

A Heuristic Approach for the Home Health Care Scheduling and Routing Problem

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Abstract

Home Health Care (HHC) is a health care service delivered by sending caregivers such as nurses or personal support workers (PSW) to visit patients in their homes. The assignment of patients to nurses as well as the sequencing of patients for each nurse is called the Home Health Care Scheduling and Routing Problem (HHCSR). This thesis proposes a heuristic approach to solve HHCSR to which it is hard and even impossible to obtain an optimal solution for relative larger instances in a reasonable amount of computational time by using an exact algorithm as HHCSR is NP hard. In the approach, this thesis developed and contributed a heuristic partition method to partition patients into a number of single nurse groups. The computational test result shows that the proposed approach can achieve good solutions which remain within 5% of the commercial solver CPLEX's best solution using an acceptable solution time on all test instances.

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1. Introduction

Home Health Care (HHC) is a health care service delivered by sending caregivers such as nurses or personal support workers (PSW) to visit patients in their homes. The health care service provided may include wound care, injection, or monitoring chronic illness. With HHC service, a patient does not have to go to a health center or stay in a hospital thus releasing hospital capacity pressure and lowering system-wide costs of delivering health care (Milburn, 2012). Therefore, HHC service is an essential component of the health care system (Milburn, 2012). HHC service is especially welcomed by the elderly, by people who are chronically ill, and individuals with disabilities (CMS, 2008). However, HHC providers are facing nurse shortages and high labor costs. A baseline scenario estimated a shortage of about 11,000RN FTEs (Registered Nurses Full Time Equivalent) in 2007 for Canada, increasing to over 60,000 by 2022. (Tomblin Murphy, 2012). Therefore, using operation research methodology to optimize the assignment of patients to nurses as well as the sequencing of patients for each nurse called the Home Health Care Scheduling and Routing Problem (HHCSR) is a practical means to address these challenges.

In this manuscript, the HHCSR problem is inspired by real-world challenges faced by a HHC provider called CareFor. Every day CareFor assigns workers (workers include nurses and personal support workers) and a pool of casual workers as well to patients and sequences visits so that patients are seen within pre-specified visit time windows while also considering the workers' availability, location, and skillset. The goal is to minimize the total travel time required by workers and the total resource cost. However, dynamic patient demand, changing staff availability, and various skill requirements make the assignment task challenging. (Home Care Services | ComForCare).

The problem in this manuscript can be considered as a Multiple Depot Vehicle Routing Problem with Time Windows (MDVRP-TW) which is one variant of the Vehicle Routing Problem (VRP). For that reason, we choose to formulate the problem as a Mixed Integer Linear Programming (MILP) model. For small instances in which the number of nurses and patients is not too large, commercial solvers can solve the MILP model and obtain an optimal solution. For example, we used CPLEX solver for instances where both the number of nurses and patients are less than 20. However, it is hard and even impossible to obtain an optimal solution for larger instances in a reasonable amount of computational time. It has been proven that the Vehicle Routing Problem with Time Windows (VRPTW) is NP-hard (Marius M. Solomon, 1988) since Solomon (1986) and Savelsbergh (1984) found it is fundamentally more difficult than VRP whose mathematical complexity has also been shown to be NP-hard. (J.K. Lenstra, 1981). To the best of our knowledge, even the best exact algorithms cannot solve vehicle routing problem (VRP) instances on the scale experienced by HHC providers such as CareFor (Subramanian, Uchoa, & Ochi, 2013) in a reasonable amount of time. For that reason, we propose a heuristic approach to solve the MILP model. In this approach, we first develop a heuristic partition method to assign a group of patients to nurses. Each nurse is assigned a set of patients forming a small single nurse group where there is only one nurse and a small number of patients to visit. Then for each single nurse group, we solve a mixed integer linear model using CPLEX to obtain the optimal visiting route for the nurse. Then, all visiting routes of the nurses constitute an initial solution to the problem. Finally, an improvement step is applied to the initial solution to achieve the final solution. The critical part of this approach is the proposed heuristic partition method. To implement the method, we define a measurement called maximum distance (MD). MD is the longest Euclidean distance directly between the nurse and a patient within a single nurse group.

When partitioning, our method features assigns a patient to a single nurse group that has the shortest MD among all eligible single nurse groups.

In the computational tests, we compare the solution time and objective function value of a straightforward mixed integer linear model (MILP) solved by the proposed approach and by CPLEX on four different randomly generated instance sets. The test results demonstrate that, for small instances in which the number of nurses is 2 or 3 and the number of patients is 5 or 10, our method can determine the same optimal solutions as CPLEX on almost all test instances. On relatively larger instances in which the number of nurses is 6 or 7 and the number of patients is 20 or 23, where CPLEX struggles to find optimal solution within 600 seconds and 1200 seconds respectively, our method can still determine good solutions within 5% of the best CPLEX solution using around 50% less solution time than CPLEX.

This manuscript is organized as follows. In section 2, a literature review is presented. The problem statement and research question are presented in sections 3 and 4. Our mathematical model and proposed approach are described in sections 5 and 6. In section 7, our proposed approach is compared to the optimal results. Finally, we discuss our research and give future research directions in sections 8.

2. Literature Review

The Home Health Care Scheduling and Routing Problem (HHCSR) is a variant of the Vehicle Routing Problem (VRP). What makes HHCSR more complicated than a general VRP are the various additional constraints applied to the problem. One of these constraints is a temporal dependency between visits. For example, a patient may need a nurse visit 3 days before another nurse visit. (Cissé, et al., 2017). Another challenge in HHCSR is the dynamic and stochastic

nature of the problem. A deterministic version of the problem is that all patient's requests are fully known (Demirbilek, Branke, & Strauss, 2019) and there is no uncertainty in such parameters as service time, travel time and patient demand. However, in the real world, patient requests arrive dynamically during the service horizon (Demirbilek, Branke, & Strauss, 2019) and it is common that, for example, travel time is uncertain due to traffic. This dynamic and stochastic aspect of the problem make the HHSCRП even more of a challenge to solve. In this literature review, we introduce some remarkable papers that demonstrate the complexity of HHCSRП and more important inspire us to use a creative methodology to solve the problem in this manuscript.

The approach developed in this manuscript is inspired by the two-stage method used in (Hiermann, Prandtstetter, Rendl, Puchinger, & Raidl, 2013). The paper presents a general framework for solving a real-world multimodal home-health-care scheduling (MHS) problem from a major Austrian home-healthcare provider. The MHS problem is a combination of the vehicle routing problem with time windows (VRPTW) and the nurse rostering problem (NRP). It aims to find an assignment of nurses to patient requests and scheduling tours for visiting the patients while minimizing violations of side constraints and the total travel time. A two-stage method is used to solve the problem. First, an initial solution is generated via a constraint programming technique or a random procedure. Then, one of four metaheuristics: variable neighborhood search, a memetic algorithm, scatter search, and a simulated annealing hyper-heuristic is used to improve the initial solution. The computational results show that the approach is capable of solving real-world instances in reasonable time and produces valid solutions within a few seconds.

