IMPROVED SOLUTIONS FOR THE RELIABLE NETWORK COMMUNICATION PROBLEM

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WHAT ARE RELIABLE NETWORKS

A reliable network is one that still allows communication if a link fails.

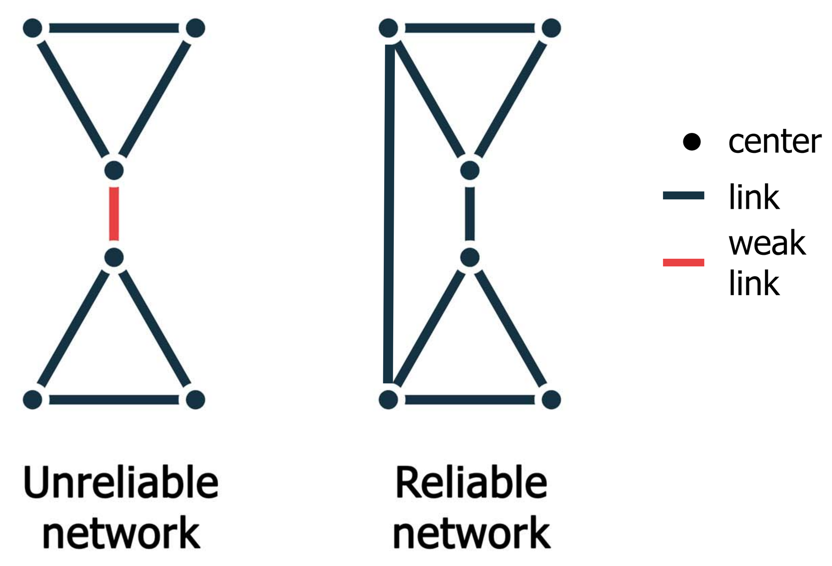


Figure 1. Reliable and unreliable networks

RELIABLE NETWORK COMMUNICATION PROBLEM

The Reliable Network Communication Problem (RNCP) consists of finding the minimum cost reliable network, given all possible links between centers, and the cost of those links.

For example, Figure 2 describes the minimum cost network that can be built from a sample set of centers.

