THE FLOW OF NEWTONIAN AND NON-NEWTONIAN FLUIDS THROUGH PACKED BEDS

by

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I-ABSTRACT

The viscous flow of a Newtonian fluid and four non-Newtonian fluids (Ostwald-de-Waele type) through a bed of spheres and a bed comprised of irregular aluminum oxide particles was investigated in order to test the validity of a proposed generalised Kozeny-Carman equation. The following relation deduced from the generalized form of the Kozeny-Carman equation and the experimental measurements was found to correlate the data for randomly packed beds of irregular particles

$$f = \frac{4^{2+n'} \left(\frac{1.14 + 1.36\, n'}{3n' + 1}\right)^{n'}}{Re''}$$

When the flow behavior index, n', is unity, in the case of Newtonian fluids, this equation reduces to the well-known Kozeny-Carman equation, as expected.

II- INTRODUCTION

The flow of fluids through packed beds has been extensively investigated in the past thirty years. Several different methods for correlation of the data have been proposed. Among them, the Kozeny-Carman equation, utilizing the porosity of the bed and the hydraulic radius as the characteristic length, has been most widely used for the determination of the relationship between pressure drop and flow rate in the flow of Newtonian fluids through porous beds.

In view of the frequency with which non-Newtonian fluids are encountered in the chemical industry, their importance in chemical engineering has been well recognized and many efforts have been made to evaluate the behavior of non-Newtonian fluids. The present need for extension of the data to non-Newtonian fluids is apparent.

The purpose of the current investigation was to modify the Kozeny-Carman equation in order to extend its applicability to both kinds of fluids. It is expected that this modified equation will prove useful in the extension of the filtration equations to separations involving slurries comprised of particles in non-Newtonian fluids.

III- THEORETICAL CONSIDERATIONS AND LITERATURE REVIEW

A. NON-NEWTONIAN FLUIDS

Fluids may be classified according to two main categories: Newtonian and non-Newtonian. The former is characterized by its viscosity, defined as the ratio of shear stress to shear rate in isothermal laminar flow, which is independent of the shear rate and constant. The latter can be further subdivided into three types: time-independent, time-dependent, and viscoelastic fluids. The time-independent non-Newtonian fluids are of most concern in current investigations.

These fluids are conventionally subdivided into three distinct groups:

(1) Bingham plastic fluids;

(2) pseudoplastic fluids;

(3) dilatant fluids.

The pseudoplastic fluids are the most commonly encountered of the above three.

Rabinowitsch (1) and Mooney (2) developed an expression based on the assumptions that no slip occurs at the wall of the conduit, flow is laminar and the fluid is time-independent. The expression is

$$\left(-\frac{du}{dr}\right)_o = \frac{3}{4}\left(\frac{8U}{D_o}\right) + \frac{1}{4}\left(\frac{8U}{D_o}\right)\frac{d\ln\left(\frac{8U}{D_o}\right)}{d\ln\tau_o} \tag{1}$$

from which it is concluded that a unique relationship between τ_o and $\frac{8U}{D_o}$ exists for any time-independent fluid flowing in a circular conduit.

Metzner and Reed (3) utilized this result by defining a coefficient n' as follows:

$$n' = \frac{d\ln\tau_o}{d\ln\left(\frac{8U}{D_o}\right)} \tag{2}$$

Eq. (1) may be rewritten in terms of n' as follows:

$$\left(-\frac{du}{dr}\right)_o = \frac{3n'+1}{4n'}\left(\frac{8U}{D_o}\right) \tag{3}$$

An additional coefficient K' is defined by

$$\tau_o = K'\left(\frac{8U}{D_o}\right)^{n'} \tag{4}$$

n' and K' are referred to as the flow behavior and fluid consistency indices, respectively. They may be evaluated with a capillary-tube viscometer. The slope of the curve obtained in a logarithmic plot of $\tau_o = \frac{D_o}{4}\left(\frac{-dp'}{dx}\right)$ vs. $\frac{8U}{D_o}$ yields n' and Eq. (4) can be used to evaluate K'. If the curve obtained in such a plot is a straight line, n' is constant. If not, n' is the slope of the tangent to the curve at the point and will, clearly, have different values at different shear rates.

The Fanning friction factor f is a factor of proportionality defined as the ratio of resisting force per unit area of solid surface to the kinetic energy of the fluid per unit volume (4) and is given as follows:

$$f = \frac{D_o}{4} \left(-\frac{dp'}{dx}\right) / \frac{\rho U^2}{2 g_c} \tag{5}$$

It is related to the Reynolds number, Re, as follows:

$$f = \frac{16}{Re} \tag{6}$$

Eq. (6) is valid for the laminar flow of Newtonian fluids in pipes. Metzner and Reed (3) used this equation to arrive at a generalized Reynolds number, Re', which may be applied to both Newtonian and time-independent non-Newtonian fluids.

Eq. (6) for the general case involving time-independent non-Newtonian fluids becomes

$$f = \frac{16}{Re'} \tag{7}$$

The expression for the generalized Reynolds number is obtained by combining Eqs. (4) and (6) to yield

$$Re' = \frac{D_o^{\;n'} U^{2-n'} \rho}{K' 8^{n'-1}} \tag{8}$$

B. VISCOMETRY

The design of the capillary-tube viscometer has not been standardised, and the researcher is free to design his own instrument. This instrument usually falls in one of two groups, according to the driving force used to cause the fluid to pass through the capillary tube

(1) Extrusion rheometer

(2) Orr-Dalla Viscometer

The viscometer used for the present work is a modification of the Orr-Dalla model which is considered to have a number of advantages. A schematic drawing of the arrangement of the viscometer is presented in Fig. 2 with the accompanying legend in Table 2.

The constant temperature bath C_4 is filled with water. The temperature of the bath is capable of being controlled to within $\pm$ 0.1°C to enable maintaining a constant temperature of the fluid inside tank C_3, which is of paramount importance. The effect and importance of temperature variations upon the determinations of n' and K' has been shown (5), (6).

A number of capillary tubes of different diameters were used. Curves of the logarithmic plots of τ_o vs. $\frac{8U}{D_o}$ are independent of tube diameter, provided there is no slip at the tube wall and the flow is laminar (7). The effect of slip on the curves obtained has been investigated (5).

Briefly, at least two determinations were made on each fluid using two capillary tubes of different diameters and the results plotted to assure that they merged on a single curve. The merged curves so obtained were used in the determinations of n' and K'.

It is known that capillary tubes of large diameters should be used to minimize the effect of slip. There is no general criterion for the determination of the optimum tube diameter, and in practice, there is a restriction on the smallest diameter of the capillary tube to be used for each individual fluid. It is, therefore, necessary that several capillary tubes be available for data taking.

The determinations of the inside diameters of the capillary tubes were made by a calibration procedure since direct measurements were impracticable (8).

The main difficulty encountered in capillary-tube viscometry is in the correction of the measured total pressure drop. Corrections (9) are necessary to take into consideration:

(1) the head of liquid over the tube;

(2) kinetic energy effects;

(3) entrance losses.

The head of liquid over the tube can be maintained constant and therefore no correction is involved in this case. The kinetic energy of the fluid is increased when the fluid flows from tank C_3 into the tube where the velocity of the fluid is greater. The entrance loss is also associated with this change in kinetic energy of the liquid.

Mooney and Black (10) observed that the entrance loss of pseudoplastic fluids is up to seven times that for comparable Newtonian fluids. If we consider the flatness of the velocity profiles of these fluids, the reverse should be true. Dodge (11) suggested that the correction for entrance and kinetic energy effects combined is approximately equal to 1.5 $\frac{\rho U^2}{g_c}$.

One way of circumventing this problem is to design the capillary tube with a high ratio of length to diameter. Winding (7) found that if the ratio is greater than 150, the correction can be safely neglected.

C. MODIFICATION OF KOZENY-CARMAN EQUATION

The Kozeny-Carman equation expresses the relationship

$$f = \frac{5}{Re} \tag{9}$$

where,

$$f = \frac{r_h g_c}{\rho U^2} \left(\frac{-dp'}{dx}\right) \tag{10}$$

and

$$Re = \frac{r_h U \rho}{\mu} \tag{11}$$

The hydraulic radius is related to the diameter of a tube of circular cross-section by

$$r_h = \frac{D_o}{4} \tag{12}$$

If Eq. (12) is substituted into Eq. (5), one obtains

$$f = \frac{2 r_h g_c}{\rho U^2} \left(- \frac{dp'}{dx}\right) \tag{13}$$

and

$$Re = \frac{D_o U \rho}{\mu} = \frac{4 r_h U \rho}{\mu} \tag{14}$$

If Eqs. (10) and (11) are compared with Eqs. (13) and (14), respectively, it is seen that the constant coefficients 2 and 4 do not appear in the former two equations.

If Eqs. (13) and (14) are used as a basis for the definitions of f and Re in the present work, the Kozeny-Carman equation then becomes

$$f = \frac{40}{Re} \tag{15}$$

The hydraulic radius for beds of irregular particles is found to be (12)

$$r_h = \frac{\epsilon}{(1-\epsilon)\; S_o} \tag{16}$$

If Eq. (16) is substituted into Eqs. (13) and (8), the Fanning friction factor f and Re' become, respectively,

$$f = \frac{2\,\epsilon\; g_c}{(1-\epsilon)\,\rho\, U^2\, S_o} \left(-\frac{dp'}{dx}\right) \tag{17}$$

and

$$Re' = \frac{\left(\frac{4\,\epsilon}{S_o\,(1-\epsilon)}\right)^{n'} U^{2-n'}\,\rho}{8^{n'-1}\,K'} \tag{18}$$

The Dupuit relation (13a) relates the velocities as follows:

$$U = \frac{U_s}{\epsilon} \tag{19}$$

If Eq. (19) is substituted into Eqs. (17) and (18), respectively, one obtains

$$f = \frac{2\,\epsilon^3\, g_c}{S_o\,(1-\epsilon)\,\rho\, U_s^2} \left(-\frac{dp'}{dx}\right) \tag{20}$$

and

$$Re'' = \left(\frac{\epsilon}{(1-\epsilon)\,S_o}\right)^{n'} \frac{U_s^{2-n'}\,\rho}{2^{n'-3}\;{}^{2-n'}K'} \tag{21}$$

It is assumed that the flow through packed beds of time-independent non-Newtonian fluids of the Ostwald-de-Waele type is described by the relation

$$f = \frac{k}{Re''} \tag{22}$$

Kozicki (15) has proposed that k is given by the following relation

$$k = 16 \left(\frac{4(a + bn)}{3n + 1}\right)^{n} \tag{23}$$

The two coefficients a, b in Eq. (23) are geometric parameters characteristic of the bed and must be evaluated experimentally.

IV-EXPERIMENTAL ASPECTS

A. DETERMINATION OF THE DENSITY OF THE LIQUIDS AND OF THE PARTICLES COMPRISING THE PACKED BEDS

The density of a liquid under investigation was determined by weighing a known volume of the liquid. The weight divided by the volume yielded the specific weight and subsequently the density.

The density of the particles was readily obtained by obtaining the weight of a given amount of the particles then putting these particles into a titration buret containing a known volume of water. The weight of the particles divided by the volume of water displaced by the particles yielded the density of the particles.

Separate determinations were made on each system of particles. The average value determined for the density of the plastic spheres was 1.261 g_m per c.c., and for the aluminum oxide particles 3.930 g_m per c.c.

B. THE DETERMINATION OF THE SPECIFIC SURFACE OF PARTICLES, $\frac{1}{S_o}$

The $\frac{1}{S_o}$ for a sphere is equal to $D_p/6$, where D_p is the diameter of the sphere. The plastic spheres used in the present work were found to have an average diameter of 0.3480 cm., which was determined by randomly picking up one hundred spheres, their diameters being subsequently measured with a micrometer.

Another bed used in this investigation was comprised of irregular aluminum oxide particles. The $\frac{1}{S_o}$ for these irregular particles may be determined by the permeability method (13b), which utilizes the following equation.

$$\left(\frac{1}{S_o}\right)^2 = \frac{5\,(1-\epsilon)^2\,\mu}{g_c\,\epsilon^3}\;\frac{U_s}{\frac{dp'}{dx}} \qquad (24)$$

Five sets of data consisting of U_s vs. dp'/dx measurements (Table 6) were used to yield an average value of 0.0245 cm for the $\frac{1}{S_o}$ of these particles.

The experimental method of obtaining this data will be described fully in this chapter.