Heuristic approach is increasingly used and heavily developed to address the challenges in solving HHCSR. This fact also inspired us to try to use heuristic approach in this manuscript (Akjiratikarl, Yenradee, & Drake, 2007) presents a new application of a meta-heuristic technique called Particle Swarm Optimization (PSO) to the scheduling of home care workers in an actual situation arising in the UK. The problem is how to dispatch care workers on a daily base to minimize the total distance traveled while satisfying a number of constraints (listed below). The problem is derived from discussion with Ceredigion local authority, one of the members of the Welsh Systems Consortium and results are based on an analysis of its home care data. The PSO-based algorithm has been tested on ‘real’ demand data and produced significantly and consistently better results across all the test problems than those obtained with the existing manual approach and those obtained by the AiMES Centre at University of Liverpool using CPLEX. (Mankowska, Meisel, & Bierwirth, 2014) presents a model for the daily planning of home health care service by staff members of a home care company. The planning takes into account the individual service requirements of the patients, individual qualifications of the staff, and possible interdependencies between different service operations. A heuristic approach is developed to solve the problem. The numerical experiments indicate that the method consistently achieves low traveling costs for caregivers, lower waiting time for patients, a fair distribution of inevitable tardiness, or a combination thereof. At the same time, very large problem instances with several hundred patients can be treated in acceptable runtime. The paper also features involving temporal interdependencies of service operations. Interdependencies of services can include, for example, temporal separation of two services as is required if drugs have to be administered at specific times before providing a meal. Other services like handling a disabled patient may require two members working together at a patient’s home.

As mentioned above one factor which complicated the HHCSR is the temporal dependency between visits. The problem in (Rasmussen, Justesen, Dohn, & Larsen, 2012) is complicated as it involves soft preference constraints and temporal dependencies between the start times of visits. When a customer needs two home care workers, temporal dependency means the sequence of the two visits is important. It includes synchronization, where two visits happen at the same time, overlap or have a minimum or maximum difference. The author models the problem as a set partitioning problem with side constraints and makes use of a branch – and – price solution algorithm to generate the optimal solution. The test results show that running times were significantly decreased with only a loss of quality in a few instances.

Another factor is the dynamic and stochastic nature of the problem. (Yuan, Liu, & Jiang, 2015) addresses a home health care (HHC) scheduling and routing problem with stochastic service times and skill requirement. Each customer has a normally distributed service time with a given mean and standard deviation. A stochastic programming model with recourse is proposed to formulate the problem, and a branch and price algorithm is developed to solve the problem. A label algorithm is used to solve the pricing problem. The results of numerical experiments prove the effectiveness of the approximate formula for the expected penalty and the proposed algorithm and demonstrate the value of accelerating strategies on the whole algorithm. (Eduyn Ramiro López-Santana, 2016) addresses the dynamic version of the problem using a multi-agent approach that allows the model to handle new requests. The problem addressed in this paper is characterized by the dynamic arrival of new patients to the system. A mixed integer program is used in the approach in order to determine the routing of the caregivers. The results show that the model works well for small problems of up to 15 patients, but for bigger problems, the authors propose heuristics and/or metaheuristics to speed up the computational time.

Although inspired by the two-stage method as well as heuristic approach used in other papers, to the best of our knowledge and limit of literature review, we have not found any paper mention, discuss and use the heuristic partition method developed in this manuscript. This may contribute to fill the gap found in the literature review to use a practical method to address the changelings home health care service providers are facing.

3. Problem Statement

In a HHCSR, a set of patients need to be visited in their homes by a set of nurses according to a prescribed weekly frequency for a prescribed number of consecutive weeks during a planning horizon (Milburn, 2012). The problem consists of determining a set of routes that cover the required HHC service visits over a planning horizon that minimizes criteria such as cost or maximizes the quality of service while ensuring no constraint is violated (Cissé, et al., 2017).

We consulted with a home health care provider throughout the project to ensure that the problem described here closely resembles reality. Therefore, we make a number of additional assumptions regarding the problem in this manuscript.

Nurses are assumed to leave from their homes to visit a set of patients and then returns home. There are full-time and casual nurses and it is preferred to assign a patient to a full-time nurse than a casual nurse. Each nurse has a different skill set but they have the same working time limit, and their real working time cannot exceed the working time limit (no overtime). The travel time of a nurse depends only on the travel distance (time of day is ignored). Thus, travel time is assumed to be known in advance.

Patients are located at different addresses and each has a viable visiting time window. Each patient has different requirements for nurse skill set. Based on the required service, each patient

requires different service times and the service time remains the same regardless of which nurse visits the patient.

Given those assumptions, we consider the following *constraints*:

- Each patient is visited once and only once per day by only one nurse
- The nurse returns to the same location where s/he began
- Assigned nurse working time does not exceed her/his working time limit
- Time windows for each patient are respected. In other words, the time when a nurse arrives at patient's home has to be after the start time and before the end time of the time window.
- Skill match: The nurse assigned to visit a patient has to have all the skills the patient requires.
- Full-time nurse preference: A full-time nurse is preferred over a casual nurse.

The *objective* is to minimize the total travel time for all nurses and the total labor cost.

4. Research Question

In an HHCSRP, 'which nurse is assigned to each patient' can be presented as binary variables but 'what time a patient is visited' can only be presented as a continuous variable in a mathematical formulation. So, a mixed integer linear programming (MILP) is used to model the problem. Therefore, the first research question is:

- How can we formulate the HHCSR problem as a MILP model?

Although in the real-world travel times are stochastic, here we assume the travel times between locations is determined by the Euclidean Distance between two location coordinates. The use of Euclidean Distance is inspired by (Demirbilek, Branke, & Strauss, 2019) where Euclidean Distance was used to represent the travel time in the calculation of the insertion penalty. Thus,

we limit our research to a deterministic model rather than a stochastic model. Nevertheless, for large instances where the number of nurses and patients is large, it is still hard to find an exact method to solve the MILP model. Therefore, we have to ask the second research question:

- What is an efficient approach that can be used to solve the MILP model?

5. Model

In this section, we present the formulation of the problem as a mixed integer linear program (MILP).

A set of nurses $k \in K = \{1, 2, \dots, K\}$ are assigned to visit a set of patients $i \in C_k = \{1, 2, \dots, N_k\}$.

