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ET POSTDOCTORALES

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**STOCHASTIC ANALYSIS OF
ROBOT-SAFETY SYSTEMS
WITH COMMON-CAUSE FAILURES**

Zhijian Li

A thesis submitted to the Faculty of Graduate and Postdoctoral Studies in partial fulfillment of the requirements for the degree of

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I dedicate this thesis to my beloved father, Qifeng Li.

ABSTRACT

This study presents reliability and availability analyses of four different types of robot-safety systems with common-cause failures. The system failure rates and the partially failed system repair rates are assumed constant, and the failed system repair time is assumed arbitrarily distributed. Markov and the supplementary variable methods were used to perform mathematical analysis of these models. Generalized expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure are developed. The models developed in this study can be applied to their corresponding robot-safety systems to predict robot-safety system reliability and availability, and to prepare appropriate maintenance scheduling policies.

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1.1 Background

The history of robots can be traced back to the fourth century B.C. in which Aristotle presented the idea of the functional robot and stated: “ If every instrument could accomplish its own work, obeying or anticipating the will of the others...”[88, 97]. In 1941, science fiction writer Isaac Asimov first used the word robotics to describe the technology of robots and predicted the rise of a powerful robot industry. In the following year (1942), Asimov wrote “Runaround”, a story about robots, which contained the “Three Laws of Robotics” [89]:

- A robot may not injure a human nor, through inaction, allow a human being to come to harm.
- A robot must obey the orders from humans except when such orders conflict with the First Law.
- A robot must protect its own existence as long as such protection does not conflict with the First or Second Law.

There are many definitions of the word “ROBOT”. For examples, Oxford English Dictionary defines robot as an apparently human automaton, intelligent but impersonal machine, while Webster English dictionary defines robot as a mechanism guided by automatic controls. However, the following two definitions are widely used in the industrial sector [90, 91]:

- An all purpose machine equipped with a memory device and terminal, capable of rotation and of replacing human labor by automatic performance of movements.
- A programmable, multifunctional manipulator designed to move materials, parts, tools, or specialized devices through various programmed motions for the performance of a variety of tasks.

In 1960 there were probably not more than a few hundred robots throughout the world. Since then the robot population has grown at a remarkable rate. By 1983 it reached around 30,000 [92]. According to the International Federation of Robotics (IFR) and the United Nations Economic Commission for Europe (UN/ECE) the worldwide industrial robot population increased to 350,000 in 1987 and 710,000 in 1997 [93, 94]. A conservative forecast for industrial robots for 2005 is 965,000 [95]. Figures 1.1 and 1.2 present a histogram for the population growth of the robots and percentage breakdowns of robot world population for various countries, respectively.

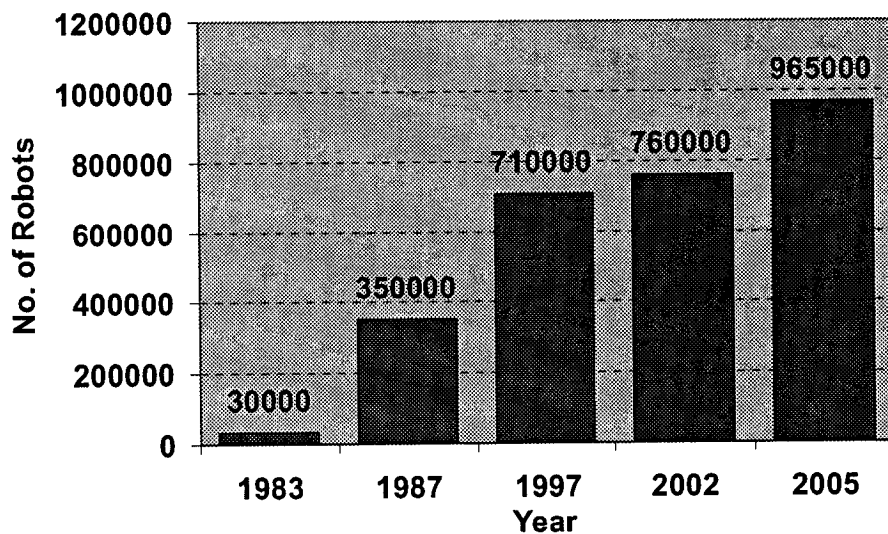


Figure 1.1 The population growth of industrial robots

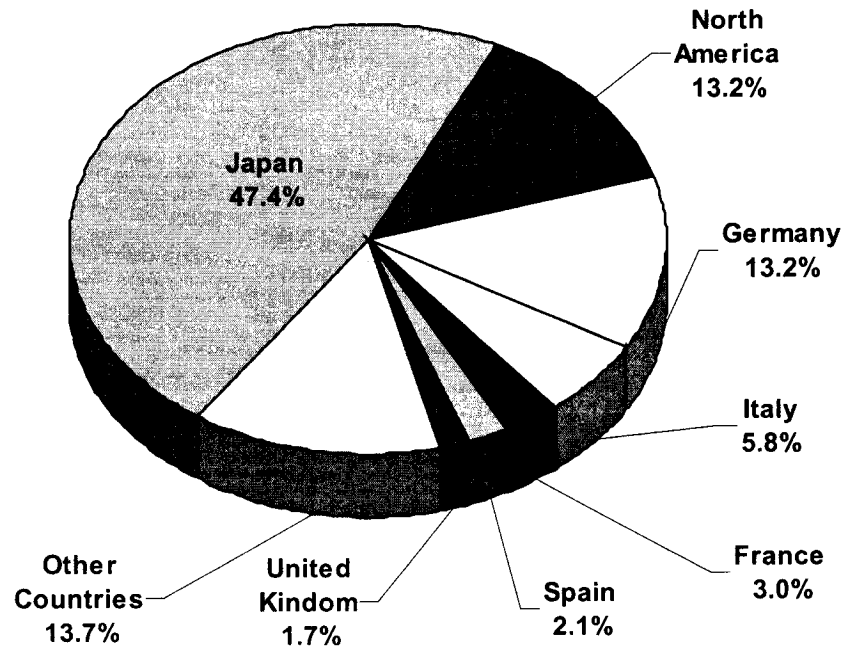


Figure 1.2 World population breakdowns of industrial robots for the year 2002

Today robots are used in various sectors of industry for purposes such as arc or spot welding, underwater exploration, outer space exploration, fire fighting, medical aid, and relieving humans from performing hazardous tasks. The ever-increasing number of robots and the large number of serious robot-related accidents indicate that safety is still the prime concern in the design, installation, operation and maintenance of robots [96, 97]. Nonetheless, a robot must be safe and reliable. An unreliable robot may lead to unsafe conditions, high maintenance costs, inconvenience, etc. Over the years, a vast number of publications on robot reliability and safety have appeared. In 1991, Dhillon listed over 460 publications on robot reliability and safety in his book entitled, “Robot reliability and safety” [97]. For the period 1973 to 1997 a comprehensive review of published literature on robot reliability and safety is presented in Ref. [87].

1.2 Literature Review: Robot Reliability and Safety

This section presents a review of published literature on robot reliability and safety for the period covering 1997-2002 [1-86]. The sources of these publications are presenting in Table 1.1.

Table 1.1: The sources of 86 publications reviewed

<u>No.</u>	<u>Source (Journal name / Conference proceedings)</u>
1	Journal of Robotic Systems
2	Proceedings of the IEEE International Symposium on Assembly and Task Planning
3	Proceedings of the IEEE International Conference on Robotics and Automation
4	IEEE Transactions on Reliability
5	Reliability Engineering & System Safety
6	Advanced Robotics
7	Accident Analysis & Prevention
8	Proceedings of the 2001 International Symposium on Micromechatronics and Human Science
9	Proceedings of the 2000 International Symposium on Micromechatronics and Human Science
10	Compliance Engineering
11	International Journal of Human Factors in Manufacturing
12	CIRP Annals
13	IEEE Transactions on Systems, Man & Cybernetics
14	Proceedings of the Annual International Conference of the IEEE on Engineering in Medicine and Biology

- 15 Operations Research Letters
- 16 Proceedings of the IEEE International Conference on Intelligent Robots
and Systems
- 17 IEEE Transactions on Information Technology in Biomedicine
- 18 Journal of Intelligent Manufacturing
- 19 Journal of Guidance Control & Dynamics
- 20 Proceedings of the American Control Conference
- 21 International Journal of Robotics Research
- 22 Journal of Quality in Maintenance Engineering
- 23 Robotics & Autonomous systems
- 24 Journal of Infrastructure Systems
- 25 Proceedings of the IEEE International Conference on Systems, Man and
Cybernetics
- 26 Proceedings of the Annual Reliability and Maintainability Symposium
- 27 IEEE/ASME Transactions on Mechatronics
- 28 Proceedings of the First Joint BMES/EMBS Conference
- 29 Journal of Intelligent and Robotic Systems
- 30 IEEE Transactions on Robotics and Automation
- 31 Proceedings of the International Conference on Human Interfaces in
Control Rooms, Cockpits and Command Centres
- 32 International Journal of Modelling & Simulation
- 32 Robotica
- 33 Robotics World
- 34 Proceedings of the IEEE International Conference on Control
Applications
- 35 Mathematical Methods in the Applied Sciences

36	International Journal of Industrial Ergonomics
37	Proceedings of the International Workshop on Advanced Motion Control
38	The Industrial Robot
39	Chinese Journal of Mechanical Engineering (English Edition)
40	Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics
41	Proceedings of the IEEE International Symposium on Industrial Electronics
42	Proceedings of the IEEE International Workshop on Robot and Human Communication
43	Proceedings of the IEEE International Conference on Intelligent Engineering Systems
44	Proceedings of the International Conference on Nuclear Engineering
45	International Journal of Systems Science
46	Microelectronics & Reliability
47	Proceedings of the ANS 7th Topical Meeting on Robotics and Remote Systems
48	Journal of Robotics and Mechatronics

1.2.1 Classification of the Literature

The publications on the subject are classified into three broad categories: (i) robot reliability, (ii) robot safety, and (iii) miscellaneous. The category of robot reliability is divided further into three subcategories: methods, evaluation, and models to assess overall robot system reliability. Similarly, the robot safety category is subdivided into classifications such as general safety, human factors, safety methods/systems, and robot

safety standards. The miscellaneous category includes published literature that discusses topics directly or indirectly related to both robot reliability and robot safety. All these three categories and their sub-classifications are presented in Table 1.2 and reviewed below.

Table 1.2: Categories of literature on Robot Reliability and Safety

(i)	Robot Reliability	
	• Methods:	[5] [29] [33] [40] [42] [58] [63] [67] [73] [82]
	• Evaluation:	[50] [55] [78] [85]
	• Models:	[19] [23] [27] [70] [77] [79] [81]
(ii)	Robot Safety	
	• General:	[7] [8] [11] [12] [22] [31] [32] [35] [47] [54] [66]
	• Human Factors:	[1] [9] [10] [16] [18] [21] [28] [36] [37] [38] [43] [45] [46] [48] [59] [65] [68] [72] [74] [84] [86]
	• Safety Methods:	[4] [6] [13] [14] [15] [26] [34] [56] [60] [62] [61] [64]
	(a) Safety Systems:	[2] [3] [17] [20] [24] [25] [41] [49] [52] [53] [71] [76]
	• Safety Standards:	[83]
(iii)	Miscellaneous	[30][39][44] [51] [69][75][80]

1.2.2 Robot Reliability

With the ever-increasing utilization of robot technology in industries, the reliability of the robot becomes an important issue. Over the past 5 years, many publications concerned with robot reliability methods, evaluation and models have appeared. Their reviews are presented below.

- **Methods**

In 1997, Cavallaro and Walker [82] described qualitative fault tree analysis by presenting in detail the major failure modes of a robot manipulator-based system used for tank waste retrieval. In the same year, Lendvay [73] presented a prediction method to demonstrate that a mathematical character recognition based on the similarity of the reliability parameters of components, can be applied for defining the reliability of the electromechanical products. In 1998, Ulrich and Borenstein [67] developed an enhanced Vector Field Histogram (VFH+) method for real-time mobile robot to improve the smoothness and the reliability of the robot trajectory. In the same year, Shi et al. [63] developed a new approach of robot on-line fault detection and diagnosis based on the neural networks model of robot controlling system in Flexible Manufacturing Systems (FMS). In 1998 and 1999, Lee et al. [58, 42] developed a new way of integrating human and the robot, for robot path generation to increase the accuracy and the reliability of teleoperation. In the following year, Carreras and Walker [40, 29] proposed a novel alternative interval-based method for reliability estimation of a robot manipulator system by using fault trees. In the same year, Dixon et al. [33] presented a prediction error based fault detection method for robot manipulators. In 2001, Carreras and Walker [5] developed further the robotic-based interval methods.

- **Evaluation**

In 1997, Monterverde and Tosunoglu [85] introduced a fault tolerance capacity measure for robot systems by using serial and parallel mechanisms, kinematic redundancy, and dual actuators. In the same year, Musto and Saridis [78] developed an entropy-based reliability assessment technique for intelligent machines and concluded that the

minimization of entropy at all levels of the hierarchical control structure can be interpreted as the maximization of a reliability bound. Two years later, Leuschen et al. [55] investigated and evaluated the reliability of hydraulic robots by using the technique of analytic redundancy. In the same year, Monterverde and Tosunoglu [50] developed further the fault tolerance measure and demonstrated its application to serial and parallel robotic structures.

- **Models**

In 1997, Dhillon and Fashandi [81] presented a general model for performing probabilistic analysis of a robot system by using the Markov method. In the same year, Harpel et al. [79] applied coverage models to the basic components of the joint of a robot and performed analysis of a three-joint kinematically redundant robot manipulator arm. In the same year, Younis and Hamed [77] developed a stochastic approach for modeling the robot-served manufacturing system by introducing two new parameters of the robot availability and the machine stoppage. In 1998, Leuschen et al. [70] developed a fuzzy Markov model for analyzing fault tolerant designs under considerable uncertainty and applied it to the MLDUA robot system. At the end, they concluded that computational complexity is the main drawback of the model. In 2000, Michaelson and Jiang [27] presented a novel model representing a redundant system in multiple mobile robots to investigate overall system reliability and fault-tolerance capability. In the same year, Savsar [23] developed a stochastic model to determine the performance of a flexible manufacturing cell under random operational conditions. In the following year, Leuschen et al. [19] presented a fuzzy Markov model to analyze a prototype hazardous waste cleanup manipulator.

1.2.3 Robot Safety

Safety is an important issue in the design, manufacture, and operation of robots. This section reviews publications that appeared after 1996.

- **General Safety**

In 1997, Varley [85] developed safety-related software for surgical robots by using the techniques of identifying potential hazards and tracing their mitigating factors throughout the project lifecycle. In the following year, Perret [66] discussed a case study for concerning developing a service robot for nuclear safety. In 1999, Fox [54] pointed out that robotic safety is everyone's responsibility, including plant managers, buyers, integrators, maintenance people, operators, and robot manufacturers. In the same year, a new approach for real-time collision avoidance of position-controlled conventional six-DOF arms and dexterous seven-DOF arms was developed and demonstrated by Seraji and Bon [47]. In 2000, Gregory and Kangari [35] introduced a new methodology for performing cost/benefit analysis of robotic system to reduce health risks and potential deaths. In the same year, Rovetta [32] discussed the progress on telerobotic surgery control and safety. Also, in the same year, Anderson et al. [31] introduced the integration of modular telerobot control architecture, called 'SMART', into the accident response mobile manipulator system (ARMMS) to improve precision, safety, and operability of manipulators. A total of four publications appeared in 2001. Das [12] discussed specific ergonomics problems of socio-psychological factors, system safety design, communications, training, and workplace design with respect to industrial robots. Armstrong [11] presented guidance for achieving functional safety of electronically controlled equipment. Chan and Courtney [8] reported a survey conducted to examine the

safety and ergonomics issues of robotic workplaces in Hong Kong. Hamada and Fujie [7] discussed robotic applications in laboratory that support social safety.

- **Human Factors**

In 1997, Yamada [86] presented a human-oriented design approach for human-robot coexistence based on human pain tolerance limit. Also, in the same year, three more publications appeared. Yamada et al. [74] further discussed and proposed a human-robot (H-R) coexistent system that allows H-R contact in the safeguarding space, and demonstrated the validity and efficient utility of the safeguarding space by two practical experiments. Kuroda [72] conducted an overall review of the influence of ethnic culture on human-robot cooperation. Morita and Sugano [68] proposed a safety design methodology for human symbiotic manipulator. In 1998, Shibata and Inooka [84] investigated a psychological evaluation of human arm motions and robot arm motions by using the rating scale method. Wakita et al. [65], in the same year, proposed physical and intelligent augmentation to the coexisting robotic systems and realization of safety through information sharing. In 1999, Lin and Wang [59] analyzed human errors and robot failures associated with a human-robot drilling system and proposed countermeasures and feasible recommendations to enhance the hybrid system's safety and performance. In the same year, Graves and Czarnecki [48] discussed the design of a man-machine interface for a mobile telerobot to reduce operator workload and error rates. Also, in 1999, Ikuta and Nokata [46] developed a safety evaluation method for human-care robots and defined safety values for devising general safety strategies for these robots. Also, in 1999, six more publications appeared. Takanobu et al. [45] studied remote interaction between human and humanoid robot. Lim and Tanie [43] presented a

study of human behaviour in a collision environment by using a human-friendly robot with passive viscoelastic trunk. Kajikawa et al. [38] analyzed human arm movements of catching a moving object for improving robot safety. Yamada et al. [37] presented Fault Tree Analysis (FTA) concerning robot-human safety. Yamamoto et al. [36] demonstrated that fuzzy control is an effective method to realize the robot's avoidance motion. Morita and Sugano [21] developed a technology map for standardizing safety measurements of human symbiotic robots. In 2000, Lim and Tanie [28] investigated the technologies of human robot symbiosis and performed research on human safety mechanisms of human-friendly robots. They also proposed a human-friendly robot with passive viscoelastic trunk and passively movable base. In the same year, Traver et al. [18] presented elusive and ergonomic strategies for making service robot human-safe and human-friendly. Song et al. [16], in the same year, proposed visual mechanism, based on space variant vision, to improve human-robot interaction in rehabilitation robots. Also, in 2000, Ikuta et al. [9] proposed a danger evaluation method for human-care robots. In the following year, Ikuta et al. [10] developed a robot simulator (SAFE-CAD) for the danger evaluation of various types of human-care robots operating under various environments. In 2001, Dario et al. [1] reviewed different approaches and technological implementations in the area of humanoid and personal robotics.

- **Safety Method**

In 1997, Caccavale and Walker [56] proposed a new fault detection scheme based on a full-state diagnostic observer for robotic manipulators. In 1998, Aghazadeh et al. [64] presented a hazard analysis method to evaluate and improve the safety of robotic work cells. Also, in 1998, Juang [62] proposed a computer-aided potential field method for

manipulator collision avoidance. In the same year, Lankenau et al. [61] addressed the development of a real-time safety layer for an electrically driven wheelchair and presented a new formal design method for safety improvement in rehabilitation robots. In 1998 [60] and later in 2000 [34], Yang and Meng proposed an efficient neural network model for real-time path planning with safety consideration in a nonstationary environment. Also, in 1998, Shen et al. [57] proposed an approach to solve the problem of robot motion planning and collision avoidance. Two years later, Roger and McInnes [26] presented a path-planning method to generate safe trajectories for a small free-flying robot camera. In the same year, Fei et al. [15] put forward a methodology of hazard identification and safety insurance control (HISIC) for the enhancement of safety of medical robot. In the following year, Fei et al. [6] developed a safety model with three axes (software, hardware and HISIC) to analyze potential hazards of medical robotics. In 2001, Zurada et al. [14] presented a neuro-fuzzy approach to robot system safety with the utilizations of an integrated sensing architecture and detection and decision logic. Burger et al. [13], in the same year, studied the design and test of a fail-safe numerical control for robotic surgery. Also, in 2001, Tunstel et al. [4] presented fuzzy logic approaches with respect to robot safety and survivability.

(a) Safety Systems

In 1997, Pegman and Reed [76] suggested that more complex electromechanical systems with competent control systems could be quite beneficial to improve safety of remote robot operations. In the same year, Rachkov [71] reviewed software and hardware measures to organize safety systems of technological climbing robots. In 1998, Lawn and Takeda [53] presented robotic-hybrid wheelchair designs for applications in barrier

present environments. A total of four publications appeared in 1999. Karlsson [52] developed a capacitance sensor for safeguarding operators of industrial robots. Kawaji et al. [49] proposed a safety system for human-machine co-operation. Kim et al. [41] developed tele-robotic safe systems for preventive maintenance and remote inspections for use in nuclear power plants. Komatsu et al. [20] introduced a safety mechanism by using a gas spring. In 2000, Karlsson et al. [25] proposed a dynamic safety system based on sensor fusion for use in industrial robots. In 2001, Beerthuisen and Kruidhof [24] presented system and software safety analysis of the European Robotic Arm (ERA). In the same year, Vanderperre [17] performed stochastic analysis with respect to availability of a robot-safety device. Also, in 2001, Engel et al. [3] introduced a safety robot system for use in craniofacial surgery. Takaoka et al. [2], in the same year, reviewed the development of a live-line maintenance robot and introduced a new intelligent system for a fully automatic live-line maintenance robot.

- **Safety Standard**

In 2000, Fryman [83] proposed a robot safety standard for the design and installation of industrial robots.

1.2.4 Miscellaneous

In 1997, Dhillon and Yang [80] presented reliability and availability analysis of a redundant robot system with one safety system. In the same year, for achieving high reliability and safety of manufacturing systems Zeng [69] studied maintenance of car assembly lines. Also, in 1997, Dhillon and Fashandi [75] presented an overview of safety and reliability assessment techniques in robotics. Two years later, Dhillon and Fashandi [51] presented reliability and availability analysis of a robot with duplicate safety units.

In 1999, Fei et al. [44] discussed a methodology of the verification & validation (V&V) to ensure the safety and reliability of a medical robot software. In the same year, Li et al. [39] proposed a Max-plus algebra model for human/machine cooperation to improve safety, reliability, and fault tolerance of operations in teleoperation systems. In 2000, Dhillon and Aleem [30] reported the results of a survey of Canadian robot manufacturers and users concerning robot reliability and safety.

1.3 Objective of the Study

Robots are complex and sophisticated machines, and many robot systems involve interaction between humans and robots, thus the reliability and safety of robots are very important. In usual robot reliability and availability analyses, the occurrence of common-cause failures is overlooked and only general failures are considered. A common-cause failure may be defined as any instance where multiple units or elements fail due to a single cause [98]. Under such conditions, the end results may not present a true picture regarding the actual system reliability and availability. The main objective of this study was to perform effective reliability and availability analyses of robot-safety systems with common-cause failures. To meet this objective, the correlation of reliability, safety, and the occurrence of common-cause failures are carefully considered for four types of robot-safety systems. Markov and the supplementary variable methods were used to perform mathematical analysis of these systems, and specific expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure were developed.

Comparing this thesis with those of previous researchers, the main contributions of the study are:

- The models in this study integrated general failures (hardware failures), common-cause failures and general repairable system.
- The models in this study considered common-cause failures to occur from any of the operable states of the system when at least two units are functioning normally and generalized the failed system repair time process with various distributions that may be of more practical significance.
- The models in this study can be applied to their corresponding robot-safety systems to predict robot-safety system reliability and availability, and to prepare appropriate maintenance scheduling policies.

1.4 Organization of the Study

This study is presented in six chapters.

Chapter 1 provides a brief background of robot reliability and safety and a detailed review of published literature for the period 1997-2002.

Chapter 2 presents stochastic analysis of a robot-safety system having one robot and n -redundant safety units with common-cause failures.

Chapter 3 presents stochastic analysis of a robot-safety system having redundant robots (k -out-of- n) and one safety unit with common-cause failures.

Chapter 4 presents stochastic analysis of a robot-safety system having n -redundant robots and m -redundant safety units with common-cause failures.

Chapter 5 presents stochastic analysis of a robot-safety system having one robot and redundant safety units (k -out-of- n) with common-cause failures.

In Chapters 2-5, the system failure rates and the partially failed system repair rates are assumed constant, and the failed system repair time is assumed arbitrarily distributed.

Markov and the supplementary variable methods were used to perform mathematical analysis of the models. Markov method is a reliability analysis approach used for systems with constant failure and repair rates, while supplementary variable method can be used to perform reliability analysis of systems with constant failure rates and time-dependent failed system repair rates. Generalized expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure are presented. Some special case models are also presented.

Chapter 6 presents conclusions and recommendations for future studies.

STOCHASTIC ANALYSIS OF A ROBOT-SAFETY SYSTEM CONTAINING ONE ROBOT AND N-REDUNDANT SAFETY UNITS WITH COMMON-CAUSE FAILURES

2.1 Introduction

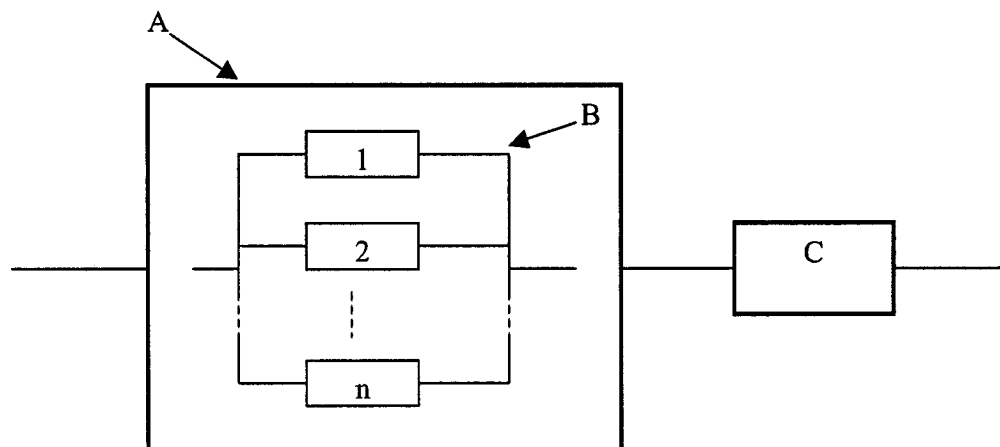
Robots are complex and sophisticated machines. Past experiences indicate that robots can constitute a source of great danger to humans. Robot safety is of utmost importance as Ramirez pointed out, “In short, safety is a concern because generally robots are extremely strong, fast, deaf, dumb, blind, automatic, and therefore dangerous” [99]. There is absolutely no doubt that a robot not only has to be reliable, but also safe. Thus, the safety unit is an important element of the overall robot system. More specifically, an overall robot system is made up of a robot and its associated safety units. In effective robot reliability and availability analyses, the coupling between reliability and safety must be studied and the occurrence of common-cause failures considered.

The concept of redundancy is widely used to increase the safety and reliability of a system. It can also be applied to the robot-safety systems, in particular to safety units. Therefore, this chapter presents reliability and availability analyses of a robot-safety system having one robot and n-redundant safety units with common-cause failures. The system failure rates and the partially failed system repair rates are assumed constant, and the failed system repair time is assumed arbitrarily distributed. Markov and the

supplementary variable methods were used to perform mathematical analysis of this model. Generalized expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure are developed. A special case model ($n=2$) is presented.

2.2 The Description of the Robot-Safety System

The block diagram of the robot-safety system having one robot and n -redundant safety units with common-cause failures is shown in Figure 2.1, and its corresponding state space diagram is given in Figure 2.2. The numerals and letter n in boxes and ellipses of Figure 2.2 denote system states.



A: the robot

B: n -identical safety units

C: common-cause failures

Figure 2.1 Block diagram of the robot-safety system with common-cause failures.

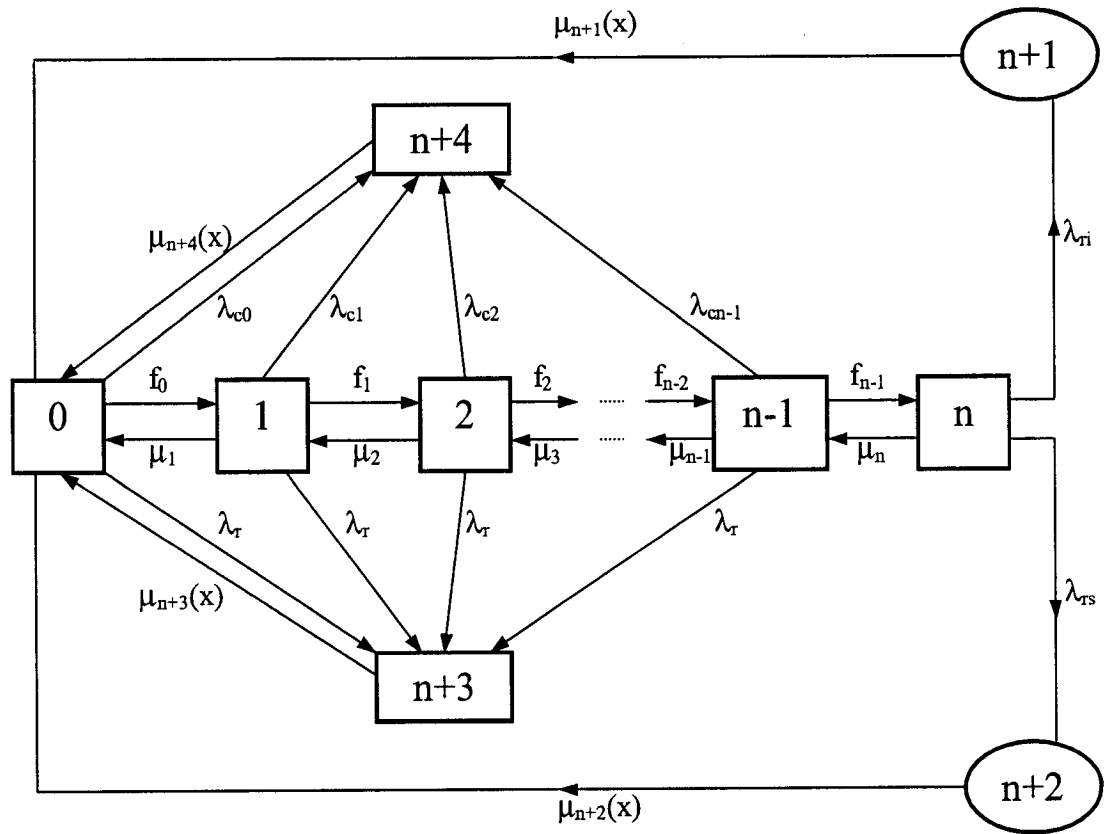


Figure 2.2 The state space diagram of the robot-safety system with common-cause failures. The numerals and letter n in squares, rectangles, and ellipses denote system states and $f_i=(n-i)\lambda_s$, for $i=0, 1, 2, \dots, n-1$.

At time $t=0$, the robot and all n safety units start operating. The robot-safety system can fail due to the failure of the robot itself or the occurrence of a common-cause failure. Nonetheless, the robot-safety system will function normally until at least one safety unit and the robot are operating normally. The system goes through $(n+1)$ distinct operating states. A common-cause failure can occur only if at least one safety unit is functioning successfully. Once all n safety units fail, the robot may continue to operate and subsequently it may fail either safely or with an incident. The degraded or fully failed robot-safety system is repaired.

Assumptions

The following assumptions are associated with this model:

- (i) The robot-safety system is composed of one robot and n identical safety units.
- (ii) The redundant safety units are operating simultaneously.
- (iii) All failures are statistically independent.
- (iv) All failure rates and the partially failed system repair rates are constant.
- (v) The failed robot-safety system repair rates can be constant or non-constant.
- (vi) The repaired robot or a safety unit is as good as new.
- (viii) The overall system fails when the active robot fails or a common-cause failure occurs.

Notation

The following symbols are associated with the model:

- i i^{th} state of the overall robot-safety system: for $i=0$, means robot and all n safety units are in perfect working condition; for $i=1$, means robot and $n-1$ safety units operating normally while one safety unit has failed; for $i=k$ (where $k=2,3,\dots,n-1$), means the robot and $n-k$ safety units operating normally while k safety units have failed; for $i=n$, means robot operating normally while all n safety units have failed.
- j j^{th} state of the failed robot-safety system: for $j=n+1$, means robot failed with an incident and all n safety units have also failed; for $j=n+2$, means robot failed safely and all n safety units have also failed; for $j=n+3$, means robot failed itself while at least one safety unit is functioning; for $j=n+4$, means robot-safety system failed due to a common-cause failure.
- t time

λ_s	Constant failure rate of the safety unit.
λ_{ri}	Constant failure rate of the robot failing with an incident.
λ_{rs}	Constant failure rate of the robot failing safely.
λ_r	Constant failure rate of the robot.
λ_{ci}	Constant common-cause failure rate of the robot-safety system in state i ; for $i = 0, 1, 2, \dots, n-1$.
μ_i	Constant repair rate of the safety unit in state i ; for $i = 1, 2, \dots, n$.
Δx	Finite repair time interval.
$\mu_j(x)$	Time-dependent repair rate when the failed robot-safety system is in state j and has an elapsed repair time of x ; for $j = n+1, n+2, n+3, n+4$.
$p_j(x,t)\Delta x$	The probability that at time t , the failed robot-safety system is in state j and the elapsed repair time lies in the interval $[x, x+\Delta x]$; for $j = n+1, n+2, n+3, n+4$.
pdf	Probability density function.
$z_j(x)$	pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x ; for $j = n+1, n+2, n+3, n+4$.
$P_i(t)$	Probability that the robot-safety system is in state i at time t ; for $i = 0, 1, \dots, n$.
$P_j(t)$	Probability that the robot-safety system is in state j at time t ; for $j = n+1, n+2, n+3, n+4$.
P_i	Steady-state probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, n$.
P_j	Steady-state probability that the robot-safety system is in state j ; for $j = n+1, n+2, n+3, n+4$.

s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, n$.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j ; for $j = n+1, n+2, n+3, n+4$.
$AV_{rs}(s)$	Laplace transform of the robot-safety system availability when the robot working with at least one safety unit.
$AV_r(s)$	Laplace transform of the robot-safety system availability when the robot working with or without the safety unit(s).
$AV_{rs}(t)$	Robot-safety system time-dependent availability when the robot working with at least one safety unit.
$AV_r(t)$	Robot-safety system time-dependent availability when the robot working with or without the safety unit(s).
$SSAV_{rs}$	Robot-safety system steady state availability when the robot working with at least one safety unit.
$SSAV_r$	Robot-safety system steady state availability when the robot working with or without the safety unit(s).
$R_{rs}(s)$	Laplace transform of the robot-safety system reliability when the robot working with at least one safety unit.
$R_r(s)$	Laplace transform of the robot-safety system reliability when the robot working with or without the safety unit(s).
$R_{rs}(t)$	Robot-safety system reliability when the robot working with at least one safety unit.

- $R_s(t)$ Robot-safety system reliability when the robot working with or without the safety unit(s).
- $MTTF_{rs}$ Robot-safety system mean time to failure when the robot working with at least one safety unit.
- $MTTF_r$ Robot-safety system mean time to failure when the robot working with or without the safety unit(s).
- σ^2 Robot-safety system variance of time to failure when the robot working with or without the safety unit(s).

2.3 Generalized Robot-Safety System Analysis

Using the supplementary method [100,101], the system of Equations associated with the diagram shown in Figure 2.2 can be expressed as follows:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{j=n+1}^{n+4} \int_0^{\infty} P_j(x,t) \mu_j(x) dx \quad (2.1)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1) \lambda_s P_{i-1}(t) + \mu_{i+1} P_{i+1}(t) \quad (\text{for } i = 1, 2, \dots, n-1) \quad (2.2)$$

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) \quad (2.3)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \quad (2.4)$$

(for $j = n+1, n+2, n+3, n+4$)

where

$$a_0 = n\lambda_s + \lambda_r + \lambda_{e0}$$

$$a_i = (n-i)\lambda_s + \lambda_r + \lambda_{ci} + \mu_i \quad (\text{for } i = 1, 2, \dots, n-1)$$

$$a_n = \lambda_{ri} + \lambda_{rs} + \mu_n$$

The associated boundary conditions are as follows:

$$P_{n+1}(0, t) = \lambda_{r_i} P_n(t) \quad (2.5)$$

$$P_{n+2}(0, t) = \lambda_{r_s} P_n(t) \quad (2.6)$$

$$P_{n+3}(0, t) = \lambda_r \sum_{i=0}^{n-1} P_i(t) \quad (2.7)$$

$$P_{n+4}(0, t) = \sum_{i=0}^{n-1} \lambda_{c_i} P_i(t) \quad (2.8)$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero.

2.3.1 Time Dependant Availability Analysis

Using the Laplace transform technique and the initial conditions in Equations (2.1) – (2.8), we get

$$(s + a_0)P_0(s) = 1 + \mu_1 P_1(s) + \sum_{j=n+1}^{n+4} \int_0^{\infty} P_j(x, s) \mu_j(x) dx \quad (2.9)$$

$$(s + a_i)P_i(s) = (n - i + 1)\lambda_s P_{i-1}(s) + \mu_{i+1} P_{i+1}(s) \quad (\text{for } i = 1, 2, \dots, n-1) \quad (2.10)$$

$$(s + a_n)P_n(s) = \lambda_s P_{n-1}(s) \quad (2.11)$$

$$sP_j(x, s) + \frac{\partial P_j(x, s)}{\partial x} + \mu_j(x)P_j(x, s) = 0 \quad (2.12)$$

(for $j = n+1, n+2, n+3, n+4$)

and the boundary conditions:

$$P_{n+1}(0, s) = \lambda_{r_i} P_n(s) \quad (2.13)$$

$$P_{n+2}(0, s) = \lambda_{r_s} P_n(s) \quad (2.14)$$

$$P_{n+3}(0, s) = \lambda_r \sum_{i=0}^{n-1} P_i(s) \quad (2.15)$$

$$P_{n+4}(0, s) = \sum_{i=0}^{n-1} \lambda_{ci} P_i(s) \quad (2.16)$$

Solving differential Equation (2.12), we get the following expression:

$$P_j(x, s) = P_j(0, s) e^{-sx} \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \quad (2.17)$$

(for $j = n+1, n+2, n+3, n+4$)

Since

$$P_j(s) = \int_0^{\infty} P_j(x, s) dx \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.18)$$

and together with Equation (2.17), we get

$$P_j(s) = P_j(0, s) \frac{1 - Z_j(s)}{s} \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.19)$$

where

$$\frac{1 - Z_j(s)}{s} = P_j(0, s) \int_0^{\infty} e^{-sx} \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] dx \quad (2.20)$$

(for $j = n+1, n+2, n+3, n+4$)

or

$$Z_j(s) = \int_0^{\infty} e^{-sx} z_j(x) dx \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.21)$$

$$z_j(x) = \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \mu_j(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

Using Equations (2.10), (2.11), and (2.19), together with

$$\sum_{i=0}^n P_i(s) + \sum_{j=n+1}^{n+4} P_j(s) = \frac{1}{s} \quad (2.22)$$

we get the following general form of state probability solutions:

$$P_i(s) = \frac{N_i(s)}{M_0(s)} \quad (\text{for } i = 0, 1, \dots, n) \quad (2.23)$$

$$P_j(s) = \frac{N_j(s)}{M_0(s)} \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.24)$$

where

$$k_1 = \frac{n\lambda_s\mu_1}{s+a_1-k_2}$$

$$k_i = \frac{(n-i+1)\lambda_s\mu_i}{(s+a_i)-k_{i+1}} \quad (\text{for } i = 1, 2, \dots, n-1)$$

$$k_n = \frac{\mu_n\lambda_s}{s+a_n}$$

$$a_{n+1} = \lambda_{r_i} \prod_{i=1}^n \frac{k_i}{\mu_i}$$

$$a_{n+2} = \lambda_{r_s} \prod_{i=1}^n \frac{k_i}{\mu_i}$$

$$a_{n+3} = \lambda_r [1 + \sum_{m=1}^{n-1} (\prod_{i=1}^m \frac{k_i}{\mu_i})]$$

$$a_{n+4} = \lambda_{c_0} + \sum_{m=1}^{n-1} (\lambda_{c_m} \prod_{i=1}^m \frac{k_i}{\mu_i})$$

$$M_0(s) = s(1 + \sum_{i=1}^n \prod_{m=1}^i \frac{k_m}{\mu_m} + \sum_{j=n+1}^{n+4} a_j \frac{1-Z_j(s)}{s}) \quad (2.25)$$

$$N_0(s) = 1 \quad (2.26)$$

$$N_i(s) = \prod_{m=1}^i \frac{k_m}{\mu_m} N_0(s) \quad (\text{for } i = 0, 1, 2, \dots, n) \quad (2.27)$$

$$N_j(s) = \frac{a_j[1-Z_j(s)]}{s} \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.28)$$

Thus, the Laplace transform of the robot-safety system availability with at least one working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{\sum_{i=0}^{n-1} N_i(s)}{M_0(s)} \quad (2.29)$$

Similarly, the Laplace transform of the robot-safety system availability with or without working safety units is given by

$$AV_r(s) = \sum_{i=0}^n P_i(s) = \frac{\sum_{i=0}^n N_i(s)}{M_0(s)} \quad (2.30)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions in Equations (2.29) and (2.30), and taking the inverse Laplace transforms of the resulting equations, we can get the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

2.3.2 Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Thus, Equations (2.1) – (2.8) reduce to Equations (2.31) – (2.38), respectively.

$$a_0 P_0 = \mu_1 P_1 + \sum_{j=n+1}^{n+4} \int_0^{\infty} P_j(x) \mu_j(x) dx \quad (2.31)$$

$$a_i P_i = (n-i+1) \lambda_s P_{i-1} + \mu_{i+1} P_{i+1} \quad (\text{for } i = 1, 2, \dots, n-1) \quad (2.32)$$

$$a_n P_n = \lambda_s P_{n-1} \quad (2.33)$$

$$\frac{dP_j(x)}{dx} + \mu_j(x) P_j(x) = 0 \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.34)$$

$$P_{n+1}(0) = \lambda_{ri} P_n \quad (2.35)$$

$$P_{n+2}(0) = \lambda_{rs} P_n \quad (2.36)$$

$$P_{n+3}(0) = \lambda_r \sum_{i=0}^{n-1} P_i \quad (2.37)$$

$$P_{n+4}(0) = \sum_{i=0}^{n-1} \lambda_{ci} P_i \quad (2.38)$$

Solving Equation (2.34), we get

$$P_j(x) = P_j(0) \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.39)$$

After a failure the robot-safety system under repair steady state condition probability is given by

$$P_j = \int_0^{\infty} P_j(x) dx \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.40)$$

Substituting Equation (2.39) into Equation (2.40), yields

$$P_j = P_j(0) E_j[x] \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.41)$$

where

$$\begin{aligned} E_j(x) &= \int_0^{\infty} \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] dx \\ &= \int_0^{\infty} x z_j(x) dx \end{aligned} \quad (2.42)$$

which is the mean time to robot-safety system repair when the failed robot-safety system is in state j and has an elapsed repair time of x .