C. DESCRIPTION, ASSEMBLY AND OPERATION OF PACKED BED

The flow diagram illustrating the packed column and its accessories is shown in Fig. 1. This diagram gives only the relative positions of the parts and is not drawn to scale. Some pieces of equipment and lines are not shown to make the diagram clearer. The dimensions and materials of construction of each part are presented in detail in Table 1.

Flange F was dismantled and a known amount of aluminum oxide particles, which were previously screened, was slowly poured into column C_2. While pouring, the wall of C_2 was subjected to vibrations, by knocking with a rubber hammer, to ensure an even packed bed. The height of the packed bed was measured so as to be able to calculate the porosity of the bed. In all cases, at least two inches of bed was provided and maintained above tap T_2. When the packing was in place, flange F was connected.

Valve V_2 was closed, V_1 opened, and about 3,000 c.c. solution was poured slowly into the spare tank C_3. V_1 was regulated so as to control the rate of filling of the column. This procedure was carried out slowly to permit the air entrapped inside the voids of the bed to be purged out. If any air bubbles were present inside manometer M and the Tygon tubing between M and taps T_1 and T_2, the flange of manometer M was loosened to release the air bubbles.

When the height of the liquid level attained inside the column was about five inches above the surface of the bed, V_1 was closed and the liquid was poured slowly into tank C_1 until the whole system was full of liquid.

During a run, V_2 was adjusted as required and a continuous flow maintained to C_1, sufficient to ensure that there was always an overflow. By shaking the manometer the time to attain steady state could be shortened. This usually required more than fifteen minutes. The manometer reading was taken and flow rate measured simultaneously with a graduated cylinder. Valve V_2 was regulated to change the flow rate, and the above procedure was repeated until sufficient measurements were again obtained.

When this was terminated, the overflow to C_1 was stopped and V_2 fully opened to empty the system. The liquid was collected in plastic containers. Flange F then dismantled, and the particles in C_2 as well as the column were then washed thoroughly with water.

The pressure drop ($-\frac{dp'}{dx}$) of the 42% sugar solution, used as a standard, or of the non-Newtonian fluids investigated was evaluated by means of the following equation for an open manometer:

$$(-\frac{dp'}{dx}) = (\frac{H_1 - H_2}{H_3}) \frac{\rho g}{g_c} \tag{25}$$

or, for a closed manometer, the following equation was used

$$(-\frac{dp'}{dx}) = \frac{H_g}{H_3} (\rho_{H_g} - \rho) \frac{g}{g_c} \tag{26}$$

Other equations used in the calculation, which were derived by replacing the D_o and U in Eq. (4) by Eqs. (12), (16), and (19) were

$$r_h (-\frac{dp'}{dx}) = \frac{\epsilon}{(1 - \epsilon) S_o} (-\frac{dp'}{dx})$$

$$= K' (\frac{2U_s}{r_h \epsilon}) = K' (\frac{2U_s (1 - \epsilon) S_o}{\epsilon^2})^{n'} \tag{27}$$

for the bed of aluminum oxide particles, and

$$0.0580 \frac{\epsilon}{1 - \epsilon} (-\frac{dp'}{dx}) = K' (34.5 \frac{1 - \epsilon}{\epsilon^2} U_s)^{n'} \tag{28}$$

for the bed of spheres, respectively.

The results obtained for the flow of non-Newtonian fluids through these two beds were calculated using the above two equations and plotted as

$$r_h (-\frac{dp'}{dx}) \text{ vs. } \frac{2U_s}{\epsilon r_h} .$$

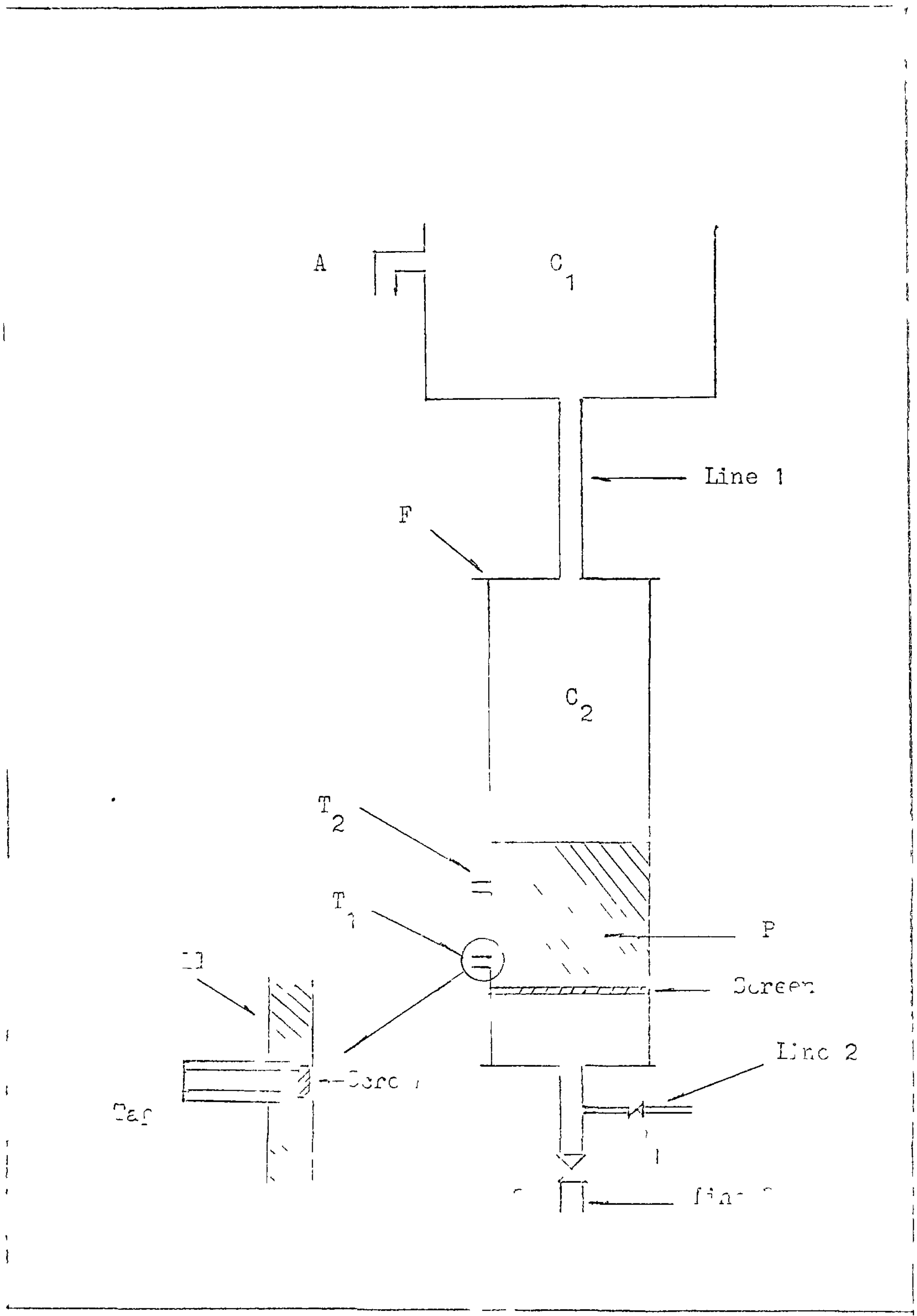

Fig. I The Packed Column

TABLE 1 KEY TO FIG. 1

Equipment	Materials of Construction	Dimensions	From	To	Remark
Feed tank C_1	.135" Steel plate	30.5 cm x 30.5 cm x 30.6 cm			
Overflow conduit A	Steel pipe	1/2" Sch. 40	C_1	drainage	
Line 1	Brass tube	1" I.D., 30' long The height measured from bottom of C_1 to Flange F is 17'	C_1	C_2	
Column C_2	Acrylic resin	113.4 cm long (measured from F to screen) 8.73 cm I.D.			
Bed packing P	Fused aluminum oxide particles, plastic spheres				Chapter IV, Section B
Tap T_1	Acrylic resin	3/8" I.D. The distance from screen to the center line of T_1 is 3.0 cm.			
Line 4	Tygon tubing	3/8" I.D.	T_1	Left arm of manometer M	Not shown
Tap T_2	Acrylic resin	3/8" I.D. H_3, the distance between T_1 and T_2 is 12.648 cm.			

TABLE 1 KEY TO FIG. 1 (Continued)

Equipment	Materials of Construction	Dimensions	From	To	Remark
Line 5	Tygon tubing	3/8" I. D.	T_2	Right arm of manometer M	Not shown
Manometer M	Mercury	54 cm long, 0.1 cm subdivision			Not shown
Tank C_3	Acrylic resin plate thickness 3/8"	12" x 12" x 12"			Not shown
Line 2	Tygon tubing	3/8" I. D.	C_3	V_1	Not shown
Line 3	Steel pipe	1" Sch. 40	C_2	drainage	
Valve V_1	Steel clamp				
Valve V_2	Brass	1" globe valve			
Graduated Cylinder	Pyrex glass	1000 c.c. Capacity 10 c.c. Subdivision			Not shown

D. THE VISCOMETER AND ITS EXPERIMENTAL SETUP

The flow diagram, dimensions, and materials of construction of the viscometer are presented in Fig. 2 and accompanying Table 2.

As the effective diameter of a capillary tube can not be measured directly, calibration is necessary (8). This calibration may be conducted by filling the bore of the capillary tube with mercury whose weight is subsequently determined. The diameter of the tube is then readily calculated.

Another method of calibration is to conduct a measurement with the viscometer on a Newtonian fluid of known viscosity. The following method of operation of the viscometer was employed.

The capillary tube P was first plugged at its inlet with a rubber stopper. Tanks C_1 and C_3 were filled with the liquid to be investigated and valve V_1 was closed.

Tank C_4 was filled with water and the temperature controller R was set to the desired temperature. The heater H and the circulator M were next turned on. The time taken to adjust the liquid in C_1 and C_3 to a constant temperature was usually in the neighbourhood of about half an hour, sometimes one hour.

Valve V_2 was closed, the vacuum pump connected to tap T_3 was turned on, and the level of vacuum in tank C_5, the separation funnel C_6, and the two-neck flask C_7 was regulated at a desired reading by adjusting the clamp attached to the rubber tubing connected to the vacuum pump and T_3. The vacuum was indicated by the manometer connected to tap T_4.

The rubber stopper was then pulled out, V_1 was regulated to permit an overflow in order to keep the liquid level in C_3 constant. The distance H_o measured from the free surface of the liquid in C_3 to the inlet of the capillary tube was measured.

Once the rubber stopper was pulled out, liquid started to accumulate in C_6. Attention was then shifted to the reading on the manometer. When this reading was constant, indicating the attainment of equilibrium within the system, the flow rate was measured by noting the time taken to collect a given volume of the liquid in a buret.

When the flow rate was determined, the liquid in C_6 was then drained into C_7 by opening V_2. The procedure to this point yielded one set of readings. V_2 was closed, the vacuum was regulated, and the same procedure described above was repeated to yield additional data.

Usually six sets of readings, each at a different flow rate, were taken with one capillary tube. When this was completed, the tube was removed, another capillary tube inserted, and the same procedure described above was repeated.

The desired pressure gradient was calculated from the experimental data using the following formula:

$$\left(-\frac{dp'}{dx}\right) = \left(\frac{H_o + L}{L}\right) \frac{g\rho}{g_c} + \left(\frac{H_g}{L}\right) \frac{g\,\rho_{Hg}}{g_c} - \frac{1.5\,\rho\,U^2}{L\,g_c} \qquad (29)$$

The quantity given by Eq. (29) was converted into equivalent shear stress.

The shear stresses so obtained and their corresponding values of $8U/D_o$ were plotted to obtain n' and K' according to Eq. (4).