We use 0 to denote the nurse departure location and $n+1$ to denote the nurse returning location.

In our problem, these two are the same location but for ease of understanding we use 2 different numbers to represent them in the formulation. After patients are assigned, each nurse k departs from their location 0_k and visits a sequence of patients before returning to the same

location $(n+1)_k$. Here $N = C_k \cup \{0_k, n+1_k\}$, $k \in K$ where C_k represents all the patients assigned to nurse k and $\{0_k, n+1_k\}$, $k \in K$ represents nurse k 's departure and return locations.

There are full-time nurses and casual nurses. A full-time nurse is preferred to a casual nurse. To reflect this constraint, we let p_k denote the visit labor cost of nurse k and set a smaller value for full-time nurse than casual nurse. There are a set of skills $\{1, 2 \dots r\}$, $r \in R$, which a nurse may have, or a patient may require. We use the following variables to denote skill status:

$$\eta_k^r = \begin{cases} 1 & \text{if nurse } k \text{ has skill } r \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_i^r = \begin{cases} 1 & \text{if patient } i \text{ requires skill } r \\ 0 & \text{otherwise} \end{cases}$$

To match skills, the nurse's skill set must include all the skills required by the patient. For example, if a nurse has skills 1, 2 and 3 and a patient requires skills 2 and 3, then the nurse matches the skill requirements of the patient though the nurse has one more skill (skill 1) than what the patient requires. However, if the patient requires skills 2, 3 and 4, then the nurse cannot be matched with the patient because the nurse does not have skill 4.

Each patient i has an allowed time window $[\alpha_i, \beta_i]$ for a visit and requires a certain service time s_i regardless of which nurse is performing the service. Each nurse has the same working time window $[\alpha_0, \beta_{n+1}]$. A travel time t_{ij} is incurred for a nurse to visit patient i and then go to patient j . If patient i is visited by nurse k , we denote it by using the following variable.

$$x_i^k = \begin{cases} 1 & \text{if nurse } k \text{ visits patient } i \\ 0 & \text{otherwise} \end{cases}$$

For a feasible solution that consists of a number of visits, we must decide which nurse takes which visit and at what time. Therefore, we use the following variables as decision variables.

$$y_{ij}^k = \begin{cases} 1 & \text{if nurse } k \text{ visits } j \text{ right after } i \\ 0 & \text{otherwise} \end{cases}$$

$$t_i^k \quad \text{time when nurse } k \text{ arrives at patient } i$$

The objective function is composed of two parts - total travel time cost and total labor cost. Here we use the second objective to reward full-time nurse utilization. To reach the objective of minimizing total labor cost, the model forces full-time nurses to be assigned first since the labor cost of a full-time nurse is lower than that of a casual nurse. This is balanced with travel time costs. We assume the travel cost is 1 dollar per travel time unit (minute) as all that matters is the relative cost of labour versus travel. Thus, the two objectives are combined to minimize total cost

which includes travel cost and labor cost. We put the weights d_1 and d_2 to the two parts of the objective respectively.

The following provides the objective function and constraints.

$$\text{Min } d_1 * \sum_{k \in K} \sum_{j \in N} \sum_{j \neq i} \sum_{i \in N} t_{ij} * y_{ij}^k + d_2 * \sum_{k \in K} \sum_{i \in C} x_i^k * p_k \quad (1)$$

$$\sum_{k \in K} \sum_{j \in N} y_{ij}^k = 1 \quad \forall i \in C \quad (2)$$

$$\sum_{i \in N} y_{ih}^k - \sum_{i \in N} y_{hj}^k = 0 \quad \forall h \in C \quad \forall k \in K \quad (3)$$

$$\sum_{j \in C} y_{0j}^k = 1 \quad \forall k \in K \quad (4)$$

$$\sum_{i \in C} y_{in+1}^k = 1 \quad \forall k \in K \quad (5)$$

$$\sum_{j \in C} y_{j0}^k = 0 \quad \forall k \in K \quad (6)$$

$$\sum_{i \in C} y_{n+1i}^k = 0 \quad \forall k \in K \quad (7)$$

$$\alpha_i * x_i^k \leq t_i^k \leq \beta_i \quad \forall k \in K, \quad \forall i \quad (8)$$

$$\alpha_0 \leq t_{n+1}^k \leq \beta_{n+1} \quad \forall k \in K \quad (9)$$

$$t_i^k + (t_{ij} + s_i^k) * x_{ij}^k \leq t_j^k + \beta_i * (1 - x_{ij}^k) \quad \forall k \in K, \forall i, j \in C, i \neq j \quad (10)$$

$$x_i^k = 0 \quad i \in C, k \in K, r \in R, \eta_k^r \neq \delta_i^r \quad (11)$$

$$x_i^k = \sum_{j \in N} y_{ij}^k \quad \forall i \in C, \forall k \in K \quad (12)$$

Constraint (2) ensures each patient is visited once and only once and by only one nurse.

Constraints (3), (4), and (5) ensure each nurse departs from their location and returns to the same location. Constraints (6) and (7) prevent a nurse departing from destination location k_{n+1} and returning to departure location k_0 . The nurse working time and time window constraints are represented in constraint sets (8), (9), and (10). Skill match constraints are represented in constraint set (11). Constraint set (12) ensures that if a patient is visited then a subsequent patient is chosen to visit next.

6. Proposed Method

Commercial solvers can solve the MILP model and obtain an optimal solution for small instances in a reasonable amount of time. However, it is hard or even impossible to obtain a solution for large instances. For that reason, we propose a heuristic approach. The heuristic approach does not guarantee the optimal solution, but it seeks to provide a good quality solution within a short computational time. For large-sized problems, a good quality solution is often the only practical and functional option.

The essential principle of our approach is to transfer a complex, hard to solve problem into many simple, solvable problems and then, by solving these smaller problems, obtain a solution for the larger, more complex one. For the problem in this manuscript, while it is hard or even impossible to obtain an optimal solution for realistic problems (such as that experienced by CareFor), we are able to solve the MILP model for small instances with, for example, 1 nurse and 3 or 4 more

patients by using CPLEX. The number of patients that one nurse can visit varies and depends on the length of the working day, the distance between patients and nurses, and the length of patients' service time. In our heuristic approach, we first develop a method to partition all patients into a number of single nurse groups that are each small enough to be solved using CPLEX. Then the combination of these smaller problems provides a feasible solution to the large one. Finally, an improvement is applied on the initial feasible solution to achieve a better solution.

6.1. A heuristic partition method to assign patients to single nurse groups

In this section, we introduce the steps of the proposed heuristic partition method.

