CAHIER DE RECHERCHE #1709E
Département de science économique
Faculté des sciences sociales
Université d’Ottawa

WORKING PAPER #1709E
Department of Economics
Faculty of Social Sciences
University of Ottawa

The Impact of Oil Price Changes in a New Keynesian Model of the U.S. Economy[*]

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June 2017

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[*] I would like to thank Pierre Brochu for his suggestions and advice. The usual disclaimer applies.

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Abstract

This paper studies the impact of a change in real oil prices on output and inflation in a New Keynesian model of the U.S. economy. The main goal of the analysis is to assess whether the cross-equation restrictions imposed by the model play a role in the transmission mechanism of exogenous oil price shocks. I find that the interactions between oil prices, domestic variables, and expectations implied by the New Keynesian framework generate responses that are quite modest, and that can depart from those emerging from a more unrestricted SVAR model. I also find that changes in oil prices that cannot be predicted based on the available information are, for the most part, exogenous to the U.S. economy. As such, augmenting the model to account for their possible endogeneity does not deliver substantially different results.

Key words: *Oil Prices, Endogeneity, New Keynesian Model, Expectations.*

JEL Classification: E12, E17, Q43, Q47.

1 Introduction

In recent years, the New Keynesian model has become the standard framework employed for analysing the interactions between macroeconomic variables, economic policy, and expectations. Following the original work of Blanchard and Gali (2007) - the first to incorporate the effects of oil prices within the New Keynesian framework - many researchers have subsequently used this model to explore the relationship between oil prices and economic outcomes.[1] Given this growing literature, it is of critical importance to understand how oil prices transmit into the economy through the relationships assumed by the equations of the model. In particular, this understanding is crucial for sound policy decision-making.

This paper studies the transmission of oil prices in the U.S. economy within the context of a New Keynesian framework. Its contributions are twofold: First, it investigates whether the cross-equation restrictions imposed by the New Keynesian framework materially affect the predicted responses of the economy to an exogenous change in oil prices. Second, it studies the endogeneity of oil price changes to the U.S. economy, and it examines whether accounting for this endogeneity alters the predictions of the model.

The analysis is based on an empirical version of the Blanchard and Gali (2007) model. Using this framework, I examine the responses of output and inflation to an exogenous change in oil prices, and contrast them to those arising from a structural VAR (SVAR) model. This VAR is identified using ordering assumptions that are fully consistent with the timing of the variables in the New Keynesian model; as such, any differences that arise across the two models can be fully attributed to the impact of the cross-equation restrictions imposed by the New Keynesian framework.

As in Blanchard and Gali (2007), this paper focuses on the responses to exogenous changes in oil prices. This implies that the empirical analysis explicitly accounts for the part of the

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[1] For instance, Blanchard and Riggi (2013) use an extension of Blanchard and Gali (2007) to investigate the reasons for the milder impact of oil prices in the 2000s relative to the 1970s, while Acurio Vásconez et al. (2015) employ a similar model to study the relationship between oil prices and capital accumulation. On the other hand, Natal (2012) and Plante (2014) use the New Keynesian framework to study the optimal response of monetary policy to oil prices.

changes in oil prices that is endogenous to the U.S. domestic variables included in the model. Thus, this work is also able to provide an overview of the degree of endogeneity of oil prices to the U.S. economy in the different subsamples considered in this analysis.[2]

The two main results of this paper are as follows: First, I find that the New Keynesian model predicts a response of the economy to an exogenous change in oil prices that is both modest and statistically insignificant. This prediction is not always consistent with the empirical responses emerging from the more unrestricted VAR model. Second, the paper shows that the changes in oil prices that cannot be anticipated based on past values of the model variables are, for the most part, exogenous to the U.S. economy. For this reason, explicitly accounting for their endogenous component does not deliver results that are substantially different from those obtained under the assumption of exogeneity.

The number of studies analysing the macroeconomic impact of oil prices is extensive.[3] This paper bridges two broad strands of this economic literature. On the one hand, there is a long history of theoretical contributions that model the different channels through which oil prices can affect the economy. The employment of the New Keynesian model is one of the last developments in this area of work. On the other hand, there exists a large empirical literature that focuses on the use of unrestricted and a-theoretical models to assess the response of real and nominal variables to oil price shocks. In this area, structural VAR models have been one of the most commonly employed frameworks.

The vast theoretical literature in the area of oil prices originates from the fact that the price of oil can affect economic activity in a number of different ways. An increase in oil prices has a direct effect on the price of goods and services that are made with oil products, and can change the general costs of production because of rising energy costs. In a standard neoclassical model, the resulting adjustment in productivity can affect employment through a change in the wage level (Rasche and Tatom, 1977; Kim and Loungani, 1992). However,

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[2] As discussed in section 3, these subsamples are designed to take into consideration the oil price shocks of the late 1970s, the "Great Moderation" period, the increase in oil prices that started around 2002, and the recent financial crisis.

[3] Detailed reviews are provided by Mork (1994), Hamilton (2008), and Kilian (2008b).

in the presence of rigidities and frictions, the impact of oil prices on the economy could be amplified through a number of additional transmission channels. On the production side, some examples that have been proposed are: changes in business markups (Rotemberg and Woodford, 1996), limits imposed by capacity utilization rates (Finn, 2000), and costs of reallocation of labor or capital between sectors (Bernanke, 1983; Hamilton, 1988). With respect to the labor markets, further effects on employment could arise because of wage rigidities (Solow, 1980; Pindyck, 1980). Finally, in the more recent literature that builds upon the New Keynesian framework, the assumption of Calvo pricing in addition to wage rigidity extends the number of channels through which the impact of oil prices can propagate into the economy (Blanchard and Gali, 2007; Blanchard and Riggi, 2013; Natal, 2012; Plante, 2014).

The empirical literature typically relies on regression or VAR models to measure the response of real variables, prices, and interest rates, to an oil price shock. Several works in this area have debated on the magnitude, and possible asymmetry, of the impact of oil prices on output (Cavallo and Wu, 2012; Hamilton, 1996, 2003, 2009a, 2011; Kilian, 2008a; Kilian and Barsky, 2002; Kilian and Vigfusson, 2011a, 2011b, 2017; Peersman and Van Robays, 2009). An important part of this literature has also discussed the role of monetary policy in the transmission of oil prices into the economy (Bernanke et al., 1997 and 2004; Hamilton and Herrera, 2004; Kilian and Lewis, 2011). Finally, some recent contributions have employed time-varying VAR models to investigate the possible changes in the impact of oil prices over time (Baumeister and Peersman, 2013; Blanchard and Gali, 2007; Clark and Terry, 2010).

