THE REGULAR MARKOV CHAIN MODEL APPLIED TO GAME-PLAYING BEHAVIOUR IN RAPOPORT'S "ARCHETYPES" OF THE 2X2 NON-ZERO-SUM GAME

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CURRICULUM STUDIORUM

Marius Andrew Wozny was born on March 27, 1940, in Warsaw, Poland. In 1961, he received the Bachelor of Science degree in chemistry and biology from Loyola College, University of Montreal and, in 1966, the Master of Arts degree in psychology from the University of Ottawa. The Master's thesis was entitled "The Clustering and Saturation of Jung's Personality Dimensions in the Gray-Wheelwright Psychological Type Questionnaire."

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INTRODUCTION

In this study, the regular Markov chain model is applied to game-playing behaviour of subjects in four 2x2 non-zero-sum games. Using principally the theory of finite Markov chains as presented by Kemeny and Snell,[1] predictions are made from a simulated Markov chain process assumed as underlying the game behaviour. The model is then tested against collected data by showing goodness of fit between predicted and obtained values.

The object of the study is to have pairs of subjects play, over a large number of consecutive trials, four symmetric 2x2 non-zero-sum games, where each player of a pair has a choice between two strategies A and B on each trial of the game, and where it is possible to observe the four possible outcomes AA, AB, BA and BB that result from the combination of both players' strategies. By attaching specific values to the two choices open to each player, incentives or "motives" can be created in the player for preferring one choice over the other. Since outcomes are directly dependent on the mutual choices of both players, this interdependency could arouse motivational conflict in the subjects as the number of trials becomes large, thus affecting choice behaviour.

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1 J.G. Kemeny and J.L. Snell, Finite Markov Chains, van Nostrand, Princeton, N.J., 1960, viii-210 p.

In essence, the whole study is based on just such an assumption, namely, that pre-decisional events of an underlying motivational process determine directly the overt responses (outcomes) in the two-person game situations (described fully in chapter two in the formulation of the problem) and that, once the process is operationally quantified, its effect on choice behaviour can be studied in terms of Markov chain theory. In other words, the pre-decisional motivational process determining the decision task is modelled as a finite-state Markov process from which predictions are made as to the actual distribution of outcomes obtained in the total sequence of trials over which the games are played.

The thesis is divided into three chapters. The first chapter presents some basic ideas about games and reviews the literature relevant to the problem. The second chapter discusses in detail the problem of the present study, the Markov chain model, and the experimental method followed in the collection of the data. Finally, the third chapter presents and analyzes the results of the study.

CHAPTER I

REVIEW OF THE RESEARCH LITERATURE

1. General Introduction.

The impetus for utilizing games as experimental paradigms for studying interperson-interactions stems primarily from the work of von Neumann and Morgenstern[1] and its reinterpretation for the behavioural sciences by Luce and Raiffa.[2] Stated simply, game theory aims at developing criteria for rational behaviour in situations that involve total or partial conflict of interests between two or more individuals.

In its simplest non-mathematical formulation, it is based on two assumptions known as rationality postulates. The first of these basic postulates, that of individual utility maximization or individual rationality, states that each player tries to maximize his own expected satisfaction (or utility). The second, known as the postulate of mutually expected rationality, states that each player expects and acts on the expectation that the player(s) will also try to

1 J. von Neumann and O. Morgenstern, Theory of Games and Economic Behavior, Princeton University Press, Princeton, N.J., 1944, x-641 p.

2 R.D. Luce and H. Raiffa, Games and Decisions, John Wiley and Sons, New York, 1957, xi-509 p.

maximize his(their) own expected utility. Essentially, these postulates describe the interaction of two or more individuals in terms of behaviour which can be defined as the rational pursuit of their own self-interests.

One of the contributions of von Neumann and Morgenstern was to provide a classification of games into zero-sum and non-zero-sum. The zero-sum game is one in which the total won by a player, or players, is equal to the total lost by another player, or players. A non-zero-sum game is one in which the gains made by one player are not necessarily made at the expense of the other. Only in the zero-sum game is there a situation of pure conflict. In the non-zero-sum game, there is usually the element of cooperation or competition operating. From a psychological point of view, the zero-sum games are uninteresting since they are games of pure conflict in which the interests of the players are diametrically opposed.

The non-zero-sum game, the so-called "mixed motive game" in Schelling's[3] terminology, is of particular psychological interest because the play may be the result of competitive motivation, cooperative motivation, or both kinds of motivation in various combinations. As a result, this type of game has

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3 T. Schelling, The Strategy of Conflict, Harvard University Press, Cambridge, Mass., 1960, vii-309 p.

served, in recent years, as *the* research paradigm for studies analyzing social (dyadic) interaction.

Particularly prominent in contemporary game research has been the Prisoner's Dilemma (PD) game described by Luce and Raiffa as a special type of "mixed-motive" game. It is mixed-motive because the player has to choose between increasing his own immediate gain or increasing the total gain of both players. The reason for its extensive use in research is that it has been considered a prototype of social situations where the interplay of mutual and divergent interests are operating.

The general form of this game can be represented by the pay-off matrix shown in Figure 1. There are two players, 1 and 2, where each must choose between A for cooperation, and B for defection on every trial. Joint decisions are indicated by A_1A_2, A_1B_2, B_1A_2 or B_1B_2. In order to simplify discussion, subscripts are usually omitted: the first letter of each joint response indicates player 1's decision in all cases. There are four entries in the pay-off matrix and, by convention, the first of the pay-offs in each entry is that accruing to the row player, or player 1.

The pay-off matrix is subject to the following rules in the PD game:

1. $2x_1 > x_2 + x_3 > 2x_4$
2. $x_3 > x_1 > x_4 > x_2$

		Player 2	
		A_1	B_2
Player 1	A_1	x_1 x_1	x_2 x_3
	B_1	x_3 x_2	x_4 x_4

Figure 1.- General Pay-Off Matrix for 2x2 Games where 1 and 2 in the row and column headings refer to the players, A and B to the two choices open to each player and where x_1, x_2, x_3 and x_4 denote different (or equal when used twice) pay-offs, and where the first pay-off value in every cell refers to player 1.

The first rule is introduced to prevent the possibility of more than the one form of tacit collusion, i.e. tacit agreement to play AA. If the inequality in the first rule is reversed, another form of tacit collusion is possible, namely, an alteration between AB and BA, assuming that the game is repeated. The second rule is introduced to motivate each player to play non-cooperatively.

In PD games, B dominates A in the game-theoretic sense. In game theory, one strategy dominates another if, and only if, in using it a player does no worse and, in general, better regardless of the strategy chosen by the other player. However, the joint choice BB results in a pay-off (x_4) to each player which is smaller than the pay-off (x_1) associated with AA, hence the dilemma. For a detailed description of the game, the sources are Luce and Raiffa,[4] Rapoport and Chammah,[5] and Rapoport.[6] Reviews of experimental studies of PD up to 1962 are given by Rapoport and Orwant,[7] and up to 1965 by Gallo and McClintock.[8]

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4 Luce and Raiffa, Op. Cit., xi-509 p.

5 A. Rapoport and A. Chammah, Prisoner's Dilemma, University of Michigan Press, Ann Arbor, 1965, xii-258 p.

6 A. Rapoport, Two-Person Game Theory: The Essential Ideas, University of Michigan Press, Ann Arbor, 1966, 229 p.

7 A. Rapoport and C. Orwant, "Experimental Games: A Review," Behavioral Science, Vol. 7, 1962, p. 1-37.

8 P.S. Gallo and C.G. McClintock, "Cooperative and Competitive Behavior in Mixed-Motive Games," Journal of Conflict Resolution, Vol. 9, 1965, p. 68-78.

2. Research Literature.

There have been three major trends in game literature which have attempted to provide an explanation for the observed fact that players do not, on the whole, play rationally in the game-theoretic sense, that is, by making their choices in terms of their dominating strategies according to the "minimax" principle. "Minimax" in game theory refers to that strategy which will minimize a player's loss, if he should suffer a loss. Attempts at a possible rationale for this observed discrepancy between what game theory prescribes and the subjects' actual game-playing behaviour have tried to clarify PD findings in terms of: 1) personality variables of the players; 2) strategies employed by the other player; and 3) such concepts as "trust" and "trustworthiness."

Studies trying to relate individual personality differences to choice behaviour in PD games have followed the generalized experimental design of separating subjects on some personality dimension (in terms of high and low scores) and then, either matching subjects with respect to the same pole of the presumed continuum, or pairing them in terms of "polar opposites." Thus, Lutzker,[9] using an "internationalism" scale and selecting subjects with extreme scores, found that

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9 D.R. Lutzker, "Internationalism as a Predictor of Cooperative Game Behavior," Journal of Conflict Resolution, Vol. 4, 1960, p. 426-435.

pairs strongly "internationalistic" in their personal orientation towards the political world were significantly more cooperative than pairs who were strongly "isolationistic."

McClintock, Harrison, Strand and Gallo[10] found similar results when "isolationist" and "internationalist" subjects were paired against programmed "other players" using strategies with markedly different levels of cooperative responses. Also using a programmed opponent, but in this case one that responded with an unconditionally cooperative choice, Marlowe[11] reported that extreme non-cooperators scored significantly higher than extremely cooperative subjects on need aggression and autonomy on the Gough Adjective Check-list; extremely cooperative subjects scored higher than non-cooperative on need abasement and need deference. For a similar game, Deutsch[12] found that authoritarianism scores as measured by the California F Scale correlated significantly with game behaviour. Subjects with low F scores tended to be trusting and trustworthy, and those with high F scores tended to be suspicious and untrustworthy in their game choices.

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10 C.G. McClintock, A. Harrison, S. Strand and P.S. Gallo, "Internationalism, Isolationism, Strategy of the Other Player and Two-Person Game Behavior," Journal of Abnormal and Social Psychology, Vol. 67, 1963, p. 631-636.

11 D. Marlowe, "Psychological Needs and Cooperation vs. Competition in a Two-Person Game," Psychological Reports, Vol. 13, 1963, p. 364.

12 M. Deutsch, "Trust, Trustworthiness, and the F Scale," Journal of Abnormal and Social Psychology, Vol. 61, 1960, p. 138-140.

More recently, Shure, Meeker, Moore and Kelley,[13] using the Shure-Meeker Personality Attitude Schedule composed of six factored-scales, found interesting personality correlates in two-person, non-zero-sum games. They reported that high "conciliators," "equalitarians" and "risk-avoiders" earned more, respectively, than pairs composed of low "conciliators," "authoritarians" and "risk-seekers."

Studies involving strategies as explanatory concepts for the observed PD game behaviour have been more interesting from the point of view of presenting the rationale in more dynamic terms than the previous (static) approach in terms of personality differences. These studies have usually been designed in such a way as to control the strategy of one of the players in some predetermined fashion, and study what influencing effect, if any, this has on the strategy of the other player. The player who plays the predetermined set of strategies is a confederate or "stooge" of the experimenter; occasionally, the preprogrammed "other" has been an electronic computer unknown to both players.

In the literature reported to date, five types of strategies have been studied as to their effect on the choice behaviour of the other player. These strategies are: 1) total

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13 G.H. Shure, R.J. Meeker, W.H. Moore and H.H. Kelley, Computer Studies of Bargaining Behavior: The Role of Threat in Bargaining, Report No. SP-2196, System Development Corporation, Santa Monica, California, 1966.

unilateral cooperative strategy; 2) total unilateral defection strategy; 3) random strategy; 4) matching strategy; and 5) partial reinforcement strategy.

In the unilateral cooperation strategy, the stooge cooperates without regard to the subject's actual moves, as can be seen from the experimental designs of Rapoport and Chammah,[14] and Solomon.[15] In unilateral defection, the stooge defects without regard to the subject's moves. Under random strategy, the stooge cooperates or defects according to a random schedule without regard to the subject's behaviour. The probability of an A or B move is present and remains the same for every trial. This type of strategy has been used by Bixenstine, Potash and Wilson,[16] and by Marlowe, Gergen and Dobb.[17]

Matching strategies have been considered more interesting: the stooge matches the actual performance of the subject

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14 Rapoport and Chammah, Op. Cit., xii-258 p.

15 L. Solomon, "The Influence of Some Types of Power Relationships and Game Strategies Upon the Development of Interpersonal Trust," Journal of Abnormal and Social Psychology, Vol. 61, 1960, p. 223-230.

16 V.E. Bixenstine, H.M. Potash and K.V. Wilson, "Effects of Level of Cooperative Choice by the Other Player on Choices in a Prisoner's Dilemma Game: Part I," Journal of Abnormal and Social Psychology, Vol. 66, 1963, p. 308-313.

17 D. Marlowe, K.J. Gergen and A.N. Dobb, "Opponent's Personality, Expectation of Social Interaction, and Interpersonal Bargaining," Journal of Personality and Social Psychology, Vol. 3, No. 2, 1966, p. 206-213.

in "tit-for-tat" fashion, where the matching is of the subject's current move or of his previous one. In the latter case, a player who defected on one trial would be faced with defection on the subsequent trial. In such strategies, the level of cooperation by the stooge is entirely contingent upon the subject's own performance as shown by Komorita.[18]

Partial reinforcement strategy lies between the random and matching strategies: the stooge's responses to particular moves are randomly selected but with one probability for a subject's cooperative moves, and a different probability for his defecting moves. Hence, the proportion of times cooperation will be rewarded or defection punished can be predetermined. For example, eighty-five per cent of subject's cooperative moves are met by A responses while sixty per cent of subject's B responses are met by defection. Such reinforcement schedules have been characteristic of the work of Pylyshyn, Agnew and Illingworth,[19] and Komorita.[20]

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18 S.S. Komorita, "Cooperative Choice in a Prisoner's Dilemma Game," Journal of Personality and Social Psychology, Vol. 2, 1965, p. 741-745.

19 Z. Pylyshyn, N. Agnew and J. Illingworth, "Comparison of Individuals and Pairs as Participants in a Mixed-Motive Game," Journal of Conflict Resolution, Vol. 10, 1966, p. 211-220.

20 Komorita, Op. Cit., p. 741-745.

In general, studies on strategies, notably by Bixenstine *et al.*,[21] McClintock *et al.*,[22] and Sermat,[23] have shown that it is difficult to influence the subject's game strategy by varying systematically the strategy of his partner. However, Scodel[24] found that, in a PD game, a stooge who played competitively for the first ten trials and then became unconditionally cooperative (total unilateral cooperation strategy) produced more cooperation in the subjects than did a stooge who played unconditionally cooperatively throughout the fifty trials of the experiment.

Bixenstine and Wilson[25] found that the stooge's initial level of cooperation had a significant effect on subject's game strategy, and also interacted with the sex of the subject and the personality measure of ethicality. The level of cooperation observed in their study was generally below fifty per cent.

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21 Bixenstine *et al.*, *Op. Cit.*, p. 308-313.

22 McClintock *et al.*, *Op. Cit.*, p. 631-636.

23 V. Sermat, "Cooperative Behavior in a Mixed-Motive Game," *Journal of Social Psychology*, Vol. 62, 1964, p. 217-239.

24 A. Scodel, "Induced Collaboration in Some Non-Zero-Sum Games," *Journal of Conflict Resolution*, Vol. 4, 1962, p. 335-340.

25 V.E. Bixenstine and K.V. Wilson, "Effects of Level of Cooperative Choice by the Other Players in a Prisoner's Dilemma Game: Part II," *Journal of Abnormal and Social Psychology*, Vol. 67, 1963, p. 139-147.

Sermat and Gregovich,[26] using a chicken matrix (a game somewhat similar to PD) and having the simulated other player match subject's choices with a one-trial lag (the "tit-for-tat" strategy), found that subjects were significantly more cooperative when their first choice coincided with that of the other player, than when it did not. A second study by Sermat,[27] employing the same chicken matrix, showed that one hundred per cent competition from the other player produced more cooperation in the subject than did one hundred per cent cooperation.

In his latest study, Sermat[28] investigated the effect of pregame treatment on subsequent performance against a tit-for-tat strategy, using a PD and a chicken matrix in four experiments. A pre-arranged program, simulating the "other player," made thirty consecutive cooperative or competitive choices during a "pretreatment" and then, for two hundred trials, reciprocated the subject's choices with a one-trial lag (the tit-for-tat strategy). A highly significant increase in cooperative behaviour was observed in all four experiments.

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26 V. Sermat and R.P. Gregovich, "The Effect of Experimental Manipulation on Cooperative Behavior in a Chicken Game," Psychonomic Science, Vol. 4, No. 12, 1966, p. 435-436.

27 Sermat, Op. Cit., p. 217-239.

28 ———, "The Effect of an Initial Cooperative or Competitive Treatment Upon a Subject's Response to Conditional Cooperation," Behavioral Science, Vol. 12, No. 4, 1967, p. 301-313.

The data suggest that either type of pretreatment may facilitate the development of a cooperative strategy, if followed by a tit-for-tat treatment, while in the absence of pretreatment, no increase in cooperation occurs.

This finding supports Sermat's earlier study.[29] Here, subjects who initially received thirty consecutive competitive choices from the other player were significantly more cooperative at the end of one hundred trials of tit-for-tat treatment than subjects for whom this pretreatment was omitted. The tit-for-tat method, when not preceded by any other treatment, produced a generally low level of cooperation without any apparent increase over one hundred trials. That a matching treatment by itself is relatively ineffective for inducing cooperation has been shown by Solomon,[30] Minas *et al.*[31] Oskamp and Perlman,[32] and by Bixenstine *et al.*[33]

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29 V. Sermat, *Cooperative Behavior in a Mixed-Motive Game*, unpublished doctoral dissertation, Yale University, 1961.

30 Solomon, *Op. Cit.*, p. 223-230.

31 J.S. Minas, A. Scodel, D. Marlowe and H. Rawson, "Some Descriptive Aspects of Two-Person Non-Zero-Sum Games: II," *Journal of Conflict Resolution*, Vol. 4, 1960, p. 193-197.

32 S. Oskamp and D. Perlman, "Factors Affecting Cooperation in a Prisoner's Dilemma Game," *Journal of Conflict Resolution*, Vol. 9, 1965, p. 359-374.

33 Bixenstine, *et al.*, *Op. Cit.*, p. 206-213.

Finally, Wozny,[34] in an unpublished paper, investigated the effects of an extreme random strategy of the stooge on the choice behavior of the subject under three matrix conditions, where the index of average advantage of competition varied from 1½, through o, to -1½ units for each game matrix, respectively. The game was played over ten blocks of 12 trials each where the stooge played competitively 87% of trials in the first block, 87% cooperatively in the second block, then 87% competitively in the third, etc., for a total of ten blocks. That is, the stooge alternately defected and cooperated without regard to the subject's response behaviour. The overall finding was that the stooge's strategy had an "inducing" effect on subjects' choice behaviour to respond in kind for all three matrix conditions. The subject matched the stooge's responses in kind, but the matching (goodness of fit) was significantly greater for the competitive responses than for the cooperative ones. There was a slight decrease in goodness of fit of the competitive responses as the index of competition decreased from 1½ to 0 to -1½, showing that structural features of the matrix influence the "inducing" effect.

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34 M.A. Wozny, The Effect of the Strategy of the Other Player and Average Advantage of Competition on Choice Behavior of Subjects in Two-Person Games, unpublished paper, 1967, 23 p.

In addition to both personality variables and strategy discussed above, the attitudinal correlate of trust has been shown to influence choice behaviour in the PD game. It is clear, from a consideration of the properties of this dilemma, that one player's choice behaviour will be very much affected by the extent to which he trusts the other player to cooperate. Deutsch[35] appears to have been the first social psychologist to recognize the implications of this game for studying cooperative behaviour and trust in a social situation.

In this study, Deutsch examined the influence of cooperative, individualistic and competitive motivational orientations upon the development of mutual trust in the PD game. He found that individualistically oriented dyads did not differ from the competitive ones with respect to the number of trusting choices made. However, contrary to common-sense prediction, the individualistic subjects were unable to act rationally (by choosing A) in the absence of mutual trust. That is, each player was unable to subordinate immediate self-interest for long-range group-gain, and the relationship tended to deteriorate into self-defeating competition.

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35 M. Deutsch, "Trust and Suspicion," Journal of Conflict Resolution, Vol. 2, 1958, p. 265-279.

Deutsch,[36] in a subsequent study, followed up these results by showing further interesting relationships. Here, subjects were given no motivational orientation. Each subject played the game twice, choosing first in the first position and supposedly having his choices announced to the second player (who was fictitious). In the second position, the subject was informed that the other player (still fictitious) had made the trusting choice, A. The results showed that subjects who were trusting when they chose first tended to be trustworthy when they chose second; they expected the same behaviour of the other player. Subjects who were suspicious and untrustworthy expected to be exploited by the other players; these subjects responded to a trusting choice by taking advantage of it.

One major theoretical question that has received attention recently concerns the motivational bases of choice in experimental games. Present research by Messick and McClintock[37] indicates that at least three motives are operative in non-zero-sum games, namely, maximizing own gain, maximizing joint gain and, maximizing the difference between own and other player's gain.

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36 M. Deutsch, "Trust, Trustworthiness, and the F Scale," Journal of Abnormal and Social Psychology, Vol. 61, 1960, p. 138-140.

37 D.M. Messick and C.G. McClintock, "Motivational Bases of Choice in Experimental Games," Journal of Experimental and Social Psychology, Vol. 4, 1968, p. 1-25.

Studies on game behaviour have usually assumed that subjects were attempting to maximize the pay-offs as defined by the experimenter and displayed in the pay-off matrix. Recent research has clearly demonstrated that this is not always the case. Results of studies conducted by McClintock and McNeel[38,39,40,41,42] strongly suggest that individuals do not exclusively try to maximize their pay-offs, but are more concerned with their scores relative to the other player than with the absolute magnitude of their own scores.

