

"A NEW MODEL FOR NON-IDEAL
TYPE II SUPERCONDUCTORS"

BY

A. Zahradnitzky
University of Ottawa
Ottawa,
Canada.

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ABSTRACT

In this treatise a New Model for hysteretic type II superconductors is formulated, based on the following three major concepts: Abrikosov diamagnetism, pinning of flux by imperfections (the critical state), and the existence of an irreversible surface barrier (or surface pinning). Three separate distinct experiments are reported on which explore these well documented phenomena.

A transport current is applied to a long ribbon of superconducting $\text{Nb}_{.25}\text{Ta}_{.75}$ wound into a solenoid in order to create asymmetric boundary conditions across the ribbon sample's cross-section. The magnetization arising along the axis of the windings is studied under this constraint when a static external field is applied along this axis, hence transverse to the impressed current. The critical state picture by itself is found to provide an adequate description only over a limited range. For a complete understanding of the observed phenomena over the full spectrum of investigations, our new model must be invoked. We verify this assertion by extracting, from experimental data, information on the relative contributions and functional dependences of the pertinent components comprising our synthesized model.

Next, we study the behaviour of the moments arising along the major and two minor axes of a long rectangular strip (semi infinite slab) of $\text{Pb}_{.86}\text{In}_{.14}$ as currents are induced in the sample in the presence of a static field applied along one of the axes. Currents are made to flow via an inducing field varied along one of the axes perpendicular to the static field. With the restriction that the direction of one of the magnetic fields (the inducing or the constant) be along the major axis of the ribbon, all possible combinations were explored and moments arising both along the transverse and longitudinal directions were monitored. We clearly demonstrate that none of the previously proposed models, relying on partial or incomplete descriptions, can fully account for the observed trends. The difficult task of a complete analysis in terms of our new model however, is not provided in light of the nature of this thesis.

In the third and last set of investigations presented, we monitor the amount of flux expelled when the temperature of a ribbon sample (V and PbIn) is lowered through the superconducting transition temperature T_c to the helium bath temperature as a function of field intensity H and angle ϕ that $\vec{H}$ subtends with the minor axis of the rectangular slab. Due to the relative simplicity of this configuration, the experimental observations are interpreted quantitatively in terms of our new composite model. The ensuing excellent agreement between experimental and calculated trends provides indisputable evidence of the validity of our proposed model.

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CHAPTER I

INTRODUCTION

Preamble

For a full description of the magnetic behaviour and current carrying properties of non ideal type II superconductors several distinct physical contributions have been proposed. Previous researchers have either endeavoured to establish the validity of a particular concept, pursued detailed features of a specific concept, or considered at most a combination of two of the several proposed contributions in their analysis of experimental observations. Since in many situations one or two contributions clearly dominate, the models employed have been adequate and successful in describing the results under scrutiny.

In this treatise a "new" model is developed which combines the essential features of the major pertinent concepts proposed to date in the literature. Although no new concepts are introduced, the model proposed is "new" inasmuch as it systematically intertwines and synthesizes the prevailing ones into a consistent picture. Further, in this thesis three separate but related experimental investigations are reported that were conducted to test the validity of the proposed synthesis. The main features of the present experimental results are accounted for by this "new" model, but cannot be understood in light of any of the previous approaches relying on incomplete or partial descriptions.

Next, the basic concepts exploited for the proposed "new model" shall be briefly reviewed with special attention to the aspects relevant to the work performed herein.

Reversible Surface Currents (Abrikosov Diamagnetism)

The magnetic behaviour of reversible (ideal) type II superconductors has been examined theoretically, both on a microscopic and a macroscopic scale, by Abrikosov¹⁾, starting with the Ginzburg-Landau phenomenological formulation of the superconducting state²⁾.

Ginzburg-Landau had shown that two types of superconductors were possible, depending on whether a parameter κ was less than or greater than $1/\sqrt{2}$. The parameter κ is referred to as the Ginzburg-Landau parameter and is approximately given by the relation $\kappa = \lambda/\xi$ where λ is the penetration depth and ξ is the coherence length. These authors also introduced the concepts of an upper critical field $H_{c2} = \sqrt{2} \kappa H_c$ above which bulk superconductivity is destroyed and a lower critical field H_{c1} below which the magnetic induction is zero in the body or bulk of the superconductor. In a type I superconductor $H_{c1} = H_c = H_{c2}$, whereas in a type II superconductor, $H_{c1} < H_c < H_{c2}$ where H_c is a thermodynamic critical field related to the energy gap and experimentally defined by the relation $H_c^2/8\pi = A$ where A is the area under the reversible magnetization curve.

Microscopically, Abrikosov predicted that a type II superconductor immersed in a magnetic field H between H_{c1} and H_{c2} would assume a configuration, called the mixed state, consisting of a uniform and regular array of flux filaments or current vortices with their normal cores parallel to the applied field. Further, Abrikosov showed that the energy is minimized when each flux filament or current vortex contains a single quantum of flux $\phi_0 = hc/2e$.

Macroscopically, Abrikosov derived a relation between the equilibrium density of vortices N (hence the average magnetic induction $\langle B \rangle$) and the applied field H . In this result, $\langle B \rangle = N\phi_0$ is always less than the applied field H . Physically, this means that a persistent current I_R is circulating in a diamagnetic (field opposing) sense in a region of thickness $\approx \lambda$ at the surface of the superconductor. The magnitude of this equilibrium surface current is linearly proportional to H in the range 0 to H_{c1} , and is sufficient to reduce B to zero in the body of the superconductor. Further, this reversible diamagnetic surface current decreases monotonically to zero with increasing H between H_{c1} and H_{c2} . Following other workers, this surface current is referred to as the reversible or equilibrium, or Abrikosov current and we denote its value per unit length of surface perpendicular to H by I_R . The magnetic moment due to this current is often referred to as intrinsic Abrikosov diamagnetism. The magnetization per unit volume in the system of units used in this thesis is written as $4\pi M_A$ (in gauss) $= \mu I_A$ where $\mu = 4\pi/10$ and I_A is in amperes/cm.

Both the microscopic and macroscopic predictions of Abrikosov have been extensively verified experimentally³⁾. The essential features of this work which pertain to the present investigation are the following:

- (i) the presence of a macroscopic current flowing in a diamagnetic (flux shielding) sense at the surface of a type II superconductor immersed in a magnetic field,
- (ii) the magnitude of this surface current as a function of H , in particular $I_R = H/\mu$ when $0 \leq H \leq H_{c1}$ and $I_R = (H_{c2} - H)/1.16\mu(2\kappa^2 - 1)$ with H in the neighbourhood of H_{c2} , for a stable triangular lattice of vortices.
- (iii) the equilibrium density of vortices (hence $\langle B \rangle$) as a function of the applied field H . (Note that in this context $\langle B \rangle$ is an average over several vortices).

Irreversible Surface Currents.

Several workers⁴⁻⁷⁾ have provided evidence that the surface region of nonideal type II superconductors can support macroscopic persistent irreversible currents of very high density (e.g. $\approx 10^6$ to 10^8 A/cm²). In contrast to the reversible Abrikosov surface current which flows only in a diamagnetic (field opposing) sense, the irreversible current can circulate in either a diamagnetic (field opposing) or paramagnetic (field aiding) sense. The sense of circulation of this current depends on whether the previous history of the specimen in the superconducting state has called for opposition to entry or exit of flux.

The maximum magnitude or critical value I_{IR} of this current depends on several quantities; superconducting parameters (hence T/T_c), surface conditions^{8,9)} the field intensity H at the surface and the angle $\vec{H}$ subtends with the plane of the surface and with the irreversible surface current density vector $\vec{j}_{IR}$. In the present work an attempt was made to provide some information on the last three aspects of this problem.

It is noted above that the reversible (Abrikosov) surface current is in equilibrium with and is determined by the applied field seen by the surface. An interesting question thus arises: what is the effective field at the surface when an irreversible surface current is flowing? Several workers have stipulated and experiments by LeBlanc and his collaborators^{10,11)}, seem to confirm that the "true applied field" in such circumstances is the external field H aided or partially screened by the irreversible surface current, hence $(H \pm \mu I_{IR})$ where the sign depends on whether I_{IR} aids (+) or opposes (-) the external field H . In this picture, the surface region in which the irreversible current I_{IR} flows is considered to be separate from, adjacent to and embracing the "surface" within which the reversible Abrikosov current exists. The work to be presented indicates this viewpoint to be correct. Pursuing this approach, the dependence of I_{IR} on field intensity and on its orientation with respect to the plane of the surface was extracted from appropriate data.

It will be seen below that the density of vortices and the Lorentz force both play an important role in imposing a ceiling on the magnitude of the macroscopic currents which can flow in the bulk of a type II superconductor. It is important to determine what role these quantities

play in setting a limit on the irreversible surface current. Our experimental results provide information on this problem but do not yield unambiguous answers to these questions.

Pinning and Irreversible Currents in the Bulk.

Critical State Concept

It is now well established that physical and chemical imperfections contribute to the irreversible behaviour of type II superconductors¹²⁾. The qualitative picture of the mechanism which gives rise to the irreversibility is straight forward. In a perfect (ideal) type II superconductor, the lattice of flux filaments (vortices) is completely free to adjust to ambient conditions (H,T). The density of vortices in equilibrium with H is uniform and no macroscopic persistent currents flow in the body of the specimen. In an imperfect (nonideal) type II superconductor, the defects provide sites which can pin individual flux filaments, or bundles of these. The most effective pinning site has geometric dimensions in the order of the coherence length (the size of the vortex core). The higher the κ value, the smaller the coherence length so that the effectiveness of pinning may be reflected by the parameter κ . This phenomenon of pinning impedes and opposes the establishing of a uniform distribution of vortices in equilibrium with the applied field. As a consequence, very inhomogeneous configurations of magnetic induction can exist in the bulk of the material and persistent macroscopic currents of high density can flow in the body of a nonideal type II superconductor.

Bean¹³⁾ and Kim et al¹⁴⁾ were the first workers to propose that the maximum persistent current density which can be sustained in the bulk of a type II superconductor is determined by the equilibrium between the Lorentz driving force density $\vec{j} \times \vec{B}$ and the pinning force density $\vec{F}_p$. Applying the laws of induction to this situation, these workers stipulated that the currents which are caused to flow under isothermal conditions must adopt the maximum value allowed. These two ideas constitute the critical state concept which can be stated mathematically as:

$$\vec{j}_c \times \vec{B} = \vec{F}_p \quad \text{or} \quad \vec{j} = 0$$

The pinning force density can depend on superconducting parameters (hence on the reduced temperature T/T_c) of the material, the intensity of the magnetic induction and on the type, number and arrangement of the defects. Two complicating features may be encountered. The distribution and the nature of the defects can lead to pinning forces which are anisotropic¹⁵⁾ and which vary over the volume of the sample. Precautions were taken in the preparation of our samples to ensure homogeneity. It is assumed, for simplicity, that pinning is isotropic. The results of the analysis suggest that this is a valid assumption.

The sense of circulation or direction of flow of the macroscopic currents present in the bulk of the specimen is determined by the previous history of the sample in the superconducting state using the Faraday-Lenz law of induction. Once the system of coordinates

is suitably specified, these bulk currents can be caused to flow in either a positive or negative (clockwise and counterclockwise) direction. Following the prevailing convention, currents flowing in the bulk are referred to as diamagnetic or paramagnetic, depending on whether the Lorentz force the current density vector experiences is directed away from or towards the neighbouring surface of the specimen. As noted above, the magnitude of the currents flowing in the bulk is determined by the critical condition $\vec{j}_c \times \vec{B} = \vec{F}_p$. The critical state concept was originally developed for, and mainly applied to situations where the current density vector $\vec{j}$ is orthogonal to the magnetic induction vector $\vec{B}$. For these circumstances one can write the critical state condition more simply as $j_c B = F_p$ since it is then understood that j_c is perpendicular to B . Various workers have proposed simple analytic functions for the pinning force density, hence for j_c and B , which lead to an adequate description of the macroscopic behaviour of the samples under scrutiny. Indeed, a considerable effort has been dedicated to the determination of the "best" analytic function which optimizes the fit between calculated and observed curves¹⁶⁾. Unfortunately, some of the "best" analytic functions exploited in the calculation of a set of results are sometimes found to be inadequate for, or inconsistent with the qualitative interpretation of other important observations on the same specimen. A simple and physically reasonable analytic function for F_p was introduced in our calculations. The emphasis in the present work, however, has been to establish whether the model proposed is valid. The ensuing qualitative analysis and conclusions

are not crucially dependent on the choice of the pinning function incorporated in the calculations.

Force-Free or Nearly Force Free Current Flow

The pattern of current flow and the configuration of the magnetic induction in the bulk of a nonideal type II superconductor can be established qualitatively and quantitatively by invoking the critical state concept and the Faraday-Lenz law of induction provided that j_c is perpendicular to B . Important situations are encountered however where this condition is not satisfied¹⁷⁾. LeBlanc and his collaborators¹⁸⁾ have pioneered in the study of such cases. These workers have examined this type of situation using two different techniques which are now described.

- a) A transport current is fed from an external supply attached to the ends of a long wire or ribbon specimen placed with its axis along an externally applied static magnetic field. Even when the magnetic field associated with the current is taken into account it is clear that the current density vector is not perpendicular to the magnetic induction vector.
- b) A ribbon (or wire) specimen is placed with its axis along an externally applied static magnetic field. Now, however, the persistent currents flowing along the length of the sample, hence along the axial magnetic field, are generated by induction, i.e. by applying a magnetic field perpendicularly to the axis of the sample. Again $\vec{j}$ is not transverse to $\vec{B}$.

The critical state condition alone is inadequate in these circumstances. This can be seen as follows. The Lorentz force $\vec{j} \times \vec{B}$ now has at least two components (which may aid or oppose each other). There is only one constraint available, the critical state condition, but there are at least two components of the current density which need to be determined in order that $\vec{j}_c$ can be specified.

LeBlanc and his collaborators¹⁸⁾ have observed that under the circumstances described in (a) and (b) above, the current density vector tends to adopt "spontaneously" a pattern of flow such that $\vec{j}$ tends to lie along $\vec{B}$. This configuration clearly tends to reduce the Lorentz force for given values of $\vec{j}$ and $\vec{B}$. Indeed if $\vec{j}$ is exactly parallel to $\vec{B}$, the Lorentz force is zero. This situation is referred to as force free flow. It is also clear that the Faraday-Lenz law is now insufficient to predict the precise direction of current flow, since the induced (or impressed) current does not flow transverse to the inducing external field change.

These phenomena were encountered and explored by LeBlanc et al in type II superconductors having large Ginzburg-Landau parameter κ (≈ 30) and exhibiting strong pinning, hence capable of supporting high current densities in the bulk. This work indicated that the strong pinning interfered with the tendency of the flux line lattice and the macroscopic current density to achieve parallel, hence force-free configurations. Indeed, the angle between $\vec{j}$ and $\vec{B}$ appeared to remain quite large. Exploiting the technique described above under (b), these phenomena were examined in a relatively weak bulk pinning and low κ

material (PbIn). This particular project was also intended to determine whether the irreversible surface currents also tend to adopt a force free, or nearly force free pattern of circulation. It is unfortunately difficult to separate bulk and surface contributions to the data.

The Composite Model for Hysteretic Superconductors

Our new model whose constituents have been just discussed combines Abrikosov diamagnetism, irreversible surface currents and the prevailing bulk critical state concepts into one systematic picture. We stress that it is new because all these surface¹⁹⁾ and bulk²⁰⁾ phenomena are well documented both experimentally and theoretically^{21,7)}, but up to now no direct rigorous analysis has been performed which involves all three of these. We neglect force free flow which seems to play a minor role in our results.

Following Silcox and Rollins²²⁾ it is assumed that the boundary value of the local magnetic field in the bulk, B_s , has the value $\langle B \rangle = f(H^*)$ where $\langle B \rangle$ is the average magnetic induction predicted by Abrikosov and H^* is the effective field seen by the bulk. Thus we write for the boundary of the bulk

$$B_s = B_s(H^*)$$

This means, as also stated earlier, that there is a reversible diamagnetic current I_R flowing within a thin layer (the dimension of λ) at the surface of the bulk. From this it follows that

$$\mu I_R = B_s(H^*) - H^* = 4\pi M_A = \delta_R$$

where $4\pi M_A$ is the reversible Abrikosov diamagnetic moment of an ideal sample at H^* .

Within the bulk, currents can flow as prescribed by the critical state conditions, i.e., the metallurgical nature of the bulk determines the pinning strength, hence the steepness of the B_z profiles, and the previous magnetic history in the superconducting state determines the sense of circulation of the bulk currents, hence the sign of the slope of the B_z profiles. We stress again that the boundary for these bulk currents (hence of the B_z profile) is B_c which is not the applied external field H .

If, as in the Silcox model, there are no irreversible surface currents circulating then $H^* = H$. However we take into account in our model that irreversible currents can flow in a thin sheath (thickness of the order of λ) embracing the surface within which Abrikosov currents flow. We extract information on the magnitude and field dependence of these surface currents from our experimental observations. In our model then $H^* = H \pm \mu I_{IR}$ (where I_{IR} is the magnitude of the irreversible current), the sign depending on the sense of circulation of I_{IR} , hence on how it was induced.

In the above description we have, for simplicity, assumed geometries where currents and fields are mutually orthogonal. Where this is not true, our model still holds but calculations become very difficult. Nevertheless, some of these obstacles have been successfully overcome as will be shown in the course of this thesis.

CHAPTER II

SIMULATED INFINITE SLAB OF NbTa
CONDUCTION AND INDUCED CURRENTS

Introduction

An infinite slab immersed in a uniform applied magnetic field H parallel to its surfaces and through which a conduction current can be caused to flow transverse to the direction of H and with uniform density along H constitutes the ideal situation for theoretical analysis of magnetization and conduction current phenomena in a type II superconductor. Chang²³⁾ has devised a sample design which simulates this ideal situation extremely well. Using this arrangement, the transport and magnetic behaviour of a type II superconductor can be investigated with currents introduced into the "infinite" slab from an external source and/or by induction. Chang pursued extensive measurements on a "simulated" infinite slab sample made with severely cold-worked Nb₃Zr ribbon, a material with strong bulk pinning, large κ parameter (≈ 35), and high ratio of H_{c2}/H_{c1} (≈ 160). In this material contributions to the magnetic phenomena and transport properties due to the surface currents (reversible and irreversible) are smothered by the large current carrying capacity of the bulk.

Using a sample construction almost identical to that developed by Chang we undertook to study a material of smaller κ

exhibiting relatively weaker bulk pinning in order to explore the behaviour in a situation where surface current and the intrinsic Abrikosov diamagnetism would not be dominated and their role made manifest. NbTa was selected since this material could be purchased in ribbon form (at a not too exorbitant price) and preliminary measurements indicated that it satisfied the desired conditions.

Although bulk currents play an important role in the material and tend to mask the other contributions, we show that the data can be consistently interpreted quantitatively and qualitatively only if we apply the synthetic model we propose and develop in this thesis.

Construction of the Sample

The material was obtained from M.R.C. in the form of severely cold worked ribbon of width and breadth .040" and .015" respectively. It was wound on a bakelite former into two concentric solenoids of equal turns such that for a current I in one ribbon there is a current $-I$ carried by the one directly below it. This was achieved by turning the material back on itself after the desired length of the first layer was attained; this ensured the continuity of the material and of the applied transport current. A thin layer (.001") of mylar separates the two coils. It was found from resistance measurements in the normal state that the oxide layer at the surface of the material provides complete insulation between the closely wound turns. Transport

currents therefore always flow perpendicular to the axis of the cylinder.

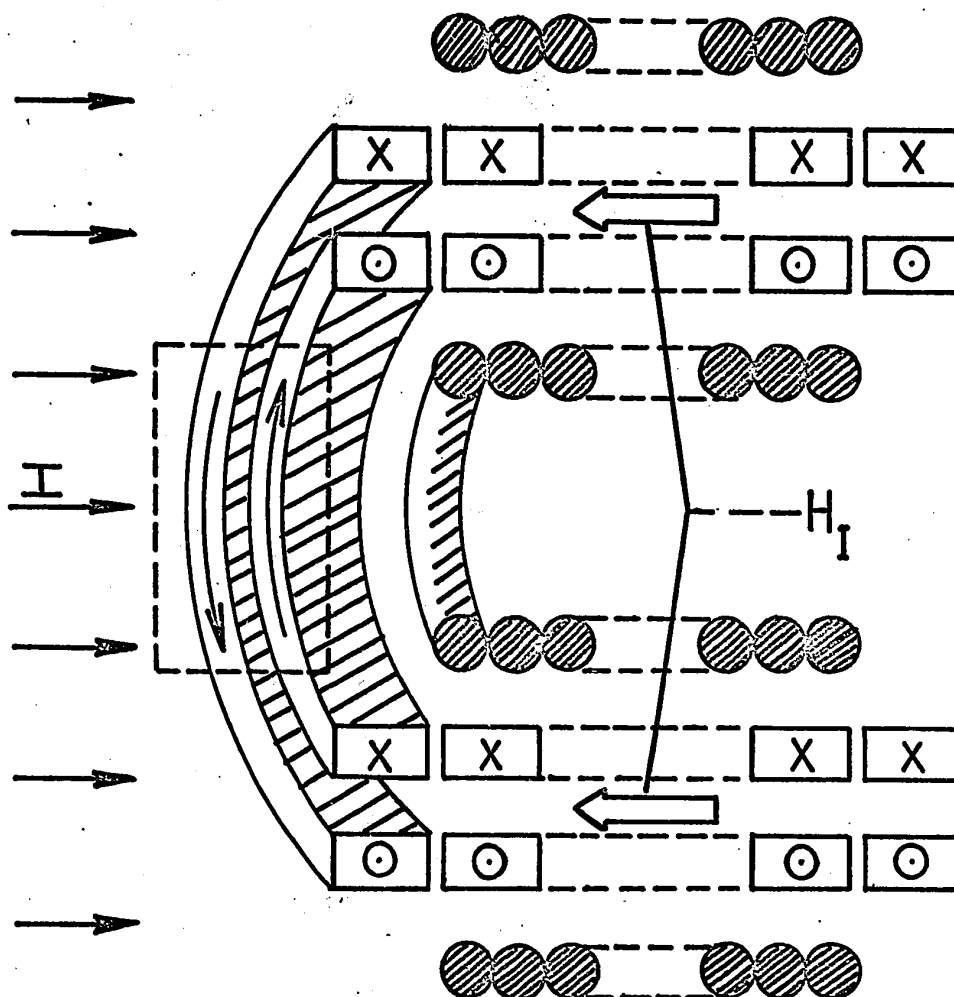
A solenoid wound from flat ribbon material simulates infinite slab geometry if the curvature is much larger than the pitch of the windings. The ratio of the latter to the former in fact was .04/.8 or 1 in 200. Thus, the plane of each turn is essentially perpendicular to the axis of the solenoid.

If a magnetic field H is applied along the axis of the solenoid, then H is perpendicular to the transport current I and the boundary conditions for the outer coil are given by H on the outer, and $H \pm \mu NI$ on the inner surface since the magnetic field produced by I is effectively zero outside (except for end effects) and has a constant value of μNI on the inside, where $\mu = 4\pi/10$, I is in amperes/cm and N is the number of turns/cm. In the present investigations, extensive use is made of these boundary conditions.

The type of concentric solenoid arrangement employed in this construction can be treated as a single solenoid since the two coils or "infinite" slabs are "mirror images" of one another with respect to boundary conditions and configuration of magnetic induction. (See schematic diagram A-1). Our arrangement has several practical advantages on which we now elaborate.

As shown in Appendix A, the solenoidal approximation ($\mu NI = H_I = \text{constant}$) is much improved chiefly because of the small gap ($\sim .002$ ") between the coils. End effects are also greatly reduced since

DIAGRAM A-1



SAMPLE ARRANGEMENT

Diagram shows schematically the sample arrangement which consists of two concentric but oppositely wound coils of Nb_{0.25}Ta_{0.75} ribbon material. A pickup coil surrounds the sample and a bucking coil is placed inside directly below it.

the field produced by the conduction current reaches $\sim 98\%$ of its central value μNI , $1/10$ the way in from the end of the solenoid. The total magnetization is doubled, and flux trapped inside the sample is always destroyed symmetrically when the sample is driven normal by raising the temperature, T , to above the transition temperature T_c ^{23,24}). This latter is only true if the sample is heated symmetrically from both sides. For this reason non inductive heaters of no. 32 manganin wire were wound directly below and above the sample, thin mylar separating them from the sample surface. These also serve to maintain the sample at a temperature higher than that of the helium bath. This is done to insure that the current is truly limited by the sample and not at the contacts with the current source.

The heaters do not extend over the entire length of the sample so that the ends can be maintained at a lower temperature than the main body. This has the result of minimizing end effects, where the direction of the self field is perpendicular to the flat face of the ribbon; the critical fields and currents increase with decreasing temperature. The inner heater had a room temperature resistance of $2.1 \text{ k}\Omega$ and was connected in parallel with the outer one ($2.2 \text{ k}\Omega$) for uniform heating. Another reason for working at elevated temperatures ($> 4.2^\circ\text{K}$) is the avoidance of undesirable flux jumps²⁵), as magnetic stability increases when the temperature approaches T_c .

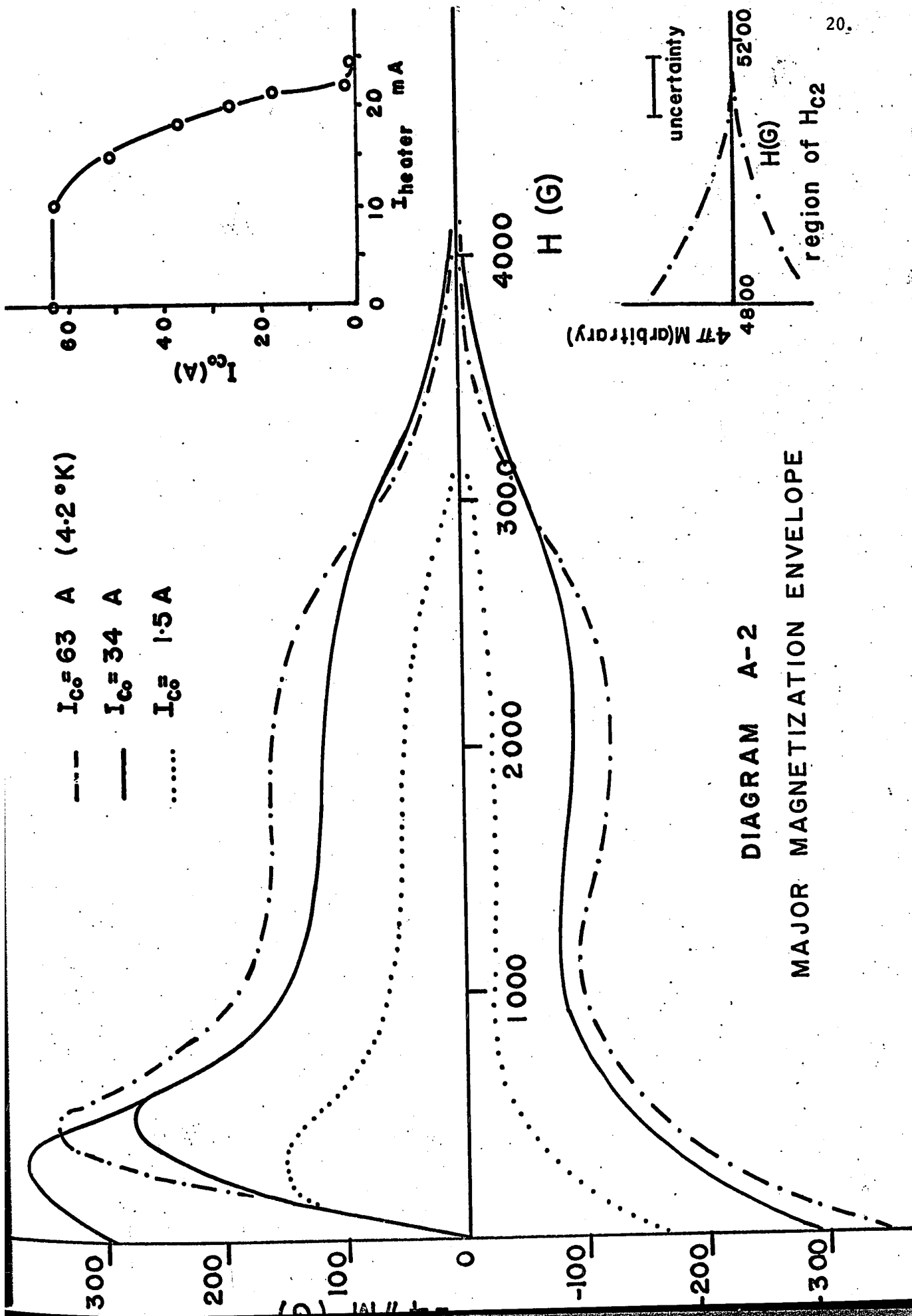
Pickup coils wound directly below and above the heater coils, of 44 B & S formvar insulated copper wire, serve to detect changes in the magnetization of the sample. These were balanced and connected in series

opposition in such a manner that in the normal state ($T > T_c$) a field H applied along the axis of the solenoidal sample (parallel to the "infinite slab") produces zero e.m.f. in the coils. It is shown in Appendix E that as a consequence the integrated voltage produced, when the pickup coil signal is fed into an operational amplifier, is directly proportional to the magnetization of the superconducting sample.

The bakelite former, cylindrical in shape, was supplied with shoulders of appropriate thickness to fit snugly into a superconducting solenoid wound of NbZr wire which provided a field H along the axis of the sample. (H was 1150 gauss/amp).

The two ends of the sample wire were carefully cleaned of the oxide layer and then sandwiched between clean copper blocks attached to the bakelite former. Indium foil placed in between the blocks, on top and underneath the sample, ensure a uniform & intimate electrical contact. Relatively high pressure for this electrical contact was provided by bolts clamping the top and bottom blocks together.

This technique of making electrical contact is used since NbTa does not solder, and any welding is difficult and undesirable, as it may partially anneal the material. (Voltage probes attached to the copper blocks and sample indicate a maximum of $\sim 40\mu\Omega$ contact resistance under such circumstances). Thus now a transport current I can be fed into the sample. A copper shunt of $.2\Omega$ was placed across the blocks since currents of the order of 100 amps were to be passed through the sample until it was driven normal in which case it has a resistance of approximately 7Ω at 4.2°K . Thus the initial dissipation I^2R may be very high without a shunt,



whereas the shunt will not divert any current from the sample when it is superconducting. Finally, voltage probes were attached directly to the superconducting sample material to detect the onset of resistance in the sample.

Experimental Results

In the following, the pertinent measurements taken on the sample are briefly described.

A. The major hysteresis loop of the sample was determined, that is, the magnetization was measured as a function of an externally applied magnetic field H in the range $H = 0$ to $H = H_{c2}$ in all four quadrants. This was done at several temperatures T where T is greater than the helium bath temperature of 4.2°K but less than T_c . A particular "working" temperature was chosen such that the critical current was definitely determined by the warm region of the sample and the measurable moments were optimized. (See figure A-2). This serves as a general reference with respect to other measurements to be reported later in this chapter.

B. The critical current I_c was measured as a function of an externally applied field H . I_c is defined here as the maximum value of the applied transport current, with H and T constant, that the superconducting sample can sustain before the onset of resistance. (The reason for this definition will become obvious).

To eliminate any persistent currents (magnetic history) present due to previous measurements, the sample was cooled through the

transition temperature T_c to the final temperature T_f in the presence of a chosen static field H for each measurement of I_c .

Since the actual temperature was not measured, the critical current in zero field denoted I_{c0} was used to establish the same elevated temperature for the sample from one run to another. Diagram A-2 also shows I_{c0} as a function of heater current.

C. Simultaneously with the measurements of $I_c(H)$ the magnetization at the critical current was monitored, hence under identical conditions.

D. Hysteresis loops of the magnetization as the transport current I is cycled between positive and negative values were traced for several externally applied fields. Again, previous history was "erased" prior to such cycling of I by cooling from T_c to T_f in the chosen H .

Here the sign of the current denotes whether the field produced by the current aids or opposes the externally applied static magnetic field H in which the sample cooled.

E. The incomplete Meissner effect or imperfect Abrikosov diamagnetism was determined as a function of H . This is the diamagnetic moment developed when a hysteretic type II superconductor cools through its transition temperature T_c to a final temperature T in the presence of a static magnetic field H . This quantity serves as a gauge of the reversibility of the material, since it can be compared with the ideal curve expected for a material of comparable H_{c1} , H_{c2} and κ . The degree of flux expulsion in any field is considerably reduced in a hysteretic sample. This is the case for our NbTa sample.

Discussion

The design of the simulated infinite slab sample allows us to introduce supercurrents in two ways: (i) by applying a transport current, and (ii) by induction, i.e. by applying (or removing) a magnetic field along the axis of the cylinder in the superconducting state; according to prevailing views the magnetic behaviour observed should be completely consistent regardless of the "origin" of the currents present.

Another way of stating this is that flux can be introduced into the sample in two different ways. However, applying a field $H > H_{c1}$ introduces flux into the sample symmetrically, whereas the application of a current introduces flux into the sample from the inner surface only since the self field produced by the solenoid sample is identically zero on the outer surface.

(i) Check on Boundary Conditions.

From considerations of symmetry and boundary conditions only, one expects the average field introduced by the two methods to obey the following relationship until flux has penetrated to the midplane of the infinite slabs.

