Using Time Series Models to Analyze Large Financial Data Sets

Over the years, finance has become more and more complex. Financial data are random and very irregular. One of the particular problems is to study which financial data have the leverage property. In particular, if the data do not have the leverage, they cannot be modeled by the commonly used GARCH process, and will guide practitioners that SV model should be used instead. In this project, we test for the presence of leverage in financial time series. We start by describing different stationary time series models used to simulate financial data. These include stochastic volatility models and GARCH models. Then we estimate stochastic volatility models using different approaches. In this project, we will use existing theoretical results data as a benchmark for our extensive GARCH model experiments We also simulate a large number of time series for the ARCH and GARCH model.

Definitions:

Time series is a random sequence, measured typically at successive equally spaced time points.

Time Series Models

Autoregressive-moving-average (ARMA) models provide a parsimonious description of a (weakly) stationary stochastic process in terms of two polynomials, one for the auto-regression and the second for the moving average. For example, AR(1) model is defined as

$$X_t = \phi X_{t-1} + Z_t,$$

where Z_t is a white noise.

A **(G)ARCH sequence** is used to model and predict volatility (of e.g. stocks, XE rates etc.), based on historical values through model fitting. Recent data is given more significance than older data. For example, ARCH(1).

$$X_t = \sigma_t Z_t,$$

where Z_t is a white noise and

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2.$$

A GARCH(1,1) sequence is defined by

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 + \beta_1 \sigma_{t-1}^2.$$

Leverage usually understood as a negative dependence between previous returns and future volatility. This means that large negative returns should usually be followed by high volatility. There are many ways to represent leverage. One can think of it as a covariance coefficient, or as the conditional tail probability:

$$L_q = E[X_{t-1}|X_t|^q].$$

If $L_q = 0$ then there is no leverage.

Conditional tail probability:

$$\psi = \lim_{x\to\infty} P(X_t > x | X_{t-1} > x).$$

If $\psi = 0$ then there is a no leverage.

We deal with the leverage using the covariance coefficient approach. We use the following test statistic:

$$V_m = \frac{\sum_{t=2}^n X_{t-1} X_t^2}{\sqrt{\sum_{t=2}^n X_{t-1}^2 X_t^4}}.$$

Under the appropriate assumptions, this test statistics is asymptotically standard normal.

Theoretical properties: for ARCH(1) or GARCH(1,1) or AR(1), there is no leverage whenever the white noise is symmetric (for example normal with mean zero).

Question :
Do different value of phi and alpha1 have impact on AR(1) and ARCH(1) respectively?

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We conducted an extensive simulation for different time series models. We test null hypothesis "there is no leverage" on the level of significance of 5%. Hence, in theory, we should reject the hypothesis about 5% of time.

For ARCH(1) we have the following results according to different values of α_1. These results indicate that the procedure works properly for ARCH(1). On the other hand, this approach to measure leverage is not good for AR(1) model, since we rejected too many times hypothesis test of "no leverage". In practice, one should not use the test for linear time series models like AR(1).

α_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Rejection	0.046	0.037	0.052	0.05	0.051	0.036	0.036	0.038	0.026