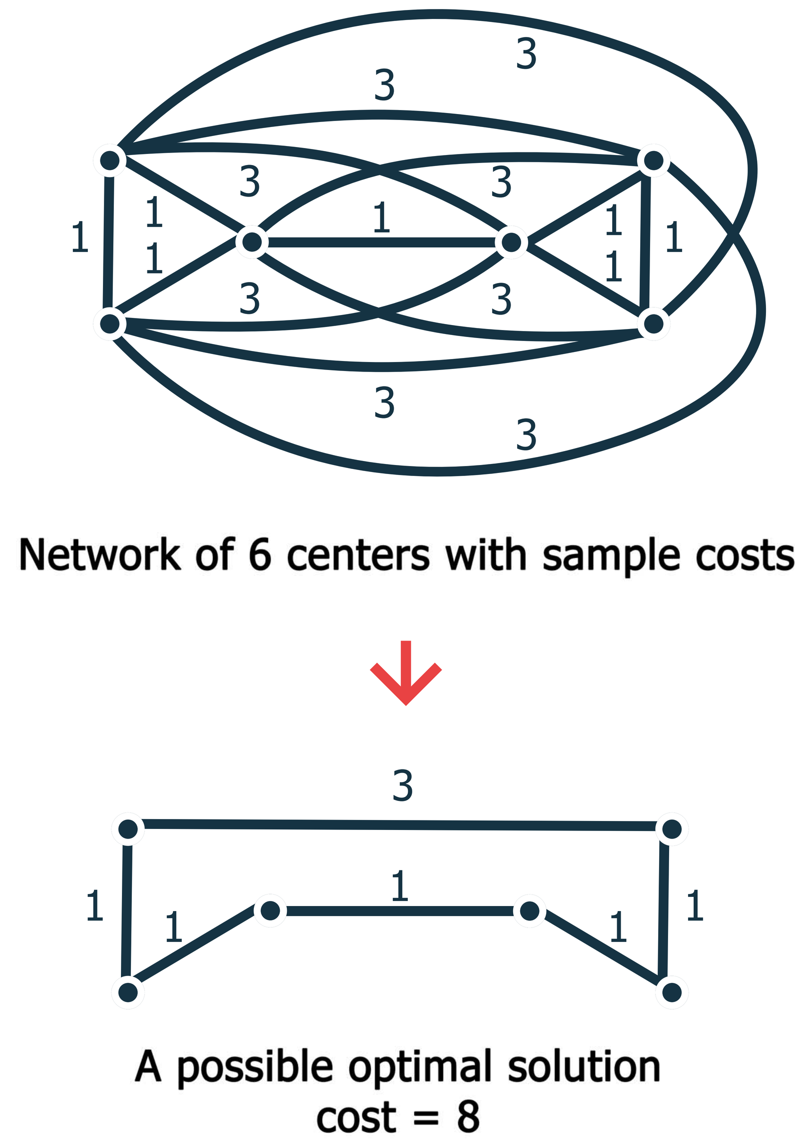


Figure 2. Reliable network of minimal cost for sample network of 6 centers

Obstacle

RNPC is known to be NP-hard, which means that it is highly unlikely that an efficient algorithm to solve it exists. For real world instances, **k**-approximation algorithms are used, which guarantee the following:

Solution cost ≤ **k** * Optimal solution cost

Improving Solution Quality

The integrality gap, **k**, is the factor of optimality of a solution produced by an approximation algorithm

Smaller **k** ⇒ Better performance guarantee

The smallest value of **k** found so far is 3/2. There exist several conjectures as to the minimal possible value of **k** [1].

- **Conjecture 1**: k = 4/3
  Open for 35 years
- **Conjecture 2**: k = 6/5
  Open for 8 years

SCOPE

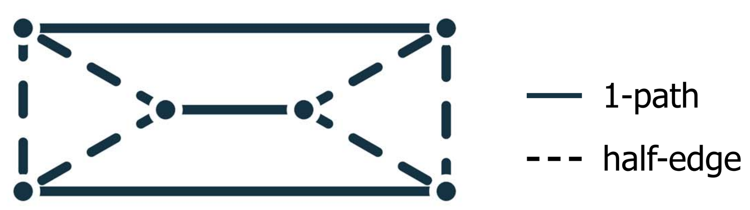
Alexander, Boyd & Elliot-Magwood identify Conjecture 2 in [1] as basis for future work:

> "This project focuses on the investigation of an integrality gap k of 6/5 for a special class of the RNCP, which we shall call half-integer triangle networks class."

Half-Integer Triangle Networks Class

Relaxing a problem means to eliminate some constraints that make it difficult to compute a solution. In the case of the RNCP, we relax the edge selection to allow selection half-links (ie. selecting links by a fraction that is 1/2).

The half-integer triangle networks consist of half-integer triangles connected between them by 1-paths, which are single chains of links and centers. Boyd and Carr identify this family of graphs as having the property of being the optimal half-integer relaxed solution when every edge is selected once [2].



Previous Work

- Carr & Ravi proved a k of 4/3 for this class [2]
- Alexander, Boyd & Elliot-Magwood proved a k of 6/5 for this class, but only for networks having less than 15 centers [1].

Methodology

Investigating is done by making convex combinations of solutions that use every 1-path **k** times and half-edge **k/2** times. One of these is guaranteed to be within **k** of the optimal solution.

One of the key problems in generating the convex combinations is keeping the connectivity property. Structures called gadgets, such as the one in figure 4, are easily disconnected by not selecting one of the edges on the sides.

