = \frac{\partial U}{\partial \dot{\epsilon}} \frac{\partial \dot{\epsilon}}{\partial \dot{\phi}'_y} = \frac{4(1+\eta+\eta^2)}{\sqrt{3}} ab F_y t \int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} \cos \beta d\beta \quad (3.40)$$

The bimoment is

$$\begin{aligned} B &= \frac{\partial U}{\partial \dot{\epsilon}} \frac{\partial \dot{\epsilon}}{\partial \dot{\Omega}'} \\ &= -\frac{2(1+\eta+\eta^2)}{\sqrt{3}} F_y t a^2 b \left(\frac{a-b}{a+b} \right) \int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} \sin(2\beta) d\beta \end{aligned} \quad (3.41)$$

The torsion due to warping is

$$T_w = \frac{\partial U}{\partial \dot{\gamma}} \frac{\partial \dot{\gamma}}{\partial \dot{\Omega}} = \frac{F_y abt}{\sqrt{3}} \int_0^{2\pi} \frac{\text{sgn}(\dot{\gamma}) \left(1 - \frac{2\pi}{p_r} a \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \right)}{\sqrt{4(1+\eta+\eta^2)(\dot{\epsilon}/\dot{\gamma})^2 + 1}} d\beta \quad (3.42)$$

and the Saint Venant torsion is

$$T_{sv} = \frac{\partial U}{\partial \dot{\gamma}} \frac{\partial \dot{\gamma}}{\partial \dot{\psi}'} = \frac{F_y abt}{\sqrt{3}} \int_0^{2\pi} \frac{\text{sgn}(\dot{\gamma})}{\sqrt{4(1+\eta+\eta^2)(\dot{\epsilon}/\dot{\gamma})^2 + 1}} d\beta \quad (3.43)$$

In Eqs. 3.38 through 3.43, one has

$$\frac{\dot{\gamma}}{\dot{\epsilon}} = \frac{\frac{1}{\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}} (b\dot{\psi}') + \left(\frac{1}{\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}} - 2\pi \frac{a}{p_r} \right) (b\dot{\Omega})}{(a\dot{\phi}'_x) \sin \beta + (b\dot{\phi}'_y) \cos \beta + \dot{w}'_0 + \left[-\frac{1}{2} ab \left(\frac{a-b}{a+b} \right) \dot{\Omega}' \right] \sin(2\beta)} \quad (3.44)$$

The six equations 3.38 through 3.43 form a system of parametric equations for the internal forces and, collectively, they describe an upper bound interaction equation for EHS.

3.6.8. Plastic Resistances

In order to make the interaction relations universally applicable, the internal forces should be normalized. In the present work, normalization is achieved by dividing the internal force by the corresponding plastic capacity of section in the absence of the other internal forces. The plastic resistances are calculated for a section away from the supports (i.e., $\eta = -0.5$)

By setting $\dot{\gamma} = 0$ and $\dot{\epsilon} = \dot{w}'_0$ in Eq. 3.38, the tensile resistance N_p of the section is recovered as

$$N_p = aF_y t \int_0^{2\pi} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (3.45)$$

The plastic moment resistance of section about the major axis is obtained by setting $\dot{\gamma} = 0$ and $\dot{\epsilon} = a\dot{\phi}'_x \sin \beta$ in Eq. 3.39 yielding

$$M_{xp} = a^2 F_y t \int_0^{2\pi} \text{sgn}(\sin \beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin \beta d\beta \quad (3.46)$$

In a similar manner, the plastic moment resistance about the minor axis is obtained from Eq. 3.40 yielding

$$M_{yp} = a^2 \left(\frac{b}{a} \right) F_y t \int_0^{2\pi} \text{sgn}(\cos \beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \cos \beta d\beta \quad (3.47)$$

The plastic bimoment resistance is obtained by setting $\dot{\gamma} = 0$ and $\dot{\varepsilon} = -\dot{\Omega}' [ab(a-b)/(a+b)] \sin(2\beta)/2$ in Eq. 3.41 yielding

$$B_p = \frac{\sqrt{3}}{2} a^3 \left(\frac{b}{a} \right) \left(\frac{1-b/a}{1+b/a} \right) F_y t \times \int_0^{2\pi} \text{sgn}(\sin(2\beta)) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta \quad (3.48)$$

The plastic resistance of the section for the warping components of the torsion is obtained by setting $\dot{\varepsilon} = 0$ in Eq. 3.42 as

$$T_{wp} = \frac{1}{\sqrt{3}} a^2 \left(\frac{b}{a} \right) t F_y \int_0^{2\pi} \text{sgn}(\dot{\gamma}) \left(1 - \frac{2\pi a}{p_r} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \right) d\beta = 0 \quad (3.49)$$

The right hand side of Eq. 3.49 vanishes since the perimeter of the section is $p_r = a \int_0^{2\pi} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta$. According to Eq.3.49, the plastic capacity of warping torsion in the absence of other internal forces vanishes. This is a normal outcome of the fact that warping torsion cannot exist without warping displacement.

The Saint Venant's torsion plastic resistance is obtained from Eq. 3.43 by setting $\dot{\varepsilon} = 0$ yielding

$$T_{svp} = \frac{abt}{\sqrt{3}} F_y \int_0^{2\pi} d\beta = \frac{2\pi}{\sqrt{3}} a^2 \left(\frac{b}{a} \right) t F_y \quad (3.50)$$

3.6.9. Dimensionless Interaction Equations

By respectively dividing Eqs. 3.38 through 3.43 by Eqs. 3.45 through 3.50, one recovers the parametric form of the dimensionless interaction equations sought as

$$N_r = \frac{N}{N_p} = \frac{4(1+\eta+\eta^2)}{\sqrt{3}} \frac{\int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} d\beta}{\int_0^{2\pi} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta} \quad (3.51)$$

$$M_{xr} = \frac{M_x}{M_{xp}} = \frac{4(1+\eta+\eta^2)}{\sqrt{3}} \frac{\int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} \sin \beta d\beta}{\int_0^{2\pi} \text{sgn}(\sin \beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin \beta d\beta} \quad (3.52)$$

$$M_{yr} = \frac{M_y}{M_{yp}} = \frac{4(1+\eta+\eta^2)}{\sqrt{3}} \frac{\int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} \cos \beta d\beta}{\int_0^{2\pi} \text{sgn}(\cos \beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \cos \beta d\beta} \quad (3.53)$$

$$B_r = \frac{B}{B_p} = \frac{4(1+\eta+\eta^2)}{\sqrt{3}} \frac{\int_0^{2\pi} \frac{\text{sgn}(\dot{\epsilon}) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\dot{\gamma}/\dot{\epsilon})^2}} \sin(2\beta) d\beta}{\int_0^{2\pi} \text{sgn}(\sin(2\beta)) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta} \quad (3.54)$$

Both the warping and St. Venant's torsions can be normalized relative to the St. Venant's torsional capacity T_{svp} , i.e.,

$$T_{wr} = \frac{T_w}{T_{svp}} = \frac{1}{2\pi} \int_0^{2\pi} \text{sgn}(\dot{\gamma}) \frac{1 - 2\pi \frac{a}{p_r} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) (\dot{\epsilon}/\dot{\gamma})^2 + 1}} d\beta \quad (3.55)$$

$$T_{svr} = \frac{T_{sv}}{T_{svp}} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\text{sgn}(\dot{\gamma})}{\sqrt{4(1+\eta+\eta^2) (\dot{\epsilon}/\dot{\gamma})^2 + 1}} d\beta \quad (3.56)$$

In Eq. 3.51 to 3.56, the expression for $\dot{\gamma}/\dot{\epsilon}$ is obtained by dividing the numerator and denominator of Eq. 3.44 by w'_0 to yield

$$\frac{\dot{\gamma}}{\dot{\epsilon}} = \frac{\frac{C_1}{\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}} + \left(\frac{1}{\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}} - 2\pi \frac{a}{p_r} \right) C_2}{C_3 \sin \beta + C_4 \cos \beta + C_5 \sin(2\beta) + 1} \quad (3.57)$$

Equations 3.51 to 3.56 are implicit functions of constants $C_1...C_5$. Conceptually, the five constants $C_1...C_5$ are to be eliminated between the six equations, resulting into a single interaction relation of the form $F(N_r, M_{xr}, M_{yr}, T_{wr}, T_{svr}, B_r) = 0$. Given the nonlinear dependence of Eqs 3.51 to 3.56 on constants $C_1...C_5$, this elimination can be performed only numerically.

3.6.10. Admissible Internal Force Combinations

An internal force combination $(N, M_x, M_y, B, T_w, T_{sv})$ is admissible when it lies within the interaction surface. For that, one could multiply the load combination $(N_r, M_{xr}, M_{yr}, B_r, T_{wr}, T_{svr})$ by an unknown scalar α , and solve Eqs. 3.51 through 3.56 and Eq. 3.57 for the six constants C_1, C_2, C_3, C_4, C_5 and α . This is equivalent to solving the dimensionless interaction equation $F(\alpha N_r, \alpha M_{xr}, \alpha M_{yr}, \alpha B_r, \alpha T_{wr}, \alpha T_{svr}) = 0$. The value of α , called load scale factor, is an indicator of the proximity of the applied load combination $(N_r, M_{xr}, M_{yr}, B_r, T_{wr}, T_{svr})$ to the yield surface. A value of unity denotes a load combination lying on the interaction surface. When $\alpha < 1$, the combination needs to be scaled down in order to lie on the yield surface, i.e., is unattainable. In contrast, $\alpha > 1$ denotes that the load combination needs to be magnified in order to lie on the yield surface (i.e., the load combination does not cause the section to fully plastify).

3.6.11. Iterative Procedure

Given the non-linearity of Eqs. 3.51 through 3.56, the solution is iterative. Different iterative solutions were attempted including grid search, simplex search, gradient descent, Newton, Gauss-Newton, trust-region and Levenberg-Marquardt. The details of these methods can be found in most optimization texts (e.g., Vlasov 1961). The first three techniques very slowly converge to the solution and are judged as unsuitable. Both Newton methods and the trust region method fail to converge in many cases. The most efficient technique, both in computational speed and stability, was found to be the

Levenberg-Marquardt method. Interested readers are referred to the Appendix I for specifics of this procedure.

3.6.12. Sections under Applied Torsion

The objective of this section is to investigate the plastic resistance of an EHS under non-uniform twisting moments (i.e., involving bi-moment effects). For this purpose, a parametric finite element analysis was conducted.

For the finite element model an EHS with $a = 100 \text{ mm}$, $b = 47 \text{ mm}$ and $t = 8 \text{ mm}$ was considered.

The member has a span of 1.0 m. It is fixed at one end and subjected to an external torsional moment at the other end. The mesh involves 60 elements circumferentially and 60 elements longitudinally. For the 100 mm above the fixed end, a fine mesh was adopted with 10 mm long elements in the longitudinal direction. For the remaining specimen, a coarser mesh with 20 mm long elements was adopted. The S4R element was used for the EHS wall. The steel material is assumed to be elastic-perfectly plastic with $E = 200 \text{ GPa}$, $\nu = 0.3$ and $F_y = 350 \text{ MPa}$. The applied twisting moment is incrementally increased until the plastic capacity of the section is attained. The stresses at the end section of the member were monitored.

It is observed that both the shear stresses induced by torsion and the longitudinal stresses due to warping linearly increase with the applied torsion up to the elastic limit of the member (i.e., when the first yield is attained). At the first yield, the shear stresses are observed (Fig. 3.6) to range between 171 MPa and 202 MPa along the circumference. As depicted in the figure, the changes in the circumferential shear stress are observed to decrease after the elastic limit is attained. When the section is fully plastified, the shear stress takes a nearly uniform distribution. After the first yield, the peak longitudinal stress is observed to decrease to about 40% of that attained in the elastic analysis (Fig. 3.7).

From Eq. 3.33 one has $\dot{\varepsilon} / \sqrt{4(1 + \eta + \eta^2)\dot{\varepsilon}^2 + \dot{\gamma}^2} = \sqrt{3} \sigma / [2(2 + \eta)F_y]$. By assuming $\eta = 0$ (i. e., section is at the support) and substituting into Eq. 3.41 one obtains

$$B = \left(\frac{a^2 bt}{2} \right) \left(\frac{a-b}{a+b} \right) \int_0^{2\pi} \sigma(\beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta \quad (3.58)$$

Also from Eq. 3.33 and $\eta = 0$, one has $\dot{\gamma} / \sqrt{4\dot{\epsilon}^2 + \dot{\gamma}^2} = \sqrt{3}\tau / F_y$. By substituting into Eq. 3.42 one obtains

$$T_w = abt \int_0^{2\pi} \tau(\beta) \left(1 - \frac{2\pi}{p_r} \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} \right) d\beta \quad (3.59)$$

In a similar manner, the Saint Venant torsion is obtained from Eqs. 3.33 and 3.43 as

$$T_{sv} = abt \int_0^{2\pi} \tau(\beta) d\beta \quad (3.60)$$

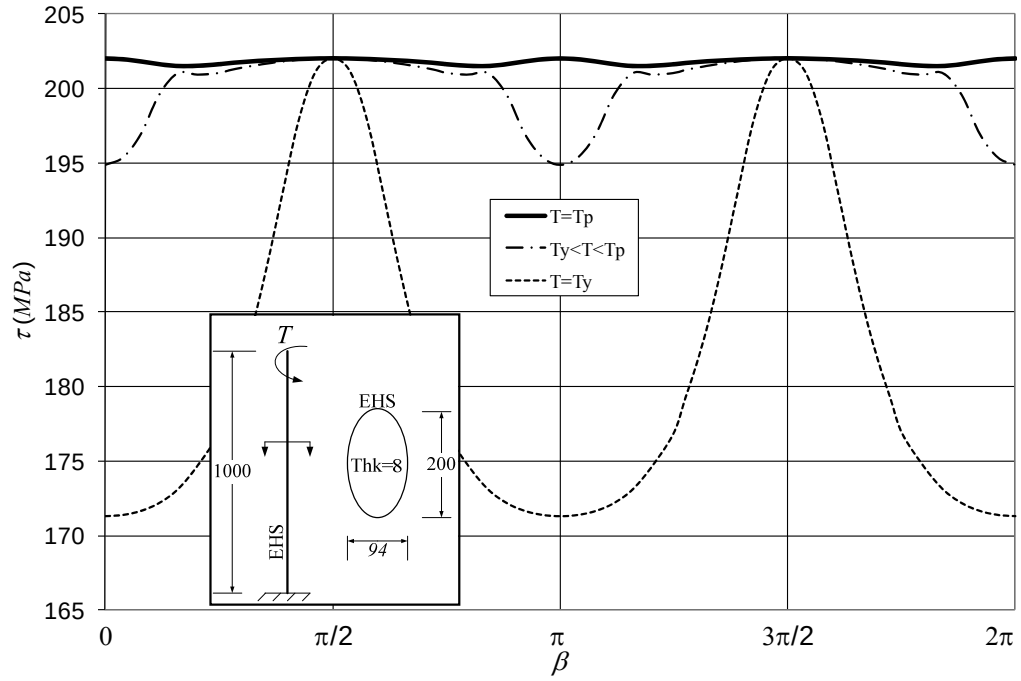


Fig. 3.6: Shear stresses along the circumference of an EHS under torsion

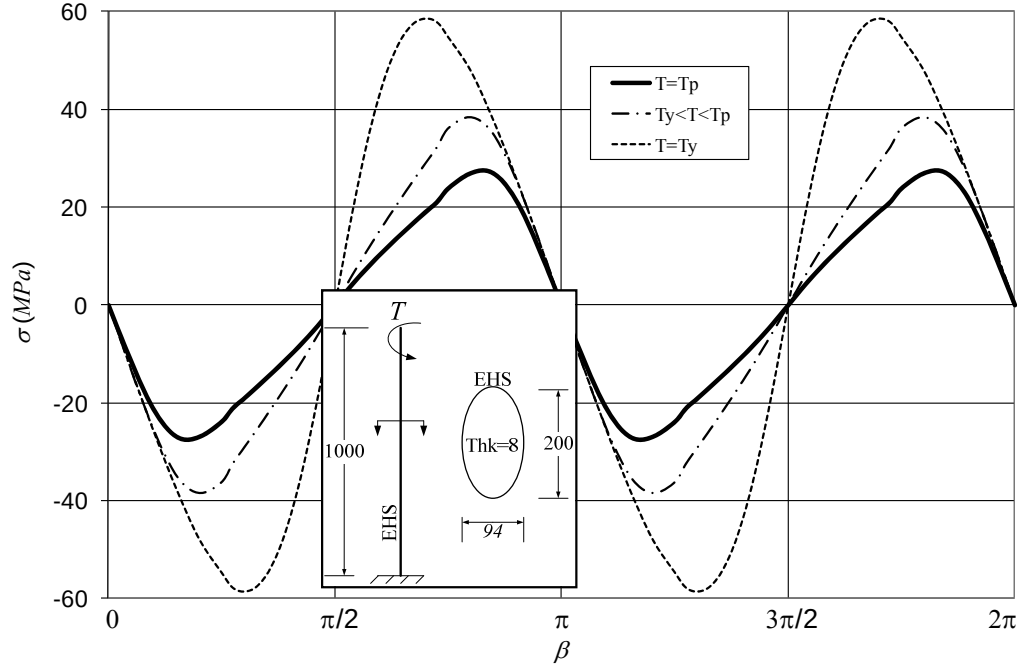


Fig. 3.7: Longitudinal stresses along the circumference of an EHS under torsion

The shear stress distribution at the base of the member is extracted from the FEA and the integrals in Eqs. 3.59 and 3.60 are numerically evaluated to obtain the warping torsion T_w and Saint Venant torsion T_{sv} and plotted on Fig. 3.8. In this example, the value of the warping torsion ratio T_{wr} is two orders of magnitude smaller than the Saint Venant torsion ratio T_{svr} . In order to clearly illustrate the progression of warping torsion ratio T_{wr} , a right hand vertical axis with a magnified scale was added to the figure in addition to the left vertical axis which is applicable to the Saint Venant ratio T_{svr} . It is observed that the warping torsion T_w proportionally increases with the applied twisting moment up to the first yield. After this point, it decreases and finally vanishes as the section becomes fully plastified.

Warping torsion takes place because of the longitudinal restraints at the base of the member. Once the section yields, it could be argued that the section becomes less restrained which causes a redistribution of warping torsion towards the Saint Venant torsion, a stiffer load carrying mechanism in the plastic regime. When the section is fully plastified, the

torsion is exclusively carried by the Saint Venant torsion mechanism with essentially a uniform shear stress distribution along the section perimeter (the solid line in Fig. 3.6).

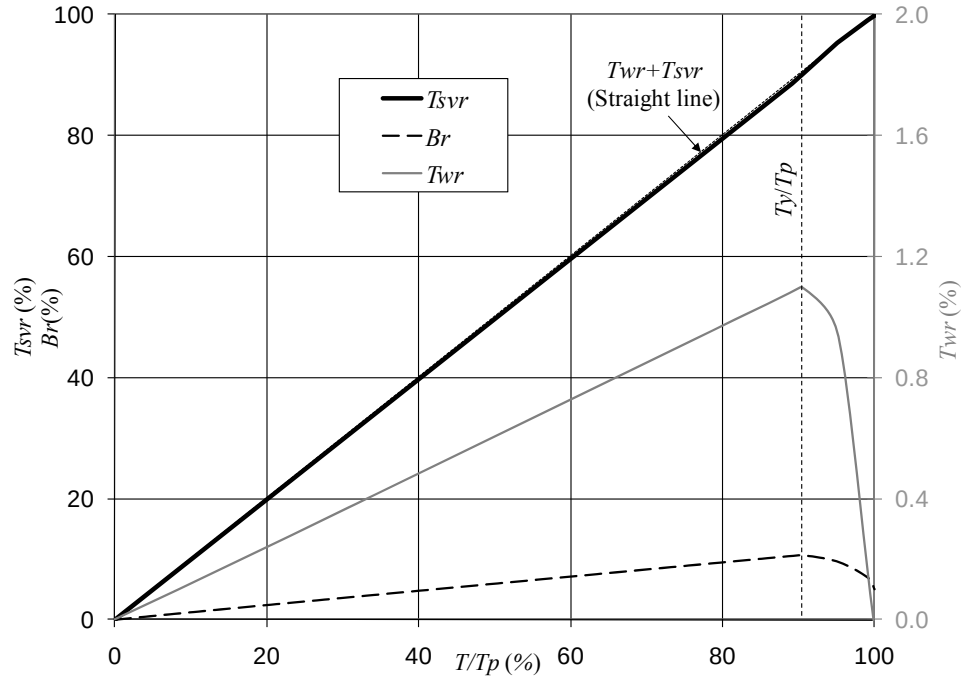


Fig. 3.8: Bimoment, Warping and Saint Venant torsions for an EHS under torsion T

3.6.13. Interactions for a Section under Torsion and Bimoments

The special case of the interaction relations involving only torsion and bi-moments is depicted in Fig. 3.9. In generating this figure, the internal forces N_r, M_{xr}, M_{yr} and T_{wr} are set to zero and the system of equations 3.51 to 3.56 is solved for two cases $\eta = 0$ and $\eta = -0.5$. On the figure, the lower bound interaction equation $T_r^2 + B_r^2 = 1.0$, in which $T_r = T_{svr}$ as derived based on statistically admissible stresses (formulated in the appendix) is superimposed for comparison.

In order to verify the findings, a series of finite element analyses is performed. The model mesh is identical to that in the previous section. Two types of loading are applied to the free end of the member; a) a tangential shear load along the section, and b) a longitudinal distributed load with distribution of a magnitude proportional to $\sin(2\beta)$. In a given analysis, the ratio of the applied tangential to longitudinal warping loads is kept constant.

Both loads are increased until the section becomes fully plastified. The longitudinal and shear stresses at the plastified section are extracted. Using Eqs. 3.58 and 3.60, the torsion and bimoments induced at the fixed end of the member are calculated and depicted in Fig. 3.9.

It is worth noting that the upper bound solution based on $\eta = 0$ yields a bimoment ratio B/B_p higher than unity (i.e., $2/\sqrt{3} \cong 1.15$) when no external torsion is applied. This is due to the fact that the plastic bimoment resistance B_p has been defined for a section that is free to deform in the radial direction (i.e., by setting $\eta = -0.5$ in Eq. 3.48 while the bi-moment value B in the numerator is computed based on a section that is restrained from deforming in the radial direction (by setting $\eta = 0$ in Eq. 3.41).

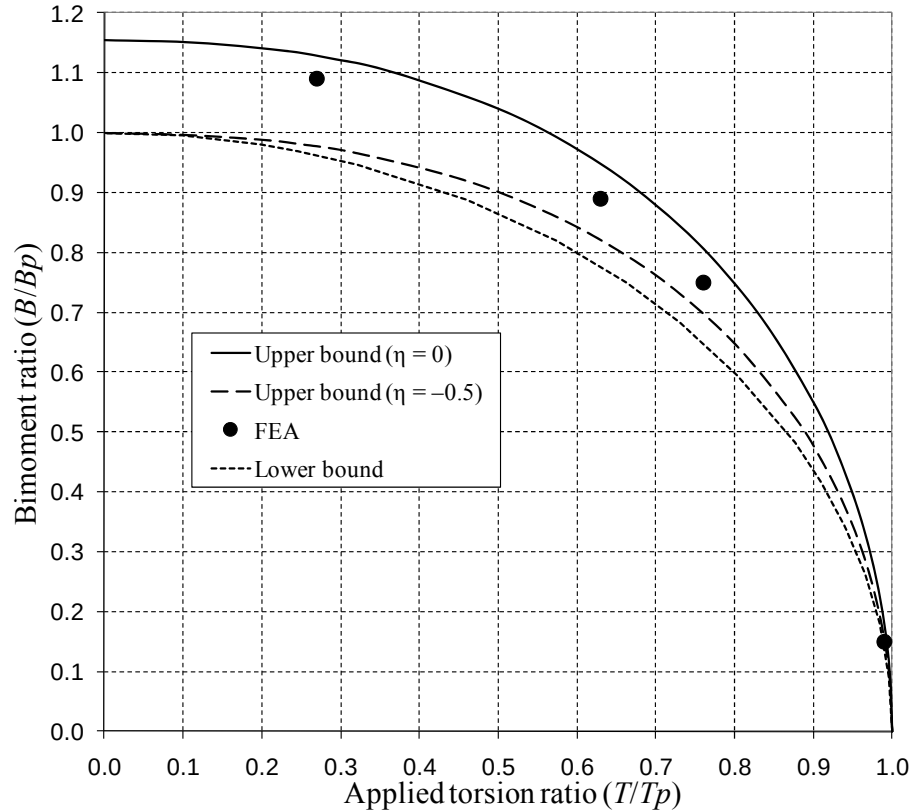


Fig. 3.9: Interaction relation for torsion and bi-moment ($b/a = 0.47$)

3.6.14. Interaction Equation under Biaxial Moments

The special case of the interaction relations involving only bending moments about major and minor axes is depicted in Fig. 3.10. In generating this figure, the internal forces N_r, B_r, T_{svr} and T_{wr} are set to zero and the system of equations 3.51 to 3.56 is solved simultaneously for two cases of $\eta = 0$ and $\eta = -0.5$. On the figure, the lower bound interaction equation derived from statistically admissible stresses (Nowzartash and Mohareb 2009) is superimposed for comparison.

In order to verify the findings, a series of finite element analyses is performed. The mesh used is identical to that used in the previous section. Two types of loading are applied to the free end of the member; a) a uniform distributed lateral load in x direction, and b) a uniform distributed lateral load in y direction. While the ratio of the two loads is kept constant in a given analysis, they both are increased until the section fully plastifies.

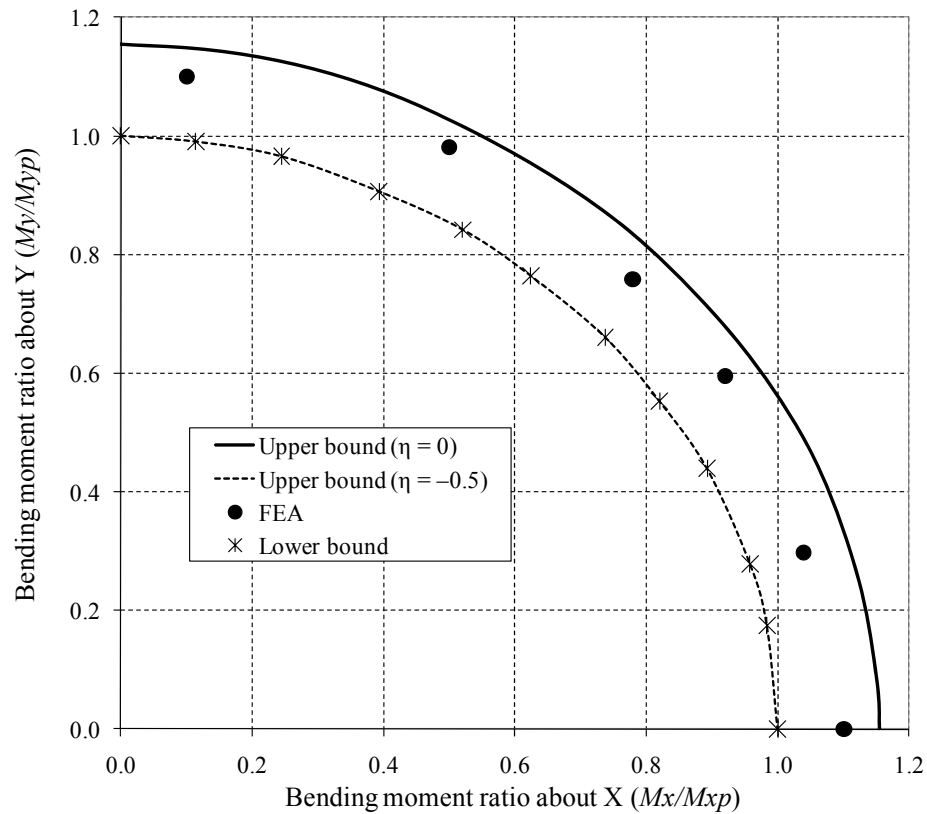


Fig. 3.10: Interaction relation for bi-axial bending moments ($b/a = 0.47$)

In all models, it is observed that the section of plastic failure is located slightly above the fixed cross-section at the base of the member. An illustration of this observation is depicted in Fig. 3.11. In this figure, a dark shell element is plastified and a light element is still in elastic stage. The longitudinal stresses at the root of the member were extracted, and the two bending moments calculated based on the expressions

$$M_x = a^2 t \int_0^{2\pi} \sigma(\beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin \beta d\beta \quad (3.61)$$

$$M_y = ab t \int_0^{2\pi} \sigma(\beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \cos \beta d\beta \quad (3.62)$$

then are divided by their respective plastic resistances and depicted in Fig. 3.10. Equations 3.61 and 3.62 are derived by assuming $\eta = 0$ and substituting Eq. 3.33 into Eqs. 3.52 and 3.53. In the figure, when $\eta = -0.5$, the present upper bound solution is observed to coincide with the lower bound solution reported in Nowzartash and Mohareb (2009). As expected, the results from FEA fall between two curves $\eta = 0$, and $\eta = -0.5$. This reflects the fact that at the failed section, located slightly above the fixed section, only partial confinement is provided by the fixed end section.

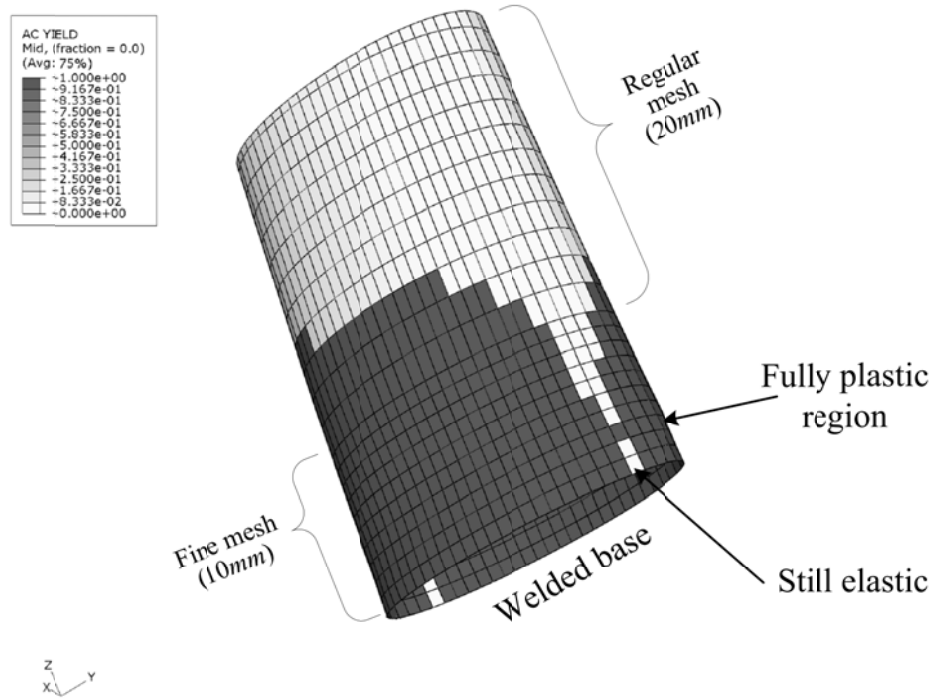


Fig. 3.11: The location of fully plastified section in $M_x - M_y$ loading FEA

3.6.15. Parametric Study

The objective of this section is to assess the validity of the results based on the present formulation by conducting a parametric study for the case of a section that is fully confined in the radial direction ($\eta = 0$). In practice, this case is approached at welded end sections. For sections away from the end with $\eta = -0.5$ and zero bimoments, the reader is referred to the interaction relations in Nowzartash and Mohareb (2009) which can be conceived as a special case of the present formulation.

For this purpose a steel EHS- $300 \times 150 \times 10$ section ($a = 145 \text{ mm}$, $b = 70 \text{ mm}$ and $t = 10 \text{ mm}$) with yield strength of 350 MPa is considered. The section is subject to various external loadings. It is required to determine the scaling factor α at the section adjacent to a welded end using FEA and compare it to the proposed formulation.

For the finite element model, a 2 m span EHS- $300 \times 150 \times 10$ cross-section is modelled. It is fixed at the bottom end and free at the top end. Lateral ring tractions are applied at the free end. Equal and opposite lateral ring tractions are applied at member mid-span in order to subject the bottom half of the model to biaxial bending moments and zero shear. Additional uniform longitudinal tractions are applied to mid-span section in order to subject the bottom half of the model to axial force. Recalling that the aim is to assess the validity of the interaction relations for the case of full confinement i.e., $\eta = 0$, loading is applied such that the bottom end of the section first undergoes full plastification. Towards this goal, constant tangential shear traction was applied 30 mm above the fixed end to subject the bottom section to additional twisting moments. For Cases 1 through 5 in Table 3.1, an internal bi-moment B_{int} naturally arises at the bottom section as shown. For Cases 6 through 10, in order to provide an additional external bimoment B_{ext} to the end of the specimen, a set of additional longitudinal tractions meeting the conditions $\int_A \sigma dA = \int_A \sigma x dA = \int_A \sigma y dA = 0$ and $\int_A \sigma \omega dA = B_{ext} \neq 0$ are applied to the model. The external traction inducing bimoments was identical for Cases 6 to 10 and the signs are selected such that B_{ext} is additive to B_{int} .

The mesh consists of 60 elements circumferentially. In the longitudinal direction, the bottom 30 mm of the model is discretized using a fine mesh with six 5 mm long elements. The remaining of the specimen is modelled using a coarser mesh with fifty 39.4 mm long elements.

The applied loads are incrementally increased until the plastic capacity of the section is attained at the fixed end. The load scaling factor α_{FEA} and reaction forces at the fixed end are extracted.

In all of the cases considered in Table 3.1, the load combination lies further beyond the yield surface. Therefore, the load combinations needs to be scaled down with α_{FEA} factors smaller than unity as stated in the section “Admissible Internal Force Combinations”. The internal forces B, T_w, T_{sv}, M_x, M_y and N in the section are respectively calculated using Eqs. 3.58 to 3.62 and

$$N = at \int_0^{2\pi} \sigma(\beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad (3.63)$$

These values are tabulated in the second column of Table 3.1 in a dimensionless form by dividing them by their respective plastic resistances and α_{FEA} . As discussed in the previous section, the warping torsion was observed to essentially vanish at the bottom end of the model (i.e., $T_r = T_{svr}$). The scaling factor α based on the present formulation is then calculated and tabulated in the dimensionless form α/α_{FEA} in column four. Two approximate interaction relations (linear and quadratic) are investigated for their ability to predict the interaction behaviour and the corresponding dimensionless scaling factors α_L/α_{FEA} and α_Q/α_{FEA} are reported in columns 5 and 6 of Table 3.1.

$$\alpha_L \left(|N_r| + |M_{xr}| + |M_{yr}| + |T_r| + |B_r| \right) = 1.0 \quad (3.64)$$

$$\alpha_Q^2 \left(N_r^2 + M_{xr}^2 + M_{yr}^2 + T_r^2 + B_r^2 \right) = 1.0 \quad (3.65)$$

When only one of the internal forces is applied and all others vanish, it is clear that present solution and the proposed approximations in Eqs. 3.64 and 3.65 provide exact predictions.

For the load combinations investigated, the proposed interaction relations provides the best approximation of the finite element results, for which the average α/α_{FEA} is 1.007 and a standard deviation of 0.0031. The linear interaction equation is overly conservative with an average α_L/α_{FEA} ratio of 0.438 and standard deviation of 0.020, while the quadratic interaction provides a reasonable compromise between simplicity and accuracy (with an average α_Q/α_{FEA} ratio of 0.915 and a standard deviation of 0.025).

3.7. Summary and Conclusions

The following are the summary and/or conclusions of the paper:

1. Upper bound plastic interaction relations were developed for elliptical hollow sections under combinations of axial force, bi-axial bending moments, twisting moments and bimoments. The resulting parametric relations consist of a set of non-dimensional integrals applicable to EHS of general cross-sectional geometries.
2. The obtained system of equations is nonlinear in the parametric variables. An efficient solution algorithm is developed to solve the equations and numerically determine whether a given internal force combination is admissible within the limitations of the formulation.
3. The plastic interaction relations successfully capture the Saint Venant and warping twisting moments, bi-moments, biaxial bending moments, and axial forces. Predictions based on the interaction relations developed agree well with elasto-plastic shell FEA.
4. The interaction relations successfully capture the confining effects present near welded ends as well as sections away from welded ends with essentially no confining effects.
5. A quick capacity analysis of sections near welded ends can be conducted by using the quadratic interaction equation. i.e., $N_r^2 + M_{xr}^2 + M_{yr}^2 + T_{svr}^2 + B_r^2 = 1.0$.

Table 3.1: Parametric study

Case No.	Reference internal forces ($N_r, M_{xr}, M_{yr}, T_r, B_r^*$)	α_{FEA}	$\frac{\alpha}{\alpha_{FEA}}$	$\frac{\alpha_L}{\alpha_{FEA}}$	$\frac{\alpha_Q}{\alpha_{FEA}}$
1	(1.0, 1.0, 1.0, 1.0, 1.05)	0.5022	1.008	0.396	0.882
2	(<u>2.5</u> , 1.0, 1.0, 1.0, 1.5)	0.3132	1.004	0.453	0.941
3	(1.0, <u>2.5</u> , 1.0, 1.0, 1.0)	0.3498	1.005	0.436	0.893
4	(1.0, 1.0, <u>2.5</u> , 1.0, 1.0)	0.3496	1.005	0.440	0.893
5	(1.0, 1.0, 1.0, <u>2.5</u> , 1.4)	0.3202	1.009	0.453	0.933
6	(1.0, 1.0, 1.0, 1.0, 1.55)	0.4460	1.002	0.410	0.892
7	(<u>2.5</u> , 1.0, 1.0, 1.0, 1.9)	0.2906	1.008	0.462	0.959
8	(1.0, <u>2.5</u> , 1.0, 1.0, 1.4)	0.3230	1.009	0.445	0.925
9	(1.0, 1.0, <u>2.5</u> , 1.0, 1.35)	0.3294	1.011	0.440	0.912
10	(1.0, 1.0, 1.0, <u>2.5</u> , 2.4)	0.2822	1.012	0.449	0.915
Average	-	-	1.007	0.438	0.915
Standard Deviation	-	-	0.0031	0.020	0.025

* Calculated after analysis based on nodal forces.

3.8. Notation

A	Area enclosed by the ellipse mid-surface
a, b	Major and minor half diameters of the EHS
N, M_x, M_y	Axial force, bending about major and minor axes
T, B	Total twisting moments and bi-moment
T_w, T_{sv}	Warping and St Venant's twisting moments
t	Thickness of EHS
s	Coordinate along the circumference of the section
u, v, w	Incremental displacement in directions X, Y and Z
u_n, u_t	Incremental normal and tangent in-plane deformations
α	Angle between normal to the EHS and the X axis
β	Parameter of ellipse
$\gamma = \gamma_{tz}(z)$	Tangential shear strain
$\varepsilon = \varepsilon_{zz}(z)$	Longitudinal strain
$\varepsilon_H = \varepsilon_{tt}(z)$	Hoop strain
η	Ratio between longitudinal and hoop strains
$\phi_x = \phi_x(z)$	Angle of rotation about X axis
$\phi_y = \phi_y(z)$	Angle of rotation about Y axis
$\psi = \psi(z)$	Angle of twist
$w_0 = w_0(z)$	Axial deformation due to axial force
$\omega(s), \omega(\beta)$	Warping function
$\Omega = \Omega(z)$	Warping intensity function
$\dot{\varepsilon}, \dot{\gamma}$	Incremental longitudinal and shear strains
σ, σ_H, τ	Normal, hoop and shear stresses

3.10. Appendix: Formulation of Lower Bound Interaction Relation for Torsion and Bimoment

Consider an EHS cross-section that is plastified under the action of torsion and bi-moments. A lower bound interaction relation is sought. Towards this goal, statically admissible stress fields are postulated and the corresponding stress resultants are determined. The tangential shear stress distribution is assumed constant over the circumference of the cross-section. The longitudinal stresses are assumed to have a step function distribution, i.e.,

$$\sigma(\beta) = \begin{cases} \sigma & 0 \leq \beta < \pi/2 \\ -\sigma & \pi/2 \leq \beta < \pi \\ \sigma & \pi \leq \beta < 3\pi/2 \\ -\sigma & 3\pi/2 \leq \beta < 2\pi \end{cases} \quad (3.66)$$

For a thin-walled section, the shearing stress τ can be assumed constant through the wall thickness t (Boresi and Sidebottom 1985), and is related to the torsion $T = 2At\tau$, in which A is the area enclosed by the section mid-surface. In the case of an EHS, $A = ab\pi$. Thus,

$$T = 2\pi abt\tau \quad (3.67)$$

The torsion plastic capacity ratio is

$$T_r = \frac{T}{T_p} = \frac{2\pi abt\tau}{2\pi abt\left(\frac{F_y}{\sqrt{3}}\right)} = \sqrt{3} \frac{\tau}{F_y} \quad (3.68)$$

The bimoment is defined as the integration of product of longitudinal stress and warping function along the section circumference, or

$$B = \int_A \sigma(s)\omega(s)dA \quad (3.69)$$

By substituting Eqs. 3.15 and 3.16 into 3.69 one has

$$\begin{aligned} B &= \int_0^{2\pi} a\sigma(\beta)\omega(\beta)\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \\ &= \left(\frac{a^2 bt}{2}\right) \left(\frac{a-b}{a+b}\right) \int_0^{2\pi} \sigma(\beta)\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta \end{aligned} \quad (3.70)$$

Considering the stepwise distribution of longitudinal stress postulated in Eq. 3.66, one has

$$B = 2a^2bt \left(\frac{a-b}{a+b} \right) \sigma \int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta \quad (3.71)$$

The section plastic bimoment resistance is simply derived from Eq. 3.71 by replacing σ with F_y . The plastic bimoment resistance ratio of the section is

$$B_r = \frac{B}{B_p} = \frac{2a^2bt \left(\frac{a-b}{a+b} \right) \sigma \int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta}{2a^2bt \left(\frac{a-b}{a+b} \right) F_y \int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta} = \frac{\sigma}{F_y} \quad (3.72)$$

Assuming that the hoop stresses vanish, the plastic potential function (Eq. 3.2) takes the form

$$\sigma^2 + 3\tau^2 = F_y^2 \quad (3.73)$$

By substituting Eqs. 3.68 and 3.72 in Eq. 3.73, a lower bound interaction equation for the case of twisting moments and bimoment is recovered

$$B_r^2 + T_r^2 = 1 \quad (3.74)$$

CHAPTER 4

Background on Thermo-Mechanical Analysis

The state of residual stresses in a hot-rolled steel body depends on its thermal history throughout the cooling process. Therefore, in order to predict the residual stresses, it is required to know the temperature history at any point within the body mass. This is achieved through either a succession of thermal and mechanical analyses or a coupled thermo-mechanical analysis. This chapter presents the fundamentals of such analyses. Discussions of key input parameters in a thermo-mechanical analysis and a discussion of FEA modeling are provided. This chapter is intended to provide the background material needed to conduct the analysis in Chapter 5.