6.1.1. Rank patients and sort the eligible nurse list

For each patient, we find all the nurses whose skill set matches the patient's skill requirements. We refer to those nurses as 'eligible nurses' for the patient. Then we count the number of eligible nurses for each patient and rank the patients in ascending order based on the number of eligible nurses; that is, we place the patient who has the least number of eligible nurses at the top of the sequence. We use this sequence to assign the patients to their eligible nurses in the following step. Using this sequence ensures that the patient who has fewer eligible nurses will be assigned to a nurse sooner.

It is obvious that the fewer eligible nurses a patient has, the more challenging it is to find an eligible nurse. By giving a higher priority to those patients with fewer eligible nurses, it allows more patients to be visited while keeping the number of nurses the same. For example, suppose there are 2 patients, P1 and P2, and 2 nurses, N1 and N2. Patient P1 has eligible nurse N1, and patient P2 has eligible nurses N1 and N2. The eligible nurse list for the patients are P1: [N1], P2:

[N1, N2]. Here and in the following we use square bracket [] to denote a list or a sequence of patients or nurses. The ranked sequence of the patients should be [P1, P2] since patient P1 has 1 eligible nurse while patient P2 has 2 eligible nurses. Since nurse N1 is the only eligible nurse for patient P1, patient P1 will be assigned to nurse N1 to create a single nurse group (P1, N1). Here and in the following we use round bracket () to denote a nurse group which includes a nurse and/or the patients to visit. Patient 2 has 2 assignment options. She/he will either be assigned to nurse N1 if nurse N1's capacity is not full or to nurse N2 if N1's capacity is full. Therefore, the possible assignment results for patient 2 are (P2, N1) or (P2, N2). The final partition results for this setting with 2 patients and 2 nurses is [(P1, P2, N1), (N2)] or [(P1, N1), (P2, N2)]. In either scenario, patients P1 and P2 are both assigned to a nurse. However, suppose the proposed sequence is not used to assign patients and the assignment sequence is instead [P2, P1]. Given the eligible nurse list of patients P2 is P2: [N1, N2], patient P2 may be assigned to nurse N1. If nurse N1's capacity is full after patient P2 is assigned, patient P1 will have no eligible assignment. In this case, the final partition result is [(P1), (P2, N1), (N2)] which is one single nurse group with nurse N1 and patient P2 as well patient P1 without an assigned nurse, and nurse N2 without a patient to visit. Although the number of nurses remains the same, the number of patients assigned and visited decreases from 2 to 1 when not using the ranked assignment sequence of the patients.

From the problem statement, there is another constraint in our problem called 'full-time nurse preference' which means that a full-time nurse is preferred over a casual nurse. To comply with this constraint, we sort the eligible nurse list by putting full-time nurses at the beginning and casual nurses at the end of the sequence. But we do not further sort either the full-time nurses or casual nurses within the eligible nurse list. We just use the order in which the nurses appear in

the data file to test the nurses' eligibilities and put them into the eligible nurse list in sequence. How to choose between multiple eligible full-time nurses or casual nurses is described below. In the following step, we follow this sequence and attempt to assign each patient to one of the eligible nurses. For the above instance where patient P2 has an eligible nurse list of [N1, N2], if nurse N2 is a full-time nurse and nurse N1 is a casual nurse, the sorted sequence of the eligible nurse list should be [N2, N1]. While assigning patients, we first attempt to assign patient P2 to nurse N2. Only if nurse N2 is not available due to nurse capacity or skill set mismatch do we assign patient P2 to nurse N1. Thus, we bias the assignment of a patient to a full-time nurse and only when all full-time nurses are not an option, do we turn to a casual nurse.

6.1.2. Determine and assign a patient to the Minimum Maximum Distance nurse group

After ranking the patients and sorting the eligible nurse list, each patient will have a sorted list of eligible nurses. If a patient has no eligible nurses, we stop the procedure as no feasible solution exists.

If a patient has more than one eligible nurse in her/his eligible nurse list, we need to decide which nurse the patient should be assigned to. To do that, two measurements we define as 'Maximum Distance' (MD) and 'Minimum Maximum Distance' (MMD) are required. MD is the longest Euclidean distance between the nurse and a patient within a single nurse group. To calculate MD, we first assign a patient to the first nurse in the sorted eligible nurse list to create a single nurse group. The eligible nurse may or may not have other patients assigned. Within this single nurse group, all distances from this nurse to every patient are calculated. The longest one is the MD for this single nurse group. Repeating this process for each nurse in the eligible nurse list, we can determine all MDs for all single nurse groups. Among those MDs, the shortest is MMD. The single nurse group with MMD is MMD nurse group. We assign the patient to this

MMD nurse group. Considering the ‘full-time nurse preference’ constraint, we first include only full-time nurse into the eligible nurse list and find the MMD nurse group. If there is no eligible full-time nurse, we then include the casual nurses.

For example, suppose patient P1 has a sorted eligible nurse list [N1, N2, N3] who are all full-time nurses and nurse N1 has assigned patient P2. To calculate the MD, we assign patient P1 to each nurse in the eligible nurse list to create a set of single nurse groups which are (P1, N1, P2), (P1, N2), (P1, N3). For the first group, assume that the distance between nurse N1 and patient P1 is 10 and the distance between nurse N1 and patient P2 is 15. Then the MD of the first group is 15. Following the same method, if we know the distance between patient P1 and nurse N2 is 10 and the distance between patient P1 and nurse N3 is 10 as well, the MD for both groups is 10. Therefore, the MD calculation results for patient P1 are: $MD(P1, N1, P2) = 15$, $MD(P1, N2) = 10$, $MD(P1, N3) = 10$. In this case, the MMD is 10.

6.1.3. Iteratively assign patients to the MMD nurse group and test the assignment with MILP model

When there are more than one nurse group that have the same MMD, any one of these single nurse groups could be assigned the patient. In this manuscript, we choose to assign the patient to the first single nurse group among the ones who have the same MMD. For the above example, we would choose the second group (P1, N2) as the MMD nurse group.

The benefit of using MMD to find the right nurse group to which a patient should be assigned is to minimize the MD among all single nurse groups. However, since MD considers only the distance between nurse and patients and does not consider the distance between patients, sometimes a patient may be assigned to an MMD nurse group, but she/he is far from other patients in the group. This may result in a solution that is not optimal.

