MODULUS OF ELASTICITY BASED METHOD FOR ESTIMATING THE VERTICAL MOVEMENT OF NATURAL UNSATURATED EXPANSIVE SOILS

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ABSTRACT

Expansive soils are widely distributed in arid and semi-arid regions around the world and are typically found in a state of unsaturated condition. These soils are constituted of the clay mineral montmorillonite that is highly active and contributes significantly to volume changes of soils due to variations in the natural water content conditions. The volume changes of expansive soils often cause damage to lightly loaded structures. The costs associated with the damage to lightly loaded structures constructed on expansive soils in the United States alone were estimated as $2.3 billion per year in 1973, which increased to $13 billion per year by 2009. In other words, these damages have increased more than five fold during the last four decades. Similar trends in damages were also reported in other countries (e.g., Australia, China, France, Saudi Arabia, United Kingdom, etc.).

Numerous methods have been proposed in the literature over the past 50 years for the prediction of the volume change movement of expansive soils. However, the focus of these methods has been towards estimating the maximum potential heave, which occurs when soils attain the saturation condition. The results of heave estimation considering saturated soil conditions are not always useful in engineering practice. This is because most of damages due to expansive soils often occur prior to reaching the saturation condition. A reliable design of structures on expansive soils is likely if the anticipated soil movements in the field can be reliably estimated over time, taking into account the influence of environmental factors. Limited studies are reported in the literature during the past decade in this direction to estimate/predict the expansive soil movements over time. The existing methods, however, suffer from the need to run expensive and time consuming tests. In addition, verification of these studies for different natural expansive soils has been rather limited.

A simple approach, which is referred to as a modulus of elasticity based method (MEBM), is proposed in this study for the prediction of the heave/shrinkage movements of natural expansive soils over time. The proposed MEBM is based on a simplified constitutive relationship used for the first time to estimate the vertical soil movements with respect to time in terms of the matric suction variations and the corresponding

values of the modulus of elasticity. The finite element program VADOSE/W (Geo-Slope 2007) for simulating the soil-atmospheric interactions is used as a tool to estimate the changes in matric suction over time. A semi-empirical model that was originally proposed by Vanapalli and Oh (2010) for fine-grained soils has been investigated and extended for unsaturated expansive soils to estimate the variation of the modulus of elasticity with respect to matric suction in the constitutive relationship of the proposed method. The MEBM has been tested for its validity in five case studies from the literature for a wide variety of site and environmental conditions, from Canada, China, and the United States. For each case study, factors influencing the volume change behavior of soils, such as climate conditions, soil cracks, lawn irrigation, and cover type (pavement, vegetation), are successfully modeled over the period of each simulation. The proposed MEBM provides good predictions of soil movements with respect to time for all the case studies. The MEBM is simple and efficient for the prediction of vertical movements of natural expansive soils underlying lightly loaded structures.

In addition, a new dimensionless model is also proposed, based on the dimensional analysis approach, for the estimation of the modulus of elasticity which can also be used in the constitutive relationship of the MEBM. The dimensional model is rigorous and takes into account the most significant influencing parameters such as matric suction, net confining stress, initial void ratio, and degree of saturation. This model provides a comprehensive characterization of the modulus of elasticity of expansive soils under unsaturated conditions for different scenarios of loading conditions (i.e., both lightly and heavily loaded structures).

The results of the present study are encouraging for proposing guidelines based on further investigations and research studies for the rational design of pavements, shallow and deep foundations placed on/in expansive soils using the mechanics of unsaturated soils.

DEDICATION

This thesis is dedicated to my beloved family members, my father (Hussin El-Saete) and my brother-in-law (Sami El-Selene), who were looking forward to seeing this accomplishment possible!

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SUBSCRIPTS

f	= final value
i	= initial value, or order
max	= maximum value
mean	= mean value
min	= minimum value
unsat	= unsaturated condition
sat	= saturated condition

ABBREVIATIONS AND SYMBOLS

a, b, c, d, f, g	= fitting parameters of the void ratio constitutive surface proposed by Vu and Fredlund (2006)
a_m	= coefficient of compressibility with respect to change in matric suction
a_t	= coefficient of compressibility with respect to change in net normal stress
a_w	= ratio of area of water-mineral and water-water contact of total area of "wavy" plane
a_1, b_1, x_1, y_1	= fitting parameters of the relationship of void ratio versus net normal stress for saturated soils proposed by Zhang (2004)
A_c	= soil activity
AEV	= air-entry value
b_m	= coefficient of water content change with respect to a change in matric suction
b_j	= components of body force vector
b_t	= coefficient of water content change with respect to a change in net normal stress

B	= slope of suction versus water content relationship
C	= clay content
C_c	= compression index
C_h	= suction index with respect to void ratio
C_H	= heave index in Nelson and Miller (1992) method
C_m	= compressive index with respect to matric suction
C_s	= swelling index
C_t	= compressive index with respect to total stress
C_w	= specific water capacity of a soil, or suction modulus ratio used by Vanapalli et al. (2010)
C_τ , C_ψ	= suction index
$COLE$	= coefficient of linear extensibility
D	= independent primary dimension in the dimensional analysis
e	= void ratio
e_0	= initial void ratio
e_i	= void ratio for the i^{th} layer
e_f	= final void ratio
E	= initial tangent modulus of elasticity
E_{unsat}	= soil modulus of elasticity under unsaturated condition
E_{sat}	= soil modulus of elasticity under the saturated condition
E_w	= water volumetric modulus associated with a change in net normal stress, or shrink-swell modulus proposed by Briaud et al. (2003)
f	= crack fabric factor proposed by Lytton et al. (2004), function, shrinkage ratio, or lateral restraint factor proposed by McKeen (1992)
$f(x,y,z,t)$	= internal source of moisture in the transient suction diffusion equation developed by Mitchell (1979)
f_i	= lateral confinement factor proposed Lytton (1977)

F_i = initial state factor in soil heave potential equation proposed by Zumrawi (2013)

FSI = free swell index

g = gravitational acceleration

G = shear modulus

G_s = specific gravity

h = soil water pressure head

h_0 = initial height of a specimen tested in triaxial shear tests

h_i, h_f = initial and final water potentials, or initial and final matric suction

h_i, H_i, Δz_i, Δt = thickness of the i^{th} soil layer

h_m = matric suction

h_s = solute suction

H = total hydraulic head, modulus of elasticity for the soil structure with respect to a change in matric suction, depth of soil, or soil layer thickness

H_w = water volumetric modulus associated with a change in matric suction

i = hydraulic gradient

I_{pt} = instability index

IL = liquidity index

k = coefficient of permeability (i.e., hydraulic conductivity)

k_{wi} = coefficient of permeability in the *i* direction

k_{sat} = saturated coefficient of permeability

K = fitting parameter in Janbu's relationship (1963) for the soil modulus of elasticity, or correction parameter used by Vanapalli et al. (2010)

K_0 = coefficient of lateral earth pressure at rest

l = number of variables in the dimensional analysis

L = dimension of length in the dimensional analysis

LL , w_L = liquid limit

LL_W = weighted liquid limit

m = number of independent primary dimensions in the dimensional analysis

m_1^s	= coefficient of total volume change with respect to change in net normal stress
m_2^s	= coefficient of volume change with respect to change in matric suction
m_1^w	= coefficient of water volume change with respect to change in net normal stress
m_2^w	= coefficient of water volume change with respect to change in matric suction
M	= dimension of mass in the dimensional analysis
MBV	= methylene blue value proposed by Cokca (2002)
n	= total number of soil layers considered, or fitting parameter in Janbu's relationship (1963) for the soil modulus of elasticity
N	= number of dimensionless parameters in the dimensional analysis
p	= unsaturated permeability in the transient suction diffusion equation developed by Mitchell (1979)
p''	= pore water pressure deficiency proposed by Donald (1956)
p''_m	= matric suction in the effective stress equations for unsaturated soils proposed by Aitchison (1973)
p''_s	= solute suction in the effective stress equation for unsaturated soils proposed by Aitchison (1973)
P	= partial pressure of pore water vapor, or surcharge pressure
P_0	= saturation pressure of water vapour over a flat surface of pure water
P_a , u_{atm}	= atmospheric pressure
P_f	= final stress state
P_s	= corrected swelling pressure
PI , I_p	= plasticity index
PL, w_p	= plastic limit
q	= surcharge pressure
q_i	= initial surcharge pressure
R	= osmotic suction in the effective stress equation for unsaturated soils proposed by Allam and Sridharan (1987), or universal gas constant (8.31432 J/(mol K))
R_h	= relative humidity

R_s = radius of curvature of the water meniscus

R^2 = coefficient of determination

s = reduction factor to account for overburden proposed by McKeen (1992)

S = degree of saturation, or heave/swell potential

S_p = heave/swell potential

S_f = surface displacement

SI = shrinkage index

SP = swell pressure applied to the soil due to overburden pressure

SW = percent swell

u = total suction

u_a = pore-air pressure

u_c = capillary pressure

u_i = components of displacement in the *i*-direction

u_w = pore-water pressure

t = time

T = temperature, or dimension of time in the dimensional analysis

T_s = surface tension

v_w = Darcy's flux (flow rate per unit area)

V = a variable in the dimensional analysis

V_0 = initial overall volume of an unsaturated soil element

V_a = volume of air in an unsaturated soil element

V_{mol} = molecular volume of water vapour (0.01802 m^3)

V_s = volume of solid particles in an unsaturated soil element

V_v = volume of soil voids in an unsaturated soil element

V_{vo} = volume of voids in an unsaturated soil element

V_w = volume of water in an unsaturated soil element

w_i , w_0 = initial water/moisture content

w_f	= final water/moisture content
w_n	= natural moisture content
w_{optm}	= optimum water content
x , y , z	= space coordinates
X	= dimensionless parameter in the dimensional analysis
Y	= elevation
z	= soil depth
α	= compressibility index proposed by Johnson and Snethen (1978), diffusion coefficient, or fitting parameter in Vanapalli and Oh (2010) model
β	= fitting parameter in Vanapalli and Oh (2010) model
β_1	= statistical factor of the same type as the contact area in the effective stress equation for unsaturated soils proposed by Jennings (1961)
β'	= holding or bonding factor in the effective stress equation for unsaturated soils proposed by Croney et al. (1958)
γ_d	= dry unit weight of soil
γ_{di}	= initial dry unit weight of soil
γ_{dmax}	= maximum dry unit weight of soil
γ_h	= suction compressibility index proposed by Wray (1984)
γ_t	= total unit weight of soil
γ_w	= unit weight of water
γ_{yz} , γ_{zx} , γ_{xy}	= shear strains on the *x*-, *y*-, and *z*-planes
γ_σ	= mean principal stress compression index
δ	= vertical shrinkage or heave
δ_{ij}	= Kroneker delta or substitution tensor in the effective stress equation for unsaturated soils proposed by Jommi (2000)
Δc	= difference in the concentration between two solutions
Δh	= total vertical movement (heave/shrink) at any depth, or change in the specimen height upon compression

Δh_i = vertical movement for each arbitrary layer

ΔH = soil heave, heave/swell potential, or surface displacement

ΔpF = change in the soil suction over a vertical increment

ΔpP = change in the soil overburden over a vertical increment

Δu = soil suction change

Δw = moisture content change

$\Delta \varepsilon_y$ = change in vertical strain

ε = axial strain, or mean-zero Gaussian random error term proposed by Türköz and Tosun (2011)

ε_{ij} = components of the strain tensor

ε_v = volumetric strain

ε_x, ε_y, ε_z = normal strains in the *x*-, *y*-, and *z*-directions respectively

ς = interface parameter on the reference plane in the effective stress equation for unsaturated soils proposed by Allam and Sridharan (1987)

θ_s = saturated volumetric water content

θ_w = volumetric water content

λ = parameter proposed by Nelson et al. (2006)

μ = Poisson's ratio

π = osmotic suction, or dimensionless parameter (i.e. Pi group) in the dimensional analysis

ρ_d = dry density of soil

ρ_i = maximum heave

ρ_w = density of water

σ = externally applied stress

σ_i , σ_f = initial and final values of mean principal stress

σ_{ij} = total stress tensor

σ_{ij}^* = Bishop's average soil skeleton stress

σ_{mean} = mean total normal stress

σ_x, σ_y, σ_z = normal stresses in the *x*-, *y*-, and *z*-directions, respectively

σ_1, σ_3 = major and minor principal stresses, respectively

σ' = effective stress

σ'_{cs} = swelling pressure from the overburden swell test

σ'_{cv} = swelling pressure from constant volume oedometer test

σ'_i = inundation stress subjected to a sample in overburden swell test

σ'_{ob} = overburden stress

φ = parameter in the effective stress equation for unsaturated soils proposed by Aitchison (1960)

χ = effective stress parameter related to the degree of soil saturation

χ_m = effective stress parameter for matric suction

χ_s = effective stress parameter for solute suction

ψ = total suction

ψ_i, ψ_f = initial and final total suction, respectively

CHAPTER 1

INTRODUCTION

1.1 Background

Expansive soils formation may probably be associated to the gradual weathering or erosion of basic igneous rocks or sedimentary rocks (Donaldson 1969). The minerals of the parent rocks from which expansive soils are derived decompose to form the highly active clay minerals (e.g., montmorillonite). These minerals have an affinity for absorbing large amounts of water between their clay sheets and therefore have a large shrink–swell potential. When potentially expansive soils become saturated, after rainfall or due to other activities (e.g., garden watering, or water pipe leaks), more water molecules are absorbed between the clay sheets, causing the bulk volume of the soil to increase or swell (heave). Conversely, when water is removed, by evaporation or gravitational forces, the water between the clay sheets is released, causing the overall volume of the soil to decrease or shrink (Jones and Jefferson 2012). When supporting structures, the effect of significant changes in the water content of expansive soils can be severe. The heave/shrink movements of expansive soils can cause tilt in trees, highway surfaces, building foundations and pipelines, and pose problems to the functionality of the infrastructure. These soils have also been referred to in the literature with various names such as the swelling soils, heaving soils, volume change soils, shrink-swell soils, problematic soils, or black cotton soils.

Expansive soils are vastly distributed in many regions around the world, and their distribution is dependent on many factors such as the geology, climate, hydrology, geomorphology, and vegetation of the region. The countries in which expansive soils have been reported are: Argentina, Australia, Burma, Canada, China, Cuba, Ethiopia, Ghana, Great Britain, India, Iran, Kenya, Mexico, Morocco, Rhodesia, South Africa, Spain, Turkey, USA, and Venezuela (Chen 1975, Fredlund and Rahardjo 1993). Approximately 60% of the world's population live in regions with expansive soils and have no choice but to construct their infrastructures on these problematic soils.

Problems associated with expansive soils were first recognized and documented by the U.S. Bureau of Reclamation at their Owyhee Project in Oregon in 1930 (Holtz and Gibbs 1954). Research interest in expansive soils to better understand them has increased as more extensive damages to structures were being documented after 1940. Jones and Holtz (1973) were the first investigators who estimated the damages associated with expansive soils. The damages to lightly loaded structures constructed in the United States alone amounts to about $2.3 billion per year. Krohn and Slosson (1980) estimates show the losses to the structures increased to about $7 billion per year. Steinberg (1998) reported further increases to these losses and reported them to be approximately $10 billion annually. More recently, Puppala and Cerato (2009) reported that the cost of damage due to expansive soils in the United States has risen dramatically to over $13 billion per year. Figure (1.1) shows that the losses associated with expansive soils in the United States alone have increased five fold during the last four decades. It is worth that the cumulative deleterious effects that expansive soils have on constructed facilities in the United States annually exceeds those of hurricanes, tornadoes, floods, and earthquakes combined (Jones and Holtz 1973). Expansive soils have been called the "hidden disaster": while they do not typically cause loss of life, economically, they are one of the United States costliest natural hazards (Snethen 1986). Similar problems have been reported in many other countries. Table (1.1) shows the annual costs associated with the infrastructure damage problems caused by the movements of expansive soils in different regions of the world. It is no wonder that these soils are considered to be a nightmare by geotechnical engineers.