In one-way coupled analysis, it is assumed that the stress/strain field in the steel member depends on the temperature field, but the temperature field can be determined without knowledge of the stress/strain response. As will be discussed later on, this assumption is reasonable when modelling the cooling process of a hot-rolled member since the dependency of the heat transfer analysis on the mechanical behaviour is weak. The assumption signifies that the temperature field obtained from a heat transfer analysis can be input into the mechanical analysis as a thermal loading.

4.1. One-way Coupled Formulation

4.1.1. Heat Transfer Analysis

The transient heat balance is governed by the well known three dimensional heat diffusion equation (e.g., Lienhard 1981)

$$\frac{\partial}{\partial x_i} \left(k_{ij}(T) \frac{\partial T}{\partial x_j} \right) = \rho \frac{\partial H(T)}{\partial t} \quad (4.1)$$

in which, T is the temperature, $H(T)$ is the isotropic temperature dependent enthalpy, $k_{ij}(T)$ is the conductivity tensor, ρ is the specific mass, t is the time, and x_i is the position vector. The specific mass of solid steel varies very little with temperature and is commonly assumed constant (EN 1993-1-2 2001). In the heat diffusion equation, the work done by expansion/shrinkage of the body is assumed negligible. Recalling that once the cooling process starts, the steel mass is in a solid state, this assumption is nearly exact.

By applying the chain rule of differentiation to the right hand side of Eq. 4.1 one obtains

$$\frac{\partial}{\partial x_i} \left(k_{ij}(T) \frac{\partial T}{\partial x_j} \right) = \rho C_p(T) \left(\frac{\partial T}{\partial t} \right) \quad (4.2)$$

in which $C_p(T) = \partial H(T) / \partial T$ is the specific heat capacity of the steel. For an isotropic material such as steel $k_{ij}(T) = \delta_{ij} k(T)$ which simplifies Eq. 4.2 to

$$\frac{\partial}{\partial x_i} \left(k(T) \frac{\partial T}{\partial x_i} \right) = \rho C_p(T) \left(\frac{\partial T}{\partial t} \right) \quad (4.3)$$

in which $k(T)$ is the isotropic conductivity of material.

Boundary conditions

The body heat loss from the boundary surface during the cooling process occurs through three mechanisms: 1) conduction, 2) radiation, and 3) convection. This means that the total surface heat flux q from a body is

$$q = q_k + q_r + q_c \quad (4.4)$$

in which q_k , q_r and q_c are heat flux due to conduction, radiation and convection, respectively.

Heat loss is defined as the component of heat flux $k(T) \partial T / \partial x_i$ along the normal n_i to the boundary surface, i.e.,

$$q = - \left(k(T) \frac{\partial T}{\partial x_i} \right) \cdot n_i \quad (4.5)$$

where the negative sign indicates that heat is dissipating from the body.

Conductive heat losses take place only when two solid bodies are in direct contact. The heat then transfers from the body with higher temperature. During the cooling process of steel members, the zone of contact between the steel body and the supports on the cooling table is very small compared to the whole surface area of the member. Thus, the effect of boundary surface conduction is commonly assumed insignificant (Boyadjiev et al. 2004b) and a close approximation of Eq. 4.4 is

$$q \cong q_r + q_c \quad (4.6)$$

Radiation is another heat transfer mechanism which takes place for a body in any physical state (i.e., gaseous, liquid, or solid) that is above the absolute zero temperature (i.e., -273°C) and can take place in a perfect vacuum. Energy dissipates from the body in the form of photons and electromagnetic waves. The amount of radiated energy emitted from a body to the surroundings is proportional to the fourth power of the absolute temperature of the body. At the same time, the body absorbs heat from the surroundings proportional to the absolute temperature of the ambient. This yields a nonlinear boundary condition for the field equation (Eq. 4.1) on the radiating surface A_r (e.g., Lienhard 1981)

$$q_r = k_e k_B \left[(T_s + 273)^4 - (T_a + 273)^4 \right] \quad \text{on} \quad A_r \quad (4.7)$$

in which T_s , T_a are respectively the surface and ambient temperatures (in Celsius) and k_B is the Stephen-Boltzmann constant. In Eq. 4.7, k_e is the emissivity factor which is a value between zero and unity depending on condition of the boundary surface. The emissivity factor is unity for a perfectly smooth black surface.

Convection is a heat transfer mechanism which occurs when a body (solid or liquid) is surrounded by a fluid. The movement of fluid molecules dissipates heat from the surface of the body. Based on Newton's law of cooling (e.g., Lienhard 1981) the thermal boundary condition on a convective surface A_c is

$$q_c = h(T)(T_s - T_a) \quad \text{on} \quad A_c \quad (4.8)$$

in which $h(T)$ is the temperature dependent convective heat transfer coefficient and depends on many other factors (Totten 2006) including

1. The cross-sectional shape and size of the body;

2. The location of the body within the surrounding fluid (i.e., the relative movement of the surrounding fluid and the body);
3. The roughness of the body surface (i.e., actual boundary surface area);
4. The physical properties of the surrounding fluid (i.e., density, latent heat, phase, specific heat capacity, dynamic viscosity and etc.), and
5. The flow rate of the surrounding fluid.

If the fluid particles move solely because of the difference in density caused by temperature difference, the movement is called natural convection (e.g., air cooling of steel on the outside cooling bed). On the other hand, when external factors increase the movement of the fluid particle, the movement is called forced convection (e.g., water jet cooling at the point of impinging) resulting in convective heat transfer coefficient h value that could be several folds higher, depending on properties and flow rate of the cooling fluid. Values of the convective heat transfer coefficient h are highly variable, as they range from minimum $0.4 \text{ Wm}^{-2}\text{K}^{-1}$ for a natural convection of a vertical wall in air to a maximum of $900,000 \text{ Wm}^{-2}\text{K}^{-1}$ for boiling water heat exchanger in optimum conditions (Lienhard 1981).

An accurate heat transfer model for a specific steel body necessitates the knowledge of the convective heat transfer coefficient h as a function of temperature. This is especially important if the cooling process employs immersion quenching in liquid, where multi phase heat transfer takes place with very high heat flux under non-stationary conditions (Totten 2006). A practical method to obtain such a curve is to measure the surface temperature of a well probed steel body placed appropriately in the cooling regime under investigation. From the history of surface temperature, the corresponding heat flux history, the convective heat transfer coefficient h vs. temperature T curve can be developed. This procedure has been used in a number of studies (e.g., Liščić et al. 1992; Baskiyar 2001; Totten 2006; Bhattacharya et al. 2009; Leocadio et al. 2009).

Given the fact that hot-rolled structural members are generally cooled down through a short period of water spraying followed by air cooling (Richardson 1996), or possibly solely by air cooling (Marcelin et al. 1986), there is no quenching process involved. Therefore, the changes in convective heat transfer h value throughout the cooling process is not as drastic

as that in a quenching process. For this reason, it is common to use a constant time averaged heat transfer coefficient, $\bar{h}$ in Eq. 4.8 instead of the temperature dependent heat transfer coefficient $h(T)$ (e.g., Landau et al. 1960; Yoshida 1984a; Richardson 1996; Baskiyar 2001; 2004; Marcelin 2007) yielding an approximate but a much simpler convective boundary condition on the convective surface A_c

$$q_c \cong \bar{h} (T_s - T_a) \quad \text{on} \quad A_c \quad (4.9)$$

Finite element formulation

The basic energy balance dictates that in a solid body, the variation of the total internal thermal energy, $\delta\Omega$ must be equal to the variation of energy loss, i.e.,

$$\delta\Omega = \delta Y + \delta Q \quad (4.10)$$

in which, δY is the heat dissipated via internal conduction and δQ is the heat dissipated from the boundary surfaces. By differentiation with respect to time, one has

$$\delta\dot{\Omega} = \delta\dot{Y} + \delta\dot{Q} \quad (4.11)$$

in which all dots represents rate of change. By integration of Eq. 4.3 over the body volume, V one recovers

$$\delta\dot{\Omega} = \int_V \rho C_p(T) \frac{\partial T}{\partial t} \delta T dV \quad (4.12)$$

$$\delta\dot{Y} = - \int_V \frac{\partial}{\partial x_i} \left(k(T) \frac{\partial T}{\partial x_i} \right) \delta T dV \quad (4.13)$$

where δT is the arbitrary variational temperature field satisfying the essential boundary conditions. The negative sign in Eq. 4.13 indicates that the heat is dissipated from the body.

By definition the heat loss from the boundary surface, A is

$$\delta\dot{Q} = \int_A q \delta T dA \quad (4.14)$$

From Eqs. 4.12-4.14, by substituting into 4.11, the variational form of the incremental form of energy balance equation is recovered as

$$\int_V \rho C_p(T) \frac{\partial T}{\partial t} \delta T dV + \int_V \frac{\partial}{\partial x_i} \left(k(T) \frac{\partial T}{\partial x_i} \right) \delta T dV = \int_A q \delta T dA \quad (4.15)$$

By discretizing the body into finite elements and defining appropriate element thermal shape functions $\{^{\text{th}}N\}_{n \times 1} = \{^{\text{th}}N(x_i)\}_{n \times 1}$, the temperature fields is interpolated as

$$T = T(x_i) = \langle^{\text{th}}N\rangle_{1 \times n}^T \{T_N\}_{n \times 1} \quad (4.16)$$

where subscript N denotes a nodal variable and n is the number of nodes per finite element.

Similarly, the specific heat capacity and heat conductivity fields are interpolated as

$$C_p(T) = \langle^{\text{th}}N\rangle_{1 \times n}^T \{C_p(T)_N\}_{n \times 1} \quad (4.17)$$

$$k(T) = \langle^{\text{th}}N\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \quad (4.18)$$

Using Eq. 4.18, the integrand of the second term (i.e., conductivity) in finite element formulation is approximated as

$$\frac{\partial}{\partial x_i} \left(k(T) \frac{\partial T}{\partial x_i} \right) \delta T \approx \frac{\partial}{\partial x_i} \left(\left[k(T) \frac{\partial \langle^{\text{th}}N\rangle^T}{\partial x_i} \right]_{3 \times n} \{T_N\}_{n \times 1} \right) \langle^{\text{th}}N\rangle_{1 \times n}^T \{\delta T_N\}_{n \times 1} \quad (4.19)$$

or

$$\begin{aligned} \frac{\partial}{\partial x_i} \left(k(T) \frac{\partial T}{\partial x_i} \right) \delta T &\approx \left(\left\langle \frac{\partial}{\partial x_i} \right\rangle_{1 \times 3} \left(\langle^{\text{th}}N\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \right) [^{\text{th}}B]_{3 \times n}^T \{T_N\}_{n \times 1} \right) \left(\langle^{\text{th}}N\rangle_{1 \times n}^T \{\delta T_N\}_{n \times 1} \right) \\ &= \left(\langle T_N \rangle_{1 \times n}^T [^{\text{th}}B]_{n \times 3} \left(\langle^{\text{th}}N\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \right) \left\{ \frac{\partial}{\partial x_i} \right\}_{3 \times 1} \right) \left(\langle^{\text{th}}N\rangle_{1 \times n}^T \{\delta T_N\}_{n \times 1} \right) \\ &= \langle T_N \rangle_{1 \times n}^T [^{\text{th}}B]_{n \times 3} \left(\langle^{\text{th}}N\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \right) [^{\text{th}}B]_{3 \times n}^T \{\delta T_N\}_{n \times 1} \\ &= \langle \delta T_N \rangle_{1 \times n}^T \left(\langle^{\text{th}}N\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \right) [^{\text{th}}B]_{n \times 3} [^{\text{th}}B]_{3 \times n}^T \{T_N\}_{n \times 1} \end{aligned} \quad (4.20)$$

Substituting Eqs. 4.16 through 4.20 into Eq. 4.15 and knowing that the variation of the temperature field δT is an arbitrary function, one obtains

$$\begin{aligned} \int_V \left(\rho \langle^{\text{th}}N\rangle_{1 \times n}^T \{C_p(T)_N\}_{n \times 1} \right) \left\{ \frac{\partial T_N}{\partial t} \right\}_{n \times 1} dV \\ + \int_V \left(\langle^{\text{th}}N\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \right) [^{\text{th}}B]_{n \times 3} [^{\text{th}}B]_{3 \times n}^T \{T_N\}_{n \times 1} dV = \int_A \{^{\text{th}}N\}_{n \times 1} \hat{q} dA \end{aligned} \quad (4.21)$$

in which $[{}^{th}B]_{n \times 3} = \partial \{ {}^{th}N \} / \partial x_i$ is the spatial derivative matrix of shape functions $\{ {}^{th}N \}$ and $\hat{q} = q(x_i, t)$ is the heat flux at the boundary surface. ABAQUS (SIMULIA 2007b) uses the backward-difference method for time integration

$$\left\{ \frac{\partial T_N}{\partial t} \right\}^{t+\Delta t} = \frac{1}{\Delta t} \left(\{T_N\}^{t+\Delta t} - \{T_N\}^t \right) \quad (4.22)$$

By substituting Eqs. 4.9 and 4.22 in 4.21, one recovers the system of equations at the current time $t + \Delta t$ as

$$\begin{aligned} & \frac{1}{\Delta t} \int_V \left\langle {}^{th}N \right\rangle_{1 \times n}^T \rho \{C_p(T)_N\}_{n \times 1} \left(\{T_N\}_{n \times 1}^{t+\Delta t} - \{T_N\}_{n \times 1}^t \right) dV \\ & + \int_V \left(\left\langle {}^{th}N \right\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \right) [{}^{th}B]_{n \times 3} [{}^{th}B]_{3 \times n}^T \{T_N\}_{n \times 1}^{t+\Delta t} dV = \\ & k_e k_B \int_{A_r} \{ {}^{th}N \}_{n \times 1} \left[\left(\left\langle {}^{th}N \right\rangle_{1 \times n}^T \{T_N\}_{n \times 1}^{t+\Delta t} + 273 \right)^4 - (T_a + 273)^4 \right] dA \\ & + \int_{A_c} \bar{h} \{ {}^{th}N \}_{n \times 1} \left(\left\langle {}^{th}N \right\rangle_{1 \times n}^T \{T_N\}_{n \times 1}^{t+\Delta t} - T_a \right) dA \end{aligned} \quad (4.23)$$

In Eq. 4.23, it is assumed that the ambient temperature is constant in the surroundings environment. Equation 4.23 is highly nonlinear, both in time and geometry domains, especially at the beginning of the cooling process. Given that the purpose of the analysis sought is to determine the temperature field in the body, the sources of nonlinearity are

1. Convection and radiation changes over time result into changes in boundary conditions;
2. The steel specific heat capacity, C_p , and the thermal conductivity, k , are functions of temperature;
3. The release of latent heat due to phase transformation introduces extra energy to the system.

ABAQUS solves the nonlinear system of equations to achieve thermal equilibrium at every time increment by utilizing modified Newton-Raphson iterative method. The temperature history for each node is stored in a result file that is subsequently used in the mechanical modeling. The uncoupled heat transfer for cooling of a steel member is represented in the

left side of Fig. 4.1. The time increments and iterations involved in the iteration process are shown in the figure.

4.1.2. Mechanical Analysis

The general governing equation for the static mechanical problem is given by the force equilibrium balance (e.g., Tall 1964) as

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \quad (4.24)$$

in which b_i is the body force density. The boundary conditions are defined either as prescribed displacements $\hat{u}_i$ on boundary surface A_u or specified tractions Φ_i on surface A_Φ , i.e.,

$$u_i = \hat{u}_i \quad \text{on} \quad A_u \quad (4.25)$$

$$\sigma_{ij} \cdot n_j = \Phi_i \quad \text{on} \quad A_\Phi \quad (4.26)$$

The cooling of hot-rolled members is assumed to involve small strains. This signifies that geometric nonlinear effects have not included in the thermo-mechanical analysis. Therefore, the total small strain rates in the Cartesian coordinate system are (e.g., Tall 1964)

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i} \right) \quad (4.27)$$

The total strain rate decomposition assumption is assumed to hold, i.e.,

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{el} + \dot{\epsilon}_{ij}^{pl} + \dot{\epsilon}_{ij}^{th} \quad (4.28)$$

where $\dot{\epsilon}_{ij}^{el}$, $\dot{\epsilon}_{ij}^{pl}$ and $\dot{\epsilon}_{ij}^{th}$ are the elastic, plastic and thermal strain rate tensors, respectively.

The corresponding stress rate tensor $\dot{\sigma}_{ij}$ is related to the elastic strain tensor through the generalized Hooke's law as

$$\dot{\sigma}_{ij} = D_{ijkl} \dot{\epsilon}_{kl}^{el} = D_{ijkl} (\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^{pl} - \dot{\epsilon}_{kl}^{th}) \quad (4.29)$$

in which the isotropic constitutive tensor D_{ijkl} is defined as (e.g., Basar and Weichert 2000)

$$D_{ijkl} = 2\mu\delta_{ij} + \lambda\delta_{ij}\delta_{kl} \quad (4.30)$$

where $\mu = E/[2(1+\nu)]$ and $\lambda = E\nu/[(1+\nu)(1-2\nu)]$ are the Lamé constants, E is the modulus of elasticity and ν is the Poisson's ratio. The second order tensor δ_{ij} is the Kronecker's delta defined by

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad (4.31)$$

Plastic Strains

The behaviour of carbon steel used in structural sections is commonly assumed to be described by isotropic plasticity. This means that the von Mises yield criterion and associated plasticity are applicable (e.g., Hill 1967), i.e.,

$$\dot{\epsilon}_{ij}^{pl} = \frac{3}{2} \dot{\bar{\epsilon}}^{pl} \frac{\sigma'_{ij}}{\bar{\sigma}} \quad (4.32)$$

where

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \quad (4.33)$$

$$\bar{\sigma} = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}} \quad (4.34)$$

$$\dot{\bar{\epsilon}}^{pl} = \sqrt{\frac{2}{3} \dot{\epsilon}_{ij}^{pl} \dot{\epsilon}_{ij}^{pl}} \quad (4.35)$$

Thermal Strains

Both temperature change and steel phase transformations cause volume changes which result in thermal strains. The thermal strains as described in Koric and Thomas (2006) are

$$\epsilon_{ij}^{th} = \int_{T_{ref}}^T \alpha(T) dT \delta_{ij} \quad (4.36)$$

where $\alpha(T)$ is temperature-dependent coefficient of thermal expansion, T_{ref} is an arbitrary reference temperature, typically the ambient temperature, $T_a = 20^\circ C$. The rate of thermal strains is calculated (SIMULIA 2007b) as

$$\left(\dot{\varepsilon}_{ij}^{th}\right)^{t+\Delta t} = \frac{1}{\Delta t} \left[\left(\varepsilon_{ij}^{th}\right)^{t+\Delta t} - \left(\varepsilon_{ij}^{th}\right)^t \right] \quad (4.37)$$

Finite Element Formulation

The displacement field of the solid is developed by using the principle of virtual work which states that the virtual work $\delta\pi$ is equal to the difference between the incremental internal strain energy δU and the incremental work done by externally applied loads δW , i.e.

$$\delta\pi = \delta U - \delta W = 0 \quad (4.38)$$

By differentiation with respect to time, one obtains

$$\delta\dot{\pi} = \delta\dot{U} - \delta\dot{W} = 0 \quad (4.39)$$

For a deformed solid $\delta\dot{U}$, the incremental rate of internal strain energy is

$$\delta\dot{U} = \int_V \sigma_{ij} \delta\dot{\varepsilon}_{ij} dV \quad (4.40)$$

From Eqs. 4.27, and 4.40, one obtains

$$\delta\dot{U} = \int_V \sigma_{ij} \left(\frac{\partial \delta\dot{u}_i}{\partial x_j} + \frac{\partial \delta\dot{u}_j}{\partial x_i} \right) dV \quad (4.41)$$

Using Eqs. 4.27, 4.28 and 4.29, the stress rate $\dot{\sigma}_{ij}$ is expressed as

$$\sigma_{ij} = D_{ijkl} \left(\frac{\partial \dot{u}_k}{\partial x_l} + \frac{\partial \dot{u}_l}{\partial x_k} - \frac{3}{2} \dot{\varepsilon}^{pl} \frac{\sigma'_{kl}}{\bar{\sigma}} - \delta_{kl} \int_{T_{ref}}^T \alpha(T) dT \right) \quad (4.42)$$

The incremental rate of work, $\delta\dot{W}$ done by body forces, b_i and external tractions Φ_i is

$$\delta\dot{W} = \int_V b_i \delta\dot{u}_i dV + \int_{A_\Phi} \Phi_i \delta\dot{u}_i dA \quad (4.43)$$

Substituting Eqs. 4.41 and 4.43 in Eq. 4.39, the variational energy balance equation is

$$\int_V \sigma_{ij} \left(\frac{\partial \delta\dot{u}_i}{\partial x_j} + \frac{\partial \delta\dot{u}_j}{\partial x_i} \right) dV - \left(\int_V b_i \delta\dot{u}_i dV + \int_{A_\Phi} \Phi_i \delta\dot{u}_i dA \right) = 0 \quad (4.44)$$

By discretizing the body into finite elements and defining appropriate element shape functions $\{^me N\}_{dn \times 1}$ to interpolate the displacement fields one obtains,

$$\{\delta u\}_{d \times 1} = \left\langle {}^{me}N \right\rangle_{d \times d \cdot n}^T \{\delta u_N\}_{d \cdot n \times 1} \quad (4.45)$$

in which subscript d denotes the the number of displacement fields (e.g., 3 for a brick element). Substituting Eq. 4.45 into Eq. 4.44 yields

$$\left\langle \delta u_N \right\rangle'_{1 \times d \cdot n} \left[\int_V [{}^{me}B]_{d \cdot n \times d \cdot n} \{\sigma_N\}_{d \cdot n \times 1} dV - \left(\int_V [{}^{me}N]_{d \cdot n \times d} \{b\}_{d \times 1} dV + \int_{A_\Phi} [{}^{me}N]_{d \cdot n \times d} \{\Phi\}_{d \times 1} dA \right) \right] = 0 \quad (4.46)$$

where

$$[{}^{me}B]_{d \cdot n \times d \cdot n} = \frac{\partial}{\partial x_j} [{}^{me}N] \quad (4.47)$$

is the matrix of spatial derivatives of the shape functions. Knowing that δu_N is an arbitrary function, Eq. 4.46 results into the following system of nonlinear equations

$$\int_V [{}^{me}B]_{d \cdot n \times d \cdot n} \{\sigma_N\}_{d \cdot n \times 1} dV - \left(\int_V [{}^{me}N]_{d \cdot n \times d} \{b\}_{d \times 1} dV + \int_{A_\Phi} [{}^{me}N]_{d \cdot n \times d} \{\Phi\}_{d \times 1} dA \right) = 0 \quad (4.48)$$

Equilibrium at the end of each time increment Δt (i.e., new temperature field) is satisfied when Eq. 4.48 is met within an acceptable tolerance. This is done in ABAQUS using Newton-Raphson method.

4.1.3. Analysis Flowchart

In the one-way coupled formulation, it is assumed that the mechanical fields (i.e., stresses, strains, displacements) are dependent upon the temperature but the thermal state can be determined independently from the stress/strain fields. This assumption allows the separation of the analysis into two separate problems; 1) a thermal analysis where the temperature field is determined; and 2) a mechanical analysis based on the temperature field changes as determined from the thermal analysis. The mechanical analysis computes the mechanical properties based on the temperature fields and user-input temperature dependent definitions of steel properties, and conducts an uncoupled mechanical analysis based on the computed mechanical properties. A flow chart for one-way coupling is depicted in Fig. 4.1. The main advantage of one-way coupling is its computational efficiency compared to two-way coupling.

In the present study, the temperature dependent properties of steel are determined based on an intensive literature review and will be presented in Section 4.3.

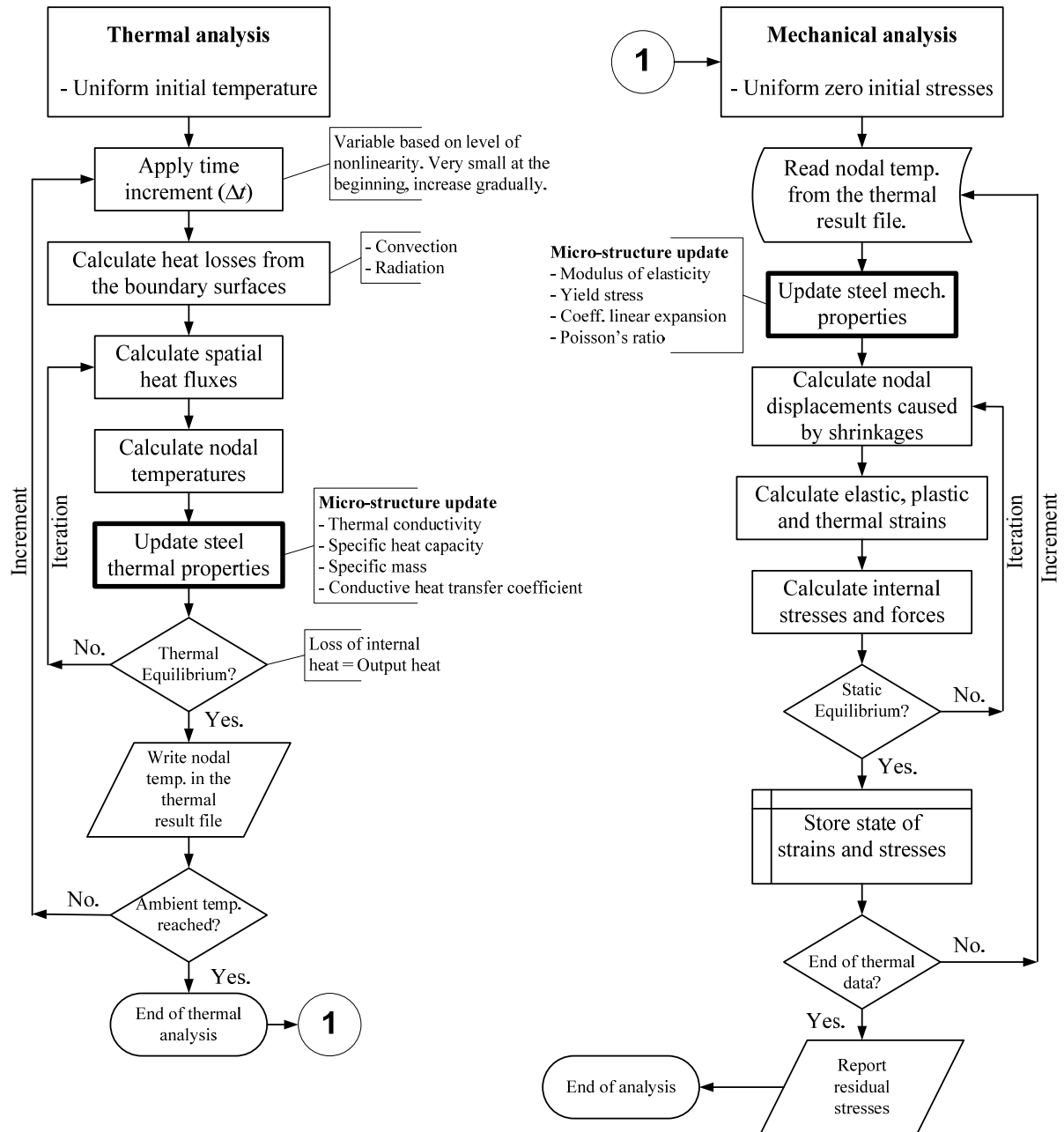


Fig. 4.1: Flow chart for one-way coupled thermo-mechanical analysis

4.2. Two-Way Coupled Analysis

In a two way coupled analysis, the effect of stress/strain fields on the heat transfer analysis is taken into account. During cooling, the solid body shrinks as well as its boundary surfaces. Therefore, the surface losing heat shrinks as the body cools down. Also, the density of the body increases with volume shrinking, which requires updating the mesh coordinates. These effects, along with those described in Section 4.1, can be captured by a two-way coupled analysis, in which the heat transfer analysis output is assumed to influence the mechanical analysis and vice-versa. A flowchart of the coupled analysis is depicted in Fig. 4.2.

The two-way coupled formulation is essentially identical to that of a one-way coupled analysis with the difference that, within every iteration, the boundary surface area in the heat transfer analysis is updated to reflect shrinking computed in the previous mechanical analysis. In principle, a two-way coupled analysis provides a better representation of the problem, but increases the computational effort involved. The accuracy gained by conducting a two-way coupled analysis (in comparison to a one-way coupled analysis) is application dependent. In the present research, initial studies have shown that the accuracy gained by conducting a two-way coupled analysis is minimal. Nevertheless, given that two-way coupling is seamlessly implemented in ABAQUS, which eliminates the need for multiple data transfer between the two analyses, a two-way coupling analysis was used in this study.

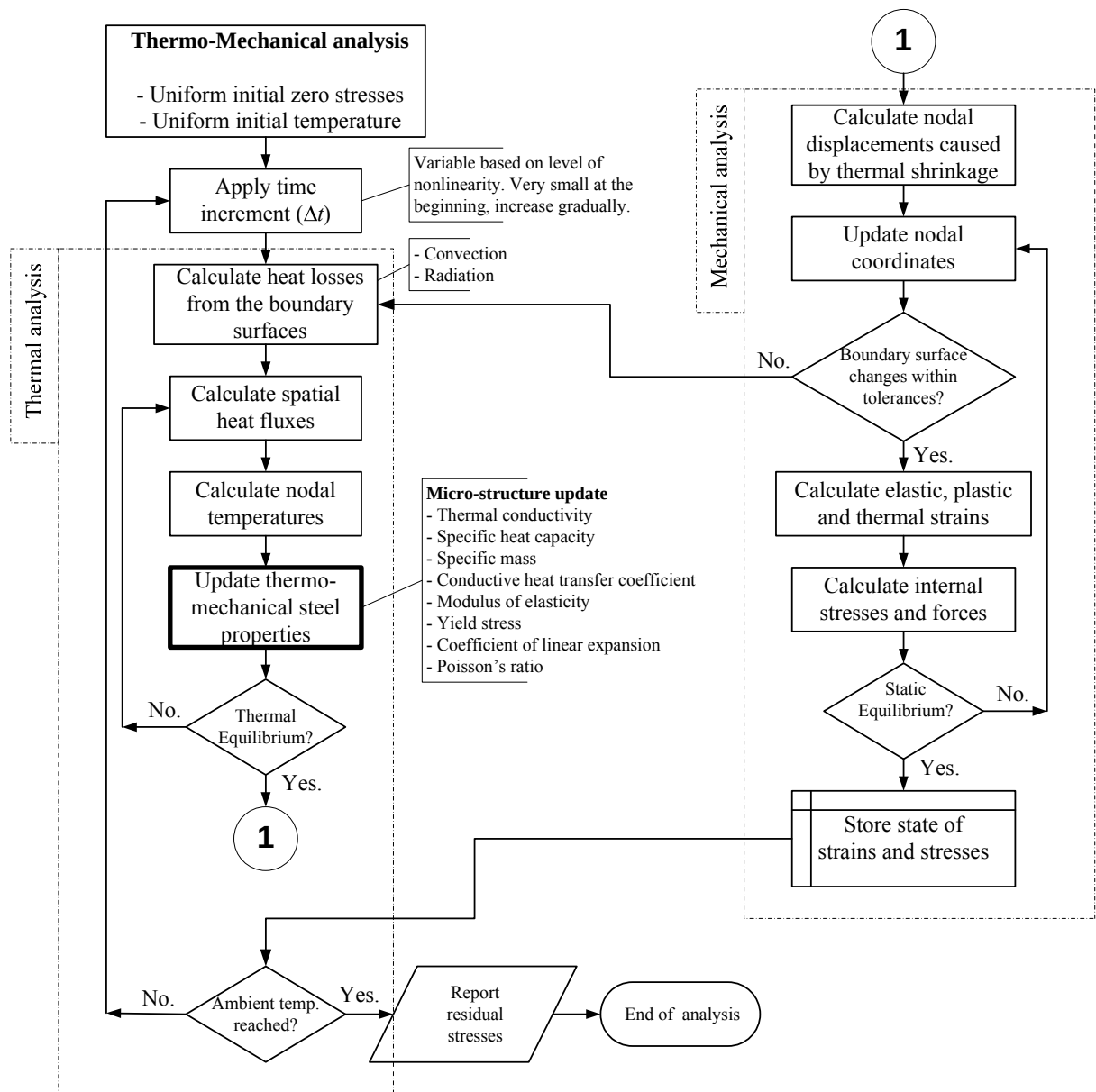


Fig. 4.2: Flow chart for two-way coupled thermo-mechanical

4.3. Input Parameters

4.3.1. Steel Type

As discussed, accurate temperature dependent properties of steel are key to conducting a reliable thermo-mechanical analysis. Steel properties are known to depend on their alloying compositions. In general, when the carbon content is more than 2% of the total mass, the composition is called cast iron while the term “steel” refers to cases where the carbon content is less than 2%. Structural steel contains much less carbon (i.e., it ranges from 0.2 to 0.3%).

Carbon steel is the most common form of structural steel. In carbon steel, the main alloying constituent is carbon. According to the American Iron and Steel Institute (AISI 2009), steel is considered as “carbon steel” when the minimum content needed to obtain a desired effect is specified only for carbon; and when the maximum content percentage specified for manganese, silicon and copper does not exceed 1.65, 0.50 and 0.60, respectively. The Canadian Standard for carbon steel called CSA G40.21-350W is similar to ASTM A572-G50 and AISI 1020. Thus, the present research focuses on ASTM A572-G50 steel which is the most common type in Canada and the USA. The component elements of this steel are iron, carbon and manganese with respective relative weights of 98.0%, 0.23% and 1.35% of the total mass. The weight percentage content of phosphorous, silicon and sulphur is limited to 0.04, 0.30 and 0.05, respectively.

4.3.2. Phase transformation

A material phase can be defined as a state of material of which properties and composition are homogeneous and distinctive from other states of material. Steel is a composition of mainly iron (Fe) and Carbon (C). This composition demonstrates a complex behaviour depending on the carbon percentage and temperature (Fig. 4.3). Composition of Iron-Carbon can have five pure phases. These are

1. Ferrite ($\alpha - Fe$), at low temperature, very low carbon content, hard and ductile
2. Austenite ($\gamma - Fe$), at high temperature, moderate carbon content, soft and ductile

3. Delta iron (δ -Fe), at temperature just lower than the melting point, low carbon content, very soft
4. Cementite (Fe_3C), at low and mid-range temperature, very hard and brittle
5. Liquid (L), at temperature above the melting point.

In regions between pure phases, there are transitional regions in which two phases exist simultaneously.

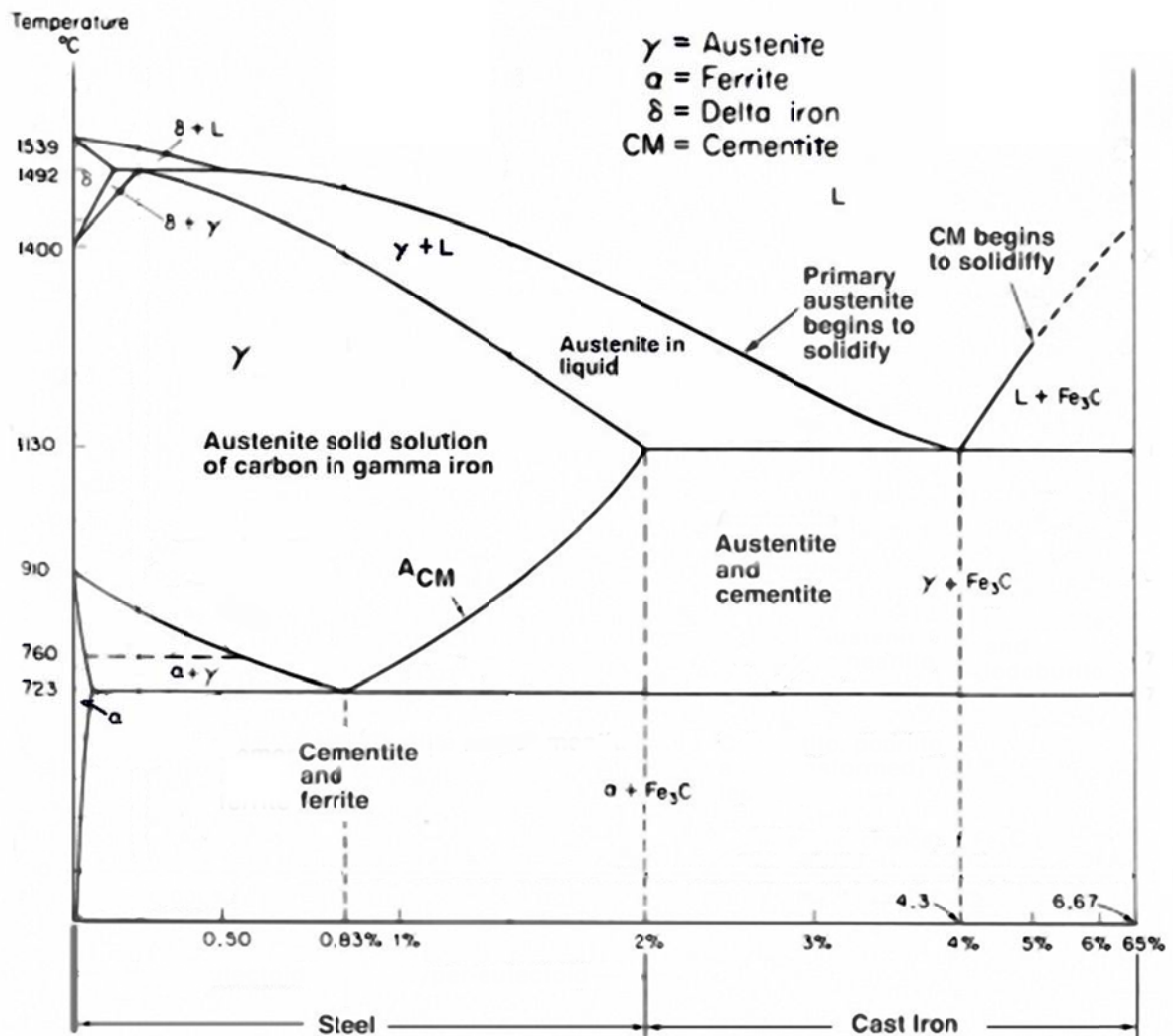


Fig. 4.3: Iron-Carbon phase diagram

(Diagram taken from www.msm.cam.ac.uk/phasetrans/images/FeC.gif)

An Fe-C diagram for steels similar to those of hot-rolled structural section is depicted in Fig. 4.4. As can be seen, for steel with more than 0.025% carbon (e.g., CSA G40.21-350W), a major phase change at about 723°C takes place (i.e., formation of Cementite). For this reason, steel properties (see following sections) exhibit behavioural changes around that temperature.

