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Estimation of the Taylor Rule for Canada Under Multiple Structural Changes[*]

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Abstract

The Taylor rule is estimated under the period $1963Q2$ to $1999Q4$ using Canadian data and the methodology proposed by Bai and Perron (1998) to estimate regression models with multiple endogenous breaks. Although monetary rules are notorious for suffering from structural instability, recent attempts at modeling it are only considering exogenous breaks which are imposed on the data generating process (e.g. Judd and Rudebusch, 1998; and Clarida, Galí and Gertler, 2000). We show that the monetary rule cannot be evaluated over this period without taking into account parameter instability and structural changes, reflecting changes in monetary policy preferences. Inflation is modeled as a Markov-Switching (MS) process to extract expectations, which we treat as the implicit inflation target. To extract the potential level of output, we also use a MS process. Modeling the rule when allowance for two breaks is made ($1978Q3$ and $1988Q2$) illustrates well the changing policy preferences of the Bank of Canada.

Keywords: Taylor Rule, Markov-Switching Models, Structural Change, Unit Root, Inflation, Interest Rate, Output Gap.

JEL: C2, E5

1 Introduction

This paper attempts to address the issue of the stability of parameter estimates of monetary rules in a context of changing policy preferences. It is now well established that a central bank's reaction function is subject to monetary policy changes that may render obsolete any macroeconomic model based on the information of previous monetary regimes. In effect, McNees (1986, 1992) argued that, for various reasons, central banks' reaction functions are unlikely to remain constant over time, saying that "policy reaction function can be fragile". And yet, only a few researchers have dealt with this important issue.[1] Although reaction functions have been estimated in the context of changing monetary policies, policy changes have always been imposed on the data generating process. In other words, policy shifts have been treated as exogenous. For example, Judd and Rudebusch (1998) and Clarida, Galí and Gertler (2000) have investigated the Fed's reaction function over different periods by estimating a monetary rule for various subsamples, according to the composition of the *Federal Open Market Committee.* In a similar approach, Nelson (2000) investigated the Bank of England's behaviour by estimating a monetary rule over five different periods.

We are considering a specification similar to that proposed by Taylor (1993) with the addition of persistence and by allowing for more general deterministic components. We specify the monetary rule as

$$i_t = z_t + \beta_1(y_t - y_t^P) + \beta_2(\pi_t - \pi_t^T) + \beta_3 i_{t-1} + \varepsilon_t, \tag{1}$$

where i_t is the overnight nominal interest rate, z_t is a set of deterministic components, y_t is the log of the actual output level,[2] y_t^P is the log of the potential output level, which is unobserved. The rate of inflation, π_t, is defined as the quarterly log difference of the Consumer Price Index multiplied by 100; π_t^T is the unobserved inflation target; finally, $\varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2)$ is a monetary policy shock. In a benchmark model, π_t^T is defined as the unconditional expectation of π_t based on its $ARMA(p,q)$ representation.

This specification for the inflation target is similar to that used by Clarida *et al.* (2000) and implies that the inflation target is time invariant. In an alternative model, π_t^T is extracted from a Markov-Switching autoregression

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[1] See, e.g. Judd and Rudebusch (1998); Clarida, Galí and Gertler (2000); and Nelson (2000).

[2] Source of the data is Statistics Canada. The CANSIM labels for the data are B114011 (overnight rate of interest), D14816 (real gross domestic product) and P100000 (Consumer Price Index).

$(MS - AR)$ framework with three regimes.[3] Hence, the constant inflation target is denoted as π^T, while the time varying one is denoted as π_t^T. Also, for simplicity, we denote the deviations from the target as $\pi^* = \pi_t - \pi^T$, $\pi_t^* = \pi_t - \pi_t^T$ and $g_t = y_t - y_t^P$. The potential level of output is estimated via a state-space representation with a Markov-Switching growth rate for the stochastic trend function, which able us to have an estimate of the output gap.

Hence, by considering monetary policy changes as an endogenous event, this paper extends the literature by estimating a naive monetary rule for Canada from $1963Q2 - 1999Q4$ under endogenous structural changes following the methodology of Bai and Perron (1998, hereafter BP). It makes possible for the analyst to estimate a model where an unknown number of structural changes are present. Among other things, we show that the hypothesis of stability is strongly rejected by the data using either definition for the inflation target. However, the estimation results for the model involving the time-varying inflation target and two structural breaks appears to be the most appropriate to describe the changing monetary policy preferences of the Bank of Canada.

The rest of this paper is organized as follows. Section 2 presents the results of the analysis of stationarity of each time series. Section 3 deals with the modeling of the potential output level and the extraction of the conditional expectation of inflation. Section 4 presents benchmark results for the estimation of (1) using π^T and allowing for an unknown number of breaks using the BP methodology. Then, we relax the assumption that the inflation target is constant through out the sample period and we use π_t^T. Finally, Section 5 briefly concludes.

2   Stationarity Analysis

In order to investigate the stationarity of each time series that we are considering in this study, we apply five tests. The first two tests are the augmented Dickey-Fuller test proposed by Dickey and Fuller (1979) and by Said and Dickey (1984) and the nonparametric test proposed by Phillips and Perron (1988). The third test is the ADF statistic based on the Generalized Least Squares detrending procedure proposed by Elliott, Rothenberg and Stock

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[3]Hereafter, we write MS(m)-AR(p) to denote an autoregressive model of order p with a Markov-Switching structure of m regimes.

(1996), hereafter ERS.[4] The other two tests allow for more complex dynamics by introducing the possibility that the series are best represented by a shifting mean, a broken trend, or both simultaneously. They are the procedures proposed by Zivot and Andrews (1992) and that of Perron (1997). These statistics are denoted as ADF, PP, ADF^{GLS}, ZA and $P97$, respectively.

