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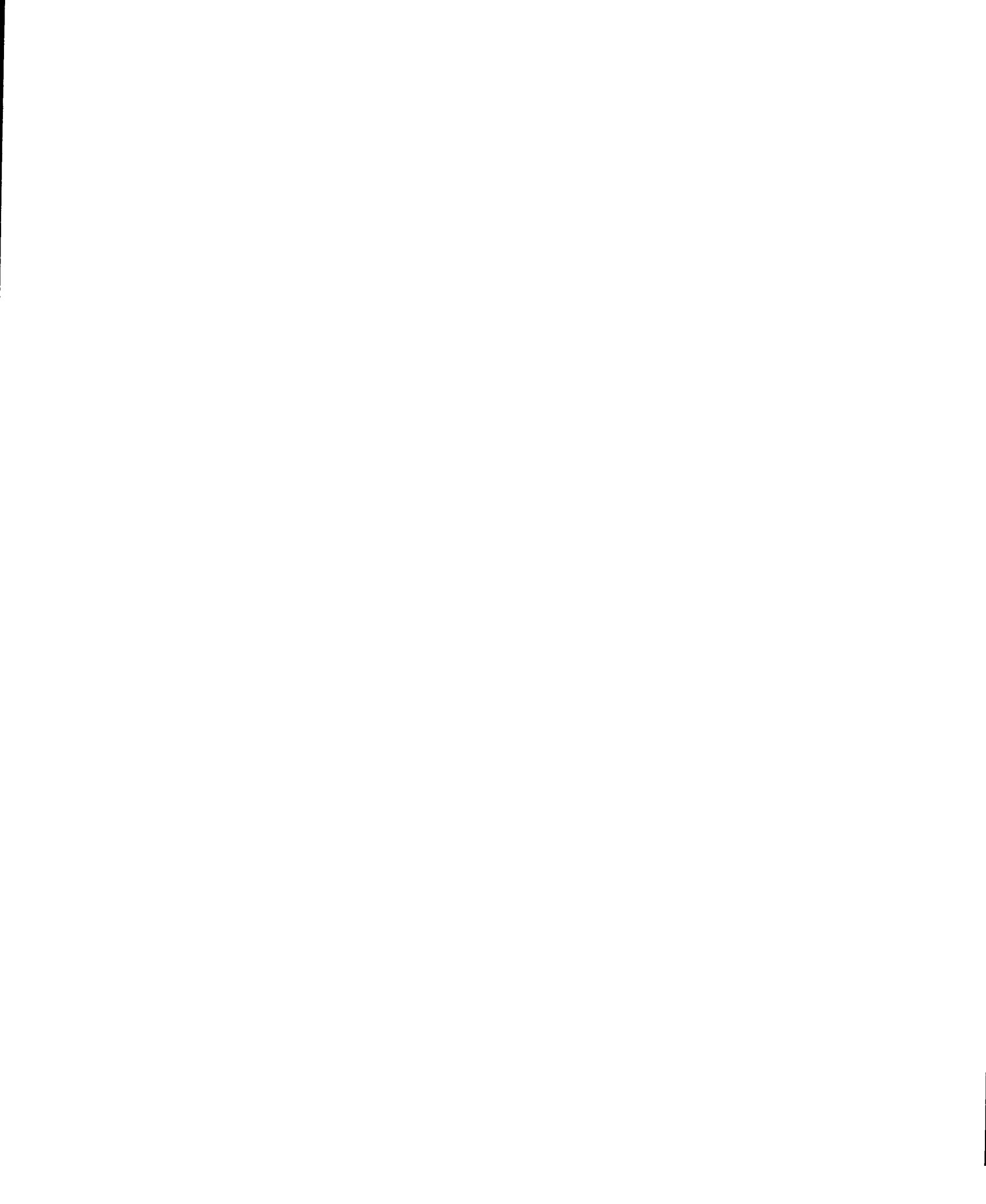
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EMPIRICAL COMPARISON ON PERFORMANCE OF RYAN AND RITCHKEN
BOUND THEORIES AND ASSESSMENT ON PRICING BIAS

by

WEIMIN SUN

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF BUSINESS ADMINISTRATION

in

THE FACULTY OF ADMINISTRATION

We accept this thesis as conforming

to the required standard

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THE UNIVERSITY OF OTTAWA

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Abstract

The focus of our empirical assessment is on studying two bound theories proposed by Ritchken (1985) and Ryan (1997). The study embarks on two tasks. One task is to answer how much tighter Ryan bounds are. The second is to propose a method to detect the pricing bias problem for a bound theory.

The test data set consists of 1910 samples recorded at 9:00am, 11:00am and 2:00pm Central Time at Chicago Board Options Exchange, in total, 20875 S&P 500 index call options. Analysis with our test data set shows that by adding in an additional constraint, Ryan bounds are approximately 22% tighter than Ritchken bounds. The empirical analysis also shows that the proposed method has successfully detected the pricing bias problem for the Ritchken's theory when a lognormal distribution is assumed.

1. Introduction

The name of derivative originates from a fact that its value is derived from the value of another asset commonly called the underlying asset. Since the seventies derivative products have become popular investment tools. Currently trillions worth of dollars of derivative products are traded very day. Investors use derivatives to hedge against risks, to speculate or to arbitrage. The class of derivatives is wide. Examples of its members are options and exotic options, futures and forwards, innovations, structured products, and option-like instruments such as convertible bonds and warrants.

Options along with their exotic variants are the best known examples of derivatives. Derivatives have been around for a long time. One early example is the so-called “Cotton Bond,” (Bhansali 1998) used by the Confederate States of America to raise desperately needed money for the Civil War. Holders of the “Cotton Bond” had the choice either to receive the loan in 40 semi-annual instalments in Paris, London, Amsterdam, or Frankfurt, at their option or cotton at the rate of sixpence per pound.

In ancient China, a marriage agreement for unborn children between two rich families was often arranged as a way to guarantee an affluent future for their children. (The actual agreement was that if both children were born to be the same sex, they became brothers or sisters depending on the sex. If they were born to be the opposite sex, a match was made.) Either family had the right to annul the agreement if the other family went to bankruptcy.

There are two basic types of options, call options and put options. Each type of option has two variants, American option and European option. A European call option allows its holder a right to buy the underlying asset at a predetermined price (called the strike price) at a maturing date. A European put option allows the holder to sell the asset at a strike price. On the other hand, an American option allows a holder to exercise the option at any time prior to or at the maturing date.

Pricing an option is difficult. It took more than a decade for financial economists to progress from the CAPM model to a workable solution for pricing an option. Black-Scholes (hereafter B-S) in their 1973 landmark paper first derived a partial differential equation (PDE) that must be satisfied by the price of an European call option contingent on an underlying asset, which pays no cash or incurs no carrying costs. The derivation of the PDE relies on the fact that at maturity the payoff of an option is perfectly correlated with value of its underlying asset. A portfolio, which consists of the underlying asset and treasury bills, can be constructed to replicate the same payoffs of the option. By subtracting the option from the portfolio, the resulted payoffs become zero. Discounting the riskless payoffs with a riskless interest rate leads to the B-S PDE. The above derivation requires two important assumptions. First, the underlying asset must be tradable so that the arbitrage-free argument is applicable. Second, one has to continuously adjust the portfolio in order to maintain a perfect replication.

The B-S PDE and its variants can be solved with proper boundary conditions. However, a closed form solution is often difficult to obtain. Frequently a numeric solution is necessary. Black and Scholes took an ingenious way to solve their equation. They observed that the B-S PDE did not contain any risk preference parameters (such as the expected return of the underlying asset). As a result, the solution of their PDE does not change if the expected return of the underlying asset is replaced with a riskless rate. In other words, one can solve the equation in a world where investors have no risk preference. This idea leads to a powerful option pricing tool, the risk-neutral valuation method. The risk-neutral method significantly simplifies the job of solving the B-S PDE, and a closed form is easily derived.

In spite of its mathematical elegance, the B-S theory is difficult for option traders and financial students to comprehend because of the mathematical subtlety involved. Technical terms such as “Stochastic Calculus” or “Diffusion Process” prevent mathematically less sophisticated people from gaining insights to the underlying ideas of the theory. Cox, Ross and Rubinstein (1979) proposed a discrete version of the B-S theory, known as the binomial tree approach. The binomial tree method provides an easy way for people to follow. In addition, the method is more flexible to accommodate further improvements. The central limit theorem (CLT) in probability theory links the binomial tree method with B-S formula. CLT claims that sum of n independent and identically distributed random variables asymptotically goes to a normal distribution as n goes to infinity.

The binomial tree method divides the life period, $[0, T]$, of an option into n equal intervals. Within each interval, price of the underlying asset moves either upward or downward. The percentage of upward movement within each interval is assumed to be the same. Similarly, the downward movements are assumed to be the same. The log-return of the underlying asset at time T is a sum of n bivalent random variables, all independent and identically distributed. By CLT, the sum goes to a normal distribution, provided that percentages of the upward and downward movement as well as the associated probabilities are properly chosen. As a result, solution of the binomial tree converges to the B-S formula as n goes to infinity. The B-S theory along with its binomial counterpart becomes a building block for pricing other options and derivatives in general.

The B-S formula requires several important assumptions. First, log-returns of underlying asset follow a Brownian motion. Second, the expected log-return of the underlying asset remains constant during the life of an option. Third, the volatility of the underlying asset keeps unchanged during the same period. Fourth, the market is perfect. Some of these assumptions may not be met in reality. It has been well known that the assumption of a constant volatility is not always true. Recently, the analysis with the post-crash index options of S&P 500 by Rubinstein (1994) confirmed the above claim. By inversely applying the B-S formula, one can solve for the volatility (called implied volatility) basing on the observed price of an option. In his analysis, Rubinstein showed the strong dependence between implied volatility of the coterminous options and their strike prices.

One such dependence is the post-crash smile, where implied volatility of deep in-the-money or out-of-the-money options is larger than the volatility of at-the-money options.

Rubinstein's analysis articulates the need to modify B-S theory, and to make it closer to the real world. In fact, since its publication in 1973, numerous efforts have been made to improve the theory. Consequently, many variants of the B-S theory are proposed. These variants mainly focus on corrections in three areas: corrections for non-continuous asset prices or the distribution of the asset returns, corrections for non-constant interest rates, and corrections for non-constant volatility.

The corrections for non-continuous asset prices are based on the fact that price change of a traded asset may sometimes be discontinuous. For example, arrival of new information such as a merger or acquisition announcement, a court decision and a newly landed contract often causes a sudden movement of a stock price. A recent infamous example is associated with Bre-X. When news of a possible fraud was made known to the public, the stock price of Bre-X plummeted more than eighty percent on a single trading day. The Brownian motion assumed by B-S theory for returns of an underlying asset can not reflect these sudden movements. The jump diffusion model and, its special case, the pure jump model, are proposed by some authors to correct the deficiency.

The pure jump model is used to price derivative on underlying assets whose prices tend to drift unidirectionally, but occasionally exhibit sporadic price jumps in the opposite direction. Specifically, during any infinitesimal interval, Δt , the asset will jump in price

from S to Su ($u > 1$) with probability $\lambda \Delta t$ and fall to price $Se^{-w\Delta t}$ with probability $1 - \lambda \Delta t$. Equations for derivative prices under the pure jump model look similar to those found with the original binomial or B-S models. For example, the value of an European call option on an underlying security, S , with a strike price of X and time t remaining to expiry can be written as

$$C = S\Psi(x; y) - e^{-r} X\Psi(x; y/u),$$

where r is the risk-free interest rate, x and y are constants that are functions of r , w , X and S , and $\Psi(x, y)$ is the complementary Poisson distribution function. J.C. Cox etc (1979) derived a similar expression by modelling the security price as tending to drift upward at a continuous rate of w , but allowing for sporadic downward price jumps to Sd with probability $\lambda \Delta t$ for time intervals of length Δt . Page and Sanders (1986) have studied the case where the security price usually remains unchanged but can jump from S to Su or Sd . Perrakis (1988) first derived a pair of bounds under a trinomial model, and then proved that the bounds converged to two distinct values as the stock returns asymptotically went to a jump process.

The idea behind the jump diffusion model is that changes in an underlying asset price can be divided into two distinct forms. Most common are small, continuous changes that are attributed to supply and demand imbalances or new information that causes only marginal adjustments to price. These continuous price changes are modelled by geometric Brownian motion with constant volatility. In addition, there will occasionally be large jumps in asset price due to situations such as the release of new and vital information. These price changes are modelled by jump processes and their causes are

referred to as Poisson processes (Merton 1976). The derivation of an equation for the price of a derivative contingent upon an underlying asset, S , described by such a process is complicated by several factors (Merton 1971). These factors lead to an expression for the derivative price, which is a nonlinear function of S . It is this nonlinearity that leads to additional problems with forming a riskless portfolio.

Ritchey (1990) and Guo (1997, 1995) proposed to use a finite mixture of lognormal distributions to replace the single lognormal distribution assumed by B-S model. These authors argued that empirical studies (for example, Rubinstein 1994) did not support a single lognormal prior. Instead, the recovered risk-neutral distributions (based on observed data) have positive skewness, significant kurtosis, fat left tail, and multiple modes. The observation leads the above authors to using a mixture of lognormal distributions.

The model used most often to correct for non-constant interest rates is Merton's stochastic interest rate model. Merton (1973) introduced a new assumption. He assumed that riskless rate followed a geometric Brownian motion. By incorporating a stochastic interest rate process into the B-S model, Merton derived a simple and elegant solution. He proved that in the presence of a stochastic interest rate, the B-S theory was still valid after some minor modifications (Hull 1997). The modifications are:

- Replacing the instantaneous interest rate, r , with the rate of interest on a riskless bond maturing at the same time as the option.

- Replacing the volatility, σ , with the average of $\{Var[S_t - P(t, T)]\}^{0.5}$ over $[t, T]$. Here, S_t is the price of an underlying asset at time t , and $P(t, T)$, price of a bond at time t with maturing date at T .

Other approaches of dealing with non-constant interest rates include the implied trees (Rubinstein 1994) and Heston's model (1993). Both models can handle non-constant volatility too.

The problem of non-constant volatility is thoroughly documented in literature and the volatility smile is a well-known example of the problem. The issue attracts the attention of many researchers and many articles on various methods to correct the problem are published. Several of these methods are summarised below.

Some authors tackled the problem by exploring the association between price of an underlying asset and its volatility. Cox and Ross (1976) suggested a model by taking the idea that stock price and the volatility of the stock returns should be negatively correlated. The displaced diffusion model (Rubinstein 1983) explored the structure of a balance sheet to price an option. One shortcoming of Rubinstein model is that it implies a positive correlation between the volatility and the stock price. Both economic and empirical evidences imply a negative correlation. In his formulation, Rubenstein (1983) assumed that the asset of a firm consisted of a risky asset and a riskless asset, and he derived a B-S formula based solution. Guo and Ryan (1996) extended the Rubenstein theory, and permitted two risky assets with one asset being less risky.