Another series of experiments reported by Messick and Thorngate[43] also demonstrate that subjects do, in fact, tend to maximize relative gain, or the difference between

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38 C.G. McClintock and S.P. McNeel, "Reward Level and Game Playing Behavior," Journal of Conflict Resolution, Vol. 10, 1966a, p. 98-102.

39 --------, "Cross-Cultural Comparisons of Interpersonal Motives," Sociometry, Vol. 29, 1966b, p. 406-427.

40 --------, "Societal Membership, Score Status and Game Behavior: A Phenomenological Analysis," International Journal of Psychology, Vol. 1, 1966c, p. 263-279.

41 --------, "Reward and Score Feedback as Determinants of Cooperative Behavior," Journal of Personality and Social Psychology, Vol. 4, 1966d, p. 606-613.

42 --------, "Prior Dyadic Experience and Monetary Reward as Determinants of Cooperative and Competitive Game Behavior," Journal of Personality and Social Psychology, Vol. 5, 1967, p. 282-294.

43 D.M. Messick and W. Thorngate, "Relative Gain Maximization in Experimental Games," Journal of Experimental and Social Psychology, Vol. 3, 1967, p. 85-101.

their pay-offs and those of the other player, under the condition of complete information about the other's pay-offs. They found that this effect is mediated more by a tendency to avoid getting less than the other player, than by a desire to surpass him. An important implication of their findings is that of the three motives operating in PD-type games, the difference between own and other's outcome is more effective in determining choice behaviour than the motives to maximize own and/or joint gain.

CHAPTER II

THE PRESENT STUDY

1. Formulation of the Problem.

The object of the present study is to have pairs of subjects play, over a large number of consecutive trials, four symmetric games discussed by Rapoport[1] and Rapoport and Guyer.[2] These four games are presented in the present text as figure 2 and are labelled as "Exploiter," "Leader," "Hero," and "Martyr."

Referring to figure 2, each player has a choice between two strategies A and B so that, on each trial of the game, one of the four possible outcomes AA, AB, BA and BB can be observed. Associated with each such outcome is a pair of numbers that denotes the value of the outcome to the players. The first number of such a pair is the value of the outcome to player 1, and the second to player 2. In this way, by attaching specific values to the two choices open to each player, an incentive or "motive" is created for preferring one choice over the other. And, as the number of trials

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1 A. Rapoport, "Exploiter, Leader, Hero and Martyr: the Four Archetypes of the 2x2 Game," Behavioral Science, Vol. 12, No. 2, 1967, p. 81-84.

2 A. Rapoport and M. Guyer, "A Taxonomy of 2x2 Games," General Systems, Vol. 11, 1966, p. 203-214.

Player 1		Player 2: A	Player 2: B
	A	1 1	-1 2
	B	2 -1	-2 -2

a) "Exploiter"

Player 1		Player 2: A	Player 2: B
	A	-1 -1	1 2
	B	2 1	-2 -2

b) "Leader"

Player 1		Player 2: A	Player 2: B
	A	-1 -1	2 1
	B	1 2	-2 -2

c) "Hero"

Player 1		Player 2: A	Player 2: B
	A	-1 -1	2 -2
	B	-2 2	1 1

d) "Martyr"

Figure 2.- Pay-Off Matrices for Rapoport's "Archetype" 2x2 Non-Zero-Sum Games, where A and B refer to the two choices open to each player and where the first number in each cell is the pay-off value of that outcome to player 1.

becomes large, it is possible to hypothesize on intuitive grounds the outcomes that should emerge during the course of the game.

From inspection of figure 2, in game Exploiter, Leader and Hero, the preference should be for each player to choose B over A, for by making this choice, each would get a larger pay-off provided the other did not choose B. If both happen to choose B, both are punished by the worst pay-off, namely, -2. In this respect, these three games are similar; however, they differ in other respects. For instance, in game Exploiter, if one player chooses B while the other does not, the one who chose B helps himself and punishes the other. Such a player could, therefore, be called an "exploiter." In game Leader, if one player chooses B while the other does not, the one choosing B rewards both himself and the other, but himself more than the other. Such a player could, then, be called a "leader." In game Hero, if one player chooses B while the other does not, the one who chooses B rewards both himself and the other, but the other more than himself. Such a player could be called a "hero." Game Martyr, however, is different from the other three because, in this game, neither player is "motivated" to choose B. If a player should choose B, he would reward the other while harming himself. Such a player could very well be called a "martyr."

It is also to be noted that in games Exploiter, Leader and Hero, if both players choose B, both get the worst pay-off. One could say that there is no room for two "exploiters," two "leaders" or two "heroes." But, in game Martyr, if both choose B, both gain. Thus, there is room for two "martyrs."

Games Exploiter and Leader can be further described by calling them "preemption games" after Rapoport. If one player succeeds in choosing B before the other has had the chance, he gets not only the largest pay-off but assures this pay-off for himself, since the other player can now by choosing B only impair his own pay-off. Thus, a choice of B by a single player results in the so-called "equilibrium" outcome in Rapoport's terminology. There are two equilibria and the player who chooses B first secures the equilibrium with the largest pay-off for himself.

In game Hero, each player can secure the equilibrium with the largest pay-off for himself, if the other makes the B choice. If both wait for the other to make the B choice, both may persist in the outcome (AA) in which each loses. However, if both try to be "heroes" by choosing B, the result is even worse for both, namely, -2 pay-off to both. Finally, in game Martyr, there is only one equilibrium. Neither player wants to choose B, that is to be a "martyr," yet both would gain if both chose B. In this respect, it is identical with the famous Prisoner's Dilemma game discussed in chapter one.

Thus, the present study can be viewed as falling within the recent research trend on motivational variables underlying game behaviour. Specifically, the problem of the present study is to translate the "motives" operating, as discussed above, into parameters of a probabilistic model from which predictions can be made about the actual playing-behaviour of the players, as the games are played over a repeated number of trials. In other words, what is aimed at in this study is the quantification of the "motives" presumed operating on choice behaviour and the possibility of describing their effect in terms of a mathematical model.

Rapoport considers these four games psychologically interesting as they bring out four distinct types of "psychological pressure" operating on each player. In game-theoretic terminology, he sees these "pressures" as the motivation of a player, in making his choice, to shift from the so-called natural (minimax) outcome of the game.

The natural outcome of each game is in the upper left-hand corner of the matrix. The natural outcome results if each player chooses his "minimax," that is, that strategy which will minimize his loss should he suffer a loss. In games Exploiter, Leader and Hero, this outcome is not an equilibrium because both players are motivated to switch unilaterally from the natural outcome. In game Martyr, on the other hand, the natural outcome is an equilibrium, an outcome from which neither player is motivated to switch unilaterally.

Following Rapoport's logical analysis of these four "archetypes," it is seen from figure 2 that the motivational pressure operating on each player in games Exploiter and Leader is to switch from the AA outcome--in other words, to preempt the other in order to secure the largest pay-off for himself. There is, then, motivation operating on each player, at the beginning of each trial, to choose B. Similarly, in game Hero, the pressure is not to switch from AA in the hope that the other player will switch, and in game Martyr where the AA outcome is an equilibrium, the pressure operating on each player is also not to switch. In terms of the two choices open to each player at the beginning of each trial, in games Hero and Martyr, this means a pressure to choose A over B.

By assuming that these motives channel choice behaviour into particular outcomes of the 2x2 matrix by directly affecting the probability of being in a given outcome on each trial of the game, it should be possible to study the dynamic interplay of these motivations in terms of a probabilistic model, and hopefully determine whether a connection does exist between Rapoport's logically deduced "pressures" and the four outcomes (AA, AB, BA, BB), actually observed, as the game is played repeatedly over a large number of trials.

2. The Markov Chain Model.

On the assumption, therefore, that the distribution of actual outcomes during the course of the games is governed by the interplay of the pressure "to switch or not to switch" at the beginning of each trial, an attempt will be made to apply Markov chain theory as a possible model to describe the game behaviour.

Following Kemeny and Snell,[3] a Markov chain is a mathematical model for describing a certain type of process that moves in a sequence of steps through a set of states. The process is a stochastic one and the number of states through which it moves may be either infinite or finite. If the number of states is finite, denoted by s_1, s_2, s_3, ... s_r, we have a finite Markov chain for which the probability of entering a certain state depends only on the last state occupied. When the process is in state s_i, there is probability p_{ij} that the next position will be state s_j. The matrix $P = [p_{ij}]$ is called the transition matrix from every state s_i to s_j; all of its entries are non-negative and its row entries sum up to 1.

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3 J.G. Kemeny and J.L. Snell, Finite Markov Chains, van Nostrand, Princeton, N.J., 1960, viii-210 p.

In general, finite Markov chains are differentiated into two special types of chains: absorbing and ergodic. A state is absorbing if, once entered, it is never left. Consequently, a chain is an absorbing chain if it has at least one absorbing state and if from every state it is possible, not necessarily in one step, to reach an absorbing state. An ergodic chain, on the other hand, is a chain in which it is possible to go from every state to every other state, not necessarily in one step. An important special case of an ergodic chain is the so-called regular chain, a chain such that for some n, P^n [4] has no zero entries. The average P-matrix for each game in this study satisfies this condition and, therefore is regular. (Cf. Table V in Chapter III, Results and Discussions.)

Matrix algebra plays a basic role in this theory. According to Kemeny and Snell,[5] in a regular Markov chain the probability of moving from state s_i to s_j in n steps is given by the ijth entry of P^n, and the fraction of time (steps) that the process can be expected to be in state s_j for a large n is given by the limiting value a_j. The value a_j is calculated from a basic theorem for regular chains:

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4 The symbol P^n means "P raised to the power n," where n can take on integer values of 1, 2, 3, ... n.

5 Kemeny and Snell, Op. Cit., p. 69-74.

$$\lim_{n \to \infty} P^n = A \qquad (1)$$

where A is a probability matrix having in each row the same probability vector α $(a_1, a_2, \ldots a_n)$ whose components add up to 1. The matrix A and vector α are then the limiting matrix and vector for the Markov chain determined by P.

Furthermore, if π is defined as a row vector whose components give the probability of being in the various states at the present moment, then $\pi \cdot P$ gives the probabilities after one step, and $\pi \cdot P^n$ after n steps. For a large n, Kemeny and Snell[6] show that

$$\lim_{n \to \infty} \pi \cdot P^n = \alpha \qquad (2)$$

where α is the same as given in (1) and is related to P by

$$\alpha P = \alpha \qquad (3)$$

This vector can have two interpretations: 1) as measuring the amount of statistical equilibrium achieved by a regular chain after a large number of steps; and 2) as giving the limiting value for different initial probabilities during the history of the chain. This unique vector is referred to as the stationary probability vector of the chain.

In order to determine such a chain completely as it moves from state s_i to state s_j over n steps, it is only

6 Kemeny and Snell, Op. Cit., p. 71.

necessary to determine the transition probability matrix P over the n steps and the initial probability vector π. As there does not appear to be a way of calculating π directly at each moment in the chain, α has been proposed as a good average approximation by Snell[7] and by Atkinson, Bower and Crothers.[8] By substituting α for π, the probability of finding the chain in each of the states at *any* time during the history of the process can be calculated.

Strictly speaking, when dealing with Markov processes, P is determined by counting the number of transitions from s_i to s_j in the complete sequence of n steps, as seen in studies by Bernbach[9] and Pike.[10] Since the interest of the present study is in studying the effect of "pressures" presumed as determining the state of the process, counting the number of transitions from state to state would not tell us anything about the underlying pressures and what effect they

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7 J.L. Snell, "Stochastic Processes," in R.D. Luce, R.R. Bush and E. Galanter (eds.), *Handbook of Mathematical Psychology*, Wiley, New York, Vol. 3, Chap. 20, 1965, p. 411-485.

8 R.C. Atkinson, G.H. Bower and E.J. Crothers, *An Introduction to Mathematical Learning Theory*, Wiley, New York, 1965, xiii-429; p. 317-322.

9 H.A. Bernbach, "Derivation of Learning Process Statistics for a General Markov Model," *Psychometrika*, Vol. 31, No. 2, 1966, p. 225-234.

10 A.R. Pike, Stochastic Models of Choice Behaviour: Response Probabilities and Latencies of Finite Markov Chain Systems," *British Journal of Mathematical and Statistical Psychology*, Vol. 19 (Part I), 1966, p. 15-32.

have, if any, on the course of the game. In order to do this, the presumed psychological pressures must, in some way, be reflected by or connected with the stationary probability vector α .

Hence, let us define P in terms of joint probabilities of the "pressures" acting on both players to switch or not to switch from the natural outcome AA as a function of the state in which the players find themselves. Finding α for such a P gives the best estimate of this "pressure" on both players at any instant of time in the chain. And, assuming from Rapoport's analysis, that this pressure operating at time n-1 in state s_i directly affects being in state s_j at time n, then α also gives the expected probabilities of actually being in each of the four states AA, AB, BA, and BB during the course of the game.

For matrix Exploiter and Leader, let us define this pressure to switch from the natural outcome (AA) as the probability of choosing B following each of AA, AB, BA and BB outcomes. For matrix Hero and Martyr, let us define the presumed pressure not to switch in terms of the probability of choosing A following each of AA, AB, BA and BB. With respect to this method of defining variables, the present study shows similarity with studies by Rapoport et. al.[11]

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11 A. Rapoport, A. Chammah, J. Dwyer and J. Gyr, "Three-Person Non-Zero-Sum Non-Negotiable Games," Behavioral Science, Vol. 7, 1962, p. 38-58.

Rapoport and Dale,[12] Rapoport and Mowshowtiz,[13] and Rapoport.[14]

That is, for matrix Exploiter and Leader, let x_i, where $i = 1, 2$ corresponding to player 1 and player 2, respectively, be defined as the probability of choosing B following AA and be expressed as $x_i = P(B_i|AA)$. Similarly, for choosing B, following AB, BA and BB, let

$$\begin{aligned} y_i &= P(B_i|AB) \\ z_i &= P(B_i|BA) \qquad (4) \\ \text{and } w_i &= P(B_i|BB) \end{aligned}$$

In a similar manner, for Hero and Martyr, let us define x_i, y_i, z_i and w_i as

$$\begin{aligned} x_i &= P(A_i|AA) \\ y_i &= P(A_i|AB) \\ z_i &= P(A_i|BA) \qquad (5) \\ \text{and } w_i &= P(A_i|BB) \end{aligned}$$

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12 A. Rapoport and P. Dale, "Models for Prisoner's Dilemma," Journal of Mathematical Psychology, Vol. 3, 1966, p. 269-286.

13 Amnon Rapoport and A. Mowshowitz, "Experimental Studies of Stochastic Models for the Prisoner's Dilemma," Behavioral Science, Vol. 11, No. 6, 1966, p. 444-458.

14 Amnon Rapoport, "Optimal Policies for the Prisoner's Dilemma," Psychological Review, Vol. 74, No. 2, 1967, p. 136-148.

From a consideration of Exploiter and Leader in figure 2, we observe that at each step in the sequence of n steps over which the games are played repeatedly B_1 is either the outcome BA or BB, and that B_2 is AB or BB. For Hero and Martyr, this gives A_1 as AA or AB, and A_2 as AA or BA. Substituting this in equations (4) and (5) above, redefines x_1, y_1, z_1 and w_1 in terms of collected data following each of the four possible states. In this way, each player is characterized by a set of response probabilities defined in terms of the "pressure" operating.

These pressure probabilities are next combined to give the sixteen transition probabilities, the parameter values of our model. Figure 3 gives the transition matrix P for Exploiter and Leader; figure 4 gives P for Hero and Martyr. To get P, as shown in both these figures, the maximum likelihood estimates of these parameters are computed for each pair of players. They are called maximum likelihood following the terminology coined by Anderson and Goodman.[15]

For example, to get $\hat{x}_1$, the maximum likelihood estimate of x_1, we count all the instances in which following AA each player chose according to the operationally observable motivational "pressure" as defined above. Dividing this

15 T.W. Anderson and L.A. Goodman, "Statistical Inference About Markov Chains," Annals of Mathematical Statistics, Vol. 28, 1957, p. 89-110.

STATE	AA	AB	BA	BB
AA	$(1-x_1)(1-x_2)$	$(1-x_1)x_2$	$x_1(1-x_2)$	x_1x_2
AB	$(1-y_1)(1-z_2)$	$(1-y_1)z_2$	$y_1(1-z_2)$	y_1z_2
BA	$(1-z_1)(1-y_2)$	$(1-z_1)y_2$	$z_1(1-y_2)$	z_1y_2
BB	$(1-w_1)(1-w_2)$	$(1-w_1)w_2$	$w_1(1-w_2)$	w_1w_2

Figure 3.- "Pressure" Transition Matrix P for Games Exploiter and Leader, where each cell entry is the joint probability of transition from the state indicated in the column at the left to the state indicated in the top row, and where the x's, y's, z's and w's have the same meaning as defined by equation (4) on page 29.

STATE		STATE: AA	AB	BA	BB
	AA	$x_1 x_2$	$x_1(1-x_2)$	$(1-x_1)x_2$	$(1-x_1)(1-x_2)$
	AB	$y_1 z_2$	$y_1(1-z_2)$	$(1-y_1)z_2$	$(1-y_1)(1-z_2)$
	BA	$z_1 y_2$	$z_1(1-y_2)$	$(1-z_1)y_2$	$(1-z_1)(1-y_2)$
	BB	$w_1 w_2$	$w_1(1-w_2)$	$(1-w_1)w_2$	$(1-w_1)(1-w_2)$

Figure 4.- "Pressure" Transition Matrix P for Games Hero and Martyr, where each cell entry is the joint probability of transition from the state indicated in the column at the left to the state indicated in the top row, and where the x's, y's, z's and w's have the same meaning as defined by equation (5) on page 29.

number by the total number of occurrences of AA gives x_1. Similarly for the other parameters. This is done for each subject of a pair. Average values are then calculated over all pairs playing the game, and the P matrices are computed according to the entries of figures 3 and 4.

Calculating the unique probability vector α gives the proportion of time that the game-playing process can be expected to be in each of the four states (outcomes). Comparison of these predicted values with the actual number of times that the grame-process is in each of the four states will show how well the presumed pressures channel the game behaviour.

In order to obtain a measure of the total deviation from equilibrium during the history of the chain, a matrix Z is calculated. Following Kemeny and Snell[16] if P is the transition matrix for a regular Markov chain, and A the limiting matrix as given by equation (1) above, then

$Z = [I - (P-A)]^{-1}$, or in its expanded geometric form,

$$Z = I + \sum_{n=1}^{\infty} (P^n - A) \qquad (6)$$

where I is the identity matrix of same order as P. Z is then the <u>fundamental matrix</u> for the Markov chain determined by P.

Finally, from Z a matrix C is calculated which, for the purposes of the present study, completes the description of

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16 Kemeny and Snell, Op. Cit., p. 75.

the behaviour of the regular Markov chain assumed to express the dynamics of the motivational pressures. Matrix C is a covariance-variance matrix for the number of times the process is in each of the four states motivated by the "pressures." According to Kemeny and Snell,[17] the off-diagonal entries of C are calculated by

$$c_{ij} = a_i z_{ij} + a_j z_{ji} - a_i d_{ij} - a_i a_j \qquad (7)$$

where a_i, a_j are components of the limiting vector of equation (3); z_{ij} the ijth entry of matrix Z; and where d_{ij} are the off-diagonal zero entries of a diagonal matrix D, whose diagonal elements $d_{ii} = 1/a_i$. The off-diagonal elements c_{ij} represent the limiting covariance for the number of times in states s_i and s_j in n steps, and the values c_{ii} give the limiting variances for the number of times in state s_i. The c_{ii} entries are calculated by

$$c_{ii} = a_j (2z_{jj} - 1 - a_j) \qquad (8)$$

where a_j is as defined above, and where z_{jj} are the diagonal entries of matrix Z.

From C, the limiting correlation for the number of times the process, as determined by the "pressures," is in s_i and s_j can then be calculated by

$$r_{ij} = \frac{c_{ij}}{\sqrt{c_{ii} \cdot c_{jj}}} \qquad (9)$$

17 *Ibid.*, p. 85.

The r_{ij}'s will show the extent and directionality of the "pressure" for channelling the process into the particular states of the chain. This completes the descriptive mathematical quantities for the model.

3. Experimental Method.

Subjects.- The sample consisted of forty male graduate students from the Faculties of Psychology and Education, University of Ottawa. The subjects were divided into dyads and randomly assigned to play one of the four games. Ten pairs were run per game matrix.

Apparatus.- The experimental apparatus used in this study was the Game Theory Apparatus designed by Schwartz,[18] which is described fully in Appendix 2. By masking one complete row and column of squares on each subject panel, a 2x2 matrix was produced. The left button on each panel was called the A choice, and the right button the B choice.

Procedure.- Pairs of subjects were seated in front of individual panels with the appropriate matrix displayed on them. The game matrices employed were those given in figure 2. The subjects were separated by a visual barrier and were not allowed to communicate with each other in any way. The

18 N.F. Schwartz, A Game Theory Apparatus for Psychological Research, ASD Technical Report 61-239, Air Force Systems Command, USAF, Wright-Patterson Air Force Base, Dayton, Ohio, 1961, 6 p.

instructions were taped and identical for each of the four groups; also, they were purposely made neutral with respect to implying cooperation or competition. Deutsch[19] notes the importance of this control, and his suggestion was followed in the present procedure. The task was explicitly stated as being an exercise in decision-making, where the goal was to accumulate points.

The games were played under conditions of perfect information where each subject knew, at the end of each trial, the amount of his pay-off and the other person's pay-off. In addition, each subject was given a continuous cumulative feedback of his own total points at the end of each trial, but no feedback as to the cumulative total of the other subject. The reason for not giving this information was <u>not</u> to have subjects "lock-in" on a competitive strategy and, hence, obscure the operation of the presumed pressures being studied. Furthermore, all subjects played under "free-play" conditions, i.e., under no predetermined strategy feedback from the experimenter.