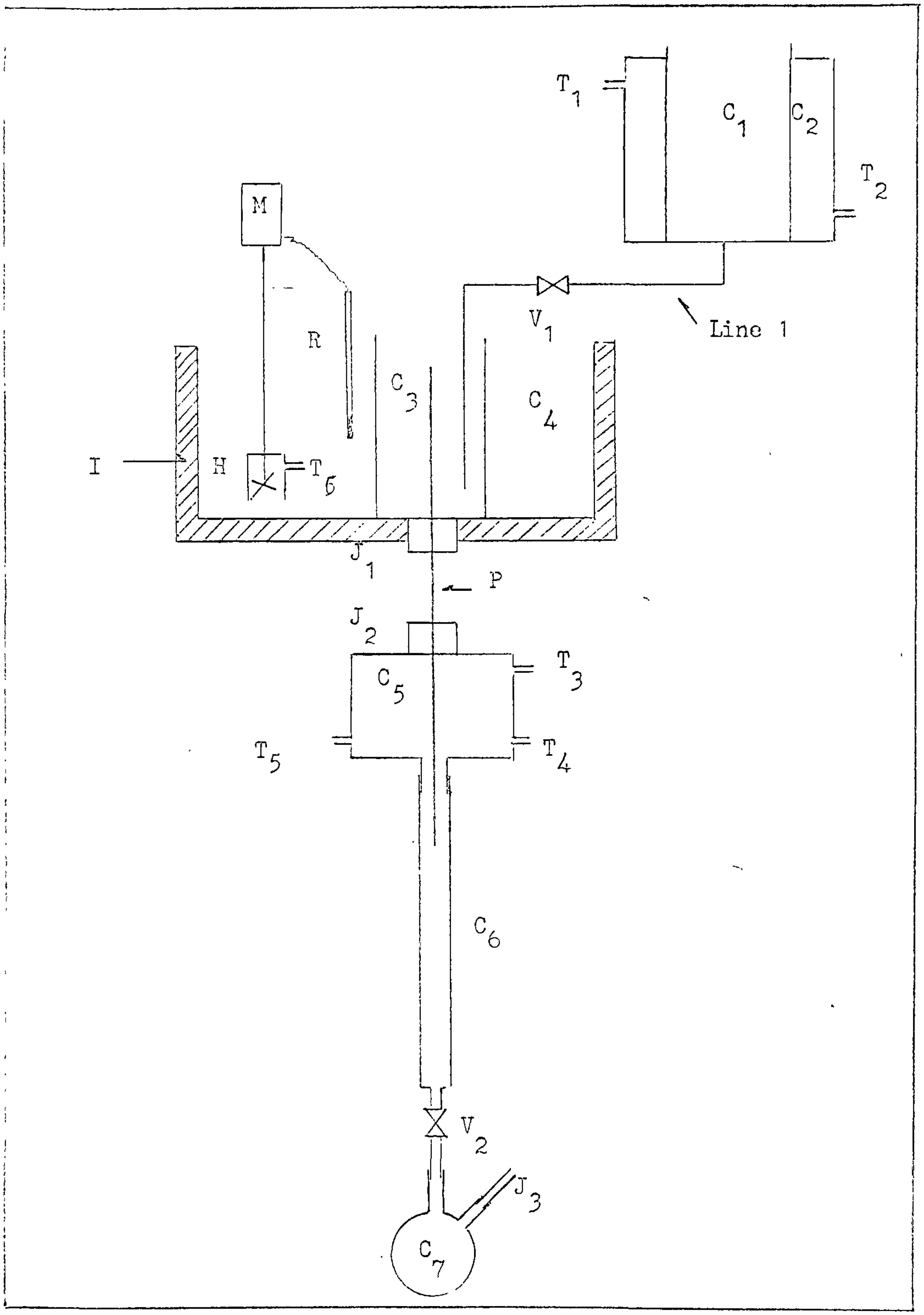

Fig. 2 The Viscometer

TABLE 2 KEY TO FIG. 2

Equipment	Materials of Construction	Dimensions	From	To	Remark
Feed tank C_1	Steel	15.2 cm I.D. 30.4 cm long			
Jacket C_2	Steel	15.4 cm I.D. 20.0 cm O.D.			
Taps T_1, T_2	Steel	0.720 cm I.D.			
Line 1	Rubber	0.720 cm I.D. The height from the bottom of C_1 to the liquid level in C_3 is 6.3 cm.	C_1	C_3	
Clamp V_1	Steel				
Tank C_3	Stainless steel	14.3 cm I.D. 19.6 cm long			
Tank C_4	Acrylic resin plate	55.3 cm x 43.9 cm x 22.8 cm			
Temperature Controller H. M. R.	Steel	Output: 3.67 gpm Motor: 2,850 r.p.m.			
Tap T_6	Steel	0.7 cm I.D.			
Line 2	Rubber	0.7 cm I.D.	T_6	T_2	Not shown
Line 3	Rubber	0.7 cm I.D.	T_1	C_4	Not shown

TABLE 2 KEY TO FIG. 2 (Continued)

Equipment	Materials of Construction	Dimensions	From	To	Remark
Seals J_1, J_2	Rubber				
Insulation I	Styrofoam	2.5 cm thick			
Capillary tube P	Glass		C_3	C_6	see Table 3
Tank C_5	Brass	14.2 cm I.D. 21.6 cm long			
Taps T_3, T_4, T_5	Brass	0.70 cm I.D.			
Line 4	Rubber	0.70 cm I.D.	T_3	Vacuum pump	Not shown
Line 5	Rubber	0.70 cm I.D.	T_4	Mano-meter	Not shown
Separation Funnel C_6	Glass	50 c.c. capacity 0.1 c.c. subdivision			
Valve V_2	Glass	Bore 0.24 cm			Part of C_6
2-neck flask C_7	Glass	1000 c.c. capacity			
Ground joint J_3	Glass	1.8 cm O.D.			
Line 6	Rubber	0.7 cm I.D.	T_5	J_3	Not shown

TABLE 2 KEY TO FIG. 2 (Continued)

Equipment	Materials of Construction	Dimensions	From	To	Remark
Manometer	Mercury	80 cm long 0.1 cm subdivision			Not shown
Stopwatch		0.1 sec. subdivision			Not shown

V-RESULTS

TABLE 3

SPECIFICATIONS OF CAPILLARY TUBES

Tube number	Length, cm	Diameter Given, cm	Diameter Calibrated, cm	Length to Diameter Ratio
1	84.30	0.08890	0.08903	947
2	84.26	0.1143	0.1150	733
3	84.36	0.1676	0.1681	500
4	84.45	0.1854	0.1865	452
5	84.20	0.2243	0.2249	374

Remark: The Calibrated tube diameter was used in calculations.

TABLE 4

VISCOMETER DATA

Liquid:	42% sugar solution
Liquid Temperature:	22° C
Room Temperature:	24.2° C
Capillary Tube:	No. 3
H_o =	13.3 cm
ρ =	1.18 g_m/cm^3

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$\frac{8U}{D_o}$ (sec^{-1})
0	0.384	0.0568	825
20.6	1.28	0.190	2,755
34.2	1.78	0.275	3,820

Result:

μ = 6.89 centipoises

TABLE 5

DATA OF FLOW OF 42% SUGAR SOLUTION THROUGH BED OF SPHERES

ϵ = 0.3800

$H_1 - H_2$ (cm of solution)	Q ($\frac{cm^3}{sec}$)	f	Re
1.65	6.28	59.2	0.630
3.62	13.9	26.4	1.40
4.62	17.8	20.6	1.80
1.46	5.83	65.1	0.584
1.92	7.38	49.8	0.741
2.33	9.45	37.1	0.948
6.53	25.3	14.4	2.54
15.2	61.2	13.6	6.14
25.8	92.7	10.1	9.30

TABLE 6

DATA OF FLOW OF 42% SUGAR SOLUTION THROUGH BED OF ALUMINUM OXIDE PARTICLES

$\epsilon = 0.3779$

$H_1 - H_2$ (cm of solution)	$Q\ (\frac{cm^3}{sec})$	f	Re
17.7	11.7	79.6	0.522
18.4	11.9	78.6	0.532
6.10	4.00*	22.8	1.79
35.2	22.3	42.4	0.996
41.5	27.8	32.3	1.24
17.1	11.6*	77.1	0.519
24.0	16.0*	56.2	0.714
4.04	2.72*	329	0.121
5.06	3.40*	263	0.152
10.9	7.42	119	0.332

Remarks:

* used to determine the $\frac{1}{S_o}$ of aluminum oxide particles

TABLE 7

DATA OF FLOW OF 0.4% CARBOPOL SOLUTION THROUGH BED OF ALUMINUM OXIDE PARTICLES

$\epsilon = 0.3779$

H_g (cm of mercury)	Q (cm^3/sec)	$r_h\left(-\frac{dp'}{dx}\right)$ (g_f/cm^2)	$2U_s/\epsilon r_h$ (sec^{-1})	f	Re''
5.40	19.3	0.0800	114	217	0.137
5.40	19.6	0.0800	117	208	0.140
5.40	19.7	0.0800	117	207	0.141
5.40	19.6	0.0800	116	210	0.140
5.40	19.4	0.0800	115	214	0.138
5.40	19.4	0.0800	115	214	0.138
13.4	84.7	0.199	503	27.9	1.09
13.4	84.2	0.199	500	28.2	1.09
13.4	83.4	0.199	495	28.7	1.07
12.2	69.4	0.181	411	37.9	0.825
12.2	70.0	0.181	416	37.0	0.838
12.2	69.8	0.181	414	37.2	0.835
12.2	70.2	0.181	417	36.8	0.841
9.20	42.9	0.136	255	74.4	0.421
9.20	43.5	0.136	260	73.1	0.427
9.20	43.2	0.136	256	73.5	0.425
4.10	10.9	0.0608	64.8	513	0.0619
4.10	10.9	0.0608	64.9	512	0.0619
4.10	11.0	0.0608	65.4	503	0.0623
6.10	21.5	0.0904	127	197	0.159
6.10	21.7	0.0904	129	192	0.162
6.10	21.5	0.0904	128	196	0.160

TABLE 7 (Continued)

H_g (cm of mercury)	Q (cm^3/sec)	$r_h\left(-\frac{dp'}{dx}\right)$ (g_f/cm^2)	$2U_s/\epsilon r_h$ (sec^{-1})	f	Re''
6.10	21.5	0.0904	128	196	0.160
10.8	51.2	0.161	304	59.9	0.540
10.8	51.8	0.161	307	58.5	0.548
10.8	51.5	0.161	306	59.2	0.544

TABLE 8

VISCOMETER DATA

Liquid: 0.4% Carbopol

Liquid Temperature: 18°C

Room Temperature: 20°C

Capillary Tube: No. 3

H_o = 11.9 cm

ρ = 1.00 g_m/cm^3

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	8U/D (sec^{-1})
5.55	0.164	0.0855	351
5.55	0.164	0.0855	351
5.55	0.163	0.0855	349
4.30	0.134	0.0771	288
4.30	0.134	0.0771	287
4.30	0.134	0.0771	286
0.000	0.0616	0.0479	132
0.000	0.0632	0.0479	136
0.000	0.0612	0.0479	131
0.000	0.0611	0.0479	131
8.00	0.214	0.102	459
8.00	0.215	0.102	461
8.00	0.215	0.102	461
8.00	0.218	0.102	467
11.9	0.321	0.128	687
11.9	0.317	0.128	680
11.9	0.319	0.128	683
15.2	0.414	0.151	888

TABLE 8 (Continued)

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$8U/D_o$ (sec^{-1})
15.2	0.420	0.151	899
15.2	0.415	0.151	890
15.2	0.417	0.151	895
15.2	0.418	0.151	897

Results:

$$n' = 0.595$$

$$K' = 2.58 \frac{g_m}{cm\ sec^{1.405}}$$

TABLE 9

DATA OF FLOW OF 0.5% CARBOPOL SOLUTION THROUGH BED OF SPHERES

$\epsilon = 0.4202$

H_g (cm of mercury)	Q (cm^3/sec)	$r_h\left(-\frac{dp'}{dx}\right)$ (g_f/cm^2)	$2U_s/\epsilon r_h$ (sec^{-1})	f	Re''
3.25	66.8	0.136	126	37.8	0.813
2.00	28.8	0.0837	54.5	125	0.242
2.00	28.4	0.0837	53.7	129	0.237
2.00	28.3	0.0837	53.5	130	0.236
2.00	28.4	0.0837	53.8	128	0.237
3.60	81.0	0.151	153	28.5	1.07
3.60	80.4	0.151	152	28.9	1.06
3.60	80.5	0.151	152	28.8	1.06
3.60	81.0	0.151	153	28.5	1.07
4.35	106	0.180	200	20.2	1.57
4.35	105	0.180	198	20.5	1.56
4.35	104	0.180	197	20.7	1.55
3.10	57.3	0.130	108	48.9	0.652
3.10	58.4	0.130	110	47.1	0.669
3.10	58.3	0.130	110	47.3	0.668
3.10	57.0	0.130	108	49.5	0.646
3.10	57.8	0.130	109	48.2	0.659
3.10	57.3	0.130	108	49.0	0.651
3.10	56.4	0.130	107	50.6	0.636
3.10	57.0	0.130	108	49.5	0.646
3.10	56.7	0.130	107	50.1	0.641
2.70	40.8	0.101	77.2	74.7	0.400
2.70	40.9	0.101	77.3	74.6	0.401
2.70	40.9	0.101	77.4	74.3	0.398

