STRESS DISTRIBUTION IN RECTANGULAR PLATES WITH INSERTS UNDER AXIAL LOADING

by

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ABSTRACT

The stress distribution in finite plates containing rectangular inserts and subjected to axial loads has been investigated in this thesis. For obtaining the solutions, the basic differential equations have been reduced in terms of finite difference form.

The influence of modulus of elasticity, shape and orientation of insert on the stress concentration produced has also been examined here.

Two-dimensional models containing inserts of rectangular shape and various orientations have been analysed further, by means of the photoelastic techniques. Experimental data have been obtained along the sections where rapid stress variations are expected. Shear difference procedure has been employed to calculate the principal stress and shear stress distributions along these sections.

NOMENCLATURE

X, Y	Rectangular co-ordinates
P_x, P_y	Normal stress in x and y direction
τ_{xy}	Shear stress
P_1, P_2	Principal stresses
u, v	Displacements in x and y directions
ϵ_x, ϵ_y	Strains in x and y directions
γ_{xy}	Shear strain
E	Young's modulus of elasticity
μ	Poisson's ratio
ϕ, Φ	Airy's stress function
N	Fringe order
f_σ	Stress optic coefficient
a, b	Dimensions of the rectangular insert
ρ	Radius of curvature
h	Step size
H	Thickness of the material

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CHAPTER 1

1. INTRODUCTION

All engineering structures and machine parts have to be designed so that they are strong and sufficiently rigid. At the same time, because of economic considerations, the quantity of the material used should be kept to a minimum. When designing structures and machines, various factors have to be taken into account. One of these is stress concentration or stress distribution in general. Stress distribution plays an important role in design problems. Its knowledge helps a designer to produce balanced designs.

The problem of stress distribution around holes, grooves, notches, shoulder fillets etc. has been treated extensively in the literature [1] but little work has been done on the stress distribution in a finite plate with inserts of different material.

In this thesis an attempt has been made to study the stress distribution in a finite plate with a rectangular insert. Both the theoretical and experimental investigations have been carried out. The object was to develop and evaluate methods, both mathematical and experimental which could then be employed in more complicated practical situations.

The well known classical solutions for the strip with central hole under tension and bending were obtained by Howland and Stevenson [2,3]. This work was later extended to the cases of eccentric circular holes and elliptical holes in infinite strips by Isida [4,5]. Other papers were published on the associated problem of stress concentration with two or more holes in infinite plates [6, 7] and the corresponding problems in strips of finite width [8,9].

The case of rectangular holes, with round corners, in infinite plates have also been studied [10]. The rectangular cut outs with different ratios of their sides and orientations have been analysed by Savin [11].

Isida [12] has studied the effects of rounding the corners of the cutout in an infinite strip which is an extension of Savin's problem.

Researchers have made attempts to determine the effect of reinforcements around the holes of various shapes [13, 14, 15] .

The work of Goodier [16] and Donell [17] also includes the problems of the stress distribution in infinite plates having circular and elliptical inclusions. The results for the case of a circular inclusion were confirmed experimentally by photoelastic method by Thibodeau and Wood [18] .

Edmonds and Beevers [19] have studied the stress distribution existing in and around hard inclusions in solids subjected to a uniaxially applied stress. Two-dimensional models containing inclusions of various shapes and orientations have been analysed by means of photoelastic technique. The influence of modulus of elasticity, shape and orientation of inclusions on the stress concentration has been further examined by a mathematical treatment. However, this study is confined mainly to the circular, elliptical and pentagonal inclusions and for particular values of modulus of elasticity.

The theoretical solution essentially consists of the evaluation of a certain stress function ϕ , which must satisfy the biharmonic equation $\nabla^4 \phi = 0$ at every point in the plate. This function is defined such that

$$P_x = \frac{\partial^2 \phi}{\partial y^2}$$

$$P_y = \frac{\partial^2 \phi}{\partial x^2}$$

$$\tau_{xy} = - \frac{\partial^2 \phi}{\partial x \, \partial y}$$

This is a mixed boundary value problem. Along the external boundary the conditions are prescribed in terms of stresses, while at the interface between the two materials the deformations as well as the stresses must be compared.

The computer program written for the solution of the problem has the flexibility in terms of any values of E, μ , different orientations and insert sizes. For experimental analysis, photoelasticity method, which gives complete exploration of stresses, is adopted. The experimental investigation has been carried out on a sophisticated portable polariscope (comparator type). The method of shear differences is employed for the separation of stresses. The modulii of elasticity for the two materials used in the experimental investigation are 340×10^3 psi and 1×10^3 psi. Inserts of different sizes and orientations are placed in a finite rectangular plate which is subjected to a uniform tension along its length.

CHAPTER 2

2. FORMULATION OF THE PROBLEM

The theoretical study of this problem mainly involves the calculation of stress function ϕ which should satisfy the biharmonic equation, $\nabla^4 \phi = 0$, at each point in the plate. Due to the complex nature of the problem, a numerical method using finite difference techniques has been employed. These techniques replace the governing equations and boundary conditions by finite number of unknown values of the dependent variables at a number of discrete points within or just outside the domain of integration.

The finite difference equations form a set of linear algebraic equations. The large number of simultaneous equations thus formed are solved on the digital computer.

Two orientations of insert with respect to the plate have been studied.

Case I : When the insert is parallel to the sides of the plate.

Case II: When the insert is inclined at an angle of 45° to the sides of the plate.

In each of the above two cases three different sizes of inserts have been studied.

In order to simplify the mathematical formulation of the problem, the insert has been kept parallel to x, y co-ordinates and the outside plate has been rotated. when required. The solution in each case has been obtained in three steps:

step 1, the generation of a system of non-homogeneous simultaneous equations at nodal points ;

step 2, the solutions of these equations, i.e. the determination of the value of the stress function ϕ at each nodal point;

step 3, calculation of the stresses from the stress function.

2.1. FUNDAMENTAL RELATIONS

In this problem of plane stress two possible formulations can be used. The problem can be set up in terms of a stress function or alternatively the governing equations could be written in terms of displacements. However, in this case use is made of Airy stress function. This is primarily because of the reason that the boundary conditions are prescribed in terms of stresses. Thus, the solution of the problem reduces to the integration of the differential equations of equilibrium together with the compatibility equations.

Neglecting the body and temperature forces, the differential equations of equilibrium for a two dimensional problem, in terms of cartesian co-ordinates are [20]

$$\frac{\partial P_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial P_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \qquad (2.1.1)$$

These two equations of equilibrium (2.1.1) are not sufficient for the determination of three unknown stress components P_x, P_y and τ_{xy}. The necessary third equation is derived by using the elastic properties of the body. In the case of two dimensional problem, the linear strain displacement relations are

$$\epsilon_x = \frac{\partial u}{\partial x}$$

$$\epsilon_y = \frac{\partial v}{\partial y} \qquad (2.1.2)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

where u and v are displacements in the x and y directions. Differentiating the first of these eqns. (2.1.2) twice with respect to y, the second twice with respect to x, and the third once with respect to x and once with respect to y, we obtain the compability equation:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \, \partial y} \qquad (2.1.3)$$

From Hooke's law, the stress-strain relations in a two dimensional problem are written as

$$\begin{aligned} \epsilon_x &= A\,(P_x - B\,P_y) \\ \epsilon_y &= A\,(P_y - B\,P_x) \\ \gamma_{xy} &= 2A\,(1+B)\,\tau_{xy} \end{aligned} \qquad (2.1.4)$$

where $A = \frac{1}{E}$ and $B = \mu$ for plane stress.

Substituting relations (2.1.4) into eqn. (2.1.3) and simplifying, we obtain the compatibility equation in terms of stress components:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (P_x + P_y) = 0 \qquad (2.1.5)$$

In order to solve eqns. (2.1.1) and (2.1.5), we introduce the stress function ϕ, which is defined such that

$$\begin{aligned} P_x &= \frac{\partial^2 \phi}{\partial y^2} \\ P_y &= \frac{\partial^2 \phi}{\partial x^2} \\ \tau_{xy} &= -\frac{\partial^2 \phi}{\partial x \, \partial y} \end{aligned} \qquad (2.1.6)$$

which identically satisfies eqn. (2.1.1).

Now substituting eqn. (2.1.6) into eqn. (2.1.5), we find that the stress function ϕ must satisfy the following equation :

$$\frac{\partial^4 \phi}{\partial x^4} + 2\,\frac{\partial^4 \phi}{\partial x^2 \, \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0 \qquad (2.1.7)$$

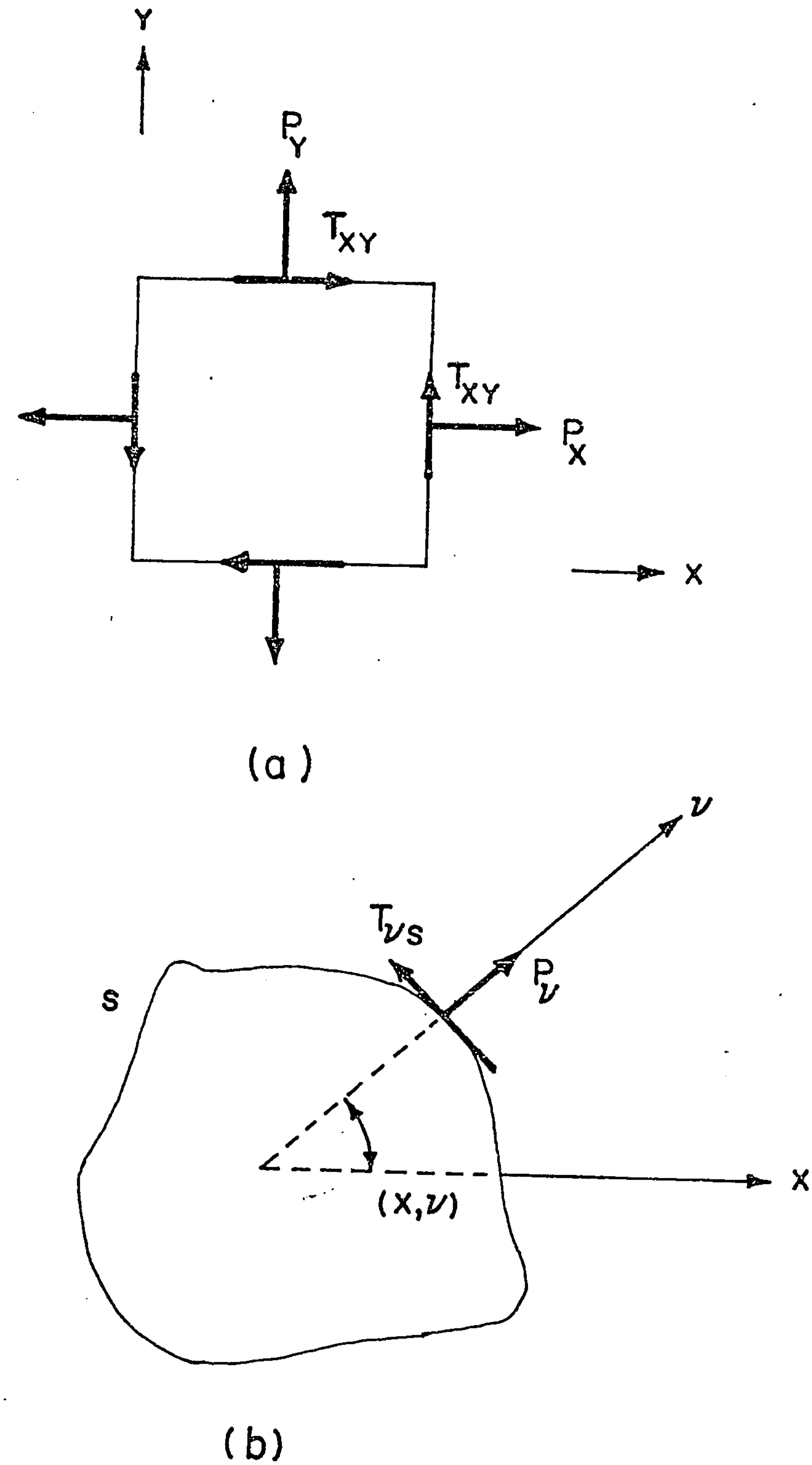


FIG.- I

which can also be written as

$$\nabla^4 \phi = 0 \tag{2.1.7}$$

where ∇^4 is the Laplacian operator

$$\nabla^4 \equiv \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

The problem is now reduced to that of finding a function ϕ which satisfies eqn. (2.1.7) at every point within the boundary and at the boundary.

2.2. BOUNDARY VALUES OF THE STRESS FUNCTION

For the complete solution of the problem, it is necessary that the boundary values of the stress function and its gradients be specified. As the stress components are represented by the second derivatives of ϕ, the addition of any linear function of co-ordinates of the form

$$C_1 x + C_2 y + C_3 = 0$$

where C_1, C_2, C_3 are constants, will not affect the stress system. Therefore it is possible, always, to assume arbitrarily convenient values of ϕ, $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ at some boundary point. If the external forces are known these values can now be determined at all points of the boundary by a simple integration [21].

Let P_ν and $\tau_{\nu s}$ represent the values of normal direct stress and the tangential shear stress respectively at a point on the boundary. The ν denotes the direction of the outward drawn normal at the edge and s the direction of the tangent in a counter clockwise circuit in fig. 1.

We know that at the boundary, the components of stresses $P_{\nu x}$ and $P_{\nu y}$ in x and y direction respectively can be given by

$$P_{\nu x} = P_\nu \cos(x, \nu) - \tau_{\nu s} \sin(x, \nu) \quad \text{and}$$

$$P_{\nu y} = P_\nu \sin(x, \nu) + \tau_{\nu s} \cos(x, \nu)$$

But also

$$P_{\nu x} = P_x \cos(x, \nu) + \tau_{xy} \sin(x, \nu) \qquad (2.2.1)$$

and

$$P_{\nu y} = P_y \sin(x, \nu) + \tau_{xy} \cos(x, \nu) \qquad (2.2.2)$$

Now substituting in the above equations for P_x and P_y in terms of ϕ and for $\cos(x, \nu)$ and $\sin(x, \nu)$, we get

$$P_{\nu x} = \frac{\partial^2 \phi}{\partial y^2} \cdot \frac{\partial y}{\partial s} + \frac{\partial^2 \phi}{\partial x \, \partial y} \cdot \frac{\partial x}{\partial s}$$

which can also be written as

$$P_{\nu x} = \frac{\partial}{\partial s} \left(\frac{\partial \phi}{\partial y} \right)$$

and similarly

$$P_{\nu y} = - \frac{\partial^2 \phi}{\partial x^2} \cdot \frac{\partial x}{\partial s} - \frac{\partial^2 \phi}{\partial x \, \partial y} \cdot \frac{\partial y}{\partial s}$$

or

$$= - \frac{\partial}{\partial s} \left(\frac{\partial \phi}{\partial x} \right)$$

Hence by line integration round the boundary we can obtain boundary values for $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ as

$$\frac{\partial \phi}{\partial x} = - \int P_{\nu y} \, ds$$

$$\frac{\partial \phi}{\partial y} = \int P_{\nu x} \, ds \qquad (2.2.3)$$

To find ϕ we use the equation

$$\frac{\partial \phi}{\partial s} = \frac{\partial \phi}{\partial x} \cdot \frac{dx}{ds} + \frac{\partial \phi}{\partial y} \cdot \frac{dy}{ds}$$

which, after integration by parts, gives

$$\phi = x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} - \int \left(x \frac{\partial \phi}{\partial s \, \partial x} + y \frac{\partial^2 \phi}{\partial s \, \partial y} \right) ds \qquad (2.2.4)$$

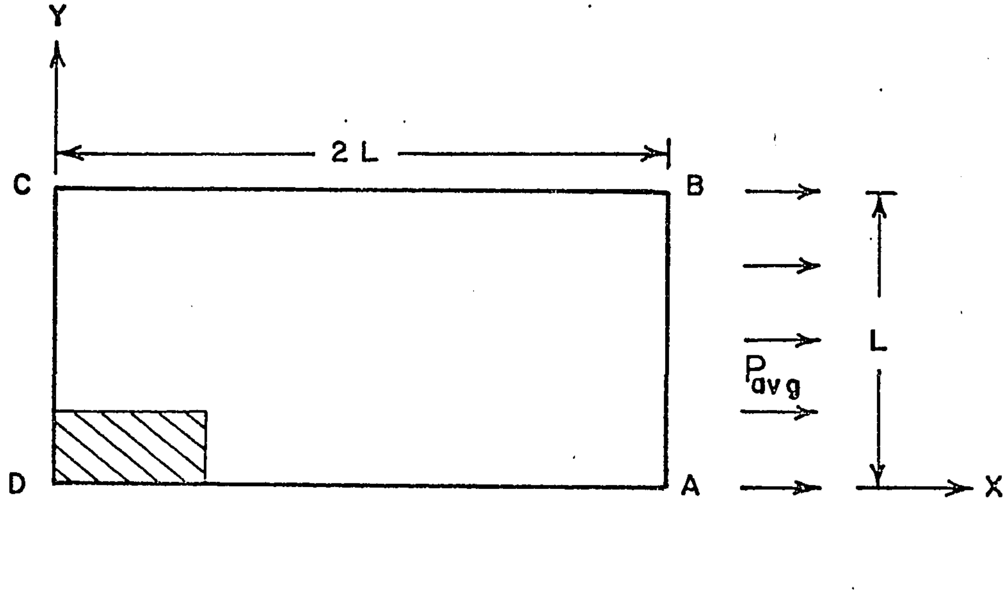


FIG.- 2

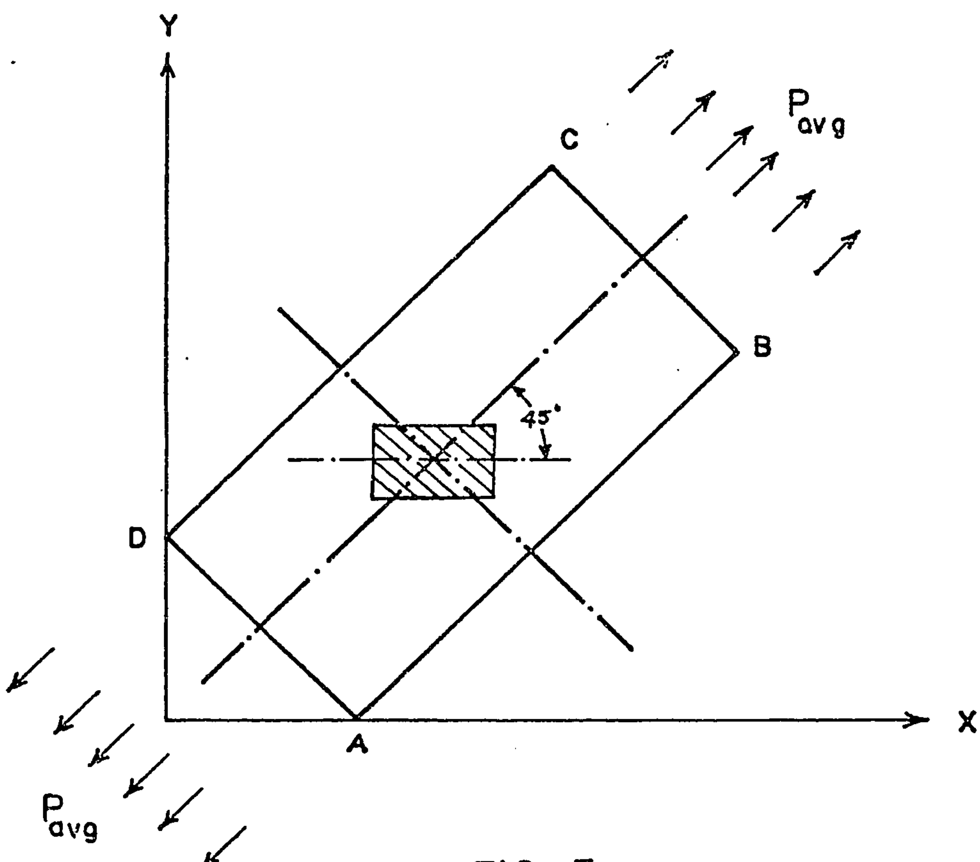


FIG.- 3

Substituting in this equation the values of the derivatives given by eqn. (2.2.3), we calculate the boundary values of ϕ .

The boundary values of the stress function for the two different orientations of insert have been given below

Case I - 0^{o} Orientation (Refer fig. 2)

In this case the plate has two-fold symmetry. Therefore it is sufficient to work with one quarter of the plate. The half width, AB, of the plate is L and the half length DA is 2L. The intensity of the applied tension along AB is 100 (a non-dimensional value).

Then the stress boundary conditions on ABC are

$$P_{vy} = 0 \quad \text{everywhere}$$

$$P_{vx} = 100 \quad \text{on } AB$$

$$= 0 \quad \text{on } BC$$

Now substituting these values in eqn. (2.2.3), we get

$$\frac{\partial \phi}{\partial x} = 0 \quad \text{everywhere } (\because \frac{\partial \phi}{\partial x} = 0 \text{ at C due to symmetry})$$

and

$$\frac{\partial \phi}{\partial y} = 100\, y + C_1 \quad \text{along } AB$$

$$= 100\, y \quad (C_1 = 0 \text{ because } \frac{\partial \phi}{\partial y} = 0 \text{ at A due to symmetry}).$$

Similarly,

$$\frac{\partial \phi}{\partial y} = 100 \quad \text{on } BC$$

Now substituting the values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ in eqn. (2.2.4) we obtain

$$\phi = 50\, y^2 \quad \text{on } AB$$

$$= 50 \quad \text{on } BC$$

where constant of integration is chosen to be such that ϕ vanishes at A.

Case II - 45° orientation (Refer fig. 3)

The orientation of the plate A B C D with respect to the x-y axes requires that the plate A B C D be taken as the working diagram. Let the length of the plate be 2L and its width be L. Again the intensity of applied tension along A D and B C is 100 (non-dimensional value). Using eqns. (2.2.3) and (2.2.4), we obtain the values of $\frac{\partial \phi}{\partial x}$, $\frac{\partial \phi}{\partial y}$ and ϕ along the boundary A B C D as follows.

Along the boundary A B :

$$\frac{\partial \phi}{\partial x} = 150\sqrt{2}$$

$$\frac{\partial \phi}{\partial y} = 100\sqrt{2}$$

$$\phi_{AB} = 150\sqrt{2}\,x + 100\sqrt{2}\,y - 325$$

Along the boundary B C :

$$\frac{\partial \phi}{\partial x} = 100\,x$$

$$\frac{\partial \phi}{\partial y} = 100\,y$$

$$\phi_{BC} = 50x^2 + 50y^2$$

Along the boundary C D :

$$\frac{\partial \phi}{\partial x} = 100\sqrt{2}$$

$$\frac{\partial \phi}{\partial y} = 150\sqrt{2}$$

$$\phi_{CD} = 100\sqrt{2}\,x + 150\sqrt{2}\,y - 325$$

Along the boundary A D :

$$\frac{\partial \phi}{\partial x} = 100\,x + 100\sqrt{2}$$

$$\frac{\partial \phi}{\partial y} = 100\,y + 100\sqrt{2}$$

$$\phi_{AD} = 50x^2 + 50y^2 + 100\sqrt{2}\,x + 100\sqrt{2}\,y - 300$$

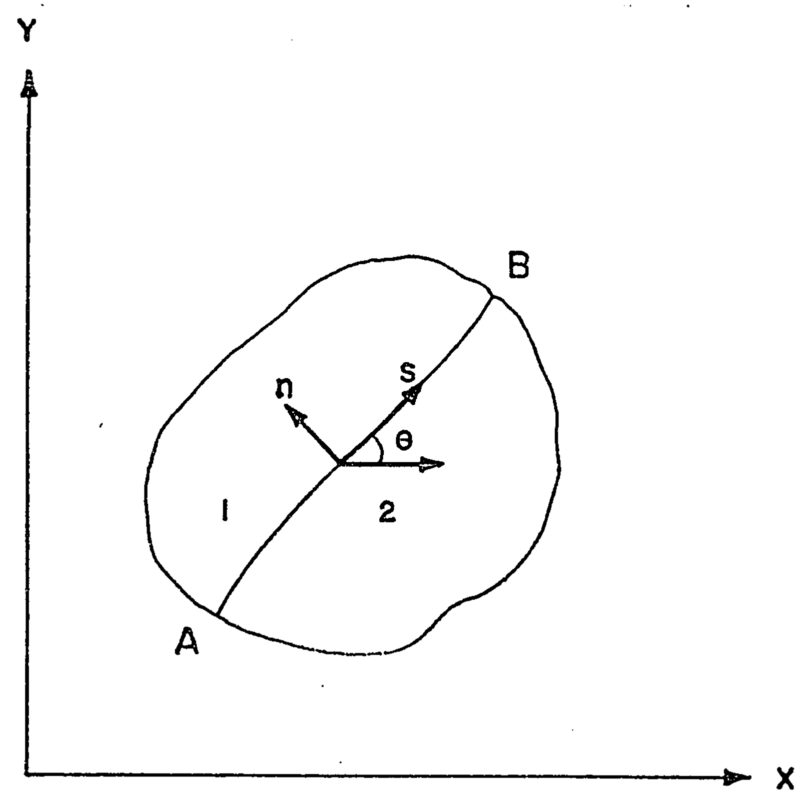


FIG.- 4

For detailed derivation of this case see appendix (I.1).

2.3 INTERFACE CONDITIONS

Some special conditions have to be imposed on the stress function for the interface separating the two materials having different elastic properties [22]. These conditions are

1. Interface displacement conditions
2. Interface stress conditions

2.3.1. Interface Displacement Conditions

Let the line AB in fig.4 denote an interface between the two regions of different elastic properties. It is assumed that on this interface no slippage is possible and therefore the stresses in each region must be such that a "perfect fit" of displacement is assured. If u_1' and v_1' represent respectively the tangential and normal components of displacement for region 1 along any surface s and u_2', v_2' represent the similar displacement for region 2, then

$$\frac{\partial u_1'}{\partial s} = \frac{\partial u_2'}{\partial s}$$

and

$$\frac{\partial^2 v_1'}{\partial s^2} = \frac{\partial^2 v_2'}{\partial s^2}$$

which ensures that the tangential 'stretching' and curvatures of the two regions are identical.

With the stresses given in terms of the appropriate stress functions, the interface conditions expressed in terms of two stress functions ϕ_1 and ϕ_2 are [22]

$$A_1\left[\frac{\partial^2\phi_1}{\partial n^2} - B_1\left(\frac{\partial^2\phi_1}{\partial s^2} - \frac{1}{\rho}\frac{\partial\phi_1}{\partial n}\right)\right] = A_2\left[\frac{\partial^2\phi_2}{\partial n^2} - B_2\left(\frac{\partial^2\phi_2}{\partial s^2} - \frac{1}{\rho}\frac{\partial\phi_2}{\partial n}\right)\right] \tag{2.3.1}$$

and

$$- A_1 \left[\frac{\partial}{\partial n} (\nabla^2 \phi_1 + (1+B_2) \frac{\partial}{\partial s} \left(\frac{\partial^2 \phi_1}{\partial n \, \partial s} \right) \right] = -A_2 \left[\frac{\partial}{\partial n} (\nabla^2 \phi_2) + (1+B_2) \frac{\partial}{\partial s} \left(\frac{\partial^2 \phi_2}{\partial n \, \partial s} \right) \right]$$

in which $\frac{1}{\rho} = \frac{\partial \theta}{\partial s}$ represents the curvature of the boundary

$$A = \frac{1}{E}$$

and $B = \mu$

with appropriate suffix.

2.3.2. Interface Stress Conditions

In addition to the displacement conditions it is necessary that the normal and tangential stress in both regions be equal on the interface which is a condition from the considerations of static equilibrium.

The value of normal stress in terms of stress function is given as

$$P_n = \frac{\partial^2 \phi}{\partial x^2} \cos^2 \theta + \frac{\partial^2 \phi}{\partial y^2} \sin^2 \theta + \frac{\partial^2 \phi}{\partial x \, \partial y} \sin 2\theta$$

$$\equiv \frac{\partial^2 \phi}{\partial s^2} - \frac{1}{\rho} \frac{\partial \phi}{\partial n}$$

and the tangential stress is given as

$$\tau_{ns} = \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) \frac{1}{2} \sin 2\theta - \frac{\partial^2 \phi}{\partial x \, \partial y} \cos 2\theta$$

$$\equiv - \frac{\partial^2 \phi}{\partial n \partial s}$$

Thus the stress equality interface conditions become

$$\frac{\partial^2 \phi_1}{\partial s^2} - \frac{1}{\rho} \frac{\partial \phi_1}{\partial n} = \frac{\partial^2 \phi_2}{\partial s^2} - \frac{1}{\rho} \frac{\partial \phi_2}{\partial n} \qquad (2.3.3)$$

and

$$\frac{\partial^2 \phi_1}{\partial n \, \partial s} = \frac{\partial^2 \phi_2}{\partial n \, \partial s} \qquad (2.3.4)$$

The eqns.(2.3.3) and (2.3.4) can be further simplified using the fact that an addition of any linear function of x and y to either stress function does not alter the stresses. Therefore, arbitrarily, the equality of the two stress functions and their slopes is imposed at some point. Hence the eqns. (2.3.3) and (2.3.4) reduce to

$$\phi_1 = \phi_2 \tag{2.3.5}$$

and

$$\frac{\partial \phi_1}{\partial n} = \frac{\partial \phi_2}{\partial n} \tag{2.3.6}$$

In addition, in each region the stress function has to satisfy the general governing equation

$$\nabla^4 \phi_1 = \nabla^4 \phi_2 = 0$$

Now, in the case of straight interface between two regions parallel to x-axis in which the stress functions are defined as $\phi_1 = \phi$ and $\phi_2 = \Phi$ and which have the elastic constants A1, B1 and A2, B2 respectively

$$\frac{1}{\rho} = 0, \quad \partial n = \partial y, \quad \partial s = \partial x \; .$$

Therefore, substituting these values in the eqns.(2.3.1), (2.3.2), (2.3.5) and (2.3.6), the following conditions are obtained:

$$\phi = \Phi \tag{2.3.7}$$

$$\frac{\partial \phi}{\partial y} = \frac{\partial \Phi}{\partial y} \tag{2.3.8}$$

$$A_1 \left[\frac{\partial^2 \phi}{\partial y^2} - B_1 \frac{\partial^2 \phi}{\partial x^2}\right] = A_2 \left[\frac{\partial^2 \Phi}{\partial y^2} - B_2 \frac{\partial^2 \Phi}{\partial x^2}\right] \tag{2.3.9}$$

$$-A_1 \left[\frac{\partial^3 \phi}{\partial y^3} + (2+B_1) \frac{\partial^2 \phi}{\partial x^2 \partial y}\right] = - A_2 \left[\frac{\partial^3 \phi}{\partial y^3} + (2+B_2) \frac{\partial^3 \Phi}{\partial x^2 \partial y}\right]$$

(2.3.10)

Similarly in the case of the straight interface parallel to y-axis we have

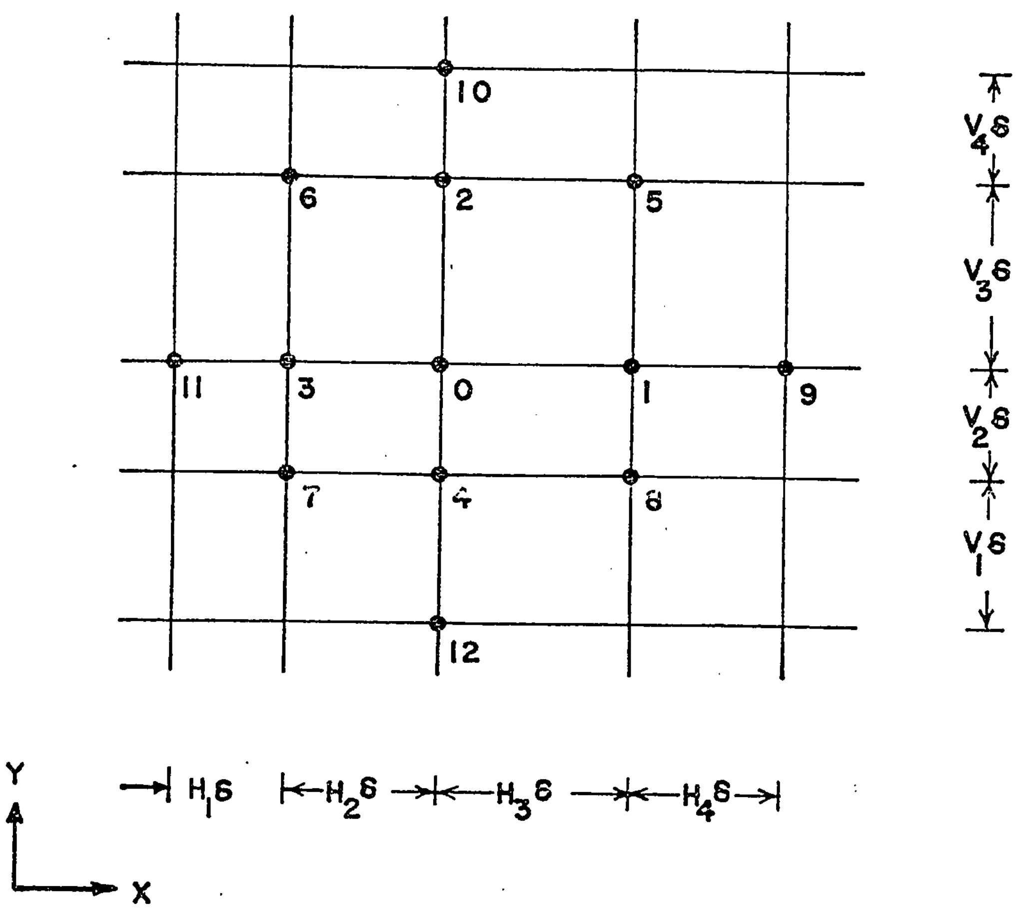

10
6
2
5
11
3
0
1
9
7
4
8
12
$V_4\delta$
$V_3\delta$
$V_2\delta$
$V_1\delta$
Y
X
$H_1\delta$
$H_2\delta$
$H_3\delta$
$H_4\delta$

FIG.- 5

$$\frac{1}{\rho} = o$$

$$\partial n = \partial x$$

$$\partial s = \partial y$$

Substitution of these values in eqns. (2.3.5), (2.3.6), (2.3.1) and (2.3.2) yields

$$\phi = \Phi \qquad (2.3.11)$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \Phi}{\partial x} \qquad (2.3.12)$$

$$A_1 \left[\frac{\partial^2 \phi}{\partial x^2} - B_1 \frac{\partial^2 \phi}{\partial y^2} \right] = A_2 \left[\frac{\partial^2 \Phi}{\partial x^2} - B_2 \frac{\partial^2 \Phi}{\partial y^2} \right] \qquad (2.3.13)$$

and

$$-A_1 \left[\frac{\partial^3 \phi}{\partial x^3} + (2+B_1) \frac{\partial^3 \phi}{\partial y^2 \partial x} \right] = -A_2 \left[\frac{\partial^3 \Phi}{\partial x^3} + (2+B_2) \frac{\partial^3 \Phi}{\partial y^3 \partial x} \right]$$

(2.3.14)

2.4. NUMERICAL PROCEDURE - FINITE DIFFERENCE TREATMENT

The numerical values of the function ϕ are obtained at the nodes of the network covering the region of integration. The size of the network is kept as small as possible in order to achieve better accuracy. Various forms of the biharmonic operators in finite difference have been used for these nodal points depending upon their position relative to other points of the network. These forms in their string pattern are given below. For detailed derivation and notations see appendix II.

2.4.1 General Form

This form of biharmonic operator has been used for the nodal points where all the points associated with the operator fall within one region. When using for points near the outside boundary, the fictitious values for the points outside are calculated as described in the Art 2.4.2. Referring to fig. 5 the string pattern for this case can be given as shown

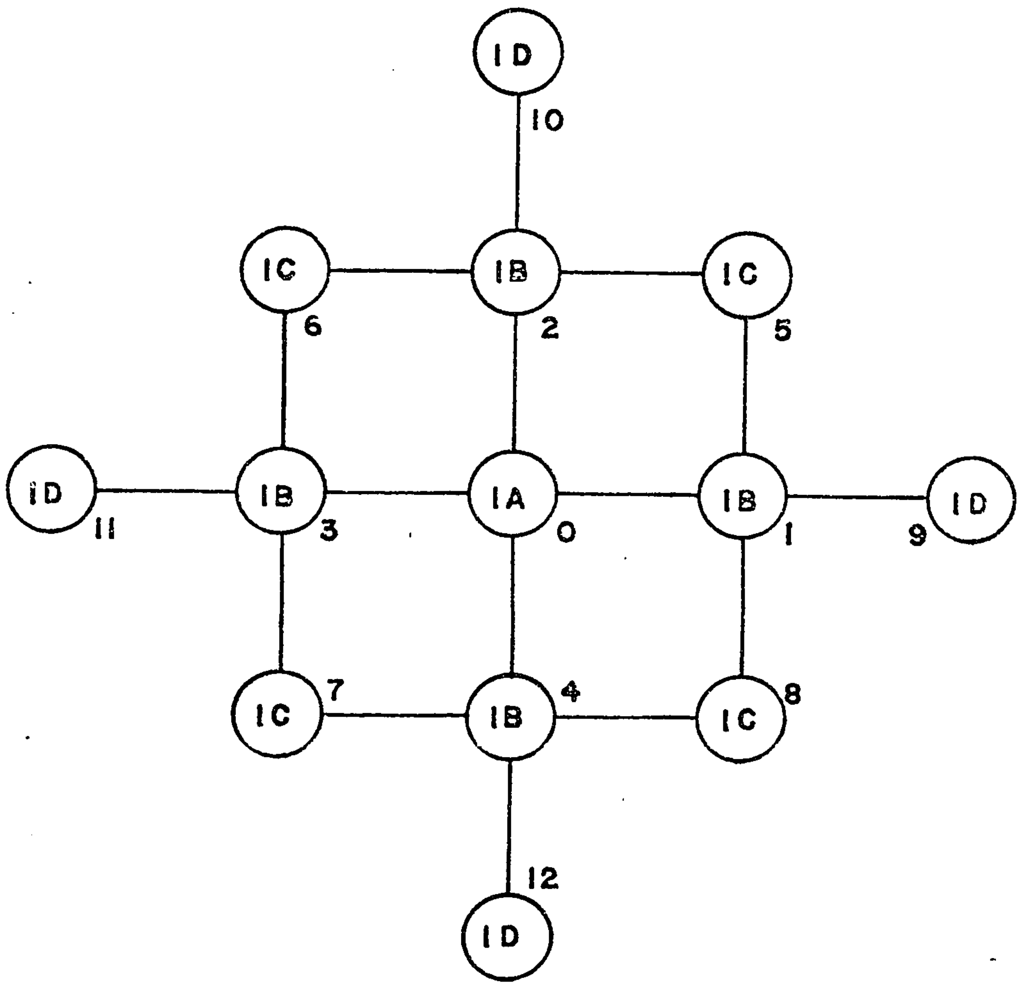


FIG.- 6

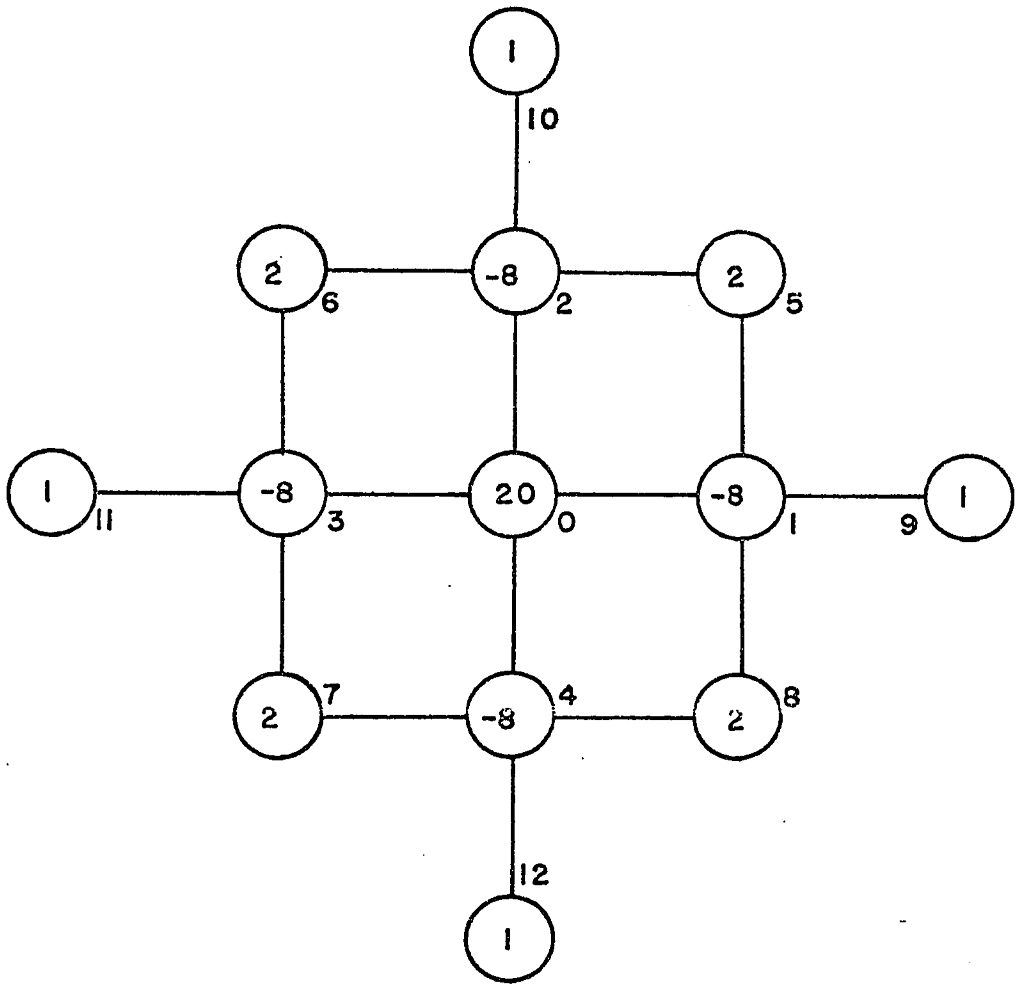


FIG.- 7

in fig. 6.

When the mesh is square, the above pattern reduces to the form shown in fig. 7.

2.4.2. Rectilinear Boundaries

a. When mesh line coincides with boundary: The two boundary conditions are appropriately associated with a biharmonic equation at all points of the boundary.

While writing the biharmonic equation at the nodal points which are only one mesh length within the boundary, a fictitious value is involved in the equation. This fictitious value can be removed by using one of the boundary conditions in finite difference form [21] .

b. When the mesh and the boundary do not coincide: When the boundary intersects the mesh at other than nodal points irregular stars exist and therefore the following method has to be used for finding the values at the fictitious nodes and at the nodes which are less than one mesh length away from the boundary. Both cases, when the points are along the increasing value of ϕ and when they are along the decreasing value of ϕ have been dealt with here [21] .

Case I

The values of ϕ can be attached to node 0 by the following relations (fig. 8)

$$\phi_9 = \frac{4\xi K}{(1+\xi)^2} + \frac{2(1-\xi)hk}{1+\xi} + \frac{(1-\xi)^2\phi_0}{(1+\xi)^2} , \tag{2.4.21}$$

$$\phi_1 = \frac{(1+2\xi)K}{(1+\xi)^2} - \frac{\xi hk}{1+\xi} + \frac{\xi^2\phi_0}{(1+\xi)^2}$$

where $\frac{\partial \phi}{\partial x}$ or $\frac{\partial \phi}{\partial y}$ at B are denoted by k and ϕ at B by K.