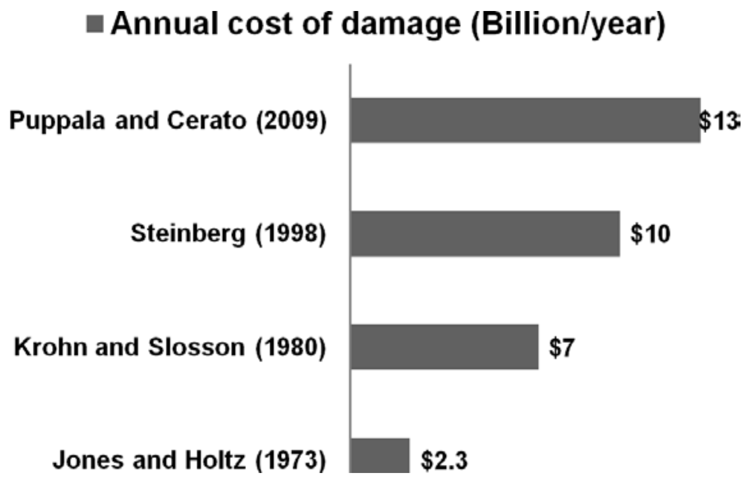

Fig. 1.1. The annual costs of damage to structures constructed on expansive soils in the United States since 1973

Table 1.1. The annual costs associated with the damage to structures constructed on expansive soils for different regions in the world

Region	Cost of damage/ year	Reference
USA	$13 billion	Puppala and Cerato (2009)
UK	£ 400 million	Driscoll and Crilly (2000)
France	€ 3.3 billion	Johnson (1973)
Saudi Arabia	$ 300 million	Ruwaih (1987)
China	¥ 100 million	Ng et al. (2003)
Victoria, Australia	$ 150 million	Osman et al. (2005)

Expansive soils are prone to heave (swelling) and shrinkage as they are sensitive to even small changes in their natural water content conditions. The volume change behavior of expansive soils is one of the key engineering properties that influence the performance of lightly loaded structures placed on expansive soils. Therefore, it is important to provide tools for the practitioners to reliably estimate the heave and shrink related volume change of expansive soils. Several methods have been proposed in the literature for the determination, or prediction of the volume change movement of expansive soils (Vanapalli and Lu 2012). The methods developed can be classified into three main categories namely; empirical methods, oedometer methods, and suction based methods. The focus has been towards estimating the maximum potential heave (i.e., the extreme condition), which occurs when soils attain the saturation condition. However, the results of heave estimation considering saturated soil conditions are not always practical or economical in engineering practice. A reliable design of lightly loaded structures on expansive soils is likely if the anticipated soil movements in the field can be estimated over time, taking into account the influence of environmental changes. Limited studies were reported in the literature to measure the soil movements in the field during the past three decades (Yoshida et al. 1983, Ching and Fredlund 1984, Clifton et al. 1984, Wray 1989, Allman et al. 1998, Erol and Dhowian 1990, Ng et al. 2003, Briaud et al. 2003, Fityus et al. 2004, Chao 2007, Tang et al. 2009). More studies in this direction are useful to better understand the behavior of expansive soils with respect to time. However, they are expensive and cumbersome and hence cannot be undertaken for routine engineering practice.

During the past decade focus of research has been towards proposing prediction procedures for estimating the expansive soil movement over time (Briaud et al. 2003, Vu and Fredlund 2004 and 2006, Zhang 2004, Wray et al. 2005, Overton et al. 2006, Nelson et al. 2007). The proposed methods, however, suffer from the need to run expensive and time consuming tests, and limited verification studies for different expansive soils. In other words, the methods validity was limited to expansive soils of local regions and has not been extended widely. The prediction of volume change movements is a challenge even to date to geotechnical engineers as heave and shrinkage behavior with respect to environmental changes cannot be well estimated.

1.2 Objective

The key objective of this research is to develop a simple and efficient method, which is referred to as a modulus of elasticity based method (MEBM), for predicting the vertical movements of natural expansive soils associated with the variations of environmental conditions. The research program focus is directed towards: (i) developing a simple constitutive relationship for estimating the vertical movements with respect to time in terms of the matric suction variations and the corresponding modulus of elasticity; (ii) modelling the soil-atmospheric interactions considering the environmental variations to simulate the matric suction changes over time; (iii) developing simple models to estimate the soil modulus of elasticity with respect to matric suction.

While all the existing prediction methods published to date were limited to localized areas, the proposed MEBM is tested for its validity in several case studies collected and gathered from the literature. These case studies, chosen to be representative of a variety of site conditions from different regions of the world, include:

- A slab-on-ground placed on Regina expansive clay subjected to a constant infiltration rate, which was originally modeled by Vu and Fredlund (2006).
- A case history of a light industrial building in North-Central Regina, Saskatchewan, Canada. History of the site and details of testing and monitoring programs were conducted by Yoshida et al. (1983). Heave analyses of the case history using laboratory oedometer data were carried out by Vu and Fredlund (2004).
- A field test site in Regina, Saskatchewan, modeled by Ito and Hu (2011) for one year. Various factors influencing soil movements such as climate changes, vegetation, watering of lawn, and soil cover type have been considered.
- A comprehensive field study previously investigated by Ng et al. (2003). This field study is a cut-slope in an expansive soil in Zao-Yang, Hubie, China, in which the effect of the soil compressibility, soil cracks, and environmental conditions on the soil movements have been investigated.
- A field experiment in Arlington, Texas, conducted by Briaud et al. (2003) for measuring the movements of four full-scale spread footings over a period of 2 years.

The MEBM approach proposed in this thesis is assessed by providing comparisons between soil movement estimates and the published results of the case studies under consideration.

1.3 Research Methodology

The research methodology can be summarized by the following:

- Review the available literature on the expansive soils to identify the most significant soil properties directly related to the volume change behavior of expansive soils.
- Develop basic theoretical understanding of the state-of-the-art prediction methods of the volume change of expansive soils, and critically review the current prediction methods in terms of their predictive capacities and their strengths and limitations for their use.
- Extend Vanapalli and Oh (2010) model, originally developed for estimating the modulus of elasticity in terms of matric suction for fine-grained soils with plasticity index I_p values lower than 16%, to be used for expansive soils (i.e., $I_p > 16\%$) and to test its validity using experimental data of triaxial shear tests for different expansive soils from the literature.
- Develop a constitutive relationship based on the volume change theory of unsaturated soils to predict the vertical movements of natural expansive soils over time.
- Simulate the time-evolution of matric suction profile within the active zone of soil based on the numerical modeling of the soil-atmospheric interactions. A finite element program VADOSE/W (Geo-Slope 2007) is used in this research study for this purpose.
- Apply the step-by-step procedure of the proposed modulus of elasticity based method (MEBM) on different case studies summarized in the preceding section to test its validity for the prediction of the heave and shrink movements of unsaturated expansive soils with respect to time.
- Propose a dimensionless model for estimating the modulus of elasticity of unsaturated expansive soils based on the dimensional analysis of experimental data of triaxial tests for different expansive soils, taking into account all the influencing parameters.
- Revisit a field study conducted by Ng et al. (2003) to evaluate the proposed MEBM based on using the new dimensionless model for estimating the modulus of elasticity of unsaturated expansive soils.

1.4 Novelty of the Research Study

The mechanics of unsaturated soils has been used as a tool to interpret the heave and shrink related volume change behavior of expansive soils since 1970's. However, there are few methods to predict the volume change behavior of unsaturated expansive soils over time. Existing prediction methods suggest that a given soil will exhibit a unique three-dimensional surface relating void ratio (i.e., volume change) to the mechanical stress and matric suction (Vu and Fredlund 2004, Zhang 2004). However, there are limitations to apply the three-dimensional constitutive surface model in practice. Current volume change constitutive surfaces are developed based on testing soils under conditions not experienced in the field such as a shrinkage test or a matric suction test at no normal stress, or a consolidation test at fully saturated conditions. Also, many conventional laboratories are not equipped to run controlled matric suction tests. These tests generally require costly, time consuming, and difficult laboratory testing. Most importantly, the prediction methods based on the volume change constitutive surface have been only validated for one case study.

The innovative aspect of the research presented in this thesis is to predict the volume change movement of natural expansive soils for different case studies using one simple approach (i.e., modulus of elasticity based method (MEBM)). The proposed MEBM is based on soil properties determined by using conventional geotechnical testing methods. This is the first time in the literature that a simplified constitutive relationship for the total volume change of soil is used to estimate the vertical soil movements in terms of the soil suction variations and the associated modulus of elasticity.

The pioneering work by Terzaghi (1925, 1926, and 1931) to understand the shrinkage and swelling behavior of clay showed that the shrinkage and swelling capacity of any soil are essentially dependent on the elastic properties of the solid phase of the soil. These fundamental studies of Terzaghi though not as widely cited as his other research studies in the conventional geotechnical literature, have significantly contributed to our present state-of-the-art interpretation of the volume change movements of expansive soils in terms of the soil modulus of elasticity. Vanapalli and Oh (2010) proposed a semi-empirical model for estimating the variation of the modulus of elasticity with respect to matric suction for soils with plasticity index I_p values lower than 16%. This model has been extended in this study to estimate the modulus of elasticity of unsaturated expansive soils (i.e., I_p > 16%). The information required for using the model include the soil-water characteristic curve (SWCC) and the modulus of elasticity of soil under saturated condition along with two fitting parameters. Experimental data of triaxial tests for three different expansive soils from the literature are used in this study to examine the validity of the model for expansive soils. Good comparisons are provided between the values of modulus of elasticity derived from triaxial tests results and from the modified Vanapalli and Oh (2010) model.

Based on the dimensional analysis of the same experimental data of triaxial tests used for the Vanapalli and Oh (2010) model, a new innovative model is proposed in this study to estimate the modulus of elasticity of unsaturated expansive soils. The new dimensionless model provides a more realistic characterization of the soil modulus of elasticity, taking account of the influence of matric suction and mechanical stress along with initial void ratio and degree of saturation.

The proposed MEBM is tested for its validity in five case studies from three countries: Canada, China, and the United States for a wide variety of site and environmental conditions. The MEBM, in comparison to other available methods, is simple and efficient for the prediction of vertical movements of natural expansive soils over time. The strength of the MEBM lies in its use of available soil properties that can be determined by using conventional geotechnical testing methods. The results of the research study are valuable to provide guidelines for design of structures constructed on expansive soils.

1.5 Layout of the Thesis

The thesis is organised into seven chapters. This chapter presents the problem definition, objective, methodology and novelty of the research study, and the layout of the thesis.

Chapter Two provides literature review that includes a comprehensive and detailed description on background of the expansive soils that is necessary for explaining the research studies presented in the thesis. The focus of the chapter has been directed to summarize the various approaches available in the literature for the prediction of volume change behavior of expansive soils.

Chapter Three provides an evidence for the validity of Vanapalli and Oh (2010) model to predict the modulus of elasticity for expansive soils under variably saturated conditions. Available experimental data of triaxial shear tests for different compacted unsaturated expansive soils is used in the chapter to examine the validity of the Vanapalli and Oh (2010) model.

Chapter Four details the fundamental concepts along with the step-by-step procedure of the proposed modulus of elasticity based method (MEBM) for predicting the vertical movement of natural expansive soils over time. Variations of soil matric suction and the corresponding modulus of elasticity with respect to matric suction variations are introduced into a volume change constitutive relationship to estimate the soil movement with respect to time. The soil-atmosphere model VADOSE/W is selected to be used for modeling the matric suction variations associated with the environmental changes over time for all case studies simulated in this research.

Chapter Five presents the validation of the MEBM approach for predicting the vertical soil movements over time using different case studies. Each case study is described, and the soil properties used in the analysis are listed. A detailed description of the simulation of each case study is also presented in this chapter. Comparisons of the results of the MEBM with the published data (measurements/estimates) are provided.

Chapter Six proposes an alternative model for estimating the unsaturated modulus of elasticity based on the dimensional analysis of the triaxial shear test results of compacted unsaturated expansive soils. The model takes into account the most significant factors influencing the value of the modulus of elasticity of unsaturated expansive soils. Comparisons are provided between the values of modulus of elasticity derived from the triaxial tests results, the extended Vanapalli and Oh (2010) model, and the new dimensionless model. In addition, a field study previously investigated by Ng et al. (2003) is revisited in this chapter to evaluate the MEBM using the dimensionless model as a tool for estimating the soil modulus of elasticity.

Finally, conclusions, and recommendations and suggestions for future research studies are presented in Chapter Seven.

Research undertaken through the present study has resulted in the following peer review journal publications:

- Adem, H.H. and Vanapalli, S.K. 2014. A state-of-the art review of methods for predicting the in situ volume change movement of expansive soil over time (tentatively accepted for publication in the Journal of Rock Mechanics and Geotechnical Engineering, revised version submitted).

- Adem, H.H. and Vanapalli, S.K. Heave prediction in a natural unsaturated expansive soil deposit under a lightly loaded structure (submitted to Geotechnical and Geological Engineering Journal).

- Adem, H.H. and Vanapalli, S.K. 2014. Elasticity moduli of expansive soils from dimensional analysis. Geotechnical Research, 1(2): 60-72, DOI: 10.1680/gr.14.00006

- Adem, H.H. and Vanapalli, S.K. 2014. Soil-environment interactions modeling for expansive soils. Environmental Geotechnics, DOI: 10.1680/envgeo.13.00089

- Adem, H.H. and Vanapalli, S.K. 2014. Prediction of the modulus of elasticity of compacted unsaturated expansive soils. International Journal of Geotechnical Engineering, DOI: http://dx.doi.org/10.1179/1939787914Y.0000000050

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CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Expansive soils absorb large quantities of water after rainfall or due to local site changes (such as leakage from water supply pipes or drains), becoming sticky and heavy. Conversely, they can also become stiff when dry, resulting in shrinking and cracking of the ground. This hardening and softening is known as 'shrink-swell' behavior (Jones and Jefferson 2012). When supporting lightly loaded structures, the effect of significant changes in moisture content on soils with a high shrink-swell potential can be severe. Hence, it is important to provide tools for practitioners to reliably estimate the volume change behavior of expansive soils in the field. Significant advances were made during the last half-a-century to understand the volume change behavior of expansive soils. This chapter reviews the literature that discusses the properties of unsaturated expansive soils. The chapter also presents the constitutive relations for the volume change of unsaturated expansive soils as well as the flow laws for the soil moisture migration. In addition, a state-of-the-art of the prediction methods of the volume change movement of expansive soils is succinctly summarized. The available prediction methods are critically reviewed in terms of their predictive capacities and their strengths and limitations. The review highlights the need for prediction methods that are conceptually simple yet efficient for use in conventional engineering practice for different types of expansive soils.

2.2 Expansive Soil Mineralogy

The expansion potential of any particular expansive soil is mainly determined by the percentage and the type of clay minerals in the soil. The shape and structure of the clay minerals are determined by their arrangement of their constituent atoms which form thin clay crystals. The three most important clay minerals are montmorillonite, illite, and kaolinite. Montmorillonite is the clay mineral that contributes to the expansive soil problems.

The clay minerals are formed through a complicated process from a variety of parent materials. These materials typically include feldspars, micas, and limestone. The alteration process that takes place on land is referred to as weathering and that on the sea floor or lake bottom as halmyrolysis. The alteration process includes disintegration, oxidation, hydration, and leaching (Chen 1975). The formation of montmorillonite requires extreme disintegration, strong hydration, and restricted leaching. When the leaching is restricted, magnesium, calcium, sodium, and iron cations may accumulate in the system. Thus, the formation of montmorillonite minerals is aided by an alkaline environment, presence of magnesium ions, and a lack of leaching (Tourtelot 1973). Such conditions are favorable in semi-arid regions, particularly where evaporation exceeds precipitation. Under these conditions, enough water is available for the alteration process, but the accumulated cations will not be removed by flush rain (Tourtelot 1973, Chen 1975). The parent minerals for the formation of montmorillonite often consist of ferromagnesium minerals, calcic feldspars, volcanic glass, and many volcanic rocks (Chen 1975). Bentonite is highly plastic, swelling clay composed primarily of montmorillonite which has been formed by the chemical weathering of volcanic ash. The properties of bentonite are familiar to most geotechnical engineers. Expansive clays are commonly referred to as bentonitic soils.

There are two fundamental molecular structures as the basic unit of the lattice structure of clay minerals. These are the silica tetrahedron and the alumina octahedron. The silica tetrahedron is composed of a silicon atom surrounded tetrahedrally by four oxygen ions as shown on Figure (2.1a). When each oxygen atom is shared by two tetrahedral, a plate-shaped layer is formed (Figure 2.1b). However, the alumina octahedron is composed of an aluminum atom surrounded octahedrally by six oxygen ions as shown in Figure (2.2a). Similarly, when each aluminum atom is shared by two octahedrons, a sheet is formed (Figure 2.2b) (Chen 1975, Mitchell 1976). The clay minerals are characterized by staking arrangement of sheets of these units and the manner in which two successive two- or three-sheet layers are held together.

