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Application of Finite Difference Method to the
Deflection of Clamped and Simply supported
Thick Rectangular Plates

By

Nasr-Eddine Bencharif

A thesis
presented to the University of Ottawa
in partial fulfillment of the
requirements for the degree of
Master of Applied Science
in
Civil Engineering

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Abstract

Variational methods are widely used for the solution of complex differential equations in mechanics, for which exact solutions are not possible. The finite difference method, although well known as an efficient numerical method was applied in the past only for the solution of thin plates.

In the present study, the suitability of the method for the solution of deflection of thick plates is studied for the first time. While there is major differences between thin plate and thick plate theories, the former can be treated as a particular case of the latter, when the centre deflection of the plate is less than or equal to 0.2-0.25 of the thickness of the plate. The finite difference method as applied here is a modified finite difference approach to the ordinary finite difference method generally used for the solution of thin plate problems. In this thesis thin plates are treated as a particular case of the corresponding thick plates when the boundary conditions of the plates are taken into account.

The method is first applied to investigate the behaviour for clamped, square isotropic homogeneous plates. After the validity of the method is established, it is then extended to the solution of similar problems for simply supported square plates.

Once a solution for a particular plate aspect ratio and boundary condition is obtained using a limited number of mesh sizes, a general solution of the problem to investigate accuracy and convergence was extended to rectangular thick plates by providing more detailed functions satisfying the

rectangular mesh sizes generated automatically by the programme.

Whenever possible results of the present method are compared with the existing solutions in the technical literature obtained by much more laborious methods and close agreements are found. Significant amounts of results presented herein are not available in the technical literature for various plate aspect ratios and Poisson's ratios. The submatrices involved in the formation of the finite difference equations from the governing differential equations forming the general system are generated directly by the programme. The two subroutines MATINV and MATMUL take care of the inversion and the multiplication of the matrices respectively.

Simplicity in formulation and quick convergence are the obvious advantages of the method found in comparison with other numerical methods requiring extensive computer facilities.

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Notations

x, y, z	rectangular co-ordinate system.
u, v, w	displacements of the nodes in x, y, and z direction respectively.
a	longitudinal dimension of the plate.
b	transverse dimension of the plate.
h	thickness of the plate.
N_x	axial forces parallel to the x-axis per unit length of a section of plate perpendicular to y-axis.
N_y	axial forces parallel to the y-axis per unit length of a section of plate perpendicular to x-axis.
Q_x, Q_y	shearing forces parallel to the z-axis per unit length of a section of plate perpendicular to x-and y-axes respectively.
N_{xy}	the inplane shearing intensities per unit length.
M_x, M_y	bending moment intensities per unit length.
M_{xy}	twisting moment intensity per unit length.
$q = q(x, y)$	transverse external force per unit area.
$\sigma_x, \sigma_y, \sigma_z$	normal stress components in x, y, z directions respectively.
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	shearing stress components in rectangular co-ordinates.
$\epsilon_x, \epsilon_y, \epsilon_z$	unit elongation in x-, y-, and z- directions.
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	shearing strain components in rectangular co-ordinates.
E	Modulus of elasticity in tension and compression.
G	modulus of elasticity in shear.
ν	Poisson's ratio.
$z = \pm h/2$	lower and upper face of a plate, respectively.

D	flexural rigidity of an isotropic plate.
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$	Laplacian operator in rectangular co-ordinates.
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$	Laplacian operator in space co-ordinates.
$[UU]$	matrix determining the participation of u in the x direction.
$[UV]$	matrix determining the participation of v in the x direction.
$[UW]$	matrix determining the participation of w in the x direction.
$[VU]$	matrix determining the participation of u in the y direction.
$[VV]$	matrix determining the participation of v in the y direction.
$[VW]$	matrix determining the participation of w in the y direction.
$[WU]$	matrix determining the participation of u in the z direction.
$[WV]$	matrix determining the participation of v in the z direction.
$[WW]$	matrix determining the participation of w in the z direction.
$[FF]$	nodal loading.
$[FOR]$	vector force.
$[DIS]$	vector displacement.
$[AA]$	general matrix.

Chapter 1

Introduction

1.1 General

Plates are used as components of large scale structures in both mechanical and civil engineering. They are plane surface structures bounded either by straight or curved lines. Plates may have free, simply-supported, or fixed boundary conditions, including elastic supports or, in some cases, point supports. The static loads carried by plates are predominantly perpendicular to the plate surface. Plates are classified by their geometrical forms or by their physical characteristics. Thick plates are a type of plates where the thickness is considerable and the approximate theories of thin plates are no longer applicable. In such cases, the thick plate theory must be applied. This theory considers the problem of thick plates as a three-dimensional problem

of elasticity. The stress analysis becomes, consequently, much more involved and up to now, the problem is solved only for a few particular cases. The difficulty of these solutions is a direct result of the complexities of the partial differential equations governing the behaviour of thick plates. Furthermore the boundary conditions for thick plates have to be satisfied in three directions. And hence, three functions determining the three displacements are required. The thick plate theory finds important application in the static analysis of heavy floor slabs and the dynamic analysis of heavy floors carrying rotating machinery in industrial buildings. Often, designers must also ensure that structures withstand both applied static and dynamic loads. An exact analysis is often required to determine the deformation of the plate, under the type of loading the plate is designed to carry. The fundamental problems in mechanics are generally governed by both differential equations and minimum energy principles. The governing differential equations and the equations derived by minimum energy principles are usually rather complex and do not lend themselves to easy exact solutions. Confronted with this problem engineers and researchers have to resort to numerical methods to effect a solution. Variational methods were exploited by engineers and scientists as a very effective tool for solving applied mechanics problems. These methods gained popularity in recent years due to the development of high speed digital computers. Finite differences methods were also used extensively for solving complex differential equations. This method can be considered as a means for obtaining the approximate solutions of differential equations, where the exact solutions are not possible. According to the method, the problem is solved only at the chosen appropriate points where



the system of linear equations was defined. In applying the finite difference method, a mesh size must first be chosen. The mesh size covers the entire domain of the plate with n points yielding n simultaneous equations. Convergence is easily investigated by increasing the mesh size covering the plate domain. In the present study the finite differences method is applied to the three dimensional problem of thick plates where the governing differential equations of equilibrium of such plates are expressed in terms of the displacements u , v , and w following the co-ordinates axes x , y and z respectively, care being taken to modify the finite difference expressions to satisfy the appropriate boundary conditions and the symmetry of a particular problem.

1.2 Object and Scope

The main object of the thesis is to study the applicability of the finite difference method to the bending of thick plates. To study the applicability of the method, the problem of the bending of a square clamped plate is first studied using a rather crude mesh size. After the validity of the method has been established, the same method is then extended to analyse the corresponding problems of rectangular clamped and simply-supported thick plates. The method can be further applied to determine the buckling and vibration of thick plates by the introduction of corresponding appropriate elasticity equations.

1.3 Outline of the thesis

Since the main objective of this thesis is to study the applicability of the finite difference method for thick plates, existing literature related to the solution of thick plates by various numerical methods is briefly reviewed in Chapter (2). Chapter (3) is devoted to the formulation of the governing equations of elasticity and the relationship between stress and strain in the three dimensional plate system and the relationship between displacements and stresses in thick plates is also derived. In chapter (4), the finite difference expressions required to analyse the deflection problem of clamped and simply-supported isotropic rectangular thick plates are formulated. The formulation of finite difference in the case of thick plates is not an easy task. For example, in the case of thin plate theory, the problem can be reduced to one governing equation in terms of the vertical displacement w , whereas for thick plates the governing differential equations require the inclusion of two additional components u and v , thus increasing significantly the complexity of the problem. To apply finite difference to solve this complicated problem, previous research did not show a direct application using this numerical solution. In thick plate theory, the requirement to satisfy the boundary conditions deserves careful consideration especially for boundary conditions applied at extreme surfaces of the plate. For this reason, the thick plate is divided in layers with appropriate boundary expressions for each layer. The modified finite difference method presented may be taken as a general method applicable to both thin and thick plate structures.

The formulation of the finite difference expression for each point is presented in detail, showing how the partial derivatives are replaced by finite difference expressions, and how the resulting equations are reduced in matrix form to yield the solution of a particular problem. In chapter (5), deflection results are shown for different plate aspect ratios for both clamped and simply-supported thick plates. Results are also tabulated for different values of Poisson's ratio showing the influence of the ratio on the height "h", and on the longitudinal dimension "a". Finally, conclusions of the present study are summarized for all cases considered and, whenever possible, comparisons are made with other investigators. Significant amounts of data presented herein are not available in the technical literature and the detailed results included in this thesis can be looked upon as an exhaustive study of the deflection of simply supported and clamped thick plates with various plate aspect ratios and Poisson's ratios. All computations involved in this thesis were programmed in FORTRAN 4 for the Amdahl mainframe computer. The general programme is presented starting with reduction of submatrices and the formulation of the general matrix describing the system is generated automatically. This is followed by the formulation of the vector force. Programme subroutines MATINV, and MATMUL are also presented. The program is included for reference in the appendix.

Chapter 2

Literature review

The classical theory of plates has been well established and widely applied since Thomson and Tait resolved the controversy over the boundary conditions of Poisson's theory in the latter half of the nineteenth century. Due to approximations inherent in their derivations, classical theories restrict themselves in their application to plates having thickness much less than their lateral dimensions. As a result, these theories cannot be applied with any guarantee of accuracy to thick plates or problems where local effects predominate such as stress concentration problems. Thus, there are cases when a more refined theory is required. This led to several significant developments in recent years in the field of analysis of thick plates.

The shear deformation theory of Reissner [1] derived on the basis of a variational criteria has been one of the foremost in the treatment of thick plates.

This theory has been applied to various rectangular plates by many investigators. Kromm [2] demonstrated the absence of corner reactions using a theory that takes into account transverse shear deformation. This point has also been discussed in detail by Marguerre and Woernle [3]. Green [4] has pointed out that the Reissner equations can be obtained directly from the three dimensional elasticity equations without recourse to variational considerations. Donnel [5] has given a three dimensional thick plate theory in which the solution is obtained in the form of infinite series with the first term representing classical thin plate theory results. Lee [6] has applied this method to simply supported rectangular plates. Friedrichs and Dressler [7] and Goldenveiser [8] have given approximate theories which could predict boundary layer effects, by the method of asymptotic integration of the governing equations of elasticity. Lur'e [9] has given a general power series method of solution of the elasticity equations. Poniotovskii [10] has employed Legendre polynomial expansion in the thickness coordinate to derive a system of two dimensional equations. A three dimensional solution for rectangular plates and laminates has been developed by Srinivas et al. [11] in which the solution for displacements is taken in the form of a double trigonometric series satisfying the equations of equilibrium in terms of displacements. Srinivas and Rao [12] solved the problem of thick plates using a three dimensional analysis by expressing the displacement functions in terms of hyperbolic functions using the techniques of collocation, orthogonalization, and orthogonalization along with collocation at corners. The generalized Levy solution [13] has been taken by David.W.Cooke to solve the problem of a statically loaded, rectangular plate which is simply

supported on two opposite edges and has arbitrary boundary conditions on the remaining edges. Kvashnina [14] presented a method based on the expressions obtained by Berdichevskii to solve the problem of bending of thick plates. Kishida [15] carried out a three dimensional elastic stress analysis of thick plates, and solved the problem of axisymmetric bending of a clamped circular plate subjected to an annularly distributed load. An analysis of an infinite elastic thick plate subjected to loads symmetrical to the axis of revolution has been developed by Nakajima, Iamsopana and Kakuzen [16]. After they have represented the part of hyperbolic functions of the integrand of stress functions in terms of exponential functions, Maclaurin's law was used to expand the denominator and each term was then integrated separately. The approximate stress components were calculated by solving the simultaneous linear algebraic equations with four unknowns by satisfying the boundary conditions at two points on the boundary surfaces. Kobayashi, Nagasawa, Ishikawa, and Hata [17] proposed an extension of Lover's moderately thick plate theory to solve the problem of a three dimensional rectangular cantilever plate. Bending of thick plates under an arbitrary load has been carried out by Gruzdev [18] using the method proposed by Lur'e, who reduces the three dimensional plate problem to the two dimensional problem using the infinite order differential operators of the displacements and rotations of the middle plane of the plate. Deshmukh and Archer [19] proposed a new method which considerably reduces the amount of numerical computations for certain classes of plate problems as compared to the alternate methods, i.e., finite element or finite difference methods. The method proposed is based on Reissner's theory

where the total unknown component is equal to the sum of particular and complimentary solutions. Mixed finite difference scheme for the analysis of simply supported thick plates was proposed by Noor [20]. His analysis is based on the linear, three dimensional theory of orthotropic elasticity and a Fourier approach is used to reduce the governing equations to six first-order ordinary differential equations in the thickness coordinate and a finite difference method is carried out to form a system of linear equations from the previous differential equations. Ryabov and Rasskazov [21] examined the bending of thick nonuniform plates through their thickness. Their assumption is based on the distribution of the shear components in the cross sections and its change over the plate thickness in accordance with a square parabola law . Finite element is the most rigorous method used to solve complicated problems during the last three decades. Here, Epstein and Huttermaier [22] made use of the bilinear isoparametric finite element concept for the numerical analysis of multilayered and thick plates. Guruswamy and Tang [23] have expressed a twenty four degree of freedom sector finite element for the static and dynamic analysis of thick circular plates. Sundara, Chandrashekhara and Sebastian [24] applied a method of initial functions (MIF) proposed by Vlassov for the analysis of thick rectangular plates. The unknowns are expanded in Maclaurin series in the thickness coordinate and hence the solution is obtained in terms of unknown initial functions on the reference plane. Many other papers dealing within this area of research had an advantage of developing some classical methods or by proposing a modified formulas in the area of buckling and free vibrations of thick plates .

Chapter 3

Formulation of the governing equations

3.1 Equation of equilibrium

The formulation of the governing differential equations of equilibrium of an isotropic rectangular thick plate under a vertical load $q(x,y)$ is presented in the three principal directions x , y and z . The equilibrium will be defined by equating the internal forces and external forces in each principal direction. Figure (1) shows all the forces on the plate, and Figure (2) shows the distribution of moments inside an element of the plate. Figure (3) shows the deformation of the plate in its general form. For small slopes, summing forces in the Z direction yields the equations:

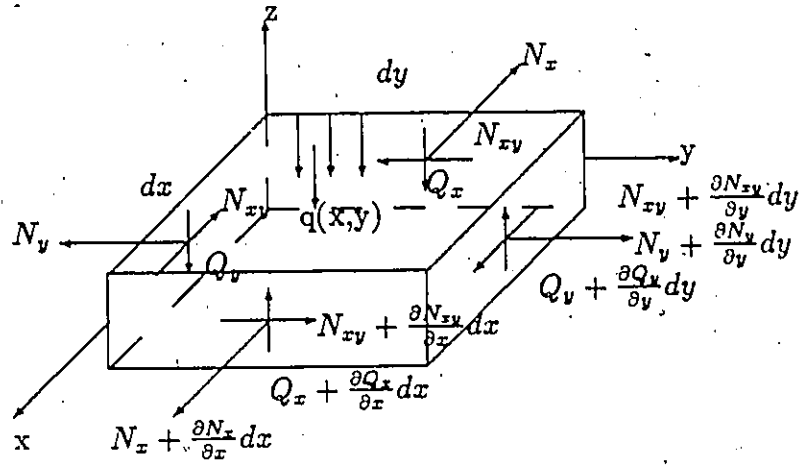


Figure (3.1) equilibrium state of the plate element

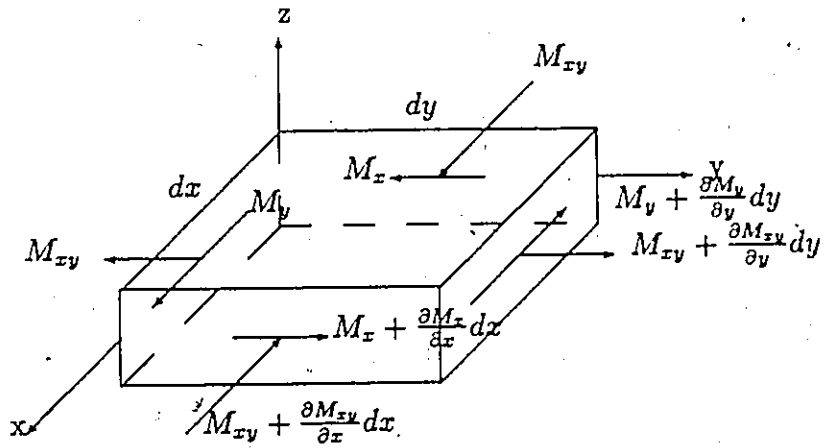


Figure (3.2) distribution of the moments in the plate element

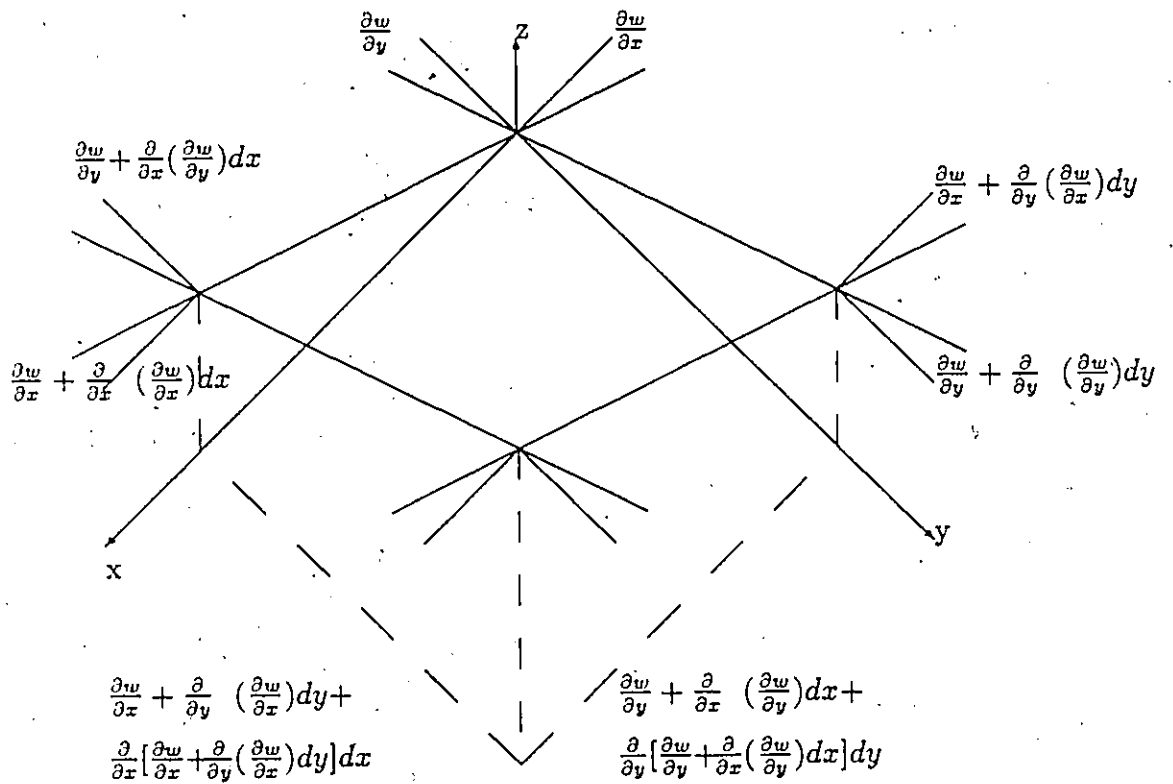


Figure (3.3) deformations of the plate

$$\begin{aligned}
& \frac{\partial Q_x}{\partial x} dx dy + \frac{\partial Q_y}{\partial y} dx dy - N_x dy \frac{\partial w}{\partial x} + \left(N_x + \frac{\partial N_x}{\partial x} dx \right) dy \left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} dx \right) \\
& - N_y dx \frac{\partial w}{\partial y} + \left(N_y + \frac{\partial N_y}{\partial y} dy \right) dx \left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} dy \right) - N_{xy} dx \frac{\partial w}{\partial x} + \\
& \quad \left(N_{xy} + \frac{\partial N_{xy}}{\partial y} dy \right) dx \left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x \partial y} dy \right) - N_{xy} dy \frac{\partial w}{\partial y} + \\
& \quad \left(N_{xy} + \frac{\partial N_{xy}}{\partial x} dx \right) dy \left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial x \partial y} dx \right) - q dx dy = 0
\end{aligned} \tag{3.1}$$

Expanding the terms involving products, and discarding the resulting third order differential terms yields,

$$\begin{aligned}
& \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - N_x dy \frac{\partial w}{\partial x} - N_y dx \frac{\partial w}{\partial y} + N_x dy \frac{\partial w}{\partial x} + N_x dy \frac{\partial^2 w}{\partial x^2} dx + \frac{\partial N_x}{\partial x} dx dy \frac{\partial w}{\partial x} + \\
& \quad \frac{\partial N_x}{\partial x} dx dy \frac{\partial^2 w}{\partial x^2} dx + N_y dx \frac{\partial w}{\partial y} + N_y dx dy \frac{\partial^2 w}{\partial y^2} + \frac{\partial N_y}{\partial y} dx dy \frac{\partial w}{\partial y} + \\
& \quad \frac{\partial N_y}{\partial y} dx dy \frac{\partial^2 w}{\partial y^2} dy - N_{xy} dx \frac{\partial w}{\partial x} + N_{xy} dx \frac{\partial w}{\partial x} + N_{xy} dx \frac{\partial^2 w}{\partial x \partial y} dy + \\
& \quad \frac{\partial N_{xy}}{\partial y} dy dx \frac{\partial w}{\partial x} + \frac{\partial N_{xy}}{\partial y} dy dx \frac{\partial^2 w}{\partial x \partial y} dy - N_{xy} dy \frac{\partial w}{\partial y} + N_{xy} dy \frac{\partial w}{\partial y} + \\
& \quad N_{xy} dy \frac{\partial^2 w}{\partial x \partial y} dx + \frac{\partial N_{xy}}{\partial x} dx dy \frac{\partial w}{\partial y} + \frac{\partial N_{xy}}{\partial x} dx dy \frac{\partial^2 w}{\partial x \partial y} - q dx dy = 0
\end{aligned} \tag{3.2}$$

Simplifying and dividing by the area $dx dy$ yields:

$$\begin{aligned}
& \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + \frac{\partial N_x}{\partial x} \frac{\partial w}{\partial x} + \frac{\partial N_x}{\partial x} \frac{\partial^2 w}{\partial x^2} dx + N_y \frac{\partial^2 w}{\partial y^2} + \frac{\partial N_y}{\partial y} \frac{\partial w}{\partial y} \\
& \quad + \frac{\partial N_y}{\partial y} \frac{\partial^2 w}{\partial y^2} dy + N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial N_{xy}}{\partial y} \frac{\partial w}{\partial x} + \frac{\partial N_{xy}}{\partial y} \frac{\partial^2 w}{\partial x \partial y} dy \\
& \quad + N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial N_{xy}}{\partial x} \frac{\partial w}{\partial y} + \frac{\partial N_{xy}}{\partial x} \frac{\partial^2 w}{\partial x \partial y} dx - q = 0
\end{aligned} \tag{3.3}$$

By adding common derivative terms the equation yields,

$$\begin{aligned} \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial}{\partial x} \left(N_x \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left(N_y \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial x} \left(N_{xy} \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial y} \left(N_{xy} \frac{\partial w}{\partial x} \right) = \\ q - \frac{\partial N_x}{\partial x} \frac{\partial^2 w}{\partial x^2} dx - \frac{\partial N_y}{\partial y} \frac{\partial^2 w}{\partial y^2} dy - \frac{\partial N_{xy}}{\partial y} \frac{\partial^2 w}{\partial x \partial y} dy - \frac{\partial N_{xy}}{\partial x} \frac{\partial^2 w}{\partial x \partial y} dx \end{aligned} \quad (3.4)$$

This equation is applicable to problems of large deflections, but for small deflection problems the equation can be reduced to :

$$\frac{\partial}{\partial x} \left(N_x \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial x} \left(N_{xy} \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial y} \left(N_y \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial y} \left(N_{xy} \frac{\partial w}{\partial x} \right) + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = q \quad (3.5)$$

The force in the x direction is expressed by the formula shown below

$$\begin{aligned} \frac{\partial N_x}{\partial x} dx dy + \frac{\partial N_{xy}}{\partial y} dx dy + Q_x dy \frac{\partial w}{\partial x} - \left(Q_x + \frac{\partial Q_x}{\partial x} dx \right) dy \left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} dx \right) \\ Q_y dx \frac{\partial w}{\partial x} - \left(Q_y + \frac{\partial Q_y}{\partial y} dy \right) dx \left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x \partial y} dy \right) = 0 \end{aligned} \quad (3.6)$$

By expanding the above equation

$$\begin{aligned} \frac{\partial N_x}{\partial x} dx dy + \frac{\partial N_{xy}}{\partial y} dx dy + Q_x dy \frac{\partial w}{\partial x} - Q_x dy \frac{\partial w}{\partial x} - Q_x dy \frac{\partial^2 w}{\partial x^2} dx - \frac{\partial Q_x}{\partial x} dx dy \frac{\partial w}{\partial x} \\ - \frac{\partial Q_x}{\partial x} dx dy \frac{\partial^2 w}{\partial x^2} dx + Q_y dx \frac{\partial w}{\partial x} - Q_y dx \frac{\partial w}{\partial x} - Q_y dx \frac{\partial^2 w}{\partial x \partial y} dy - \frac{\partial Q_y}{\partial y} dx dy \frac{\partial w}{\partial x} \\ - \frac{\partial Q_y}{\partial y} dy dx \frac{\partial^2 w}{\partial x \partial y} dy = 0 \end{aligned} \quad (3.7)$$

Dividing by dx dy and simplifying yields:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{\partial}{\partial x} \left(Q_x \frac{\partial w}{\partial x} \right) - \frac{\partial}{\partial y} \left(Q_y \frac{\partial w}{\partial x} \right) = \frac{\partial Q_x}{\partial x} \frac{\partial^2 w}{\partial x^2} dx + \frac{\partial Q_y}{\partial y} \frac{\partial^2 w}{\partial x \partial y} dy \quad (3.8)$$

Again, this equation is applicable to the problem of large deflections, but for small deflection the equation changes to .