To choose the order of the autoregression (k) in the ADF autoregressions, we follow Campbell and Perron (1991) and Ng and Perron (1995), to use a data-dependent method based on a general to specific recursive procedure. Starting from a maximal order of k (say $k\max$), the method tests if the last lag included is significant, and if not, the order of the autoregression is decreased by one and the coefficient of the last lag is again examined. This is repeated until a rejection occurs or the lower bound 0 is reached. In our case, we consider $k\max = \text{integer}(12 \times \frac{T}{100})^{1/4}$.

For the unit root tests in the presence of structural change, we consider three possible cases according to the composition of the deterministic components, denoted by z_t. In the first model, $z_t = \{1, t, DU_t\}$ with $DU_t = \mathbf{1}(t > T_B)$, where $\mathbf{1}(\cdot)$ is the indicator function. This model considers the possibility that there is a break in the intercept, that is, a "crash model, in the terminology of Perron (1989). The second model considers $z_t = \{1, t, DT_t\}$ where $DT_t = t\mathbf{1}(t > T_B)$, indicating that a change in the slope of the trend function is allowed. Finally, a third model where $z_t = \{1, t, DU_t, DT_t\}$ is also considered. In this model both changes are allowed. In all cases, T_B is the date of the unknown break point, which is also defined as $\lambda = T_B/T$. In order to choose T_B, we estimate the ADF statistic above through $t = k+1, k+2, ..., T-1$, and we select the date, $\widehat{T}_B$, as the point associated to the infimum value of the $t_{\hat{\alpha}}$. This method is called the 'infimum' procedure.

Results are presented in Table 1. For interest rate, we estimated the ADF test using only an intercept and, for the lag structure, we have selected $k^* = 7$. Given this parameterization, we were not able to reject the null hypothesis since $\hat{\alpha} = 0.94$ and $t_{\hat{\alpha}} = -2.01$. When applying the PP test, we arrived at the same conclusion that the interest rate is nonstationary with $\hat{\alpha} = 0.94$ and $Z_{t_{\hat{\alpha}}} = -2.33$. Following the GLS detrending procedure and

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[4] The ERS approach consists of first locally removing the deterministic components of $\{y_t\}$ via GLS. Denoting $\{y_t^{\bar{\alpha}}\}$ and $\{z_t^{\bar{\alpha}}\}$ as $y_t^{\bar{\alpha}} = y_1, (1 - \bar{\alpha}L)y_t$ and $z_t^{\bar{\alpha}} = z_1, (1 - \bar{\alpha}L)z_t$, for $t = 2, 3, ..., T$; where $\bar{\alpha} = 1 + \bar{c}T^{-1}$, with $\bar{c} = -7.0$ for the case where $z_t = \{1\}$, and $\bar{c} = -13.5$ when $z_t = \{1, t\}$. If we define $\hat{\psi}$ as the estimator obtained from a regression between $y_t^{\bar{\alpha}}$ and $z_t^{\bar{\alpha}}$, then, we this estimate to construct the detrended series $\widetilde{y}_t = y_t - \hat{\psi}' z_t$ and we apply an ADF test on $\widetilde{y}_t$.

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selecting $k^* = 9$, we have a $t_{\hat{\alpha}}$ of -1.26 and an $\hat{\alpha}$ of 0.97, which is not statistically different from one when we compare the calculated statistic to the critical values.[5] However, using the ZA approach with a $k^* = 9$, we found that the series suffers from a structural break in both the intercept and the slope, with a $\hat{\alpha} = 0.51$ and a $t_{\hat{\alpha}} = -6.10$, which is well below the 1% asymptotic critical value. According to this test, the break corresponds to the observation $1981Q1$. Because this date is located near the major turn-around in the Canadian monetary policy, we can argue that the break is in fact capturing this important historical event. Similarly, the $P97$ approach test led us to the same conclusion that the series is $I(0)$ with the break point at the observation $1979Q3$.

For $\{\pi_t\}$, the ADF test, including only an intercept and selecting a $k^* = 8$, gives us $\hat{\alpha} = 0.89$ and $t_{\hat{\alpha}} = -1.96$, which is well below the 10% critical value. The PP test first led us to believe that the inflation rate was following a stationary process since the test statistic was equal to -4.01, thus we reject the null at the 1% critical level.[6] Using the ADF^{GLS}, we conclude in favour of the presence of a unit root in $\{\pi_t\}$. However, with the ZA test, we find that $\{\pi_t\}$ is $I(0)$ at the 2.5% significance level, with a break in the intercept and the slope at $1982Q4$. In the case of the $P97$ test, we arrive at the same conclusion at the 1% level.

Lastly, for the output series, $\{y_t\}$, applying the ADF test, we find $\hat{\alpha} = 0.97$ and $t_{\hat{\alpha}} = -2.31$.[7] With the PP test, we cannot reject the null hypothesis. Results from the ADF^{GLS} statistic give us same conclusion. Using either ZA or $P97$, it is impossible to reject the null that the series is integrated of order one.

3   A Markov-Switching Models for Output and Inflation

Our specification for the measure of the potential output level draws from the algorithm of Kim (1994), which uses the approximations proposed by Lam (1990) as an extension to the method originally proposed by Hamilton

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[5] Because in the GLS detrending approach the presence of the intercept is asymptotically negligible (see the condition B of Elliott, Rothenberg and Stock, 1996), the critical values for $z_t = \{1\}$ are the same as those for $z_t = \{0\}$.

[6] When a $MA(1)$ for $\{\Delta\pi_t\}$ was estimated, we found a coefficient $\hat{\theta} = -0.59$, which we consider to be a highly negative moving average component which can be the reason why the PP test is rejecting the null hypothesis of a unit root.

[7] For this variable, a time trend was also included in the ADF, PP and ADF^{GLS} statistics.