In studies using VAR models, a central part of the debate concerns the assumptions to be implemented in order to identify exogenous oil price shocks (for a discussion, see Kilian and Murphy, 2012). In this paper, the identification of the structural VAR model is not an issue, as the New Keynesian framework employed in the analysis implies a specific ordering of the variables that can be used to identify the structural VAR. On the other hand, it is worth noting that while I employ an econometric model to forecast the short term pattern of oil prices, I do not aim at characterizing the demand and supply of oil, or more generally the behavior of the oil markets. In this sense, the approach adopted in this paper departs from the frameworks of Kilian (2009), Baumeister and Kilian (2012), Lippi and Nobili (2012), and Baumeister and Peersman, (2013), and is more in the spirit of the (unrestricted) forecasting models discussed in Hamilton (2009b) and Alquist et al. (2013).

The reminder of the paper is organized as follows. Section 2 outlines the New Keynesian and structural VAR models, and the approach used to deal with the endogeneity of oil prices. In section 3, I describe the data and the implementation of the models. Section 4 reports the results of the empirical analysis, and section 5 concludes.

2 The model

The model under study is an empirical interpretation of the equilibrium equations of Blanchard and Gali (2007); essentially, it is a version of the standard New Keynesian framework used in the Macro-econometric literature,[4] augmented to incorporate oil prices.

The model is composed of the following two equations:

$$y_t = \alpha_0 E_{t-1}(y_t) + (1-\alpha_0)\alpha_y(L)\, y_{t-1} + \alpha_i\left[i_{t-1} - E_{t-1}(\pi_t)\right] + \alpha_s(L)\, s_t + \omega_{y,t} \tag{1}$$

$$\pi_t = \beta_0 E_{t-1}(\pi_t) + (1-\beta_0)\beta_\pi(L)\, \pi_{t-1} + \beta_y(L)\, y_t + \beta_s(L)\, s_t + \omega_{\pi,t} \tag{2}$$

where y_t is the output gap, π_t is core inflation, i_t is the nominal interest rate, and s_t is a measure of oil price changes. The shocks $\omega_{y,t}$ and $\omega_{\pi,t}$ are assumed to be normally distributed, with mean zero and variances σ_y^2 and σ_π^2, respectively. The data is assumed to be measured quarterly. Equations (1) and (2) are typically interpreted as an IS curve and a Phillips curve, respectively. In equation (2), I impose the restriction: $\beta_0 + (1-\beta_0)\sum_{j=1}^{J}\beta_{\pi j} = 1$, where J is the number of lags of the polynomial $\beta_\pi(L)$. This assumption is standard in the literature,

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[4] Examples of this framework can be found in Rudebusch and Svensson (1999), Clarida et al. (2000), and Rudebusch (2002).

and guarantees that the long run Phillips curve is vertical.

Equations (1) and (2) allow oil prices to affect the output gap and inflation directly through the lag polynomials $\alpha_s(L)$ and $\beta_s(L)$. The parameters in these polynomials can thus be interpreted as measuring the direct effects of oil prices on the economy, potentially due to their impact on the costs of production and on the general price level. As in Blanchard and Gali (2007), inflation is included in the model as a core measure, i.e. a measure that excludes the price of energy. This implies that we should not necessarily expect the value of π_t to be affected by changes in the price of oil, by construction.

In addition to their direct impact on the variables of interest, changes in oil prices will transmit into the economy through the relationships implied by (1) and (2). Any effects of oil prices on the output gap will be transferred to the inflation rate, contemporaneously and with lags, through the parameters of the lag polynomial $\beta_y(L)$. Moreover, additional persistence in the impact of oil prices might emerge from the fact that both y_t and π_t are a function of their own lagged terms. Finally, both (1) and (2) include expectation terms, which could introduce further dynamics in the transmission of oil prices into the economy.

In the model specification employed in this paper, expectations are assumed to be rational and to be formed using information up to time $t-1$. In order to be able to estimate equations (1) and (2), I approximate $E_{t-1}(y_t)$ and $E_{t-1}(\pi_t)$ using a reduced-form model with recursively updated parameters. However, in studying the responses of the variables to a change in oil prices, I fully incorporate the cross-equations restrictions implied by rational expectations. Further details are provided in the Estimation section and in the Appendix.

For the model to be complete, I need to specify laws of motion for the nominal interest rate i_t, and for the real price of oil s_t. As the purpose of this paper is not to investigate the role of monetary policy or to study how prices are formed in the oil markets, I adopt more general and a-theoretical assumptions for these two variables. For the nominal interest rate, I employ a Taylor-type policy reaction function of the form:

$$i_t = f_y y_t + f_\pi \pi_t + f_i i_{t-1} + \omega_{i,t} \tag{3}$$

The policy parameters in (3) will be estimated from the data; this implies that they can be interpreted as incorporating the average response of monetary policy in the sample period under analysis. As i_t is included in (1), the patterns of the variables y_t and π_t originating from the model will also embody the effects of this average policy response.

The process for the real price of oil is not modelled explicitly. Instead, I use a reduced-form linear model to forecast s_t in each period using the available data. The specific equation that I use follows Hamilton (2009b), and is defined as:

$$s_t = \gamma' X_{t-1} + e_{s,t} \tag{4}$$

where X_{t-1} is a vector containing lags of all the variables included in the model (s_t, y_t, π_t, and i_t), with the most recent lags being those for time $t-1$. The innovation $e_{s,t}$ can be interpreted as a one period ahead forecast error. Notice that the forecasting equation (4) is able to capture the possible endogeneity of oil prices to the U.S. economy, as lagged values of y_t, π_t, and i_t are included as predictors for s_t, and the forecast error $e_{s,t}$ is not necessarily assumed to be orthogonal to the structural shocks of the model. A more extensive discussion of this issue will be provided below.

As previously mentioned, the approach adopted in this paper departs from the empirical literature that aims at modelling the behavior of the international oil markets. It also departs from the New Keynesian frameworks of Blanchard and Gali (2007) and Blanchard and Riggi (2013), which assume an exogenous AR(1) process for the real price of oil, and from those of Natal (2012) and Plante (2014), which characterize the demand and supply of oil in the economy. The choice of employing a forecasting equation in the form of (4) was motivated by the following three reasons: First, this equation makes use of a relatively large information set, and it does not impose any a priori restrictions on the way in which the variables affect each other. Second, this approach is in the spirit of parsimony, as (4) only uses the domestic data included in the New Keynesian model, and it does not require the introduction of new variables (for instance, global variables that capture the behavior of the international oil markets). Finally, equation (4) is able to account for the possible endogeneity of oil prices to the U.S. economy, and it is thus more encompassing than the exogenous autoregressive process used in some of the previous literature.

2.1 Endogeneity of oil prices

The forecasting equation for oil prices only employs lagged values of the variables as predictors; this implies that (4) can be estimated by OLS. For the monetary policy rule (3), the paper assumes that the nominal interest rate reacts contemporaneously (i.e. within the quarter) to the variables in the model, but only affects them with a one period delay. This assumption is common in the monetary policy literature. The innovation $\omega_{i,t}$ is typically interpreted as a monetary policy shock, which is orthogonal to the right-hand side variables in (3). Thus, (3) can also be estimated using OLS. Equations (1) and (2), on the other hand, require a little more discussion. In particular, for these equations to be estimated, the relationships between the innovations in (1) and (2), and the forecast error in (4), need to be further specified.