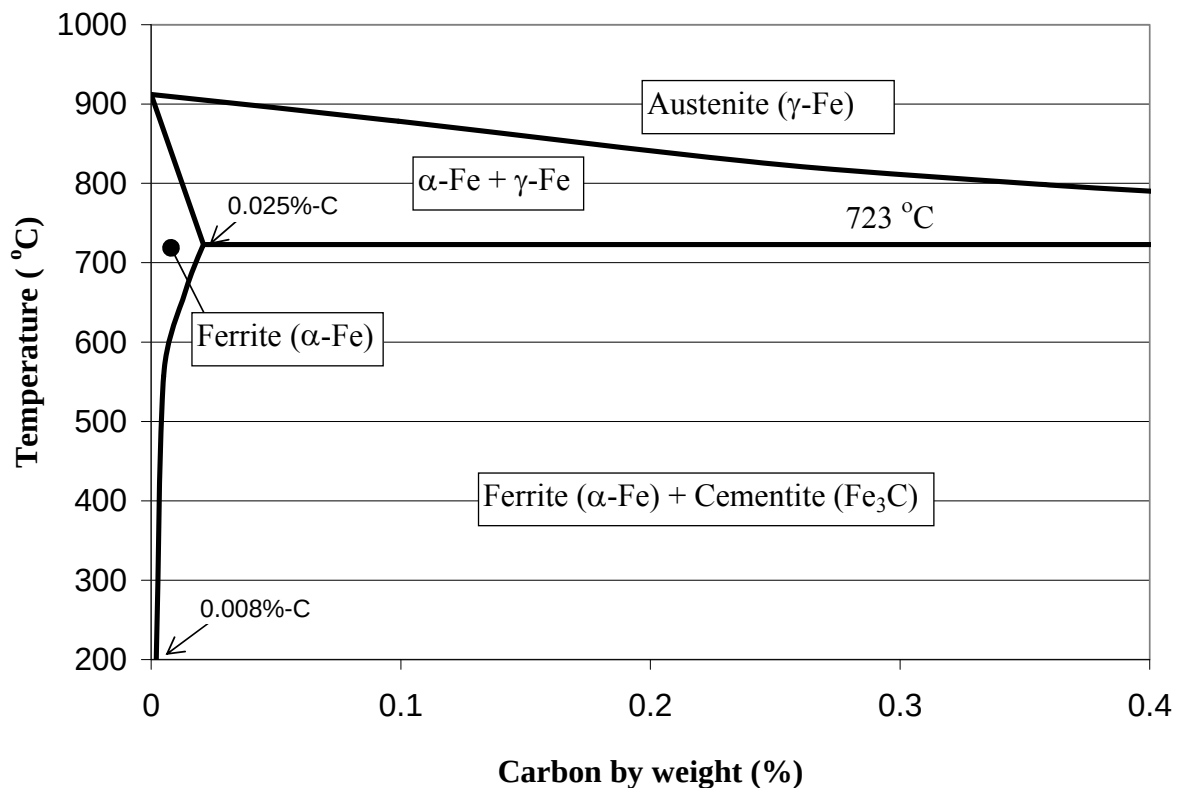


Fig. 4.4: Low carbon steel phase transformation (e.g., Krauss 1990)

4.3.3. Steel Thermo-Mechanical Properties

Modulus of Elasticity

Modulus of elasticity E is a function of temperature. For most solids, including steel, it reduces when temperature T increases. The typical value of E for steel is 200 GPa at room temperature. Experiments (Marcelin et al. 1986) and (Zhou et al. 2003) show that the modulus of elasticity reduces to 50 GPa at about 1050 $^{\circ}C$ as depicted in Fig. 4.5. Therefore, in coupled thermo-mechanical FEA models, it is taken as

$$E[GPa] = \begin{cases} 200 - 0.1T & 0 \leq T < 700 \text{ }^{\circ}C \\ 130 - 0.25(T - 700) & 700 \leq T < 1100 \text{ }^{\circ}C \end{cases} \quad (4.49)$$

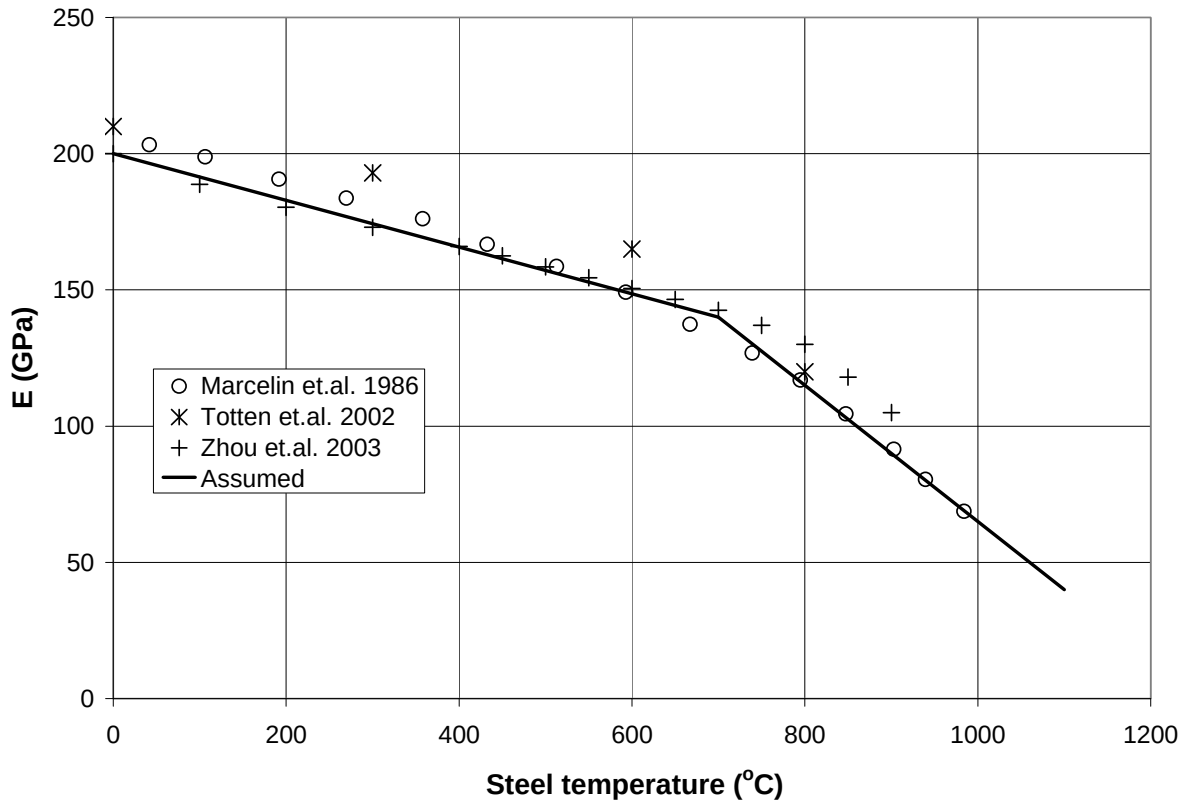


Fig. 4.5: Modulus of elasticity dependency on temperature

Poisson's Ratio

The typical value for Poisson's ratio ν is 0.3 at room temperature (e.g., Landau et al. 1960; diractdelta 2009). The value of Poisson's ratio for AISI 1020 steel at 0 °C is 0.295 (Matweb 2009). Measurements (Marcelin et al. 1986; Totten et al. 2002) showed that Poisson's ratio increases from about 0.295 at zero degree Celsius to over 0.350 at 900 °C (Fig. 4.6). Thus, in the FEA models, the Poisson ratio is taken as

$$\nu = \begin{cases} 0.295 + 5.4 \times 10^{-5} T & 0 \leq T < 700 \text{ }^{\circ}\text{C} \\ 0.333 + 8.0 \times 10^{-5} (T - 700) & 700 \leq T < 1100 \text{ }^{\circ}\text{C} \end{cases} \quad (4.50)$$

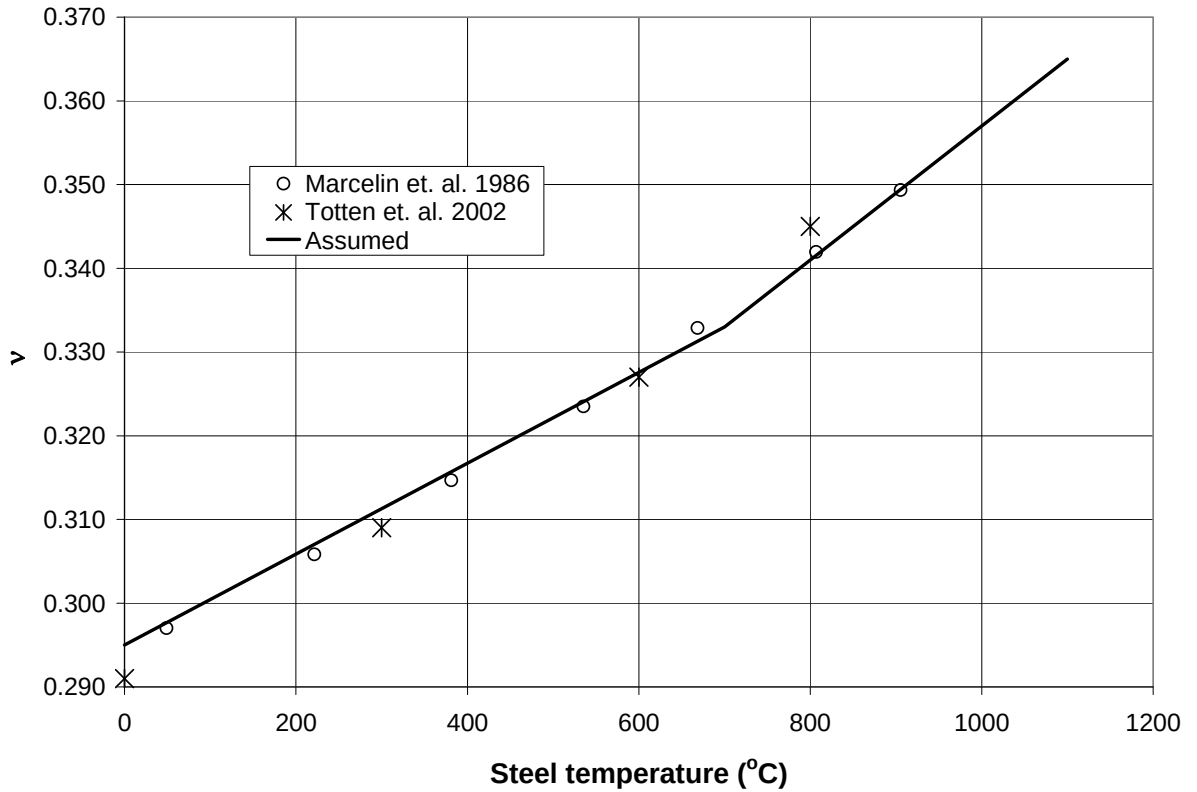


Fig. 4.6: Poisson's ratio dependency on temperature

Yield Strength

The yield strength (F_y) for carbon steel depends on the percentage of carbon and the process of steel making, especially during the cooling process. Marcelin et al. (1986) reported the measured yield strength of carbon steel at various temperatures (Fig. 4.7) for railroad sections. The yield strength of this steel is similar to that of structural steel under investigation, and thus was selected as one of the relationships plotted in Fig. 4.7. Also shown are the measurements by Totten et.al. (2002) and the Eurocode (EN 1993-1-2 2001) curve. In the FEA model, the yield strength (F_y) is assumed to be

$$F_y [MPa] = \begin{cases} 350 - 0.14T & 0 < T \leq 350 \text{ } ^\circ\text{C} \\ 300 - 0.96(T - 350) & 350 < T \leq 600 \text{ } ^\circ\text{C} \\ 60 - 0.12(T - 600) & 600 < T \leq 1100 \text{ } ^\circ\text{C} \end{cases} \quad (4.51)$$

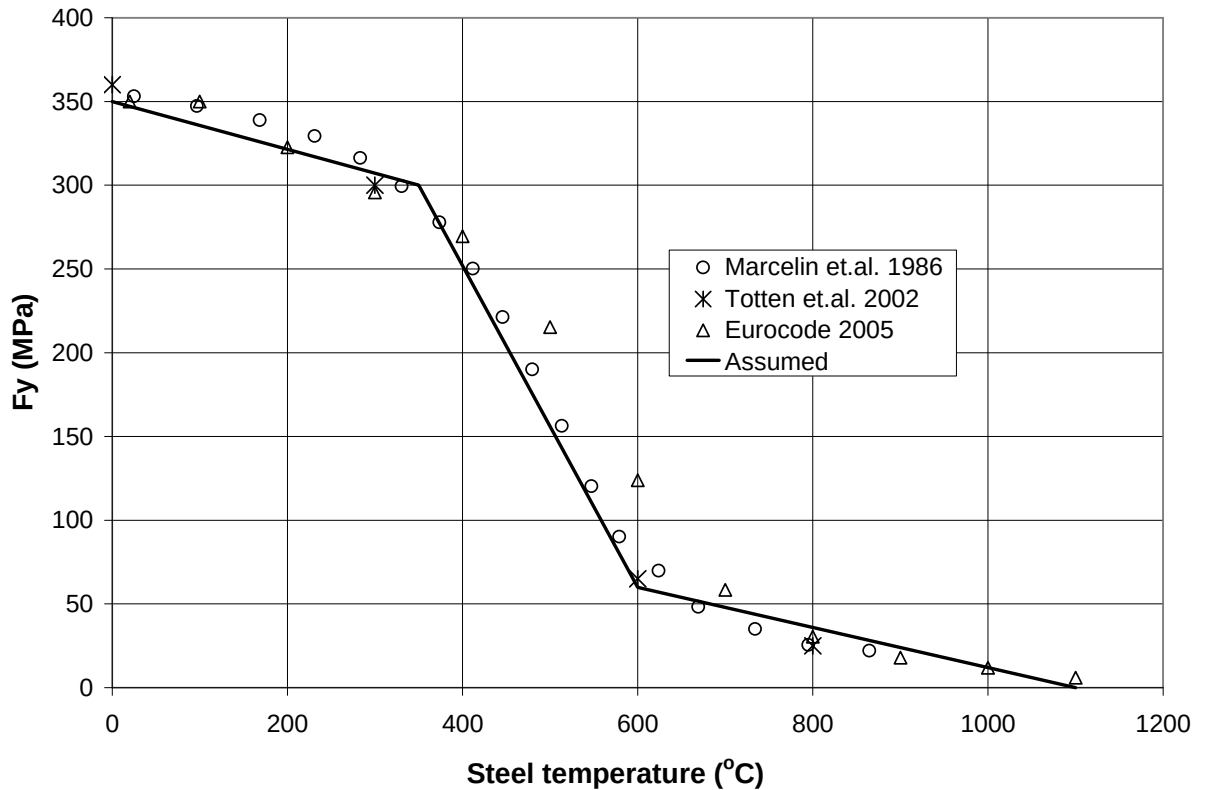


Fig. 4.7: Tensile strength dependency on temperature

Specific Mass

The typical value for the specific mass (ρ) for carbon steel at room temperature is $7,850 \text{ kg/m}^3$ (Matweb 2009). The change in specific mass due to temperature is very small and is usually disregarded in the calculations (EN 1993-1-2). Marcelin (1986) proposed a specific mass-temperature diagram based on changes in volume of the steel body due to temperature while keeping the mass constant (Fig. 4.8). In the FEA model, the specific mass is assumed to be

$$\rho [\text{kg/m}^3] = \begin{cases} 7860 - 0.37T & 0 \leq T < 700^\circ\text{C} \\ 7700 - 0.55(T - 700) & 700 \leq T < 1100^\circ\text{C} \end{cases} \quad (4.52)$$

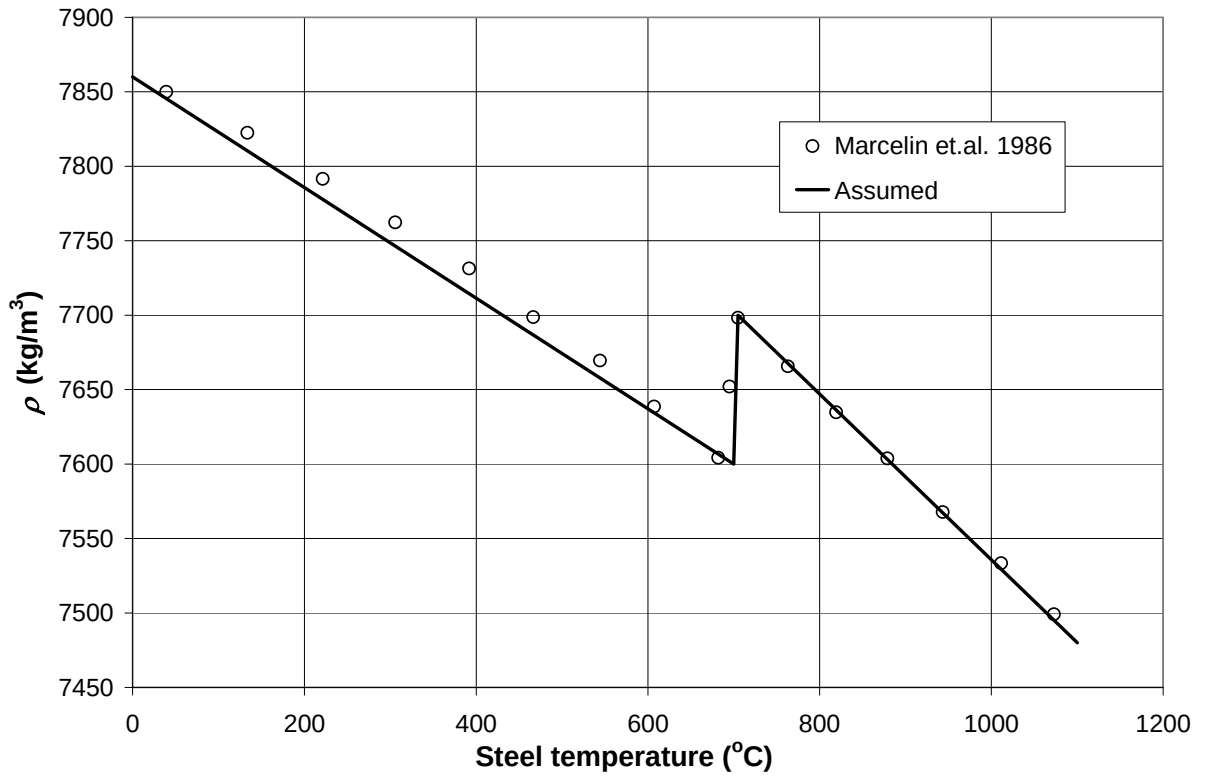


Fig. 4.8: Specific mass dependency on temperature

Specific Heat Capacity

The specific heat capacity for carbon steel C_p is reported to be $420 \text{ J kg}^{-1} \text{ K}^{-1}$ at room temperature (e.g. diractdelta 2009). The materials web database (Matweb 2009) reports the specific heat values for AISI 1020 steel starting from room temperature up to about 400°C (Fig. 4.9). Both Yoshida (1984a) and the Eurocode (EN 1993-1-2) provide the specific heat capacity versus temperature for structural steel. A constant average value of $600 \text{ J kg}^{-1} \text{ K}^{-1}$ independent of temperature can also be taken in simpler calculations (Landau et al. 1960; Purkiss 2007). In the FEA model, the specific heat capacity is assumed as follows

$$C_p [J / kgK] = \begin{cases} 420 + 0.543T & 0 \leq T < 700^\circ\text{C} \\ 800 + 30.0(T - 700) & 700 \leq T < 730^\circ\text{C} \\ 1700 - 14.3(T - 730) & 730 \leq T < 800^\circ\text{C} \\ 700 - 0.333(T - 800) & 800 \leq T < 1100^\circ\text{C} \end{cases} \quad (4.53)$$

The spike in the C_p indicates a latent heat, when phase change takes place.

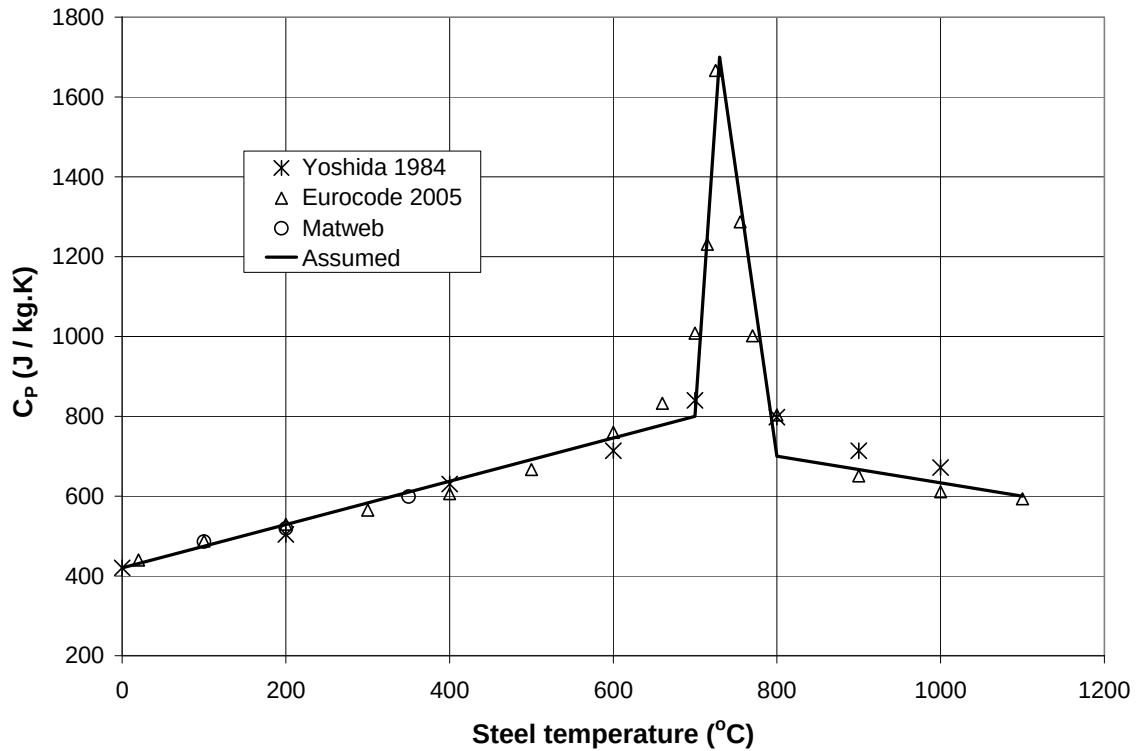


Fig. 4.9: Specific heat capacity dependency on temperature

Thermal Conductivity

The thermal conductivity k is reported to be affected by steel strength (Purkiss 2007). Different constant values for thermal conductivity of carbon steel at room temperature have been suggested in the literature such as $58.8 \text{ Wm}^{-1}\text{K}^{-1}$ (Landau et al. 1960), $45.0 \text{ Wm}^{-1}\text{K}^{-1}$ (Purkiss 2007) and $43.0 \text{ Wm}^{-1}\text{K}^{-1}$ (Enginnering_toolbox 2009). For AISI 1020 steel, the thermal conductivity value is reported to be $51.9 \text{ Wm}^{-1}\text{K}^{-1}$ at room temperature (Matweb 2009). Totten (2006) provided a temperature dependent graph of heat conductivity for pure iron, unalloyed and alloyed steel. By using this graph and starting with a thermal conductivity $k = 52 \text{ Wm}^{-1}\text{K}^{-1}$ at room temperature, the conductivity versus temperature relationship depicted in (Fig. 4.10) is adopted for ASTM-A572 steel.

$$k[W / mK] = \begin{cases} 52 - 0.029T & 0 \leq T < 800^\circ\text{C} \\ 28.8 - 0.003(T - 800) & 800 \leq T < 1100^\circ\text{C} \end{cases} \quad (4.54)$$

This is in good agreement with what is suggested in the Eurocode (EN 1993-1-2 2001).

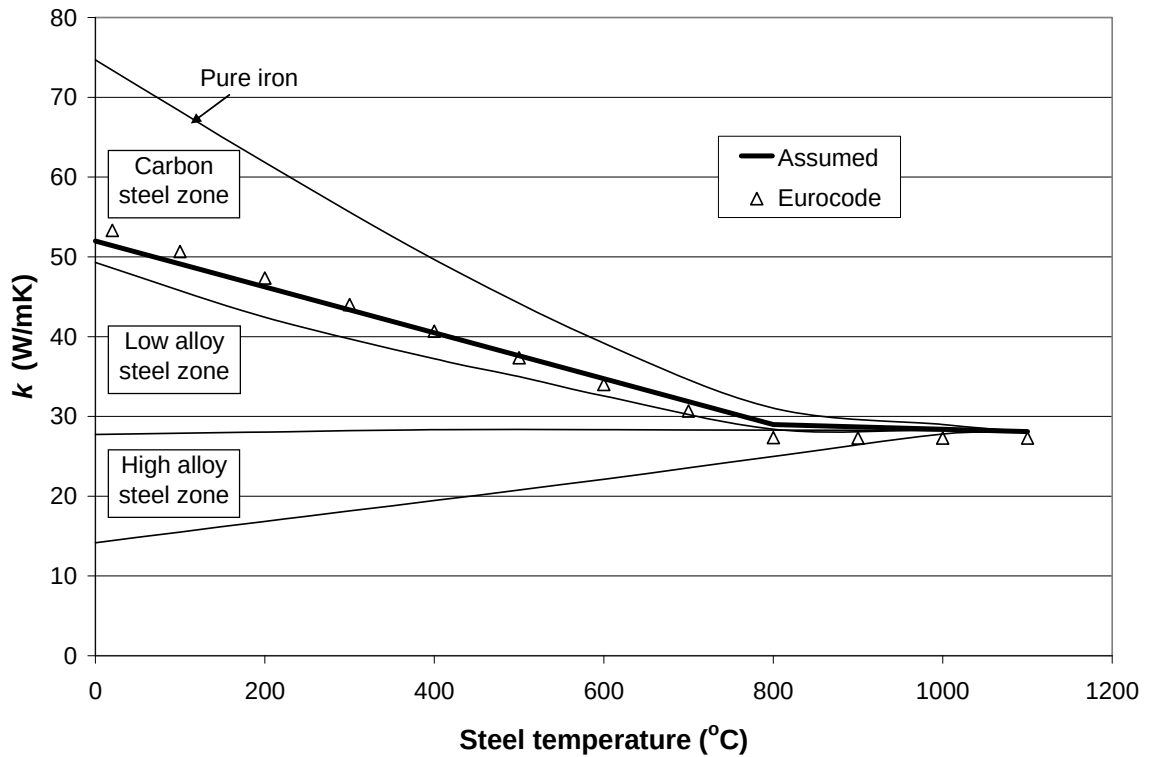


Fig. 4.10: Heat conductivity dependency on temperature in steel (Totten 2006)

Coefficient of Linear Expansion

The coefficient of linear expansion, α , for steel is commonly taken as $1.1 \times 10^{-5} K^{-1}$ at room temperature (e.g., diractdelta 2009; Matweb 2009). This coefficient increases with temperature up to the steel phase change (approximately $700^\circ C$) then decreases to about $1.25 \times 10^{-5} K^{-1}$ at $850^\circ C$ and again starts to increase with temperature until the melting point of steel (Fig. 4.11). The value of coefficient of linear expansion in the FEA model is taken as

$$\alpha [10^{-5} / K] = \begin{cases} 1.15 + 0.0005T & 0 \leq T < 700^\circ C \\ 1.5 - 0.0013(T - 700) & 700 \leq T < 850^\circ C \\ 1.3 + 0.0008(T - 850) & 850 \leq T < 1100^\circ C \end{cases} \quad (4.55)$$

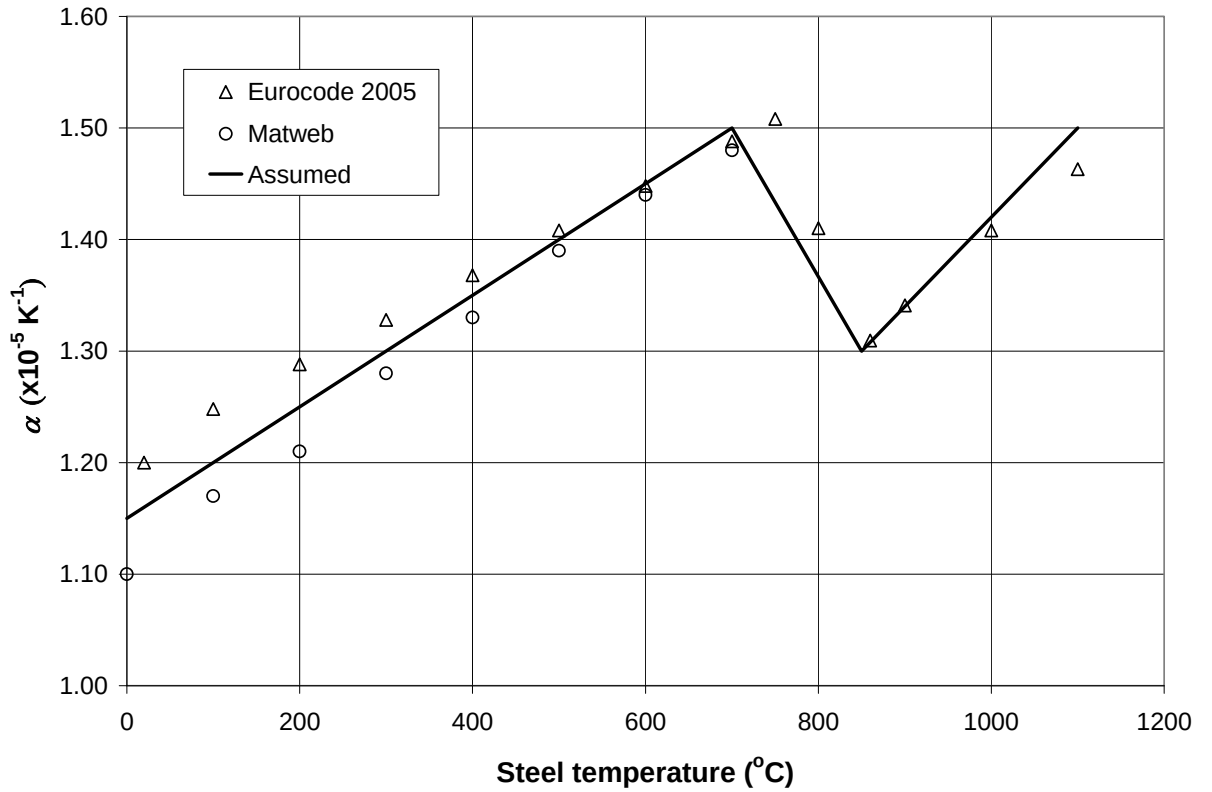


Fig. 4.11: Coefficient of linear expansion

4.3.4. Other parameters

The other parameters needed for the thermo-mechanical analysis are the Stephen-Boltzmann constant: $k_b = 5.67 \times 10^{-8} \text{ W / m}^2 \text{ K}^4$, the emissivity factor for structural steel $k_e = 0.7 - 0.9$ (Yoshida 1984a), and the average convective heat transfer coefficient (Totten 2006).

Table 4.1: Average convective heat transfer

Surrounding fluid	$\bar{h} [\text{W / mK}]$
Entrapped air	10
Still air	30
Moving air	40
Compressed moving air	80
Air-water mist	520
Water spray	3500

4.4. Modeling Aspects

The objective of this section is to describe the specifics of the thermo-mechanical models. The ABAQUS/Standard is capable of performing both coupled thermo-mechanical and uncoupled heat transfer analysis. It has capabilities of modeling solid body heat conduction with general temperature-dependent conductivity; internal energy (including latent heat effects); and quite general convection and radiation boundary conditions. For these reasons, the problem has been modelled using ABAQUS. A few problems will be modeled and the predicted residual stresses will be compared with available experimental results.

4.4.1. Selection of Finite Element

ABAQUS is equipped with a number of shell, solid, and generalized plane strain elements, all capable of performing thermo-mechanical analysis.

Shell Elements

Shell elements are computationally more efficient than solid elements when modeling the load-deformation analysis component of the problem. However, they cannot capture the through thickness heat transfer and concomitant through thickness residual stresses. Therefore, they were judged not suitable for the thermo-mechanical analysis sought in the present study.

Solid Elements

Solid elements are able to provide better three dimensional modelling of heat transfer mechanism and the corresponding residual stresses in hot-rolled EHS. Therefore, they were adopted in the present study. The solid element library in ABAQUS includes 8-node and 20-node isoparametric brick elements. According to ABAQUS (SIMULIA 2007b), these elements are computationally more cost-effective relative to others such as triangular, tetrahedron, and wedge elements. ABAQUS recommends using isoparametric 3D elements everywhere in a given continuum except when filling awkward parts of the model mesh. Since this study deals with structural members with simple cross sections (i.e., W-sections, EHS, and HSS), this recommendation is followed and only the brick element will be used for the thermo-mechanical analysis.

The ABAQUS library has two families of thermo-mechanical brick elements which are C3D8T and C3D20T. The former is a linearly interpolated 8-node brick element and the later is a quadratically interpolated 20 node one (Fig. 4.12). Other variations of the elements are their reduced integration counterparts C3D8RT and C3D20RT.

The choice between linearly and quadratic interpolated elements should be made based on the application. Linearly interpolated elements are essentially constant strain elements. In a case where the strain fields vary smoothly, such as elastic bending or conductivity, these elements should either be avoided or used with very refined meshes. In general, quadratic interpolated elements provide C1 continuity and thus provide a smoother approximation to the response.

For the thermo-mechanical modeling in the present study, it was decided to consider C3D20T in favour of C3D8T for the following reasons

1. There is no difference between both elements in the thermal analysis since the nodal temperatures are defined only at the corner nodes;
2. Based on preliminary analyses, for a given three dimensional problem, models based on C3D20T were found to yield approximately the same mechanical results of model based on C3D8T with less than one thousandth of the number of elements (i.e., one tenth in each direction);
3. This element has been frequently used in ABAQUS examples manual for thermo-mechanical analysis.

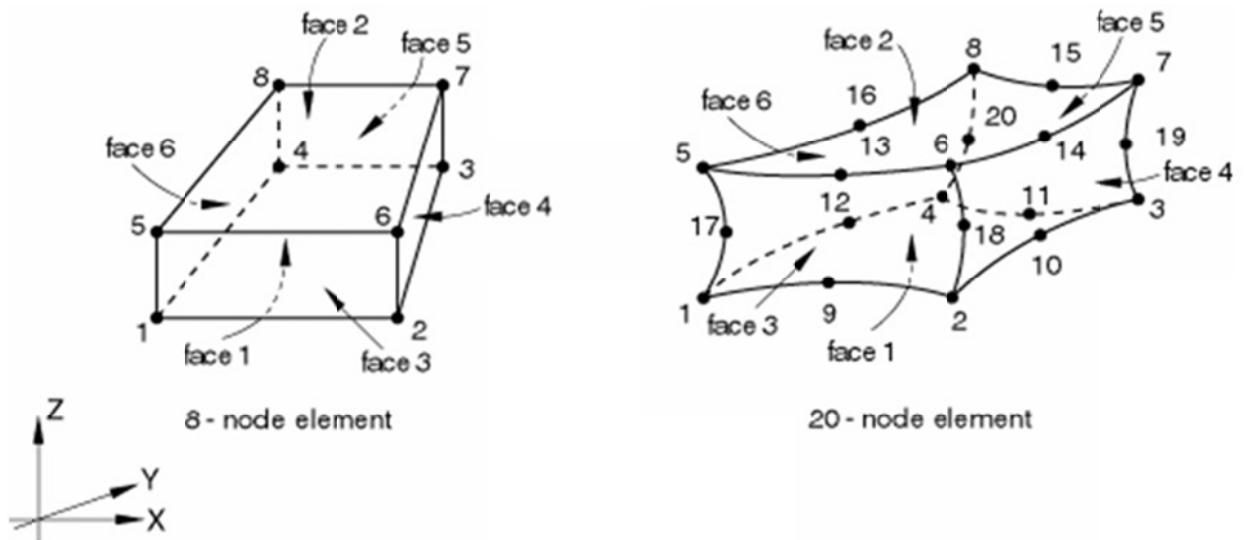


Fig. 4.12: Linear vs. quadratic brick elements in ABAQUS (SIMULIA 2007b)

Generalized Plane Strain Elements

Unlike common plane strain elements, the generalized plane strain elements have a non-zero through thickness strain. These elements are intended to be used for conducting 2D analysis to obtain the essential behavioural characteristics of 3D problem when the through thickness strains have a spatially linear distribution. Thus, the through thickness strains are defined by two boundary planes which move as rigid planes relative to each other.

A four node generalized plane strain element is depicted in Fig. 4.13. The through thickness strains are defined by two boundary planes which move as rigid planes relative to each other. A fixed reference point (i.e., $R(x_0, y_0)$) must be assigned to the element and located

on the boundary plane 1. Each element node can move in the element plan (i.e., u_x and u_y) but its displacement u_z along the z direction, is restrained by the movement of the boundary planes. The deformed location of boundary plane 2 is defined by the reference point displacement in the z direction, u_z its rotation about x axis, ϕ_x and its rotation about y axis, ϕ_y . Therefore, the element thickness at an arbitrary point P (x, y) on the element is

$$t = t_0 + u_z + \phi_x (y - y_0) - \phi_y (x - x_0) \quad (4.56)$$

in which t_0 is the initial thickness of the element.

In general, the well known plane strain elements are a special case of generalized plane strain element where the $u_z = \phi_x = \phi_y = 0$. In this special case, it is assumed that the thickness of the element remains unchanged after deformation. Thus, the through thickness strains vanishes.

In the cooling problem to be modelled in the present study, the longitudinal temperature gradient of a long steel member is negligible, except for localized effects towards both ends. As explained in Chapter 1, the ends affected by steep thermal gradients are cut and discarded. Thus, the modelling of localized effects near the ends is of no interest to the present study. A generalized plane strain analysis thus provides another efficient alternative for modelling the problem. This approach can realistically model the residual stress in the entire length of the final hot-rolled product with a fraction of computation cost when compared to a 3D brick element model.

The ABAQUS library is equipped with a family of generalized plane strain thermal capable element CPEGnT where n is the number of nodes per element and can take the values 3, 4 or 8.

Theoretically, these elements are suitable for the type of thermo-mechanical analysis of this study. Residual stresses predicted by models based on generalized plane strain elements for plates, pipes and EHS were found to agree very well with the results of full length 3D models. For a reason unknown to the author, it was found that when there is a kink in the cross-section (e.g, flange to web connection in W-sections) these elements yield unreliable mechanical behaviours. For this reason, it was decided to simulate the generalized plane

strain analysis using brick elements with appropriate constraints. This type of model will subsequently be called the “slice model”.

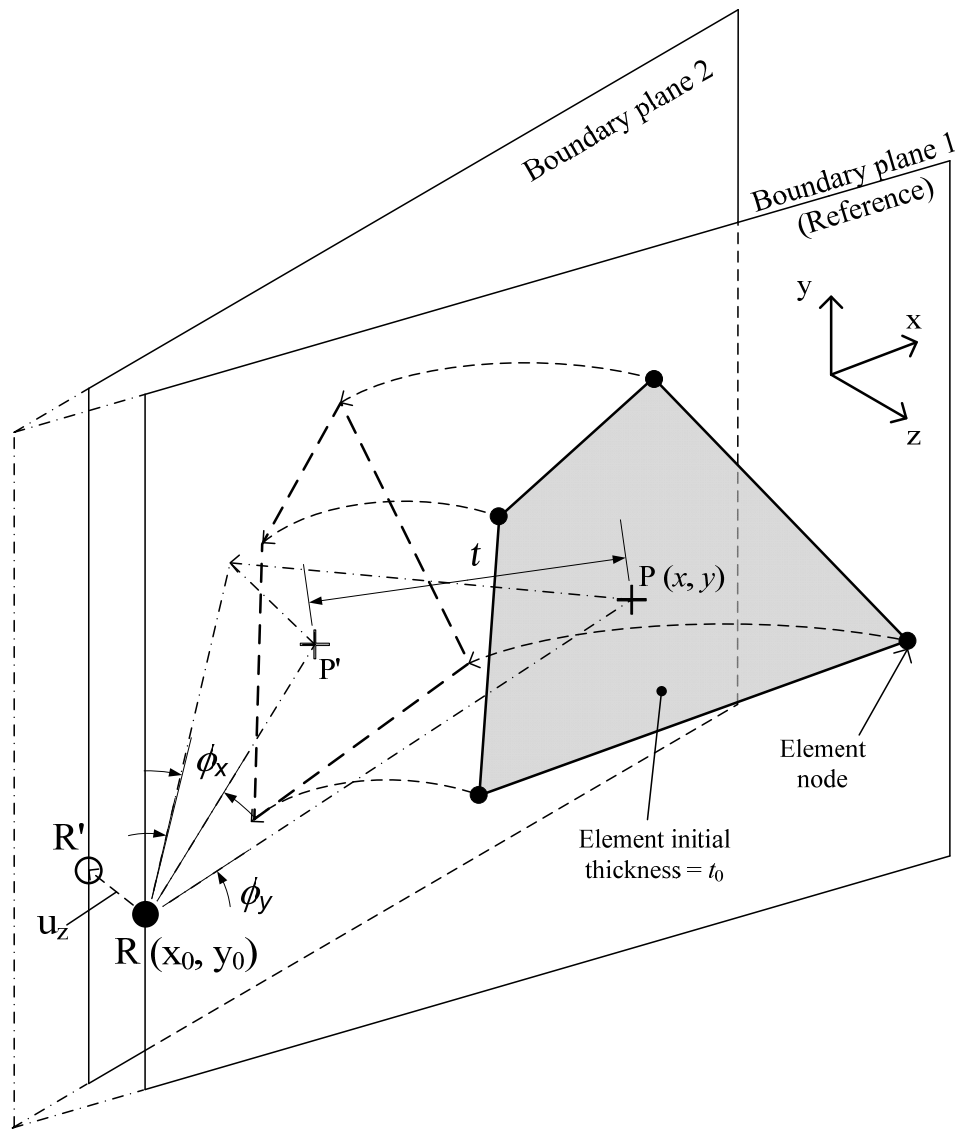


Fig. 4.13: A generalized plane strain element

C3D20T Element

The C3D20T element is a three dimensional brick element with 20 nodes (Fig. 4.12). It can be used both for coupled and uncoupled thermo-mechanical analyses. Each corner node has four degrees of freedom which are three displacements and the temperature. Each mid-edge node has three displacement degrees of freedom. The element has twenty seven integration

points in three layers inside the element (Fig. 4.14). The reduced integration element (i.e., C3D20RT) has eight integration points in two layers. The integration points on second and third layers are numbered consecutively.

The location of integration points for thermal analysis is identical to that for mechanical analysis. The main advantage of the reduced integration element is that it does not suffer from shear locking (i.e., overly stiffed behaviour). Since no external forces are applied to the system during the cooling analysis, the shear locking phenomenon does not appear. On the other hand, the full integration element provides a smoother heat transfer distribution due to its higher number of integration points. Thus, it was decided to use the C3D20T element.