Another approach of dealing with non-constant volatility is to assume a stochastic volatility. Numerous pricing models that treat the volatility as a stochastic variable have been proposed over the past decade. The introduction of a second stochastic process presents two difficulties. First, the differential equation governing the price of a derivative will involve the correlation between the process governing the volatility and the process describing the price of the underlying security. Many of the models take this correlation to be zero in order to greatly simplify the PDE that needs to be solved. To date, there is no known analytic solution to the general problem where the correlation is nonzero, but many tractable numerical evaluation schemes have been proposed to treat these cases. The second difficulty presented by these models is related to the risk-neutral valuation concept. When the volatility of the underlying asset varies stochastically, it is not possible to construct a riskless portfolio to imitate the derivation of B-S formula. Scott (1987) got around the problem by formulating the model in terms of ξ , the risk premium associated with the stochastic volatility. When ξ is taken to be zero, simple models are derived. Another method of dodging the risk-neutral valuation problem is to assume the existence of an asset that is instantaneously perfectly correlated with the price of the asset of interest (Johnson and Shanno 1987).

The Hull and White (1987) model is unique among the stochastic volatility models because it leads to a relatively simple analytic expression for the price of a derivative. It is also interesting because their derivation is the only one that uses an obscure equation presented by Garman (1976). Heston (1993) and many other authors did not use

Garman's equation to derive the PDE that governs changes in derivative prices. One can use the traditional B-S analysis with risk-neutral valuation to come up with a PDE analogous to that presented by Hull and White (1987) by simply assigning a "price of volatility risk" that corresponds to the variable ξ . Other papers dealing with the topic of stochastic volatility include Merton (1976), Geske (1979), J. B. Wiggins (1987) and Stein & Stein (1991).

Other authors used time series models to model volatility. A well-known model is the generalised autoregressive conditional heteroskedastic (GARCH, Bollerslev 1986) model. GARCH is inspired by a time series model called autoregressive integrated moving average (ARIMA) model. ARIMA models a stationary time series, whose unconditional variances are identical. While maintaining a constant unconditional variance, the GARCH model allows the conditional variance at time t to be a function of previous values. GARCH generalises ARCH in the same way as ARMA extending MA models; that is, GARCH adds an autoregressive component into its formulation.

B-S formula and other theories mentioned above offer a single price for an option traded at a perfect market. Some researchers proposed to derive a pair of prices, and the true price of an option should fall within these two prices. Call the larger price upper bound, and the smaller price, the lower bound. In a real market, the quoted bid-ask prices should fall within these bounds. Violation of the requirement could theoretically mean an arbitrage opportunity.

Section 2 summarises two theories of pricing option with bounds proposed by Ritchken (1985) and Ryan (1997). The subsequent two sections empirically assess the theories. Section 3 uses a test data set consisting of 1910 call option samples for evaluating the improvement of Ryan bounds over Ritchken bounds. Section 4 examines Ritchken theory for the pricing bias problem. Since the two theories share the same idea, results in section 4 are also applicable to Ryan theory. In section 5, we draw some conclusions, and discuss potential improvements for our analyses.

2. The Upper and Lower Pricing Bounds

2.1 Review of Theories for Pricing Options with Bounds

The B-S theory along with its variants are designed to work in a perfect market. Here, a perfect market refers to a market where there are no trade costs or commissions (therefore no bid-ask prices), no arbitrage opportunities, no tax implications. A perfect market also assumes that investors can borrow and lend money at the same rate. Furthermore, the derivation of B-S formula relies on the fact that, with continuous trading opportunities, the payoffs of the option can be perfectly replicated by a self-financing strategy involving continual revisions in a portfolio consisting of riskless bond and the underlying asset.

However, a real market is, in fact, imperfect since trading often involves significant costs. It is obvious that even a negligible transaction cost occurs during a revision of the portfolio within an infinitesimal interval, and the aggregated cost tends to infinity as the number of intervals goes to infinity. In other words, continuous revision is infeasible in reality. In addition, when the underlying asset is thinly traded such as in some Canadian stock markets, only a finite number of revisions can be made to the portfolio. Similarly, if the underlying asset is a physical asset like a building or a car, trading on the physical asset rarely takes place prior to the expiry of a contingent claim on the asset. The assumption of borrowing and lending at the same rate presents another problem. Commonly, investors have to borrow money at a higher rate, and lend at a lower rate.

The problems existing in an imperfect market propelled a group of researchers seeking alternative ways to the B-S and binomial option models. For instance, Bergman (1995) studied the case when the borrowing and lending rates were different. He formulated the option pricing bounds as the solutions of an optimal problem subject to three constraints, and derived upper and lower bounds to price an option instead of giving a single price. The review in this section, however, mainly focuses on the recent studies in dealing with the problem of having only a finite number of revision opportunities, as these results are closely related to the bound theories that are investigated in the rest of the document.

In their studies, many papers discussed below share some common features. First, many of them derived their bounds with a discrete time model. Using a discrete model has the advantage of being able to incorporate taxes, transactions and commission costs etc. When there is a discrete number of trading opportunities, some authors explored these opportunities to make revisions to, for example, the arbitrage portfolio. Their theories are then upgraded from single-period theories to multiple-period theories. Because of the additional revision opportunities, multiple-period theories generally yield tighter bounds than bounds implied by single-period theories. Some multiple-period theory papers further studied the limiting behaviour of their bounds when the number of infinitesimal intervals goes to infinity. Many papers also assumed the return of an underlying asset could only take a finite number of states at the end of an elementary time period. The solutions for the bounds of a continuous distribution can, then, be derived by letting the number of states go to infinity. In deriving the bounds, the papers discussed by us used

mainly one of three methods: (1) stochastic dominance, (2) arbitrage portfolios, and (3) linear or nonlinear programming.

Deriving from general arbitrage considerations, Merton (1973) gave a pair of upper and lower bounds, in which the value of an option should lie. Merton's derivation does not assume any form of distribution for the underlying asset, and, therefore, his bounds are wider than the bounds presented by the authors mentioned below. By assuming a known form of distribution for the asset return along with a set of mild assumptions, Perrakis and Ryan (1984 hereafter PR) obtained a tighter lower bound than that of Merton. Perrakis and Ryan (1984) first derived their bounds without allowing any revision during the life of an option (a single-period model), and, then, extended the results to allow for a finite number of revision opportunities (a multiple-period model). The PR bounds are adjustable for commissions and dividends.

Levy (1985) explored stochastic dominance approach in deriving his bounds. He obtained his bounds under the rule of first order stochastic dominance and second stochastic dominance as well. A disadvantage of Levy's theory is its difficulty in tightening the bounds by adopting a multiple-period model.

Following footsteps of Perrakis and Ryan (1984), Ritchken (1985) yielded tighter bounds than PR. He adopted a linear programming approach. He mainly assumed two conditions: (i) investor prefers more wealth to less; (ii) investors are risk averse. Ritchken (1985) did not require the asset return distribution to be known. Under

condition (i) only, Ritchken's (1985) lower bound is the same as Merton's lower bound, and smaller upper bound than that of Merton's. Under the additional condition (ii) and on state probabilities, tighter bounds are developed. Like Levy (1985), Ritchken (1985) adopted a single-period model. Ryan (1997) followed the same formulation of Ritchken (1985), and introduced an additional constraint, which reflects the information from a coterminous option. His bounds are intuitively tighter than those of Ritchken (1985) because of the additional constraint.

Perrakis (1986) generalised and significantly tightened the PR bounds for European options. He developed similar bounds for American puts. Like PR, Perrakis (1986) used the arbitrage portfolio approach and derived the bounds under both the single- and multiple-period models. Furthermore, Perrakis (1986) proved that the new, improved bounds converged (as number of periods goes to infinity) to a single value as in Cox et al. (1979) and Rendleman & Barter (1979) when the single-period distribution is binomial. Thus, Perrakis (1986) showed his bounds to be a valid generalisation of the two-state option models in Cox et al. (1979) and Rendleman & Barter (1979).

Ritchken and Kuo (1988) generalised the single-period linear-programming bounds to multiple-period linear-programming bounds. Initially, they assumed a multiplicative multinomial asset return distribution and derived tighter bounds. When the process is binomial, then it turns out that the upper and lower are the same, and the bound is equal to the value of binomial option-pricing model. Under restrictive assumptions on

probabilities and risk aversion, their upper bounds were shown to be identical to those of Perrakis (1986), while the lower bounds are generally higher.

By assuming a trinomial distribution, Perrakis (1988) adopted the linear programming approach to yield upper and lower bounds. He identified four types of relationship among the three discount factors (see next section for the explanation of discount factors), and derived bounds for each type. Like his previous papers, the model used here is multiple-period. He investigated the limiting behaviour of the bounds as the length of time between two successive trades goes to zero. When the trinomial distribution goes to a diffusion process, under the assumption of monotone ordering for the discount factors he showed that his upper and lower bounds converged to the same B-S value. In contrast, under a jump process both bounds do not converge to a single price.

Ritchken and Kuo (1989) provided option bounds under higher order stochastic dominance. By retaining the linear programming approach, They showed that the lower bound under third order was larger than the second order lower bound, and the upper bound remains the same as the second order upper bound. When moving to n^{th} order dominance, the upper bound is still unchanged, and the lower bound improves. Under a nonlinear programming model, they derived the decreasing absolute risk aversion (DARA) bounds. DARA implies that the investor's willingness to engage in small gambles of a fixed size increases with wealth. Although no simple analytical solution is available to the DARA bounds, Ritchken and Kuo (1989) showed that the DARA upper bound was the same as the third order dominance upper bound, and the lower bound is

tighter than the third order dominance lower bound. The theory of Ritchken and Kuo (1989) is a single-period theory.

Perrakis (1993) extended results of Perrakis (1988) from the trinomial asset return distribution to a multinomial distribution, and examined the behaviour of these bounds in the continuous trading case for both the diffusion and jump process. The paper showed that both the n -period upper and lower bounds tended to the B-S option price under the diffusion process while the sufficient condition was the monotone (discount factor) ordering assumption. Under the jump process, the paper concluded that the bounds converged to two distinct values that bracket both the B-S and Merton option prices.

Other related papers in the area include Levy and Levy (1988) that derived results under a multinomial distribution. Madan et al. (1989) and Naik & Lee (1990) study the outcomes under jump and mixed processes. Basing on the binomial model, Boyle & Vorst (1992) and Merton (1989) investigated bounds when transaction costs are explicitly introduced. Leland (1985) incorporated the transactions under a B-S model. Cochrane and Saá-Requejo (1998) developed their theory in a multiple-period context, and derived, in a limit case, a set of PDEs satisfied by bounds.

Since the empirical study on comparing performance described in this document involves the bounds derived by Ritchken (1985) and Ryan (1997) only, the two subsections below outline the bounds givens by these two papers.

2.2 Ritchken's Bounds

Ritchken (1985) assumed the support for possible values of an underlying asset at time, T , is a finite set of nonnegative real numbers. Here, T is the maturity date of an option. Ritchken (1985) developed his theory by treating the time from now to T as a single period.

Denote values of the underlying asset, in an ascending order, $\{s_1, \dots, s_n\}$ with the corresponding probabilities of occurrence, $\{\pi_1, \dots, \pi_n\}$. Associated with each state, s_j , is a discount factor, d_j . To understand the meaning of discount factors, let us restate a fact shown by Rubinstein (1976) and Brennan (1979). They proved that the value of a contingent claim $h(Z)$ on the asset was given by the expectation, $E(Y(Z)h(Z)/(1 + r))$, where r is the instantaneous riskless rate, and Z , the random price change of the asset per period. Then $Y(Z)/(1 + r)$ are the state-contingent discount factors, assuming Z takes discrete values. (The discussion here is also valid for continuous Z .) According to Rubinstein (1976), $Y(Z) = E(U'(c) | Z = z) / E(U'(c))$, where $U'(c)$ is the marginal utility for consumption of a representative investor. In other words, the discount factor can be explained as the conditional marginal utility of consumption of a representative investor, normalised by the expectation of the marginal utility. By assuming investors prefer more wealth to less, we require d_j to be nonnegative. Other useful notations are:

- $S_0 (S_T)$: the current (end of period) underlying asset price
- $C_0 (C_T)$: the current (end of period) option price

- X : strike price
- $B_0 = \exp(-rt)$

Under the assumption of one period and no intermediate dividends, the law of one price concludes the following arbitrage-free equation,

$$S_0 = \sum_{j=1}^n \pi_j s_j d_j. \quad (1)$$

Let B_0 be the current value of a portfolio consisting of all state securities (i.e. a riskless bond), then

$$B_0 = \sum_{j=1}^n \pi_j d_j, \quad e_j \geq 0. \quad (2)$$

Since the portfolio is riskless, $B_0 = \exp(-rt)$. In addition, let $\{c_1, \dots, c_n\}$ denote the payouts of an option at time T . Ritchken formulated the derivation of the lower (upper) bound of an option as a linear programming problem; that is, find

$$\min (\max) C = \sum_{j=1}^n \pi_j d_j c_j$$

subject to (1) and (2).

To achieve tighter bounds, Ritchken introduced another constraint by assuming investors are risk averse. The new assumption implies that d_j is monotonically decreasing in j . Define $x_j = d_j - d_{j-1} \geq 0$, and let y_j be x_j times the sum of π_i over $i = 1, \dots, j$. Then, with the new constraint, the lower and upper bound of an option price is given by solutions of the following linear programming problem:

$$\min (\max) C = \sum_{j=1}^n y_j \bar{c}_j \quad P1(P2)$$

subject to:

$$B_0 = \sum_{j=1}^n y_j, \quad S_0 = \sum_{j=1}^n \bar{s}_j y_j, \quad y_j \geq 0$$

where

$$\bar{s}_j = \sum_{i=1}^j \pi_i s_i / \sum_{i=1}^j \pi_i = E(\tilde{S}_T | \tilde{S}_T \leq s_j), \text{ and}$$

$$\bar{c}_j = \sum_{i=1}^j \pi_i c_i / \sum_{i=1}^j \pi_i = E(\tilde{C}_T | \tilde{S}_T \leq s_j).$$

Proposition 3 of Ritchken (1985) states:

- (i) The optimal solution to the maximization problem given by P1 (P2) is:

$$C_{\max} = E(\tilde{C}_T)S_0 / E(\tilde{S}_T) \quad (3)$$

- (ii) The optimal solution to the minimization problem given by P1 (P2) is:

$$C_{\min} = \frac{(\bar{c}_{j^*} \bar{s}_{j^*+1} - \bar{c}_{j^*+1} \bar{s}_{j^*})B_0}{\bar{s}_{j^*+1} - \bar{s}_{j^*}} + \frac{(\bar{c}_{j^*+1} - \bar{c}_{j^*})S_0}{\bar{s}_{j^*+1} - \bar{s}_{j^*}} \quad (4)$$

where S_{j^*} is chosen such that

$$\bar{s}_{j^*} \leq S_0 / B_0 < \bar{s}_{j^*+1}.$$

- (iii) For the continuous state problem, the form of the upper bound remains the same and the lower bound converges to:

$$C_{\min} = E(\tilde{C}_T | \tilde{S}_T \leq s_{j^*})B_0 \quad (5)$$

where to find s_{j^*} , solve

$$E(\tilde{S}_T | \tilde{S}_T \leq s_{j^*}) = S_0 / B_0.$$

2.3 Ryan's Bounds

Ryan (1997) derived tighter bounds than those of Ritchken by introducing an additional constraint. He pointed out that *"In the case of standardised market-traded options, the procedures for issuance of options will usually cause the co-existence of at least two calls on the same stock with different exercise prices and the same maturity. Currently, traders are likely to react to the pricing of these coexisting options by a Black-Scholes implicit variance approach to determine relative overpricing between the option alternatives."* Therefore, he concluded that the option pricing should reflect the prices of coterminous options.