Subjects were run for two hundred consecutive trials. The beginning of each trial was indicated to each subject by the on-set of a green light on his panel. The session started

19 M. Deutsch, "Trust and Suspicion," Journal of Conflict Resolution, Vol. 2, 1958, p. 265-279.

on the experimenter's signal and, after both subjects had made a choice, the outcome light lit up for the appropriate cell in the matrix. The indicated pay-offs remained illuminated for approximately five seconds before the beginning of the next trial. The experimenter recorded the consecutive occurrence of the four possible outcomes so as to obtain a continuous history of the process for analysis.

CHAPTER III

RESULTS AND DISCUSSION

The maximum likelihood estimates of the psychological "pressures" operating in each of the games are presented in Tables I to IV inclusively. The tables show values for each pair playing the game as well as average statistics from which the transition matrices have been calculated. The "pressure" transition matrices are shown in Table V. From these matrices, α vectors were calculated predicting the proportions of the actual outcomes that should occur in the two hundred repeated trials of the game.

Table VI shows the observed and predicted distribution of the four outcomes for each of the games. By inspection, the fit between obtained and predicted proportions is close for Exploiter, Leader and Martyr. For game Hero, the observed values do not approach the predicted ones, the discrepancy between them being, again by inspection, rather large when compared with the discrepancies for the other three games.

These results are most interesting. For three of the four games, the transition probabilities of the assumed motivational matrix P do generate the actual sequence of outcomes observed in the course of the game. The results clearly show that the x, y, z and w variables are the determining factors in the prediction of the observed proportion of outcomes. These

Table I.-

Calculated Joint Probabilities from Observed Data for Game Exploiter as Defined by the Transition Matrix of Figure 3 on page 31, for Each Pair of Players Separately and also the Average Values for the Whole Group (N=10). (The letters x, y, z and w are defined on page 29; the digits 1 and 2 refer to the two players.)

Pair No.	$(1-x_1)(1-x_2)$	$(1-x_1)x_2$	$x_1(1-x_2)$	x_1x_2	$(1-y_1)(1-z_2)$	$(1-y_1)z_2$	$y_1(1-z_2)$	y_1z_2
1	(.637)(.409)	(.637)(.591)	(.363)(.409)	(.363)(.591)	(.667)(.366)	(.667)(.634)	(.333)(.366)	(.333)(.634)
2	(.358)(.286)	(.358)(.714)	(.642)(.286)	(.642)(.714)	(.429)(.500)	(.429)(.500)	(.571)(.500)	(.571)(.500)
3	(.500)(.000)	(.500)(1.000)	(.500)(.000)	(.500)(1.000)	(.616)(.250)	(.616)(.750)	(.384)(.250)	(.384)(.750)
4	(.319)(.728)	(.319)(.272)	(.681)(.728)	(.681)(.272)	(.000)(.541)	(.000)(.459)	(1.000)(.541)	(1.000)(.459)
5	(.223)(.667)	(.223)(.333)	(.777)(.667)	(.777)(.333)	(.059)(.821)	(.059)(.179)	(.941)(.821)	(.941)(.179)
6	(.667)(.584)	(.667)(.416)	(.333)(.584)	(.333)(.416)	(.421)(.500)	(.421)(.500)	(.579)(.500)	(.579)(.500)
7	(1.000)(.000)	(1.000)(1.000)	(.000)(.000)	(.000)(1.000)	(.458)(.000)	(.458)(1.000)	(.542)(.000)	(.542)(1.000)
8	(.267)(.467)	(.267)(.533)	(.733)(.467)	(.733)(.533)	(.663)(.362)	(.663)(.638)	(.337)(.362)	(.337)(.638)
9	(.320)(.680)	(.320)(.320)	(.680)(.680)	(.680)(.320)	(.261)(.443)	(.261)(.557)	(.739)(.443)	(.739)(.557)
10	(.778)(.556)	(.778)(.444)	(.222)(.556)	(.222)(.444)	(.417)(.400)	(.417)(.600)	(.583)(.400)	(.583)(.600)
Sum	1.908279	3.160721	2.468721	2.462279	1.39399	2.59701	2.76901	3.21999
Mean	.191	.316	.247	.246	.139	.260	.279	.322

Pair No.	$(1-z_1)(1-y_2)$	$(1-z_1)y_2$	$z_1(1-y_2)$	z_1y_2	$(1-w_1)(1-w_2)$	$(1-w_1)w_2$	$w_1(1-w_2)$	w_1w_2
1	(.317)(.367)	(.317)(.633)	(.683)(.367)	(.683)(.633)	(.208)(.234)	(.208)(.766)	(.792)(.234)	(.792)(.766)
2	(.260)(.477)	(.260)(.523)	(.740)(.477)	(.740)(.523)	(.070)(.218)	(.070)(.782)	(.930)(.218)	(.930)(.782)
3	(.500)(.020)	(.500)(.980)	(.500)(.020)	(.500)(.980)	(.445)(.045)	(.445)(.955)	(.555)(.045)	(.555)(.955)
4	(.149)(.250)	(.149)(.750)	(.851)(.250)	(.851)(.750)	(.080)(.390)	(.080)(.610)	(.920)(.390)	(.920)(.610)
5	(.076)(.412)	(.076)(.588)	(.924)(.412)	(.924)(.588)	(.236)(.221)	(.236)(.779)	(.764)(.221)	(.764)(.779)
6	(.167)(.264)	(.167)(.736)	(.833)(.264)	(.833)(.736)	(.072)(.108)	(.072)(.892)	(.928)(.108)	(.928)(.892)
7	(.000)(.029)	(.000)(.971)	(1.000)(.029)	(1.000)(.971)	(.125)(.032)	(.125)(.968)	(.875)(.032)	(.875)(.968)
8	(.278)(.205)	(.278)(.795)	(.722)(.205)	(.722)(.795)	(.440)(.228)	(.440)(.772)	(.560)(.228)	(.560)(.772)
9	(.200)(.435)	(.200)(.565)	(.800)(.435)	(.800)(.565)	(.256)(.464)	(.256)(.536)	(.744)(.464)	(.744)(.536)
10	(.200)(.250)	(.200)(.750)	(.800)(.250)	(.800)(.750)	(.046)(.023)	(.046)(.977)	(.954)(.023)	(.954)(.977)
Sum	.556999	1.590001	2.152001	5.700999	.399251	1.578749	1.563749	6.458251
Mean	.056	.159	.215	.570	.040	.158	.156	.646

Table II.-

Calculated Joint Probabilities from Observed Data for Game Leader, as Defined by the Transition Matrix of Figure 3 on page 31, for Each Pair of Players Separately and also the Average Values for the Whole Group (N=10). (The letters x, y, z and w are defined on page 29; the digits 1 and 2 refer to the two players.)

Pair No.	$(1-x_1)(1-x_2)$	$(1-x_1)x_2$	$x_1(1-x_2)$	x_1x_2	$(1-y_1)(1-z_2)$	$(1-y_1)z_2$	$y_1(1-z_2)$	y_1z_2
11	(.950)(.000)	(.950)(1.000)	(.050)(.000)	(.050)(1.000)	(.311)(.322)	(.311)(.678)	(.689)(.322)	(.689)(.678)
12	(.715)(.143)	(.715)(.857)	(.285)(.143)	(.285)(.857)	(.000)(.623)	(.000)(.377)	(1.000)(.623)	(1.000)(.377)
13	(.500)(.000)	(.500)(1.000)	(.500)(.000)	(.500)(1.000)	(.417)(.334)	(.417)(.666)	(.583)(.334)	(.583)(.666)
14	(.125)(.125)	(.125)(.875)	(.875)(.125)	(.875)(.875)	(.355)(.223)	(.355)(.777)	(.645)(.223)	(.645)(.777)
15	(.348)(.392)	(.348)(.608)	(.652)(.392)	(.652)(.608)	(.102)(.112)	(.102)(.888)	(.898)(.112)	(.898)(.888)
16	(.572)(.215)	(.572)(.785)	(.428)(.215)	(.428)(.785)	(.816)(.091)	(.816)(.909)	(.184)(.091)	(.184)(.909)
17	(1.000)(.000)	(1.000)(1.000)	(.000)(.000)	(.000)(1.000)	(.167)(.000)	(.167)(1.000)	(.833)(.000)	(.833)(1.000)
18	(.539)(.077)	(.539)(.923)	(.461)(.077)	(.461)(.923)	(.237)(.072)	(.237)(.928)	(.763)(.072)	(.763)(.928)
19	(.500)(.228)	(.500)(.772)	(.500)(.228)	(.500)(.772)	(.491)(.345)	(.491)(.655)	(.509)(.345)	(.509)(.655)
20	(.425)(.485)	(.425)(.515)	(.575)(.485)	(.575)(.515)	(.565)(.250)	(.565)(.750)	(.435)(.250)	(.435)(.750)
Sum	[illegible]	4.935106	.926106	3.399894	.731974	2.729026	1.640026	4.898974
Mean	.074	.493	.093	.340	.073	.273	.164	.490

Pair No.	$(1-z_1)(1-y_2)$	$(1-z_1)y_2$	$z_1(1-y_2)$	z_1y_2	$(1-w_1)(1-w_2)$	$(1-w_1)w_2$	$w_1(1-w_2)w$	w_1w_2
11	(.733)(.271)	(.733)(.729)	(.267)(.271)	(.267)(.729)	(.240)(.780)	(.240)(.220)	(.760)(.780)	(.760)(.220)
12	(.044)(.200)	(.044)(.800)	(.956)(.200)	(.956)(.800)	(.041)(.663)	(.041)(.337)	(.959)(.663)	(.959)(.337)
13	(.334)(.014)	(.334)(.986)	(.666)(.014)	(.666)(.986)	(.342)(.033)	(.342)(.967)	(.658)(.033)	(.658)(.967)
14	(.556)(.226)	(.556)(.774)	(.444)(.226)	(.444)(.774)	(.433)(.135)	(.433)(.865)	(.567)(.135)	(.567)(.865)
15	(.973)(.849)	(.973)(.151)	(.027)(.849)	(.027)(.151)	(.616)(.462)	(.616)(.538)	(.384)(.462)	(.384)(.538)
16	(.546)(.076)	(.546)(.924)	(.454)(.076)	(.454)(.924)	(.393)(.233)	(.393)(.767)	(.607)(.233)	(.607)(.767)
17	(.000)(.167)	(.000)(.833)	(1.000)(.167)	(1.000)(.833)	(.028)(.044)	(.028)(.956)	(.972)(.044)	(.972)(.956)
18	(.215)(.606)	(.215)(.394)	(.785)(.606)	(.785)(.394)	(.243)(.262)	(.243)(.738)	(.757)(.262)	(.757)(.738)
19	(.345)(.302)	(.345)(.698)	(.655)(.302)	(.655)(.698)	(.303)(.219)	(.303)(.781)	(.697)(.219)	(.697)(.781)
20	(.465)(.291)	(.465)(.709)	(.535)(.291)	(.535)(.709)	(.442)(.260)	(.442)(.740)	(.558)(.260)	(.558)(.740)
Sum	1.575143	2.635857	1.42665	4.362143	.906460	2.17454	[illegible]	4.73446
Mean	.157	.264	.143	.436	.091	.217	.216	[illegible]

Table III.-

Calculated Joint Probabilities from Observed Data for Game Hero, as Defined by the Transition Matrix of Figure 4 on page 32, for Each Pair of Players Separately and also the Average Values for the Whole Group (N=10). (The letters x, y, z and w are defined on page 29; the digits 1 and 2 refer to the two players.)

Pair No.	x_1x_2	$x_1(1-x_2)$	$(1-x_1)x_2$	$(1-x_1)(1-x_2)$	y_1z_2	$y_1(1-z_2)$	$(1-y_1)z_2$	$(1-y_1)(1-z_2)$
21	(.712)(.946)	(.712)(.054)	(.288)(.946)	(.288)(.054)	(.333)(.990)	(.333)(.010)	(.667)(.990)	(.667)(.010)
22	(.789)(.736)	(.789)(.264)	(.211)(.736)	(.211)(.264)	(.076)(.300)	(.076)(.700)	(.924)(.300)	(.924)(.700)
23	(.904)(.931)	(.904)(.069)	(.096)(.931)	(.096)(.069)	(.884)(.705)	(.884)(.295)	(.116)(.705)	(.116)(.295)
24	(.500)(.250)	(.500)(.750)	(.500)(.250)	(.500)(.750)	(.010)(.020)	(.010)(.980)	(.990)(.020)	(.990)(.980)
25	(.660)(.547)	(660)(.453)	(.340)(.547)	(.340)(.453)	(.636)(.440)	(.636)(.560)	(.364)(.440)	(.364)(.560)
26	(.920)(.440)	(.920)(.560)	(.080)(.440)	(.080)(.560)	(.923)(1.000)	(.923)(.000)	(.077)(1.000)	(.077)(.000)
27	(.264)(1.000)	(.264)(.000)	(.736)(1.000)	(.736)(.000)	(.000)(.994)	(.000)(.006)	(1.000)(.994)	(1.000)(.006)
28	(.810)(.684)	(.810)(.316)	(.190)(.684)	(.190)(.316)	(.000)(.645)	(.000)(.355)	(1.000)(.645)	(1.000)(.355)
29	(.904)(.714)	(.904)(.286)	(.096)(.714)	(.096)(.286)	(.011)(.056)	(.011)(.944)	(.989)(.056)	(.989)(.944)
30	(.855)(.447)	(.855)(.553)	(.145)(.447)	(.145)(.553)	(.923)(.571)	(.923)(.429)	(.077)(.571)	(.077)(.429)
Sum	4.832381	2.485619	1.862619	.819381	2.706379	1.089621	3.014621	3.189379
Mean	.483	.249	.186	.082	.271	.109	.301	.319

Pair No.	z_1y_2	$z_1(1-y_2)$	$(1-z_1)y_2$	$(1-z_1)(1-y_2)$	w_1w_2	$w_1(1-w_2)$	$(1-w_1)w_2$	$(1-w_1)(1-w_2)$
21	(.277)(1.000)	(.277)(.000)	(.723)(1.000)	(.723)(.000)	(.000)(1.000)	(.000)(.000)	(1.000)(1.000)	(1.000)(.000)
22	(.971)(.038)	(.971)(.062)	(.029)(.938)	(.029)(.062)	(.500)(.250)	(.500)(.750)	(.500)(.250)	(.500)(.750)
23	(.647)(.384)	(.647)(.616)	(.353)(.384)	(.353)(.616)	(.500)(.400)	(.500)(.600)	(.500)(.400)	(.500)(.600)
24	(.968)(.978)	(.968)(.022)	(.032)(.978)	(.032)(.022)	(.142)(.857)	(.142)(.143)	(.858)(.857)	(.858)(.143)
25	(.360)(.527)	(.360)(.473)	(.640)(.527)	(.640)(.473)	(.476)(.547)	(.476)(.453)	(.524)(.547)	(.524)(.453)
26	(.833)(.051)	(.833)(.949)	(.167)(.051)	(.167)(.949)	(.692)(.384)	(.692)(.616)	(.308)(.384)	(.308)(.616)
27	(.144)(.000)	(.144)(1.000)	(.856)(.000)	(.856)(1.000)	(.000)(.000)	(.000)(1.000)	(1.000)(.000)	(1.000)(1.000)
28	(1.000)(.907)	(1.000)(.093)	(.000)(.907)	(.000)(.093)	(.857)(.571)	(.857)(.429)	(.143)(.571)	(.143)(.429)
29	(.988)(.988)	(.988)(.012)	(.012)(.988)	(.012)(.012)	(.500)(1.000)	(.500)(.000)	(.500)(1.000)	(.500)(.000)
30	(.857)(.432)	(.857)(.568)	(.143)(.432)	(.143)(.568)	(1.000)(.500)	(1.000)(.500)	(.000)(.500)	(.000)(.500)
Sum	4.868521	2.176479	1.336479	1.618521	2.462141	2.204859	3.046859	2.286141
Mean	.487	.218	.134	.162	.246	.220	.305	.229

Table IV.-

Calculated Joint Probabilities from Observed Data for Game Martyr, as Defined by the Transition Matrix of Figure 4 on page 32, for each Pair of Players Separately and also the Average Values for the Whole Group (N=10). (The letters x, y, z and w are defined on page 29; the digits 1 and 2 refer to the two subjects.)

Pair No.	x_1x_2	$x_1(1-x_2)$	$(1-x_1)x_2$	$(1-x_1)(1-x_2)$	y_1z_2	$y_1(1-z_2)$	$(1-y_1)z_2$	$(1-y_1)(1-z_2)$
31	(.[illegible])(.339)	(.[illegible])(.661)	(.102)(.339)	(.102)(.661)	(.754)(.654)	(.754)(.346)	(.246)(.654)	(.246)(.346)
32	(.753)(.823)	(.753)(.1[illegible])	(.[illegible]7)(.[illegible])	(.247)(.177)	(.750)(.[illegible])	(.7[illegible])(.[illegible])	(.250)(.800)	(.250)(.200)
33	(.[illegible])(.[illegible]44)	(.856)(.056)	(.1[illegible]4)(.[illegible]4[illegible])	(.144)(.056)	(.615)(.500)	(.615)(.500)	(.385)(.500)	(.385)(.500)
34	(.6[illegible])(.[illegible]20)	(.6[illegible])(.180)	(.372)(.820)	(.372)(.180)	(.508)(.470)	(.508)(.530)	(.492)(.470)	(.492)(.530)
35	(.[illegible])(.687)	(.[illegible])(.313)	(.[illegible]1)(.68[illegible])	(.021)(.313)	(1.000)(.000)	(1.[illegible])(1.000)	(.000)(.000)	(.000)(1.000)
36	(.814)(.732)	(.[illegible]14)(.268)	(.1[illegible])(.73[illegible])	(.186)(.268)	(.541)(.711)	(.541)(.289)	(.459)(.711)	(.459)(.289)
37	(.[illegible])(.662)	(.[illegible])(.338)	(.[illegible])(.662)	(.2[illegible])(.338)	(.[illegible]36)(.655)	(.736)(.345)	(.264)(.655)	(.264)(.345)
38	(.[illegible]43)(.[illegible])	(.[illegible]43)(.0[illegible])	(.[illegible])(.[illegible])	(.057)(.[illegible]12)	(.250)(.[illegible]44)	(.250)(.056)	(.750)(.944)	(.750)(.056)
39	(.017)(.663)	(.017)(.33[illegible])	(.1[illegible]3)(.66[illegible])	(.1[illegible])(.337)	(.[illegible]43)(.812)	(.[illegible]43)(.188)	(.057)(.812)	(.057)(.188)
40	(.[illegible])(.[illegible])	(.81[illegible])(.2[illegible])	(.1[illegible])(.[illegible])	(.190)(.2[illegible])	(.641)(.718)	(.641)(.[illegible]82)	(.359)(.718)	(.359)(.282)
Sum	6.090761	2.117[illegible]	1.[illegible]23	.444761	3.968061	2.[illegible]939	2.295939	.966061
Mean	.609	.212	.13[illegible]	.044	.39[illegible]	.277	.229	.097

Pair No.	z_1y_2	$z_1(1-y_2)$	$(1-z_1)y_2$	$(1-z_1)(1-y_2)$	w_1w_2	$w_1(1-w_2)$	$(1-w_1)w_2$	$(1-w_1)(1-w_2)$
31	(.580)(.698)	(.580)(.302)	(.420)(.698)	(.420)(.302)	(.000)(.210)	(.000)(.790)	(1.000)(.210)	(1.000)(.790)
32	(.333)(.464)	(.333)(.536)	(.667)(.464)	(.667)(.536)	(.30[illegible])(.262)	(.309)(.738)	(.691)(.262)	(.691)(.738)
33	(.687)(.769)	(.687)(.231)	(.313)(.769)	(.313)(.231)	(.353)(.176)	(.353)(.824)	(.647)(.176)	(.647)(.824)
34	(.970)(.322)	(.970)(.678)	(.030)(.322)	(.030)(.678)	(.862)(.448)	(.862)(.552)	(.138)(.448)	(.138)(.552)
35	(1.000)(.291)	(1.000)(.709)	(.000)(.291)	(.000)(.709)	(.000)(.000)	(.000)(1.000)	(1.000)(.000)	(1.000)(1.000)
36	(.440)(.916)	(.440)(.084)	(.560)(.916)	(.560)(.084)	(.000)(.580)	(.000)(.420)	(1.000)(.580)	(1.000)(.420)
37	(.655)(.473)	(.655)(.527)	(.345)(.473)	(.345)(.527)	(.645)(.354)	(.645)(.646)	(.355)(.354)	(.355)(.646)
38	(.666)(.500)	(.666)(.500)	(.334)(.500)	(.334)(.500)	(.000)(1.000)	(.000)(.000)	(1.000)(1.000)	(1.000)(.000)
39	(.937)(.464)	(.937)(.536)	(.063)(.464)	(.063)(.536)	(.777)(.555)	(.777)(.445)	(.223)(.555)	(.223)(.445)
40	(.512)(.641)	(.512)(.359)	(.408)(.641)	(.408)(.359)	(.545)(.363)	(.545)(.637)	(.455)(.363)	(.455)(.637)
Sum	3.49981	3.28019	2.03819	1.18181	1.386662	2.104338	2.561338	3.947662
Mean	.350	.328	.204	.118	.139	.210	.256	.395

Table V.-

Transition Probability Matrices P for Games Exploiter and Leader as Given by the Average Values of Tables I and II, and for Games Hero and Martyr as Given by the Average Values of Tables III and IV (N=200).
(A cell entry gives the joint probability of going from the state expressed by the row to the state expressed by the column.)