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{\partial}{\partial x} \left(Q_x \frac{\partial w}{\partial x} \right) - \frac{\partial}{\partial y} \left(Q_y \frac{\partial w}{\partial x} \right) = 0 \quad (3.9)$$

For the force in the y direction .

$$\begin{aligned} \frac{\partial N_y}{\partial y} dy dx + \frac{\partial N_{xy}}{\partial x} dx dy + Q_x dy \frac{\partial w}{\partial y} - \left(Q_x + \frac{\partial Q_x}{\partial x} dx \right) dy \left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial x \partial y} dx \right) \\ - Q_y dx \frac{\partial w}{\partial y} - \left(Q_y + \frac{\partial Q_y}{\partial y} dy \right) dx \left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} dy \right) = 0 \end{aligned} \quad (3.10)$$

After all simplifications the equation yields

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} - \frac{\partial}{\partial x} \left(Q_x \frac{\partial w}{\partial y} \right) - \frac{\partial}{\partial y} \left(Q_y \frac{\partial w}{\partial y} \right) = \frac{\partial Q_x}{\partial x} \frac{\partial^2 w}{\partial x \partial y} dx + \frac{\partial Q_y}{\partial y} \frac{\partial^2 w}{\partial y^2} dy \quad (3.11)$$

and the equation for small deflection becomes after simplification

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} - \frac{\partial}{\partial x} \left(Q_x \frac{\partial w}{\partial y} \right) - \frac{\partial}{\partial y} \left(Q_y \frac{\partial w}{\partial y} \right) = 0 \quad (3.12)$$

The general form in stress components yields three equation of equilibrium for the x,y,and z directions respectively

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \quad (3.13)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \quad (3.14)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \quad (3.15)$$

The inplane forces and bending moments are obtained by integrating the inplane stresses over the thickness. In the case of homogeneous plates, these

integrals are :

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz \quad (3.16)$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y dz \quad (3.17)$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} dz \quad (3.18)$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x z dz \quad (3.19)$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y z dz \quad (3.20)$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z dz \quad (3.21)$$

3.2 Stress and strain functions

From elasticity the general Hook's law stress functions taking care of the strain ϵ_x , and rotation deformations γ_{xz} , γ_{yz} are expressed by the following equations .

$$\sigma_x = \frac{E}{1-\nu^2} [\epsilon_x + \nu(\epsilon_y + \epsilon_z)] \quad (3.22)$$

$$\sigma_y = \frac{E}{1-\nu^2} [\epsilon_y + \nu(\epsilon_x + \epsilon_z)] \quad (3.23)$$

$$\sigma_z = \frac{E}{1-\nu^2} [\epsilon_z + \nu(\epsilon_x + \epsilon_y)] \quad (3.24)$$

$$\tau_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy} \quad (3.25)$$

$$\tau_{yz} = \frac{E}{2(1+\nu)} \gamma_{yz} \quad (3.26)$$

$$\tau_{zx} = \frac{E}{2(1+\nu)} \gamma_{zx} \quad (3.27)$$

The stress-strain relationships from the six previous equations are defined as :

$$\epsilon_x = \frac{1}{E} [\sigma_x - \nu (\sigma_y + \sigma_z)] \quad (3.28)$$

$$\epsilon_y = \frac{1}{E} [\sigma_y - \nu (\sigma_z + \sigma_x)] \quad (3.29)$$

$$\epsilon_z = \frac{1}{E} [\sigma_z - \nu (\sigma_x + \sigma_y)] \quad (3.30)$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E} \tau_{xy} \quad (3.31)$$

$$\gamma_{yz} = \frac{2(1+\nu)}{E} \tau_{yz} \quad (3.32)$$

$$\gamma_{zx} = \frac{2(1+\nu)}{E} \tau_{zx} \quad (3.33)$$

The strain relations for small deformations are ;

$$\epsilon_x = \frac{\partial u}{\partial x} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad (3.34)$$

$$\epsilon_y = \frac{\partial v}{\partial y} \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad (3.35)$$

$$\epsilon_z = \frac{\partial w}{\partial z} \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \quad (3.36)$$

with respect to the equilibrium equation .By differentiation ,

$$\frac{\partial \sigma_x}{\partial x} = \frac{E}{1-\nu^2} \left(\frac{\partial \epsilon_x}{\partial x} - \nu \left(\frac{\partial \epsilon_y}{\partial x} + \frac{\partial \epsilon_z}{\partial x} \right) \right) \quad (3.37)$$

$$\frac{\partial \tau_{xy}}{\partial y} = \frac{E}{2(1+\nu)} \frac{\partial \gamma_{xy}}{\partial y} \quad (3.38)$$

$$\frac{\partial \tau_{xz}}{\partial z} = \frac{E}{2(1+\nu)} \frac{\partial \gamma_{xz}}{\partial z} \quad (3.39)$$

$$\frac{\partial \tau_{yx}}{\partial x} = \frac{E}{2(1+\nu)} \frac{\partial \gamma_{yx}}{\partial x} \quad (3.40)$$

$$\frac{\partial \sigma_y}{\partial y} = \frac{E}{1-\nu^2} \left(\frac{\partial \epsilon_y}{\partial y} - \nu \left(\frac{\partial \epsilon_x}{\partial y} + \frac{\partial \epsilon_z}{\partial y} \right) \right) \quad (3.41)$$

$$\frac{\partial \tau_{yz}}{\partial z} = \frac{E}{2(1+\nu)} \frac{\partial \gamma_{yz}}{\partial z} \quad (3.42)$$

$$\frac{\partial \tau_{zx}}{\partial x} = \frac{E}{2(1+\nu)} \frac{\partial \gamma_{zx}}{\partial x} \quad (3.43)$$

$$\frac{\partial \tau_{zy}}{\partial y} = \frac{E}{2(1+\nu)} \frac{\partial \gamma_{zy}}{\partial y} \quad (3.44)$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{E}{1-\nu^2} \left(\frac{\partial \epsilon_z}{\partial z} - \nu \left(\frac{\partial \epsilon_x}{\partial z} + \frac{\partial \epsilon_y}{\partial z} \right) \right) \quad (3.45)$$

If we calculate the differential of the strains for small deformation the strains can be expressed as functions of the partial derivatives of u,v,and w :

$$\frac{\partial \epsilon_x}{\partial x} = \frac{\partial^2 u}{\partial x^2} \quad \frac{\partial \epsilon_y}{\partial x} = \frac{\partial^2 v}{\partial x \partial y} \quad \frac{\partial \epsilon_z}{\partial x} = \frac{\partial^2 w}{\partial x \partial z} \quad (3.46)$$

$$\frac{\partial \epsilon_x}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} \quad \frac{\partial \epsilon_y}{\partial y} = \frac{\partial^2 v}{\partial y^2} \quad \frac{\partial \epsilon_z}{\partial y} = \frac{\partial^2 w}{\partial y \partial z} \quad (3.47)$$

$$\frac{\partial \epsilon_x}{\partial z} = \frac{\partial^2 u}{\partial x \partial z} \quad \frac{\partial \epsilon_y}{\partial z} = \frac{\partial^2 v}{\partial y \partial z} \quad \frac{\partial \epsilon_z}{\partial z} = \frac{\partial^2 w}{\partial z^2} \quad (3.48)$$

$$\frac{\partial \gamma_{xy}}{\partial x} = \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \quad \frac{\partial \gamma_{xy}}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \quad (3.49)$$

$$\frac{\partial \gamma_{xz}}{\partial x} = \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial x^2} \quad \frac{\partial \gamma_{xz}}{\partial z} = \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \quad (3.50)$$

$$\frac{\partial \gamma_{yz}}{\partial y} = \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial y^2} \quad \frac{\partial \gamma_{yz}}{\partial z} = \frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 w}{\partial z \partial y} \quad (3.51)$$

The equilibrium equations in the displacement components become :

$$\frac{E}{1-\nu^2} \left[\frac{\partial^2 u}{\partial x^2} - \nu \left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial z \partial x} \right) \right] + \frac{E}{2(1+\nu)} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) + \frac{E}{2(1+\nu)} \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) = 0 \quad (3.52)$$

$$\frac{E}{2(1+\nu)} \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) + \frac{E}{1+\nu^2} \left[\frac{\partial^2 v}{\partial y^2} - \nu \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial z \partial y} \right) \right] + \frac{E}{2(1+\nu)} \left(\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 w}{\partial z \partial y} \right) = 0 \quad (3.53)$$

$$\frac{E}{2(1+\nu)} \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial x^2} \right) + \frac{E}{2(1+\nu)} \left(\frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial y^2} \right) + \frac{E}{1-\nu^2} \left[\frac{\partial^2 w}{\partial z^2} - \nu \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} \right) \right] = 0 \quad (3.54)$$

After multiplying the three equations by $2(1+\nu)$, and adding them together we get ,

$$\begin{aligned} & \frac{2}{1-\nu} \left\{ \frac{\partial^2 u}{\partial x^2} - \nu \frac{\partial^2 v}{\partial x \partial y} - \nu \frac{\partial^2 w}{\partial x \partial z} + \frac{\partial^2 v}{\partial y^2} - \nu \frac{\partial^2 u}{\partial x \partial y} - \nu \frac{\partial^2 w}{\partial y \partial z} \right\} + \\ & \frac{2}{1-\nu} \left\{ \frac{\partial^2 w}{\partial z^2} - \nu \frac{\partial^2 u}{\partial x \partial z} - \nu \frac{\partial^2 v}{\partial y \partial z} \right\} + \\ & \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial w^2} + \frac{\partial^2 w}{\partial z \partial y} + \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial x^2} + \\ & \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial y^2} = 0 \end{aligned} \quad (3.55)$$

By making simplification the general governing equation takes the following form :

$$\nabla^2 \{u, v, w\} + \frac{1}{1+2\nu} \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \quad (3.56)$$

3.3 Conditions of compatibility

Reverting now to the expressions of the strain components in terms of the components of displacement, eqs (3.34), (3.35), (3.36), we observe that the six strain components are expressed in terms of three displacement components. The equations may be regarded as a system of partial differential equations for the determination of the displacements u, v, w when the strain components $\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz},$ and γ_{zx} are expressed as functions of x, y, z . Since there are six equations for three unknown functions, we cannot expect in general that these equations will possess a solution if the strain components are arbitrarily prescribed. Thus, there must be some conditions to be imposed on the strain components in order that these six equations will give a set of single-valued continuous solutions for the three displacement components. The fact that the strain components cannot be prescribed arbitrarily can be seen from the following geometrical considerations: Imagine that an elastic body is subdivided into a number of small cubic elements before deformation. Now, suppose that each element is subjected to an arbitrary deformation. After the deformation, these elements become parallelepipeds, and it may happen that it is impossible to arrange the parallelepipeds to form a continuous body, the strain components for each element must satisfy certain relations. The determination of the six components of strain at each point is completely satisfied by the three functions u, v, w defining the components of displacement. From the previous

relations, the following equations can be written as :

$$\frac{\partial^2 \epsilon_x}{\partial y^2} = \frac{\partial^3 u}{\partial x \partial y^2}; \quad \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^3 v}{\partial y \partial x^2}; \quad \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial x^2 \partial y} \quad (3.57)$$

then ;

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \quad (3.58)$$

$$\frac{\partial^2 \epsilon_x}{\partial z^2} = \frac{\partial^3 u}{\partial x \partial z^2}; \quad \frac{\partial^2 \epsilon_z}{\partial x^2} = \frac{\partial^3 w}{\partial x^2 \partial z}; \quad \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} = \frac{\partial^3 u}{\partial x \partial z^2} + \frac{\partial^3 w}{\partial x^2 \partial z} \quad (3.59)$$

then ;

$$\frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} \quad (3.60)$$

$$\frac{\partial^2 \epsilon_y}{\partial z^2} = \frac{\partial^3 v}{\partial y \partial z^2}; \quad \frac{\partial^2 \epsilon_z}{\partial y^2} = \frac{\partial^3 w}{\partial z \partial y^2}; \quad \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = \frac{\partial^3 v}{\partial y \partial z^2} + \frac{\partial^3 w}{\partial y^2 \partial z} \quad (3.61)$$

then ;

$$\frac{\partial^2 \epsilon_y}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} \quad (3.62)$$

from the derivatives

$$\frac{\partial^2 \epsilon_x}{\partial y \partial z} = \frac{\partial^3 u}{\partial x \partial y \partial z}; \quad \frac{\partial \gamma_{xz}}{\partial y} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 w}{\partial x \partial y} \quad (3.63)$$

$$\frac{\partial \gamma_{yz}}{\partial x} = \frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y}; \quad \frac{\partial \gamma_{xy}}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 v}{\partial x \partial z} \quad (3.64)$$

we find that ,

$$2 \frac{\partial^2 \epsilon_x}{\partial y \partial z} = \frac{\partial}{\partial x} \left(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right) \quad (3.65)$$

$$2 \frac{\partial^2 \epsilon_y}{\partial x \partial z} = \frac{\partial}{\partial y} \left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right) \quad (3.66)$$

$$2 \frac{\partial^2 \epsilon_z}{\partial y \partial x} = \frac{\partial}{\partial z} \left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right) \quad (3.67)$$

where ;

$$\frac{\partial^2 \epsilon_y}{\partial x \partial z} = \frac{\partial^3 v}{\partial x \partial y \partial z}; \quad \frac{\partial^2 \epsilon_z}{\partial x \partial y} = \frac{\partial^3 w}{\partial x \partial y \partial z} \quad (3.68)$$

for the last two equations (3.66), (3.67)

3.4 Formulation of elasticity problem

If we are interested in finding the stress components only in a body, we may reduce the system of equations to six equations with six unknown stress components. Since the displacement components are not to be found in this case, the compatibility equations must be satisfied to ensure the existence of single valued displacements. By using the notation

$$\theta = \sigma_x + \sigma_y + \sigma_z \quad (3.69)$$

the first three strain stress relations are

$$\epsilon_x = \frac{1}{E} ((1 + \nu) \sigma_x - \nu \theta) \quad (3.70)$$

$$\epsilon_y = \frac{1}{E} ((1 + \nu) \sigma_y - \nu \theta) \quad (3.71)$$

$$\epsilon_z = \frac{1}{E} ((1 + \nu) \sigma_z - \nu \theta) \quad (3.72)$$

Now if we take :

$$\frac{\partial^2 \epsilon_y}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} \quad (3.73)$$

and deriving the equation (3.71),(3.72),(3.32) with respect to the differential

equation above we find

$$(1 + \nu) \left(\frac{\partial^2 \sigma_y}{\partial z^2} + \frac{\partial^2 \sigma_z}{\partial y^2} \right) - \nu \left(\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial^2 \theta}{\partial y^2} \right) = 2(1 + \nu) \frac{\partial^2 \tau_{yz}}{\partial y \partial z} \quad (3.74)$$

Again, if we take

$$\frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} \quad (3.75)$$

and deriving the equations (3.70), (3.72), (3.33) with respect to the second differential equation we find

$$(1 + \nu) \left(\frac{\partial^2 \sigma_x}{\partial z^2} + \frac{\partial^2 \sigma_z}{\partial x^2} \right) - \nu \left(\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial^2 \theta}{\partial x^2} \right) = 2(1 + \nu) \frac{\partial^2 \tau_{xz}}{\partial x \partial z} \quad (3.76)$$

Similarly, using

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \quad (3.77)$$

and deriving the equation (3.70), (3.71), (3.31) with respect to the third differential equation we find :

$$(1 + \nu) \left(\frac{\partial^2 \sigma_x}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} \right) - \nu \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) = 2(1 + \nu) \frac{\partial^2 \tau_{xy}}{\partial x \partial y} \quad (3.78)$$

From the equilibrium system, we form three couple of equations; where each couple corresponds to a simplification of the above expressions respectively.

$$\frac{\partial \tau_{yz}}{\partial y} = -\frac{\partial \sigma_z}{\partial z} - \frac{\partial \tau_{xz}}{\partial x} - F_z \quad (3.79)$$

$$\frac{\partial \tau_{yz}}{\partial z} = -\frac{\partial \sigma_y}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - F_y \quad (3.80)$$

$$\frac{\partial \tau_{xz}}{\partial z} = -\frac{\partial \sigma_x}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} - F_x \quad (3.81)$$

$$\frac{\partial \tau_{xz}}{\partial x} = -\frac{\partial \sigma_z}{\partial z} - \frac{\partial \tau_{yz}}{\partial y} - F_z \quad (3.82)$$

$$\frac{\partial \tau_{xy}}{\partial y} = -\frac{\partial \sigma_x}{\partial x} - \frac{\partial \tau_{xz}}{\partial z} - F_x \quad (3.83)$$

$$\frac{\partial \tau_{xy}}{\partial x} = -\frac{\partial \sigma_y}{\partial y} - \frac{\partial \tau_{yz}}{\partial z} - F_y \quad (3.84)$$

Differentiating the first of these equations of each couple with respect to i and the second with respect to j ; and adding the equations of each differentiated couple together. i.e. ,

$$\frac{\partial \tau_{ij}}{\partial i \partial j}; \quad \frac{\partial \tau_{ji}}{\partial j \partial i} \quad (3.85)$$

and then

$$\frac{\partial \tau_{ij}}{\partial i \partial j} + \frac{\partial \tau_{ji}}{\partial j \partial i} = 2 \frac{\partial \tau_{ij}}{\partial i \partial j} \quad (3.86)$$

the first couple gives

$$2 \frac{\partial^2 \tau_{yz}}{\partial y \partial z} = -\frac{\partial^2 \sigma_x}{\partial z^2} - \frac{\partial^2 \sigma_y}{\partial y^2} - \frac{\partial}{\partial x} \left(\frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial y} \right) - \frac{\partial F_z}{\partial z} - \frac{\partial F_y}{\partial y} \quad (3.87)$$

using the 1st relation from equilibrium equations

$$\frac{\partial \sigma_x^2}{\partial x^2} + \frac{\partial F_x}{\partial x} = -\frac{\partial}{\partial x} \left(\frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial y} \right) \quad (3.88)$$

then

$$2 \frac{\partial^2 \tau_{yz}}{\partial y \partial z} = \frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2} - \frac{\partial^2 \sigma_z}{\partial z^2} + \frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} - \frac{\partial F_z}{\partial z} \quad (3.89)$$

the second couple gives

$$2 \frac{\partial^2 \tau_{xz}}{\partial x \partial z} = -\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_z}{\partial z^2} - \frac{\partial}{\partial y} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} \right) - \frac{\partial F_z}{\partial z} - \frac{\partial F_x}{\partial x} \quad (3.90)$$

using the 2nd relation from equilibrium equation

$$\frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial F_y}{\partial y} = -\frac{\partial}{\partial y} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} \right) \quad (3.91)$$

then

$$2 \frac{\partial^2 \tau_{xz}}{\partial x \partial z} = -\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2} - \frac{\partial^2 \sigma_z}{\partial z^2} - \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} - \frac{\partial F_z}{\partial z} \quad (3.92)$$

the third couple gives

$$2 \frac{\partial^2 \tau_{xy}}{\partial x \partial y} = -\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2} - \frac{\partial}{\partial z} \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \right) - \frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} \quad (3.93)$$

using the 3rd relation from equilibrium equation

$$\frac{\partial^2 \sigma_z}{\partial z^2} + \frac{\partial F_z}{\partial z} = -\frac{\partial}{\partial z} \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \right) \quad (3.94)$$

then

$$2 \frac{\partial^2 \tau_{xy}}{\partial x \partial y} = -\frac{\partial^2 \sigma_x}{\partial x^2} - \frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \sigma_z}{\partial z^2} - \frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \quad (3.95)$$

using the notation below to simplify the writing

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (3.96)$$

substituting equation (3.95) in equation (3.78) and using the symbol ∇^2

we find :

$$(1 + \nu) \left(\nabla^2 \theta - \nabla^2 \sigma_x - \frac{\partial^2 \theta}{\partial x^2} \right) - \nu \left(\nabla^2 \theta - \frac{\partial^2 \theta}{\partial x^2} \right) = (1 + \nu) \left(\frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} - \frac{\partial F_z}{\partial z} \right) \quad (3.97)$$

Two analogous equation can be obtained by substituting (3.89) in (3.74)

and (3.92) in (3.76)

$$(1 + \nu) \left(\nabla^2 \theta - \nabla^2 \sigma_y - \frac{\partial^2 \theta}{\partial y^2} \right) - \nu \left(\nabla^2 \theta - \frac{\partial^2 \theta}{\partial y^2} \right) = (1 + \nu) \left(\frac{\partial F_y}{\partial y} - \frac{\partial F_x}{\partial x} - \frac{\partial F_z}{\partial z} \right) \quad (3.98)$$

and

$$(1 + \nu) \left(\nabla^2 \theta - \nabla^2 \sigma_z - \frac{\partial^2 \theta}{\partial z^2} \right) - \nu \left(\nabla^2 \theta - \frac{\partial^2 \theta}{\partial z^2} \right) = (1 + \nu) \left(\frac{\partial F_z}{\partial z} - \frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y} \right) \quad (3.99)$$

adding together all the three equations (3.97), (3.98), (3.99) we find

$$(1 - \nu) \nabla^2 \theta = -(1 + \nu) \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) \quad (3.100)$$

substituting this expression for $\nabla^2 \theta$ in (3.97), (3.98) and (3.99) gives

$$\nabla^2 \sigma_x + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial x^2} = \frac{-\nu}{1 - \nu} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) - 2 \frac{\partial F_x}{\partial x} \quad (3.101)$$

$$\nabla^2 \sigma_y + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial y^2} = \frac{-\nu}{1 - \nu} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) - 2 \frac{\partial F_y}{\partial y} \quad (3.102)$$

$$\nabla^2 \sigma_z + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial z^2} = \frac{-\nu}{1 - \nu} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) - 2 \frac{\partial F_z}{\partial z} \quad (3.103)$$

In the same manner the remaining three conditions (3.74), (3.76), (3.78) can be transformed into equations of the following kinds,

$$\nabla^2 \tau_{yz} + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial y \partial z} = - \left(\frac{\partial F_z}{\partial y} + \frac{\partial F_y}{\partial z} \right) \quad (3.104)$$

$$\nabla^2 \tau_{xz} + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial x \partial z} = - \left(\frac{\partial F_z}{\partial x} + \frac{\partial F_x}{\partial z} \right) \quad (3.105)$$

$$\nabla^2 \tau_{xy} + \frac{1}{1 + \nu} \frac{\partial^2 \theta}{\partial x \partial y} = - \left(\frac{\partial F_x}{\partial y} + \frac{\partial F_y}{\partial x} \right) \quad (3.106)$$

If there are no body forces or if the body forces are constant, the (3.101), (3.102), (3.103) equations and (3.104), (3.105), (3.106) equations become

$$(1 + \nu) \nabla^2 \sigma_x + \frac{\partial^2 \theta}{\partial x^2} = 0 \quad (3.107)$$

$$(1 + \nu) \nabla^2 \sigma_y + \frac{\partial^2 \theta}{\partial y^2} = 0 \quad (3.108)$$

$$(1 + \nu) \nabla^2 \sigma_z + \frac{\partial^2 \theta}{\partial z^2} = 0 \quad (3.109)$$

$$(1 + \nu) \nabla^2 \tau_{yz} + \frac{\partial^2 \theta}{\partial y \partial z} = 0 \quad (3.110)$$

$$(1 + \nu) \nabla^2 \tau_{xz} + \frac{\partial^2 \theta}{\partial x \partial z} = 0 \quad (3.111)$$

$$(1 + \nu) \nabla^2 \tau_{yz} + \frac{\partial^2 \theta}{\partial x \partial y} = 0 \quad (3.112)$$

In addition to the equilibrium equation and the boundary conditions the stress components in an isotropic body must satisfy the six conditions of compatibility (3.58), (3.60), (3.62) and (3.65), (3.66), (3.67). Now the generalized Hook's of stress components (3.22), (3.23), (3.24), (3.25), (3.26), (3.27) expressed in terms of strain components become,

$$\sigma_x = \lambda e + 2G\epsilon_x \quad (3.113)$$

$$\sigma_y = \lambda e + 2G\epsilon_y \quad (3.114)$$

$$\sigma_z = \lambda e + 2G\epsilon_z \quad (3.115)$$

$$\tau_{xy} = G\gamma_{xy} \quad (3.116)$$

$$\tau_{yz} = G\gamma_{yz} \quad (3.117)$$

$$\tau_{zx} = G\gamma_{zx} \quad (3.118)$$

where, $e = \epsilon_x + \epsilon_y + \epsilon_z$, $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$, $G = \frac{E}{2(1+\nu)}$, Substituting the relation $\sigma_x, \tau_{xy}, \tau_{zx}$ into the first equation of equilibrium (3.13) we get

$$\lambda \frac{\partial e}{\partial x} + G \left(2 \frac{\partial \epsilon_x}{\partial x} + \frac{\partial \gamma_{xy}}{\partial y} + \frac{\partial \gamma_{zx}}{\partial z} \right) = 0 \quad (3.119)$$

and if we substitute for the strain component the expressions (3.46), (3.49), (3.50) we find that (3.119) can be written in the form

$$(\lambda + G) \frac{\partial e}{\partial x} + G \nabla^2 u = 0 \quad (3.120)$$

The other two equations can be transformed in a similar manner.

Thus the three equations of equilibrium, expressed in terms of displacements are

$$(\lambda + G) \frac{\partial e}{\partial x} + G \nabla^2 u = 0 \quad (3.121)$$

$$(\lambda + G) \frac{\partial e}{\partial y} + G \nabla^2 v = 0 \quad (3.122)$$

$$(\lambda + G) \frac{\partial e}{\partial z} + G \nabla^2 w = 0 \quad (3.123)$$

The governing differential equation can be obtained by adding the three equations of equilibrium together

$$(\lambda + G) \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} e + G \nabla^2 \{u, v, w\} = 0 \quad (3.124)$$

and by rearranging and simplifying :

$$\nabla^2 \{u, v, w\} + \frac{1}{1 - 2\nu} \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = 0 \quad (3.125)$$

Now, we can extend the equations of equilibrium in terms of displacement components to their explicit forms in terms of the derivatives in the case where no body forces exist. The first equation after the extension gives

$$(\lambda + G) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) + G \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = 0 \quad (3.126)$$

Similarly, the other two equations can be derived and the general system of equations is transformed to the following.

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + G \frac{\partial^2 u}{\partial y^2} + G \frac{\partial^2 u}{\partial z^2} + (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} + (\lambda + G) \frac{\partial^2 w}{\partial x \partial z} = 0 \quad (3.127)$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} + G \frac{\partial^2 v}{\partial x^2} (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + G \frac{\partial^2 v}{\partial x^2} + (\lambda + G) \frac{\partial^2 w}{\partial y \partial z} = 0 \quad (3.128)$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} + (\lambda + G) \frac{\partial^2 v}{\partial y \partial z} + G \frac{\partial^2 w}{\partial x^2} + G \frac{\partial^2 w}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} = 0 \quad (3.129)$$

and in matrix form the system of equation of equilibrium can be represented

$$\text{by : } \begin{bmatrix} UU & UV & UW \\ VU & VV & VW \\ WU & WV & WW \end{bmatrix}$$

The explanation of these notations will be shown at chapter [4]. if we remark the matrix notations we conclude that for thin plate theory, only one element is considered which is WW, where this element takes care of the vertical deflection w. The other terms are neglected by the fact that their quantities are very small and their contributions are insignificant, whereas in thick plate theory the nine elements are involved and their contributions are well considered in solving the problem. Then we can see the difficulty of the problem from this matrix interpretation and the difference between the used finite difference method and the modified finite difference method presented herein. It is obvious once thick plate theory is a complicate problem, the solution must be in the same range.

3.5 Boundary conditions

The differential equations of equilibrium which have been derived previously for stresses and displacements within the plate must also be such as to accommodate the conditions of equilibrium with respect to prescribed forces or displacements at the boundary. In thin plate theory the boundary conditions need only be satisfied at the longitudinal and transverse dimensions but for thick plates, an additional dimension must be taken into account. In other words, for thick plates there are three boundaries to satisfy. If we consider that the origin of the plate at mid-plate, then the two boundary surfaces should be at $z = \pm h/2$ with respect to the z axis upward positive. In this thesis we are considering the case of static deflection due to uniformly distributed loads acting perpendicular to the surface of the plate. For rectangular clamped and simply supported plates, the top and bottom surfaces boundaries conditions are,

$$\text{at : } z = \pm \frac{h}{2}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0, \quad \sigma_z = \pm \frac{q}{2} \quad (3.130)$$

then the system of equilibrium equations reduces itself to the system of plane stress where,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad (3.131)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \quad (3.132)$$

we know from the theory of thin plates that this system can be transformed to the well known differential equation governing the deflection of thin

plates :viz.,

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad (3.133)$$

Rewriting this using the Laplacian operator, we have,

$$\nabla^4 w = \frac{q}{D} \quad (3.134)$$

3.6 Clamped plate

For the clamped plate, the contour of the plate is fully fixed and all the displacements are equal to zero,

$$\text{at : } x = \pm \frac{a}{2}, \quad u = v = w = 0 \quad (3.135)$$

$$\text{at : } y = \pm \frac{b}{2}, \quad u = v = w = 0 \quad (3.136)$$

$$\text{at : } z = \pm \frac{h}{2}, \quad \sigma_z = \pm \frac{q}{2}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0 \quad (3.137)$$

3.7 Simply supported plate

In the case of simply supported plates the choice is more complex and the boundary conditions are satisfied by both displacements and stresses:

at

$$\text{at : } x = \pm \frac{a}{2}, \quad \sigma_x = 0, \quad \tau_{xy} = 0, \quad w = 0 \quad (3.138)$$

and the system of equilibrium in the general form is equal to

$$\frac{\partial \tau_{xz}}{\partial z} = 0 \quad (3.139)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \quad (3.140)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \quad (3.141)$$

but at the boundary surfaces

$$\frac{\partial \sigma_y}{\partial y} = 0 \quad (3.142)$$

which means that $\sigma_y = \text{constant}$,

$$\text{at: } y = \pm \frac{b}{2}, \quad \sigma_y = 0, \quad \tau_{xy} = 0, \quad w = 0 \quad (3.143)$$

and the system of equilibrium in the general form is equal to

$$\frac{\partial \sigma_x}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \quad (3.144)$$

$$\frac{\partial \tau_{yz}}{\partial z} = 0 \quad (3.145)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \quad (3.146)$$

but at the boundary surfaces

$$\frac{\partial \sigma_x}{\partial x} = 0 \quad (3.147)$$

which means $\sigma_x = \text{constant}$

For simply supported plates at the corners there would be reactions acting upward resulting from the sum of the twisting moment. At the corners the partial elements of the equilibrium system are equal to zero which means,

$$\text{at: } z = \pm \frac{h}{2} \quad (3.148)$$

all the stresses are equal to zero except $\sigma_z = \pm q/2$, at $-h/2 < z < h/2$
all the stresses are equal to zero except $\tau_{xz}, \tau_{yz}, \sigma_z$ are equal to constants.

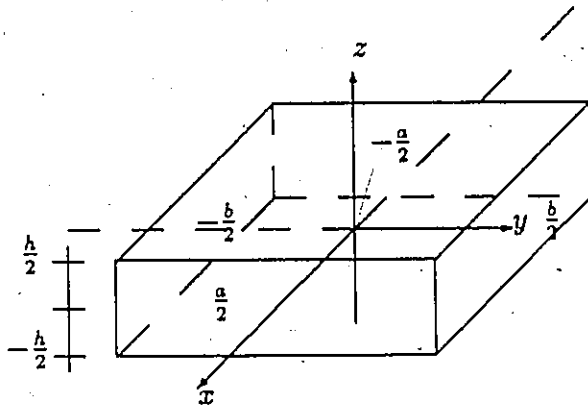


Figure (3.4) Position of the coordinate system.

Chapter 4

Formulation of the finite differences

4.1 Finite difference method

The derivation of the equations of equilibrium and deformation in the analysis of plates leads to a number of partial differential equations in which the unknowns are the internal forces and the deformations. The unknowns occur in differential form by considering the equilibrium and deformation of an infinitesimal plate element. This makes the problem to be solved much more complex than those which can be solved by simply considering elementary strength of materials where the expressions for the internal forces may be directly established by studying a finite cross-section. The differ-

ential relations have in their turn a well defined physical meaning. They permit the local study of forces or deformations. The complete determination of these phenomena involves the integration of the equations.

In most cases, the solutions of differential or partial differential equations cannot be obtained by means of elementary functions. This arises because in the most general case, when the radii of curvature of the middle surface, as well as the external loads, are not given explicitly, the form of the differential equation or partial differential equation is not known and the general solution cannot be obtained.

In general, solutions to the governing differential equations are difficult to obtain and numerical computations are to be recommended. The approximations in numerical calculation may be made as small as we wish by suitable choice of the initial scheme for the calculation. The basis of any method of numerical computation lies in not employing infinitely small quantities, but in using very small finite quantities. For this operation however, the form of the problem is modified.

In the first case, the analytical solution leads to a continuous expression for the unknowns at least within distinct intervals; in other words, having the expression for an unknown as a function of the independent variables, we can determine directly the value of this unknown at any point on the middle surface. On the other hand, the numerical calculation leads to the determination of the values of the unknowns only at the points of a previously established network. In Figure (4.1) we show how the network of modified finite difference is assembled with respect to the origin at the mid-plate. To make everything clear Figure (4.2) shows the three principal planes xy ,

yz , zx , which are the symmetrical planes of the plate. The same notation is used to simplify the interpretation of the partial derivatives defining the finite difference network within the plate.