Substituting Equations (2.35) - (2.38) into Equation (2.41), we get:

$$P_{n+1} = \lambda_{ri} P_n E_{n+1}[x] \quad (2.43)$$

$$P_{n+2} = \lambda_{rs} P_n E_{n+2}[x] \quad (2.44)$$

$$P_{n+3} = \lambda_r \sum_{i=0}^{n-1} P_i E_{n+3}[x] \quad (2.45)$$

$$P_{n+4} = \sum_{i=0}^{n-1} \lambda_{ci} P_i E_{n+4}[x] \quad (2.46)$$

Solving Equations (2.32), (2.33), and (2.43) - (2.46), together with

$$\sum_{i=0}^n P_i + \sum_{j=n+1}^{n+4} P_j = 1 \quad (2.47)$$

lead to the following general form of the steady state probability solutions:

$$P_0 = (L + \sum_{j=n+1}^{n+4} L_j E_j[x])^{-1} = \frac{1}{G} \quad (2.48)$$

$$P_i = \frac{L_i}{\mu_i} P_{i-1} = \prod_{m=1}^i \frac{L_m}{\mu_m} P_0 \quad (\text{for } i = 1, 2, \dots, n-1) \quad (2.49)$$

$$P_n = \frac{L_n}{\mu_n} P_{n-1} = \prod_{i=1}^n \frac{L_i}{\mu_i} P_0 \quad (2.50)$$

$$P_j = L_j E_j[x] P_0 \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.51)$$

where

$$L = 1 + \sum_{m=1}^n \prod_{i=1}^m \frac{L_i}{\mu_i}$$

$$L_i = \frac{(n-i+1)\lambda_s \mu_i}{a_i - L_{i+1}} \quad (\text{for } i = 1, 2, \dots, n-1)$$

$$L_n = \frac{\lambda_s \mu_n}{a_n}$$

$$L_{n+1} = \lambda_{r1} \prod_{i=1}^n \frac{L_i}{\mu_i}$$

$$L_{n+2} = \lambda_{rs} \prod_{i=1}^n \frac{L_i}{\mu_i}$$

$$L_{n+3} = \lambda_r (1 + \sum_{m=1}^{n-1} \prod_{i=1}^m \frac{L_i}{\mu_i})$$

$$L_{n+4} = \lambda_{c0} + \sum_{m=1}^{n-1} \lambda_{cm} \prod_{i=1}^m \frac{L_i}{\mu_i}$$

$$G = L + \sum_{j=n+1}^{n+4} L_j E_j[x] \quad (2.52)$$

The generalized steady state availability of the robot-safety system with at least one working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^{n-1} P_i = \frac{L - \prod_{i=1}^n \frac{L_i}{\mu_i}}{G} \quad (2.53)$$

Similarly, the generalized steady state availability of the robot-safety system with or without working safety units is given by

$$SSAV_r = \sum_{i=0}^n P_i = \frac{L}{G} \quad (2.54)$$

For different failed system repair time distributions, the values of G are obtained as follows:

- (i) When the failed robot-safety system repair time x is exponentially distributed, the probability density function of the repair time is

$$z_j(x) = \mu_j e^{-\mu_j x} \quad (\mu_j > 0, \quad j = n+1, n+2, n+3, n+4) \quad (2.55)$$

where x is the repair time, and μ_j is the constant repair rate of state j . Thus, the mean time to robot-safety system repair, $E_j[x]$, for the exponential distribution is

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \frac{1}{\mu_j} \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.56)$$

Substituting Equation (2.56) into Equation (2.52), we get

$$G = G_e = L + \sum_{j=n+1}^{n+4} (L_j \frac{1}{\mu_j}) \quad (2.57)$$

- (ii) When the failed robot-safety system repair time x is gamma distributed, the probability density function of the repair time is

$$z_j(x) = \frac{\mu_j^\beta x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \quad (\beta > 0) \quad (2.58)$$

(for $j = n+1, n+2, n+3, n+4$)

where x is the repair time, $\Gamma(\beta)$ is the gamma function, and β and μ_j are the shape and scale parameters, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the gamma distribution is

$$E_j[x] = \int_0^\infty x z_j(x) dx = \frac{\beta}{\mu_j} \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.59)$$

Substituting Equation (2.59) into Equation (2.52), we get

$$G = G_g = L + \sum_{j=n+1}^{n+4} (L_j \frac{\beta}{\mu_j}) \quad (2.60)$$

(iii) When the failed robot-safety system repair time x is Weibull distributed, the probability density function of the repair time is expressed by

$$z_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j(x)^\beta} \quad (\beta > 0) \quad (2.61)$$

(for $j = n+1, n+2, n+3, n+4$)

where x is the repair time, and β and μ_j are the shape and scale parameters of the Weibull distribution, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Weibull distribution is given by

$$E_j[x] = \int_0^\infty x z_j(x) dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \quad (2.62)$$

(for $j = n+1, n+2, n+3, n+4$)

Substituting Equation (2.62) into Equation (2.52), we get

$$G = G_w = L + \sum_{j=n+1}^{n+4} \left[L_j \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \right] \quad (2.63)$$

(iv) When the failed robot-safety system repair time x is Rayleigh distributed, the probability density function of the Rayleigh distribution is expressed by

$$z_j(x) = \mu_j x e^{-\mu_j x^2/2} \quad (\mu_j > 0) \quad (2.64)$$

(for $j = n+1, n+2, n+3, n+4$)

where x is the repair time, and μ_j is the scale parameter. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Rayleigh distribution is

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \sqrt{\frac{\pi}{4\mu_j}} \quad (2.65)$$

(for $j = n+1, n+2, n+3, n+4$)

Substituting Equation (2.65) into Equation (2.52), we get

$$G = G_r = L + \sum_{j=n+1}^{n+4} (L_j \sqrt{\frac{\pi}{4\mu_j}}) \quad (2.66)$$

(v) When the robot-safety system repair time x is lognormal distributed, the probability density function of the repair time is

$$z_j(x) = \frac{1}{x \sigma_{y_j} \sqrt{2\pi}} e^{\frac{-(\ln x - \mu_{y_j})^2}{2\sigma_{y_j}^2}} \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.67)$$

where x is the repair time, and $\ln x$ is the natural logarithm of x with a mean and variance μ and σ^2 , respectively. The conditions on parameters are as follows:

$$\sigma_{y_j} = \ln \sqrt{1 + \left(\frac{\sigma_{x_j}}{\mu_{x_j}}\right)^2},$$

$$\mu_{y_j} = \ln \sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \quad (2.68)$$

(for $j = n+1, n+2, n+3, n+4$)

Hence, the failed robot-safety system mean time to repair, $E_j[x]$, for the lognormal distribution is

$$E_j[x] = e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.69)$$

Substituting Equation (2.69) into Equation (2.52), we get

$$G = G_l = L + \sum_{j=n+1}^{n+4} [L_j e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})}] \quad (2.70)$$

2.3.3 System Reliability, MTTF, and Variance of Time to Failure

Setting $\mu_{n+1}(x) = \mu_{n+2}(x) = \mu_{n+3}(x) = \mu_{n+4}(x) = 0$ in Figure 2.2. With the aid of Markov method, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_1 P_1(t) \quad (2.71)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_s P_{i-1}(t) + \mu_{i+1} P_{i+1}(t) \quad (2.72)$$

(for $i = 1, 2, \dots, n-1$)

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) \quad (2.73)$$

$$\frac{dP_{n+1}(t)}{dt} = \lambda_{r1} P_n(t) \quad (2.74)$$

$$\frac{dP_{n+2}(t)}{dt} = \lambda_{rs} P_n(t) \quad (2.75)$$

$$\frac{dP_{n+3}(t)}{dt} = \lambda_r \sum_{i=0}^{n-1} P_i(t) \quad (2.76)$$

$$\frac{dP_{n+4}(t)}{dt} = \sum_{i=0}^{n-1} \lambda_{ci} P_i(t) \quad (2.77)$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero. Taking the Laplace transforms of Equations (2.71) - (2.77) and solving the resulting set of Equations, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^n \prod_{m=1}^i \frac{k_m}{\mu_m} + \sum_{j=n+1}^{n+4} \frac{a_j}{s})]^{-1} \quad (2.78)$$

$$P_i(s) = \prod_{m=1}^i \frac{k_m}{\mu_m} P_0(s) \quad (\text{for } i = 1, 2, \dots, n) \quad (2.79)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = n+1, n+2, n+3, n+4) \quad (2.80)$$

The Laplace transform of the robot-safety system reliability with at least one working safety unit is

$$R_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = (1 + \sum_{m=1}^{n-1} \prod_{i=1}^m \frac{k_i}{\mu_i}) P_0(s) \quad (2.81)$$

Utilizing Equation (2.81), the robot-safety system mean time to the failure is obtained as follows [103]:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + \sum_{m=1}^{n-1} \prod_{i=1}^m \frac{L_i}{\mu_i}}{\sum_{j=n+1}^{n+4} L_j} \quad (2.82)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without the working safety units is

$$R_r(s) = \sum_{i=0}^n P_i(s) = (1 + \sum_{m=1}^n \prod_{i=1}^m \frac{k_i}{\mu_i}) P_0(s) \quad (2.83)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + \sum_{m=1}^n \prod_{i=1}^m \frac{L_i}{\mu_i}}{\sum_{j=n+1}^{n+4} L_j} \quad (2.84)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (2.81) and (2.83).

The system variance of time to failure is expressed by [103]

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + \sum_{m=1}^n \prod_{i=1}^m \frac{L_i}{\mu_i})(1 + \sum_{m=1}^n \prod_{i=1}^m \frac{L_i}{\mu_i} + \sum_{j=n+1}^{n+4} a_{dj})}{(\sum_{j=n+1}^{n+4} L_j)^2} - \frac{2 \sum_{m=1}^n k_{dm}}{\sum_{j=n+1}^{n+4} L_j} - (MTTF_r)^2 \end{aligned} \quad (2.85)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$k_{dm} = \lim_{s \rightarrow 0} \left(\prod_{i=1}^m \frac{k_i}{\mu_i} \right)' \quad (\text{for } m = 1, 2, \dots, n)$$

$$a_{dj} = \lim_{s \rightarrow 0} a_j' \quad (\text{for } j = n+1, n+2, n+3, n+4)$$

$$a_{dn+1} = \lim_{s \rightarrow 0} a_{n+1}' = \lambda_{ri} k_{dn}$$

$$a_{dn+2} = \lim_{s \rightarrow 0} a_{n+2}' = \lambda_{rs} k_{dn}$$

$$a_{dn+3} = \lim_{s \rightarrow 0} a_{n+3}' = \lambda_r \sum_{m=1}^{n-1} k_{dm}$$

$$a_{dn+4} = \lim_{s \rightarrow 0} a_{n+4}' = \sum_{m=1}^{n-1} \lambda_{cm} k_{dm}$$

$\left(\prod_{i=1}^m \frac{k_i}{\mu_i} \right)'$ denotes the derivative of $\prod_{i=1}^m \frac{k_i}{\mu_i}$ with respect to s .

a_j' denotes the derivative of a_j with respect to s .

The number of safety units to be incorporated within the robot-safety system is the matter of desired level of safety. More safety units we use, the better system reliability and MTTF we achieve. For the specified values of model parameters, the plots of Equations (2.81) (i.e., after taking inverse Laplace transform), and (2.82) are shown in Figures 2.3 and 2.4, respectively.

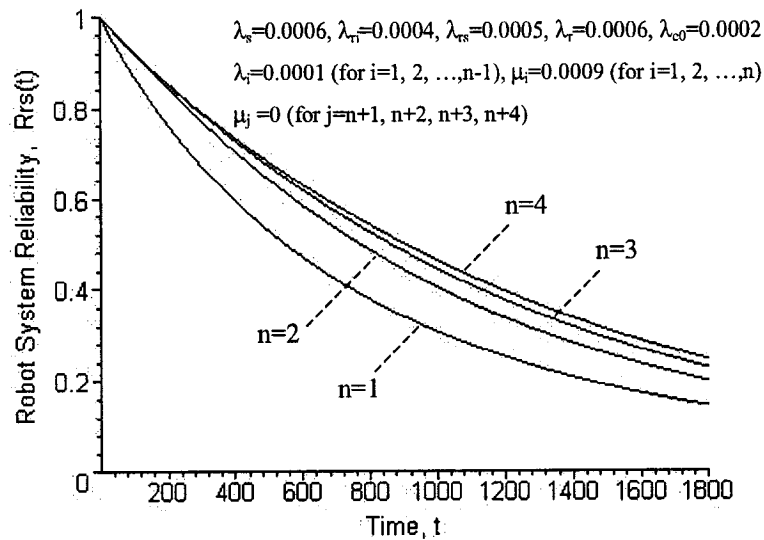


Figure 2.3 Reliability plots of an irreparable robot-safety system with at least one working safety unit.

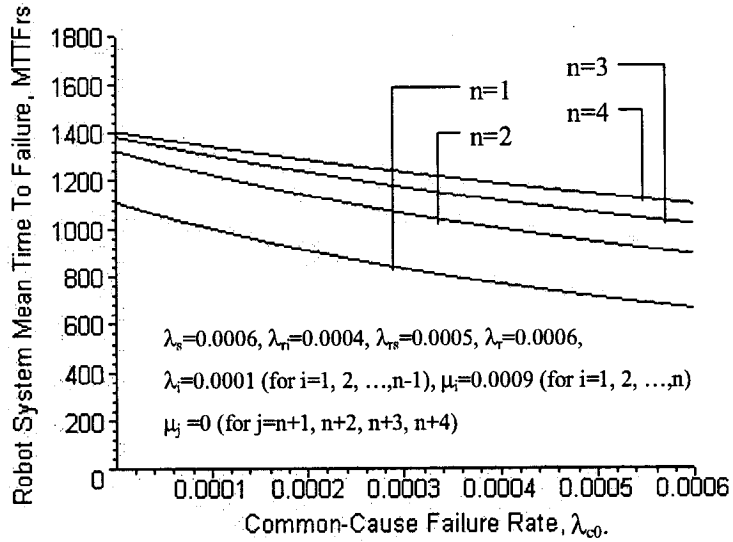


Figure 2.4 Mean time to failure plots of an irreparable robot-safety system with at least one working safety unit.

2.4 Special Case Model 2.I: (n=2)

For n=2 in Figures 2.1 and 2.2, the model becomes for a system having one robot and two redundant safety units*. The corresponding system of Equations can be obtained from Equations (2.1)-(2.8) by setting n=2. Furthermore, robot-safety system state probabilities $[P_i(t), P_j(t), P_i, P_j]$, availabilities $[AV_{rs}(t), AV_r(t), SSAV_{rs}, SSAV_r]$, reliabilities $[R_{rs}(t), R_r(t)]$, mean time to failure $[MTTF_{rs}, MTTF_r]$, and variance of time to failure $[\sigma^2]$ can also be obtained by inserting n=2 into the corresponding generalized Equations.

2.4.1 Time Dependant Availability Plots for n=2

Setting:

$$\lambda_s=0.0006, \quad \lambda_{rn}=0.0004, \quad \lambda_{rs}=0.0005, \quad \lambda_r=0.0006, \quad \lambda_{c0}=0.0002, \quad \lambda_{c1}=0.0001,$$

$$\mu_1=\mu_2=0.0009, \quad \mu_3=0.0010, \quad \mu_4=0.0011, \quad \mu_5=0.0012, \quad \mu_6=0.0006$$

* Detailed analysis is provided in *Appendix A*.

in Equations (2.23)-(2.24) and (2.29)-(2.30), and for gamma distributed failed system repair times using Maple computer program [102], the time-dependant plots of robot-safety system state probabilities and availabilities are shown in Figures 2.5 and 2.6, respectively.

2.4.2 Steady State Availability Plots for n=2

Setting:

$$\lambda_s=0.0006, \quad \lambda_{ri}=0.0004, \quad \lambda_{rs}=0.0005, \quad \lambda_r=0.0006, \quad \lambda_{c1}=0.0001,$$

$$\mu_1=\mu_2=0.0009, \quad \mu_3=0.0010, \quad \mu_4=0.0011, \quad \mu_5=0.0012, \quad \mu_6=0.0006$$

in Equation (2.53), and for gamma and Weibull distributed failed system repair times using Maple computer program [102] plots for SSAV_{rs} are shown in Figures 2.7 and 2.8, respectively.

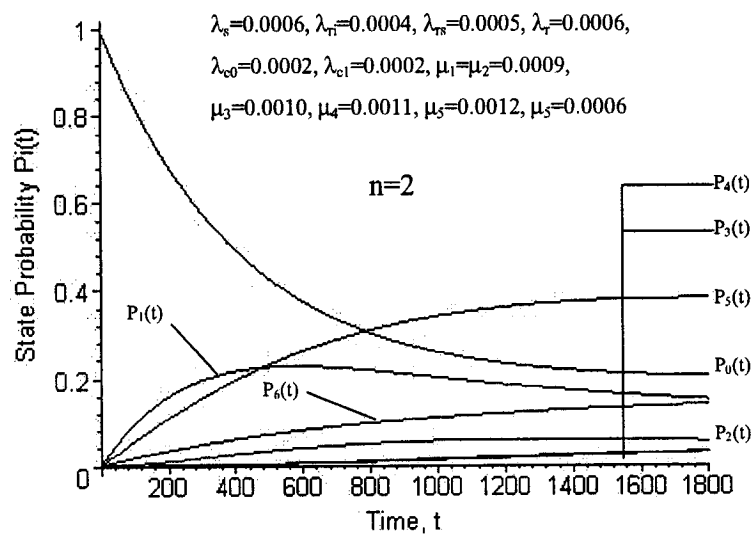


Figure 2.5 Time-dependent probability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

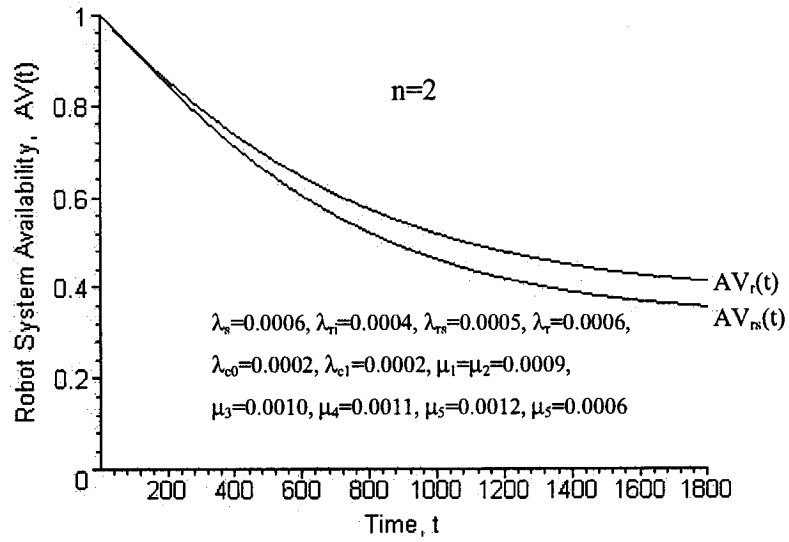


Figure 2.6 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

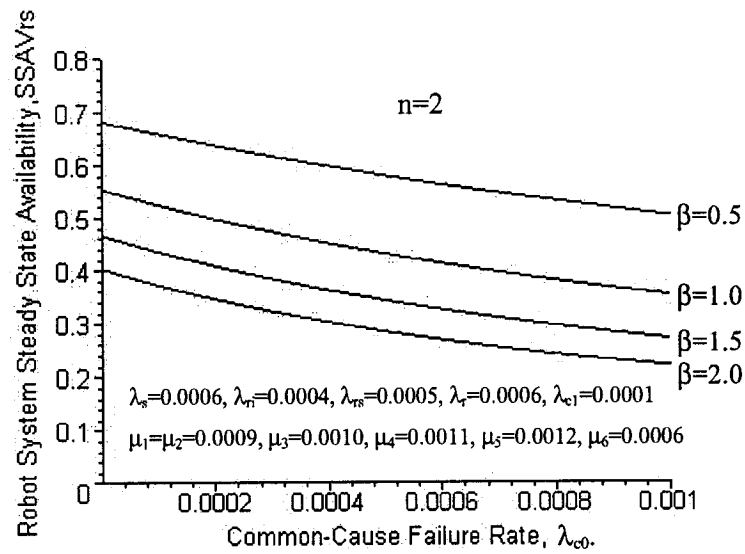


Figure 2.7 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with gamma distributed ($\beta=0.5, 1, 1.5, 2$) failed system repair time distribution.

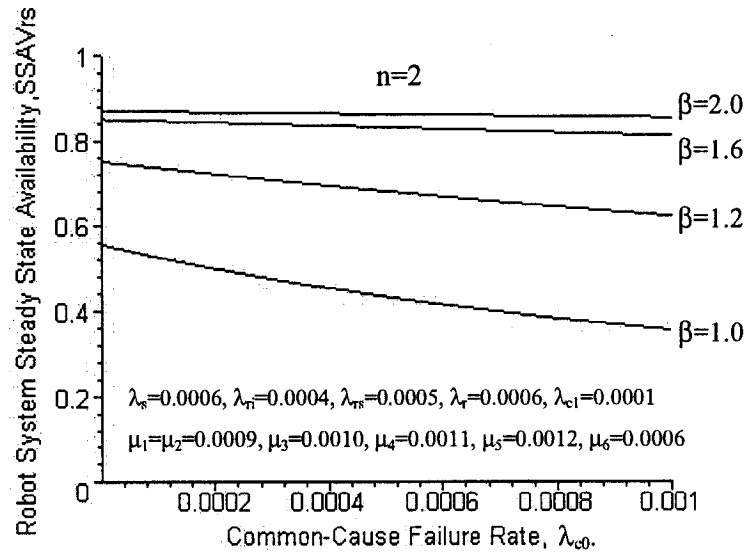


Figure 2.8 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair time distribution.

STOCHASTIC ANALYSIS OF A ROBOT-SAFETY SYSTEM CONTAINING K-OUT-OF-N REDUNDANT ROBOTS AND ONE BUILT-IN SAFETY UNIT WITH COMMON-CAUSE FAILURES

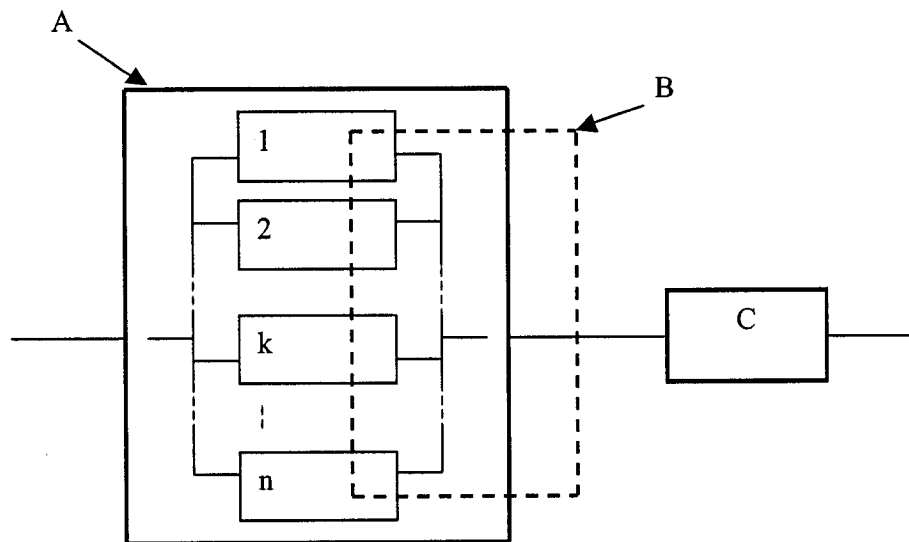
3.1 Introduction

In Chapter 2, the redundancy of safety units is used to increase the safety and reliability of a robot-safety system. In this chapter, the redundancy of n robots is considered along with one built-in safety unit to perform same or similar tasks. At least k robots must be operative for the robot-safety system success. More specifically, the robot system consists of n identical independent robots of which at least k ($<n$) robots must function normally for the robot system to operate successfully.

The chapter presents reliability and availability analyses of a mathematical model representing a robot-safety system having n -redundant robots and one built-in safety unit with common-cause failures. Expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure are developed. Two special case models (i.e., 1-out-of-2 and 2-out-of-3) are presented.

3.2 The Description of the Robot-Safety System

The block diagram of this robot-safety system having n -redundant robots and one built-in safety unit with common-cause failures is shown in Figure 3.1 and its corresponding state space diagram is given in Figure 3.2. The numerals and letters n and k in the boxes and ellipses of Figure 3.2 denote system states.



A: k -out-of- n robots

B: the safety unit

C: common-cause failures

Figure 3.1 Block diagram of the robot-safety system with common-cause failures.

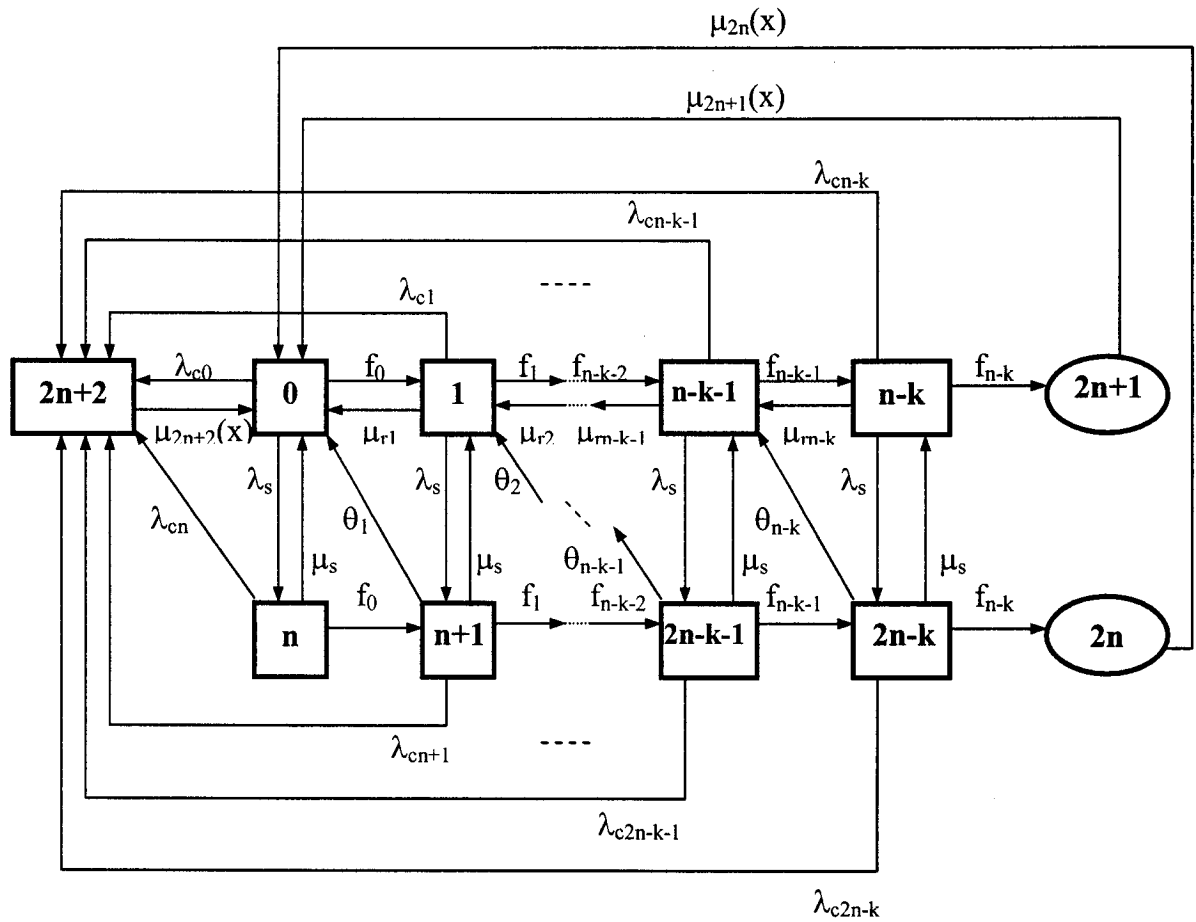


Figure 3.2 The state space diagram of the robot-safety system with common-cause failures. The numerals and letters n and k in squares, rectangles, and ellipses denote system states, and $f_i=(n-i)\lambda_r$, for $i=0, 1, \dots, n-k$.

At time $t=0$, all n -redundant robots and the safety unit start operating. The robot-safety system can fail due to the failure of the $(n-k+1)^{th}$ robot or the occurrence of a common-cause failure. Nonetheless, the robot-safety system will function normally until at least k robots and one safety unit are operating normally. The system goes through $(2n-2k+2)$ distinct operating states. A common-cause failure can occur only if at least two units (including at least one robot) are functioning successfully. Once the safety unit fails, the robots may continue to operate until the failure of the $(n-k+1)^{th}$ robot. The robot-safety system has a total of $(2n-2k+5)$ distinct states. It means the array of numerals

representing system states may be discontinuous. For example, for a 2-out-of-4 robots, the array of numerals representing system states are 0, 1, 2, 4, 5, 6, 8, 9, 10. More specifically, in this array of numerals, numerals 3 and 7 are missing. The degraded or fully failed robot-safety system may be repaired.

Assumptions

The following assumptions are associated with this model:

- (i) The robot-safety system is composed of n identical robots and one built-in safety unit.
- (ii) The redundant robots and the safety unit are operating simultaneously.
- (iii) All failures are statistically independent.
- (iv) All failure rates and the partially failed system repair rates are constant.
- (v) The repair of the safety unit has the priority over the repair of the robot when the overall system is in the partially failed operating state.
- (vi) The failed robot-safety system repair rates can be constant or non-constant.
- (vii) A repaired robot or safety unit is as good as new.
- (viii) The overall system fails when the $(n-k+1)^{\text{th}}$ robot fails or a common-cause failure occurs.

Notation

The following symbols are associated with the model:

- i i^{th} state of the overall robot-safety system: for $i=0$ means all n robots and the safety unit are in perfect working condition; for $i=mn+q$ (where $m=0, 1$, and $q=0, 1, \dots, n-k$) means $(n-q)$ robots and $(1-m)$ safety unit operating normally while q robots and m (where $m=0$ or 1) safety unit have failed; for $i=2n-k$

means k robots operating normally while $(n-k)$ robots and the safety unit have failed.

j j^{th} state of the failed robot-safety system: for $j=2n$ means the robot-safety system failed due to the malfunction of the $(n-k+1)^{\text{th}}$ robot while the safety unit has also failed; for $j=2n+1$ means the robot-safety system failed due to the failure of the $(n-k+1)^{\text{th}}$ robot while the safety unit is functioning; for $j=2n+2$ means the robot-safety system failed due to a common-cause failure.

t time

λ_s Constant failure rate of the safety unit.

λ_r Constant failure rate of a robot.

λ_{ci} Constant common-cause failure rate of the robot-safety system in state i ; for $i = 0, 1, \dots, n-k, n, \dots, 2n-k$, if $k=1$, $\lambda_{c2n-1}=0$.

μ_s Constant repair rate of the safety unit in state i ; for $i = n, n+1, \dots, 2n-k$.

μ_{ri} Constant repair rate of the robot in state i ; for $i = 1, 2, \dots, n-k$.

θ_i Constant repair rate from state $q=n+i$ to state $i-1$; for $i = 1, 2, \dots, n-k$.

Δx Finite repair time interval.

$\mu_{rj}(x)$ Time-dependent repair rate when the failed robot-safety system is in state j and has an elapsed repair time of x ; for $j=2n, 2n+1, 2n+2$.

$p_j(x,t)\Delta x$ The probability that at time t , the failed robot-safety system is in state j and the elapsed repair time lies in the interval $[x, x+\Delta x]$; for $j=2n, 2n+1, 2n+2$.

pdf Probability density function.

$z_j(x)$ pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x ; for $j=2n, 2n+1, 2n+2$.

$P_i(t)$	Probability that the robot-safety system is in state i at time t ; for $i = 0, 1, \dots, n-k, n, \dots, 2n-k$.
$P_j(t)$	Probability that the robot-safety system is in state j at time t ; for $j=2n, 2n+1, 2n+2$.
P_i	Steady-state probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, n-k, n, \dots, 2n-k$.
P_j	Steady-state probability that the robot-safety system is in state j ; for $j=2n, 2n+1, 2n+2$.
s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, n-k, n, \dots, 2n-k$.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j ; for $j=2n, 2n+1, 2n+2$.
$AV_{rs}(s)$	Laplace transform of the robot-safety system availability when the robot-safety system working with the safety unit.
$AV_r(s)$	Laplace transform of the robot-safety system availability when the robot-safety system working with or without the safety unit.
$AV_{rs}(t)$	Robot-safety system time-dependent availability when the robot-safety system working with the safety unit.
$AV_r(t)$	Robot-safety system time-dependent availability when the robot-safety system working with or without the safety unit.
$SSAV_{rs}$	Robot-safety system steady state availability when the robot-safety system working with the safety unit.

- SSAV_r Robot-safety system steady state availability when the robot-safety system working with or without the safety unit.
- R_{rs}(s) Laplace transform of the robot-safety system reliability when the robot-safety system working with the safety unit.
- R_r(s) Laplace transform of the robot-safety system reliability when the robot-safety system working with or without the safety unit.
- R_{rs}(t) Robot-safety system reliability when the robot-safety system working with the safety unit.
- R_r(t) Robot-safety system reliability when the robot-safety system working with or without the safety unit.
- MTTF_{rs} Robot-safety system mean time to failure when the robot-safety system working with the safety unit.
- MTTF_r Robot-safety system mean time to failure when the robot-safety system working with or without the safety unit.
- σ² Robot-safety system variance of time to failure when the robot-safety system working with or without the safety unit.

3.3 Generalized Robot-Safety System Analysis

Using the supplementary method [100,101], the system of Equations associated with the model in Figure 3.2 can be expressed as follows:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_r P_1(t) + \mu_s P_n(t) + \theta_1 P_{n+1}(t) + \sum_{j=2n}^{2n+2} \int_0^{\infty} P_j(x,t) \mu_{r_j}(x) dx \quad (3.1)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1) \lambda_r P_{i-1}(t) + \mu_{r+1} P_{i+1}(t) + \mu_s P_{n+i}(t) + \theta_{i+1} P_{n+i+1}(t) \quad (3.2)$$

(for $i = 1, 2, \dots, n-k-1$)

$$\frac{dP_{n-k}(t)}{dt} + a_{n-k}P_{n-k}(t) = (k+1)\lambda_r P_{n-k-1}(t) + \mu_s P_{2n-k}(t) \quad (3.3)$$

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_0(t) \quad (3.4)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (2n-i+1)\lambda P_{i-1}(t) + \lambda_s P_{i-n}(t) \quad (3.5)$$

(for $i = n+1, n+2, \dots, 2n-k-1$)

$$\frac{dP_{2n-k}(t)}{dt} + a_{2n-k} P_{2n-k}(t) = (k+1)\lambda_r P_{2n-k-1}(t) + \lambda_s P_{n-k}(t) \quad (3.6)$$

where

$$a_0 = n\lambda_r + \lambda_{c_0} + \lambda_s$$

$$a_i = (n-i)\lambda_r + \lambda_{c_i} + \lambda_s + \mu_{r_i} \quad (\text{for } i = 1, 2, \dots, n-k-1)$$

$$a_{n-k} = k\lambda_r + \lambda_{c_{n-k}} + \lambda_s + \mu_{r_{n-k}}$$

$$a_n = n\lambda_r + \lambda_{c_n} + \mu_s$$

$$a_i = (2n-i)\lambda_r + \theta_{i-n} + \mu_s + \lambda_{c_i} \quad (\text{for } i = n+1, n+2, \dots, 2n-k-1)$$

$$a_{2n-k} = k\lambda_r + \theta_{n-k} + \mu_s + \lambda_{c_{2n-k}} \quad (\text{If } k=1, \lambda_{c_{2n-1}} = 0)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_{r_j}(x)P_j(x,t) = 0 \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.7)$$

The associated boundary conditions are as follows:

$$P_{2n}(0,t) = k\lambda_r P_{2n-k}(t) \quad (3.8)$$

$$P_{2n+1}(0,t) = k\lambda_r P_{n-k}(t) \quad (3.9)$$

$$P_{2n+2}(0,t) = \sum_{i=0}^{n-k} \lambda_{c_i} P_i(t) + \sum_{i=n}^{2n-k} \lambda_{c_i} P_i(t) \quad (\text{If } k=1, \lambda_{c_{2n-1}} = 0) \quad (3.10)$$

At time $t=0$, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

Unfortunately, it is very difficult to obtain general formulas for robot-safety system reliability and availability using Equations (3.1)-(3.10). However, for special values of k and n , Equations (3.1)-(3.10) can be solved. This is demonstrated for ($k=1, n=2$) and ($k=2, n=3$) subsequently as special case models.

Setting $\theta_i=0$ (for $i = 1,2,\dots,n-k$) in Figure 3.2, which means the repair of the safety unit has the priority over the repair of the robot during the robot-safety system up states, Equations (3.1)-(3.2) and (3.5)-(3.6) become:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_n(t) + \sum_{j=2n}^{2n+2} \int_0^{\infty} P_j(x,t) \mu_{rj}(x) dx \quad (3.11)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1) \lambda_r P_{i-1}(t) + \mu_{n+1} P_{i+1}(t) + \mu_s P_{n+i}(t) \quad (3.12)$$

(for $i = 1,2,\dots,n-k-1$)

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (2n-i+1) \lambda P_{i-1}(t) + \lambda_s P_{i-n}(t) \quad (3.13)$$

(for $i = n+1, n+2,\dots,2n-k-1$)

$$\frac{dP_{2n-k}(t)}{dt} + a_{2n-k} P_{2n-k}(t) = (k+1) \lambda_r P_{2n-k-1}(t) + \lambda_s P_{n-k}(t) \quad (3.14)$$

where

$$a_0 = n\lambda_r + \lambda_{c0} + \lambda_s$$

$$a_i = (n-i)\lambda_r + \lambda_{ci} + \lambda_s + \mu_{ri} \quad (\text{for } i = 1,2,\dots,n-k-1)$$

$$a_i = (2n-i)\lambda_r + \mu_s + \lambda_{ci} \quad (\text{for } i = n+1, n+2,\dots,2n-k-1)$$

$$a_{2n-k} = k\lambda_r + \mu_s + \lambda_{c2n-k} \quad (\text{If } k=1, \lambda_{c2n-1} = 0)$$

Equations (3.3)-(3.4) and (3.7)-(3.10) remain the same.