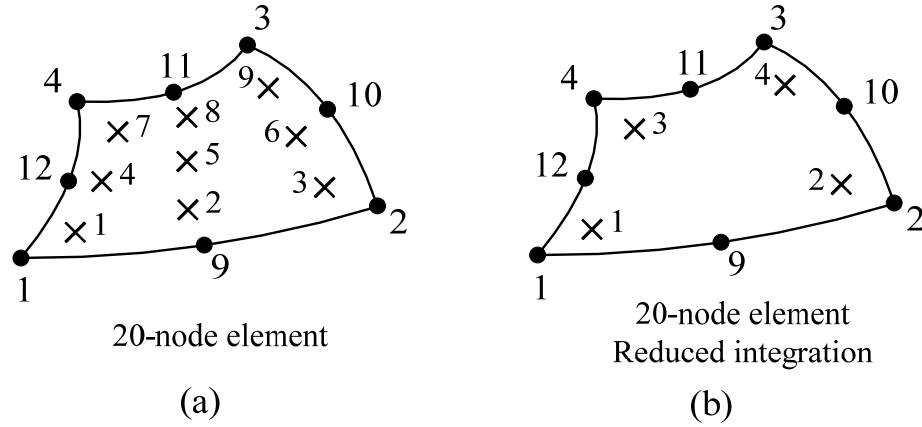


Fig. 4.14: Integration points close to face 1 of (a) C3D20T, (b) C3D20RT elements

4.4.2. Slice model

The thermo-mechanical analysis of the cooling process involves an extensive computational effort. Therefore, every measure is required to be taken to reduce the computational effort while maintaining the level of accuracy. One important step towards this goal is to reduce the size of model.

Since the longitudinal heat transfer is negligible everywhere away from the ends of a hot-rolled member and these ends are cut after cooling, the longitudinal strain variation is generally negligible. This signifies that a slice of single elements in the longitudinal direction would be enough for predicting the typical residual stress within the member if all nodes of each end are constrained to move equally in the longitudinal direction.

Comparing Full and Slice Models

The intent of this section is to demonstrate that the performance of slice model is very close to that of full length models in the prediction of residual stresses at sections away from the member ends. This is done through modeling a 100 by 10 mm cross-section plate and 400 mm length (Fig. 4.15). Steel is CSA G40.21-350W and its thermo-mechanical properties are assumed to be those defined in section 4.3. The initial temperature of the member is taken as 1,000 °C and is cooled to the room temperature (i.e., 20 °C) by free moving air (i.e., $\bar{h} = 40 \text{ W/mK}$). The heat loss due to radiation is also included with emissivity assumed as 0.8 (i.e., $k_e = 0.8$). It is required to compare a) the predicted residual stress patterns based on the two models and b) the computation time involved in both cases.

Since the problem has three axes of symmetry, only one eighth of the plate is modelled in the full model (i.e., the gray section in Fig. 4.15). In this figure, the slice model is bounded by two dark gray planes. In principle, the longitudinal dimension of the slice model can be arbitrarily determined. However, brick elements are known to behave best when their three dimensions are of the same order. For this model the longitudinal dimension was taken as 5mm.

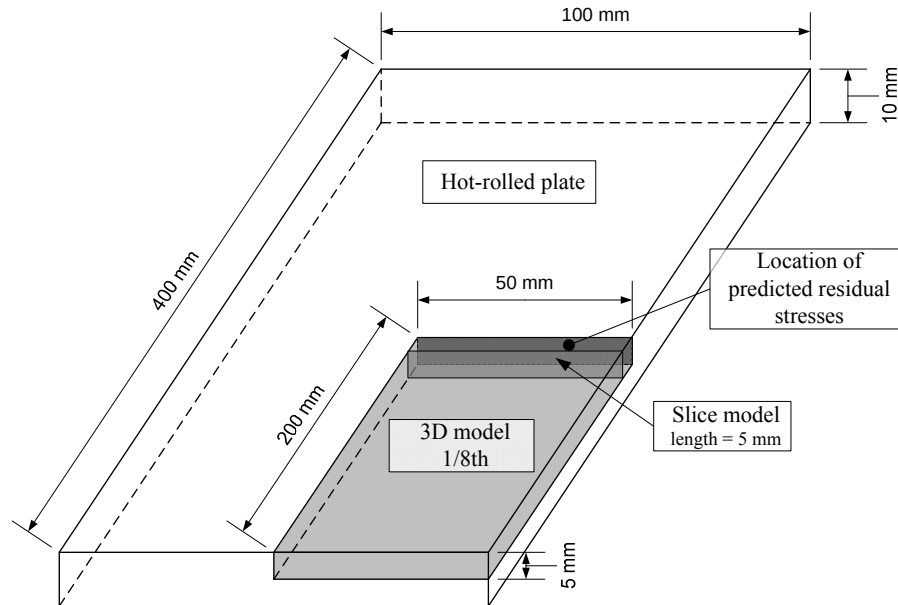


Fig. 4.15: Geometry of the hot-rolled plate thermo-mechanical model

For both models, the plate half-width was modelled by 10 elements, each element being 5 mm wide, and the half-thickness of the plate by 3 elements, each element being 1.67 mm thick. For the full length model, half of the plate was modelled using 25 elements.

The predicted residual stress in the plate using the full length model is depicted in Fig. 4.16. As can be seen, the end effect dissipates at about three times the half-width of the plate and residual stress pattern remains constant for all throughout the remaining length of the plate. Since the length of the plate modeled is four times its width, the residual stress pattern at the end of half-length (Fig. 4.17) must be free of end effects.

In the slice model, all the nodes on the surface coinciding with the mid-span of the plate, as depicted in Fig. 4.15, are restrained from movement along the longitudinal direction and all nodes on the opposite surface are constrained to move equally. This creates a uniform longitudinal strain similar to what happens in the full model away from the ends (Fig. 4.18). As depicted in this figure, in the middle of plate, the changes in the longitudinal strain are less than 0.01% which is practically negligible. Therefore it is reasonable to assume that the longitudinal strain is constant. The predicted residual stress in the plate using the slice model is depicted in Fig. 4.19. The residual stress results from the two models match almost exactly.

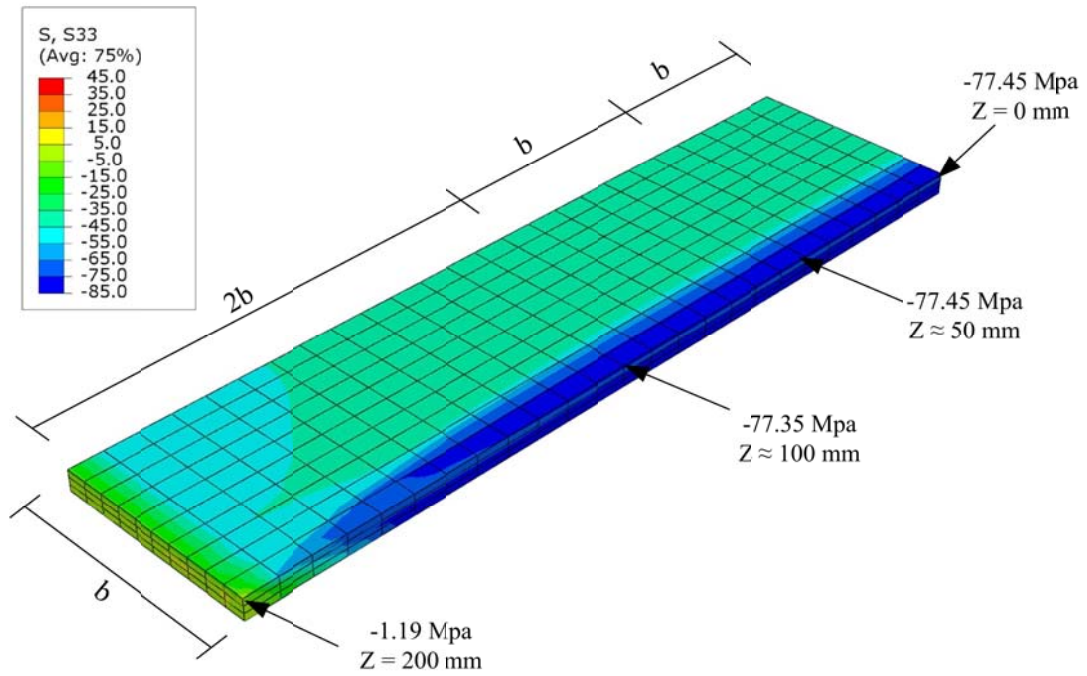


Fig. 4.16: Longitudinal residual stress in the hot-rolled plate (full model)

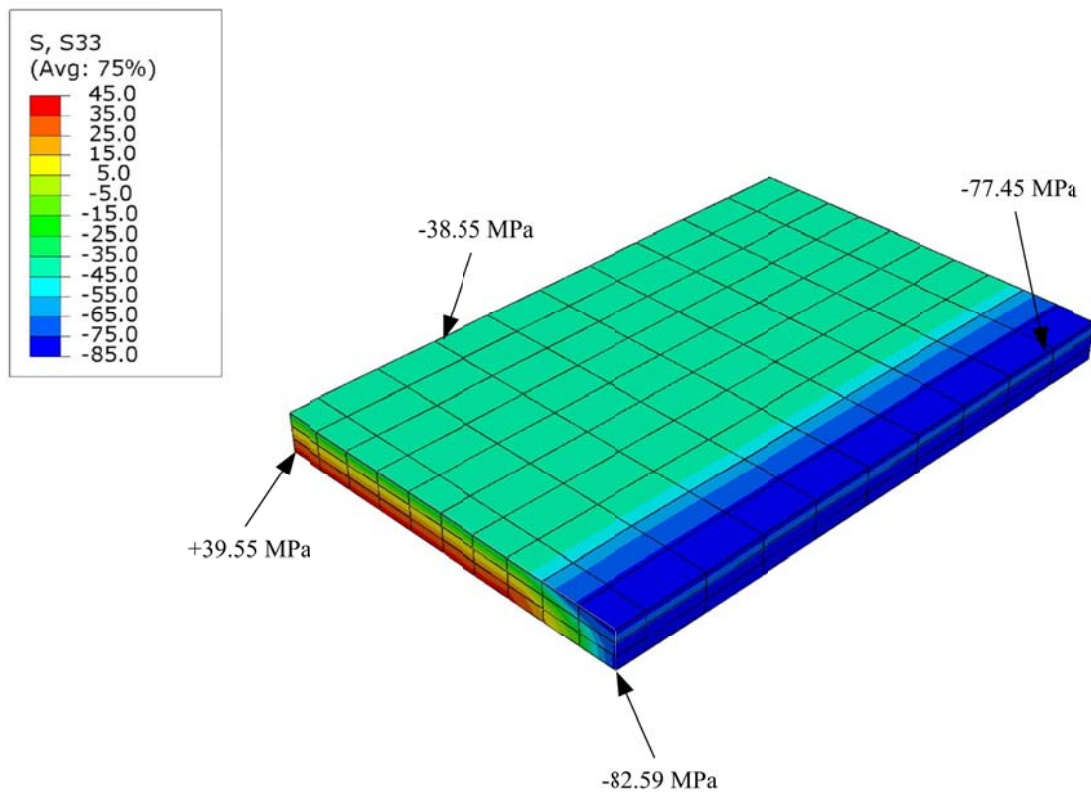


Fig. 4.17: Longitudinal residual stress away from ends by the full model

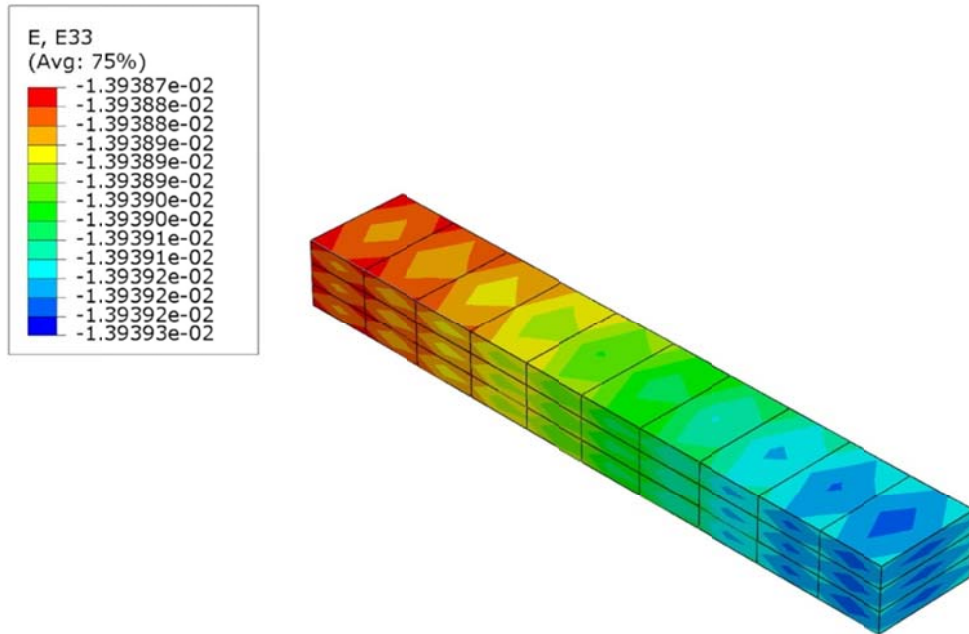


Fig. 4.18: Longitudinal residual strain away from ends by the full model

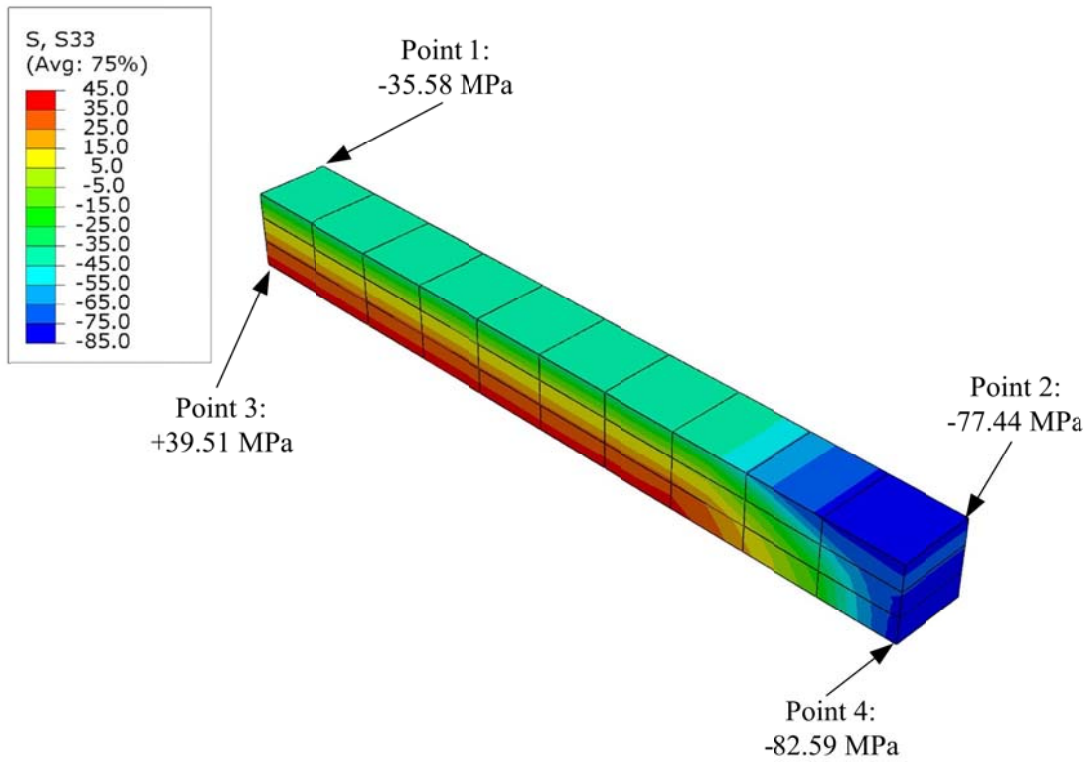


Fig. 4.19: Longitudinal residual stresses by the slice model

The worse agreement takes place at point 1 at the top corner of the plate where the difference between both models is less than 0.1%. On an Intel(R)-D 3.4 GHz desktop computer, the full model run time was about 70 minutes. This compares to less than 4 minutes for the slice model. Based on this comparison, it is decided to use slice model for the prediction of residual stresses in the hot-rolled EHS in the following part of the study.

4.4.3. Mesh sensitivity

The objective of this section is to find a proper mesh size for the thermo-mechanical analysis based on element C3D20T. The same problem is solved with various mesh configurations. The meshing configurations and the corresponding longitudinal residual stresses at the four corner points (Fig. 4.19) are summarized in Table 4.2.

Table 4.2: Mesh sensitivity analysis

No. of elements× size in mm		Longitudinal residual stresses in MPa (compared to the finest mesh)			
Width	Thickness	Point 1	Point 2	Point 3	Point 4
5×50	1×5.0	−23.39 (57.2%)	−81.62 (105.5%)	+53.47 (143.4%)	−71.46 (90.7%)
10×5.0	2×2.5	−35.26 (86.2%)	−74.31 (96.0%)	+42.51 (114.0%)	−82.39 (104.6%)
25×2.0	2×2.5	−35.56 (86.9%)	−74.39 (96.1%)	+42.25 (113.3%)	−80.26 (101.9%)
25×2.0	5×1.0	−40.07 (97.9%)	−75.12 (97.0%)	+37.96 (101.8%)	−80.63 (102.3%)
50×1.0	5×1.0	−40.10 (98.0%)	−75.64 (97.7%)	+37.92 (101.7%)	−79.21 (100.6%)
100×0.5	10×0.5	−40.53 (99.1%)	−76.48 (98.8%)	+37.33 (100.1%)	−78.88 (100.2%)
200×0.25	20×0.25	−40.90 (-)	−77.37 (-)	+37.28 (-)	−78.75 (-)

Based on the sensitivity analysis and other preliminary studies it is recommended

1. To have at least three elements across the section thickness;
2. To use elements with approximate in-plane size of 2 mm;
3. To have longitudinal length of elements equal to 2 mm.

4.5. Notation:

$C_p(T)$	Specific heat capacity
$E = E(T)$	Steel modulus of elasticity
F_e	Euler's column stress
$H(T)$	Isotropic temperature dependent enthalpy
$h = h(T)$	Convective heat transfer coefficient
$\bar{h}$	Averaged over time of convective heat transfer coefficient
$i, j = 1, 2, 3$	Cartesian tensor indices
$k = k(T)$	Isotropic conductivity
k_B	Stephen-Boltzmann constant
k_e	Emissivity factor
$k_{ij}(T)$	Conductivity tensor
$q_k = q_k(T)$	Conductive heat flux
$q_r = q_r(T)$	Radiative heat flux
$q_c = q_c(T)$	Convective heat flux
$T = T(x_i)$	Temperature field
$T_s = T_s(x_i)$	Surface temperature
T_a	Ambient temperature
t	Time
$x_i \equiv (x, y, z)$	Position tensor
$\alpha = \alpha(T)$	Coefficient of linear expansion
ρ	Specific mass
δY	Heat dissipated via internal conduction
δQ	Heat dissipated from the boundary surfaces
$\delta \Omega$	Total energy loss
$\dot{\epsilon}_{ij}^{el}, \dot{\epsilon}_{ij}^{pl}, \dot{\epsilon}_{ij}^{th}$	Elastic, plastic and thermal stress rate tensors
μ, λ	Lamé constants
$\nu = \nu(T)$	Steel Poisson's ratio
δ_{ij}	Kronecker's delta
$\dot{\sigma}_{ij}$	Stress rate tensor

CHAPTER 5

Column Curves and Interaction Relations for Elliptical Hollow Section Members³

5.1. Abstract

A thermo-mechanical finite element analysis model is developed to predict residual stress patterns in hot-rolled sections. The model is first verified against experimental measurements for residual stresses reported for I-sections reported in the literature. The method is then used to predict residual stresses in elliptical hollow sections. A sensitivity analysis is then conducted to assess the influence of various input parameters of the model on the predicted residual stress patterns. The effects of cross-section geometric parameters on the residual stress distribution are then investigated.

A series of column curves is generated providing the compressive capacity of a column as a function of slenderness. The column curves are generated based on a) elasto-plastic geometrically nonlinear analyses, b) including the effect of residual stresses as predicted from thermo-mechanical analyses and c) incorporating initial geometric out-of straightness according to the fundamental buckling mode as predicted from an elastic buckling eigen value analysis. Generated column curves are then compared to those in current design codes. A best fit for the numeric results obtained is conducted to cast them in a format similar to that in the current codes.

Keywords: Elliptical Hollow Sections, Plasticity, Residual Stresses, Initial Imperfection

³ Nowzartash, F., and Mohareb, M. (2011). "Column curves for elliptical hollow section members " *J. Const. Steel Res.*, In press, DOI: 10.1016/j.jcsr.2011.03.026

5.2. Introduction and Scope

A lower bound interaction relation for EHS subject to combination of axial force, bi-axial bending moments and torsion was developed by Nowzartash and Mohareb (2009). They also developed upper bound interaction relations for EHS subject to these loads in addition of bimoments caused by non-circular shape of the section at fixed ends (Nowzartash and Mohareb 2010). The lower bound interaction relations developed are based on the cross-sectional capacity resistance and do not include member stability effects. In this context, this paper is intended to modify the interaction relations already developed to include slenderness effects, residual stresses, and geometric initial imperfections.

In addition to column slenderness, the compressive capacity of columns is known to be influenced by 1) material plasticity, 2) residual stresses and 3) initial out-of-straightness. Other factors such as material variability and load eccentricity can also play a role (Beer and Schulz 1970) on column strength, but will not be investigated in the present study. A methodology is described and verified against available experimental results to predict the residual stress in EHS members. Initial imperfections are determined using elastic buckling analysis. Given the residual stresses and out-of-straightness predicted, a series of nonlinear load deformation FEA including material and geometric nonlinearity is to be conducted to generate points on the column curves.

5.3. Literature Review

5.3.1. Residual stresses in hot rolled sections

In hot rolled sections, residual stresses are mainly induced by the uneven cooling of the different parts of a section. Wide flange sections are most widely used in the construction industry. Also, they are more susceptible to uneven cooling compared to closed sections. Hence, most residual stress investigations were devoted to wide flange sections. Residual stresses were measured and reported in a number of publications (e.g., Fujita 1955; Huber and Beedle 1959; Beedle and Tall 1960; Gala 1962; Brozzetti et al. 1970; Nyashin et al. 1978). Galambos and Ketter (1959) and Young (1972) proposed simplified self-balanced

residual stress patterns for I-sections based on curves conservatively fitted to the measured data. Later Szalai and Papp (2005) modified these patterns to additionally satisfy the torsional equilibrium of the cross section.

The works by Weiner (1956) and Landau et al. (1960) are among the earliest attempts to analytically model residual stress in hot-rolled sections. Given the complexity of behaviour of steel during the cooling process, one-dimensional analytical models have been proposed to describe the residual stress in hot-rolled long beams (Yoshida 1984a; b). Two dimensional analyses were then conducted (e.g., Abouaf et al. 1983; Marcelin et al. 1986; Zhou et al. 2003) and more recently three dimensional finite element analyses were used to predict residual stresses (e.g., Boyadjiev et al. 2004a; Li and Thomas 2004; Koric and Thomas 2006). Although the thermo-mechanical behaviour of structural steel sections under different cooling regimes was extensively studied both experimentally and analytically in the past, the modeling of this behaviour is still an interesting topic among investigators. Studies by Marcelin (2007), Leocadio et al. (2009) and Bhattacharya et al. (2009) are a few examples.

5.3.2. Column curves and residual stresses

After the introduction of hot-rolled steel sections in the early twentieth century, it was observed that, due to residual stresses, the critical load of columns with moderate slenderness is considerably below the Euler formula. This was the beginning of extensive research aimed at developing a modified column curve which incorporate the effect of residual stresses in addition to plasticity and initial imperfection. A comprehensive literature review of the relevant studies can be found in the works of Fukumoto and Itoh (1983), and Johnston (1983). The ultimate result of the present study is various column curve equations such as the single column curve equation in Canadian Steel Standard (Loov 1996; CAN/CSA-S16 2009), the step-wise column curve equations of the Structural Stability Research Council (SSRC) (Bjorhovde 1972; AISC 1993), the newly adopted equations in the American steel specifications (AISC-360 2005), and the European standard (EN 1993-1-1 2005). Column curves including residual stresses for hot-finished hollow sections were developed by Stamenkovic and Gardner (1983). Adluri and Madugula (1996) developed column curves for hot-rolled angles.

Bradford et al. (1987) developed an efficient method for modeling the inelastic lateral buckling of beam-columns including residual stress effects. Recently, Chan and Gardner (2009) experimentally investigated the flexural buckling of EHS columns and compared the results against column curves in the European (EN 1993-1-1 2005), American (AISC-360 2005) and Australian (Standards Australia 1998) codes.

5.3.3. Interaction relations and residual stresses

The American Specification (AISC-360 2005) and Canadian Steel Standard (CAN/CSA-S16 2009) do not directly include the effects of residual stresses in their interaction relations. These effects are indirectly incorporated in the column curve relation that consequently affects the interaction relations via compressive capacity of the member. The European code (EN 1993-1-1 2005) takes a more elaborate approach in which the effects of residual stresses are taken not only in the compressive capacity of the member but also in the bi-axial bending moment components of the interaction relations.

5.4. Assumptions

The major assumptions made in this study are

- The steel stress-strain relationship is elastic perfectly plastic (i.e. strain hardening is neglected).
- The section yields in accordance with the maximum distortional energy density yield criterion (i.e., von Mises yield surface).
- No distinction is made between the true and engineering stresses.
- Steel is non-alloyed (i.e., carbon steel).
- The work done by shrinkage and/or expansion of steel during cooling is negligible.
- Heat loss due to conduction through the cooling bed is negligible.
- The cooling regime does not involve quenching.
- The convective heat transfer coefficient is constant throughout the cooling process.
- The section is stress free at the beginning of the cooling process.

5.5. Formulation – An Overview

The modelling of residual stresses induced during cooling involves a coupled thermo-mechanical analysis. Thus, presented in the following sections are the key aspects of finite element formulation relating to heat transfer analysis, stress deformation analysis, and coupled thermo-mechanical analysis.

5.5.1. Finite Element Analysis

If the nodal temperatures in any finite element at time $\tilde{t}$, $\{T_N\}_{n \times 1}^{\tilde{t}}$ are known, the nodal temperatures at the time $\tilde{t} + \Delta\tilde{t}$, $\{T_N\}_{n \times 1}^{\tilde{t} + \Delta\tilde{t}}$ are given through (i.e., Chapter 4)

$$\begin{aligned}
 & \frac{1}{\Delta\tilde{t}} \int_V \left\langle {}^{th}N \right\rangle_{1 \times n}^T \rho \{C_p(T)_N\}_{n \times 1} \left(\{T_N\}_{n \times 1}^{\tilde{t} + \Delta\tilde{t}} - \{T_N\}_{n \times 1}^{\tilde{t}} \right) dV \\
 & + \int_V \left(\left\langle {}^{th}N \right\rangle_{1 \times n}^T \{k(T)_N\}_{n \times 1} \right) \left[{}^{th}B \right]_{n \times 3} \left[{}^{th}B \right]_{3 \times n}^T \{T_N\}_{n \times 1}^{\tilde{t} + \Delta\tilde{t}} dV = \\
 & k_e k_B \int_{A_r} \left\langle {}^{th}N \right\rangle_{n \times 1} \left[\left(\left\langle {}^{th}N \right\rangle_{1 \times n}^T \{T_N\}_{n \times 1}^{\tilde{t} + \Delta\tilde{t}} + 273 \right)^4 - (T_a + 273)^4 \right] dA \\
 & + \int_{A_c} \bar{h} \left\langle {}^{th}N \right\rangle_{n \times 1} \left(\left\langle {}^{th}N \right\rangle_{1 \times n}^T \{T_N\}_{n \times 1}^{\tilde{t} + \Delta\tilde{t}} - T_a \right) dA
 \end{aligned} \tag{5.1}$$

where ρ is the specific mass, n is the number of nodes in each finite element, $\{C_p(T)_N\}_{n \times 1}$ is the nodal specific heat capacity, $\{k(T)_N\}_{n \times 1}$ is the nodal thermal conductivity, $[{}^{th}B]_{n \times 3}$ is the spatial derivative matrix of the thermal shape functions $\{^{th}N\}_{n \times 1}$, k_B is the Stephen-Boltzmann constant, k_e is the emissivity factor, T_a is the ambient temperature in Celsius and $\bar{h}$ is the average convective heat transfer coefficient. In Eq. 4.23, V , A_r and A_c are volume, radiation boundary surface and convective boundary surface of the finite element, respectively.

After each time increment the state of stress-deformation inside the element is given by (i.e., Chapter 4)

$$\int_V [{}^{me}B]_{d \cdot n \times d \cdot n} \{\sigma_N\}_{d \cdot n \times 1} dV - \left(\int_V [{}^{me}N]_{d \times d \cdot n} \{b_N\}_{d \cdot n \times 1} dV + \int_{A_\Phi} [{}^{me}N]_{d \times d \cdot n} \{\Phi_N\}_{d \cdot n \times 1} dA \right) = 0 \quad (5.2)$$

in which $[{}^{me}B]_{d \cdot n \times d \cdot n}$ is the spatial derivative matrix of the shape functions $[{}^{me}N]_{d \cdot n \times d}$, d is the number of mechanical degrees of freedom for each node (i.e., 3 for brick elements), $\{b_N\}_{d \cdot n \times 1}$ is the vector of nodal internal forces and $\{\Phi_N\}_{d \cdot n \times 1}$ is the vector of externally applied nodal tractions. Equilibrium is reached when Eq. 4.48 is satisfied within an acceptable tolerance

5.5.2. Coupled Thermal -Stress Deformation Analysis

In an uncoupled analysis, it is assumed that the stress and deformation fields are independent of the temperature field within the body.

In a one-way coupled analysis, the stress and deformation fields are considered dependent on the temperature field within the body. The temperature field distribution within the body is assumed to vary with time. At a given time $\tilde{t}$, the temperature field is first determined from a transient heat transfer analysis (i.e., Eq. 5.1). The temperature field is used to determine the thermal strains and magnitude of the constitutive parameters then a stress-deformation analysis (i.e., Eq. 5.2) is performed to determine the stress and deformation field. If the deformation of the body can be assumed not to influence subsequent heat transfer analyses, a one-way transient analysis is sufficient. If, on the other hand, the deformation of the body is significant enough so that it influences subsequent heat transfer analyses, a two-way coupled analysis needs to be conducted. In such a case, the shrinking of body is modeled by updating the coordinates of the finite element mesh after each stress-deformation analysis, prior conducting the following thermal analysis. This involves an additional computational cost. A two-way coupled thermal–stress deformation analysis is adopted in this research.

During the cooling process, the stress-deformation state of the solid body can affect the temperature field in only one way, which is the boundary surfaces change due to the whole body shrinkage. Therefore, the heat losing surface area shrinks as the body cools down. This effect can be captured by coupled analysis.

The formulation for coupled analysis is essentially identical to that of uncoupled analysis with the difference that, within each increment, the boundary surface area of the heat transfer model is updated after the mechanical analysis. Therefore a two-way coupled analysis is computationally more expensive. In addition to providing a more accurate representation to the problem, a two-way coupled analysis seamlessly models the problem in a single integrated model, in which the results do not need to be transferred between thermal and mechanical analyses. However, a coupled analysis involves an additional computational cost. A two-way coupled thermal–stress deformation analysis is adopted in this research.

5.6. Finite Element Modeling

5.6.1. Steel Thermo-Mechanical Properties

Carbon steel is most common in structural applications. In carbon steel, the main alloying constituent is carbon. The standards ASTM A572-G50 (2007) in the US, CSA-G40.21-350W (2009) in Canada, and EN 10025-S355 (2004) in Europe are similar and define this type of steel. According to ASTM A572-G50 the component elements of this steel are iron, carbon and magnesium with respective relative weights of 98.0%, 0.23% and 1.35% of the total mass. The weight percentage content of phosphorous, silicon and sulphur is limited to 0.04, 0.30 and 0.05, respectively.

The thermo-mechanical properties of Carbon steel as a function of temperature were extracted from literature (i.e., Yoshida 1984a; Marcelin et al. 1986; Krauss 1990; Totten et al. 2002; Zhou et al. 2003; EN 1993-1-2 2005; Totten 2006; Matweb 2009). The normalized value of each property (Fig. 5.1) has been normalized with respect to its own value at room temperature. The related thermo mechanical properties of steel considered are the modulus of elasticity, E , Poisson's ratio, ν , yield stress, F_y , specific mass, ρ , specific heat capacity, C_p , thermal conductivity, k and coefficient of linear expansion, α . Their assumed respective values at room temperature are 200 GPa, 0.3, 350 MPa, 7,850 kg/m³, 420 J/kg°C, $k = 52 \text{ W/m}^\circ\text{C}$ and $1.15 \times 10^{-5} \text{ 1/}^\circ\text{C}$. Because of a major steel phase transformation at a temperature about 730 °C, there is a spike in the value of

specific heat capacity indicating a latent heat. In the modeling, the specific heat capacity is assumed to have a peak value of $1700 \text{ J/kg}^\circ\text{C}$ at 730°C .

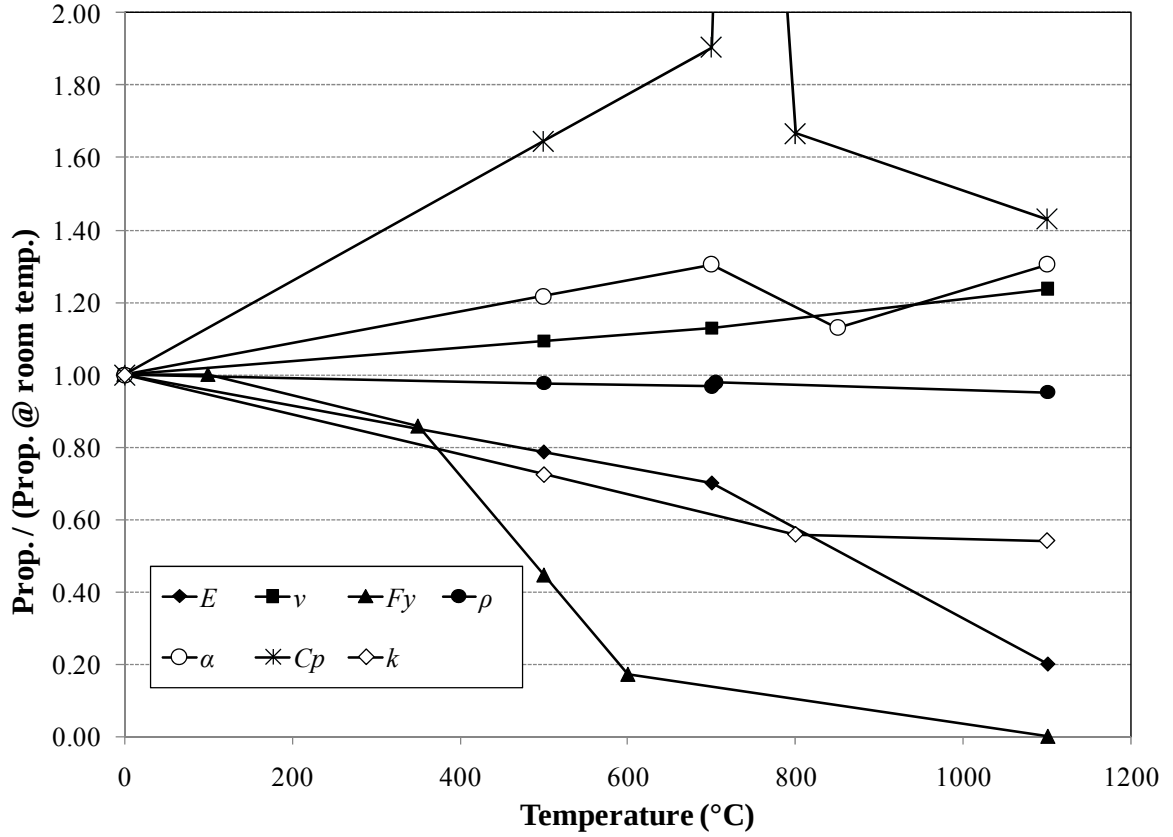


Fig. 5.1: Normalized thermo-mechanical properties of steel

Other parameters needed for modeling the residual stresses are the Stephen-Boltzmann constant: $k_b = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ }^\circ\text{C}^4$ and the emissivity factor for structural steel: $k_e = 0.7 - 0.9$ (Yoshida 1984a).

5.6.2. Selection of the Finite Element and Mesh

ABAQUS is equipped with a number of shell and solid elements capable of performing thermo-mechanical analyses. Shell elements cannot capture the through thickness heat transfer and concomitant through thickness residual stresses. Therefore, they were judged unsuitable for the thermo-mechanical analysis sought in the present study. Solid elements are able to provide a better three dimensional modelling of the heat transfer mechanism and

the corresponding residual stresses in hot-rolled EHS and therefore were adopted in the present study. The C3D20RT element was adopted in the analysis which is thermo-mechanical quadratically interpolated 20 node 3D brick element.

The thermo-mechanical analysis of the cooling process involves an extensive computational task. Therefore, every measure needs to be taken to reduce the computational effort while maintaining the desired level of accuracy. One important step towards this goal is to reduce the size of model.

A hot-rolled member experiences a uniform cooling regime along its length except for the end effects. Moreover, the two end segments are cut after cooling (VMTubes 2008). Therefore, the longitudinal residual strain variation is generally negligible. This allows the modeling to be reliably performed using a single element in the longitudinal direction for predicting the residual stress in the member, while accurately assuming the heat flux to vanish in the longitudinal direction. All nodes are constrained to move equally in the longitudinal direction.

Through a mesh sensitivity analysis (i.e., Chapter 4) it was observed that in thermo-mechanical modeling, the optimum size of the solid elements is about 2 mm in all three directions. It was also observed that at least three elements through the section thickness are required to achieve a reliable result.

5.7. Model Verification

The intent of this section is to verify the proposed technique for predicting residual stresses. This is done by modeling a few sections and comparing the results with the experimental data available in the literature. For this purpose, a wide flange section (i.e., W200×46) and a hollow square section (i.e., HSS-180×180×6.3) will be considered.

5.7.1. Problem 1: Residual Stresses in a W200x46 Section

Many residual stress measurements were conducted on wide flange sections in late '50s and early '60s. Among those are the studies at Lehigh University (Huber and Beedle 1959) and Cambridge University (Gala 1962). Both studies used the method of sectioning (e.g., Totten et al. 2002) for the measurement. One of the hot-rolled sections investigated in both

studies was the W200×46 (section height, d is 203 mm, flange width, b is 203 mm, flange thickness, t is 11.0 mm and web thickness, w is 7.2 mm).

Steel was assumed to have the properties given under section 5.6.1. Based on yield strength, this steel is similar to that in Gala (1962). The initial temperature of the member is taken as 1100°C and is cooled to the room temperature (i.e., 20°C) using moving air. The heat losing surfaces are divided into two sub-domains; 1) the outside surface including the outside face and the tip of flanges and 2) inside surface of the flanges and the side of the web. The reason for this distinction between surfaces is the way hot-rolled wide section members are placed on the cooling bed. In general, the moving air comes in contact more easily to the outside surfaces of flanges than the inside surfaces. Therefore if the average convective heat transfer coefficient for the outside surface is taken as $\bar{h}_{out} = 40 \text{ W/m}^2\text{C}$ (Totten 2006), $\bar{h}_{in}$ is taken as $30 \text{ W/m}^2\text{C}$ for the inside surfaces. Moreover, the mutual radiation between the inside surfaces of the flanges and the web reduce the absolute energy loss due to radiation from the side of web and the inside surfaces of flanges. Therefore, the emissivity factor for the outside surface is taken as $k_{e_out} = 0.8$ while that of the inside surface is assumed to be $k_{e_in} = 0.6$.

One quarter of the section was modeled using six elements through the flange thickness, 50 elements along half of flange width, two elements through the half web and 45 elements along half of the web height. As stated under section 5.6.2. , only a single element is used longitudinally. The time increment was chosen as one millisecond at the beginning of the process and was allowed to automatically increase as long as the temperature difference at any point within the continuum between two consecutive times remained smaller than 20°C. A total cooling allowance time was chosen as 10,000 seconds or about 2:45 hours. After this time, the heat flux from the section is assumed to be practically zero.

The predicted longitudinal residual stresses and those measured by Gala (1962) are depicted in Fig. 5.2. The method of sectioning shows a stress average over the piece cut. Therefore the average of stresses at the centroid of elements over the corresponding area of the measuring pieces is depicted in Fig. 5.2. Given the nature of residual stress and uncertainties/approximations existing both in the model and measurements, very good

agreement is obtained. At the tip, the measured residual stress was -75 MPa while that predicted by the analysis is -78.2 MPa, a 4% difference. Also, the measured stresses at the middle of the web is -88 MPa and at intersection of flange and web is +55 MPa. The corresponding FEA predicted stresses are -83.6 MPa and 53.9 MPa, representing 11% and 2% differences, respectively.