Using MMD, only travel time is addressed. Other constraints in our problem have not been taken into consideration. Therefore, after assigning a patient to a nurse group, we apply the MILP model developed in section 5 to test if an optimal or even a feasible routing can be obtained for this single nurse group. If a solution is obtained, then all other constraints set up in the problem are satisfied. If no solution is obtained, it indicates that the assignment violates a particular constraint. We therefore discard this assignment by ignoring this eligible nurse in the eligible nurse list and repeat step 6.1.2 to find the next MMD nurse group and test with the MILP model until we obtain a feasible solution. If all assignments for a given patient are tested without a feasible solution being found among the full-time nurses, then the eligible casual nurses are considered. If this too yields no solution, then the patient is not visited. We stop the procedure and state that no solution to the problem exists.

6.2. Apply MILP model on each single nurse group to get the final solution

Once all patients are assigned to nurses, we have partitioned all patients and nurses into a set of single nurse groups. Then we can apply the MILP model presented in section 5 to each group and get a solution. By combining all solutions, we obtain an initial complete solution to our problem.

6.3. Apply improvement on initial solution to achieve better solution

After construction of an initial solution, an improvement to the heuristic approach is applied to further minimize the initial objective function value. This improvement consists of 5 steps:

6.3.1. Minimize the use of casual nurses

6.3.1.1. Transfer patients assigned to a casual nurse to a full time one

Since casual nurse visiting cost is higher than full time nurse visits, minimizing the use of casual nurses by transferring their patients into a full time nurse group may reduce the objective value if there is casual nurse in the initial solution.

In detail, the improvement heuristic transfers a patient in a casual nurse group into every full-time nurse group and repeats this move for all patients in casual nurse groups.

Monitor the objective value of each move and select the smallest one as the initial objective value. Repeat this process until the initial objective value does not decrease any more.

6.3.1.2. Transfer a patient in one casual nurse group into another casual nurse group.

If 2 or more casual nurses are still required after step 1.1, try concentrating the patients into fewer casual nurse groups to reduce the objective value. To accomplish this, transfer a patient in a casual nurse group into each of the other casual nurse groups and repeat this move for all patients in casual nurse groups. Monitor all the objective values and take the smallest one as the initial objective value. Repeat this process until the initial objective value does not decrease any more.

6.3.2. *Minimize Maximum Distance (MD) of each single nurse groups*

MD is an important element determining travel distance. Here we try to further minimize the MDs to reduce the objective value. Find the patient in a single nurse group who has the Maximum Distance and then assign this patient to the nurse who is nearest to her/him. Apply these steps to all single nurse groups and find the smallest objective value as the initial objective

value. Repeat this process until the initial objective value stops decreasing.

6.3.3. Swap 2 nearest patients from every 2 single nurse groups

Find the closest two patients between two nurse groups. Swap the patients' nurse group. Do the same swap for every two nurse groups and select the smallest objective value as the initial objective value. Repeat this process until no improvement on the initial objective value.

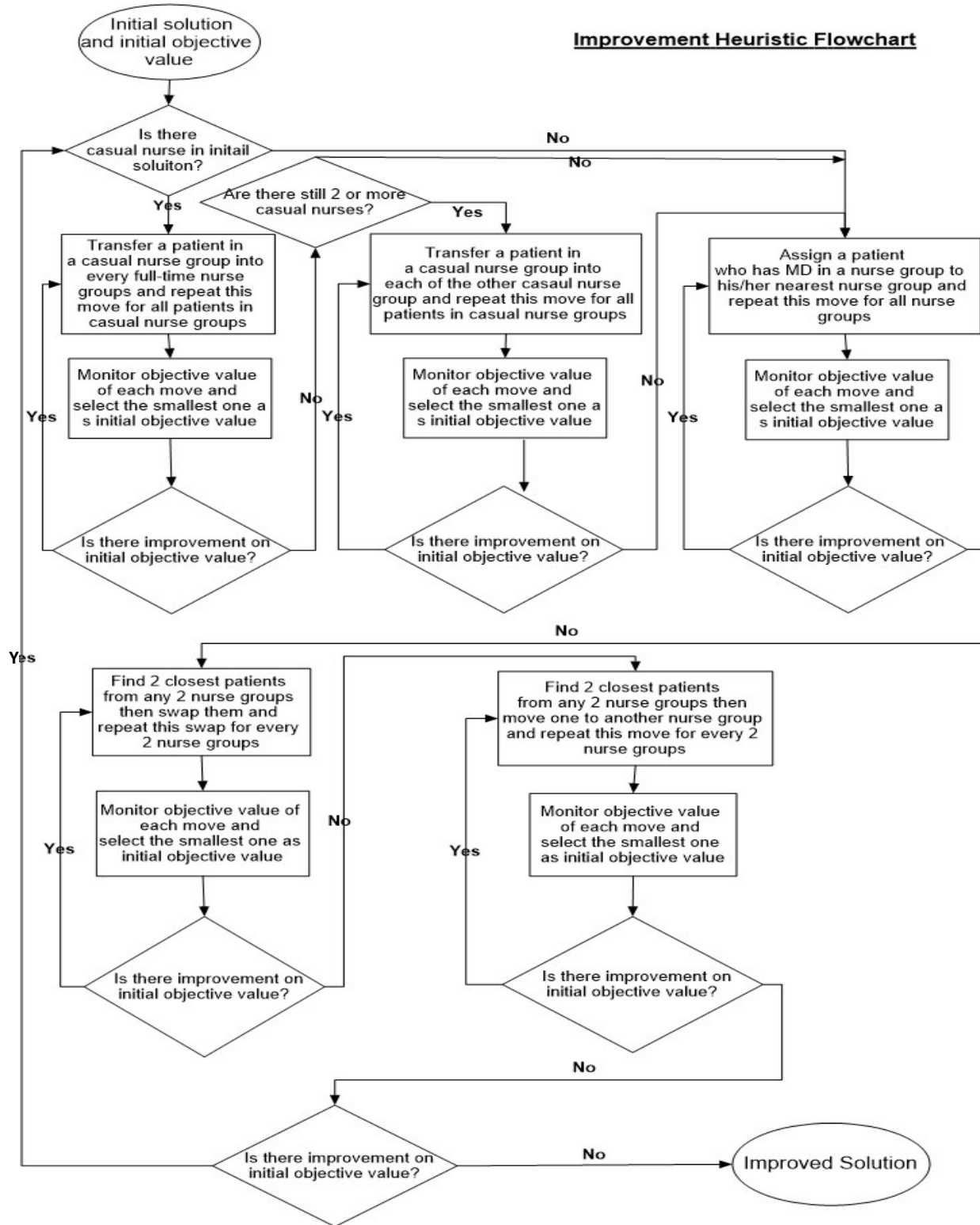
6.3.4. Move nearest patient from one nurse group to its closest neighbour.

Similar to step 6.3.3, instead of swapping the 2 patients, move one patient into the other nurse group. Do the same move for every two nurse groups and select the smallest objective value as the initial objective value. Repeat this process until there is no further improvement on the initial objective value.