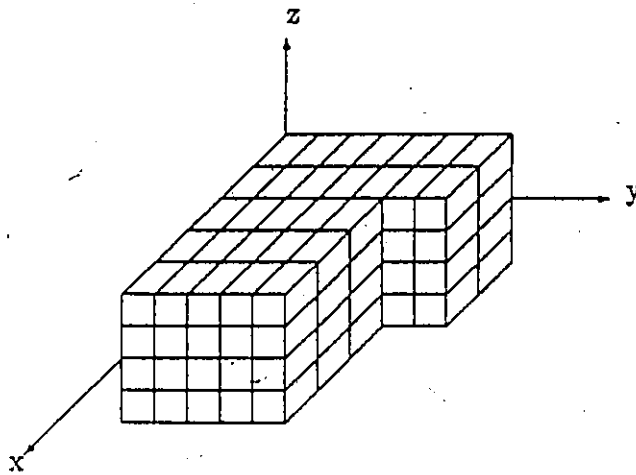


Figure (4.1) Three dimensional finite difference network

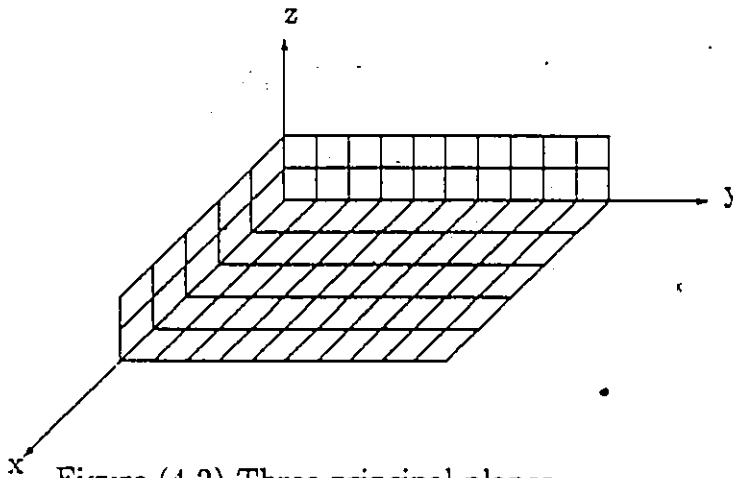
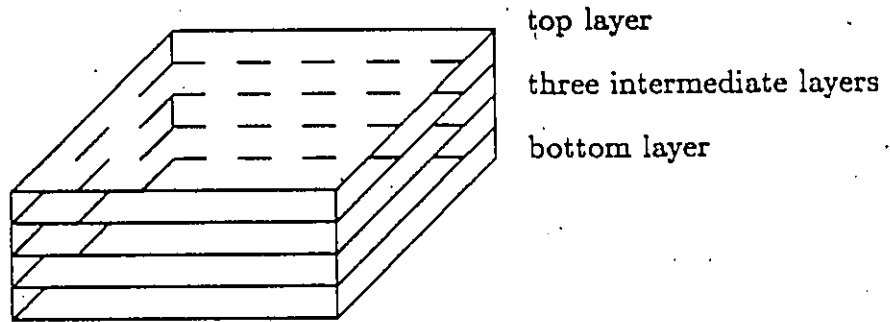


Figure (4.2) Three principal planes



Figure(4.3) Horizontal partition showing layers of the plate

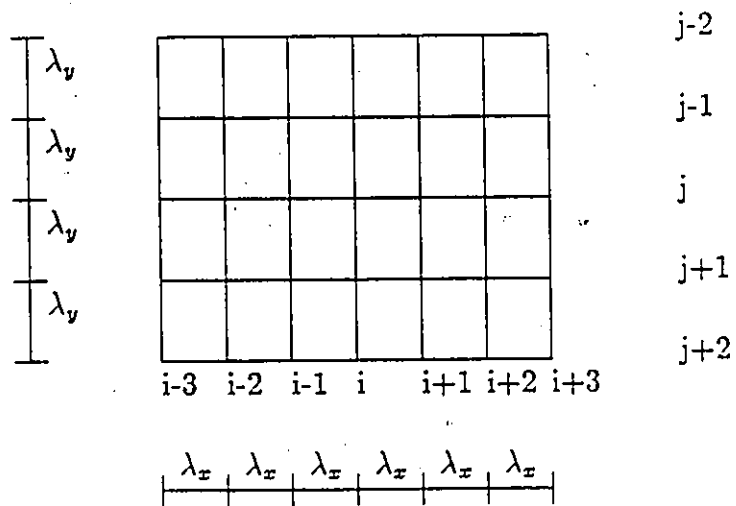


Figure (4.4) The finite difference mesh-size in the xy plane

While defining the system of notations u, v, w , for the partial derivatives in terms of finite difference equations, in the x, y and z directions respectively, the plate is also divided in layers as shown in Figure (4.3). The equations of equilibrium are divided into sub-matrices defining the participation of each function in one principal direction. Here,

UU : represents the participation of the u function in the x direction.

UV : represents the participation of the v function in the x direction.

UW : represents the participation of the w function in the x direction.

VU : represents the participation of the u function in the y direction.

VV : represents the participation of the v function in the y direction.

VW : represents the participation of the w function in the z direction.

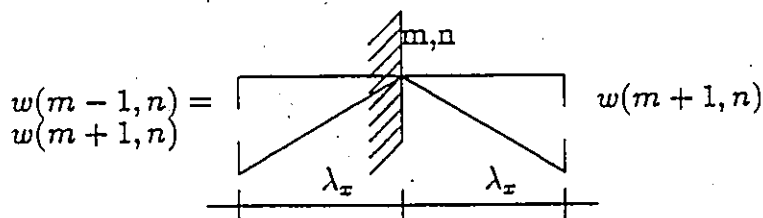
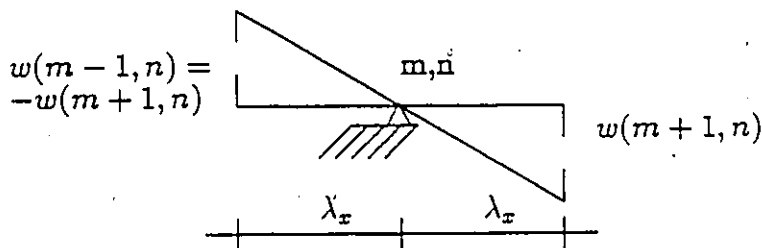
WU : represents the participation of the u function in the z direction.

WV : represents the participation of the v function in the z direction.

WW : represents the participation of the w function in the z direction.

For the notation of our grid network, only a quarter of the plate is considered due to the symmetry in the xy plane, while in the z direction the advantage of the symmetry is not considered for the general use of the program, in case we consider subsequently a plate with variable thickness. The increment in the notation is considered in a manner to simplify the notation. For example, in the x direction the increment is always unity, in the y direction the increment is equal to the term IJ and in the z direction we have in the plane xz the term IK , in the plane yz the term JK . These two terms IK, JK are equal in the case of the plate with constant thickness. As it was described in chapter (3) the top and bottom layers are determined by equation (3.133) governing thin plate theory. Then for this purpose it

is obvious that we need to show the representation of that governing equation in terms of finite difference for the general case where the mesh sizes is rectangular. The diagram in Figure (4.7) shows the development of this equation and it can be employed at the boundary. As it is known for the boundary conditions of clamped plates, points are added beyond the boundary limit similar to the point inside the plate affected with a plus sign and with a minus sign for the simply supported plate, see Figures (4.5) and (4.6).



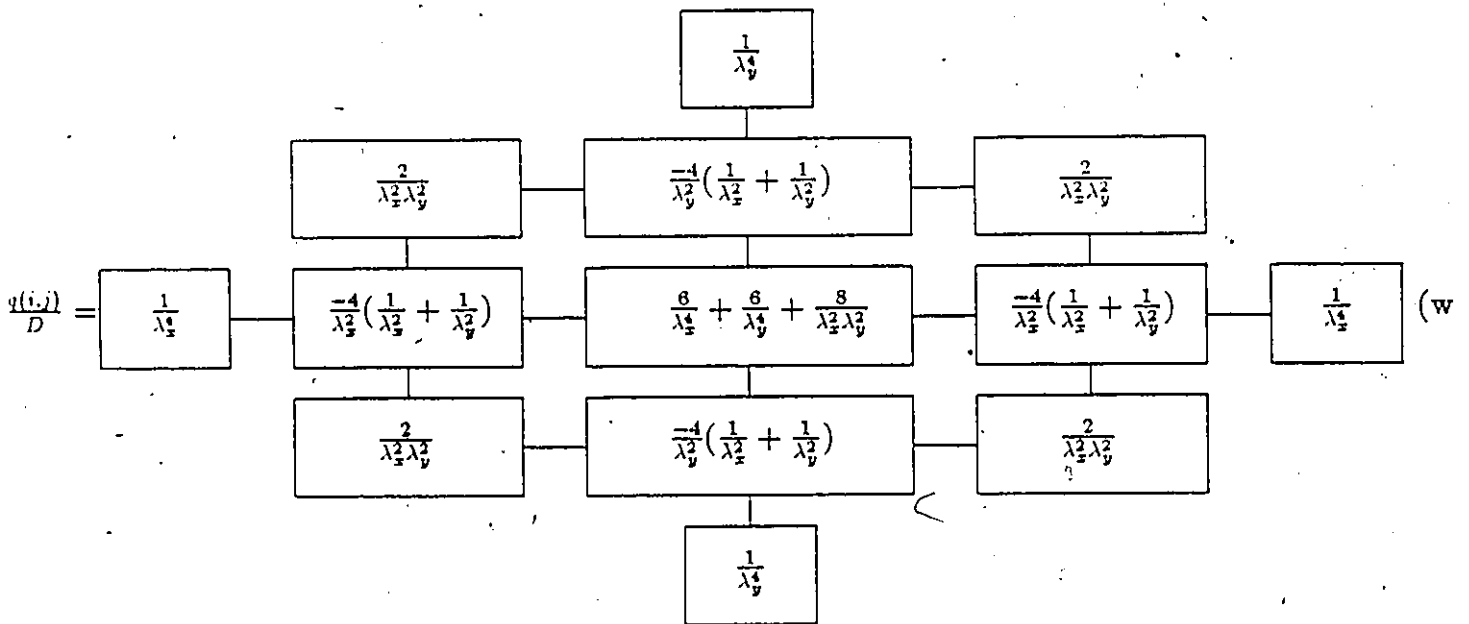
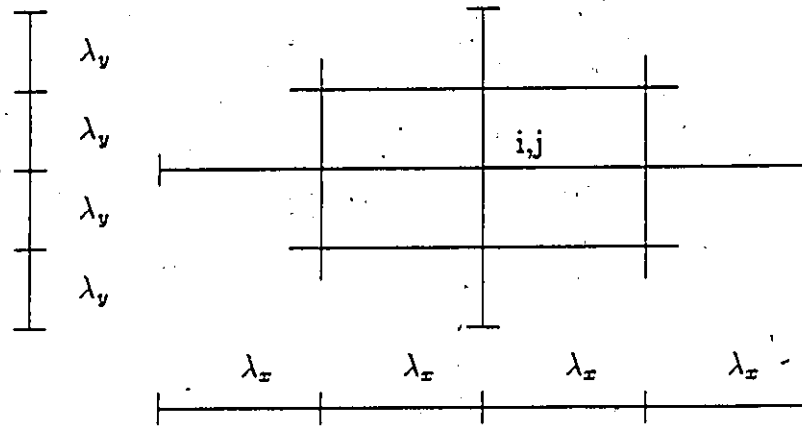


Figure (4.7) Finite difference scheme for thin plate

For the general case we represent the partial derivatives determining the equilibrium equations (3.127,3.128,3.129) by the following [101]

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [U(I-1) - 2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 u}{\partial y^2} &= [U(I-IJ) - 2U(I) + U(I+IJ)] \frac{G}{\lambda_y^2} \\
 G \frac{\partial^2 u}{\partial z^2} &= [U(I- IK) - 2U(I) + U(I+ IK)] \frac{G}{\lambda_z^2} \\
 (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 (\lambda + G) \frac{\partial^2 w}{\partial x \partial z} &= [W(I) - W(I+1) - W(I+ IK) + W(I+ IK+1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 G \frac{\partial^2 v}{\partial x^2} &= [U(I-1) - 2U(I) + U(I+1)] \frac{G}{\lambda_x^2} \\
 (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_y^2} \\
 G \frac{\partial^2 v}{\partial z^2} &= [V(I- IK) - 2V(I) + V(I+ IK)] \frac{G}{\lambda_z^2} \\
 (\lambda + G) \frac{\partial^2 w}{\partial y \partial z} &= [W(I) - W(I+IJ) - W(I+ JK) + W(I+IJ+ JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial z} &= [U(I) - U(I+1) - U(I+ IK) + U(I+ IK+1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
 (\lambda + G) \frac{\partial^2 v}{\partial y \partial z} &= [V(I) - V(I+IJ) - V(I+ JK) + V(I+IJ+ JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
 G \frac{\partial^2 w}{\partial x^2} &= [W(I-1) - 2W(I) + W(I+1)] \frac{G}{\lambda_x^2} \\
 G \frac{\partial^2 w}{\partial y^2} &= [W(I-IJ) - 2W(I) + W(I+IJ)] \frac{G}{\lambda_y^2} \\
 (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} &= [W(I- IK) - 2W(I) + W(I+ IK)] \frac{(\lambda + 2G)}{\lambda_z^2}
 \end{aligned}$$

For the points on the y axis [102]

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [2U(I-1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 u}{\partial y^2} &= [U(I-IJ) - 2U(I) + U(I+IJ)] \frac{G}{\lambda_y^2} \\
 G \frac{\partial^2 u}{\partial z^2} &= [U(I- IK) - 2U(I) + U(I+ IK)] \frac{G}{\lambda_z^2} \\
 (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I-1) - V(I+IJ) + V(I+IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 (\lambda + G) \frac{\partial^2 w}{\partial x \partial z} &= [W(I) - W(I-1) - W(I+IK) + W(I+IK-1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I-1) - U(I+IJ) + U(I+IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 G \frac{\partial^2 v}{\partial x^2} &= [2V(I-1) - 2V(I)] \frac{G}{\lambda_x^2} \\
 (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_y^2} \\
 G \frac{\partial^2 v}{\partial z^2} &= [V(I- JK) - 2V(I) + V(I+ JK)] \frac{G}{\lambda_z^2} \\
 (\lambda + G) \frac{\partial^2 w}{\partial y \partial z} &= [W(I) - W(I+IJ) - W(I+ JK) + W(I+IJ+ JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial z} &= [U(I) - U(I-1) - U(I+IK) + U(I+IK-1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
 (\lambda + G) \frac{\partial^2 v}{\partial y \partial z} &= [V(I) - V(I+IJ) - V(I+ JK) + V(I+IJ+ JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
 G \frac{\partial^2 w}{\partial x^2} &= [2W(I-1) - 2W(I)] \frac{G}{\lambda_x^2} \\
 G \frac{\partial^2 w}{\partial y^2} &= [W(I-IJ) - 2W(I) + W(I+IJ)] \frac{G}{\lambda_y^2} \\
 (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} &= [W(I- IK) - 2W(I) + W(I+ IK)] \frac{(\lambda + 2G)}{\lambda_z^2}
 \end{aligned}$$

For the points on the x axis [103]

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [U(I-1) - 2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 u}{\partial y^2} = [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_y^2}$$

$$G \frac{\partial^2 u}{\partial z^2} = [U(I-IK) - 2U(I) + U(I+IK)] \frac{G}{\lambda_z^2}$$

$$(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} = [V(I) - V(I+1) - V(I-IJ) + V(I-IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}$$

$$(\lambda + G) \frac{\partial^2 w}{\partial x \partial z} = [W(I) - W(I+1) - W(I+IK) + W(I+IK+1)] \frac{(\lambda + G)}{\lambda_x \lambda_z}$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I+1) - U(I-IJ) + U(I-IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}$$

$$G \frac{\partial^2 v}{\partial x^2} = [V(I-1) - 2V(I) + V(I+1)] \frac{G}{\lambda_x^2}$$

$$(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = [2V(I-IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_y^2}$$

$$G \frac{\partial^2 v}{\partial z^2} = [V(I-JK) - 2V(I) + V(I+JK)] \frac{G}{\lambda_z^2}$$

$$(\lambda + G) \frac{\partial^2 w}{\partial y \partial z} = [W(I) - W(I-IJ) - U(I+JK) + U(I-IJ+JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z}$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} = [U(I) - U(I+1) - U(I+IK) + U(I+IK+1)] \frac{(\lambda + G)}{\lambda_x \lambda_z}$$

$$(\lambda + G) \frac{\partial^2 v}{\partial y \partial z} = [V(I) - V(I-IJ) - V(I+IK) + V(I-IJ+JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z}$$

$$G \frac{\partial^2 w}{\partial x^2} = [W(I-1) - 2W(I) + W(I+1)] \frac{G}{\lambda_x^2}$$

$$G \frac{\partial^2 w}{\partial y^2} = [2W(I-IJ) - 2W(I)] \frac{G}{\lambda_y^2}$$

$$(\lambda + 2G) \frac{\partial^2 w}{\partial z^2} = [W(I-IK) - 2W(I) + W(I+IK)] \frac{(\lambda + 2G)}{\lambda_z^2}$$

For the central point [104]

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [2U(I-1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 u}{\partial y^2} = [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_y^2}$$

$$G \frac{\partial^2 u}{\partial z^2} = [U(I-IK) - 2U(I) + U(I+IK)] \frac{G}{\lambda_z^2}$$

$$(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} = [V(I) - V(I-1) - V(I-IJ) + V(I-IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}$$

$$(\lambda + G) \frac{\partial^2 w}{\partial x \partial z} = [W(I) - W(I-1) - W(I+IK) + W(I+IK-1)] \frac{(\lambda + G)}{\lambda_x \lambda_z}$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I-1) - U(I-IJ) + U(I-IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}$$

$$G \frac{\partial^2 v}{\partial x^2} = [2V(I-1) - 2V(I)] \frac{G}{\lambda_x^2}$$

$$(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = [2V(I-IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_y^2}$$

$$G \frac{\partial^2 v}{\partial z^2} = [V(I-IK) - 2V(I) + V(I+IK)] \frac{G}{\lambda_z^2}$$

$$(\lambda + G) \frac{\partial^2 w}{\partial y \partial z} = [W(I) - W(I-IJ) - U(I+JK) + U(I-IJ+JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z}$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} = [U(I) - U(I-1) - U(I+IK) + U(I+IK-1)] \frac{(\lambda + G)}{\lambda_x \lambda_z}$$

$$(\lambda + G) \frac{\partial^2 v}{\partial y \partial z} = [V(I) - V(I-IJ) - V(I+JK) + V(I-IJ+JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z}$$

$$G \frac{\partial^2 w}{\partial x^2} = [2W(I-1) - 2W(I)] \frac{G}{\lambda_x^2}$$

$$G \frac{\partial^2 w}{\partial y^2} = [2W(I-IJ) - 2W(I)] \frac{G}{\lambda_y^2}$$

$$(\lambda + 2G) \frac{\partial^2 w}{\partial z^2} = [W(I-IK) - 2W(I) + W(I+IK)] \frac{(\lambda + 2G)}{\lambda_z^2}$$

For the points on the top and bottom layers, the system of the equilibrium equations(3.131,3.132) expressed in terms of the partial derivatives yields :

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + G \frac{\partial^2 u}{\partial y^2} + (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} + \lambda \frac{\partial^2 w}{\partial x \partial z} = 0 \quad (4.1)$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} + G \frac{\partial^2 v}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + \lambda \frac{\partial^2 w}{\partial y \partial z} = 0 \quad (4.2)$$

$$(4.3)$$

The above system of equations with three unknowns is statically indeterminate. To proceed to solve the third function, we create another equation from the plane stress equilibrium system giving $\nabla^4 w = q/D$ which how gives a statically determinate system with $\frac{\partial^2 w}{\partial x \partial z}, \frac{\partial^2 w}{\partial y \partial z}, \frac{\partial^2 u}{\partial x \partial z}, \frac{\partial^2 v}{\partial y \partial z}$, all equal to zero, i.e.,

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + G \frac{\partial^2 u}{\partial y^2} + (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} = 0 \quad (4.4)$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} + G \frac{\partial^2 v}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = 0 \quad (4.5)$$

$$\nabla^4 w = \frac{q}{D} \quad (4.6)$$

A general point on the boundary layer will be defined by the expressions below [105] :

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [U(I - 1) - 2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 u}{\partial y^2} = [U(I - IJ) - 2U(I) + U(I + IJ)] \frac{G}{\lambda_y^2}$$

$$(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} = [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}$$

$$\begin{aligned}
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [V(I-1) - 2V(I) + V(I+1)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_y^2}
\end{aligned}$$

For the case when the point is on the y axis [106]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [2U(I-1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial y^2} &= [U(I-IJ) - 2U(I) + U(I+IJ)] \frac{G}{\lambda_y^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I-1) - V(I+IJ) + V(I+IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I-1) - U(I+IJ) + U(I+IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [2V(I-1) - 2V(I)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_y^2}
\end{aligned}$$

and when the point is on the x axis [107]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [U(I-1) - 2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial y^2} &= [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_y^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I+1) - V(I-IJ) + V(I-IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I+1) - U(I-IJ) + U(I-IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}
\end{aligned}$$

$$G \frac{\partial^2 v}{\partial x^2} = [V(I-1) - 2V(I) + V(I+1)] \frac{G}{\lambda_x^2}$$

$$(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = [2V(I-IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_y^2}$$

and for the central point [108]

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [2U(I-1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$G \frac{\partial^2 u}{\partial y^2} = [2U(I-IJ) - 2U(I)] \frac{G}{\lambda_y^2}$$

$$(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} = [V(I) - V(I-1) - V(I-IJ) + V(I-IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}$$

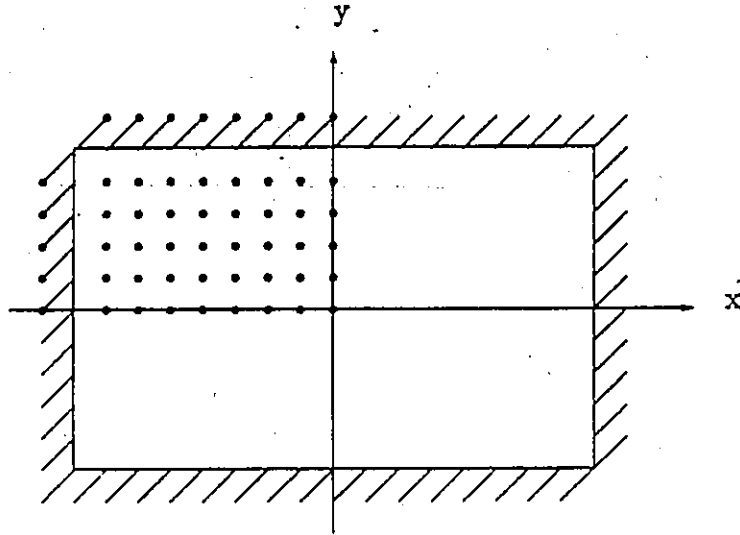
$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I-1) - U(I-IJ) + U(I-IJ-1)] \frac{(\lambda + G)}{\lambda_x \lambda_y}$$

$$G \frac{\partial^2 v}{\partial x^2} = [2V(I-1) - 2V(I)] \frac{G}{\lambda_x^2}$$

$$(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = [2V(I-IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_y^2}$$

Now after describing the four cases common for all plates we now attempt to determine the particular points near the boundary. The application is limited to two cases, clamped and simply supported.

4.2 Clamped plate



For the clamped plate all the components on the boundary are equal to zero, and therefore for points near the boundary, the backward components are equal to zero from the boundary conditions.

4.2.1 Intermediate surfaces

For the corner point, [100]

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [-2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 u}{\partial y^2} &= [-2U(I) + U(I+1)] \frac{G}{\lambda_y^2}
 \end{aligned}$$

$$\begin{aligned}
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 w}{\partial x \partial z} &= [W(I) - W(I + 1) - W(I + IK) + W(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [-2U(I) + U(I + 1)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [-2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - IK) - 2V(I) + V(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 w}{\partial y \partial z} &= [W(I) - W(I + IJ) - W(I + JK) + W(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} &= [U(I) - U(I + 1) - U(I + IK) + U(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 v}{\partial y \partial z} &= [V(I) - V(I + IJ) - V(I + JK) + V(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
G \frac{\partial^2 w}{\partial x^2} &= [-2W(I) + W(I + 1)] \frac{G}{\lambda_x^2} \\
G \frac{\partial^2 w}{\partial y^2} &= [-2W(I) + W(I + IJ)] \frac{G}{\lambda_y^2} \\
(\lambda + 2G) \frac{\partial^2 w}{\partial z^2} &= [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_z^2}
\end{aligned}$$

For points parallel to x axis, [99]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [U(I - 1) - 2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial y^2} &= [-2U(I) + U(I + IJ)] \frac{G}{\lambda_y^2}
\end{aligned}$$

$$\begin{aligned}
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 w}{\partial x \partial z} &= [W(I) - W(I + 1) - W(I + IK) + W(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [U(I - 1) - 2U(I) + U(I + 1)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [-2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - IK) - 2V(I) + V(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 w}{\partial y \partial z} &= [W(I) - W(I + IJ) - W(I + JK) + W(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} &= [U(I) - U(I + 1) - U(I + IK) + U(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 v}{\partial y \partial z} &= [V(I) - V(I + IJ) - V(I + JK) + V(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
G \frac{\partial^2 w}{\partial x^2} &= [W(I - 1) - 2W(I) + W(I + 1)] \frac{G}{\lambda_x^2} \\
G \frac{\partial^2 w}{\partial y^2} &= [-2W(I) + W(I + IJ)] \frac{G}{\lambda_y^2} \\
(\lambda + 2G) \frac{\partial^2 w}{\partial z^2} &= [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_z^2}
\end{aligned}$$

For points parallel to the x axis and on the y axis, [97]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [2U(I - 1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial y^2} &= [-2U(I) + U(I + IJ)] \frac{G}{\lambda_y^2}
\end{aligned}$$

$$\begin{aligned}
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I - 1) - V(I + IJ) + V(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 w}{\partial x \partial z} &= [W(I) - W(I - 1) - W(I + IK) + W(I + IK - 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I - 1) - U(I + IJ) + U(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [2V(I - 1) - 2V(I)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [-2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 w}{\partial y \partial z} &= [W(I) - W(I + IJ) - W(I + JK) + W(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} &= [U(I) - U(I - 1) - U(I + IK) + U(I + IK - 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 v}{\partial y \partial z} &= [V(I) - V(I + IJ) - V(I + JK) + V(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
G \frac{\partial^2 w}{\partial x^2} &= [2W(I - 1) - 2W(I)] \frac{G}{\lambda_x^2} \\
G \frac{\partial^2 w}{\partial y^2} &= [-2W(I) + W(I + IJ)] \frac{G}{\lambda_y^2} \\
(\lambda + 2G) \frac{\partial^2 w}{\partial z^2} &= [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_z^2}
\end{aligned}$$

For the points parallel to the y axis [98]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [-2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial y^2} &= [U(I - IJ) - 2U(I) + U(I + IJ)] \frac{G}{\lambda_y^2}
\end{aligned}$$

5.

$$\begin{aligned}
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 w}{\partial x \partial z} &= [W(I) - W(I + 1) - W(I + IK) + W(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [-2U(I) + U(I + 1)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [V(I - IJ) - 2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - IK) - 2V(I) + V(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 w}{\partial y \partial x} &= [W(I) - W(I + IJ) - W(I + JK) + W(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_x} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} &= [U(I) - U(I + 1) - U(I + IK) + U(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 v}{\partial y \partial z} &= [V(I) - V(I + IJ) - V(I + JK) + V(I + IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
G \frac{\partial^2 w}{\partial x^2} &= [-2W(I) + W(I + 1)] \frac{G}{\lambda_x^2} \\
G \frac{\partial^2 w}{\partial y^2} &= [W(I - IJ) - 2W(I) + W(I + IJ)] \frac{G}{\lambda_y^2} \\
(\lambda + 2G) \frac{\partial^2 w}{\partial z^2} &= [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_z^2}
\end{aligned}$$

For a point parallel to y axis and on the x axis, [96]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [-2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial y^2} &= [2U(I - IJ) - 2U(I)] \frac{G}{\lambda_y^2}
\end{aligned}$$

$$\begin{aligned}
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I - IJ) + V(I - IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 w}{\partial x \partial z} &= [W(I) - W(I + 1) - W(I + IK) + W(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I + 1) - U(I - IJ) + U(I - IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [-2V(I) + V(I + 1)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [2V(I - IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_z^2} \\
(\lambda + G) \frac{\partial^2 w}{\partial y \partial z} &= [W(I) - W(I - IJ) - U(I + JK) + U(I - IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} &= [U(I) - U(I + 1) - U(I + IK) + U(I + IK + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_z} \\
(\lambda + G) \frac{\partial^2 v}{\partial y \partial z} &= [V(I) - V(I - IJ) - V(I + IK) + V(I - IJ + JK)] \frac{(\lambda + G)}{\lambda_y \lambda_z} \\
G \frac{\partial^2 w}{\partial x^2} &= [-2W(I) + W(I + 1)] \frac{G}{\lambda_x^2} \\
G \frac{\partial^2 w}{\partial y^2} &= [2W(I - IJ) - 2W(I)] \frac{G}{\lambda_y^2} \\
(\lambda + 2G) \frac{\partial^2 w}{\partial z^2} &= [W(I - IK) - 2W(I) + W(I + IK)] \frac{(\lambda + 2G)}{\lambda_z^2}
\end{aligned}$$

4.2.2 Boundary surfaces

For the corner point, [109]

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [-2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 u}{\partial y^2} &= [-2U(I) + U(I+IJ)] \frac{G}{\lambda_y^2} \\
 (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 G \frac{\partial^2 v}{\partial x^2} &= [-2V(I) + V(I+1)] \frac{G}{\lambda_x^2} \\
 (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [-2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_y^2}
 \end{aligned}$$

For points parallel to the x axis, [110]

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [U(I-1) - 2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 u}{\partial y^2} &= [-2U(I) + U(I+IJ)] \frac{G}{\lambda_y^2} \\
 (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 G \frac{\partial^2 v}{\partial x^2} &= [V(I-1) - 2V(I) + V(I+1)] \frac{G}{\lambda_x^2} \\
 (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [-2V(I) + V(I+IJ)] \frac{(\lambda + 2G)}{\lambda_y^2}
 \end{aligned}$$

For points parallel to the x axis and on the y axis, [112]

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [2U(I-1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 u}{\partial y^2} &= [-2U(I) + U(I + IJ)] \frac{G}{\lambda_y^2} \\
 (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I-1) - V(I + IJ) + V(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I-1) - U(I + IJ) + U(I + IJ - 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 G \frac{\partial^2 v}{\partial x^2} &= [2V(I-1) - 2V(I)] \frac{G}{\lambda_x^2} \\
 (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [-2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_y^2}
 \end{aligned}$$

For points parallel to the y axis, [111]

$$\begin{aligned}
 (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [-2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
 G \frac{\partial^2 u}{\partial y^2} &= [U(I - IJ) - 2U(I) + U(I + IJ)] \frac{G}{\lambda_y^2} \\
 (\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 (\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
 G \frac{\partial^2 v}{\partial x^2} &= [-2V(I) + V(I + 1)] \frac{G}{\lambda_x^2} \\
 (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [V(I - IJ) - 2V(I) + V(I + IJ)] \frac{(\lambda + 2G)}{\lambda_y^2}
 \end{aligned}$$

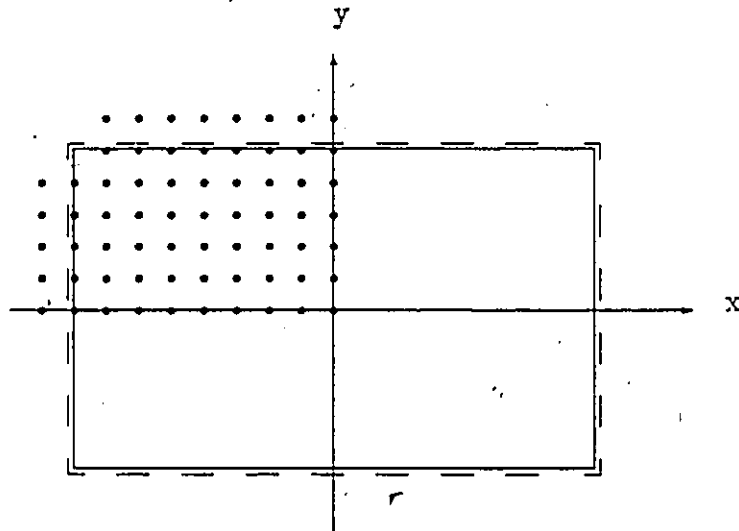
For the point parallel to the y axis and on the x axis, [113]

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [-2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$\begin{aligned}
G \frac{\partial^2 u}{\partial y^2} &= [2U(I - IJ) - 2U(I)] \frac{G}{\lambda_y^2} \\
(\lambda + G) \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I - IJ) + V(I - IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
(\lambda + G) \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I + 1) - U(I - IJ) + U(I - IJ + 1)] \frac{(\lambda + G)}{\lambda_x \lambda_y} \\
G \frac{\partial^2 v}{\partial x^2} &= [-2V(I) + V(I + 1)] \frac{G}{\lambda_x^2} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [2V(I - IJ) - 2V(I)] \frac{(\lambda + 2G)}{\lambda_y^2}
\end{aligned}$$

4.3 simply supported

For plates simply supported only the deflection w at the boundary is zero and, as a result, we have to analyse the layers differently from the boundary conditions shown before.