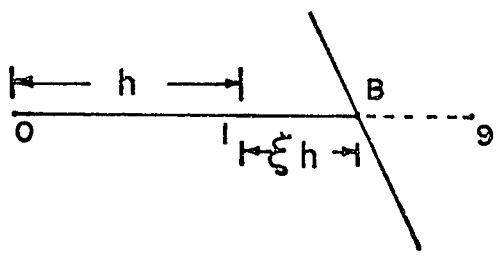


FIG- 8

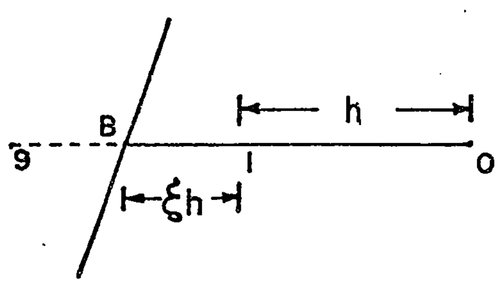


FIG.- 9

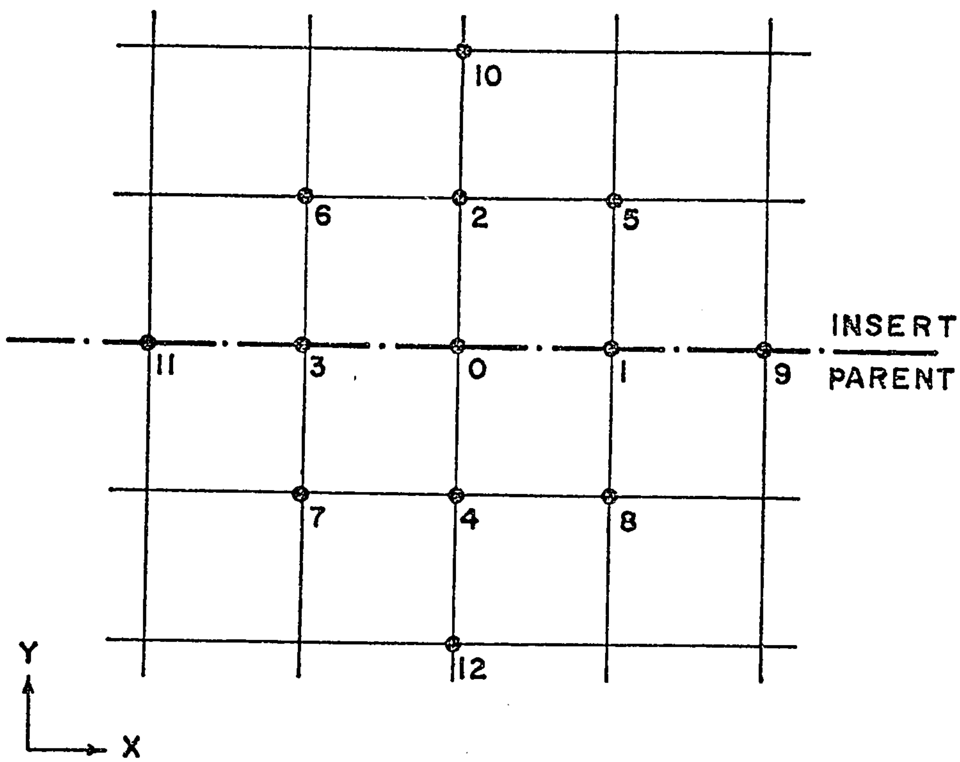


FIG.- 10

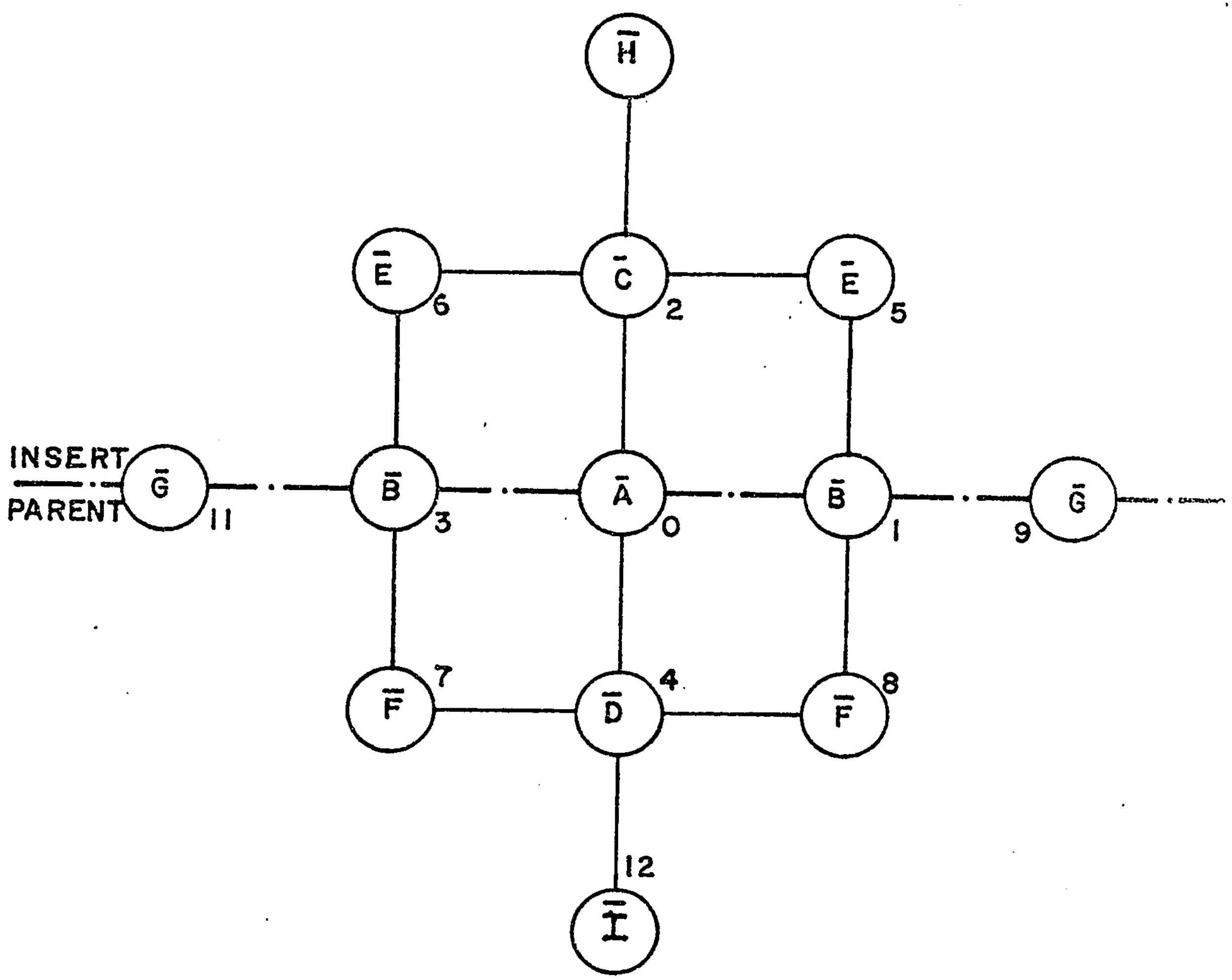


FIG.-11

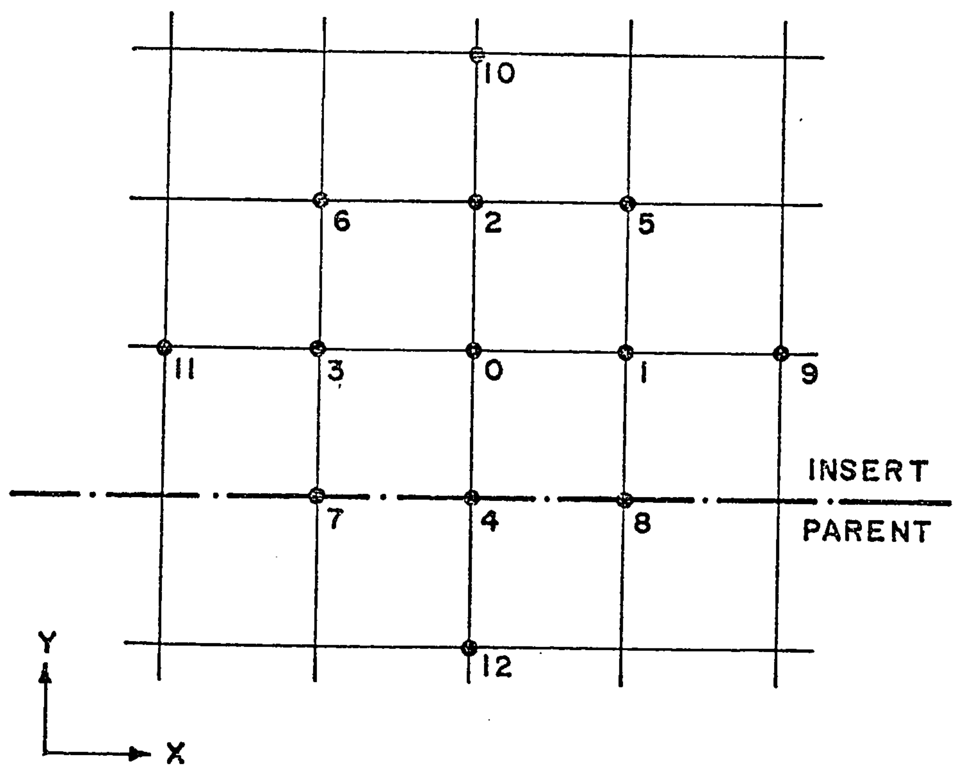


FIG.- 12

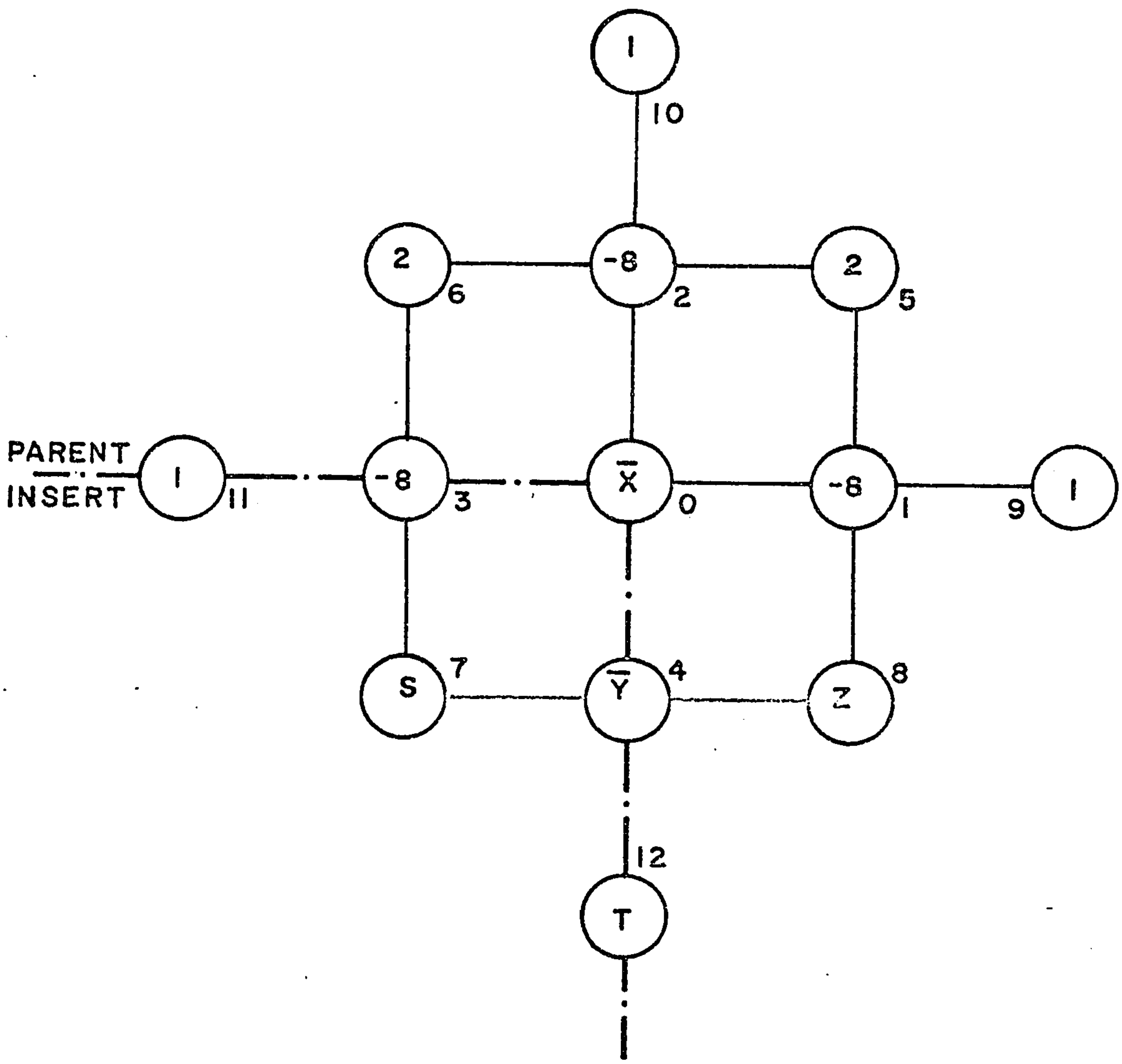


FIG.-13

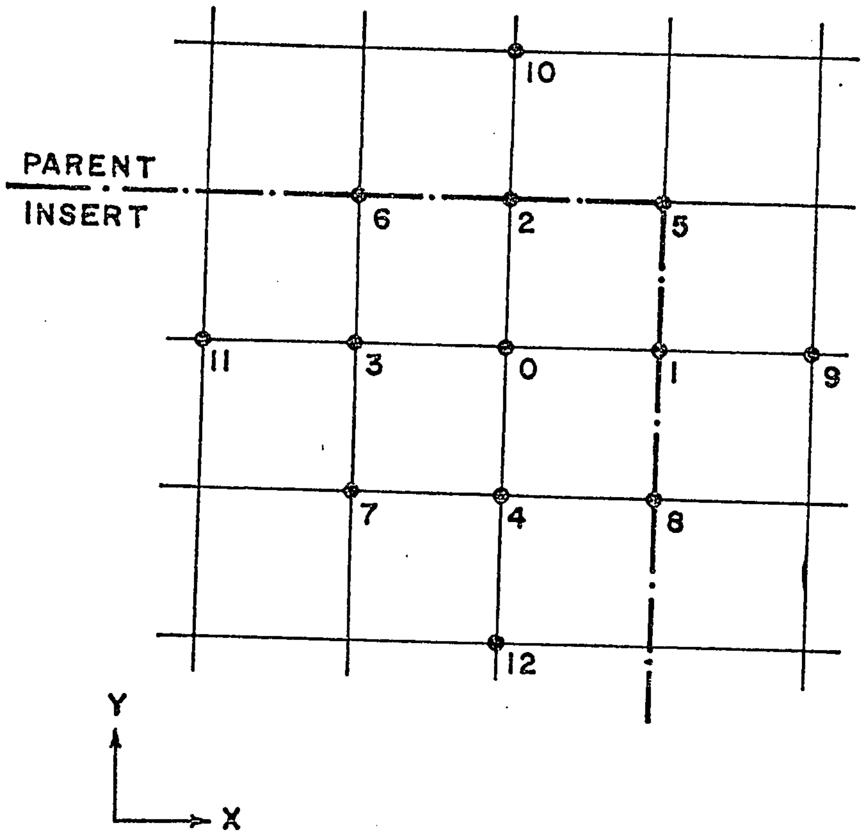


FIG.- 14

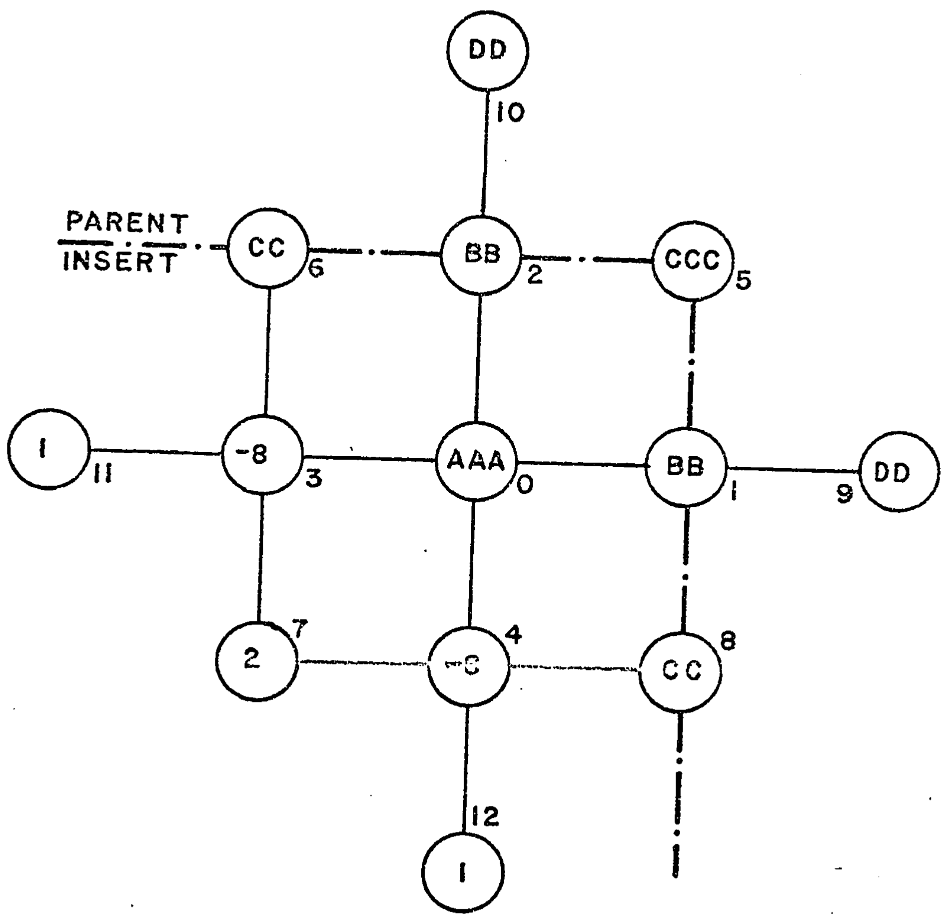

DD
10
PARENT
INSERT
CC
6
BB
2
CCC
5
1
11
-8
3
AAA
0
BB
1
9
DD
2
7
-8
4
CC
8
12
1

FIG.-15

After the values of ϕ have been attached to all other internal nodes, such as the node 0 in fig. 8, using the pair of relations (2.4.2I), the consistent values of ϕ satisfying the boundary conditions can be evaluated at those internal points such as 1, which lie within one mesh length of the boundary and also at points, like 9, which are fictitious.

Case II

This case shown in fig. 9 is same as before except the orientation is different i.e. the points lie along decreasing value of ϕ. In this case ϕ_9 and ϕ_1 are given by the following relations

$$\phi_9 = \frac{4\ \xi K}{(1+\xi)^2} - \frac{2(1-\xi)\,hk}{(1+\xi)} + \frac{(1-\xi)^2\phi_0}{(1+\xi)^2}$$

and (2.4. 2II)

$$\phi_1 = \frac{(1+2\xi)K}{(1+\xi)^2} + \frac{\xi\,hk}{(1+\xi)} + \frac{\xi^2}{(1+\xi)^2}\phi_0$$

2.4.3. Operator for the Points on the Interface

Referring to fig. 10, the string pattern for the point 0 on the interface can be given as shown in fig. 11.

2.4.4. Operator for the Points one Mesh away from the Interface

Referring to fig. 12, the string pattern of the operator for the points which are one mesh length away from the interface can be given as shown in fig. 13.

The operator for the points below the interface is obtained by reversing the notation appropriately.

2.4.5. Operator for the Point one Mesh away from two Perpendicular Interfaces

Referring to fig. 14, the string pattern of the operator for point 0 can be given as shown in fig. 15.

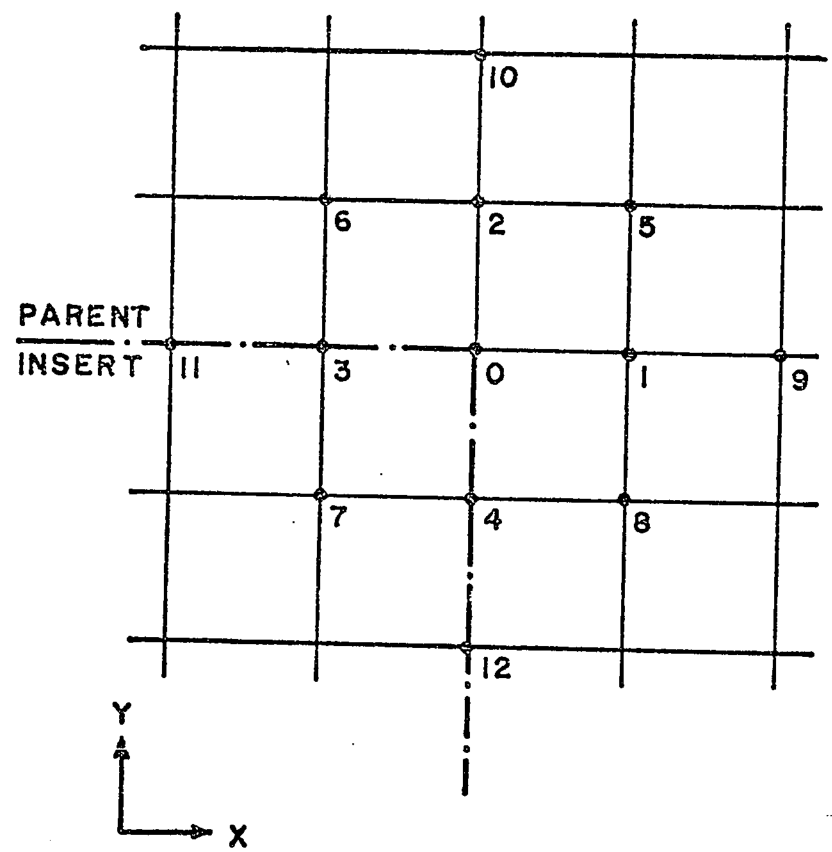


FIG.- 16

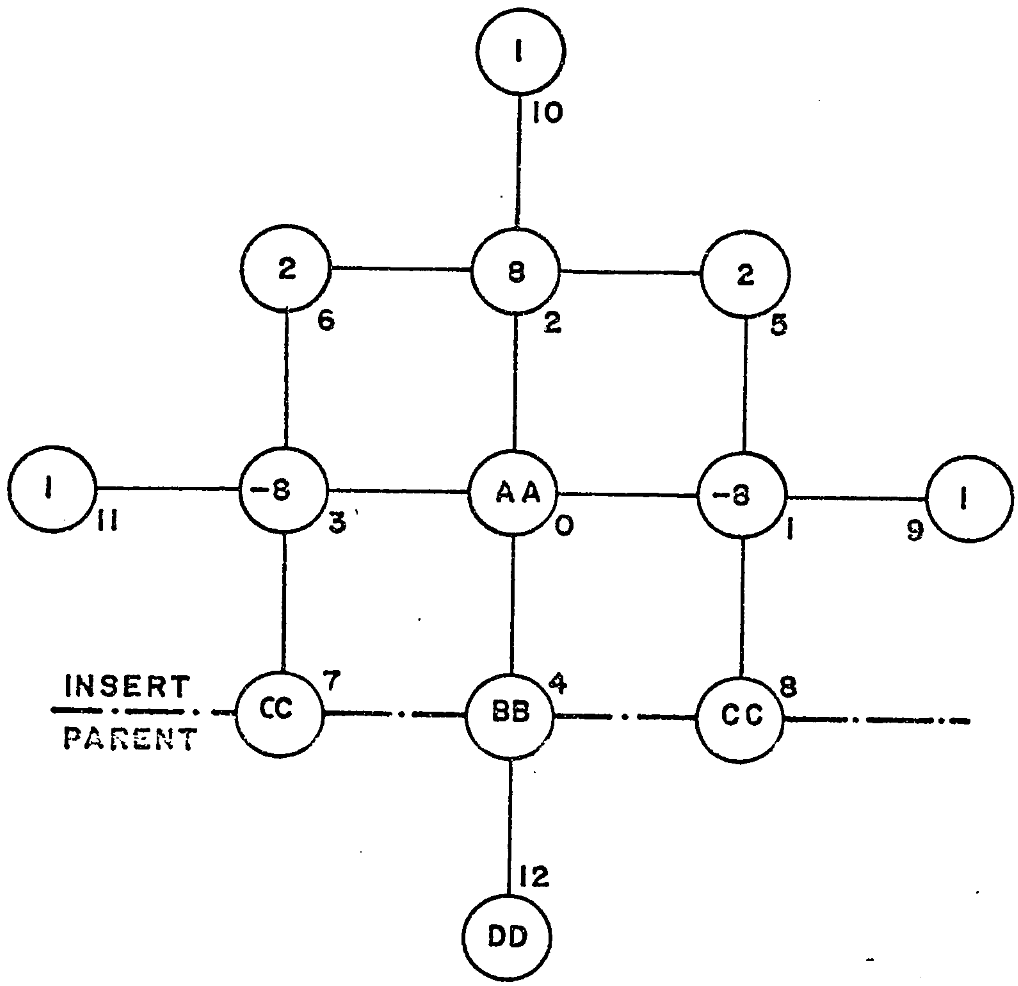


FIG.-17

2.4.6. Operator for the Point at the Intersection of two Perpendicular Interfaces

Referring to fig. 16, the operator for the point 0 at the intersection of two perpendicular interfaces can be given by the string pattern shown in fig. 17.

2.5. EQUATIONS OF BOUNDARY

Referring to fig. 3, the equations of the lines constituting the outside boundary of the plate are as follows:

Case II

$$y_{AB} = x - \frac{1}{\sqrt{2}}$$

$$y_{BC} = -x + \frac{5}{\sqrt{2}}$$

$$y_{CD} = x + \frac{1}{\sqrt{2}}$$

$$y_{AD} = -x + \frac{1}{\sqrt{2}}$$

3. COMPUTATION

To provide a mathematical tool of fairly wide scope, a three step computer program of sufficient flexibility is written for the solution of this problem.

Step 1

1. The first operation consists of formation of a grid on the plate. In case I, since the center axes of the plate are parallel to the co-ordinate axes, as shown in fig. 2, the square mesh is so formed that the outer points coincide exactly with the boundary.

In case II, the full plate is considered and its axes are inclined at 45° to the co-ordinate axes. For simplification, the whole plate is placed in the first quadrant, so that two of its corners lie on the co-ordinate axes, as shown in fig. 3. All the nodal points are numbered. For convenience, the finite difference mesh is so chosen that the interface coincides with a mesh line. The mesh so formed is antisymmetric.

2. The position of the insert is identified in terms of the nodal points.

3. The ϕ values at the points which fall on the boundary are computed and also the fictious points which are to be used in the biharmonic operator are written in terms of the real points.

4. The biharmonic equation $\nabla^4 \phi = 0$ is written in finite difference form. This is done for each nodal point, using the appropriate operator depending upon the position of the point in the plate. The set of simultaneous equations thus generated is stored on the tape.

Step 2

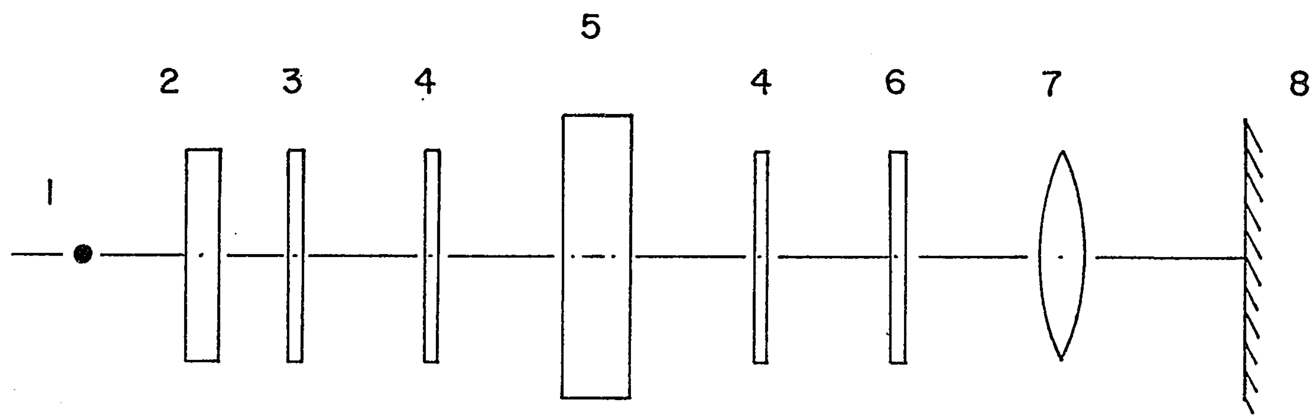
This step consists of the actual solution of the previously obtained set of equations. Several methods may be used. In our case, the methods chosen were matrix inversion and Gauss-Siedel iterative technique. The former was employed in case I, when the number of equations was not too great, while the latter was preferred in case II, in which there were as many as 549 equations. The reason for choosing matrix inversion is that it takes less machine time; it does, however, require a large core storage. On the other hand, the slower Gauss-Siedel method can be programmed with a considerable storage economy, whenever a large proportion of elements are zeros [23].

Step 3

After the stress function values at each nodal point have been determined, this step is used to compute the normal and shear stresses. Once the stresses with respect to the x and y axes are obtained the principal stresses can easily be calculated.

The listings of the programs are given in appendix III.

SCHEMETIC DIAGRAM OF A PHOTOELASTIC BENCH



1 Light Source
2 Condenser Lens
3 Polarizer
4 Quarter Wave Plate
5 Loading frame and model
6 Analyzer
7 Focusing
8 Screen

FIG.-18

CHAPTER 4

4. EXPERIMENTAL INVESTIGATION

The object of experimental stress analysis is to deduce the state of stress in a structural element subjected to some specified loading either by observation of the physical changes brought about in it by the applied loads or by measurements made on a model or analogue. The experimental stress analysis is more than just a counterpart of theoretical methods of stress analysis. It maintains direct contact with the true physical characteristics of the problem under consideration.

Although there are usually several possible methods available for carrying out an experimental investigation of stresses, it is important however to select the technique so as to obtain the most satisfactory results with the minimum of time, effort and money.

Under the circumstances, the photoelastic stress analysis appears best suited. The method of photoelasticity has the advantage of giving a complete picture of stress field under investigation. Further it makes use of comparatively small models which in turn require light and relatively inexpensive loading gear.

4.1 THEORY

The photoelastic method is based on the phenomenon of double refraction exhibited by transparent materials when subjected to stress. Loaded models are observed in a beam of polarized light. A simple arrangement is shown in fig. 18. Point light source is used, and a parallel beam is obtained by means of a condenser lens. The polarizer and analyzer are either natural crystals or "Polaroid" discs. The quarter-wave plates are of mica. Their thickness being related to the wavelength of the light source. The model is carried in a loading frame. Light and dark colored bands are produced on the screen and may be photographed for subsequent analysis. An outline of the effects obtained and their interpretation is given below.

If the quarter-wave plates are removed, plane polarized light is produced, and with the axes of the polarizer and analyzer at 90° no light is transmitted to the screen. When the specimen is loaded, however, some light is passed through, except at points where the direction of one principal stress is in the plane of polarization. The result is a dark band across the model indicating the locus of all points at which the principal stresses are in the same directions. These bands are called isoclinics, and by rotating the polarizer and analyzer together a number of isoclinics can be obtained for various directions of principal stress.

The quarter-wave plates, placed with their axes at 45° to those of the polarizer and analyzer, produce circularly polarized light, and serves to cut out the isoclinics.

Under this arrangement, a color map of lines of equal color appear on the screen. These lines are called isochromatics. Every equal color line represent a constant level line of stress. Beginning at the lower level and progressing to areas of higher level, the color sequence observed will be black, yellow red, blue, yellow, red, green, yellow, red, green etc. .

At a particular point on the model the difference of the principal stresses is given by [24]

$$P_1 - P_2 = \frac{N f_\sigma}{H}$$

H of the material is known and f_σ is determined by calibration. Thus the remaining number to be determined is N. The fringes are related to increasing order as follows:

Along the black fringe	N = 0
Along the first fringe (Red - Blue)	N = 1
Along the second fringe (first Red - Green)	N = 2
Along the third fringe (second Red - Green)	N = 3
Along the fourth fringe (third Red - Green)	N = 4

4.2 SEPARATION TECHNIQUES

The optical data from the polariscope give only the difference of the principal stresses and their directions. The separate values of each stress can be obtained by different methods and the choice of the method determines the experimental procedures, which must be adopted.

With our polariscope we have the choice of using either shear difference method or oblique incidence method but the shear difference method is preferred for following reasons:

1. In case of oblique incidence method the difficulty can arise where the stress gradient is very steep.

2. In the case of oblique incidence method it is essential that the plane stress conditions be attained. Any stress present in any other plane, which normally is not detected when observed at normal incidence, would greatly affect the result when studied under oblique incidence.

Shear difference method is based on the equations of equilibrium, for the two-dimensional stress state. The equations of equilibrium in the absence of body forces are

$$\frac{\partial P_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \qquad (4.2.1)$$

$$\frac{\partial P_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \qquad (4.2.2)$$

The solutions of these equations in terms of the normal stresses

P_x and P_y can be obtained from :

$$(P_x)_{x_1} = (P_x)_{x_0} - \int_{x_0}^{x_1} \frac{\partial \tau_{xy}}{\partial y} \, dx \qquad (4.2.3)$$

$$(P_y)_{y_1} = (P_y)_{y_0} - \int_{y_0}^{y_1} \frac{\partial \tau_{xy}}{\partial x} \, dy \qquad (4.2.4)$$

where $(P_x)_{x_0}$ and $(P_y)_{y_0}$ represent the values of P_x and P_y at a given location (for which the stress values are known). Usually these are the values at the free boundary. In the eqns. (4.2.3) and (4.2.4), the value of the integral can be represented by the area under a curve for which the ordinates are the rate of change of shear in a direction normal to the direction of integration and the abscissas are distances along the path of integration.

Equation (4.2.4) can be evaluated numerically by employing finite difference method , as shown below

$$(P_y)_{y_1} = (P_y)_{y_0} - \Delta y \left(\frac{\Delta \tau_{xy}}{\Delta x} \right)_{\frac{(y_0+y_1)}{2}} \qquad (4.2.5)$$

where y , y_1 etc. represent positions along y.

The value of $\Delta \tau_{xy}$ can be computed using the isoclinic and isochromatic data obtained in the following relation.

$$\tau_{xy} = \frac{P_1 - P_2}{2} \cdot \sin 2\theta$$

$$= \frac{N f_\sigma}{2 H} \cdot \sin 2\theta$$

where θ is the angle between the x-axis and the direction of P_2 as given by isoclinic parameter.

Hence it is possible to determine P_y at the interior point y_1.
If the procedure is continued stepwise, the value of P_y can be determined across the entire length of the line.

$$P_y|_{y_2} = P_y|_{y_1} - \Delta_y \left(\frac{\Delta \tau_{xy}}{\Delta x} \right)_{\frac{(y_1+y_2)}{2}}$$

$$P_y|_{y_3} = P_y|_{y_2} - \Delta_y \left(\frac{\Delta \tau_{xy}}{\Delta x} \right)_{\frac{(y_2+y_3)}{2}}$$

$$P_y|_{y_4} = P_y|_{y_3} - \Delta_y \left(\frac{\Delta \tau_{xy}}{\Delta x} \right)_{\frac{(y_3+y_4)}{2}}$$

Having obtained P_y along the required line, we proceed further to evaluate P_x using either Mohr's Circle or the analytical expressions, as follows

$$P_x = P_y - (P_1 - P_2) \cos 2\theta \qquad (4.2.6)$$

Substituting $(P_1 - P_2)$ from eqn. (4.1.1) into eqn. (4.2.6), we get

$$P_x = P_y - \frac{N f_\sigma}{H} \cdot \cos 2\theta \qquad (4.2.7)$$

Now from the first invariant of stress, we have

$$P_x + P_y = P_1 + P_2 \qquad (4.2.8)$$

Solving eqns. (4.1.1) and (4.2.8), we get P_1 and P_2 as

$$P_1 = \frac{1}{2} \left(P_x + P_y + \frac{N f_\sigma}{H} \right)$$

$$P_2 = \frac{1}{2} \left(P_x + P_y - \frac{N f_\sigma}{H} \right)$$

In this manner, using the method of shear difference we can separate the stresses along any arbitrary line.

4.3 MATERIALS FOR MODELS

For the success of this investigation, it is necessary that the material should meet the prime characteristics such as complete freedom from internal stresses, excellent transparency, very low time edge effect, higher sensitivity etc. .

PSM 1 and PSM 4 are the trade names of the plastics used in this case which have the above-mentioned characteristics. The elastic properties of the materials are as follows:

(i) PSM-1

This is a hard polyester sheet with a nominal thickness of 0.25 in.

The elastic properties of PSM-1 are

$E = 340{,}000$ psi

$\mu = 0.38$

$f_\sigma = 36$ psi per fringe per inch

(ii) PSM-4

This is a soft polyurethone sheet,

The elastic properties of PSM-4 are

$E = 1000$ psi

$\mu = 0.4$

$f_\sigma = 3 - 5$ psi per fringe per inch

This is used as an insert.

A special cement has been used to get a bond as free from slippage as possible. The cement has the same elastic properties as PSM-4.