Ryan examined the case when only one coterminous option existed. He adopted a graphic method to derive the bounds for an option. The main result of Ryan (1997) is summarised below for completeness.

Assume we wish to derive bounds for a call (C^1) with exercise price X^1 . A coexisting option (C^2) is priced at C_0 with strike price, X^2 . Option pricing bounds of C^1 are solutions of the following linear programming problem in the presence of investor risk aversion,

$$\min (\max) C = \sum_{j=i}^n y_j \bar{c}_j^1 \quad P3(P4)$$

subject to:

$$B_0 = \sum_{j=1}^n y_j, \quad S_0 = \sum_{j=1}^n \bar{s}_j y_j, \quad C_0 = \sum_{j=1}^n \bar{c}_j^2 y_j, \quad y_j \geq 0$$

where all notations are defined similarly as before. The only difference is the superscripts. The variables with superscript one refer to characteristics of the call option to be priced and superscript two, to characteristics of the coexisting call option.

To present solutions of P3(P4), we need the following notations,

$$\begin{aligned} \rho &= E(\tilde{S}_T / S_0) & \rho_l &= E(\tilde{S}_T / S_0 \mid \tilde{S}_T \leq s_l) \\ \rho^1 &= E(\tilde{C}_T^1 / C^*) & \rho_l^1 &= E(\tilde{C}_T^1 / C^* \mid \tilde{S}_T \leq s_l) \\ \rho^2 &= E(\tilde{C}_T^2 / C_0) & \rho_l^2 &= E(\tilde{C}_T^2 / C_0 \mid \tilde{S}_T \leq s_l) \end{aligned}$$

In addition, $r=1/B_0$. $C^* = \min C^l$ is the price of C^l at its revised lower bound. Theorem 4 of Ryan (1997) concluded that

- i) For $X^l < X^2$, the minimum price of C^l is determined by taking conditional expectations over the range $[0, s_l]$ defined by equating risk premium ratios:

$$\frac{\rho_l - r}{\rho - r} = \frac{\rho_l^2 - r}{\rho^2 - r} = \frac{\rho_l^1 - r}{\rho^1 - r}$$

The lower bound is defined as:

$$C^* = \frac{1}{r} \left(\frac{\rho^2 - r}{\rho^2 - \rho_l^2} E(\tilde{C}_T^1 \mid \tilde{S}_T \leq s_l) - \frac{\rho_l^2 - r}{\rho^2 - \rho_l^2} E(\tilde{C}_T^2) \right). \quad (6)$$

- ii) For $X^1 < X^2$, the maximum price of C^1 is determined by taking conditional expectations over the range $[0, s_k]$, where s_k is defined by equating returns:

$$\rho_k = \rho_k^2 = \rho_k^1.$$

The upper bound is defined as:

$$C^* = \frac{1}{\rho_k} E(\bar{C}_T^1 | \bar{S}_T \leq s_k). \quad (7)$$

- iii) For $X^1 > X^2$, the bounds are reversed.

3. Comparing Bounds of Ritchken and Ryan

The discussion in section 2 implies that with the addition of a new constraint, Ryan bounds should be heuristically tighter than Ritchken bounds. The reason is clear. The addition of a new constraint shrinks the set, to which the discount factor belongs. Since the optimisation algorithm searches a smaller set for optimal solutions of P3(P4), the upper bound of P3(P4) becomes smaller, and lower bound, larger. In other words, the spread of bounds solved for P3(P4) is smaller than that for P1(P2). However, the heuristic argument does not give a quantitative value of the reduction. The focus of this section is to use a real data set to evaluate the quantitative reduction.

This section first describes the test data set, and, then, discusses estimates of parameters used for the lognormal distribution in our analysis. Since identification of s_j , s_b , s_k for equation (3), (5), (6) and (7) plays an important role in calculating Ryan and Ritchken bounds, an ad hoc search procedure is described in details. Due to poor quality of the test data set, further checking is made to eliminate problematic samples, which violate Ryan's arbitrage bounds. At the end, main results of the analysis are described.

3.1 The Test Data Set

The initial data set consists of all intraday S&P 500 index call option samples observed at 9:00, 11:00 and 14:00 Central Time in 1993. The observed option prices are taken as the mean of the ask and bid prices. The S&P 500 index is traded at Chicago Board Options

Exchange. Originally, there are 2,477 samples. After checking and deleting for vertical and butterfly spread arbitrage violation (convexity violation), only 1910 sample are left (see Guo 1997 for more details). This is the test data set used for the analysis below.

The number of options embedded in each sample varies from 3 to 26. In total, there are 20875 call options. Each call option contains information of bid-ask prices, time, strike price, risk free interest rate, and the S&P 500 index level at the sampled time. The risk free interest rate is extracted from the interest term structure since it is not directly observable.

To empirically assess the reduction, we need to find Ryan bounds. Ryan method calls for a pair of options. To fit this requirement, we choose two options from each sample. To make the empirical results cross samples comparable, we opt for choosing the two options, whose strike prices are closest to the recorded S&P 500 index level. Therefore, the final test data set used for the empirical study in this section comprises 1910 pairs of call options, which are at-the-money calls. Within each pair of call options, for simplicity we take the option with the smaller strike price as C^1 , and the other option as the coexisting option, C^2 . In the remaining of this section, without further explanation the bounds of C^1 are the study object, and the term “1910 options” specifically refers to the 1910 call options of C^1 .

3.2 Estimates of Model Parameters

Both Ritchken and Ryan theories do not assume a particular form of distribution for returns of an underlying asset. For the purpose of empirical evaluation, we have to assume a probability distribution. By following B-S theory, we choose a lognormal distribution. More specifically, if S_T denotes the index level at time T , S_T follows a lognormal distribution,

$$\ln S_T \sim \phi\left[\ln(S) + \left(r - \frac{\sigma^2}{2}\right)(T-t), \sigma\sqrt{T-t}\right], \quad (8)$$

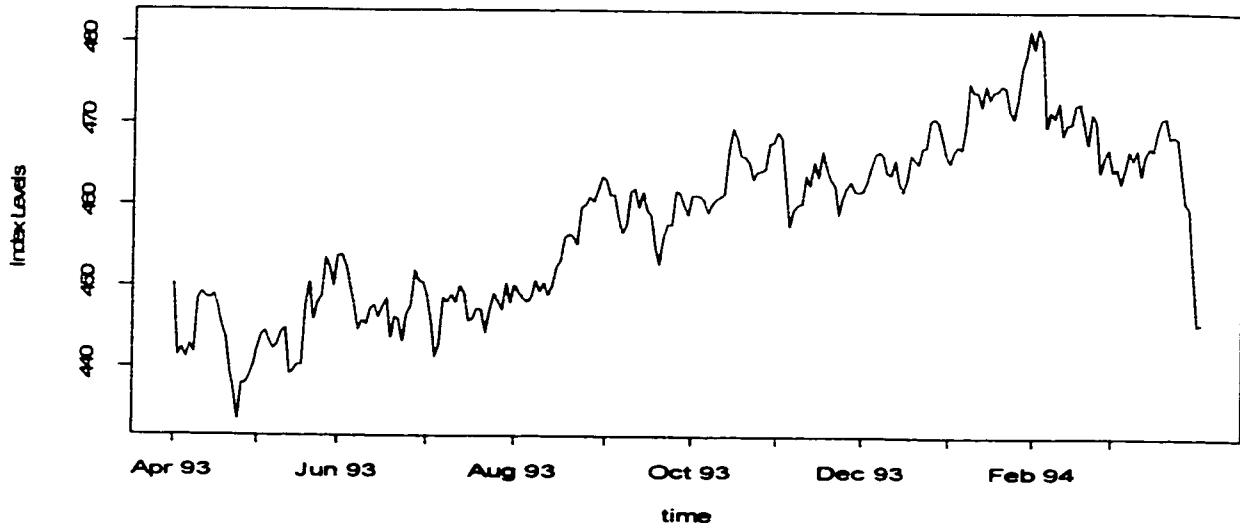
where r is the yearly riskless interest rate. S is the index level at the trading time t . T is the expiration time of an S&P 500 call option, and ϕ represents a normal distribution. In addition, σ is the volatility of the S&P 500 index. The above model says that theoretically $\ln(S_T/S)$ grows in average at a rate of $\mu = (r - \sigma^2/2)(T-t)$. The actual growth rate will be affected by dividend payments of the index. The S&P 500 index consists of a portfolio of many stocks and these stocks do not pay dividends at the same time. It is reasonable to assume the index pays a continuous dividend at a rate $q(t)$ (Rubinstein 1994). Like the B-S formula, we assume constant μ and σ during the life of an option.

To evaluate Ryan and Ritchken bounds under distribution (8), we need to know two parameters: μ and σ . These values are unknown. There are at least two ways to estimate them. First, with the observed option prices, which are the means of bid-ask prices, we can apply B-S formula to derive the implied volatility, σ . To find the mean, we set $\mu = (r - \sigma^2/2)(T-t)$. Using the implied estimates has two disadvantages. First, both the implied volatility and mean of S&P 500 index tend to overestimate the true mean and volatility (see Figure of “S&P 500 Implied and Historical Volatility Comparison”, Clark 1994, P. 39). Second, we have to solve a nonlinear equation.

The second method is to estimate the parameters empirically. We use daily closed S&P 500 index levels to estimate μ and σ . The purpose is to ensure that the empirical estimates can be applied to options sampled at different times. Originally, daily settled prices of the index are available (see NASDQ 1993-94) from April 1, 1993 to March 30, 1994. Based on this data set, the estimated mean of daily returns is negative. A close examination of the data set (see figure 1) reveals that the index experienced a correction during the February-March period of 1994. During the last nine months of 1993 the index exhibited an increasing pattern. The correction in early 1994 results in a negative mean. This correction, therefore, distorts the upward pattern showed by the index in 1993. There is a change point somewhere in early 1994. In other words, the distribution of the index returns in 1993 will not be the same as that in 1994. Since all sampled options are traded in 1993, their prices were affected by market's perception of the future index movement basing on the information in 1993. Thus, only the 1993 index levels are relevant. We drop index data in 1994, and use the daily settled index levels of nine-months in 1993 to estimate the mean and volatility.

With the nine month index data set, the annualised estimated mean return of S&P 500 index is 7.24%, and the empirical volatility, 9.725%. These two estimates are used as inputs of the lognormal distribution for our empirical study below.

Figure 1: Time Series of the Daily S&P 500 Settled Prices From April 93 To March 94
S&P500 Index Levels From April 1, 1993 To March 31, 1994



3.3 The Search Procedure

To make the empirical assessment on both sets of bounds as close to reality as possible, we should compare their performances in the continuous case. Equation (3) and (5) of section 2 give Ritchken bounds in the continuous case and equation (6) and (7) give Ryan bounds. These equations call for a search procedure to locate s_{j^*} , s_l , s_k . One likely search procedure is applying a numeric procedure, the Newton-Raphson method (J. Hull 1997), to solve for s_{j^*} , s_l , s_k . Some initial trials on the procedure showed that the algorithm was not stable in this application, and we have to carefully choose initial values in order to find right solutions. Therefore, we choose using an alternative search procedure to identify s_{j^*} , s_l , s_k .

For explaining the alternative procedure, suppose we need find s_l , which is the root for equating risk premium ratios (see section 2). Here is the rough idea of the procedure. First in the discrete case for a large enough n , suppose the set of possible index states at expiry of an option is $\{s_1, \dots, s_n\}$. Within the set $\{s_1, \dots, s_n\}$, we search for two consecutive values, a and b such that $|f(a)|$ and $|f(b)|$ are the minimal of $\{|f(s_1)|, \dots, |f(s_n)|\}$ subject to $f(a) \cdot f(b) < 0$, where,

$$f(s_l) = \frac{\rho_l - r}{\rho - r} - \frac{\rho_l^2 - r}{\rho^2 - r}.$$

Without loss of generality let $a < b$. Next, we choose an interpolation procedure to interpolate for s_l basing on a, b . For instance, a simple linear interpolation can be used to improve the result. When $f(a) < 0$ (i.e. $f(b) > 0$), the linear interpolator gives a solution by $f(b) - b(f(b)-f(a))/(b-a)$. If $f(a) > 0$, the interpolated solution should be $f(a) + a(f(b) - f(a))/(a-b)$. However, our test shows that when n is large, 5000 say, the interpolation step is not necessary. Thus, for simplicity, we choose either a or b , whichever makes $f(s_l)$ closest to zero, as an approximate root for s_l .

The next issue is how to make distribution (8) discrete in order to identify a and b . The problem is equivalent to how to choose s_1, \dots, s_n , and how to assign probabilities to them. We use the following ad hoc method.

For given n and a pre-determined positive value c , let $\ln(s_i) = \mu - q - c \cdot \sigma + h \cdot i$ for $i=1, 2, \dots, n$, where $h_i = (2 \cdot c \cdot \sigma) \cdot i$, and define $\ln(s_i) = 0$. Since, with almost probability one, a normal random variable takes a value within four standard deviations of the mean, a

value 4 for c should be sufficient. The probability associated with $\ln(s_i)$ is defined to be $P\{\ln(s_i) < \ln(S_T) < \ln(s_{i+1})\}$. Refinements or more formal procedures are available, but they are not needed for this analysis. Our test shows that the ad hoc method works well for the empirical assessment.

Before finding roots, s_l , s_k , s_{k^*} , of

$$\frac{\rho_l - r}{\rho - r} = \frac{\rho_l^2 - r}{\rho^2 - r}, \quad (3.1)$$

$$\rho_k = \rho_k^2, \quad (3.2)$$

$$E(\tilde{S}_T \mid \tilde{S}_T \leq s_{j^*}) = S_0 / B_0. \quad (3.3)$$

we should ensure the existence of nontrivial s_l , s_k , s_{k^*} . Here a nontrivial solution means a non-zero solution. The existence of the roots can be easily shown in a formal way. Fact 1 and 2 below give a heuristic proof for the existence of, s_k and s_{k^*} . (Proof for existence of s_l is not short and is not pursued here.) The facts are given for completeness of discussion in the next subsection.

Fact 1: Under assumptions

- Risk of an underlying asset is smaller than the option on it
- Investors are risk averse; that is, investors demand a higher return on more risky securities
- The strike price of an option is positive

Then, a nontrivial solution, s_k , of equation (3.2) exists.