The New Keynesian model described by equations (1) and (2), together with the assumptions on the timing of expectations, imply a precise direction of causality between the output gap and inflation. More specifically, the output gap is assumed to affect the inflation rate contemporaneously, while inflation only enters the output gap equation in the form of expectations based on time $t-1$ data. This implies that only lagged values of inflation will affect the output gap. With respect to the structural shocks $\omega_{y,t}$ and $\omega_{\pi,t}$, they are usually interpreted as a domestic demand shock and a domestic supply ("markup") shock respectively, with the standard assumption that $E\left(\omega_{y,t}\omega_{\pi,t}\right)=0$.

The contemporaneous relationships between y_t and π_t are important to understand the role of the oil price variable s_t in (1) and (2), and the possible correlations between the structural shocks $\omega_{y,t}$ and $\omega_{\pi,t}$ and the forecast error $e_{s,t}$. Let $\omega_{s,t}$ denote the component of s_t that is due to shocks exogenous to the U.S. economy (for instance, shocks to the supply or demand of oil that originate in the world markets and are not related to events in the U.S.). Since $\omega_{y,t}$ and $\omega_{\pi,t}$ are domestic shocks, one can reasonably assume that $E\left(\omega_{s,t}\omega_{y,t}\right) = 0$ and $E\left(\omega_{s,t}\omega_{\pi,t}\right) = 0$. Now, it is possible that the forecast error $e_{s,t}$ in (4) is entirely due to oil price shocks that are exogenous to the U.S. economy, i.e.:

$$e_{s,t} = \omega_{s,t} \tag{5}$$

If this is the case, then $\omega_{y,t}$ is orthogonal to all of the right-hand side variables in (1), and $\omega_{\pi,t}$ is orthogonal to all of the right-hand side variables in (2), so both equations can be estimated by OLS.

Unfortunately, there is no reason to believe that the forecast error $e_{s,t}$ is only due to shocks that are exogenous to the U.S. On the contrary, in the global economy shocks are often correlated across countries, and they can ultimately have an impact on the demand or supply of oil in the international markets. In this paper, I assume that the U.S. domestic shocks that can be contemporaneously correlated with the price of oil are the demand shocks $\omega_{y,t}$. This choice is motivated by the structure of the New Keynesian model under analysis. As previously discussed, (1) and (2) imply that realizations of the shock $\omega_{y,t}$ affect y_t directly, and π_t through y_t, while realizations of the shock $\omega_{\pi,t}$ have an impact on π_t, but not on y_t. As s_t enters equation (1), assuming a correlation between $\omega_{\pi,t}$ and the forecast error $e_{s,t}$ would imply that the shocks $\omega_{\pi,t}$ can somehow affect y_t contemporaneously through s_t, which would alter the original structure of the New Keynesian model.[5] In addition, it seems reasonable from an economic perspective to assume that demand shocks, which are

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[5] Alternatively, the assumption that s_t can only be affected by current realizations of $\omega_{y,t}$, but not $\omega_{\pi,t}$, can be interpreted as imposing the restriction that domestic shocks can only affect the price of oil contemporaneously through their impact on the output gap.

often quite correlated across different countries, can affect the global demand of oil and oil prices quite rapidly. On the other hand, supply shocks seem to have effects that are more domestically localized, or that might take longer than a quarter to alter the demand or supply of oil (as, for instance, in the case of technology shocks). In the U.S., the extraction and production of oil has significantly increased in recent years (see Kilian, 2016), but the country is still a net importer of oil, so U.S. domestic oil supply shocks are unlikely to affect the world oil markets to a large extent, especially within the same quarter.

Based on the above discussion, in the presence of an endogenous component the expression for the forecast error $e_{s,t}$ can be written as:

$$e_{s,t} = \omega_{s,t} + \rho \omega_{y,t} \tag{6}$$

Clearly, this expression simplifies to (5) in the special case in which $\rho = 0$. However, if $\rho \neq 0$ then (1) cannot be estimated by OLS anymore, because under (6) we have that: $E\left(s_t \omega_{y,t}\right) \neq 0$. This implies that a different approach needs to be adopted. The strategy that I employ involves instrumenting s_t in (1) using the variable $\widehat{s}_t$, computed as:

$$\widehat{s}_t = \widehat{\gamma}' X_{t-1} \tag{7}$$

This expression employs the forecasting equation (4), and the vector of parameters $\widehat{\gamma}$ estimated from this equation, to provide predicted values for s_t. As X_{t-1} only includes variables up to time $t-1$, the instrument $\widehat{s}_t$ is orthogonal to all the structural shocks included in the model, which allows (1) to be estimated by OLS. The parameters of (1) obtained using this strategy, together with the actual values of s_t and the other right-hand side variables of the equation, can then be used to recover the shocks $\omega_{y,t}$. Finally, the series of the shocks $\omega_{y,t}$ and forecast errors $e_{s,t}$ can be used to estimate ρ in (6), and to obtain $\omega_{s,t}$ as the residuals from this regression. Notice that, under this approach, $\omega_{s,t}$ is computed as the portion of the forecast error $e_{s,t}$ that is orthogonal to the domestic shock $\omega_{y,t}$.

As previously mentioned, the papers of Kilian (2009), Baumeister and Kilian (2012), Lippi and Nobili (2012), and Baumeister and Peersman, (2013) aim at identifying different types of structural shocks that can affect oil prices in the international oil markets. The more unrestricted approach adopted in this paper implies that the forecast error $e_{s,t}$ in (4) has no structural interpretation, and is likely to be due to a combination of different underlying structural shocks. However, if these structural shocks are all approximately exogenous to the U.S. economy, as suggested for instance by Kilian (2009), then $e_{s,t}$ would be essentially exogenous to the U.S. economy as well, in which case (5) could be a reasonable assumption. As the results will show, (5) is indeed delivering results that are very similar to those obtained under the more general assumption (6), supporting the idea that the structural shocks that compose $e_{s,t}$ are approximately exogenous to the U.S. variables included in the model.

3 Estimation

The model is estimated using U.S. quarterly data for the period $1977 : I - 2016 : IV$. The output gap (y_t) is computed as the log difference between real GDP and the CBO's estimate of real potential GDP, both measured in chain-weighted dollars. The inflation rate (π_t) is the annualized log difference in the core PCE index (Personal Consumption Expenditures excluding food and energy). The nominal interest rate (i_t) is the Wu-Xia Shadow Federal Funds Rate.[6] Oil price changes (s_t) are measured as the log difference in the real price of oil, where the real price of oil is the U.S. crude oil imported acquisition cost by refiners, deflated by the same core PCE index used to obtain π_t.[7]

The use of core inflation is consistent with the New Keynesian model of Blanchard and

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[6] This variable is equal to the effective Federal Funds Rate (measured at the end of the month) in the entire sample except for the period December 2008 − December 2015. During this period, the effective Federal Funds Rate is replaced by the shadow short-term interest rate computed using the Wu and Xia's (2016) model.