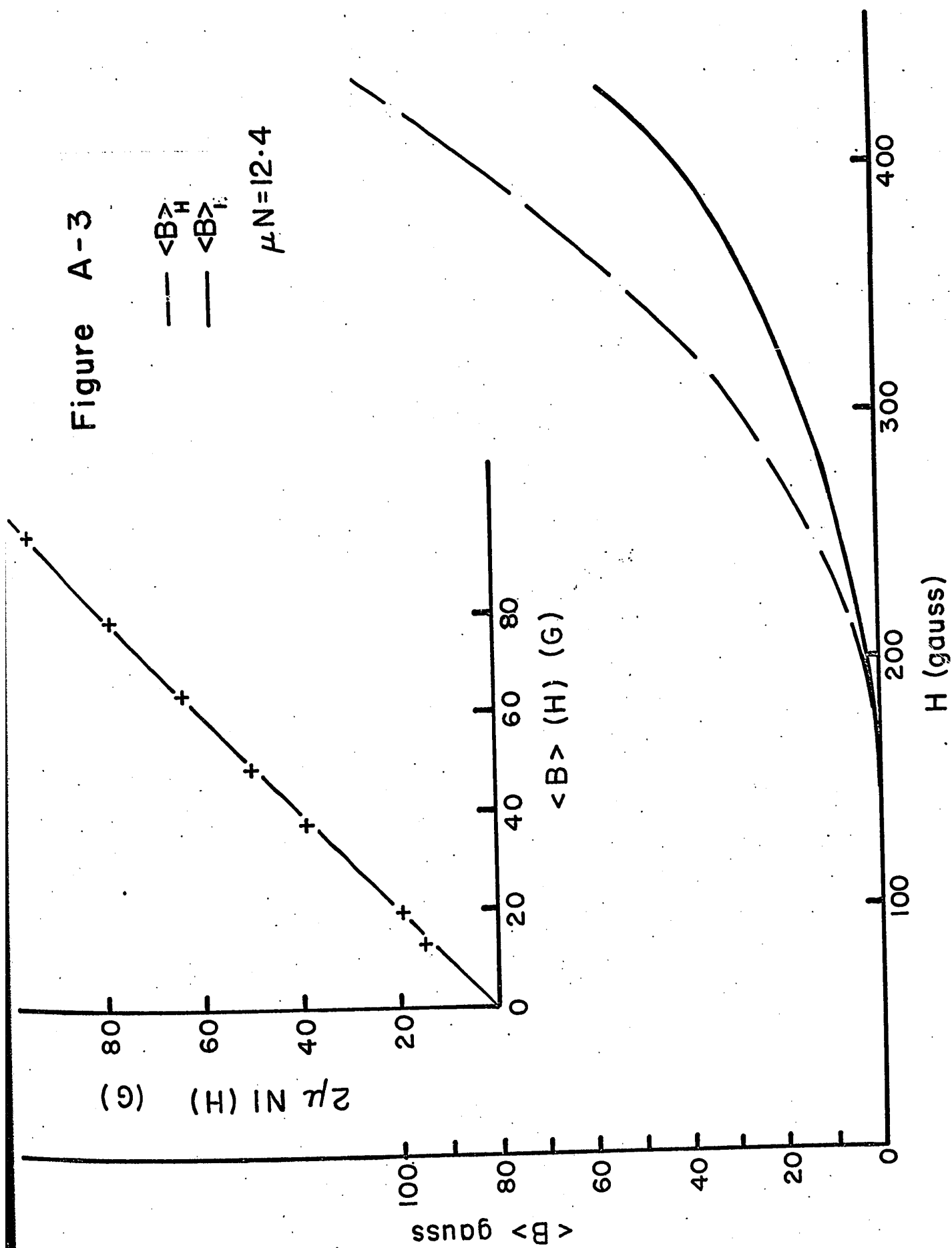
$$\frac{\langle B \rangle_I}{\langle B \rangle_H} = \frac{1}{2}$$

where $\langle B \rangle_H$ is the average magnetic induction inside the sample produced by an externally applied field H (in gauss) and $\langle B \rangle_I$ is the average magnetic induction inside the sample produced by a current I flowing in the ribbon. Here H_I (in gauss) = μNI with I in amperes, $\mu = 4\pi/10$ and N is the number of turns/cm. The experimentally observed $\langle B \rangle_I$ and $\langle B \rangle_H$ are shown

Figure A-3

$\langle B \rangle_H$
 $\langle B \rangle_i$

$$\mu N = 12.4$$



in figure A-3. The observed ratio of $1/2$ verifies that the boundary conditions $H_I = \mu NI$ inside (zero outside) are correct (see Appendix A) and thus paves the way for postulating identical flux profiles with an added degree of certainty whether the currents are fed into or induced in the samples.

(ii) Estimate of κ .

From the main magnetization curve (Diagram A-2) the critical fields H_{c1} and H_{c2} can be estimated and a value for κ for our sample can be calculated. It should be pointed out that calculations of κ from magnetization curves are approximate only, when highly hysteretic samples are involved.

If H_{c1} is taken to be that value of H where the magnetization curve breaks away from the perfect diamagnetic slope (that is, where flux penetration first occurs) then the value of H_{c1} at $4.2^\circ K$ is 165 ± 25 gauss, estimating $H_{c2} \approx 5200$ gauss yields a value of $\kappa \approx 6$.

(iii) I_c vs H .

It is clear that using an external current source to feed a current into the sample, two choices of polarity or direction of current flow are possible. Because of the design of the sample, the magnetic field produced by the current aids the externally applied field for one sense of flow and opposes it for the other sense of flow. Figure B-1 shows I_c , the maximum lossless conduction current the sample can support, vs H for the two possibilities. The main features of these two curves can be readily understood on the basis that the density of both irreversible surface and bulk currents flowing $\perp$ to a magnetic induction decreases as B increases.

For I aiding H , both contribute to increasing the magnetic induction, hence the monotonic decrease of the critical current with H is expected. For the case where I opposes H , a large peak in I_c is expected

GRAPH OF I_c vs. H
experimental

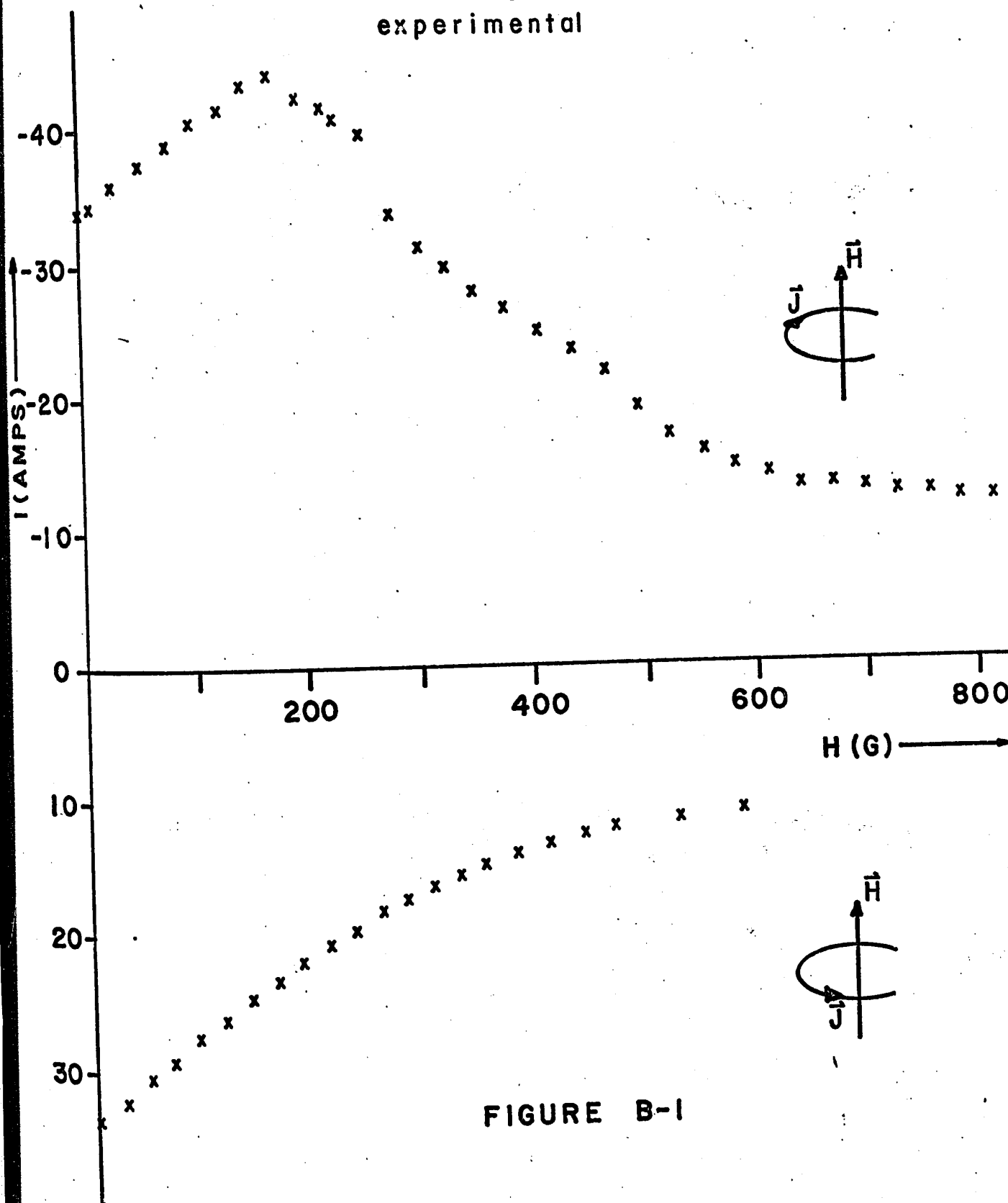
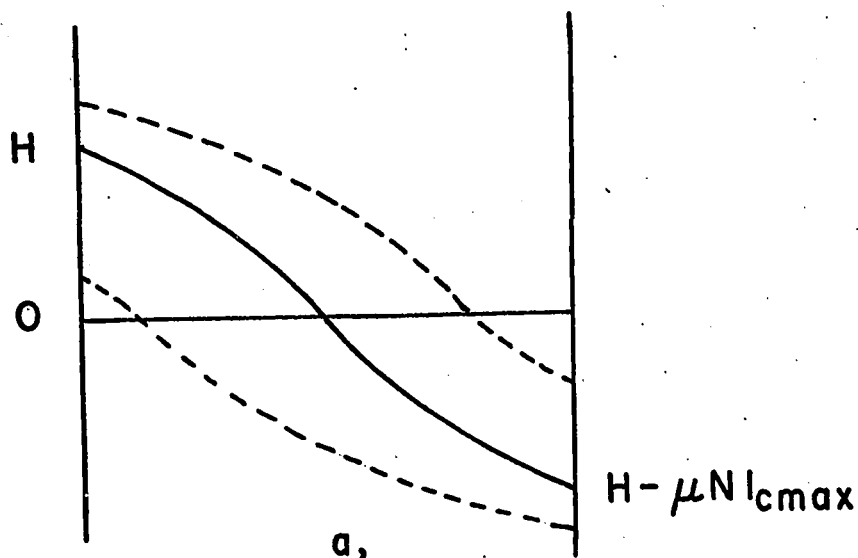
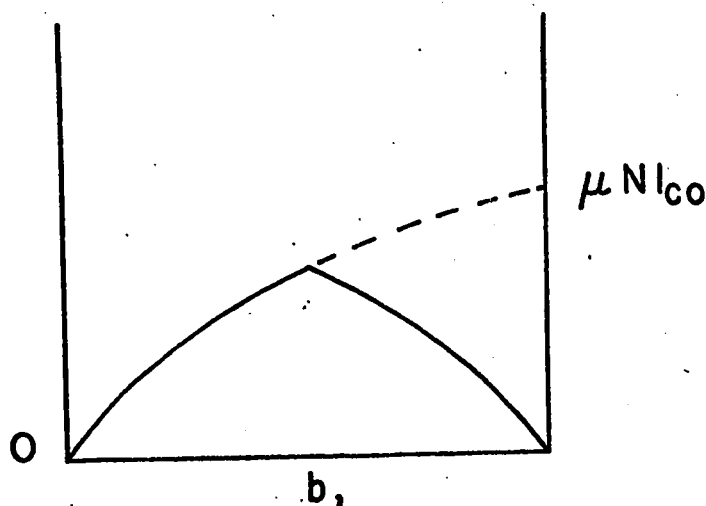


FIGURE B-1

FIGURE B-2



- a) This shows that $|H - \mu N I_c| = 2H$ when the critical curve passes through the center of the specimen. It is easy to see that if $\frac{dB}{dx} \rightarrow 0$ for increasing B as it must, the above defined value of I_c is $I_{c \max}$. The dotted lines show cases to either sides of this maximum.



- b) This demonstrates that the remanent magnetization (area under solid curve) can be obtained either by half cycling I_c in $H = 0$ (dashed curve) or by reducing H from a high value to zero (I being kept zero)

SCHEMATIC $I_{c \max}$

before the monotonic decline. The reason why this behaviour is expected is presented schematically in Figure B-2. Further, solely from considerations of symmetry, of the appropriate boundary conditions and of the decrease of bulk current density with B, we can obtain the relationship (see Figure B-2)

$$2 H_{\max} = \mu N I_{c \max} \quad 28-1$$

where $H_{\max}$ is the externally applied field in which the sample can support the largest critical current $I_{c \max}$. We note however, that introducing the observed values for $H_{\max}$ and $I_{c \max}$ in equation 28-1 yields a value for $\mu N = 7.6$ whereas we know from construction of the sample (we have checked by the measurements described in (i) above that no error occurred) that $\mu N = 12.3$. We shall return to this disturbing result later.

(iv) $4\pi M$ at I_c vs H

Figure C-1 presents the measurements of the magnetization of the sample when the conduction current flowing in it attains the values of $\pm I_c$ as a function of the externally applied field H . We note that, as expected, these curves reflect and indeed emphasize the main feature of the corresponding I_c vs H curves (see figure B-1).

From inspection of the magnetization curves as H is swept up to and down from H_{c2} (see Figure A-2) and from the observation that scarcely any flux is expelled from the sample during cooling from T_c in a static H we may conclude that pinning in the bulk, hence macroscopic bulk currents, play a major role in the behaviour of our sample. Accordingly, as a first step, we have applied the critical state concept alone to the analysis of our results for $4\pi M$ at $\pm I_c$ vs. H . In other words we calculate

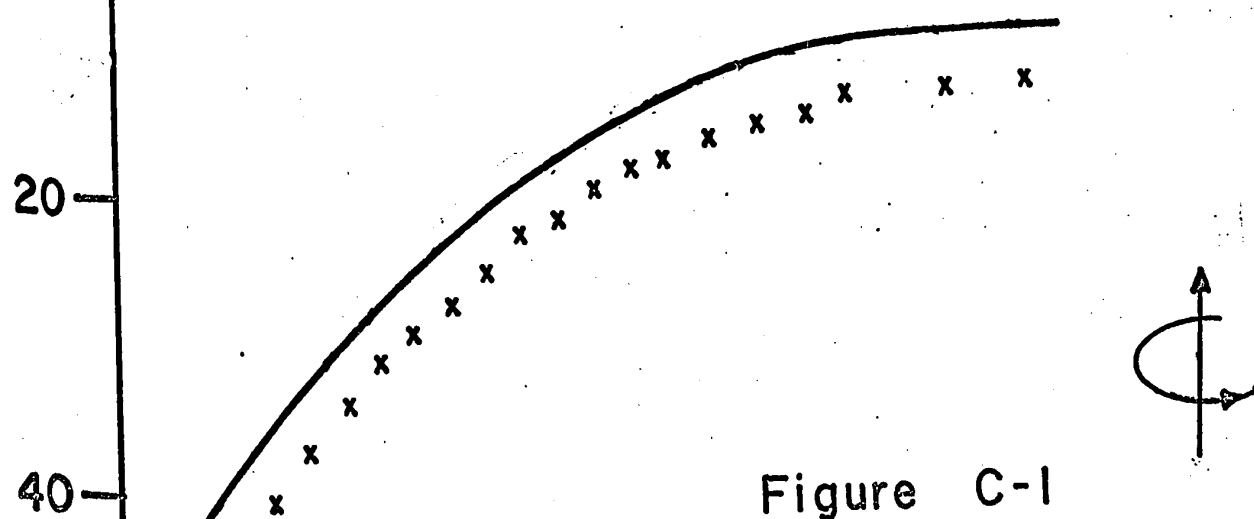
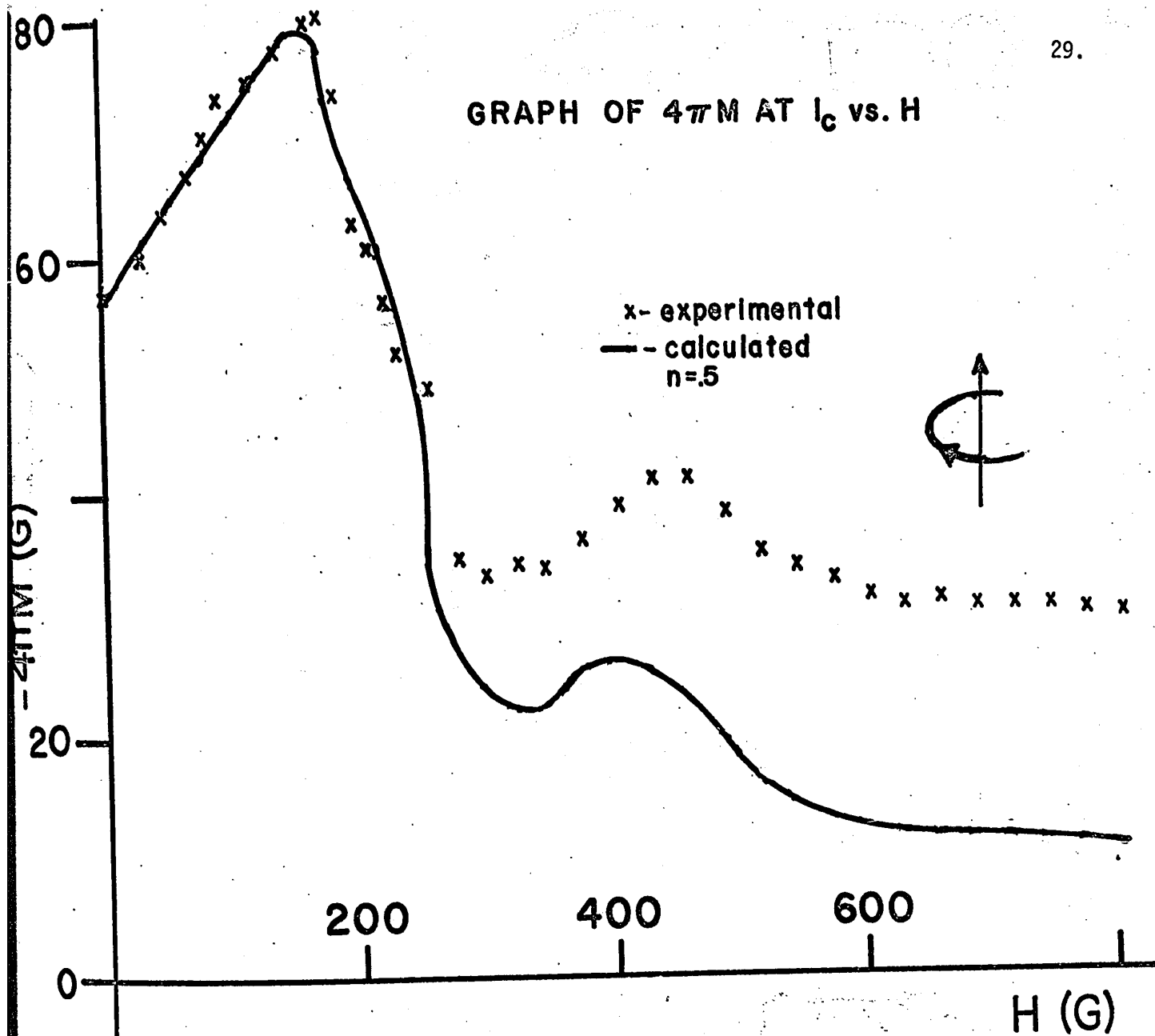
GRAPH OF $4\pi M$ AT I_c vs. H 

Figure C-1

$4\pi M$ at $\pm I_c$ assuming that the irreversible and reversible surface currents do not exist and the intrinsic Abrikosov diamagnetism can be completely ignored. In this calculation we used the pinning function

$$F_p = \alpha B^n \quad 30-1$$

where we determine the pinning coefficient $\alpha = 6.6 \times 10^4$ (having arbitrarily chosen $n = 0.5$) from the measured $4\pi M$ at I_c when $H = 0$.

The pertinent equation used in this calculation is derived in appendix A and reproduced here

$$4\pi M = \frac{B_s^{3-n} H^{3-n}}{(3-n)\mu\alpha L} - H \frac{(B_s^{2-n} H^{2-n})}{(2-n)\mu\alpha L} \quad 30-2$$

where

$$B_s = H \pm \mu N I_c$$

The solid curves in Figure C-1 show the results of this calculation. We emphasize that the critical currents $\pm I_c$ introduced in this calculation serve only to establish the boundary condition $H \pm \mu N I_c$.

Since the calculated curves reproduce the main features of the data qualitatively and semi-quantitatively, we may conclude that our basic assumption that bulk pinning plays a significant role is correct. Further, we have explored other choices of n in this calculation and note that choices of $n > 0.5$ definitely lead to less satisfactory results.

At present we cannot provide any physical explanation of the second peak occurring in the curve of $4\pi M$ vs H with I opposing H (upper curve in Figure C-1)

(v) Meaning of Measured I_c

The distance of penetration of the conduction current into the thickness of the infinite slab is given by the expression

$$x_0 = \frac{B_s^{2-n} H^{2-n}}{(2-n) \alpha L}$$

Calculations of this quantity reveal that x_0 is appreciably smaller than the thickness L of the ribbon. In other words, the conduction current does not fill the cross-section of the entire sample when I_c is attained. This result indicates that I_c is determined by a particular length of the sample whose current carrying capacity is significantly lower than the rest of the sample.

Further we note that the remanent magnetization generated by induction (i.e. by reducing H from H_{c2} to 0) at the chosen temperature is several times greater (≈ 6.5) than the remanent magnetization generated by cycling I to near I_c and back to zero in $H = 0$. If the conduction current I were to truly fill the entire sample at I_c we expect, independently of any particular model chosen, these two remanent moments be equal. The ratio of ≈ 6.5 encountered substantiates the conclusion that the onset of resistance is determined by a region (a weak spot) with a current carrying capacity considerably lower than the rest of the sample. The presence of lengths of different current carrying capacity in a cold-rolled ribbon of several feet is not surprising and it is unfortunately the weakest length, or weakest spot which will set the ceiling on I hence determine the observed I_c . It is disturbing and annoying that this weakest spot

is so much weaker than the rest (or the average) of the specimen. Indeed using the model and the quantities given above we calculate that the ribbon should support a maximum current of 120 amperes when $H = 0$.

We can however, infer from the observed behaviour of I_c vs H shown in Figure B-1 that the weakest region has effectively the same geometry as the rest of the ribbon sample. Indeed we are led to suspect from this data, that the length of the ribbon which is responsible for limiting the conduction current is a section, effectively or actually significantly thinner than the rest of the ribbon, which is part of the inner coil of the sample and hence placed with its outer surface recessed with respect to the outer surface of that coil. The total field "seen" by that surface of the "weakest" link would then be a constant fraction of $H \pm \mu NI$.

An observation of interest and which is consistent with our diagnosis of the nature of the weakest section is the following. Our calculations of x_0 , the actual depth filled by the conduction current at I_c , versus H for the case aiding and opposing reveal that x_0 is approximately constant. This conclusion is confirmed by another observation which we will discuss later.

We emphasize that although the conduction current does not fill the volume of the sample at the observed I_c , this feature does not in any way invalidate any of our experimental results. Our analysis of magnetization data is not affected either since the conduction current enters only as a boundary condition in the interpretation. Hence it is not important whether I is truly sub-critical or critical. The presence of a weak link however does mean that we cannot analyse the I_c vs H

curves to provide an additional check of the validity of any model proposed for the interpretation of the magnetization data.

(vi) Magnetization Loops Generated by Cycling I

Graphs (D-1) through (D-4) present hysteresis loops of the magnetization as I is cycled in the presence of various applied fields H frozen into the sample (i.e. H is present when the sample cools from T_c). Clearly we may vary I between any convenient extreme values less than $\pm I_c$ for the given H . In one set of measurements we varied I between its maximum positive and negative values i.e. between $+I_c$ and $-I_c$. In a second set of measurements the amplitude of $+I$ was chosen equal to the amplitude of $-I$ and we picked I_c aiding for the amplitude. The different sets of hysteresis loops will be referred to as asymmetric and symmetric for obvious reasons. In figures D1-D4 we present some of the experimental loops obtained in low, intermediate and high fields H . The calculations were based on the critical state concept only. In other words, the possible existence of any surface current was ignored as a first approximation. The power of B was chosen to be $n = .5$ while $\alpha = 6.6 \times 10^4$, was chosen from the magnitude of the magnetization at I_c in zero applied field. On any model where x_0 , the distance of penetration of the currents in the bulk is the same for $+I$ as for $-I$ the asymmetric curves are expected to form a closed loop after one complete cycle. This indeed is found experimentally (see Figures D1-D2) and thus provides evidence that the flux profiles penetrate the sample to the same depth x_0 in both I_c aiding and I_c opposing directions for a chosen value of H .

Fig. D-1

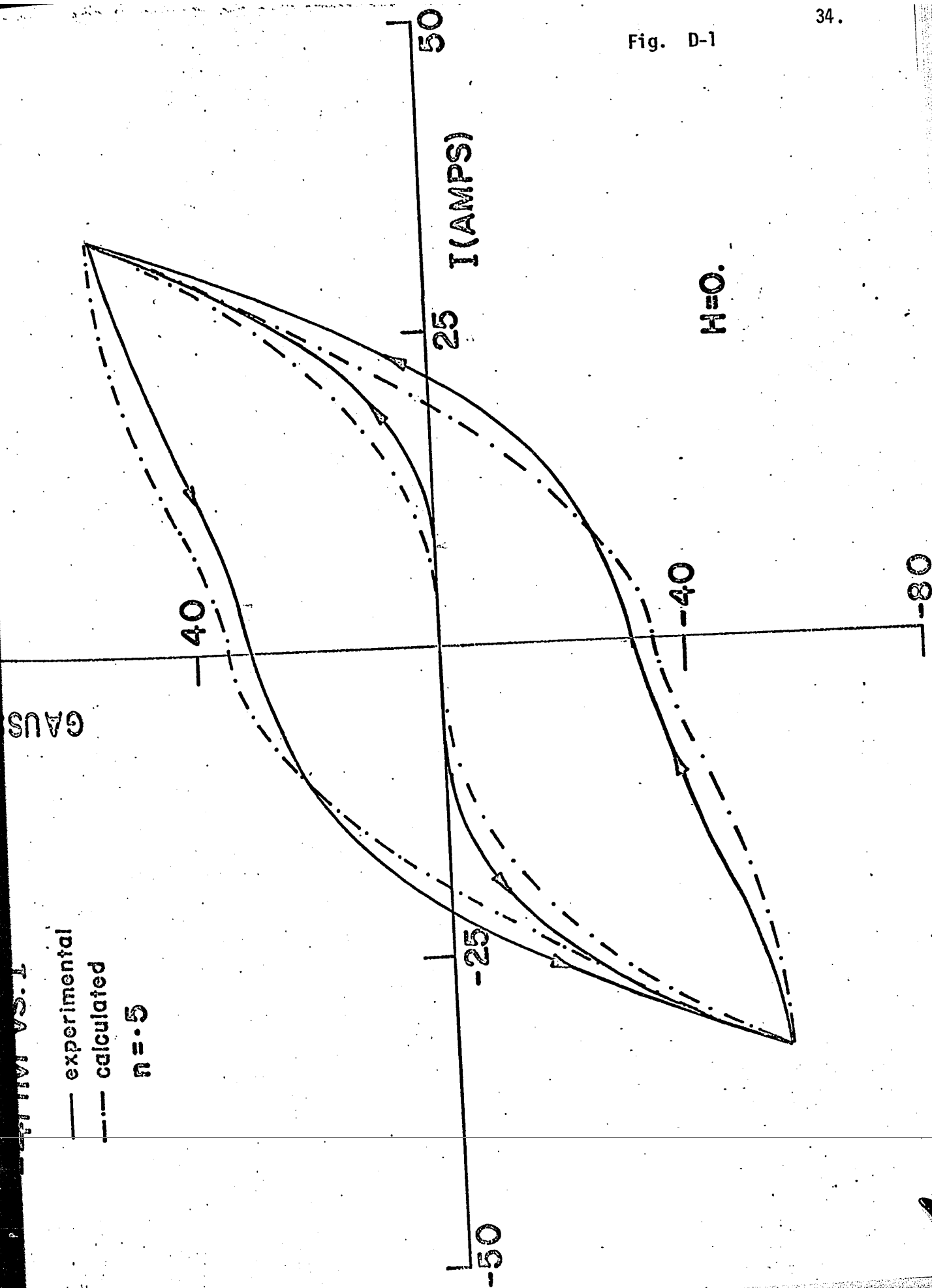


Fig. D-2

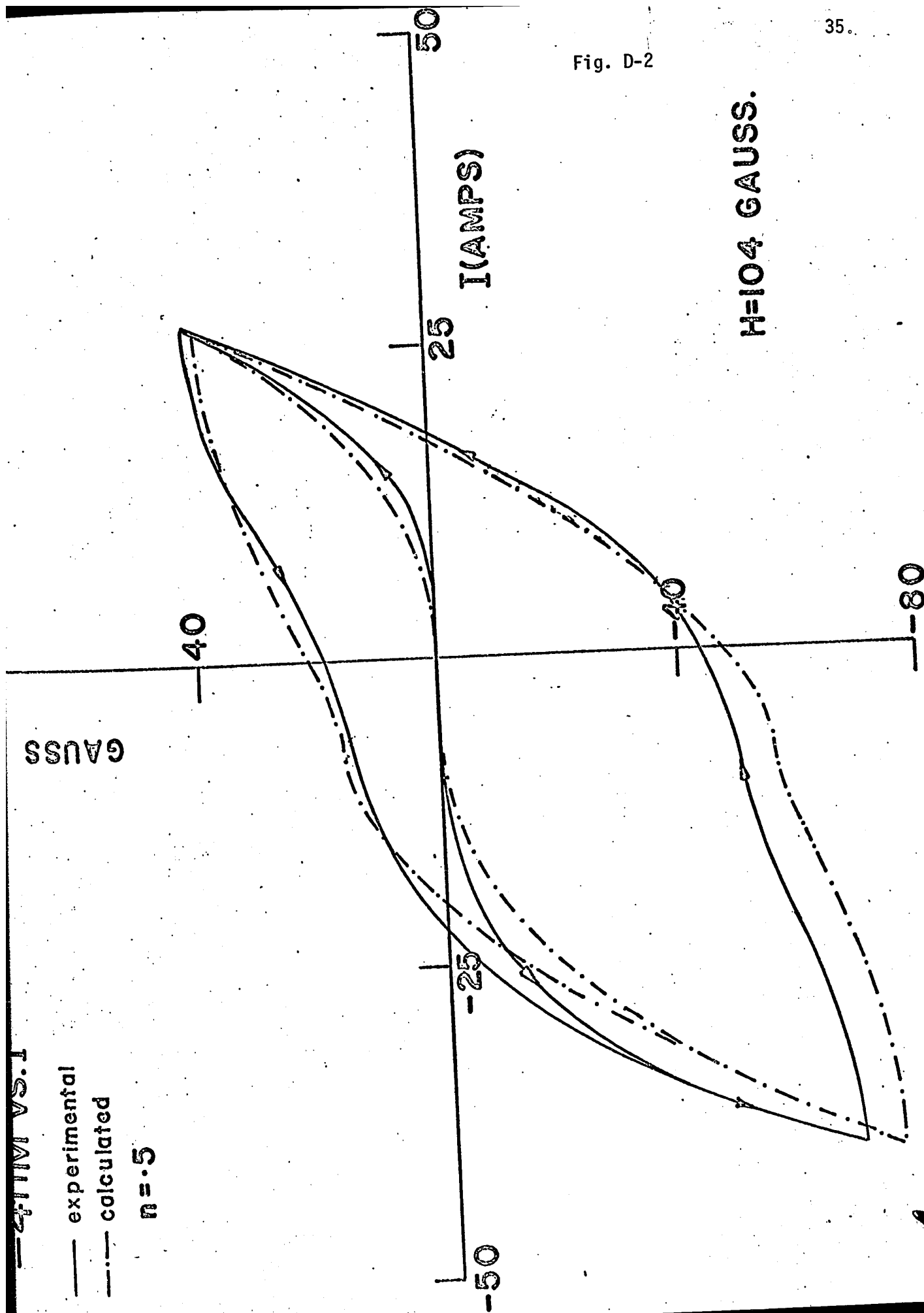


Fig. D-3

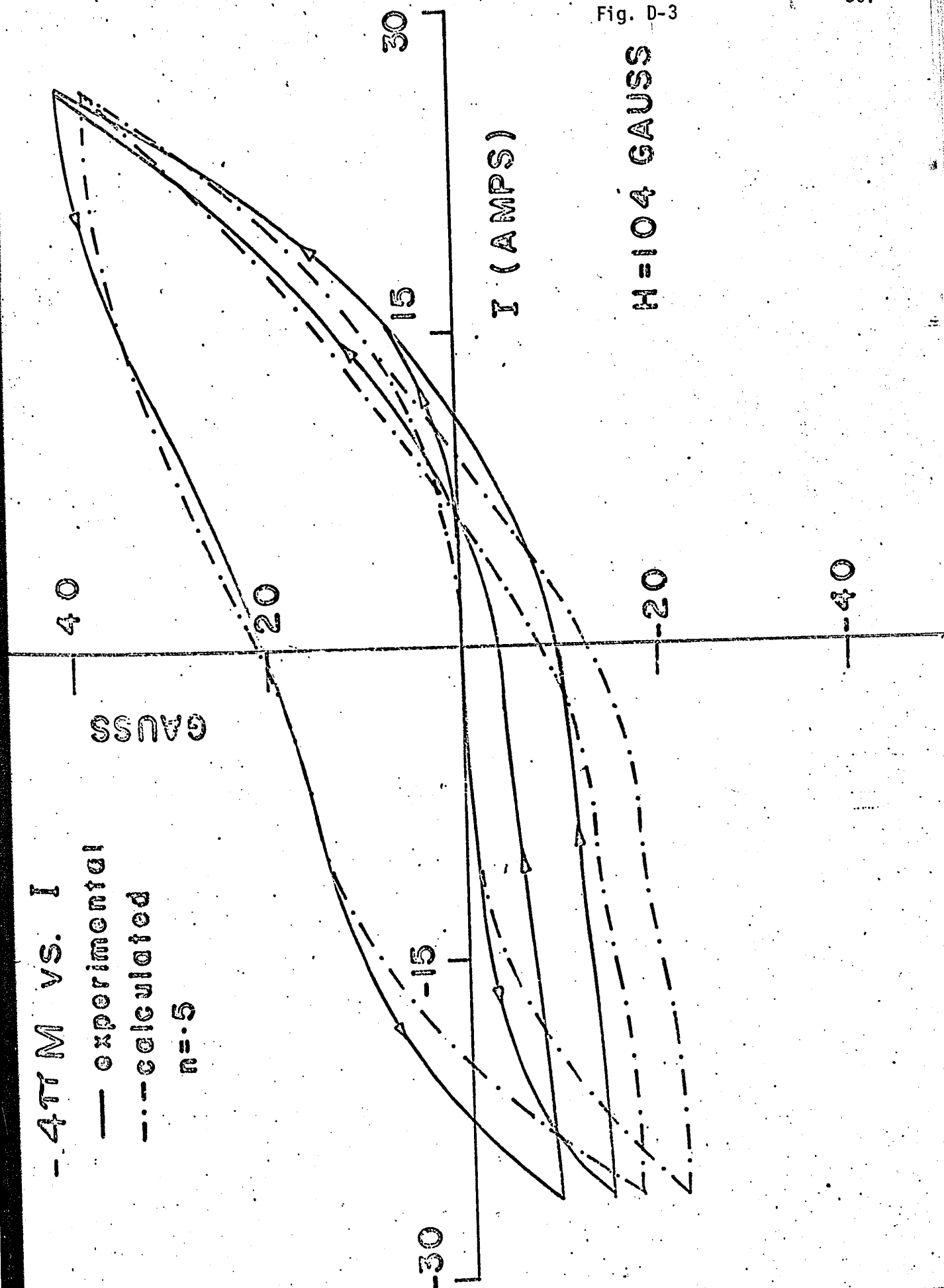
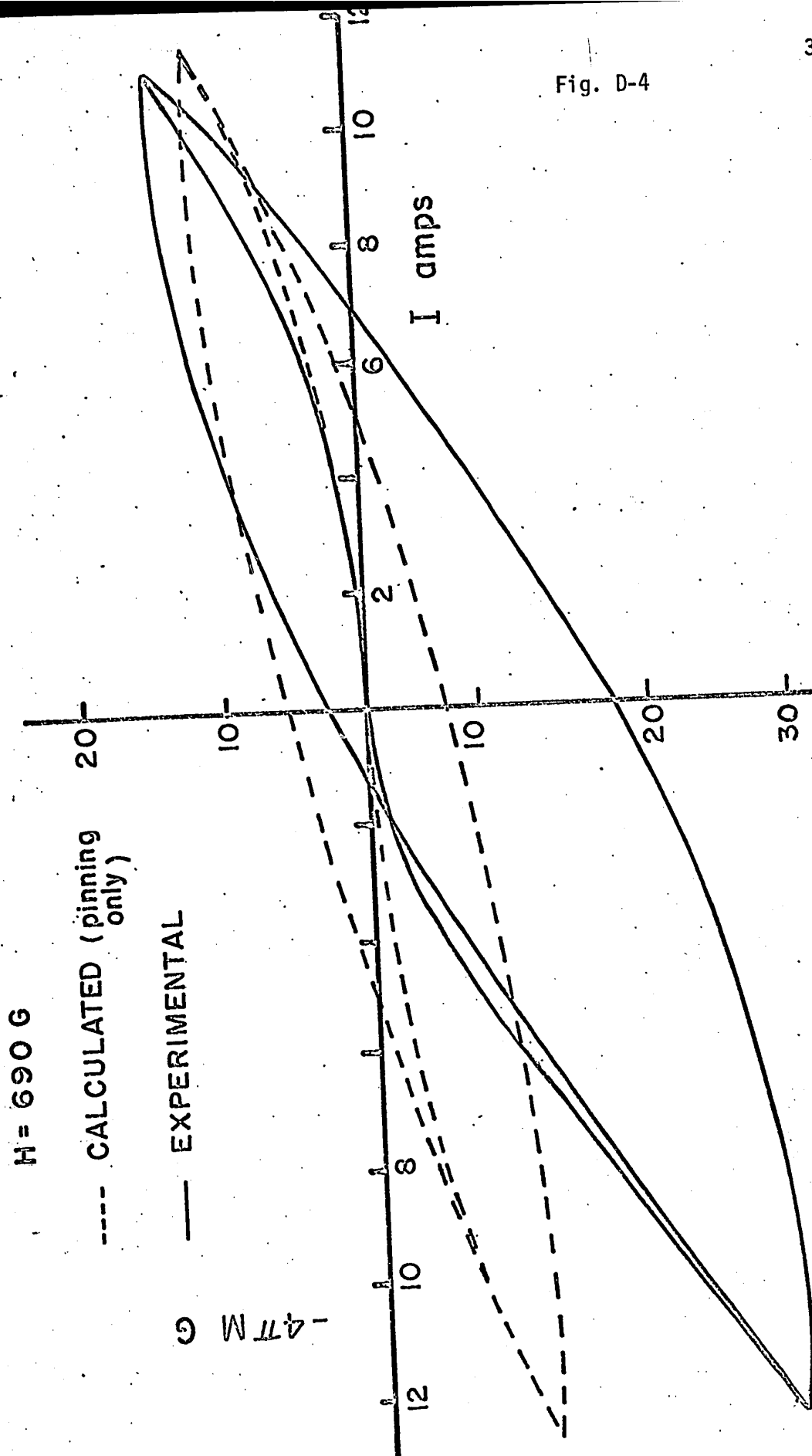


Fig. D-4



The behaviour of the symmetric curves is more complicated. If the initial direction of I opposes H , fully closed loops are expected after one and one quarter cycle. If the initial direction of I aids H then a closed repeating loop is only expected after two and one quarter cycles. (See Appendix A). This indeed is what is experimentally observed (Fig. D-3). It is particularly these details that fortify the critical state concept. For low values of H qualitative and quantitative agreement between the approximations and experiment is reasonable. However when H becomes large the crude model cannot predict the experimental results either qualitatively or quantitatively. From Figure C1, $4\pi M$ at I_c vs H , we can see that the α chosen for the magnetization loops as I is cycled (from the first point of C1) will not account for the magnitude of the magnetization in high fields as I is raised near $\pm I_c$.