Exploiter

State	AA	AB	BA	BB
AA	.191	.316	.247	.246
AB	.139	.260	.279	.322
BA	.056	.159	.215	.570
BB	.040	.158	.156	.646

Leader

State	AA	AB	BA	BB
AA	.074	.493	.093	.340
AB	.073	.273	.164	.490
BA	.157	.264	.143	.436
BB	.091	.217	.218	.473

Hero

State	AA	AB	BA	BB
AA	.483	.249	.186	.082
AB	.271	.109	.301	.319
BA	.487	.218	.134	.162
BB	.264	.220	.305	.229

Martyr

State	AA	AB	BA	BB
AA	.609	.212	.135	.044
AB	.397	.277	.229	.097
BA	.350	.328	.204	.118
BB	.139	.210	.256	.395

Table VI.-

Comparison of Observed and Predicted Distribution of the Four Outcomes for Each of the Games Over 200 Successive Trials. (The data are given in proportions.*)

Game		Outcome AA	AB	BA	BB
Exploiter					
	Observed	.066	.189	.209	.535
	Predicted (α)	.073	.189	.197	.541
Leader					
	Observed	.077	.285	.189	.449
	Predicted (α)	.096	.267	.178	.459
Hero					
	Observed	.303	.316	.328	.052
	Predicted (α)	.398	.208	.219	.174
Martyr					
	Observed	.497	.232	.152	.119
	Predicted (α)	.457	.249	.184	.109

* The observed proportions were calculated by counting the actual number that each outcome occurred in 200 successive trials; the predicted proportions were calculated by computing the stationary probability vector α for each transition matrix.

results strongly support the conclusion that Rapoport's logically deduced motives determine the overt behaviour of the players.

In game Hero, it was not possible to predict the game behaviour from the motivational matrix defining the psychological "pressures." The relatively large discrepancy between the predicted and observed distributions leads to the hypothesis of a more complicated motivational process underlying the game behaviour than Rapoport's suggested motivational polarity "to switch or not to switch" from the AA outcome. From Table VI, it is seen that the predicted proportions are higher than the observed proportions for the AA and BB outcomes, and are lower than the observed proportions for the AB and BA outcomes. A possible reason for this discrepancy could have been due to interaction effects and lack of independence among the x, y, z and w variables, leading to overlap in determining particular outcomes.

In order to find this out, [illegible] matrices measuring the total deviation from equilibrium during the history of the chain were calculated from which variance-covariance matrices could be derived for the original ρ matrices. These matrices are presented in Tables VII and VIII, respectively. From the variance-covariance matrices of Table VIII, limiting correlations were calculated for the number of times the x, y, z and w "pressure" variables determined the transition of the

Table VII.-

Fundamental Matrices Z for the Four Games Calculated from Their Respective "Pressure" Transition Probabilities of Table V by Means of Equation 6 on Page 33.

Exploiter

State	AA	AB	BA	BB
AA	1.158	.172	.094	-.427
AB	.068	1.097	.111	-.300
BA	-.029	-.037	1.013	.045
BB	.017	-.041	-.052	1.135

Leader

State	AA	AB	BA	BB
AA	.969	.225	-.007	-.111
AB	-.023	.996	-.011	.032
BA	.058	.012	.096	-.032
BB	-.001	-.004	.040	1.007

Hero

State	AA	AB	BA	BB
AA	1.101	.040	-.037	-.099
AB	-.142	.907	.086	.154
BA	.093	.012	.921	-.017
BB	-.169	.006	.093	1.076

Martyr

State	AA	AB	BA	BB
AA	1.233	-.047	-.070	-.111
AB	-.076	1.037	.052	-.009
BA	-.145	.090	1.035	.025
BB	-.557	-.024	.135	1.452

Table VIII.-

Variance-Covariance Matrices C* for the Limiting Number of Times in Each State for the Four Games as Determined by Their Respective "Pressure" Transition Probabilities of Table V (N=200).

Exploiter

State	AA	AB	BA	BB
AA	.091	.015	-.013	-.016
AB	.015	.190	-.023	-.181
BA	-.013	-.023	.163	-.126
BB	-.016	-.181	-.126	.394

Leader

State	AA	AB	BA	BB
AA	.081	-.010	-.015	-.055
AB	-.010	.195	-.048	-.116
BA	-.015	-.048	.132	-.069
BB	-.055	-.116	-.069	.255

Hero

State	AA	AB	BA	BB
AA	.320	-.096	-.082	-.136
AB	-.096	.126	-.025	-.033
BA	-.082	-.025	.136	-.026
BB	-.136	-.003	-.026	.170

Martyr

State	AA	AB	BA	BB
AA	.461	-.133	-.143	-.161
AB	-.133	.205	-.016	-.032
BA	-.143	-.016	.163	-.001
BB	-.161	-.032	-.001	.196

* The c_{ij} entries give the limiting covariance for the number of times in states s_i and s_j in 200 trials; the c_{ii} entries give the limiting variance for the number of times in state s_i. These symbols have been previously defined by equations (7) and (8) on page 34.

game-playing process from one state (outcome) to another. These correlations are presented in Table IX. From this table it is readily seen that the "pressure" variables are relatively independent, showing no overlap (except for one small positive correlation between AA and AB in game Exploiter), but rather substantial "pressuring" effects in the expected directions. On the basis of this information, it would seem that the failure of the model to predict behaviour in game Hero is not due to lack of independence in the "pressure" parameters of the model.

One possible explanation of the results obtained for Hero could perhaps be that one or all of the x, y, z and w variables are subject to learning. Since the model did not take learning into consideration, it therefore grossly underestimated the AB and BA outcomes, and overestimated AB and BB. It would, nevertheless, be difficult to answer why a learning variable needs to be introduced into the model as an added parameter in order to explain the behaviour for this particular game situation, and not for the other three. In any case, it would involve a parametric study which is beyond the scope of the present design. It is suggested here as a worthwhile direction in which to expand the present study.

Table IX.-

Matrices of Limiting Correlations for the Number of Times in States s_i and s_j for the Four Games Calculated from Their Corresponding C Matrices of Table VIII by the Formula Given on Page 34.

Exploiter

State	AA	AB	BA	BB
AA		.114	-.107	-.322
AB			-.130	-.662
BA				-.497
BB				

Leader

State	AA	AB	BA	BB
AA		-.079	-.145	-.363
AB			-.299	-.520
BA				-.376
BB				

Hero

State	AA	AB	BA	BB
AA		-.470	-.393	-.591
AB			-.191	-.020
BA				-.171
BB				

Martyr

State	AA	AB	BA	BB
AA		-.433	-.522	-.536
AB			-.088	-.160
BA				-.006
BB				

Along somewhat similar lines, Rapoport and Dale[1] suggested for the PD game, a four-state Markov chain model with one of their "state-conditioned" propensities subject to learning. Unfortunately, their model did not capture the dynamics of the process as well as the stochastic learning model of Bush and Mosteller[2] and Estes.[3] Rapoport and Dale discovered for their PD data that a Markov chain with absorbing states and/or the Bush-Mosteller stochastic learning model gave the best fits between predicted and observed time courses of the outcomes. As one possible extension of the present study, it would be interesting to attempt working out a model for Hero which would combine the stochastic characteristics of the regular Markov chain with the principles of the Bush-Mosteller learning model.

As to the contribution of this study, it accomplished the aims presented in chapter two under the formulation of the problem. It reduced to quantification Rapoport's presumed motivational dynamics underlying the overt game-behaviour in terms of observable data and, secondly, it showed that the effects of this pre-decisional motivational process can be

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1 A. Rapoport and P. Dale, "Models for Prisoner's Dilemma," Journal of Mathematical Psychology, Vol. 3, No. 2, 1966, p. 269-286.

2 R.R. Bush and F. Mosteller, Stochastic Models for Learning, Wiley, New York, 1955.

3 W.K. Estes, "Toward a Statistical Theory of Learning," Psychological Review, Vol. 57, 1950, p. 94-107.

adequately modelled, as a first approximation, by a finite-state Markov chain model, a purely mathematical deductive system.

SUMMARY AND CONCLUSIONS

In this study, an attempt was made to apply the regular Markov chain model to the underlying interplay of motives operating in four of Rapoport's seventy-eight non-equivalent games. Predictions were made as to the overt playing-behaviour of the subjects.

The presumed motivational "pressures" were quantified in terms of actual choices made and outcomes observed as state-conditioned variables x, y, z and w, forming a motivational matrix P with transitional probabilities p_{ij} (from state i to state j) limited by $0 \leq p_{ij} \leq 1$, and where for each row of P the sum of the transitional probabilities equalled 1. Assuming that P was a matrix of an ergodic Markov process which underlied the overt game-playing behaviour, predictions were made as to the actual distribution of outcomes in each game by calculating the limiting probability vector α for the chain.

The actual game-playing behaviour approximated the predicted behaviour of the players for games Exploiter, Leader and Martyr, but the players' game-behaviour for Hero could not be predicted from the Markov chain model. It was suggested that the motives operating in Hero were much more complicated than Rapoport's view of a polar "pressure" to switch or not to switch from the natural outcome of the game, and that the fit between the predicted and observed values could perhaps be more closely approximated by a model which would combine the stochastic characteristics of the regular Markov chain with the principles of the Bush-Mosteller learning model. This was suggested as a possible direction in which to expand the present study.

BIBLIOGRAPHY

Kemeny, J.G. and J.L. Snell, Finite Markov Chains, van Nostrand, Princeton, N.J., 1960, viii-210 p.

This book provides a fairly complete treatment, in english, of the basic ideas of finite Markov chains. It was important for the present study in providing the mathematics for the solution of the problem. This book is basic for the understanding of this branch of mathematics.

Rapoport, A. and M. Guyer, "A Taxonomy of 2x2 Games," General Systems, Vol. 11, 1966, p. 203-214.

A very important theoretical paper presenting the development of a complete typology of 2x2 non-zero-sum games derived from game-theoretical assumptions. On the basis of the discussion presented here, the four games used in the study were chosen for being "psychologically interesting."

Rapoport, A., "Exploiter, Leader, Hero and Martyr: The Four Archetypes of the 2x2 Games," Behavioral Science, Vol. 12, No. 2, 1967. p. 81-84.

In this paper, Rapoport gives a thorough motivational analysis of the four "psychologically interesting" games discussed in reference two above. His presumed motivational pressures operating in each game were translated into parameters of the Markov chain model from which predictions could be made.

Snell, J.L., "Stochastic Processes," in R.D. Luce, R.R. Bush and E. Galanter (eds.), Handbook of Mathematical Psychology, Vol. 3, Chapter 20, Wiley, New York, p. 411-485

This chapter presents a good discussion of stochastic processes and their application in psychology, particularly in statistical learning theory. It was important for the present study in providing a good introduction to this whole topic.

APPENDIX 1

RAW DATA

APPENDIX 1

RAW DATA

To facilitate computer analysis of the data, the four outcomes AA, AB, BA and BB were punched onto IBM cards as integer numbers 1, 2, 3 and 4 respectively. Forty pairs of subjects were run for a total of two hundred consecutive trials. In the pages presented here, data set 1 to 10 corresponds to the ten pairs playing game Exploiter, set 11 to 20 to the pairs playing game Leader, set 21 to 30 to the pairs playing game Hero and set 31 to 40 to the ten pairs playing game Martyr. The consecutive outcomes observed during the course of the games are given for each pair under the heading "Input Data was." Immediately below this are the total numbers of each outcome occurring in the two hundred trials run and the number of times that each outcome occurred following each of the four possible ones. From these frequencies the x, y, z and w values were computed and the transition matrices calculated.

```
IEF285I   SYS1.[illegible]LIB                                          KEPT
IEF285I   VOL SER NOS= UUDRUM.
IEF285I   SYS69[illegible].T115908.V000.FAC2961.GOSET                  PASSED
IEF285I   VOL SER NOS= LIBVOL.
IEF285I   SYS69049.T115908.SV000.FAC2961.R0000010                      SYSOUT
IEF285I   VOL SER NOS= LIBVOL.
IEF285I   SYS69049.T115908.RV000.FAC2961.R0000011                      DELETED
IEF285I   VOL SER NOS= LIBVOL.
IEF285I   SYS69049.T115908.RV000.FAC2961.LOADSET                       DELETED
IEF285I   VOL SER NOS= LIBVOL.
$STEP 69.049 12:02:14.72 FAC2961 LKED       POREBSKI,S.C
XXGO EXEC PGM=*.LKED.SYSLMOD,COND=((4,LT,FORT),(4,LT,LKED))                  002
XXFT01F001  DD  DDNAME=SYSIN                                                 003
XXFT06F001 DD  SYSOUT=A                                                      003
XXFT03F001  DD   SYSOUT=A,DCB=(RECFM=FBA,BLKSIZE=1330,LRECL=133)             003
//GO.SYSIN DD *                                                       DATA NEXT
IEF236I ALLOC. FOR FAC2961  GO
IEF237I PGM=*.DD ON 131
IEF237I FT01F001 ON 131
IEF237I FT06F001 ON 130
IEF237I FT03F001 ON 130


THE RESULTS FOR DATA SET  1


INPUT DATA WAS

 442234313331232112213332123344311243322133211433 21
 2213443434311234121434344344222213223444432243434 4
 4422224444222222444444442442232222234344444221212 2
 144234+22444442212333443342242444442444443244424 44


OCCURRENCE OF  1 IS   22

OCCURRENCE OF  1 FOLLOWED     1 IS     4
OCCURRENCE OF  1 FOLLOWED     2 IS    10
OCCURRENCE OF  1 FOLLOWED     3 IS     5
OCCURRENCE OF  1 FOLLOWED     4 IS     3


OCCURRENCE OF  2 IS   60
```

Handwritten annotations: "BY" above the "FOLLOWED" column; $x_1 = .363$ (corrected from .364); $x_2 = .591$

OCCURRENCE OF [illegible] FOLLOWED [illegible] IS 2?
OCCURRENCE OF [illegible] FOLLOWED 3 IS 9
OCCURRENCE OF [illegible] FOLLOWED 4 IS 11

$y_1 = .333$ $y_2 = .6$

$y = .633$

OCCURRENCE OF 3 IS 41

OCCURRENCE OF 3 FOLLOWED BY 1 IS 4
OCCURRENCE OF 3 FOLLOWED 2 IS 9
OCCURRENCE OF 3 FOLLOWED 3 IS 11
OCCURRENCE OF 3 FOLLOWED 4 IS 17

$z_1 = .683$ $z_2 = .63$

$z_2 = .630$

OCCURRENCE OF 4 IS 77

OCCURRENCE OF 4 FOLLOWED BY 1 IS 1
OCCURRENCE OF 4 FOLLOWED 2 IS 14
OCCURRENCE OF 4 FOLLOWED 3 IS 16
OCCURRENCE OF 4 FOLLOWED 4 IS 45

$w_1 = .742$ $w_2 = .76$

THE RESULTS FOR DATA SET 2

INPUT DATA WAS

14444213324332242123444344231443324412234444444414
44244144221444344434444444444444444444444444444444
32444444444444433444444444444443744332433433712443
33432133233333343334433433214343444444443143423331 1

OCCURRENCE OF 1 IS 14

OCCURRENCE OF 1 FOLLOWED 1 IS 1
OCCURRENCE OF 1 FOLLOWED 2 IS 3
OCCURRENCE OF 1 FOLLOWED 3 IS 2
OCCURRENCE OF 1 FOLLOWED 4 IS 7

$x_1 = ~~.643~~$ .642 $x_2 = .714$

OCCURRENCE OF 2 IS 21

OCCURRENCE OF 2 FOLLOWED 1 IS 6

```
OCCURRENCE OF  2 FOLLOWED   2 IS    3
OCCURRENCE OF  2 FOLLOWED   3 IS    4
OCCURRENCE OF  2 FOLLOWED   4 IS    8
```

$y_1 = .571$ $y_2 = \sout{.524}$ .523

```
OCCURRENCE OF  3 IS   50

OCCURRENCE OF  3 FOLLOWED   1 IS    3
OCCURRENCE OF  3 FOLLOWED   2 IS   10
OCCURRENCE OF  3 FOLLOWED   3 IS   22
OCCURRENCE OF  3 FOLLOWED   4 IS   15
```

$z_1 = .740$ $z_2 = .500$

```
OCCURRENCE OF  4 IS  115

OCCURRENCE OF  4 FOLLOWED   1 IS    3
OCCURRENCE OF  4 FOLLOWED   2 IS    5
OCCURRENCE OF  4 FOLLOWED   3 IS   22
OCCURRENCE OF  4 FOLLOWED   4 IS   85
```

$w_1 = .930$ $w_2 = \sout{.783}$ .782

```
THE RESULTS FOR DATA SET  3

INPUT DATA WAS

 43214424342244224422434242442444422441224422222442
 44444422224422244442242424242244442244222444444422442
 22424422244222224444222224422242222442424222242422
 222244422244222444222222222244422224442242242244442

OCCURRENCE OF  1 IS    2

OCCURRENCE OF  1 FOLLOWED   1 IS    0
OCCURRENCE OF  1 FOLLOWED   2 IS    1
OCCURRENCE OF  1 FOLLOWED   3 IS    0
OCCURRENCE OF  1 FOLLOWED   4 IS    1
```

$x_1 = .500$ $x_2 = 1.000$

```
OCCURRENCE OF  2 IS  104

OCCURRENCE OF  2 FOLLOWED   1 IS    1
OCCURRENCE OF  2 FOLLOWED   2 IS   62
```

OCCURRENCE OF	2 FOLLOWED	3 IS	0	$y_1 = .38\overline{5}$ .384	$y_2 =$ .981 .980
OCCURRENCE OF	2 FOLLOWED	4 IS	40		

OCCURRENCE OF 3 IS 4

OCCURRENCE OF	3 FOLLOWED	1 IS	0		
OCCURRENCE OF	3 FOLLOWED	2 IS	2		
OCCURRENCE OF	3 FOLLOWED	3 IS	1	$z_1 = .500$	z_2 .750
OCCURRENCE OF	3 FOLLOWED	4 IS	1		

OCCURRENCE OF 4 IS 90

OCCURRENCE OF	4 FOLLOWED	1 IS	1		
OCCURRENCE OF	4 FOLLOWED	2 IS	39		
OCCURRENCE OF	4 FOLLOWED	3 IS	3		
OCCURRENCE OF	4 FOLLOWED	4 IS	47	$w_1 = .555$	$w_2 = .955$

THE RESULTS FOR DATA SET 4

INPUT DATA WAS

11413411143131431113333433343344334341313131431324
41314114411334334334343333334334334344334343343333
34434334343334334344444444444443343344434444444444
44443324443433233444443444444444344444444444424444

OCCURRENCE OF 1 IS 22

OCCURRENCE OF	1 FOLLOWED	1 IS	7		
OCCURRENCE OF	1 FOLLOWED	2 IS	0	.681	.272
OCCURRENCE OF	1 FOLLOWED	3 IS	9	$x_1 =$ .682	$x_2 =$ .273
OCCURRENCE OF	1 FOLLOWED	4 IS	6		

OCCURRENCE OF 2 IS 4

OCCURRENCE OF	2 FOLLOWED	1 IS	0
OCCURRENCE OF	2 FOLLOWED	2 IS	0
OCCURRENCE OF	2 FOLLOWED	3 IS	1

OCCURRENCE OF 2 FOLLOWED 4 IS [illegible] y_1 = 1.000 y_2 = [illegible]

5

OCCURRENCE OF 3 IS 74

OCCURRENCE OF 3 FOLLOWED 1 IS 8
OCCURRENCE OF 3 FOLLOWED 2 IS 3
OCCURRENCE OF 3 FOLLOWED 3 IS 32
OCCURRENCE OF 3 FOLLOWED 4 IS 31 z_1 = .851 z_2 = .459

OCCURRENCE OF 4 IS 100

OCCURRENCE OF 4 FOLLOWED 1 IS 6
OCCURRENCE OF 4 FOLLOWED 2 IS 1
OCCURRENCE OF 4 FOLLOWED 3 IS 32
OCCURRENCE OF 4 FOLLOWED 4 IS 60 w_1 = .920 w_2 = .610

THE RESULTS FOR DATA SET 5

INPUT DATA WAS

3333344444244244333344444343334444433333444444333
3442443333444423331444442333333133424331442433333 4
4233342342344344124344324342344424333343133333333
3133333333311333333333333331333333333333334424444242

OCCURRENCE OF 1 IS 9

OCCURRENCE OF 1 FOLLOWED 1 IS 1
OCCURRENCE OF 1 FOLLOWED 2 IS 1
OCCURRENCE OF 1 FOLLOWED 3 IS 5
OCCURRENCE OF 1 FOLLOWED 4 IS 2 x_1 = ~~.778~~ .777 x_2 = .333

OCCURRENCE OF 2 IS 17

OCCURRENCE OF 2 FOLLOWED 1 IS 0
OCCURRENCE OF 2 FOLLOWED 2 IS 0
OCCURRENCE OF 2 FOLLOWED 3 IS 6
OCCURRENCE OF 2 FOLLOWED 4 IS 10 y_1 = .941 y_2 = .588

OCCURRENCE OF 3 IS 106

OCCURRENCE OF 3 FOLLOWED 1 IS 7
OCCURRENCE OF 3 FOLLOWED 2 IS 1
OCCURRENCE OF 3 FOLLOWED 3 IS 80
OCCURRENCE OF 3 FOLLOWED 4 IS 18

$z_1 = .424$ $z_2 = .179$

OCCURRENCE OF 4 IS 68

OCCURRENCE OF 4 FOLLOWED 1 IS 1
OCCURRENCE OF 4 FOLLOWED 2 IS 15
OCCURRENCE OF 4 FOLLOWED 3 IS 14
OCCURRENCE OF 4 FOLLOWED 4 IS 38

