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ESSAYS ON STOCHASTIC EXCHANGE RATE REGIME SWITCHING

A Thesis
Presented to the School of Graduate Studies and Research
of the University of Ottawa

In Partial Fulfilment of the Requirements for the Degree
of Doctor of Philosophy (Economics)

by
Patrick Georges

July, 1995



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À mes parents.

ABSTRACT

The objective of this thesis is to study the operation of exchange rate regimes in the context of possible regime switches. An intuitive survey of the regime switching literature is given in Chapter 1, and three independent essays are presented in Chapters 2, 3, and 4.

Using the theory of regulated Brownian motion, Chapter 2 derives the nonlinear relationship between the nominal exchange rate and its fundamentals during the transition period from the current free-float to a specified target zone (TZ), triggered at an announced state-dependent switch. It is shown that the derived nonlinear relationship is in general different from the relationship corresponding to a return to a fixed exchange rate regime. This is due to a "reflecting effect", the mirror image of the well known "honeymoon effect" of a target zone, and a "bandwidth effect". Also derived is a locus of "benchmark cases" demarcating the factors that lead to an immediate appreciation or depreciation of the domestic currency at the announcement of the future TZ.

The target zone model developed in Chapter 3 incorporates the possibility of a future change in the trend in the fundamentals of the exchange rate as policy reaction to specific events (e.g., impending speculative attacks and nominal anchor debates). The market has subjective expectations about this possible trend revision, which can affect the exchange rate level even if the change in trend is not implemented. These expectations are treated, first, as entirely exogenous and, second, as state-varying. In both cases various correlation patterns between the exchange rate and interest rate differentials are possible. This result is consistent with the observed behaviour of the exchange rate and interest rate differentials within the European Monetary System.

Chapter 4 studies an announced (state-dependent) regime shift from a free-floating to a permanently fixed exchange rate regime and introduces specific welfare considerations. The welfare issues introduced here try to explain why a specific exchange rate target level would be chosen instead of another one, if a return to a fixed exchange rate were on the public policy agenda and a given range for its pegged value had already been proposed. While this model does not provide a theory of choice between free-float and fixed rate regimes, it proposes a criterion to choose between fixed exchange rate regimes, taking into account the transition period from the free-float to the implementation of the fixed regime.

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0 INTRODUCTION

The major objective of this thesis is to study the operation of exchange rate regimes in the context of possible regime switches. To understand how an exchange rate regime works, one must also understand how it ends. This basic idea was fundamental to the development of the theory of exchange rate regime switching. The "switch" of regime, defined here very generally as the transformation of one particular regime to another, could be market-driven, as is the case with speculative attacks on fixed exchange rate regimes. The switching could also reflect an inherent characteristic of an exchange rate arrangement, as is the case in a target zone (TZ) regime (transitory regime switching from a free-float to a fixed rate and vice versa). Finally, the switch could be the result of a decision by the monetary authorities to switch permanently to another regime (permanent regime switching).

In the speculative attack literature [e.g., Krugman (1979) and Flood and Garber (1984)], the central problem lies in connecting the fixed rate regime to the post-collapse floating regime. In these models the fixed rate is permanently abandoned for a new free-float regime. Often, however, a central bank faced with a speculative attack withdraws from the foreign exchange market temporarily and repeg the rate at a new higher level after a short period of transitional free-float. This realignment and devaluation theory [e.g., Obstfeld (1984) and Miller and Weller (1989)], tries to connect a fixed rate regime to a possible post-collapse devaluated exchange rate. Here, there is no permanent abandoning of the fixed exchange rate regime.

In a target zone [e.g., Krugman (1991, 1992), Froot and Obstfeld (1991b)], the problem is to connect a flexible exchange rate with future attempts to peg the exchange rate if a particular state

of the economy were to be reached (i.e., the lower or upper bound of a TZ band). This particular intervention-triggering state is called a **reflecting** barrier, which means that, as soon as the exchange rate moves back inside the zone, the regime becomes a free-float again.

Finally, the permanent regime switching literature [e.g., Flood and Garber (1983), Froot and Obstfeld (1991a)] explores the effects, on the exchange rate, of prospective regime changes at an announced state or time. It could be for example, a planned return from flexible to fixed rates at a particular state or date. But in this case, the new fixed exchange rate regime would be perceived as a **permanent** regime **whatever** the future state of the economy. In contrast to the TZ regime, the permanent regime switching literature refers to the concept of **absorbing** barriers which means that, once the new regime is implemented, no return to the former regime is allowed.

The basic idea of trying to study how a particular exchange rate regime ends, was important in generating this field of research. But the present enthusiasm is probably more due to the introduction of a new analytical approach to tackle this research question, in particular the use of stochastic calculus and the introduction of **regulated** (or controlled) Brownian motions. A complete treatment of this stochastic process is given in Harrison (1985). A less advanced presentation is in Dixit (1991). Note that the use of (unregulated) Brownian motion and stochastic calculus has been intensively used in financial economics [e.g., Merton (1969, 1971), Fischer (1975) and in the work of Black and Scholes (1973) on option pricing]. But the techniques of **regulated** Brownian motion are more recent and were explicitly introduced in the exchange rate target zone literature by Froot and Obstfeld (1991a, 1991b).

The outline of the thesis is as follows. Chapter 1 presents an intuitive survey of some

canonical models of exchange rate regime switching, essentially models from the target zone literature but also from the permanent regime switching literature. The emphasis is deliberately put on the target zone model for its role in the renewed interest for the stochastic regime switching theory. Technical details of these models are given in Appendixes A1 to A10. Chapters 2, 3 and 4 are three independent essays that represent the contribution of this thesis to the existing research.

In Chapter 2, a model of state-dependent target-zone entry is built. Using the theory of regulated Brownian motion, this chapter derives the nonlinear relationship between the nominal exchange rate and its "fundamentals" during the transition period from the current free-float to a specified TZ, triggered at an announced state-dependent switch. It is shown that the derived nonlinear relationship is in general different from the relationship corresponding to a return to a fixed exchange rate regime, conjectured by Flood and Garber (1983), and derived, with different methods, by Smith (1991) and Froot and Obstfeld (1991a). This difference is due to a "reflecting effect", the mirror image of the well known "honeymoon effect" of a target zone, and a "bandwidth effect", both absent in the fixed-rate entry problem.

The second objective of Chapter 2 is to understand the determinants of the immediate jump in the exchange rate due to the announcement of a major exchange rate regime switch to a TZ. Indeed, the announcement provides information on the future monetary policy relative to the present stance. As should be expected with an asset theory of exchange rate determination, this new information implies an immediate jump in the exchange rate due to the market expectation of an important policy shift. A locus of "benchmark cases" demarcating the factors that lead to an immediate appreciation or depreciation of the domestic currency at the announcement of the

future TZ is also derived.

Chapter 3 presents a TZ model that incorporates the possibility of a future change in the trend in the fundamentals of the exchange rate as monetary policy may be altered in reaction to specific events (e.g., impending speculative attacks and nominal anchor debates). One rationale for including the possibility of a trend revision into a TZ model is as follows. As is well known from Krugman and Rotemberg (1992) and Delgado and Dumas (1993), a regime of TZ with trend is particularly sensitive to speculative attacks. If the TZ regime is viewed as essential for the domestic country (that experiences a depreciating trend), speculative attack rumours could introduce sufficient incentives for the domestic central bank to revise its current positive growth rate of the money supply to a constant money supply (i.e., revising its current positive trend in the fundamentals to a zero trend). In other words, instead of accepting the forced return to a free-float with the current positive trend, the domestic country could decide to revise its depreciating trend in order to prevent the speculative attack.

A second reason for including a possible trend revision in a TZ model is as follows. A revision from the current zero trend to a positive trend in the fundamentals could be rationalized by the situation of a country facing domestic pressure to return to a more expansionary monetary policy because of some internal problems (e.g., unemployment). As recently suggested by Ball (1994), monetary expansion, in general, does not mean a rise in the **level** of the money supply (i.e., a one-time shock to the level), but an increase in money **growth rate** (trend). In the TZ regime, such a monetary expansion could be viewed as a change from a zero trend to a positive trend in the fundamentals, and would possibly be observed during periods where the monetary policy of the leader country (the nominal anchor country) seems inappropriate for other countries

of the zone.

The market has subjective expectations about these possible trend revisions, which can affect the exchange rate level even if the change in trend is not implemented. These expectations are treated, first, as entirely exogenous and, second, as state-varying. In both cases, various correlation patterns between the exchange rate and interest rate differentials are possible. This result is consistent with the observed behaviour of the exchange rate and interest rate differentials within the European Monetary System.

Chapter 4 studies an announced (state-dependent) regime shift from a free-floating to a permanently fixed exchange rate regime. Regime switches from free-float to permanent fixed rate have been analyzed by Flood and Garber (1983), Smith and Smith (1990), Smith (1991), Froot and Obstfeld (1991a), and Miller and Sutherland (1992). These models describe a state-dependent switch (i.e., a switch that occurs when "the conditions are right"). Ichikawa, Miller, and Sutherland (1992) consider a regime switch from a free-float to a target zone based on a time-dependent switch (i.e., irrespective of the state of the market at the time of the switch).

Chapter 4 introduces specific welfare considerations in the model of Froot and Obstfeld (1991a). The welfare issues introduced here try to explain the choice of a specific exchange rate target level if a return to a fixed exchange rate were on the public policy agenda and a given **range** for its pegged value had already been proposed. While this model does not provide a theory of choice between free-float and fixed rate regimes, it proposes a criterion to choose between fixed exchange rate regimes, taking into account the transitory period from the free-float to the implementation of the fixed exchange rate.

Chapters 2, 3, and 4 are written as independent essays. In consequence, some repetition may

exist, but has been kept to a minimum.

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1 STOCHASTIC REGIME SHIFTS: A SELECTIVE SURVEY OF THE LITERATURE

1.0 Introduction to the target zone theory

This chapter surveys some models of exchange rate regime shifts. The target zone model of Krugman (1991) and some well-known extensions are presented in sections 1.1 to 1.6. The TZ model may be understood as a transitory regime shift from free-floating to a fixed exchange rate and vice versa. The notion of permanent regime shift is introduced in section 1.7. The objectives of section 1.7 are, first, to contrast the notions of permanent relative to transitory regime switching but, also, to underline the basic similar framework of analysis between both types of regime switching. Indeed, the renewed interest for the theory of permanent regime switching is probably due to the framework of analysis developed in the target zone theory.

A target zone differs from a pure fixed exchange rate regime in allowing some range of variation of the exchange rate around a reference rate (i.e., a central parity). A TZ can then be understood as a (nonlinear) compromise between a fixed exchange rate and a freely flexible exchange rate regime. With respect to a TZ, two main questions can be raised, one positive and one normative.

The positive question is about the dynamics of exchange rates, interest rates and central bank interventions within the exchange rate bounds. A naive view would suppose that the exchange rate behaves as if the regime were one of free floating until the rate hits the edge of the band, whereupon the regime switches to a fixed rate. Krugman (1991) has accurately shown that this cannot be the case. The existence of bounds constrains possible future paths of the exchange rate;

exchange markets, knowing this, should behave differently than they would if there were no bounds. In other words, the existence of a band of variation for the exchange rate should affect exchange rate behaviour even when the exchange rate is inside the band and the zone is not actively being defended. The Krugman model has clear testable implications for exchange rates and interest rate differentials. These implications have been tested, in particular, on data from the European Monetary System (E.M.S.). Those tests have consistently rejected the model [see Flood, Rose, and Mathieson (1991) and Lindberg and Söderlind (1994)]. Nevertheless, different theoretical extensions of the basic model have been proposed that can potentially resolve the empirical difficulties.

The basic TZ model has been extended in five different directions by relaxing several key assumptions.

- 1) The basic underlying model of exchange rate determination in the TZ literature is the flexible-price monetary model. This model presumes a constant real exchange rate. But, as mentioned by Miller and Weller (1991a, 1991b), such a model is not able to take into account the sharp distinction [as underlined by Williamson (1985)] between a real TZ and the nominal TZ of Krugman. Hence, Miller and Weller use the Dornbush sticky-price exchange-rate overshooting model as the basis for their model of the TZ.
- 2) In the original TZ model, the interventions are infinitesimal (very small) and marginal (i.e., at the edges of the band). In the terminology of Harrison (1985), we have to solve an **instantaneous** reflecting barrier control problem. Possibilities of finite (discrete) interventions at the edges of the band have been introduced by Flood and Garber (1991, 1992). This is an **impulse** or **discrete** reflecting barrier control problem. Intra-marginal

interventions (i.e., interventions inside the band) have been introduced by Delgado and Dumas (1992). These intra-marginal interventions can be considered as of the "leaning against the wind" type, which is an important characteristic of central bank intervention behaviour in the E.M.S.

- 3) Other important characteristics of the E.M.S. are the frequent realignments of the central parities, and the existence of devaluation risk. These characteristics, together with the role of realignment expectations and other credibility problems, not considered in the original TZ model, have been analyzed in two papers by Bertola and Caballero (1992a, 1992b), and also by Svensson (1991), Bertola and Svensson (1993), and Tristani (1994). The inherent exchange rate stability of the standard (Krugman) TZ appears weaker in these models and may, indeed, be reversed (instability) depending on the expected probability of realignments.
- 4) The E.M.S. events during Summer of 1993 should remind us that TZ are also subject to speculative attacks. This topic, in the specific framework of a TZ model, has been studied by Krugman and Rotemberg (1992) and by Delgado and Dumas (1993).
- 5) It is not easy to justify why a TZ should be introduced. Following Krugman and Miller (1993), this is largely due to the way the exchange rate market is assumed to behave in a free-float regime (i.e., efficient market assumption). But, as shown by these authors, introducing new tendencies like taking into account inefficiencies and irrational runs (e.g., stop-loss strategies) in the exchange rate market, provides a stronger justification for a TZ.

These extensions provide a better understanding of the mechanism of a TZ and, hopefully, better empirical results [see Svensson (1992)]. What is left, nevertheless, is the normative aspect

of a TZ. The normative question considers the optimality of such a regime relative to other theoretical regimes of exchange rates. Also, if such a TZ is to be implemented what is the optimal bandwidth? Curiously enough, nothing has been done in this field of research on basis of the Krugman model. This point is also stressed in Svensson's survey (1992) of the TZ literature. To tackle this problem, the theory of optimal regulation of Brownian motion should be used. This stochastic optimal control theory has been considerably simplified by Dixit (1991) and makes the search for an optimal TZ bandwidth a promising field of research. Nevertheless, we have to be realistic relative to any welfare analysis of this topic: as underlined by Krugman (1992), welfare analysis is a general failure of international monetary economics. [Exceptions being Helpman and Razin (1979, 1982) and Helpman (1981).] Target zones are an hybrid combination of fixed and floating rates and, even if different schemes to limit the exchange rate variation have new respectability, it is still not known if a TZ combines the best or the worst of both regimes.

The basic Krugman target zone model (KTZ) examines the dynamic of the exchange rate in a TZ with defined bounds on the **exchange rate**. But, as often with new concepts, a re-interpretation of the basic idea of Krugman gave rise to a second target zone model where bands were defined on the "**fundamentals**" of the exchange rate rather than on the exchange rate itself. This model can be referred to as the bounded fundamentals target zone (FTZ), pioneered by Flood and Garber (1991) and Froot and Obstfeld (1991b). This re-interpretation has led to some confusion, essentially because a majority of FTZ authors claim an essentially identical approach with the KTZ model, whereas Krugman and Rotemberg (1992) disagree. Actually, the major difference between these approaches lies in the interpretation of boundary conditions necessary

to solve the TZ problem.

Given this brief outline of the target zone literature, some specific topics are more thoroughly discussed in subsequent sections. In section 1.1, the canonical model of a target zone, as presented by Krugman (1991), is described in detail. Sections 1.2, 1.3, 1.4 and 1.5 provide an intuitive presentation of one representative paper for extensions 2, 3, 4 and 5 as given above. Section 1.6 reviews the testable implications of the Krugman model and the fact that those predictions have not been verified with data observations on the E.M.S. The notion of a permanent regime switching is introduced in section 1.7 and, finally, section 1.8 concludes.

1.1 The canonical model of a target zone [Krugman (1991)]

The first necessary step to model a TZ is to start from a theory of exchange rate determination. In the short term the level of exchange rate is usually explained using a general asset market model of exchange rate determination that can be written in logarithmic form as:

$$(1.1) \quad s(t) = f(t) + \theta E_t[ds(t)|I(t)]/dt$$

where $s(t)$ is the log of the spot exchange rate at time t (i.e., the domestic currency price of the foreign currency), $E_t[ds(t)|I(t)]/dt$ is the expected rate of depreciation conditional upon the information set available at time t , and $f(t)$ represents other contemporaneous determinants --the fundamentals of the exchange rate.

The asset market view of exchange rate determination is essentially captured by the positive discount factor θ . The exchange rate is an asset price; thus, like other asset prices its current value depends on the expected value in the future. This is in the nature of assets that do not

deplete: since the asset will still exist in the future, and therefore will have a price in the future, this price must affect today's price.¹

This general formulation, in which the first right hand side term in equation (1.1) can be thought of as fundamental component and the second term as speculative factors, has been used by Mussa (1976), Frenkel and Mussa (1980), and Frankel (1985) among others.

Monetary and portfolio-balance models have gone one step further by specifying the economic structure underlying $f(t)$. With monetary models, the exchange rate is viewed as the relative price of the domestic and foreign currencies. Portfolio-balance models explain the exchange rate as the relative price of domestic and foreign assets, normally government bonds and claims on real capital, that are imperfect substitutes. An interpretation of equation (1.1) in terms of the flexible price monetary model is given in Appendix A2.1. Depending on which model (monetary or portfolio-balance models) is used, f , θ and $E_t[ds(t)|I(t)]/dt$ will take different values, but the key point behind equation (1.1), common throughout the asset market theory of exchange rates, is that an increase in the expected rate of future depreciation of the domestic currency will reduce the demand for this currency today, and therefore will cause it to depreciate today.²

With this very basic theory of exchange rate determination in mind it is now possible to explain the target zone model of Krugman (1991). The basic objective of the Krugman model is to demonstrate that a credible target zone will stabilize the exchange rate relative to its fundamentals. This stabilization effect inside the target zone is due to the announced central bank intervention at the edges of the band to prevent the exchange rate from going outside the TZ. This nominal anchoring (of the money supply) stabilizes the market expectations of future changes in the exchange rates and thus the exchange rate itself relative to its fundamentals.

The starting point of Krugman model is equation (1.1) with the flexible price monetary interpretation. As a consequence, f is given by $f = m + v$; m is a log-measure of the domestic money supply and v is a shift term representing velocity shocks. [The log-measure of the foreign money supply is set constant at 0.] Dropping the time index, equation (1.1) can be rewritten as:

$$(1.1') \quad s = m + v + \theta E[ds|I]/dt.$$

Krugman assumes that v follows a continuous-time random walk without drift (i.e., a Brownian motion).³ This process for v is known to the market; it is included in the information set, I .

Krugman specifies two particular exchange rate regimes: a free-float regime and a target zone. The assumptions for the free-float are that v follows a continuous random walk and m is kept constant.⁴ This means that when v increases or decreases, through equation (1.1'), s will increase (a depreciation of the domestic currency) or decrease (an appreciation). Now, if v follows a random walk without drift and m remains fixed, the market cannot rationally expect any change from the current exchange rate level and, in this case, $E[ds|I]/dt$ is equal to 0. Thus, the path of the exchange rate (or exchange rate saddle-path) in this free-float regime is given by:

$$(1.2) \quad s = m + v = f$$

where $E[ds|I]/dt$ is set equal to 0 in equation (1.1').⁵ Because m is kept constant and v follows a random walk, the exchange rate in a free-float regime is also a random walk. This matches the empirical observations that free-float exchange rates seem to behave like random walks [Meese and Rogoff (1983)]. The free-float path is drawn, in figure 1.1, in (v,s) space.

<figure 1.1 here>

As we can notice from figure 1.1, the exchange rate walks randomly along a 45-degree line. The relevant 45-degree line depends on the chosen money supply level. The vertical intercept indicates the level at which (the log of) the money supply measure has been kept constant. A lower level for the chosen money supply would imply that the exchange rate would walk along a lower 45-degree line.⁶

Krugman compares this very basic free-float model with the functioning of a credible target-zone regime. The shift term v is still assumed to follow a random walk without drift. In addition, a band of variation for the exchange rate, $[s_l, s_u]$, is announced to the market. s_l is the lower bound (the strong bound for the domestic currency), whereas s_u is the upper (weak) bound. To enforce this band of variation, it is also announced that the money supply will be kept constant for $s_l < s < s_u$, but will be reduced (unsterilised intervention) when s hits s_u , to prevent the domestic currency from depreciating furthermore. The money supply will be increased when s hits s_l to prevent the domestic currency from appreciating.

A naive (false) view of such a regime is described in figure 1.2.

<figure 1.2 here>

In figure 1.2, suppose that the domestic currency depreciates from point 1 to point 2. In the next instant, in a free-float regime, the market could be either at point 1 or at point 3, depending on the change in v . In a target zone regime, however, the exchange rate will be prevented from reaching point 3 because if, from point 2, v continues to increase, the money supply will be reduced correspondingly. Instead of point 3, the market would be at point 3' on a lower 45-

degree line (to take into account the fact that the money supply has been reduced).

This naive view would suggest that the exchange rate dynamics in a target zone regime would be a **linear** combination of the free-float dynamic inside a target zone (on a 45-degree line), and of a fixed exchange rate rule at the edges of the band of variation. If this view were correct, a target zone would not stabilize the exchange rate relatively to its fundamentals, even if the exchange rate were prevented from leaving the band. Krugman's main point is that the actual path of the exchange rate inside a target zone would be given by the S-shaped curve drawn in figure 1.2.

To understand this particular path, recall equation (1.1') and note that setting the money supply arbitrarily equal to $m = 0$, obtains:

$$(1.3) \quad s = v + \theta E[ds|I]/dt.$$

In figure 1.3, different paths have been drawn.

<figure 1.3 here>

Along the 45-degree line (labelled $s=v$) s is set equal to v , which implies by equation (1.3) that $E[ds]/dt$ is constant and equal to 0. The market does not expect any increase or decrease in the level of the exchange rate. s is always larger than v along the convex path labelled $s>v$; this indicates that the market is expecting a depreciation of the domestic currency: $E[ds]/dt > 0$. Note also that along this path the expected rate of depreciation is not constant. Actually, as v and s increase, the expected rate of depreciation increases. The paths labelled $s<v$ and S-S are concave (in the north-east quadrant) and drawn with s smaller than v . Thus, these two paths incorporate

an expected rate of appreciation of the domestic currency: $E[ds]/dt < 0$. Once more, as v and s increase, the corresponding expected rate of appreciation increases (in absolute value).

Now, suppose that no band has been specified. In terms of figure 1.3, this means that the horizontal lines at s_l and s_u can be ignored. This situation represents a pure free-float and the path of the exchange rate is the 45-degree line labelled $s=v$. Starting from point 2, in the next instant point 1 or point 3 could be reached. Because the exchange rate has the same probability of increasing or decreasing, the expected rate of depreciation is 0.

Now, introduce the bounds at s_l and s_u and the intervention rules as explained before. In this case, once point 2 is reached, the exchange rate can only stay at this level (s_u) or decrease (if point 1 is reached at the next instant). Indeed, there is no chance for the exchange rate to increase because of the intervention rule. In consequence, the likely decrease in s implies a market expectation of a future appreciation of the domestic currency. But point 2, located on the 45-degree line, is associated with $E[ds]/dt = 0$ and thus cannot be an equilibrium point in the TZ model and so, path $s=v$ cannot be the equilibrium path of the TZ. Point 2', on path labelled $s < v$, seems a more appropriate candidate because an expected rate of appreciation is associated with this point. Nevertheless, path $s < v$ does not seem to be consistent with an equilibrium path at point 4. Basically this path expresses the idea that if point 4 is reached, the market expects next period to be at point 2' or at point 5. But with the upper bound, point 5 is not attainable. The expected rate of appreciation associated with point 4 is too low in absolute value, such that point 4 is not an equilibrium and $s < v$ is not an equilibrium path for the TZ.

In conclusion, it is clear that the equilibrium path should not cross the upper bound. Thus, we are left with only one possible path, the one just tangent to the upper bound (i.e., path S-S). The

same kind of reasoning in the lower part of the zone gives finally the S-shaped equilibrium path (or saddle-path) of the target zone, as drawn in figure 1.3. This path is tangent to the upper and lower bounds of the zone. The tangency condition has been named "smooth-pasting" condition in reference to an analogous result in option pricing theory. (Actually, as mentioned in Appendix A5.2, there is an option pricing interpretation of a TZ exchange rate level.) The S-curve describing the path of the exchange rate inside the TZ shows clearly the difference from a free-float (45-degree line) path. Even if there is no intervention inside the zone [m is kept constant for $s_l < s < s_u$], the expectation of future intervention must be taken into account to understand the path of the exchange rate inside the zone.

To analyze the money supply behaviour inside such a model, consider figure 1.4.

<figure 1.4 here>

In figure 1.4, the central bank has chosen a constant money supply set at the standardized level $m = 0$. Successive positive shocks of equal magnitude push the market from point 1 to points 2 and 3 on S_l - S_l . At point 3, the tangency point with the upper edge of the band is reached. If v continues to increase, the market knows that the central bank has to intervene to prevent s from increasing: m has to decrease. Remember that when m decreases, the 45-degree line of the free-float shifts downwards and to the right by the decrease of the money supply. The associated S-shaped curve, with this lower level of money supply but same exchange rate band, is given by S_2 - S_2 in figure 1.4. Now, to prevent s from increasing, the reduction in the money supply has to be equal to the increase in v . This guarantees to be exactly at point 4, new point of tangency

with the upper bound. If v continues to increase, an equal decrease in m will drive the market to point 5.

Suppose now that, from point 5, v starts to decrease. Do we go back to point 4? The answer is a definite no. Remember that from point 4 to point 5, the money supply was decreased to prevent the domestic currency to depreciate even more. Then, to go from point 5 to point 4 when v decreases implies that the central bank has increased its money supply. But at point 5, the decrease in v generates an automatic appreciation of the domestic currency that moves the domestic currency inside the band. The domestic central bank certainly does not want to prevent such a move by increasing the money supply. Actually, when v decreases from point 5, the central bank keeps the level of money supply constant and the market goes to point 6 on S_3 - S_3 .

Now, it is interesting to observe that, from point 1 to points 2 and 3, m is kept constant and v increases, such that the fundamentals, $f = m + v$, increase. From point 3 to points 4 and 5, m decreases by the same amount as the increase in v , such that f stays constant. Finally, from point 5 to point 6, v decreases and m is kept constant, which implies a decrease in f . This suggests that a graph in (f, s) space could be useful.

By equation (1.2) we know that the free-float path is $s = f = m + v$. This path is drawn in figure 1.5 as the F - F path.

<figure 1.5 here>

Note the difference between the unique free-float path in (f, s) space whatever the chosen level of money supply, and the free-float paths in (v, s) space of figure 1.1, depending on the chosen

level of m . In the same way, there is a unique S-shaped curve in (f,s) space, in contrast to the numerous (money supply dependent) target zone paths represented in figure 1.4. Observe points 1 to 6 in figure 1.5, which correspond to the same points in figure 1.4: from 1 to 3, f increases; from 3 to 5, f stays constant; and from 5 to 6, f decreases.

Figure 1.5 suggests two interesting results of Krugman model:

- a) To set an explicit band on the exchange rate, $[s_l, s_u]$, implies an implicit band on its fundamentals, $[f_l, f_u]$. This implies also that the upper and lower edges of the band on the fundamentals can be understood as **reflecting** barriers with **instantaneous** controls: the money supply is changed **just enough** to prevent f and thus s from going outside their respective bands: this is called an infinitesimal intervention.
- b) The S-shaped curve has everywhere a slope less than 1. This implies that the exchange rate moves less than proportionally to its fundamentals and, as a consequence, the band on the exchange rate, $[s_l, s_u]$, is smaller than the band on the fundamentals, $[f_l, f_u]$. This result is Krugman's main point: a credible target zone on the exchange rate will stabilize the exchange rate, not only in the obvious sense that the exchange rate is kept inside a specified zone, but more fundamentally, that, inside this zone, the exchange rate moves less than proportionally to its fundamentals. This effect has been dubbed the "honeymoon effect" of a target zone.

1.2 Discrete intervention in the target zone model [Flood and Garber (1991)]

Flood and Garber (1991) provide an interpretation of the smooth-pasting (tangency) condition

derived by Krugman (1991) and introduce discrete interventions in a TZ. .

In the Flood and Garber model, the central bank tries to defend an explicit band on the fundamental, $[f_l, f_u]$. In contrast to the Krugman's (implicit) assumption of infinitesimal interventions, the defence of the band is implemented, here, by **discrete** interventions. In other words, Flood and Garber suppose that the monetary authorities announce a discrete intervention rule which specifies: 1) the lower and upper limits of the fundamentals (f_l and f_u , respectively) at which intervention will occur; and 2) the magnitude of intervention at each bound: ($\Delta m = \pm I$). That means that if the fundamentals reach f_u (or f_l), the intervention adjusts m instantaneously such that f jumps to $f_u - I$ (or $f_l + I$). This intervention is fully anticipated and understood as credible by the private sector.

Note the shift of concept from Krugman's model to Flood and Garber's model. In Krugman (1991), there is an explicit exchange rate band giving rise to an implicit fundamental band. This latter band turns out to be a reflecting barrier with **instantaneous** control, meaning that the money supply is adjusted slightly to prevent the fundamentals from going out of its band. Flood and Garber (1991) start with an explicit band on the fundamentals. This fundamental band is a reflecting barrier with **impulse** or **discrete** control, which means that the money supply is adjusted to move the fundamentals well **inside** their band of variation.

Figure 1.6 gives an intuitive interpretation of their analysis. (See Appendix A6 for a more technical explanation of their approach.)

<figure 1.6 here>

In figure 1.6, a family of exchange rate paths S , S' , and S'' , is drawn. At the same time a band on the fundamentals is superimposed: the vertical lines at $f = f_l$ and $f = f_u$. When the fundamentals hit f_l or f_u , a discrete intervention has to be implemented. For example, consider two possible discrete interventions of magnitude I' and I'' ; if f hits f_u , the central bank implements a discrete intervention such that the fundamentals are driven respectively to $f_u - I'$ or to $f_u - I''$. (Of course, the central bank announces one policy or the other, but not both!) Note that for the time being, the horizontal lines can be ignored.

Now suppose that the discrete intervention I'' is announced initially. The problem is to determine which member of the family of paths (S , S' , and S'') is the saddle-path solution (i.e., the relevant target zone path). To solve this problem, Flood and Garber propose an arbitrage condition analogous to the familiar requirement of no-expected arbitrage profits used in the speculative attack literature of the fixed exchange rate system. In a fixed exchange rate system, speculators, anticipating the possibility of the exchange rate's moving by a finite amount at the instant of intervention, compete away the arbitrage profits and ensure that the attack is of precisely the size such that there is no anticipated exchange rate jump at the time of the attack. In the TZ regime, the speculators know the size of the finite intervention (I'') and select the exchange rate path such that they anticipate that the jump in f , due to the intervention, should not generate any change in the exchange rate level.⁷

Basically, in figure 1.6, when the fundamentals hit f_u (at point 1, 1', or 1''), the market knows that the central bank will implement a discrete intervention such that, just after the intervention, the fundamentals are driven to $f_u - I''$ and the market would be at point 0. In this particular case, it is obvious that the market selects path S'' initially. In other words, point 1'' is reached just

before the intervention, and a return to point 0 is triggered by the intervention, with no implied jump in the exchange rate. Paths S or S' would have generated foreseeable jumps in the exchange rate, which is inconsistent with the condition of no foreseeable jump in the exchange rate at the moment of the intervention.

Of course, the choice of the saddle-path solution depends on the announced intervention magnitude. For example, with the smaller discrete intervention I' , the saddle-path solution would have been S' , slightly above S'' . But we could continue the experimentation, choosing smaller and smaller discrete interventions. New saddle-paths would be selected always slightly above the previous equilibrium path. At the limit, when the intervention tends toward zero, the market would stay at $f = f_u$ as long as v increases. This case is actually the limit of the infinitesimal intervention case as studied by Krugman (1991). Now, it seems obvious that in this latter case, the market should select the path S from the start, which is the path originally derived by Krugman (1991).

Figure 1.6 has another piece of information given by horizontal lines. These are the bands on the exchange rate as **implied** by the band on the fundamentals, $[f_l, f_u]$, and the corresponding discrete interventions (I'' , I' , and the limit as I tends to 0). Observe that, in the case of a discrete intervention, let say when I'' is announced, the events of hitting the implicit exchange rate band and having an intervention will not coincide: we have intra-marginal interventions (i.e., interventions inside the implicit exchange rate band). In consequence, if the edge of the implicit exchange rate band is reached, like at point $3''$, and if the fundamentals of the exchange rate continue to increase, we would nevertheless observe an appreciation of the exchange rate. Intuitively, when f increases, the domestic currency is depreciating (s increases). The private

sector, however, knows that the time of an intervention of size I'' is approaching and, due to its expectations of this large discrete intervention, the domestic currency begins to appreciate (s decreases) even before the implementation of the intervention and then, even if f continues to increase for a while.

Now, if we consider the case of intervention when I tends to 0, the events of hitting the implicit exchange rate band and having an intervention do coincide: the intervention takes place when the exchange rate hits the band (see point 1 in figure 1.6). We are thus in the context of figure 1.5. Observe that, **at the time of the intervention** (at point 1), the S-curve is just tangent to the implicit exchange rate bound, s_u . This means that we recover the smooth-pasting result!

Here, the only assumption of the model necessary to determine the appropriate saddle-path of a target zone, is the no-foreseeable jump condition. Also, here, the smooth-pasting is observed only when the intervention, I , tends to 0. From that, it seems obvious that the smooth-pasting condition, for Flood and Garber, is simply the limit of the no-foreseeable jump (in the exchange rate) condition when an infinitesimal intervention takes place (see Appendix A6).

Note that this paper, by introducing explicitly the defence of a band on the **fundamentals** of the exchange rate, is in part responsible for the shift in the literature from the concept of a zone on the **exchange rate**. Froot and Obstfeld (1991b) pursue this route more deeply, solving the barrier control problem with the method of regulated Brownian motions. This literature can be referred to as the bounded fundamentals target zone (FTZ).

The model of Flood and Garber (1991) is sometimes presented in (v, s) space. Figure 1.7 shows a TZ defended with discrete interventions of size I' and the implied shift, down and to the right, of the path S' at the moment of the intervention.

<figure 1.7 here>

1.3 Target zones and realignments [Bertola and Caballero (1992a)]

Bertola and Caballero (1992a) introduce the possibility of central parity realignments and analyze their effects on the exchange rate path. It is shown that, when the probability of realignment is large (greater than $1/2$), the exchange rate path becomes an **inverted** S-shaped curve, describing a destabilizing behaviour for the exchange rate inside the target zone relative to its fundamentals.

Until now, when the fundamentals hit the edges of the band there is an intervention which could be either infinitesimal or discrete, but in both cases the intervention is announced, believed, and effectively implemented to defend the bounds of the fundamentals, $[f_l, f_u]$, with central parity, $c = 0$. Nevertheless, it seems that in real world target zones, bounds are not always actively defended. Sometimes a simple shift in the bounds is implemented. To understand the effects of possible realignments on the TZ model and, more precisely, on the honeymoon effect, let us tackle the problem intuitively, considering three different types of parity realignments implemented by the central bank.

Case 1. Suppose that the monetary authority defines a band, $[f_l, f_u]$, centred around a central parity, $c = 0$, so that at this central parity $s = f = 0$. The central bank also declares that when the fundamentals hit the upper bound, the relevant band is changed to $[0, 2f_u]$ centred around $c = f_u$. The problem is to determine the effects of this policy on the dynamics of the exchange rate. In this case, the exchange rate behaves similar to a flexible exchange rate. The private sector knows that when f hits f_u there will be no active intervention of the central bank, but simply an

announcement that f_u has now to be interpreted as the new central parity. This case is illustrated in figure 1.8.

<figure 1.8 here>

Case 2. Now suppose that the central bank announces a band for the fundamentals, $[c-f_u, c+f_u]$, centred around $f = c$. The bank also declares that if the fundamentals hit the edge of the band, say, at $f = c+f_u$, a new relevant band, $[c+f_u, c+3f_u]$, centred around a new central parity, $c+2f_u$, will be implemented. Implicitly the bank is announcing that when the fundamentals are at $f = c+f_u$, it will implement a discrete (finite) intervention; but this intervention is not to sustain the initial band [so that the fundamentals are not driven to or towards the initial central parity, as is the case in the paper by Flood and Garber (1991)]. Basically, the objective of this intervention is to drive the fundamentals to a new higher central parity, $c+2f_u$, triggering a devaluation of the domestic currency. What is the effect of such a policy on the dynamics of the exchange rate? Intuitively, because the intervention "leans with the wind" of the fundamentals, we should expect a destabilizing effect of such interventions on the exchange rate. The S-shaped curve of the TZ model is now inverted and labelled $s(f)$, as shown in figure 1.9.

<figure 1.9 here>

This inverted S-shaped curve reflects the fact that the announced realignment policy provides a destabilizing nominal anchor. Intuitively, it is also easy to determine the level of the exchange

rate when the fundamentals hit the band at $f = c + f_u$. Basically, because agents expect a jump in the fundamentals when f equals $c + f_u$, and because they also know precisely what shall be the new value of the fundamentals after the jump, an arbitrage condition plays such that the exchange rate before the jump in the fundamentals is equal to the new exchange rate associated with the new fundamentals. And because the new fundamentals are at the new central parity ($c + 2f_u$), the associated exchange rate must be the one corresponding to the free float exchange rate at that level of the fundamentals. Thus, given that in a free-float $s = f = m + v$ [equation (1.2)], the exchange rate just after and then just before the jump in the fundamentals has to be $s = c + 2f_u$. This is illustrated in figure 1.9.

Case 3. Finally suppose still another intervention rule is announced by the central bank. The initial band on the fundamentals is given by $[c - f_u, c + f_u]$, centred on the initial central parity, c . The central bank also announces that if the fundamentals hit $c + f_u$, there is a probability of $1/2$ that it will intervene to drive the fundamentals to $c + 2f_u$, announced as the new central parity of the new band, $[c + f_u, c + 3f_u]$ (as in case 2), and a probability of $1/2$ that it will intervene to drive the fundamentals back to the current central parity c (as a particular case of Flood and Garber's context).

When the fundamentals hit $c + f_u$, there is now a probability of $1/2$ that the central bank implements the new band and, based on case 2, that would imply an exchange rate of $s = c + 2f_u$. But, there is a probability of $1/2$ that the central bank decides to defend the current band by driving the fundamentals back to c , such that, based on Flood and Garber's case, the exchange rate would be equal to $s = c$. So now, when the fundamentals hit $c + f_u$, due to the arbitrage condition, there will be no expected profit if the market sets the current exchange rate at:

$$s = 1/2(c+2f_u) + 1/2(c) = c+f_u.$$

In consequence, when the fundamentals hit the upper bound, $c+f_u$, the exchange rate has to be equal to the fundamentals at this upper band: this is once again the dynamics involved by a flexible exchange rate.

But now it is easy to observe that, if the probability of changing the band relatively to the one of defending the current band were higher (say $3/4;1/4$), we would have an inverted S-shaped curve, but less pronounced than in case 2. In the same vein, if the announced probability was $1/4;3/4$, an S-shaped curve (i.e., a honeymoon effect) would be observed, as shown in figure 1.10. [See Appendix A7 for a formal derivation of the exchange rate saddle-path in the context of likely devaluation/realignment.]

<figure 1.10 here>

The major point to be understood from this analysis is that if some uncertainty exists about the monetary policy implemented at the edges of the band, a target zone regime could imply more volatility in the exchange rate relative to its fundamentals, reversing the main point of Krugman's analysis.

The problem with this analysis lies in the fact that the choice of realignment is modeled as if it were a randomised policy. Everything happens as if the central bank flips a coin to see if it decides to realign or not. It might be preferable to model the idea of realignment as the result of an uncertain economic environment instead of an uncertain policy choice. This idea has been explored by Svensson (1991), Bertola and Svensson (1993), and Tristani (1994), who model

discrete changes in the fundamentals as the result of an exogenous jump (Poisson) process. [See Appendix A8 for a description of Svensson's model.]

1.4 Speculative attacks on a target zone [Krugman and Rotemberg (1992)]

The paper by Krugman and Rotemberg (1992) introduces the mechanisms of a speculative attack in a target zone and re-explains the smooth-pasting result in an optimization framework. Smooth-pasting conditions are generally viewed as conditions arising from optimization (for example in option theory). Some economists have argued that a condition that arises from optimization is hard to justify when optimizing behaviour is at best implicit. Based on this criticism, Krugman and Rotemberg (1992) try to show that the smooth-pasting conditions, in the TZ literature, can also be seen as being implied by arbitrage. In their model, smooth-pasting emerges as the limit of the "no foreseeable jump" condition of a speculative model (essentially an arbitrage condition) when reserves are sufficiently large.*

Basically, a TZ is subject to the same problem as any fixed exchange rate system; without monetary discipline, a TZ is ultimately doomed to collapse. Indeed, after a period of intervention to sustain the domestic currency, **exhaustion** of foreign reserves will occur, and a return to a regime that does not require foreign reserves (e.g., a free-float regime) is inevitable. This would be a **natural** collapse of a TZ. If the market is at a point like 3, in figure 1.4, and if v continues to increase, the intervention of the central bank to sustain the domestic currency implies a loss of foreign reserves (the 45-degree line and the S-shaped curve shift down and to the right) and the new equilibrium is at point 4. If a series of positive shocks in v occurs, exhaustion of foreign

reserves could force the bank to return to a free-float regime.⁹

To illustrate this natural collapse, let us write the domestic money supply as:

$$(1.4) \quad m = \ln (D + R)$$

where D is domestic credit and R represents foreign exchange reserves held by the central bank.

The money supply after exhaustion of reserves ($R = 0$) is then given by:

$$(1.5) \quad m_c = \ln (D).$$

In this case, using equation (1.2), we can derive a free-float equation which describes the new free-float regime after the collapse of the TZ (i.e., the regime that the central bank is forced to accept because of the lack of foreign reserves):

$$(1.6) \quad s = m_c + v.$$

This equation is drawn in the right part of figures 1.11a,b,c, as the 45-degree line F_c-F_c .

<figure 1.11 here>

Note that the 45-degree line F_c-F_c is called a "shadow" regime in the speculative attack literature, to express the idea that this regime will be implemented when the current TZ regime is no longer viable.

It is important to note, however, that the collapse of a TZ regime, and the transition to the free-float, is not likely to happen without crisis (i.e., a speculative attack). It is obvious that if the exchange rate is controlled to remain within a particular band, its level is probably different from its implicit (or shadow) free-float level. As a consequence, a discrete jump in the exchange rate at the time of the natural collapse to a free-float has to be expected.

But now, this discrete jump in the exchange rate expected by forward looking agents motivates, **before** total exhaustion of central bank reserves, a speculative attack on a TZ. It is obvious that if the market considers the domestic currency overvalued relative to its shadow free-float level, and if the TZ regime is not considered viable in the short run, speculators would immediately buy foreign reserves from the central bank to exploit any profit opportunity that would arise later on, at the time of the collapse of the regime. This speculative activity transforms a natural collapse into a speculative crisis.

Two interesting problems immediately arise: 1) when is a speculative attack triggered? In a state dependent model like the Krugman TZ this question amounts to asking: where (in which state) would the speculative attack be triggered?; 2) what is the magnitude of the speculative attack (i.e., is there a maximum speculative attack)? Although both problems are linked, it is useful to assume initially an arbitrary magnitude for a speculative attack such that the only question left is to determine the triggering state of the attack. Thereafter, the maximum speculative attack problem can be easily solved.

Consider initially that a speculative attack of $R^*/2$ (an arbitrary amount) is planned in this model.¹⁰ This means that, when the level of foreign reserves of the domestic central bank falls to $R^*/2$, the market will precipitate a speculative attack at the **next** future intervention of the central bank (i.e., at the next passage of s at the upper bound, s_u). This speculative attack, by definition, decreases the level of foreign reserves held by the domestic central bank from $R^*/2$ to 0, which forces the return to a free-float regime and implies thereafter a path for the exchange rate along the schedule F_c-F_c in figure 1.11a.

Recall that the path F_c-F_c corresponds to a free-float regime with no foreign reserve, whereas

the 45-degree line drawn immediately to its left corresponds to a free-float regime with an amount of foreign reserves given by $R^*/2$.¹¹ S_3 corresponds to a traditional TZ path drawn in the upper part of the exchange rate band and for which the level of foreign reserve held by the domestic central bank is set equal to $R^*/2$. Observe that, along S_3 , the next intervention to sustain the domestic currency takes place at point 3 such that the level of reserves held by the domestic central bank at this point will fall slightly below $R^*/2$. Point 3 may thus be considered as a potential state which could trigger a speculative attack of amount $R^*/2$.

Observe, though, that a speculative attack triggered at point 3 would imply a forced return to a free-float (with zero reserve) and an exchange rate's jump to point 3' on F_c-F_c . This discrete jump in the exchange rate implies large positive profits for the speculators who bought the foreign reserves. But speculators compete to capture these profits. This means that an individual speculator expecting a full-scale attack at point 3, has incentives to pre-empt his competitors by purchasing all the reserves an instant before point 3. Therefore, the attack will occur at a state prior to point 3. Basically, whenever agents expect a discrete exchange rate increase, they will precipitate an attack on reserves prior to the increase. In this sense, having an attack of amount $R^*/2$ at point 3 and at the same time an increase in the exchange rate is not consistent with arbitrage behaviour.

If a speculative attack of magnitude $R^*/2$ is not triggered at point 3, where is it triggered? Because the speculative attack will be triggered at the next passage of s at s_u , points 0, 1, 2, are other potential candidates. So, let suppose that, as soon as one of these points is reached, a speculative attack of magnitude $R^*/2$ is triggered such that the new relevant regime is the free-float line F_c-F_c . Given that v does not change during the instantaneous shift of regime, this

implies a (vertical) jump in the exchange rate to points $0'$, $1'$, or 2, depending on the initial position at the instant just before the attack. We can thus observe that the only point which does not involve a jump in the exchange rate at the time of the attack, is point 2. If the speculative attack were triggered when the market reaches points 0 or 1, an appreciation of the domestic currency would be observed just after the attack, to points $0'$ and $1'$ respectively. This type of attack would imply a loss for speculators (who bet on a depreciation of the domestic currency) and as a consequence, there is no reason for any speculative attack to be triggered at these points.

After determining where the TZ regime ends if an arbitrary speculative attack of $R^*/2$ is triggered (point 2), the path of the TZ just before the speculative attack is easily determined as path S_2 because this is the only path which drives the market to point 2.¹² A key observation is the **absence** of any smooth-pasting condition in this analysis. In other words, the speculative attack argument, in itself, does not imply the smooth-pasting condition at the time of the speculative attack. The saddle-path solution of this regime is not the tangent path (i.e., path S_1) as it would be if the TZ was sustainable. But this should not be too surprising: the smooth-pasting paths of figures 1.3 and 1.4 were derived on the basis of a fully credible zone with a market expectation of an intervention at the upper edge of the band, whereas here, there is a market expectation of central bank incapacity to intervene.

Now, remember that the magnitude of the speculative attack was arbitrarily set at the level $R^*/2$. But if the market can do a speculative attack of $R^*/2$, why would it not try a larger speculative attack? After all, the larger the speculative attack, the larger the potential speculative profit for speculators. So suppose that the market tries a speculative attack when the reserves of the central bank are R^* (i.e., larger than $R^*/2$). For clarity, let us draw a speculative attack of

magnitude R^* in figure 1.11b.¹³ This time, the speculative attack could be triggered at points 0, 1, 2 or 3. We observe that, relative to the corresponding shadow free-float line, points 0, 1 and 2 would involve an appreciation of the domestic currency, at the moment of the attack, respectively to points 0', 1' and 2'. This would imply a loss for speculators, preventing them from attempting a speculative attack at these points. The only possible candidate is point 3. Now, if point 3 is the transitory point between the target zone regime and the new free-float, this means that the path before the attack is S_3 (i.e., the smooth-pasting path of the TZ). As a consequence, there is a particular level of speculative attack (R^*) which implies the same path as the traditional TZ exchange rate path.

In the process of arbitrarily increasing the magnitude of the speculative attack a smooth-pasting path has been derived. But would it be possible to generate still larger speculative attacks (i.e., larger than R^*) by continually and arbitrarily increasing the amount of reserves initially held by the central bank at the moment just before the attack? The answer is a definite no. When the reserves of the bank are greater, this gives the opportunity for speculators to launch a larger speculative attack, but this also gives more credibility to the announced plan to defend the upper edge $s = s_u$. If foreign reserves are sufficiently great, the central bank will be able to intervene before any speculative attack can be triggered.

For example, suppose that foreign reserves held by the central bank were $3R^*/2$, and thus larger than R^* . Would it be possible for the speculators to launch an attack of magnitude $3R^*/2$? Let us figure out the answer of this question with the help of figure 1.11c. In this graph, points 0, 1, 2, 3, and 5 would trigger an appreciation of the domestic currency which prevents any reason for a speculative attack at these points. We are left with point 4, the only point that does

not generate a jump in the exchange rate at the time of the (would-be) speculative attack. The only path which drives the market to this point before the attack is path S_4 . But does this really correspond to a successful speculative attack of magnitude $3R^*/2$? No, simply because this path is not sustainable! Basically a successful speculative attack of magnitude $3R^*/2$ has to be triggered at point 4. But before reaching point 4, the market reaches point 5 (the next future passage of s at s_{ii}) on path S_4 and, at this point, the central bank will implement an intervention to keep the exchange rate inside the band. As mentioned before, there is no reason for the speculators to trigger an attack at point 5. As a consequence, the TZ regime is not in danger because the central bank can still sell foreign reserves (to decrease its level of foreign reserves below $3R^*/2$) to sustain the TZ regime. The implication of this last result is as follows. Speculators know that they cannot do speculative attacks of any arbitrary magnitude. As long as the central bank has reserves larger than R^* , the TZ system is viable and any speculative attack would fail.

The conclusion to be drawn from figure 1.11 is that the smooth-pasting path S_3 is the path corresponding to the largest speculative attack in the model (R^*). But remember that the smooth-pasting path is also the solution of a credible TZ regime. How can we motivate the use of the smooth-pasting path in a viable TZ ($R > R^*$) where no possible speculative attack could be triggered? Basically, even if initial foreign reserves are sufficiently large to sustain the current TZ, after a period of interventions (i.e., after a period of loss of reserves), the level of these reserves will fall at or below R^* , such that the current TZ regime will not be viable anymore. Thus, again, room for speculative attacks exists. But because speculators want to be ready to do the largest speculative attack to generate the largest potential profit, they choose from the start

(even when there is no room for an attack) the smooth-pasting path which corresponds to the largest speculative attack, R^* in this model. This strategy guarantees speculators that, when the reserves of the central bank fall to $R = R^*$, they will be along the path giving rise to the largest speculative attack possible.

In other words, a country that starts with a high level of reserves will go through a smooth-pasting phase where small interventions succeed in defending the TZ regime. But as interventions are required, there will be a drain on reserves and, as in the conventional speculative attack literature, there will eventually be a crisis once reserves have dropped to a critical level. As explained by Krugman and Rotemberg (1992), when $R > R^*$, the central bank is able to defend the TZ with an infinitesimal intervention that slightly reduces the money supply, shifting the relationship between v and s down. But, due to the gradual loss of reserves, eventually the level of reserves will fall to the critical level where a speculative attack becomes possible; at that point, the next time that the exchange rate drifts up to $s = s_m$, there will be a full-scale speculative attack that eliminates all remaining reserves.

Because the TZ literature refers very often to (f,s) space, it is important to show how to represent the maximum speculative attack in this space. Figure 1.12 explains the passage from (v,s) space to (f,s) space in the context of speculative attack.

<figure 1.12 here>

Figure 1.12a represents the maximum speculative attack path (i.e., path S_3 of figure 1.11). The level of reserves is assumed to be R^* . If we are currently at point 2, next period we could be

at point 1 or at point 3 depending on the move in v . If the economy reaches point 3, there is a speculative attack of magnitude R^* such that the domestic central bank has no more reserves and has to accept the ensuing free-float regime. From then on, any move in v will imply an equivalent move in s along the 45-degree line: at the next instant we will be at either point 4 or 4'. In figure 1.12b, the same result is drawn in (f,s) space. If foreign reserves are equal to R^* , a speculative attack of magnitude R^* would be triggered as soon as point 3 is reached. Because the central bank loses R^* of reserves, a discrete jump in the fundamentals, from f_u to f_s , is observed, and the market moves to point 3' on the 45-degree line. At the next instant, depending on the move in v , the market could be at either point 4 or 4' on the 45-degree line.

If we compare the results of section 1.4 [Krugman and Rotemberg (1992)], with those of section 1.2 [Flood and Garber (1991)], we see that both papers give a very related interpretation of smooth-pasting, essentially drawing their interpretation from principles of the speculative attack literature. But, it seems that smooth-pasting in the FTZ model is more like an implicit result of the way the model is built. In the limiting case of a discrete intervention tending toward 0, smooth-pasting emerges if the assumption of no foreseeable jump in the exchange rate is made. Smooth-pasting is thus, in the FTZ model, essentially the result of the assumption of no foreseeable jump in the exchange rate. In other words, smooth-pasting in the FTZ model is the result of a **technique** used in the speculative attack literature (the no foreseeable jump in the exchange rate) even if the **logic** of the speculative attack is nowhere in the model. Indeed, when speculators choose a particular exchange rate path such that there is no jump in the exchange rate at the time of the intervention, they do it because they really believe that the TZ regime is viable, and not because, at some particular state of the market, the regime could collapse.