4.3.1 Intermediate surfaces

At the limit $x = \pm \frac{a}{2}$ the expression showing the equilibrium state in stress functions may be expressed in displacement functions by the following :

$$\lambda \frac{\partial^2 u}{\partial x \partial y} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + G \frac{\partial^2 v}{\partial z^2} + (\lambda + G) \frac{\partial^2 w}{\partial y \partial z} = 0 \quad (4.7)$$

$$\lambda \frac{\partial^2 u}{\partial x \partial z} + (\lambda + G) \frac{\partial^2 v}{\partial y \partial z} + G \frac{\partial^2 w}{\partial y^2} + (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} = 0 \quad (4.8)$$

Here, all the terms $\frac{\partial^2 w}{\partial y \partial z}$, $\frac{\partial^2 w}{\partial y^2}$, $\frac{\partial^2 w}{\partial z^2}$, will be zero at the boundary. Knowing all moments are equal to zero, then the two terms in equation (4.8) containing w can be omitted since their sum is zero,

$$\frac{\partial^2 u}{\partial x \partial z} + \frac{(\lambda + G)}{\lambda} \frac{\partial^2 v}{\partial y \partial z} = 0 \quad (4.9)$$

By changing the form of the equation we get,

$$\frac{\partial}{\partial z} (\epsilon_x + \nu \epsilon_y) = 0 \quad (4.10)$$

then,

$$\epsilon_x + \nu \epsilon_y = \text{Constant} \quad (4.11)$$

From the expression giving σ_x we have

$$\sigma_x = \frac{1}{E} [\epsilon_x + \nu (\epsilon_y + \epsilon_z)] \quad (4.12)$$

In our case we have $\sigma_x = 0$, and $w = 0$ which means that

$$\epsilon_x + \nu \epsilon_y = 0 \quad (4.13)$$

Thus the system of two equations ends with only one expression which is,

$$\lambda \frac{\partial^2 u}{\partial x \partial y} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + G \frac{\partial^2 v}{\partial z^2} = 0 \quad (4.14)$$

By the same analogy we give the corresponding expressions in terms of the displacement components at $y = \pm \frac{b}{2}$

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + G \frac{\partial^2 u}{\partial z^2} \lambda \frac{\partial^2 v}{\partial x \partial y} + (\lambda + G) \frac{\partial^2 w}{\partial x \partial z} = 0 \quad (4.15)$$

$$(\lambda + G) \frac{\partial^2 u}{\partial x \partial z} + \lambda \frac{\partial^2 v}{\partial y \partial z} + G \frac{\partial^2 w}{\partial x^2} + (\lambda + 2G) \frac{\partial^2 w}{\partial z^2} = 0 \quad (4.16)$$

in this case

$$\frac{\partial^2 v}{\partial y \partial z} + \frac{(\lambda + G)}{\lambda} \frac{\partial^2 u}{\partial x \partial z} = 0 \quad (4.17)$$

then after simplification to the condensed form we get,

$$\frac{\partial}{\partial z} (\epsilon_y + \nu \epsilon_x) = 0 \quad (4.18)$$

Thus,

$$\epsilon_y + \nu \epsilon_x = \text{Constant} \quad (4.19)$$

From the expression giving σ_y we have

$$\sigma_y = \frac{1}{E} [\epsilon_y + \nu (\epsilon_x + \epsilon_z)] \quad (4.20)$$

At the limit we have $\sigma_y = 0, w = 0$ which verifies that,

$$\epsilon_y + \nu \epsilon_x = 0 \quad (4.21)$$

At the end, the system reduces to one equation

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + G \frac{\partial^2 u}{\partial z^2} + \lambda \frac{\partial^2 v}{\partial x \partial y} = 0 \quad (4.22)$$

Point on the limit $x = \pm \frac{a}{2}, y \neq 0, [115]$

$$\lambda \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{\lambda}{\lambda_x \lambda_y}$$

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [V(I - IJ) - 2V(I) + V(I + IJ)] \frac{\lambda + 2G}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_z^2}
\end{aligned}$$

Point on the limit $x = \pm \frac{a}{2}, y = 0$, [116]

$$\begin{aligned}
\lambda \frac{\partial^2 u}{\partial x \partial y} &= [U(I) - U(I + 1) - U(I - IJ) + U(I - IJ + 1)] \frac{\lambda}{\lambda_x \lambda_y} \\
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [2V(I - IJ) - 2V(I)] \frac{\lambda + 2G}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_z^2}
\end{aligned}$$

Point on the limit $y = \pm \frac{b}{2}, x \neq 0$, [117]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [U(I - 1) - 2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
\lambda \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{\lambda}{\lambda_x \lambda_y}
\end{aligned}$$

Point on the limit $y = \pm \frac{b}{2}, x = 0$, [118]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [2U(I - 1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
\lambda \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I - 1) - V(I + IJ) + V(I + IJ - 1)] \frac{\lambda}{\lambda_x \lambda_y}
\end{aligned}$$

Points near the corner on $x = \pm \frac{a}{2}$, [125]

$$\lambda \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I + 1) - U(I + IJ) + U(I + IJ + 1)] \frac{\lambda}{\lambda_x \lambda_y}$$

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} &= [-2V(I) + V(I + IJ)] \frac{\lambda + 2G}{\lambda_y^2} \\
G \frac{\partial^2 v}{\partial z^2} &= [V(I - JK) - 2V(I) + V(I + JK)] \frac{G}{\lambda_z^2}
\end{aligned}$$

Points near the corner on $y = \pm \frac{b}{2}$, [114]

$$\begin{aligned}
(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} &= [-2U(I) + U(I + 1)] \frac{(\lambda + 2G)}{\lambda_x^2} \\
G \frac{\partial^2 u}{\partial z^2} &= [U(I - IK) - 2U(I) + U(I + IK)] \frac{G}{\lambda_z^2} \\
\lambda \frac{\partial^2 v}{\partial x \partial y} &= [V(I) - V(I + 1) - V(I + IJ) + V(I + IJ + 1)] \frac{\lambda}{\lambda_x \lambda_y}
\end{aligned}$$

4.3.2 Boundary surfaces

At the limit $x = \pm \frac{a}{2}$ the expression (3.142), expressed in partial derivative gives,

$$\frac{\partial \sigma_y}{\partial y} = \lambda \frac{\partial^2 u}{\partial x \partial y} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} + \lambda \frac{\partial^2 w}{\partial y \partial z} \quad (4.23)$$

knowing $w = 0$ then the expression reduces to

$$\frac{\partial \sigma_y}{\partial y} = \lambda \frac{\partial^2 u}{\partial x \partial y} + (\lambda + 2G) \frac{\partial^2 v}{\partial y^2} \quad (4.24)$$

For the same case on the limit $y = \pm \frac{b}{2}$ the expression (3.147) gives,

$$\frac{\partial \sigma_x}{\partial x} = (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + \lambda \frac{\partial^2 v}{\partial x \partial y} + \lambda \frac{\partial^2 w}{\partial x \partial z} \quad (4.25)$$

and dropping the null term we get

$$\frac{\partial \sigma_x}{\partial x} = (\lambda + 2G) \frac{\partial^2 u}{\partial x^2} + \lambda \frac{\partial^2 v}{\partial x \partial y} \quad (4.26)$$

Point on the limit $x = \pm \frac{a}{2}, y \neq 0$, [121]

$$\lambda \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{\lambda}{\lambda_x \lambda_y}$$

$$(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = [V(I-IJ) - 2V(I) + V(I+IJ)] \frac{\lambda + 2G}{\lambda_y^2}$$

Point on the limit $x = \pm \frac{a}{2}, y = 0$, [122]

$$\lambda \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I+1) - U(I-IJ) + U(I-IJ+1)] \frac{\lambda}{\lambda_x \lambda_y}$$

$$(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = [2V(I-IJ) - 2V(I)] \frac{\lambda + 2G}{\lambda_y^2}$$

Point on the limit $y = \pm \frac{b}{2}, x \neq 0$, [119]

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [U(I-1) - 2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$\lambda \frac{\partial^2 v}{\partial x \partial y} = [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{\lambda}{\lambda_x \lambda_y}$$

Point on the limit $y = \pm \frac{b}{2}, x = 0$, [120]

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [2U(I-1) - 2U(I)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$\lambda \frac{\partial^2 v}{\partial x \partial y} = [V(I) - V(I-1) - V(I+IJ) + V(I+IJ-1)] \frac{\lambda}{\lambda_x \lambda_y}$$

Points near the corner on $x = \pm \frac{a}{2}$, [124]

$$\lambda \frac{\partial^2 u}{\partial x \partial y} = [U(I) - U(I+1) - U(I+IJ) + U(I+IJ+1)] \frac{\lambda}{\lambda_x \lambda_y}$$

$$(\lambda + 2G) \frac{\partial^2 v}{\partial y^2} = [-2V(I) + V(I+IJ)] \frac{\lambda + 2G}{\lambda_y^2}$$

Points near the corner on $y = \pm \frac{b}{2}$, [123]

$$(\lambda + 2G) \frac{\partial^2 u}{\partial x^2} = [-2U(I) + U(I+1)] \frac{(\lambda + 2G)}{\lambda_x^2}$$

$$\lambda \frac{\partial^2 v}{\partial x \partial y} = [V(I) - V(I+1) - V(I+IJ) + V(I+IJ+1)] \frac{\lambda}{\lambda_x \lambda_y}$$

Chapter 5

Results and conclusions

5.1 Comparison and Discussion of results

5.1.1 general

The static formulation outlined have been applied to two types of plates. It is the intention of this thesis to give an exhaustive record of results. The results presented will give great confidence in the analysis method presented herein. A complete study of the effects of various parameters is investigated and extensive results are included.

Both clamped and simply supported cases have been analysed for the same plate aspect ratios and the same type of mesh sizes, except the number of nodes used has been larger for the analysis of the simply supported plates

in order to satisfy the boundary conditions at the edges. Four mesh sizes have been chosen, namely, the surface of the plate was divided into 4x4, 6x6, 8x8, and 10x10 mesh sizes. With the number of layers kept unchanged during the analysis, the number of degrees of freedom involved is three per node. For example, for 20 nodes included, the general matrix is 60x60 and for 125 nodes we have a matrix of 375x375 which means that the number of simultaneous equations that have to be solved is greatly increased. Table (1) shows the four mesh sizes and the corresponding number of equations required for the two types of plates. While the analysis using 125 nodes yields the best results, it has been found that analysis with only 20 nodes yields acceptable results for many of the cases analysed.

TABLE 1

mesh size	clamped plate		simply-supported plate	
	no. of nodes	no. of equations	no. of nodes	no. of equations
4x4	20	60	40	120
6x6	45	135	75	225
8x8	80	240	120	360
10x10	125	375	175	525

In general, the convergence for simply supported plates and clamped plates is slightly different. The accuracy is dependent on how the boundary conditions are satisfied. In the case of clamped plates, it was easy to satisfy the boundary conditions by taking the displacements equal to zero. But in the case of simply supported plates we have to deal with stress functions and transformations as described in chapter (4).

5.1.2 convergence and accuracy

To improve the convergence and the accuracy of the method four mesh sizes were investigated for each case. For different ratio of $\frac{h}{a}$, the results of the displacement w seem to be very satisfactory for square plates. Tables (2-7) show how the results change with variation of Poisson's ratio.

The error due to the finite difference method can generally be considered to result from a combination of discretisation errors and rounding off errors. The rounding off error is the error associated with the accuracy with which the numbers are manipulated in the computations. The discretisation error occurs irrespective of the accuracy of the numerical calculations, is a result of approximating a continuum which has an infinite number of degrees of freedom with a model having a finite number of degrees of freedom. The discretisation error may approach zero as a limit when the element size approaches to zero. In other words the approximate solution

TABLE 2

Clamped plate, for $\nu = 0.30$

Ratio w/q , at $X=Y=Z=0$					
a/b	h/a	number of nodes			
		20	45	80	125
1.0	0.05	20.9200	17.8300	16.5541	15.9200
	0.10	2.6010	2.2150	2.0555	1.9760
	0.14	0.9412	0.8010	0.7428	0.7139
	0.20	0.3182	0.2703	0.2505	0.2417
1.2	0.05	13.7400	11.7000	10.8649	10.4510
	0.10	1.7060	1.4520	1.3471	1.2950
	0.14	0.6165	0.5240	0.4860	0.4671
	0.20	0.2079	0.1763	0.1637	0.1572
1.5	0.05	7.1590	6.0720	-5.6410	5.4300
	0.10	0.8869	0.7514	0.6976	0.6713
	0.14	0.3197	0.2705	0.2509	0.2410
	0.20	0.1072	0.0904	0.0838	0.0748
1.8	0.05	3.8540	3.2490	3.0168	2.9060
	0.10	0.4762	0.4008	0.3719	0.3580
	0.14	0.1711	0.1438	0.1331	0.1286
	0.20	0.0571	0.0479	0.0442	0.0425
2.0	0.05	2.6280	2.2050	2.0458	1.9700
	0.10	0.3241	0.2714	0.2516	0.2427
	0.14	0.1162	0.0971	0.0900	0.0865
	0.20	0.0386	0.0323	0.0295	0.0285

TABLE 3

Clamped plate, for $\nu = 0.25$

Ratio w/q , at $X=Y=Z=0$					
a/b	h/a	number of nodes			
		20	45	80	125
1.0	0.05	21.5500	18.3700	17.0479	16.3918
	0.10	2.6760	2.2790	2.1144	2.0325
	0.14	0.9673	0.8229	0.7631	0.7333
	0.20	0.3263	0.2770	0.2709	0.2451
1.2	0.05	14.1500	12.0500	11.1881	10.7604
	0.10	1.7550	1.4930	1.3853	1.3320
	0.14	0.6332	0.5380	0.4989	0.3706
	0.20	0.2129	0.1804	0.1687	0.1244
1.5	0.05	7.3710	6.2520	5.8080	4.4930
	0.10	0.9118	0.7724	0.7170	0.6899
	0.14	0.3280	0.2774	0.2570	0.2471
	0.20	0.1096	0.0925	0.0858	0.0824
1.8	0.05	3.9680	3.3440	3.1055	2.9910
	0.10	0.4893	0.4117	0.3820	0.3673
	0.14	0.1754	0.1473	0.1365	0.1313
	0.20	0.0583	0.0875	0.0447	0.0429
2.0	0.05	2.7060	2.2690	2.1057	2.0280
	0.10	0.3329	0.2786	0.2592	0.2484
	0.14	0.1190	0.0995	0.0920	0.0918
	0.20	0.0395	0.0325	0.0299	0.0294

TABLE 4

Clamped plate, for $\nu = 0.18$

Ratio w/q , at $X=Y=Z=0$					
a/b	h/a	number of nodes			
		20	45	80	125
1.0	0.05	22.2300	18.9500	17.5874	16.9100
	0.10	2.7580	2.3480	2.1784	2.0940
	0.14	0.9954	0.8466	0.7850	0.7518
	0.20	0.3348	0.2843	0.2632	0.2525
1.2	0.05	14.6000	12.4300	11.5409	11.1000
	0.10	1.8080	1.5380	1.4267	1.3720
	0.14	0.6513	0.5531	0.5124	0.4922
	0.20	0.2182	0.1848	0.1713	0.1643
1.5	0.05	7.6030	6.4490	5.9901	5.7660
	0.10	0.9388	0.7950	0.7378	0.7097
	0.14	0.3370	0.2846	0.2642	0.2539
	0.20	0.1124	0.0937	0.0874	0.0837
1.8	0.05	4.0920	3.4490	3.2022	3.0840
	0.10	0.5034	0.4234	0.3923	0.3782
	0.14	0.1800	0.1512	0.1392	0.1340
	0.20	0.0590	0.0492	0.0458	0.0440
2.0	0.05	2.7900	2.3400	2.1709	2.0910
	0.10	0.3423	0.2860	0.2655	0.2543
	0.14	0.1221	0.1027	0.0922	0.0869
	0.20	0.0398	0.0332	0.0303	0.0293

TABLE 5

S.Support plate, for $\nu = 0.30$

Ratio w/q , at $X=Y=Z=0$					
a/b	h/a	number of nodes			
		40	75	120	175
1.0	0.05	46.8489	47.0804	47.1543	47.1861
	0.10	5.8290	5.8572	5.8662	5.8701
	0.14	2.1118	2.1218	2.1249	2.1263
	0.20	0.7155	0.7187	0.6790	0.7207
1.2	0.05	31.4175	31.5698	31.6191	31.6409
	0.10	3.9050	3.9235	3.9295	3.9321
	0.14	1.4130	1.4194	1.4215	1.3412
	0.20	0.4775	0.4795	0.4611	0.4803
1.5	0.05	17.5889	17.6658	17.6922	17.7042
	0.10	2.1822	2.1914	2.1945	2.1960
	0.14	0.7878	0.7909	0.7540	0.7925
	0.20	0.2650	0.2488	0.2661	0.2662
1.8	0.05	10.2358	10.2707	10.2840	10.2903
	0.10	1.2671	1.2712	1.2727	1.2484
	0.14	0.4561	0.4575	0.4579	0.4582
	0.20	0.1526	0.1529	0.1529	0.1529
2.0	0.05	7.3074	7.3256	7.3332	7.3340
	0.10	0.9030	0.9051	0.8701	0.9064
	0.14	0.3244	0.3163	0.3253	0.3254
	0.20	0.1081	0.1080	0.1081	0.1080

TABLE 6

S.Support plate, for $\nu = 0.25$

Ratio w/q ,at $X=Y=Z=0$					
a/b	h/a	number of nodes			
		40	75	120	175
1.0	0.05	48.2521	48.4903	48.5663	48.5998
	0.10	5.9989	6.0278	6.0370	6.0410
	0.14	2.1712	2.1813	2.1845	2.0679
	0.20	0.7341	0.7372	0.6504	0.7384
1.2	0.05	32.3566	32.5133	32.5640	32.5865
	0.10	4.0179	4.0368	4.0428	4.0456
	0.14	1.4521	1.4586	1.4607	1.4031
	0.20	0.4895	0.4914	0.4720	0.4922
1.5	0.05	18.1128	18.1918	18.2190	18.2313
	0.10	2.2443	2.2537	2.2570	2.1216
	0.14	0.8089	0.7257	0.8131	0.8137
	0.20	0.2712	0.2722	0.2657	0.2723
1.8	0.05	10.5393	10.5752	10.5888	10.5953
	0.10	1.3025	1.3066	1.2602	1.3088
	0.14	0.4679	0.4263	0.4699	0.4700
	0.20	0.1559	0.1560	0.1561	0.1562
2.0	0.05	7.5233	7.5420	7.5498	7.5537
	0.10	0.9279	0.9300	0.9059	0.9312
	0.14	0.3325	0.3331	0.3333	0.3335
	0.20	0.1102	0.1102	0.1101	0.1102

TABLE 7

S.Support plate, for $\nu = 0.18$

Ratio w/q , at $X=Y=Z=0$					
a/b	h/a	number of nodes			
		40	75	120	175
1.0	0.05	49.7859	50.0313	50.1096	50.1441
	0.10	6.1838	6.2135	6.2229	5.9646
	0.14	2.2355	2.2458	2.0903	2.2504
	0.20	0.7539	0.6568	0.7579	0.7583
1.2	0.05	33.3829	33.5442	33.5964	33.6196
	0.10	4.1407	4.1600	4.1663	4.1690
	0.14	1.4943	1.5009	1.5031	1.5039
	0.20	0.5022	0.4690	0.5046	0.5049
1.5	0.05	18.6849	18.7662	18.7942	18.8069
	0.10	2.3117	2.3213	2.3247	2.3258
	0.14	0.8316	0.8349	0.8358	0.8363
	0.20	0.2780	0.2577	0.2787	0.2788
1.8	0.05	10.8705	10.9074	10.9214	10.9281
	0.10	1.3408	1.3449	1.3468	1.3474
	0.14	0.4805	0.4815	0.4822	0.4823
	0.20	0.1588	0.1593	0.1595	0.1595
2.0	0.05	7.7589	7.7780	7.7861	7.7901
	0.10	0.9547	0.8977	0.9578	0.9581
	0.14	0.3414	0.3416	0.3419	0.3420
	0.20	0.1122	0.1123	0.1123	0.1124

is said to converge to the exact solution. To investigate the convergence and accuracy of the method, centre deflection of various thick plates were obtained and shown in Figures 1 to 6. It can be observed on these figures that for the ratio $\frac{a}{b} = 1$, the convergence of the solution in each case is quite satisfactory for various Poisson's ratios. For the clamped plate the convergence of the results for different number of nodes (i.e. different mesh sizes) is shown in Figures (1-3) and, in the case of simply supported plates, the convergence of the results due to different mesh sizes are shown in Figures(4-6). Figure (7-8) show the influence of the plate ratio $\frac{h}{a}$, for both clamped, and simply supported thick plates.

5.1.3 Comparison of results

In order to show that the finite difference approach is a good numerical scheme for the analysis of thick rectangular plates, a few examples of analysis of rectangular plates of uniform thickness were performed and results were compared with existing solutions to build up the confidence in using the finite difference approach used herein. Once the validity and accuracy of the finite difference approach was established, the technique was then extended to analyse both simply supported and clamped thick plates of different plate aspect ratios.