TABLE 10

DATA OF FLOW OF 0.5% CARBOPOL SOLUTION THROUGH BED OF ALUMINUM OXIDE PARTICLES

$\epsilon = 0.3779$

H_g (cm of mercury)	Q (cm^3/sec)	$r_h(-\frac{dp'}{dx})$ (g_f/cm^2)	$2U_s/\epsilon r_h$ (sec^{-1})	f	Re''
5.85	12.9	0.0867	76.5	525	0.0492
5.85	13.0	0.0867	77.0	518	0.0496
5.85	12.7	0.0867	75.5	539	0.0485
5.85	12.6	0.0867	75.0	546	0.0477
9.80	30.6	0.145	182	156	0.171
9.80	31.1	0.145	185	151	0.175
9.80	31.0	0.145	184	152	0.173
9.80	30.8	0.145	183	153	0.172
9.80	30.9	0.145	184	153	0.173
9.80	30.9	0.145	184	153	0.173
7.20	17.0	0.107	101	368	0.0763
7.20	17.0	0.107	101	368	0.0763
7.20	17.0	0.107	101	370	0.0736
7.20	17.1	0.107	102	367	0.0737
12.7	44.1	0.188	262	97.1	0.289
12.7	43.7	0.188	260	99.0	0.285
12.7	43.8	0.188	260	98.4	0.286
12.7	43.8	0.188	260	98.7	0.286
7.30	17.6	0.108	105	350	0.0772
7.30	17.7	0.108	105	345	0.0781
7.30	17.7	0.108	105	345	0.0781
7.30	17.7	0.108	105	347	0.0774

TABLE 10 (Continued)

H_g (cm of mercury)	Q (cm^3/sec)	$r_h\ (-\frac{dp'}{dx})$ (g_f/cm^2)	$2U_s/\epsilon\ r_h$ (sec^{-1})	f	Re''
15.6	63.1	0.231	368	58.3	0.484
15.6	63.1	0.231	375	58.3	0.484
15.6	62.6	0.231	375	59.3	0.479
15.6	62.3	0.231	372	59.9	0.475
15.6	63.0	0.231	374	58.5	0.483
7.95	20.0	0.118	119	294	0.0926

TABLE 11

VISCOMETER DATA

Liquid:	0.5% Carbopol
Liquid Temperature:	18°C
Room Temperature:	19.2°C
Capillary Tube	No. 2
H_o =	13.6 cm
ρ =	1.00 g_m/cm^3

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$8U/D$ (sec^{-1})^o
23.7	0.0653	0.143	441
23.7	0.0654	0.143	441
23.7	0.0650	0.143	439
12.9	0.0315	0.0931	213
12.9	0.0318	0.0931	215
12.9	0.0321	0.0931	217

Capillary Tube No. 3

H_o = 13.2 cm

TABLE 11 (Continued)

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$8U/D_o$ (sec^{-1})
9.30	0.133	0.112	286
9.30	0.133	0.112	284
9.30	0.136	0.112	291
9.30	0.133	0.112	286
12.4	0.179	0.134	383
12.4	0.182	0.134	390
12.4	0.182	0.134	390
17.2	0.276	0.166	591
17.2	0.273	0.166	585
17.2	0.274	0.166	587
4.05	0.0661	0.0759	142
4.05	0.0662	0.0759	142
4.05	0.0661	0.0759	142
22.7	0.403	0.211	864
22.7	0.401	0.211	860
22.7	0.405	0.211	867

Results:

$n' = 0.560$

$$K' = 4.645 \frac{g_m}{cm\ sec^{1.44}}$$

TABLE 12

DATA OF FLOW OF CMC SOLUTION THROUGH BED OF SPHERES

$\epsilon = 0.3869$

H_g (cm of mercury)	Q (cm^3/sec)	$r_h\ (-\frac{dp'}{dx})$ g_f/cm^2	$2U_s/\epsilon\, r_h$ (sec^{-1})	f	Re''
3.80	46.7	0.145	102	68.6	0.551
3.80	46.6	0.145	102	68.9	0.508
3.80	45.8	0.145	99.9	71.1	0.534
3.60	41.5	0.137	90.4	82.3	0.481
3.60	42.6	0.137	92.8	78.1	0.497
3.60	42.8	0.137	93.2	78.5	0.497
2.17	23.7	0.0826	51.6	152	0.251
2.17	23.9	0.0826	52.2	149	0.254
2.17	24.1	0.0826	52.5	147	0.256
1.35	13.7	0.0514	29.8	285	0.133
1.35	13.8	0.0514	30.2	278	0.135
1.35	13.7	0.0514	30.1	279	0.134
4.30	50.4	0.164	110	66.5	0.603
4.30	51.5	0.164	112	63.7	0.621
4.30	51.3	0.164	112	64.3	0.614
3.75	43.1	0.143	93.9	79.4	0.503
3.75	44.2	0.143	96.3	75.6	0.517
3.75	43.1	0.143	94.0	79.4	0.503
2.57	28.2	0.0978	61.5	127	0.312
2.57	28.6	0.0978	62.3	124	0.313
2.57	28.7	0.0978	62.5	123	0.313

TABLE 13

VISCOMETER DATA

Liquid: CMC solution

Liquid Temperature: 21°C

Room Temperature: 24°C

Capillary Tube No. 3

H_o = 12.9 cm

ρ = 1.08 g_m/cm^3

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$8U/D_o$ (sec^{-1})
0.000	0.0403	0.0523	86.4
0.000	0.0401	0.0523	85.9
4.95	0.0701	0.0859	150
4.95	0.0701	0.0859	150
9.40	0.100	0.116	215
9.40	0.101	0.116	219
16.7	0.164	0.166	351
16.7	0.166	0.166	355
23.8	0.215	0.214	461

Capillary Tube No. 5

H_o = 11.1 cm

TABLE 13 (Continued)

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$8U/D_o$ (sec^{-1})
0.000	0.128	0.0688	114
0.000	0.128	0.0688	114
5.37	0.241	0.118	216
5.37	0.249	0.118	223
7.55	0.309	0.137	277
7.55	0.307	0.137	275
9.65	0.363	0.156	325
9.65	0.364	0.156	325

Results:

$$n' = 0.842$$

$$K' = 1.20 \ \frac{g_m}{cm\ sec^{1.158}}$$

TABLE 14

DATA OF FLOW OF 0.4% POLYOX SOLUTION THROUGH BED OF SPHERES

$\epsilon = 0.4143$

H_g (cm of mercury)	Q (cm^3/sec)	$r_h\left(-\frac{dp'}{dx}\right)$ (g_f/cm^2)	$2U_s/\epsilon r_h$ (sec^{-1})	f	Re''
0.850	31.1	0.0347	61.5	42.8	0.867
0.850	31.2	0.0347	61.3	43.1	0.863
0.850	31.1	0.0347	61.0	43.5	0.857
1.25	53.6	0.0511	105	21.5	1.76
1.25	53.3	0.0511	105	21.9	1.73
1.25	52.9	0.0511	104	22.0	1.72
0.950	35.7	0.0388	70.1	36.8	1.03
0.950	35.8	0.0388	70.3	38.6	1.03
0.950	35.5	0.0388	69.8	37.1	1.02
0.450	13.2	0.0184	25.9	128	0.0278
0.450	13.2	0.0184	26.0	127	0.0280
0.450	13.3	0.0184	26.2	125	0.0283
1.05	39.4	0.0429	77.4	33.3	1.17
1.05	39.2	0.0429	77.0	33.7	1.16
1.05	39.4	0.0429	77.4	33.4	1.17
2.45	119	0.0989	235	8.37	5.02
2.45	121	0.0989	237	8.18	5.09

TABLE 15

DATA OF FLOW OF 0.4% POLYOX SOLUTION THROUGH BED OF ALUMINUM OXIDE PARTICLES

$\epsilon = 0.3779$

H_g (cm of mercury)	Q (cm^3/sec)	$r_h\left(-\frac{dp'}{dx}\right)$ (g_f/cm^2)	$2U_s/\epsilon\, r_h$ (sec^{-1})	f	Re''
4.00	25.5	0.0593	151	91.9	0.369
4.00	25.1	0.0593	149	94.9	0.363
4.00	25.6	0.0593	152	91.1	0.371
6.25	42.7	0.0927	253	51.1	0.728
6.25	42.5	0.0927	252	51.6	0.723
6.25	42.6	0.0927	253	51.2	0.726
8.92	63.2	0.133	375	33.4	1.22
8.92	63.3	0.133	376	33.3	1.22
6.86	45.7	0.102	271	48.9	0.796
6.86	45.7	0.102	271	49.0	0.796
6.95	43.0	0.103	256	55.9	0.735
6.95	45.1	0.103	268	50.8	0.782
7.05	45.0	0.105	267	51.8	0.780
7.05	44.8	0.105	266	52.4	0.774
7.05	45.1	0.105	268	51.6	0.782
5.25	30.6	0.0778	182	83.6	0.470
5.25	30.3	0.0778	180	84.9	0.465
5.25	30.5	0.0778	181	84.2	0.468
5.25	30.5	0.0778	181	83.7	0.470

TABLE 16

VISCOMETER DATA

Liquid: 0.4% Polyox

Liquid Temperature: 18°C

Room Temperature: 21.8°C

Capillary Tube No. 3

H_o = 12.1 cm

ρ = 1.00 g_m/cm^3

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$8U/D_o$ (sec^{-1})
0.000	0.159	0.0480	341
0.000	0.158	0.0480	339
3.10	0.266	0.0689	571
3.10	0.264	0.0689	566
6.00	0.383	0.0884	821
6.00	0.383	0.0884	821
8.50	0.500	0.105	1,072
8.50	0.497	0.105	1,065

Capillary Tube No. 2

H_o = 13.2 cm

TABLE 16 (Continued)

H_g (cm of mercury)	Q (cm^3/sec)	τ_o (g_f/cm^2)	$8U/D_o$ (sec^{-1})
25.2	0.264	0.150	1,781
25.2	0.264	0.150	1,781
20.1	0.208	0.126	1,404
20.1	0.207	0.126	1,395
17.1	0.180	0.112	1,217
17.1	0.181	0.112	1,222
17.1	0.181	0.112	1,222
11.1	0.119	0.0846	803
11.1	0.119	0.0846	803
11.1	0.118	0.0846	797
5.50	0.0716	0.0587	484
5.50	0.0712	0.0587	481
0.000	0.0308	0.0332	207

Results:

$n' = 0.688$

$$K' = 0.865 \; \frac{g_m}{cm\ sec^{1.312}}$$

TABLE 17

THE DETERMINATION OF COEFFICIENTS a, b IN EQUATION (23)

n'	$\frac{3n'+1}{4}$	k	$\frac{k}{16}$	$\left(\frac{k}{16}\right)^{\frac{1}{n'}}$	$\frac{3n'+1}{4}\left(\frac{k}{16}\right)^{\frac{1}{n'}}$
1.00	1.00	40.0	2.50	2.50	2.50
0.688	0.766	38.0	2.37	3.49	2.67
0.595	0.696	30.4	1.90	2.94	2.05
0.560	0.670	28.3	1.77	2.78	1.86

Results:

$a = 1.14$

$b = 1.36$

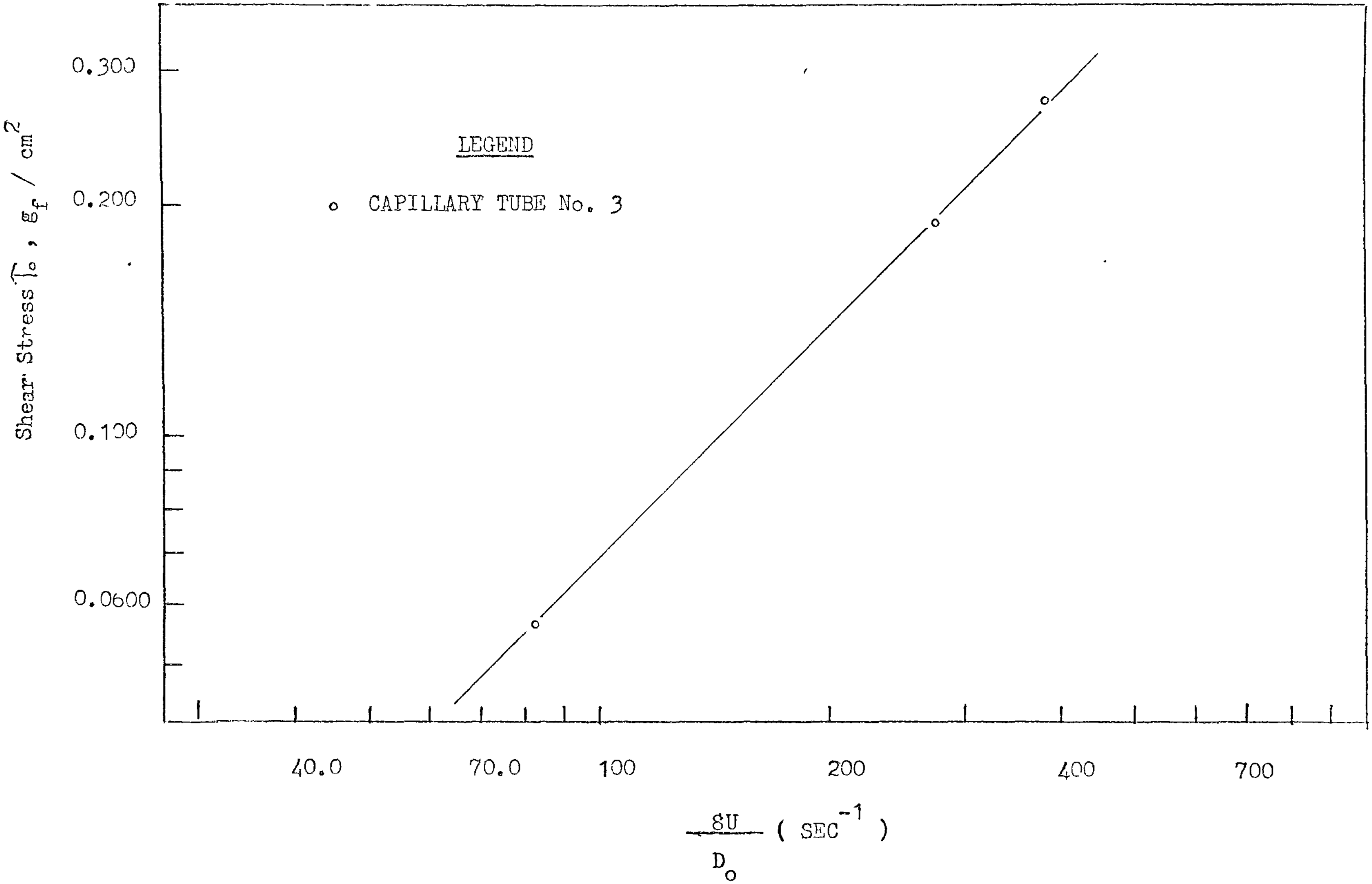

Fig. 3 Flow of Aqueous 42% Sugar Solution through Capillary Tube No.3

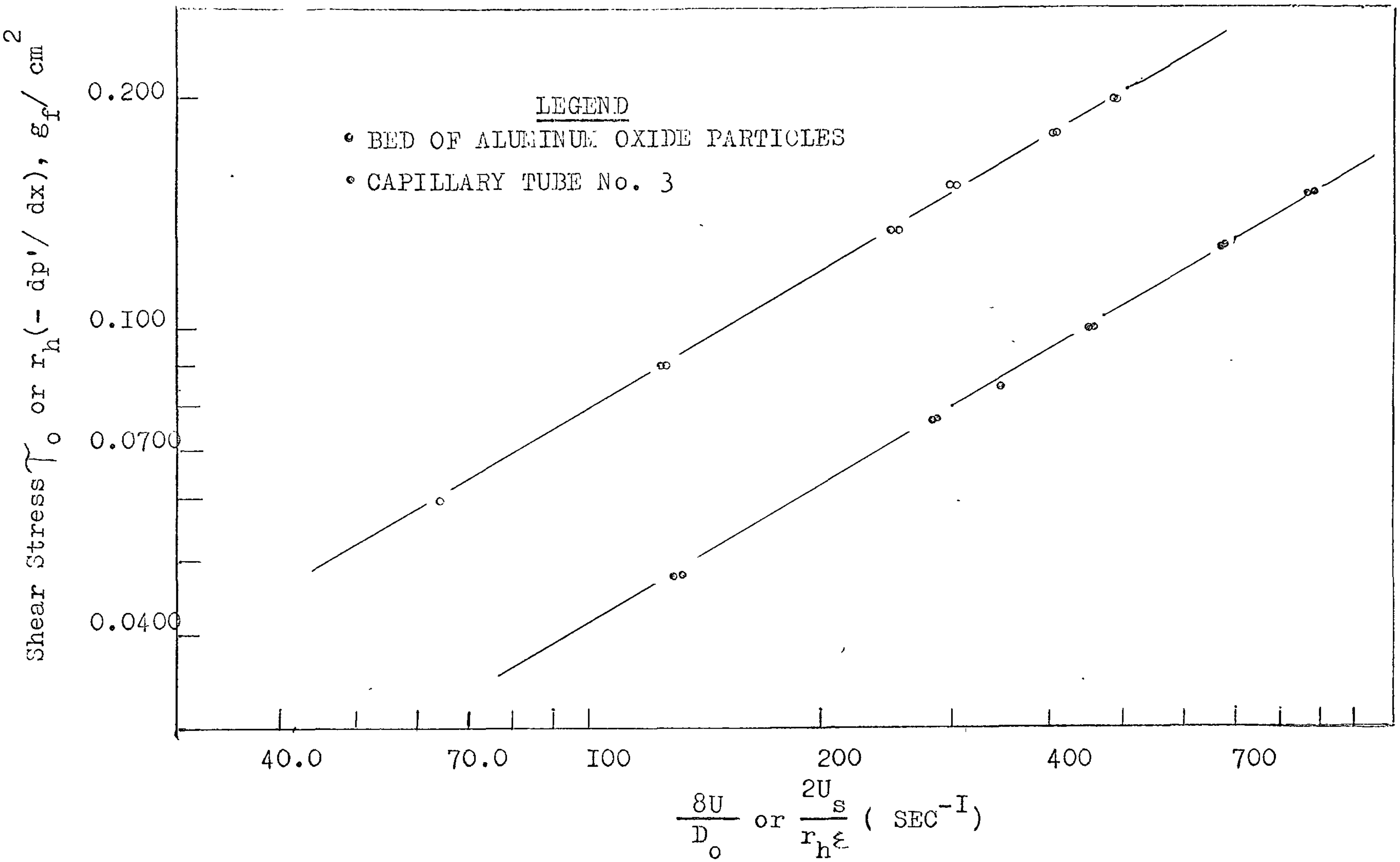

Fig. 4 Flow of 0.4% Carbopol Solution through Bed of Aluminum Oxide Particles and Capillary Tube No. 3