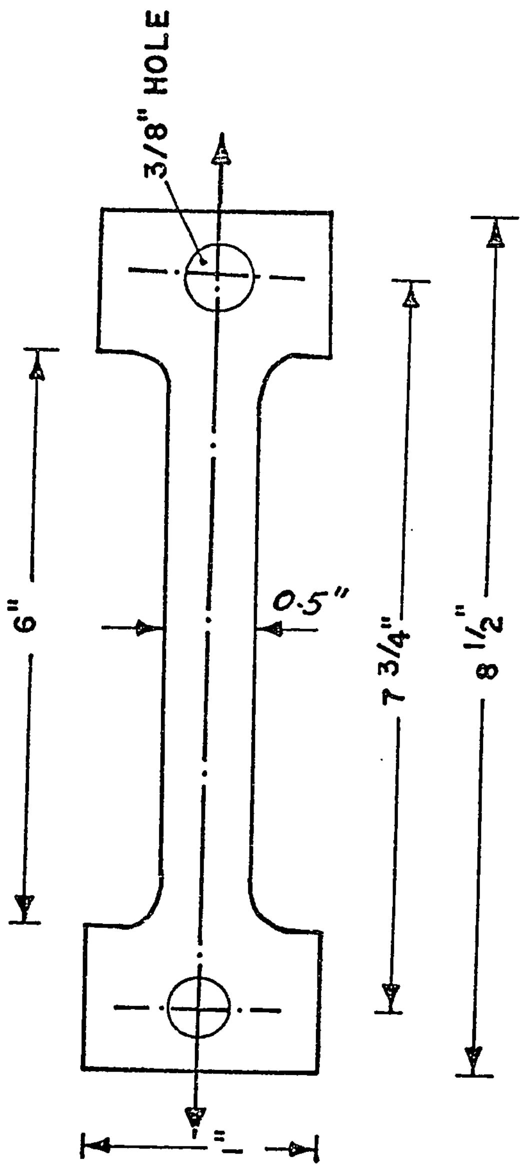


FIG.-19

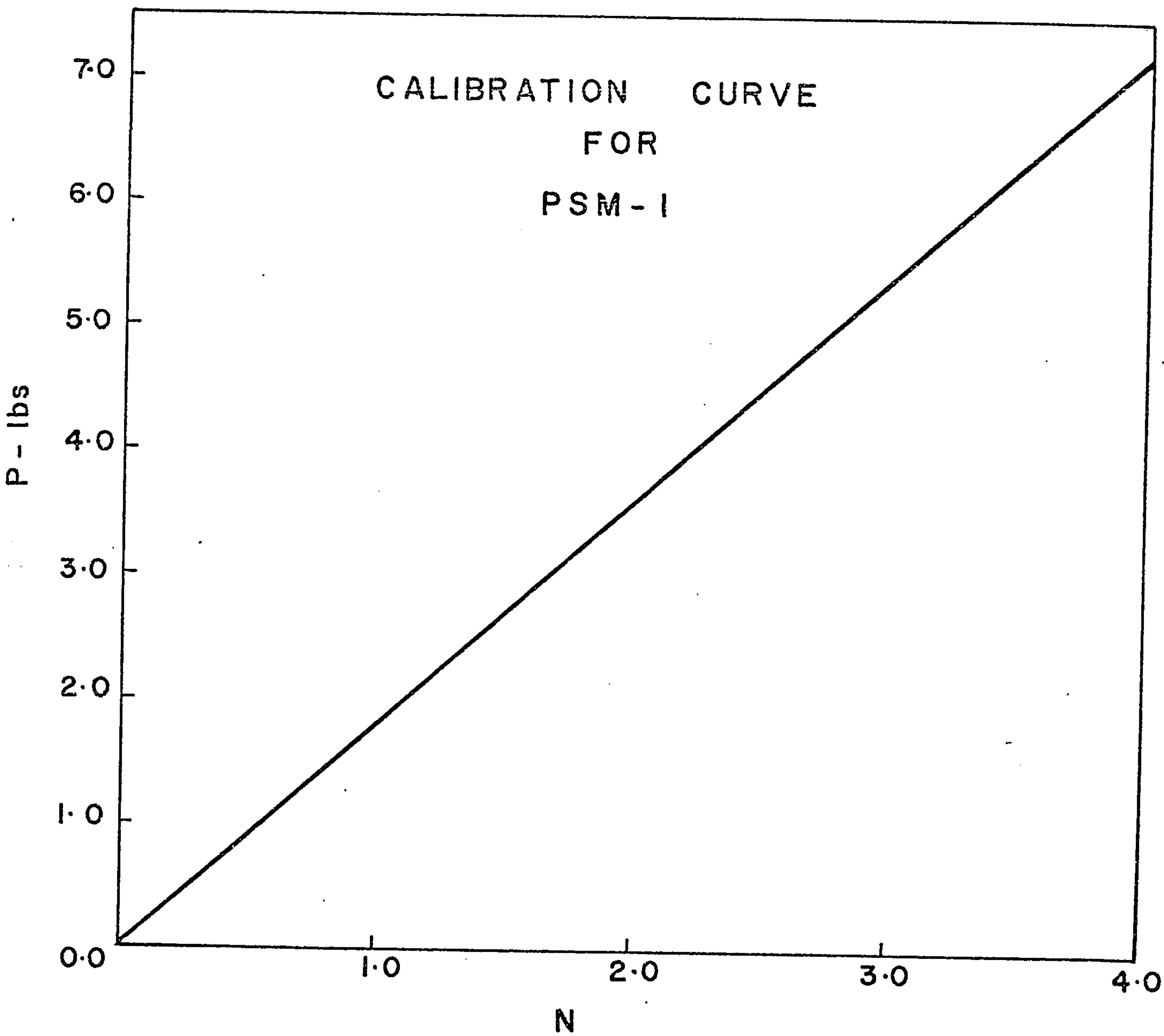

CALIBRATION CURVE
FOR
PSM - I
P - lbs
7·0
6·0
5·0
4·0
3·0
2·0
1·0
0·0
1·0
2·0
3·0
4·0
N

FIG.- 20

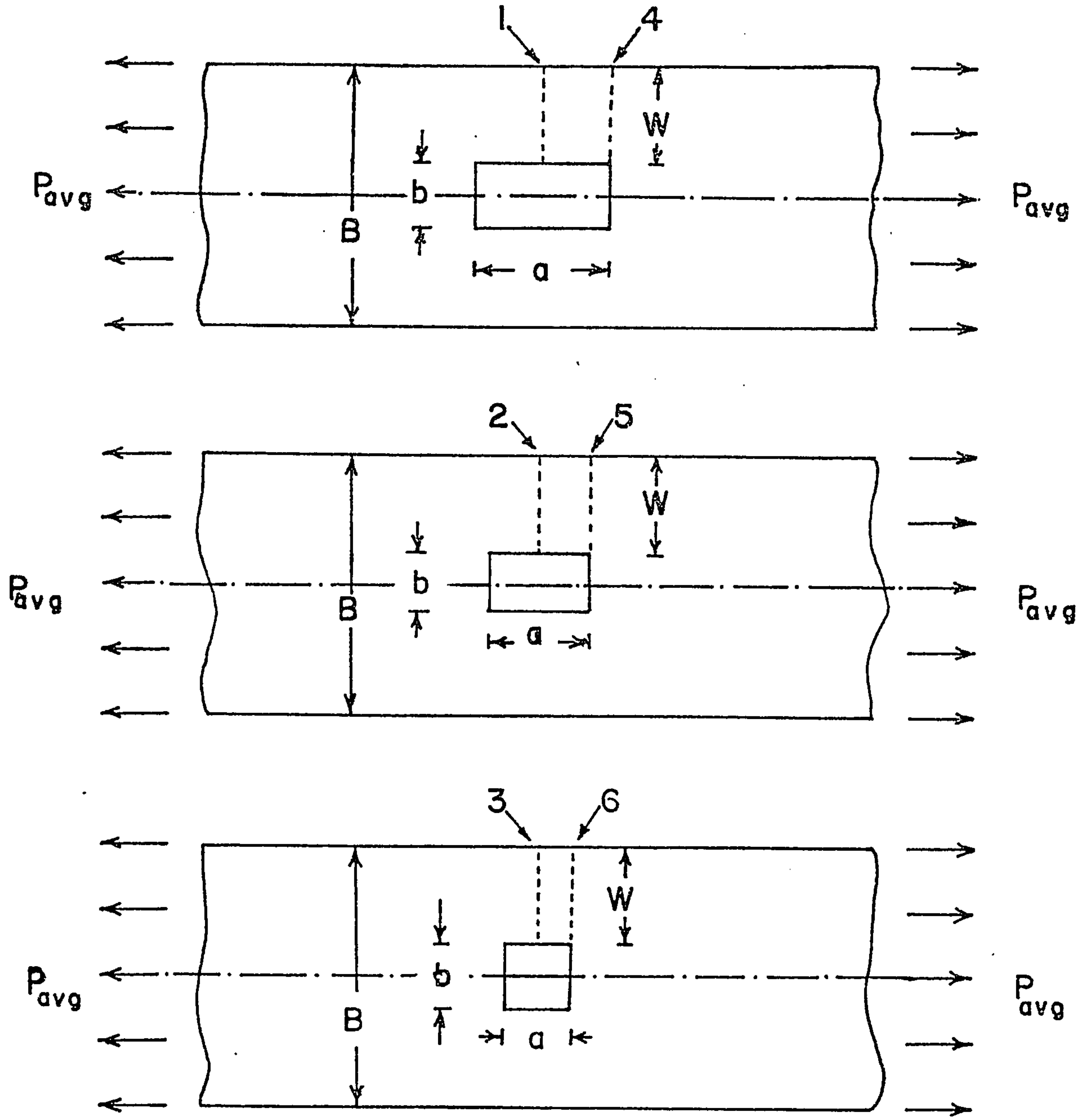


FIG.-21

SHEET-A

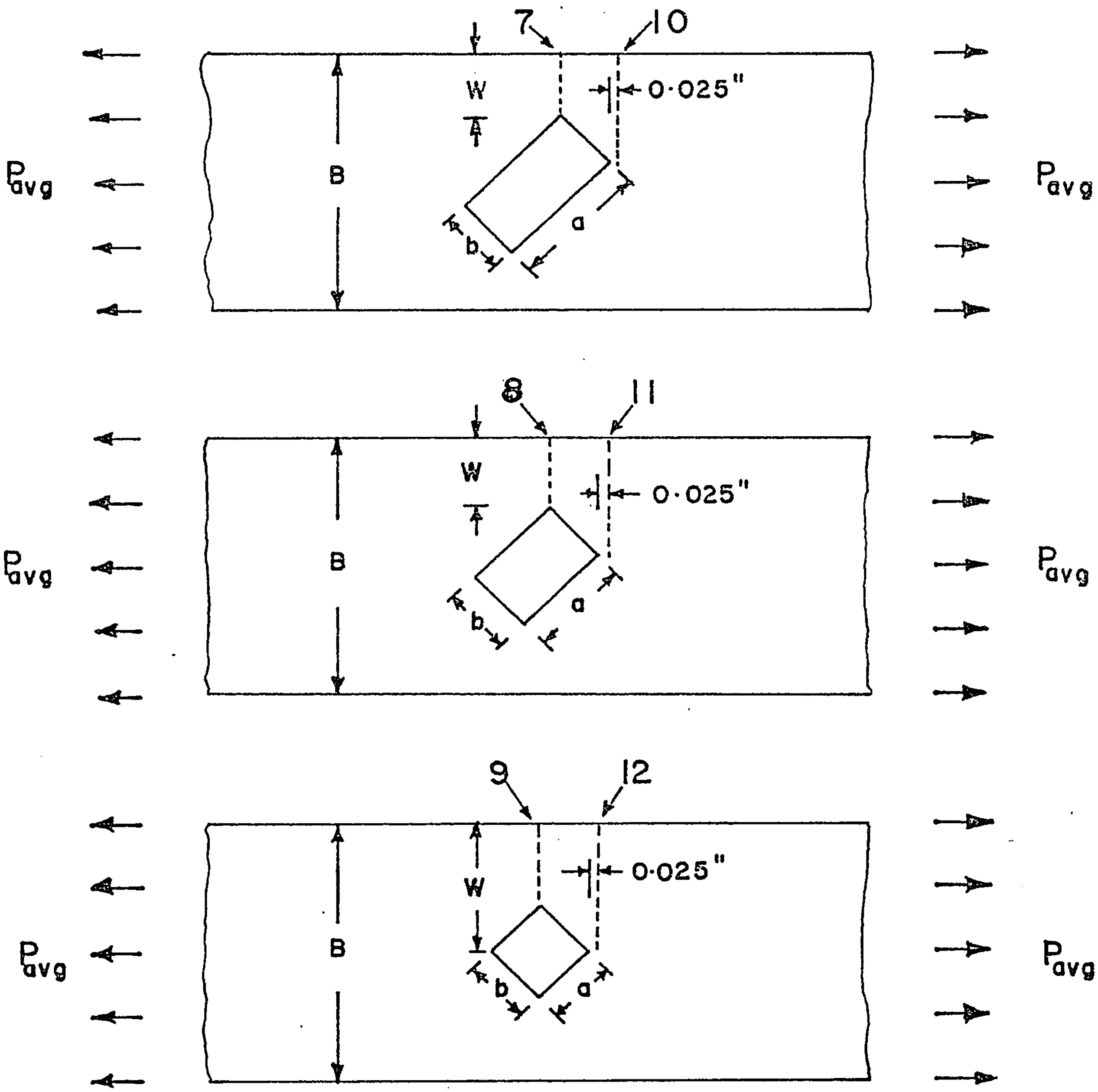


FIG.- 21

SHEET- B

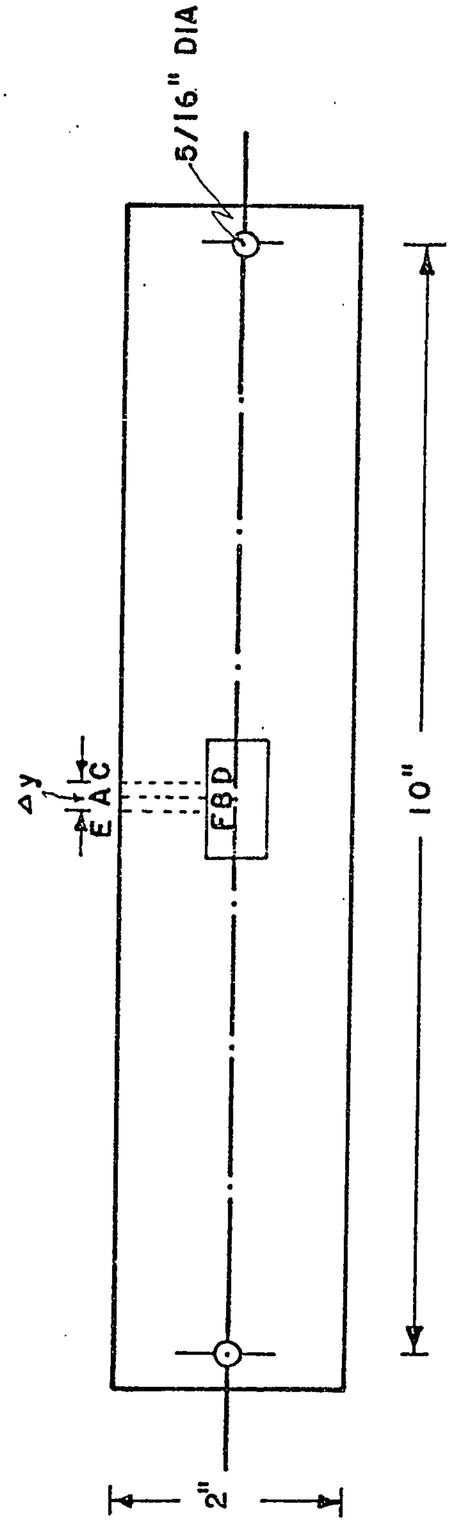


FIG.- 22

4.4 CALIBRATION OF MATERIAL

The accuracy of the evaluation of stress distribution depends mainly on material fringe value of the material. Though this value is often provided by the supplier, it is apt to vary due to the effects of time and temperature. Therefore, calibration of the material is necessary in order to obtain accurate results.

A tension bar shown in fig. 19 is employed for the calibration of the material. The calibration model is loaded in steps. The corresponding fringe orders and loads are noted.

The axial stress induced in the necked region of the tensile specimen by load P may be expressed as

$$P_1 = \frac{P}{wH} \quad \text{and} \quad P_2 = 0$$

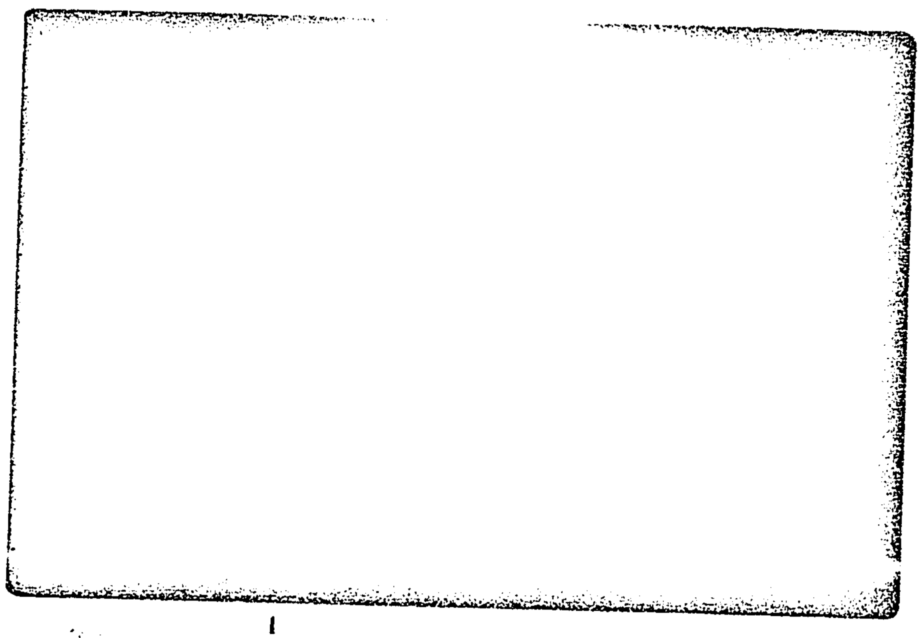
where w is the width at the necked region and H is the thickness of the specimen. Substituting this in eqn. (3.1.1), we get

$$f_\sigma = \frac{P}{wH}$$

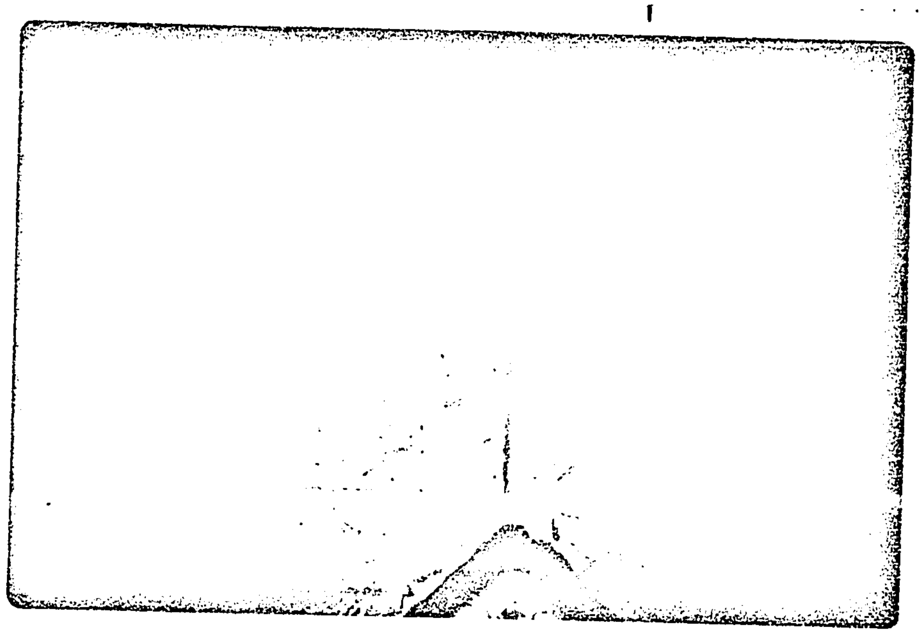
A curve of the load P is plotted as function of N for four different points which give a straight line. The slope of this straight line is used for the value of P/N. Figure 20 shows the calibration curve of material PSM-1.

4.5 EXPERIMENTAL PROCEDURE

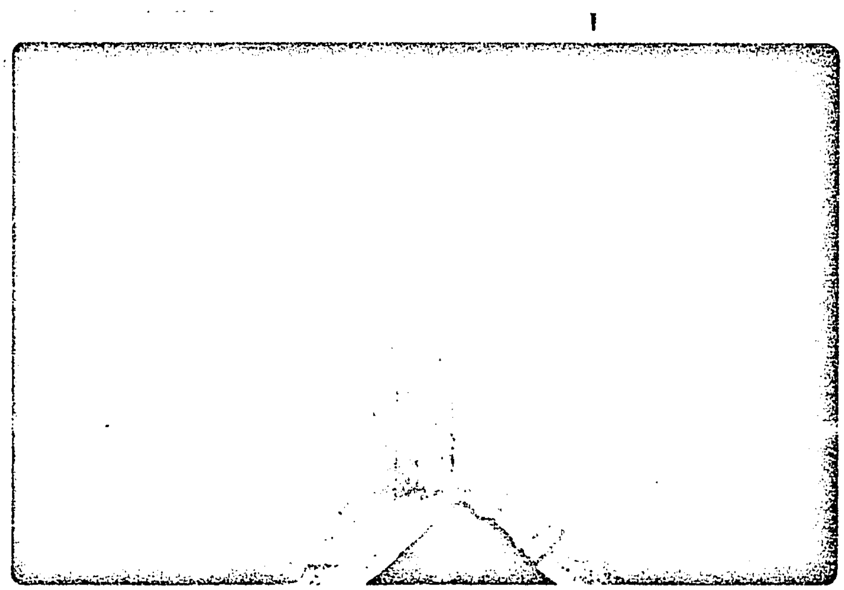
For each specimen, the stress distribution has been studied along two different lines. These lines in case of each specimen are shown in fig. 21. Two more lines are drawn distant $\frac{\Delta x}{2}$ on either side of the line under consideration as shown in fig. 22.



(a) Constant fringe order across a cross-section away from the insert.



(b) Isochromatics of specimen with 45° inclined insert.



(c) Isoclinics of specimen with 45° inclined insert.

The readings of isoclinics and isochromatics are taken along these lines at various points starting from the boundary and proceeding inwards. A smaller step size is used in the region where the change of stress is rapid.

A few photographs representative of the stress pattern in specimen are given on pages 48 - 50 with appropriate comments.

4.6 COMPUTATION

A computer program based on shear difference method has been written for the calculation of principal stresses and shear stresses from the experimental data. A listing of the program is given in appendix III.

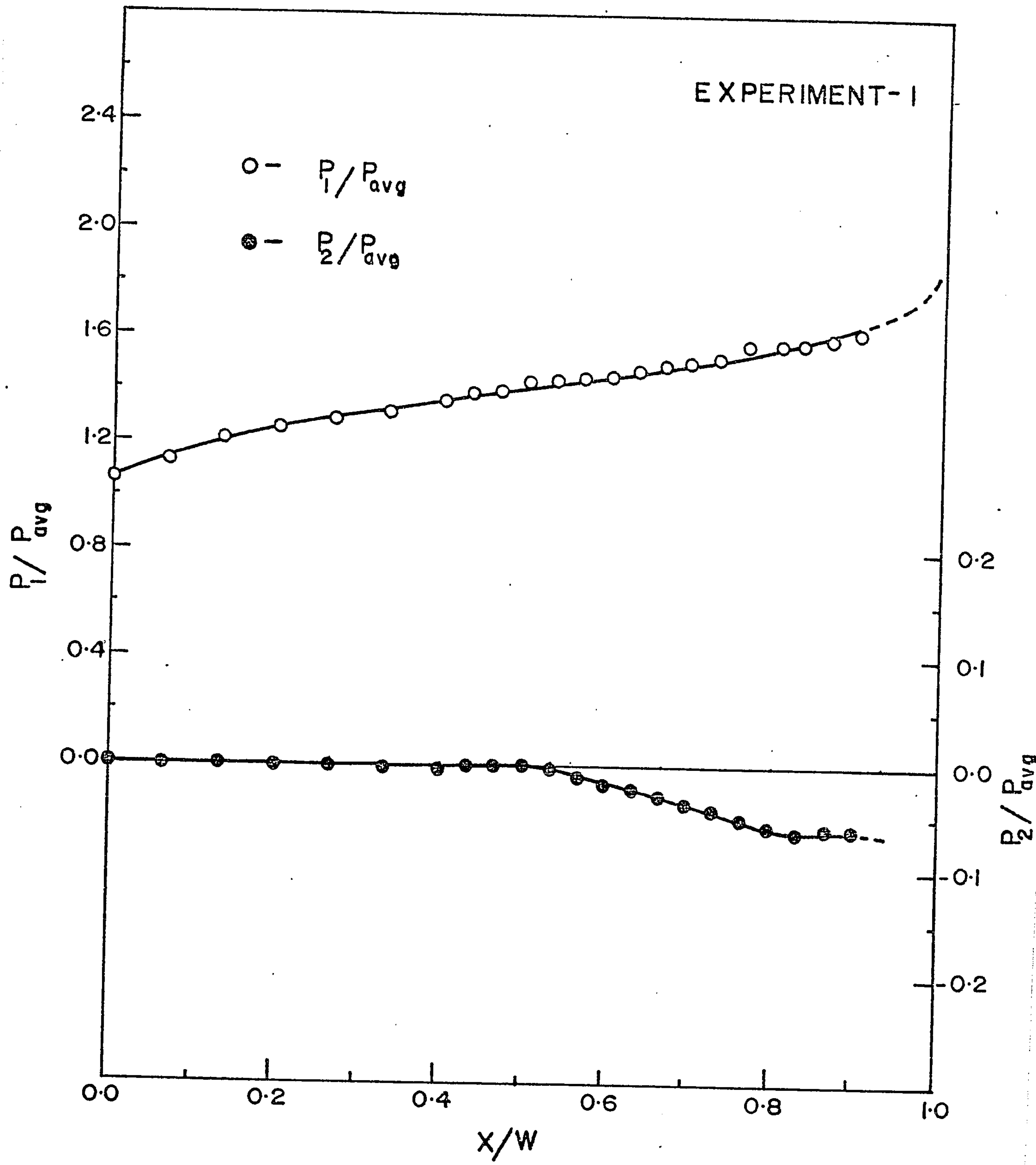
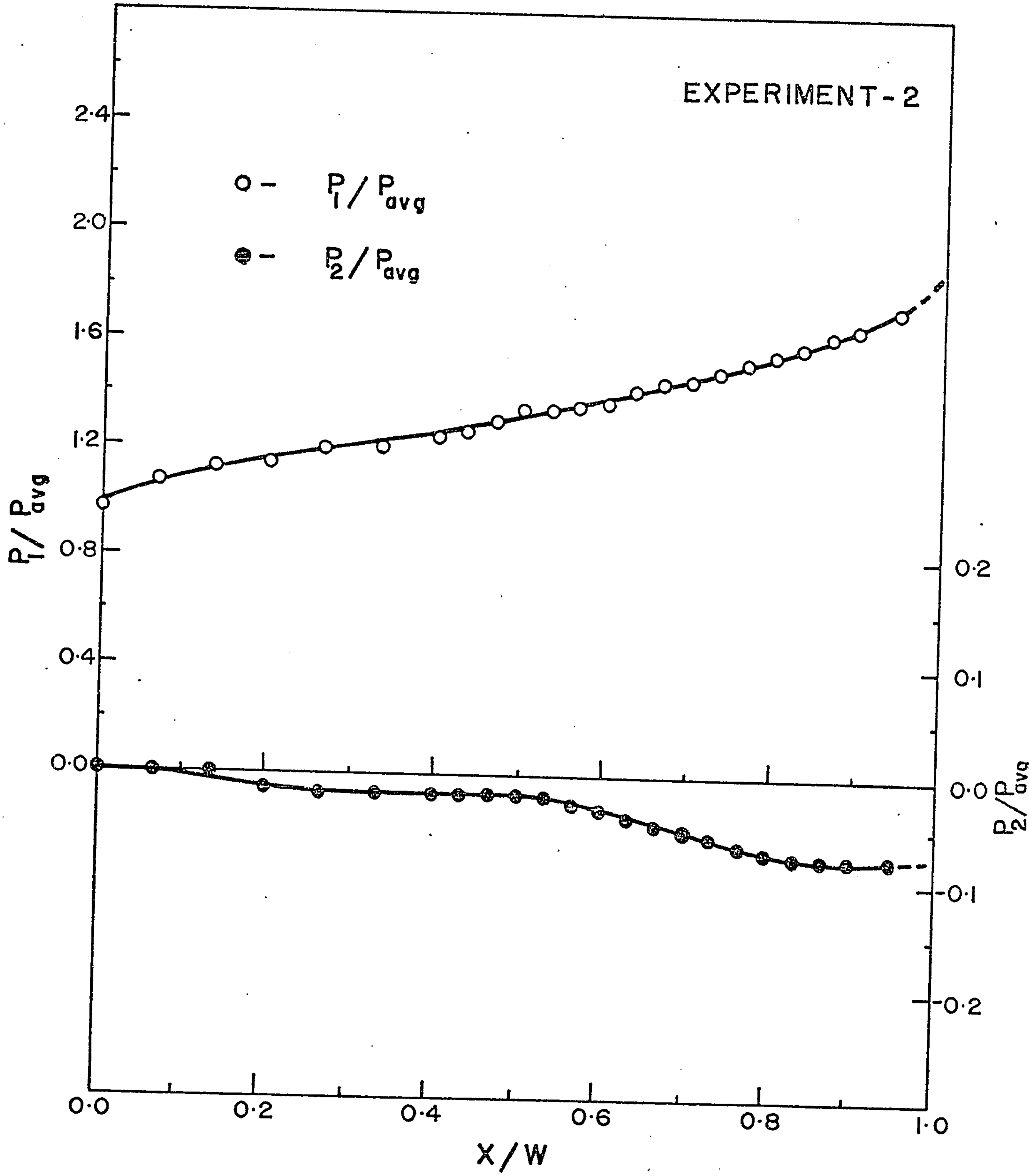

EXPERIMENT-1
○ – P_1/P_{avg}
● – P_2/P_{avg}
P_1/P_{avg}
2·4
2·0
1·6
1·2
0·8
0·4
0·0
0·2
0·1
0·0
-0·1
-0·2
P_2/P_{avg}
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.- 23



EXPERIMENT-2
○ - P_1/P_{avg}
● - P_2/P_{avg}
2·4
2·0
1·6
1·2
0·8
0·4
0·0
P_1/P_{avg}
0·2
0·1
0·0
-0·1
-0·2
P_2/P_{avg}
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.-24

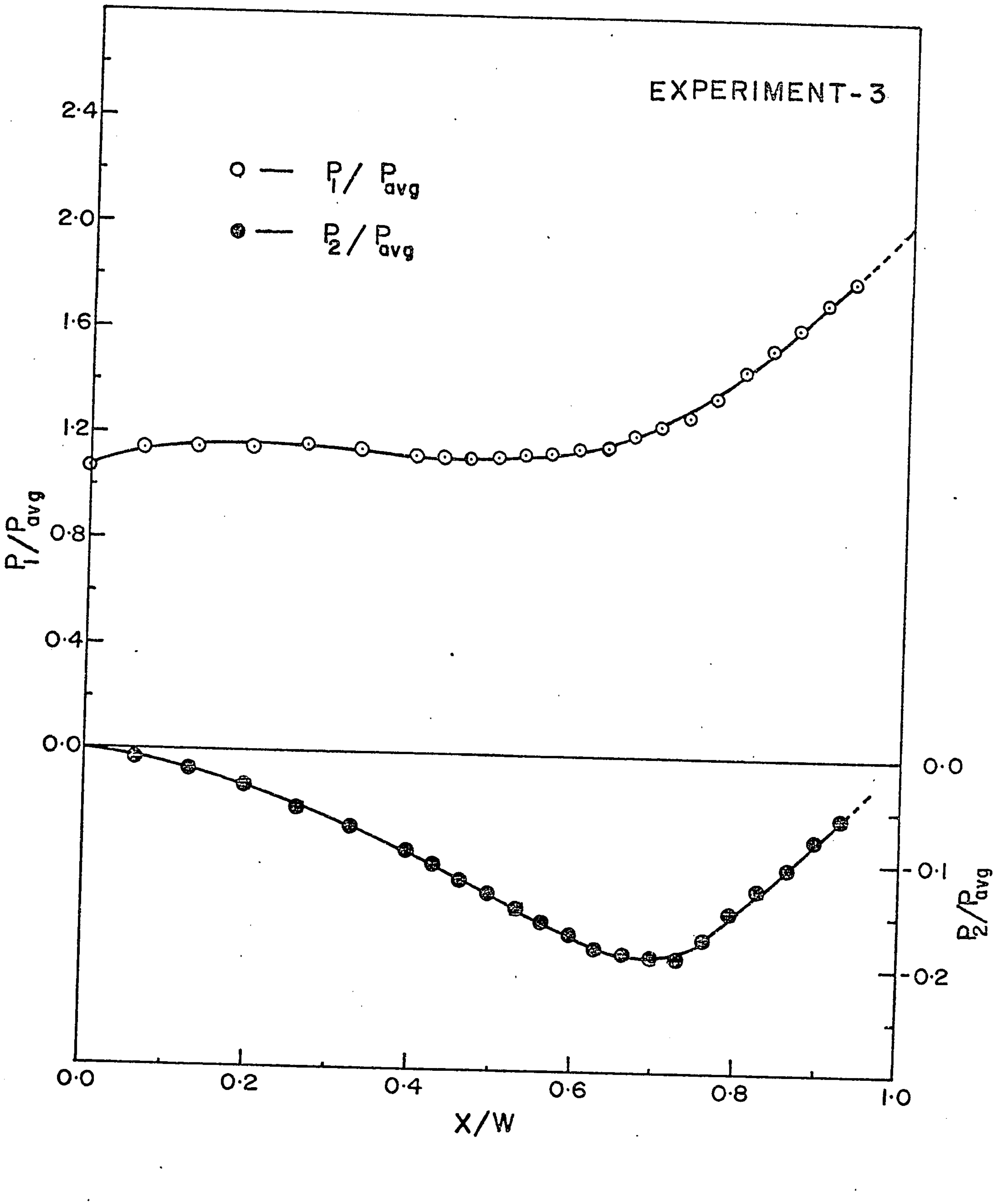

EXPERIMENT-3
P_1/P_{avg}
P_2/P_{avg}
2·4
2·0
1·6
1·2
0·8
0·4
0·0
0·0
-0·1
-0·2
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.- 25

CHAPTER 5

5. DISCUSSION OF RESULTS AND CONCLUSIONS

The experimental work reported in this thesis consists of twelve experiments performed on six specimens shown in fig. 21. Each specimen is a rectangular plate with a centrally located rectangular insert. The dimensions of the specimens are given in table 1.

The distribution of principal stresses and shear stresses has been studied in each case. The plots are made for P_1/P_{avg}, P_2/P_{avg} and τ_{xy}/P_{avg} against a non-dimensional variable x/w where

P_{avg} = the stress at a distance far away from the insert.

x = the distance of any point along the line from the free boundary.

w = the distance of the insert along the line from the free boundary.

The value $x/w = 0$ represents the free boundary and the value $x/w = 1$ represents the interface between the insert and the parent material.

As would be noticed in figs. 23 - 34, that the experimental curves are not plotted upto the interface. This is due to the following reasons:

Though every care has been taken in the preparation of the specimens, the perfection is hard to achieve. A slight imperfection in the finish of the outer boundary of the specimen is magnified ten times on the screen of the polariscope. Further, dispersion of light takes place along the edge of the specimen and it is difficult to locate the points close to the boundary. The starting point is therefore, shifted away from the boundary whereas in the calculations the distance is taken as $x = 0$. Moreover an interface should be a thin line but in the specimen it shows up

TABLE 1

GROUP	EXP. NO.	DIMENSIONS*						$\frac{P_{1\ max.}}{P_{avg}}$	FRINGE ORDER+
		A	B	a	b	w	θ		
I	1	10	2	1	0.5	0.75	0^o	1.86	1.92
	2	10	2	0.75	0.5	0.75	0^o	2.00	2.35
	3	10	2	0.5	0.5	0.75	0^o	2.04	2.67
II	4	10	2	1	0.5	0.75	0^o	2.56	2.78
	5	10	2	0.75	0.5	0.75	0^o	2.60	2.84
	6	10	2	0.5	0.5	0.75	0^o	2.70	2.92
III	7	10	2	1	0.5	0.47	45^o	-	7
	8	10	2	0.75	0.5	0.56	45^o	-	5
	9	10	2	0.5	0.5	0.65	45^o	-	4
IV	10	10	2	1	0.5	0.82	45^o	1.52	2
	11	10	2	0.75	0.5	0.91	45^o	1.42	1.84
	12	10	2	0.5	0.5	0.1	45^o	1.20	1.3

* All dimensions are in inches.

\+ Fringe order at the interface or the corner depending upon the position of the line along which the stresses are being taken at 100 lbs load.

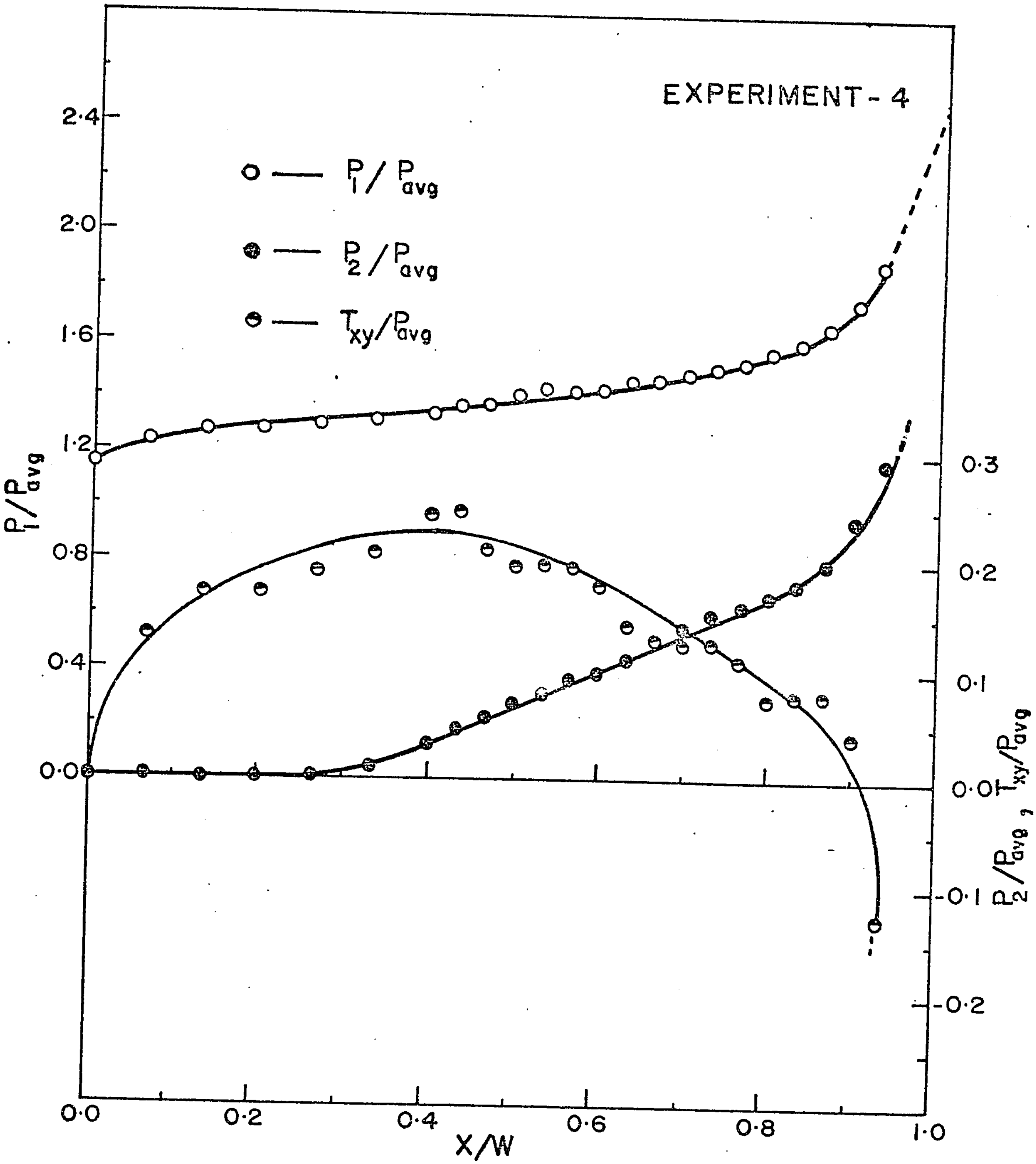

EXPERIMENT - 4
P_1/P_{avg}
P_2/P_{avg}
T_{xy}/P_{avg}
2·4
2·0
1·6
1·2
0·8
0·4
0·0
P_1/P_{avg}
0·3
0·2
0·1
0·0
-0·1
-0·2
P_2/P_{avg}, T_{xy}/P_{avg}
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.-26

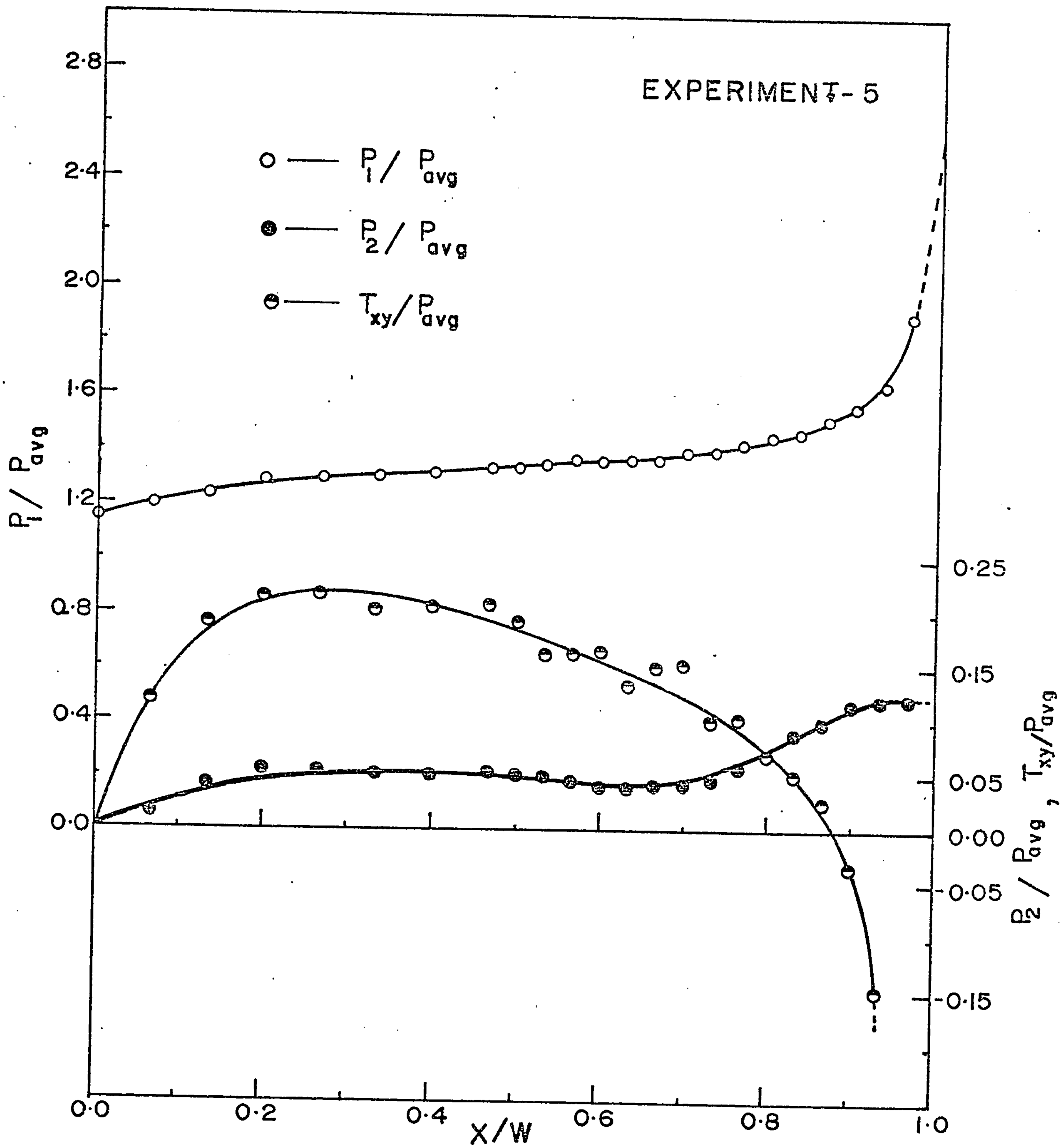

EXPERIMENT- 5
P_1 / P_{avg}
P_2 / P_{avg}
T_{xy} / P_{avg}
2·8
2·4
2·0
1·6
1·2
0·8
0·4
0·0
P_1 / P_{avg}
0·25
0·15
0·05
0·00
-0·05
-0·15
P_2 / P_{avg}, T_{xy} / P_{avg}
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.- 27

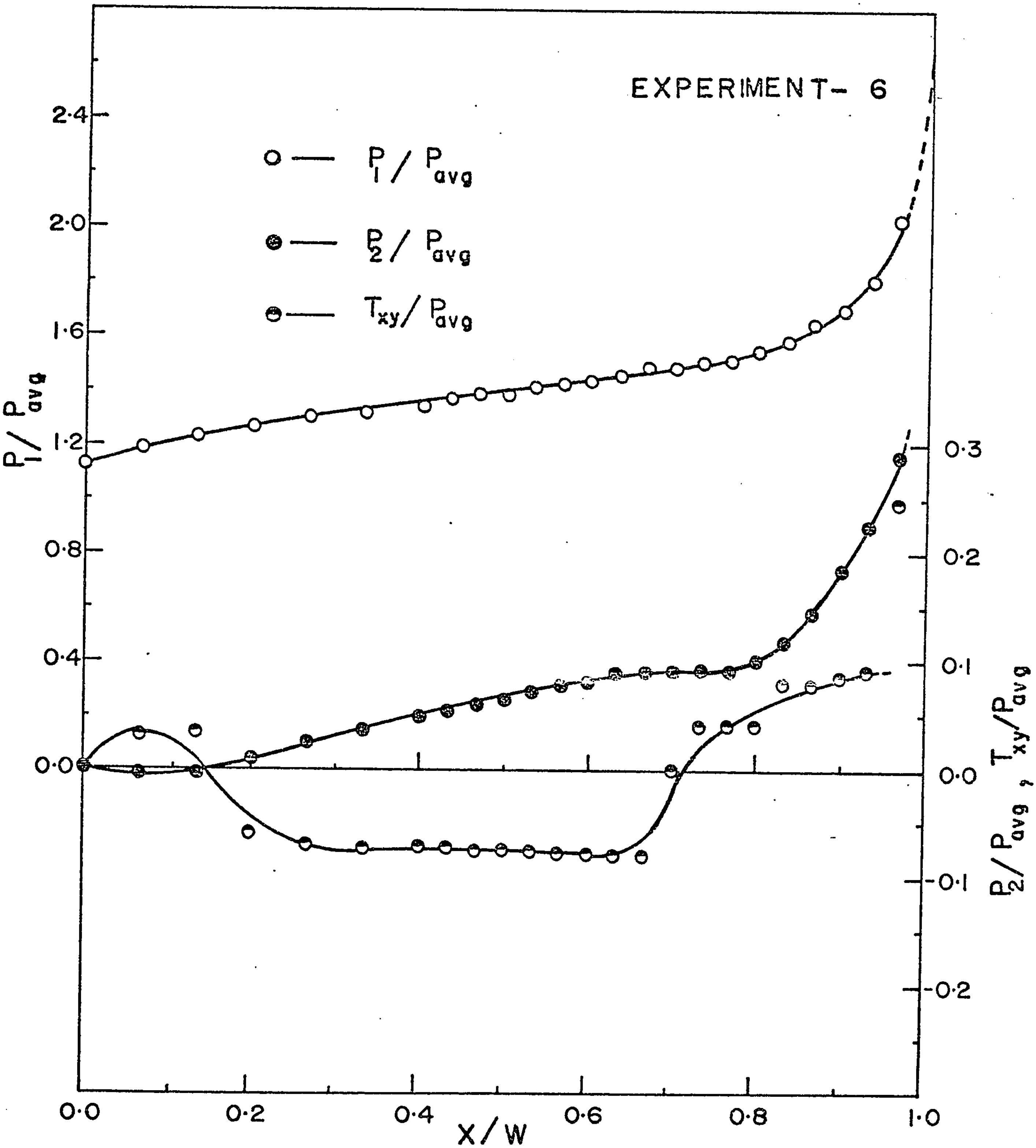

EXPERIMENT- 6
P_1/P_{avg}
P_2/P_{avg}
T_{xy}/P_{avg}
P_1/P_{avg}
2·4
2·0
1·6
1·2
0·8
0·4
0·0
0·3
0·2
0·1
0·0
-0·1
-0·2
P_2/P_{avg} , T_{xy}/P_{avg}
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.- 28