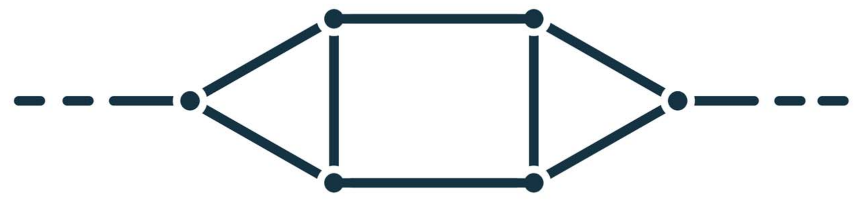


Figure 4. Sample gadget with 6 centers

RESULTS

Convex combinations for a **k** of 6/5 exist, for small instances of half-integer triangle networks. For instance, Figure 5 shows a convex combination allowing for an integrality gap **k** of 6/5 for the gadget of Figure 4.

As for 3-edge connected networks of this class, a promising series of patterns has been found, which is thought to ensure an integrality gap of 5/4. Such result cannot be extended to 2-edge connected networks.

CONCLUSION & FUTURE WORK

An integrality gap **k** of

- 6/5 can be proven for small instances of half-integer triangle networks.
- 5/4 can be shown for small gadgets.

Future work would be to extend the integrality gap of 5/4 or 6/5 to all graphs of the class.

ACKNOWLEDGEMENTS

Dr Sylvia Boyd

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REFERENCES

[1] Alexander, A., Boyd, S., & Elliot-Magwood, P. (2006). On the Integrality Gap of the 2-Edge Connected Subgraph Problem. Technical Report TR-2006-04, SITE, University of Ottawa, Ottawa, Canada, 12 pages.
[2] Carr, R., & Ravi, R. A New Bound for the 2-Edge Connected Subgraph Problem. in: Proceedings of Integer Programming and Combinatorial Optimization (IPCO), in: Lecture Notes in Computer Science, Springer, 1998, pp. 112-125.

Sample Convex Combination

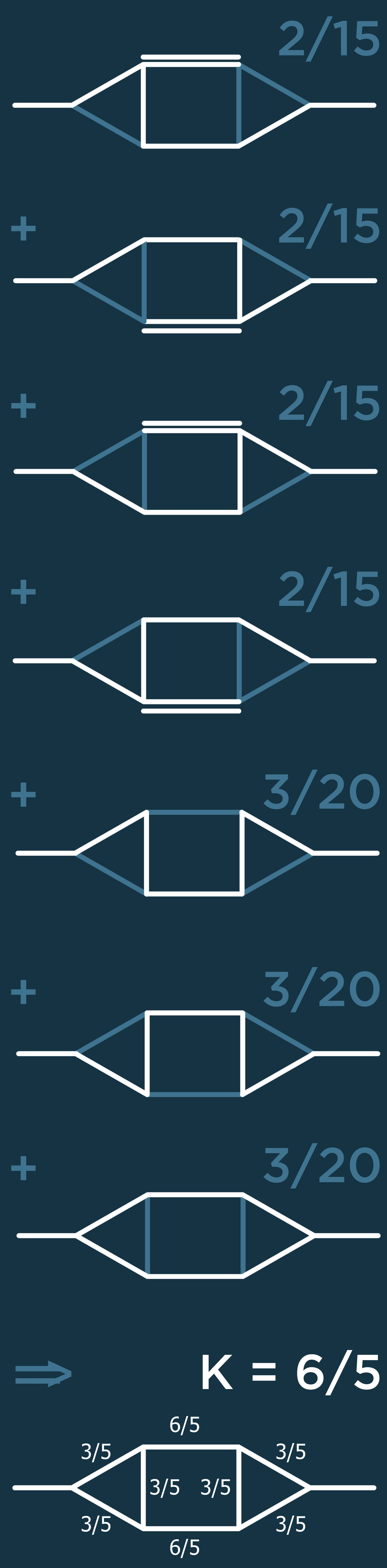

Figure 5. Convex combination of the gadget of figure 4, allowing for an integrality gap of 6/5.