Table 1: ARCH(1) with $\alpha_0 = 1$ and different values of α_1

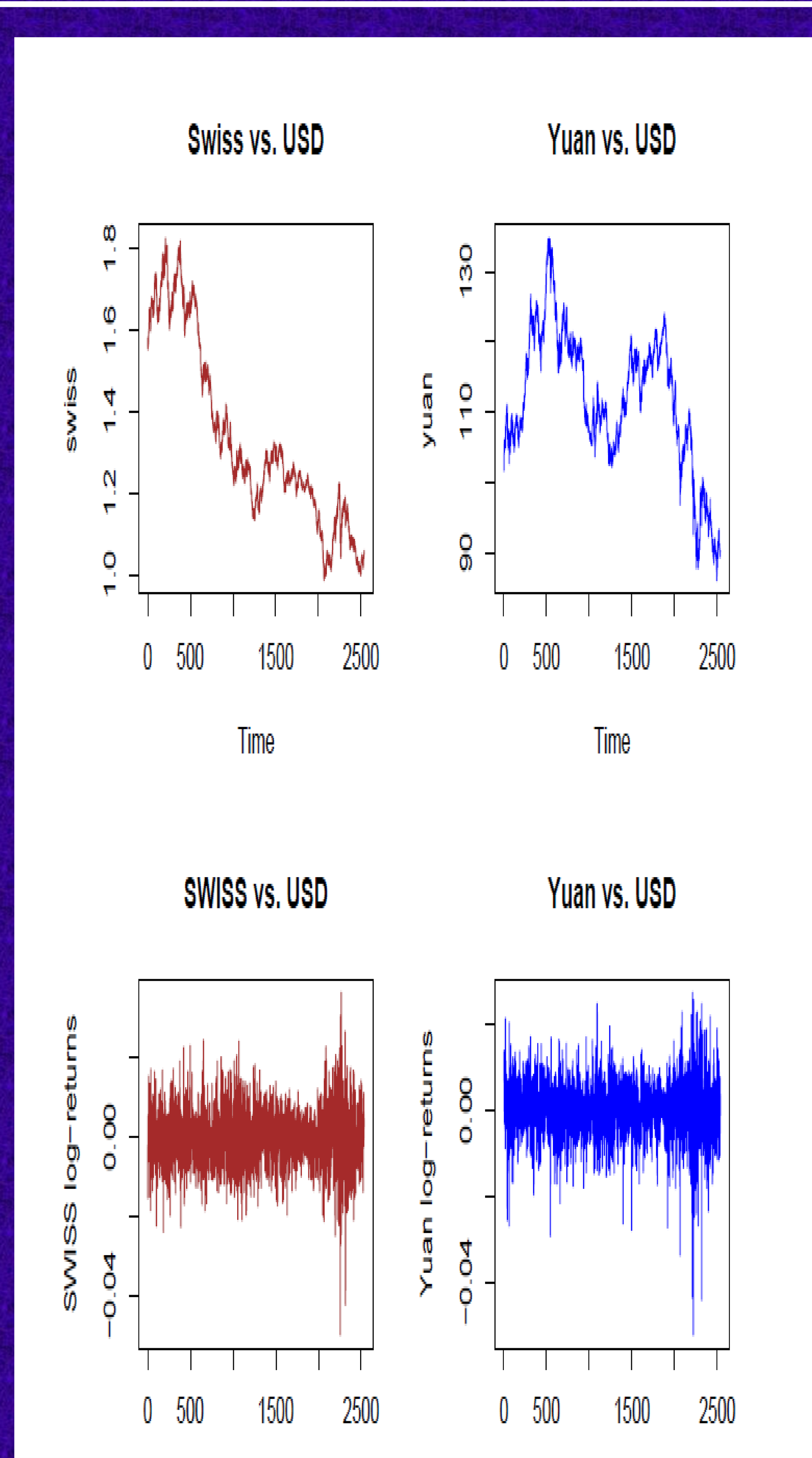
ϕ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Rejection	0.068	0.087	0.124	0.17	0.243	0.301	0.386	0.476	0.655

Table 2: AR(1) with different values of ϕ

The conclusion we get from the two tables above I that the results indicate that the procedure works properly on the AR(1) model. However, not so good for the ARCH(1) for we have rejected too many times as phi gets bigger. Therefore, we should not use the test for models such like AR(1)

Apply the knowledge into real life:

We applied our test to two data sets: Swiss/USD exchange rates and Yuan/USD exchange rates. Specifically, for two data sets displayed, we computed log-returns. Log-returns can be modeled using GARCH processes, hence we are allowed to use our test. In both cases we rejected the presence of leverage.



CONCLUSIONS:

- **The conclusion we got from the theoretical part is that for different values of alpha1, we have small rejection probability (less than significance level) for ARCH(1), However, we have large rejection probability on the AR(1) model. This indicates that one should not use the test for linear time series models like AR(1).**
- **From the exchange rates data, we specifically computed log-returns for both data sets. The results we got indicate that there is no leverage in both data sets..**

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