FIGURE 1, COMPARISON OF THE DEFLECTION
OF CLAMPED THICK PLATES
FOR FOUR MESH SIZES

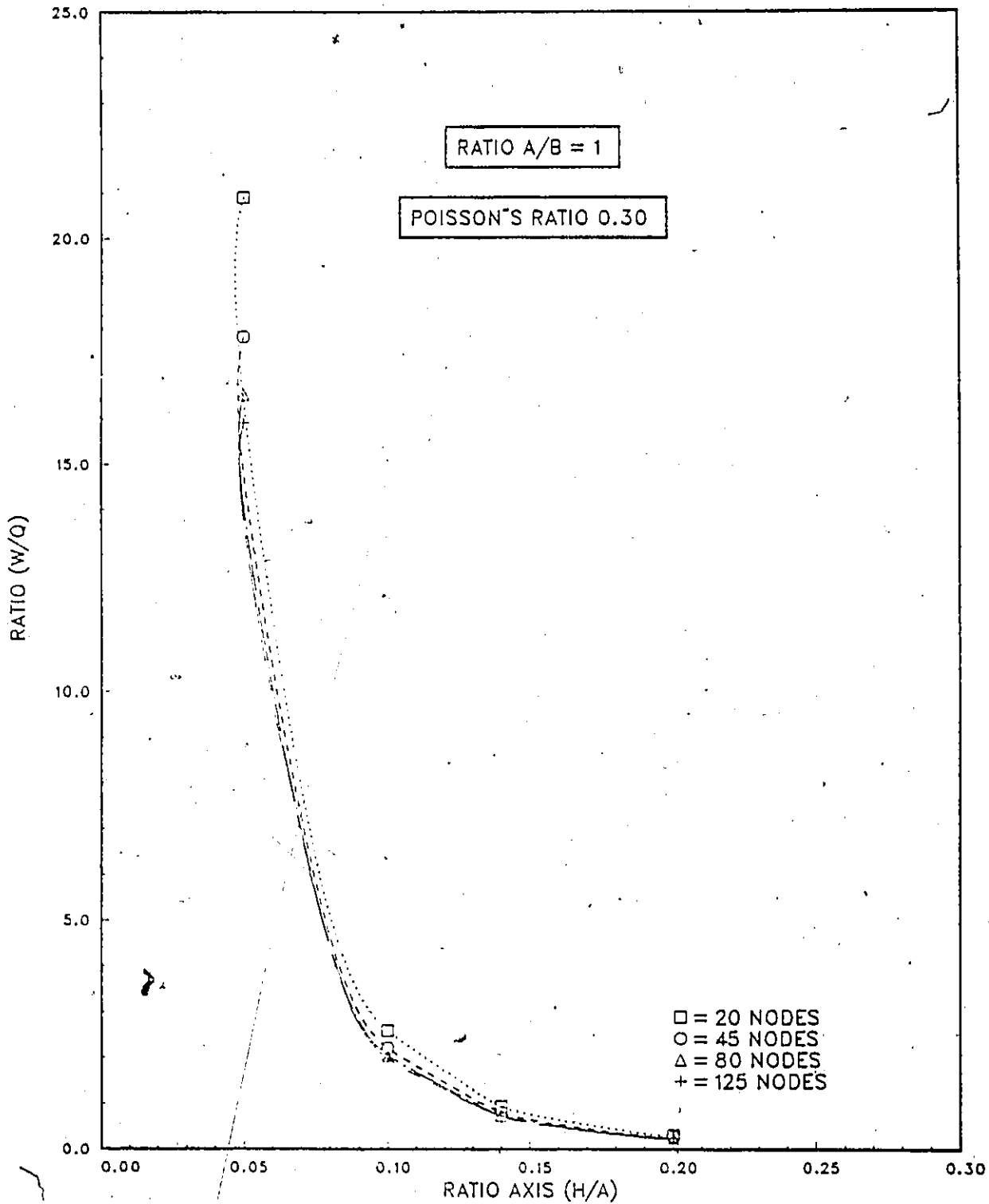


FIGURE 2, COMPARISON OF THE DEFLECTION
OF CLAMPED THICK PLATES
FOR FOUR MESH SIZES

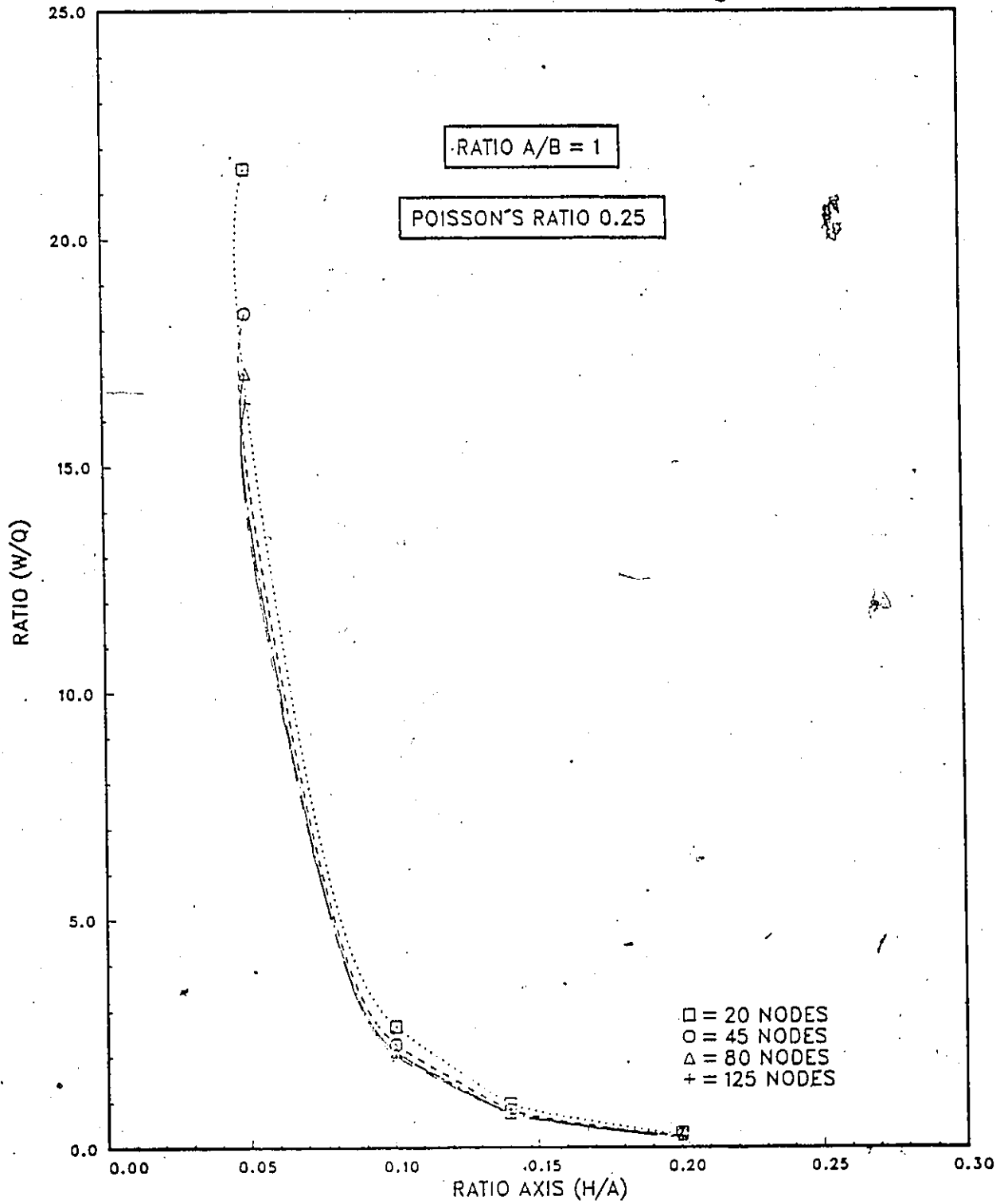


FIGURE 3, COMPARISON OF THE DEFLECTION
OF CLAMPED THICK PLATES
FOR FOUR MESH SIZES

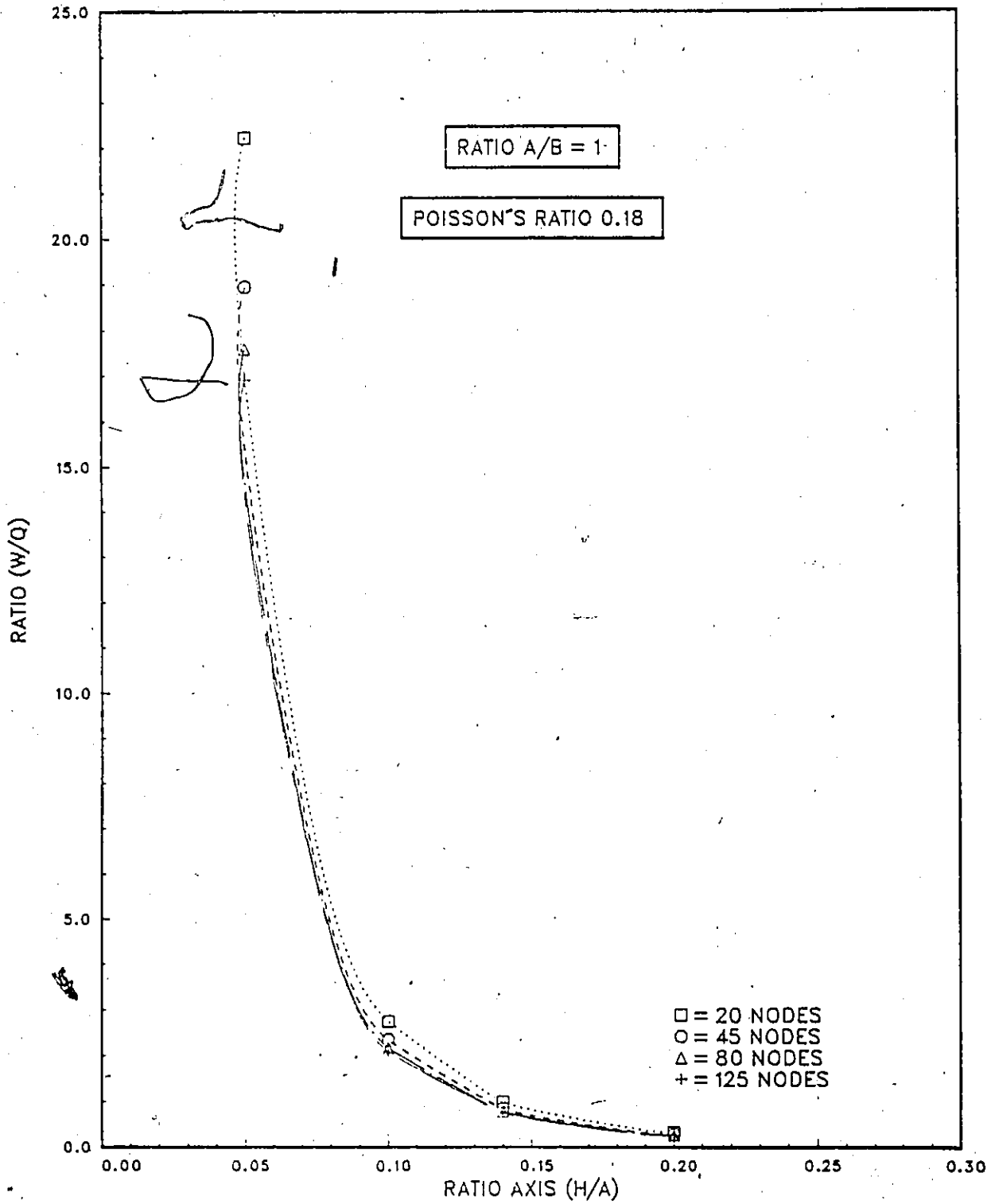


FIGURE 4, COMPARISON OF THE DEFLECTION
OF SIMPLY SUPPORTED THICK PLATES
FOR FOUR MESH SIZES

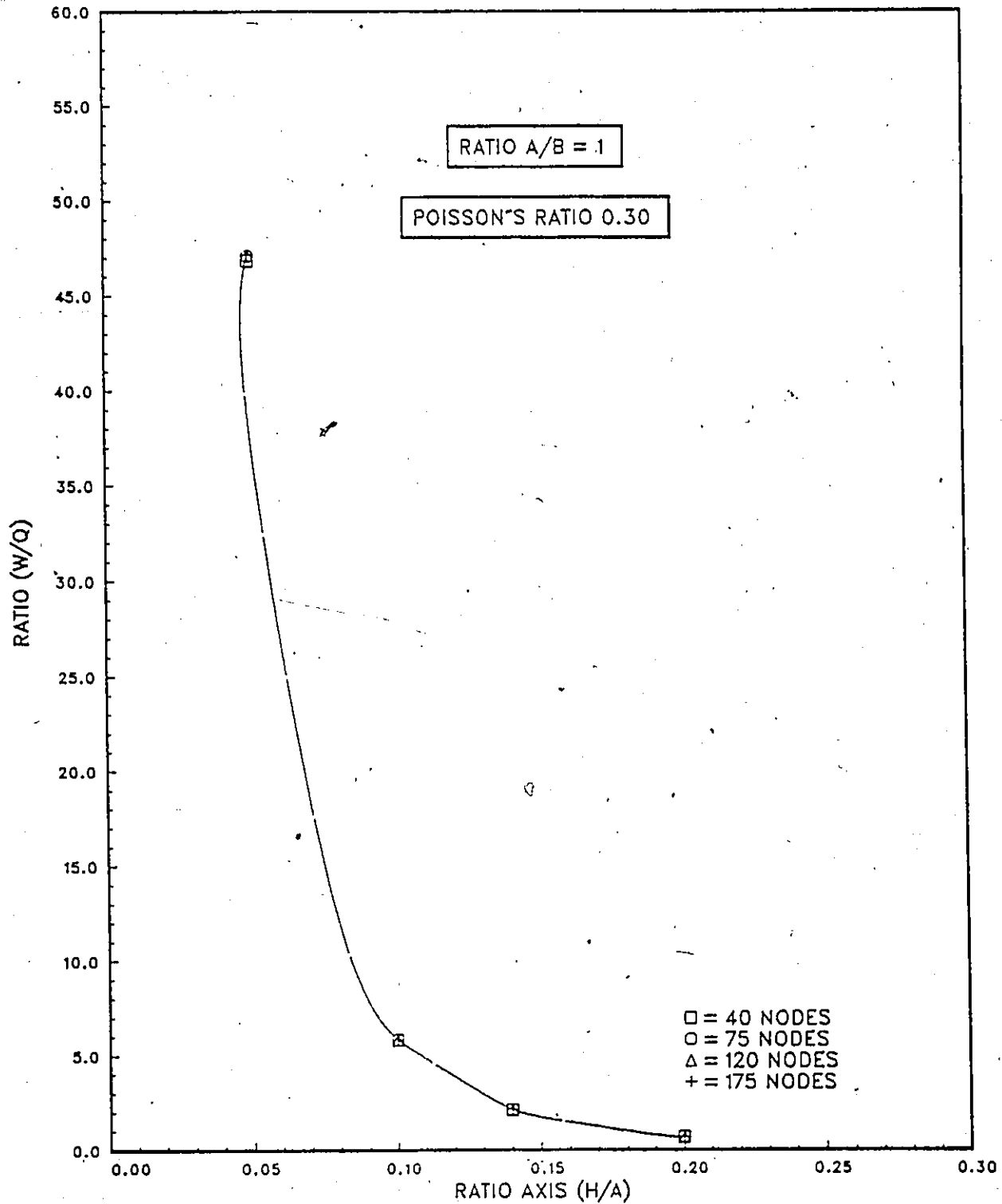


FIGURE 5, COMPARISON OF THE DEFLECTION
OF SIMPLY SUPPORTED THICK PLATES
FOR FOUR MESH SIZES

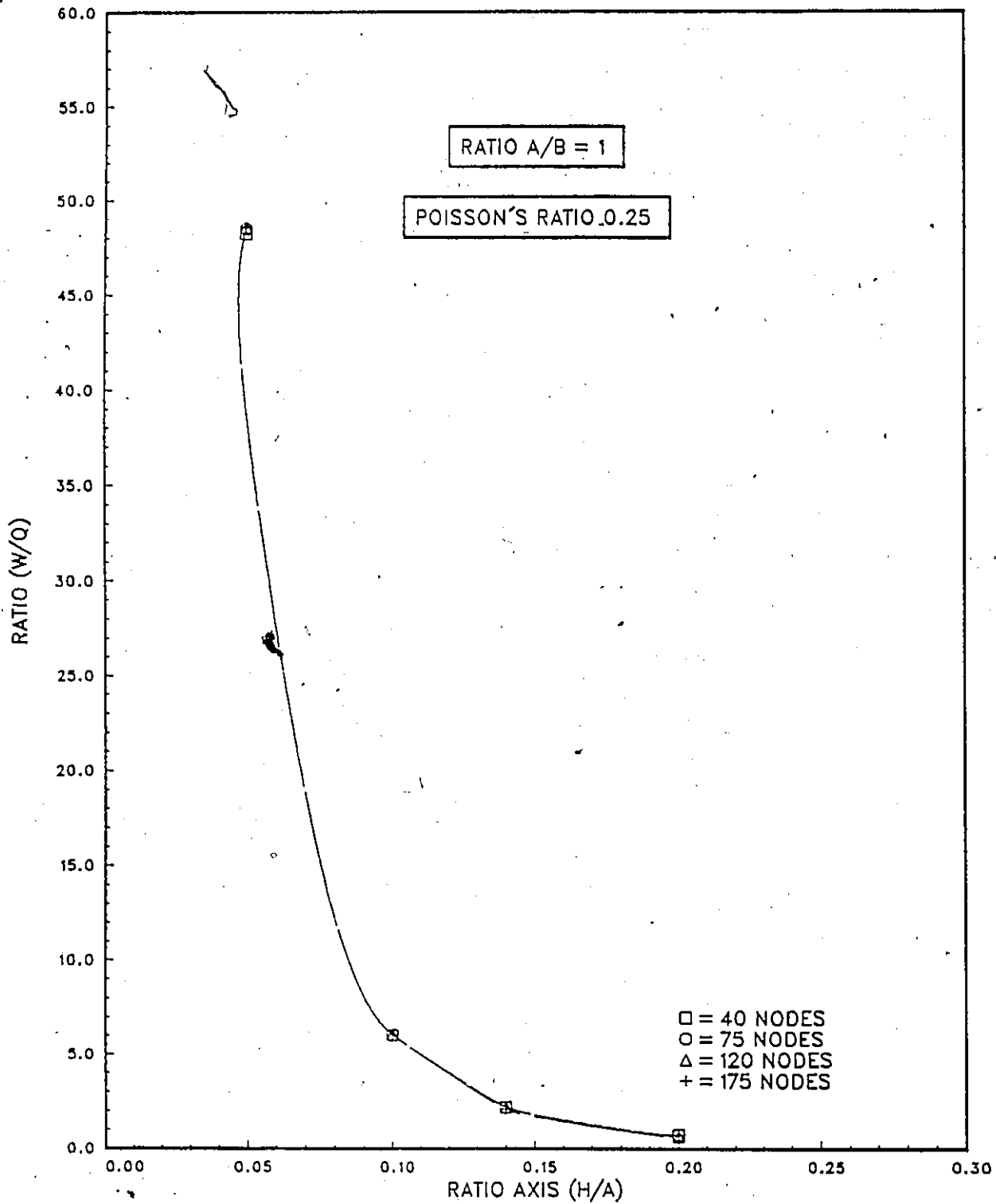


FIGURE 6, COMPARISON OF THE DEFLECTION
OF SIMPLY SUPPORTED THICK PLATES
FOR FOUR MESH SIZES

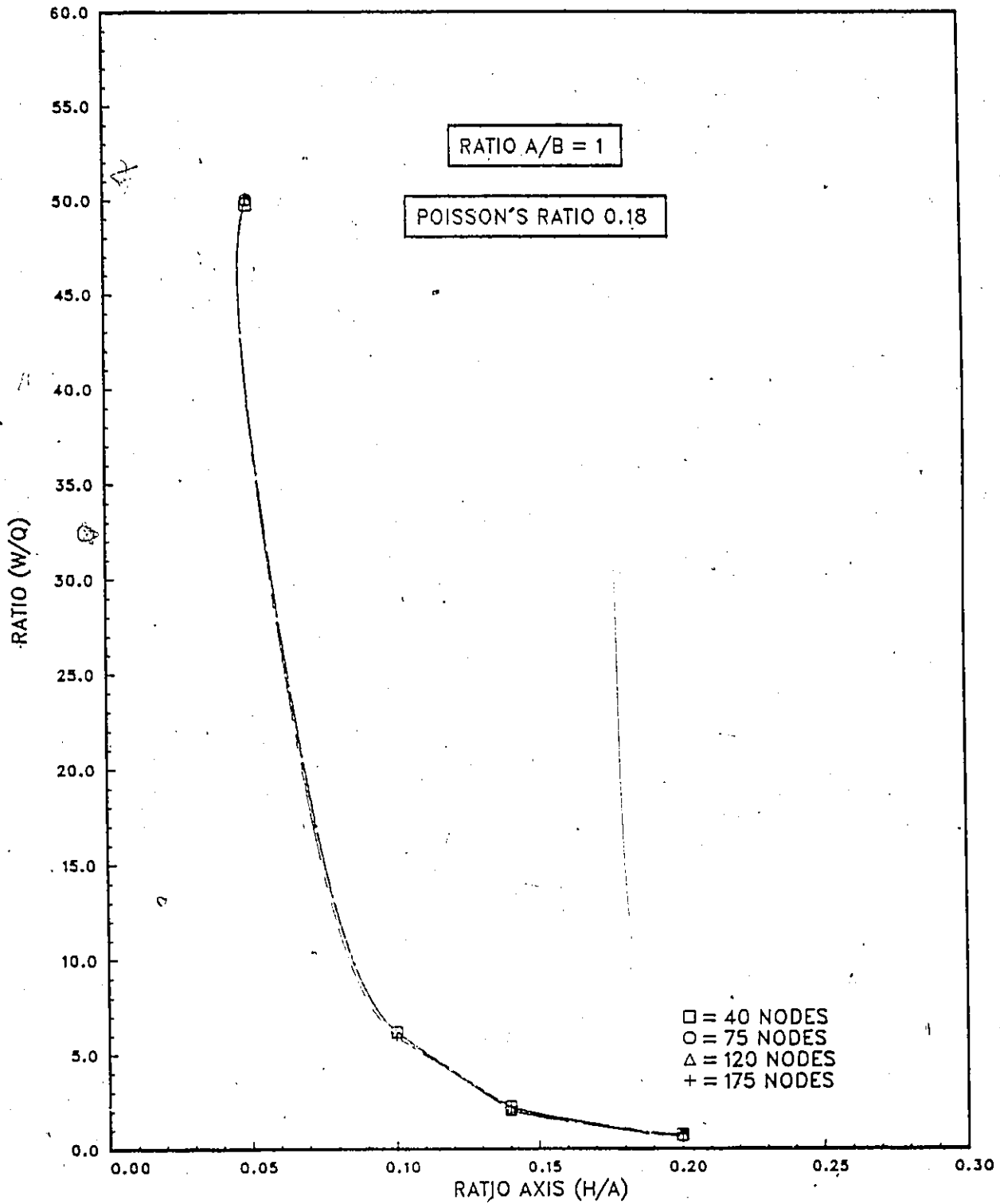


FIGURE 7, INFLUENCE OF THE PLATE ASPECT RATIO ON THE DEFLECTION OF CLAMPED THICK PLATES

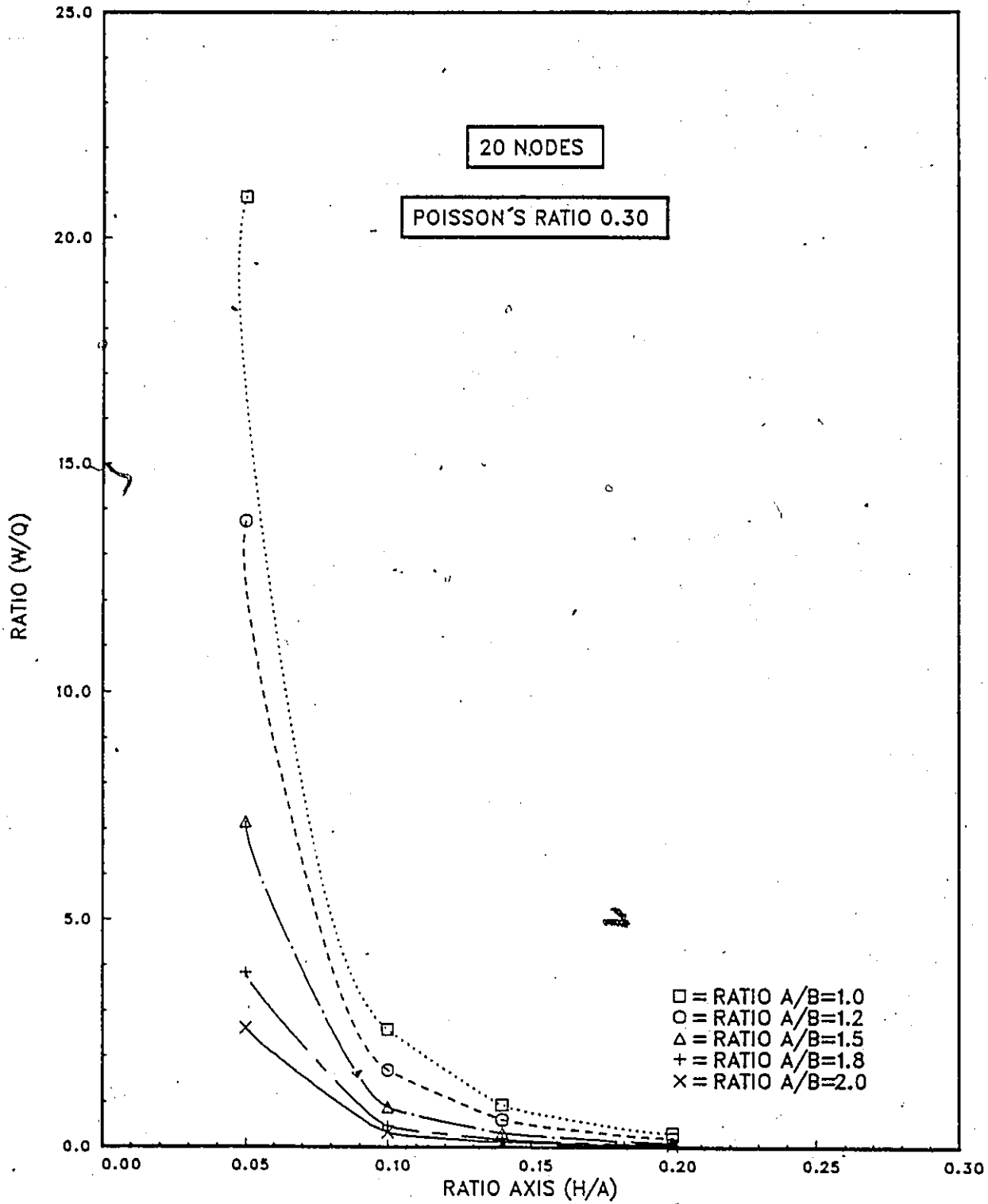


FIGURE 8, INFLUENCE OF THE PLATE ASPECT RATIO ON THE DEFLECTION OF SIMPLY SUPPORTED THICK PLATES

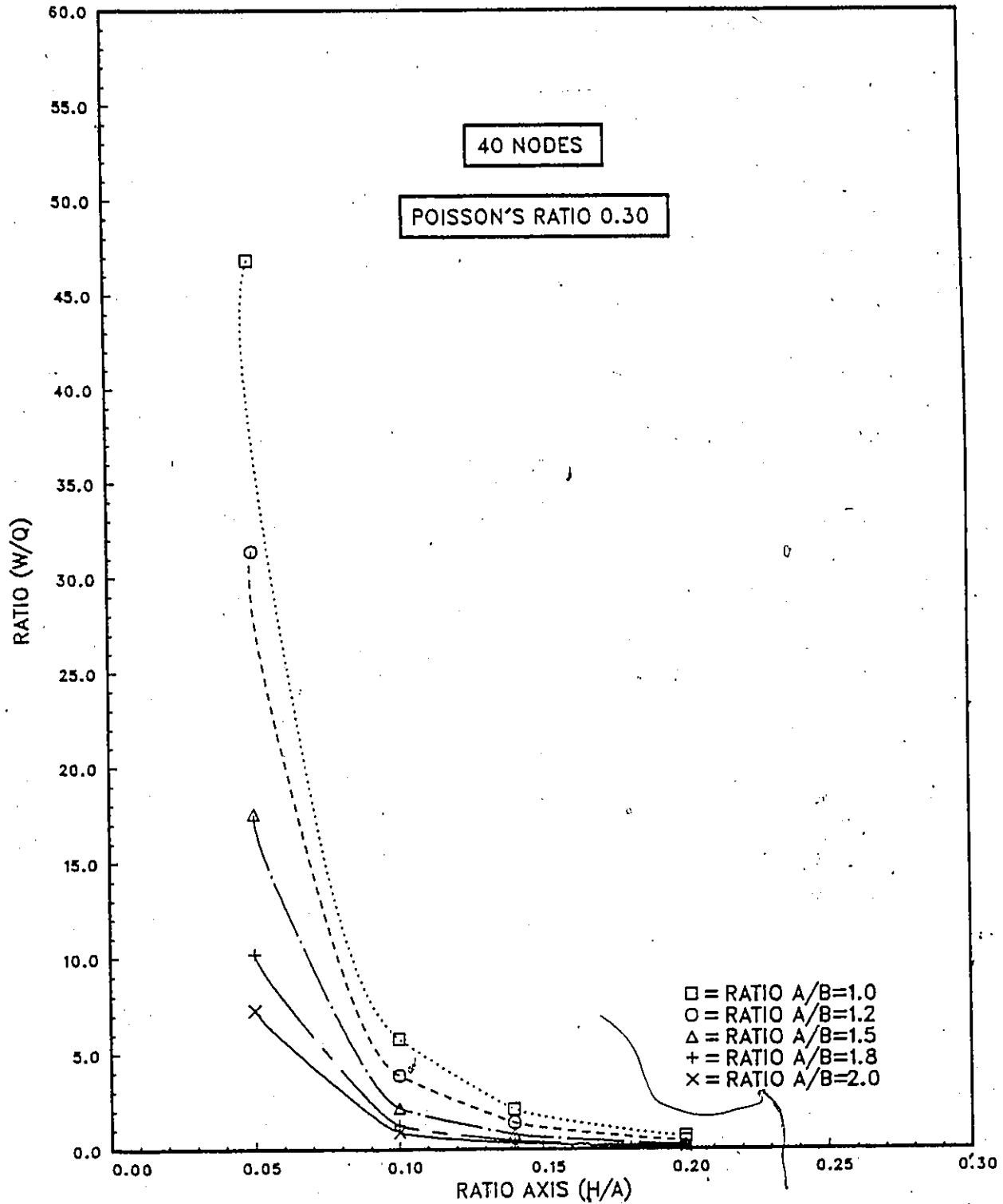


Table (8) shows results obtained by different methods for the case of clamped, square plates whereas Table (9) shows the corresponding results for simply supported thick square plates. For the purpose of demonstrating the convergence and accuracy of our results Figures (9-10) show excellent agreement between the finite difference method used herein and results obtained by other researchers.

5.2 Conclusion

A finite difference method for computing the displacements of thick rectangular plates has been presented. Equilibrium equations in terms of displacements have been derived. The numerical examples indicate that the method presented herein gives an excellent approximation for the displacements. Even with a crude mesh size of 4x4, the method yields results of acceptable accuracy for both clamped and simply supported moderately thick plates. In general, the agreement between the finite difference results and those obtained by other methods was well shown in the Tables (8-9) and Figures (9-10). As a result of this investigation the following conclusions may be drawn:

- 1-The modified finite difference method presented in this thesis yields excellent convergence characteristics.
- 2-For square thick plates, results for moderately thick plates are in excellent agreement with those obtained by other investigators.
- 3-Results obtained by other researchers for deflection of thick plates are

meagre, and significant amount of results obtained in this thesis are not available in the technical literature.

4-The method is simple to apply and fairly accurate results are obtained even with a relatively crude mesh size for moderately thick plates.

5-With the computer programme included in the appendix and the facility of flexibility of input, the designer can easily input various loadings, plate aspect ratios and boundary conditions.

5.2.1 Improvement of the method

While the use of ordinary finite difference techniques is very simple and the method is quite general, it is sometimes characterized by slow convergence depending on the plate boundary conditions. Furthermore, when higher-order derivatives and a large number of mesh points are involved, the solution seems impracticable. One of the reasons for slow convergence characteristics is that the collocating polynomials used in deriving the finite difference expressions agree only in value with the exact function at the mesh points. Additional sources of error are the approximation of boundary conditions and the use of the usually coarse load averaging rules. Furthermore, an extremely fine mesh can result in large numbers of simultaneous equations, thereby creating round-off errors in computer solutions that can adversely affect the accuracy and economy of the method. When high accuracy in the finite difference solution of plate problems is required, improved

finite difference techniques should be applied.

