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FACULTY OF GRADUATE AND POSDOCTORAL STUDIES

Song Guo
AUTEUR DE LA THÈSE / AUTHOR OF THESIS

Ph.D. (Computer Science)
GRADE / DEGREE

School of Information Technology and Engineering
FACULTÉ, ÉCOLE, DÉPARTEMENT / FACULTY, SCHOOL, DEPARTMENT

Energy-Efficient Broadcast and Multicast Algorithms in Wireless Ad Hoc Networks

TITRE DE LA THÈSE / TITLE OF THESIS

Oliver Yang
DIRECTEUR (DIRECTRICE) DE LA THÈSE / THESIS SUPERVISOR

CO-DIRECTEUR (CO-DIRECTRICE) DE LA THÈSE / THESIS CO-SUPERVISOR

EXAMINATEURS (EXAMINATRICES) DE LA THÈSE / THESIS EXAMINERS

Michel Barbeau

Azzedine Boukerche

Victor Leung

Ivan Stojmenovic

Gary W. Slater
LE DOYEN DE LA FACULTÉ DES ÉTUDES SUPÉRIEURES ET POSTDOCTORALES /
DEAN OF THE FACULTY OF GRADUATE AND POSTDOCORAL STUDIES

Energy-Efficient Broadcast and Multicast Algorithms in Wireless Ad Hoc Networks

by

Song Guo

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Abstract

This thesis systematically explores two fundamental classes of energy-efficient broadcast/multicast problem: the BPM/MPM (Broadcast/Multicast Power Minimization) problem and the BLM/MLM (Broadcast/Multicast Lifetime Maximization) problem.

For the first class of problem, we have developed a general analytical MILP (Mixed Integer Linear Programming) model for the MPM problem in an ad hoc network with adaptive antennas, and also implemented a group of polynomial-time algorithms practically to handle significantly large networks for which the MILP model may not be computationally efficient.

For the second problem, we have first used theoretical analysis to derive two polynomial-time optimal algorithms for the MLM/BLM problem in the omni-directional antenna scenarios. We then study the same problem with directional antennas, and conjecture that this problem is NP-hard. Based on the same analytical model, we have developed a constraint formulation for the MLM/BLM problem in terms of MILP, and also implemented a group of polynomial-time algorithms practically.

Finally, we consider the same problems in mobile ad hoc networks (MANETs) and propose several distributed algorithms. Several localized operations are presented for our distributed algorithms, in which each node requires only the knowledge of its distance to all neighboring nodes and distances between its neighboring nodes. Through extensive simulation study, we show that these distributed algorithms are very efficient both in terms of energy (power or lifetime) and operation overhead.

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ABAM	Associativity-Based Ad hoc Multicast	2.3
ACP	Antenna Coverage Property	3.4
ACS	Ant Colony System	2.1.1.2
ADDM	Adaptive Demand-Driven Multicast	2.3
AMRoute	Ad hoc Multicast Routing protocol	2.3
AMRIS	Ad hoc Multicast Routing protocol utilizing Increasing id-numberS	2.3
AODV	Ad-hoc On-demand Distance Vector	2.3
AODVM	Ad-hoc On-demand Distance Vector Multicast	2.3
BEEM	Basic Energy Efficient Multicast	6.2
BAIP	Broadcast Average Incremental Power	2.1.1.2
BIDP	Broadcast Incremental-Decremental Power Algorithm	2.1.1.4
BIP	Broadcast Incremental Power Algorithm	2.1.1.2
BLM	Broadcast Lifetime Maximization	1.1
BLMST	Broadcast on the Local Minimum Spanning Tree	2.1.1.3
BPM	Broadcast Power Minimization	1.1
CAMP	Core-Assisted Mesh Protocol	2.3
COBRA	Center Oriented Broadcast Routing Algorithm	2.1.1.2
D-BIP	Directional BIP Algorithm	2.1.3
D-BIDP	Directional BIDP Algorithm	1.5
DDM	Differential Destination Multicast	2.3
D-DPBT	Dynamic-weight DPBT Algorithm	1.5
D-DPMT	Dynamic-weight DPMT Algorithm	1.5
D-MIP	Directional MIP Algorithm	2.1.3
D-MIDP	Directional MIDP Algorithm	1.5
DMLM	Distributed Maximum Lifetime Multicast Algorithm	6.4
DMPM	Distributed Minimum Power Multicast Algorithm	6.3
DMST	Directed Minimum Spanning Tree Algorithm	2.2.1
DPBT	Directed Prim Broadcast Tree Algorithm	1.5
DPMT	Directed Prim Multicast Tree Algorithm	1.5

DPT	Directed Prim Tree	5.3.2.1
DSR	Dynamic Source Routing	1.1
ESO	Energy Saving Operation	6.3
EWMA	Embedded Wireless Multicast Advantage	2.1.1.4
FGMP	Forwarding Group Multicast Protocol	2.3
GPBE	Greedy Perimeter Broadcast Efficiency	2.1.1.2
IMBM	Iterative Maximum-Branch Minimization	2.1.1.4
LAM	Lightweight Adaptive Multicast	2.3
LEO	Lifetime Enhancement Operation	6.4
LESS	Largest Expanding Sweep Search	2.1.1.4
LI	Linear Insertion	2.1.3
LMST	Local Minimum Spanning Tree	2.1.1.3
L-REMiT	Lifetime-Refining Energy efficiency of Multicast Trees	2.2.1
MA	MULTICAST-ALIVE message	6.2.3
MANET	Mobile Ad Hoc Network	1.1
MCEDAR	Multicast Core Extraction Distributed Ad-Hoc Routing	2.3
MCL	Multicast Communication Lifetime	6.1
MCP	Multicast Communication Power	6.1
MDLT	Minimum Decremental Lifetime	2.2.1
MIDP	Multicast Incremental-Decremental Power Algorithm	1.5
MILP	Mixed Integer Linear Programming	1.4
MIP	Multicast Incremental Power Algorithm	1.5
MIPF	Minimum Incremental Path First	2.1.2.2
MJREP	Multicast-Join-Reply message	6.2.3
MJREQ	Multicast-Join-Request message	6.2.3
MJS	Multicast-Join-Solicitation message	6.2.5
ML	Multicast-Leave message	6.2.5
MLM	Multicast Lifetime Maximization	1.1
MLR-MD	Maximum Lifetime Routing for Multicast with Directional antennas	2.2.2
MMBT	Min-Max Broadcast Tree / Min-Max Spanning Tree	5.2
MMMT	Min-Max Multicast Tree / Min-Max Steiner Tree	5.2
MPM	Multicast Power Minimization	1.1
MST	Minimum Spanning Tree	2.1.1.2
ODMRP	On-Demand Multicast Routing Protocol	2.3

P-MST	Pruned MST	2.1.2.1
RB-BIP	Reduced Beam BIP Algorithm	2.1.3
RB-BIDP	Reduced Beam BIDP Algorithm	1.5
RB-MIP	Reduced Beam MIP Algorithm	2.1.3
RB-MIDP	Reduced Beam MIDP Algorithm	1.5
RBOP	Related Neighborhood Graph based Broadcast Oriented Protocol	2.1.1.3
RF	Radio Frequency	1.2
RNG	Relative Neighborhood Graph	2.1.1.3
RTP	Rooted Tree Property	3.4
S-DPBT	Static-weight DPBT Algorithm	1.5
S-DPMT	Static-weight DPMT Algorithm	1.5
S-REMiT	Refining Energy-Efficient Source-shared Multicast Tree	2.1.2.3
SL	Simple Linear	2.1.3
SPF	Shortest Path First	2.1.2.2
SPT	Shortest Path Tree	2.1.1.2
TLT	Tree LifeTime	5.1
TPC	Tree Power Consumption	4.1
WANET	Wireless Ad Hoc Network	1
WMA	Wireless Multicast Advantage	3.2.3

Table of Notations

		Section of First Appearance
A	an arc set corresponding to the unidirectional wireless communication link	3.1
$A(T_s)$	the arc set of a multicast tree T_s	3.4
$\mathrm{ESO}_1(v, i)$	a node set defined in (5-14)	6.3.2
$\mathrm{ESO}_2(v, i)$	a node set defined in (5-14)	6.3.3
F_{vu}	the nonnegative variables, which represent the amount of flow, produced by the multicast initiator going through (v, u);	4.2.1
G	a directed graph modeling the wireless ad hoc network	3.1
K	the number of discrete transmission power levels	6.2.1
$\mathrm{LEO}_1(v, i)$	a node set defined in (5-23)	6.4.2
$\mathrm{LEO}_2(v, i)$	a node set defined in (5-27)	6.4.3
M	a multicast group with $\|M\| = m$ members, $M \subseteq N$	3.4
N	a node set in a two-dimensional plane with $\|N\| = n$ nodes	3.1
$N(T_s)$	the node set of a multicast tree T_s	4.2.1
Q_i	the total power required by a multicast tree obtained by algorithm i	4.3.1.2
Q'_i	the normalized power required by a multicast tree obtained by algorithm i	4.4.2
$S_{i,j}$	the power saving ratio defined as the ratio of the difference of total tree power using algorithm-i from algorithm-j to the total tree power using algorithm-i	4.4.2
$S^{(i)}$	the new node chosen at the i-th iteration using Prim's algorithm	5.3.2.1
T_{pause}	the node pause time in the random waypoint mobility model	3.5
T_s	a multicast tree of $G(N, A)$ rooted at a source node s	3.4
$T_s^{(i)}$	the intermediate tree constructed at the i-th iteration using our DPBT algorithm	5.3.2.1
TN_v	a tree node set in which each node belongs to the multicast tree T_s and lies in the maximum transmission range of node v	6.2.2
TN_v^2	a node set that includes node v's all tree neighbors and the fathers of the tree neighbors	6.4.2
$V_{\min}$	the minimum node mobility speed in the random waypoint	3.5

	mobility model;	
$V_{\max}$	the maximum node mobility speed in the random waypoint mobility model;	3.5
$\overline{V}$	the steady-state average speed in the random waypoint mobility model;	3.5
Z_{vu}	a binary decision variable which is equal to one if the arc (v, u) is in T_s^* of G, and zero otherwise	4.2.1
c_v	the only one farthest child from node v in T_s	6.3
e_i	the transmission power at level i	6.2.1
f_v	the father of v in the tree T_s	3.4
f_v^i	the new father of node v if v is added in the tree T_s and uses the transmission power level e_i after a non-tree initiated ESO or LEO	6.3.3
l_v	the farthest layer that node v can cover using a certain transmission power required by the tree T_s	6.2.1
l'_v	the farthest layer that node v can cover after an ESO or LEO initiated by node v	6.3.2
l_{vu}	the layer in which node u is located within the transmission range of node $(1 \leq l_{vu} \leq K)$	6.2.1
m	the size of a multicast group M, i.e. $m = \|M\|$	3.4
n	the number of nodes in the network, i.e. $n = \|N\|$	3.1
$p_v(T_s)$	the actual RF transmission power assigned to node v in a multicast tree T_s	3.4
p_{vu}	the RF transmission power needed for the link from node v to node u	3.1
p_{max}	the maximum RF transmission power level that a node can choose	3.1
p_{recv}	the minimum power needed for reception processing	3.3
p_{tran}	the minimum power needed for transmission processing	3.3
p_v^R	the actual power component for reception processing at node v	4.2
p_v^{RF}	the actual RF power component at node v	4.2
p_v^T	the actual power component for transmission processing at node v	4.2
q_{vu}	the signal strength measured at node u for the transmissions from node v	6.2.1
r_i	the transmission range at power level i	6.2.1
r_{vu}	the distance between node v and node u	3.2.1

s	the source node of multicast group M	3.4
w	a nonnegative continuous variable that represents the weight of bottleneck arc in T_s^*	5.4.2.1
w_{vu}	the weight for an arc (v, u) in graph G	5.2
$w_M(T_s)$	the maximum arc weight of a tree $T_s \in \Omega_M$	5.2
$(.)^*$	an optimized solution	4.2.1
α	the propagation loss exponent	3.2.1
α_{vu}	the angle measured counter-clockwise from the horizontal axis to the vector from node v pointing to node u ($0 \leq \alpha_{vu} < 2\pi$)	4.2.3.2
β	a parameter that reflects the importance assigned to the impact of residual energy	5.4
$\delta^-(v, u)$	the amount of reduced power by removing link (v, u) from the tree T_s;	6.3
$\delta^+(v, u)$	the amount of increased power by adding link (v, u) into the tree T_s	6.3
ε_u	the residual energy of node u	5.2
θ_v	the antenna beamwidth of node v ($\theta_{\min} \leq \theta_v \leq \theta_{\max}$)	3.2.2
$\theta_v(T_s)$	the antenna beamwidth applied by node v in the tree T_s	3.4
φ_v	the antenna orientation of node v ($0 \leq \theta_v < 2\pi$)	3.2.2
τ_{vu}	the maximal lifetime of an arc $(v, u) \in A$ at a given energy supply	5.2
$\tau_M(T_s)$	the lifetime of a multicast tree $T_s \in \Omega_M$	5.2
τ^*_M	the maximum of $\tau_M(T_s)$ over Ω_M	5.2
τ^i_M	the actual multicast lifetime obtained using heuristic algorithm-i	5.4.2
π_v	a source route $(s, \ldots, v)$ from s to v in the tree T_s	6.2.2
$\psi^-(v, u)$	the lifetime of node v after removing link (v, u)	6.4
$\psi^+(v, u)$	the lifetime of node v after adding link (v, u)	6.4
$\Delta(v, i)$	the tree power reduction by changing node v's power level to e_i and performing a corresponding tree update	6.3.2
Σ	a given energy supply $\Sigma = \{\varepsilon_u \mid u \in N\}$ associating to each node in G	5.4
Ω_M	the family of trees T_s of G spanning all the nodes in M	4.1
Ψ_v	the lifetime of a node set $\{v\} \cup TN^2_v$	6.4.2
$\Psi(v, i)$	the lifetime of a node set $\{v\} \cup TN^2_v$ if node v extends its transmission power level to e_i and performs a corresponding tree update	6.4.2

Chapter 1. Introduction

WANET (Wireless Ad-hoc Network) is expected to be deployed in a wide range of civil and military applications. The communicating nodes in these networks might be distributed randomly and are assumed to have packet-forwarding capability in order to communicate with each other over a shared and limited radio channel. Building such networks usually poses a significant technical challenge because of the constraints imposed by the energy consumption characteristics of the networks. Energy supplied by batteries is likely to be a scarce resource, and in some applications energy is entirely non-renewable [Ephr02, Perk00]. In order to increase the longevity of such networks, it is imperative that we find solutions to either increase the battery power or alternatively to optimize the use of the battery power via energy-efficient algorithms and mechanisms. Obviously, the first solution is technology dependent, and we focus on the second one that is of much interest in network research.

1.1 Background

Ad hoc networks are multi-hop wireless networks where all nodes cooperatively maintain network connectivity. Wireless networks in the form of ad hoc networks and sensor networks have received significant attention in recent years due to their wide range of potential civil and military applications where temporary network connectivity is needed. Sensor networks find their application in healthcare (e.g., health monitoring and coordination among doctors and nurses), aircraft flight control, weather forecasting, home appliance control, and protection against bioterrorism [WaAk02]. Ad hoc networks can be used for communication in ad hoc settings such as in conferences or classrooms [Perk00]. Unlike wired networks or cellular networks, no wired backbone infrastructure is installed in ad hoc wireless networks. The communicating nodes in these networks act as a router as well as a host with packet-forwarding capability in order to communicate with each other over a shared and limited radio channel. A communication session is achieved either through a single-hop transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise.

Since a node in an ad hoc network is typically a laptop, a personal digital assistant or any mobile device, energy supplied by batteries is likely to be a scarce resource, and in some applications energy is entirely non-renewable [Ephr02]. The network usability is limited by the battery energy in wireless devices. Sensor networks will stress power sources because of their need for long operating lifetimes and high energy density [WaAk02]. Furthermore, the lifetime of batteries has not been improved as fast as processing speed of microprocessors. Therefore, energy conservation is of paramount importance for the wide deployment of wireless ad hoc networks.

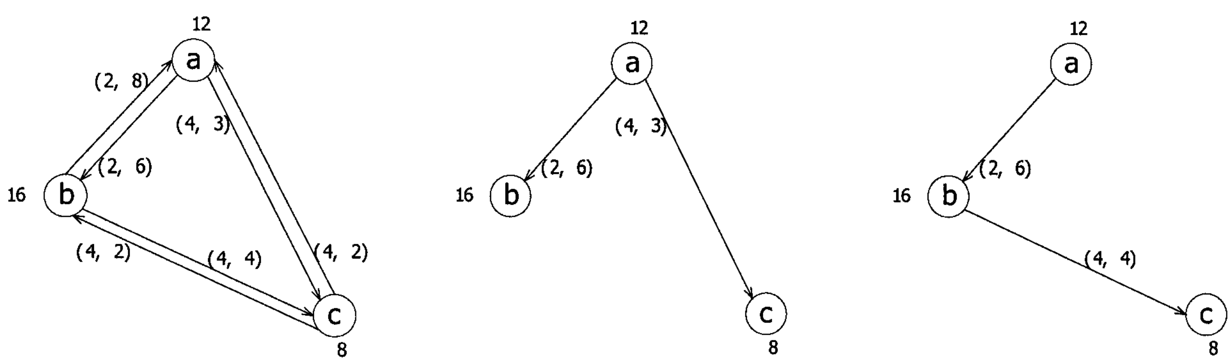
Energy conservation techniques for ad hoc networks can be broadly classified into two categories: power mode control and transmission power control. A power mode control protocol, e.g. in [ChJa02, SiRa99, YeHe02], aims to put wireless nodes into periodical sleep state in order to reduce the power consumption in the idle listening mode. Transmission power control, e.g. in [RaHa00], manages energy consumption by adjusting transmission ranges. The work in our survey deals with conserving power by employing transmission power control.

Multicasting is a service for disseminating data to a group of hosts. As a fundamental generalization of both unicasting and broadcasting, it is critical in applications where close collaboration of network hosts is required to carry out a given task. A multicast operation involves sending the piece of same information, called multicast packet, to all the members of the multicast group. The set of network nodes which may generate a multicast packet to be distributed to a multicast group are referred to as source nodes. Multicasting plays an important role in typical multi-hop ad hoc networks where bandwidth is scarce and hosts have limited battery power [ObTs98]. In addition, many routing protocols for wireless ad-hoc networks need a broadcast/multicast as a communication primitive to update their states and maintain the routes between nodes. Multicast is also widely used in sensor networks to disseminate information about environmental changes to other nodes in the network. Therefore, it is essential to develop efficient broadcast/multicast protocols that are optimized for energy consumption.

Few studies (e.g. [GuYa06a]) have addressed this optimization problem for MANETs (Mobile Ad Hoc Networks). By contract, the same optimization problem has been thoroughly studied for static ad hoc networks in recent few years. Since mobility adds a whole new dimension to the problem and it is out of the scope of this article, we shall only review the proposed solutions for static ad hoc networks in recent literature, most of which have assumed that any node is permitted to initiate a multicast and once a multicast delivery structure (e.g. a tree) is established at the beginning of a multicast session, the same structure is used for the whole session duration. If energy efficiency is to be considered, ad hoc networks require an energy-aware metric in their routing algorithms. Typically, there are two main energy-aware metrics and their corresponding problem formulations for multicasting in static ad hoc networks:

- Minimizing the total transmission power consumption of all nodes involved in the multicast session.
- Maximizing the operation time until the battery depletion of the first node involved in the multicast session.

Notice that these two metrics are not reciprocal of each other [ChTa00, GuYa04b]. In the following, we use a simple example to show that these two metrics cannot be optimized at the same time.



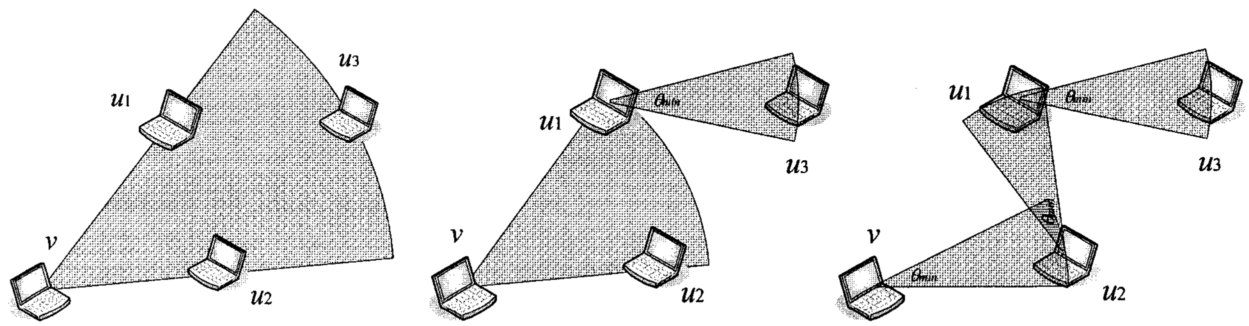
(a) a network topology (b) the minimum power tree (c) the maximum lifetime tree

Figure 1-1. An example of minimum power tree and maximum lifetime tree

Fig. 1-1 (a) is an example network with three nodes $\{a, b, c\}$, and the source at Node a. The value beside a node represents the residual battery energy of this node. We also list (p_{vu}, t_{vu}) beside each arc (v, u), where the value p_{vu} represents the power required for data transmission from node v to node u, and the value t_{vu} represents the maximum time that the arc (v, u) could last before the battery depletion in node v. The goal is to construct a broadcast tree based on these two metrics. The minimum power broadcast tree is given in Fig. 1-1 (b) with total tree power $\max\{p_{ab}, p_{ac}\} = p_{ac} = 4$ and tree lifetime $\min\{p_{ab}, p_{ac}\} = p_{ac} = 3$, and the maximum lifetime broadcast tree is given in Fig. 1-1 (c) with total tree power $p_{ab} + p_{bc} = 2 + 4 = 6$ and tree lifetime $\min\{p_{ab}, p_{bc}\} = p_{bc} = 4$.

In parallel to the algorithmic research on these energy-aware optimization problems, recent use of directional antennas in wireless communication has further enabled new approaches for energy saving in static ad hoc networks. Indeed, use of directional antennas allows concentration of the beam toward the intended destination without wasting energy in unwanted directions. It has been shown in [YuHu92, BaHa01a, BaHa01b, ChYa02, TaMa02] that the use of directional antennas in the context of WANETs can largely reduce the radio interference, thereby improving the utilization of wireless media and consequently the network performance. Some papers, e.g. [KoSh00, NaYe00], suggest the use of multiple directional antennas per node (or multiple beam antennas) to increase the throughput of 802.11 media access control protocol [MAC99]. Beam forming antennas has been explored [Rama01] to improve both throughput and delay in ad-hoc networks. Another paper [NaMa00] has suggested the use of multiple directional antennas to reduce the routing overhead of on-demand routing protocols for ad-hoc networks like DSR (Dynamic Source Routing) [JoMa01] and AODV (Ad-hoc On Demand Distance Vector) [PeBe01]. Over the last few years, energy efficient multicast in WANETs with directional antennas has received more and more attention, e.g. [CaSi04, DaWu04, DaWu05, KaPo03b, WiNg02c, WiNg02d], since nodes are battery-operated and frequent recharging or replacement of batteries may be undesirable or even impossible.

Using directional antennas, only the nodes located within the transmitting node's antenna beam can receive the signal. We consider an example as shown in Fig. 1-2, in which node v transmits information to nodes u_1, u_2, and u_3, all using directional antennas. A simple solution is that node v generates a single beam to reach all nodes in a single hop as shown in Fig. 1-2 (a). However, this approach is not energy efficient because energy consumption increases when the distance and beamwidth increase. As a result, it is essential to explore a multihop relaying approach to minimize energy consumption and to extend network lifetime, e.g. Figs. 1-2 (b) and 1-2 (c).



(a) (b) (c)

Figure 1-2. Communication examples using directional antennas

The energy-aware multicast problems for static ad hoc networks in the recent literature can therefore be classified into four categories according to the use of different energy metrics and antenna types:

(P1) The BPM/MPM (Broadcast/Multicast Power Minimization) problem in networks with omni-directional antennas

(P2) The BPM/MPM problem in networks with directional antennas

(P3) The BLM/MLM (Broadcast/Multicast Lifetime Maximization) problem in networks with omni-directional antennas

(P4) The BLM/MLM problem in networks with directional antennas

Each problem has received much attention, for example the proposals [WiNg00, WiNg01, LiNi01, WaCa01, WiNg02a, CaHu02, CaSi03, GuYa03, GuYa04a, GuYa04d, WaCa04, GuYa06c] for (P1), [WiNg02c, GuYa04c, GuYa04e, DaWu04, DaWu05, GuYa05b, GuYa06b, GuYa06d] for (P2), [ChSu03, DaMa03b, KaPo03a, WaGu03b, FlKa03, Geor03, GuYa04b] for (P3), and [WiNg02c, GuYa04b, GuYa05a, HoSh05] for (P4).

1.2 Motivations

As discussed before, the key constraint in the wireless ad hoc networks is energy. For example in military ad hoc networks, to recharge or replace the individual user's battery is impossible, so a decrease in the number of nodes due to battery depletion could degrade network performance dramatically, or even fail to provide any service when the network partitions into smaller pieces. Therefore, the limited battery lifetime imposes a constraint on the network performance. Battery capacity limits the advancement and applications of these networks. In the last eleven years battery capacity has increased only by a factor of 2.7 [WoVe02]. Communication is the dominating energy consumption component in WANETs. As a result, the communication related factors, such as the selection of relay nodes and their RF (Radio Frequency) transmission power, are the major considerations in routing algorithm design.

The multicast is an important communication primitive for distributed systems because they are required to update the states and to maintain the routes between nodes in wireless ad-hoc networks. As reviewed before, multicasting is a fundamental generalization of both unicasting and broadcasting, and a very efficient and useful means of supporting group-oriented applications, especially in wireless scenarios where bandwidth is scarce and hosts have limited battery power. Furthermore, since wireless medium is a broadcast medium in nature, a single transmission is sufficient to reach all the nodes within the transmission range, which is called the Wireless Multicast Advantage (WMA) property that will be discussed in detail in Section 2.2.3.

As the major part of our motivations and ambitions, we attempt to fill the work not done or to improve the best results known so far for the energy-efficient multicasting problem in both static and mobile networks. The absence and weakness in the recent literature for this problem have been reviewed before and are summarized as follows.

(1) Although [WiNg00, WiNg02c] have initially explored the WMA property for the BPM/MPM problem, they are heuristic in nature and not optimal even for small or moderately sized networks.

(2) None of recent work provides a general and complete global optimal solution of the

MLB/MLM problem.

(3) None of the recent energy-efficient distributed algorithms either studied the dynamic tree reconstruction in a mobile environment or the node mobility effect to the performance of various algorithms.

Table 1-1. Classification of energy-aware multicasting optimization problems

	BPM	**MPM**	**MLB**	**MLM**
Omni-directional Antenna	NP-complete	NP-hard	P	P
Directional Antenna	NP-hard	NP-hard	NP-hard	NP-hard

The known energy efficient multicast problem can be classified into two main categories as discussed in Section 1.1, most of which are NP-complete or NP-hard with respect to the scenarios of broadcasting and multicasting using omni-directional or directional antennas. The classification of such problems based on the complexity theory [GaJo79] is summarized in Table 1-1. Their difficulties have so far inhibited researchers from providing optimal solutions for even small- or moderately-sized networks. This challenge has inspired us to explore the formal formulations for these problems, which would provide insight to the inner-structure of the problems, and therefore can be used to derive a complete solution of the problems. To solve such difficult problems in large networks, the heuristic algorithms may be the only feasible way. In fact, the recent heuristic algorithms sometimes perform far from optimal. For example, [WiNg00] includes just one node at each iteration step of the tree construction, the one that can be added at minimum additional cost, but does not use all available information of the network. In such case, we should provide more efficient heuristic algorithms.

In the case of mobile nodes, the applicability of maintaining a multicast tree is contingent upon the cost of maintaining the distribution structure in face of disruption due to frequent mobility of nodes. All existing solutions are centralized and globalized, meaning that each node needs global network information. Mobility of nodes or changes in their activity status may cause global changes in any tree based structure. Therefore topology changes must be propagated throughout the network for any globalized solution. This may result in extreme and unacceptable communication overhead for ad-hoc networks. We shall focus on the distributed energy-efficient multicast algorithms for MANETs. Particularly, we are interested in a type of distributed algorithm called the Localized Algorithm [StLi00, CaSi03], in which nodes make decisions based solely on the knowledge of and distances to its 1-hop or 2-hop neighbors.

1.3 Objectives

The research subject of our work concentrates the traditional wireless ad hoc networks instead of the sensor networks. The general objective of this thesis is to investigate the construction of an energy efficient multicast tree in a WANET, by either minimizing the total power consumption of all nodes in the multicast group, or maximizing the multicast lifetime. Specifically, we would like to

(1) formulate a series of fundamental energy efficient multicast problems in WANETs,
(2) provide efficient algorithms for each optimization problem,
(3) develop distributed implementation for the practical usage of our proposed solutions,
(4) carry out performance evaluation in order to understand their capabilities.

1.4 Approaches and Methodologies

We shall systematically explore two fundamental energy-efficient broadcast/multicast problems: the BPM/MPM problem and the BLM/MLM problem. The focus in our study is on the "logical" problem of establishing energy-efficient trees for broadcast/multicast communications, rather than on the development of practical protocols for full-fledged broadcast/multicast implementation. For example, we consider a multicast group that is assigned the needed amount of interference-free bandwidth throughout the duration of the session, but the study of TDMA- and CDMA-based systems is not pursued here since we want to place emphasis on the energy constraint, especially exploiting the "wireless multicast advantage" property, with as little complication from the MAC layer as possible.

1.4.1 The MPM/MLM Problems in Static Networks

For those NP-complete and NP-hard problems listed in Table 1-1, we shall use optimization theory like MILP (mixed integer linear programming) to formulate each problem, upon which the MILP solver could obtain the optimal solutions in a timely manner at least for small or moderately sized networks using the branch-and-bound technique. In addition to the problem formulations, we shall also make use of some graph theory to design the polynomial-time optimal algorithms (for P problems) or heuristic algorithms (for NP-complete or NP-hard problems) to handle larger networks.

We have implemented a simulation tool using C++ language for the special purpose of evaluating the performance of our energy efficient multicast algorithms while the lower MAC layer is assumed to be ideal. In our simulation experiments, we shall use various parameters and measures like the minimum tree power and the normalized tree power for the BPM/MPM problem, and the maximum tree lifetime and the normalized tree lifetime for the BLM/MLM problem. We shall also compare them with other most competitive energy-efficient multicast algorithms over a wide range of randomly generated network examples.

For the NP problem, we implemented the optimal algorithms based on our MILP models using the optimization software CPLEX [CPLX05], which is a linear, integer and quadratic programming package using simplex method written in C language. It can provide optimal solutions for small/mediate scale problems in a timely manner. We evaluated such optimal algorithms through "simulation" by testing it on many topology variations (e.g. 100 for each scenario) so that we can obtain the mean and variance of the performance metrics. We also implemented the polynomial-time optimal algorithms (for the P-problem) and heuristic algorithms (for the NP-problem) using C++ language. Again, we evaluated them by "simulation". Both types of simulations have been run on an MS WIN2000 workstation with a PIII 800-MHz processor and 128 MB memory. The typical running time based on an MILP model ranges from 0.1 sec for a small size (e.g. 10 nodes) to more than one hour for bigger networks (e.g. 50 nodes). Under the same platform, the execution time of our polynomial-time algorithms (optimal or heuristic) is less than 1 sec for all cases.

Through our optimal/heuristic algorithms, we attempt to provide a complete solution to the BPM/MPM and BLM/MLM problems for all combinational cases in terms of optimization metrics power/lifetime, broadcast/multicast, and use of omni-directional/directional antennas. More specifically, the following shows how we approach these problems systematically.

For the BPM/MPM problem, we shall develop a general analytical MILP (Mixed Integer Linear Programming) model for the MPM problem in an ad hoc network with adaptive antennas. Some special cases can be easily deduced from this MILP model, including:

(1) broadcast communication,

(2) use of omni-directional antennas, and

(3) use of beamwidth-fixed directional antennas.

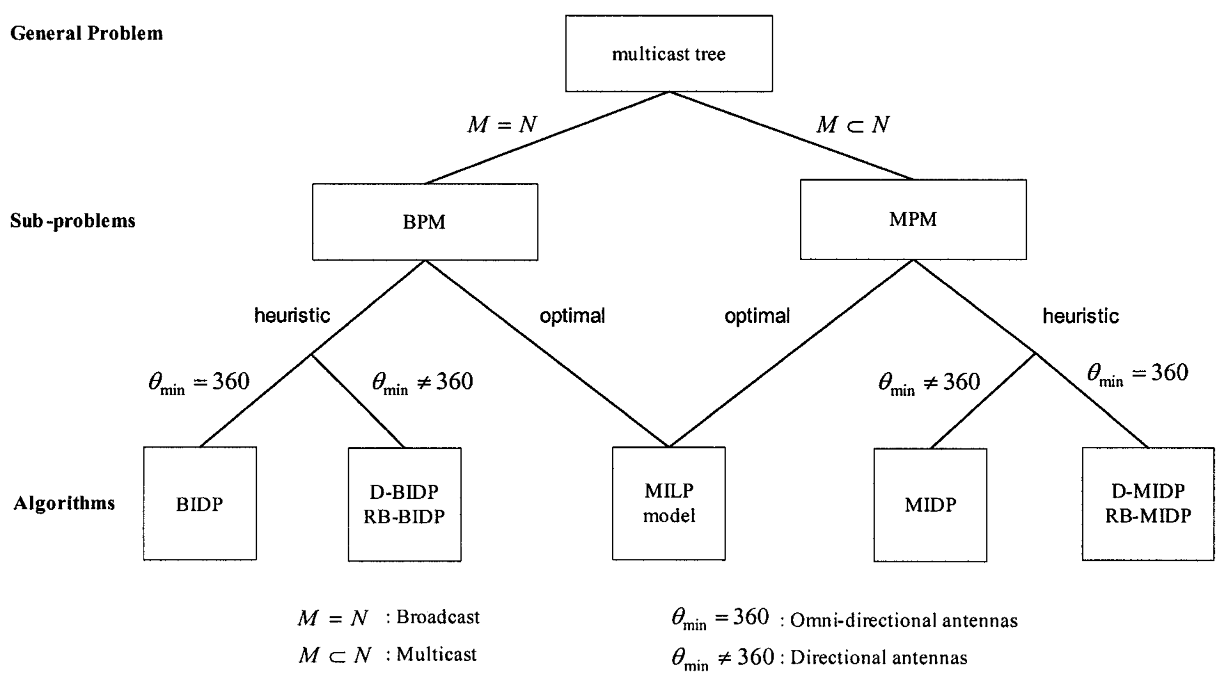


Figure 1-3. The framework of the BPM/MPM problem formulation and solutions

In addition to the theoretical effort for the MPM problem, we shall also use a practical approach by implementing a group of polynomial-time algorithms to handle significantly large networks. All scenarios under various conditions have been solved in the thesis based on the structure of the BPM/MPM problem formulations given in Figure 1-3.

For the BLM/MLM problem, we first study the MLM/BLM problem in the omni-directional antenna scenarios because it would provide insights into more general case of using directional antennas. In order to solve this P problem in networks with omni-directional antenna, we shall develop an analytical model in which the MLM/BLM can be formulated as a general min-max tree problem. We shall use theoretical analysis to derive two polynomial-time algorithms, which would allow us to obtain global optimal solutions for the BLM and MLM problems respectively. We then study the same problem with directional antennas. Some special cases can still be solved using our approach proposed for omni-directional antennas. For general cases, however, we conjecture that this problem is NP-complete. Based on the same analytical model, we shall develop a constraint formulation for the MLM/BLM problem in terms of MILP (Mixed Integer Linear Programming). In order to handle significantly large networks with directional antennas for which the MILP model may not be computationally efficient, we shall also implement a group of polynomial-time algorithms practically.

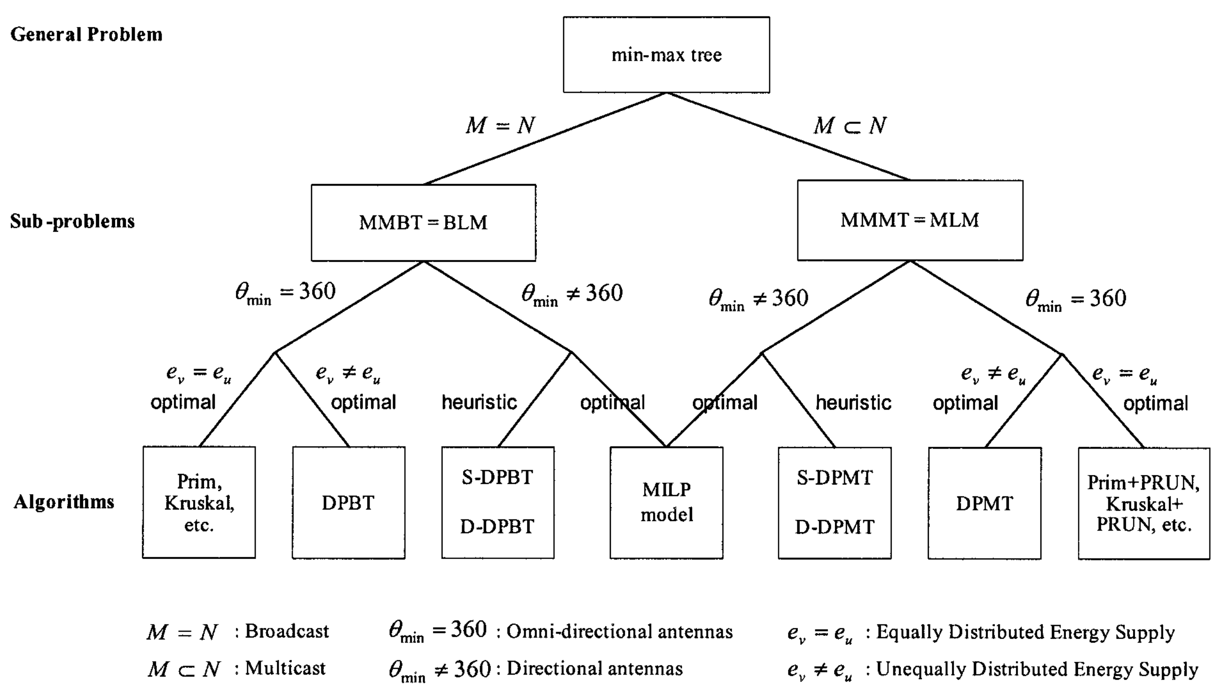


Figure 1-4. The framework of the BLM/MLM problem formulation and solutions

Figure 1-4 depicts all scenarios under various conditions and their BLM/MLM problem formulations that have been solved by this research work.

1.4.2 The MPM/MLM Problems in Mobile Networks

Having completed the analysis of the above "static" problems, we shall extend our experience and knowledge to tackle the dynamic issue. We shall focus on the distributed energy-efficient multicast algorithms for MANETs. Because of the limited resources of mobile nodes, localized operations are ideal that each node can decide on its own behavior based only on the information from all nodes within a constant hop distance. We therefore consider the distributed and localized implementation of our energy efficient algorithms for both MPM and MLM problems and study the effects of mobility on their performance.

Note that in a mobile as hoc network, the distance between any two nodes may keep changing, and then the corresponding minimum transmission power varies continuously since each node has the power control capability[1]. However, it is impractical (or even impossible) to continuously adjust the transmission power when transmitter and receiver move freely. We shall apply a quantization approach such that the transmission power adjustment is only allowed for a series of discrete levels. Any two nodes separated among a certain range use the same transmission power to communicate with each other, and the power should be adjusted only when their distance changes beyond this range due to mobility.

We have used the same simulation tool in Section 1.4.1 with an extension to support node mobility to evaluate the performance of localized operations for energy efficient multicast algorithms. Considering the transient tree disconnections in mobile networks, we shall extend the performance metrics (tree power and tree lifetime) used in static networks and define two new metrics in mobile networks: MCP (Multicast Communication Power) and MCL (Multicast Communication Lifetime). We shall also evaluate and compare the performance of our algorithms using these metrics under various mobilities.

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[1] The power control capability means that, by adjusting the transmission power level, the sender could reach destination nodes located at different distances.

1.5 Contributions

The contributions of our research work are summarized as follows:

(1) A general analytical MILP model for the BPM/MPM problem in static ad hoc networks with adaptive antennas.

(2) A group of heuristic algorithms for the BPM/MPM problem to handle larger static ad hoc networks with omni-directional / directional antennas. We shall show that our heuristic algorithm MIDP (Multicast Incremental-Decremental Power) compares well with MIP [WiNg00] in omni-directional antenna applications, and our algorithms D-MIDP (Directional MIDP) and RB-MIDP (Reduced Beam MIDP) outperform the best algorithms RB-MIP and D-MIP [WiNg02c, WiNg02d] known so far in adaptive antenna applications.

(3) Two polynomial-time optimal algorithms DPMT (Directed Prim Multicast tree) and DPBT (Directed Prim Broadcast tree) for the BLM and MLM problems respectively, in static ad hoc networks with omni-directional antennas, with lower complexity than those proposed in previous work like [KaPo03a, FlKa03].

(4) A general analytical MILP model for the BLM/MLM problem in static ad hoc networks with adaptive antennas.

(5) A group of heuristic algorithms for the BLM/MLM problem to handle larger static ad hoc networks with directional antennas. Our experimental results show that the S-DPMT (Static-weight DPMT) algorithm can construct a maximum lifetime path between any two nodes, and in all other cases, the D-DPMT/D-DPBT (Dynamic-weight DPMT/DPBT) algorithm exploits the property of minimizing the bottleneck arc weight, and thus provides much better performance than other proposals (RB-MIP-β and D-MIP-β given in [WiNg02c, WiNg02d]) over a wide range of network examples.

(6) A group of distributed energy-efficient multicast algorithms with localized operations for both BPM/MPM and BLM/MLM problems in static/mobile ad hoc networks. As various extensions of the BEEM (Basic Energy Efficient Multicast) algorithm, two

distributed algorithms DMPM (Distributed Minimum Power Multicast) and DMLM (Distributed Maximum Lifetime Multicast) perform very well in terms of energy saving and lifetime enhancement respectively under different node mobilities.

(7) Performance evaluation of our algorithms using various parameters and measures like minimum tree power and normalized tree power in both static and mobile ad hoc networks over a wide range of randomly generated network examples.

1.6 Thesis Organization

The remaining of the thesis is organized as follows.

- Chapter 2 gives a detailed literature survey of related work.
- Chapter 3 presents the system model, terminology and assumptions used throughout this thesis.
- Chapters 4 develops the formulation of the BPM/MPM problem in a form of Mixed Integer Linear Programming in networks with omni-directional antennas and adaptive antennas, and presents a group of heuristic algorithms that can easily handle larger networks.
- Chapter 5 addresses the globally optimal solutions for the BLM/MLM problem in the special case of using omni-directional antennas, and then extends the results for the same problem into the networks with adaptive antennas by developing an Integer Programming formulation and presenting a group of heuristic algorithms that can easily handle larger networks.
- Chapter 6 attempts to provide the distributed implementation of our energy efficient algorithms and evaluates the effects of mobility on their performance.
- Chapter 7 compares the performance among our proposed algorithms and provides a design guideline.
- Chapter 8 summarizes our findings and points out several future research problems.
- Our Appendix contains some algorithm descriptions and long proofs in Chapters 4 to 6.

1.7 Publications

Various results from the original work of this thesis have been published in the refereed journals and conference proceedings. We summarize these papers in the sequence of Chapters to which they are related.

Chapter 4

[1] Song Guo and Oliver Yang, "Minimum-Energy Multicast in Wireless Ad Hoc Networks with Adaptive Antennas: MILP Formulations and Heuristic Algorithms", to appear in *IEEE Transaction on Mobile Computing* in 2006.

[2] Song Guo and Oliver Yang, "A Constraint Formulation for Minimum-Energy Multicast Routing in Wireless Multi-hop Networks", to appear in *ACM Wireless Networks Journal* in February 2006.

[3] Song Guo and Oliver Yang, "Joint Optimization of Energy Consumption and Antenna Orientation for Multicasting in Static Ad Hoc Wireless Networks", accepted by *IEEE Transaction on Wireless Communications* on November 2005.

[4] Song Guo and Oliver Yang, "Minimum Energy Multicast Routing for Wireless Ad-hoc Networks with Adaptive Antennas", *Proceedings IEEE ICNP*, Berlin, Germany, October 2004, pp. 151 - 160.

[5] Song Guo and Oliver Yang, "Minimum-Energy Multicast Routing in Static Wireless Ad Hoc Networks", *Proceedings IEEE VTC*, Los Angeles, USA, September 2004, pp. 3989 - 3993.

[6] Song Guo and Oliver Yang, "Improving Energy Efficiency for Multicasting in Ad-hoc Networks with Directional Antennas", *Proceedings IEEE WiMob*, Montreal, Canada, August 2005, pp. 344 - 351.

[7] Song Guo and Oliver Yang, "Minimum-Energy Multicast Tree Construction in Wireless Ad Hoc Networks", *Proceedings Biennial Symposium on Communications*, Kingston, Canada, June 2004, pp. 156-158.

[8] Song Guo and Oliver Yang, "Antenna Orientation Optimization for Minimum-Energy Multicast Tree Construction in Wireless Ad Hoc Networks with Directional Antennas", *Proceedings ACM MobiHoc*, Tokyo, Japan, May 2004, pp.234 - 243.

[9] Song Guo and Oliver Yang, "A Dynamic Multicast Tree Reconstruction Algorithm for Minimum-Energy Multicasting in Wireless Ad Hoc Networks", *Proceedings IEEE IPCCC*, Phoenix, USA, April 2004, pp. 637 - 642.

[10] Song Guo and Oliver Yang, "Minimum Energy Broadcast Routing in Wireless

Multi-hop Networks", *Proceedings IEEE IPCCC*, Phoenix, USA, April 2003, pp. 273 - 280.

Chapter 5

[11] Song Guo and Oliver Yang, "A Framework for the Multicast Lifetime Maximization Problem in Energy-Constrained Wireless Ad-hoc Networks", submitted to *ACM Wireless Networks Journal* on August 2004.

[12] Song Guo and Oliver Yang, "On upper bound and heuristics for multicast lifetime maximization using dynamic routing in energy-limited wireless ad hoc networks", *Proceedings IEEE Mobiquitous*, San Diego, USA, July 2005, pp. 426 - 431.

[13] Song Guo and Oliver Yang, "Optimal Tree Construction for Maximum Lifetime Multicasting in Wireless Ad-hoc Networks with Adaptive Antennas", *Proceedings IEEE ICC*, Seoul, Korea, May 2005.

[14] Song Guo and Oliver Yang, "Multicast Lifetime Maximization for Energy-Constrained Wireless Ad-hoc Networks with Directional Antennas", *Proceedings IEEE Globecom*, Dallas, USA, December 2004, pp. 4120 - 4124.

Chapter 6

[15] Song Guo and Oliver Yang, "Distributed Energy Efficient Multicast Algorithm in Mobile Ad Hoc Networks", accepted by *IEEE Transaction on Parallel and Distributed Systems* on November 2005.

[16] Song Guo and Oliver Yang, "Maximizing Multicast Communication Lifetime in Wireless Mobile Ad Hoc Networks", submitted to *IEEE Journal on Selected Areas in Communications* on June 2005.

[17] Song Guo and Oliver Yang, "Performance of Distributed Algorithms for Maximizing Multicast Lifetime in Mobile Ad Hoc Networks", *Proceedings ACM Workshop on Performance Evaluation of Wireless Ad Hoc, Sensor, and Ubiquitous Networks*, 2006, pp. 152 - 159.

Chapter 2. Literature Survey

We shall provide a taxonomy of recent literature for minimum-energy and maximum-lifetime problems over static ad hoc networks, and review the main proposals for each sub-problem. We also compare and discuss these proposals. Other related work on MANET multicast protocols and energy efficiency issues in sensor networks shall also be briefly reviewed in this chapter.

2.1 Minimum Power Multicast Algorithms

This class of optimization problem was first studied in its simplest form as the BPM problem under omni-directional antenna networks. Later on, some results have been extended to more general problems, e.g. for the multicast or/and directional antenna scenarios. We shall provide below the evolution of the recent research.