3.3.1 Time Dependant Availability Analysis (i.e., $\theta_i=0$, for $i=1, \dots, n-k$)

Using the Laplace transform technique and the initial conditions in Equations (3.3)-(3.4) and (3.7)-(3.14), we get

$$(s + a_0)P_0(s) = 1 + \mu_s P_n(s) + \mu_{r1} P_1(s) + \sum_{j=2n}^{2n+2} \int_0^\infty P_j(x, s) \mu_{rj}(x) dx \quad (3.15)$$

$$(s + a_i)P_i(s) = (n - i + 1)\lambda_r P_{i-1}(s) + \mu_{r+i} P_{i+1}(s) + \mu_s P_{n+i}(s) \quad (3.16)$$

(for $i = 1, 2, \dots, n - k - 1$)

$$(s + a_{n-k})P_{n-k}(s) = (k + 1)\lambda_r P_{n-k-1}(s) + \mu_s P_{2n-k}(s) \quad (3.17)$$

$$(s + a_n)P_n(s) = \lambda_s P_0(s) \quad (3.18)$$

$$(s + a_i)P_i(s) = (2n - i + 1)\lambda_r P_{i-1}(s) + \lambda_s P_{i-n}(s) \quad (3.19)$$

(for $i = n + 1, n + 2, \dots, 2n - k - 1$)

$$(s + a_{2n-k})P_{2n-k}(s) = (k + 1)\lambda_r P_{2n-k-1}(s) + \lambda_s P_{n-k}(s) \quad (3.20)$$

$$sP_j(x, s) + \frac{\partial P_j(x, s)}{\partial x} + \mu_{rj}(x)P_j(x, s) = 0 \quad (\text{for } j = 2n, 2n + 1, 2n + 2) \quad (3.21)$$

$$P_{2n}(0, s) = k\lambda_r P_{2n-k}(s) \quad (3.22)$$

$$P_{2n+1}(0, s) = k\lambda_r P_{n-k}(s) \quad (3.23)$$

$$P_{2n+2}(0, s) = \sum_{i=0}^{n-k} \lambda_{ci} P_i(s) + \sum_{i=n}^{2n-k} \lambda_{ci} P_i(s) \quad (\text{if } k = 1, \lambda_{c2n-1} = 0) \quad (3.24)$$

Solving differential Equation (3.21), we get the following expression:

$$P_j(x, s) = P_j(0, s)e^{-sx} \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] \quad (3.25)$$

(for $j = 2n, 2n + 1, 2n + 2$)

Since

$$P_j(s) = \int_0^{\infty} P_j(x,s) dx \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.26)$$

and together with Equation (3.25), we get

$$P_j(s) = P_j(0,s) \frac{1 - Z_j(s)}{s} \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.27)$$

where

$$\frac{1 - Z_j(s)}{s} = P_j(0,s) \int_0^{\infty} e^{-sx} \exp[-\int_0^x \mu_{r_j}(\delta) d\delta] dx \quad (3.28)$$

(for $j = 2n, 2n+1, 2n+2$)

$$Z_j(s) = \int_0^{\infty} e^{-sx} z_j(x) dx \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.29)$$

$$z_j(x) = \exp[-\int_0^x \mu_{r_j}(\delta) d\delta] \mu_{r_j}(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

The Laplace transforms of the probabilities of all the system states add up to $1/s$, i.e.,

$$\sum_{i=0}^{n-k} P_i(s) + \sum_{i=n}^{2n-k} P_i(s) + \sum_{j=2n}^{2n+2} P_j(s) = \frac{1}{s} \quad (3.30)$$

Solving Equations (3.16)-(3.20), (3.22)-(3.24), (3.27), and (3.30), we get

$$P_0(s) = [s(1 + \sum_{i=1}^{n-k} W_i + \frac{\lambda_s}{s + a_n} + \sum_{i=n+1}^{2n-k} V_{i-n} + \sum_{j=2n}^{2n+2} a_j \frac{1 - Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (3.31)$$

$$P_i(s) = W_i P_0(s) \quad (\text{for } i = 1, 2, \dots, n-k) \quad (3.32)$$

$$P_n(s) = \frac{\lambda_s}{s + a_n} P_0(s) \quad (3.33)$$

$$P_i(s) = V_{i-n} P_0(s) \quad (\text{for } i = n+1, n+2, \dots, 2n-k) \quad (3.34)$$

$$P_j(s) = \frac{a_j[1-Z_j(s)]}{s} P_0(s) \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.35)$$

where

$$k_i = \frac{(n-i+1)\lambda_r}{\left[s + a_i - \frac{\lambda_s \mu_s}{s + a_{n+i}} - \mu_{r+1} \left(k_{i+1} + \frac{\lambda_s L_{i+1}}{s + a_{n+i}}\right)\right]} \quad (\text{for } i = 1, 2, \dots, n-k-1)$$

$$L_i = \frac{\mu_s + \mu_{r+1} L_{i+1}}{s + a_{n+i}} k_i \quad (\text{for } i = 1, 2, \dots, n-k-1)$$

$$k_{n-k} = \frac{(k+1)\lambda_r}{s + a_{n-k} - \frac{\lambda_s \mu_s}{s + a_{2n-k}}}$$

$$L_{n-k} = \frac{\mu_s}{s + a_{2n-k}} k_{n-k}$$

$$M_i = \frac{\lambda_s k_i}{s + a_{n+i}} \quad (\text{for } i = 1, 2, \dots, n-k-1)$$

$$N_i = \frac{(n-i+1)\lambda_r + \lambda_s L_i}{s + a_{n+i}} \quad (\text{for } i = 1, 2, \dots, n-k-1)$$

$$M_{n-k} = \frac{\lambda_s k_{n-k}}{s + a_{2n-k}}$$

$$N_{n-k} = \frac{(k+1)\lambda_r + \lambda_s L_{n-k}}{s + a_{2n-k}}$$

$$W_1 = k_1 + \frac{\lambda_s L_1}{s + a_n}$$

$$V_1 = M_1 + \frac{\lambda_s N_1}{s + a_n}$$

$$W_i = k_i W_{i-1} + L_i V_{i-1} \quad (\text{for } i = 2, 3, \dots, n-k)$$

$$V_i = M_i W_{i-1} + N_i V_{i-1} \quad (\text{for } i = 2, 3, \dots, n-k)$$

$$a_{2n} = k\lambda_r V_{n-k}$$

$$a_{2n+1} = k\lambda_r W_{n-k}$$

$$a_{2n+2} = \lambda_{c0} + \sum_{i=1}^{n-k} \lambda_{ci} W_i + \lambda_{cn} \frac{\lambda_s}{s + a_n} + \sum_{i=n+1}^{2n-k} \lambda_{ci} V_{i-n} \quad (\text{if } k=1, \lambda_{c2n-1} = 0)$$

$$H = s \left(1 + \sum_{i=1}^{n-k} W_i + \frac{\lambda_s}{s + a_n} + \sum_{i=n+1}^{2n-k} V_{i-n} + \sum_{j=2n}^{2n+2} a_j \frac{1 - Z_j(s)}{s} \right) \quad (3.36)$$

Thus, the Laplace transform of the robot-safety system availability with the working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^{n-k} P_i(s) = \frac{1 + \sum_{i=1}^{n-k} W_i}{H} \quad (3.37)$$

Similarly, the Laplace transform of the robot-safety system availability with or without the working safety unit is given by

$$AV_r(s) = \sum_{i=0}^{n-k} P_i(s) + \sum_{i=n}^{2n-k} P_i(s) = \frac{1 + \frac{\lambda_s}{s + a_n} + \sum_{i=1}^{n-k} (W_i + V_i)}{H} \quad (3.38)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions into Equations (3.37) and (3.38), and taking the inverse Laplace transforms of the resulting equations, we can get the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

3.3.2 Steady State Availability Analysis (i.e., $\theta_i=0$, for $i=1, \dots, n-k$)

As time approaches infinity, state probabilities reach the steady state. Thus, Equations (3.3)-(3.4) and (3.7)-(3.14) reduce to Equations (3.39) – (3.48), respectively.

$$a_0 P_0 = \mu_{r1} P_1 + \mu_s P_n + \sum_{j=2n}^{2n+2} \int_0^\infty P_j(x) \mu_{rj}(x) dx \quad (3.39)$$

$$a_i P_i = (n-i+1)\lambda_r P_{i-1} + \mu_{r(i+1)} P_{i+1} + \mu_s P_{n+i} \quad (\text{for } i=1, \dots, n-k-1) \quad (3.40)$$

$$a_{n-k}P_{n-k} = (k+1)\lambda_r P_{n-k-1} + \mu_s P_{2n-k} \quad (3.41)$$

$$a_n P_n = \lambda_s P_0 \quad (3.42)$$

$$a_i P_i = (2n-i+1)\lambda_r P_{i-1} + \lambda_s P_{i-n} \quad (\text{for } i = n+1, \dots, 2n-k-1) \quad (3.43)$$

$$a_{2n-k} P_{2n-k} = (k+1)\lambda_r P_{2n-k-1} + \lambda_s P_{n-k} \quad (3.44)$$

$$\frac{dP_j(x)}{dx} + \mu_{rj}(x)P_j(x) = 0 \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.45)$$

$$P_{2n}(0) = k\lambda_r P_{2n-k} \quad (3.46)$$

$$P_{2n+1}(0) = k\lambda_r P_{n-k} \quad (3.47)$$

$$P_{2n+2}(0) = \sum_{i=0}^{n-k} \lambda_{ci} P_i + \sum_{i=n}^{2n-k} \lambda_{ci} P_i \quad (\text{if } k=1, \lambda_{c2n-1} = 0) \quad (3.48)$$

Solving Equation (3.45), we get

$$P_j(x) = P_j(0) \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.49)$$

After a failure the robot-safety system under repair steady state condition probability is given by

$$P_j = \int_0^{\infty} P_j(x) dx \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.50)$$

Substituting Equation (3.49) into Equation (3.50), yields

$$P_j = P_j(0) E_j[x] \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.51)$$

where

$$\begin{aligned} E_j(x) &= \int_0^{\infty} \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] dx \\ &= \int_0^{\infty} x z_j(x) dx \end{aligned} \quad (3.52)$$

which is the mean time to robot-safety system repair when the failed robot-safety system is in state j and has an elapsed repair time of x.

Substituting Equations (3.46) - (3.48) into Equation (3.51), yields:

$$P_{2n} = k\lambda_r P_{2n-k} E_{2n}[x] \quad (3.53)$$

$$P_{2n+1} = k\lambda_r P_{n-k} E_{2n+1}[x] \quad (3.54)$$

$$P_{2n+2} = \sum_{i=0}^{n-k} \lambda_{ci} P_i E_{2n+2}[x] + \sum_{i=n}^{2n-k} \lambda_{ci} P_i E_{2n+2}[x] \quad (\text{if } k=1, \lambda_{c2n-1} = 0) \quad (3.55)$$

Using Equations (3.40)-(3.44), and (3.53) - (3.55), together with

$$\sum_{i=0}^{n-k} P_i + \sum_{i=n}^{2n-k} P_i + \sum_{j=2n}^{2n+2} P_j = 1 \quad (3.56)$$

we get the following general steady state probability solutions:

$$P_0 = \left(1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n} + \sum_{j=2n}^{2n+2} a_{yj} E_j[x]\right)^{-1} = \frac{1}{G} \quad (3.57)$$

$$P_i = Q_i P_0 \quad (\text{for } i=1, \dots, n-k) \quad (3.58)$$

$$P_n = \frac{\lambda_s}{a_n} P_0 \quad (i=n) \quad (3.59)$$

$$P_i = U_{i-n} P_0 \quad (i=n+1, \dots, 2n-k) \quad (3.60)$$

$$P_j = a_{yj} E_j[x] P_0 \quad (\text{for } j=2n, 2n+1, 2n+2) \quad (3.61)$$

where

$$C_i = \lim_{s \rightarrow 0} k_i \quad (\text{for } i=1, 2, \dots, n-k)$$

$$D_i = \lim_{s \rightarrow 0} L_i \quad (\text{for } i=1, 2, \dots, n-k)$$

$$C_{n-k} = \frac{(k+1)\lambda_r}{a_{n-k} - \frac{\lambda_s \mu_s}{a_{2n-k}}}$$

$$D_{n-k} = \frac{\mu_s}{a_{2n-k}} C_{n-k}$$

$$C_i = \frac{(n-i+1)\lambda_r}{\left[a_i - \frac{\lambda_s \mu_s}{a_{n+i}} - \mu_{r+1} \left(C_{i+1} + \frac{\lambda_s D_{i+1}}{a_{n+i}}\right)\right]} \quad (\text{for } i=1,2,\dots,n-k-1)$$

$$D_i = \frac{\mu_s + \mu_{r+1} D_{i+1}}{a_{n+i}} C_i \quad (\text{for } i=1,2,\dots,n-k-1)$$

$$E_i = \lim_{s \rightarrow 0} M_i \quad (\text{for } i=1,2,\dots,n-k)$$

$$F_i = \lim_{s \rightarrow 0} N_i \quad (\text{for } i=1,2,\dots,n-k)$$

$$E_{n-k} = \frac{\lambda_s C_{n-k}}{a_{2n-k}}$$

$$F_{n-k} = \frac{(k+1)\lambda_r + \lambda_s D_{n-k}}{a_{2n-k}}$$

$$E_i = \frac{\lambda_s C_i}{a_{n+i}} \quad (\text{for } i=1,2,\dots,n-k-1)$$

$$F_i = \frac{(n-i+1)\lambda_r + \lambda_s D_i}{a_{n+i}} \quad (\text{for } i=1,2,\dots,n-k-1)$$

$$Q_i = \lim_{s \rightarrow 0} W_i \quad (\text{for } i=1,2,\dots,n-k)$$

$$U_i = \lim_{s \rightarrow 0} V_i \quad (\text{for } i=1,2,\dots,n-k)$$

$$Q_1 = C_1 + \frac{\lambda_s D_1}{a_n}$$

$$U_1 = E_1 + \frac{\lambda_s F_1}{a_n}$$

$$Q_i = C_i Q_{i-1} + D_i U_{i-1} \quad (\text{for } i=2,3,\dots,n-k)$$

$$U_i = E_i Q_{i-1} + F_i U_{i-1} \quad (\text{for } i=2,3,\dots,n-k)$$

$$a_{s2n} = k\lambda_r U_{n-k}$$

$$\begin{aligned}
a_{s2n+1} &= k\lambda_r Q_{n-k} \\
a_{s2n+2} &= \lambda_{c0} + \sum_{i=1}^{n-k} \lambda_{ci} Q_i + \lambda_{cn} \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} \lambda_{ci} U_{i-n} \quad (\text{if } k=1, \lambda_{c2n-1} = 0) \\
G &= 1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n} + \sum_{j=2n}^{2n+2} a_{sj} E_j[x] \quad (3.62)
\end{aligned}$$

The general expression for steady state availability of the robot-safety system with the working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^{n-k} P_i = \frac{1 + \sum_{i=1}^{n-k} Q_i}{G} \quad (3.63)$$

Similarly, the general expression for steady state availability of the robot-safety system with or without the working safety unit is given by

$$SSAV_r = \sum_{i=0}^{n-k} P_i + \sum_{i=n}^{2n-k} P_i = \frac{1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n}}{G} \quad (3.64)$$

For different failed system repair time distributions, the values of G are obtained as follows:

- (i) When the failed robot-safety system repair time x is exponentially distributed, the probability density function of the repair time is

$$z_j(x) = \mu_{rj} e^{-\mu_{rj}x} \quad (\mu_{rj} > 0, j = 2n, 2n+1, 2n+2) \quad (3.65)$$

where x is the repair time, and μ_j is the constant repair rate of state j. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the exponential distribution is given by

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \frac{1}{\mu_{rj}} \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.66)$$

Substituting Equation (3.66) into Equation (3.62), we get

$$G = G_e = 1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n} + \sum_{j=2n}^{2n+2} (a_{sj} \frac{1}{\mu_{\eta_j}}) \quad (3.67)$$

- (ii) When the failed robot-safety system repair time x is gamma distributed, the probability density function of the repair time is

$$z_j(x) = \frac{\mu_{\eta_j} (\mu_{\eta_j} x)^{\beta-1} e^{-\mu_{\eta_j} x}}{\Gamma(\beta)} \quad (\beta > 0, \quad j = 2n, 2n+1, 2n+2) \quad (3.68)$$

where x is the repair time, $\Gamma(\beta)$ is the gamma function, and β and μ_j are the shape and scale parameters, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the gamma distribution is given by

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \frac{\beta}{\mu_{\eta_j}} \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.69)$$

Substituting Equation (3.69) into Equation (3.62), we get

$$G = G_g = 1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n} + \sum_{j=2n}^{2n+2} (a_{sj} \frac{\beta}{\mu_{\eta_j}}) \quad (3.70)$$

- (iii) When the failed robot-safety system repair time x is Weibull distributed, the probability density function of the repair time is expressed by

$$z_j(x) = \mu_{\eta_j} \beta x^{\beta-1} e^{-\mu_{\eta_j} (x)^\beta} \quad (\beta > 0, \quad j = 2n, 2n+1, 2n+2) \quad (3.71)$$

where x is the repair time, and β and μ_j are the shape and scale parameters of the Weibull distribution, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Weibull distribution is given by

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \left(\frac{1}{\mu_{\eta_j}}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.72)$$

Substituting Equation (3.72) into Equation (3.62), yields

$$G = G_w = 1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n} + \sum_{j=2n}^{2n+2} [a_{sj} \left(\frac{1}{\mu_{rj}}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right)] \quad (3.73)$$

(iv) When the failed robot-safety system repair time x is Rayleigh distributed, the probability density function of the Rayleigh distribution is expressed by

$$z_j(x) = \mu_{rj} x e^{-\mu_{rj} x^2 / 2} \quad (\mu_{rj} > 0, \quad j = 2n, 2n+1, 2n+2) \quad (3.74)$$

where x is the repair time, and μ_j the scale parameter. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Rayleigh distribution is

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \sqrt{\frac{\pi}{4\mu_{rj}}} \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.75)$$

Substituting Equation (3.75) into Equation (3.62), we get

$$G = G_r = 1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n} + \sum_{j=2n}^{2n+2} (a_{sj} \sqrt{\frac{\pi}{4\mu_{rj}}}) \quad (3.76)$$

(v) When the robot-safety system repair time x is lognormal distributed, the probability density function of the repair time is

$$z_j(x) = \frac{1}{x \sigma_{y_j} \sqrt{2\pi}} e^{\left[\frac{-(\ln x - \mu_{y_j})^2}{2\sigma_{y_j}^2}\right]} \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.77)$$

where x is the repair time, and $\ln x$ is the natural logarithms of x with a mean and variance μ and σ^2 , respectively. The conditions on parameters are as follows:

$$\begin{aligned} \sigma_{y_j} &= \ln \sqrt{1 + \left(\frac{\sigma_{x_j}}{\mu_{x_j}}\right)^2}, \\ \mu_{y_j} &= \ln \sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \end{aligned} \quad (3.78)$$

(for $j = 2n, 2n+1, 2n+2$)

Hence, the failed robot-safety system mean time to repair, $E_j[x]$, for the lognormal distribution is

$$E_j[x] = e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \quad (\text{for } j = 2n, 2n+1, 2n+2) \quad (3.79)$$

Substituting Equation (3.79) into Equation (3.62), we get

$$G = G_i = 1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n} + \sum_{j=2n}^{2n+2} [a_{y_j} e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})}] \quad (3.80)$$

3.3.3 System Reliability, MTTF, and Variance of Time to Failure (i.e., $\theta_i=0$, for $i=1, \dots, n-k$)

Setting $\mu_{2n}(x)=\mu_{2n+1}(x)=\mu_{2n+2}(x)=0$ in Figure 3.2 and with the aid of Markov method, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_n(t) \quad (3.81)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_r P_{i-1}(t) + \mu_{r(i+1)} P_{i+1}(t) + \mu_s P_{n+i}(t) \quad (3.82)$$

(for $i = 1, 2, \dots, n-k-1$)

$$\frac{dP_{n-k}(t)}{dt} + a_{n-k} P_{n-k}(t) = (k+1)\lambda_r P_{n-k-1}(t) + \mu_s P_{2n-k}(t) \quad (3.83)$$

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_0(t) \quad (3.84)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (2n-i+1)\lambda P_{i-1}(t) + \lambda_s P_{i-n}(t) \quad (3.85)$$

(for $i = n+1, n+2, \dots, 2n-k-1$)

$$\frac{dP_{2n-k}(t)}{dt} + a_{2n-k} P_{2n-k}(t) = (k+1)\lambda_r P_{2n-k-1}(t) + \lambda_s P_{n-k}(t) \quad (3.86)$$

$$\frac{dP_{2n}(t)}{dt} = k\lambda_r P_{2n-k}(t) \quad (3.87)$$

$$\frac{dP_{2n+1}(t)}{dt} = k\lambda_r P_{n-k}(t) \quad (3.88)$$

$$\frac{dP_{2n+2}(t)}{dt} = \sum_{i=0}^{n-k} \lambda_{ci} P_i(t) + \sum_{i=n}^{2n-k} \lambda_{ci} P_i(t) \quad (\text{if } k=1, \lambda_{c2n-1} = 0) \quad (3.89)$$

At time $t=0$, $P_0(0)=1$, and all other initial state probabilities are equal to zero. Taking the Laplace transforms of Equations (3.82) - (3.89), and solving the resulting set of Equations, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^{n-k} W_i + \frac{\lambda_s}{s + a_n} + \sum_{i=n+1}^{2n-k} V_{i-n} + \sum_{j=2n}^{2n+2} \frac{a_j}{s})]^{-1} \quad (3.90)$$

$$P_i(s) = W_i P_0(s) \quad (\text{for } i = 1, 2, \dots, n-k) \quad (3.91)$$

$$P_n(s) = \frac{\lambda_s}{s + a_n} P_0(s) \quad (3.92)$$

$$P_i(s) = V_{i-n} P_0(s) \quad (\text{for } i = n+1, n+2, \dots, 2n-k) \quad (3.93)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (3.94)$$

The Laplace transform of the robot-safety system reliability with the working safety unit is

$$R_{rs}(s) = \sum_{i=0}^{n-k} P_i(s) = (1 + \sum_{i=1}^{n-k} W_i) P_0(s) \quad (3.95)$$

Utilizing Equation (3.95), the robot-safety system mean time to the failure is obtained as follows [103]:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{(1 + \sum_{i=1}^{n-k} Q_i)}{\sum_{j=2n}^{2n+2} a_{sj}} \quad (3.96)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without the working safety unit is

$$R_r(s) = \sum_{i=0}^{n-k} P_i(s) + \sum_{i=n}^{2n-k} P_i(s) = (1 + \sum_{i=1}^{n-k} W_i + \frac{\lambda_s}{s + a_n} + \sum_{i=n+1}^{2n-k} V_{i-n}) P_0(s) \quad (3.97)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{(1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n})}{\sum_{j=2n}^{2n+2} a_{sj}} \quad (3.98)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (3.95) and (3.97).

The robot-safety system variance of time to failure is expressed by [103]

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-n})(1 + \sum_{i=1}^{n-k} Q_i + \frac{\lambda_s}{a_n} + \sum_{i=n+1}^{2n-k} U_{i-3} + \sum_{j=2n}^{2n+2} a_{dj})}{(\sum_{j=2n}^{2n+2} a_{sj})^2} \\ &\quad - \frac{2(\sum_{i=1}^{n-k} W_{di} - \frac{\lambda_s}{a_n^2} + \sum_{i=n+1}^{2n-k} V_{di-n})}{\sum_{j=2n}^{2n+2} a_{sj}} - (MTTF_r)^2 \end{aligned} \quad (3.99)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$W_{di} = \lim_{s \rightarrow 0} W_i' \quad (\text{for } i = 1, 2, \dots, n-k)$$

$$V_{di-n} = \lim_{s \rightarrow 0} V_{i-n}' \quad (\text{for } i = n+1, n+2, \dots, 2n-k)$$

$$a_{d2n} = \lim_{s \rightarrow 0} a_{2n}' = k\lambda_r V_{dn-1}$$

$$a_{d2n+1} = \lim_{s \rightarrow 0} a_{2n+1}' = k\lambda_r W_{dn-1}$$

$$a_{d2n+2} = \lim_{s \rightarrow 0} a_{2n+2}' = \sum_{i=1}^{n-k} \lambda_{ci} W_{di} - \lambda_{cn} \frac{\lambda_s}{a_n^2} + \sum_{i=n+1}^{2n-k} \lambda_{ci} V_{di-n} \quad (\text{if } k=1, \lambda_{c2n-1}=0)$$

W_i' denotes the derivative of W_i with respect to s .

V_{i-n}' denotes the derivative of V_{i-n} with respect to s .

a_j' denotes the derivative of a_j with respect to s .

3.4 Special Case Model 3.I: (k=1, n=2, $\theta_1 \neq 0$)

For $k=1$ and $n=2$ in Figures 3.1 and 3.2, the model becomes for a system having two redundant robots and one built-in safety unit. Since $k=1$, it is a parallel redundancy, the corresponding system of Equations can be extracted from Equations (3.1)-(3.10) by setting $k=1$ and $n=2^*$.

3.4.1 Time Dependant Availability Analysis (k=1, n=2, $\theta_1 \neq 0$)

With the aid of Laplace transforms and initial conditions, from Equations (3.1)-(3.10) for $k=1$ and $n=2$, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + Y_1 + \frac{\lambda_s}{s + a_2} + Y_2 + \sum_{j=4}^6 a_j \frac{1 - Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (3.100)$$

$$P_1(s) = Y_1 P_0(s) \quad (3.101)$$

$$P_2(s) = \frac{\lambda_s}{s + a_2} P_0(s) \quad (3.102)$$

$$P_3(s) = Y_2 P_0(s) \quad (3.103)$$

* Detailed analysis is provided in *Appendix B*.

$$P_j(s) = \frac{a_j[1 - Z_j(s)]}{s} P_0(s) \quad (\text{for } j = 4, 5, 6) \quad (3.104)$$

where

$$a_0 = 2\lambda_r + \lambda_{c0} + \lambda_s$$

$$a_1 = \lambda_r + \lambda_{c1} + \lambda_s + \mu_{r1}$$

$$a_2 = 2\lambda_r + \lambda_{c2} + \mu_s$$

$$a_3 = \lambda_r + \theta_1 + \mu_s$$

$$Y_1 = \frac{2\lambda_r(s + a_3 + \frac{\lambda_s\mu_s}{s + a_2})}{(s + a_1)(s + a_3) - \lambda_s\mu_s}$$

$$Y_2 = \frac{2\lambda_r\lambda_s(1 + \frac{s + a_1}{s + a_2})}{(s + a_1)(s + a_3) - \lambda_s\mu_s}$$

$$a_4 = \lambda_r Y_2$$

$$a_5 = \lambda_r Y_1$$

$$a_6 = \lambda_{c0} + \lambda_{c1} Y_1 + \frac{\lambda_{c2}\lambda_s}{s + a_2}$$

$$H = s(1 + Y_1 + \frac{\lambda_s}{s + a_2} + Y_2 + \sum_{j=4}^6 a_j \frac{1 - Z_j(s)}{s})$$

Thus, the Laplace transform of the robot-safety system availability with the working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^1 P_i(s) = \frac{(1 + Y_1)}{H} \quad (3.105)$$

Similarly, the Laplace transform of the robot-safety system availability with or without the working safety unit is given by

$$AV_r(s) = \sum_{i=0}^3 P_i(s) = \frac{1 + Y_1 + \frac{\lambda_s}{s + a_2} + Y_2}{H} \quad (3.106)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions in Equations (3.105) and (3.106), and taking the inverse Laplace transforms of the resulting equations, we can obtain the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

Setting:

$$\begin{aligned} \lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c0}=0.0002, \quad \lambda_{c1}=\lambda_{c2}=0.0001, \quad \theta_1=0.0010 \\ \mu_s=0.0009, \quad \mu_{r1}=0.0010, \quad \mu_{r4}=0.0011, \quad \mu_{r5}=0.0012, \quad \mu_{r6}=0.0006 \end{aligned}$$

in Equations (3.100)-(3.104), and (3.105)-(3.106), for exponential and gamma distributed failed system repair times, and using Maple computer program [102], the time-dependant plots of robot-safety system state probabilities and availabilities are shown in Figures 3.3-3.6, respectively.

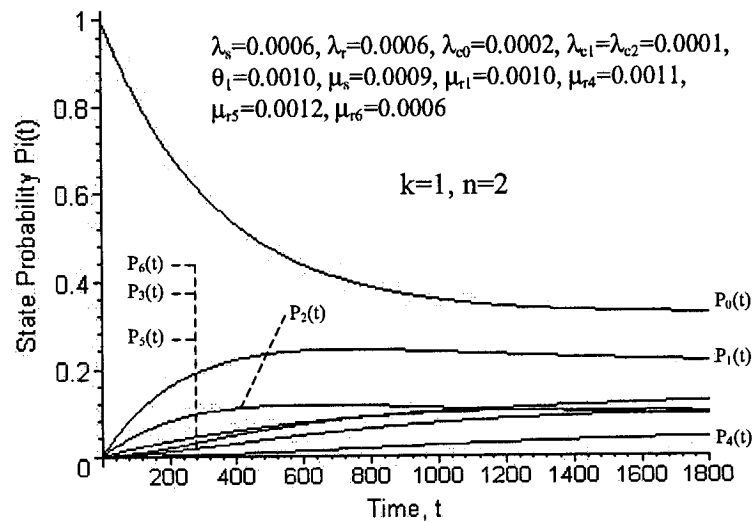


Figure 3.3 Time-dependent probability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

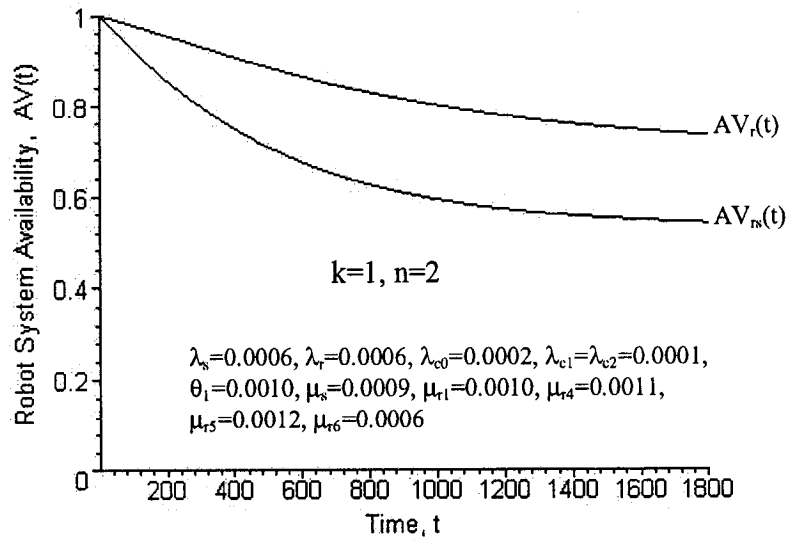


Figure 3.4 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

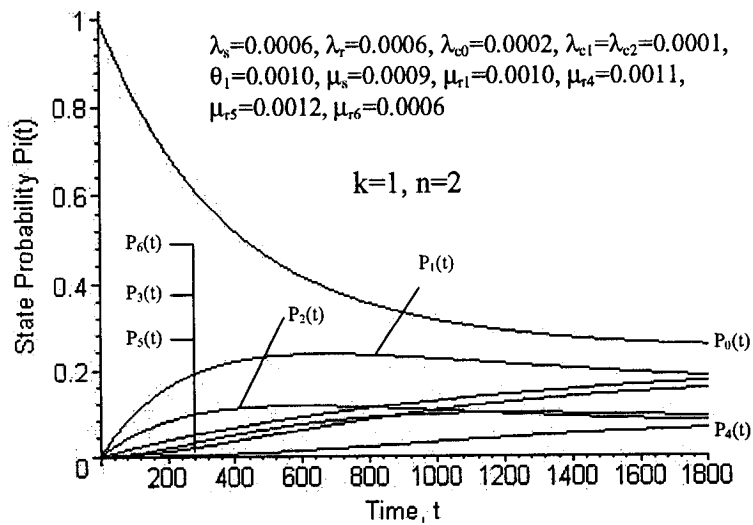


Figure 3.5 Time-dependent probability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

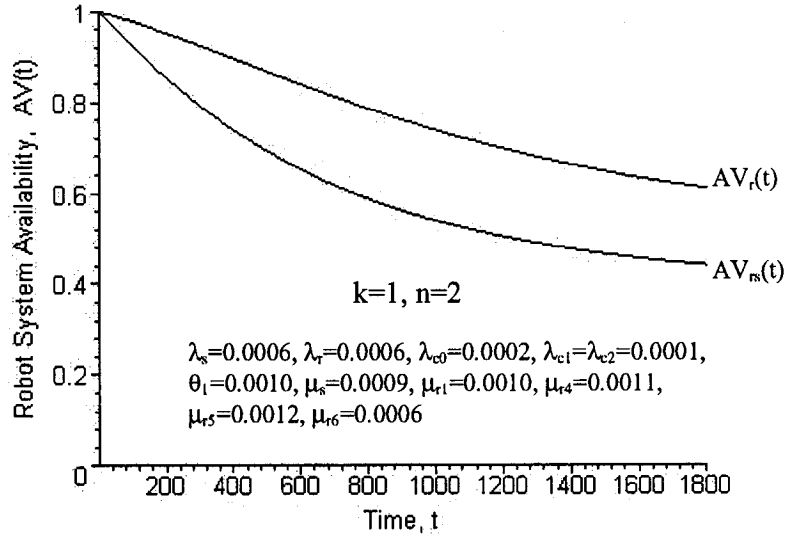


Figure 3.6 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

3.4.2 Steady State Availability Analysis ($k=1, n=2, \theta_1 \neq 0$)

As time approaches infinity, state probabilities reach the steady state. Thus, the steady state probabilities from Equations (3.1)-(3.10) for this special case model ($k=1, n=2$) are as follows:

$$P_0 = (1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4 + \sum_{j=4}^6 a_{y_j} E_j[x])^{-1} = \frac{1}{G} \quad (3.107)$$

$$P_1 = Y_3 P_0 \quad (3.108)$$

$$P_2 = \frac{\lambda_s}{a_2} P_0 \quad (3.109)$$

$$P_3 = Y_4 P_0 \quad (3.110)$$

$$P_j = a_{y_j} E_j[x] P_0 \quad (\text{for } j = 4, 5, 6) \quad (3.111)$$

where

$$Y_3 = \lim_{s \rightarrow 0} Y_1 = \frac{2\lambda_r(a_3 + \frac{\lambda_s \mu_s}{a_2})}{a_1 a_3 - \lambda_s \mu_s}$$

$$Y_4 = \lim_{s \rightarrow 0} Y_2 = \frac{2\lambda_r \lambda_s (1 + \frac{a_1}{a_2})}{a_1 a_3 - \lambda_s \mu_s}$$

$$a_{s4} = \lambda_r Y_4$$

$$a_{s5} = \lambda_r Y_3$$

$$a_{s6} = \lambda_{c0} + \lambda_{c1} Y_3 + \frac{\lambda_{c2} \lambda_s}{a_2}$$

$$G = 1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4 + \sum_{j=4}^6 a_{sj} E_j[x] \quad (3.112)$$

The steady state availability of the robot-safety system with the working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^1 P_i = \frac{1 + Y_3}{G} \quad (3.113)$$

Similarly, the steady state availability of the robot-safety system with or without the working safety unit is given by

$$SSAV_r = \sum_{i=0}^3 P_i = \frac{1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4}{G} \quad (3.114)$$

Setting:

$$\lambda_s = 0.0006, \quad \lambda_r = 0.0006, \quad \lambda_{c1} = \lambda_{c2} = 0.0001, \quad \theta_1 = 0.0010$$

$$\mu_s = 0.0009, \quad \mu_{r1} = 0.0010, \quad \mu_{r4} = 0.0011, \quad \mu_{r5} = 0.0012, \quad \mu_{r6} = 0.0006$$

in Equation (3.113) and for Weibull distributed failed system repair times, the plots for $SSAV_{rs}$ are shown in Figure 3.7.

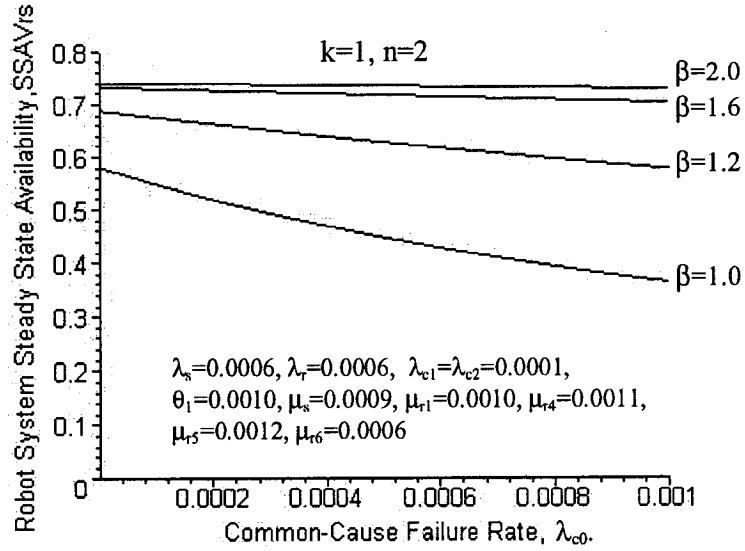


Figure 3.7 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair time distribution.

3.4.3 Robot-Safety System Reliability, MTTF, and Variance of time to failure ($k=1, n=2, \theta_1 \neq 0$)

Setting $\mu_{r4}(x)=\mu_{r5}(x)=\mu_{r6}(x)=0$ in this special case model (i.e., $k=1, n=2$), we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + Y_1 + \frac{\lambda_s}{s + a_2} + Y_2 + \sum_{j=4}^6 \frac{a_j}{s})]^{-1} \quad (3.115)$$

$$P_1(s) = Y_1 P_0(s) \quad (3.116)$$

$$P_2(s) = \frac{\lambda_s}{s + a_2} P_0(s) \quad (3.117)$$

$$P_3(s) = Y_2 P_0(s) \quad (3.118)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 4, 5, 6) \quad (3.119)$$

Thus, the Laplace transform of the robot-safety system reliability with the working safety unit is

$$R_{rs}(s) = \sum_{i=0}^1 P_i(s) = (1 + Y_1)P_0(s) \quad (3.120)$$

Using Equation (3.120), we get the following expression for the robot-safety system mean time to failure:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + Y_3}{\sum_{j=4}^6 a_{sj}} \quad (3.121)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without the working safety unit is

$$R_r(s) = \sum_{i=0}^3 P_i(s) = (1 + Y_1 + \frac{\lambda_s}{s + a_2} + Y_2)P_0(s) \quad (3.122)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4}{\sum_{j=4}^6 a_{sj}} \quad (3.123)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (3.120) and (3.122).

The robot-safety system variance of time to failure is expressed by

$$\begin{aligned}
\sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\
&= \frac{2(1+Y_3 + \frac{\lambda_s}{a_2} + Y_4)(1+Y_3 + \frac{\lambda_s}{a_2} + Y_4 + \lambda_r Y_{d1} + \lambda_r Y_{d2} + \lambda_{c1} Y_{d1} - \frac{\lambda_{c2} \lambda_s}{a_2^2})}{(\sum_{j=4}^6 a_{sj})^2} \quad (3.124) \\
&\quad - \frac{2(Y_{d1} - \frac{\lambda_s}{a_2^2} + Y_{d2})}{\sum_{j=4}^6 a_{sj}} - (MTTF_r)^2
\end{aligned}$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$Y_{d1} = \lim_{s \rightarrow 0} Y_1' = \frac{2\lambda_r(1 - \frac{\lambda_s \mu_s}{a_2^2})}{a_1 a_3 - \lambda_s \mu_s} - \frac{2\lambda_r(a_3 + \frac{\lambda_s \mu_s}{a_2})(a_3 + a_1)}{(a_1 a_3 - \lambda_s \mu_s)^2}$$

$$Y_{d2} = \lim_{s \rightarrow 0} Y_2' = \frac{2\lambda_r \lambda_s (\frac{1}{a_2} - \frac{a_1}{a_2^2})}{a_1 a_3 - \lambda_s \mu_s} - \frac{2\lambda_r \lambda_s (1 + \frac{a_1}{a_2})(a_3 + a_1)}{(a_1 a_3 - \lambda_s \mu_s)^2}$$

Y_1' denotes the derivative of Y_1 with respect to s .

Y_2' denotes the derivative of Y_2 with respect to s .

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c1}=\lambda_{c2}=0.0001,$$

$$\theta_1=0.0010 \quad \mu_s=0.0009, \quad \mu_{r1}=0.0010, \quad \mu_{r4}=\mu_{r5}=\mu_{r6}=0$$

in Equations (3.121) and (3.123), the plots of the robot-safety system mean time to failure, as a function of common-cause failure rate (λ_{c0}), are shown in Figure 3.8.

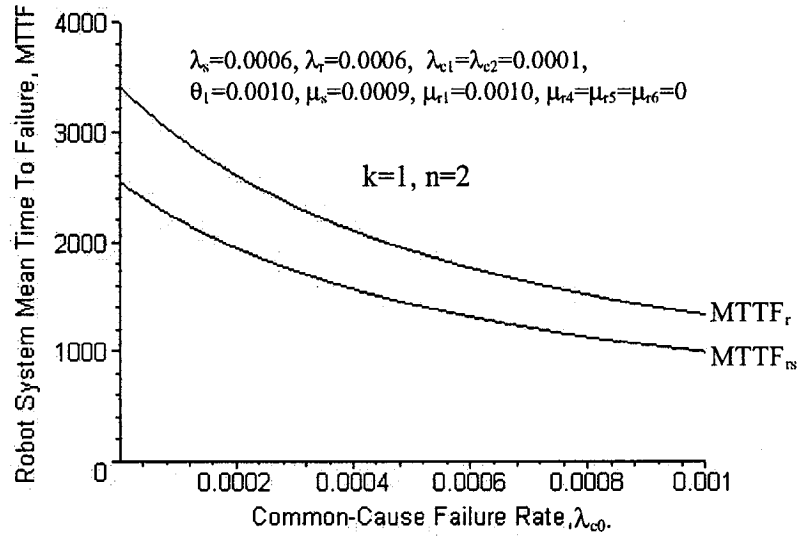


Figure 3.8 Mean time to failure plots of an irreparable robot-safety system as a function of common-cause failure rate(λ_{c0}).