Smooth-pasting in FTZ is more the result of the way this model is built, whereas, smooth-pasting in the Krugman and Rotemberg model is more the result of an eventual crisis logic: in this model, smooth-pasting is the consequence of a future constraint that speculators have to take into account now, if at some point in time they want to be able to do the largest speculative attack on the TZ. In conclusion, even if both models refer to one technique of the speculative attack literature (i.e., the no foreseeable jump in the exchange rate) to explain the smooth-pasting condition, only Krugman and Rotemberg (1992) employ the logic of the speculative attack literature to justify the use of the smooth-pasting condition. The logic of an **eventual** speculative attack is essential to their model, but not required (and in fact, absent), in the FTZ model.

We can then classify the different models encountered according to the following major characteristics:

- 1) The **explicit** band to be defended: $[s_l, s_u]$ (the exchange rate target zone) or $[f_l, f_u]$ (the bounded fundamentals target zone, FTZ).
- 2) The kind of intervention at the bounds: infinitesimal intervention [control problem with instantaneous reflecting barrier] or discrete intervention [control problem with discrete (impulse) reflecting barrier].
- 3) The way smooth-pasting is justified, either in an ad hoc manner (the no kink assumption) as in Krugman (1991), or with the no-jump assumption, or finally, with the **logic** of a speculative attack.

Figure 1.13 locates, in a three-dimension graph, the models of Krugman (1991), Krugman and Rotemberg (1992), Froot and Obstfeld (1991b) (not surveyed here but developed in Appendixes A4 and A5), and Flood and Garber (1991). This latter model is located when the intervention

is discrete and also at the limit, when the intervention tends toward 0. In this limiting case, the model of Froot and Obstfeld (1991b) and the one by Flood and Garber (1991) have the same location in figure 1.13.

<figure 1.13 here>

1.5 Why have a target zone? [Krugman and Miller (1993)]

The traditional economic arguments for choosing fixed rate regimes or target zone regimes instead of free-float regimes are generally based on three lines of reasoning: 1) the theory of optimal currency areas [cost-benefit analysis of abandoning the exchange rate as an instrument of adjustment in face of real shocks (i.e., tradeoff between the macroeconomic advantages of flexible exchange rates and the microeconomic advantages of fixed rates or a common currency)]; 2) the nominal anchor provided by a fixed exchange rate regime (and the related argument that an inflation-prone country could gain credibility in its anti-inflation commitment, by fixing its currency to the one of a well-known anti-inflation country); and 3) the alleged speculative moves of the exchange rates in a free-float regime (i.e., the assumption that the foreign asset market is not efficient and as a consequence, irrational and unstable market behaviour could develop in a free-float regime).

Krugman and Miller (1993) emphasize the third line of reasoning as the most practical reason for introducing a target zone. They argue that the main motivation for establishing a target zone, in practice (for example, the Louvre Accord in 1987) has been the desire to avoid destabilizing

speculative and irrational runs on the exchange rate. In this sense, the objective of a target zone is not only to induce rational stabilising expectations by providing a nominal anchor (s moves less than f because of the well understood money supply behaviour in a target zone regime), but to keep asset prices (the exchange rate) from fluctuating enough to generate irrational speculative selling. The objective of the authors is then to start from models of excessive volatility in exchange rate (i.e., the real concern of policy-makers), and to derive the effect of target zones in this context. In particular, they introduce the idea of potential exchange rate crashes provoked by the presence of significant groups of investors following stop-loss strategies that lead them to sell, not buy, when prices fall.

Suppose that an American investor has bought Canadian bonds (B) for a value of $CS\ 1000.00$. If, at the time of the buying, the Canadian price of one $US\$$ is $S = 1.2$, the US investor has then effectively paid $B/S = US\$ 833.33$. But if thereafter S increases (i.e., if the Canadian dollar depreciates to $S = 1.3$), the value of his Canadian bonds would fall to $US\$ 769.23$, and the American investor would incur a loss of capital. The idea of a stop-loss strategy for this investor would be to set a particular threshold for S , say, $S_v = 1.35$, at which he would automatically sell his Canadian bonds for US Bonds (F) in order to limit (here, to stop) his losses. The same kind of behaviour could also exist for Canadian investors. Having bought for a value of $US\$ 1000.00$ in US bonds (F) at a rate of $S = 1.2$, the Canadian investor has then effectively paid $SF = CS\ 1200.00$. If S decreases, the value of his US bonds would decrease. He could then set, a priori, a particular threshold, say, $S_k = 1$, at which he would sell his American bonds for Canadian bonds.

Suppose that there is a class of domestic and foreign investors who adopt the stop-loss

strategies described above. In addition to these stop-loss investors, there is a pool of rational investors. Assume also that stop-loss investors follow an all-or-nothing strategy, and that they all have the same threshold point. (These last two assumptions can be easily relaxed without altering the following results.)

Note that, at the time of the selling (selling of Canadian bonds for US bonds at S_c , or selling of US bonds for Canadian bonds at S_k), the relative money supplies do not change. This means that this kind of financial transaction is not supposed to influence the exchange rate level, if the underlying model of exchange rate determination is a flexible price monetary model (i.e., if domestic and foreign bonds are considered as perfect substitutes). Nevertheless, these financial transactions will have an effect on S if the underlying model of exchange rate determination is a portfolio model, such that domestic and foreign bonds are not perfect substitutes. As is well known from the portfolio model of exchange rate determination, an asset preference shift from domestic to foreign bonds has the same effect as a sterilised intervention against the domestic currency: the domestic interest rate, i , and S will increase. An asset preference shift from foreign to domestic bonds has the same effect as a sterilised intervention to sustain the domestic currency: i and S will decrease.

To incorporate these effects let us rewrite the basic model of exchange rate determination as:

$$(1.7) \quad s = f + \theta(E[ds]/dt + \pi - \pi^*)$$

where $s = \log(S)$, π is the risk premium associated to domestic bonds, and π^* is the risk premium associated to foreign bonds.¹⁴ When foreign investors get rid of their domestic bonds, there is an excess supply of these bonds such that π increases, depreciating the domestic currency. Also, when domestic investors get rid of their foreign bonds, an excess supply of foreign bonds

develops and π^* increases such that s decreases.

The impact of the exit of stop-loss traders is shown in figure 1.14 in (f, s) space.

<figure 1.14 here>

The locus $F-F$ represents the free-float schedule when investors (domestic and foreign) are permanently in the market. The locus $X-X$ ($Y-Y$) is the free-float schedule when foreign (domestic) stop-loss traders are permanently out (i.e., when they do not have domestic (foreign) bonds in their portfolio).¹⁵

The path of the exchange rate in such a model will ultimately depend on the rational investors' awareness of the presence of stop-loss investors. If these investors are unaware of the existence of a pool of stop-loss investors, the economy would walk randomly along $F-F$ as long as the threshold levels, s_v and s_k , represented in figure 1.14, are not reached. Once a threshold level is reached, say s_v (s_k), a selling wave of domestic (foreign) bonds will lead to a jump in the price of the foreign currency to s_w (s_j). If no re-entry is assumed, the economy will then randomly walk along $X-X$ ($Y-Y$).¹⁶ This story can easily be extended to any number of different groups of stop-loss traders with different trigger prices and more gradual disinvestment strategies. [See Krugman and Miller (1993).] The message of this model is that possible exchange rate crashes (jumps) in a free float regime could happen if the presence of stop-loss investors is not foreseen by rational investors.

If rational investors were aware of the existence, the size, and the thresholds of stop-loss investors, the outcome would be considerably less dramatic: in this case, to prevent any profit

opportunity, there would be no jump in the exchange rate when stop-loss investors leave the market. This is illustrated in figure 1.15.

<figure 1.15 here>

The outcome of this analysis, during the period before exit, is drawn as the schedule $G-G$, in figure 1.15. This schedule implies no jump in the exchange rate at the time of the exit. The shape of the $G-G$ schedule relative to the $F-F$ schedule is due to the existence of risk premiums π (π^*), which rise as the trigger points s_v (s_k) are approached. This means that the adjusted expected rate of depreciation (adjusted by the risk premium), increases as s increases: the market expectations tend to destabilize rather than stabilize exchange rates. Note, here, that exchange rate crashes (jumps) will happen only if rational investors underestimate the size of the pool of stop-loss investors. But, even without crashes in the exchange rate market, the known presence of stop-loss investors will increase volatility of the exchange rate. The shape of the $G-G$ curve implies that the exchange rate moves more than proportionally to its fundamentals.

The point of the Krugman and Miller argument is to show that a free-float regime can generate excessive volatility of the exchange rate (relative to its fundamentals) because of the known existence of stop-loss traders. They continue by suggesting that introducing a target zone in such a context would have the impact of reducing this volatility, as shown in figure 1.16.

<figure 1.16 here>

Here, a target zone $[s_l, s_u]$ has been introduced with the characteristic that $s_l > s_k$ and $s_u < s_v$. Now, the dynamics of the exchange rate will shift from $G-G$ to $S-S$. The reason is the following: once informed investors are assured that the exchange rate will not be allowed to vary enough to trigger the exit of stop-loss traders, their own speculation will shift from destabilizing to stabilizing.

For Krugman and Miller, the fear of excess volatility during free-float (the $G-G$ schedule) represents the real motivation for policy-makers to introduce a target zone. The gain of a target zone can be decomposed in two parts: the move from $G-G$ to $F-F$ represents the guarantee that there will not be any selling waves by stop-loss investors, whereas the move from $F-F$ to $S-S$ represents the usual honeymoon effect of a target zone.

Although this paper may provide one reason for a TZ, it does not explain the **bandwidth** of the target zone; in particular, it does not explain why it is not desirable to go all the way to a pure fixed exchange rate. The topic of an optimal bandwidth for a TZ has never been studied. To tackle this problem we need to formulate a flow reward function of the exchange rate fundamentals that captures the "honeymoon effect" of any TZ (relative to what would happen in the free-float regime). We need also to specify regulation costs involved by particular interventions at the upper and lower edges of the band. These costs depend on the type of control implemented: if there is a lump-sum component to the cost of intervention, we should use an impulse control (i.e., to implement a discrete intervention to move the exchange rate fundamentals inside their band). If there is no lump-sum cost, a **minimum** amount of control (instantaneous control) is exercised to keep the fundamentals from going outside their band; this implies an infinitesimal intervention at the edges of the band.

The objective is then to evaluate the expected present value of the net benefits (flow reward minus regulation costs) of given controls, and then to choose the values of these controls which maximize this expected present value. From the point of view of techniques needed to solve this problem, the theory of optimal regulation of Brownian motion should be used. This stochastic optimal control theory has been considerably simplified by Dixit (1991) and makes the search for an optimal TZ bandwidth a realistic and promising field of research.

1.6 Testable implications and empirical evidence of the target zone model [Lindberg and Söderlind (1994)]

1.6.1 Testable implications of the Krugman (1991) target zone model

The target zone theory determines a well specified nonlinear relationship between the exchange rate and the fundamentals, as well as some general stylized facts based on the statistical characteristics of the exchange rate and of the interest rate differentials. Some of these stylized facts are:

- 1) The asymptotic (unconditional) probability distribution of the exchange rate is U-shaped.
- 2) There is a negative relationship between the exchange rate and the interest rate differentials.
- 3) The instantaneous (conditional) standard deviations of the exchange rate and the interest rate differentials are variable (i.e., s and $id = i - i^*$ are heteroscedastic) and a trade-off exists between the two conditional variabilities.

The intuition for the first two stylised facts is as follows [stylised fact 3 is analyzed in

Appendix A5.6].

1) To understand that the probability distribution of the exchange rate is U-Shaped, let us note that the slope of the S - S curve has a tendency to decrease from the central parity to the edges of the band, where the slope equals zero (the smooth-pasting condition). This means that, near the bounds, the exchange rate is very insensitive to the fundamentals, and at the upper and lower edges of the band, the exchange rate is completely insensitive: $dS(f)/df|_{f=f_l;f_u} = 0$. At the same time, because the fundamentals of the exchange rate are assumed to follow a random walk with zero drift (actually a regulated Brownian motion with zero drift), their asymptotic (unconditional) density distribution is uniform. This means that the fundamentals move with constant speed between their bounds.

Now, because the exchange rate is very insensitive to the fundamentals near the edges of the band, it means that the exchange rate will move slowly near the edges of the band. As a consequence it will appear often near the edges of the band and the probability distribution of the exchange rate is U-shaped (i.e., bimodal with more probability mass towards the edges of the band than the uniform distribution). (A more rigorous explanation of this U-shaped distribution is given in Appendix A9.)

2) The Krugman target zone model implies a negative relationship between the exchange rate, s , and the interest rate differential, $id = i - i^*$. Recall that along a particular S-shaped curve like the one represented in figure 1.5, when f increases there is a decrease in the expected rate of depreciation of the domestic currency because the market expects a future intervention to sustain the domestic currency (there is an increase in the expected rate of appreciation of the domestic currency). This increase in the expected rate of appreciation of the domestic currency as f rises

implies, through the uncovered interest rate parity condition, that i has to decrease relative to i^* . The interest rate differential, id , is thus a negative function of the fundamental, and this function is represented in figure 1.17 as the schedule $I(f)$.

<figure 1.17 here>

As shown in figure 1.17, as f increases, we observe a move down along $I(f)$: $id = i - i^*$ decreases. But when f increases, we also know that s increases: this is a move up along an S-shaped curve such as the one in figure 1.5. In consequence, Krugman's model predicts a deterministic negative correlation between s and id .

1.6.2 The empirical evidence

The empirical evidence has taken two major directions: to test the stylised facts implied by the theory and to test directly the relationship between the exchange rate and the fundamentals.

- 1) Direct tests of the model have been performed with fundamentals explicitly or implicitly obtained. Parametric and nonparametric techniques have been used. But, in general, the results are unsupportive of the basic target zone model. See Lindberg and Söderlind (1994).
- 2) The U-shaped distribution of the exchange rate implies that the exchange rate should be more frequently observed near the bounds. This is clearly rejected by the data on the E.M.S. and the Swedish Krona [e.g., Flood, Rose, and Mathieson (1991) and Lindberg and Söderlind (1994)]. The data show that the exchange rate distribution is hump-shaped, with most of the probability mass in the interior of the band and very little near the edges. One explanation for this hump-

shaped distribution could be that intra-marginal interventions (leaning against the wind interventions) occur inside the exchange rate band. Intra-marginal interventions are modeled by Delgado and Dumas (1992) and Klein and Lewis (1993).

3) The negative correlation between s and id has not been verified in the data on the E.M.S. and on the Swedish krona [e.g., Flood, Rose, and Mathieson (1991) and Lindberg and Söderlind (1994)]. Svensson (1991) and Bertola and Svensson (1993) show that a positive correlation between s and id could develop when an expected rate of devaluation/realignment is introduced in the model. The second essay (Chapter 3) of this thesis provides another explanation of this positive correlation.

1.7 Permanent regime switching [Froot and Obstfeld (1991a)]

As mentioned in the Introduction of the thesis, a target zone may be understood as a **transitory** regime switching from free-floating to a fixed exchange rate and vice versa. The bounds of the zone are reflecting barriers that trigger the regime switching. Sections 1.1 to 1.6 surveyed intuitively the TZ theory.

In this section, an intuitive presentation of **permanent** regime switching is given. The objectives of the section are first, to contrast the notion of permanent relative to transitory regime switching, but also to underline the basic similar framework of analysis between both types of regime switching. Indeed, the renewed interest for the theory of permanent regime switching is probably due to the framework of analysis developed in the target zone theory. Finally, because chapters 2 and 4 examine permanent regime switchings from a free-float to a TZ (Chapter 2) and

to a fixed exchange rate regime (Chapter 4), it may also be useful to state the problem intuitively. Permanent regime switching may be state- or time-dependent; in this section some intuition is given for the problem of a (state-dependent) permanent regime switching from a free-float to a fixed exchange rate.

In an example inspired by Britain's return to the gold standard in 1925, Flood and Garber (1983) study the case in which the exchange rate floats freely until it reaches a pre-announced level, at which time the government intervenes to keep it fixed thereafter. The problem is to derive the exchange rate path during the transition period from the announcement of the future fixed rate regime to its effective implementation.

An intuition of the solution is easily stated. Again, we start from equation (1.1). In a free-float without drift we saw that $s = f$ [equation (1.2)]. To provide a more general solution we can assume that the domestic currency has a tendency to depreciate, which would represent a free-float with drift [i.e., the fundamentals (and thus the exchange rate) follow a continuous random walk with drift, $\mu > 0$]. In this case, it is obvious that the expected rate of depreciation should be equal to the drift:

$$(1.8) \quad E_t[ds(t)|I(t)]/dt = E_t[df(t)|I(t)]/dt = \mu$$

and the free-float solution is thus:¹⁷

$$(1.9) \quad s = f + \theta\mu.$$

The schedules F_0-F_0 and F_1-F_1 in figure 1.18 show the free-float paths without and with a positive drift.¹⁸

<figure 1.18 here>

Suppose now that the domestic authorities announce a return to a fixed exchange rate regime as soon as s hits s_2 . The exchange rate path during the transition period (i.e., path S_{fre} in figure 1.18) is derived using two boundary conditions.

The first condition concerns the level of the fundamentals when s is permanently fixed at s_2 . Basically, if the exchange rate is not expected to change (i.e., if $E_t[ds(t)|I(t)]/dt = 0$), equation (1.1) implies that $s = f$. As a result, if the authorities wish to peg the exchange rate at some level s_2 , they must strictly maintain the fundamentals at a level $f = s_2 = f_2$. [A fixed exchange rate regime will always be depicted by **one** point on F_0 - F_0]. Now, to preclude excess anticipated profit opportunities at the moment of the transition (i.e., as soon as f hits f_2) there should be no jump in the exchange rate (i.e., s should be equal to s_2). This implies that (f_2, s_2) is a boundary condition of the model.¹⁹ In other words the transition path should bend away from F_1 - F_1 as f increases, as shown in figure 1.18.

The second boundary condition is as follows. When the exchange rate (and thus the fundamentals) is far from s_2 (f_2) the regime switching is unlikely, such that the expectation of the regime change is low and the exchange rate level should approximately correspond to its free-float level; the transition path should tend asymptotically to F_1 - F_1 for values of s farther from s_2 .

The transition path is represented in figure 1.18 as S_{fre} . This is the path conjectured by Flood and Garber (1983) and derived by Froot and Obstfeld (1991a) using the theory of regulated Brownian motion.

1.8 Conclusion

Chapter 1 is an intuitive survey of the theory of stochastic regime switching, in particular the TZ theory. Appendixes A1 to A10 present the same theory in a slightly more formal way. This chapter does not pretend to survey exhaustively the literature but instead, to give some basic insights of the problems and solution methods encountered in this literature. Other surveys are given in Bertola (1994), De Arcangelis (1994), and Svensson (1992).

The TZ literature has grown to a worldwide program of research in the 1990s and as mentioned by Krugman (1992, p.13), "the target zone literature has expanded so rapidly that one hesitates to offer any suggestions for further work -- indeed, anything one might suggest is probably already being worked on somewhere in the world!" The following chapters represent the marginal contribution of this thesis to the field.

Endnotes

- 1) The parameter θ also measures the speed with which the asset depletes. The faster an asset depletes, the smaller is θ . In other words, when the asset depletes quickly, the future price will have a smaller effect on the current price.
- 2) The parameter θ is interpreted, in monetary models, as the semi-elasticity of money demand with respect to the rate of return on the alternative asset whereas, in portfolio-balance models, θ depends inversely on the variance of the exchange rate and the coefficient of relative risk-aversion.
- 3) See Appendix A1 for an intuitive introduction to Brownian motion and Wiener processes.
- 4) In other words, in this simple free-float model, m is not chosen or revised in function of the level of the exchange rate.
- 5) See Appendix A3 for the solution of a free-float with drift.
- 6) Note also, following Miller and Weller (1991a) that, since the purchasing power parity condition is always preserved in this model, the price level must move in line with the exchange rate. So it is obvious that constancy of the money supply is no guarantee of price stability. Despite its name, the monetary model underlying this flexible exchange rate regime does not lead here to "monetarist" conclusions. On the contrary, the assumed behaviour of velocity implies that active intervention, not a fixed rule for money, is needed to secure price stability.
- 7) Note that Flood and Garber (1991) use the condition of no-expected arbitrage profit, familiar in the speculative attack literature, to solve the TZ model. However, a speculative attack against the TZ is not considered by the market (i.e., the sustainability of the TZ is taken for granted).
- 8) In other words, smooth-pasting emerges, not as the general solution of the model, but as its solution when the central bank's reserves are sufficiently large. The main difference with the smooth-pasting interpretation of Flood and Garber (1991) is the concept of foreign reserves introduced by Krugman and Rotemberg (1992) in a TZ model and hence, a possible depletion of these reserves.
- 9) Observe that a sequence of numerous positive shocks in v is more consistently modelled with a continuous random walk **with positive drift**, as is assumed in Krugman and Rotemberg (1992). But, to keep the current level of explanation as simple as possible, it has been assumed, here, that v follows a Brownian motion without drift. Also, the presentation is given, here, in the framework of a TZ, whereas Krugman and Rotemberg present an one-sided TZ model.

- 10) The amount of the maximum speculative attack is denoted as R^* . In other words, we start, here, with a relatively small speculative attack magnitude given by $R^*/2$.
- 11) This 45-degree line is drawn for $m = \ln(D+R^*/2) = m'$. As a consequence, using equation (1.2), this 45-degree line is given by: $s = \ln(D+R^*/2) + v$. The horizontal distance between the two 45-degree lines is thus given by: $(m' - m_c) = \ln(D+R^*/2) - \ln(D)$. This horizontal distance may thus be understood as a log-measure of the arbitrary speculative attack, $R^*/2$.
- 12) Recall that path S_2 embodies a lack of attempt to pre-empt potential positive profits. Path S_2 is drawn slightly above path S_3 , reflecting the process of pre-empting speculative attacks. Speculators have an excess demand of foreign currency that the domestic central bank does not accommodate. This implies a depreciation of the domestic currency relative to path S_3 . As a consequence, the equilibrium path during this process must be slightly above S_3 .
- 13) The 45-degree line immediately to the left of F_c-F_c in figure 1.11b is drawn for $m = \ln(D+R^*) = m''$. The horizontal distance between the two 45-degree lines is thus given by: $(m'' - m_c) = \ln(D+R^*) - \ln(D) = \ln(1+R^*/D)$. Observe that this distance is larger than the one between the two 45-degree lines in figure 1.11.a and derived in note 11.
- 14) Strictly speaking, we can say that the risk premium is $\pi' = (\pi - \pi^*)$. In other words, the decomposition is not necessary.
- 15) The fundamentals f , in equation (1.7), are still assumed to follow a continuous-time random walk. Also, when investors are permanently in the market, the risk premium is assumed equal to 0. This gives rise to the locus $F-F$. When foreign stop-loss traders exit the market (i.e., sell their domestic currency-denominated assets), this generates an increase in π and/or a decrease in π^* , since the remaining investors must make an offsetting move from foreign to domestic assets. This explains the location of the locus $X-X$ above $F-F$ [i.e., the vertical intercept shift up by $\theta(\pi - \pi^*)$]. An analogous reasoning applies to the schedule $Y-Y$.
- 16) This means that the exit by stop-loss traders is assumed irreversible. In the Appendix of their paper, Krugman and Miller (1993) study a model with re-entry (i.e., reversible exit given market recovery).
- 17) The solution method for a permanent free-float with drift is derived in Appendix A3.2.
- 18) Points on F_1-F_1 have a tendency to move north-east along the path because of the positive drift but moves to the south-west cannot be excluded because this is a stochastic model.
- 19) One crucial assumption, here, is that fundamentals cannot jump at the instant of pegging.

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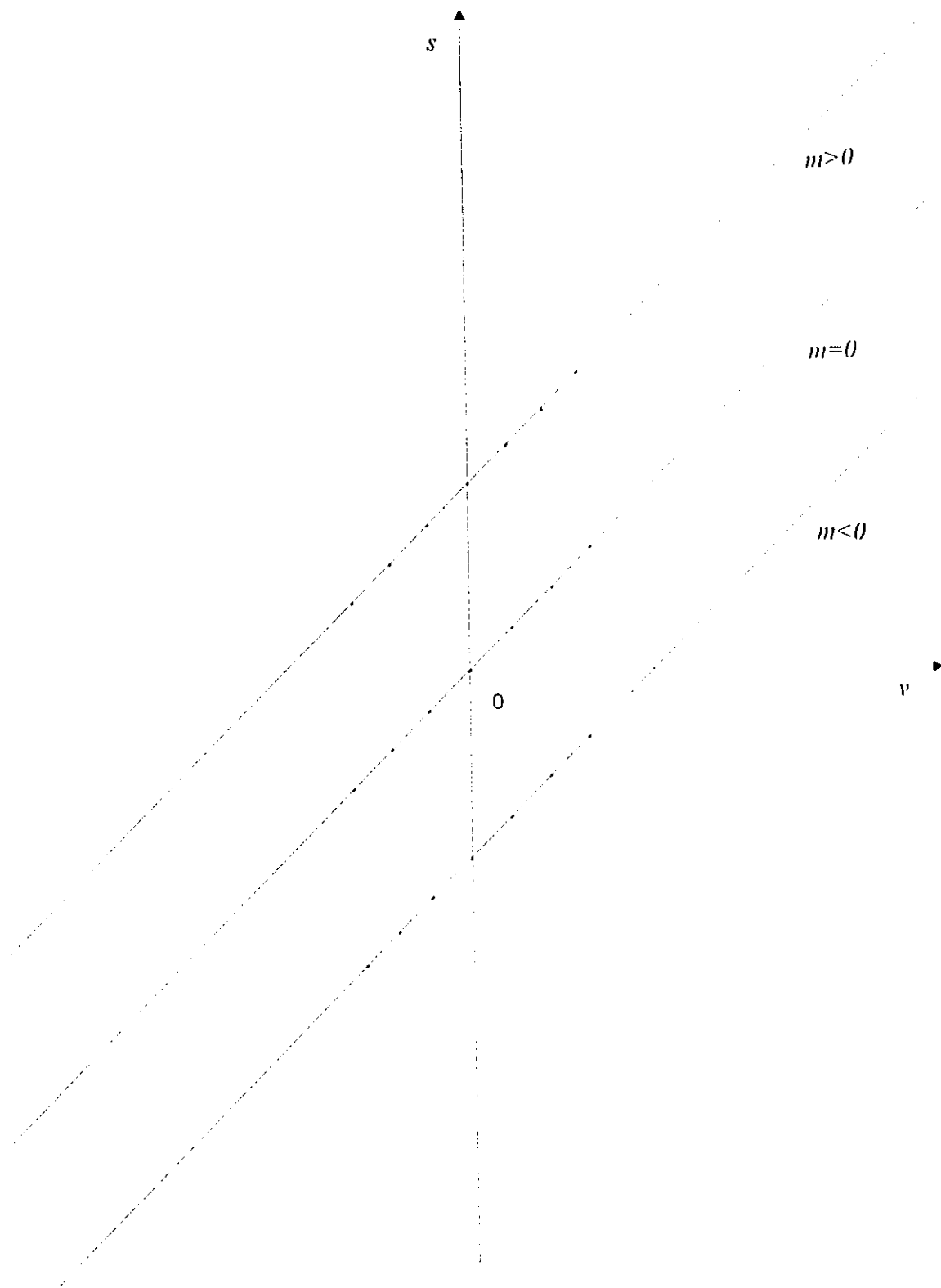


Figure 1.1 Free-float regime in (v, s) space

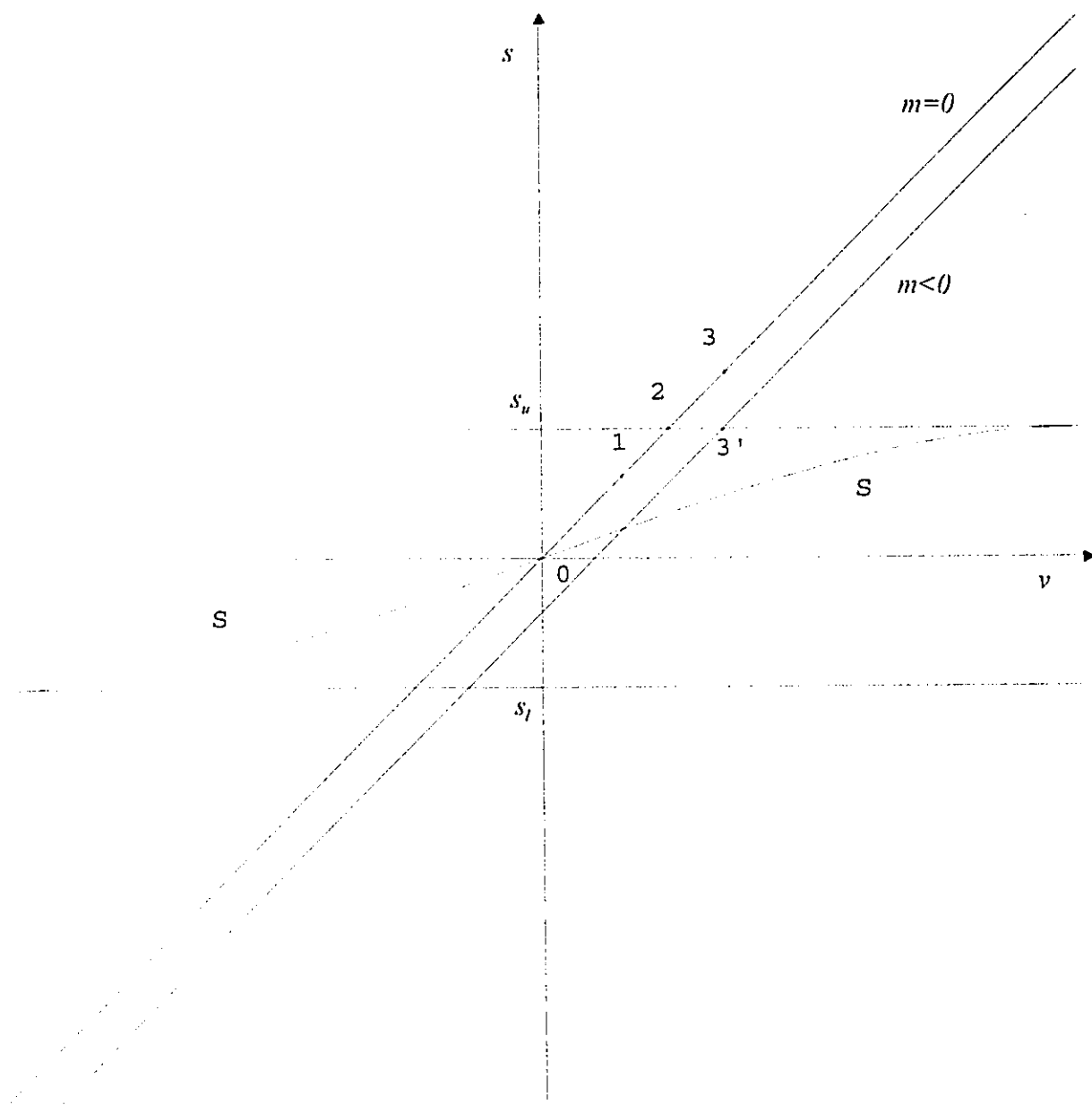


Figure 1.2 The target zone solution

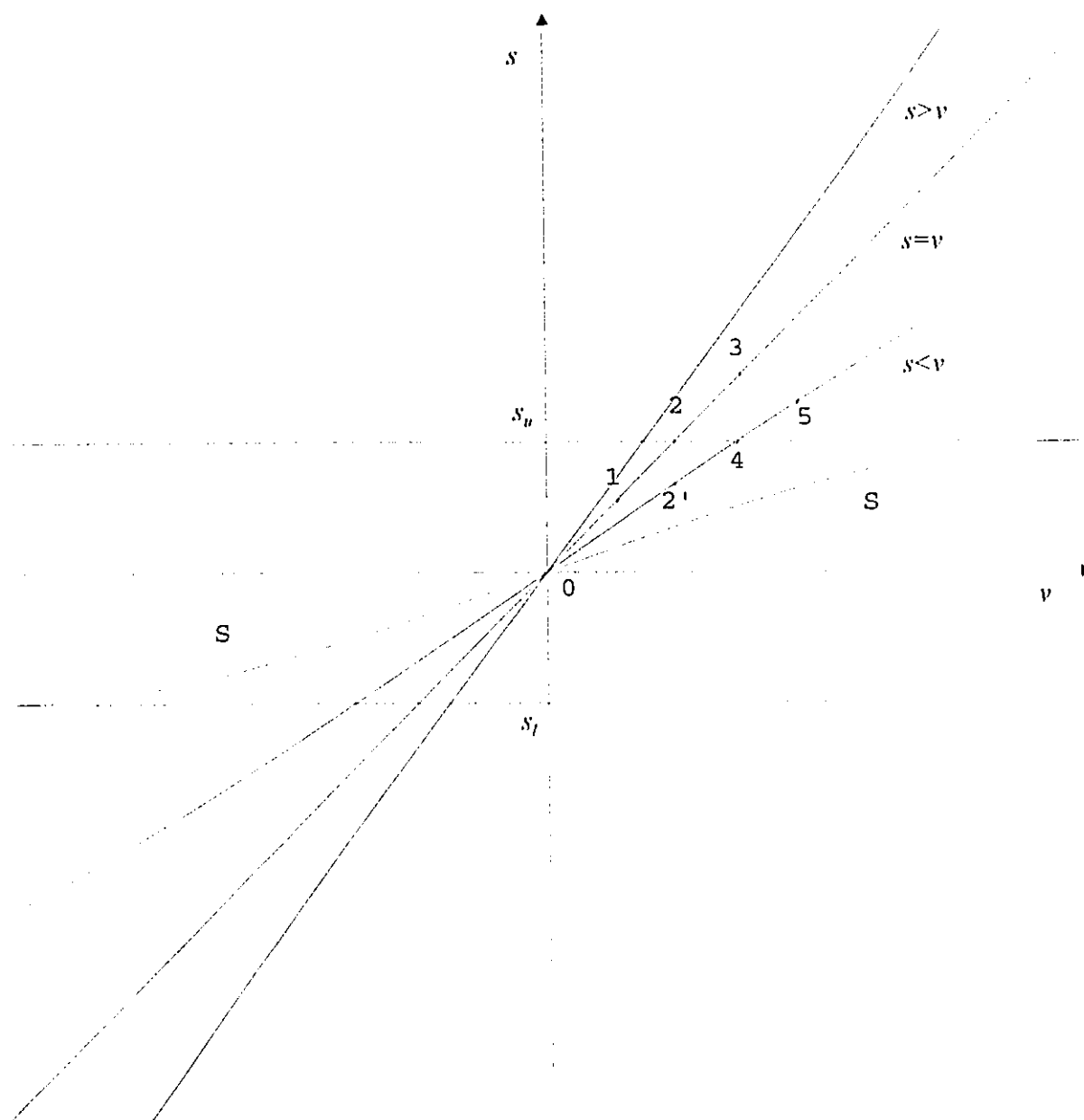


Figure 1.3 Generating the target zone path

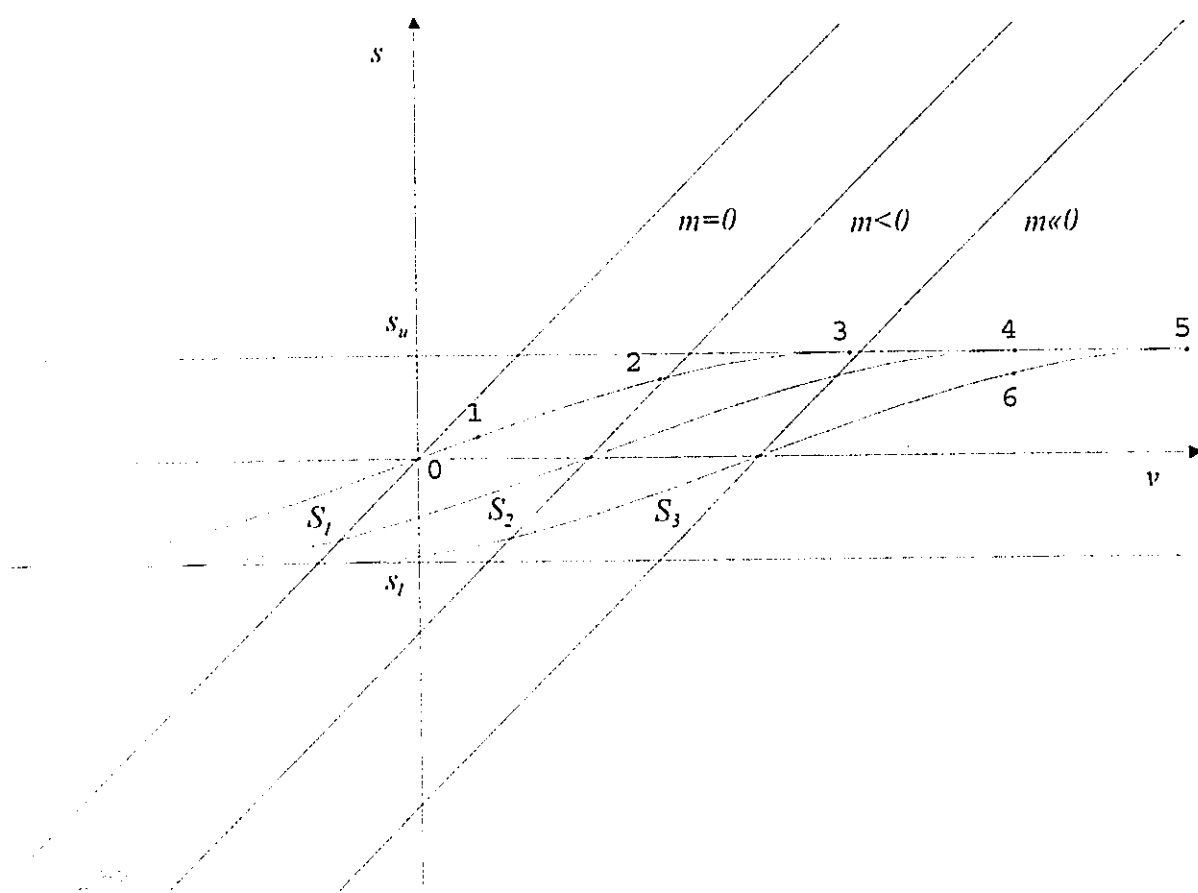


Figure 1.4 Money supply behaviour in the target zone model

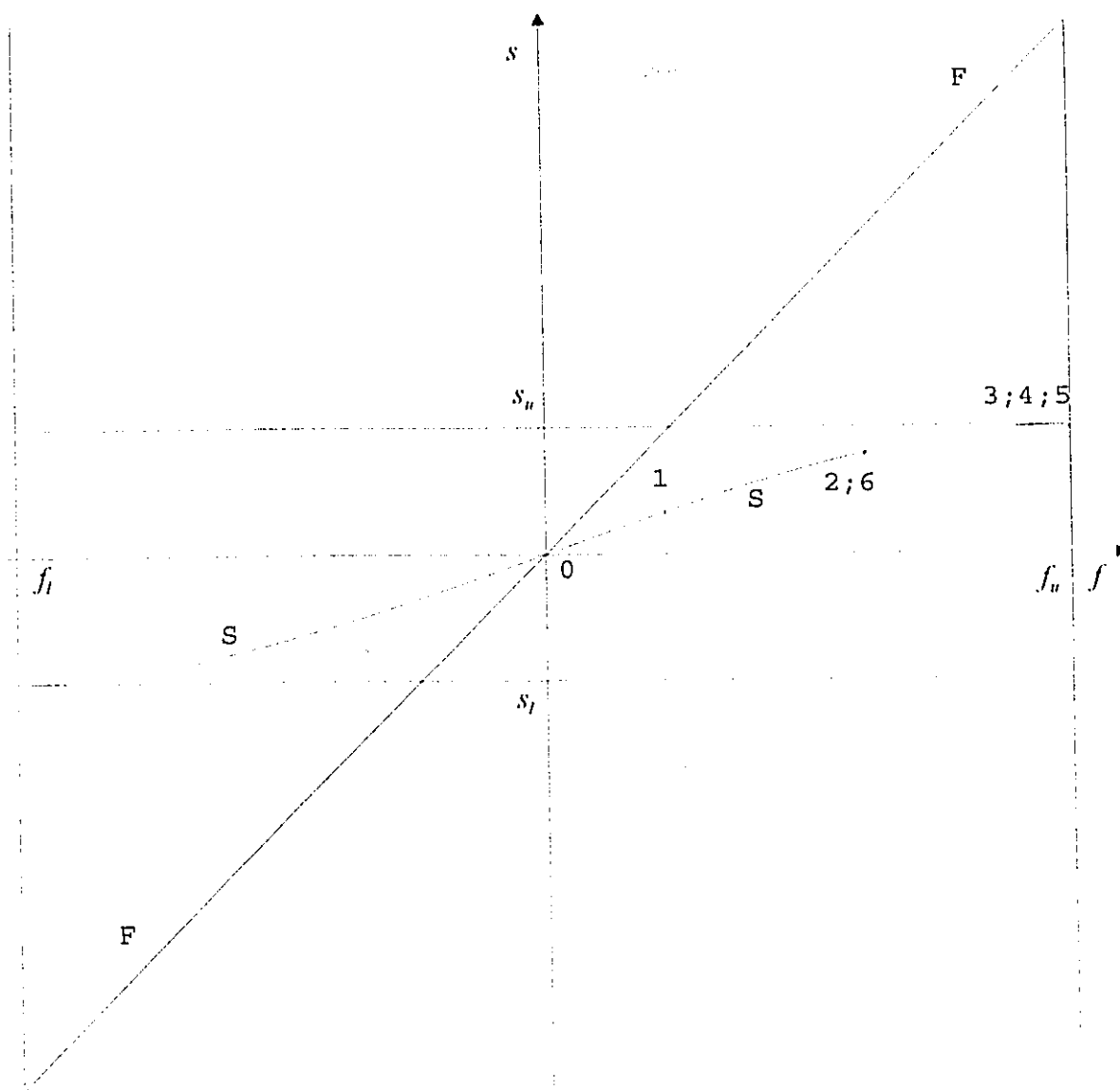


Figure 1.5 The implicit fundamental zone in (f, s) space

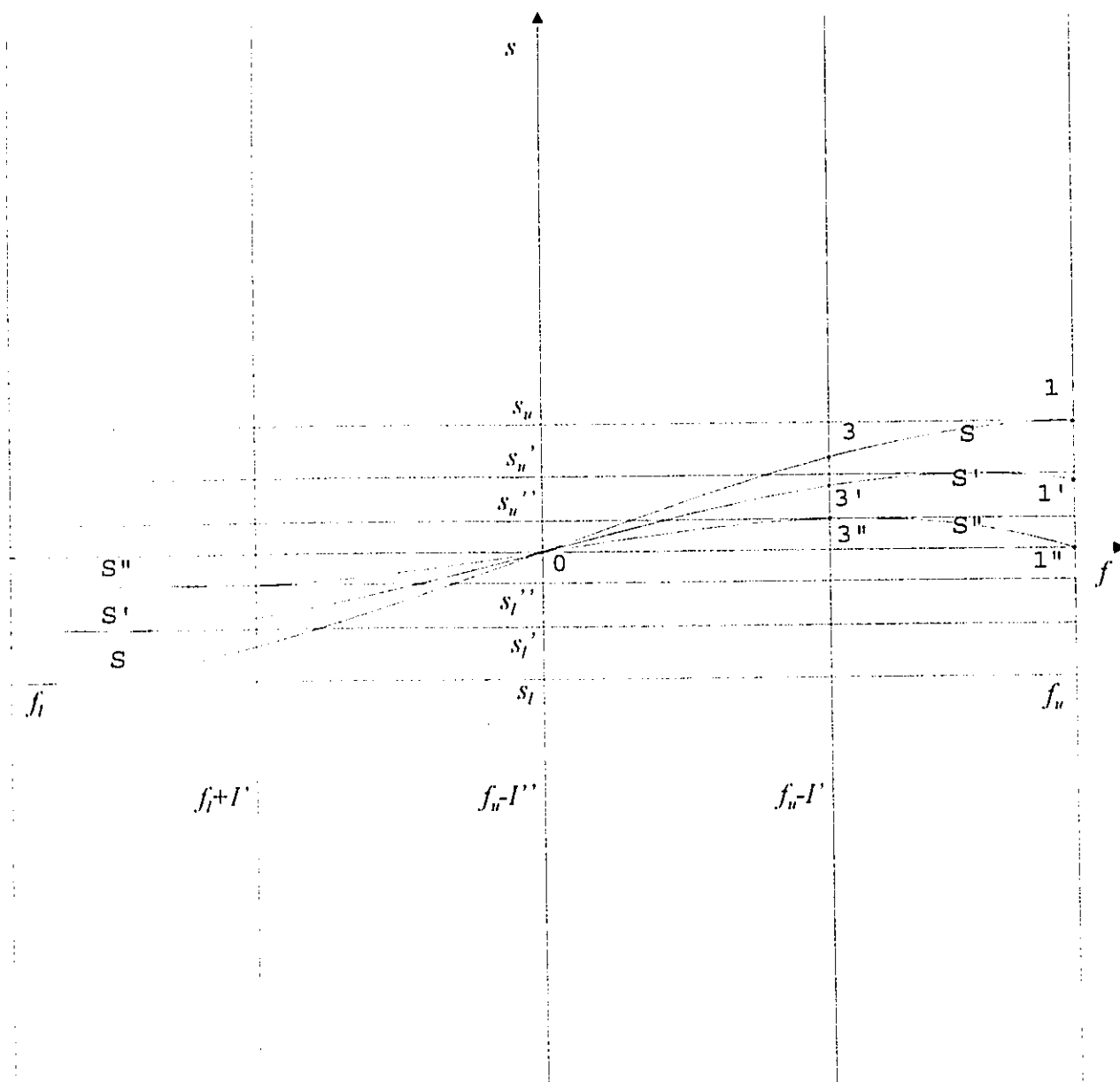


Figure 1.6 Flood and Garber target zone in (f,s) space

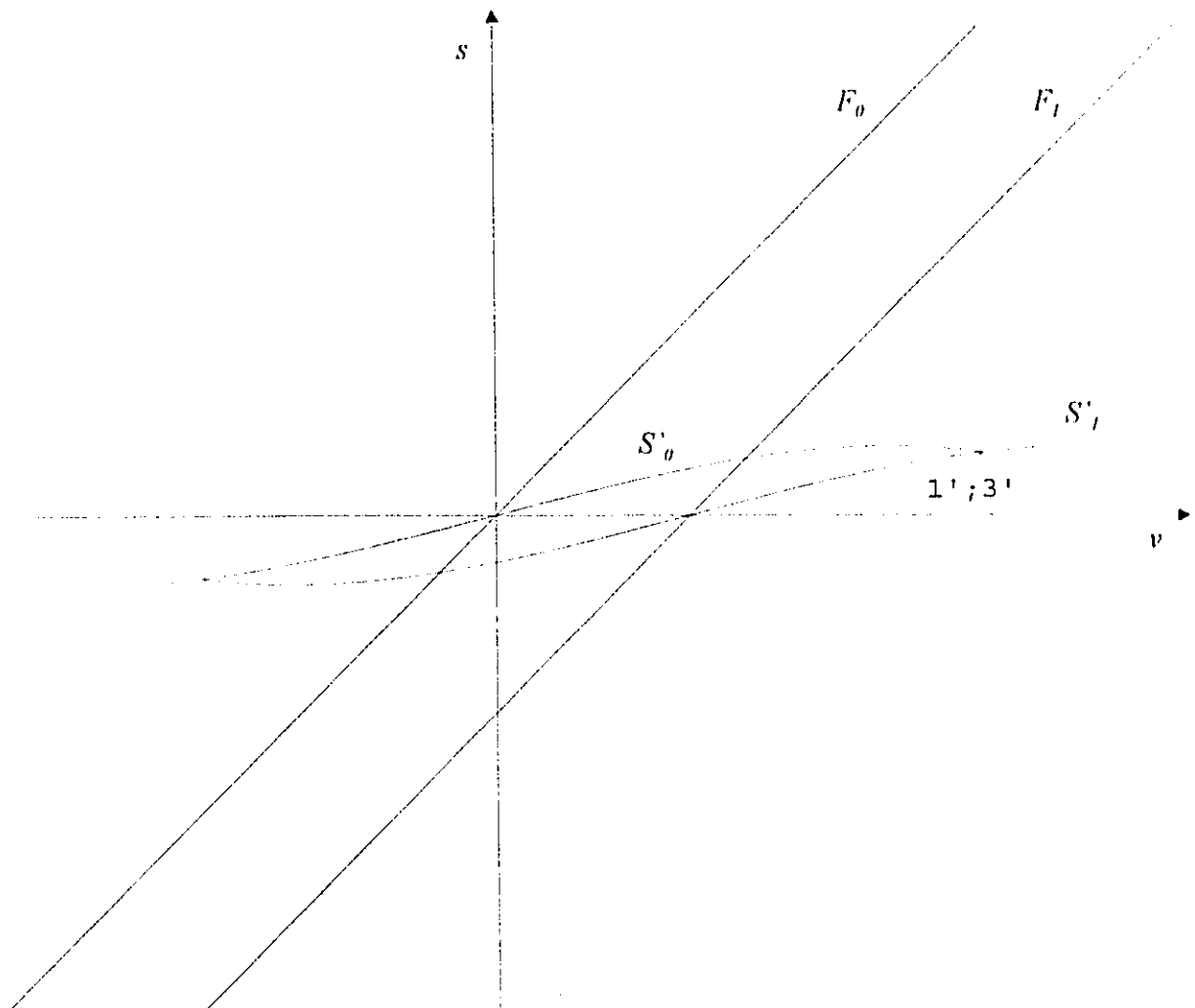


Figure 1.7 Flood and Garber model in (v, s) space

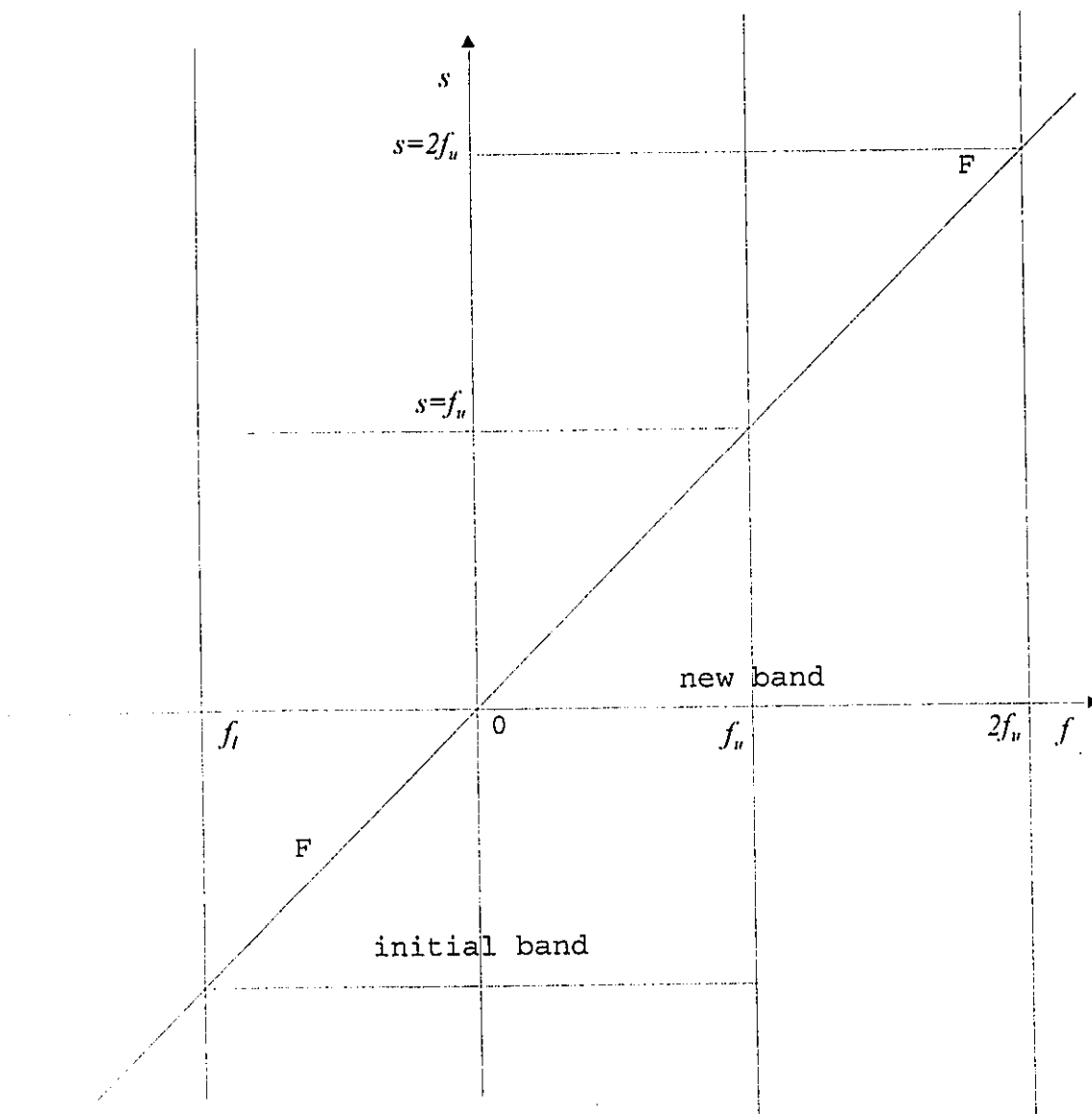


Figure 1.8 Realignment, case 1

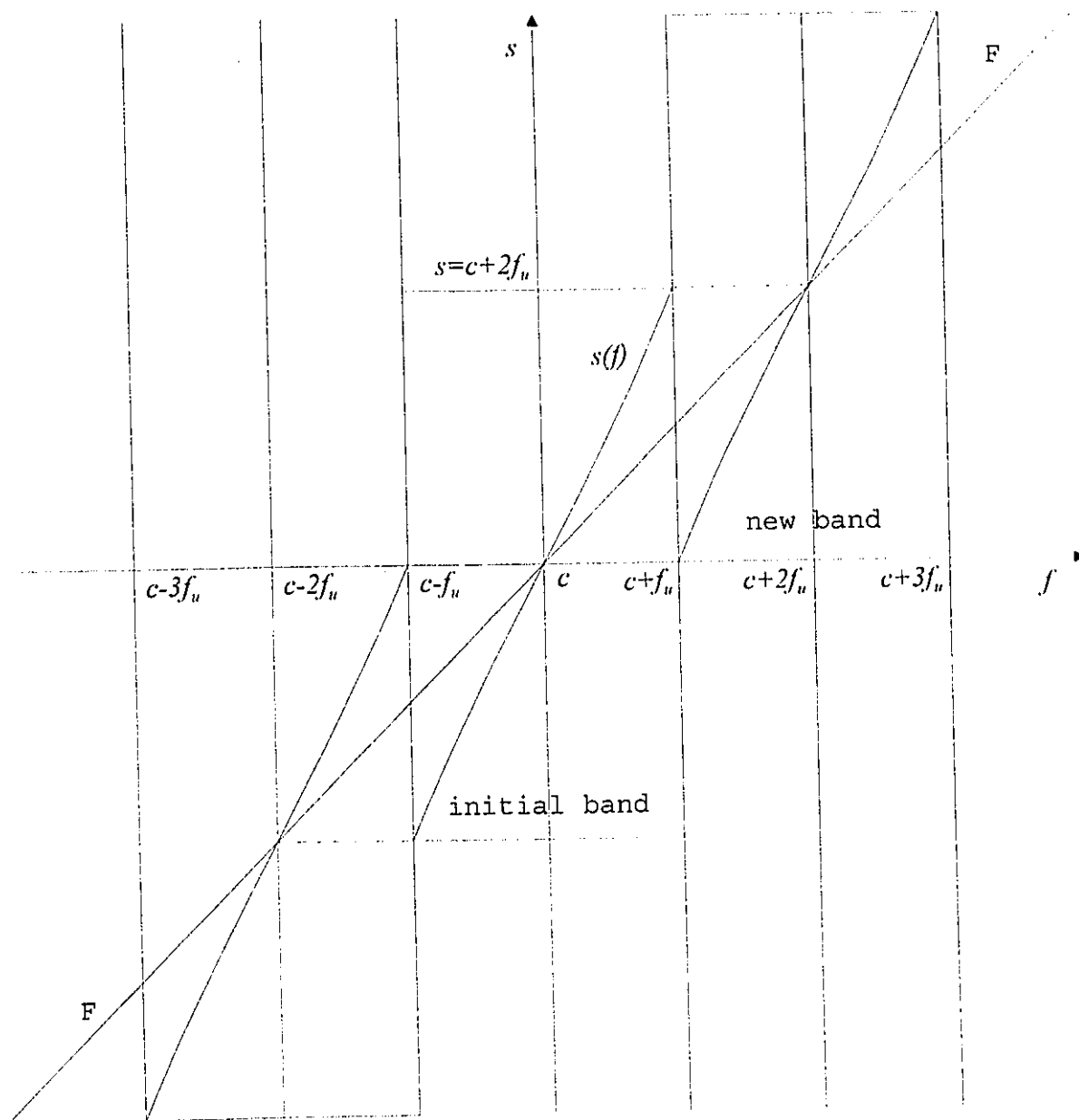


Figure 1.9 Realignment, case 2 ($p=1$)