$w_1 =$ ~~.765~~ .764 $w_2 = .179$

THE RESULTS FOR DATA SET 6

INPUT DATA WAS

13343144211132332442244344112234434421112122424414
4434224244444432444434444444444444444444444444444
24444444444442244444444444444444444444444444444444
444444444444444333333333433443443434344444444444333

OCCURRENCE OF 1 IS 12

OCCURRENCE OF 1 FOLLOWED 1 IS 5
OCCURRENCE OF 1 FOLLOWED 2 IS 3
OCCURRENCE OF 1 FOLLOWED 3 IS 2
OCCURRENCE OF 1 FOLLOWED 4 IS 2

$x_1 = .333$ $x_2 =$ ~~.417~~ .416

OCCURRENCE OF 2 IS 19

OCCURRENCE OF 2 FOLLOWED 1 IS 3
OCCURRENCE OF 2 FOLLOWED 2 IS 5
OCCURRENCE OF 2 FOLLOWED 3 IS 2
OCCURRENCE OF 2 FOLLOWED 4 IS 9

$y_1 = .579$ $y_2 =$ ~~.739~~ .736

7

OCCURRENCE OF 3 IS 30

OCCURRENCE OF 3 FOLLOWED	1 IS	1
OCCURRENCE OF 3 FOLLOWED	2 IS	3
OCCURRENCE OF 3 FOLLOWED	3 IS	13
OCCURRENCE OF 3 FOLLOWED	4 IS	1[illegible]

$z_1 = .433$ $z_2 = .500$

OCCURRENCE OF 4 IS 139

OCCURRENCE OF 4 FOLLOWED	1 IS	[illegible]
OCCURRENCE OF 4 FOLLOWED	2 IS	[illegible]
OCCURRENCE OF 4 FOLLOWED	3 IS	1[illegible]
OCCURRENCE OF 4 FOLLOWED	4 IS	110

$w_1 = .428$ $w_2 = .792$

THE RESULTS FOR DATA SET 7

INPUT DATA WAS

4122242124434422244244224444422242224242244444442224
24442242442442444444442422244434444442444422444444
44444444444444444444444444444444444444444444444444
44443444444444444444444444444444444444444444444444

OCCURRENCE OF 1 IS 2

OCCURRENCE OF 1 FOLLOWED	1 IS	0
OCCURRENCE OF 1 FOLLOWED	2 IS	2
OCCURRENCE OF 1 FOLLOWED	3 IS	0
OCCURRENCE OF 1 FOLLOWED	4 IS	[illegible]

$x_1 = .000$ $x_2 = 1.000$

OCCURRENCE OF 2 IS 35

OCCURRENCE OF 2 FOLLOWED	1 IS	1
OCCURRENCE OF 2 FOLLOWED	2 IS	15
OCCURRENCE OF 2 FOLLOWED	3 IS	0
OCCURRENCE OF 2 FOLLOWED	4 IS	19

.542

$y_1 = .543$ $y_2 = .471$

6

```
OCCURRENCE OF  3 IS    3

OCCURRENCE OF  3 FOLLOWED     1 IS    0
OCCURRENCE OF  3 FOLLOWED     2 IS    0
OCCURRENCE OF  3 FOLLOWED     3 IS    0
OCCURRENCE OF  3 FOLLOWED     4 IS    3
```

$z_1 = 1.000$ $z_2 = 1.000$

```
OCCURRENCE OF  4 IS  160

OCCURRENCE OF  4 FOLLOWED     1 IS    1
OCCURRENCE OF  4 FOLLOWED     2 IS   18
OCCURRENCE OF  4 FOLLOWED     3 IS    3
OCCURRENCE OF  4 FOLLOWED     4 IS  137
```

$w_1 = .675$ $w_2 = .968$ (~~.964~~)

```
THE RESULTS FOR DATA SET  8

INPUT DATA WAS

224443443134133311313333342222222143424212233342
3424423414214124344223244424343232332434432343443
2243444442134214422334221344144224212244422442222
2222242222224222444222224222442222424222244444222222
```

```
OCCURRENCE OF  1 IS   15

OCCURRENCE OF  1 FOLLOWED     1 IS    1
OCCURRENCE OF  1 FOLLOWED     2 IS    3
OCCURRENCE OF  1 FOLLOWED     3 IS    6
OCCURRENCE OF  1 FOLLOWED     4 IS    5
```

$x_1 = .733$ $x_2 = .533$

```
OCCURRENCE OF  2 IS   83

OCCURRENCE OF  2 FOLLOWED     1 IS    7
OCCURRENCE OF  2 FOLLOWED     2 IS   47
OCCURRENCE OF  2 FOLLOWED     3 IS    7
OCCURRENCE OF  2 FOLLOWED     4 IS   19
```

$y_1 = .337$ $y_2 = .795$

4

OCCURRENCE OF 3 IS 36

OCCURRENCE OF 3 FOLLOWED 1 IS 3
OCCURRENCE OF 3 FOLLOWED 2 IS 7
OCCURRENCE OF 3 FOLLOWED 3 IS 10
OCCURRENCE OF 3 FOLLOWED 4 IS 16

$z_1 = .722$ $z_2 = .639$ .638

OCCURRENCE OF 4 IS 66

OCCURRENCE OF 4 FOLLOWED 1 IS 4
OCCURRENCE OF 4 FOLLOWED 2 IS 25
OCCURRENCE OF 4 FOLLOWED 3 IS 11
OCCURRENCE OF 4 FOLLOWED 4 IS 26

$w_1 = .561$ .560 $w_2 = .773$ .772

THE RESULTS FOR DATA SET 9

INPUT DATA WAS

```
41114133243332341433422444334214234433444434224344
33123+44333113423342444344433342443443444433324434
243443+4343434422344433311133411342334133223334424
43434321434314234442444411334131434343434313141313
```

OCCURRENCE OF 1 IS 25

OCCURRENCE OF 1 FOLLOWED 1 IS 7
OCCURRENCE OF 1 FOLLOWED 2 IS 1
OCCURRENCE OF 1 FOLLOWED 3 IS 10
OCCURRENCE OF 1 FOLLOWED 4 IS 7

$x_1 = .680$ $x_2 = .320$

OCCURRENCE OF 2 IS 23

OCCURRENCE OF 2 FOLLOWED 1 IS 2
OCCURRENCE OF 2 FOLLOWED 2 IS 4
OCCURRENCE OF 2 FOLLOWED 3 IS 8
OCCURRENCE OF 2 FOLLOWED 4 IS 9

$y_1 = .739$ $y_2 = .565$

OCCURRENCE OF 3 IS 70

10

OCCURRENCE OF 3 FOLLOWED 1 IS 8
OCCURRENCE OF 3 FOLLOWED 2 IS 57
OCCURRENCE OF 3 FOLLOWED 3 IS 22
OCCURRENCE OF 3 FOLLOWED 4 IS 34 $z_1 = .660$ $z_2 = .557$

OCCURRENCE OF 4 IS 82

OCCURRENCE OF 4 FOLLOWED 1 IS 2
OCCURRENCE OF 4 FOLLOWED 2 IS 137
OCCURRENCE OF 4 FOLLOWED 3 IS 39
OCCURRENCE OF 4 FOLLOWED 4 IS 31 $w_1 = .714$ $w_2 = .530$

THE RESULTS FOR DATA SET 10

INPUT DATA WAS

1142422111114422124231224444443343444444444444442
434444444444442444444444444442444444444444444444444
444444444444444444444444444444444444444444444444444
44444444444444444444444444444444444444444444444444

OCCURRENCE OF 1 IS 9

OCCURRENCE OF 1 FOLLOWED 1 IS 5
OCCURRENCE OF 1 FOLLOWED 2 IS 21
OCCURRENCE OF 1 FOLLOWED 3 IS 0
OCCURRENCE OF 1 FOLLOWED 4 IS 22 $x_1 = .222$ $x_2 = .444$

OCCURRENCE OF 2 IS 12

OCCURRENCE OF 2 FOLLOWED 1 IS 2
OCCURRENCE OF 2 FOLLOWED 2 IS 37
OCCURRENCE OF 2 FOLLOWED 3 IS 1
OCCURRENCE OF 2 FOLLOWED 4 IS 0 $y_1 = .583$ $y_2 = .750$

OCCURRENCE OF 3 IS 5

11

OCCURRENCE OF	3 FOLLOWED	1 IS	1		
OCCURRENCE OF	3 FOLLOWED	2 IS	0		,600
OCCURRENCE OF	3 FOLLOWED	3 IS	1		
OCCURRENCE OF	3 FOLLOWED	4 IS	3	$z_1 = .800$	$z_2 = .375$

OCCURRENCE OF 4 IS 174

OCCURRENCE OF	4 FOLLOWED	1 IS	0		
OCCURRENCE OF	4 FOLLOWED	2 IS	8		
OCCURRENCE OF	4 FOLLOWED	3 IS	3		
OCCURRENCE OF	4 FOLLOWED	4 IS	163	$w_1 = .954$	$w_2 = .977$

THE RESULTS FOR DATA SET 11

INPUT DATA WAS

41233431221443344334334434122224434122222222244324
32423122312241224124224312222224432433412431241241
24324324312432431243124312243334124324432431243244
32432323243242432432323243232323232323232323232323

$(1-.493)(1-.562)$ (1-

$(1-.601)(1-.582)$ (1-

$(1-.785)(1-.729)$ (1-

$(1-.802)(1-.804)$ (1-

OCCURRENCE OF 1 IS 20

OCCURRENCE OF	1 FOLLOWED	1 IS	0		
OCCURRENCE OF	1 FOLLOWED	2 IS	19		
OCCURRENCE OF	1 FOLLOWED	3 IS	0		
OCCURRENCE OF	1 FOLLOWED	4 IS	1	$x_1 = .050$ ✓	$x_2 = 1.000$ ✓

OCCURRENCE OF 2 IS 74

OCCURRENCE OF	2 FOLLOWED	1 IS	1		
OCCURRENCE OF	2 FOLLOWED	2 IS	22		
OCCURRENCE OF	2 FOLLOWED	3 IS	19		
OCCURRENCE OF	2 FOLLOWED	4 IS	32	$y_1 = .689$ ✓	$y_2 = .729$ ✓

OCCURRENCE OF 3 IS 56

```
OCCURRENCE OF  3 FOLLOWED      1 IS    10
OCCURRENCE OF  3 FOLLOWED      2 IS    30
OCCURRENCE OF  3 FOLLOWED      3 IS     7
OCCURRENCE OF  3 FOLLOWED      4 IS     8
```

11 $z_1 = .267$ ✓ $z_2 = .678$ ✓

```
OCCURRENCE OF  4 IS   50

OCCURRENCE OF  4 FOLLOWED      1 IS     9
OCCURRENCE OF  4 FOLLOWED      2 IS     3
OCCURRENCE OF  4 FOLLOWED      3 IS    30
OCCURRENCE OF  4 FOLLOWED      4 IS     8
```

$w_1 = .760$ ✓ $w_2 = .220$ ✓

```
THE RESULTS FOR DATA SET 12

INPUT DATA WAS

 43123443312434411433244431243443441434341243334423
 44334333344333443334334333343334433334333344333 3
 44344333433343344334333433343444343343334334434
 43344343333333433433334433443333433343443334343333

OCCURRENCE OF  1 IS    7

OCCURRENCE OF  1 FOLLOWED      1 IS     1
OCCURRENCE OF  1 FOLLOWED      2 IS     4
OCCURRENCE OF  1 FOLLOWED      3 IS     0
OCCURRENCE OF  1 FOLLOWED      4 IS     2
```

$x_1 = .285$ ✓ $x_2 = .857$ ✓

```
OCCURRENCE OF  2 IS    5

OCCURRENCE OF  2 FOLLOWED      1 IS     0
OCCURRENCE OF  2 FOLLOWED      2 IS     0
OCCURRENCE OF  2 FOLLOWED      3 IS     1
OCCURRENCE OF  2 FOLLOWED      4 IS     4
```

$y_1 = 1.000$ ✓ $y_2 = .800$

```
OCCURRENCE OF  3 IS  114

OCCURRENCE OF  3 FOLLOWED      1 IS     3
```

OCCURRENCE OF 3 FOLLOWED	2 IS	1	
13 OCCURRENCE OF 3 FOLLOWED	3 IS	67	
OCCURRENCE OF 3 FOLLOWED	4 IS	42	z_1 = .956 ✓ z_2 .317 ✓

OCCURRENCE OF 4 IS 74

OCCURRENCE OF 4 FOLLOWED	1 IS	3	
OCCURRENCE OF 4 FOLLOWED	2 IS	0	
OCCURRENCE OF 4 FOLLOWED	3 IS	46	
OCCURRENCE OF 4 FOLLOWED	4 IS	25	w_1 = .954 ✓ w_2 = .337 ✓

THE RESULTS FOR DATA SET 13

INPUT DATA WAS

21431242222422242222224222222242422224442424424442
42224242222224424424242422444424444442434242442442
424442442444442444244442442444444444443424442444
44244444242444244424442444444444244442444444442 44

OCCURRENCE OF 1 IS 2

OCCURRENCE OF 1 FOLLOWED	1 IS	0	
OCCURRENCE OF 1 FOLLOWED	2 IS	1	
OCCURRENCE OF 1 FOLLOWED	3 IS	0	
OCCURRENCE OF 1 FOLLOWED	4 IS	1	x_1 = .500 ✓ x_2 = 1.000

OCCURRENCE OF 2 IS 72

OCCURRENCE OF 2 FOLLOWED	1 IS	1	
OCCURRENCE OF 2 FOLLOWED	2 IS	29	
OCCURRENCE OF 2 FOLLOWED	3 IS	0	
OCCURRENCE OF 2 FOLLOWED	4 IS	42	y_1 = .583 ✓ y_2 = .986

OCCURRENCE OF 3 IS 3

OCCURRENCE OF 3 FOLLOWED	1 IS	1
OCCURRENCE OF 3 FOLLOWED	2 IS	0

14

```
OCCURRENCE OF  3 FOLLOWED      3 IS   0
OCCURRENCE OF  3 FOLLOWED      4 IS   2
```

$z_1 = .666$ ✓ $z_2 = .666$

```
OCCURRENCE OF  4 IS  123

OCCURRENCE OF  4 FOLLOWED      1 IS   0
OCCURRENCE OF  4 FOLLOWED      2 IS  41
OCCURRENCE OF  4 FOLLOWED      3 IS   3
OCCURRENCE OF  4 FOLLOWED      4 IS  79
```

$w_1 = .618$ ✓ $w_2 = .967$

```
THE RESULTS FOR DATA SET 14

INPUT DATA WAS

32242313424223224142442314242312423232442442142414
42142244244244334422414242244214224424324214424423
24424444144144242234242444324444244444344224443442
14444443444442241142221444444434422422444444224224

OCCURRENCE OF  1 IS  16

OCCURRENCE OF  1 FOLLOWED      1 IS   1
OCCURRENCE OF  1 FOLLOWED      2 IS   1
OCCURRENCE OF  1 FOLLOWED      3 IS   1
OCCURRENCE OF  1 FOLLOWED      4 IS  13
```

$x_1 = .875$ ✓ $x_2 = .87$

```
OCCURRENCE OF  2 IS  62

OCCURRENCE OF  2 FOLLOWED      1 IS   6
OCCURRENCE OF  2 FOLLOWED      2 IS  16
OCCURRENCE OF  2 FOLLOWED      3 IS   8
OCCURRENCE OF  2 FOLLOWED      4 IS  32
```

$y_1 = .645$ ✓ $y_2 = .77$

```
OCCURRENCE OF  3 IS  18

OCCURRENCE OF  3 FOLLOWED      1 IS   3
OCCURRENCE OF  3 FOLLOWED      2 IS   7
OCCURRENCE OF  3 FOLLOWED      3 IS   1
```

OCCURRENCE OF 3 FOLLOWED 4 IS 7

$z_1 = .444$ $z_2 = .777$

OCCURRENCE OF 4 IS 104

OCCURRENCE OF 4 FOLLOWED	1 IS	6
OCCURRENCE OF 4 FOLLOWED	2 IS	39
OCCURRENCE OF 4 FOLLOWED	3 IS	[illegible]
OCCURRENCE OF 4 FOLLOWED	4 IS	[illegible]

$w_1 = .567$ ✓ $w_2 = .63$

THE RESULTS FOR DATA SET 15

INPUT DATA WAS

24214422132424333124224441411143223232413142414214
14431314414111123131424124232323232323232323232323
23232423214432323232323232323242323232323232323232
32323232323232323232323232323232323232323232323232

OCCURRENCE OF 1 IS 23

OCCURRENCE OF 1 FOLLOWED	1 IS	5
OCCURRENCE OF 1 FOLLOWED	2 IS	3
OCCURRENCE OF 1 FOLLOWED	3 IS	4
OCCURRENCE OF 1 FOLLOWED	4 IS	11

$x_1 = .652$ ✓ $x_2 = .60$

$x_2 = .608$

OCCURRENCE OF 2 IS 79

OCCURRENCE OF 2 FOLLOWED	1 IS	4
OCCURRENCE OF 2 FOLLOWED	2 IS	3
OCCURRENCE OF 2 FOLLOWED	3 IS	62
OCCURRENCE OF 2 FOLLOWED	4 IS	0

$y_1 = .994$ ✓ $y_2 = .15$

OCCURRENCE OF 3 IS 72

OCCURRENCE OF 3 FOLLOWED	1 IS	6
OCCURRENCE OF 3 FOLLOWED	2 IS	64
OCCURRENCE OF 3 FOLLOWED	3 IS	2
OCCURRENCE OF 3 FOLLOWED	4 IS	0

$z_2 = .11$

$z_1 = .027$ ✓ $z_2 =$

16

OCCURRENCE OF 4 IS 26

OCCURRENCE OF 4 FOLLOWED 1 I.
OCCURRENCE OF 4 FOLLOWED 2 IS [illegible]
OCCURRENCE OF 4 FOLLOWED 3 IS 4
OCCURRENCE OF 4 FOLLOWED 4 IS [illegible]

$w_1 = .384$ ✓ $w_2 =$

THE RESULTS FOR DATA SET 16

$w_2 = .538$

INPUT DATA WAS

24431212421434432411223422124121444143223434143213
24344321122244142242244444242442224442422222242422
22222224444224444444444222222222222422222222222222
22224222222222222224244222222222222222222222222222

OCCURRENCE OF 1 IS 14

OCCURRENCE OF 1 FOLLOWED 1 IS 2
OCCURRENCE OF 1 FOLLOWED 2 IS 6
OCCURRENCE OF 1 FOLLOWED 3 IS 1
OCCURRENCE OF 1 FOLLOWED 4 IS 5

$x_1 = .428$ ✓ $x_2 =$

$x_2 = .785$

OCCURRENCE OF 2 IS 119

OCCURRENCE OF 2 FOLLOWED 1 IS 6
OCCURRENCE OF 2 FOLLOWED 2 IS 91
OCCURRENCE OF 2 FOLLOWED 3 IS 2
OCCURRENCE OF 2 FOLLOWED 4 IS 20

$y_1 = .184$ ✓ $y_2 =$

$y_2 = .924$

OCCURRENCE OF 3 IS 11

OCCURRENCE OF 3 FOLLOWED 1 IS 1
OCCURRENCE OF 3 FOLLOWED 2 IS 5
OCCURRENCE OF 3 FOLLOWED 3 IS 0
OCCURRENCE OF 3 FOLLOWED 4 IS 5

$z_2 = .909$

$z_1 = .454$ z_2

OCCURRENCE OF 4 IS 56

OCCURRENCE OF 4 FOLLOWED	1 IS	[illegible]	
OCCURRENCE OF 4 FOLLOWED	2 IS	17	
OCCURRENCE OF 4 FOLLOWED	3 IS	[illegible]	
OCCURRENCE OF 4 FOLLOWED	4 IS	[illegible]	$w_1 = .607$ ✓ $w_2 = .107$

THE RESULTS FOR DATA SET 17

INPUT DATA WAS

124444412342243412444244434344444444444444444443444
44444444443444444444444444444444444444444444444444
44444444444444444444444444444444444444444444444444
44444444444444444444444444444444444444444444444444

OCCURRENCE OF 1 IS 3

OCCURRENCE OF 1 FOLLOWED	1 IS	0	
OCCURRENCE OF 1 FOLLOWED	2 IS	3	
OCCURRENCE OF 1 FOLLOWED	3 IS	0	
OCCURRENCE OF 1 FOLLOWED	4 IS	0	$x_1 = .000$ ✓ $x_2 = 1.0$

OCCURRENCE OF 2 IS 6

OCCURRENCE OF 2 FOLLOWED	1 IS	0	
OCCURRENCE OF 2 FOLLOWED	2 IS	1	
OCCURRENCE OF 2 FOLLOWED	3 IS	1	
OCCURRENCE OF 2 FOLLOWED	4 IS	4	$y_1 = .833$ ✓ $y_2 = .$

$y_2 = .833$

OCCURRENCE OF 3 IS 6

OCCURRENCE OF 3 FOLLOWED	1 IS	0	
OCCURRENCE OF 3 FOLLOWED	2 IS	0	
OCCURRENCE OF 3 FOLLOWED	3 IS	0	
OCCURRENCE OF 3 FOLLOWED	4 IS	6	$z_1 = 1.000$ ✓ $z_2 =$

$z_2 = 1.000$

OCCURRENCE OF 4 IS 133

OCCURRENCE OF 4 FOLLOWED 1 IS 6
OCCURRENCE OF 4 FOLLOWED 2 IS 5
OCCURRENCE OF 4 FOLLOWED 3 IS 4
OCCURRENCE OF 4 FOLLOWED 4 IS 17

THE RESULTS FOR DATA SET 13

INPUT DATA WAS

14424442141332443442224244434234142242342224243 44
44443421243432123234341234343144444442124341232344
443442444423234242444424444443444444444444444423 4
424443444442142344342344434343432343434343412323412

OCCURRENCE OF 1 IS 15			
OCCURRENCE OF	1 FOLLOWED	1 IS	
OCCURRENCE OF	1 FOLLOWED	2 IS	7
OCCURRENCE OF	1 FOLLOWED	3 IS	1
OCCURRENCE OF	1 FOLLOWED	4 IS	5

$x_1 = .401$ $x_2 = .423$

OCCURRENCE OF 2 IS 33			
OCCURRENCE OF	2 FOLLOWED	1 IS	5
OCCURRENCE OF	2 FOLLOWED	2 IS	3
OCCURRENCE OF	2 FOLLOWED	3 IS	17
OCCURRENCE OF	2 FOLLOWED	4 IS	12

$y_1 = .763$ $y_2 = .394$

OCCURRENCE OF 3 IS 42			
OCCURRENCE OF	3 FOLLOWED	1 IS	1
OCCURRENCE OF	3 FOLLOWED	2 IS	8
OCCURRENCE OF	3 FOLLOWED	3 IS	2
OCCURRENCE OF	3 FOLLOWED	4 IS	31

$z_1 = .781$ $z_2 = .428$

OCCURRENCE OF 4 IS 1?7

OCCURRENCE OF 4 FOLLOWED 1 IS [illegible]
OCCURRENCE OF 4 FOLLOWED 2 IS [illegible]
OCCURRENCE OF 4 FOLLOWED 3 IS 2?
OCCURRENCE OF 4 FOLLOWED 4 IS 5?