3.5 Special Case Model 3.II: ($k=2, n=3, \theta_1 \neq 0$)

For $k=2$ and $n=3$ in Figures 3.1 and 3.2, the model becomes for a system having three redundant robots and one built-in safety unit. At least two robots must function successfully for the robot system success. The corresponding system of Equations can be extracted from Equations (3.1)-(3.10) by setting $k=2$ and $n=3$ *

3.5.1 Time Dependant Availability Analysis ($k=2, n=3, \theta_1 \neq 0$)

With the aid of Laplace transforms and initial conditions, from Equations (3.1)-(3.10) for $k=2$ and $n=3$, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + Y_5 + \frac{\lambda_s}{s + a_3} + Y_6 + \sum_{j=6}^8 a_j \frac{1 - Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (3.125)$$

$$P_1(s) = Y_5 P_0(s) \quad (3.126)$$

* Detailed analysis is provided in *Appendix C*.

$$P_3(s) = \frac{\lambda_s}{s+a_3} P_0(s) \quad (3.127)$$

$$P_4(s) = Y_6 P_0(s) \quad (3.128)$$

$$P_j(s) = \frac{a_j [1 - Z_j(s)]}{s} P_0(s) \quad (\text{for } j = 6, 7, 8) \quad (3.129)$$

where

$$a_0 = 3\lambda_r + \lambda_{c0} + \lambda_s$$

$$a_1 = 2\lambda_r + \lambda_{c1} + \lambda_s + \mu_{r1}$$

$$a_3 = 3\lambda_r + \lambda_{c3} + \mu_s$$

$$a_4 = 2\lambda_r + \theta_1 + \lambda_{c4} + \mu_s$$

$$Y_5 = \frac{3\lambda_r (s + a_4 + \frac{\lambda_s \mu_s}{s + a_3})}{(s + a_1)(s + a_4) - \lambda_s \mu_s}$$

$$Y_6 = \frac{3\lambda_r \lambda_s (1 + \frac{s + a_1}{s + a_3})}{(s + a_1)(s + a_4) - \lambda_s \mu_s}$$

$$a_6 = 2\lambda_r Y_6$$

$$a_7 = 2\lambda_r Y_5$$

$$a_8 = \lambda_{c0} + \lambda_{c1} Y_5 + \frac{\lambda_{c3} \lambda_s}{s + a_3} + \lambda_{c4} Y_6$$

$$H = s(1 + Y_5 + \frac{\lambda_s}{s + a_3} + Y_6 + \sum_{j=6}^8 a_j \frac{1 - Z_j(s)}{s})$$

Thus, the Laplace transform of the robot-safety system availability with the working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^1 P_i(s) = \frac{(1+Y_5)}{H} \quad (3.130)$$

Similarly, the Laplace transform of the robot-safety system availability with or without the working safety unit is given by

$$AV_r(s) = \sum_{i=0}^1 P_i(s) + \sum_{i=3}^4 P_i(s) = \frac{1+Y_5 + \frac{\lambda_s}{s+a_3} + Y_6}{H} \quad (3.131)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions in Equations (3.130) and (3.131), and taking the inverse Laplace transforms of the resulting equations, we can obtain the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c0}=0.0002, \quad \lambda_{c1}=\lambda_{c3}=\lambda_{c4}=0.0001, \quad \theta_1=0.0010$$

$$\mu_s=0.0009, \quad \mu_{r1}=0.0010, \quad \mu_{r6}=0.0011, \quad \mu_{r7}=0.0012, \quad \mu_{r8}=0.0006$$

in Equations (3.125)-(3.129), and (3.130)-(3.131), for exponential and gamma distributed failed system repair times, and using Maple computer program [102], the time-dependant plots of robot-safety system state probabilities and availabilities are shown in Figures 3.9-3.12, respectively.

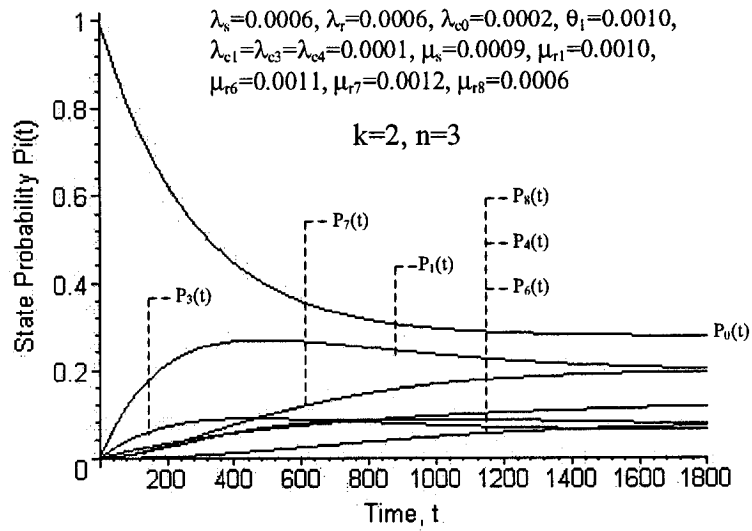


Figure 3.9 Time-dependent probability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

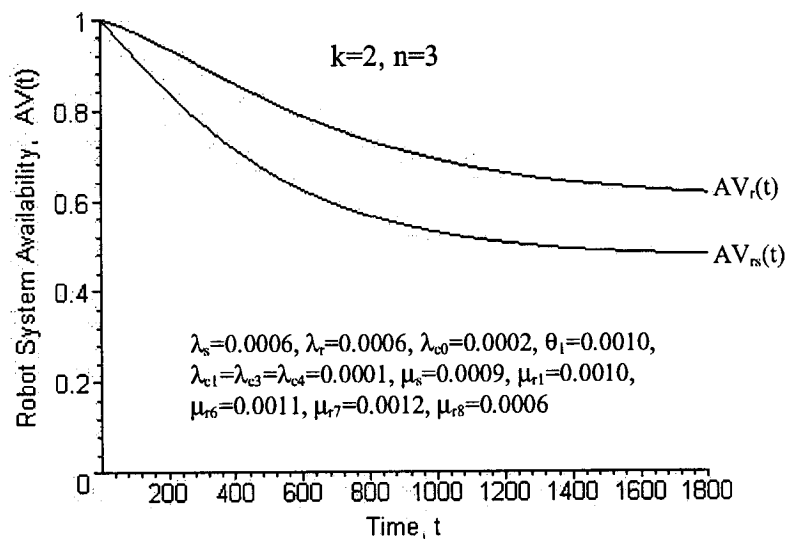


Figure 3.10 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

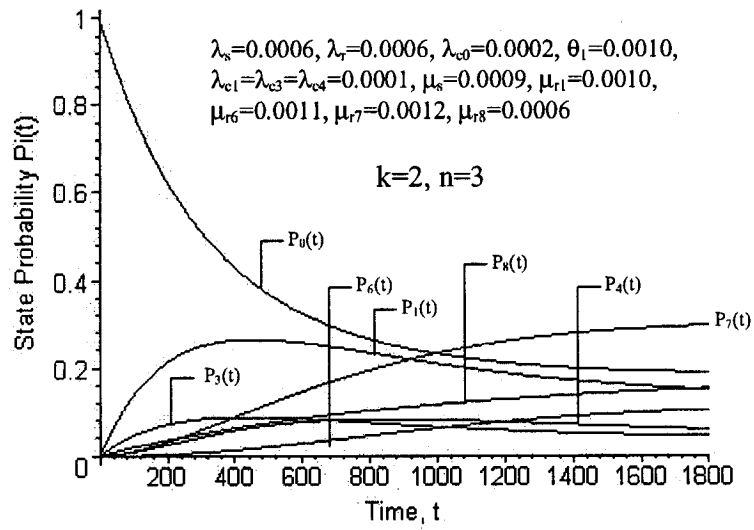


Figure 3.11 Time-dependent probability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

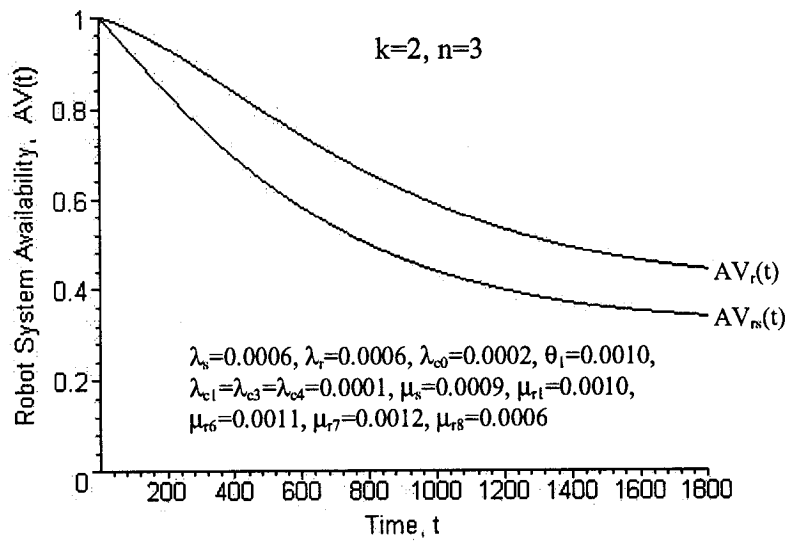


Figure 3.12 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

3.5.2 Steady State Availability Analysis ($k=2, n=3, \theta_1 \neq 0$)

As time approaches infinity, state probabilities reach the steady state. Thus, the steady state probabilities from Equations (3.1)-(3.10) for this special case model (k=2, n=3) are as follows:

$$P_0 = (1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8 + \sum_{j=6}^8 a_{sj} E_j[x])^{-1} = \frac{1}{G} \quad (3.132)$$

$$P_1 = Y_7 P_0 \quad (3.133)$$

$$P_3 = \frac{\lambda_s}{a_3} P_0 \quad (3.134)$$

$$P_4 = Y_8 P_0 \quad (3.135)$$

$$P_j = a_{sj} E_j[x] P_0 \quad (\text{for } j = 6, 7, 8) \quad (3.136)$$

where

$$Y_7 = \lim_{s \rightarrow 0} Y_5 = \frac{3\lambda_r (a_4 + \frac{\lambda_s \mu_s}{a_3})}{a_1 a_4 - \lambda_s \mu_s}$$

$$Y_8 = \lim_{s \rightarrow 0} Y_6 = \frac{3\lambda_r \lambda_s (1 + \frac{a_1}{a_3})}{a_1 a_4 - \lambda_s \mu_s}$$

$$a_{s6} = 2\lambda_r Y_8$$

$$a_{s7} = 2\lambda_r Y_7$$

$$a_{s8} = \lambda_{c0} + \lambda_{c1} Y_7 + \frac{\lambda_{c3} \lambda_s}{a_3} + \lambda_{c4} Y_8$$

$$G = 1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8 + \sum_{j=6}^8 a_{sj} E_j[x] \quad (3.137)$$

The steady state availability of the robot-safety system with the working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^1 P_i = \frac{1+Y_7}{G} \quad (3.138)$$

Similarly, the steady state availability of the robot-safety system with or without the working safety unit is given by

$$SSAV_r = \sum_{i=0}^1 P_i + \sum_{i=3}^4 P_i = \frac{1+Y_7 + \frac{\lambda_s}{a_3} + Y_8}{G} \quad (3.139)$$

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c1}=\lambda_{c3}=\lambda_{c4}=0.0001, \quad \theta_1=0.0010$$

$$\mu_s=0.0009, \quad \mu_{r1}=0.0010, \quad \mu_{r6}=0.0011, \quad \mu_{r7}=0.0012, \quad \mu_{r8}=0.0006$$

in Equation (3.138) and for Weibull distributed failed system repair times, the plots for $SSAV_{rs}$ are shown in Figure 3.13.

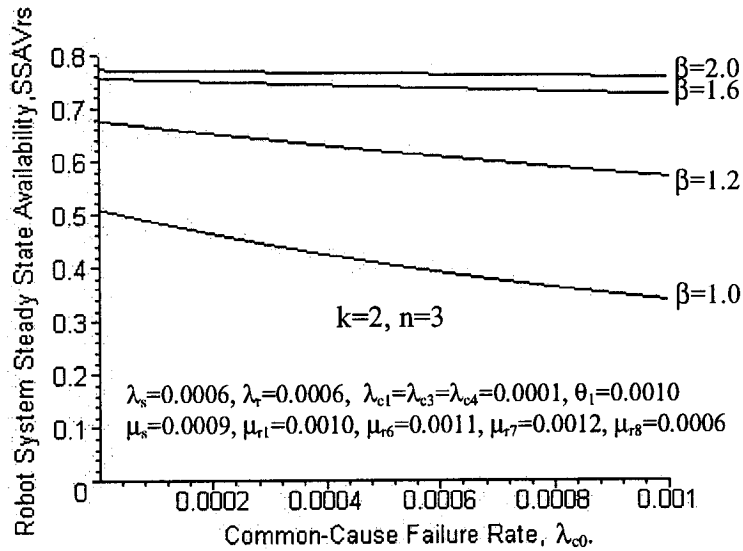


Figure 3.13 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair time distribution.

3.5.3 Robot-Safety System Reliability, MTTF, and Variance of time to failure

$$(k=2, n=3, \theta_1 \neq 0)$$

Setting $\mu_{r6}(x)=\mu_{r7}(x)=\mu_{r8}(x)=0$ in this special case model ($k=2, n=3$), we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + Y_5 + \frac{\lambda_s}{s + a_3} + Y_6 + \sum_{j=6}^8 \frac{a_j}{s})]^{-1} \quad (3.140)$$

$$P_1(s) = Y_5 P_0(s) \quad (3.141)$$

$$P_3(s) = \frac{\lambda_s}{s + a_3} P_0(s) \quad (3.142)$$

$$P_4(s) = Y_6 P_0(s) \quad (3.143)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 6, 7, 8) \quad (3.144)$$

Thus, the Laplace transform of the robot-safety system reliability with the working safety unit is

$$R_{rs}(s) = \sum_{i=0}^1 P_i(s) = (1 + Y_5) P_0(s) \quad (3.145)$$

Using Equation (3.145), we get the following expression for the robot-safety system mean time to failure:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + Y_5}{\sum_{j=6}^8 a_{sj}} \quad (3.146)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without the working safety unit is

$$R_r(s) = \sum_{i=0}^1 P_i(s) + \sum_{i=3}^4 P_i(s) = (1 + Y_5 + \frac{\lambda_s}{s + a_3} + Y_6) P_0(s) \quad (3.147)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8}{\sum_{j=6}^8 a_{sj}} \quad (3.148)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (3.145) and (3.147).

The robot-safety system variance of time to failure is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8)(1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8 + 2\lambda_r Y_{d5} + 2\lambda_r Y_{d6} + \lambda_{c1} Y_{d5} - \frac{\lambda_{c3} \lambda_s}{a_3^2} + \lambda_{c4} Y_{d6})}{(\sum_{j=6}^8 a_{sj})^2} \\ &\quad - \frac{2(Y_{d5} - \frac{\lambda_s}{a_3^2} + Y_{d6})}{\sum_{j=6}^8 a_{sj}} - (MTTF_r)^2 \end{aligned} \quad (3.149)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$Y_{d5} = \lim_{s \rightarrow 0} Y_5' = \frac{3\lambda_r(1 - \frac{\lambda_s \mu_s}{a_3^2})}{a_1 a_4 - \lambda_s \mu_s} - \frac{3\lambda_r(a_4 + \frac{\lambda_s \mu_s}{a_3})(a_4 + a_1)}{(a_1 a_4 - \lambda_s \mu_s)^2}$$

$$Y_{d6} = \lim_{s \rightarrow 0} Y_6' = \frac{3\lambda_r \lambda_s (\frac{1}{a_3} - \frac{a_1}{a_3^2})}{a_1 a_4 - \lambda_s \mu_s} - \frac{3\lambda_r \lambda_s (1 + \frac{a_1}{a_3})(a_4 + a_1)}{(a_1 a_4 - \lambda_s \mu_s)^2}$$

Y_5' denotes the derivative of Y_5 with respect to s .

Y_6' denotes the derivative of Y_6 with respect to s .

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c1}=\lambda_{c3}=\lambda_{c4}=0.0001,$$

$$\theta_1=0.0010 \quad \mu_s=0.0009, \quad \mu_{r1}=0.0010, \quad \mu_{r6}=\mu_{r7}=\mu_{r8}=0$$

in Equations (3.146) and (3.148), the plots of the robot-safety system mean time to failure, as a function of common-cause failure rate (λ_{c0}), are shown in Figure 3.14.

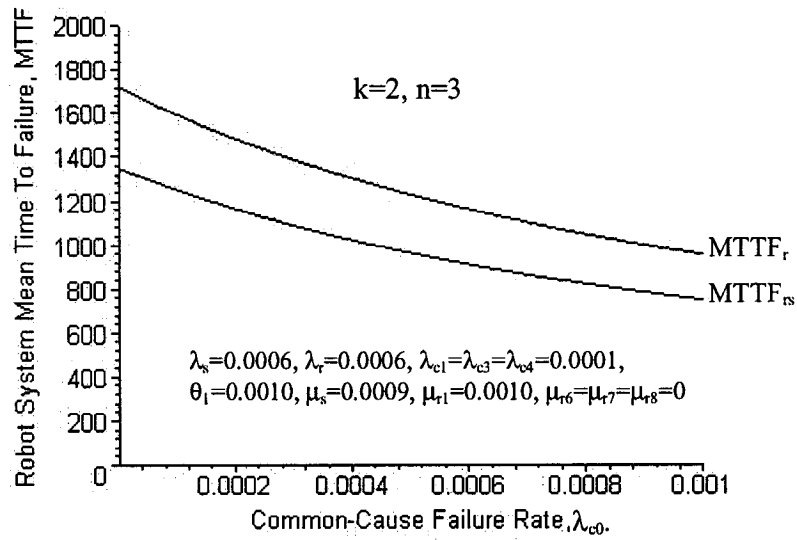


Figure 3.14 Mean time to failure plots of an irreparable robot-safety system as a function of common-cause failure rate(λ_{c0}).

**STOCHASTIC ANALYSIS OF A ROBOT-SAFETY SYSTEM
CONTAINING N-REDUNDANT ROBOTS AND
M-REDUNDANT BUILT-IN SAFETY UNITS
WITH COMMON-CAUSE FAILURES**

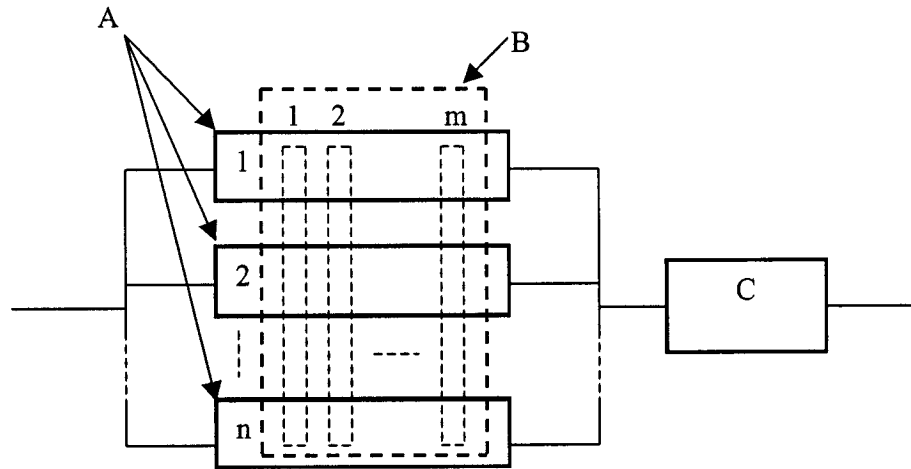
4.1 Introduction

In previous two chapters, either redundancy of safety units or redundancy of robots in a robot-safety system is considered. In this chapter, the redundancy of both robots and safety units together in a robot-safety system is studied.

Thus, the chapter presents a mathematical model to perform reliability and availability analyses of a robot-safety system having n -redundant robots and m -redundant built-in safety units with common-cause failures. As in Chapters 2 and 3, Markov and supplementary variable methods were used to develop generalized expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure. A special case model (i.e., $m=2$, $n=2$) is presented.

4.2 The Description of the Robot-Safety System

The block diagram of this robot-safety system having n -redundant robots and m -redundant built-in safety units with common-cause failures is shown in Figure 4.1 and its corresponding state space diagram is given in Figure 4.2. The numerals and letters n and m in the boxes and ellipses of Figure 4.2 denote system states.



A: n-identical robots

B: m-identical safety units

C: common-cause failures

Figure 4.1 Block diagram of the robot-safety system with common-cause failures.

At time $t=0$, all n -redundant robots and m -redundant safety units start operating. The robot-safety system can fail due to the failure of all n robots or the occurrence of a common-cause failure. Nonetheless, the robot-safety system will function normally until at least one safety unit and one robot are operating normally. The system goes through $[(m+1)n]$ distinct operating states. A common-cause failure can occur only if at least two units (including at least one robot) are functioning successfully. Once all m safety units fail, the robots may continue to operate until the failure of the n^{th} robot. The degraded or fully failed robot-safety system may be repaired.

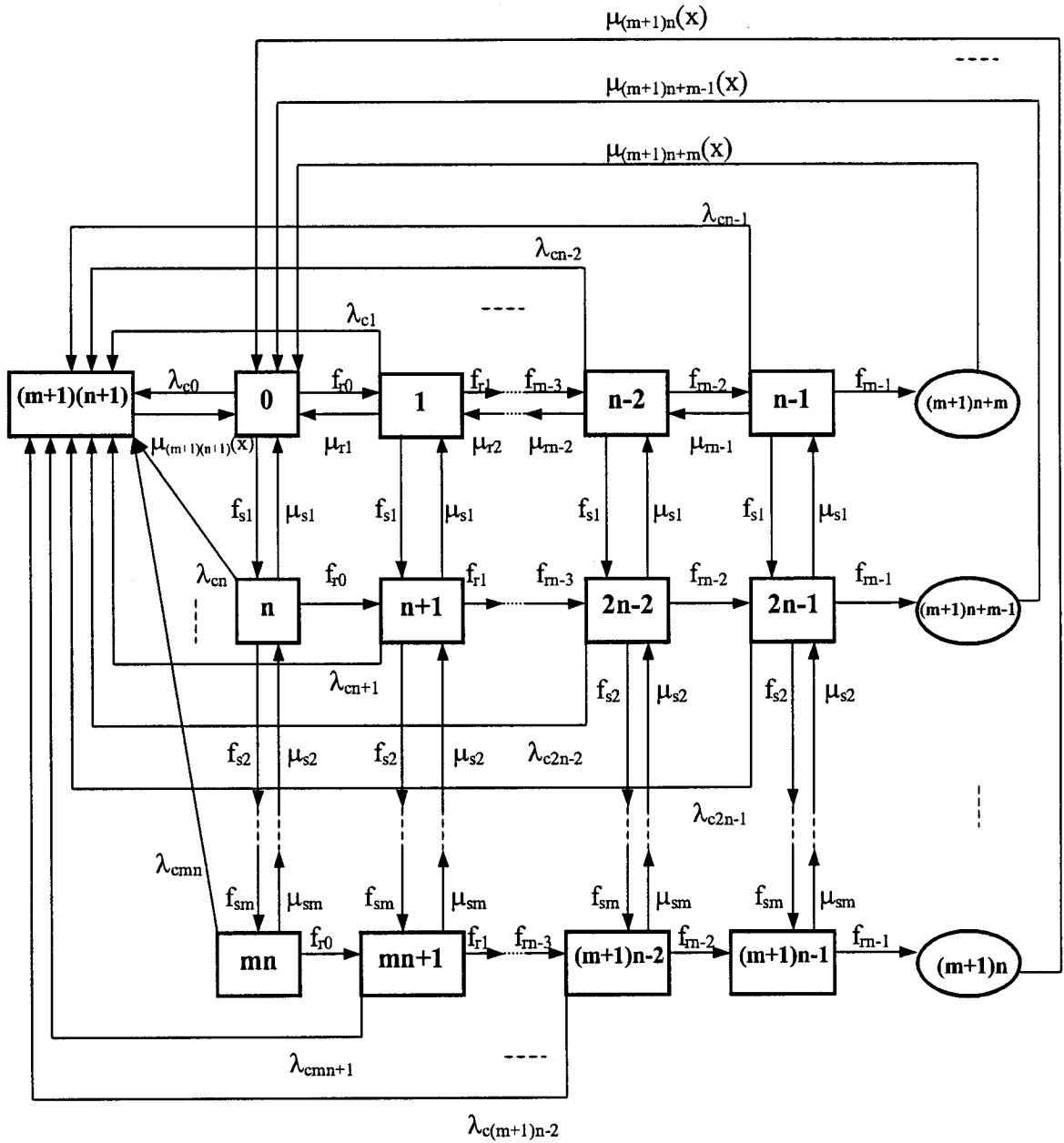


Figure 4.2 The state space diagram of the robot-safety system with common-cause failures. The numerals and letters n and m in squares, rectangles, and ellipses denote system states, and $f_{sk}=(m-k+1)\lambda_s$, for $k=1, 2, \dots, m$; $f_{rq}=(n-q)\lambda_r$, for $q=0, 1, 2, \dots, n-1$.

Assumptions

The following assumptions are associated with this model:

- (i) The robot-safety system is composed of n identical robots and m identical safety units.
- (ii) The redundant robots and safety units are operating simultaneously.
- (iii) All failures are statistically independent.
- (iv) All failure rates and the partially failed system repair rates are constant.
- (v) The repair of the safety unit has the priority over the repair of the robot when the overall system is in the partially failed operating state.
- (vi) The failed robot-safety system repair rates can be constant or non-constant.
- (vii) A repaired robot or safety unit is as good as new.
- (viii) The overall system fails when all the active robots fail or a common-cause failure occurs.

Notation

The following symbols are associated with the model:

- i i^{th} state of the overall robot-safety system: for $i=0$ means all n robots and m safety units are in perfect working condition; for $i=kn+q$ (where $k=0, 1, \dots, m$, and $q=0, 1, \dots, n-1$) means $(n-q)$ robots and $(m-k)$ safety units operating normally while q robots and k safety units have failed; for $i=(m+1)n-1$ means only one robot operating normally while $(n-1)$ robots and all m safety units have failed.
- j j^{th} state of the failed robot-safety system: for $j=(m+1)n$ means n robots and m safety units have failed; for $j=(m+1)n+k$ (where $k=0, 1, \dots, m$) means n robots

failed while k safety units are functioning; for $j=(m+1)(n+1)$ means the robot-safety system failed due to a common-cause failure.

t	time
λ_s	Constant failure rate of a safety unit.
λ_r	Constant failure rate of a robot.
λ_{ci}	Constant common-cause failure rate of the robot-safety system in state i ; for $i = 0, 1, 2, \dots, (m+1)n-2$.
μ_{sk}	Constant repair rate of the safety unit in state $i=kn+q$; for $k = 1, 2, \dots, m$ and $q=0, 1, \dots, n-1$.
μ_{ri}	Constant repair rate of the robot in state i ; for $i = 1, 2, \dots, n-1$.
Δx	Finite repair time interval.
$\mu_j(x)$	Time-dependent repair rate when the failed robot-safety system is in state j and has an elapsed repair time of x ; for $j=(m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$.
$p_j(x,t)\Delta x$	The probability that at time t , the failed robot-safety system is in state j and the elapsed repair time lies in the interval $[x, x+\Delta x]$; for $j=(m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$.
pdf	Probability density function.
$z_j(x)$	pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x ; for $j=(m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$.
$P_i(t)$	Probability that the robot-safety system is in state i at time t ; for $i = 0, 1, \dots, (m+1)n-1$.
$P_j(t)$	Probability that the robot-safety system is in state j at time t ; for $j=(m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$.

- P_i Steady-state probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, (m+1)n-1$.
- P_j Steady-state probability that the robot-safety system is in state j ; for $j=(m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$.
- s Laplace transform variable.
- $P_i(s)$ Laplace transform of the probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, (m+1)n-1$.
- $P_j(s)$ Laplace transform of the probability that the robot-safety system is in state j ; for $j=(m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$.
- $AV_{rs}(s)$ Laplace transform of the robot-safety system availability when the robot-safety system working with at least one safety unit.
- $AV_r(s)$ Laplace transform of the robot-safety system availability when the robot-safety system working with or without the safety unit(s).
- $AV_{rs}(t)$ Robot-safety system time-dependent availability when the robot-safety system working with at least one safety unit.
- $AV_r(t)$ Robot-safety system time-dependent availability when the robot-safety system working with or without the safety unit(s).
- $SSAV_{rs}$ Robot-safety system steady state availability when the robot-safety system working with at least one safety unit.
- $SSAV_r$ Robot-safety system steady state availability when the robot-safety system working with or without the safety unit(s).
- $R_{rs}(s)$ Laplace transform of the robot-safety system reliability when the robot-safety system working with at least one safety unit.

- $R_r(s)$ Laplace transform of the robot-safety system reliability when the robot-safety system working with or without the safety unit(s).
- $R_{rs}(t)$ Robot-safety system reliability when the robot-safety system working with at least one safety unit.
- $R_r(t)$ Robot-safety system reliability when the robot-safety system working with or without the safety unit(s).
- $MTTF_{rs}$ Robot-safety system mean time to failure when the robot-safety system working with at least one safety unit.
- $MTTF_r$ Robot-safety system mean time to failure when the robot-safety system working with or without the safety unit(s).
- σ^2 Robot-safety system variance of time to failure when the robot-safety system working with or without the safety unit(s).

4.3 Generalized Robot-Safety System Analysis

Using the supplementary method [100,101], the system of Equations associated with the Figure 4.2 model can be expressed as follows:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_{s1} P_n(t) + \sum_{j=(m+1)n}^{(m+1)(n+1)} \int_0^\infty P_j(x,t) \mu_j(x) dx \quad (4.1)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1) \lambda_r P_{i-1}(t) + \mu_{ri+1} P_{i+1}(t) + \mu_{si} P_{n+i}(t) \quad (4.2)$$

(for $i = 1, 2, \dots, n-2$)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = 2\lambda_r P_{n-2}(t) + \mu_{s1} P_{2n-1}(t) \quad (\text{for } i = n-1) \quad (4.3)$$

$$\frac{dP_{kn}(t)}{dt} + a_{kn} P_{kn}(t) = (m-k+1) \lambda_s P_{kn-n}(t) + \mu_{sk+1} P_{kn+n}(t) \quad (4.4)$$

(for $k = 1, 2, \dots, m-1$)

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(k+1)n - i + 1] \lambda_r P_{i-1}(t) + (m-k+1) \lambda_s P_{i-n}(t) + \mu_{sk+1} P_{i+n}(t) \quad (4.5)$$

(for $\begin{cases} i = kn+1, kn+2, \dots, (k+1)n-2 \\ k = 1, 2, \dots, m-1 \end{cases}$)

$$\frac{dP_{(k+1)n-1}(t)}{dt} + a_{(k+1)n-1} P_{(k+1)n-1}(t) = 2\lambda_r P_{(k+1)n-2}(t) + (m-k+1) \lambda_s P_{kn-1}(t) + \mu_{sk+1} P_{(k+2)n-1}(t) \quad (4.6)$$

(for $k = 1, 2, \dots, m-1$)

$$\frac{dP_{mn}(t)}{dt} + a_{mn} P_{mn}(t) = \lambda_s P_{(m-1)n}(t) \quad (\text{for } i = mn) \quad (4.7)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(m+1)n - i + 1] \lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) \quad (4.8)$$

(for $i = mn+1, mn+2, \dots, (m+1)n-2$)

$$\frac{dP_{(m+1)n-1}(t)}{dt} + a_{(m+1)n-1} P_{(m+1)n-1}(t) = 2\lambda_r P_{(m+1)n-2}(t) + \lambda_s P_{mn-1}(t) \quad (4.9)$$

(for $i = (m+1)n-1$)

where

$$a_0 = n\lambda_r + \lambda_{c0} + m\lambda_s$$

$$a_i = (n-i)\lambda_r + \lambda_{ci} + m\lambda_s + \mu_{ri} \quad (\text{for } i = 1, 2, \dots, n-2)$$

$$a_{n-1} = \lambda_r + \lambda_{cn-1} + m\lambda_s + \mu_{m-1} \quad (\text{for } i = n-1)$$

$$a_{kn} = n\lambda_r + \lambda_{ckn} + (m-k)\lambda_s + \mu_{sk} \quad (\text{for } k = 1, 2, \dots, m-1)$$

$$a_i = [(k+1)n - i] \lambda_r + \lambda_{ci} + (m-k)\lambda_s + \mu_{sk}$$

$$\left(\text{for } \begin{cases} i = kn+1, kn+2, \dots, (k+1)n-2 \\ k = 1, 2, \dots, m-1 \end{cases} \right)$$

$$a_{(k+1)n-1} = \lambda_r + \lambda_{c(k+1)n-1} + (m-k)\lambda_s + \mu_{sk} \quad (\text{for } k = 1, 2, \dots, m-1)$$

$$a_{mn} = n\lambda_r + \lambda_{cmn} + \mu_{sm} \quad (\text{for } i = mn)$$

$$a_i = [(m+1)n-i]\lambda_r + \lambda_{ci} + \mu_{sm}$$

$$(\text{for } i = mn+1, mn+2, \dots, (m+1)n-2)$$

$$a_{(m+1)n-1} = \lambda_r + \mu_{sm} \quad (\text{for } i = (m+1)n-1)$$

$$\frac{\partial P_j(x, t)}{\partial t} + \frac{\partial P_j(x, t)}{\partial x} + \mu_j(x)P_j(x, t) = 0 \quad (4.10)$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1))$$

The associated boundary conditions are as follows:

$$P_j(0, t) = \lambda_r P_{[m+1-j+(m+1)n]n-1}(t) \quad (4.11)$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)n+m)$$

$$P_{(m+1)(n+1)}(0, t) = \sum_{i=0}^{(m+1)n-2} \lambda_{ci} P_i(t) \quad (4.12)$$

At time $t=0$, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

Unfortunately, it is very difficult to obtain general formulas for robot-safety system reliability and availability using Equations (4.1)-(4.12). However, for special values of n and m , Equations (4.1)-(4.12) can be solved. This is demonstrated for $(n=2, m=2)$ as a special case model, subsequently.

Setting $\mu_{ri}=0$ (for $i = 1, 2, \dots, n-1$) in Figure 4.2, which means robots are irreparable at the operable state of the robot-safety system, generalized expressions are developed. Thus, Equations (4.1)-(4.3) become:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{s1} P_n(t) + \sum_{j=(m+1)n}^{(m+1)(n+1)} \int_0^\infty P_j(x,t) \mu_j(x) dx \quad (4.13)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1) \lambda_r P_{i-1}(t) + \mu_{s1} P_{n+i}(t) \quad (4.14)$$

(for $i = 1, 2, \dots, n-2$)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = 2\lambda_r P_{n-2}(t) + \mu_{s1} P_{2n-1}(t) \quad (\text{for } i = n-1) \quad (4.15)$$

where

$$a_0 = n\lambda_r + \lambda_{c0} + m\lambda_s$$

$$a_i = (n-i)\lambda_r + \lambda_{ci} + m\lambda_s \quad (\text{for } i = 1, 2, \dots, n-2)$$

$$a_{n-1} = \lambda_r + \lambda_{cn-1} + m\lambda_s \quad (\text{for } i = n-1)$$

Equations (4.4)-(4.12) remain the same.

4.3.1 Time Dependant Availability Analysis (i.e., $\mu_{ri}=0$, for $i=1, \dots, n-1$)

Using the Laplace transform technique and the initial conditions in Equations (4.4) – (4.15), we get

$$(s + a_0)P_0(s) = 1 + \mu_{s1} P_n(s) + \sum_{j=(m+1)n}^{(m+1)(n+1)} \int_0^\infty P_j(x,s) \mu_j(x) dx \quad (4.16)$$

$$(s + a_i)P_i(s) = (n-i+1)\lambda_r P_{i-1}(s) + \mu_{s1} P_{n+i}(s) \quad (4.17)$$

(for $i = 1, 2, \dots, n-2$)

$$(s + a_{n-1})P_{n-1}(s) = 2\lambda_r P_{n-2}(s) + \mu_{s1} P_{2n-1}(s) \quad (\text{for } i = n-1) \quad (4.18)$$

$$(s + a_{kn})P_{kn}(s) = (m-k+1)\lambda_s P_{kn-n}(s) + \mu_{sk+1} P_{kn+n}(s) \quad (4.19)$$

(for $k = 1, 2, \dots, m-1$)

$$(s + a_i)P_i(s) = [(k+1)n - i + 1]\lambda_r P_{i-1}(s) + (m - k + 1)\lambda_s P_{i-n}(s) + \mu_{sk+1} P_{i+n}(s) \quad (4.20)$$

$$(for \begin{cases} i = kn + 1, kn + 2, \dots, (k+1)n - 2 \\ k = 1, 2, \dots, m - 1 \end{cases})$$

$$(s + a_{(k+1)n-1})P_{(k+1)n-1}(s) = 2\lambda_r P_{(k+1)n-2}(s) + (m - k + 1)\lambda_s P_{kn-1}(s) + \mu_{sk+1} P_{(k+2)n-1}(s) \quad (4.21)$$

$$(for \ k = 1, 2, \dots, m - 1)$$

$$(s + a_{mn})P_{mn}(s) = \lambda_s P_{(m-1)n}(s) \quad (for \ i = mn) \quad (4.22)$$

$$(s + a_i)P_i(s) = [(m+1)n - i + 1]\lambda_r P_{i-1}(s) + \lambda_s P_{i-n}(s) \quad (4.23)$$

$$(for \ i = mn + 1, mn + 2, \dots, (m+1)n - 2)$$

$$(s + a_{(m+1)n-1})P_{(m+1)n-1}(s) = 2\lambda_r P_{(m+1)n-2}(s) + \lambda_s P_{mn-1}(s) \quad (4.24)$$

$$sP_j(x, s) + \frac{\partial P_j(x, s)}{\partial x} + \mu_j(x)P_j(x, s) = 0 \quad (4.25)$$

$$(for \ j = (m+1)n, (m+1)n + 1, \dots, (m+1)(n+1))$$

$$P_j(0, s) = \lambda_r P_{[m+1-j+(m+1)n]n-1}(s) \quad (4.26)$$

$$(for \ j = (m+1)n, (m+1)n + 1, \dots, (m+1)n + m)$$

$$P_{(m+1)(n+1)}(0, s) = \sum_{i=0}^{(m+1)n-2} \lambda_{ci} P_i(s) \quad (4.27)$$

Solving differential Equation (4.25), we get the following expression:

$$P_j(x, s) = P_j(0, s)e^{-sx} \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \quad (4.28)$$

$$(for \ j = (m+1)n, (m+1)n + 1, \dots, (m+1)(n+1))$$

Since

$$P_j(s) = \int_0^\infty P_j(x, s) dx \quad (\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)) \quad (4.29)$$

and together with Equation (4.28), we get

$$P_j(s) = P_j(0, s) \frac{1 - Z_j(s)}{s} \quad (4.30)$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1))$$

where

$$\frac{1 - Z_j(s)}{s} = P_j(0, s) \int_0^\infty e^{-sx} \exp[-\int_0^x \mu_j(\delta) d\delta] dx \quad (4.31)$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1))$$

$$Z_j(s) = \int_0^\infty e^{-sx} z_j(x) dx \quad (4.32)$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1))$$

$$z_j(x) = \exp[-\int_0^x \mu_j(\delta) d\delta] \mu_j(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

From now onward, we let $i=kn+q$ (for $k=0, 1, \dots, m$, and $q=0, 1, \dots, n-1$) (it denotes the i^{th} state of the robot-safety system), the letters k and q denote the number of failed safety units and failed robots, respectively.