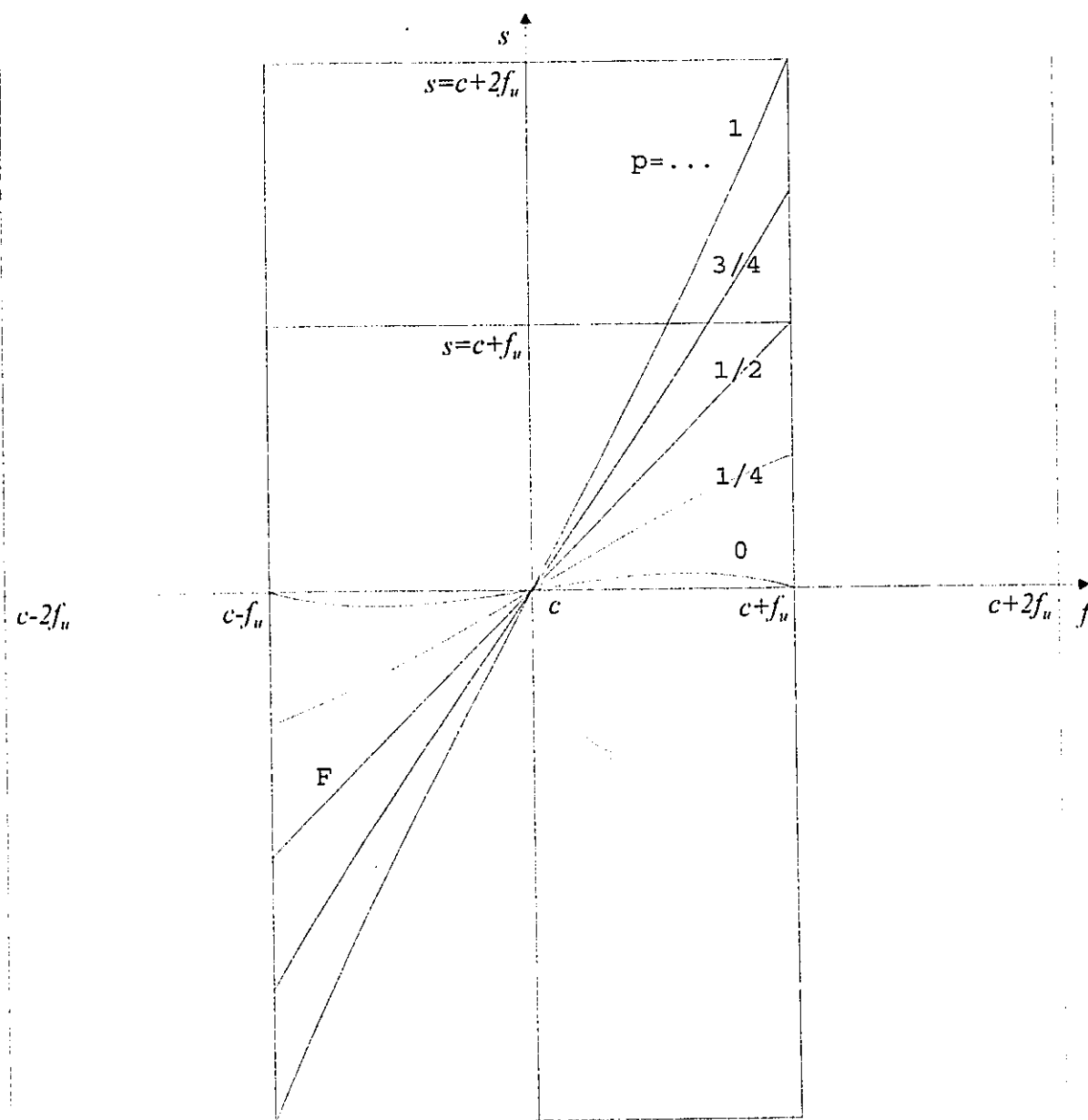


Figure 1.10 Bertola and Caballero realignment theory

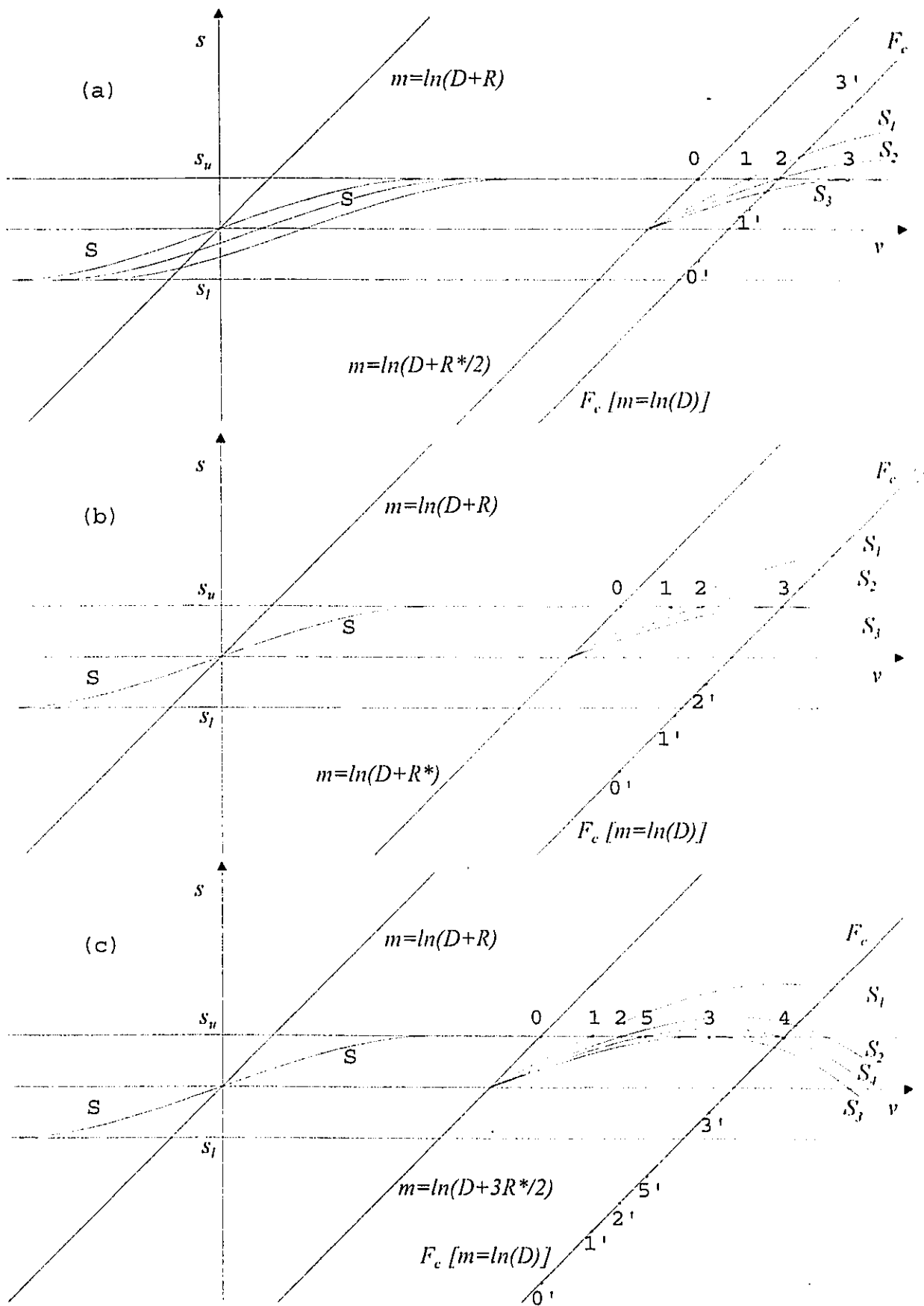


Figure 1.11 Speculative attacks in a target zone

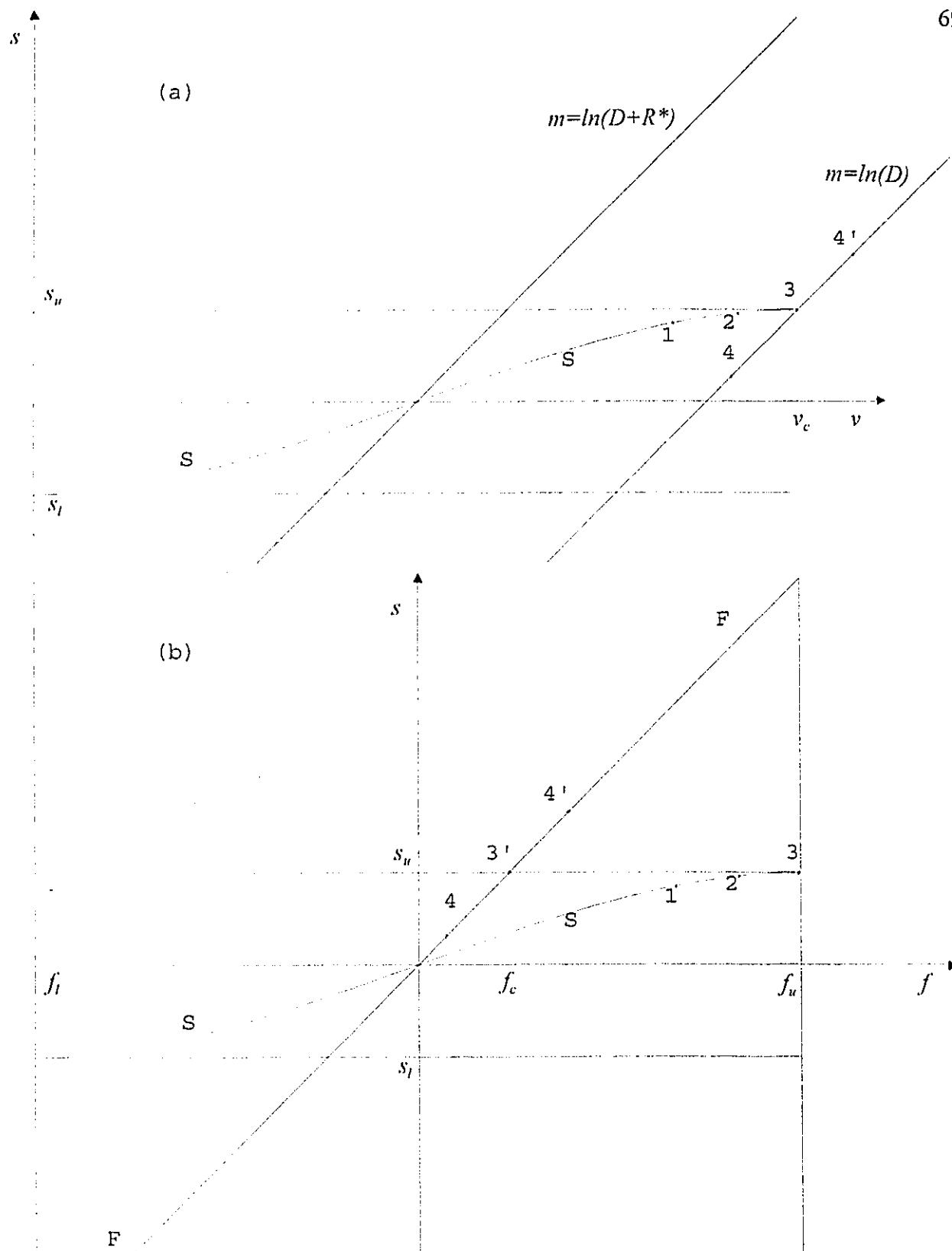
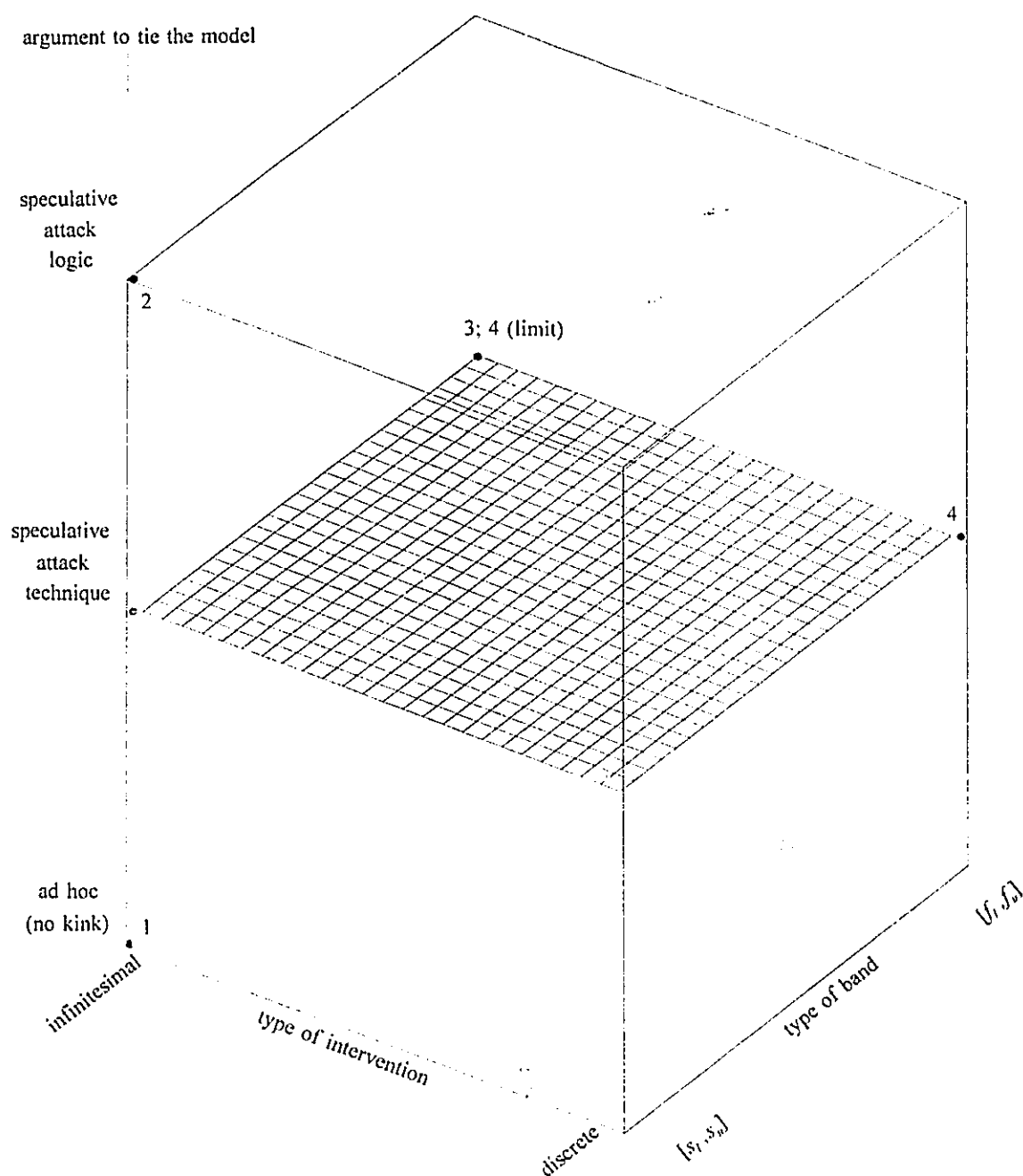


Figure 1.12 Speculative attacks in (v, s) and (f, s) spaces



1: Krugman (1991)
 2: Krugman and Rotemberg (1992)

3: Froot and Obstfeld (1991b)
 4: Flood and Garber (1991)

Figure 1.13 A classification of the literature

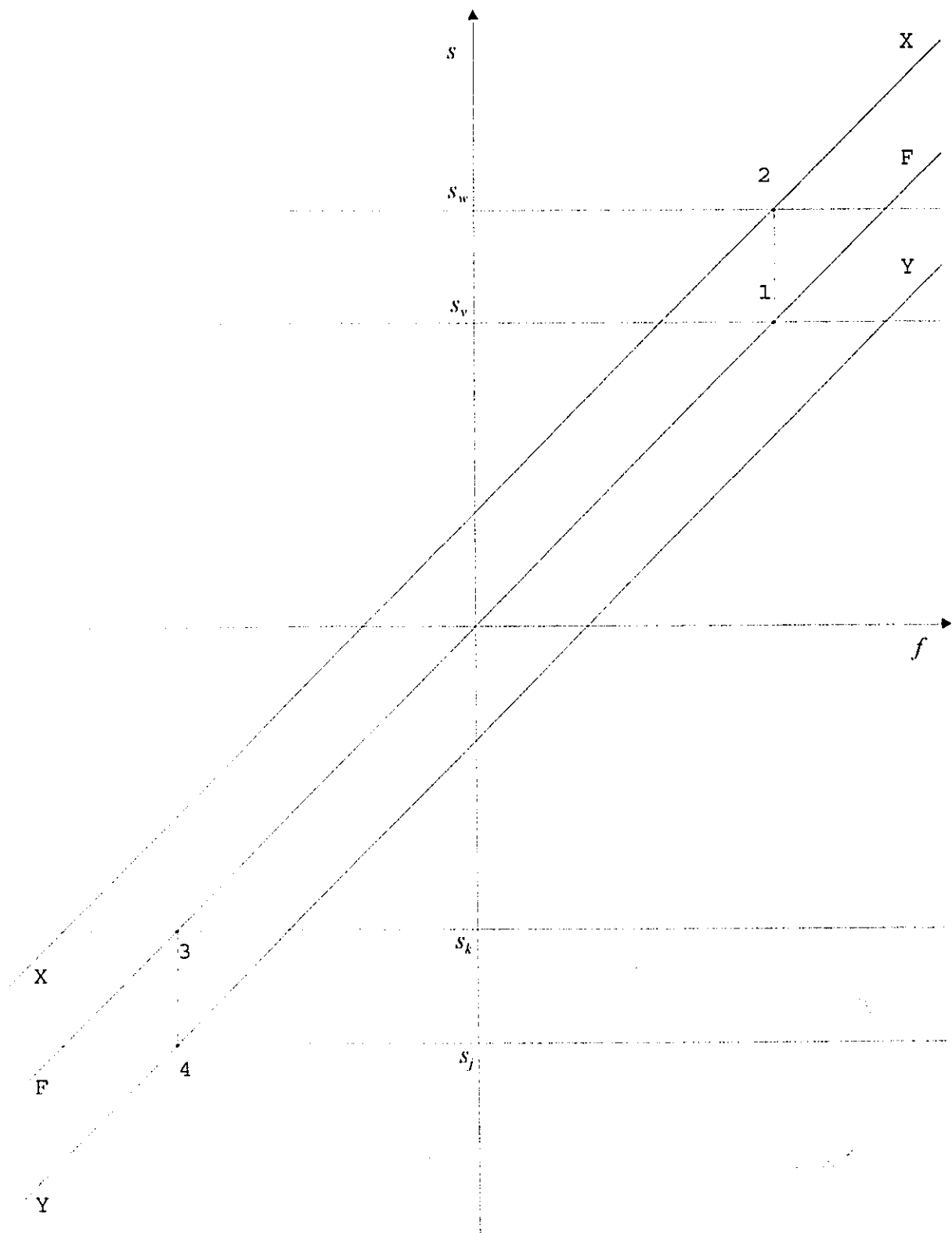


Figure 1.14 Stop-loss strategies and exchange rate crashes

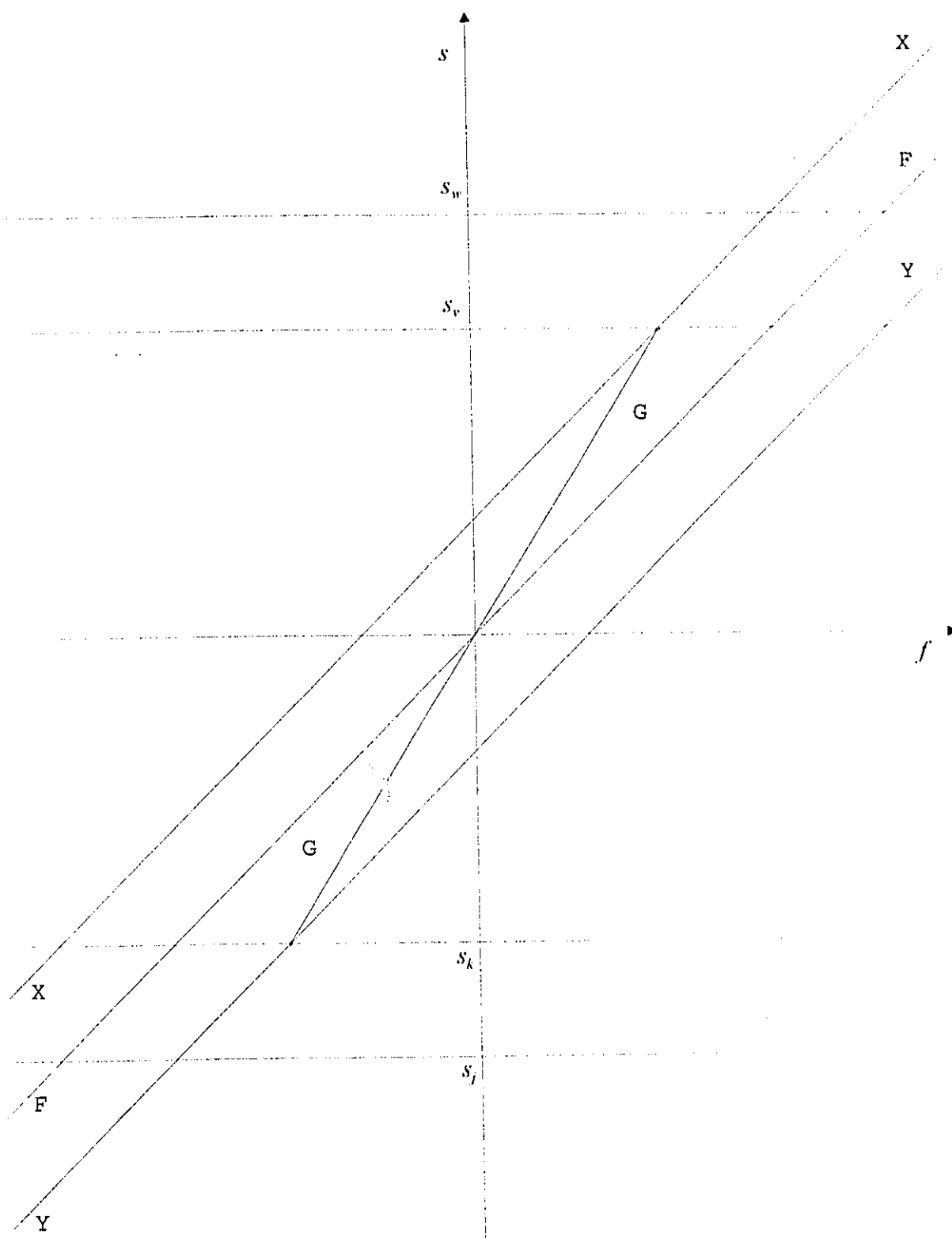


Figure 1.15 Free-float with fully expected stop-loss strategies

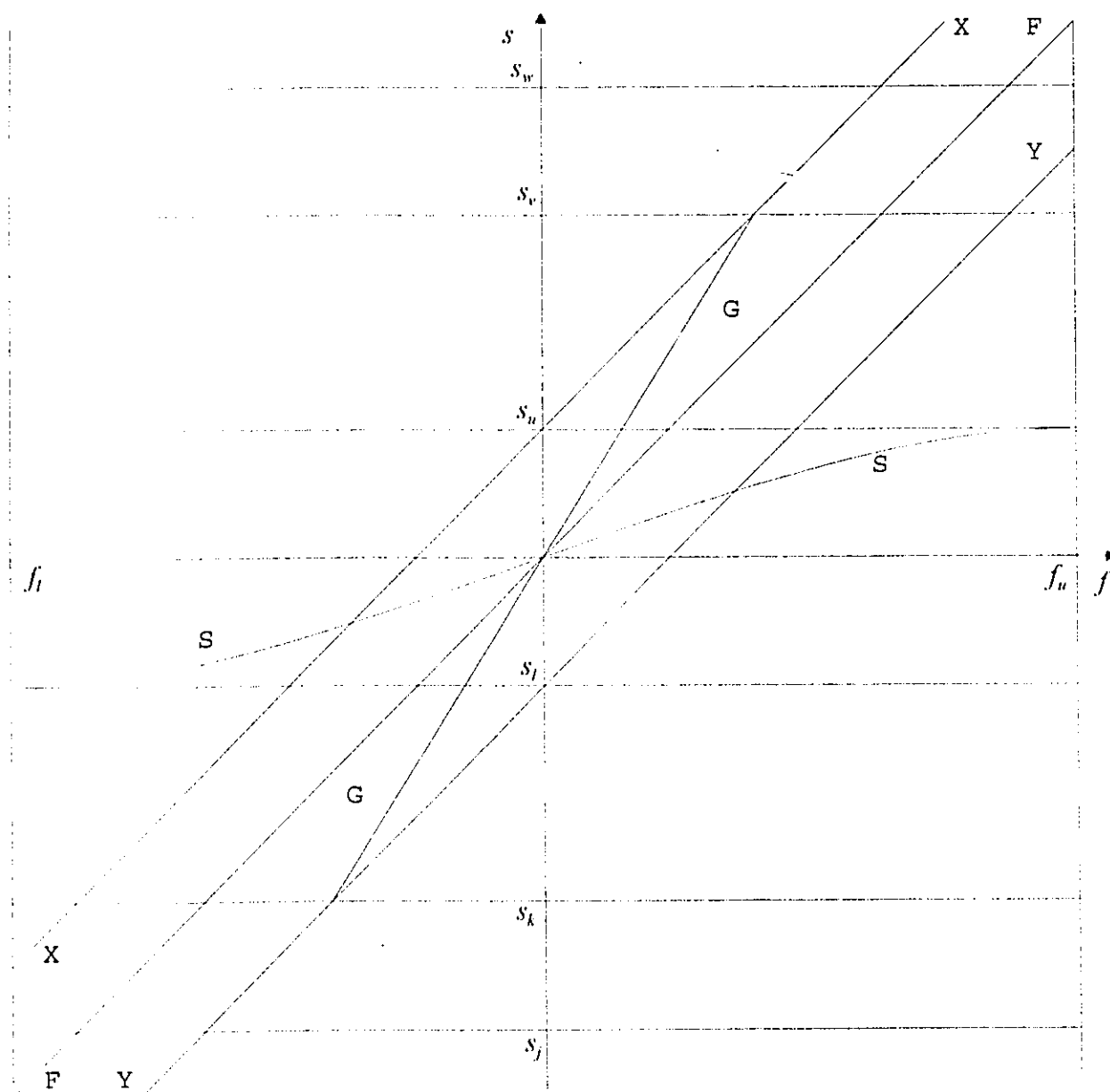


Figure 1.16 The full stabilisation effect of a target zone

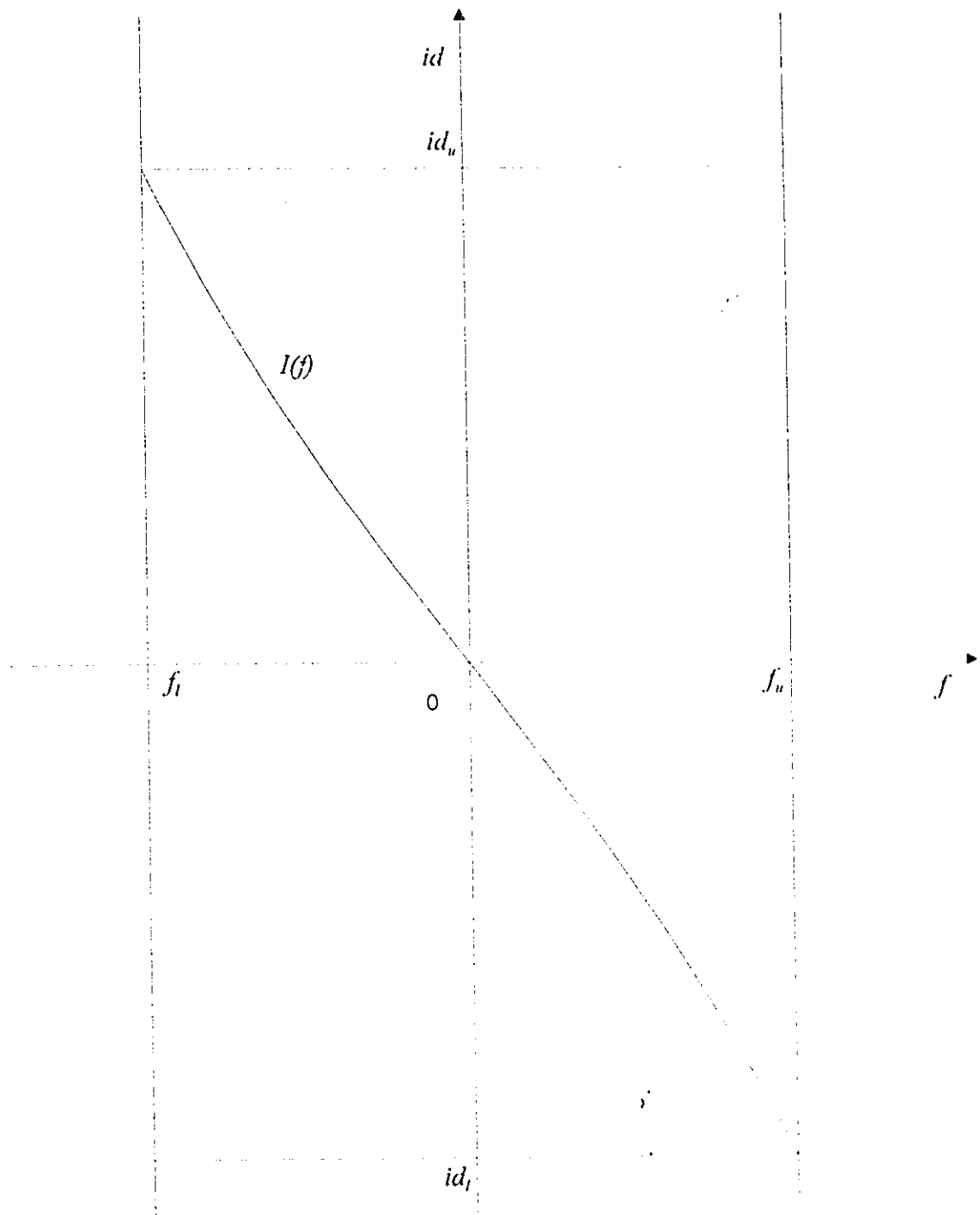


Figure 1.17 Interest rate differentials in a target zone

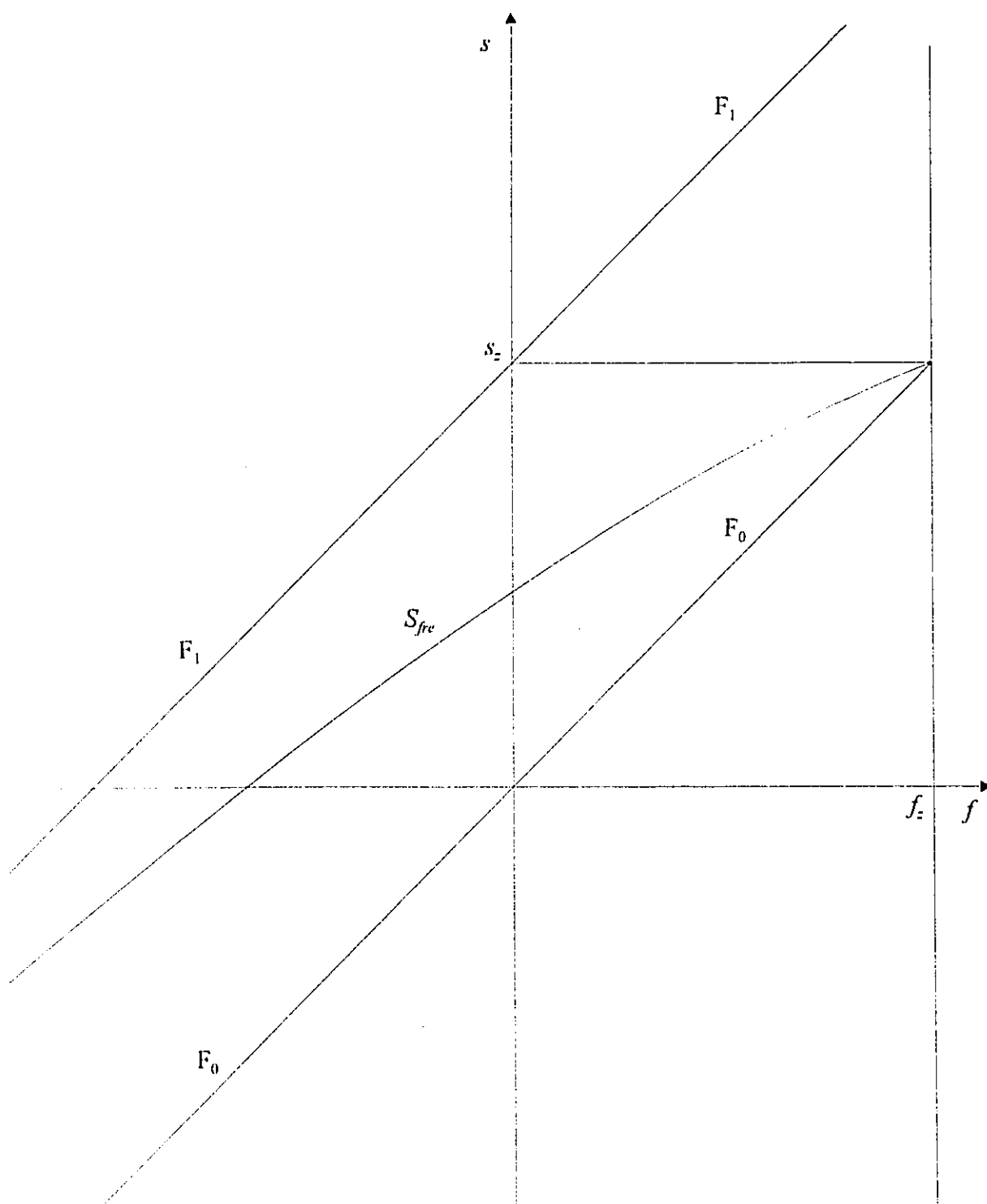


Figure 1.18 Fixed exchange rate entry path

2 A MODEL OF STATE-DEPENDENT TARGET-ZONE ENTRY

Abstract. Using the theory of regulated Brownian motion, this chapter derives the nonlinear relationship between the nominal exchange rate and its fundamentals during the transition period from the current free-float to a specified target zone (TZ), triggered at an announced state-dependent switch. It is shown that the derived nonlinear relationship is in general different from the relationship corresponding to a return to a fixed exchange rate regime. This is due to a "reflecting effect", the mirror image of the well known "honeymoon effect" of a target zone, and a "bandwidth effect". Also derived is a locus of "benchmark cases" demarcating the factors that lead to an immediate appreciation or depreciation of the domestic currency at the announcement of the future TZ.

2.0 Introduction

From time to time, countries fundamentally change their exchange rate regime. For example, the establishment of the European Monetary System (E.M.S.) in 1979 signalled a major re-orientation from a free-float (or "floating snake") to a target zone exchange rate regime. It came as a reaction to the large variability of Community exchange rates during the 1970s, which was seen as endangering the process of economic and political integration in Europe. [Sterling joined the Exchange-Rate Mechanism in October 1990 and left it in September 1992.] After the collapse of the U.S.S.R., the newly-independent countries also faced the difficult decision of choosing the most suitable exchange rate regime for their economy. As mentioned by Mundell (1995, p.25): "A major problem in these countries is to establish a price system that corresponds to scarcity relationships. The best way to do this is to import scarcity relationships of a stable foreign country and this can be achieved best by fixed exchange rates." Even in Canada, a country with a long history of a freely floating currency, proposals for a target zone with the U.S. dollar have

been advocated by some economists as a logical consequence of the North-American Free-Trade Agreement (NAFTA) [see Harris (1993)].

Since the exchange rate is determined partly by agents' beliefs about future events, policy makers deliberations and announcements inject the probabilities that exchange rate regime switches will occur in the future into current exchange rate determination. The present chapter studies the behaviour of the nominal exchange rate during the transition period from a free-float to a future target zone regime (TZ), triggered by an announced state-dependent switch.¹ [For a model of time-dependent TZ entry, see Ichikawa, Miller and Sutherland (1992).]

The chapter has two objectives. The first is to show that the derived nonlinear relationship between the exchange rate and the "fundamentals" during the transition period to a TZ is in general different from the relationship corresponding to a return to a fixed exchange rate regime, conjectured by Flood and Garber (1983), and derived, with different methods, by Smith (1991), Smith and Smith (1990), and Froot and Obstfeld (1991a).^{2,3}

The second objective of the chapter is to understand the determinants of the immediate jump in the exchange rate due to the announcement of a major exchange rate regime switch to a TZ. Indeed, the announcement provides information on the future monetary policy relative to the present stance. As should be expected with an asset theory of exchange rate determination, this new information implies an immediate jump in the exchange rate due to the market expectation of an important monetary policy shift.

The outline of the chapter is the following. Section 2.1 derives a closed-form solution for the exchange rate path during the transition period, (i.e., from the announcement of the future TZ to its effective implementation). Section 2.2 analyses the TZ-entry path in terms of two effects: the

reflecting and trend effects, on the exchange rate, of a target zone announcement. Section 2.3 studies the relative weight of the two effects by deriving a locus of "benchmark cases" where they are exactly offsetting. Section 2.4 examines the effect of the announced TZ bandwidth on the relative impact of the reflecting and trend effects. In section 2.5, the possibility of a discrete jump in the fundamentals of the exchange rate at the moment of the regime switching is envisaged. Section 2.6 explains the difference between entering a target zone with infinitesimal interventions and entering a target zone with discrete controls at the bounds. Convergence issues are introduced in section 2.7, and concluding comments are made in section 2.8.

2.1 The target zone entry path

Let $s(t)$ denote the logarithm of the exchange rate at time t , measured in units of home currency per unit foreign currency. As is commonly assumed in the literature, the exchange rate satisfies the relationship:

$$(2.1) \quad s(t) = f(t) + \theta E_t[ds(t)]/dt$$

where $f(t)$ denotes the fundamental determinants of the exchange rate, $E_t[ds(t)]/dt$ is the expected rate of depreciation of the domestic currency, and θ (a positive parameter) measures the extent to which the exchange rate level depends on its own expected rate of change.

$f(t)$ is assumed to follow an exogenously given stochastic process, more precisely, a Brownian motion with instantaneous variance, σ^2 , and trend (drift), μ (positive, equal to zero or negative).

The increments in f are then given by a stochastic differential equation:

$$(2.2) \quad df = \mu dt + \sigma dz$$

where z is a standard Wiener process. With this process, as shown by Froot and Obstfeld (1991b), the saddle path for the exchange rate will be a particular member of the **family** of paths, $G(f)$, given by:

$$(2.3) \quad s = G(f) = f + \theta\mu + A_1 \exp[r_1 f] + A_2 \exp[r_2 f]$$

where r_1 and r_2 , the roots of the characteristic equation implied by the model are:

$$(2.4) \quad r_1, r_2 = \{-\mu \pm [\mu^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2 ; r_1 < 0 < r_2$$

and A_1 and A_2 are two constants of integration to be defined through boundary conditions specific to the announced regime switch. Note that for $\mu = 0$, (i.e., the zero trend case), equations (2.3) and (2.4) become:

$$(2.3') \quad s = G(f) = f + A_1 \exp[-rf] + A_2 \exp[rf];$$

and:

$$(2.4') \quad r = r_2 = -r_1 = [2/\theta]^{1/2}/\sigma > 0.$$

If the free-float regime is perceived as permanent (i.e., no anticipated regime switch), A_1 and A_2 are simply set equal to 0 in equation (2.3) through the "no speculative bubbles" condition. The resulting exchange rate saddle-path in a permanent free-float, $S_f(f)$ [a specific member of the family $G(f)$], is thus:

$$(2.5) \quad s = S_f(f) = f + \theta\mu.$$

Different values for the trend in the fundamentals, imply different 45-degree paths, as shown in figure 2.1, where F_1 and F_2 correspond to positive (depreciating) trends, whereas F_3 is the free-float path corresponding to a zero trend and F_4 corresponds to an appreciating trend ($\mu < 0$).⁴

<figure 2.1 here>

Suppose now that a country, currently in a free-float regime, is considering a switch to a permanent target zone without drift, $[s_l, s_u]$, symmetrically centred around a central parity, c , normalised to 0. s_l is the lower (strong) bound for the domestic currency, whereas s_u is the upper (weak) bound. The analysis of the behaviour of the exchange rate in the transition period to the permanent target zone consists of the following steps:

- i) derive the exchange rate path for the planned TZ;
- ii) locate the current free-float situation relative to the planned TZ and specify a particular state of the exchange rate, s^e , enforced by a particular level of the fundamentals, f^e , that corresponds to the TZ entry-triggering state; and
- iii) derive the corresponding exchange rate TZ-entry path during the transition period with the use of the particular TZ entry-triggering state, (f^e, s^e) , as a boundary condition.

The first step essentially concerns the standard target zone model. This model is briefly reviewed in subsection 2.1.1. The second and third steps are examined in subsections 2.1.2 and 2.1.3.

2.1.1 The target zone model

Since the initial TZ model of Krugman (1991), different methods to derive the exchange rate path inside a target zone have been introduced. The method of regulated Brownian motion is reviewed here.

As emphasized by Flood and Garber (1991), Froot and Obstfeld (1991b), and Svensson (1991), it is not sufficient just to specify that interventions will occur when the exchange rate reaches the edges of the target zone, $[s_l, s_u]$. This is because there are several ways to intervene to sustain the exchange rate band. In order to obtain a determinate solution, it is necessary to specify

exactly how interventions are undertaken.

In light of the above, assume that infinitesimal and unsterilised interventions are made to prevent the fundamentals from moving outside a specified band, $[f_l, f_u]$. Specific controls are applied at f_l and f_u , respectively the lower and upper edges of the band, symmetrically positioned around $c = 0$ ($f_l = -f_u$). The increments in f are given by:

$$(2.6) \quad df = \sigma dz + dL - dU$$

where dL and dU are the smallest unsterilised interventions that confine f to $[f_l, f_u]$. These interventions are zero except at f_l where $dL > 0$ and at f_u where $dU > 0$. In the terminology of Harrison (1985), the situation described above is a control problem with instantaneous reflecting barriers. Equation (2.6) specifies that the fundamentals follow a Brownian motion without trend ($\mu = 0$) in the open interval (f_l, f_u) . This allows us to model an exchange rate TZ without trend.⁵

Using the boundary conditions relevant to this regime (the much-discussed smooth-pasting/value-matching conditions), the constants of integration A_1 and A_2 of equation (2.3') are easily derived:⁶

$$(2.7) \quad A_1 = -A_2 = A = 1/2rcosh(rf_u) > 0.$$

Substituting these values of A_1 and A_2 in equation (2.3'), the (symmetric) target zone path, S_{tz} , is obtained:

$$(2.8) \quad s = S_{tz}(f) = f - \sinh(rf)/rcosh(rf_u).$$

$S_{tz}(f)$, like $S_r(f)$ derived above, is another member of the family of paths, $G(f)$. This TZ path is represented in figure 2.1 as the S-S path. Observe that the band, $[f_l, f_u]$, defended by infinitesimal interventions, implies the exchange rate band $[s_l, s_u]$. Note also that $[s_l, s_u]$ is smaller than $[f_l, f_u]$, implying a stabilisation effect of the exchange rate relative to the value of the fundamentals.⁷

This specific target zone will henceforth be called TZ_0 , where the subscript, "0", means that the central parity of the zone is normalised to $c = 0$.

2.1.2 The target zone-triggering state and the current free-float situation

Suppose that the monetary authority today announces a future switch from the free-float regime to the permanent TZ_0 described in subsection 2.1.1, as soon as the exchange rate reaches s^* . One way to enforce this policy is to place a limit, f^* , on the fundamentals such that [by equation (2.8)]:

$$(2.9) \quad s^* = f^* - \sinh(rf^*) / r \cosh(rf_u).$$

Equation (2.9) guarantees that the announced TZ-triggering state is a point on the schedule $S_z(f)$. This equation will be used in subsection 2.1.3 to derive one boundary condition (for the TZ-entry problem) and to solve for one of the two constants of integration in equation (2.3). In figure 2.1, it is assumed that the TZ-triggering state is $(f^*, s^*) = (f_l, s_l)$ (i.e., the lower edge of TZ_0).

Note that, at the moment right before announcing the future TZ_0 , the domestic currency could be experiencing a free-float period with a depreciating or appreciating trend, and that the current level of the fundamentals could be either "higher" or "lower" than the TZ-triggering value, f^* , assumed equal to f_l in figure 2.1. Four scenarios are plausible. Scenario A is called the depreciating-convergent case. Points 1 or 2 in figure 2.1 belong to this scenario. This is called a depreciating case because points 1 and 2 are located on paths F_1 and F_2 , which represent two depreciating trends (i.e., $\mu > 0$). Also, points on F_1 and F_2 have a tendency to move north-east (see note 4). This means that points 1 and 2, associated presently with a level of the fundamentals, f_0 , lower than f_l , have a tendency to be heading for the announced TZ-entry

triggering state, f_t . Scenarios B, C, and D are called respectively the appreciating-convergent case (point 5 in figure 2.1), the depreciating non-convergent case (point 6), and the appreciating non-convergent case (point 4). Obviously, with scenarios A and B, the current state is converging towards the announced TZ-triggering state and, as a consequence, no trend correction is required during the transition period to the TZ regime. Scenarios C and D, on the other hand, will require a policy-induced trend revision during the same period to guarantee or speed up the entry process. Scenario A is analyzed in the following sections; scenario B is its mirror case. The non-convergent scenarios (and the policy-induced trend revisions) are studied in section 2.7.

2.1.3 The target-zone entry path

In the third step, the TZ-entry path is derived using two boundary conditions. The first boundary condition (for scenario A) is a familiar asymptotic condition: when s (f) is "small" and thus far from s^e (f^e), the event of a change of regime from a free-float to the announced TZ_0 is very unlikely. Because r_1 is the negative root, A_1 can simply be set equal to 0. Thus, the TZ-entry path is asymptotic to the current depreciating ($\mu > 0$) free-float line for "lower" values of the exchange rate, but diverges from the free-float line as s approaches the TZ entry-triggering state (f^e, s^e).

The second boundary condition is based on a continuity argument invoked to preclude excess anticipated profit opportunities. At (f^e, s^e) , even if the process governing f under a free-float [equation (2.2)] is replaced by the process governing f under a TZ [equation (2.6)], $s^e = G(f^e)$ will still be given by equation (2.3), such that:

$$(2.10) \quad s^e = G(f^e) = f^e + \theta\mu + A_2 \exp[r_2 f^e]$$

where A_1 has been explicitly set equal to 0. At (f^*, s^*) , however, the TZ is triggered such that the exchange rate is also given by equation (2.9). Equations (2.9) and (2.10) provide the boundary condition necessary to solve for A_2 . In particular, (2.9) and (2.10) must be equal to preclude excess profit opportunities, such that:

$$(2.11) \quad A_2 = -\{\theta\mu + [\sinh(rf^*)]/\text{rcosh}(rf_u)\}/\exp(r_2 f^*).$$

Substituting A_2 and $A_1 (=0)$ in equation (2.3), the TZ-entry path, $S_{ze}(f)$, is obtained:⁸

$$(2.12) \quad s = S_{ze}(f) = f + \theta\mu(1 - \exp[r_2(f - f^*)]) - \{\sinh(rf^*)/\text{rcosh}(rf_u)\}\exp[r_2(f - f^*)].$$

2.2 The reflecting and trend effects

To analyze the TZ entry path given by equation (2.12), specific entry-triggering states may be considered. If the triggering state is announced to be the lower edge of the target zone band, (f^*, s^*) is equal to (f_l, s_l) and equation (2.12) can be rewritten as:

$$(2.13) \quad s = S_{ze}(f) = f + \theta\mu(1 - \exp[r_2(f - f_l)]) - \{\sinh(rf_l)/\text{rcosh}(rf_u)\}\exp[r_2(f - f_l)].$$

As a result of the two boundary conditions imposed in subsection 2.1.3, observe from equation (2.13) that when $f = f_l$, $s = f_l - \sinh(rf_l)/\text{rcosh}(rf_u) = s_l$ [by equation (2.8)]. This means that when f hits f_l , the exchange rate is exactly at the lower edge of the TZ band. Also, when f is very "low", (much lower than f_l), $\exp[r_2(f - f_l)]$ tends to zero, such that $S_{ze}(f)$ tends asymptotically to $(f + \theta\mu)$.

In figure 2.1, different TZ-entry paths have been drawn for different values of the initial free-float fundamental trend, $\mu \geq 0$. Notice from (2.13) that if $\theta\mu > |\sinh(rf_l)/\text{rcosh}(rf_u)|$, S_{ze} is below the corresponding free-float line. This is indeed the case for S_{ze1} , which is below F_l in

figure 2.1.⁹ If $\theta\mu$ is equal to (or is less than) $|\sinh(rf_1)/r\cosh(rf_u)|$, S_{te} is on (or above) the corresponding free-float line, as it is the case for S_{te2} , overlapping F_2 (or S_{te3} , above F_3).

The intuition of these analytical results is as follows. Suppose that, at time t_0 , the economy is at point 1 in figure 2.1, with a value of the fundamentals of $f_0 < f^* = f_t$, as required in scenario A. At t_0 , there is an announcement of the future TZ_0 (described in subsection 2.1.1) to be implemented when s hits s_t . Based on figure 2.1, this announcement has an immediate impact on the exchange rate: an initial appreciation of the domestic currency is observed, such that the exchange rate jumps from point 1 to point 1'. If the economy was initially at point 2, there would be no initial jump in the exchange rate at time t_0 . Finally, if the economy was at point 3, we would observe, at t_0 , an initial depreciation of the domestic currency (i.e., an initial increase in s from point 3 to point 3'). How is it that the same TZ announcement can generate different initial jumps in the exchange rate? Basically, these jumps are the result of two different effects that sometimes are offsetting and sometimes reinforcing.