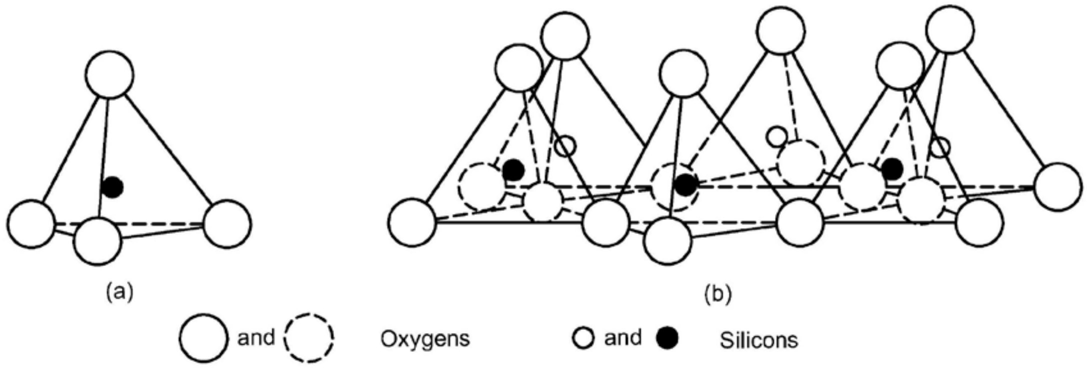


Fig. 2.1. A single silica tetrahedron and the sheet structure of silica tetrahedrons arranged in a hexagonal network (Mitchell and Soga 2005)

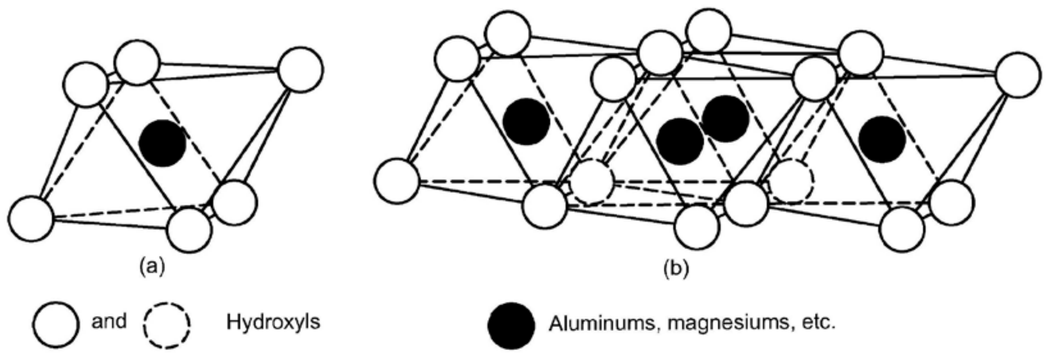


Fig. 2.2. A single octahedral unit and the sheet structure of the octahedral units (Mitchell and Soga 2005)

Kaolinite is a typical two layer mineral composed of alternating silica and octahedral sheets to form what is called a 1 to 1 lattice structure as shown in Figure (2.3a). Bonding between successive layers is by both van der Waals forces and hydrogen bonds. The bonding is sufficiently strong that there is no interlayer swelling in the presence of water (Mitchell and Soga 2005).

Montmorillonite is a three-layer mineral having a single octahedral sheet sandwiched between two silica sheets to give a 2 to 1 lattice structure as shown in Figure (2.3b).

Bonding between successive layers is by van der Waals forces and by cations that balance charge deficiencies in the structure. These bonds are weak and easily separated by adsorption of water (Mitchell and Soga 2005).

Iillite has similar structure to that of montmorillonite, but some of silicon atoms are replaced by aluminum; and, in addition, potassium ions are present between the silica sheet and adjacent crystals (Figure 2.3c). Because of these differences, the illite structural unit layers are relatively fixed in position, so that polar ions cannot enter readily between them and cause expansion. Also the potassium ions between layers are not easily exchangeable. In other words, the interlayer bonding by potassium is sufficiently strong that the basal spacing of illite remains fixed in the presence of water (Mitchell and Soga 2005).

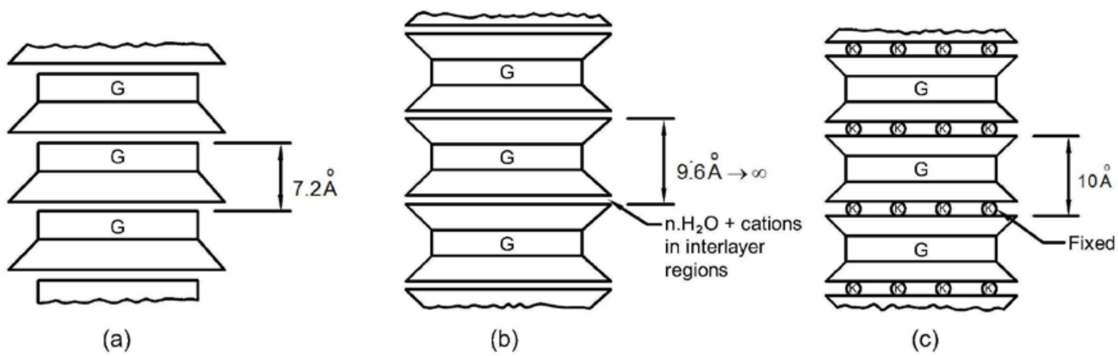


Fig. 2.3. Schematic diagrams of the structures of (a) kaolinite, (b) montmorillonite, (c) illite (Mitchell and Soga 2005)

Absorption of water by clays leads to expansion. Generally, the amount of expansion depends on percentages and types of clay minerals in the soil (Chen 1975, Bell and Culshaw 2001). For example, the presence of a relatively large amount of montmorillonite tends to increase water intrusion within the mass of soil and then create swelling, while illite and kaolinite are largely inert and resistant to water penetration. Mitchell and Soga (2005) and Grim (1968) provide extensive details of mineralogy of clay minerals and their influence on the engineering behavior of soils.

2.3 Mechanism of Soil Swelling

The mechanism of soil swelling has been described by several researchers (e.g., Anderson et al. 1973, Chen 1975, Schafer and Singer 1976, Nelson and Miller 1992, Stavridakis 2006). One proposed mechanism of soil swelling is associated with the interlayer expansion of clay mineral montmorillonite. The molecular structure of montmorillonite has a particular affinity to attract and hold water molecules between the clay crystal sheets. When potentially expansive soils become saturated, the clay mineral montmorillonite can absorb large amounts of water molecules into gaps between its clay sheets. As more water is absorbed, the sheets are forced further apart, leading to an increase in soil pressure or an expansion of soil volume (Figure 2.4). On the other hand, intraparticle swelling due to orientation of water films around high charge density clays may also contribute to swelling of soils. Clay particles are mainly flat and electrically charged (usually negative). This high potential is concentrated on the surface of the clay particles causing attraction of bipolar water molecules. Thus, an orientation of water molecules on the clay surface is achieved. As water molecules are attracted to the clay particles by negative charges, they push the clay particles apart, causing an expansion or swelling of the soil (Figure 2.5).

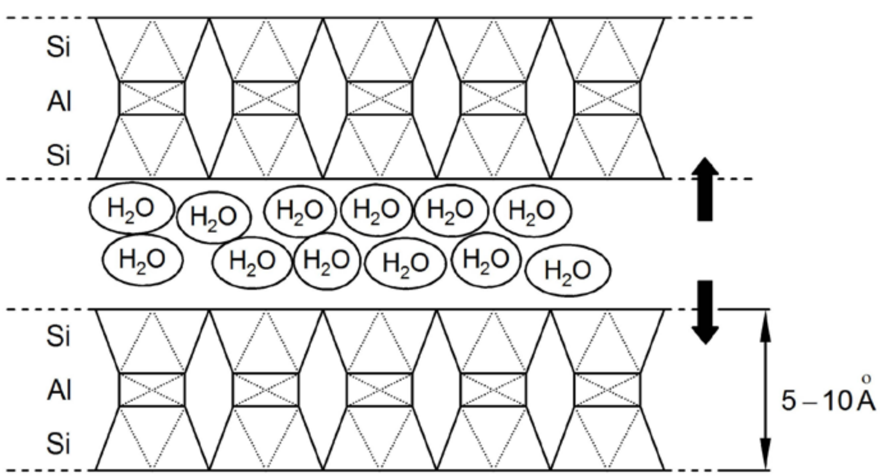


Fig. 2.4. The intercalation of water molecules in the inter-plane space of montmorillonite (Taboada 2003)

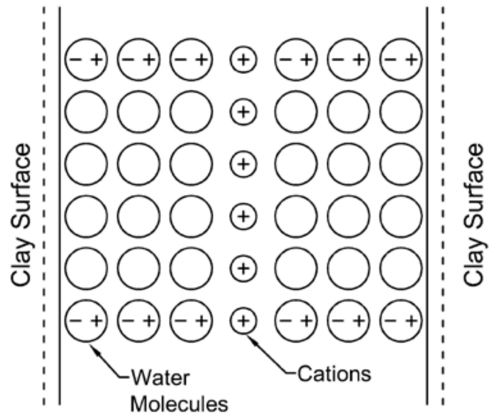


Fig. 2.5. The orientation of water films around high charge density clays (Mitchell and Soga 2005)

2.4 Volume Change Movement of Expansive Soil

Expansive soils experience extensive volume changes (heave/shrink) when there is an environmental change such as water content increase due to introduction of moisture, pressure release due to excavation, and desiccation caused by temperature increase. If the increase in soil volume is not restrained and soil has an opportunity to swell in all the directions, the soil increases its volume in all the directions with the same amount (Figure 2.6a). However, for the soil in the field near the ground surface, the lateral expansion might be to some extent prevented by the adjacent or the surrounding soil. The soil expansion or swell predominantly occurs in the vertical direction (Figure 2.6b).

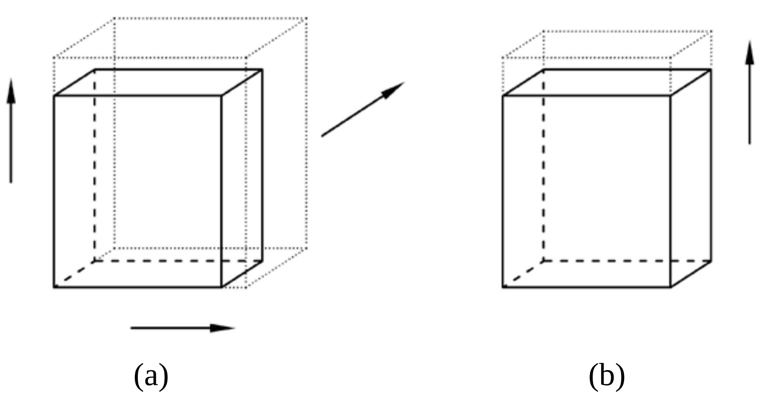


Fig. 2.6. Type of expansive soil movements: (a) soil volumetric expansion in three directions when there is no restriction, (b) soil heave when the lateral movement is restricted

Drying shrinkage of soils is caused by particles movements resulting from a severe loss of pore water. Pore water could be removed from soil by a wide variety of mechanisms. For example, excavation or other works that lower ground water levels, prolonged periods of low rainfall, and low rainfall in combination with high water demand mature trees can lead to a reduction in pore water, and then a reduction in soil volume (Chen 1975). The soil decreases its volume in the lateral and vertical directions. The volume decrease in the vertical direction causes the soil surface to go down (shrink); the lateral decrease in volume causes the soil to crack, if it is large enough.

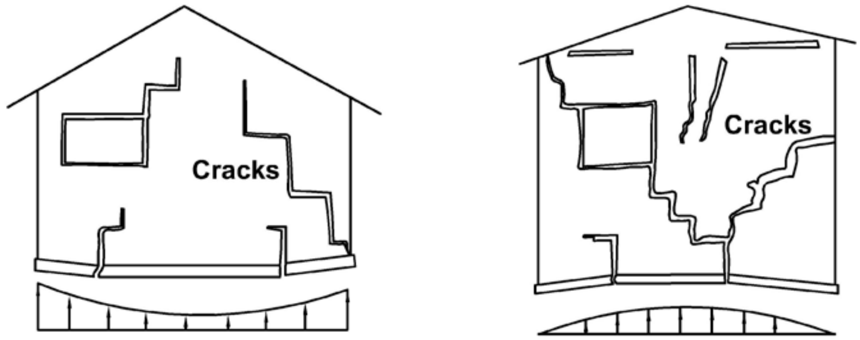
On the other hand, based on the water infiltration and the type of structure, there are two main types of expansive soil movements that could occur after construction and affect the structure performance:

- *Lateral movement*

  For many retaining and basement walls, especially if the clay backfill is compacted below the optimum moisture content, seepage of water into the clay backfill causes horizontal swelling pressures well in excess of at-rest values. Lateral thrust of expansive soil with a horizontal force approaching the passive earth pressure can cause bulging and fracture of basement and retaining walls. While it is possible that a large amount of swelling pressure can be exerted horizontally against a wall, generally backfill is so loosely compacted that distress caused by lateral expansion of backfill is unlikely (Chen 1975).

- *Vertical movement*

  If a structure having a large area, such as a pavement or foundation, is constructed on top of expansive soil, there are usually two main types of soil movements. The first is the cyclic heave and shrinkage around the perimeter of the structure, related to the amount of drainage and the frequency and the amount of rainfall and evaporation (edge uplift) (Figure 2.7a). The second is the long-term progressive swell beneath the center of the structure, which can occur as an upward long term dome shaped movement (center heave) (Figure 2.7b). Moisture can accumulate underneath structures either by thermal osmosis or capillary action.



(a) Edge uplift (b) Doming heave

Fig. 2.7. Vertical movement of expansive foundation soils: (a) edge uplift, (b) doming heave (modified after Department of the Army USA 1983)

2.5 Factors Affecting Soil Volume Change

The mechanism of soil volume change is complex and is influenced by a number of factors which are not easy to quantify. The major factors can be classified into two broad categories, as shown in Table (2.1): (i) in-situ soil properties and site conditions, (ii) environmental influence arising from proposed land use (Holtz and Gibbs 1956, Seed et al. 1962, Jennings 1969, Chen 1975, Johnson and Snethen 1978, Holland and Cameron 1981, Jones and Jefferson 2012).

Table 2.1. Factors influencing the magnitude and the rate of soil volume change (modified after Holtz and Gibbs 1956, Seed et al. 1962, Jennings 1969, Chen 1975, Johnson and Snethen 1978, Holland and Cameron 1981, Jones and Jefferson 2012)

Factor	Description
Soil properties	
Type and amount of clay minerals	The basic mineral and the fabric of particular clay, together with the salt concentration of the soil water, determine soil potential for volume change or heave.
Initial soil moisture conditions	A dry expansive soil will have higher an affinity for water and swell more than the same soil at higher water content. Conversely, a wet soil profile will lose water more readily on exposure to drying influences, and shrink more than a relatively dry initial profile.
Soil permeability	The permeability of soil determines the flow rate into the soil by either gravitational flow or diffusion. Soils with higher permeability, particularly due to fissures and cracks, allow faster migration of water

	and promote faster rates of swell.
Plasticity	Soils that exhibit plastic behavior over a wide range of moisture content and that have high liquid limit have greater potential for swelling and shrinkage.
Soil density	Higher densities usually indicate closer particles spacing, which may mean greater repulsive forces between particles and larger swelling potential. Dense soils will swell more when they become wetted compared with the same soil at the same initial water content and a lower density.
Thickness of expansive strata	The thickness and location of potentially expansive soil layers in the profile considerably influence the potential movement. The greatest movement will occur when expansive soils extend down to a great depth. However, if the expansive soil is overlain by a layer of non-expansive topsoil, or overlies bedrock at a shallow depth, the movement of the soil will be greatly reduced.
Environmental conditions	
Climate	Climatic conditions such as precipitation and evaporation greatly influence the moisture availability and depth of seasonal moisture fluctuation in the soil profile. Expansive soils will swell and shrink if the prevailing climatic conditions lead to major moisture changes. The greatest heaves will occur under semi-arid climatic conditions that have pronounced short wet period after long dry periods.
Depth of groundwater table	Shallow and fluctuating water tables provide a source of moisture changes in the active zone near the ground surface which primarily define soil movements.
Vegetation	Trees, shrubs, and grasses deplete moisture from the soil through transpiration, and have the ability to dry out soils within the zone of influence of their root systems. The drying out of soil in the vicinity of vegetation alters the pattern of soil movements by extending the drying cycle. Also, plants cause the soil to be differently wetted, the subsoil is usually subjected to nonuniform movements, and then substantial superstructure distress can take place.
Localized moisture excess	Soil heave patterns may be appreciably altered in areas where localized sources of moisture exist. Excessive garden watering and leak from broken pipe are two examples of moisture source.
Site drainage	Poor surface drainage of a site leads to moisture accumulations or ponding, so significant wetting up of the soil will occur, leading to substantial heave. Improvement of site drainage can therefore dramatically reduce the magnitude of soil heave.
Surcharge pressure	The application of surcharge load on potentially expansive soil will act to balance interparticle repulsive forces and reduce soil swell. Therefore, damage due to underlying soil heave is generally associated with lightly loaded structures.

All of the above factors should be taken into consideration in judging the usefulness of any method for predicting the amount and the rate of soil volume change. A summary of the most common prediction methods are provided later in this chapter.

2.6 Soil Suction in Unsaturated Soils

Soils above the groundwater table have an affinity for water, which, either partially or fully, fills in the space within the soil pores. This affinity that a soil has for water can be expressed through the relative humidity of the ambient air close to the soil. This affinity is called the total suction ψ which, by definition, is the total free energy of the soil water determined as the ratio of the partial pressure of the water vapor in equilibrium with a solution identical in composition to the soil water, to the partial pressure of the water vapor in equilibrium with a pool of free pure water (Aitchison 1965). The total suction can be calculated based on the principles of thermodynamics using Kelvin equation (Richards 1965)

$$\psi = -\frac{RT}{V_{mol}} \ln(R_h) \tag{2.1}$$

where R_h is the relative humidity, which equals the ratio between the partial pressure of pore water vapor P and the saturation pressure of water vapor over a flat surface of pure water at the same temperature P_0, R is the universal gas constant (8.31432 J/(mol K)), V_{mol} is the molecular volume of water vapor (0.01802 m^3), and T the absolute temperature (°K).

The total suction can also be calculated as the sum of two components; namely, the matric suction $(u_a - u_w)$ and the osmotic suction π as shown in equation (2.2) (Buckingham 1907, Bolt and Miller 1958, Aitchison 1965, Fredlund and Rahardjo 1993). The matric suction component is related to the capillary phenomenon arising from the surface tension of water (or air-water interface), and the osmotic suction component is related to the dissolve salt in soil pore water.

$$\psi = (u_a - u_w) + \pi \qquad (2.2)$$

where u_a is the pore air pressure, and u_w is the pore water pressure.