Simplifying Equations (4.17) – (4.18), (4.20) – (4.21), and (4.23) – (4.24), we get

$$P_{kn+q}(s) = \sum_{g=k}^m \frac{P_{gn+q-1}(s)}{\mu_{sk} [(n-q+1)\lambda_r]^{g-k} \prod_{l=k}^g K_{r[q]l}} + \sum_{h=0}^{k-1} \left(\prod_{f=h+1}^k U_{r[q]f} \right) \left(\sum_{g=h}^m \frac{P_{gn+q-1}(s)}{\mu_{sh} [(n-q+1)\lambda_r]^{g-h} \prod_{l=h}^g K_{r[q]l}} \right) \quad (4.33)$$

$$(\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 1, 2, 3, \dots, n-1 \end{cases})$$

where

$$K_{r[q]m} = \frac{(n-q+1)\lambda_r \mu_{sm}}{s + a_{mn+q}} \quad (\text{for } q = 1, 2, \dots, n-1)$$

$$U_{r[q]m} = \frac{\lambda_s}{s + a_{mn+q}} \quad (\text{for } q = 1, 2, \dots, n-1)$$

$$K_{r[q]k} = \frac{(n-q+1)\lambda_r \mu_{sk}}{s + a_{kn+q} - \mu_{sk+1} U_{r[q]k+1}} \quad (\text{for } \begin{cases} k = 1, 2, \dots, m-1 \\ q = 1, 2, \dots, n-1 \end{cases})$$

$$U_{r[q]k} = \frac{(m-k+1)\lambda_s}{s + a_{kn+q} - \mu_{sk+1} U_{r[q]k+1}} \quad (\text{for } \begin{cases} k = 1, 2, \dots, m-1 \\ q = 1, 2, \dots, n-1 \end{cases})$$

$$\mu_{s0} = 1$$

$$K_{r[q]0} = \frac{(n-q+1)\lambda_r \mu_{s0}}{s + a_q - \mu_{s1} U_{r[q]1}} \quad (\text{for } q = 1, 2, \dots, n-1)$$

Similarly, simplifying Equations (4.19) and (4.22), we obtain

$$P_{kn}(s) = \frac{K_{sk}}{\mu_{sk}} P_{(k-1)n}(s) = \prod_{q=1}^k \frac{K_{sq}}{\mu_{sq}} P_0(s) \quad (\text{for } \begin{cases} k = 1, 2, \dots, m \\ q = 0 \end{cases}) \quad (4.34)$$

where

$$K_{sm} = \frac{\lambda_s \mu_{sm}}{s + a_{mn}} \quad (\text{for } k = m)$$

$$K_{sk} = \frac{(m-k+1)\lambda_s \mu_{sk}}{s + a_{kn} - K_{sk+1}} \quad (\text{for } k = 1, 2, \dots, m-1)$$

From Equations (4.33) and (4.34), the Laplace transform of the i^{th} state probability can be expressed as

$$P_i(s) = Y_i P_0(s) \quad (\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 0, 1, 2, \dots, n-1 \end{cases}) \quad (4.35)$$

where Y_i is the function of the Laplace transform variable, s .

$$Y_0 = 1$$

$$Y_{kn} = \prod_{q=1}^k \frac{K_{sq}}{\mu_{sq}} \quad (\text{for } \begin{cases} k = 1, 2, \dots, m \\ q = 0 \end{cases}) \quad (4.36)$$

$$Y_{kn+q} = \sum_{g=k}^m \frac{Y_{gn+q-1}}{\mu_{sg} [(n-q+1)\lambda_r]^{g-k}} \prod_{l=k}^g K_{r[q]l} \\ + \sum_{h=0}^{k-1} \left(\prod_{f=h+1}^k U_{r[q]f} \right) \left(\sum_{g=h}^m \frac{Y_{gn+q-1}}{\mu_{sh} [(n-q+1)\lambda_r]^{g-h}} \prod_{l=h}^g K_{r[q]l} \right) \quad (4.37)$$

$$(\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 1, 2, 3, \dots, n-1 \end{cases})$$

The Laplace transforms of the probabilities of all the system states add up to $1/s$, i.e.,

$$\sum_{i=0}^{(m+1)n-1} P_i(s) + \sum_{j=(m+1)n}^{(m+1)(n+1)} P_j(s) = \frac{1}{s} \quad (4.38)$$

Solving Equations (4.33)-(4.38), we get

$$P_0(s) = [s(1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)(n+1)} a_j \frac{1 - Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (4.39)$$

$$P_i(s) = \frac{Y_i}{H} \quad (\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 0, 1, 2, \dots, n-1 \end{cases}) \quad (4.40)$$

$$P_j(s) = \frac{a_j [1 - Z_j(s)]}{sH} \quad (4.41)$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1))$$

where

$$a_j = \lambda_r Y_{[m+1-j+(m+1)n]n-1}$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)n+m)$$

$$a_{(m+1)(n+1)} = \lambda_{c0} + \sum_{i=1}^{(m+1)n-2} \lambda_{ci} Y_i$$

$$H = s(1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)(n+1)} a_j \frac{1 - Z_j(s)}{s})$$

Thus, the Laplace transform of the robot-safety system availability with at least one working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^{mn-1} P_i(s) = \frac{1 + \sum_{i=1}^{mn-1} Y_i}{H} \quad (4.42)$$

Similarly, the Laplace transform of the robot-safety system availability with or without working safety units is given by

$$AV_r(s) = \sum_{i=0}^{(m+1)n-1} P_i(s) = \frac{1 + \sum_{i=1}^{(m+1)n-1} Y_i}{H} \quad (4.43)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions into Equations (4.42) and (4.43), and taking the inverse Laplace transforms of the resulting equations, we can get the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

4.3.2 Steady State Availability Analysis (i.e., $\mu_{ri}=0$, for $i=1, \dots, n-1$)

As time approaches infinity, state probabilities reach the steady state. Thus, Equations (4.4) – (4.15) reduce to Equations (4.44) – (4.55), respectively.

$$a_0 P_0 = 1 + \mu_{s1} P_n(t) + \sum_{j=(m+1)n}^{(m+1)(n+1)} \int_0^{\infty} P_j(x) \mu_j(x) dx \quad (4.44)$$

$$a_i P_i = (n-i+1) \lambda_r P_{i-1} + \mu_{s1} P_{n+i} \quad (\text{for } i=1, 2, \dots, n-2) \quad (4.45)$$

$$a_{n-1} P_{n-1} = 2 \lambda_r P_{n-2} + \mu_{s1} P_{2n-1} \quad (\text{for } i=n-1) \quad (4.46)$$

$$a_{kn} P_{kn} = (m-k+1) \lambda_s P_{kn-n} + \mu_{sk+1} P_{kn+n} \quad (\text{for } k=1, 2, \dots, m-1) \quad (4.47)$$

$$a_i P_i = [(k+1)n-i+1] \lambda_r P_{i-1} + (m-k+1) \lambda_s P_{i-n} + \mu_{sk+1} P_{i+n} \quad (4.48)$$

$$\left(\text{for } \begin{cases} i = kn + 1, kn + 2, \dots, (k + 1)n - 2 \\ k = 1, 2, \dots, m - 1 \end{cases}\right)$$

$$a_{(k+1)n-1}P_{(k+1)n-1} = 2\lambda_r P_{(k+1)n-2} + (m - k + 1)\lambda_s P_{kn-1} + \mu_{sk+1}P_{(k+2)n-1} \quad (4.49)$$

$$\left(\text{for } k = 1, 2, \dots, m - 1\right)$$

$$a_{mn}P_{mn} = \lambda_s P_{(m-1)n} \quad \left(\text{for } i = mn\right) \quad (4.50)$$

$$a_i P_i = [(m + 1)n - i + 1]\lambda_r P_{i-1} + \lambda_s P_{i-n} \quad (4.51)$$

$$\left(\text{for } i = mn + 1, mn + 2, \dots, (m + 1)n - 2\right)$$

$$a_{(m+1)n-1}P_{(m+1)n-1} = 2\lambda_r P_{(m+1)n-2} + \lambda_s P_{mn-1} \quad (4.52)$$

$$\frac{dP_j(x)}{dx} + \mu_j(x)P_j(x) = 0 \quad (4.53)$$

$$\left(\text{for } j = (m + 1)n, (m + 1)n + 1, \dots, (m + 1)(n + 1)\right)$$

$$P_j(0) = \lambda_r P_{[m+1-j+(m+1)n]n-1} \quad (4.54)$$

$$\left(\text{for } j = (m + 1)n, (m + 1)n + 1, \dots, (m + 1)n + m\right)$$

$$P_{(m+1)(n+1)}(0) = \sum_{i=0}^{(m+1)n-2} \lambda_{ci} P_i \quad (4.55)$$

Solving Equation (4.53), we get

$$P_j(x) = P_j(0) \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \quad (4.56)$$

After a failure the robot-safety system under repair steady state condition probability is given by

$$P_j = \int_0^\infty P_j(x) dx \quad \left(\text{for } j = (m + 1)n, (m + 1)n + 1, \dots, (m + 1)(n + 1)\right) \quad (4.57)$$

Substituting Equation (4.56) into Equation (4.57), yields

$$P_j = P_j(0)E_j[x] \quad \left(\text{for } j = (m + 1)n, (m + 1)n + 1, \dots, (m + 1)(n + 1)\right) \quad (4.58)$$

where

$$\begin{aligned} E_j(x) &= \int_0^\infty \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] dx \\ &= \int_0^\infty xz_j(x) dx \end{aligned} \quad (4.59)$$

which is the mean time to robot-safety system repair when the failed robot-safety system is in state j and has an elapsed repair time of x .

Substituting Equation (4.54) - (4.55) into Equation (4.58), yields:

$$P_j = \lambda_r P_{[m+1-j+(m+1)n]n-1} E_j[x] \quad (4.60)$$

$$(for \quad j = (m+1)n, (m+1)n+1, \dots, (m+1)n+m)$$

$$P_{(m+1)(n+1)} = \sum_{i=0}^{(m+1)n-2} \lambda_{ci} P_i E_{(m+1)(n+1)}[x] \quad (4.61)$$

From now onward, we let $i=kn+q$ (for $k=0, 1, \dots, m$, and $q=0, 1, \dots, n-1$) (it denotes the i^{th} state of the robot-safety system), the letters k and q denote the number of failed safety units and failed robots, respectively.

Simplifying Equations (4.45) – (4.46), (4.48) – (4.49), and (4.51) – (4.52), we obtain

$$\begin{aligned} P_{kn+q} &= \sum_{g=k}^m \frac{P_{gn+q-1}}{\mu_{sk} [(n-q+1)\lambda_r]^{g-k}} \prod_{l=k}^g L_{r[q]l} \\ &+ \sum_{h=0}^{k-1} \left(\prod_{f=h+1}^k V_{r[q]f} \right) \left(\sum_{g=h}^m \frac{P_{gn+q-1}}{\mu_{sh} [(n-q+1)\lambda_r]^{g-h}} \prod_{l=h}^g L_{r[q]l} \right) \end{aligned} \quad (4.62)$$

$$(for \quad i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 1, 2, 3, \dots, n-1 \end{cases})$$

where

$$L_{r[q]m} = \lim_{s \rightarrow 0} K_{r[q]m} = \frac{(n-q+1)\lambda_r \mu_{sm}}{a_{mn+q}} \quad (for \quad q = 1, 2, \dots, n-1)$$

$$V_{r[q]m} = \lim_{s \rightarrow 0} U_{r[q]m} = \frac{\lambda_s}{a_{mn+q}} \quad (\text{for } q = 1, 2, \dots, n-1)$$

$$L_{r[q]k} = \lim_{s \rightarrow 0} K_{r[q]k} = \frac{(n-q+1)\lambda_r \mu_{sk}}{a_{kn+q} - \mu_{sk+1} V_{r[q]k+1}} \quad (\text{for } \begin{cases} k = 1, 2, \dots, m-1 \\ q = 1, 2, \dots, n-1 \end{cases})$$

$$V_{r[q]k} = \lim_{s \rightarrow 0} U_{r[q]k} = \frac{(m-k+1)\lambda_s}{a_{kn+q} - \mu_{sk+1} V_{r[q]k+1}} \quad (\text{for } \begin{cases} k = 1, 2, \dots, m-1 \\ q = 1, 2, \dots, n-1 \end{cases})$$

$$\mu_{s0} = 1$$

$$L_{r[q]0} = \lim_{s \rightarrow 0} K_{r[q]0} = \frac{(n-q+1)\lambda_r \mu_{s0}}{a_q - \mu_{s1} V_{r[q]1}} \quad (\text{for } q = 1, 2, \dots, n-1)$$

Similarly, simplifying Equations (4.47) and (4.50), we get

$$P_{kn} = \frac{L_{sk}}{\mu_{sk}} P_{(k-1)n} = \prod_{q=1}^k \frac{L_{sq}}{\mu_{sq}} P_0 \quad (\text{for } \begin{cases} k = 1, 2, \dots, m \\ q = 0 \end{cases}) \quad (4.63)$$

where

$$L_{sm} = \lim_{s \rightarrow 0} K_{sm} = \frac{\lambda_s \mu_{sm}}{a_{mn}} \quad (\text{for } k = m)$$

$$L_{sk} = \lim_{s \rightarrow 0} K_{sk} = \frac{(m-k+1)\lambda_s \mu_{sk}}{a_{kn} - K_{sk+1}} \quad (\text{for } k = 1, 2, \dots, m-1)$$

From Equations (4.62) and (4.63), the Laplace transform of the i^{th} state probability can be expressed as

$$P_i = Y_{si} P_0 \quad (\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 0, 1, 2, \dots, n-1 \end{cases}) \quad (4.64)$$

where

$$Y_{si} = \lim_{s \rightarrow 0} Y_i \quad (\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 0, 1, 2, \dots, n-1 \end{cases}) \quad (4.65)$$

Using Equations (4.62) - (4.65), together with

$$\sum_{i=0}^{(m+1)n-1} P_i + \sum_{j=(m+1)n}^{(m+1)(n+1)} P_j = 1 \quad (4.66)$$

we get the following general form of the steady state probability solutions:

$$P_0 = \left(1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} a_{sj} E_j[x]\right)^{-1} = \frac{1}{G} \quad (4.67)$$

$$P_i = \frac{Y_{si}}{G} \quad (\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 1, 2, 3, \dots, n-1 \end{cases}) \quad (4.68)$$

$$P_j = a_{sj} E_j[x] P_0 \quad (\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)) \quad (4.69)$$

where

$$a_{sj} = \lim_{s \rightarrow 0} a_j = \lambda_r Y_{s\{(m+1-j+(m+1)n)n-1\}} \\ (\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)n+m)$$

$$a_{s\{(m+1)(n+1)\}} = \lim_{s \rightarrow 0} a_{(m+1)(n+1)} = \lambda_{c0} + \sum_{i=1}^{(m+1)n-2} \lambda_{ci} Y_{si}$$

$$G = 1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} a_{sj} E_j[x] \quad (4.70)$$

The generalized steady state availability of the robot-safety system with at least one working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^{mn-1} P_i = \frac{1 + \sum_{i=1}^{mn-1} Y_{si}}{G} \quad (4.71)$$

Similarly, the generalized steady state availability of the robot-safety system with or without working safety units is given by

$$SSAV_r = \sum_{i=0}^{(m+1)n-1} P_i = \frac{1 + \sum_{i=1}^{(m+1)n-1} Y_{si}}{G} \quad (4.72)$$

For different failed system repair time distributions, the values of G are obtained as follows:

- (i) When the failed robot-safety system repair time x is exponentially distributed, the probability density function of the repair time is

$$z_j(x) = \mu_j e^{-\mu_j x} \quad (\mu_j > 0, \quad j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)) \quad (4.73)$$

where x is the repair time, and μ_j is the constant repair rate of state j . Thus, the mean time to robot-safety system repair, $E_j[x]$, for the exponential distribution is given by

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \frac{1}{\mu_j} \quad (4.74)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

Substituting Equation (4.74) into Equation (4.70), we get

$$G = G_e = 1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} (a_{sj} \frac{1}{\mu_j}) \quad (4.75)$$

- (ii) When the failed robot-safety system repair time x is gamma distributed, the probability density function of the repair time is

$$z_j(x) = \frac{\mu_j^\beta x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \quad (\beta > 0) \quad (4.76)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

where x is the repair time, $\Gamma(\beta)$ is the gamma function, and β and μ_j are the shape and scale parameters, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the gamma distribution is given by

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \frac{\beta}{\mu_j} \quad (4.77)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

Substituting Equation (4.77) into Equation (4.70), we get

$$G = G_g = 1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} (a_{sj} \frac{\beta}{\mu_j}) \quad (4.78)$$

(iii) When the failed robot-safety system repair time x is Weibull distributed, the probability density function of the repair time is expressed by

$$z_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j(x)^\beta} \quad (\beta > 0) \quad (4.79)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

where x is the repair time, and β and μ_j are the shape and scale parameters of the Weibull distribution, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Weibull distribution is given by

$$E_j[x] = \int_0^\infty x z_j(x) dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \quad (4.80)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

Substituting Equation (4.80) into Equation (4.70), yields

$$G = G_w = 1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} [a_{sj} \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right)] \quad (4.81)$$

(iv) When the failed robot-safety system repair time x is Rayleigh distributed, the probability density function of the Rayleigh distribution is expressed by

$$z_j(x) = \mu_j x e^{-\mu_j x^2/2} \quad (\mu_j > 0) \quad (4.82)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

where x is the repair time, and μ_j the scale parameter. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Rayleigh distribution is

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \sqrt{\frac{\pi}{4\mu_j}} \quad (4.83)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

Substituting Equation (4.83) into Equation (4.70), we get

$$G = G_r = 1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} (a_{sj} \sqrt{\frac{\pi}{4\mu_j}}) \quad (4.84)$$

(v) When the robot-safety system repair time x is lognormal distributed, the probability density function of the repair time is

$$z_j(x) = \frac{1}{x\sigma_{y_j}\sqrt{2\pi}} e^{\left[\frac{-(\ln x - \mu_{y_j})^2}{2\sigma_{y_j}^2}\right]} \quad (4.85)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

where x is the repair time, and $\ln x$ is the natural logarithms of x with a mean and variance μ and σ^2 , respectively. The conditions on parameters are as follows:

$$\sigma_{y_j} = \ln \sqrt{1 + \left(\frac{\sigma_{x_j}}{\mu_{x_j}}\right)^2},$$

$$\mu_{y_j} = \ln \sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \quad (4.86)$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$)

Hence, the failed robot-safety system mean time to repair, $E_j[x]$, for the lognormal distribution is

$$E_j[x] = e^{\left(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2}\right)} \quad (for \ j = (m+1)n, (m+1)n+1, \dots, (m+1)(n+1)) \quad (4.87)$$

Substituting Equation (4.87) into Equation (4.70), we get

$$G = G_l = 1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} [a_{sj} e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})}] \quad (4.88)$$

4.3.3 System Reliability, MTTF, and Variance of Time to Failure (i.e., $\mu_{ri}=0$, for $i=1, \dots, n-1$)

Setting $\mu_j(x)=0$ [for $j=(m+1)n, (m+1)n+1, \dots, (m+1)(n+1)$] in Figure 4.2 and with the aid of Markov method, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{s1} P_n(t) \quad (4.89)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_r P_{i-1}(t) + \mu_{s1} P_{n+i}(t) \quad (4.90)$$

(for $i = 1, 2, \dots, n-2$)

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = 2\lambda_r P_{n-2}(t) + \mu_{s1} P_{2n-1}(t) \quad (\text{for } i = n-1) \quad (4.91)$$

$$\frac{dP_{kn}(t)}{dt} + a_{kn} P_{kn}(t) = (m-k+1)\lambda_s P_{kn-n}(t) + \mu_{sk+1} P_{kn+n}(t) \quad (4.92)$$

(for $k = 1, 2, \dots, m-1$)

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(k+1)n-i+1]\lambda_r P_{i-1}(t) + (m-k+1)\lambda_s P_{i-n}(t) + \mu_{sk+1} P_{i+n}(t) \quad (4.93)$$

(for $\begin{cases} i = kn+1, kn+2, \dots, (k+1)n-2 \\ k = 1, 2, \dots, m-1 \end{cases}$)

$$\frac{dP_{(k+1)n-1}(t)}{dt} + a_{(k+1)n-1} P_{(k+1)n-1}(t) = 2\lambda_r P_{(k+1)n-2}(t) + (m-k+1)\lambda_s P_{kn-1}(t) + \mu_{sk+1} P_{(k+2)n-1}(t) \quad (4.94)$$

(for $k = 1, 2, \dots, m-1$)

$$\frac{dP_{mn}(t)}{dt} + a_{mn}P_{mn}(t) = \lambda_s P_{(m-1)n}(t) \quad (\text{for } i = mn) \quad (4.95)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(m+1)n - i + 1]\lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) \quad (4.96)$$

(for $i = mn + 1, mn + 2, \dots, (m+1)n - 2$)

$$\frac{dP_{(m+1)n-1}(t)}{dt} + a_{(m+1)n-1} P_{(m+1)n-1}(t) = 2\lambda_r P_{(m+1)n-2}(t) + \lambda_s P_{mn-1}(t) \quad (4.97)$$

(for $i = (m+1)n - 1$)

$$\frac{dP_j(t)}{dt} = \lambda_r P_{[m+1-j+(m+1)n]n-1}(t) \quad (4.98)$$

(for $j = (m+1)n, (m+1)n + 1, \dots, (m+1)n + m$)

$$\frac{dP_{(m+1)(n+1)}(t)}{dt} = \sum_{i=0}^{(m+1)n-2} \lambda_{ci} P_i(t) \quad (4.99)$$

At time $t=0$, $P_0(0)=1$, and all other initial state probabilities are equal to zero. Taking the Laplace transforms of Equations (4.90) – (4.99), and solving the resulting set of Equations, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)(n+1)} \frac{a_j}{s})]^{-1} \quad (4.100)$$

$$P_i(s) = Y_i P_0(s) \quad (\text{for } i = kn + q, \left\{ \begin{array}{l} k = 0, 1, 2, \dots, m \\ q = 0, 1, 2, \dots, n-1 \end{array} \right.) \quad (4.101)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = (m+1)n, (m+1)n + 1, \dots, (m+1)(n+1)) \quad (4.102)$$

where

Y_i is the function of the Laplace transform variable, s ; which can be derived using Equations (4.36) and (4.37).

The Laplace transform of the robot-safety system reliability with at least one working safety unit is

$$R_{rs}(s) = \sum_{i=0}^{mn-1} P_i(s) = (1 + \sum_{i=1}^{mn-1} Y_i) P_0(s) \quad (4.103)$$

Utilizing Equation (4.103), the robot-safety system mean time to the failure is obtained as follows [103]:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{(1 + \sum_{i=1}^{mn-1} Y_{si})}{\frac{(m+1)(n+1)}{\sum_{j=(m+1)n} a_{sj}}} \quad (4.104)$$

where

$$Y_{si} = \lim_{s \rightarrow 0} Y_i \quad (\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 1, 2, 3, \dots, n-1 \end{cases})$$

$$a_{sj} = \lim_{s \rightarrow 0} a_j = \lambda_r Y_{s\{(m+1)-j+(m+1)n\}n-1}$$

$$(\text{for } j = (m+1)n, (m+1)n+1, \dots, (m+1)n+m)$$

$$a_{s\{(m+1)(n+1)\}} = \lim_{s \rightarrow 0} a_{(m+1)(n+1)} = \lambda_{c0} + \sum_{i=1}^{(m+1)n-2} \lambda_{ci} Y_{si}$$

Similarly, the Laplace transform of the robot-safety system reliability with or without working safety units is

$$R_r(s) = \sum_{i=0}^{(m+1)n-1} P_i(s) = (1 + \sum_{i=1}^{(m+1)n-1} Y_i) P_0(s) \quad (4.105)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{(1 + \sum_{i=1}^{(m+1)n-1} Y_{si})}{\sum_{j=(m+1)n} a_{sj}} \quad (4.106)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (4.103) and (4.105).

The robot-safety system variance of time to failure is expressed by [103]

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + \sum_{i=1}^{(m+1)n-1} Y_{si})(1 + \sum_{i=1}^{(m+1)n-1} Y_{si} + \sum_{j=(m+1)n}^{(m+1)(n+1)} a_{dj})}{(\sum_{j=(m+1)n}^{(m+1)(n+1)} a_{sj})^2} - \frac{2 \sum_{i=1}^{(m+1)n-1} Y_{di}}{\sum_{j=(m+1)n}^{(m+1)(n+1)} a_{sj}} - (MTTF_r)^2 \end{aligned} \quad (4.107)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$Y_{di} = \lim_{s \rightarrow 0} Y_i' \quad (\text{for } i = kn + q, \begin{cases} k = 0, 1, 2, \dots, m \\ q = 0, 1, 2, \dots, n-1 \end{cases})$$

$$a_{dj} = \lim_{s \rightarrow 0} a_j' = \lambda_r Y_{d\{(m+1)-j+(m+1)n\}}$$

(for $j = (m+1)n, (m+1)n+1, \dots, (m+1)n+m$)

$$a_{d\{(m+1)(n+1)\}} = \lim_{s \rightarrow 0} a_{(m+1)(n+1)}' = \sum_{i=1}^{(m+1)n-2} \lambda_{ci} Y_{di}$$

Y_i' denotes the derivative of Y_i with respect to s .

a_j' denotes the derivative of a_j with respect to s .

4.4 Special Case Model 4.I: ($n=2, m=2, \mu_{r1} \neq 0$)

For $n=2$ and $m=2$ in Figures 4.1 and 4.2, the model becomes for a system having two redundant robots and two redundant built-in safety units. The corresponding system of Equations can be extracted from Equations (4.1)-(4.12) by setting $n=2$ and $m=2^*$.

* Detailed analysis is provided in *Appendix D*.

4.4.1 Time Dependant Availability Analysis (n=2, m=2, $\mu_{r1} \neq 0$)

With the aid of Laplace transforms and initial conditions, from Equations (4.1)-(4.12) for n=2 and m=2, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^5 Y_i + \sum_{j=6}^9 a_j \frac{1 - Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (4.108)$$

$$P_i(s) = Y_i P_0(s) \quad (\text{for } i = 1, 2, 3, 4, 5) \quad (4.109)$$

$$P_j(s) = \frac{a_j [1 - Z_j(s)]}{s} P_0(s) \quad (\text{for } j = 6, 7, 8, 9) \quad (4.110)$$

where

$$a_0 = 2\lambda_r + \lambda_{c0} + 2\lambda_s$$

$$a_1 = \lambda_r + \lambda_{c1} + 2\lambda_s + \mu_{r1}$$

$$a_2 = 2\lambda_r + \lambda_{c2} + \lambda_s + \mu_{s1}$$

$$a_3 = \lambda_r + \lambda_{c3} + \lambda_s + \mu_{s1}$$

$$a_4 = 2\lambda_r + \lambda_{c4} + \mu_{s2}$$

$$a_5 = \lambda_r + \mu_{s2}$$

$$K_{s2} = \frac{\lambda_s \mu_{s2}}{s + a_4}$$

$$K_{s1} = \frac{2\lambda_s \mu_{s1}}{s + a_2 - K_{s2}}$$

$$\mu_{s0} = 1$$

$$K_{r\{1\}2} = \frac{2\lambda_r \mu_{s2}}{s + a_5}$$

$$K_{r\{1\}1} = \frac{2\lambda_r \mu_{s1}}{s + a_3}$$

$$K_{r[1]0} = \frac{2\lambda_r \mu_{s0}}{s + a_1}$$

$$Y_1 = K_{r[1]0} + \frac{\prod_{l=0}^1 K_{r[1]l} K_{s1}}{2\lambda_r \mu_{s1}} + \frac{\prod_{l=0}^2 K_{r[1]l}}{(2\lambda_r)^2} \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$Y_2 = \frac{K_{s1}}{\mu_{s1}}$$

$$Y_3 = \frac{K_{r[1]1} K_{s1}}{\mu_{s1} \mu_{s1}} + \frac{\prod_{l=1}^2 K_{r[1]l}}{\mu_{s1} 2\lambda_r} \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$Y_4 = \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$Y_5 = \frac{K_{r[1]2}}{\mu_{s2}} \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$a_6 = \lambda_r Y_5$$

$$a_7 = \lambda_r Y_3$$

$$a_8 = \lambda_r Y_1$$

$$a_9 = \lambda_{c0} + \sum_{i=1}^4 \lambda_{ci} Y_i$$

$$H = s \left(1 + \sum_{i=1}^5 Y_i + \sum_{j=6}^9 a_j \frac{1 - Z_j(s)}{s} \right)$$

Thus, the Laplace transform of the robot-safety system availability with at least one working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^3 P_i(s) = \frac{1 + \sum_{i=1}^3 Y_i}{H} \quad (4.111)$$

Similarly, the Laplace transform of the robot-safety system availability with or without working safety units is given by

$$AV_r(s) = \sum_{i=0}^5 P_i(s) = \frac{1 + \sum_{i=1}^5 Y_i}{H} \quad (4.112)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions in Equations (4.111) and (4.112), and taking the inverse Laplace transforms of the resulting equations, we can obtain the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

Setting:

$$\begin{aligned} \lambda_s &= 0.0006, & \lambda_r &= 0.0006, & \lambda_{c0} &= 0.0002, & \lambda_{c1} &= \lambda_{c2} = \lambda_{c3} = \lambda_{c4} = 0.0001, \\ \mu_{s1} &= \mu_{s2} = 0.0009, & \mu_{r1} &= 0.0010, & \mu_6 &= 0.0011, & \mu_7 &= 0.0012, \\ \mu_8 &= 0.0013, & \mu_9 &= 0.0006 \end{aligned}$$

in Equations (4.108)-(4.110), and (4.111)-(4.112), for exponential and gamma distributed failed system repair times, and using Maple computer program [102], the time-dependant plots of robot-safety system state probabilities and availabilities are shown in Figures 4.3-4.6, respectively.

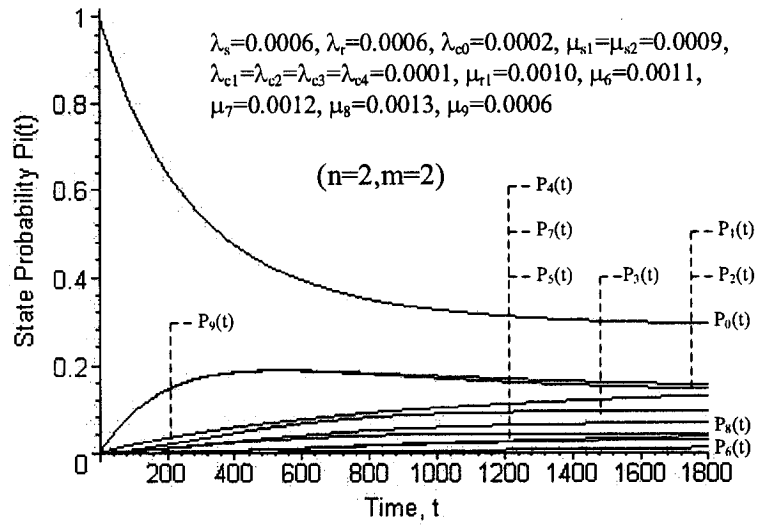


Figure 4.3 Time-dependent probability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

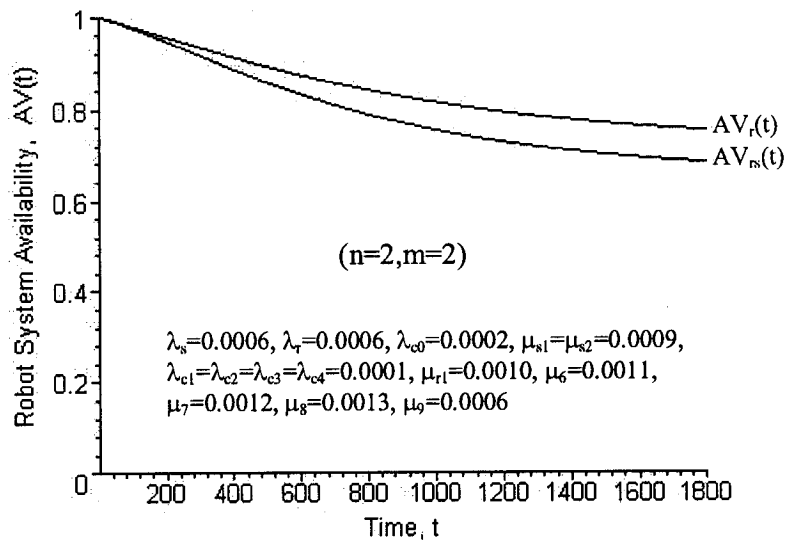


Figure 4.4 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

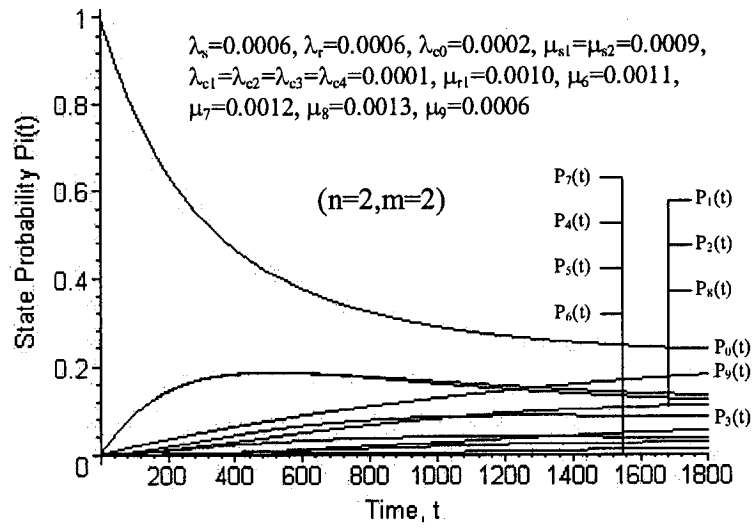


Figure 4.5 Time-dependent probability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

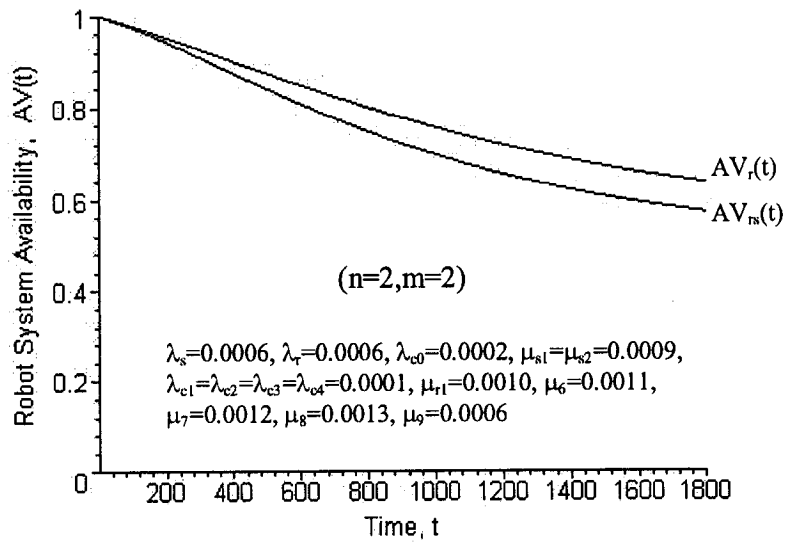


Figure 4.6 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

4.4.2 Steady State Availability Analysis (n=2, m=2, $\mu_{r1} \neq 0$)

As time approaches infinity, state probabilities reach the steady state. Thus, the steady state probabilities for this special case model (n=2, m=2) are as follows:

$$P_0 = (1 + \sum_{i=1}^5 Y_{si} + \sum_{j=6}^9 a_{sj} E_j[x])^{-1} = \frac{1}{G} \quad (4.113)$$

$$P_i = Y_{si} P_0 \quad (\text{for } i = 1, 2, 3, 4, 5) \quad (4.114)$$

$$P_j = a_{sj} E_j[x] P_0 \quad (\text{for } j = 6, 7, 8, 9) \quad (4.115)$$

where

$$L_{s2} = \lim_{s \rightarrow 0} K_{s2} = \frac{\lambda_s \mu_{s2}}{a_4}$$

$$L_{s1} = \lim_{s \rightarrow 0} K_{s1} = \frac{2\lambda_s \mu_{s1}}{a_2 - L_{s2}}$$

$$\mu_{s0} = 1$$

$$L_{r[1]2} = \lim_{s \rightarrow 0} K_{r[1]2} = \frac{2\lambda_r \mu_{s2}}{a_5}$$

$$L_{r[1]1} = \lim_{s \rightarrow 0} K_{r[1]1} = \frac{2\lambda_r \mu_{s1}}{a_3}$$

$$L_{r[1]0} = \lim_{s \rightarrow 0} K_{r[1]0} = \frac{2\lambda_r \mu_{s0}}{a_1}$$

$$Y_{s1} = \lim_{s \rightarrow 0} Y_1 = L_{r[1]0} + \frac{\prod_{l=0}^1 L_{r[1]l}}{2\lambda_r} \frac{L_{s1}}{\mu_{s1}} + \frac{\prod_{l=0}^2 L_{r[1]l}}{(2\lambda_r)^2} \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$Y_{s2} = \lim_{s \rightarrow 0} Y_2 = \frac{L_{s1}}{\mu_{s1}}$$

$$Y_{s3} = \lim_{s \rightarrow 0} Y_3 = \frac{L_{r(1)1} L_{s1}}{\mu_{s1} \mu_{s1}} + \frac{\prod_{l=1}^2 L_{r(1)l}}{\mu_{s1} (2\lambda_r)} \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$Y_{s4} = \lim_{s \rightarrow 0} Y_4 = \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$Y_{s5} = \lim_{s \rightarrow 0} Y_5 = \frac{L_{r(1)2}}{\mu_{s2}} \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$a_{s6} = \lambda_r Y_{s5}$$

$$a_{s7} = \lambda_r Y_{s3}$$

$$a_{s8} = \lambda_r Y_{s1}$$

$$a_{s9} = \lambda_{c0} + \sum_{i=1}^4 \lambda_{ci} Y_{si}$$

$$G = 1 + \sum_{i=1}^5 Y_{si} + \sum_{j=6}^9 a_{sj} E_j[x] \quad (4.116)$$

The steady state availability of the robot-safety system with at least one working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^3 P_i = \frac{1 + \sum_{i=1}^3 Y_{si}}{G} \quad (4.117)$$

Similarly, the steady state availability of the robot-safety system with or without working safety units is given by

$$SSAV_r = \sum_{i=0}^5 P_i = \frac{1 + \sum_{i=1}^5 Y_{si}}{G} \quad (4.118)$$

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c1}=\lambda_{c2}=\lambda_{c3}=\lambda_{c4}=0.0001, \quad \mu_{s1}=\mu_{s2}=0.0009,$$

$$\mu_{r1}=0.0010, \quad \mu_6=0.0011, \quad \mu_7=0.0012, \quad \mu_8=0.0013, \quad \mu_9=0.0006$$

in Equation (4.117) and for Weibull distributed failed system repair times, the plots for $SSAV_{rs}$ are shown in Figure 4.7.

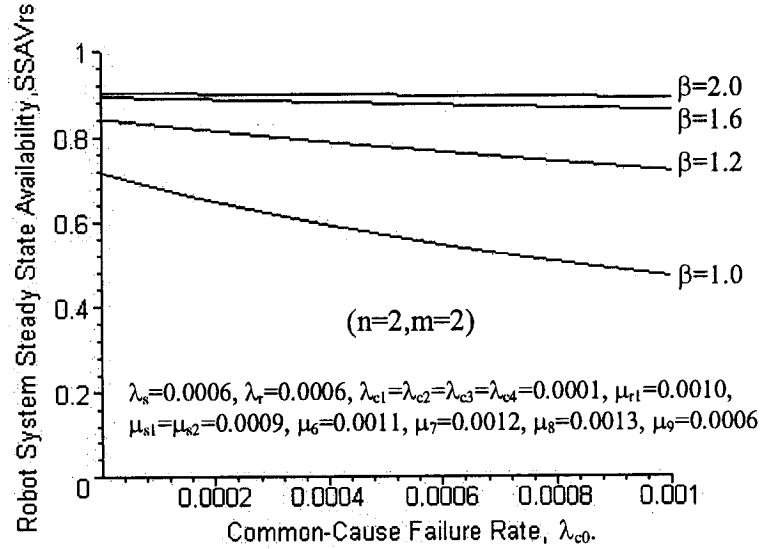


Figure 4.7 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair time distribution.

4.4.3 Robot-Safety System Reliability, MTTF, and Variance of time to failure (n=2, m=2, $\mu_{r1} \neq 0$)

Setting $\mu_6(x)=\mu_7(x)=\mu_8(x)=\mu_9(x)=0$ in this special case model (n=2, m=2), we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^5 Y_i + \sum_{j=6}^9 \frac{a_j}{s})]^{-1} \quad (4.119)$$

$$P_i(s) = Y_i P_0(s) \quad (\text{for } i = 1, 2, 3, 4, 5) \quad (4.120)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 6, 7, 8, 9) \quad (4.121)$$

Thus, the Laplace transform of the robot-safety system reliability with at least one working safety unit is

$$R_{rs}(s) = \sum_{i=0}^3 P_i(s) = (1 + \sum_{i=1}^3 Y_i) P_0(s) \quad (4.122)$$

Using Equation (4.122), we get the following expression for the robot-safety system mean time to failure:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + \sum_{i=1}^3 Y_{si}}{\sum_{j=6}^9 a_{sj}} \quad (4.123)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without working safety units is

$$R_r(s) = \sum_{i=0}^5 P_i(s) = (1 + \sum_{i=1}^5 Y_i) P_0(s) \quad (4.124)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + \sum_{i=1}^5 Y_{si}}{\sum_{j=6}^9 a_{sj}} \quad (4.125)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (4.122) and (4.124).

The robot-safety system variance of time to failure is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + \sum_{i=1}^5 Y_{si})(1 + \sum_{i=1}^5 Y_{si} + \sum_{j=6}^9 a_{dj})}{(\sum_{j=6}^9 a_{sj})^2} - \frac{2 \sum_{i=1}^5 Y_{di}}{\sum_{j=6}^9 a_{sj}} - (MTTF_r)^2 \end{aligned} \quad (4.126)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$Y_{di} = \lim_{s \rightarrow 0} Y_i' \quad (\text{for } i = 1, 2, 3, 4, 5)$$

$$a_{dj} = \lim_{s \rightarrow 0} a_j' \quad (\text{for } j = 6, 7, 8, 9)$$

$$a_{d6} = \lim_{s \rightarrow 0} a_6' = \lambda_r Y_{d5}$$

$$a_{d7} = \lim_{s \rightarrow 0} a_7' = \lambda_r Y_{d3}$$

$$a_{d8} = \lim_{s \rightarrow 0} a_8' = \lambda_r Y_{d1}$$

$$a_{d9} = \lim_{s \rightarrow 0} a_9' = \sum_{i=1}^4 \lambda_{ci} Y_{di}$$

Y_i' denotes the derivative of Y_i with respect to s .

a_j' denotes the derivative of a_j with respect to s .

Setting:

$$\lambda_s = 0.0006, \quad \lambda_r = 0.0006, \quad (\lambda_{c0} = 0.0002), \quad \lambda_{c1} = \lambda_{c2} = \lambda_{c3} = \lambda_{c4} = 0.0001,$$

$$\mu_{s1} = \mu_{s2} = 0.0009, \quad \mu_{r1} = 0.0010, \quad \mu_6 = \mu_7 = \mu_8 = \mu_9 = 0$$

in Equations (4.123) and (4.125), the plots of the robot-safety system mean time to failure, as a function of common-cause failure rate (λ_{c0}), are shown in Figure 4.8.