$w_1 = .15?$ $w_2 =$

THE RESULTS FOR DATA SET 1?

w_2 .738

INPUT DATA WAS

42212114224212444421221442144221231223212321244322
14433443344344344434444434444324424232424443114444 1
44242144442443134424433442434242444433444344444122
24334444113424414444422422244222423142224424342424 3

OCCURRENCE OF 1 IS 22

OCCURRENCE OF 1 FOLLOWED 1 IS 3
OCCURRENCE OF 1 FOLLOWED 2 IS 3[illegible]
OCCURRENCE OF 1 FOLLOWED 3 IS [illegible]2
OCCURRENCE OF 1 FOLLOWED 4 IS [illegible]

$x_1 = .500$ ✓ $x_2 = .$

$x_2 = .772$

OCCURRENCE OF 2 IS 53

OCCURRENCE OF 2 FOLLOWED 1 IS 11
OCCURRENCE OF 2 FOLLOWED 2 IS 15
OCCURRENCE OF 2 FOLLOWED 3 IS 5
OCCURRENCE OF 2 FOLLOWED 4 IS 22

$y_1 = .509$ ✓ $y_2 =$

$y_2 = .648$

OCCURRENCE OF 3 IS 2?

OCCURRENCE OF 3 FOLLOWED 1 IS 4
OCCURRENCE OF 3 FOLLOWED 2 IS 5
OCCURRENCE OF 3 FOLLOWED 3 IS 5
OCCURRENCE OF 3 FOLLOWED 4 IS 14

$z_1 = .655$ ✓ z_2

$z_2 = .655$

OCCURRENCE OF 4 IS 96

OCCURRENCE OF	4 FOLLOWED	1 IS	4
OCCURRENCE OF	4 FOLLOWED	2 IS	25
OCCURRENCE OF	4 FOLLOWED	3 IS	17
OCCURRENCE OF	4 FOLLOWED	4 IS	50

$w_1 = .497$ ✓ w

$w_2 = .781$

THE RESULTS FOR DATA SET 2:

INPUT DATA WAS

```
32111144232244443322242113342442122222123142444334
44444444414243422441423133444414344442334242442441
42141134134222212422441324232441443442444214134322
22213243232121444444242414224112223222411122213224
```

OCCURRENCE OF 1 IS 33

OCCURRENCE OF	1 FOLLOWED	1 IS	8
OCCURRENCE OF	1 FOLLOWED	2 IS	6
OCCURRENCE OF	1 FOLLOWED	3 IS	8
OCCURRENCE OF	1 FOLLOWED	4 IS	11

$x_1 = .575$ ✓ $x_2 =$

$x_2 = .515$

OCCURRENCE OF 2 IS 62

OCCURRENCE OF	2 FOLLOWED	1 IS	11
OCCURRENCE OF	2 FOLLOWED	2 IS	24
OCCURRENCE OF	2 FOLLOWED	3 IS	7
OCCURRENCE OF	2 FOLLOWED	4 IS	20

$y_1 = .435$ ✓ $y_2 =$

$y_2 = .709$

OCCURRENCE OF 3 IS 28

OCCURRENCE OF	3 FOLLOWED	1 IS	2
OCCURRENCE OF	3 FOLLOWED	2 IS	11
OCCURRENCE OF	3 FOLLOWED	3 IS	5
OCCURRENCE OF	3 FOLLOWED	4 IS	10

$z_1 = .535$ ✓ z_2

$z_2 = .750$

OCCURRENCE OF 4 IS 77

21 OCCURRENCE OF 4 FOLLOWED 1 IS 12
OCCURRENCE OF 4 FOLLOWED 2 IS 21
OCCURRENCE OF 4 FOLLOWED 3 IS 7
OCCURRENCE OF 4 FOLLOWED 4 IS 36

$w_1 = .558$ $w_2 = .7$

THE RESULTS FOR DATA SET 21

$w_2 = .740$

INPUT DATA WAS

```
11233112134331433331311231133331131113133313311111
13333331113111331133333331333311113333333333333133
13313331333311133333311111111111111111113333333333
33333333333111113113131331113313311111111111111111
```

OCCURRENCE OF 1 IS 94

OCCURRENCE OF 1 FOLLOWED 1 IS 64
OCCURRENCE OF 1 FOLLOWED 2 IS 3
OCCURRENCE OF 1 FOLLOWED 3 IS 25
OCCURRENCE OF 1 FOLLOWED 4 IS 1

$x_1 = .712$ $x_2 = .946$

$x_2 =$

OCCURRENCE OF 2 IS 3

OCCURRENCE OF 2 FOLLOWED 1 IS 1
OCCURRENCE OF 2 FOLLOWED 2 IS 0
OCCURRENCE OF 2 FOLLOWED 3 IS 2
OCCURRENCE OF 2 FOLLOWED 4 IS 0

$y_1 = .333$ $y_2 = 1.000$

$y_2 =$

OCCURRENCE OF 3 IS 101

OCCURRENCE OF 3 FOLLOWED 1 IS 28
OCCURRENCE OF 3 FOLLOWED 2 IS 0
OCCURRENCE OF 3 FOLLOWED 3 IS 72
OCCURRENCE OF 3 FOLLOWED 4 IS 1

$z_1 = .277$ $z_2 = .490$

$z_2 =$

OCCURRENCE OF 4 IS 2

OCCURRENCE OF 4 FOLLOWED 1 IS 0
OCCURRENCE OF 4 FOLLOWED 2 IS 0
OCCURRENCE OF 4 FOLLOWED 3 IS 2
OCCURRENCE OF 4 FOLLOWED 4 IS 1

THE RESULTS FOR DATA SET 22

INPUT DATA WAS

31211322311111422331144442411421131111231121131111
31311123131232311113124311113231232312323111323231
23232311111232323232323232323232323232323232323232
32323232323111231123232323232323232323232323232323

OCCURRENCE OF 1 IS 57

OCCURRENCE OF 1 FOLLOWED 1 IS 33
OCCURRENCE OF 1 FOLLOWED 2 IS 12
OCCURRENCE OF 1 FOLLOWED 3 IS 9
OCCURRENCE OF 1 FOLLOWED 4 IS 3

$x_1 = .789$ $x_2 = .136$

OCCURRENCE OF 2 IS 65

OCCURRENCE OF 2 FOLLOWED 1 IS 3
OCCURRENCE OF 2 FOLLOWED 2 IS 2
OCCURRENCE OF 2 FOLLOWED 3 IS 58
OCCURRENCE OF 2 FOLLOWED 4 IS 2

$y_1 = .076$ $y_2 = .938$

OCCURRENCE OF 3 IS 70

OCCURRENCE OF 3 FOLLOWED 1 IS 20
OCCURRENCE OF 3 FOLLOWED 2 IS 48
OCCURRENCE OF 3 FOLLOWED 3 IS 1
OCCURRENCE OF 3 FOLLOWED 4 IS 0

$z_1 = .971$ $z_2 = 300$

OCCURRENCE OF 4 IS 8

OCCURRENCE OF 4 FOLLOWED 1 IS 1

OCCURRENCE OF 4 FOLLOWED 2 IS 3
OCCURRENCE OF 4 FOLLOWED 3 IS 1
OCCURRENCE OF 4 FOLLOWED 4 IS 3

$\omega_1 = .560$ ω_2

THE RESULTS FOR DATA SET 23

$\omega_2 = .550$

INPUT DATA WAS

11321444441214411334133122413322231122231122213222
22142222213111131111111112111111111111111112111111
11112111111111111111111111111111111111111111111111
11111111111111111111111111111111111111113131311133

OCCURRENCE OF 1 IS 147

OCCURRENCE OF 1 FOLLOWED 1 IS 126
OCCURRENCE OF 1 FOLLOWED 2 IS 7
OCCURRENCE OF 1 FOLLOWED 3 IS 11
OCCURRENCE OF 1 FOLLOWED 4 IS 3

$x_1 = .906$ $x_2 =$

$x_2 = .931$

OCCURRENCE OF 2 IS 26

OCCURRENCE OF 2 FOLLOWED 1 IS 8
OCCURRENCE OF 2 FOLLOWED 2 IS 15
OCCURRENCE OF 2 FOLLOWED 3 IS 2
OCCURRENCE OF 2 FOLLOWED 4 IS 1

$y_1 = .884$ $y_2 =$

$y_2 = .384$

OCCURRENCE OF 3 IS 17

OCCURRENCE OF 3 FOLLOWED 1 IS 8
OCCURRENCE OF 3 FOLLOWED 2 IS 3
OCCURRENCE OF 3 FOLLOWED 3 IS 4
OCCURRENCE OF 3 FOLLOWED 4 IS 1

$z_1 = .647$ $z_2 =$

$z_2 = .705$

OCCURRENCE OF 4 IS 10

OCCURRENCE OF 4 FOLLOWED 1 IS 4
OCCURRENCE OF 4 FOLLOWED 2 IS 1

OCCURRENCE OF 4 FOLLOWED 3 IS 7
OCCURRENCE OF 4 FOLLOWED 4 IS , $w = .500$ w_1

THE RESULTS FOR DATA SET 24 $w_2 = .400$

INPUT DATA WAS

4343421432324314323232323232323232311232323232323232
32323232323232323232323232332323232323232323232323232
3232323232323232323232323232323232323232323232324323
2323232323232323232323232323232323232323232323232323

OCCURRENCE OF 1 IS 4

OCCURRENCE OF 1 FOLLOWED 1 IS 1
OCCURRENCE OF 1 FOLLOWED 2 IS 1
OCCURRENCE OF 1 FOLLOWED 3 IS 0
OCCURRENCE OF 1 FOLLOWED 4 IS 2

$x_1 = .500$ $x_2 = .250$ x

OCCURRENCE OF 2 IS 93

OCCURRENCE OF 2 FOLLOWED 1 IS 1
OCCURRENCE OF 2 FOLLOWED 2 IS 0
OCCURRENCE OF 2 FOLLOWED 3 IS 90
OCCURRENCE OF 2 FOLLOWED 4 IS 2

$y_1 = .010$ $y_2 = .978$

OCCURRENCE OF 3 IS 96

OCCURRENCE OF 3 FOLLOWED 1 IS 2
OCCURRENCE OF 3 FOLLOWED 2 IS 91
OCCURRENCE OF 3 FOLLOWED 3 IS ^
OCCURRENCE OF 3 FOLLOWED 4 IS 2

$z_1 = .968$

$z_2 = .020$

OCCURRENCE OF 4 IS 7

OCCURRENCE OF 4 FOLLOWED 1 IS 0
OCCURRENCE OF 4 FOLLOWED 2 IS 1
OCCURRENCE OF 4 FOLLOWED 3 IS 0

$w_1 = .142$ $w_2 = .857$

OCCURRENCE OF 4 FOLLOWED 4 IS 0

THE RESULTS FOR DATA SET 25

INPUT DATA WAS

```
4433443441142231243421221344122232213122134341232
41233343443431111123324121323213413112123431412343
33441422211424411131342211112131342422134121134434
42313324213333224434143321223331222322111221221122
```

OCCURRENCE OF 1 IS 53

OCCURRENCE OF 1 FOLLOWED 1 IS 16
OCCURRENCE OF 1 FOLLOWED 2 IS 19
OCCURRENCE OF 1 FOLLOWED 3 IS 13
OCCURRENCE OF 1 FOLLOWED 4 IS 5

$x_1 = .660$ $x_2 = .547$ $x_2 =$

OCCURRENCE OF 2 IS 55

OCCURRENCE OF 2 FOLLOWED 1 IS 17
OCCURRENCE OF 2 FOLLOWED 2 IS 18
OCCURRENCE OF 2 FOLLOWED 3 IS 12
OCCURRENCE OF 2 FOLLOWED 4 IS 7

$y_1 = .636$ $y_2 = .527$ $y_2 =$

OCCURRENCE OF 3 IS 50

OCCURRENCE OF 3 FOLLOWED 1 IS 9
OCCURRENCE OF 3 FOLLOWED 2 IS 9
OCCURRENCE OF 3 FOLLOWED 3 IS 13
OCCURRENCE OF 3 FOLLOWED 4 IS 19

$z_1 = .360$ $z_2 = .440$ z_2

OCCURRENCE OF 4 IS 42

OCCURRENCE OF 4 FOLLOWED 1 IS 11
OCCURRENCE OF 4 FOLLOWED 2 IS 9
OCCURRENCE OF 4 FOLLOWED 3 IS 12
OCCURRENCE OF 4 FOLLOWED 4 IS 10

$w_1 = .476$ $w_2 = .547$ w

26 THE RESULTS FOR DATA SET 26

INPUT DATA WAS

```
33121241131311111222112222222212222222222222222222
22224112221221222224211222222222222241222222222222
42222222222424431222222222222222222222222222222222
22224431122222221222422422222222122222222222222422
```

```
OCCURRENCE OF  1 IS   25

OCCURRENCE OF  1 FOLLOWED  1 IS   9
OCCURRENCE OF  1 FOLLOWED  2 IS  14
OCCURRENCE OF  1 FOLLOWED  3 IS   2
OCCURRENCE OF  1 FOLLOWED  4 IS   0
```

$x_1 = .920$ $x_2 = .440$

```
OCCURRENCE OF  2 IS  156

OCCURRENCE OF  2 FOLLOWED  1 IS   8
OCCURRENCE OF  2 FOLLOWED  2 IS 136
OCCURRENCE OF  2 FOLLOWED  3 IS   0
OCCURRENCE OF  2 FOLLOWED  4 IS  11
```

$y_1 = .923$ $y_2 = .051$

```
OCCURRENCE OF  3 IS    6

OCCURRENCE OF  3 FOLLOWED  1 IS   5
OCCURRENCE OF  3 FOLLOWED  2 IS   0
OCCURRENCE OF  3 FOLLOWED  3 IS   1
OCCURRENCE OF  3 FOLLOWED  4 IS   0
```

$z_1 = .833$ $z_2 = 1.000$

```
OCCURRENCE OF  4 IS   13

OCCURRENCE OF  4 FOLLOWED  1 IS   3
OCCURRENCE OF  4 FOLLOWED  2 IS   6
OCCURRENCE OF  4 FOLLOWED  3 IS   2
OCCURRENCE OF  4 FOLLOWED  4 IS   2
```

$w_1 = .692$ $w_2 = .384$

THE RESULTS FOR DATA SET 27

27

INPUT DATA WAS

```
11333313111133333133333313313333113131333333313333
33331333333331333333333133333333333331333333333133 3
33133333313333333333333333333333333333133333333131
33311111333333333333133333333331333331333333331 3333
```

OCCURRENCE OF	1 IS	34		
OCCURRENCE OF	1 FOLLOWED	1 IS	9	
OCCURRENCE OF	1 FOLLOWED	2 IS	0	
OCCURRENCE OF	1 FOLLOWED	3 IS	25	
OCCURRENCE OF	1 FOLLOWED	4 IS	0	$x_1 = .264$ $x_2 = 1$ $x_2 = 1.000$
OCCURRENCE OF	2 IS	0		
OCCURRENCE OF	2 FOLLOWED	1 IS	0	
OCCURRENCE OF	2 FOLLOWED	2 IS	0	
OCCURRENCE OF	2 FOLLOWED	3 IS	0	
OCCURRENCE OF	2 FOLLOWED	4 IS	0	$y_1 = .000$ $y_2 =$ $y_2 = .000$
OCCURRENCE OF	3 IS	166		
OCCURRENCE OF	3 FOLLOWED	1 IS	24	
OCCURRENCE OF	3 FOLLOWED	2 IS	0	
OCCURRENCE OF	3 FOLLOWED	3 IS	141	$z_1 = .144$ $z_2 =$.
OCCURRENCE OF	3 FOLLOWED	4 IS	0	$z_2 = .994$
OCCURRENCE OF	4 IS	0		
OCCURRENCE OF	4 FOLLOWED	1 IS	0	
OCCURRENCE OF	4 FOLLOWED	2 IS	0	
OCCURRENCE OF	4 FOLLOWED	3 IS	0	$w_1 = .000$
OCCURRENCE OF	4 FOLLOWED	4 IS	0	$w_2 = .000$ $w_2 =$

THE RESULTS FOR DATA SET 28

2'

INPUT DATA WAS

23241414431423231423141423242314241314132314132323
13231323231313231111231123124123123112311231123132
32323111111231111231111231111231111123111111231111
1111231232323111111231123232312311112311111111111

OCCURRENCE OF 1 IS 95

OCCURRENCE OF	1 FOLLOWED	1 IS	57
OCCURRENCE OF	1 FOLLOWED	2 IS	20
OCCURRENCE OF	1 FOLLOWED	3 IS	3
OCCURRENCE OF	1 FOLLOWED	4 IS	9

$x_1 = .810$ $x_2 =$

$x_2 = .684$

OCCURRENCE OF 2 IS 43

OCCURRENCE OF	2 FOLLOWED	1 IS	0
OCCURRENCE OF	2 FOLLOWED	2 IS	0
OCCURRENCE OF	2 FOLLOWED	3 IS	39
OCCURRENCE OF	2 FOLLOWED	4 IS	4

$y_1 = .000$ y_2

$y_2 = .407$

OCCURRENCE OF 3 IS 48

OCCURRENCE OF	3 FOLLOWED	1 IS	31
OCCURRENCE OF	3 FOLLOWED	2 IS	17
OCCURRENCE OF	3 FOLLOWED	3 IS	0
OCCURRENCE OF	3 FOLLOWED	4 IS	0

$z_1 = 1.000$ z_2

$z_2 = .645$

OCCURRENCE OF 4 IS 14

OCCURRENCE OF	4 FOLLOWED	1 IS	7
OCCURRENCE OF	4 FOLLOWED	2 IS	5
OCCURRENCE OF	4 FOLLOWED	3 IS	1
OCCURRENCE OF	4 FOLLOWED	4 IS	1

$w_1 = .857$ w_2

$w_2 = .571$

THE RESULTS FOR DATA SET 29

29 INPUT DATA WAS

31112312411111123111231131111211432323232323232323
23232323232323232323232323232323232323232323232323
23232323232323232323232323232323232323232323232323
23232323232323232323232323232323232323232323232323

OCCURRENCE OF 1 IS 21

OCCURRENCE OF	1	FOLLOWED	1	IS	14
OCCURRENCE OF	1	FOLLOWED	2	IS	5
OCCURRENCE OF	1	FOLLOWED	3	IS	1
OCCURRENCE OF	1	FOLLOWED	4	IS	1

$x_1 = .400$ $x_2 = .7$

$x_2 = .714$

OCCURRENCE OF 2 IS 88

OCCURRENCE OF	2	FOLLOWED	1	IS	1
OCCURRENCE OF	2	FOLLOWED	2	IS	0
OCCURRENCE OF	2	FOLLOWED	3	IS	86
OCCURRENCE OF	2	FOLLOWED	4	IS	1

$y_1 = .011$ $y_2 = .$

$y_2 = .488$

OCCURRENCE OF 3 IS 89

OCCURRENCE OF	3	FOLLOWED	1	IS	5
OCCURRENCE OF	3	FOLLOWED	2	IS	83
OCCURRENCE OF	3	FOLLOWED	3	IS	0
OCCURRENCE OF	3	FOLLOWED	4	IS	0

$z_1 = .488$ $z_2 =$

$z_2 = .056$

OCCURRENCE OF 4 IS 2

OCCURRENCE OF	4	FOLLOWED	1	IS	1
OCCURRENCE OF	4	FOLLOWED	2	IS	0
OCCURRENCE OF	4	FOLLOWED	3	IS	1
OCCURRENCE OF	4	FOLLOWED	4	IS	0

$w_1 = .500$ $w_2 =$

$w_2 = 1.000$

THE RESULTS FOR DATA SET 30

INPUT DATA WAS

30

23132132342231311322213322114113121321112122311131
12212112212211212122112121122212212222222211212111
22124122112241231111112122121421212211212212222121
22222222222212122121222212222211112114221122222222

OCCURRENCE OF 1 IS 76

OCCURRENCE OF 1 FOLLOWED	1 IS	20	
OCCURRENCE OF 1 FOLLOWED	2 IS	39	
OCCURRENCE OF 1 FOLLOWED	3 IS	3	
OCCURRENCE OF 1 FOLLOWED	4 IS	3	$x_1 = .455$ $x_2 =$