The first effect comes from the comparison (done by the market) of the current free-float trend, μ , with the trend of the target zone, assumed to be zero. For example, if the initial trend is positive (as is the case at points 1 or 2), the market expects a future attempt, by the domestic country, to reduce its money supply trend **during** the TZ_0 regime (to keep the domestic money supply in line with the foreign money supply, which is assumed to remain constant). The expectation of this future (coordination of) monetary policy generates an appreciation of the domestic currency at time t_0 : there is a decrease in s . This effect can be called the "trend" effect of a TZ entry problem. The analytical expression of this trend effect is given by:

$$(-\theta\mu)\exp(r_2(f_0-f_t)),$$

where $-\theta\mu (= \theta - \theta\mu)$ compares the current free-float trend, μ , with the zero-trend of the TZ_0 , and $\exp(r_2(f_0 - f_t))$ is a weighting term measuring the relative proximity of the regime shift.^{10;11} Accordingly, this effect is zero if the initial trend of the free-float is $\mu = 0$, as is the case at point 3, but is negative (s decreases) at points 1 and 2 ($\mu > 0$).

The trend effect cannot entirely explain the initial jump in the exchange rate at t_0 . Indeed, when the (domestic) central bank is announcing the future TZ_0 , the market has to take into account not only the promise of a future domestic monetary policy in line with the foreign stance, but also a set of precise monetary rules to defend the announced TZ_0 . This set of rules, in particular the infinitesimal and unsterilised interventions at the lower and upper edges of the band, was outlined in subsection 2.1.1. This second effect can be called the "reflecting" effect and its analytical expression is given, for the announced entry at f_t , by:

$$\{-\sinh(rf_t)/\cosh(rf_u)\}\exp(r_2(f_0 - f_t))$$

which is positive (note: $f_t < 0$) such that s has to increase at time t_0 .^{12;13} This generates a depreciating effect on the exchange rate at time t_0 , whatever the initial situation of the economy (i.e., whatever the initial point 1, 2, or 3 in figure 2.1).

Both the trend and reflecting effects have to be taken into account to understand the initial jump in the exchange rate at time t_0 . This explains the variety of jumps illustrated in figure 2.1. For example, at point 1, the trend effect is larger, in absolute value, than the reflecting effect; thus, the exchange rate has to decrease. At point 2, the effects are exactly offsetting, such that the exchange rate does not jump. Finally, at point 3, the trend effect is zero and the reflecting effect is positive, such that s increases.

Until now, we have taken $(f', s') = (f_t, s_t)$ as the TZ entry-triggering state. But, actually, any

point on the TZ path could have been selected. For example, another natural candidate could be the central parity of the TZ. In this case, $(f^e, s^e) = (0, 0)$ and equation (2.12) becomes:

$$(2.14) \quad s = S_{tz}(f) = f + \theta\mu(1 - \exp[r_2 f]).$$

The graph of this path is given, in figure 2.2, for alternative values of $\mu \geq 0$.

<figure 2.2 here>

As explained in notes 11 and 13, the reflecting effect is always zero for an entry at the central parity, whereas the trend effect is negative for a current $\mu > 0$ (implying an appreciation of the domestic currency at points 1 and 2 in figure 2.2), and is equal to zero for $\mu = 0$ (implying no initial jump at point 3).

This TZ entry case at the central parity is interesting because it parallels previous results from the stochastic switching literature from a free-float to a fixed exchange rate regime [e.g., Flood and Garber (1983), Froot and Obstfeld (1991a), and Smith and Smith (1990)]. For example, if the announcement is a regime switch to a permanent fixed exchange rate at $s = s^e = f^e$ (the question raised by Flood and Garber and Froot and Obstfeld), the well-known transition path is:¹⁴

$$(2.15) \quad s = S_{fc}(f) = f + \theta\mu(1 - \exp[r_2(f - f^e)]).$$

The exchange rate path for the fixed rate entry problem, $S_{fc}(f)$, obviously reduces to equation (2.14) when $f^e = 0$. This means that the TZ- and fixed rate entry paths are the same when the entry-triggering state is $s^e = f^e = 0$ (i.e., the central parity).¹⁵ However, if the TZ entry-triggering state is below and to the left of the central parity, as at (f_l, s_l) , the TZ entry path will be consistently above the corresponding fixed rate entry path derived for the switching at $s^e = f_l$.

The reason being the presence of a positive reflecting effect for a TZ entry-triggering state to the "left" of the central parity (i.e., for $f^e < 0$). Also, if the TZ entry-triggering state is above and to the right of the central parity, as for example at (f_u, s_u) , the reflecting effect is negative, and the TZ entry path will consistently be below the corresponding fixed rate entry path derived for the switching at $s^e = f_u$.

This model shows that announcing an entry into a **symmetric** TZ without trend or announcing a return to a fixed parity would generate the same dynamics during the transition period as long as the entry-triggering state corresponds to the central parity of the TZ. In all other cases, the dynamics will differ, essentially because of the reflecting effect at the time of the TZ announcement.

2.3 The locus of benchmark cases

Observe from section 2.2 that for each possible TZ triggering state, there exists a particular situation where the trend effect is exactly offset by the reflecting effect, and refer to the set of such situations as "benchmark cases". It is then possible to draw the locus of benchmark cases as the state of entry (f^e, s^e) is changed, which appears as $B-B$ in figure 2.3 and is given by:^{16;17}

$$-\sinh(rf^e)/r\cosh(rf_u).$$

<figure 2.3 here>

This locus shows the direction (appreciation or depreciation) of the initial exchange rate jump due

to the announcement of a particular TZ-entry triggering state. For example, as shown in figure 2.3, an announced specific entry state, f^* , implies a critical value of:

$$cv(f^*) = -\sinh(rf^*)/\cosh(rf_u).$$

In consequence, if the current $\theta\mu$ is larger than $cv(f^*)$, an immediate appreciation of the domestic currency will be observed at the announcement of the future TZ, whereas for $\theta\mu < cv(f^*)$, an initial depreciation will be experienced.

Note once again the difference between entering a TZ and entering a fixed rate regime. For entry into a fixed rate regime, the $B-B$ locus would actually be the horizontal axis.¹⁸ In this case, the critical value is $cv(f^*) = 0$ for all f^* . If the current $\theta\mu > 0$, we would expect an initial appreciation, whereas if $\theta\mu < 0$, there should be an initial depreciation. This comparison in the trend of the relative money supply, before and after the implementation of the new regime, is actually the full trend effect, as mentioned above. This is another way to see that the major difference between entering a TZ without drift and entering a permanent fixed rate regime is due to the reflecting effect. This latter effect (abstracting from the weighting term representing the relative proximity of the TZ implementation) is represented graphically, in figure 2.3, as the vertical distance between the locus $B-B$ and the horizontal axis.

2.4 The bandwidth effect

Until now we have considered a given bandwidth $[f_l, f_u]$ or $[s_l, s_u]$ for the announced TZ. But it is also important to understand the impact of the TZ **bandwidth** on the relative importance of the reflecting and trend effects.

It seems reasonable to argue that the reflecting effect has to be taken into account for small bands (as long as we do not enter at the central parity of the band). However, is the reflecting effect really significant for entry into larger bands? An increase in the bandwidth of the announced TZ₀ implies an increase in f_u (and a symmetric decrease in f_l). Consequently, for the **same** entry-triggering state, f^c , the critical value given by $cv(f^c) = -\sinh(rf^c)/rcosh(rf_u)$ has to decrease (in absolute value), implying a flattening of the B - B locus as the TZ bandwidth is increased, as illustrated in figure 2.4.

<figure 2.4 here>

The flattening of the B - B locus implies a decreasing weight for the reflecting effect (relative to the trend effect) on the initial jump in the exchange rate. In other words, as long as we are entering a **large** TZ at a triggering state not too far away from its central parity, the reflecting effect should have a very small impact (relative to the impact of the trend effect) on the immediate jump in the exchange rate.

Also, we get the asymptotic result that the largest (in absolute value and non weighted) reflecting effect in this model (which always happens when the TZ triggering state is the lower or upper edge of the band) is $\pm 1/r$. This result comes from the fact that, as the bandwidth is increased (and thus as f_u is increased and f_l is decreased), the critical value at the lower edge of the band, $cv(f_l) = -\sinh(rf_l)/rcosh(rf_u)$, tends rapidly to $1/r$ (and the critical value at the upper edge of the band, $cv(f_u) = -\sinh(rf_u)/rcosh(rf_u)$, tends rapidly to $-1/r$). The following proposition (valid for scenarios A and B) can be obtained:

Proposition: When the current $\theta\mu$ is larger than $1/r$, an immediate appreciation of the domestic currency is experienced at the announcement time, t_0 , whatever the TZ triggering state and whatever the bandwidth of the announced (zero trend) TZ. If $\theta\mu < -1/r$, an initial depreciation of the domestic currency is to be expected. Finally, if $-1/r < \theta\mu < 1/r$, the immediate depreciation or appreciation crucially depends on the relationship between the current $\theta\mu$, the bandwidth of the announced TZ, and the location of the triggering state inside the TZ.¹⁹

2.5 Preventing the exchange rate's jump at the TZ announcement

The derivation of the target zone entry path in subsection 2.1.3 (equation 2.12) and the analysis presented in sections 2.2 to 2.4 depend on a crucial assumption: at the moment of the regime switching to a TZ, the fundamentals (and thus the domestic money supply level) do not jump discretely. To understand the essence of this assumption, suppose that the TZ entry problem is generalised by assuming that the authorities announce a discrete jump in the fundamentals, by an amount H , at the moment of the regime switching (i.e., as soon as s reaches s^*).

The derivation of the TZ entry path under this condition follows the procedure explained in subsection 2.1.3 (i.e., it requires the use of two boundary conditions). The first boundary condition is, again, to set A_1 equal to 0 in equation (2.3) in order to approximate the linear solution as long as the regime switching is unlikely to happen.

The second boundary condition is as follows. As s reaches s^* , an intervention instantaneously raises the fundamentals by an amount H to guarantee that equation (2.9) be satisfied (i.e., to guarantee that the market be on the TZ path at the time of the regime switching). The

intervention that brings f to f^e occurs at the moment s reaches s^e so that fundamentals jump discreetly from $f^e - H$ to f^e . The market fully anticipates the change in the fundamentals as the exchange rate hits s^e such that the exchange rate should not jump. As a consequence, we have a continuity condition to prevent excess profit opportunities:

$$(2.16) \quad G(f^e - H) = s^e = f^e - \sinh(rf^e)/\cosh(rf_u),$$

where $G(f^e - H)$ is also given by equation (2.3) where A_1 is set equal to 0:

$$(2.17) \quad G(f^e - H) = f^e - H + \theta\mu + A_2 \exp[r_2(f^e - H)].$$

Using equations (2.16) and (2.17), we get:

$$(2.18) \quad A_2 = \{H - \{\theta\mu + [\sinh(rf^e)/\cosh(rf_u)]\}\} / \exp(r_2 f^e).$$

Substituting A_2 and $A_1 (=0)$ in equation (2.3), the TZ-entry path, $S_{he}(f)$, is obtained:

$$(2.19) \quad s = S_{he}(f) = f + \theta\mu(1 - \exp[r_2(f - f^e + H)]) + \{H - \sinh(rf^e)/\cosh(rf_u)\} \exp[r_2(f - f^e + H)].$$

The TZ entry path given in equation (2.19) is a generalisation of path $S_{he}(f)$ given in (2.12). Indeed, observe that by setting $H = 0$, we get equation (2.12). The implicit assumption of subsection 2.1.3 should now be clear (i.e., no discrete jump in the fundamentals at the moment of regime switching).

Another case is interesting to analyze. Assume that H is announced to be:

$$(2.20) \quad H = \theta\mu + \sinh(rf^e)/\cosh(rf_u).$$

Substituting H by its assumed value in equation (2.19), we get that:

$$(2.21) \quad s = S_{he}(f) = f + \theta\mu.$$

Under this particular case the transition path is identical to the current free-float path. This implies no jump in the exchange rate at the moment of the TZ announcement. There is thus a trade off between the immediate jump in the exchange rate at the TZ announcement (as explained

in sections 2.2 to 2.4) and the discrete jump in the fundamentals at the regime switching. From the previous sections it was derived that if, say, $\theta\mu > \sinh(rf^c)/\text{rcosh}(rf_u)$, an initial appreciation of the domestic currency at the TZ announcement should be expected. But the domestic central bank could also prevent the initial appreciation by simply announcing that the fundamentals will be increased by $H = \theta\mu + \sinh(rf^c)/\text{rcosh}(rf_u) > 0$ at the moment of the regime switching.²⁰

2.6 Entering a target zone with discrete controls

In subsection 2.1.1, the target zone rules of interventions at f_l and f_u are infinitesimal increases/decreases in the money supply. In this section, we explore the possibility of discrete changes in the money supply at f_l and f_u to defend the zone. This is a control problem with discrete (impulse) reflecting barriers and was introduced by Flood and Garber (1991) to give an intuitive interpretation of the smooth-pasting conditions in a TZ model. The exchange rate path inside a TZ with discrete interventions of size I , $S_i(f)$, is easily derived as:^{21,22}

$$(2.22) \quad s = S_i(f) = f - I \sinh(rf) / \{ \sinh(rf_u) - \sinh(r(f_u - I)) \}.$$

Using the procedure presented in subsections 2.1.2 and 2.1.3, it is straightforward to derive the target zone entry path (for scenario A) when a specific TZ entry-triggering state (f^c, s^c) is announced to the market. This path, $S_{ie}(f)$ is given by the following equation:

$$(2.23) \quad s = S_{ie}(f) = f + \theta\mu(1 - \exp[r_2(f - f^c)]) - \{ I \sinh(rf^c) / [\sinh(rf_u) - \sinh(r(f_u - I))] \} \exp[r_2(f - f^c)].$$

Using L'Hôpital's rule, observe that, at the limit, when I tends to zero (infinitesimal interventions), the term in brace brackets tends to $\{ \sinh(rf^c) / \text{rcosh}(rf_u) \}$, and equation (2.23) is reduced to equation (2.12): the results of sections 2.2 to 2.4 still hold.

For discrete interventions, however, new results are obtained. Figure 2.5 shows the locus of benchmark cases, $I-I$, for discrete interventions of size I , and the $B-B$ locus derived assuming infinitesimal interventions.²³

<figure 2.5 here>

For $f'' < 0$ the $I-I$ locus is above $B-B$, whereas for $f'' > 0$, $I-I$ is below $B-B$. This means that the reflecting effect is always larger (in absolute value) for an announced entry into a TZ defended with discrete interventions (relative to a TZ with infinitesimal interventions). Basically, the market expectations of discrete interventions at f_u (f_l) imply a larger expected rate of appreciation (depreciation) and a larger divergence between the exchange rate and the level of its fundamentals inside the band, relative to the infinitesimal interventions case. As explained in note 12, the (non-weighted) reflecting effect is the mirror image (i.e., seen from outside the TZ) of this expectation effect, and will also be larger (in absolute value) with discrete interventions. However, the vertical distance between $I-I$ (and $B-B$) and the X -axis is the reflecting effect, which explains the position of $I-I$ relative to $B-B$.

2.7 Convergence issues

Until now the initial trend was considered as purely exogenous ($\mu \geq 0$ in scenario A and $\mu \leq 0$ in scenario B).²⁴ However, this trend could also be viewed as the result of a policy action to speed up entry into TZ_0 or, at least, to ensure the convergence to the TZ-entry triggering state.

This was suggested by Miller and Sutherland (1992) in the framework of a fixed rate entry, but can also be introduced here.

Let us go back to figure 2.1 with the initial fundamentals f_0 and the announced TZ-entry triggering state, $(f^*, s^*) = (f_i, s_i)$. Depending on the current free-float trends, the economy is at points 1, 2, 3, or 4. If the economy is currently at points 1 or 2 in figure 2.1, the market will eventually converge toward s_i because the respective trends are positive.

However, if the current trend is zero as at point 3, or worse, negative, like at point 4 (scenario D), the convergence toward the entry-triggering state (f_i, s_i) is more problematic because it would depend essentially on the stochastic part of the process followed by the fundamentals. In such cases, to guarantee convergence toward (f_i, s_i) , monetary policy must be altered to reverse the current trend **before** the implementation of the TZ. The monetary authority can effectively reverse the negative or zero trends by announcing, at time t_0 , a positive rate of increase in the domestic money supply (which makes $\mu > 0$), as long as the TZ entry-triggering state is not yet reached. Such a policy of trend reversal causes the exchange rate eventually to converge to the TZ entry-triggering state, and speeds up the convergence process to this state.

The announcement at time t_0 , of the future TZ_0 and of a immediate trend reversing monetary policy to ensure the convergence, would initially depreciate the domestic currency. Figure 2.1 can help to understand this result. Assume that the market is at point 3.²⁵ At time t_0 , the central bank announces the future TZ_0 and the particular increase (from $\mu = 0$ to $\mu > 0$) in the growth rate of the domestic money supply to accelerate the process of TZ entry. Assume that this new positive μ is the trend in the fundamentals, which generates F_i if we were in a permanent free-float. If this new positive trend in the money stock were viewed as permanent, the exchange rate

would jump to point 1 from point 3 in figure 2.1. But actually, what is announced is a positive trend of μ as long as f is **below** f_l . Thereafter the growth rate of the money supply is set equal to 0. Thus, the initial depreciation in the exchange rate must be smaller: the exchange rate jumps to point 1' on the saddle-path S_{rel} .

This story is quite different from what happens to the initial jump in the exchange rate if the current positive growth rate, μ , were initially exogenous. As observed in section 2.2, the domestic country would experience an initial appreciation: the exchange rate would jump from point 1 to point 1' on the same saddle-path S_{rel} .

In conclusion, when the economy is not heading for the planned TZ_0 , the monetary authority can announce an **immediate** trend revision, to ensure or speed up convergence. In this case, part of the initial jump in the exchange rate is due to the immediate reversal of the trend.

2.8 Conclusion

This chapter applies the two-step method of regulated Brownian motion, developed by Froot and Obstfeld (1991a/b), to analyze a state-dependent TZ-entry from a free-float regime. The problem is more complex than a state-dependent fixed rate entry because of the necessity of taking into account three effects: the trend, the reflecting, and the bandwidth effects. The reflecting and bandwidth effects are simply non-existent in a fixed rate entry problem. In consequence, the exchange rate dynamics for the transition period are in general different in both problems, except in the particular case of an announced TZ-triggering state at the central parity of a symmetric TZ.

A second objective of the chapter has been to understand the immediate impact, on the

exchange rate, of the announcement of a future target zone. A locus of "benchmark cases" where the trend and reflecting effects are exactly offset has been derived. This locus is interesting because of the demarcation it draws between situations that would imply an immediate appreciation of the domestic currency, and situations that would imply an initial depreciation. It was also shown that the benchmark locus is bandwidth-dependent: a flattening of the locus is observed as the TZ bandwidth is enlarged. This implies a decrease in the significance of the reflecting effect, relative to the trend effect, in the initial jump of the exchange rate.

Endnotes

- 1) What is announced to the market is: 1) a future TZ with a central parity and a particular bandwidth; 2) the future rules of interventions to sustain the zone; and 3) a specific TZ-triggering state. The announcement of the future TZ implementation, and the TZ itself, are assumed to be fully credible. Loosely speaking, a state-dependent switch implies a change of regime when the "conditions are right". In the present chapter, the change of regime from a free-float to a target zone is triggered when a specific exchange rate level, enforced by a corresponding level of the fundamentals, is reached. The specific announced target zone entry-triggering state could be one particular exchange rate level from the target zone, not necessarily the central parity or the lower/upper edges of the band. The period between the actual announcement of the future TZ, and the TZ-triggering when a specific exchange rate is reached, is referred to as the transition period.
- 2) This nonlinear relationship implies that, during a period when agents are anticipating a stochastic exchange rate regime switch, it is not appropriate to estimate an exchange rate equation by typical linear methods.
- 3) The derivation of the nonlinear relationship between the exchange rate and the fundamentals is the focus of the exchange rate regime switching literature. The return to a fixed exchange rate regime is taken for granted, without questioning its relevance. The same position is also adopted here. However, issues related to the choice between free-float and fixed regimes have generated a great deal of literature [see the papers in Bhandari (1985)]. A result that emerges is that, for a small country, a fixed (free-float) exchange rate regime is superior to a free-float (fixed regime) when nominal (real) disturbances are dominant. More recently, target zone regimes have also been graded relative to fixed and flexible regimes [see De Arcangelis (1993) and Sutherland (1995)]. This literature derives the optimal degree of exchange rate intervention, (or the optimal regime). But because the underlying economic structure is assumed time-invariant, the optimal regime is chosen once and for all. An interesting paper by Flood, Bhandari and Horne (1989) develops a model that enables an exchange rate regime switch to be viewed as an optimal response by policymakers to the evolving state of the world.
- 4) Note that along F_3 the expected rate of depreciation equals zero such that the probability of a future increase in s is just equal to the probability of a future decrease in s . Along F_2 or F_1 , the expected rate of depreciation is positive: a future depreciation is more likely than a future appreciation: points on F_2 and F_1 have a **tendency** (a trend) to move north-east along their respective free-float lines. However, moves to the south-west cannot be excluded because this is a stochastic model. Finally, points along F_1 have a tendency to move south-west.
- 5) A TZ with zero trend implies that the domestic money supply growth rate, [in (f_1, f_n)], is identical to the foreign one, assumed equal to 0. This implies some coordination of monetary policies between both countries (the so-called "liquidity/nominal anchor"

- problem). Target zones with positive (depreciating) trends have been modelled, but are necessarily more liable to speculative attacks. See Krugman and Rotemberg (1992) and Delgado and Dumas (1993).
- 6) The smooth-pasting/value-matching conditions in a TZ are given by: $G_f(f_l) = G_f(f_u) = 0$. These conditions were initially called smooth-pasting or high-order contact (due to the close connection with smooth-pasting conditions used in option theory and in the analysis of irreversible investment), even if the term value-matching is more appropriate. See Froot and Obstfeld (1991b) and Bartolini (1993).
 - 7) This stabilisation effect has been dubbed the honeymoon effect from a reference in Krugman (1987) to a "target zone honeymoon", and is given [in equation (2.8)] by the analytical expression: $-\sinh(rf)/rcosh(rf_u)$. The honeymoon effect expresses the idea that the exchange rate progressively differs from the value of its fundamentals as the bounds are approached. This is due to the market expectations of a future intervention (to defend the TZ), which becomes more likely near the bounds. For example, in the upper part of the band, the exchange rate is always smaller than its fundamentals, due to a positive expected rate of appreciation which the market turns into an immediate appreciation. This expected rate of appreciation is triggered by the market expectations of a future infinitesimal decrease in the domestic money supply to defend the zone. As the upper bound is approached, there is an increase in the expected rate of appreciation because the intervention becomes more likely, implying an increasing over-appreciation of the exchange rate relative to its fundamentals. From a technical perspective, the **concavity** of $S_{\pi}(f) = G(f)$ in the upper part of the zone implies a **negative** expected rate of depreciation $[(1/dt)E_d G(f) = (1/2)G_{ff}\sigma^2]$, by application of Itô's lemma] given by: $-\sinh(rf)/\theta rcosh(rf_u)$ and this, even though the expected rate of change in the fundamentals **inside** the TZ is zero.
 - 8) Note that for scenario B (the appreciating-convergent case), A_2 is equal to zero, A_1 is given by: $A_1 = -\{\theta\mu + [\sinh(rf^c)]/rcosh(rf_u)\}/\exp(r_1 f^c)$, and the TZ entry path is: $s = S_{\pi}(f) = f + \theta\mu(1 - \exp[r_1(f - f^c)]) - \{\sinh(rf^c)/rcosh(rf_u)\}\exp[r_1(f - f^c)]$, with $\mu < 0$.
 - 9) Note that $\sinh(rf_l)/rcosh(rf_u) < 0$ because $f_l < 0$.
 - 10) Because $f_0 < f_l$ (in scenario A), the weighting term, $\exp(r_2(f_0 - f_l))$, is less than 1. If at t_0 , f_0 were equal to f_l (i.e., if the TZ were announced to begin immediately), the full trend effect ($-\theta\mu$) would take place. Nevertheless, for f_0 smaller than f_l the somewhat reduced trend effect, $-\theta\mu\exp(r_2(f_0 - f_l))$, should be expected.
 - 11) In the general case of TZ entry at (f^c, s^c) , the trend effect is given by $-\theta\mu\exp(r_2(f_0 - f^c))$. This means that for $\mu > 0$ ($\mu < 0$), the trend effect is negative (positive), and implies an initial appreciation (depreciation) of the exchange rate at time t_0 . For $\mu = 0$, this effect is zero (no jump in s). Observe that the trend effect depends on f^c through the weighting term only.

- 12) The second effect has been named the "reflecting" effect, essentially because it is the mirror image of the honeymoon effect of a TZ. Basically, the (non-weighted) reflecting effect is the honeymoon effect seen from **outside** the TZ. Since the honeymoon effect is considered to be an important characteristic of the TZ model, the reflecting effect has to be taken into account in a TZ entry problem.
- 13) In the general case of TZ entry at (f^e, s^e) , the reflecting effect is given by: $\{-sinh(rf^e)/rcosh(rf_u)\}exp(r_2(f_0-f^e))$. In contrast to the trend effect, the reflecting effect depends on f^e through the weighting term, but also through the term in brace brackets. This means that the sign of the reflecting term would change depending on the position of f^e relative to the central parity, 0. Consider three cases: a) if $f^e < 0$ (i.e., if TZ₀ is triggered at a level below the central parity of the zone), the reflecting effect is positive, implying an immediate depreciation of the domestic currency. The market believes that there is a higher probability for a first intervention at f_l than a first intervention at f_u . In other words, in the near future, it is more likely to see an infinitesimal money supply increase to prevent s from going below s_b than to observe an infinitesimal money supply decrease to prevent s from going above s_u . This effect is taken immediately into account at time t_0 , implying an immediate depreciation. b) If $f^e > 0$, we would get a negative reflecting effect (i.e., an immediate appreciation). Finally, c) if $f^e = 0$, the probabilities of intervention are completely offsetting, such that the reflecting effect is zero.
- 14) In a fixed exchange rate regime, the expected rate of depreciation equals zero. Therefore, by equation (2.1), we have that $s = f$. If the exchange rate is fixed at s^e , f will be fixed at $f^e = s^e$. The equality between s and f is used to derive one of the two boundary conditions for the fixed rate-entry problem. The path given in equation (2.15) has been derived by Froot and Obstfeld (1991a) with the (two-step) method of regulated Brownian motion.
- 15) Observe that equation (2.9) is used to derive one boundary condition for the TZ-entry problem. With (2.9), s^e equals f^e only if f^e is set to zero (i.e., if the TZ-triggering switch is announced to be the central parity of the symmetric zone). In this case, the TZ-entry problem and the fixed rate-entry problem have the same "entry relationship" between s^e and f^e , ($s^e = f^e = 0$), leading to the same boundary condition and the same transition path.
- 16) For entry at (f^e, s^e) , the reflecting effect is given by $\{-sinh(rf^e)/rcosh(rf_u)\}exp(r_2(f_0-f^e))$ and the trend effect is given by $-\theta\mu exp(r_2(f_0-f^e))$. The two effects are offsetting if $\theta\mu = -sinh(rf^e)/rcosh(rf_u)$. B-B in figure 2.3 is the locus of points where this equality is verified. Observe that: a) when $f^e < 0$, there is a "critical" initial situation represented by a **positive** free-float trend, $\mu_c = -sinh(rf^e)/\theta rcosh(rf_u)$, which would generate no jump in the exchange rate at t_0 ; b) if $f^e = 0$, a free-float trend of $\mu_c = 0$ would generate no jump at t_0 ; and c) if $f^e > 0$, there is an initial situation represented by a **negative** free-float trend, $\mu_c = -sinh(rf^e)/\theta rcosh(rf_u)$, which would generate no jump at t_0 . Results a), b), and c) are theoretically valid for scenarios A and B. Note, however, that the derivation of a (theoretical) critical value for μ_c does not necessarily imply that this value belongs to the

set of plausible current free-float trends assumed for the two scenarios. For example, with case c), f^c is positive and μ_c is negative. In this case, scenario A (where the current free-float trend is assumed to be positive) is not likely to deliver a no-jump result because the positive free-float trend is always larger than μ_c : an initial appreciation of the domestic currency has to be expected. Symmetrically, for case a), where $f^c < 0$ and $\mu_c > 0$, scenario B (where a current negative free-float trend is assumed) is not likely to deliver a no-jump result: an initial depreciation is to be expected.

- 17) In this chapter, we are interested in a TZ-triggering state given by (f^c, s^c) . This state is summarized by f^c , on the X -axis, in figure 2.3. Observe that [by equation (2.9)] there is a monotone non-decreasing relationship between s^c and f^c . For this reason, if s^c were on the X -axis instead of f^c , we would still get a downward-sloping B - B locus. The analytical solution for this new locus would be different, but the critical values derived in note 16 would be the same.
- 18) In a fixed rate entry problem, the trend effect is given by $-\theta\mu(\exp(r_2(f_0 - f^c)))$. The reflecting effect is always zero because the upper and lower controls are both located at f^c . In other words, infinitesimal increases or decreases in the domestic money supply to keep s at the fixed rate $s^c = f^c$ are always equally likely. In consequence, to foresee the direction of the initial jump in the exchange rate at the announcement of a future fixed exchange rate, when s hits $s^c = f^c$, one simply has to compare the current trend, μ , with the zero trend of a fixed exchange rate regime. For a current $\mu > 0$, an immediate appreciation would be experienced, whereas an initial depreciation would happen if the current trend μ were negative. The benchmark case, or critical value, of no-jump at time t_0 turns out to be $\mu_c = 0$ for all f^c ; the B - B locus is an horizontal in $\theta\mu = 0$ (the X -axis) in figure 2.3.
- 19) In other words, $-1/r < \theta\mu < 1/r$ is a condition delimiting a zone of uncertainty concerning the direction (i.e., the sign) of the exchange rate's jump at the moment of the announcement of the future TZ. Note that r is given by equation (2.4'). As a result, this condition may be rewritten as: $-\sigma/[2\theta]^{1/2} < \mu < \sigma/[2\theta]^{1/2}$. If the parameter σ is arbitrarily increased or if θ is decreased, the bandwidth of the zone of uncertainty increases.
- 20) Observe the analogy with the literature on how to stop hyperinflation [e.g., Sargent (1982)]. One stylised fact of this literature is the tendency for the level of the money supply to increase once inflation has been controlled. Indeed, rational expectations' models show that simply stopping printing money (i.e., revising the positive growth rate of the money supply to a zero rate) would involve an immediate deflation. To prevent the decrease in the domestic price level the money supply should be discreetly increased at the time of the regime switch.
- 21) In the discrete barriers control problem, the value-matching conditions are given by: $G(f_i) - G(f_i + I) = G(f_u) - G(f_u - I) = 0$. These arbitrage conditions require that there be no expected move of the exchange rate in response to an anticipated finite intervention of size I . Note that, dividing by I and taking the limit as $I \rightarrow 0$, we get, say at f_u ,

$\lim_{I \rightarrow 0} [G(f_u) - G(f_u - I)]/I = G'(f_u) = 0$ (i.e., we get the value-matching conditions of note 6). This interpretation of smooth-pasting conditions (no-foreseeable jump in the exchange rate at the time of the interventions) was suggested by Flood and Garber (1991). With these conditions and the assumption of a symmetric TZ, we get that $A_1 = -A_2 = A > 0$, with $A = 1/2\{\sinh(rf_u) - \sinh[r(f_u - I)]\}$. Substituting A_1 and A_2 by their respective values in equation (2.3') gives equation (2.22).

- 22) Defending the band $[f_l, f_u]$ with discrete interventions of size I implies a corresponding exchange rate zone, $[s_l, s_u]_{(I)}$. Note that in subsection 2.1.1, the same band $[f_l, f_u]$ is defended with infinitesimal interventions, and gives rise to an implicit exchange rate band $[s_l, s_u]$ larger than $[s_l, s_u]_{(I)}$. Observe that in the discrete interventions case, when I tends to zero, the value for A derived in note 21 is reduced, at the limit (using L'Hôpital's rule), to the value for A given by equation (2.7), and the TZ path given by (2.22) turns out to be the TZ path of equation (2.8). As a result, defending the band $[f_l, f_u]$ with discrete interventions of size I implies a corresponding exchange rate zone $[s_l, s_u]_{(I)}$ smaller than $[s_l, s_u]$, but which turns out to be $[s_l, s_u]$, at the limit, for infinitesimal interventions.
- 23) In figure 2.5, as in figure 2.3, the TZ-triggering state (f^e, s^e) is summarized by f^e , on the X -axis. As explained in note 17, as long as the TZ path is monotonically non-decreasing, substituting f^e by s^e does not alter the basic downward-sloping characteristic of the B - B locus. This is not true in figure 2.5 because the TZ path with discrete interventions [equation (2.22)] is **not** monotonic. With the exchange rate on the X -axis, the I - I locus would actually appear like a steep "S".
- 24) $\mu = 0$ is a limit case for the different scenarios.
- 25) This is actually the "limit" ($\mu = 0$) of scenario D, as described in subsection 2.1.2, where $\mu \leq 0$ and $f_0 < f^e$.

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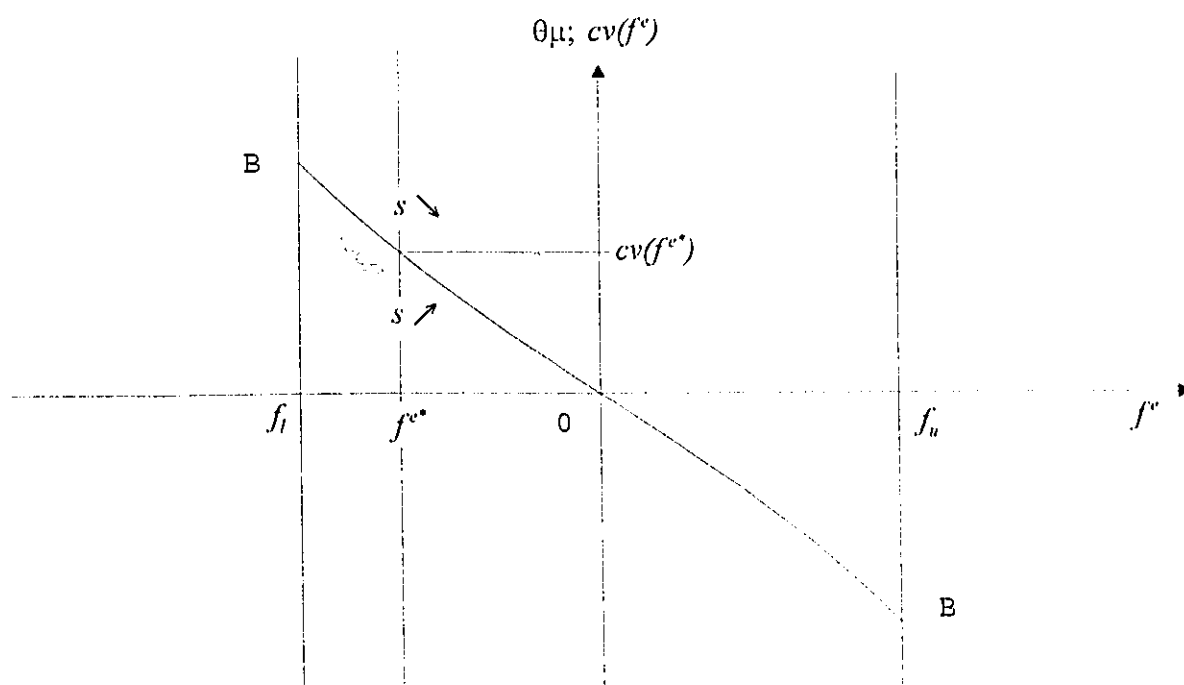


Figure 2.3 The locus of benchmark cases

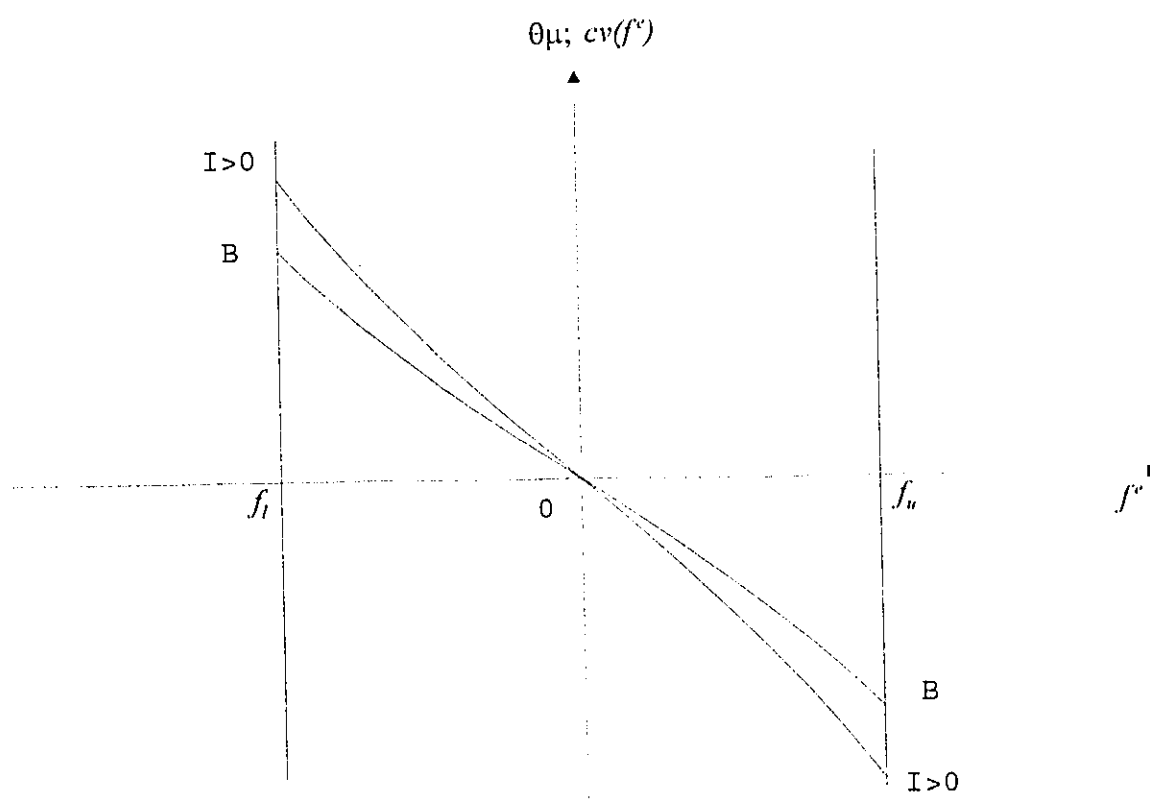


Figure 2.5 Benchmark locus with discrete interventions of size I

3 A TARGET ZONE MODEL WITH EXPECTED TREND REVISION

Abstract. The target zone model developed in this chapter incorporates the possibility of a future change in the trend in the underlying fundamentals of the exchange rate as policy reaction to specific events (e.g., impending speculative attacks and nominal anchor debates). The market has subjective expectations about this possible trend revision, which can affect the exchange rate level even if the change in trend is not implemented. These expectations are treated, first, as entirely exogenous and, second, as state-varying. In both cases various correlation patterns between the exchange rate and interest rate differentials are possible. This result is consistent with the observed behaviour of the exchange rate and interest rate differentials within the European Monetary System.

3.0 Introduction

The existing research on exchange rate target zones (TZ) has made a specific assumption about the relative monetary stance of participating countries.¹ In particular, a constant depreciating trend for the domestic currency is usually assumed, reflecting, say, a larger growth rate of the domestic (relative to foreign) money supply. Froot and Obstfeld (1991) model this trend by assuming that the fundamentals of the exchange rate follow a Brownian motion with trend (or drift). Krugman (1991) assumes that both countries have the same relative monetary stance so that the trend is set equal to zero.

The first objective of the present chapter is to introduce a TZ model with a possible trend revision (i.e., a possible change in the relative monetary stance). One rationale for including the possibility of a trend revision into a TZ model is as follows. As is well known from Krugman and Rotemberg (1992) and Delgado and Dumas (1993), a regime of TZ with trend is particularly sensitive to speculative attacks. If the TZ regime is viewed as essential for the domestic country

(that experiences a depreciating trend), speculative attack rumours could introduce sufficient incentives for the domestic central bank to revise its current positive growth rate of the money supply to a constant money supply (i.e., revising its current positive trend in the fundamentals to a zero trend). In other words, instead of accepting the forced return to a free-float with the current positive trend, the domestic country could decide to revise its depreciating trend in order to prevent the speculative attack.

A second reason for including a possible trend revision in a TZ model is as follows. A revision from the current zero trend to a positive trend in the fundamentals could be rationalized by the situation of a country facing domestic pressure to return to a more expansionary monetary policy because of some internal problems (e.g., unemployment). As recently suggested by Ball (1994), monetary expansion, in general, does not mean a rise in the **level** of the money supply (i.e., a one-time shock to the level), but an increase in money **growth rate** (trend). In the TZ regime, such a monetary expansion could be viewed as a change from a zero trend to a positive trend in the fundamentals, and would possibly be observed during periods where the monetary policy of the leader country (the nominal anchor country) seems inappropriate for other countries of the zone.

The second objective of the chapter is to show that a TZ model that allows for a trend revision can generate a realistic correlation pattern between the exchange rate and interest rate differentials. As is well known, the initial TZ model of Krugman (1991) or, in general, any TZ model with constant trend, predicts a negative relationship between the exchange rate and interest rate differentials. As shown by Flood, Rose, and Mathieson (1991) and Lindberg and Söderlind (1994), this prediction has not been verified in data observations from the European Monetary

System (E.M.S.) and from the Swedish krona (for a period when there was a unilateral target zone for the Swedish krona vis-à-vis a trade weighted currency basket). The realignment models by Svensson (1991a) and Bertola and Svensson (1993) try to produce various correlation patterns between the exchange rate and interest rate differentials. In these models, the fundamentals of the exchange rate are assumed to follow a jump-diffusion process, implying a one-time increase in the **level** of the money supply at the moment of the realignment. The present chapter focuses on a change in the **growth rate** (trend) of the money supply as an alternative way to produce a realistic correlation pattern between the exchange rate and interest rate differentials.

The outline of the chapter is as follows. Section 3.1 introduces the model of a TZ with expected trend revision. Section 3.2 provides a possible explanation of the fact that the deterministic negative correlation between the exchange rate and interest rate differentials, as predicted by standard TZ models, may not hold in practice. Section 3.3 considers the domestic central bank's revision of the current trend as a potential policy response to some economic problems (e.g., impending speculative attacks and unemployment periods). In section 3.4, state-varying market expectations of trend changes are introduced. Concluding comments are presented in section 3.5.

3.1 A target zone model with expected trend revision

Let $s(t)$ denote the logarithm of the exchange rate at time t , measured in units of home currency per unit foreign currency. As is commonly assumed in the literature, the exchange rate satisfies the relationship:

$$(3.1) \quad s(t) = f(t) + \theta E_t[ds(t)]/dt$$

where $f(t)$ denotes the fundamental determinants of the exchange rate, $E_t[ds(t)]/dt$ is the expected rate of depreciation of the domestic currency, and θ (a positive parameter) measures the extent to which the exchange rate level depends on its own expected rate of change.²

$f(t)$ is assumed to follow a controlled Brownian motion with instantaneous variance, σ^2 , and trend, $(\mu_c - D)$, inside a band, $[f_l, f_u]$, where f_l and f_u are the lower and upper edges of the fundamental band. μ_c is the trend in the fundamentals, observed by the market in the previous periods up to now, and D is a decision variable controlled by the domestic central bank. Increments in f are given by a stochastic differential equation:

$$(3.2) \quad df = (\mu_c - D)dt + \sigma dz + dL - dU$$

where z is a standard Wiener process, and dL and dU are the smallest unsterilized interventions that confine f to $[f_l, f_u]$. These interventions are zero except at f_l where $dL > 0$ and at f_u where $dU > 0$. At any instant, the domestic central bank can decide to keep the trend μ_c constant, by setting $D = 0$ (such that $df = \mu_c dt + \sigma dz + dL - dU$), or to change the current trend by deciding on a specific value for D different from 0.

As a particular example, assume that the current trend is $\mu_c = 0$. In this case, the central bank can always change the zero trend to a positive value by setting $D = -\mu$, with μ positive and assumed of fixed and known magnitude. The problem faced by the market is to form expectations about the central bank's setting of D . In other words, the market assigns subjective probabilities on the value taken by the decision variable, D , during the next time interval dt . Assume the following market beliefs:

$$\begin{aligned}
 (3.3) \quad & D = 0 \text{ (i.e., no change in } \mu_c = 0) \text{ with probability } 1-v; \text{ and} \\
 & D = -\mu < 0 \text{ (i.e., a change in the trend to } \mu > 0) \text{ with probability } v.
 \end{aligned}$$

The expected value for D is thus $-\mu v$ and the expected trend for the next time interval is: $\mu_c - (-\mu v) = \mu v$.

In this particular case, conjecturing that s is a twice continuously differentiable function of f [$s=G(f)$] and using Itô's lemma to calculate the expected rate of depreciation, we get:

$$(3.4) \quad E_t[ds]/dt = E_t[dG(f)]/dt = [\sigma^2/2]G_{ff}(f) + \mu v G_f(f).$$

Substituting equation (3.4) into (3.1) and remembering that $s = G(f)$, a second order differential equation is obtained:

$$(3.5) \quad G_{ff}(f) + [2\mu v/\sigma^2]G_f(f) - [2/\theta\sigma^2]G(f) = [-2/\theta\sigma^2]f.$$

The solution to this differential equation is:

$$(3.6) \quad s = G(f) = f + \theta\mu v + A_1 \exp[\lambda_1 f] + A_2 \exp[\lambda_2 f]$$

where λ_1 and λ_2 , the roots of the characteristic equation implied by the model are:

$$(3.7) \quad \lambda_1, \lambda_2 = \{-\mu v \pm [(\mu v)^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2; \lambda_1 < 0 < \lambda_2.$$

The values for A_1 and A_2 are defined with the use of two boundary conditions. In the case where no band on the fundamentals is specified, dL and dU are zero for all f in equation (3.2), and the exchange rate regime is a permanent free-float. In this regime, the well-known condition of "no-speculative bubbles" implies that $A_1 = A_2 = 0$, which gives the specific free-float path with possible trend revision, $S_{fv}(f)$, dependent on the value taken by v :

$$(3.8) \quad s = S_{fv}(f) = f + \theta\mu v.$$

If $v = 0$ (1), the expected trend for the next time interval is $\mu v = 0$ (μ) and the economy is somewhere on F_0 (F_1) in figure 3.1.

<figure 3.1 here>

For the target zone model, as was specified above, dL (dU) is positive at f_l (f_u), implying infinitesimal and unsterilised interventions to confine f to $[f_l, f_u]$. In this regime, we use the well-known "smooth-pasting" conditions given by $G_f(f_l) = G_f(f_u) = 0$. These conditions imply specific values for A_l and A_u that, when substituted into equation (3.6), give the target zone solution with possible trend revision, $S_{\nu}(f)$:³

$$(3.9) \quad s = S_{\nu}(f) = f + \theta\mu\nu + (K + K') / \{\lambda_l\lambda_u(\exp[\lambda_l f_u + \lambda_u f] - \exp[\lambda_l f_l + \lambda_u f])\}$$

where $K = \lambda_u \exp[\lambda_u f_u + \lambda_l f] - \lambda_l \exp[\lambda_l f_l + \lambda_u f]$ and $K' = \lambda_l \exp[\lambda_l f_l + \lambda_u f] - \lambda_u \exp[\lambda_u f_u + \lambda_l f]$.

If f_l is set equal to $-f_u$ such that the bounds are symmetrically located around $f = 0$, equation (3.9) reduces to:⁴

$$(3.10) \quad s = S_{\nu}(f) = f + \theta\mu\nu + \{\sinh(\lambda_u f_u) \exp[\lambda_l f]\} / \{\lambda_l \sinh(\lambda_l f_u - \lambda_u f_u)\} \\ - \{\sinh(\lambda_l f_u) \exp[\lambda_u f]\} / \{\lambda_u \sinh(\lambda_l f_u - \lambda_u f_u)\}.$$

The target zone path represented by equation (3.10) is dependent on the specific value taken by ν (i.e., dependent on the market's subjective probability of a possible trend revision in the next time interval dt). These target zone paths are represented in figure 3.1 by $S_j(f)$ for $0 \leq \nu = j \leq 1$. (Note that the subscript j in $S_j(f)$ indicates that the probability of a trend revision is constant at $\nu = j$ along this path.)^{5,6}

In consequence, if $\nu = 0$ (i.e., if the market is sure that the central bank has no intention to change the current zero trend to its positive value, μ), the exchange rate is somewhere on the $S_0(f)$ curve of figure 3.1, say point 1. On the other hand, if the market is certain that the domestic central bank will change its trend to $\mu > 0$ during the next time interval dt , then $\nu = 1$ and the

exchange rate jumps to point 2 on $S_l(f)$, and this happens **before** any change in the trend from $\mu_c = 0$ to μ occurs. If, however, some uncertainty exists concerning the future trend, v could take some intermediate value, say $v = 0.4$. The exchange rate would be on a S-curve between $S_0(f)$ and $S_l(f)$, like $S_{0.4}(f)$ in figure 3.1. This means that if the exchange rate is currently at point 1 in figure 3.1, but the market believes, with subjective probability, $v = 0.4$, in a potential change of $\mu_c = 0$ to $\mu > 0$, the exchange rate would immediately jump to point 4, and this would occur even if the current trend has not changed. Therefore, the upward or downward shifts of S-curves reflect market updates of a possible trend revision. In the next period, new information confirming the increased likelihood of a trend revision could imply a change in v to $v = 0.8$, which would trigger a jump to point 3 on $S_{0.8}(f)$.⁷ The following sections motivate the reasons why such a TZ model with possible trend revision may be interesting.

3.2 Exchange rate and interest rate differentials

As mentioned in the introduction of this chapter, the target zone model of Krugman (1991) implies a deterministic negative relationship between the exchange rate, s , and the nominal interest rate differential, $id = i - i^*$ (where i^* is the foreign interest rate). Given the lack of empirical support for this prediction, it is interesting to point out that the TZ model presented in section 3.1 suggests a correlation pattern between s and id which may be more consistent with the observations from the E.M.S.

Introducing the uncovered interest rate parity condition given by:

$$(3.11) \quad id(t) = E_t [ds(t)] / dt,$$

and combining equations (3.1) and (3.11), we obtain:

$$(3.12) \quad id = [s - f]/\theta.$$

Substituting s , given by equation (3.10), into equation (3.12), the path for the interest rate differential inside a target zone, $I_{\pi\nu}(f)$, is obtained:

$$(3.13) \quad id = I_{\pi\nu}(f) = \mu\nu + \{sinh(\lambda_2 f_u) exp[\lambda_1 f]\} / \{\theta\lambda_1 sinh(\lambda_1 f_u - \lambda_2 f_u)\} \\ - \{sinh(\lambda_1 f_u) exp[\lambda_2 f]\} / \{\theta\lambda_2 sinh(\lambda_1 f_u - \lambda_2 f_u)\}.$$

Again, the path $I_{\pi\nu}(f)$ depends on the specific values taken by ν . Setting $\nu = 1$ in equations (3.13) and (3.7), we would get the specific path:

$$(3.14) \quad id = I_1(f) = \mu + \{sinh(\lambda_2 f_u) exp[\lambda_1 f]\} / \{\theta\lambda_1 sinh(\lambda_1 f_u - \lambda_2 f_u)\} \\ - \{sinh(\lambda_1 f_u) exp[\lambda_2 f]\} / \{\theta\lambda_2 sinh(\lambda_1 f_u - \lambda_2 f_u)\},$$

with: $\lambda_1, \lambda_2 = \{-\mu \pm [\mu^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2$; $\lambda_1 < 0 < \lambda_2$.

Setting $\nu = 0$ in equations (3.13) and (3.7) we would get, after straightforward manipulations:

$$(3.15) \quad id = I_0(f) = -sinh(\lambda f)/\theta\lambda cosh(\lambda f_u),$$

with: $\lambda = \lambda_2 = -\lambda_1 = [2/\theta]^{1/2}/\sigma > 0$.

Figure 3.2 represents the interest rate differential paths for different values of ν .

<figure 3.2 here>

The negative slope of these paths is explained as follows. Observe in figure 3.1 that **along** $S_0(f)$, when f is in the upper part of the band and increases, an infinitesimal intervention to sustain the domestic currency becomes more likely, implying an increase in the positive expected rate of appreciation of the domestic currency (i.e., $E_t [ds(t)]/dt$ decreases). Thus, through the

uncovered interest rate parity condition, as f increases, i has to decrease relative to i^* , implying a decrease in the interest rate differential, id . Consequently $I_0(f)$ has a negative slope in figure 3.2.

Because $id = I_{\pi\nu}(f)$ and $s = S_{\pi\nu}(f)$ [given by equations (3.13) and (3.10)] are functions of f , a parametric plotting between s and id , for $f_l \leq f \leq f_u$, is possible. Figure 3.3 presents the parametric plotting for ν **constant** at $\nu = 0$.

<figure 3.3 here>

Figure 3.3 is explained as follows. Assume that the economy is at point 1 in figures 3.1 and 3.2. Suppose f increases and the market does not expect any change in the zero trend of the domestic money supply (ν stays constant at 0). The economy would thus go somewhere like point 5 in both figures: this is a move up along $S_0(f)$ in figure 3.1 and a move down along $I_0(f)$ in figure 3.2. In consequence, the increase in f implies an increase in s and a decrease in id . The negative relationship between s and id for ν constant at 0 is represented with the locus P_0 of figure 3.3: the economy moves from point 1 to point 5 along P_0 . The literature refers to the locus P_0 as the deterministic negative correlation between the exchange rate and the interest rate differentials in a TZ model. This is a well-known prediction of the traditional (Krugman) target zone model.