(1989). We therefore have the following representation:

$$y_t = n_t + g_t \tag{2}$$
$$n_t = n_{t-1} + \mu_{s_t} \tag{3}$$
$$\mu_{s_t} = \mu_0 s_t + \mu_1 s_t + \mu_2 s_t \tag{4}$$
$$g_t = \phi_1 g_{t-1} + \phi_2 g_{t-2} + e_t, \tag{5}$$

which means that y_t is composed of two components: n_t, a stochastic trend function, and g_t, a cyclical deviations from the stochastic trend apecified as an AR(2) process. This term is the output gap. The coefficient μ_{s_t} is the growth rate of output following a first order Markov-Switching process, subject to discrete shift, with $\mu_0 < 0$, $\mu_1 > 0$, and $\mu_0 < \mu_1 < \mu_2$. The unobservable state of the economy is $s_t = 1$ if $s_{t-1} = j$, and $s_t = 0$ otherwise; and the stochastic process for the unobservable states is defined as $\Pr[s_t = i \mid s_{t-1} = j] = p_{ij}$. The error term, e_t, is an $i.i.d.$ $N(0, \sigma_e^2)$ sequence. In general, we have $i, j = 1, 2, ..., m$; with m being the number of regimes. Here, we have selected $m = 3$. To estimate the potential output level, we used the data for the period going from $1961Q1$ until $1999Q4$.

For the inflation rate, we directly modelled π_t^e as also being governed by a three state Markov-Switching model, equivalent to how Garcia and Perron (1996) modeled the U.S. ex-ante real interest rate. We estimated the following equation:

$$\left(\pi_t - \mu_{s_t} \right) = \phi_1 \left(\pi_{t-1} - \mu_{s_{t-1}} \right) + \phi_2 \left(\pi_{t-2} - \mu_{s_{t-2}} \right) + v_t, \tag{6}$$

where $v_t \sim i.i.d.N(0, \sigma_v^2)$. Under rational expectation, π_t^e is equivalent to π_t^T, our implicit inflation target. In addition, following Ricketts and Rose (1995), we interpret the filtered probabilities as reflecting the relative beliefs of agents that the central bank will actually follow one of the three policy regime with respect to the rate of inflation it will tolerate. Moreover, given this interpretation of the filtered probabilities, we can also interpret π_t^T as being the rate anticipated by agents, or, in other words, as being an implicit inflation target. This point is very important since Canada has been using an explicit target only since February 1991. However, the Bank's authorities evidently paid a lot of attention to inflation in the past despite the fact there was no public announcement made relative to the inflation rate it would set as a target.

3.1 Estimation Results

Estimations were done on Gauss and Kim and Nelson's (1999) source codes were used as a basis to construct the codes for the MS(m)-AR(p) models. In

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Figure 1, we show the estimated potential output and output gap measures. Since the 60s, we notice that there are three major periods where the actual output has been well below its potential level. First, during the period of the late 70s. Second, during the severe economic times of the early 80s, when Canada's monetary policy became tight. Finally, it appears as if the recession of the early 90s and the major cutbacks in government spending during the mid 90s caused the Canadian output to remain well below its potential level during most of the last decade. Given our estimates of the output gap, we have only recently entered a phase where the production level would be above a sustainable potential level. The nature of these results can partly explain why inflation has remained relatively low during the last decade, while it can also explain its recent resurgence as the strong current expansion brought y_t above y_t^P, thus creating excess demand.

The maximum likelihood estimation results are presented in Table 2 and the transition probabilities are shown in Figure 2. The values for the three mean growth rates of output are $\hat{\mu}_0 = 2.63$, $\hat{\mu}_1 = 0.78$, and $\hat{\mu}_2 = -0.75$. The autoregressive coefficients are $\hat{\phi}_1 = 1.35$ and $\hat{\phi}_2 = -0.46$. Our model appears to be well able to capture the main dynamics and cyclical movements of output as the filtered probabilities of being in one of the three regimes allowed are in accordance with times of recessions or booms. Moreover, the probability of being in the state of high growth remains near zero after the economic boom of 1987. This is consistent with the Golden Age period, which lasted through the sixties until the oil price shock of 1973, where output grew at a much faster pace than what we have been observing since (see Figure 1). The filtered probabilities of being in a state of contraction illustrate quite well the periods which are significantly marked with quarters of negative growth, especially from $1981Q3$ to $1983Q4$, where they rose to nearly 100%. A similar behaviour is observed from $1990Q2$ to $1991Q2$, as they again rose from 26% to peak at nearly 100% during $1990Q4$ until $1991Q1$. Given the transition probabilities, we find that the expected duration of being in high growth, moderate growth, and negative growth are 1.5, 6.5 and 3.1 quarters, respectively.

For the inflation rate, maximum likelihood results for the three regimes specification are presented in Table 3. For the autoregressive coefficients we have $\hat{\phi}_1 = 0.24$, and $\hat{\phi}_2 = -0.19$. These results contrast very much with those estimated under a linear specification where the sum of the $\hat{\phi}$'s is near unity. For the mean of π_t, we have $\hat{\mu}_0 = 9.24$, $\hat{\mu}_1 = 4.01$ and $\hat{\mu}_2 = 1.56$. The estimated variances for the three regimes are $\hat{\sigma}_0^2 = 4.64$, $\hat{\sigma}_1^2 = 2.39$ and $\hat{\sigma}_2^2 = 1.17$, respectively. All these parameters are significant at the 1% significance level. An interesting feature of the estimated coefficients is that

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$\hat{\mu}_2$, the mean rate of inflation corresponding to a monetary policy concerned with keeping prices under control, is about half a point below the official mid-range target announced by the Bank of Canada since 1994. Moreover, these results are in line with those of Ruge-Murcia (1998), who showed that since the beginning of the official policy of inflation targeting by the Bank, inflation rates in Canada have been more likely to be below the mid-range of the target then above. The results of Ruge-Murcia (1998) are also suggesting that the Bank of Canada has asymmetric preferences with respect to inflation. Furthermore, our results also highlight quite well previous allegations (e.g. Ball and Cecchetti, 1990) which stated that inflation volatility increased with higher rates of inflation. In effect, the estimated variance increases significantly as we go from a regime of low inflation to a regime of medium inflation, while it increases even more when we are under the regime of high inflation, being nearly twice as high as that observed in regime of moderate inflation. The expected durations for the regime of high, moderate and low inflation are 18.48, 25.38 and 81.96 quarters, respectively.