One particular loop, that of $H = 690$ gauss (Fig. D4) is chosen to explore what reasonable amendments should be made. (For Figure D4 we have used $\alpha = 3.2 \times 10^4$ chosen from $4\pi M$ at I_c aiding in $H = 690$ gauss in order to emphasize the qualitative differences between experiment and calculation).

The first thing we note is that experimentally the magnitude of I_c is approximately the same both for the aiding and opposing directions. Our simplistic model predicts that therefore the moment at I_c aiding should be somewhat larger than at I_c opposing H as long as $n < 1$. Experimental results of Fig. D4 show just the opposite; indeed, $4\pi M$ at I_c opposing is twice as large as $4\pi M$ at I_c aiding.

The first physically obvious correction seems to be the introduction of a reversible surface current. Then, if the external boundary at $x = L$ is $H + \mu NI$ then just within the surface it is $B_s = H + \mu NI - \delta_R$ which represents the boundary magnetic induction of the bulk.

The simplest choice is to pick δ_R to be a constant whose magnitude we have estimated by close inspection of the experimental curve of $4\pi M$ vs I . Assuming δ_R to be a constant is a good approximation since the variation in $\langle B \rangle$ is small over the cycle compared to H and H is appreciably greater than H_{c1} . This follows from the following considerations.

When $H \gg H_{c1}$ (See for example the book by de Gennes²⁶) the Abrikosov diamagnetic moment is given by

$$4\pi M_A = \frac{H - H_{c2}}{\beta(2\kappa^2 - 1)}$$

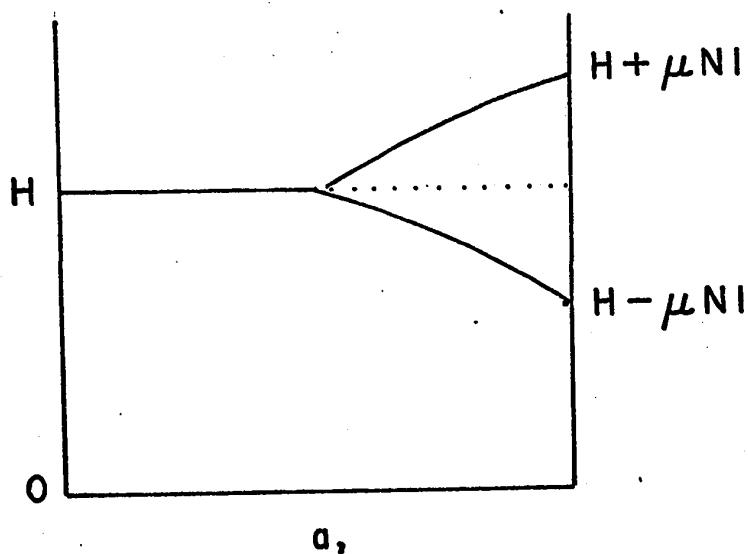
where β is a fluxoid lattice constant (~ 1.69 for a triangular array) then

$$|\Delta(4\pi M_A)| = \frac{|\Delta H|}{\beta(2\kappa^2 - 1)}$$

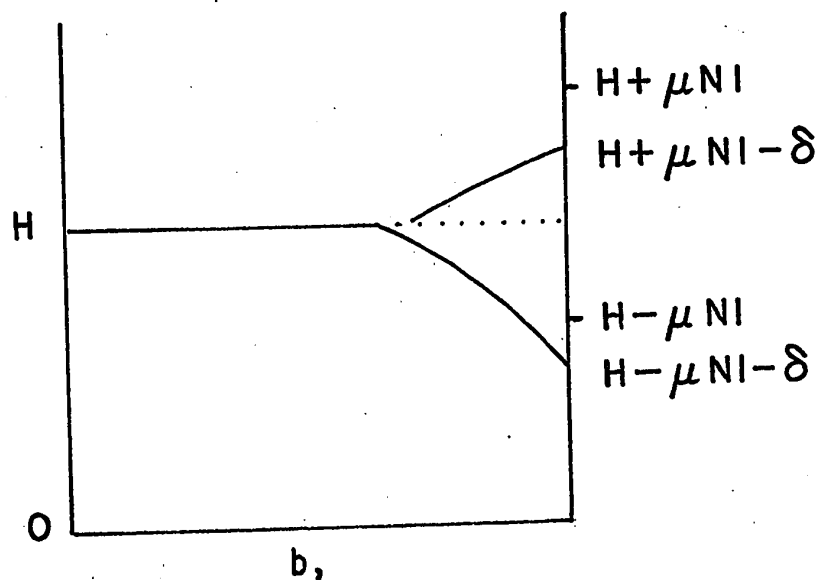
Here $H = 690$ gauss ($\sim 4H_{c1}$) while $|\Delta H|_{\max} \sim 300$ gauss. A reasonable minimum value of $\beta(2\kappa^2 - 1)$ is ~ 100 which gives the maximum variation of

$$|\Delta(4\pi M_A)| = |\Delta\delta_R| \approx \frac{300}{100} = 3 \text{ gauss}$$

FIGURE D-5



Graph a shows schematically $4\pi M$ as I is at either maxima for high value of H assuming $H \pm \mu NI$ to be the true boundary condition



Graph b shows the inclusion of a constant Abrikosov-like surface step.

SCHEME FOR INCLUDING SURFACE STEP

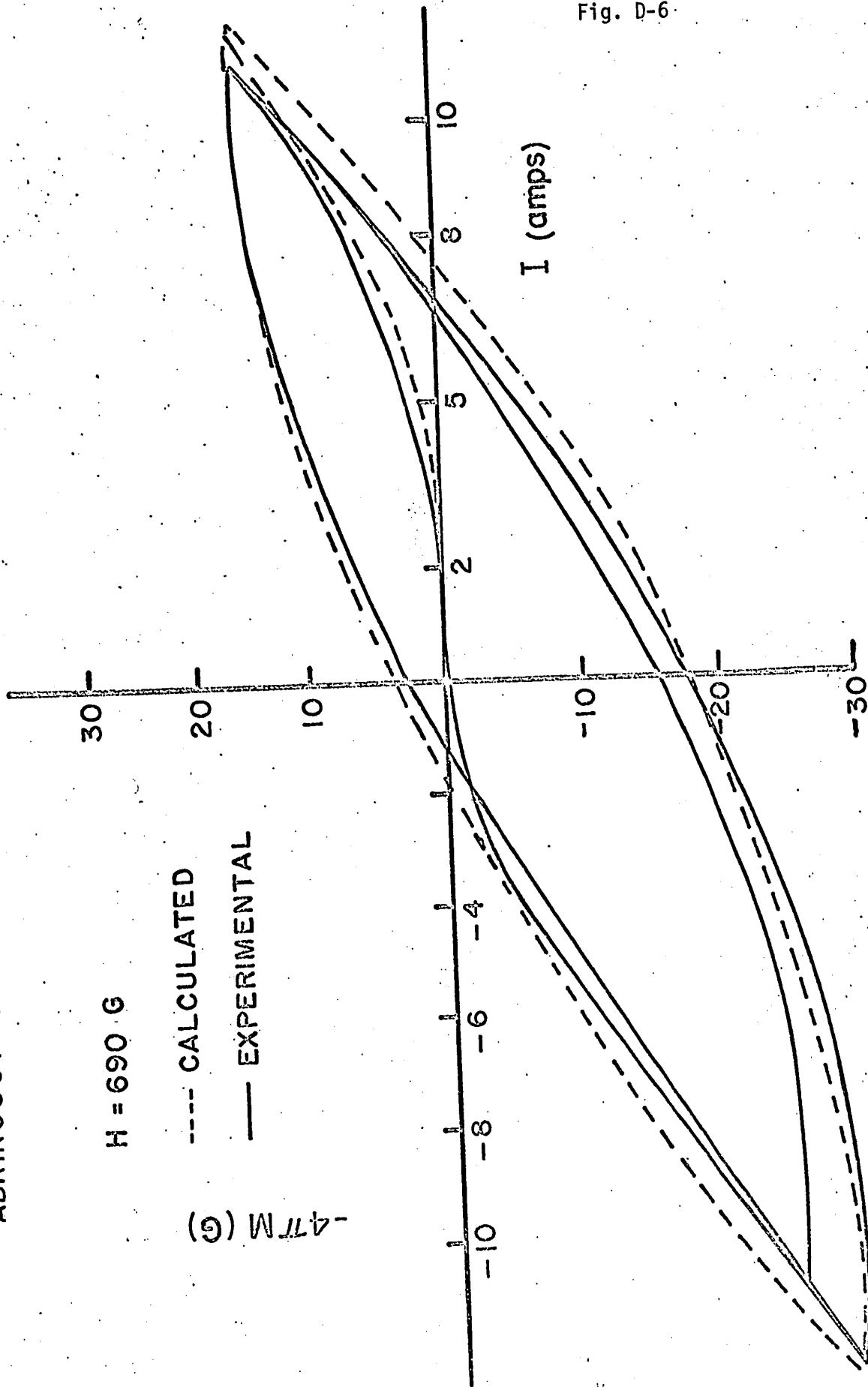
Fig. D-6

ABRIKOSOV SURFACE CURRENT INCLUDED

 $H = 690 \text{ G}$

---- CALCULATED

— EXPERIMENTAL

 -47 M (G) $I \text{ (amps)}$ 

Choosing δ_R to be 25 gauss the assumption of it being constant over the range considered is correct within $\pm 6\%$.

Consideration of diagram D-5 may assist the reader in seeing why the introduction of δ_R leads to improved agreement with observation.

Graph D-6 compares the agreement between the experimental loop and the one calculated using the improved model above described. $\alpha = 3.2 \times 10^4$ was determined by fitting $4\pi M$ at I_c aiding for this particular value of $H = 690$ gauss.

It is important to note that this value of α corresponds more closely to the one derived from the remanent moment of the main magnetization curve. (See table D).

As we shall see in more detail in the next section, if δ , the correct surface current is introduced, it is then possible with one adjustable parameter α , selected once and for all from any experimental information, to obtain excellent agreement between calculations and observation.

(vii) The Field Dependence of the Irreversible Current

In this final section we apply our composite model to arrive at the variation with magnetic field of δ_{IR} , the irreversible surface current, from experimental observation. First δ , the net effective surface current must be determined.

In light of its relative success the choice of the pinning function to be used for this analysis remains $\alpha B^{1/2}$. We choose the adjustable parameter $\alpha = 3.2 \times 10^4$ (amp-gauss^{1/2}/cm²) obtained in the previous section where a constant reversible surface current was introduced and lead to an

TABLE D

Method of Calculation	α_L gauss ^{3/2}	α amp gauss ^{1/2/cm²}	$\delta(H_{eff})$ gauss
Remanent Magnetization			N.A.
$4\pi M$ at I_{co}	1.6×10^4	4.3×10^4	0.
$4\pi M$ at I_c in $H = 690$ gauss	2.5×10^4	6.7×10^4	25
$4\pi M$ at I_{co}	1.2×10^4	3.2×10^4	105

excellent account of the magnetization loop as I is cycled in a static field $H = 690$ gauss. We wish to point out that once these two parameters ($n = 1/2$, the power law of the pinning function and α , the strength of pinning) are specified, a measured magnetic moment combined with a current flowing in the sample (this defines a boundary condition) uniquely determines the total net surface current $\delta = \mu I_s$.

To obtain δ equation 30-2 of this chapter must be solved for the unknown quantity B_s , the magnetic induction at the surface of the bulk.

This equation is obtained from equation 7 of appendix A by removing the restriction that the applied external boundary represent the field experienced by the bulk.

Although some difficult cases are encountered, equation 30-2 was solved numerically for B_s . We note that δ is just the difference between the experimentally imposed boundary $H \pm \mu NI$ and the corresponding computed value for B_s .

$$\delta = (H \pm \mu NI) - B_s$$

Accordingly δ was calculated using the data obtained for the magnetization at the maximum applied current I_c . (See section iv of this chapter). Figure D-8 shows δ versus the appropriate boundary field ($B_s + \delta/2$) or ($B_s + \delta$) depending on which is more suitable.

Figure D-7

Nb-Ta

$$\delta_o = 104.55 \text{ (G)}$$

x, • I opposing H

o I aiding H

+ I opposing H

δ and B have
opposite signs

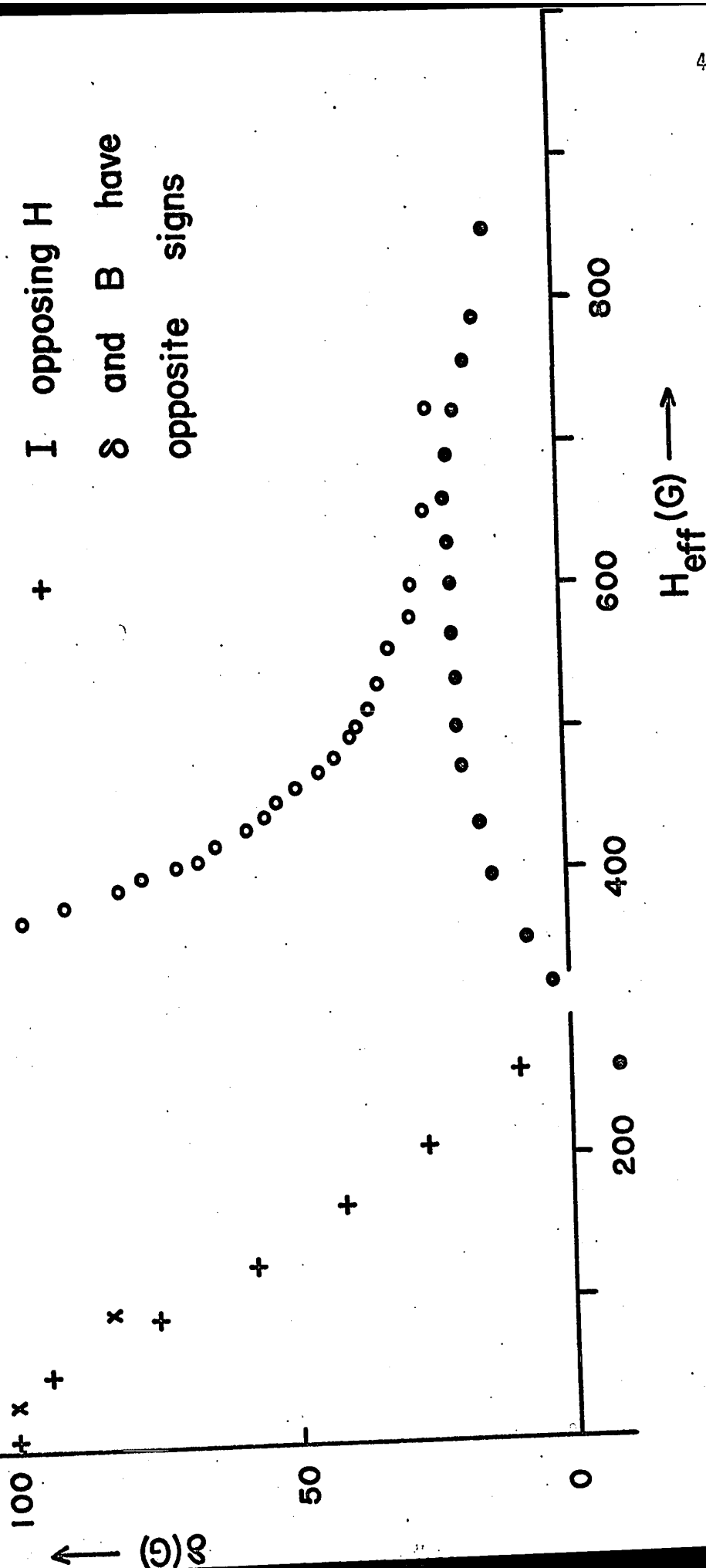


Figure shows δ calculated from $4\pi M$ at I_c data

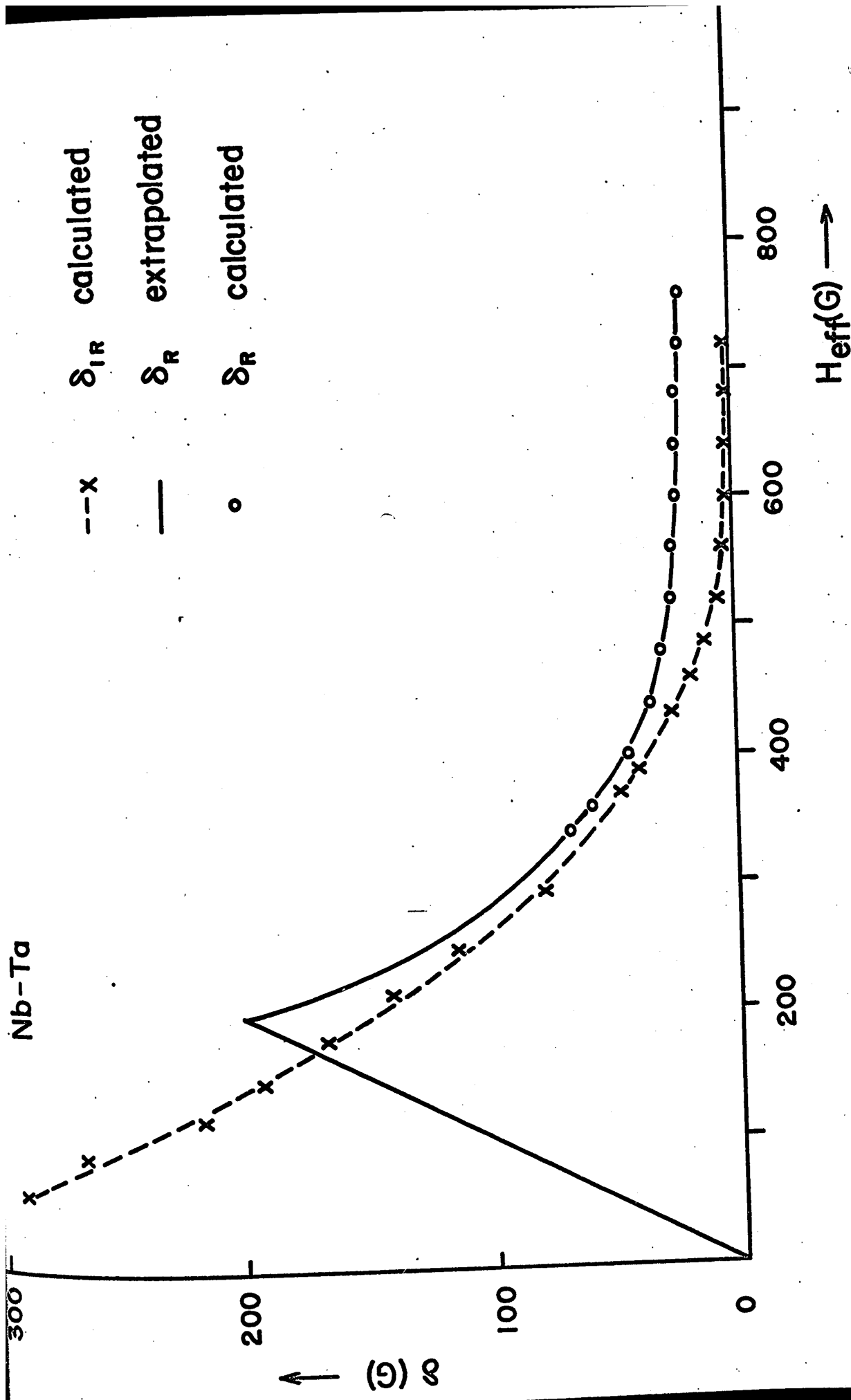


Figure D-8

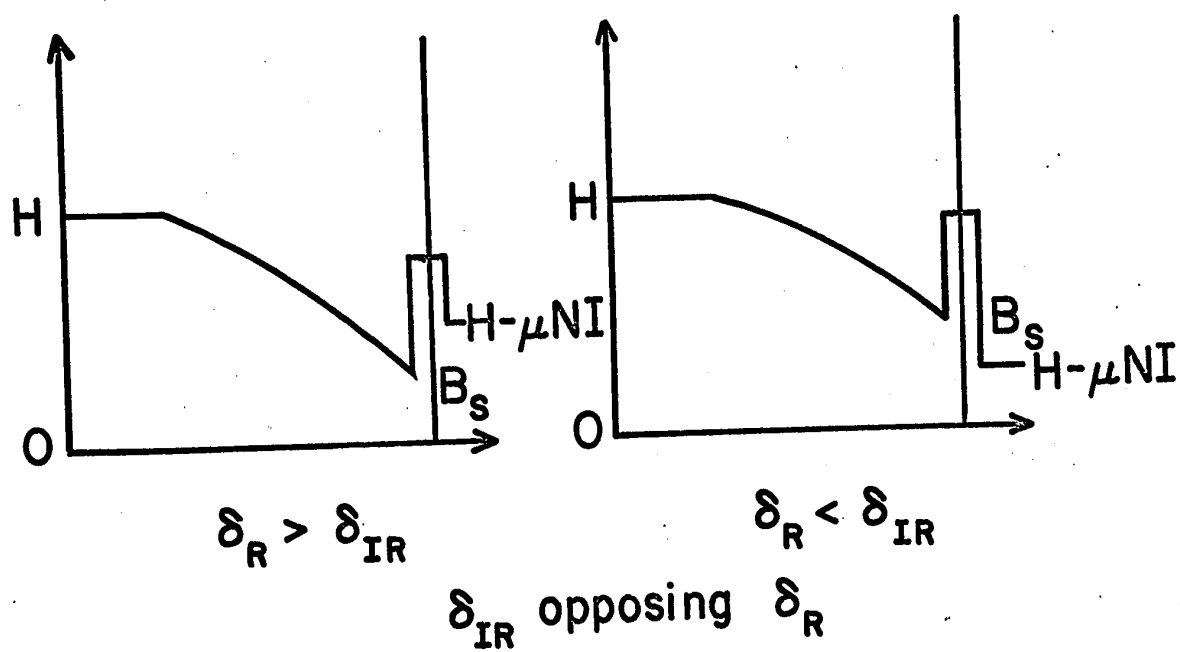
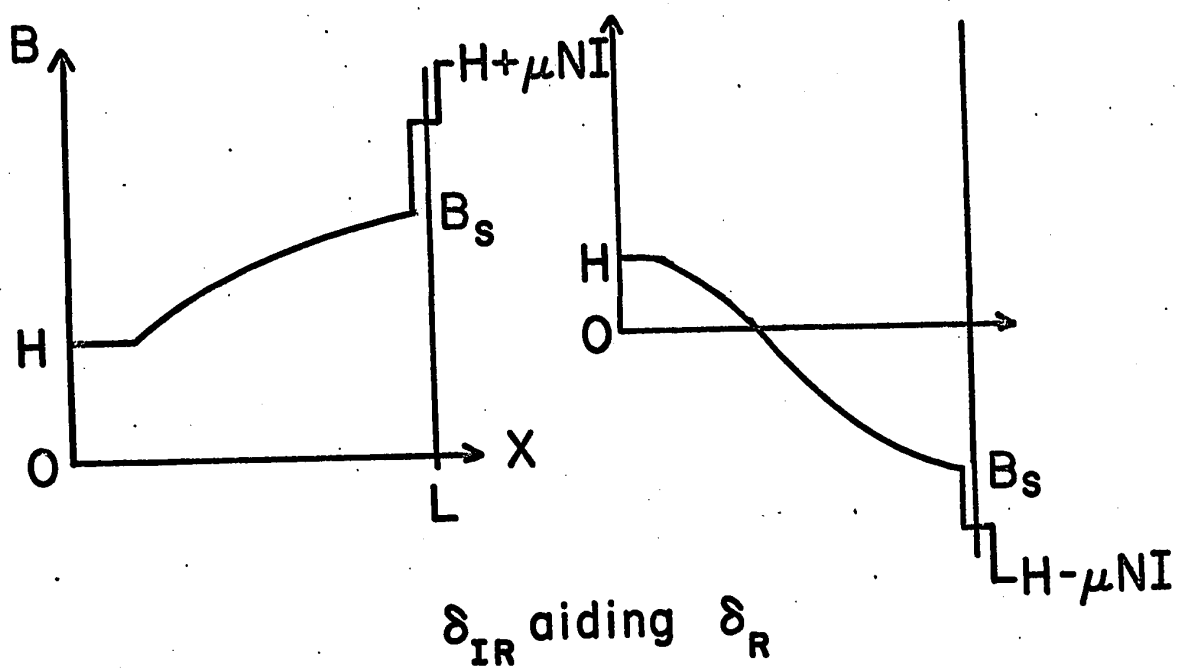


Figure D - 9

Schematic Boundary Conditions

In accordance with our model, irreversible currents may flow at the surface of the specimen in a region different from and embracing the reversible Abrikosov surface current. As was seen, the sense of circulation of the irreversible surface current may aid or oppose the Abrikosov current which is always diamagnetic with respect to the external field with which it is in equilibrium.

$$\delta = |\delta_R| \pm |\delta_{IR}|$$

Whether the irreversible current I_{IR} is paramagnetic (opposes I_R) or diamagnetic (aids I_R) depends on the magnetic history of the sample in the superconducting state. The possible flux profiles thus arising are depicted in the schematic drawings of figure D-9.

Although not rigorously correct, it is a good approximation, for high values of the boundary field, to combine δ for the cases of δ_{IR} aiding and opposing δ_R , thereby obtaining the high field dependence of δ_R and δ_{IR} . The error introduced by this approach is less than 5% within the range where this approximation was applied. The curve obtained for δ_R in this manner from experimental values was then extrapolated to H_{C1} , which was taken to be 200 gauss. Figure D-8 shows δ_R as a function of the boundary field.

Since for each remaining experimental point δ is known and δ_R has been estimated, δ_{IR} can now be readily deduced using the equation given above. On figure D-8 δ_{IR} is plotted as a function of $H_{ext} - \delta_{IR}/2$ where H_{ext} is the external boundary field.

Summarizing this briefly, within the framework of our model we have postulated a pinning function $F_p = \alpha B^n$. Having fixed the parameters α and n , we have determined the boundary field of the bulk B_s , and hence δ from the data on the magnetization at I_c . The well documented Abrikosov ¹⁾ surface current δ_R was then extracted from this information. The irreversible surface current δ_{IR} is then the excess surface current (paramagnetic or diamagnetic) in δ not accounted for by δ_R . In this manner we find a curve of δ_{IR} of reasonable magnitude which varies monotonically with boundary field. Similar results have been reported by other workers ²⁷⁾ in the literature.

In conclusion we wish to assert that, of existing models, only our new model is capable of accounting for the magnetization curves of the NbTa sample when a conduction current is present. We find that the bulk critical state by itself can account only for a small part of the data; in fact for the particular magnetic moments discussed in section vi, it predicts relative magnitudes that are opposite to that experimentally observed!

The Silcox model, we find, seems to account for this particular feature. This, however, is fortuitous since for high values of H the irreversible current turns out to be negligible; the Silcox model is incapable of explaining the results for $4\pi M$ at I_c at low fields. But our composite model, correctly synthesizing the three well documented mechanisms introduced for understanding the behaviour of type II superconductors, does explain, systematically, all the essential experimental features encountered in our investigations.

CHAPTER III

PbIn Ribbon Sample; Induced CurrentsIntroduction

It is now well established that the pinning force density in the bulk or body of strong pinning type II superconductors exhibits a resonance form of dependence on magnetic induction²⁸⁾. The maximum of the pinning occurs when B/H_{c2} lies in the range between 0.3 to 0.7. Timms and LeBlanc²⁹⁾ have shown that, as a direct consequence of this fact, the peak magnetic moments induced in a ribbon (or wire) of a nonideal type II superconductor, when an external field is applied perpendicular to the sample axis and varied between $\pm H_{c2}$, can be dramatically increased by a factor of ~ 3 when a static magnetic field $\sim 0.5 H_{c2}$ is also present and directed along the ribbon axis. Further, these workers have shown that the persistent currents induced by the varying transverse field adopt a "tilted" pattern of circulation and tend to flow along the lines of total magnetic induction.

Exploiting the critical state concept and a simple "resonant" pinning function, these workers quantitatively accounted for the dramatically enhanced moments they observed. These authors applied the laws of induction to this problem complemented by the postulate that the pattern of currents

generated in the specimen will tend to flow so as to reduce the net Lorentz force. In this way they also accounted qualitatively for the main features of the evolution of magnetic moments along the static field directed parallel to the axis of the ribbon as the transverse field component was cycled between $\pm H_{c2}$.

Since in the sample these workers studied pinning in the bulk swamped any role played by reversible and irreversible surface currents, their results are relatively uncomplicated. In their case the concept of bulk pinning and the tendency for nearly force free flow of currents in the bulk sufficed to account for their observations. For these same reasons, however, their work could not shed light on several important basic questions:

i) Does the magnitude of irreversible surface currents also exhibit a resonance type dependence on magnetic induction?

Stated differently: are the concepts of a critical state with a driving Lorentz force opposed by surface pinning applicable to considerations of the limit or ceiling for irreversible surface currents?

ii) Do irreversible surface currents also exhibit the capacity to adjust their pattern of circulation so as to reduce the Lorentz force they experience? Or again stated in another way: will irreversible surface currents also flow along field lines, hence in nearly force free configurations?

iii) What role, if any, does the intrinsic Abrikosov diamagnetism play in determining the pattern of current flow and the

magnitude or ceiling of macroscopic persistent currents in the bulk?

To provide information on these questions we undertook an extensive investigation of PbIn along the lines followed by Timms and LeBlanc on NbTa, and Mattes and LeBlanc on Nb₃Zr and NbTi. This material has a low κ parameter, and preliminary studies by ourself and others³⁰⁾ indicated that the pinning in the bulk was sufficiently weak so as not to overwhelm the contributions of irreversible surface currents and intrinsic Abrikosov diamagnetism to the magnetic behaviour. Further, the maximum reversible and irreversible surface currents appear to be comparable.

In this chapter we present some typical results of this extensive series of measurements. Data was taken with the static field directed along the ribbon axis and the transverse varying (current inducing) field along the flat face of the ribbon and vice versa since these two situations are physically equivalent. As shown in the next chapter, the capacity of the surface to support irreversible currents is practically quenched when a magnetic field pierces the flat face of the ribbon. For this reason, we have also taken measurements with the static field directed into the flat face and the axial field cycled between $\pm H_{c2}$ (and vice versa). Unfortunately, significant demagnetization effects in this case also appear with a magnetic field directed into the plane of the rectangular slab sample and complicate the interpretation of the data.