Unfortunately, the predicted negative relationship between s and id has typically not been verified in the data from the E.M.S. and the Swedish krona [e.g., Svensson (1992), Flood, Rose, and Mathieson (1991), and Lindberg and Söderlind (1994)]. Plots of interest rate differentials against exchange rates result in a wide scatter of observations. The correlations between

exchange rates and interest rate differentials are often positive or zero, and only occasionally negative, depending upon the sample and the sample period. These observations raise doubts about the validity of the (Krugman) TZ model. The model of section 3.1 has the potential to explain why a deterministic negative correlation between s and id should not always be expected in a TZ model.

Assume that the economy is initially at point 1 in figures 3.1 and 3.2 with $v = 0$. At that instant, the market receives new information about a possible trend revision to $\mu > 0$. The market evaluates this revision with probability, say, $v = 0.8$. In consequence, the economy jumps immediately to point 3 in both figures. An increase in s and in id (i.e., a **positive** relationship between s and id) is thus observed at the time of the change in v . Figure 3.4 presents the parametric plotting between s and id for $f_l \leq f \leq f_u$ and for different levels of v .

<figure 3.4 here>

A change in v (from 0 to 0.8) for the same level of the fundamentals implies a jump to point 3 (on $P_{0.8}$) from point 1 (on P_0) in figure 3.4, and, in consequence, a positive correlation between s and id .

Observe that figure 3.4 can theoretically explain the empirical fact that plots of interest rate differentials against exchange rates result in a wide scatter of observations. In consequence, the observed plots from the E.M.S. data could be explained with the "augmented" Krugman TZ model proposed in section 3.1, that is Krugman's model "augmented" by the market expectations of a possible trend revision.

This model thus predicts that during periods of stable expectations (about the future value of the trend), the economy will have a tendency to move **along** particular $S_{\pi,\nu}(f)$ and $I_{\pi,\nu}(f)$ curves, and (as predicted by the Krugman model) a negative correlation between s and id should be expected. Nevertheless, during periods of ambiguous information and conflicting signals about possible revisions of the trend, the market could very often update its subjective probability, ν , which could lead to a period of positive correlation between s and id because of numerous **shifts** of the $S_{\pi,\nu}(f)$ and $I_{\pi,\nu}(f)$ curves and in consequence, numerous **shifts** of the locus P , in figure 3.4.

3.3 Change of trend as a policy response to particular events

News about a possible trend revision will affect the information set of market participants and imply changes in ν . In this section, two specific periods plagued with ambiguous signals and new information about a possible trend revision are analyzed. The objective is to show that these periods that involve large updates in ν will imply numerous shifts of the $S_{\pi,\nu}(f)$ and $I_{\pi,\nu}(f)$ curves. Thus, during these periods, a positive correlation between s and id becomes more likely if the fundamentals are relatively stable over the same period.

3.3.1 *Speculative attacks*

The introduction of an inflationary trend for the domestic currency is common in the speculative attack literature on fixed exchange rate regimes [e.g., Flood and Garber (1984)]. The objective of this literature is to show that without monetary discipline, a fixed exchange rate regime is ultimately doomed to collapse due to speculative attacks. The reasoning is as follows: if the

domestic central bank has an inflationary monetary policy (relative to the foreign stance), this creates a tendency for the domestic currency to depreciate. To maintain a fixed parity, the domestic central bank has no other choice but to buy its own money by selling, and thereby, depleting its foreign reserves. When these reserves are exhausted, the central bank will probably let the exchange rate float. This would be a natural collapse for a fixed exchange rate regime. The speculative attack literature [e.g., Krugman (1979), and Flood and Garber (1984)] has shown that the anticipation of this scenario by rational agents would create a speculative attack by the market well before the natural collapse of the regime. An inflationary monetary policy (i.e., a positive μ), which captures the lack of domestic monetary discipline relative to the foreign stance, is explicitly built into these models in order to understand the mechanism of speculative attacks. The same methodology has been applied to an exchange rate target zone regime to explain its sensitivity to these attacks. Krugman and Rotemberg (1992) and Delgado and Dumas (1993) have explained the mechanism of speculative attacks in target zone models with positive trends.

In subsection 3.3.1 (in contrast to section 3.1), the current trend is assumed to be $\mu_c = \mu > 0$. A possible revision of the current trend to $\mu = 0$ is introduced as a policy response of the domestic central bank to an impending speculative attack. For example, suppose that a speculative attack on the domestic currency is likely to occur. If the target zone regime is viewed as essential for the domestic country, speculative attack rumours could introduce sufficient incentives for the domestic central bank to revise its current monetary policy. In other words, instead of accepting the forced return to a free float with the current positive trend, the domestic country could decide to revise its depreciating trend in order to prevent the speculative attack.⁸ One objective of this subsection is to show that a speculative attack could be postponed if the

domestic country could convince the market of its willingness to accept the discipline of a target zone regime by choosing a non-inflationary monetary policy (i.e., a zero trend).

It is doubtful that any announcement of stricter monetary policy would be entirely credible during such a period. A revised version of the model of section 3.1 can thus be used to introduce expectations of trend revisions during the period prior to a likely speculative attack. The beliefs of the market over the next time interval dt are:

$$(3.16) \quad \begin{aligned} D = 0 \text{ (i.e., no change in } \mu_c = \mu > 0) \text{ with probability } 1-v; \text{ and} \\ D = \mu \text{ (i.e., a change in the current positive trend to a zero trend) with probability } v. \end{aligned}$$

Equations (3.4), (3.8), (3.10), and (3.7) become respectively:

$$(3.17) \quad E_t[ds]/dt = E_t[dG(f)]/dt = [\sigma^2/2]G_{ff}(f) + \mu(1-v)G_f(f),$$

$$(3.18) \quad s = S_{fv}(f) = f + \theta\mu(1-v),$$

$$(3.19) \quad s = S_{fv}(f) = f + \theta\mu(1-v) + \{ \sinh(\lambda_2 f_u) \exp[\lambda_2 f] \} / \{ \lambda_1 \sinh(\lambda_1 f_u - \lambda_2 f_u) \} \\ - \{ \sinh(\lambda_1 f_u) \exp[\lambda_2 f] \} / \{ \lambda_2 \sinh(\lambda_1 f_u - \lambda_2 f_u) \}$$

with:

$$(3.20) \quad \lambda_1, \lambda_2 = \{-\mu(1-v) \pm [(\mu(1-v))^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2; \lambda_1 < 0 < \lambda_2.$$

Again, $S_{fv}(f)$ and $S_{fv}(f)$ are dependent on the specific value taken by v as shown in figure 3.5.

<figure 3.5 here>

When $v = 0$ the market is somewhere on $S_0(f)$ in figure 3.5, say point 1. (The upper $S(f)$ curve has been labelled with the subscript "0" to indicate that $v = 0$: the market believes with probability one in the constancy of the current trend μ_c at $\mu > 0$.) If $v = 1$, the exchange rate is

somewhere on $S_1(f)$, say point 2. Any intermediate value for v will reflect some market ambiguity about what the domestic central bank is going to do, as is the case at point 3 or 4.

During the period just prior to a speculative attack, the domestic central bank may have incentives to try to influence the market beliefs of trend revision (i.e., to influence the value of v) by generating statements reaffirming its commitment to a target zone regime (instead of a free float), and thereby announcing a less inflationary monetary policy as a sign of its commitment. The reason for this central bank behaviour may be found in a "minimum foreign reserve requirement", necessary to sustain a TZ (i.e., to prevent a speculative attack), that depends on the market expectation of a trend revision. This point is established as follows.

The central problem in finding the minimum reserve requirement in a TZ regime lies in connecting the TZ regime with the post-collapse regime. This connection is established in three steps: 1) A crisis scenario specifies, a priori, a **shadow** exchange rate regime, that is the regime that would prevail if reserves were exhausted (i.e., if the domestic central bank runs out of foreign reserves or out of borrowing capacity). In other words, the shadow regime is the regime that would prevail following a successful speculative attack. Assume that this shadow regime is the free-float with possible trend revision as given in equation (3.18); 2) By definition of a speculative attack, agents buy all foreign reserves held by the domestic central bank when the TZ regime is perceived as "unsustainable". The (short-term) sustainability of a TZ is defined relative to a minimum reserve requirement: the TZ regime is considered as sustainable as long as this level of reserve is not reached yet. When this level is reached, a speculative attack of the amplitude of this minimum reserve requirement will be triggered at the **next** domestic central bank intervention likely to decrease the foreign reserves stock below this minimum level (i.e., at

the next time $dU > 0$ or $f = f_u$); 3) Discrete jumps (increase) in the exchange rate at the moment of transition between the TZ regime and the free-float can never be expected by the market. Indeed, predicted jumps would precipitate an attack on reserves **prior** to the increase, due to the existence of profitable speculative opportunities. In conclusion, a speculative attack and a discrete jump in the exchange rate involves a contradiction.

By step 1) and equation (3.18), we know that:

$$(3.21) \quad s_+ = f + \theta\mu(1-\nu),$$

where s_+ represents the level of the exchange rate just after the speculative attack. By step 2) and equation (3.19), we know that at the moment just before a speculative attack, the exchange rate is given by:

$$(3.22) \quad s_- = f_u + \theta\mu(1-\nu) + \{sinh(\lambda_1 f_u)exp[\lambda_1 f_u]\}/\{\lambda_1 sinh(\lambda_1 f_u - \lambda_2 f_u)\} \\ - \{sinh(\lambda_2 f_u)exp[\lambda_2 f_u]\}/\{\lambda_2 sinh(\lambda_1 f_u - \lambda_2 f_u)\},$$

where s_- is the exchange rate just before the attack. Finally, by step 3), set s_- [given by equation (3.22)], equal to s_+ [given by (3.21)], such that:

$$(3.23) \quad f_u - f = -[\{sinh(\lambda_1 f_u)exp[\lambda_1 f_u]\}/\{\lambda_1 sinh(\lambda_1 f_u - \lambda_2 f_u)\} - \{sinh(\lambda_2 f_u)exp[\lambda_2 f_u]\}/\{\lambda_2 sinh(\lambda_1 f_u - \lambda_2 f_u)\}],$$

where λ_1, λ_2 are given by equation (3.20). The difference, $f_u - f$, represents the loss of (log) foreign reserves incurred by the domestic central bank at the time of the speculative attack. This also represents the (log) minimum reserve requirement of the TZ regime. Observe that the minimum reserve requirement depends on the subjective probability ν through the roots λ_1 and λ_2 . The larger is ν , the larger is the minimum reserve requirement. To make this dependence more explicit, let us call the (log) minimum reserve requirement r_ν [given by $(f_u - f)|_\nu$].

In the case where $\nu = 0$, $r_{\nu=0}$ is given by the distance BA in figure 3.5.⁹ If this level of foreign

reserves were to be reached, there would be a speculative attack against the domestic currency as soon as point A, on $S_0(f)$, is attained (i.e., as soon as a new intervention to sustain the zone is required). The market would sell the domestic currency, buying all the domestic central bank's foreign reserves. The economy would thus jump from point A to point B on the free-float line F_0 , reflecting the one-time loss of the remaining foreign reserves. From then on, the new regime would be a free-float (with the current depreciating trend, $\mu_c = \mu > 0$): the exchange rate would move along F_0 in figure 3.5.

For $v = 1$, $r_{v=1}$ is given by the distance $DC < BA$.¹⁰ In consequence, if the domestic central bank is able to fully convince the market of a trend revision to $\mu = 0$, this will reduce the minimum reserve requirement necessary to sustain the current TZ to $r_{v=1} < r_{v=0}$ and will prevent a speculative attack. Because $r_{v=1} < r_{v=j} < r_{v=0}$ for $j \in (0, 1)$, statements by the domestic central bank about a potential trend revision will have a beneficial impact in postponing the speculative attack (i.e., prolonging the sustainability of the TZ), if these statements have some impact on market beliefs, v .

As shown in section 3.2, quick changes in v for a relatively stable level of fundamentals imply a positive correlation between the exchange rate and interest rate differentials. As a result, this positive relationship may arise during the period just prior to a speculative attack if the statements of the domestic central bank about a potential trend revision have some bearing on market beliefs of this trend revision.

3.3.2 Unemployment and the nominal anchor debate in the E.M.S. in July 1993

As suggested by Ball (1994), monetary expansion (contraction), in general, does not mean a rise

(a fall) in the **level** of the money supply (i.e., a one-time shock to the level), but an increase (decline) in money **growth rate** (trend). Consequently, assume that the domestic country has currently a zero trend monetary policy ($\mu_c = 0$), while facing domestic pressure to return to a more expansionary monetary policy ($\mu > 0$), because of some internal problem (e.g., unemployment). This case can be directly analyzed using the framework of section 3.1.

Suppose that the domestic currency is at point 1 on $S_0(f)$ and $I_0(f)$, in figures 3.1 and 3.2. Point 1 could illustrate the situation of the French franc relative to the Deutschmark in June 1993, before speculative pressures against the franc emerged at the end of July 1993. Point 1 can be understood as a situation where the market finally believed ($v = 0$) in the "strong franc" policy (understood here as a zero trend monetary policy). In other words, after announcing this policy for a while, the market ultimately believed it, such that v decreased to 0. Point 1 also depicts a situation where the franc is near its central parity with the mark.

At that time, though, the market received new conflicting signals: 1) the unemployment situation in France was considered as critically bad; and 2) suggestions were made by the French political elite that the franc had become the new nominal anchor of the E.M.S.^{11:12} As the nominal anchor country, France could theoretically impose its preferred monetary policy on the other countries of the E.M.S.^{*} Hardly anybody believed in the "franchor" but the suggestion of this pretention could have sent the message to the market that France was prone to revise its current "strong franc" policy to improve its unemployment situation. Due to the unemployment situation in France, the market believed that this country would actually prefer a more expansionary monetary policy (i.e., an increase in the growth rate of the French money supply), as opposed to the official position. The market began to revise its subjective probability of a

possible positive trend in the near future. That is, v increased as the beliefs of the market changed to reflect a possible positive future trend. So, even without any change in the monetary trend, the expectation of a possible revision of the trend generated a persistent depreciation of the franc from point 1 to points 4, 3, and 2 in figure 3.1.

In terms of interest rate differentials, the expectation of a trend revision implied an increase in the French interest rate required by the market to compensate for the higher expected rate of depreciation of the franc: the interest rate differential jumped from point 1 to points 4, 3, and 2 in figure 3.2.

It is worth repeating that these jumps in the exchange rate are triggered by market expectations, and not by an actual change in the monetary trend of the domestic country (here, France). These increases in the exchange rate, **for the same fundamentals**, are typical of what happened in July 1993. France had a lower inflation rate and a relatively stronger economy than the one of the newly unified Germany, suggesting, if not an appreciation of the franc relative to the mark, then at least a stable value. But discussions surrounding the unemployment situation in France and the new role attributed to the franc by French politicians (i.e., the "franchor") generated the type of market expectations leading to a depreciation of the franc relative to the mark.

The implication of this analysis is as follows. During periods of tensions in the E.M.S. about the nominal anchor and the type of monetary policy to be conducted, updates in v are likely to be frequent. The volatility of v can thus imply (according to figure 3.4) a positive correlation between the exchange rate and the interest rate differentials.

3.4 State-varying expectations

As developed to this point, the model explains the consequences of exogenous changes in the parameter v . This section discusses a possible way of endogenizing v in the framework of the model of section 3.1.

Consider a state-varying parameter $v = pf^q$, where p and q are positive constants and $f \geq 0$. In this case, any increase in f would imply an increase in v . In other words, as f is increasing towards the upper edge of the target zone band, the market believes that a trend change from $\mu_c = 0$ to $\mu > 0$ is more likely to happen.¹³ Also, when $f = 0$, $v = 0$ (i.e., the market expects with probability one that no change in the current zero trend will happen). For symmetry, in the lower edge of the band (where f is negative), set $v = p|f|^q$ so that the probability v remains positive. When f decreases below zero, the market fears of a trend revision from $\mu_c = 0$ to $-\mu$ is increasing.

For the interval $[0, f_u]$, equations (3.4) and (3.5) become:

$$(3.24) \quad E_t[ds/dt] = E_t[dG(f)]/dt = [\sigma^2/2]G_{ff}(f) + \mu pf^q G_f(f)$$

$$(3.25) \quad G_{ff}(f) + [2\mu pf^q/\sigma^2]G_f(f) - [2/\theta\sigma^2]G(f) = [-2/\theta\sigma^2]f.$$

Equation (3.25) has no simple solution, but it is straightforward to determine several points verifying (3.25). Suppose that $q = 1/3$ and $p = 1/(f_u)^{1/3}$. Then, when $f = f_u$, $v = pf_u^{1/3} = 1$ and the economy is at point 1 on $S_1(f)$, in figure 3.6, where $S_1(f)$ satisfies equation (3.10) with $v = 1$.

<figure 3.6 here>

When $f = 0$, $v = 0$ and the economy is at point 0 on $S_0(f)$ where $S_0(f)$ satisfies equation (3.10) with $v = 0$. Other points can be generated as well. For specific values of f (e.g., $f = f_u/2$, $f_u/3, \dots$), specific values for v can be computed (e.g., $v = j = (1/2)^{1/3}$, $(1/3)^{1/3}, \dots$), which in turn allows us to identify specific points $(f, s = S_j(f))$ in figure 3.6. Each point will be located on a specific S-curve, $S_j(f)$, bounded by $S_0(f)$ and $S_1(f)$, and satisfying equation (3.10) with v set equal to j . The locus of these points is given by $E-E$ in figure 3.6. The same methodology in the lower part of the band generates the locus $E'-E'$. Finally, a **steeper** S-curve is obtained instead of a family of S-curves dependent on different values of v .

Using the same approach, an interest rate differential curve, $F-F$, has been drawn in figure 3.7 for the case where v is state-varying.

<figure 3.7 here>

Observe that we get an interest rate differential curve with a positive slope in the neighbourhood of the central parity of the zone, but with a negative slope towards the edges. The intuition of this result is as follows. At each positive f there is: 1) a positive expected rate of **appreciation** due to the future intervention to sustain the band; and 2) a positive expected rate of **depreciation** due to the possibility of a trend revision. When f is small and near the central parity of the band, the first effect is smaller than the second. Thus, through the uncovered interest rate parity condition, a positive interest rate differential appears. Near the upper edge of the band, the relative strength of both effects tends to reverse, such that a negative interest rate differential develops. A similar type of reasoning can be applied to the lower part of the band.

Observe that, due to the sign-reversing slope of the curve $F-F$, a positive relationship between s and id appears near the central parity of the zone, whereas it reverses itself towards the edges. Again, we get the proposition that various correlation patterns between the exchange rate and interest rate differentials are possible in a target zone regime.

3.5 Conclusion

The target zone model developed in this chapter incorporates the possibility of a future change in the trend in the fundamentals of the exchange rate. The market has subjective expectations about possible trend revisions. It was shown that this type of expectations can affect the exchange rate level even without an actual change in the trend. These expectations have been treated, first, as entirely exogenous and, second, as state-varying. In both cases, it was shown that various correlation patterns between the exchange rate and the interest rate differentials are possible. That appears to be consistent with observations taken from the E.M.S.

Endnotes

- 1) See Krugman and Miller (1992) for a sample of papers on this new target zone literature.
- 2) The differential equation given by (3.1) governs exchange rate behaviour under the following **sufficient** conditions: 1) Commodity prices are flexible and the purchasing power parity condition prevails; 2) The demand for real balances in both countries is log-linear with the same interest semi-elasticity, θ , in both countries; 3) Capital flows are perfectly mobile between the two countries: the uncovered interest rate parity condition prevails. Under these conditions, the fundamentals of the exchange rate are given by $f = m - m^* + v$, where m and m^* are domestic and foreign measures of money supply, and v is an exogenous monetary shock.
- 3) In the model of section 3.1, observe that, if $v = 1$ (i.e., if the market expects with probability one a revision of the current zero trend to a trend $\mu > 0$), λ_1 and λ_2 take the particular values [given equation (3.7)]: $\lambda_1, \lambda_2 = \{-\mu \pm [\mu^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2$; $\lambda_1 < 0 < \lambda_2$. Thus, the TZ solution given by equation (3.9) reduces to the well-known target zone path with (constant) trend derived, for example, in equation (16) in Froot and Obstfeld (1991).
- 4) In section 3.1, observe that if $v = 0$ (i.e., if the market expects, with probability one, no revision in the current zero trend), λ_1 and λ_2 take the particular value [given equation (3.7)]: $\lambda = \lambda_2 = -\lambda_1 = [2/\theta]^{1/2}/\sigma > 0$. Thus, the solution derived in equation (3.10) reduces to: $s = S_{\pm 0}(f) = f - \sinh(\lambda f)/\lambda \cosh(\lambda f_u)$. This is actually the well-known target zone solution with zero trend as derived, for example, in equation (18c) in Svensson (1991b).
- 5) Observe in figure 3.1 that for $v = 0$, the band on the fundamentals $[f_l, f_u]$ implies an implicit exchange rate band given by $[S_0(f_l), S_0(f_u)]$ (not drawn). When $v = 1$, the same band $[f_l, f_u]$ implies a new implicit exchange rate band, $[S_1(f_l), S_1(f_u)]$. If the domestic country wants to have a more expansionary monetary policy inside a TZ model, it could decide to revise its current zero trend to a positive trend. Such a divergence between the domestic and the foreign monetary policies should ultimately imply a revision of the **exchange rate** band (i.e., a realignment). In other words, the model of section 3.1 can also serve as a model of realignment where v , the probability of trend revision, could be viewed as the probability of exchange rate realignment. An alternative interpretation of the exchange rate band in this model is as follows. Countries sometimes announce a relatively large band of variation for the exchange rate around the central parity, say $\pm 6\%$, but also announce their intentions to stay as long as possible in a smaller band, say, $\pm 2.25\%$. This announcement gives an indication to the market that the domestic country may decide to revise the trend in the fundamentals of the exchange rate in the future. The small band in the model would be given by $[S_0(f_l), S_0(f_u)]$ and the large band would be $[-S_1(f_u), S_1(f_u)]$.

- 6) Different models of realignment exist in the literature. In Bertola and Caballero (1992), realignments are triggered by a discrete and substantial change in the level of the money supply. This change is permitted to happen only when the fundamentals hit their upper or lower bounds. Also, the market has a subjective (and exogenous) probability about central bank behaviour at the bounds (i.e., to keep the same band or to realign). In Svensson (1991a) realignments are also triggered by a discrete change in the level of the money supply, but this change is not restricted to occur when the fundamentals are at the lower or upper edges of their band. Indeed, realignments are modelled as occurring with some given constant probability regardless of where in the band the exchange rate lies and hence, expectations of realignment are constant throughout. Bertola and Svensson (1993) extend Svensson's initial model by treating expectations as exogenous, but following a stochastic process. These models by Bertola and Caballero, by Svensson, and by Bertola and Svensson have one point in common: positive (negative) realignments are triggered by a one-shot increase (decrease) in the level of the money supply. The model of section 3.1 could serve as a way to model realignments triggered by a change in the **growth rate** of the money supply. As in Svensson (1991a), realignments would occur with a **constant** probability, regardless of where in the band the exchange rate lies. Section 3.4 presents a model with **state-varying** probabilities of trend revision/realignment, along the lines of Tristani (1994).

- 7) In this model, v is exogenous and updates in v are introduced exogenously to analyze their impact on the exchange rate and interest rate differentials. Changes in v are the market responses to new ambiguous signals. Suppose that Inf_v is a subset of the information set of the market. Inf_v is bounded by two unambiguous levels: 1) Inf_0 is a signal representing a firm commitment by the domestic central bank to the current trend, implying thus a value of $v = 0$; and 2) Inf_1 is a signal representing an immediate change in the current trend, implying a value of $v = 1$. In between these two levels of information lie ambiguous information about possible changes in the trend of the monetary policy. Assume that: 1) a variable N is a Poisson process such that $dN = 0$ with probability $1 - \epsilon dt$ and $dN = 1$ with probability ϵdt ; and 2) changes in Inf_v are given by a Poisson differential equation: $dInf_v = g dN$ where g is a constant. When $dN = 0$, Inf_v stays constant (no news). When $dN = 1$, Inf_v changes discretely, which represents news available to the market. Finally, assume that: 3) $v = h(Inf_v)$ (i.e., assume that changes in v depend on new information about a possible trend revision). Consequently, changes in v over the interval dt are given by: $dv = dh(Inf_v) = [h(Inf_v + g; N+1) - h(Inf_v; N)]$ with probability ϵ and $dv = 0$ with probability $1 - \epsilon$. This is a simple example to illustrate the likely changes in v as new information becomes available to the market over the interval dt .

- 8) Countries from continental Europe have a strong attachment to some form of target zone regime. As argued by Giavazzi and Giovannini (1989, p.1): "Europeans dislike exchange rate fluctuations for three reasons. First, they all live in relatively open countries. Second, many of them hold the floating rates of the 1920s and 1930s responsible for the ensuing collapse of national economies and of the international trading and monetary systems.

Third, postwar European institutions, particularly the common agricultural market, depend for their survival on exchange rate stability." In consequence, leaving the E.M.S. is perceived in Europe as a move that would endanger other spheres of cooperation as well.

- 9) Observe in figure 3.5 that, by construction, distance $BE = \theta\mu$ and distance $EA = f_u - s_u|_{v=0}$ where $s_u|_{v=0}$ is given by equation (3.19) with v set equal to 0 and $f = f_u$. Once the substitution for s_u is done we get that distance $BA = BE + EA = \theta\mu + (f_u - s_u|_{v=0})$. Doing this, we get the same value as the one obtained by setting $v = 0$ in equation (3.23). In other words, distance BA represents the amplitude of the speculative attack or the minimum reserve requirement when $v = 0$ (i.e., $r_{v=0} = (f_u - f)|_{v=0}$).
- 10) Distance DC in figure 3.5 is given, by construction, as $f_u - s_u|_{v=1}$. Substituting s_u by its value given in equation (3.19) with v set equal to 1 and $f = f_u$, we get that $DC = \sinh(\lambda f_u) / \lambda \cosh(\lambda f_u)$, with $\lambda = -\lambda_1 = \lambda_2 = [2/\theta]^{1/2} / \sigma > 0$. This is the value we would obtain by setting $v = 1$ in equation (3.23) (i.e., $r_{v=1} = (f_u - f)|_{v=1}$). Observe also that, in general, the minimum reserve requirement is given by $r_v = (f_u - f)|_v = \theta\mu(1-v) + f_u - s_u|_v$. This expression obviously decreases as v increases. As a result, r_v is a negative function of v such that: $r_{v=1} < r_{v=j} < r_{v=0}$ for $j \in (0,1)$.
- 11) The liquidity or nominal anchor problem is well known in the literature on fixed exchange rate regimes. Because two countries are linked by a fixed bilateral exchange rate, the expected rate of depreciation is equal to zero, implying an equalisation of domestic and foreign interest rates. There is a fundamental indeterminacy in this system (the liquidity / nominal anchor problem) because a large variety of global monetary policies can equalize the nominal interest rates of both countries. A negotiated choice has to be reached. The solution to this problem could be symmetric or asymmetric. In the symmetric solution with repeated negotiations, countries usually face a chainstore paradox such that the cooperative solution is not an equilibrium. The asymmetric solution where the leader, say the foreign country, imposes its own monetary policy (the so-called anchor of the system) on the domestic country that has to adjust, is the most usual scheme in practice (e.g., the United States during Bretton Woods and Germany for the E.M.S.), but gives rise to the so-called "n-1 problem": does the leader pursue the right monetary policy? The question turns out to be the selection of the leader; in other words, what are the fundamental characteristics of a currency considered to be a nominal anchor? The anchor should be the currency which, due to its long history of low and stable inflation, is the less likely to depreciate, implying that this country (the leader) has a lower interest rate. Implementation of a credible fixed rate regime after the choice of the leader should then equalize the domestic interest rate to the level of the foreign interest rate (i.e., that of the leader). Pressures on the foreign country to decrease its interest rate will occur when unemployment problems arise in the domestic country. If the domestic country were to decrease its own interest rate by implementing an expansionary monetary policy, this would imply a depreciation of the domestic currency that the domestic country would have to prevent by selling foreign reserves, thereby countering the initial expansionary policy. As a result, the domestic country can get lower interest rates only if the foreign

country agrees to lower its rates. Again, note that the country with the nominal anchor currency imposes its monetary policy on the system.

- 12) The strict equalization of domestic and foreign interest rates is relaxed in a TZ regime because the bilateral exchange rate is authorised to fluctuate somewhat: in consequence, the interest rate can fluctuate inside an interest rate differential band. For $\mu_c = 0$ and $v = 0$, this band is given by $[I_0(f_u), I_0(f_l)]$ in figure 3.2 (not drawn). In other words, with $\mu_c = 0$ and $v = 0$, the domestic (e.g., French) interest rate would be lower than the foreign (e.g., German) interest rate if the French franc were in the upper (weak) part of its exchange rate band. Point 1 in figures 3.1 and 3.2 represents such a situation. With the increased market belief in the "strong franc" policy, v decreased to zero: the French economy moved from a point like 3 to one like 4, and finally to point 1 in figures 3.1 and 3.2 as the strong franc policy became fully credible. The interest rate differential was shrinking, until it finally became negative. As mentioned in *The Economist* (26 June 1993, p.82), the key intervention rate in France was 7% on June 21st whereas the discount rate, its German equivalent, stood at $7\frac{1}{4}\%$. Even though French long-term interest rates were still above Germany's, the gap was closing fast. At the same time, France had a lower inflation rate and a stronger economy than Germany's. All this could have suggested that the franc was actually the new anchor of the E.M.S., if this system were a pure fixed exchange rate regime as described in note 11. Nevertheless, due to the authorized exchange rate band of fluctuation inside a TZ, the slight negative interest rates differential simply reflected the fact that the French franc was slightly above its central parity with the Deutschemark. Describing point 1 in figures 3.1 and 3.2 as a situation leading to the emergence of the franc as the new nominal anchor of the E.M.S. is a misunderstanding of the functioning of a TZ regime.
- 13) This way of endogenizing v was initially introduced by Tristani (1994) in a realignment model. In section 3.4, this assumption captures the idea that the market loses confidence in the monetary authority to keep up with the current zero trend as the fundamentals are moving away from the central parity. This does not mean that the authorities are actually more willing to change the current zero trend, but it does mean that increasing market fears of trend revision appear as the fundamentals move away from their central parity.

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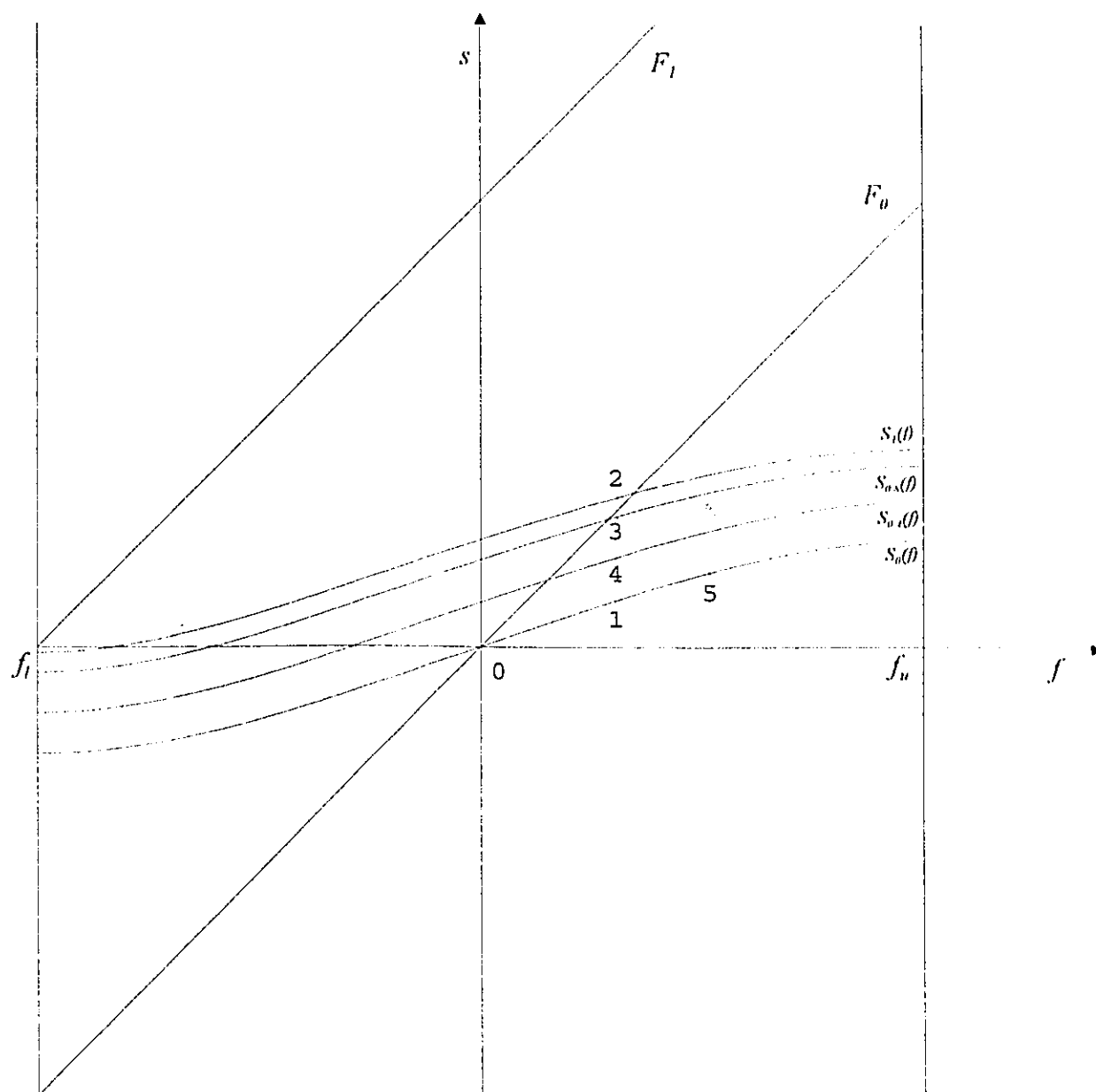


Figure 3.1 A target zone model with expected trend revision

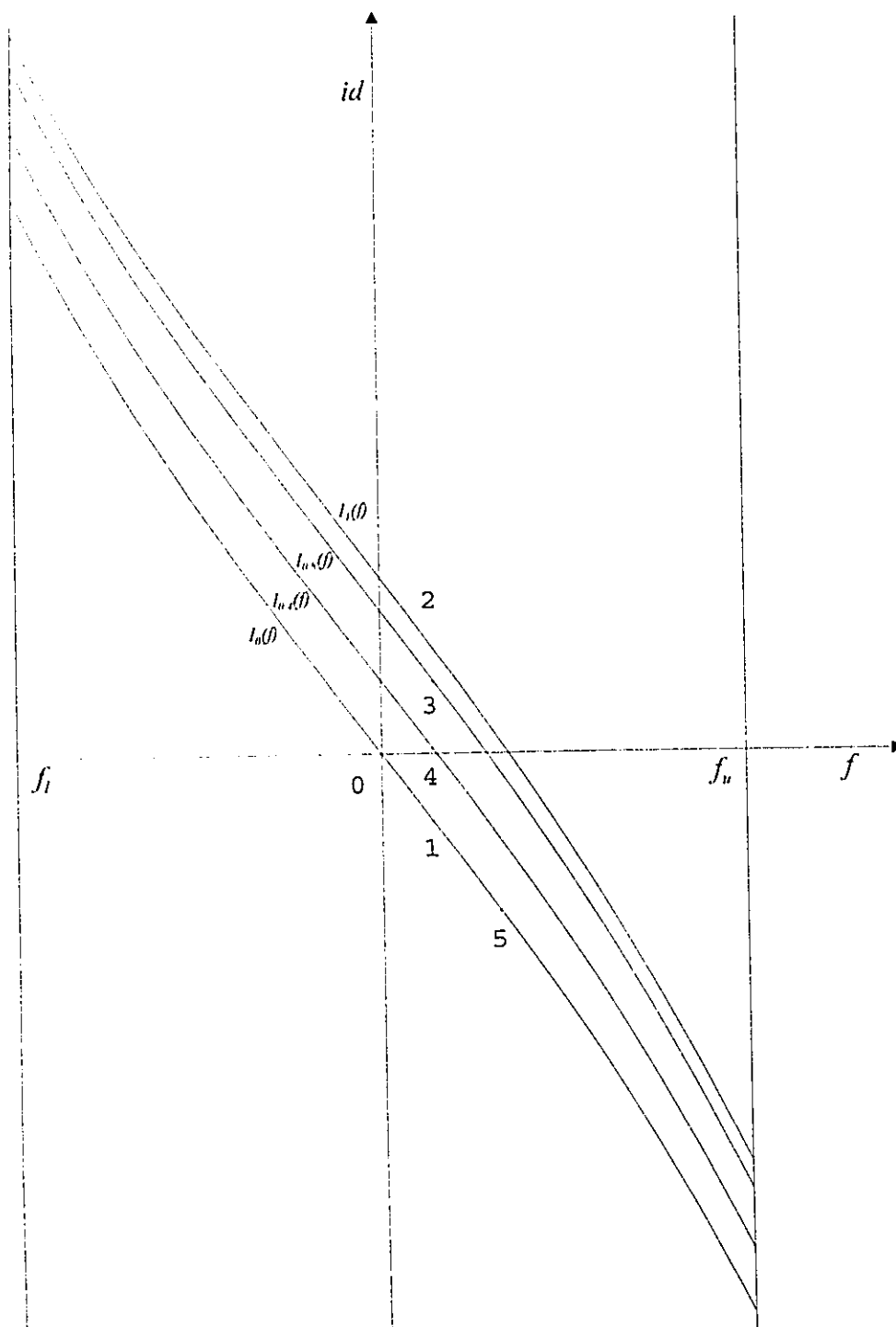


Figure 3.2 Interest rate differentials

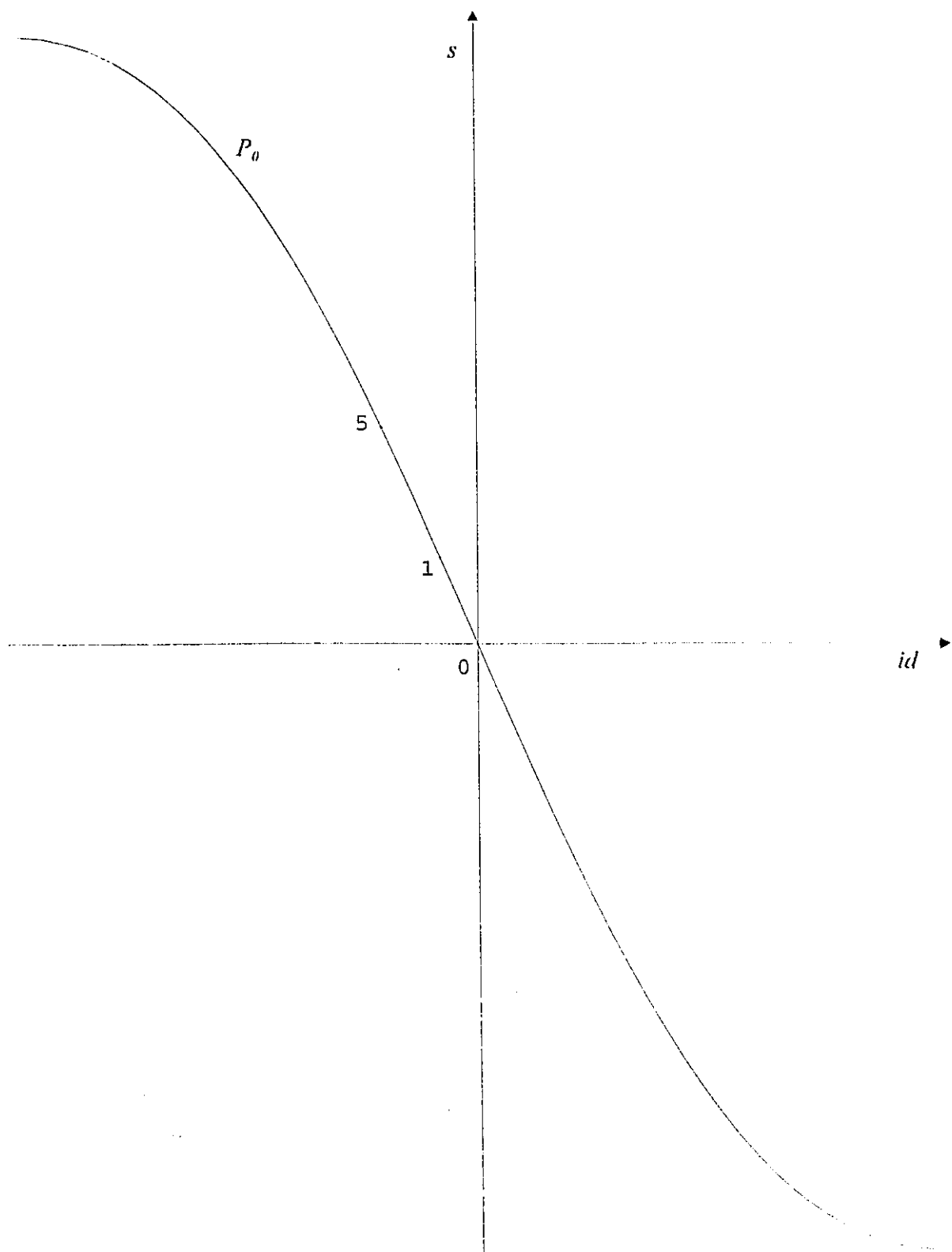


Figure 3.3 Parametric plotting of id against s for $f_l \leq f \leq f_u$ and $v=0$

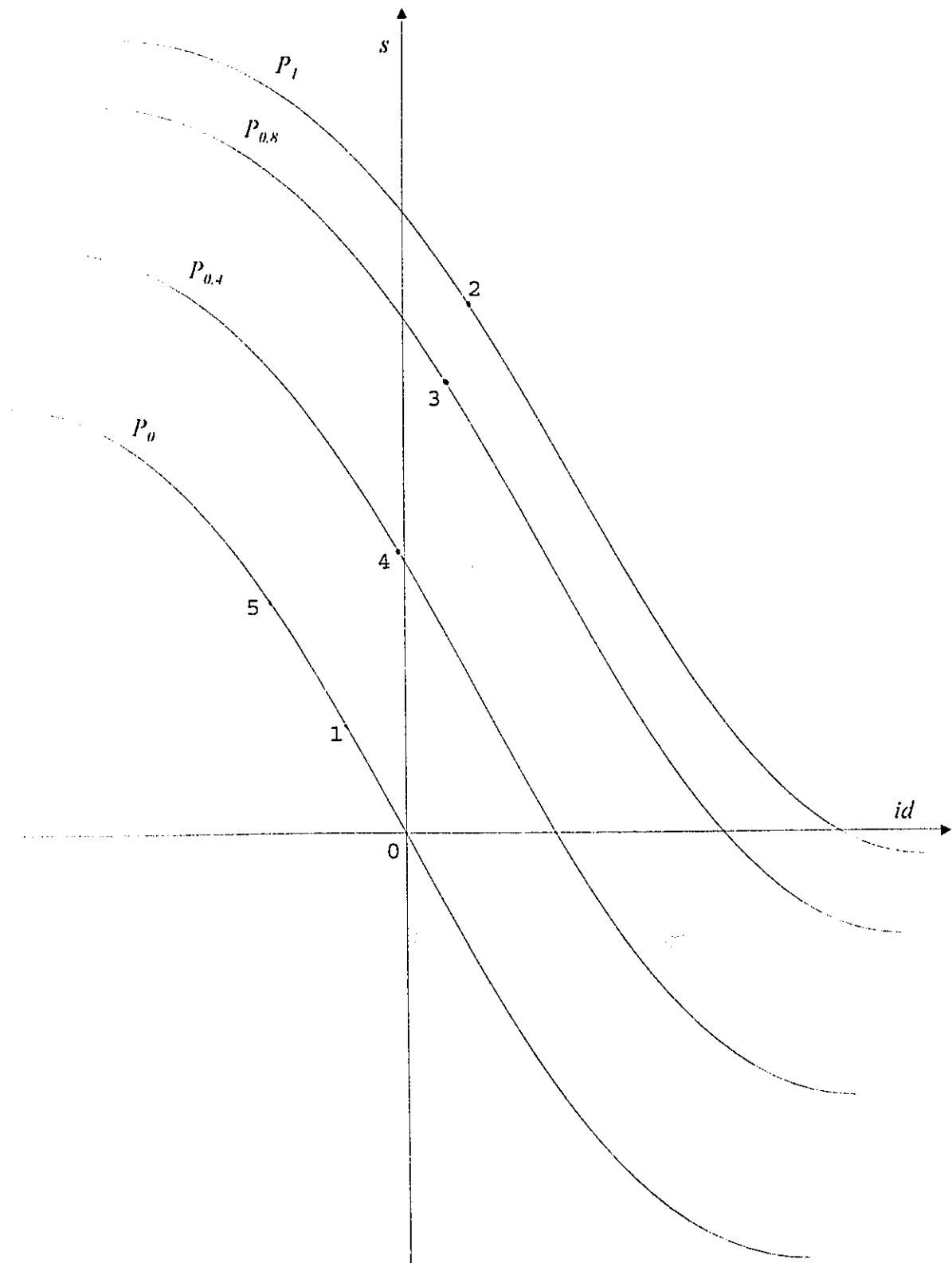


Figure 3.4 Plot of the id - s relationship for different values of v

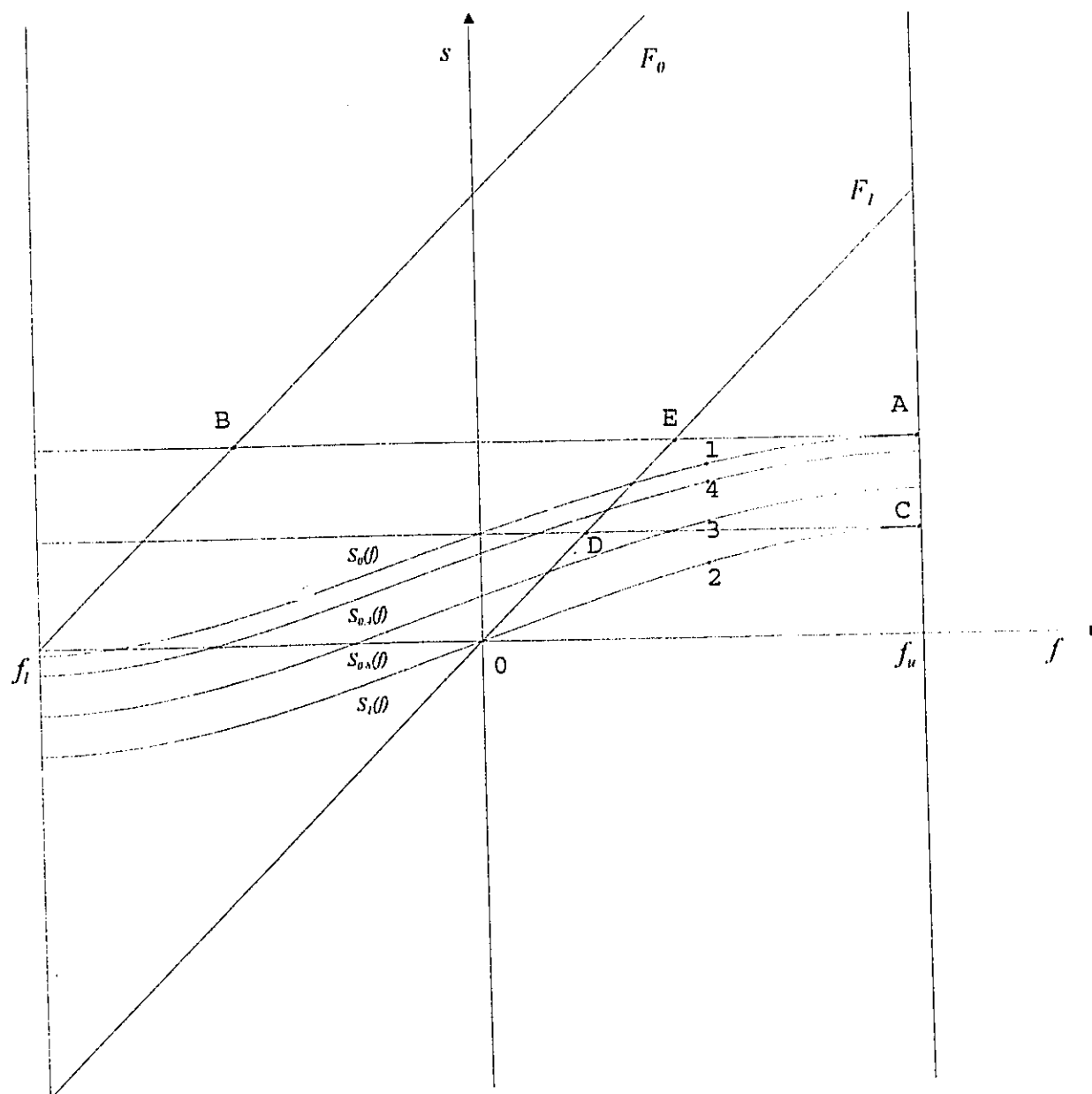


Figure 3.5 Speculative attacks and trend revisions

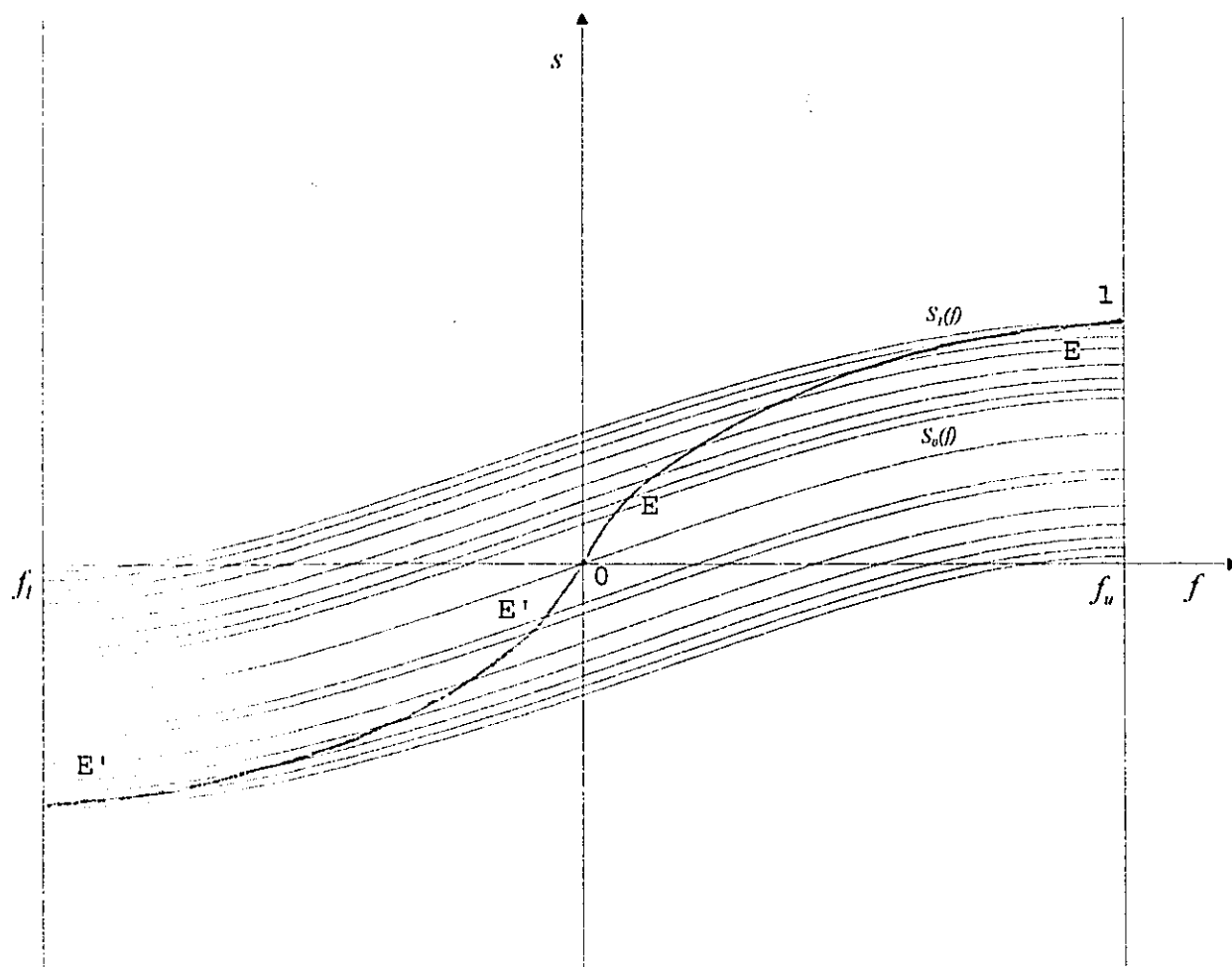


Figure 3.6 The steeper S-curve with state-varying expectations

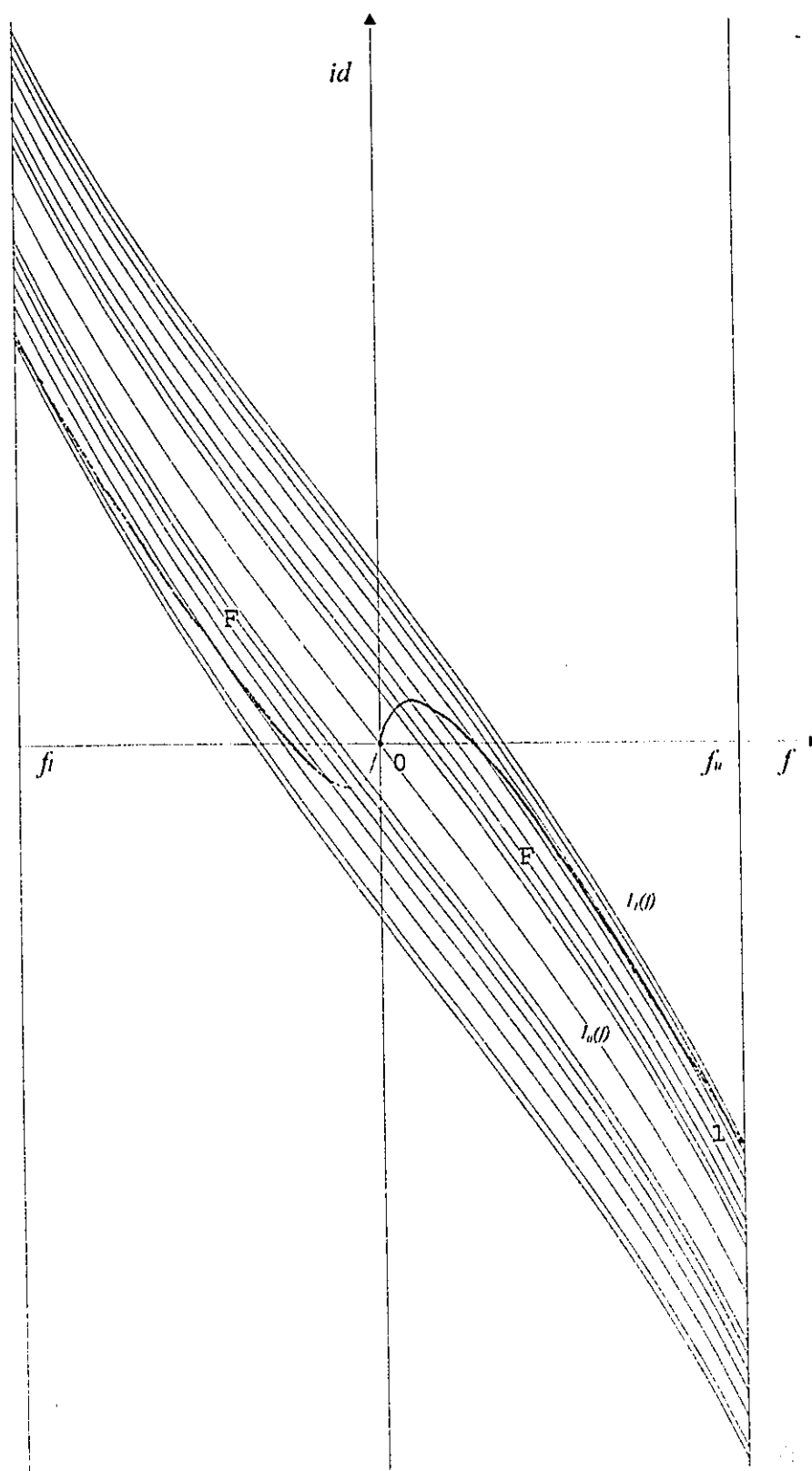


Figure 3.7 Interest rate differentials and state-varying expectations

4 FIXED EXCHANGE RATE ENTRY: SOME WELFARE ISSUES

*Abstract. This chapter studies an announced (state-dependent) regime shift from a free-floating to a permanently fixed exchange rate regime and introduces specific welfare considerations. The welfare issues introduced here try to explain why a specific exchange rate target level would be chosen instead of another one, if a return to a fixed exchange rate were on the public policy agenda and a given **range** for its pegged value had already been proposed. While this model does not provide a theory of choice between free-float and fixed rate regimes, it proposes a criterion to choose between fixed exchange rate regimes, taking into account the transition period from the free-float to the implementation of the fixed regime.*

4.0 Introduction

This chapter examines a state-dependent regime shift from a free-float to a permanently fixed exchange rate regime and introduces specific welfare considerations. An ambitious welfare program would be to analyze the motivations underlying this regime switch. In this chapter, a less ambitious problem is addressed. The welfare considerations introduced here motivate the choice of a specific exchange rate target level, if a return to a fixed exchange rate was on the public policy agenda. If the central bank of a particular country is considering a return to such a regime (whatever the reason invoked for this return), what should the pegged exchange rate target be?