2.6.1 Matric suction

In the literature of unsaturated soils, soil suction usually refers to the matric suction which is expressed as the excess of pore air pressure u_a over pore water pressure u_w (i.e., $(u_a - u_w)$). Matric suction has also been defined from a thermodynamics point of view as the ratio of partial pressure of the water vapor in equilibrium with the soil water, to the partial pressure of the water vapor in equilibrium with a solution identical in composition with the soil water (Aitchison 1965).

It is more appropriate to consider the matric suction as a variable that expresses quantitatively the degree of attachment of water to solid particles which results from the general solid/water/interface interaction (Gens 2010). Because water is attracted to soil particles and because water can develop surface tension, matric suction develops inside the pore fluid when a saturated soil begins to dry. The action associated with matric suction is similar to vacuum and will directly contribute to the effective stress or skeletal forces (Mitchell 1993). Analogous to a case of the capillary tube shown in Figure (2.8), the matric suction is related to the surface tension and the curvature of the water meniscus and it can be calculated as

$$(u_a - u_w) = \frac{2T_s}{R_s} \qquad (2.3)$$

where T_s is the surface tension of water, and R_s is the radius of curvature. The later can be considered analogous to the pore radius in a soil (Fredlund and Rahardjo 1993). The smaller the pore radius of a soil, the higher the soil matric suction can be. Figure (2.9) shows the relationship among the pore radius, matric suction, and capillary height.

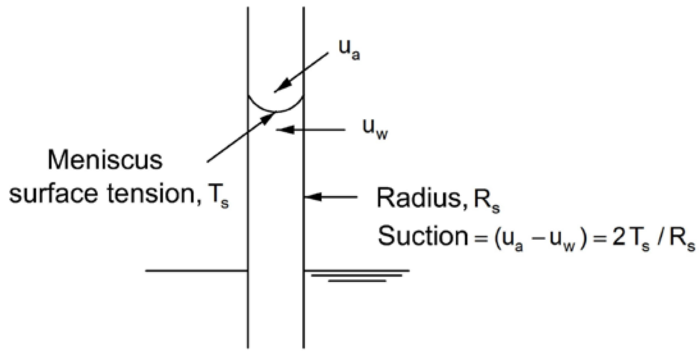


Fig. 2.8. The capillary phenomenon contributing to the matric suction (Mitchell and Soga 2005)

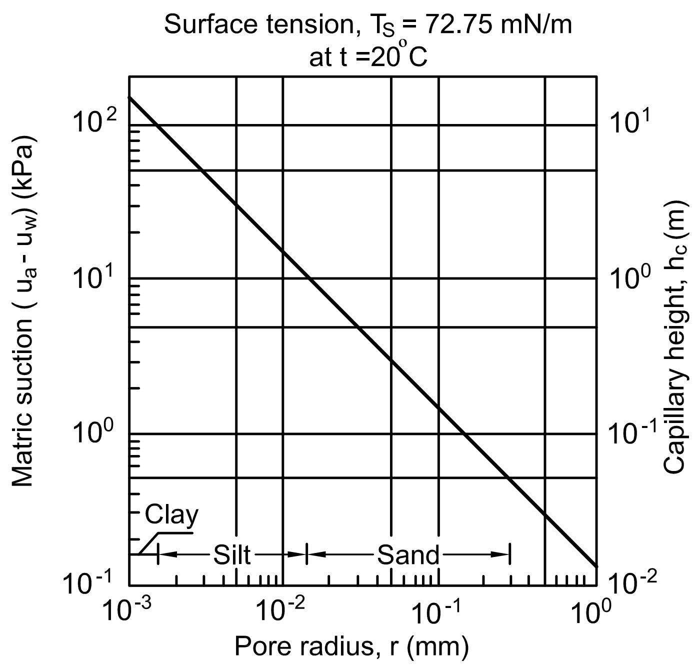
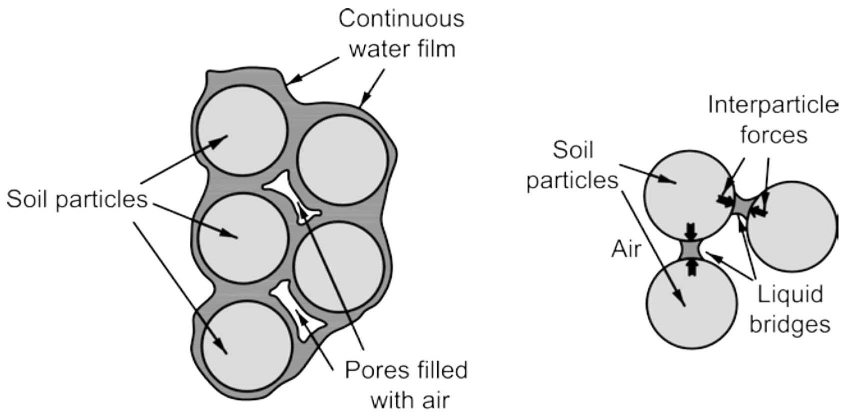


Fig. 2.9. Relationship among the pore radius, matric suction, and capillary height (Fredlund and Rahardjo 1993)

Since the water acts like a membrane with negative pressure, the suction force contributes directly to the skeletal forces like the water pressure (Figure 2.10a). As the soil continues to dry, the water phase becomes disconnected and remains in the form of menisci or liquid bridges at the interparticle contacts (Figure 2.10b). The curved air-water interface produces pore water tension, which, in turn, generates interparticle compressive forces (suction forces) that only act at particle contacts. This suction force generally depends on the separation between the two particles, the radius of liquid bridge, interfacial tension, and contact angle (Lian et al. 1993, Mitchell and Soga 2005).



(a) (b)

Fig. 2.10. Microscopic water-soil interaction in unsaturated soils: (a) negative pore pressure acts all around the particles, (b) suction forces act only at particles contact (Mitchell and Soga 2005)

In this thesis, u_w is used to represent the pore water pressure, which can be either positive (saturated soils) or negative (unsaturated soils), and $(u_a - u_w)$ is used to represent the matric suction, which is the absolute value of the negative pore water pressure at an atmospheric air pressure $(u_a = 0)$. This is consistent with the continuum mechanics principles and has been used by several investigators (Fredlund and Rahardjo 1993).

2.6.2 Osmotic suction

Osmotic suction arises from differences in the salt concentration within the soil pore water from that of pure water. The osmotic suction is defined by Aitchison (1965) as the equivalent suction derived from the measurement of the partial pressure of the water vapor in equilibrium with a solution identical in composition with the soil water, relative to the partial pressure of water vapor in equilibrium with free pure water. The osmotic suction can be calculated using van't Hoff equation as

$$\pi = R_s T_s \Delta c \quad (2.4)$$

where R_s and T_s are defined in equation (2.3), Δc is the difference in the concentration between two solutions.

The presence of osmotic suction gives some additional affinity for water of the soil. The osmotic suction is related to the tendency of water to move from the region of low salt concentration to high concentration. For example, when a pool of pure water is placed in contact with a salt solution through a membrane, which allows only the water to flow through, an osmotic potential will develop due to the difference in the concentration of salt solution and water will flow through membrane. Thus, changes in osmotic suction have no effect on the mechanical behavior (i.e., volume change and shear strength) of the soil (Fredlund and Rahardjo 1993).

The results of the experimental research conducted by Miller and Nelson (2006) to evaluate the effects of osmotic suction on suction measurements shows, although osmotic suction dominates the total suction measurements, its influence on soil behavior is relatively small compared to the effect of the changes in matric suction. Miller and Nelson (2006) also studied the effect of salt concentration on the soil-water characteristic curve and the soil compressibility in terms of matric suction. It was concluded that adding salt did not result in a substantially different soil with respect to its volume change response to changes in matric suction. Since soil osmotic suction is relatively constant at various water contents, Krahn and Fredlund (1972) suggested that the osmotic suction can be assumed as a constant value and subtracted from the total suction measurements. Alonso et al. (1987) and Fredlund and Rahardjo (1993) suggested that the matric suction is only taken into account as a relevant variable in the interpretation of unsaturated soils, assuming that the ionic concentration of liquid in osmotic suction remains unchanged. Hence, only matric suction has been closely related to the engineering properties of soils, and it has been used as a tool by several researchers to estimate the properties of the soil such as swelling behavior, shear strength, and soil compressibility (e.g., van Genuchten 1980, Alonso et al. 1990, Vanapalli et al. 1996, Rassam and Williams 1999, Rampino et al. 2000, Vanapalli and Oh 2010).

2.6.3 Matric suction profile

The potential change in matric suction is generally attributed to environmental condition, human imposed irrigation, influence of vegetation, and accidental wetting due to broken pipelines. Figure (2.11) shows the effect of the environmental conditions on the in-situ profile of pore water pressure (i.e., matric suction). Dry and wet seasons cause variations in the matric suction profile, particularly close to the ground surface. During a dry session, the evaporation rate is high, and it results in a net loss of water from the soil. As consequence, soil shrinkage may occur. However, if evaporation is eliminated due to any reason such as the precipitation during wet session or covering the area, the opposite condition may occur and soil swelling may take place (Fredlund and Rahardjo 1993). The matric suction fluctuations are slow; however, even small suction changes may cause significant volume changes.

The expected depth of matric suction changes in the soil profile is particularly important as it is used to estimate soil volume change by integrating the soil strain produced over the zone in which matric suctions change. By understanding the interaction between the matric suction and soil volume change, a reasonable estimation of the volume change movement of expansive soils associated with the environmental variations is possible.

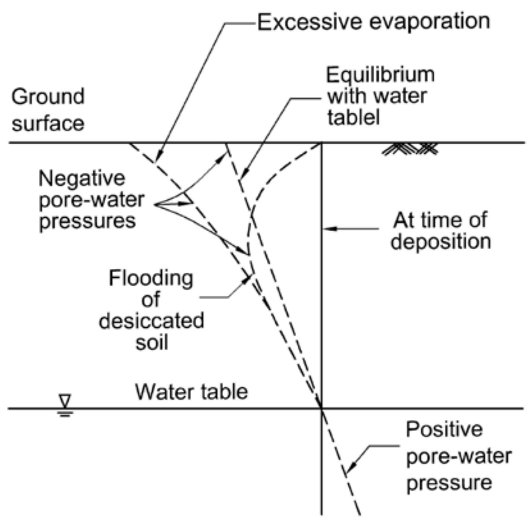


Fig. 2.11. Typical pore-water pressure profiles (modified after Fredlund and Rahardjo 1993)

The prediction of matric suction variations is complex because of soil heterogeneity, nonlinear unsaturated soil properties, and volume change characteristics of expansive soils (Dye 2008). Several commercial programs, summarized in a later section, are available for estimating the matric suction profiles considering both water flow in unsaturated soils and soil-atmospheric interactions. Such techniques are valuable, simple, and economical in comparison to the direct measurement of in-situ suction.

2.7 Modeling of Unsaturated Flow and Atmospheric Interactions

Water flow in the unsaturated zones is significantly influenced by the atmospheric interactions and impacts the engineering behavior of the soil. Water flow through soils in both saturated and unsaturated conditions can be described by Darcy's law (Richards 1931). Darcy's law for vertical flow is stated as follows

$$v_w = -ki = -k\frac{\partial H}{\partial z} = -k\frac{\partial}{\partial z}(h - z) \tag{2.5}$$

where v_w is the flow rate per unit area, i is the hydraulic gradient, H is the total hydraulic head, h is the soil water pressure head, and z is the vertical distance from the soil surface downward (i.e., the soil depth), k is the hydraulic conductivity (i.e., coefficient of permeability). The hydraulic conductivity for unsaturated soils is not a constant as for saturated soils, but it is a function of soil suction (or volumetric water content θ_w) (Figure 2.12). The capillary model shown in Figure (2.8) illustrates how suction can be related to the effective radius of a fluid-filled pore, and helps to explain why the relationship between hydraulic conductivity and suction is highly non-linear. As suction increases, the largest pores drain, decreasing the open area of flow, decreasing the effective radius of fluid filled pores, increasing the tortuosity of structure of the flow path, and decreasing the hydraulic conductivity (see Figure 2.12).

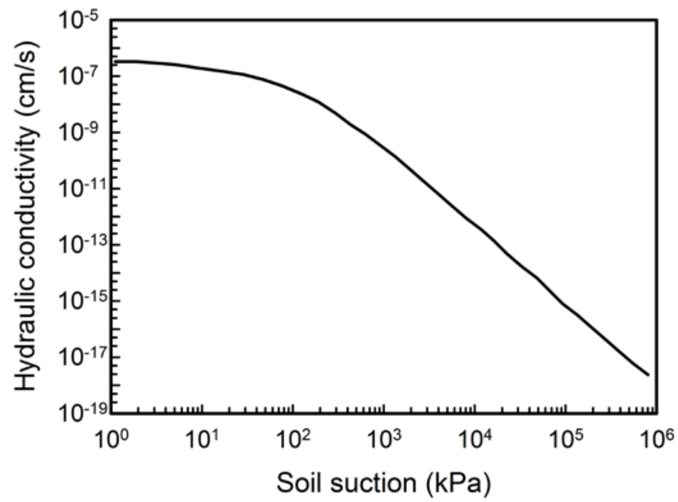


Fig. 2.12. Hydraulic conductivity as a function of soil suction (Benson 2007)

By combining the formulation of Darcy equation (2.5) with the continuity equation $\partial\theta_w/\partial t = -\partial v_w/\partial z$, and assuming isothermal conditions, isotropic hydraulic conductivity, incompressible water phase and pore space, and static and continuous vapor phase, the general flow equation can be written as

$$\frac{\partial\theta_w}{\partial t} = \frac{\partial}{\partial z}(k\frac{\partial H}{\partial z}) = \frac{\partial}{\partial z}(k\frac{\partial h}{\partial z}) - \frac{\partial k}{\partial z} \tag{2.6}$$

If soil volumetric water content θ_w and pressure head h are uniquely related, then the left-hand side of equation (2.6) can be written as $\partial\theta_w/\partial t = \partial\theta_w/\partial h \cdot \partial h/\partial t$ which transforms equation (2.6) into Richards equation (Equation 2.7)

$$C_w\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}(k\frac{\partial h}{\partial z}) - \frac{\partial k}{\partial z} \tag{2.7}$$

where C_w $(= \partial\theta_w/\partial h)$ is the specific water capacity, which is defined as the change in water content in a unit volume of soil per unit change in matric potential (i.e., the slope of the relation between the volumetric water content and soil suction described through the use of the soil-water characteristic curve (SWCC) (Figure 2.13)). The specific water capacity varies between -∞ and 0, and can approach negative values for soils with uniform pore size distribution (Benson 2007). The non-linearity of equation (2.7) arises because the equation coefficients (k and C_w) are functions of the dependent variables, and the exact analytical solution for specific boundary conditions is extremely difficult to obtain. In addition, hysteresis in k and C_w further complicate the solution.

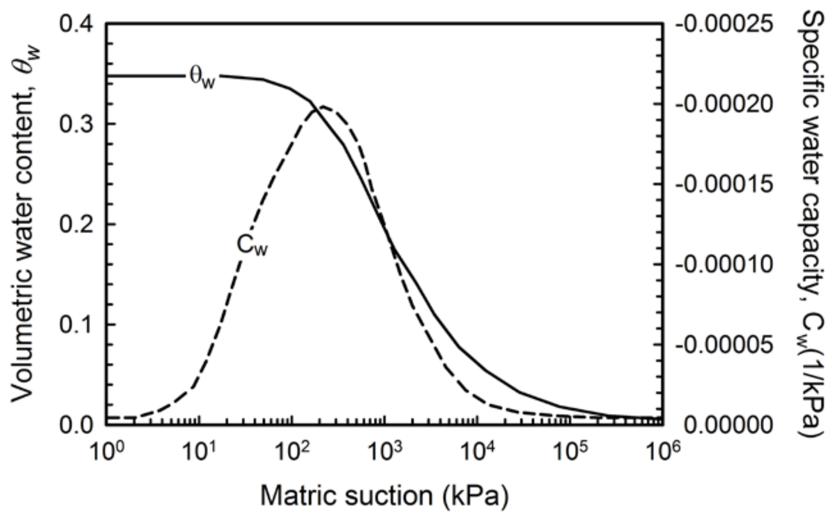


Fig. 2.13. Soil-water characteristic curve and specific water capacity (Benson 2007)

Several software packages are available for solving Richards equation (Equation 2.7). Examples of commercially available and commonly used packages are listed in Table (2.2). Each of these packages uses numerical methods to solve Richards equation, considering climate boundary to simulate atmospheric interactions and root-water uptake functions to simulate plant transpiration. Factors such as solute transport, heat transfer and thermally driven flow, and vapor flow are also incorporated in these programs using modified versions of equation (2.7). Each of these packages is available with a graphical user interface and runs in the Windows (TM) operating system. The packages are similar in conceptually as well as in functionality. However, the codes use different algorithms, and therefore yield slightly different predictions for the same input (Benson 2007). No definitive or universal recommendation can be provided with respect to a program that would ensure water flow predictions are accurate (Bohnhoff et al. 2009).