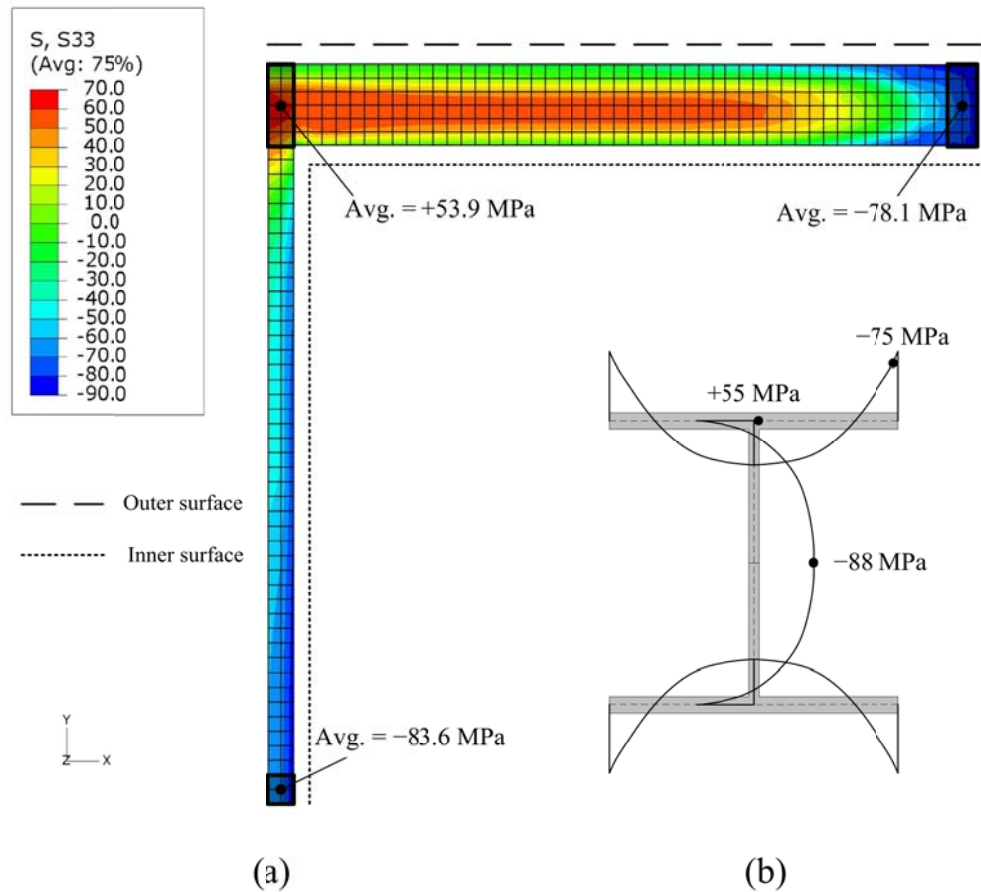


Fig. 5.2: Predicted residual stresses in a W200 $\times$ 46 section a) Prediction, b) Measurements

5.7.2. Problem 2: HSS 180x180x6.3

The second verification problem is a hot-rolled HSS-180 $\times$ 180 $\times$ 6.3 section as manufactured by VMTubes which reported measured residual stresses at the outside surface of HSS-180 $\times$ 180 $\times$ 6.3 (VMTubes 2008).

An initial steel temperature of 1100°C and a room temperature of 20°C were assumed. Steel is assumed to conform to S355 (EN 10025-2 2004). The cooling regime was assumed to consist of two second water spraying followed by 10,000 seconds cooling with moving air. For the outside surface, the average heat transfer coefficient was assumed to be $\bar{h}_{out_W} = 3500 \text{ W/m}^2\text{C}$ during water spraying and $\bar{h}_{out_A} = 40 \text{ W/m}^2\text{C}$ during air cooling periods as proposed in Totten (2006). Since the air inside the section is almost entrapped and sprayed water cannot come in contact with it, $\bar{h}_{in} = 10 \text{ W/m}^2\text{C}$ is assumed throughout the whole cooling process. The outside surface emissivity factor is assumed to be $k_{e_out} = 0.8$. The radiation heat loss from the inside surface is assumed negligible, since radiation takes place in a closed environment in which thermal energy cannot dissipate.

Element size is taken approximately as 2 mm by 2 mm, i.e., three elements were used across the thickness and an average of 44 elements along the width and height. The longitudinal stresses predicted and the experimentally measured values are depicted in Fig. 5.3. As can be seen, there is a very good agreement between the predicted values and the experimental results.

The transverse residual stresses are observed to range from minimum of -51 MPa on the outside surface to maximum of $+53 \text{ MPa}$ on the inside surface which are comparable in magnitude to the longitudinal stresses.

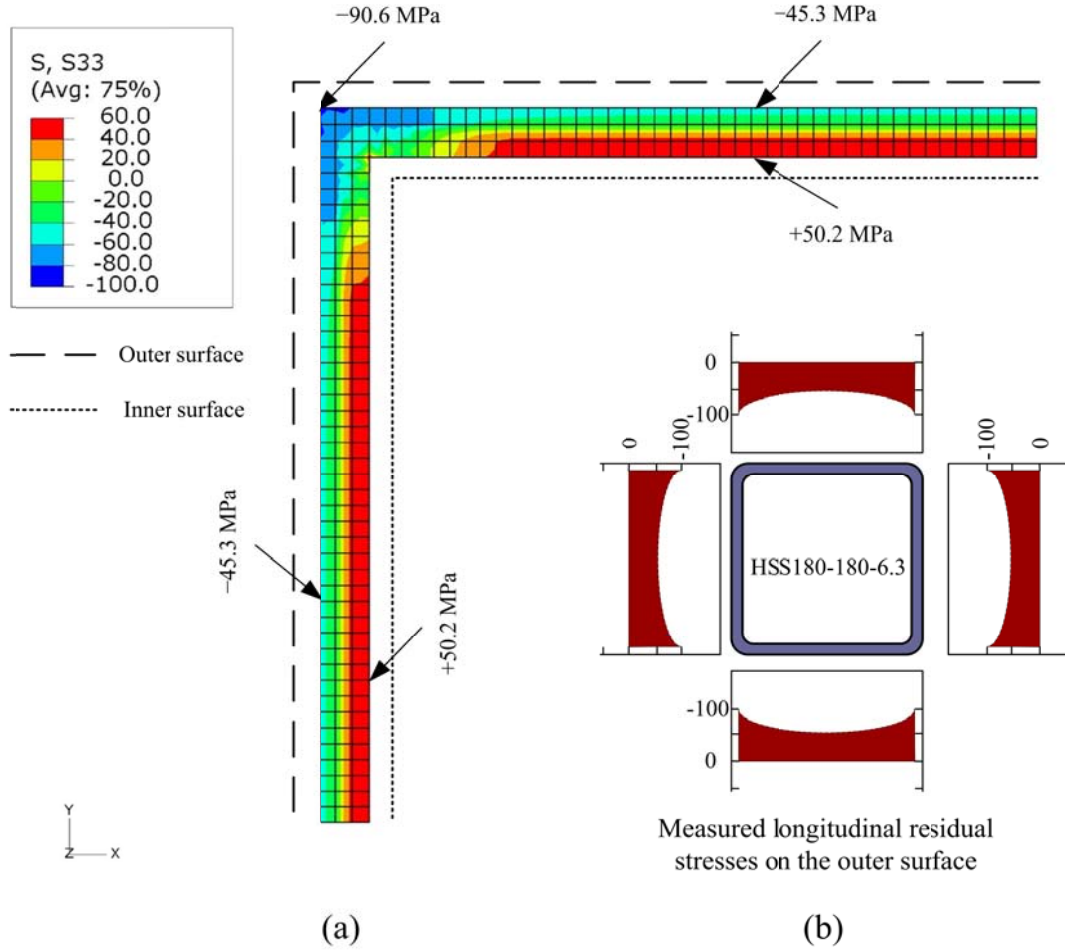


Fig. 5.3: Longitudinal residual stresses in a HSS-180 $\times$ 180 $\times$ 6.3

a) Prediction b) Measurements (Adapted from VMTubes 2008)

5.8. Residual Stresses Prediction in EHS

The thermo-mechanical model developed in Section 5.6. and validated in Section 5.7. is used for predicting the residual stresses in steel elliptical hollow sections.

5.8.1. Residual Stresses for Elliptical Sections

The EHS-200 $\times$ 100 $\times$ 10 cross-section is considered in the investigation. A thermo-mechanical analysis is conducted to predict the residual stresses in the section. The input thermo-mechanical parameters are identical to those in the verification problem for section HSS-180 $\times$ 180 $\times$ 6.3. The residual stresses predicted are depicted in Fig. 5.4. In this

example, the longitudinal stresses on the outside surface are found to be always compressive; with a lowest absolute value of 33 MPa at the intersection with the x-axis and a highest absolute value of 68 MPa at the y-axis. On the inside surface, longitudinal stresses were observed to always be tensile and range from 8 MPa at y-axis to 56 MPa at the x-axis. The presence of compressive longitudinal stresses on the outside surface and self balancing tensile stresses on the inside surface is in agreement with (Beer and Schulz 1970).

The contour of in-plane residual stresses is also obtained and are found to vary from -41 to -51 MPa on the outside surface, and from +36 to +53 MPa on the inside surface (Fig. 5.5).

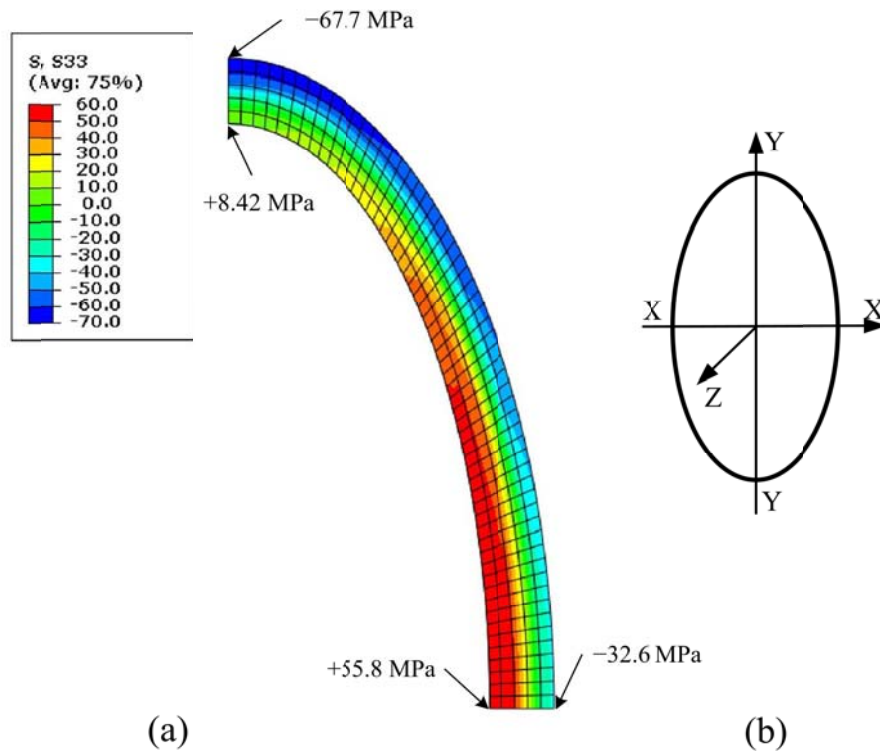


Fig. 5.4: a) Predicted longitudinal residual stresses in EHS – $200 \times 100 \times 10$ b) EHS axes

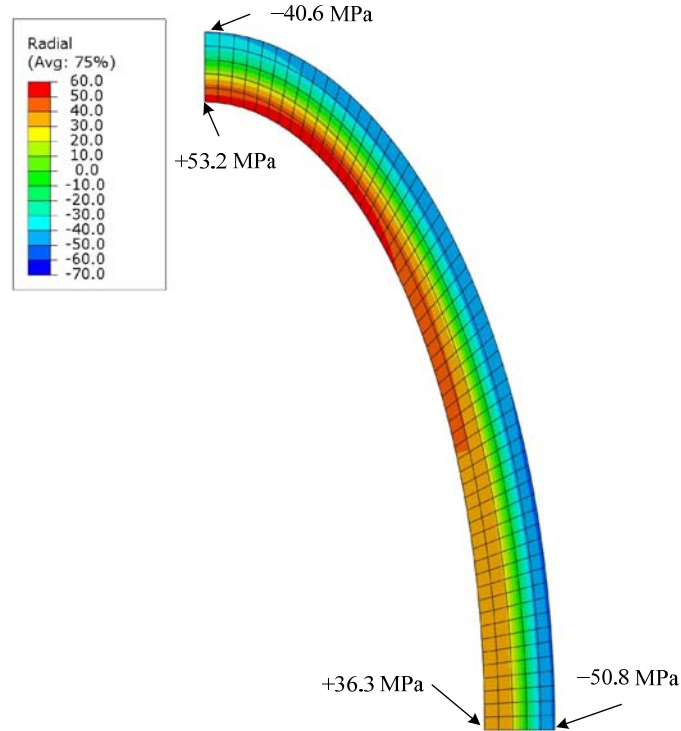


Fig. 5.5: Predicted transverse residual stresses in EHS – $200 \times 100 \times 10$

5.8.2. Effects of Geometric Parameters on Residual Stresses

In order to investigate the effect of EHS measurements on the residual stresses in the cross section, a thermo-mechanical analysis was conducted on the following EHS sections

- EHS – $500 \times 250 \times 16$, the largest and heaviest available section ($a/t = 15.1$)
- EHS – $500 \times 250 \times 10$, the thinnest section ($a/t = 24.5$)
- EHS – $300 \times 150 \times 16$, the thickest section ($a/t = 8.9$)
- EHS – $200 \times 100 \times 10$, a mid-range, thick section ($a/t = 9.5$)
- EHS – $200 \times 100 \times 5$, a mid-range, thin section ($a/t = 19.5$)
- EHS – $150 \times 75 \times 5$, a small section with some available experimental column loading results ($a/t = 14.5$)

The model parameters are identical to those for the EHS – $200 \times 100 \times 10$ section. The longitudinal residual stress vs. the normalized circumferential coordinate along the ellipse perimeter is provided in Fig. 5.6 for the inside and outside surfaces. The circumferential

coordinate is normalized with respect to $\frac{1}{4}$ of the perimeter s and takes the values 0, and 1 at the minor and major axes respectively. The longitudinal stresses are not normalized with respect to the yield stress, F_y , since, as will be discussed under Section 5.8.4. , it is observed that the value of the yield strength at room temperature has a minor effect on the residual stress. It is observed that for the currently available EHS geometries (i.e. $a / b \cong 2$) the residual stress pattern is weakly dependent on the size and thickness of the cross section. The residual stress pattern predicted for all cross sections examined is very similar on the inside and outside surfaces. The only exception is EHS-200×100×10 which exhibits a slightly different pattern on the inside surface near the y-axis.

The predicted residual stresses on the outside surface approximately range from -40 ± 5 MPa at the x-axis to about -70 ± 10 MPa at the y-axis. These respectively correspond to 11% and 20% of the yield stress of the steel when $F_y = 350$ MPa . On the inside surface, the residual stresses range from $+55 \pm 2$ MPa and $+35 \pm 5$ MPa representing 16% and 10% of $F_y = 350$ MPa , respectively.

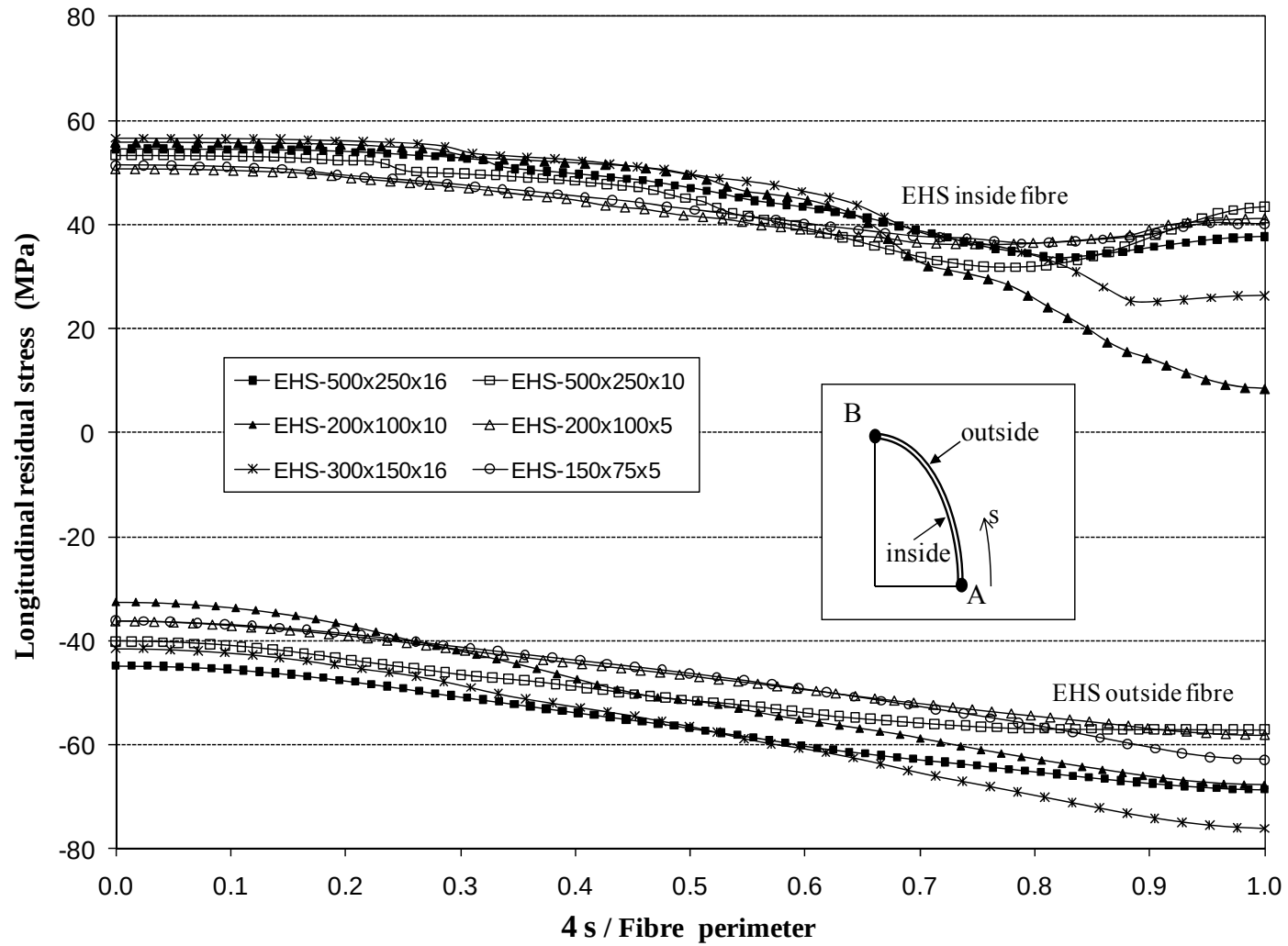


Fig. 5.6: Predicted longitudinal residual stresses vs. normalized tangential coordinate

5.8.3. Sensitivity Analysis

As discussed, the formation of residual stress depends on steel properties, shape of the cross section, rolling and cooling processes. Among these factors, the cooling process is quite arbitrary and depends on the manufacturer, its facilities and capabilities. Although there are restrictions involved in the cooling process to achieve the properties of the final product, each manufacturer uses his techniques to cool down hot-rolled sections. It is thus of interest to conduct a sensitivity analysis to assess effect of possible variations in each parameter on the residual stresses predicted.

A reference model is created by assuming all parameters at their most likely value given the information available. A series of analyses is then performed by changing one of the parameters to its high or low value while keeping all other parameters at their reference values. The reference value, as well as high and low values for each of the parameters considered are provided in Table 5.1.

Table 5.1: Parameter changes in the sensitivity analysis

Parameter	Init. Temp.	Spraying time	$\bar{h}_{out_W}$	$\bar{h}_{out_A}$	k_{e_out}
Unit	$^{\circ}C$	s	W/mK	W/mK	-
High value	1200	3.0	5000	60	0.9
Reference	1100	2.0	3500	40	0.8
Low value	1000	1.0	2000	20	0.7

In order to compare the effect of each parameter on the predicted residual stress, a sensitivity index, $S.I.$ is defined as

$$S.I. = \frac{\frac{1}{n} \sum_{i=1}^n |(RS)_i - (RS_R)_i|}{\frac{1}{P_R} (P - P_R)} \quad (5.3)$$

in which, n is the number of nodes on the outside and inside fibres. Symbols $(RS_R)_i$ and $(RS)_i$ denote the longitudinal residual stress at node i of the reference and the parameter

high/low value case, respectively. P is the value of the input parameter under investigation and P_R is the reference value of the same parameter. This index provides an indication on the sensitivity of the residual stresses predicted to the changes in the input parameter. A higher number indicates a more sensitive parameter. The sensitivity indices for the parameter are provided in Table 5.2. The effects of the first three parameter changes on the predicted residual stresses are illustrated in Fig. 5.7 through Fig. 5.9.

Table 5.2: Sensitivity index for thermo-mechanical parameters

Parameter	Initial Temp.	Water spraying time	Outside water heat transfer $\bar{h}_{out_W}$	Outside air heat transfer $\bar{h}_{out_A}$	Outside emissivity k_{e_out}
High value	4.59	0.55	0.49	0.03	0.06
Low value	2.35	0.48	0.54	0.03	0.06

It is clear that the assumed initial temperature of the hot-rolled section is key to the proper prediction of residual stresses. The duration of water spraying and its intensity have a moderate effect. Special care is required in estimating these values.

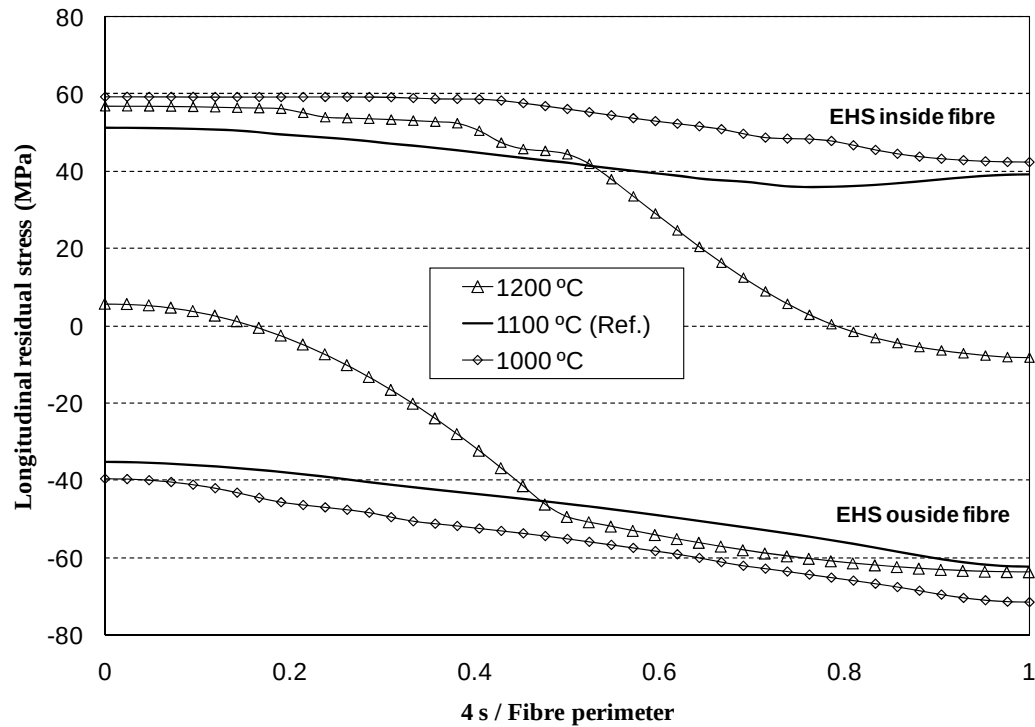


Fig. 5.7: Effect of initial temperature

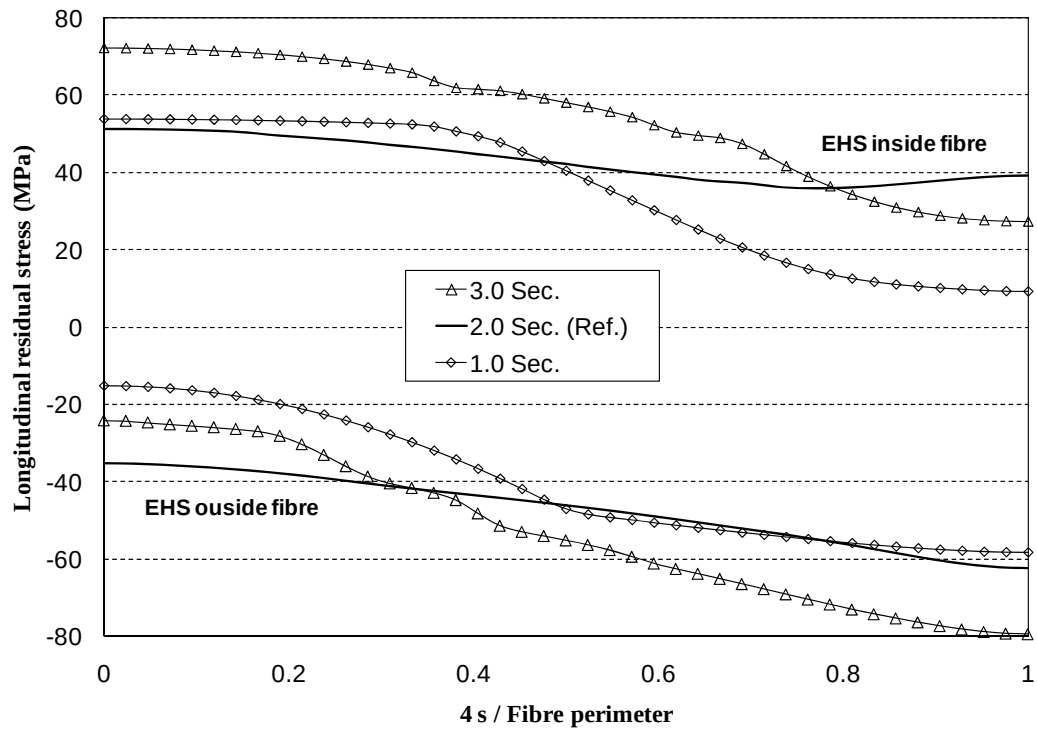


Fig. 5.8: Effect of water spraying time

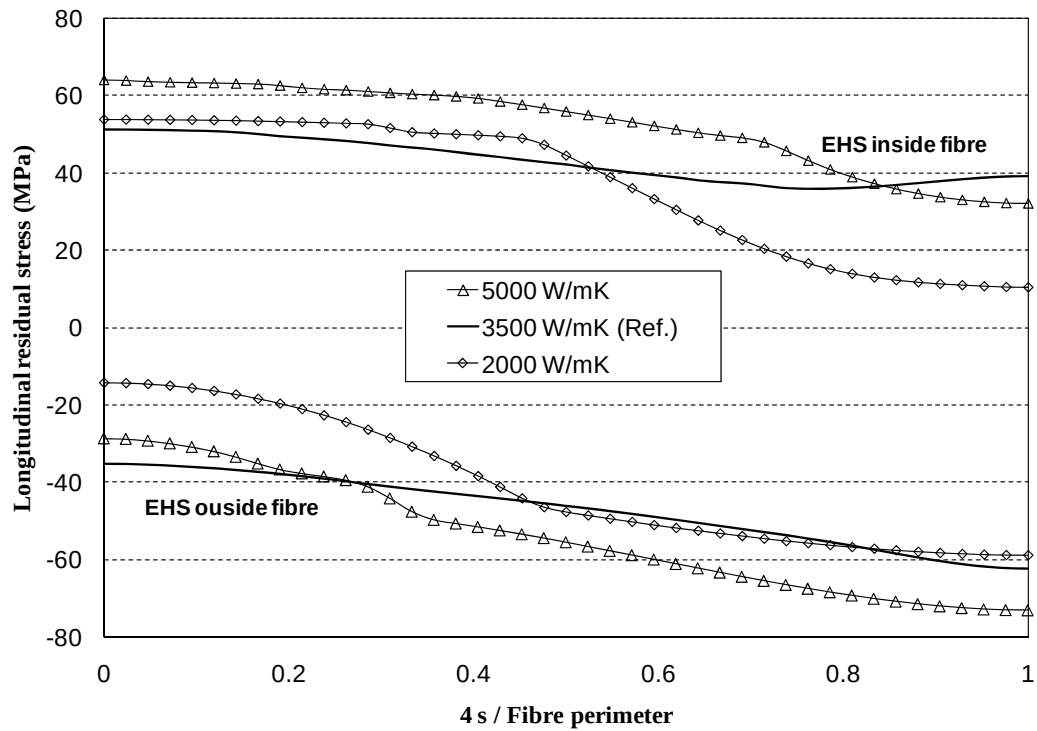


Fig. 5.9: Effect of average heat transfer coefficient during water spraying

5.8.4. Effect of Yield Stress on Residual Stresses

Three yield stress-temperature curves (Fig. 5.10) were investigated. The assumed yield strengths at zero temperature were 250, 350, and 450 MPa while those at 600°C were assumed to have 60 MPa for all steels. The three cases are representative of carbon-steel under various cooling regimes (e.g., Krauss 1990; Totten 2006). The residual stresses predicted based on these three models show no significant differences (i.e., $S.I. < 0.02$). It is concluded that steel strength at room temperature has a rather minor effect on residual stresses in elliptical hollow sections. This observation is in agreement with the experimental findings of Tall (1964) for hot rolled I-sections.

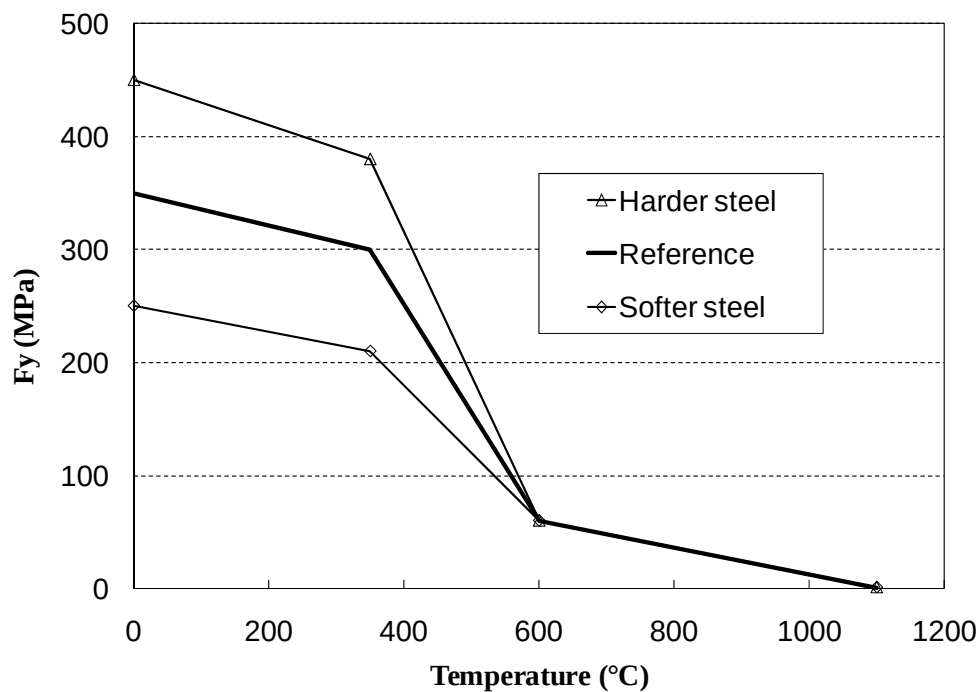


Fig. 5.10: Yield stress of three types of steel

5.9. Prediction of Initial Imperfection

In the absence of initial geometric imperfection measurements, their pattern is unknown. However, the out-of-straightness maximum amplitude can be selected to match the out-of-

straightness tolerance given in design standards. For instance, the Canadian Standards (CAN/CSA-S16 2001) specifies a threshold out-of-straightness of $1/1000$ of the member length (i.e., $L/1000$) but less than 10mm for W-sections used as column. No guidance is provided for columns of other cross-sections under the Canadian Standard.

Both in the Eurocode (EN 1993-1-1 2005) and the American Standards (AISC-360 2005), a fabrication imperfection tolerance of $1/1000$ of the member length is specified for all cross-sections. The Eurocode (EN 10210-2 2006), under which EHS are manufactured, allows an out of straightness of $L/500$ for cross-sections larger than 250mm. This limit is relaxed to $L/250$ for smaller sections. Corus (2005) and VMTubes (2008), the manufacturers of EHS, report maximum out-of-straightness for their products of $L/500$ regardless of section size. Therefore, for the current study the maximum out-of-straightness is assumed to be $L/500$.

Conceptually, a given geometric imperfection pattern can be decomposed into a linear combination of eigen-modes (as determined from a linearized eigenvalue buckling analysis). It is well accepted (e.g., Chajes 1974) that the first buckling mode has the most predominant contribution. As the structure deforms, the influence of higher modes tends to dampen out while that of the first mode tends to amplify. Thus, it is conservative to assume an initial geometric imperfection pattern according to the first buckling mode as determined from a linearized buckling eigen analysis with a maximum amplitude of $1/1000$ of the member length.

Conceptually, a given out-of-straightness imperfection pattern can be decomposed into a linear combination of eigen-modes (as determined from a linearized eigenvalue buckling analysis). It is well accepted (e.g., Chajes 1974) that the first buckling mode has the most predominant contribution. As the member deforms, the influence of higher modes tends to dampen out while that of the first mode tends to amplify. Thus, it is conservative to assume an initial out-of-straightness pattern according to the first buckling mode as determined from a linearized buckling eigen-analysis. This type of buckling analysis was conducted under ABAQUS FEA program. The analysis yields the buckling loads and corresponding mode shapes. The predicted mode shape corresponding to the lowest eigenvalue is then conservatively assumed to represents the out-of-straightness pattern within the member. This approach has been used by others (e.g., Bjorhovde 1972; Gardner and Ministro 2004)..

A pin-pin column with an EHS- $150 \times 75 \times 5$ cross-section is subjected to a pure axial force. The first and second buckling modes of the column are determined using the static perturbation buckling procedure in ABAQUS. As expected, the first mode corresponds to the flexural buckling mode about the y axis and the second one corresponds to the flexural buckling mode about the x-axis. The results are shown in Fig 5.11.

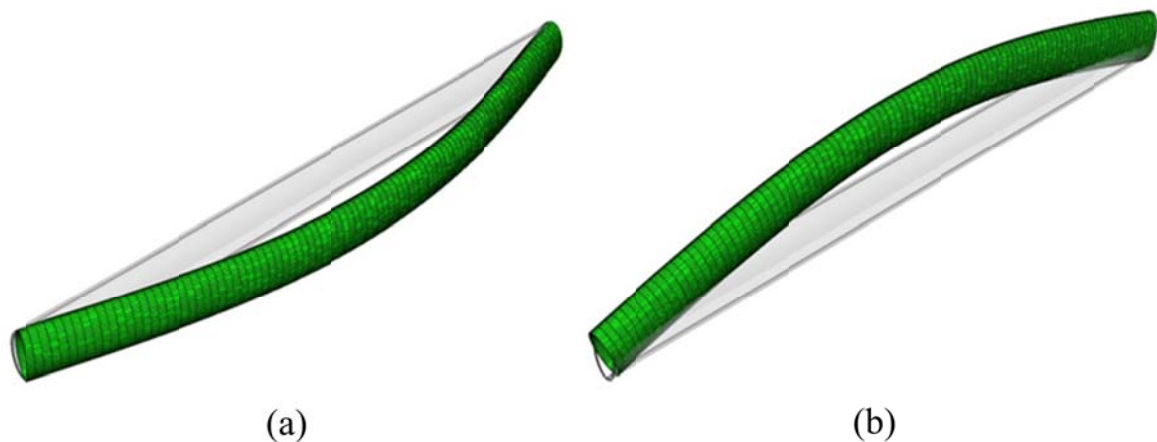


Fig 5.11: EHS elastic buckling about a) y-axis; b) x-axis

5.10. Column Curves

A series of finite element analyses are performed for EHS- $150 \times 75 \times 5$ columns with different lengths. Column curves were generated for flexural buckling. The analyses account for residual stresses, initial geometric imperfections, and material and geometric nonlinear effects. Three types of analyses are performed. These are:

1. A thermal stress-deformation analysis to predict residual stresses as described in section 5.8. ,
2. An elastic buckling analysis to predict the eigen modes as described in section 5.9. ,
3. An elasto-plastic buckling analysis including geometric nonlinearity.

The residual stresses predicted from the first analysis and out-of-straightness patterns matching the first buckling mode from the second analysis with amplitude of $L/500$ were input into the third analysis. A nonlinear incremental load deformation analysis was then conducted and the peak compressive load was obtained. The compressive resistance of the members is depicted as a function of slenderness in Fig. 5.12 and Fig. 5.13 for flexural

buckling about the x and y axes. On the figures, additional analysis runs for members with L/1000 out-of-straightness are provided for comparison. Also, superimposed on the plots are the experimental results from Chan and Gardner (2009) and column curves from Canadian (CAN/CSA-S16 2009), American (AISC-360 2005) and European (EN 1993-1-1 2005) steel codes of practice.

According to the Canadian standard (CAN/CSA-S16 2009) the nominal compressive resistance $C_n = C_r / \phi$ due to flexural buckling is

$$C_n = C_r / \phi = AF_y (1 + \lambda^{2n})^{-1/n}, \quad \lambda = \sqrt{F_y / F_e} \quad (5.4)$$

where $n = 2.24$ for hot-rolled hollow structural sections such as EHS and F_e is the Euler stress and defined as $F_e = \pi^2 E / (KL/r)^2$. In Eq. 5.4, C_r is the factored compressive resistance and ϕ is the resistance factor.

The American Standard (AISC-360 2005) gives the following equations as compressive strength for flexural buckling

$$\begin{aligned} \text{When } F_e \geq 0.44F_y \quad C_r &= \left(0.658^{\frac{F_y}{F_e}} \right) F_y A \\ \text{When } F_e < 0.44F_y \quad C_r &= 0.877F_e A \end{aligned} \quad (5.5)$$

The corresponding equations according to Eurocode (EN 1993-1-1 2005) is

$$C_n = AF_y \left(\frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} \right), \quad \Phi = 0.5(1 + \alpha(\lambda - 0.2) + \lambda^2) \quad (5.6)$$

in which the imperfection factor α is taken as 0.21 for square, rectangle, and circular hollow structural sections (i.e., curve 'a'). It is assumed that the same value is applicable for EHS. All the axial resistances have been normalized with respect to the axial yield load, $P_y = AF_y$ in the figures.

The FEA results obtained from ABAQUS were used to fit an equation similar to that of the Canadian standard column (i.e., Eq. 5.4). The ' n ' value to fit the data obtained from ABAQUS. The best fit was found to correspond $n = 1.57$ for buckling about y-axis and $n = 1.32$ for buckling about x-axis.

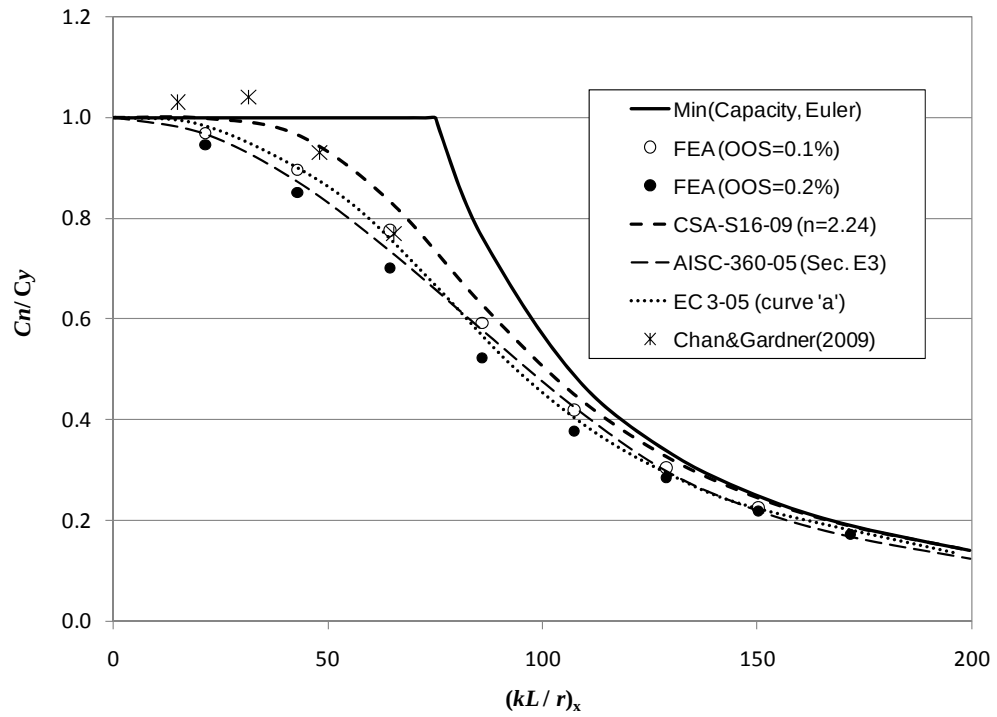


Fig. 5.12: Column curves for EHS buckling about x-axis

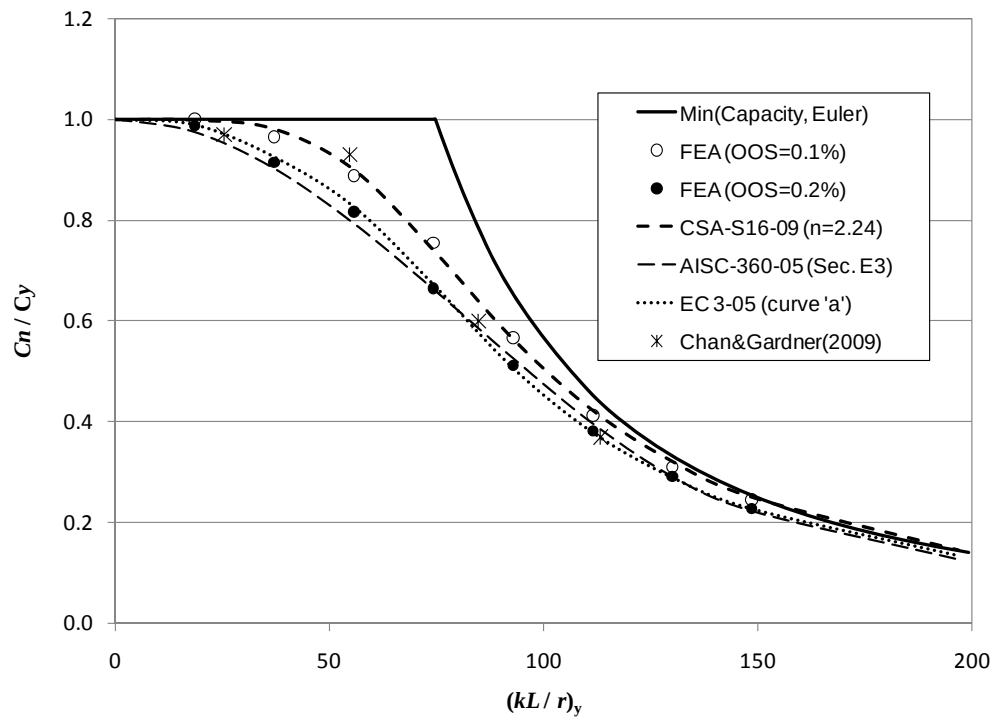


Fig. 5.13: Column curves for EHS buckling about y-axis

The proposed column curves are shown in Fig. 5.14. On this figure the Eurocode (EN 1993-1-1 2005) column buckling curves ‘a’ (i.e., $\alpha = 0.21$) and ‘b’ (i.e., $\alpha = 0.34$) are superimposed. It is apparent that Eurocode curve ‘a’ provides a very good approximation for column curve about the y-axis while curve ‘b’ provides a very good approximation for column buckling about the x-axis.