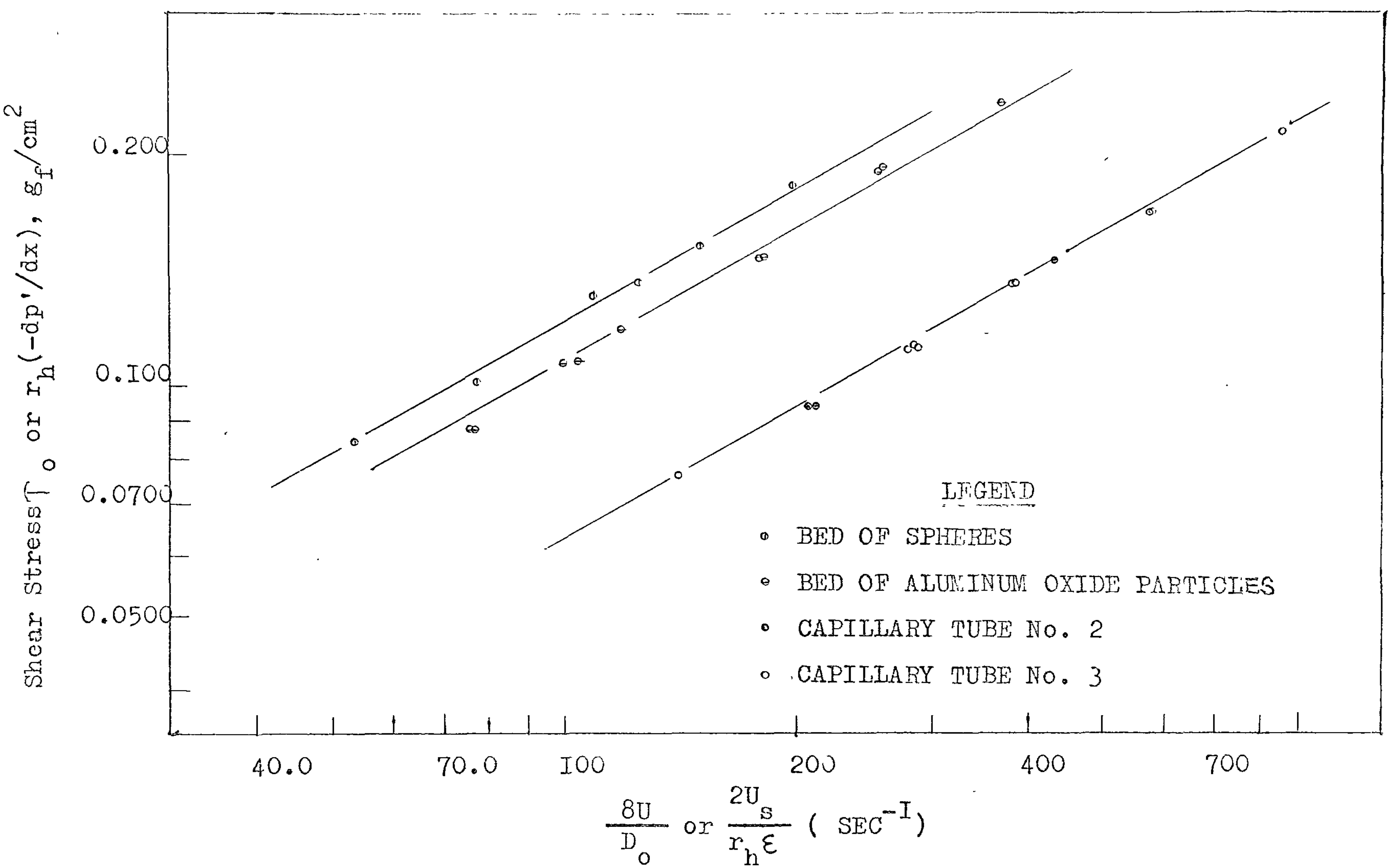

Fig. 5 Flow of 0.5% Carbopol Solution through Beds and Capillary Tubes

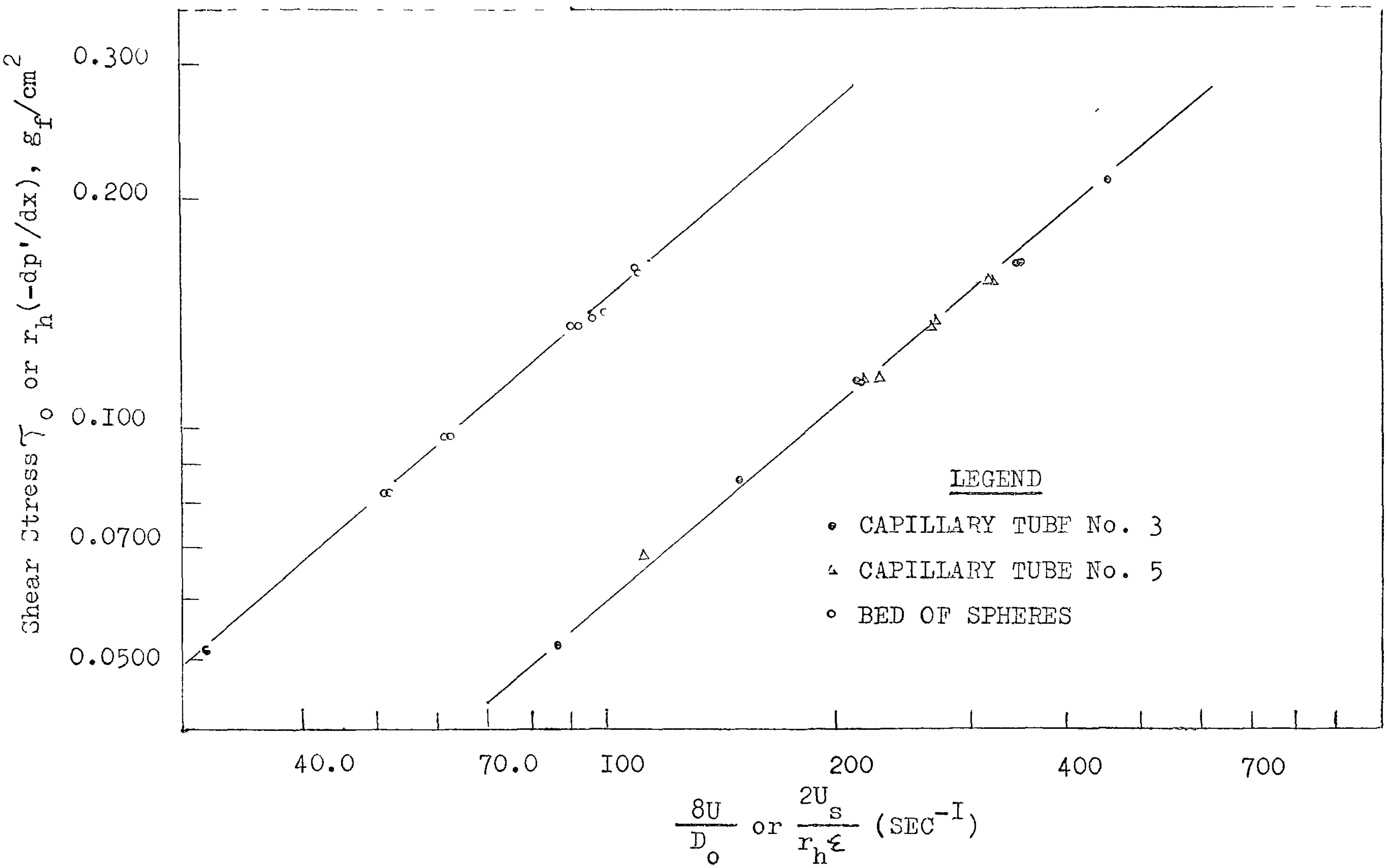

Fig. 6 Flow of CMC Solution through Bed of Spheres and Capillary Tubes

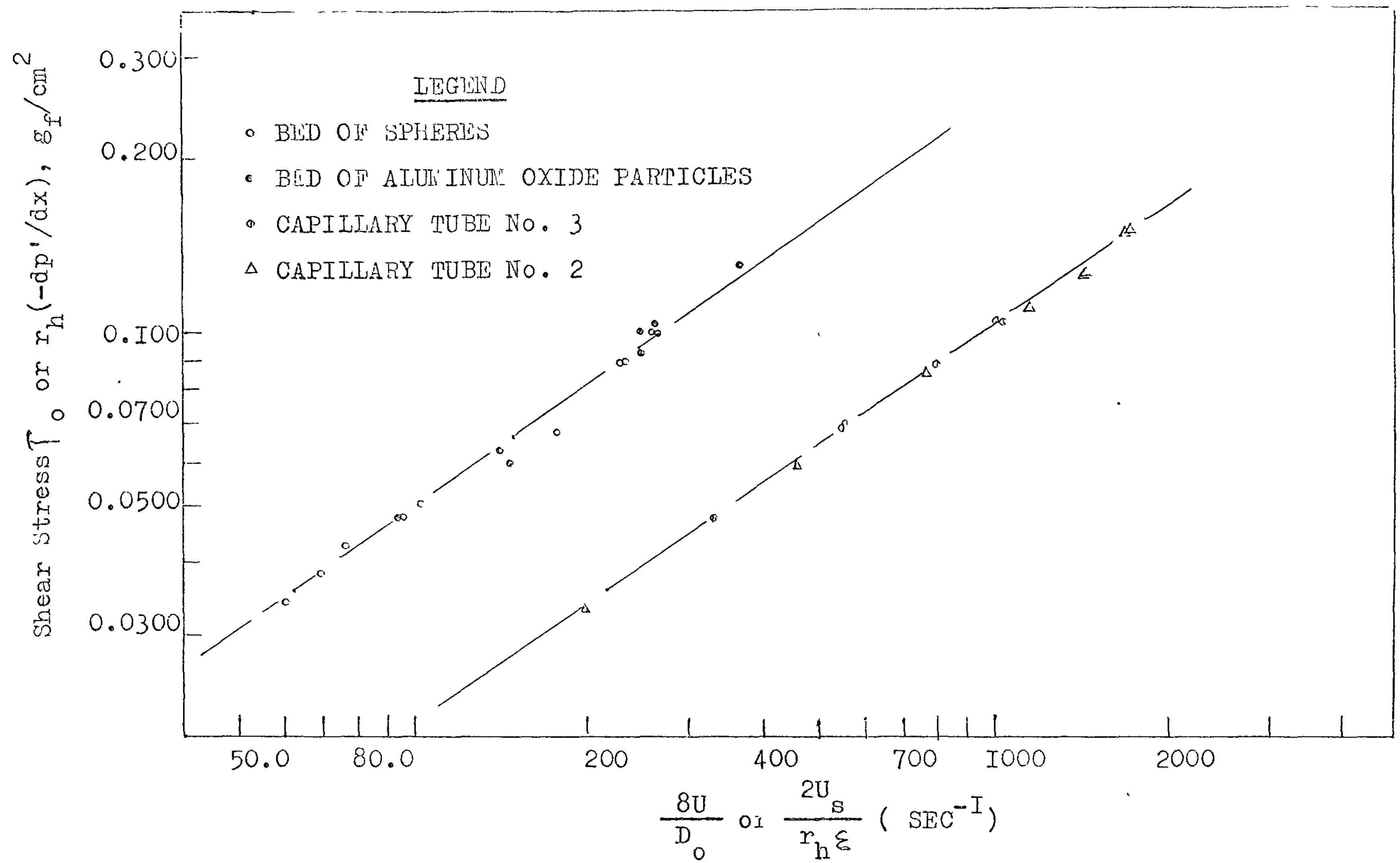

Fig. 7 Flow of 0.4% Polyox Solution through Beds and Capillary Tubes

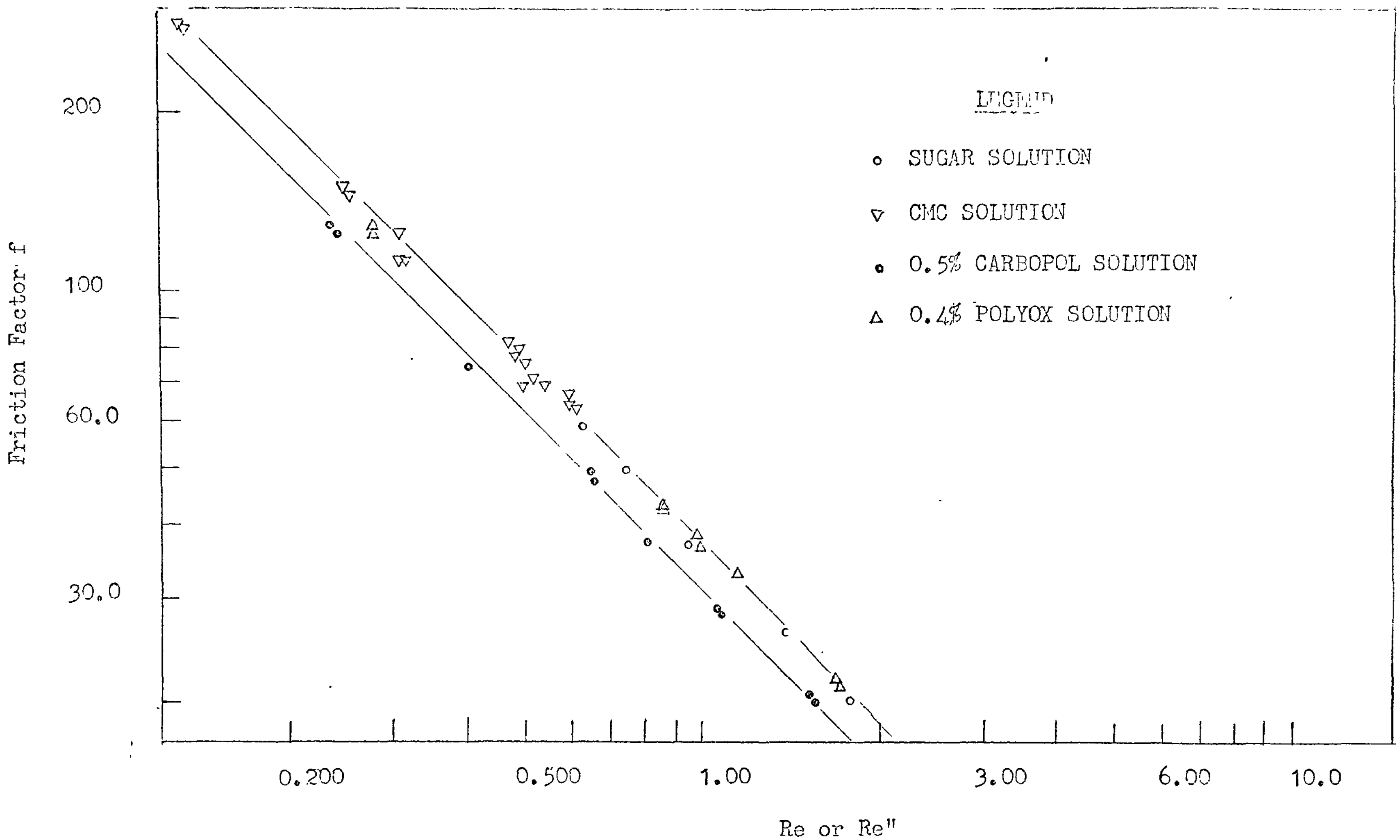

Fig. 8 f vs. Re and Re" for Beds of Spheres

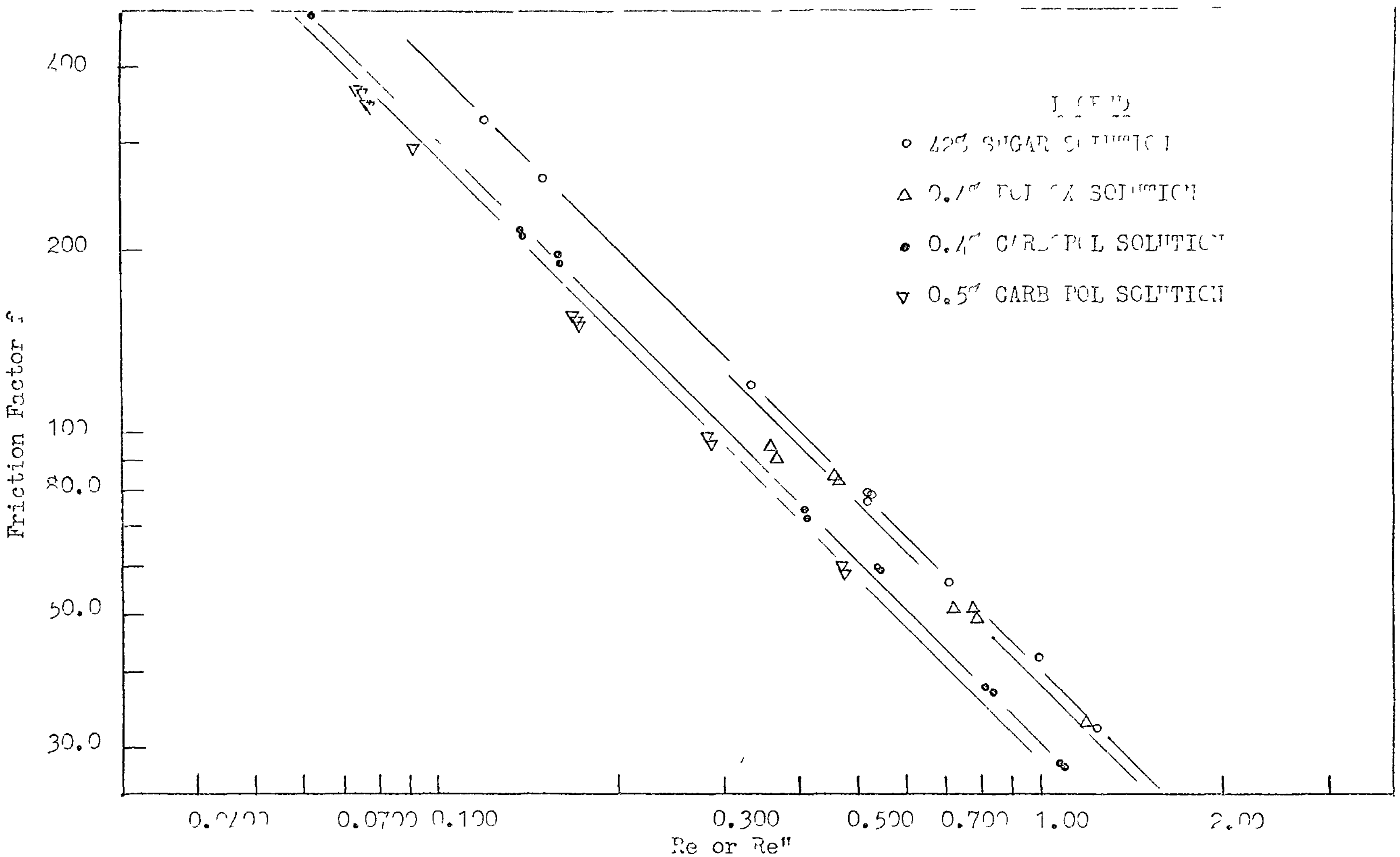

Fig. 9 f vs. Re and Re" for Beds of Aluminum Oxide Particles

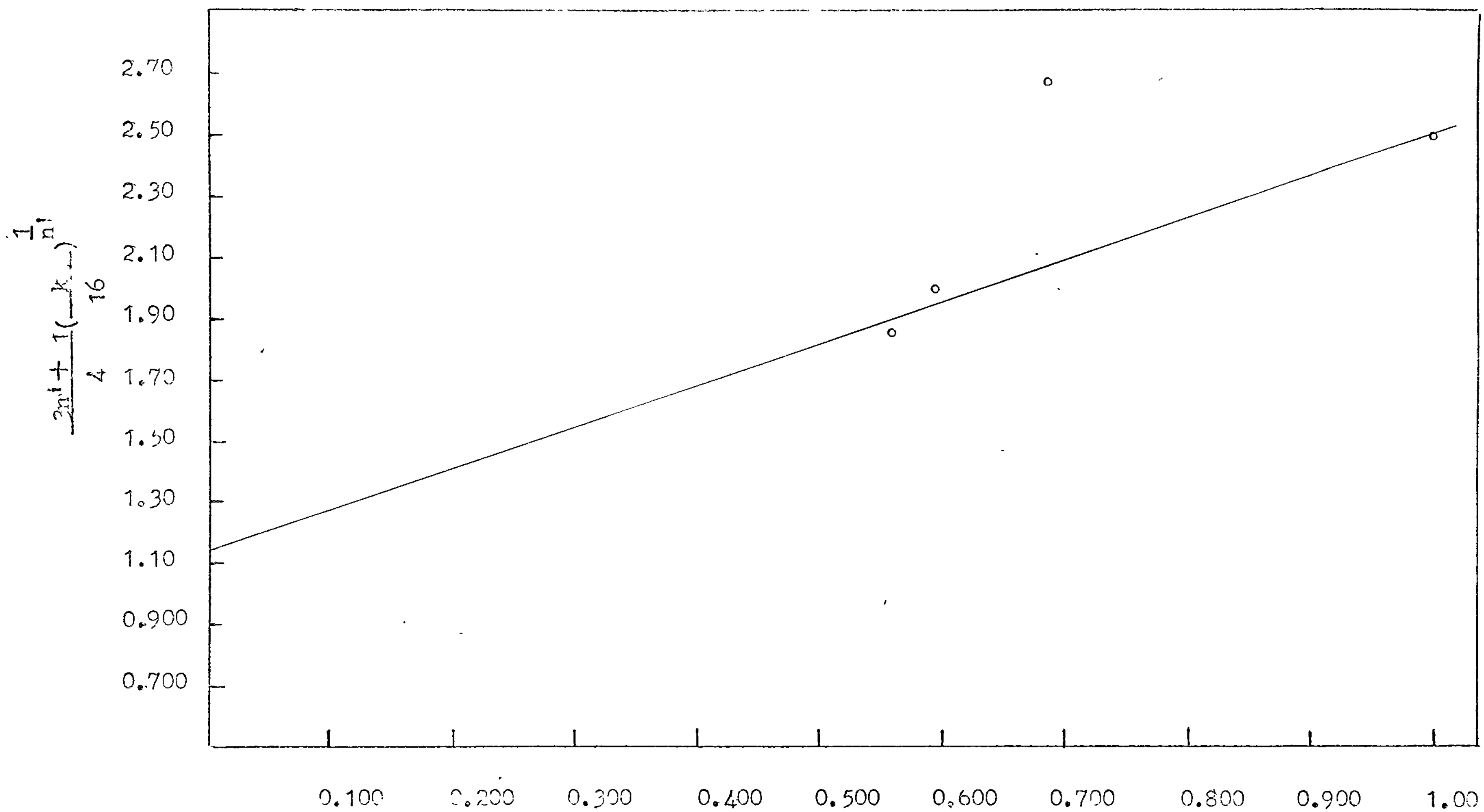

Fig. 10 The Determination of Coefficients a and b in Equation (23)

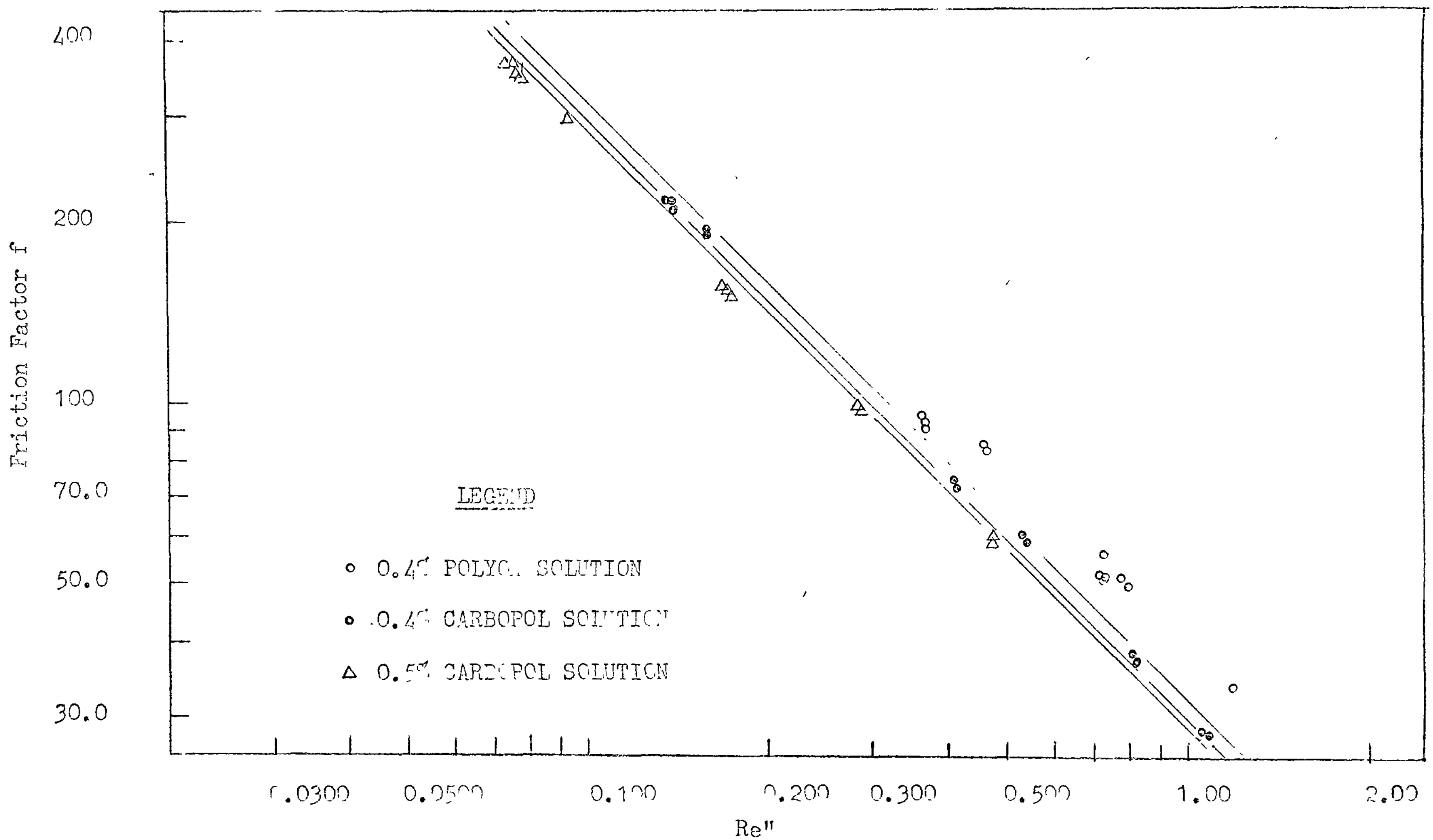

Fig. 11 The Comparasion of Predicted and Experimental Results of Flow of Non-Newtonian Fluids through Beds of Aluminum Oxide Particles

VI-DISCUSSION OF RESULTS

The results of the density determinations conducted on the liquids are tabulated in Tables 4, 8, 11, 13, and 16. The instruments used to measure the density were a precise balance and a graduated cylinder. Because of the use of the cylinder, the accuracy was limited to two significant figures. However, to avoid round-off errors, three significant figures were used to express the densities of all liquids.

The instruments used to measure the density of particles were more precise. As a result, these determinations were accurate to at least three significant figures. Six separate determinations of the density of the spheres were conducted. The max. deviation from the mean value is 0.6% and the average deviation from the mean is 0.26%. Four separate determinations were made for the mean density of the aluminum oxide particles. The average deviation from the mean is 0.26%.

The porosity of a bed is established with the following information:

(1) Weight of particles in the bed;

(2) Density of the particles;

(3) Diameter of column;

(4) Length of the bed.

Due to the limitation in the accuracy of the measurement of the length of the bed, the porosity was only accurate to three significant figures. Again, to avoid round-off errors, four significant figures were used to express the porosity of the beds. These are given in Tables 5, 6, 7, 9, 10, 12, 14, and 15.