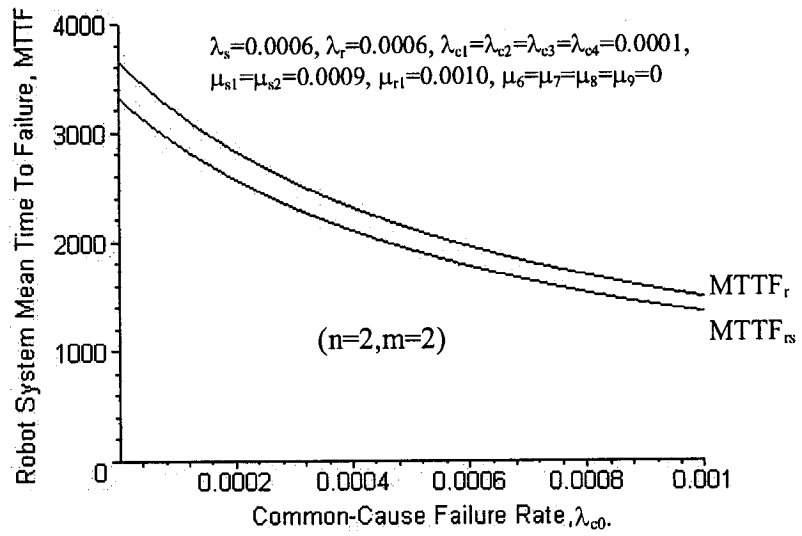


Figure 4.8 Mean time to failure plots of an irreparable robot-safety system as a function of common-cause failure rate(λ_{c0}).

STOCHASTIC ANALYSIS OF A ROBOT-SAFETY SYSTEM CONTAINING ONE ROBOT AND K-OUT-OF-N REDUNDANT SAFETY UNITS WITH COMMON-CAUSE FAILURES

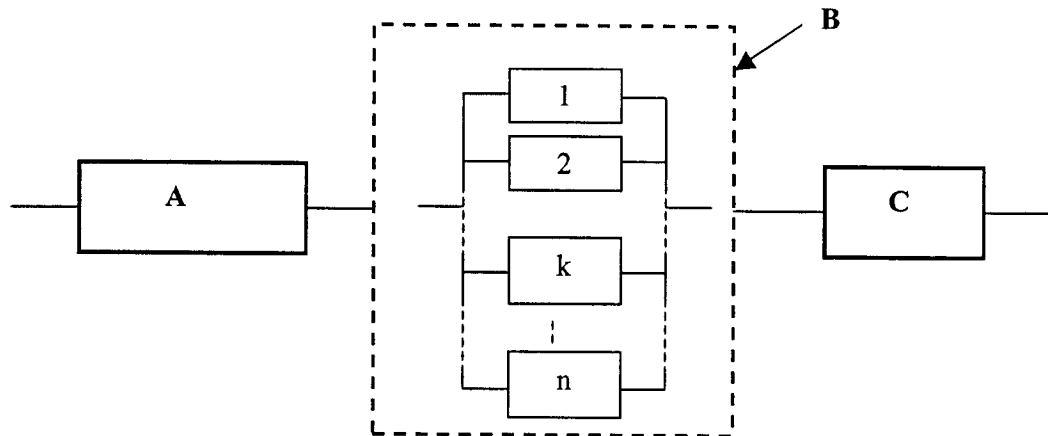
5.1 Introduction

The robot in the model of Chapter 2 is allowed to work without safety units. However, in some critical situations, the robot cannot be permitted to work under any unsafe condition (e.g., a life support medical robot), because a failure of that robot may cause injury or death to human beings.

Therefore, this chapter presents reliability and availability analyses of a model representing a system having one robot and n -redundant safety units with common-cause failures. At least k safety units must function successfully for the robot-safety system success. Markov and supplementary variable methods were used to perform mathematical analysis of this model. Generalized expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure are developed. A special case model (i.e., for $k=2$, $n=3$) is presented.

5.2 The Description of the Robot-Safety System

The block diagram of this robot-safety system having one robot and n -redundant safety units with common-cause failures is shown in Figure 5.1, and its corresponding state space diagram is given in Figure 5.2. The numerals and letters n and k in the boxes and ellipse of Figure 5.2 denote system states.



A: the robot

B: k -out-of- n safety units

C: common-cause failures

Figure 5.1 Block diagram of the robot-safety system with common-cause failures.

At time $t=0$, the robot and all n safety units start operating. The robot-safety system can fail either due to the failure of the robot itself, the malfunction of the $(n-k+1)^{\text{th}}$ safety unit, or the occurrence of a common-cause failure. Nonetheless, the robot-safety system will function successfully until at least k safety units and the robot are operating normally. The system goes through $(n-k+1)$ distinct operating states. A common-cause failure can

occur only if at least k safety units and the robot are functioning successfully. The robot-safety system has a total of $(n-k+4)$ distinct states. It means the array of numerals representing system states may be discontinuous. For example, for a 2-out-of-4 safety units, the array of numerals representing system states are 0, 1, 2, 5, 6, 7. More specifically, in this array of numerals, numerals 3 and 4 are missing. The degraded or fully failed robot-safety system is repaired.

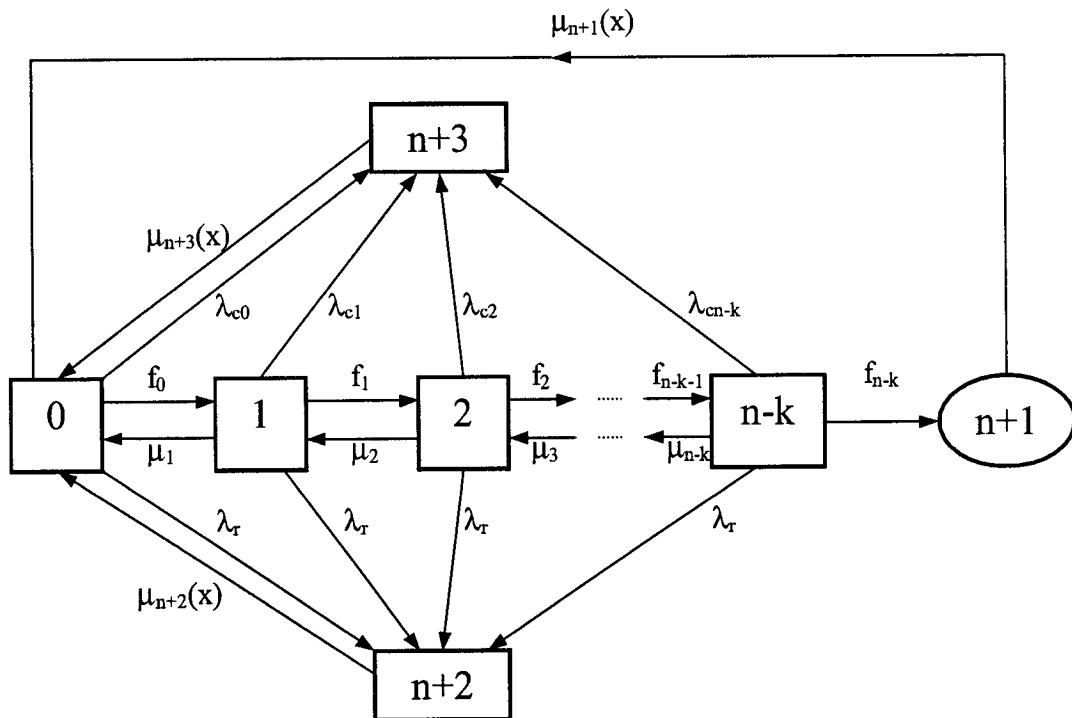


Figure 5.2 The state space diagram of the robot-safety system with common-cause failures. The numerals and letters n and k in squares, rectangles, and ellipse denote system states and $f_i=(n-i)\lambda_s$, for $i=0, 1, 2, \dots, n-k$.

Assumptions

The following assumptions are associated with this model:

- (i) The robot-safety system is composed of one robot and n identical safety units.

- (ii) The robot and redundant safety units are operating simultaneously.
- (iii) All failures are statistically independent.
- (iv) All failure rates and the partially failed system repair rates are constant.
- (v) The failed robot-safety system repair rates can be constant or non-constant.
- (vi) The repaired robot or a safety unit is as good as new.
- (vii) The overall robot-safety system fails when the active robot fails, a common-cause failure occurs, or the $(n-k+1)^{\text{th}}$ safety unit fails.

Notation

The following symbols are associated with the model:

- i i^{th} state of the overall robot-safety system: for $i=0$, means robot and all n safety units are in perfect working condition; for $i=1$, means robot and $n-1$ safety units operating normally while one safety unit has failed; for $i=m$ (where $m=2,3,\dots,n-k-1$ and $k=1,2,\dots,n-1$), means the robot and $n-m$ safety units operating normally while m safety units have failed; for $i=n-k$ (where $k=1,2,\dots,n$), means robot and k safety units operating normally while $n-k$ safety units have failed.
- j j^{th} state of the failed robot-safety system: for $j=n+1$, means robot-safety system failed due to the malfunction of the $(n-k+1)^{\text{th}}$ safety unit ; for $j=n+2$, means robot-safety system failed due to the failure of the robot itself; for $j=n+3$, means robot-safety system failed due to a common-cause failure.
- t time
- λ_s Constant failure rate of the safety unit.
- λ_r Constant failure rate of the robot.

λ_{ci}	Constant common-cause failure rate of the robot-safety system in state i ; for $i = 0, 1, 2, \dots, n-k$.
μ_i	Constant repair rate of the safety unit in state i ; for $i = 1, 2, \dots, n-k$.
Δx	Finite repair time interval.
$\mu_j(x)$	Time-dependent repair rate when the failed robot-safety system is in state j and has an elapsed repair time of x ; for $j = n+1, n+2, n+3$.
$p_j(x,t)\Delta x$	The probability that at time t , the failed robot-safety system is in state j and the elapsed repair time lies in the interval $[x, x+\Delta x]$; for $j = n+1, n+2, n+3$.
pdf	Probability density function.
$z_j(x)$	pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x ; for $j = n+1, n+2, n+3$.
$P_i(t)$	Probability that the robot-safety system is in state i at time t ; for $i = 0, 1, \dots, n-k$.
$P_j(t)$	Probability that the robot-safety system is in state j at time t ; for $j = n+1, n+2, n+3$.
P_i	Steady-state probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, n-k$.
P_j	Steady-state probability that the robot-safety system is in state j ; for $j = n+1, n+2, n+3$.
s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i ; for $i = 0, 1, \dots, n-k$.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j ; for $j = n+1, n+2, n+3$.

- $AV_{rs}(s)$ Laplace transform of the robot-safety system availability when the robot working with at least k safety units.
- $AV_{rs}(t)$ Robot-safety system time-dependent availability when the robot working with at least k safety units.
- $SSAV_{rs}$ Robot-safety system steady state availability when the robot working with at least k safety units.
- $R_{rs}(s)$ Laplace transform of the robot-safety system reliability when the robot working with at least k safety units.
- $R_{rs}(t)$ Robot-safety system reliability when the robot working with at least k safety units.
- $MTTF_{rs}$ Robot-safety system mean time to failure when the robot working with at least k safety units.
- σ^2 Robot-safety system variance of time to failure when the robot working with at least k safety units.

5.3 Generalized Robot-Safety System Analysis

Using the supplementary method [100,101], the system of Equations associated with Figure 5.2 can be expressed as follows:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{j=n+1}^{n+3} \int_0^{\infty} P_j(x,t) \mu_j(x) dx \quad (5.1)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1) \lambda_s P_{i-1}(t) + \mu_{i+1} P_{i+1}(t) \quad (5.2)$$

(for $i = 1, 2, \dots, n-k-1$)

$$\frac{dP_{n-k}(t)}{dt} + a_{n-k} P_{n-k}(t) = (k+1) \lambda_s P_{n-k}(t) \quad (5.3)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x)P_j(x,t) = 0 \quad (5.4)$$

(for $j = n+1, n+2, n+3$)

where

$$a_0 = n\lambda_s + \lambda_r + \lambda_{c_0}$$

$$a_i = (n-i)\lambda_s + \lambda_r + \lambda_{c_i} + \mu_i \quad (\text{for } i = 1, 2, \dots, n-k-1)$$

$$a_{n-k} = k\lambda_s + \lambda_r + \lambda_{c_{n-k}} + \mu_{n-k}$$

The associated boundary conditions are as follows:

$$P_{n+1}(0,t) = k\lambda_s P_{n-k}(t) \quad (5.5)$$

$$P_{n+2}(0,t) = \lambda_r \sum_{i=0}^{n-k} P_i(t) \quad (5.6)$$

$$P_{n+3}(0,t) = \sum_{i=0}^{n-k} \lambda_{c_i} P_i(t) \quad (5.7)$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero.

5.3.1 Time Dependant Availability Analysis

Using the Laplace transform technique and the initial conditions in Equations (5.1) – (5.7), we get

$$(s + a_0)P_0(s) = 1 + \mu_1 P_1(s) + \sum_{j=n+1}^{n+3} \int_0^{\infty} P_j(x,s) \mu_j(x) dx \quad (5.8)$$

$$(s + a_i)P_i(s) = (n-i+1)\lambda_s P_{i-1}(s) + \mu_{i+1} P_{i+1}(s) \quad (5.9)$$

(for $i = 1, 2, \dots, n-k-1$)

$$(s + a_{n-k})P_{n-k}(s) = (k+1)\lambda_s P_{n-k-1}(s) \quad (5.10)$$

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)P_j(x,s) = 0 \quad (5.11)$$

(for $j = n+1, n+2, n+3$)

$$P_{n+1}(0, s) = k\lambda_s P_{n-k}(s) \quad (5.12)$$

$$P_{n+2}(0, s) = \lambda_r \sum_{i=0}^{n-k} P_i(s) \quad (5.13)$$

$$P_{n+3}(0, s) = \sum_{i=0}^{n-k} \lambda_{ci} P_i(s) \quad (5.14)$$

Solving differential Equation (5.11), we get the following expression:

$$P_j(x, s) = P_j(0, s) e^{-sx} \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \quad (5.15)$$

(for $j = n+1, n+2, n+3$)

Since

$$P_j(s) = \int_0^{\infty} P_j(x, s) dx \quad (for \ j = n+1, n+2, n+3) \quad (5.16)$$

and together with Equation (5.15), we get

$$P_j(s) = P_j(0, s) \frac{1 - Z_j(s)}{s} \quad (for \ j = n+1, n+2, n+3) \quad (5.17)$$

where

$$\frac{1 - Z_j(s)}{s} = P_j(0, s) \int_0^{\infty} e^{-sx} \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] dx \quad (5.18)$$

(for $j = n+1, n+2, n+3$)

$$Z_j(s) = \int_0^{\infty} e^{-sx} z_j(x) dx \quad (for \ j = n+1, n+2, n+3) \quad (5.19)$$

$$z_j(x) = \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \mu_j(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

Using Equations (5.9) – (5.10), and (5.17), together with

$$\sum_{i=0}^n P_i(s) + \sum_{j=n+1}^{n+4} P_j(s) = \frac{1}{s} \quad (5.20)$$

we get the following Laplace Transforms of state probability solutions:

$$P_i(s) = \frac{N_i(s)}{M_0(s)} \quad (\text{for } i = 0, 1, \dots, n-k) \quad (5.21)$$

$$P_j(s) = \frac{N_j(s)}{M_0(s)} \quad (\text{for } j = n+1, n+2, n+3) \quad (5.22)$$

where

$$k_1 = \frac{n\lambda_s\mu_1}{s+a_1-k_2}$$

$$k_i = \frac{(n-i+1)\lambda_s\mu_i}{(s+a_i)-k_{i+1}} \quad (\text{for } i = 1, 2, \dots, n-k-1)$$

$$k_{n-k} = \frac{(k+1)\lambda_s\mu_{n-k}}{s+a_{n-k}}$$

$$a_{n+1} = k\lambda_s \prod_{i=1}^{n-k} \frac{k_i}{\mu_i}$$

$$a_{n+2} = \lambda_s [1 + \sum_{m=1}^{n-k} (\prod_{i=1}^m \frac{k_i}{\mu_i})]$$

$$a_{n+3} = \lambda_{c0} + \sum_{m=1}^{n-k} (\lambda_{cm} \prod_{i=1}^m \frac{k_i}{\mu_i})$$

$$M_0(s) = s(1 + \sum_{i=1}^{n-k} \prod_{m=1}^i \frac{k_m}{\mu_m} + \sum_{j=n+1}^{n+3} a_j \frac{1-Z_j(s)}{s}) \quad (5.23)$$

$$N_0(s) = 1 \quad (5.24)$$

$$N_i(s) = \prod_{m=1}^i \frac{k_m}{\mu_m} N_0(s) \quad (\text{for } i = 0, 1, 2, \dots, n-k) \quad (5.25)$$

$$N_j(s) = \frac{a_j[1 - Z_j(s)]}{s} \quad (\text{for } j = n+1, n+2, n+3) \quad (5.26)$$

Thus, the Laplace transform of the robot-safety system availability with at least k working safety units is

$$AV_{rs}(s) = \sum_{i=0}^{n-k} P_i(s) = \frac{\sum_{i=0}^{n-k} N_i(s)}{M_0(s)} \quad (5.27)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions in Equation (5.27), and taking the inverse Laplace transform of the resulting equation, we can get the time-dependent robot-safety system availability, $AV_{rs}(t)$.

5.3.2 Steady State Availability Analysis

As time t approaches infinity, state probabilities reach the steady state. Thus, Equations (5.1) – (5.7) reduce to Equations (5.28) – (5.34), respectively.

$$a_0 P_0 = \mu_1 P_1 + \sum_{j=n+1}^{n+3} \int_0^{\infty} P_j(x) \mu_j(x) dx \quad (5.28)$$

$$a_i P_i = (n-i+1) \lambda_s P_{i-1} + \mu_{i+1} P_{i+1} \quad (\text{for } i = 1, 2, \dots, n-k-1) \quad (5.29)$$

$$a_{n-k} P_{n-k} = k \lambda_s P_{n-k} \quad (5.30)$$

$$\frac{dP_j(x)}{dx} + \mu_j(x) P_j(x) = 0 \quad (\text{for } j = n+1, n+2, n+3) \quad (5.31)$$

$$P_{n+1}(0) = k \lambda_s P_{n-k} \quad (5.32)$$

$$P_{n+2}(0) = \lambda_r \sum_{i=0}^{n-k} P_i \quad (5.33)$$

$$P_{n+3}(0) = \sum_{i=0}^{n-k} \lambda_{ci} P_i \quad (5.34)$$

Solving Equation (5.31), we get

$$P_j(x) = P_j(0) \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \quad (\text{for } j = n+1, n+2, n+3) \quad (5.35)$$

After a failure the robot-safety system under repair steady state condition probability is given by

$$P_j = \int_0^\infty P_j(x) dx \quad (\text{for } j = n+1, n+2, n+3) \quad (5.36)$$

Substituting Equation (5.35) into Equation (5.36), yields

$$P_j = P_j(0) E_j[x] \quad (\text{for } j = n+1, n+2, n+3) \quad (5.37)$$

where

$$\begin{aligned} E_j(x) &= \int_0^\infty \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] dx \\ &= \int_0^\infty x z_j(x) dx \end{aligned} \quad (5.38)$$

which is the mean time to robot-safety system repair when the failed robot-safety system is in state j and has an elapsed repair time of x .

Substituting Equations (5.32) - (5.34) into Equation (5.37), we get:

$$P_{n+1} = k \lambda_s P_{n-k} E_{n+1}[x] \quad (5.39)$$

$$P_{n+2} = \lambda_r \sum_{i=0}^{n-k} P_i E_{n+2}[x] \quad (5.40)$$

$$P_{n+3} = \sum_{i=0}^{n-k} \lambda_{ci} P_i E_{n+3}[x] \quad (5.41)$$

Solving Equations (5.29), (5.30), and (5.39) - (5.41), together with

$$\sum_{i=0}^n P_i + \sum_{j=n+1}^{n+4} P_j = 1 \quad (5.42)$$

yield the following steady state probabilities:

$$P_0 = \left(L + \sum_{j=n+1}^{n+3} L_j E_j[x]\right)^{-1} = \frac{1}{G} \quad (5.43)$$

$$P_i = \frac{L_i}{\mu_i} P_{i-1} = \prod_{m=1}^i \frac{L_m}{\mu_m} P_0 \quad (\text{for } i = 1, 2, \dots, n-k-1) \quad (5.44)$$

$$P_{n-k} = \frac{L_{n-k}}{\mu_{n-k}} P_{n-k-1} = \prod_{i=1}^{n-k} \frac{L_i}{\mu_i} P_0 \quad (5.45)$$

$$P_j = L_j E_j[x] P_0 \quad (\text{for } j = n+1, n+2, n+3) \quad (5.46)$$

where

$$\begin{aligned} L &= 1 + \sum_{m=1}^{n-k} \prod_{i=1}^m \frac{L_i}{\mu_i} \\ L_i &= \frac{(n-i+1)\lambda_s \mu_i}{a_i - L_{i+1}} \quad (\text{for } i = 1, 2, \dots, n-k-1) \\ L_{n-k} &= \frac{(k+1)\lambda_s \mu_{n-k}}{a_{n-k}} \\ L_{n+1} &= k\lambda_s \prod_{i=1}^{n-k} \frac{L_i}{\mu_i} \\ L_{n+2} &= \lambda_r \left(1 + \sum_{m=1}^{n-k} \prod_{i=1}^m \frac{L_i}{\mu_i}\right) \\ L_{n+3} &= \lambda_{c0} + \sum_{m=1}^{n-k} \lambda_{cm} \prod_{i=1}^m \frac{L_i}{\mu_i} \\ G &= L + \sum_{j=n+1}^{n+3} L_j E_j[x] \end{aligned} \quad (5.47)$$

The steady state availability of the robot-safety system with at least k working safety units is

$$SSAV_{rs} = \sum_{i=0}^{n-k} P_i = \frac{L}{G} \quad (5.48)$$

For different failed system repair time distributions, the values of G are obtained as follows:

- (i) When the failed robot-safety system repair time x is exponentially distributed, the probability density function of the repair time is

$$z_j(x) = \mu_j e^{-\mu_j x} \quad (\mu_j > 0, \quad j = n+1, n+2, n+3) \quad (5.49)$$

where x is the repair time, and μ_j is the constant repair rate of state j . Thus, the mean time to robot-safety system repair, $E_j[x]$, for the exponential distribution is

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \frac{1}{\mu_j} \quad (\text{for } j = n+1, n+2, n+3) \quad (5.50)$$

Substituting Equation (5.50) into Equation (5.47), we get

$$G = G_e = L + \sum_{j=n+1}^{n+3} (L_j \frac{1}{\mu_j}) \quad (5.51)$$

- (ii) When the failed robot-safety system repair time x is gamma distributed, the probability density function of the repair time is

$$z_j(x) = \frac{\mu_j (\mu_j x)^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \quad (\beta > 0, \quad j = n+1, n+2, n+3) \quad (5.52)$$

where x is the repair time, $\Gamma(\beta)$ is the gamma function, and β and μ_j are the shape and scale parameters, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the gamma distribution is

$$E_j[x] = \int_0^{\infty} x z_j(x) dx = \frac{\beta}{\mu_j} \quad (\text{for } j = n+1, n+2, n+3) \quad (5.53)$$

Substituting Equation (5.53) into Equation (5.47), we get

$$G = G_g = L + \sum_{j=n+1}^{n+3} (L_j \frac{\beta}{\mu_j}) \quad (5.54)$$

- (iii) When the failed robot-safety system repair time x is Weibull distributed, the probability density function of the repair time is expressed by

$$z_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j(x)^\beta} \quad (\beta > 0, j = n+1, n+2, n+3) \quad (5.55)$$

where x is the repair time, and β and μ_j are the shape and scale parameters of the Weibull distribution, respectively. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Weibull distribution is given by

$$E_j[x] = \int_0^\infty x z_j(x) dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \quad (5.56)$$

(for $j = n+1, n+2, n+3$)

Substituting Equation (5.56) into Equation (5.47), we get

$$G = G_w = L + \sum_{j=n+1}^{n+3} \left[L_j \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \right] \quad (5.57)$$

- (iv) When the failed robot-safety system repair time x is Rayleigh distributed, the probability density function of the Rayleigh distribution is expressed by

$$z_j(x) = \mu_j x e^{-\mu_j x^2/2} \quad (\mu_j > 0, j = n+1, n+2, n+3) \quad (5.58)$$

where x is the repair time, and μ_j is the scale parameter. Thus, the mean time to robot-safety system repair, $E_j[x]$, for the Rayleigh distribution is

$$E_j[x] = \int_0^\infty x z_j(x) dx = \sqrt{\frac{\pi}{4\mu_j}} \quad (5.59)$$

(for $j = n+1, n+2, n+3$)

Substituting Equation (5.59) into Equation (5.47), we get

$$G = G_r = L + \sum_{j=n+1}^{n+3} \left(L_j \sqrt{\frac{\pi}{4\mu_j}} \right) \quad (5.60)$$

- (v) When the robot-safety system repair time x is lognormal distributed, the probability density function of the repair time is

$$z_j(x) = \frac{1}{x\sigma_{y_j}\sqrt{2\pi}} e^{\frac{-(\ln x - \mu_{y_j})^2}{2\sigma_{y_j}^2}} \quad (\text{for } j = n+1, n+2, n+3) \quad (5.61)$$

where x is the repair time, and $\ln x$ is the natural logarithms of x with a mean and variance μ and σ^2 , respectively. The conditions on parameters are as follows:

$$\begin{aligned} \sigma_{y_j} &= \ln \sqrt{1 + \left(\frac{\sigma_{x_j}}{\mu_{x_j}}\right)^2}, \\ \mu_{y_j} &= \ln \sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \end{aligned} \quad (5.62)$$

(for $j = n+1, n+2, n+3$)

Hence, the failed robot-safety system mean time to repair, $E_j[x]$, for the lognormal distribution is

$$E_j[x] = e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \quad (\text{for } j = n+1, n+2, n+3) \quad (5.63)$$

Substituting Equation (5.63) into Equation (5.47), we get

$$G = G_t = L + \sum_{j=n+1}^{n+3} [L_j e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})}] \quad (5.64)$$

5.3.3 System Reliability, MTTF, and Variance of Time to Failure

Setting $\mu_{n+1}(x) = \mu_{n+2}(x) = \mu_{n+3}(x) = 0$ in Figure 5.2 and applying the Markov method, we get the following differential equations:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_1 P_1(t) \quad (5.65)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_s P_{i-1}(t) + \mu_{i+1} P_{i+1}(t) \quad (5.66)$$

(for $i = 1, 2, \dots, n-k-1$)

$$\frac{dP_{n-k}(t)}{dt} + a_{n-k}P_{n-k}(t) = (k+1)\lambda_s P_{n-k-1}(t) \quad (5.67)$$

$$\frac{dP_{n+1}(t)}{dt} = k\lambda_s P_{n-k}(t) \quad (5.68)$$

$$\frac{dP_{n+2}(t)}{dt} = \lambda_r \sum_{i=0}^{n-k} P_i(t) \quad (5.69)$$

$$\frac{dP_{n+3}(t)}{dt} = \sum_{i=0}^{n-k} \lambda_{ci} P_i(t) \quad (5.70)$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero. Taking the Laplace transforms of Equations (5.65) - (5.70) and solving the resulting set of equations, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^{n-k} \prod_{m=1}^i \frac{k_m}{\mu_m} + \sum_{j=n+1}^{n+3} \frac{a_j}{s})]^{-1} \quad (5.71)$$

$$P_i(s) = \prod_{m=1}^i \frac{k_m}{\mu_m} P_0(s) \quad (\text{for } i = 1, 2, \dots, n-k) \quad (5.72)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = n+1, n+2, n+3) \quad (5.73)$$

The Laplace transform of the robot-safety system reliability with at least k working safety units is

$$R_{rs}(s) = \sum_{i=0}^{n-k} P_i(s) = (1 + \sum_{m=1}^{n-k} \prod_{i=1}^m \frac{k_i}{\mu_i}) P_0(s) \quad (5.74)$$

Using Equation (5.74), the robot-safety system mean time to the failure is obtained as follows [103]:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + \sum_{m=1}^{n-k} \prod_{i=1}^m \frac{L_i}{\mu_i}}{\sum_{j=n+1}^{n+3} L_j} \quad (5.75)$$

The time-dependant robot-safety system reliability, $R_{rs}(t)$, can be obtained by taking the inverse Laplace transform of Equation (5.74).

The robot-safety system variance of time to failure is expressed by [103]

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_{rs}'(s) - (MTTF_{rs})^2 \\ &= \frac{2(1 + \sum_{m=1}^{n-k} \prod_{i=1}^m \frac{L_i}{\mu_i})(1 + \sum_{m=1}^{n-k} \prod_{i=1}^m \frac{L_i}{\mu_i} + \sum_{j=n+1}^{n+3} a_{dj})}{(\sum_{j=n+1}^{n+3} L_j)^2} - \frac{2 \sum_{m=1}^{n-k} k_{dm}}{\sum_{j=n+1}^{n+3} L_j} - (MTTF_{rs})^2 \end{aligned} \quad (5.76)$$

where

$R_{rs}'(s)$ denotes the derivative of $R_{rs}(s)$ with respect to s .

$$k_{dm} = \lim_{s \rightarrow 0} \left(\prod_{i=1}^m \frac{k_i}{\mu_i} \right)' \quad (\text{for } m = 1, 2, \dots, n-k)$$

$$a_{dj} = \lim_{s \rightarrow 0} a_j' \quad (\text{for } j = n+1, n+2, n+3)$$

$$a_{dn+1} = \lim_{s \rightarrow 0} a_{n+1}' = k \lambda_s k_{dn-k}$$

$$a_{dn+2} = \lim_{s \rightarrow 0} a_{n+2}' = \lambda_r \sum_{m=1}^{n-k} k_{dm}$$

$$a_{dn+3} = \lim_{s \rightarrow 0} a_{n+3}' = \sum_{m=1}^{n-k} \lambda_{cm} k_{dm}$$

$$\left(\prod_{i=1}^m \frac{k_i}{\mu_i} \right)' \text{ denotes the derivative of } \prod_{i=1}^m \frac{k_i}{\mu_i} \text{ with respect to } s.$$

a_j' denotes the derivative of a_j with respect to s .

The number of safety units incorporated within the robot-safety system is the matter of desired level of safety. More safety units we use, the better system safety, reliability, and MTTF we can achieve.

5.4 Special Case Model 5.I: (k=2, n=3)

For k=2 and n=3 in Figures 5.1 and 5.2, the model becomes for a system having one robot and three redundant safety units*. However, at least two safety units must function successfully for the robot-safety system success. The corresponding system of Equations can be obtained from Equations (5.1)-(5.7) by setting k=2 and n=3. Furthermore, robot-safety system state probabilities $[P_i(t), P_j(t), P_i, P_j]$, availabilities $[AV_{rs}(t), SSAV_{rs}]$, reliability $[R_{rs}(t)]$, mean time to failure $[MTTF_{rs}]$, and variance of time to failure $[\sigma^2]$ for the special case model can also be obtained by inserting k=2 and n=3 into the corresponding generalized Equations.

5.4.1 Time Dependant Availability Plots for k=2 and n=3

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c0}=0.0002, \quad \lambda_{c1}=0.0001,$$

$$\mu_1=0.0009, \quad \mu_4=0.0011, \quad \mu_5=0.0012, \quad \mu_6=0.0006$$

in Equations (5.21)-(5.22) and (5.27), and for gamma distributed failed system repair times using Maple computer program [102], the time-dependant plots of robot-safety system state probabilities and availability are shown in Figures 5.3 and 5.4, respectively.

* Detailed analysis is provided in *Appendix E*.

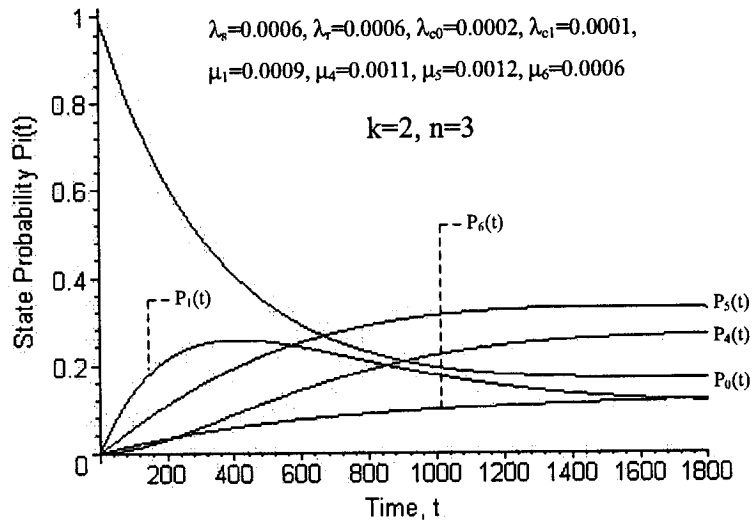


Figure 5.3 Time-dependent probability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair times.

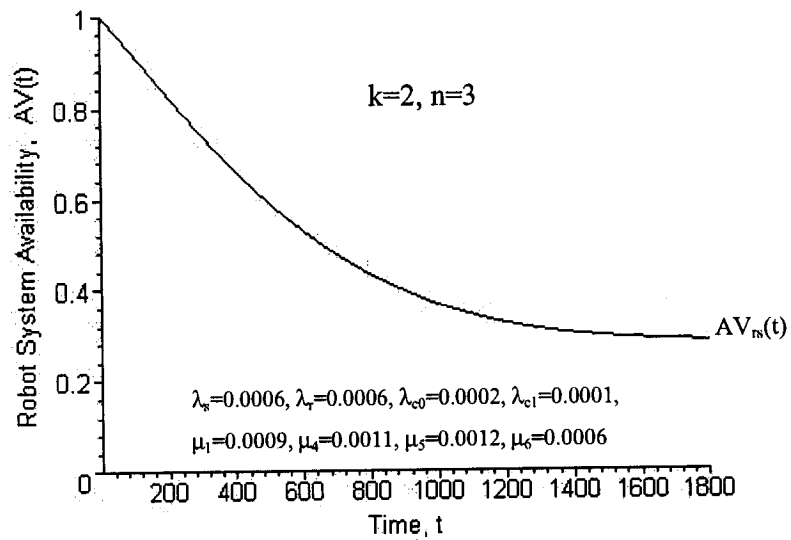


Figure 5.4 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair times.

5.4.2 Steady State Availability Plots for k=2 and n=3

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad \lambda_{c1}=0.0001,$$

$$\mu_1=0.0009, \quad \mu_4=0.0011, \quad \mu_5=0.0012, \quad \mu_6=0.0006$$

in Equation (5.48), and for gamma and Weibull distributed failed system repair times using Maple computer program [102] plots for $SSAV_{rs}$ are shown in Figures 5.5 and 5.6, respectively.

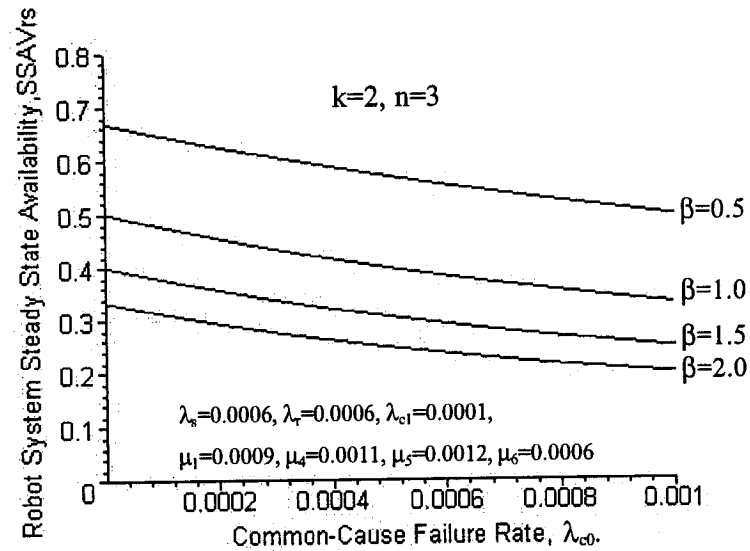


Figure 5.5 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with gamma distributed ($\beta=0.5, 1, 1.5, 2$) failed system repair times.

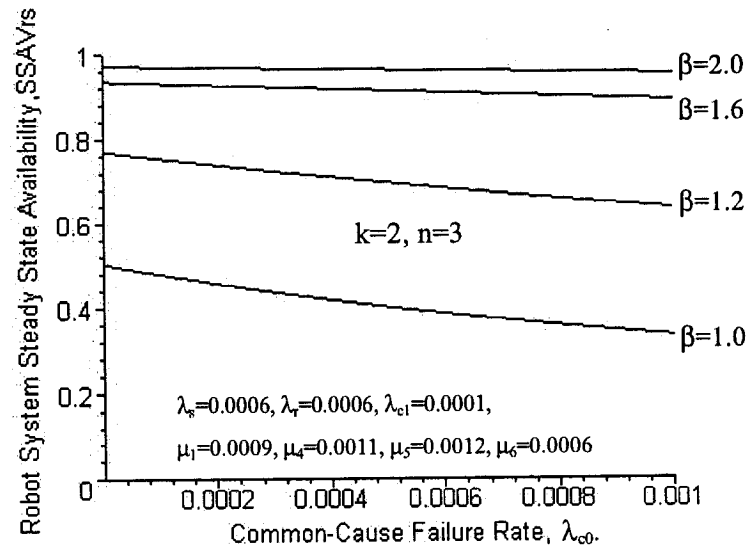


Figure 5.6 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair times.

5.4.3 Reliability and MTTF Plots for $k=2$ and $n=3$

Setting:

$$\lambda_s=0.0006, \quad \lambda_r=0.0006, \quad (\lambda_{c0}=0.0002), \quad \lambda_{c1}=0.0001,$$

$$\mu_4=\mu_5=\mu_6=0$$

in Equation (5.74) and using Maple computer program [102], the time-dependant reliability plots of the robot-safety system are shown in Figure 5.7. Similarly, plots of the robot-safety system mean time to failure, using Equation (5.75), as a function of common-cause failure rate (λ_{c0}), are shown in Figures 5.8.

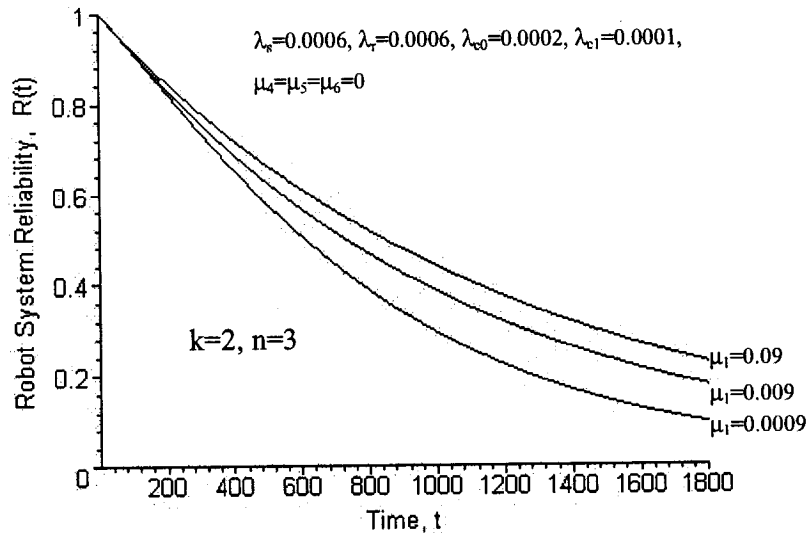


Figure 5.7 Reliability plots of the robot-safety system.

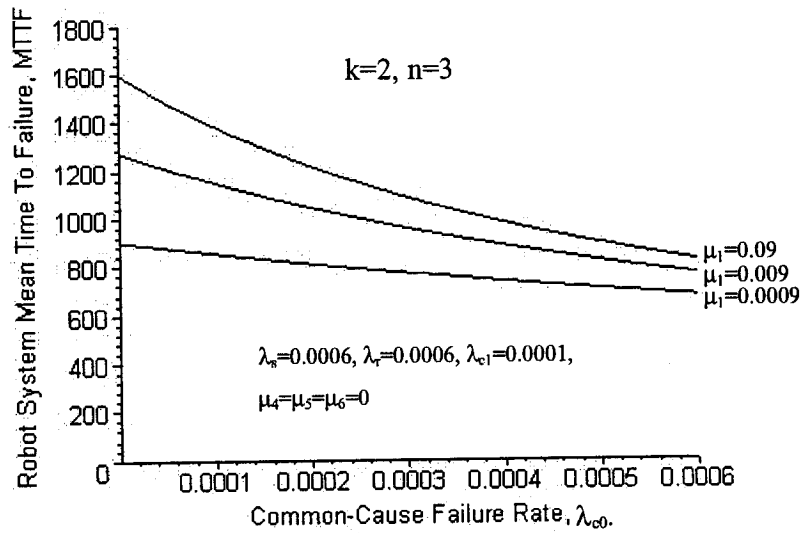


Figure 5.8 Mean time to failure plots of the robot-safety system as a function of common-cause failure rate(λ_{e0}).

CONCLUSIONS AND RECOMMENDATIONS

6.1 Discussion

This study presented a review of published literature on robot reliability and safety for the period 1997-2002 and a reliability and availability analyses of various robot-safety systems with common-cause failures.

The models developed in the study incorporate elements of hardware failures (failures of robots and safety units), common-cause failures and general repairable systems. The analysis considers robot-safety systems with constant robot and safety unit failure rates, constant common-cause failure rates, and constant partially failed system repair rates, and arbitrarily distributed failed system repair times. Generalized expressions for state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure are presented. Some plots of these expressions are shown.