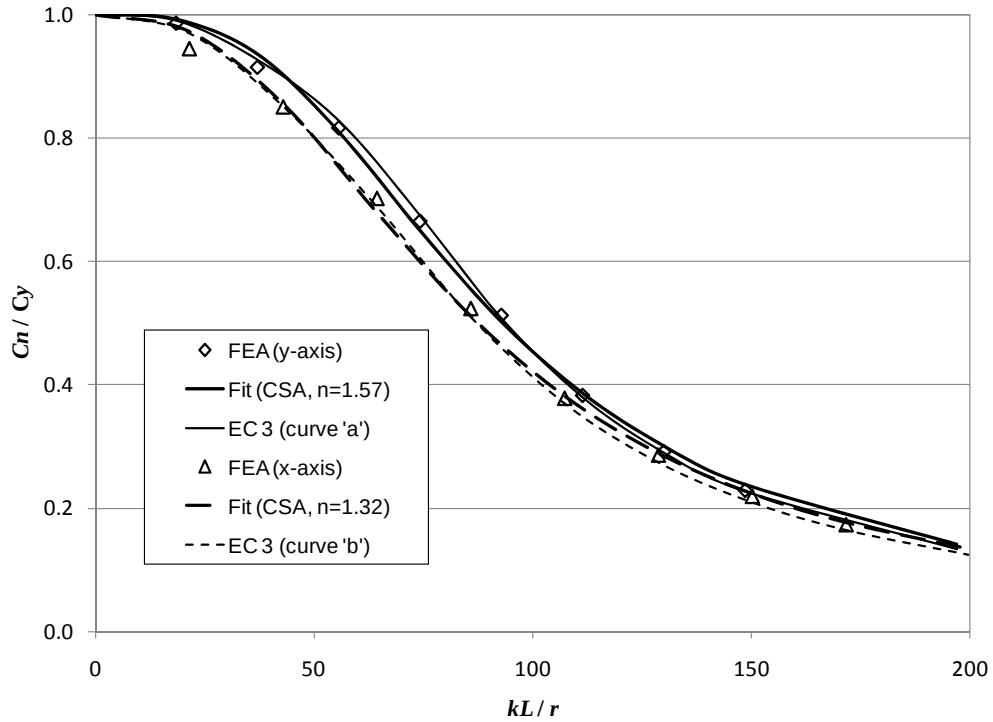


Fig. 5.14: Best representative column curves

An interesting characteristic of EHS column curves is the considerable difference between column curves for buckling about minor and major axes. This difference stems from the residual stress pattern in EHS. For instance for an EHS $150 \times 75 \times 5$, the residual stress at the short radius (i.e., at $s=0$ coordinate in Fig. 5.6) is about +55 MPa (tensile) on the inside fibre and -35 MPa (i.e., compressive) at the outside fibre. These stresses significantly differ from those at the long radius (i.e., at $4s/L=1.0$). They are +40 MPa at the inside fibre and -63 MPa outside fibre. As a result, the through-thickness averaged tensile residual stresses are predominantly tensile at short radius and predominantly compressive at the long radius.

Thus, they tend to reduce the flexural buckling strength about x-axis but increase the buckling strength about the y-axis.

5.11. Conclusions

The major conclusions of this study are:

1. In addition to longitudinal residual stresses in steel sections, the thermo-mechanical FEA developed in this study predicts the presence of transverse residual stresses with magnitudes comparable to those of longitudinal stresses. An experimental verification of this observation would be of interest.
2. The inside fibre of EHS is always subject to longitudinal tensile stresses and the outside fibre is subject to compressive longitudinal residual stresses.
3. The through-thickness average longitudinal residual stresses in EHS are tensile at the short radius and compressive at the long radius of the section.
4. For a given steel and manufacturing process, the residual stress pattern in currently available EHS is weakly dependent on the section geometric parameters and cross section thickness.

The predicted residual stresses are highly dependent upon the initial temperature and moderately dependent on water spraying intensity and duration. Special care is required to estimate these values for realistic predictions of residual stresses.

5. Steel yield strength at room temperature has a negligible influence on the residual stress patterns in EHS members.
6. Based on the current limit of out-of-straightness in the production of EHS, the majority of the steel code column curves overestimate the compressive capacity of EHS columns.
7. The Canadian standard column capacity relationship with $n=2.24$ over-estimates the buckling capacity of EHS columns. The curve with $n=1.34$ seems more appropriate for design.

8. Eurocode 3 buckling curve ‘b’ provides very good approximation for the EHS buckling about major axis. Also buckling curve ‘a’ provides a very close approximation for buckling about minor axis.
9. The column curves developed in the present study in conjunction with the plastic interaction relations developed in (Nowzartash and Mohareb 2009) can be combined to develop design interaction relations for EHS members which account for stability effects. This has been done in Appendix II.

5.12. Notations:

A	Cross sectional area
a	EHS centerline major half diameter
b	EHS centerline minor half diameter
$C_p = C_p(T)$	Specific heat capacity
C_r	Column axial compressive capacity
$E = E(T)$	Steel modulus of elasticity
F_y	Steel yield stress
$\bar{h}$	Averaged over temperature of convective heat transfer coefficient
$\bar{h}_{out_W}$	$\bar{h}$ on the outside surface of EHS during water cooling
$\bar{h}_{out_A}$	$\bar{h}$ on the outside surface of EHS during air cooling
$\bar{h}_{in}$	$\bar{h}$ on the inside surface of EHS
$k = k(T)$	Isotropic conductivity
k_B	Stephen-Boltzmann constant
k_e	Emissivity factor
k_{e_out}	Emissivity factor on the EHS outside surface
r_x, r_y	Major and minor radius of gyration
Z_x, Z_y	Major and minor plastic section modulus
s	Arc length coordinate axis along EHS perimeter
T	Temperature field
T_a	Ambient temperature
t	EHS thickness
$\tilde{t}$	Time
λ	Unitless column slenderness ratio
ρ	Specific mass

CHAPTER 6

Plastic Interaction Relations for Semi-Elliptical Hollow Sections⁴

6.1. Abstract

General plastic interaction relationships are developed for semi-elliptical hollow structural sections subjected to general combinations of normal force, twisting moments, and biaxial bending moments. The lower bound theorem of plasticity is employed to obtain the fully plastic resistance of the section in conjunction with the maximum distortional energy density criterion.

The developments are expressed as universal, non-dimensional relationships suitable for limit states design. An iterative scheme to solve the parametric form of the interaction relations is developed and a grid of admissible stress resultant combination is generated. A series of trial functions are fitted to the grid of internal force combinations and two simplified and conservative interaction equations are proposed. The interaction relations relate axial force, biaxial bending and twisting moment combinations for semi-elliptical hollow sections of common geometries. The applicability of the newly proposed interaction equations for conducting the cross-sectional interaction check is illustrated through practical examples.

Keywords:

Steel, Semi-Elliptical Hollow Section, Plasticity, Yield Surface

⁴ Nowzartash, F., and Mohareb, M. (2010). "Plastic Interaction Relations for Semi-Elliptical Hollow Sections." *Thin-Walled Structures*, 48, 42-54

6.2. Introduction and Scope

The family of structural hollow sections has substantially grown since it started with circular, square and rectangular hollow sections. Recently, elliptical hollow sections (EHS) were introduced. The newest member of this family, the semi-elliptical hollow sections (SEHS) is depicted in Fig. 6.1. Semi-elliptical sections are aesthetically pleasing and are gaining popularity among architects in exposed steel works. From a structural viewpoint, SEHS sections offer the structural advantages of closed sections (i.e., high resistance to torsion and lateral torsional buckling). Their rather close major and minor elastic and plastic moduli provide an efficient use of material when the bending moments about the major and minor axes have comparable magnitudes. As compression members, the SEHS are economically comparable to square hollow sections for members with nearly equal effective slenderness ratios about both principal axes. Despite the recent interest in adopting SEHS in structural designs, the lack of design procedures specifically geared towards SEHS design forces engineers either to a) conduct their design in an overly conservative manner or b) resort to complex and time consuming finite element solutions. Within this context, the present paper aims at developing interaction relations specific to SEHS subject to general loading combinations involving axial force, biaxial bending, and twisting moments. The interaction relations are applicable to sections able to attain their plastic resistance prior undergoing local buckling.



Fig. 6.1: Actual shape of SEHS

(Courtesy from Ancofer)

6.3. Literature Review

Early work on the development of interaction relations for pipes was provided in a summary by Gerard and Becker (1957) who focused on thin walled pipes which undergo the local buckling prior to attaining their plastic resistance. Chen and Atsuta (1972) formulated lower and upper bound plastic interaction relationships for tubular sections under biaxial bending moments and axial force. Chen and Atsuta (1972) and Hodge (1981) provide the principles for developing lower and upper bound plastic interaction relations. As recognized in Canadian structural steel standard (e.g., CAN/CSA-S16 2001) and steel pipeline standards (CAN/CSA-Z662 2007), stockier pipes are known to attain their fully plastic resistance prior the occurrence of local buckling. Such plastic interaction relations were developed (e.g., Hu et al. 1993; Mohareb and Murray 1999; Mohareb 2002; 2003) and experimentally verified (Ozkan and Mohareb 2003; 2009). The interaction relations formulated by Mohareb (Mohareb 2003) were utilized in developing a pipe finite element which efficiently modelled the elasto-plastic behaviour of pipelines (Nowzartash and Mohareb 2004).

Gaydon and Nuttall (1957) developed an upper bound interaction relation for rectangular hollow structural sections subjected to twisting and uniaxial bending moments. Lower bound plastic interaction relations subjected to combinations of biaxial bending, torsion, and axial force were established by Morris and Fenves (1969). Pillai and Ellis (1971) conducted an experimental study on rectangular hollow structural sections subjected to combined axial force and uniaxial bending moments and proposed a simplified interaction relationship based on their work. Interaction relationships for square hollow structural beam columns subjected to biaxial moments and axial forces were proposed by Pillai (1974) and later verified by an experimental investigation (Pillai and Kurian 1977). The effects of bi-axial shear and torsion were incorporated into the interaction relations by Mohareb and Ozkan (2004).

Recently, Chan and Gardner (2008a; b) developed analytical expressions for the yield compressive strength and plastic bending resistances of hot-rolled EHS and verified them against experimental results. The plastic interaction relations for EHS under combined loading of axial force, bi-axial bending moments and torsion were developed by

Nowzartash and Mohareb (2009). It is observed that all interaction relations reviewed focused on doubly symmetric sections. In contrast, the study reported here aims at developing plastic interaction relations for a mono-symmetric section.

6.4. Statement of Problem

Plastic interaction relations for SEHS subject to the combined action of biaxial bending, torsion and axial force are sought. It is assumed that the fully plastic resistance of the cross section will be attained. The fully plastic resistance is based on the maximum resistance that can be attained under an idealized bilinear elastic perfectly plastic stress-strain curve and is commonly referred to as the cross-sectional capacity in steel design codes. The formulation is based on the lower bound theorem of plasticity (e.g., Hodge 1981) in which stress distributions consistent with the material constitutive law are postulated and lower bound interaction relations are recovered. Such a lower bound solution underestimates the capacity of the cross-section and thus is suitable for cross-sectional checks conducted as part of the design process. A steel semi elliptical hollow section subject to axial force N (positive when tensile), bending moments M_x , M_y acting about x and y axes respectively and twisting moment T is considered (Fig. 6.2). The figure also shows the positive directions of M_x , M_y and T . It is required to determine whether the semi elliptical section is able to withstand the action of the internal forces while assuming the fully plastic resistance of the section is reached. For this purpose, it is required to derive a series of piecewise n interaction relationships of the form $f_i(N, M_x, M_y, T) = 0$; ($i = 1, \dots, n$) such that the conditions $f_i < 0$ for all $i = 1, \dots, n$ are met for any physically possible combination of internal forces. The condition $f_i \leq 0$, in which the equality holds true for at least one of the relations $i = 1, \dots, n$, corresponds to a fully plastic state of the cross section. The condition of $f_i > 0$ for any i corresponds to a loading combination that is unattainable under the assumptions of the formulation.

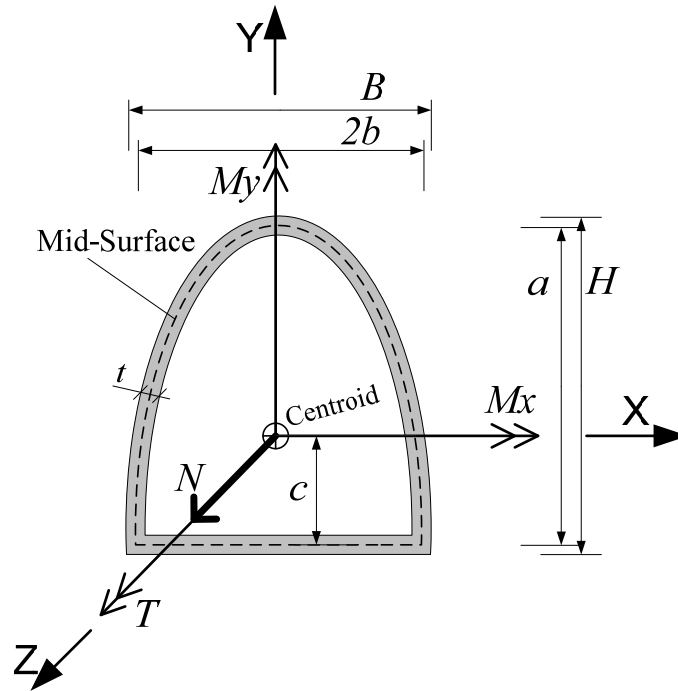


Fig. 6.2: Idealized SEHS and internal forces

6.5. Assumptions

6.5.1. Cross-Sectional Distortion

The cross section is assumed to remain undistorted under the action of internal forces induced. This assumption is justifiable for stockier SEHS, where the cross-sectional ovalization/distortion is negligible. Consequently, the fully plastic resistance can normally be attained, or nearly attained, before the occurrence of local buckling. Such sections are identified as Class 2 in the Eurocode 3 (EN 1993-1-1 2005) and the Canadian Standard (CAN/CSA-S16 2001). Currently, no criteria are available to specifically classify SEHS based on their cross-sectional dimensions. In the absence of such criteria, the designer may refer to the classification rules for elliptical hollow sections (EHS) (Gardner and Chan 2007) which account for the local buckling of the curved portion of SEHS and those reported in steel design Codes/Standard (CAN/CSA-S16 2001; AISC-360 2005; EN 1993-1-1 2005) which account for the local buckling of the flat portion of SEHS. A thorough

study for classification of SEHS is of practical interest but is outside the scope of the present work.

6.5.2. Idealized Stress-Strain Relationship

In this study, a bilinear elastic-perfectly plastic stress versus strain representation is adopted. The first line, representing the linear range of deformation, passes through the origin and has a slope identical to the initial slope of a tension coupon test. The second line is assumed to have zero slope and aligned with the yield plateau of the stress-strain relationship for a tension coupon test. The additional capacity of the steel material due to strain hardening is neglected, leading to a lower bound approximation for the section plastic resistance.

6.5.3. Yield Criterion

Semi-elliptical hollow section steel is assumed to yield in accordance with the maximum distortional energy density yield criterion (e.g., Boresi and Sidebottom 1985), i.e., a given point on an EHS (Fig. 6.3) will attain yielding when the following condition is met

$$\frac{1}{2} \left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{11} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2) \right] = F_y^2 \quad (6.1)$$

where $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}$ and σ_{23} are stress tensor components at the point considered and F_y is the yield strength of the SEHS material.

For an infinitesimal element of volume, the relevant stress components are the normal stress $\sigma = \sigma_{11}$ and the tangential shearing stress $\tau = \sigma_{12}$. Longitudinal stress σ is induced by the axial forces and/or bending moments. Shearing stress τ is caused by the Saint Venant torsion twisting moments. Normal stresses due to warping are neglected in the present formulation. This assumption is accurate where the section is free to warp and is expected to lead to a conservative lower bound interaction relation when section warping is restrained. As a simplification, the through thickness shear stresses σ_{13} and σ_{23} are assumed negligible. Also, the stress normal to the mid-surface σ_{33} and the circumferential stress σ_{22}

are assumed negligible. Based on these simplifications, the maximum distortional energy density yield criterion simplifies to $\sigma^2 + 3\tau^2 = F_y^2$ or

$$\sigma = \pm\sigma^* = \pm\sqrt{F_y^2 - 3\tau^2} \quad (6.2)$$

where σ^* is the maximum longitudinal stress that can be attained in the presence of a shear stress τ .

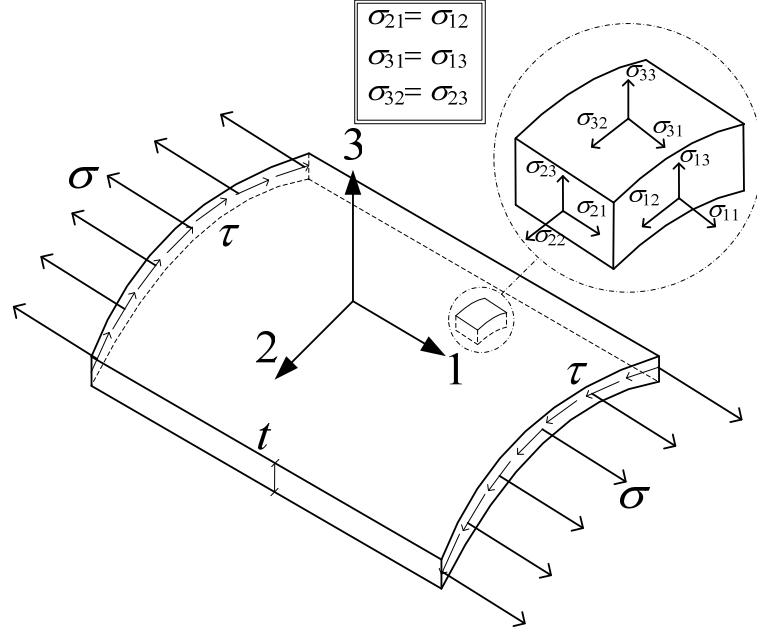


Fig. 6.3: Stress components acting on a SEHS element

6.5.4. Other Assumptions

No distinction is made between the true stress, based on the deformed infinitesimal element of area, and the engineering stress, based on the un-deformed infinitesimal element of area. Also, the logarithmic strain is assumed to be nearly equal to the engineering strain as the formulation is restricted to small strains. Residual stresses are not incorporated into the present study. The formulation does not include global buckling effects and thus is not intended to predict the resistance of slender members governed by the overall buckling mode of failure.

6.6. Formulation

Consider an SEHS with a mid-surface height a , mid-surface base length $2b$ and thickness t (Fig. 6.2). For a thin-walled closed section, the shearing stress τ can be assumed constant through the wall thickness t (Boresi and Sidebottom 1985), and is related to the torsion T through $T = 2A_s t \tau$, in which A_s is the area enclosed by the section mid-surface. In the case of SEHS, $A_s = \pi ab/2$. Thus the relationship between shearing stress τ and the applied twisting moment T , is

$$T = \pi ab t \tau \quad (6.3)$$

For a section to fully plastify under bending moments, a portion of the cross-section (A^+) has to attain the maximum tensile stress σ^* while the remaining portion of the cross-section (A^-) has to attain the maximum compressive stress $-\sigma^*$. This may happen in six different ways (i.e. cases I-1, I-2, II-1, II-2, III-1 and III-2) as depicted in Fig. 6.4. In the figure, the dotted lines along the mid-surface denote tensile stress σ^* and the solid line denote compression stress $-\sigma^*$.

In the following, we focus on the fundamental cases I-1, II-1 and III-1, i.e.,

- Case I-1: The plastic neutral axis (PNA) intercepts the curved portion of the section twice.
- Case II-1: The PNA intercepts the curved portion and the flat portion.
- Case III-1: The PNA intercepts the section flat portion twice.

Once the interaction relations have been developed for the above cases, those for the secondary cases I-2, II-2 and III-2 can be easily deducted by changing the signs of the stresses. The idealized locations of PNA on the section corresponding to the above cases are shown in Fig. 6.5. In the figure, the dashed line along the mid-surface denotes the tensile region and the solid line denotes the compressive region. The origin of the coordinate system is located at the section centroid.

Cases III-1 and III-2, provide an approximate solution for the various possible PNA locations depicted in Fig. 6.6. Since the formulation sought here is dealing with relatively thin sections, cases III-1 and III-2 yield very good approximation of the interaction surface. This approximation reduces the number of interaction equations associated with the

different scenarios for the PNA location illustrated in Fig. 6.6 while preserving the continuity of the interaction relations at the borderline of different cases. A similar approach was successfully adopted by Mohareb and Ozkan (2004) for square hollow structural sections and yielded a close approximation of the “exact” interaction equations.

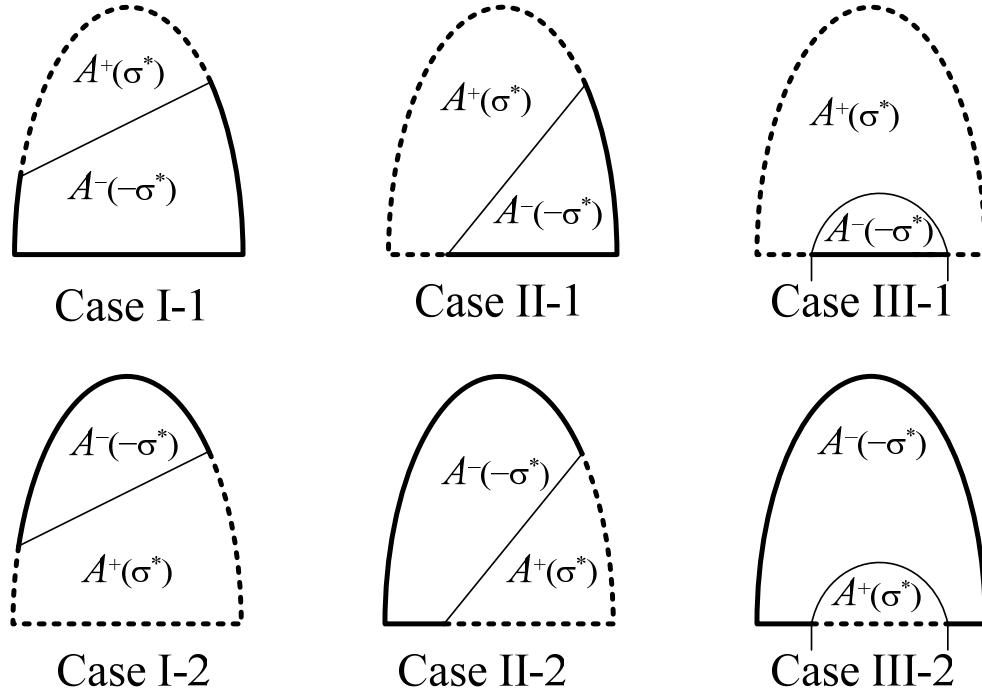


Fig. 6.4: Possible stress patterns in the idealized section

6.6.1. Case I-1: PNA intercepts the curved portion twice

For a section under the combined action of axial force N (tension is positive), biaxial bending moments M_x, M_y and twisting moments T , one has:

$$\begin{aligned}
 N = \int_A \sigma dA &= \int_{A^+} \sigma^+ dA + \int_{A^-} \sigma^- dA = \int_{A^+} \sigma^* dA - \int_{A-A^+} \sigma^* dA = 2\sigma^* \int_{A^+} dA - \sigma^* \int_A dA = \\
 &\sigma^* t \left[2 \int_{\beta_1}^{\beta_2} \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta - \int_0^\pi \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta - 2b \right]
 \end{aligned} \tag{6.4}$$

where the angles β_1 and β_2 (subtending an arc of the circumferential circle of radius a) are illustrated in Fig. 6.5. They are related to ϕ_1 and ϕ_2 (subtending an arc of the semi-elliptical contour) through the relation $\tan \beta_{1,2} = (b/a) \tan \phi_{1,2}$ (Fig. 6.7).

The bending moment about major axis M_x is obtained by

$$M_x = \int_A \sigma y dA = \int_{A^+} \sigma^+ y dA + \int_{A^-} \sigma^- y dA = 2\sigma^* \int_{A^+} y dA - \sigma^* \int_A y dA =$$

$$2\sigma^* t \int_{\beta_1}^{\beta_2} (a \sin \beta + c) \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta \quad (6.5)$$

in which c (Fig. 6.2) is the distance between the base mid-surface and the section centroid and is given as

$$c = - \frac{a \int_0^\pi \sin \beta \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta}{\int_0^\pi \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta + 2b} \quad (6.6)$$

In Eq. 2.5, the identity $\int_A y dA = 0$ was used. In a similar manner, the bending moment about minor axis, M_y is given as

$$M_y = - \int_A \sigma x dA = - \left(\int_{A^+} \sigma^+ x dA + \int_{A^-} \sigma^- x dA \right) = -2\sigma^* \int_{A^+} x dA + \sigma^* \int_A x dA =$$

$$-2\sigma^* t \int_{\beta_1}^{\beta_2} (b \cos \beta) \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta \quad (6.7)$$

In Eq. 2.6, the identity $\int_A x dA = 0$ was used. In Eqs. 2.4, 2.5 and 2.6, angles β_1 and β_2 are bound by the relation $0 \leq \beta_1 \leq \beta_2 \leq \pi$.

6.6.2. Case II-1: PNA intercepts the curved and flat portions

For Case II-1, the axial force N is

$$N = \int_A \sigma dA = \int_{A^+} \sigma^+ dA + \int_{A^-} \sigma^- dA = 2\sigma^* \int_{A^+} dA - \sigma^* \int_A dA =$$

$$\sigma^* t \left[2 \int_{\beta_1}^{\beta_2} \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta + 2(b + \xi b) \right] \quad (6.8)$$

The bending moment about major axis, M_x , is

$$\begin{aligned}
M_x &= \int_A \sigma y dA = \int_{A^+} \sigma^+ y dA + \int_{A^-} \sigma^- y dA = 2\sigma^* \int_{A^+} y dA = \\
&2\sigma^* t \left[\int_{\beta}^{\pi} (a \sin \beta + c) \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta + c(b + \xi b) \right]
\end{aligned} \tag{6.9}$$

and the bending moment about minor axis, M_y , is

$$\begin{aligned}
M_y &= -\int_A \sigma x dA = -\int_{A^+} \sigma^+ x dA - \int_{A^-} \sigma^- x dA = -2\sigma^* \int_{A^+} x dA = \\
&2\sigma^* t \left[-\int_{\beta}^{\pi} (b \cos \beta) \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta + 0.5(b + \xi b)(b - \xi b) \right]
\end{aligned} \tag{6.10}$$

In Eqs. 6.8 to 6.10, angle β and parameter ξ are respectively bound by $0 \leq \beta \leq \pi$ and $-1 \leq \xi \leq 1$.

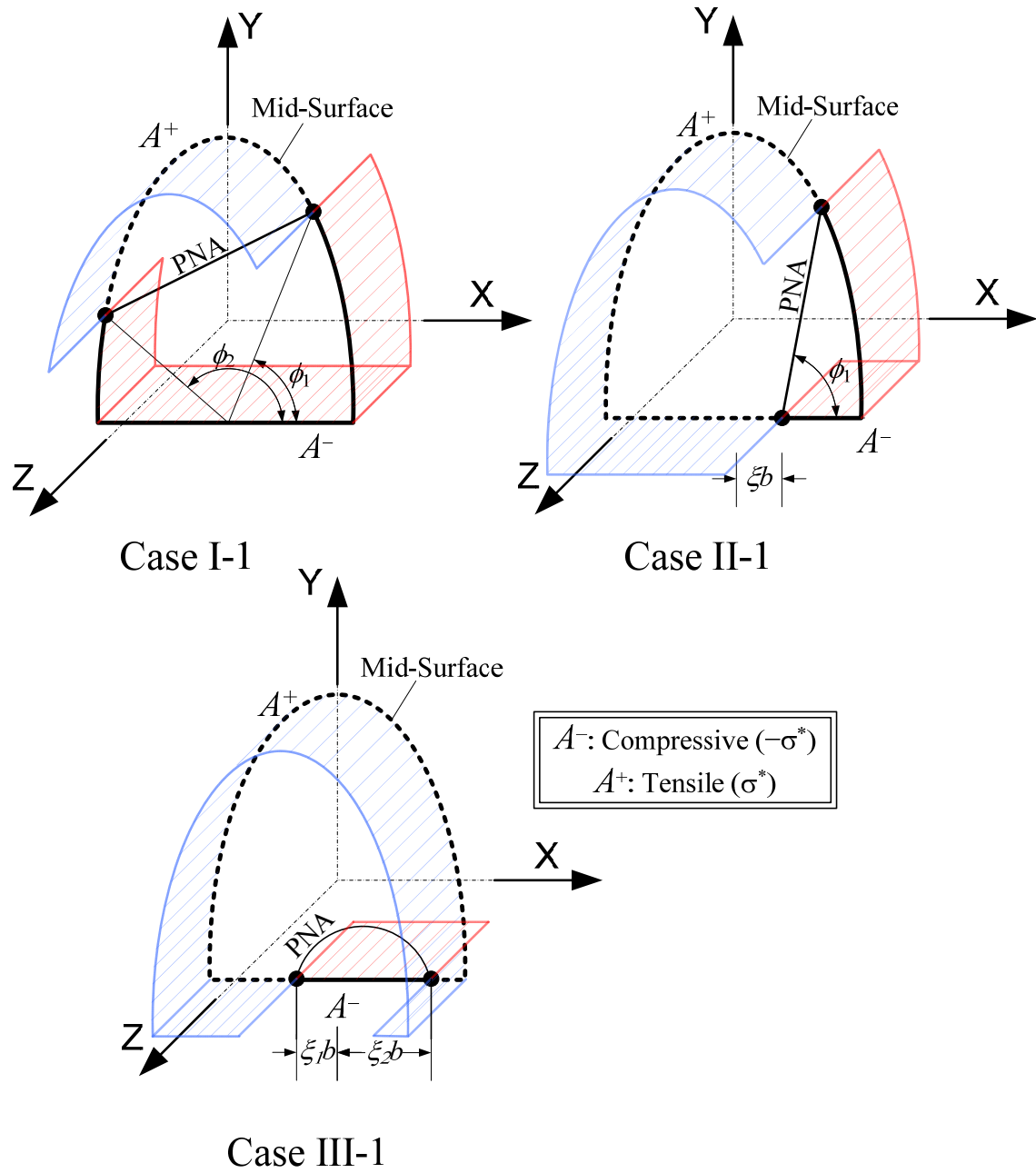


Fig. 6.5: Stress Distributions

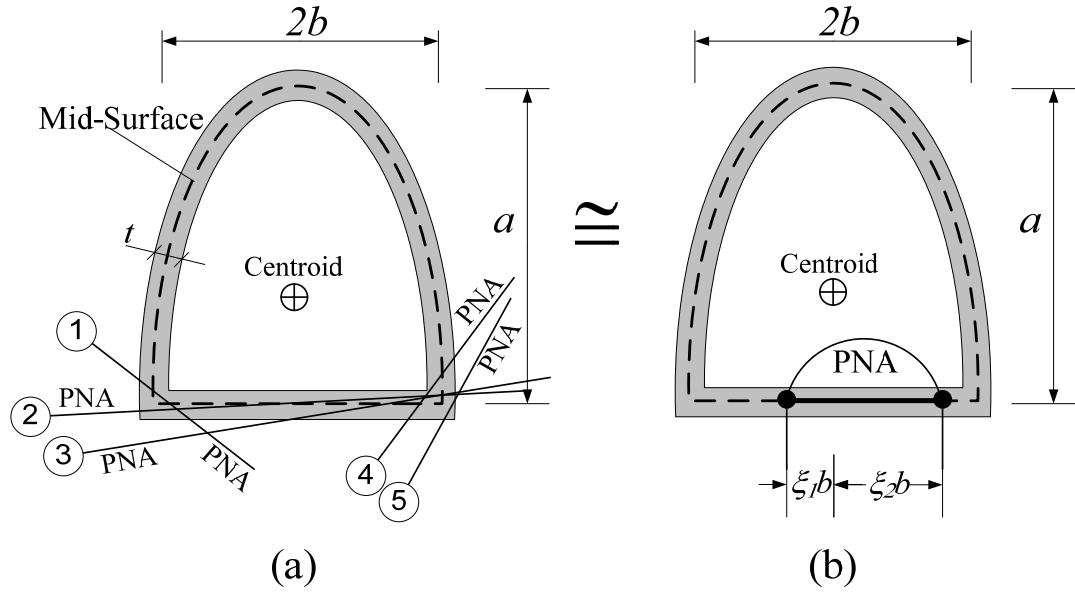


Fig. 6.6: Actual (a) and approximate (b) locations of Plastic Neutral Axis in case III-1

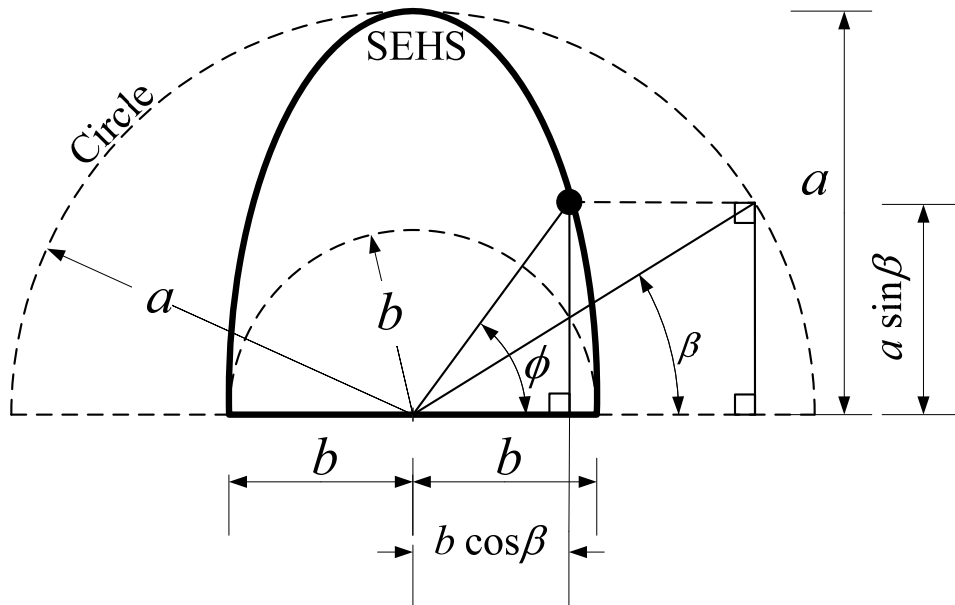


Fig. 6.7: Relation between ϕ and β in SEHS

6.6.3. Case III-1: PNA intercepts the flat portion twice

For Case III-1, the axial force N is

$$\begin{aligned} N &= \int_A \sigma dA = \sigma^* \int_{2A^+} dA - \sigma^* \int_A dA = \\ &= \sigma^* t \left[\int_0^\pi \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} d\beta + 2b(1 + \xi_1 - \xi_2) \right] \end{aligned} \quad (6.11)$$

The bending moment about major axis, M_x , is

$$M_x = \int_A \sigma y dA = \int_{A^+} \sigma^+ y dA + \int_{A^-} \sigma^- y dA = -2\sigma^* \int_{A^-} y dA = -2\sigma^* t (\xi_2 b - \xi_1 b) c \quad (6.12)$$

and the bending moment about minor axis, M_y is

$$M_y = -\int_A \sigma x dA = -\int_{A^+} \sigma^+ x dA - \int_{A^-} \sigma^- x dA = -2\sigma^* \int_{A^-} x dA = -\sigma^* t (\xi_2 b - \xi_1 b) (\xi_2 b + \xi_1 b) \quad (6.13)$$

In Eqs. 6.11 to 6.13 parameters ξ_1 and ξ_2 are bounded by $-1 \leq \xi_1 \leq \xi_2 \leq 1$.

6.6.4. Expressions for Plastic Resistances

It is desirable to normalize the interaction relations sought to make them universally applicable to any EHS geometry. Towards this goal, the limiting resistance value for each of the internal forces in the absence of all other internal forces is formulated. These limiting resistances are special cases of Eqs. 2.3, 2.4, 2.6 and 6.9. From Eq. 2.3, knowing that the maximum shear stress occurs when the longitudinal stress vanishes, one can express the plastic torsional capacity of the cross section, T_p as

$$T_p = \pi a b t \frac{F_y}{\sqrt{3}} \quad (6.14)$$

The limiting tensile axial force, N_y is attained when the twisting moments vanishes (i.e., $\tau = 0$ and $\sigma^* = F_y$) and when no bending moments are acting on the cross section. This is possibly obtained by setting $\xi_1 = 0$ and $\xi_2 = 0$ in Eq. 6.11, yielding

$$N_y = 2F_y t a \left[\int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + (b/a) \right] \quad (6.15)$$

The plastic resisting moment about the major axis, M_{xp} , occurs in the absence of torsion, minor axis bending and axial force. Under this scenario, the plastic neutral axis is the horizontal line which subdivides the cross-section into two equal parts, i.e.,

$$\int_{\beta_p}^{\pi-\beta_p} \sqrt{a^2 \sin^2 \beta + b^2 \cos^2 \beta} \, d\beta = \frac{1}{2} \left(\int_0^\pi \sqrt{a^2 \sin^2 \beta + b^2 \cos^2 \beta} \, d\beta + 2bt \right) \quad (6.16)$$

Using the symmetry of the section with respect to the y axis, Eq. 6.16 may be simplified as

$$2 \int_0^{\beta_p} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta - \int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta + (b/a) = 0 \quad (6.17)$$

Equation 6.17 is solved for β_p and substituted into Eq. 2.5 yielding the plastic resistance moment

$$M_{xp} = 2F_y t a^2 \int_{\beta_p}^{\pi-\beta_p} (\sin \beta + c/a) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta \quad (6.18)$$

For the plastic resisting moment about the minor axis M_{yp} , $\beta = \pi/2$ and $\xi = 0$ are substituted into 6.10 to yield

$$M_{yp} = 2F_y t a^2 (b/a) \left[\int_0^{\pi/2} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta + 0.5(b/a) \right] \quad (6.19)$$

For case I-1, Eqs. 2.3, 2.4, 2.5 and 2.6 can be respectively divided by Eqs. 2.7, 2.8, 6.18 and 6.19 resulting in a set of non-dimensional equations. The equations recovered are valid for all semi-elliptical sections irrespective of their geometry.

$$T_r = \frac{T}{T_p} = \frac{\sqrt{3}\tau}{F_y} = \tau_r \quad (6.20)$$

in which τ_r is the ratio of shear stress τ to the yield shear stress $F_y/\sqrt{3}$.

$$N_r = \frac{N}{N_y} = \sqrt{1 - \tau_r^2} \left(\frac{\int_{\beta_1}^{\beta_2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta}{\int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta + (b/a)} - 1 \right) \quad (6.21)$$

$$M_{rx} = \frac{M_x}{M_{xp}} = \sqrt{1 - \tau_r^2} \frac{\int_{\beta_1}^{\beta_2} (\sin \beta + c/a) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta}{\int_{\beta_p}^{\pi-\beta_p} (\sin \beta + c/a) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \, d\beta} \quad (6.22)$$

$$M_{ry} = \frac{M_y}{M_{yp}} = \sqrt{1 - \tau_r^2} \frac{-\int_{\beta_1}^{\beta_2} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta}{\int_0^{\pi/2} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + 0.5(b/a)} \quad (6.23)$$

In Eqs. 6.20 through 6.23, T_r , N_r , M_{rx} and M_{ry} respectively are the ratios of torsion, axial force, bending moment about x-axis and bending moment about y-axis relative to their respective plastic resistances (in the absence of other internal forces). They constitute the parametric equations of the interaction relation sought for case I-1.