[7] The data used to compute y_t and π_t was obtained from the FRED (Federal Reserve Bank of St. Louis) webpage. The data for the Wu-Xia Federal Funds Rate was retrieved from the Federal Reserve Bank of Atlanta webpage. The data for the U.S. crude oil acquisition cost by refiners was obtained from the U.S. Energy Information Administration webpage. Monthly series were transformed into quarterly data by taking averages of the months in the quarter.

Gali (2007) and the VAR model of Clark and Terry (2010). The choice of employing the PCE index in the baseline analysis follows again Clark and Terry (2010), and is motivated by the central role of this measure in economic policy.[8] As in Kilian and Vigfusson (2011a, 2011b), nominal oil prices are the U.S. crude oil imported acquisition cost by refiners. Kilian and Vigfusson (2011b) suggest that this measure conforms to models of the U.S. economy in which oil is considered as an imported good, and oil prices are determined in the global oil markets. As discussed in Kilian (2008b) and Kilian and Vigfusson (2011b), it is still debated whether nominal or real oil prices should be used in empirical analysis. This paper employs real oil prices as the variable of choice, but a robustness exercise performed using the nominal measure produced almost identical results.

The data on the U.S. crude oil imported acquisition cost by refiners is available from 1968 onwards on an annual basis, and starting from January 1974 at the monthly frequency. For the 1968 to 1973 years, the data was thus linearly interpolated to obtain quarterly observations. While this procedure increases the sample available for the analysis, it does introduce some arbitrariness in the first few years of the oil price data. In addition, Kilian and Vigfusson (2011b) explain that before 1974 oil prices were strictly regulated, and suggest that data before this year should not be included in the estimation of economic models. Finally, as discussed below, the framework adopted in this paper requires some initial observations to be employed as a training sample. For all of these reasons, the main analysis is performed using data from $1977 : I$ to $2016 : IV$.

Additional exercises use parameter estimates obtained from alternative sample periods. The economic literature has long debated whether the response of the U.S. economy to oil price shocks has structurally changed starting from the mid 1980s.[9] For this reason, I consider $1985 : I$, i.e. the year which is often denoted as the start of the "Great Moderation", as an alternative starting point for the analysis. With respect to the end of the sample, two

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[8] For instance, the Federal Reserve has announced that this is the main price index that they monitor for policy decisions.

[9] See Kilian (2008b) for a discussion.

additional dates are examined: $2007 : IV$, which corresponds to the start of the "Great Recession", and $2002 : IV$, which corresponds to the beginning of the steady and large increase in oil prices that characterized the years before the "Great Recession".[10]

To be able to estimate the parameters of (1) and (2), I approximate the unobservable variables $E_{t-1}(y_t)$ and $E_{t-1}(\pi_t)$ by the measures $y^e_{t|t-1}$ and $\pi^e_{t|t-1}$, defined as:

$$y^e_{t|t-1} = \widehat{\phi}'_{y,t-1} X_{t-1} \tag{8}$$

$$\pi^e_{t|t-1} = \widehat{\phi}'_{\pi,t-1} X_{t-1} \tag{9}$$

where X_t is the same vector used in (4), and $\widehat{\phi}'_{y,t-1}$ and $\widehat{\phi}'_{y,t-1}$ are estimated vectors of parameters, that are updated in each period using the new observations that become available. For this purpose, I use the following Recursive Least Squares updating formulas:

$$R_t = R_{t-1} + \frac{1}{t}\left(X_t X'_t - R_{t-1}\right)$$

$$\widehat{\phi}_{y,t} = \widehat{\phi}_{y,t-1} + \frac{1}{t} R_t^{-1} X_t \left(y_t - X'_t \widehat{\phi}_{y,t-1}\right)$$

$$\widehat{\phi}_{\pi,t} = \widehat{\phi}_{\pi,t-1} + \frac{1}{t} R_t^{-1} X_t \left(\pi_t - X'_t \widehat{\phi}_{\pi,t-1}\right)$$

The values $\widehat{\phi}_{y,0}$, $\widehat{\phi}_{\pi,0}$, and R_0 are computed from an initial training sample, which goes from $1968 : I$ to $1976 : IV$.[11]

Equations (8) and (9) are very general, and they use all the information available at each point in time to approximate the values of $E_{t-1}(y_t)$ and $E_{t-1}(\pi_t)$.[12] In addition, it

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[10] Several researchers have argued that this increase in oil prices was due to an increase in the world demand of oil, rather than to changes in the supply side of the oil market (see Kilian, 2008b, for more details). If this is the case, then we should expect oil price changes to be more endogenous to the U.S. economy after the early 2000s.

[11] The vectors $\widehat{\phi}_{y,0}$ and $\widehat{\phi}_{\pi,0}$ are obtained from the regressions of y_t on X_{t-1} and π_t on X_{t-1}, respectively, using data from the training sample. The matrix R_0 is defined as: $R_0 = \sum_{t=1}^{T_0} X_t X'_t$, where T_0 is the length of the training sample. These initial values imply that the vectors $\widehat{\phi}_{y,t}$ and $\widehat{\phi}_{\pi,t}$ computed from the Recursive Least Squares formulas correspond to OLS estimates.

[12] Possible alternatives that have been employed in the literature to approximate expectations are adaptive

is important to remark that $y^e_{t|t-1}$ and $\pi^e_{t|t-1}$ are only employed in the estimation of the parameters of the model, while rational expectations are imposed in the analysis of the responses to an exogenous oil price shock. The Appendix explains how I derive the law of motion of the variables of the model under the assumption of rational expectations.

The exact specification of the right-hand side variables in equations (1) and (2) was chosen based on the BIC criterion. Specifically, (1) was estimated using $y^e_{t|t-1}$, one lag of y_t, the term $\left[i_{t-1} - \pi^e_{t|t-1}\right]$, and the contemporaneous value of s_t. On the other hand, (2) was estimated using $\pi^e_{t|t-1}$, four lags of π_t, the contemporaneous value of y_t and four of its lags, and the contemporaneous value of s_t. Finally, the vector X_t employed in (4), (8), and (9) included four lags of all the variables.

The responses arising from the New Keynesian framework described by (1), (2), (3), and (4) are compared to the responses obtained from the following structural VAR model:

$$Y_t = D_0 + D_1 Y_{t-1} + ... + D_4 Y_{t-4} + C_V \varepsilon_t \tag{10}$$

where $Y_t = [s_t \quad y_t \quad \pi_t \quad i_t]'$, and ε_t is a vector of structural shocks that satisfies $E\left(\varepsilon_t \varepsilon_t'\right) = I$. The VAR includes four lags of the vector Y_t, in line with the maximum number of lags of the variables in the equations of the New Keynesian model.