Other measurements involved in the investigation of the flow of liquids through packed beds were the manometer readings and flow rates. These measurements were good only to two significant figures. Three significant figures were used to compensate for round-off errors.

The $\frac{1}{S_o}$ of a sphere is easily determined when the diameter of the sphere is known. The average diameter of the spheres used in the present work is presented in Chapter IV, Section B. Four significant figures were used to express this quantity, as a micrometer was used in the measurements.

The flow of 42% sugar solution, a Newtonian fluid of known viscosity, through the bed of spheres was first investigated. The result is given in Table 5 and Fig. 8. The k of Eq. (22), obtained in this experiment, has a value of 37.3, which is acceptable (13c). This result attests to the validity of the experimental procedure and the method of calculation.

The $\frac{1}{S_o}$ for irregular particles, such as the aluminum oxide particles, is more difficult to determine. The permeability method was adopted. This method involves using the Kozeny-Carman equation, provided that the premises on which this equation is based are not violated. These premises are (13d):

(1) an unconsolidated bed with a random structure;

(2) uniformity of pore size.

The method of filling the column, Chapter IV, Section C, assured that the beds were unconsolidated and random in texture. The particles were screened before transferring to the column, and the sise and shape of the particles were similar, so that uniformity of the beds in the present work was attained.

In addition, the ratio of diameter of column to diameter of particles was bigger than 32, so that the wall effect did not play a role here. The length of column measured from F (Fig. 1) to the surface of the bed was always greater than 71 cm, a few centimeters of bed was situated above the first pressure tap connected to the manometer, and all flow rates involved were less than 2 cm per second, as a result of which, the entrance effect was not important here.

The result for the flow of 42% sugar solution through the bed of aluminum oxide particles is given in Table 6. Part of the result was used, in conjunction with Eq. (24), to calculate the $\frac{1}{S_o}$

for these particles. This calculation yielded an average value of 0.0245 cm, with an average deviation of 0.9%.