As for the case of the potential output estimations, the filtered probabilities, shown in Figure 3, are well distributed. Notice that in $1973Q2$, the filtered probabilities of going into the regime of high inflation suddenly increased, to decrease only in $1983Q1$. Furthermore, with a lag of one quarter, our model is capturing the sudden and brief rise in inflation that occurred in $1991Q1$. For the regime of moderate inflation, the filtered probabilities are high from $1966Q2$ to $1972Q3$ and from $1983Q3$ to $1991Q1$. Finally, the filtered probabilities for the regime of low inflation are close to zero form the beginning of the sample until $1966Q1$, while they remain close to zero until $1991Q3$. Afterwards, the filtered probabilities remain high until $1999Q4$. This period corresponds to the era of the official inflation targeting policy by the Bank of Canada.

3.2 Selecting the MS*(m)*-AR*(p)* Model

The estimated model that yielded the most attractive results for the output estimations was the MS(3)-AR(2). The model considered in Kim (1994), a MS(2)-AR(2), did not prove to be able to provide us with parameters that made any economic sense. For example, the filtered probabilities of being in one of the two regimes were always equal to unity for the first half of the sample while they were equal to zero for the second half. The consequences of such results are that once we enter the second regime, we are to remain there for ever as $1/(1-p_{jj})$ diverges to infinity. These results were robust to the choice of initial values used. Finally, another problem with the MS(2)-

AR(2) model is that we were confronted with having either a model where the filtered probabilities captured the main features of the known business cycles well but where the estimated potential output was not well behaved at all, or a model with opposite characteristics.

When testing Markov-Switching specifications, the commonly used tests such as the likelihood ratio or the Wald test cannot be applied as usual since a number of parameters (denote them by κ) are not identified under the null hypothesis. When some parameters are not identified under the null hypothesis, the distribution then depends on nuisance parameters present under the alternative, leading to non-standard testing procedures and the LR test does not follow a χ^2_κ distribution. Therefore, to be able to verify if our model is truly justified despite its attractive results, we used the testing procedure proposed by Davies (1987), where the null hypothesis consists of $m - 1$ regimes and the alternative consists of m regimes.[8]

For the specifications of the output, we have the following results. Testing for a linear specification (i.e. a single regime) against a non-linear specification with two regimes ($\kappa = 3$), we can reject the null hypothesis with a p-value of 0.001. Finally, testing for the null of two regimes against the alternative of three ($\kappa = 5$), we can still reject the null hypothesis in favour of the alternative (of three regimes) as the p-value equal to 0.0. Therefore, our model for the output which yields the most attractive results is also very well supported by Davies' (1987) test.

Testing the specifications of π_t, for one regime against two ($\kappa = 4$) gives a p-value equal to 0.023. Thus, we are able to reject the null that π_t is generated by a linear autoregressive process. Testing for the case of two regimes against three regimes ($\kappa = 6$), we are once again able to reject the null hypothesis with a p-value equal to 0.0. Thus, our three regimes Markov-Switching specification for π_t appears to be strongly justified according to the test of Davies (1987).

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[8] See the technical appendix in Garcia and Perron (1996) for a brief description of the test of Davies (1987).

4 Estimating the Taylor Rule Under Multiple Structural Changes

Following BP and using similar notation, we consider the following multiple linear regression with m breaks ($m+1$ regimes)

$$\left\{ \begin{array}{ll} i_t = z_t'\gamma_1 + x_t'\beta + \varepsilon_t, & t = 1, 2, ..., T_1 \\ i_t = z_t'\gamma_2 + x_t'\beta + \varepsilon_t, & t = T_1 + 1, ..., T_2 \\ \vdots & \\ i_t = z_t'\gamma_{m+1} + x_t'\beta + \varepsilon_t, & t = T_m + 1, ..., T \end{array} \right\}. \qquad (7)$$

According to this specification, i_t is the observed dependent variable at time t; z_t ($q \times 1$) and x_t ($p \times 1$) are vectors of covariates and γ_j ($j = 1, 2, ..., m+1$) and β are the corresponding vectors of coefficients; and ε_t is the disturbance term at time t. The indices $(T_1, ..., T_m)$, or the break points, are explicitly treated as unknown. The purpose is to estimate the unknown regression coefficients together with the break points when T observations on (i_t, x_t, z_t) are available.

The method of estimation considered is that based on the least squares principle. For each $m-$partition $(T_1, ..., T_m)$, the associated least-squares estimates of β and γ_j are obtained by minimizing the sum of squared residuals

$$(I - X\beta - \bar{Z}\gamma)'(I - X\beta - \bar{Z}\gamma) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [i_t - x_t'\beta - z_t'\gamma_i]^2. \qquad (8)$$

The estimation of (8) able us to obtain $\hat{\beta}(\{\hat{T}_{B_j}\})$ and $\hat{\gamma}_j(\{\hat{T}_{B_j}\})$ which can be considered as the resulting estimates based on the given $m-$partition $(T_1, ..., T_m)$ denoted by $\{T_j\}$. Substituting these estimates in the objective function and denoting the resulting sum of squared residuals as $S_T(T_1, ..., T_m)$, the estimated break points $(\hat{T}_1, ..., \hat{T}_m)$ are such that $(\hat{T}_1, ..., \hat{T}_m) = \arg\min_{\{T_1,...,T_m\}} S_T(T_1, ..., T_m)$, where the minimization is taken over all partitions $(T_1, ..., T_m)$. Then, the break-point estimators are global minimizers of the objective function. Finally, the regression parameter estimates are obtained using the associated least-squares estimates at the estimated m-partition $\{\hat{T}_j\}$, i.e. $\hat{\beta} = \hat{\beta}(\{\hat{T}_j\})$ and $\hat{\gamma} = \hat{\gamma}(\{\hat{T}_j\})$.