2.1.1 Proposals for Broadcast and Omni-directional Antenna Scenarios

The BPM problem in static ad hoc networks with omni-directional antennas was shown to be NP-complete [ClCr01, CaHu02]. To solve such hard combinatorial optimization problems, there are usually two approaches to take. One is to use the exact method to obtain globally optimal solutions, which often may not be known for a given problem. For an NP-complete or NP-hard problem, we can get optimal solutions by using mixed integer linear programming (MILP) formulation. Albeit valuable for theoretical reasons, practical usage of MILP is usually limited to a small network size, as is the usual case for many integer programming problems. Therefore, to deal with instances in real world applications, we still have to rely on the heuristics, which have received more attention. An efficient and effective heuristic usually produces satisfactory results in polynomial time, but does not always guarantee global optimality. The heuristic approximate algorithms can be broadly categorized into the following three classes.

1) Spanning Tree Algorithms – A class of greedy heuristics that obtains feasible solutions by constructing a rooted spanning tree.

2) Topology Control Algorithms – Based on transmission power adjustment, topology control algorithms assign transmission powers to the nodes such that the resulting topology (not necessarily being a tree) achieves certain connectivity properties, and such that certain objective functions of the transmission powers are optimized.

3) Local Search Algorithms – This class heuristics iteratively improve an initial feasible solution (e.g. obtained by a spanning tree algorithm or topology control algorithm) using local search technology [PaSt82].

The classification for the solutions to the BPM problem in static ad hoc networks with omni-directional antennas is summarized in Fig. 2-1. Under each category, we shall review several typical proposals in the recent literature.

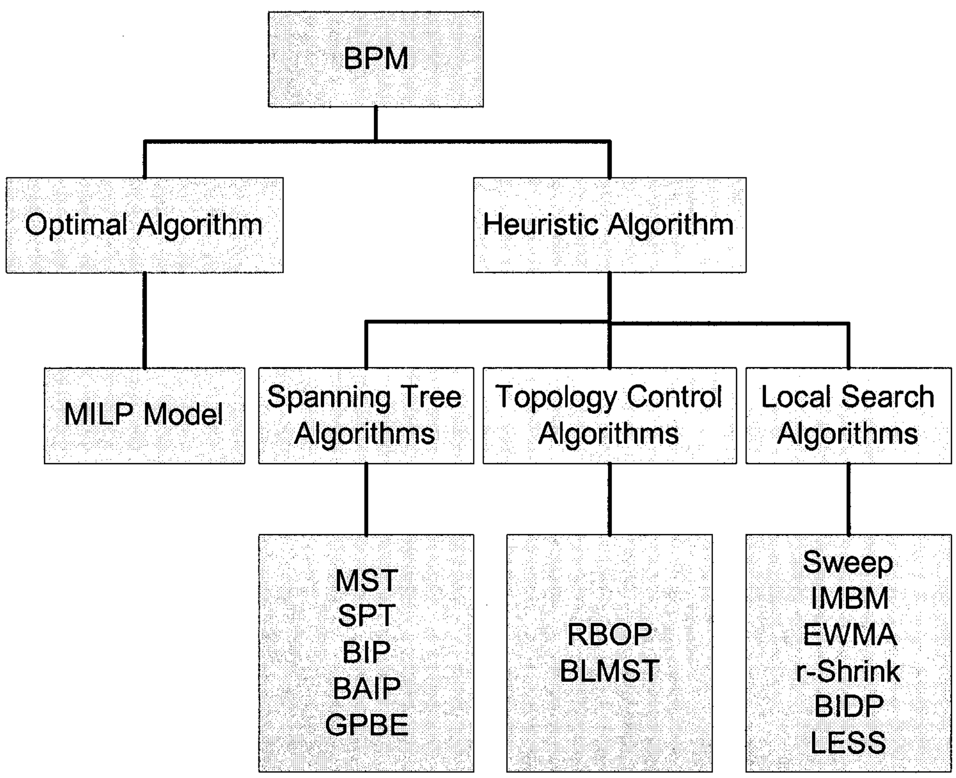


Figure 2-1. A classification for the solutions to the BPM problem in static ad hoc networks with omni-directional antennas

2.1.1.1 Optimal Solution

Approaches to solve the BPM problem to optimality do not appear extensively in the literature. Das *et al.* [DaMa03a] proposed three different MILP models for the BPM problem. One of these is a network flow model with binary design variables, which was also briefly discussed in [KlNa04]. At the same time, Guo *et al.* [GuYa03] presented another form of MILP model based on the new concept "virtual relay". Altinkemer *et al.* [AlSa04] reformulate the same problem as an integer programming model of set covering type. Recently, a multi-commodity flow model [Yuan05] has been presented for the MPM problem. The advantage of this new formulation is that both the LP (linear programming) relaxation and the Lagrangian relaxation of the integer programming formulation yield a very good approximation to the integer optimum, and thus provide us with an effective tool for evaluating the performance of heuristics for minimum-energy broadcast routing.

2.1.1.2 Spanning Tree Algorithms

One efficient paradigm for achieving multicast involves building source-based multicast trees which connect all the group members and routing multicast packets along the tree links. Only the relay nodes (also known as forwarding nodes) in the multicast tree forward the multicast packet, while all other nodes do not. A multicast tree is an acyclic directed graph with a source node called the root with no incoming arcs, and each other node v has exactly one incoming arc. In this way, the multicast tree rooted at the source spans all the other multicast members who are receivers. Many spanning tree algorithms apply a theme that iteratively "grows" a set S of covered nodes starting from the source. At each step, according to some objective function S is expanded to cover one or more new nodes until all nodes in the network are in S.

Over the years, several spanning tree algorithms for the BPM problem have been developed, e.g. MST (Minimum Spanning Tree) [WiNg00], SPT (Shortest Path Tree) [WiNg00], BIP (Broadcast Incremental Power) [WiNg00], BAIP (Broadcast Average Incremental Power) [WaCa02], GPBE (Greedy Perimeter Broadcast Efficiency) [KaPo03b], COBRA (Center Oriented Broadcast Routing Algorithm) [KaPo04], Steiner tree based heuristics [Lian02], Ant

Colony System (ACS) [DaMa02], etc.

MST The MST (Minimum Spanning Tree) is a straight greedy approach based on the use of minimum spanning tree. The MST heuristic first applies Prim's algorithm [Prim57] to obtain a MST, and then to orient it as a tree rooted at the source node. For a graph of n nodes and m edges, the complexity of MST is $O(n^3)$ when a straightforward implementation of Prim's algorithm is used [AhMa93]. However, a more sophisticated implementation using a Fibonacci heap yields complexity $O(m + n\log n) = O(n^2)$ for a fully connected network. The distributed MST algorithm proposed by Gallager *et al.* [GaHu83] to construct a minimum-weight spanning tree. The total number of messages required is at most $2m + 5n\log_2 n$. Theoretical performance of the heuristic in terms of approximation ratio[1] has been analyzed by Wan *et al.* [WaCa02]. The approximation ratio of MST is between 6 and 12. Its upper bound has been improved to 7.6 by Flammini *et al.* [FlNa04] and further improved to 6.33 recently by Navarra [Nava05].

SPT The SPT (Shortest Path Tree) is a spanning tree algorithm that makes use of a shortest-path tree formed from the best unicast paths to each individual destination from the source node (broadcast session initiator). This heuristic first applies Dijkstra's algorithm [Dijk59] to obtain a SPT, and then to orient it as a tree rooted at the source node. The complexity of SPT is $O(n^2)$ when implemented by means of the Dijkstra's algorithm. The approximation ratios of SPT is at least $n/2$.

BIP The BIP (Broadcast Incremental Power) heuristic algorithm [WiNg00, WiNg02a] is similar in principle to the standard Prim's algorithm for the formation of minimum spanning trees. It maintains a single tree rooted at the source node throughout its execution and exploits the WMA property in the formation of the broadcast trees. The tree starts from the source node,

[1] An algorithm for a problem has an approximation ratio of $\rho(n)$ if, for any input of size n, the expected cost C of the solution produced by the randomized algorithm is within a factor of $\rho(n)$ of the cost C^* of an optimal solution: $\max\{\frac{C}{C^*}, \frac{C^*}{C}\} \le \rho(n)$.

and new nodes are added to the tree one at a time on the minimum incremental cost basis, instead of the minimum cost in Prim's algorithm, until all nodes are included in the tree. The incremental cost of adding a new node to the tree is the minimum additional power increased by some node in the current tree to reach this new node. The complexity of BIP is $O(n^3)$ and the analytical performance has showed that the approximation ratio of BIP is between 13/3 and 12 [WaCa02]. Wieselthier *et al.* [WiNg02b] extended their work by proposing two distributed versions of BIP: Dist-BIP-A (Distributed-BIP-All) and Dist-BIP-G (Distributed-BIP-Gateways). In both schemes, each node first calculates its local BIP tree. Then the global tree can be constructed by combining the local trees in certain temporal manner. After the global tree is constructed, the sweep operation (which is essentially a centralized procedure) can be applied. The performance of both distributed versions without the sweep operation is degraded 30% - 50% as compared to that of BIP.

BAIP The BAIP (Broadcast Average Incremental Power) is an algorithm [WaCa02] as a slight variation of BIP. Like BIP, a rooted tree, which starts with the source node, is maintained throughout the execution of the algorithm. However, unlike BIP, many new nodes can be added one at a time. The new nodes added are chosen to have the minimal average incremental cost, which is defined as the ratio of the minimum additional power increased by some node in the current arborescence to reach these new nodes to the number of these new nodes. The approximation ratios of BAIP is at least $4n/\ln n - o(1)$ [WaCa02], where n is the number of nodes.

GPBE The GPBE (Greedy Perimeter Broadcast Efficiency) algorithm [KaPo03b] applies the same tree formation procedure as the BIP algorithm, but based on another greedy decision metric *broadcast efficiency*, which is defined as the number of newly covered nodes reached per unit transmission power. The introduction of such metric is based on the observation that, over a whole deployed region, the wireless broadcast advantage is the most beneficial in a sub-region where nodes are most densely deployed because more nodes can be simultaneously reached with the same amount of transmit power. Both theoretical analysis and simulation have shown that the average broadcast efficiency is location dependent and the center of symmetric deploy region is the optimal place where the broadcast efficiency can be probabilistically best utilized. Furthermore, when the source is located near the center of deploy region, the broadcast efficiency plays an important role and GPBE performs better than BIP.

The comparison of the above algorithms is summarized as in Table 2-1.

Table 2-1. Comparison of the spanning tree algorithms for the BPM problem in static ad hoc networks with omni-directional antennas

	Implementation fashion	**Complexity**	**Criteria of new node inclusion**	**Approximation ratio**
MST	centralized distributed	time: $O(n^2)$ communication: $O(n^2)$	minimum cost	$6 \le \rho_{MST} \le 6.33$
SPT	centralized	time: $O(n^2)$	minimum distance to the source	$n/2 \le \rho_{SPT}$
BIP	centralized	time: $O(n^3)$	minimum incremental cost	$13/3 \le \rho_{BIP} \le 12$
BAIP	centralized	time: $O(n^3)$	minimal average incremental cost	$4n/\ln n - o(1) \le \rho_{BAIP}$
GPBE	centralized	time: $O(n^3)$	maximum broadcast efficiency	unknown

2.1.1.3 Topology Control Algorithms

Topology control problems are concerned with the assignment of power values to the nodes of an ad hoc network so that the power assignment leads to a graph topology satisfying some specified properties. Topology control algorithms let each wireless device *locally* adjust its transmission range and select certain neighbors for communication, while maintaining a structure that can support energy efficient routing and improve the overall network performance. It has been shown in [Li03] that the total power required on such underlying structure could be as large as $O(n)$ times of the optimal solution.

RBOP The RBOP (Relative Neighborhood Graph based Broadcast Oriented Protocol) algorithm [CaSi03] is based on the concept of a relative neighborhood graph (RNG), proposed by Toussaint [Tous80]. RNG consists of all edges uv such that uv is not the longest edge in any triangle uvw. That is, uv belongs to RNG if there is no node w such that $uw < uv$ and $vw < uv$. The RNG structure can be obtained locally with linear communication complexity [BoJe02]. In RBOP, the broadcast is initiated at the source and propagated, following the rules of neighbor elimination, on the topology derived under RNG. Simulation results show that the performance degradation could be as high as 100% as compared to BIP.

BLMST The BLMST (Broadcast on the Local Minimum Spanning Tree) algorithm [LiHo04] first derives an underlying topology by using LMST [LiHo03]. It is a localized topology control algorithm that aims to assign the transmission power of each node so as to maintain network connectivity while consuming the minimum possible power. In LMST, each node calculates local minimum spanning tree of itself and its 1-hop neighbors. A link is in LMST if and only if both ending nodes select each other in their respective trees. BLMST is a flooding algorithm applied to the network topology derived by LMST, with one optimization that if a node has received a broadcast message from all its neighbors (in the LMST-derived topology) or has known that all its neighbors have received the message by overhearing, it will not relay the message. This reduces the extent to which flooding is performed. BLMST requires only local (rather than global) information and achieves 100 % broadcast coverage. The simulation results indicate that the performance of BLMST is significantly better than the performance of RBOP and comparable to BIP. If the communication overheads incurred in collecting all the information is considered, BLMST can outperform BIP in large-scale wireless networks where the network topology changes from time to time.

The comparison of the above algorithms is summarized as in Table 2-2.

Table 2-2. Comparison of the topology control algorithms for the BPM problem in static ad hoc networks with omni-directional antennas

	Implementation fashion	Communication complexity	Underlying topology	Approximation ratio
RBOP	localized	$O(n)$	Relative Neighborhood Graph (RNG)	$O(n)$ for the underlying structure
BLMST	localized	$O(n)$	Local Minimum Spanning Tree (LMST)	

2.1.1.4 Local Search Algorithms

Local search algorithms perform a *walk* on multicast forwarding structures. The walk starts from an initial multicast topology obtained by algorithm X (any spanning tree algorithm or topology control algorithm). In each step, a local search algorithm moves to a new multicast topology obtained by removing some of the links of the previous one and adding new links, so that the necessary connectivity properties are maintained. The rule used in each move for selecting the next multicast topology is related to power and the algorithm terminates when there is no further improvement within the neighborhood[1] [PaSt82]. Therefore, the final tree must have a lower power than the initial feasible tree. In other words, the approximation ratio of a local search algorithm is in general less than or equal to ρ_X. The design of neighborhood is one of the most crucial steps for a successful local search heuristic. There are mainly two types of neighborhood, tree based and power assignment based, in the recent literature.

[1] A neighborhood is the set of all possible multicast topologies that can be obtained using a local research from a given multicast topology.

Sweep The Sweep algorithm was proposed in [WiNg00] as a simple heuristic for improving performance the BIP algorithm by detecting transmissions that are redundant or can be reduced in power. Clearly, it can be used on any spanning tree as well. The Sweep algorithm starts from the initial given tree and proceeds in steps. It examines each node v in the tree as follows. If the current transmission power of node v can reach some non-ancestor node (e.g. node u) in the tree, then remove the incoming link of node u and add a new link from v to u. Note that such operation keeps the tree topology and would reduce the total energy required by the tree. The algorithm terminates when all nodes have been examined. Typically, a single application of the sweep operation provides significant improvement (it can reduce total tree power by 5% or more); small further improvement can often be obtained by repeating the sweep once more, but little improvement has been found by additional applications of this procedure.

IMBM The IMBM (Iterative Maximum-Branch Minimization) algorithm [LiNi01] was another effort to construct power efficient broadcast trees. It begins with a basic broadcast tree in which the source directly transmits to all other nodes. Then it attempts to approximate the minimum-energy broadcast tree by iteratively replacing the maximum branch (the longest link in the tree) with a less-power 2-hop path that connects the ending nodes of this branch. Such operation is applied for each transmitting node until the total required power cannot be further minimized. The simulation results show that when the propagation constant α is 2, the IMBM algorithm demonstrates a considerable better performance than the BIP algorithm, but when the propagation constant is getting larger, the advantage of IMBM shrinks.

EWMA The EWMA (Embedded Wireless Multicast Advantage) algorithm [CaHu02] modifies a minimum spanning tree to form a better power assignment. It is clear that the algorithm can start with any spanning tree. The basic idea is to start from the root node, check to see if expanding the root's power to cover the children of one of its children would save some power. The saving comes from the fact that once the children of a node are covered, the power assignment at that node could be reduced to 0. This process propagates itself until all nodes are covered. One advantage of EWMA is that it is a better modification of MST, and therefore its performance ratio is at least as good as MST. The authors also showed by simulations that it works better than BIP on average. Additionally, the authors also gave a distributed protocol to implement this algorithm. The distributed MST is run first, and then the distributed EWMA is run by exchanging information between 2-hop neighbors. The disadvantage of this distributed protocol is that it is still highly complicated, requiring two rounds of computations, and may not perform well under frequent topological changes.

***r*-Shrink** The *r*-shrink procedure [DaMa03c] is a heuristic for improving the solutions obtained using fast sub-optimal algorithms (a minimum spanning tree algorithm or the BIP algorithm, for example). Given a transmission from node v to node u, with nodes w_0, w_1, …, and w_{k-1} covered implicitly in an ordering of the nodes with respect to their distances from node v. The *r*-shrink operation applied to node v implies a reduction of its transmission power level (or, shrinkage of its transmission radius) by r notch, such that the farthest node now reached is w_{k-r} instead of u. Some children may be temporarily disconnected from the tree as a result of the shrinkage operation, and should be accommodated from any of their non-descendants (excluding the current parent) in order to keep the tree topology. If the new tree obtained from such operation has a lower tree power, the *r*-shrink algorithm shall perform further improvement on this updated tree, otherwise check other transmitting node on the previous tree. The transmission radii of the transmitting nodes in the tree are shrunk sequentially until no further improvement is possible. Simulation results show that considerably better solutions can be obtained if the 1-shrink procedure is applied to trees grown using the BIP algorithm. Similar improvements are possible if it is applied to improve directed minimum spanning trees of the networks.

BIDP The BIDP (Broadcast Incremental-Decremental Power) algorithm [GuYa04a] is a further improvement than the post-sweep operation because it improves the performance not only by eliminating unnecessary transmissions, but also by globally searching and reconstructing the broadcast. In fact, the sweep operation can be seen as a special case of the post-processing of

BIDP. The BIDP algorithm improves a broadcast tree obtained by the BIP algorithm. It then iteratively reconstructs the broadcast tree by switching tree arcs in order to obtain maximal power saving using global topology information. It then iteratively reconstruct this tree towards a better solution in a power-decremental manner that the total power required for the broadcast tree is continuously decreased at each tree reconstruction operation until no further power savings or the number of iterations reaches a pre-defined constant number.

LESS The Largest Expanding Sweep Search (LESS) algorithm [KaPo04b] is a heuristic centralized algorithm within the framework of local search that improves upon some of the shortcomings of the EWMA algorithm. The EWMA algorithm is always to remove a transmitting node at each improvement operation, while the LESS algorithm generalizes this operation in such a way that in exchange for slight increase in transmit power of a node, multiple nodes can eliminate or reduce their transmission powers, and hence the overall transmit power can be reduced. This operation is called the generalized Expanding Sweep Search (ESS). The LESS algorithm iteratively finds the largest power saving using the generalized ESS operation and performs a corresponding tree update until there is no further gain. It has been observed that there is still a significant gain over the EWMA tree as initial feasible solution, but not vice-versa. Applying EWMA on a LESS tree has no effect, because LESS finds every possible gain. The performance of LESS is further strengthened (the average performance is within 1.12% of the optimal solution) by using iterated local optimization (ILO) techniques at the cost of additional computational complexity [KaPo05].

The comparison of the above algorithms is summarized as in Table 2-3.

2.1.2 Approaches for Extension to Multicast Scenarios

Some MILP models for the BPM problem have been extended in order to obtain the optimal solution of the more general MPM problem. Such an extension can be found in [GuYa04e], in which a new group of linear constraints are incorporated for the more general multicast scenarios. Similarly, the multi-commodity flow model [Yuan05] can be easily adapted to minimum-energy multicast routing.

Table 2-3. Comparison of the local search algorithms for the BPM problem in static ad hoc networks with omni-directional antennas

	Implementation fashion	Complexity of single round improvement	Search neighborhood	Approximation ratio
Sweep	centralized	time: $O(n)$	Tree-based	Less or equal to ρ_X, where X is the algorithm used to obtain the initial feasible solution.
IMBM	centralized	time: $O(n)$	Tree-based	
EWMA	centralized	time: $O(n^3)$	Tree-based	
	distributed	communication: unknown		
***r*-Shrink**	centralized	time: $O(n^2)$	Power assignment based	
BIDP	centralized	time: $O(n^3)$	Tree-based	
LESS	centralized	time: $O(n^3)$	Power assignment based	

For the MPM problem, there are mainly three different heuristic approaches as follows:

1) Pruning
2) Steiner minimum Tree
3) Local Search

2.1.2.1 Pruning Approach

In most of the literature [WiNg00, WiNg01, WiNg02a, WiNg02e], the MPM problem was studied using the same approach as the BPM problem except that the final minimum power multicast tree is obtained by pruning from the minimum power broadcast structure (obtained from the spanning tree algorithms or topology control algorithms, for example) all transmissions that are not needed to reach the member of the multicast group.

We use a prefix "P-" before a broadcast algorithm's name to denote the multicast algorithm based on pruning. For example, corresponding to MST, SPT, and BIP, the three heuristics for the MPM problem are referred to as pruned minimum spanning tree (P-MST), pruned shortest-path tree (P-SPT), and pruned broadcasting incremental power (P-BIP), respectively. Note that the P-BIP algorithm is originally denoted as MIP (Multicast Incremental Power) in [WiNg00]. Theoretical analysis in terms of approximation ratio has shown that the pruning operation is not good enough [WaCa04]. The approximation ratios of these three heuristics are all at least as large as $\Omega(n)$, where n is the number of nodes in the network. In fact, P-SPT has an approximation ratio of at least $(n - 1)/2$, P-MST has an approximation ratio of at least $n - 1$, and P-BIP has an approximation ratio of at least $n - 1 - o(1)$.

2.1.2.2 Steiner Minimum Tree Approach

This approached has been studied on the existence of heuristics for the MPM problem with constant approximation ratio in [WaCa04], in which Wan *et al.* first prove that any heuristic of ρ-approximation algorithm for Steiner minimum tree [HwRi92] gives rise to a heuristic for the MPM problem of approximation ratio at most $c\rho$, where is the approximation ratio of MST. The best result is $6 \leq c \leq 6.33$ given in [Nava05]. To our best knowledge, the best-known approximation algorithm so far is Robins–Zelikovsky Steiner tree heuristic [RoZe00] with an approximation ratio approaching $\rho = 1 + \ln 3/2$. In the following, we present another two 2-approximation algorithms.

SPF The Takahashi–Matsuyama Steiner tree heuristic [TaMa80] leads to SPF (Shortest Path First) algorithm [WaCa04]. This algorithm can be regarded as an adaptation of MST for the MPM problem. Throughout the execution, it maintains a directed tree rooted at the source node. Initially, the tree contains only the source node. At each iterative step, the tree is grown by one path from with least total power that can reach a destination not yet in the tree. This path can be found by collapsing the entire tree into one artificial node and then applying the single-source shortest-path algorithm. This procedure is repeated until all the destinations are included in the tree. Indeed, when the communication session is a broadcast, it acts the same way as MST. The approximation ratio of SPF is between 6 and $2c$.

MIPF The Minimum Incremental Path First (MIPF) algorithm [WaCa04] is an adaptation of BIP for the MPM problem. MIPF is implemented as follows. Similarly, it maintains a directed tree rooted at the source node throughout the execution. Initially, the tree contains only the source node. At each iterative step, the algorithm first finds a path from the tree with the least incremental power that can reach a destination not yet in the tree, where the incremental power of a path is defined as its total power minus the transmission power in the tree of its first node. After this path is found, the entire path is attached to the tree. This procedure is repeated until all destinations are included in the tree. The approximation ratio of MIPF is between 13/3 and $2c$.

2.1.2.3 Local Search Approach

Another approach is using the local search technology to refine a multicast tree iteratively. In general, the local search algorithms described in Section 2.1.1.4 can be revised for the MPM problem.

S-REMiT The S-REMiT (Refining Energy-Efficient Source-shared Multicast Tree) algorithm [WaGu03a] tries to minimize the total tree power of the initial multicast tree using local search technology. As a distributed algorithm, the S-REMiT algorithm uses MST or SPT as the initial solution and improves the multicast tree energy efficiency by switching some tree nodes from their respective parent nodes to new corresponding parent nodes. This distributed algorithm is not scalable because the message complexity of one round in which each node performs at most one parent changing is $O(n^2)$ in a fully connected network.

MIDP The centralized MIDP (Multicast Incremental-Decremental Power) algorithm [GuYa04a] is an extension of BIDP. It is similar to the S-REMiT algorithm but with different search neighborhood. The search neighborhood of S-REMiT is a multicast tree with the same set of nodes as the initial given multicast tree, while the search neighborhood of MIDP has not such constraint. In the MIDP algorithms, the pruning operation is embedded in the iterative multicast tree reconstruction, where the objective is to minimize the total tree power required by the multicast tree, instead of the broadcast tree.

The comparison of the reviewed algorithms in this section is summarized as in Table 2-4.

Table 2-4. Comparison of the algorithms for the MPM problem in static ad hoc networks with omni-directional antennas

	Extension Approach	Implementation fashion	Complexity	Approximation ratio
P-SPT	pruning	centralized	additional time: $O(n)$	$(n-1)/2 \leq \rho_{\text{P-SPT}}$
P-MST	pruning	centralized	additional time: $O(n)$	$n-1 \leq \rho_{\text{P-MST}}$
MIP	pruning	centralized	additional time: $O(n)$	$n-1-o(1) \leq \rho_{\text{MIP}}$
SPF	Steiner minimum tree	centralized	time: $O(n^3)$	$6 \leq \rho_{\text{SPF}} \leq 12.66$
MIPF	Steiner minimum tree	centralized	time: $O(n^3)$	$13/3 \leq \rho_{\text{MIPF}} \leq 12.66$
S-REMiT	Local search	distributed	$O(n^2)$ communication complexity for one round	$\rho_{\text{S-REMiT}} \leq \rho_{\text{P-MST}}$ or $\rho_{\text{S-REMiT}} \leq \rho_{\text{P-SPT}}$
MIDP	Local search	centralized	$O(n^3)$ time complexity for one round	$\rho_{\text{MIDP}} \leq \rho_{\text{MIF}}$

2.1.3 Approaches for Extension to Directional Antenna Scenarios

Since the minimum power broadcast in network with omni-directional antenna problem is NP-complete [ClCr01, CaHu02], the more general multicast problem in network with directional antenna appears to be at least as hard. The MILP model presented in [GuYa04d] is the first formulation that is for the directional antenna scenarios and can be used to obtain the optimal solution. On the other hand, heuristic algorithms do not appear extensively in the literature.

RB-MIP The RB-MIP (Reduced Beam MIP) algorithm [WiNg02c] is a straightforward approach to readjust the beamwidth to the minimum for each node in the tree obtained by an algorithm for omni-direction antenna scenario. It uses this approach. Initially, a low-cost broadcast or multicast tree is formed using MIP under the assumption that the transmitting antennas are omni-directional. Then, after the tree is constructed in this manner, each transmitting node's antenna beamwidth is reduced to the smallest possible value that provides coverage of the node's downstream neighbors, subject to the constraint that the beamwidth is within θ_{min} and 360. We use RB-BIP (Reduced Beam BIP) to denote the version for the broadcast. The time complexity of the centralized algorithm RB-MIP is $O(n^3)$ [WiNg02d].

D-MIP The D-MIP (Directional MIP) algorithm [WiNg02c] was proposed as an extension of the MIP algorithm for the situation of using adaptive antennas. It utilizes wireless multicast advantage property in the core of the algorithm while building a routing tree. At each step of the tree-construction process, a single node is added, as in MIP algorithm. However, whereas the only variable involved in computing the incremental power in the omni-directional case is the transmission range, the directional-antenna case involves the choices of antenna orientation and beamwidth as well. The time complexity of the centralized algorithm D-MIP is $O(n^3 \log n)$ [WiNg02d]. We use D-BIP (Directional BIP) to denote the version for the broadcast. The analytical performance in terms of approximation ratio of both RB-MIP and D-MIP are unknown so far, while the simulation results [WiNg02c] show that D-MIP outperforms RB-MIP for all randomly generated instances for directional antenna networks.

D-MIDP The D-MIDP (Directional MIDP) algorithm [GuYa05b] is similar in principle to the MIDP algorithm. It iteratively reconstructs multicast tree in the direction of possibly maximizing the total power saving using global topology information, while the beamwidth constraint is taken into account at each improvement round. It has a time complexity of $O(n^3)$, which is a bit lower than that of the D-MIP algorithm. We use D-BIDP (Directional BIDP) to denote the version for the broadcast. Experimental results have also shown that D-MIDP provides much better performance than D-MIP in directional antenna applications.

Some other proposals like Simple-Linear (SL) and Linear-Insertion (LI) have been developed in [DaWu04, DaWu05]. They have improved the performance of D-BIP only for the situations when the minimum antenna beamwidth is very small.

2.2 Maximum Lifetime Multicast Algorithms

This class of optimization problem has received similar attentions.

2.2.1 Solutions for Omni-directional Antenna Scenarios

In the case of using omni-directional antennas, a few algorithms [KaPo02, KaPo03a, ChSu03, WaGu03b, FlKa03, DaMa03b] use the maximum broadcast/multicast lifetime as an explicit optimization metric to build a maximum-lifetime broadcast/multicast tree.

For the MLB problem, it has been presented in [KaPo02] that the network lifetime for a multicast session can be significantly extended by additionally considering residual battery energy as a parameter in cost metric function for constructing a power efficient routing tree. Recently, this problem has been shown to belong to P-problem class, and several polynomial-time algorithms MDLT (Minimum Decremental Lifetime) [DaMa03b], DMST (Directed Minimum Spanning Tree) [KaPo03a], and DPBT (Directed Prim Broadcast Tree) algorithm [GuYa04b] have been proposed. All of them are essentially various implementations of Prim's algorithm running on a general directed graph and thus could be a time complexity of $O(n^2)$.

For the more general MLM problem, Wang *et al.* [WaGu03b] proposed a heuristic algorithm L-REMiT (Lifetime-Refining Energy efficiency of Multicast Trees) that uses the minimum-weight spanning tree as an initial tree and improves its lifetime by switching (reassigning) the children of a "bottleneck" node to another node in the tree. Finally, a multicast tree is obtained from the "refined" MST (after all possible refinements have been performed) by pruning the tree to reach only multicast group nodes. Similar to the BLM problem, Floreen [FlKa03] has proved that the MLM problem is also a P-problem. Two polynomial-time algorithms have been presented recently. In [Geor03], the author proposed an implementation of Dijkstra's algorithm for solving the MLM problem with a time complexity $O(n^2)$ and proved that the optimal path tree obtained by this process contains a maximum-lifetime multicast tree that can be easily obtained using pruning. Guo *et al.* [GuYa04b] proposed another optimal algorithm DPMT (Directed Prim Multicast Tree) with the same complexity. They proved that the pruned spanning tree obtained from Prim's algorithm on a directed graph has maximum lifetime.

2.2.2 Proposals for Directional Antenna Scenarios

Just recently the MLB/MLM problem for directional antenna scenarios has been proven a NP-hard problem [HoSh05]. The only optimum solution for such difficult problem in the recent literature is the MILP formulation presented in [GuYa05a]. The heuristic algorithm design has received more attention. The following algorithms are for the multicast and the broadcast can be seen as a special case.

D-MIP-β The D-MIP-β (Directional-MIP-β) algorithm [WiNg02c, WiNg02d] extends the total power metric by incorporating residual battery energy into the cost metric in a similar manner as in [ChTa00]. This is a better cost metric than those only using the total power required to maintain the tree, because rapid energy depletion may occur at some nodes. In other words, the cost of a link between Node v and Node u is defined as $c_{vu} = p_{vu} \cdot (E_v(0) / E_v(t))^{\beta}$, where p_{vu} is the RF transmission power needed for the link from node v to node u, $E_v(t)$ is the residual energy at Node v at time t, and β is a parameter that reflects the importance we assign to the impact of residual energy. We use D-MIP-β to denote algorithm D-MIP with different values of β. The algorithm has the same complexity of $O(n^3 \log n)$ as the D-MIP algorithm. Experimental results in [WiNg02d] show the beneficial effects of using β in the range [0.5, 2]. In this way, the heuristic algorithms discourage the inclusion of energy-poor nodes as transmitting nodes in the multicast tree by increasing the cost associated with their use, and therefore provide long-lived multicast trees.

MLR-MD The MLR-MD (for Maximum Lifetime Routing for Multicast with Directional antennas) algorithm [HoSh05] starts with a multicast routing solution first (e.g., a single beam from the source covering all multicast destination nodes) and then iteratively improves lifetime performance of the current solution by identifying the node with the smallest lifetime and revising routing topology as well as corresponding beam-forming behavior for an increased network lifetime. Simulation results have demonstrated that MLR-MD offers consistent performance improvement over the D-MIP algorithm.

2.3 MANET Multicast Protocols

MANET (Mobile Ad Hoc Networks) is another class of WANET as opposed to the *static* ad hoc networks. Some algorithms, e.g. BIP, reviewed in the previous sections require that each node should know global network information (including distances between any two neighboring nodes in the network) in order to decide its own transmission radius. They are suited only for WANETs with no node mobility, and thus cannot be directly applied for multicast protocols in MANETs.

Table 2-5. Literature classification of multicast protocols in mobile ad hoc networks

Proactive Multicast Protocols	AMRIS : Ad hoc Multicast Routing protocol utilizing Increasing id-numberS CAMP : Core-Assisted Mesh Protocol MCEDAR : Multicast Core Extraction Distributed Ad-Hoc Routing AMRoute : Ad-hoc Multicast Routing Protocol FGMP : Forwarding Group Multicast Protocol
On-Demand Multicast Protocols	ODMRP : On-Demand Multicast Routing Protocol AODVM : Ad-hoc On-demand Distance Vector Multicast LAM : Lightweight Adaptive Multicast DDM : Differential Destination Multicast ABAM : Associativity-Based Ad hoc Multicast ADDM : Adaptive Demand-Driven Multicast

There are a few MANET multicast routing (using basic routing algorithms and techniques) protocols developed in the past few years, e.g. ODMRP [LeGe99, LeSu99a, LeSu99b], AODVM [RoPe99], DDM [JiCo01], LAM [JiCo98], ABAM [ToGu00], ADDM [JeHu01, JeJo01], AMRIS [WuTa98], CAMP [GaMa99a, GaMa99b, MaGa99], MCEDAR [SiSi99], AMRoute [BoLi98], and FGMP [ChGe98a, ChGe98b]. They can be categorized into two groups as summarized in

Table 2-5.

In proactive multicast protocols e.g. [WuTa98, GaMa99a, GaMa99b, MaGa99, SiSi99, BoLi98, ChGe98a, ChGe98b], each node maintains one or more tables containing routing information to every other node in the network. All nodes update these tables so as to maintain a consistent and up-to-date view of the network. When the network topology changes the nodes propagate update messages throughout the network in order to maintain consistent and up-to-date routing information about the whole network. There are some attempts e.g. [LeGe99, LeSu99a, LeSu99b, RoPe99, JiCo01, JiCo98, ToGu00, JeHu01, JeJo01] to operate in an on-demand fashion driven by the presence of data packets sent rather than by continuous or periodic background activity in the proactive protocols, only significant portions of the protocol operation are active only for active multicast groups, thus reducing routing protocol's overhead and improving its ability to react quickly to topology changes in the network. All of these have the same feature of maintaining one or more multicast structures (e.g., trees) to effectively and efficiently deliver packets from sources to destinations in the multicast group. However, none of them studied the energy efficiency of multicasting.

Energy efficient broadcast/multicast algorithms in MANETs require constructing an energy-efficient broadcast/multicast tree in a distributed and self-reconfiguring fashion. Moreover, the scheme should be adaptive to mobility, and it should scale well because each node can make its decision based on its local information only. This research topic is still developing in a preliminary stage, and very little work e.g. [LiWu02] has been proposed to study either the dynamic tree reconstruction in a mobile environment or the node mobility effect to the performance of various algorithms.

The core technology of an energy-efficient multicast protocol for MANETs is the distributed topology control algorithm for power efficient operation. However, to our best knowledge, none of the recent energy-efficient distributed algorithms like Dist-BIP, EWMA, and RBOP (reviewed in Section 1.1.1) either studied the dynamic tree reconstruction in a mobile environment, or the node mobility effect to the performance of various algorithms.

2.4 Energy Efficiency Issues in Sensor Networks

One cannot help to notice there is a lot of work done in sensor networks nowadays, e.g. [ChTa99, BhCh01, BhCh02, BlPa02, KaDa02, HoSh04, ZhHo04], which cover similar problems. Although sensor network has been also seen as a type of WANET from the general viewpoint of infrastructureless property, the research subject of our work concentrates the traditional wireless ad hoc networks. A few significant differences between them are summarized as follows:

(1) The number of sensor nodes in a sensor network can be several orders of magnitude higher than the nodes in our ad hoc network, which normally has couple hundreds of nodes.

(2) Sensor nodes are cheap devices (limited computational capacities, memory, and low-cost antenna), while the nodes in our ad hoc network could be equipped with advanced antennas facilities.

(3) Our ad hoc network provides the typical unicast/broadcast/multicast routing protocols, while sensor network normally uses data aggregation as routing scheme to collect information and send back to a sink e.g. [AkSu02].

(4) An ad hoc network uses power control to conserve energy, while sensor network normally uses scheduling technique by turning off the non-used transceivers for a period of time e.g. [HoDo04].

We shall see in the following chapters that these differences differentiate our work from sensor networks. For example, we are especially interested in the multicast applications in networks with directional antennas.

Chapter 3. Modeling, Definitions and Assumptions

In this chapter, we define the system model, terminology, and assumptions used throughout this thesis.

3.1 The General WANET Model

We consider a wireless ad hoc network modeled by a simple weighted directed graph $G(N, A, p)$. The directed graph G has a finite node set N with $|N| = n$ nodes and each node is labeled with a node ID $\in \{1, 2, \ldots, n\}$. An arc set A corresponds to the unidirectional wireless communication links. The arc weight function p: $A \rightarrow R^+$ assigns power to each arc, where R^+ denotes the positive real number set. That is, for each arc $(v, u) \in A$, p_{vu} is the RF power needed for the link from node v to node u.

We assume that each node has the same maximum transmission power p_{max}. Further, each node can dynamically change its transmission power level, not to exceed the value p_{max}. Therefore in our model, the connectivity in the network depends on the transmission power of the nodes, and any directed arc $(v, u) \in A$ if and only if $p_{vu} \leq p_{max}$. Neighbors of node v are the nodes with an arc to it.

3.2 The Antenna Propagation Model

The antenna propagation model is an important consideration for energy consumption in wireless network communications because this model determines how RF power is expended.

3.2.1 Omni-directional Antenna

Omni-directional antennas are used such that all nodes at the same communication distance r can receive the same signal power proportional to $r^{-\alpha}$, where α is a parameter that takes on a typical value between 2 and 4, depending on the characteristics of the communication medium. Without loss of generality, we can assume all receivers to have the same power threshold for signal detection. If we further normalize this threshold to one, and the separation distance to $r > 1$, then the transmitted power p_{vu} required to support the communication between node v and node u will be $p_{vu} = r_{vu}^{\alpha}$.

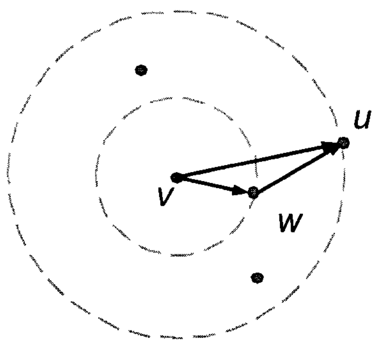


Figure 3-1. Omni-directional antenna propagation model

Due to the non-linear power attenuation, relaying signal on a path between two end nodes may require a lower RF transmission power than direct communication between the same two end nodes. Take Fig. 3-1 as a simple illustration, and assume $\alpha = 2$. Node v can transmit the signal directly to node u, with a power consumption of p_{vu}. Alternatively, it can request node w to relay the same message to node u, with a total power consumption of $p_{vw} + p_{wu} = r_{vw}^2 + r_{wu}^2$. If the angle $\angle vwu$ is obtuse, the second option consumes less total power, i.e., $r_{vw}^2 + r_{wu}^2 < r_{vu}^2$.

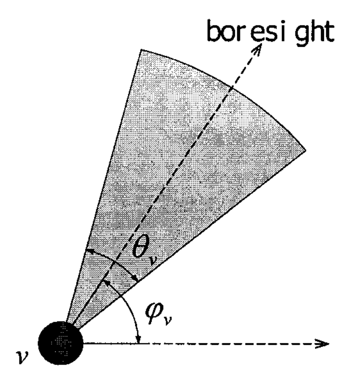


Figure 3-2. Directional antenna propagation model

3.2.2 Directional Antenna

Directional antennas permit energy savings by concentrating RF transmission energy where it is needed. Adaptive array antennas [StTh81, SaWi96] are a set of antenna elements arranged in space whose outputs are combined to give an overall antenna pattern that can differ from the pattern of the individual elements. By varying the phase and amplitude of the individual element outputs before combining, the overall array pattern can be steered in the desired user's direction without physically moving any of the individual elements. We use an idealized directional antenna propagation model as shown in Fig 3-2, where two important parameters, antenna orientation and antenna directionality, are given as follows.

Definition 3-1.
The *antenna orientation* φ_v ($0 \leq \varphi_v < 2\pi$) of node v is defined as the angle measured counter-clockwise from the horizontal axis to the antenna boresight.

Definition 3-2.
The *antenna directionality* θ_v ($\theta_{\min} \leq \theta_v \leq \theta_{\max}$) of node v is defined as its beamwidth.

In this ideal model, the possibility of sidelobe interference is ignored, and all of the transmitted energy is concentrated uniformly in a beamwidth. Similar to the omni-directional antenna propagation model, all receivers have the same signal detection power threshold, which is normalized to one. Therefore, the RF transmission power of node v to transmit to node u in its antenna beam coverage using beamwidth θ_v is

$$p_{vu} = \frac{\theta_v}{2\pi} \cdot r_{vu}^{\alpha} \quad . \tag{3-1}$$

Consequently, the use of narrow beams allows energy saving for a given communication range or extends the antenna range for a given RF transmission power level when compared to the use of omni-directional antennas.

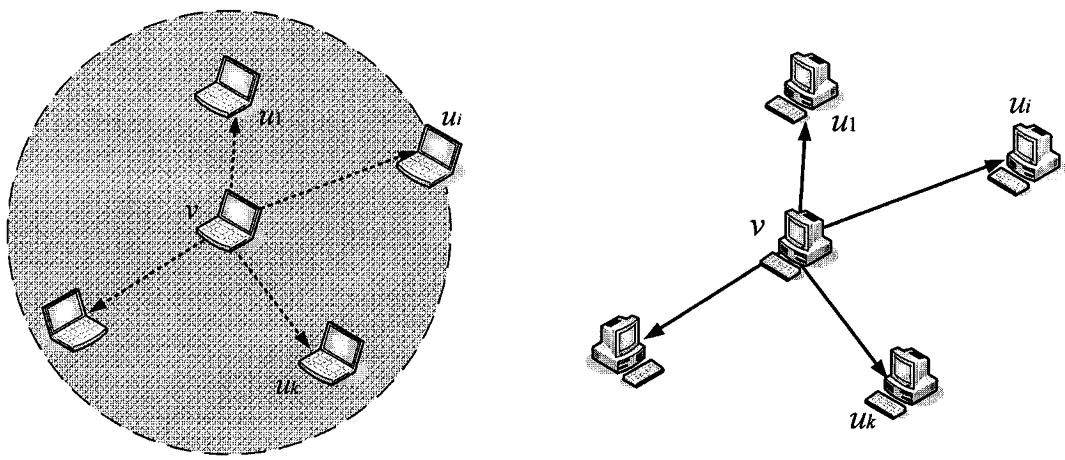
3.2.3 The Wireless Multicast Advantage

Unlike wired networks, broadcasting is an inherent characteristic of wireless transmission. Signal propagation occurs in all directions if networking units are equipped with omni-directional antennas. In such a networking environment, a certain transmission power corresponds to an area of coverage, and a single transmission delivers a message to all nodes within the area. In a wireless network as shown in Fig. 3-3 (a), node v can transmit the same unit message to nodes u_1, u_2, .., u_k, using the maximum of the power for reaching any of them, i.e.

$$p_v^{wireless} = \max\{p_{vu_i}^{wireless} \mid 1 \le i \le k\}.$$

By contrast, in a wired network as shown in Fig. 3-3 (b), so long as there is a wire connecting any two nodes, the transmission power of node v required to communicate with the same set of nodes would be the sum of the power to the individual nodes, i.e.

$$p_v^{wired} = \sum_{i=1}^{k} p_{vu_i}^{wired}.$$



(a) wireless network

(b) wired network

Figure 3-3. Illustration of the wireless multicast advantage

Depending on the relative magnitude of $p_{vu_i}^{wireless}$ and $p_{vu_i}^{wired}$, we can have $p_v^{wireless} \le p_v^{wired}$ or $p_v^{wireless} \ge p_v^{wired}$. In the case of $p_v^{wireless} \le p_v^{wired}$, the reduction in energy needed to broadcast from one node to several others in a wireless network over that needed in a wired network is referred to the *wireless multicast advantage* (WMA) [WiNg00]. It has an obvious benefit for broadcast and multicast applications. However, such property of wireless communication may result in the energy-aware multicast problem even harder. For example, recent study has shown that the BPM problem is NP-compete [ClCr01, CaHu02]. By contrast, the analogue for the similar optimization problem in wired networks can be formulated as the well-known, and easily solved, minimum-cost spanning tree (MST) problem.

3.2.4 Antenna Classification

Table 3-1. Antenna classification

	Omni-directional	Modestly directional	Highly directional
Antenna Directionality	fixed beamwidth	fixed beamwidth	variable beamwidth
Antenna Orientation	unsteerable	steerable	steerable

Based on our signal propagation model, antenna can be loosely classified into omni-directional, modestly directional (switched antenna), and highly directional (adaptive antenna), which are listed in Table 3-1. There have been a number of antenna products[SaSi99] in each of the category. In our study, we shall focus on the adaptive antennas. The omni-directional antenna can be specialized from this idealized directional antenna propagation model letting $\theta_v = 2\pi$ for any node v.

3.3 Energy Consumption Components

There are three major causes of energy depletion in a node v:

(1) power expended for RF propagation p_v^{RF},
(2) power expended in the transmitting p_v^T for operations such as encoding and modulation,
(3) power expended in the receiving p_v^R for operations such as demodulation and decoding.

Let p_{tran} and p_{recv} denote the power expended for transmission and reception processing respectively. We neglect any energy consumption that occurs when the node is simply "on" (idling), although it would be easy to incorporate it into this model.

$$p_v^{RF} = \max\{p_{vu} \mid u \text{ is one of } v\text{'s desired recievers}\} \tag{3-2}$$

$$p_v^T = \begin{cases} 0 & v \text{ is a transmistting node;} \\ p_{tran} & \text{otherwise.} \end{cases} \tag{3-3}$$

$$p_v^R = \begin{cases} 0 & v \text{ is a recieving node;} \\ p_{recv} & \text{otherwise.} \end{cases} \tag{3-4}$$

3.4 Multicast Communication and Multicast Tree

A multicast is a selective broadcast in the sense that only a subset of the system may be interested in receiving the broadcast messages. The goal is to efficiently deliver messages originating from a single or multiple sources to multiple recipients. In wireless ad hoc networks where energy is at a premium, multicast services employ energy-efficient multicast delivery structures such as a *multicast tree*. Therefore, the energy-efficient multicast optimization problem is to construct optimal multicast tree in terms of various multicast tree costs.

We consider a source-initiated multicast operation where any node is permitted to initiate a multicast session, and once a multicast tree is established at the beginning of a multicast session, the same tree is used for the whole session duration. All the nodes involved in the multicast form a directed tree rooted at the source s, i.e. a rooted tree, with a node set $N(T_s) \subseteq N$, and an arc set $A(T_s) \subseteq A$.

Definition 3-3.
A *rooted tree* T_s is an acyclic directed graph with a source node s called the root with no incoming arcs, and each other node v has exactly one incoming arc (f_v, v) where f_v denotes the father of v in the tree.

A node with no out-going arcs is called a leaf node. All other nodes are internal nodes, also called relay nodes. Each multicast session is supported by a set of multicast members M ($m = |M|$), including the source node, all destination nodes, and other relay nodes. Note that a relay node u may belong to the multicast group ($u \in M$) or may not. An important property of a rooted tree can be obtained straightforward from Definition 3-3 and given in Lemma 3-1.

Lemma 3-1.
For any node $v \in N(T_s)$, there exists a single directed path from s to v in the rooted tree T_s.

Finally, a formal multicast tree definition for a wireless ad hoc network with directional antennas is given below.