Even if all economic advisers had the same theory in mind, it is likely that an equilibrium real exchange rate **range** would be proposed, instead of a specific **level**. For example, suppose that economists could agree on the view that the equilibrium exchange rate is the one that generates a balanced current account.

<figure 4.1 here>

In figure 4.1, the schedule T_1-T_1 represents the usual positive relationship between the current account and the real exchange rate. If the economy is currently at point 1, on T_1-T_1 , with a current account deficit, the equilibrium exchange rate would be s_{z1} . But, depending on other empirical estimates of (import and export) elasticities, the relationship between the current account and the real exchange rate could also be believed to be T_2-T_2 , delivering the equilibrium exchange rate s_{z2} .

If T_1-T_1 and T_2-T_2 are the two polar estimates, we would get an arguably continuous range of possible equilibrium exchange rates, $[s_{z1}, s_{z2}]$. In this case, it would be interesting to have a welfare criterion to choose the optimal exchange rate level inside this closed interval. Two specific steps are necessary to answer this question. In section 4.1, we consider the impact on the paths of the exchange rate and the interest rate of a credible announcement by the central bank of a return to a (future) fixed exchange rate regime. Through such announcements, the central bank can effectively control the corresponding exchange rate and interest rate paths during the transition period from the current free-float to the future peg. Section 4.2 introduces a welfare criterion that permits to choose the optimal exchange rate level inside the proposed interval. Concluding comments and limitations of the model are presented in section 4.3.

4.1 A state dependent fixed rate entry

Flood and Garber (1983) discuss the following problem. The domestic monetary authorities wish

to let the exchange rate float until it reaches a particular level, say, s_2 , at which time they plan to fix the exchange rate permanently at this level. If this regime switching from a free-float to a fixed exchange rate regime is announced to the market in advance, what will be the dynamic path of the exchange rate during the transition period (i.e., as long as the exchange rate is still floating)?¹ Despite a very simple underlying model and extensive derivations, Flood and Garber (1983) were unable to produce a closed-form solution to their problem.

Froot and Obstfeld (1991), however, derive a solution for this problem by using the techniques of regulated Brownian motions in a two-step method. Their solution is as follows. Let $s(t)$ denote the logarithm of the exchange rate at time (t), measured in units of home currency per unit foreign currency. As is commonly assumed in the literature, the exchange rate satisfies the relationship:

$$(4.1) \quad s(t) = f(t) + \theta E_t[ds(t)]/dt$$

where $f(t)$ denotes the fundamental determinants of the exchange rate, $E_t[ds(t)]/dt$ is the expected rate of depreciation of the domestic currency and θ (a positive parameter) measures the extent to which the exchange rate level depends on its own expected rate of change.

The fundamentals represented by $f(t)$ are assumed to follow a Brownian motion with instantaneous variance, σ^2 , and (depreciating) trend, $\mu > 0$. Increments in f are given by a stochastic differential equation:

$$(4.2) \quad df = \mu dt + \sigma dz$$

where z is a standard Wiener process. With this process, as shown by Froot and Obstfeld (1991), a permanent free-float would satisfy:

$$(4.3) \quad s = S_r(f) = f + \theta\mu.$$

This path is represented in figure 4.2 as the F-F path.

<figure 4.2 here>

When it is announced that the current free-float will be replaced by a **permanent** fixed exchange rate regime as soon as the exchange rate (the fundamental) hits s_2 (f_2), the saddle-path for the exchange rate during the transition period is given by:

$$(4.4) \quad s = S_{fe}(f) = f + \theta\mu(1 - \exp[r_2(f-f_2)])$$

where:

$$(4.5) \quad r_2 = \{-\mu + [\mu^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2 > 0.$$

Equation (4.4) is the closed-form solution of the problem posed by Flood and Garber (1983) and obtained by Froot and Obstfeld (1991). Path 1 in figure 4.2 represents this saddle-path when it is known that a return to a fixed exchange rate regime will be triggered as soon as s hits s_2 .

The path of the interest rate during this transition period can be derived as follows. First, by comparing equation (4.1) and (4.4) we get that:

$$(4.6) \quad E[ds]/dt = \mu(1 - \exp[r_2(f-f_2)]).$$

Then, assuming that the uncovered interest rate parity condition holds, it must be the case that:

$$(4.7) \quad i = i^* + \mu(1 - \exp[r_2(f-f_2)]).$$

This path is drawn as path 1 in figure 4.3, for the return at $s_{2f} = f_{2f}$.

<figure 4.3 here>

Had the central bank announced a fixed rate entry at $s = s_{z2}$, the paths for the exchange rate and the interest rate would have been given by paths 2 in figures 4.2 and 4.3, respectively.

4.2 Welfare issues

The problem raised in this section is the following: is there a preferred fixed exchange rate entry for the domestic central bank? For example, does the central bank prefer announcing a return at s_{z1} or s_{z2} or at some other level? To solve this problem, we have to introduce a welfare function that the central bank tries to maximize:

$$(4.8) \quad W(t) = \int_t^{\infty} E_t y(\tau) \exp[-\delta(\tau-t)] d\tau$$

where δ is the central bank discount rate and y is output. This form of objective function has been used by Ozkan and Sutherland (1994) and implies that the government dislikes economic recessions and prefers expansions. We assume that the price of goods is fixed and that output is determined by aggregate demand, itself a function of the exchange rate and the interest rate:

$$(4.9) \quad y = \gamma s - \eta i.$$

(Because prices are assumed to be fixed, s , i and y can be viewed as real variables.)²

Note that, in figure 4.2, the exchange rate takes on a higher value along path 2 than along path 1. Thus, along path 2, aggregate demand and the level of welfare will be higher during this transition period. But, as far as the interest rate is concerned, observe in figure 4.3 that the interest rate is higher along path 2 than along path 1, implying a lower aggregate demand and a lower level of welfare during the transition period. The welfare during the transition period will thus depend critically on the relative importance of parameters γ and η in equation (4.9).

Note also that once the exchange rate is permanently fixed either at s_{21} or at s_{22} , the interest rate is fixed at i^* . This means that when the exchange rate is permanently pegged (after the transition period), point 2 would always be preferred to point 1 because $s_{22} > s_{21}$ and thus, the present value of the perpetual flow of welfare associated with s_{22} is higher than the one for s_{21} .

The objective is then to choose the path that maximizes the welfare associated with the new permanent fixed regime plus the welfare associated with the transition period. To solve this problem we first have to calculate the welfare associated with a permanently fixed exchange rate at $s = s_2 = f_2$. Substituting equations (4.4) and (4.7) into (4.9), we obtain, after normalizing i^* to 0:

$$(4.10) \quad y = \mu(\gamma\theta - \eta) + \gamma f - \mu(\gamma\theta - \eta) \exp[r_2(f - f_2)].$$

Substituting f by f_2 in equation (4.10) we get the perpetual flow of output associated with a fixed exchange rate at s_2 :

$$(4.11) \quad y_2 = \gamma f_2.$$

Substituting this value for y in equation (4.8) and integrating, we get:

$$(4.12) \quad W_2 = \gamma f_2 / \delta$$

where W_2 is the present value of the perpetual flow of welfare for a fixed exchange rate at s_2 .

As a result, observe that as f_2 increases, W_2 increases.

Next we have to find the welfare function during the transition period. Differentiating equation (4.8) with respect to t , we obtain:

$$(4.13) \quad E[dW]/dt = \delta W - y.$$

To find $E[dW]/dt$, assume that W is a function of f . Because f follows a Brownian motion [described by equation (4.2)], we use Itô's lemma to get:

$$(4.14) \quad E[dW(f)]/dt = W(f) \mu + (\sigma^2/2) W_{ff}(f).$$

Substituting equations (4.14) and (4.10) in equation (4.13), a second-order differential equation is obtained:

$$(4.15) \quad W_{ff}(f) + (2\mu/\sigma^2)W_f(f) - (2\delta/\sigma^2)W(f) = -(2\mu/\sigma^2)(\gamma\theta-\eta)-(2\gamma/\sigma^2)f + (2\mu/\sigma^2)(\gamma\theta-\eta)\exp[r_2(f-f_2)].$$

The general solution to (4.15) is:

$$(4.16) \quad W(f) = (\gamma/\delta)f + [\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2 + \{[2\mu(\gamma\theta - \eta)]/(\sigma^2 r_2^2 + 2\mu r_2 - 2\delta)\} \exp[r_2(f - f_2)] \\ + A_1 \exp(\lambda_1 f) + A_2 \exp(\lambda_2 f)$$

where λ_1 and λ_2 are given by:

$$(4.17) \quad \lambda_1, \lambda_2 = \{-\mu \pm [\mu^2 + 2\sigma^2\delta]^{1/2}\}/\sigma^2, \quad \lambda_1 < 0 < \lambda_2.$$

The values for A_1 and A_2 are easily found through two boundary conditions. The first is an asymptotical condition: when s is far from s_2 (or, equivalently, when f is far from f_2), a return to the fixed rate regime is very unlikely in the near future and the level of welfare is asymptotically the one that would be obtained in a free float [i.e., the first two terms in equation (4.16)]. Thus A_1 can be set equal to 0.

The second boundary condition is given by equation (4.12) (i.e., the present value of the perpetual flow of welfare at $s_2 = f_2$). This comes from the assumption of a continuous exchange rate function (to prevent one-way arbitrage). The continuity of the exchange rate function implies the continuity of the interest rate function, the aggregate demand function and finally the welfare function. As a result, at f_2 , there cannot be any jump in W , such that at f_2 , W must be equal to $\gamma f_2/\delta$.

Evaluating $W(f)$ at f_2 in equation (4.16) and remembering that $A_1 = 0$, we obtain:

$$(4.18) \quad W(f_2) = (\gamma/\delta)f_2 + [\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2 + [2\mu(\gamma\theta - \eta)]/(\sigma^2 r_2^2 + 2\mu r_2 - 2\delta) + A_2 \exp[\lambda_2 f_2] = \gamma f_2/\delta.$$

Solving for A_2 , we have that:

$$(4.19) \quad A_2 = \{-[\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2 - 2\mu(\gamma\theta - \eta)/(\sigma^2 r_2^2 + 2\mu r_2 - 2\delta)\} \exp[-\lambda_2 f_2].$$

Substituting $A_1 (= 0)$ and A_2 given by equation (4.19) in (4.16) gives:

$$(4.20) \quad W(f) = (\gamma/\delta)f + [\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2 + \{2\mu(\gamma\theta - \eta)/(\sigma^2 r_2^2 + 2\mu r_2 - 2\delta)\} \exp[r_2(f - f_2)] \\ + \{-[\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2 - 2\mu(\gamma\theta - \eta)/(\sigma^2 r_2^2 + 2\mu r_2 - 2\delta)\} \exp[\lambda_2(f - f_2)].$$

To analyze this welfare function it is convenient to assume that $r_2 = \lambda_2$, which implies that $\delta = 1/\theta$ [compare equations (4.5) and (4.17)].³ In this case the welfare function during the transition period reduces to:

$$(4.21) \quad W(f) = [\gamma/\delta]f + \{[\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2\}(1 - \exp[\lambda_2(f - f_2)]).$$

Taking the first and second derivatives of W with respect to f , we get respectively:

$$(4.22) \quad W_f(f) = dW/df = [\gamma/\delta] - \lambda_2 \{[\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2\} \exp[\lambda_2(f - f_2)]$$

$$(4.23) \quad W_{ff}(f) = d^2W/df^2 = -\lambda_2^2 \{[\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2\} \exp[\lambda_2(f - f_2)].$$

Based on the above, we can study two cases.

CASE 1: $\mu\gamma + \delta(\gamma\theta - \eta)\mu > 0$, implying (given that $\delta = 1/\theta$) that $\gamma > \eta/2\theta$.⁴

In this case, analyzing the first and second derivatives [given in equations (4.22) and (4.23)], we conclude that the welfare function during the transition period [equation (4.21)] is **concave**. The graph of this function is given in figure 4.4 for the entry at $s_{z1} = f_{z1}$ (path 1) and the entry at $s_{z2} = f_{z2}$ (path 2).⁵

<figure 4.4 here>

Now, a welfare comparison of re-entry at $s_{z1} = f_{z1}$ versus at $s_{z2} = f_{z2}$ can be made. In figure 4.4, note that a return at s_{z2} would imply, relative to a return at s_{z1} , a higher present value of the perpetual flow of welfare once the exchange rate is fixed (i.e., $\gamma f_{z2}/\delta > \gamma f_{z1}/\delta$), and a higher value of welfare during the transition period (i.e., path 2 is always above path 1).

This result can also be obtained by differentiating total welfare, $W(f) + W_z$, with respect to f_z :

$$(4.24) \quad dW/df_z|_{f_{cm}} = \{\lambda_2[\mu\gamma + \delta(\gamma\theta - \eta)\mu]/\delta^2\} \exp[\lambda_2(f - f_z)] + \gamma/\delta.$$

The first term in equation (4.24) corresponds to the increase in welfare during the transition period due to an increase in $s_z = f_z$ (i.e., a postponement of the implementation of the fixed rate regime). This term is assumed positive in Case 1. The second term in equation (4.24) is the increase in the present value of the perpetual flow of welfare due to a pegging at a higher exchange rate (i.e., a more depreciated value for the domestic currency). Figure 4.5 shows the (positive) marginal increase in total welfare due to a fixed exchange rate postponement.

<figure 4.5 here>

Therefore, if the central bank observes that $\gamma > \eta/2\theta$ (i.e., Case 1) and has to choose between a return at s_{z1} or at $s_{z2} > s_{z1}$, it will prefer s_{z2} because postponing the entry always marginally raises welfare. Typically, when a return to a fixed exchange rate is advocated, a range of possible exchange rate targets, say $[s_{z1}, s_{z2}]$ is proposed. If Case 1 is relevant, a return to the higher proposed exchange rate target, s_{z2} , is recommended: the central bank now has a specific criterion to motivate its choice.

CASE 2: $\mu\gamma + \delta(\gamma\theta - \eta)\mu < 0$, implying (given that $\delta = 1/\theta$) that $\gamma < \eta/2\theta$.⁶

In this case, the first and second derivatives, given respectively by equations (4.22) and (4.23), are positive. This generates a **convex** welfare function for the transition period, as represented by path 1, in figure 4.6, for an announced return to a fixed exchange rate at $s_{z,1} = f_{z,1}$.

<figure 4.6 here>

Observe that if a return at $s_{z,2} = f_{z,2}$ is announced, path 2 is obtained.

We see that during the transition period, welfare path 1 is above welfare path 2, revealing a preference for path 1 (in contrast to Case 1). But, when the fixed exchange rate is triggered, the permanent flow associated with $s_{z,2}$ is higher than the one associated with $s_{z,1}$ (as in Case 1). A trade-off is emerging; announcing a re-entry at $s_{z,1}$ instead of $s_{z,2}$ involves a higher welfare during the transition period [the first term in equation (4.24) is now negative], but a lower permanent flow of welfare once the exchange rate is pegged [the second term in equation (4.24) is positive]. In other words, a country that fulfils Case 2 would permanently benefit from a higher fixed exchange rate but transition costs exist and thus must be compared with the present value of the future gains obtained once the exchange rate is pegged.⁷

The problem is thus to find a particular rule for the central bank to decide when the fixed exchange rate should be triggered. Setting equation (4.24) equal to 0 obtains:

$$(4.25) \quad f_z^* = f - [1/\lambda_2] \ln\{-\gamma\delta/\lambda_2(\mu\gamma + \delta(\gamma\theta - \eta)\mu)\}.$$

The calculated value of the second derivative indicates that $f_z^*(=s_z^*)$ is actually a minimum.

<figure 4.7 here>

Figure 4.7 shows that for $s_z(=f_z) < s_z^*(=f_z^*)$, any postponement of the fixed rate would generate decreases in welfare (i.e., the marginal welfare is negative); it is always better to fix the exchange rate as soon as possible. But, for $s_z > s_z^*$, postponement of the entry is preferable because it generates marginal increases in welfare. The choice of a specific pegged exchange rate in the interval $[s_{z1}, s_{z2}]$ is obvious: if $s_{z1} < s_{z2} < s_z^*$, it is always better for the central bank to announce a return at s_{z1} . In the opposite situation (i.e., when $s_z^* < s_{z1} < s_{z2}$), announcing a return at s_{z2} is preferable.

4.3 Conclusion

This chapter applies the two-step method of regulated Brownian motions to analyze a state-dependent switch to a fixed exchange rate regime from a situation of free float. The two-step method was initially introduced by Froot and Obstfeld (1991) to solve for the problem raised by Flood and Garber (1983): what is the path of the exchange rate when an announcement is made to switch from the current free-float regime to a fixed rate at some future state of the market?

The contribution of this chapter is to introduce some welfare issues in the choice of a specific exchange rate pegging level once the return to a fixed exchange rate is on the public policy agenda and a given **range** for the pegging has already been proposed. This model does not propose a theory of choice between free-float and fixed rate regimes, but rather gives a criterion to choose between fixed exchange rate levels, taking into account the transition period from the

free-float to the implementation of the fixed regime.

This analysis has been kept as simple as possible but can be extended by analyzing the case where $r_2 \neq \lambda_2$. Other extensions could be carried out by analyzing an appreciating trend for the exchange rate during the transition period, introducing a less extreme form of price stickiness, as in a Calvo contract for example [Calvo (1983)], modifying the welfare function, and introducing lump-sum costs of implementing a fixed exchange rate regime. These extensions would increase the realism of this model.

Finally, if a country wants to enter a target zone instead of a fixed exchange rate regime, it was shown in Chapter 2 that target zone entry paths during the transition period will depend on the announced target zone entry-triggering state. Welfare considerations as introduced in Chapter 4 should help to choose between entering a target zone at the central parity of the zone, or, say, at the lower (upper) edge of the zone.

Endnotes

- 1) Regime switches from the free-float to a permanent fixed rate have been analyzed by Flood and Garber (1983), Smith and Smith (1990), Froot and Obstfeld (1991), Miller and Sutherland (1992) among others. These models describe a state-dependent switch (i.e., a switch that occurs when "the conditions are right"). Ichikawa, Miller, and Sutherland (1992) describe a regime switch from a free-float to a target zone regime based on a time-dependent switch (i.e., irrespective of the state of the market at the time of the switch).
- 2) If prices were fully flexible and the purchasing power parity condition held at each moment, there would be no real effects in this model and y would stay constant at θ . To generate some real effects, we need to assume some price stickiness. Assuming that the price of goods is fixed is an extreme assumption (as is the purchasing power parity condition). A more realistic assumption would be to model price stickiness as in a Calvo contract [see Calvo (1983)].
- 3) Integrating equation (4.1) forward, we get: $s(t) = (1/\theta) \int_t^\infty E f(\tau) \exp[(-1/\theta)(\tau-t)] d\tau$. $1/\theta$ may thus be understood as the rate of discount on future values of the fundamentals. As seen in equation (4.8), δ is the rate of discount on future values of the output. Setting $1/\theta = \delta$ simply means that the two rates of discount are assumed identical. Setting $1/\theta \neq \delta$ would not change fundamentally the **logic** of the argument, in particular the derivation of two cases as given shortly.
- 4) Based on equation (4.9), the condition $\gamma > \eta/2\theta$ indicates a relatively large sensitivity of output with respect to the exchange rate. This condition is more likely to be fulfilled in a small open economy.
- 5) Setting equation (4.22) equal to 0 gives an extremum for $W(f)$, which is a maximum because the second derivative [equation (4.23)] is negative under Case 1. The maximum value for $W(f)$ is obtained when f reaches: $f^* = f_z + (1/\lambda_2) \ln\{\gamma\delta/\lambda_2(\mu\gamma + \delta(\gamma\theta - \eta)\mu)\}$. Depending on the value of $\ln\{ \cdot \}$ we can observe three situations:
 - a) if $\gamma\delta < \lambda_2(\mu\gamma + \delta(\gamma\theta - \eta)\mu)$, $\ln\{ \cdot \} < 0$ and $f^* < f_z$.
 - b) if $\gamma\delta = \lambda_2(\mu\gamma + \delta(\gamma\theta - \eta)\mu)$, $\ln\{ \cdot \} = 0$ and $f^* = f_z$.
 - c) if $\gamma\delta > \lambda_2(\mu\gamma + \delta(\gamma\theta - \eta)\mu)$, $\ln\{ \cdot \} > 0$ and $f^* > f_z$.
 Figure 4.4 depicts the particular case where $f^* = f_z$ (i.e., situation b).
- 6) This condition is more likely to be fulfilled by a large economy with a small degree of openness.
- 7) Note the analogy with the debate on controlling inflation. A country would permanently benefit from a lower (zero) and stable inflation rate. But the transition costs involved in the decrease of the current positive rate of inflation may be considered as too high relative to the present value of future gains of having a stable zero inflation rate.

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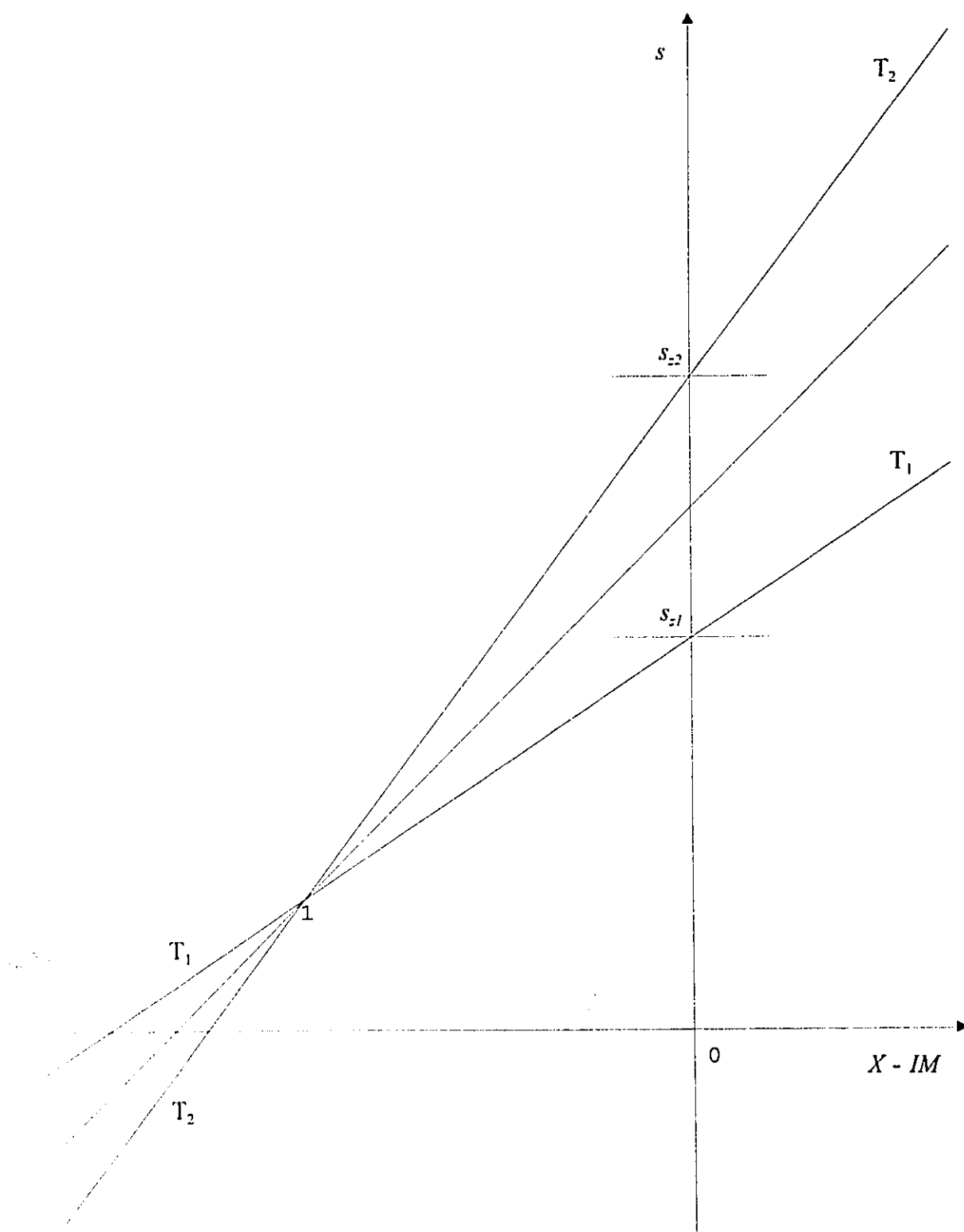


Figure 4.1 A range for the equilibrium exchange rate

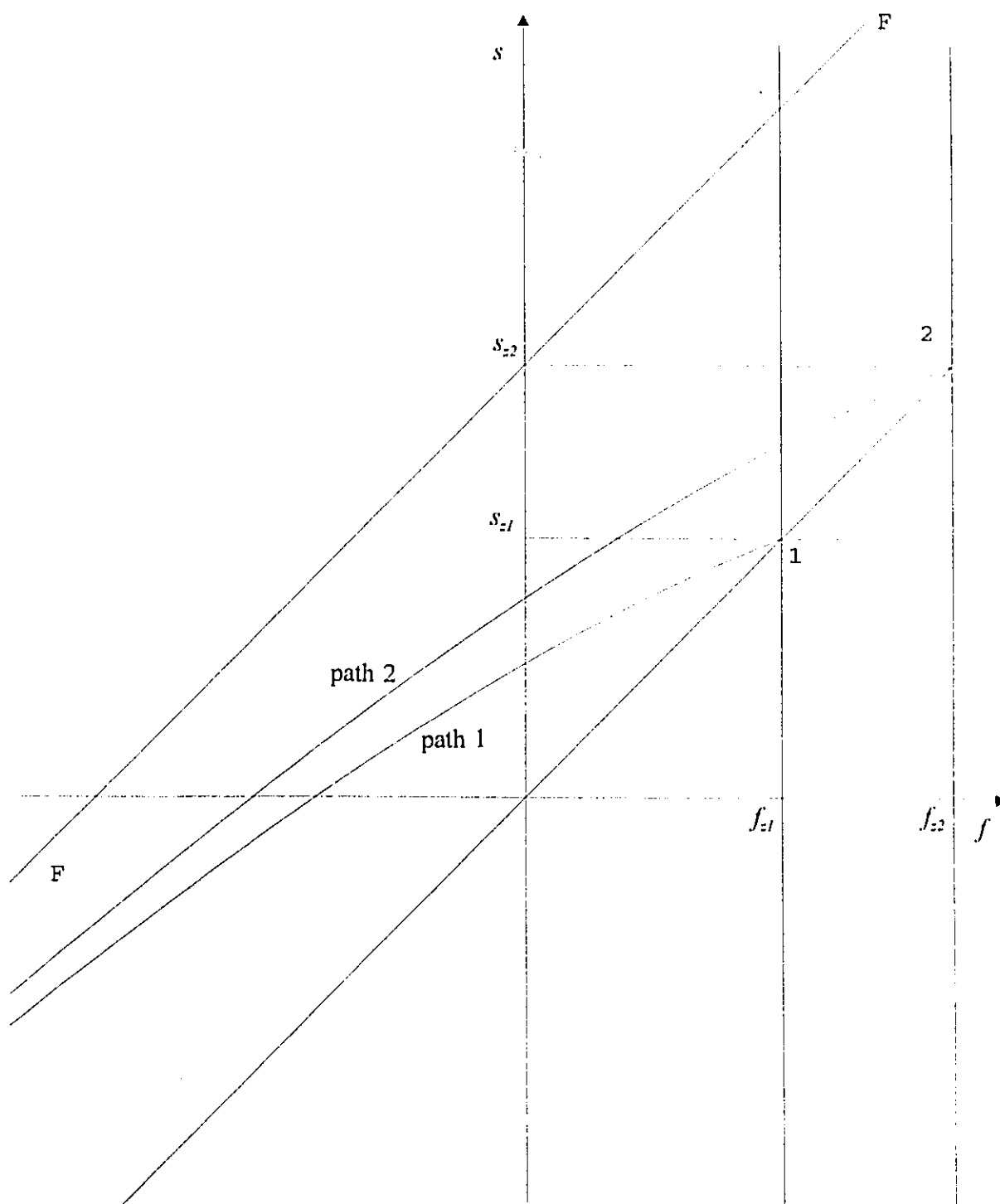


Figure 4.2 Fixed exchange rate entry paths

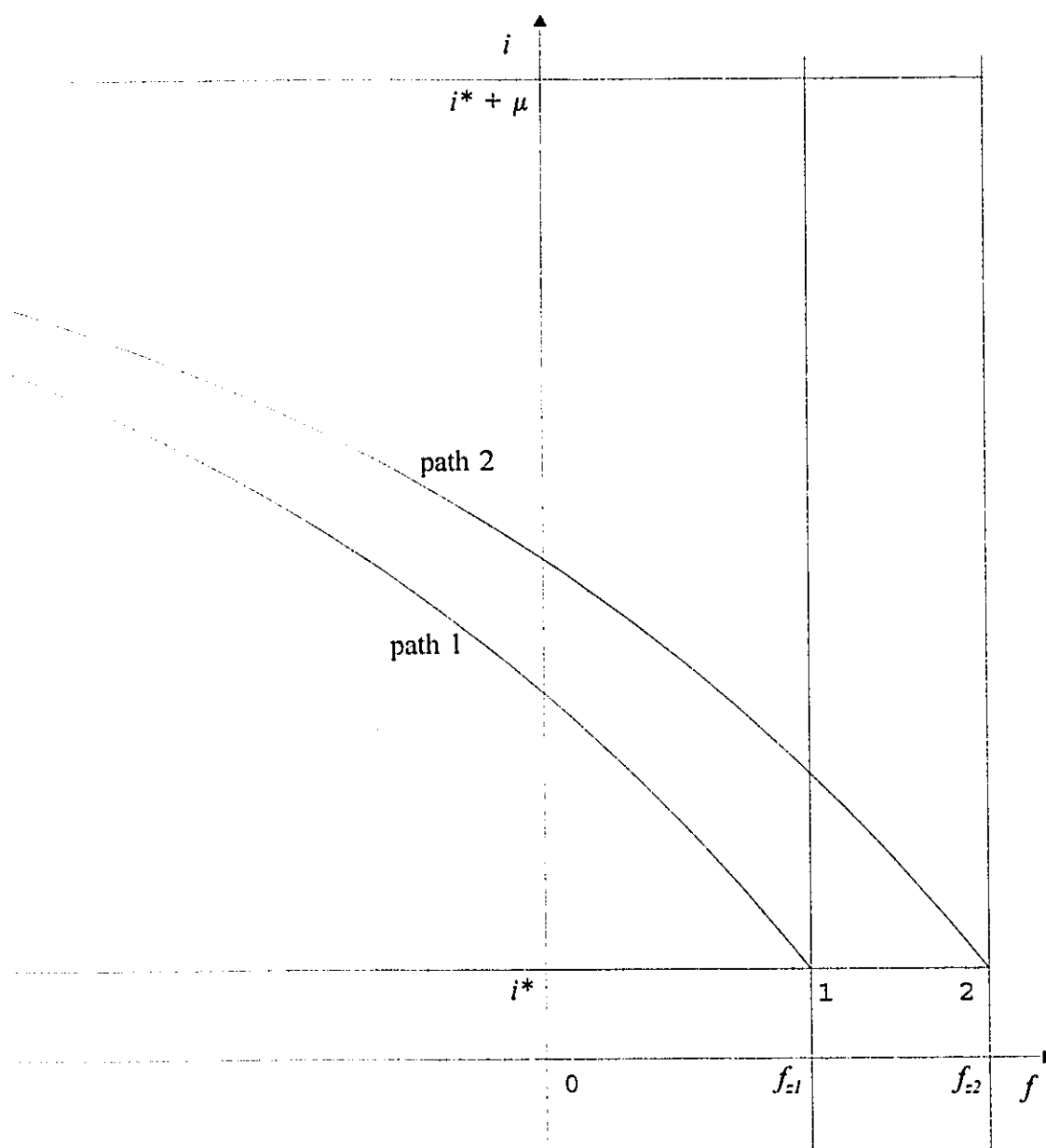


Figure 4.3 Interest rate entry paths

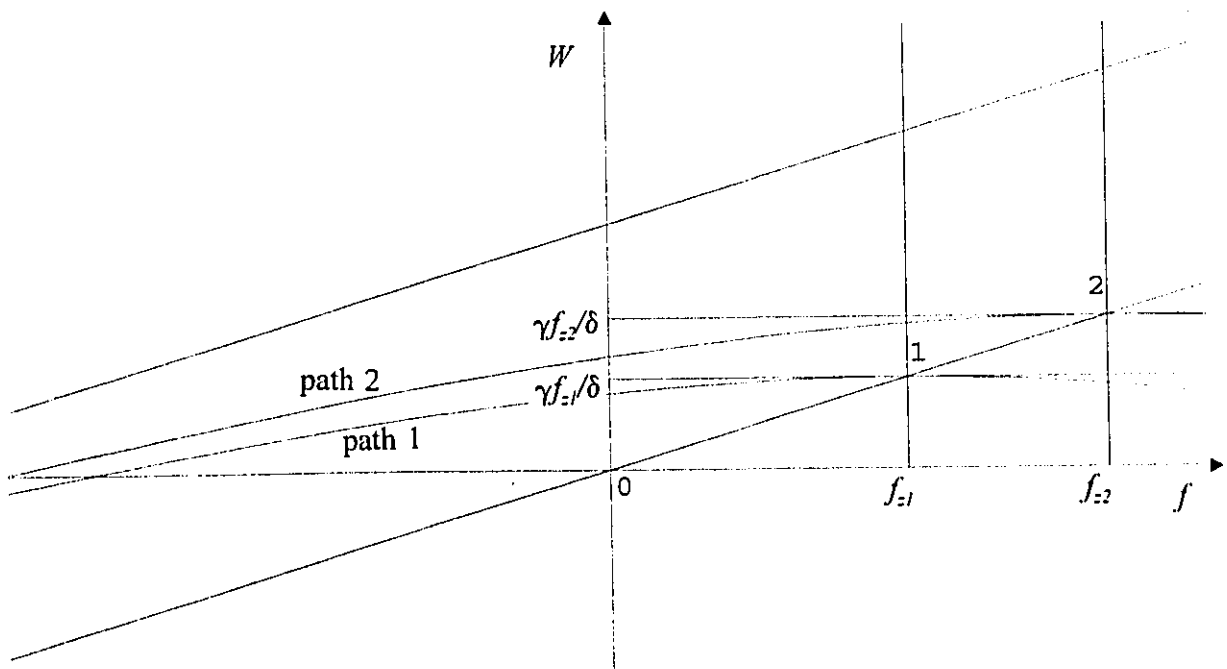


Figure 4.4 Transition welfare paths, Case 1

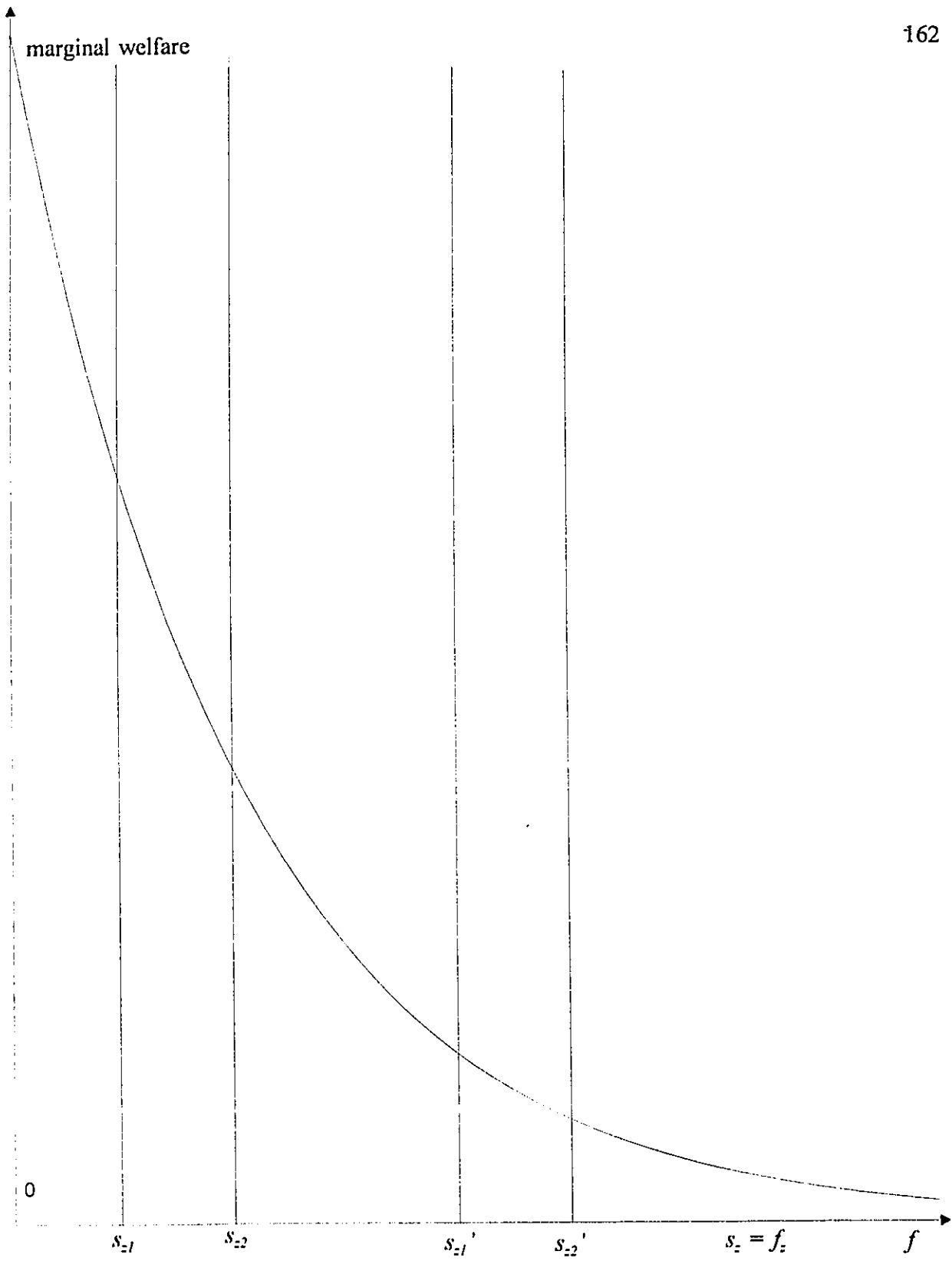


Figure 4.5 Marginal welfare curve, Case 1

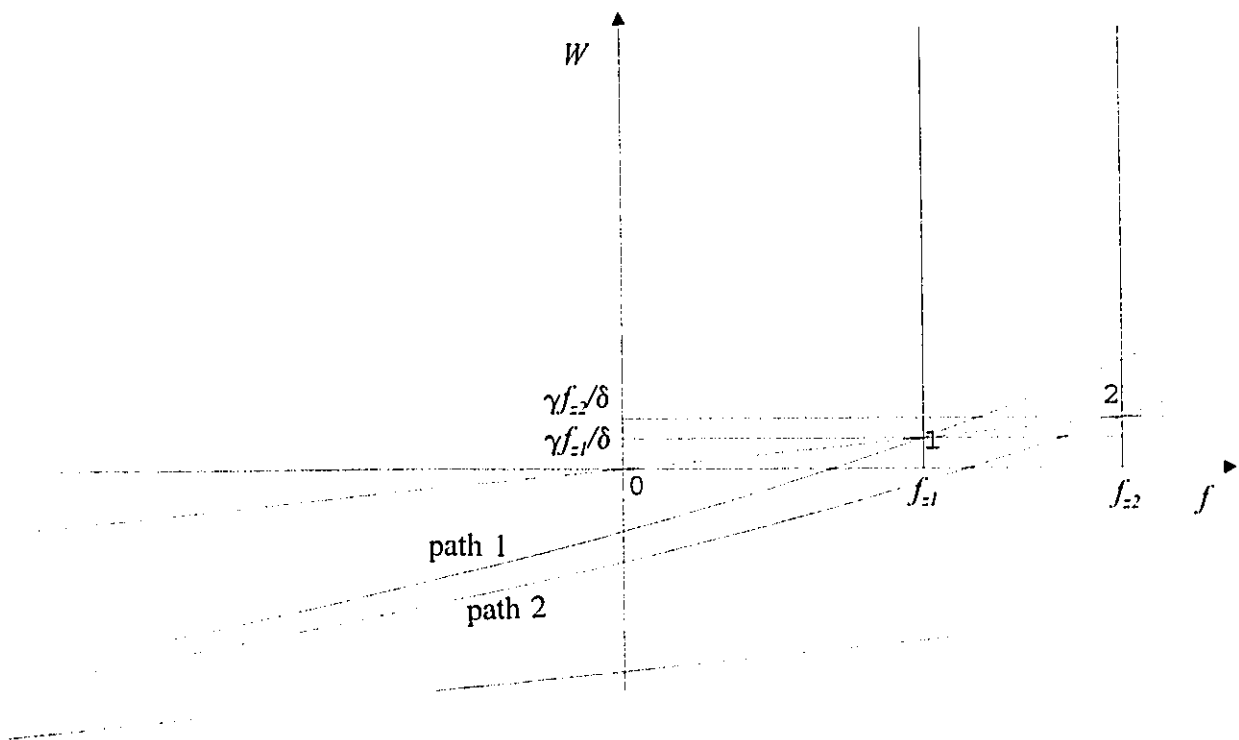


Figure 4.6 Transition welfare paths, Case 2

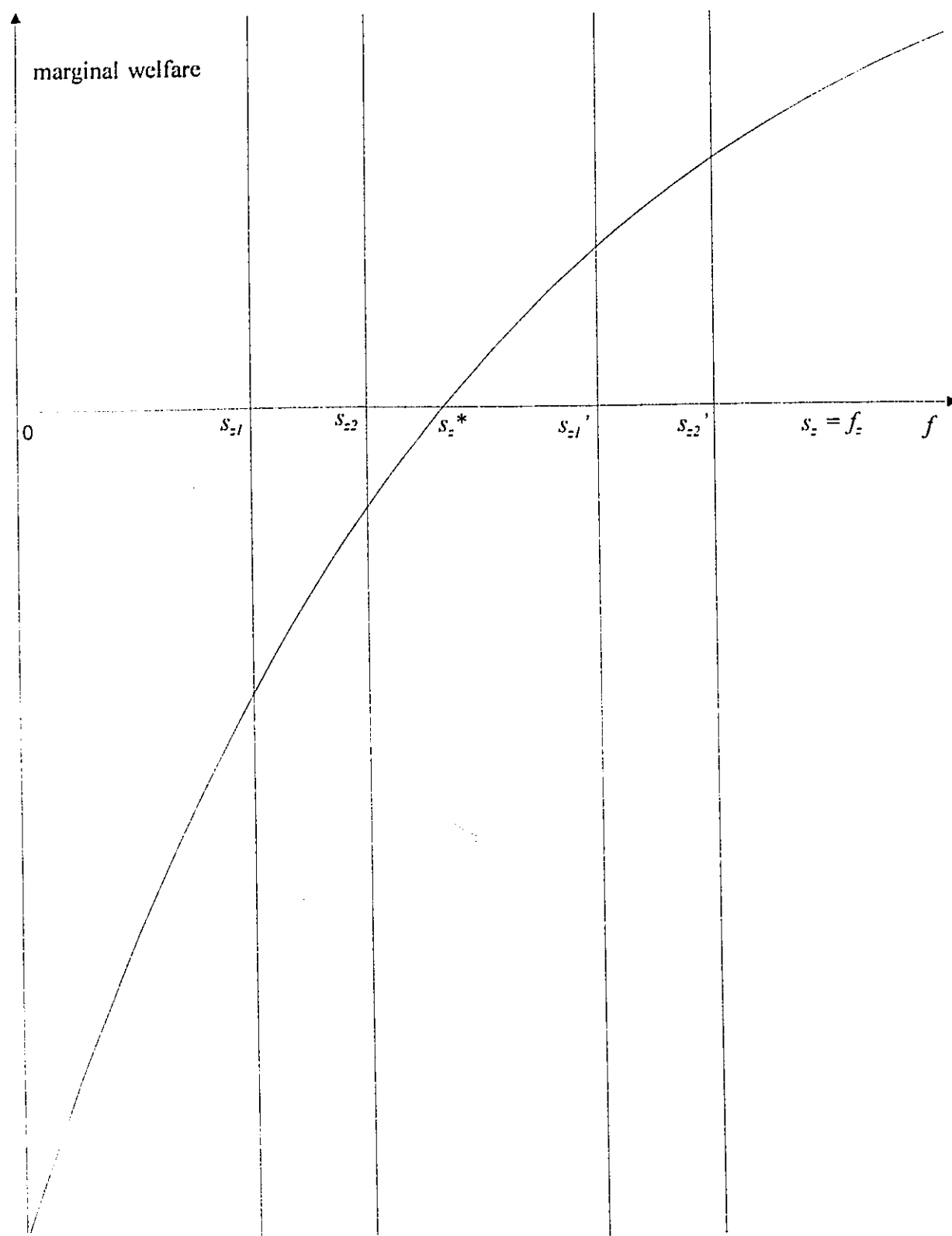


Figure 4.7 Marginal welfare curve, Case 2

5 CONCLUSION

This thesis studies the operation of exchange rate regimes in the context of possible regime switches. Since the exchange rate is determined partly by agents' beliefs about future events, deliberations and announcements of policy makers inject the probabilities that exchange rate regime switches will occur in the future into current exchange rate determination. After an intuitive introduction to this literature, three independent essays have been presented in Chapters 2, 3, and 4.

Chapter 2 describes a model of target zone entry. This model is a generalisation of the model by Froot and Obstfeld (1991a). Basically, instead of announcing a return from the current free-float to a pure fixed exchange rate regime [as envisaged by Froot and Obstfeld (1991a)], the domestic central bank announces a return to a TZ. As shown by Krugman (1991), a TZ is a nonlinear compromise between a fixed exchange rate and a freely flexible regime. As a result, the dynamics of the exchange rate during the transition to a TZ must differ from the ones involved for the return to a pure fixed rate. Indeed, it is shown in Chapter 2 that the derived nonlinear relationship between the exchange rate and its fundamentals during the transition to a TZ is in general different from the relationship corresponding to a return to a fixed exchange rate regime. This is essentially due to a "reflecting effect", the mirror image of the well-known "honeymoon effect" of a TZ, and to a "bandwidth effect".

The problem envisaged in Chapter 2 is a **state-dependent** TZ entry; this underlines an approach based on macro-economic "convergence". Politicians tend to emphasize convergence in inflation rates and/or convergence in living standards as a **precondition** for successful target

zone regimes. Convergence in inflation rates as a precondition to a TZ entry does not make much sense because a TZ is in itself the provider of anti-inflation credibility. Indeed, a TZ regime confronts domestic firms and workers with the certainty that inflationary wages and prices would affect their competitiveness, rather than be accommodated by a domestic currency depreciation. That knowledge would make such inflationary pressure less likely in the first place. As a result, the higher a country's inflation rate, the more it needs a TZ regime. In this context, convergence in inflation rates is a consequence of a successful TZ regime and should not be imposed as a precondition to the entry. Convergence in living standards as a precondition to a TZ does not make much sense either. Indeed, many of the world's most successful developing countries have fixed their currency to the dollar, the French franc, or a basket of rich country currencies.

What matters for a TZ entry is initial convergence in price levels. In other words, a TZ should be triggered when the exchange rate broadly equalizes price levels in both countries. To enter a TZ at too high a rate (a more depreciated domestic currency) improves the competitiveness of the domestic country, but forces domestic prices up. Entering at a too low a rate does the opposite. Thus, TZ entry is assumed, in Chapter 2, to be triggered as soon as a specific (purchasing power) level for the exchange rate is reached. This guarantees the least initial strain of adjustment on the domestic country, while applying an immediate anti-inflation policy. Thereafter, free trade and fixed exchange rates look like the best way to keep prices in step.

Chapter 3 presents a model of target zone with expected trend revisions. Relative to a fixed exchange rate, a TZ regime grants policymakers a certain degree of independence in setting their monetary policy. This independence is not necessarily strictly limited to a level of the money

supply but also to a trend; in other words, changes in the trend of the fundamentals of the exchange rate may exist and may thus be anticipated by the market. A target zone model with positive trend is sensitive to speculative attacks, as shown by Delgado and Dumas (1993). The domestic central bank may prefer to stop or postpone any speculative moves from the market by announcing a return to a TZ with zero trend, implying a more disciplined domestic monetary policy. This announcement, even if not fully credible, will have an impact on the short-run sustainability of the TZ. As a result, speculative attacks may be postponed.

A target zone regime is also sensitive to the traditional liquidity / nominal anchor problem (i.e., the way global monetary policies are determined in the two countries forming the TZ). This problem is most often solved in an asymmetric way; that is, the leader, say the foreign country, imposes its own monetary policy on the domestic country. With time, the domestic country may become unsatisfied with the monetary policy of the foreign country, and may decide to revise its monetary policy, implying a change in the trend of the fundamentals.

The market expectations of a trend revision has an immediate impact on the current level of the exchange rate as well as on interest rate differentials. The expectation of a trend revision implies a positive relationship between the exchange rate and the interest rate differential. As a result, the well-known negative relationship between the exchange rate and the interest rate differentials of the basic TZ model is shown to be unstable during periods where a revision of the trend is expected by the market.