If the failed robot-safety system repair times is governed by the exponential distribution, then the transition rate from one state to another state of a system is constant and does not depend on how long the system spends in a given state nor does it depend on how it arrived at a particular state. This assumption leads to Markovian process, and Markov method was used to perform mathematical analysis of the system. However, when the distribution is not exponential (e.g. gamma, Weibull, Rayleigh, or lognormal), then the repair process becomes non-Markovian, and the supplementary variable method was used to perform mathematical analysis of the system.

Due to the computational complexity, additional condition that robots are irreparable at the operable state of the robot-safety system is assumed to obtain the generalized formulas pertaining to the model presented in Chapter 4. Moreover, for the models in Chapter 3 and 4 it is assumed that the repair of the safety unit has the priority over the repair of the robot when the overall system is in the partially failed operating state.

6.2 Conclusions

The main results obtained in this study can be summarized as follows:

- Based on certain assumptions, generalized formulas for the robot-safety system state probabilities, system availabilities, reliability, mean time to failure, and variance of time to failure were developed.
- The generalized formulas developed in this study can be used to calculate robot-safety system reliability and availability, and to develop appropriate maintenance scheduling policies.
- The number of safety units incorporated within the robot-safety system is the matter of desired level of safety. More safety units use, the better system safety, reliability and MTTF achieve.
- The robot-safety system's availabilities (steady state availability and time-dependant availability), reliability, mean time to failure decrease with increasing values of common-cause failures. This is true regardless of whether the common-cause failure rates are time-dependent or constant, the system units are identical or non-identical, the system is repairable or non-repairable.
- Comparing all the models developed in this study, it can be observed that an increase in the number of redundant robots would improve system reliability and

mean time to failure more significantly than that of an increase in the number of redundant safety units. On the other hand, an increase in the number of redundant safety units would also improve system safety tremendously.

6.3 Recommendations for Further Study

1. The time-dependent system availability was studied for special cases when failed robot-safety system repair times were assumed exponentially or gamma distributed. It is still difficult to get the time-dependent system availability expressions for Weibull, Rayleigh, or lognormally distributed failed system repair times since the Laplace transforms of the probability density functions of those system repair time distributions don't exist. For those distributions, different techniques need to be developed in further studies to solve this problem.
2. Critical common-cause failures that cause the entire robot-safety system failure were considered in this thesis. The models could be studied further by considering both critical and non-critical common-cause failures.
3. The models discussed in this thesis assumed constant robot and safety unit failure rates, constant common-cause failure rates, and constant partially failed system repair rates, which can be studied further with time-dependent failure rates and partially failed system repair rates.

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APPENDIX A

Special Case Model 2.1

Setting $n=2$ in Equations (2.1)-(2.8). The model becomes for a system having one robot and two redundant safety units. The corresponding system of Equations become

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{j=3}^6 \int_0^{\infty} P_j(x,t) \mu_j(x) dx \quad (\text{A.1})$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_s P_0(t) + \mu_2 P_2(t) \quad (\text{A.2})$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = \lambda_r P_1(t) \quad (\text{A.3})$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \quad (\text{for } j = 3,4,5,6) \quad (\text{A.4})$$

$$P_3(0,t) = \lambda_{ri} P_2(t) \quad (\text{A.5})$$

$$P_4(0,t) = \lambda_{rs} P_4(t) \quad (\text{A.6})$$

$$P_5(0,t) = \lambda_r \sum_{i=0}^1 P_i(t) \quad (\text{A.7})$$

$$P_6(0,t) = \sum_{i=0}^1 \lambda_{ci} P_i(t) \quad (\text{A.8})$$

where

$$a_0 = 2\lambda_s + \lambda_r + \lambda_{c0}$$

$$a_1 = \lambda_s + \lambda_r + \lambda_{c1} + \mu_1$$

$$a_2 = \lambda_{ri} + \lambda_{rs} + \mu_2$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero.

A.1 Time Dependant Availability Analysis

Setting $n=2$ in Equations (2.23) and (2.24), we get

$$P_i(s) = \frac{N_i(s)}{M_0(s)} \quad (\text{for } i = 0,1,2) \quad (\text{A.9})$$

$$P_j(s) = \frac{N_j(s)}{M_0(s)} \quad (\text{for } j = 3,4,5,6) \quad (\text{A.10})$$

where

$$M_0(s) = s(1 + \sum_{m=1}^2 \prod_{i=1}^m \frac{k_i}{\mu_i} + \sum_{j=3}^6 \frac{a_j}{s + \mu_j})$$

$$N_0(s) = 1$$

$$N_i(s) = \prod_{m=1}^i \frac{k_m}{\mu_m} N_0(s) \quad (\text{for } i = 1,2)$$

$$N_3(s) = \frac{a_3}{s + \mu_3}$$

$$N_4(s) = \frac{a_4}{s + \mu_4}$$

$$N_5(s) = \frac{a_5}{s + \mu_5}$$

$$N_6(s) = \frac{a_6}{s + \mu_6}$$

$$k_1 = \frac{2\lambda_s \mu_1}{s + a_1 - k_2}$$

$$k_2 = \frac{\mu_2 \lambda_s}{s + a_2}$$

$$a_3 = \lambda_{ri} \prod_{i=1}^2 \frac{k_i}{\mu_i}$$

$$a_4 = \lambda_{rs} \prod_{i=1}^2 \frac{k_i}{\mu_i}$$

$$a_5 = \lambda_r \left(1 + \frac{k_1}{\mu_1}\right)$$

$$a_6 = \lambda_{c0} + \lambda_{c1} \frac{k_1}{\mu_1}$$

From Equation (2.29), the Laplace transform of the robot-safety system availability with at least one working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^1 P_i(s) = \frac{\sum_{i=0}^1 N_i(s)}{M_0(s)} \quad (\text{A.11})$$

Similarly, from Equation (2.30) the Laplace transform of the robot-safety system availability with or without the working safety units is given by

$$AV_r(s) = \sum_{i=0}^2 P_i(s) = \frac{\sum_{i=0}^2 N_i(s)}{M_0(s)} \quad (\text{A.12})$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions in Equations (A.11) and (A.12), and taking the inverse Laplace transforms of the resulting equations, we can get the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

A.2 Steady State Availability Analysis

Setting $n=2$ in Equations (2.48) - (2.51), we get

$$P_0 = (L + \sum_{j=3}^6 L_j E_j[x])^{-1} = \frac{1}{G} \quad (\text{A.13})$$

$$P_1 = \frac{L_1}{\mu_1} P_0 \quad (\text{A.14})$$

$$P_2 = \frac{L_2}{\mu_2} P_1 = \prod_{i=1}^2 \frac{L_i}{\mu_i} P_0 \quad (\text{A.15})$$

$$P_j = L_j E_j[x] P_0 \quad (\text{for } j = 3, 4, 5, 6) \quad (\text{A.16})$$

where

$$L = 1 + \sum_{m=1}^2 \prod_{i=1}^m \frac{L_i}{\mu_i}$$

$$L_1 = \frac{2\lambda_s \mu_1}{a_1 - L_2}$$

$$L_2 = \frac{\lambda_s \mu_2}{a_2}$$

$$L_3 = \lambda_{ri} \prod_{i=1}^2 \frac{L_i}{\mu_i}$$

$$L_4 = \lambda_{rs} \prod_{i=1}^2 \frac{L_i}{\mu_i}$$

$$L_5 = \lambda_r \left(1 + \frac{L_1}{\mu_1}\right)$$

$$L_6 = \lambda_{e0} + \lambda_{e1} \frac{L_1}{\mu_1}$$

$$G = L + \sum_{j=3}^6 L_j E_j[x] \quad (\text{A.17})$$

From Equation (2.53), the steady state availability of the robot-safety system with at least one working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^1 P_i = \frac{L - \prod_{i=1}^2 \frac{L_i}{\mu_i}}{G} \quad (\text{A.18})$$

Similarly, from Equation (2.54) the steady state availability of the robot-safety system with or without working safety units is given by

$$SSAV_r = \sum_{i=0}^n P_i = \frac{L}{G} \quad (\text{A.19})$$

A.3 System Reliability, MTTF, and Variance of Time to Failure

Setting $n=2$ in Equations (2.78) - (2.80), we get

$$P_0(s) = [s(1 + \sum_{i=1}^2 \prod_{m=1}^i \frac{k_m}{\mu_m} + \sum_{j=3}^6 \frac{a_j}{s})]^{-1} \quad (\text{A.20})$$

$$P_i(s) = \prod_{m=1}^i \frac{k_m}{\mu_m} P_0(s) \quad (\text{for } i = 1, 2) \quad (\text{A.21})$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 3, 4, 5, 6) \quad (\text{A.22})$$

From Equation (2.81), the Laplace transform of the robot-safety system reliability with at least one working safety unit is

$$R_{rs}(s) = \sum_{i=0}^1 P_i(s) = (1 + \frac{k_1}{\mu_1}) P_0(s) \quad (\text{A.23})$$

From Equation (2.82), the robot-safety system mean time to the failure is obtained as follows:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + \frac{L_1}{\mu_1}}{\sum_{j=3}^6 L_j} \quad (\text{A.24})$$

Similarly, from Equation (2.83) the Laplace transform of the robot-safety system reliability with or without the working safety units is

$$R_r(s) = \sum_{i=0}^2 P_i(s) = (1 + \sum_{m=1}^2 \prod_{i=1}^m \frac{k_i}{\mu_i}) P_0(s) \quad (\text{A.25})$$

From Equation (2.84), the mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + \sum_{m=1}^2 \prod_{i=1}^m \frac{L_i}{\mu_i}}{\sum_{j=3}^6 L_j} \quad (\text{A.26})$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (A.23) and (A.25).

From Equation (2.85), the system variance of time to failure is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + \sum_{m=1}^2 \prod_{i=1}^m \frac{L_i}{\mu_i})(1 + \sum_{m=1}^2 \prod_{i=1}^m \frac{L_i}{\mu_i} + \sum_{j=3}^6 a_{dj})}{(\sum_{j=3}^6 L_j)^2} - \frac{2 \sum_{m=1}^2 k_{dm}}{\sum_{j=3}^6 L_j} - (MTTF_r)^2 \end{aligned} \quad (\text{A.27})$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$k_{dm} = \lim_{s \rightarrow 0} \left(\prod_{i=1}^m \frac{k_i}{\mu_i} \right)' \quad (\text{for } m = 1, 2)$$

$$a_{d3} = \lim_{s \rightarrow 0} a_3' = \lambda_{r1} k_{d2}$$

$$a_{d4} = \lim_{s \rightarrow 0} a_4' = \lambda_{rs} k_{d2}$$

$$a_{d5} = \lim_{s \rightarrow 0} a_5' = \lambda_r k_{d1}$$

$$a_{d6} = \lim_{s \rightarrow 0} a_6' = \lambda_{c1} k_{d1}$$

$$\left(\prod_{i=1}^m \frac{k_i}{\mu_i} \right)' \text{ denotes the derivative of } \prod_{i=1}^m \frac{k_i}{\mu_i} \text{ with respect to } s.$$

a_j' denotes the derivative of a_j with respect to s .

APPENDIX B

Special Case Model 3.I

Setting $k=1$ and $n=2$ in Equations (3.1)-(3.10). The model becomes for a system having two redundant robots and one built-in safety unit. Since $k=1$, it is a parallel redundancy.

The corresponding system of Equations become

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_2(t) + \theta_1 P_3(t) + \sum_{j=4}^6 \int_0^{\infty} P_j(x,t) \mu_{rj}(x) dx \quad (\text{B.1})$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) + \mu_s P_3(t) \quad (\text{B.2})$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = \lambda_s P_0(t) \quad (\text{B.3})$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = 2\lambda_r P_2(t) + \lambda_s P_1(t) \quad (\text{B.4})$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_{rj}(x) P_j(x,t) = 0 \quad (\text{for } j = 4,5,6) \quad (\text{B.5})$$

$$P_4(0,t) = \lambda_r P_3(t) \quad (\text{B.6})$$

$$P_5(0,t) = \lambda_r P_1(t) \quad (\text{B.7})$$

$$P_6(0,t) = \sum_{i=0}^2 \lambda_{ci} P_i(t) \quad (\text{B.8})$$

where

$$a_0 = 2\lambda_r + \lambda_{c0} + \lambda_s$$

$$a_1 = \lambda_r + \lambda_{c1} + \lambda_s + \mu_{r1}$$

$$a_2 = 2\lambda_r + \lambda_{c2} + \mu_s$$

$$a_3 = \lambda_r + \theta_1 + \mu_s$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero.

B.1 Time Dependant Availability Analysis

Using Laplace Transform technique and the initial conditions in Equations (B.1)-(B.8),

we obtain

$$(s + a_0)P_0(s) = 1 + \theta_1 P_3(s) + \mu_s P_2(s) + \mu_{r1} P_1(s) + \sum_{j=4}^6 \int_0^{\infty} P_j(x, s) \mu_{rj}(x) dx \quad (\text{B.9})$$

$$(s + a_1)P_1(s) = 2\lambda_r P_0(s) + \mu_s P_3(s) \quad (\text{B.10})$$

$$(s + a_2)P_2(s) = \lambda_s P_0(s) \quad (\text{B.11})$$

$$(s + a_3)P_3(s) = 2\lambda_r P_2(s) + \lambda_s P_1(s) \quad (\text{B.12})$$

$$sP_j(x, s) + \frac{\partial P_j(x, s)}{\partial x} + \mu_{rj}(x)P_j(x, s) = 0 \quad (\text{for } j = 4, 5, 6) \quad (\text{B.13})$$

$$P_4(0, s) = \lambda_r P_3(s) \quad (\text{B.14})$$

$$P_5(0, s) = \lambda_r P_1(s) \quad (\text{B.15})$$

$$P_6(0, s) = \sum_{i=0}^2 \lambda_{ci} P_i(s) \quad (\text{B.16})$$

Solving differential Equation (B.13), we get the following expression:

$$P_j(x, s) = P_j(0, s) e^{-sx} \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] \quad (\text{for } j = 4, 5, 6) \quad (\text{B.17})$$

Since

$$P_j(s) = \int_0^{\infty} P_j(x, s) dx \quad (\text{for } j = 4, 5, 6) \quad (\text{B.18})$$

and together with Equation (B.17), we get

$$P_j(s) = P_j(0, s) \frac{1 - Z_j(s)}{s} \quad (\text{for } j = 4, 5, 6) \quad (\text{B.19})$$

where

$$\frac{1-Z_j(s)}{s} = P_j(0,s) \int_0^\infty e^{-sx} \exp[-\int_0^x \mu_{r_j}(\delta) d\delta] dx \quad (\text{B.20})$$

(for $j = 4,5,6$)

$$Z_j(s) = \int_0^\infty e^{-sx} z_j(x) dx \quad (\text{for } j = 4,5,6) \quad (\text{B.21})$$

$$z_j(x) = \exp[-\int_0^x \mu_{r_j}(\delta) d\delta] \mu_{r_j}(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

The Laplace transforms of the probabilities of all the system states add up to $1/s$, i.e.,

$$\sum_{i=0}^3 P_i(s) + \sum_{j=4}^6 P_j(s) = \frac{1}{s} \quad (\text{B.22})$$

Solving Equations (B.10)-(B.12), (B.14)-(B.16), (B.19), and (B.22), we get

$$P_0(s) = [s(1 + Y_1 + \frac{\lambda_s}{s+a_2} + Y_2 + \sum_{j=4}^6 a_j \frac{1-Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (\text{B.23})$$

$$P_1(s) = Y_1 P_0(s) \quad (\text{B.24})$$

$$P_2(s) = \frac{\lambda_s}{s+a_2} P_0(s) \quad (\text{B.25})$$

$$P_3(s) = Y_2 P_0(s) \quad (\text{B.26})$$

$$P_j(s) = \frac{a_j [1-Z_j(s)]}{s} P_0(s) \quad (\text{for } j = 4,5,6) \quad (\text{B.27})$$

where

$$Y_1 = \frac{2\lambda_r (s+a_3 + \frac{\lambda_s \mu_s}{s+a_2})}{(s+a_1)(s+a_3) - \lambda_s \mu_s}$$

$$Y_2 = \frac{2\lambda_r\lambda_s(1 + \frac{s+a_1}{s+a_2})}{(s+a_1)(s+a_3) - \lambda_s\mu_s}$$

$$a_4 = \lambda_r Y_2$$

$$a_5 = \lambda_r Y_1$$

$$a_6 = \lambda_{c0} + \lambda_{c1} Y_1 + \frac{\lambda_{c2}\lambda_s}{s+a_2}$$

$$H = s(1 + Y_1 + \frac{\lambda_s}{s+a_2} + Y_2 + \sum_{j=4}^6 a_j \frac{1-Z_j(s)}{s})$$

Thus, the Laplace transform of the robot-safety system availability with the working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^1 P_i(s) = \frac{(1+Y_1)}{H} \quad (B.28)$$

Similarly, the Laplace transform of the robot-safety system availability with or without the working safety unit is given by

$$AV_r(s) = \sum_{i=0}^3 P_i(s) = \frac{1 + Y_1 + \frac{\lambda_s}{s+a_2} + Y_2}{H} \quad (B.29)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions into Equations (B.28) and (B.29), and taking the inverse Laplace transforms of the resulting equations, we can obtain the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

B.2 Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Thus, Equations (B.1)-(B.8) reduce to Equations (B.30)-(B.37), respectively.

$$a_0 P_0 = \mu_r P_1 + \mu_s P_2 + \theta_1 P_3 + \sum_{j=4}^6 \int_0^{\infty} P_j(x) \mu_{rj}(x) dx \quad (\text{B.30})$$

$$a_1 P_1 = 2\lambda_r P_0 + \mu_s P_3 \quad (\text{B.31})$$

$$a_2 P_2 = \lambda_s P_0 \quad (\text{B.32})$$

$$a_3 P_3 = 2\lambda_r P_2 + \lambda_s P_1 \quad (\text{B.33})$$

$$\frac{dP_j(x)}{dx} + \mu_{rj}(x) P_j(x) = 0 \quad (\text{for } j = 4, 5, 6) \quad (\text{B.34})$$

$$P_4(0) = \lambda_r P_3 \quad (\text{B.35})$$

$$P_5(0) = \lambda_r P_1 \quad (\text{B.36})$$

$$P_6(0) = \sum_{i=0}^2 \lambda_{ci} P_i \quad (\text{B.37})$$

Solving Equation (B.34), we get

$$P_j(x) = P_j(0) \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] \quad (\text{for } j = 4, 5, 6) \quad (\text{B.38})$$

After a failure the robot-safety system under repair steady state condition probability is given by

$$P_j = \int_0^{\infty} P_j(x) dx \quad (\text{for } j = 4, 5, 6) \quad (\text{B.39})$$

Substituting Equation (B.38) into Equation (B.39), yields

$$P_j = P_j(0) E_j[x] \quad (\text{for } j = 4, 5, 6) \quad (\text{B.40})$$

where

$$\begin{aligned} E_j(x) &= \int_0^{\infty} \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] dx \\ &= \int_0^{\infty} x z_j(x) dx \end{aligned} \quad (\text{B.41})$$

which is the mean time to robot-safety system repair when the failed robot-safety system is in state j and has an elapsed repair time of x .

Substituting Equations (B.35) - (B.37) into Equation (B.40), yields:

$$P_4 = \lambda_r P_3 E_4[x] \quad (\text{B.42})$$

$$P_5 = \lambda_r P_1 E_5[x] \quad (\text{B.43})$$

$$P_6 = \sum_{i=0}^2 \lambda_{ci} P_i E_6[x] \quad (\text{B.44})$$

Using Equations (B.31)-(B.33), (B.42) - (B.44), together with

$$\sum_{i=0}^3 P_i + \sum_{j=4}^6 P_j = 1 \quad (\text{B.45})$$

we get the following general steady state probability solutions:

$$P_0 = (1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4 + \sum_{j=4}^6 a_{sj} E_j[x])^{-1} = \frac{1}{G} \quad (\text{B.46})$$

$$P_1 = Y_3 P_0 \quad (\text{B.47})$$

$$P_2 = \frac{\lambda_s}{a_2} P_0 \quad (\text{B.48})$$

$$P_3 = Y_4 P_0 \quad (\text{B.49})$$

$$P_j = a_{sj} E_j[x] P_0 \quad (\text{for } j = 4, 5, 6) \quad (\text{B.50})$$

where

$$Y_3 = \lim_{s \rightarrow 0} Y_1 = \frac{2\lambda_r (a_3 + \frac{\lambda_s \mu_s}{a_2})}{a_1 a_3 - \lambda_s \mu_s}$$

$$Y_4 = \lim_{s \rightarrow 0} Y_2 = \frac{2\lambda_r \lambda_s (1 + \frac{a_1}{a_2})}{a_1 a_3 - \lambda_s \mu_s}$$

$$a_{s4} = \lambda_r Y_4$$

$$a_{s5} = \lambda_r Y_3$$

$$a_{s6} = \lambda_{c0} + \lambda_{c1} Y_3 + \frac{\lambda_{c2} \lambda_s}{a_2}$$

$$G = 1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4 + \sum_{j=4}^6 a_{sj} E_j[x] \quad (\text{B.51})$$

The steady state availability of the robot-safety system with the working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^1 P_i = \frac{1 + Y_3}{G} \quad (\text{B.52})$$

Similarly, the steady state availability of the robot-safety system with or without the working safety unit is given by

$$SSAV_r = \sum_{i=0}^3 P_i = \frac{1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4}{G} \quad (\text{B.53})$$

B.3 System Reliability, MTTF, and Variance of Time to Failure

Setting $\mu_{r4}(x) = \mu_{r5}(x) = \mu_{r6}(x) = 0$, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_2(t) + \theta_1 P_3(t) \quad (\text{B.54})$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) + \mu_s P_3(t) \quad (\text{B.55})$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = \lambda_s P_0(t) \quad (\text{B.56})$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = 2\lambda_r P_2(t) + \lambda_s P_1(t) \quad (\text{B.57})$$

$$\frac{dP_4(t)}{dt} = \lambda_r P_3(t) \quad (B.58)$$

$$\frac{dP_5(t)}{dt} = \lambda_r P_1(t) \quad (B.59)$$

$$\frac{dP_6(t)}{dt} = \sum_{i=0}^2 \lambda_{ci} P_i(t) \quad (B.60)$$

At time $t=0$, $P_0(0)=1$, and all other initial state probabilities are equal to zero. Taking the Laplace transforms of Equations (B.55) - (B.60), and solving the resulting set of Equations, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + Y_1 + \frac{\lambda_s}{s + a_2} + Y_2 + \sum_{j=4}^6 \frac{a_j}{s})]^{-1} \quad (B.61)$$

$$P_1(s) = Y_1 P_0(s) \quad (B.62)$$

$$P_2(s) = \frac{\lambda_s}{s + a_2} P_0(s) \quad (B.63)$$

$$P_3(s) = Y_2 P_0(s) \quad (B.64)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 4,5,6) \quad (B.65)$$

The Laplace transform of the robot-safety system reliability with the working safety unit is

$$R_{rs}(s) = \sum_{i=0}^1 P_i(s) = (1 + Y_1) P_0(s) \quad (B.66)$$

Utilizing Equation (B.66), the robot-safety system mean time to the failure is obtained as follows:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + Y_3}{\sum_{j=4}^6 a_{sj}} \quad (B.67)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without the working safety unit is

$$R_r(s) = \sum_{i=0}^3 P_i(s) = (1 + Y_1 + \frac{\lambda_s}{s + a_2} + Y_2)P_0(s) \quad (\text{B.68})$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4}{\sum_{j=4}^6 a_{sj}} \quad (\text{B.69})$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (B.66) and (B.68).

The robot-safety system variance of time to failure is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4)(1 + Y_3 + \frac{\lambda_s}{a_2} + Y_4 + \lambda_r Y_{d1} + \lambda_r Y_{d2} + \lambda_{c1} Y_{d1} - \frac{\lambda_{c2} \lambda_s}{a_2^2})}{(\sum_{j=4}^6 a_{sj})^2} \quad (\text{B.70}) \\ &\quad - \frac{2(Y_{d1} - \frac{\lambda_s}{a_2} + Y_{d2})}{\sum_{j=4}^6 a_{sj}} - (MTTF_r)^2 \end{aligned}$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$Y_{d1} = \lim_{s \rightarrow 0} Y_1' = \frac{2\lambda_r(1 - \frac{\lambda_s \mu_s}{a_2^2})}{a_1 a_3 - \lambda_s \mu_s} - \frac{2\lambda_r(a_3 + \frac{\lambda_s \mu_s}{a_2})(a_3 + a_1)}{(a_1 a_3 - \lambda_s \mu_s)^2}$$

$$Y_{d2} = \lim_{s \rightarrow 0} Y_2' = \frac{2\lambda_r \lambda_s \left(\frac{1}{a_2} - \frac{a_1}{a_2^2} \right)}{a_1 a_3 - \lambda_s \mu_s} - \frac{2\lambda_r \lambda_s \left(1 + \frac{a_1}{a_2} \right) (a_3 + a_1)}{(a_1 a_3 - \lambda_s \mu_s)^2}$$

Y_1' denotes the derivative of Y_1 with respect to s .

Y_2' denotes the derivative of Y_2 with respect to s .

Special Case Model 3.II

Setting $k=2$ and $n=3$ in Equations (3.1)-(3.10). The model becomes for a system having three redundant robots and one built-in safety unit. At least two robots must function successfully for the robot system success. The corresponding system of Equations become

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_3(t) + \theta_1 P_4(t) + \sum_{j=6}^8 \int_0^{\infty} P_j(x,t) \mu_{rj}(x) dx \quad (C.1)$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 3\lambda_r P_0(t) + \mu_s P_4(t) \quad (C.2)$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = \lambda_s P_0(t) \quad (C.3)$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = 3\lambda_r P_3(t) + \lambda_s P_1(t) \quad (C.4)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_{rj}(x) P_j(x,t) = 0 \quad (\text{for } j = 6,7,8) \quad (C.5)$$

$$P_6(0,t) = 2\lambda_r P_4(t) \quad (C.6)$$

$$P_7(0,t) = 2\lambda_r P_1(t) \quad (C.7)$$

$$P_8(0,t) = \sum_{i=0}^1 \lambda_{ci} P_i(t) + \sum_{i=3}^4 \lambda_{ci} P_i(t) \quad (C.8)$$

where

$$a_0 = 3\lambda_r + \lambda_{c0} + \lambda_s$$

$$a_1 = 2\lambda_r + \lambda_{c1} + \lambda_s + \mu_{r1}$$

$$a_3 = 3\lambda_r + \lambda_{c3} + \mu_s$$

$$a_4 = 2\lambda_r + \theta_1 + \lambda_{c4} + \mu_s$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero.

C.1 Time Dependant Availability Analysis

Using Laplace Transform technique and the initial conditions in Equations (C.1)-(C.8), we obtain

$$(s + a_0)P_0(s) = 1 + \theta_1 P_4(s) + \mu_s P_3(s) + \mu_{r1} P_1(s) + \sum_{j=6}^8 \int_0^{\infty} P_j(x, s) \mu_{rj}(x) dx \quad (C.9)$$

$$(s + a_1)P_1(s) = 3\lambda_r P_0(s) + \mu_s P_3(s) \quad (C.10)$$

$$(s + a_3)P_3(s) = \lambda_s P_0(s) \quad (C.11)$$

$$(s + a_4)P_4(s) = 3\lambda_r P_3(s) + \lambda_s P_1(s) \quad (C.12)$$

$$sP_j(x, s) + \frac{\partial P_j(x, s)}{\partial x} + \mu_{rj}(x)P_j(x, s) = 0 \quad (\text{for } j = 6, 7, 8) \quad (C.13)$$

$$P_6(0, s) = 2\lambda_r P_4(s) \quad (C.14)$$

$$P_7(0, s) = 2\lambda_r P_1(s) \quad (C.15)$$

$$P_8(0, s) = \sum_{i=0}^1 \lambda_{ci} P_i(s) + \sum_{i=3}^4 \lambda_{ci} P_i(s) \quad (C.16)$$

Solving differential Equation (C.13), we get the following expression:

$$P_j(x, s) = P_j(0, s) e^{-sx} \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] \quad (\text{for } j = 6, 7, 8) \quad (C.17)$$

Since

$$P_j(s) = \int_0^{\infty} P_j(x, s) dx \quad (\text{for } j = 6, 7, 8) \quad (C.18)$$

and together with Equation (C.17), we get

$$P_j(s) = P_j(0, s) \frac{1 - Z_j(s)}{s} \quad (\text{for } j = 6, 7, 8) \quad (C.19)$$

where

$$\frac{1-Z_j(s)}{s} = P_j(0,s) \int_0^\infty e^{-sx} \exp[-\int_0^x \mu_{rj}(\delta) d\delta] dx \quad (C.20)$$

(for $j = 6,7,8$)

$$Z_j(s) = \int_0^\infty e^{-sx} z_j(x) dx \quad (for \ j = 6,7,8) \quad (C.21)$$

$$z_j(x) = \exp[-\int_0^x \mu_{rj}(\delta) d\delta] \mu_{rj}(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

The Laplace transforms of the probabilities of all the system states add up to $1/s$, i.e.,

$$\sum_{i=0}^1 P_i(s) + \sum_{i=3}^4 P_i(s) + \sum_{j=6}^8 P_j(s) = \frac{1}{s} \quad (C.22)$$

Solving Equations (C.10)-(C.12), (C.14)-(C.16), (C.19), and (C.22), we get

$$P_0(s) = [s(1 + Y_5 + \frac{\lambda_s}{s+a_3} + Y_6 + \sum_{j=6}^8 a_j \frac{1-Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (C.23)$$

$$P_1(s) = Y_5 P_0(s) \quad (C.24)$$

$$P_3(s) = \frac{\lambda_s}{s+a_3} P_0(s) \quad (C.25)$$

$$P_4(s) = Y_6 P_0(s) \quad (C.26)$$

$$P_j(s) = \frac{a_j [1-Z_j(s)]}{s} P_0(s) \quad (for \ j = 6,7,8) \quad (C.27)$$

where

$$Y_5 = \frac{3\lambda_r (s+a_4 + \frac{\lambda_s \mu_s}{s+a_3})}{(s+a_1)(s+a_4) - \lambda_s \mu_s}$$

$$Y_6 = \frac{3\lambda_r \lambda_s (1 + \frac{s+a_1}{s+a_3})}{(s+a_1)(s+a_4) - \lambda_s \mu_s}$$

$$a_6 = 2\lambda_r Y_6$$

$$a_7 = 2\lambda_r Y_5$$

$$a_8 = \lambda_{c0} + \lambda_{c1} Y_5 + \frac{\lambda_{c3} \lambda_s}{s+a_3} + \lambda_{c4} Y_6$$

$$H = s(1 + Y_5 + \frac{\lambda_s}{s+a_3} + Y_6 + \sum_{j=6}^8 a_j \frac{1-Z_j(s)}{s})$$

Thus, the Laplace transform of the robot-safety system availability with the working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^1 P_i(s) = \frac{(1+Y_5)}{H} \quad (C.28)$$

Similarly, the Laplace transform of the robot-safety system availability with or without the working safety unit is given by

$$AV_r(s) = \sum_{i=0}^1 P_i(s) + \sum_{i=3}^4 P_i(s) = \frac{1 + Y_5 + \frac{\lambda_s}{s+a_3} + Y_6}{H} \quad (C.29)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions into Equations (C.28) and (C.29), and taking the inverse Laplace transforms of the resulting equations, we can obtain the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

C.2 Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Thus, Equations (C.1)-(C.8) reduce to Equations (C.30)-(C.37), respectively.

$$a_0 P_0 = \mu_{r1} P_1 + \mu_s P_3 + \theta_1 P_4 + \sum_{j=6}^8 \int_0^{\infty} P_j(x) \mu_{rj}(x) dx \quad (C.30)$$

$$a_1 P_1 = 3\lambda_r P_0 + \mu_s P_4 \quad (C.31)$$

$$a_3 P_3 = \lambda_s P_0 \quad (C.32)$$

$$a_4 P_4 = 3\lambda_r P_3 + \lambda_s P_1 \quad (C.33)$$

$$\frac{dP_j(x)}{dx} + \mu_{rj}(x) P_j(x) = 0 \quad (\text{for } j = 6, 7, 8) \quad (C.34)$$

$$P_6(0) = 2\lambda_r P_4 \quad (C.35)$$

$$P_7(0) = 2\lambda_r P_1 \quad (C.36)$$

$$P_8(0) = \sum_{i=0}^1 \lambda_{ci} P_i + \sum_{i=3}^4 \lambda_{ci} P_i \quad (C.37)$$

Solving Equation (C.34), we get

$$P_j(x) = P_j(0) \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] \quad (\text{for } j = 6, 7, 8) \quad (C.38)$$

After a failure the robot-safety system under repair steady state condition probability is given by

$$P_j = \int_0^{\infty} P_j(x) dx \quad (\text{for } j = 6, 7, 8) \quad (C.39)$$

Substituting Equation (C.38) into Equation (C.39), yields

$$P_j = P_j(0) E_j[x] \quad (\text{for } j = 6, 7, 8) \quad (C.40)$$

where

$$\begin{aligned} E_j(x) &= \int_0^{\infty} \exp\left[-\int_0^x \mu_{rj}(\delta) d\delta\right] dx \\ &= \int_0^{\infty} x z_j(x) dx \end{aligned} \quad (C.41)$$

which is the mean time to robot-safety system repair when the failed robot-safety system is in state j and has an elapsed repair time of x .

Substituting Equations (C.35) - (C.37) into Equation (C.40), yields:

$$P_6 = 2\lambda_r P_4 E_6[x] \quad (C.42)$$

$$P_7 = \lambda_r P_1 E_7[x] \quad (C.43)$$

$$P_8 = \sum_{i=0}^1 \lambda_{ci} P_i E_8[x] + \sum_{i=3}^4 \lambda_{ci} P_i E_8[x] \quad (C.44)$$

Using Equations (C.31)-(C.33), (C.42) - (C.44), together with

$$\sum_{i=0}^1 P_i + \sum_{i=3}^4 P_i + \sum_{j=6}^8 P_j = 1 \quad (C.45)$$

we get the following general steady state probability solutions:

$$P_0 = (1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8 + \sum_{j=6}^8 a_{sj} E_j[x])^{-1} = \frac{1}{G} \quad (C.46)$$

$$P_1 = Y_7 P_0 \quad (C.47)$$

$$P_3 = \frac{\lambda_s}{a_3} P_0 \quad (C.48)$$

$$P_4 = Y_8 P_0 \quad (C.49)$$

$$P_j = a_{sj} E_j[x] P_0 \quad (\text{for } j = 6, 7, 8) \quad (C.50)$$

where

$$Y_7 = \lim_{s \rightarrow 0} Y_5 = \frac{3\lambda_r (a_4 + \frac{\lambda_s \mu_s}{a_3})}{a_1 a_4 - \lambda_s \mu_s}$$

$$Y_8 = \lim_{s \rightarrow 0} Y_6 = \frac{3\lambda_r \lambda_s (1 + \frac{a_1}{a_3})}{a_1 a_4 - \lambda_s \mu_s}$$

$$\begin{aligned}
a_{s6} &= 2\lambda_r Y_8 \\
a_{s7} &= 2\lambda_r Y_7 \\
a_{s8} &= \lambda_{c0} + \lambda_{c1} Y_7 + \frac{\lambda_{c3} \lambda_s}{a_3} + \lambda_{c4} Y_8 \\
G &= 1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8 + \sum_{j=6}^8 a_{sj} E_j[x]
\end{aligned} \tag{C.51}$$

The steady state availability of the robot-safety system with the working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^1 P_i = \frac{1 + Y_7}{G} \tag{C.52}$$

Similarly, the steady state availability of the robot-safety system with or without the working safety unit is given by

$$SSAV_r = \sum_{i=0}^1 P_i + \sum_{i=3}^4 P_i = \frac{1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8}{G} \tag{C.53}$$

C.3 System Reliability, MTTF, and Variance of Time to Failure

Setting $\mu_{r6}(x)=\mu_{r7}(x)=\mu_{r8}(x)=0$, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_3(t) + \theta_1 P_4(t) \tag{C.54}$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 3\lambda_r P_0(t) + \mu_s P_4(t) \tag{C.55}$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = \lambda_s P_0(t) \tag{C.56}$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = 3\lambda_r P_3(t) + \lambda_s P_1(t) \tag{C.57}$$

$$\frac{dP_6(t)}{dt} = \lambda_r P_4(t) \quad (C.58)$$

$$\frac{dP_7(t)}{dt} = \lambda_r P_1(t) \quad (C.59)$$

$$\frac{dP_8(t)}{dt} = \sum_{i=0}^1 \lambda_{ci} P_i(t) + \sum_{i=3}^4 \lambda_{ci} P_i(t) \quad (C.60)$$

At time $t=0$, $P_0(0)=1$, and all other initial state probabilities are equal to zero. Taking the Laplace transforms of Equations (C.55) - (C.60), and solving the resulting set of Equations, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + Y_5 + \frac{\lambda_s}{s + a_3} + Y_6 + \sum_{j=6}^8 \frac{a_j}{s})]^{-1} \quad (C.61)$$

$$P_1(s) = Y_5 P_0(s) \quad (C.62)$$

$$P_3(s) = \frac{\lambda_s}{s + a_3} P_0(s) \quad (C.63)$$

$$P_4(s) = Y_6 P_0(s) \quad (C.64)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 6,7,8) \quad (C.65)$$

The Laplace transform of the robot-safety system reliability with the working safety unit is

$$R_{rs}(s) = \sum_{i=0}^1 P_i(s) = (1 + Y_5) P_0(s) \quad (C.66)$$

Utilizing Equation (C.66), the robot-safety system mean time to the failure is obtained as follows:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + Y_7}{\sum_{j=6}^8 a_{sj}} \quad (C.67)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without the working safety unit is

$$R_r(s) = \sum_{i=0}^1 P_i(s) + \sum_{i=3}^4 P_i(s) = (1 + Y_5 + \frac{\lambda_s}{s + a_3} + Y_6)P_0(s) \quad (C.68)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8}{\sum_{j=6}^8 a_{sj}} \quad (C.69)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (C.66) and (C.68).

The robot-safety system variance of time to failure is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8)(1 + Y_7 + \frac{\lambda_s}{a_3} + Y_8 + 2\lambda_r Y_{d5} + 2\lambda_r Y_{d6} + \lambda_{c1} Y_{d5} - \frac{\lambda_{c3} \lambda_s}{a_3^2} + \lambda_{c4} Y_{d6})}{(\sum_{j=6}^8 a_{sj})^2} \\ &\quad - \frac{2(Y_{d5} - \frac{\lambda_s}{a_3^2} + Y_{d6})}{\sum_{j=6}^8 a_{sj}} - (MTTF_r)^2 \end{aligned} \quad (C.70)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$Y_{d5} = \lim_{s \rightarrow 0} Y_5' = \frac{3\lambda_r(1 - \frac{\lambda_s \mu_s}{a_3^2})}{a_1 a_4 - \lambda_s \mu_s} - \frac{3\lambda_r(a_4 + \frac{\lambda_s \mu_s}{a_3})(a_4 + a_1)}{(a_1 a_4 - \lambda_s \mu_s)^2}$$

$$Y_{d6} = \lim_{s \rightarrow 0} Y_6' = \frac{3\lambda_r \lambda_s \left(\frac{1}{a_3} - \frac{a_1}{a_3^2} \right)}{a_1 a_4 - \lambda_s \mu_s} - \frac{3\lambda_r \lambda_s \left(1 + \frac{a_1}{a_3} \right) (a_4 + a_1)}{(a_1 a_4 - \lambda_s \mu_s)^2}$$

Y_5' denotes the derivative of Y_5 with respect to s .

Y_6' denotes the derivative of Y_6 with respect to s .

APPENDIX D

Special Case Model 4.I

Setting $n=2$ and $m=2$ in Equations (4.1)-(4.12). The model becomes for a system having two redundant robots and two redundant built-in safety units. The corresponding system of Equations become

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_{s1} P_2(t) + \sum_{j=6}^9 \int_0^{\infty} P_j(x,t) \mu_j(x) dx \quad (D.1)$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) + \mu_{s1} P_3(t) \quad (D.2)$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = 2\lambda_s P_0(t) + \mu_{s2} P_4(t) \quad (D.3)$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = 2\lambda_r P_2(t) + 2\lambda_s P_1(t) + \mu_{s2} P_5(t) \quad (D.4)$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = \lambda_s P_2(t) \quad (D.5)$$

$$\frac{dP_5(t)}{dt} + a_5 P_5(t) = 2\lambda_r P_4(t) + \lambda_s P_3(t) \quad (D.6)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \quad (\text{for } j = 6,7,8,9) \quad (D.7)$$

$$P_j(0,t) = \lambda_r P_{17-2j}(t) \quad (\text{for } j = 6,7,8) \quad (D.8)$$

$$P_9(0,t) = \sum_{i=0}^4 \lambda_{ci} P_i(t) \quad (D.9)$$

where

$$a_0 = 2\lambda_r + \lambda_{c0} + 2\lambda_s$$

$$a_1 = \lambda_r + \lambda_{c1} + 2\lambda_s + \mu_{r1}$$

$$a_2 = 2\lambda_r + \lambda_{c2} + \lambda_s + \mu_{s1}$$

$$a_3 = \lambda_r + \lambda_{c3} + \lambda_s + \mu_{s1}$$

$$a_4 = 2\lambda_r + \lambda_{c4} + \mu_{s2}$$

$$a_5 = \lambda_r + \mu_{s2}$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero.