Table 2.2. Examples of software packages commonly used for unsaturated flow modeling with atmospheric interactions (Benson 2007)

Code	Source	Dimensionality	Other Features
HYDRUS	pc-progress.com	1, 2, or 3D	Solute and colloid transport, heat transfer, dual porosity, hysteresis, snow hydrology, runoff, stochastic soil properties
SVFLUX	soilvision.com	1, 2, or 3D	Heat transfer, ground freezing, stochastic soil properties, runoff
UNSAT-H	hydrology.pnl.gov (or) uwgeosoft.org	1D	Vapor flow, heat transfer, thermally driven flow, run off, hysteresis
VADOSE/W	geoslope.com	1D or 2D	Oxygen transport, snow hydrology, ground freezing, run off and down slope infiltration, heat transfer, vapor flow

2.8 Volume Change Theory of Unsaturated Soils

The volume change behavior of expansive soils can be explained using the mechanics of unsaturated soils through the use of the constitutive relationships that relate the deformation state variables to the stress state variables. The deformation state variables for an unsaturated soil element are the changes in total volume (i.e., soil structure) and the changes in water volume. The volume change behavior of an unsaturated soil primarily involves two processes; namely, the transient water flow process and the soil volume change process. The coupled consolidation (i.e., volume change) theory of unsaturated soils links these two processes to each other. A brief literature review of the stress state variables and the volume change (i.e., coupled consolidation) theory of unsaturated soils is presented in this section.

2.8.1 Stress state variables

During the early years of the development of soil mechanics, Terzaghi (1936) introduced the concept of effective stress for saturated soils followed by other major contributions to the definition of effective stress (Skempton 1961, Nur and Byerlee 1971). Since these early works, the effective stress principle has been widely used in modeling geotechnical engineering applications, giving a simple link between elastic deformation and stress acting in the soil, the later being proportional to the observed deformation in saturated soils (Nuth and Laloui 2008). The effective stress σ' is a combination of both the externally applied stress σ and the internal pressure of pore water u_w, and it is expressed as

$$\sigma' = \sigma - u_w \tag{2.8}$$

The use of Terzaghi's effective stress has been well accepted and experimentally verified for saturated soils (Rendulic 1936, Bishop and Eldin 1950, Laughton 1955, Skempton 1961). The success associated with the use of the Terzaghi's effective stress principle for saturated soils has prompted early investigators to extend the effective stress principle to unsaturated soils and provide unified formulations of the effective stress as shown in Table (2.3).

Table 2.3. Effective stress equations for unsaturated soils (modified after Lu 2010)

Equation	Notations	Reference
$\sigma' = \sigma + p''$	p'' : pore-water pressure deficiency	Donald (1956)
$\sigma' = \sigma - (u_a + u_c)$	u_c : capillary pressure	Hilf (1956)
$\sigma' = \sigma - \beta' u_w$	β' : holding or bonding factor which is a measure of the number of bonds under tension effective in contributing to shear strength of the soil	Croney et al. (1958)
$\sigma' = (\sigma - u_a) + \chi (u_a - u_w)$	χ : effective stress parameter related to the soil degree of saturation	Bishop (1959)
$\sigma' = \sigma - \varphi p''$	φ : parameter varying between zero to one depending on the degree of saturation	Aitchison (1960)
$\sigma' = \sigma - \beta p''$	β : statistical factor of the same type as the contact area	Jennings (1961)
$\sigma' = \sigma_i - (u_a - u_w) + \chi(\sigma - u_a)$	σ_i : intrinsic stress arising from inter-particle forces	Newland (1965)
$\sigma' = \sigma - u_a + \chi_m (h_m + u_a)$ $+ \chi_s (h_s + u_a)$	h_m : matric suction h_s : solute suction χ_m : effective stress parameter for matric suction χ_s : effective stress parameter for solute suction	Richards (1965)

$\sigma' = \sigma + \chi_m p''_m + \chi_s p''_s$	p''_m : matric suction p''_s : solute suction χ_m, χ_s : soil parameters which are dependent upon the stress path	Aitchison (1973)
$\sigma' = (\sigma - u_a) + a_w(u_a - u_w)$	a_w : ratio of area of water-mineral and water-water contact of total area of "wavy" plane	Lambe and Whitman (1979)
$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) - R - \varsigma T_s$	χ : parameter representing the proportion of the total void area occupied the water on a reference plane R : osmotic suction ς : interface parameter on the reference plane	Allam and Sridharan (1987)
$\sigma' = (\sigma - u_a) + \chi(u_a - u_w),$ $where: \quad \chi = \left[\frac{(u_a - u_w)}{(u_a - u_w)_b}\right]^{-0.55}$	χ : effective stress parameter $(u_a - u_w)_b$: air entry value	Khalili and Khabbaz (1998)
$\sigma^*_{ij} = \sigma_{ij} - (S\,u_w + (1-S)\,u_a)\,\delta_{ij}$	σ_{ij} : total stress tensor δ_{ij} : Kroneker delta or substitution tensor σ^*_{ij} : Bishop's average soil skeleton stress S : degree of saturation	Jommi (2000)

The need for a unified formulation of the stress variables rose from the complexity of modeling the behavior of porous materials (saturated and unsaturated soils). The single-valued effective stress principle converts the analysis of a multiphase porous media into a mechanically equivalent, single-phase, single-stress state continuum comprehension (Nuth and Laloui 2008). As noted by Khalili and Khabbaz (1998), the advantage of the effective stress approach is that the change in the shear strength with changes in total stress, pore water pressure, and pore air pressure can be related to a single stress variable. As a result, a complete characterization of the soil strength requires matching of a single stress history rather than two or three independent stress variables. Furthermore, the approach requires very limited testing of soils in an unsaturated state.

However, limitations of the single-valued effective stress principle have been cited in the literature by many researchers (Coleman 1962, Aitchison 1965, Blight 1965, Burland 1965, Matyas and Radhakrishna 1968, Barden et al. 1969, Brackley 1971, Fredlund and

Morgenstern 1977, Gens et al. 1995). All the formulations shown in Table (2.3) incorporate a soil parameter in order to form a single valued effective stress variable. Jennings and Burland (1962) and many subsequent authors have shown that whatever the relationship chosen for soil parameter, there is no unique relationship between volumetric strain and effective stress, and that single-valued effective stress principle is not valid. Jennings and Burland (1962) stated that the volume change and shear strength of unsaturated soils cannot be related to a single effective stress. Rather, more than one stress state variable should be used to describe the behavior of unsaturated soils. Fredlund and Morgenstern (1977) and Fung (1977) further argued that the variables used for the description of a stress should be independent of the material properties. Due to the experimental difficulties to evaluate the soil parameter, and the philosophical difficulties to justify the use of soil properties in the description of a stress state, the single-valued effective stress equations have not received much attention in describing the mechanical behavior of unsaturated soils. However, in another recent viewpoint, Khalili et al. (2004) criticized the theoretical basis for this argument. They pointed out that in a multiphase porous medium, such as saturated and unsaturated soils, the stress state within each phase will naturally be a function of the properties of that phase as well as the other phases within the system. This is required in order to ensure deformation compatibility between the phases. In addition, Nuth and Laloui (2008) suggested that soil parameter might also be related to the current stress and stress history; hence, a unique relationship between the soil parameter and the ratio of matric suction over the air entry value can be obtained for most soils.

Fredlund and Morgenstern (1977) proposed two independent stress variables to describe the constitutive behavior of unsaturated soils. The identification of state variables can be based on multiple continuum mechanics, leading to the conclusion that any two of the three possible state variables (σ, u_w, u_a) can be used to define the stress state (Fredlund and Morgenstern 1977), possible combinations being:

$(\sigma - u_a)$ and $(u_a - u_w)$, e.g., used in Alonso et al. (1990) (2.9a)

$(\sigma - u_w)$ and $(u_a - u_w)$, e.g., used in Geiser et al. (2000) (2.9b)

$$(\sigma - u_a) \text{ and } (\sigma - u_w) \quad (2.9c)$$

For physical and practical reasons, the most frequently used stress variables in the two independent stress state variables approach are the net normal stress $(\sigma - u_a)$ and the matric suction $(u_a - u_w)$ (Equation 2.9a) (e.g., Matyas and Radhakrishna 1968, Alonso et al. 1990). The matric suction $(u_a - u_w)$ has a definite physical meaning, while most of the time the air pressure may be considered constant and equal to the atmospheric pressure $(u_a = u_{atm} = 0)$. Under this assumption, the net normal stress is simplified to the total normal stress and the matric suction is equal to the absolute value of the negative pore water pressure. Moreover, this choice is adapted to the axis translation technique, consisting in the application of $(u_a > 0)$ (Nuth and Laloui 2008). Fredlund and Morgenstern (1977) experimentally validated the principle of independent stress state variables for unsaturated soils using null tests. The components of the proposed stress state variables (σ, u_a, u_w) were varied equally in order to maintain constant values for stress state variables (i.e., $(\sigma - u_a)$ and $(u_a - u_w)$). As there are no overall changes in the state of the soil due to changing in the components of the stress state variables, the stress state variables are valid well for unsaturated soils. Tarantino et al. (2000) investigated these stress state variables using a new laboratory apparatus designed and constructed to test unsaturated soils in a broad range of degree of saturation and negative pore water pressures. The results confirmed the use of the net normal stress and matric suction as stress state variables.

Re-examination of the independent stress state variables proposed by Fredlund and Morgenstern (1977) has led several ongoing attempts to develop modified stress variables to describe the behavior of unsaturated soils (e.g., Alonso et al. 1990, Kohgo et al. 1993, Kato et al. 1995, Wheeler and Sivakumar 1995, Wheeler et al. 2003, Blatz and Graham 2003, Gallipoli et al. 2003). There are several state-of-the-art reports and review papers over the last years; for examples, Gens (1996), Wheeler and Karube (1996), Kohgo (2003), Gens et al. (2006), Sheng and Fredlund (2008), Sheng et al. (2008), Gens (2009), Cui and Sun (2009), Gens (2010), and Sheng (2011). These papers may serve as good references for studying the alternative stress state variables or constitutive variables that can be used to establish various models for unsaturated soils. The recent proposed stress variables are clearly more complex than the traditional variables of net normal stress and matric suction.

Wheeler and Karube (1996) discussed the justification of using the more complicated stress variables and discussed some disadvantages that might arise from using that approach. First, it would be more difficult for practicing engineers to think in terms of the new stress variables even when describing a relatively simple stress path, e.g., drying/wetting under constant applied load. Second, it would be more difficult to devise simple experiments to obtain the model parameters. Only if the new stress variables have a strong physical significance, resulting in considerable improvement in modeling capacity and more simplicity in stress-strain relationships, the use of new stress variables would be justified (Jotisankasa 2005).

2.8.2 Volume change constitutive relationships

In developing the analysis of volume change behavior of unsaturated expansive soils, it is necessary to express soil volume change in terms of stress state variables and appropriate strain variables through specified constitutive relations. Different constitutive relationships have been proposed to interpret the volume change behavior of the soil. Several researchers (Biot 1941, Coleman 1962, Matyas and Radhakrisha 1968, Barden et al. 1969, Aitchison and Woodburn 1969, Brackley 1971, Aitchison and Martin 1973, Fredlund and Morgenstern 1976, 1977, Fredlund and Rahardjo 1993, Zhang 2004) developed volume change constitutive relationships based on the assumption that the soil is elastic in nature for a large range of loading conditions. Two constitutive relationships have been suggested for describing the deformation state of unsaturated soil. One constitutive relationship is formulated for soil structure (in terms of void ratio or volumetric strain) and the other constitutive relationship is formulated for water phase (in terms of degree of saturation or water content). Two independent stress variables (i.e., net normal stress and matric suction) are used in the formulations. In total four volumetric deformation coefficients are required to link the stress and deformation states. The constitutive relations can be graphically presented in the form of a deformation state variable versus two independent stress state variables, and they can be formulated in different forms, namely soil mechanics formulation, compressibility formulation, and elasticity formulation (Fredlund et al. 2012).

In another viewpoint, a variety of elasto-plastic constitutive relationships or models has been introduced and studied (e.g., Alonso et al. 1987, Karube 1988, Alonso et al. 1990, Gens and Alonso 1992, Kohgo et al. 1993, Modaressi and Abou-Beker 1994, Bolzon et al. 1996, Cui et al. 1995, Delage and Graham 1995, Kato et al. 1995, Wheeler and Sivalumar 1995, Wheeler et al. 2003, Blatz and Graham 2003, Chiu and Ng 2003, Tamagnini 2004, Thu et al. 2007, Sheng et al. 2008). Lloret and Alonso (1980) established that the constitutive models based on the concept of elastoplasticity provide a better understanding and explanation of expansive soil behavior, on particular, those features concerning stress path dependency and soil collapse upon wetting. However, Alonso et al. (1990) argued that, for pavements or shallow foundations built on expansive soils, the assumption that expansive soils are elastic is considered as a reasonable assumption where drying–wetting process can cause an unsaturated expansive soil to yield. Similar arguments were also put forward by Zhang and Briaud (2010) since it is reasonable to assume the soil has experienced the maximum wetness and dryness in the past. In other words, if an expansive soil has some plasticity, this kind of plasticity could have been eliminated by a long history of wetting-drying cycles. This may be also the reason why most expansive soils are usually heavily overconsolidated. Since the volume of the expansive soil is influenced by the mechanical stress, one may argue that the soil will yield under a combination of mechanical stress and matric suction variations. However, for pavements and light residential or commercial buildings where the majority of the soil volume change problems are likely to occur, the mechanical stress due to repeated traffic or superstructure load is very small that it will not cause soil yielding. As a result, for engineering practice applications, expansive soils can be assumed to be elastic (Zhang and Briaud 2010). Consequently, the details of elasto-plastic constitutive relationships will not be covered here. Only Fredlund and Morgenstern (1976, 1977) constitutive relationships for volume change behavior that are employed in the thesis will be discussed in detail in the following.

The unsaturated soil is considered as a four-phase mixture (Fredlund 1979), with two phases that come to equilibrium under applied stress (i.e., soil particle and contractile skin) and two phases that flow under applied pressure (i.e., air and water). The total volume change of a soil element must be equal to the sum of volume changes associated with each phase. If the soil particles are assumed incompressible and the volume change of the contractile skin are assumed internal to the element, the continuity requirement for an element of unsaturated soil is (Fredlund and Morgenstern 1976)

$$\frac{\Delta V_v}{V_0} = \frac{\Delta V_w}{V_0} + \frac{\Delta V_a}{V_0} \tag{2.10}$$

where V_0 is initial overall volume of an unsaturated soil element, V_v is volume of soil voids, V_w is volume of water in the soil element, and V_a is volume of air in the soil element.

The above continuity requirement shows, in order to describe the volume change behavior in an unsaturated soil, the volume changes associated with any two of the three above volume variables must be measured or predicted, while the third volume change can be computed. In practice, the overall volume change $(\Delta V_v / V_0)$ and the water volume change $(\Delta V_w / V_0)$ are usually measured, while the air volume change $(\Delta V_a / V_0)$ is calculated as the difference between the volume change of soil structure and water phase (Fredlund and Rahradjo 1993).

By assuming the soil behaves as an incrementally isotropic, linear elastic material, the soil structure constitutive relations associated with the strains can be written as

$$d\varepsilon_x = \frac{d(\sigma_x - u_a)}{E} - \frac{\mu}{E}\left[d(\sigma_y - u_a) + d(\sigma_z - u_a)\right] + \frac{d(u_a - u_w)}{H} \tag{2.11a}$$

$$d\varepsilon_y = \frac{d(\sigma_y - u_a)}{E} - \frac{\mu}{E}\left[d(\sigma_x - u_a) + d(\sigma_z - u_a)\right] + \frac{d(u_a - u_w)}{H} \tag{2.11b}$$

$$d\varepsilon_z = \frac{d(\sigma_z - u_a)}{E} - \frac{\mu}{E}\left[d(\sigma_x - u_a) + d(\sigma_y - u_a)\right] + \frac{d(u_a - u_w)}{H} \tag{2.11c}$$

$$d\gamma_{yz} = \frac{d\tau_{yz}}{G}, \quad d\gamma_{zx} = \frac{d\tau_{zx}}{G}, \quad d\gamma_{xy} = \frac{d\tau_{xy}}{G} \tag{2.11d}$$

where σ_x, σ_y, and σ_z are normal stresses in the *x*-, *y*-, and *z*-directions, respectively, ε_x, ε_y, and ε_z are normal strains in the *x*-, *y*-, and *z*-directions, respectively, $(\sigma_x - u_a)$, $(\sigma_y - u_a)$, and $(\sigma_z - u_a)$ are net normal stresses in the *x*-, *y*-, and *z*-directions, respectively, μ is Poisson's ratio, *E* is modulus of elasticity for the soil structure with respect to a change in net normal stress, *H* is modulus of elasticity for the soil structure with respect to a change in matric suction, γ_{yz}, γ_{zx}, and γ_{xy} are shear strains on the *x*-, *y*-, and *z*-planes, respectively, and *G* is shear modulus.