Definition 3-4.
A directed tree T_s is a *multicast tree* if and only if the following properties are satisfied:

(1) **RTP** (Rooted Tree Property) requires that T_s can span all the multicast members from node s, i.e. $M \subseteq N(T_s)$, and

(2) **ACP** (Antenna Coverage Property) requires that node u must be located within the antenna beam of node v, for any $(v, u) \in A(T_s)$.

Given a multicast tree T_s, let $p_v(T_s)$ be the minimal RF transmission power required at node v to reach its farthest child in the tree T_s, and $\theta_v(T_s) \in [\theta_{min}, \theta_{max}]$ be the minimal antenna beamwidth that can cover all its children in T_s. Considering the wireless multicast advantage property, we obtain the actual RF power $p_v(T_s)$ assigned to node v in T_s as follows:

$$p_v(T_s) = \max\{p_{vu} \mid (v,u) \in A(T_s)\} = \frac{\theta_v(T_s)}{2\pi} \cdot \max\{r_{vu}^{\alpha} \mid (v,u) \in A(T_s)\}. \tag{3-5}$$

3.5 The Mobility Model

In a mobile environment, we shall use a variation of the best-known mobility model called the Random Waypoint Model [JoMa96]. For simplicity, we describe and denote it by the notation $V_{min}/V_{max}/T_{pause}$. In this model, each node starts its journey from a random location to a random destination with a randomly chosen speed between V_{min} and V_{max}. Once the destination is reached, another random destination is targeted after a pause time T_{pause}. The *steady-state average speed* $\overline{V}$ are used to measure the level of mobility in the network, where $\overline{V}$ is given by [YoLi03].

$$\overline{V} = \frac{V_{max} - V_{min}}{\ln V_{max} - \ln V_{min}}. \tag{3-6}$$

Two interesting results [YoLi03] could be derived from the equation above: (1) $\overline{V} \le (V_{max} + V_{min})/2$ (equality holds when $V_{max} = V_{min}$), and (2) $\overline{V} = 0$ when $V_{min} = 0$. The latter is known as the speed decay problem[1]. Since the most common mobility scenario $0/V_{max}/T_{pause}$ would fail due to the speed decay problem, and since node speed is shown to be a significant factor to affect the performance of routing protocols (while pause time is not), our simulations shall use the same model $0^+/V_{max}/0$ used in [JoMa96] with continuous motion and non-zero minimum speed (e.g. 1m/sec) in order to achieve non-zero mobility level $\overline{V}$. We also use different maximum speeds

[1] When $V_{min} = 0$, the longer we run the simulation, the further our results would deviate from the real performance because the level of mobility continuously decreases as the simulation evolves.

to evaluate performance of a number of algorithms with various mobility level $\overline{V}$.

3.6 Assumptions

All the assumptions through the whole study are summarized as follows. Most of these are commonly used in the literature.

(1) Each node can dynamically change its transmission power level from 0 to p_{max}.

(2) Beamwidth of each antenna is fixed $\theta_v = 2\pi$ if using omni-directional antennas. Otherwise, it can be adjusted continuously subject to $\theta_{\min} \leq \theta_v \leq \theta_{\max}$.

(3) Orientation of each antenna can be shifted to any desired direction to provide connectivity to a subset of the nodes that are within communication range when using adaptive antennas.

(4) A single antenna beam is provided for a broadcast / multicast session in which a node participates.

(5) The possibility of sidelobe interference is ignored and all of the transmitted energy is concentrated uniformly in a beamwidth.

(6) For simplicity but without loss of generality, each node has the same maximum transmission power level p_{max}, the transmission processing power p_{tran}, the reception processing power p_{recv}, and the signal detection power threshold.

(7) The MAC and PHY layer characteristics do not influence the performance of the proposed algorithms.

Chapter 4. Broadcast and Multicast Power Minimization Problems

This chapter applies Graph Theory to derive a set of useful theorems which can be used to obtain a general analytical MILP model for the BPM/MPM problem in an ad hoc network with adaptive antennas. We also deduce from this model some special cases involving (1) broadcast communication, (2) the use of omni-directional antennas, and (3) the use of beamwidth-fixed directional antennas. We also use a practical approach by implementing a group of polynomial-time algorithms to handle significantly large networks for which the MILP model may not be computationally efficient.

4.1 Problem Statement

We shall first give the following definition, from which we can describe the problem of minimizing power consumption of the multicast tree.

Definition 4-1.
Given a multicast tree T_s rooted at source s, the Tree Power Consumption $TPC(T_s)$ is defined as the summation of the power consumptions on each node in the tree.

Let Ω_M denote the set of all possible multicast trees for a fixed multicast group M, then the minimum power consumption multicast tree T_s^* is

$$T_s^* = \arg\min_{T_s}\{TPC(T_s) \mid \forall T_s \in \Omega_M\} \quad [1] \quad . \tag{4-1}$$

This cost requires a careful choice of intermediate transceivers, or equivalently their RF transmission power levels, such that the overall power consumed for the multicasting is minimized.

4.2 MILP Formulation

Definition 3-4 on multicast tree allows us to formulate the BPM/MPM Problem as a MILP

[1] The expression $\arg\min_x f(x)$ means "the x such that $f(x)$ is minimized", i.e. the argument x that achieved the smallest $f(x)$.

(Mixed Integer Linear Programming) model. The main idea is to extract a sub-graph T_s^* from the original graph G, such that T_s^* is a multicast tree rooted at node s with minimum energy consumption. In this section, we shall systematically study the optimization variables, objective function, linear constraints, and MILP model for the general BPM/MPM Problem in a WAHN with adaptive antennas.

In the following, we shall provide several lemmas and theorems using graph theory to elaborate how to construct a set of linear constraints for our MILP model.

4.2.1 Optimization Variables

In order to formulate the problem, we define the following variables:

(i) p_v^{RF} is a nonnegative continuous variable which represents the RF power of the node v required by the multicast tree T_s^*;

(ii) p_v^T is a nonnegative continuous variable which represents the transmission processing power of the node v required by the multicast tree T_s^*;

(iii) p_v^R is a nonnegative continuous variable which represents the reception processing power of the node v required by the multicast tree T_s^*;

(iv) Z_{vu} is a binary decision variable which is equal to one if the arc (v, u) is in the sub-graph T_s^* of G, and zero otherwise;

(v) F_{vu} is a nonnegative continuous variable that only represents fictitious flow produced by the multicast initiator s going through arc (v, u), and thus helps prevent loops;

(vi) θ_v is the antenna beamwidth of node v ($\theta_{\min} \le \theta_v \le \theta_{\max}$);

(vii) φ_v is the antenna orientation of node v ($0 \le \theta_v < 2\pi$).

We shall prove that if $(x)^*$ is the optimal solution of variable x obtained from this MILP model, then the graph T_s^* is the optimal tree associated with this solution. In other words, T_s^* is a multicast tree of G with minimum tree power consumption. Let $A(T_s^*) = \{(v, u) | Z^*_{vu} = 1\}$ be the arc set of multicast tree T_s^* and $N(T_s^*) = \{u \mid \exists\ (v, u) \in A(T_s^*) \text{ or } (u, v) \in A(T_s^*)\}$ its node set. A straightforward enumeration method for $|N(T_s^*)|$ and $|A(T_s^*)|$ can be directly obtained from the variable definitions and given in Lemma 4-1.

Lemma 4-1.

$$\left|N(T_s^*)\right| = \sum_{v\in N}\sum_{u\in N} Z^*_{vu} + 1 \qquad (4\text{-}2a)$$

$$\left|A(T_s^*)\right| = \sum_{v\in N} \sum_{u\in N} Z_{vu}^* \qquad . \qquad (4\text{-}2b)$$

Proof:

Note that there are totally $|N(T_s^*)|$ number of nodes and $|A(T_s^*)|$ number of receivers in the minimum power multicast tree T_s^*. It follows that $\left|A(T_s^*)\right| = \sum_{v\in N} \sum_{u\in N} Z_{vu}^*$. Since T_s^* is a directed tree, we have $|N(T_s^*)| = |A(T_s^*)| + 1 = \sum_{v\in N} \sum_{u\in N} Z_{vu}^* + 1$. ■

4.2.2 Objective Function

The objective function of the BPM/MPM Problem can be easily obtained by substituting the optimization variables into Equation (4-1).

$$\textit{Minimize:} \ \sum_{v\in N} (p_v^{RF} + p_v^T + p_v^R) \qquad . \qquad (4\text{-}3a)$$

Note that there are exactly $|A(T_s^*)|$ number of arcs in the optimal multicast tree, and each of arcs corresponds to one and only one receiver. Therefore, the objective function can be equivalently rewritten as

$$\textit{Minimize:} \ \sum_{v\in N} (p_v^{RF} + p_v^T) + p_{recv} \cdot \sum_{v\in N} \sum_{u\in N} Z_{vu} \qquad . \qquad (4\text{-}3b)$$

4.2.3 Linear Constraints

We shall provide in this section a set of linear constraints that would guarantee that T_s^* obtained from the formulation satisfies both the rooted tree property (RTP) and the antenna coverage property (ACP).

4.2.3.1 RTP Constraints

T_s^* is called a rooted tree spanning all the multicast members, i.e., $M \subseteq N(T_s^*)$, if it obeys the following constrains:

RTP (a): Every node $u \in N(T_s^*) \backslash \{s\}$[1] has exactly one incoming arc, and node s has no incoming arcs;

RTP (b): T_s^* does not contain cycles.

[1] Here, the backslash symbol is used to denote a set difference.

The construction and interpretation of the linear constraints for the rooted tree property are elaborated in the following theorems.

Theorem 4-2.
T_s^* is a directed graph in which node s has no incoming arcs, and each other node has exactly one incoming arc, provided Problem MPM satisfies the following constraints:

$$\sum_{v\in N} Z_{vs} = 0; \tag{4-4}$$

$$\sum_{v\in N} Z_{vu} = 1; \ \forall u \in M\setminus\{s\} \tag{4-5}$$

$$\sum_{v\in N} Z_{vu} \le 1; \ \forall u \in N\setminus M \tag{4-6}$$

$$\sum_{v\in N} Z_{uv} \le (n-1)\sum_{v\in N} Z_{vu}; \ \forall u \in N\setminus M \quad . \tag{4-7}$$

Proof:
See Appendix B.1 for details. ■

Note that if $\sum_{v\in N} Z^*_{vu} = 1$ for any non-multicast member u in T_s^*, constraints (4-7) become redundant since the out-degree of node u is at most $n-1$. From constraints (4-4), (4-5), and (4-6), we obtain the following conclusion.

Corollary 4-3.

$$\sum_{v\in N} Z_{vu} \in \{0,1\}, \ \forall u\in N \quad . \tag{4-8}$$

Theorem 4-4.
T_s^* does not contain cycles, if Problem MPM satisfies the constraints (4-4) – (4-6) and the following constraints:

$$\sum_{v\in N} F_{vu} - \sum_{v\in N} F_{uv} = \sum_{v\in N} Z_{vu}; \ \forall u \in N\setminus\{s\} \tag{4-9}$$

$$Z_{vu} \le F_{vu} \le (n-1)\, Z_{vu}; \ \forall u \in N\setminus\{s\}, \forall v \in N \quad . \tag{4-10}$$

Proof:
See Appendix B.2 for details. ■

4.2.3.2 ACP Constraints

A fine-grain characterization for ACP can be as follows.

<u>ACP (a):</u> This property requires that for any $(v, u) \in A(T_s^*)$, the distance r_{vu} must be within the

transmission range of node v's antenna. This is also called the transmission range requirement.

ACP (b): This property requires that for any $(v, u) \in A(T_s^*)$, the antenna orientation of node v must point to a proper position such that node u can be essentially covered by the antenna beam of node v (by increasing node v's RF power). This is also called the antenna orientation requirement.

Note that the antenna orientation requirement is trivially satisfied in a network with omni-directional antennas, where ACP degenerates to the transmission range requirement. In the following sections, we shall reconstruct the ACP constraints for more general scenario of using adaptive antennas.

Lemma 4-5.

T_s^* satisfies ACP (a), if the formulation of Problem MPM includes the following constraints.

$$p_v^{RF} + p_v^T \geq (p_{vu} + p_{tran}) \cdot Z_{vu};\ \forall v, u \in N \tag{4-11}$$

$$p_v^{RF} \leq p_{\max};\ \forall v \in N \quad . \tag{4-12}$$

Proof:

For any node v in T_s^*, if v is a leaf node, i.e., $Z^*_{vu} = 0$ and $(p_v^T)^* = 0$ for all $u \in N(T_s^*)$, then $(p_v^{RF})^* = (p_v^{RF})^* + (p_v^T)^* \geq (p_{vu} + p_{tran}) \cdot Z_{vu}^* = 0$; if v is an internal node, , i.e., $Z^*_{vu} = 1$ and $(p_v^T)^* = p_{tran}$, then $(p_v^{RF})^* \geq p_{vu}$ for all $u \in N(T_s^*)$, i.e., $(p_v^{RF})^* \geq \mathrm{Min}\{p_{vu} \mid (v,u) \in A(T_s^*)\}$.

The equalities are achieved in the inequations above when the summation of the variables p_v^{RF} is minimized in the objective function (Equation 4-3a). Thus Equ. (3-5) must be held by T_s^*. ■

Let us reinvestigate Constraint (4-11) by substituting Equation (3-1) into it, and obtain a new constraint

$$p_v^{RF} + p_v^T \geq \frac{r_{vu}^{\alpha}}{2\pi} \cdot \theta_v \cdot Z_{vu} + p_{tran} \cdot Z_{vu} \quad . \tag{4-13}$$

We observe that the multiplication form in Constraint (4-13) appears nonlinear because θ_v is also an optimization variable. The following theorem illustrates that this constraint can be linearized by introducing a new variable.

Theorem 4-6.
T_s^* satisfies ACP (a), if the formulation of Problem MPM includes Constraint (4-12) and the following constraints.

$$p_v^{RF} + p_v^T \geq r_{vu}^{\alpha} \cdot Y_{vu} / 2\pi + p_{tran} \cdot Z_{vu}\,;\ \forall v,\, u \in N \tag{4-14}$$

$$0 \leq Y_{vu} \leq 2\pi Z_{vu};\ \forall v,\, u \in N \tag{4-15}$$

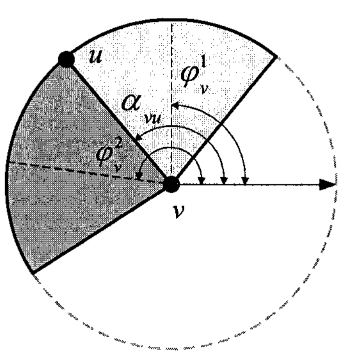
$$Y_{vu} \leq \theta_v;\ \forall v,\, u \in N \tag{4-16}$$

$$\theta_v - Y_{vu} + 2\pi Z_{vu} \leq 2\pi;\ \forall v,\, u \in N \tag{4-17}$$

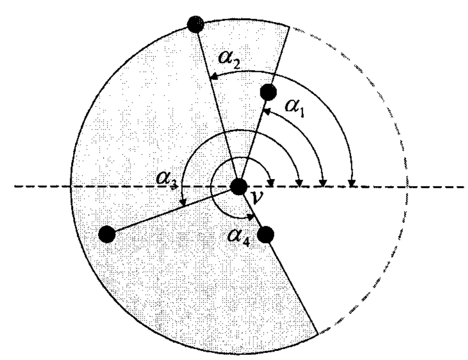
Proof:
In order to prove Constraints (4-14) to (4-17) are equivalent to Constraint (4-13), we only need to verify that variable Y_{vu} must satisfy the following conditions:

$$Y_{vu} = \theta_v Z_{vu} = \begin{cases} 0, & Z_{vu} = 0 \\ \theta_v, & Z_{vu} = 1 \end{cases} \tag{4-18}$$

When $Z_{vu} = 0$, from Constraint (4-15) $0 \leq Y_{vu} \leq 2\pi Z_{vu} = 0$ we have $Y_{vu} = 0$, and Constraints (4-16) and (4-17) become trivially true. When $Z_{vu} = 1$, Constraint (4-17) is simplified as $\theta_v \leq Y_{vu}$, from which we conclude that $Y_{vu} = \theta_v$, considering Constraint (4-16) $Y_{vu} \leq \theta_v$ at the same time. ■



(a)

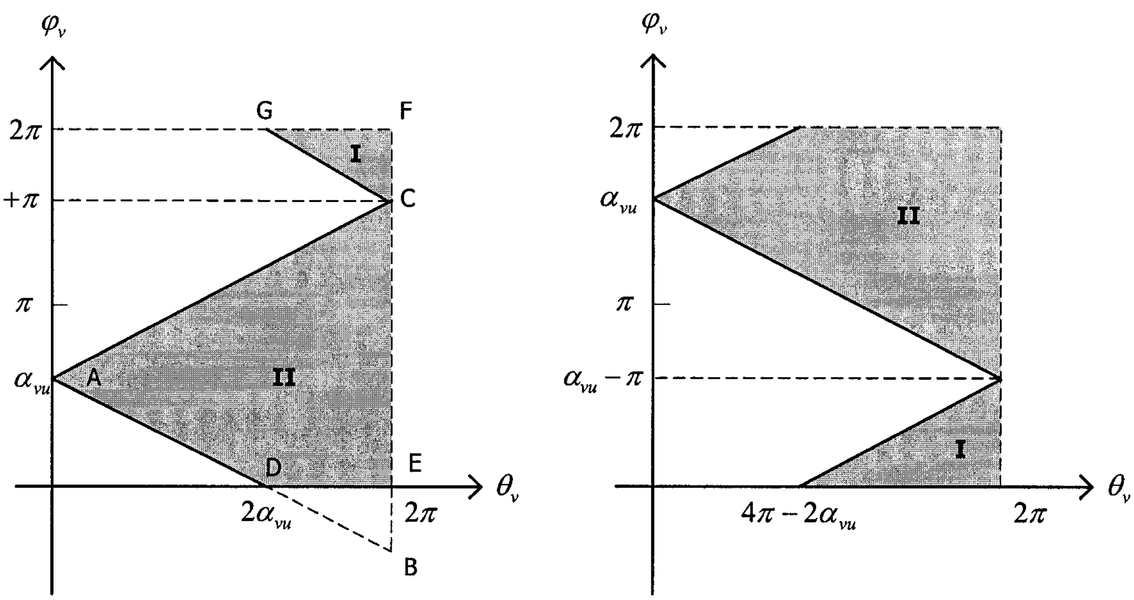


(b)

Figure 4-1. Choosing the sector to be covered by the beam
(a) The range of antenna orientation for a given transmission (v, u); (b) The calculation of minimum angle of beamwidth θ_v at node v by using sorted lists $A_v = (\alpha_1<\alpha_2<\alpha_3<\alpha_4)$

Before discussing the construction of linear constraints for ACP (b), we first investigate in more detail the relationship among the variables: antenna orientation φ_v, beamwidth θ_v, and Z_{vu} indicating if the arc (v, u) exists in the multicast tree T_s^*.

Let α_{vu} ($0 \le \alpha_{vu} < 2\pi$) be the angle measured counter-clockwise from the horizontal axis to the vector $\overrightarrow{vu}$ as shown in Fig. 4-1 (a). Then the angle α_{vu} ($v, u \in N$) can be obtained once their positions are given. In Fig. 4-1 (a), the lighter shaded area is the space covered by the antenna beam of node v when it is about entering the position of node u (i.e., for v making contact with u), and the darker shaded area is the space just before the beam is leaving the position of node u (i.e., for v losing contact with u). Thus it is clear that the wireless link (v, u) exists in the multicast tree T_s^*, i.e., $Z_{vu} = 1$, only if the antenna orientation φ_v is bounded by the two pointing directions $\varphi^1_v = \alpha_{vu}-\theta_v/2$ and $\varphi^2_v = \alpha_{vu}+\theta_v/2$, indicated by the dotted lines as shown in Fig. 4-1 (a).



(a) $0 \le \alpha_{vu} \le \pi$ (b) $\pi < \alpha_{vu} < 2\pi$

Figure 4-2. Illustration of linear constraint construction for property ACP

In order to simplify our analysis, we first relax the constraint $\theta_{\min} \le \theta_v \le \theta_{\max}$ into $0 \le \theta_v \le 2\pi$. In a φ_v–θ_v plane as shown in Fig. 4-2 (a), the points (φ_v, θ_v) that satisfy the constraint $\alpha_{vu}-\theta_v/2 \le \varphi_v \le \alpha_{vu}+\theta_v/2$ must be within the area bounded by line AB, line AC, and line BC, where $A = (0, \alpha_{vu})$, $B = (2\pi, \alpha_{vu}-\pi)$, and $C = (2\pi, \alpha_{vu}+\pi)$. When we consider $0 \le \alpha_{vu} \le \pi$ and $0 \le \varphi_v < 2\pi$, this area must be mapped into the shaded area in Fig. 4-2 (a) since φ_v and $\varphi_v+2\pi$ denote the same physical direction. Let I denote the area covered by triangle *CFG*, and II the area covered by quadrangle *ADEC*, where $D = (2\alpha_{vu}, 0)$, $E = (2\pi, 0)$, $F = (2\pi, 2\pi)$, and $G = (2\alpha_{vu}, 2\pi)$. Recall that ACP (b) requires node u to be located within the antenna beam of node v, for any (v, u) included in the multicast tree T_s^*. Based on our analysis above, this property can be rewritten as $Z_{vu} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{I} \cup \mathrm{II}$. Since I and II are disjoint sets, Z_{vu} can be decomposed into a summation of two new binary variables Z_{vu1} and Z_{vu2}, where $Z_{vu1} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{I}$, and $Z_{vu2} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{II}$. Theorem 4-7 explains how property ACP (b) can be satisfied by a set of linear constraints.

Theorem 4-7.
For any (v, u) included in the multicast tree T_s^* and $0 \le \alpha_{vu} \le \pi$, T_s^* satisfies ACP (b) if the following constraints hold.

$$2\varphi_v + \theta_v - (4\pi + 2\alpha_{vu})Z_{vu1} \ge 0 \tag{4-19}$$

$$2\varphi_v - \theta_v + (4\pi - 2\alpha_{vu})Z_{vu2} \le 4\pi \tag{4-20}$$

$$2\varphi_v + \theta_v - 2\alpha_{vu}Z_{vu2} \ge 0 \quad . \tag{4-21}$$

Proof:
See Appendix B.3 for details. ■

So far, we only consider the case $0 \le \alpha_{vu} \le \pi$. A similar constraint construction for property ACP (b) can be made under condition $\pi < \alpha_{vu} < 2\pi$. Fig. 4-2 (b) shows the shaded area, only in which the value of Z_{vu} can be equal to 1. The corresponding linear constraints that characterize property ACP (b) for $\pi < \alpha_{vu} < 2\pi$ are summarized in the theorem below.

Theorem 4-8.
For any (v, u) included in the multicast tree T_s^* and $\pi \le \alpha_{vu} \le 2\pi$, T_s^* satisfies ACP (b) if the following constraints hold.

$$2\varphi_v - \theta_v + (8\pi - 2\alpha_{vu})Z_{vu1} \le 4\pi \tag{4-22}$$

$$2\varphi_v - \theta_v + (4\pi - 2\alpha_{vu})Z_{vu2} \le 4\pi \tag{4-23}$$

$$2\varphi_v + \theta_v - 2\alpha_{vu}Z_{vu2} \ge 0 \quad . \tag{4-24}$$

Proof:
The constraints for the case of $\pi \le \alpha_{vu} \le 2\pi$ can be verified in a very similar way as in Theorem

4-7, and thus the proof is omitted. ∎

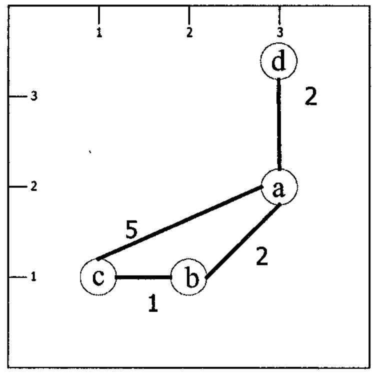


Figure 4-3. Example 4-node network G^4

4.2.3.3 An Example

A generic example of a 4-node network G^4 that we consider is shown in Figure 4-3, where multicast group is {a, b, c} and node a is the source The weight for each arc represents the power required transmitting packets on it when using omni-directional antennas. We assume the channel loss exponent $\alpha = 2$, $p_{tran} = p_{recv} = 0$, and $p_{max} = 5$.

Table 4-1. RTP linear constraints for example network G^4

RTP (a)	RTP (b)
$Z_{ba} = 0$	$F_{ab} + F_{cb} - F_{ba} - F_{bc} = Z_{ab} + Z_{cb}$
$Z_{ab} + Z_{cb} = 1$	$F_{ac} + F_{bc} - F_{cb} = Z_{ac} + Z_{bc}$
$Z_{ac} + Z_{bc} = 1$	$F_{ad} - F_{da} = Z_{ad}$
$Z_{ad} \leq 1$	$Z_{ab} \leq F_{ab} \leq 3Z_{ab}$
$3Z_{ad} \geq 0$	$Z_{cb} \leq F_{cb} \leq 3Z_{cb}$
	$Z_{ac} \leq F_{ac} \leq 3Z_{ac}$
	$Z_{bc} \leq F_{bc} \leq 3Z_{bc}$
	$Z_{ad} \leq F_{ad} \leq 3Z_{ad}$

Referring to G^4, we can now list in Table 4-1 a set of constraints corresponding to each property RTP (a) and RTP (b) based on Constraints (4-4)-(4-7) and (4-9)-(4-10) respectively.

Table 4-2. ACP linear constraints for example network G^4

	ACP (a)	ACP (b)
Constraints on arc (a, b)	$p_a^{RF} + p_a^T \geq \frac{1}{\pi} Y_{ab}$ $0 \leq Y_{ab} \leq 2\pi \cdot (Z_{ab1} + Z_{ab2})$ $Y_{ab} \leq \theta_a$ $\theta_a - Y_{ab} + 2\pi \cdot (Z_{ab1} + Z_{ab2}) \leq 2\pi$	$2\varphi_a - \theta_a + \frac{11\pi}{2} Z_{ab1} \leq 4\pi$ $2\varphi_a - \theta_a + \frac{3\pi}{2} Z_{ab2} \leq 4\pi$ $2\varphi_a + \theta_a - \frac{5\pi}{2} Z_{ab2} \geq 0$
Constraints on arc (b, a)	$p_b^{RF} + p_b^T \geq \frac{1}{\pi} Y_{ba}$ $0 \leq Y_{ba} \leq 2\pi \cdot (Z_{ba1} + Z_{ba2})$ $Y_{ba} \leq \theta_b$ $\theta_b - Y_{ba} + 2\pi \cdot (Z_{ba1} + Z_{ba2}) \leq 2\pi$	$2\varphi_b + \theta_b - \frac{9\pi}{2} Z_{ba1} \geq 0$ $2\varphi_b - \theta_b + \frac{7\pi}{2} Z_{ba2} \leq 4\pi$ $2\varphi_b + \theta_a - \frac{\pi}{2} Z_{ba2} \geq 0$

For the ACP constraints, we only discuss the linear constraint formulation on arcs (a, b) and (b, a) as examples. We list these constraints in Table 4-2. For constraints on arc (a, b), since $\alpha_{ab} = 5\pi/4$ ($\pi \leq \alpha_{ab} \leq 2\pi$), this belongs to the case in Theorem 4-8. For constraints on arc (b, a), since $\alpha_{ba} = \pi/4$ ($0 \leq \alpha_{ba} \leq \pi$), it belongs to the case in Theorem 4-7.

Table 4-3. Values of coefficients

	$0 \leq \alpha_{vu} \leq \pi$	$\pi < \alpha_{vu} < 2\pi$
A_{vu}	-2	2
B_{vu}	$4\pi + 2\alpha_{vu}$	$8\pi - 2\alpha_{vu}$
C_{vu}	0	4π

4.2.4 MILP Model for the BPM/MPM Problem

Our previous derivation on the linear constraints can now allow us to formulate the BPM/MPM Problem as a MILP model. There are all together four cases.

$$\textit{Minimize:}\ \sum_{v\in N}(p_v^{RF}+p_v^{T})+p_{recv}\cdot\sum_{v\in N}\sum_{u\in N}(Z_{vu1}+Z_{vu2})$$

Subject to:

$$\text{RTP (a):}\quad \begin{cases} \sum_{v\in N}(Z_{vs1}+Z_{vs2})=0; \\ \sum_{v\in N}(Z_{vu1}+Z_{vu2})=1;\forall u\in M\setminus\{s\} \\ \sum_{v\in N}(Z_{vu1}+Z_{vu2})\le 1;\forall u\in N\setminus M \\ \sum_{v\in N}(Z_{uv1}+Z_{uv2})\le (n-1)\sum_{v\in N}(Z_{vu1}+Z_{vu2});\forall u\in N\setminus M \end{cases}$$

$$\text{RTP (b):}\quad \begin{cases} \sum_{v\in N}F_{vu}-\sum_{v\in N}F_{uv}=\sum_{v\in N}(Z_{vu1}+Z_{vu2});\forall u\in N\setminus\{s\} \\ Z_{vu1}+Z_{vu2}\le F_{vu}\le (n-1)\cdot(Z_{vu1}+Z_{vu2});\forall u\in N\setminus\{s\},\forall v\in N \end{cases}$$

$$\text{ACP (a):}\quad \begin{cases} p_v^{RF}+p_v^{T}\ge Y_{vu}\cdot r_{vu}^{\alpha}/2\pi+p_{tran}\cdot(Z_{vu1}+Z_{vu2});\forall v,u\in N \\ 0\le Y_{vu}\le 2\pi\cdot(Z_{vu1}+Z_{vu2});\forall v,u\in N \\ Y_{vu}\le\theta_v;\forall v,u\in N \\ \theta_v-Y_{vu}+2\pi\cdot(Z_{vu1}+Z_{vu2})\le 2\pi;\forall v,u\in N \\ p_v^{RF}\le p_{\max};\forall v\in N \end{cases}$$

$$\text{ACP (b):}\quad \begin{cases} A_{vu}\varphi_v-\theta_v+B_{vu}Z_{vu1}\le C_{vu};\forall v,u\in N \\ 2\varphi_v-\theta_v+(4\pi-2\alpha_{vu})Z_{vu2}\le 4\pi;\forall v,u\in N \\ 2\varphi_v+\theta_v-2\alpha_{vu}Z_{vu2}\ge 0;\forall v,u\in N \\ 0\le\varphi_v<2\pi;\forall v\in N \\ \theta_{\min}\le\theta_v\le\theta_{\max};\forall v\in N \end{cases}$$

$$\text{Integrality:}\quad Z_{vu1},\ Z_{vu2}\in\{0,1\},\ \forall v,\ u\in N$$

Figure 4-4. MILP model for problem MPM under adaptive antenna networks

4.2.4.1 Adaptive Antenna Case

The general MILP model for the MPM problem under adaptive antenna networks is shown in Fig.

4-4, in which the coefficients A_{vu}, B_{vu}, and C_{vu} are given in Table 4-3. In this formulation, Z_{vu1} and Z_{vu2} are binary variables; others are continuous variables. The number of variables in the formulation is approximately $3n^2 + 5n$, and the number of constraints is of the order of $O(n^2)$.

$$\text{Minimize: } \sum_{v\in N}(p_v^{RF}+p_v^T)+p_{recv}\cdot\sum_{v\in N}\sum_{u\in N}(Z_{vu1}+Z_{vu2}) \quad .$$

Subject to:

$$\text{RTP (a): } \begin{cases} \sum_{v\in N}(Z_{vs1}+Z_{vs2})=0; \\ \sum_{v\in N}(Z_{vu1}+Z_{vu2})=1; \forall u\in M\setminus\{s\} \\ \sum_{v\in N}(Z_{vu1}+Z_{vu2})\le 1; \forall u\in N\setminus M \\ \sum_{v\in N}(Z_{uv1}+Z_{uv2})\le (n-1)\sum_{v\in N}(Z_{vu1}+Z_{vu2}); \forall u\in N\setminus M \end{cases}$$

$$\text{RTP (b): } \begin{cases} \sum_{v\in N}F_{vu}-\sum_{v\in N}F_{uv}=\sum_{v\in N}(Z_{vu1}+Z_{vu2}); \forall u\in N\setminus\{s\} \\ Z_{vu1}+Z_{vu2}\le F_{vu}\le (n-1)\cdot(Z_{vu1}+Z_{vu2}); \forall u\in N\setminus\{s\}, \forall v\in N \end{cases}$$

$$\text{ACP (a): } \begin{cases} p_v^{RF}+p_v^T\ge (r_{vu}^{\alpha}\cdot\theta_v/2\pi+p_{tran})\cdot(Z_{vu1}+Z_{vu2}); \forall v,u\in N \\ p_v^{RF}\le p_{\max}; \forall v\in N \end{cases}$$

$$\text{ACP (b): } \begin{cases} A_{vu}\varphi_v+B_{vu}Z_{vu1}\le C_{vu}+\theta_v; \forall v,u\in N \\ 2\varphi_v+(4\pi-2\alpha_{vu})Z_{vu2}\le 4\pi+\theta_v; \forall v,u\in N \\ 2\varphi_v-2\alpha_{vu}Z_{vu2}\ge -\theta_v; \forall v,u\in N \\ 0\le\varphi_v<2\pi; \forall v\in N \end{cases}$$

Integrality: $Z_{vu1}, Z_{vu2} \in \{0, 1\}, \forall v, u \in N$

Figure 4-5. MILP model for problem MPM under switched antenna networks

4.2.4.2 Switched Antenna Case

Switched antenna (beamwidth-fixed directional antenna) scenario can be considered as a special case of adaptive antenna model when the optimized variable θ_v for each node v is set to a constant between 0 and 2π. Therefore, Constraints (4-14) to (4-17) can be replaced by Constraint (4-13), since it is already in a linear form. The complete formulation is given in Fig. 4-5.

$$\text{Minimize: } \sum_{v\in N}(p_v^{RF}+p_v^{T})+p_{recv}\cdot\sum_{v\in N}\sum_{u\in N}Z_{vu}$$

Subject to:

RTP (a):
$$\begin{cases}\sum_{v\in N}Z_{vs}=0;\\ \sum_{v\in N}Z_{vu}=1;\forall u\in M\setminus\{s\}\\ \sum_{v\in N}Z_{vu}\le 1;\forall u\in N\setminus M\\ \sum_{v\in N}Z_{uv}\le (n-1)\sum_{v\in N}Z_{vu};\forall u\in N\setminus M\end{cases}$$

RTP (b):
$$\begin{cases}\sum_{v\in N}F_{vu}-\sum_{v\in N}F_{uv}=\sum_{v\in N}Z_{vu};\forall u\in N\setminus\{s\}\\ Z_{vu}\le F_{vu}\le (n-1)Z_{vu};\forall u\in N\setminus\{s\},\forall v\in N\end{cases}$$

ACP:
$$\begin{cases}p_v^{RF}+p_v^{T}\ge (r_{vu}^{\alpha}+p_{tran})\cdot Z_{vu};\forall v,u\in N\\ p_v^{RF}\le p_{\max};\forall v\in N\end{cases}$$

Integrality: $Z_{vu}\in\{0,1\}, \forall v, u\in N$

Figure 4-6. MILP model for problem MPM under omni-directional antenna networks

4.2.4.3 Omni-directional Antenna Case

Omni-directional antenna can be also obtained easily by setting $\theta_{\min} = \theta_{\max} = 2\pi$. Further, the antenna orientation requirement is trivially satisfied, and thus the entire constraints for ACP (b) disappear. The resulting formulation is given in Fig. 4-6.

$$\text{Minimize: } \sum_{v\in N}(p_v^{RF}+p_v^{T})+(n-1)\cdot p_{recv}$$

Subject to:

RTP (a):
$$\begin{cases}\sum_{v\in N}Z_{vs}=0;\\ \sum_{v\in N}Z_{vu}=1;\forall u\in M\setminus\{s\}\end{cases}$$

RTP (b):
$$\begin{cases}\sum_{v\in N}F_{vu}-\sum_{v\in N}F_{uv}=1;\forall u\in N\setminus\{s\}\\ Z_{vu}\le F_{vu}\le (n-1)Z_{vu};\forall u\in N\setminus\{s\},\forall v\in N\end{cases}$$

ACP:
$$\begin{cases}p_v^{RF}+p_v^{T}\ge (r_{vu}^{\alpha}+p_{tran})\cdot Z_{vu};\forall v,u\in N\\ p_v^{RF}\le p_{\max};\forall v\in N\end{cases}$$

Integrality: $Z_{vu}\in\{0,1\}, \forall v, u\in N$

Figure 4-7. MILP model for problem BPM under omni-directional antenna networks

4.2.4.4 Broadcast Case

The previous MILP models for the multicast with various types of antenna can be easily transformed to the corresponding models for the broadcast. Since broadcast can be considered as a special case of multicast when $M = N$, the Constraints (4-6) and (4-7) disappear and some of the constraints like Constraint (4-9) can be simplified.

As example, the MILP model for Problem BPM under omni-directional antenna networks is shown in Fig. 4-7. The rest of BPM problem formulations can be obtained in a similar way.

4.3 Heuristic Algorithm

The amount of time required to solve the Problem BPM/MPM based on the MILP model for significantly large networks might be excessive. In order to handle such networks, we now present a group of heuristic algorithms for both cases of using omni-directional antennas and adaptive antennas, which have been summarized in Fig. 1-3. Since the broadcast can be seen as a special case of multicast, the solutions for broadcast problem are thus obtained by setting the multicast member group as the whole node set. In the following, we only discuss the multicast algorithms.

4.3.1 Algorithms for Omni-directional Antenna Networks

Our algorithms here extend the BIP and MIP algorithms reviewed before, and the descriptions of BIP and MIP in pseudo-code are given in Appendix A.2 for completeness and for later comparison. We do this by using more global topology information in a more "intelligent " way throughout the tree construction in order to achieve better performance.

4.3.1.1 The MIDP Algorithm

We now improve the performance of algorithm MIP by presenting a new heuristic algorithm called MIDP (Multicast Incremental-Decremental Power). MIDP first dynamically constructs a broadcast tree based on the minimal incremental power by adding a new node, just like BIP algorithm. It then iteratively reconstructs the broadcast tree based on the maximal decremental power by switching a tree arc with a non-tree arc. Note that in order to tradeoff the complexity, this operation can be performed at most K times. Finally, it prunes the broadcast tree to be a multicast tree. Let POW(T) be the total power a tree T required, and PRUN(T) be a multicast tree after being pruned from a broadcast tree T. Our algorithm MIDP can be described as follows.

The MIDP Algorithm

Step 1. A broadcast tree T_s is constructed first using BIP.

Step 2. For each node $v \in N \backslash \{s\}$ whose uplink node in T_s is u, if there exists $u' \in N$ such that $T'_s = T_s - (u, v) + (u', v)$ is a spanning tree, POW(PRUN(T_s)) > POW(PRUN(T_s')), and POW(PRUN(T_s)) – POW(PRUN(T'_s)) is maximized, then update T_s to be T_s'.

Step 3. Repeat Step 2 until POW(PRUN(T_s)) can not be decreased further or the number of iterations of Step 2 reaches a pre-defined constant number K.

Step 4. Return the final multicast tree PRUN(T_s).

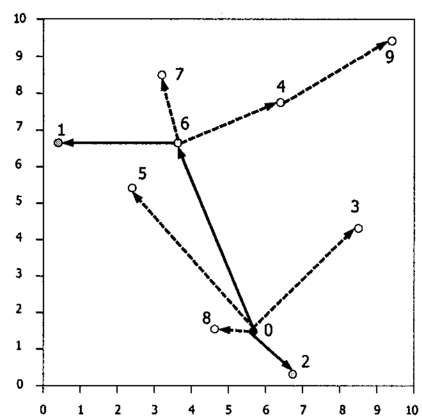
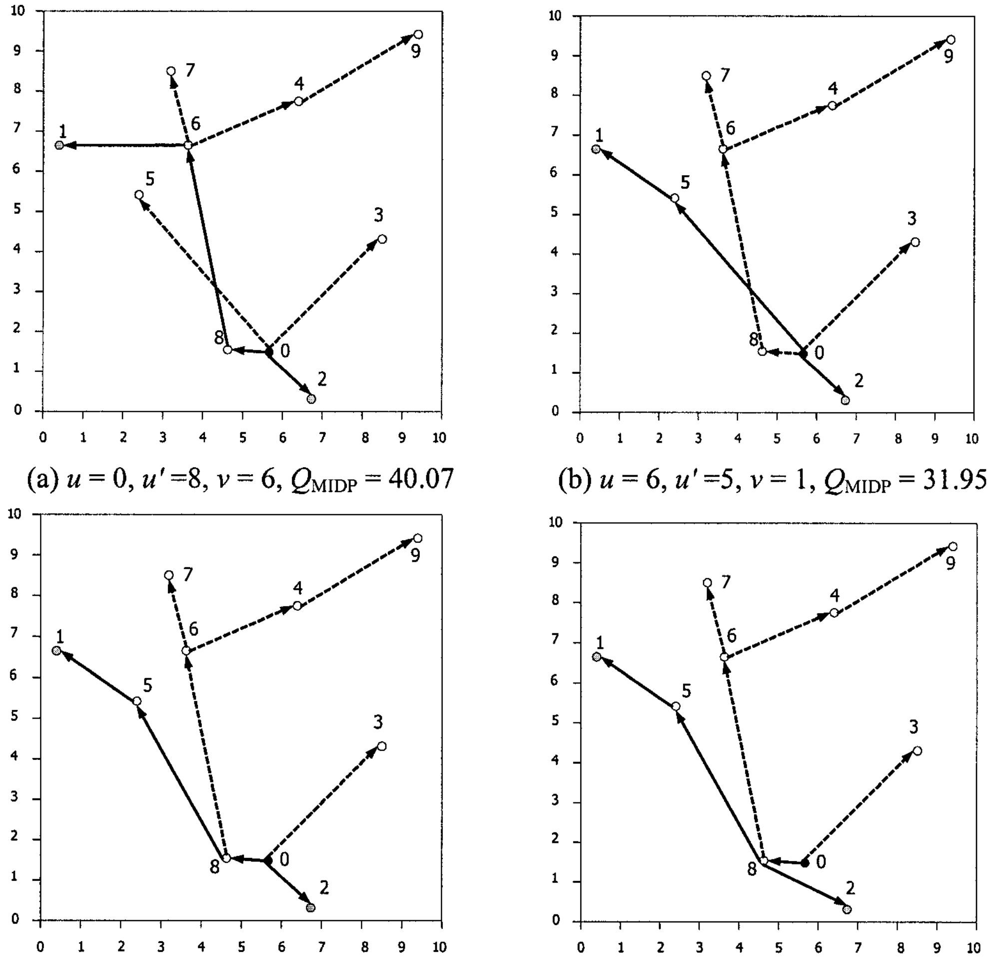


Figure 4-8. Example of multicast tree obtained using MIP algorithm in a 10-node network with multicast members {0, 1, 2} and source 0

4.3.1.2 An Example

We explain the basic operation of MIDP here by presenting a simple example of tree construction in a 10×10 square region with ten nodes, in which nodes {0, 1, 2} are multicast members and Node 0 is the source. The objective is to construct a minimum-energy multicast tree. Let Q_i be the total power of the multicast tree obtained using algorithm-i where $i \in$ {MIP, MIDP}. Initially, the broadcast tree is constructed from the BIP, and then unnecessary transmissions are eliminated as shown in Fig. 4-8, where the dotted lines indicate the arcs pruned from the broadcast trees, and the solid links indicate the arcs in the corresponding multicast trees.

The total power to maintain the multicast session is $Q_{\mathrm{MIP}} = 41.72$.



(a) $u = 0$, $u' = 8$, $v = 6$, $Q_{\mathrm{MIDP}} = 40.07$

(b) $u = 6$, $u' = 5$, $v = 1$, $Q_{\mathrm{MIDP}} = 31.95$

(c) $u = 0$, $u' = 8$, $v = 5$, $Q_{\mathrm{MIDP}} = 29.75$

(d) $u = 0$, $u' = 8$, $v = 2$, $Q_{\mathrm{MIDP}} = 29.15$

Figure 4-9. Examples of multicast tree construction steps using MIDP algorithm in a 10-node network with multicast members {0, 1, 2} and source 0

We then determine which tree arc can be switched with a non-tree arc to reconstruct a new spanning tree, whose corresponding multicast tree has smaller total power. Fig. 4-9 (a) shows a tree reconstruction by switching arc (0, 6) and arc (8, 6) to decrease total power to 40.07. Such an operation can be iterated until the total power cannot be decreased further as shown in Figs. 4-9 (a)-(d). Actually, the final multicast tree obtained by MIDP in Fig. 4-9 (d) is the optimal solution, i.e., $Q_{\mathrm{MIDP}} = Q_{\mathrm{OPT}} = 29.15$.

We note that the post-sweep in BIP/MIP [WiNg00, WiNg02a] can be seen as a special case of the operation in Step 2 of MIDP, since MIDP improves the performance not only by eliminating unnecessary transmissions as like post-sweep operation, but also by globally searching and reconstructing the broadcast/multicast tree. The tree reconstruction shown in Fig. 4-9 (d) is an example of sweep operation, since Node 8's transmitted power is sufficient to reach Node 2; thus Node 0 can reduce its power so that it reach only Node 8.

4.3.1.3 Time Complexity

The time complexity of MIDP is $O(n^3+Kn^3) = O(n^3)$, where K is the maximum iteration number of Step 2 in MIDP. This can be explained as follows. Step 1 in just BIP, which has complexity $O(n^3)$ [WiNg00]. In each iteration of Step 2, at most n^2 operations of POW() and PRUN() are performed, and each POW() or PRUN() operation has complexity $O(n)$. Step 4 in MIDP has complexity $O(n)$.

4.3.2 Algorithms for Adaptive Antenna Networks

In a similar manner as in Section 4.3.1, we shall present two heuristic algorithms called D-MIDP (Directional Multicast Incremental-Decremental Power) and RB-MIDP (Reduced Beam Multicast Incremental -Decremental Power) to improve the performance of algorithms D-MIP and RB-MIP [WiNg02c] respectively. The descriptions of D-MIDP and RB-MIDP in pseudo-code are given in Appendix A.4 and A.3 respectively.

4.3.2.1 Reimplementation of the D-MIP Algorithm

Before presenting our algorithms, we first give a variant implementation of the D-MIP algorithm using a new sector-choosing algorithm, which involves finding the minimum beamwidth that reaches all children. We shall see later that this new implementation results in a lower time complexity.

Notice that if we maintain a sorted list $A_v = \{\alpha_{vu} \mid (v, u) \in A_T\}$ (α_{vu} is defined in Section 4.2.3.2) for each node v throughout the formation of the broadcast tree T, the beam reduction operation can be reduced to $O(c)$. Without loss of generality, we assume that $\alpha_1 < \alpha_2 < \ldots < \alpha_c$ are all element of this list. The minimum beamwidth of the node v to cover all its children, denoted as $\theta_v(A_v)$, can be obtained straightforward with complexity $O(c)$ as follows

$$\theta_v(A_v) = \min\{\theta_{\min}, 2\pi - \max\{\alpha_2 - \alpha_1, \alpha_3 - \alpha_2, \ldots, \alpha_c - \alpha_{c-1}, 2\pi + (\alpha_1 - \alpha_c)\}\}. \tag{4-25}$$

In the example of Fig. 4-1 (b), node v has four children ($c = 4$) and maintains a list $A_v = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ sorted in an increasing order. Its minimum beamwidth thus can be obtained using Equation (4-25) and denoted as shaded area in Fig. 4-1 (b).

Let ADD(A_v, α_{vu}) be a procedure that returns a new sorted list after the element α_{vu} is added into the list A_v with c elements. Since A_v is already sorted, the complexity of ADD() is only $O(c)$. The description of the D-BIP algorithm based on the new data structure A_v is given in Appendix A.4, in which the sector-choosing algorithm is imbedded into the steps of tree formation.

4.3.2.2 The D-MIDP and RB-MIDP Algorithms

Following a similar tree reconstruction philosophy and referring to the same notations used in the MIDP algorithm pseudo code (Section 4.3.1.1), our D-MIDP algorithm can be described as follows.