The matrix C_V is a lower triangular matrix obtained from the Cholesky decomposition of the covariance matrix of the reduced-form residuals. This implies that the variables are assumed to affect each other in the specific ordering in which they are included in the vector Y_t. This ordering is consistent with the structure of the New Keynesian framework, under the assumption that the forecast error $e_{s,t}$ satisfies (5), i.e. it is exogenous to the U.S. economy. In this case, s_t can be expressed as a linear function of past values of all the variables, with an error $e_{s,t}$ which is orthogonal to the other shocks of the model. This structure is consistent

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expectations formulas, or the use of survey data. However, the former imposes much stronger restrictions compared to the approach used in this paper, while survey data on expected core inflation is not available for a long enough period of time, and as such is not an available option here.

with s_t being ordered first in the Cholesky VAR.[13] In the case of the output gap, equation (1) includes the current value of s_t, while for the inflation rate, equation (2) includes current values of both s_t and y_t. Finally, in the estimated Taylor rule (3), the nominal interest rate i_t is assumed to respond to the contemporaneous values of the output gap and inflation.[14] These relationships fully match the ordering implied by a lower triangular matrix C_V in (10).

Apart from the ordering assumptions imposed by C_V, all the other relationships between variables are left unrestricted in the VAR model, and are just estimated from the data. On the other hand, the New Keynesian framework incorporates a number of additional restrictions, which are due to the specific equations of the model, and to the interconnections induced by expectations. As the assumptions about the contemporaneous relationships between variables are the same in the two models, then a different response of the economy to an exogenous oil price shock can be attributed to the additional cross-equation restrictions that characterize the New Keynesian framework.

4 Results

I first study the response of the domestic U.S. variables to an exogenous change in real oil prices. More specifically, I focus on the responses of the output gap and core inflation to a shock $\omega_{s,t}$, normalized to increase s_t by 10%. In the baseline scenario, the responses are obtained under the (more general) assumption that the forecast error $e_{s,t}$ in (4) can be partially due to the domestic demand shock $\omega_{y,t}$, as defined by (6). These responses are compared to those obtained under the stronger assumption that $e_{s,t}$ is fully exogenous, as specified by (5), and to those obtained using the VAR model described by (10). As a reminder, the VAR model is identified imposing ordering restrictions that are consistent with the New Keynesian model under the assumption of exogeneity of $e_{s,t}$.

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[13] Actually, given the way in which the matrix X_t is specified, the forecasting equation (4) gives exactly the same expression for s_t as the VAR model (10).

[14] This implies that i_t also responds contemporaneously to changes in oil prices through their impact on the output gap and inflation.

The results are reported in figures 1 through 3. Each figure shows the median response of s_t (top left panel), y_t (top right panel), π_t (bottom left panel), and i_t (bottom right panel). The error bands are the 5th and 95th percentile responses for the baseline scenario, i.e. for the model given by (1), (2), (3), (4), and (6). These bands are obtained from 10,000 replications of a bootstrap procedure that accounts for the uncertainty in the estimation of equation (4), and consequently in the procedure used to estimate (1) and (6).[15] In this sense, the error bands reported in the figures capture the estimation uncertainty that is more directly related to the equations that describe the behavior of oil prices.[16]

Figure 1 refers to the full sample, ranging from $1977 : I$ to $2016 : IV$. For the New Keynesian framework, most of the responses to an exogenous oil price shock are very small and insignificant.[17] For all the variables, the median responses obtained under the assumption of a fully exogenous forecast error $e_{s,t}$ are almost identical to those arising under the more general assumption of potential endogeneity. On the other hand, the median responses obtained from the VAR model show a somehow different behavior, particularly the response of the output gap starting from about 6 quarters after the impact.

The recent financial crisis is often interpreted as a structural break in the relationships between variables in the U.S. data. On this account, the second exercise that I performed employed a reduced sample ending in the last quarter of 2007. The responses for the $1977 : I - 2007 : IV$ sample are similar to those arising from the full sample, and are reported in figure 2. For the inflation rate, the general tendency of the responses is now negative, but the magnitude is very small and again insignificant in most periods.[18] Again, the median responses are different in the VAR model, for both inflation (that increases after an initial

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[15] A different set of parameters in (4) implies a different value of the instrument $\widehat{s}_t$, which in turn gives a different estimate for the parameters in (1). As the histories of $e_{s,t}$ and $\omega_{y,t}$ are recovered from (4) and (1), respectively, the estimated value of ρ in (6) will be different as well.

[16] Adding the uncertainty about the estimated parameters in the other equations of the model would make the error bands larger, but it would not change the main message of the paper.

[17] Note that, in all the figures, the responses are reported in percentage points, so a 0.02 increase denotes a change from, say, 2% to 2.02%.

[18] Recall that the measure of inflation employed in the paper excludes energy prices, so one should not necessarily expect an increase in inflation following an increase in the real price of oil.

decline) and the output gap (which shows a even more marked decline compared to figure 1).

The behavior of the nominal interest rate deserves a separate discussion. It is true that the patterns reported in figure 2 show a significantly different response of i_t in the VAR model relative to the New Keynesian model. However, I believe that this result is less relevant compared to those obtained for the output gap and inflation. Part of the differences are due to the fact that the New Keynesian framework employs the specific Taylor rule described by (3), while the VAR model estimates an unrestricted equation for i_t. In addition, both (3) and (10) presume a reaction of the nominal interest rate to the contemporaneous values of the other variables, so differences in the responses of y_t and π_t in the two models will be reflected on i_t. Finally, the figures seem to suggest that, especially in the VAR model, the estimated equation for i_t might be affected by the unusual behavior of the Federal funds rate during the "Great Recession" period. In all, the study of the response of monetary policy to changes in oil prices is not the main interest of this paper, and the variable i_t is included only for completeness of the New Keynesian framework.

I performed one last impulse-response exercise using data for the period $1985 : I - 2016 : IV$. In this case, the analysis excludes the first few years of the sample, which were characterized by large changes in oil prices. The results of this exercise are shown in figure 3. For both the New Keynesian and VAR models, the responses are very similar to those reported in figure 1, except for the median response of inflation in the VAR model, which is slightly more negative.

For the structural VAR, the results reported in the figures are very similar to those found in the previous literature using similar frameworks (for instance, Clark and Terry, 2010, and Cavallo and Wu, 2012). As explained above, the VAR model imposes no restrictions other than the ordering in which the variables are determined. This implies that the responses emerging from the VAR model are fully guided by the empirical relationships estimated from the data. On the other hand, the equations of the New Keynesian model impose stronger assumptions on the way in which the variables affect each other, as detailed by the equations of the model. Figures 1 to 3 seem to suggest that these assumptions have repercussions on the predicted responses of the variables to an exogenous change in oil prices. In particular, they appear to have the effect of mitigating the transmission of oil price shocks into the economy.