The deformation variable associated with the overall or total volume change (dV_v/V_0) can be written as the sum of the normal strains (Equation 2.12).

$$\frac{dV_v}{V_0} = d\varepsilon_v = d\varepsilon_x + d\varepsilon_y + d\varepsilon_z = 3(\frac{1-2\mu}{E})d(\sigma_{mean} - u_a) + \frac{3}{H}d(u_a - u_w) \tag{2.12}$$

where ε_v is volumetric strain, σ_{mean} is mean total normal stress ($\sigma_{mean} = (\sigma_x + \sigma_y + \sigma_z)/3$), in which σ_x, σ_y, and σ_z are normal stresses in the *x*-, *y*-, and *z*-directions, respectively. The total volume change refers to the volume change of the soil structure, and equation (2.12) is the constitutive equation for soil structure.

The constitutive equation for the water phase defines the water volume change in the soil element for any change in the total stress and matric suction. By assuming water is incompressible, the constitutive equation for the water phase can be formulated as a linear combination of the stress state variables changes as below (Fredlund and Rahardjo 1993).

$$\begin{aligned}\frac{dV_w}{V_0} &= \frac{d(\sigma_x - u_a)}{E_w} + \frac{d(\sigma_y - u_a)}{E_w} + \frac{d(\sigma_z - u_a)}{E_w} + \frac{d(u_a - u_w)}{H_w} \\ &= \frac{3}{E_w}d(\sigma_{mean} - u_a) + \frac{1}{H_w}d(u_a - u_w)\end{aligned} \tag{2.13}$$

where E_w is water volumetric modulus associated with a change in net normal stress, and H_w is water volumetric modulus associated with a change in matric suction.

Fredlund and Morgenstern (1976) proposed the following constitutive relationships for volume change of soil structure and water phase in a compressibility form

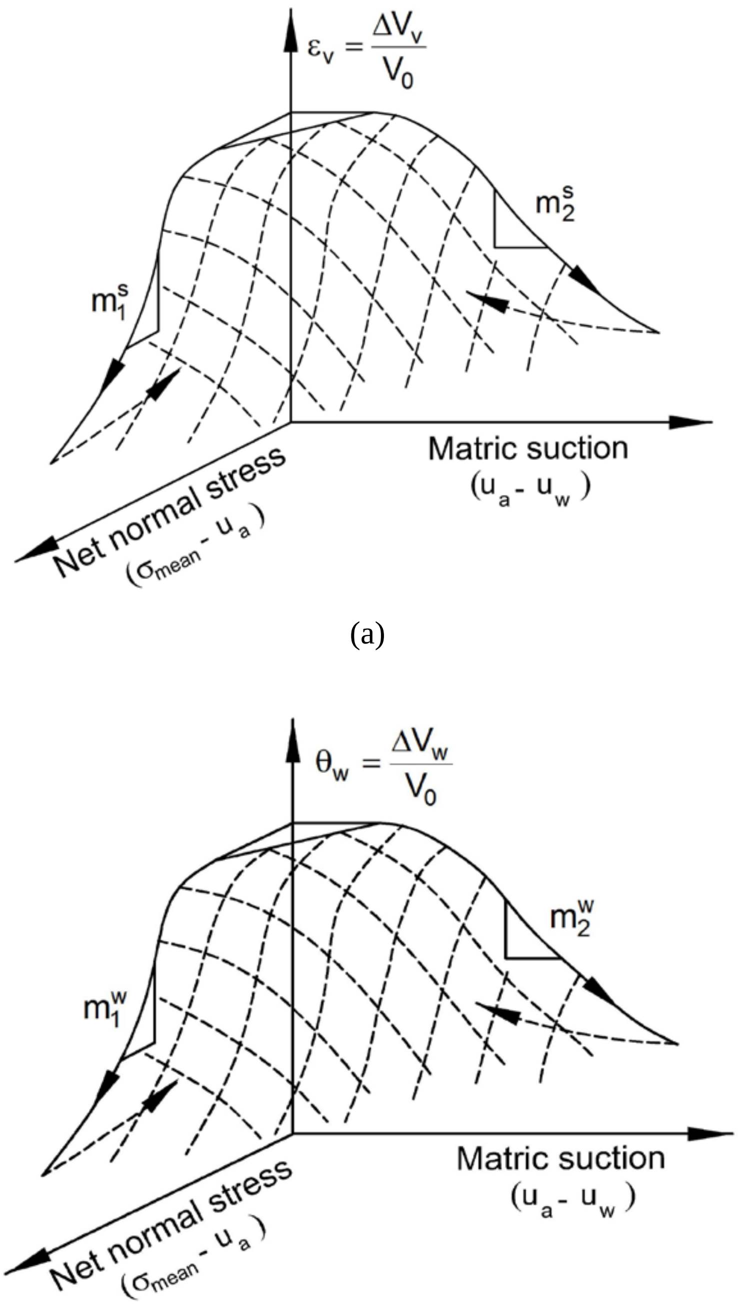
$$d\varepsilon_v = \frac{dV_v}{V_0} = m_1^s d(\sigma_{mean} - u_a) + m_2^s d(u_a - u_w) \tag{2.14}$$

$$d\theta = \frac{dV_w}{V_0} = m_1^w d(\sigma_{mean} - u_a) + m_2^w d(u_a - u_w) \tag{2.15}$$

where m_1^s is coefficient of total volume change with respect to change in net normal stress, m_2^s is coefficient of total volume change with respect to change in matric suction, m_1^w is coefficient of water volume change with respect to change in net normal stress, and m_2^w is coefficient of water volume change with respect to change in matric suction. Comparing equations (2.14) and (2.15) with (2.12) and (2.13), the volume change coefficients can be related to the elastic moduli E and H, volumetric modulus E_w and H_w, and Poisson's ratio μ as follows

$$m_1^s = 3(\frac{1-2\mu}{E}),\ m_2^s = \frac{3}{H},\ m_1^w = \frac{3}{E_w},\ \text{and}\ m_2^w = \frac{1}{H_w} \tag{2.16}$$

The constitutive relationships for soil structure and water phase of an unsaturated soil can be presented graphically in the form of constitutive surfaces (Figure 2.14). The deformation state is plotted with respect to the stress state variables $(\sigma_{mean} - u_a)$ and $(u_a - u_w)$. All the volume change coefficients in equations (2.14) and (2.15) can be determined from the constitutive surfaces as shown in Figure (2.14), which are the slopes of the constitutive surface at a point.



(a)

(b)

Fig. 2.14. Three-dimensional constitutive surfaces for unsaturated soil: (a) soil structure constitutive surface, (b) water phase constitutive surface (modified after Fredlund et al. 2012)

Conventional soil mechanics terminology makes the use of void ratio *e*, gravimetric water content *w*, and degree of saturation *S* to define the volume-mass properties of unsaturated soils. Therefore, the constitutive equations for unsaturated soils can be written in terms of void ratio and gravimetric water content as the deformation state variables for soil structure and water phase, respectively.

$$de = a_t\, d(\sigma_{mean} - u_a) + a_m\, d(u_a - u_w) \tag{2.17}$$

$$dw = b_t\, d(\sigma_{mean} - u_a) + b_m\, d(u_a - u_w) \tag{2.18}$$

where a_t is coefficient of compressibility with respect to change in net normal stress, a_m is coefficient of compressibility with respect to change in matric suction, b_t is coefficient of water content change with respect to a change in net normal stress, and b_m is coefficient of water content change with respect to a change in matric suction. Equations (2.17) and (2.18) can also be visualized as constitutive surface on a three-dimensional plot. Each abscissa represents one of the stress state variables and the ordinate represents the soil volume-change properties (Figure 2.15). The compressibility coefficients a_t, a_m, b_t, and b_m are another form of the volume change coefficients, which can be determined as the slopes of the void ratio and water content constitutive surfaces at a point as shown in Figure (2.15).

The volume change coefficients of equations (2.14) and (2.15) (m_1^s, m_2^s, m_1^w, m_2^w) can be expressed in terms of the coefficients of equations (2.17) and (2.18) (a_t, a_m, b_t, b_m), and then the volume change coefficients of soil can be obtained from void ratio and water content constitutive surfaces (Figure 2.15) (Fredlund and Rahardjo 1993) as follows

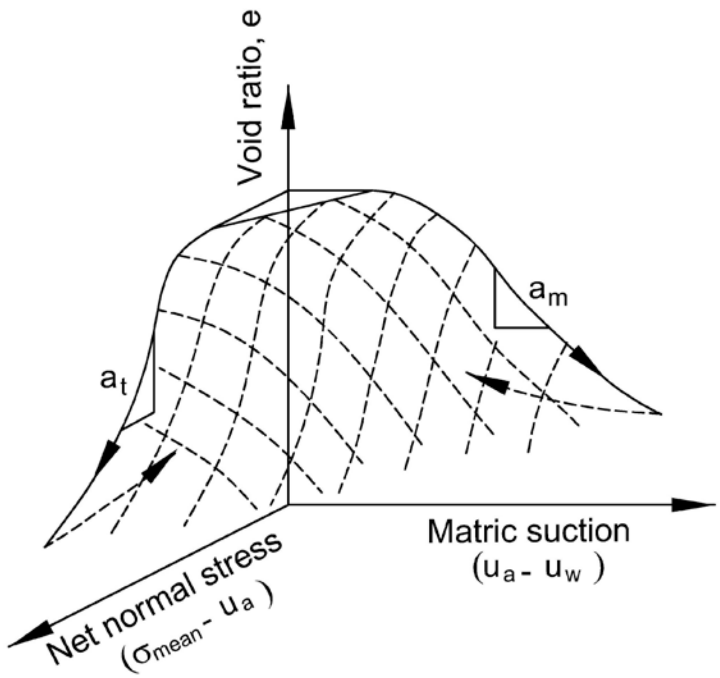
$$m_1^s = \frac{1}{1+e_0}\frac{de}{d(\sigma_{mean} - u_a)} = \frac{1}{1+e_0} a_t \tag{2.19}$$

$$m_2^s = \frac{1}{1+e_0}\frac{de}{d(u_a - u_w)} = \frac{1}{1+e_0} a_m \tag{2.20}$$

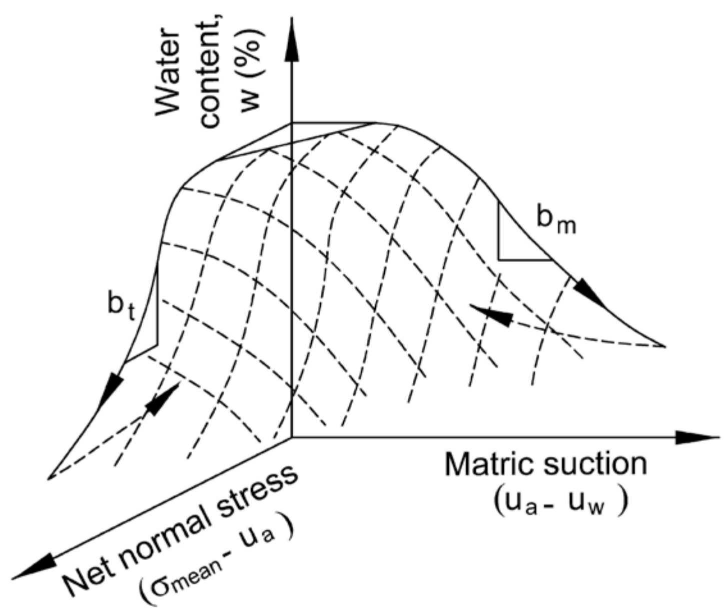
$$m_1^w = \frac{G_s}{1+e_0}\frac{dw}{d(\sigma_{mean} - u_a)} = \frac{G_s}{1+e_0} b_t \tag{2.21}$$

$$m_2^w = \frac{G_s}{1+e_0}\frac{dw}{d(u_a - u_w)} = \frac{G_s}{1+e_0} b_m \tag{2.22}$$

where e_0 is initial void ratio prior to deformation, and G_s is the specific gravity of the soil solids.



(a)



(b)

Fig. 2.15. Three-dimensional constitutive surfaces for unsaturated soil expressed using soil mechanics terminology: (a) void ratio constitutive surface, (b) water content constitutive surface (modified after Fredlund et al. 2012)

The volume change coefficients can, in general, be obtained from the consolidation tests or triaxial tests with suction control. However, such tests are usually time consuming, costly, and may not be reasonable in engineering practice (Fredlund and Raharajo 1993). Vu and Fredlund (2006) proposed a method to calculate the four coefficients of soil volume change. The void ratio constitutive surface of unsaturated soil is estimated in terms of the compressive indices obtained from the conventional oedometer tests. However, this method estimates unreasonably large soil deformations at low net normal stresses and/or low suctions. More details about Vu and Fredlund (2006)'s method are provided in a later section.

2.8.3 Coupled consolidation theory for unsaturated soils

The rigorous formulation for consolidation (i.e., volume change) of unsaturated soils requires that the continuity equation be coupled with the equilibrium equations (Fredlund and Hasan 1979, Dakshanamurthy and Fredlund 1980, Lloret and Alonso 1980, Dakshanamurthy et al. 1984, Lloret et al. 1987, Fredlund and Rahardjo 1993, Wong et al. 1998, Vu 2002). In a three-dimensional consolidation problem, there are five unknowns of deformation and volumetric variables to be solved. These unknowns are the displacements in the *x*-, *y*-, and *z*-directions and the water volume change and air volume change. The displacements in the *x*-, *y*-, and *z*-directions are used to compute the total volume change. The five unknowns can be obtained from three equilibrium equations for the soil structure and two continuity equations (water and air phase continuities). These equations require constitutive relations for the volume change of unsaturated soils as well as flow laws for fluid phases (air and water phases). However, the pore air pressure is generally assumed to be atmospheric and remains unchanged during the consolidation process. In this case, only stress equilibrium condition and water flow continuity need to be considered in the analysis.

2.8.3.1 Equilibrium equations for soil structure

The stress state for an unsaturated soil element should satisfy the following equilibrium conditions

$$\sigma_{ij,j} + b_j = 0 \tag{2.23}$$

where $\sigma_{ij,j}$ are components of the net total stress tensor, and b_j are components of body force vector.

The strain-displacement equations (Cauchy's Equation) for soil structure of an unsaturated soil are given as follows

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2.24}$$

where ε_{ij} are components of the strain tensor, and u_i are components of displacement in the *i*-direction.

By substituting the strain-displacement equation (Equation 2.24) and the stress-strain relationship (Equation 2.12) into the equilibrium equation (Equation 2.23), the differential equations for soil structure for general three-dimensional problems can be written as

$$(\lambda + G)\frac{\partial \varepsilon_v}{\partial x} + G\nabla^2 u - (3\lambda + 2G)\frac{1}{H}\frac{\partial (u_a - u_w)}{\partial x} + b_x = 0 \tag{2.25a}$$

$$(\lambda + G)\frac{\partial \varepsilon_v}{\partial y} + G\nabla^2 v - (3\lambda + 2G)\frac{1}{H}\frac{\partial (u_a - u_w)}{\partial y} + b_y = 0 \tag{2.25b}$$

$$(\lambda + G)\frac{\partial \varepsilon_v}{\partial z} + G\nabla^2 w - (3\lambda + 2G)\frac{1}{H}\frac{\partial (u_a - u_w)}{\partial z} + b_z = 0 \tag{2.25c}$$

where $\lambda = \mu E / [(1+\mu)(1-2\mu)]$, *u*, *v*, and *w* are displacements in the *x*-, *y*-, and *z*-directions, respectively, and b_x, b_y, and b_z are body force in the *x*-, *y*-, and *z*-directions, respectively.

2.8.3.2 Water continuity equation

The water continuity equation for unsaturated soils, assuming that water is incompressible and deformations are incrementally infinitesimal, can be written as (Freeze and Cherry 1979)

$$\frac{\partial(V_w/V_0)}{\partial t} = \frac{\partial v_w^x}{\partial x} i + \frac{\partial v_w^y}{\partial y} j + \frac{\partial v_w^z}{\partial z} k \tag{2.26}$$

where $\partial(V_w/V_0)/\partial t$ is net flux of water per unit volume of the soil, t is time, and $v_w = v_w^x i + v_w^y j + v_w^z k$ is Darcy's flux which relates to the hydraulic head (i.e., pressure head plus elevation head) using Darcy's law

$$v_{wi} = -k_{wi} \frac{\partial}{\partial x_i}\left(\frac{u_w}{\rho_w g} + Y\right) \tag{2.27}$$

where v_{wi} is Darcy's flux in the i direction, k_{wi} is hydraulic conductivity in the i direction which is a function of matric suction, u_w is pore water pressure, ρ_w is density of water, g is gravitational acceleration, and Y is elevation.