The D-MIDP Algorithm

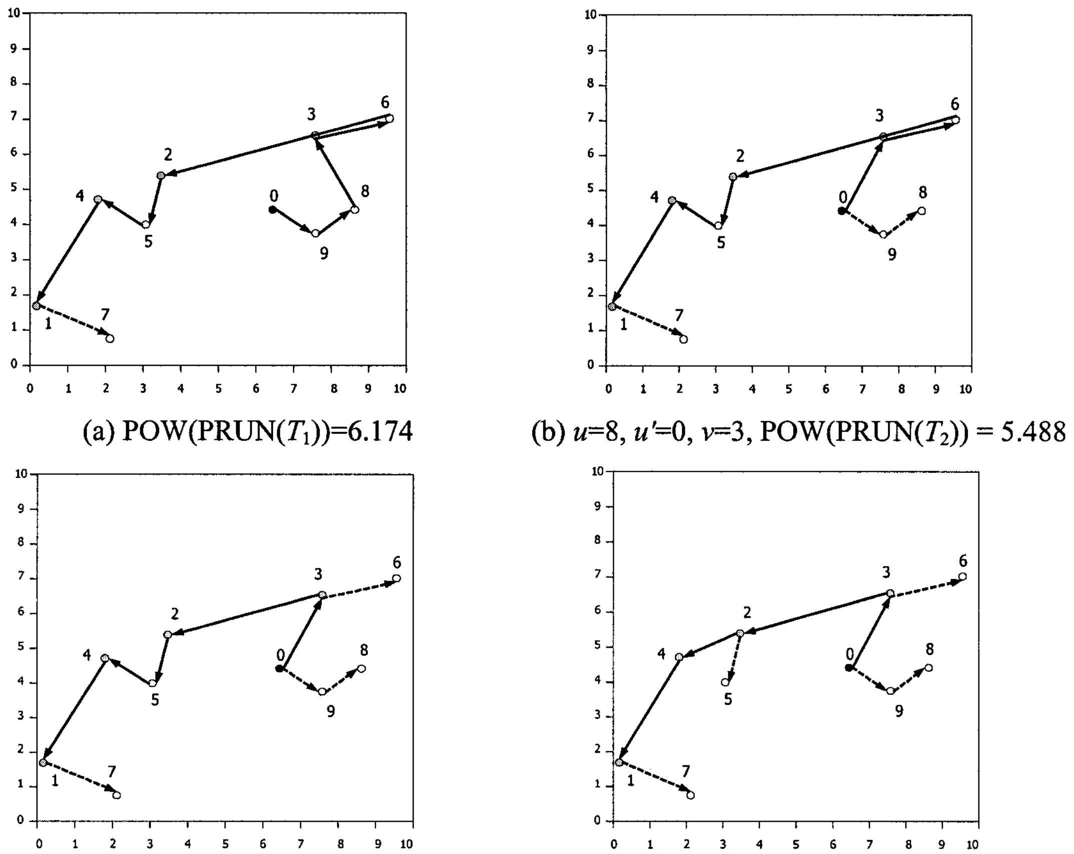
Step 1 A broadcast tree T is constructed first using D-BIP.

Step 2 For each node $u \in N\backslash\{s\}$ whose uplink node in T is v, if there exists $v' \in N$ such that by adjusting $\varphi_{v'}$ and $\theta_{v'}$, $T' = T - (v, u) + (v', u)$ is still a spanning tree which satisfies ACP, the inequality POW(PRUN(T)) > POW(PRUN(T')), and maximizes the power saving POW(PRUN(T)) – POW(PRUN(T')), then update T to be T'.

Step 3 Repeat Step 2) until POW(PRUN(T)) cannot be decreased further, or the number of iterations of Step 2) reaches a pre-defined constant number K.

Step 4 Return the final multicast tree PRUN(T).

Note that at each tree reconstruction operation of switching arc from (v, u) to (v', u), the antenna coverage property should be checked at the same time. This may involve updating the transmission range, antenna beamwidth and antenna orientation for both node v and v'. In a similar manner, we can formulate another algorithm RB-MIDP to improve the RB-MIP algorithm. The only difference of RB-MIDP from D-MIDP is in the first step where RB-BIP is used instead of D-BIP.



(a) POW(PRUN(T_1))=6.174 (b) u=8, u'=0, v=3, POW(PRUN(T_2)) = 5.488

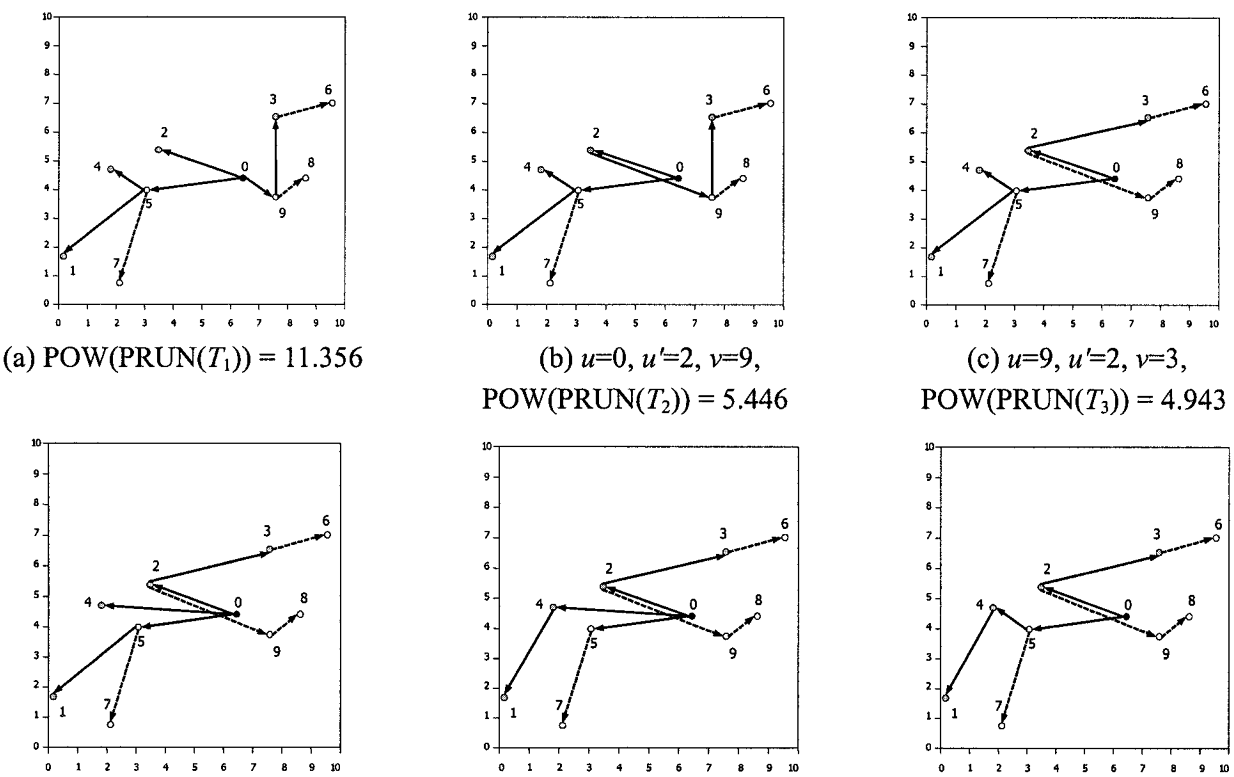
(c) u=6, u'=3, v=2, POW(PRUN(T_3)) = 3.329 (d) u=5, u'=2, v=4, POW(PRUN(T_4)) = 3.264

Figure 4-10. Examples of multicast tree construction using the D-MIDP algorithm on a 10-node network with multicast nodes {0, 1, 2, 3, 4} and source node 0 for $\theta_{\min} = 30°$

4.3.2.3 An Example

We shall explain the basic operation of D-MIDP here by presenting a simple example of tree construction in a ten-node network, in which nodes {0, 1, 2, 3, 4} are multicast members and Node 0 is the source. Each node is equipped with a directional antenna with minimum beamwidth 30°. The maximum number of iterations of Step 2) in the D-MIDP algorithm is 5.

Initially, the broadcast tree T_1 is constructed from the D-BIP as shown in Fig. 4-10 (a), and the total power to maintain the multicast session is POW(PRUN(T_1)) = 6.174. We then determine which tree arc can be switched with a non-tree arc to reconstruct a new spanning tree T_2, whose corresponding multicast tree PRUN(T_2) can save total power to the utmost extent when compared to the original multicast tree PRUN(T_1). In this example, T_2 is obtained by switching arc (8, 3) and arc (0, 3) as shown in Fig. 4-10 (b), and the total power decreases to POW(PRUN(T_2)) = 5.488. The power saving from T_1 to T_2 can be calculated as POW(PRUN(T_1)) – POW(PRUN(T_2)) = $r_{09}^2 \cdot \frac{30}{360} + r_{98}^2 \cdot \frac{30}{360} + r_{83}^2 \cdot \frac{30}{360} - r_{03}^2 \cdot \frac{30}{360}$. The advantage of this tree reconstruction operation is to help the avoidance of power inefficient paths in the multicast tree. For example, in this step, the path (0, 9, 8, 3) is replaced by (0, 3) in T_2. Such an operation can be iterated until the total power can not be decreased further as shown in Figs. 4-10 (b)-(d), where the dotted lines indicate the arcs pruned from the broadcast trees, and the solid links indicate the arcs in the corresponding multicast trees. We have also noticed that for the topology in Fig. 4-10, D-BIP actually obtains the optimal tree, which can be verified by our MILP model.



(a) POW(PRUN(T_1)) = 11.356

(b) u=0, u'=2, v=9, POW(PRUN(T_2)) = 5.446

(c) u=9, u'=2, v=3, POW(PRUN(T_3)) = 4.943

(d) u=5, u'=0, v=4 POW(PRUN(T_4))) = 4.242

(e) u=5, u'=4, v=1 POW(PRUN(T_5)) = 4.127

(f) u=0, u'=5, v=4 POW(PRUN(T_6)) = 3.538

Figure 4-11. Examples of multicast tree construction using the RB-MIDP algorithm on a 10-node network with multicast nodes {0, 1, 2, 3, 4} and source node 0 for $\theta_{\min}$ = 30°

The tree construction steps for the RB-MIDP on the same network topology in Fig. 4-10 are illustrated in Figs. 4-11 (a)-(f). We note that the tree is always reconstructed using global information to save power required for the final multicast tree. Take Fig. 4-11 (b) as an example. The tree is reconstructed by switching arc (0, 9) and arc (2, 9). The power saving from T_1 to T_2 can be calculated as POW(PRUN(T_1)) – POW(PRUN(T_2)) = $r_{05}^2 \cdot \angle 209/360 - (r_{05}^2 \cdot \angle 205/360 + r_{29}^2 \cdot 30/360)$[1].

4.3.2.4 Time Complexity

The time complexities of D-MIP and RB-MIP are $O(n^3 \log n)$ and $O(n^3)$ respectively [WiNg02d] using their sector-choosing algorithm[2], while our new D-MIP given in Appendix A.4 has a lower complexity of $O(n^3)$. This can be explained as follows. In Step 2-i), a minimum-cost node must be chosen, and the complexity of this search is at most $O(n^2)$. Then in Step 2-ii), a new node u is added to the tree, and accordingly, its parent node v must insert α_{vu} into its sorted list A_v. The complexity of this operation is at most $O(n)$. Finally in Step 2-iii), the costs that relate to the new child's parent v must be updated. Considering the calculation of $\theta_v(A_v)$ using Equation (4-25) and the procedure ADD() both have a complexity $O(n)$, we conclude that for a given outside node x, the complexity of the cost update operation is $O(n)$. Since the number of children outside the tree the node v may have is at most $n - 1$, the overall complexity of Step 2-iii) is $O(n^2)$. Therefore, the complexity at each step (Step 2) is at most $O(n^2)$. Since there are n steps, the overall complexity is at most $O(n^3)$.

The time complexity of D-MIDP is $O(n^3 + Kn^3) = O(n^3)$, where K is the maximum number of iterations of Step 2 in the D-MIDP algorithm. This can be explained as follows. Step 1 is just D-BIP, which has complexity $O(n^3)$ as explained above. In each iteration of Step 2, at most n^2 operations of POW() and PRUN() are performed, and each POW() or PRUN() operation has complexity $O(n)$. Step 4 in D-MIDP has complexity $O(n)$. Similarly, the time complexity of

[1] The symbol $\angle$abc indicates the degree of angle between arc (b, a) and arc (b, c).
[2] The complexity of the sector-choosing algorithm for a node with c children is $O(c \log c)$ [WiNg02d].

RB-MIDP is $O(n^3)$ since RB-BIP has complexity $O(n^3)$ [WiNg02d].

4.3.3 Innovative Features of Our Heuristic Algorithms

The distinctive features of our MIDP-group algorithms (MIDP for omni-directional antenna case and RB-MIDP/D-MIDP for adaptive antenna case) described in this section as opposed to many known algorithms can now be summarized as follows.

(1) Our algorithms take the multicast tree obtained from the existing algorithms as an initial solution, and consequently reconstruct this tree towards a better solution in a power-decremental manner that the total power required for the multicast tree is continuously decreased at each tree reconstruction operation until no further power savings.

(2) In many known algorithms [WiNg00, WiNg02a, CaHu02, WiNg02e], a central theme has been used to iteratively "grow" a set S of covered nodes starting from the source. At each step, according to some objective function S is expanded to cover one or more new nodes until all nodes in the multicast (broadcast) group are in S. These heuristic algorithms disregard the fact that a global topology update might actually be better. Our MIDP-group algorithms iteratively reconstructs multicast tree in the direction of possibly maximizing the total power saving using global topology information.

(3) Most of the current MPM algorithms use the obvious idea of pruning some broadcast tree [WiNg00, WiNg01, WiNg02a, WiNg02e]. However, pruning broadcast tree at the last step does not give good performance once the multicast group sizes start to drop below a certain threshold, as will be demonstrated later in Section 4.4. In our algorithm, the pruning operation is embedded in the iterative multicast tree reconstruction, where the objective is to minimize the total tree power required by the multicast tree, instead of the broadcast tree.

4.4 Performance Evaluation

We have evaluated the performance of the four heuristic algorithms (RB-MIP, D-MIP, RB-MIDP, and D-MIDP) for many network examples. We have specifically compared the performance between MIDP and MIP when using omni-directional antennas, since in this case both D-MIDP and RB-MIDP degenerate to MIDP. Further, we shall use the improved MIP algorithm using the "sweep" operation [WiNg00, WiNg02a] that would scan the tree, identify all redundant transmissions, and eliminate them so that power can be reduced. This operation is considered to be the real challenge to our MIDP algorithm.

In order to investigate the performance of these algorithms in different network scales, we specify two typical network configurations: small networks with 20 nodes and large networks with 100 nodes. For both network sizes, the maximum number of multicast tree reconstruction in our algorithm MIDP is set to 5.

In each network example, nodes are randomly generated within a square region 1000 meters × 1000 meters. Each node is equipped with an adaptive antenna, which can point to any desired direction with an antenna beamwidth subject to $\theta_{\min} \le \theta_v \le 2\pi$. We have considered a finite number of minimum antenna beamwidth $\theta_{\min}$ = 10°, 30°, 60°, 90°, 180°, and 360°. The maximum transmission power is restricted by the maximum radio propagation range of 300 meters when a maximum beamwidth is used. We have only considered $p_{recv} = 0$, $p_{tran} = 0$, and $\alpha = 2$.

One of the nodes is randomly chosen to be the source. Multicast groups of a specified size are also chosen randomly from the overall set of nodes. For the small networks with 20 nodes, we have considered the multicast group size $m \in \{5, 10, 15, 20\}$, and for large networks with 100 nodes, $m \in \{10, 25, 50, 75, 100\}$.

In all cases, (i.e., for a specified network size n, multicast group size m, minimum antenna beamwidth $\theta_{\min}$, and tree algorithm $i \in I$ = {RB-MIP, D-MIP, RB-MIDP, D-MIDP}), our results are based on the performance of 100 randomly generated networks.

We use CPLEX to solve the MPM Problem based on our MILP model (in Fig. 4-4). The optimal solution for a typical wireless ad hoc network with not more than 50 nodes can be always obtained in a timely manner. When running on a MS WIN2000 workstation with a PIII 800-MHz processor and 128 MB memory, the CPU time is about couple of seconds on each 20-node network example, and less than 10 minutes on each 50-node network example. Under the same platform, the execution time of each heuristic algorithm is less than 1 sec for all cases.

4.4.1 Performance Metrics

We have used the following metrics in evaluating and comparing algorithm performance.

Definition 4-2.
Minimum Tree Power Q_{BEST} is the minimum total power of the multicast tree required.

The *minimum tree power* metric shows the advantage offered by the use of narrow beams compared to the use of omni-directional antennas. For small network examples, the best solution is the optimal solution, i.e., $Q_{\text{BEST}} = Q_{\text{OPT}}$. For large network examples, the best solution can only be obtained by choosing the minimum total power from all heuristic algorithms, i.e. $Q_{\text{BEST}} = min\{Q_i \mid i \in \{\text{MIP, MIP-S, MIDP}\}\}$, since OPT is not computationally efficient in this case.

Definition 4-3.
Normalized tree power is defined the ratio of actual tree power Q_i required using heuristic algorithm-i to the minimum tree power Q_{BEST}, i.e., $Q'_i = Q_i/Q_{\text{BEST}}$, where $Q_{\text{BEST}} = min\{Q_i \mid i \in I\}$.

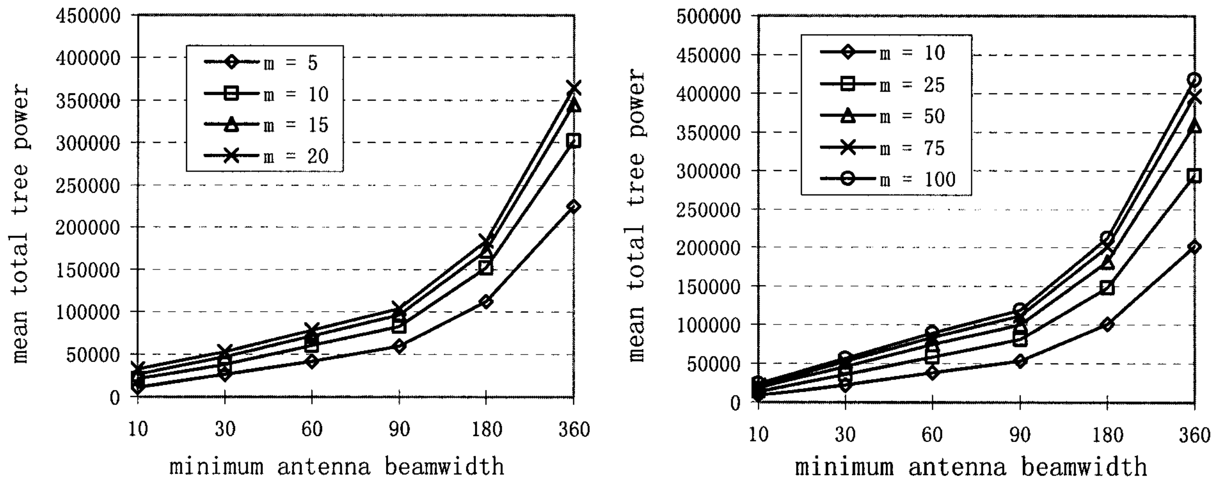
The *normalized tree power* metric provides a measure of how close each algorithm comes to providing the lowest-power tree. Thus it allows us to facilitate the comparison of different algorithms over a wide range of network examples.

Definition 4-4.
Power Saving Ratio $S_{i,j} = (Q_i - Q_j)/Q_i$ - the ratio of the difference of total tree power using algorithm-i from algorithm-j to the total tree power using algorithm-i.

The *power saving ratio* metric presents the total tree power saving percentage obtained by using algorithm-j when compared to algorithm-i. In our experiments, we are especially interested in the following two ratios: $S_{\text{RB-MIP, RB-MIDP}}$ and $S_{\text{D-MIP,D-MIDP}}$, which show how our algorithms improve the performance of the algorithms in [WiNg02c].

4.4.2 Minimum Tree Power

The first set of experiments explores how the *minimum tree power* changes with different minimum antenna beam widths and multicast group sizes.



(a) Q_{OPT} in 20-node networks (b) Q_{BEST} in 100-node networks

Figure 4-12. Minimum tree power

Figure 4-12 depicts graphically the mean total tree power of the algorithms we have studied over different connected network topologies with 20 and 100 nodes respectively. The x-axis represents the minimum antenna beamwidth, and the y-axis presents the mean of total tree power using different algorithms.

Referring to the m = 5 members in Fig. 4-12 (a), we observe that the mean of the total tree power is an increasing function of minimum antenna beamwidth. This trend remains the same for m = 10, 15, and 20 members. For a fixed minimum antenna beamwidth, the mean total tree power is an increasing function of m. However, the additional power required appears to be diminishing. Fig. 4-12 (b) shows the same performance curves for 100-node networks. All observations are similar to 20-node network except for the higher total power requirement. For each algorithm, the experiment results verify the advantages offered by the use of directional antennas.

For all the cases, we also have the following observations:

(1) When the minimum antenna beamwidth is relatively small (θ_{min} < 90°), the total tree power increases slowly as θ_{min} and m increase.

(2) When the minimum antenna beamwidth is relatively large (θ_{min} > 90°), the total tree power

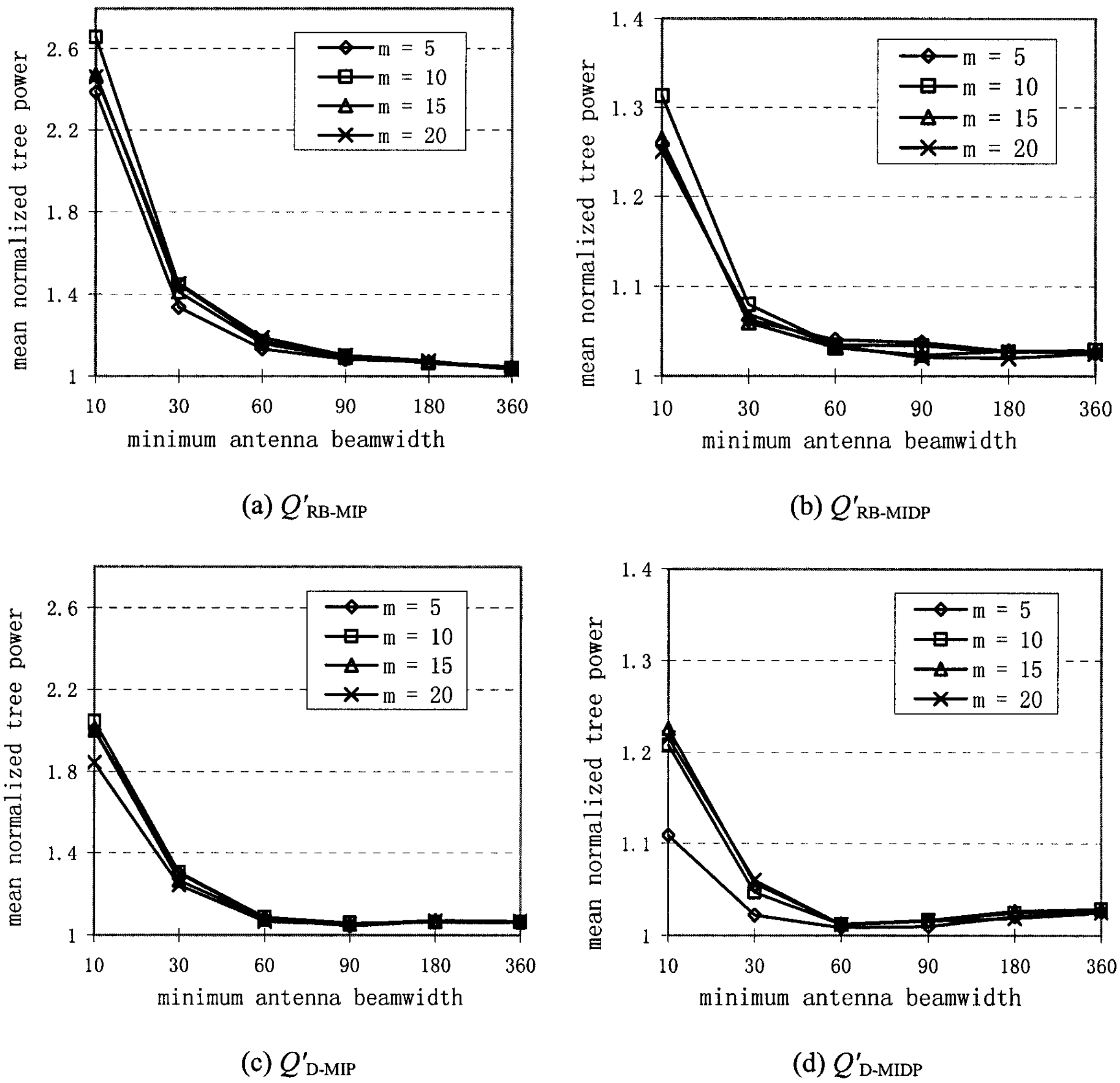
increases much as $\theta_{\min}$ and m increase.

Table 4-4. Normalized tree power in 20-node networks with adaptive antennas

$\theta_{\min}$	$\lvert M\rvert$	$Q'_{\text{RB-MIP}}$	$Q'_{\text{RB-MIDP}}$	$Q'_{\text{D-MIP}}$	$Q'_{\text{D-MIDP}}$
	5	(2.389, 2.668)	(1.260, 0.362)	(2.007, 1.019)	(1.109, 0.065)
10°	10	(2.659, 2.398)	(1.313, 0.185)	(2.049, 0.832)	(1.208, 0.076)
	15	(2.476, 1.311)	(1.265, 0.102)	(2.002, 0.626)	(1.226, 0.072)
	20	(2.464, 1.071)	(1.251, 0.111)	(1.845, 0.549)	(1.217, 0.078)
	5	(1.336, 0.230)	(1.063, 0.033)	(1.299, 0.102)	(1.022, 0.002)
30°	10	(1.449, 0.212)	(1.080, 0.015)	(1.306, 0.082)	(1.047, 0.005)
	15	(1.413, 0.093)	(1.060, 0.009)	(1.268, 0.056)	(1.056, 0.006)
	20	(1.456, 0.098)	(1.070, 0.009)	(1.244, 0.045)	(1.060, 0.006)
	5	(1.133, 0.050)	(1.041, 0.009)	(1.080, 0.012)	(1.009, 0.001)
60°	10	(1.172, 0.040)	(1.035, 0.004)	(1.087, 0.010)	(1.012, 0.000)
	15	(1.159, 0.024)	(1.032, 0.003)	(1.077, 0.007)	(1.013, 0.001)
	20	(1.189, 0.026)	(1.034, 0.002)	(1.067, 0.005)	(1.012, 0.000)
	5	(1.080, 0.017)	(1.038, 0.007)	(1.046, 0.007)	(1.010, 0.001)
90°	10	(1.100, 0.016)	(1.034, 0.005)	(1.061, 0.008)	(1.017, 0.002)
	15	(1.089, 0.009)	(1.024, 0.002)	(1.055, 0.005)	(1.017, 0.002)
	20	(1.103, 0.012)	(1.021, 0.001)	(1.053, 0.003)	(1.016, 0.001)
	5	(1.070, 0.009)	(1.028, 0.006)	(1.068, 0.008)	(1.021, 0.002)
180°	10	(1.064, 0.009)	(1.027, 0.006)	(1.064, 0.008)	(1.025, 0.005)
	15	(1.072, 0.005)	(1.028, 0.003)	(1.072, 0.005)	(1.027, 0.003)
	20	(1.074, 0.005)	(1.020, 0.001)	(1.068, 0.004)	(1.019, 0.001)
	5	(1.044, 0.007)	(1.026, 0.006)	(1.044, 0.007)	(1.026, 0.006)
360°	10	(1.040, 0.006)	(1.029, 0.006)	(1.040, 0.006)	(1.029, 0.006)
	15	(1.036, 0.004)	(1.029, 0.003)	(1.036, 0.004)	(1.029, 0.003)
	20	(1.033, 0.002)	(1.025, 0.002)	(1.033, 0.002)	(1.025, 0.002)

4.4.3 Normalized Tree Power

Table 4-4 summarizes the *normalized tree power* for the four algorithms on networks with 20 nodes, various multicast group sizes, and various minimum antenna beamwidth. We list mean and variance of the normalized tree power as (mean, variance) for each algorithm-*i* in the table. As noted above, the normalization is taken with respect to OPT. We observe from Table 4-4 that, for all the cases, D-MIDP and RB-MIDP provide much better performance than D-MIP and RB-MIP respectively, both in terms of mean and variance.



(a) $Q'_{\text{RB-MIP}}$ (b) $Q'_{\text{RB-MIDP}}$

(c) $Q'_{\text{D-MIP}}$ (d) $Q'_{\text{D-MIDP}}$

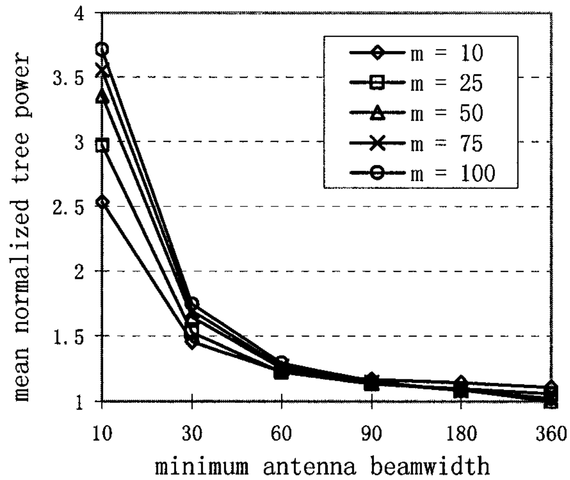
Figure 4-13. Normalized tree power in 20-node networks with adaptive antennas

Figure 4-13 illustrates graphically the mean normalized tree power as a function of multicast group size and minimum antenna beamwidth on small network examples. Referring to the m = 5 members in Fig. 4-13 (a), we observe that the mean of the normal tree power is a decreasing function of minimum antenna beamwidth. This trend remains the same for m = 10, 15, and 20 members. For a fixed minimum antenna beamwidth, the normal total tree powers are close for different m and become closer when the minimum antenna beamwidth increases. All observations are similar in Figs. 4-13 (b)-(d) except that the D-MIDP has a much lower normal tree power when both m and θ_{min} are small (m = 5 and θ_{min} < 60°). We also have the following observations:

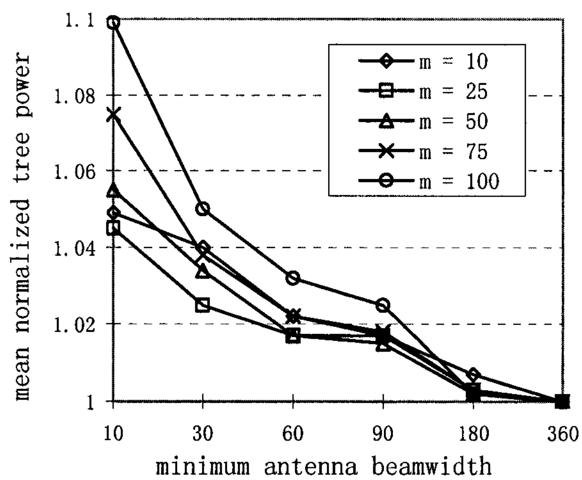
(1) A significant improvement can be achieved by RB-MIDP and D-MIDP algorithms. Both of them perform very close (within 10%) to the optimum when θ_{min} > 30° for any multicast group size.

(2) For each of the four algorithms, the relative performance of each algorithm with respect to the optimal solution degrades rapidly when the minimum antenna beam-width becomes smaller than 60°.

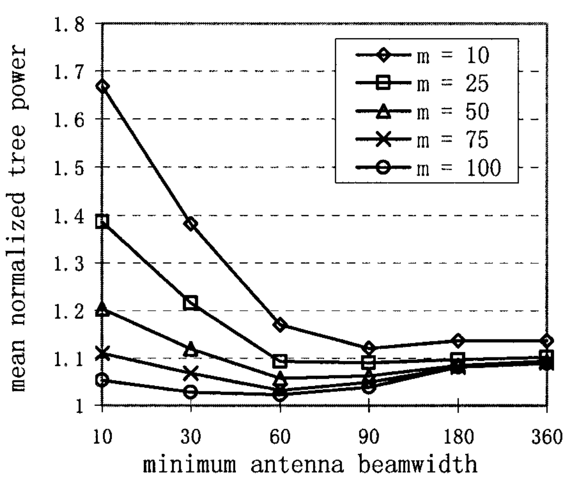
(3) For most cases the multicast group size has only minor effects on the mean of normalized tree power for a given minimum antenna beam-width.



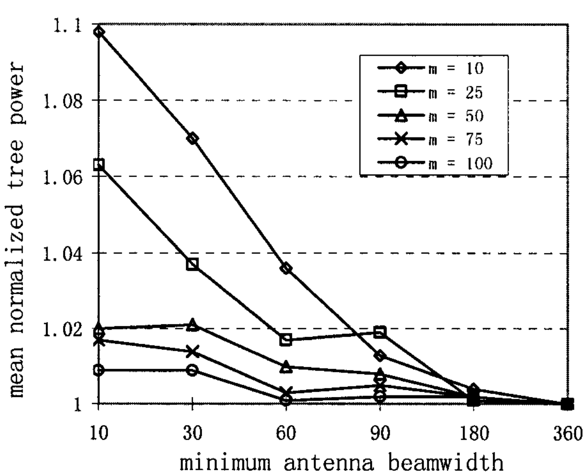
(a) $Q'_{\text{RB-MIP}}$



(b) $Q'_{\text{RB-MIDP}}$



(c) $Q'_{\text{D-MIP}}$



(d) $Q'_{\text{D-MIDP}}$

Figure 4-14. Normalized tree power in 100-node networks with adaptive antennas

Table 4-5. Normalized tree power in 100-node networks with adaptive antennas

θ_{min}	$\|M\|$	$Q'_{RB\text{-}MIP}$	$Q'_{RB\text{-}MIDP}$	$Q'_{D\text{-}MIP}$	$Q'_{D\text{-}MIDP}$
	10	(2.530, 0.437)	(1.049, 0.008)	(1.668, 0.112)	(1.098, 0.018)
	25	(2.971, 0.215)	(1.045, 0.006)	(1.386, 0.028)	(1.063, 0.010)
10°	50	(3.358, 0.232)	(1.055, 0.005)	(1.204, 0.008)	(1.020, 0.002)
	75	(3.557, 1.118)	(1.075, 0.026)	(1.111, 0.020)	(1.017, 0.011)
	100	(3.713, 0.191)	(1.099, 0.007)	(1.054, 0.003)	(1.009, 0.001)
	10	(1.455, 0.046)	(1.040, 0.004)	(1.382, 0.050)	(1.070, 0.014)
	25	(1.526, 0.027)	(1.025, 0.002)	(1.217, 0.009)	(1.037, 0.003)
30°	50	(1.641, 0.034)	(1.034, 0.002)	(1.120, 0.003)	(1.021, 0.002)
	75	(1.690, 0.027)	(1.038, 0.002)	(1.068, 0.001)	(1.014, 0.001)
	100	(1.744, 0.026)	(1.050, 0.002)	(1.028, 0.001)	(1.009, 0.001)
	10	(1.228, 0.016)	(1.022, 0.002)	(1.171, 0.021)	(1.036, 0.004)
	25	(1.222, 0.006)	(1.017, 0.002)	(1.094, 0.003)	(1.017, 0.001)
60°	50	(1.254, 0.008)	(1.017, 0.001)	(1.058, 0.001)	(1.010, 0.000)
	75	(1.267, 0.007)	(1.022, 0.001)	(1.032, 0.001)	(1.003, 0.000)
	100	(1.291, 0.007)	(1.032, 0.001)	(1.023, 0.000)	(1.001, 0.000)
	10	(1.170, 0.010)	(1.017, 0.001)	(1.121, 0.012)	(1.013, 0.001)
	25	(1.134, 0.003)	(1.017, 0.001)	(1.091, 0.003)	(1.019, 0.001)
90°	50	(1.138, 0.003)	(1.015, 0.000)	(1.063, 0.001)	(1.008, 0.000)
	75	(1.143, 0.003)	(1.018, 0.001)	(1.050, 0.001)	(1.005, 0.000)
	100	(1.151, 0.002)	(1.025, 0.0005)	(1.039, 0.000)	(1.002, 0.000)
	10	(1.142, 0.010)	(1.007, 0.002)	(1.138, 0.008)	(1.004, 0.000)
	25	(1.101, 0.003)	(1.003, 0.000)	(1.097, 0.002)	(1.001, 0.000)
180°	50	(1.090, 0.001)	(1.002, 0.000)	(1.086, 0.001)	(1.002, 0.000)
	75	(1.085, 0.001)	(1.003, 0.000)	(1.082, 0.001)	(1.002, 0.000)
	100	(1.086, 0.001)	(1.002, 0.000)	(1.081, 0.001)	(1.002, 0.000)
	10	(1.111, 0.008)	(1.000, 0.000)	(1.111, 0.008)	(1.000, 0.000)
	25	(1.060, 0.002)	(1.000, 0.000)	(1.060, 0.002)	(1.000, 0.000)
360°	50	(1.041, 0.001)	(1.000, 0.000)	(1.041, 0.001)	(1.000, 0.000)
	75	(1.024, 0.000)	(1.000, 0.000)	(1.024, 0.000)	(1.000, 0.000)
	100	(1.024, 0.002)	(1.000, 0.000)	(1.024, 0.002)	(1.000, 0.000)

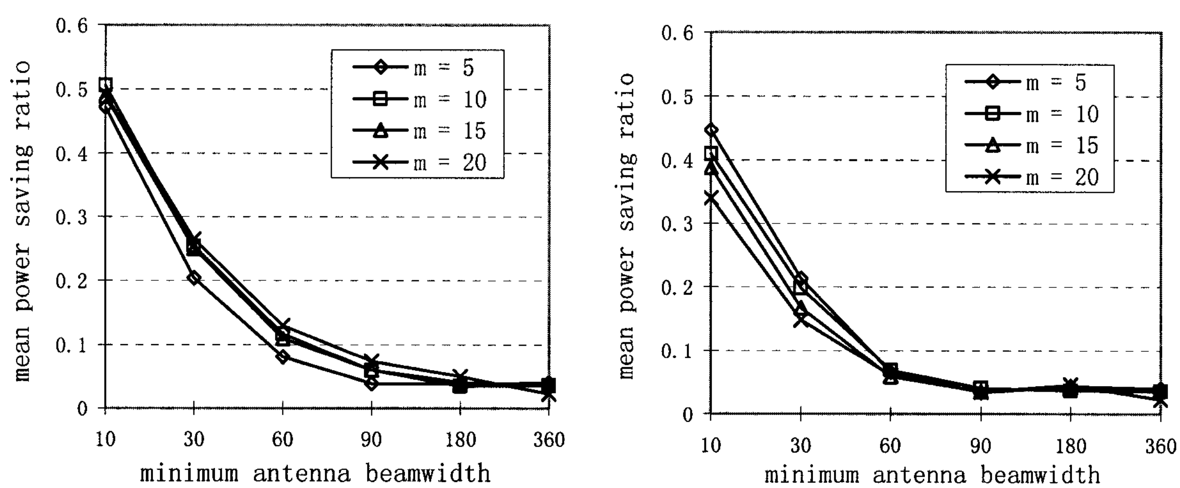
Figure 4-14 illustrates the mean normalized tree power of the four algorithms for the network topologies with 100 nodes. Referring to the m = 10 members in Fig. 4-14 (a), we observe that the mean of the normal tree power of RB-MIP is a decreasing function of minimum antenna beamwidth. This trend remains the same for m = 25, 50, 75, and 100 members. For a fixed minimum antenna beamwidth, the mean total tree power is an increasing function of m.

However, the increased normalized power appears to be diminishing. The observations are similar in Fig. 4-13 (b) except that the normal tree power for the minimum multicast group size (m = 10) is not always the worst at a fixed minimum antenna beamwidth. Referring to the m = 10 members in Fig. 4-14 (c), the mean of the normal tree power of D-MIP first drops dramatically as $\theta_{\min}$ increases while slightly increases later on when $\theta_{\min}$ increases further. This trend remains the same for m = 25, 50, 75, and 100 members. For a fixed minimum antenna beamwidth, the mean total tree power is a decreasing function of m. However, the decreased normalized power appears to be diminishing. The similar observations can be taken in Fig. 4-13 (d) except that the normal tree power converges to one for all various group sizes when the minimum antenna beamwidth increases. We also have the following observations based on Table 4-5 and Fig. 4-14:

(1) Both RB-MIDP and D-MIDP significantly improve the performance of RB-MIP and D-MIP respectively. For all the cases, they perform very close (within 10%) to the best solutions.

(2) When $\theta_{\min}$>90°, both RB-MIDP and D-MIDP converge to the best solutions; when $\theta_{\min}$<90°, RB-MIDP performs better than D-MIDP for smaller multicast size, but worse for larger multicast size.

(3) D-MIP performs much better than RB-MIP when $\theta_{\min}$ < 180° for any multicast group size, and both converge to the same performance when $\theta_{\min}$ > 180°. This is just as expected since RB-MIP and D-MIP degenerate to MIP when using omni-directional antennas.

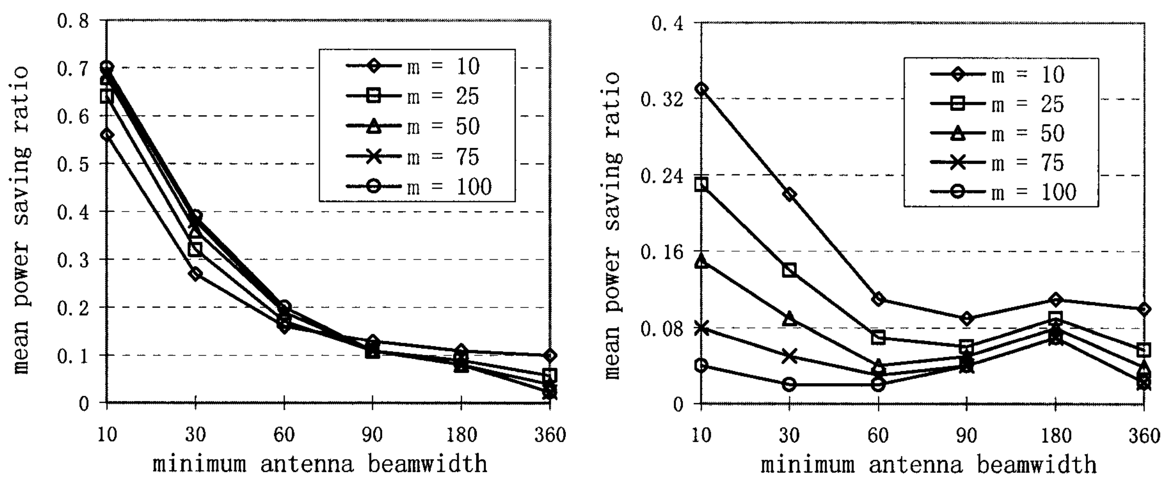


(a) $S_{\text{RB-MIP, RB-MIDP}}$ (b) $S_{\text{D-MIP, D-MIDP}}$

Figure 4-15. Power saving ratio in 20-node networks with adaptive antennas

4.4.4 Power Saving Ratio

Figure 4-15 depicts the performance enhancement achieved by our algorithms RB-MIDP/D-MIDP compared to RB-MIP/D-MIP [WiNg02c] on network topologies with 20 nodes. Referring to the m = 5 members in Fig. 4-15 (a), we observe that the power saving ratio is a decreasing function of minimum antenna beamwidth. This trend remains the same for m = 10, 15, and 20 members. However the group size has minor effect on this metrics. The similar observations can be taken in Fig. 4-15 (b). Therefore we can conclude that RB-MIDP / D-MIDP improving RB-MIP/D-MIP mainly depends on the factor of minimum antenna beamwidth. The smaller the minimum antenna beamwidth is, the more the performance enhancement can be achieved by RB-MIDP/D-MIDP.



(a) $S_{\text{RB-MIP, RB-MIDP}}$ (b) $S_{\text{D-MIP, D-MIDP}}$

Figure 4-16. Power saving ratio in 100-node networks with directional antennas

In 100-node networks as shown in Fig. 4-16, similar observations can be made for RB-MIDP. For D-MIDP, however, the major performance enhancement compared to D-MIP only concentrates the area of small minimum antenna beamwidth ($\theta_{\min} \leq 60°$) and small multicast group size ($m \leq 50$). Another interesting phenomenon in Fig. 4-16 (b) is the power saving ratio becomes an increasing function of the minimum antenna beamwidth when the minimum antenna beamwidth goes beyond a certain value, e.g. 60° when m = 100, for various multicast group sizes.

The increased power saving ratio appears to be more significant in larger group sizes. However, such power saving ratio will finally drop in the case of using omni-directional antennas ($\theta_{\min}$ = 360°). This is because BIP with the "sweep" operation [WiNg00, WiNg02a] already provides nearly optimal performance, and further improvement is very limited. In this situation for both small and large networks, MIDP improves the performance of MIP about 5% - 10% for small multicast group size, but only slightly (about 2%) for broadcast.

4.5 Summary

In this chapter we have presented a general formulation for the minimum power broadcast/multicast problem in wireless ad hoc networks. Based on the analysis on the properties of minimum energy multicast tree, the problem can be characterized as a form of mixed integer linear programming problem. To our best knowledge, these are the first work using mixed integer linear programming to formulate the problem in a context of adaptive antenna applications. Many application scenarios for small networks can be solved efficiently based on this formulation using branch-and-cut or cutting planes techniques. The optimal solutions can be used to assess the performance of heuristic algorithms for mobile networks by running them at discrete time instances. We have also presented a group of novel heuristic algorithms for handling larger networks. The experiment results show that RB-MIDP/D-MIDP provides much better performance than RB-MIP/D-MIP respectively in adaptive antenna applications, and MIDP compares well with MIP in omni-directional antenna applications.

Chapter 5. Broadcast and Multicast Lifetime Maximization Problems

This chapter systematically explores the BLM/MLM problem by translating it to an equivalent min-max tree problem for both cases of using omni-directional and directional antennas. We first study the omni-directional antenna scenarios and use theoretical analysis to derive two polynomial-time global optimal algorithms. We conjecture that the same problem for the directional antenna scenarios is NP-complete and then develop an MILP formulation in a similar manner as the previous chapter. We also use a practical approach by implementing a group of polynomial-time algorithms to handle significantly large networks for which the MILP model may not be computationally efficient.

5.1 Problem Statement

We shall first give the following definition, from which we can describe the problem of maximizing lifetime of multicast tree.

Definition 5-1.
Given a multicast tree T_s rooted at source s, the Tree Lifetime $TLT(T_s)$ can be defined as the time duration starting from beginning of communication supported by the tree until the first node in the tree fails due to battery energy exhaustion.

Similar to Equation (4-1), the maximum lifetime multicast tree T_s^* is

$$T_s^* = \arg\max_{T_s}\{TLT(T_s) \mid \forall T_s \in \Omega_M\}^{1} \qquad . \qquad (5\text{-}1)$$

Optimizing this cost is also very challenge, because we need to select the nodes involved in multicasting in such a way that the energy of all the nodes are depleted uniformly. In other words, given a network, we need to determine all nodes that are bottleneck nodes where energy gets depleted faster than other nodes. The lifetime of the bottleneck nodes determines the lifetime of the entire multicast tree.

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[1] The expression $\arg\max_{x} f(x)$ means "the x such that $f(x)$ is maximized".

5.2 Problem Formulation

Let the energy supply set $\Sigma = \{\varepsilon_u \mid u \in N\}$ be the energy level associated with each node in G. The maximal lifetime τ_{vu} of an arc $(v, u) \in A$ is therefore

$$\tau_{vu} = \frac{\varepsilon_v}{p_{vu} + p_{tran} + p_v^R} \quad . \tag{5-2}$$

Given a multicast tree T_s and the RF transmission power assignment $p_v(T_s)$ for each multicast node v in the tree, the multicast lifetime $\tau_M(T_s)$ related to the lifetime of a tree arc can be obtained using Definition 5-1 as follows:

$$\tau_M(T_s) \equiv \min_{v \in N(T_s)} \begin{cases} \dfrac{\varepsilon_v}{\max_{(v,u)\in A(T_s)} \{p_{vu}\} + p_{tran}}; & v = s \\ \dfrac{\varepsilon_v}{\max_{(v,u)\in A(T_s)} \{p_{vu}\} + p_{tran} + p_{recv}}; & v \text{ is a relay node, but } v \neq s \\ \dfrac{\varepsilon_v}{p_{recv}}; & v \text{ is a leaf node} \end{cases} \tag{5-3}$$

$$= \min_{v \in N(T_s)} \left\{ \min_{(v,u)\in A(T_s)} \{\frac{\varepsilon_v}{p_{vu} + p_{tran} + p_v^R}\} \right\}$$

$$= \min_{(v,u)\in A(T_s)} \{\tau_{vu}\} \quad .$$

What remains is the choice of a multicast tree to maximize the multicast lifetime. That is, we should focus on the selection of multicast routes, which in the wireless environment would translate to choosing the RF transmission power and the set of receiving neighbor nodes in the multicast tree. Let Ω_M be the family of the trees T_s of G spanning all the nodes in M. The objective of our multicast lifetime maximization (MLM) problem is to find a multicast tree with the maximum multicast lifetime τ_M^* defined as

$$\tau_M^* \equiv \max_{T_s \in \Omega_M} \{\tau_M(T_s)\} = \max_{T_s \in \Omega_M} \left\{ \min_{(v,u)\in A(T_s)} \{\tau_{vu}\} \right\} = \frac{1}{\min_{T_s \in \Omega_M} \left\{ \max_{(v,u)\in A(T_s)} \{\frac{1}{\tau_{vu}}\} \right\}} \quad . \tag{5-4}$$

As a special case of MLM problem when $M = N$, the BLM problem is to find a spanning tree with the maximum broadcast lifetime. The statement of the problem now allows us to formulate it as a *min-max tree* problem.