Proof: Let

$$g(s_k) = \rho_k - \rho_k^2 = E(\tilde{S}_T / S_0 | \tilde{S}_T \leq s_k) - E(\tilde{C}_T / C_0 | \tilde{S}_T \leq s_k).$$

When $s_k = 0$, $g(s_k)$ is zero. As s_k increases slightly, the second term of $g(s_k)$ remains zero. On the other hand, the first term of $g(s_k)$ becomes positive. Consequently, $g(s_k)$ becomes positive. When $g(s_k)$ is infinite, the conditional returns are equivalent to unconditional returns. Because the risk of an option is higher than the underlying asset, under the assumption of risk aversion investors demand a higher return on option. In other words, $g(+\infty)$ is negative. Note that, $g(s_k)$ is continuous in s_k . Thus, we have proved the existence of s_k .

Fact 2: Under the following assumptions

- Investors are risk averse
- The underlying asset is a risky asset

Then, a nontrivial solution, s_{j^*} , for equation (3.3) exists.

Proof: Let

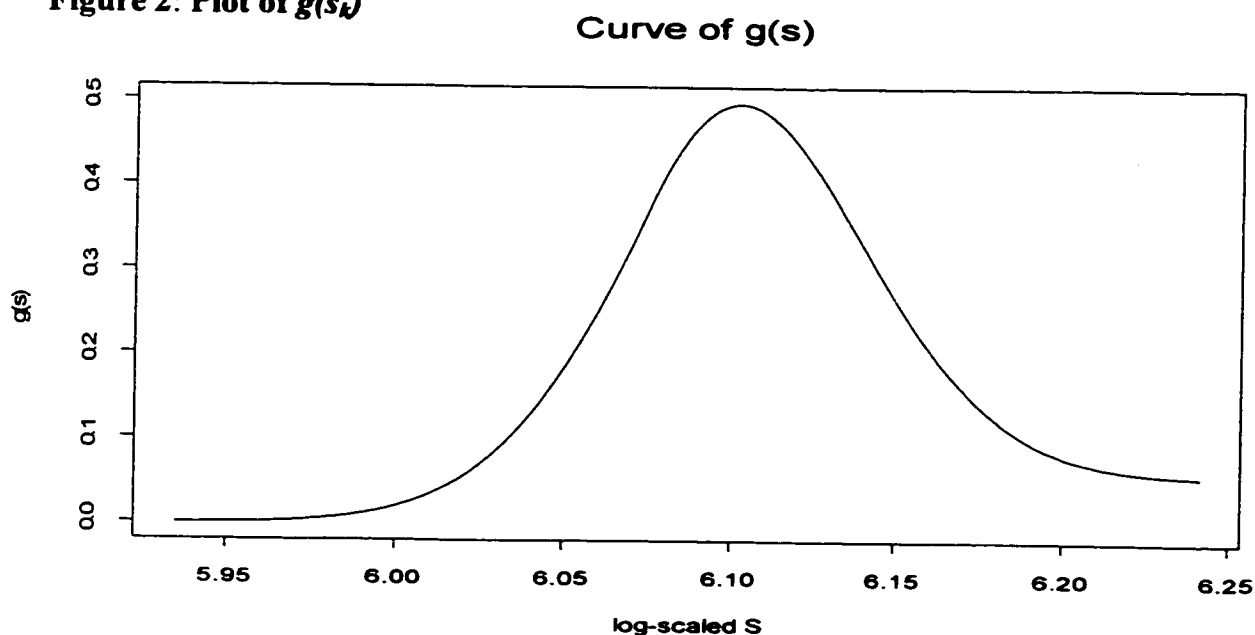
$$h(s_{j^*}) = E(\tilde{S}_T / S_0 | \tilde{S}_T \leq s_{j^*}) - 1 / B_0.$$

The solution for equation (3.3) is equivalent to the solution of $h(s_{j^*}) = 0$. Note that $h(0) = -1/B_0 < 0$. When the underlying asset is risky, under the condition of risk aversion the expected return of underlying asset is larger than the risk free interest rate. Therefore, $h(+\infty) > 0$. Continuity of $h(s_{j^*})$ in s_{j^*} implies the existence of a nontrivial root.

3.4 Additional Check for Arbitrage Violation

Tested on our option data set, the search program successfully identifies nontrivial solutions of s_b, s_{j^*} for all 1910 options. In the case of s_k , twenty-nine of solutions are trivial. A zero solution of equation (3.2) means that $g(s_k) > 0$ whenever $s_k > 0$. One example of these 29 cases is the call option C^1 recorded at 14:00 Central Time on January 4, 1993. The index level at that time was 434.57, and strike price of the option was 430. (The coexisting option had a strike price, 435, and price $C_0 = 11.5$.) Figure 2 shows the curve of $g(s_k)$.

Figure 2: Plot of $g(s_k)$



To study figure 2, we note that Fact 1 implies the existence of a non-trivial solution if all assumptions are satisfied. Since a call option is riskier than the S&P 500 index, the only reasonable explanation for a trivial solution is that investors are not risk averse. Computation shows that the expected return of the January 4, 1993 option was 0.9574, and the expected return of the index was 1.0138. Obviously investors asked for a lower return of option than that of the index. Even worse, investors were expected to lose money by buying the January 4, 1993 call option. The analysis clearly indicates that the option was over-priced. An over-priced option market implies an arbitrage opportunity in a perfect market, although it may not be true in a real market. To maintain a sound comparison between Ryan and Ritchken bounds, we exclude these twenty-nine option pairs from further analysis.

By plugging in the non-trivial solutions, we obtain Ryan and Ritchken bounds for the remaining 1881 options. A quick check shows that 194 option prices lie outside the Ritchken bounds with 186 below the lower bound and 8 above the upper bounds. Prices of 833 options lie outside Ryan bounds with 565 below the lower bound and 268 above the upper bound. Thus, according to Ryan bounds 833 call options provide a theoretical arbitrage opportunity.

Because of trading costs, in reality the number of options providing arbitrage opportunities should be smaller. In a real market, violating one of the following conditions could lead to an arbitrage opportunity:

- i) Bid price of an option is larger than the upper bound plus any commission
- ii) Ask price of an option is smaller than the lower bound minus any commission
- iii) Bid price is larger than the ask price

Without actually including the commission in our calculation, among the 1881 options, 117 pairs of Ritchken bounds violate the condition that the ask price is smaller than the lower bound. In the case of Ryan bounds, 335 pairs violate the above condition, and 92 pairs violate the condition that the bid price is larger than the upper bound. In total, according to Ryan bounds, 427 call options potentially provide arbitrage opportunities.

Samples offering a theoretical arbitrage opportunity may distort outcomes of the empirical study below. We discard the 833 pairs of options, and study the reduction of Ryan bounds over Ritchken bounds with the remaining 1084 pairs.

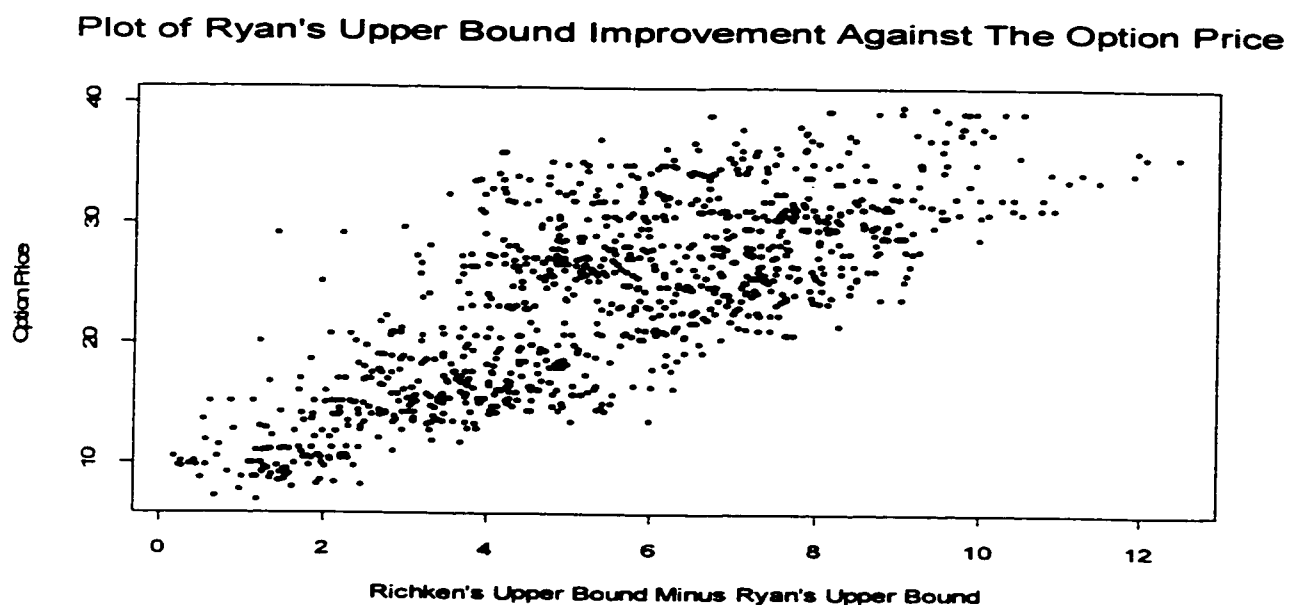
3.5 Comparing Ryan's and Ritchken's Bounds

Care is needed for selecting comparison criteria since the choice influences conclusions. First, a relative reduction should be chosen for a reason that will be clear below. Second, the reduction of Ryan upper (lower) bound over Ritchken upper (lower) bound should be studied as well.

Because of the scale effect, we expect that the absolute reduction will be positively correlated with the underlying asset price. Figure 3 shows that the absolute reduction and

the option price are positively correlated as well (correlation = 0.76). (Here, each dot in the plot is determined by the absolute reduction of Ryan upper bound over Ritchken upper bound and the corresponding option price. In other words, using the absolute reduction to evaluate the performance of Ryan bound theory is not meaningful since a large reduction does not imply good performance. Hence, we should choose the relative reduction as a comparison criterion.

Figure 3: The Positive Correlation Between Reduction and Option Price



In summary, the following three comparison criteria are chosen:

- a) Overall relative reduction = $(\text{Ryan Upper Bound} - \text{Ryan Lower Bound}) / (\text{Ritchken Upper Bound} - \text{Ritchken Lower Bound})$

- b) Upper bound relative reduction = $(\text{Ryan Upper Bound} - \text{Option Price}) / (\text{Ritchken Upper Bound} - \text{Option Price})$
- c) Lower bound relative reduction = $(\text{Option Price} - \text{Ryan Lower Bound}) / (\text{Option Price} - \text{Ritchken Lower Bound})$

The three sampling hours of the test data set represent three periods of a trading day at a market: the opening, middle and ending period. Volatility of the S&P 500 index may behave differently at each of these three periods. To study for possible affects of the trading periods on bounds, we compute the relative reductions for each trading period and for the whole data set too. Table 1 reports the results.

Table 1: Relative Reductions of Ryan's Bounds Over Ritchken's Bounds

	All Time			9:00 AM			11:00 AM			2:00 PM		
	Over All	Lower	Upper	Over All	Lower	Upper	Over All	Lower	Upper	Over All	Lower	Upper
Max	38.4	68.4	45.1	38.4	66.6	44	38.2	66.7	45.1	38.3	68.4	42.8
Min	2.3	0.02	0.08	2.8	0.05	0.15	2.9	0.03	0.08	2.3	0.02	0.15
Mean	22.1	27.5	19.8	21.2	27.8	18.5	22.5	27.1	20.3	22.6	27.6	20.6

In the table, under subtitle "All Time" relative reductions for the whole test data set are reported. Similarly, the relative reductions for each sampling hour are reported under a properly labelled column.

Some quick conclusions drawn from the table are:

- The results show no direct effect of trading periods on relative reductions. However, the conclusion is not as affirmative as it sounds. In the analysis, the same empirical volatility is used. The trading period factor exerts its influence on the relative reductions through the volatility. Therefore, a more reliable conclusion will be based on the results of using a trading period specific volatility. Lack of proper data currently prevents the implementation of such a study.
- The lower bound relative reduction dominates that of the upper bound. In average, Ryan lower bound improves by 7.7 percent more than the upper bound. The maximum improvement by Ryan lower bound is almost 70 percent, comparing to the best reduction of 45.1% by the upper bound.
- On average, the range of Ryan bounds is about 22%, approximately one-fifth, tighter than Ritchken bounds. In other words, the introduction of a coexisting option helps to reduce the spread of Ritchken arbitrage bounds by more than one-fifth. As pointed out later on, this reduction may not be enough to derive bid-ask prices basing on Ryan bounds, since the average spread of Ryan bounds is still much larger than the typical spread of bid-ask prices observed at option exchange markets.

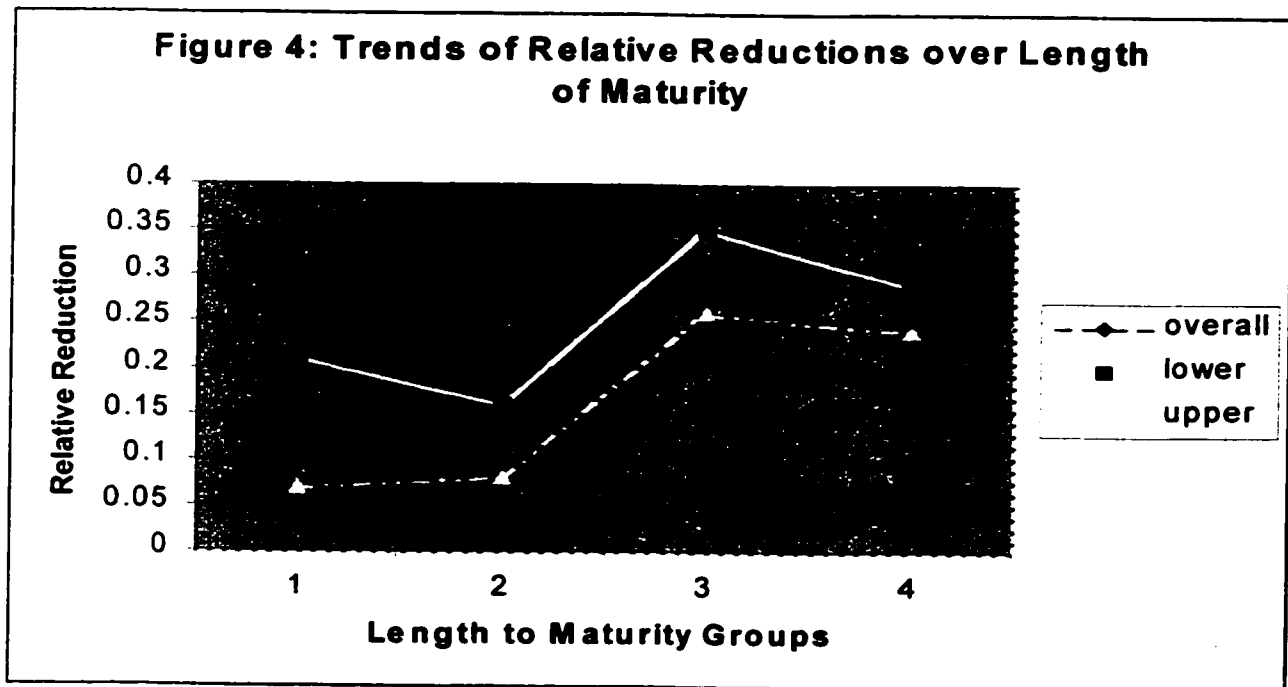
While table 1 summarizes the relative reductions for options regardless of their days to maturity, it is interesting to examine trends of these reductions over the time dimension. To this purpose, we group all options into four groups based on the days to maturity. All options with days to maturity between 1 and 90 days (1 ~ 3 month) belong to the first group. Similarly, these between 91 and 180 days (4 ~ 6 months) belong to the second group; these between 181 and 270 days (7 ~ 9 months) belong to the third group; and

these between 271 and 365 days (10 ~ 12 months) belong to the fourth group. Then, relative reductions are computed for each of three criteria and each group. Table 2 lists the outcomes. Among all the options, 14% belong to group 1; 21% belong to group 2; 33% belong to group 3; and 32% belong to the fourth group.