An analysis of the main features and the details of our observations on this material is intrinsically extremely difficult.

To start with, we are exploring a completely uncharted area of physical phenomena. Further, the challenge undertaken is by its very nature formidable since the role played by the several contributions (bulk pinning, nearly force free current flow, intrinsic Abrikosov diamagnetism and irreversible surface currents) are all of comparable importance. Nevertheless such an analysis by the author is in progress and the prospects of success are encouraging. The analysis when completed will be presented in a separate treatise and for publication.

In this chapter of my M.Sc. thesis I propose to confine myself to a presentation of the extensive data and a discussion of various features of the results which are more easily amenable to interpretation using only one or two of the basic pertinent concepts.

Experimental Arrangement

The PbIn sample (13.92 at % In) was rolled from a wire of .25 cm diameter into a slab of dimensions .36 x .05 x 5.7 cm. A bifilar heater of formvar insulated no. 38 manganin wire was wound around a sandwich of the soft PbIn ribbon and thin sheets of mica. Hairpin heaters of the same manganin wire on the large face of the slab were included within the sandwich. The sample assembly was embedded within a bakelite holder designed for accurate aligning of the major axis of the slab along the axis of the solenoid wound of superconducting (NbTi) wire. The sample holder was also equipped with positive location in the transverse direction where a pair of Helmholtz coils, wound with the same superconducting wire (NbTi) produces a magnetic field perpendicular to the direction of the solenoid. The bakelite holder can be rotated about its major axis in such a way that the field produced by the Helmholtz pair is either perpendicular to or is in the face of the slab. The solenoid and Helmholtz pair produce 134 and 121 gauss/amp. respectively.

A pickup coil of no. 42 formvar coated copper wire wound cylindrically around a bakelite former embraces the entire length of the sample. Bucking coils at both ends of the former were wound of the same wire material and were connected in series opposition to the pickup coil in such a manner that when the magnetization of the sample is zero, the integrated signal is also zero when a magnetic field $H_{//}$ along the axis of the sample is changed by ΔH . That is, the coil arrangement must satisfy the requirement

$$\left| \int V dt \right| = |A \Delta H (N_{B1} + N_{B2} - N_p)| \quad (1)$$

where A is the cross-sectional area of the former, N_{B1} and N_{B2} are the number of turns of the two bucking coils and N_p is the number of turns of the pickup coil.

When the magnetization of the sample, seen only by the pickup coil is non zero then

$$\left| \int V dt \right| = A |N_p \Delta B - (N_{B1} + N_{B2}) \Delta H| \quad (2)$$

which combined with (1) gives

$$\left| \int V dt \right| = |N_p A \Delta(4\pi M)|$$

that is, the integrated signal thus developed is directly proportional to the magnetic moment developed along the major axis of the sample.

In a similar manner, rectangular pickup and bucking coils of no. 52 formvar insulated copper wire, with the appropriate turn-area relationship were constructed to detect changes in the magnetization of the sample in the transverse direction (i.e. along the axis of the Helmholtz pair). The analysis given above also holds here. The pickup coils and sample holder once assembled together were fitted snugly and rigidly inside the pair of superconducting magnets.

The main feature of our experimental arrangement is that magnetic fields can be applied independently along the major and minor axes of the rectangular slab material, while moments developed along the respective axes, as fields are swept, can be monitored.

Experimental Results

The type of experimental arrangement discussed in the previous section, enables us to study the pattern of flow of currents that circulate in the surface and within the bulk of a superconducting ribbon sample in the presence of a static magnetic field. We point out that currents are induced in the sample via the varying field, so that the changing moments subsequently observed yield information on the changing flow pattern and the forces that arise as a result of the interaction of fields and currents. By judiciously altering the magnetic and geometric configuration of our specimen it is possible to separate the three distinct critical currents that comprise the effective behaviour of the sample. Accordingly, the following measurements were taken.

(The subscript // shall be reserved for the direction along the solenoid and therefore along the major axis of our rectangular slab, and the subscript $\perp$ for the direction of the field produced by the Helmholtz coils, thus along either the width or breadth of our sample.)

- 1, The superconducting sample is allowed to cool through T_c in a constant $H_{//}$ and
 - i) $H_{\perp}$ is varied while $4\pi M_{\perp}$ is monitored (i.e., along the field inducing the currents)
 - ii) $H_{\perp}$ is varied while $4\pi M_{//}$ is monitored (i.e., along the static field)
- 2, The sample is cooled through T_c in a constant $H_{\perp}$ and
 - i) $H_{//}$ is varied while $4\pi M_{//}$ is monitored (i.e. along the field inducing the currents)
 - ii) $H_{//}$ is varied while $4\pi M_{\perp}$ is monitored (i.e., along the static field)

A complete set of measurements was taken with $H_{\perp}$ oriented into the narrow and the flat faces of the ribbon alternately. Figures (E-1) to (E-8) show the corresponding representative families of curves obtained.

Explanatory notes to Figures E and F.

The ribbon geometry of the sample employed in these sections are schematically sketched on the graphs showing the experimentally observed curves.

All arrows placed on this refer to applied or resultant magnetic quantities, in the appropriate directions.

More precisely, the straight arrow represents the direction of the applied static field, the wiggly arrow represents the field being varied while the solid wide arrow on one face of the ribbon represents the direction in which the magnetization is monitored.

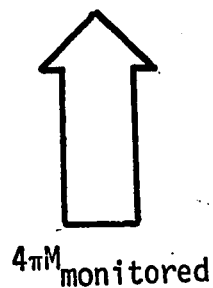
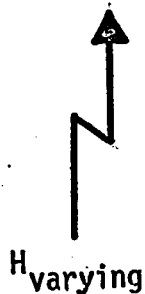


Figure E-1

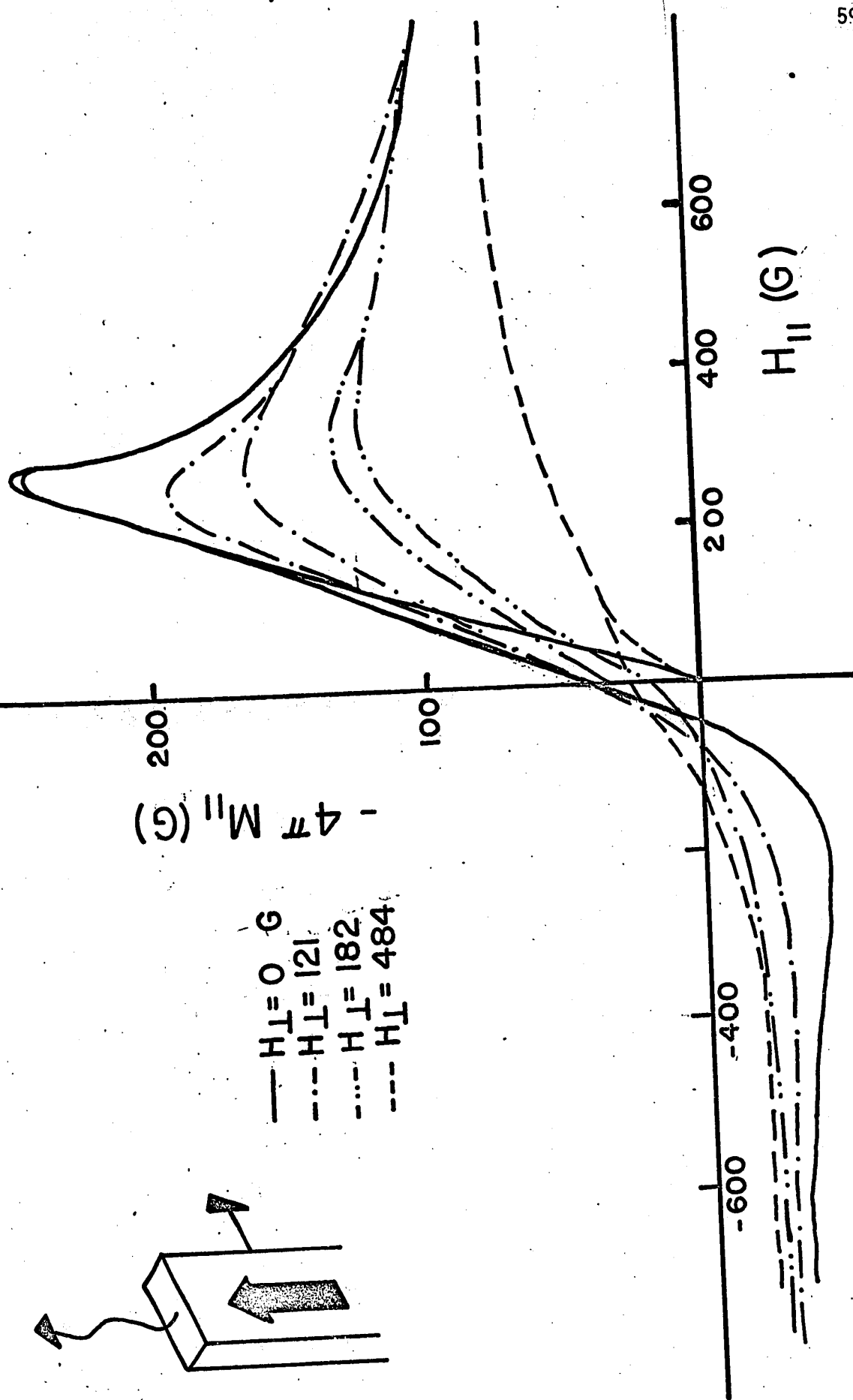


Figure E-2

$H_I = 60 \text{ G}$
 $H_I = 121$
 $H_I = 726$

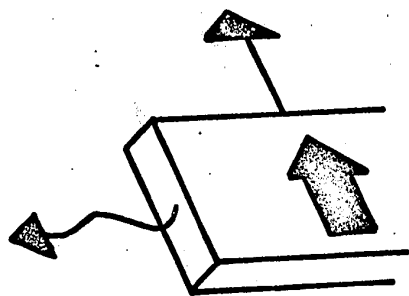
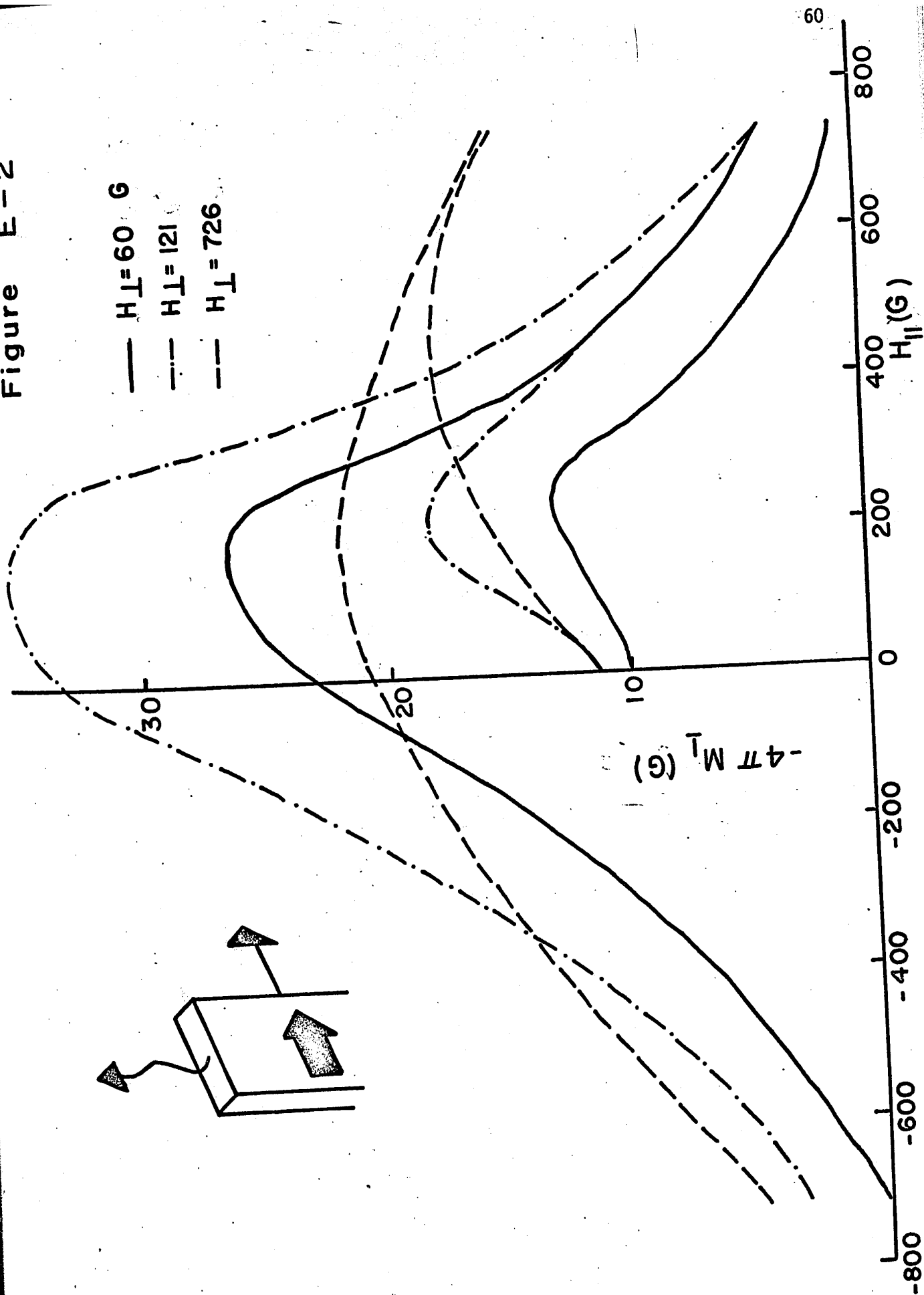
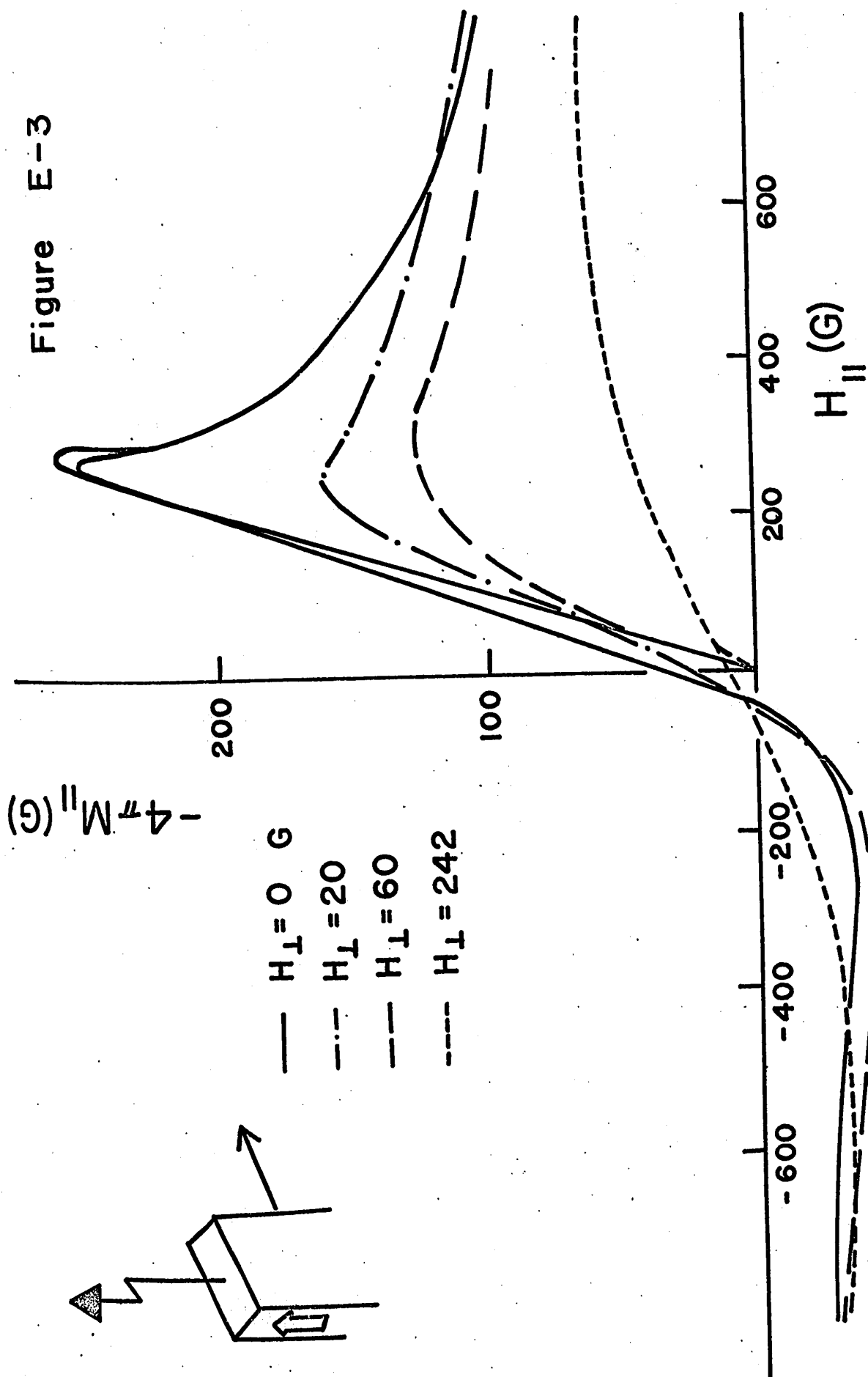