Definition 5-2.
Given a source node s and a subset M of nodes ($s \in M \subseteq N$), the min-max Steiner or multicast tree (MMMT) problem is to determine a directed tree rooted at node s that spans all nodes in M (i.e., there is a directed path on the tree from s to any node $u \in M$), such that the maximum weight of the tree arc is minimized.

We observe from Equation (5-4) that if we define

$$w_{vu} = \frac{1}{\tau_{vu}} = \frac{p_{vu} + p_{tran} + p_v^R}{\varepsilon_v} \quad , \tag{5-5}$$

as the weight for each arc (v, u), then the MLM problem is equivalent to the MMMT problem. The optimal min-max multicast tree T_s^*, corresponding to the optimal solution of the MMMT problem, is therefore a multicast tree with the maximum multicast lifetime. Similarly, the BLM problem is finding a min-max spanning or broadcast tree (MMBT) of G. In this paper, we shall first consider the general min-max tree problem, that is

$$\min_{T_s \in \Omega_M} \left\{ \max_{(v,u)\in A(T_s)} \{w_{vu}\} \right\}, M \subseteq N \quad . \tag{5-6}$$

Let $w_M(T_s)$ denote the maximum arc weight of a tree $T_s \in \Omega_M$, i.e.

$$w_M(T_s) = w_{xy} = \max_{(v,u)\in A(T_s)} \{w_{vu}\}, M \subseteq N, \tag{5-7}$$

where (x, y) is called the bottleneck arc of the tree T_s. Therefore the optimal solution of problem (5-6) w_M^* is the minimum of $w_M(T_s)$ over Ω_M, i.e.,

$$w_M^* = \min_{T_s \in \Omega_M} \{w_M(T_s)\} = w_M(T_s^*), M \subseteq N. \tag{5-8}$$

The structure of the problem formulations is given in Figure 1-3. One can see that the $M = N$ case is the MMBT problem, while the $M \subset N$ case is the MMMT problem. We also include the solutions in Fig. 1-3 for each sub-problem, which will be discussed further in the following sections.

5.3 Maximizing Lifetime in Omni-directional Antenna Networks

In this section, we investigate the BLM/MLM problem in energy-constrained wireless ad-hoc networks with omni-directional antennas. We have provided a few theorems that would allow us to solve the BLM/MLM problem in the case of equally distributed energy supply and the case of unequally distributed energy supply.

5.3.1 Case of Equally Distributed Energy Supply

This is a special case when all the nodes have identical initial battery energy levels, i.e., $\varepsilon_v = \varepsilon$ for any node $v \in N$. If we define the weight on each arc as follows,

$$\begin{cases} w_{su} = w_{us} = \dfrac{r_{su}^{\alpha} + p_{tran}}{\varepsilon}; & \forall (s,u) \in A \\ w_{vu} = w_{uv} = \dfrac{r_{vu}^{\alpha} + p_{tran} + p_{recv}}{\varepsilon}; & v,u \neq s, \forall (v,u) \in A \end{cases} , \tag{5-9}$$

then the original graph G can be simply considered as an undirected graph.

5.3.1.1 The BLM Problem

As discussed in the last section, the BLM problem is equivalent to the MMBT problem. Another problem related to the MMBT problem is the following[1]:

$$\min_{T \in \Omega_N} \left\{ \sum_{(v,u) \in A(T)} w_{vu} \right\}, \tag{5-10}$$

which is the well-known minimum spanning tree (MST) problem. Problem MMBT is related to problem (5-10) because if T_{MST} is a minimum spanning tree of G, T_{MST} is also a min-max broadcast tree of G (the converse is obviously not true) [KaPo03a]. For completeness, we include the following theorem.

Theorem 5-1.
In an undirected graph G, if T_{MST} is a minimum spanning tree of G, T_{MST} is also a min-max broadcast tree of G, i.e. $w_N^* = w_N(T_{MST})$.

[1] In the following, we use T to indicate an undirected tree in an undirected graph, while T_S a directed tree rooted at node s in a directed graph.

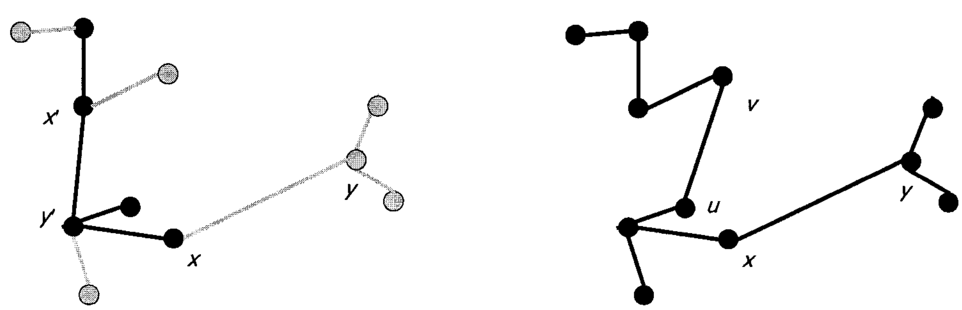
Proof:
The proof can be found in [KaPo03a] and is omitted. ■

Problem (5-10) has been studied extensively, and many algorithms have been proposed for solving minimum spanning tree problems e.g. [ChTa76, Gabo77, Prim57]. In comparison, the MMBT problem receives much less attention, and therefore subject of our study in Section 5.3.2.1.

5.3.1.2 The MLM Problem

Similar to BLM, the MLM problem can be formulated as an MMMT problem. We provide in the following a theorem that would give us a global optimal solution of MLM problem in energy equally distributed networks with omni-directional antennas.

Theorem 5-2.
In an undirected graph G, if $T_{p\text{-}MST}$ is a pruned minimum spanning tree[1] of G, then $T_{p\text{-}MST}$ is also a min-max multicast tree of G, i.e. $w_M^* = w_M(T_{p-MST})$.



(a) (b)

Figure 5-1. Illustration of proof for Theorem 5-2

Proof:
We assume that (x, y) and (x', y') are the arcs of maximum weight of T_{MST} and $T_{p\text{-}MST}$ respectively, i.e. $w_{xy} = w_N(T_{MST})$ and $w_{x'y'} = w_M(T_{p-MST})$. We note that (x, y) may not be the same as (x', y'), as shown in Fig. 5-1 (a), where the light nodes and arcs indicate the branches pruned. Let N_1 and N_2 be the two subsets of nodes which become disconnected if arc (x', y') is removed from

[1] A pruned spanning tree is obtained by deleting all arcs that are not needed to reach the multicast nodes.

T_{MST}, where $x' \in N_1$ and $y' \in N_2$. We further assume that $s \in N_1$. Then the weight of any arc (v, u) of G connecting a node of N_1 with a node of N_2 is not less than $w_{x'y'}$, i.e.

$$w_{x'y'} \leq w_{vu}, \text{ for } \forall v \in N_1,\ \forall u \in N_2, \text{ and } (v, u) \in A \quad . \tag{5-11}$$

Otherwise, if there exists an arc (v, u) connecting N_1 and N_2 and satisfying $w_{x'y'} > w_{vu}$, then a new spanning tree T' as shown in Fig. 5-1 (b) of G can be constructed from T_{MST} by replacing (x', y') with (v, u). Thus we have $\sum_{(v,u)\in A(T')} w_{vu} < \sum_{(v,u)\in A(T_{MST})} w_{vu}$, which leads to a contradiction with the definition of MST.

We note that there is at least one multicast member z belonging to N_2, i.e. $z \in N_2 \cap M$, since otherwise any node in N_2 should be pruned, and then it contradicts the assumption $y' \in N_2$. For any multicast tree $T \in \Omega_M$, there must exist an arc $(a, b) \in A(T)$, $a \in N_1$ and $b \in N_2$, that connects N_1 and N_2 in order to satisfy the property that there exists a directed path from s to the multicast member z (as stated in Lemma 3-1). From the conclusion in (5-11), we have $w_M(T_{p-MST}) = w_{x'y'}$ $\leq w_{ab} \leq \max\{w_{vu} \mid (v,u) \in A(T)\} = w_M(T)$, $\forall\ T \in \Omega_M$. This is equivalent to say that $T_{p\text{-}MST}$ is a min-max multicast tree of G, i.e. $w_M(T_{p-MST}) = w_M^*$. ■

To realize our optimal solution, we provide a simple procedure PRUN whose pseudo code is given below in recursive form. The inputs are a broadcast tree T, a multicast group M and the source node s. The procedure PRUN(T, M, s) returns the corresponding multicast tree spanning all the nodes in M by pruning from a broadcast tree T all transmissions that are not needed to reach the nodes in M.

The PRUN(T, M, v) Algorithm

1) For each child u of v in the tree T, update the tree T = PRUN(T, M, u).
2) If node v is a leave node of T and $v \notin M$, then update the tree $T = T - \{(p(v), v)\}$, where $p(v)$ is the uplink tree node of v.
3) Return T.

We note from Theorems 5-1 and 5-2 that the bottleneck arc weight of a broadcast/multicast tree in an undirected is minimized by an MST/pruned-MST.

5.3.1.3 Time Complexity

From the view point of time complexity, when G is a complete graph, one of the best known algorithms for problem (5-10) is due to Prim [Prim57] with an $O(|N|^2)$ complexity. Kruskal's algorithm [Kru56] is better for sparse graphs. Its running time is asymptotically dominated by the time for sorting the edges by weight. When G is even moderately sparse, a much more sophisticated method [ChTa76] is preferable, yielding an $O(|A|\log\log|N|)$ complexity.

To obtain the optimal solution of the MLM problem based on a min-max broadcast tree, our approach only yields an additional $O(|N|)$ complexity on the procedure PRUN.

5.3.2 Case of Unequally Distributed Energy Supply

In the general distribution of the initial energy supply (i.e. $\varepsilon_v \neq \varepsilon_u$), the graph is no longer undirected. The weight on each arc is given below.

$$\begin{cases} w_{su} = \dfrac{r_{su}^{\alpha} + p_{tran}}{\varepsilon_s}; & \forall (s,u) \in A \\ w_{vu} = \dfrac{r_{vu}^{\alpha} + p_{tran} + p_{recv}}{\varepsilon_v}; & v \neq s, \forall (v,u) \in A \end{cases} \qquad (5\text{-}12)$$

Again, we need to solve the related BLM and MLM problems.

5.3.2.1 The BLM Problem

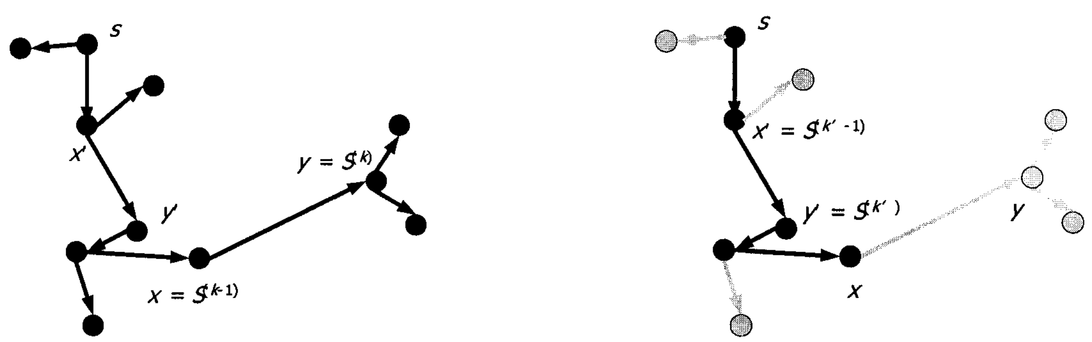
Unlike the BLM problem in an energy equally distributed network, we like to point out that the minimum spanning tree of a directed graph G is not necessarily a min-max broadcast tree of G (and vice versa), because the *minimum* and *min-max* problems have been shown to be not the same as in the undirected case [KaPo03a]. However, we shall observe later that a min-max broadcast tree of a directed graph G can be simply obtained using the well-known Prim's algorithm, which was originally designed to find an MST in an undirected graph.

We define a Directed Prim Tree (DPT) as the spanning rooted tree constructed by Prim's algorithm. In fact, if T_{DPT} is a directed prim tree, T_{DPT} is also a min-max broadcast tree. In order to prove this, we first recall the standard Prim algorithm for the formation of a minimum spanning tree. It maintains throughout its execution a single tree rooted at the source node.

Initially, the rooted tree only includes the source node. Subsequently, new nodes are added to the tree iteratively one at a time on a minimum weigh basis until all nodes are included in the tree.

Let $S^{(i)}$ be the new node chosen at the i-th iteration of the tree incremental formation, where $i = 1, 2, \ldots, |N|$-1, and $S^{(0)} = s$. At the same time, a new arc $(S^{(i-1)}, S^{(i)})$ is added in the tree with the minimum weigh, i.e.

$$w_{s^{(i-1)}s^{(i)}} \le w_{vu}, \text{ for } \forall v \in N_1^i \equiv \bigcup_{j=0}^{i} S^{(j)}, \ \forall u \in N_2^i \equiv N - N_1^i, \text{ and } (v, u)\in A. \qquad (5\text{-}13)$$



(a) T_{DPT} (b) $T_{p\text{-}DPT}$

Figure 5-2. Illustration of proof for Theorems 5-3 and 5-4
(a) a directed prim tree, (b) the corresponding pruned directed prim tree

Theorem 5-3.
In a directed graph G, if T_{DPT} is a directed prim tree of G, then T_{DPT} is also a min-max broadcast tree of G, i.e. $w_N^* = w_N(T_{DPT})$.

Proof:
We assume that (x, y) is the arc with maximum weight of T_{DPT} as shown in Fig. 5-2 (a), and it is added into the tree at the k-th iteration, i.e. $w_{xy} = w_{s^{(k-1)}s^{(k)}} = w_N(T_{DPT})$. For any broadcast tree $T_s \in \Omega_N$, there must exist an arc $(a, b) \in A(T_s)$, $a \in N_1^i$ and $b \in N_2^i$, to connect two node-disjoint sets N_1^k and N_2^k to keep the connectivity of the tree. From Constraint (5-13), we have $w_N(T_{DPT}) = w_{xy} \le w_{ab} \le \max_{(v,u)\in A(T_s)} \{w_{vu}\} = w_N(T_s)$, $\forall\ T_s \in \Omega_N$. Hence T_{DPT} is a min-max broadcast tree of G, i.e. $w_N(T_{DPT}) = w_N^*$. ■

We shall extend the standard Prim algorithm into the directed graph, and denote the modified version as the DPBT (Directed Prim Broadcast Tree) algorithm. Let $T_s^{(i)}$ be the intermediate tree constructed at the i-th iteration using our DPBT algorithm. The DPBT algorithm described below has two input parameters: the given directed graph G and a source node s.

The DPBT(G, s) Algorithm

1) Initialize $T_s^{(0)}$ by setting $N(T_s^{(0)})= \{s\}$ and $A(T_s^{(0)}) =\varnothing$, and initialize weight w_{vu} using Equation (16) for any $(v, u) \in A$.
2) For (i = 1 to $|N|$-1)
   i) Find the arc (x, y) connecting tree node and outside node such that the value w_{xy} is minimized, i.e. $w_{xy} = \min\{w_{vu} \mid v \in N(T_s^{(i-1)}),\ u \in N\text{-}N(T_s^{(i-1)}), \text{ and } (v, u)\in A\}$.
   ii) Construct $T_s^{(i)}$ by setting $N(T_s^{(i)}) = N(T_s^{(i-1)})\cup\{y\}$ and $A(T_s^{(i)}) = A(T_s^{(i-1)})\cup\{(x, y)\}$.
3) Return the final broadcast tree $T_s^{(|N|-1)}$.

5.3.2.2 The MLM Problem

In Section 5.3.1.2, we showed that a min-max multicast tree of an undirected graph can be obtained by simply pruning a minimum spanning tree. It is natural to ask whether a min-max multicast tree of a directed graph can be simply obtained by pruning a DPT. The answer is yes and is stipulated by the following theorem.

Theorem 5-4.
In a directed graph G, if $T_{p\text{-}DPT}$ is a pruned directed prim tree of G, then $T_{p\text{-}DPT}$ is also a min-max multicast tree of G, i.e. $w_M^* = w_M(T_{p-DPT})$.

Proof:
We assume that (x, y) and (x', y') are the arcs with maximum weight in T_{DPT} and $T_{p\text{-}DPT}$ respectively, i.e. $w_{xy} = w_N(T_{DPT})$ and $w_{x'y'} = w_M(T_{p-DPT})$, as shown in Fig. 5-2 (b). We further assume $x = s^{(k-1)}$, $y = s^{(k)}$, $x' = s^{(k'-1)}$, and $y' = s^{(k')}$. Note that there is at least one multicast member z belonging to $N_2^{k'}$, i.e. $z \in N_2^{k'}\cap M$, since otherwise any node in $N_2^{k'}$ should be pruned, which contradicts the fact $y' \in N_2^{k'}$. For any multicast tree $T_s \in \Omega_M$, there must exist an arc $(a, b) \in A(T_s)$ that connects $N_1^{k'}$ and $N_2^{k'}$ ($a \in N_1^{k'}$ and $b \in N_2^{k'}$) in order to satisfy the property that there exists a directed path from s to the multicast member z (Lemma 3-1). Therefore, from

Constraint (5-13) we have $w_M(T_{p-DPT}) = w_{x'y'} \le w_{ab} \le \max_{(v,u)\in A(T_s)} \{w_{vu}\} = w_M(T_s)$, $\forall\ T_s \in \Omega_M$, i.e.

$w_M(T_{p-DPT}) = w_M^*$. ■

Theorem 5-4 immediately suggests a solution for our MLM problem, which we shall call the DPMT (Directed Prim Multicast Tree) algorithm. Its procedure is given below.

The DPMT(G, M, s) Algorithm

1) Initialize a directed prim tree T_s by setting T_s = DPBT(G, s).
2) Return the final multicast tree PRUN(T_s, M, s).

Similar to the Equally Distributed Energy Supply in Section 5.3.1, we would argue that the bottleneck arc weight of a broadcast/multicast tree in a directed graph is minimized by a DPT/pruned-DPT.

5.3.2.3 Time Complexity

Our DPBT algorithm has the same complexity as Prim's algorithm [Prim57]. The advantage of our approach using DPBT algorithm is that it has only an $O(|N|^2)$ complexity. This can be compared to the complexity of $\Theta((|N|+|A|)\log|A|) = \Theta(|N|^2\log|N|)$ in the min-max broadcast tree algorithm [KaPo03a]. Again, the DPMT algorithm has only an $O(|N|^2)$ complexity, which is better than the algorithm of finding min-max multicast tree proposed in [FlKa03] with a complexity $O(|N|^2\log|N|)$.

5.4 Maximizing Lifetime in Directional Antenna Networks

We now turn our attention to the more difficult task of multicast time maximization using directional antennas. We first consider the form of weight on each arc, which can be easily obtained by substituting Equation (3-1) into Equation (5-5):

$$w_{vu} = \frac{1}{\tau_{vu}} = \begin{cases} \dfrac{r_{vu}^{\alpha} \cdot \theta_v}{360 \cdot \varepsilon_v} + \dfrac{p_{tran}}{\varepsilon_v} & v = s \text{ and } \forall (v,u) \in A \\ \dfrac{r_{vu}^{\alpha} \cdot \theta_v}{360 \cdot \varepsilon_v} + \dfrac{p_{tran} + p_{recv}}{\varepsilon_v} & v \neq s \text{ and } \forall (v,u) \in A \end{cases} \qquad (5\text{-}14)$$

5.4.1 Special Case Studies

In fact, in an energy-limited wireless ad-hoc network, it is desirable to be able to precompute the min-max broadcast/multicast tree for any given M. However our approaches in Section 5.3 are only for fixed weight in the case of omni-direction antennas (see Equations 5-9 and 5-12), but not necessarily valid for directional antennas (see Equation 5-14) since the value of $\theta_v = \theta_v(T_s)$ normally remains unknown until the tree T_s is completely constructed. Fortunately, there are couple special cases that still can be solved using the same approach discussed in the last section if $\theta_v(T_s)$ could degenerate to a constant[1]. They are the following.

Corollary 5-5.
If network nodes are equipped with fixed-beamwidth antennas ($\theta_{min} = \theta_{max} = \theta$), and the condition $\theta_v(T_{DPT}) \leq \theta$ can be satisfied for any node $v \in N$, then $w^*_N = w_N(T_{DPT})$.

Corollary 5-6.
If network nodes are equipped with fixed-beamwidth antennas ($\theta_{min} = \theta_{max} = \theta$), and the condition $\theta_v(T_{p\text{-}DPT}) \leq \theta$ can be satisfied for any node $v \in N(T_{p\text{-}DPT})$, then $w^*_M = w_M(T_{p-DPT})$.

Corollary 5-7.
If network nodes are equipped with adaptive antennas ($\theta_{min} \neq \theta_{max}$) and if $|M| = 2$ (unicast case[2]), then $w^*_M = w_M(T_{p-DPT})$, where $T_{p\text{-}DPT}$ degenerates to a min-max unicast path.

The solutions to the above special cases have a polynomial time complexity. Unfortunately, we do not have any scalable solutions for the general MLM/BLM problem ($|M| > 2$) using adaptive antennas. In fact, we suspect that this problem is NP-complete.

5.4.2 MILP Formulation

The definition of multicast tree given in the context of directional antenna applications allows us to rewrite the formulation of the min-max tree problem (5-10) in terms of MILP. The main idea is to extract a sub-graph T_s^* from the original graph G such that T_s^* is a multicast tree rooted at

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[1] In the case of using omni-directional antennas, the value of $\theta_v(T_s)$ is 360.
[2] In the case of unicast, the value of $\theta_v(T_s)$ is θ_{min}.

node s with maximum lifetime. The MILP approach would provide optimal solutions in a timely manner for moderately sized networks.

5.4.2.1 Optimization Variables

In order to formulate the problem, we use the same set of variables introduced in Section 4.2.1, and an additional variable w, which is a nonnegative continuous variable that represents the weight of bottleneck arc in T_s^*. The variable w must satisfy the definition of bottleneck given in Equation (5-7).

5.4.2.2 Objective Function

The objective function is therefore

$$\textit{Minimize:} \quad w \quad . \tag{5-15}$$

By substituting Equation (5-14) into Equation (5-7), w can be written as follows.

$$w = \max_{(v,u)\in A(T_s^*)} \begin{cases} \dfrac{r_{vu}^{\alpha}}{360\cdot\varepsilon_v}\cdot\theta_v + \dfrac{p_{tran}}{\varepsilon_v}; & v = s \\ \dfrac{r_{vu}^{\alpha}}{360\cdot\varepsilon_v}\cdot\theta_v + \dfrac{p_{tran}+p_{recv}}{\varepsilon_v}; & v \neq s \end{cases} \quad . \tag{5-16}$$

5.4.2.3 Linear Constraints

In Section 4.2.3, we have already provided a set of linear constraints that would guarantee that T_s^* obtained from the MILP formulation to be a multicast tree, i.e., satisfying both the rooted tree property (RTP) and the antenna coverage property (ACP).

We also note that the "max" form in Constraint (5-16) is nonlinear. In order to finalize the MILP formulation of the BLM/MLM problem, it remains to construct additional constraints that could linearize Constraint (5-16). This will be accomplished by the following theorem.

Theorem 5-8.
The variable w represents the weight of bottleneck arc in T_s^*, if the formulation of Problem MLM includes Constraints (5-17a) and (5-17b), where γ is a relatively large number.

$$w-\left(\frac{r_{su}^{\alpha}}{360\cdot\varepsilon_s}\cdot\theta_s+\frac{p_{tran}}{\varepsilon_s}\right)\geq\gamma(Z_{su}-1);u\in N \tag{5-17a}$$

$$w-\left(\frac{r_{vu}^{\alpha}}{360\cdot\varepsilon_v}\cdot\theta_v+\frac{p_{tran}+p_{recv}}{\varepsilon_v}\right)\geq\gamma(Z_{vu}-1);v\in N\setminus\{s\},u\in N \quad . \tag{5-17b}$$

Proof:

See Appendix B.4 for details. ■

$$\textit{Minimize}: w \quad .$$

Subject to:

$$w-\left(\frac{r_{su}^{\alpha}}{360\cdot\varepsilon_s}\cdot\theta_s+\frac{p_{tran}}{\varepsilon_s}\right)\geq\gamma(Z_{su1}+Z_{su2}-1);u\in N$$

$$w-\left(\frac{r_{vu}^{\alpha}}{360\cdot\varepsilon_v}\cdot\theta_v+\frac{p_{tran}+p_{recv}}{\varepsilon_v}\right)\geq\gamma(Z_{vu1}+Z_{vu2}-1);v\in N\setminus\{s\},u\in N$$

RTP (a):

$$\begin{cases}\sum_{v\in N}(Z_{vs1}+Z_{vs2})=0;\\ \sum_{v\in N}(Z_{vu1}+Z_{vu2})=1;\forall u\in M\setminus\{s\}\\ \sum_{v\in N}(Z_{vu1}+Z_{vu2})\leq 1;\forall u\in N\setminus M\\ \sum_{v\in N}(Z_{uv1}+Z_{uv2})\leq(n-1)\sum_{v\in N}(Z_{vu1}+Z_{vu2});\forall u\in N\setminus M\end{cases}$$

RTP (b):

$$\begin{cases}\sum_{v\in N}F_{vu}-\sum_{v\in N}F_{uv}=\sum_{v\in N}(Z_{vu1}+Z_{vu2});\forall u\in N\setminus\{s\}\\ Z_{vu1}+Z_{vu2}\leq F_{vu}\leq(n-1)\cdot(Z_{vu1}+Z_{vu2});\forall u\in N\setminus\{s\},\forall v\in N\end{cases}$$

ACP (a):

$$\begin{cases}p_v^{RF}+p_v^{T}\geq Y_{vu}\cdot r_{vu}^{\alpha}/2\pi+p_{tran}\cdot(Z_{vu1}+Z_{vu2});\forall v,u\in N\\ 0\leq Y_{vu}\leq 2\pi\cdot(Z_{vu1}+Z_{vu2});\forall v,u\in N\\ Y_{vu}\leq\theta_v;\forall v,u\in N\\ \theta_v-Y_{vu}+2\pi\cdot(Z_{vu1}+Z_{vu2})\leq 2\pi;\forall v,u\in N\\ p_v^{RF}\leq p_{\max};\forall v\in N\end{cases}$$

ACP (b):

$$\begin{cases}A_{vu}\varphi_v-\theta_v+B_{vu}Z_{vu1}\leq C_{vu};\forall v,u\in N\\ 2\varphi_v-\theta_v+(4\pi-2\alpha_{vu})Z_{vu2}\leq 4\pi;\forall v,u\in N\\ 2\varphi_v+\theta_v-2\alpha_{vu}Z_{vu2}\geq 0;\forall v,u\in N\\ 0\leq\varphi_v<2\pi;\forall v\in N\\ \theta_{\min}\leq\theta_v\leq\theta_{\max};\forall v\in N\end{cases}$$

Integrality: $Z_{vu1}, Z_{vu2}\in\{0, 1\}, \forall v, u\in N$

Figure 5-3. MILP model for problem MLM under adaptive antenna networks

The constant γ in Constraint (5-17) should be large enough to make the inequality always tenable when $Z_{vu} = 0$. A possible value is given below.

$$\gamma = \max_{(v,u)\in A} \left\{ \frac{r_{vu}^{\alpha} + p_{tran} + p_{recv}}{\varepsilon_v} \right\} \quad . \tag{5-18}$$

5.4.2.4 MILP Model

Our previous derivation on the linear constraints can now allow us to formulate the BLM/MLM Problem as a MILP model. The general MILP model for the MLM problem under adaptive antenna networks is shown in Fig. 5-3.

A number of MILP models for various scenarios can be derived from such general MILP model in Fig. 5-3 in a similar way as in Section 4.2.4. The coefficients A_{vu}, B_{vu}, and C_{vu} in Fig. 5-3 are also given in Table 4-3. In this formulation, Z_{vu1} and Z_{vu2} are binary variables; others are continuous variables. The number of variables in the formulation is approximately $3n^2 + 5n + 1$, and the number of constraints is of the order of $O(n^2)$.

5.4.3 Heuristic Algorithms

In order to handle significantly large networks with directional antennas for which the MILP model may not be computationally efficient, we shall design the following heuristic algorithms for the MLM/BLM problem to explore the properties that we have studied in Section 5.3. We have considered two approaches for broadcasting and multicasting with adaptive antennas: (1) construct the tree by using an algorithm designed for omni-directional antennas as if the graph has a fixed (static) weight, and then reduce each antenna beam to the minimum possible width that can support the tree; (2) incorporate the directional antennas properties into the tree construction process by updating the weight dynamically.

5.4.3.1 Static Weight Approach

In this approach, we assume that the transmitting antennas are omni-directional. Therefore, we can ignore the term $\frac{\theta_v(T_s)}{360}$ in Equation (5-14) and directly apply the DPBT or the DPMT algorithm. After the tree is constructed, each internal node reduces its antennas beamwidth to the smallest possible value that can still provide a coverage of the node's downstream neighbors in the tree, subject to the constraint $\theta_{\min} \leq \theta_v(T_s) \leq 360°$. Thus in the process of the tree formation, the tree structure is independent of $\theta_{\min}$. When applied to the DPBT algorithm, we shall call this scheme Static-weight DPBT (S-DPBT). Likewise, when applied to the DPMT algorithm, the resulting scheme is called Static-weight DPMT (S-DPMT). The descriptions are given below.

The S-DPBT(G, s) Algorithm

1) Initialize a directed prim tree T_s by setting T_s = DPBT(G, s).
2) Each internal node of T_s reduces its antennas beamwidth to the smallest possible value that provides coverage of the node's downstream neighbors in the tree T_s.
3) Return the final broadcast tree T_s.

S-DPMT(G, M, s) Algorithm

1) Initialize a pruned directed prim tree T_s by setting T_s = DPMT(G, M, s).
2) Each internal node of T_s reduces its antennas beamwidth to the smallest possible value that provides coverage of the node's downstream neighbors in the tree T_s.
3) Return the final multicast tree T_s.

5.4.3.2 Dynamic Weight Approach

D-DPBT is similar in principle to DPBT algorithm for the formation of directed prim tree, in the sense that new nodes are added to the tree one at a time on a minimum weight basis until all nodes are included in the tree.

Unlike the input arc weights $w_{vu} = \frac{r_{vu}^{\alpha}}{\varepsilon_v}$ in the DPBT, which are precomputed and can remain unchanged throughout the execution of the algorithm, D-DPBT must dynamically update the arc weights $w_{vu} = \frac{r_{vu}^{\alpha} \cdot \theta_v(T_s)}{360 \cdot \varepsilon_v}$ at each step to reflect the value $\theta_v(T_s)$. A pseudo code of the D-DPBT algorithm is given below where $T_s^{(i)}$ is the intermediate tree constructed at the i-th iteration using our D-DPBT algorithm.

The D-DPBT(G, s) Algorithm

1) Initialize $T_s^{(0)}$ by setting $N(T_s^{(0)})= \{s\}$ and $A(T_s^{(0)}) =\varnothing$, and initialize weight w_{xu} as follows

$$w_{vu} = \begin{cases} \dfrac{r_{vu}^{\alpha}\cdot\theta_{\min}}{360\cdot\varepsilon_v} + \dfrac{p_{tran}}{\varepsilon_v} & v = s \text{ and } \forall(v,u)\in A; \\ \dfrac{r_{vu}^{\alpha}\cdot\theta_{\min}}{360\cdot\varepsilon_v} + \dfrac{p_{tran}+p_{recv}}{\varepsilon_v} & v \neq s \text{ and } \forall(v,u)\in A. \end{cases} \tag{5-19}$$

2) For (i = 1 to $|N|$-1)
   i) Find the arc (x, y) connecting tree node and outside node such that the value w_{xy} is minimized, i.e. $w_{xy} = \min\{w_{vu} \mid v \in N(T_s^{(i-1)}),\ u \in N\text{-}N(T_s^{(i-1)}),\ \text{and } (v, u)\in A\}$.
   ii) Construct $T_s^{(i)}$ by setting $N(T_s^{(i)}) = N(T_s^{(i-1)})\cup\{y\}$ and $A(T_s^{(i)}) = A(T_s^{(i-1)})\cup\{(x, y)\}$.
   iii) Update w_{xu} using Equation (26) for any node $u \in N\text{-}N(T_s^{(i)})$ and $(x, u)\in A$:

$$w_{xu} = \begin{cases} \dfrac{r_{xu}^{\alpha}\cdot\theta_x(T_s')}{360\cdot\varepsilon_x} + \dfrac{p_{tran}}{\varepsilon_x} & x = s; \\ \dfrac{r_{xu}^{\alpha}\cdot\theta_x(T_s')}{360\cdot\varepsilon_x} + \dfrac{p_{tran}+p_{recv}}{\varepsilon_x} & x \neq s \end{cases}, \tag{5-20}$$

   where T'_s is determined by $N(T'_s) = N(T_s^{(i)})\cup\{u\}$ and $A(T'_s) = A(T_s^{(i)})\cup\{(x, u)\}$.

3) Return the final broadcast tree $T_s^{(|N|-1)}$.

Similar to the approach we have applied in Section 5.3, the D-DPMT (Dynamic-weight DPMT) algorithm simply prunes the broadcast tree obtained using D-DPBT. The description of the algorithm is given below.

The D-DPMT(G, M, s) Algorithm

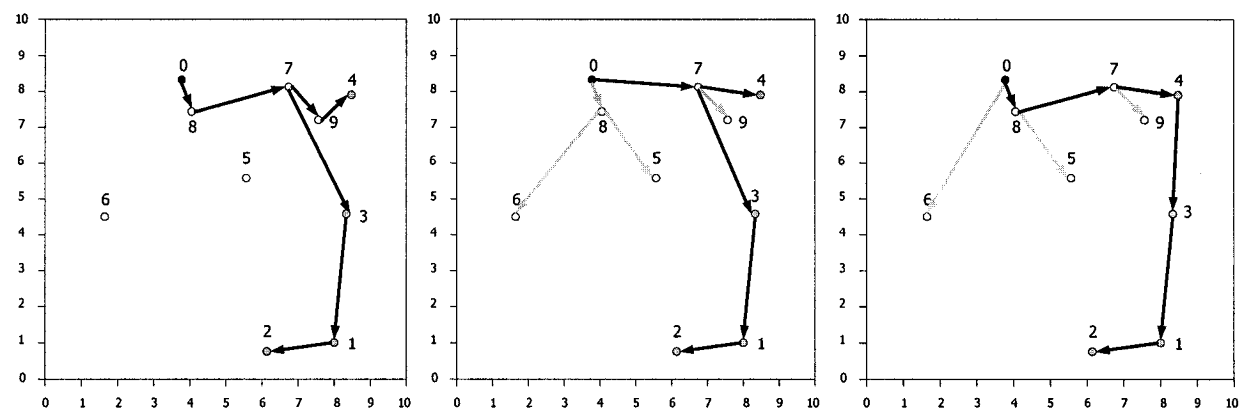
1) Initialize a broadcast tree T_s by setting T_s = D-DPBT(G, s).
2) Return the final multicast tree PRUN(T_s, M, s).

5.4.3.3 An Example

We consider a generic example of a 10-node network, in which nodes {0, 1, 2, 3, 4} are multicast members and Node 0 is the source. Each node is equipped a directional antenna with minimal beamwidth 30°. The node placement (x_i, y_i) and initial energy supply e_i are listed in Table 5-1. For simplicity, no restrictions are placed on the maximum transmission power ($p_{max} = \infty$), power consumptions on transmission and reception are ignored ($p_{tran} = p_{recv} = 0$), and a propagation-loss exponent of $\alpha = 2$ is assumed.

Table 5-1. Network configuration and values of parameters

Network configuration											Parameters	
Node i	0	1	2	3	4	5	6	7	8	9	$\theta_{\min}$	30°
x_i	3.8	8.0	6.1	8.2	8.4	5.5	1.6	6.8	4.0	7.1	p_{tran}	0
y_i	8.3	1.0	0.8	4.6	7.9	5.6	4.5	8.1	7.4	7.8	p_{recv}	0
ε_i	854	593	415	695	560	274	662	1001	698	244	α	2



(a) OPT (lifetime = 641) (b) S-DPMT (lifetime = 415) (c) D-DPMT (lifetime = 615)

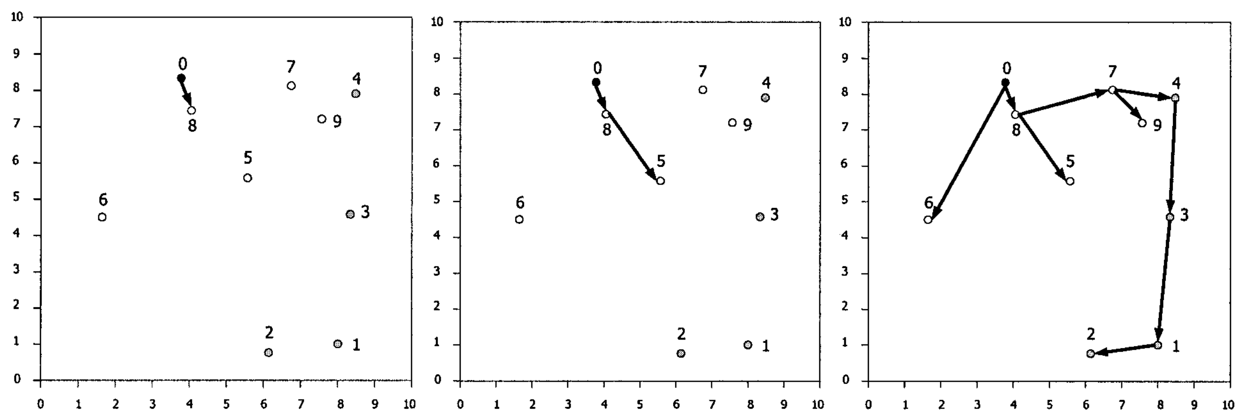
Figure 5-4. Multicast trees produced by variant algorithms based on the network configuration listed in Table 1 (Solid arcs indicate multicast tree arcs; light arcs indicate the pruned branches from the broadcast tree.)

The optimal min-max multicast tree is shown in Fig. 5-4 (a), where OPT denotes the branch-and-bound algorithm based on our MILP model given in Figure 5-3. Figures 5-4 (b) and 5-4 (c) show the multicast trees produced by S-DPMT and D-DPMT for the same network configuration. In this example, we observe that D-DPMT is better than S-DPMT and performs very close (within 95%) to the optimal solution.

Specially, we would also like to describe the basic operation of D-DPBT by presenting a simple example of tree construction using the network configuration given in Table 5-1.

Step 1: Initially, the tree consists of only the source node and the initial weight value w_{vu} on each arc (v, u) is set as $\frac{[(x_v - x_u)^2 + (y_v - y_u)^2]\cdot 30}{\varepsilon_v \cdot 360}$. We then determine the node u that Node

0 can reach with minimum weight w_{0u} (u = 1, …, 9). Based on the information in Table 1, Node 8 node is chosen and arc (0, 8) is added to the tree. Thus $N(T_0^{(1)})$ = {0, 8} and $A(T_0^{(1)})$ = {(0, 8)} as shown in Fig. 5-5 (a), where 0→8 denotes the arc chosen to be included in the tree at this step. We then update weigh for each arc (0, u) ($u \in$ {1, 2, 3, 4, 5, 6, 7, 9}). For example, the value of w_{07} is changed accordingly from $\frac{[(x_0-x_7)^2+(y_0-y_7)^2]\cdot 30}{\varepsilon_0 \cdot 360}$ to $\frac{[(x_0-x_7)^2+(y_0-y_7)^2]\cdot \angle 807}{\varepsilon_0 \cdot 360}$, where $\angle 807$ indicates the degree of angle between arc (0, 8) and arc (0, 7).



(a) Step 1: 0 → 8 (b) Step 2: 8 → 5 (c) final tree

Figure 5-5. Example of tree construction using D-DPBT

Step 2: We then again determine the next new node to be added into the tree. There are two alternatives. Either Node 0 or Node 8 adjusts its transmission power level and antenna beamwidth to reach a new node outside {0, 8}. Since $w_{85} = \min_{u \neq 0,8}\{w_{0u}, w_{8u}\}$, Node 5 is chosen at this time and a new arc (8, 5) is added into the tree. At this point, we have obtained $N(T_0^{(2)})$ = {0, 5, 8} and $A(T_0^{(2)})$ = {(0, 8), (8, 5)} as shown in Figure 5-5 (b). Consequently, we update the weigh for each arc (8, u), $u \in$ {1, 2, 3, 4, 6, 7, 9}. For example, the value of w_{87} is changed accordingly from $\frac{[(x_8-x_7)^2+(y_8-y_7)^2]\cdot 30}{\varepsilon_8 \cdot 360}$ to

$$\frac{[(x_8-x_7)^2+(y_8-y_7)^2]\cdot\angle 587}{\varepsilon_8\cdot 360}.$$

The following steps: This procedure is continued until all nodes are included in the tree, as shown in Fig. 5-5 (c). The order in which the nodes were added in step 3 through 9 is: 8→7, 7→9, 7→4, 4→3, 3→1, 1→2, 0→6.

5.4.3.4 Time Complexity

The time complexity of S-DPBT is $O(|N|^2\log|N|)$. In the S-DPBT algorithm, Step 1 is just DPBT, which has a complexity of $O(|N|^2)$ as explained in Section 5.3. In Step 2, we apply the sector-choosing algorithm introduced in [WiNg02d] to find the minimum beamwidth that reaches all children. The overall complexity of the beam reduction operation in Step 2 for all transmitting nodes is $O(|N|^2\log|N|)$ [WiNg02d]. A similar complexity analysis can be made for the S-DPMT algorithm.

The time complexity of D-DPBT is $O(|N|^3\log|N|)$. This can be explained as follows. In Step 2-i), a minimum-weight arc (x, y) must be chosen, and the complexity of this search is at most $O(|N|^2)$. Then in Step 2-ii), the new arc (x, y) is added into the tree with a complexity of $O(1)$. Finally, in Step 2-iii) the weights that relate to the new child's parent x must be updated. For each outside node u, the complexity of the weight update operation is $O(|N|\log|N|)$ using the sector-choosing algorithm [WiNg02d], which involves to calculate the minimum beamwidth of node x after the new child u is added. Since the number of children outside the tree the node x may have is at most $|N| - 1$, the overall complexity of Step 2-iii) is $O(|N|^2\log|N|)$. Therefore, the complexity at each iteration of Step 2) is at most $O(|N|^2\log|N|)$. Since there are $|N|$ steps, the overall complexity is at most $O(|N|^3\log|N|)$. Similarly, D-DPMT has the same complexity of $O(|N|^3\log|N|)$ as D-DPBT.

5.5 Performance Evaluation

We have evaluated the performance of a set of heuristic algorithms I = {S-DPMT, D-DPMT, RB-MIP-β, D-MIP-β} for many network examples, where β is a parameter that reflects the importance assigned to the impact of residual energy [WiNg02c, WiNg02d] and we use RB-MIP-β and D-MIP-β (reviewed in Section 2.1.3) to denote algorithms RB-MIP and D-MIP with different values of β, respectively. We have only considered $\beta = 0$, 1, and 2.

We have applied a similar experiment environment as in Chapter 4. In each network example, a number of nodes are randomly generated within a 1000 meters × 1000 meters region. One of the nodes is randomly chosen to be the source. Multicast groups of a specified size are chosen randomly from the overall set of nodes. Each antenna can point to any desired direction with an antenna beamwidth subject to $\theta_{\min} \leq \theta_v \leq 360°$. We further assume that the initial energy supply is the same for all the nodes, and the residual energy ε has a normal distribution with a mean $E(\varepsilon)$ and a variance $D(\varepsilon)$. The values of parameters used in simulation are given in Table 5-2. In all cases, (i.e., network size n, multicast group size m, minimal antenna beamwidth $\theta_{\min}$, and tree algorithm $i \in I$), our results are based on the performance of 100 randomly generated network examples. The execution time of each scenario is similar to those given in Section 4.4.

Table 5-2. Parameter values for simulation

Parameters	Values	
n	20 (small networks)	100 (large networks)
m	{5, 10, 15, 20}	{5, 25, 50, 100}
$\theta_{\min}$	{10°, 30°, 60°, 90°, 180°, 360°}	
p_{max}	100	
p_{tran}	0.1	
p_{recv}	1	
$E(\varepsilon)$	500[1]	
$D(\varepsilon)$	200	
α	2	

5.5.1 Performance Metrics

We have used the following two metrics in evaluating and comparing algorithm performance.

[1] It uses the minimum transmission power for an omni-directional antenna between two nodes separated with 100 meters as the unit which is consistent with the unit of distance.

Definition 5-3.
Maximum tree lifetime τ_{BEST} is the maximum lifetime that a multicast tree can survive.

The *maximum tree lifetime* metric shows the advantage offered by the use of narrow beams compared to the use of omni-directional antennas. Let τ'^{i}_{M} be the actual multicast lifetime obtained using heuristic algorithm-*i*. For small network examples, the best solution is the optimal solution (i.e., $\tau^{BEST}_{M} = \tau^{OPT}_{M}$), which can be obtained in a timely manner using an MILP solver CPLEX. Here OPT denotes the branch-and-bound algorithm based on our MILP model. For large network examples, the best solution can be only obtained by choosing the maximum multicast lifetime from all heuristic algorithms, i.e. $\tau^{BEST}_{M} = \max_{i \in I}\{\tau'^{i}_{M}\}$.

Definition 5-4.
Normalized tree lifetime is defined the ratio of actual tree lifetime τ'^{i}_{M} required using algorithm-*i* to the best solution τ_{BEST}.

The *normalized tree power* metric provides a measure of how close each algorithm comes to provide the longest multicast lifetime. Thus allows us to facilitate the comparison of different algorithms over a wide range of network examples.

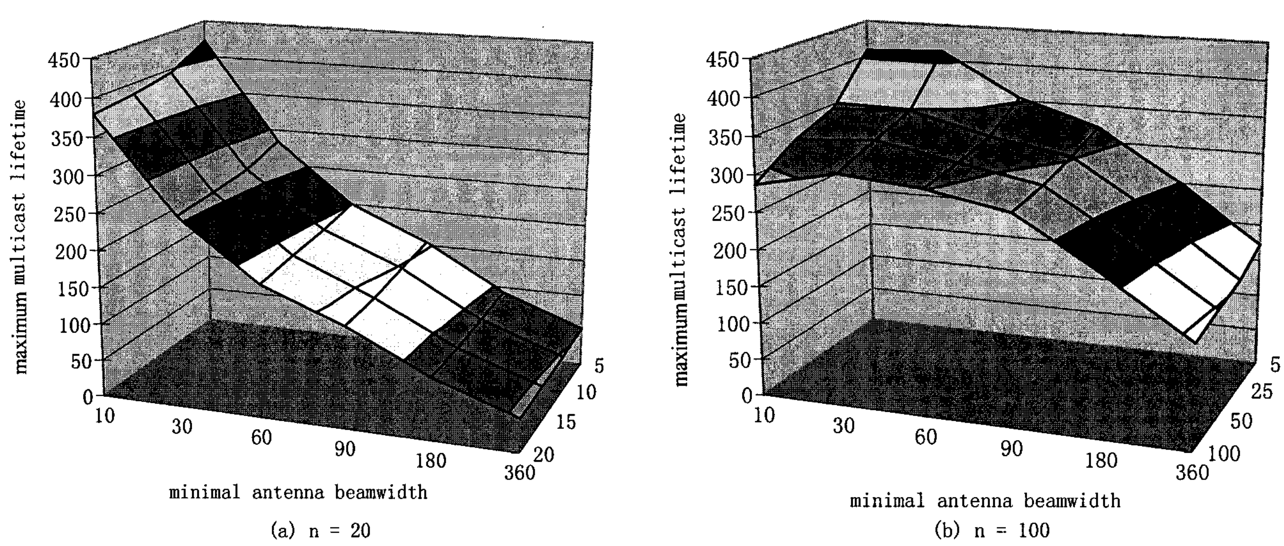


Figure 5-6. Maximum multicast lifetime (z-axis) as a function of multicast group size (y-axis) and minimal antenna beamwidth (x-axis) (a) in 20-node networks, (b) in 100-node networks

5.5.2 Maximum Tree Lifetime

The first set of experiments explores how the mean *maximum multicast lifetime* changes with different minimal antenna beamwidth and multicast group size. Figures 5-6 (a) and 5-6 (b) depict graphically the mean values of maximum multicast lifetime over different connected network topologies with 20 and 100 nodes respectively. The x-axis represents the minimal antenna beamwidth $\theta_{\min}$, the y-axis presents the multicast group size m, and the z-axis is the mean of maximum multicast lifetime $E(\tau_{M}^{BEST})$.