6.3.5. Repeatedly apply the above 4 steps on the initial solution until the objective values does not decrease any more.

The following Figure 1 describes the above improvement procedure through a flow chart.

Figure 1 Flow Chart of Improvement procedure



7. Results

7.1. Case Study

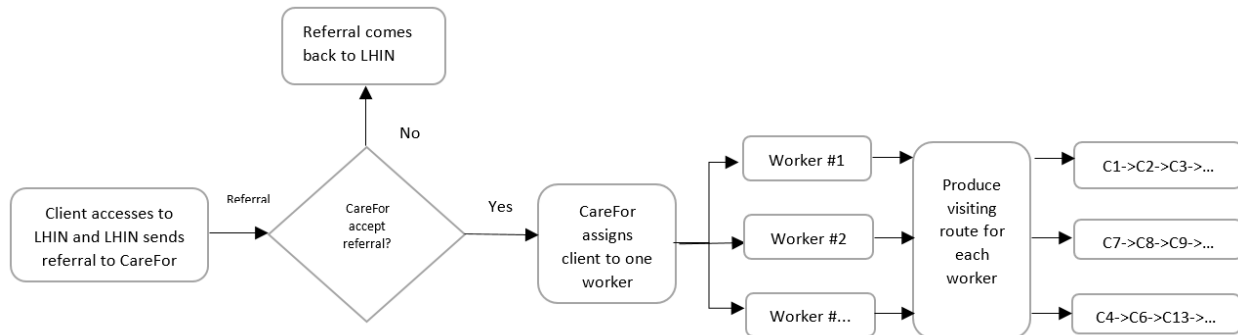
The HHCSRP problem described in this thesis is inspired by real-world challenges faced by a HHC provider called CareFor. The following problem description is based on the report (*Home Care Services | ComForCare*, n.d.) provided by CareFor.

CareFor is a not-for-profit organization that delivers home health care services to patients within the Ottawa region. Patients' demand for care comes in the form of referrals from the Champlain Local Health Integration Network (LHIN). The referral provides the mandatory start date, the type of care need, the length of time care is expected to last, any medication that will need to be administered, visit time window, visit frequency (such as once a week, twice a week) and the service time of each visit. While CareFor has the right to reject a referral from the LHIN based on staff availability, it must meet the requirements of the contract with the LHIN to accept a certain percentage of referrals a year. For that reason, CareFor employs not only specialized full-time workers but also casual workers who are an alternative when no full-time worker is available. Full-time nurse is paid a salary regardless of workload. Casual nurse is paid if used. Consequently, it is encouraged to send a full-time worker rather than a casual worker to visit a patient if possible.

Once referrals have been accepted, the daily task for CareFor is to assign all the patients who require a visit that day to its full-time workers (mixture of nurses and personal support workers) as well as casual workers (if required) and plan the visit route for each worker. The goal is to ensure the right care at the right time while minimizing the total travel time required by the workers.

Figure 2 outlines the steps taken by CareFor.

Figure 2 Flow Chart of CareFor work procedure



C in C1, C2... represents patient (client)

Several challenges complicate the task:

- **Staff Availability:** A worker may call in sick the night before, causing CareFor to struggle to cover the suddenly unassigned visits. Fortunately, there is a pool of casual workers who can be called upon.
- **Length of Service Uncertainty:** A patient may suddenly no longer require service due to death or entry into a hospital, or long-term care facility. Conversely, their length of service may be increased from what was initially anticipated.
- **Demand Movement:** A patient may move leading to the need to re-assign the patient to a different worker.

Given these challenges, the problem is to design a mathematical approach which will run in a timely fashion to determine the staff assignment and routing. More specifically, given a set of

patients and workers, we look to determine the optimal assignment or at least a ‘good’ assignment of a patient to a worker and the optimal sequence of visits for each worker.

7.2. Instance and Data Generation:

The organization we partnered with, as a result of unanticipated difficulties with their database, was unable to provide us with the necessary data. As a result, we simulate the real-world data and randomly generate a set of instances for the resulting version of the HHCSRP to evaluate the performance of the proposed approach. We compare the results obtained from our proposed heuristic approach with ones obtained from a direct solution using CPLEX. The main points of comparison are the objective function value and the running time of the solution method.

To do the test, we need determine couple of parameters in the model described in section 5. First, we set casual nurse visit labor coast $p_k = 10$ while full-time nurse one $p_k = 1$ in order to limit the use of casual nurse. Second, based on the literature (Darlene, 2013), we found the ratio between travel cost and labor cost is roughly 1:9. Therefore, we put the weights $d_1 = 0.1$ and $d_2 = 0.9$ to the two parts of the objective respectively.

To evaluate the performance of the proposed approach in different scenarios, four instance sets, called A, B, C, and D, are generated by the method described in the following section. In the four instance sets, the number of nurses varies from 2 to 6 and the number of patients varies from 5 to 20. Each instance set consists of 10 instances that have the same number of nurses and patients but are different in terms of the locations of patients and nurses, service times, nurses’ skills, working time and nurses’ types, patients’ skills requirements and allowed service windows.

Table 1 gives the description of the instance sets.

Table 1 Description of four instance sets

Instance set	Number of instances	Number of nurses	Number of patients
A	10	2	5
B	10	3	10
C	10	6	20
D	10	7	23

To generate the above instance sets, a series of data such as nurse location and travel time are required. This section presents the method used to generate those data.

1. Nurse

- Nurse location

For each nurse, two random integers from a “discrete uniform” distribution in the half-open interval $[-60, 60)$ are generated to represent nurse location coordinates. Here the measurement unit is kilometer.

- Nurse type

One of “full time” or “casual” is randomly picked and applied to each nurse. To make sure the full-time nurses are the majority of the employees, the probability that a “full time” designation is chosen is set to 0.9 and “casual” to 0.1.

- Nurse skill

For any of 6 skills a nurse may have, we use “0” to represent that a nurse does not have the skill and “1” to represent that a nurse has the skill. “0” and “1” are randomly assigned to each skill with the same probability 0.5. If there is any nurse who has an empty skill set, we assign “1” to her/his skill 1 to ensure each nurse has at least one skill.

2. Patient

- Patient location

For each patient, two random integers from “discrete uniform” distributions in the half-open interval $[-60, 60)$ are generated to represent its location coordinate. Here the measurement unit is kilometer.

- Time window

The start time of any time window has 2 options: $[540, 660]$ which represent 9:00 a.m. and 11:00 a.m. The end time also has 2 options: $[780, 900]$ which represent 13:00 p.m. and 15:00 p.m. The choice of time option all follows uniform distributions. For each patient, a start time and an end time is randomly selected from the start time options and end time options to construct her/his time window.