Figure E-3



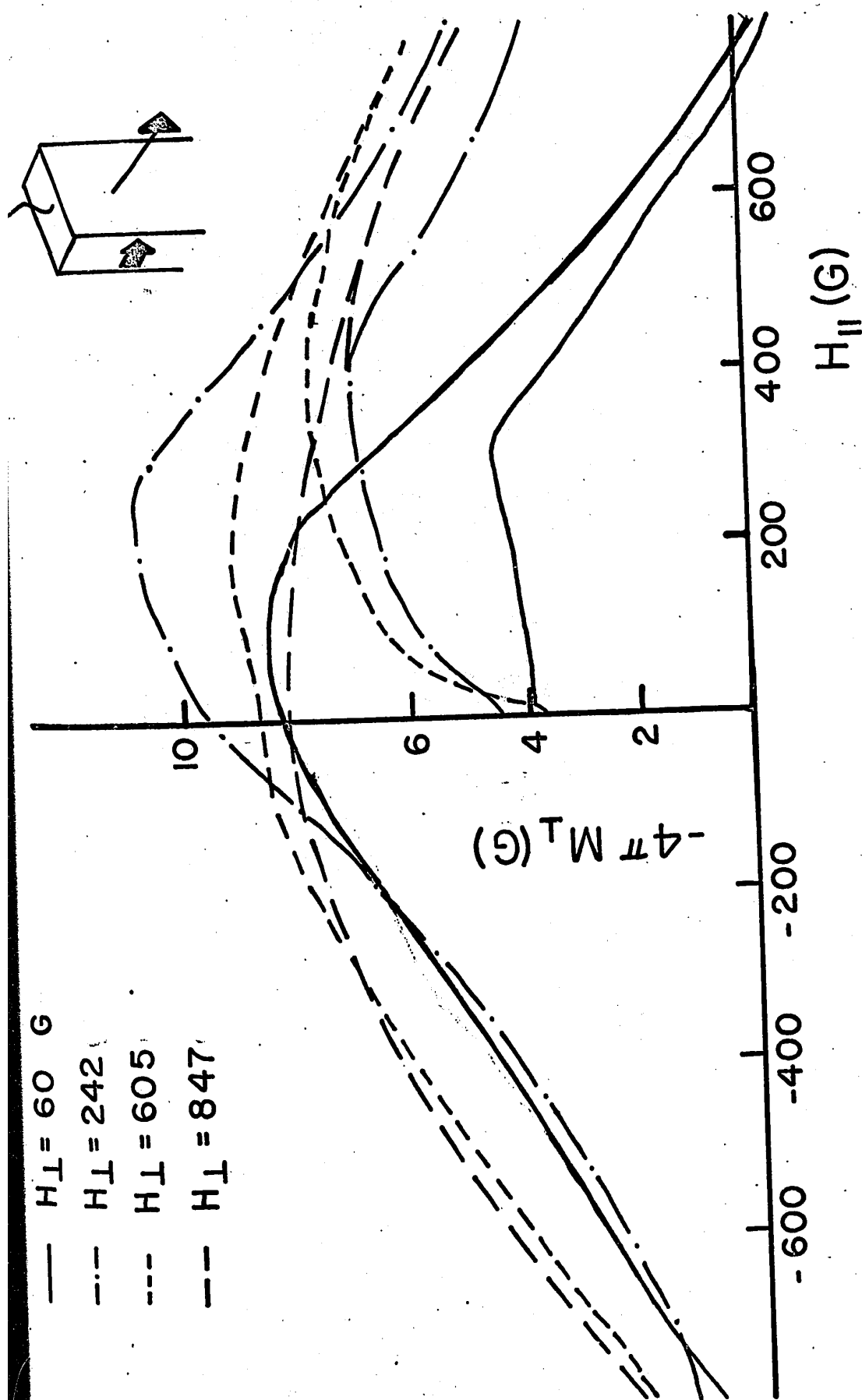


Figure E-4

Figure E - 5

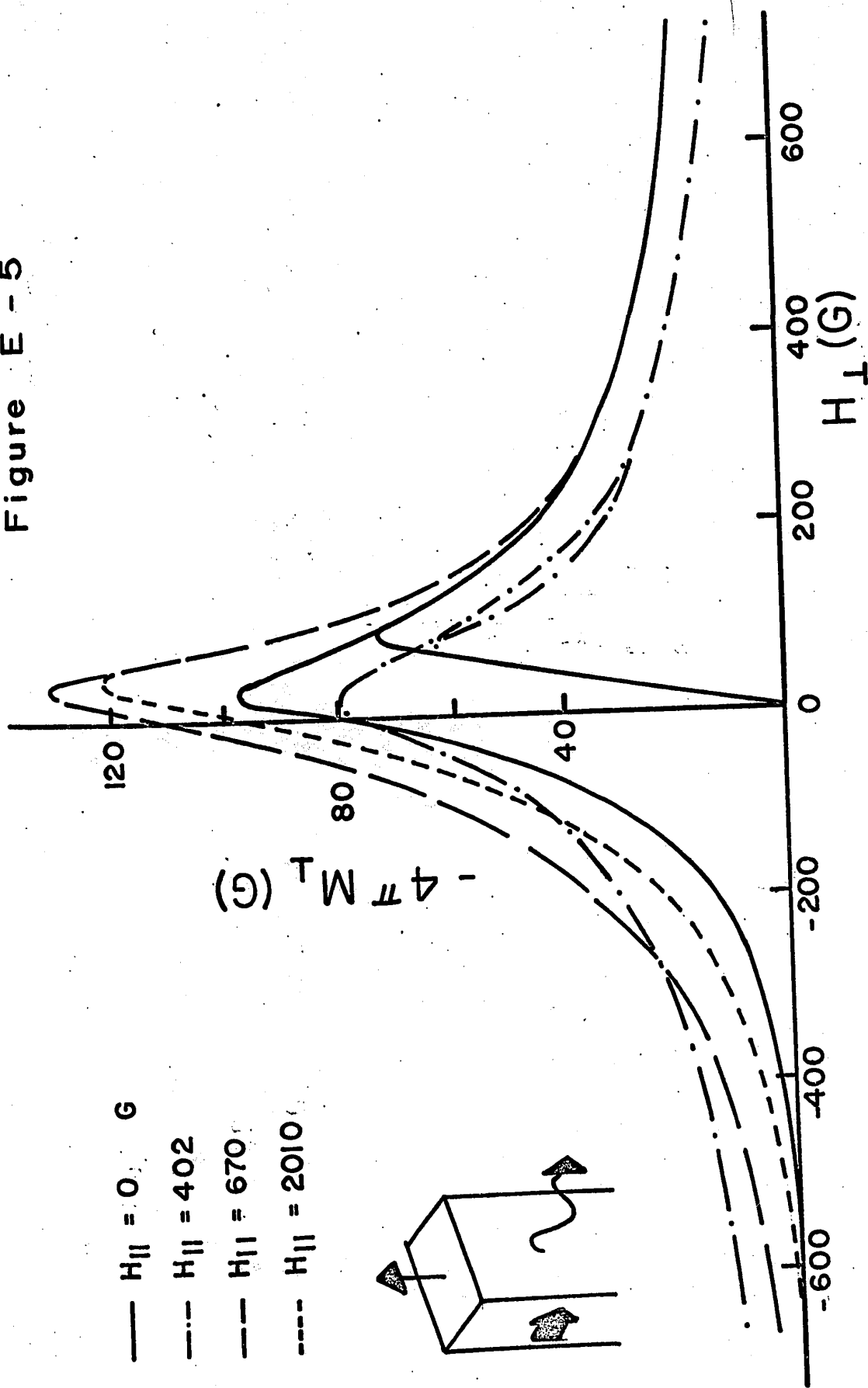


Figure E-6

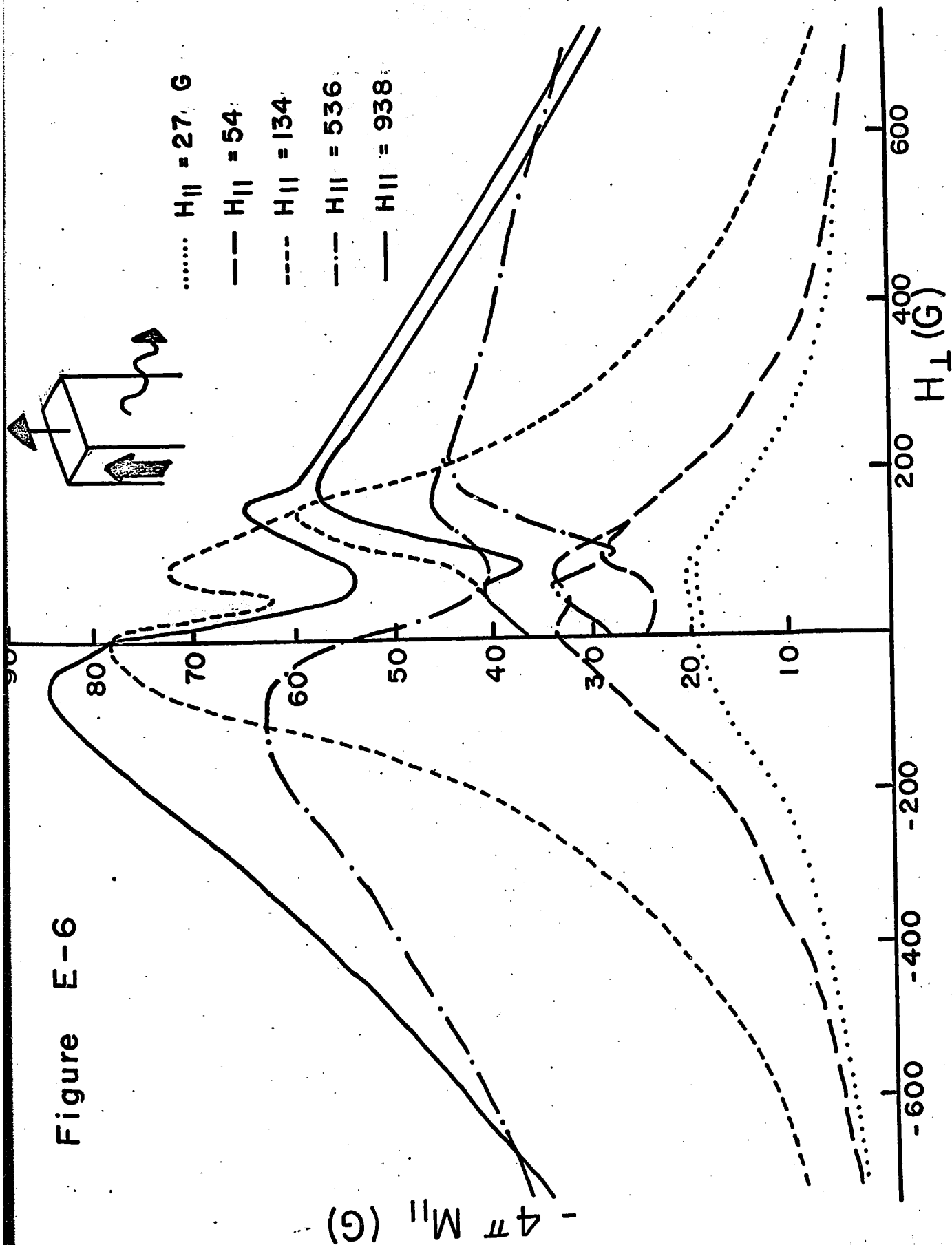


Figure E-6

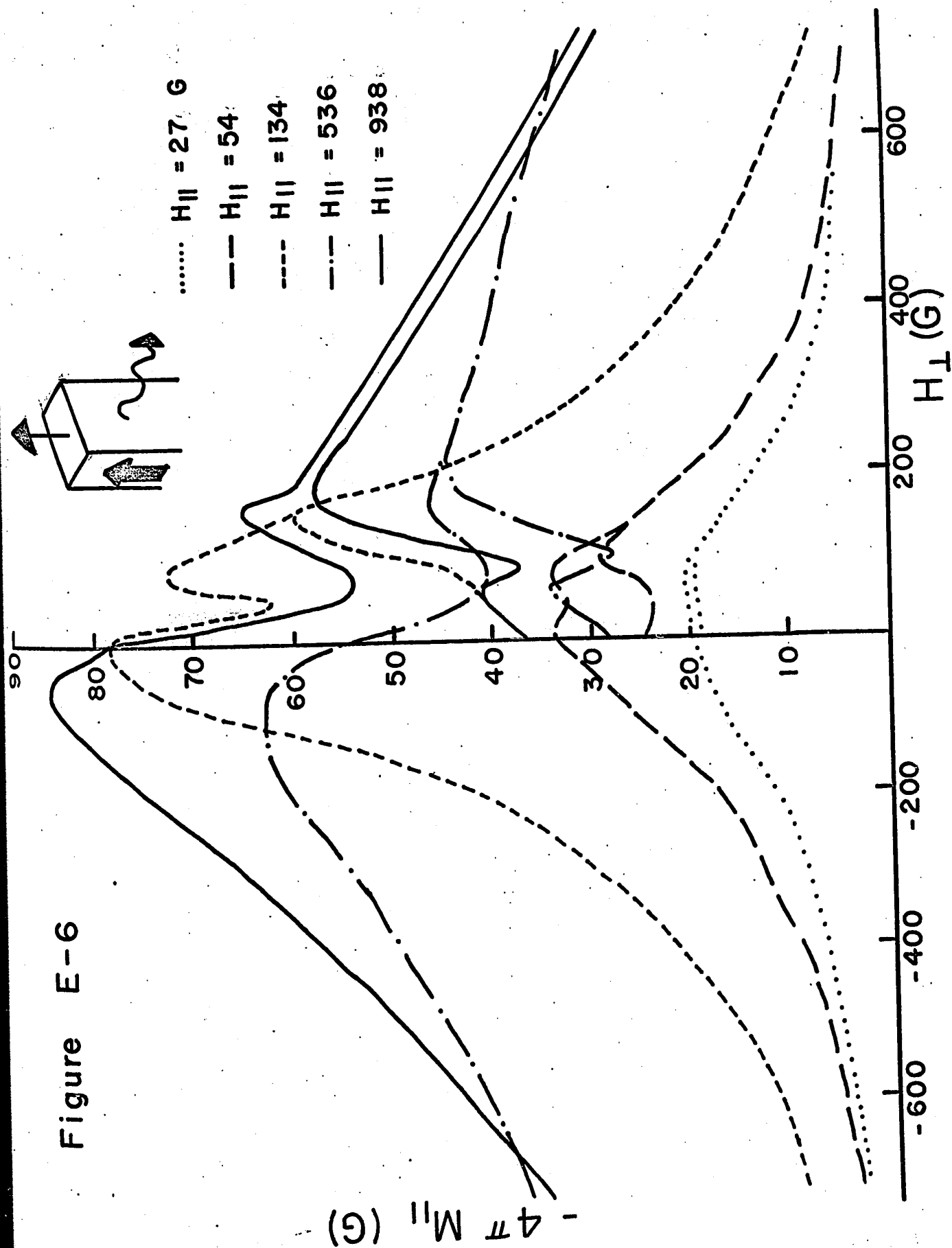


Figure E-7

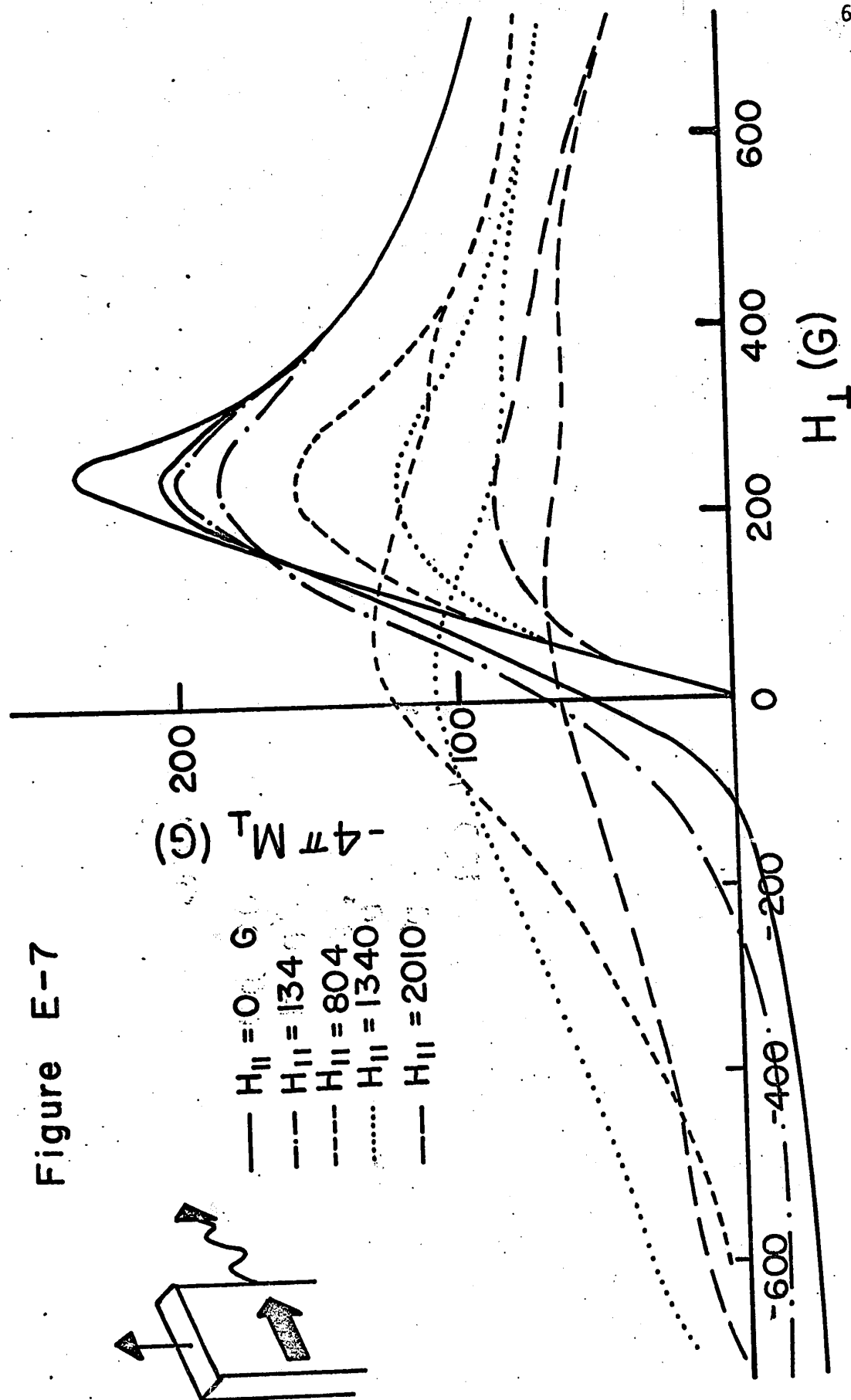
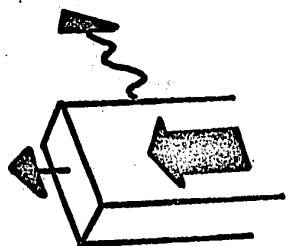
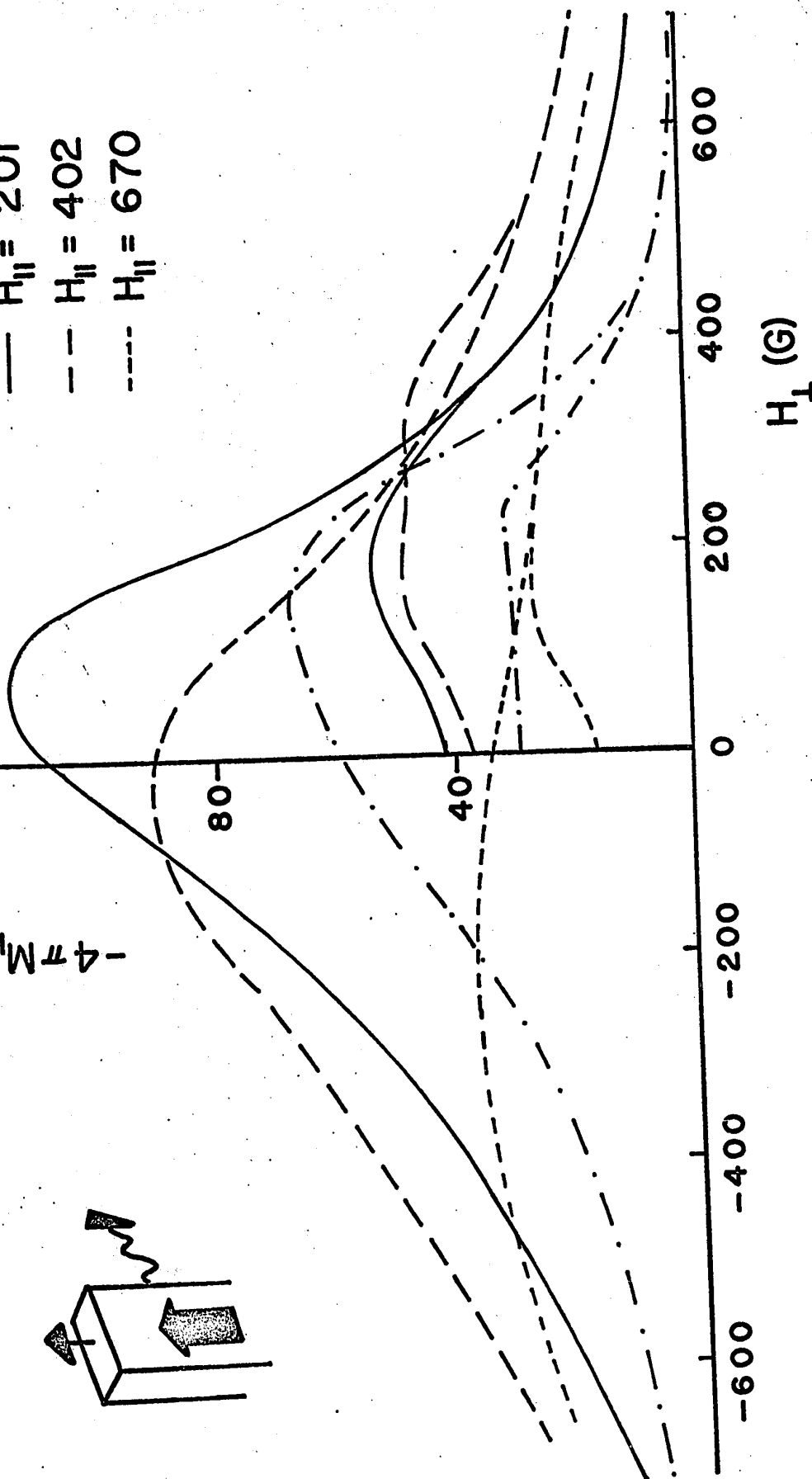


Figure E-8

 $-4\pi M_{II} \text{ (G)}$ --- $H_{II} = 67 \text{ G}$ — $H_{II} = 201$ --- $H_{II} = 402$ --- $H_{II} = 670$ $H_I \text{ (G)}$ 

Discussion

A detailed application of the composite model developed in this thesis to the interpretation of the variety of complex behaviour encountered in our PbIn sample is beyond the scope of an M.Sc. thesis. This analysis presents a formidable task because the several constituents of the model make comparable contribution to the observed phenomena. A necessary initial step for the analysis of the results consists of assessing the evidence that each component must be taken into account and of estimating their relative importance. Accordingly, in this section we consider those features of our data which reveal the presence of bulk pinning, intrinsic Abrikosov diamagnetism and irreversible surface currents.

i) Bulk Pinning

Both pinning in the bulk and irreversible surface currents contribute to trapping of magnetic flux in type II superconductors. In particular, the remanent flux, i.e., the flux trapped in zero field after an excursion to a high value of the field, is an easily measured quantity and is frequently used to evaluate the degree of irreversibility or hysteresis of a specimen. Since, however, either or both bulk pinning and irreversible surface currents may be responsible for flux trapping and the remanent magnetization, this piece of information alone is ambiguous.

There is, fortunately, a simple procedure for sorting out the relative importance of these two mechanisms to flux trapping. This consists of separately measuring and then comparing the remanent magnetization along the length, width and thickness of the ribbon sample. These results enable us to distinguish between these two contributions for the following reasons.

The remanent magnetization due to irreversible surface currents δ_{IR} (amp/cm) is linearly proportional to this quantity. This can be seen from considering that the magnetic moment $\mu_M = I_s(AL)$ where A is the area embraced by the current and L is the dimension through which it flows, whereas the magnetization is the magnetic moment per unit volume, hence μ_M/AL . It then follows that the magnetization due to surface currents is size independent and thus the same whether measured along the

length, width or breadth of the sample.

The remanent magnetization due to persistent currents flowing in the bulk (hence associated with bulk pinning) by contrast is size dependent. In this case, the magnetic moment increases more rapidly than linearly with the area filled by these currents. The reason for this is that μ_M is now an integral over the concentric areas embraced by the individual elements of currents. It follows that changing the "effective" cross-section will cause the remanent magnetization due to bulk pinning to change.

A comparison of figures E1, E3 and E5 shows the following;

a) the sample traps only $\sim 2\%$ of H_{c2} along the major axis of the ribbon. This indicates that the specimen is not highly hysteretic; the bulk pinning is not very strong and the irreversible surface currents are not considerable.

b) the sample also traps only $\sim 2\%$ of H_{c2} along the width of the ribbon. This indicates that the capacity of the surface and bulk to support irreversible currents is fairly isotropic.

c) the remanent magnetization along the thickness is appreciably larger than along the other two dimensions. This observation along with the foregoing enables us to deduce that it is mainly the bulk which contributes to the remanent magnetization, and thus to estimate the strength of the bulk pinning.

It is important to indicate here that the data presented in Figures E5 and E6 have not been adjusted for demagnetization effects.

Corrections to the horizontal scale are non linear since they depend on the magnetization, and are rigorously applicable only when the magnetization is uniform. We know however that the magnetization in fact is very inhomogeneous except over the part of the curve where H is increased from zero to H_{c1} with the sample in the virgin state. Fortunately, it is a straight forward matter to correct the vertical scale. It is sufficient in this instance to multiply the "raw" data by the appropriate demagnetization coefficient D , which can be estimated from geometric considerations as well as from our data.

Considering the ribbon as an oblate spheroid and taking m , the ratio of the major and minor axes to be given by the ratio of the width to the thickness of the slab, D can be calculated from the following expression given in the literature ³¹⁾.

$$1 - 1/D = 1/2 \left[\frac{m^3}{(m^2-1)^{3/2}} \sin^{-1} \left(\frac{\sqrt{m^2-1}}{m} \right) - \frac{1}{m^2-1} \right]$$

This computation gives $D = 4.7$.

In view of the fact that our ribbon is nearly rectangular, this is probably a crude estimate.

An estimate of D from our magnetization data directly is probably more reliable. This is done in the following way. Clearly, the demagnetization coefficient along the length is negligible, therefore it is reasonable to take H_{c1} obtained in this direction to be the true value of the material. Now, since this quantity is a property of the material independent of geometry, it follows that the demagnetization factor

should be

$$D = \frac{(H_{c1})_{//}}{(H_{c1})_{\perp}} \quad (1)$$

where the subscripts indicate the direction of the applied magnetic field with respect to the ribbon axis. Unfortunately, the onset of the mixed state, (H_{c1}) , is not always unambiguously manifested in magnetization curves. Here we take H_{c1} to be the value of the applied field where the maximum diamagnetic moment occurs as H is applied from zero to a virgin sample. We are justified in this choice since this maximum value is sharply defined within a small range of H . In this manner D turns out to be $3.3 \pm .1$.

The remanent magnetization in figure E5 can now be corrected accordingly and is seen to be 260 gauss or $\sim 10\% H_{c2}$. We are now in a position to quantitatively assess the contribution of bulk pinning and trapped flux to the remanent magnetization.

If we assume that surface currents play no role then it follows from the critical state concept, using pinning functions of the form $F_p = \alpha B^n$, that the ratio of the remanent magnetizations is given by (see appendix A)

$$\frac{4\pi M_{//}}{4\pi M_{\perp}} = \left(\frac{L_{//}}{L_{\perp}} \right)^{\frac{1}{2-n}} \quad (2)$$

where $L_{//}$ is the thickness and $L_{\perp}$ is the width of the ribbon. Note that the ratio is independent of the pinning coefficient α . Taking $n = 1$,

or in other words assuming constant flux gradients

$$\frac{4\pi M_{//}}{4\pi M_{\perp}} = \frac{L_{//}}{L_{\perp}} = \frac{.02''}{.14''} = \frac{1}{7}$$

The experimentally observed ratio (using $4\pi M$ corrected by the demagnetization coefficient) is $\sim 1/6.5$. We therefore conclude that the starting assumption is valid and that surface currents contribute negligibly to the remanent magnetization. In view of this we can next estimate the bulk pinning with some degree of confidence using the remanent magnetization.

Employing this estimate and from inspection of the magnetization envelopes (Figures E1, E3) in fields above H_{c1} we infer that bulk pinning does not play a dominant role in this region and for these orientations. Unfortunately, however, it is significant enough that it cannot be completely neglected.

ii) Abrikosov Diamagnetism

When a magnetic field applied to a type II superconductor is raised beyond a critical value, flux enters the specimen in form of quantized filaments. At this point, if there are few or no pinning centers within the bulk to impede the migration of fluxoids towards the center of the superconductor, a sudden drop in the absolute value of the diamagnetic moment occurs as predicted by Abrikosov's ¹⁾ theory, and first observed by Shubnikov ³²⁾. A modest increase in H beyond this value produces a rapid diminution of the diamagnetic moment (more vortices in the bulk) until the

density of flux tubes becomes large enough for these to experience appreciable mutual repulsion at which stage the diamagnetic moment becomes a slowly varying function of applied field, decreasing monotonically to zero as $H \rightarrow H_{c2}$.

The qualitative behaviour of the initial diamagnetic moment obtained for our PbIn slab in all three orientations corresponds to that described above. For this reason it is instructive to calculate families of curves expected for an ideal Abrikosov specimen subjected to the external conditions in effect in our investigations.

We consider an infinite slab in an external field oriented along the surfaces of the slab. Since the material of the slab is taken to be ideal (hence reversible), the magnetization is uniquely determined by the magnitude of the external field H and the pertinent parameters (H_{c1} , H_{c2} , β , κ)

Abrikosov diamagnetism can be represented by the following equation (19), for values of total field $H > H_{c1}$

$$4\pi M = \gamma \left(1 - \frac{H}{H_{c2}} \right) \quad (1)$$

where γ is a constant related to the fluxoid lattice constants β and the κ of the material.

If H is to be applied in two orthogonal directions (denoted $\perp$ and $//$) then the moments developed along the respective axes are

$$4\pi M_{\perp} = \gamma \left(1 - \frac{H}{H_{c2}}\right) \cos \theta \quad (2)$$

$$4\pi M_{//} = \gamma \left(1 - \frac{H}{H_{c2}}\right) \sin \theta$$

where θ is the angle between H and the perpendicular direction, then

$$\cos \theta = H_{\perp}/H = H_{\perp} / \sqrt{H_{\perp}^2 + H_{//}^2} \quad (3)$$

$$\sin \theta = H_{//} / \sqrt{H_{\perp}^2 + H_{//}^2}$$

so that (2) becomes

$$4\pi M_{\perp} = \gamma \left(1 - \frac{\sqrt{H_{\perp}^2 + H_{//}^2}}{H_{c2}}\right) \frac{H_{\perp}}{\sqrt{H_{\perp}^2 + H_{//}^2}} \quad (4)$$

$$4\pi M_{//} = \gamma \left(1 - \frac{\sqrt{H_{\perp}^2 + H_{//}^2}}{H_{c2}}\right) \frac{H_{//}}{\sqrt{H_{\perp}^2 + H_{//}^2}}$$

For the sake of argument $H_{//}$ is considered constant and $H_{\perp}$ variable.

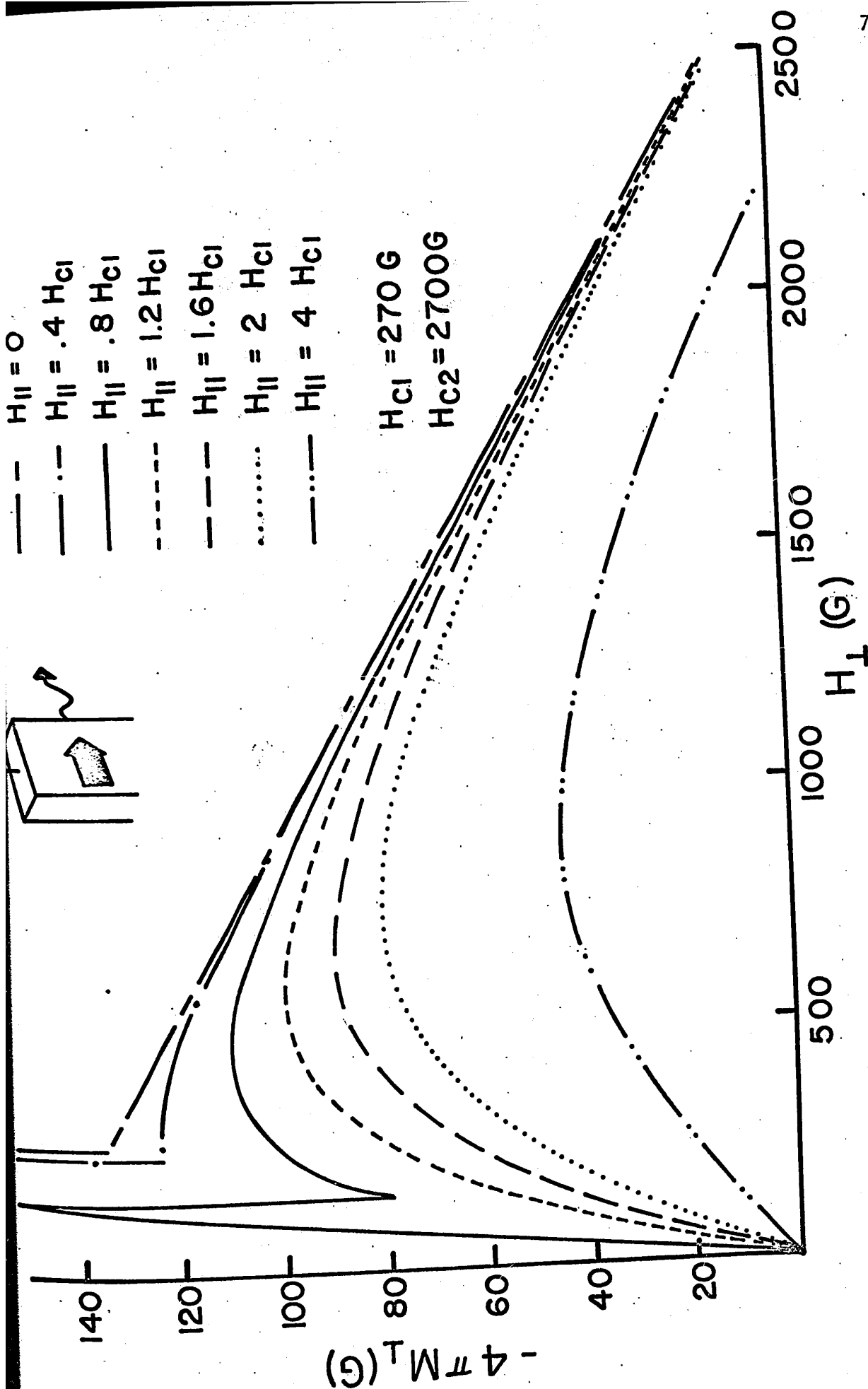


Figure E-9

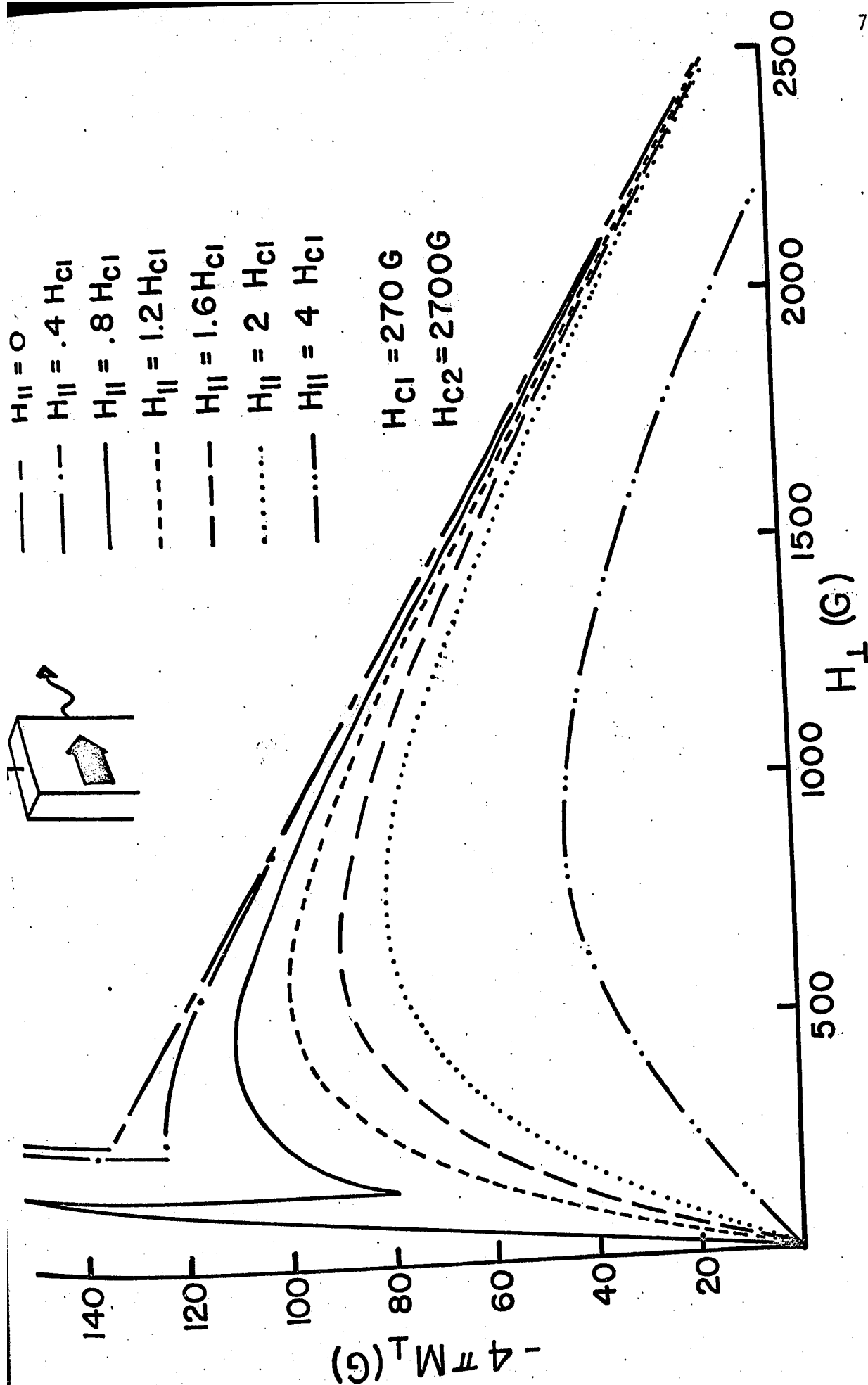
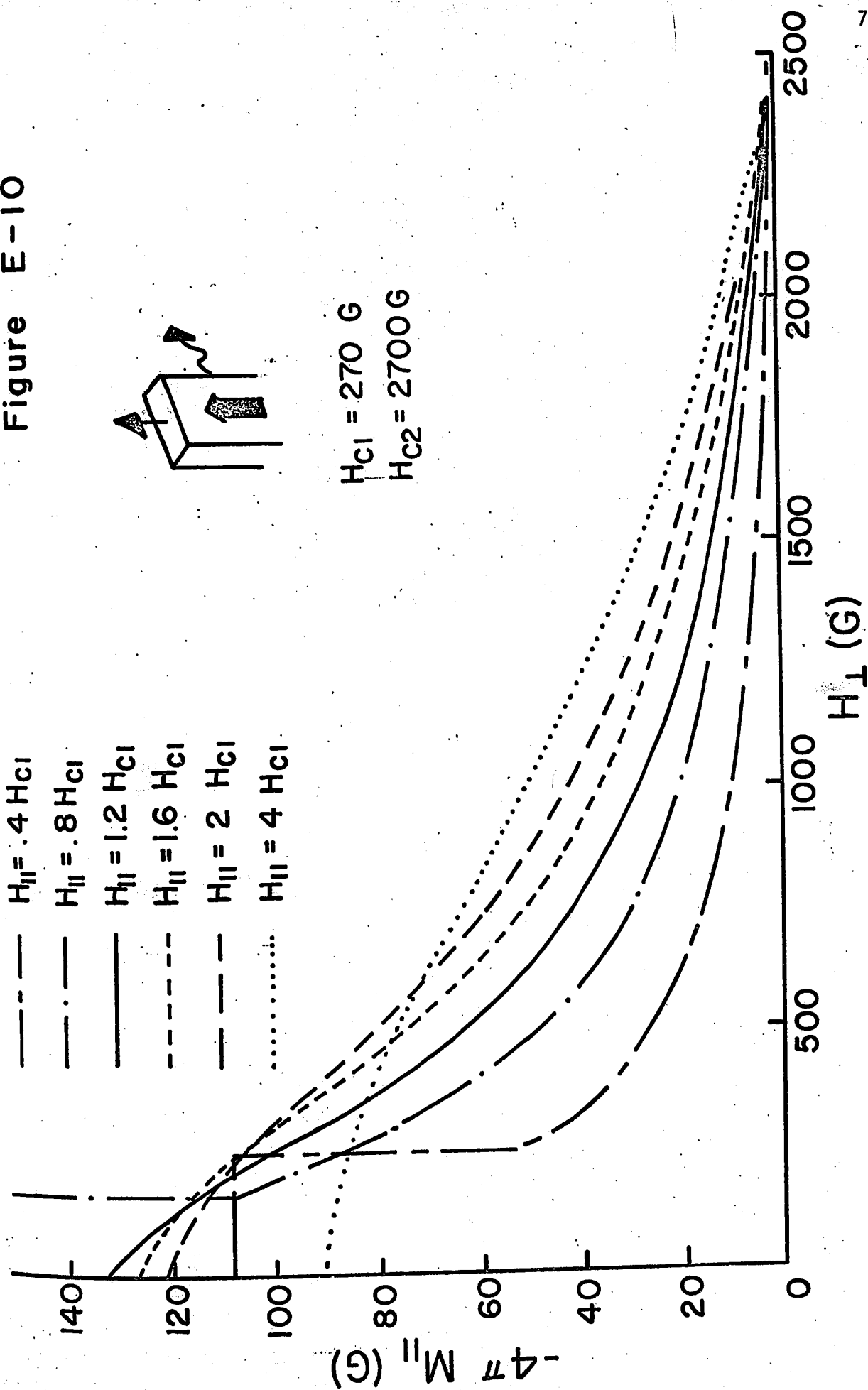


Figure E-9

Figure E-10



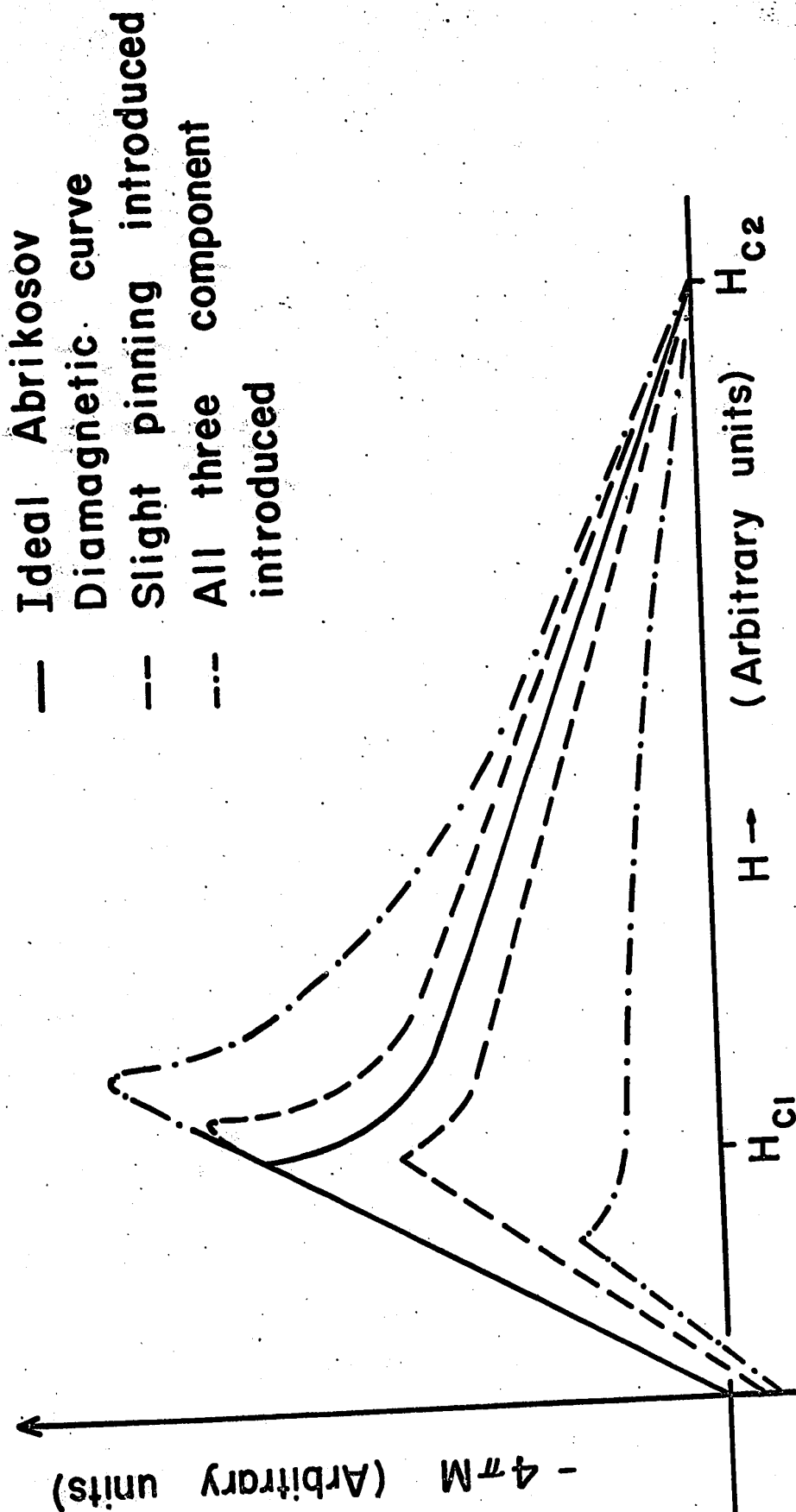
Families of curves calculated for a slab of ideal Abrikosov material are shown in figures E9 and E10. The corresponding measured curves in figures E1 through E8 are those where H is increased from zero for a "virgin" sample. (The comparison is then to be made between these families of curves in figures E1, E3, E5, E7 with figure E9 and figures E2, E4, E6, E8 with figure E10. It emerges from this comparison that the ideal Abrikosov slab exhibits general features and trends of behaviour which correspond closely to that encountered in our specimen. We can thus conclude that the reversible component of our specimen in these circumstances plays the dominant role and the contribution of bulk pinning and irreversible currents are not appreciable when H is applied from zero to a virgin sample.

iii) Irreversible Surface Currents

Irreversible surface currents play a significant role in the hysteretic behaviour encountered in our PbIn ribbon particularly when H is swept from a high field through zero to a high field of opposite polarity. We now present three sets of observations which provide evidence for the existence of these currents and yield information on their magnitude in the PbIn sample.

a) The main features of the magnetization curve of a type II superconductor with weak pinning as H is swept from 0 to H_{c2} and back to 0 can be readily established using the Silcox model. The result is shown in Fig. E-11. If this material can also support a modest irreversible surface current, it is also fairly straight forward to predict its behaviour using

Figure E-II



CURVE PREDICTED BY THE COMPOSITE MODEL

our three-component composite model. This is also shown in Fig. E-11. From inspection we note immediately the two main differences between the sets of curves

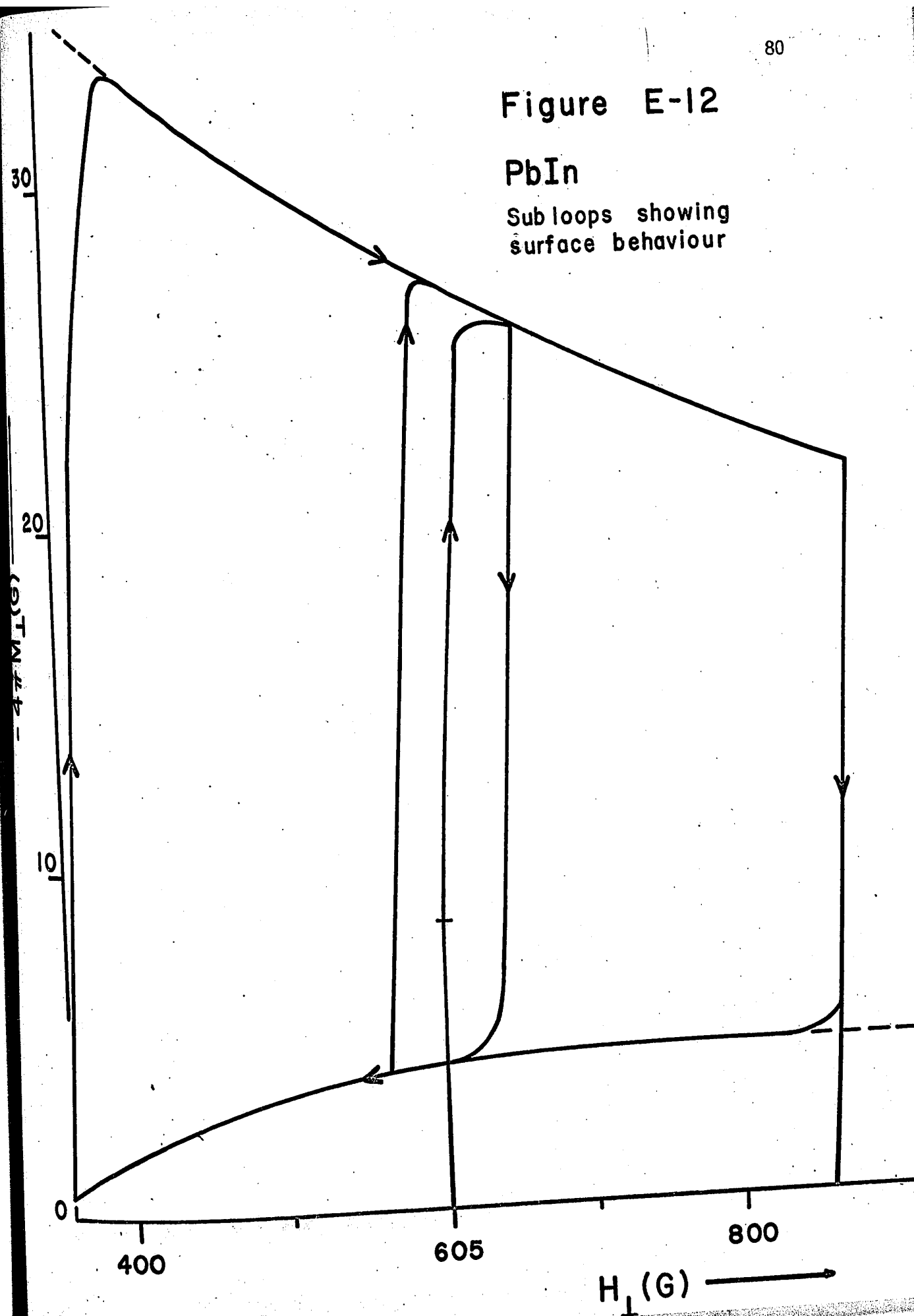
(i) the shift in the relative position of the peaks along the field axis.