TABLE 8 comparison of results CC/CC

Ratio $w/q, \frac{a}{b} = 1, \nu = 0.30$						
h/a	Reissner Ref [4]	Reissner Ref [19]	Srinivas Ref [11]	Srinivas Ref [12]	MIF Ref [24]	Present Study
0.05		15.4747		15.4319		15.9200
0.10		2.1907		2.1781		1.9760
0.14				0.9047		0.7139
0.20		0.3943		0.3885		0.2417

TABLE 9 comparison of results SS/SS

Ratio $w/q, \frac{a}{b} = 1, \nu = 0.30$						
h/a	Reissner Ref [4]	Reissner Ref [19]	Srinivas Ref [11]	Srinivas Ref [12]	MIF Ref [24]	Present Study
0.05	47.8392	49.8808	47.8591	49.8888	47.8660	47.1861
0.10	6.1754	6.6755	6.1863	6.6799	6.1858	5.8701
0.14	2.3417		2.3491	2.5927	2.3490	2.1263
0.20		1.0004		0.9855		0.7207

FIGURE 9, COMPARISON OF DEFLECTION
OF CLAMPED THICK PLATES

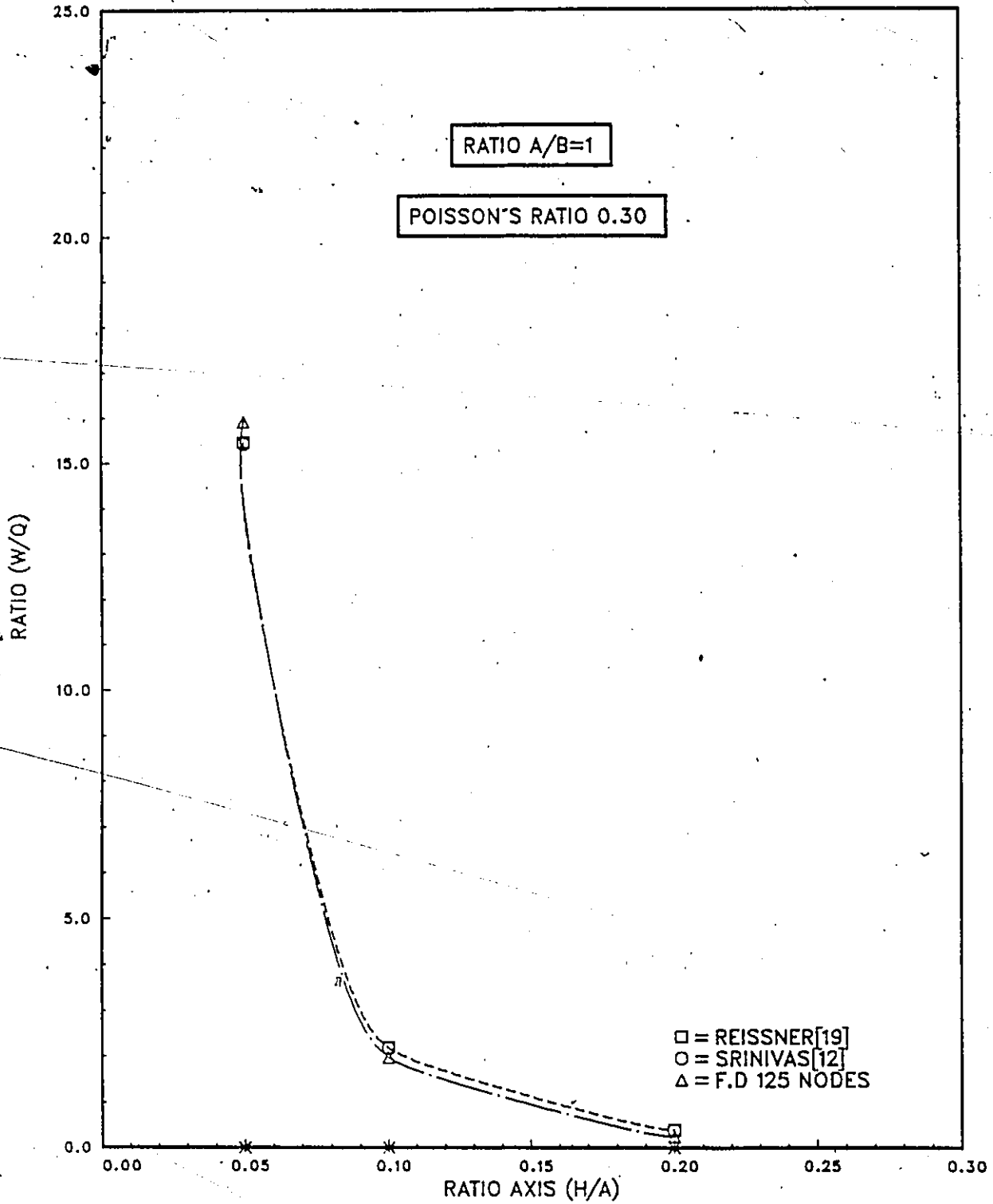
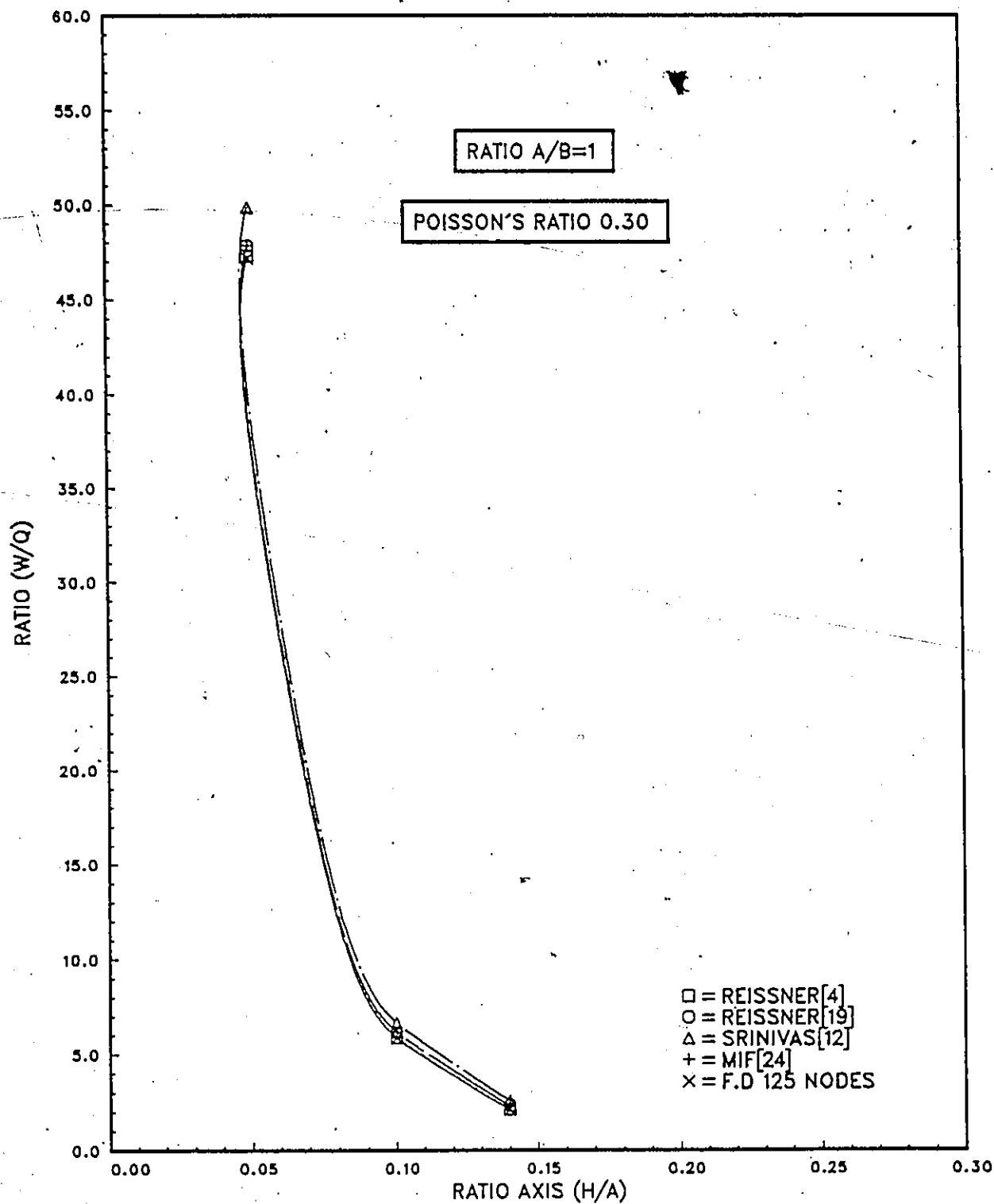


FIGURE 10, COMPARISON OF DEFLECTION
OF SIMPLY SUPPORTED THICK PLATES



Refinements in the finite difference method usually come from three sources:

1-Refinement of the individual derivatives by expressions obtained from osculating polynomials (which also collocate, up to some higher order, the derivatives of the exact function).

2-Refined representation of the actual boundary conditions.

3-Imposed load-averaging techniques.

The general tool for derivation of improved derivatives is the Taylor expression, which not only collocates a function by going through the same points but has the same geometrical properties, i.e, tangent, curvature, etc., in the vicinity of a given point. To obtain improved finite difference expressions, we consider additional mesh points in the vicinity of the pivotal point i .

Here we name some methods of higher approximation :

1-Multilocal method (Hermitian method).

2-Funicular polygon method (A variation of the Hermitian method).

3-Successive approximations.

If one of these methods can be applied to the theory of thick plates, the results will be much better and more accurate.

5.2.2 Recommendations for further study

This investigation on thick plate theory using finite difference method for the case of clamped and simply supported boundary conditions can be extended for other boundary conditions for the static analysis. At the same time, it can be used to investigate the problem of buckling and vibration

of thick rectangular plates.

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Appendix A

Program Elements

This program is written for the three dimensional finite difference method described in chapter [4]. The program is used to solve clamped and simply supported thick plates.

This program performs three major tasks:

1- The submatrices forming the general matrix are generated first and stored in full mode with its number of rows, and columns equal to the number of nodes.

2- The equilibrium equations for the total system are formed by storing the submatrices in the general matrix in full mode where the number of rows and columns is equal to three times the number of nodes.

3- The set of matrix equations is solved for the nodal point displacements by calling two subroutines MATINV to invert the global matrix and MATMUL to multiply the inverted global matrix with the force matrix.

In this program, double precision arithmetic is used for the submatrices and the global matrix.

A.1 Description of Input Data

1-Title Card (10A8)

Columns 1-80: Arbitrary problem Identification.

2-Master Control Card (I5,F10.4,F5.2,F10.5,2F5.3,2I5)

One Card to define the number of nodes, the modulus of elasticity, Poisson's ratio, the principal length, the ratio a/b, the ratio h/a, number of mesh

sizes in x,y directions, number of mesh sizes in z direction.

Columns 1-5 : number of nodes in the plate [NOD].

6-15 : Modulus of elasticity (E).

16-20 : Poisson's ratio (P).

21-30 : The principal length (XA).

31-35 : The ratio a/b (AB).

36-40 : The ratio h/a (AH).

41-45 : Number of mesh sizes in the x and y direction (NTX).

46-50 : Number of mesh sizes in z direction (NTZ).

3-Element Data (F7.4,I3,3I5,13I4).

One Card for each node in order.

Columns 1-7 : Load factor [FF(NOD,1)].

FF(NOD,1)=1 loaded.

FF(NOD,1)=0 not loaded.

8-10 : Incrementation in the Y direction (IJ).

11-15 : Incrementation in the plan XZ (IK).

16-20 : Incrementation in the plan YZ (JK).

21-25 : Label of the functions related to each node (IJKL).

26-78 : Nodes relationship for the boundary layers [NP(13)].

For clamped plate the points beyond the edges are affected with a sign (+).

For simply supported plate the points beyond the edges are affected with a sign (-).

Appendix B

Input-Output

B.1 Input-Output: Clamped Plate 20 nodes

B.2 Input-Output: Simply supp₂ Plate 40
nodes

CLAMPED ISOTROPIC THICK RECTANGULAR PLATE ;

	20	3000.0000	.30	400.0000	1.0	0.050												
							4	4										
1.0000	2	4	4	109	1	0	0	0	1	0	1	2	1	0	3	4	1	
1.0000	2	4	4	112	2	0	0	0	0	1	2	1	0	3	4	3	2	
1.0000	2	4	4	113	0	0	1	2	3	0	3	4	3	0	1	2	0	
1.0000	2	4	4	108	0	1	2	1	0	3	4	3	0	1	2	1	0	
0.0000	2	4	4	100														
0.0000	2	4	4	97														
0.0000	2	4	4	96														
0.0000	2	4	4	104														
0.0000	2	4	4	100														
0.0000	2	4	4	97														
0.0000	2	4	4	96														
0.0000	2	4	4	104														
0.0000	2	4	4	100														
0.0000	2	4	4	97														
0.0000	2	4	4	96														
0.0000	2	4	4	104														
1.0000	2	-4	-4	109	17	0	0	0	17	0	17	18	17	0	19	20	17	
1.0000	2	-4	-4	112	18	0	0	0	0	17	18	17	0	19	20	19	18	
1.0000	2	-4	-4	113	0	0	17	18	19	0	19	20	19	0	17	18	0	
1.0000	2	-4	-4	108	0	17	18	17	0	19	20	19	0	17	18	17	0	

CLAMPED ISOTROPIC THICK RECTANGULAR PLATE ;

NUMBRE OF NODES - 20
 YOUNG MODULOUS -3000.00000
 POISSON S RATIO - 0.30
 LONGITUDINAL LENGTH - 400.00000
 RATIO A/B -1.000
 RATIO A/H -0.050
 MESH NUMBER ON X,Y - 4
 MESH NUMBER ON Z - 4
 MESH SIZE ON X AXIS - 100.00000
 MESH SIZE ON Y AXIS - 100.00000
 MESH SIZE ON Z AXIS - 5.00000

NODE X-DIRECTION Y-DIRECTION Z-DIRECTION

1	0.0002	0.0002	9.5218
2	-0.0002	0.0004	14.0590
3	0.0004	-0.0002	14.0590
4	-0.0004	-0.0004	20.9607
5	0.0004	0.0004	9.5110
6	-0.0004	0.0007	14.0416
7	0.0007	-0.0004	14.0416
8	-0.0007	-0.0007	20.9312
9	0.0009	0.0009	9.5075
10	-0.0009	0.0016	14.0358
11	0.0016	-0.0009	14.0358
12	-0.0016	-0.0016	20.9214
13	0.0011	0.0011	9.5111
14	-0.0011	0.0021	14.0416
15	0.0021	-0.0011	14.0416
16	-0.0020	-0.0020	20.9311
17	0.0006	0.0006	9.5218
18	-0.0006	0.0011	14.0590
19	0.0011	-0.0006	14.0590

20

-0.0010

-0.0010

20.9607

SIMPLY SUPPORTED ISOTROPIC THICK RECTANGULAR PLATE ;

40	3000.0000	.30	400.0000	1.0	0.050	4	4										
1.0000	3	8	8	123													
1.0000	3	8	8	122													
1.0000	3	8	8	124													
1.0000	3	8	8	105	-4	0	0	0	-4	0	4	5	4	0	7	8	4
1.0000	3	8	8	106	-5	0	0	0	0	4	5	4	0	7	8	7	5
1.0000	3	8	8	120													
1.0000	3	8	8	107	0	0	4	5	-7	0	7	8	7	0	4	5	0
1.0000	3	8	8	108	0	4	5	4	0	7	8	7	0	4	5	4	0
0.0000	3	8	8	125													
0.0000	3	8	8	118													
0.0000	3	8	8	114													
0.0000	3	8	8	101													
0.0000	3	8	8	102													
0.0000	3	8	8	116													
0.0000	3	8	8	103													
0.0000	3	8	8	104													
0.0000	3	8	8	125													
0.0000	3	8	8	118													
0.0000	3	8	8	114													
0.0000	3	8	8	101													
0.0000	3	8	8	102													
0.0000	3	8	8	116													
0.0000	3	8	8	103													
0.0000	3	8	8	104													
0.0000	3	8	8	125													
0.0000	3	8	8	118													
0.0000	3	8	8	114													
0.0000	3	8	8	101													
0.0000	3	8	8	102													
0.0000	3	8	8	116													
0.0000	3	8	8	103													
0.0000	3	8	8	104													
0.0000	3	8	8	125													
0.0000	3	8	8	118													
0.0000	3	8	8	114													
0.0000	3	8	8	101													
0.0000	3	8	8	102													
0.0000	3	8	8	116													
0.0000	3	8	8	103													
0.0000	3	8	8	104													
1.0000	3	-8	-8	123													
1.0000	3	-8	-8	122													
1.0000	3	-8	-8	124													
1.0000	3	-8	-8	105	-36	0	0	0	-36	0	36	37	36	0	39	40	36
1.0000	3	-8	-8	106	-37	0	0	0	0	36	37	36	0	39	40	39	37
1.0000	3	-8	-8	120													
1.0000	3	-8	-8	107	0	0	36	37	-39	0	39	40	39	0	36	37	0
1.0000	3	-8	-8	108	0	36	37	36	0	39	40	39	0	36	37	36	0



SIMPLY SUPPORTED ISOTROPIC THICK RECTANGULAR PLATE ;

NUMBRE OF NODES = 40
YOUNG MODULOUS -3000.00000
POISSON S RATIO = 0.30
LONGITUDINAL LENGTH = 400.00000
RATIO A/B -1.000
RATIO A/E -0.080
MESH NUMBER ON X,Y = 4
MESH NUMBER ON Z = 4
MESH SIZE ON X AXIS = 100.00000
MESH SIZE ON Y AXIS = 100.00000
MESH SIZE ON Z AXIS = 5.00000

0 ZERO PIVOT

NODE X-DIRECTION Y-DIRECTION Z-DIRECTION

1	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000
4	0.0003	0.0003	24.8828
5	-0.0003	0.0004	34.1250
6	0.0000	0.0000	0.0000
7	0.0004	-0.0003	34.1250
8	-0.0004	-0.0004	46.9219
9	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000
12	0.0005	0.0005	24.8493
13	-0.0005	0.0007	34.0824
14	0.0000	0.0000	0.0000
15	0.0007	-0.0005	34.0824
16	-0.0007	-0.0007	46.8672
17	0.0000	0.0000	0.0000
18	0.0000	0.0000	0.0000
19	0.0000	0.0000	0.0000

20	0.0012	0.0012	24.8382
21	-0.0012	0.0016	34.0682
22	0.0000	0.0000	0.0000
23	0.0016	-0.0012	34.0682
24	-0.0016	-0.0016	46.8489
25	0.0001	0.0000	0.0000
26	0.0000	0.0000	0.0000
27	0.0000	0.0001	0.0000
28	0.0015	0.0015	24.8494
29	-0.0015	0.0021	34.0824
30	0.0000	0.0000	0.0000
31	0.0021	-0.0015	34.0824
32	-0.0020	-0.0020	46.8670
33	0.0001	0.0000	0.0000
34	0.0000	0.0000	0.0000
35	0.0000	0.0001	0.0000
36	0.0008	0.0008	24.8828
37	-0.0008	0.0011	34.1250
38	0.0000	0.0000	0.0000
39	0.0011	-0.0008	34.1250
40	-0.0010	-0.0010	46.9219

Appendix C

Computer Programme


```

VU(I,J)-0.0
VV(I,J)-0.0
VW(I,J)-0.0
WU(I,J)-0.0
WV(I,J)-0.0
WW(I,J)-0.0
1 CONTINUE
DO 2 I=1,NOD
READ(5,1000) FF(I,1),IJ,IK,JK,IJKL,(NP(II),II-1,13)
IF(IJKL.EQ.96) GO TO 96
IF(IJKL.EQ.97) GO TO 97
IF(IJKL.EQ.98) GO TO 98
IF(IJKL.EQ.99) GO TO 99
IF(IJKL.EQ.100) GO TO 100
IF(IJKL.EQ.101) GO TO 101
IF(IJKL.EQ.102) GO TO 102
IF(IJKL.EQ.103) GO TO 103
IF(IJKL.EQ.104) GO TO 104
IF(IJKL.EQ.105) GO TO 105
IF(IJKL.EQ.106) GO TO 106
IF(IJKL.EQ.107) GO TO 107
IF(IJKL.EQ.108) GO TO 108
IF(IJKL.EQ.109) GO TO 109
IF(IJKL.EQ.110) GO TO 110
IF(IJKL.EQ.111) GO TO 111
IF(IJKL.EQ.112) GO TO 112
IF(IJKL.EQ.113) GO TO 113
IF(IJKL.EQ.114) GO TO 114
IF(IJKL.EQ.115) GO TO 115
IF(IJKL.EQ.116) GO TO 116
IF(IJKL.EQ.117) GO TO 117
IF(IJKL.EQ.118) GO TO 118
IF(IJKL.EQ.119) GO TO 119
IF(IJKL.EQ.120) GO TO 120
IF(IJKL.EQ.121) GO TO 121
IF(IJKL.EQ.122) GO TO 122
IF(IJKL.EQ.123) GO TO 123
IF(IJKL.EQ.124) GO TO 124
IF(IJKL.EQ.125) GO TO 125

96 UU(I,I)-UU(I,I)-2.0*G1/TX**2
UU(I,I+1)-UU(I,I+1)+G1/TX**2
UU(I,I-IJ)-UU(I,I-IJ)+2.0*G/TY**2
UU(I,I)-UU(I,I)-2.0*G/TY**2
UU(I,I-IK)-UU(I,I-IK)+G/TZ**2
UU(I,I)-UU(I,I)-2.0*G/TZ**2
UU(I,I+IK)-UU(I,I+IK)+G/TZ**2

UV(I,I)-UV(I,I)+G2/(TX*TY)
UV(I,I+1)-UV(I,I+1)-G2/(TX*TY)
UV(I,I-IJ)-UV(I,I-IJ)-G2/(TX*TY)
UV(I,I-IJ+1)-UV(I,I-IJ+1)+G2/(TX*TY)

UW(I,I)-UW(I,I)+G2/(TX*TZ)