D.1 Time Dependant Availability Analysis

Using Laplace Transform technique and the initial conditions in Equations (D.1)-(D.9), we obtain

$$(s + a_0)P_0(s) = 1 + \mu_{r1}P_1(s) + \mu_{s1}P_2(s) + \sum_{j=6}^9 \int_0^{\infty} P_j(x,s)\mu_j(x)dx \quad (D.10)$$

$$(s + a_1)P_1(s) = 2\lambda_r P_0(s) + \mu_{s1}P_3(s) \quad (D.11)$$

$$(s + a_2)P_2(s) = 2\lambda_s P_0(s) + \mu_{s2}P_4(s) \quad (D.12)$$

$$(s + a_3)P_3(s) = 2\lambda_r P_2(s) + 2\lambda_s P_1(s) + \mu_{s2}P_5(s) \quad (D.13)$$

$$(s + a_4)P_4(s) = \lambda_s P_2(s) \quad (D.14)$$

$$(s + a_5)P_5(s) = 2\lambda_r P_4(s) + \lambda_s P_3(s) \quad (D.15)$$

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)P_j(x,s) = 0 \quad (\text{for } j = 6,7,8,9) \quad (D.16)$$

$$P_j(0,s) = \lambda_r P_{17-2j}(s) \quad (\text{for } j = 6,7,8) \quad (D.17)$$

$$P_9(0,s) = \sum_{i=0}^4 \lambda_{ci} P_i(s) \quad (D.18)$$

Solving differential Equation (D.16), we get the following expression:

$$P_j(x,s) = P_j(0,s)e^{-sx} \exp\left[-\int_0^x \mu_j(\delta)d\delta\right] \quad (\text{for } j = 6,7,8,9) \quad (D.19)$$

Since

$$P_j(s) = \int_0^{\infty} P_j(x,s) dx \quad (\text{for } j = 6,7,8,9) \quad (\text{D.20})$$

and together with Equation (D.19), we get

$$P_j(s) = P_j(0,s) \frac{1 - Z_j(s)}{s} \quad (\text{for } j = 6,7,8,9) \quad (\text{D.21})$$

where

$$\frac{1 - Z_j(s)}{s} = P_j(0,s) \int_0^{\infty} e^{-sx} \exp[-\int_0^x \mu_j(\delta) d\delta] dx \quad (\text{D.22})$$

$$(\text{for } j = 6,7,8,9)$$

$$Z_j(s) = \int_0^{\infty} e^{-sx} z_j(x) dx \quad (\text{for } j = 6,7,8,9) \quad (\text{D.23})$$

$$z_j(x) = \exp[-\int_0^x \mu_{r_j}(\delta) d\delta] \mu_{r_j}(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

The Laplace transforms of the probabilities of all the system states add up to $1/s$, i.e.,

$$\sum_{i=0}^5 P_i(s) + \sum_{j=6}^9 P_j(s) = \frac{1}{s} \quad (\text{D.24})$$

Solving Equations (D.11)-(D.15), (D.17)-(D.18), (D.21), and (D.24), we get

$$P_0(s) = [s(1 + \sum_{i=1}^5 Y_i + \sum_{j=6}^9 a_j \frac{1 - Z_j(s)}{s})]^{-1} = \frac{1}{H} \quad (\text{D.25})$$

$$P_i(s) = Y_i P_0(s) \quad (\text{for } i = 1,2,3,4,5) \quad (\text{D.26})$$

$$P_j(s) = \frac{a_j [1 - Z_j(s)]}{s} P_0(s) \quad (\text{for } j = 6,7,8,9) \quad (\text{D.27})$$

where

$$K_{s2} = \frac{\lambda_s \mu_{s2}}{s + a_4}$$

$$K_{s1} = \frac{2\lambda_s \mu_{s1}}{s + a_2 - K_{s2}}$$

$$\mu_{s0} = 1$$

$$K_{r[1]2} = \frac{2\lambda_r \mu_{s2}}{s + a_5}$$

$$K_{r[1]1} = \frac{2\lambda_r \mu_{s1}}{s + a_3}$$

$$K_{r[1]0} = \frac{2\lambda_r \mu_{s0}}{s + a_1}$$

$$Y_1 = K_{r[1]0} + \frac{\prod_{l=0}^1 K_{r[1]l} K_{s1}}{2\lambda_r \mu_{s1}} + \frac{\prod_{l=0}^2 K_{r[1]l}}{(2\lambda_r)^2} \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$Y_2 = \frac{K_{s1}}{\mu_{s1}}$$

$$Y_3 = \frac{K_{r[1]1} K_{s1}}{\mu_{s1} \mu_{s1}} + \frac{\prod_{l=1}^2 K_{r[1]l}}{\mu_{s1} 2\lambda_r} \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$Y_4 = \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$Y_5 = \frac{K_{r[1]2}}{\mu_{s2}} \prod_{q=1}^2 \frac{K_{sq}}{\mu_{sq}}$$

$$a_6 = \lambda_r Y_5$$

$$a_7 = \lambda_r Y_3$$

$$a_8 = \lambda_r Y_1$$

$$a_9 = \lambda_{c0} + \sum_{i=1}^4 \lambda_{ci} Y_i$$

$$H = s(1 + \sum_{i=1}^5 Y_i + \sum_{j=6}^9 a_j \frac{1 - Z_j(s)}{s})$$

Thus, the Laplace transform of the robot-safety system availability with at least one working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^3 P_i(s) = \frac{1 + \sum_{i=1}^3 Y_i}{H} \quad (D.28)$$

Similarly, the Laplace transform of the robot-safety system availability with or without working safety units is given by

$$AV_r(s) = \sum_{i=0}^5 P_i(s) = \frac{1 + \sum_{i=1}^5 Y_i}{H} \quad (D.29)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions into Equations (D.28) and (D.29), and taking the inverse Laplace transforms of the resulting equations, we can obtain the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

D.2 Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Thus, Equations (D.1)-(D.9) reduce to Equations (D.30)-(D.38), respectively.

$$a_0 P_0 = \mu_{r1} P_1 + \mu_{s1} P_2 + \sum_{j=6}^9 \int_0^{\infty} P_j(x) \mu_j(x) dx \quad (D.30)$$

$$a_1 P_1 = 2\lambda_r P_0 + \mu_{s1} P_3 \quad (D.31)$$

$$a_2 P_2 = 2\lambda_s P_0 + \mu_{s2} P_4 \quad (D.32)$$

$$a_3 P_3 = 2\lambda_r P_2 + 2\lambda_s P_1 + \mu_{s2} P_5 \quad (D.33)$$

$$a_4 P_4 = \lambda_s P_2 \quad (D.34)$$

$$a_5 P_5 = 2\lambda_r P_4 + \lambda_s P_3 \quad (D.35)$$

$$\frac{dP_j(x)}{dx} + \mu_j(x)P_j(x) = 0 \quad (\text{for } j = 6,7,8,9) \quad (D.36)$$

$$P_j(0) = \lambda_r P_{17-2j} \quad (\text{for } j = 6,7,8) \quad (D.37)$$

$$P_9(0) = \sum_{i=0}^4 \lambda_{ci} P_i \quad (D.38)$$

Solving Equation (D.36), we get

$$P_j(x) = P_j(0) \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] \quad (\text{for } j = 6,7,8,9) \quad (D.39)$$

After a failure the robot-safety system under repair steady state condition probability is given by

$$P_j = \int_0^{\infty} P_j(x) dx \quad (\text{for } j = 6,7,8,9) \quad (D.40)$$

Substituting Equation (D.39) into Equation (D.40), yields

$$P_j = P_j(0) E_j[x] \quad (\text{for } j = 6,7,8,9) \quad (D.41)$$

where

$$\begin{aligned} E_j(x) &= \int_0^{\infty} \exp\left[-\int_0^x \mu_j(\delta) d\delta\right] dx \\ &= \int_0^{\infty} x z_j(x) dx \end{aligned} \quad (D.42)$$

which is the mean time to robot-safety system repair when the failed robot-safety system is in state j and has an elapsed repair time of x .

Substituting Equations (D.37) - (D.38) into Equation (D.41), yields:

$$P_j = \lambda_r P_{17-2j} E_j[x] \quad (\text{for } j = 6,7,8) \quad (D.43)$$

$$P_9 = \sum_{i=0}^4 \lambda_{ci} P_i E_9[x] \quad (\text{D.44})$$

Using Equations (D.31)-(D.35), (D.43) - (D.44), together with

$$\sum_{i=0}^5 P_i + \sum_{j=6}^9 P_j = 1 \quad (\text{D.45})$$

we get the following general steady state probability solutions:

$$P_0 = (1 + \sum_{i=1}^5 Y_{si} + \sum_{j=6}^9 a_{sj} E_j[x])^{-1} = \frac{1}{G} \quad (\text{D.46})$$

$$P_i = Y_{si} P_0 \quad (\text{for } i = 1, 2, 3, 4, 5) \quad (\text{D.47})$$

$$P_j = a_{sj} E_j[x] P_0 \quad (\text{for } j = 6, 7, 8, 9) \quad (\text{D.48})$$

where

$$L_{s2} = \lim_{s \rightarrow 0} K_{s2} = \frac{\lambda_s \mu_{s2}}{a_4}$$

$$L_{s1} = \lim_{s \rightarrow 0} K_{s1} = \frac{2\lambda_s \mu_{s1}}{a_2 - L_{s2}}$$

$$\mu_{s0} = 1$$

$$L_{r[1]2} = \lim_{s \rightarrow 0} K_{r[1]2} = \frac{2\lambda_r \mu_{s2}}{a_5}$$

$$L_{r[1]1} = \lim_{s \rightarrow 0} K_{r[1]1} = \frac{2\lambda_r \mu_{s1}}{a_3}$$

$$L_{r[1]0} = \lim_{s \rightarrow 0} K_{r[1]0} = \frac{2\lambda_r \mu_{s0}}{a_1}$$

$$Y_{s1} = \lim_{s \rightarrow 0} Y_1 = L_{r[1]0} + \frac{\prod_{l=0}^1 L_{r[1]l}}{2\lambda_r} \frac{L_{s1}}{\mu_{s1}} + \frac{\prod_{l=0}^2 L_{r[1]l}}{(2\lambda_r)^2} \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$Y_{s2} = \lim_{s \rightarrow 0} Y_2 = \frac{L_{s1}}{\mu_{s1}}$$

$$Y_{s3} = \lim_{s \rightarrow 0} Y_3 = \frac{L_{r[1]1}}{\mu_{s1}} \frac{L_{s1}}{\mu_{s1}} + \frac{\prod_{l=1}^2 L_{r[1]l}}{\mu_{s1} (2\lambda_r)} \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$Y_{s4} = \lim_{s \rightarrow 0} Y_4 = \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$Y_{s5} = \lim_{s \rightarrow 0} Y_5 = \frac{L_{r[1]2}}{\mu_{s2}} \prod_{q=1}^2 \frac{L_{sq}}{\mu_{sq}}$$

$$a_{s6} = \lambda_r Y_{s5}$$

$$a_{s7} = \lambda_r Y_{s3}$$

$$a_{s8} = \lambda_r Y_{s1}$$

$$a_{s9} = \lambda_{c0} + \sum_{i=1}^4 \lambda_{ci} Y_{si}$$

$$G = 1 + \sum_{i=1}^5 Y_{si} + \sum_{j=6}^9 a_{sj} E_j[x] \quad (D.49)$$

The steady state availability of the robot-safety system with at least one working safety unit is expressed by

$$SSAV_{rs} = \sum_{i=0}^3 P_i = \frac{1 + \sum_{i=1}^3 Y_{si}}{G} \quad (D.50)$$

Similarly, the steady state availability of the robot-safety system with or without working safety units is given by

$$SSAV_r = \sum_{i=0}^5 P_i = \frac{1 + \sum_{i=1}^5 Y_{si}}{G} \quad (D.51)$$

D.3 System Reliability, MTTF, and Variance of Time to Failure

Setting $\mu_6(x)=\mu_7(x)=\mu_8(x)=\mu_9(x)=0$, the system of differential equations becomes

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_{s1} P_2(t) \quad (D.52)$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 2\lambda_r P_0(t) + \mu_{s1} P_3(t) \quad (D.53)$$

$$\frac{dP_2(t)}{dt} + a_2 P_2(t) = 2\lambda_s P_0(t) + \mu_{s2} P_4(t) \quad (D.54)$$

$$\frac{dP_3(t)}{dt} + a_3 P_3(t) = 2\lambda_r P_2(t) + 2\lambda_s P_1(t) + \mu_{s2} P_5(t) \quad (D.55)$$

$$\frac{dP_4(t)}{dt} + a_4 P_4(t) = \lambda_s P_2(t) \quad (D.56)$$

$$\frac{dP_5(t)}{dt} + a_5 P_5(t) = 2\lambda_r P_4(t) + \lambda_s P_3(t) \quad (D.57)$$

$$\frac{dP_j(t)}{dt} = \lambda_r P_{17-2j}(t) \quad (\text{for } j = 6,7,8) \quad (D.58)$$

$$\frac{dP_9(t)}{dt} = \sum_{i=0}^4 \lambda_{ci} P_i(t) \quad (D.59)$$

At time $t=0$, $P_0(0)=1$, and all other initial state probabilities are equal to zero. Taking the Laplace transforms of Equations (D.53) - (D.59), and solving the resulting set of Equations, we obtain the following Laplace transforms of state probabilities:

$$P_0(s) = [s(1 + \sum_{i=1}^5 Y_i + \sum_{j=6}^9 \frac{a_j}{s})]^{-1} \quad (D.60)$$

$$P_i(s) = Y_i P_0(s) \quad (\text{for } i = 1,2,3,4,5) \quad (D.61)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 6,7,8,9) \quad (D.62)$$

Thus, the Laplace transform of the robot-safety system reliability with at least one working safety unit is

$$R_{rs}(s) = \sum_{i=0}^3 P_i(s) = (1 + \sum_{i=1}^3 Y_i) P_0(s) \quad (D.63)$$

Using Equation (D.63), we get the following expression for the robot-safety system mean time to failure:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + \sum_{i=1}^3 Y_{si}}{\sum_{j=6}^9 a_{sj}} \quad (D.64)$$

Similarly, the Laplace transform of the robot-safety system reliability with or without working safety units is

$$R_r(s) = \sum_{i=0}^5 P_i(s) = (1 + \sum_{i=1}^5 Y_i) P_0(s) \quad (D.65)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \rightarrow 0} R_r(s) = \frac{1 + \sum_{i=1}^5 Y_{si}}{\sum_{j=6}^9 a_{sj}} \quad (D.66)$$

The time-dependant robot-safety system reliabilities, $R_{rs}(t)$ and $R_r(t)$, can be obtained by taking the inverse Laplace transforms of the resulting Equations (D.63) and (D.65).

The robot-safety system variance of time to failure is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_r'(s) - (MTTF_r)^2 \\ &= \frac{2(1 + \sum_{i=1}^5 Y_{si})(1 + \sum_{i=1}^5 Y_{si} + \sum_{j=6}^9 a_{dj})}{(\sum_{j=6}^9 a_{sj})^2} - \frac{2 \sum_{i=1}^5 Y_{di}}{\sum_{j=6}^9 a_{sj}} - (MTTF_r)^2 \end{aligned} \quad (D.67)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$Y_{di} = \lim_{s \rightarrow 0} Y_i' \quad (\text{for } i = 1, 2, 3, 4, 5)$$

$$a_{dj} = \lim_{s \rightarrow 0} a_j' \quad (\text{for } j = 6, 7, 8, 9)$$

$$a_{d6} = \lim_{s \rightarrow 0} a_6' = \lambda_r Y_{d5}$$

$$a_{d7} = \lim_{s \rightarrow 0} a_7' = \lambda_r Y_{d3}$$

$$a_{d8} = \lim_{s \rightarrow 0} a_8' = \lambda_r Y_{d1}$$

$$a_{d9} = \lim_{s \rightarrow 0} a_9' = \sum_{i=1}^4 \lambda_{ci} Y_{di}$$

Y_i' denotes the derivative of Y_i with respect to s .

a_j' denotes the derivative of a_j with respect to s .

Special Case Model 5.I

Setting $k=2$ and $n=3$ in Equations (5.1)-(5.7). The model becomes for a system having one robot and three redundant safety units. However, at least two safety units must function successfully for the robot-safety system success. The corresponding system of Equations become

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{j=4}^6 \int_0^{\infty} P_j(x,t) \mu_j(x) dx \quad (E.1)$$

$$\frac{dP_1(t)}{dt} + a_1 P_1(t) = 3\lambda_s P_1(t) \quad (E.2)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x) P_j(x,t) = 0 \quad (\text{for } j = 4,5,6) \quad (E.3)$$

$$P_4(0,t) = 2\lambda_s P_1(t) \quad (E.4)$$

$$P_5(0,t) = \lambda_r \sum_{i=0}^1 P_i(t) \quad (E.5)$$

$$P_6(0,t) = \sum_{i=0}^1 \lambda_{ci} P_i(t) \quad (E.6)$$

where

$$a_0 = 3\lambda_s + \lambda_r + \lambda_{c0}$$

$$a_1 = 2\lambda_s + \lambda_r + \lambda_{c1} + \mu_1$$

At time $t=0$, $P_0(0)=1$, and all other initial condition state probabilities are equal to zero.

E.1 Time Dependant Availability Analysis

Setting $k=2$ and $n=3$ in Equations (5.21) and (5.22), we get

$$P_i(s) = \frac{N_i(s)}{M_0(s)} \quad (\text{for } i = 0,1) \quad (\text{E.7})$$

$$P_j(s) = \frac{N_j(s)}{M_0(s)} \quad (\text{for } j = 4,5,6) \quad (\text{E.8})$$

where

$$M_0(s) = s \left(1 + \frac{k_1}{\mu_1} + \sum_{j=4}^6 a_j \frac{1 - Z_j(s)}{s} \right)$$

$$N_0(s) = 1$$

$$N_1(s) = \frac{k_1}{\mu_1} N_0(s)$$

$$N_4(s) = a_4 \frac{1 - Z_4(s)}{s}$$

$$N_5(s) = a_5 \frac{1 - Z_5(s)}{s}$$

$$N_6(s) = a_6 \frac{1 - Z_6(s)}{s}$$

$$k_1 = \frac{3\lambda_s \mu_1}{s + a_1}$$

$$a_4 = 2\lambda_s \frac{k_1}{\mu_1}$$

$$a_5 = \lambda_r \left(1 + \frac{k_1}{\mu_1} \right)$$

$$a_6 = \lambda_{c0} + \lambda_{c1} \frac{k_1}{\mu_1}$$

From Equation (5.27), the Laplace transform of the robot-safety system availability with at least two working safety units is

$$AV_{rs}(s) = \sum_{i=0}^1 P_i(s) = \frac{\sum_{i=0}^1 N_i(s)}{M_0(s)} \quad (E.9)$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions into Equation (E.9), and taking the inverse Laplace transform of the resulting equation, we can get the time-dependent robot-safety system availability, $AV_{rs}(t)$.

E.2 Steady State Availability Analysis

Setting $k=2$ and $n=3$ in Equations (5.43) - (5.46), we get

$$P_0 = (L + \sum_{j=4}^6 L_j E_j[x])^{-1} = \frac{1}{G} \quad (E.10)$$

$$P_1 = \frac{L_1}{\mu_1} P_0 \quad (E.11)$$

$$P_4 = L_4 E_4[x] P_0 \quad (E.12)$$

$$P_5 = L_5 E_5[x] P_0 \quad (E.13)$$

$$P_6 = L_6 E_6[x] P_0 \quad (E.14)$$

where

$$L = 1 + \frac{L_1}{\mu_1}$$

$$L_1 = \frac{3\lambda_s \mu_1}{a_1}$$

$$L_4 = 2\lambda_s \frac{L_1}{\mu_1}$$

$$L_5 = \lambda_r \left(1 + \frac{L_1}{\mu_1}\right)$$

$$L_6 = \lambda_{c0} + \lambda_{e1} \frac{L_1}{\mu_1}$$

$$G = L + \sum_{j=4}^6 L_j E_j[x] \quad (E.15)$$

From Equation (5.48), the steady state availability of the robot-safety system with at least two working safety units is expressed by

$$SSAV_{rs} = \sum_{i=0}^1 P_i = \frac{L}{G} \quad (E.16)$$

E.3 System Reliability, MTTF, and Variance of Time to Failure

Setting $k=2$ and $n=3$ in Equations (5.71) - (5.73), we get

$$P_0(s) = [s(1 + \frac{k_1}{\mu_1} + \sum_{j=4}^6 \frac{a_j}{s})]^{-1} \quad (E.17)$$

$$P_1(s) = \frac{k_1}{\mu_1} P_0(s) \quad (E.18)$$

$$P_j(s) = \frac{a_j}{s} P_0(s) \quad (\text{for } j = 4,5,6) \quad (E.19)$$

From Equation (5.74), the Laplace transform of the robot-safety system reliability with at least two working safety units is

$$R_{rs}(s) = \sum_{i=0}^1 P_i(s) = (1 + \frac{k_1}{\mu_1}) P_0(s) \quad (E.20)$$

From Equation (5.75), the robot-safety system mean time to the failure is obtained as follows:

$$MTTF_{rs} = \lim_{s \rightarrow 0} R_{rs}(s) = \frac{1 + \frac{L_1}{\mu_1}}{\sum_{j=4}^6 L_j} \quad (E.21)$$

The time-dependant robot-safety system reliability, $R_{rs}(t)$, can be obtained by taking the inverse Laplace transform of Equation (E.20).

From Equation (5.76), the system variance of time to failure is expressed by

$$\begin{aligned} \sigma^2 &= -2 \lim_{s \rightarrow 0} R_{rs}'(s) - (MTTF_{rs})^2 \\ &= \frac{2(1 + \frac{L_1}{\mu_1})(1 + \frac{L_1}{\mu_1} + \sum_{j=4}^6 a_{dj})}{(\sum_{j=4}^6 L_j)^2} - \frac{2k_{d1}}{\sum_{j=4}^6 L_j} - (MTTF_{rs})^2 \end{aligned} \quad (E.22)$$

where

$R_r'(s)$ denotes the derivative of $R_r(s)$ with respect to s .

$$k_{d1} = \lim_{s \rightarrow 0} \left(\frac{k_1}{\mu_1} \right)' = -\frac{3\lambda_s}{a_1^2}$$

$$a_{dj} = \lim_{s \rightarrow 0} a_j' \quad (\text{for } j = 4, 5, 6)$$

$$a_{d4} = \lim_{s \rightarrow 0} a_4' = 2\lambda_s k_{d1}$$

$$a_{d5} = \lim_{s \rightarrow 0} a_5' = \lambda_r k_{d1}$$

$$a_{d6} = \lim_{s \rightarrow 0} a_6' = \lambda_{c1} k_{d1}$$

$\left(\frac{k_1}{\mu_1} \right)'$ denotes the derivative of $\frac{k_1}{\mu_1}$ with respect to s .

a_j' denotes the derivative of a_j with respect to s .

Miscellaneous Plots

M.1 Models 3.I and II

Setting $\theta_i=0$ (for $i = 1, 2, \dots, n-k$) in Figure 3.2, for $k=1$ and $n=2$, the model becomes for a system having two redundant robots and one built-in safety unit. For the specified values of this model parameters, the plots of Equations (3.37)-(3.38) (after taking inverse Laplace transforms), (3.63), and (3.96) are shown in Figures F.1-F.4, respectively. Similarly, for $k=2$ and $n=3$, the model becomes for a system having three redundant robots and one built-in safety unit. At least two robots must function successfully for the robot system success. For the specified values of this model parameters, the plots of Equations (3.37)-(3.38) (after taking inverse Laplace transforms), (3.63), and (3.96) are shown in Figures F.5-F.8, respectively.

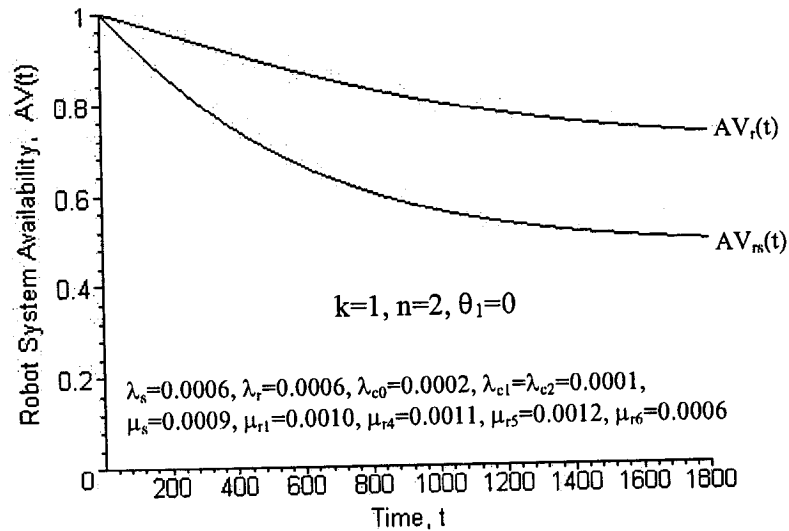


Figure F.1 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

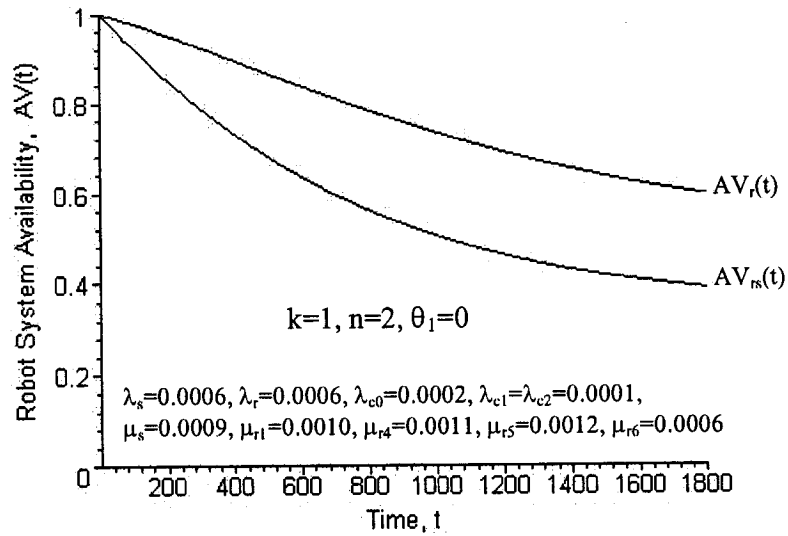


Figure F.2 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

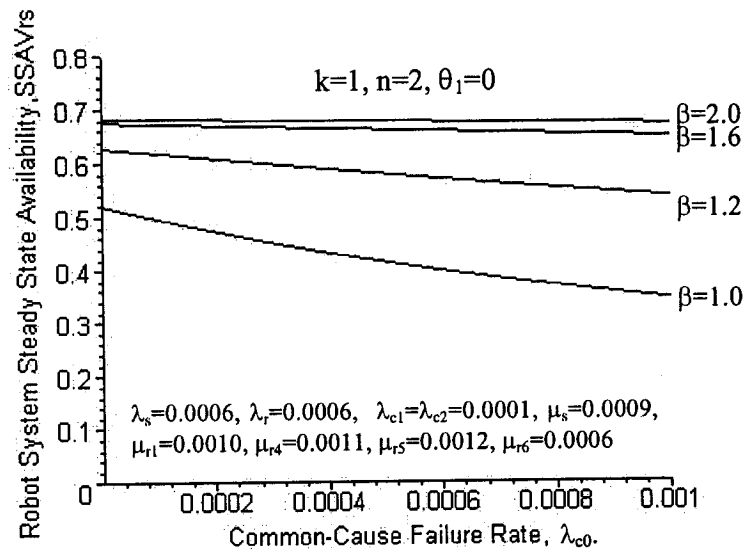


Figure F.3 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair time distribution.

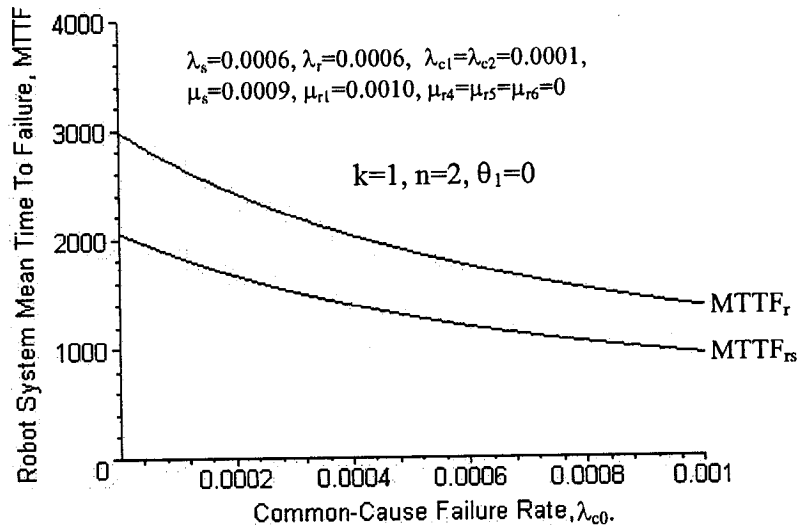


Figure F.4 Mean time to failure plots of an irreparable robot-safety system as a function of common-cause failure rate(λ_{c0}).

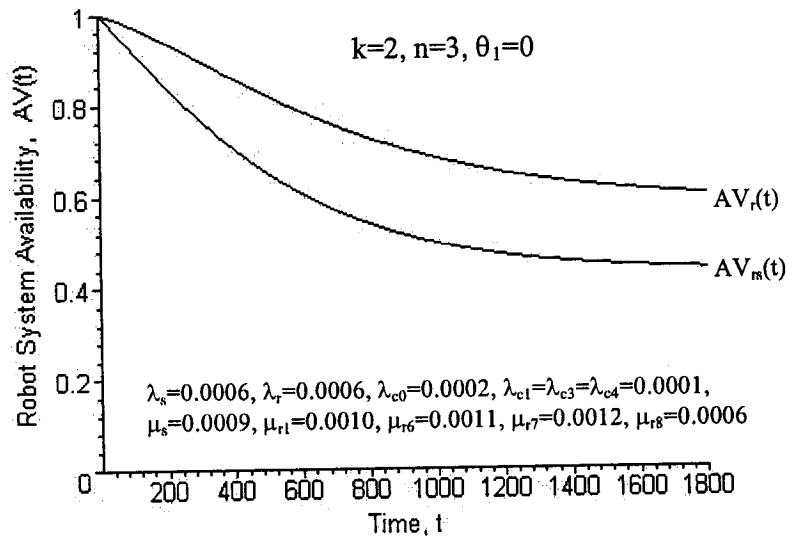


Figure F.5 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

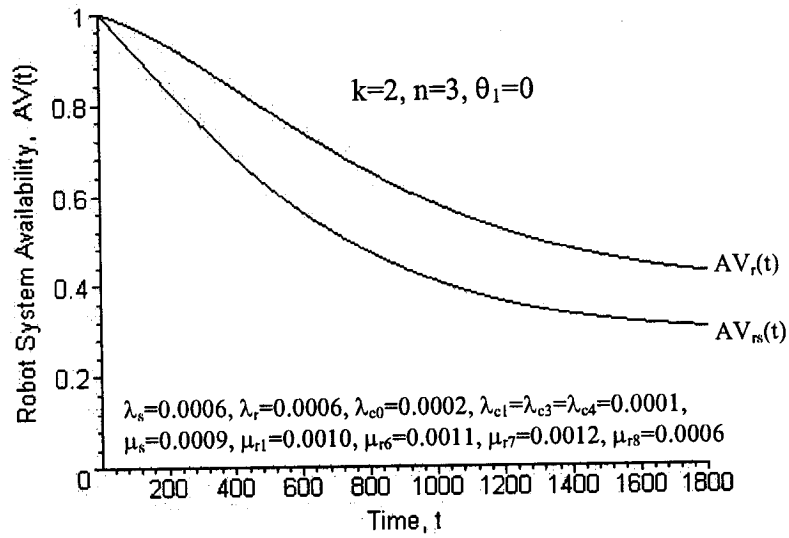


Figure F.6 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

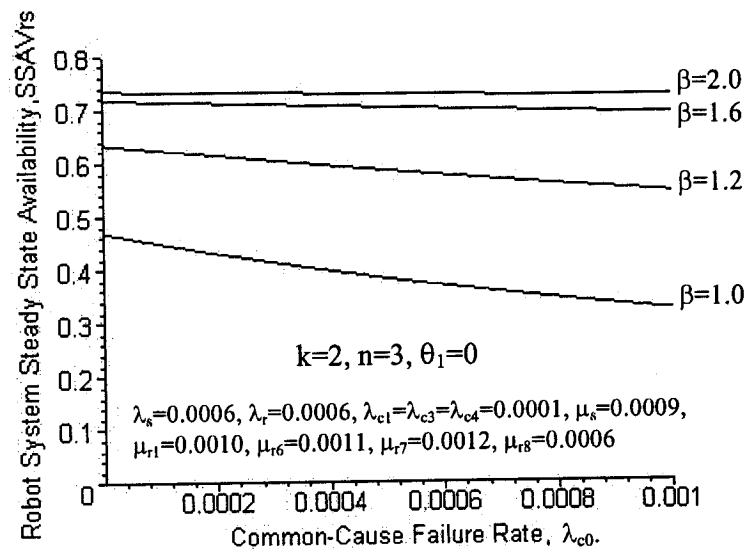


Figure F.7 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair time distribution.

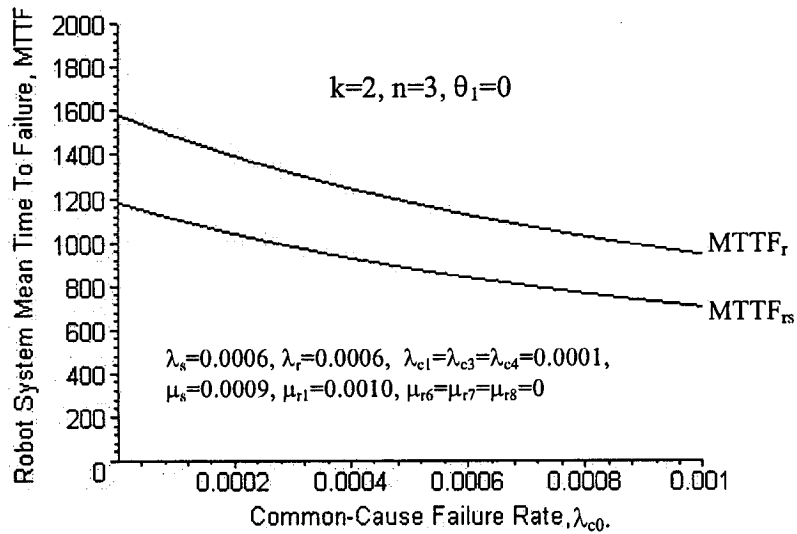


Figure F.8 Mean time to failure plots of an irreparable robot-safety system as a function of common-cause failure rate(λ_{c0}).

M.2 Models 4.I

Setting $\mu_{ri}=0$ (for $i = 1, 2, \dots, n-1$) in Figure 4.2, for $m=2$ and $n=2$, the model becomes for a system having two redundant robots and two redundant built-in safety units. For the specified values of this model parameters, the plots of Equations (4.42)-(4.43) (after taking inverse Laplace transforms), (4.71), and (4.104) are shown in Figures F.9-F.12, respectively.

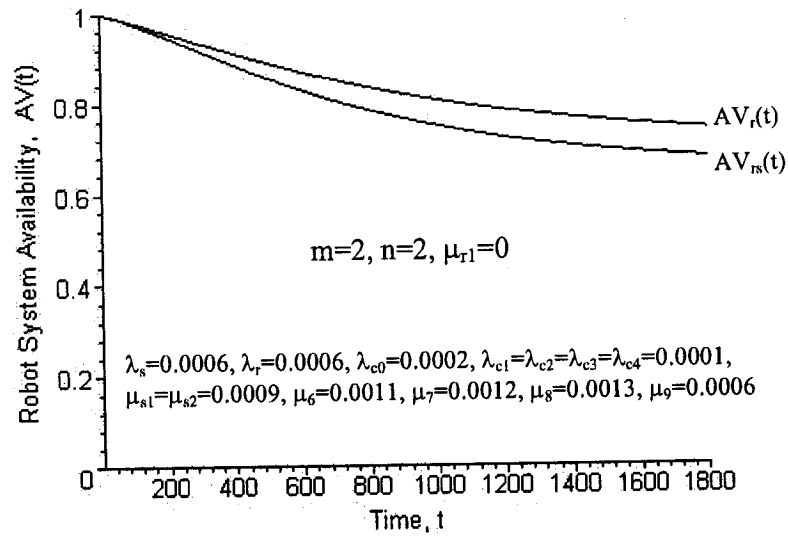


Figure F.9 Time-dependent availability plots for a robot-safety system with exponentially distributed failed system repair time distribution.

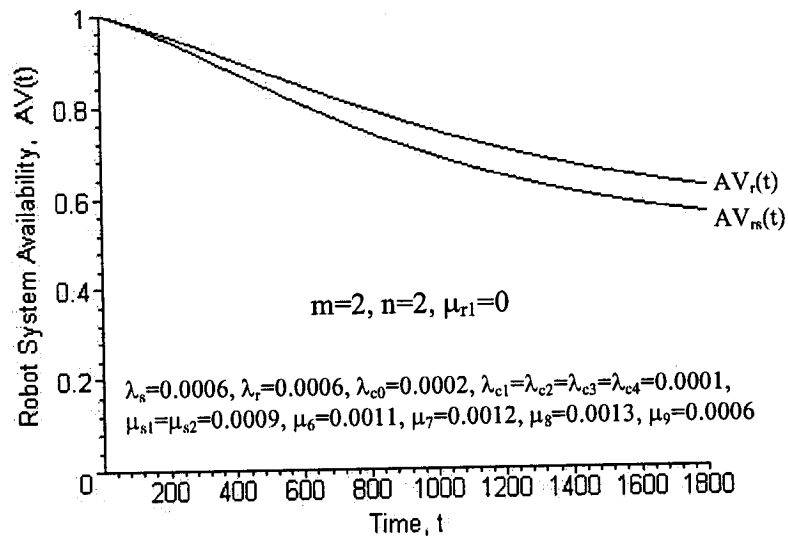


Figure F.10 Time-dependent availability plots for a robot-safety system with gamma distributed ($\beta=2$) failed system repair time distribution.

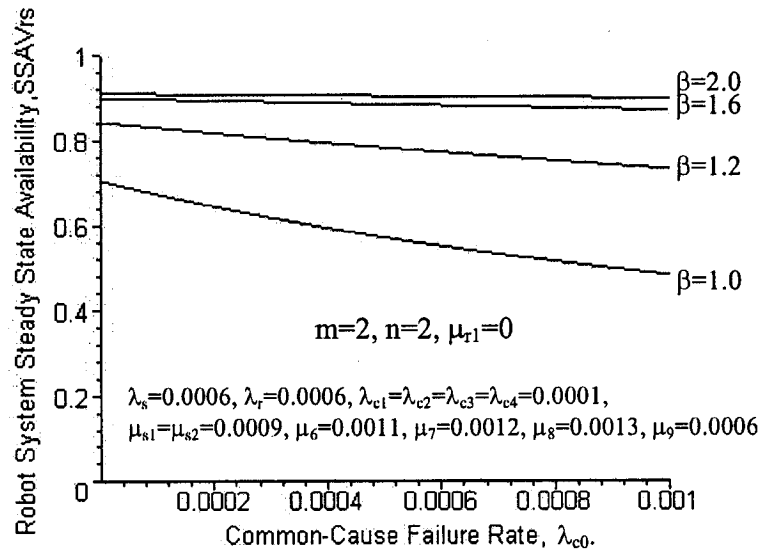


Figure F.11 Robot-safety system steady state availability versus common-cause failure rate (λ_{c0}) plots with Weibull distributed ($\beta=1.0, 1.2, 1.6, 2$) failed system repair time distribution.

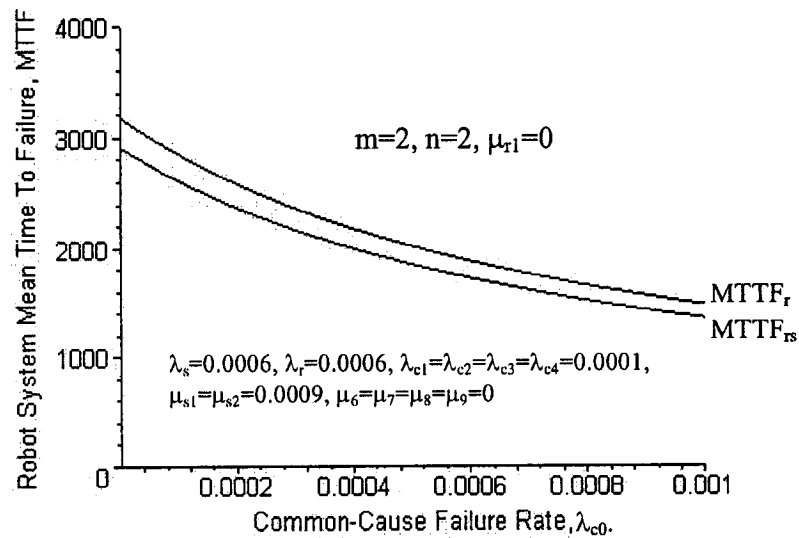


Figure F.12 Mean time to failure plots of an irreparable robot-safety system as a function of common-cause failure rate(λ_{c0}).