In a sense, the results can be interpreted as showing that the Keynesian framework employed in this paper is not able to fully reproduce the empirical responses obtained from the VAR model. One could further advance that these results suggest that the cross-equation restrictions imposed by the New Keynesian model might be too strong. In this direction, the main culprit could be expectations. Following most of the literature in this area, this paper adopts the assumption of rational expectations; however, this assumption might not be reflective of the way in which private agents form expectations in the real world. This conclusion would be in line with the recent results of Coibion and Gorodnichenko (2015), who show that the way in which expectations react to changes in oil prices is important for the ability of macroeconomic models (in their case a Phillips curve model) to reproduce the data.

With respect to the potential endogeneity of the forecast error $e_{s,t}$, figures 1 to 3 show that the transmission of exogenous oil price changes is almost the same under assumptions (5) or (6). In other words, allowing $e_{s,t}$ to be endogenous to the U.S. economy does not materially alter the results relative to the assumption of full exogeneity, so that instrumenting the value of s_t in (1) seems almost unnecessary. To further investigate the degree of endogeneity of $e_{s,t}$, I examined the correlation between the exogenous shocks $\omega_{s,t}$ defined by (5), and those obtained from the more general assumption (6). Table 1 reports the median correlation, together with the 16th and 84th percentiles (corresponding to one standard deviation), for different time periods. As shown in this table, the shocks $\omega_{s,t}$ identified under the assumption of full exogeneity of the forecast error are very highly correlated with the same shocks identified under assumption (5). The correlation seems to increase if a longer sample is employed, reflecting perhaps a better fit of the model. The correlation is a little lower in the subsamples starting from 1985, indicating that the endogenous component of the forecast error $e_{s,t}$ became slightly more important after this year. This result is consistent with the generally accepted belief that the large oil price changes of the late 1970s and early 1980s were exogenous to the U.S. economy, whereas demand driven changes have been more common in recent years.

Figures 4 and 5 confirm that the exogenous shocks $\omega_{s,t}$ are remarkably similar whether they are computed from (5) or from (6). Figure 4 shows the median of the series of $\omega_{s,t}$ identified using assumption (6), together with its 5th and 95th percentiles, and the median of the same series obtained under assumption (5). It is evident from the figure that these two series are almost overlapping. Figure 5 provides further details by reporting the predicted change in real oil prices $\widehat{s}_t$, together with the actual series for s_t (top panel), the exogenous shocks $\omega_{s,t}$ (middle panel), and the endogenous component of the forecast error $e_{s,t}$ (bottom panel). This figure confirms that the part of $e_{s,t}$ that is due to the domestic demand shock $\omega_{y,t}$ is very small compared to the part that is due to $\omega_{s,t}$.

Finally, table 2 reports the one period ahead forecast error variance decomposition for the model estimated under assumption (6). The table clearly shows that the contribution of the shock $\omega_{y,t}$ to the variance of the forecast error $e_{s,t}$ is very small, reaching a maximum of only 2.29%. The table also indicates that this contribution is not much different across subsamples. With respect to the one period ahead forecast error variance for the output gap y_t, the contribution of $\omega_{s,t}$ is higher in the samples including more recent data, but it still remains minor. The impact of the shock $\omega_{s,t}$ on y_t reported in figures 1 to 3 is indeed slightly larger in magnitude in the more recent samples, but its transmission to the economy does not seem to have changed in any significant way.

Overall, the results of the analysis seem to suggest that the forecast error in the oil price equation (4) is essentially exogenous to the U.S. economy. This conclusion is consistent with previous contributions in this area. As already mentioned, Kilian (2009) argues that the structural oil price shocks identified using a VAR model of the international oil markets are approximately exogenous to the U.S. economy. In this paper, oil prices are not modelled, they are simply forecasted based on the information available in the model. Nonetheless, if the structural shocks that affect the oil markets are in fact exogenous to the U.S. economy, then the forecast error $e_{s,t}$, which is function of these shocks, will be as well.[19]

In terms of robustness of the results, substituting the PCE index with the core CPI index does not alter the main conclusions of the paper, even if the inflation rate seems to remain affected by the exogenous oil price shock for longer in this case. The results are also similar if the GDP deflator is used instead of the PCE index but, as expected, the response of inflation in this case is positive, as the deflator includes prices that are directly affected by energy costs. Finally, the responses of the variables are almost unchanged if the nominal price of oil is employed instead of the real.

5 Conclusions

This paper examined the responses of the U.S. economy to an exogenous change in oil prices within the context of a New Keynesian framework. I showed that the impact of oil prices predicted by the New Keynesian model is modest, and this is true even if the sample period used for the estimation includes the large oil price shocks of the late 1970s and early 1980s. For some of the variables considered in the analysis, in particular the output gap, the responses can be qualitatively and quantitatively different from those arising from a more unrestricted VAR model.

The results reported in this paper, in addition to those highlighted by Coibion and Gorodnichenko (2015), seem to suggest that private agents do account for oil prices when they form expectations about future developments in the economy, in a way that might not

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[19] In a previous version of the paper, I was dealing with the possible endogeneity of the forecast error $e_{s,t}$ by employing the instrument $\widehat{z}_t = \widehat{s}_t + \varepsilon_t^s$, instead of $\widehat{s}_t$, where ε_t^s were oil supply shocks obtained from a VAR of the world oil markets as in Kilian (2009). The results were almost the same as those reported in this paper for the simpler instrument $\widehat{s}_t$.

always correspond to the rational expectations outcome. I believe that this conclusion can be used as a starting point for a thorough investigation of the links between oil prices, private agents' expectations, and the transmission of oil prices into the economy through the channel of expectations. Clearly, a deep understanding of these links has key policy implications.

The paper also shows that the changes in oil prices that cannot be anticipated based on past data are, for the most part, exogenous to the U.S. economy. This result confirms previous findings on the exogeneity of oil price shocks to U.S. output and inflation; however, its implications are slightly stronger. Indeed, this paper suggests that it might not be necessary to explicitly model the international oil markets if one is not interested in recovering the structural oil price shocks. A simple forecasting equation in the form of (4) might be enough to obtain predictions for the real price of oil in which the error is essentially uncorrelated with the current values of U.S. output and inflation.

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Appendix

State space representation of the model

The model described by equations (1), (2), (3) and (4) can be written in state space form as:

$$A_0 Y_t = B_0 E_{t-1}(Y_t) + B_1 Z_{t-1} + \widetilde{C}\omega_t \tag{11}$$

where

$$Y_t = [s_t \quad y_t \quad \pi_t \quad i_t]'$$

$$\omega_t = [\omega_{s,t} \quad \omega_{y,t} \quad \omega_{\pi,t} \quad \omega_{i,t}]'$$

and the vector Z_{t-1} includes all the non-contemporaneous right-hand side variables in the equations. The matrices A_0, B_0, B_1, and $\widetilde{C}$ contain all the coefficients of the model, which are estimated using the procedure explained in the main text. Notice that the matrix $\widetilde{C}$ will be different depending on whether assumption (5) or (6) is used.