Using this result, the result in Table 6 was calculated for f and Re using Eqs. (13), (14), and (16). They are plotted in Fig. 9. The max. deviation involved is less than 5% and the average deviation is less than 0.5%.

The quantities to be determined in the operation of the viscometer were the liquid head H_o, manometer reading H_g, flow rate Q, and the dimensions of the capillary tubes. The first three quantities, due to the inherent limitations of the instruments employed, yielded an accuracy in the final measurement to two significant figures only. Three significant figures were retained however to eliminate round-off errors.

The length of a capillary tube could be measured to an accuracy of three significant figures.

Two methods were used to calibrate the diameter of a capillary tube. The first involved filling the tube with mercury. The instruments used in this method were quite precise, and four significant figures were used to report the diameter of a tube. The results are shown in Table 3.

The second method involved conducting a determination with a viscometer on a 42% sugar solution of known viscosity. The result is shown in Table 4 and plotted in Fig. 3.

This determination had a three-fold purpose:

(1) to check the calculation equation (29);

(2) to check the operation of the viscometer;

(3) to provide an additional calibration of the diameter of the tube as a check.

The measured viscosity deviates from the published value (14) by 1.19%. This deviation is attributed entirely to the uncertainty associated in preparation of the 42% sugar solution due to the large quantity involved, viz., 200,000 c.c. It could not be ascertained that the solution was exactly 42%. However the deviation is small.

The flow of non-Newtonian fluids through the beds of spheres and aluminum oxide particles was conducted in the same way as with the Newtonian fluid. The results were calculated as $r_h\left(-\frac{dp'}{dx}\right)$ and $2U_s/r_h\epsilon$ through the use of Eqs. (27) and (28), and are shown in Tables 7, 9, 10, 12, 14, and 15, and Figs. 4, 5, 6, and 7.

It is known that for any fluid the shear stress is a function of the shear rate. The classification of time-independent, non-Newtonian fluids as mentioned in Chapter III, Section A may be considered idealized. A real fluid which behaves as a pseudoplastic fluid in a certain range of shear rates may behave as a Bingham plastic fluid in some other range of shear rates. The above considerations raise the question as to over what range of shear rates should the current experimental measurements with the viscometer be taken.

The problem was resolved by operating the viscometer within the range of $8U/D_o$ corresponding to the range of $2U_s/r_h \epsilon$ utilized in the flow of non-Newtonian fluids through the beds, shown in Figs. 4, 5, 6, and 7. This was accomplished by the selection of capillary tubes with appropriate diameters.

Both the viscometer and the packed column were operated at the same temperature of the liquid, so that the effect of difference in temperature could not play a role.

Two capillary tubes were usually employed to obtain the six to eight sets of data taken with the viscometer for each non-Newtonian fluid. Only the data for 0.4% Carbopol solution were taken with a single capillary tube, because the min. diameter requirement for this liquid had already been established.

The experimental results obtained with the viscometer using non-Newtonian fluids are shown in Tables 8, 11, 13, and 16, and Figs. 4, 5, 6, and 7. Straight lines were drawn through the points on these figures. The deviations of these points from the straight lines are not appreciable. The max. deviation is less than 2.5% and the average deviation less than 1%. These straight lines were used to calculate n' and K' for each non-Newtonian fluid. The values of these two indices are also presented in Tables 8, 11, 13, and 16.

Once the values of n' and K' for a given non-Newtonian fluid were established, these results together with the corresponding data for the flow of the same liquid through beds were used to calculate the values of *f*, using Eq. (20), and Re'', using Eq. (21). The relationships between *f* and Re'' for all of the non-Newtonian fluids investigated are shown in Tables 7, 9, 10, 12, 14, and 15, and Figs. 8, and 9. It is seen in the figures that a number of parallel lines were obtained. Only the line representing the data for 0.4% Polyox solution is quite uncertain (see Fig. 7).

Efforts were made, without success, to maintain the bed of spheres at a constant porosity, 0.3800. The cause of the variation in the porosity of the bed is believed to be due to the low density of the spheres. However, the variations were still within the limits usually encountered with spheres (13c). But the results could not be used for testing the validity of Eq. (23). However, they still do serve to show that k in Eq. (22) is a function of n'.

The porosity of the beds of aluminum oxide particles on the other hand was kept constant, so as to be able to eliminate the effect of change in porosity upon k in Eq. (22).

The n in Eq. (23) is equal to n', defined by Eq. (4), in view of the linearity of the plots in Figs. 4, 5, 6, and 7, characterizing power law behavior. Equation (23) thus may be written

$$k = 16 \left(\frac{4\,(a + b\,n')}{3n' + 1}\right)^{n'}$$

This equation can also be written

$$\frac{3n' + 1}{4} \left(\frac{k}{16}\right)^{\frac{1}{n'}} = a + b\,n'$$

For each fluid, a value of n' was specified and a value of k was read from Fig. 9. A plot of $\frac{3n' + 1}{4} \left(\frac{k}{16}\right)^{\frac{1}{n'}}$ vs. n' gave the values of a and b characterizing the bed. The result is shown in Table 17 and Fig. 10. The line in this figure was drawn from the point at n' equal to unity and passed nearer the two points that were considered reliable. Eq. (22) thus takes on the following form:

$$f = \frac{4^{2 + n'} \left(\frac{1.14 + 1.36\,n'}{3n' + 1}\right)^{n'}}{Re''}$$

The three lines drawn in Fig. 11 were based upon this equation. Results obtained from flow measurements with the three non-Newtonian fluids through the bed of aluminum oxide particles were also plotted on the same figure for comparison. The average deviation is 7.3%.

Clearly, additional data for flow of non-Newtonian fluids through the same bed are required to obtain the location of the line in Fig. 10 with more certainty and thereby yield a more reliable result. It should be noted, however, that the average deviation involved is well within engineering accuracy.

VII-CONCLUSIONS

A generalised Kozeny-Carman equation has been developed for the flow of fluids through unconsolidated beds comprised of irregular particles. The relationship is applicable to Newtonian and time-independent non-Newtonian fluids that follow the power law.

This relationship between the friction factor and generalised Reynolds number is given by

$$f = \frac{4^{2 + n'} \left(\frac{1.14 + 1.36\, n'}{3n' + 1}\right)^{n'}}{Re''}$$

It is seen that the relation reduces to the Kozeny-Carman Equation, when n' is unity.

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IX-NOMENCLATURE

a	coefficient of porous bed (see Eq. (23))	
b	coefficient of porous bed (see Eq. (23))	
d	differential operator	
D_o	diameter	cm
D_p	diameter of a sphere	cm
f	Fanning friction factor	
g	acceleration of gravity	cm/sec^2
g_c	Newton's-law Conversion factor	$cm\text{-}g_m/g_f\text{-}sec^2$
g_f	gram force	
g_m	gram mass	
h	elevation relative to a datum plane	cm
H_1	length of liquid	cm
H_2	length of liquid	cm
H_3	distance between the two pressure taps (see Fig. 1)	cm
H_g	manometer reading	cm
H_o	liquid head above the inlet of a capillary tube	cm
k	constant of the Kozeny-Carman equation (see Eqs. (22) and (23))	
K'	flow consistency index	$\frac{g_m}{cm\text{-}sec^{2-n'}}$ or $\frac{g_f\ sec^{n'}}{cm^2}$

L	length of a capillary tube	cm
n	flow behavior index	
n'	flow behavior index (see Eq. (2))	
p	pressure	g_f/cm^2
p'	potential $(p + \sigma h)$	g_f/cm^2
Q	volumetric flow rate	cm^3/sec
r	radial distance from axis or center of a pipe	cm
r_h	hydraulic radius	cm
Re	Reynolds number (see Eqs. (11) and (14))	
Re'	generalized Reynolds number (see Eq. (8))	
Re''	generalized Reynolds number (see Eq. (21))	
S_o	specific surface of the particles per unit volume of the particles	cm^{-1}
u	point velocity	cm/sec
U	bulk velocity in a tube or average velocity in the pore space of a packed bed	cm/sec
U_s	bulk velocity based on the cross-sectional area of the bed	cm/sec
x	position variable in direction of flow	cm
ϵ	porosity	
μ	viscosity	$g_m/cm\text{-}sec$
ρ	density	g_m/cm^3
ρ_{H_g}	density of mercury	g_m/cm^3
σ	specific weight	g_f/cm^3
τ_o	shear stress at wall	g_f/cm^2

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XI-APPENDIX: SAMPLE CALCULATIONS

A. The calculation of the relationship of $r_h \left(-\frac{dp'}{dx}\right)$ vs. $\frac{2U_s}{r_h \epsilon}$ for the flow of 0.4% Carbopol solution through bed of aluminum oxide particles

ρ of aluminum oxide particles	=	$3.930\ g_m/cm^3$
weight of particles used	=	$3{,}000\ g_m$
length of packed bed	=	20.50 cm
cross-sectional area of bed	=	$59.87\ cm^2$

$$\epsilon = 1 - \frac{3{,}000}{3.930 \times 59.87 \times 20.50} = 0.3779$$

$$\rho_{H_g} = 13.6\ g_m/cm^3$$

$$\rho \text{ of the solution} = 1.00\ g_m/cm^3$$

$$H_3 = 12.648\ cm$$

$$H_g = 6.10\ cm \text{ of mercury}$$

From Eq. (26),

$$\left(-\frac{dp'}{dx}\right) = \frac{6.10}{12.648}(13.6 - 1.00) = 6.08\ g_f/cm^3$$

and $$\frac{1}{S_o} = 0.0245\ cm$$

From Eq. (27),

$$r_h\left(-\frac{dp'}{dx}\right) = \frac{0.0245 \times 0.3779 \times 6.08}{1 - 0.3779} = 0.0904\ \frac{g_f}{cm^2}$$

The corresponding flow rate was 21.5 cm^3/sec.

$$U_s = \frac{21.5}{59.87} = 0.360 \text{ cm/sec}$$

$$\frac{2U_s}{r_h \epsilon} = \frac{2 \times 0.360 \times (1 - 0.3779)}{0.0245 \times 0.3779^2} = 127 \text{ sec}^{-1}$$

The result of $r_h(-\frac{dp'}{dx})$ vs. $\frac{2U_s}{r_h \epsilon}$ is shown in Table 7 and Fig. 4.

B. The calculation of the relationship of τ_o vs. $\frac{8U}{D_o}$ for the flow of the same liquid through a capillary tube

capillary tube no. 3

capillary tube length	=	84.36 cm
capillary tube diameter	=	0.1681 cm
H_g	=	4.30 cm of mercury
H_o	=	11.9 cm
Q	=	0.134 cm^3/sec
U	=	$\frac{0.134}{\frac{\pi}{4}\, 0.1681^2}$ = 6.04 cm/sec

From Eq. (29),

$$(-\frac{dp'}{dx}) = (\frac{11.9 + 84.36}{84.36}) \times 1.00 + \frac{4.30}{84.36} \times 13.6$$

$$- \frac{1.5 \times 1.00 \times 6.04^2}{84.36 \times 980}$$

$$= 1.835 - 0.000661 = 1.835 \ g_f/cm^3$$

$$\tau_o = \frac{D_o}{4}\left(-\frac{dp'}{dx}\right) = \frac{0.1681}{4} \times 1.835 = 0.0771\ g_f/cm^2$$

$$\frac{8U}{D_o} = \frac{8 \times 6.04}{0.1681} = 288\ sec^{-1}$$

The result of τ_o vs. $\frac{8U}{D_o}$ is shown in Table 8 and Fig. 4. The straight line in this figure has a slope of 0.595, or n' is 0.595. When τ_o is 0.0740 g_f/cm^2, the value of $8U/D_o$ read from this line is 270 sec^{-1}.

From Eq. (4),

$$K' = \frac{0.0740 \times 980}{270^{0.595}} = 2.58\ \frac{g_m}{cm\text{-}sec^{1.405}}$$

C. The calculation of f and Re'' using Eqs. (20) and (21)

$$f = \frac{2 \times 0.3779^3 \times 0.0245 \times 980 \times 6.08}{(1 - 0.3779) \times 1.00 \times 0.360^2} = 197$$

$$Re'' = \left(\frac{0.3779 \times 0.0245}{1 - 0.3779}\right)^{0.595} \frac{1.00 \times 0.36^{1.405}}{2^{-2.405} \times 0.3779^{1.405} \times 2.58}$$

$$= 0.159$$

The result of f vs. Re'' is shown in Table 7 and Fig. 9. The line in this figure gives k a value of 30.4.