```

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DIF0058
DIF0059
DIF0060
DIF0061
DIF0062
DIF0063
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DIF0065
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DIF0070
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DIF0105
DIF0106
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DIF0109
DIF0110
DIF0111

```

$UW(I, I+1) = UW(I, I+1) - G2 / (TX * TZ)$
 $UW(I, I+IK) = UW(I, I+IK) - G2 / (TX * TZ)$
 $UW(I, I+IK+1) = UW(I, I+IK+1) + G2 / (TX * TZ)$

$VU(I, I) = VU(I, I) + G2 / (TX * TY)$
 $VU(I, I+1) = VU(I, I+1) - G2 / (TX * TY)$
 $VU(I, I-IJ) = VU(I, I-IJ) - G2 / (TX * TY)$
 $VU(I, I-IJ+1) = VU(I, I-IJ+1) + G2 / (TX * TY)$

$VV(I, I) = VV(I, I) - 2.0 * G / TX ** 2$
 $VV(I, I+1) = VV(I, I+1) + G / TX ** 2$
 $VV(I, I-IJ) = VV(I, I-IJ) + 2.0 * G1 / TY ** 2$
 $VV(I, I) = VV(I, I) - 2.0 * G1 / TY ** 2$
 $VV(I, I-JK) = VV(I, I-JK) + G / TZ ** 2$
 $VV(I, I) = VV(I, I) - 2.0 * G / TZ ** 2$
 $VV(I, I+JK) = VV(I, I+JK) + G / TZ ** 2$

$VW(I, I) = VW(I, I) + G2 / (TY * TZ)$
 $VW(I, I-IJ) = VW(I, I-IJ) - G2 / (TY * TZ)$
 $VW(I, I+JK) = VW(I, I+JK) - G2 / (TY * TZ)$
 $VW(I, I+JK-IJ) = VW(I, I+JK-IJ) + G2 / (TY * TZ)$

$WU(I, I) = WU(I, I) + G2 / (TX * TZ)$
 $WU(I, I+1) = WU(I, I+1) - G2 / (TX * TZ)$
 $WU(I, I+IK) = WU(I, I+IK) - G2 / (TX * TZ)$
 $WU(I, I+IK+1) = WU(I, I+IK+1) + G2 / (TX * TZ)$

$WV(I, I) = WV(I, I) + G2 / (TY * TZ)$
 $WV(I, I-IJ) = WV(I, I-IJ) - G2 / (TY * TZ)$
 $WV(I, I+JK) = WV(I, I+JK) - G2 / (TY * TZ)$
 $WV(I, I-IJ+JK) = WV(I, I-IJ+JK) + G2 / (TY * TZ)$

$WW(I, I) = WW(I, I) - 2.0 * G / TX ** 2$
 $WW(I, I+1) = WW(I, I+1) + G / TX ** 2$
 $WW(I, I-IJ) = WW(I, I-IJ) + 2.0 * G / TY ** 2$
 $WW(I, I) = WW(I, I) - 2.0 * G / TY ** 2$
 $WW(I, I-IK) = WW(I, I-IK) + G1 / TZ ** 2$
 $WW(I, I) = WW(I, I) - 2.0 * G1 / TZ ** 2$
 $WW(I, I+IK) = WW(I, I+IK) + G1 / TZ ** 2$

GO TO 2

97 $UU(I, I-1) = UU(I, I-1) + 2.0 * G1 / TX ** 2$
 $UU(I, I) = UU(I, I) - 2.0 * G1 / TX ** 2$
 $UU(I, I) = UU(I, I) - 2.0 * G / TY ** 2$
 $UU(I, I+IJ) = UU(I, I+IJ) + G / TY ** 2$
 $UU(I, I-IK) = UU(I, I-IK) + G / TZ ** 2$
 $UU(I, I) = UU(I, I) - 2.0 * G / TZ ** 2$
 $UU(I, I+IK) = UU(I, I+IK) + G / TZ ** 2$

$UV(I, I) = UV(I, I) + G2 / (TX * TY)$
 $UV(I, I-1) = UV(I, I-1) - G2 / (TX * TY)$
 $UV(I, I+IJ) = UV(I, I+IJ) - G2 / (TX * TY)$
 $UV(I, I+IJ-1) = UV(I, I+IJ-1) + G2 / (TX * TY)$

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 DIF01650

UW(I,I)-UW(I,I)+G2/(TX*TZ)
 UW(I,I-1)-UW(I,I-1)-G2/(TX*TZ)
 UW(I,I+IK)-UW(I,I+IK)-G2/(TX*TZ)
 UW(I,I+IK-1)-UW(I,I+IK-1)+G2/(TX*TZ)

 VU(I,I)-VU(I,I)+G2/(TX*TY)
 VU(I,I-1)-VU(I,I-1)-G2/(TX*TY)
 VU(I,I+IJ)-VU(I,I+IJ)-G2/(TX*TY)
 VU(I,I+IJ-1)-VU(I,I+IJ-1)+G2/(TX*TY)

 VV(I,I-1)-VV(I,I-1)+2.0*G/TX**2
 VV(I,I)-VV(I,I)-2.0*G/TX**2
 VV(I,I)-VV(I,I)-2.0*G1/TY**2
 VV(I,I+IJ)-VV(I,I+IJ)+G1/TY**2
 VV(I,I-JK)-VV(I,I-JK)+G/TZ**2
 VV(I,I)-VV(I,I)-2.0*G/TZ**2
 VV(I,I+JK)-VV(I,I+JK)+G/TZ**2

 VW(I,I)-VW(I,I)+G2/(TY*TZ)
 VW(I,I+IJ)-VW(I,I+IJ)-G2/(TY*TZ)
 VW(I,I+JK)-VW(I,I+JK)-G2/(TY*TZ)
 VW(I,I+IJ+JK)-VW(I,I+IJ+JK)+G2/(TY*TZ)

 WU(I,I)-WU(I,I)+G2/(TX*TZ)
 WU(I,I-1)-WU(I,I-1)-G2/(TX*TZ)
 WU(I,I+IK)-WU(I,I+IK)-G2/(TX*TZ)
 WU(I,I+IK-1)-WU(I,I+IK-1)+G2/(TX*TZ)

 WV(I,I)-WV(I,I)+G2/(TY*TZ)
 WV(I,I+IJ)-WV(I,I+IJ)-G2/(TY*TZ)
 WV(I,I+JK)-WV(I,I+JK)-G2/(TY*TZ)
 WV(I,I+IJ+JK)-WV(I,I+IJ+JK)+G2/(TY*TZ)

 WW(I,I-1)-WW(I,I-1)+2.0*G/TX**2
 WW(I,I)-WW(I,I)-2.0*G/TX**2
 WW(I,I)-WW(I,I)-2.0*G/TY**2
 WW(I,I+IJ)-WW(I,I+IJ)+G/TY**2
 WW(I,I-IK)-WW(I,I-IK)+G1/TZ**2
 WW(I,I)-WW(I,I)-2.0*G1/TZ**2
 WW(I,I+IK)-WW(I,I+IK)+G1/TZ**2

 GO TO 2
 98 UU(I,I) -UU(I,I)-2.0*G1/TX**2
 UU(I,I+1)-UU(I,I+1)+G1/TX**2
 UU(I,I-IJ)-UU(I,I-IJ)+G/TY**2
 UU(I,I)-UU(I,I)-2.0*G/TY**2
 UU(I,I+IJ)-UU(I,I+IJ)+G/TY**2
 UU(I,I-IK)-UU(I,I-IK)+G/TZ**2
 UU(I,I)-UU(I,I)-2.0*G/TZ**2
 UU(I,I+IK)-UU(I,I+IK)+G/TZ**2

 UV(I,I)-UV(I,I)+G2/(TX*TY)
 UV(I,I+1)-UV(I,I+1)-G2/(TX*TY)
 UV(I,I+IJ)-UV(I,I+IJ)-G2/(TX*TY)

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 DIF02190

UV(I, I+IJ+1)-UV(I, I+IJ+1)+G2/(TX*TY)

UW(I, I)-UW(I, I)+G2/(TX*TZ)

UW(I, I+1)-UW(I, I+1)-G2/(TX*TZ)

UW(I, I+IK)-UW(I, I+IK)-G2/(TX*TZ)

UW(I, I+IK+1)-UW(I, I+IK+1)+G2/(TX*TZ)

VU(I, I)-VU(I, I)+G2/(TX*TY)

VU(I, I+1)-VU(I, I+1)-G2/(TX*TY)

VU(I, I+IJ)-VU(I, I+IJ)-G2/(TX*TY)

VU(I, I+IJ+1)-VU(I, I+IJ+1)+G2/(TX*TY)

VV(I, I)-VV(I, I)-2.0*G/TX**2

VV(I, I+1)-VV(I, I+1)+G/TX**2

VV(I, I-IJ)-VV(I, I-IJ)+G1/TY**2

VV(I, I)-VV(I, I)-2.0*G1/TY**2

VV(I, I+IJ)-VV(I, I+IJ)+G1/TY**2

VV(I, I-JK)-VV(I, I-JK)+G/TZ**2

VV(I, I)-VV(I, I)-2.0*G/TZ**2

VV(I, I+JK)-VV(I, I+JK)+G/TZ**2

VW(I, I)-VW(I, I)+G2/(TY*TZ)

VW(I, I+IJ)-VW(I, I+IJ)-G2/(TY*TZ)

VW(I, I+JK)-VW(I, I+JK)-G2/(TY*TZ)

VW(I, I+JK+IJ)-VW(I, I+JK+IJ)+G2/(TY*TZ)

WU(I, I)-WU(I, I)+G2/(TX*TZ)

WU(I, I+1)-WU(I, I+1)-G2/(TX*TZ)

WU(I, I+IK)-WU(I, I+IK)-G2/(TX*TZ)

WU(I, I+IK+1)-WU(I, I+IK+1)+G2/(TX*TZ)

WV(I, I)-WV(I, I)+G2/(TY*TZ)

WV(I, I+IJ)-WV(I, I+IJ)-G2/(TY*TZ)

WV(I, I+JK)-WV(I, I+JK)-G2/(TY*TZ)

WV(I, I+JK+IJ)-WV(I, I+JK+IJ)+G2/(TY*TZ)

WW(I, I)-WW(I, I)-2.0*G/TX**2

WW(I, I+1)-WW(I, I+1)+G/TX**2

WW(I, I-IJ)-WW(I, I-IJ)+G/TY**2

WW(I, I)-WW(I, I)-2.0*G/TY**2

WW(I, I+IJ)-WW(I, I+IJ)+G/TY**2

WW(I, I-IK)-WW(I, I-IK)+G1/TZ**2

WW(I, I)-WW(I, I)-2.0*G1/TZ**2

WW(I, I+IK)-WW(I, I+IK)+G1/TZ**2

GO TO 2

99 UU(I, I-1)-UU(I, I-1)+G1/TX**2

UU(I, I) -UU(I, I)-2.0*G1/TX**2

UU(I, I+1)-UU(I, I+1)+G1/TX**2

UU(I, I)-UU(I, I)-2.0*G/TY**2

UU(I, I+IJ)-UU(I, I+IJ)+G/TY**2

UU(I, I-IK)-UU(I, I-IK)+G/TZ**2

UU(I, I)-UU(I, I)-2.0*G/TZ**2

UU(I, I+IK)-UU(I, I+IK)+G/TZ**2

DIF02200

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DIF02230

DIF02240

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DIF02290

DIF02300

DIF02310

DIF02320

DIF02330

DIF02340

DIF02350

DIF02360

DIF02370

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DIF02650

DIF02660

DIF02670

DIF02680

DIF02690

DIF02700

DIF02710

DIF02720

DIF02730

UV(I,I)-UV(I,I)+G2/(TX*TY)
 UV(I,I+1)-UV(I,I+1)-G2/(TX*TY)
 UV(I,I+IJ)-UV(I,IJ)-G2/(TX*TY)
 UV(I,I+IJ+1)-UV(I,I+IJ+1)+G2/(TX*TY)

UW(I,I)-UW(I,I)+G2/(TX*TZ)
 UW(I,I+1)-UW(I,I+1)-G2/(TX*TZ)
 UW(I,I+IK)-UW(I,I+IK)-G2/(TX*TZ)
 UW(I,I+IK+1)-UW(I,I+IK+1)+G2/(TX*TZ)

VU(I,I)-VU(I,I)+G2/(TX*TY)
 VU(I,I+1)-VU(I,I+1)-G2/(TX*TY)
 VU(I,I+IJ)-VU(I,I+IJ)-G2/(TX*TY)
 VU(I,I+IJ+1)-VU(I,I+IJ+1)+G2/(TX*TY)

VV(I,I-1)-VV(I,I-1)+G/TX**2
 VV(I,I)-VV(I,I)-2.0*G/TX**2
 VV(I,I+1)-VV(I,I+1)+G/TX**2
 VV(I,I)-VV(I,I)-2.0*G1/TY**2
 VV(I,I+IJ)-VV(I,I+IJ)+G1/TY**2
 VV(I,I-JK)-VV(I,I-JK)+G/TZ**2
 VV(I,I)-VV(I,I)-2.0*G/TZ**2
 VV(I,I+JK)-VV(I,I+JK)+G/TZ**2

VW(I,I)-VW(I,I)+G2/(TY*TZ)
 VW(I,I+IJ)-VW(I,I+IJ)-G2/(TY*TZ)
 VW(I,I+JK)-VW(I,I+JK)-G2/(TY*TZ)
 VW(I,I+JK+IJ)-VW(I,I+JK+IJ)+G2/(TY*TZ)

WU(I,I)-WU(I,I)+G2/(TX*TZ)
 WU(I,I+1)-WU(I,I+1)-G2/(TX*TZ)
 WU(I,I+IK)-WU(I,I+IK)-G2/(TX*TZ)
 WU(I,I+IK+1)-WU(I,I+IK+1)+G2/(TX*TZ)

WV(I,I)-WV(I,I)+G2/(TY*TZ)
 WV(I,I+IJ)-WV(I,I+IJ)-G2/(TY*TZ)
 WV(I,I+JK)-WV(I,I+JK)-G2/(TY*TZ)
 WV(I,I+JK+IJ)-WV(I,I+JK+IJ)+G2/(TY*TZ)

WW(I,I-1)-WW(I,I-1)+G/TX**2
 WW(I,I)-WW(I,I)-2.0*G/TX**2
 WW(I,I+1)-WW(I,I+1)+G/TX**2
 WW(I,I)-WW(I,I)-2.0*G/TY**2
 WW(I,I+IJ)-WW(I,I+IJ)+G/TY**2
 WW(I,I-1K)-WW(I,I-1K)+G1/TZ**2
 WW(I,I)-WW(I,I)-2.0*G1/TZ**2
 WW(I,I+1K)-WW(I,I+1K)+G1/TZ**2

GO TO 2

100 UU(I,I) -UU(I,I)-2.0*G1/TX**2
 UU(I,I+1)-UU(I,I+1)+G1/TX**2
 UU(I,I)-UU(I,I)-2.0*G/TY**2
 UU(I,I+IJ)-UU(I,I+IJ)+G/TY**2

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 DIF03270

$UU(I, I-1K) - UU(I, I-1K) + G/TZ^{**2}$
 $UU(I, I) - UU(I, I) - 2.0 * G/TZ^{**2}$
 $UU(I, I+1K) - UU(I, I+1K) + G/TZ^{**2}$

$UV(I, I) - UV(I, I) + G2/(TX * TY)$
 $UV(I, I+1) - UV(I, I+1) - G2/(TX * TY)$
 $UV(I, I+1J) - UV(I, IJ) - G2/(TX * TY)$
 $UV(I, I+1J+1) - UV(I, I+1J+1) + G2/(TX * TY)$

$UW(I, I) - UW(I, I) + G2/(TX * TZ)$
 $UW(I, I+1) - UW(I, I+1) - G2/(TX * TZ)$
 $UW(I, I+1K) - UW(I, I+1K) - G2/(TX * TZ)$
 $UW(I, I+1K+1) - UW(I, I+1K+1) + G2/(TX * TZ)$

$VU(I, I) - VU(I, I) + G2/(TX * TY)$
 $VU(I, I+1) - VU(I, I+1) - G2/(TX * TY)$
 $VU(I, I+1J) - VU(I, I+1J) - G2/(TX * TY)$
 $VU(I, I+1J+1) - VU(I, I+1J+1) + G2/(TX * TY)$

$VV(I, I) - VV(I, I) - 2.0 * G/TX^{**2}$
 $VV(I, I+1) - VV(I, I+1) + G/TX^{**2}$
 $VV(I, I) - VV(I, I) - 2.0 * G1/TY^{**2}$
 $VV(I, I+1J) - VV(I, I+1J) + G1/TY^{**2}$
 $VV(I, I-1JK) - VV(I, I-1JK) + G/TZ^{**2}$
 $VV(I, I) - VV(I, I) - 2.0 * G/TZ^{**2}$
 $VV(I, I+1JK) - VV(I, I+1JK) + G/TZ^{**2}$

$VW(I, I) - VW(I, I) + G2/(TY * TZ)$
 $VW(I, I+1J) - VW(I, I+1J) - G2/(TY * TZ)$
 $VW(I, I+1JK) - VW(I, I+1JK) - G2/(TY * TZ)$
 $VW(I, I+1JK+1J) - VW(I, I+1JK+1J) + G2/(TY * TZ)$

$WU(I, I) - WU(I, I) + G2/(TX * TZ)$
 $WU(I, I+1) - WU(I, I+1) - G2/(TX * TZ)$
 $WU(I, I+1K) - WU(I, I+1K) - G2/(TX * TZ)$
 $WU(I, I+1K+1) - WU(I, I+1K+1) + G2/(TX * TZ)$

$WV(I, I) - WV(I, I) + G2/(TY * TZ)$
 $WV(I, I+1J) - WV(I, I+1J) - G2/(TY * TZ)$
 $WV(I, I+1JK) - WV(I, I+1JK) - G2/(TY * TZ)$
 $WV(I, I+1JK+1J) - WV(I, I+1JK+1J) + G2/(TY * TZ)$

$WW(I, I) - WW(I, I) - 2.0 * G/TX^{**2}$
 $WW(I, I+1) - WW(I, I+1) + G/TX^{**2}$
 $WW(I, I) - WW(I, I) - 2.0 * G1/TY^{**2}$
 $WW(I, I+1J) - WW(I, I+1J) + G1/TY^{**2}$
 $WW(I, I-1K) - WW(I, I-1K) + G1/TZ^{**2}$
 $WW(I, I) - WW(I, I) - 2.0 * G1/TZ^{**2}$
 $WW(I, I+1K) - WW(I, I+1K) + G1/TZ^{**2}$

GO TO 2

101 $UU(I, I-1) - UU(I, I-1) + G1/TX^{**2}$
 $UU(I, I) - UU(I, I) - 2.0 * G1/TX^{**2}$
 $UU(I, I+1) - UU(I, I+1) + G1/TX^{**2}$

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$UU(I, I-IJ) = UU(I, I-IJ) + G/TY^{**2}$
 $UU(I, I) = UU(I, I) - 2.0 * G/TY^{**2}$
 $UU(I, I+IJ) = UU(I, I+IJ) + G/TY^{**2}$
 $UU(I, I- IK) = UU(I, I- IK) + G/TZ^{**2}$
 $UU(I, I) = UU(I, I) - 2.0 * G/TZ^{**2}$
 $UU(I, I+ IK) = UU(I, I+ IK) + G/TZ^{**2}$

$UV(I, I) = UV(I, I) + G2/(TX * TY)$
 $UV(I, I+1) = UV(I, I+1) - G2/(TX * TY)$
 $UV(I, I+IJ) = UV(I, IJ) - G2/(TX * TY)$
 $UV(I, I+IJ+1) = UV(I, I+IJ+1) + G2/(TX * TY)$

$UW(I, I) = UW(I, I) + G2/(TX * TZ)$
 $UW(I, I+1) = UW(I, I+1) - G2/(TX * TZ)$
 $UW(I, I+ IK) = UW(I, I+ IK) - G2/(TX * TZ)$
 $UW(I, I+ IK+1) = UW(I, I+ IK+1) + G2/(TX * TZ)$

$VU(I, I) = VU(I, I) + G2/(TX * TY)$
 $VU(I, I+1) = VU(I, I+1) - G2/(TX * TY)$
 $VU(I, I+IJ) = VU(I, I+IJ) - G2/(TX * TY)$
 $VU(I, I+IJ+1) = VU(I, I+IJ+1) + G2/(TX * TY)$

$VV(I, I-1) = VV(I, I-1) + G/TX^{**2}$
 $VV(I, I) = VV(I, I) - 2.0 * G/TX^{**2}$
 $VV(I, I+1) = VV(I, I+1) + G/TX^{**2}$
 $VV(I, I-IJ) = VV(I, I-IJ) + G1/TY^{**2}$
 $VV(I, I) = VV(I, I) - 2.0 * G1/TY^{**2}$
 $VV(I, I+IJ) = VV(I, I+IJ) + G1/TY^{**2}$
 $VV(I, I- JK) = VV(I, I- JK) + G/TZ^{**2}$
 $VV(I, I) = VV(I, I) - 2.0 * G/TZ^{**2}$
 $VV(I, I+ JK) = VV(I, I+ JK) + G/TZ^{**2}$

$VW(I, I) = VW(I, I) + G2/(TY * TZ)$
 $VW(I, I+IJ) = VW(I, I+IJ) - G2/(TY * TZ)$
 $VW(I, I+ JK) = VW(I, I+ JK) - G2/(TY * TZ)$
 $VW(I, I+ JK+ IJ) = VW(I, I+ JK+ IJ) + G2/(TY * TZ)$

$WU(I, I) = WU(I, I) + G2/(TX * TZ)$
 $WU(I, I+1) = WU(I, I+1) - G2/(TX * TZ)$
 $WU(I, I+ IK) = WU(I, I+ IK) - G2/(TX * TZ)$
 $WU(I, I+ IK+1) = WU(I, I+ IK+1) + G2/(TX * TZ)$

$WV(I, I) = WV(I, I) + G2/(TY * TZ)$
 $WV(I, I+IJ) = WV(I, I+IJ) - G2/(TY * TZ)$
 $WV(I, I+ JK) = WV(I, I+ JK) - G2/(TY * TZ)$
 $WV(I, I+ JK+ IJ) = WV(I, I+ JK+ IJ) + G2/(TY * TZ)$

$WW(I, I-1) = WW(I, I-1) + G/TX^{**2}$
 $WW(I, I) = WW(I, I) - 2.0 * G/TX^{**2}$
 $WW(I, I+1) = WW(I, I+1) + G/TX^{**2}$
 $WW(I, I-IJ) = WW(I, I-IJ) + G/TY^{**2}$
 $WW(I, I) = WW(I, I) - 2.0 * G/TY^{**2}$
 $WW(I, I+IJ) = WW(I, I+IJ) + G/TY^{**2}$
 $WW(I, I- IK) = WW(I, I- IK) + G1/TZ^{**2}$

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$WW(I, I) - WW(I, I) - 2.0 * G1 / TZ ** 2$
 $WW(I, I + IK) - WW(I, I + IK) + G1 / TZ ** 2$

GO TO 2

102 $UU(I, I - 1) - UU(I, I - 1) + 2.0 * G1 / TX ** 2$
 $UU(I, I) - UU(I, I) - 2.0 * G1 / TX ** 2$
 $UU(I, I - IJ) - UU(I, I - IJ) + G / TY ** 2$
 $UU(I, I) - UU(I, I) - 2.0 * G / TY ** 2$
 $UU(I, I + IJ) - UU(I, I + IJ) + G / TY ** 2$
 $UU(I, I - IK) - UU(I, I - IK) + G / TZ ** 2$
 $UU(I, I) - UU(I, I) - 2.0 * G / TZ ** 2$
 $UU(I, I + IK) - UU(I, I + IK) + G / TZ ** 2$

$UV(I, I) - UV(I, I) + G2 / (TX * TY)$
 $UV(I, I - 1) - UV(I, I - 1) - G2 / (TX * TY)$
 $UV(I, I + IJ) - UV(I, I + IJ) - G2 / (TX * TY)$
 $UV(I, I + IJ - 1) - UV(I, I + IJ - 1) + G2 / (TX * TY)$

$UW(I, I) - UW(I, I) + G2 / (TX * TZ)$
 $UW(I, I - 1) - UW(I, I - 1) - G2 / (TX * TZ)$
 $UW(I, I + IK) - UW(I, I + IK) - G2 / (TX * TZ)$
 $UW(I, I + IK - 1) - UW(I, I + IK - 1) + G2 / (TX * TZ)$

$VU(I, I) - VU(I, I) + G2 / (TX * TY)$
 $VU(I, I - 1) - VU(I, I - 1) - G2 / (TX * TY)$
 $VU(I, I + IJ) - VU(I, I + IJ) - G2 / (TX * TY)$
 $VU(I, I + IJ - 1) - VU(I, I + IJ - 1) + G2 / (TX * TY)$

$VV(I, I - 1) - VV(I, I - 1) + 2.0 * G / TX ** 2$
 $VV(I, I) - VV(I, I) - 2.0 * G / TX ** 2$
 $VV(I, I - IJ) - VV(I, I - IJ) + G1 / TY ** 2$
 $VV(I, I) - VV(I, I) - 2.0 * G1 / TY ** 2$
 $VV(I, I + IJ) - VV(I, I + IJ) + G1 / TY ** 2$
 $VV(I, I - JK) - VV(I, I - JK) + G / TZ ** 2$
 $VV(I, I) - VV(I, I) - 2.0 * G / TZ ** 2$
 $VV(I, I + JK) - VV(I, I + JK) + G / TZ ** 2$

$VW(I, I) - VW(I, I) + G2 / (TY * TZ)$
 $VW(I, I + IJ) - VW(I, I + IJ) - G2 / (TY * TZ)$
 $VW(I, I + JK) - VW(I, I + JK) - G2 / (TY * TZ)$
 $VW(I, I + IJ + JK) - VW(I, I + IJ + JK) + G2 / (TY * TZ)$

$WU(I, I) - WU(I, I) + G2 / (TX * TZ)$
 $WU(I, I - 1) - WU(I, I - 1) - G2 / (TX * TZ)$
 $WU(I, I + IK) - WU(I, I + IK) - G2 / (TX * TZ)$
 $WU(I, I + IK - 1) - WU(I, I + IK - 1) + G2 / (TX * TZ)$

$WV(I, I) - WV(I, I) + G2 / (TY * TZ)$
 $WV(I, I + IJ) - WV(I, I + IJ) - G2 / (TY * TZ)$
 $WV(I, I + JK) - WV(I, I + JK) - G2 / (TY * TZ)$
 $WV(I, I + IJ + JK) - WV(I, I + IJ + JK) + G2 / (TY * TZ)$

$WW(I, I - 1) - WW(I, I - 1) + 2.0 * G / TX ** 2$
 $WW(I, I) - WW(I, I) - 2.0 * G / TX ** 2$

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$WW(I, I-IJ) - WW(I, I-IJ) + G/TY^{**2}$
 $WW(I, I) - WW(I, I) - 2.0 * G/TY^{**2}$
 $WW(I, I+IJ) - WW(I, I+IJ) + G/TY^{**2}$
 $WW(I, I-IK) - WW(I, I-IK) + G1/TZ^{**2}$
 $WW(I, I) - WW(I, I) - 2.0 * G1/TZ^{**2}$
 $WW(I, I+IK) - WW(I, I+IK) + G1/TZ^{**2}$

GO TO 2

103 $UU(I, I-1) - UU(I, I-1) + G1/TX^{**2}$
 $UU(I, I) - UU(I, I) - 2.0 * G1/TX^{**2}$
 $UU(I, I+1) - UU(I, I+1) + G1/TX^{**2}$
 $UU(I, I-IJ) - UU(I, I-IJ) + 2.0 * G/TY^{**2}$
 $UU(I, I) - UU(I, I) - 2.0 * G/TY^{**2}$
 $UU(I, I-IK) - UU(I, I-IK) + G/TZ^{**2}$
 $UU(I, I) - UU(I, I) - 2.0 * G/TZ^{**2}$
 $UU(I, I+IK) - UU(I, I+IK) + G/TZ^{**2}$

$UV(I, I) - UV(I, I) + G2/(TX*TY)$
 $UV(I, I+1) - UV(I, I+1) - G2/(TX*TY)$
 $UV(I, I-IJ) - UV(I, I-IJ) - G2/(TX*TY)$
 $UV(I, I-IJ+1) - UV(I, I-IJ+1) + G2/(TX*TY)$

$UW(I, I) - UW(I, I) + G2/(TX*TZ)$
 $UW(I, I+1) - UW(I, I+1) - G2/(TX*TZ)$
 $UW(I, I+IK) - UW(I, I+IK) - G2/(TX*TZ)$
 $UW(I, I+IK+1) - UW(I, I+IK+1) + G2/(TX*TZ)$

$VU(I, I) - VU(I, I) + G2/(TX*TY)$
 $VU(I, I+1) - VU(I, I+1) - G2/(TX*TY)$
 $VU(I, I-IJ) - VU(I, I-IJ) - G2/(TX*TY)$
 $VU(I, I-IJ+1) - VU(I, I-IJ+1) + G2/(TX*TY)$

$VV(I, I-1) - VV(I, I-1) + G/TX^{**2}$
 $VV(I, I) - VV(I, I) - 2.0 * G/TX^{**2}$
 $VV(I, I+1) - VV(I, I+1) + G/TX^{**2}$
 $VV(I, I-IJ) - VV(I, I-IJ) + 2.0 * G1/TY^{**2}$
 $VV(I, I) - VV(I, I) - 2.0 * G1/TY^{**2}$
 $VV(I, I-JK) - VV(I, I-JK) + G/TZ^{**2}$
 $VV(I, I) - VV(I, I) - 2.0 * G/TZ^{**2}$
 $VV(I, I+JK) - VV(I, I+JK) + G/TZ^{**2}$

$VW(I, I) - VW(I, I) + G2/(TY*TZ)$
 $VW(I, I-IJ) - VW(I, I-IJ) - G2/(TY*TZ)$
 $VW(I, I+JK) - VW(I, I+JK) - G2/(TY*TZ)$
 $VW(I, I+JK-IJ) - VW(I, I+JK-IJ) + G2/(TY*TZ)$

$WU(I, I) - WU(I, I) + G2/(TX*TZ)$
 $WU(I, I+1) - WU(I, I+1) - G2/(TX*TZ)$
 $WU(I, I+IK) - WU(I, I+IK) - G2/(TX*TZ)$
 $WU(I, I+IK+1) - WU(I, I+IK+1) + G2/(TX*TZ)$

$WV(I, I) - WV(I, I) + G2/(TY*TZ)$
 $WV(I, I-IJ) - WV(I, I-IJ) - G2/(TY*TZ)$
 $WV(I, I+JK) - WV(I, I+JK) - G2/(TY*TZ)$

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$$WV(I, I-IJ+JK) - WV(I, I-IJ+JK) + G2 / (TY * TZ)$$

$$WW(I, I-1) - WW(I, I-1) + G / TX ** 2$$

$$WW(I, I) - WW(I, I) - 2.0 * G / TX ** 2$$

$$WW(I, I+1) - WW(I, I+1) + G / TX ** 2$$

$$WW(I, I-IJ) - WW(I, I-IJ) + 2.0 * G / TY ** 2$$

$$WW(I, I) - WW(I, I) - 2.0 * G / TY ** 2$$

$$WW(I, I-IK) - WW(I, I-IK) + G1 / TZ ** 2$$

$$WW(I, I) - WW(I, I) - 2.0 * G1 / TZ ** 2$$

$$WW(I, I+IK) - WW(I, I+IK) + G1 / TZ ** 2$$

GO TO 2

104
$$UU(I, I-1) - UU(I, I-1) + 2.0 * G1 / TX ** 2$$

$$UU(I, I) - UU(I, I) - 2.0 * G1 / TX ** 2$$

$$UU(I, I-IJ) - UU(I, I-IJ) + 2.0 * G / TY ** 2$$

$$UU(I, I) - UU(I, I) - 2.0 * G / TY ** 2$$

$$UU(I, I-IK) - UU(I, I-IK) + G / TZ ** 2$$

$$UU(I, I) - UU(I, I) - 2.0 * G / TZ ** 2$$

$$UU(I, I+IK) - UU(I, I+IK) + G / TZ ** 2$$

$$UV(I, I) - UV(I, I) + G2 / (TX * TY)$$

$$UV(I, I-1) - UV(I, I-1) - G2 / (TX * TY)$$

$$UV(I, I-IJ) - UV(I, I-IJ) - G2 / (TX * TY)$$

$$UV(I, I-IJ-1) - UV(I, I-IJ-1) + G2 / (TX * TY)$$

$$UW(I, I) - UW(I, I) + G2 / (TX * TZ)$$

$$UW(I, I-1) - UW(I, I-1) - G2 / (TX * TZ)$$

$$UW(I, I+IK) - UW(I, I+IK) - G2 / (TX * TZ)$$

$$UW(I, I+IK-1) - UW(I, I+IK-1) + G2 / (TX * TZ)$$

$$VU(I, I) - VU(I, I) + G2 / (TX * TY)$$

$$VU(I, I-1) - VU(I, I-1) - G2 / (TX * TY)$$

$$VU(I, I-IJ) - VU(I, I-IJ) - G2 / (TX * TY)$$

$$VU(I, I-IJ-1) - VU(I, I-IJ-1) + G2 / (TX * TY)$$

$$VV(I, I-1) - VV(I, I-1) + 2.0 * G / TX ** 2$$

$$VV(I, I) - VV(I, I) - 2.0 * G / TX ** 2$$

$$VV(I, I-IJ) - VV(I, I-IJ) + 2.0 * G1 / TY ** 2$$

$$VV(I, I) - VV(I, I) - 2.0 * G1 / TY ** 2$$

$$VV(I, I-JK) - VV(I, I-JK) + G / TZ ** 2$$

$$VV(I, I) - VV(I, I) - 2.0 * G / TZ ** 2$$

$$VV(I, I+JK) - VV(I, I+JK) + G / TZ ** 2$$

$$VW(I, I) - VW(I, I) + G2 / (TY * TZ)$$

$$VW(I, I-IJ) - VW(I, I-IJ) - G2 / (TY * TZ)$$

$$VW(I, I+JK) - VW(I, I+JK) - G2 / (TY * TZ)$$

$$VW(I, I+JK-IJ) - VW(I, I+JK-IJ) + G2 / (TY * TZ)$$

$$WU(I, I) - WU(I, I) + G2 / (TX * TZ)$$

$$WU(I, I-1) - WU(I, I-1) - G2 / (TX * TZ)$$

$$WU(I, I+IK) - WU(I, I+IK) - G2 / (TX * TZ)$$

$$WU(I, I+IK-1) - WU(I, I+IK-1) + G2 / (TX * TZ)$$

$$WV(I, I) - WV(I, I) + G2 / (TY * TZ)$$