Fredlund and Rahardjo (1993) derived the differential equation for water phase (Equation 2.28) by substituting the time derivative of the water phase constitutive equation (Equation 2.13 or 2.15) and Darcy's law (Equation 2.27) into the water phase continuity equation (Equation 2.26).

$$m_1^w \frac{\partial(\sigma_{mean} - u_a)}{\partial t} + m_2^w \frac{\partial(u_a - u_w)}{\partial t} = \frac{\partial}{\partial x}\left[k_w^x \frac{\partial}{\partial x}\left(\frac{u_w}{\rho_w g} + Y\right)\right] + \frac{\partial}{\partial y}\left[k_w^y \frac{\partial}{\partial y}\left(\frac{u_w}{\rho_w g} + Y\right)\right] + \frac{\partial}{\partial z}\left[k_w^z \frac{\partial}{\partial z}\left(\frac{u_w}{\rho_w g} + Y\right)\right] \tag{2.28}$$

Fredlund and Rahardjo (1993) further derived equation (2.28) by extending Biot's consolidation theory for saturated soils (Biot 1941). Equation (2.12) (or 2.14) was solved for $d(\sigma_{mean} - u_a)$ in terms of $d\varepsilon_v$ and $d(u_a - u_w)$, and then $d(\sigma_{mean} - u_a)$ was substituted into equation 2.13 (or 2.15). The volumetric water content variations can be expressed as

$$d\theta = \frac{dV_w}{V_0} = \beta_{w1}\, d\varepsilon_v + \beta_{w2}\, d(u_a - u_w) \tag{2.29}$$

where $\beta_{w1} = \frac{m_1^w}{m_1^s}$, and $\beta_{w2} = m_2^w - \frac{m_1^w\, m_2^s}{m_1^s}$.

By substituting equation (2.29) into the left-handed side of equation (2.28), the differential equation for water phase can be obtained as

$$\beta_{w1}\frac{\partial \varepsilon_v}{\partial t} + \beta_{w2}\frac{\partial (u_a - u_w)}{\partial t} = \frac{\partial}{\partial x}\left[k_w^x \frac{\partial}{\partial x}\left(\frac{u_w}{\rho_w g} + Y\right)\right] + \frac{\partial}{\partial y}\left[k_w^y \frac{\partial}{\partial y}\left(\frac{u_w}{\rho_w g} + Y\right)\right] + \frac{\partial}{\partial z}\left[k_w^z \frac{\partial}{\partial z}\left(\frac{u_w}{\rho_w g} + Y\right)\right] \tag{2.30}$$

Equations (2.25) and (2.30) together are the differential equations for the coupled consolidation for unsaturated soils that can be used to predict the volume change behavior of unsaturated soils (Fredlund and Rahardjo 1993).

2.9 Volume Change Predictions

The uncertainty in the estimation of volume change behavior of expansive soils can be of concern for geotechnical engineering practitioners as it may contribute to several undesirable outcomes: (i) expensive foundation systems due to overestimation of volume change; (ii) litigation due to underestimation of volume change; (iii) growth in a number of local protocols that have limited applicability; and (iv) lack of confidence in the future performance of existing and newly designed structures on expansive soils (Singhal 2010). Hence, it is important to provide tools for practitioners to reliably estimate the volume change behavior of expansive soils in the field. Significant advances were made during the last half-a-century towards prediction of the heave and the shrink related volume change behavior of expansive soils. This section presents a critical review of the state-of-the-art of methods for predicting the volume change movement of expansive soils.

2.9.1 Methods for predicting heave potential

The focus of most prediction methods proposed in the literature has been towards estimating the swelling characteristics; namely, heave/swell potential and swelling pressure. The heave potential is defined as the ratio of increase in thickness ΔH to the original thickness H of a laterally confined sample on soaking under 7 kPa surcharge, after being compacted to the maximum density at the optimum water content in the standard AASHTO compaction test (Seed et al. 1962). However, the swelling pressure is the pressure required to hold the soil, or restore the soil, to its initial void ratio when given access to water (Shuai 1996). The estimation of the swelling pressure was beyond the scope of this study, thus only methods for predicting the heave potential are briefly described in this section. The available methods can be categorised into: (i) oedometer methods, (ii) empirical methods, and (iii) suction-based methods.

2.9.1.1 Oedometer methods

Considerable research has been conducted to predict the soil heave potential based on the results of oedometer tests (e.g., Jennings and Knight 1957, Salas and Serratosa 1957, Lambe and Whitman 1959, Clisby 1963, Sullivan and McClelland 1969, Aitchison et al. 1973, Smith 1973, Fredlund et al. 1980, Weston 1980, Justo and Saetersdal 1981, Dhowian 1990, Nelson and Miller 1992, Abdullah 2002, Nelson et al. 2006, Nelson et al. 2012). The oedometer based methods require representative undisturbed samples collected from the active zone depth typically in a dry season. The samples are then restrained laterally and loaded axially in a consolidometer with access to free water to saturation. The magnitude of the heave potential in oedometer testing can be estimated by applying the consolidation theory in reverse (Wanyan et al. 2008)

$$\frac{\Delta H}{H} = \frac{|e_f - e_0|}{1 + e_0} \tag{2.31}$$

where ΔH is soil heave, H is soil layer thickness, e_f and e_0 are initial and final void ratios, respectively.

In oedometer tests, it is vital to follow as closely as possible the expected stress sequence to which the soils will be subjected in the field. Therefore, there are different opinions in the published methods concerning the simulation of field conditions in the oedometer tests (Dhowian 1990). A list of various methods utilizing the oedometer test results in estimating the heave potential is presented in Vanapalli and Lu (2012). The most common oedometer methods along with the interpretation of the actual stress path that is being followed in each method are reviewed here.

Direct method (Texas Highway Department Method TEX-124-E)

The direct method is based on a free swell oedometer test conducted on an undisturbed specimen to model the field behavior with a zero additional applied surface loading (Figure 2.16). In the free swell test, undisturbed specimens of the soils are inundated while only a token load (seating pressure = 1 to 7 kPa) is applied and vertical deformations are recorded. The common modification to the free swell test is to apply the field overburden plus structural load stress, the specimen and then to inundate and observe swell. The stress path followed when using this test procedure is shown in Figure (2.16).

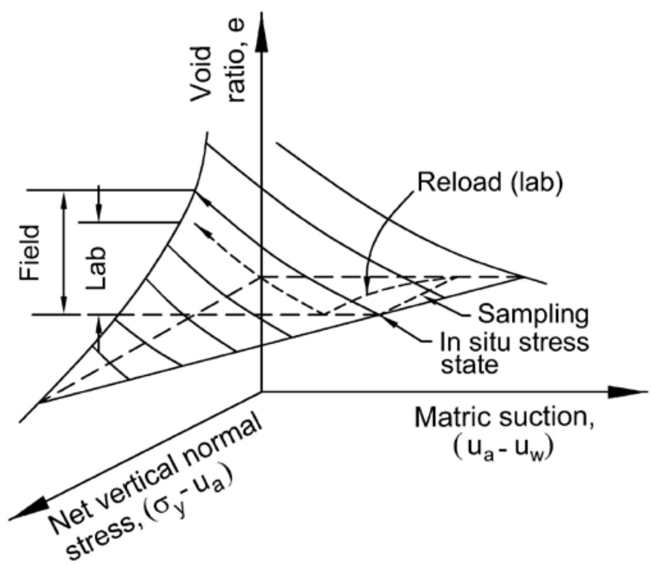


Fig. 2.16. Stress path followed when using the direct method (Fredlund et al. 1980)

NAVFAC (1971) outlined the procedure of the direct method for estimating the magnitude of heave that may occur when footings are built on expansive soils. Undisturbed, unsaturated soil specimens are extracted from the subsoil at different elevations up to a depth of zero swell. Each specimen is subjected to a free swell test with the sum of the field overburden pressure and the anticipated structural load that is applied as surcharge load on the specimen. The test results are plotted as percent swell versus depth as shown in Figure (2.17a). The area under the percent swell versus depth curve, integrated upward from the depth of zero swell, represents the total heave/swell. This total swell is plotted versus depth to predict the swell at any depth (Figure 2.17b). Smith (1973) presented a similar procedure, and provided an example to illustrate how the direct method works for the determination of heave potential in soil strata. This is valuable in deciding the methods of construction to be employed and the remedial procedures to use in securing the greatest value for construction money.

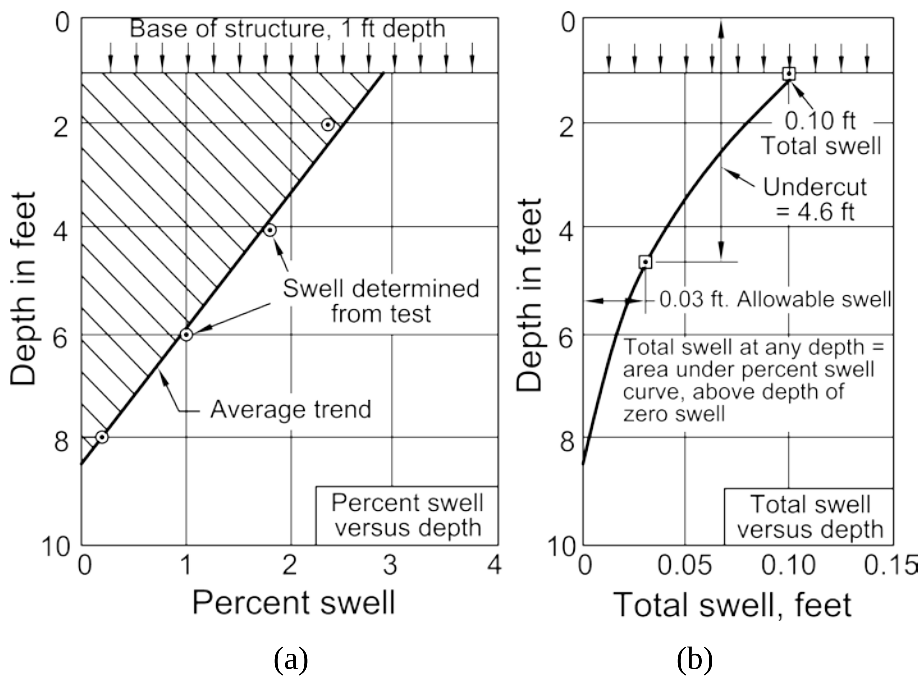


Fig. 2.17. Heave calculations using the direct method (NAVFAC 1971)

The greatest virtue of the direct method is its simplicity yet applicability to the field condition. However, Fredlund et al. (1980) found that the predicted heave is significantly below the actual heave experienced in the field. It was anticipated that the underestimation of the amount of field heave would be primarily due to a lack of accounting for sampling disturbance and a great difficulty in securing full water-uptake in the oedometer specimen. Abdullah (2002) experimentally showed that the direct method overestimates the field heave because the oedometer test allows simulation of soil heave in the vertical direction only and does not account for the reduction in the vertical heave due to the lateral soil swelling in the field. Abdullah (2002) introduced a heave reduction factor to adjust the predicted heave in order to represent the real in situ vertical heave.

Sullivan and McClelland (1969) method

Sullivan and McClelland (1969) proposed a heave prediction method based on constant volume oedometer tests on undisturbed specimens. The undisturbed specimen is initially subjected to in situ overburden pressure and allowed to come to equilibrium (Figure 2.18). The specimen is then allowed free access to water and maintained at constant volume by adding loads until no more swelling tendency is observed (i.e., the swelling pressure is reached). The sample is then unloaded and allowed to swell by decreasing the loads in small increments.

The method can be used to estimate the soil heave occurred due to reduction in overburden pressure (unloading). However, Fredlund et al. (1980) mentioned that this method is expected to underestimate the actual heave if the sampling disturbance will not be taken into consideration. It was suggested that the results of constant volume oedometer tests should be adjusted for the effects of compressibility of the apparatus prior to their interpretation. Figure (2.19) shows the manner in which an adjustment should be applied to the laboratory data. The (uncorrected) swelling pressure must also be corrected for sampling disturbance as shown in Figure (2.20). The correction procedure was similar to the Casagrande's construction used for determining the preconsolidation pressure of a saturated soil. Details of the correction procedure are explained in Fredlund and Rahardjo (1993).

The constant volume oedometer test method has been used widely and considered to be one of the most reliable methods for the determination of swell characteristics (i.e., heave potential and swell pressure). Meanwhile, it is one of the most difficult and cumbersome methods of measuring the swell characteristics. This is probably due to the difficult and somewhat impossible restrictions for the constant volume test such as controlling the vertical deformation by 0.005-0.01 mm, which requires computer control and also careful adjustments for apparatus compliance (Abbaszadeh 2011). Also, it could be due to the difficulty to secure water entry when the specimen is under high applied loadings which are necessary for the test (Jennings 1969).

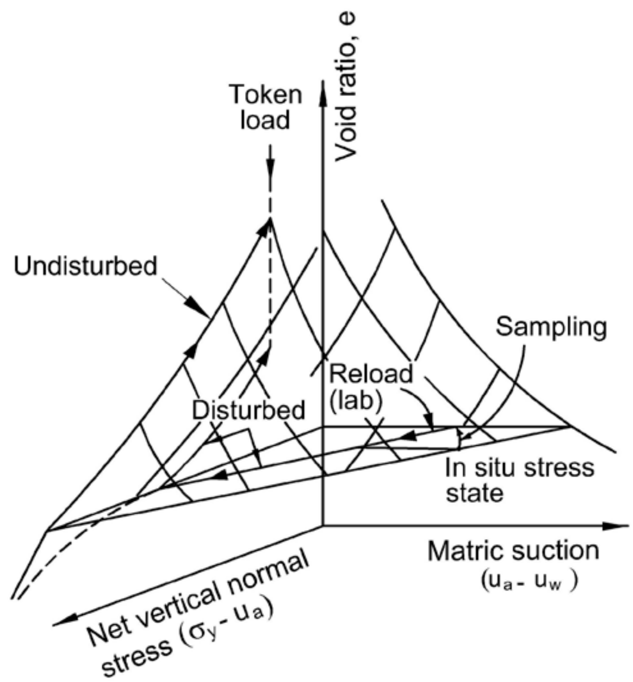


Fig. 2.18. Stress path followed when using Sullivan and McClelland method (Fredlund et al. 1980)

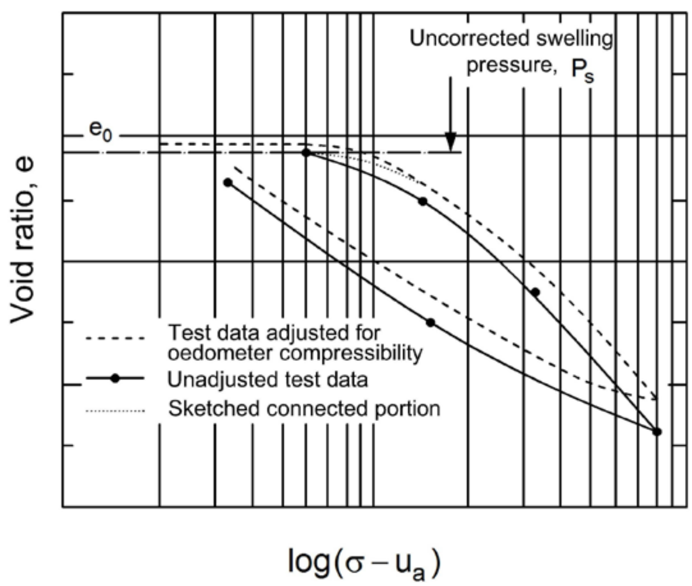


Fig. 2.19. Adjustment of laboratory test data to compensate for compressibility of oedometer apparatus (Fredlund and Rahardjo 1993)

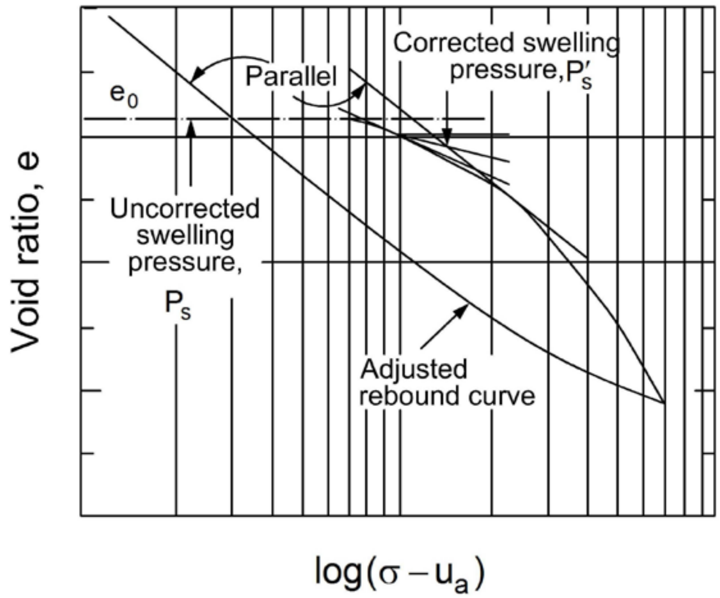


Fig. 2.20. Construction procedure to correct for sampling disturbance (Fredlund and Rahardjo 1993)

Double-oedometer method

Jennings and Knight (1957) proposed the double odometer method based on the results of two oedometer tests, namely, the free-swell oedometer test and the natural water content oedometer test. The two tests were conducted, as explained in the direct method, on identical specimens initially subjected to a token load of 1 kPa. However, no water is added to the oedometer pot during the natural water content test. To compute the soil heave, the data of the natural water content oedometer test are adjusted vertically to match the results of free swell test at high applied loads. The stress paths followed by the double odometer tests in terms of net normal stress and matric suction are shown in Figure (2.21).

Jennings and Knight (1957) and other researchers (Weston 1980, Justo and Saetersdal 1981, Abdullah 2002) found that the double odometer method overestimates the actual heave. This overestimation appears to be primarily due to the dependency of the method on the one-dimensional oedometer tests which assume the heave potential is manifested only in the vertical direction. In addition, the specimens in the double oedometer tests are allowed to swell under the token load only, thus, producing a higher value of soil heave

(Abdullah 2002). Another drawback is that the difficulty to obtain identical undisturbed specimens in order to conduct the odometer tests. However, Fredlund et al. (1980) suggested that the heave prediction using the double oedometer method is generally satisfactory since the data analysis takes into consideration the effect of sample disturbance.