Referring to the $m = 5$ members in Fig. 5-6 (a), we observe that the mean of the minimum tree lifetime is a decreasing function of minimum antenna beamwidth and it drops very fast. This trend remains the same for $m = 10$, 15, and 20 members. For a fixed minimum antenna beamwidth, the mean of the minimum tree lifetime is a decreasing function of m while the decreasing amplitude is marginal.

Fig. 5-6 (b) shows curves for the same metrics in 100-node networks. The effect of multicast group size on the performance is very similar to Fig. 5-6 (a). However, we observe a notable difference from Fig. 5-6 (a) that the maximum multicast lifetime decreases very slowly when the minimum antenna beamwidth is less than 90° for all various multicast group sizes.

For each algorithm, the experiment results verify the advantages offered by the use of directional antennas that using directional antennas would increase the multicast lifetime significantly with the decrement of $\theta_{\min}$.

5.5.3 Normalized Tree Lifetime

We first evaluate the performance on small networks (with 20 nodes) for all algorithms {S-DPMT, D-DPMT, RB-MIP-0, RB-MIP-1, RB-MIP-2, D-MIP-0, D-MIP-1, D-MIP-2}. As noted before, the normalization is taken with respect to OPT.

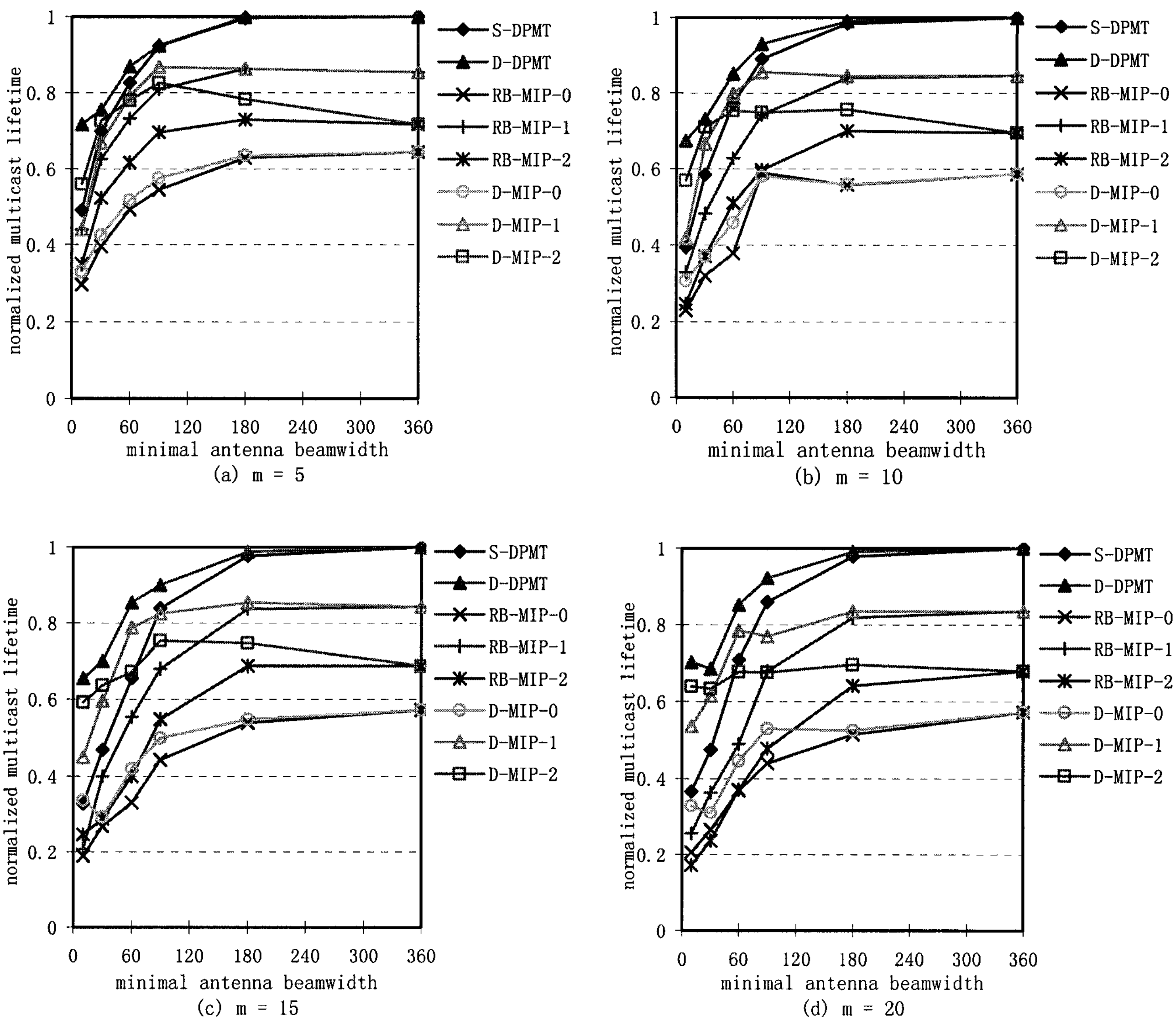


Figure 5-7. Normalized multicast lifetime for 20-node networks.

Figure 5-7 illustrates the mean *normalized multicast lifetime* as a function of multicast group size and minimal antenna beamwidth. Referring to the m = 5 members in Fig. 5-7 (a), we observe that the normalized minimum tree lifetime of both D-DPMT and S-DPMT algorithms is an increasing function of minimum antenna beamwidth and converges to optimal when $\theta_{\min}$ becomes greater than 180°. This trend remains the same for various group sizes (m = 10, 15 and 20) as shown in Figs. 5-7 (b) – (c). However the multicast group size has minor effects on the performance for both D-DPMT and D-DPMT algorithms. We have also observed that in all cases

D-DPMT provides much better performance than other algorithms, and its superiority is even greater in network examples with larger θ_{min}, e.g. always within 15% close to the optimum when $\theta_{min} \geq 60°$. In fact, as guaranteed by the theorems in Section 5.2, both S-DPMT and D-DPMT degenerate into DPMT and therefore achieve the globally optimal solutions for the case of using omni-directional antennas (θ_{min}=θ_{max}=360°).

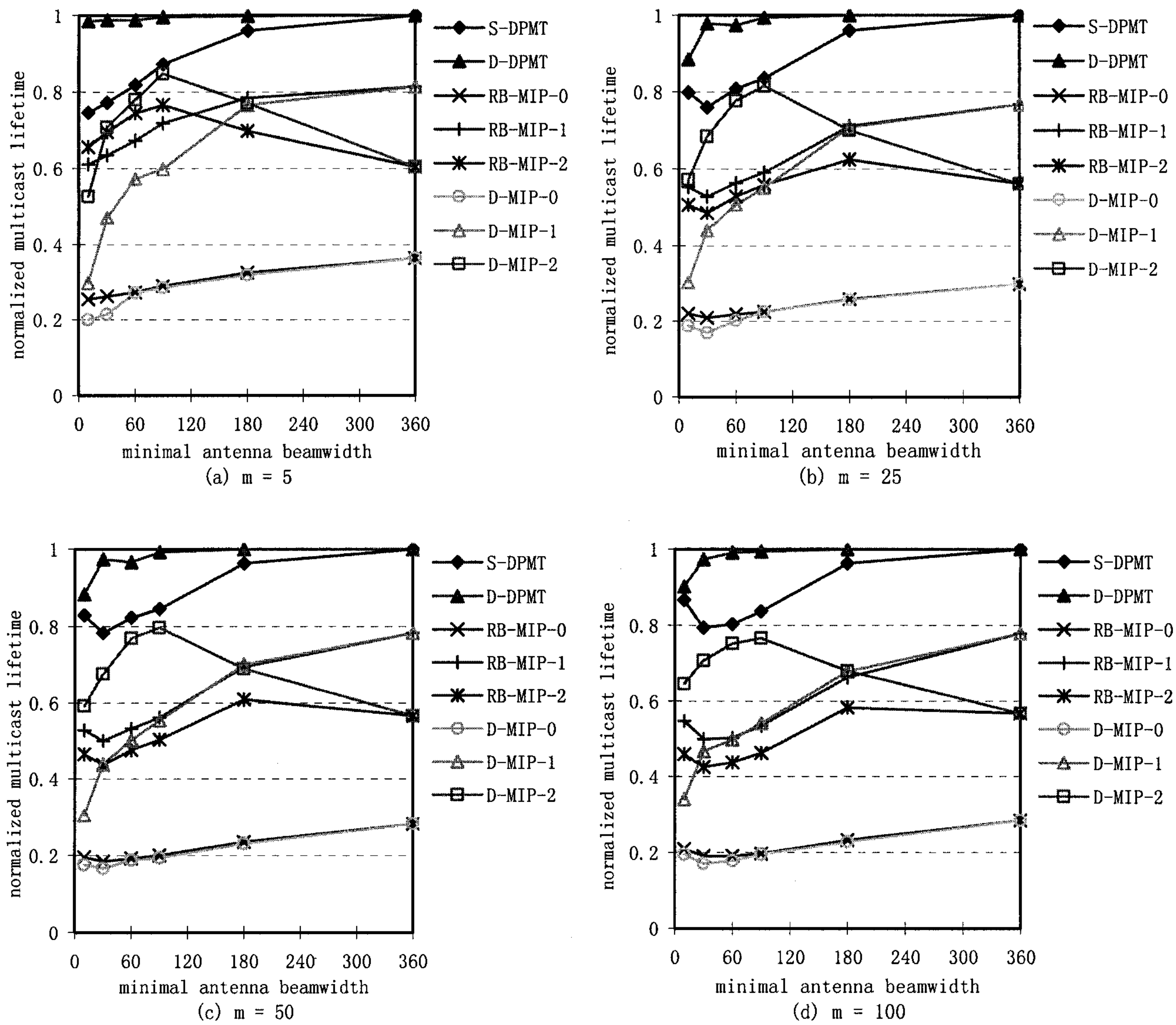


Figure 5-8. Normalized multicast lifetime for 100-node networks.

When evaluating the performance of algorithms RB-MIP-β and D-MIP-β (β = 0, 1, 2), we have observed that when the minimum antenna beamwidth θ_{min} is less than 90°, the normalized multicast lifetime is an increasing function of $\theta_{\min}$ for all algorithms with various multicast group sizes except the D-MIP-0 algorithm, which has a little longer lifetime when the minimum antenna beamwidth is extremely small ($\theta_{\min} = 10°$) and the group size $m \geq 15$. On the other hand in larger $\theta_{\min}$ cases ($\theta_{\min} > 90°$), the normalized multicast lifetime is increasing slightly with the minimum antenna beamwidth except the D-MIP-2 algorithm, which is decreasing instead. Under a fixed minimum antenna beamwidth, we have found that the multicast group size has little effect on the performance. The detailed discussion on performance evaluation among the group of algorithms {RB-MIP-β, D-MIP-β} can be found in [WiNg02d].Their performance comparisons based on our simulation experiments are summarized in Tables 5-3 for 20-node networks below.

Table 5-3. Best algorithms in {RB-MIP-β, D-MIP-β} ($\beta = 0, 1, 2$) for 20-node networks

	$\theta_{\min} \leq 30°$	$30° < \theta_{\min} \leq 360°$
$n = 20$	D-MIP-2	D-MIP-1

Performance results for 100-node networks are shown in Fig. 5-8. Similar results can be observed for the optimal performance of our D-DPMT algorithm in omni-directional antenna case. A major difference from the performance in 20-node networks, S-DPMT performs quite well even in the case of using antennas with small minimal beamwidth. In all cases, both D-DPMT and S-DPMT have better performance than other algorithms, and D-DPMT is even better and always performs very close (within 10%) to the best solutions. On the average, D-DPMT could provide over 35% longer lifetime when compared to the best performance produced by algorithms in {RB-MIP-β, D-MIP-β}.

We have taken the similar observations for the algorithms RB-MIP-β and D-MIP-β ($\beta = 0$, 1, 2) in 100-node networks. The major difference is that the performance gaps between algorithms in the same group are much larger than in 20-node networks. Similarly, their performance comparisons based on our simulation experiments are summarized in Table 5-4 below for 100-node networks.

Table 5-4. Best algorithms in {RB-MIP-β, D-MIP-β} ($\beta = 0, 1, 2$) for 100-node networks

		$\theta_{\min} < 30°$	$30° \le \theta_{\min} < 180°$	$180° \le \theta_{\min} \le 360°$
$n = 100$	$m \le 5$	RB-MIP-2	D-MIP-2	RB-MIP-1
	$m > 5$	D-MIP-2	D-MIP-2	D-MIP-1

We summarize key conclusions of this section as follows.

(1) The performances of S-DPMT and D-DPMT are complementary to each other. In networks with directional antennas, the S-DPMT algorithm guarantees to obtain the optimal solutions for unicast (as stated in Corollary 5-7), while the D-DPMT algorithm always achieves the best performance over all the other algorithms for multicast and broadcast. We attribute their superior performance to the DPMT algorithm's property of finding min-max multicast tree. In fact, as variant extensions of DPMT, they partially possess this property.

(2) The minimal total energy consumption does not guarantee maximum lifetime for a network either for broadcast or multicast. This can be observed from Figure 5-8, where D-MIP-0 performs the worst in terms of multicast lifetime, while it has the best performance on the total energy consumption. The revised minimum-energy multicast algorithms, like RB-MIP-1/RB-MIP-2 and D-MIP-1/D-MIP-2, by incorporating residual energy into the cost metric could provide longer lifetime as shown in Figures 5-7 and 5-8.

5.6 Summary

In this chapter, we have systematically studied some of the fundamental issues associated with maximum lifetime broadcasting and multicasting in energy-constrained wireless ad-hoc networks, and we have presented several efficient algorithms for cases of both omni-directional antennas and adaptive antennas. Our theoretical analysis and experimental results show that:

(1) Our DPMT/DPBT algorithm always maximizes the unicast/multicast/broadcast lifetime when using omni-directional antennas;

(2) The S-DPMT algorithm can construct a maximum lifetime path between any two nodes in the case of using adaptive antennas;

(3) Our MILP formulation provides optimal solutions for moderately sized networks with

adaptive antennas;

(4) In all other cases, D-DPMT/D-DPBT algorithm exploits the property of minimizing the bottleneck arc weight, and thus provides much better performance than other proposals over a wide range of network examples.

Chapter 6. Distributed Energy Efficient Multicast Algorithms

This chapter develops several distributed multicast algorithms for MANETs. We first present a Basic Energy Efficient Multicast (BEEM) algorithm, which uses a tree flood[1] mechanism to allow real-time transmission power adjustment such that BEEM could significantly save energy compared to those MANET multicast algorithms only applying a single level of transmission power. We then propose two algorithms as various extensions of BEEM: Distributed Minimum Power Multicast (DMPM) algorithm and Distributed Maximum Lifetime Multicast (DMLM) algorithm for the MPM and MLM problems respectively. We also design several localized operations that require only the knowledge of and the distance to all neighboring nodes.

6.1 Problem Statement

We shall reconsider the two basic problems MPM and MLM in the situations where each node in the network can move freely. In such networks, the objective of each optimization problem has a little difference from that in static ad hoc networks as discussed already in previous chapters.

In the duration of a multicast communication, a multicast tree may become disconnected temporarily due to link/node failures (caused by mobility or energy exhaustion) and be repaired by a multicast routing algorithm later. During these short disconnections, the multicast session is still active, but the multicast packets can be only delivered to a subset of all destinations. A multicast communication is considered as terminated only if the partition of a multicast tree persists for longer than a predefined period, which is called the disconnection time.

Definition 6-1.
The Multicast Communication Lifetime (MCL) is defined as the connection time in the active duration starting from beginning of communication supported by a multicast tree until a disconnection timer expires.

[1] We refer to the flood of a packet constrained to the nodes in the multicast forwarding tree as a tree flood, and to the more general type of flood of a packet through all nodes as a network flood.

Definition 6-2.
The Multicast Communication Power (MCP) is defined as the ratio of total energy consumed on multicast traffic over the active duration to its multicast communication lifetime.

In a MANET, the MLM and MPM problems are to maximize MCL and to minimize MCP respectively. They are more challenging than the same problems in static networks, because in addition to all our previous efforts, we also need to implement the energy efficient multicast algorithms in a distributed manner in such bandwidth-scarce networks.

6.2 Basic Energy Efficient Multicast Algorithm

We shall present a distributed Basic Energy Efficient Multicast (BEEM) algorithm for wireless ad hoc networks, which can be very useful in case of nodes moving freely and randomly in the network. In this section, a framework for the design of our BEEM algorithm are systematically described, covering from the discrete power level management, energy efficient tree construction process, tree flood mechanism, multicast traffic forwarding process, to localized operations for tree maintenance.

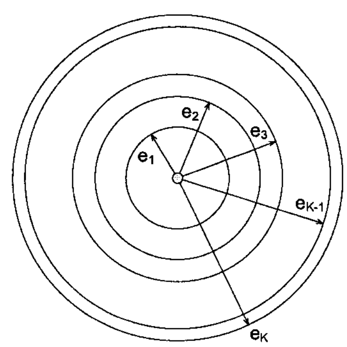


Figure 6-1. Discrete power level management

6.2.1 Discrete Power Level Management

As described in Section 3.1, each node's transmitter has power control capability. That is, by adjusting the transmission power level, the sender could reach destination nodes located at different distances. However, it is impractical to continuously adjust the transmission power when the transmitter v and the receiver u move freely, because a lot of bandwidth would be required to exchange messages between nodes v and u in order to maintain the on-line distance information r_{vu}. This would not leave enough bandwidth to transmit the multicast traffic in a node processor that is limited in power and transmission bandwidth.

As a quantization approach, the transmission power adjustment is only allowed for a series of discrete levels (shown in Fig. 6-1). Generally, the transmission power has at most K discrete levels e_i ($1 \le i \le K$) with the corresponding transmission ranges r_i, where e_K and r_K are the maximal transmission power and transmission range respectively. Note that any two nodes separated by range r ($r_{i-1} < r \le r_i$) use the same power e_i to communicate with each other, and the power should be adjusted only when their distance changes beyond this range due to mobility. Compared to those traditional MANET multicast algorithms in Section 2.3, which only apply single level of transmission power, our discrete power level management allows real-time transmission power adjustment at each relay node in the tree and thus BEEM could significantly save energy. Without loss of generality, the transmission power level threshold is evenly but discretely distributed, i.e.

$$e_i = \frac{i}{K} \cdot r_K^{\alpha},\ 1 \le i \le K \tag{6-1}$$

$$r_i = \left(\frac{i}{K}\right)^{1/\alpha} \cdot r_K,\ 1 \le i \le K \quad . \tag{6-2}$$

The antenna propagation space is therefore separated into K concircular layers by the boundaries r_i ($1 \le i \le K$). Let l_{vu} ($1 \le l_{vu} \le K$) be the layer in which node u is located within the transmission range of node v, i.e.

$$l_{vu} = \left\lceil K \cdot \frac{r_{vu}^{\alpha}}{r_K^{\alpha}} \right\rceil = \left\lceil \frac{r_{vu}^{\alpha}}{e_1} \right\rceil \quad , \tag{6-3}$$

where $\lceil x \rceil$ denotes the ceiling operator (the minimum integer that is not less than x). In addition to using distance information, the following lemma also allows using signal strength to identify l_{vu} for any wireless link (v, u).

Lemma 6-1.
If the signal strength attenuates from e_i ($1 \le i \le K$) to q_{vu} along the wireless link (v, u), then

$$l_{vu} = \left\lceil \frac{i}{q_{vu}} \right\rceil \quad . \tag{6-4}$$

Proof:

By substituting $e_i = r_{vu}^{\alpha} \cdot q_{vu}$ into Equ. (6-3), we have $l_{vu} = \left\lceil \frac{r_{vu}^{\alpha}}{e_1} \right\rceil = \left\lceil \frac{i \cdot r_{vu}^{\alpha}}{i \cdot e_1} \right\rceil = \left\lceil \frac{i \cdot r_{vu}^{\alpha}}{e_i} \right\rceil = \left\lceil \frac{i}{q_{vu}} \right\rceil$. ■

Based on this power management mechanism, the transmitted power p_{vu} required to support a link between two nodes v and u is

$$p_{vu} = \frac{l_{vu}}{K} \cdot r_K^{\alpha} = \left\lceil K \cdot \frac{r_{vu}^{\alpha}}{r_K^{\alpha}} \right\rceil \cdot \frac{r_K^{\alpha}}{K} = \left\lceil \frac{r_{vu}^{\alpha}}{e_1} \right\rceil \cdot e_1 \quad . \tag{6-5}$$

Similarly, let l_v ($0 \le l_v \le K$) be the farthest layer that the transmission power of node v applied in the tree T_s can reach, i.e.

$$l_v = \begin{cases} 0 & v \text{ is a leaf node;} \\ \max\{l_{vu} \mid (v,u) \in A(T_s)\} & v \text{ is a relay node.} \end{cases} \tag{6-6}$$

The transmission power p_v of the node v required by the multicast tree T_s is therefore

$$p_v = \frac{l_v}{K} \cdot r_K^{\alpha} = l_v \cdot e_1 \quad . \tag{6-7}$$

6.2.2 Data Structure

We have formulated a data structure to maintain locally the multicast forwarding state for BEEM at each tree node v. For each multicast session identified by the source address and destination address (the multicast group address), this data structure has the following components.

(1) Membership Status – This is a flag that indicate if this node is a *source*, *receiver*, or *forwarder*. Note that a node can be both a receiver and forwarder.

(2) Source Route π_v – This is a directed path from the source to node v along the tree links. We shall see later that this information is used to avoid loops in the tree reconstruction process.

(3) Tree Neighborhood Table TN_v – It contains one entry for each tree neighbor u within it maximum transmission range, i.e. $TN_v = \{u \mid 0 < l_{vu} \le K\}$. Each entry in the table includes a flag to indicate if the node u is a *father*, *child*, or *other* (neither a father nor a child) of the node v and the distance information l_{vu}.

6.2.3 Energy Efficient Multicast Tree Construction

In BEEM, whenever there is at least one source and one receiver in the network but no route information is known, a source-based shortest-path tree is created by performing a network-wide flood to propagate the MJREQ (Multicast-Join-Request) messages initiated by the source.

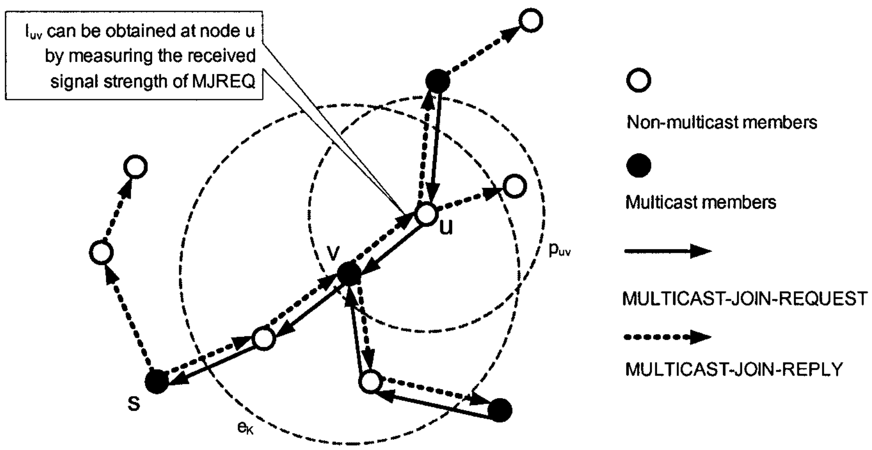


Figure 6-2. Multicast tree construction

When receiving a new MJREQ message, each intermediate node v would broadcast MJREQ to all its one-hop neighbors and record the previous hop address from which this message was received. All subsequent duplicate MJREQs (with the same request ID) are simply dropped. In addition, if node v is a multicast destination, it will set its membership status as *receiver*, set its previous hop node as *father* in its neighborhood table for this multicast session, and then unicast a MJREP (Multicast-Join-Reply) message back to its previous hop node. When an intermediate node v receives a MJREP message from node u, it will set its membership status as *forwarder* and set node u as *child* in its neighborhood table for this multicast session. Furthermore, if v is the source, it also sets its membership status as *source*; otherwise it sets its previous hop node as *father* and unicasts a MJREP message to its previous hop node. Any node v not equal to the source s should forward MJREP only once, and the following received MJREPs will not be forwarded.

A multicast forwarding tree is finally created according to this on-demand manner. The message exchange in the multicast tree construction is depicted in Fig. 6-2, where the solid nodes indicate multicast members (source and all destinations) and hollow nodes indicate non-multicast members. We note that the MJREQ messages propagate at the maximum transmission power e_K, while the MJREP messages only at p_{uv} (that is $l_{uv} \cdot e_1$) for any wireless link (u, v) to save power, where l_{uv} can have been obtained at node u by measuring the signal strength of MJREQ received from node v using Lemma 6-1. Without explicit indication to the contrary, this rule is also applied to other unicast control messages.

A network-wide flood is performed only once in the multicast tree construction process and any multicast packet is thus sent along the tree established. When a node receives such a packet, it checks its membership status entry for this group and source. If it is a *forwarder*, it should forward the packet to all its children by a single transmission. If it is a *receiver*, it passes the packet up within the protocol stack to allow the packet to be processed as a received multicast packet.

6.2.4 Tree Flood Mechanism

Our BEEM algorithm uses a tree flood mechanism to allow real-time transmission power adjustment at each relay node in the multicast tree. In the whole period of the multicast session, the source will initiate a tree flood periodically (e.g. 10/s) by sending MA (Multicast-Alive) messages. An MA message contains the source address, the multicast group address, the sequence number, and the source route (which is used to avoid loops as discussed later on). The MA messages are constrained to propagate in the multicast tree nodes to keep refreshing the forwarding state for each node involved in the tree, i.e. a node will forward an MA message only when it received an MA message from its father. If a tree node fails to receive a number of successive expected MAs from its father or child, it will set the flag to *other* for each corresponding entry in its neighborhood table. When the source has no more data to send, the MA messages stop and the entire tree silently expires.

The MA messages propagating with the maximum transmission power allow updating the neighborhood table in each tree node. The distance information l_{uv} between any tree link (u, v) can be refreshed by measuring the signal strength of received MA message or by exchanging coordinates included in the message using positioning system like GPS, and therefore it allows the relay nodes updating their transmission power promptly. Furthermore, the neighborhood information enables monitoring for potential link breaks in the multicast forwarding tree, and then BEEM performs a local operation to avoid tree disconnections in advance.

6.2.5 Multicast Tree Maintenance

Occasionally, the multicast tree may become disconnected because a node has moved out of the maximum transmission range of its father, thus resulting in a link breakage. Each forwarder or receiver v determines that it has become disconnected from the multicast forwarding tree when it fails to receive a number of successive expected MAs (e.g., 3) from source s. It then initiates a local repair of the multicast tree after a short delay which is proportional to the node's hop count from the source $|\pi_v|$. This allows the node closest to a broken link to initiate a local repair first.

In a local operation, node v first sends MAs on behalf of node s to the other nodes in the subtree rooted at node v in the multicast tree. The source route in the MA message is initialized as (v) instead of the broken directed path $(s, \ldots, v)$. The propagation of MAs on the subtree allows that each node u in the subtree would notice that a local repair at node v is in progress (by identifying the source route changes for the same multicast session in the MA message), and therefore it should not initiate their own local repair until the disconnection is fixed. In this duration, any node in the subtree rooted at v will not perform any energy saving or lifetime enhancement operation (which shall be discussed later) because it would let other nodes join the subtree, resulting in more nodes parted from the tree rooted at the source. For the same reason, any non-tree node will not choose the node in the subtree as its father for a non-tree node initiated the ESO (Energy Saving Operation), which shall be discussed later.

Node v then sends a hop-limited MJS (Multicast-Join-Solicitation) message as a form of network flood. The hop count limit for the MJS message (e.g., 3 hops) limits this flood to only reaching nodes near v. When an intermediate node u receives an MJS, the process is performed as follows: (1) if node u is in the subtree rooted at v, i.e. $v \in \pi_u$, it will drop it quietly; (2) if node u is a non-tree node, it will decrease the hop count by one and then forward it if the hop count limit does not reach zero; (3) if node u is in the subtree rooted at s, i.e. $s \in \pi_u$, it will use unicast to send an MA message back to v along the reverse route and drop the MJS message.

If an MA reaches node v, it only responses the first received MA message by sending an MJREP to node u. The processing for a node receiving an MJREP message is discussed in Section 6.2.3. Therefore, the multicast forwarding tree will be reconnected, and node u will forward MAs as normal. If node v does not receive any MA after a period of time, this is an indication that the local repair has probably failed, perhaps because the mobility level in the network has been too great to allow the type of hop-limited repair attempted. In this case, node v will stop sending MAs on behalf of source, and then some nodes in the subtree rooted at v will initiate the same local operations by their own due to the absence of a number of successive expected MAs. Node v itself has to perform a network-wide flood to join the multicast as the last resort.

6.2.6 Multicast Traffic Forwarding

Unlike the traditional MANET multicast algorithms, in which nodes use the same transmission power, in our BEEM algorithm each node in the multicast tree monitors the distances of its children and updates its transmission power to a minimum level dynamically. After collecting the MAs from all its children, each tree node v should update the distance information l_{vu} (u is a child of v) in its corresponding neighborhood table and re-adjust its transmission power using Equations (6-6) and (6-7) based on the new distance information.

6.3 Distributed Minimum Power Multicast Algorithm

The DMPM (Distributed Minimum Power Multicast) algorithm extends the BEEM algorithm by performing an extra operation called ESO (Energy Saving Operation) to achieve significant energy savings.

Before discussing the details of the ESO, let us examine the possibility of how a relay node in the tree could reduce its transmission power. A key observation is that if a tree node v has only one child u in its farthest layer (i.e. $l_{vu} = l_v$ and $l_{vw} < l_v$ for any child $w \neq u$), the removal of node u can thus reduce the transmission power of node v. We denote such a node u as c_v and the amount of reduced power by removing link (v, c_v) as $\delta^-(v, c_v)$. Therefore,

$$\delta^-(v,c_v) = p_v - \max\{p_{vu} \mid (v,u) \in A(T_s) \wedge u \neq c_v\} \quad . \tag{6-8}$$

Similarly, let $\delta^+(v, u)$ be the amount of increased transmission power at node v by adding a new child u, and therefore we have

$$\delta^+(v,u) = \max\{p_{vu}, p_v\} - p_v \quad . \tag{6-9}$$

6.3.1 ESO Header Option

Compared to the BEEM algorithm, the DMPM algorithm allows energy saving in a more sophisticated manner by exploiting the tree flood. An MA header option called ESO option would be included in the MA message, but at a lower frequency (e.g. 1/s) in order to limit the overhead on the ESO operations. A simplified data structure for the MA message with an ESO header option forwarded by node v can be presented as a tuple $[\pi_v, l_v, c_v, \delta^-(v, c_v), \delta^-(f_v, v)]$, where l_v, c_v, and $\delta^-(v, c_v)$ can be obtained locally using the distance information in its neighborhood table. Initially the source sends MA$[(s), l_s, c_s, \delta^-(s, c_s), 0]$ with the source route $\pi_s = (s)$ including only node s. Subsequently, each intermediate node v would forward MA$[\pi_v, l_v, c_v, \delta^-(v, c_v), \delta^-(f_v, v)]$ to all its neighbors by a single transmission after receiving a MA$[\pi_u, l_u, c_u, \delta^-(u, c_u), \delta^-(f_u, u)]$ message from its father u, in which π_v is obtained by appending its own address to π_u, i.e. $\pi_v = \pi_u + (v) = (s, \ldots, u, v)$ and $\delta^-(f_v, v)$ by

$$\delta^-(f_v,v) = \begin{cases} 0 & v \neq c_u \\ \delta^-(u,c_u) & v = c_u \end{cases} \quad . \tag{6-10}$$

In this way, any node (inside/outside the multicast tree) in the network could initiate an ESO to reduce total tree power by monitoring MAs received from all its 1-hop away tree neighbors.

6.3.2 Tree Node Initiated ESO

Consider a tree node v that can listen to all the MA messages with an ESO header option propagated by the nodes within its maximum transmission range r_K. Then this node would only record a copy of the message MA[π_u, l_u, c_u, $\delta^-(u, c_u)$, $\delta^-(f_u, u)$] from node u if (i) u is a tree neighbor of v, (ii) u is not a child of v, (iii) u is not an ancestor of v, and (iv) the link removal (f_u, u) can save tree power. We shall see later that these conditions guarantee the tree topology after the ESO and with a lower tree power.

We use $\mathrm{ESO}_1(v, i)$ ($l_v \le i \le K$) to denote a node set that satisfies all above conditions and each node element lies within layer-i of v's transmission range, i.e.

$$\mathrm{ESO}_1(v,i) \equiv \{u \mid (u \in TN_v) \wedge (f_u \ne v) \wedge (u \notin \pi_v) \wedge (\delta^-(f_u,u) > 0) \wedge (l_{vu} \le i)\},\ l_v \le i \le K. \qquad (6\text{-}11)$$

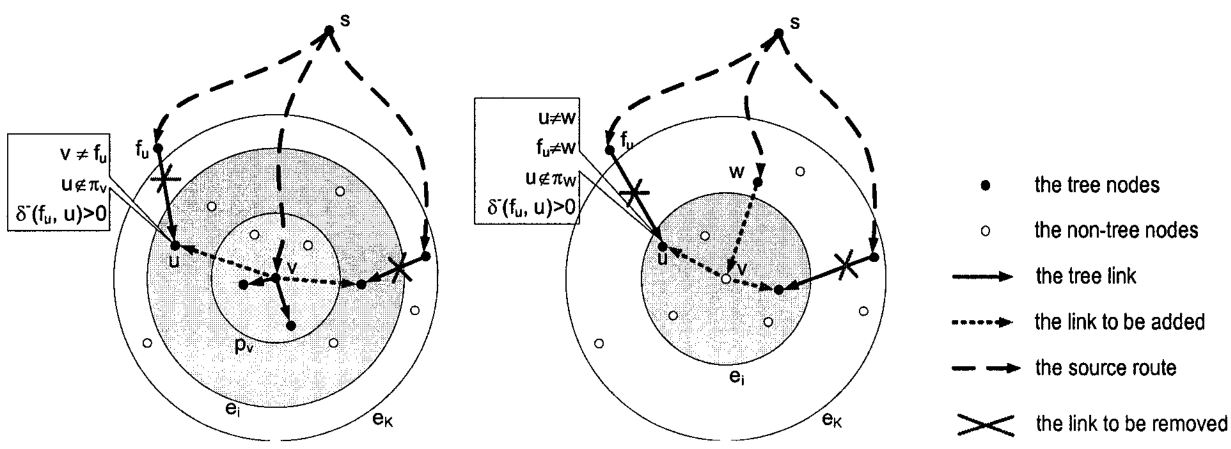


Figure 6-3. Illustration of ESO in the multicast tree reconstruction
(a) The ESO initiated by a tree node, (b) the ESO initiated by a non-tree node

Let $\Delta(v, i)$ be the tree power reduction by changing node v's transmission power level to e_i. It can be calculated using

$$\Delta(v,i) = -(e_i - p_v) + \sum_{u \in \mathrm{ESO}_1(v,i)} \delta^-(f_u,u) \qquad . \qquad (6\text{-}12)$$

The above equation can be explained as follows: the term ($e_i - p_v$) is the amount of power increased by expanding v's transmission power from p_v to e_i, while the second term is the power that can be saved by removing all crossed links (f_u, u) as shown in Fig. 6-3 (a) since node u can be covered by v and node f_u no longer needs to reach node u. Note that if $e_i = p_v$, node u can be reached without additional cost due to the wireless multicast advantage. If node v finds that it can reduce the tree power by extending its transmission range from l_v to l'_v, where

$$l'_v = \arg\max_i \{\Delta(v,i) \mid \Delta(v,i) > 0 \wedge l_v \le i \le K\}, \qquad (6\text{-}13)$$

node v would request to initiate a tree update by unicasting an ESO-REQUEST message back to the source with the energy saving information $\Delta(v, l'_v)$. The following process of tree update for the purpose of energy saving will be discussed in Section 6.3.4.

6.3.3 Non-tree Node Initiated ESO

A non-tree node v would also initiate an energy saving operation. The only difference from a tree node initiated ESO is that a non-tree node not only needs to choose a transmission power level to maximize the tree power reduction, but also to find a father in order to join the multicast tree. Similar to the previous section, we define node sets $\mathrm{ESO}_2(v, i)$ $(l_v \le i \le K)$ as follows

$$\mathrm{ESO}_2(v,i) \equiv \left\{ u \left| \begin{array}{l} (u \in TN_v) \wedge (\exists w \in TN_v) \wedge (w \ne u) \wedge (w \ne f_u) \\ \wedge (u \notin \pi_w) \wedge (\delta^-(f_u,u) > 0) \wedge (l_{vu} \le i \le K) \end{array} \right. \right\} . \qquad (6\text{-}14)$$

Let $N_{SF}(v, i, w) = \{u \mid (u \in \mathrm{ESO}_2(v, i)\text{-}\{w\}) \wedge (f_u \ne w) \wedge (\pi_w \ne u)\}$ be the node set that can switch their father to node v when node v applies power level i and chooses node w as its father. As shown in Fig. 6-3 (b), the tree power reduction by expanding node v's power to e_i can be calculated using

$$\Delta(v,i) = -e_i - \delta^+(f_v^i, v) + \sum_{u \in N_{SF}(v,i,f_v^i)} \delta^-(f_u,u),\ l_v \le i \le K, \qquad (6\text{-}15a)$$

$$\text{where } f_v^i = \arg\max_w \{-e_i - \delta^+(w,v) + \sum_{u \in N_{SF}(v,i,w)} \delta^-(f_u,u) \mid w \in TN_v\}. \qquad (6\text{-}15b)$$

The term e_i is the amount of power increased by expanding v's transmission power from 0 to e_i. The second term $\delta^+(f_v^i, v)$ is the transmission power that may be increased if a new child v (not in the tree yet) is added to such a tree node f_v^i that the final energy saving is maximized as shown in Equation (6-15b). Actually, this increased power can be obtained locally as

$$\delta^+(w,v) = \max\{\left\lceil \frac{K}{q_{wv}} \right\rceil \cdot e_1, l_w \cdot e_1\} - l_w \cdot e_1 \ , \qquad (6\text{-}16)$$

by measuring the signal strength q_{wv} of the MA[π_w, l_w, c_w, $\delta^-(w, c_w)$, $\delta^-(f_w, w)$] message sent by node w and received at node v. The last term is the amount of power that can be saved by removing all crossed links (f_u, u) as shown in Fig. 6-3 (b) since node u can be covered by v and node f_u no longer needs to reach node u.

Similarly, if there exists a new transmission range l'_v that satisfies the condition in (6-13), node v would request to initiate a tree reconstruction.

6.3.4 Tree Reconstruction

After the local calculation of the energy saving information $\Delta(v, l'_v)$ at each node across the network, several ESO-REQUEST messages may traverse different routes to reach the source node. From this information, the source would send back an ESO-REPLY message to the node with the maximum energy saving using the reverse route. Finally, only one node (e.g. node v) receives the ESO-REPLY message.

If node v is a tree node, it would perform tree updates immediately by expanding its transmission power to $l'_v \cdot e_1$. In order to exchange the tree links (f_u, u) and (v, u) for any $u \in$ $ESO_1(v, l'_v)$, node v sets u as *child* in its tree neighbor table, and node u sets v as *father* and simply sends a ML (Multicast-Leave) message to leave its previous father f_u. If node v is a non-tree node, in addition to expanding its transmission power to $l'_v\, e_1$, and exchanging the links between (f_u, u) and (v, u) for any $u \in ESO_2(v, l'_v)$, it also need to add a link ($f_v^{l'_v}$, v) to include the new tree node v, where $f_v^{l'_v}$ is obtained from Equation 6-15b by substituting i with l'_v. This can be done by setting its member status as *forwarder*, node $f_v^{l'_v}$ as *father*, and sending an MJREP to node $f_v^{l'_v}$. Once a node receives an MJREP message, it will execute the procedure already discussed in Section 6.2.3. Whenever any node v receives an ML message from node u, it will update u from *child* to *other* in its tree neighborhood table. Furthermore, if node v is not a receiver and has no children in the multicast tree, it will also forward ML to its father f_v and then update f_v from *father* to *other* in its neighborhood table.

Finally, it remains to show that the forwarding state maintained for the multicast still constitutes a multicast tree after an ESO operation initiated by a tree node or a non-tree node. This is stipulated in Lemma 6-2 as follows.

Lemma 6-2.
An energy saving operation retains a multicast tree structure.
Proof:
We first show that the topology T'_s evolved from the initial multicast tree T_s after a tree node initiated ESO operation still holds the following conditions: (1) node s has no incoming arcs, (2) every node $u \in N(T'_s) \backslash \{s\}$ has exactly one incoming arc, and (3) it does not contain cycles. Since $s \in \pi_v$ for any $v \in N(T_s)$, s can not be included in any set $ESO_1(v)$ and thus there is no such a link (v, s) that would be added in T'_s by the operation. This guarantees that node s still has no incoming arcs in T'_s. Any link exchange between (f_u, u) and (v, u) performed by the operation holds the second condition. Finally, the condition $u \notin \pi_v$ in the definition of $ESO_1(v)$ (given in 6-11a and 6-11b) avoids loops in T'_s. For the non-tree node initiated ESO operation, it is easy to obtain the same conclusion by following the above steps using a similar reasoning. ■

6.4 Distributed Maximum Lifetime Multicast Algorithm

The DMLM (Distributed Maximum Lifetime Multicast) algorithm is another extension of the BEEM algorithm. Recall that the removal of node c_v can reduce the transmission power of node v. In other words, it can extend the lifetime of node v. Based on this fact, the DMLM algorithm performs the LEO (Lifetime Enhancement Operation) to prolong the tree lifetime.

We use $\psi^-(v, c_v)$ to denote the lifetime of node v after removing link (v, c_v). It can be calculated as

$$\psi^-(v,c_v) = \begin{cases} \infty & \max\{p_{vu} \mid (v,u) \in A(T_s) \wedge u \neq c_v\} = 0; \\ \dfrac{\varepsilon_v}{\max\{p_{vu} \mid (v,u) \in A(T_s) \wedge u \neq c_v\}} & \text{otherwise.} \end{cases} \tag{6-17}$$

Similarly, let $\psi^+(v, u)$ be the lifetime of node v after adding a new child u, and therefore

$$\psi^+(v,u) = \frac{\varepsilon_v}{\max\{p_{vu}, p_v\}} \quad . \tag{6-18}$$

6.4.1 LEO Header Option

The DMPM algorithm also exploits the tree flood by propagating another header option called LEO option along with the MA messages to prolong the tree lifetime. Similar to the ESO header option, it would be included in MAs at a lower frequency (e.g. 1/s) in order to limit the overhead on the LEO operations. A simplified data structure for the MA message with an LEO header option forwarded by node v can be presented as $[\pi_v, l_v, c_v, \varepsilon_v, \tau_{f_v}, \psi^-(v, c_v), \psi^-(f_v, v)]$, where ε_v is its residual energy, while l_v, c_v, and $\psi^-(v, c_v)$ can be obtained locally using the distance information in its tree neighborhood table. The other parameters τ_{f_v} and $\psi^-(f_v, v)$ are obtained from the MA$[\pi_u, l_u, c_u, \varepsilon_u, \tau_{f_u}, \psi^-(u, c_u), \psi^-(f_u, u)]$ message received from its father u as follows. The lifetime $\tau_{f_v} = \tau_u$ of node f_v can be calculated using

$$\tau_{f_v} = \tau_u = \begin{cases} \infty & l_u = 0 \\ \varepsilon_u / (l_u \cdot e_1) & l_u > 0 \end{cases} , \qquad (6\text{-}19)$$

and the lifetime $\psi^-(f_v, v)$ of node f_v after removing link (f_v, v) will increase to $\psi^-(u, c_u)$ if v is in the farthest layer of u, and will not change otherwise, i.e.

$$\psi^-(f_v, v) = \begin{cases} \tau_{f_v} & v \neq c_u \\ \psi^-(u, c_u) & v = c_u \end{cases} . \qquad (6\text{-}20)$$

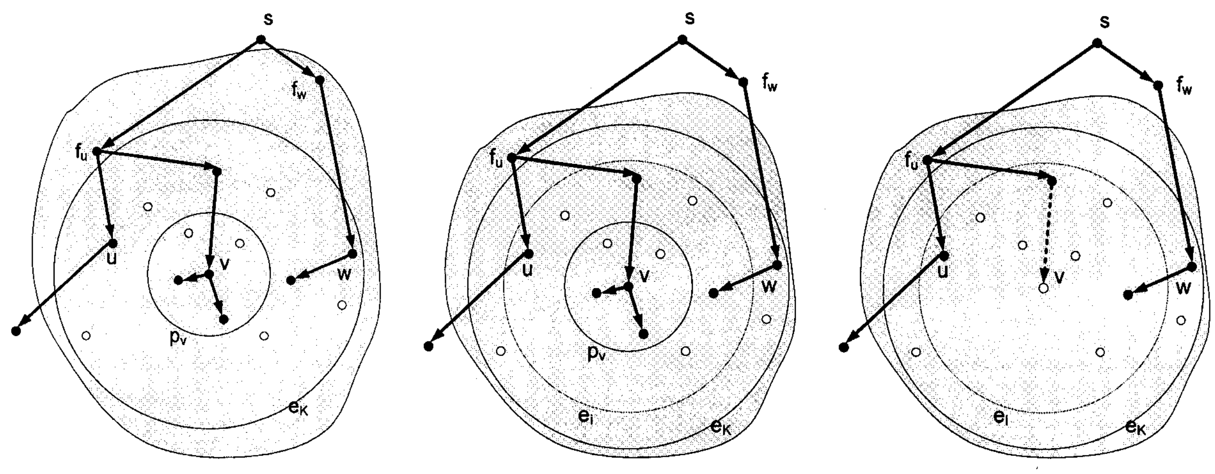


Figure 6-4. Examples of node sets (a) TN^2v, (b) $LEO_1(v, i)$ and (c) $LEO_2(v, i)$ (Each set is covered by a shaded area to indicate their different elements.)

In the following sections, we shall explore how to use this information at each node (inside/outside the multicast tree) to prolong the tree lifetime by monitoring LEO options received from all tree neighbors within its maximum transmission range.

6.4.2 Tree Node Initiated LEO

The objective of a LEO is to extend the lifetime of a subgroup of tree nodes only using local computation. Let TN_v^2 be a node set that includes all its tree neighbors and their fathers, i.e.

$$TN_v^2 \equiv \{u \mid u \in TN_v\} \cup \{f_u \mid (u \in TN_v) \wedge (f_u \neq v)\} \qquad . \tag{6-21}$$

Fig. 6-4 (a) gives an example of the node set TN_v^2 which is covered by a shaded area. Note that it may contain some nodes (like f_u and f_w) that are outside the maximum transmission range of node v. Let Ψ_v be the lifetime of a node set $\{v\} \cup TN_v^2$, and therefore it can be obtained locally

$$\Psi_v = \min\{\tau_v,\ \tau_u \mid u \in TN_v^2\} \qquad , \tag{6-22}$$

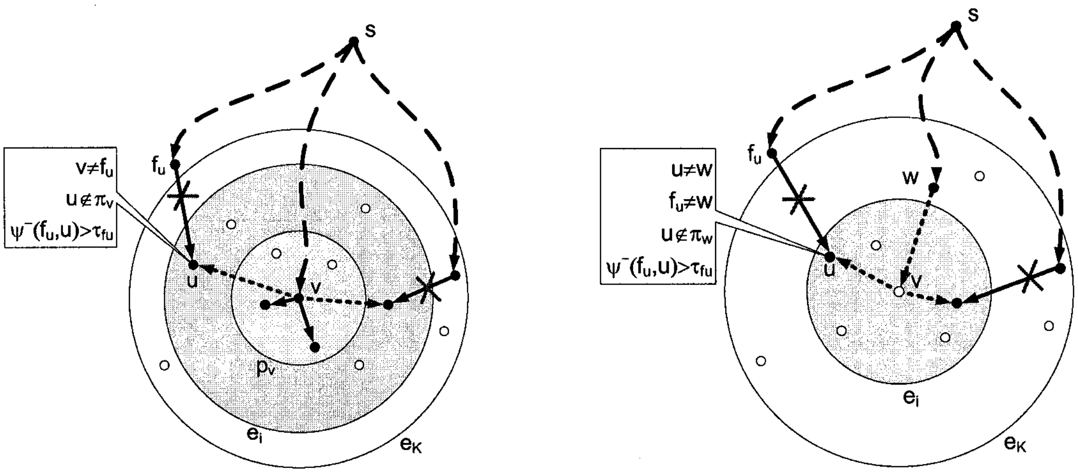
since each node lifetime τ_u or τ_{f_u} can be obtained from the information in the LEO option received from node u.