- Skill required by a patient

For any of the 6 skills a patient may require, we use “0” to represent that a patient does not require the skill and “1” to represent that a patient requires the skill. To increase the probability that all patients’ required skills are provided by the nurse, we set the probability that “0” is assigned to a skill at 0.9. If there is any patient who has no skill requirement, we assign “1” to skill 1 to ensure each patient has at least one skill requirement.

3. Travel time

The travel time between any two patients will be represented by the Euclidean Distance calculated using the two patients’ coordinates while assuming the travel speed is one kilometer per minute.

The travel time between any patient and any nurse will be represented by the Euclidean Distance calculated using the patient’s and the nurse’s coordinates and assuming the travel speed is one kilometer per minute.

4. Service time

For each patient, a random integer from a “discrete uniform” distribution in the half-open interval [10, 40) is generated to represent the service time required by the patient. The measurement unit is minutes corresponding to the travel time unit. Service time remains the same for the patient regardless of which nurse visits this patient.

7.3. Test Results

The main points of comparison are the objective function value and solution time of the MILP model by using the proposed approach and CPLEX. A gap is defined and used to measure the difference between the two methods. It is calculated using the following formula:

$$Gap = \frac{\text{proposed approach result} - \text{CPLEX result}}{\text{CPLEX result}} * 100$$

In our test, 600 seconds on set A, B and C and 1200 seconds on set D are set as the maximum solution time with CPLEX. If solution time reaches the maximum before CPLEX can arrive at an optimal solution, it returns the best solution achieved so far with an optimality gap.

7.3.1. Performance of proposed approach on set A

Table 2 outlines the test results on instance set A where the number of nurses is 2 and the number of patients is 5. 10 different instances are indexed as A1 to A10. The first objective value, solution time, and optimality gap columns present the results using CPLEX. The results with our method are presented in the following objective and solution time columns. In the last 2 columns, the gap between the two methods (both in objective function value and solution) is calculated indicating the performance of our method. Based on these results, we can state the following:

- The solution time is longer with our method compared to the solution time of the MILP model solved by CPLEX.

The average solution time with our method is 9.4638 seconds while with CPLEX it is 0.0451 seconds on instance set A.

- Our model achieves the same optimal objective value as CPLEX.

On all 10 instances of set A, our model provides the same objective value as CPLEX.

Table 2 Test results on instance set A

Instance index	Nbr. Of nurses	Nbr. Of patients	CPLEX			Proposed Approach		Gap between CPLEX and Proposed Approach	
			Objective Value	Solution Time (s)	Optimality Gap	Objective Value	Solution Time (s)	Objective Value %	Solution Time %
A1	2	5	63.8639	0.0310	0%	63.8639	8.9363	0%	28727%
A2	2	5	37.3664	0.0630	0%	37.3664	8.2260	0%	12957%
A3	2	5	41.2592	0.0460	0%	41.2592	12.9158	0%	27978%
A4	2	5	40.7763	0.0310	0%	40.7763	10.7707	0%	34644%
A5	2	5	38.7293	0.0310	0%	38.7292	10.0872	0%	32439%
A6	2	5	31.9919	0.0620	0%	31.9919	3.7914	0%	6015%
A7	2	5	31.2419	0.0460	0%	31.2419	8.3164	0%	17979%
A8	2	5	28.2675	0.0310	0%	28.2675	11.3367	0%	36470%
A9	2	5	40.5016	0.0630	0%	40.5016	10.3736	0%	16366%
A10	2	5	48.9967	0.0470	0%	48.9967	9.8838	0%	20929%
Average			40.2995	0.0451	0%	40.2995	9.4638	0%	20884%

7.3.2. Performance of proposed approach on set B

Table 3 presents the test results on instance set B where each instance has 3 nurses and 10 patients. The results with CPLEX including the objective value, solution time, and optimality gap are recorded in the table. Following that, the results with our method are also presented. The last 2 columns provide the gap between the two methods.

Based on these results, we can state the following:

- The solution time with our method is getting closer to the solution time with CPLEX on set B compared to set A. The average solution time with our method is 42.3568 seconds while with CPLEX it is 4.8517 seconds on instance set B. Compared to set A, the solution time gap is dramatically reduced from 20884% to 773%.
- Our model almost always achieves the same optimal objective value as CPLEX. On 8 out of 10 instances, our model achieves the same optimal objective value as CPLEX. On the remaining 2 instances, our model provides good solutions with an objective gap of only 1.7% and 0.9% respectively.

Table 3 Test results on instance set B

Instance index	Nbr. Of nurses	Nbr. Of patients	CPLEX			Proposed Approach		Gap between CPLEX and Proposed Approach	
			Objective Value	Solution Time (s)	Optimality Gap	Objective Value	Solution Time (s)	Objective Value %	Solution Time %
B1	3	10	46.3009	16.0160	0%	46.3009	71.9155	0.0%	349%
B2	3	10	78.3849	1.3440	0%	78.3849	22.5971	0.0%	1581%
B3	3	10	63.0304	8.8120	0%	63.0304	42.9823	0.0%	388%
B4	3	10	41.6337	0.5780	0%	41.6337	46.2807	0.0%	7907%
B5	3	10	57.6609	0.8590	0%	57.6609	43.1874	0.0%	4928%
B6	3	10	98.1297	6.7660	0%	99.8237	25.4873	1.7%	277%
B7	3	10	82.7954	7.2500	0%	82.7954	57.5230	0.0%	693%
B8	3	10	61.1332	0.8910	0%	61.1332	38.7935	0.0%	4254%
B9	3	10	58.9534	4.6880	0%	59.4707	12.5970	0.9%	169%
B10	3	10	50.0712	1.3130	0%	50.0712	62.2038	0.0%	4638%
Average			63.8094	4.8517	0%	64.0305	42.3568	0.3%	773%

7.3.3. Performance of proposed approach on set C

The test results on set C are summarized in Table 4. The number of nurses and patients are double than in instances set B reaching to 6 and 10 respectively. All the CPLEX results recorded in Table 4 are sub-optimal and represent the best solutions CPLEX can achieve within the

running time limit of 600 seconds. The solution time gap in the last column is negative when our method uses less solution time than CPLEX.

The following observations result from Table 4:

- The solution time with our method is shorter compared to the solution time with CPLEX.

The average solution time with our method is 313.8890 seconds while CPLEX reaches the solution time limit of 600 seconds on all instance. Our model uses 48% less solution time to get the results.

- Our model achieves an objective value with an average gap of - 0.8%.