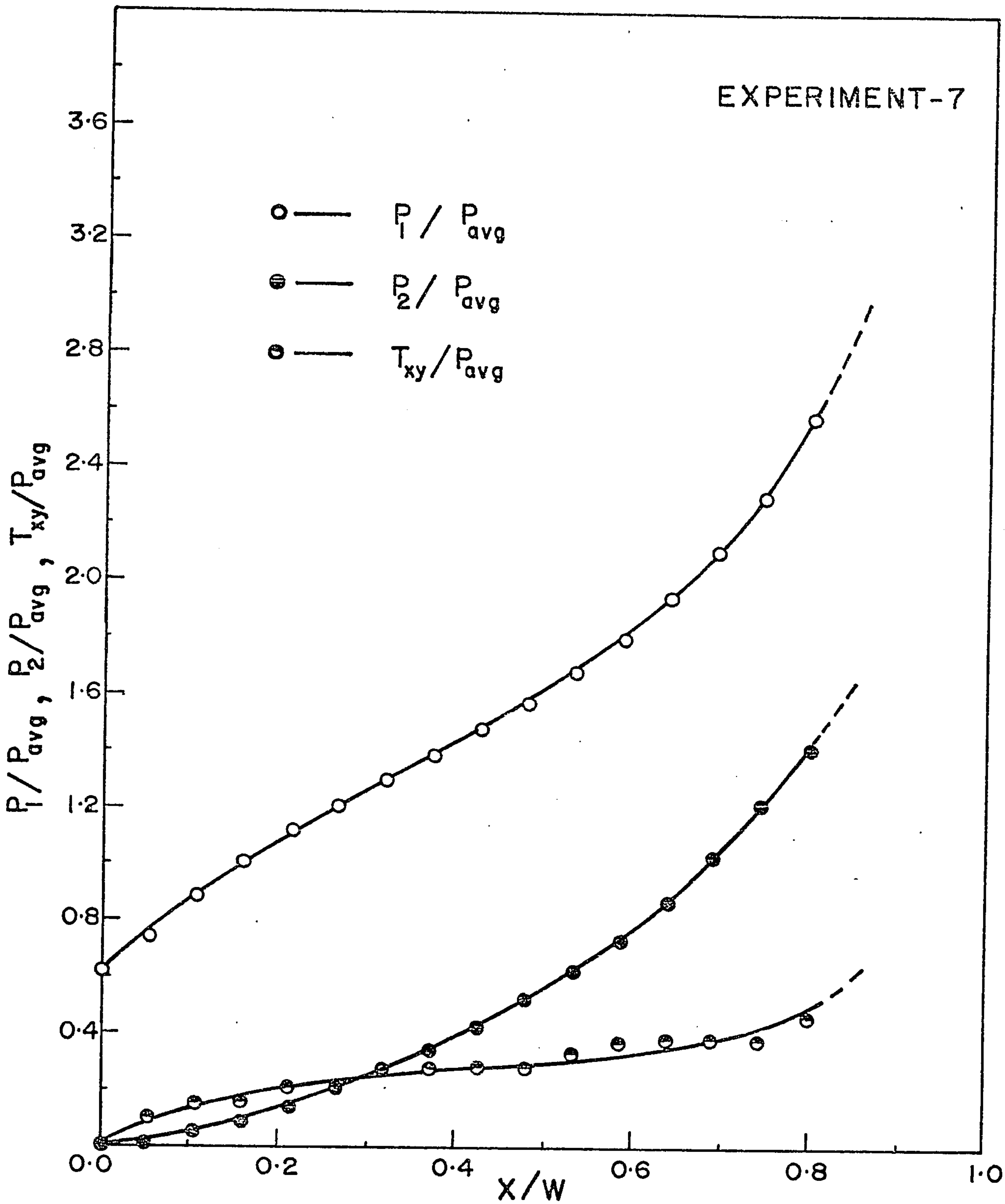

EXPERIMENT-7
P_1/P_{avg}
P_2/P_{avg}
T_{xy}/P_{avg}
P_1/P_{avg}, P_2/P_{avg}, T_{xy}/P_{avg}
3·6
3·2
2·8
2·4
2·0
1·6
1·2
0·8
0·4
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.-29

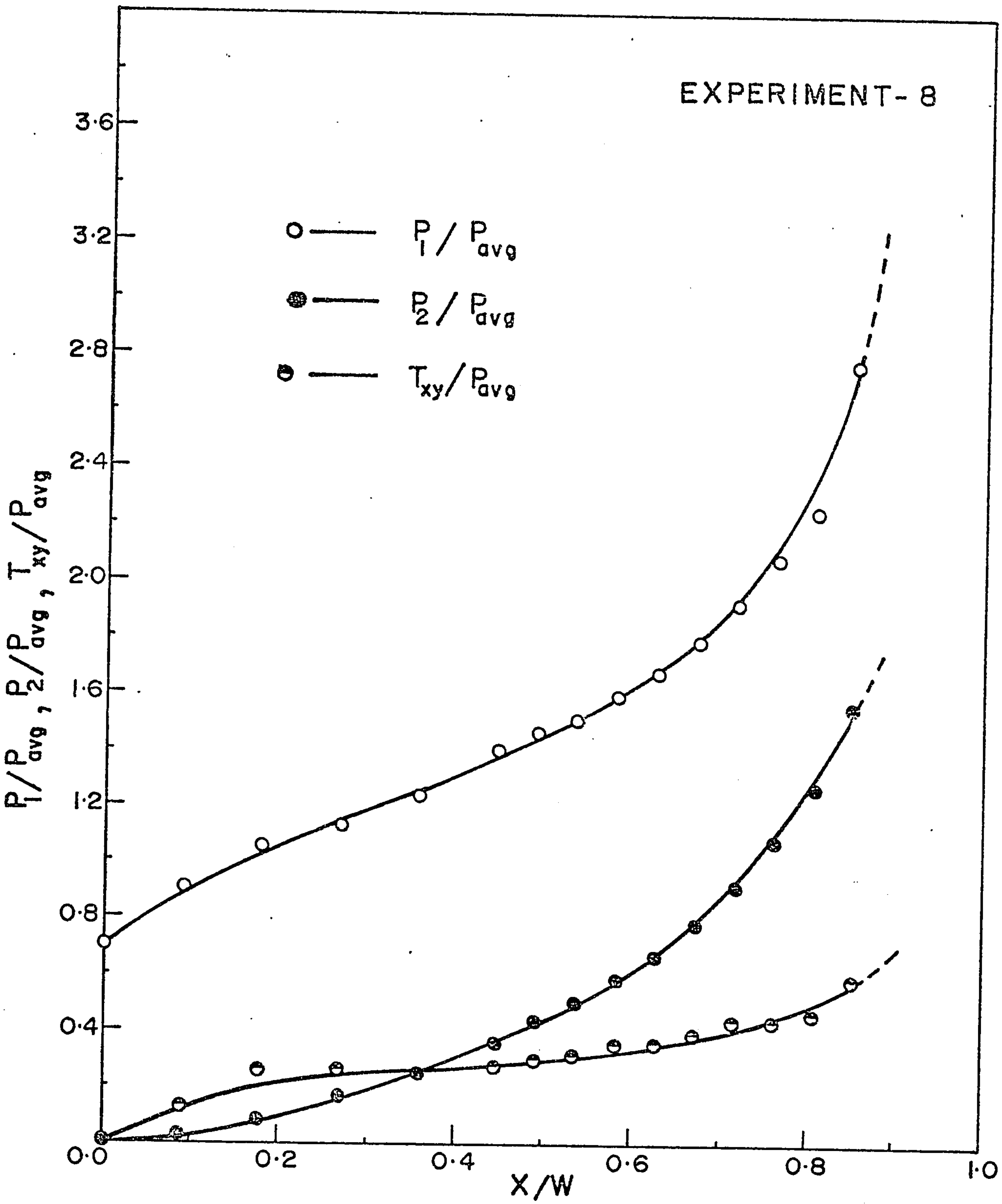

EXPERIMENT- 8
P_1/P_{avg}
P_2/P_{avg}
T_{xy}/P_{avg}
P_1/P_{avg}, P_2/P_{avg}, T_{xy}/P_{avg}
X/W
0·0
0·2
0·4
0·6
0·8
1·0
1·2
1·6
2·0
2·4
2·8
3·2
3·6

FIG.-30

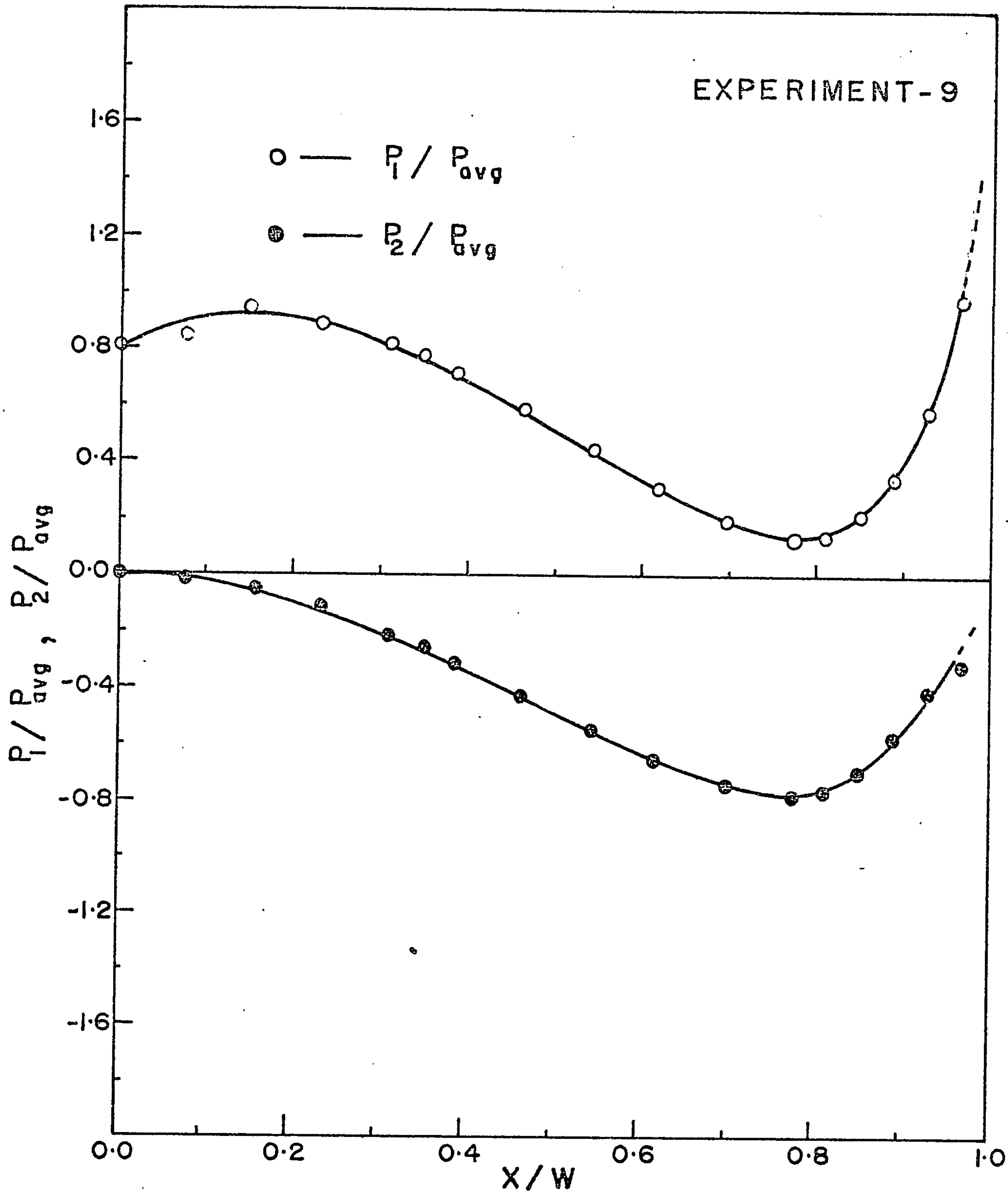

EXPERIMENT-9
P_1/P_{avg}
P_2/P_{avg}
1·6
1·2
0·8
0·4
0·0
-0·4
-0·8
-1·2
-1·6
P_1/P_{avg} , P_2/P_{avg}
0·0
0·2
0·4
0·6
0·8
1·0
X / W

FIG.- 31

as a thick balck line. This is due to the presence of the cement between the parent material and the insert even though a 'see through' type cement has been used. Further, due to high concentration of stresses near the interface it is very difficult to identify the fringes clearly. Therefore the readings in this region could not be taken.

In order to have some idea of stress distribution near the interface, a simple extrapolation has been used. This is shown by dotted lines. A comparative idca about the stresses can be had from the fringe order at the point of interest in the specimens under tension.

The twelve experiments have been divided in four groups as shown in table I for a comparative study.

In group I (figs. 23 - 25), the shear stress is zero. This is in agreement with the theory that shear stress is always zero on the line of symmetry. In all the three cases the maximum stress occurs at the interface and the minimum stress occurs at the outer boundary. The stress P_1 at the interface increases with the decrease in the size of the insert and the rate of increase is greater as we approach the interface in the case of smalles insert (case 3). For all the cases, the magnitude of the stress P_2 on the interface is very small.

In group II (figs. 25 - 27), the stress P_1 follows the same pattern as in group I.

In group III (figs. 29 - 31), the maximum stress concentration occurs at the corner in all the three cases but the pattern is slightly different from the other two groups. The stress P_1 at the interface has the largest value in the case of largest insert (case 7) and decreases with the decrease in size of insert. Also the rate of increase of slope decreases with the decrease in size of insert. The minimum value of stress P_1 in the case of smallest insert (case 9) occurs not at the outer boundary but somewhere in the middle. The effect of the corner on the outer boundary is also noted in this group. The shear stress in the case of smallest insert is zero,

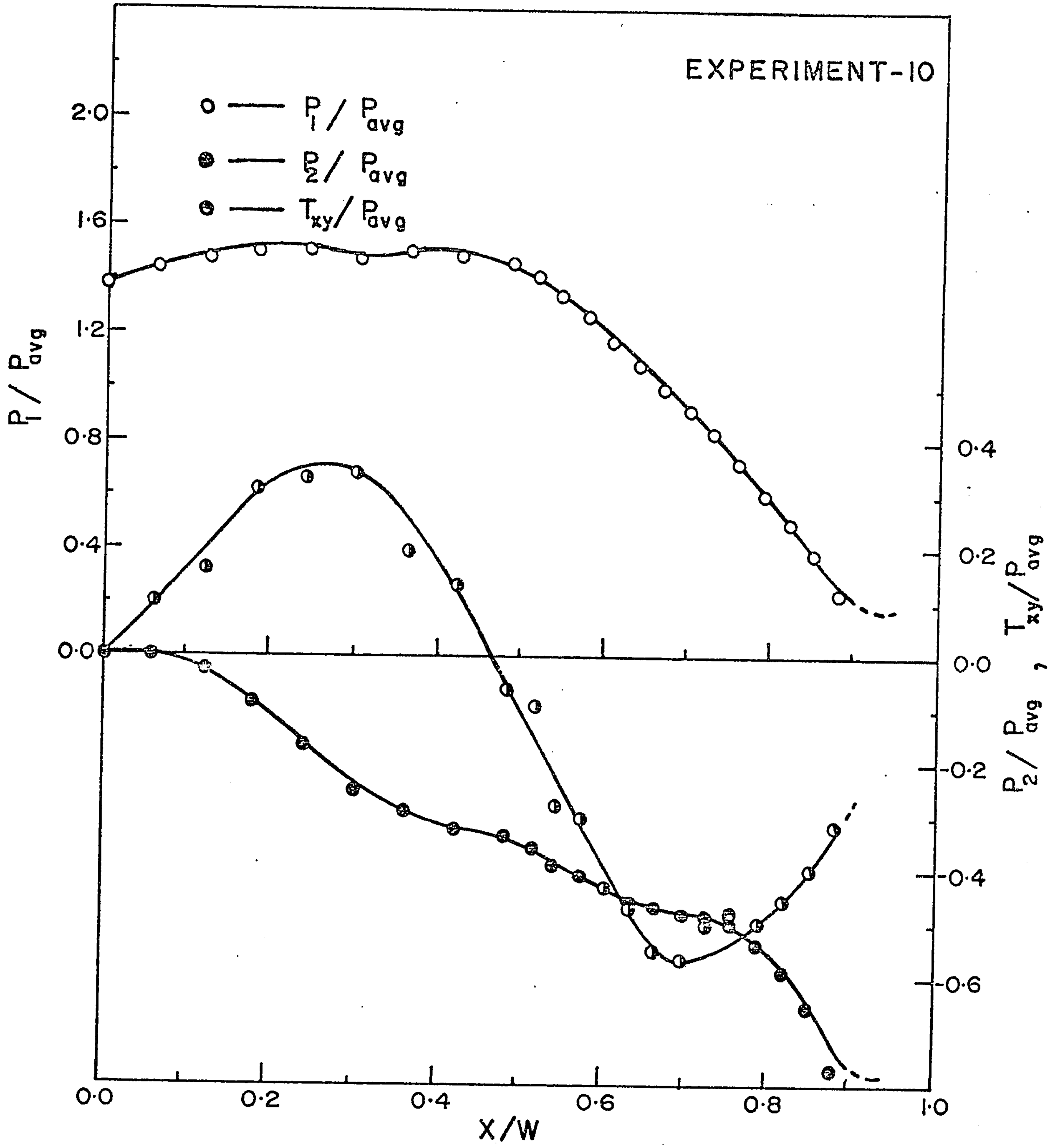

EXPERIMENT-10
P_1 / P_{avg}
P_2 / P_{avg}
T_{xy} / P_{avg}
2·0
1·6
1·2
0·8
0·4
0·0
P_1 / P_{avg}
0·4
0·2
0·0
-0·2
-0·4
-0·6
T_{xy}/P_{avg} , P_2/P_{avg}
0·0
0·2
0·4
0·6
0·8
1·0
X/W

FIG.-32

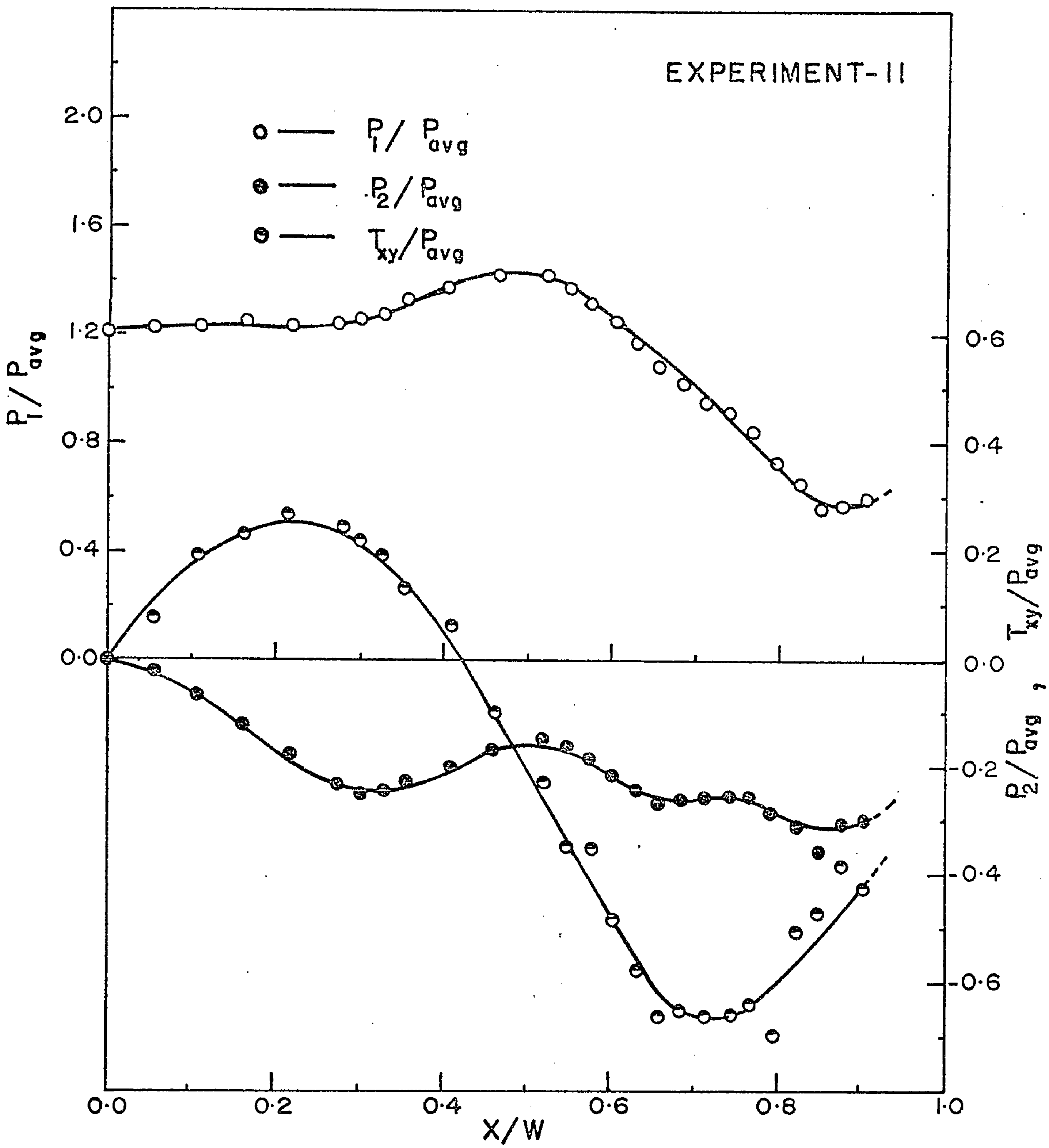

EXPERIMENT-II
P_1/P_{avg}
P_2/P_{avg}
T_{xy}/P_{avg}
P_1/P_{avg}
T_{xy}/P_{avg}, P_2/P_{avg}
X/W
2·0
1·6
1·2
0·8
0·4
0·0
0·6
0·4
0·2
0·0
-0·2
-0·4
-0·6
0·0
0·2
0·4
0·6
0·8
1·0

FIG.-33

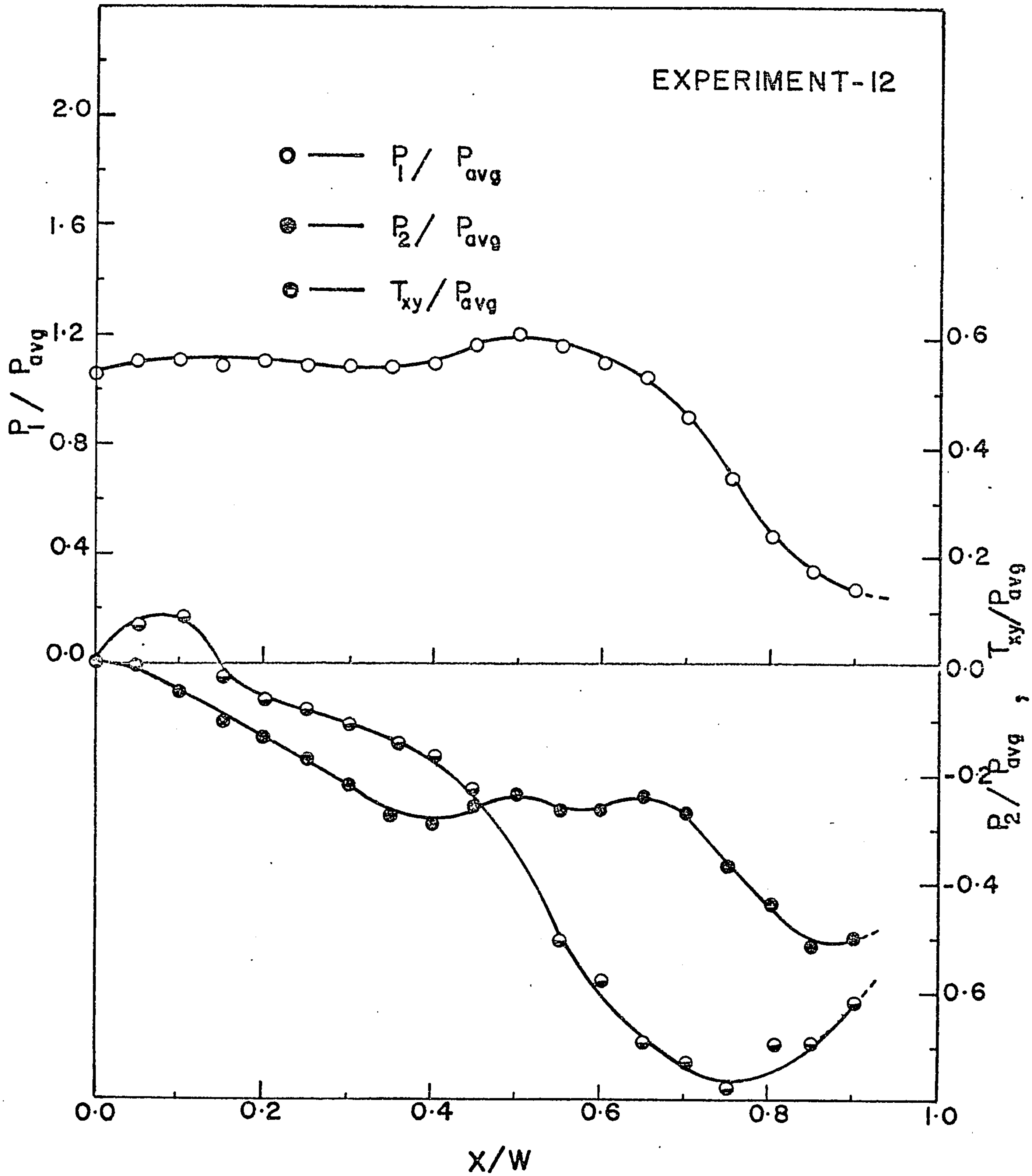

EXPERIMENT-12
P_1/P_{avg}
P_2/P_{avg}
T_{xy}/P_{avg}
2.0
1.6
1.2
0.8
0.4
0.0
0.6
0.4
0.2
0.0
-0.2
-0.4
0.6
0.0
0.2
0.4
0.6
0.8
1.0
X/W
P_1/P_{avg}
T_{xy}/P_{avg}, P_2/P_{avg}

FIG.-34

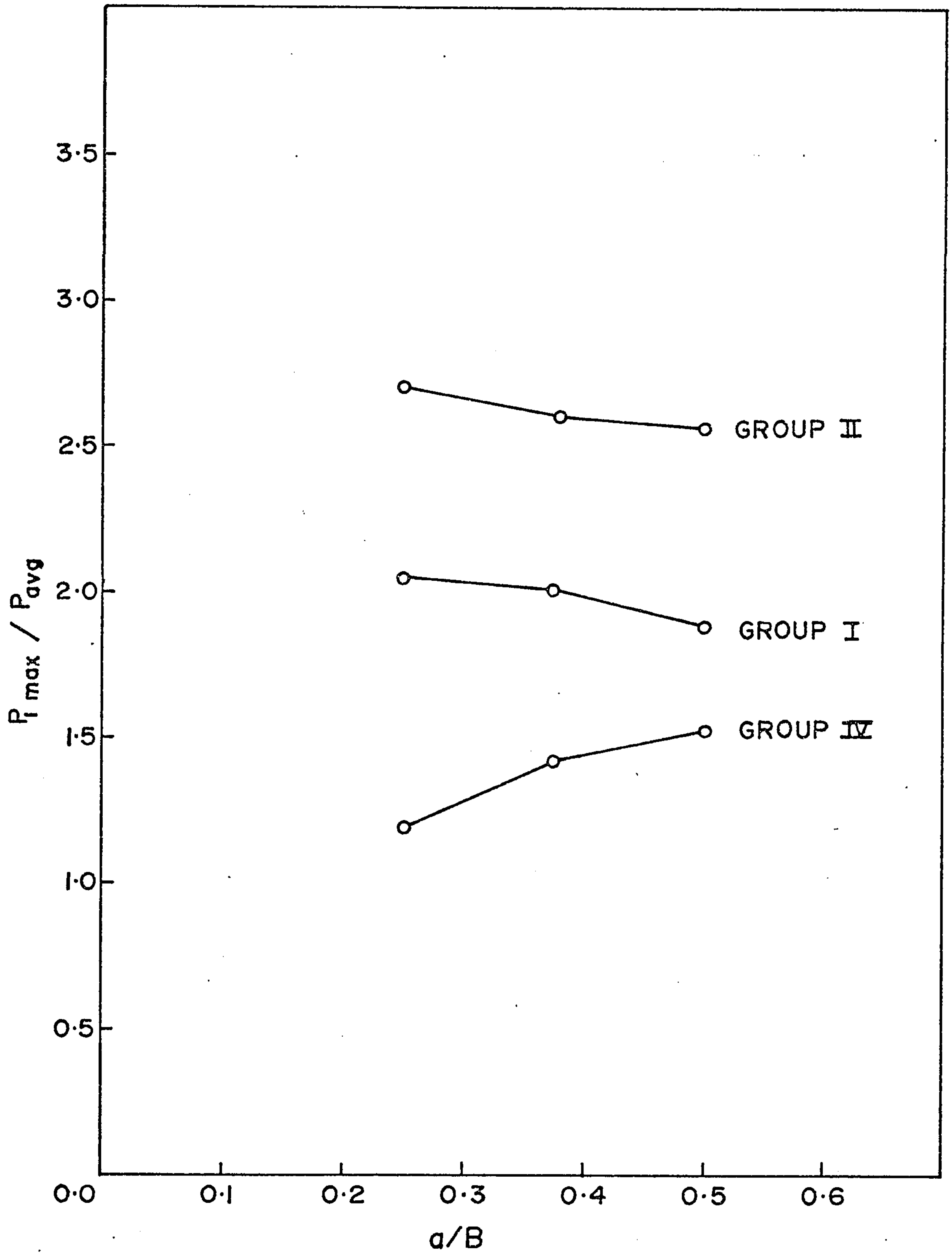

3·5
3·0
2·5
2·0
1·5
1·0
0·5
0·0
0·1
0·2
0·3
0·4
0·5
0·6
$P_{i\,max} / P_{avg}$
a/B
GROUP II
GROUP I
GROUP IV

FIG. 35

because of symmetry.

In group IV, (figs. 32 - 34) the line along which the stress distribution is studied is 0.025" away from the corner. In this case, the maximum stress occurs somewhere in the middle and then it drops down rapidly, as we proceed towards the insert. The stress P_2 remains compressive in all three cases. The magnitude of shear stress is quite high as compared to the other groups.

A cross plot between the maximum stress P_1 and the various sizes and orientations of the insert is plotted as shown in fig. 35. From figs. 23 - 35 the following conclusions are made.

1. The maximum stress concentration occurs at the corner closest to the outside boundary.

2. In case of insert with 0° orientation, the stress concentration increases with the decrease in the size of the insert where as in the case of 45° orientation, the stress concentration decreases with the decrease in the size of the insert.

3. The rate of increase of stress P_1 in the region near the interface or corner is much higher than in any other region.

4. The stress P_1 at the outer boundary increases with the increase in distance between the corner and the outer boundary. Since no information is available in this field, it is not possible to compare these results with any other. The theoretical results obtained in this investigation are unsatisfactory and therefore are not presented in this thesis. However an attempt has been made here to lay out the reasons as to why it is so and a few suggestions have been made.

The finite difference approximation as other numerical approximation, is liable to error in region where stresses are subject to very rapid changes such as exist near singularities caused by corners in this case.

Elimination of singularities due to reentrant corners has not yet been successfully treated and the usual way of dealing with the difficulties is to use finer mesh in such regions as suggested by Zienkiewicz [25] . This approach has also been tried but without success. The smallest mesh size tried here is $h = 1/16$ which generatesa mesh with 512 nodal points in case of insert with 0° orientation and 549 nodal points in case of insert with 45° orientation. This poses another problem of the solution of a large set of simultaneous equations which has, however, been overcome with the development of a special technique of economical storage and the solution of large matrices [23] , using Gauss Sidel iterative technique. During the numerical calculations, it has been found that the matrix generated is ill conditioned . With a slight alteration in the value of any element of the matrix, the end result changes rapidly and the convergence is also very slow. It has also been noted that the interface conditions obtained in Reference 3 are not valid for piecewise continuous boundaries.

It is therefore suggested that a further study is needed to find a suitable finite difference operator for the re-entrant corner. Once the operator has been found,these programs can be used to determine the values of the stresses in this problem.

APPENDICES

I.1. DERIVATION OF BOUNDARY CONDITIONS FOR CASE II - 45° ORIENTATION

Referring to fig. 3, the length of the plate is 2L and its breadth is L. The intensity of applied tension along AD and BC is 100 (non dimensional value). The values of $\frac{\partial \phi}{\partial x}$, $\frac{\partial \phi}{\partial y}$ and ϕ along the boundary can be calculated as follows.

Along boundary BC, we have.

$$P_\nu = 100$$

$$(x, \nu) = 45^\circ$$

and $$ds = -\sqrt{2}\,dx = \sqrt{2}\,dy$$

Therefore using these values in eqns. (2.2.1) and (2.2.2), we get

$$P_{\nu x} = 50\sqrt{2}$$

$$P_{\nu y} = 50\sqrt{2}$$

and substituting in pair of eqns. (2.2.3), we obtain

$$\frac{\partial \phi}{\partial x} = 100\,x + C_1$$

$$\frac{\partial \phi}{\partial y} = 100\,y + C_2$$

Now, from eqn. (2.2.4) and using the values obtained above we have

$$\phi_{BC} = 100x^2 + C_1 x + 100y^2 + C_2 y - 50x^2 - 50y^2 + C_3$$

where C_1, C_2 and C_3 are constants.

Let $C_1 = C_2 = C_3 = 0$ [see Art 2.2.]

Therefore

$$\phi_{BC} = 50x^2 + 50y^2$$

The same procedure is followed for the boundaries AB, CD and AD.

Now for boundary CD, $P_\nu = 100$

Therefore from eqns. (2.2.1) and (2.2.2)

$$P_{\nu x} = 0$$

and

$$P_{\nu y} = 0$$

The values of $\frac{\partial \phi}{\partial x}$, $\frac{\partial \phi}{\partial y}$ and ϕ should be continuous all along the boundary, therefore

$$\frac{\partial \phi}{\partial x} = 100\sqrt{2}$$

$$\frac{\partial \phi}{\partial y} = 150\sqrt{2}$$

and substituting in eqn. (2.2.4) we have

$$\phi_{CD} = 100\sqrt{2}\,x + 150\sqrt{2}\,y + C_4$$

To evaluate C_4, we equate the values of ϕ_{CD} and ϕ_{BC} at the common point C, Thus,

$$(\phi_{CD})_C = (\phi_{BC})_C$$

or

$$(100\sqrt{2}\,x + 150\sqrt{2}\,y + C_4)_C = (50\,x^2 + 50\,y^2)_C$$

Therefore $C_4 = -325$

Hence

$$\phi_{CD} = 100\sqrt{2}\,x + 150\sqrt{2}\,y - 325$$

Now along boundary AB, we have

$$P_\nu = 0$$

Therefore, using eqns. (2.2.1) and (2.2.2)

$$P_{\nu x} = 0$$

$$P_{\nu y} = 0$$

and using the same reasoning as mentioned in case of boundary CD, we have

$$\left(\frac{\partial \phi}{\partial x}\right)_{AB} = 150\sqrt{2}$$

$$\left(\frac{\partial \phi}{\partial y}\right)_{AB} = 100\sqrt{2}$$

From eqn. (2.2.4), we get

$$\phi_{AB} = 150\sqrt{2}\,x + 100\sqrt{2}\,y + C_5$$

Now to evaluate C_5, put

$$(\phi_{AB})_B = (\phi_{BC})_B$$

Therefore

$$C_5 = -325$$

Hence

$$\phi_{AB} = 150\sqrt{2}\,x + 100\sqrt{2}\,y - 325$$

Now for boundary AD, we have

$$P_\nu = 100$$

$$ds = \sqrt{2}\,dx = -\sqrt{2}\,dy$$

Therefore

$$P_{\nu x} = -50\sqrt{2}$$

$$P_{\nu y} = -50\sqrt{2}$$

Now using pair of eqns. (2.2.3), we have

$$\frac{\partial \phi}{\partial x} = 100x + a_1$$

$$\frac{\partial \phi}{\partial y} = 100y + a_2$$

where a_1 and a_2 are constants which are evaluated by putting

$$\left(\frac{\partial \phi}{\partial x}\right)_{\text{Along AD at D}} = \left(\frac{\partial \phi}{\partial x}\right)_{\text{Along CD at D}}$$

Therefore

$$a_1 = 100\sqrt{2}$$

Similarly

$$\left(\frac{\partial \phi}{\partial y}\right)_{\text{Along AD at D}} = \left(\frac{\partial \phi}{\partial y}\right)_{\text{Along CD at D}}$$

Therefore

$$a_2 = 100\sqrt{2}$$

Hence

$$\frac{\partial \phi}{\partial x} = 100\,x + 100\sqrt{2}$$

and

$$\frac{\partial \phi}{\partial y} = 100\,y + 100\sqrt{2}$$

Now substituting these values in eqn.(2.2.4), we have

$$\phi_{AD} = 50x^2 + 100\sqrt{2}\,x + 50y^2 + 100\sqrt{2}\,y + a_3$$

To evaluate the constant a_3, we use the fact that in order to maintain the continuity of function, ϕ_{AD} and ϕ_{CD} should have the same value at point D i.e.

$$(\phi_{AD})_D = (\phi_{CD})_D$$

Therefore

$$a_3 = -300$$

Check :

The value of ϕ_{AB} and ϕ_{AD} should also be same at the common point A, therefore

$$(\phi_{AD})_A = (\phi_{AB})_A$$

$$\therefore \quad a_3 = -300$$

Hence

$$\phi_{AD} = 50x^2 + 50y^2 + 100\sqrt{2}\,x + 100\sqrt{2}\,y - 300$$

II. 1 GENERAL FORM OF BIHARMONIC OPERATOR

The size of the mesh is as shown in the fig. 5. The biharmonic equation at point 0 is given by

$$(\nabla^4 \phi)_0 = 0 \equiv \left(\frac{\partial^4 \phi}{\partial x^4}\right)_0 + 2\left(\frac{\partial^4 \phi}{\partial x^2 \partial y^2}\right)_0 + \left(\frac{\partial^4 \phi}{\partial y^2}\right)_0 \qquad (2.1.1a)$$

Now

$$\left(\frac{\partial^4 \phi}{\partial x^4}\right)_0 = \left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0\right]_0 \qquad (2.1.2a)$$

and

$$\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0 = 4\,\frac{\phi_3 - 2\phi_0 + \phi_1}{\delta^2 (H_2 + H_3)^2}$$

Substituting this in eqn. (2.1.2a), we have

$$\left(\frac{\partial^4 \phi}{\partial x^4}\right)_0 = \frac{4}{\delta^2 (H_2+H_3)^2}\left[\left(\frac{\partial^2 \phi}{\partial x^2}\right)_1 - 2\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0 + \left(\frac{\partial^2 \phi}{\partial x^2}\right)_3\right]$$

$$= \frac{4}{\delta^2 (H_2+H_3)^2}\left[\frac{4}{\delta^2 (H_3+H_4)^2}(\phi_9 - 2\phi_1 + \phi_0) - \frac{2.4}{\delta^2 (H_2+H_3)^2}(\phi_1 - 2\phi_0 + \phi_3)\right.$$

$$\left. + \frac{4}{\delta^2 (H_i+H_2)^2}(\phi_0 - 2\phi_3 + \phi_{11})\right] \qquad (2.1.3a)$$

Similarly $\dfrac{\partial^4 \phi}{\partial y^4}$ can be written as

$$= \frac{4}{\delta^2 (v_2+v_3)}\left[\frac{4}{\delta^2 (v_3+v_4)^2}(\phi_{10} - 2\phi_2 + \phi_0) - \frac{2.4}{\delta^2 (v_2+v_3)^2}(\phi_2 - 2\phi_0 + \phi_4)\right.$$

$$\left. + \frac{4}{\delta^2 (v_1+v_2)^2}(\phi_0 - 2\phi_4 + \phi_{12})\right] \qquad (2.1.4a)$$

and $\dfrac{\partial^4 \phi}{\partial x^2 \, \partial y^2}$

$$= \frac{4}{\delta^2 (v_2+v_3)^2} \left[\frac{4}{\delta^2 (H_2+H_3)^2} (\phi_5 - 2\phi_2 + \phi_6) - \frac{2.4}{\delta^2 (H_2+H_3)^2} (\phi_1 - 2\phi_0 + \phi_3) \right.$$

$$\left. + \frac{4}{\delta^2 (H_2+H_3)^2} (\phi_8 - 2\phi_4 + \phi_7) \right] \qquad (2.1.5a)$$

Now substituting the values in eqn.(2.1.1a) from eqns.(2.1.3a),(2.1,4a) and (2.1.5a), we get

$$\nabla^4 \phi = \phi_0 [IA] + \phi_1 [IB] + \phi_2 [IC] + \phi_3 [ID] + \phi_4 [IE]$$

$$+ (\phi_5 + \phi_6 + \phi_7 + \phi_8) [IF] + \phi_9 [IG] + \phi_{10} [IH]$$

$$+ \phi_{11} [II] + \phi_{12} [IK] \qquad (2.1.6a)$$

where

$$IA = \left[\frac{16}{(H_2+H_3)^2 (H_3+H_4)^2} + \frac{64}{(H_2+H_3)^4} + \frac{16}{(H_2+H_3)^2 (H_1+H_2)^2} \right.$$

$$\left. + \frac{16}{(v_2+v_3)^2 (v_3+v_4)^2} + \frac{64}{(v_2+v_3)^4} + \frac{16}{(v_2+v_3)^2 (v_1+v_2)^2} + \frac{128}{(v_2+v_3)^2 (H_2+H_3)^2} \right]$$

$$IB = \left[- \frac{32}{(H_2+H_3)^2 (H_3+H_4)^2} - \frac{32}{(H_2+H_3)^4} - \frac{64}{(v_2+v_3)^2 (H_2+H_3)^2} \right]$$

$$IC = \left[\frac{32}{(v_2+v_3)^2 (v_3+v_4)^2} - \frac{32}{(v_2+v_3)^4} - \frac{64}{(v_2+v_3)^2 (H_2+H_3)^2} \right]$$

$$ID = \left[\frac{32}{(H_2+H_3)^4} - \frac{32}{(H_2+H_3)^2 (H_1+H_2)^2} - \frac{64}{(v_2+v_3)^2 (H_2+H_3)^2} \right]$$

$$IE = \left[\frac{32}{(v_2+v_3)^4} - \frac{32}{(v_2+v_3)^2 (v_1+v_2)^2} - \frac{64}{(v_2+v_3)^2 (H_2+H_3)^2} \right]$$

$$IF = \left[\frac{32}{(v_2+v_3)^2 (H_2+H_3)^2} \right]$$

$$IG = \left[\frac{16}{(H_2+H_3)^2 (H_3+H_4)^2} \right]$$

$$IH = \left[\frac{16}{(v_2+v_3)^2 (v_3+v_4)^2} \right]$$

$$II = \left[\frac{16}{(H_2+H_3)^2 (H_1+H_2)^2} \right]$$

$$IK = \left[\frac{16}{(v_2+v_3)^2 (v_1+v_2)^2} \right]$$

When the mesh is square i.e.

$H_1 = H_2 = H_3 = H_4 = v_1 = v_2 = v_3 = v_4$, eqn. (2.1.6 a)

reduces to

$$\nabla^4 \phi = 20\,\phi_0 - 8(\phi_1+\phi_2+\phi_3+\phi_4) + 2(\phi_5+\phi_6+\phi_7+\phi_8)$$

$$+ \phi_9 + \phi_{10} + \phi_{11} + \phi_{12} = 0$$

II.2 OPERATOR FOR THE POINTS ON THE INTERFACE

The orientation of the point is shown in fig. 10.