$x_2 = .447$

OCCURRENCE OF 2 IS 104

OCCURRENCE OF 2 FOLLOWED	1 IS	40	
OCCURRENCE OF 2 FOLLOWED	2 IS	56	
OCCURRENCE OF 2 FOLLOWED	3 IS	5	
OCCURRENCE OF 2 FOLLOWED	4 IS	2	$y_1 = .423$ $y_2 =$

$y_2 = .432$

OCCURRENCE OF 3 IS 14

OCCURRENCE OF 3 FOLLOWED	1 IS	7	
OCCURRENCE OF 3 FOLLOWED	2 IS	5	
OCCURRENCE OF 3 FOLLOWED	3 IS	1	
OCCURRENCE OF 3 FOLLOWED	4 IS	1	$z_1 = .857$ z_2

$z_2 = .571$

OCCURRENCE OF 4 IS 6

OCCURRENCE OF 4 FOLLOWED	1 IS	3	
OCCURRENCE OF 4 FOLLOWED	2 IS	3	
OCCURRENCE OF 4 FOLLOWED	3 IS	0	
OCCURRENCE OF 4 FOLLOWED	4 IS	0	$w_1 = 1.000$ w_2

THE RESULTS FOR DATA SET 31

$w_2 = .500$

INPUT DATA WAS

31 12212121212121131212111222111211122212121212121112
2111124312122321332112333212121212312121134444444
4443312331214444443233121234444444444344343244312
2331143112244312114444444444444443124444323443122 2

OCCURRENCE OF 1 IS 59

OCCURRENCE OF 1 FOLLOWED	1 IS	17
OCCURRENCE OF 1 FOLLOWED	2 IS	36
OCCURRENCE OF 1 FOLLOWED	3 IS	3
OCCURRENCE OF 1 FOLLOWED	4 IS	3

$x_1 = .898$ $x_2 = .339$

OCCURRENCE OF 2 IS 53

OCCURRENCE OF 2 FOLLOWED	1 IS	29
OCCURRENCE OF 2 FOLLOWED	2 IS	11
OCCURRENCE OF 2 FOLLOWED	3 IS	9
OCCURRENCE OF 2 FOLLOWED	4 IS	4

$y_1 = .754$ $y_2 = .698$

OCCURRENCE OF 3 IS 31

OCCURRENCE OF 3 FOLLOWED	1 IS	12
OCCURRENCE OF 3 FOLLOWED	2 IS	6
OCCURRENCE OF 3 FOLLOWED	3 IS	8
OCCURRENCE OF 3 FOLLOWED	4 IS	5

$z_1 = .580$ $z_2 = .654$

OCCURRENCE OF 4 IS 57

OCCURRENCE OF 4 FOLLOWED	1 IS	0
OCCURRENCE OF 4 FOLLOWED	2 IS	0
OCCURRENCE OF 4 FOLLOWED	3 IS	12
OCCURRENCE OF 4 FOLLOWED	4 IS	45

$w_1 = .000$ $w_2 = .210$

THE RESULTS FOR DATA SET 32

INPUT DATA WAS

44224433412212412231421111222131313111111112222233
3v 33344444444444444421444214212444132131114321111444
44242413414341143221111133111111333131312131111111
13111111121111111111111111111111334333333313333333

OCCURRENCE OF 1 IS 85

OCCURRENCE OF	1 FOLLOWED	1 IS	56		
OCCURRENCE OF	1 FOLLOWED	2 IS	8		
OCCURRENCE OF	1 FOLLOWED	3 IS	14		
OCCURRENCE OF	1 FOLLOWED	4 IS	7	$x_1 = .753$	$x_2 = .823$

OCCURRENCE OF 2 IS 28

OCCURRENCE OF	2 FOLLOWED	1 IS	11		
OCCURRENCE OF	2 FOLLOWED	2 IS	10		
OCCURRENCE OF	2 FOLLOWED	3 IS	2		
OCCURRENCE OF	2 FOLLOWED	4 IS	5	$y_1 = .750$	$y_2 = .464$

OCCURRENCE OF 3 IS 45

OCCURRENCE OF	3 FOLLOWED	1 IS	12		
OCCURRENCE OF	3 FOLLOWED	2 IS	3		
OCCURRENCE OF	3 FOLLOWED	3 IS	24		
OCCURRENCE OF	3 FOLLOWED	4 IS	5	$z_1 = .333$	$z_2 = .800$

OCCURRENCE OF 4 IS 42

OCCURRENCE OF	4 FOLLOWED	1 IS	6		
OCCURRENCE OF	4 FOLLOWED	2 IS	7		
OCCURRENCE OF	4 FOLLOWED	3 IS	5		
OCCURRENCE OF	4 FOLLOWED	4 IS	24	$w_1 = .309$	$w_2 = .262$

THE RESULTS FOR DATA SET 33

INPUT DATA WAS

44444223111334132131444323211313231111131343133312

33

111313321111312111111112111113421113311112111113211
1111113121111111111111111111111111111122111111111
11111111111111111111111113232444211132322432234234

OCCURRENCE OF 1 IS 125

OCCURRENCE OF 1 FOLLOWED	1 IS	101
OCCURRENCE OF 1 FOLLOWED	2 IS	6
OCCURRENCE OF 1 FOLLOWED	3 IS	17
OCCURRENCE OF 1 FOLLOWED	4 IS	1

$x_1 = .?56$ $x_2 = .94?$

OCCURRENCE OF 2 IS 26

OCCURRENCE OF 2 FOLLOWED	1 IS	12
OCCURRENCE OF 2 FOLLOWED	2 IS	4
OCCURRENCE OF 2 FOLLOWED	3 IS	8
OCCURRENCE OF 2 FOLLOWED	4 IS	2

$y_1 = .615$ $y_2 = .769$

OCCURRENCE OF 3 IS 32

OCCURRENCE OF 3 FOLLOWED	1 IS	11
OCCURRENCE OF 3 FOLLOWED	2 IS	11
OCCURRENCE OF 3 FOLLOWED	3 IS	5
OCCURRENCE OF 3 FOLLOWED	4 IS	5

$z_1 = .687$ $z_2 = .500$

OCCURRENCE OF 4 IS 17

OCCURRENCE OF 4 FOLLOWED	1 IS	1
OCCURRENCE OF 4 FOLLOWED	2 IS	5
OCCURRENCE OF 4 FOLLOWED	3 IS	2
OCCURRENCE OF 4 FOLLOWED	4 IS	9

$w_1 = .353$ $w_2 = .17?$

THE RESULTS FOR DATA SET 34

INPUT CATA WAS

44242424321121221211214111111311111132224113222242
13211322424241322441132241313222242242424242411313

11113111321232142311132241311131113111311134113241
34 32224142323231113114131213243231212311211113122242

OCCURRENCE OF 1 IS 78

OCCURRENCE OF 1 FOLLOWED	1 IS	39	
OCCURRENCE OF 1 FOLLOWED	2 IS	10	
OCCURRENCE OF 1 FOLLOWED	3 IS	25	
OCCURRENCE OF 1 FOLLOWED	4 IS	4	$x_1 = .628$ $x_2 = .8$

$x_2 = .820$

OCCURRENCE OF 2 IS 59

OCCURRENCE OF 2 FOLLOWED	1 IS	12	
OCCURRENCE OF 2 FOLLOWED	2 IS	18	
OCCURRENCE OF 2 FOLLOWED	3 IS	7	
OCCURRENCE OF 2 FOLLOWED	4 IS	21	$y_1 = .508$ $y_2 = .3$

$y_2 = .322$

OCCURRENCE OF 3 IS 34

OCCURRENCE OF 3 FOLLOWED	1 IS	16	
OCCURRENCE OF 3 FOLLOWED	2 IS	17	
OCCURRENCE OF 3 FOLLOWED	3 IS	0	
OCCURRENCE OF 3 FOLLOWED	4 IS	1	$z_1 = .970$ $z_2 = .4$

$z_2 = .470$

OCCURRENCE OF 4 IS 29

OCCURRENCE OF 4 FOLLOWED	1 IS	11	
OCCURRENCE OF 4 FOLLOWED	2 IS	14	
OCCURRENCE OF 4 FOLLOWED	3 IS	2	
OCCURRENCE OF 4 FOLLOWED	4 IS	2	$w_1 = .862$ $w_2 =$

THE RESULTS FOR DATA SET 35

$w_2 = .448$

INPUT DATA WAS

11113222222222222212111221111122222222121111112221 1
11111111112222211211222211221221212112222221122112
11111111111111111111112222122222111122222212222112

122211112222122222222221111212222112221122121222211

3

OCCURRENCE OF 1 IS 96

OCCURRENCE OF 1 FOLLOWED 1 IS 65
OCCURRENCE OF 1 FOLLOWED 2 IS 29
OCCURRENCE OF 1 FOLLOWED 3 IS 1
OCCURRENCE OF 1 FOLLOWED 4 IS 0 $x_1 = .479$ $x_2 = .118$

OCCURRENCE OF 2 IS 103

OCCURRENCE OF 2 FOLLOWED 1 IS 30
OCCURRENCE OF 2 FOLLOWED 2 IS 73
OCCURRENCE OF 2 FOLLOWED 3 IS 0
OCCURRENCE OF 2 FOLLOWED 4 IS 0 $y_1 = 1.000$ $y_2 = .291$

OCCURRENCE OF 3 IS 1

OCCURRENCE OF 3 FOLLOWED 1 IS 0
OCCURRENCE OF 3 FOLLOWED 2 IS 1
OCCURRENCE OF 3 FOLLOWED 3 IS 0
OCCURRENCE OF 3 FOLLOWED 4 IS 0 $z_1 = 1.000$ $z_2 = .00$

OCCURRENCE OF 4 IS 0

OCCURRENCE OF 4 FOLLOWED 1 IS 0
OCCURRENCE OF 4 FOLLOWED 2 IS 0
OCCURRENCE OF 4 FOLLOWED 3 IS 0
OCCURRENCE OF 4 FOLLOWED 4 IS 0 $w_1 = .000$ $w_2 = .0$

THE RESULTS FOR DATA SET 36

INPUT DATA WAS

2433244333231312113143131111433311211211112111123
4331123123431321121113121133111131123111114331123 1
14312111211133323331211111111214333444434434434434
4333444334433443344332313231333112111121111111111

3,6

OCCURRENCE OF 1 IS 86

OCCURRENCE OF 1 FOLLOWED 1 IS 53
OCCURRENCE OF 1 FOLLOWED 2 IS 17
OCCURRENCE OF 1 FOLLOWED 3 IS 10
OCCURRENCE OF 1 FOLLOWED 4 IS 5

$x_1 = .814$ $x_2 = .73$

OCCURRENCE OF 2 IS 24

OCCURRENCE OF 2 FOLLOWED 1 IS 13
OCCURRENCE OF 2 FOLLOWED 2 IS 0
OCCURRENCE OF 2 FOLLOWED 3 IS 9
OCCURRENCE OF 2 FOLLOWED 4 IS 2

$y_1 = .541$ $y_2 = .916$

OCCURRENCE OF 3 IS 59

OCCURRENCE OF 3 FOLLOWED 1 IS 20
OCCURRENCE OF 3 FOLLOWED 2 IS 6
OCCURRENCE OF 3 FOLLOWED 3 IS 22
OCCURRENCE OF 3 FOLLOWED 4 IS 11

$z_1 = .440$ $z_2 = .711$

OCCURRENCE OF 4 IS 31

OCCURRENCE OF 4 FOLLOWED 1 IS 0
OCCURRENCE OF 4 FOLLOWED 2 IS 0
OCCURRENCE OF 4 FOLLOWED 3 IS 18
OCCURRENCE OF 4 FOLLOWED 4 IS 13

$w_1 = .000$ $w_2 = .58$

THE RESULTS FOR DATA SET 37

INPUT DATA WAS

4311344443134424224342113332422242242311412113111
3113224222331211313144211131134222414243214133314 2
3111112122421111111311221223221111111223141211211 2
2221421222141111121432211114421212123411112131221 1

37

OCCURRENCE OF 1 IS 83

OCCURRENCE OF 1 FOLLOWED 1 IS 42
OCCURRENCE OF 1 FOLLOWED 2 IS 17
OCCURRENCE OF 1 FOLLOWED 3 IS 13
OCCURRENCE OF 1 FOLLOWED 4 IS 10

$x_1 = .710$ $x_2 = .662$

OCCURRENCE OF 2 IS 57

OCCURRENCE OF 2 FOLLOWED 1 IS 21
OCCURRENCE OF 2 FOLLOWED 2 IS 21
OCCURRENCE OF 2 FOLLOWED 3 IS 6
OCCURRENCE OF 2 FOLLOWED 4 IS 9

$y_1 = .736$ $y_2 = .473$

OCCURRENCE OF 3 IS 29

OCCURRENCE OF 3 FOLLOWED 1 IS 14
OCCURRENCE OF 3 FOLLOWED 2 IS 5
OCCURRENCE OF 3 FOLLOWED 3 IS 5
OCCURRENCE OF 3 FOLLOWED 4 IS 5

$z_1 = .655$ $z_2 = .655$

OCCURRENCE OF 4 IS 31

OCCURRENCE OF 4 FOLLOWED 1 IS 6
OCCURRENCE OF 4 FOLLOWED 2 IS 14
OCCURRENCE OF 4 FOLLOWED 3 IS 5
OCCURRENCE OF 4 FOLLOWED 4 IS 6

$w_1 = .645$ $w_2 = .354$

THE RESULTS FOR DATA SET 38

INPUT DATA WAS

24311331233222331111111113133111111113313131111111111
11111111311111111111111111111111111111111111111111
111111111113111111331111111111111111111111111111111
1111111111111111111111111111111111111111111111111111

38

OCCURRENCE OF 1 IS 177

OCCURRENCE OF 1 FOLLOWED 1 IS 116
OCCURRENCE OF 1 FOLLOWED 2 IS 1
OCCURRENCE OF 1 FOLLOWED 3 IS [illegible]
OCCURRENCE OF 1 FOLLOWED 4 IS [illegible]

$x_1 = .943$ x_2 .730

OCCURRENCE OF 2 IS 4

OCCURRENCE OF 2 FOLLOWED 1 IS 1
OCCURRENCE OF 2 FOLLOWED 2 IS 1
OCCURRENCE OF 2 FOLLOWED 3 IS 2
OCCURRENCE OF 2 FOLLOWED 4 IS 1

$y_1 = .250$ $y_2 = .500$

OCCURRENCE OF 3 IS 18

OCCURRENCE OF 3 FOLLOWED 1 IS 11
OCCURRENCE OF 3 FOLLOWED 2 IS 1
OCCURRENCE OF 3 FOLLOWED 3 IS 6
OCCURRENCE OF 3 FOLLOWED 4 IS 0

$z_1 = .666$ $z_2 = .944$

OCCURRENCE OF 4 IS 1

OCCURRENCE OF 4 FOLLOWED 1 IS 0
OCCURRENCE OF 4 FOLLOWED 2 IS 0
OCCURRENCE OF 4 FOLLOWED 3 IS 1
OCCURRENCE OF 4 FOLLOWED 4 IS 0

$w_1 = .000$ $w_2 = 1.000$

THE RESULTS FOR DATA SET 39

INPUT DATA WAS

22212222213223113141311111312122214414121424131211
31131331131222221211211442113111122321413111111111
31121111121322111122212111222211221111211222212211
21121112222111121122221222112221221121111111211112

OCCURRENCE OF 1 IS 104

OCCURRENCE OF 1 FOLLOWED	1 IS	56
OCCURRENCE OF 1 FOLLOWED	2 IS	29
OCCURRENCE OF 1 FOLLOWED	3 IS	13
OCCURRENCE OF 1 FOLLOWED	4 IS	6

$x_1 = .817$ $x_2 = .663$

OCCURRENCE OF 2 IS 71

OCCURRENCE OF 2 FOLLOWED	1 IS	31
OCCURRENCE OF 2 FOLLOWED	2 IS	35
OCCURRENCE OF 2 FOLLOWED	3 IS	[illegible]
OCCURRENCE OF 2 FOLLOWED	4 IS	1

$y_1 = .943$ $y_2 = .464$

OCCURRENCE OF 3 IS 16

OCCURRENCE OF 3 FOLLOWED	1 IS	12
OCCURRENCE OF 3 FOLLOWED	2 IS	3
OCCURRENCE OF 3 FOLLOWED	3 IS	1
OCCURRENCE OF 3 FOLLOWED	4 IS	0

$z_1 = .937$ $z_2 = .812$

OCCURRENCE OF 4 IS 9

OCCURRENCE OF 4 FOLLOWED	1 IS	5
OCCURRENCE OF 4 FOLLOWED	2 IS	2
OCCURRENCE OF 4 FOLLOWED	3 IS	0
OCCURRENCE OF 4 FOLLOWED	4 IS	2

$w_1 = .777$ $w_2 = .555$

THE RESULTS FOR DATA SET 4C

INPUT DATA WAS

2312132111341434132112311121121213111214413241 1411
3221111131223112112322311111312111131122111111312
1212112113111131111111121111133331113333333333111
11111111123111122442444241324442442334422341222134

OCCURRENCE OF 1 IS 100

40

OCCURRENCE OF 1 FOLLOWED 1 IS 62
OCCURRENCE OF 1 FOLLOWED 2 IS 17
OCCURRENCE OF 1 FOLLOWED 3 IS 16
OCCURRENCE OF 1 FOLLOWED 4 IS 3

$x_1 = .810$ $x_2 = .7$

OCCURRENCE OF 2 IS 39

OCCURRENCE OF 2 FOLLOWED 1 IS 17
OCCURRENCE OF 2 FOLLOWED 2 IS 8
OCCURRENCE OF 2 FOLLOWED 3 IS 3
OCCURRENCE OF 2 FOLLOWED 4 IS 6

$y_1 = .641$ $y_2 = .64$

OCCURRENCE OF 3 IS 39

OCCURRENCE OF 3 FOLLOWED 1 IS 14
OCCURRENCE OF 3 FOLLOWED 2 IS 6
OCCURRENCE OF 3 FOLLOWED 3 IS 14
OCCURRENCE OF 3 FOLLOWED 4 IS 5

$z_1 = .512$ $z_2 = .718$

OCCURRENCE OF 4 IS 22

OCCURRENCE OF 4 FOLLOWED 1 IS 7
OCCURRENCE OF 4 FOLLOWED 2 IS 5
OCCURRENCE OF 4 FOLLOWED 3 IS 1
OCCURRENCE OF 4 FOLLOWED 4 IS 9

$w_1 = .545$ $w_2 = .36$

APPENDIX 2

THE APPARATUS

ASD TECHNICAL REPORT 61-239

A GAME THEORY APPARATUS FOR PSYCHOLOGICAL RESEARCH

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JULY 1961

PROJECT No. 7183
TASK No. 71618

AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

ABSTRACT

This report describes an electrical apparatus designed to facilitate psychological research in games of strategy for game matrices no larger than 3 x 3.

The description includes an operational procedure and an explanation of the circuitry with an accompanying photograph of the equipment and simplified schematic.

The apparatus provides push-button selection of matrix columns by one subject and rows by his opponent who may be another subject or the experimenter. These selections determine numerical payoff values of varying desirability for the players. This conflict situation allows the psychologist to study various aspects of human behavior, such as strategy formation.

PUBLICATION REVIEW

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A GAME THEORY APPARATUS FOR PSYCHOLOGICAL RESEARCH

INTRODUCTION

The matrix game apparatus is an electrically operated device designed for use in psychological research involving games of strategy. It consists of two subject panels connected by electrical cables to an experimenter's console (figure 1). In normal use the subjects would be isolated from each other and from the experimenter. Each subject panel has a matrix display consisting of nine squares arranged in a 3 x 3 configuration, that is, 3 rows and 3 columns. The experimenter's console has two such matrices.

Each square of a matrix contains a payoff value. Three push buttons are located below each matrix so that one player selects which of the 3 columns the payoff shall be in and the other player selects one of the three rows. The square at the intersection of opponent A's column and opponent B's row is the payoff for this particular pair of selections. This square then lights up on each player's panel and on the experimenter's console. This shows each player and the experimenter the outcome and allows the experimenter to total this payoff with the cumulative score from previous payoffs and display the new score on each player's panel by means of the "nixie" numerical indicators.

Two modes of play are possible. In the S (Subject's) mode the subjects play each other. In the E (Experimenter's) mode each subject plays the experimenter. Modes may be changed without the subject's knowledge by means of a switch on the experimenter's console.

As an example of play in the S mode, subject A may be told to attempt to accumulate as large a positive score as possible and subject B a negative score. Therefore, A's first choice might be column 3 (figure 2) with a good chance of a positive payoff and the possibility of only a small negative payoff. B's first choice might be to try for the -4 by choosing row 2. To make the panels just alike and for simplicity of instruction, the columns of the B matrices are actually rows of the A matrices so that, although both A and B appear to select columns, B is actually selecting rows on A's matrix. Column 3 and row 2 intersect at the +3 which lights up and shows A to be the winner of this round.

The +3 remains backlighted for a period determined by the built-in timer, T_1 (left timer viewed from the front). The other timer determines the lapse after display of the payoff before the next round of competition may begin as signaled by the ready lights.

The game values are easily changed as described later.

Silent or nearly silent switches are used throughout so that audible cues do not give away the fact that the experimenter has entered the game. For this same reason the relays are shock-mounted in a sound-deadened compartment.