After collecting all the LEO options from its tree neighbors at a certain tree-flood round, each tree node v ($v \in N(T_s)$) will construct a serial of node sets as follows. They would be used for the Lifetime Enhancement Operation later on.

$$\mathrm{LEO}_1(v,i) \equiv \{f_u \mid (u \in TN_v) \wedge (f_u \neq v) \wedge (u \notin \pi_v) \wedge (\psi^-(f_u,u) > \tau_{f_u}) \wedge (l_{vu} \leq i)\},\ l_v \leq i \leq K. \tag{6-23}$$

As illustrated in Fig. 6-4 (b), the node set $\mathrm{LEO}_1(v, i)$ is covered by a shared area to indicate the difference from TN_v^2. For example, node f_w is not included in $\mathrm{LEO}_1(v, i)$ since $l_{vw} > i$ does not satisfy the conditions in (6-24).

If the set $\mathrm{LEO}_1(v, K)$ is not empty, node v continues to find a new transmission power level in order to prolong the lifetime of a tree node subset $\{v\} \cup TN_v^2$. Let $\Psi(v, i)$ be the lifetime of this node set after node v extends its transmission power level to e_i and performs link exchanges between (f_u, u) and (v, u) for any node $u \in \mathrm{LEO}_1(v, i)$. In the following, we consider the lifetimes of three subsets: $\{v\}$, $\mathrm{LEO}_1(v, i)$, and $TN_v^2 - \mathrm{LEO}_1(v, i)$ respectively.



(a) (b)

Figure 6-5. Illustration of LEO in the multicast tree reconstruction
(a) The LEO initiated by a tree node, (b) the LEO initiated by a non-tree node.

Obviously the residual lifetime of node v is ε_v/e_i if it expands its transmission power from p_v to e_i. For any node $f_u \in \mathrm{LEO}_1(v, i)$, its lifetime could increase from τ_{f_u} to $\psi^-(f_u, u)$ after exchanging links between (f_u, u) and (v, u) as shown in Fig. 6-5 (a). Finally, the lifetimes of all the other nodes are given by Equation (6-19). Therefore, $\Psi(v, i)$ can be calculated using

$$\Psi(v,i) = \min \begin{cases} \varepsilon_v / e_i & \text{node } v; \\ \psi^-(f_u, u) & \text{node } f_u \in \mathrm{LEO}_1(v,i); \\ \tau_x & \text{node } x \in TN_v^2 - \mathrm{LEO}_1(v,i) \end{cases}, \quad l_v \le i \le K. \tag{6-24}$$

Node v would request to initiate a tree update if the following condition is satisfied.

$$\max\{\Psi(v,i) \mid l_v \le i \le K\} > \Psi_v \tag{6-25}$$

In this situation, node v will extend its transmission range from l_v to l'_v, resulting in the lifetime of node set $\{v\} \cup TN_v^2$ increasing to $\Psi(v, l'_v)$, where

$$l'_v = \arg\max_i \{\Psi(v,i) \mid \Psi(v,i) > \Psi_v \wedge l_v \le i \le K\} \quad . \tag{6-26}$$

Node v will also unicast a LEO-REQUEST message back to the source with the information of

Ψ_v and $\Psi(v, l'_v)$. The following process of tree update for the purpose of lifetime enhancement will be discussed in Section 6.4.4.

6.4.3 Non-tree Node Initiated LEO

In order to increase the chances of prolonging the lifetime of node set $\{v\} \cup TN_v^2$, the non-tree nodes are also allowed to perform a lifetime enhancement operation. Any non-tree node v ($v \notin N(T_s)$) should reconstruct the following sets each time it receives the LEO options from all its tree neighbors.

$$\mathrm{LEO}_2(v,i) \equiv \left\{ f_u \middle| \begin{array}{l} (u \in TN_v) \wedge (\exists w \in TN_v) \wedge (w \neq u) \wedge (w \neq f_u) \\ \wedge (u \notin \pi_w) \wedge (\psi^-(f_u,u) > \tau_{f_u}) \wedge (l_{vu} \leq i) \end{array} \right\} . \tag{6-27}$$

Fig. 6-4 (c) gives an example of the node set $\mathrm{LEO}_2(v, i)$. In the construction of $\mathrm{LEO}_2(v, i)$, the node like f_u would have been chosen if it has a longer lifetime after node u changes its father to node v, i.e. $u \in TN_v \wedge \psi^-(f_u,u) > \tau_{f_u}$. Node v must also be able to find a father from its tree neighbors to join the multicast tree, like node w in Fig. 6-5 (b). Furthermore, the inclusion of this potential father node will not generate any cycle after the lifetime enhancement operation in order to keep the tree topology. Actually, this is guaranteed by the condition $(\exists w \in TN_v) \wedge (w \neq u) \wedge (w \neq f_u) \wedge (u \notin \pi_w)$ in Equation (6-27).

If the set $\mathrm{LEO}_2(v, K)$ is not empty, the non-tree node v then initiates an LEO. It will use the following equations to calculate each lifetime $\Psi(v, i)$ of the node set $\{v\} \cup TN_v^2$ for the case that node v extends its transmission power level to e_i and performs tree updates as shown in Fig. 6-5 (b).

$$\Psi(v,i) = \min \begin{cases} \varepsilon_v / e_i & \text{node } v; \\ \psi^+(f_v^i, v) & \text{node } f_v^i \in TN_v; \\ \psi^-(f_u,u) & \text{node } f_u \in \mathrm{LEO}_2(v,i) \wedge f_u \neq f_v^i \wedge u \neq f_v^i \wedge u \notin \pi_{f_v^i}; \\ \tau_x & \text{other node } x \text{ in } TN_v^2. \end{cases} \tag{6-28a}$$

$$
f_v^i \equiv \arg\max_w \min \begin{cases} \varepsilon_v / e_i & \text{node } v; \\ \psi^+(w,v) & \text{node } w \in TN_v; \\ \psi^-(f_u,u) & \text{node} f_u \in \mathrm{LEO}_2(v,i) \wedge f_u \neq w \wedge u \neq w \wedge u \notin \pi_w; \\ \tau_x & \text{other node } x \text{ in } TN_v^2. \end{cases} \qquad \text{(6-28b)}
$$

Compared to Equation (6-24), the only difference is that the lifetime of a potential father node w should be also considered. The inclusion of the new link (w, v) would reduce node w's lifetime, which can be calculated locally as

$$
\psi^+(w,v) = \frac{\varepsilon_w}{\max\{\left\lceil \frac{K}{q_{wv}} \right\rceil \cdot e_1, l_w \cdot e_1\}} \quad , \qquad \text{(6-29)}
$$

by measuring the signal strength q_{wv} of the MA[π_w, l_w, c_w, ε_w, τ_{f_w}, $\psi^-(w, c_w)$, $\psi^-(f_w, w)$] message received at node v. Similarly, if there exists a new transmission range l'_v such that satisfies the condition in (6-26), node v would request to initiate a tree reconstruction.

6.4.4 Tree Reconstruction

After the local calculation of the lifetime enhancement information (Ψ_u, $\Psi(u, l'_u)$) at each node across the network, several LEO-REQUEST messages may traverse different routes to reach the source node. From this information, the source will only respond to the node v that satisfies the following condition:

$$
\forall u \neq v, (\Psi_v \leq \Psi_u) \wedge ((\Psi_v = \Psi_u) \rightarrow (\Psi(v,l'_v) > \Psi(u,l'_u))) \quad , \qquad \text{(6-30)}
$$

by sending back an LEO-REPLY message. Once node v receives the LEO-REPLY message, it will adjust its transmission range and initiate tree update in a similar manner as discussed in Section 6.3.4. The LEO can also retain the tree structure as given in Lemma 6-3.

Lemma 6-3.
A lifetime enhancement operation retains a multicast tree structure.
Proof:
We omit the proof since it can be derived in a very similar way as in the proof of Lemma 6-2. ■

6.5 Performance Evaluation

We have extended the simulation tool used in static networks to support node mobility. In mobile networks, we have studied the performance of three distributed algorithms BEEM, DMPM and DMLM through detailed simulation in terms of power, lifetime, and overhead. In particular, we have also evaluated their performance in static networks by comparing them with our centralized algorithms proposed in previous chapters.

6.5.1 Network Configuration and Parameters

An ad hoc network with 100 nodes in a physical area of size 1,000 meters × 1,000 meters has been simulated. The radio interface is modeled as a shared-media radio with a nominal radio range of 250 meters. We consider $K = 10$ discrete transmission power levels, and choose specific multicast group size $m = |M| \in \{5, 25, 50, 100\}$ randomly from the overall set of nodes. One of the nodes is randomly chosen to be the source. Tree-flooding to propagate MA messages is initiated every 100ms, and a header option (ESO option or LEO option) is included once every 10 MAs. The hop counter for the local repair process is set to 3 and the disconnection timer is set to 5 seconds. Other simulation parameters are the same as those presented in Section 5.4.1.

In a static network, simulations are run until the first tree partition occurs. In order to compare with other centralized algorithms, our distributed algorithms do not perform any tree maintenance process during the course of simulation. For each of these multicast group sizes and various multicast algorithms, our results are based on the performance of 50 randomly generated network examples.

For the mobile networks, the mobility model uses $0^{+}/V_{max}/0$ (discussed in Section 3.5) with different maximum speeds (1, 5, 10, 15, and 20 m/s) and a minimum speed of 1 m/s. In all cases, (i.e., multicast group size m, maximum node movement speed V_{max}, and tree algorithm), we randomly generated 50 different scenarios, and we present here the average over those 50 scenarios. For each scenario, simulation for multicast algorithms starts to run after the initial 300-second warm-up period and terminates until the disconnection timer expires.

6.5.2 Static Network Performance

We have evaluated the performance of our algorithms in static networks using the same metrics *Normalized Tree Power* (defined in Section 4.4.2) and *Normalized Tree Lifetime* (defined in Section 5.4.2) before to compare with other centralized algorithms from the previous chapters.

6.5.2.1 Normalized Tree Power

We have evaluated the performance of four heuristic algorithms I = {MST, MIP, MIDP, DMPM} for many network examples. The MIP algorithm evaluated here includes the "sweep" operation [WiNg00, WiNg02a] (also explained in Section 4.4) which could improve its performance a lot. We choose the MIDP algorithm presented in Section 4.3.1.1 since it also uses a tree reconstruction mechanism and has shown a better performance than MIP.

Table 6-1. Normalized tree power in 100-node networks

m	MST	MIP	MIDP	DMPM
5	(1.3023, 0.0449)	(1.2909, 0.0311)	(1.0348, 0.0033)	(1.1001, 0.0213)
25	(1.2587, 0.0065)	(1.1422, 0.0083)	(1.0609, 0.0055)	(1.0079, 0.0003)
50	(1.2930, 0.0053)	(1.1172, 0.0038)	(1.0772, 0.0028)	(1.0048, 0.0002)
100	(1.2601, 0.0026)	(1.0574, 0.0023)	(1.0536, 0.0021)	(1.0052, 0.0002)

Table 6-1 summarizes the *normalized tree power* for the four algorithms on networks with 100 nodes and various multicast group sizes. We list mean and variance of the normalized tree power as (mean, variance) for each algorithm-i ($i \in I$) in the table. We observe from all cases in Table 6-1 that MIDP and DMPM provide better performance than others, both in terms of mean and variance. Especially, the DMPM algorithm performs the best and very close (within 1%) to the best solution for medium and large group size scenarios ($m \geq 25$), while the MIDP algorithm performs the best for the small group size scenarios ($m = 5$).

6.5.2.2 Normalized Tree Lifetime

We have also evaluated the performance of four algorithms I = {MST, MIP, DPMT, DMLM} for many network examples, in which DPMT is the global optimal algorithm while others are heuristic algorithms. The MIP algorithm evaluated here includes the "sweep" operation [WiNg00, WiNg02a].

Table 6-2. Normalized tree lifetime in 100-node networks

m	MST	MIP	DPMT	DMLM
5	(0.5415, 0.0447)	(0.4417, 0.0513)	(1.0000, 0.0000)	(0.8782, 0.0177)
25	(0.4574, 0.0323)	(0.3929, 0.0348)	(1.0000, 0.0000)	(0.8988, 0.0128)
50	(0.4789, 0.0388)	(0.4012, 0.0439)	(1.0000, 0.0000)	(0.9038, 0.0147)
100	(0.4699, 0.0408)	(0.4004, 0.0456)	(1.0000, 0.0000)	(0.9195, 0.0109)

Table 6-2 summarizes the *normalized tree lifetime* for the four algorithms on networks with 100 nodes and various multicast group sizes. As expected, the DPMT performs the best for all the cases. Our distributed algorithm DMLM provides much better performance than other heuristic algorithms, both in terms of mean and variance. In fact, it always performs very close to the optimal solution for all group size scenarios. We have also observed that its performance would increase slightly when the multicast group size becomes larger.

6.5.3 Mobile Network Performance

To evaluate the performance of mobile networks, the metrics for static networks are no longer appropriate because they only measure the power or lifetime of a static and always connected tree. Therefore we have used the following metrics instead to evaluate algorithm performance.

(i) *Multicast Communication Power* (MCP) – given earlier in Definition 6-2.

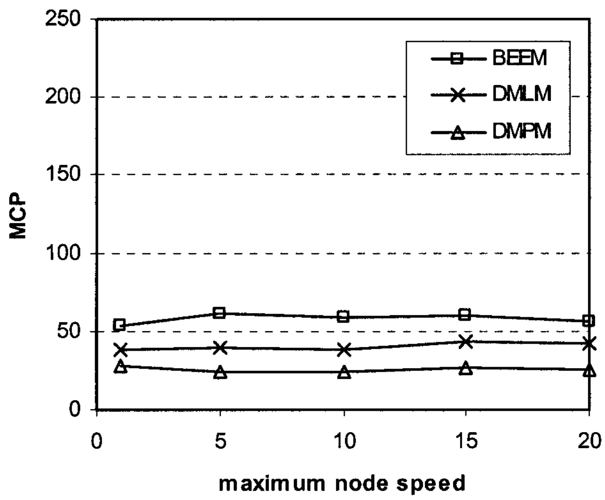
(ii) *Normalized MCP* – the ratio of actual MCP using heuristic algorithm-i ($i \in I$) to the minimum multicast *communication* power that would be obtained by a group of heuristic algorithms I.

(iii) *Multicast Communication Lifetime* (MCL) – given earlier in Definition 6-1.

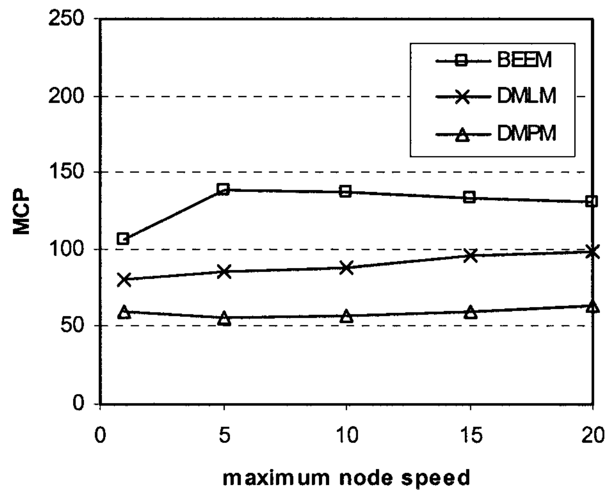
(iv) *Normalized MCL* – the ratio of actual *MCL* using heuristic algorithm-i ($i \in I$) to the maximum multicast communication lifetime that would be obtained by a group of heuristic algorithms I.

(v) *Average Overhead* – the ratio of total number of control messages sent across the network to the multicast communication lifetime.

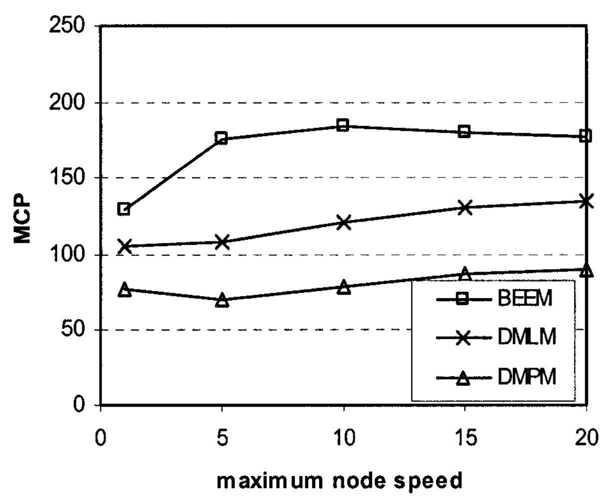
The metrics *MCP* and *MCL* represent the energy efficiency of the basic multicast forwarding in a mobile network in terms of energy consumption and communication lifetime respectively. Their corresponding normalized metrics provide a measure of how close each algorithm comes to providing the lowest-power or longest-lifetime multicast communication. Thus they allow us to facilitate the comparison of different algorithms over a wide range of network examples. The last metric represents the multicast algorithm efficiency on operations for tree repairs and updates.



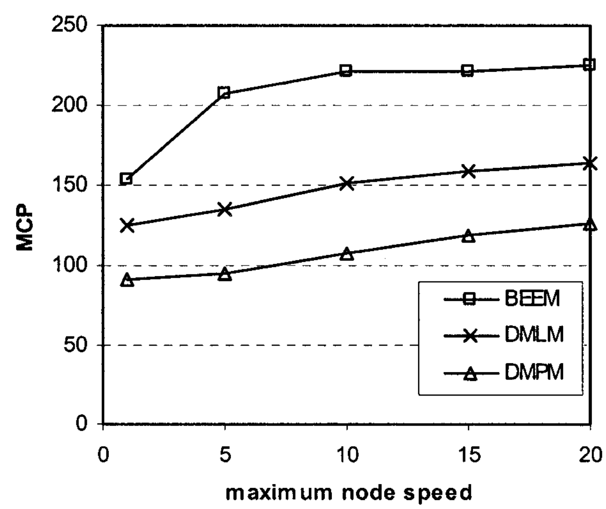
(a) $m = 5$



(b) $m = 25$



(c) $m = 50$



(d) $m = 100$

Figure 6-6. Multicast communication power in 100-node networks

6.5.3.1 Multicast Communication Power

Figure 6-6 depicts the multicast communication power as a function of maximum node speed for algorithms BEEM, DMLM, and DMPM under different multicast group size. Referring to the $m = 5$ members in Fig. 6-6 (a), we observe that the maximum node speed has little effect on the MCP performance. However the MCP appears to be an increasing function of the maximum node speed for $m = 25$, 50, and 100 as illustrated in Figs. 6-6 (b) – (d). Moreover, the increasing amplitude is more notable in lower mobility (e.g. $V_{\max} = 5$) for the BEEM algorithm. By comparing Figs. 6-6 (a) – (d), we observe that the MCP is an increasing function of m at a fixed maximum node speed for all algorithms. Finally, comparing to the performance of BEEM, both DMPM and DMLM can improve performance in terms of energy saving while the improved performance by DMPM is much more significant. We conclude that both ESO and LEO operations can reduce total power consumption while the ESO operation appears more efficient in terms of multicast communication power because the objective of such operation is to save power as much as possible.

Table 6-3. Normalized multicast communication power in 100-node networks

$V_{\max}$	BEEM	DMPM	DMLM	BEEM	DMPM	DMLM
		$m = 5$			$m = 25$	
1	(1.941, 0.153)	(1.000, 0.000)	(1.406, 0.044)	(1.788, 0.057)	(1.000, 0.000)	(1.360, 0.015)
5	(2.574, 0.401)	(1.000, 0.000)	(1.618, 0.128)	(2.512, 0.126)	(1.000, 0.000)	(1.541, 0.016)
10	(2.400, 0.198)	(1.000, 0.000)	(1.586, 0.073)	(2.443, 0.097)	(1.000, 0.000)	(1.569, 0.015)
15	(2.240, 0.140)	(1.000, 0.000)	(1.607, 0.045)	(2.229, 0.050)	(1.000, 0.000)	(1.595, 0.014)
20	(2.181, 0.148)	(1.000, 0.000)	(1.650, 0.077)	(2.065, 0.034)	(1.000, 0.000)	(1.553, 0.016)
		$m = 50$			$m = 100$	
1	(1.692, 0.045)	(1.000, 0.000)	(1.380, 0.015)	(1.674, 0.038)	(1.000, 0.000)	(1.362, 0.009)
5	(2.482, 0.053)	(1.000, 0.000)	(1.537, 0.011)	(2.180, 0.075)	(1.000, 0.000)	(1.526, 0.006)
10	(2.321, 0.045)	(1.000, 0.000)	(1.531, 0.009)	(2.067, 0.059)	(1.000, 0.000)	(1.506, 0.009)
15	(2.063, 0.035)	(1.000, 0.000)	(1.489, 0.011)	(1.857, 0.029)	(1.000, 0.000)	(1.431, 0.005)
20	(1.987, 0.018)	(1.000, 0.000)	(1.505, 0.010)	(1.792, 0.013)	(1.000, 0.000)	(1.407, 0.007)

6.5.3.2 Normalized Multicast Communication Power

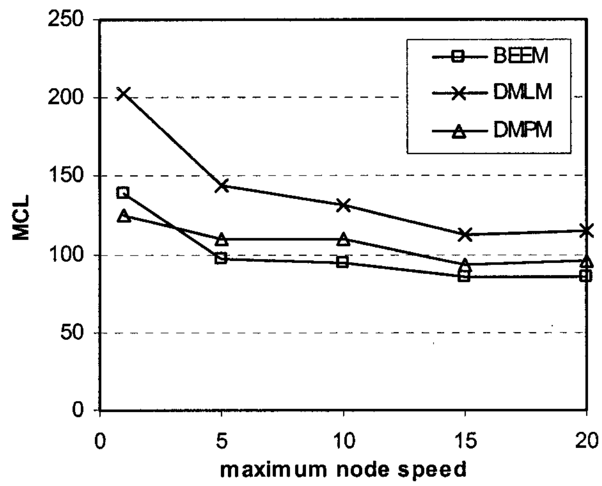
Table 6-3 summarizes the *normalized MCP* for the three distributed algorithms on networks with 100 nodes, various multicast group sizes, and various maximum node speeds. We list mean and variance of the normalized MCP as (mean, variance) for each algorithm-*i* in the table.

In all scenarios, the DMPM algorithm performs the best (with mean of 1 and variance of 0) as shown in Table 6-3. Using the energy saving operation, it could save energy on multicast traffic up to 40% - 60% compared to BEEM algorithm, and 20% - 40% to DMLM algorithm for all cases of various multicast group size and maximum node speed. The DMLM algorithm has a better performance than BEEM in all cases. This implies that the lifetime enhancement operation can also help to save energy, but not as significantly as the energy saving operation does.

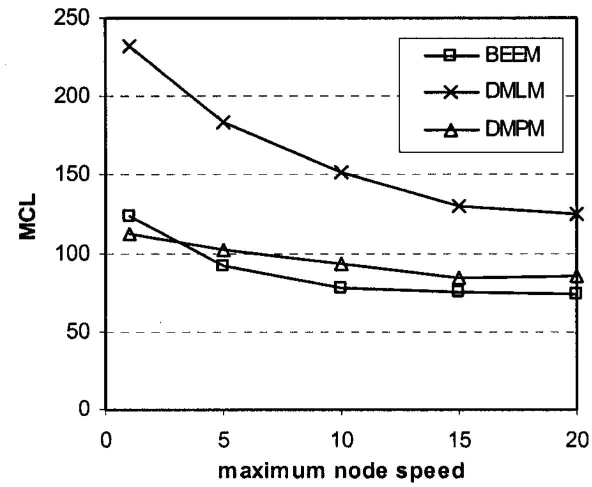
At a fixed mobility level, the performance of both BEEM and DMLM algorithms increases slightly when there are more multicast members. When considering the effect of mobility, we also observed that they both have the best performance in the almost static networks (the maximum speed is 1m/s). When the mobile nodes are moving a little bit faster (the maximum speed is about 5m/s), their performance decreases dramatically, but it would increase again slowly when the mobility level increases further.

6.5.3.3 Multicast Communication Lifetime

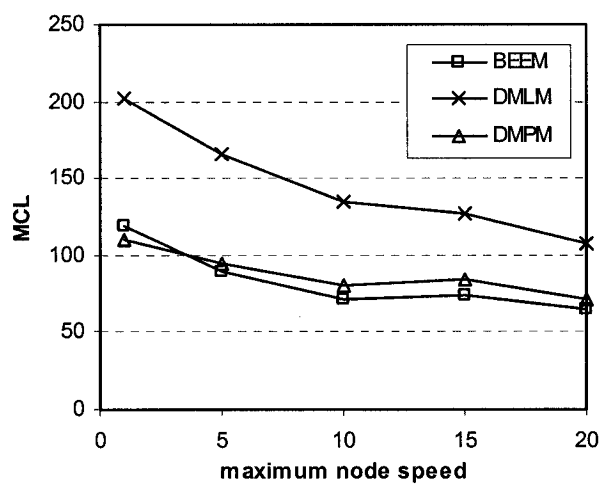
Figure 6-7 shows how the multicast communication lifetime changes as the maximum node speed varies under different multicast group size for algorithms BEEM, DMLM, and DMPM. Referring to the $m = 5$ members in Fig. 6-7 (a), we observe that the MCL is a decreasing function of maximum node speed for all three algorithms. In fact, it decreases rapidly only in the beginning when the mobile nodes are moving faster (e.g. the maximum node speed is between 1 and 5 m/s). This is because high mobility scenarios result in a greater chance that the multicast tree becomes disconnected and cannot be repaired promptly. This trend remains the same for m = 25, 50, and 100 members as shown in Figs. 6-7 (b) – (d).



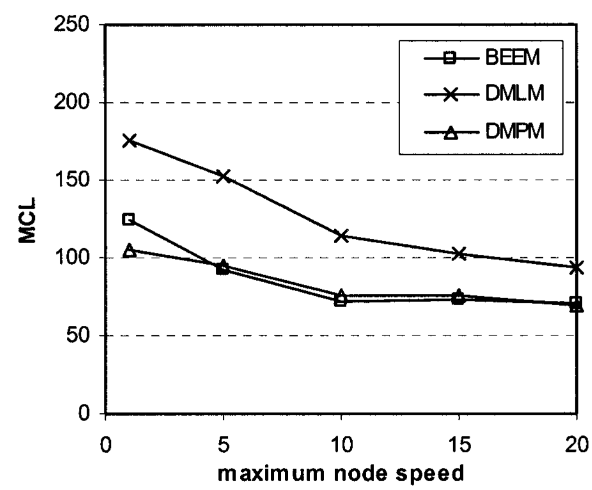
(a) $m = 5$



(b) $m = 25$



(c) $m = 50$



(d) $m = 100$

Figure 6-7. Multicast communication lifetime in 100-node networks

We have also observed that the multicast group has a significant effect on the MCL performance of the algorithm DMLM. For a fixed mobility level, a larger multicast group would benefit to have a longer multicast communication lifetime than very small multicast group as shown by comparing Figs. 6-7 (a) and (b). In such scenarios larger multicast groups increase the success possibilities of the hop-limited local repair operations. However, the lifetime will drop eventually when the multicast group size continues increasing since the number of tree partitions will become a dominant factor as shown in Figs. 6-7 (c) and (d). On the other hand, we have observed little effect of the multicast group on the performance of both BEEM and DMPM algorithms.

Another interesting observation in Fig. 6-7 is that the DMPM algorithm performs slightly better than the BEEM algorithm in higher mobility levels ($V_{max} \geq 5$), while a little bit worse only in more stable networks ($V_{max} = 1$). This is because in very low mobility scenarios, minimizing the communication power sometimes conflicts with maximizing the communication lifetime. In other words, the ESO operation would result in earlier energy depletion at certain nodes and thus reduce the communication lifetime. However, when the mobility increases, such conflict diminishes and the ESO operation becomes to help increase the communication lifetime. This increased lifetime appears marginal compared to that obtained by the LEO operation.

Finally, we observed that the DMLM algorithm can provide much longer multicast communication lifetime by exploiting the lifetime enhancement operation compared to other algorithms. This increased performance becomes more significant in the networks with low mobility level.

Table 6-4. Normalized multicast communication lifetime in 100-node networks

V_{max}	BEEM	DMPM	DMLM	BEEM	DMPM	DMLM
		$m = 5$			$m = 25$	
1	(0.727, 0.055)	(0.643, 0.069)	(0.958, 0.008)	(0.576, 0.046)	(0.520, 0.050)	(0.987, 0.002)
5	(0.716, 0.044)	(0.776, 0.042)	(0.958, 0.010)	(0.571, 0.037)	(0.621, 0.059)	(0.985, 0.002)
10	(0.731, 0.038)	(0.828, 0.035)	(0.933, 0.013)	(0.543, 0.026)	(0.635, 0.053)	(0.996, 0.000)
15	(0.769, 0.028)	(0.804, 0.026)	(0.966, 0.004)	(0.610, 0.026)	(0.682, 0.038)	(0.995, 0.000)
20	(0.744, 0.031)	(0.810, 0.023)	(0.959, 0.008)	(0.603, 0.019)	(0.707, 0.033)	(0.986, 0.004)
		$m = 50$			$m = 100$	
1	(0.627, 0.034)	(0.570, 0.052)	(0.993, 0.002)	(0.724, 0.033)	(0.622, 0.047)	(0.992, 0.000)
5	(0.576, 0.033)	(0.596, 0.052)	(0.988, 0.001)	(0.627, 0.035)	(0.642, 0.043)	(0.982, 0.011)
10	(0.556, 0.039)	(0.622, 0.045)	(0.994, 0.000)	(0.643, 0.027)	(0.655, 0.044)	(0.994, 0.001)
15	(0.619, 0.030)	(0.691, 0.038)	(1.000, 0.000)	(0.730, 0.033)	(0.751, 0.040)	(0.985, 0.001)
20	(0.607, 0.021)	(0.666, 0.029)	(1.000, 0.000)	(0.748, 0.029)	(0.740, 0.029)	(0.993, 0.000)

6.5.3.4 Normalized Multicast Communication Lifetime

We have listed the mean and variance of the *normalized MCL* in the Table 6-4 for each distributed algorithm on networks with 100 nodes, under various multicast group sizes and various maximum node speeds. In all combinations of maximum node speed and multicast group size, the DMLM algorithm can always provide the longest MCL which is actually within 5% to the best performance that we can obtain so far.

Using the lifetime enhancement operation, the DMLM algorithm could prolong MCL up to 35% - 80% compared to BEEM algorithm, and 25% - 90% to the DMPM algorithm for all cases. In very low mobility scenarios, the DMPM algorithm performs the worst. However, it has a slightly better performance (less than 20% improvement) than BEEM when the maximum speed is larger than 1m/s. This implies that the energy saving operation can also help to prolong lifetime in a mobile network.

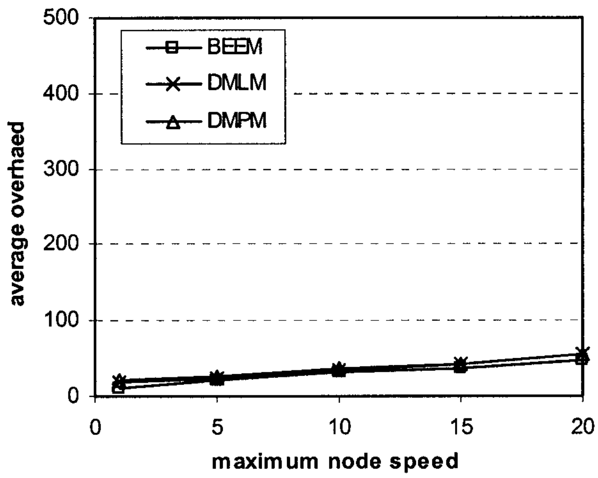
At a fixed multicast group size, the normalized performance of both BEEM and DMPM algorithms increases slightly when the mobile nodes are moving faster. But the multicast group size has little effects on such performance metrics.

6.5.3.5 Average Overhead

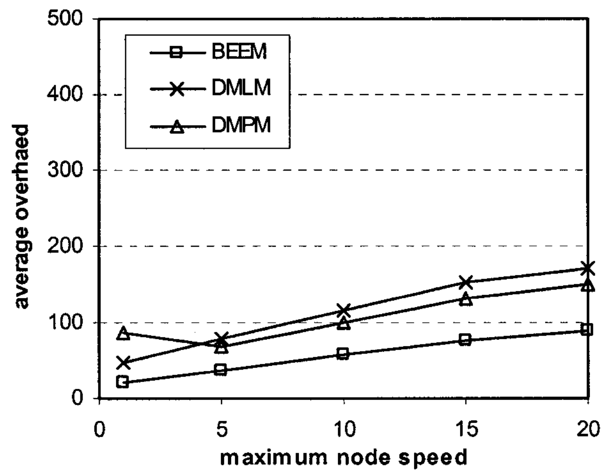
Figure 6-8 compares the performance of the average overhead as a function of maximum node speed for algorithms BEEM, DMLM, and DMPM under different multicast group size. Referring to the $m = 5$ members in Fig. 6-8 (a), the average overhead is an increasing function of maximum node speed for all three algorithms. This trend remains almost the same for $m = 25$, 50, and 100 members as shown in Figs. 6-8 (b) – (d). The only exception is the DMPM algorithm because it tends to generate slightly more control messages at the pseudo-static scenario ($V_{\max} = 1$) than those at low mobility scenario ($V_{\max} = 5$). We conclude that the efficiency of all algorithms decreases because of the increase in link breakage when the mobile nodes are moving faster.

For a fixed mobility level, we also observe that the average overhead is an increasing function of m for all three algorithms. In fact, all algorithms perform similar for groups with $m =$

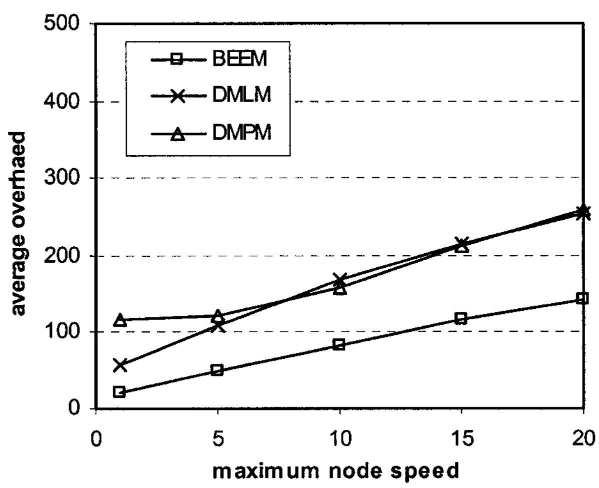
5 members in Fig. 6-8 (a). This is because in such scenarios, a multicast tree only includes a small number of nodes and thus there would be enough for few number of ESO or LEO to update a tree for the purposes of energy saving or lifetime enhancement. The dominant factor affecting the performance is therefore the overhead on tree maintenance. When there are more multicast members in the network, the DMPM and DMLP algorithms generate more control messages than BEEM does as shown in Figs. 6-8 (b) - (d). Furthermore, the amount of overhead increase appears to be more significant for the DMPM algorithm as shown in Figs. 6-8 (b) – (d). We conclude that the overhead on ESO or LEO dominates in this situation.



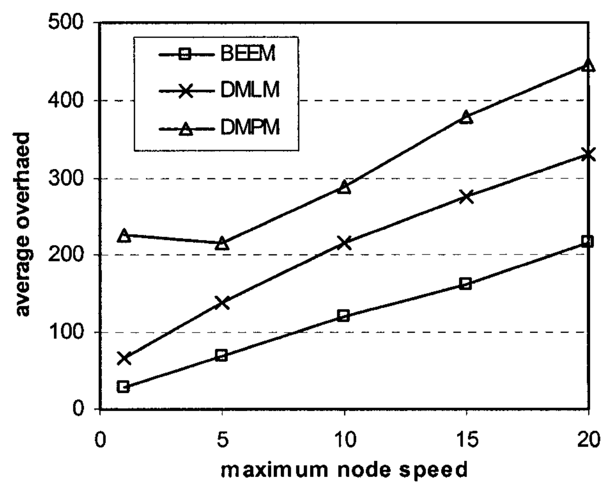
(a) $m = 5$



(b) $m = 25$



(c) $m = 50$



(d) $m = 100$

Figure 6-8. Average overhead in 100-node networks

6.6 Summary

We have presented several distributed energy efficient multicast algorithms for wireless ad hoc networks, which can be very useful in case of nodes in the network moving freely and randomly. In most of existing solutions each node requires global network information (including distances between any two neighboring nodes in the network) in order to decide its own transmission radius. In this chapter, we have systematically described a framework for the design of our distributed algorithms, covering from the discrete power level management, energy efficient tree construction process, tree flood mechanism, to localized operations for energy saving, lifetime enhancement, and tree maintenance. In these operations, each node requires only the knowledge of its distance to all neighboring nodes and distances between its neighboring nodes. We have proved their correctness in keeping a tree structure all the time as the network topology continuously changing.

The effectiveness of our distributed algorithms has been also validated using the simulation experiments. In static networks, the distributed algorithm DMLM performs very close (between 87% and 92%) to our optimal centralized algorithm DPMT in terms of multicast communication lifetime, and the distributed algorithm DMPM performs comparable to our centralized algorithm MIDP (better in larger group size and worse in small group size) in terms of multicast communication power. In mobile networks, our distributed algorithms DMLM and DMPM can improve performance significantly in terms of multicast communication lifetime and power respectively.

Chapter 7. Design Guideline

Having provided a complete solution suite for the above two basic classes of energy-efficient multicast problems in both static and mobile networks, we shall compare our proposed algorithms from different perspectives and recommend solutions for various applications and scenarios.

We first consider the solutions for the MPM and MLM problems in static networks. Some comparison criteria are especially useful to evaluate the solutions proposed in Chapters 4 and 5. These criteria include:

1) Optimization metrics: It determines what algorithms we should choose for different applications. For example, the algorithms with the TLT optimization metrics should be more appropriate for the multicast requests with lifetime requirement (a kind of QoS multicasting).
2) Optimality: It represents if an algorithm can provide a global optimal solution or not.
3) Complexity: It shows how fast an algorithm can find the solution from a theoretical point of view. Normally, an MILP model with lower order of complexity (variables and constraints) can be solved faster. An algorithm with lower time complexity can find the solution faster.
4) Implementation fashion: It characterizes an algorithm obtaining the solution in a centralized, distributed, or localized manner. A centralized algorithm is acceptable for the applications that the problems are static[1] and the solutions can be deployed into the network in advance. It can also be used to obtain the optimal solutions as a performance benchmark. For other cases like in mobile networks, a distributed or localized algorithm is preferred.
5) Usability: It represents if the algorithm is computationally feasible for a certain size of the

[1] A static problem means that the solution of the problem can be used all the time after it is found.

networks. Normally, an MILP based approach can be used only for small or moderately sized networks, while the heuristic algorithms with polynomial time complexity for large sized networks.

6) Performance: It shows which algorithm is better in a set of candidate algorithms which are all appropriate for a certain application (this means all the other criteria are the same for these candidate algorithms).

Table 7-1. Comparison of the solutions in static networks with omni-directional antennas

	MIDP	MILP model[1]	DPMT
Objective problem	MPM		MLM
Problem difficulty	NP-Complete		P
Optimization metrics	TPC		TLT
Optimality	sub-optimal	global optimal	global optimal
Complexity	running time: $O(n^3)$	variable: $2n^2+3n$ constraint: $O(n^2)$	running time: $O(n^2)$
Implementation fashion	centralized	centralized	centralized
Usability	for any sized networks	for small or moderate networks	for any sized networks

These criteria are thoroughly compared for all the algorithms we proposed in the thesis and are summarized for static networks with omni-directional antennas and adaptive antennas in Tables 7-1 and 7-2 respectively. For the MPM problem, we observe that

(1) For small or moderate networks, we can use our MILP model to obtain the optimal solution for both omni-directional antenna and adaptive antenna applications in a timely manner.

(2) In large size networks (e.g. 100-node networks), the best polynomial time algorithms are

[1] The MILP model for the MPM problem in omni-directional antenna networks is given in Fig. 3-6.

summarized in Table 7-3 based on our simulation results (given in Section 4.4).

Similarly, for the MLP problem, we have the following conclusions.

(1) Our DPMT is the best algorithm for omni-directional antennas for any sized networks.

(2) For small or moderate networks with adaptive antennas, we can use our MILP model to obtain the optimal solution in a timely manner.

(3) In large size networks (e.g. 100-node networks) with adaptive antennas, the best polynomial time algorithm is D-DPMT based on our simulation results (given in Section 5.4).

Table 7-2. Comparison of the solutions in static networks with adaptive antennas

	MILP model[1]	**RB-MIDP**	**D-MIDP**	**MILP model**[2]	**S-DPMT**	**D-DPMT**
Objective problem	MPM			MLM		
Problem difficulty	NP-hard			NP-hard		
Optimization metrics	TPC			TLT		
Optimality	global optimal	sub-optimal	sub-optimal	global optimal	sub-optimal or optimal[3]	sub-optimal
Complexity	variable: $3n^2+5n$ constraint: $O(n^2)$	running time: $O(n^3)$	running time: $O(n^3)$	variable: $3n^2+5n+1$ constraint: $O(n^2)$	running time: $O(n^3)$	running time: $O(n^3 \log n)$
Implementation fashion	centralized	centralized	centralized	centralized	centralized	centralized
Usability	small networks	any sized network	any sized network	small networks	any sized network	any sized network

[1] The MILP model for the MPM problem in directional antenna networks is given in Fig. 3-4.
[2] The MILP model for the MLM problem in directional antenna networks is given in Fig. 4-3.
[3] It provides optimal solutions for some special cases (given in Section 4.4.1).

Table 7-3. Algorithms with the best performance for the MPM problem in static networks (with 100 nodes)

The MPM Problem		
	$0 < \theta_{min} \leq 90°$	$90° < \theta_{min} \leq 360°$
small groups ($0 < m \leq 25$)	RB-MIDP	D-MIDP
large groups ($25 < m \leq 100$)	D-MIDP	RB-MIDP or D-MIDP
The MLM Problem		
$m = 2$	$\theta_{max} = \theta_{min}$	other scenarios
S-DPMT	S-DPMT[1]	D-DPMT

We consider an example of a 100-node static network, in which there are a certain number m of multicast members, node s is the source and each node is equipped a directional antenna with maximal beamwidth θ_{max} = 360° and minimal beamwidth θ_{min}. Given a set of multicast requests with different requirement, we shall provide the best solutions based on our previous analysis.

Scenario 1: The source node s only transfers a small quantity of multicast traffic to a group with 50 members. The minimal beamwidth is 45°.

Analysis: Since the multicast traffic is small, the connection time is not a major concern, and thus we choose the TPC as the optimization metrics. For 100-node networks (large networks) with directional antennas, our choices narrow to only heuristic algorithms RB-MIDP and D-MIDP since MILP solver is not computational feasible in this case as shown in Table 7-2. Referring to m = 50 and θ_{min} = 45° in Table 7-3, we choose the best algorithm D-MIDP for this request.

Scenario 2: The source node s requests to connect to a specific node t and transfers data for at least 2 mins. The minimal beamwidth is 60°.

[1] In this scenario, the S-DPMT would provide optimal solution (the detail can be found in Section 4.4.1).

Analysis: Since the connection request specifies a lifetime lower bound, we choose the TLT as the optimization metrics. Although the general maximum-lifetime multicast problem may be NP-hard in directional antenna networks, this request is actually a special case of unicast and S-DPMT can provide global optimal solution as shown in Table 7-2.

Scenario 3: The source node s needs to transfer a large quantity of multicast traffic to a group with 5 members. The minimal beamwidth is 30°.

Analysis: Since the multicast traffic is large (i.e. the connection time for traffic transmission is long), we choose the TLT as the optimization metrics to provide the longest lifetime. For 100-node networks (large networks) with directional antennas, our choices narrow to only heuristic algorithms S-DPMT and D-DPMT since MILP solver is not computational feasible in this case as shown in Table 7-2. We choose D-DPMT because it always performs better than S-DPMT at this situation ($m > 2$ and $\theta_{\max} \neq \theta_{\min}$) as shown in Table 7-3.

Table 7-4. Comparison of the solutions in mobile networks

	BEEM	**DMPM**	**DMLM**
Objective problem	No specific objective	MPM	MLM
Problem difficulty	NP-hard		
Optimization metrics	No specific metrics	MCP	MCL
Optimality	sub-optimal		
Implementation fashion	distributed	distributed and localized	distributed and localized
Usability	for any sized networks		

We consider the same problems for in mobile networks using the same set of comparison criteria. Based on the simulation results (Section 6.5), we have the following observations:

(1) Using the energy saving operation, it could save energy on multicast traffic up to 40% - 60% compared to BEEM algorithm, and 20% - 40% to DMLM algorithm for all cases of

various multicast group size and maximum node speed.

(2) Using the lifetime enhancement operation, it could prolong MCL up to 35% - 80% compared to BEEM algorithm, and 25% - 90% to the DMPM algorithm for all cases.

Finally, we reconsider the example of 100-node network for mobility scenarios. Based on previous analysis, we conclude that at various node mobilities, the DMPM algorithm is our best choice for transferring a small quantity of multicast traffic, while the DMLM algorithm the best for transferring a large quantity of multicast traffic.

Chapter 8. Conclusion

In this thesis, we have systematically explored and solved two classes of energy-efficient broadcast/multicast problem: the BPM/MPM (Broadcast/Multicast Power Minimization) problem and the BLM/MLM (Broadcast/Multicast Lifetime Maximization) problem.

For the first class of problem, so far a general analytical MILP (Mixed Integer Linear Programming) model has been developed for the MPM problem in an ad hoc network with adaptive antennas. This model can be easily applied for some special cases, e.g., (1) broadcast communication, (2) use of omni-directional antennas, and (3) use of beamwidth-fixed directional antennas. To our best knowledge, these are the first work using mixed integer linear programming to formulate the problem in a context of adaptive antenna applications. Many application scenarios can be solved efficiently based on this formulation using branch-and-cut or cutting planes techniques. Our simulation results show that an optimal solution of the BPM/MPM problem using our model can always be obtained in a timely manner for moderately sized networks typically with 50 nodes. In addition to the theoretical effort, we have also implemented a group of polynomial-time algorithms practically to handle significantly large networks for which the MILP model may not be computationally efficient. These algorithms include the BIDP/MIDP algorithm for omni-directional antenna networks, and the RB-BIDP/RB-MIDP and D-BIDP/D-MIDP algorithms for directional antenna networks. The experimental results show that RB-MIDP/D-MIDP provides much better performance than RB-MIP/D-MIP respectively in adaptive antenna applications, and MIDP compares well with MIP in omni-directional antenna applications.

For the second problem, we first studied the MLM/BLM problem in the omni-directional antenna scenarios to provide insights into a more general case of using directional antennas. In order to solve this problem, this thesis has developed an analytical model in which the MLM/BLM can be formulated as a general min-max tree problem, which can derive two polynomial-time algorithms, called DPBT and DPMT, using theoretical analysis. They allow us to obtain global optimal solutions for the BLM and MLM problems respectively. We studied the same problem with directional antennas. Some special cases have been proved solvable using our approach proposed for omni-directional antennas. For general cases, however, we conjecture that this problem is NP-hard. Based on the same analytical model, we develop a constraint formulation for the MLM/BLM problem in terms of MILP. In order to handle significantly large networks with directional antennas, we have also implemented a group of polynomial-time algorithms S-DPBT/S-DPMT and D-DPBT/D-DPMT. Our experimental results show that D-DPMT/D-DPBT algorithm exploits the property of minimizing the bottleneck arc weight, and thus provides much better performance than other proposals over a wide range of network examples.