Table 2: Breakdown for Relative Reductions on Days to Maturity of Options

	1 ~ 3 months			4 ~ 6 months			7 ~ 9 months			10 ~ 12 months		
	Over All	Lower	Upper	Over All	Lower	Upper	Over All	Lower	Upper	Over All	Lower	Upper
Max	0.18	0.44	0.25	0.25	0.43	0.32	0.38	0.68	0.45	0.32	0.62	0.39
Min	0.02	0.003	0	0.03	0	0	0.03	0.001	0.001	0.09	0.003	0.061
Mean	0.11	0.21	0.07	0.11	0.16	0.08	0.29	0.35	0.26	0.25	0.29	0.24

Figure 4 graphically represents trends of mean relative reductions over the four groups of maturity length. An interesting observation is that mean of relative reductions for the longer options (with maturity length higher than six months) groups are larger than mean reductions for short length options (with length less than six months).



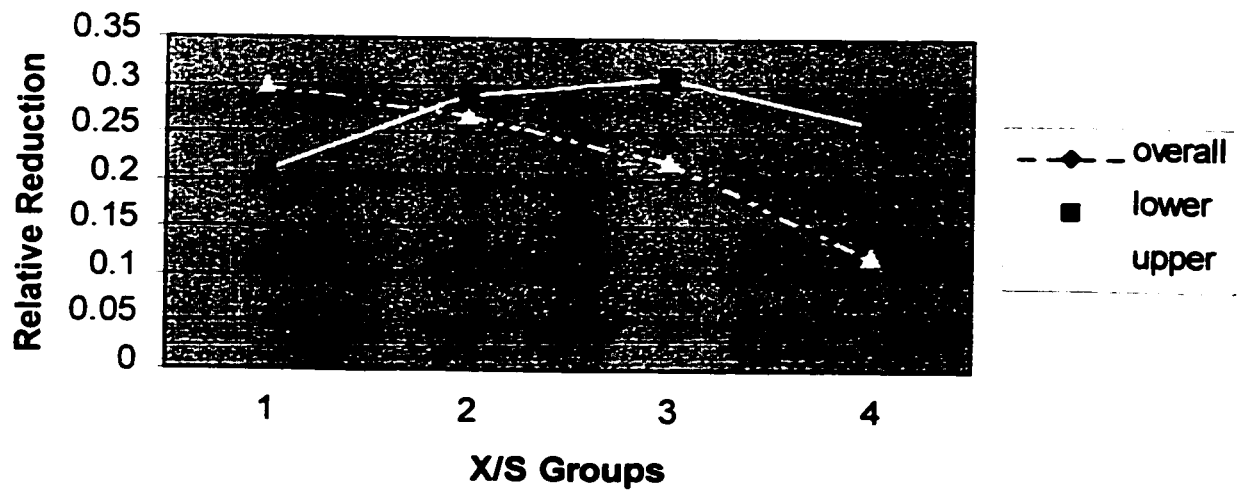
Another good way of examining trends of mean reductions is to group options according to their ratios of strike price over index level. Since range of the ratios for the 1084 pairs is from 0.94 to 1, we divide this range into four equal groups: 0.94 ~ 0.958 (group 1), 0.958 ~ 0.972 (group 2), 0.972 ~ 0.986 (group 3) and 0.986 ~ 1 (group 4). Then, relative reductions are computed for each of three criteria and each group. Table 3 lists the findings.

Table 3: Breakdown for Relative Reductions on Ratio of Strike Price over Index Level

	Group 1			Group 2			Group 3			Group 4		
	Over All	Lower	Upper	Over All	Lower	Upper	Over All	Lower	Upper	Over All	Lower	Upper
Max	0.37	0.60	0.42	0.36	0.61	0.45	0.37	0.66	0.43	0.38	0.68	0.35
Min	0.19	0.001	0.18	0.09	0.003	0.06	0.03	0.003	0.001	0.02	0	0
Mean	0.29	0.21	0.30	0.28	0.29	0.27	0.24	0.31	0.22	0.17	0.26	0.12

Figure 5 graphically represents trends of mean relative reductions over the four groups of ratios. The interesting observation is that mean reductions of relative reductions overall and upper bounds decrease as the options moves from in the money to at the money.

Figure 5: Trends of Relative Reductions over X/S



4. Assessing Pricing Bias of the Bound Theory

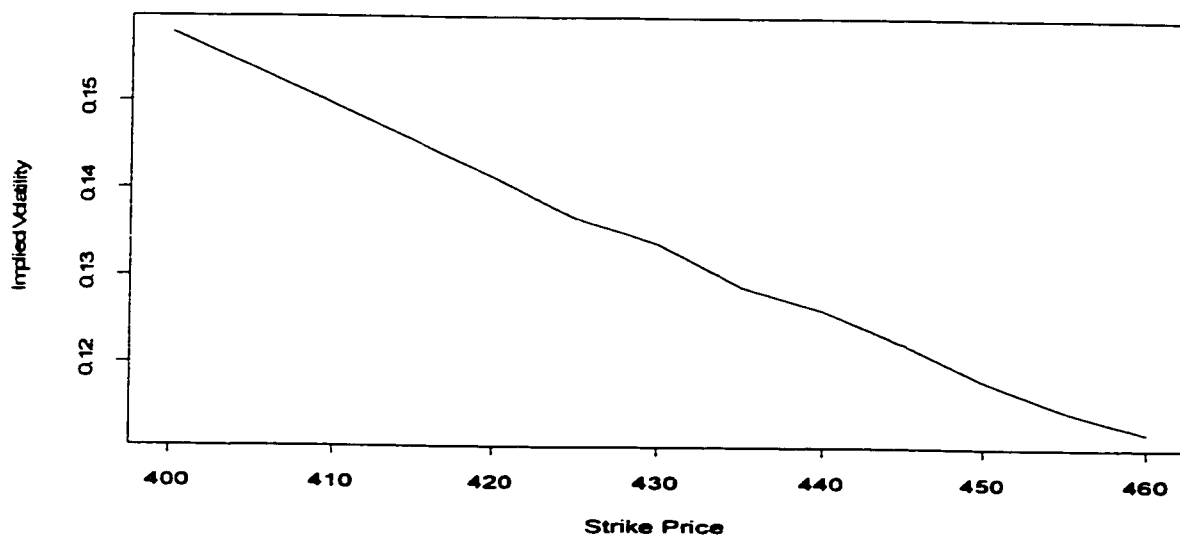
4.1 The “Volatility Smile”

Volatility smile is a well-publicized problem of B-S theory. B-S theory assumes a constant volatility. If the constant volatility assumption of B-S formula is correct, the implied volatility should be independent of any other variables such as strike price and time to maturity. However, researchers found that the implied volatility actually depended on some of these variables.

Rubinstein (1994) examined S&P 500 index options both before and after the market crash of October 1987. He discovered that starting from 1988, the constant volatility assumption no longer held. *“Out-of-the money puts (and hence in-the-money calls by perforce put-call parity) became valued much more highly,”* he observed, *“eventually leading to the 1990 to 1992 (as well as current) situation where low striking price options had significantly higher implied volatilities than high striking price options.”*

What he described is the correlation between volatility and strike price of coterminous options. The problem now is well documented in literature. To confirm Rubinstein’s finding, we show an example of such a correlation in figure 6. The sample used in figure 6 is observed on January 4, 1994 at 9:00am Central Time.

Figure 6: Plot of Implied Volatility Vs. Strike Price With a Sample of S&P 500 Index Options Observed on January 4, 1993 (9:00Am Central Time, $S=434.57$, $r=3.3\%$, Maturity Date = June 6, 1993)



By its narrow definition, the term, volatility smile, refers to a volatility curve, where the implied volatility of both deep in-the-money and out-of-the-money options is higher than the volatility of at-the-money options. In literature, the meaning of volatility is extended to referring to any systematic dependence between the implied volatility and the strike price with a common maturity date.

Since the concept of implied volatility is invalid for a bound theory, directly studying the volatility smile problem of a bound theory is meaningless. However, we can study the pricing bias problem of the bound theory. If we take the mean of upper and lower bounds as the implied price of a bound theory, and subtract it from the observed price, we can then plot the price difference (Y-axis) against the strike price (X-axis). If the bound theory implied price does not systematically depend on the strike price, we expect that the price difference should be parallel to the X-axis. Any deviation from a parallel line

implies a possible dependent relationship between the (bound theory) implied price and the strike price. We call the dependence the pricing bias problem of a bound theory.

In the remaining of this section, first we intuitively argue that the bias problem of a bound theory is a counterpart of the volatility smile problem for the B-S theory. Second, we develop a measurement to quantitatively assess the bias problem on the Ritchken's bound theory. As pointed out by Jackwerth and Rubinstein (1996), the pricing bias problem is closely related to the lognormal distribution assumption and the only way to eliminate the problem is to use a non-lognormal distribution. Thus, we expect that under the lognormal distribution, Ritchken's bound theory (similarly, the Ryan theory) can not eliminate the pricing bias problem. Nevertheless, we opt for using the lognormal distribution to demonstrate how to apply our methodology to study the bias problem. The same empirical study can be easily extended to a non-lognormal distribution.

4.2 D-Measurement

A common way to show the volatility smile problem of the B-S theory is to plot implied volatility of options against their strike prices (calling it the implied volatility plot) while other variables (such as days to maturity) are fixed. We now argue that shape on a price difference plot (by plotting observed price minus the B-S implied price against the strike price) is similar to the shape on the implied volatility plot. Here the implied B-S price refers to the option price calculated by plugging in the implied volatility into the B-S formula. According to the B-S pricing model, option price is a monotonically increasing

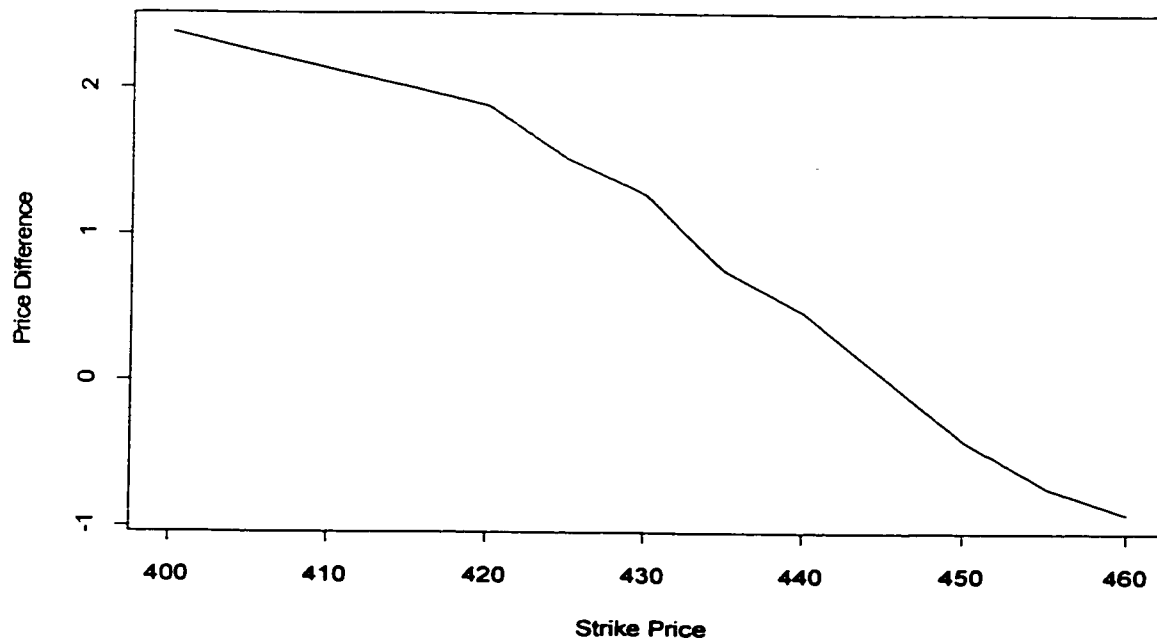
function of the volatility variable. When an implied volatility is larger (smaller) than the constant volatility assumed by the B-S theory, the corresponding B-S implied price is larger (smaller) than the true price. Thus, the volatility difference plot (by plotting the assumed constant volatility minus the implied volatility against the strike price) looks similar to the price difference plot. The implied volatility curve is simply a shift of the volatility difference curve. Thus, we have heuristically shown that under the B-S theory, the concept of pricing bias is closely related to the volatility smile.

Figure 7 provides an empirical evidence to support the above argument. In figure 7, the difference of observed and B-S implied price (based on the same sample of figure 6) is plotted against strike price. The curves in both figures are similar.

The following steps show how to construct price difference plots for the Ritchken's bound theory. For each option sample,

- a) Find the proper parameters of the assumed distributions. For example, if a lognormal distribution is used, we can use the B-S implied volatility of the at the money option as the volatility.
- b) Compute Ritchken bounds by using his limiting formulae
- c) Calculate the mean of the two bounds, and use it as the central, neutral price (or the implied price)
- d) Plot the price difference (i.e. true price - implied price) against the strike price

Figure 7: Plot of Price Difference Vs. Strike Price With a Sample of S&P 500 Index Options Observed on January 4, 1993 (9:00Am Central Time)



The price difference plot is useful for studying the price bias problem, but examining 1910 plots is a tedious work. More importantly, how to summarise information from 1910 plots is unresolved yet. Thus, a summary statistic is desirable.

One good candidate for the summary statistics is the correlation coefficient, ρ . ρ measures a linear correlation between two variables. When two variables are linearly independent, ρ is zero. When two variables are linearly dependent, $|\rho|$ is close to one. The correlation coefficient in figure 6 is -0.998 (in figure 7, $\rho = -0.978$), which means that the implied volatility and strike price are highly correlated. However, ρ can not

measure non-linear dependence. For instance when two variables are quadratically dependent, their ρ could be zero. (As an example, the ρ between x and $y = x^2$ is zero while y quadratically depends on x).

The narrowly defined volatility smile is a quadratic dependence. To include quadratic dependence in our study, the newly developed measurement should be able to detect such dependence too.

To explain the intuition of our new measurement, let us reexamine the $y = x^2$ example.

Let

$$y^* = \begin{cases} x^2 & x < 0 \\ -x^2 & x > 0 \end{cases}$$

The correlation between y^* and x becomes 0.9682 (an empirical value basing on $x = -100, -99, \dots, 100$). The correlation is now significantly different from zero.