Writing the biharmonic equation in finite difference form at point 0, we have

$$20\,\phi_0 - 8(\phi_1+\phi_2+\phi_3+\phi_4) + 2(\phi_5+\phi_6+\phi_7+\phi_8)+\phi_9+\phi_{10}+\phi_{11}+\phi_{12} = 0$$

in the above equation ϕ_7, ϕ_4, ϕ_8 and ϕ_{12} are fictitious and would have to be replaced by real values .

Using conditions (2.3.8) and (2.3.9) at point 0 and writing in finite difference form, we have

$\phi_2 - \phi_4 = \Phi_2 - \Phi_4$ (from 2.3.8) (2.2.1a) and

$$A_1[\phi_2+\phi_4-2\phi_0-B_1(\phi_1+\phi_3-2\phi_0)] = A_2[\Phi_2+\Phi_4-2\Phi_0-B_2(\Phi_1+\Phi_3-2\Phi_0)]$$

(from 2.3.9)

(2.2.2a)

Now using the condition that on the interface

$$\phi = \Phi$$

from eqns. (2.2.1a) and (2.2.2a), we get

$$\phi_4\left[\frac{1}{A_1+A_2}\right]\left[2A_2\Phi_4-(\phi_1+\phi_3)\{-A_1B_1+A_2B_2\}-\phi_2\{A_1-A_2\}\right.$$
$$\left.-\{-2A_1+2A_1B_1+2A_2-2A_2B_2\}\right]$$

Again writing eqns.(2.3.8) and (2.3.9) in finite difference for point 3, we have

$$\phi_6-\phi_7 = \Phi_6-\Phi_7 \qquad (2.2.3a)$$

and

$$A_1[\phi_6-2\phi_3+\phi_7-B_1(\phi_0-2\phi_3+\phi_{11})] = A_2[\Phi_6-2\Phi_3+\Phi_7-B_2(\Phi_0-2\Phi_3+\Phi_{11})]$$

(2.2.4a)

From eqns.(2.2.3a) and (2.2.4a) and also using eqn.(2.3.7), we get

$$\phi_7 = \left[\frac{1}{A_1+A_2}\right]\left[2A_2\Phi_7-(A_1-A_2)\phi_6-\phi_3(+2A_1B_1-2A_1+A_2-2A_2B_2)-\phi_0\right.$$
$$\left.(-A_1B_1+A_2B_2)-\phi_{11}(-A_1B_1+A_2B_2)\right]$$

Similarly using eqns.(2.3.7), (2.3.8) and (2.3.9) at point 1, we get

$$\phi_8 = \left[\frac{1}{A_1+A_2}\right]\left[2A_2\Phi_8-(A_1-A_2)\phi_5-\phi_1(-2A_1+2A_1B_1+A_2-2A_2B_2)\right.$$
$$\left.-(\phi_9+\phi_0)\{-A_1B_1+A_2B_2\}\right]$$

Now writing eqn. (2.3.10) in finite difference for point 0, we have

$$- A_1 [\phi_{10} - 2\phi_2 + 2\phi_4 - \phi_{12} + (2+B_1)(\phi_5 - \phi_8 - 2\phi_2 + 2\phi_4 + \phi_6 - \phi_7)]$$

$$= - A_1 [\Phi_{10} - 2\Phi_2 + 2\Phi_4 - \Phi_{12} + (2+B_2)(\Phi_5 - \Phi_8 - 2\Phi_2 + 2\Phi_4 + \Phi_6 - \Phi_7)]$$

(2.2.5a)

The condition that at the interface

$$\nabla^4 \phi = \nabla^4 \Phi = 0$$

can be written in finite difference as follows

$$20\phi_0 - 8(\phi_1 + \phi_2 + \phi_3 + \phi_4) + 2(\phi_5 + \phi_6 + \phi_7 + \phi_8) + \phi_9 + \phi_{10} + \phi_{11} + \phi_{12}$$

$$= 20\Phi_0 - 8(\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4) + 2(\Phi_5 + \Phi_6 + \Phi_7 + \Phi_8) + \Phi_9 + \Phi_{10} + \Phi_{11} + \Phi_{12}$$

Now stubstituting for Φ_2, Φ_5 and Φ_6 in terms of real values using eqn. (2.3. 8) and simplifying we get

$$\Phi_{10} = - 16\phi_4 + 4\phi_7 + 4\phi_8 + \phi_{10} + \phi_{12} + 16\Phi_4 - 4\Phi_8 - 4\Phi_7 - \Phi_{12}$$

Again in eqn. (2.2.5a) substituting for Φ_2, Φ_5, Φ_6 and Φ_{10} in terms of real values and rearranging we get

$$\frac{1}{A_1+A_2} \{ \phi_2[-2A_1 - 2(2+B_1)A_1 + 2A_2 + 2A_2(2+B_2)] + \phi_4[2A_1 + 2A_1(2+B_1) + 16A_2 - 2A_2$$

$$- 2A_2(2+B_2)] + (\phi_5 + \phi_6)[A_1(2+B_1) - A_2(2+B_2)] + (\phi_7+\phi_8)[-A_1(2+B_1) - 4A_2$$

$$+ (2+B_2)A_2] + \phi_{10}(A_1 - A_2) - \Phi_4(16A_2) + \Phi_7(4A_2) + \Phi_8(4A_2) \} = \phi_{12}$$

Now substituting the values of fictitious points ϕ_4, ϕ_7, ϕ_8 and ϕ_{12} in the finite difference form of $\nabla^4 \phi = 0$ and rearranging we get

$$\phi_0 [20 + \frac{1}{(A_1+A_2)^2} \{(A_2B_2 + A_1B_1)\ 8(A_2 - A_1) - 6(A_2B_2 - A_1B_1)^2 - 4(A_2 - A_1)^2 \}]$$

$$+ (\phi_1 + \phi_3)\ [-8 + \frac{4(A_2B_2 - A_1B_1)}{(A_1+A_2)^2} \{(A_2B_2 - A_1B_1) - (A_2 - A_1)\}]$$

$$+ \phi_2 \left[-8 + \frac{1}{(A_1+A_2)^2} \{ 4A_1(A_2B_2-A_1B_1) + (A_2-A_1)(8A_2+4A_1) \} \right]$$

$$+ \Phi_4 \left[-8 - \frac{1}{(A_1+A_2)^2} \{ 4A_2(A_2B_2-A_1B_1) + (A_2-A_1)(4A_2+8A_1) \} \right]$$

$$+ (\phi_5+\phi_6) \left[2 - \frac{2}{(A_1+A_2)^2} \{ (A_2B_2 - A_1B_1)A_1 + (A_2-A_1)(A_1+A_2) \} \right]$$

$$+ (\Phi_7+\Phi_8) \left[2 + \frac{2}{(A_1+A_2)^2} \{ (A_2B_2 - A_1B_1)A_2 + (A_2-A_1)(A_1+A_2) \} \right]$$

$$+ (\phi_9+\phi_{11}) \left\{ 1 - \frac{1}{(A_1+A_2)^2} (A_2B_2 - A_1B_1)^2 \right\}$$

$$+ \phi_{10} \left\{ 1 - \frac{A_2-A_1}{A_2+A_1} \right\} + \Phi_{12} \left\{ 1 + \frac{A_2-A_1}{A_2+A_1} \right\} = 0$$

or can be written as

$$\phi_0 [\bar{A}] + (\phi_1+\phi_3)[\bar{B}] + \phi_2[\bar{C}] + \phi_4[\bar{D}] + (\phi_5+\phi_6)[\bar{E}]$$

$$+ (\Phi_7+\Phi_8)[\bar{F}] + (\phi_9+\phi_{11})[\bar{G}] + \phi_{10}[\bar{H}] + \Phi_{12}[\bar{I}] = 0$$

where

$$\bar{A} = \left[20 + \frac{1}{(A_1+A_2)^2} \{ (A_2B_2-A_1B_1)\, 8(A_2-A_1) - 6(A_2B_2-A_1B_1)^2 - 4(A_2-A_1)^2 \} \right]$$

$$\bar{B} = \left[-8 + \frac{4(A_2B_2-A_1B_1)}{(A_1+A_2)^2} \{ (A_2B_2-A_1B_1) - (A_2-A_1) \} \right]$$

$$\bar{C} = \left[-8 + \frac{1}{(A_1+A_2)^2} \{ 4A_1(A_2B_2-A_1B_1) + (A_2-A_1)(8A_2+4A_1) \} \right]$$

$$\bar{D} = [-8 - \frac{1}{(A_1+A_2)^2} \{ 4A_2(A_2B_2-A_1B_1) + (A_2-A_1)(4A_2+8A_1) \}]$$

$$\bar{E} = [\ 2 - \frac{2}{(A_1+A_2)^2} \{ (A_2B_2 - A_1B_1)A_1 + (A_2-A_1)(A_1+A_2) \}]$$

$$\bar{F} = [\ 2 + \frac{2}{(A_1+A_2)^2} \{ (A_2B_2 - A_1B_1)A_2 + (A_2-A_1)(A_1+A_2) \}]$$

$$\bar{G} = [\ 1 - \frac{1}{(A_1+A_2)^2} (A_2B_2 - A_1B_1)^2 \ ]$$

$$\bar{H} = [\ 1 - \frac{A_2 - A_1}{A_2 + A_1} \ ]$$

$$\bar{I} = [\ 1 + \frac{A_2 - A_1}{A_2 + A_1} \ ]$$

The string pattern for this is given in fig. 11.

II.3 OPERATOR FOR THE POINTS ONE MESH AWAY FROM THE INTERFACE

The $\nabla^4 \phi = 0$ in finite difference from at point 0 as shown in fig.12 can be written as

$$20\,\phi_o - 8(\phi_1+\phi_2+\phi_3+\phi_4) + 2(\phi_5+\phi_6+\phi_7+\phi_8) + \phi_9+\phi_{10}+\phi_{11}+\phi_{12} = 0 \qquad (2.3.1a)$$

in which ϕ_{12} is a fictious which have to be replaced by the real value.

Now writing eqns.(2.3.8) and (2.3.9) in finite difference form for points 4, we have

$$\phi_{12} - \phi_0 = \Phi_{12} - \Phi_0 \qquad (2.3.2a)$$

and

$$A_1 [\phi_{12} - 2\phi_4 + \phi_0 - B_1(\phi_8 - 2\phi_4 + \phi_7)] = A_2 [\Phi_{12} - 2\Phi_4 + \Phi_0 - B_2(\Phi_8 - 2\Phi_4 + \Phi_7)]$$

$$(2.3.3a)$$

Using eqns. (2.3.2a), (2.3.3a) and the condition

$$\phi = \Phi$$

At the interface, we get

$$\phi_{12} = (1 + \frac{A_2 - A_1}{A_1 + A_2} \Phi_{12} + (\frac{A_2 - A_1}{A_1 + A_2}) \phi_0 + \frac{2(A_2B_2 - A_1B_1 - A_2 + A_1)}{A_1 + A_2} \phi_4$$

$$- \frac{A_2B_2 - A_1B_1}{A_1 + A_2}) (\phi_7 + \phi_8)$$

Substituting ϕ_{12} in eqn. (2.3.1a) and rearranging we get

$$\phi_0 [20 + \frac{A_2 - A_1}{A_2 + A_1}] - 8(\phi_1 + \phi_2 + \phi_3) + \phi_4 [-8 + \frac{2}{A_1 + A_2} \{ A_2B_2 - A_1B_1 + A_1 - A_2 \}]$$

$$+ 2(\phi_5 + \phi_6) - (\phi_7 + \phi_8) [2 - \frac{A_2B_2 - A_1B_1}{A_1 + A_2}] + \phi_9 + \phi_{10} + \phi_{11}$$

$$+ \Phi_{12} [1 + \frac{A_2 - A_1}{A_1 + A_2}] = 0$$

or this can be written as

$$\phi_0 [AA] - (\phi_1 + \phi_2 + \phi_3) [8] + \phi_4 [BB] + (\phi_5 + \phi_6) [2]$$

$$+ (\phi_7 + \phi_8) [CC] + \phi_9 + \phi_{10} + \phi_{11} + \Phi_{12} [DD] = 0$$

where

$$AA = [20 + \frac{A_2 - A_1}{A_2 + A_1}]$$

$$BB = [-8 + \frac{2}{A_1 + A_2} \{A_2 B_2 - A_1 B_1 + A_1 - A_2\}]$$

$$CC = [2 - \frac{A_2 B_2 - A_1 B_1}{A_1 + A_2}]$$

$$DD = [1 + \frac{A_2 - A_1}{A_1 + A_2}]$$

The string pattern for this is given in fig. 13.

II.4. OPERATOR FOR THE POINT ONE MESH AWAY FROM TWO PERPENDICULAR INTERFACES

The biharmonic equation in the finite difference form at point 0 as shown in fig. 14, can be given as

$$20 \phi_0 - 8(\phi_1 + \phi_2 + \phi_3 + \phi_4) + (2 \phi_5 + \phi_6 + \phi_7 + \phi_8) + \phi_9 + \phi_{10} + \phi_{11} + \phi_{12} = 0 \quad (2.3.1a)$$

Where ϕ_9 and ϕ_{10} are fictitious values. So by using the boundary conditions, these values have to be replaced by real values.

From eqn. (2.3.7), at the interface

$$\phi = \Phi \tag{2.3.2a}$$

Writing eqns. (2.3.8) and (2.3.9) in the finite difference form at point 2, we have

$$\phi_{10} - \phi_0 = \Phi_{10} - \Phi_0 \tag{2.3.3a}$$

and

$$A_1 \left[\phi_{10} - 2\phi_2 + \phi_0 - B_1(\phi_6 - 2\phi_2 + \phi_5) \right] = A_2 \left[\Phi_{10} - 2\Phi_2 + \Phi_0 - B_2(\Phi_6 - 2\Phi_2 + \Phi_5) \right] \tag{2.3.4a}$$

From eqns. (2.3.2a), (2.3.3a), and (2.3.4a), we get

$$\phi_{10} = \left(1 + \frac{A_2 - A_1}{A_2 + A_1}\right) \Phi_{10} + \left(\frac{A_2 - A_1}{A_1 + A_2}\right) \phi_0 + \frac{2(A_2B_2 - A_1B_1 - A_2 + A_1)}{A_1 + A_2} \phi_2 - \left(\frac{A_2B_2 - A_1B_1}{A_1 + A_2}\right)(\phi_5 + \phi_6)$$

Similarly from eqns. (2.3.11), (2.3.12), (2.3.13) and (2.3.14) for the face parallel to y-axis we get

$$\phi_9 = \left(1 + \frac{A_2 - A_1}{A_1 + A_2}\right) \Phi_9 + \left(\frac{A_2 - A_1}{A_1 + A_2}\right) \phi_0 + \frac{2(A_2B_2 - A_1B_1 - A_2 + A_1)}{(A_1 + A_2)} \phi_1 - \left(\frac{A_2B_2 - A_1B_1}{A_1 + B_2}\right)(\phi_5 + \phi_8)$$

Now substituting the values of ϕ_{10} and ϕ_9 in eqn. (2.3.1a) and rearranging we have

$$\phi_0 \left[20 + \frac{2(A_2-A_1)}{(A_1+A_2)} \right] + (\phi_1+\phi_2) \left[-8 + \frac{2(A_2B_2-A_1B_1-A_2+A_1)}{(A_1+A_2)} \right]$$

$$-8(\phi_3+\phi_4) + \phi_5 \left[2 - \frac{2(A_2B_2-A_1B_1)}{A_1+A_2} \right] + (\phi_6+\phi_8) \left[2 - \frac{2(A_2B_2-A_1B_1)}{A_1+A_2} \right]$$

$$+ 2\phi_7 + (\Phi_9+\Phi_{10}) \left[1 + \frac{A_2-A_1}{A_1+A_2} \right] + \phi_1 + \phi_{12} = 0$$

or

$$\phi_0 [AAA] + (\phi_1+\phi_2) [BB] - (\phi_3+\phi_4) 8 + \phi_5 [CCC] + (\phi_6+\phi_8) [CC]$$

$$+ 2\phi_7 + (\Phi_9+\Phi_{10}) [DD] + \phi_{11} + \phi_{12} = 0$$

where

$$AAA = \left[20 + \frac{2(A_2-A_1)}{(A_1+A_2)} \right]$$

$$BB = \left[-8 + \frac{2(A_2B_2-A_1B_1-A_2+A_1)}{(A_1+A_2)} \right]$$

$$CC = \left[2 - \frac{A_2B_2-A_1B_1}{(A_1+A_2)} \right]$$

$$CCC = \left[2 - \frac{2(A_2B_2-A_1B_1)}{(A_1+A_2)} \right]$$

$$DD = \left[1 + \frac{A_2-A_1}{A_1+A_2} \right]$$

The string pattern for this is given in fig. 15.

II.5. OPERATOR FOR THE POINT ONE MESH AWAY FROM TWO PERPENDICULAR INTERFACES

At point 0, as shown in fig. 16, the biharmonic equation in finite difference form can be written as

$$20\,\Phi_0 - 8(\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4) + 2(\Phi_5 + \Phi_6 + \Phi_7 + \Phi_8) + \Phi_9 + \Phi_{10} + \Phi_{11} + \Phi_{12} = 0 \tag{2.3.1a}$$

Where Φ_7 is a fictitious value which has to be replaced by real value using the interface conditions.

Using eqns. (2.3.11), (2.3.12), (2.3.13) and (2.3.14) in their finite difference form at point 4, we get

$$\Phi_7 = \left(\frac{A_1 - A_2}{A_1 + A_2}\right)\Phi_8 + \frac{(2A_2 - 2A_1 + 2A_1B_1 - 2A_2B_2)}{(A_1 + A_2)}\Phi_4$$
$$+ \frac{(A_2B_2 - A_1B_1)}{(A_1 + A_2)}\Phi_o + \frac{(A_2B_2 - A_1B_1)}{(A_1 + A_2)}\Phi_{12} + \frac{2A_1}{A_1 + A_2}\phi_7 \tag{2.3.2a}$$

Substituting in eqn. (2.3.1a) and rearranging we get

$$\Phi_0\left[20 + \frac{2(A_2B_2 - A_1B_1)}{A_1 + A_2}\right] - 8(\Phi_1 + \Phi_2 + \Phi_3) + \Phi_4\left[-8 + \frac{2(2A_2 - 2A_1 + 2A_1B_1 - 2A_2B_2)}{A_1 + A_2}\right]$$
$$+ 2(\Phi_5 + \Phi_6) + \phi_7\left[\frac{4A_1}{A_1 + A_2}\right] + \phi_8\left[2 + \frac{2(A_1 - A_2)}{A_1 + A_2}\right]$$
$$+ \Phi_9 + \Phi_{10} + \Phi_{11} + \Phi_{12}\left[1 + \frac{2(A_2B_2 - A_1B_1)}{A_1 + A_2}\right] = 0$$

or

$$\Phi_0[X] - \delta(\Phi_1 + \Phi_2 + \Phi_3) + \Phi_4[Y] + 2(\Phi_5 + \Phi_6)$$
$$+ \phi_7[S] + \Phi_8[Z] + \Phi_9 + \Phi_{10} + \Phi_{11} + \Phi_{12}[T] = 0$$

where

$$X = \left[20 + \frac{2(A_2B_2 - A_1B_1)}{(A_1 + A_2)} \right]$$

$$Y = \left[-8 + \frac{4(A_2 - A_1 + A_1B_1 - A_2B_2)}{(A_1 + A_2)} \right]$$

$$Z = \left[2 + \frac{2(A_1 - A_2)}{(A_1 + A_2)} \right]$$

$$S = \left[\frac{4A_1}{A_1 + A_2} \right]$$

$$T = \left[1 + \frac{2(A_2B_2 - A_1B_1)}{(A_1 + A_2)} \right]$$

The string pattern for this is given in fig. 16.