The state space form (11) can be rewritten as:

$$Y_t = AE_{t-1}(Y_t) + \widetilde{B}Z_{t-1} + C\omega_t$$

where $A = (A_0)^{-1} B_0$, $\widetilde{B} = (A_0)^{-1} B_1$, and $C = (A_0)^{-1} \widetilde{C}$. This expression can be used to compute expectations as:

$$\begin{aligned} E_{t-1}(Y_t) &= AE_{t-1}(Y_t) + \widetilde{B}Z_{t-1} \\ &= (I - A)^{-1}\widetilde{B}Z_{t-1} \end{aligned}$$

thus rewriting:

$$
\begin{aligned}
Y_t &= A(I-A)^{-1}\widetilde{B}Z_{t-1} + \widetilde{B}Z_{t-1} + C\omega_t \\
&= BZ_{t-1} + C\omega_t
\end{aligned}
$$

with $B = (I-A)^{-1}\widetilde{B}$.

The expression $Y_t = BZ_{t-1} + C\omega_t$ is used to produce all the responses reported in the paper.

Estimation of the parameters in the endogenous model

The specific equation for the output gap estimated in the paper is:

$$
y_t = \alpha_0 E_{t-1}\left(y_t\right) + (1-\alpha_0)\alpha_y y_{t-1} + \alpha_i\left[i_{t-1} - E_{t-1}\left(\pi_t\right)\right] + \alpha_s s_t + \omega_{y,t} \tag{12}
$$

where oil prices are forecasted using the expression $s_t = \gamma' X_t + e_{s,t}$. If (5) is the true model, then $E\left(s_t\omega_{y,t}\right) = 0$ and (12) can consistently be estimated by OLS. However, if (6) is in fact the true model, then $E\left(s_t\omega_{y,t}\right) \neq 0$. Substituting $s_t = \widehat{s}_t + e_{s,t}$, one has:

$$
y_t = \alpha_0 E_{t-1}\left(y_t\right) + (1-\alpha_0)\alpha_y y_{t-1} + \alpha_i\left[i_{t-1} - E_{t-1}\left(\pi_t\right)\right] + \alpha_s \widehat{s}_t + \widehat{\omega}_{y,t}
$$

where $\widehat{s}_t = \gamma' X_t$, $\widehat{\omega}_t = \alpha_s e_{s,t} + \omega_{y,t}$, and $e_{s,t} = \omega_{s,t} + \rho\omega_{y,t}$. Thus, $E\left(\widehat{s}_t\widehat{\omega}_{y,t}\right) = 0$, and the parameters of the model can now be consistently estimated by OLS.

Tables and Figures

	16th	median	84th
1977 – 2002	0.8767	0.9093	0.9351
1977 – 2007	0.8898	0.9214	0.9463
1977 – 2016	0.9044	0.9357	0.9561
1985 – 2007	0.8451	0.8837	0.9150
1985 – 2016	0.8809	0.9174	0.9425

Table 1: Correlations between the exogenous component of oil price changes estimated using (5) and (6), computed for different time periods.

		$\omega_{s,t}$	$\omega_{y,t}$
1977 – 2002	s_t	98.16%	1.84%
	y_t	1%	99%
1977 – 2007	s_t	98%	2%
	y_t	1.23%	98.77%
1977 – 2016	s_t	97.81%	2.19%
	y_t	2.64%	97.36%
1985 – 2016	s_t	97.71%	2.29%
	y_t	3.51%	96.49%

Table 2: One period ahead forecast error variance decomposition in the case oil price changes are allowed to be endogenous according to (6).

Responses for the period 1977 - 2016

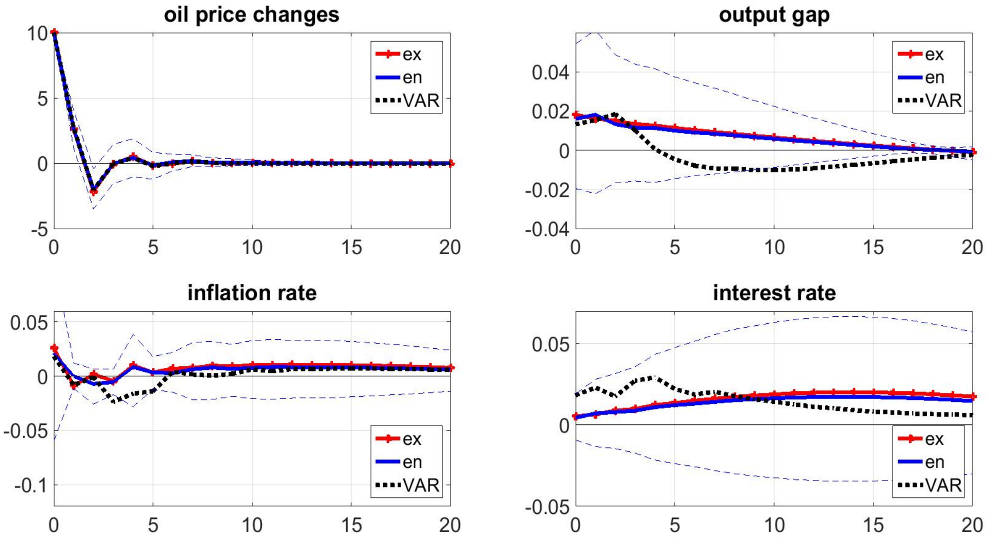

Figure 1: Responses of the variables to an exogenous change in oil prices normalized to increase s_t by 10%. For the model described by (6), the median response (blue continuous line) and the 5th and 95th percentile bands are reported. For the model described by (5) and the VAR model, the median response is reported (red marked line and black dotted line respectively).

Responses for the period 1977 - 2007

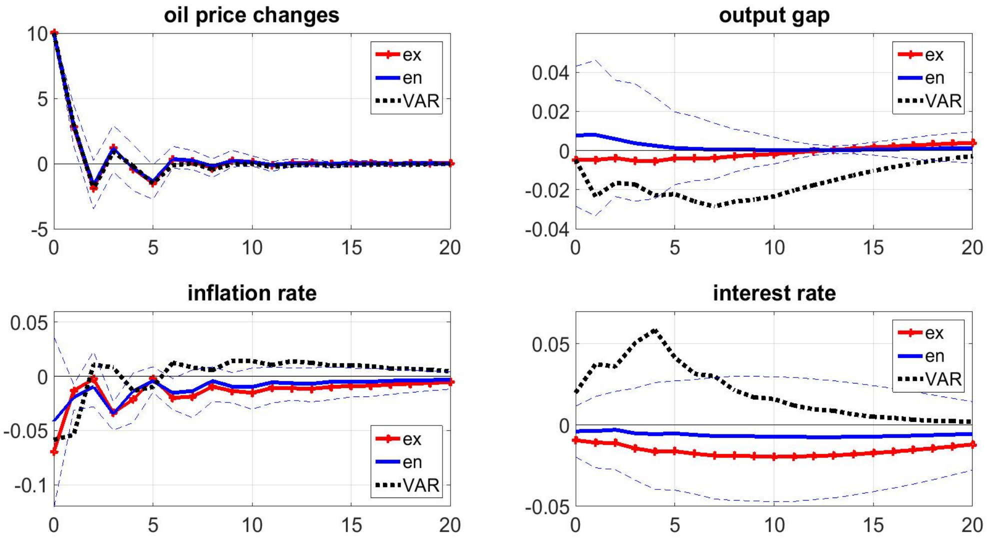

Figure 2: Responses of the variables to an exogenous change in oil prices normalized to increase s_t by 10%. For the model described by (6), the median response (blue continuous line) and the 5th and 95th percentile bands are reported. For the model described by (5) and the VAR model, the median response is reported (red marked line and black dotted line respectively).