This example inspires us a way to detect the quadratic dependence between the price difference and the strike price. Let u denote the price difference and v the strike price. To implement the above idea, we take v_0 as the median of v . Then, $u^*(v)$ equals $-u(v)$ for v lying at the right side of v_0 , and otherwise, $u^*(v)$ equals $u(v)$. The choice of median (instead of at the minimum point) as the reflection point is based on the fact that most of the price difference curves (showed by figure 8 and 9 at the end of this section) are hockey stick shaped. The modified correlation coefficient, M_d , is defined to be the correlation coefficient between u^* and v .

A perfect quadratic dependence such as the dependence between $y = x^2$ and x implies a high $|M_d|$. The opposite does not necessarily hold. For instance, $|M_d| = 1$ between variables $y = x$ and x . The problem that there is no x_0 exists, where y reaches its minimal. In this case, M_d is equivalent to ρ .

To quantify both the linear and quadratic dependence between price difference and strike price, define $D = \max \{|\rho|, |M_d|\}$. D is the measurement developed for studying the pricing bias problem of bound theory. Our D -measurement can only detect the linear or quadratic association between price difference and strike price.

To meaningfully interpret the computed D -measurements of a bound theory, we need a base for comparison. The D -measurements of B-S theory serve as a good base. If D -measurements of a bound theory are much smaller than those of B-S theory, we can conclude that the bound theory avoids the pricing bias problem. Otherwise, the bound theory does have the problem. To this end, in the remaining study we first compute D -measurements of B-S theory, and, then, D -measurements of a bound theory.

4.3 D-Measurements of B-S Theory

An empirically estimated volatility is used for the analysis described in the previous section. As a result, the same volatility is applied to all samples. For assessing the pricing bias problem we should use a sample-specific volatility. Some authors in literature recommended using the mean implied volatility of the two at-the-money

options as the sample-specific volatility. Instead, we use the implied volatility of the closest-to-the-money option. To calculate for the implied volatility, a dividend yield of the S&P 500 index is required. By annualising the mean of monthly dividend yields showed in a graph of Z. Bodie *etc.* (1997), the yield is taken to be 2.4%. The Newton-Raphson algorithm is used to solve the B-S formula for implied volatility.

The procedure for computing *D*-measurements of B-S theory is as follows. For each given call option sample:

- I. Find the implied volatility of the closest-to-the-money option, and use it as an estimate of the true volatility for returns of underlying asset
- II. Compute the B-S implied prices
- III. Calculate price differences (= true price – B-S implied price)
- IV. Compute its *D*-measurements

Table 4 below summarises *D*-measurements of B-S theory along with its two components of the measurement.

Table 4 *D*-Measurement of B-S Theory

	$ \rho $	$ M_d $	<i>D</i> -Measurement
Maximum	0.997	0.995	0.998
Minimum	0.032	0.0021	0.19
Mean	0.873	0.671	0.882

Main observations from table 4 are:

- The maximum D -measurement of 1910 samples is almost one, and the average D -measurement is 0.88. In other words, the analysis indicates a strong “volatility smile” problem suffered by the B-S theory. This observation confirms the observation reported by Rubenstein (1994).
- Table 4 reveals a stronger linear dependence between price difference and strike price. However, in most of cases $|M_d|$ is high as well. A close examination shows that most of the high $|M_d|$ values are due to the nearly perfect linear dependence not to the quadratic dependence. Only in a small number of cases, the quadratic dependence is more significant than the linear dependence. In conclusion, our analysis shows that the constant volatility assumption of B-S theory is violated mainly because of the linear dependence.

4.4 D-Measurements of Bound-theory

We have the choice to compute D -measurements of Ritchken or Ryan theory. Conceptually, the choice of the theory will not affect conclusions since both theories share the same basic idea. However, operationally the Ritchken theory is easier for the analysis. Ryan theory requires a decision made on choosing a call option 1 and 2 when there are several coterminous options are available. Such a decision may affect the results. To avoid this additional complication, we select Ritchken bound-theory for the analysis.

The following is the procedure for computing D -measurements of Ritchken theory. For each call option sample

- Find the implied volatility of the closest-to-the-money option, and use it as an estimate of the true volatility for returns of underlying asset
- Compute Ritchken option pricing bounds of each option, and use the mean of the two bounds as the implied price
- Calculate price differences (= observed option price - implied price)
- Compute D -measurements

Table 5 summarises the computed D -measurements of Ritchken theory.

Table 5 D -Measurement of Bound Theory

	$ \rho $	$ M_d $	D -Measurement
Maximum	0.9999	0.993	0.9999
Minimum	0.0018	0.036	0.287
Mean	0.883	0.85	0.933

To examine the sensitivity of D -measurement under different choices of the constant volatility, we work on several more scenarios. In scenario one, the constant volatility for a sample is chosen to be the mean of all implied volatility derived from the sample. The results are almost identical to what reported in table 5. In scenario two, the empirical volatility is used for all samples. The results are listed in table 6.

Table 6 Use of Empirical Volatility for Computing D -Measurement of Bound Theory

	$ \rho $	$ M_d $	D -Measurement
Maximum	0.982	0.9997	0.9998
Minimum	0.003	0.023	0.023
Mean	0.771	0.892	0.918

From the above two tables, we have the following observations:

- Sensitivity analysis shows that in the case where a sample-specific volatility is used, the method used to compute the sample specific volatility has little impact on the final results.
- When the common (empirical) volatility is used for all samples, the results are somewhat different. The overall D -measurement drops. Since the empirical volatility is smaller than implied volatility, the observation indicates that a smaller volatility dampens the dependence between price difference and strike price. However, the drop is small.
- The linear dependence dominates the quadratic dependence when the sample-specific volatility is used. When the empirical volatility is used, the dominance by the linear dependence disappears.
- As expected, under the lognormal distribution assumption, the bound theory has the pricing bias problem. This result is encouraging, since it empirically demonstrates the power of our methodology in detecting the pricing bias for a bound theory.

An alternative way to study the pricing bias problem is to plot the upper and lower bounds implied by a bound theory against the ratio of strike price over underlying asset price. We have examined many samples and the shapes of these plots are strikingly similar, a hockey stick shape. For explanation purpose, we give two representative plots, one plot (figure 8) for a long sample (containing 25 options) and another plot (figure 9) for a short sample (containing 8 options). These shapes are expected. Another way to plot the graph is to plot the difference of the upper and the lower bound against the ratio. It will be interesting to examine the shape of such a plot.

Figure 8: Plot of Ritchken Bounds vs. X/S Ratio of S&P 500 Options (Calculation Based on 25 Options Sampled on February 25, 1993 at 11:00AM)

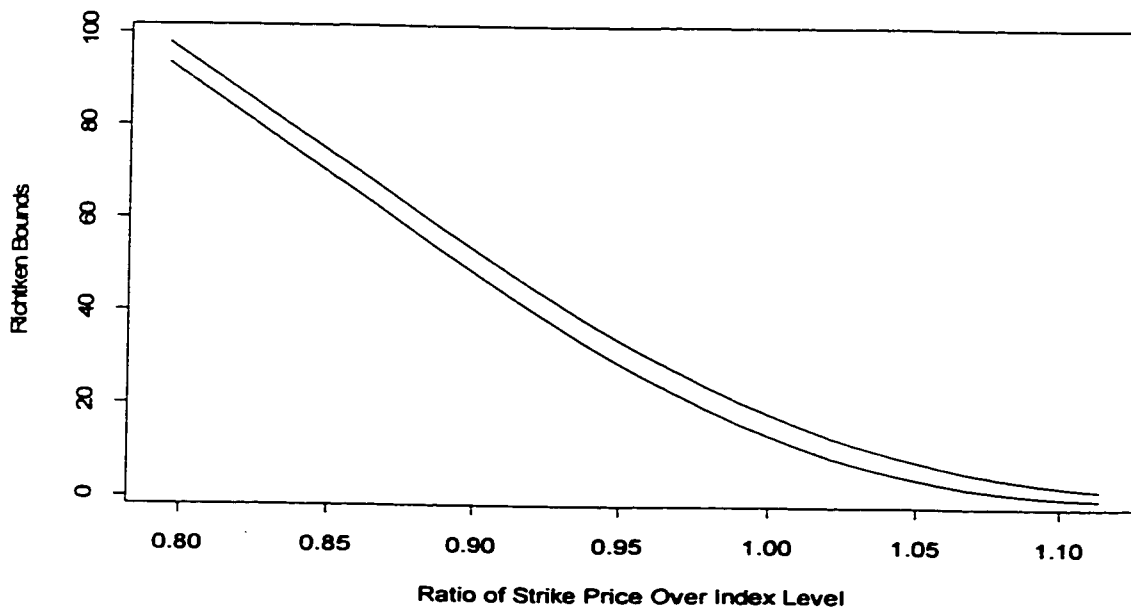
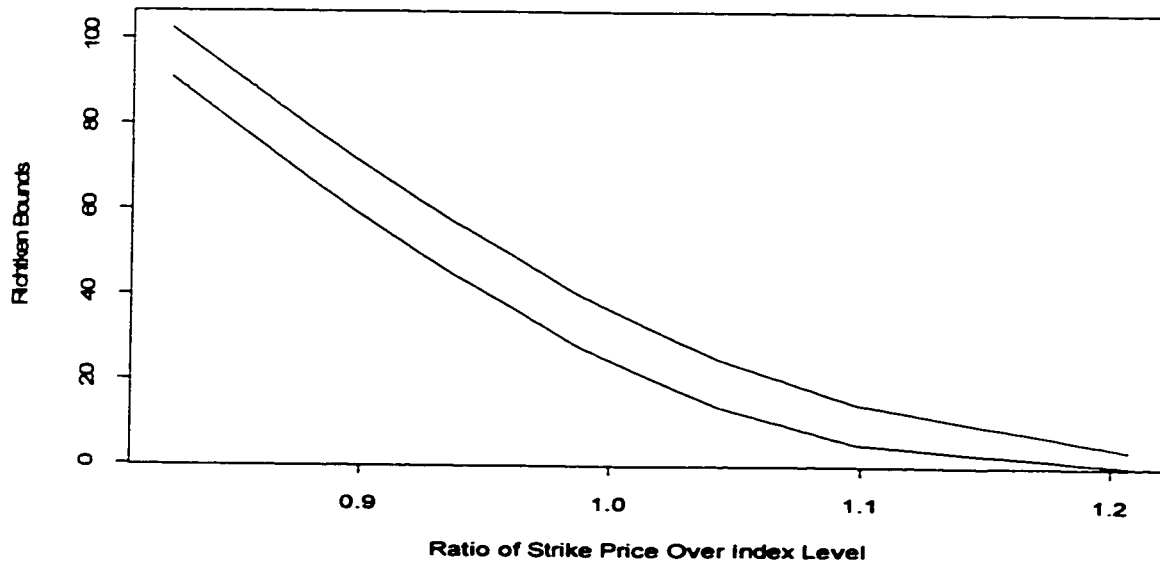


Figure 9: Plot of Ritchken Bounds vs. X/S Ratio of S&P 500 Options (Calculation Based on 8 Options Sampled on August 20, 1993 at 9:00AM)



5. Concluding Remarks And Discussion

Pricing an asset with a single price is useful, but in reality, the value of a traded security is quoted often with two prices: the bid price and the ask price. Options are examples of traded securities. Hence, bound theories on option pricing are useful. A basic idea of bound theories is to find the largest allowable range for the value of an option. A price falling inside the range will not violate the arbitrage-free requirement. No arbitrage opportunity is a minimal requirement for a well-functioning financial market. Therefore, bound theories are built upon a minimal requirement.

The emphasis of our empirical assessment is on studying two bound theories proposed by Ritchken (1985) and Ryan (1997). Our study focuses on two tasks. One task is to answer how much tighter Ryan bounds are. The second is to propose a methodology to assess the pricing bias problem on a bound theory. The proposed methodology is demonstrated with a real data set under the lognormal distribution assumption.

Analysis with our test data set shows that by adding in an additional constraint, Ryan bounds are approximately 22% tighter than Ritchken bounds. Furthermore, segmentation analyses on mean reductions by maturity and ratio of strike price over index level reveal two results. First, mean reductions of longer options (longer than six months) are larger than mean reductions of shorter options (six months or less). Second, means of overall reduction decrease as options move from in-the-money to at-the-money. From Jackwerth and Rubinstein (1996), we know that under the lognormal distribution assumption, the Ritchken bound theory has the pricing bias problem. We use this fact to test the power of

our D-measurement theory that is developed for detecting (linear and quadratic) bias. The analysis shows that our methodology has successfully detected the bias problem of both the B-S theory and the Ritchken bound theory. This is a positive sign that our method is valuable for detecting the bias problem of a bound theory when a non-lognormal distribution is assumed.

A bound theory provides theoretical bounds for analysts to evaluate market bid-ask prices, and to search for arbitrage opportunities. Similarly, a market maker can use the theory to ensure that the published bid-ask prices do not provide arbitrage opportunities. In that sense, a tighter pair of bounds is more useful in real application. Compared to a spread of the published bid-ask prices, the range of Ryan bounds is still too wide. For example, the average spread of Ryan bounds for 1048 options analysed in section 3 is \$2.24. The maximum is \$4.21 and minimum, \$0.057. The average spread, based on the observed bid-ask prices of our test data set (20875 call options), is \$0.66. This observation signifies a need of extending Ryan method to including all coexisting options. Conceptually, such extension is straightforward. An additional coexisting option means an extra constraint in the linear programming formulation of Ryan theory.

There is a non-negligible result from our analysis. Among the 1910 pairs of options used for empirical assessment in section 3, over 800 pairs violate Ryan arbitrage bounds. The original data set consists of 2477 samples. The test data set (1910 samples) is the remaining of the data set after samples which violate vertical spread and butterfly spread arbitrage constraints, were deleted. In other words, even with the cleaned data set, the

percentage of samples that violate arbitrage-free requirement is alarmingly high. The discussion below intends to offer some clues to this result. Further study is needed for uncovering a satisfactory answer.

Although theoretically over 800 samples violate Ryan arbitrage bounds, the number of samples, which provide a real arbitrage opportunity, should be much smaller. As shown in section 3, when the implicit option trading cost is considered, the number drops by half. If other factors such as the transactions costs, and different lending and borrowing rates are considered, the number will be even smaller.

The data set used for the analysis may play a role to the result. The test data set consists of option samples recorded at 9:00am, 11:00am, and 2:00pm, Central Time. One immediate problem may come from the samples recorded at 9:00am. Commonly, S&P 500 index is very unstable during the first half-hour of the market opening. The explanation is that member companies of the index gradually enter the market during that period. This fact indicates that we probably should not use the samples observed at 9:00am. The use of a common volatility for all samples, in some way, offsets the market opening influence. Quick analysis shows that the samples that violate arbitrage-free requirement of Ryan bounds evenly spread at the three sampling points of time. Nevertheless, if the study purpose is to search for an arbitrage opportunity, sample-specific volatility should be used, and the options sampled at 9:00am should be treated differently from other samples, and ideally should be excluded from the analysis.

There are several possible refinements for the analyses described in early sections. If implemented, these refinements will improve the quality of our analyses. Some of the potential refinements are stated below.