III. The Listings of the programs are as follows :

```
            C
            C
            C      ****************************************************************************
            C      THIS PROGRAMME IS THE STEP ONE OF THE THREE STEPS SOLUTION OF
            C      CASE I ( 0 --- INCLINATION ).
            C      ****************************************************************************
            C
            C
            C      ****************************************************************************
            C      THIS PROGRAMME GENERATES THE BIHARMONIC EQUATION AT EACH NODAL
            C      POINT USING APPROPRIATE SUBROUTINE.
            C      ****************************************************************************
            C
            C
            C      THE VARIABLES ARE DEFINED AS FOLLOWS :
            C
            C
            C      M               NO. OF STEPS IN X - DIRECTION
            C      N               NO. OF STEPS IN Y - DIRECTION
            C      MC              LENGTH OF THE INSERT
            C      NC              WIDTH OF THE INSERT
            C      E1,E2           MODULII OF INSERT AND PARENT MATERIAL RESPECTIVELY.
            C      B1,B2           POISSIONS RATIO OF INSERT AND PARENT MATERIAL
            C                      RESPECTIVELY.
            C      P(KK)           ROW OF A MATRIX
            C      RT(KK)          RIGHT HAND SIDE VECTOR OF THE MATRIX.
            C      DEL             STEP SIZE
            C      BY(J)           PHI VALUE ALONG AB ( REF. FIG.2 )
            C      ****************************************************************************
            C
0001                IMPLICIT REAL*8(A-H,O-Z)
0002               COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
0003               DIMENSION KR(128)
            C
            C
0004               READ(1,10)MM,NN,E1,E2,B1,B2
0005               WRITE(3,10)MM,NN,E1,E2,B1,B2
0006            10 FORMAT(2I10,4F15.8)
0007               M=16
0008               N=8
0009               HD=0.125 0D0
0010               MC=5
```

```
0011                NC=3
            C
            C
            C       *************************************************************************
            C       CALCULATE THE COEFFIENTS OF THE VARIOUS OPERATORS.
            C       *************************************************************************
            C
0012             11 A1 = 1./E1
0013                A2 = 1./E2
0014                AA1 = (A1+A2)**2
0015                A = (20.+((A2*B2-A1*B1)*8.*(A2-A1)-6.*((A2*B2-A1*B1)**2)-4.*((A2-
               1A1)**2))/AA1)
0016                BI= (-8.+4.*(A2*B2-A1*B1)*((A2*B2-A1*B1)-(A2- A1))/AA1)
0017                C1 = (-8.+(4.*A1*(A2*B2-A1*B1)+(A2-A1)*(8.*A2+4.*A1))/AA1)
0018                D = (-8.-(4.*A2*(A2*B2-A1*B1)+(A2-A1)*(4.*A2+8.*A1))/AA1)
0019                E = (2.-2.*((A2*B2-A1*B1)*A1+(A2-A1)*(A1+A2))/AA1)
0020                F=(2.+2.*((A2*B2-A1*B1)*A2+(A2-A1)*(A1+A2))/AA1)
0021                G=(1.-((A2*B2-A1*B1)*(A2*B2-A1*B1))/AA1)
0022                H=(1.-(A2-A1)/(A2+A1))
0023                TI=(1.+(A2-A1)/(A2+A1))
0024                AR=A+(-A1*B1+A2*B2)/(A1+A2)
0025                BR=BI+2.*(-A1+A1*B1+A2-A2*B2)/(A1+A2)
0026                ER=E+(A1-A2)/(A1+A2)
0027                FR=F-2.*A2/(A1+A2)
0028                GR=G+(-A1*B1+A2*B2)/(A1+A2)
0029                AA=(20.+(A2-A1)/(A2+A1))
0030                AA1=(20.-(A2-A1)/(A2+A1))
0031                BB=(-8.+2.*(A2*B2-A1*B1+A1-A2)/(A1+A2))
0032                BB1=(-8.-2.*(A2*B2-A1*B1+A1-A2)/(A1+A2))
0033                CC=(2.-(A2*B2-A1*B1)/(A1+A2))
0034                CC1=(2.+(A2*B2-A1*B1)/(A1+A2))
0035                DD=(1.+(A2-A1)/(A1+A2))
0036                DD1=(1.-(A2-A1)/(A1+A2))
0037                AAA=(20.+2.*(A2-A1)/(A1+A2))
0038                CCC=(2.-2.*(A2*B2-A1*B1)/(A1+A2))
0039                XX=(20.-2.*(A1*B1-A2*B2)/(A1+A2))
0040                YY=(-8.-4.*(A1-A2-A1*B1+A2*B2)/(A1+A2))
0041                Z=(2.-2.*(A2-A1)/(A1+A2))
0042                S=(4.*A1/(A1+A2))
0043                T=(1.-2.*(A1*B1-A2*B2)/(A1+A2))
0044                WRITE(3,12)A,BI,C1,D,E,F,G,H,TI,AA,AA1,BB,BB1,CC,CC1,DD,DD1,XX,YY,
               1Z,S,T ,AAA,CCC
```

```
   12 FORMAT(3X,10F12.5)
C
C
C     ************************************************************************
C     THE GENERATION OF MATRIX STARTS FROM HERE.
C     ************************************************************************
C
C
      L=0
      NU=0
      DO 7 J=1,N
      DO 7 I=1,M
      NU=NU+1
      LA(I,J)=NU
    7 CONTINUE
      DEL=0.0D0
      BY(1)=0.0D0
      DO 5 J=2,N
      DEL=DEL+HD
      BY(J)=50.*(DEL**2)
    5 CONTINUE
      DO910 J=1,N
      DO910 I=1,M
      DO 55  K5=1,NU
   55 P(K5)=0.0D0
      B=0.0D0
      K=LA(I,J)
      IF(J.LT.(NC-1)) GO TO 20
      IF(J.EQ.(NC-1) ) GO TO 30
      IF(J.EQ.NC) GO TO 40
      IF(J.EQ.(NC+1)) GO TO 50
   60 CALL STAND
      CALL GAIN(I,J)
      GO TO 150
   20 IF(I.EQ.(MC-1))GO TO 70
      IF(I.EQ.MC)GO TO 80
      IF(I.EQ.(MC+1))GO TO 90
      GO TO 60
   30 IF(I.LT.(MC-1))GO TO 100
      IF(I.EQ.(MC-1))GO TO 110
      IF(I.EQ.MC)GO TO 80
      IF(I.EQ.(MC+1)) GO TO 90
      GO TO 60
```

```
   40 IF(I.LT.(MC+1))GO TO 120
      IF(I.EQ.MC)GO TO 130
      IF(I.EQ.(MC+1))GO TO 90
      GO TO 60
   50 IF(I.LT.(MC+1))GO TO 140
      GO TO 60
   70 CALL FOUR(AA,BB,CC,DD)
      CALL GAIN(I,J)
      GO TO 150
   80 CALL ONFOUR(A,BI,C1,D,E,F,G,H,TI)
      CALL GAIN(I,J)
      GO TO 150
   90 CALL OFOUR(AA1,BB1,CC1,DD1)
      CALL GAIN(I,J)
      GO TO 150
  100 CALL TWO(AA,BB,CC,DD)
      CALL GAIN(I,J)
      GO TO 150
  110 CALL ICFOUR(AAA,BB,CCC,CC,DD)
      CALL GAIN(I,J)
      GO TO 150
  120 CALL ONTWO(A,BI,C1,D,E,F,G,H,TI)
      CALL GAIN(I,J)
      GO TO 150
  130 CALL COFOUR(XX,YY,S,Z,T)
      CALL GAIN(I,J)
      GO TO 150
  140 CALL OTWO(AA1,BB1,CC1,DD1)
      CALL GAIN(I,J)
  150 L=L+1
      KR(L)=K
      WRITE(7)(P(KK),KK=1,NU)
      RT(K)=B
  910 CONTINUE
      WRITE(3,500)((KR(KK),RT(KK)),KK=1,NU)
      WRITE(7)(RT(KK),KK=1,NU)
   88 FORMAT(4D20.10)
  500 FORMAT(1H1,5X,8(I6,F8.3))
      RETURN
      END
```

```
0001                SUBROUTINE GAIN(I,J)
          C
          C
          C         ******************************************************************************
          C         THIS SUBROUTINE PUTS THE COEFFICIENTS OF THE VARIOUS OPERATORS IN
          C         APPROPRIATE PLACES.
          C         ******************************************************************************
          C
          C
0002                 IMPLICIT REAL*8(A-H,O-Z)
0003                COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
0004                BX=50.0D0
0005                FY=100.0D0*2.0D0*HD
0006                L1=LA(I,J)
0007                P(L1)=C(1)+P(L1)
0008                IF(I.EQ.M )GO TO 10
0009                L1=LA(I+1,J)
0010                P(L1)=C(2)+P(L1)
0011                GO TO 20
0012             10 B=B-BY(J)*C(2)
0013             20 IF(J.EQ.N )GO TO 30
0014                L1=LA(I,J+1)
0015                P(L1)=C(3)+P(L1)
0016                GO TO 40
0017             30 B=B-BX*C(3)
0018             40 IF(I.EQ.1)GO TO 50
0019                L1=LA(I-1,J)
0020                P(L1)=C(4)+P(L1)
0021                GO TO 60
0022             50 L1=LA(I+1,J)
0023                P(L1)=C(4)+P(L1)
0024             60 IF(J.EQ.1)GO TO 70
0025                L1=LA(I,J-1)
0026                P(L1)=C(5)+P(L1)
0027                GO TO 80
0028             70 L1=LA(I,J+1)
0029                P(L1)=C(5)+P(L1)
0030             80 IF(J.EQ.N )GO TO 100
0031                IF(I.EQ.M )GO TO 90
0032                L1=LA(I+1,J+1)
0033                P(L1)=C(6)+P(L1)
0034                GO TO 110
0035             90 B=B-BY(J+1)*C(6)
```

```
      GO TO 110
  100 B=B-BX*C(6)
  110 IF(J.EQ.N )GO TO 120
      IF(I.EQ.1)GO TO 130
      L1=LA(I-1,J+1)
      P(L1)=C(7)+P(L1)
      GO TO 140
  120 B=B-BX*C(7)
      GO TO 140
  130 L1=LA(I+1,J+1)
      P(L1)=C(7)+P(L1)
  140 IF(I.EQ.1)GO TO 150
      IF(J.EQ.1)GO TO 160
      L1=LA(I-1,J-1)
      P(L1)=C(8)+P(L1)
      GO TO 180
  150 IF(J.EQ.1)GO TO 170
      L1=LA(I+1,J-1)
      P(L1)=C(8)+P(L1)
      GO TO 180
  160 IF(I.EQ.1)GO TO 170
      L1=LA(I-1,J+1)
      P(L1)=C(8)+P(L1)
      GO TO 180
  170 L1=LA(I+1,J+1)
      P(L1)=C(8)+P(L1)
  180 IF(J.EQ.1)GO TO 190
      IF(I.EQ.M )GO TO 200
      L1=LA(I+1,J-1)
      P(L1)=C(9)+P(L1)
      GO TO 220
  190 IF(I.EQ.M )GO TO 210
      L1=LA(I+1,J+1)
      P(L1)=C(9)+P(L1)
      GO TO 220
  200 IF(J.EQ.1)GO TO 210
      B=B-BY(J-1)*C(9)
      GO TO 220
  210 B=B-BY(J+1)*C(9)
  220 IF(I.EQ.(M-1))GO TO 230
      IF(I.EQ.M )GO TO 240
      L1=LA(I+2,J)
      P(L1)=C(10)+P(L1)
```

```
                GO TO 250
          230   B=B-BY(J)*C(10)
                GO TO 250
          240   L1=LA(I,J)
                P(L1)=P(L1)+C(10)
          250   IF(J.EQ.(N-1))GO TO 260
                IF(J.EQ.N )GO TO 270
                L1=LA(I,J+2)
                P(L1)=P(L1)+C(11)
                GO TO 280
          260   B=B-BX*C(11)
                GO TO 280
          270   L1=LA(I,J)
                P(L1)=P(L1)+C(11)
                B=B-FY*C(11)
          280   IF(I.EQ.1)GO TO 290
                IF(I.EQ.2)GO TO 300
                L1=LA(I-2,J)
                P(L1)=C(12)+P(L1)
                GO TO 310
          290   L1=LA(I+2,J)
                P(L1)=C(12)+P(L1)
                GO TO 310
          300   L1=LA(I,J)
                P(L1)=P(L1)+C(12)
          310   IF(J.EQ.1)GO TO 320
                IF(J.EQ.2)GO TO 330
                L1=LA(I,J-2)
                P(L1)=C(13)+P(L1)
                GO TO 340
          320   L1=LA(I,J+2)
                P(L1)=P(L1)+C(13)
                GO TO 340
          330   L1=LA(I,J)
                P(L1)=P(L1)+C(13)
          340   RETURN
                END
```

```
0001               SUBROUTINE STAND
            C
            C      *****************************************************************************
            C      STANDARD OPERATOR WITH SQUARE MESH. REF. ART.--2..4.1
            C      *****************************************************************************
            C
0002                IMPLICIT REAL*8(A-H,O-Z)
0003               COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
            C
            C
0004           111 C(1)=20.0D0
0005               C(2)=-8.0D0
0006               C(3)=-8.0D0
0007               C(4)=-8.0D0
0008               C(5)=-8.0D0
0009               C(6)=2.0D0
0010               C(7)=2.0D0
0011               C(8)=2.0D0
0012               C(9)=2.0D0
0013               C(10)=1.0D0
0014               C(11)=1.0D0
0015               C(12)=1.0D0
0016               C(13)=1.0D0
0017               RETURN
0018               END
```

```
0001              SUBROUTINE TWO(AA,BB,CC,DD)
            C
            C     ******************************************************************************
            C     OPERATOR FOR THE POINT ON THE INSERT , ONE MESH LENGTH
            C     BELOW THE INTERFACE . REF ART. 2..4.4
            C     ******************************************************************************
            C
0002               IMPLICIT REAL*8(A-H,O-Z)
0003              COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
            C
            C
0004              C(1)=AA
0005              C(2)=-8.0D0
0006              C(3)=BB
0007              C(4)=-8.0D0
0008              C(5)=-8.0D0
0009              C(6)=CC
0010              C(7)=CC
0011              C(8)=2.0D0
0012              C(9)=2.0D0
0013              C(10)=1.0D0
0014              C(11)=DD
0015              C(12)=1.0D0
0016              C(13)=1.0D0
0017              RETURN
0018              END
```

```
 0001                 SUBROUTINE FOUR(AA,BB,CC,DD)
             C
             C        ******************************************************************
             C        OPERATOR FOR THE POINT ON THE INSERT , ONE MESH LENGTH
             C        LEFT OF INTERFACE. REF. ART.--2..4.4
             C        ******************************************************************
             C
 0002                  IMPLICIT REAL*8(A-H,O-Z)
 0003                 COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
             C
             C
 0004                 C(1)=AA
 0005                 C(2)=BB
 0006                 C(3)=-8.0D0
 0007                 C(4)=-8.0D0
 0008                 C(5)=-8.0D0
 0009                 C(6)=CC
 0010                 C(7)=2.0D0
 0011                 C(8)=2.0D0
 0012                 C(9)=CC
 0013                 C(10)=DD
 0014                 C(11)=1.0D0
 0015                 C(12)=1.0D0
 0016                 C(13)=1.0D0
 0017                 RETURN
 0018                 END
```

```
 0001                    SUBROUTINE ICFOUR(AAA,BB,CCC,CC,DD)
               C
               C         ***********************************************************************
               C         OPERATOR FOR THE POINT ON INSERT ONE MESH LENGTH AWAY FROM THE
               C         UPPER RIGHT CORNER OF INTERFACE. REF. ART.--2..4.5
               C         ***********************************************************************
               C
 0002                     IMPLICIT REAL*8(A-H,O-Z)
 0003                    COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
               C
               C
 0004                    C(1)=AAA
 0005                    C(2)=BB
 0006                    C(3)=BB
 0007                    C(4)=-8.0D0
 0008                    C(5)=-8.0D0
 0009                    C(6)=CCC
 0010                    C(7)=CC
 0011                    C(8)=2.0D0
 0012                    C(9)=CC
 0013                    C(10)=DD
 0014                    C(11)=DD
 0015                    C(12)=1.0D0
 0016                    C(13)=1.0D0
 0017                    RETURN
 0018                    END
```

```
0001          SUBROUTINE OTWO(AA1,BB1,CC1,DD1)
        C
        C     ************************************************************************
        C     OPERATOR FOR THE POINT ON THE PARENT MATERIAL ONE MESH LENGTH
        C     ABOVE THE INTERFACE. REF. ART.--2..4.4
        C     ************************************************************************
        C
0002           IMPLICIT REAL*8(A-H,O-Z)
0003          COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
        C
        C
0004          C(1)=AA1
0005          C(2)=-8.0D0
0006          C(3)=-8.0D0
0007          C(4)=-8.0D0
0008          C(5)=BB1
0009          C(6)=2.0D0
0010          C(7)=2.0D0
0011          C(8)=CC1
0012          C(9)=CC1
0013          C(10)=1.0D0
0014          C(11)=1.0D0
0015          C(12)=1.0D0
0016          C(13)=DD1
0017          RETURN
0018          END
```

```
0001                  SUBROUTINE OFOUR(AA1,BB1,CC1,DD1)
                C
                C     ****************************************************************************
                C     OPERATOR FOR THE POINT ON THE PARENT MATERIAL ONE MESH LENGTH
                C     RIGHT OF INTERFACE. REF.ART.--2..4.4
                C     ****************************************************************************
                C
0002                   IMPLICIT REAL*8(A-H,O-Z)
0003                  COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
                C
0004                  C(1)=AA1
0005                  C(2)=-8.0D0
0006                  C(3)=-8.0D0
0007                  C(4)=BB1
0008                  C(5)=-8.0D0
0009                     C(6)=2.0D0
0010                  C(7)=CC1
0011                  C(8)=CC1
0012                  C(9)=2.0D0
0013                  C(10)=1.0D0
0014                  C(11)=1.0D0
0015                  C(12)=DD1
0016                  C(13)=1.0D0
0017                  RETURN
0018                  END
```

```
 0001                SUBROUTINE ONTWO(A,BI,C1,D,E,F,G,H,TI)
               C
               C     **********************************************************************
               C     OPERATOR FOR THE POINT ON THE
               C     UPPER INTERFACE . REF. ART.--2..4.3
               C     **********************************************************************
 0002                 IMPLICIT REAL*8(A-H,O-Z)
 0003                COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
               C
               C
 0004                C(1)=A
 0005                C(2)=BI
 0006                C(3)=D
 0007                C(4)=BI
 0008                C(5)=C1
 0009                C(6)=F
 0010                C(7)=F
 0011                C(8)=E
 0012                C(9)=E
 0013                C(10)=G
 0014                C(11)=TI
 0015                C(12)=G
 0016                C(13)=H
 0017                RETURN
 0018                END
```

```
0001          SUBROUTINE ONFOUR(A,BI,C1,D,E,F,G,H,TI)
        C
        C     OPERATOR FOR THE POINT ON THE
        C     ***********************************************************************
        C     RIGHT INTERFACE . REF. ART.--2..4.3
        C     ***********************************************************************
        C
0002           IMPLICIT REAL*8(A-H,O-Z)
0003          COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
        C
        C
0004          C(1)=A
0005          C(2)=D
0006          C(3)=BI
0007          C(4)=C1
0008          C(5)=BI
0009          C(6)=F
0010          C(7)=E
0011          C(8)=E
0012          C(9)=F
0013          C(10)=TI
0014          C(11)=G
0015          C(12)=H
0016          C(13)=G
0017          RETURN
0018          END
```

```
0001                SUBROUTINE COFOUR(AR,BI,BR,C1,D,E,ER,F,FR,G,GR,H,TI)
              C
              C     ***************************************************************************
              C     OPERATOR FOR THE POINT ON THE
              C     UPPER RIGHT CORNER OF THE INTERFACE . REF. ART .--2..4.6
              C     ***************************************************************************
              C
0002                 IMPLICIT REAL*8(A-H,O-Z)
0003                COMMON P(128),RT(128),C(13),BY(8),B,HD,LA(16,8),M,N
              C
0004                C(1)=X
0005                C(2)=-8.0D0
0006                C(3)=-8.0D0
0007                C(4)=-8.0D0
0008                C(5)=Y
0009                C(6)=2.0D0
0010                C(7)=2.0D0
0011                C(8)=S
0012                C(9)=Z
0013                C(10)=1.0D0
0014                C(11)=1.0D0
0015                C(12)=1.0D0
0016                C(13)=T
0017                RETURN
0018                END
```

```
C
C
C     ************************************************************
C     THIS PROGRAMME IS THE SECOND STEP IN THE THREE STEPS SOLUTION OF
C     THE PROBLEM
C     ************************************************************
C
C
C     ************************************************************
C     THIS SOLVES THE MATRIX, GENERATED BY STEP-1 , USING      GAUSSIAN
C     ELIMINATION METHOD.
C     ************************************************************
C
C
       IMPLICIT REAL*8(A-H,O-Z)
      COMMON Q(128,128)
      DIMENSION QQ(128),X(128),P(128),PP(128)
      READ(1,5)N
      DO 45 I=1,N
      READ(7)(P(K),K=1,N)
      DO 45 J=1,N
   45 Q(I,J)=P(J)
      READ(7,END=123)(PP(I),I=1,N)
  123 CONTINUE
      DO 75 I=1,N
   75 QQ(I)=PP(I)
      L1=1
      L=16
   40 WRITE(3,25)L1,L
      DO 35 I=1,N
   35 WRITE(3,110)(I,(Q(I,J),J=L1,L))
      IF(L.EQ.N)GO TO 20
      L1=L+1
      L=L+16
      IF(L.GT.N)L=N
      GO TO 40
   20 CONTINUE
      WRITE(3,10)(QQ(I),I=1,N)
   10 FORMAT(8F10.4)
      CALL MATINV(N)
      DO525 I=1,N
      X(I)=0.0
      DO525 J=1,N
```

```
  525 X(I)=X(I)+Q(I,J)*QQ(J)
      WRITE(3,57)((I, X(I)),I=1,N)
      WRITE(2,15)( X(I),I=1,N)
   15 FORMAT(4E20.10)
    5 FORMAT(I5)
   25 FORMAT(//,5X,'COLUMN NO.',I5,3X,I5)
   57 FORMAT(//,3X,6(I5,F12.6,2X))
  110 FORMAT(2X,I3,1X,16(F6.2,1X))
      RETURN
      END
```

```
      SUBROUTINE MATINV(   N)
       IMPLICIT REAL*8(A-H,O-Z)
      COMMON A(128,128)
        DIMENSION INDEX(128,2)
      DO 108 I=1,N
108   INDEX(I,1)=0
      II=0
109   AMAX=-1. 0D0
      DO 110 I=1,N
      IF (INDEX(I,1)) 110,111,110
111   DO 112 J=1,N
      IF (INDEX(J,1)) 112,113,112
113   TEMP=DABS(A(I,J))
      IF (TEMP-AMAX) 112,112,114
114   IROW=I
      ICOL=J
      AMAX=TEMP
112   CONTINUE
110   CONTINUE
      IF(AMAX) 225,115,116
116   INDEX (ICOL,1)=IROW
      IF (IROW-ICOL) 119,118,119
119   DO 120 J=1,N
      TEMP=A(IROW,J)
      A(IROW,J)=A(ICOL,J)
120   A(ICOL,J)=TEMP
      II=II+1
      INDEX (II,2)=ICOL
118   PIVOT=A(ICOL,ICOL)
      A(ICOL,ICOL)=1.0D0
      PIVOT=1.0D0/PIVOT
      DO 121 J=1,N
121   A(ICOL,J)=A(ICOL,J)*PIVOT
      DO 122 I=1,N
      IF (I-ICOL) 123,122,123
123   TEMP=A(I,ICOL)
      A(I,ICOL)=0.0D0
      DO 124 J=1,N
124   A(I,J)=A(I,J)-A(ICOL,J)*TEMP
122   CONTINUE
      GO TO 109
125   ICOL=INDEX(II,2)
      IROW=INDEX(ICOL,1)
```

```
          DO 126 I=1,N
          TEMP=A(I,IROW)
          A(I,IROW)=A(I,ICOL)
      126 A(I,ICOL)=TEMP
          II=II-1
      225 IF(II) 125,127,125
      115 WRITE (3,100)
      100 FORMAT(1H0,2X;' ZERO  PIVOT')
      127 CONTINUE
          RETURN
          END
```

```
C
C
C
C     ************************************************************************
C     THIS PROGRAMME IS THE THIRD STEP IN THE THREE STEPS SOLUTION OF
C     THE PROBLEM ( CASE I --- INSERT INCLINED 0 ) .
C     ************************************************************************
C
C
C     ************************************************************************
C     THIS PROGRAMME CALCULATES THE DIRECT AND SHEAR STRESSES FROM THE
C     PHI VALUES AT THE NODAL POINTS OBTAINED FROM STEP 2 .
C     ************************************************************************
C
C
C     ************************************************************************
C     THE VARIABLES ARE DEFINED AS FOLLOWS : -----
C
C
C     I               NUMBER COUNTER IN Y - DIRECTION
C     J               NUMBER COUNTER IN X - DIRECTION
C     M               NO . OF STEPS IN Y - DIRECTION
C     N               NO . OF STEPS IN X - DIRECTION
C     E1 , E2         MODULII OF ELASTICITY OF INSERT AND PARENT MATERIAL
C                     RESPECTIVELY .
C     B1 , B2         THE POISSIONS RATIO OF INSERT AND PARENT MATERIAL
C                     RESPECTIVELY .
C     C               PHI VALUE ALONG BC ( REF. FIG. 2 )
C     BI(I,J)         THE PHI VALUE OF A FICTITIOUS POINT ON THE INSERT
C                     IN TERMS OF REAL POINTS.
C     BO(I,J)         THE PHI VALUE OF A FICTITIOUS POINT ON THE PARENT
C                     MATERIAL IN TERMS OF REAL POINTS.
C     PX , PY         DIRECT STRESSES
C     TXY             SHEAR  STRESS
C     PX1 , PY1       MAJOR AND MINOR PRINCIPAL STRESSES
C     PX2 , PY2       MAJOR AND MINOR PRINCIPAL STRESSES AT THE INTERFACE
C                     ON THE PARENT MATERIAL.
C     ************************************************************************
C
C
C     THE INSERT SIZE IS DEFINED AS FOLLOWS :
C     LENGTH
C                     LARGEST        NC = 5
```

```
              C                    MEDIUM          NC = 4
              C                    SMALLEST        NC = 3'
              C      WIDTH
              C                    FOR ALL CASES   MC = 3
              C      ***************************************************************************
              C
              C
              C
              C
              C
0001                 IMPLICITREAL*8(A-H,O-Z)
0002                 DIMENSION A(16,32),BI(16,32),BO(16,32),P(16),PXX(17,32),PPXX(8),
                    1PYY(17,32),PPYY(8),TXY(17,32),P1(17,32),PP1(8),PP2(8),P2(17,32),
                    2PX1(8),PX2(8),PY1(4),PY2(4)
0003                 READ(1,10)M,N,DEL
0004                 WRITE(3,10)M,N,DEL
0005              10 FORMAT(2I5,F10.3)
0006                 READ(1,16)E1,E2,B1,B2
0007                 WRITE(3,16)E1,E2,B1,B2
0008              16 FORMAT(2D20.8,2F10.6)
0009                 MC=3
0010                 NC=3
0011                 MC1=MC-1
0012                 NC1=NC-1
0013                 DE=0.0
0014                 P(1)=0.0
0015                 DO 25 I=2,M
0016                 DE=DE+0.125
0017              25 P(I)=50.*(DE **2)
0018                 WRITE(3,26)(P(I),I=1,M)
0019              26 FORMAT(8F10.6)
0020                 READ(1,20)((A(I,J),J=1,N),I=1,M)
0021                 WRITE(3,20)((A(I,J),J=1,N),I=1,M)
0022              20 FORMAT(4D20.12)
0023                 M1=M+1
0024                 DO 5 I=1,M1
0025                 DO 5 J=1,N
0026                 PXX(I,J)=0.
0027                 PYY(I,J)=0.
0028                 TXY(I,J)=0.
0029               5 CONTINUE
0030                 A1=1./E1
0031                 A2=1./E2
0032                 O1=(A2-A1)/(A2+A1)
```

```
      02=((A2*B2-A1*B1+A1-A2)*2.)/ (A1+A2)
      03=-((A2*B2-A1*B1)/(A1+A2))
      04=(1+(A2-A1)/(A1+A2))
      001=-(A2-A1)/(A2+A1)
      002=-((A2*B2-A1*B1+A1-A2)*2.)/(A1+A2)
      003=((A2*B2-A1*B1)/(A1+A2))
      004=(1-(A2-A1)/(A1+A2))
C
C
C     ******************************************************************
C     CALCULATION OF PXX
C     ******************************************************************
C
C
      DO 85 K=1,NC1
      PPXX(K)=0.0
   85 PPYY(K)=0.0
      I=1
      J=1
   40 PXX(I,J)=(A(I+1,J)-A(I,J))*2.*DEL
      J=J+1
      IF(J.LT.(N+1))GOTO40
      I=I+1
   60 J=1
   50 PXX(I,J)=(A(I+1,J)+A(I-1,J)-2.*A(I,J))*DEL
      WRITE(3,81)I,J,A(I+1,J),A(I,J),A(I-1,J),PXX(I,J)
   81 FORMAT(5X,I5,5X,I5,5X,4F10.4)
      J=J+1
      IF(J.LT.(N+1))GOTO50
      I=I+1
      IF(I.EQ.MC) GO TO 70
      IF(I.LT.M)GO TO 60
      GOTO90
   70 J=1
      BI(I+1,J)=A(I-1,J)*01+A(I,J)*02+A(I,J+1)*2.*03+A(I+1,J)*04
      BO(I-1,J)=A(I+1,J)*001+A(I,J)*002+A(I,J+1)*2.*003+A(I-1,J)*004
      GO TO 80
   75 BI(I+1,J)=A(I-1,J)*01+A(I,J)*02+(A(I,J+1)+A(I,J-1))*03+A(I+1,J)*04
      BO(I-1,J)=A(I+1,J)*001+A(I,J)*002+(A(I,J-1)+A(I,J+1))*003+A(I-1,J)
     1*004
   80 PXX(I,J)=(BI(I+1,J)+A(I-1,J)-A(I,J)*2.)*DEL
      PPXX(J)=(A(I+1,J)+BO(I-1,J)-A(I,J)*2.)*DEL
      J=J+1
```

```
0068              IF(J.LT.NC) GO TO 75
0069              GO TO 50
0070          90  C=50.
0071              J=1
0072         100  PXX(I,J)=(C+A(I-1,J)-2.*A(I,J))*DEL
0073              PXX(I+1,J)=(A(I,J)*2.+12.5-2.*C)*DEL
0074              J=J+1
0075              IF(J.LT.(N+1))GO TO 100
            C
            C
            C     ******************************************************************************
            C     CALCULATION OF PYY
            C     ******************************************************************************
            C
            C
0076              J=1
0077              I=1
0078         110  PYY(I,J)=(A(I,J+1)-A(I,J))*2.*DEL
0079              I=I+1
0080              IF(I.LT.(M+1))GO TO 110
0081              I=1
0082         120  J=2
0083         130  PYY(I,J)=(A(I,J+1)+A(I,J-1)-2.*A(I,J))*DEL
0084              J=J+1
0085              IF(J.EQ.NC.AND.I.EQ.1)GO TO 140
0086              IF(J.EQ.NC.AND.I.LT.MC) GO TO 150
0087              IF(J.LT.N)GO TO 130
0088              GO TO 160
0089         140  BI(I,J+1)=A(I,J-1)*O1+A(I,J)*O2+A(I+1,J)*2.*O3+A(I,J+1)*O4
0090              BO(I,J-1)=A(I,J+1)*OO1+A(I,J)*OO2+A(I+1,J)*2.*OO3+A(I,J-1)*OO4
0091              GO TO 170
0092         150  BI(I,J+1)=A(I,J-1)*O1+A(I,J)*O2+(A(I-1,J)+A(I+1,J))*O3+A(I,J+1)*O4
0093              BO(I,J-1)=A(I,J+1)*OO1+A(I,J)*OO2+(A(I-1,J)+A(I+1,J))*OO3+A(I,J-1)
                 1*OO4
0094         170  PYY(I,J)=(BI(I,J+1)+A(I,J-1)-A(I,J)*2.)*DEL
0095              PPYY(I)=(A(I,J+1)+BO(I,J-1)-A(I,J)*2.)*DEL
0096              J=J+1
0097              GO TO 130
0098         160  J=N
0099              PYY(I,J)=(P(I)+A(I,J-1)-2.*A(I,J))*DEL
0100              I=I+1
0101              IF(I.LT.(M+1))GO TO 120
            C
```

```
C
C        **********************************************************************
C        CALCULATION OF TXY
C        **********************************************************************
C
C
         DO 180 I=1,M1
         DO 180 J=1,N
     180 TXY(I,J)=0.0
         I=2
         J=2
     190 IF(I.EQ.MC.AND.J.EQ.NC) GO TO 200
         TXY(I,J)=-(A(I+1,J+1)-A(I+1,J-1)+A(I-1,J-1)-A(I-1,J+1))*DEL/4.
     195 J=J+1
         IF(J.LT.N)GO TO 190
         J=2
         I=I+1
         IF(I.LT.M)GO TO 190
         GO TO 205
     200 TXY(I,J)=-(A(I+1,J+1)-A(I+1,J-1)+A(I-1,J-1)-BI(I-1,J+1))*DEL/4.
         GO TO 195
     205 I=M
         J=2
     210 TXY(I,J)=-(A(I-1,J-1)-A(I-1,J+1))*DEL/4.
         J=J+1
         IF(J.LT.N)GO TO 210
         J=N
         I=2
     220 TXY(I,J)=-(P(I+1)-A(I+1,J-1)+A(I-1,J-1)-P(I-1))*DEL/4.
         I=I+1
         IF(I.LT.M)GO TO 220
         I=M
         J=N
         TXY(I,J)=-(A(I-1,J-1)-P(I-1))*DEL/4.
         WRITE(3,199)
         DO 300 I=1,M1
         DO 300J=1,N
         PC=(PXX(I,J)+PYY(I,J))/2.
         PR=DSQRT(((PXX(I,J)-PYY(I,J))/2.)**2+(TXY(I,J))**2)
         P1(I,J)=PC+PR
         P2(I,J)=PC-PR
     300 CONTINUE
         I=3
```

```
          DO 700 J=1,NC1
          PC=(PPXX(J)+PYY(I,J))/2.
          PR=DSQRT(((PPXX(J)-PYY(I,J))/2.)**2+(TXY(I,J))**2)
          PX1(J)=PC+PR
          PX2(J)=PC-PR
      700 CONTINUE
          J=NC
           DO 705 I=1,MC1
          PC=(PXX(I,J)+PPYY(I))/2.
          PR=DSQRT(((PXX(I,J)-PPYY(I))/2.)**2+(TXY(I,J))**2)
          PY1(I)=PC+PR
          PY2(I)=PC-PR
      705 CONTINUE
          WRITE(3,246)
      246 FORMAT(5X,'RECT.  INSERT  A/AA=1/4,  B/BB=1/8,  BB/AA=1/2,  STRESS
        1=100PSI')
          WRITE(3,249)
      249 FORMAT(45X,'STRESS  PXX')
          WRITE(3,250)((PXX(I,J),J=1,N),I=1,M1)
      250 FORMAT(//,1X,16F8.0)
      199 FORMAT(1H1)
          WRITE(3,259)
      259 FORMAT(//,2X,'STRESS ON OUTSIDE OF INTERFACE')
          WRITE(3,260)(PPXX(K),K=1,8)
      260 FORMAT(//,1X,4F8.0)
          WRITE(3,199)
          WRITE(3,248)
      248 FORMAT(45X,'STRESS  PYY')
          WRITE(3,250)((PYY(I,J),J=1,N),I=1,M)
          WRITE(3,259)
          WRITE(3,260)(PPYY(K),K=1,MC1)
          WRITE(3,199)
          WRITE(3,247)
      247 FORMAT(45X,'STRESS  TXY')
          WRITE(3,250)((TXY(I,J),J=1,N),I=1,M)
          WRITE(3,199)
          K1=1
          K=10
      755 WRITE(3,716)
      716 FORMAT(1H1,20X,'PRINCIPAL  STRESSES  P1 (LARGER ) AT INTERFACE ')
          WRITE(3,15)K1,K
          DO 725 I=1,M
      725 WRITE(3,35)(I,(  P1(I,J),J=K1,K))
```

```
      IF(K.EQ.N )GO TO 745
      K1=K+1
      K=K+10
      IF(K.GT.N )K=N
      GO TO 755
  745 CONTINUE
      WRITE(3,259)
      WRITE(3,800)(PX1(J),J=1,8),(PY1(I),I=1,4)
  800 FORMAT(//2X,'PX1 ',8F 9.0,10X,'PY2 ',2X,4F 9.0)
      WRITE(3,199)
      K1=1
      K=10
  955 WRITE(3,916)
  916 FORMAT(1H1,20X,' PRINCIPAL STRESSES  P2 (SMALLER )')
      WRITE(3,15)K1,K
      DO 925 I=1,M
  925 WRITE(3,35)(I,(P2 (I,J),J=K1,K))
      IF(K.EQ.N )GO TO945
      K1=K+1
      K=K+10
      IF(K.GT.N )K=N
      GO TO 955
  945 CONTINUE
   35 FORMAT(/,5X,I10,10F10.0)
   15 FORMAT(5X,2I10)
      WRITE(3,259)
      WRITE(3,801)(PX2(J),J=1,NC1),(PY2(I),I=1,MC1)
  801 FORMAT(//2X,'PX2 ',8F 9.0,10X,'PY2 ',2X,4F 9.0)
      RETURN
      END
```

```
C
C
C
C       ****************************************************************************
C       THIS IS THE FIRST STEP OF THE THREE STEPS PROGRAM FOR THE SOLUTION
C       OF CASE-2 (INSERT INCLINED AT AN ANGLE OF 45 )
C       ****************************************************************************
C
C
C
C
C
C       ****************************************************************************
C       THE VARIABLES ARE DEFINED AS FOLLOWS
C
C
C
C
C       I               NUMBER COUNTER IN X DIRECTION.
C       J               NUMBER COUNTER IN Y DIRECTION.
C       MM              NUMBER OF STEPS ALONG X DIRECTION.
C       NN              NUMBER OF STEPS ALONG Y DIRECTION.
C       DEL(I)          STEP SIZE ALONG X DIRECTION.
C       DEL1(J)         STEP SIZE ALONG Y DIRECTION.
C       DEK(I)          RATIO OF ACTUAL TO STANDARD STEP SIZE ALONG
C                       ALONG X DIRECTION.
C       DEK1(J)         RATIO OF ACTUAL TO STANDARD STEP SIZE ALONG
C                       ALONG Y DIRECTION.
C       E1,E2           THE MODULII OF ELASTICITY OF INSERT AND PARENT
C                       MATERIAL RESPECTIVELY.
C       B1,B2           THE PISSION
C       B1,B2           THE POISSION RATIO OF INSERT AND PARENT MATERIAL
C                       RESPECTIVELY.
C       X,Y             THE DISTANCE OF THE NODAL POINT FROM THE CO-ORDINATE
C                       AXIS.
C       LA(I,J)         THE NUMBER OF NODAL POINTS.
C       DX(I,J),        THE DISTANCES IN X AND Y DIRECTIONS RESPECTIVELY
C       DY(I,J)         FROM THE BOUNDARY OF THE PLATE OF THE POINTS LESS
C                       THAN ONE LENGTH AWAY.
C       M1              I-TH POSITION OF THE NODAL POINTS.
C       N1              J-TH POSITION OF THE NODAL POINTS.
C       P(KK)           ROW OF THE CO-EFFICENT MATRIX.
C       B               THE RIGHT HAND VECTOR.
C
```

```
             C      ****************************************************************************
             C
             C
             C
0001                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),RY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                   2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
                   3,M1(500),N1(500),RT(500)
             C
             C
0002                READ(1,10)MM,NN,E1,E2,B1,B2
0003                WRITE(3,10)MM,NN,E1,E2,B1,B2
0004                READ(1,20)  (DEL(I),I=1,MM),(DEL1(J),J=1,NN)
0005                READ(1,20)(DEK(I),I=1,MM),(DEK1(J),J=1,NN)
0006                WRITE(3,20)((DEL(I),I=1,MM),(DEL1(J),J=1,NN))
0007                WRITE(3,20)((DEK(I),I=1,MM),(DEK1(J),J=1,NN))
0008             10 FORMAT(2I10,4F15.8)
0009             20 FORMAT(8F10.6)
0010                DO 65I=1,MM
0011                DO 65J=1,NN
             C
             C
             C      ****************************************************************************
             C      INITIAL THE VARIABLES.
             C      ****************************************************************************
             C
             C
0012                LA(I,J)=0
0013                DX(I,J)=0.0
0014                DY(I,J)=0.0
0015                BX(I,J)=0.0
0016                BY(I,J)=0.0
0017                BFX(I,J)=0.0
0018                BFY(I,J)=0.0
0019                BOFX (I,J)=0.0
0020                BOFY(I,J)=0.0
0021                FX(I,J)=0.
0022                FFX(I,J)=0.
0023                FY(I,J)=0.
0024                FFY(I,J)=0.
0025             65 CONTINUE
             C
             C
```

```
              C
              C      ***********************************************************************
              C      DRAW THE MESH.
              C      NUMBER THE NODAL POINTS.
              C      CALCULATE THE FICITITIOUS POINTS FALLING LESS THAN ONE LENGTH
              C      AWAY FROM THE BOUNDARY, IN TERMS OF REAL VALUES USING SUBROUTINES
              C      XONE , YONE , BOONE , XTWO , YTWO , BOTWO , XTHREE , YTHRE,
              C      BOTHRE , XFOUR , YFOUR , BOFOUR .
              C      ***********************************************************************
              C
              C
 0026                CALL MESH(MM,NN)
              C
              C
              C      ***********************************************************************
              C      THE GENERATION OF THE MATRIX STARTS FROM HERE.
              C      THE SIZE OF THE INSERTS ARE DEFINED AS FOLLOWS:-
              C                LENGTH OF THE INSERT
              C
              C                LARGEST      MI=14   MO=22
              C                MEDIUM       MI=15   MO=21
              C                SMALLEST     MI=16   MO=20
              C
              C                WIDTH OF THE INSERT
              C                WIDTH OF THENI=16    NO=20
              C                IN ALL THE CASES
              C      THE APPROPRIATE SUBROUTINES ARE USED AT THE NODAL POINTS DEPENDING
              C      UPON THEIR LOCATION. EACH ROW IS GENRATED AND STORED ON THE TAPE.
              C      ***********************************************************************
              C
              C
              C
              C
              C
              C      ***********************************************************************
              C      CALCULATE THE COEFFIENT OF VARIOS OPERATORS.
              C      ***********************************************************************
              C
              C
 0027             11 A1 = 1./E1
 0028                A2 = 1./E2
 0029                AA1 = (A1+A2)**2
 0030                A = (20.+((A2*B2-A1*B1)*8.*(A2-A1)-6.*((A2*B2-A1*B1)**2)-4.*((A2-
                   1A1)**2))/AA1)
```

```
0031          BI= (-8.+4.*(A2*B2-A1*B1)*((A2*B2-A1*B1)-(A2- A1))/AA1)
0032          C1 =  (-8.+(4.*A1*(A2*B2-A1*B1)+(A2-A1)*(8.*A2+4.*A1))/AA1)
0033          D =  (-8.-(4.*A2*(A2*B2-A1*B1)+(A2-A1)*(4.*A2+8.*A1))/AA1)
0034          E =  (2.-2.*((A2*B2-A1*B1)*A1+(A2-A1)*(A1+A2))/AA1)
0035          F=(2.+2.*((A2*B2-A1*B1)*A2+(A2-A1)*(A1+A2))/AA1)
0036          G=(1.-((A2*B2-A1*B1)*(A2*B2-A1*B1))/AA1)
0037          H=(1.-(A2-A1)/(A2+A1))
0038          TI=(1.+(A2-A1)/(A2+A1))
0039          AA=(20.+(A2-A1)/(A2+A1))
0040          AA1=(20.-(A2-A1)/(A2+A1))
0041          BB=(-8.+2.*(A2*B2-A1*B1+A1-A2)/(A1+A2))
0042          BB1=(-8.-2.*(A2*B2-A1*B1+A1-A2)/(A1+A2))
0043          CC=(2.-(A2*B2-A1*B1)/(A1+A2))
0044          CC1=(2.+(A2*B2-A1*B1)/(A1+A2))
0045          DD=(1.+(A2-A1)/(A1+A2))
0046          DD1=(1.-(A2-A1)/(A1+A2))
0047          AAA=(20.+2.*(A2-A1)/(A1+A2))
0048          CCC=(2.-2.*(A2*B2-A1*B1)/(A1+A2))
0049          XX=(20.-2.*(A1*B1-A2*B2)/(A1+A2))
0050          YY=(-8.-4.*(A1-A2-A1*B1+A2*B2)/(A1+A2))
0051          Z=(2.-2.*(A2-A1)/(A1+A2))
0052          S=(4.*A1/(A1+A2))
0053          T=(1.-2.*(A1*B1-A2*B2)/(A1+A2))
0054          WRITE(3,12)A,BI,C1,D,E,F,G,H,TI,AA,AA1,BB,BB1,CC,CC1,DD,DD1,XX,YY,
             1Z,S,T ,AAA,CCC
0055       12 FORMAT(3X,10F12.5)
0056          DO 451 K=1,N
0057          DO 453 K5=1,N
0058      453 P(K5)=0.0
0059          B=0.0
0060      111 I=M1(K)
0061          J=N1(K)
0062          IF(I.LT.(MI-1).OR.I.GT.(MO-1)) GO TO 170
0063          IF(J.LT.(NI-1).OR.J.GT.(NO+1))GO TO 170
0064          IF(I.LT.(MI+1).OR.I.GT.(MO-1))GO TO 180
0065          IF(J.LT.(NI+1).OR.J.GT.(NO-1))GO TO 180
        C     POINTS FALL INSIDE THE BOUNDRY.
0066      195 IF(I.GT.(MI+1).AND.I.LT.(MO-1))GO TO 200
0067          IF(I.EQ.(MI+1).AND.J.EQ. (NI+1))GO TO 230
0068          IF(I.EQ.(MO-1).AND.J.EQ.(NI+1)) GO TO 250
0069          IF(I.EQ.(MI+1).AND.J.EQ.(NO-1))GO TO 260
0070          IF(I.EQ.(MO-2).AND.J.EQ.(NO-2))GO TO 225
0071          IF(I.EQ.(MI+1).AND.J.EQ.(NO-2)) GO TO 215
```

```
            WRITE(3,498)I,J,N
            RETURN
      200   IF(J.EQ.(NO-2))  GO TO 190
            IF(J.EQ.(NI+1)) GO TO 210
            IF(J.EQ.(NO -1))GO  TO 220
            WRITE(3,498)I,J,N
            RETURN
      190   CALL             STAND
            CALL GAIN(I,J)
            GO TO 450
      210   CALL             ONE(AA,BB,CC,DD)
            CALL GAIN(I,J)
            GO TO 450
      220   CALL             TWO(AA,BB,CC,DD)
            CALL GAIN(I,J)
            GO TO 450
      215   CALL              THREE(AA,BB,CC,DD)
            CALL   GAIN(I,J)
            GO TO 450
      225   CALL              FOUR(AA,BB,CC,DD)
            CALL  GAIN(I,J)
            GO TO 450
      230   CALL             ICONE(AAA,BB,CCC,CC,DD)
            CALL GAIN(I,J)
            GO TO 450
      240   CALL             ICTWO(AAA,BB,CCC,CC,DD)
            CALL GAIN(I,J)
            GO TO 450
      250   CALL             ICTHRE(AAA,BB,CCC,CC,DD)
            CALL GAIN(I,J)
            GO TO 450
      260   CALL             ICFOUR(AAA,BB,CCC,CC,DD)
            CALL GAIN(I,J)
            GO TO 450
C           THE POINTS FALL ON THE BOUNDRY OR OUDSIDE THE INSERT BUT WITH
C           REGULAR MESH.
      180   IF(I.GT.MI.AND.I.LT.MO)GO TO 270
            IF(J.GT.NI.AND.J.LT.NO)GO TO 280
            IF(I.EQ.MI.AND.J.EQ.NI)GO TO 370
            IF(I.EQ.MI.AND.J.EQ.NO)GO TO 380
            IF(I.EQ.MO.AND.J.EQ.NI)GO TO 390
            IF(I.EQ.MO.AND.J.EQ.NO)GO TO 400
            CALL             STAND
```

```
      CALL GAIN(I,J)
      GO TO 450
  270 IF(J.EQ.(NI-1)) GO TO 290
      IF(J.EQ.(NO+1)) GO TO 300
      IF(J.EQ.NI)GO TO 330
      IF(J.EQ.NO)GO TO 340
      WRITE(3,498)I,J,N
      RETURN
  280 IF(I.EQ.(MI-1)) GO TO 310
      IF(I.EQ.(MO+1)) GO TO 320
      IF(I.EQ.MI)GO TO 350
      IF(I.EQ.MO)GO TO 360
      WRITE(3,498)I,J,N
      RETURN
  290 CALL          OONE(AA1,BB1,CC1,DD1)
      CALL GAIN(I,J)
      GO TO 450
  300 CALL          OTWO(AA1,BB1,CC1,DD1)
      CALL GAIN(I,J)
      GO TO 450
  310 CALL         OTHREE(AA1,BB1,CC1,DD1)
      CALL GAIN(I,J)
      GO TO 450
  320 CALL          OFOUR(AA1,BB1 ,CC1,DD1)
      CALL GAIN(I,J)
      GO TO 450
  330 CALL           ONONE(A,BI,C1,D,E,F,G,H,TI)
      CALL GAIN(I,J)
      GO TO 450
  340 CALL          ONTWO(A,BI,C1,D,E,F,G,H,TI)
      CALL GAIN(I,J)
      GO TO 450
  350 CALL          ONTHRE(A,BI,C1,D,E,F,G,H,TI)
      CALL GAIN(I,J)
      GO TO 450
  360 CALL          ONFOUR(A,BI,C1,D,E,F,G,H,TI)
      CALL GAIN(I,J)
      GO TO 450
  370 CALL          COONE(XX,YY,S,Z,T)
      CALL GAIN(I,J)
      GO TO 450
  380 CALL          COTWO(XX,YY,S,Z,T)
      CALL GAIN(I,J)
```

```
      GO TO 450
  390 CALL             COTHRE(XX,YY,S,Z,T)
      CALL GAIN(I,J)
      GO TO 450
  400 CALL             COFOUR(XX,YY,S,Z,T)
      CALL GAIN(I,J)
      GO TO 450
  170 CALL GNERAL(I,J,B,K)
  450 WRITE(3,499)K
      WRITE(7)(P(KK),KK=1,N)
      WRITE(3,501)B
      RT(K)=B
  499 FORMAT(//,5X,I5)
  500 FORMAT(3X,10(F10.4,2X))
  498 FORMAT(5X,3I10)
  501 FORMAT(/,5X,F10.4)
  451 CONTINUE
      WRITE(7)(RT(KK),KK=1,N)
      RETURN
      END
```

```
      SUBROUTINE MESH(MM,NN)
      COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
     2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
     3,M1(500),N1(500),RT(500)
C
C
      Y=0.0
      X1=0.0
      Y1=0.0
      N=0
      DO 60 J=1,35
      Y=Y+DEL1(J)
      X=0.0
      DO 60 I=1,34
      X=X+DEL(I)
      EE=1./SQRT(2.)
      IF(X.GT.EE)GO TO 70
      IF(Y.GT.EE)GO TO 130
      Y1=-X+EE
      X1=-Y+EE
      IF(Y1.GT.Y)GO TO 60
      IF((Y-Y1).GT.DEL1(J))GOTO40
      IF((X-X1).GT.DEL(I))GO TO 90
      CALL          BOONE(I,J,X,X1,Y,Y1)
      GO TO 60
   90 CALL          YONE(I,J,X,Y,Y1)
      GO TO 60
   40 IF((X-X1).LT.DEL(I))GO TO 100
  130 Y1=X+EE
      X1=Y-EE
      IF(Y1.LT.Y)GO TO 60
      IF((Y1-Y).GT.DEL1(J+1))GO TO 50
      IF((X-X1).GT.DEL(I))GO TO 55
      CALL          BOTWO(I,J,X,X1,Y,Y1)
      GO TO 60
   55 CALL          YTWO(I,J,X,Y,Y1)
      GO TO 60
  100 CALL          XONE(I,J,X,X1,Y)
      GO TO 60
   50 IF((X-X1).GT.DEL(I))GO TO 80
      CALL          XTWO(I,J,X,X1,Y)
      GO TO 60
```

```
 70 Y1=X-EE
    X1=Y+EE
    IF(Y1.GT.Y)GO TO 60
    IF((Y-Y1).GT.DEL1(J))GO TO 110
    IF((X1-X).GT.DEL(I+1))GO TO 120
    CALL            BOTHRE(I,J,X,X1,Y,Y1)
    GO TO 60
120 CALL            YTHRE(I,J,X,Y,Y1)
    GO TO 60
110 IF((X1-X).GT.DEL(I+1))GO TO 140
    CALL           XTHREE(I,J,X,X1,Y)
    GO TO 60
140 IF(X.LT.(2./SQRT(2.)))GO TO 130
    EE1=5.*EE
    Y1=-X+EE1
    X1=-Y+EE1
    IF(Y1.LT.Y)GO TO 60
    IF((Y1-Y).GT.DEL1(J+1))GO TO 150
    IF((X1-X).GT.DEL(I+1))GO TO 160
    CALL            BCFOUR(I,J,X,X1,Y,Y1)
    GO TO 60
160 CALL            YFOUR(I,J,X,Y,Y1)
    GO TO 60
150 IF((X1-X).GT.DEL(I+1))GO TO 80
    CALL            XFOUR(I,J,X,X1,Y)
    GO TO 60
 80 N=N+1
    M1(N)=I
    N1(N)=J
    LA(I,J)=N
 60 CONTINUE
    WRITE(3,81)N
    WRITE(3,87)
    WRITE(3,88)((DX(I,J),I=1,MM),J=1,NN)
    WRITE(3,87)
    WRITE(3,88)((DY(I,J),I=1,MM),J=1,NN)
    WRITE(3,87)
    WRITE(3,88)((BX(I,J),I=1,MM),J=1,NN)
    WRITE(3,87)
    WRITE(3,88)((BY(I,J),I=1,MM),J=1,NN)
    WRITE(3,87)
    WRITE(3,88)((BFX(I,J),I=1,MM),J=1,NN)
    WRITE(3,87)
```

```
      WRITE(3,88)((BFY(I,J),I=1,MM),J=1,NN)
      WRITE(3,87)
      WRITE(3,88)((BOFX(I,J),I=1,MM),J=1,NN)
      WRITE(3,87)
      WRITE(3,88)((BOFY(I,J),I=1,MM),J=1,NN)
      WRITE(3,87)
      WRITE(3,87)
      WRITE(3,89)((LA(I,J),I=1,MM),J=1,NN)
      WRITE(3,87)
   87 FORMAT(///)
   88 FORMAT(5X,11F11.5)
   89 FORMAT(5X,21I5)
   81 FORMAT(5X,I5)
      RETURN
       END
```

```
 0001                 SUBROUTINE XONE (I,J,X,X1,Y)
             C
             C        ***********************************************************************
             C        THIS SUBROUTINE IS USED ALONG LINE AD USING POINTS IN X DIRECTION.
             C         REF. -ART.2..4.2
             C        ***********************************************************************
             C
 0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                     2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
             C
 0003            11   DD=X-X1
 0004                 IF(I.EQ.2)GO TO 10
 0005                 DU=DEL(I)-DD
 0006                 GO TO 20
 0007            10   DU=.0625-DD
 0008            20   DELTA=0.06250
 0009                 DX(I,J)=DD/DELTA
 0010                 SLOX= 100.0*X1 + 100.0*SQRT(2.0)
 0011                 BX(I-1,J)= 50.0*(X1**2)+50.0*(Y*Y)
                     1+100.0*SQRT(2.0) *X1 + 100.0*SQRT(2.0) *Y-300.0
 0012                 D1 = 1+DX(I,J)                                                          1
 0013                 D2 = D1**2
 0014                 D3 = 1-DX(I,J)
 0015                 BFX(I,J) = (1+2.*DX(I,J))*BX(I-1,J)/D2+DD*SLOX/D1
 0016                 BOFX(I-1,J) =  4.*DX(I,J)*BX(I-1,J)/D2-2.*DU*SLOX/D1
 0017                 FX(I+1,J)=DX(I,J)**2/D2
 0018                 FFX(I+1,J)=D3**2/D2
 0019                 RETURN
 0020                 END
```

```
0001                 SUBROUTINE YONE (I,J,X,Y,Y1)
               C
               C     ************************************************************************
               C     THIS SUBROUTINE IS USED ALONG LINE AD USING POINTS IN Y DIRECTION.
               C      REF. -ART.2..4.2
               C     ************************************************************************
               C
0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                    2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
               C
               C
0003                 DD=Y-Y1
0004                 IF(J.EQ.2)GO TO 10
0005                 DU=DEL1(J)-DD
0006                 GO TO 20
0007              10 DU=.0625-DD
0008             20  DELTA=0.06250
0009                 DY(I,J)=DD/DELTA
0010                 SLOY=100.0*Y1+100.0*SQRT(2.0)
0011                 BY(I,J-1)= 50.0*(X*X)+50.0*(Y1**2) +100.0*SQRT(2.0)*X
                    1+100.*SQRT(2.0)*Y1 -300.0
0012                 D1 = 1+DY(I,J)
0013                 D2 = D1**2                                                        2
0014                 D3 = 1-DY(I,J)
0015                 BFY(I,J) = (1+2.*DY(I,J))*BY(I,J-1)/D2+DD*SLOY/D1
0016                 BOFY(I,J-1) = 4.*DY(I,J)*BY(I,J-1)/D2-2.*DU*SLOY/D1
0017                 FY(I,J+1)=DY(I,J)**2/D2
0018                 FFY(I,J+1)=D3**2/D2
0019                 RETURN
0020                 END
```

```
0001                SUBROUTINE BOONE (I,J,X,X1,Y,Y1)
           C
           C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                   2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
           C
           C
0003                CALL          YONE (I,J,X,Y,Y1)
0004       10       CALL          XONE (I,J,X,X1,Y)
0005       20       RETURN
0006                END
```

```
0001            SUBROUTINE XTWO (I,J,X,X1,Y )
           C
           C    ************************************************************************
           C    THIS SUBROUTINE IS USED ALONG LINE CD USING POINTS IN X DIRECTION.
           C     REF. -ART.2..4.2
           C    ************************************************************************
           C
0002            COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
               1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
               2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
           C
           C
0003            DD=X-X1
0004            IF(I.EQ.2)GO TO 10
0005            DU=DEL(I)-DD
0006            GO TO 20
0007         10 DU=.0625-DD
0008         20 DELTA=0.06250
0009            DX(I,J)=DD/DELTA
0010            SLOX = 100.0*SQRT(2.0)
0011            BX (I-1,J) = 100.0*SQRT(2.0)*X1 + 150.0 *SQRT(2.0)*Y -325.0
0012            D1 = 1+DX(I,J)
0013            D2 = D1**2
0014            D3 = 1-DX(I,J)
0015            BFX(I,J) = (1+2.*DX(I,J))*BX(I-1,J)/D2+DD*SLOX/D1
0016            BOFX(I-1,J)=4.*DX(I,J)*BX(I-1,J)/D2-2.*DU*SLOX/D1
0017            FX(I+1,J)=DX(I,J)**2/D2
0018            FFX(I+1,J)=D3**2/D2
0019            RETURN
0020            END
```