Our model provides smaller objective values in 4 out of 10 instances compared to CPLEX's best result within 600 seconds solution time and remains within 5% of the best CPLEX solution on all remaining instances.

Table 4 Test results on instance set C

Instance index	Nbr. Of nurses	Nbr. Of patients	CPLEX			Proposed Approach		Gap between CPLEX and Proposed Approach	
			Objective Value	Solution Time (s)	Optimality Gap	Objective Value	Solution Time (s)	Objective Value %	Solution Time %
C1	6	20	83.1641	600	15.6386%	83.4418	268.2832	0.3%	-55%
C2	6	20	100.5770	600	38.0328%	99.6301	160.0096	-0.9%	-73%
C3	6	20	63.2601	600	14.2801%	65.8718	397.5755	4.1%	-34%
C4	6	20	86.1990	600	30.6978%	81.7953	194.1101	-5.1%	-68%
C5	6	20	122.0260	600	40.5300%	122.3205	614.7603	0.2%	2%
C6	6	20	72.4081	600	25.9897%	75.4498	293.9323	4.2%	-51%
C7	6	20	111.5670	600	45.1757%	105.7508	344.7855	-5.2%	-43%
C8	6	20	93.0830	600	27.8161%	94.5605	407.9990	1.6%	-32%
C9	6	20	89.1208	600	29.5114%	90.4066	229.3868	1.4%	-62%
C10	6	20	96.5748	600	36.7478%	91.0379	228.0472	-5.7%	-62%
Average			91.7980	600	30.4420%	91.0265	313.8890	-0.8%	-48%

7.3.4. Performance of proposed approach on set D

Table 5 presents the test results on instance set D. The number of nurses and number of patients increase to 7 and 23 respectively for each instance. The CPLEX results are the best solutions CPLEX can get within the running time limit of 1200 seconds. The gap of solution time in the last column remains negative since our method uses less solution time than CPLEX.

- The solution time with our method continues to be shorter than the solution time with CPLEX.

The average solution time with our method increases to 615.7210 seconds while CPLEX reaches to a running time limit of 1200 seconds on all instances. Average solution time of our model is 49% faster than CPLEX.

- Our model achieves an objective value with an average gap of -9%.

Our model achieved a smaller objective value on 7 out of 10 instances compared to the best result achieved by the CPLEX within 1200 seconds and achieved the same objective value on the 3 remaining instances.

- It is worth pointing out that when the number of nurses increase to 8, CPLEX is unable to find a solution within 3600 seconds.

Table 5 Test results on instance set D

Instance index	Nbr. Of nurses	Nbr. Of patients	CPLEX			Proposed Approach		Gap between CPLEX and Proposed Approach	
			Objective Value	Solution Time (s)	Optimality Gap	Objective Value	Solution Time (s)	Objective Value %	Solution Time %
D1	7	23	100.439	1200	30.8606%	95.9842	957.2212	-4.4%	-20%
D2	7	23	102.438	1200	30.9313%	97.662	1019.2161	-4.7%	-15%
D3	7	23	76.7809	1200	17.3042%	76.7809	388.885	0.0%	-68%
D4	7	23	100.614	1200	36.8228%	83.1989	405.5655	-17.3%	-66%
D5	7	23	121.687	1200	15.5492%	121.6872	596.4318	0.0%	-50%
D6	7	23	90.9754	1200	27.4311%	90.9754	568.6320	0.0%	-53%
D7	7	23	154.749	1200	58.0638%	101.7947	449.2793	-34.2%	-63%
D8	7	23	95.3074	1200	23.8528%	92.5128	519.9835	-2.9%	-57%
D9	7	23	83.2350	1200	20.7029%	81.3829	455.6276	-2.2%	-62%
D10	7	23	87.6531	1200	29.8467%	81.3899	796.3677	-7.1%	-34%
Average			101.3879	1200	29.1365%	92.3369	615.7210	-9%	-49%

In summary, on small instance sets A and B in which the number of nurses is 2 and 3 and the number of patients is 5 and 10 respectively, our method almost always achieves the same optimal objective value as CPLEX. Although the solution time required by our method is longer than the solution time required by CPLEX, the average solution time of 9 seconds and 42 seconds are still acceptable in practice. On larger instance sets C and D in which the number of nurses is 6 and 7 and the number of patients is 20 and 23 respectively, our method achieves smaller objective values on more than half of the instances and remains within 5% of the best CPLEX solution on the remaining instances. The average objective values are 0.8% and 9% better than CPLEX, and the solution time is around 50% faster compared to CPLEX. Table 6 describes the average test results on the four instance sets.

Table 6 Average test results on instance set A, B, C, and D

Instance set	Nbr. Of nurses	Nbr. Of patients	CPLEX Average Value			Proposed Approach Average Value		Gap between CPLEX and Proposed Approach	
			Objective Value	Solution Time (s)	Optimality Gap	Objective Value	Solution Time (s)	Objective Value %	Solution Time %
A	2	5	40.2995	0.0451	0%	40.2995	9.4638	0%	20884%
B	3	10	63.8094	4.8517	0%	64.0305	42.3568	0%	773%
C	6	20	91.7980	600	30.4420%	91.0265	313.8890	-0.8%	-48%
D	7	23	101.3879	1200	29.1365%	92.3369	615.7210	-9%	-49%

8. Future Research

The importance of Home health care (HHC) is increasing in our society as our population ages and the demands on hospitals exceed their capacity. However, the home health care service provider is faced with challenges in staff scheduling and routing. We propose a heuristic approach in which we use a heuristic partition method to partition patients into a number of single nurse groups. After each assignment, we use the single nurse MILP model to test the assignment to ensure the assignment satisfies all constraints and to determine the optimal route. By combining the solutions obtained from all single nurse groups, we get the initial solution to

the research problem. A number of improvement steps are applied to the initial solution to achieve the final solution. We tested the proposed approach with simulated data. The results show that the proposed approach can achieve good solutions which remain within 5% of CPLEX's best solution using an acceptable solution time on all test instances.

One limit in our study is that we did not use real data to test our proposed approach. All data used to test is generated by simulating the real-world data. This may result in variance between our test result and the result with real data. In a real setting there may be more than 6 patient skill requirements and related nurse skills. This may cause an issue on skill matching and result in different performance by the heuristic. One thing is certain thought – in such a setting solving to optimality will not be possible leaving the heuristic as the sole option.

In future research, real-world data and larger instances could be used to test the approach further. Besides that, an alternative direction would be to transform the travel time from deterministic travel times to dynamic travel times. Travel time depends on several elements such as the mode of transportation, the traffic on the road at that time, etc. This stochastic aspect has not yet been heavily studied.

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