Changes of member companies: Members of the S&P 500 index are often replaced by other companies. Here is an example of change needed for the index. S&P 500 index consists of American companies only. A foreign controlled company is not permitted to be a member. Before the Daimler-Benz and Chrysler merger, Chrysler is a member company of the S&P 500 index. After the merger, Chrysler has to be replaced by another company since Daimler-Benz owns a simple majority share of the merged company. The new company is considered to be foreign, and Chrysler has to be replaced. The impact of a change made to the index is obvious. Often, the index breaks between the market closing on the announcement day and the opening of the next day. Our analysis does not take the fact into consideration in estimating the index volatility.

Dividends: Through the study period, a constant rate of dividend yield is assumed. This simplified treatment of dividends is sometime questionable. The study period is one year, and the life of options under study is frequently several months long. The monthly dividend yield varies widely in 1993 (see Z. Bodie 1997). For instance, companies traditionally pay out the largest dividends of a year in March and April. Thus, using a constant yield for a year is not realistic. A better approach is to either find the effective dividend yield for an option or to account for each dividend payment made during the life

of the option. The refinement calls for additional data of dividend payments, and they are currently not available to the analysis.

Empirical Volatility: The empirical volatility used in analyses of early sections is based on the daily closed S&P 500 index level. A desirable way is to use the index levels recorded when an option is sampled, and to estimate an empirical volatility for each sampling point of time (i.e. one each for 9:00am, 11:00am and 2:00pm). However, the set of recorded index levels is not complete due to some deleted samples. When a complete test data set is available, this improvement can be easily implemented.

Distribution: Through our analyses, a lognormal distribution is assumed for returns of an underlying asset. Unlike the B-S theory, both Ryan and Ritchken theories do not require a particular form of distribution. Studies with other plausible distributions will offer additional insights to both theories. It is conceivable to eliminate the pricing bias problem by using a properly chosen distribution for the Ryan and Ritchken theories. Therefore, it will be interesting to use the proposed methodology to assess the pricing bias problem for both bound theories when a non-lognormal distribution such as the mixture of lognormal distributions is assumed.

D-Measurement: The theoretical properties of the *D*-measurement developed for our study is not well studied yet. A study on its properties will provide a means to make more insightful interpretations about the results in section 4. An alternative way to measure nonlinear correlation was proposed by Doksum et al (1994).

Here are highlights of the discussion in the section.

- Ryan bounds improve Ritchken bounds by 22%. However, tighter bounds are needed for more realistic application.
- Our proposed methodology is empirically successful in detecting the pricing bias problem for the Ritchken's bound theory
- Several refinements are possible for our analyses. Some of the refinements call for obtaining additional information and a good quality set of test data. Other refinements include further studies for gaining better insights to the bound theories.

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Appendix A: Splus Codes

The analyses described in the main body are mainly done with programming in S+. Some I/O operations on the original data set to create an input file for the S+ codes are done with C+. The follow is the main part of S+ codes used for our analyses.

```
#1. reading in files and create a data as desired
#two relevant data files are saved under the directory of #"c:/users/sunweim/ou/the"
#file option.dat contains information such as expiration time, the time of the #option etc
#file callp.dat gives information such as strike price, trading hour etc.
#definition for columns of the option matrix are
# Strike CBid Cj CAsk Spread Date Hour ExpDate DaysToExp Index Interest #-of-#calls
# Since the solution is sensitive to the chosen initial value, different values #are tried and
# only the workable initial values are used as showed in the function below.
#find the implied volatility standard deviation, first define the function that #use Newton-Raphson method
#to
#find the implied volatility standard deviation.
#the following function computes implied volatility
```

```
option_scan("c:/users/sunweim/ou/the/option.dat")
option_matrix(option,1910,7,byrow=T)
tmp_scan("c:/users/sunweim/ou/the/callp.dat")
tmp_matrix(tmp,20875,5,byrow=T)
count_0
tmp1_matrix(0,20875,7)
for (i in 1:dim(option)[1])
  for (j in 1:option[i,7]) {
    count_count + 1
    tmp1[count,]_option[i,]
  }
option_cbind(tmp,tmp1)
rm(tmp,tmp1)
```

#2. Converting date format of the trading date and expiry to day format (from 1 #to 365)

```
date2day_function(year,month,day) {
if (year==94) month_month+12
daysofmonth_c(0,cumsum(c(31,28,31,30,31,30,31,31,30,31,30,31,31,28,31,30,31,30,31,31,30,31,30,31)))
days_daysofmonth[month]+day
days
}
option_scan("c:/users/sunweim/ou/the/option.dat")
option_matrix(option,1910,7,byrow=T)
tmpind1_rep(1,365*2)
tmpind2_c(5:7,rep(1:7,103),1:6)
tmpind1[c(1:730)*((tmpind2==6)+(tmpind2==7))]_0 #setting weekends as no trading days
hodindx_c(date2day(93,2,15),date2day(93,4,9),date2day(93,5,31),date2day(93,7,5), date2day(93,9,6))
hodindx_c(hodindx,date2day(93,11,25),date2day(93,12,24),date2day(94,2,21),date2day(94,4,1))
hodindx_c(hodindx,date2day(94,6,5),date2day(94,7,4),date2day(94,9,5),date2day(94,11,25),date2day(94,1
2,26))
tmpind1[hodindx]_0
TradeDay_as.character(option[,1])
TDyear_as.numeric(substring(TradeDay,1,2))
```

```

TDmonth_as.numeric(substring(TradeDay,3,4))
TDday_as.numeric(substring(TradeDay,5,6))
ExpiryDay_as.character(option[,3])
EDyear_as.numeric(substring(ExpiryDay,1,2))
EDmonth_as.numeric(substring(ExpiryDay,3,4))
EDday_as.numeric(substring(ExpiryDay,5,6))
tradeday_0
expiryday_0
for (i in 1:dim(option)[1]) {
tmp_tmpind1[date2day(EDyear[i],EDmonth[i],EDday[i]):date2day(TDyear[i],TDmonth[i],TDday[i])]
  tradeday_c(tradeday,sum(tmp)) # no. of trading days
  expiryday_c(expiryday,date2day(EDyear[i],EDmonth[i],EDday[i]))
}
tradeday_tradeday[-1]
expiryday_expiryday[-1]
#replace the date with day. 1=Jan. 1,1993 and 365=Dec. 31,93. Expiry date (in days)-period
(=option[,4])=trade date (in days)
option[,3]_expiryday
option_cbind(option,tradeday)

```

#3. For each recorded time (day+recorded hour) pick two call options whose strikes are closest to the stock level

```

tmp_scan("c:/users/sunweim/ou/the/callp.dat")
tmp_matrix(tmp,20875,5,byrow=T)
count_0
tmp1_matrix(0,20875,dim(option)[2])
for (i in 1:dim(option)[1])
  for (j in 1:option[i,7]) {
    count_count + 1
    tmp1[count,]_option[i,]
  }
option1_cbind(tmp,tmp1)
rm(tmp,tmp1)
ptr_1
callpair_rep(0,dim(option1)[2])
for (i in 1:dim(option)[1]) {
  indextmp_option[i,5]
  ptrdelta_option[i,7]
  striketmp_option1[ptr:(ptr+ptrdelta-1),1]
  ranktmp_rank(abs(striketmp-indextmp))
  position_ranktmp<3
  optiontmp_option1[ptr:(ptr+ptrdelta-1),]
  tmp_optiontmp[position,]
  if (dim(tmp)[1]>2) {
    tmp1_tmp[1,]
    for (j in 2:dim(tmp)[1]) {
      if (tmp[j,1] != tmp[1,1]) {
        tmp2_tmp[j,]
        break
      }
    }
  }
  tmp_rbind(tmp1,tmp2)
}
ptr_ptr+ptrdelta
callpair_rbind(callpair,tmp)

```

```

    cat(",i=",i,"dimension=",dim(tmp)[1],"n")
}

callpair_callpair[-1,]

#4. Compute the implied volatility

#function used to compute the implied volatility with Newton-Raphson method
# Since the solution is sensitive to the chosen initial value, different values are tried and
# only the workable initial values are used as showed in the function below.
#find the implied volatility standard deviation, first define the function that use Newton-Raphson method to
#find the implied volatility standard deviation.

nrsigma_function(C,S,q,T1,X,r) {
  sqsigma0_0
  sqsigma1_0.3
  difference_abs(sqsigma0-sqsigma1)
  j_0
  while (difference>0.0000000001 && j < 1000) {
    sqsigma0_sqsigma1
    a1_(log(S/X)+(r-q)*T1)/sqrt(T1)
    a2_sqrt(T1)/2
    d1_a1/sqsigma0 + a2*sqsigma0
    d2_d1-sqsigma0*sqrt(T1)
    b1_a2-a1/sqsigma0**2
    b2_b1-sqrt(T1)
    f0_S*exp(-q*T1)*pnorm(d1)-X*exp(-r*T1)*pnorm(d2)-C
    f0p1_S*exp(-q*T1)*(exp(-d1**2)/sqrt(2*pi))*b1 -X*exp(-r*T1)*(exp(-d2**2)/sqrt(2*pi))*b2
    sqsigma1_sqsigma0-f0/f0p1
    difference_abs(sqsigma0-sqsigma1)
    j_j+1
  }
  sqsigma1
}

#5. Find the implied volatilities based on 1993 option quotes
#Column 1 of option is the call price (meand of ask and bid)
#Column 2 of option is the trading hour, one of 9, 11, 14 central time.
#Column 3 of option is strike price
#Column 4 of option is time to expiration
#Column 5 of option is the current index level of S&P 500
#Column 6 of option is the interest rate corresponding to the option period
#Column 7 of option will be the implied volatility, which will be added in #later.

tmp_(callpair[,2]+callpair[,4])/2
tmp_matrix(tmp,length(tmp),1)
callpair1_cbind(tmp,callpair[,c(1,7,9,10,11)])

impvol_0
q_0.02
for (i in 1:length(tmp)) {
  C_callpair1[i,1]
  S_callpair1[i,5]
  T1_callpair1[i,4]/365
  X_callpair1[i,2]
  r_callpair1[i,6]
}

```

```

    impvol_c(impvol,nrsigma(C,S,q,T1,X,r))
    cat("i=",i,",")
}
impvol_impvol[-1]
callpair1_cbind(callpair1,matrix(impvol,length(impvol),1))
callpair1[,4]-callpair1[,4]/365 #annualize the time to expiration

```

#6. Use Daily S&P500 observations to estimate annualized mean and variance
 #The observed period is from April, 1993 - March, 1994.
 #First estimate sample mean and variance for the index returns
 #sp500.dat contains all daily from April, 93 - Mrch, 94. sp500tmp.dat only from #April 1, 93 to Dec. 31, 93.

```

sp500_scan("c:/users/sunweim/ou/the/SP500tmp.dat")
tmp_length(sp500)
spreturns_(sp500[2:tmp]-sp500[1:(tmp-1)]/sp500[1:(tmp-1)])
#there are 254 trading days in 1993
rmean_mean(spreturns)*365 #there are 254 trading days in 93
rstd_sqrt(var(spreturns)*365)

```

#7. plot daily S&P500 return against time

```

indexp_scan("c:/users/sunweim/ou/the/SP500.dat")
tmonths_c("Apr 93","May 93","Jun 93","Jul 93","Aug 93","Sep 93","Oct 93","Nov 93","Dec 93","Jan
94","Feb 94","Mar 94")
tdays_c(21,20,22,21,22,21,21,21,22,21,19,23)
molab_rep(tmonths,tdays)
win.printer(format="metafile", file="clipboard")
plot(c(1:length(indexp)),indexp,xlab="time",axes=F,type="l",ylab="Index Levels")
axis(2)
axis(1,at=c(1,22,42,64,85,107,128,149,170,192,213,232),labels=tmonths)
box()
title("S&P500 Index Levels From April 1, 1993 To March 31, 1994")
dev.off()

```

#8. The search procedure for Ryan's tighter bounds under monotonic preference
 #First, for given n, discretise the log-normal dist. at s_1, \dots, s_n with $s_1=0$
 #n = no. of discrete points, μ =mean, σ =std, index= S&P500 level at time #t, $\Delta T = T-t$
 #return value is a two column matrix, column1= s_1, \dots, s_n , column2=prob. at s_1, \dots, s_n

```

distri_function(n,mu,sigma,index,deltaT){
  mu1_log(index)+(mu-0.5*sigma**2)*deltaT
  sigma1_sigma*sqrt(deltaT)
  rsigma_7 #change the sigma range
  intvl_rsigma*sigma1/n
  SnT_exp(mu1-0.5*rsigma*sigma1+intvl*c(0:(n-1)))
  SnT[1]_0 #set s1=0
  logS_log(SnT[2:length(SnT)]/index)
  mean1_(mu-0.5*sigma**2)*deltaT
  sd1_sqrt(deltaT)*sigma
  probdis_pnorm(logS,mean=mean1,sd=sd1)[2:length(logS)]-
  pnorm(logS,mean=mean1,sd=sd1)[1:(length(logS)-1)]
  tmp_pnorm(logS[1],mean=mean1,sd=sd1)
  tmp1_1-pnorm(logS[length(logS)],mean=mean1,sd=sd1)
}

```

```

    probdis_c(tmp,probdis,tmp1)
    cbind(SnT,probdis)
}

#9. search for rho1 and rho2 for both Ryan and Ritchken's methods and find the #bounds
# parameters: x1=strike price of option 1, x2=strike price of option 2 (always #let X1 < x2 for convenience)
# td= no. of trading days of an option to expiration,ra=annualized riskless rate
# n = no. of discrete points
# il=index level of S&P500 at time 0,
# cp0=call price of the second option
# mu and sigma are the annualized mean and standard deviation of S&P500
#(estimated for year 1993)
# return value: l,k

search1_function(n,x1,x2,td,ra,il,cp0,mu,sigma) {
  td1_td/365 #converting trading days to year and there are 254 trading days in 1993
  r_exp(ra*td1)
  tmp_distri(n,mu,sigma,il,td1)
  ci_apply(cbind(tmp[,1]-x2,rep(0,length(tmp[,1]))),1,max)
  tmp1_cumsum(tmp[,1]*tmp[,2])
  tmp2_cumsum(ci*tmp[,2])
  tmp3_cumsum(tmp[,2])
  rho_tmp1[length(tmp1)]/il
  rho2_tmp2[length(tmp2)]/cp0
  a1_1/((rho-r)*il)
  a2_1/((rho2-r)*cp0)
  a3_r/(rho-r) - r/(rho2-r)
  mu1_td1*(mu-0.5*(sigma**2))+log(il)
  sigma1_sigma*sqrt(td1)
  t1_(a1*tmp1-a2*tmp2-a3*tmp3)[-length(tmp3)]
  truncate_as.integer(0.1*n)
  t2_(tmp1/il-tmp2/cp0)[-c(1:truncate)]
  t3_tmp1/il-r*tmp3
  t1_tmp[c(1:length(t1))[c(1:length(t1))*(abs(t1)==min(abs(t1)))],1]
  t2_tmp[truncate+c(1:length(t2))[c(1:length(t2))*(abs(t2)==min(abs(t2)))],1]
  t3_tmp[c(1:length(t3))[c(1:length(t3))*(abs(t3)==min(abs(t3)))],1]
  ci1_apply(cbind(tmp[,1]-x1,rep(0,length(tmp[,1]))),1,max)
  tmp4_cumsum(ci1*tmp[,2])
  indx1_sum(tmp[,1]<=t1)
  rho21_tmp2[indx1]/(cp0*tmp3[indx1])
  b1_(rho2-r)/((rho2-rho21)*tmp3[indx1])
  b2_(rho21-r)/(rho2-rho21)
  rylb_(b1*tmp4[indx1]-b2*tmp4[length(tmp4)])/r #ryan's lower bound
  indx1_sum(tmp[,1]<=t2)
  ryub_il*tmp4[indx1]/tmp1[indx1] #ryan's upper bound
  indx1_sum(tmp[,1]<=t3)
  rilb_tmp4[indx1]/(r*tmp3[indx1]) #ritchken's lower bound
  riub_il*tmp4[length(tmp4)]/tmp1[length(tmp1)] #ritchken's upper bound
c(rylb,ryub,rilb,riub)
}
i_1
bounds_rep(0,4)
while (i <= 1910) {
  x1_callpair[i,1]
  x2_callpair[i+1,1]
  td_callpair[i,9]
}