Finally, Chapter 4 introduces some welfare issues into the initial model of fixed exchange rate regime entry of Froot and Obstfeld (1991a). If a return to a fixed exchange rate is on the public policy agenda and economists propose, a priori, a given range for the real exchange rate, which

level of the exchange rate from this range should be selected as the fixed rate regime triggering state? Depending on specific parameters of the model, Chapter 4 shows that either the higher or the lower exchange rate level from the proposed range should be announced to the market as the fixed rate triggering state (i.e., a corner solution is obtained). The lower value of the exchange rate's range comes as a surprise in a model which has a strong bias for depreciation. This is the result of a transition period involving transition costs that must be compared with the present value of the future gains obtained once the exchange rate is pegged.

The nature of the solution (i.e., a corner solution) comes from the assumption of the initially given range for the exchange rate. Without this assumption, the model would collapse. It would be interesting to improve the model in order to get rid of this initial range and still get a solution. The assumption of fixed prices is certainly not satisfactory; a sticky price model as in Calvo (1983) should be considered. The welfare function is also too simple. In other words, the model of Chapter 4 must be understood as a first step to be developed further.

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APPENDIXES

A1 Introduction to Brownian motions and Poisson processes

A1.0 Introduction

The objective of this Appendix is to introduce intuitively the notion of a generalised Wiener process or Brownian motion (section A1.1) as well as the rule (Itô's lemma) to totally differentiate a function of a random variable following such a Brownian motion (section A1.2). Whereas sections A1.1 and A1.2 study processes which are everywhere continuous (diffusion processes), section A1.3 introduces processes that make infrequent but discrete jumps (i.e, Poisson processes). Appendix A1 draws heavily on Hull (1989), Pindyck (1991), Fisher (1975), and Cox and Miller (1968). A more advanced presentation is in Dixit (1991). A mathematical approach is given in Harrison (1985).

A1.1 A generalised Wiener process

The increments of the exchange rate fundamentals are assumed, in the literature surveyed in Chapter 1, to be given by the following stochastic differential equation:

$$(A1.1) \quad df = \mu dt + \sigma dz$$

where μ and σ are constant parameters. The stochastic process described by equation (A1.1) is called a **generalised Wiener process** or **Brownian motion with drift**. Note that any variable

whose value changes over time in an uncertain way is said to follow a stochastic process. The dz component, which makes this process stochastic, is called a **standard** Wiener process. In a small time interval Δt , the change in f , Δf , is given by a stochastic difference equation:

$$(A1.2) \quad \Delta f = \mu \Delta t + \sigma \Delta z.$$

The first objective of this section is to explain the stochastic part of equation (A1.2) (i.e., to explain Δz). Once Δz is understood, by taking the limit as Δt tends to zero we will obtain the Wiener process dz . The second objective is to explain the generalised Wiener process or Brownian motion with drift given in equation (A1.1). Finally, a discrete-time discrete-state random walk representation of a Brownian motion with drift is presented.

i) A standard Wiener process

Consider the following discrete-time continuous-state random walk:

$$(A1.3) \quad z(t+1) = z(t) + \epsilon(t+1); \quad z(0) = z_0; \quad \epsilon(t+1) \sim i.i.d.N(0,1).$$

The variable t represents time and is measured in *discrete* integer increments from $-\infty$ to $+\infty$. This justifies the word *discrete-time* random walk. The random variable $\epsilon(t+1)$ is serially chosen from a normal distribution with mean 0 and variance 1. Also, the draws through time are independent of each other and identically distributed: $\epsilon(t+1) \sim i.i.d.N(0,1)$. This justifies the word *continuous-state* random walk. [If $\epsilon(t+1)$ were a random variable that could only take the values, say, $+1$ and -1 , each with probability $1/2$ and for $t = 0, 1, 2, \dots$, we would have an example of a discrete-time *discrete-state* random walk.]

Introducing the unit of time, suppose that time, in equation (A1.3), increases at monthly increments (i.e., $+1$ represents a monthly increment). Obviously, we could be interested by

smaller time increments, for example weeks. A stochastic process that "mimics" the process given in (A1.3) would be:

$$(A1.3') \quad z(t+T) = z(t) + \epsilon(t+T); \quad z(0) = z_0; \quad \epsilon(t+T) \sim i.i.d.N(0, T).$$

where $T = 1/m$. If $+1$ represents monthly increments and $m = 4$, thus $+T$ represents weekly increments. "Mimicking" means that the stochastic properties of periodic samples from a more frequently observed process would be the same as those observed in the less frequently observed process, provided the observation interval were the same. Thus, observe that:

$$E[\sum_{i=1}^m \epsilon_i(t+T)] = 0 = E[\epsilon(t+1)],$$

and:

$$Var[\sum_{i=1}^m \epsilon_i(t+T)] = mT = 1 = Var[\epsilon(t+1)].$$

We can continue to choose smaller time intervals. For example, break T into n small intervals Δt such that $n\Delta t = T$. Again, a stochastic process that mimics (A1.3') is:

$$(A1.3'') \quad z(t+\Delta t) = z(t) + \epsilon(t+\Delta t); \quad z(0) = z_0; \quad \epsilon(t+\Delta t) \sim i.i.d.N(0, \Delta t).$$

The small change in z , Δz , during the interval Δt is thus given by:

$$(A1.4) \quad \Delta z = z(t+\Delta t) - z(t); \quad \Delta z = \epsilon(t+\Delta t) \sim i.i.d.N(0, \Delta t).$$

Observe that, by setting:

$$\Delta z = \epsilon(t+\Delta t) = \epsilon(t+1)(\sqrt{\Delta t}),$$

we get, as should be:

$$E(\Delta z) = E[\epsilon(t+\Delta t)] = (\sqrt{\Delta t})E[\epsilon(t+1)] = 0,$$

and:

$$Var(\Delta z) = Var[\epsilon(t+\Delta t)] = \Delta t \quad Var[\epsilon(t+1)] = \Delta t.$$

Thus, there are two basic properties of Δz :

Property 1: Δz is related to Δt by the equation:

$$(A1.5) \quad \Delta z = \epsilon \sqrt{\Delta t}; \epsilon \sim i.i.d.N(0,1),$$

such that, as shown in (A1.4), $\Delta z \sim i.i.d.N(0, \Delta t)$.

Property 2: Because ϵ is serially uncorrelated (i.e., $E[\epsilon(t)\epsilon(s)] = 0$ for $t \neq s$), the values of Δz for any two different short intervals of time Δt are independent. This implies that z follows a Markov process with independent increments. Note that a Markov process is a particular type of stochastic process where only the present state of the process is relevant for predicting the future. The past history of the process and the way in which the present has emerged from the past are irrelevant.

A Wiener process is the limit, as Δt tends to dt , of the process for z , described in equations (A1.4) and (A1.5). dt is heuristically defined as the smallest positive real number such that $(dt)^\alpha = 0$ whenever $\alpha > 1$. The limiting case of equations (A1.4) and (A1.5) are given by:

$$(A1.4') \quad dz = z(t+dt) - z(t); dz = \epsilon(t+dt) \sim i.i.d.N(0, dt),$$

and:

$$(A1.5') \quad dz = \epsilon \sqrt{dt}; \epsilon \sim i.i.d.N(0,1).$$

This is the standard Wiener process. The state space of this process is the continuum of real numbers (continuous-state stochastic process) and the changes of state are occurring all the time (continuous-time stochastic process).

Observe the properties that follow by construction:

1. $E[dz] = 0$.
2. $E[dzdt] = dtE[dz] = 0$.
3. $E[dz]^2 = Var[dz] = dt$.

4. $Var[dz]^2 = E[(dz)^2 - dt]^2 = E[(\epsilon \sqrt{dt})^2 - dt]^2 = 0$ [given that $(dt)^2 = 0$ by heuristic definition of dt].
5. $E[dzdt]^2 = 0$.
6. $Var[dzdt] = 0$.

Note that if the variance of a random variable is zero, the expectation operator is redundant. In consequence, properties 2 and 3 imply, given properties 6 and 4, the following multiplication rules:

7. $dzdt = 0$.
8. $(dz)^2 = dt$.

Other important properties are:

1. $z(t)$ is continuous in t . Indeed, although dz is a random variable, it is of infinitesimal magnitude.
2. $z(t)$ is not differentiable (there is no time derivative) since the left and right differentials are not the same (the dz 's are independently drawn). Observe that, by equation (A1.5), $\Delta z/\Delta t = (\Delta t)^{-1/2}\epsilon$, which becomes infinite as Δt approaches zero.
3. The conditional distribution of $z(u)$ given $z(t)$, for $u > t$, is normal with mean $z(t)$ and variance $(u-t)$. $z(\cdot)$ is an integral of random variables dz . The sum of normally distributed random variables is also normal: the mean of the sum is the sum of the means, and the variance of the sum equals the sum of the variances if the correlations are all zero.
4. The variance of a forecast $z(u)$ increases indefinitely as $u \rightarrow \infty$: the Wiener process is nonstationary.

Basically, this process can be viewed as a continuous-state, continuous-time random walk [see

Cox and Miller (1968, pp. 203-10)]. In a small time interval, such a process can only undergo a small displacement or change of state. We may then expect realizations of such processes to be continuous functions. It has been used in physics to describe the motion of a particle, suspended in a fluid, that is subject to the rapid, successive, random impacts of neighbouring particles. If, for such a particle, the displacement in a given direction were plotted against time, we would expect to obtain a continuous, if somewhat erratic, graph which would in fact be a realization of a stochastic process in continuous-time with continuous-state space. As an approximation to this Wiener process, we may divide the time axis and the state axis each into a very large number of small intervals and consider a particle undergoing a simple random walk, moving one small step up or down in each small time interval.

ii) A generalised Wiener process or Brownian motion with drift

Consider a discrete-time random walk with drift μ and variance σ^2 :

$$(A1.6) \quad f(t+1) = f(t) + \mu + \sigma\epsilon(t+1); f(0) = f_0; \epsilon(t+1) \sim i.i.d.N(0,1).$$

Choosing a subinterval $T = 1/m$ that mimics the behaviour of this process, we can write:

$$(A1.6') \quad f(t+T) = f(t) + \mu T + \sigma\epsilon(t+T); f(0) = f_0; \epsilon(t+T) \sim i.i.d.N(0,T).$$

Observe that:

$$E[f(t+T)-f(t)] = \mu T,$$

and:

$$Var[f(t+T)-f(t)] = E[\mu T + \sigma\epsilon(t+T) - \mu T]^2 = \sigma^2 E[\epsilon(t+T)]^2 = \sigma^2 T.$$

Choosing yet a smaller interval $\Delta t = T/n$ that mimics the behaviour of (A1.6'), we can write:

$$(A1.6'') \quad f(t+\Delta t) = f(t) + \mu\Delta t + \sigma\epsilon(t+\Delta t); f(0) = f_0; \epsilon(t+\Delta t) \sim i.i.d.N(0,\Delta t).$$

Given equation (A1.4), equation (A1.6'') can be rewritten as:

$$(A1.7) \quad \Delta f = f(t+\Delta t) - f(t) = \mu\Delta t + \sigma\Delta z,$$

or also, using (A1.5):

$$(A1.7') \quad \Delta f = \mu\Delta t + \sigma\epsilon\sqrt{\Delta t}; \epsilon \sim i.i.d.N(0,1).$$

As we let $\Delta t \rightarrow dt$ in equation (A1.6''), we get:

$$(A1.6''') \quad f(t+dt) = f(t) + \mu dt + \sigma\epsilon(t+dt); f(0) = f_0; \epsilon(t+dt) \sim i.i.d.N(0,dt).$$

Given equation (A1.4'), (A1.6''') can also be rewritten as:

$$(A1.8) \quad df = f(t+dt) - f(t) = \mu dt + \sigma dz.$$

This is a **generalised** Wiener process or an (arithmetic) Brownian motion with drift for a variable f . The term dz (i.e., a standard Wiener process) is a normally distributed random variable, with mean zero and variance dt . Observe in equation (A1.8) that df is a linear function of a normal random variable. In consequence, df is also a random variable which is normally distributed. Note that:

$$E[df] = \mu dt + \sigma E[dz] = \mu dt,$$

and:

$$Var[df] = E[df - \mu dt]^2 = E[\sigma dz]^2 = \sigma^2 E[dz]^2 = \sigma^2 var[dz] = \sigma^2 dt.$$

In consequence, we have that:

$$(A1.9) \quad df = \mu dt + \sigma dz; df \sim i.i.d.N(\mu dt, \sigma^2 dt).$$

To understand fully the stochastic differential equation given in (A1.8), it is intuitively appealing to consider the two components of the right hand side separately. Suppose that the stochastic term σdz was not present in equation (A1.8). Thus we could rewrite (A1.8) as: $df = \mu dt$. This means that f grows at a constant drift rate μ per unit time (i.e., in a time interval of

length T , f increases by an amount μT . The term σdz (i.e., σ times a Wiener process), can thus be considered as adding noise or variability to the path followed by f .

Some important characteristics of the process described by (A1.8) are:

1. f may be positive or negative.
2. $f(t)$ is continuous in t . Indeed, although df is a random variable, it is of infinitesimal magnitude.
3. $f(t)$ is nowhere differentiable. Indeed, the increments in f are independent random variables. In consequence the left and right differentials are not the same. Thus, although the sample paths are continuous, this suggests that they are very kinky, and their derivatives exist nowhere. In other words, df/dt does not exist, and we cannot speak of $E(df/dt)$. However, $E(df)$ will in general exist, and so will $(1/dt)E(df)$.
4. The conditional distribution of $f(u)$ given $f(t)$, for $u > t$, is normal with mean: $f(t) + \mu(u-t)$ and variance: $\sigma^2(u-t)$.
5. The variance of a forecast $f(u)$ increases indefinitely as $u \rightarrow \infty$. A Brownian motion is nonstationary.

Note that what is called an Itô process, is a generalised Wiener process where the parameters μ and σ are functions of the value of the underlying variable, f , and time, t . The Itô process can thus be written as:

$$(A1.10) \quad df = \mu(f,t)dt + \sigma(f,t)dz.$$

The expected drift rate and variance rate of an Itô process are liable to change over time.

Now, a hypothetical example based on equation (A1.7') is given. Suppose that the variation of f is normally distributed with mean $\mu\Delta t$ and s.d. $\sigma\sqrt{\Delta t}$. Assume that the expected variation

of f is 2% per year and the s.d. of this variation (the volatility of f) is 20% per year. Assume that $\Delta t = 0.01$, so that we are considering changes in f in time interval of length 0.01 year or 3.65 days. It follows that:

$$\begin{aligned}\Delta f &= \mu \Delta t + \sigma \epsilon \sqrt{\Delta t} \\ &= 0.02 * 0.01 + \epsilon * 0.2 * \sqrt{0.01} = 0.0002 + \epsilon * 0.02.\end{aligned}$$

A realization path for f can thus be simulated by sampling for ϵ from a standardized normal distribution. For example we could obtain:

Initial f	random sample for ϵ	Δf	new f
1.2452	0.52	0.0106	1.2558
1.2558	1.44	0.029	1.2848
1.2848	-0.86	-0.017	1.2678
1.2678	1.46	0.0294	1.2972
1.2972	-0.69	-0.0136	1.2836
1.2836	-0.74	-0.0146	1.2690
1.2690	0.21	0.0044	1.2734
1.2734	-1.10	-0.0218	1.2516
1.2516	0.73	0.0148	1.2664
1.2664	1.16	0.0234	1.2898
1.2898	2.56	0.0514	1.3412

In this example 10 small intervals Δt are considered. $f(t) = 1.2452$ and $f(t+10\Delta t) = 1.3412$ (i.e., a realization). Based on the current value of the process, $f(t)$, we can also construct an optimal forecast for $f(t+10\Delta t)$ given by $\hat{f}(t+10\Delta t)$, and a confidence interval for the forecast. The

following calculations are based on property 4 above [i.e., the conditional distribution of $f(u)$ given $f(t)$, for $u > t$, is normal with mean: $f(t) + \mu(u-t)$ and variance: $\sigma^2(u-t)$]. The forecast of $f(t+10\Delta t)$ given that $f(t) = 1.2452$ is thus:

$$\begin{aligned}\underline{f}(t+10\Delta t) &= \underline{f}(u) = f(t) + \mu(u-t) = f(t) + \mu(10\Delta t) \\ &= 1.2452 + 0.02*10*0.01 = 1.2472.\end{aligned}$$

A 66-percent confidence interval is given by the forecast plus or minus one standard deviation, such that:

$$\begin{aligned}f(t) + \mu(u-t) \pm \sigma \sqrt{(u-t)} &= 1.2472 \pm 0.2 * \sqrt{(10\Delta t)} \\ &= 1.2472 \pm 0.06324 = [1.1840; 1.3104].\end{aligned}$$

A 95-percent confidence interval is given by the forecast plus or minus 1.96 standard deviations, such that:

$$\begin{aligned}f(t) + \mu(u-t) \pm 1.96 [\sigma \sqrt{(u-t)}] &= 1.2472 \pm 1.96 * 0.2 * \sqrt{(10\Delta t)} \\ &= 1.2472 \pm 0.12396 = [1.1232; 1.3712].\end{aligned}$$

Thus, observe that the realization $f(t+10\Delta t) = 1.3412$ is indeed in the 95-percent confidence interval, but not in the 66-percent confidence interval.

iii) Random walk representation of a Brownian motion

In subsection (i), we saw that a standard Wiener process can be viewed as the limit of a discrete-time continuous-state random walk. In subsection (ii), a Brownian motion with drift was considered as the limit of a discrete-time continuous-state random walk with drift. Now, we consider a Brownian motion with drift as the limit of a discrete-time and discrete-state random walk with drift.

Let us divide time up into discrete periods of length Δt , and assume that in each period the variable f moves up or down by an amount Δh (i.e., $\Delta f = \pm \Delta h$). In other words, Δf is a random variable defined over a sample space $S = \{\text{up}, \text{down}\}$ with value in R ($+\Delta h, -\Delta h$). Let the probability that it moves up be p and the probability that it moves down be $q = 1 - p$. These probabilities are assumed independent of what happened in previous periods (i.e., f follows a Markov process with independent increments).

If the random experiment of observing Δf is repeated only once, we get a Bernoulli trial such that:

$$(A1.11) \quad E(\Delta f) = (p-q)\Delta h,$$

and:

$$(A1.12) \quad \text{Var}(\Delta f) = [1-(p-q)^2](\Delta h)^2.$$

If we are considering an interval of length T with $n = T/\Delta t$ discrete and independent steps, the cumulative change in f , $[f(t+T)-f(t)]$, is a binomial variable with mean:

$$(A1.13) \quad E[f(t+T)-f(t)] = n(p-q)\Delta h = T(p-q)\Delta h/\Delta t$$

and variance:

$$(A1.14) \quad \text{Var}[f(t+T)-f(t)] = n[1-(p-q)^2](\Delta h)^2.$$

Introducing more structure in this model, suppose that the relationship between Δh and Δt is given by:

$$(A1.15) \quad \Delta h = \sigma \sqrt{\Delta t},$$

and p and q are given by:

$$(A1.16a) \quad p = (1/2)[1 + (\mu/\sigma)\sqrt{\Delta t}]$$

and:

$$(A1.16b) \quad q = (1/2)[1 - (\mu/\sigma)\sqrt{\Delta t}].$$

Substitute these expressions for Δh , p , and q , into equations (A1.13) and (A1.14) and let Δt approach 0 (i.e., $\Delta t \rightarrow dt$). For any finite length of time T , the number of steps, n , goes to infinity and the binomial distribution converges to a normal distribution with mean:

$$(A1.17) \quad E[f(t+T)-f(t)] = \mu T,$$

and variance:

$$(A1.18) \quad Var[f(t+T)-f(t)] = \sigma^2 T.$$

Compare these results with the ones derived just after equation (A1.6'). With the structure imposed in equations (A1.15) and (A1.16a/b), observe that in the limit as $\Delta t \rightarrow 0$, both the mean and the variance are independent of Δh and Δt as opposed to the equations (A1.13) and (A1.14). Equations (A1.17) and (A1.18) are exactly the values we need for a Brownian motion: μ is the drift, and σ^2 the variance, **per unit of time**. Thus, a Brownian motion is the limit of a random walk, when the time interval and step length go to zero together while preserving the relationship of equation (A1.15).

Again note that the time derivative of a Brownian motion does not exist. The differentiability condition of a function $f(t)$ is given by:

$$df/dt = \lim_{\Delta t \rightarrow 0} (\Delta f / \Delta t) = \lim_{\Delta t \rightarrow 0} [f(t+\Delta t) - f(t)] / \Delta t.$$

Given equation (A1.15), we have that:

$$\Delta f / \Delta t = \pm \Delta h / \Delta t = \pm [\sigma \sqrt{\Delta t}] / \Delta t = \pm \sigma / \sqrt{\Delta t},$$

and thus:

$$\lim_{\Delta t \rightarrow 0} (\Delta f / \Delta t) = \lim_{\Delta t \rightarrow 0} (\pm \sigma / \sqrt{\Delta t}) = \pm \infty.$$

This means that $\Delta f / \Delta t$ has no limit. Indeed, as $\Delta t \rightarrow 0$, $(\Delta f / \Delta t) \rightarrow \infty$ and if $\Delta f / \Delta t$ takes ever increasing

values (in absolute value) as $\Delta t \rightarrow 0$, it would be contradictory to say that $\Delta f/\Delta t$ has a limit. In consequence, the differentiability condition is not fulfilled and df/dt does not exist. However, we will often want to work with functions of f (or z), and we will need to find the differentials of such functions. To do this, we make use of Itô's lemma.

A1.2 Itô's lemma

Assume the following function at least twice differentiable in f and t :

$$(A1.19) \quad G(f,t)$$

where f is a random variable following a generalised Wiener process:

$$(A1.20) \quad df = \mu dt + \sigma dz.$$

Note that alternative processes examined in subsequent Appendixes are: a) a Brownian motion without drift (i.e., $\mu = 0$); b) a Brownian motion with lower regulator; and c) a Brownian motion with upper regulator. These processes are given respectively by:

$$(A1.20a) \quad df = \sigma dz,$$

$$(A1.20b) \quad df = \mu dt + \sigma dz + dL,$$

and:

$$(A1.20c) \quad df = \mu dt + \sigma dz - dU.$$

The problem asked in this section is how to differentiate such a function G . The answer is to be found using Itô's lemma. This lemma gives the rule for finding the total differential of $s = G(f,t)$, noted dG . The objective of this section is not to prove Itô's lemma but to show that, using analogies with **ordinary** calculus, we can obtain Itô's lemma, also called the fundamental

theorem of the **stochastic** calculus.

Let us begin with the familiar result of ordinary calculus with a continuous and differentiable function G of two variables, x and y . If Δx and Δy are two small changes in x and y and ΔG is the resulting small change in G , we can approximately say that:

$$(A1.21) \quad \Delta G \approx G_x \Delta x + G_y \Delta y$$

where $G_z = \delta G / \delta z$, $z=x,y$.

The error in (A1.21) is due to terms of second order. If more precision is required, a Taylor series expansion of ΔG can be used:

$$(A1.22) \quad \Delta G = G_x \Delta x + G_y \Delta y + (1/2)G_{xx}(\Delta x)^2 + (1/2)G_{yy}(\Delta y)^2 + G_{xy}\Delta x \Delta y + \dots$$

In the limit as Δx and Δy tend to zero, terms of second and higher order are ignored (because second and higher order terms go to zero faster than Δx and Δy as Δx and Δy become infinitesimally small), such that equation (A1.22) gives:

$$(A1.23) \quad dG = G_x dx + G_y dy.$$

With these basic results of ordinary calculus in mind, let us come back to the original problem of differentiating the function given in equation (A1.19) when f follows a generalised Wiener process given in (A1.20). By analogy with equation (A1.22) we can write:

$$(A1.24) \quad \Delta G = G_f \Delta f + G_t \Delta t + (1/2)G_{ff}(\Delta f)^2 + (1/2)G_{tt}(\Delta t)^2 + G_{ft} \Delta f \Delta t + \dots$$

Now, remember that in section A1.1 [equation (A1.7')], (A1.20) was discretized as:

$$(A1.25) \quad \Delta f = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t}; \quad \epsilon \sim i.i.d.N(0,1).$$

Then, taking the square of Δf , we obtain:

$$(A1.26) \quad (\Delta f)^2 = \mu^2 (\Delta t)^2 + 2\mu \Delta t \sigma \epsilon \sqrt{\Delta t} + \sigma^2 \epsilon^2 \Delta t.$$

This shows that the term involving $(\Delta f)^2$ in equation (A1.24) has a component $(\sigma^2 \epsilon^2 \Delta t)$ which is

of order Δt and then cannot be ignored in limiting arguments. Taking limits as Δf and Δt tend to zero, we have to be a little more cautious than going from equation (A1.22) to (A1.23): we have to take into account the term $\sigma^2 \epsilon^2 \Delta t$. From equation (A1.25) we know that $E(\epsilon)^2 = 1$. Hence, the expected value of $\epsilon^2 \Delta t$ is given by Δt . It can also be shown that the variance of $\epsilon^2 \Delta t$ is of order Δt^2 and that, as a result of this, $\epsilon^2 \Delta t$ becomes non-stochastic and equal to its expected value of Δt as Δt tends to 0. Thus, it follows that $\sigma^2 \epsilon^2 \Delta t$ becomes non-stochastic and equal to $\sigma^2 dt$ as Δt tends to zero.

We are now ready to take limits as Δf and Δt tend to 0 in equation (A1.26). We obtain, using the last result, and ignoring second order terms:

$$(A1.27) \quad (df)^2 = \sigma^2 dt.$$

Now, taking the limit as Δf and Δt tend to 0 in equation (A1.24), and using the last result given in (A1.27), we get:

$$(A1.28) \quad dG = G_\mu df + G_t dt + (1/2)[G_{ff}]\sigma^2 dt,$$

which is Itô's lemma. Substituting for df from equation (A1.20), we also obtain:

$$(A1.29) \quad dG = \{G_\mu + G_t + (1/2)[G_{ff}]\sigma^2\}dt + G_\sigma dz.$$

Thus G also follows a generalised Wiener process with drift rate of $G_\mu + G_t + 1/2[G_{ff}]\sigma^2$, and a variance rate of $(G_\sigma)^2\sigma^2$.

At this stage, it is interesting to compare the total differential, dG , given by Itô's lemma, with the wrong result we would have obtained using ordinary calculus. In this case $dG = G_\mu df + G_t dt$, and substituting df by its value given in equation (A1.20), we would get:

$$(A1.29') \quad dG = \{G_\mu + G_t\}dt + G_\sigma dz.$$

The difference between equations (A1.29) and (A1.29') is given by the term $(1/2)[G_{ff}]\sigma^2 dt$. This

term is of order dt and should not be ignored [as is explicit in the discussion surrounding equations (A1.26) and (A1.27)]. In other words, this implies that, when calculating the differential of G , the effect of the convexity ($G_{ff} > 0$) or concavity ($G_{ff} < 0$) of the function $G(f)$ are of order dt and cannot be ignored.

If, instead of $G(f,t)$ we had simply $G(f)$, Itô's lemma would be written as:

$$(A1.30) \quad dG = G_f df + (1/2)[G_{ff}]\sigma^2 dt.$$

Once more, replacing df by its value given in (A1.20), we obtain:

$$(A1.31) \quad dG = [G_f \mu + (1/2)G_{ff}\sigma^2]dt + G_f \sigma dz.$$

Alternative processes for df are presented in equations (A1.20a,b,c). With these processes, dG is given by:

$$(A1.31a) \quad dG = [(1/2)G_{ff}\sigma^2]dt + G_f \sigma dz,$$

$$(A1.31b) \quad dG = [G_f \mu + (1/2)G_{ff}\sigma^2]dt + G_f \sigma dz + G_f dL,$$

and:

$$(A1.31c) \quad dG = [G_f \mu + (1/2)G_{ff}\sigma^2]dt + G_f \sigma dz - G_f dU.$$

Note that another concept useful for working with generalised Wiener processes is the differential generator of $G(f,t)$. A heuristic method for finding this differential generator is, first, to find dG using Itô's lemma, then to take the conditional expectation of dG , and finally to "divide" by dt . This will give the differential generator of $G(f,t)$ and can be interpreted as the average or expected time rate of change of the function $G(f,t)$. As such, this is the natural generalization of the ordinary time derivative for deterministic functions. In the particular case of $G(f)$, starting from equation (A1.31) we take the conditional expectation of $dG(f)$:

$$(A1.32) \quad E[dG(f)] = [G_f \mu + (1/2)G_{ff}\sigma^2]dt$$

and dividing by dt , we obtain:

$$(A1.33) \quad E[dG(f)] / dt = G_f \mu + (1/2) G_{ff} \sigma^2.$$

For the alternative processes discussed earlier, we get:

$$(A1.33a) \quad E[dG(f)] / dt = (1/2) G_{ff} \sigma^2,$$

$$(A1.33b) \quad E[dG(f)] / dt = [G_f \mu + (1/2) G_{ff} \sigma^2] + G_f dL/dt,$$

and:

$$(A1.33c) \quad E[dG(f)] / dt = [G_f \mu + (1/2) G_{ff} \sigma^2] - G_f dU/dt.$$

A1.3 Poisson processes

The Poisson process is a process that changes discretely at infrequent intervals. Let us denote by N a Poisson process such that N increases by steps of u every time a Poisson event occurs.

We thus have:

$$(A1.34) \quad \begin{aligned} dN &= 0, \text{ with probability } 1 - \nu dt \\ dN &= u, \text{ with probability } \nu dt, \end{aligned}$$

where ν is a constant intensity parameter called a mean arrival rate.

If a variable f follows a Poisson process, its increments are given by a Poisson differential equation:

$$(A1.35) \quad df = \mu dt + g dN.$$

It could also be the case that a variable f follows a combination of a Brownian motion and a Poisson process. In this case, the increments in f are given by:

$$(A1.36) \quad df = \mu dt + \sigma dz + g dN.$$

If we have a function $G(\cdot)$ of a variable f following (A1.36), the extension of Itô's lemma gives:

$$(A1.37) \quad EdG(f;N) = [G_f(f;N)\mu + (\sigma^2/2)G_{ff}(f;N)]dt + \nu[G(f+ug;N+u) - G(f;N)]dt.$$

A2 Asset market view of exchange rate determination

A2.1 Monetary approach

In this section, an interpretation of equation (1.1) is given in term of a minimalist flexible price monetary model of exchange rate determination. However, it is important to underline the fact that the monetary model is sufficient but not necessary for the interpretation of (1.1). In the last few years, the monetary interpretation has lost popular favour and has been replaced by a general agnostic interpretation of equation (1.1). This new preferred agnosticism regarding the "fundamentals" of the exchange rate is probably due to the fact that economists still do not know the necessary conditions for equilibrium models to give rise to equation (1.1).

Expressing all variables in logarithms, the basic monetary model can be written as:

$$(A2.1a) \quad m(t) - p(t) = \psi y(t) - \theta i(t) + \epsilon(t), \quad \psi \text{ and } \theta \text{ positive}$$

$$(A2.1b) \quad m^*(t) - p^*(t) = \psi y^*(t) - \theta i^*(t) + \epsilon^*(t)$$

$$(A2.2) \quad s(t) = p(t) - p^*(t)$$

$$(A2.3) \quad i(t) = i^*(t) + E_t[ds|I(t)]/dt.$$

Equation (A2.1a) is a money market equilibrium relationship, at time t , where m is the (log of) domestic money supply, p is the (log of) domestic price level, y is the (log of) real domestic income, i is the domestic nominal interest rate (expressed as a level), and ϵ is a money demand/supply disturbance. Equation (A2.1b) is the foreign money market equilibrium relationship. A "*" indicates a foreign variable. Equation (A2.2) is the purchasing power parity (PPP) condition where s is the spot (domestic-currency) price of foreign currency. Equation

(A2.3) is the uncovered interest rate parity condition where $E_t[ds|I(t)]/dt$ represents the expected rate of depreciation of the domestic currency given a time- t information set which is to be defined shortly.

Substituting equations (A2.2) and (A2.3) in the result of the subtraction of (A2.1b) from (A2.1a), the exchange rate at time t is given by:

$$(A2.4) \quad s(t) = f(t) + \theta E_t[ds|I(t)]/dt, \quad \theta > 0$$

where $f(t) = m(t) - m^*(t) + v(t)$, are the "fundamentals" of the exchange rate and $v(t)$ is a velocity or composite money demand/supply shock given by:

$$(A2.5) \quad v(t) = \psi[y^*(t) - y(t)] + \epsilon^*(t) - \epsilon(t).$$

The time- t information set, $I(t)$, includes the current value of the fundamentals, $f(t)$, as well as any explicit or implicit restrictions the authorities have set on the future evolution of the exchange rate fundamentals. For example, the authorities may have announced that they will keep the exchange rate from moving outside certain limits. Or they may have announced a particular exchange rate regime shift. Such information about future known policies would be incorporated into $I(t)$.

A2.2 Integral representation of the saddle-path exchange rate

Consider a scalar variable t , taking values on the real line, and two scalar-valued functions, $s(\cdot)$ and $f(\cdot)$, linked by the relationship:

$$(A2.6) \quad s(t) = f(t) + \theta ds(t)/dt$$

for all t . The problem is that of finding $\{s(t)\}$ such that equation (A2.6) is satisfied over some

range of t values, for a given scalar θ and an exogenous $\{f(t)\}$. The functional solution $s(\cdot)$ can be found by integration of (A2.6). Separating the endogenous function and its derivative from the exogenous $f(\cdot)$ and multiplying by $(1/\theta)\exp[(-1/\theta)(\tau-t)]$, we get:

$$(A2.7) \quad -s(\tau)(1/\theta)\exp[(-1/\theta)(\tau-t)] + \exp[(-1/\theta)(\tau-t)]ds(\tau)/d\tau = -\exp[(-1/\theta)(\tau-t)]f(\tau)/\theta.$$

Note that if we set $y(\tau) = s(\tau)\exp[(-1/\theta)(\tau-t)]$, then $dy(\tau)/d\tau = y'(\tau)$ is equal to the left hand side of equation (A2.7). We can then use the fundamental theorem of calculus:

$$(A2.8) \quad \int_t^T y'(\tau)d\tau = \int_t^T dy(\tau) = y(T) - y(t)$$

to obtain:

$$(A2.9) \quad \int_t^T -\exp[(-1/\theta)(\tau-t)]f(\tau)/\theta d\tau = s(T)\exp[(-1/\theta)(T-t)] - s(t),$$

or, after rearranging:

$$(A2.10) \quad s(t) = (1/\theta) \int_t^T f(\tau)\exp[(-1/\theta)(\tau-t)] d\tau + s(T)\exp[(-1/\theta)(T-t)].$$

As T diverges to infinity, we get:

$$(A2.11) \quad s(t) = (1/\theta) \int_t^\infty f(\tau)\exp[(-1/\theta)(\tau-t)] d\tau + \lim_{T \rightarrow \infty} s(T)\exp[(-1/\theta)(T-t)].$$

If s and f are understood as the exchange rate and its fundamentals, respectively, the solution in (A2.11) consists of two parts: an economically-meaningful (forward looking) market fundamental component (the first term on the right-hand-side) and an economically arbitrary (self-fulfilling) speculative bubbles component (the second term on the right-hand-side). The market fundamental solution is an exponentially-declining weighted average of current and future f . The speculative bubbles component reflects a self-fulfilling belief on the part of agents, that the exchange rate does not conform to its market fundamental value. This phenomena can be ruled out by imposing a transversality condition:

$$(A2.12) \quad \lim_{T \rightarrow \infty} s(T)\exp[(-1/\theta)(T-t)] = 0.$$

In this particular case, we get:

$$(A2.13) \quad s(t) = (1/\theta) \int_t^\infty f(\tau) \exp[(-1/\theta)(\tau-t)] d\tau.$$

Starting from equation (A2.6) and integrating forward, we get equation (A2.13). It is also of interest to know how to go from (A2.13) to (A2.6), by differentiating (A2.13) with respect to t , using Leibniz's rule:

$$(A2.14) \quad ds(t)/dt = (1/\theta) \int_t^\infty (1/\theta) f(\tau) \exp[(-1/\theta)(\tau-t)] d\tau - f(t)/\theta$$

where the second term in the right hand side of (A2.14) comes from the fact that t is the lower limit of integration. Combining equations (A2.13) and (A2.14), we get:

$$(A2.15) \quad ds(t)/dt = (1/\theta)s(t) - f(t)/\theta$$

or also:

$$(A2.16) \quad s(t) = f(t) + \theta ds(t)/dt$$

which is the same as equation (A2.6).

The above results have been derived in the context of certainty. In the presence of uncertainty, a linkage is posited among realizations of three processes rather than three functions of time and equation (A2.6) is rewritten as:

$$(A2.17) \quad s(t) = f(t) + \theta E_t[ds(t)|I(t)]/dt.$$

Taking the process $\{f(\cdot)\}$ as given, we want to solve for a process $\{s(\cdot)\}$ whose level and conditional expected differential satisfy (A2.17). The solution is given by:

$$(A2.18) \quad s(t) = (1/\theta) \int_t^\infty E f(\tau) \exp[(-1/\theta)(\tau-t)] d\tau.$$

Even if (A2.18) seems an "obvious" extension of (A2.13), the passage from (A2.13) to (A2.14) is not trivial. The reason is that the steps followed to derive (A2.13) cannot be used in the presence of uncertainty. In particular, the fundamental theorem of calculus, as set in (A2.8),

cannot be applied here. This theorem is to be replaced by the fundamental theorem of stochastic calculus, or Itô's lemma. For an explicit passage from (A2.17) to (A2.18), see Bertola (1994).

In conclusion, in a rational expectations equilibrium with no speculative bubbles, there is a unique exchange-rate path that satisfies equation (1.1). This path has the integral representation:

$$(A2.19) \quad s(t) = (1/\theta) \int_t^{\infty} E_t[f(\tau)|I(t)] e^{-(1/\theta)(\tau-t)} d\tau.$$

Equation (A2.19) shows that the current exchange rate is equal to the present value of all future expected fundamentals, where $1/\theta$ is the rate of discount. This equation is the continuous discounting present value analogous to the well-known discrete time rational expectations solutions as derived in Bilson (1976, equation 13) or Mussa (1976, equation 5). The equilibrium exchange rate given by (A2.19) is called, thereafter, the saddle-path exchange rate. This integral representation is valid under **any** exchange rate regime. But its solution will depend on the particular process assumed for the fundamentals which, in itself, reflects the particular exchange rate regime in place.

A3 A permanent free-float regime

A3.1 Stochastic forcing processes for the fundamentals

Without specifying a process for f , equation (A2.19) is "vacuous". Thus, at this point, let us describe the assumed dynamics of the fundamentals $f = (m - m^*) + v$ in a pure free-float regime. The component, v , given by equation (A2.5), is exogenous and stochastic. Assume that v is driven by a Brownian motion with drift. As shown in Appendix A1, the realized sample paths for v are continuous over time and do not include discrete jumps; changes in v over any fixed time interval, Δt , are distributed as a normal random variable with mean $\mu\Delta t$ and variance $\sigma^2\Delta t$. The increments in v are given by the following stochastic differential equation:

$$(A3.1) \quad dv = \mu dt + \sigma dz$$

where μ and σ are constant and dz , which makes this process stochastic, is a standard Wiener process.

The other component of the fundamental is the (relative) money supply, $(m - m^*)$, supposed to be constant to express the idea that monetary policy will not change in the future (i.e., the money supply is not chosen in function of the exchange rate). As a result, the increments in f are also given by:

$$(A3.2) \quad df = \mu dt + \sigma dz, \quad df \sim N(\mu dt, \sigma^2 dt).$$

A3.2 Solution method

In a permanent free-float, as described in Appendix A3.1, the conditional expectations of f in (A2.19) are easy to evaluate, since they depend exclusively on current fundamentals, and not on possible future regime shifts. In this case, the information at time $\tau = t$ is limited to the information on the fundamentals of the exchange rate at this time, such that $I(t) = f(t)$. Thus equation (A2.19) can be rewritten as:

$$(A3.3) \quad s(t) = (1/\theta) \int_t^{\infty} E_t[f(\tau)|f(t)] e^{l-1/\theta|(\tau-t)} d\tau.$$

In this permanent free-float case, it is easy to evaluate the conditional expectation, $E_t[f(\tau)|f(t)]$. Starting from the initial fundamentals, $f(t)$, and because f follows a Brownian motion with drift, the conditional expectation is given by:

$$(A3.4) \quad \begin{aligned} E_t[f(\tau)|f(t)] &= f(t) + E_t\Delta f \\ &= f(t) + \mu\Delta t \\ &= f(t) + \mu(\tau-t). \end{aligned}$$

Substituting (A3.4) in (A3.3) we get:

$$(A3.5) \quad s(t) = (1/\theta) \int_t^{\infty} [f(t) + \mu(\tau-t)] e^{l-1/\theta|(\tau-t)} d\tau.$$

Taking note that t is the initial (given) time, and not the time index, τ , we can rewrite (A3.5) as:

$$(A3.6) \quad s(t) = (1/\theta)(f(t)-\mu t)e^{l/\theta} \int_t^{\infty} e^{-\tau/\theta} d\tau + \mu e^{l/\theta} \int_t^{\infty} (\tau/\theta)e^{-\tau/\theta} d\tau.$$

Call $A = \int_t^{\infty} e^{-\tau/\theta} d\tau$ and $B = \int_t^{\infty} (\tau/\theta)e^{-\tau/\theta} d\tau$.

To solve for A , note that the primitive of a function $y' = e^{-\tau/\theta}$ is given by $y = -\theta e^{-\tau/\theta}$, such that:

$$(A3.7) \quad \begin{aligned} A &= \lim_{\tau \rightarrow \infty} (-\theta e^{-\tau/\theta}) + \theta e^{-t/\theta} \\ &= [-\theta e^{-\tau/\theta}]_t^{\infty} = \theta e^{-t/\theta}. \end{aligned}$$

B can be solved in parts: let us set $v = -\tau$ such that $dv = -d\tau$. Also let us set $u = e^{-\tau/\theta}$, such that $du = (-1/\theta)e^{-\tau/\theta}d\tau$. Thus, following the rule $\int_i^x vdu = [uv]_i^x - \int_i^x u dv$, we get:

$$\begin{aligned}
 (A3.8) \quad B &= \int_i^x (\tau/\theta)e^{-\tau/\theta}d\tau = [-\tau e^{-\tau/\theta}]_i^x + \int_i^x e^{-\tau/\theta}d\tau \\
 &= [-\tau e^{-\tau/\theta}]_i^x + A \\
 &= t e^{-t/\theta} + \theta e^{-t/\theta} \\
 &= e^{-t/\theta} (t + \theta).
 \end{aligned}$$

Substituting (A3.7) and (A3.8) into (A3.6) we have that:

$$(A3.9) \quad s(t) = (1/\theta)(f(t) - \mu t)e^{t/\theta}\theta e^{-t/\theta} + \mu e^{t/\theta}e^{-t/\theta}(t + \theta),$$

which after simplifications, gives finally that:

$$(A3.10) \quad s(t) = f(t) + \theta\mu$$

or simply:

$$s = f + \theta\mu.$$

This permanent free-float is represented in figure A3.1 for the drift and no-drift cases.

<figure A3.1 here>

A4 Regulated Brownian motions: an application to the TZ theory

A4.1 Process for the fundamentals

As described in Appendix A3.1, the fundamentals are given by $f = (m - m^*) + v$. Setting m^* arbitrarily equal to 0, we get $f = m + v$. The process followed by v is still the one specified in equation (A3.1) and rewritten here as:

$$(A4.1) \quad dv = \mu dt + \sigma dz.$$

Foreign exchange interventions, which directly affect the money supply (unsterilised interventions), are undertaken to prevent the fundamentals to move outside a specified band. This will imply a well-defined band for the exchange rate. Hence, assume that there are lower and upper bounds for f , f_l and f_u , such that:

$$(A4.2) \quad f_l \leq f \leq f_u.$$

With interventions affecting the money supply, the stochastic process for f obeys:

$$(A4.3) \quad df = dv + dm.$$

Inside the band for the fundamentals, there is no interventions such that $dm = 0$ and hence:

$$(A4.4) \quad df = dv + dm = \mu dt + \sigma dz.$$

Observe that this process is identical to the process followed by f in a pure free-float regime, as given in equation (A3.2).

At the edges of the band, there are infinitesimal interventions to prevent the fundamentals from moving outside their band. Let L be the integral of all intervention purchases of foreign exchange, at the lower bound (these are infinitesimal increases in m), and let U be the integral

of infinitesimal decreases in m (intervention sales) at the upper bound. L and U are thus lower and upper regulators, such that:

$$(A4.5) \quad dm = dL - dU$$

where dL and dU are nonnegative, and dL represents increases in the domestic money supply and is positive only if $f = f_b$, whereas dU represents decreases in the domestic money supply and is positive only if $f = f_u$. In consequence, at the edges of the band, the fundamentals follow the process:

$$(A4.6) \quad df = dv + dm = \mu dt + \sigma dz + dL - dU.$$

Such a regime is called a target zone with instantaneous reflecting barrier. Note that, as mentioned in Chapter 1, this is the approach of the "fundamental" target zone theory, which is not the initial approach of Krugman (1991). Krugman starts initially with an explicit target on the exchange rate and concludes that this implies an implicit target on the fundamentals. (Basically, both models give the same saddle-path solution for the exchange rate, but the major difference lies in the **interpretation** of the smooth pasting or boundary conditions; see section 1.4.)

A4.2 Solution method

If the market expects that the authorities will alter (A4.4) **in the future**, the exchange rate need not satisfy (A3.10), even while allowed **presently** to float. In such a case, direct computation of the sequence of conditional expectations in the present-value formula (A2.19) is likely to be difficult. [See for example Flood and Garber (1983) who were unable to obtain a closed-form

solution to their particular problem.] Froot and Obstfeld (1991a,1991b) propose a two-step method to determine the saddle-path exchange rate when a regime switch, from (A4.4) to (A4.6) is expected.

Initially, given (A4.4) and the type of regime change considered [in particular (A4.6)], the saddle-path exchange rate can be written as a twice continuously differentiable function of a single variable --the current fundamentals:

$$(A4.7) \quad s(t) = S_{\varepsilon}(f(t)).$$

The reason is as follows. The fundamentals are characterized by a Brownian motion, which is a Markov process. Hence, in equation (A2.19), each of the conditional expectations $E_t[f(\tau)]$ for $\tau > t$, can be expressed as a function of $f(t)$. As a consequence, the l.h.s. term of (A2.19) can also be written in terms of $f(t)$, such that $s(t) = S_{\varepsilon}(f(t))$. In other words, we can express the dependence of s on t through f , bypassing the t dimension of the problem and work with $S(f)$ instead of $s(t)$. For this reason, we can also drop the time variable and write equation (A4.7) as:

$$s = S_{\varepsilon}(f),$$

which is time independent and represents a state relationship. The precise form of the functional $S_{\varepsilon}(\cdot)$, to be derived later on, depends on the regime shift explained in section A4.1. Observe that equation (A4.7) is a conjecture to be verified.

The first step of the two-step method is to characterize the family of functions that satisfies the equilibrium condition (A2.4) so long as fundamentals evolve according to (A4.4), as:

$$(A4.8) \quad s = G(f)$$

where $G(f)$ is assumed to be continuous twice differentiable in f (i.e., G_f and G_{ff} exist, where $G_f = dG(f)/df$ and $G_{ff} = d^2G(f)/df^2$). In the second step, we have to find the member of this family

that satisfies boundary conditions appropriate to the stochastic regime switch under consideration. This latter function is the saddle-path solution $S_{\varepsilon}(f)$ (i.e., the exchange rate path in a TZ regime).

i) First step

As given in equation (A4.8), the exchange rate is a function of the fundamentals which, in the open interval (f_l, f_u) , follow the stochastic process given by equation (A4.4). In consequence, as explained in Appendix A1.2, stochastic calculus has to be used. Hence, use Itô's lemma and equations (A2.4) and (A4.8) to express the expected depreciation rate as:

$$(A4.9) \quad E(ds)/dt = E(dG(f))/dt = \mu G(f) + (\sigma^2/2) G_{ff}(f).$$

Combining (A2.4), (A4.8) and (A4.9) yields a second order differential equation that (A2.4) and (A2.19) must satisfy:

$$(A4.10) \quad G(f) = f + \theta \mu G(f) + (\theta \sigma^2/2) G_{ff}(f).$$

This is a second order linear differential equation with constant coefficients but a variable term, which can be rewritten as:

$$(A4.11) \quad G_{ff}(f) + [2\mu/\sigma^2] G_f(f) - [2/\theta\sigma^2] G(f) = [-2/\theta\sigma^2] f.$$

The reduced equation of (A4.11) is given by:

$$(A4.12) \quad G_{ff}(f) + [2\mu/\sigma^2] G_f(f) - [2/\theta\sigma^2] G(f) = 0,$$

and the complementary function, G_c , is the general solution of this reduced equation.

As a trial solution, set $G(f) = Ae^{rf}$, such that $G_f(f) = rAe^{rf}$ and $G_{ff}(f) = r^2Ae^{rf}$. Substituting these values in equation (A4.12) gives:

$$r^2Ae^{rf} + [2\mu/\sigma^2] rAe^{rf} - [2/\theta\sigma^2] Ae^{rf} = 0,$$

and rearranging:

$$(A4.13) \quad Ae^{rf} \{ r^2 + [2\mu/\sigma^2] r - 2/\theta\sigma^2 \} = 0.$$

The characteristic equation is then given by:

$$(A4.14) \quad r^2 + [2\mu/\sigma^2] r - 2/\theta\sigma^2 = 0,$$

and the two characteristic roots (distinct real roots) are:

$$(A4.15) \quad r_1, r_2 = \{-\mu \pm [\mu^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2, \quad r_1 < 0 < r_2.$$

The complementary function is thus written as:

$$(A4.16) \quad G_c = A_1 \exp(r_1 f) + A_2 \exp(r_2 f)$$

where A_1 and A_2 are two arbitrary constants.

The particular integral is to be found using the method of undetermined coefficients. Set:

$$(A4.17) \quad G(f) = C_1 f^2 + C_2 f + C_3,$$

such that $G_f(f) = 2C_1 f + C_2$ and $G_{ff}(f) = 2C_1$. Substituting these values in equation (A4.11) gives:

$$(A4.18) \quad 2C_1 + [2\mu/\sigma^2](2C_1 f + C_2) - [2/\theta\sigma^2](C_1 f^2 + C_2 f + C_3) = [-2/\theta\sigma^2] f.$$

Collecting terms in f, f^2 and independent of f in the left hand side of (A4.18), we obtain:

$$(A4.19) \quad -[2/\theta\sigma^2] C_3 f^2 + \{[4\mu/\sigma^2]C_1 - 2C_2/\theta\sigma^2\} f + \{2C_1 + [2\mu/\sigma^2]C_2 - [2/\theta\sigma^2]C_3\} = [-2/\theta\sigma^2] f.$$

Equating l.h.s. with r.h.s terms in (A4.19), we obtain the following system of three equations with three unknowns, C_1, C_2 and C_3 :

$$(A4.20) \quad \begin{aligned} -[2/\theta\sigma^2] C_3 &= 0, \\ [4\mu/\sigma^2]C_1 - 2C_2/\theta\sigma^2 &= -2/\theta\sigma^2, \\ 2C_1 + [2\mu/\sigma^2]C_2 - [2/\theta\sigma^2]C_3 &= 0. \end{aligned}$$

Solving the system given in (A4.20), we finally obtain the value of the coefficients:

$$C_1 = 0; C_2 = 1; C_3 = \theta\mu.$$

Substituting these values in equation (A4.17), we finally obtain the particular integral:

$$(A4.21) \quad G_p = f + \theta\mu.$$

The general solution of the complete equation is thus $G = G_p + G_c$, such that:

$$(A4.22) \quad G(f) = f + \theta\mu + A_1 \exp(r_1 f) + A_2 \exp(r_2 f)$$

where A_1 and A_2 are two arbitrary constants to be defined with the help of two initial conditions.

The next step is to choose, from the family of functions given by (A4.22), the most relevant member, given the particular exchange rate regime announced by the central bank. The problem is then to define A_1 and A_2 with some initial or final conditions implied by the exchange rate regime. Once the particular values for A_1 and A_2 are found, the family of functions given by (A4.22) is reduced to one specific function. This specific function will turn out to be equivalent to the present-value formula for s in (A2.19), and is the saddle-path, $S_c(f)$.

Notice that the general solution (A4.22) consists of two components: one is linear, and the other is non linear, in f . The linear part, $f + \theta\mu$, is the linear saddle-path solution for the permanent free-float as derived in Appendix A3 (using the direct method), in equation (A3.10). This means that, in the framework of the two-step method, once the general solution (A4.22) is found, the boundary conditions (the second step) associated with a permanent free-float should be $A_1 = A_2 = 0$, which is the usual "no-speculative bubbles" condition. These boundary conditions for a permanent free-float make sense because equation (A4.22) is the result of two components as shown above: the particular integral $[G_p = f + \theta\mu]$, which provides the equilibrium value for the exchange rate; and the complementary function $[G_c = A_1 \exp(r_1 f) + A_2 \exp(r_2 f)]$, which reveals the deviation of the state-path G from the equilibrium. Setting the arbitrary constants equal to 0, then simply allows us to get rid of any speculative bubbles such that the market is always at its equilibrium as given by the particular integral G_p . Under prospective

regime switches, however, the saddle-path value of the exchange rate will depend on the nonlinear terms in equation (A4.22). Step 2 shows how to derive this saddle-path value.

ii) Step two

The particular process switching for the fundamentals from equations (A4.4) to (A4.6) at the lower or upper edges of the band implies specific boundary conditions that permit to find which member of the family defined by (A4.22) is the saddle-path exchange rate solution for the TZ regime.