For cases II-1 and III-1, Eq. 6.20 remains valid. Equations 6.8, 6.9 and 6.10 are respectively divided by Eqs. 2.8, 6.18 and 6.19 resulting in the following set of non-dimensional relations

$$N_r = \frac{N}{N_y} = \sqrt{1 - \tau_r^2} \left(\frac{\int_{\beta}^{\pi} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + (1 + \xi)(b/a)}{\int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + (b/a)} - 1 \right) \quad (6.24)$$

$$M_{rx} = \frac{M_x}{M_{xp}} = \sqrt{1 - \tau_r^2} \frac{\int_{\beta}^{\pi} (\sin \beta + c/a) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + (1 + \xi)(b/a)(c/a)}{\int_{\beta_p}^{\pi - \beta_p} (\sin \beta + c/a) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta} \quad (6.25)$$

$$M_{ry} = \frac{M_y}{M_{yp}} = \sqrt{1 - \tau_r^2} \frac{-\int_{\beta}^{\pi} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + 0.5(1 - \xi^2)(b/a)}{\int_0^{\pi/2} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + 0.5(b/a)} \quad (6.26)$$

Also, Eqs. 6.11, 6.12 and 6.13 are respectively divided by Eqs. 2.8, 6.18 and 6.19 yielding

$$N_r = \frac{N}{N_y} = \sqrt{1 - \tau_r^2} \left(1 - \frac{(b/a)(\xi_2 - \xi_1)}{\int_0^{\pi/2} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + (b/a)} \right) \quad (6.27)$$

$$M_{rx} = \frac{M_x}{M_{xp}} = \sqrt{1 - \tau_r^2} \frac{(b/a)(c/a)(\xi_1 - \xi_2)}{\int_{\beta_p}^{\pi - \beta_p} (\sin \beta + c/a) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta} \quad (6.28)$$

$$M_{ry} = \frac{M_y}{M_{yp}} = \sqrt{1 - \tau_r^2} \frac{0.5(\xi_1^2 - \xi_2^2)(b/a)}{\int_0^{\pi} \cos \beta \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta + (b/a)} \quad (6.29)$$

Conceptually, in order to recover an explicit form of the interaction relation, one needs to eliminate constants τ_r , β_1 and β_2 from Eqs. 6.20 through 6.23; constants τ_r , β and ξ

from Eqs. 6.20 and 6.24 to 6.26 or constants τ_r , ξ_1 and ξ_2 from Eqs. 6.20 and 6.27 to 6.29. It is noted however, that the integrals for cases I-1 and II-1 cannot be explicitly evaluated. Thus, an exact closed form expression for the sought interaction relation is unattainable. For a given section, knowing any three of the four internal forces applied N , M_x and M_y , and T , the relevant four relations in Eqs. 6.20 to 6.29 will provide $(\tau_r, \beta_1, \beta_2)$, (τ_r, β, ξ) or (τ_r, ξ_1, ξ_2) as well as the internal force sought.

6.7. Admissible Internal Force Combinations

In order to determine whether a given internal force combination (N, M_x, M_y, T) is admissible, one could multiply the load combination $(N_r, M_{rx}, M_{ry}, T_r)$ by an unknown scalar α_i and solve the parametric form of the interaction relation $f_i(\alpha_i N_r, \alpha_i M_{rx}, \alpha_i M_{ry}, \alpha_i T_r) = 0$, $i = 1, 2, 3$ for constant α_i . It is noted here that, conceptually, $f_1 = 0$ denotes the interaction equation based on the parametric Eq. 6.20 to 6.23 which are valid for Case I. Also, $f_2 = 0$ denotes the interaction equation based on the parametric Eqs 6.20, 6.24 to 6.26 which are valid for Case II and $f_3 = 0$ denotes the interaction equation based on the parametric Eqs. 6.20, 6.27 to 6.29 which are valid for Case III. We recall that no closed form expressions for functions f_i are attainable and thus the parametric form of equations $f_i(\alpha_i N_r, \alpha_i M_{rx}, \alpha_i M_{ry}, \alpha_i T_r) = 0$, $i = 1, 2, 3$ (Eqs. 6.20 to 6.29) must be iteratively solved.

The value of α_i is an indicator of the proximity of the internal force combination $(N_r, M_{rx}, M_{ry}, T_r)$ from the piecewise definition f_i of the interaction surface. When the condition $|\alpha_i| = 1$ is met, the load combination is known to lie on the segment of interaction surface described by $f_i = 0$. When $|\alpha_i| < 1$ for any i , the combination needs to be scaled down in order to lie on the segment of interaction surface described by $f_i = 0$, i.e., the given load combination is unattainable. Conversely, when $|\alpha_i| > 1$, the load combination needs to be magnified in order to lie on the piecewise definition of the interaction surface.

Only when the condition $f_i = 0$ is met for all $i = 1, 2, 3$, the load combination is admissible. It is noted that when solving $f_i = 0$, one obtains a positive root and another negative root for α_i . One of the roots would correspond to the fundamental case (such as Case I-1) and the other one corresponds to the secondary case (such as Case I-2).

For a given internal force combination $(N_r, M_{rx}, M_{ry}, T_r)$, the location of the PNA axis is not known apriori. Thus, one has no means of determining which set of interaction equations will yield the smallest scaling factor α_i . Thus, one must solve all three interaction relations $f_1 = 0$, $f_2 = 0$, and $f_3 = 0$ for the scaling factors α_1, α_2 , and α_3 . The scaling factor with the smallest absolute value is the governing one (i.e., $|\alpha| = \min(|\alpha_1|, |\alpha_2|, |\alpha_3|)$) and will correspond to the relevant case. i.e., will determine whether the PNA location is according to Case I-1, II-1, or III-1 if α is positive, or whether PNA location is according to Case I-2, II-2, or III-2 if α is negative.

6.8. Yield Surface for SEHS of Common Geometries

Currently manufactured semi elliptical hollow sections (Corus 2005; ANCOFER 2008) have a height H to width B ratio of about 0.90. The height and width are measured from outer surface of the cross-section. The smallest available section is SEHS-203 $\times$ 223 $\times$ 5 and the largest one is SEHS-324 $\times$ 375 $\times$ 14.2, where the first two numbers indicate the height H and width B respectively and the third number is the thickness t , all in millimetres. For the range of available profiles, the value $b/a = 0.5(B-t)/(H-t)$ varies from 0.55 to 0.58. In order to err on the conservative side, the results reported in the subsequent developments are based on $b/a = 0.55$. By setting $b/a = 0.55$ in Eqs. 2.8, 6.18 and 6.19, and using Simpson's rule for numerical integration, one recovers the following equations

$$N_y = 2F_y t a \left[\int_0^{\pi/2} \sqrt{(0.55)^2 \sin^2 \beta + \cos^2 \beta} d\beta + 0.55 \right] = 1.7932 (2F_y t a) \quad (6.30)$$

$$M_{xp} = 2F_y t a^2 \int_{\beta_p}^{\pi-\beta_p} (\sin \beta - c/a) \sqrt{(0.55)^2 \sin^2 \beta + \cos^2 \beta} d\beta = 0.5984 (2F_y t a^2) \quad (6.31)$$

$$\begin{aligned}
M_{yp} &= 2F_y t a^2 (b/a) \left[\int_0^{\pi/2} \cos \beta \sqrt{(0.55)^2 \sin^2 \beta + \cos^2 \beta} d\beta + 0.5(0.55) \right] \\
&= 0.6280 (2F_y t a^2)
\end{aligned} \tag{6.32}$$

By assuming $b/a = 0.55$, using Eqs. 6.20 through 6.29 in conjunction with Eq. 6.30 to 6.32 the limiting interaction coefficients for SEHS can be computed (Table 6.1 and Table 6.2). These are also depicted in Fig. 6.8 to Fig. 6.10. In the figures and tables, the normalized internal forces modified for torsion N_r^* , M_{rx}^* and M_{ry}^* are defined as

$$N_r^* = N_r / \sqrt{1 - T_r^2} \tag{6.33}$$

$$M_{rx}^* = M_{rx} / \sqrt{1 - T_r^2} \tag{6.34}$$

$$M_{ry}^* = M_{ry} / \sqrt{1 - T_r^2} \tag{6.35}$$

Through the introduction of the normalized internal forces defined in Eqs. 6.33 to 6.35, the number of parameters in the interaction relations is reduced from four to three.

Table 6.1: Interaction surface for SEHS with $b/a = 0.55$ ($M_{rx}^* \geq 0$, $M_{ry}^* \geq 0$ and $N_r^* \geq 0$)*

N_r^*		M_{rx}^*																				
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
M_{ry}^*	0.00	1.000	0.962	0.918	0.877	0.837	0.796	0.755	0.714	0.673	0.632	0.591	0.551	0.510	0.470	0.428	0.385	0.340	0.288	0.226	0.148	-
	0.05	0.967	0.950	0.920	0.877	0.837	0.796	0.755	0.714	0.673	0.632	0.591	0.551	0.510	0.469	0.428	0.385	0.338	0.287	0.225	0.144	-
	0.10	0.930	0.936	0.918	0.878	0.837	0.796	0.755	0.714	0.673	0.632	0.592	0.551	0.510	0.468	0.426	0.382	0.335	0.282	0.220	0.133	-
	0.15	0.894	0.900	0.900	0.878	0.837	0.796	0.755	0.714	0.673	0.632	0.592	0.551	0.509	0.467	0.423	0.377	0.328	0.274	0.209	0.121	-
	0.20	0.858	0.865	0.870	0.865	0.835	0.796	0.755	0.714	0.673	0.632	0.591	0.550	0.507	0.464	0.418	0.370	0.320	0.263	0.196	0.094	-
	0.25	0.823	0.830	0.835	0.836	0.822	0.792	0.753	0.713	0.672	0.631	0.589	0.547	0.503	0.458	0.411	0.361	0.308	0.246	0.173	0.052	-
	0.30	0.787	0.794	0.800	0.803	0.797	0.778	0.745	0.708	0.668	0.627	0.585	0.541	0.497	0.450	0.401	0.349	0.292	0.227	0.143	-	-
	0.35	0.751	0.758	0.764	0.768	0.767	0.753	0.729	0.696	0.659	0.619	0.577	0.533	0.487	0.439	0.388	0.333	0.271	0.199	0.097	-	-
	0.40	0.714	0.722	0.728	0.733	0.733	0.724	0.706	0.678	0.644	0.605	0.564	0.520	0.473	0.423	0.370	0.312	0.244	0.162	-	-	-
	0.45	0.677	0.685	0.692	0.696	0.697	0.691	0.677	0.653	0.623	0.586	0.546	0.502	0.455	0.404	0.348	0.286	0.209	0.107	-	-	-
	0.50	0.639	0.648	0.655	0.659	0.661	0.656	0.644	0.624	0.596	0.562	0.523	0.480	0.431	0.378	0.320	0.250	0.157	-	-	-	-
	0.55	0.601	0.610	0.617	0.622	0.623	0.618	0.608	0.590	0.565	0.533	0.495	0.451	0.402	0.346	0.283	0.202	0.055	-	-	-	-
	0.60	0.563	0.571	0.578	0.583	0.583	0.579	0.569	0.552	0.528	0.498	0.460	0.416	0.366	0.306	0.231	0.083	-	-	-	-	-
	0.65	0.523	0.531	0.538	0.542	0.542	0.537	0.527	0.511	0.488	0.458	0.420	0.374	0.320	0.252	0.148	-	-	-	-	-	-
	0.70	0.481	0.490	0.496	0.498	0.498	0.492	0.482	0.465	0.442	0.411	0.372	0.323	0.261	0.154	-	-	-	-	-	-	-
	0.75	0.437	0.445	0.450	0.452	0.450	0.444	0.432	0.414	0.389	0.356	0.316	0.260	0.163	-	-	-	-	-	-	-	-
	0.80	0.389	0.396	0.400	0.401	0.398	0.390	0.377	0.356	0.329	0.291	0.234	0.121	-	-	-	-	-	-	-	-	-
	0.85	0.336	0.342	0.345	0.344	0.339	0.329	0.311	0.289	0.251	0.195	-	-	-	-	-	-	-	-	-	-	-
	0.90	0.274	0.278	0.280	0.277	0.270	0.252	0.228	0.191	0.054	-	-	-	-	-	-	-	-	-	-	-	-
	0.95	0.191	0.196	0.194	0.185	0.166	0.133	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

*) May also be used when

$$M_{rx}^* \geq 0, M_{ry}^* \leq 0, N_r^* \geq 0$$

$$M_{rx}^* \leq 0, M_{ry}^* \geq 0, N_r^* \leq 0$$

$$M_{rx}^* \leq 0, M_{ry}^* \leq 0, N_r^* \leq 0$$

Table 6.2: Interaction surface for SEHS with $b/a = 0.55$ ($M_{rx}^* \leq 0$, $M_{ry}^* \geq 0$ and $N_r^* \geq 0$)*

N_r^*		$-M_{rx}^*$																				
		0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
M_{ry}^*	0.00	1.000	0.973	0.945	0.917	0.888	0.860	0.831	0.801	0.770	0.738	0.705	0.671	0.635	0.597	0.557	0.513	0.466	0.412	0.349	0.268	-
	0.05	0.967	0.952	0.931	0.907	0.881	0.854	0.825	0.796	0.766	0.735	0.702	0.668	0.633	0.595	0.555	0.511	0.463	0.409	0.346	0.267	-
	0.10	0.930	0.919	0.903	0.884	0.861	0.837	0.811	0.784	0.755	0.724	0.693	0.659	0.624	0.587	0.547	0.503	0.455	0.401	0.337	0.255	-
	0.15	0.894	0.884	0.871	0.854	0.835	0.813	0.790	0.764	0.737	0.708	0.678	0.645	0.611	0.574	0.534	0.491	0.443	0.388	0.322	0.237	-
	0.20	0.858	0.849	0.837	0.822	0.804	0.785	0.763	0.740	0.714	0.687	0.658	0.626	0.593	0.556	0.517	0.474	0.425	0.369	0.302	0.171	-
	0.25	0.823	0.813	0.802	0.788	0.772	0.754	0.734	0.712	0.688	0.662	0.634	0.603	0.570	0.534	0.495	0.451	0.402	0.344	0.273	0.153	-
	0.30	0.787	0.778	0.766	0.753	0.738	0.721	0.702	0.681	0.658	0.633	0.606	0.576	0.544	0.508	0.469	0.425	0.374	0.313	0.236	-	-
	0.35	0.751	0.741	0.730	0.717	0.703	0.686	0.668	0.648	0.626	0.602	0.575	0.546	0.514	0.478	0.439	0.393	0.340	0.275	0.160	-	-
	0.40	0.714	0.705	0.694	0.681	0.667	0.651	0.633	0.614	0.592	0.568	0.542	0.513	0.481	0.445	0.404	0.357	0.300	0.226	0.069	-	-
	0.45	0.677	0.667	0.657	0.644	0.630	0.614	0.597	0.578	0.556	0.533	0.506	0.477	0.445	0.408	0.365	0.315	0.251	0.142	-	-	-
	0.50	0.639	0.630	0.619	0.606	0.592	0.576	0.559	0.540	0.519	0.495	0.468	0.439	0.405	0.366	0.320	0.264	0.188	-	-	-	-
	0.55	0.601	0.591	0.580	0.567	0.553	0.537	0.520	0.500	0.479	0.455	0.427	0.396	0.361	0.319	0.269	0.199	0.086	-	-	-	-
	0.60	0.563	0.552	0.541	0.528	0.513	0.497	0.479	0.459	0.437	0.412	0.383	0.351	0.312	0.265	0.205	0.104	-	-	-	-	-
	0.65	0.523	0.513	0.500	0.487	0.472	0.455	0.437	0.416	0.393	0.366	0.335	0.300	0.256	0.200	0.115	-	-	-	-	-	-
	0.70	0.481	0.471	0.459	0.445	0.430	0.412	0.392	0.370	0.345	0.316	0.282	0.241	0.190	0.110	-	-	-	-	-	-	-
	0.75	0.437	0.427	0.415	0.401	0.385	0.366	0.346	0.322	0.294	0.262	0.222	0.169	0.084	-	-	-	-	-	-	-	-
	0.80	0.389	0.380	0.368	0.353	0.336	0.316	0.293	0.266	0.235	0.196	0.144	0.053	-	-	-	-	-	-	-	-	-
	0.85	0.336	0.326	0.314	0.299	0.281	0.259	0.233	0.201	0.160	-	-	-	-	-	-	-	-	-	-	-	-
	0.90	0.274	0.263	0.251	0.234	0.214	0.188	0.155	0.108	0.022	-	-	-	-	-	-	-	-	-	-	-	-
	0.95	0.191	0.183	0.169	0.148	0.118	0.072	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

*) May also be used when

$$M_{rx}^* \geq 0, M_{ry}^* \leq 0, N_r^* \leq 0$$

$$M_{rx}^* \geq 0, M_{ry}^* \geq 0, N_r^* \leq 0$$

$$M_{rx}^* \leq 0, M_{ry}^* \leq 0, N_r^* \geq 0$$

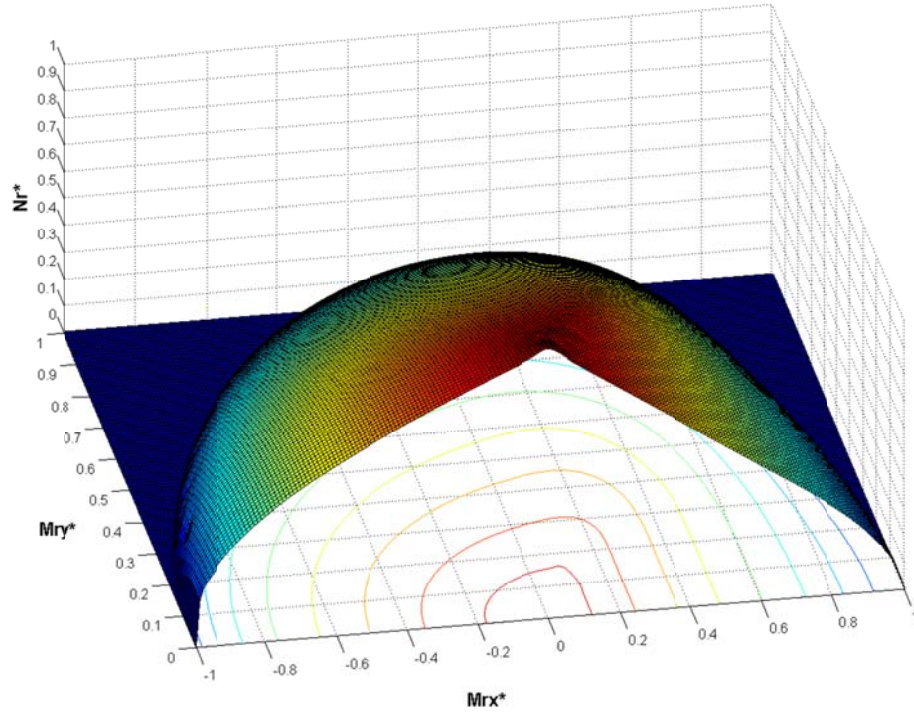


Fig. 6.8: A quarter of the interaction surface of SEHS

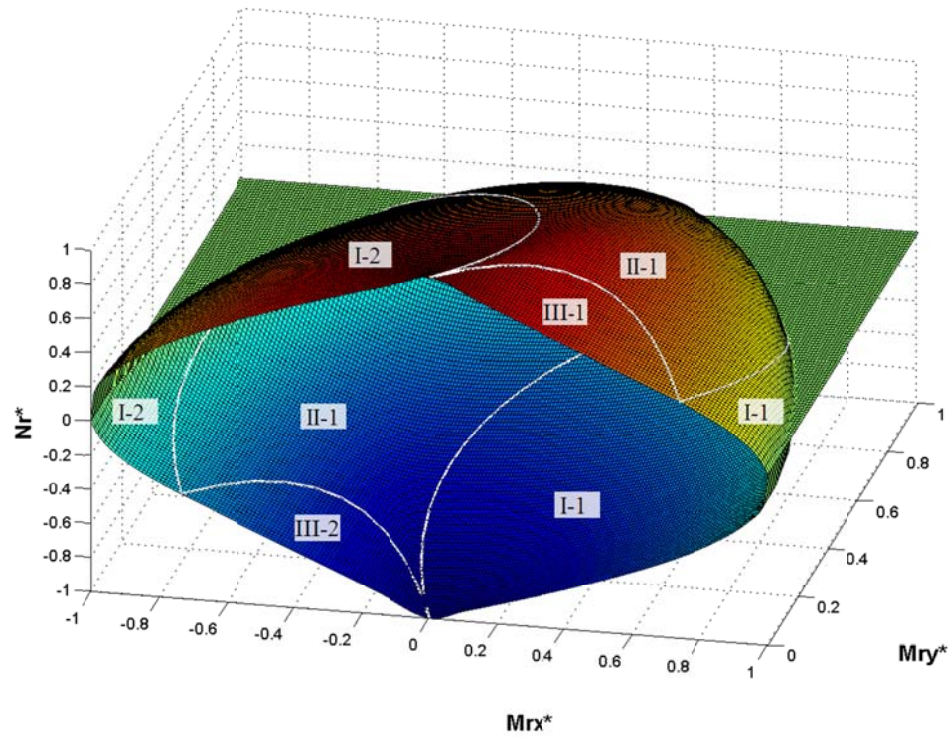


Fig. 6.9: Half interaction surface of SEHS for $M_{ry}^* \geq 0$

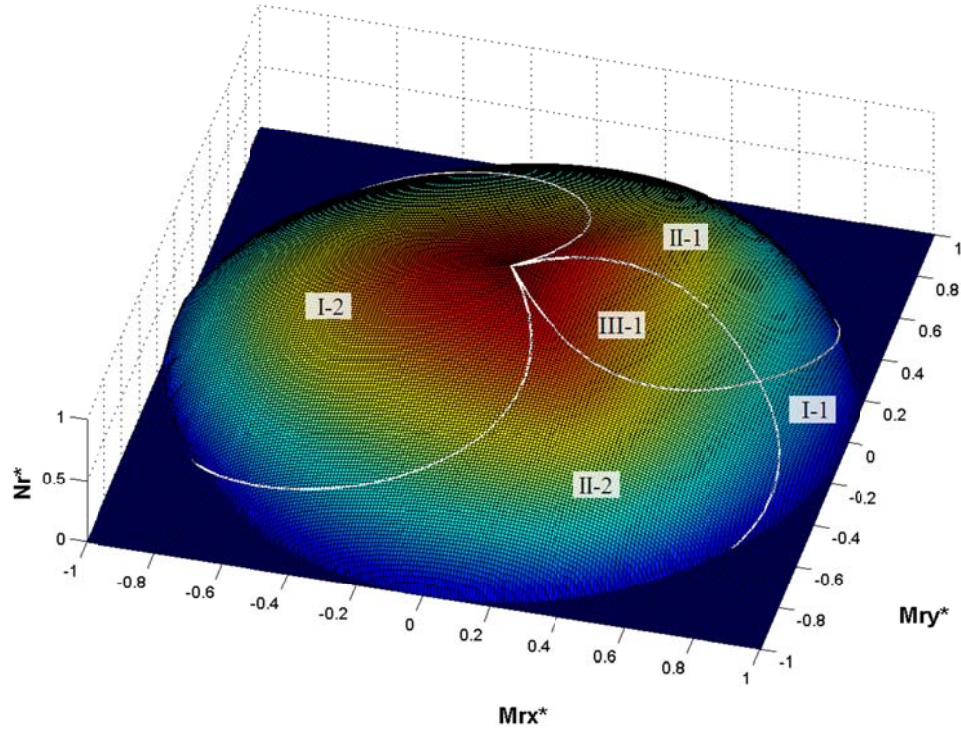


Fig. 6.10: Half interaction surface of SEHS for $N_r^* \geq 0$

6.9. Approximate Interaction Relations

The objective now is to find a simple function describing the interaction between the modified internal forces N_r^*, M_{rx}^* and M_{ry}^* from 6.33 to 6.35. It was observed that the interaction relations have three properties of interest. These are:

1. They are skew symmetric with respect to the origin, i.e.,

$$f(M_{rx}^*, M_{ry}^*, N_r^*) = 0 \Rightarrow f(-M_{rx}^*, -M_{ry}^*, -N_r^*) = 0$$
2. They are symmetric with respect to the $M_{rx}^* - N_r^*$ plane at $M_{ry}^* = 0$, i.e.,

$$f(M_{rx}^*, M_{ry}^*, N_r^*) = 0 \Rightarrow f(M_{rx}^*, -M_{ry}^*, N_r^*) = 0$$
3. They are independent of the direction of torsion

These characteristics of the interaction relations, imply that the complete interaction surface can be fully described once determined in two octants: a) ($M_{rx}^* \geq 0, M_{ry}^* \geq 0, N_r^* \geq 0$) and b) ($M_{rx}^* \leq 0, M_{ry}^* \geq 0, N_r^* \geq 0$). When the interaction relation is known in these two octants (Fig. 6.8), its value for the remaining six octants is determined from properties 1 and 2.

The numerical values in Table 6.1 or Table 6.2 were used to find a best fit surface for the points. Five candidate functions $g_j(a_k^j)$, ($j=1...5$) were examined for their ability to fit the numerical values. These are:

$$\begin{aligned}
 g_1(a_k^1) &= M_{rx}^{*a_1^1} + M_{ry}^{*a_2^1} + N_r^{*a_3^1} - 1 = 0 \\
 g_2(a_k^2) &= a_5^2 M_{rx}^{*a_1^2} + a_6^2 M_{ry}^{*a_2^2} + N_r^{*a_3^2} + (1-a_4^2)M_{rx}^* + (1-a_5^2)M_{ry}^* - 1 = 0 \\
 g_3(a_k^3) &= a_4^3 M_{rx}^{*a_1^3} + a_5^3 M_{ry}^{*a_2^3} + a_6^3 N_r^{*a_3^3} + \\
 &\quad (1-a_4^3)M_{rx}^* + (1-a_5^3)M_{ry}^* + (1-a_6^3)N_r^* - 1 = 0 \\
 g_4(a_k^4) &= \sqrt{M_{rx}^{*a_1^4} + M_{ry}^{*a_2^4}} + N_r^{*a_3^4} - 1 = 0 \\
 g_5(a_k^5) &= \left(M_{rx}^{*a_1^5} + M_{ry}^{*a_2^5} \right)^{a_4^5} + N_r^{*a_3^5} - 1 = 0
 \end{aligned} \tag{6.36a-e}$$

where a_k^j are fitting parameters for the candidate function j which are to be determined from nonlinear regression analysis. A common feature between the candidate functions is the fact that regardless of a_k^j values, they all meet the condition that when one of N_r^*, M_{rx}^* and M_{ry}^* has a unity value, the other two vanish. The form of the proposed functions (2.27 a-e) are similar to those attempted in Nowzartash and Mohareb (2009) for fitting the interaction surface for elliptical hollow sections. For each candidate function, the sum of the squares of errors (SSE)

$$E_j(a_k^j) = \sum_{p=1}^{p_{\max}} \left[g_j^2(a_k^j) \right]_p \tag{6.37}$$

was minimized in the $\{a_k^j\}$ space. In Eq. 2.29, $p_{\max}$ is the number of data points for the nonlinear regression (i.e., 323 in each table). The sum of the squares of errors E_j for each candidate function g_j is minimized by enforcing the conditions

$$\left\{ \frac{\partial E_j(a_k^j)}{\partial a_l^j} \right\} = \left\{ \sum_{p=1}^{p_{\max}} 2 \left[g_j(a_k^j) \frac{\partial g_j(a_k^j)}{\partial a_l^j} \right]_p \right\} = \{0\} \quad (6.38)$$

in which, index p denotes a given point on the yield surface and the summation is performed for all points $p_{\max}$. By using Newton's method, and given a guess solution vector for the fitting parameters $\{a_k^j\}_n$ based on the n^{th} iteration, an improved solution vector $\{a_k^j\}_{n+1}$ is given by

$$\{a_k^j\}_{n+1} = \{a_k^j\}_n - \left[\frac{\partial}{\partial a_l^j} \left(g_j(a_k^j) \frac{\partial g_j(a_k^j)}{\partial a_k^j} \right) \right]_n^{-1} \left\{ g_j(a_k^j) \frac{\partial g_j(a_k^j)}{\partial a_k^j} \right\}_n \quad (6.39)$$

In Eq. 2.31, the terms of matrix $\left[\partial(g_j \partial g_j / \partial a_k^j) / \partial a_l^j \right]_n$ are numerically computed through the central finite difference technique and the matrix obtained is then inverted. Equation 2.27e was found to provide the best fit to the numeric value in Table 6.1 or Table 6.2 while Eq. 2.27d provides a close second best fit. Equation 2.27d is simpler and allows isolating the effect of torsion after simple manipulations, and was thus adopted in the following steps.

6.9.1. Best Fit Interaction Relations

The best-fit function for the octant $M_{rx}^* \geq 0, M_{ry}^* \geq 0$ and $N_r^* \geq 0$ was found to be

$$\sqrt{|M_{rx}^*|^{1.9} + |M_{ry}^*|^{2.6}} + |N_r^*|^{1.5} - 1.0 = 0 \quad (6.40)$$

In Eq. 6.40, SSE is 0.61 for the points in Table 6.1. This equation may be used also for the octants a) $M_{rx}^* \geq 0, M_{ry}^* \leq 0$ and $N_r^* \geq 0$, b) $M_{rx}^* \leq 0, M_{ry}^* \geq 0$ and $N_r^* \leq 0$, and c) $M_{rx}^* \leq 0, M_{ry}^* \leq 0$ and $N_r^* \leq 0$.

The best fit function for the case $M_{rx}^* \leq 0, M_{ry}^* \geq 0$, and $N_r^* \geq 0$ is

$$\sqrt{|M_{rx}^*|^{2.1} + |M_{ry}^*|^{1.55}} + |N_r^*|^{1.95} - 1.0 = 0 \quad (6.41)$$

In Eq. 6.41, SSE is 0.18 for the points in Table 6.2. This equation may be used also for the octants $M_{rx}^* \geq 0, M_{ry}^* \leq 0$ and $N_r^* \leq 0$, b) $M_{rx}^* \geq 0, M_{ry}^* \geq 0$ and $N_r^* \leq 0$, and c) $M_{rx}^* \leq 0, M_{ry}^* \leq 0$ and $N_r^* \geq 0$.

6.9.2. Conservative Fit Interaction Relations

Piecewise Surface Fit

Whilst Eqs. 6.40 and 6.41 provide a very good representation of the yield surface, they lead to slightly un-conservative results half of the time. In order to obtain a conservative curve fit suitable for design, a weighted regression analysis based on the same form of the function is performed. The error weight for un-conservative prediction was taken as 100 time larger than that of the error weight for a conservative prediction. The conservative expression was found to be

$$\sqrt{|M_{rx}^*|^{2.0} + |M_{ry}^*|^{1.6}} + |N_r^*|^{1.8} - 1.0 = 0 \quad (6.42)$$

for the region where $M_{rx}^* \geq 0, M_{ry}^* \geq 0$ and $N_r^* \geq 0$. The SSE for the points in Table 6.1 is 3.50, and

$$\sqrt{|M_{rx}^*|^{1.7} + |M_{ry}^*|^{2.0}} + |N_r^*|^{1.55} - 1.0 = 0 \quad (6.43)$$

for the region where $M_{rx}^* \leq 0, M_{ry}^* \geq 0$, and $N_r^* \geq 0$ and three other similar regions. The SSE for the points in Table 6.2 is 0.62.

Equations 6.42 and 2.33 provide conservative fit for the interaction surface. However, they are not continuous at the planes $M_{rx}^* = 0$, $M_{ry}^* = 0$ or $N_r^* = 0$. In a design context, when either $M_{rx}^* = 0$, $M_{ry}^* = 0$ or $N_r^* = 0$, this issue can be addressed by applying both Equations and adopting the more conservative result.

Single Function Fit

In order to obtain a single and continuous interaction relation, one could fit the whole surface by a single equation in the form of Eq. 2.27d. Evidently, the level of conservatism

in this approach is higher. The conservative interaction relation applicable for any kind of applied load is found to be

$$\sqrt{|M_{rx}^*|^{1.65} + |M_{ry}^*|^{1.95}} + |N_r^*|^{1.5} - 1.0 = 0 \quad (6.44)$$

The SSE for points in both Table 6.1 and Table 6.2 is 7.51. Equation 6.44 can be expressed in terms of M_{rx} , M_{ry} , N_r and T_r by substituting M_{rx}^* , M_{ry}^* and N_r^* with their definitions provided in Eqs. 6.33 to 6.35 as

$$\left(\frac{|M_{rx}|}{\sqrt{1-T_r^2}} \right)^{1.65} + \left(\frac{|M_{ry}|}{\sqrt{1-T_r^2}} \right)^{1.95} + 2 \left(\frac{|N_r|}{\sqrt{1-T_r^2}} \right)^{1.5} - \left(\frac{|N_r|}{\sqrt{1-T_r^2}} \right)^{3.0} - 1 = 0 \quad (6.45)$$

The interaction relation in 2.34 explicitly describes the effect of torsion ratio.

6.10. Design Examples

6.10.1. Example 1

A simply supported beam (Fig. 6.11a) has a 2.0 m span. The beam is made of steel SEHS-225×259×8 section with a yield strength of 350 MPa. The beam is subjected to axial force and bending moments about x-axis. For the given cross-section $a = 217 \text{ mm}$, $b = 126 \text{ mm}$, $t = 8 \text{ mm}$ and $b/a = 0.581$. It is required to plot the $N_r - M_{rx}$ interaction relation for the mid-span section of the beam.

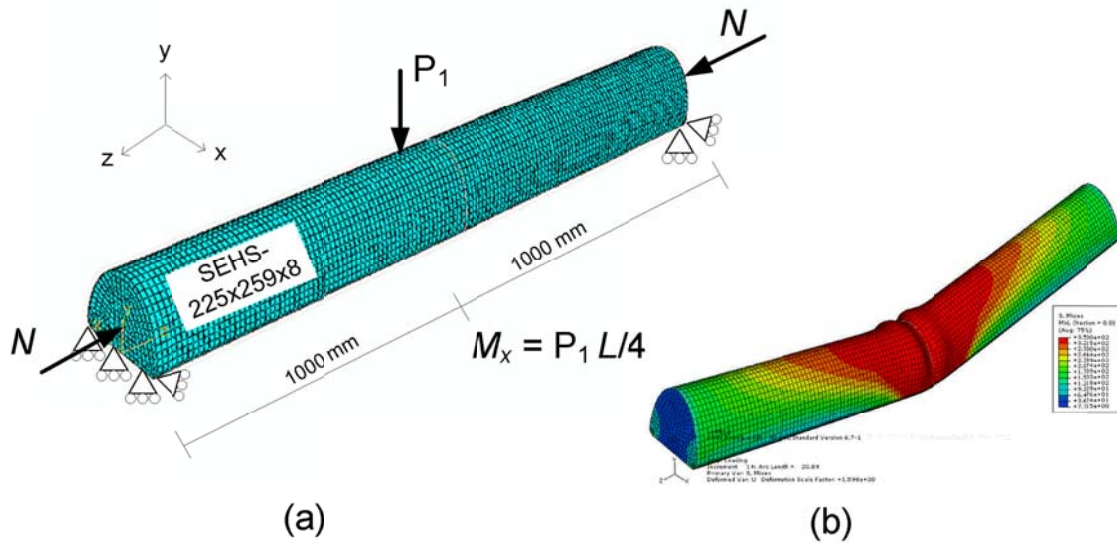


Fig. 6.11: Example 1: a) FEA model; b) failure mechanism

Solution:

The problem is solved based on the interaction surfaces developed in the present study. In addition, a series of elasto-plastic finite element analyses is conducted for the problem as a verification of the analytical solutions developed.

Details of the Finite Element Model

The finite element analysis program ABAQUS 6-7.1 (SIMULIA 2007a) is used to model the member. Longitudinally, the member has a uniform mesh consisting of 100 elements, each having a length of 20 mm. The model has 28 elements in the semi-elliptical portion of the section, 14 in the flat portion and 7 in each of the fillets between the curved and the flat portions, equal to 56 elements in total, circumferentially. The S4R element, a four-node shell element with reduced integration, is used to model the section wall. Steel material is assumed linearly elastic-perfectly plastic with a modulus of elasticity $E = 200$ GPa, Poisson's ratio $\nu = 0.3$ and yield strength of $F_y = 350$ MPa. The axial force is applied as two longitudinal ring loads at the ends of member. The bending moments are induced by a transverse ring load applied at the mid-span of the member. Only the resultants of the loads applied are depicted in Fig. 6.11a. In order to eliminate local deformations, three stiffener

plates are embedded into the section at the location of loads and reactions. The longitudinal and transverse loads were simultaneously applied to model and the analysis was incrementally progressed until full plastification of the middle cross-section has taken place. The analysis is based on static Riks method with the displacement control option. The beam mid-span deflection is used as the controlling parameter. The plastic resistance moment M_{xp} for the model is obtained by removing the axial force and loading the member until full plastification has occurred in the middle section. The deformed configuration of the member and the contour of the von-Mises stresses are illustrated in Fig. 6.11b.

Comparison of Results

The interaction relations based on FEA are plotted on Fig. 6.12. The interaction relations based on the solution of Eqs. 6.20 through 6.29 are superimposed on the same plot. The simplified solution (i.e., Table 6.1 or Table 6.2) is observed to coincide with the “exact” solution and thus was removed from the figure. On the same plot, are superimposed a) the conservative piecewise fit (Eqs. 6.42 and 2.33) and b) the single function fit (Eq. 6.44). The figure illustrates the degree of conservatism induced by both fits. This figure shows an excellent match between the finite element analysis results and the solution provided by Eqs. 6.20 through 6.29. The interaction relations developed yield slightly more conservative predictions for points (N_r, M_{rx}) of (0.43, 0.68) and (−0.43, −0.68) compared to other points. This is due to the geometric approximation made for the location of the PNA in cases III-1 and III-2 compared to the more exact depiction of the PNA illustrated in Fig. 6.6.

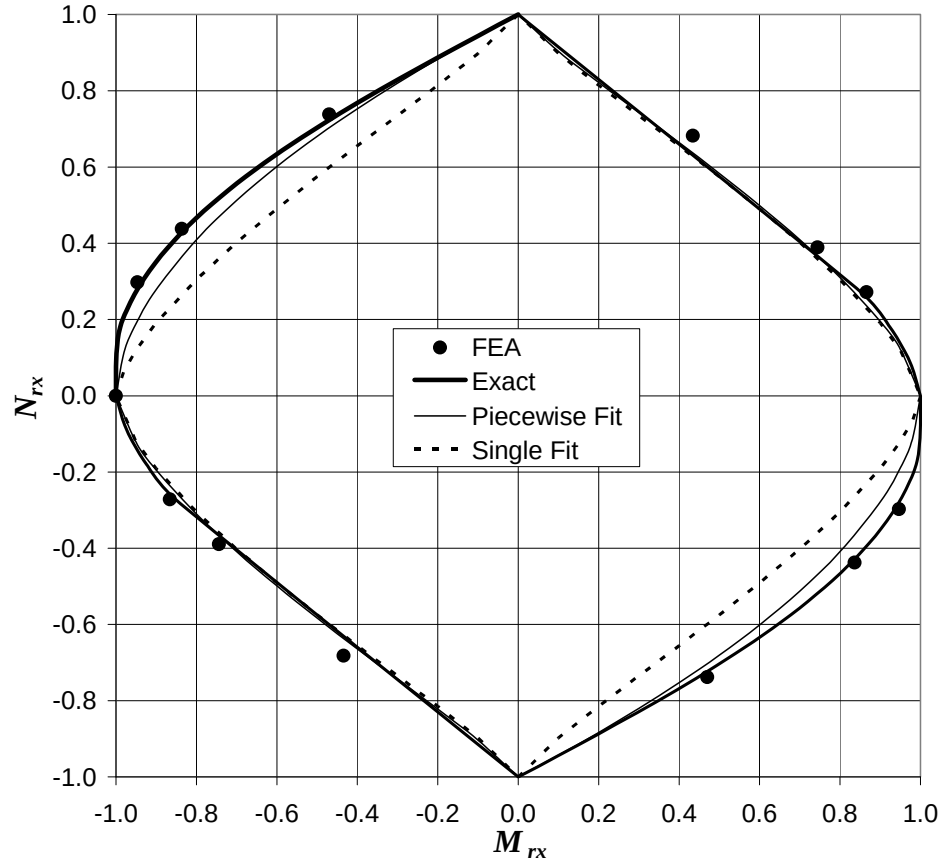


Fig. 6.12: Comparison of solutions for $N_r - M_{rx}$ interaction relation (example 1)

6.10.2. Example 2

A 3.0m span steel SEHS-225×259×8 member (Fig. 6.13a) has a yield stress of 350 MPa. The member is subject to a tensile force N , two 90 kN loads each acting at the third span points, inducing a major bending moment in the middle section of 90 kNm. Also it is subject to 90 kN loads acting at the same point inducing a minor bending moment of 90 kNm. Two 30 kNm twisting moment are applied to the member as shown. Under the given torsional moments the twisting moment in the middle section is 20 kNm. It is required to determine, based on a cross-sectional resistance limit state, the magnitude of the tensile force that can be safely applied to the member in addition of the above combination of internal forces (i.e, $M_x = 90$ kNm, $M_y = 90$ kNm and $T = 20$ kNm). Other limit states

involving possible local buckling induced by compressive stresses due to bending and the presence of residual stresses are outside the scope of the present problem.

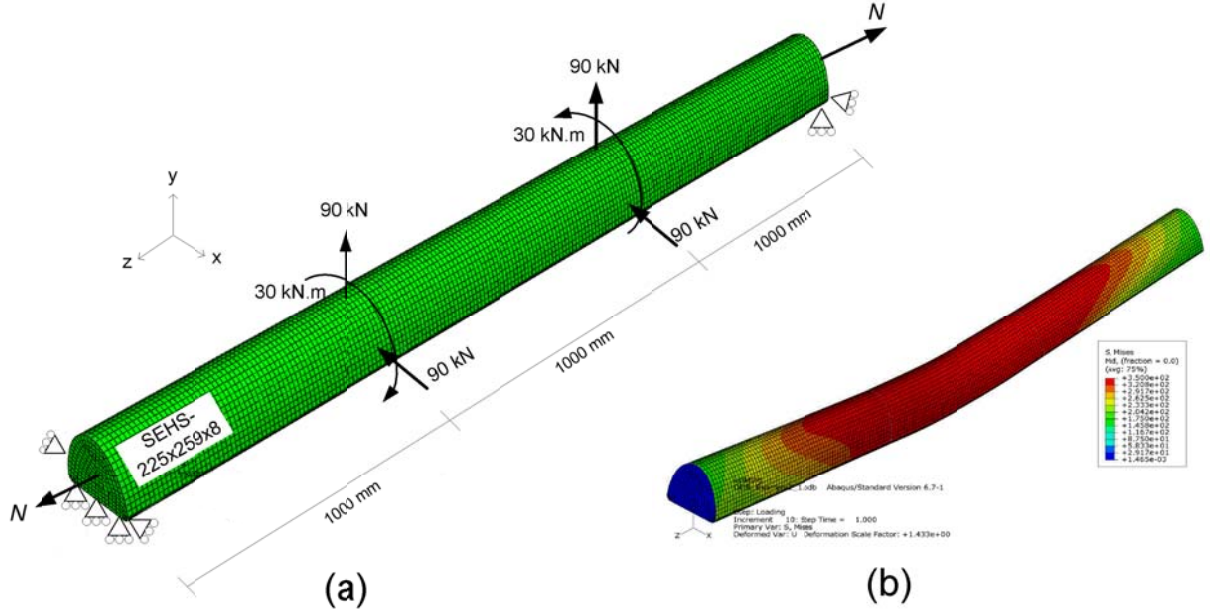


Fig. 6.13: Example 2: a) FEA model; b) failure mechanism

Solution 1

A finite element model was developed in ABAQUS 6-7.1 (SIMULIA 2007a). The model consists of one middle and two end segments. Longitudinally, each segment was modelled by a uniform mesh consisting of 50 elements. The definition of steel properties, circumferential division of the members, ring loads, boundary conditions and element type were identical to Example 1. The maximum axial load predicted by the model was 1,040 *kN*. The deformed configuration of the member and the contour of the von-Mises stresses are illustrated in Fig. 6.11b.