(ii) the increased vertical separation between the upper and lower curves: Now from inspection of the corresponding curves for our sample (Figs. E-1 and E-3) we readily recognize these two features. We may then conclude that a modest (relative to H_{c1}) irreversible surface current flows in our ribbon under these conditions.

b) The onset of entry of flux in an increasing field indicates that the capacity of the surface to support an irreversible current is attained. Here the current is flowing in a flux shielding or diamagnetic sense. If now the field is decreased until exit of flux just commences, the capacity of the surface to sustain an irreversible current is again on the verge of being exceeded. In this case the current is flowing in a flux retaining or paramagnetic sense. Accordingly, the total range over which the external field can be varied between onset of flux entry and onset of flux exit provides a measure of the sum of the two currents. Several workers³³⁾ have exploited this viewpoint to determine critical surface currents using various techniques to monitor the onset of flux entry and exit. We have performed similar experiments. Sub-loops of the magnetization envelope were traced after the PbIn ribbon was cooled through T_c to the Helium

Figure E-12

PbIn

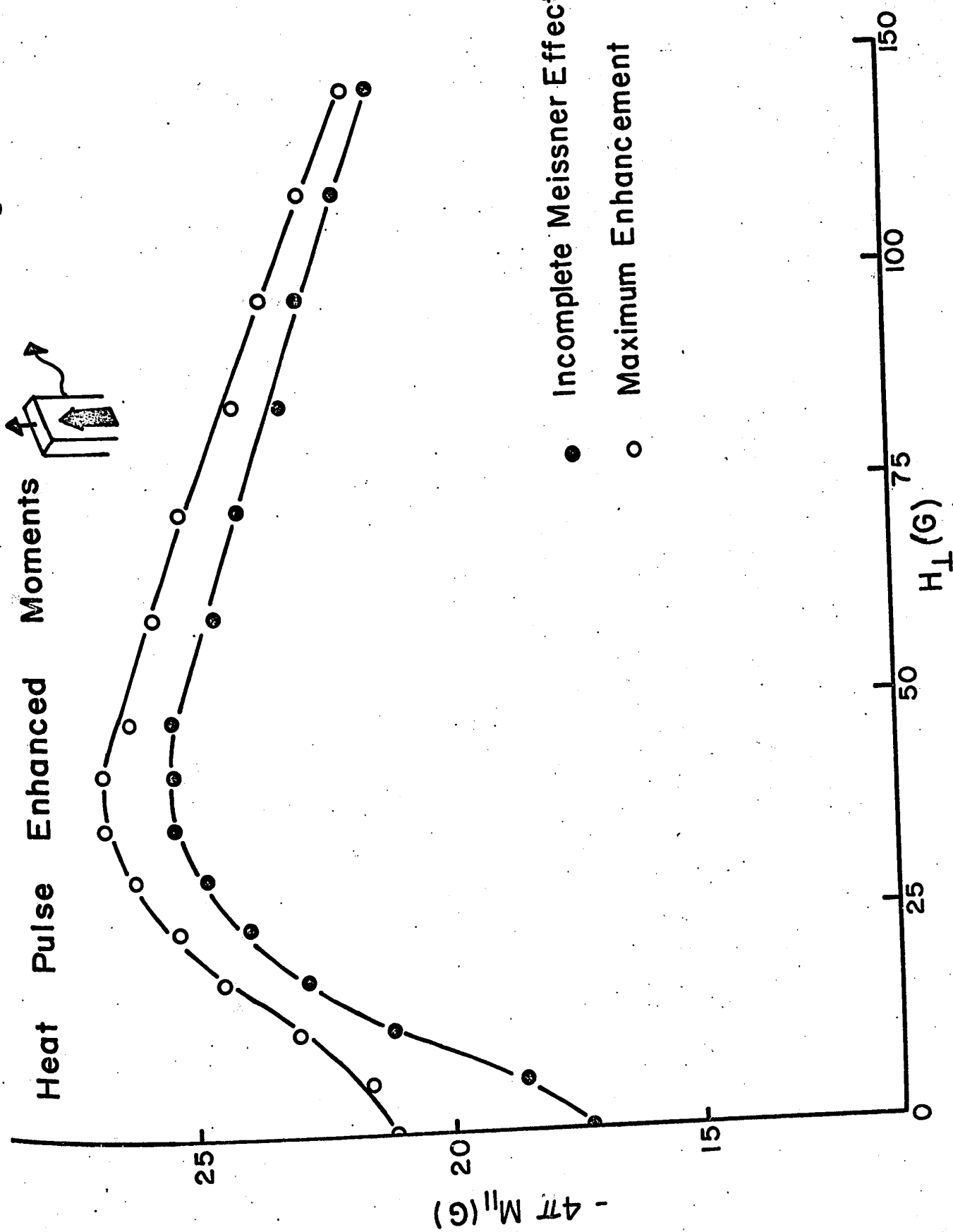
Subloops showing
surface behaviour

bath temperature in the presence of a static field H which was subsequently cycled between $H + \Delta H_f$. Here ΔH_f is an arbitrarily chosen value. A typical resultant sub-loop is displayed on Fig. E12. For this particular curve ($H = 600$ gauss) we deduce from ΔH , the change in field necessary to bring the irreversible surface current from critical diamagnetic to critical paramagnetic (and vica versa), an order of magnitude for δ_{IR} of 10-20 gauss which is in agreement with other observations on this material to be presented in chapter IV.

c) LeBlanc and his co-workers¹⁰⁾ have also reported the enhancement of the Meissner effect in non-ideal type II superconductors by raising the temperature of the surface only towards T_c . This phenomena is seen to arise as a result of the destruction of the irreversible, paramagnetic surface current induced when the sample immersed in a static magnetic field is cooled through T_c . We have performed similar experiments on our PbIn slab preferentially heating the sample surface by discharging a capacitor through the hairpin heaters. The enhancement observed is again a manifestation of the presence of irreversible surface currents. Typical results are shown on figure E13. It is noted that the degree of enhancement achieved for our sample is considerably smaller than that reported by the above authors. We next show this feature to be consistent with the observation that the pinning strength is weaker in our sample.

If we for simplicity assume linear flux gradients then a decrease in the irreversible surface current by ΔI_{IR} (via heat flashing the surface) produces an increase in the magnetic moment which is proportional to $(\Delta I_{IR})^2$

Figure E-13



when the bulk pinning is sufficiently strong. On the other hand, if the sample, due to weak pinning, is near ideal within the bulk, the change in magnetization is roughly proportional to ΔI_{IR} only. (This is the case for our sample). Furthermore, as is the case in the work of these authors, if the critical irreversible current is large the unenhanced moment will be small and the relative enhancement large. For our sample the critical irreversible current seems to be smaller thus allowing a greater fraction of flux to leave on initial cooling. The proportional increase in flux expulsion is consequently reduced when the irreversible current is destroyed by suitable heat flashing.

CHAPTER IV

Meissner Effect PhenomenaIntroduction

Timms and LeBlanc³⁴⁾ have studied flux expulsion in a ribbon sample of NbTa allowed to cool from T_c in different weak static magnetic fields oriented at various angles with respect to the flat face of the specimen. These workers discovered that the amount of flux which the specimen expels is largest when the external field H is $\sim H_{c1}$ and is oriented at an angle of $\sim 45^\circ$. Further, they noted that it is the component of the magnetic field along the axis of the ribbon which is preferentially expelled.

We have pursued similar studies in a ribbon of Vanadium and find that in this material, the enhancement in flux expulsion at the optimum field level and orientation is even greater than that encountered in NbTa. We have also investigated this phenomena in the ribbon of PbIn for which other measurements were reported in chapter III. In this material the optimum enhancement in flux expulsion is not appreciable.

In this chapter we show that our composite model can account for all of these observations: the remarkable enhancement encountered in V and NbTa as well as the weak enhancement found in PbIn. Further, in this chapter we show that this model qualitatively and

quantitatively reproduces the complicated families of curves of flux expulsion measured experimentally for the three samples studied to date. This success of the composite model constitutes the strongest evidence yet available that it is essentially correct and complete.

Experimental Procedure

A sample of pure Vanadium received in a wire form of .20 cm in diameter was cold rolled into a long thin slab of dimensions .038 x .325 x 6.25 cm. As Vanadium is harder than PbIn, it was possible to wind a bifilar heater of no. 38 formvar coated manganin wire along the entire length of the specimen. This allows faster heating with less current through the heater coils compared with the type of arrangement used for the softer PbIn slab that was sandwiched between sheets of mica. Also, paramagnetic impurities encountered in mica are avoided. The Vanadium slab was then placed in the sample holder previously housing the PbIn ribbon.

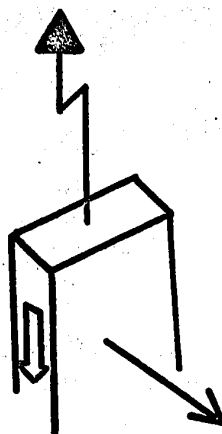
Results and Discussion

The experimental observation consists of determining the Meissner moment $4\pi M_{//m}$ arising along the length of the sample, denoted as the parallel direction. By the Meissner effect we mean the expulsion of flux, hence the appearance of a diamagnetic moment as a superconducting sample cools from T_c in the presence of a static magnetic field. This moment was also monitored in the perpendicular direction and was found to be nearly independent of field orientation.

Figure F-1

V

Meissner Effect



perfect
diamagnetic line

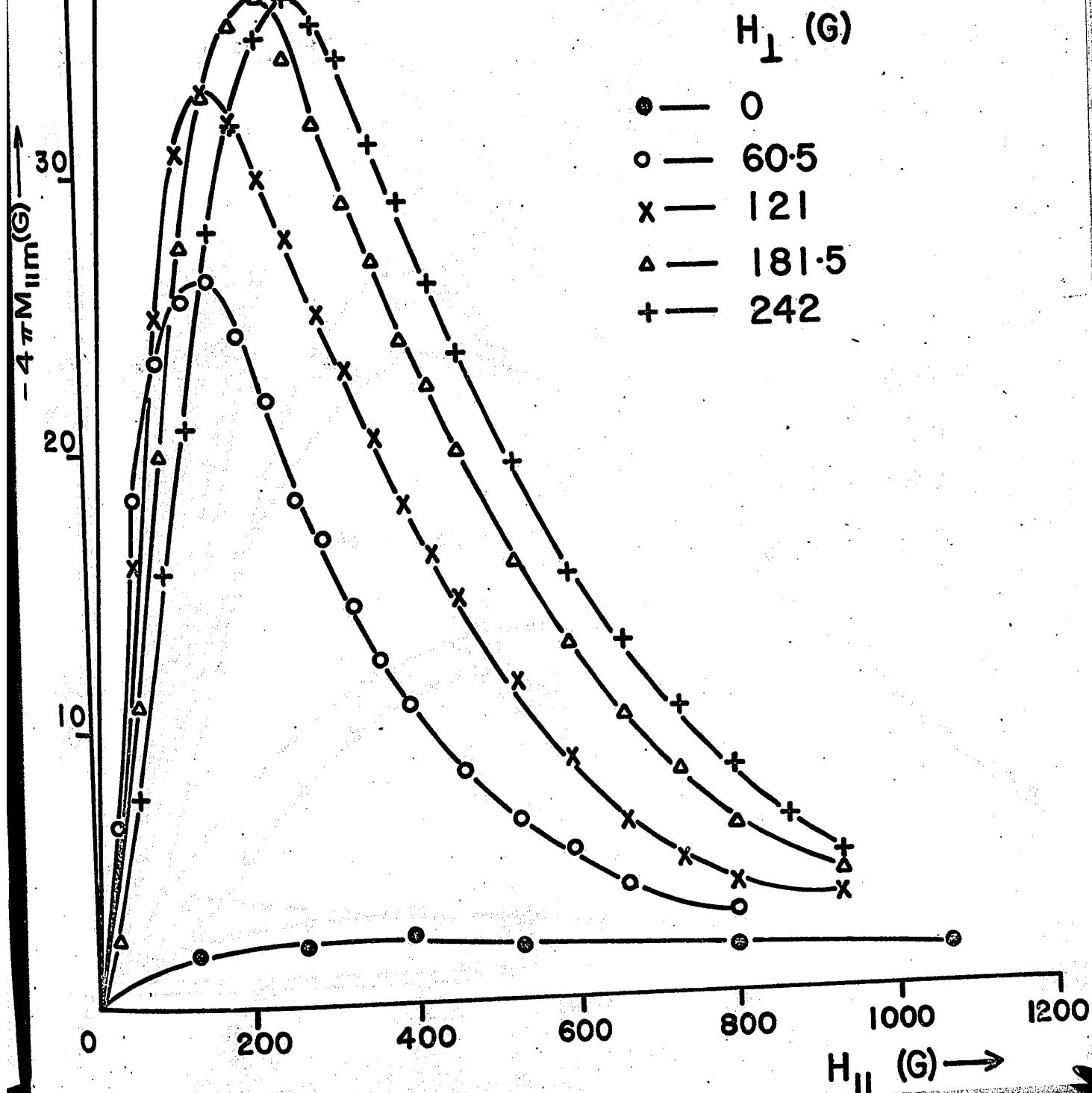
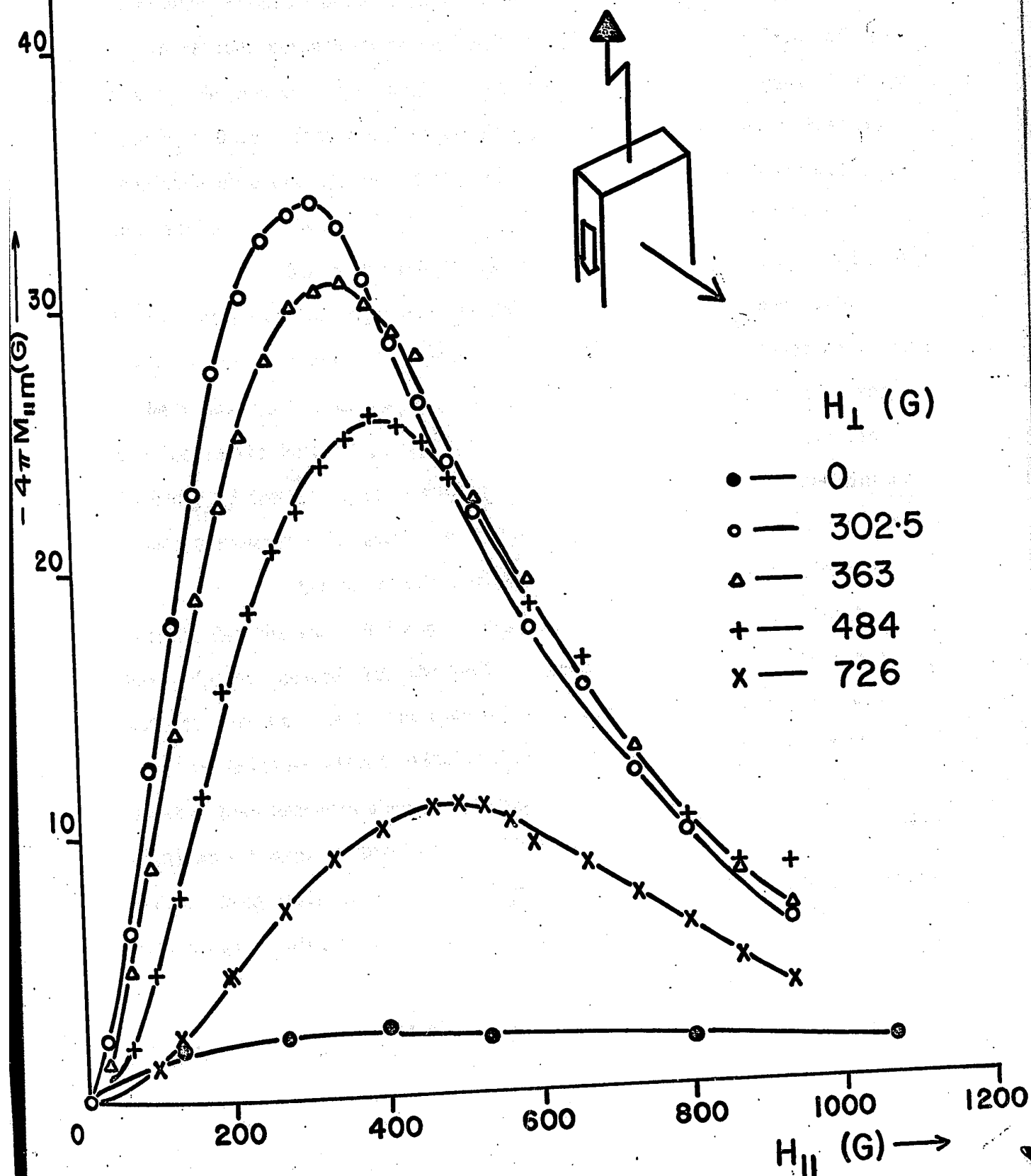


Figure F-2

Meissner Effect

V



In figures F1-F2 we present families of curves of $4\pi M_{//}/m$ as a function of $H_{//}$ in various perpendicular fields H applied into the flat face of the ribbon. We notice a spectacular enhancement of a small (~ 2 gauss) moment when $H_{\perp} = 0$ by a factor of nearly twenty (to ~ 40 gauss) when H has the magnitude of about H_{c1} and is oriented approximately 45° with respect to the sample.

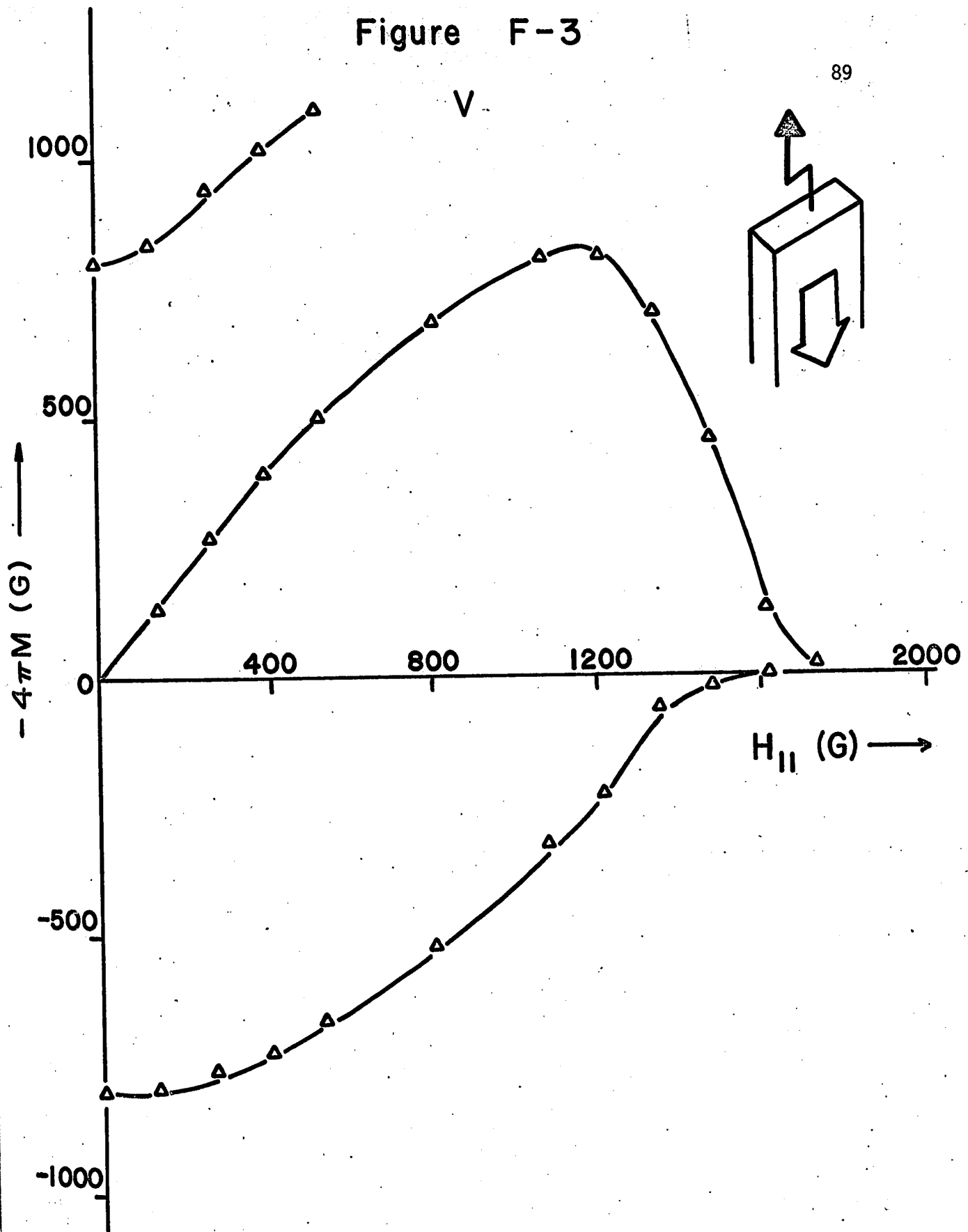
Our cold rolled sample is highly irreversible as can be seen by inspection of the magnetization curves in increasing and decreasing fields shown in figure F3. In particular the remanent magnetization is found to be $\sim 50\% H_{c2}$. This large amount of trapped flux is evidence of strong pinning in the bulk of the specimen. This indicates that large defects ($\sim$ order of the coherence length) are present in the bulk since the Ginzburg Landau parameter κ is small. We estimate κ to be 2.5 with 20% uncertainty.

Although bulk pinning is strong it is not sufficient to account for the small Meissner effect observed. Calculations using the bulk pinning deduced from the maximum remanent moment and a reversible surface current (the Silcox model) are displayed on figure F4. We note that the Meissner effect calculated with this model is a factor of 25 greater than measured when H is along the sample axis and the maximum attainable moment is monotonically decreasing with increasing perpendicular field. Once again we are lead to believe that irreversible surface currents play a major role; therefore our new model must be applied.

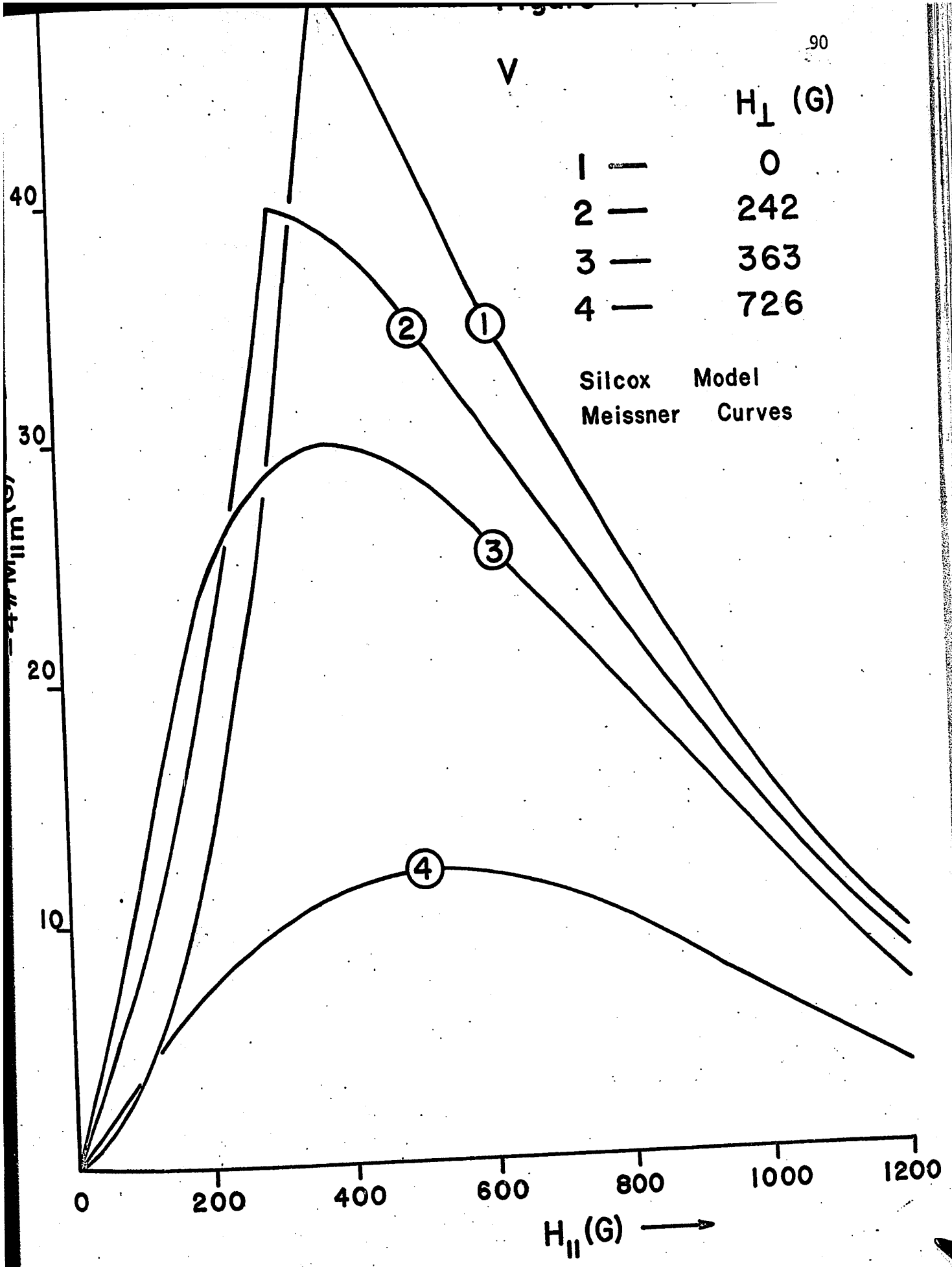
Figure F-3

V

89



MAJOR HYSTERESIS CURVE



V

 H_L (G)

1 —

0

2 —

242

3 —

363

4 —

726

Silcox

Model

Meissner

Curves

— $4\pi M_{eff}$ —

40

30

20

10

0

200

400

600

800

1000

1200

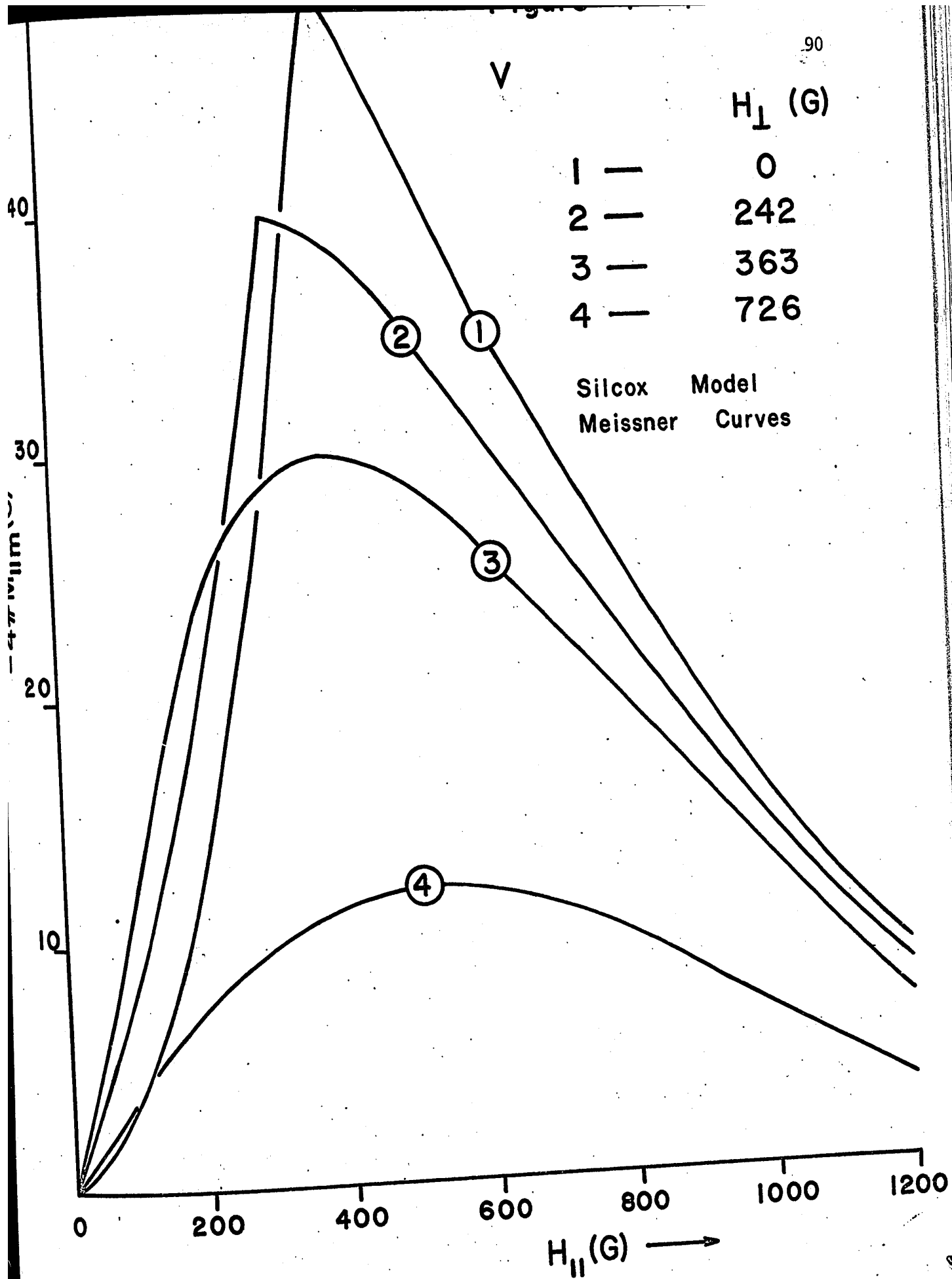
 H_{II} (G) →

②

①

③

④



In figures F5-F6 we present families of curves of flux expulsion calculated using our composite model. A comparison of these figures with the measured ones displayed on figures F1-F2 reveals an excellent agreement. We now describe the application of the composite model to this situation and the procedure followed in the calculations.

As the ambient temperature falls just below T_c superconductivity arises in the bulk. At this juncture the homogeneous configuration of flux in the bulk is modified and a redistribution into quantized flux filaments takes place. As T further decreases, migration of these fluxoids towards the surface occurs when the net outward force they experience exceeds the pinning strength. The laws of induction require that a paramagnetic current simultaneously arise in the multiply connected surface layer to oppose exit of flux³⁵). This paramagnetic (irreversible) surface current will grow until it attains some maximum value or until the outward forces experienced by the flux filaments are balanced. The development of a diamagnetic moment as T decreases indicates that this balance occurs after the surface current has attained a critical state and excess flux has been expelled, and that the reversible currents exceed the irreversible (paramagnetic) current.

The schematic diagrams of figure F7 may assist the reader in visualizing the final flux profiles that ensue from this sequence of events according to our model.