$G(f)$, given by equation (A4.22) and calculated with the process given in equation (A4.4), is valid in the TZ problem for $f \in (f_l, f_u)$ (open interval), but should also be valid at f_l and f_u . The reason is the following: the saddle-path exchange rate, $S_{tz}(f)$, has to be a continuous function on the closed interval $[f_l, f_u]$, because we have to preclude anticipated excess profit opportunities at f_l or f_u . If the exchange rate jumped at the edge of the band due to the intervention, there would be room for one-way arbitrage and thus anticipated excess profit opportunities. But, if the function $S_{tz}(f)$ is continuous on the closed interval $[f_l, f_u]$, it cannot coincide with a function of the form $G(f)$ on the open interval (f_l, f_u) unless it also coincides with the same function at the edges of the band. Now, if (A4.22) is to be satisfied at, say, f_u , it also means that (A4.10) and (A4.9) have to be satisfied at f_u , which means that:

$$(A4.23) \quad G(f_u) = f_u + \theta[\mu G_f(f_u) + (\sigma^2/2)G_{ff}(f_u)]$$

and:

$$(A4.24) \quad G(f_u) = f_u + \theta E[dG(f_u)|I]/dt.$$

Subtracting (A4.23) from (A4.24) gives:

$$(A4.25) \quad E[dG(f_u)] = [\mu G_f(f_u) + (\sigma^2/2) G_{ff}(f_u)] dt.$$

Nevertheless, we also know that, at f_u , the process followed by f is not (A4.4) but (A4.6) with dL set equal to 0. Now, we should use Itô's lemma and equation (A4.8) while (A4.6) holds, in order to express the expected rate of depreciation. The result (based on equation A1.33c) is given by:

$$(A4.26) \quad E[dG(f_u)] = [\mu G_f(f_u) + (\sigma^2/2) G_{ff}(f_u)] dt - G_f(f_u) dU.$$

Intuitively, because the market expects an appreciation of the exchange rate at f_u , this decreases the expected change in $G(f)$ in proportion to $G_f(f_u)$.

In conclusion, at f_u , both equations (A4.25) and (A4.26) must hold and dU is positive, which finally gives that $G_f(f_u)$ is to be set equal to 0. A symmetric reasoning at f_l would give that $G_f(f_l)$ is to be set equal to 0. In conclusion, the conditions involved by the regime switching process explained in subsection A4.1 are the following "smooth-pasting/value-matching" conditions:

$$(A4.27a) \quad G_f(f_l) = 0,$$

and:

$$(A4.27b) \quad G_f(f_u) = 0.$$

Speculators set A_1 and A_2 in equation (A4.22) such that conditions (A4.27a) and (A4.27b) are satisfied. Then, if (A4.27a/b) are taken as initial conditions, A_1 and A_2 can be determined.

Differentiating (A4.22) we have:

$$(A4.28) \quad G(f) = 1 + A_1 r_1 \exp[r_1 f] + A_2 r_2 \exp[r_2 f].$$

Using the conditions given by (A4.27a/b) we get the following system of two equations and two unknowns, A_1 and A_2 :

$$(A4.29a) \quad G(f_u) = 1 + A_1 r_1 \exp[r_1 f_u] + A_2 r_2 \exp[r_2 f_u] = 0,$$

and:

$$(A4.29b) \quad G(f_l) = 1 + A_1 r_1 \exp[r_1 f_l] + A_2 r_2 \exp[r_2 f_l] = 0.$$

This system provides the following values for A_1 and A_2 :

$$(A4.30) \quad A_1 = (\exp[r_2 f_u] - \exp[r_2 f_l]) / r_1 (\exp[r_1 f_u + r_2 f_l] - \exp[r_1 f_l + r_2 f_u]) > 0$$

and:

$$(A4.31) \quad A_2 = (\exp[r_1 f_l] - \exp[r_1 f_u]) / r_2 (\exp[r_1 f_u + r_2 f_l] - \exp[r_1 f_l + r_2 f_u]) < 0.$$

Substituting these values for A_1 and A_2 in equation (A4.22), get:

$$(A4.32) \quad s = S_e(f) = f + \theta \mu + (W + W') / \{r_1 r_2 (\exp[r_1 f_u + r_2 f_l] - \exp[r_1 f_l + r_2 f_u])\}$$

where $W = r_2 \exp[r_2 f_u + r_1 f_l] - r_2 \exp[r_2 f_l + r_1 f_u]$ and $W' = r_1 \exp[r_1 f_l + r_2 f_u] - r_1 \exp[r_1 f_u + r_2 f_l]$.

Equation (A4.32) is the saddle-path exchange rate solution for a TZ with drift.

A5 A target zone model with zero drift

A5.1 The dynamic path of the exchange rate inside a target zone with zero drift

A target zone model with zero drift can be easily derived from Appendix A4 by setting the value for μ equal to 0 in the results of that section. In this case, the process followed by v is given by:

$$(A5.1) \quad dv = \sigma dz.$$

Inside the band (where $dm = 0$), the process for f is thus:

$$(A5.2) \quad df = dv = \sigma dz$$

whereas, at the bounds of the band, the process for f switches to:

$$(A5.3) \quad df = dv + dm = \sigma dz + dL - dU$$

where dL and dU have the same definitions as in Appendix A4.

The first step of the solution method is obtained by setting $\mu = 0$ in equation (A4.15) such that:

$$(A5.4) \quad r_2 = -r_1 = [2/\theta]^{1/2}/\sigma > 0.$$

Setting $r = r_2 = -r_1$ in equation (A4.22) and remembering that μ is set equal to 0, we get:

$$(A5.5) \quad G(f) = f + A_1 \exp(-rf) + A_2 \exp(rf).$$

For the second step, use the smooth-pasting/value-matching condition given in Appendix A4 and rewritten here as:

$$(A5.6) \quad G_f(f_l) = G_f(f_u) = 0.$$

For simplification, it can also be assumed that the TZ is symmetric such that:

$$(A5.7) \quad f_l = -f_u.$$

Differentiating (A5.5), get:

$$(A5.8) \quad G_f(f) = 1 - rA_1 \exp(-rf) + rA_2 \exp(rf).$$

Given equations (A5.6), (A5.7) and (A5.8), a system of two equations with two unknowns, A_1 and A_2 , is obtained:

$$(A5.9a) \quad G_f(f_1) = G_f(-f_u) = 1 - rA_1 \exp[rf_u] + rA_2 \exp[-rf_u] = 0$$

and:

$$(A5.9b) \quad G_f(f_u) = 1 - rA_1 \exp[-rf_u] + rA_2 \exp[rf_u] = 0.$$

Solving the system given in (A5.9a/b), we get that:

$$(A5.10) \quad A_1 = -A_2.$$

Setting $A_1 = A$ and thus $A_2 = -A$ and substituting in equation (A5.9a), we get, using the definition of a hyperbolic cosine:

$$(A5.11) \quad A = A_1 = -A_2 = 1/[2r \cosh(rf_u)] > 0.$$

Substituting A_1 and A_2 by their value, in equation (A5.5), we get:

$$(A5.12) \quad s = S_z(f) = f - \sinh(rf)/[r \cosh(rf_u)]$$

where $\sinh(rf)/[r \cosh(rf_u)]$ has a positive (negative) value for $f > 0$ ($f < 0$). This is the TZ saddle-path solution for the exchange rate as represented in figure 1.5.

The results are in essence the following:

1. $dS_z(f)/df = 1 - [\cosh(rf)/\cosh(rf_u)] = S_f < 1$.
2. $s_u - s_l < f_u f_l$; the bandwidth of the fundamentals is larger than the bandwidth of the exchange rate.

Intuitively, these results imply that a TZ system stabilises the exchange rate. In a flexible exchange rate system, we have that $dS(f)/df = 1$ [see equation (A3.10)]. Every variation in the

fundamentals is directly and entirely reflected in the exchange rate. In a TZ regime, the transmission is only partial ($dS_e(f)/df < 1$). This effect has been named the honeymoon effect. Graphically, this implies that the S-curve of the TZ, representing the exchange rate for each value of the fundamentals, is flatter than the F-curve of the free float exchange rate. The intuition, as given in Svensson (1992, p.124), behind the honeymoon effect is straightforward.

"When the exchange rate is higher (the domestic currency is weaker) and closer to the upper edge of the exchange rate band, the probability that it will, within a given finite time, reach the upper edge is higher. As a result, the probability of a future intervention to reduce the money supply and strengthen the currency is higher. This means that a future currency appreciation is expected, which the market turns into an immediate appreciation and a lower exchange rate. In this case the exchange rate is less than the rate predicted by the current fundamental alone, because an expected currency appreciation is being taken into account. In other words, the TZ exchange rate is less than the free-float exchange rate for a given level of the fundamental".

The honeymoon effect leads to the important insight that a perfectly credible TZ is inherently stabilizing: the expectation of future interventions makes the exchange rate more stable than the underlying fundamentals. As mentioned by Svensson, a TZ means stabilizing the fundamentals (between the vertical lines at f_l and f_u in figure 1.5), but the exchange rate stabilizes even more (between the horizontal lines at s_l and s_u). Some exchange rate stability is thus obtained ($s_u - s_l < f_u - f_l$).

A5.2 Option pricing interpretation of a target zone exchange rate level

The conditions given by equation (A5.6) are closely related to the "high-order contact" or "smooth-pasting" conditions that occur in option pricing theory and in the analysis of irreversible investment. As suggested by Krugman (1991), there is indeed an option pricing interpretation

of a target zone.

Given that $f = m + v$, equation (A2.19) can be rewritten as:

$$(A5.13) \quad s(t) = (1/\theta) \int_t^{\infty} E_t[m(\tau) + v(\tau) | I(t)] e^{-(1-\theta)(\tau-t)} d\tau.$$

The exchange rate is thus a present discounted value of future realizations of $(m + v)$.

Now, suppose that we were to consider an imaginary asset whose price is the present discounted value of $m + v$ holding m constant at m_0 . The value of this asset would thus be:

$$(A5.14) \quad \hat{s}(t) = (1/\theta) \int_t^{\infty} E_t[m_0 + v(\tau) | I(t)] e^{-(1-\theta)(\tau-t)} d\tau.$$

As a result, the exchange rate may be viewed as the price of a compound asset consisting of the imaginary asset whose price is determined by (A5.14), plus the right to sell the asset at a price s_h , plus the obligation to sell at the price s_l . The deviation of the S-curve from the F-line may now be viewed as the combined price of the two options. The requirement to sell at s_l becomes more important the higher is $\hat{s}$, so that the price of the compound asset falls below $\hat{s}$ at high values of v ; conversely, the right to sell at s_h supports the value of the asset at low values of v .

A5.3 The dynamic path of the interest rate inside a target zone

The exchange rate in a target zone seems more stable, less variable than in the flexible exchange rate system. But what is the implication for interest rates? The uncovered interest rate parity is given by equation (A2.3). Call the interest rate differential $id = i - i^*$, such that equation (A2.3) can be rewritten as:

$$(A5.15) \quad id = E_t[ds | I(t)] / dt.$$

Combining equations (A2.4), (A4.8) and (A5.5), we obtain that:

$$(A5.16) \quad id = [G(f) - f] / \theta.$$

In a permanent free-float, we saw that $s = G(f) = f + \theta\mu$ [equation (A3.10)]. Substituting $G(f)$ by its value, in equation (A5.16), the interest rate differential in a free float system becomes:

$$(A5.17) \quad id = \mu$$

that is, the interest rate differential is equal to the drift in the fundamental. If this drift is zero, then we have the result that $id = 0$, which means that $i = i^*$.

In the case of a target zone with infinitesimal intervention, substituting $G(f)$ given by equation (A5.12), in (A5.16), we get:

$$(A5.18) \quad id = I(f) = -\sinh(rf) / \theta \cosh(rf_u)$$

with $d[I(f)]/d(f) < 0$. This path is presented in figure 1.17.

As long as the fundamentals increase, we observe a decrease in the interest rate differential and, as a consequence, when f hits f_u , the differential id hits id_l . This means that, in the case of an infinitesimal intervention, a band on the fundamentals $[f_l, f_u]$ gives rise to both an exchange rate band $[s_l, s_u]$ and an interest rate differential band $[id_l, id_u]$. Interventions occur only when the fundamentals reach the edges of their band, which coincides with the exchange rate and the interest rate differential reaching the edges of their respective bands. The interventions that prevent the fundamentals and the exchange rate from moving outside their bands, at the same time, prevent the interest rate differential from moving outside its band. In this case, an exchange rate TZ is equivalent to an interest rate differential TZ. Defending the exchange rate at the edges of its band is the same thing as defending the interest rate differential at the edges of its band.

Observe also that in a permanent free-float $id = 0$, whereas in a TZ id fluctuates: a TZ is not a free lunch. The gain in stability for the exchange rate is paid in terms of a loss in interest rate

stability.

A5.4 Small and large target zones

Until now we have studied a target zone of given fundamental width, $[f_l, f_u]$, giving rise to a particular exchange rate width, $[s_l, s_u]$. This section shows that an increase in the band on the fundamentals will generate an increase in the exchange rate band: there is a monotonically non-decreasing relationship between the positioning of the bounds on the fundamentals and the positioning of the bounds on the exchange rate. This relation authorizes us to define a target zone either on the fundamentals or on the exchange rate.

Remember that, at f_u , by equation (A5.12), we have that:

$$(A5.19) \quad s_u = f_u - [\sinh(rf_u)]/\cosh(rf_u)$$

with $\{\sinh(rf_u)/\cosh(rf_u)\} < 1$. This ratio increases and tends rapidly to 1 for f_u large (for f_l negative and small, this ratio tends to -1). This means that for large TZ (i.e, for large positive f_u and small negative f_l), the exchange rates at the lower and upper edges of the band are approximately given by the relations:

$$(A5.20a) \quad s_l \approx f_l + 1/r$$

$$(A5.20b) \quad s_u \approx f_u - 1/r.$$

Target zones of different sizes are presented in figure A5.1.

<figure A5.1 here>

This figure is constructed by changing the width of the band around $s = f = 0$. The monotonically increasing relationship between bounds on f and those on s , as mentioned above, is represented by the locus $A-A$, which is a locus of intervention points. Points on the locus, above $s = 0$, represent pairs (f_u, s_u) for different bandwidths. Points below $s = 0$ represent pairs (f_l, s_l) .

A5.5 A target zone model in deviation form

An useful extension of the TZ model described in section A5.1 is a deviation TZ model with central parity, c . This model is shown in figure A5.2.

<figure A5.2 here>

In this deviation form model, set:

$$(A5.21a) \quad \underline{f} = f - c$$

$$(A5.21b) \quad \underline{s} = s - c$$

where an underlined variable expresses a variable in deviation from its central parity value, c .

Observe in figure A5.2 that when $\underline{f} = \underline{s} = 0$, the exchange rate is at its central parity and, when

$\underline{f} = f_u$ (f_l), the fundamentals are at their upper (lower) bound. In this model, equations (A2.4)

and (A4.8) are replaced respectively by:

$$(A5.22) \quad \underline{s}(t) = \underline{f}(t) + \theta E_t[d\underline{s}|I(t)]/dt$$

and:

$$(A5.23) \quad \underline{s} = G(\underline{f}; c).$$

Substituting (A5.23) in (A5.22) and solving the differential equation using Itô's lemma, we get:

$$(A5.24a) \quad \underline{s} = G(\underline{f}; c) = \underline{f} - 2A \sinh(r\underline{f}),$$

or alternatively:

$$(A5.24b) \quad s - c = G(f-c; c) = f - c - 2A \sinh[r(f-c)],$$

or:

$$(A5.24c) \quad s = G(f; c) = f - 2A \sinh[r(f-c)].$$

One boundary condition to define A is:

$$(A5.25) \quad G_f(\underline{f}=f_u; c) = 1 - 2A \cosh(rf_u) = 0.$$

Once the value for A is derived from (A5.25), it can be substituted in (A5.24a/b/c) to get:

$$(A5.26a) \quad \underline{s} = S_{\underline{f}}(\underline{f}; c) = \underline{f} - \sinh(r\underline{f})/\cosh(rf_u),$$

$$(A5.26b) \quad s - c = S_{\underline{f}}(f-c; c) = f - c - \sinh[r(f-c)]/\cosh(rf_u),$$

or:

$$(A5.26c) \quad s = S_{\underline{f}}(f; c) = f - \sinh[r(f-c)]/\cosh(rf_u).$$

The interest rate differential is given by:

$$(A5.27) \quad id = [s - f]/\theta.$$

Finally, substituting (A5.26.c) in (A5.27), we get:

$$(A5.28) \quad id = - \sinh[r(f-c)]/\theta \cosh(rf_u).$$

A5.6 Instantaneous (conditional) variability

This section examines the instantaneous (conditional) variability of the exchange rate and interest

rate differentials.

When the fundamentals of the exchange rate follow a Brownian motion without drift we know by equation (A1.31a) that:

$$(A5.29) \quad ds = dG = [(1/2)G_{ff}(f)\sigma^2]dt + G_f(f)\sigma dz.$$

Taking expectations and using equation (A5.15), we get:

$$(A5.30) \quad (1/2)G_{ff}(f)\sigma^2 = EdG(f)/dt = id.$$

As a result, the exchange rate follows a process given by:

$$(A5.31) \quad ds = dG = (id)dt + G_f(f)\sigma dz.$$

In a free float regime without drift $id = 0$ and $G_f(f) = 1$, such that $ds = \sigma dz$; if the fundamentals follow a Brownian motion, the exchange rate also follows a Brownian motion.

In a TZ regime, the exchange rate follows an Itô process. Indeed, the drift of the exchange rate process is then $\mu^s(f) = id = I(f)$ [where $I(f)$ is given by equation (A5.18)], and thus variable inside the TZ band. The instantaneous standard deviation is heteroscedastic and given by $\sigma^s(f) = \sigma G_f(f)$. Because $G_f(f)$ is the slope of the TZ exchange rate function, we can conclude that a plot of the instantaneous standard deviation of the exchange rate against the fundamentals is $\cap$ -shaped; the variability of the exchange rate is decreasing when approaching its boundaries. Intuitively, the exchange rate is affected by increasingly probable interventions of the official authorities when it is approaching the band margins. This triggers not only appreciation expectations (hence, affecting the first moment), but also decreases the volatility (i.e., there is an effect on the second moment).

Following the same procedure, we know that, when the fundamentals follow a Brownian motion without drift, increments in the TZ interest rate differentials are given by:

$$(A5.32) \quad d(id) = dI(f) = [(1/2)I_{ff}(f)\sigma^2]dt + I_f(f)\sigma dz.$$

But, by equation (A5.16) we have that $dI(f) = [dG(f) - df]/\theta$, such that:

$$E(dI)/dt = [E(dG)/dt - E(df)/dt]/\theta,$$

or:

$$(A5.33) \quad \mu^{id}(f) = [I(f) - 0]/\theta.$$

As a result, in a TZ, id follows an Itô process given by:

$$(A5.34) \quad d(id) = dI(f) = [I(f)/\theta]dt + I_f(f)\sigma dz.$$

In other words, the interest rate differential follows an Itô process with variable drift, $\mu^{id} = I(f)/\theta$ and heteroscedastic standard deviation $\sigma^{id}(f) = -I_f(f)\sigma$. [The minus sign guarantees a positive standard deviation.] A plot of $\sigma^{id}(f)$ against the fundamental is U-shaped; the volatility of the interest rate increases as the band margins are approached.

Observe that there is a linear trade-off between the exchange rate's and the interest rate differential's instantaneous variability. Indeed, by equation (A5.16), we know that $I_f = [G_f - 1]/\theta$.

As a consequence we have that:

$$\theta\sigma^{id}(f) = -\sigma[G_f - 1],$$

and thus:

$$(A5.35) \quad \sigma^*(f) + \theta\sigma^{id}(f) = \sigma G_f(f) - \sigma[G_f - 1] = \sigma.$$

The exchange rate's instantaneous standard deviation and θ times the interest rate differential's instantaneous standard deviation sum to the (constant) fundamental's instantaneous standard deviation. This trade-off has an intuitive economic interpretation: when the official authorities set a TZ they can achieve less volatile exchange rates at the margins of the band by simply announcing the defense of the band. However, the announcement of a credible band is also going

to trigger more volatile instantaneous interest rate differentials as the band margins are approached. The central bank can effectively decrease the volatility of the exchange rate by means of a TZ only at the expense of the higher volatility in the domestic instantaneous interest rate differentials.

A6 The model of Flood and Garber (1991)

The objective of this Appendix is to derive mathematically the saddle-path exchange rate for the model of Flood and Garber (1991), presented in section 1.2. The starting point (the first step) is equation (A5.5) rewritten here as:

$$(A6.1) \quad G(f) = f + A_1 \exp(-rf) + A_2 \exp(rf).$$

As seen in Appendix A5, a symmetric TZ model implies that $A_1 = -A_2$. Setting $A = A_1 = -A_2$ in equation (A6.1), we get, using the definitions of hyperbolic sine and cosine:

$$(A6.2) \quad G(f) = f - 2A \sinh(rf).$$

In the second step, a particular member of this family has to be selected with the help of an initial condition which defines A . In the TZ system, the speculators know the size of the finite intervention I and select the member of the family $G(f)$ by setting A such that they anticipate that the jump in f , due to the intervention, should not generate any change in the exchange rate. In other words, the speculators set A such that:

$$(A6.3a) \quad G(f_l) - G(f_l + I) = 0$$

and:

$$(A6.3b) \quad G(f_u) - G(f_u - I) = 0.$$

These arbitrage conditions require that there be no expected move of the exchange rate in response to an anticipated finite intervention.

Now, according to equation (A6.2), when $f = f_u$ we have that:

$$(A6.4) \quad G(f_u) = f_u - 2A \sinh(rf_u),$$

and when $f = f_u - I$ we have:

$$(A6.5) \quad G(f_u - I) = f_u - I - 2A \sinh[r(f_u - I)].$$

But, following the upper-bound arbitrage condition, $G(f_u)$ is also equal to $G(f_u - I)$. Equalising the right hand sides of equations (A6.4) and (A6.5) we get the following defined value for A , named A_d :

$$(A6.6) \quad A_d = I / 2\{\sinh(rf_u) - \sinh[r(f_u - I)]\}.$$

Substituting A in (A6.2) by its value defined in (A6.6) gives rise to the saddle-path solution of the target zone model of Flood and Garber:

$$(A6.7) \quad s = S_d(f) = f - I \sinh(rf) / \{\sinh(rf_u) - \sinh[r(f_u - I)]\}$$

where the subscript "d" is introduced to remember that this saddle-path depends on the particular discrete intervention I announced by the central bank.

In the target zone model of Appendix A5 (the infinitesimal intervention case), we had the following results for A and the saddle-path exchange rate:

$$(A6.8) \quad A = 1/[2r \cosh(rf_u)]$$

$$(A6.9) \quad s = S(f) = f - \sinh(rf) / [r \cosh(rf_u)].$$

Comparing equations (A6.8) and (A6.6), it is easy to see that A_d is larger than A . Also, as I decreases, we can observe that A_d decreases [the numerator decreases more rapidly than the denominator in (A6.6)], and, at the limit when I tends to 0, we get, using L'Hôpital's rule:

$$(A6.10) \quad \lim_{I \rightarrow 0} A_d = \lim_{I \rightarrow 0} [dI/dI] / [d2\{\sinh(rf_u) - \sinh(r(f_u - I))\}/dI] \\ = 1/2r \cosh(rf_u) = A.$$

As a result, for a discrete intervention, $S_d(f)$ will be slightly below $S(f)$. But, for a smaller discrete intervention, the market would select a higher path, which will turn out to be $S(f)$ when I tends to 0.

As shown in section 1.2, the smooth-pasting condition is observed only when the intervention I tends to 0. From that, it seems obvious that the smooth-pasting condition in the Flood and Garber model is simply the limit of the no-foreseeable jump (in the exchange rate) condition. In particular, take the left member of equation (A6.3b) [the no-foreseeable jump condition] and divide it by I :

$$(A6.11) \quad [G(f_u) - G(f_u - I)] / I = 0.$$

Take the limit of this expression for I tending to 0:

$$(A6.12) \quad \lim_{I \rightarrow 0} [G(f_u) - G(f_u - I)] / I = 0.$$

Equation (A6.12) means that the derivative of the function G evaluated at f_u is to be set equal to 0. An alternative notation is:

$$(A6.13) \quad G_f(f_u) = 0,$$

which is simply the smooth-pasting condition as given in equation (A5.6). The smooth-pasting condition in the FTZ model is then really the result of the assumption of no jump in the exchange rate at the time of the intervention.

A7 The model of Bertola and Caballero (1992a)

In this Appendix, a saddle-path exchange rate is derived for the case when there is a probability p that the new band, defined as in case 3 of section 1.3, is implemented, and a probability $1-p$ that the actual band is defended by a discrete intervention driving the fundamentals back to the (initial) central parity c .

As derived in equation (A5.24c), the family of functions for a TZ regime when the fundamentals are given by f and the central parity is c is given by:

$$(A7.1) \quad s = G(f;c) = f - 2A \sinh[r(f-c)].$$

Now the arbitrage condition should express the fact that, when the fundamentals hit the upper bound (when $f = c + f_u$), the exchange rate before the intervention should be equal to the **expected** exchange rate after the intervention. This condition is given by the following equation:

$$(A7.2) \quad G(c+f_u;c) = pG(c+2f_u;c+2f_u) + (1-p)G(c;c).$$

Adapting equation (A7.1) to the left and right sides of equation (A7.2), it is easy to solve for A . We find that:

$$(A7.3) \quad A = (1-2p)f_u / 2\sinh(rf_u).$$

Substituting A by its value in equation (A7.1), we get:

$$(A7.4) \quad s = f - (1-2p)f_u \sinh(rf) / \sinh(rf_u).$$

If $p = 0$ (i.e., if the central bank announces a credible policy of defending the current band by driving the fundamentals back to its previous central parity c), A is positive: we have a particular case of Flood and Garber's analysis with $I = f_u$ [see equations (A6.6) and (A6.7)] and a full honeymoon effect with an S-shape exchange rate function. If $p = 1/2$, then $A = 0$ and we have

the free-float result with no speculative bubbles: the exchange rate function is the 45 degree line. As p increases beyond $1/2$, A takes a negative value and the exchange rate function begins to have an inverted-S shape.

A8 The model of Svensson (1991)

In contrast to Bertola and Caballero (1992), Svensson (1991) advocates that some real world devaluations seem to occur when the exchange rate and thus the fundamentals are in the **interior** of their band. He then tries to model devaluations as occurring with some given constant probability regardless of where the exchange rate lies within its band.

Basically, Svensson models devaluations as occurring according to a Poisson process $N(t)$, as described in Appendix A1.3, where u is set equal to 1 in equation (A1.34). Then, $N(t)$ can be interpreted as the number of devaluations that occurred up to and including time t . The central parity of the target zone, c , is supposed to follow a Poisson differential equation:

$$(A8.1) \quad dc = g dN$$

such that g reflects the fixed magnitude of the devaluation.

The expected rate of realignment/devaluation is then given by:

$$(A8.2) \quad E(dc)/dt = gE(dN)/dt = gv.$$

This expected rate of realignment is exogenous and constant everywhere inside the band. Note that the band around c does not change, such that after N devaluation, the target zone band is translated from $[f_l, f_u]$ to $[f_l + gN, f_u + gN]$.

The fundamentals are supposed to follow a combination of a Brownian motion without drift and a Poisson process:

$$(A8.3) \quad df = \sigma dz + g dN.$$

This assumption about the fundamentals process implies that a devaluation maintains the fundamentals' position relative to their own band and, as a consequence, the exchange rate's

position relative to the exchange rate band.

Using the techniques described in Appendixes A.4 and A.5, we can derive the target zone saddle-path as:

$$(A8.4) \quad s(f;N) = G(f;N) = f + \theta \nu g - \sinh(r(f-gN))/r \cosh r f_u.$$

When no devaluation has happened yet, $N = 0$ and (A8.4) reduces to:

$$(A8.5) \quad s(f;0) = G(f;0) = f + \theta \nu g - \sinh(rf)/r \cosh r f_u.$$

Comparing (A8.5) with the TZ solution without devaluation risk [equation (A5.12)], observe that the only difference is the addition of the term $\theta \nu g$ (i.e., θ times the rate of expected change in the central parity of the TZ). Figure A8.1 shows an S-curve when no devaluation risk is considered.

<figure A8.1 here>

When a devaluation risk is introduced the S-curve shifts up by $\theta \nu g$. When the devaluation effectively happens, the S-curve shifts to the North-East: every point on the graph is translated by g , to the North-East, along a 45-degree line.

The model of Svensson (1991) has been extended by Tristani (1994). In this model, Tristani endogenizes the expected rate of realignment, $E[dc]/dt$, by setting ν equal to $p \underline{f}$, where p is a constant and $\underline{f}$ is the deviation of f from its central parity. This gives a model which lies somewhere between Bertola and Caballero (1992) and Svensson (1991). Bertola and Svensson (1993) extend the model of Svensson (1991) by treating the expected rate of realignment as exogenous, but randomly variable: they assume that $E[dc]/dt$ follows a Brownian motion with drift.

A9 Asymptotic (unconditional) exchange rate distribution in a TZ model

The first step to compute the asymptotic (unconditional) exchange rate distribution in a target zone is to find the asymptotic distribution of the fundamentals, denoted $\phi(f)$. From this latter distribution we will find the former by the rule of transformation of random variables.

If f follows a regulated Brownian motion with drift (with control at f_l and f_u) for a long time, f will settle down to a stationary long run process. Let us use the random walk approximation of a Brownian motion as explained in Appendix A1.1. Consider any three adjacent points, say $f-\Delta h$, f , and $f+\Delta h$. In any small interval of time, Δt , the process can reach f in one of two ways, by increasing from $f-\Delta h$, or by decreasing from $f+\Delta h$. In other words, during the interval of time Δt , the probability mass $\phi(f-\Delta h)$ moves up with probability p and the probability mass $\phi(f+\Delta h)$ moves down with probability q , where p and q are given by equations (A1.16a/b). These moves constitute the probability mass at f after the end of the time interval, such that:

$$(A9.1) \quad \phi(f) = p\phi(f-\Delta h) + q\phi(f+\Delta h).$$

Equation (A9.1) means that the density function at point f is a linear combination of the density at points $f-\Delta h$ and $f+\Delta h$, weighted by the probabilities p and q .

Substituting for p and q and expanding the right hand side member around f , by Taylor's theorem, we get:

$$\begin{aligned} \phi(f) = & 1/2[1+(\mu/\sigma^2)(\Delta h)][\phi(f)-(\Delta h)\phi'(f)+(1/2)\phi''(f)(\Delta h)^2...] + \\ & 1/2[1-(\mu/\sigma^2)(\Delta h)][\phi(f)+(\Delta h)\phi'(f)+(1/2)\phi''(f)(\Delta h)^2...]. \end{aligned}$$

Cancelling $\phi(f)$ from both sides, dividing by $(\Delta h)^2$ and taking limits as $\Delta h \rightarrow 0$, we get a differential equation:

$$(A9.2) \quad \phi''(f) - (2\mu/\sigma^2)\phi'(f) = 0.$$

Equation (A9.2) is actually the Kolmogorov transition equation satisfied by the time-invariant density function of a Brownian motion with drift. Because this is a homogenous equation, the complementary function, ϕ_c , is indeed the general solution of this equation and is given by $\phi_c = A_1 \exp[(2\mu/\sigma^2)f] + A_2$, such that the general solution is:

$$(A9.3) \quad \phi(f) = A_1 \exp[(2\mu/\sigma^2)f] + A_2.$$

To determine A_1 and A_2 we need two boundary conditions. The first boundary condition comes from the reflecting nature of the two bounds, f_l and f_u . For example, at the upper bound, equation (A9.1) becomes:

$$(A9.4) \quad \phi(f_u - \Delta h) = p\phi(f_u - 2\Delta h) + p\phi(f_u - \Delta h).$$

Note that $(f_u - \Delta h)$ can (obviously) be reached from $(f_u - 2\Delta h)$ but also from $(f_u - \Delta h)$ itself. Actually, if f tries to take an upward step from $(f_u - \Delta h)$, it is moved right back to $(f_u - \Delta h)$ due to the reflecting nature of f_u . Hence, equation (A9.4) means that in any time interval Δt , the probability mass $\phi(f_u - 2\Delta h)$ moves up with probability p and the probability mass $\phi(f_u - \Delta h)$ moves **up** with probability p , but is reflected right back to $f_u - \Delta h$. Expanding the first term on the right hand side member of equation (A9.4) around $(f_u - \Delta h)$, dividing by Δh and taking limits as $\Delta h \rightarrow 0$, we get the first boundary condition:

$$(A9.5) \quad \phi'(f_u) - (2\mu/\sigma^2)\phi(f_u) = 0.$$

Using (A9.3) and (A9.5), we finally get that $A_2 = 0$.

The second boundary condition comes from the fact that the whole probability mass must also satisfy the summing-up condition:

$$(A9.6) \quad \int_{f_l}^{f_u} \phi(f) df = 1.$$

Substituting equation (A9.3) in (A9.6), with A_2 set equal to 0, and integrating, we get:

$$(A9.7) \quad A_1 = [2\mu/\sigma^2] / \{ \exp[(2\mu/\sigma^2)f_u] - \exp[(2\mu/\sigma^2)f_l] \}.$$

Now, substituting A_1 and A_2 in (A9.3), we finally get the asymptotic density function of the exchange rate fundamentals inside the target zone:

$$(A9.8) \quad \phi(f) = [2\mu/\sigma^2] \exp[(2\mu/\sigma^2)f] / \{ \exp[(2\mu/\sigma^2)f_u] - \exp[(2\mu/\sigma^2)f_l] \}.$$

This is a truncated exponential function, leaning in the direction of the drift: if the process for f has a positive drift rate ($\mu > 0$), the exponential should be rising toward f_u , and if it has a negative drift rate, the density function should be falling toward f_u . In the particular case of zero drift, we would get a uniform distribution given by:

$$(A9.9) \quad \phi^f(f) = 1/[f_u - f_l].$$

[The superscript f has been added to prevent confusion in the rule of transformation given shortly.]

Now that we have found the unconditional (asymptotic) density function for the fundamentals inside a target zone, we would like to find the corresponding asymptotic density function for the exchange rate inside the target zone. The function relating exchange rates and fundamentals, given by equation (A5.12), is single-valued and admits an inverse, $f = h(s)$, (in the interior of the band). Thus the long-run density of exchanges rates within the band, $\phi^s(s)$, can be derived by a transformation [see Ramanathan (1993), chapter 3, theorem 3.5]:

$$(A9.10) \quad \begin{aligned} \phi^s(s) &= \phi^f(h(s)) |dh(s)/ds| \\ &= \{1/[h(s_u) - h(s_l)]\} |dh(s)/ds| \\ &= \{1/[f_u - f_l]\} |dh(s)/ds|, \end{aligned}$$

for $s_l < s < s_u$.

The function $f = h(s)$ has a slope that reaches a minimum at $s = 0$ but that increases and tends to $+\infty$ as s tends to s_l and s_u . As a result, through equation (A9.10), the exchange rate distribution must tend to a spike at both bounds: the probability distribution of the exchange rate is U-shaped (i.e., bimodal with more probability mass towards the edges of the band than the uniform distribution).

This Krugman target zone model prediction (the U-shaped distribution of the exchange rate) is clearly rejected by the data on the E.M.S. [see Flood, Rose, and Mathieson (1991)]. Actually, the data show that the distribution is hump-shaped, with most of the probability mass in the interior of the band and very little near its edges. One explanation for this hump-shaped distribution could be that intra-marginal interventions (leaning against the wind interventions) occur inside the exchange rate band. To model these interventions, Delgado and Dumas (1992) assume that the fundamentals follow an exogenous Ornstein-Uhlenbeck mean-reverting process [continuous time AR(1) process].

A10 Permanent regime switching [Froot and Obstfeld (1991a)]

Froot and Obstfeld (1991a) derive, in a two-step method, the solution for a regime switching from the current free-float to a fixed exchange rate, using the techniques of controlled Brownian motion. Their solution is as follows.

The starting point of the analysis is again the exchange rate equation:

$$(A10.1) \quad s(t) = f(t) + \theta E_t[ds(t)]/dt$$

where $f(t)$ is assumed to follow a Brownian motion with instantaneous variance, σ^2 , and (depreciating) trend, $\mu > 0$, such that increments in f are given by a stochastic differential equation:

$$(A10.2) \quad df = \mu dt + \sigma dz$$

where z is a standard Wiener process.

Under these conditions it was shown, in Appendix A4.2, that the saddle path for the exchange rate is a particular member of the family of paths given by:

$$(A10.3) \quad G(f) = f + \theta\mu + A_1 \exp[r_1 f] + A_2 \exp[r_2 f]$$

where r_1, r_2 are given by:

$$(A10.4) \quad r_1, r_2 = \{-\mu \pm [\mu^2 + 2\sigma^2/\theta]^{1/2}\}/\sigma^2, \quad r_1 < 0 < r_2$$

and A_1 and A_2 are two constants of integration to be defined, in the second step, through boundary conditions peculiar to the announced regime switch. [The identical first step for a permanent regime switching or a target zone model underlines the basic similar framework shared by the two models.]

When it is announced that the free-float will be replaced by a permanent fixed exchange rate

as soon as s hits s_z , we have to look for two specific boundary conditions. One condition is obvious: as long as s is relatively far from s_z , the market considers the return to a fixed exchange rate as an unlikely event for the near future. In this case, we should expect that the dynamic of the exchange rate should be asymptotically the one of the free-float [as given in equation A3.10]: A_1 is to be set equal to 0.

The second boundary condition is based on a continuity argument invoked to preclude excess anticipated profit opportunities. When the exchange rate is permanently fixed at s_z , the expected rate of depreciation is zero and, by equation (A10.1), it must be the case that $s_z = f_z$. Now, to prevent jump in the exchange rate (and thus anticipated profit opportunities) at the moment of entry, $G(f)$ must be a continuous function such that, at f_z , we have:

$$(A10.5) \quad G(f_z) = f_z + \theta\mu + A_2 \exp[r_z f_z] = s_z,$$

which gives that:

$$(A10.6) \quad A_2 = -\theta\mu / \exp[r_z f_z].$$

Substituting $A_1 (= 0)$ and A_2 by their values in equation (A10.3) gives finally the saddle-path for the exchange rate during the transition period:

$$(A10.7) \quad s = S_{fre}(f) = f + \theta\mu(1 - \exp[r_z(f - f_z)]).$$

This is the closed-form solution of the problem set by Flood and Garber (1983) and derived by Froot and Obstfeld (1991a).

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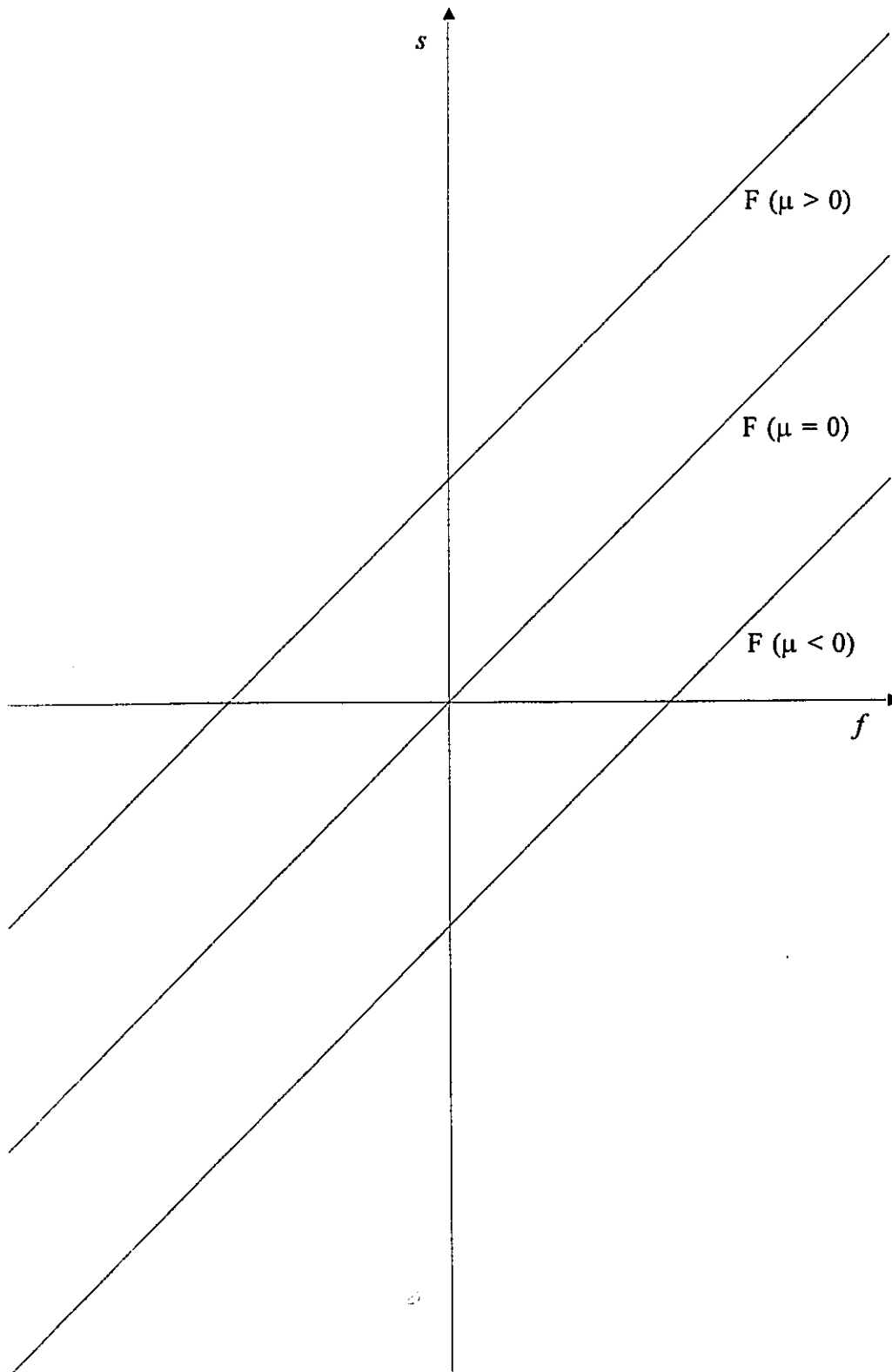


Figure A3.1 Permanent free-float paths in (f, s) space

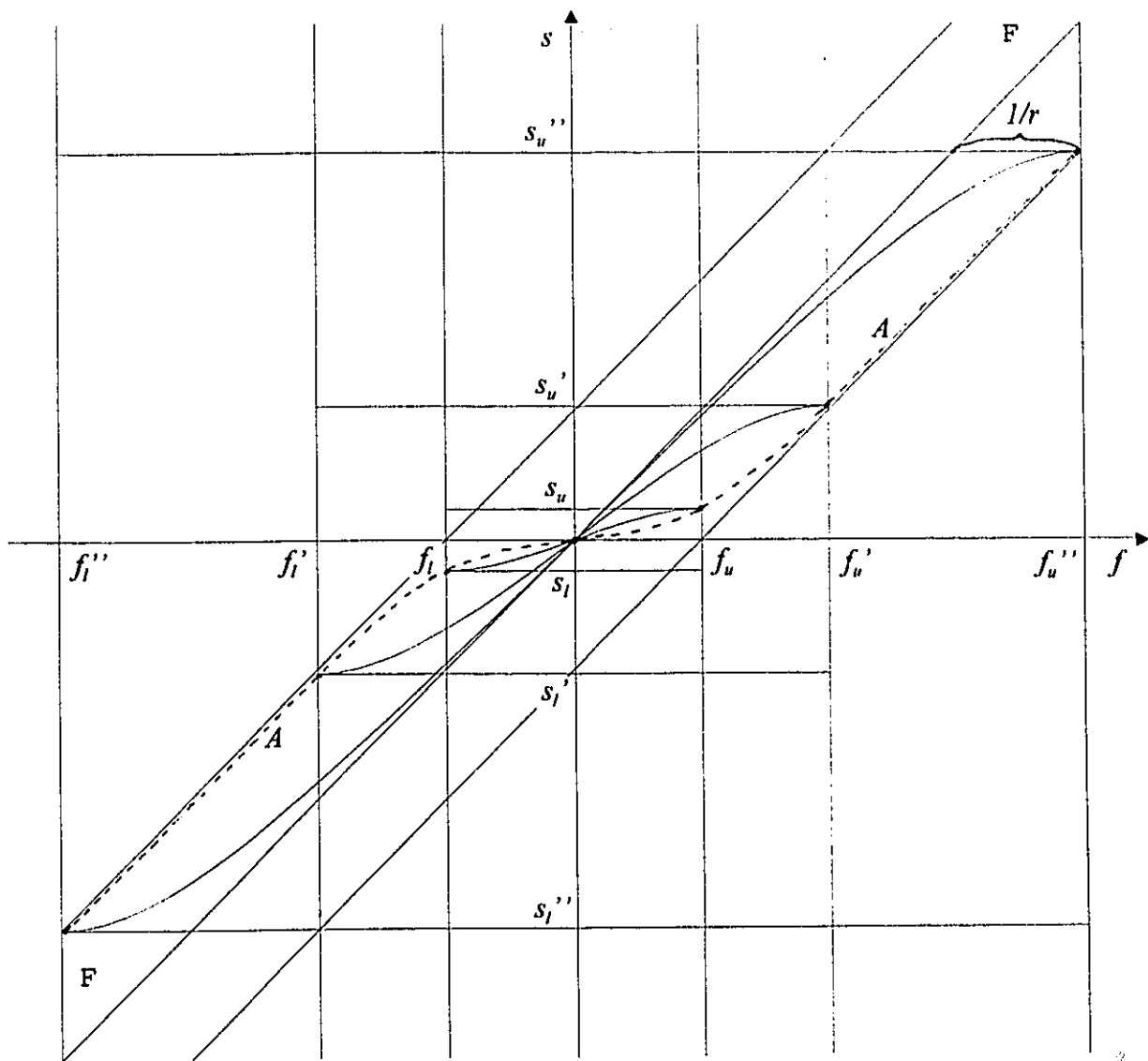


Figure A5.1 Small and large target zones

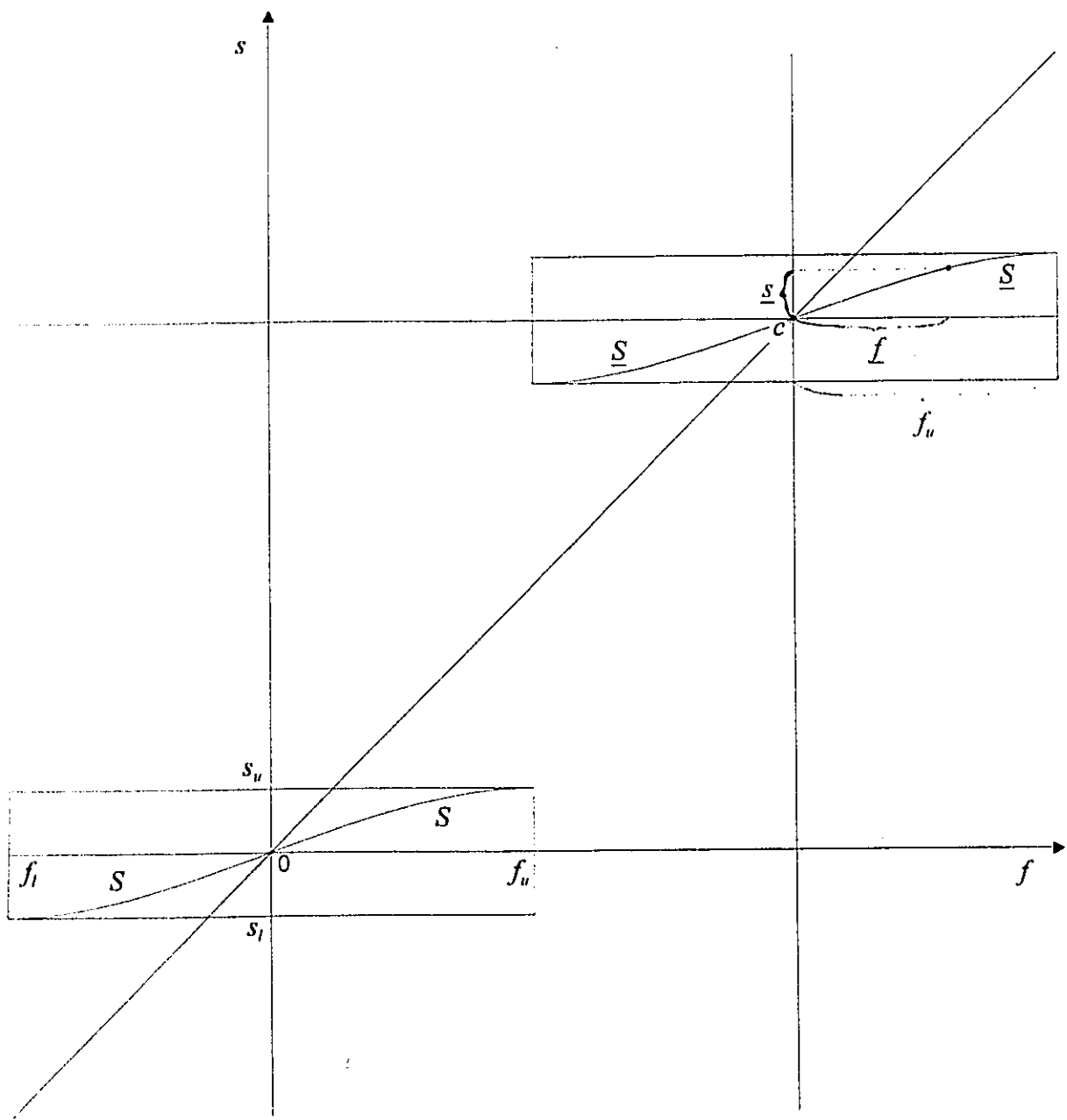


Figure A5.2 A target zone model in deviation form

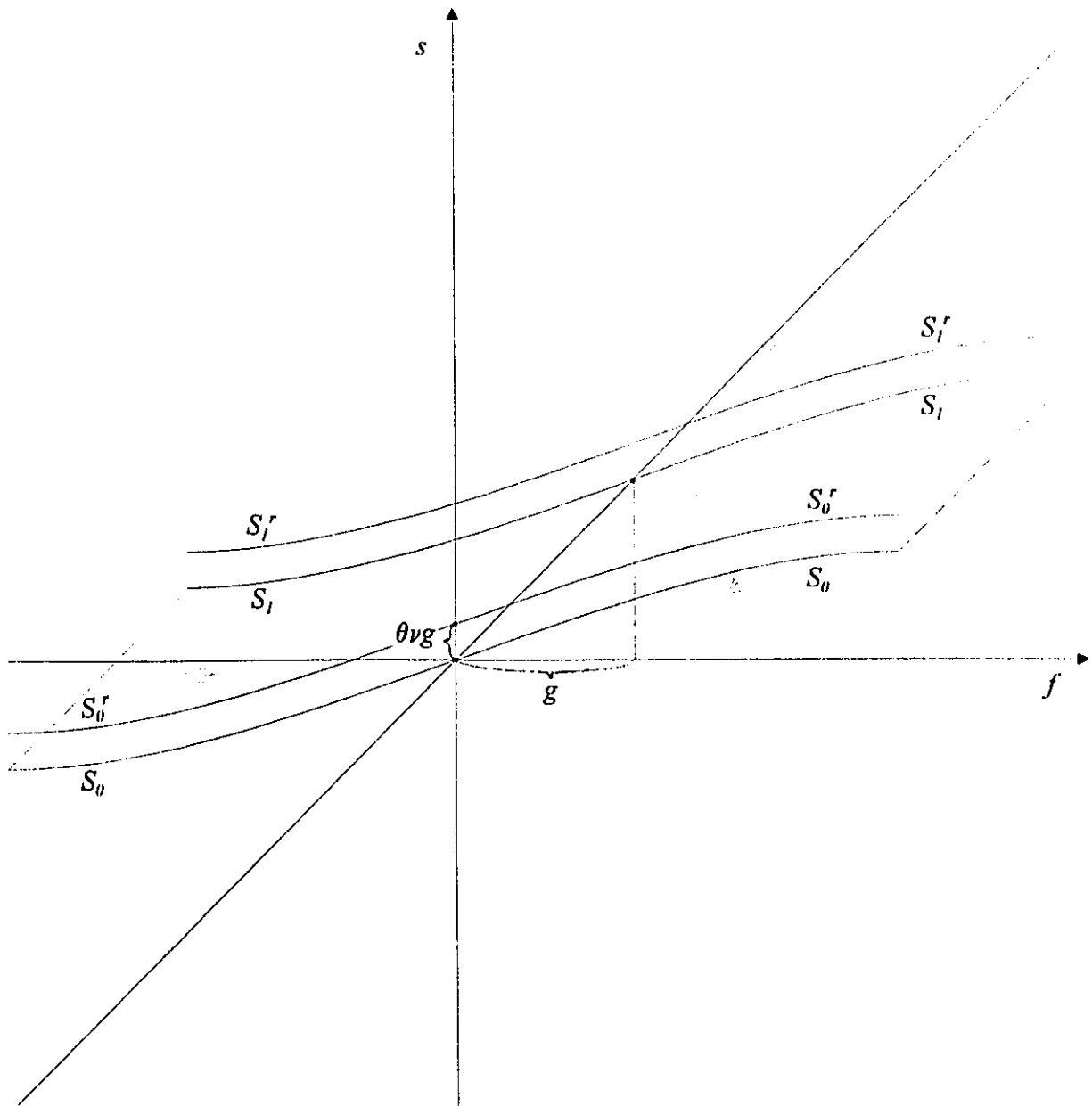


Figure A8.1 Realignment as a jump process