Solution 2

As discussed, the location of the PNA and thus the governing case for the combination of the applied loads is not known a-priori. Thus it is required to solve all possible six cases. From Eq. 6.20 one has $\tau_r = 0.144$. Cases I-1 to III-2 are solved consecutively and tabulated in Table 6.3. The only admissible answer is case II-1 which is $N_r = 0.459$, i.e.,

$N = 1,028 kN$, representing 98.8% of solution 1. As expected in a lower bound solution, the predicted axial load capacity is slightly lower than solution 1.

Table 6.3: Solution 2 of example 2

Case	System of Eqs.	Results	Substitute into Eq.	Result	Comments
I-1	(22) and (23)	$\beta_1 = 0.283$, $\beta_2 = 1.687$	(21)	$N_r = -0.426$	Rejected: Not a tensile force.
I-2	(22) and (23)	$\beta_1 = 1.687$, $\beta_2 = 6.566$	-	-	Rejected: β_2 violates applicability limits (i.e., $0 \leq \beta_1 \leq \beta_2 \leq 2\pi$)
II-1	(25) and (26)	$\beta = 0.320$, $\xi = -0.159$	(24)	$N_r = 0.459$	Admissible answer.
II-2	(25) and (26)	$\beta = 6.603$, $\xi = -1.841$	-	-	Rejected: β and ξ violate applicability limits (i.e., $0 \leq \beta \leq 2\pi$ and $-1 \leq \xi \leq 1$)
III-1	(28) and (29)	$\xi_1 = -0.057$, $\xi_2 = 1.431$	-	-	Rejected: ξ_1 violates applicability limits (i.e., $-1 \leq \xi_1 \leq \xi_2 \leq 1$)
III-2	(28) and (29)	$\xi_1 = 1.431$, $\xi_2 = -0.057$	-	-	Rejected: ξ_2 violates applicability limits (i.e., $-1 \leq \xi_1 \leq \xi_2 \leq 1$)

Solution 3

The exact aspect ratio b/a of the cross-section section is 0.581. This is slightly higher from the value $b/a \approx 0.55$ adopted in developing Eqs 6.30 to 6.32. The use of the interaction relations developed will introduce a slight approximation on the conservative side. From Eqs. 2.7 and 6.30 to 6.32, one can determine the section plastic resistances in the absence of other internal forces. These are $T_p = 139 \text{ kNm}$, $N_y = 2,179 \text{ kN}$, $M_{xp} = 157 \text{ kNm}$, $M_{yp} = 165 \text{ kNm}$. The corresponding dimensionless internal force ratios are

$$T_r = 20 / 139 = 0.144 , \quad M_{rx}^* = 90 / \left(157 \sqrt{1 - 0.144^2} \right) = 0.579 \quad \text{and}$$

$$M_{ry}^* = 90 / \left(165 \sqrt{1 - 0.144^2} \right) = 0.545 . \quad \text{A linear interpolation on the grid in Table 6.1}$$

provides the prediction $N_r^* = 0.432$ or $N = 941kN$, which is 9.5% less than that of solution 1.

Solution 4

A last solution is provided by applying the conservative interaction relations. By substituting $M_{rx}^* = 0.579$ and $M_{ry}^* = 0.545$ into Eq. 6.42 and solving for the modified axial force, one obtains $N_r^* = 0.371$, which correspond to $N = 725kN$.

When Eq. 2.34 is used, the modified axial force ratio predicted is $N_r^* = 0.300$ corresponding to an axial force of $N = 586kN$. As expected, the last solution provides the most conservative prediction.

6.11. Summary

Interaction relations for semi-elliptical hollow sections under combination of axial force, bi-axial bending moments and twisting moment were developed. The relations provide cross-sectional strength design criteria. The relations are based on the fully plastic condition and will govern the design in cases where local buckling does not occur before the fully plastic resistance of the section is mobilized, and where members are short enough not to be influenced by global buckling considerations. The solution has been verified against elasto-plastic finite element analysis. Tabulated interaction coefficients, simple best fit interaction relations, as well as conservative interaction equations were developed. Upon experimental verification, the conservative interaction relations developed are suitable for conducting cross-sectional checks commonly performed in limit states design in steel design standards.

6.12. Notation:

A	Cross sectional area
a	SEHS center-line height
b	Half of SEHS center-line base
E	Modulus of elasticity
F_y	Yield strength
M_x	Bending moments about X axis
M_{xp}	Plastic capacity of section for bending about X axis
M_{rx}	Bending moment ratio about X axis
M_y	Bending moments about Y axis
M_{yp}	Plastic capacity of section for bending about Y axis
M_{ry}	Bending moment ratio about Y axis
N	Axial force
N_p	Plastic capacity of section for axial force
N_r	Axial force ratio
T	Twisting moment
T_p	Plastic capacity for twisting moment
T_r	Torsional ratio
t	Thickness
β	Ellipse parameter [i.e., $x = b \cos(\beta)$, $y = a \sin(\beta)$]
σ	Longitudinal stress
σ^*	Maximum longitudinal stress in the presence of shear stress τ
τ	Shear stress
ξ	Parameter locating the intersection of PNA with the base

CHAPTER 7

Summary and Recommendations

7.1. Summary of Original Contributions

Following is the summary of the thesis:

1. Lower bound interaction relations for elliptical hollow sections under combination of axial force, bi-axial bending moments and twisting moment were developed. The relations provide cross-sectional strength design criteria to assess whether a section can withstand a combination of given internal forces. The relations are based on the fully plastic condition and will govern the design in cases where localized strains do not occur before the fully plastic resistance of the section is mobilized. The solution has been verified by using nonlinear finite element analysis. Previously established interaction relations for pipes are recovered as a special case of the obtained relations.
2. More simplified interaction equations were developed for commercially available sections and design table and diagrams were developed. These simplified interaction relations developed conservative and thus suitable for design purposes.
3. Upper bound plastic interaction relations were developed for elliptical hollow sections under combinations of axial force, bi-axial bending moments, twisting moments and bi-moments. The resulting parametric relations consist of a set of non-dimensional integrals applicable to EHS of general cross-sectional geometries. An efficient solution algorithm is developed to solve the non-linear equations and numerically determine whether a given internal force combination is admissible within the limitations of the formulation. The plastic interaction relations successfully capture the Saint Venant and warping twisting moments, bi-moments, biaxial bending moments, and axial forces. Predictions based on the interaction relations developed agree well with elasto-plastic shell FEA. The interaction relations successfully capture the confining effects present near welded ends as well as

sections away from welded ends with essentially no confining effects. A conservative relationship was proposed to perform a quick capacity analysis of sections near welded ends.

4. A thermo-mechanical finite element model was developed to predict residual stresses in the hot-rolled steel sections including hot-rolled EHS. The validity of the FEA model is established through comparisons with measured residual stresses by other researchers.

5. In addition to longitudinal residual stresses, the FEA model the presence of transverse residual stresses with magnitudes of the same order as longitudinal stresses. The inside fibre of EHS is always in tension and the outside fibre in compression. On average, the residual stresses in EHS are tensile near the minor axis and compressive near the major axis.

6. It is observed that for a given steel and manufacturing process, the residual stress pattern and magnitude depends mostly on EHS aspect ratio. Moreover, the predicted residual stresses are highly dependent upon the initial temperature and moderately dependent on water spraying intensity and duration. Steel yield stress at room temperature has a negligible impact on the residual stress patterns in EHS members.

7. Column curves for EHS members which include the effects of predicted residual stresses have been developed. It was found that the Canadian Standard column capacity relation with $n=2.24$ overestimates buckling capacity of EHS columns. It can be improved by using $n=1.57$ for minor axis buckling and $n=1.32$ for major axis buckling. Moreover, Eurocode 3 buckling curve 'b' provides very good approximation for the EHS buckling about major axis while buckling curve 'a' provides a close approximation for buckling about minor axis.

8. An illustration for the potential use of the present analysis in developing interaction relations was provided in Appendix II.

9. Interaction relations for semi-elliptical hollow sections under combination of axial force, bi-axial bending moments and twisting moment were developed. The relations provide cross-sectional strength design criteria. The relations are based on the fully plastic condition and govern the design in cases where local buckling does not occur before the fully plastic resistance of the section is mobilized, and where members are short enough not

to be influenced by global buckling considerations. The solution has been verified by against elasto-plastic finite element analysis. Tabulated interaction coefficients, simple best fit interaction relations, as well as conservative interaction equations were developed. Upon experimental verification, the conservative interaction relations developed are suitable for conducting cross-sectional checks commonly performed in limit state design in steel design standards.

7.2. Recommendations for Future Research

1. All interaction relations developed in this study exclude shear forces. One interesting area of investigation is to include shear forces into the interaction relations for EHS and SEHS. Another possible area of investigation is the upper bound interaction relations for semi elliptical section in a manner similar to that adopted for the upper bound solution already developed for EHS.
2. The residual stress investigation in this study was confined to the hot-rolled sections. Thus, they are not applicable to SEHS which are cold-formed. In these sections, the residual stresses are mainly introduced because of the plastification of the plate during pressing. Therefore, the prediction of residual stresses in such sections would be an interesting area of study. These residual stresses then could be utilized to develop column curves for members of semi elliptical cross section. Furthermore, the developed interaction relations could be modified to take into account the effects of residual stresses and initial imperfection.
3. The elasto-plastic buckling analysis technique developed in this thesis can be used in investigating the lateral torsional buckling of EHS and SEHS members under uni-axial or bi-axial bending moments.
4. In the current study, the parametric studies performed were aimed at developing reasonable understanding in the nature of residual stresses in elliptical hollow sections. This understanding could be further deepened by conducting more parametric studies on different EHS aspect ratios, steel properties, initial steel temperature, and different cooling regimes.

5. As a by-product of the current research, it was observed that there are in-plane residual stresses in the section in the same order of the well-known longitudinal stresses. These stresses have received little attention in past research. Investigating effects of in-plane stresses on the buckling resistance of members with hot-rolled cross sections could be an interesting area of research.

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Appendix I: Solution Procedure for EHS Upper Bound Interaction Relations

Formulation and Algorithm

In order to numerically determine the interaction function $F(N_r, M_{rx}, M_{ry}, T_{rw}, T_{rsv}, B_r) = 0$, an EHS is assumed to be subjected to a given a set of internal forces $(N_r, M_{rx}, M_{ry}, T_{rw}, T_{rsv}, B_r)$. Under the internal force expressions in Eqs. 3.51 through 3.56, it is required to determine whether the given load combination lies within the interaction surface defined by the parametric equations 3.51 through 3.56 or outside interaction surface (i.e., inadmissible under the assumptions of the formulation). Towards this goal, one scales the internal force combination by a factor α . It is now required to determine the six constants $\alpha, C_1, \dots, C_5$ so that $\alpha(N_r, M_{rx}, M_{ry}, T_{rw}, T_{rsv}, B_r)$ lies on the interaction surface. Equations 3.51 through 3.56 are re-written for the scaled load combination so that

$$\begin{aligned}
 R_1 &= \alpha N_r - \frac{2\sqrt{1+\eta+\eta^2}}{I_1} \int_0^{2\pi} \frac{\text{sgn}(\Upsilon) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\Gamma/\Upsilon)^2}} d\beta \\
 R_2 &= \alpha M_{xr} - \frac{2\sqrt{1+\eta+\eta^2}}{I_2} \int_0^{2\pi} \frac{\text{sgn}(\Upsilon) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\Gamma/\Upsilon)^2}} \sin \beta d\beta \\
 R_3 &= \alpha M_{yr} - \frac{2\sqrt{1+\eta+\eta^2}}{I_3} \int_0^{2\pi} \frac{\text{sgn}(\Upsilon) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\Gamma/\Upsilon)^2}} \cos \beta d\beta \\
 R_4 &= \alpha B_r - \frac{2\sqrt{1+\eta+\eta^2}}{I_4} \int_0^{2\pi} \frac{\text{sgn}(\Upsilon) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2) + (\Gamma/\Upsilon)^2}} \sin(2\beta) d\beta \\
 R_5 &= \alpha T_{rw} - \frac{1}{2\pi} \int_0^{2\pi} \text{sgn}(\Gamma) \frac{1 - 2\pi \frac{a}{p_r} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}}{\sqrt{4(1+\eta+\eta^2)(\Upsilon/\Gamma)^2 + 1}} d\beta \\
 R_6 &= \alpha T_{rsv} - \frac{1}{2\pi} \int_0^{2\pi} \frac{\text{sgn}(\Gamma)}{\sqrt{4(1+\eta+\eta^2)(\Upsilon/\Gamma)^2 + 1}} d\beta
 \end{aligned} \tag{I.1}$$

in which

$$Y = C_3 \sin \beta + C_4 \cos \beta + C_5 \sin(2\beta) + 1,$$

$$\Gamma = \frac{C_1}{\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}} + \left(\frac{1}{\sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta}} - 2\pi \frac{a}{p_r} \right) C_2.$$

It is required to vanish the residuals $R_i = R_i(\alpha, C_1, C_2, C_3, C_4, C_5), i = 1, \dots, 6$ defined in Eqs.

I.1. The following definitions are made

$$I_1 = \int_0^{2\pi} \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} d\beta \quad I_2 = \int_0^{2\pi} \text{sgn}(\sin \beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin \beta d\beta,$$

$$I_3 = \int_0^{2\pi} \text{sgn}(\cos \beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \cos \beta d\beta \quad \text{and}$$

$$I_4 = \int_0^{2\pi} \text{sgn}(2\beta) \sqrt{(b/a)^2 \sin^2 \beta + \cos^2 \beta} \sin(2\beta) d\beta.$$

The Levenberg-Marquardt method (Theofanous et al. 2009) is a nonlinear least squares minimization solution technique. In this method, the function to be minimized has the form

$$g(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^n R_i^2(\mathbf{x}). \quad \text{In the present problem, } \mathbf{x}^T = \langle \alpha \quad C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5 \rangle^T \quad \text{and}$$

$R_i (i = 1, \dots, n)$ are the residuals to be minimized.

In a least square minimization, the size of vector $\mathbf{x}^T$ is generally less than the number of residuals n to be minimized. In contrast, in the present problem, the size vector $\mathbf{x}^T$ is equal to the number of residuals defined by Eqs. I.1, with the direct implication that $g(\mathbf{x})$ should vanish when minimized, i.e, the techniques yields the solution of the nonlinear system of equations within the specified tolerance.

Using Levenberg-Marquardt method and starting with a guess solution vector $\mathbf{x}_n$ based on the n th iteration, an improved solution vector $\mathbf{x}_{n+1}$ for the $(n+1)$ th iteration is determined from the updating rule

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \left(\mathbf{H}(\mathbf{x}_n) + \mu \text{diag}[\mathbf{H}(\mathbf{x}_n)] \right)^{-1} \nabla(\mathbf{x}_n) \quad (\text{I.2})$$

in which $\mathbf{H}$ is the Hessian matrix defined as $\mathbf{H}(\mathbf{x}) \cong \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x})$, in which

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial R_1}{\partial \alpha} & \frac{\partial R_1}{\partial C_1} & \frac{\partial R_1}{\partial C_2} & \frac{\partial R_1}{\partial C_3} & \frac{\partial R_1}{\partial C_4} & \frac{\partial R_1}{\partial C_5} \\ \frac{\partial R_2}{\partial \alpha} & \frac{\partial R_2}{\partial C_1} & \frac{\partial R_2}{\partial C_2} & \frac{\partial R_2}{\partial C_3} & \frac{\partial R_2}{\partial C_4} & \frac{\partial R_2}{\partial C_5} \\ \frac{\partial R_3}{\partial \alpha} & \frac{\partial R_3}{\partial C_1} & \frac{\partial R_3}{\partial C_2} & \frac{\partial R_3}{\partial C_3} & \frac{\partial R_3}{\partial C_4} & \frac{\partial R_3}{\partial C_5} \\ \frac{\partial R_4}{\partial \alpha} & \frac{\partial R_4}{\partial C_1} & \frac{\partial R_4}{\partial C_2} & \frac{\partial R_4}{\partial C_3} & \frac{\partial R_4}{\partial C_4} & \frac{\partial R_4}{\partial C_5} \\ \frac{\partial R_5}{\partial \alpha} & \frac{\partial R_5}{\partial C_1} & \frac{\partial R_5}{\partial C_2} & \frac{\partial R_5}{\partial C_3} & \frac{\partial R_5}{\partial C_4} & \frac{\partial R_5}{\partial C_5} \\ \frac{\partial R_6}{\partial \alpha} & \frac{\partial R_6}{\partial C_1} & \frac{\partial R_6}{\partial C_2} & \frac{\partial R_6}{\partial C_3} & \frac{\partial R_6}{\partial C_4} & \frac{\partial R_6}{\partial C_5} \end{bmatrix} \quad (\text{I.3})$$

and the gradient $\nabla(\mathbf{x})$ is the gradient and defined as $\nabla(\mathbf{x}) = \mathbf{J}(\mathbf{x})^T \mathbf{R}(\mathbf{x})$. In the updating rule, μ is a small positive number (e.g., $10^{-8}, 10^{-7} \dots 1$), $\text{diag}[\mathbf{H}]$ is a 6×6 diagonal matrix with diagonal entries identical to those of the Hessian matrix while all off diagonal entries are zero.

The algorithm used is as follows:

1. Assume initial values of vector $\mathbf{x}$ (i.e., $\alpha^0, C_1^0, C_2^0, C_3^0, C_4^0$ and C_5^0) for the interaction $n = 0$
2. Select a mid-range number for μ (e.g., $\mu = 10^{-3}$)
3. Evaluate $g(\mathbf{x}_n)$.
4. Calculate the updated $\mathbf{x}_{n+1}$ using the updating rule I.2.
5. Evaluate $g(\mathbf{x}_{n+1})$
6. If $g(\mathbf{x}_{n+1}) > g(\mathbf{x}_n)$, retract the iteration (i.e., reset all the parameters to the previous values) and increase η by a factor of 10. Go to step 4.
7. If $g(\mathbf{x}_{n+1}) < g(\mathbf{x}_n)$ accept the iteration and decrease μ by a factor of 10.
8. Increase the iteration counter n by one.

9. If $g(\mathbf{x}_{n+1})$ is smaller than the convergence criteria specified (e.g., 10^{-6}), terminate the procedure and print results, otherwise go to step 4.

Based on extensive numeric experimentation, the entries of the following recommendations are made for the values of the initial guess vector $\mathbf{x}_0$:

1. Assume an initial value for α as 0.50
2. If $N_r < 0.001$ let $N_r = 0.001$ in order to reduce the possibility of numeric instability. In the case of zero applied axial force, the assumption $N_r = 0.001$ has a negligible effect on the solution.
3. Assume an initial value for C_1 as $C_1^0 = T_{rsv} / N_r$ in which $0.5 \leq C_1^0 \leq 100$
4. Assume an initial value for C_2 as $C_2^0 = T_{rw} / N_r$ in which $0.5 \leq C_2^0 \leq 100$
5. Assume an initial value for C_3 as $C_3^0 = M_{xr} / N_r$, in which $0.5 \leq C_3^0 \leq 100$
6. Assume an initial value for C_4 as $C_4^0 = M_{yr} / N_r$ in which $0.5 \leq C_4^0 \leq 100$
7. Assume an initial value for C_5 as $C_5^0 = B_r / N_r$ in which $0.5 \leq C_5^0 \leq 100$

The lower limit of 0.5 for parameters $C_1^0, C_2^0, C_3^0, C_4^0$ and C_5^0 is selected to reduce the possibility of numerical instability when solving the equations. The ceiling value of 100 is arbitrarily selected as it was observed in practical cases that higher values are rarely needed. Additionally, the procedure was found to converge to the higher values when needed without numerical difficulties.

Main Program Code

```
function EHS_UB_solve
clc
clear all
global bONa Fp Fr HPara;

%% ===== given values
Nr = .3; Trsv = 0.4; Trw = 0.1; Mrx = 0.1; Mry = 0.5; Br = 0.5;

eta = -0.5;          % between -0.5 (no hoop stress) & 0 (no hoop strain)
bONa = 0.47;
%
%=====
% Initial values
% alpha, C(1...5)
%=====

HPara = 4*(1 + eta + eta^2);

alpha=0.5;

C(1) = max(min(Trsv/Nr,100),0.5);
C(2) = max(min(Trw/Nr,100),0.5);
C(3) = max(min(Mrx/Nr,100),0.5);
C(4) = max(min(Mry/Nr,100),0.5);
C(5) = max(min(Br/Nr,100),0.5);

V = [alpha; C'];

%% ===== Plastic resistances =====
% Capacity in absence of all others forces.
% Note: missing a,b,Fy and t terms

S = ['sqrt((',num2str(bONa,15), '*sin(beta)).^2 + cos(beta).^2)'];

Np = quad(S, 0, 2*pi);
Mxp = quad(['sign(sin(beta)).*', S, '.*sin(beta)'],0, 2*pi);
Myp = quad(['sign(cos(beta)).*', S, '.*cos(beta)'],0, 2*pi);
Bp = quad(['sign(sin(2*beta)).*', S, '.*sin(2*beta)'],0, 2*pi);
Tp = 2*pi;

Fp = [Np; Mxp; Myp; Bp; Tp; Tp];
Fr = [Nr; Mrx; Mry; Br; Trw; Trsv];

%% ===== Main procedure =====

options.Display = 1;          % print of iterations
options.MaxIter = 100;        % maximum number of iterations allowed
options.FunTol = 1e-8;        % tolerance for final function value
options.XTol = 1e-4;          % tolerance on difference of x-solutions

x = LMFSolve(@fun,V,options); % Calls the solving program

disp(' ');
disp('The solution values are: ');
disp(['Alpha = ', num2str(x(1))]);
disp(['C1 = ', num2str(x(2))]);
disp(['C2 = ', num2str(x(3))]);
disp(['C3 = ', num2str(x(4))]);
disp(['C4 = ', num2str(x(5))]);
disp(['C5 = ', num2str(x(6))]);

Res = integrals(x)./Fp/x(1) - Fr;
end

%% ===== Used for LMFSolve
function [R]= fun (V)
global Fp Fr;
```

```

    R = V(1)*Fr - integrals(V)./Fp;
end

%% ===== Calculating integrals

function [interF] = integrals(V)
global bONa HPara;

C(1)=V(2); C(2)=V(3); C(3)=V(4); C(4)=V(5); C(5)=V(6);

S=[ '(sqrt((' ,num2str(bONa,15),'*sin(beta)).^2 + cos(beta).^2))' ];

prONa = quad(S, 0, 2*pi);

gama = [ '(' ,num2str(C(1),15),'./',S,'-(2*pi/',num2str(prONa,15),'-
1./',S,')'*',num2str(C(2),15),'')' ];
epsil =
[ '(' ,num2str(C(3),15),'*sin(beta)+' ,num2str(C(4),15),'*cos(beta)+1+' ,num2str(C(5)
,15),'*sin(2*beta))' ];

FN= [ 'sign',epsil,'./sqrt(' ,num2str(HPara,15),'+' ,gama,'./',epsil,')).^2).*',S];
N = (HPara/sqrt(3))*quad(FN,0,2*pi);

FMx=
[ 'sign',epsil,'./sqrt(' ,num2str(HPara,15),'+' ,gama,'./',epsil,')).^2).*',S,'.*sin
(beta)' ];
Mx = (HPara/sqrt(3))*quad(FMx,0,2*pi);

FMy=
[ 'sign',epsil,'./sqrt(' ,num2str(HPara,15),'+' ,gama,'./',epsil,')).^2).*',S,'.*cos
(beta)' ];
My = (HPara/sqrt(3))*quad(FMy,0,2*pi);

FB=
[ 'sign',epsil,'./sqrt(' ,num2str(HPara,15),'+' ,gama,'./',epsil,')).^2).*',S,'.*sin
(2*beta)' ];
B = (HPara/sqrt(3))*quad(FB,0,2*pi);

FTsv = [ 'sign',gama,'./sqrt(' ,num2str(HPara,15),'*(' ,epsil,'./',gama,')).^2+1)' ];
Tsv = quad(FTsv,0,2*pi);

FTw=
[ 'sign',gama,'./sqrt(' ,num2str(HPara,15),'*(' ,epsil,'./',gama,')).^2+1).*',S];
Tw = Tsv - 2*pi*quad(FTw,0,2*pi)/prONa;

interF = [N; Mx; My; B; Tw; Tsv];

end

```

Levenberg-Maquardt Solution Program

```

function [xf, S, cnt] = LMFSolve(varargin)
% LMFSOLVE Solve a Set of Nonlinear Equations in Least-Squares Sense.
% =====
% A solution is obtained by Levenberg-Maquardt algorithm and minimizing
% sum of squares of 'n' equation residuals.
%
% [Xf, Ssq, CNT] = LMFSolve(FUN,Xo,Options)
%
% FUN      is a function handle or a function M-file name that returns
%           a vector of 'n' equation residuals,
% Xo       is a vector of 'n' initial guesses of solution,
% Options  is a structure to control program parameters
%           The function may be called without options to select the default
%           values
%
%           Options = LMFSolve('Name',Value, ... ), or updating the Options
%                   set by calling
%           Options = LMFSolve(Options,'Name',Value, ...).
%
% Name      Values [default]      Description
% 'Display'  integer              Display iteration information
%           [0] no display
%           k display initial and every k-th iteration;
% 'FunTol'   [1e-7]               norm(FUN(x),1) stopping tolerance;
% 'XTol'     [1e-7]               norm(x-xold,1) stopping tolerance;
% 'MaxIter'  [100]               Maximum number of iterations;
% No defined field of the "Options" structure is replaced by the
% default values.
%
% Output Arguments:
% Xf         final solution approximation
% Ssq        sum of squares of residuals
% Cnt        >0 count of iterations
%           -MaxIter, did not converge in MaxIter iterations
% =====

FUN=varargin{1}; % function handle
if ~(isvarname(FUN) || isa(FUN,'function_handle'))
    error('FUN Must be a Function Handle or M-file Name.')
end

xc=varargin{2}; % Xo

if nargin == 2 % No OPTIONS included
    options.Display = 1; % print of iterations
    options.MaxIter = 100; % maximum number of iterations allowed
    options.FunTol = 1e-7; % tolerance for final function value
    options.XTol = 1e-4; % tolerance on difference of x-solutions
end

if nargin > 2 % OPTIONS
    options=varargin{3};
end

x = xc(:);
lx = length(x);

R = feval(FUN,x); % Residuals at starting point
%=====
S = R'*R;
epsx = options.XTol(:);
epsf = options.FunTol(:);
if length(epsx)<lx, epsx=epsx*ones(lx,1); end
J = CalJacobian(FUN,R,x,epsx);
%=====
H = J.'*J; % System matrix
Nabla = J.'*R;

```

```

D = diag(diag(H)); % automatic scaling

Lambda=1; % Initial value
cnt = 0;
ipr = options.Display;
printit(ipr,-1); % Print the table header
d = options.XTol; % vector for the first cycle
maxit = options.MaxIter; % maximum number of iterations

while cnt<maxit && ... % MAIN ITERATION CYCLE
    any(abs(d) >= epsx) && ...
    any(abs(R) >= epsf)
    d = (H+Lambda*D)\Nabla; % negative solution increment
    xd = x-d;
    Rd = feval(FUN,xd);

    Sd = Rd.'*Rd;

    if Sd < S
        cnt = cnt+1;
        Lambda = Lambda/10;
        if ipr~=0
            printit(ipr,cnt,S,Lambda)
        end
        S = Sd;
        x = xd;
        R = Rd;
        J = CalJacobian(FUN,R,x,epsx);
        H = J'*J;
        Nabla = J'*R;
    elseif Sd > S
        Lambda = Lambda*10;
    end
end

xf=x; % final solution
if cnt==maxit
    cnt = -cnt;
end % maxit reached

%=====
% Numerical approximation of Jacobi matrix
%
function J = CalJacobian(FUN,r,x,epsx)

lx=length(x);
J=zeros(length(r),lx);
for k=1:lx
    dx=.25*epsx(k);
    xd=x;
    xd(k)=xd(k)+dx;
    rd=feval(FUN,xd);

    J(:,k)=(rd-r)/dx;
end

%=====
% Printing the intermediate results
%
function printit(ipr,cnt,SS,Lambda)
%
% ipr > 0 print every (ipr)th iteration
% = 0 do not print
% cnt = -1 prints the header
% >0 prints SSR and Lambda
if ipr~=0
    if cnt == -1 % table header
        disp('')
        disp(char(' '*ones(1,40)))
        fprintf(' iter SUM(Res.^2) Lambda\n');
        disp(char(' '*ones(1,40)))
        disp('')
    end
end

```

```

else                                     % iteration output
    f='%12.3e  ';
    fprintf(['%4.0f      ' f f '\n'],cnt,SS,Lambda);
end
end
end

```

Output of Program

For the case of

Nr = .3; Trsv = 0.4; Trw = 0.1; Mrx = 0.1; Mry = 0.5; Br = 0.5;

The program output is

```

*****
Iter      SUM(Res.^2)      Lambda
*****
  1         3.545e-001      1.000e-001
  2         1.059e-001      1.000e-002
  3         3.753e-002      1.000e-003
  4         3.136e-003      1.000e-004
  5         1.389e-005      1.000e-005
  6         1.200e-008      1.000e-006
  7         4.716e-016      1.000e-007

```

The solution values are:

```

Alpha = 0.95075
C1 = 0.55566
C2 = 2.5038
C3 = 1.2553
C4 = 1.3365
C5 = 1.2903

```

Appendix II: Modifying the Lower Bound Interaction Relations for EHS

The objective of this appendix is to illustrate how the column curves derived in chapter 5 along with the plastic interaction relations derived in chapter 2 can be combined to formulate interaction relations for design purposes. A complete development of the process is code dependent, but the ideas presented here remain valid.

The interaction equation for EHS of common geometry subject to axial tensile load N , bending moment about major axis M_x , bending moment about minor axis M_y and torsion T as given is

$$\left[\frac{\frac{M_x}{M_{xp}}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{2.0} + \left[\frac{\frac{M_y}{M_{yp}}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{1.7} + 2 \left[\frac{\frac{N}{N_p}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{1.75} - \left[\frac{\frac{N}{N_p}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{3.5} = 1 \quad (\text{II.1})$$

where $M_{xp} = 2.695F_y t a^2$ is the major axis bending moment plastic capacity, $M_{yp} = 1.594F_y t a^2$ is the minor axis bending moment plastic capacity, $T_p = 2\pi a b t F_y / \sqrt{3}$ is the torsional plastic capacity and $N_p = 4.770F_y t a$ is the axial plastic capacity, all in absence of any other load.

When a member is subject to axial compressive load C instead of tension, two modifications need to be made to Eq. II.1

- 1- The plastic capacity of the section under tension, N_p needs to be replaced by the member resistance under compression C_r .
- 2- The applied bending moments, M_x and M_y need to be replaced by the corresponding second order bending moments $\hat{M}_x$ and $\hat{M}_y$ to incorporate the $p-\delta$ and $P-\Delta$ effects.

Applying these two modifications, one obtains the modified interaction equation as

$$\left[\frac{\frac{\hat{M}_x}{M_{xp}}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{2.0} + \left[\frac{\frac{\hat{M}_y}{M_{yp}}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{1.7} + 2 \left[\frac{\frac{C}{C_r}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{1.75} - \left[\frac{\frac{C}{C_r}}{\sqrt{1 - \left(\frac{T}{T_p}\right)^2}} \right]^{3.5} = 1 \quad (\text{II.2})$$

In Eq. II.2, the value of C_r is to be determined from column curves provided in Section 5.10. The values of modified bending moments need to be determined using either (1) a finite element analysis including geometric and material nonlinear effects or (2) a code approach. Code approaches are generally based on increasing the applied bending moments by a magnification factor (e.g., AISC-360 2005; CAN/CSA-S16 2009) or reducing the plastic capacity of the section by a reduction factor (e.g., EN 1993-1-1 2005). Given the variety of code approaches and the illustrative nature of this appendix, it is decided to use the FEA approach. In the FEA approach, the second order bending moments at the section of interest are calculated using the internal stresses and the deformed configuration of that section.

Design Example

The problem described in the Nowzartash and Mohareb (2009) (Fig. II.1) is solved here. We recall that the forces applied to the design section (i.e., middle portion of the member) are $C = 1000 \text{ kN}$, $M_x = 50 \text{ kNm}$, $M_y = 15 \text{ kNm}$ and $T = 20 \text{ kNm}$. Member length is 3 m and it is simply supported with respect to bending moments.

A difference from the solution presented in Nowzartash and Mohareb (2009) is the inclusion of the geometric nonlinearity. This enabled the ABAQUS model to capture second order resistance reduction caused by the P-delta effect. The required sectional properties of EHS250×125×8 as extracted from Corus tubes (2005) are, radii of gyration $r_x = 77.8 \text{ mm}$, $r_y = 44.9 \text{ mm}$, plastic section moduli $Z_x = 307 \times 10^3 \text{ mm}^3$, $Z_y = 186 \times 10^3 \text{ mm}^3$ and cross sectional area $A = 4510 \text{ mm}^2$.

When calculating the compressive resistance C_r , it is conservative to use the weak axis buckling resistance. Under the Canadian Standards, for example, one has

$$\lambda_y = \frac{K_y L}{\pi r_y} \sqrt{\frac{F_y}{E}} = 0.89$$

$$C_r = C_{ry} = A F_y \left(1 + \lambda_y^{2 \times 1.34}\right)^{-1/1.34} = 1048 \text{ kN}$$

Other cross section plastic capacities are

$$M_{xp} = Z_x F_y = 307 \times 10^3 \times 350 = 107.4 \text{ kNm}$$

$$M_{yp} = Z_y F_y = 186 \times 10^3 \times 350 = 65.1 \text{ kNm}$$

$$T_p = 2\pi abtF_y / \sqrt{3} = 2 \times \pi \times 121 \times 58.5 \times 8 \times 350 / \sqrt{3} = 71.8 \text{ kNm}$$

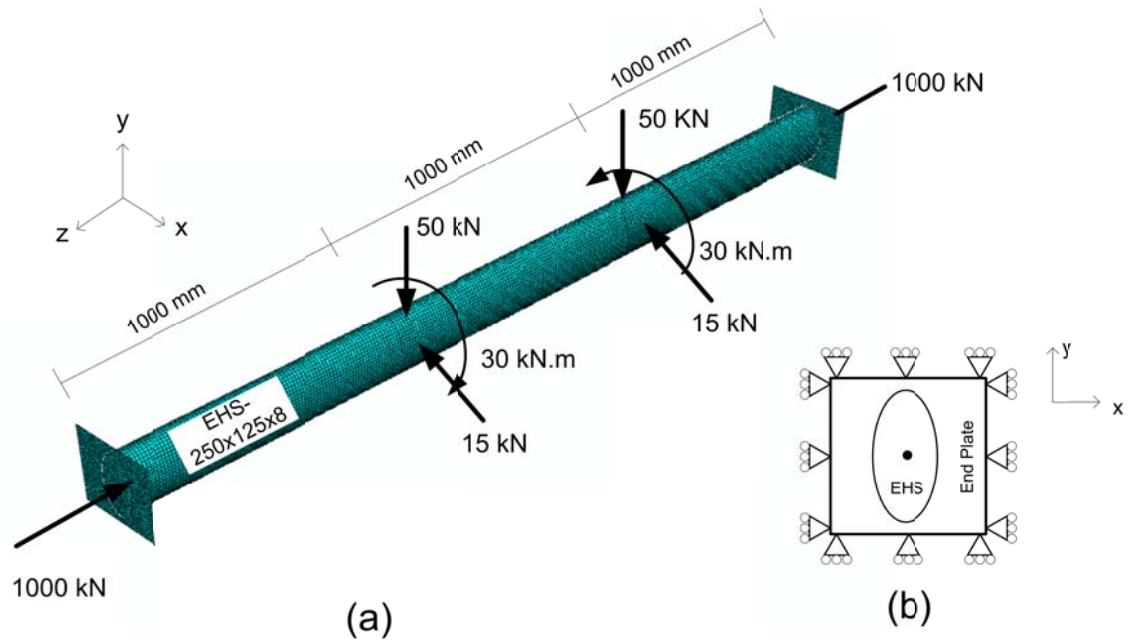


Fig. II.1: (a) FEA model, (b) End plates and boundary conditions

From the FEA model, the maximum bending moments at the middle section of the member were found as $\hat{M}_x = 51.5 \text{ kNm}$ and $\hat{M}_y = 16.4 \text{ kNm}$. By using the modified interaction equation one may calculate the load proportionality factor α from

$$\left(\alpha \frac{51.5/107.4}{\sqrt{1-(\alpha \times 20/71.8)^2}} \right)^{2.0} + \left(\alpha \frac{16.4/65.1}{\sqrt{1-(\alpha \times 20/71.8)^2}} \right)^{1.7} + 2 \left(\alpha \frac{1000/1038}{\sqrt{1-(\alpha \times 20/71.8)^2}} \right)^{1.75} - \left(\alpha \frac{1000/1038}{\sqrt{1-(\alpha \times 20/71.8)^2}} \right)^{3.5} = 1 \quad (\text{II.3})$$

Equation II.3 is solved using the Newton-Raphson method (e.g. the solver feature of MS Excel) resulting into $\alpha = 0.734$. This value compares to 0.808 as predicted from FEA.

The load proportionality factor, α , versus mid-span deflection is depicted in Fig. II.2. The dotted line is based on an elasto-plastic analysis of the member excluding second order effects (geometric nonlinearity) while the solid line is based on an analysis including geometric and material nonlinearity. Also shown on the figure are the values based on the Canadian Standard (CAN/CSA-S16 2009) and Eurocode 3 (EN 1993-1-1 2005) interaction relations.

The interaction relation clause used in the Canadian Standard is 13.8.3 and it is 6.3.3(4) in the European code. For the Eurocode interaction relations, the second method (i.e., Annex B, table B.1) has been used. Since the load pattern of this example does not exist in the Eurocode, the values of C_{my} , C_{mz} and C_{mLT} in Annex B were conservatively assumed as 1.0.

As can be seen, in this example, the proposed interaction relation approximates the buckling load of the member as 91% of that predicted by the FEA. This is an improvement compared to the current Canadian Standard which predicts 68% of the FEA solution and the Eurocode which predicts 79% of the FEA solution.

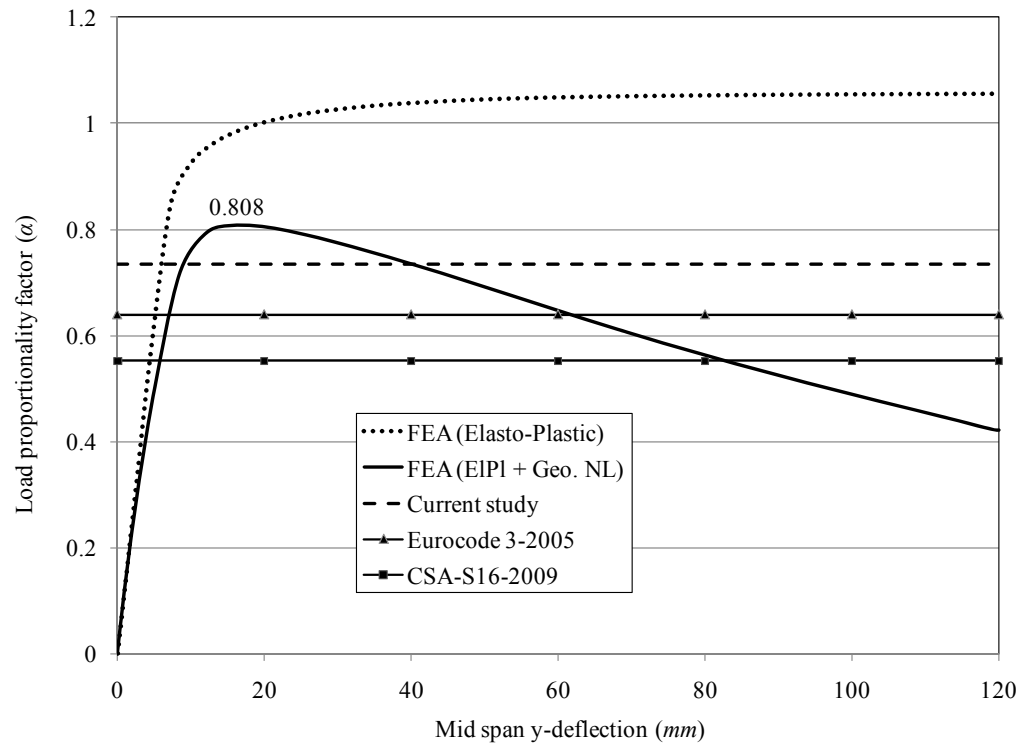


Fig. II.2: Load vs. Deflection curve for the design example