The functional dependence of the reversible current δ_R is the well known Abrikosov diamagnetism. In our work for simplicity

V

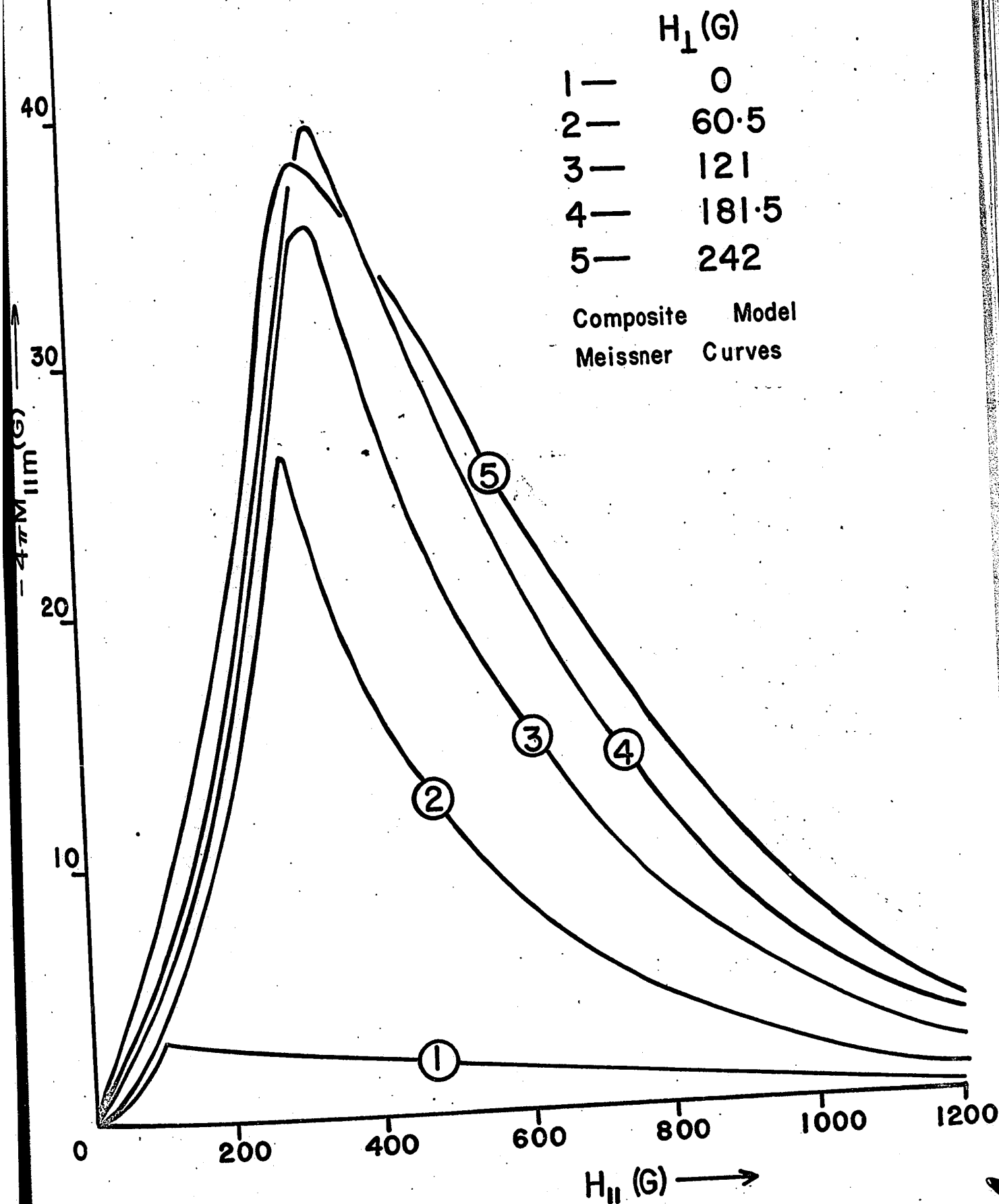


Figure F-6

V

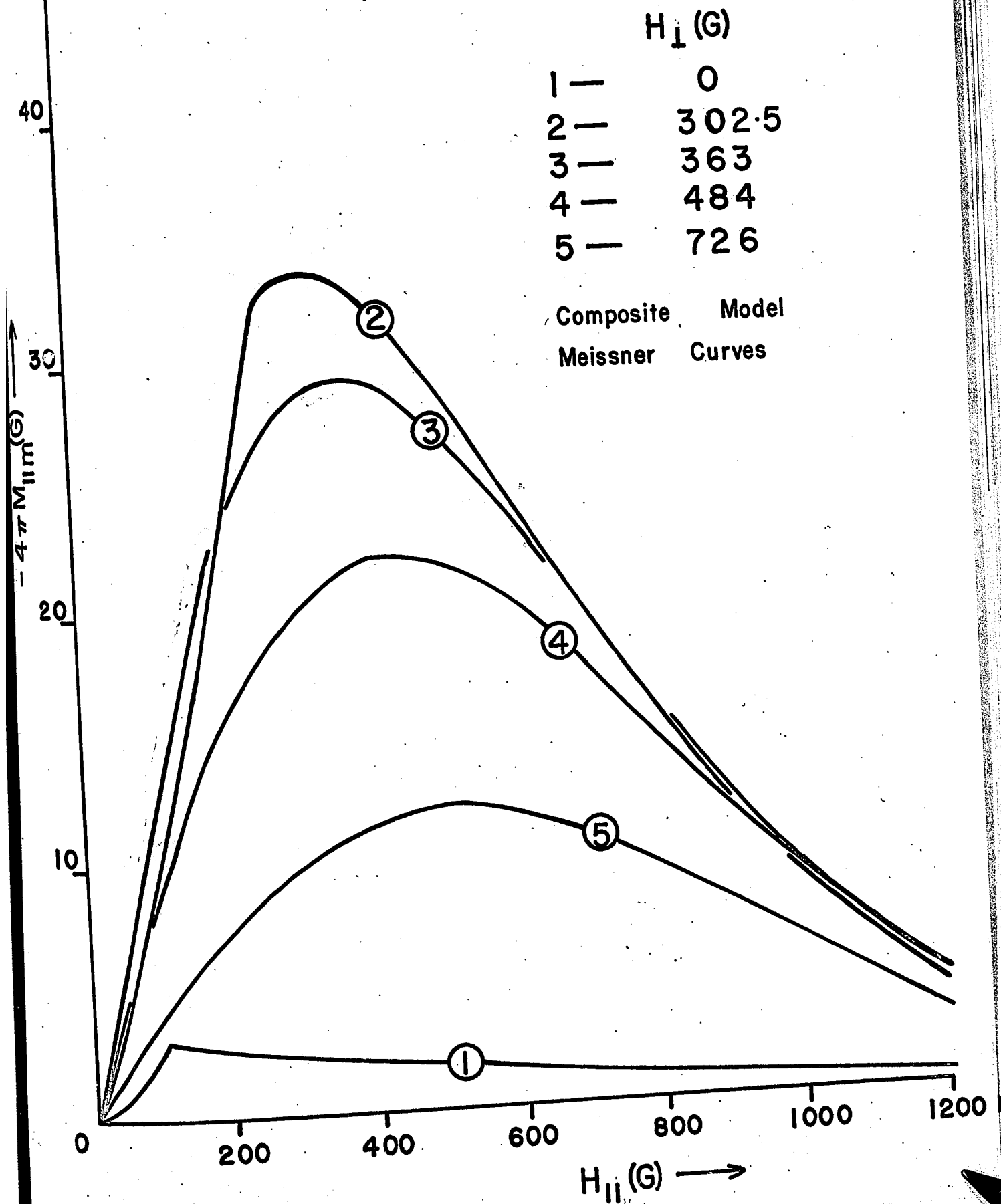
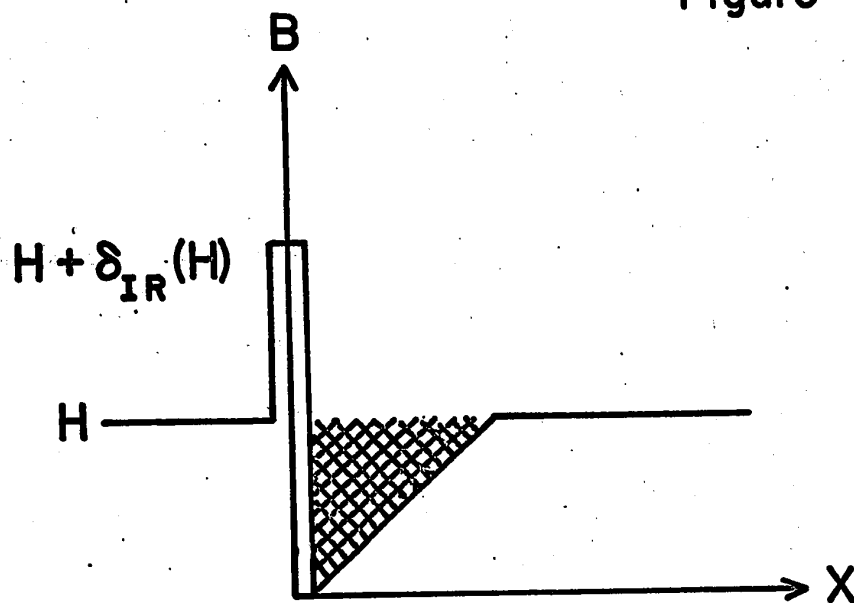
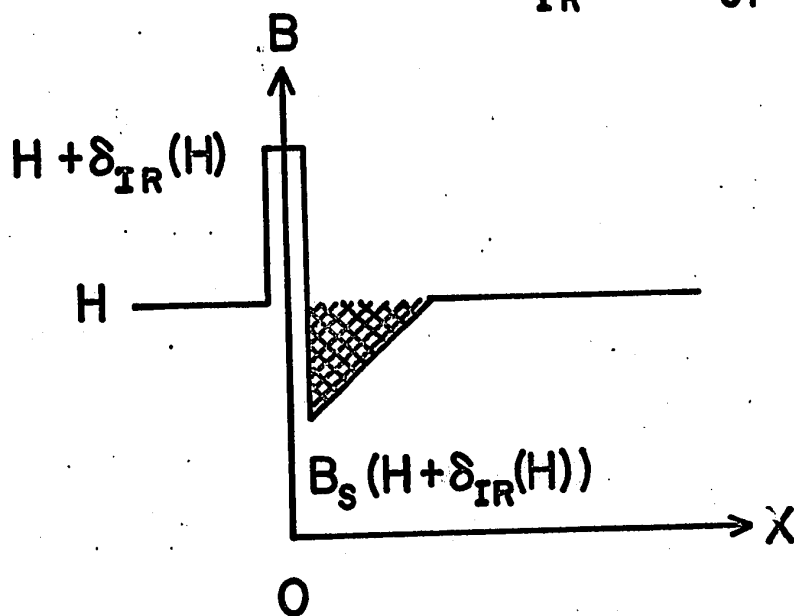


Figure F-7



$$H + \delta_{IR} \leq H_{CI}$$



$$H + \delta_{IR} > H_{CI}$$

Schematic Flux Expulsion

Shaded area represents resultant magnetic moment.

we employ a triangular²²⁾ approximation to the Abrikosov diamagnetic curve. Also for simplicity we assume a linear dependence of δ_{IR} on field intensity H namely

$$\delta_{IR}(H) = S(H_{c2} - H) \quad (1)$$

which also implies that $\delta_{IR} \rightarrow 0$ as $H \rightarrow H_{c2}$. Although this latter is probably incorrect it is not significant in our analysis since the enhanced Meissner effect phenomenon is confined to low fields only ($\sim H_{c1}$). The measured value of the Meissner moment at H_{c1} was used to determine the parameter s once a value for α was obtained. Here α is the pinning strength in the bulk critical state equation which, again for simplicity is assumed to be

$$F_p = \alpha B \quad (2)$$

and is calculated from the maximum remanent moment.

Next, it is necessary to guess at the angular variation

of δ_{IR}

$$\delta_{IR}(H, \phi) = \delta_{IR}(H) * f(\phi) \quad (3)$$

Physically this means the weakening or total destruction of the irreversible surface barrier by the flux piercing the surface. ϕ is defined as the angle $\vec{H}$ subtends with the direction normal to the face of the slab. We expect that a field piercing the surface at small angles will have a pronounced effect on the surface barrier. Tinkham³⁶⁾ proposed an angular

dependence of critical currents in the surface layer with magnetic field orientation with respect to the surface, involving sinusoidal functions. To yield satisfactory results, our requirement for the angular dependence is a rapidly varying function vanishing at $\phi = 0$. In fact $\delta_{IR}(H, \phi)$ must become negligibly small when $H_{\perp} \sim H_{c1}$ since the Silcox model seems sufficient to reproduce the quantitative behaviour of the Meissner effect for perpendicular fields greater than H_{c1} . It is thought to be quite significant that an angular dependence of this nature was derived by Saint James and de Gennes³⁷⁾ for the critical field at which a superconducting surface sheath can exist above H_{c2} .

With all the parameters fixed and the functional dependence of δ_{IR} on H judiciously chosen (these are summarized in table F) the scheme of calculation is as follows. For a given value of H and its orientation (ϕ), δ_{IR} is defined by equation (1). Now, since $H + \delta_{IR}$ is the effective field seen by the boundary of the bulk, δ_R is determined using the triangular approximation for the Abrikosov diamagnetism and this effective field $H + \delta_{IR}$. The magnetic induction at the surface of the bulk B_s is then specified since

$$B_s = (H + \delta_{IR}) - \delta_R \quad (4)$$

B_s together with the assumed flux profile then determines the resulting Meissner moment (See figure F7).

In light of the success of the composite model in explaining the Meissner Effect phenomena for Vanadium, we applied the same analysis to

TABLE F

Specimen	Parameters				
	$H_{c1}(G)$	$H_{c2}(G)$	κ	$\alpha L/H_{c2}$	S
Vanadium	400	1800	$2.6 \pm 20\%$	.9	.176
PbIn	200	2700	$4.1 \pm 20\%$	.033	.043

results obtained for the PbIn ribbon which is shown in figure F8. As can be seen, due to a different value of the Ginzburg Landau parameter κ , and a different metallurgical condition, PbIn exhibits a strikingly dissimilar family of curves to the one observed in Vanadium for the parallel Meissner effect in perpendicular fields. Table F lists the parameters and functional dependences for both PbIn and Vanadium, while the calculated curves shown on figure F9 indicate that the model we are proposing is just as excellent for PbIn as for Vanadium.

In closing it is gratifying to note that the Meissner effect results of Timms and LeBlanc on NbTa, which are very similar to our results for the Vanadium sample, are also well accounted for by our synthesized model. Work is presently under way on refinements to the approximations employed in this analysis, however, these are expected to improve only slightly on some qualitative aspects since the major features of our experiments have already been well explained.

Figure F-8
Pb-In

99.

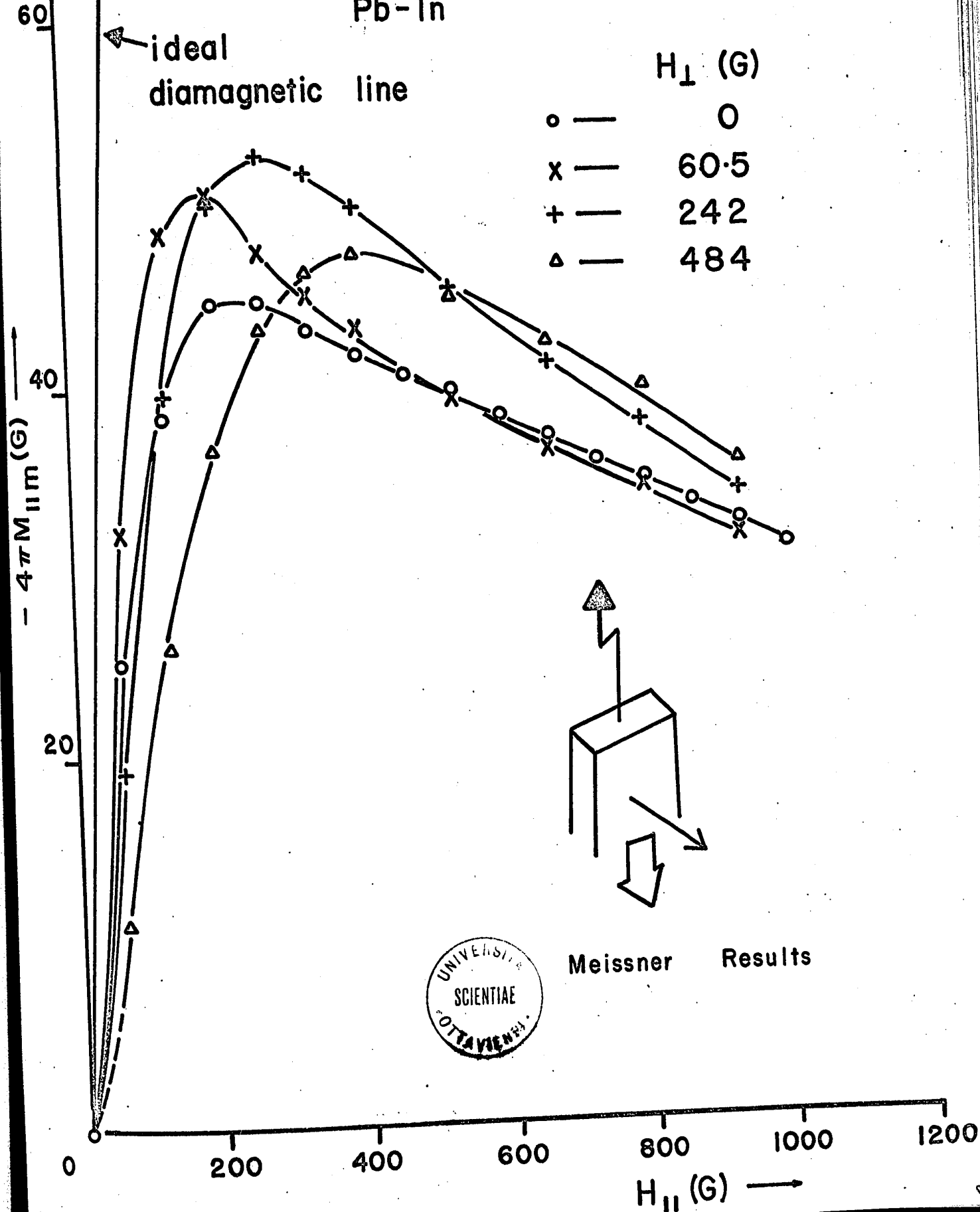


Figure F-9

100.

Pb - In

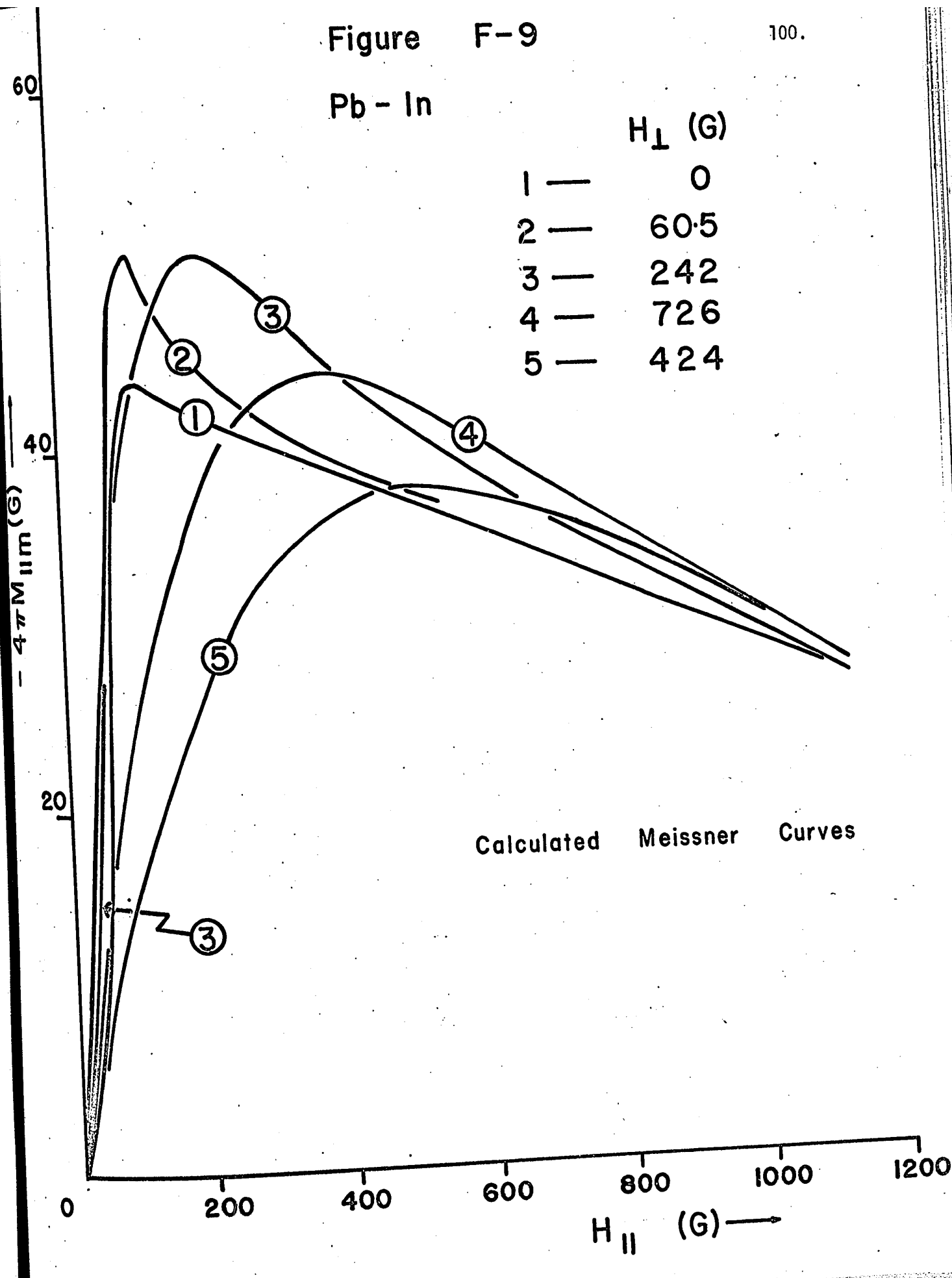
$H_{\perp}$ (G)

- | | |
|-----|------|
| 1 — | 0 |
| 2 — | 60.5 |
| 3 — | 242 |
| 4 — | 726 |
| 5 — | 424 |

$4\pi M_{||}$ (G)

Calculated Meissner Curves

$H_{||}$ (G) →



CONCLUSIONS

In the introduction we have outlined the essential concepts developed in the literature for the understanding of macroscopic behaviour of non ideal type II superconductors: bulk pinning, intrinsic Abrikosov diamagnetism, and irreversible surface currents. Each of these concepts has been well documented experimentally and explored theoretically by numerous workers and form the corner stones or building blocks of the new composite model we propose and develop in this thesis. Our model is a careful synthesis into a consistent picture of these three basic concepts.

Much of the work in the literature has been focussed on the bulk pinning property since this is the major feature in the highly hysteretic materials used for superconducting magnets. Following the lead of Silcox and Rollins, the concept that thermodynamic equilibrium could apply in non-ideal type II superconductors (intrinsic Abrikosov diamagnetism) evolved slowly and rarely explicitly. Indeed, to this day the concept seems not to have been applied rigorously to describe phenomena encountered in low fields (below H_{c1}). Several authors³⁸⁻⁴⁰ have also previously pointed out the necessity of including a surface barrier in devising a model adequate to explain all hysteretic phenomena in type II superconductors. To our knowledge, however, this is the first time that a systematic approach has been taken to mould all the pertinent concepts into a single unified scheme.

We have conducted three distinct and extensive series of investigations on three different samples and report on these in separate chapters of this thesis. In each case, existing models were applied to the interpretation and analysis of the data. In all three cases the complete set of observations cannot be accounted for by these models although they are adequate in describing some features of the behaviour over limited ranges of field and under special sets of conditions. We show that, however, in two of the three cases the complex spectrum of phenomena are fully explained qualitatively and quantitatively by employing our composite model. In the third case (chapter III on PbIn) we have not applied the model systematically because of the intrinsic difficulty of the program, and have confined ourselves to setting the stage (defining the parameters) for such an undertaking.

In summary, we have measured and accounted for the behaviour of the magnetization at the limiting transport current in a simulated infinite slab, constructed with NbTa ribbon, over the full range of applied magnetic fields. We have also extracted data on the dependence of the surface currents in this material on magnetic field.

In Vanadium and PbIn ribbons we have investigated a new phenomenon recently discovered by Timms and LeBlanc; this is the considerable increase in flux expulsion along the axis of the ribbon when the ambient field is oriented at an angle to the axis of the ribbon. Exploiting our composite model we have reproduced the main features of the full variety of experimental curves in detail.

Finally, we mention a further recent success of our model, not reported in the text of this thesis as it was achieved during its preparation. We have applied our model and successfully accounted for a substantial part of the vast assortment of magnetic behaviour encountered in non-ideal type II superconductors in the range of low positive and negative fields ($\lesssim H_{c1}$) as the external field is swept from H_{c2} through zero to $-H_{c2}$. These phenomena hitherto remained unexplained in the literature. Indeed most workers have previously limited their analysis to fields outside this region. We believe that we have pierced much of the mystery enveloping this "forbidden" domain.

APPENDICES

Appendix A

Calculation of $4\pi M$ as I is varied

It was earlier established that, from geometrical considerations equation A(1) can be put in the following form

$$B \frac{dB}{dx} = \alpha B^n \quad (1)$$

The magnetization in general is defined as the average magnetic induction inside the superconductor, minus the external field, so that

$$4\pi M = \langle B \rangle - H \quad (2)$$

$$\text{Now} \quad \langle B \rangle = \frac{1}{L} \int_0^L B dx \quad (3)$$

where L is the width of the infinite slab considered here.

Equation (3) can be written in the following form with the use of equation (1)

$$\langle B \rangle = \frac{1}{\alpha L} \int_{B(0)}^{B(L)} B^{2-n} dB \quad (4)$$

which can be solved if the boundary conditions $B(0)$ and $B(L)$ are known.

If only one boundary conditions is known exactly,
the other may be calculated by integrating equation (1) whence

$$B^{2-n} = \pm (2-n)\alpha x + \text{const.} \quad (5)$$

With the aid of (4) and (5) it is possible to compute the magnetization for any sequence of profiles.

Consider a cycle of $4\pi M$ vs I with the initial direction of I aiding H . From Figure AA-1.

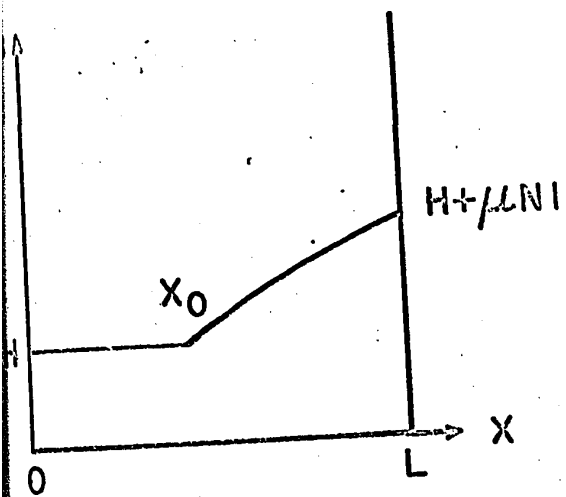
$$\begin{aligned} \langle B \rangle &= \frac{1}{\alpha L} \int_{B(x_0)}^{B(L)} B^{2-n} dx + \frac{x_0}{L} H - \\ &= \frac{1}{(3-n)\alpha L} [(H + \mu NI)^{3-n} - H^{3-n}] + \frac{x_0}{L} H \end{aligned}$$

and x_0 can be found from (5)

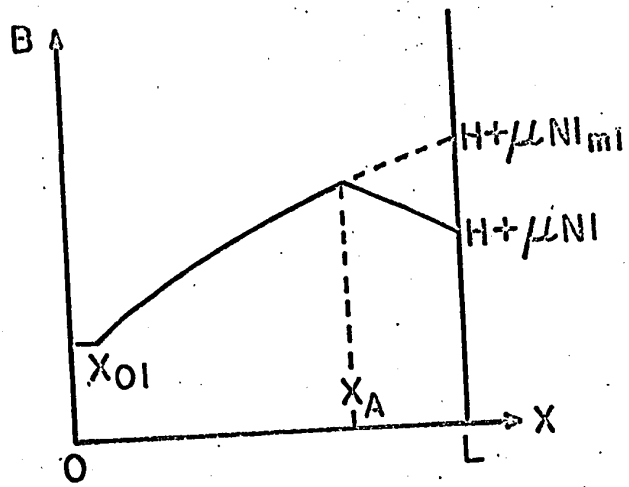
$$B^{2-n} \Big|_H^{H+\mu NI} = (2-n) \alpha x \Big|_{x_0}^L$$