BP propose some test statistics to detect multiple breaks. The first statistic is the sup$F$ type test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there are $m = k$ breaks. Maximizing the F-statistic is equivalent to minimizing the global sum of

squared residuals. They showed that this procedure is asymptotically equivalent since the break dates are consistent even in the presence of serial correlation. However, the asymptotic distribution still depends on the selected minimal length of the segments, call it h, with $\epsilon = h/T$.

When the investigator wishes not to pre-specify a particular number of breaks to make inference, BP propose two tests of the null hypothesis of no structural break against an unknown number of breaks given some upper bound M. These are called the double maximum tests (UDmax $F_T(M, q)$ and WDmax $F_T(M, q)$). Similarly, they propose a test of the null of no break versus an alternative of k breaks, k being fixed. This test is denoted as the sup$F_T(k; q)$. BP also propose a test for l versus $l + 1$ breaks which is applied sequentially. This test is labelled as the sup$F_T(l + 1|l)$. The method amounts to the application of $l + 1$ tests of the null hypothesis of no structural change versus the alternative hypothesis of a single change. The test is applied to each segment containing the observation $\widehat{T}_{j-1}$ to $\widehat{T}_j$ with $j = 1, ..., l + 1$. We conclude for a rejection in favor of a model with $l + 1$ breaks if the overall minimal value of the sum of squared residuals (over all segments where an additional break is included) is sufficiently smaller than the sum of squared residuals from the l breaks model. The break date is selected as the one associated with this overall minimum.

Other criteria to select for the number of breaks presented by BP are the BIC and LWZ methods. Using different data generating processes, Bai and Perron (2000) performed Monte Carlo experiments to investigate the behavior of each method. Their main conclusions are that "[...] the BIC works well when breaks are present but less so under the null hypothesis" (Bai and Perron, 1998). According to them, if we suspect that no break are present in the model, then the LWZ method is more appropriate. Finally, they argue that the sequential procedure works best when compared with the other methods they investigated.

For the estimation, we assumed that the data have different distributions across segments. This implies that the standard errors and the LM tests were calculated separately for each segment. Finally, since the potential output level and the inflation target are both unobservable, they need to be generated (see above) prior to the estimation of the monetary rule. We are therefore confronted to the problem of using generated regressors. In other words, the parameters of the first stage equation may not be orthogonal to those of the second stage, which can lead to non-spherical errors (Pagan, 1984). Given the non-sphericity of the variance-covariance matrix, we are using a robust standard error technique in order to achieve estimates of the

standard errors.[9]

Since our goal is to capture the possible monetary policy changes, with respect to all the population parameters, we only consider the case of a pure structural change model. We investigated the model using different sizes for the trimming of the sample. That is, $\epsilon = 0.05, 0.1, 0.15, 0.2$ and 0.25, for which the asymptotic critical values are tabulated in BP.[10] We imposed a maximum of seven breaks for ϵ's of 0.05 and 0.1; five when $\epsilon = 0.15$; three when $\epsilon = 0.2$; and two when $\epsilon = 0.25$. Because many possibilities arise as one sets different trimming sizes, we only report the most pertinent and illustrative results that we obtained.

Finally, as i_t can be said to evolve around a deterministically broken time trend, we included a time trend to the vector of deterministic components to capture this important characteristic of the dependent variable.[11]

4.1 The Benchmark Model: A Time Invariant Inflation Target

In this section, we present a benchmark model which uses π^T with $\epsilon = 0.2$ ($h = 29$) and $M = 3$. Estimation results of (8) along with some diagnostic tests are reported in Table 4. A single break located in $1978Q3$ can be detected by either the $\sup F_T(k)$ or the $\sup F_T(l+1|l)$ test.[12] In the first regime, the estimated long-run response of i_t to a change in g_t is 1.37, which is larger than that of π^* which stands at 0.47. During the second regime, the long-run response of g_t falls to 0.51 and that of π^* increases to 0.81. This model is perhaps still misspecified since first order serial correlation remains in the residuals during the first regime, implying that the Bank is smoothing interest over a period of six months. In addition, ARCH effects of order 2 are present in the second regime. However, it is now well known that ignored additive outliers[13] or structural changes can generate leptokurtik errors and hence, generate spurious ARCH effects and apparent non-normality. We thus tested for the presence of additive outliers using the test recently proposed by Perron and Rodríguez (2002) and applied a ro-

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[9] The parametric estimator proposed by den Haan and Levin (1996) was used. The number of lag is selected by AIC and is allowed to vary across variables.

[10] The estimations were done using the accompanying Gauss codes of Bai and Perron (1998).

[11] The unit root tests performed above are clearly providing support for this argument.

[12] According to the sequential procedure, the results were the same when using $\epsilon = 0.15$ and $\epsilon = 0.25$; while when using $\epsilon = 0.1$, three breaks are found at 10% and none at 5%. Results are unreported, but available form the authors upon request.

[13] For the remaining of the paper, we freely employ the word outlier for 'additive outlier'.

bust LM test on the residuals, as suggested by Franses, van Dijk and Lucas (1998).[14] Using the finite critical values ($T = 100$) tabulated in Perron and Rodríguez (2002), significant outliers were found in both segment.[15] Then, using data corrected for additive outliers, the null hypothesis of no serial correlation and homoskedasticity is again rejected.