Finally, we have designed several distributed multicast algorithms for energy efficient operation, and studying how the node mobility affects to the performance of various algorithms. In our distributed algorithms, each node requires only the knowledge of its distance to all neighboring nodes and distances between its neighboring nodes. Distances can be measured by using signal strength or more sophisticated techniques like GPS. We have also designed two localized operations the Energy Saving Operation and the Lifetime Enhancement Operation for different optimization objectives. The extensive simulation results have shown that these operations are very efficient both in terms of energy saving and lifetime enhancement.

The work on my PhD thesis over these past few years has been a valuable experience that can be shared with people working in this area. One of the most difficult parts in our work is to build the MILP model for the MPM and MLM problems in static networks. We have tried various methods of formulating these problems, including the graph theory as presented in Section 4.2.3.1 and the constraint linearization as presented in Sections 4.2.3.2 and 4.4.2.3. Our techniques may be useful and suggestive in solving similar problems. Another major difficulty we must face is how to extend our initial results to more challenging mobile networks. Unlike the previous work which is more like a pure theoretical experience, it needs more careful efforts to design a series of subtle and well organized distributed mechanisms to achieve energy efficient algorithms. Some ideas in designing the distributed/localized operations and their corporations would elicit some new efficient algorithms. In my research, the experiments sometimes could inspire us to achieve important and productive results. For example, at the beginning on the MLM problem in static networks, we have never thought about its complexity and supposed it the same complexity as the MPM problem. We were working along a similar approach as in the MPM algorithms to refine a minimum spanning tree to increase its lifetime. We found an interesting phenomenon that the tree obtained by Prim's algorithm cannot be further refined for every network instance. This made us suspect that it is a P-problem and the Prim's algorithm provides the optimum solution. With this confidence, we have obtained the proof eventually. We have learned that the experiment not only can validate our thoughts but also can lead to some new discoveries.

8.1 Future work

Our work has laid a foundation for more important and interesting work to come, and it can be used to explore a number of new directions for further research which are discussed briefly in the following.

The antenna model described in the paper makes a set of assumptions listed in Section 3.6. We can expect that these restrictions can be surmounted in real world and in some applications. Applying more realistic and general antenna model, our MILP formulation for both MPM and MLM problems would be remodeled and turned into nonlinear problems. Significant advances have been made in recent years in solving nonlinear programs, and a number of these solvers are now readily available[1]. We shall examine the feasibility of using these nonlinear solvers for such problems. For the same reason, we shall revise or redesign our heuristic algorithms accordingly when considering the new issues in the physical layer.

For the future work, it will be an important research direction to explore architectural and cross-layer protocol design (especially the interactions between MAC layer and IP layer) for

[1] http://www-neos.mcs.anl.gov/neos/server-solvers.html

energy conservation in ad hoc networks. Most recent energy-efficient multicast algorithms assume an ideal MAC layer, e.g. [WiNg00, DaMa03b], and thus they can hardly be deployed in the real network infrastructure. We also observe that the cross-layer approach can be extended horizontally to allow the cooperation among multiple nodes. Virtual MIMO systems [Jaya04] can be constructed via node cooperation on both the transmitting side and the receiving side. In this way, energy saving and delay reduction can be achieved at the same time within a certain transmission range by such cooperative MIMO systems, even when we consider the extra energy/delay penalty caused by the local information exchange and collection.

In addition to energy-efficient algorithm design for ad hoc networks, it would be also desirable to extend our work to sensor networks, which are different from traditional ad hoc networks in that they are energy constrained and require scalable protocols because of the vast quantities of nodes. All these have posed new challenges to researchers. For example, self-awareness and self-organization in protocols are mandatory to achieve automation with minimal or no human intervention once such networks are deployed. In my future work, I am especially interested in some fundamental optimization problems like optimal sensor node scheduling [HoDo04] for energy optimization.

Reference

[AgCh04] M. Agarwal, J.H. Cho, and et al, "Energy efficient broadcast in wireless ad hoc networks with hitch-hiking", *Proceedings IEEE INFOCOM*, Hong Kong, China, March 7–11, 2004, pp. 2096 - 2107.

[AhMa93] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, "Network Flows", Englewood Cliffs: Prentice Hall, 1993.

[AkSu02] I. F. Akyildiz, Weilian Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks", *IEEE Communications Magazine*, vol. 40, issue 8, Aug. 2002, pp. 102 - 114.

[AlLi03] Khaled Alzoubi, Xiang-Yang Li, Yu Wang, Peng-Jun Wan, and Ophir Frieder, "Geometric spanners for wireless ad hoc networks", *IEEE Transactions on Parallel and Distributed Processing*, Vol. 14, Issue 4, April 2003, pp. 408 - 421.

[AlSa04] K. Altinkemer, F. S. Salman, P. Bellur, "Solving the minimum energy broadcasting problem in ad hoc wireless networks by integer programming", *Proceedings IEEE WiOpt* (workshop on modeling and optimization in mobile, ad hoc, and wireless networks), April 2004, pp. 48 - 54.

[BaHa01a] S. Bandyopadhyay, K. Hasuike, S. Horisawa, S. Tawara, "An Adaptive MAC and Directional Routing Protocol for Ad Hoc Wireless Network Using Directional ESPAR Antenna", *Proceedings ACM MobiHoc*, Long Beach, California, USA, October 4 - 5, 2001, pp. 243 - 246.

[BaHa01b] S. Bandyopadhyay, K. Hasuike, S. Horisawa, S. Tawara, "An Adaptive MAC Protocol for Wireless Ad Hoc Community Network (WACNet) Using Electronically Steerable Passive Array Radiator Antenna", *Proceedings IEEE Globecom*, Texas, USA, November 2001, pp. 2896 - 2900.

[BhCh01] M. Bhardwaj and A. P. Chandrakasan, "Upper bounds on the lifetime of sensor networks", *Proceedings IEEE ICC*, June 11-14, 2001, pp. 785 - 790.

[BiFe99] Robert E. Bixby, Mary Fenelon, and et al, "MIP: Theory and Practice - Closing the Gap", the 19th IFIP TC7 Conference on System Modeling and Optimization, July 12-16, 1999, Cambridge, UK, pp. 19-50.

[BhCh02] M. Bhardwaj and A.P. Chandrakasan, "Bounding the lifetime of sensor networks via optimal role assignments", *Proceedings IEEE Infocom*, New York, USA, June 23 - 27, 2002, pp. 1587 - 1596.

[BlPa02] Douglas M. Blough and Santi Paolo, "Investigating upper bounds on network lifetime extension for cell-based energy conservation techniques in stationary ad hoc networks", *Proceedings ACM MobiCom*, Atlanta, USA, Sept. 23 - 28, 2002, pp. 183 - 192.

[BoLi98] E. Bommaiah, M. Liu, A. McAuley, and R. Talpade, "AMRoute: Adhoc Multicast Routing Protocol", Internet-Draft, draft-talpade-manetamroute-00.txt, August 1998.

[BoJe02] S. Borbash and E. Jennings, "Distributed topology control algorithm for multihop wireless networks", *Proceedings WCCI* (World Congress on Computational Intelligence), Honolulu, Hawaii, 2002, pp. 355 - 360.

[CaHu02] Mario Cagalj, Jean-Pierre Hubaux, and Christian Enz, "Minimum-energy broadcast in all-wireless networks: NP-completeness and distribution issues", *Proceedings ACM MobiCom*, Atlanta, Georgia, USA, 2002, pp. 172 - 182.

[CaSi03] J. Cartigny, D. Simplot, and I. Stojmenovic, "Localized minimum-energy broadcasting in ad-hoc networks", *Proceedings IEEE Infocom*, April 2003, pp. 2210 - 2217.

[CaSi04] J. Cartigny, D. Simplot-Ryl, and I. Stojmenovic, "An adaptive localized scheme for energy-efficient broadcasting in ad hoc networks with directional antennas", *Proceedings PWC* (International Conference on Personal Wireless Communications), September 21 - 23, 2004, Delft, Netherlands.

[ChGe98a] C. C. Chiang, Mario Gerla, and Lixia Zhang, "Forwarding Group Multicast Protocol (FGMP) for Multihop, Mobile Wireless Networks", *ACM Journal of Cluster Computing*: Special Issue on Mobile Computing, 1(2), 1998, pp. 187 – 196.

[ChGe98b] C. C. Chiang, M. Gerla, L. Zhang, "Adaptive shared tree multicast in mobile wireless networks", *Proceedings IEEE Globecom*, Sydney, Australia, 1998, pp. 1817 – 1822.

[ChJa02] B. Chen, K. Jamieson, H. Balakrishnan, and R. Morris, "SPAN: an energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks", *ACM Wireless Networks Journal*, 8 (5): 481 - 494, 2002.

[ChSu03] M. X. Cheng, J. Sun, and et al, "Energy-efficient Broadcast and Multicast Routing in Ad Hoc Wireless Networks", *Proceedings IEEE IPCCC*, Phoenix, Arizona, April 2003, pp.87 - 94.

[ChTa99] J. H. Chang and L. Tassiulas, "Routing for maximum system lifetime in wireless ad-hoc networks", *Proceedings Allerton Conference on Communications, Control, and Computing*, Monticello, USA, Sept. 1999.

[ChTa00] J. H. Chang and L. Tassiulas, "Energy conserving routing in wireless ad-hoc networks", *Proceedings IEEE Infocom*, New York, USA, 2000, pp. 22 - 31.

[ChTa76] D. Cheriton and R.E. Tarjan, "Finding minimum spanning trees", *SIAM Journal on Computing*, vol. 5, 1976, pp. 419 - 429.

[ChYa02] Romit Roy Choudhury, Xue Yang, Nitin H. Vaidya, and Ram Ramanathan, "Media Access Control for Ad Hoc Networks: Using directional antennas for medium access control in ad hoc networks", *Proceedings ACM MobiCom*, Atlanta, USA, September 2002, pp. 59 - 70.

[ClCr01] A. Clementi, P. Crescenzi, and et al, "On the Complexity of Computing Minimum

Energy Consumption Broadcast Subgraphs", *Symposium on Theoretical Aspects of Computer Science*, vol. 2010 of LNCS, 2001, pp. 121-131.

[CPLX05] http://www.ilog.com/download/docs/DS-CPLEX2005.pdf, downloaded on 2005.

[DaMa02] A.K. Das, R.J. Marks, M. El-Sharkawi, P. Arabshahi, and A. Gray, "The minimum power broadcast problem in wireless networks: an ant colony system approach", *Proceedings IEEE CAS Workshop on Wireless Communications and Networking*, Pasadena, CA, September 5-6, 2002.

[DaMa03a] A.K. Das, R.J. Marks, M. El-Sharkawi, P. Arabshahi, and A. Gray, "Minimum power broadcast trees for wireless networks: Integer programming formulations," in *Proceedings IEEE Infocom,* San Francisco, April 2003, pp. 2210 - 2217.

[DaMa03b] A.K. Das, R.J. Marks II, M.A. El-Sharkawi, P. Arabshahi and A. Gray, "MDLT: a polynomial time optimal algorithm for maximization of time-to-first-failure in energy-constrained broadcast wireless networks", *Proceedings IEEE Globecom*, San Francisco, December 2003, pp. 362 - 366.

[DaMa03c] Arindam K. Das, Robert J. Marks, Mohamed El-Sharkawi, Payman Arabshahi, Andrew Gray, "r-shrink: A heuristic for improving minimum power broadcast trees in wireless networks", *Proceedings IEEE Globecom*, San Francisco, December 2003, pp. 523-527.

[DaWu04] Q. Dai and J. Wu, "Construction of Power Efficient Routing Tree for Ad Hoc Wireless Networks using Directional Antenna", *Proceedings WWAN* (International Workshop on Wireless Ad Hoc Networking), Tokyo, Japan, March 2004, pp.718 - 722.

[DaWu05] F. Dai, Q. Dai, and J. Wu, "Power Efficient Routing Trees for Ad Hoc Wireless Networks using Directional Antenna", *Ad Hoc Networks* (*Elsevier*), vol. 3, no. 5, Sep. 2005, pp. 621-628.

[Dijk59] E. W. Dijkstra, "A Note on Two Problems in Connection with Graphs", *Numerische Math.* 1, 1959, pp. 269 - 271.

[Ephr02] Anthony Ephremides, "Energy concerns in wireless networks", *IEEE Wireless Communications*, vol. 9, no. 4, pp. 48–59, Aug 2002.

[FlKa03] B. Floréen, P. Kaski, and et al, "Multicast time maximization in energy constrained wireless networks", *Proceedings Workshop on Foundations of Mobile Computing*, New York, NY, 2003, pp. 50–58.

[FlNa04] M. Flammini, A. Navarra, R. Klasing, S.Perennes, "Improved approximation results for the Minimum Energy Broadcasting Problem", *Proceedings ACM DIALM-POMC*, 2004, pp. 85 - 91.

[Gabo77] H.N. Gabow, "Two algorithms for generating weighted spanning trees in order", *SIAM Journal on Computing*, vol. 6, 1977, pp. 139 - 150.

[GaHu83] R. Gallager, P. Humblet, and P. Spira, "A distributed algorithm for minimum-weight spanning trees", *ACM Transactions on Programming Languages and Systems*, 5(1), January 1983, pp. 66 - 77.

[GaJo79] Michael R. Garey and David S. Johnson, Computers and intractability, W. H. Freeman and Co., San Francisco, Calif., 1979, A guide to the theory of NP-completeness, A Series of Books in the Mathematical Sciences.

[GaMa99a] J.J. Garcia-Luna-Aceves and E.L. Madruga, "The Core-Assisted Mesh Protocol", *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 8, Aug. 1999, pp. 1380-1394.

[GaMa99b] J.J. Garcia-Luna-Aceves and E.L. Madruga, "A Multicast Routing Protocol for Ad-Hoc Networks", *Proceedings IEEE Infocom*, New York, NY, March 1999, pp. 784-792.

[Geor03] L. Georgiadis, "Bottleneck multicast trees in linear time", *IEEE Communications Letters*, 7(11), Nov. 2003, pp. 564 – 566.

[GuYa03] Song Guo and Oliver Yang, "Minimum Energy Broadcast Routing in Wireless Multi-hop Networks", *Proceedings IEEE IPCCC*, Phoenix, USA, April 2003, pp. 273 - 280.

[GuYa04a] Song Guo and Oliver Yang, "A Dynamic Multicast Tree Reconstruction Algorithm for Minimum-Energy Multicasting in Wireless Ad Hoc Networks", *Proceedings IEEE IPCCC*, Phoenix, USA, April 2004, pp. 637 - 642.

[GuYa04b] Song Guo and Oliver Yang, "Multicast Lifetime Maximization for Energy-Constrained Wireless Ad-hoc Networks with Directional Antennas", *Proceedings IEEE Globecom*, Dallas, USA, December 2004.

[GuYa04c] Song Guo and Oliver Yang, "Antenna Orientation Optimization for Minimum-Energy Multicast Tree Construction in Wireless Ad Hoc Networks with Directional Antennas", *Proceedings ACM MobiHoc*, Tokyo, Japan, May 2004, pp. 234 - 243.

[GuYa04d] Song Guo and Oliver Yang, "Minimum Energy Multicast Routing for Wireless Ad-hoc Networks with Adaptive Antennas", *Proceedings IEEE ICNP*, Berlin, Germany, October 2004, pp. 151 - 160.

[GuYa04e] Song Guo and Oliver Yang, "Minimum-Energy Multicast Routing in Static Wireless Ad Hoc Networks", *Proceedings IEEE VTC*, Los Angeles, USA, September 2004, pp. 3989 - 3993.

[GuYa05a] Song Guo and Oliver Yang, "Optimal Tree Construction for Maximum Lifetime Multicasting in Wireless Ad-hoc Networks with Adaptive Antennas", *Proceedings IEEE ICC*, Seoul, Korea, May 2005, pp. 3370 - 3374.

[GuYa05b] Song Guo and Oliver Yang, "Improving Energy Efficiency for Multicasting in Ad-hoc Networks with Directional Antennas", *Proceedings IEEE WiMob* (Wireless and Mobile Computing, Networking and Communications), Montreal, Canada, August 2005, pp. 344 - 351.

[GuYa06a] Song Guo and Oliver Yang, "Performance of Distributed Algorithms for

Maximizing Multicast Lifetime in Mobile Ad Hoc Networks", *Proceedings ACM Workshop on Performance Evaluation of Wireless Ad Hoc, Sensor, and Ubiquitous Networks*, Montreal, Canada, October 2006.

[GuYa06b] Song Guo and Oliver Yang, "Minimum-Energy Multicast in Wireless Ad Hoc Networks with Adaptive Antennas: MILP Formulations and Heuristic Algorithms", to appear in *IEEE Transaction on Mobile Computing* in 2006.

[GuYa06c] Song Guo and Oliver Yang, "A Constraint Formulation for Minimum-Energy Multicast Routing in Wireless Multi-hop Networks", to appear in *ACM Wireless Networks Journal* on February 2006.

[GuYa06d] Song Guo and Oliver Yang, "Joint Optimization of Energy Consumption and Antenna Orientation for Multicasting in Static Ad Hoc Wireless Networks", to appear in *IEEE Transaction on Wireless Communications* in 2006.

[HoDo04] B. Hohit, L. Doherty, and E. Brewer, "Flexible power scheduling for sensor networks", *Proceedings International Symposium on Information Processing in Sensor Networks*, April 26-27, 2004, pp. 205 - 214.

[HoSh04] Y. Thomas Hou, Yi Shi, and Hanif D. Sherali, "Rate Allocation in Wireless Sensor Networks with Network Lifetime Requirement", *Proceedings ACM MobiHoc*, Tokyo, Japan, May 24 -26, 2004, pp. 67 - 77.

[HoSh05] Y. Thomas Hou, Yi Shi, Hanif D. Sherali, and Jeffrey E. Wieselthier, "Online lifetime-centric multicast routing for ad hoc networks with directional antennas", *Proceedings IEEE Infocom*, March 2005, pp. 761 – 772.

[HwRi92] F. K. Hwang, D. S. Richards, and P. Winter, "The Steiner Tree Problem. Amsterdam", The Netherlands: North-Holland, 1992.

[Jaya04] S. K. Jayaweera, "An energy-efficient virtual MIMO architecture based on V-BLAST processing for distributed wireless sensor networks", *Proceedings IEEE SECON*, October 2004, pp. 299 - 308.

[JeHu01] Jorjeta G. Jetcheva, Yih-Chun Hu, David A. Maltz, and David B. Johnson, "A Simple Protocol for Multicast and Broadcast in Mobile Ad Hoc Networks", Internet Draft, draft-ietf-manet-simple-mbcast-01.txt, July 2001.

[JeJo01] Jorjeta G. Jetcheva and David B. Johnson, "Adaptive Demand-Driven Multicast Routing in Multi-Hop Wireless Ad Hoc Networks", *Proceedings ACM MobiHoc*, October 2001, pp. 33 - 44.

[JiCh02] H. Jiang, S. Cheng, Y. He, B. Sun, "Multicasting along energy-efficient meshes in mobile ad hoc networks", *Proceedings IEEE WCNC*, March 2002, pp. 807–811.

[JiCo98] L. Ji and M. S. Corson, "A Lightweight Adaptive Multicast Algorithm", *Proceedings IEEE Globecom*, December 1998, pp. 1036–1042.

[JiCo01] L. Ji and M. Corson, "Differential destination multicast - a manet multicast routing protocol for small groups", *Proceedings IEEE Infocom*, April 2001, pp. 1192 - 1201.

[JoSi01] C. E. Jones, K. M. Sivalingam, P. Agrawal, and J. C. Chen, "A survey of energy efficient network protocols for wireless networks", *Wireless Networks*, 7(4) , 2001, pp. 343 - 358.

[JoMa96] D.B. Johnson and D.A. Maltz, "Dynamic Source Routing in Ad Hoc Wireless Networks," *Mobile Computing*, T. Imielinski and H. Korth, eds. Kluwer Academic, 1996, pp. 153-181.

[JoMa01] D. Johnson, D. Maltz, Yih-Chun Hu, and Jorjeta G. Jetcheva, "Dynamic Source Routing", IETF Internet Draft, draft-ietf-manet-dsr-06.txt, November 2001.

[KaDa02] K. Kalpakis, K. Dasgupta, and P. Namjoshi, "Maximum lifetime data gathering and aggregation in wireless sensor networks", *Proceedings IEEE ICN* (International Conference on Networking), Aug. 26 - 29, 2002, Atlanta, USA, pp. 685 - 696.

[KaPo02] Intae Kang and Radha Poovendran, "On the Lifetime Extension of Energy-Efficient Multihop Broadcast Networks", *Proceedings World Congress on Computational Intelligence*, Honolulu, USA, May 2002, pp. 365 - 370.

[KaPo03a] Intae Kang and Radha Poovendran, "Maximizing Static Network Lifetime of Wireless Broadcast Adhoc Networks", *Proceedings IEEE ICC*, Alaska, May 2003, pp. 2256 -2261.

[KaPo03b] I. Kang and R. Poovendran, "A novel power-efficient broadcast routing algorithm exploiting broadcast efficiency", *Proceedings IEEE VTC*, Orlando, October, 2003, pp. 2926 - 2930.

[KaPo04a] I. Kang and R. Poovendran, "COBRA: Center-oriented broadcast routing algorithms for wireless ad hoc networks", *Proceedings IEEE WCNC*, Atlanta, March, 2004, pp. 813 - 818.

[KaPo04b] Intae Kang and Radha Poovendran, "Broadcast with heterogeneous node capability", *Proceedings IEEE Globecom*, Dallas, December 2004, pp. 4114 - 4119.

[KaPo05] Intae Kang and Radha Poovendran, "Iterated local optimization for minimum energy broadcast", *Proceedings IEEE WiOpt*, Trentino, Italy, April 2005, pp. 332 - 341.

[KaPo06] I. Kang and R. Poovendran, "Maximizing network lifetime of broadcast over wireless stationary ad hoc networks", *ACM/Kluwer MONET*, Vol. 11, No. 2, April 2006.

[KlNa04] R. Klasing, A. Navarra, A. Papadopoulos, S. Perennes, "Adaptive Broadcast Consumption (ABC), a new heuristic and new bounds for the minimum energy broadcast routing problem", N. Mitrou et al. (eds.): *Networking* 2004, Lecture Notes in Computer Science 3042, 2004, pp. 866 - 877.

[KoSh00] Y. B. Ko, V. Shankarkumar, and N. H. Vaidya, "Medium access control protocols using directional antennas in ad-hoc networks", *Proceedings IEEE Infocom*, Tel-Aviv, Israel, March 2000, pp. 13 - 21.

[Kru56] J.B. Kruskal, "On the shortest spanning subtree of a graph and the traveling salesman problem", *Proceedings of the American Mathematical Society*, Vol. 7, 1956, pp. 48-50.

[LeGe99] S.-J. Lee, M. Gerla, and C.-C. Chiang, "On-Demand Multicast Routing Protocol", *Proceedings IEEE WCNC*, New Orleans, LA, September 1999, pp. 1298-1304.

[LeSu99a] S.-J. Lee, W. Su, and M. Gerla, "Ad hoc Wireless Multicast with Mobility Prediction", *Proceedings IEEE ICCCN*, Boston, MA, Oct. 1999, pp. 4-9.

[LeSu99b] S.-J. Lee, W. Su, and M. Gerla, "On-Demand Multicast Routing Protocol (ODMRP) for Ad Hoc Networks," Internet-Draft, draft-ietf-manet- odmrp-01.txt, June 1999.

[LeSu00] S. J. Lee, W. Su, J. Hsu, M. Gerla, R. Bagrodia, "A performance comparison study of ad hoc wireless multicast protocol", *Proceedings IEEE Infocom*, Tel Aviv, ISRAEL, 2000, pp. 565 - 574.

[Li03] Xiang-Yang Li, "Approximate MST for UDG locally", *Proceedings COCOON* (International Conference on Computing and Combinatorics), Big Sky, MT, 2003, pp. 364 - 373.

[Lian02] W. Liang, "Constructing minimum-energy broadcast trees in wireless ad hoc networks", *Proceedings ACM MobiHoc*, Lausanne, Switzerland, 2002, pp. 112 - 122.

[LiCa03] Xiang-Yang Li, Gruia Calinescu, Peng-Jun Wan, and Yu Wang, "Localized delaunay triangulation with application in wireless ad hoc networks", *IEEE Transaction on Parallel and Distributed Processing*, Vol. 14, Issue 10, Oct. 2003, pp. 1035 - 1047.

[LiHo03] Ning Li, Jennifer C. Hou, and Lui Sha, "Design and analysis of an MST-based topology control algorithm", *Proceedings IEEE Infocom*, San Francisco, USA, April 2003, pp. 1702 - 1712.

[LiHo04] Ning Li and Jennifer C. Hou, "BLMST: A Scalable, Power-Efficient Broadcast Algorithm for Wireless Networks", *Proceedings IEEE QShine* (Quality of Service in Heterogeneous Wired/Wireless Networks), Oct. 2004, pp. 44- 51.

[LiHo05] Ning Li, Jennifer C. Hou, and Lui Sha, "Design and Analysis of an MST-Based Topology Control Algorithm", *IEEE Transactions on Wireless Communications*, Vol. 4, No. 3, May 2005, pp. 1195 - 06.

[LiNi01] F. Li and I. Nikolaidis, "On minimum-energy broadcasting in all-wireless networks", *Proceedings IEEE LCN*, Tampa, November, 2001, pp. 14 -16.

[LiSo05] XiangYang Li, WenZhan Song, and Weizhao Wang, "A Unified Energy Efficient Topology for Unicast and Broadcast", *Proceedings ACM MobiCom*, Cologne, Germany, August 28 - September 02, 2005, pp. 1 - 15.

[LiWa03] Xiang-Yang Li, Peng-Jun Wan, and Ophir Frieder, "Coverage in wireless ad-hoc sensor networks", *IEEE Transactions on Computers*, Vol. 52, Issue 6, June 2003 pp. 753 – 763.

[LiWu02] Fulu Li and Kui Wu, "Reliable, distributed and energy-efficient broadcasting in multi-hop mobile ad hoc networks", *Proceedings IEEE LCN*, Nov. 6 - 8, 2002, pp. 761 - 769.

[MAC99] IEEE Local and Metropolitan Area Network Standards Committee, Wireless LAN medium access control (MAC) and physical layer (PHY) specifications, IEEE standard 802.11, 1999.

[MaDa02] R. J. Marks, A. K. Das, M. El-Sharkawi, P. Arabshahi, and A. Cray, "Minimum power broadcast trees for wireless networks: optimizing using the viability lemma", *Proceedings IEEE International Symposium on Circuits and Systems*, Scottsdale, USA, May 2002, pp. 273 - 276.

[MaGa99] E.L. Madruga and J.J. Garcia-Luna-Aceves, "Multicasting Along Meshes in Ad-Hoc Networks", *Proceedings IEEE ICC*, Vancouver, Canada, June 1999, pp. 314-318.

[NaMa00] A. Nasipuri, J. Mandava, H. Manchala, and R. E. Hiromoto, "On-demand routing using directional antennas in mobile ad hoc networks", *Proceedings IEEE ICCCN*, Las Vegas, USA, September 2000, pp. 535 - 541.

[Nava05] A. Navarra, "Tighter bounds for the minimum energy broadcasting problem", *Proceedings IEEE WiOpt* (Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks), April 2005, pp. 313 - 322.

[NaYe00] A. Nasipuri, S. Ye, J. You, and R. E. Hiromoto, "A MAC protocol for mobile ad hoc networks using directional antennas", *Proceedings IEEE WCNC*, Chicago, USA, September 2000, pp. 1214 - 1219.

[ObTs98] Katia Obraczka and Gene Tsudik, "Multicast routing issues in ad hoc networks", *Proceedings IEEE UPC* (International Conference on Universal Personal Communications), October 1998, pp. 751-756.

[PaSt82] C. H. Papadimitriou and K. Steiglitz, "Combinatorial optimization: algorithms and complexity", Englewood Cliffs, Prentice Hall, 1982.

[PeBe01] C. Perkins, E. Belding-Royer, and S. Das, "Ad Hoc On Demand Distance Vector (AODV) Routing", IETF Internet draft, draft-ietf-manet-aodv-09.txt, November 2001.

[Perk00] Charles E. Perkins, Ad Hoc Networking, Pearson Education, New Jersey, USA, Dec 2000.

[Prim57] R.C. Prim, "Shortest connection networks and some generalizations", the *Bell System Technical Journal*, vol. 26, 1957, pp. 1389 - 1401.

[RaHa00] R. Ramanthan and R. Hain, "Topology control of multihop wireless networks using transmit power adjustment", *Proceedings IEEE Infocom*, 2000, pp. 404 - 413.

[Rama01] R. Ramanathan, "On the Performance of Ad Hoc Networks with Beamforming Antennas", *Proceedings ACM MobiHoc*, Long Beach, USA, May 2001, pp. 95 - 105.

[Rapp96] T. Rappaport, "Wireless Communications: Principles and Practices", Prentice Hall, 1996.

[RoPe99] Elizabeth M. Royer and Charles E. Perkins, "Multicast Operation of the Ad-hoc On-Demand Distance Vector Routing Protocol", *Proceedings ACM MobiCom*, August 1999, pp. 207–218.

[RoZe00] G. Robins and A. Zelikovsky, "Improved Steiner tree approximation in graphs", *Proceedings ACM/SIAM SODA* (Symposium on Discrete Algorithms), San Francisco, 2000, pp. 770 - 779.

[SaSi99] Simon R. Saunders and Saunders R Simon, "Antennas and Propagation for Wireless Communication Systems", John Wiley and Sons, 1999.

[SaWi96] T.K. Sarkar, M.C.Wicks, M. Salazar-Palma, and R.J. Bonneau, "Smart Antennas", JohnWiley and Sons, Hoboken, NJ, 1996.

[SiCh00] K. M. Sivalingam, J. C. Chen, P. Agrawal, and M. B. Srivastava, "Design and analysis of low-power access protocols for wireless and mobile ATM Networks", *Wireless Networks*, 6(1), 2000, pp. 73 - 87.

[SiRa99] S. Singh, C. S. Raghavendra, and J. Stepanek, "Power-aware broadcasting in mobile ad hoc networks", *Proceedings IEEE PIMRC*, Osaka, Japan, September 1999, pp. 22 - 31.

[SiSi99] P. Sinha, R. Sivakumar, and V. Bharghavan, "MCEDAR: Multicast Core Extraction Distributed Ad-Hoc Routing", *Proceedings IEEE WCNC*, September 1999, pp. 1313 - 1317.

[SoWa04] Wen-Zhan Song, Yu Wang, Xiang-Yang Li "Localized Algorithms for Energy Efficient Topology in Wireless Ad Hoc Networks", *Proceedings ACM MobiHoc*, 2004, pp. 98 - 108.

[StLi00] I. Stojmenovic and X. Lin, "Power-aware Localized Routing in Wireless Networks", *Proceedings IEEE IPDPS* (International Parallel and Distributed Processing Symposium), 2000, pp. 371 - 376.

[StTh81] W. L. Stutzman and G. A. Thiele, "Antenna Theory and Design", John Wiley and Sons, New York, 1981.

[TaMa80] H. Takahashi and A. Matsuyama, "An approximate solution for the Steiner problem in graphs", *Mathematica Japonica*, vol. 24, pp. 573–577, 1980.

[TaMa02] M. Takai, J. Martin, R. Bagrodia and A. Ren, "Directional Virtual Carrier Sensing for Directional Antennas in Mobile Ad Hoc Networks", *Proceedings ACM MobiHoc*, Lausanne, Switzerland, June 2002, pp. 183 - 193.

[ToGu00] C. K. Toh, Guillermo Guichala, and Santithorn Bunchua, "ABAM: On-Demand Associativity-Based Multicast Routing for Ad Hoc Mobile Networks", *Proceedings IEEE VTC*, September 2000, pp. 987 - 993.

[Tous80] G. Toussaint, "The relative neighborhood graph of finite planar set", *Pattern*

Recognition, vol. 12, no. 4, pp. 261 - 268, 1980.

[WaAk02] X. Wang and I. F. Akilydiz, "A survey on sensor networks", *IEEE Communication Magazine*, 2002, pp. 102 - 114.

[WaCa01] P. J. Wan, G. Calinescu, X.-Y. Li, and O. Frieder, "Minimum-energy broadcast routing in static ad hoc wireless networks", *Proceedings IEEE Infocom*, Anchorage, Alaska, April 2001, pp. 1162 - 1171.

[WaCa02] Peng-Jun Wan, G. Calinescu, Xiang-Yang Li, and Ophir Frieder, "Minimum-energy broadcast routing in static ad hoc wireless networks", *ACM Wireless Networks*, Volume 8, Issue 6, November 2002, pp. 607 - 617.

[WaCa04] Peng-Jun Wan, G. Calinescu, Xiang-Yang Li, and Ophir Frieder, "Minimum-energy multicast routing in static ad hoc wireless networks", *IEEE/ACM Transactions on Networking*, Volume 12, No. 3, June 2004, pp. 507 - 514.

[WaGu03a] B. Wang and Sandeep K. S. Gupta, "S-REMiT: a distributed algorithm for source-based energy efficient multicasting in wireless ad hoc networks", *Proceedings IEEE Globecom*, Dec. 2003, pp. 3519 - 3524.

[WaGu03b] Bin Wang and Sandeep K. S. Gupta, "On Maximizing Lifetime of Multicast Trees in Wireless Ad hoc Networks", *Proceedings ICPP* (International Conference on Parallel Processing), Kaohsiung, Taiwan, October 2003, pp. 333 – 340.

[WaGu03c] Bin Wang and Sandeep K. S. Gupta, "G-REMiT: An Algorithm for Building Energy Efficient of Multicast Trees in Wireless Ad Hoc Networks", *Proceedings IEEE NCA* (International Symposium on Network Computing and Applications), Cambridge, MA, April 2003, pp. 265-272.

[WaLi03] Yu Wang and Xiang-Yang Li, "Localized construction of bounded degree planar spanner for wireless networks," in *Proceedings ACM DialM-POMC*, 2003, pp. 59 - 68.

[WiNg00] J. E. Wieselthier, G. D. Nguyen, et al, "On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks", *Proceedings IEEE Infocom*, March 2000, pp.585-594.

[WiNg01] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "Algorithms for energy-efficient multicasting in static ad hoc wireless networks", *ACM MONET*, 6(3): pp. 251 –263, June 2001.

[WiNg02a] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "Energy-Efficient Broadcast and Multicast Trees in Wireless Networks", *ACM MONET*, 7(6), December 2002, pp. 481 – 492.

[WiNg02b] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "The energy efficiency of distributed algorithms for broadcasting in ad hoc networks," *Proceedings IEEE WPMC* (International Symposium on Wireless Personal Multimedia Communications), October 2002, pp. 499-503.

[WiNg02c] J. E. Wieselthier, G. D. Nguyen, et al, "Energy-Limited Wireless Networking with

Directional Antennas: The Case of Session-Based Multicasting", *Proceedings IEEE Infocom*, 2002, pp. 190-199.

[WiNg02d] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "Energy-Aware Wireless Networking with Directional Antennas: The Case of Session-Based Broadcasting and Multicasting", *IEEE Transactions on Mobile Computing*, July-September 2002 (Vol. 1, No. 3), pp. 176-191.

[WiNg02e] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "Resource Management in Energy-Limited, Bandwidth-Limited, Transceiver-Limited Wireless Networks for Session-Based Multicasting", *Elsevier Computer Networks*, vol. 39, 2002, pp. 113 – 131.

[WoVe02] J.L. Wong and G. Veltri and M. Potkonjak, "Energy-efficient Data Multicast in Multi-Hop Wireless Networks", *Proceedings IEEE IMPACCT* (Workshop on Integrated Management of Power Aware Communications, Computing and Networking), May, 2002.

[WuTa98] C.W. Wu, Y.C. Tay, and C.-K. Toh, "Ad hoc Multicast Routing protocol utilizing Increasing id-numberS (AMRIS) Functional Specification", Internet-Draft, draft-ietf-manet-amris-spec-00.txt, Nov. 1998.

[YeHe02] W. Ye, J. Heidemann, and D. Estrin, "An energy-efficient mac protocol for wireless sensor networks", *Proceedings IEEE Infocom*, 2002, pp. 1567 - 1576.

[YoLi03] Jungkeun Yoon, Mingyan Liu, and Brian Noble, "Random Waypoint Considered Harmful", *Proceedings IEEE Infocom*, 2003, pp. 1312-1321.

[Yuan05] Di Yuan, "Computing optimal or near-optimal trees for minimum-energy in wireless networks", *Proceedings IEEE WiOpt*, Trentino, Italy, April 2005, pp. 323 - 331.

[YuHu92] T. S. Yum and K. W. Hung, "Design algorithms for multihop packet radio networks with multiple directional antennas stations", *IEEE Transactions on communications*, 40(11), 1992, pp. 1716 - 1724.

[ZhHo04] Honghai Zhang and Jennifer Hou, "On Deriving the Upper Bound of α-Lifetime for Large Sensor Networks", *Proceedings ACM MobiHoc*, Tokyo, Japan, May 24 - 26, 2004, pp. 121 - 132.

[ZoRo97] M. Zorzi and R. R. Rao, "Energy constrained error control for wireless channels", *IEEE Personal Communications,* vol. 4, December 1997, pp. 27 - 33.

Appendix A. Summary of Various Algorithms

This Appendix summarizes various algorithms used here for extension and comparison. The details can be found in [WiNg00, WiNg02c].

A.1 Inputs and Outputs

In general, each algorithm has the following inputs and outputs.

Inputs

- G a given network graph with node set N and arc set A;
- r_{vu} the distance between node v and node u;
- M a set of multicast members;
- s the source node of multicast group M;
- α the propagation loss exponent;

Outputs

- T a broadcast/multicast tree of G rooted at node s with node set N_T and arc set A_T.

A.2 Pseudo code of BIP/MIP Algorithms

The BIP Algorithm

Variables:

p'_{vu} the incremental cost associated with adding node u to the set of nodes to which node v already transmits;

1) Initialize T by setting $N_T = \{s\}$ and $A_T = \varnothing$, and initialize cost $p'_{vu} = r_{vu}^{\alpha}$ for each arc $(v, u) \in A$.
2) Repeat Step 2) until $N_T = N$.
   i) Find the node u that can be added with the least incremental power, i.e. $p'_{vu} = \min\{p'_{xy} \mid x \in N_T, y \in N \backslash N_T\}$.
   ii) Update T by setting $N_T = N_T \cup \{u\}$ and $A_T = A_T \cup \{(v, u)\}$.
   iii) Recompute the costs $p'_{vx} = \max\{r_{vy}^{\alpha} \mid (v,y) \in A_T \cup \{(v,x)\}\} - \max\{r_{vy}^{\alpha} \mid (v,y) \in A_T\}$ for the arcs from node v to all outside nodes $x \in N \backslash N_T$.
3) Return the broadcast tree T.

The MIP Algorithm

1) A broadcast tree T is constructed first using BIP.
2) Return the final multicast tree by pruning from T all arcs that are not needed.

A.3 Pseudo code of RB-BIP/RB-MIP Algorithms

The RB-BIP Algorithm

1) A broadcast tree T is constructed first using BIP.
2) The beamwidth of antenna is reduced to fit minimum possible angle to cover all child nodes of each node.

The RB-MIP Algorithm

1) A multicast tree T is constructed first using MIP.
2) The beamwidth of antenna is reduced to fit minimum possible angle to cover all child nodes of each node.

A.4 Pseudo code of D-BIP/D-MIP Algorithms

The D-BIP Algorithm

Variables:

A_v a sorted list $\{\alpha_{vu} \mid (v, u) \in A_T\}$ maintained by node v;

p'_{vu} the incremental cost associated with adding node u to the set of nodes to which node v already transmits;

$\theta_v(A_v)$ the minimum possible beamwidth of node v obtained using Equation (39);

1) Initialize T by setting $N_T = \{s\}$ and $A_T = \varnothing$, and initialize cost $p'_{vu} = \theta_{\min} \cdot r_{vu}^{\alpha} / 2\pi$ for each arc $(v, u) \in A$.
2) Repeat Step 2) until $N_T = N$.
   i) Find the node u that can be added with the least incremental power, i.e. $p'_{vu} = \min\{p'_{xy} \mid x \in N_T, y \in N \backslash N_T\}$.
   ii) Update T by setting $N_T = N_T \cup \{u\}$ and $A_T = A_T \cup \{(v, u)\}$, and add α_{vu} into the list A_v=ADD (A_v, α_{vu}).
   iii) Recompute $p'_{vx} = \frac{\theta_v(A'_v)}{2\pi}\max\{r_{vy}^{\alpha} \mid (v, y) \in A_T \cup \{(v, x)\}\} - \frac{\theta_v(A_v)}{2\pi}\max\{r_{vy}^{\alpha} \mid (v, y) \in A_T\}$ for the arcs from node v to all outside nodes $x \in N \backslash N_T$, where A'_v = ADD(A_v, α_{vx}).
3) Return the broadcast tree T.

The D-MIP Algorithm

1) A broadcast tree T is constructed first using D-BIP.
2) Return the final multicast tree by pruning from T all arcs that are not needed.

Appendix B. Proofs of Theorems

In order to improve the coherence of the thesis, the long proofs for several theorems in Chapters 4 to 6 are provided in this Appendix. They lead to the final MILP formulations for both MPM and MLM problems.

B.1 Proof of Theorem 4-2

Note that $\sum_{v \in N} Z^*_{vu}$ and $\sum_{v \in N} Z^*_{uv}$ are the in-degree and out-degree of node u in T_s^* respectively. Therefore, the root node s and the other multicast members satisfy this statement directly from the Constraints (4-4) and (4-5) respectively. It remains to prove that any non-multicast member in T_s^* supporting the multicast communications must have exactly one incoming arc.

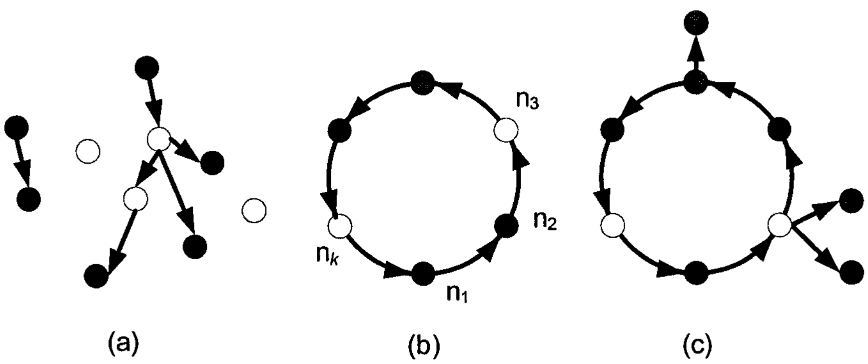


Figure B-1. Illustration of linear constraints for rooted tree property
(a) Any non-multicast member in T_s^* must have exactly one incoming arc, (b) a connected component of T_s^* may be a simple cycle, (c) a cycle with sub tree leaving out of it. (Solid nodes indicate multicast members, and hollow nodes indicate non-multicast members.)

Assume $u \in N(T_s^*)$ is a non-multicast member in T_s^*, indicated by a hollow node in Fig. B-1 (a), its incoming degree must be 1 or 0 from Constraint (4-6). If $\sum_{v \in N} Z^*_{vu} = 0$, from constraints (4-7), it follows that $\sum_{v \in N} Z^*_{uv} = 0$. That means u must be an isolated node as shown in Fig. B-1 (a), thus $u \notin N(T_s^*)$. This contradicts the original assumption. Therefore node u has exactly one incoming arc. ■

B.2 Proof of Theorem 4-4

From Constraints (4-4), (4-5) and (4-6), it follows that the only connected components in T_s^* that might contain cycles could be composed of either a simple cycle shown in Fig. B-1 (b), or a simple cycle with sub tree leaving out of it as shown in Fig. B-1 (c). We will show in the following that such topologies are not feasible for Problem MPM. Assume that the nodes (n_1, n_2, …, n_k, $n_{k+1} = n_1$), $k > 1$, form a simple cycle in T_s^*. Then from Constraint (4-4), node s will never be included in such a cycle. Constraint (4-10) implies that F^*_{vu} could be positive if and only if $(v, u) \in A'$. Letting $F^*_{n_1n_2} = f$, then from Constraint (4-9) it follows that $F^*_{n_rn_{r+1}} = F^*_{n_1n_2} - \sum_{i=1}^{r-1} Z^*_{n_in_{i+1}}$ for $r = 1, …, k$. Each node n_r ($r = 1, …, k$) is in $A(T_s^*)$ as stated in the assumption, i.e., $Z^*_{n_rn_{r+1}} = 1$. Therefore $F^*_{n_rn_{r+1}} = F^*_{n_1n_2} - \sum_{i=1}^{r-1} Z^*_{n_in_{i+1}} = f - (r - 1)$ for $r = 1, …, k$. After substituting $F^*_{n_kn_1} = f - (k - 1)$ into Constraint (4-9), for $u = n_1$, we obtain $\sum_{v \in N} F^*_{vn_1} - \sum_{v \in N} F^*_{n_1v} = f - (k-1) - f = 1 - k < 0$. On the other hand, $\sum_{v \in N} F^*_{vn_1} - \sum_{v \in N} F^*_{n_1v} = \sum_{v \in N} Z^*_{vn_1} \geq 0$ from Constraints (4-8) in Corollary 4-3. Thus Constraint (4-9) are violated, and therefore simple cycles are not possible in T_s^*. Similar reasoning shows that the topology in Fig. B-1 (c) also violates Constraint (4-9), and therefore T_s^* cannot contain cycles. ■

B.3 Proof of Theorem 4-7

We only need to prove that the statement "$Z_{vu} = 1$ only if (φ_v, θ_v) $\in$ I $\cup$ II" is equivalent to the Constraints (4-19) to (4-21). Since $Z_{vu} = Z_{vu1} + Z_{vu2}$, and they are all binary variables, $Z_{vu} = 1$ if and only if just one of the Boolean expressions ($Z_{vu1} = 1$ and $Z_{vu2} = 0$) and ($Z_{vu1} = 0$ and $Z_{vu2} = 1$) is true. We first consider the case $Z_{vu1} = 1$ and $Z_{vu2} = 0$. Thus Constraints (4-19) to (4-21) become $2\varphi_v \geq \theta_v/2 + 2\pi + \alpha_{vu}$, $\varphi_v \leq \theta_v/2 + 2\pi$, and $\varphi_v \geq -\theta_v/2$ respectively. Considering the boundary conditions $0 \leq \theta_v \leq 2\pi$ and $0 \leq \varphi_v < 2\pi$, we observe that these constraints just define the area I as shown in Fig 4-3 (a). Similarly, after substituting $Z_{vu1} = 0$ and $Z_{vu2} = 1$ into Constraints (4-19) to

(4-21), we can easily verify that the resulting constraints $\varphi_v \geq -\theta_v/2$, $\varphi_v \leq \alpha_{vu} + \theta_v/2$, and $\varphi_v \geq \alpha_{vu} - \theta_v/2$ define the area II. Combing the two cases, we conclude that Constraints (4-19) to (4-21) characterize the statement "$Z_{vu} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{I} \cup \mathrm{II}$" correctly. ■

B.4 Proof of Theorem 5-8

In the following, we shall show that Constraint (5-17) is equivalent to Constraint (5-16) and in a linear form. For any arc (v, u) in the tree T_s^*, i.e. $Z_{vu} = 1$, we obtain $w \geq \frac{r_{su}^{\alpha}}{360 \cdot e_s} \cdot \theta_s + \frac{p_{tran}}{e_s}$ and $w \geq \frac{r_{vu}^{\alpha}}{360 \cdot e_v} \cdot \theta_v + \frac{p_{tran} + p_{recv}}{e_v}$ for $v \neq s$ straightforward from Constraint (5-17a) and Constraint (5-17a) respectively. Therefore, w must be not less than the maximum among $\{ \left. \frac{r_{vu}^{\alpha}}{360 \cdot e_v} \cdot \theta_v + \frac{p_{tran}}{e_v} \right|_{v=s}, \left. \frac{r_{vu}^{\alpha}}{360 \cdot e_v} \cdot \theta_v + \frac{p_{tran} + p_{recv}}{e_v} \right|_{v \neq s} \}$ for all $(v, u) \in A(T_s^*)$, i.e., $w \geq \max_{(v,u) \in A(T_s^*)} \{ \left. \frac{r_{vu}^{\alpha}}{360 \cdot e_v} \cdot \theta_v + \frac{p_{tran}}{e_v} \right|_{v=s}; \left. \frac{r_{vu}^{\alpha}}{360 \cdot e_v} \cdot \theta_v + \frac{p_{tran} + p_{recv}}{e_v} \right|_{v \neq s} \}$. The equality must be achieved in the inequation above when the variables w is minimized. On the other hand, for the arc (v, u) that is not in the tree T_s^*, i.e. $Z_{vu} = 0$, Constraint (5-17) becomes redundant. ■