$$(H + \mu NI)^{2-n} - H^{2-n} = (2-n)\alpha L \left(1 - \frac{x_0}{L}\right)$$

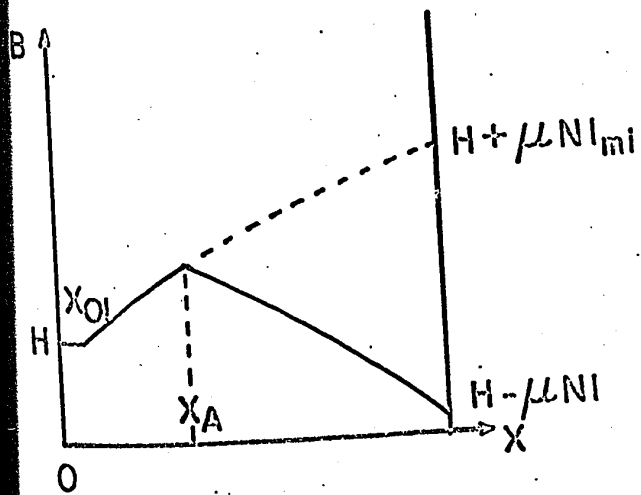
figures AA



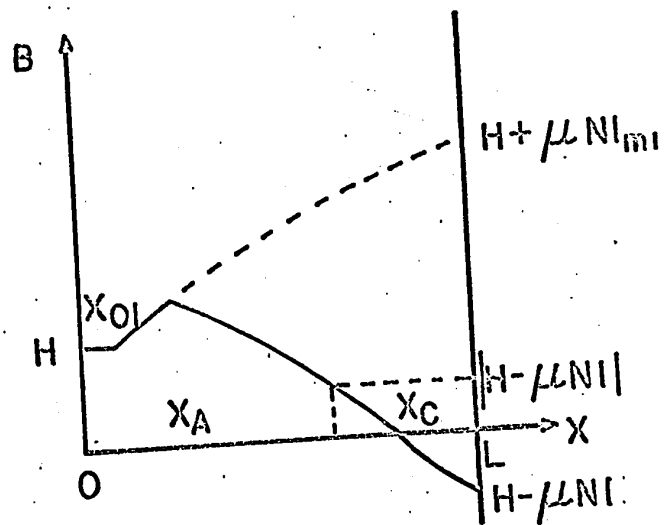
1



2

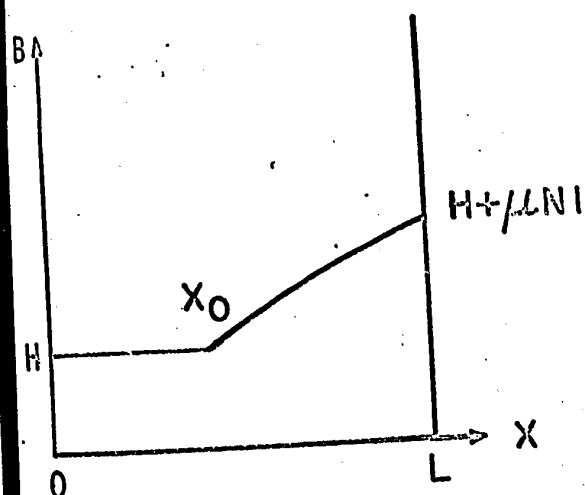


3A

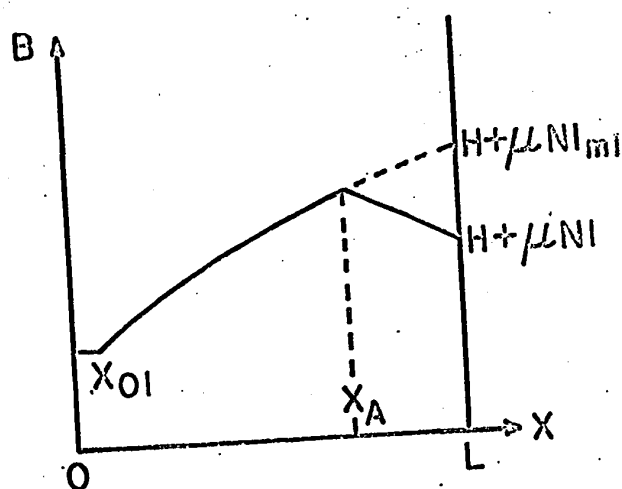


3B

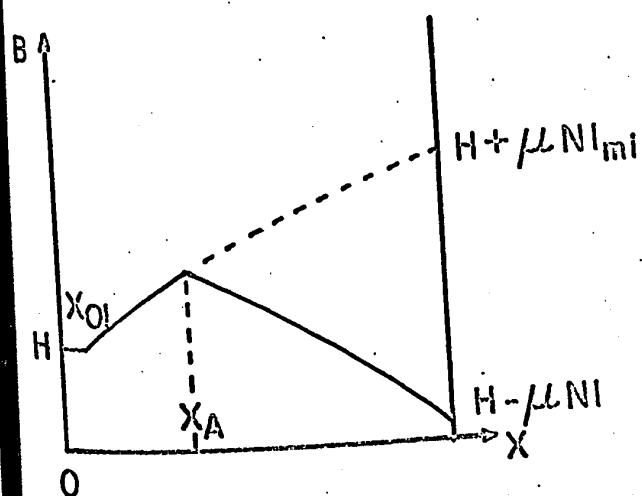
figures AA



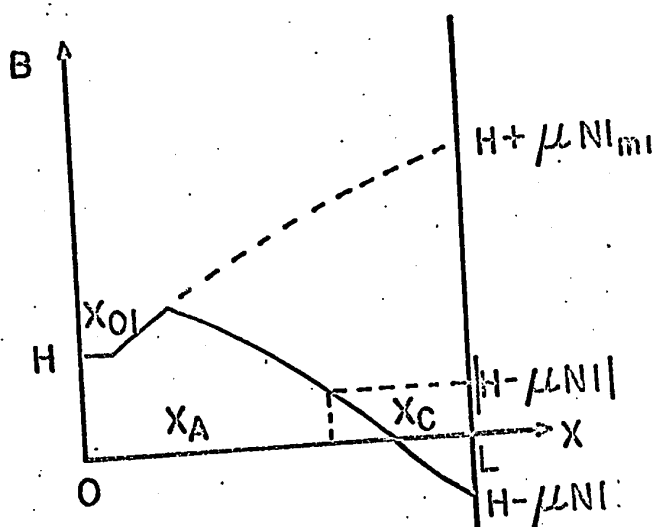
1



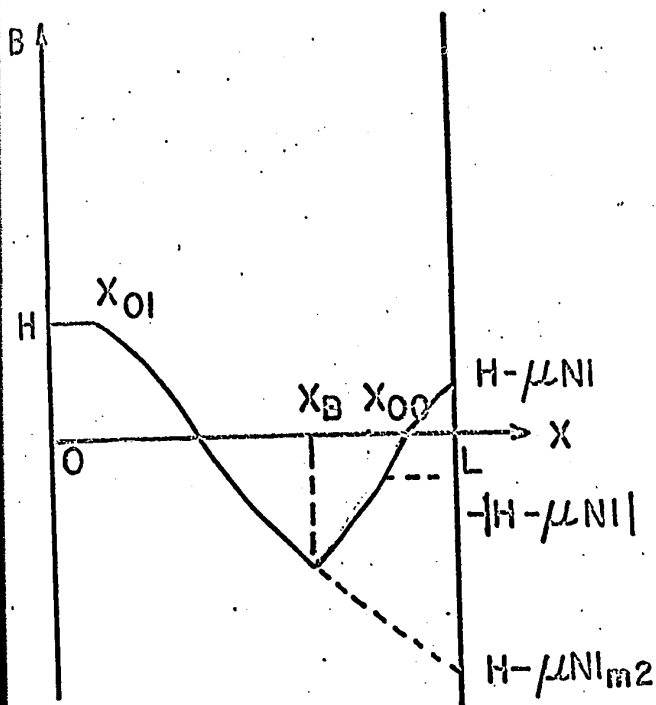
2



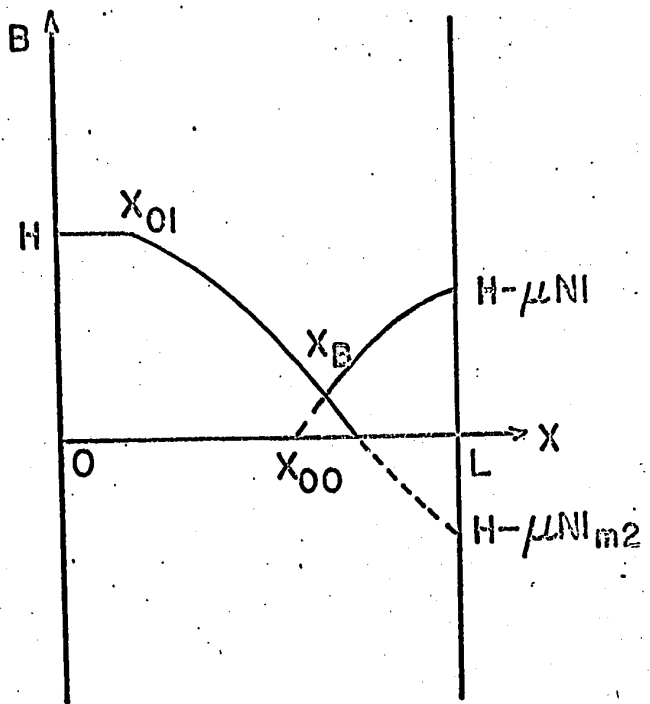
3A



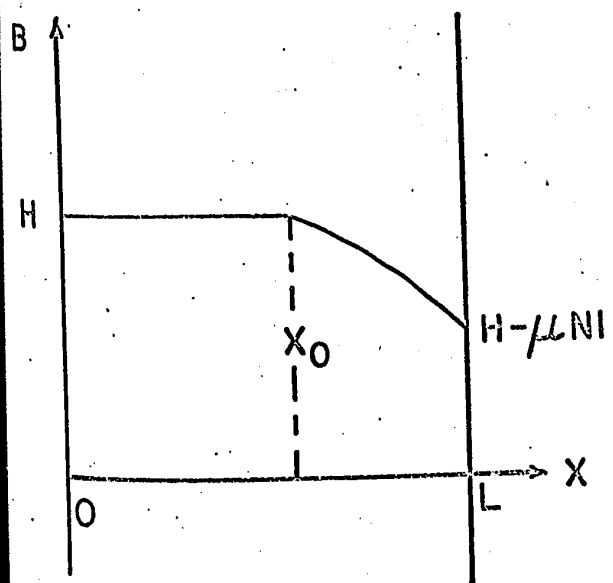
3B



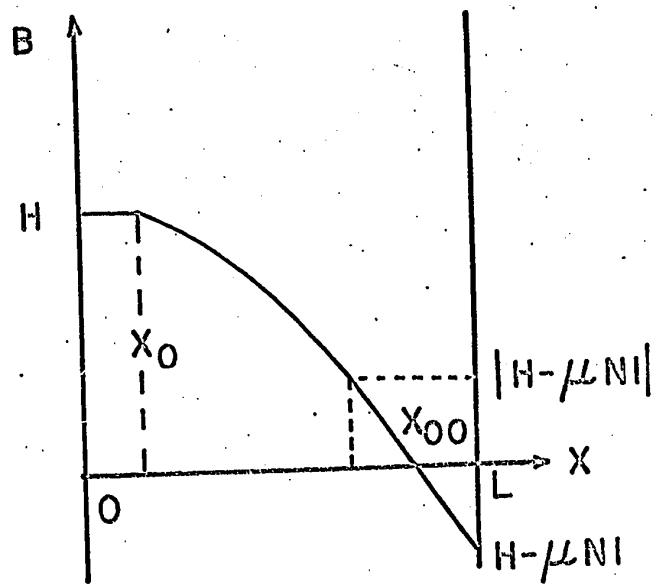
4 C



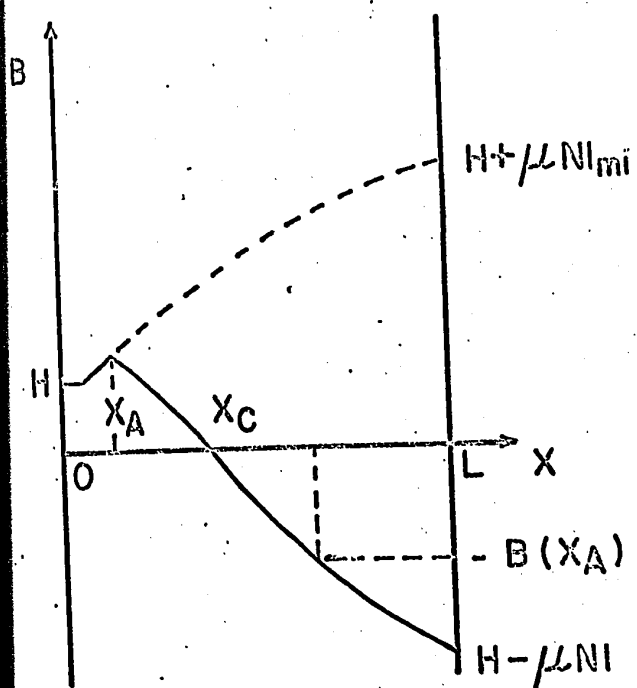
4 D



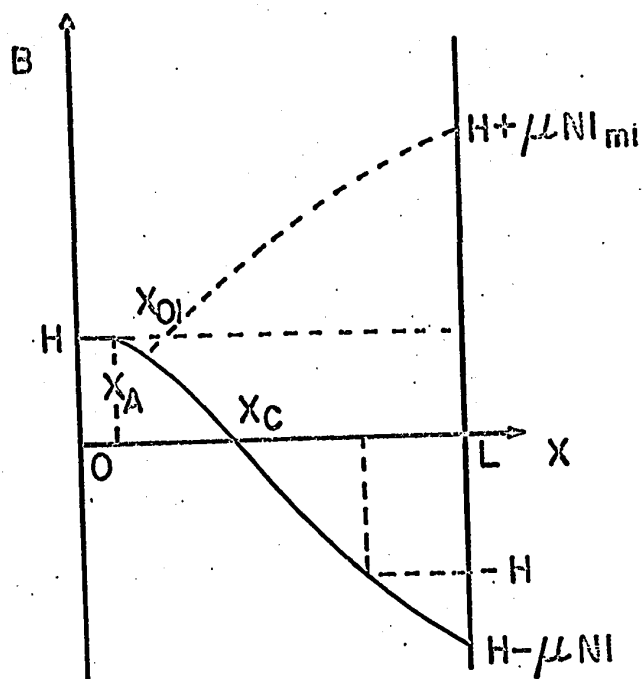
5A



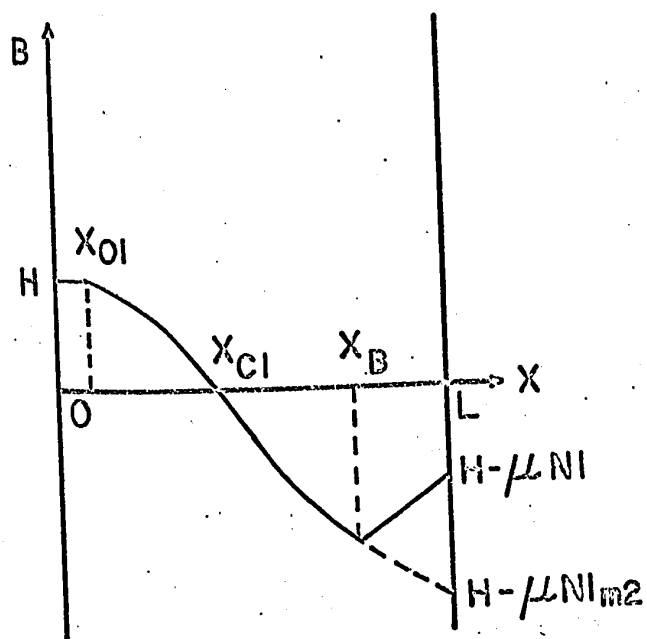
5 B



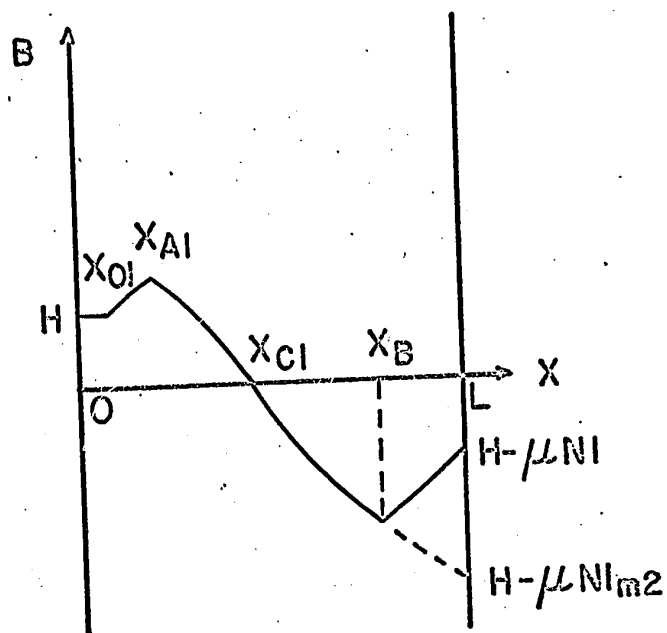
3 C



3 D



4 A



4 B

or

$$\frac{x_0}{L} = 1 - \frac{1}{(2-n)\alpha L} [(H + \mu NI)^{2-n} - H^{2-n}] \quad (6)$$

whence

$$\begin{aligned} 4\pi M &= \langle B \rangle - H \\ &= \frac{1}{(3-n)\alpha L} [(H + \mu NI)^{3-n} - H^{3-n}] \\ &\quad - \frac{H}{(2-n)\alpha L} [(H + \mu NI)^{2-n} - H^{2-n}] \end{aligned} \quad (7)$$

Note that (7) can be used for calculating $4\pi M$ at I_c where I is replaced by I_c ; however, it is generally assumed that, on this picture the critical profile occurs when $x_0 = 0$. In that case

$$4\pi M = \frac{1}{(3-n)\alpha L} [(H + \mu NI)^{3-n} - H^{3-n}] - H \quad (8)$$

And although this is not strictly true as mentioned in the main text, it is true for the weak cross section.

From AA2 one can next deduce that

$$\begin{aligned} \langle B \rangle &= \frac{1}{(3-n)\alpha L} [2(B(x_A))^{3-n} - (H + \mu NI)^{3-n}] + (H + \mu NI)^{3-n} \\ &\quad - H^{3-n} - \frac{x_0}{L} H \end{aligned} \quad (9)$$

where $B(x_A)$ is the intersection of the critical curves with initial conditions $H + \mu NI_{m1}$ and a positive gradient, where I_{m1} is the maximum value of the current in the initial direction, and $H + \mu NI$ with a negative gradient; x_{01} is the value of x_0 when $I = I_{m1}$

Thus from (5)

$$(H + \mu NI_{m1})^{2-n} - B(x_A)^{2-n} = (2-n) \alpha (L - x_A) \quad i)$$

$$(H + \mu NI)^{2-n} - B(x_A)^{2-n} = -(2-n) \alpha (L - x_A) \quad ii)$$

adding these one obtains

$$B(x_A) = \{ 1/2[(H + \mu NI_{m1})^{2-n} + (H + \mu NI)^{2-n}] \}^{\frac{1}{2-n}}$$

From (6)

$$\frac{x_{01}}{L} = 1 - \frac{1}{(2-n)\alpha L} [(H + \mu NI_{m1})^{2-n} - H^{2-n}]$$

Combining these with (9) the value of $4\pi M$ is

$$4\pi M = \frac{1}{(3-n)\alpha L} [2\{1/2[H + \mu NI_{m1}]^{2-n} + (H + \mu NI)^{2-n}\}]^{\frac{3-n}{2-n}} - (H + \mu NI)^{3-n} - H^{3-n} - \frac{H}{(2-n)\alpha L} [(H + \mu NI_{m1})^{2-n} - H^{2-n}] \quad (10)$$

When I goes negative so that $H - \mu NI < H$ as shown in figure AA 3B the calculation is as follows

$$\begin{aligned} \langle B \rangle &= \frac{1}{(3-n)\alpha L} [2B(x_A)^{3-n} - |H - \mu NI|^{3-n} - H^{3-n}] \\ &\quad + \frac{x_0}{L} H \end{aligned} \quad (11)$$

since equal areas below and above the line $H=0$ cancel. AA3-C shows that this value of $\langle B \rangle$ is valid even when the effective area is below the line for then

$$\begin{aligned} \langle B \rangle &= \frac{1}{(3-n)\alpha L} [B(x_A)^{3-n} - H^{3-n} - \{ |H - \mu NI|^{3-n} - |B(x_A)|^{3-n} \}] \\ &= \frac{1}{(3-n)\alpha L} [2B(x_A)^{3-n} - H^{3-n} - |H - \mu NI|^{3-n}] \end{aligned}$$

Thus it remains to find $B(x_A)$

From (5) as before

$$|H - \mu NI|^{2-n} = (2-n)\alpha(L - x_C) \quad \text{i)}$$

$$B(x_A)^{2-n} = -(2-n)\alpha(x_A - x_C) \quad \text{ii)}$$

$$(H \pm \mu NI_{ml})^{2-n} - B(x_A)^{2-n} = (2-n)\alpha(L - x_A) \quad \text{iii)}$$

Adding i, and ii, and subtracting iii, one obtains:

$$2B(X_A)^{2-n} - (H + \mu NI_{m1})^{2-n} + |H - \mu NI|^{2-n} = 0$$

$$B(X_A) = \{1/2[H + \mu NI_{m1}]^{2-n} - |H - \mu NI|^{2-n}\} \frac{1}{2-n}$$

Whence combining with (11)

$$\begin{aligned} 4\pi M &= \frac{1}{(3-n)} \alpha L \left[2 \frac{1}{2} [(H + \mu NI_{m1})^{2-n} - |H - \mu NI|^{2-n}] \frac{3-n}{2-n} \right. \\ &\quad \left. - |H - \mu NI|^{3-n} - H^{3-n} \right] \\ &\quad - \frac{H}{(2-n) \alpha L} [(H + \mu NI_{m1})^{2-n} - H^{2-n}] \end{aligned} \quad (12)$$

Figure 3D shows the possibility that $B(X_{01}) > H$ in which case $B(X_{01}) = H$ is of the only interest whence

$$4\pi M = \frac{1}{(3-n)} \alpha L [H^{3-n} - |(H - \mu NI)|^{3-n}] - (1 - \frac{X_{01}}{L}) H$$

From (5)

$$H^{2-n} = (2-n) \alpha L \left(\frac{X_{C1} - X_{01}}{L} \right)$$

$$|H - \mu NI|^{2-n} = (2-n) \alpha L \left(1 - \frac{X_{C1}}{L} \right)$$

$$1 - \frac{X_{01}}{L} = \frac{H^{2-n} + |H - \mu NI|^{2-n}}{(2-n) \alpha L}$$

Whence

$$4\pi M = \frac{1}{(3-n)\alpha L} [H^{3-n} - |H - \mu NI|^{3-n}] - \frac{H}{(2-n)\alpha L} [H^{2-n} + |H - \mu NI|^{2-n}] \quad (13)$$

Next on the sequence of profiles is figure 4A or 4B depending on whether 3C or 3D was reached by $H - \mu NI_{m2}$.

For 4A

$$\begin{aligned} 4\pi M &= \frac{1}{3-n\alpha L} [-2x (B(X_B))^{3-n} - |H - \mu NI|^{3-n} + H^{3-n} \\ &\quad - |H - \mu NI|^{3-n}] - (1 - \frac{x_{01}}{L})H \\ &= \frac{1}{(3-n)\alpha L} \{ |H - \mu NI|^{3-n} + H^{3-n} - 2B(X_B)^{3-n} \} \\ &\quad - \frac{H}{(2-n)\alpha L} [H^{2-n} + |H - \mu NI_{m2}|^{2-n}] \end{aligned}$$

Since x_{01} is now a constant determined by I_{m2} in (13).

For $B(X_B)$ from (5)

$$\begin{aligned} B(X_B)^{2-n} - |H - \mu NI|^{2-n} &= (2-n)\alpha L (1 - \frac{x_B}{L}) \\ |H - \mu NI_{m2}|^{2-n} - B(X_B)^{2-n} &= (2-n)\alpha L (1 - \frac{x_B}{L}) \\ B(X_B)^{2-n} &= \frac{1}{2} [|H - \mu NI|^{2-n} + |H - \mu NI_{m2}|^{2-n}] \end{aligned}$$

Thus

$$\begin{aligned}
 4\pi M &= \frac{1}{(3-n)\alpha L} \left[2 \left\{ \frac{1}{2} \left[|H - \mu NI|^{2-n} + |H - \mu NI_{m2}|^{2-n} \right] \right\}^{\frac{3-n}{2-n}} \right. \\
 &\quad \left. - |H - \mu NI|^{3-n} - H^{3-n} \right] \\
 &\quad - \frac{H}{(2-n)\alpha L} \left[H^{2-n} + |H - \mu NI_{m2}|^{2-n} \right] \quad (14)
 \end{aligned}$$

For the case 4B the magnetization is slightly more complicated.

To (14) the term

$$\frac{2}{(3-n)\alpha L} (B^{3-n}(\chi_{01}) - H^{3-n})$$

must be added where $B(\chi_{01})$ is found from (12) whence,

$$\begin{aligned}
 4\pi M &= \frac{1}{(3-n)\alpha L} \left[2 \left\{ \left[\frac{1}{2} \left(|H - \mu NI|^{2-n} + |H - \mu NI_{m2}|^{2-n} \right) \right]^{\frac{3-n}{2-n}} \right. \right. \\
 &\quad \left. \left. + \left[\frac{1}{2} \left(|H + \mu NI_{m1}|^{2-n} - |H - \mu NI_{m2}|^{2-n} \right) \right]^{\frac{3-n}{2-n}} \right\} \right. \\
 &\quad \left. - 3 H^{3-n} \right] \\
 &\quad - \frac{H}{(2-n)\alpha L} \left[H^{2-n} + |H - \mu NI_{m2}|^{2-n} \right] \quad (15)
 \end{aligned}$$

When $H - \mu NI$ is again greater than zero as depicted in Figure 4C

$B(x_B)$ must be redefined again from (5)

$$(H - \mu NI)^{2-n} = (2-n) \alpha L \left(1 - \frac{x_{00}}{L}\right) \quad i)$$

$$B(x_B)^{2-n} = (2-n) \alpha L \left(\frac{x_{00} - x_B}{L}\right) \quad ii)$$

$$|H - \mu NI_{m2}|^{2-n} - B(x_B)^{2-n} = (2-n) \alpha L \left(1 - \frac{x_B}{L}\right) \quad iii)$$

Combining the three find

$$B(x_B) = \left| \frac{1}{2} \left[|H - \mu NI_{m2}|^{2-n} - (H - \mu NI)^{2-n} \right] \right|^{\frac{1}{2-n}}$$

Whence for the case corresponding to 4A

$$4\pi M = \frac{1}{(3-n) \alpha L} \left[2 \left| \frac{1}{2} \left[|H - \mu NI_{m2}|^{2-n} - (H - \mu NI)^{2-n} \right] \right|^{\frac{3-n}{2-n}} \right.$$

$$\left. - |H - \mu NI|^{3-n} - H^{3-n} \right]$$

$$- \frac{H}{(2-n) \alpha L} \left[H^{2-n} + |H - \mu NI_{m2}|^{2-n} \right]$$

(16)

And for the case corresponding to 4B

$$\begin{aligned}
 4\pi M = & \frac{1}{(3-n)\alpha L} \left[2 \left| \frac{1}{2} \left[|H - \mu N I_{m2}|^{2-n} - (H - \mu N I)^{2-n} \right] \right|^{\frac{3-n}{2-n}} \right. \\
 & + 2 \left| \frac{1}{2} \left[|H + \mu N I_{m1}|^{2-n} - |H - \mu N I_{m2}|^{2-n} \right] \right|^{\frac{3-n}{2-n}} - 3H^{3-n} \Big] \\
 & - \frac{H}{(2-n)\alpha L} \left[H^{2-n} + |H - \mu N I_{m2}|^{2-n} \right]
 \end{aligned} \tag{17}$$

Figure D-4 depicts the situation when $B(X_B) > 0$.

Here

$$(H - \mu N I)^{2-n} - B(X_B)^{2-n} = (2-n)\alpha L \left(1 - \frac{X_B}{L} \right)$$

$$|H - \mu N I_{m2}|^{2-n} = (2-n)\alpha L \left(1 - \frac{X_{00}}{L} \right)$$

$$\frac{2-n}{B(X_B)} = (2-n)\alpha L \frac{(X_{00} - X_B)}{L}$$

Combining the three gives the same equation for $B(X_B)$ namely

$$B(X_B) = \left| \frac{1}{2} \left[(H - \mu N I)^{2-n} - (H - \mu N I_{m2})^{2-n} \right] \right|^{\frac{1}{2-n}}$$

and

$$\begin{aligned}
 4\pi M &= \frac{1}{(3-n)\alpha_L} [H^{3-n} - 2B(X_B)^{3-n} + (H - \mu NI)^{3-n}] - \frac{(1-X_0)H}{L} \\
 &= \frac{1}{(3-n)\alpha_L} [H^{3-n} + (H - \mu NI)^{3-n-2} \frac{1}{2} \{ (H - \mu NI)^{2-n} - |H - \mu NI_{m2}|^{2-n} \}] \\
 &\quad - \frac{H}{(2-n)\alpha_L} [H^{2-n} + |H - \mu NI_{m2}|^{2-n}] \quad (18)
 \end{aligned}$$

for case corresponding to 4A

$$\begin{aligned}
 4\pi M &= \frac{1}{(3-n)\alpha_L} [H^{3-n} + (H - \mu NI)^{3-n-2} \frac{1}{2} \{ (H - \mu NI)^{2-n} - |H - \mu NI_{m2}|^{2-n} \}^{\frac{3-n}{2-n}}] \\
 &\quad + 2 \left[\frac{1}{2} (|H + \mu NI_{m1}|^{2-n} - |H - \mu NI_{m2}|^{2-n})^{\frac{3-n}{2-n}} - 3H^{3-n} \right] \\
 &\quad - \frac{H}{(2-n)\alpha_L} [H^{2-n} + |H - \mu NI_{m2}|^{2-n}] \quad (19)
 \end{aligned}$$

for the case corresponding to 4B

Now 16 or 17, or 18 or 19 can be used till I changes sign until it again reaches its maximum value I_{m1} .

One more distinct case must be considered, that of initial direction I opposing H

For Figure 5A

$$4\pi M = \frac{1}{(3-n)\alpha_L} [H^{3-n} - |H - \mu NI|^{3-n}] - \frac{H}{(2-n)\alpha_L} [H^{2-n} - |H - \mu NI|^{2-n}] \quad (20)$$

and for 5B

$$4\pi M = \frac{1}{(3-n)\alpha_L} [H^{3-n} - |H - \mu NI|^{3-n}] - \frac{H}{(2-n)\alpha_L} [H^{2-n} + |H - \mu NI|^{2-n}] \quad (21)$$

as these are analogous to 11 and 13 respectively.

whence at $I_{\max 2}$ the formula 20 or 21 gives $4\pi M$ at I_c opposing.

When $\pm I$ is again decreased (in magnitude) the formulas are given by

4A and its ensuing cases, that is by equations 14, 16, 18.

Appendix B

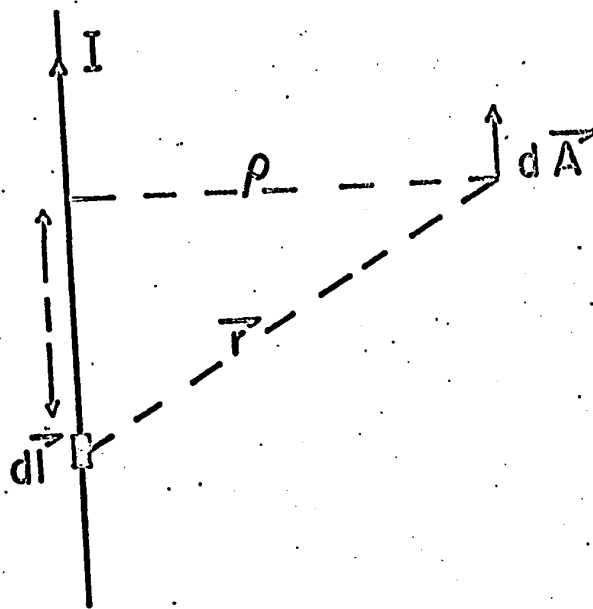
"Concentric Solenoids"

The vector potential $\vec{A}$ for a straight wire of length $2b$ at a distance $\rho \ll L$ from the wire can be found from the equation

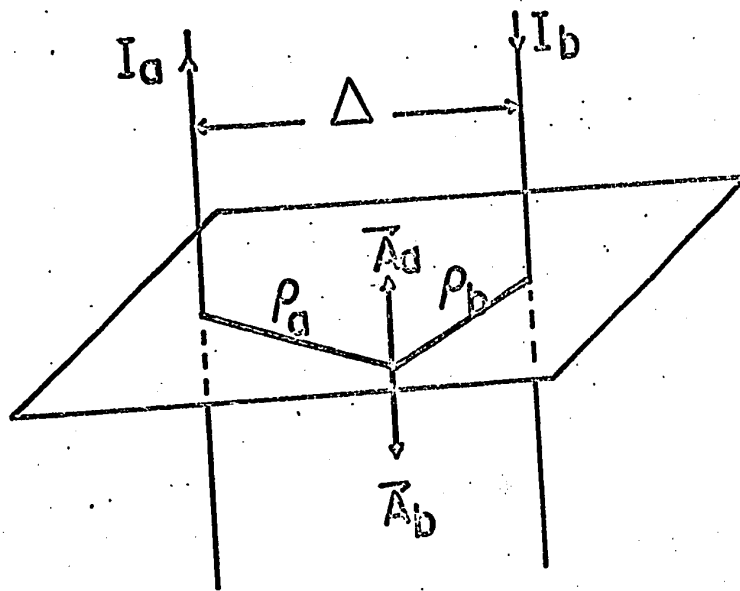
$$\begin{aligned} d\vec{A} &= \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell}}{r} \\ \vec{A} &= A_z \vec{k}; \text{ if } \vec{H} \text{ is assumed to be along the } z \text{ axis} \\ A_z &= \frac{\mu_0 I}{4\pi} \int_{-L}^L \frac{d\ell}{r} = \frac{\mu_0 I}{4\pi} \int_{-L}^L \frac{d\ell}{\rho^2 + \ell^2} \\ &= \frac{\mu_0 I}{4\pi} \ln [\ell + (\rho^2 + \ell^2)^{1/2}] \Big|_{-L}^L \\ &\approx \frac{\mu_0 I}{4\pi} \ln \left[1 + \frac{4L^2}{\rho^2} \right] \end{aligned}$$

For 2 wires carrying opposite signs of currents

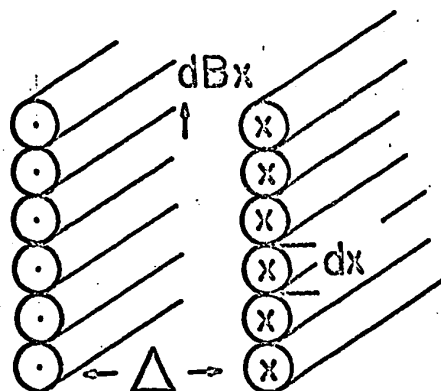
$$\begin{aligned} I_a &= -I_b \\ A_z &= A_{za} - A_{zb} \\ &= \frac{\mu_0 I}{4\pi} \ln \left(1 + \frac{4L^2}{\rho_a^2} \right) - \frac{\mu_0 I}{4\pi} \ln \left(1 + \frac{4L^2}{\rho_b^2} \right) \\ &= \frac{\mu_0 I}{4\pi} \ln \left(1 + \frac{4L^2}{\rho_a^2} \times \frac{1}{1 + \frac{4L^2}{\rho_b^2}} \right) \end{aligned}$$



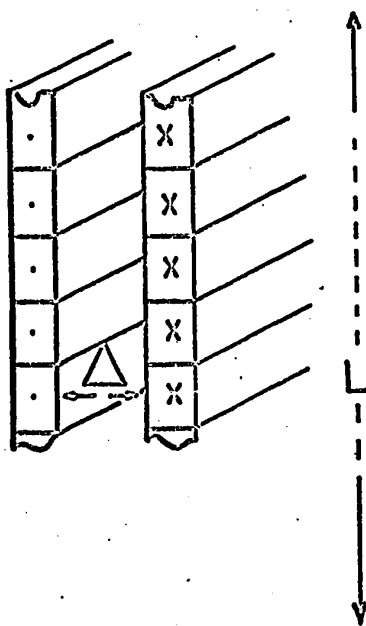
Vector Potential $\vec{A}$ for a Single Wire
Carrying Current I



Vector Potential for Pair of Wires
Carrying Opposite Currents



Cross section of infinite sheet of pairs of infinite wires carrying equal and opposite currents.



Sample cross section; $\Delta \ll R$, R is the radius of the solenoidal sample of length L .

$$= \frac{\mu_0 I}{4\pi} \ln \left(\frac{\rho_b^2}{\rho_a^2} \right) \left(\frac{\rho_a^2 + 4L^2}{\rho_b^2 + 4L^2} \right)$$

$$A_z = \frac{\mu_0 I}{4\pi} \ln \frac{\rho_a^2}{\rho_b^2}$$

whence one can compute B component wise

$$B_x = \frac{\partial A_z}{\partial y} \quad B_y = - \frac{\partial A_z}{\partial x} \quad B_z = 0$$

again

$$A_z = \frac{\mu_0 I}{4\pi} \ln \left(\frac{x^2 + (\Delta - y)^2}{x^2 + y^2} \right)$$

where Δ is the separation
of the wires

$$B_x = \frac{\mu_0 I}{2\pi} \left(- \frac{\Delta - y}{\rho_b^2} - \frac{y}{\rho_a^2} \right)$$

$$B_y = \frac{\mu_0 I}{2\pi} \left(\frac{x}{\rho_b^2} - \frac{x}{\rho_a^2} \right) = \frac{\mu_0 I x}{2\pi} \frac{(\rho_a - \rho_b)(\rho_a + \rho_b)}{(\rho_a^2) \rho_b^2}$$

When $\rho_a \sim \rho_b$ i.e. $\rho_a - \rho_b \rightarrow 0$ $B_y = 0$ as is the case
in these concentric solenoids.

One is left with B_x . Essentially one has N pairs of "wires"
as above, so that, as each carries a current element $dI = \frac{NI}{L} dx$ if it
is quasi continuous (see diagram on page 18).

then

$$dB_x = -\frac{\mu_0}{2\pi} \left(\frac{\Delta-y}{\rho_b^2} + \frac{y}{\rho_a^2} \right) dI$$

$$= -\frac{\mu_0 NI}{2\pi L} \left(\frac{\Delta-y}{\rho_b^2} + \frac{y}{\rho_a^2} \right) dx$$

L is the length of the solenoid

$$B_x = -\frac{\mu_0 NI}{2\pi L} \int_{-L/2}^{L/2} \left(\frac{\Delta-y}{\rho_b^2} + \frac{y}{\rho_a^2} \right) dx$$

Consider the integral $I_1 = \int_{-L/2}^{L/2} \frac{\Delta-y}{\rho_b^2} dx = \int_{-L/2}^{L/2} \frac{\Delta-y}{x^2+y^2} dx$

Now $\int \frac{dx}{x^2+y^2} = \frac{1}{y} \tan^{-1} \frac{x}{y}$

$$\begin{aligned}
I_1 &= \int_{-L/2}^{L/2} \frac{\Delta-y}{x^2+(\Delta-y)^2} dx \\
&= \frac{\Delta-y}{\Delta-y} \tan^{-1} \frac{x}{\Delta-y} \Big|_{-L/2}^{L/2} \\
&= \tan^{-1} \left(\frac{L/2}{\Delta-y} \right) - \tan^{-1} \left(\frac{-L/2}{\Delta-y} \right) \\
I_2 &= \int_{-L/2}^{L/2} \frac{y}{x^2+y^2} dx = \frac{y}{y} \tan^{-1} \left(\frac{x}{y} \right) \Big|_{-L/2}^{L/2} \\
&= \tan^{-1} (L/2y) - \tan^{-1} (-L/2y)
\end{aligned}$$

Therefore

$$\begin{aligned}
B_x &= \frac{\mu_0 NI}{2\pi L} \left[\tan^{-1} (L/2 / \Delta-y) - \tan^{-1} (-L/2 / (\Delta-y)) \right. \\
&\quad \left. + \tan^{-1} \left(\frac{L}{2y} \right) - \tan^{-1} \left(-\frac{L}{2y} \right) \right]
\end{aligned}$$

Now of interest y has a max. value of Δ and $L/2 \gg \Delta$

so that

$$B_x \sim \frac{\mu_0 NI}{2\pi L} \left[\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} \right]$$

$$= \mu_0 n I \quad \text{where } n = N/L \text{ is the turns/unit length.}$$

That is B_x in the center.

One can write a general expression for $B(x)$ from p.(21),

I_1 and I_2

$$B(x) = \frac{\mu_0 NI}{2\pi L} \left(\tan^{-1} \frac{x}{y} + \tan^{-1} \frac{x}{\Delta - y} + c \right) \quad (17-1)$$

when $x = 0$, $B(x) = \frac{\mu_0 NI}{2L} \therefore C = \pi$

(i.e. if the integration is between 0, L)

Furthermore since Δ is small ($\Delta_{\max} = .01''$) and $y_{\max} = \Delta$
 $y_{\min} = 0$ we shall use the special position $y = \Delta/2$, that is in
 the center of the gap. Note that (17-1) sort of "averages" the
 value of B over y .

$$B(x) = \frac{\mu_0 NI}{2\pi L} \left(2 \tan^{-1} \left(\frac{2x}{\Delta} + \pi \right) \right)$$

Now consider $B(.1L)$ that is $B, 1/10$ of the way from the end
 ($L \sim 2''$)

$$B(.1L) = \frac{\mu_0 NI}{2\pi L} \left(2 \tan^{-1} \left(\frac{.2}{.01} \right) + \pi \right)$$

$$\text{Now } \tan (.48) \sim 15$$

Substituting this above

$$B(L/10) \geq \frac{\mu_0 NI}{2\pi L} (1.96\pi) = \frac{\mu_0 NI}{L} (.98)$$

So that within 1/10 of the length of the solenoid B attains more than 98% of its value approximated by $\frac{\mu_0 NI}{L} = \mu_0 nI$

From (17-1) One can also see that B is uniform across the gap

$$B \text{ at the surface on the left is } \frac{\mu_0 NI}{2\pi L} (\tan^{-1}(\frac{x}{\Delta}) + c + \frac{\pi}{2}) = B_L$$

$$B_R = \frac{\mu_0 NI}{2\pi L} (\frac{\pi}{2} + c + \tan^{-1}(\frac{x}{\Delta})) \quad \text{on the right}$$

$$B_c = \frac{\mu_0 NI}{2\pi L} (c + 2 \tan^{-1}(\frac{2L}{\Delta})) \quad \text{in the center}$$

$$\text{and since for all approximations } \tan^{-1}(\frac{L}{\Delta}) \sim \tan^{-1}(\frac{2L}{\Delta}) = \frac{\pi}{2}$$

B is uniform across.

Appendix C

INTEGRATED SIGNALS

It shall be briefly demonstrated that the integrated signal produced by the pickup coils, as shown in schematic diagram (A-1), is directly proportional to the magnetization of the sample.

Letting N_i , A_i , and N_o , A_o be the number of turns and the area seen by the inner and outer pickup coils, from Faraday's induction law one can write

$$\left| \int V dt \right| = \left| - \int N A dB \right| = |N_i A_i - N_o A_o| \Delta B \quad (1)$$

The turn areas are so chosen that when $\Delta M = 0$, $\left| \int V dt \right| = 0$, that is $\Delta B = \Delta H$

$$(N_i A_i - N_o A_o) \Delta B = (N_i A_i - N_o A_o) \Delta H = 0 \quad (2)$$

If the magnetization of the sample is non zero, then since only the outer coil sees the sample, (1) is written as

$$\left| \int V dt \right| = |N_o A_o \Delta B - N_i A_i \Delta H| \quad (3)$$

Whence combining with (2) one gets

$$\left| \int V dt \right| = |N_o A_o \Delta(B-H)| = |N_o A_o \Delta(4\pi M)| \quad (4)$$

If the pickup coil balance is not ideal, (2) is written as

$$\begin{aligned} (N_o A_o - N_i A_i) \Delta H &= \pm \delta \\ N_i A_i &= N_o A_o \mp \delta / \Delta H \end{aligned} \quad (5)$$

So that (4) becomes

$$|\int v dt| = |N_o A_o (4\pi M)| \pm \delta \quad (6)$$

From (5) it is clear that δ depends only on H and is linear with H , consequently it can be easily eliminated electronically by feeding a signal directly proportional to the solenoid current $I = \frac{H}{\mu N}$, equal and of opposite sign to δ , into the integrator until condition (2) is satisfied.

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