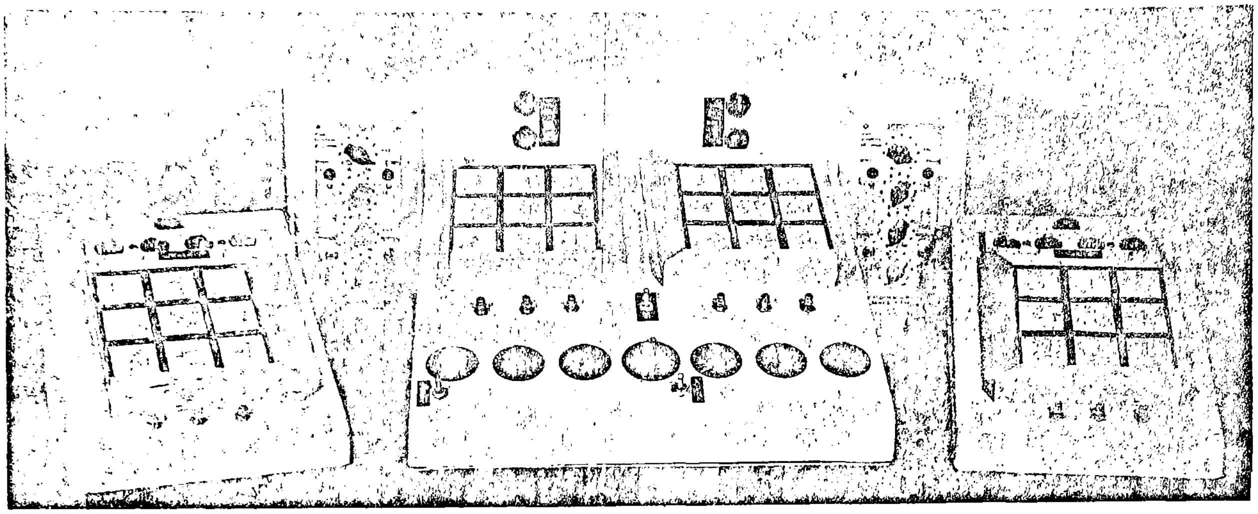

Figure 1. Matrix Game Apparatus

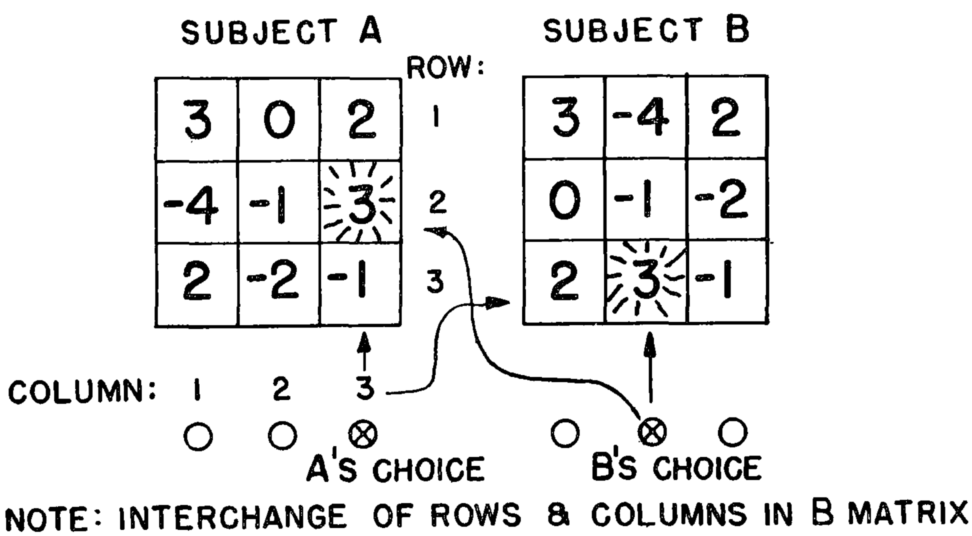

Figure 2. Example of Selection and Payoff For One Round of Play

OPERATIONAL PROCEDURE

POWER

The on-off switch is located on the experimenter's console. This switch controls power to the entire apparatus, including the timers' power switches which may therefore be left on. The timer interval switch is left off since control is external to the timer.

MODES

The two-position mode switch on the experimenter's console determines whether the subjects play each other (S) or whether they play the experimenter (E).

Mode S - to begin play, one subject merely presses one of the push buttons to select which column he chooses. After a column has been selected, the ready light for that particular matrix goes out and no changes of choice can be made due to lockout circuitry. When the other subject has made his choice, the payoff square is illuminated on each subject's panel and on the left-hand matrix of the console. The period of illumination is set on timer T_1. At the end of T_2 the ready lights come back on and another round of play may begin.

Mode E - with the mode switch in position, mode E play is similar to mode S, except the subjects are each playing the experimenter and a response must be made at all four matrices to start the timers and display the payoff.

SCORING

Subject mode during the interval T_1 the experimenter adds the payoff shown on the illuminated square of his matrix to the accumulated score shown on each set of scoring switches on the experimenter's console. This score is relayed to the "nixie" tubes on each subject's panel so each may know his cumulative score.

Experimenter mode - the only difference is that there are two different scores to keep. Subject A's payoff is shown on the left-hand matrix of the console and subject B's on the right-hand matrix.

Scores may be accumulated up to 2,999 for each subject. To eliminate negative values the game may be started with a "pot," say 1,000 points, on the nixie tubes. Scoring of ± 999 is possible with a slight wiring change and a ± nixie substitution.

TIMING

T_1 - the time set on timer 1 determines how long the payoff square shall be illuminated (see figure 3). This time period is the information feedback interval.

T_2 - the time set on timer 2 (which must always be longer than T_1) determines how long all ready lights shall be out after the last player has made his choice, that is, the time between the end of one round and the beginning of another.

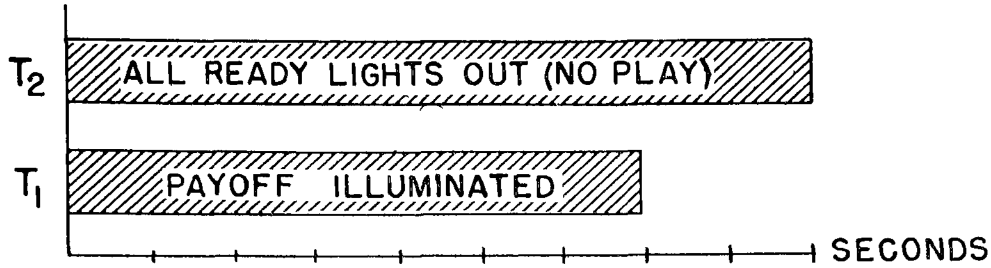

Figure 3. Timer Relationships

MATRIX DISPLAY

The matrix values are easily changed by sliding the display out of its slotted frame and changing or replacing the vellum sheet on which the payoff values are printed. By masking unused squares, 2 x 2 and 2 x 3 games can be played.

CIRCUITRY

Each matrix has associated with it three push-button switches, one for each column; three double-pole, double-throw, 115-volt A.C. relays; and two 24-volt D.C. D.P.D.T. relays connected in parallel to form in effect one 4.P.D.T. relay (figure 4). The push buttons are connected in such a way that pushing more than one button causes only the left-most button pushed to perform. Each 115-volt relay is operated by one pole of its associated push button. The other pole of the push-button switch supplies power to the 24-volt relays.

The 115-volt relay holds itself with one set of contacts and with the other set completes the circuit to: (a) one side of a column of lights and (b) one side of a row of lights in the opponent's matrix.

The 24-volt relays: (a) break the circuit to the contacts of the push buttons which energize the 115-volt relays so a player may not respond again (the capacitor delays this action allowing the 115-volt relay time to make and hold), (b) turn off the ready light, (c) complete this portion of the timer start circuit, and (d) hold themselves.

The timers are commercial models 111C made by the Hunter Mfg. Co. To start a timer, terminal 1 is connected to terminal 2. This connection for timer 1 is made through a series of 4 relay contacts (timer ready contacts), one at each matrix. In mode E a response must be made at all four matrices to complete this connection. However, in mode S the timer ready contacts in the console are bypassed by the mode switch and only the two subject responses are required. The start relay on the first timer starts the second timer so both start nearly simultaneously. At the end of T_1 the finish relay in the first timer operates. This relay opens the circuit to all 115-volt relay coils, thus releasing those which were held which in turn extinguishes the payoff lights.

At the end of T_2 the finish relay in the second timer: (a) releases six of the eight 24-volt relays, discharging their respective capacitors, and (b) energizes the auxiliary finish relay.

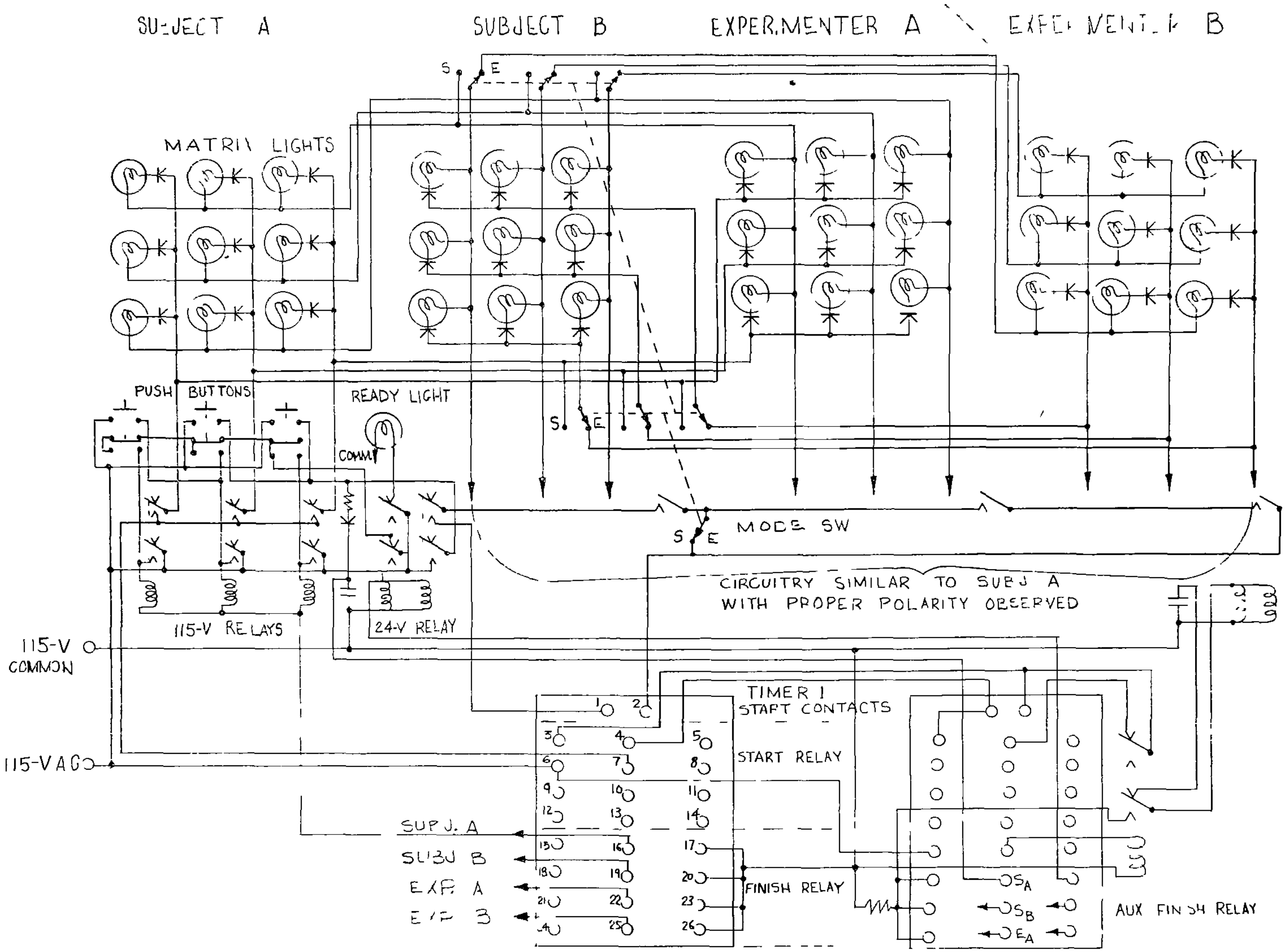

Figure 4. Simplified Schematic Diagram

The auxiliary relay: (a) releases the remaining two 24-volt relays and (b) opens the second timer hold circuit (held through the start relay and normally closed contacts of the auxiliary relay) to ready it for another cycle.

Mode switching is accomplished by means of a 7-pole, 2-position switch. Three poles switch subject A's matrix from subject B's response to experimenter A's response, and three switch subject B's matrix from subject A's response to experimenter B's response. The remaining pole bypasses the timer ready contacts in the console during mode S play.

Series-parallel paths, allowing lights other than the one at the intersection of the selected row and column to light dimly, are avoided by routing the current for each light in a matrix through a silicon diode.

The nixie indicators on each subject panel are wired through a cable to a group of three rotary switches and a toggle switch on the experimenter's console. These switches route the B+ voltage from a full-wave, voltage-doubler power supply to the appropriate electrodes of the nixie tubes. The first nixie on the left is wired only for 1 and 2. Plus and minus indications can be obtained rather than 1 and 2 by substituting a ± nixie and adding a jumper.

APPENDIX 3

ABSTRACT OF

The Regular Markov Chain Model Applied to Game-Playing Behaviour in Rapoport's "Archetypes" of the 2x2 Non-Zero-Sum Game

APPENDIX 3

ABSTRACT OF

<u>The Regular Markov Chain Model Applied to Game-Playing Behaviour in Rapoport's "Archetypes" of the 2x2 Non-Zero-Sum Game</u>[1]

In this study, an attempt was made to apply the regular Markov chain model to the underlying interplay of motives operating in four of the 2x2 non-zero-sum games discussed by Rapoport as "Exploiter," "Leader," "Hero" and "Martyr." On the assumption that the underlying motivational process was directly responsible in determining the overt responses or outcomes on each trial of the game, the presumed motivational "pressures" were quantified in terms of state-conditioned variables forming a motivational matrix P with transition probabilities. Assuming that P was a matrix of an ergodic Markov process, the whole motivational process was then modelled as a finite Markov chain process from which predictions were made as to the actual distribution of outcomes in each game by calculating the limiting probability vector α for the chain.

The sample consisted of forty male graduate students from the Faculties of Psychology and Education, University of

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1. M.A. Wozny, doctoral thesis presented to the Faculty of Psychology of the University of Ottawa, Ontario, April, 1969, ix-60 p.

Ottawa. The subjects were divided into dyads and randomly assigned to play one of the four games. Ten pairs were run for a total of two hundred consecutive trials on an electrically designed game theory apparatus, consisting of two subject panels connected to an experimenter's console and providing each subject with a two-choice push-button selection of matrix rows and columns.

The games were played under condition of perfect information where each subject knew, at the end of each trial, the amount of his pay-off and the other person's pay-off. In addition, each subject was given a continuous comulative feedback of his sum total score at the end of each trial, but no feedback as to the cumulative total of the other subject. All subjects played under "free-play" conditions; that is, under no predetermined strategy coming from the experimenter. Consecutive occurrence of the choice-outcomes were recorded so as to obtain a continuous history of the process for analysis.

The actual game-playing behaviour approximated the predicted behaviour of the players for games Exploiter, Leader and Martyr, but the players' game-behaviour for Hero could not be predicted from the Markov chain model. It was suggested that the motives operating in Hero were much more complicated than Rapoport's view of a polar "pressure" to switch or not to switch from the natural outcome of the game, and that the fit between predicted and observed values could perhaps be more closely approximated by a model which would combine the stochastic characteristics of the regular Markov chain with the principles of the Bush-Mosteller learning model. This was suggested as a possible direction in which to expand the present study.

ADDENDUM TO EQUATIONS 4 AND 5 (p.29)
AND FIGURES 3 AND 4 (p.31)
OF CHAPTER II

Note that both equations (4) and (5) as shown in the text are in their <u>general</u> form and, therefore, their connection with figures 3 and 4 respectively may not be obvious on first glance. Note, however, that for player 1, equation (4) is actually the following:

$$x_1 = \Pr(B_1 | A_1 A_2)$$
$$y_1 = \Pr(B_1 | A_1 B_2)$$
$$z_1 = \Pr(B_1 | B_1 A_2)$$
$$w_1 = \Pr(B_1 | B_1 B_2)$$

and for player 2, it is:

$$x_2 = \Pr(B_2 | A_2 A_1)$$
$$y_2 = \Pr(B_2 | A_2 B_1)$$
$$z_2 = \Pr(B_2 | B_2 A_1)$$
$$w_2 = \Pr(B_2 | B_2 B_1)$$

Since a transition from state to state is conceptualized as depending on what both players do, the entries of figure 3 are constructed in terms of the above equations. The transitions from state AA to any other state are quite straight forward and obvious. So are the transition probabilities from BB to each of the other three. The transitions from AB and BA (rows 2 and 3, respectively) are not so obvious and need to be demonstrated. For example, to go from AB to BB, the probability of player 1 playing B following AB (that is, A_1B_2) is y_1, and the probability of player 2 playing B following AB (that is, A_1B_2) is z_2. Hence, the joint probability of transition is, therefore, y_1z_2 (the last cell entry in the second row of figure 3). Similarly, in calculating the other transition probabilities in row two. Now, to go from BA to BB, player 1 must play B following BA (i.e., B_1A_2) with probability z_1, and player 2 must play B following BA (i.e., B_1A_2) with probability y_2. Hence, the joint transition probability from state BA to BB is z_1y_2 (the last cell entry in the third row of figure 3). Similarly for the other entries of row three.

Equation (5) can be similarly expanded for player 1 and player 2. For player 1, equation (5) is:

$$x_1 = Pr\ (A_1 | A_1 A_2)$$
$$y_1 = Pr\ (A_1 | A_1 B_2)$$
$$z_1 = Pr\ (A_1 | B_1 A_2)$$
$$w_1 = Pr\ (A_1 | B_1 B_2)$$

For player 2, this is:

$$x_2 = Pr\ (A_2 | A_2 A_1)$$
$$y_2 = Pr\ (A_2 | A_2 B_1)$$
$$z_2 = Pr\ (A_2 | B_2 A_1)$$
$$w_1 = Pr\ (A_2 | B_2 B_1)$$

Thus, for example, to go from AB to AA the probability of player 1 playing A following AB (i.e., A_1B_2) is y_1, and the probability of player 2 playing B following AB (i.e., A_1B_2) is z_2. Therefore, the joint transition probability from AB to AA is y_1z_2 (the first cell entry in row two of figure 4). One further example: to go from BA to BB, player 1 plays B following BA (or B_1A_2) with probability that is the complement of z_1, that is, $(1 - z_1)$ and player 2 plays B following BA (or B_1A_2) with probability that is the complement of y_2, that is, $(1 - y_2)$. Hence, the joint transition probability from BA to BB is $(1 - z_1)(1 - y_2)$ as given by the last entry in row three of figure 4. Similar reasoning is used to obtain the other entries.

A specific reference which uses a transition matrix somewhat akin to that of figure 3 and 4, but without presenting the logic of how it was constructed, is the study by Amnon Rapoport and Abbe Mowshowitz, "Experimental Studies of Stochastic Models for the Prisoner's Dilemma", Behavioral Science, Vol. 11, No. 6, 1966, p. 444-458.

THE REGULAR MARKOV CHAIN MODEL APPLIED TO GAME-PLAYING BEHAVIOUR IN RAPOPORT'S "ARCHETYPES" OF THE 2X2 NON-ZERO-SUM GAME

by M.A. Wozny

ERRATA

p.2 line 4: Morgentern should read Morgenstern

p.4 for Player 2: A_1 should read A_2

p.9 in reference 17: Dobb should read Doob

p.29 in eq'ns. (4) and (5): P should read Pr

p.30 line 3: B_i should read B_1

line 5: A_i should read A_1

p.35 line 6: "forty"should read'eighty"

p.58 Appendix 3, line 16: "forty" should read "eighty"

line 3: before "2x2" insert "seventy-eight"

p.59 line 2: insert "per game matrix" after "Ten pairs were run"

line 11: "comulative" should read "cumulative"

ADDENDUM TO EQUATIONS 4 & 5 (p.29) AND FIGURES 3 AND 4 (p.31) OF CHAPTER II

In equations (4) and (5), we are letting a B response (the tendency to switch from the AA outcome) and an A response (not to switch) of a player, respectively, depend only on what happened on the previous play. Under this condition for a somewhat similar two-choice two-person situation as that of the present study, Rapoport and Chammah[1] show that it is possible to conceptualize this dependence in either of three ways: (1) on what the player in question did, (2) on what the other player did, and (3) on what both of them did.

If the dependence is conceptualized as either (1) or (2), Rapoport and Chammah use "response-conditioned propensities" on pages 67-69 and 116-119 to define their transition matrices. If, on the other hand, the dependence is conceptualized in terms of (3), as it is in this study, then it is better to use the so-called "state-conditioned propensities" to define the transition matrix. The state-conditioned variables x_i, y_i, z_i and w_i defining equations(4) and (5) of the present study are similar in concept to the "state-conditioned propensities" of these authors, though without the "psychological" meanings they attach to them. For comparison, see pages 71-73 of their book.

With respect to the transition matrices of figures 3 and 4, the 16 transition probabilities of figure 3 are defined in terms of the tendency to switch from AA and those of figure 4, in terms of the tendency <u>not</u> to switch from the AA outcome. Both figures represent in matrix form a system of four simultaneous equations in four unknowns and are <u>identical</u> in form to equations (61), (62), (63) and (64) derived by Rapoport and Chammah for their general case, four-state Markov model as given on page 121 of their book. For discussion and derivation, see pages 105-123. The entries of figure 4 are the respective terms of Rapoport and Chammah's equations, since both are defined in terms of the same response A; for figure 3 the entries are the complement values of these terms, since the definition is in terms of the other response, that is, B.

<u>References</u>:

1 A. Rapoport and A. Chammah, <u>Prisoner's Dilemma</u>: A Study in Conflict and Cooperation, University of Michigan Press, Ann Arbor, 1965, xii-258 p.

A.W.