Alternatively, it is possible that using a simple time invariant inflation target is not an appropriate way to describe agents beliefs regarding inflation expectations, and hence, consists of a misspecification of the monetary rule. Thus, for the rest of this section, we analyze the rule using π_t^*, as estimated above by Markov-Switching approach.

4.2 A Time-Varying Inflation Target

When using this time π_t^T and imposing an $\epsilon = 0.25$, the dates which are minimizing the squared sum of residuals are $1978Q3$ and $1988Q2$. The $\sup F_T(k)$ is significant for all k, implying that there is at least one break present in the model. The UDmax, WDmax and the test of $\sup F_T(2|1)$, are all significant at the 1% significance level.[16] However, both information criterion are selecting the linear model. Given that the $\sup F_T(2|1)$ test provides strong evidence in favour of a model with two breaks, we selected this model, for which the results are presented in Table 5.[17]

The least squares estimation results for this model are suggesting that the long-run responses during the first monetary regime the Bank of Canada was moving interest rates in a Keynesian fashion by responding mainly to output movements ($\widehat{\beta}_{1,1}/(1 - \widehat{\beta}_{3,1}) = 1.44$). Meanwhile, inflation was trending up during most of the second half of this regime, generating only a mild ($\widehat{\beta}_{2,1}/(1 - \widehat{\beta}_{3,1}) = 0.14$) long-run response from the Bank. With respect to the LM tests, significant (5%) order two serial correlation is detected, but the conditional variance is $i.i.d.$. Also, the removal of an outlier ($1975Q2$) is enough to make the ARCH-LM test fail to reject the null hypothesis.

During the second monetary regime, with a $\beta_{1,2}/(1 - \beta_{3,2})$ of -0.61, we see that the Bank reacted strongly and pro-cyclically to the accumulated excess demand pressures in the aftermath of the second oil price shock. Whenever the response of i_t to output gap fluctuations is negative, we can

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[14] In effect, Franses *et al.* have shown that the ARCH-LM test suffers from important size distortion when outliers are present in the data.

[15] The dates are: $1975Q2$ and $1980Q4$.

[16] The date of the break for the model with a single structural change is $1978Q3$.

[17] The dates are almost identical for $\varepsilon = 0.2$, while as many as six breaks can be found when $\varepsilon = 0.05$.

say that the Central Bank is trying push output either beyond or above its potential capacity level pro-cyclically, an idea which goes against general wisdom, although sometimes necessary. In effect, Bernanke, Laubach, Mishkin and Posen (1999) argued that this strong but deemed unavoidable reaction in face of accelerating inflation caused an important recession during the period of the early 80s. In this segment, the errors appear to be well behaved, although an outlier is detected ($1980Q4$).

During the current monetary regime, characterized by an explicit inflation target, the long-run response to output ($\widehat{\beta}_{1,3}/(1 - \widehat{\beta}_{3,3})$) is 2.38 while that of inflation is only 0.22, statistically insignificant. Once again, the residuals of this regime are well behaved with a single outlier in $1993Q1$.

For all the regimes, we see that the persistence of i_t is significantly below unity when we allow for the monetary rule to shift, indicating that i_t is stationary around a deterministic trend, a result in line with the ZA and $P97$ unit root tests applied above.

Two possible explanations can help understand the results observed in the last regime, characterized by a zero response to inflation deviations. Firstly, if we compare the actual rate of inflation with the estimated target, we see that inflation was not in line with the target during the 60s and 70s, whereas from the late 80s and during the 90s we can see that inflation fluctuated around its target. This implies that as long as inflation remains within the target band, it is then possible to have a coefficient on inflation equal to zero as agents strongly believe that the monetary policy is credible. The argument over the 'credibility' of the Bank of Canada has been made many times by the Canadian monetary authorities in publications such as the semi-annual *Monetary Policy Report*. Lastly, in the context of the Phillips curve, future rates of inflation can be expressed as a function of current demand and supply conditions, an argument also raised by Judd *et al.* (1998). Hence, in such a framework it is therefore natural that the contemporaneous output gap will transmit partial information regarding which direction π_t is likely to lean to during the following quarters.

5 Conclusion

This paper has investigated the stability of a simple monetary rule for Canada from $1963Q2 - 1999Q4$. We estimated a time-varying inflation target which is based on a Markov-Switching model with three regimes for the mean and variance. The potential output level estimations were done using a state-space model with a Markov-Switching process for the stochastic

trend function, and we also allowed for three different regimes of growth.

We estimated the monetary rule using the BP methodology where we allowed for an unknown number breaks. In this framework, we first showed that when using a constant inflation target, the residuals were ill behaved, an indication of possible misspecification. Then, by using a time-varying inflation target, we showed that up to six structural breaks could be detected and that the residuals were generally well behaved after making allowance for two pure structural breaks. This last specification appears, in our view, to be the most appropriate in providing a simple description of how the monetary policy is conducted in Canada.

Hence, this paper offered an investigation of the well known assumption that reaction functions are unlikely to remain stable over time (e.g. Judd *et al.*, 1998; Clarida *et al.,* 2000; and Nelson, 2000). One implication of our results is that macroeconomic model analysis should be concerned with not only parameter instability, but also with different generating processes for the parameters (i.e. changing variances).

A natural extension would be to analyze a system describing the economy and the transmission mechanism using the vector autoregression (VAR) framework and to analyze the impulse response functions when structural breaks are present in the VAR representation. To that effect, the recent work of Ng and Vogelsang (2000) offers some interesting grounds for future research to build on.