3

```
0001          SUBROUTINE YTWO (I,J,X,Y,Y1)
       C
       C      ************************************************************************
       C      THIS SUBROUTINE IS USED ALONG LINE CD USING POINTS IN Y DIRECTION.
       C       REF. -ART.2..4.2
       C      ************************************************************************
       C
0002          COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
             1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
             2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
       C
       C
0003          DD=Y1-Y
0004          IF(J.EQ.2)GO TO 10
0005          DU=DEL1(J+1)-DD
0006          GO TO 20
0007       10 DU=.0625-DD
0008      20  DELTA=0.06250
0009          DY(I,J+1)=DD/DELTA
0010          SLOY = 150.0 *SQRT (2.0)
0011          BY (I,J+1) = 100.0 *SQRT (2.0)*X+ 150.0*SQRT (2.0) *Y1 -325.0
0012          D1=1+DY(I,J+1)
0013          D2=D1**2
0014          D3=1-DY(I,J+1)
0015          BFY(I,J)=(1.+2.*DY(I,J+1))*BY(I,J+1)/D2-DD*SLOY/D1
0016          BOFY(I,J+1)=4.*DY(I,J+1)*BY(I,J+1)/D2+2.*DU*SLOY/D1
0017          FY(I,J-1)=DY(I,J+1)**2/D2
0018          FFY(I,J-1)=D3**2/D2
0019          RETURN
0020          END
```

```
0001                SUBROUTINE  BOTWO  (I,J,X,X1,Y,Y1)
            C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                   2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
            C
            C
0003                CALL           YTWO (I,J,X,Y,Y1)
0004        10      CALL           XTWO (I,J,X,X1,Y)
0005        20      RETURN
0006                END
```

```
0001                 SUBROUTINE XTHREE(I,J,X,X1,Y)
              C
              C      ******************************************************************************
              C      THIS SUBROUTINE IS USED ALONG LINE AB USING POINTS IN X DIRECTION.
              C       REF. -ART.2..4.2
              C      ******************************************************************************
              C
0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                    2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
              C
              C
0003                 DD=X1-X
0004                 IF(I.EQ.2)GO TO 10
0005                 DU=DEL(I+1)-DD
0006                 GO TO 20
0007              10 DU=.0625-DD
0008              20 DELTA=0.06250
0009                 DX(I+1,J)=DD/DELTA
0010                 SLOX=150.*SQRT(2.)
0011                 BX(I+1,J)=150.*SQRT(2.)*X1+100.*SQRT(2.)*Y-325.
0012                 D1=1.+DX(I+1,J)
0013                 D2=D1**2
0014                 D3=1.-DX(I+1,J)
0015                 BFX(I,J)=(1.+2.*DX(I+1,J))*BX(I+1,J)/D2-DD*SLOX/D1
0016                 BOFX(I+1,J)=4.*DX(I+1,J)*BX(I+1,J)/D2+2.*DU*SLOX/D1
0017                 FX(I-1,J)=DX(I+1,J)**2/D2
0018                 FFX(I-1,J)=D3**2/D2
0019                 RETURN
0020                 END
```

```
      SUBROUTINE YTHRE (I,J,X,Y,Y1)
C
C     ***********************************************************************
C     THIS SUBROUTINE IS USED ALONG LINE AB USING POINTS IN Y DIRECTION.
C      REF. -ART.2..4.2
C     ***********************************************************************
C
      COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
     2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
C
C
      DD=Y-Y1
      IF(J.EQ.2)GO TO 10
      DU=DEL1(J)-DD
      GO TO 20
   10 DU=.0625-DD
   20 DELTA=0.06250
      DY(I,J)=DD/DELTA
      SLOY = 100.*SQRT(2.)
      BY(I,J-1) = 150.*SQRT(2.)*X+100.*SQRT(2.)*Y1-325.
      D1=1.+DY(I,J)
      D2=D1**2
      D3=1.-DY(I,J)
      BFY(I,J)=(1.+2.*DY(I,J))*BY(I,J-1)/D2+DD*SLOY/D1
      BOFY(I,J-1)=4.*DY(I,J)*BY(I,J-1)/D2-2.*DU*SLOY/D1
      FY(I,J+1)=DY(I,J)**2/D2
      FFY(I,J+1)=D3**2/D2
      RETURN
      END
```

```
 0001                 SUBROUTINE  BOTHRE (I,J,X,X1,Y,Y1)
                C
                C
 00C2                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                     2DEK(35),DEK1(35),P(500),FX(451),FFX(451),FY(451),FFY(451)
                C
                C
 0003                 CALL            YTHRE (I,J,X,Y,Y1)
 0004              10 CALL            XTHREE(I,J,X,X1,Y)
 00C5            20   RETURN
 0006                 END
```

```
      SUBROUTINE XFOUR(I,J,X,X1,Y)
C
C     ************************************************************************
C     THIS SUBROUTINE IS USED ALONG LINE BC USING POINTS IN X DIRECTION.
C      REF. -ART.2..4.2
C     ************************************************************************
C
      COMMON DEL(35),DELI(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
     2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
C
C
      DD=X1-X
      DU=DEL(I+1)-DD
      DELTA=0.06250
      DX(I+1,J)=DD/DELTA
      SLOX=100.*X1
      BX(I+1,J)=50.*X1**2.+50.*Y**2.
      D1 = 1+DX(I+1,J)
      D2 = D1**2
      D3 = 1-DX(I+1,J)
      BFX(I,J) = (1.+2.*DX(I+1,J))*BX(I+1,J)/D2-DD*SLOX/D1
      BOFX(I+1,J) =4.*DX(I+1,J)*BX(I+1,J)/D2+2.*DU*SLOX/D1
      FX(I-1,J)=DX(I+1,J)**2/D2
      FFX(I-1,J)=D3**2/D2
      RETURN
      END
```

```
0001                 SUBROUTINE YFOUR(I,J,X,Y,Y1)
               C
               C     ***********************************************************************
               C     THIS SUBROUTINE IS USED ALONG LINE BC USING POINTS IN Y DIRECTION.
               C      REF. -ART.2..4.2
               C     ***********************************************************************
               C
0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                    2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
               C
               C
0003                 DD=Y1-Y
0004                 DU=DEL1(J+1)-DD
0005                 DELTA=0.06250
0006                 DY(I,J+1)=DD/DELTA
0007                 SLOY=100.*Y1
0008                 BY(I,J+1)=50.*X**2.+50.*Y1**2.
0009                 D1=1.+DY(I,J+1)
0010                 D2 = D1**2
0011                 D3 = 1. - DY(I,J+1)
0012                 BFY(I,J) = (1.+2.*DY(I,J+1))*BY(I,J+1)/D2-DD*SLOY/D1
0013                 BOFY(I,J+1) = 4.*DY(I,J+1)*BY(I,J+1)/D2+2.*DU*SLOY/D1
0014                 FY(I,J-1)=DY(I,J+1)**2/D2
0015                 FFY(I,J-1)=D3**2/D2
0016                 RETURN
0017                 END
```

```
 0001                 SUBROUTINE BOFOUR(I,J,X,X1,Y,Y1)
            C
            C
 0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                     2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
            C
            C
 0003                 CALL             XFOUR(I,J,X,X1,Y)
 0004              10 CALL             YFOUR(I,J,X,Y,Y1)
 0005              20 RETURN
 0006                 END
```

```
      SUBROUTINE GAIN(I,J)
C
C     ************************************************************************
C     THIS SUBROUTINE PUTS THE COEFFICENTS OF THE OPERATOR IN PROPER
C     LOCATION.
C     ************************************************************************
C
      COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
     2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
C
C
  111 L1=LA(I,J)
      P(L1)=C(1)
      L1=LA(I+1,J)
      P(L1)=C(2)
      L1=LA(I,J+1)
      P(L1)=C(3)
      L1=LA(I-1,J)
      P(L1)=C(4)
      L1=LA(I,J-1)
      P(L1)=C(5)
      L1=LA(I+1,J+1)
      P(L1)=C(6)
      L1=LA(I-1,J+1)
      P(L1)=C(7)
      L1=LA(I-1,J-1)
      P(L1)=C(8)
      L1=LA(I+1,J-1)
      P(L1)=C(9)
      L1=LA(I+2,J)
      P(L1)=C(10)
      L1=LA(I,J+2)
      P(L1)=C(11)
      L1=LA(I-2,J)
      P(L1)=C(12)
      L1=LA(I,J-2)
      P(L1)=C(13)
      RETURN
      END
```

```
0001              SUBROUTINE STAND
             C
             C    ***********************************************************************
             C    STANDARD OPERATOR WITH SQUARE MESH. REF. ART.--2..4.1
             C    ***********************************************************************
             C
0002              COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                 1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                 2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
             C
             C
0003          111 C(1)=20.
0004              C(2)=-8.
0005              C(3)=-8.
0006              C(4)=-8.
0007              C(5)=-8.
0008              C(6)=2.
0009              C(7)=2.
0010              C(8)=2.
0011              C(9)=2.
0012              C(10)=1.
0013              C(11)=1.
0014              C(12)=1.
0015              C(13)=1.
0016              RETURN
0017              END
```

```
 0001                SUBROUTINE ONE(AA,BB,CC,DD)
              C
              C      ************************************************************************
              C      OPERATOR FOR THE POINT ON THE INSERT , ONE MESH LENGTH
              C      ABOVE THE INTERFACE.  REF ART.--2..4.4
              C      ************************************************************************
              C
 0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                    2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
              C
              C
 0003                C(1)=AA
 0004                C(2)=-8.
 0005                C(3)=-8.
 0006                C(4)=-8.
 0007                C(5)=BB
 0008                C(6)=2.
 0009                C(7)=2.
 0010                C(8)=CC
 0011                C(9)=CC
 0012                C(10)=1.
 0013                C(11)=1.
 0014                C(12)=1.
 0015                C(13)=DD
 0016                RETURN
 0017                END
```

```
 0001              SUBROUTINE TWO(AA,BB,CC,DD)
             C
             C     ***********************************************************************
             C     OPERATOR FOR THE POINT ON THE INSERT , ONE MESH LENGTH
             C     BELOW THE INTERFACE . REF ART. 2..4.4
             C     ***********************************************************************
             C
 0002              COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                  1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                  2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
             C
             C
 0003              C(1)=AA
 0004              C(2)=-8.
 0005              C(3)=BB
 0006              C(4)=-8.
 0007              C(5)=-8.
 0008              C(6)=CC
 0009              C(7)=CC
 0010              C(8)=2.
 0011              C(9)=2.
 0012              C(10)=1.
 0013              C(11)=DD
 0014              C(12)=1.
 0015              C(13)=1.
 0016              RETURN
 0017              END
```

```
 0001               SUBROUTINE THREE(AA,BB,CC,DD)
              C
              C     **********************************************************************
              C     OPERATOR FOR THE POINT ON THE INSERT , ONE MESH LENGTH
              C     RIGHT OF INTERFACE. REF. ART.--2..4.4
              C     **********************************************************************
              C
 0002               COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                   2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
              C
              C
 0003               C(1)=AA
 0004               C(2)=-8.
 0005               C(3)=-8.
 0006               C(4)=BB
 0007               C(5)=-8.
 0008               C(6)=2.
 0009               C(7)=CC
 0010               C(8)=CC
 0011               C(9)=2.
 0012               C(10)=1.
 0013               C(11)=1.
 0014               C(12)=DD
 0015               C(13)=1.
 0016               RETURN
 0017               END
```

```
0001                 SUBROUTINE FOUR(AA,BB,CC,DD)
               C
               C     **********************************************************************
               C     OPERATOR FOR THE POINT ON THE INSERT , ONE MESH LENGTH
               C     LEFT OF INTERFACE. REF. ART.--2..4.4
               C     **********************************************************************
               C
0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                    2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
               C
               C
0003                 C(1)=AA
0004                 C(2)=BB
0005                 C(3)=-8.
0006                 C(4)=-8.
0007                 C(5)=-8.
0008                 C(6)=CC
0009                 C(7)=2.
0010                 C(8)=2.
0011                 C(9)=CC
0012                 C(10)=DD
0013                 C(11)=1.
0014                 C(12)=1.
0015                 C(13)=1.
0016                 RETURN
0017                 END
```

```
0001               SUBROUTINE ICONE(AAA,BB,CCC,CC,DD)
            C
            C      ****************************************************************************
            C      OPERATOR FOR THE POINT ON INSERT ONE MESH LENGTH AWAY FROM THE
            C      LOWER CORNER OF INTERFACE. REF. ART.--2..4.5
            C      ****************************************************************************
            C
0002               COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                  1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                  2DEK(35),CEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
            C
            C
0003               C(1)=AAA
0004               C(2)=-8.
0005               C(3)=-8.
0006               C(4)=BB
0007               C(5)=BB
0008               C(6)=2.
0009               C(7)=CC
0010               C(8)=CCC
0011               C(9)=CC
0012               C(10)=1.
0013               C(11)=1.
0014               C(12)=DD
0015               C(13)=DD
0016               RETURN
0017               END
```

```
0001             SUBROUTINE ICTWO(AAA,BB,CCC,CC,DD)
           C
           C     **********************************************************************
           C     OPERATOR FOR THE POINT ON INSERT ONE MESH LENGTH AWAY FROM THE
           C     UPPER LEFT CORNER OF THE INTERFACE.REF. ART.--2..4.5
           C     **********************************************************************
           C
0002             COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
           C
           C
0003             C(1)=AAA
0004             C(2)=BB
0005             C(3)=-8.
0006             C(4)=-8.
0007             C(5)=BB
0008             C(6)=CC
0009             C(7)=2.
0010             C(8)=CC
0011             C(9)=CCC
0012             C(10)=DD
0013             C(11)=1.
0014             C(12)=1.
0015             C(13)=DD
0016             RETURN
0017             END
```

```
0001               SUBROUTINE ICTHRE(AAA,BB,CCC,CC,DD)
             C
             C     ************************************************************************
             C     OPERATOR FOR THE POINT ON INSERT ONE MESH LENGTH AWAY FROM THE
             C     LOWER RIGHT CORNER OF THE INTERFACE. REF. ART.--2..4.5
             C     ************************************************************************
             C
0002               COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                  1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                  2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
0003               C(1)=AAA
0004               C(2)=-8.
0005               C(3)=BB
0006               C(4)=BB
0007               C(5)=-8.
0008               C(6)=CC
0009               C(7)=CCC
0010               C(8)=CC
0011               C(9)=2.
0012               C(10)=1.
0013               C(11)=DD
0014               C(12)=DD
0015               C(13)=1.
0016               RETURN
0017               END
```

```
0001                SUBROUTINE ICFOUR(AAA,BB,CCC,CC,DD)
          C
          C         ****************************************************************
          C         OPERATOR FOR THE POINT ON INSERT ONE MESH LENGTH AWAY FROM THE
          C         UPPER RIGHT CORNER OF INTERFACE. REF. ART.--2..4.5
          C         ****************************************************************
          C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                   2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
          C
          C
0003                C(1)=AAA
0004                C(2)=BB
0005                C(3)=BB
0006                C(4)=-8.
0007                C(5)=-8.
0008                C(6)=CCC
0009                C(7)=CC
0010                C(8)=2.
0011                C(9)=CC
0012                C(10)=DD
0013                C(11)=DD
0014                C(12)=1.
0015                C(13)=1.
0016                RETURN
0017                END
```

```
0001                 SUBROUTINE OONE(AA1,BB1,CC1,DD1)
            C
            C        **************************************************************************
            C        OPERATOR FOR THE POINT ON THE PARENT MATERIAL ONE MESH LENGTH
            C        BELOW THE INTERFACE. REF. ART.2..4.4
            C        **************************************************************************
            C
0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                    2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
            C
            C
0003                 C(1)=AA1
0004                 C(2)=-8.
0005                 C(3)=BB1
0006                 C(4)=-8.
0007                 C(5)=-8.
0008                 C(6)=CC1
0009                 C(7)=CC1
0010                 C(8)=2.
0011                 C(9)=2.
0012                 C(10)=1.
0013                 C(11)=DD1
0014                 C(12)=1.
0015                 C(13)=1.
0016                 RETURN
0017                 END
```

```
0001                SUBROUTINE OTWO(AA1,BB1,CC1,DD1)
           C
           C        ********************************************************************************
           C        OPERATOR FOR THE POINT ON THE PARENT MATERIAL ONE MESH LENGTH
           C        ABOVE THE INTERFACE. REF. ART.--2..4.4
           C        ********************************************************************************
           C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                   2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
           C
           C
0003                C(1)=AA1
0004                C(2)=-8.
0005                C(3)=-8.
0006                C(4)=-8.
0007                C(5)=BB1
0008                C(6)=2.
0009                C(7)=2.
0010                C(8)=CC1
0011                C(9)=CC1
0012                C(10)=1.
0013                C(11)=1.
0014                C(12)=1.
0015                C(13)=DD1
0016                RETURN
0017                END
```

```
0001                SUBROUTINE OTHREE(AA1,BB1,CC1,DD1)
              C
              C     ************************************************************************
              C     OPERATOR FOR THE POINT ON THE PARENT MATERIAL ONE MESH LENGTH
              C     LEFT OF THE INTERFACE. REF. ART.-- 2..4.4
              C     ************************************************************************
              C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                   2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
              C
              C
0003                C(1)=AA1
0004                C(2)=BB1
0005                C(3)=-8.
0006                C(4)=-8.
0007                C(5)=-8.
0008                C(6)=CC1
0009                C(7)=2.
0010                C(8)=2.
0011                C(9)=CC1
0012                C(10)=DD1
0013                C(11)=1.
0014                C(12)=1.
0015                C(13)=1.
0016                RETURN
0017                END
```

```
0001               SUBROUTINE OFOUR(AA1,BB1,CC1,DD1)
            C
            C      **********************************************************************
            C      OPERATOR FOR THE POINT ON THE PARENT MATERIAL ONE MESH LENGTH
            C      RIGHT OF INTERFACE. REF.ART.--2..4.4
            C      **********************************************************************
            C
0002               COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                  1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                  2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
            C
0003               C(1)=AA1
0004               C(2)=-8.
0005               C(3)=-8.
0006               C(4)=BB1
0007               C(5)=-8.
0008               C(6)=2.
0009               C(7)=CC1
0010               C(8)=CC1
0011               C(9)=2.
0012               C(10)=1.
0013               C(11)=1.
0014               C(12)=DD1
0015               C(13)=1.
0016               RETURN
0017               END
```

```
0001              SUBROUTINE ONONE(A,B,C1,D,E,F,G,H,TI)
            C
            C     ***********************************************************
            C     OPERATOR FOR THE POINT ON THE
            C     LOWER INTERFACE. REF. ART.2..4.3
            C     ***********************************************************
            C
0002              COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                 1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                 2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
            C
            C
0003              C(1)=A
0004              C(2)=B
0005              C(3)=C1
0006              C(4)=B
0007              C(5)=D
0008              C(6)=E
0009              C(7)=E
0010              C(8)=F
0011              C(9)=F
0012              C(10)=G
0013              C(11)=H
0014              C(12)=G
0015              C(13)=TI
0016              RETURN
0017              END
```

```
0001              SUBROUTINE ONTWO(A,B,C1,D,F,F,G,H,TI)
           C
           C      ******************************************************************
           C      OPERATOR FOR THE POINT ON THE
           C      UPPER INTERFACE . REF. ART.--2..4.3
           C
           C      ******************************************************************
0002              COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                 1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                 2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
           C
           C
0003              C(1)=A
0004              C(2)=B
0005              C(3)=D
0006              C(4)=B
0007              C(5)=C1
0008              C(6)=F
0009              C(7)=F
0010              C(8)=E
0011              C(9)=E
0012              C(10)=G
0013              C(11)=TI
0014              C(12)=G
0015              C(13)=H
0016              RETURN
0017              END
```

```
0001          SUBROUTINE ONTHRE(A,B,C1,D,E,F,G,H,TI)
        C
        C     ****************************************************************************
        C     OPERATOR FOR THE POINT ON THE
        C     LEFT INTERFACE . REF . ART.--2..4.3
        C     ****************************************************************************
        C
0002          COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
             1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
             2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
        C
        C
0003          C(1)=A
0004          C(2)=C1
0005          C(3)=B
0006          C(4)=D
0007          C(5)=B
0008          C(6)=E
0009          C(7)=F
0010          C(8)=F
0011          C(9)=E
0012          C(10)=H
0013          C(11)=G
0014          C(12)=TI
0015          C(13)=G
0016          RETURN
0017          END
```

```
0001          SUBROUTINE ONFOUR(A,B,C1,D,E,F,G,H,TI)
        C
        C     OPERATOR FOR THE POINT ON THE
        C     ********************************************************************************
        C     RIGHT INTERFACE . REF. ART.--2..4.3
        C     ********************************************************************************
        C
0002          COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
             1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
             2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
        C
        C
0003          C(1)=A
0004          C(2)=D
0005          C(3)=B
0006          C(4)=C1
0007          C(5)=B
0008          C(6)=F
0009          C(7)=E
0010          C(8)=E
0011          C(9)=F
0012          C(10)=TI
0013          C(11)=G
0014          C(12)=H
0015          C(13)=G
0016          RETURN
0017          END
```

```
 0001                SUBROUTINE COONE(X,Y,S,Z,T)
           C
           C         **********************************************************************
           C         OPERATOR FOR THE POINT ON THE
           C         LOWER LEFT CORNER OF THE INTERFACE .REF ART.--2..4.6
           C         **********************************************************************
           C
 0002                COMMON DEL(35),DELI(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                    2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
           C
           C
 0003                C(1)=X
 0004                C(2)=-8.
 0005                C(3)=Y
 0006                C(4)=-8.
 0007                C(5)=-8.
 0008                C(6)=S
 0009                C(7)=Z
 0010                C(8)=2.
 0011                C(9)=2.
 0012                C(10)=1.
 0013                C(11)=T
 0014                C(12)=1.
 0015                C(13)=1.
 0016                RETURN
 0017                END
```

```
0001              SUBROUTINE COTWO(X,Y,S,Z,T)
             C
             C    **********************************************************************
             C    OPERATOR FOR THE POINT ON THE
             C    UPPER LEFT CORNER OF THE INTERFACE . REF. ART.--2..4.6
             C    **********************************************************************
             C
0002              COMMON DEL(35),DELI(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                 1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                 2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
             C
             C
0003              C(1)=X
0004              C(2)=Y
0005              C(3)=-8.
0006              C(4)=-8.
0007              C(5)=-8.
0008              C(6)=Z
0009              C(7)=2.
0010              C(8)=2.
0011              C(9)=S
0012              C(10)=T
0013              C(11)=1.
0014              C(12)=1.
0015              C(13)=1.
0016              RETURN
0017              END
```

```
0001                  SUBROUTINE COTHRE(X,Y,S,Z,T)
             C
             C        **************************************************************************
             C        OPERATOR FOR THE POINT ON THE
             C        LOWER RIGHT CORNER OF THE INTERFACE. REF. ART --2..4.6
             C        **************************************************************************
             C
0002                  COMMON DEL(35),DELI(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                     2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
             C
             C
0003                  C(1)=X
0004                  C(2)=-8.
0005                  C(3)=-8.
0006                  C(4)=Y
0007                  C(5)=-8.
0008                  C(6)=2.
0009                  C(7)=S
0010                  C(8)=Z
0011                  C(9)=2.
0012                  C(10)=1.
0013                  C(11)=1.
0014                  C(12)=T
0015                  C(13)=1.
0016                  RETURN
0017                  END
```

```
FORTRAN IV G LEVEL  18               COFOUR               DATE = 70278          18/30/45                PAGE 0001
 0001                  SUBROUTINE COFOUR(X,Y,S,Z,1)
              C
              C        **********************************************************************
              C        OPERATOR FOR THE POINT ON THE
              C        UPPER RIGHT CORNER CF THE INTERFACE . REF. ART .--2..4.6
              C        **********************************************************************
              C
 0002                  COMMON DEL(35),DELI(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                      1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                      2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
              C
 0003                  C(1)=X
 0004                  C(2)=-8.
 0005                  C(3)=-8.
 0006                  C(4)= -8.
 0007                  C(5)= Y
 0008                  C(6)=2.
 0009                  C(7)= 2.
 0010                  C(8)=S
 0011                  C(9)= Z
 0012                  C(10)=1.
 0013                  C(11)=1.
 0014                  C(12)= 1.
 0015                  C(13)= 1.
 0016                  RETURN
 0017                  END
```

```
0001              SUBROUTINE GNERAL(I,J,B,N)
            C
            C     ******************************************************************
            C     THIS SUBROUTINE USES THE GENERAL OPERATOR . REF. ART.--2..4.1
            C     ******************************************************************
            C
0002              COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                 1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),C(13),LA(35,35),
                 2DEK(35),DEK1(35),P(451),FX(35,35),FFX(35,35),FY(35,35),FFY(35,35)
            C
            C
0003              H3=DEK(I+1)
0004              V3=DEK1(J+1)
0005            1 H2=DEK(I)
0006              V2=DEK1(J)
0007           20 H4=DEK(I+2)
0008           50 V4=DEK1(J+2)
0009           80 H1=DEK(I-1)
0010          110 V1=DEK1(J-1)
0011          120 F1=(H2+H3)**2.
0012              F2=(H3+H4)**2.
0013              F3=(H1+H2)**2.
0014              F4=(V2+V3)**2.
0015              F5=(V3+V4)**2.
0016              F6=(V1+V2)**2.
0017              F7=(H2+H3)**4.
0018              F8=(V2+V3)**4.
0019              PH0=16./(F1*F2)+64./F7+16./(F1*F3)+16./(F4*F5)+64./F8+16./(F4*F6)+
                 1128./(F4*F1)
0020              PH1=-32./(F1*F2)-32./F7-64./(F4*F1)
0021              PH2=-32./(F4*F5)-32./F8-64./(F4*F1)
0022              PH3=-32./(F7)-32./(F1*F3)-64./(F4*F1)
0023              PH4=-32./F8-32./(F4*F6)-64./(F4*F1)
0024              PH5=32./(F4*F1)
0025              PH9=16./(F1*F2)
0026              PH10=16./(F4*F5)
0027              PH11=16./(F1*F3)
0028              PH12=16./(F4*F6)
0029              P(N) = PH0
0030              IF ( LA(I+1 , J)) 140 , 130 , 140
0031        140   L1 = LA(I+1,J)
0032              P(L1) = P(L1) + PH1
0033              GO TO 150
```

```
 130   B = B - BFX(I+1 , J) * PH1
       L1 = LA (I,J)
       P(L1)=P(L1)+FX(I,J)*PH1
 150   IF ( LA (I,J+1)) 170 , 160 , 170
 160   B = B - BFY(I,J+1) * PH2
       L1 = LA (I,J)
       P(L1)=P(L1)+FY(I,J)*PH2
       GO TO 180
 170   L1 = LA(I,J+1)
        P(L1)=P(L1)+PH2
 180   IF ( LA(I-1 , J))200 , 190 , 200
 190   B = B - BFX(I - 1 , J) * PH3
       L1 = LA(I,J)
       P(L1)=P(L1)+FX(I,J)*PH3
       GO TO 210
 200   L1 = LA(I-1,J)
       P(L1) = P(L1) + PH3
 210   IF(LA ( I,J-1))230, 220,230
 220   B = B - BFY ( I,J-1) *  PH4
       L1 = LA(I,J)
       P(L1)=P(L1)+FY(I,J)*PH4
       GO TO 240
 230   L1 = LA ( I,J-1)
       P(L1) = P(L1) + PH4
   240 IF(LA(I+1,J+1))250,260,250
 250   L1=LA(I+1,J+1)
       P(L1)=P(L1)+PH5
       GO TO 290
   260 IF(I.EQ.33.AND.J.EQ.23)GO TO 275
       IF(I.EQ.33.AND.J.EQ.24)GO TO 285
       IF(I.EQ.23.AND.J.EQ.33)GO TO 295
       IF(I.EQ.24.AND.J.EQ.33)GO TO 305
       IF(BFX(I+1,J+1))270,280,270
   275 B=B-BFX(I+1,J+1)*PH5
       L1=LA(I,J+1)
       P(L1)=P(L1)+FX(I,J+1)*PH5
       GO TO 290
   285 B=B-BOFX(I+1,J+1)*PH5
       L1=LA(I-1,J+1)
       P(L1)=P(L1)+FFX(I-1,J+1)*PH5
       GO TO 290
   295 B=B-BFY(I+1,J+1)*PH5
       L1=LA(I+1,J)
```

```
          P(L1)=P(L1)+FY(I+1,J)*PH5
          GO TO 290
      305 B=B-BOFY(I+1,J+1)*PH5
          L1=LA(I+1,J-1)
          P(L1)=P(L1)+FFY(I+1,J-1)*PH5
          GO TO 290
  270     B=B-(BFX(I+1,J+1)+BFY(I+1,J+1))*0.5*PH5
          L1=LA(I,J+1)
          P(L1)=P(L1)+0.5*FX(I,J+1)*PH5
          L1=LA(I+1,J)
          P(L1)=P(L1)+0.5*FY(I+1,J)*PH5
          GO TO 290
  280     B=B-(BOFX(I+1,J+1)+BOFY(I+1,J+1))*0.5*PH5
          L1=LA(I-1,J+1)
          P(L1)=P(L1)+.5*FFX(I-1,J+1)*PH5
          L1=LA(I+1,J-1)
          P(L1)=P(L1)+.5*FFY(I+1,J-1)*PH5
  290     IF(LA(I-1,J+1))300,310,300
  300     L1=LA(I-1,J+1)
          P(L1)=P(L1)+PH5
          GO TO 340
      310 IF(I.EQ.23.AND.J.EQ.33)GO TO 304
          IF(I.EQ.24.AND.J.EQ.33)GO TO 315
          IF(I.EQ. 3.AND.J.EQ.12)GO TO 325
          IF(I.EQ. 3.AND.J.EQ.13)GO TO 335
          IF(BFX(I-1,J+1))320,330,320
      304 B=B-BOFY(I-1,J+1)*PH5
          L1=LA(I-1,J-1)
          P(L1)=P(L1)+FFY(I-1,J-1)*PH5
          GO TO 340
      315 B=B-BFY(I-1,J+1)*PH5
          L1=LA(I-1,J)
          P(L1)=P(L1)+FY(I-1,J)*PH5
          GO TO 340
      325 B=B-BFX(I-1,J+1)*PH5
          L1=LA(I,J+1)
          P(L1)=P(L1)+FX(I,J+1)*PH5
          GO TO 340
      335 B=B-BOFX(I-1,J+1)*PH5
          L1=LA(I+1,J+1)
          P(L1)=P(L1)+FFX(I+1,J+1)*PH5
          GO TO 340
  320     B=B-(BFX(I-1,J+1)+BFY(I-1,J+1))*0.5*PH5
```

```
              L1=LA(I-1,J)
              P(L1)=P(L1)+.5*FY(I-1,J)*PH5
              L1=LA(I,J+1)
              P(L1)=P(L1)+.5*FX(I,J+1)*PH5
              GO TO 340
  330         B=B-(BOFX(I-1,J+1)+BOFY(I-1,J+1))*0.5*PH5
              L1=LA(I-1,J-1)
              P(L1)=P(L1)+.5*FFY(I-1,J-1)*PH5
              L1=LA(I+1,J+1)
              P(L1)=P(L1)+.5*FFX(I+1,J+1)*PH5
  340         IF(LA(I-1,J-1))350,360,350
  350         L1=LA(I-1,J-1)
              P(L1)=P(L1)+PH5
              GO TO 390
    360       IF(I.EQ.3.AND.J.EQ.12)GO TO 365
              IF(I.EQ.3.AND.J.EQ.13)GO TO 375
              IF(I.EQ.12.AND.J.EQ.3)GO TO 385
              IF(I.EQ.13.AND.J.EQ.3)GO TO 395
              IF(BFX(I-1,J-1))370,380,370
    375       B=B-BFX(I-1,J-1)*PH5
              L1=LA(I,J-1)
              P(L1)=P(L1)+FX(I,J-1)*PH5
              GO TO 390
    365       B=B-BOFX(I-1,J-1)*PH5
              L1=LA(I+1,J-1)
              P(L1)=P(L1)+FFX(I+1,J-1)*PH5
              GO TO 390
    385       B=B-BOFY(I-1,J-1)*PH5
              L1=LA(I-1,J+1)
              P(L1)=P(L1)+FFY(I-1,J+1)*PH5
              GO TO 390
    395       B=B-BFY(I-1,J-1)*PH5
              L1=LA(I-1,J)
              P(L1)=P(L1)+FY(I-1,J)*PH5
              GO TO 390
  370         B=B-(BFX(I-1,J-1)+BFY(I-1,J-1))*0.5*PH5
              L1=LA(I,J-1)
              P(L1)=P(L1)+.5*FX(I,J-1)*PH5
              L1=LA(I-1,J)
              P(L1)=P(L1)+.5*FY(I-1,J)*PH5
              GO TO 390
  380         B=B-(BOFX(I-1,J-1)+BOFY(I-1,J-1))*0.5*PH5
              L1=LA(I+1,J-1)
```

```
            P(L1)=P(L1)+ .5*FFX(I+1,J-1)*PH5
            L1=LA(I-1,J+1)
            P(L1)=P(L1)+.5*FFY(I-1,J+1)*PH5
     390    IF(LA(I+1,J-1))400,410,400
     400    L1=LA(I+1,J-1)
            P(L1)=P(L1)+PH5
            GO TO 440
        410 IF(I.EQ.12.AND.J.EQ.3)GO TO 415
            IF(I.EQ.13.AND.J.EQ.3)GO TO 425
            IF(I.EQ.33.AND.J.EQ.23)GO TO 435
            IF(I.EQ.33.AND.J.EQ.24)GO TO 445
            IF(BFX(I+1,J-1))420,430,420
        415 B=B-BFY(I+1,J-1)*PH5
            L1=LA(I+1,J)
            P(L1)=P(L1)+FY(I+1,J)*PH5
            GO TO 440
        425 B=B-BOFY(I+1,J-1)*PH5
            L1=LA(I+1,J+1)
            P(L1)=P(L1)+FFY(I+1,J+1)*PH5
            GO TO 440
        435 B=B-BOFX(I+1,J-1)*PH5
            L1=LA(I-1,J-1)
            P(L1)=P(L1)+FFX(I-1,J-1)*PH5
            GO TO 440
        445 B=B-BFX(I+1,J-1)*PH5
            L1=LA(I,J-1)
            P(L1)=P(L1)+FX(I,J-1)*PH5
            GO TO 440
     420    B=B-(BFX(I+1,J-1)+BFY(I+1,J-1))*0.5*PH5
            L1=LA(I+1,J)
            P(L1)=P(L1)+.5*FY(I+1,J)*PH5
            L1=LA(I,J-1)
            P(L1)=P(L1)+.5*FX(I,J-1)*PH5
            GO TO 440
     430    B=B-(BOFX(I+1,J-1)+BOFY(I+1,J-1))*0.5*PH5
            L1=LA(I+1,J+1)
            P(L1)=P(L1)+.5*FFY(I+1,J+1)*PH5
            L1=LA(I-1,J-1)
            P(L1)=P(L1)+.5*FFX(I-1,J-1)*PH5
     440    IF(LA(I+2,J))450,460,450
     450    L1 = LA ( I+2 , J)
            P(L1) = P(L1) + PH9
            GO TO 490
```

```
 460   IF ( BFX (I+2,J)) 470,480,470
 470   B = B -BFX ( I+2 , J) * PH9
       L1 = LA(I+1 , J)
       P(L1)=P(L1)+FX(I+1,J)*PH9
       GO TO 490
 480   B = B- BCFX ( I+2,J) *PH9
       L1 = LA (I,J)
       P(L1)=P(L1)+FFX(I,J)*PH9
 490   IF(LA(I,J+2))500,510,500
 500   L1 = LA(I,J+2)
       P(L1) = P(L1) + PH10
       GO TO 540
 510   IF(BFY(I,J+2))520,530,520
 520   B = B-BFY(I,J+2)*PH10
       L1 = LA(I,J+1)
       P(L1)=P(L1)+FY(I,J+1)*PH10
       GO TO 540
 530   B = B-BOFY(I,J+2)*PH10
       L1 = LA(I,J)
       P(L1)=P(L1)+FFY(I,J)*PH10
 540   IF(LA(I-2,J))550,560,550
 550   L1 = LA(I-2,J)
       P(L1) = P(L1)+PH11
       GO TO 590
 560   IF(BFX(I-2,J))570,580,570
 570   B = B-BFX(I-2,J)*PH11
       L1 = LA(I-1,J)
       P(L1)=P(L1)+FX(I-1,J)*PH11
       GO TO 590
 580   B = B-BOFX(I-2,J)*PH11
       L1 = LA(I,J)
       P(L1)=P(L1)+FFX(I,J)*PH11
 590   IF(LA(I,J-2))600,610,600
 600   L1 = LA(I,J-2)
       P(L1) = P(L1)+PH12
       GO TO 640
 610   IF(BFY(I,J-2))620,630,620
 620   B = B-BFY(I,J-2)*PH12
       L1 = LA(I,J-1)
       P(L1)=P(L1)+FY(I,J-1)*PH12
       GO TO 640
 630   B = B-BOFY(I,J-2)*PH12
       L1 = LA(I,J)
```

```
 0249              P(L1)=P(L1)+FFY(I,J)*PH12
 025C          640 RETURN
 0251              END
```

```
      C
      C
      C      ********************************************************************
      C      THIS IS THE SECOND STEP OF THE THREE STEPS PROGRAM FOR THE
      C      SOLUTION OF THE PROBLEM
      C      ********************************************************************
      C
      C
      C
      C      THIS PROGRAM IS USED FOR THE SOLUTION OF SET OF SIMULTANEOUS
      C      EQUATIONS GENERATED BY STEP_1.
      C
      C      THIS IS BASED ON THE GAUSS - SIEDEL ITERATIVE TECHNIQUE .
      C      THIS PROGRAM IS NOST USEFUL WHERE THE LARGE MATRICES ARE
      C      ENCOUNTERED.
      C
      C      THIS METHODPERFORMS OPERATIONS ONLY ON NON - ZERO ELEMENTS.
      C
      C
      C      ********************************************************************
      C      THE VARIABLESUSED ARE DEFINED AS FOLLOWS .
      C
      C
      C      N              NO. OF ROWS
      C      EPS            TOLERANCE LIMIT
      C      MAXIT          ITERATION LIMIT
      C      J              COLUMN COUNTER
      C      I              ROW COUNTER
      C      P(J)           ELEMENTS OF ANY ROW
      C      KK             COUNTER FOR TOTAL NO. OF NON-ZERO ELEMENTS EXCEPT
      C                     DIAGONAL ELEMENTS IN MATRIX
      C      K              COUNTER FOR TOTAL NO. OF NON-ZERO ELEMENTS EXCEPT
      C                     DIAGONAL ELEMENTS IN EACH ROW
      C      C(I)           DIAGONAL ELEMENTS OF THE MATRIX
      C      D(KK)          ALL THE NON-ZERO ELEMENTSOF THE MATRIX
      C      M(KK)          JTH POSITION FOR THE NON-ZERO ELEMENTS OF MATRIX
      C      NN(I)          NO. OF NON-ZERO ELEMENTS EXCEPT DIAGONAL ELEMENTS
      C                     IN EACH ROW
      C      PP(I)          RIGHT HAND SIDE VECTOR
      C      ********************************************************************
      C
      C
0001         IMPLICIT REAL*8(A-H,O,Q-Z)
```

```
 0002                 DIMENSION P(512),PP(512),PPP(512),D(7000),C(512),X(512),B(512),
                     1M(7000),NN(512)
              C       READ N,EPSILON AND THE MAXIMUM NUMBER OF ITERATIONS.
 0003                 READ(1,20)N,EPS,MAXIT
 0004                 WRITE(3,20)N,EPS,MAXIT
 0005              20 FORMAT(I4,E10.3,I4)
              C
              C
              C       *************************************************************************
              C          THIS PART OF THE PROGRAM STORES THE MATRIX IN ECONOMICAL FORM
              C          BY STORING ONLY NON-ZERO ELEMENTS
              C       *************************************************************************
              C
              C
              1
 0006                 KK=0
 0007                 DO 130  I=1,N
 0008                 K=0
 0009                 READ(7)(P(J),J=1,N)
 0010                 DO 130 J=1,N
 0011                 IF (I-J) 60,50,60
 0012             60  IF(P(J))120,130,120
 0013            120  KK=KK+1
 0014                 D(KK)=P(J)
 0015                 K=K+1
 0016                 NN(I)=K
 0017                 M(KK)=J
 0018                 GO TO 130
 0019             50  C(I)=P(J)
 0020            130  CONTINUE
 0021                 WRITE(3,160)
 0022            160  FORMAT('1',5X,'C(I)',7X,'NN(I)')
 0023                 WRITE(3,170)((I,C(I),NN(I)),I=1,N)
 0024            170  FORMAT(5X,4(I5,5X,F10.5,I8))
 0025                 WRITE(3,162)((D(KL),M(KL)),KL=1,KK)
 0026            162  FORMAT(5X,8(F8.3,4X,I3))
 0027                 WRITE(3,165)KK
 0028            165  FORMAT(////,I10)
 0029                 READ(7)(PP(I),I=1,N)
              C       INTIAL VALUES OF X(I).
 0030                 READ(1,132)(PPP(I),I=1,N)
 0031                 WRITE(3,57)((I,PPP(I)),I=1,N)
 0032            132  FORMAT(4E20.10)
 0033                 DO 75I=1,N
```

```
0034            X(I)=PPP(I)
0035         75 B(I)=PP(I)
         C
         C
         C      ************************************************************
         C         THE  GAUSS SEIDEL ITERATION STARTS FROM HERE
         C      ************************************************************
         C
         C
0036            ITER=1
0037         99 BIG=0.0
0038            LL=0
0039            K2=1
0040            DO 105I=1,N
0041            SUM=0.0
0042            LL=LL+NN(I)
0043            DO 100KK=K2,LL
0044            L=M(KK)
0045        100 SUM=SUM+D(KK)*X(L)
0046            TEMP=(B(I)-SUM)/C(I)
0047            K2=LL+1
         C
         C      AT END OF SWEEP OF THIS STATEMENT HAS PUT LARGEST RESIDUAL IN BIG.
         C
0048            RESID=DABS(TEMP-X(I))
0049            IF(RESID.GT.BIG) BIG=RESID
0050        105 X(I)=TEMP
         C
         C
         C      ONE SWEEP HAS NOW BEEN COMPLETED- PRINT VARIABLES.
         C      IF LARGEST RESIDUAL IS LESS THAN EPSILON THE PROCESS HAS CONVERGED
         C
         C
0051            IF(BIG.LT.EPS)GO TO 190
         C      IF ITERATION COUNTER EXCEEDS MAXIMUM ALLOWABLE, STOP ITERATING.
0052            IF(ITER.GT.MAXIT)GO TO 190
0053            ITER=ITER+1
0054            GO TO 99
0055        190 WRITE(3,57)((I,X(I)),I=1,N)
0056            WRITE(3,111)ITER
0057       111  FORMAT(5X,I5)
0058         57 FORMAT(//,3X,6(I5,F12.6,2X))
0059            WRITE(2,15)( X(I),I=1,N)
```

```
0060            15 FORMAT(4E20.10)
0061               RETURN
0062               END
```

```
C
C
C      ********************************************************************
C      THIS IS THIRD   STEP OF THREE STEPS PROGRAM FOR THE SOLUTION OF
C      CASE 2 (45 DEG.INCLINENED INSERT)
C      ********************************************************************