```

```

ra_log(1+callpair[i,11])
il_callpair[i,10]
cp0_(callpair[i+1,2]+callpair[i+1,4])/2
n_5000
mu_rmean
sigma_rstd
tmpbds_search1(n,x1,x2,td,ra,il,cp0,mu,sigma)
bounds_rbind(bounds,tmpbds,tmpbds)
cat(",i=",i)
i_i+2
}
i_1911
while (i <= 3000) {
x1_callpair[i,1]
x2_callpair[i+1,1]
td_callpair[i,9]
ra_log(1+callpair[i,11])
il_callpair[i,10]
cp0_(callpair[i+1,2]+callpair[i+1,4])/2
n_5000
mu_rmean
sigma_rstd
tmpbds_search1(n,x1,x2,td,ra,il,cp0,mu,sigma)
bounds_rbind(bounds,tmpbds,tmpbds)
cat(",i=",i)
i_i+2
}
i_3001
while (i <= dim(callpair)[1]) {
x1_callpair[i,1]
x2_callpair[i+1,1]
td_callpair[i,9]
ra_log(1+callpair[i,11]) #treat the annualized interest rate as a discrete rate
il_callpair[i,10]
cp0_(callpair[i+1,2]+callpair[i+1,4])/2
n_5000
mu_rmean
sigma_rstd
tmpbds_search1(n,x1,x2,td,ra,il,cp0,mu,sigma)
bounds_rbind(bounds,tmpbds,tmpbds)
cat(",i=",i)
i_i+2
}
bounds_bounds[-1,]
tp_c(1:3820)[c(1:3820)*(bounds[,1]<bounds[,3])]
bounds1_bounds
bounds_bounds[-tp,] #there are a few (3) anomalous cases that ritcken's lower bounds are lower than
ryan's. We remove them here.

#10. Study on reduction of Ryan bounds on Ritchken bounds
#the four columns are: col1=ryan's lower bound, col2=ryan's upper bound
#col3=ritchken's lower bound, col4=ritchken's upper bound
#col5=price of the call option

bounds_bounds[seq(1,dim(bounds)[1],2),]
tmp_((callpair[,2]+callpair[,4])/2)[-tp]

```

```

tmp_tmp[seq(1,length(tmp),2)]
tmp1_callpair[,2][-tp][seq(1,2*length(tmp),2)]
tmp2_callpair[,4][-tp][seq(1,2*length(tmp),2)]
tmp3_callpair[,7][-tp][seq(1,2*length(tmp),2)]
bounds_cbind(bounds,tmp,tmp1,tmp2,tmp3) #adding in the price of call option of interest, ask and bid
price as well
sum((bounds[,2])==0) #check how many NA (i.e. 0 upper bound for Ryan)
naindx_c(1:length(bounds[,2]))[(bounds[,2])!=0]
bounds_bounds[naindx,]
sum(bounds[,1]<=bounds[,5] & bounds[,5]<=bounds[,2])
sum(bounds[,3]<=bounds[,5] & bounds[,5]<=bounds[,4])
sum(bounds[,1]<=bounds[,7] & bounds[,6]<=bounds[,2])
sum(bounds[,3]<=bounds[,7] & bounds[,6]<=bounds[,4])
outindx_c(1:dim(bounds)[1])[bounds[,7]<bounds[,3]]
outindx_c(1:dim(bounds)[1])[bounds[,1]>bounds[,5] | bounds[,5]>bounds[,2]]
bounds_bounds[-outindx,] #comparing improvement in %
compana_function(bds){
reduction1_(bds[,2]-bds[,1])/(bds[,4]-bds[,3])
reduction2_(bds[,5]-bds[,1])/(bds[,5]-bds[,3]) #lower
reduction3_(bds[,2]-bds[,5])/(bds[,4]-bds[,5]) #upper
tmp1_range(reduction1)
tmp2_mean(reduction1)
tmp3_mean(reduction2)
tmp4_range(reduction2)
tmp5_mean(reduction3)
tmp6_range(reduction3)
list(bothrange=tmp1,bothmean=tmp2,lowrange=tmp3,lowmean=tmp4,uprange=tmp5,upmean=tmp6)
}
bds_bounds
compana(bds)
hindx_c(1:dim(bounds)[1])[bounds[,8]==9]
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[bounds[,8]==11]
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[bounds[,8]==14]
bds_bounds[hindx,]
compana(bds)
#cross-sectional study comparison, comparing at 1- 3, 3-6, 6-9 , 9-12 months
hindx_c(1:dim(bounds)[1])[(bounds[,9]>0) & (bounds[,9]<91)] # 1-3 month
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[(bounds[,9]>90) & (bounds[,9]<181)] # 3-6 month
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[(bounds[,9]>180) & (bounds[,9]<271)] # 6-9 month
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[(bounds[,9]>270) & (bounds[,9]<366)] # 9-12 month
bds_bounds[hindx,]
compana(bds)
#temporal analysis: relative reduction vs. (strike price)/(index level)
xovers_bounds[,10]/bounds[,11]
intvl_(range(xovers)[2]-range(xovers)[1])/4
bd1_range(xovers)[1] + intvl

```

```

bd2_range(xovers)[1] + 2*intvl
bd3_range(xovers)[1] + 3*intvl
hindx_c(1:dim(bounds)[1])[xovers<bd1]          # group 1
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[xovers<bd2 & xovers>=bd1] # group 2
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[xovers<bd3 & xovers>=bd2] # group 3
bds_bounds[hindx,]
compana(bds)
hindx_c(1:dim(bounds)[1])[xovers>=bd3]          # group 4
bds_bounds[hindx,]
compana(bds)

```

#11. Build Figure 2, and 3.

```

win.printer(format= "metafile", file= "clipboard")
plot(log(tmp[-1,1]),t2[-1],type="l",xlab="log-scaled S", ylab="g(s)")
title("Curve of g(s)")
dev.off()
win.printer(format= "metafile", file= "clipboard")
plot(bounds[,4]-bounds[,2],bounds[,5],type="p",xlab="Ritchken's Upper Bound Minus Ryan's Upper Bound", ylab="Option Price")
title("Plot of Ryan's Upper Bound Improvement Against The Option Price")
dev.off()

```

#12. compute implied volatilities and implied B-S prices

```

q1_c(0.04,0.345,0.285,0.09,0.043,0.25,0.065,0.38,0.26,0.1,0.35,0.225)/100
q_mean(q1)*12
impsig_0
for (i in 1:dim(callpair)[1]){
r_callpair[i,11]
X_callpair[i,1]
T1_callpair[i,9]/365
C_(callpair[i,4]+callpair[i,2])/2
S_callpair[i,10]
sigma1_nrsigma(C,S,q,T1,X,r)
impsig_c(impsig,sigma1)
cat("i=",i)
}
impsig_impsig[-1]
tmpindx_seq(1,length(impsig),2)
impsig1_(impsig[tmpindx]+impsig[tmpindx+1])/2
option2_cbind(option1,rep(impsig1,option[,7]))
T1_option2[,9]/365
d1_log(option2[,10]/option2[,1])+(option2[,11]-q+option2[,14]/2)*T1
d1_d1/(option2[,14]*sqrt(T1))
d2_d1-(option2[,14]*sqrt(T1))
cpri_option2[,10]*exp(-q*T1)*pnorm(d1)-option2[,1]*exp(-option2[,11]*T1)*pnorm(d2)
option2_cbind(option2,cpri)

```

#13. Build Figure 6, and 7

```

tmp_option2[20:29,]
tmp1_0
for (i in 1:10){
  r_tmp[i,1]
  X_tmp[i,1]
  T1_tmp[i,9]/365
  C_(tmp[i,4]+tmp[i,2])/2
  S_tmp[i,10]
  sigma1_nrsigma(C,S,q,T1,X,r)
  tmp1_c(tmp1,sigma1)
  cat("i=",i)
}
tmp1_tmp1[-1]
win.printer(format= "metafile", file= "clipboard")
plot(tmp[,1],tmp1,xlab="Strike Price", ylab="Implied Volatility",type="l")
dev.off()
win.printer(format= "metafile", file= "clipboard")
plot(option2[20:29,1],(option2[20:29,2]+option2[20:29,4])/2-option2[20:29,15],type="l",xlab="Strike
Price",ylab="Price Difference")
dev.off()

```

#14. Compute D-measurements of B-S theory

```

tmpcor_0
modcor_0
i_1
j_1
while (i <= dim(option2)[1]) {
  deltai_option2[i,12]
  strike_option2[i:(i+deltai-1),1]
  tmp2_(option2[i:(i+deltai-1),2]+option2[i:(i+deltai-1),4])/2
  pricediff_tmp2-option2[i:(i+deltai-1),15]
  tmpcor_c(tmpcor,cor(strike,pricediff))
  #find the modified correlation
  indx_sum(strike-option2[i:(i+deltai-1),10]<0)
  pricediff_c(pricediff[1:indx],-pricediff[(indx+1):deltai])
  modcor_c(modcor,cor(pricediff,strike))
  cat("j="j)
  j_j+1
  i_i+deltai
}
tmpcor_tmpcor[-1]
modcor_modcor[-1]
sum(tmpcor<modcor)
BSd_apply(cbind(abs(tmpcor),abs(modcor)),1,max)
range(BSd)
mean(BSd)
bstmpcor_tmpcor
bsmodcor_modcor

```

#15. Compute implied prices for both ritchken's and ryan's methods

```

ryanpri_function(a,b){
  tmpmatbds_matrix(0,1,4)
  n_1000
  mu_rmean

```

```

samsize_c(0,cumsum(matrix(scan("c:/users/sunweim/ou/the/option.dat"),1910,7,byrow=T)[,7]))
for (j in a:b) {
cindx1_samsize[j]+1
cindx2_samsize[j+1]
td_option2[cindx2,9]
ra_log(1+option2[cindx1,11])
il_option2[cindx1,10]
for (i in cindx1:(cindx2-1)){
sigma_option2[i,14]
x1_option2[i,1]
x2_option2[i+1,1]
cp0_(option2[i+1,2]+option2[i+1,4])/2
tmpbds_search1(n,x1,x2,td,ra,il,cp0,mu,sigma)
tmpmatbds_rbind(tmpmatbds,tmpbds)
}
x2_option2[cindx2-1,1]
cp0_(option2[cindx2-1,2]+option2[cindx2-1,4])/2
sigma_option2[cindx2,14]
x1_option2[cindx2,1]
tmpbds_search1(n,x1,x2,td,ra,il,cp0,mu,sigma)
tmpmatbds_rbind(tmpmatbds,c(min(tmpbds[1:2]),max(tmpbds[1:2]),tmpbds[3],tmpbds[4]))
cat("j=",j)
}
matbds_tmpmatbds[-1,]
matbds
}
ryribounds_matrix(0,1,4)
ryribounds_rbind(ryribounds,ryanpri(1,1910))
ryribounds_ryribounds[-1,]

```

#16. Compute D-measurements of Ritchken theory

```

tmpcor_0
modcor_0
i_1
j_1
while (i <= dim(option2)[1]) {
deltai_option2[i,12]
strike_option2[i:(i+deltai-1),1]
tmp2_(option2[i:(i+deltai-1),2]+option2[i:(i+deltai-1),4])/2
pricediff_tmp2-((ryribounds[,3]+ryribounds[,4])/2)[i:(i+deltai-1)]
tmpcor_c(tmpcor,cor(strike,pricediff))
#find the modified correlation
indx_sum(strike-option2[i:(i+deltai-1),10]<0)
pricediff_c(pricediff[1:indx],-pricediff[(indx+1):deltai])
modcor_c(modcor,cor(pricediff,strike))
cat("j="j)
j_j+1
i_i+deltai
}
tmpcor_tmpcor[-1]
modcor_modcor[-1]
sum(tmpcor<modcor)
Richd_apply(cbind(abs(tmpcor),abs(modcor)),1,max)
range(Richd)
mean(Richd)

```

#17 Figure 8 and 9

#Figure 8

cons1_25

tmp1_option2[option2[,12]==cons1,]

tm1_ryribounds[option2[,12]==cons1,]

i_1

deltai_tmp1[i,12]

strike_tmp1[i:(i+deltai-1),1]

tmp2_(tmp1[i:(i+deltai-1),2]+tmp1[i:(i+deltai-1),4])/2

uptmp_(tm1[,4])[i:(i+deltai-1)]

lwtmp_(tm1[,3])[i:(i+deltai-1)]

xovers_(tmp1[,1]/tmp1[,10])[i:(i+deltai-1)]

win.printer(format="metafile", file="clipboard")

par(mfrow=c(1,1))

plot(xovers,uptmp,xlab="Ratio of Strike Price Over Index Level", ylab="Richtken Bounds",type="l")

points(xovers,lwtmp,type="l")

#title("Plot of Ritchken Bounds Vs. X/S Ratio on S&P 500 Option on February 25, 1993 at 11:00AM (25
Options in the Sample)")

dev.off()

#Figure 9

cons1_8

tmp1_option2[option2[,12]==cons1,]

tm1_ryribounds[option2[,12]==cons1,]

i_169

deltai_tmp1[i,12]

strike_tmp1[i:(i+deltai-1),1]

tmp2_(tmp1[i:(i+deltai-1),2]+tmp1[i:(i+deltai-1),4])/2

uptmp_(tm1[,4])[i:(i+deltai-1)]

lwtmp_(tm1[,3])[i:(i+deltai-1)]

xovers_(tmp1[,1]/tmp1[,10])[i:(i+deltai-1)]

win.printer(format="metafile", file="clipboard")

par(mfrow=c(1,1))

plot(xovers,uptmp,xlab="Ratio of Strike Price Over Index Level", ylab="Richtken Bounds",type="l")

points(xovers,lwtmp,type="l")

#title("Plot of Ritchken Bounds Vs. X/S Ratio on S&P 500 Option on February 25, 1993 at 11:00AM (25
Options in the Sample)")

dev.off()