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Table 1. Unit Root Tests: 1961Q1-1999Q4

Output

Parameter	$\hat{\alpha}$	$t_{\hat{\alpha}}$	k^*	$\hat{T}_B$
ADF	0.97	-2.31	9	$-$
ADF^{GLS}	0.99	-0.72	14	$-$
PP	0.92	-2.43	9	$-$
ZA	0.90	-3.97	9	$1972Q2$
$P97$	0.83	-4.25	10	$1979Q2$

Inflation

Parameter	$\hat{\alpha}$	$t_{\hat{\alpha}}$	k^*	$\hat{T}_B$
ADF	0.89	-1.96	8	$-$
ADF^{GLS}	0.96	-0.88	15	$-$
PP	0.76	-4.01^a	8	$-$
ZA	0.15	-5.31^b	12	$1982Q4$
$P97$	0.59	-5.01^a	12	$1982Q4$

Interest Rate

Parameter	α	t_{α}	k^*	$\widehat{T}_B$
ADF	0.94	-2.01	7	$-$
ADF^{GLS}	0.96	-1.58	7	$-$
PP	0.94	-2.33	7	$-$
ZA	0.51	-6.10^a	9	$1981Q1$
$P97$	0.61	-5.12^b	7	$1979Q3$

a, b, c, d denotes significance levels at 1%, 2.5%, 5.0% and 10%, respectively.

Table 2. Maximum Likelihood Estimates of State-Space Models for Output:
1961Q1-1999Q4*

Parameter	AR(2)		MS(2)-AR(2)		MS(3)-AR(2)	
p_{00}	—	—	0.997^{a}	(0.005)	0.352^{a}	(0.089)
p_{01}	—	—	—	—	0.645^{a}	(0.089)
p_{10}	—	—	0.985^{a}	(0.017)	0.121^{a}	(0.028)
p_{11}	—	—	—	—	0.846^{a}	(0.031)
p_{20}	—	—	—	—	0.276^{c}	(0.135)
p_{21}	—	—	—	—	0.046	(0.054)
ϕ_{1}	0.28^{a}	(0.079)	1.212^{a}	(0.079)	1.352^{a}	(0.025)
ϕ_{2}	0.059	(0.081)	-0.277^{a}	(0.079)	-0.457^{a}	(0.017)
μ_{0}	0.898^{a}	(0.109)	1.322^{a}	(0.11)	2.623^{a}	(0.089)
μ_{1}	—	—	-0.663^{a}	(0.106)	0.7852^{a}	(0.076)
μ_{2}	—	—	—	—	-0.7535^{a}	(0.034)
σ^{2}	0.798^{a}	(0.046)	0.832^{a}	(0.049)	0.446^{a}	(0.024)
Log Lik	-197.814		-186.176		-170.962	

a, b ,c ,d denotes significance levels at 1%, 2.5%, 5.0% and 10%, respectively.

*standard errors in parentheses

Table 3. Maximum Likelihood Estimates of MS(m)-AR(p) Models for the Inflation: 1961Q1-1999Q4*

Parameter	AR(2)		MS(2)-AR(2)		MS(3)-AR(2)	
p_{00}	-		0.946^{a}	(0.034)	0.95^{a}	(0.04)
p_{01}	-		-		0.02	(0.03)
p_{10}	-		0.983^{a}	(0.012)	0.09^{a}	(0.03)
p_{11}	-		-		0.96^{a}	(0.03)
p_{20}	-		-		0.0	(0.0)
p_{21}	-		-		0.011^{a}	(0.01)
ϕ_1	0.597^{a}	(0.079)	0.458^{a}	(0.088)	0.24^{a}	(0.09)
ϕ_2	0.228^{a}	(0.079)	0.008	(0.105)	-0.19^{b}	(0.09)
μ_0	4.701^{b}	(0.993)	4.332^{a}	(1.043)	1.56^{a}	(0.20)
μ_1	-		2.93^{a}	(0.418)	4.01^{a}	(0.22)
μ_2	-		-		9.24^{a}	(0.37)
σ_0^2	4.618		9.035^{a}	(0.609)	4.64^{a}	(1.11)
σ_1^2	-		2.929^{a}	(0.306)	2.39^{a}	(0.49)
σ_2^2	-		-		1.17^{a}	(0.35)
Log lik	-334.85		-325.94		-310.14	

a, b, c, d denotes significance levels at 1%, 2.5%, 5.0% and 10%, respectively.

*standard errors in parentheses.

Table 4: Estimation Results of the Multiple Structural Change Model for the Time Invariant Inflation Target: 1963Q2-1999Q4[*]

Specifications	$M = 3$	$h = 29$	$\epsilon = 0.2$
Tests			
$UD\max$	$\sup_T F(1)$	$\sup_T F(2)$	$\sup_T F(3)$
21.158^b	17.300^d	13.174	21.158^b
$WD\max(1\%)$	$\sup_T F(2|1)$	$\sup_T F(3|2)$	
31.519^a	17.274^d	11.142	

Number of Breaks Selected	
Sequential procedure	1
BIC	0
LWZ	0

Estimates with One Break[*]

$T_B = 1978Q3$	μ_1	μ_2	LM Tests for Each Segment j		
$\bar{R}^2 = 0.905$	0.930^a	4.352^a	Raw Data		
$\sigma_1 = 0.679$	(0.390)	(1.307)	j	AR**	ARCH**
$\sigma_2 = 1.303$	$\beta_{1,1}$	$\beta_{1,2}$	1	1^d	0
	0.285^a	0.155	2	0	2^c
	(0.088)	(0.127)	Filtered Data		
	$\beta_{2,1}$	$\beta_{2,2}$	j	AR**	ARCH**
	0.098^a	0.246^a	1	1^c	0
	(0.031)	(0.127)	2	1^d	2^c
	$\beta_{3,1}$	$\beta_{3,2}$			
	0.792^a	0.695^a			
	(0.094)	(0.092)			
	δ_1	δ_2			
	0.006	-0.014^d			
	(0.007)	(0.009)			

a, b, c, d denotes significance levels at 1%, 2.5%, 5.0% and 10%, respectively.