C
C
C      ********************************************************************
C      THIS PROGRAM CALCULATES THE VA LUES OF STRESS FUNCTION
C      OBTAINED FROM STEP-2.
C      ********************************************************************
C
C      THE VARIABLES ARE DEFINED AS FOLLOWS-
C
C
C
C      I               NUMBER COUNTER IN X DIRECTION.
C      J               NUMBER COUNTER IN Y DIRECTION.
C      MM              NUMBER OF STEPS ALONG X DIRECTION.
C      NN              NUMBER OF STEPS ALONG Y DIRECTION.
C      DEL(I)          STEP SIZE ALONG X DIRECTION.
C      DEL1(J)         STEP SIZE ALONG Y DIRECTION.
C      DEK(I)          RATIO OF ACTUAL TO STANDARD STEP SIZE ALONG
C                      ALONG X DIRECTION.
C      DEK1(J)         RATIO OF ACTUAL TO STANDARD STEP SIZE ALONG
C                      ALONG Y DIRECTION.
C      E1,E2           THE MODULII OF ELASTICITY OF INSERT AND PARENT
C                      MATERIAL RESPECTIVELY.
C      B1,B2           THE PISSION
C      B1,B2           THE POISSION RATIO OF INSERT AND PARENT MATERIAL
C                      RESPECTIVELY.
C      X,Y             THE DISTANCE OF THE NODAL POINT FROM THE CO-ORDINATE
C                      AXIS.
C      LA(I,J)         THE NUMBER OF NODAL POINTS.
C      DX(I,J),        THE DISTANCES IN X AND Y DIRECTIONS RESPECTIVELY
C      DY(I,J)         FROM THE BOUNDARY OF THE PLATE OF THE POINTS LESS
C                      THAN ONE LENGTH AWAY.
C      M1              I-TH POSITION OF THE NODAL POINTS.
C      N1              J-TH POSITION OF THE NODAL POINTS.
C      P(KK)           ROW OF THE CO-EFFICENT MATRIX.
C      B               THE RIGHT HAND VECTOR.
C      BI(I,J)         THE PHI VALUE OF THE FICTITIOUS POINT ON THE
```

```
              C      INSERT IN TERM OF REAL VALUES.
              C      BO(I,J)         THE PHI VALUE OF THE FICTIOUS POINT ON THE PARENT
              C      MATERIALIN TERM OF REAL POINTS.
              C      PP(I,J)         VALUE OF STRESS FUNCTION AT THE NODAL POINTS
              C      PX,PY           DIRECT STRESSES
              C      PPX,PPY         DIRECT STRESSES ON THE INTERFACE IN THE PARENT MATERIAL
              C      TXY             SHEAR STRESS
              C      P1,P2           MAJOR AND MINOR PRINCIPAL STRESSES
              C
              C
              C      THE INSERT SIZE IS DEFINED AS FOLLOWS :
              C      LENGTH
              C                      LARGEST          MI = 14      MO = 22
              C                      MEDIUM           MI = 15      MO = 21
              C                      SMALLEST         MI = 16      MO = 20
              C      WIDTH
              C                      FOR ALL CASES  NI = 16      NO = 20
              C      ****************************************************************************
              C
              C
0001                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                    2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                    3FFY(35,35),PP(35,35)
              C
              C
0002                 READ(1,10)MM,NN,E1,E2,B1,B2
0003                 WRITE(3,10)MM,NN,E1,E2,B1,B2
0004                 READ(1,20) (DEL(I),I=1,MM),(DEL1(J),J=1,NN)
0005                 READ(1,20)(DEK(I),I=1,MM),(DEK1(J),J=1,NN)
0006              10 FORMAT(2I10,4F15.6)
0007                 WRITE(3,20)((DEL(I),I=1,MM),(DEL1(J),J=1,NN))
0008                 WRITE(3,20)((DEK(I),I=1,MM),(DEK1(J),J=1,NN))
0009              20 FORMAT(8F10.6)
              C
              C
              C      ****************************************************************************
              C      INITIALIZE THE VARIABLES.
              C      ****************************************************************************
              C
              C.
0010                 Y=0.0
0011                 X1=0.0
```

```
      Y1=0.0
      N=0
      DO 65I=1,MM
      DO 65J=1,NN
      LA(I,J)=C
      KA(I,J)=0
      DX(I,J)=0.0
      DY(I,J)=0.0
      BX(I,J)=0.0
      BY(I,J)=C.0
      BFX(I,J)=0.0
      BFY(I,J)=0.0
      BOFX (I,J)=0.0
      BOFY(I,J)=0.0
      PP(I,J)=0.
      FX(I,J)=0.
      FFX(I,J)=0.
       FY(I,J)=0.
      FFY(I,J)=0.
   65 CONTINUE
C
C     ***********************************************************************
C     DRAW THE MESH.
C     NUMBER THE NODAL POINTS.
C     CALCULATE THE FICITITIOUS POINTS FALLING LESS THAN ONE LENGTH
C     AWAY FROM THE BOUNDARY, IN TERMS OF REAL VALUES USING SUBROUTINES
C     XONE , YONE , BOONE , XTWO , YTWO , BOTWO , XTHREE , YTHRE,
C     BOTHRE , XFOUR , YFOUR , BOFOUR .
C     ***********************************************************************
      DO 60 J=1,35
      Y=Y+DEL1(J)
      X=0.0
      DO 60 I=1,34
      X=X+DEL(I)
      EE=1./SQRT(2.)
      IF(X.GT.EE)GO TO 70
      IF(Y.GT.EE)GO TO 130
      Y1=-X+EE
      X1=-Y+EE
      IF(Y1.GT.Y)GO TO 60
      IF((Y-Y1).GT.DEL1(J))GOTO40
      IF((X-X1).GT.DEL(I))GO TO 90
      CALL            BOONE(I,J,X,X1,Y,Y1)
```

```
      KA(I,J)=1
      GO TO 60
   90 CALL          YONE(I,J,X,Y,Y1)
      KA(I,J)=1
      GO TO 60
   40 IF((X-X1).LT.DEL(I))GO TO 100
  130 Y1=X+EE
      X1=Y-EE
      IF(Y1.LT.Y)GO TO 60
      IF((Y1-Y).GT.DEL1(J+1))GO TO 50
      IF((X-X1).GT.DEL(I))GO TO 55
      CALL          BOTWO(I,J,X,X1,Y,Y1)
      KA(I,J)=1
      GO TO 60
   55 CALL          YTWO(I,J,X,Y,Y1)
      KA(I,J)=1
      GO TO 60
  100 CALL          XONE(I,J,X,X1,Y)
      KA(I,J)=1
      GO TO 60
   50 IF((X-X1).GT.DEL(I))GO TO 80
      CALL          XTWO(I,J,X,X1,Y)
      KA(I,J)=1
      GO TO 60
   70 Y1=X-EE
      X1=Y+EE
      IF(Y1.GT.Y)GO TO 60
      IF((Y-Y1).GT.DEL1(J))GO TO 110
      IF((X1-X).GT.DEL(I+1))GO TO 120
      CALL          BOTHRE(I,J,X,X1,Y,Y1)
      KA(I,J)=1
      GO TO 60
  120 CALL          YTHRE(I,J,X,Y,Y1)
      KA(I,J)=1
      GO TO 60
  110 IF((X1-X).GT.DEL(I+1))GO TO 140
      CALL         XTHREE(I,J,X,X1,Y)
      KA(I,J)=1
      GO TO 60
  140 IF(X.LT.(2./SQRT(2.)))GO TO 130
      EE1=5.*EE
      Y1=-X+EE1
      X1=-Y+EE1
```

```
0089          IF(Y1.LT.Y)GO TO 60
0090          IF((Y1-Y).GT.DEL1(J+1))GO TO 150
0091          IF((X1-X).GT.DEL(I+1))GO TO 160
0092          CALL          BOFOUR(I,J,X,X1,Y,Y1)
0093          KA(I,J)=1
0094          GO TO 60
0095      160 CALL          YFOUR(I,J,X,Y,Y1)
0096          KA(I,J)=1
0097          GO TO 60
0098      150 IF((X1-X).GT.DEL(I+1))GO TO 80
0099          CALL          XFOUR(I,J,X,X1,Y)
0100          KA(I,J)=1
0101          GO TO 60
0102       80 N=N+1
0103          LA(I,J)=N
0104       60 CONTINUE
0105          READ(1,35)(P(I),I=1,N)
0106          WRITE(3,25)(P(I),I=1,N)
0107       25 FORMAT(5X,10F12.7)
0108       35 FORMAT(4E20.10)
0109          WRITE(3,81)N
0110          WRITE(3,87)
0111       87 FORMAT(///)
0112       88 FORMAT(5X,11F11.5)
0113       89 FORMAT(5X,21I5)
0114       81 FORMAT(5X,I5)
        C
        C
        C     CALCULATION OF STRESSES STARTS FROM HERE.
        C
        C
0115          CALL  PHI
0116          WRITE(3,88)((PP(I,J),I=1,MM),J=1,NN)
0117          CALL      STRESS(MM,NN,E1,E2,B1,B2,N)
0118          RETURN
0119          END
```

```
0001            SUBROUTINE STRESS(MM,NN,E1,E2,B1,B2,N)
          C
          C
          C     ****************************************************************
          C        THIS SUBROUTINE CALCULATES THE VALUES OF DIRECT STRESSES AND
          C        SHEAR STRESSES ISING EQUATION 2,1,6 IN FINITE DIFFERENCE FORM.
          C        THEN USING MOHR'S CIRCLE THE PRINCIPAL STRESSES ARE CALCULATED.
          C     ****************************************************************
          C
          C
0002            COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
               1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
               2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
               3FFY(35,35),PP(35,35)
0003            DIMENSION PX(35,35),PPX(35,35),PY(35,35),PPY(35,35),TXY(35,35),
               1TTXY(35,35),PXX(35,35),PPXX(35,35),PYY(35,35),PPYY(35,35),
               2TXXYY(35,35),TTXXYY(35,35),BI(35,35),BO(35,35)
          C     STRESSES PX,PPX,PY,PPY,TXY,TTXY ARE IN ACTUAL CO-ORDINATES.
          C     STRESSES PXX,PPXX,PYY,PPYY,TXXYY,TTXXYY ARE IN ORIENTION WITH THE
          C     PLATE.
0004         11 A1 = 1./E1
0005            A2 = 1./E2
0006            O1=(A2-A1)/(A1+A2)
0007            O2=((A2*B2-A1*B1+A1-A2)*2.)/ (A1+A2)
0008            O3=-((A2*B2-A1*B1)/(A1+A2))
0009            O4=(1+(A2-A1)/(A1+A2))
0010            OO1=-(A2-A1)/(A2+A1)
0011            OO2=-((A2*B2-A1*B1+A1-A2)*2.)/(A1+A2)
0012            OO3=((A2*B2-A1*B1)/(A1+A2))
0013            OO4=(1-(A2-A1)/(A1+A2))
0014            DO 31J=1,NN
0015            DO 31I=1,MM
0016            PX(I,J)=0.
0017            PPX(I,J)=0.
0018            PY(I,J)=0.
0019            PPY(I,J)=0.
0020            TXY(I,J)=0.
0021            TTXY(I,J)=0.
0022            PXX(I,J)=0.
0023            PPXX(I,J)=0.
0024            PYY(I,J)=0.
0025            PPYY(I,J)=0.
0026            TXXYY(I,J)=0.
```

```
            TTXXYY(I,J)=0.
            MI =14
            MO =22
            NI =16
            NO =20
         31 CONTINUE
            MM1=MM-1
            NN1=NN-1
C           CALCULATION OF PXX STARTS FROM HERE.
            DO 10 J=2,NN1
            DO 10 I = 2 , MM1
            IF(J.EQ.2)GO TO121
            D2=DELI(J)
            GO TO 141
  121       D2=.0625
  141       D1=DELI(J+1)
            IF (LA (I,J)) 20,10,20
  20        L1 = LA(I,J)
            F1 = P(L1)
            GO TO 70
  70        IF (LA ( I,J + 1)) 90,80,90
  90        L1 = LA (I , J + 1 )
            F2 = P ( L1)
            GO TO 160
         80 F2=PP(I,J+1)
  160       IF(LA(I,J-1)) 180, 170,180
  180       L1 = LA(I,J-1)
            F3 = P(L1)
            GO TO 250
        170 F3=PP(I,J-1)
        250 IF((I.GT.MI).AND.(I.LT.MO)) GO TO 270
  260       PX(I,J) = (F2-2.*F1+F3)*4./(D1+D2)**2
            GO TO 10
  270       L1 = LA(I-1,J)
            F4 = P(L1)
            L1 = LA(I+1,J)
            F5 = P(L1)
            IF (J.EQ.16) GO TO 281
            IF (J.EQ.20) GO TO 291
            GO TO 260
  281       BI(I,J-1) = F2*O1+F1*O2+(F4+F5)*O3+F3*O4
            BO(I,J+1) = F3*OO1+F1*OO2+(F4+F5)*OO3+F2*OO4
            WRITE(3,7)O1,O2,O3,O4,OO1,OO2,OO3,OO4,BI(I,J-1),BO(I,J+1)
```

```
   7    FORMAT(//,5X,5E20.8)
        DELTA=(D1+D2)**2.
        PX(I,J) = (F2-2.*F1+BI(I,J-1))*4./DELTA
        PPX(I,J) = (BO(I,J+1)-2.*F1+F3)*4./DELTA
        GO TO 10
 291    BI(I,J+1) = F3*O1+F1*O2+(F4+F5)*O3+F2*O4
        BO(I,J-1) = F2*OO1+F1*OO2+(F4+F5)*OO3+F3*OO4
        DELTA=(D1+D2)**2.
        PX(I,J) = (BI(I,J+1)-2.*F1+F3)*4./DELTA
        PPX(I,J) = (F2-2.*F1+BO(I,J-1))*4./DELTA
  10    CONTINUE
 C      CALCULATION OF  PYY STARTS FROM HERE.
        DO 280 J = 2,NN1
        DO 280 I=2,MM1
        IF(I.EQ.2)GO TO 21
        D2=DEL(I)
        GO TO 32
  21    D2=.0625
  32    D1=DEL(I+1)
        IF(LA(I,J)) 300,280,300
 300    L1 = LA(I,J)
        F1 = P(L1)
        GO TO 340
 340    IF(LA(I+1,J)) 360,350,360
 360    L1 = LA(I+1,J)
        F2 = P(L1)
        GO TO 430
    350 F2=PP(I+1,J)
 430    IF(LA(I-1,J)) 450,440,450
 450    L1 = LA(I-1,J)
        F3 = P(L1)
        GO TO 520
    440 F3=PP(I-1,J)
    520 IF(J.GT.NI.AND.J.LT.NO)GO TO 540
    530 DELTA=(D1+D2)**2.
        PY(I,J)=(F2-2.*F1+F3)*4./DELTA
        GO TO 280
 540    L1=LA(I,J-1)
        F4=P(L1)
        L1=LA(I,J+1)
        F5=P(L1)
        IF(I.EQ.16) GO TO 550
        IF (I.EQ.20) GO TO 560
```

```
                GO TO 530
          550   BI(I-1,J)=F2*01+F1*02+(F4+F5)*03+F3*04
                BO(I+1,J)=F3*001+F1*002+(F4+F5)*003+F2*004
                DELTA=(D1+D2)**2.
                PY(I,J)=(F2-2.*F1+BI(I-1,J))*4./DELTA
                PPY(I,J)=(BO(I+1,J)-2.*F1+F3)*4./DELTA
                GO TO 280
          560   BI(I + 1,J) = F3*01+ F1*02 + (F4 + F5)*03 + F2*04
                BO(I-1,J) = F2*001 + F1*002 + (F4 + F5)*003 + F3*004
                DELTA=(D1+D2)**2.
                PY(I,J)=(BI(I+1,J)-2.*F1+F3)*4./DELTA
                PPY(I,J) = (F2 - 2.0*F1 + BO (I - 1,J))*4./DELTA
          280   CONTINUE
          C     CALCULATION OF TXY STARTS FROM HERE.
                DO 580 J=2,NN1
                DO 580 I=2,MM1
                IF(I.EQ.2)GO TO221
                D2=DEL(I)
                GO TO 231
          221   D2=.0625
          231   IF(J.EQ.2)GO TO241
                D4=DEL1(J)
                GO TO 252
            241 D4=.0625
          252   D1=DEL(I+1)
                D3=DEL1(J+1)
                DELTA=(D1+D2)*(D3+D4)
                IF(LA(I,J))590,580,590
            590 IF(LA(I+1,J+1))600,610,600
            600 L1=LA(I+1,J+1)
                F1=P(L1)
                GO TO 700
            610 F1=PP(I+1,J+1)
            700 IF(LA(I-1,J+1))720,710,720
            720 L1=LA(I-1,J+1)
                F2=P(L1)
                GO TO 790
            710 F2=PP(I-1,J+1)
            790 IF(LA(I-1,J-1))810,800,810
            810 L1=LA(I-1,J-1)
                F3=P(L1)
                GO TO 880
            800 F3=PP(I-1,J-1)
```

```
  880 IF(LA(I+1,J-1))900,890,900
  900 L1=LA(I+1,J-1)
      F4=P(L1)
      GO TO 970
  890 F4=PP(I+1,J-1)
  970 IF(I.EQ.MI.AND.J.EQ.NI)GO TO 980
      IF(I.EQ.MO.AND.J.EQ.NI)GO TO 990
      IF(I.EQ.MI.AND.J.EQ.NO)GO TO 1000
      IF(I.EQ.MO.AND.J.EQ.NO)GO TO 1100
      TXY(I,J)=-(F1-F2+F3-F4)/DELTA
      GO TO 580
  980  TXY(I,J)=-(BO(I+1,J+1)-F2+F3-F4)/DELTA
      GO TO 580
  990  TXY(I,J)=-(F1-BO(I-1,J+1)+F3-F4)/DELTA
      GO TO 580
 1000  TXY(I,J)=-(F1-F2+F3-BO(I+1,J-1))/DELTA
      GO TO 580
 1100  TXY(I,J)=-(F1-F2+BO(I-1,J-1)-F4)/DELTA
  580 CONTINUE
      DO 1124J=1,NN
      DO 1124I=1,MM
      PXX(I,J)=(PX(I,J)+PY  (I,J))*0.5+TXY(I,J)
      IF(PPX(I,J).EQ.0.)GO TO 581
      PPXX(I,J)=(PPX(I,J)+ PY(I,J))*.5+ TXY(I,J)
581   PYY(I,J)=(PX(I,J)+PY  (I,J))*0.5-TXY(I,J)
      IF(PPY(I,J).EQ.0.)GO TO 582
      PPYY(I,J)=(  PX(I,J)+PPY(I,J))*0.5-  TXY(I,J)
  582 TXXYY(I,J)=(PY(I,J)-PX(I,J))*.5
 1124 CONTINUE
      WRITE(3,1123)
      WRITE(3,1122)((  PX(I,J),I=1,MM),J=1,NN)
      WRITE(3,1123)
      WRITE(3,1122)((PPX(I,J),I=1,MM),J=1,NN)
      WRITE(3,1123)
      WRITE(3,1122)((  PY(I,J),I=1,MM),J=1,NN)
      WRITE(3,1123)
      WRITE(3,1122)((PPY(I,J),I=1,MM),J=1,NN)
      WRITE(3,1123)
      WRITE(3,1122)((TXY(I,J),I=1,MM),J=1,NN)
      WRITE(3,1123)
      WRITE(3,1122)((TTXY(I,J),I=1,MM),J=1,NN)
      DO 1300J=1,NN
      DO 1300I=1,MM
```

```
      PXX(I,J)=0.
      PYY(I,J)=0.
      PPXX(I,J)=0.
      PPYY(I,J)=0.
      PC=(PX(I,J)+PY(I,J))/2.
      PR=SQRT(((PX(I,J)-PY(I,J))/2.)**2+TXY(I,J)**2)
      PXX(I,J)=PC+PR
      PYY(I,J)=PC-PR
 1300 CONTINUE
      DO 1400 J=16,20,4
      DO 1400 I=17,19
      PC=(PPX(I,J)+PY(I,J))/2.
      PR=SQRT(((PPX(I,J)-PY(I,J))/2.)**2+TXY(I,J)**2)
      PPXX(I,J)=PC+PR
      PPYY(I,J)=PC-PR
 1400 CONTINUE
      DO 1500 I=16,20,4
      DO 1500 J=17,19
      PC=(PX(I,J)+PPY(I,J))/2.
      PR=SQRT(((PX(I,J)-PPY(I,J))/2.)**2+TXY(I,J)**2)
      PPXX(I,J)=PC+PR
      PPYY(I,J)=PC-PR
 1500 CONTINUE
      WRITE(3,1123)
      K1=1
      K=10
   55 WRITE(3,16)
   16 FORMAT(1H1,20X,'STRESSES  IN    THE    DIRECTION    OF    LOAD.')
      WRITE(3,15)K1,K
      DO 25 J=1,NN
   25 WRITE(3,35)(J,(PXX(I,J),I=K1,K))
      IF(K.EQ.MM)GO TO 45
      K1=K+1
      K=K+10
      IF(K.GT.MM)K=MM
      GO TO 55
   45 CONTINUE
      K1=1
      K=10
  155 WRITE(3,114)
  114 FORMAT(1H1,25X,'INTERFACE  STRESSES  IN  THE  DIRECTION OF  LOAD')
      WRITE(3,15)K1,K
      DO 125 J=1,NN
```

```
  125 WRITE(3,35)(J,(PPXX(I,J),I=K1,K))
      IF(K.EQ.MM)GO TO 135
      K1=K+1
      K=K+10
      IF(K.GT.MM)K=MM
      GO TO 155
  135 CONTINUE
      K1=1
      K=10
  255 WRITE(3,216)
  216 FORMAT(1H1,20X,'STRESSES IN THE DIRECTION PARPENDICULAR TO LOAD.')
      WRITE(3,15)K1,K
      DO 225 J=1,NN
  225 WRITE(3,35)(J,(PYY(I,J),I=K1,K))
      IF(K.EQ.MM)GO TO245
      K1=K+1
      K=K+10
      IF(K.GT.MM)K=MM
      GO TO 255
  245 CONTINUE
      K1=1
      K=10
  355 WRITE(3,316)
  316 FORMAT(1H1,20X,'INTERFACES STRESSES PARPENDICULAR TO THE LOAD')
      WRITE(3,15)K1,K
      DO 325 J=1,NN
  325 WRITE(3,35)(J,(PPYY(I,J),I=K1,K))
      IF(K.EQ.MM)GO TO345
      K1=K+1
      K=K+10
      IF(K.GT.MM)K=MM
      GO TO 355
  345 CONTINUE
      K1=1
      K=10
  455 WRITE(3,416)
  416 FORMAT(1H1,20X,'SHEAR STRESSES.')
      WRITE(3,15)K1,K
      DO 425 J=1,NN
  425 WRITE(3,35)(J,(TXXYY(I,J),I=K1,K))
      IF(K.EQ.MM)GO TO445
      K1=K+1
      K=K+10
```

```
                IF(K.GT.MM)K=MM
                GO TO 455
          445   CONTINUE
                K1=1
                K=10
        555     WRITE(3,556)
          556   FORMAT(1H1,20X,'PRINCIPAL  STRESSES P1 (LARGER )')
                WRITE(3,15)K1,K
                DO 557J=1,NN
          557   WRITE(3,35)(J,(PXX(I,J),I=K1,K))
                IF(K.EQ.MM)GO TO 559
                K1=K+1
                K=K+10
                IF(K.GT.MM)K=MM
                GO TO 555
          559   CONTINUE
                K1=1
                K=10
          755   WRITE(3,716)
          716   FORMAT(1H1,20X,'PRINCIPAL  STRESSES  P1 (LARGER ) AT  INTERFACE ')
                WRITE(3,15)K1,K
                DO 725 J=1,NN
          725   WRITE(3,35)(J,(PPXX(I,J),I=K1,K))
                IF(K.EQ.MM)GO TO 745
                K1=K+1
                K=K+10
                IF(K.GT.MM)K=MM
                GO TO 755
          745   CONTINUE
                K1=1
                K=10
          955   WRITE(3,916)
          916   FORMAT(1H1,20X,' PRINCIPAL STRESSES  P2 (SMALLER )')
                WRITE(3,15)K1,K
                DO 925 J=1,NN
          925   WRITE(3,35)(J,(PYY(I,J),I=K1,K))
                IF(K.EQ.MM)GO TO945
                K1=K+1
                K=K+10
                IF(K.GT.MM)K=MM
                GO TO 955
          945   CONTINUE
                K1=1
```

```
      K=10
 1055 WRITE(3,1016)
 1016 FORMAT(1H1,20X,' PRINCIPAL  STRESSES  P2(SMALLER) AT INTERFACE')
      WRITE(3,15)K1,K
      DO1025 J=1,NN
 1025 WRITE(3,35)(J,(PPYY(I,J),I=K1,K))
      IF(K.EQ.MM)GO TO 1045
      K1=K+1
      K=K+10
      IF(K.GT.MM)K=MM
      GO TO 1055
 1045 CONTINUE
   15 FORMAT(5X,2I10)
   35 FORMAT(/,5X,I10,10F10.0)
 1122 FORMAT(6X,10(F10.0,2X))
 1123 FORMAT(///)
      RETURN
      END
```

```
0001             SUBROUTINE XONE (I,J,X,X1,Y)
           C
           C     ************************************************************************
           C     THIS SUBROUTINE IS USED ALONG LINE AD USING POINTS IN X DIRECTION.
           C      REF. -ART.2..4.2
           C     ************************************************************************
           C
0002             COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                3FFY(35,35),PP(35,35)
           C
0003             DD=X-X1
0004             IF(I.EQ.2)GO TO 10
0005             DU=DEL(I)-DD
0006             GO TO 20
0007          10 DU=.0625-DD
0008       20    DX(I,J)=DD*16
0009             SLOX= 100.0*X1 + 100.0*SQRT(2.0)
0010             BX(I-1,J)= 50.0*(X1**2)+50.0*(Y*Y)
                1+100.0*SQRT(2.0) *X1 + 100.0*SQRT(2.0) *Y-300.0
0011             D1 = 1+DX(I,J)
0012             D2 = D1**2
0013             D3 = 1-DX(I,J)
0014             BFX(I,J) = (1+2.*DX(I,J))*BX(I-1,J)/D2+DD*SLOX/D1
0015             BOFX(I-1,J) =  4.*DX(I,J)*BX(I-1,J)/D2-2.*DU*SLOX/D1
0016             FX(I+1,J)=DX(I,J)**2/D2
0017             FFX(I+1,J)=D3**2/D2
0018             RETURN
0019             END
```

```
ORTRAN IV G LEVEL  18                    XONE                DATE = 70278           18/32/15              PAGE 0002

                              COMMON BLOCK /           / MAP SIZE    12858
MBCL        LOCATION         SYMBOL     LOCATION       SYMBOL      LOCATION         SYMBOL      LOCATION          SYMBOL      LOCAT
L                  0         DEL1             8C       DX               118         DY              143C          BX           276
                3A84         BFX            4DA8       BFY             60CC         BOFX            73F0          BOFY         871
K               9A38         DEK1           9AC4       LA              9B50         KA              AE74          P            C19
                C8A4         FFX            DBC8       FY              EEEC         FFY            10210          PP          1153

                              SUBPROGRAMS CALLED
MBCL        LOCATION         SYMBOL     LOCATION       SYMBOL      LOCATION         SYMBOL      LOCATION          SYMBOL      LOCAT
RT                E0

                              SCALAR MAP
MBCL        LOCATION         SYMBOL     LOCATION       SYMBOL      LOCATION         SYMBOL      LOCATION          SYMBOL      LOCAT
                  F0         X                F4       X1                F8         I                 FC          DU           10
                 104         SLOX            108       Y                10C         D1               110          D2           11
                 118

 *OPTIONS IN EFFECT*   ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
 *OPTIONS IN EFFECT*   NAME = XONE     , LINECNT =       45
 *STATISTICS*    SOURCE STATEMENTS =        19,PROGRAM SIZE =      1256
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001                SUBROUTINE YONE (I,J,X,Y,Y1)
           C
           C        ****************************************************************************
           C        THIS SUBROUTINE IS USED ALONG LINE AD USING POINTS IN Y DIRECTION.
           C         REF. -ART.2..4.2
           C        ****************************************************************************
           C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                   2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                   3FFY(35,35),PP(35,35)
           C
           C
0003                DD=Y-Y1
0004                IF(J.EQ.2)GO TO10
0005                DU=DEL1(J)-DD
0006                GO TO 20
0007             10 DU=.0625-DD
0008           20   DY(I,J)=DD*16
0009                SLOY=100.0*Y1+100.0*SQRT(2.0)
0010                BY(I,J-1)= 50.0*(X*X)+50.0*(Y1**2) +100.0*SQRT(2.0)*X
                   1+100.*SQRT(2.0)*Y1 -300.0
0011                D1 = 1+DY(I,J)
0012                D2 = D1**2
0013                D3 = 1-DY(I,J)
0014                BFY(I,J) = (1+2.*DY(I,J))*BY(I,J-1)/D2+DD*SLOY/D1
0015                BOFY(I,J-1) = 4.*DY(I,J)*BY(I,J-1)/D2-2.*DU*SLOY/D1
0016                FY(I,J+1)=DY(I,J)**2/D2
0017                FFY(I,J+1)=D3**2/D2
0018                RETURN
0019                END
```

```
ORTRAN IV G LEVEL  18                    YONE                DATE = 70278          18/32/15              PAGE 0002

                          COMMON BLOCK /          / MAP SIZE   12858
MBCL      LOCATION       SYMBOL     LOCATION      SYMBOL     LOCATION       SYMBOL      LOCATION       SYMBOL     LOCAT
L             0          DEL1          8C         DX            118         DY            143C         BX          276
           3A84          BFX         4DA8         BFY          60CC         BOFX          73F0         BOFY        871
K          9A38          DEK1        9AC4         LA           9B50         KA            AE74         P           C19
           C8A4          FFX         DBC8         FY           EEEC         FFY          10210         PP         1153

                          SUBPROGRAMS CALLED
MBCL      LOCATION       SYMBOL     LOCATION      SYMBOL     LOCATION       SYMBOL      LOCATION       SYMBOL     LOCAT
RT           E0

                          SCALAR MAP
MBOL      LOCATION       SYMBOL     LOCATION      SYMBOL     LOCATION       SYMBOL      LOCATION       SYMBOL     LOCAT
             F0          Y             F4         Y1             F8         J               FC         .DU         10
            104          SLOY         108         X             10C         D1             110         D2          11
            118

 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
 *OPTIONS IN EFFECT*  NAME = YONE    , LINECNT =       45
 *STATISTICS*    SOURCE STATEMENTS =        19,PROGRAM SIZE =      1276
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001                 SUBROUTINE BOONE (I,J,X,X1,Y,Y1)
              C
              C
0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                    2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                    3FFY(35,35),PP(35,35)
              C
              C
0003                 CALL            YONE (I,J,X,Y,Y1)
0004          10     CALL            XONE (I,J,X,X1,Y)
0005          20     RETURN
0006                 END
```

```
0001                SUBROUTINE XTWO (I,J,X,X1,Y )
              C
              C     ************************************************************************
              C     THIS SUBROUTINE IS USED ALONG LINE CD USING POINTS IN X DIRECTION.
              C      REF. -ART.2..4.2
              C     ************************************************************************
              C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                   2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                   3FFY(35,35),PP(35,35)
              C
              C
0003                DD=X-X1
0004                IF(I.EQ.2)GO TO 10
0005                DU=DEL(I)-DD
0006                GO TO 20
0007             10 DU=.0625-DD
0008          20    DX(I,J)=DD*16
0009                SLOX = 100.0*SQRT(2.0)
0010                BX (I-1,J) = 100.0*SQRT(2.0)*X1 + 150.0 *SQRT(2.0)*Y -325.0
0011                D1 = 1+DX(I,J)
0012                D2 = D1**2
0013                D3 = 1-DX(I,J)
0014                BFX(I,J) = (1+2.*DX(I,J))*BX(I-1,J)/D2+DD*SLOX/D1
0015                BOFX(I-1,J)=4.*DX(I,J)*BX(I-1,J)/D2-2.*DU*SLOX/D1
0016                FX(I+1,J)=DX(I,J)**2/D2
0017                FFX(I+1,J)=D3**2/D2
0018                RETURN
0019                END
```

```
0001                SUBROUTINE YTWO (I,J,X,Y,Y1)
              C
              C     ****************************************************************************
              C     THIS SUBROUTINE IS USED ALONG LINE CD USING POINTS IN Y DIRECTION.
              C      REF. -ART.2..4.2
              C     ****************************************************************************
              C
0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                   1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                   2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                   3FFY(35,35),PP(35,35)
              C
              C
0003                DD=Y1-Y
0004                IF(J.EQ.2)GO TO10
0005                DU=DEL1(J+1)-DD
0006                GO TO 20
0007             10 DU=.0625-DD
0008          20    DY(I,J+1)=DD*16
0009                SLOY = 150.0 *SQRT (2.0)
0010                BY (I,J+1) = 100.0 *SQRT (2.0)*X+ 150.0*SQRT (2.0) *Y1 -325.0
0011                D1=1+DY(I,J+1)
0012                D2=D1**2
0013                D3=1-DY(I,J+1)
0014                BFY(I,J)=(1.+2.*DY(I,J+1))*BY(I,J+1)/D2-DD*SLOY/D1
0015                BOFY(I,J+1)=4.*DY(I,J+1)*BY(I,J+1)/D2+2.*DU*SLOY/D1
0016                FY(I,J-1)=DY(I,J+1)**2/D2
0017                FFY(I,J-1)=D3**2/D2
0018                RETURN
0019                END
```

```
 0001              SUBROUTINE  BOTWO  (I,J,X,X1,Y,Y1)
             C
 0002              COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                  1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                  2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                  3FFY(35,35),PP(35,35)
             C
             C
 0003              CALL            YTWO (I,J,X,Y,Y1)
 0004        10    CALL            XTWO (I,J,X,X1,Y)
 0005        20    RETURN
 0006              END
```

```
0001             SUBROUTINE  BOTHRE (I,J,X,X1,Y,Y1)
          C
          C
0002             COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                3FFY(35,35),PP(35,35)
          C
          C
0003             CALL           YTHRE (I,J,X,Y,Y1)
0004          10 CALL           XTHREE(I,J,X,X1,Y)
0005          20 RETURN
0006             END
```

```
FORTRAN IV G LEVEL  18                 YTHRE              DATE = 70278          18/32/15              PAGE 0001

 0001                 SUBROUTINE YTHRE (I,J,X,Y,Y1)
               C
               C      ***********************************************************************
               C      THIS SUBROUTINE IS USED ALONG LINE AB USING POINTS IN Y DIRECTION.
               C       REF. -ART.2..4.2
               C      ***********************************************************************
               C
 0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                     1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                     2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                     3FFY(35,35),PP(35,35)
               C
               C
 0003                 DD=Y-Y1
 0004                 IF(J.EQ.2)GO TO10
 0005                 DU=DEL1(J)-DD
 0006                 GO TO 20
 0007              10 DU=.0625-DD
 0008          20     DY(I,J)=DD*16
 0009                 SLOY = 100.*SQRT(2.)
 0010                 BY(I,J-1) = 150.*SQRT(2.)*X+100.*SQRT(2.)*Y1-325.
 0011                 D1=1.+DY(I,J)
 0012                 D2=D1**2
 0013                 D3=1.-DY(I,J)
 0014                 BFY(I,J)=(1.+2.*DY(I,J))*BY(I,J-1)/D2+DD*SLOY/D1
 0015                 BOFY(I,J-1)=4.*DY(I,J)*BY(I,J-1)/D2-2.*DU*SLOY/D1
 0016                 FY(I,J+1)=DY(I,J)**2/D2
 0017                 FFY(I,J+1)=D3**2/D2
 0018                 RETURN
 0019                 END
```

```
0001            SUBROUTINE XTHREE(I,J,X,X1,Y)
          C
          C     ***********************************************************************
          C     THIS SUBROUTINE IS USED ALONG LINE 1B&USING POINTS IN X DIRECTION.
          C      REF. -ART.2..4.2
          C     ***********************************************************************
          C
0002            COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
               1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
               2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
               3FFY(35,35),PP(35,35)
          C
          C
0003            DD=X1-X
0004            IF(I.EQ.2)GO TO 10
0005            DU=DEL(I+1)-DD
0006            GO TO 20
0007         10 DU=.0625-DD
0008      20    DX(I+1,J)=DD*16
0009            SLOX=150.*SQRT(2.)
0010            BX(I+1,J)=150.*SQRT(2.)*X1+100.*SQRT(2.)*Y-325.
0011            D1=1.+DX(I+1,J)
0012            D2=D1**2
0013            D3=1.-DX(I+1,J)
0014            BFX(I,J)=(1.+2.*DX(I+1,J))*BX(I+1,J)/D2-DD*SLOX/D1
0015            BOFX(I+1,J)=4.*DX(I+1,J)*BX(I+1,J)/D2+2.*DU*SLOX/D1
0016            FX(I-1,J)=DX(I+1,J)**2/D2
0017            FFX(I-1,J)=D3**2/D2
0018            RETURN
0019            END
```

```
 0001                SUBROUTINE BOFOUR(I,J,X,X1,Y,Y1)
            C
            C
 0002                COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                    2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                    3FFY(35,35),PP(35,35)
            C
            C
 0003                CALL            XFOUR(I,J,X,X1,Y)
 0004             10 CALL            YFOUR(I,J,X,Y,Y1)
 0005             20 RETURN
 0006                END
```

```
0001              SUBROUTINE YFOUR(I,J,X,Y,Y1)
           C
           C      ***********************************************************************
           C      THIS SUBROUTINE IS USED ALONG LINE BC USING POINTS IN Y DIRECTION.
           C       REF. -ART.2..4.2
           C      ***********************************************************************
           C
0002              COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                 1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                 2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                 3FFY(35,35),PP(35,35)
           C
           C
0003              DD=Y1-Y
0004              DU=DEL1(J+1)-DD
0005              DY(I,J+1)=DD*16
0006              SLOY=100.*Y1
0007              BY(I,J+1)=50.*X**2.+50.*Y1**2.
0008              D1=1.+DY(I,J+1)
0009              D2 = D1**2
0010              D3 = 1. - DY(I,J+1)
0011              BFY(I,J) = (1.+2.*DY(I,J+1))*BY(I,J+1)/D2-DD*SLOY/D1
0012              BOFY(I,J+1) = 4.*DY(I,J+1)*BY(I,J+1)/D2+2.*DU*SLOY/D1
0013              FY(I,J-1)=DY(I,J+1)**2/D2
0014              FFY(I,J-1)=D3**2/D2
0015              RETURN
0016              END
```

```
0001          SUBROUTINE XFOUR(I,J,X,X1,Y)
        C
        C     ************************************************************************
        C     THIS SUBROUTINE IS USED ALONG LINE BC USING POINTS IN X DIRECTION.
        C      REF. -ART.2..4.2
        C     ************************************************************************
        C
0002          COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
             1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
             2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
             3FFY(35,35),PP(35,35)
        C
        C
0003          DD=X1-X
0004          DU=DEL(I+1)-DD
0005          DX(I+1,J)=DD*16
0006          SLOX=100.*X1
0007          BX(I+1,J)=50.*X1**2.+50.*Y**2.
0008          D1 = 1+DX(I+1,J)
0009          D2 = D1**2
0010          D3 = 1-DX(I+1,J)
0011          BFX(I,J) = (1.+2.*DX(I+1,J))*BX(I+1,J)/D2-DD*SLOX/D1
0012          BOFX(I+1,J) =4.*DX(I+1,J)*BX(I+1,J)/D2+2.*DU*SLOX/D1
0013          FX(I-1,J)=DX(I+1,J)**2/D2
0014          FFX(I-1,J)=D3**2/D2
0015          RETURN
0016          END
```

```
0001                 SUBROUTINE PHI
           C
           C
           C         ******************************************************************************
           C            THIS SUBROUTINE PUTS THE VALUES OF PHI OF THE NODAL POINTS LESS
           C            THAN ONE MESH AWAY FROM THE BOUNDARY IN ORDER
           C         ******************************************************************************
           C
           C
0002                 COMMON DEL(35),DEL1(35),DX(35,35),DY(35,35),BX(35,35),BY(35,35),
                    1BFX(35,35),BFY(35,35),BOFX(35,35),BOFY(35,35),DEK(35),DEK1(35),
                    2LA(35,35),KA(35,35),P(451),FX(35,35),FFX(35,35),FY(35,35),
                    3FFY(35,35),PP(35,35)
0003                 DO 10 J = 3,12
0004                 DO 10 I = 3, 12
0005                 IF ( KA(I,J)) 20,10,20
0006         20      L1 = LA ( I + 1,J)
0007                 L2 = LA ( I , J + 1)
0008                 L3=LA(I-1,J+2)
0009                 PP(I,J)=(BFX(I,J)+BFY(I,J)+FX(I+1,J)*P(L1)+FY(I,J+1)*P(L2)
                    1)*0.5
0010                 IF (I.EQ.3.AND.J.EQ.11) GOTO 10
0011                 PP( I-1 ,J) = (BOFX(I-1,J) + BOFY (I-1,J) + FFX(I+1,J) * P(L1)
                    1+FFY(I-1,J+2)*P(L3))*.5
0012         10      CONTINUE
0013                 DO 30 J = 13,33
0014                 DO 30 I = 3,23
0015                 IF (KA(I,J)) 40,30,40
0016         40      L1 = LA (I+ 1 ,J)
0017                 L2 = LA(I,J-1)
0018                 L3=LA(I-1,J-2)
0019                 PP ( I,J ) = (BFX(I,J ) + BFY (I,J) + FX (I+1,J) *P(L1) +
                    1FY (I,J-1) * P(L2))*0.5
0020                 IF ( I. EQ.3 .AND.J. EQ.14  ) GO TO 30
0021                 PP(I-1,J   ) = ( BOFX(I-1,J    ) + BOFY(I-1,J    ) + FFX (I+1,J   )*
                    1P(L1)+FFY(I-1,J-2)*P(L3))*.5
0022         30      CONTINUE
0023                 DO 50 J = 24 ,33
0024                 DO 50 I = 24 , 33
0025                 IF (KA(I,J)) 60 , 50 , 60
0026         60      L1 = LA(I-1 , J)
0027                 L2 = LA ( I,J-1)
0028                 L3=LA(I+1,J-2)
```

```
0029              PP(I   ,   J ) = (BFX(I   ,  J   ) + BFY(I   ,  J   ) + FX(I-1,J     ) *
                 1P(L1) + FY(I,J-1   ) * P(L2)) * 0.5
0030              IF(I.EQ.33.AND.J.EQ.25)GO TO 50
0031              PP( I+1,J   ) = ( BOFX( I+1,J   ) + BOFY( I+1,J   ) + FFX ( I-1,J )*
                 1P(L1)+FFY(I+1,J-2)*P(L3))*.5
0032           50 CONTINUE
0033              DO 70 J=3,23
0034              DO 70 I=13,33
0035              IF(KA(I,J))80,70,80
0036           80 L1=LA(I-1,J)
0037              L2=LA(I,J+1)
0038              L3=LA(I+1,J+2)
0039              PP(I,J)=(BFX(I,J)+BFY(I,J)+FX(I-1,J)*P(L1)+FY(I,J+1)*P(L2))*.5
0040              IF(I.EQ.33 .AND.J.EQ.22)GO TO 70
0041              PP(I+1,J)=(BOFX(I+1,J)+BOFY(I+1,J)+FFX(I-1,J)*P(L1)+FFY(I+1,J+2)
                 1*P(L3))*.5
0042           70 CONTINUE
0043              DO 90 J=2,3
0044              DO 90 I=11,14
0045              IF(KA(I,J))100,90,100
0046          100 L2=LA(I,J+1)
0047              IF(I.EQ.11.AND.J.EQ.3)GO TO 95
0048              IF(I.EQ.14.AND.J.EQ.3)GO TO 95
0049              PP(I,J)=BFY(I,J)+FY(I,J+1)*P(L2)
0050          95  PP(I,J-1)=BOFY(I,J-1)+FFY(I,J+1)*P(L2)
0051           90 CONTINUE
0052              DO 110 J=11,14
0053              DO 110 I=2,3
0054              IF(KA(I,J))120,110,120
0055          120 L1=LA(I+1,J)
0056              IF(I.EQ.3.AND.J.EQ.11)GO TO 130
0057              IF(I.EQ.3.AND.J.EQ.14)GO TO 130
0058              PP(I,J)=BFX(I,J)+FX(I+1,J)*P(L1)
0059          130 PP(I-1,J)=BOFX(I-1,J)+FFX(I+1,J)*P(L1)
0060          110 CONTINUE
0061              DO 140 J=22,25
0062              DO 140 I=33,34
0063              IF(KA(I,J))150,140,150
0064          150 L1=LA(I-1,J)
0065              IF(I.EQ.33.AND.J.EQ.22)GO TO 160
0066              PP(I,J)=BFX(I,J)+FX(I-1,J)*P(L1)
0067          160 PP(I+1,J)=BOFX(I+1,J)+FFX(I-1,J)*P(L1)
0068          140 CONTINUE
```

```
          DO 170 J=33,34
          DO 170 I=22,25
          IF(KA(I,J))180,170,180
      180 L2=LA(I,J-1)
          IF(I.EQ.22.AND.J.EQ.33)GO TO 190
          IF(I.EQ.25.AND.J.EQ.33)GO TO 190
          PP(I,J)=BFY(I,J)+FY(I,J-1)*P(L2)
      190 PP(I,J+1)=BOFY(I,J+1)+FFY(I,J-1)*P(L2)
      170 CONTINUE
          RETURN
          END
```

```
              C
              C
              C       ****************************************************************************
              C       THIS PROGRAM CALCULATES THE PRINCIPAL STRESSES AND SHEAR STRESSES
              C       FROM THE EXPERIMENTAL DATA,USING SHEAR DIFFERENCE METHOD.
              C       FOR THEORY REF ART. 4.2
              C       ****************************************************************************
              C
              C
              C
              C       ****************************************************************************
              C       THE VARIABLES ARE DEFINED AS FOLLOWS.
              C       P              APPLIED TENSILE LOAD.
              C       NL             NUMBER OF READINGS.
              C       X              DISTANCE FROM THE FREE BOUNDARY.
              C       B              WIDTH OF THE SPECIMEN.
              C       H              THICKNESS OF THE SPECIMEN.
              C       DL             DISTANCE OF THE INTERFACE FROM THE FREE BOUNDARY.
              C       DL=0.75                                        EXPT.-- 1-6
              C       DL=1.-0.75/SQRT(2.)                            EXPT.--7
              C       DL=1.-(3./8. +0.25)/SQRT(2.)                   EXPT--8
              C       DL=1.-0.5 /SQRT(2.)                            EXPT.--9
              C       DL=1.-0.25/SQRT(2.)                            EXPT--10
              C       DL=1.-(3./8. -0.25)/SQRT(2.)                   EXPT.--11
              C       DL=1.                                          EXPT--12
              C       REFER.  FIG. -22
              C       AN,ANN         FRINGE ORDER ALONG LINE AB.
              C       AN1,ANN1       FRINGE ORDER ALONG LINE CD.
              C       AN2,ANN2       FRINGE ORDER ALONG LINE EF.
              C       C              STRESS OPTIC COEFFICIENT.
              C       DELY           DISTANCE BETWEEN LINE CD AND EF.
              C       ****************************************************************************
              C
              C
0001                  DIMENSION PXX(30),PYY(30),TAB(30),TCD(30),TEF(30),P1(30),P2(30),
                     1DIST(30)
0002                  WRITE(3,160)
0003                  DL=1.
0004                  P=125.
0005                  NL=19
0006                  AN=2.-27./180.
0007                  AN1=2.-30./180.
0008                  AN2=2.-26./180.
0009                  X=0.0
```

```
      B1=2.-2.*DL
      DIST(1)=0.0
      B=2.
      H=0.254
      DELY=.05
      A=(B-B1)*H
      A1=B*H
      PNET=P/A
      PNET1=P/A1
      PI=3.1415927
      H1=2.*H
      THEATA=0.0
      THETA1=0.0
      THETA2=0.0
      THETA1=THETA1*PI/180.
      THETA2=THETA2*PI/180.
      C=36.
      PXX(1)=0.0
      PYY(1)=(AN*C)/H
      TAB(1)=(AN1*C*SIN(2.*THETA1))/H1
      TCD(1)=(AN2*C*SIN(2.*THETA2))/H1
      TEF(1)=0.0
      CONFAC=PYY(1)/PNET
      CONFA1 =PYY(1)/PNET1
      WRITE(3,150)PNET,PNET1,PXX(1),PYY(1),TAB(1),TCD(1),CONFAC,CONFA1,X
      I=1
      WRITE(3,400)
  200 READ(1,100)DELX,AN1,ANN1,THETA1,AN,ANN,THETA,AN2,ANN2,THETA2
      X=X+DELX
      I=I+1
      DIST(I)=X/DL
      I1=I-1
      THETA=THETA*PI/180.
      THETA1=THETA1*PI/180.
      THETA2=THETA2*PI/180.
      TAB(I)=((AN1+ANN1/180.)*C*SIN(2.*THETA1))/H1
      TCD(I)=((AN2+ANN2/180.)*C*SIN(2.*THETA2))/H1
      TEF(I)=((AN+ANN/180.)*C*SIN(2.*THETA))/H1
      TABX=(TAB(I)+TAB(I1))/2.
      TCDX=(TCD(I)+TCD(I1))/2.
      DETXY=TABX-TCDX
      PXX(I)=PXX(I1)-(DETXY*DELX/DELY)
      PYY(I)=PXX(I)+((AN+ANN/180.)*C*COS(2.*THETA))/H
```

```
0053            AB=PXX(I)+PYY(I)
0054            CD=((AN+ANN/180.)*C)/H
0055            P1(I)=(AB+CD)/2.
0056            P2(I)=(AB-CD)/2.
0057            CONFAC=P1(I)/PNET
0058            CONFA1 =P1(I)/PNET1
0059            WRITE(3,101)I,CONFA1,          P1(I),P2(I),PXX(I),PYY(I),CONFAC,CD,X
0060            IF(I-NL)200,300,300
0061        300 FAC=PNET1/100.
0062            WRITE(3,45)
0063            P1(1)=PYY(1)/FAC
0064            P2(1)=PXX(1)/FAC
0065            DO 70 I=2,NL
0066            P1(I)=P1(I)/FAC
0067            P2(I)=P2(I)/FAC
0068         70 CONTINUE
0069            WRITE(3,46)((I,DIST(I),P1(I),P2(I),TEF(I)),I=1,NL)
0070        160 FORMAT(1H1,10X,'PNET',10X,'PNET1',10X,'PXX(1)',10X,'PYY(1)',8X,'TA
               1B(1)',10X,'TCD(1)',8X,'CONFAC',8X,'CONFA1 ',8X,'X')
0071        150 FORMAT(/,1X,8F15.4,F9.3)
0072        400 FORMAT(1H1,2X,'I',12X,'CONFAC1',              9X,'P1(I)',9X,'P2(I)',
               19X,'PXX(I)', 9X,'PYY(I)', 9X,'CONFAC',9X,'P1-P2',5X,'X')
0073        100 FORMAT(10F8.4)
0074        101 FORMAT(/,1X,I2,8(F13.4,2X))
0075         45 FORMAT(1H1,12X,'I',15X,'DIST(I)',22X,'P1(I)',19X,'P2(I)',16X,'
               1SHEAR STRESS')
0076         46 FORMAT(//,10X,I3,15X,F10.6,15X,F10.2,15X,F10.2,15X,F10.2)
0077            RETURN
0078            END
```

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