Responses for the period 1985 - 2016

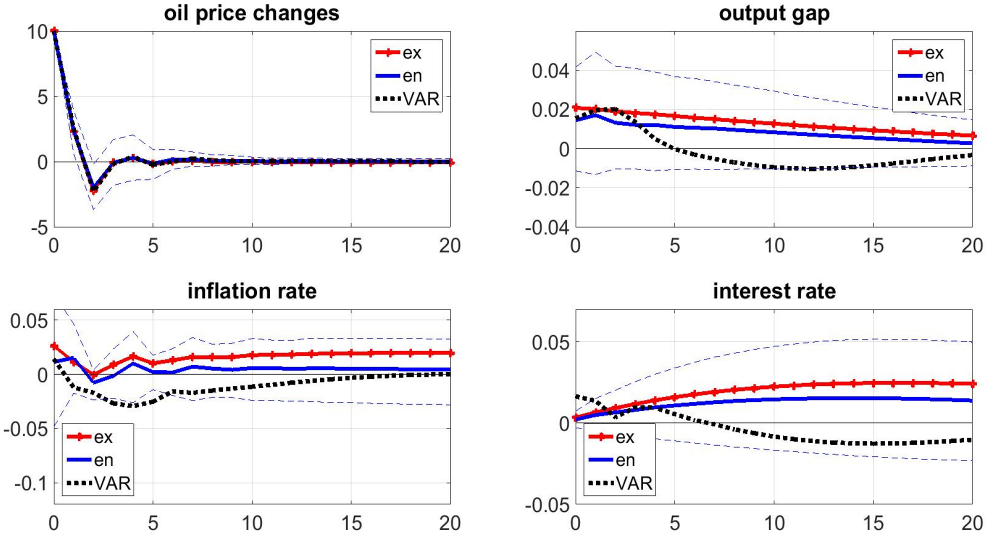

Figure 3: Responses of the variables to an exogenous change in oil prices normalized to increase s_t by 10%. For the model described by (6), the median response (blue continuous line) and the 5th and 95th percentile bands are reported. For the model described by (5) and the VAR model, the median response is reported (red marked line and black dotted line respectively).

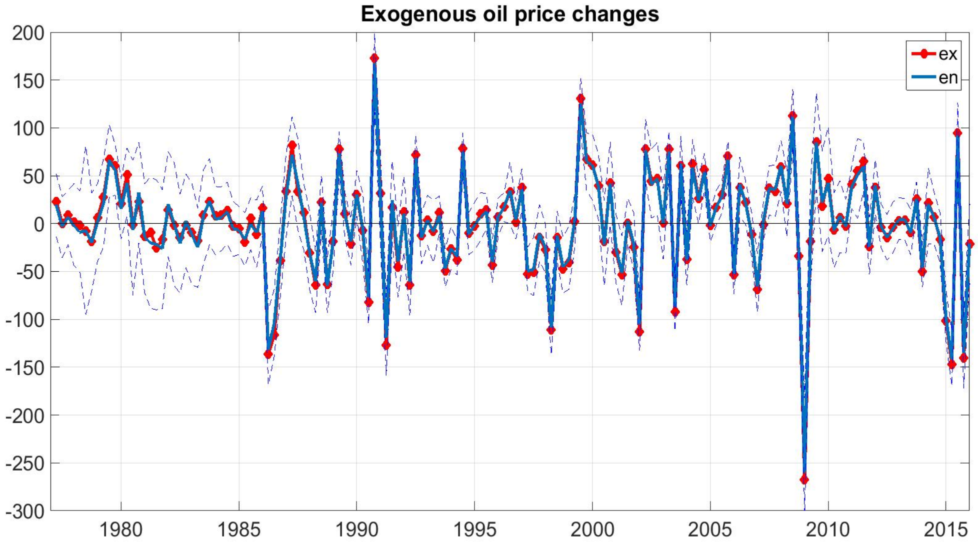

Figure 4: The exogenous component of the forecast error in the oil equation (4), obtained from (5) (red line), and (6) (blue line). The error bands are the 5th and 95th percentiles for (6).

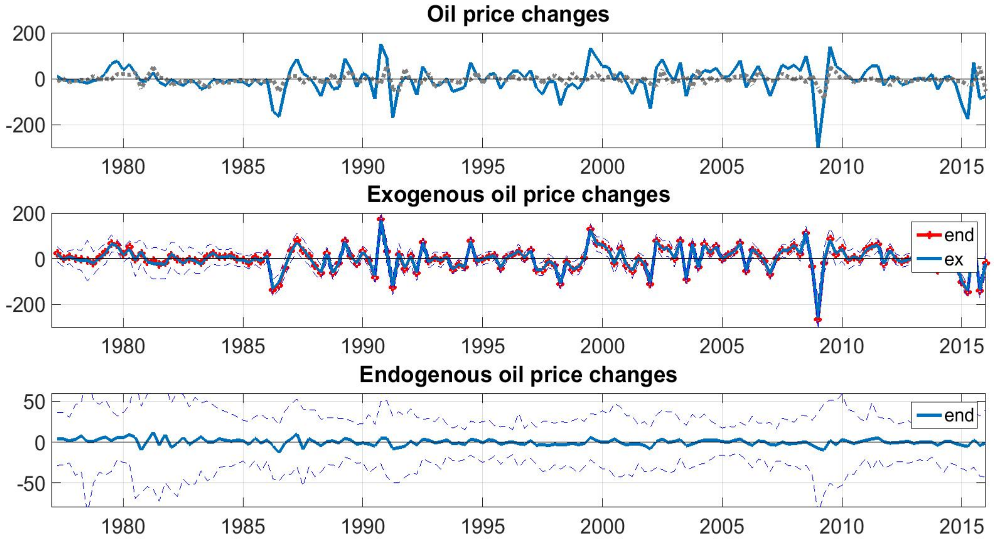

Figure 5: Shocks analysis for the period 1977-2016. The top panel reports the data for s_t (blue continuous line) and $\widehat{s_t}$ (grey dotted line). The middle panel reports the same information as in figure 4, i.e. the exogenous component of the forecast error in the oil equation (4), under both assumption (5) and assumption (6). The bottom panel reports the endogenous component of the forecast error in the oil equation, obtained from assumption (6).