[*]standard errors in parentheses.

** Lag order selection

Table 5: Estimation Results of the Multiple Structural Change Model for the
Time-Varying Inflation Target: 1963Q2-1999Q4*

Specifications	$M = 2$	$h = 36$	$\epsilon = 0.25$			
Tests						
$UD\max$	$\sup_T F(1)$	$\sup_T F(2)$				
22.22^a	20.99^b	22.22^a				
$WD\max(1\%)$	$\sup_T F(2|1)$					
29.54^a	23.98^a					

Number of Breaks Selected	
Sequential procedure	2
BIC	0
LWZ	0

Estimates with One Break*						
$T_{B_1} = 1978Q3$	μ_1	μ_2	μ_3	LM Tests for Each Segment j		
$T_{B_2} = 1988Q2$	0.559	9.182^a	5.991^b	Raw Data		
$\bar{R}^2 = 0.918$	(0.375)	(1.835)	(2.452)	j	AR**	ARCH**
$\sigma_1 = 0.716$	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$	1	2^c	0
$\sigma_2 = 1.546$	0.286^a	-0.237	0.533^a	2	0	0
$\sigma_3 = 0.750$	(0.085)	(0.310)	(0.167)	3	0	0
	$\beta_{2,1}$	$\beta_{2,2}$	$\beta_{2,3}$	Filtered Data		
	0.029	0.516^a	0.050	j	AR**	ARCH**
	(0.0482)	(0.130)	(0.068)	1	2^a	0
	$\beta_{3,1}$	$\beta_{3,2}$	$\beta_{3,3}$	2	0	0
	0.802^a	0.612^a	0.777^a	3	0	0
	(0.096)	(0.114)	(0.098)			
	δ_1	δ_2	δ_2			
	0.018^a	-0.059^d	-0.033^c			
	(0.007)	(0.017)	(0.016)			

a,b,c,d denotes significance levels at 1%, 2.5%, 5.0% and 10%, respectively.

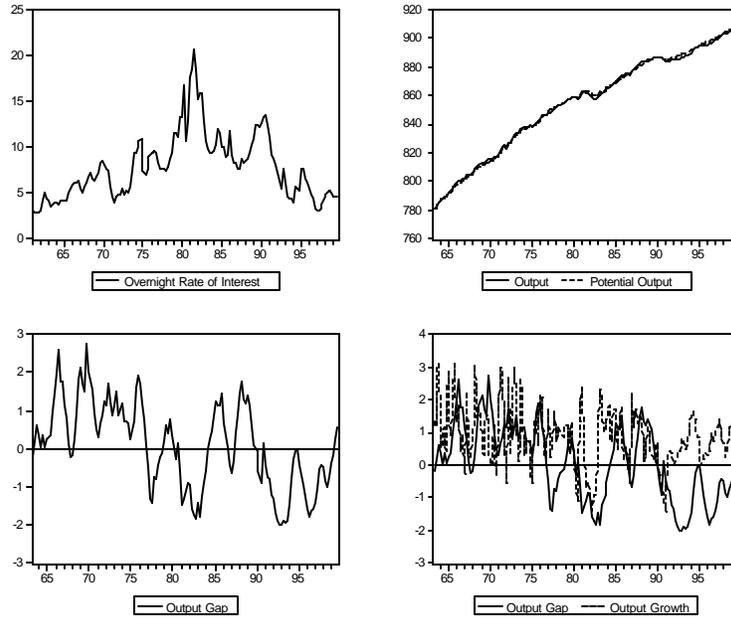*Standard errors in parentheses.

** Lag order selection.

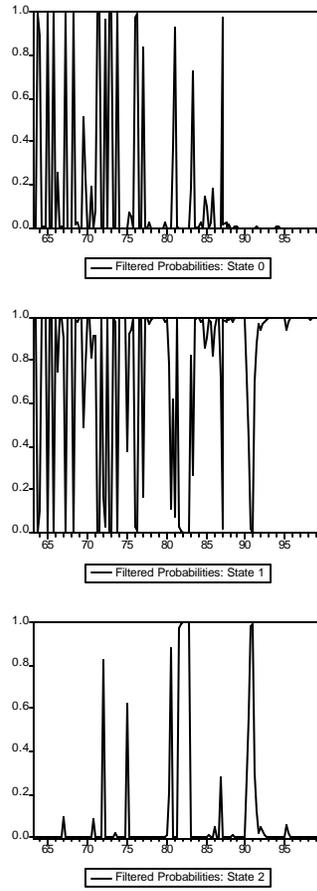Figure 1. Interest Rate, Output, Potential Output, Output Gap and Growth Rate of Output

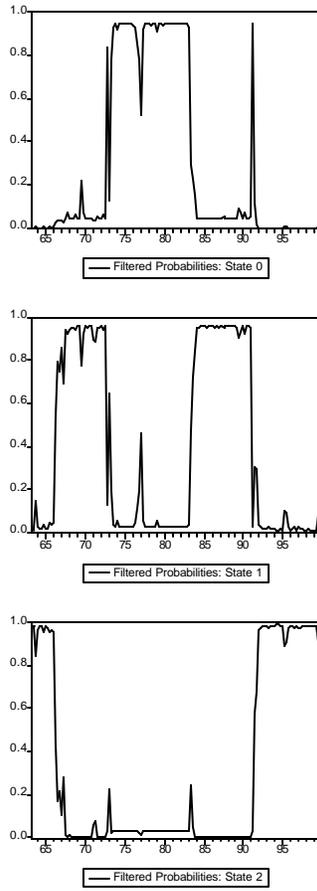Figure 2. Transition Filtered Probabilities for the Output Process

Figure 3. Transition Filtered Probabilities for the Inflation Process