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DIF05900

DIF05910

DIF05920

DIF05930

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DIF05960

DIF05970

$WV(I, I-IJ) = WV(I, I-IJ) - G2 / (TY * TZ)$
 $WV(I, I+JK) = WV(I, I+JK) - G2 / (TY * TZ)$
 $WV(I, I+JK-IJ) = WV(I, I+JK-IJ) + G2 / (TY * TZ)$

$WW(I, I-1) = WW(I, I-1) + 2.0 * G / TX ** 2$
 $WW(I, I) = WW(I, I) - 2.0 * G / TX ** 2$
 $WW(I, I-IJ) = WW(I, I-IJ) + 2.0 * G / TY ** 2$
 $WW(I, I) = WW(I, I) - 2.0 * G / TY ** 2$
 $WW(I, I-1K) = WW(I, I-1K) + G1 / TZ ** 2$
 $WW(I, I) = WW(I, I) - 2.0 * G1 / TZ ** 2$
 $WW(I, I+1K) = WW(I, I+1K) + G1 / TZ ** 2$

GO TO 2

105 $UU(I, I-1) = UU(I, I-1) + G1 / TX ** 2$
 $UU(I, I) = UU(I, I) - 2.0 * G1 / TX ** 2$
 $UU(I, I+1) = UU(I, I+1) + G1 / TX ** 2$
 $UU(I, I-IJ) = UU(I, I-IJ) + G / TY ** 2$
 $UU(I, I) = UU(I, I) - 2.0 * G / TY ** 2$
 $UU(I, I+IJ) = UU(I, I+IJ) + G / TY ** 2$
 $UU(I, I) = UU(I, I) + 2.0 * G3 / TZ ** 2$
 $UU(I, I+1K) = UU(I, I+1K) - 1.0 * G3 / TZ ** 2$

$UV(I, I) = UV(I, I) + G2 / (TX * TY)$
 $UV(I, I+1) = UV(I, I+1) - G2 / (TX * TY)$
 $UV(I, I+IJ) = UV(I, I+IJ) - G2 / (TX * TY)$
 $UV(I, I+IJ+1) = UV(I, I+IJ+1) + G2 / (TX * TY)$

$VU(I, I) = VU(I, I) + G2 / (TX * TY)$
 $VU(I, I+1) = VU(I, I+1) - G2 / (TX * TY)$
 $VU(I, I+IJ) = VU(I, I+IJ) - G2 / (TX * TY)$
 $VU(I, I+IJ+1) = VU(I, I+IJ+1) + G2 / (TX * TY)$

$VV(I, I-1) = VV(I, I-1) + G / TX ** 2$
 $VV(I, I) = VV(I, I) - 2.0 * G / TX ** 2$
 $VV(I, I+1) = VV(I, I+1) + G / TX ** 2$
 $VV(I, I-IJ) = VV(I, I-IJ) + G1 / TY ** 2$
 $VV(I, I) = VV(I, I) - 2.0 * G1 / TY ** 2$
 $VV(I, I+IJ) = VV(I, I+IJ) + G1 / TY ** 2$
 $VV(I, I) = VV(I, I) + 2.0 * G3 / TZ ** 2$
 $VV(I, I+JK) = VV(I, I+JK) - 1.0 * G3 / TZ ** 2$

DO 61 II=1,13

NPA(II) = ABS(NP(II))

IF(NP(II).EQ.0) GO TO 61

IF(NP(II).NE.0) GO TO 62

62 $WW(I, NPA(II)) = WW(I, NPA(II)) + (NP(II) / NPA(II)) * R(II)$

61 CONTINUE

GO TO 2

106 $UU(I, I-1) = UU(I, I-1) + 2.0 * G1 / TX ** 2$
 $UU(I, I) = UU(I, I) - 2.0 * G1 / TX ** 2$
 $UU(I, I-IJ) = UU(I, I-IJ) + G / TY ** 2$
 $UU(I, I) = UU(I, I) - 2.0 * G / TY ** 2$
 $UU(I, I+IJ) = UU(I, I+IJ) + G / TY ** 2$

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 DIF070400
 DIF070500

UU(I,I)-UU(I,I)+2.0*G3/TZ**2
 UU(I,I+IK)-UU(I,I+IK)-1.0*G3/TZ**2

UV(I,I)-UV(I,I)+G2/(TX*TY)
 UV(I,I-1)-UV(I,I-1)-G2/(TX*TY)
 UV(I,I+IJ)-UV(I,I+IJ)-G2/(TX*TY)
 UV(I,I+IJ-1)-UV(I,I+IJ-1)+G2/(TX*TY)

VU(I,I)-VU(I,I)+G2/(TX*TY)
 VU(I,I-1)-VU(I,I-1)-G2/(TX*TY)
 VU(I,I+IJ)-VU(I,I+IJ)-G2/(TX*TY)
 VU(I,I+IJ-1)-VU(I,I+IJ-1)+G2/(TX*TY)

VV(I,I-1)-VV(I,I-1)+2.0*G/TX**2
 VV(I,I)-VV(I,I)-2.0*G/TX**2
 VV(I,I-IJ)-VV(I,I-IJ)+G1/TY**2
 VV(I,I)-VV(I,I)-2.0*G1/TY**2
 VV(I,I+IJ)-VV(I,I+IJ)+G1/TY**2
 VV(I,I)-VV(I,I)+2.0*G3/TZ**2
 VV(I,I+JK)-VV(I,I+JK)-1.0*G3/TZ**2

DO 63 II=1,13

NPA(II)-ABS(NP(II))

IF(NP(II).EQ.0) GO TO 63

IF(NP(II).NE.0) GO TO 64

64 WW(I,NPA(II))-WW(I,NPA(II))+(NP(II)/NPA(II))*R(II)

63 CONTINUE

GO TO 2

107 UU(I,I-1)-UU(I,I-1)+G1/TX**2
 UU(I,I)-UU(I,I)-2.0*G1/TX**2
 UU(I,I+1)-UU(I,I+1)+G1/TX**2
 UU(I,I-IJ)-UU(I,I-IJ)+2.0*G/TY**2
 UU(I,I)-UU(I,I)-2.0*G/TY**2
 UU(I,I)-UU(I,I)+2.0*G3/TZ**2
 UU(I,I+IK)-UU(I,I+IK)-1.0*G3/TZ**2

UV(I,I)-UV(I,I)+G2/(TX*TY)
 UV(I,I+1)-UV(I,I+1)-G2/(TX*TY)
 UV(I,I-IJ)-UV(I,I-IJ)-G2/(TX*TY)
 UV(I,I-IJ+1)-UV(I,I-IJ+1)+G2/(TX*TY)

VU(I,I)-VU(I,I)+G2/(TX*TY)
 VU(I,I+1)-VU(I,I+1)-G2/(TX*TY)
 VU(I,I-IJ)-VU(I,I-IJ)-G2/(TX*TY)
 VU(I,I-IJ+1)-VU(I,I-IJ+1)+G2/(TX*TY)

VV(I,I-1)-VV(I,I-1)+G/TX**2
 VV(I,I)-VV(I,I)-2.0*G/TX**2
 VV(I,I+1)-VV(I,I+1)+G/TX**2
 VV(I,I-IJ)-VV(I,I-IJ)+2.0*G1/TY**2
 VV(I,I)-VV(I,I)-2.0*G1/TY**2
 VV(I,I)-VV(I,I)+2.0*G3/TZ**2
 VV(I,I+JK)-VV(I,I+JK)-1.0*G3/TZ**2

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DO 65 II-1,13
NPA(II)=ABS(NP(II))
IF(NP(II).EQ.0) GO TO 65
IF(NP(II).NE.0) GO TO 66
66 WW(I,NPA(II))=WW(I,NPA(II))+(NP(II)/NPA(II))*R(II)
65 CONTINUE

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GO TO 2
108 UU(I,I-1)=UU(I,I-1)+2.0*G1/TX**2
UU(I,I)=UU(I,I)-2.0*G1/TX**2
UU(I,I-IJ)=UU(I,I-IJ)+2.0*G/TY**2
UU(I,I)=UU(I,I)-2.0*G/TY**2
UU(I,I)=UU(I,I)+2.0*G3/TZ**2
UU(I,I+IK)=UU(I,I+IK)-1.0*G3/TZ**2

UV(I,I)=UV(I,I)+G2/(TX*TY)
UV(I,I-1)=UV(I,I-1)-G2/(TX*TY)
UV(I,I-IJ)=UV(I,I-IJ)-G2/(TX*TY)
UV(I,I-IJ-1)=UV(I,I-IJ-1)+G2/(TX*TY)

VU(I,I)=VU(I,I)+G2/(TX*TY)
VU(I,I-1)=VU(I,I-1)-G2/(TX*TY)
VU(I,I-IJ)=VU(I,I-IJ)-G2/(TX*TY)
VU(I,I-IJ-1)=VU(I,I-IJ-1)+G2/(TX*TY)

VV(I,I-1)=VV(I,I-1)+2.0*G/TX**2
VV(I,I)=VV(I,I)-2.0*G/TX**2
VV(I,I-IJ)=VV(I,I-IJ)+2.0*G1/TY**2
VV(I,I)=VV(I,I)-2.0*G1/TY**2
VV(I,I)=VV(I,I)+2.0*G3/TZ**2
VV(I,I+JK)=VV(I,I+JK)-1.0*G3/TZ**2

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DO 67 II-1,13
NPA(II)=ABS(NP(II))
IF(NP(II).EQ.0) GO TO 67
IF(NP(II).NE.0) GO TO 68
68 WW(I,NPA(II))=WW(I,NPA(II))+(NP(II)/NPA(II))*R(II)
67 CONTINUE

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GO TO 2
109 UU(I,I)=UU(I,I)-2.0*G1/TX**2
UU(I,I+1)=UU(I,I+1)+G1/TX**2
UU(I,I)=UU(I,I)-2.0*G/TY**2
UU(I,I+IJ)=UU(I,I+IJ)+G/TY**2
UU(I,I)=UU(I,I)+2.0*G3/TZ**2
UU(I,I+IK)=UU(I,I+IK)-1.0*G3/TZ**2

UV(I,I)=UV(I,I)+G2/(TX*TY)
UV(I,I+1)=UV(I,I+1)-G2/(TX*TY)
UV(I,I+IJ)=UV(I,I+IJ)-G2/(TX*TY)
UV(I,I+IJ+1)=UV(I,I+IJ+1)+G2/(TX*TY)

VU(I,I)=VU(I,I)+G2/(TX*TY)

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VU(I,I+1)-VU(I,I+1)-G2/(TX*TY)
VU(I,I+IJ)-VU(I,I+IJ)-G2/(TX*TY)
VU(I,I+IJ+1)-VU(I,I+IJ+1)+G2/(TX*TY)

VV(I,I)-VV(I,I)-2.0*G/TX**2
VV(I,I+1)-VV(I,I+1)+G/TX**2
VV(I,I)-VV(I,I)-2.0*G1/TY**2
VV(I,I+IJ)-VV(I,I+IJ)+G1/TY**2
VV(I,I)-VV(I,I)+2.0*G3/TZ**2
VV(I,I+JK)-VV(I,I+JK)-1.0*G3/TZ**2

DO 69 II-1,13
NPA(II)-ABS(NP(II))
IF(NP(II).EQ.0) GO TO 69
IF(NP(II).NE.0) GO TO 70
70 WW(I,NPA(II))-WW(I,NPA(II))+(NP(II)/NPA(II))*R(II)
69 CONTINUE

GO TO 2

110 UU(I,I-1)-UU(I,I-1)+G1/TX**2
UU(I,I)-UU(I,I)-2.0*G1/TX**2
UU(I,I+1)-UU(I,I+1)+G1/TX**2
UU(I,I)-UU(I,I)-2.0*G/TY**2
UU(I,I+IJ)-UU(I,I+IJ)+G/TY**2
UU(I,I)-UU(I,I)+2.0*G3/TZ**2
UU(I,I+IK)-UU(I,I+IK)-1.0*G3/TZ**2

UV(I,I)-UV(I,I)+G2/(TX*TY)
UV(I,I+1)-UV(I,I+1)-G2/(TX*TY)
UV(I,I+IJ)-UV(I,I+IJ)-G2/(TX*TY)
UV(I,I+IJ+1)-UV(I,I+IJ+1)+G2/(TX*TY)

VU(I,I)-VU(I,I)+G2/(TX*TY)
VU(I,I+1)-VU(I,I+1)-G2/(TX*TY)
VU(I,I+IJ)-VU(I,I+IJ)-G2/(TX*TY)
VU(I,I+IJ+1)-VU(I,I+IJ+1)+G2/(TX*TY)

VV(I,I-1)-VV(I,I-1)+G/TX**2
VV(I,I)-VV(I,I)-2.0*G/TX**2
VV(I,I+1)-VV(I,I+1)+G/TX**2
VV(I,I)-VV(I,I)-2.0*G1/TY**2
VV(I,I+IJ)-VV(I,I+IJ)+G1/TY**2
VV(I,I)-VV(I,I)+2.0*G3/TZ**2
VV(I,I+JK)-VV(I,I+JK)-1.0*G3/TZ**2

DO 71 II-1,13
NPA(II)-ABS(NP(II))
IF(NP(II).EQ.0) GO TO 71
IF(NP(II).NE.0) GO TO 72
72 WW(I,NPA(II))-WW(I,NPA(II))+(NP(II)/NPA(II))*R(II)
71 CONTINUE

GO TO 2

111 UU(I,I)-UU(I,I)-2.0*G1/TX**2

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DIF0813

UU(I, I+1)-UU(I, I+1)+G1/TX**2	DIF0814
UU(I, I-IJ)-UU(I, I-IJ)+G/TY**2	DIF0815
UU(I, I)-UU(I, I)-2.0*G/TY**2	DIF0816
UU(I, I+IJ)-UU(I, I+IJ)+G/TY**2	DIF0817
UU(I, I)-UU(I, I)+2.0*G3/TZ**2	DIF0818
UU(I, I+IK)-UU(I, I+IK)-1.0*G3/TZ**2	DIF0819
UV(I, I)-UV(I, I)+G2/(TX*TY)	DIF0820
UV(I, I+1)-UV(I, I+1)-G2/(TX*TY)	DIF0821
UV(I, I+IJ)-UV(I, I+IJ)-G2/(TX*TY)	DIF0822
UV(I, I+IJ+1)-UV(I, I+IJ+1)+G2/(TX*TY)	DIF0823
VU(I, I)-VU(I, I)+G2/(TX*TY)	DIF0824
VU(I, I+1)-VU(I, I+1)-G2/(TX*TY)	DIF0825
VU(I, I+IJ)-VU(I, I+IJ)-G2/(TX*TY)	DIF0826
VU(I, I+IJ+1)-VU(I, I+IJ+1)+G2/(TX*TY)	DIF0827
VV(I, I)-VV(I, I)-2.0*G/TX**2	DIF0828
VV(I, I+1)-VV(I, I+1)+G/TX**2	DIF0829
VV(I, I-IJ)-VV(I, I-IJ)+G1/TY**2	DIF0830
VV(I, I)-VV(I, I)-2.0*G1/TY**2	DIF0831
VV(I, I+IJ)-VV(I, I+IJ)+G1/TY**2	DIF0832
VV(I, I)-VV(I, I)+2.0*G3/TZ**2	DIF0833
VV(I, I+JK)-VV(I, I+JK)-1.0*G3/TZ**2	DIF0834
DO 73 II-1, 13	DIF0835
NPA(II)-ABS(NP(II))	DIF0836
IF(NP(II).EQ.0) GO TO 73	DIF0837
IF(NP(II).NE.0) GO TO 74	DIF0838
74 WW(I, NPA(II))-WW(I, NPA(II))+(NP(II)/NPA(II))*R(II)	DIF0839
73 CONTINUE	DIF0840
GO TO 2	DIF0841
112 UU(I, I-1)-UU(I, I-1)+2.0*G1/TX**2	DIF0842
UU(I, I)-UU(I, I)-2.0*G1/TX**2	DIF0843
UU(I, I)-UU(I, I)-2.0*G/TY**2	DIF0844
UU(I, I+IJ)-UU(I, I+IJ)+G/TY**2	DIF0845
UU(I, I)-UU(I, I)+2.0*G3/TZ**2	DIF0846
UU(I, I+IK)-UU(I, I+IK)-1.0*G3/TZ**2	DIF0847
UV(I, I)-UV(I, I)+G2/(TX*TY)	DIF0848
UV(I, I-1)-UV(I, I-1)-G2/(TX*TY)	DIF0849
UV(I, I+IJ)-UV(I, I+IJ)-G2/(TX*TY)	DIF0850
UV(I, I+IJ-1)-UV(I, I+IJ-1)+G2/(TX*TY)	DIF0851
VU(I, I)-VU(I, I)+G2/(TX*TY)	DIF0852
VU(I, I-1)-VU(I, I-1)-G2/(TX*TY)	DIF0853
VU(I, I+IJ)-VU(I, I+IJ)-G2/(TX*TY)	DIF0854
VU(I, I+IJ-1)-VU(I, I+IJ-1)+G2/(TX*TY)	DIF0855
VV(I, I-1)-VV(I, I-1)+2.0*G/TX**2	DIF0856
VV(I, I)-VV(I, I)-2.0*G/TX**2	DIF0857
VV(I, I)-VV(I, I)-2.0*G1/TY**2	DIF0858
VV(I, I+IJ)-VV(I, I+IJ)+G1/TY**2	DIF0859

VV(I,I)-VV(I,I)+2.0*G3/TZ**2
 VV(I,I+JK)-VV(I,I+JK)-1.0*G3/TZ**2

DO 75 II-1,13
 NPA(II)-ABS(NP(II))
 IF(NP(II).EQ.0) GO TO 75
 IF(NP(II).NE.0) GO TO 76

76 WW(I,NPA(II))-WW(I,NPA(II))+(NP(II)/NPA(II))*R(II)
 75 CONTINUE

GO TO 2

113 UU(I,I)-UU(I,I)-2.0*G1/TX**2
 UU(I,I+1)-UU(I,I+1)+G1/TX**2
 UU(I,I-IJ)-UU(I,I-IJ)+2.0*G/TY**2
 UU(I,I)-UU(I,I)-2.0*G/TY**2
 UU(I,I)-UU(I,I)+2.0*G3/TZ**2
 UU(I,I+IK)-UU(I,I+IK)-1.0*G3/TZ**2

UV(I,I)-UV(I,I)+G2/(TX*TY)
 UV(I,I+1)-UV(I,I+1)-G2/(TX*TY)
 UV(I,I-IJ)-UV(I,I-IJ)-G2/(TX*TY)
 UV(I,I-IJ+1)-UV(I,I-IJ+1)+G2/(TX*TY)

VU(I,I)-VU(I,I)+G2/(TX*TY)
 VU(I,I+1)-VU(I,I+1)-G2/(TX*TY)
 VU(I,I-IJ)-VU(I,I-IJ)-G2/(TX*TY)
 VU(I,I-IJ+1)-VU(I,I-IJ+1)+G2/(TX*TY)

VV(I,I)-VV(I,I)-2.0*G/TX**2
 VV(I,I+1)-VV(I,I+1)+G/TX**2
 VV(I,I-IJ)-VV(I,I-IJ)+2.0*G1/TY**2
 VV(I,I)-VV(I,I)-2.0*G1/TY**2
 VV(I,I)-VV(I,I)+2.0*G3/TZ**2
 VV(I,I+JK)-VV(I,I+JK)-1.0*G3/TZ**2

DO 77 II-1,13
 NPA(II)-ABS(NP(II))
 IF(NP(II).EQ.0) GO TO 77
 IF(NP(II).NE.0) GO TO 78

78 WW(I,NPA(II))-WW(I,NPA(II))+(NP(II)/NPA(II))*R(II)
 77 CONTINUE

GO TO 2

114 UU(I,I-IK)-UU(I,I-IK)+G/TZ**2
 UU(I,I)-UU(I,I)-2.*G/TZ**2
 UU(I,I+IK)-UU(I,I+IK)+G/TZ**2

VU(I,I)-VU(I,I)+G3/(TX*TY)
 VU(I,I+1)-VU(I,I+1)-G3/(TX*TY)
 VU(I,I+IJ)-VU(I,I+IJ)-G3/(TX*TY)
 VU(I,I+IJ+1)-VU(I,I+IJ+1)+G3/(TX*TY)

VV(I,I)-VV(I,I)-2.*G1/TY**2
 VV(I,I+IJ)-VV(I,I+IJ)+G1/TY**2

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VV(I, I-JK)-VV(I, I-JK)+G/TZ**2
 VV(I, I)-VV(I, I)-2.*G/TZ**2
 VV(I, I+JK)-VV(I, I+JK)+G/TZ**2

GO TO 2

115 UU(I, I-IK)-UU(I, I-IK)+G/TZ**2
 UU(I, I)-UU(I, I)-2.*G/TZ**2
 UU(I, I+IK)-UU(I, I+IK)+G/TZ**2

VU(I, I)-VU(I, I)+G3/(TX*TY)
 VU(I, I+1)-VU(I, I+1)-G3/(TX*TY)
 VU(I, I+IJ)-VU(I, I+IJ)-G3/(TX*TY)
 VU(I, I+IJ+1)-VU(I, I+IJ+1)+G3/(TX*TY)

VV(I, I-IJ)-VV(I, I-IJ)+G1/TY**2
 VV(I, I)-VV(I, I)-2.*G1/TY**2
 VV(I, I+IJ)-VV(I, I+IJ)+G1/TY**2
 VV(I, I-JK)-VV(I, I-JK)+G/TZ**2
 VV(I, I)-VV(I, I)-2.*G/TZ**2
 VV(I, I+JK)-VV(I, I+JK)+G/TZ**2

GO TO 2

116 UU(I, I-IK)-UU(I, I-IK)+G/TZ**2
 UU(I, I)-UU(I, I)-2.*G/TZ**2
 UU(I, I+IK)-UU(I, I+IK)+G/TZ**2

VU(I, I)-VU(I, I)+G3/(TX*TY)
 VU(I, I+1)-VU(I, I+1)-G3/(TX*TY)
 VU(I, I-IJ)-VU(I, I-IJ)-G3/(TX*TY)
 VU(I, I-IJ+1)-VU(I, I-IJ+1)+G3/(TX*TY)

VV(I, I-IJ)-VV(I, I-IJ)+2.*G1/TY**2
 VV(I, I)-VV(I, I)-2.*G1/TY**2
 VV(I, I-IK)-VV(I, I-IK)+G/TZ**2
 VV(I, I)-VV(I, I)-2.*G/TZ**2
 VV(I, I+IK)-VV(I, I+IK)+G/TZ**2

GO TO 2

117 UU(I, I-1)-UU(I, I-1)+G1/TX**2
 UU(I, I)-UU(I, I)-2.*G1/TX**2
 UU(I, I+1)-UU(I, I+1)+G1/TX**2
 UU(I, I-IK)-UU(I, I-IK)+G/TZ**2
 UU(I, I)-UU(I, I)-2.*G/TZ**2
 UU(I, I+IK)-UU(I, I+IK)+G/TZ**2

UV(I, I)-UV(I, I)+G3/(TX*TY)
 UV(I, I+1)-UV(I, I+1)-G3/(TX*TY)
 UV(I, I+IJ)-UV(I, I+IJ)-G3/(TX*TY)
 UV(I, I+IJ+1)-UV(I, I+IJ+1)+G3/(TX*TY)

VV(I, I-JK)-VV(I, I-JK)+G/TZ**2
 VV(I, I)-VV(I, I)-2.*G/TZ**2
 VV(I, I+JK)-VV(I, I+JK)+G/TZ**2

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 DIF0975

GO TO 2

118 $UU(I, I-1) = UU(I, I-1) + 2 * G1 / TX ** 2$
 $UU(I, I) = UU(I, I) - 2 * G1 / TX ** 2$
 $UU(I, I-1K) = UU(I, I-1K) + G / TZ ** 2$
 $UU(I, I) = UU(I, I) - 2 * G / TZ ** 2$
 $UU(I, I+1K) = UU(I, I+1K) + G / TZ ** 2$

$UV(I, I) = UV(I, I) + G3 / (TX * TY)$
 $UV(I, I-1) = UV(I, I-1) - G3 / (TX * TY)$
 $UV(I, I+1J) = UV(I, I+1J) - G3 / (TX * TY)$
 $UV(I, I+1J-1) = UV(I, I+1J-1) + G3 / (TX * TY)$

$VV(I, I-1JK) = VV(I, I-1JK) + G / TZ ** 2$
 $VV(I, I) = VV(I, I) - 2 * G / TZ ** 2$
 $VV(I, I+1JK) = VV(I, I+1JK) + G / TZ ** 2$

GO TO 2

119 $VU(I, I) = VU(I, I) + G3 / (TX * TY)$
 $VU(I, I+1) = VU(I, I+1) - G3 / (TX * TY)$
 $VU(I, I+1J) = VU(I, I+1J) - G3 / (TX * TY)$
 $VU(I, I+1J+1) = VU(I, I+1J+1) + G3 / (TX * TY)$

$VV(I, I-1J) = VV(I, I-1J) + G1 / TY ** 2$
 $VV(I, I) = VV(I, I) - 2 * G1 / TY ** 2$
 $VV(I, I+1J) = VV(I, I+1J) + G1 / TY ** 2$

GO TO 2

120 $VU(I, I) = VU(I, I) + G3 / (TX * TY)$
 $VU(I, I+1) = VU(I, I+1) - G3 / (TX * TY)$
 $VU(I, I-1J) = VU(I, I-1J) - G3 / (TX * TY)$
 $VU(I, I-1J+1) = VU(I, I-1J+1) + G3 / (TX * TY)$

$VV(I, I-1J) = VV(I, I-1J) + 2 * G1 / TY ** 2$
 $VV(I, I) = VV(I, I) - 2 * G1 / TY ** 2$

GO TO 2

121 $UU(I, I-1) = UU(I, I-1) + G1 / TX ** 2$
 $UU(I, I) = UU(I, I) - 2 * G1 / TX ** 2$
 $UU(I, I+1) = UU(I, I+1) + G1 / TX ** 2$

$UV(I, I) = UV(I, I) + G3 / (TX * TY)$
 $UV(I, I+1) = UV(I, I+1) - G3 / (TX * TY)$
 $UV(I, I+1J) = UV(I, I+1J) - G3 / (TX * TY)$
 $UV(I, I+1J+1) = UV(I, I+1J+1) + G3 / (TX * TY)$

GO TO 2

122 $UU(I, I) = UU(I, I) - 2 * G1 / TX ** 2$
 $UU(I, I-1) = UU(I, I-1) + 2 * G1 / TX ** 2$

$UV(I, I) = UV(I, I) + G3 / (TX * TY)$
 $UV(I, I-1) = UV(I, I-1) - G3 / (TX * TY)$
 $UV(I, I+1J) = UV(I, I+1J) - G3 / (TX * TY)$
 $UV(I, I+1J-1) = UV(I, I+1J-1) + G3 / (TX * TY)$

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DIF10250
DIF10260
DIF10270
DIF10280
DIF10290

GO TO 2
 123 UU(I,I)-UU(I,I)-2.*G1/TX**2
 UU(I,I+1)-UU(I,I+1)+G1/TX**2

 UV(I,I)-UV(I,I)+G3/(TX*TY)
 UV(I,I+1)-UV(I,I+1)-G3/(TX*TY)
 UV(I,I+IJ)-UV(I,I+IJ)-G3/(TX*TY)
 UV(I,I+IJ+1)-UV(I,I+IJ+1)+G3/(TX*TY)

 GO TO 2
 124 VU(I,I)-VU(I,I)+G3/(TX*TY)
 VU(I,I+1)-VU(I,I+1)-G3/(TX*TY)
 VU(I,I+IJ)-VU(I,I+IJ)-G3/(TX*TY)
 VU(I,I+IJ+1)-VU(I,I+IJ+1)+G3/(TX*TY)

 VV(I,I)-VV(I,I)-2.*G1/TY**2
 VV(I,I+IJ)-VV(I,I+IJ)+G1/TY**2

 GO TO 2
 125 UU(I,I)-UU(I,I)-2.*G1/TX**2
 UU(I,I+1)-UU(I,I+1)+G1/TX**2
 UU(I,I-1K)-UU(I,I-1K)+G/TZ**2
 UU(I,I)-UU(I,I)-2.*G/TZ**2
 UU(I,I+1K)-UU(I,I+1K)+G/TZ**2

 UV(I,I)-UV(I,I)+G3/(TX*TY)
 UV(I,I+1)-UV(I,I+1)-G3/(TX*TY)
 UV(I,I+IJ)-UV(I,I+IJ)-G3/(TX*TY)
 UV(I,I+IJ+1)-UV(I,I+IJ+1)+G3/(TX*TY)

 VV(I,I-1K)-VV(I,I-1K)+G/TZ**2
 VV(I,I)-VV(I,I)-2.*G/TZ**2
 VV(I,I+1K)-VV(I,I+1K)+G/TZ**2

 GO TO 2
 2 CONTINUE
 DO 3 M-1,3*NOD
 DO 3 N-1,3*NOD
 AA(M,N)-0.0
 3 CONTINUE
 DO 4 M-1,3*NOD
 IF(M.GT.2*NOD) GO TO 30
 IF(M.GT.NOD) GO TO 20
 IF(M.LE.NOD) GO TO 10
 10 DO 5 N-1,3*NOD
 IF(N.GT.2*NOD) GO TO 31
 IF(N.GT.NOD) GO TO 21
 IF(N.LE.NOD) GO TO 11
 11 AA(M,N)-UU(M,N)
 GO TO 5
 21 AA(M,N)-UV(M,N-NOD)
 GO TO 5
 31 AA(M,N)-UW(M,N-2*NOD)
 GO TO 5

DIF10300
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 DIF10830

5	CONTINUE	DIF10840
	GO TO 4	DIF10850
20	DO 6 N-1,3*NOD	DIF10860
	IF(N.GT.2*NOD) GO TO 32	DIF10870
	IF(N.GT.NOD) GO TO 22	DIF10880
	IF(N.LE.NOD) GO TO 12	DIF10890
12	AA(M,N)-VU(M-NOD,N)	DIF10900
	GO TO 6	DIF10910
22	AA(M,N)-VV(M-NOD,N-NOD)	DIF10920
	GO TO 6	DIF10930
32	AA(M,N)-VW(M-NOD,N-2*NOD)	DIF10940
	GO TO 6	DIF10950
6	CONTINUE	DIF10960
	GO TO 4	DIF10970
30	DO 7 N-1,3*NOD	DIF10980
	IF(N.GT.2*NOD) GO TO 33	DIF10990
	IF(N.GT.NOD) GO TO 23	DIF11000
	IF(N.LE.NOD) GO TO 13	DIF11010
13	AA(M,N)-WU(M-2*NOD,N)	DIF11020
	GO TO 7	DIF11030
23	AA(M,N)-WV(M-2*NOD,N-NOD)	DIF11040
	GO TO 7	DIF11050
33	AA(M,N)-WW(M-2*NOD,N-2*NOD)	DIF11060
	GO TO 7	DIF11070
7	CONTINUE	DIF11080
	GO TO 4	DIF11090
4	CONTINUE	DIF11100
C	PRINT 1012	DIF11110
C	WRITE(6,1002)((AA(M,N),N-1,3*NOD),M-1,3*NOD)	DIF11120
	DO 8 K-1,3*NOD	DIF11130
	FOR(K,1)-0.0	DIF11140
8	CONTINUE	DIF11150
	DO 9 K-1,3*NOD	DIF11160
	IF(K.GT.2*NOD) GO TO 14	DIF11170
	IF(K.LE.2*NOD) GO TO 15	DIF11180
14	FOR(K,1)-FF(K-2*NOD,1)	DIF11190
	GO TO 9	DIF11200
15	FOR(K,1)-0.0	DIF11210
	GO TO 9	DIF11220
9	CONTINUE	DIF11230
	CALL MATINV(AA,60)	DIF11240
C	PRINT 1012	DIF11250
C	WRITE(6,1002)((AA(M,N),N-1,3*NOD),M-1,3*NOD)	DIF11260
	CALL MATMUL(AA,60,60,FOR,1,DIS)	DIF11270
C	WRITE(6,1014) (FF(I,1),I-1,NOD)	DIF11280
C	WRITE(6,1014) (I,FOR(I,1),FOR(I+NOD,1),FOR(I+2*NOD,1),I-1,NOD)	DIF11290
C	WRITE(6,*)((AA(M,N),N-1,3*NOD),M-1,3*NOD)	DIF11300
C	PRINT 1003	DIF11310
C	WRITE(6,1001)((UU(I,J),J-1,20),I-1,20)	DIF11320
C	PRINT 1004	DIF11330
C	WRITE(6,1001)((UV(I,J),J-1,20),I-1,20)	DIF11340
C	PRINT 1005	DIF11350
C	WRITE(6,1001)((UW(I,J),J-1,20),I-1,20)	DIF11360
C	PRINT 1006	DIF11370

IF (INDEX(J,1)) 112,113,112	DIF1192
113 TEMP=DABS(AA(I,J))	DIF1193
IF (TEMP-AMAX) 112;112,114	DIF1194
114 IROW-I	DIF1195
ICOL-J	DIF1196
AMAX-TEMP	DIF1197
112 CONTINUE	DIF1198
110 CONTINUE	DIF1199
IF (AMAX) 225,115,116	DIF1200
116 INDEX(ICOL,1)-IROW	DIF1201
IF (IROW-ICOL) 119,118,119	DIF1202
119 DO 120 J-1,NN	DIF1203
TEMP-AA(IROW,J)	DIF1204
AA(IROW,J)-AA(ICOL,J)	DIF1205
120 AA(ICOL,J)-TEMP	DIF1206
II-II+1	DIF1207
INDEX(II,2)-ICOL	DIF1208
118 PIVOT-AA(ICOL,ICOL)	DIF1209
AA(ICOL,ICOL)-1.	DIF1210
PIVOT-1./PIVOT	DIF1211
DO 121 J-1,NN	DIF1212
121 AA(ICOL,J)-AA(ICOL,J)*PIVOT	DIF1213
DO 122 I-1,NN	DIF1214
IF (I-ICOL) 123,122,123	DIF1215
123 TEMP-AA(I,ICOL)	DIF1216
AA(I,ICOL)-0.	DIF1217
DO 124 J-1,NN	DIF1218
124 AA(I,J)-AA(I,J)-AA(ICOL,J)*TEMP	DIF1219
122 CONTINUE	DIF1220
GO TO 109	DIF1221
125 ICOL=INDEX(II,2)	DIF1222
IROW=INDEX(ICOL,1)	DIF1223
DO 126 I-1,NN	DIF1224
TEMP-AA(I,IROW)	DIF1225
AA(I,IROW)-AA(I,ICOL)	DIF1226
126 AA(I,ICOL)-TEMP	DIF1227
II-II-1	DIF1228
225 IF(II) 125,127,125	DIF1229
115 WRITE (6,1001)	DIF1230
1001 FORMAT(1H0,2X,11H ZERO PIVOT)	DIF1231
127 CONTINUE	DIF1232
RETURN	DIF1233
END	DIF1234
*****	DIF1235
*SUBROUTINE PROGRAM FOR CALCULATING MATRIX *	DIF1236
* MULTIPLICATION *	DIF1237
*****	DIF1238
C	DIF1239
C	DIF1240
C	DIF1241
C	DIF1242
C	DIF1243
C	DIF1244
C	DIF1245
SUBROUTINE MATMUL(AA, NN, MM, BB, LL, CC)	
IMPLICIT REAL*8 (A-H, O-Z)	
DIMENSION AA(NN, MM), BB(MM, LL), CC(NN, LL)	
DO 100 I-1, NN	
DO 200 J-1, LL	

CC(I,J)=0.0
DO 300 K1=1,MM
CC(I,J)=CC(I,J)+AA(I,K1)*BB(K1,J)

300 CONTINUE
200 CONTINUE
100 CONTINUE
RETURN
END

DIF1246
DIF1247
DIF1248
DIF1249
DIF1250
DIF1251
DIF1252
DIF1253