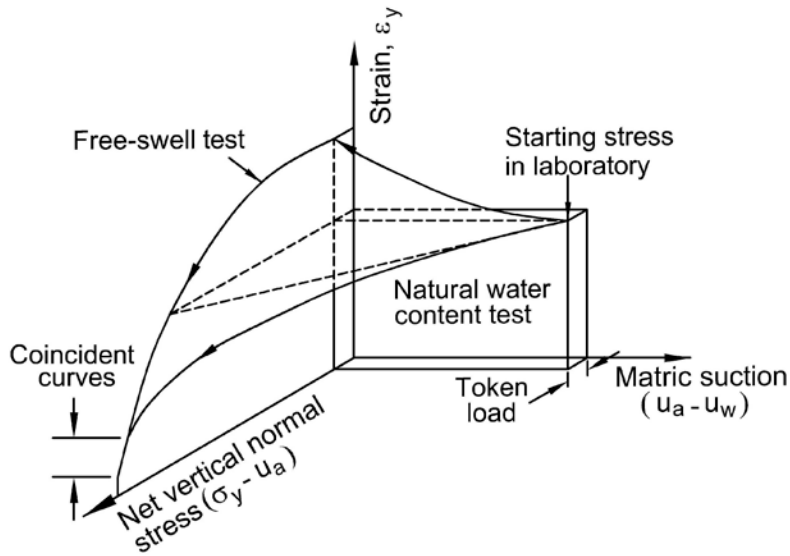


Fig. 2.21. Stress paths followed when using double oedometer method (Jennings and Knight 1957)

Nelson and Miller (1992) method

Nelson and Miller (1992) method is based on data obtained from the overburden swell test and the constant volume oedometer test. The two tests are conducted on identical samples obtained from the same depth. Typical test results for both types of oedometer tests are shown in Figure (2.22). In the overburden swell test, the percent swell *S* (%) corresponds to the particular value of the vertical stress applied at the time of inundation $\sigma_i^{'}$ for the conditions under which heave is being computed. In the constant-volume test, the percent swell is zero at an inundation pressure of $\sigma_{cv}^{'}$. Thus, points *B* and *D*, as shown in Figure (2.22), fall on the line representing the desired relationship between $\sigma_i^{'}$ and *S* (%). This relationship is a straight line (i.e., heave line *BD*) on a semi-logarithmic plot. At any depth in the soil, the percent swell %*S* will fall along the straight line *BD*. The slope of that line is defined as the heave index C_H and is given by

$$C_H = \frac{S\,(\%)}{\log\left(\frac{\sigma'_{cv}}{\sigma'_i}\right)} \tag{2.32}$$

where C_H is heave index, %*S* is percent swell, σ'_{cv} is swelling pressure from the constant-volumetric oedometer test, and σ'_i is inundation stress subjected to a sample in the overburden swell test which equals to the overburden stress in the field for the conditions under which heave is being computed. If values of C_H and σ'_{cv} are known, the vertical strain or percent swell that will occur during inundation at any depth *z* in a soil profile can be determined from equation (2.32). For the case of free field heave, when the soil at a depth *z* is inundated, the stress on the soil is the overburden stress $(\sigma'_{ob})_z$. This value is, therefore, the inundation stress σ'_i in the field and equation (2.32) can be rewritten as equation (2.33).

$$(\varepsilon_v)_z = S_z\,(\%) = C_H \log(\frac{\sigma'_{cv}}{(\sigma'_{ob})_z}) \tag{2.33}$$

For a layer of soil of thickness Δz_i that exists at a depth *z* to its midpoint, the maximum heave ρ_i, that will occur due to the expansion of that layer during a complete inundation, can be obtained by multiplying the vertical strain $(\varepsilon_v)_z$ (Equation 2.33) by the layer thickness Δz_i; thus,

$$\rho_i = C_H\,\Delta z_i \log(\frac{\sigma'_{cv}}{(\sigma'_{ob})_z}) \tag{2.34}$$

In the actual application of equation (2.34), a soil profile will be divided into layers of thickness Δz_i, the value of heave for each layer will be computed, and the incremental values will be added to determine the total heave in the field (Nelson et al. 2011).

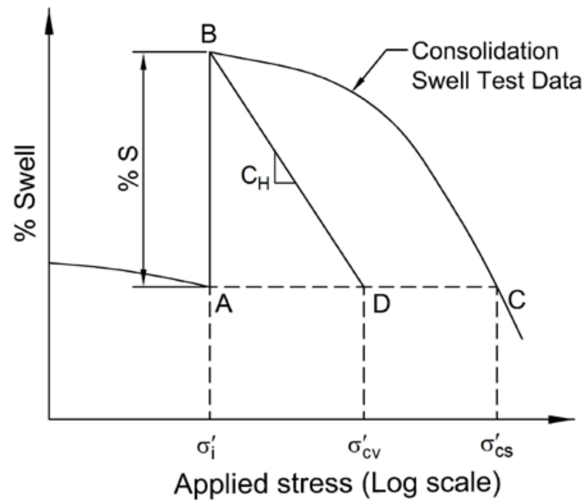


Fig. 2.22. Hypothetical oedometer test results (modified after Nelson and Miller 1992)

Nelson and Miller method utilizes the mechanical stress as the controlling stress state variable to calculate the heave potential, and gives conservative estimates of soil heave (Nelson and Miller 1992). However, it has been indicated that the determination of the heave index C_H by conducting the overburden swell test and the constant volume oedometer test on identical samples is generally not practical, mainly because it is almost impossible to obtain two identical samples from the field (Nelson et al. 2012). To accurately determine C_H, Nelson et al. (2006) suggested that several overburden swell tests at different inundation pressures along with a constant volume oedometer test would be required. This is also neither practical nor economical for engineering practice; therefore, a relationship between the swell pressure from the constant volume oedometer test σ'_{cv} and the swell pressure from the overburden swell test σ'_{cs} is proposed so that the value of the heave index C_H can be determined from a single overburden swell test (Nelson et al. 2006). The relationship is represented as

$$\sigma'_{cv} = \sigma'_i + \lambda(\sigma'_{cs} - \sigma'_i) \quad (2.35)$$

where λ is a parameter. The rationale behind this equation is that the value of σ'_{cv} must fall between σ'_i and σ'_{cs} by proportionality defined by the value of λ (Nelson et al.

2012). Nelson et al. (2006) suggested that a reasonable value of λ for the clay soil in the Front Range area of Colorado, USA, is 0.6. However, the actual value of λ to be used for a soil should be investigated for that soil.

Nelson and Miller (1992) method suffers from severe shortcomings as it is based on a depth potential heave (i.e., a single depth below which no heave occurs) and a single swell pressure value for the complete depth of soil, which is a topic of debate (Singhal 2010).

The oedometer test results are widely used in practice for estimating the heave potential; however, the environmental factors such as drainage conditions similar to in situ conditions, and the effects of lateral pressures cannot be simulated well in the oedometer tests. Attention is also drawn to the possible difficulty in determining a unique swelling pressure as described by Fredlund et al. (1980) since it is sensitive to the testing procedure. In addition, this single value of swelling pressure may not be a representative value over the entire depth of the active zone and for the area considered for expansive soils heave. The other disadvantage of oedometer methods is the extremely long time periods required (up to 100 days) for achieving equilibration conditions, which makes these methods both costly and tedious for use in practice (Holland and Cameron 1981).

2.9.1.2 Empirical methods

To reduce the amount of time required to conduct oedometer tests for estimating the heave potential, many attempts have been made to correlate oedometer test data with soil properties. The result of the correlation studies are empirical equations for predicting the heave potential, accounting both soil state and soil type representative parameters. The soil state is reflected by placement conditions factors namely moisture content, dry density, void ratio, and surcharge pressure, while the soil type is reflected by the compositional parameters namely plasticity index, and clay content (Zumrawi 2013). Rao et al. (2011) and Vanapalli and Lu (2012) summarized the most common empirical relations proposed in the literature to correlate the heave potential to the soil properties (Table 2.4).

Table 2.4. The most common empirical methods for the determination of soil heave potential (modified after Rao et al. 2011 and Vanapalli and Lu 2012)

Relationship	Reference
$S = 3.6\times10^{-5} A_c^{2.44} C^{3.44}$	Seed et al. (1962)
$S = 2.16\times10^{-3} PI^{2.44}$	Seed et al. (1962)
$S = 4.13\times10^{-4} SI^{2.67}$	Ranganatham and Satyanarayan (1965)
$S = 4.57\times10^{-5} [SI/(C-13)]^{2.67} C^{3.44}$	Ranganatham and Satyanarayan (1965)
$S_p = 2.29\times10^{-2} PI^{1.45}(C/w_i)+6.38$	Nayak and Christensen (1971)
$Log(S) = (1/12)(0.4LL - w_n + 5.5)$	Vijayvergiya and Ghazzally (1973)
$Log(S) = (1/19.5)(\gamma_d + 0.65LL - 130.5)$	Vijayvergiya and Ghazzally (1973)
$Log(S_p) = 0.9(PI/w_0) - 1.19$	Schneider and Poor (1974)
$S = 0.2558e^{0.0838PI}$	Chen (1975)
$S = 7.5 - 0.8w_0 + 0.203C$	McCormack and Wilding (1975)
$S_p = (5.3 - (147e/PI) - \log P)\times(0.525PI + 4.1 - 0.85w_0)$	Brackely (1975)
$S = 2.77 + 0.131LL - 0.27w_n$	O'Neil and Ghazzally (1977)
$S_p = 23.82 + 0.7346PI - 0.1458H - 1.7w_0 + 0.0025PI\,w_0 - 0.00884PI\,H, \quad for\ PI \geq 40$	Johnson (1978)
$S_p = -9.18 + 1.5546PI + 0.08424H + 0.1w_0 - 0.0432PI\,w_0 - 0.01215PI\,H, \quad for\ PI \leq 40$	Johnson (1978)
$S_p = 0.00041(LL_W)^{4.17}(P)^{-0.386}(w_0)^{-2.33}$	Weston (1980)
$S = 0.0000114A_c^{2.559} C^{3.44}$	Bandyopadhyay (1981)
$S = -121.807 + (12.1696MBV) + (27.6579\log(\psi_i))$	Cokca (2002)
$S = 4.24\gamma_{di} - 0.47w_i - 0.14q_i - 0.06FSI - 55$	Rao et al. (2004)
$S_P = 0.6(PI)^{1.188}$	Azam (2007)
$S = 2.0981e^{-1.7169IL}$	Yilmaz (2009)
$\Delta H = C_s \frac{H}{1+e_0}\log\left\{\frac{K P_f}{10^{\left(\frac{C_s}{C_w}\Delta w\right)}}\right\}$	Vanapalli et al. (2010)
$S_P = -57.965 + 37.076\rho_d + 0.524MBV + \varepsilon$	Türköz and Tosun (2011)
$S_P = 24.5(q)^{-0.26}(PI\,C)^{1.26}[F_i - 7.1(q)^{0.22}(PI\,C)^{0.78}]$	Zumrawi (2013)

where:

A_c : soil activity
C : clay content
C_s : swelling index
C_w : suction modulus ratio
e : void ratio
e_0 : initial void ratio
F_i : initial state factor
FSI : free swell index
H : depth of soil
IL : liquidity index
K : correction parameter
LL : liquid limit
LL_W : weighted liquid limit
MBV : methylene blue value
P , q : surcharge
P_f : final stress state
q_i : initial surcharge
PI : plasticity index
S , S_p , ΔH : heave/swell potential
SI : shrinkage index
w_i , w_0 : initial moisture content
w_n : natural moisture content
Δw : change in water content
ρ_d : in situ dry density
γ_d : dry unit weight
γ_{di} : initial dry unit weight
ε : mean-zero Gaussian random error term
ψ_i : initial soil suction

The comparative studies undertaken by Noble (1966) and Zein (1987) clearly show that the empirical equations for predicting soil heave potential, while seemingly adequate for known conditions in the regions where they were developed, have some limitations when used for other regions. In other words, the empirical relationships are suggested based on a limited amount of data from a specific region, and it is not appropriate to extend the usage of these site-specific prediction relationships toward more generalized analysis.

Also, these methods evaluate the heave potential for certain defined conditions. For example, the heave potential of a remolded soil is commonly evaluated for a confining pressure of 7 kPa and a saturated (zero suction) soil profile (Johnson and Snethen 1978). Most of the available empirical methods don't consider some key parameters that influence the swell behavior such as the soil structure, clay mineralogy, and environmental factors to list a few. Furthermore, the variation of clay property over the same site or different sites makes it a challenge to obtain representative soil samples and determine reliable data from laboratory tests.

Recently, Vanapalli et al. (2010) proposed an empirical method for estimating the maximum heave potential of natural expansive soils occurring as a response to the water content variations. The method is based on empirical relationships that have been developed from the published data of various regions of the world. The information required for these relationships can be obtained from simple laboratory tests; thus, this method eliminates the need for difficult and time consuming experimental tests. Vanapalli et al. (2010) method was tested in seven case studies published in the literature; however, the method appears to overestimate the field heave for some conditions.

It is suggested that the application of the empirical methods should be used with caution and should be considered only as indicator for heave.

2.9.1.3 Suction-based methods

Most researchers in the geotechnical engineering field since 1960's described moisture movement in unsaturated expansive soils in terms of soil suction (e.g., Richards 1965, Lytton and Kher 1970, Mitchell 1979, Pufahl and Lytton 1992, Fredlund 1997, Wray 1998, Fredlund and Vu 2001). Richards (1974) suggested that soil suction can be used to represent the state of the soil water much more effectively than the water content for two reasons. Firstly, soil suction is primarily controlled by the soil environment and not by the soil itself, and it tends to not exhibit discontinuous trends. The soil suction profile tends towards an equilibrium value at a particular depth under a particular climatic condition while water content is highly sensitive to the soil material variables (e.g., soil type, clay content, soil density, and soil structure). Secondly, the correlation of soil parameters (i.e., permeability or hydraulic conductivity, diffusivity, and shear strength) with water content is poor unless other soil properties such as density and clay content are considered, but these parameters can be conveniently correlated with soil suction.

In suction-based methods, the movement associated with volume change of expansive soils can be evaluated by measuring the present in situ suction condition and estimating (or predicting) possible future suction condition under a certain environment. The basic concept of those methods is that the volume change of unsaturated soils (usually void ratio or vertical strain) is linearly proportional to the soil suction in a logarithm scale over the moisture content range between shrinkage limit and plastic limit (Johnson and Snethen 1978, Mitchell and Avalle 1984, Hamberg 1985). Figure (2.23) shows an idealized relationship between void ratio and suction for a representative soil sample. The available suction-based methods differ mainly in the definition of the soil suction parameter which represents the slope of void ratio versus soil suction plot (e.g., soil suction index (Johnson 1977, Johnson and Snethen 1978, Snethen 1980, Hamberg 1985, Dhowian 1990), instability index (Aitchison 1973, Mitchell and Avalle 1984), or suction compression index (McKeen and Nielsen 1978, McKeen 1980, 1981, 1992, Wray 1984, 1997). The different names for the soil suction parameter arise from the concept that the unit volume change (i.e., void ratio change) is related linearly to either the soil suction change or the moisture content change within the range of field conditions. Table (2.5) lists the most common representatives of suction-based methods.

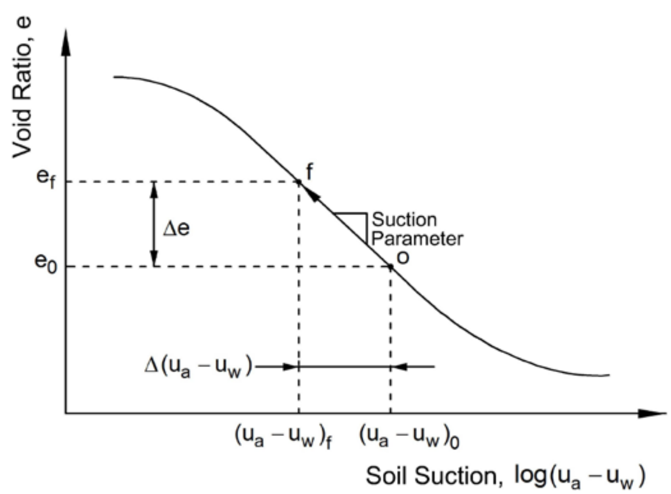


Fig. 2.23. Idealized void ratio versus logarithm of suction relationship for a representative sample (modified after Hamberg 1985)

Table 2.5. Suction-based methods for predicting heave potential (modified after Vanapalli and Lu 2012)

Equation	Reference
$\Delta H = \sum_{\text{layers}} \frac{H}{3} \frac{(w_f - w_i) G_s}{(100 + w_i G_s)}$ where: ΔH : soil heave H : soil layer thickness w_i : initial water content (measured) w_f : final water content (estimated in terms of the equilibrium matric suction) G_s : specific gravity	Richards (1967)
$\delta = \sum_{i=1}^{n} (I_{pt}\, \Delta u\, \Delta z_n)$ where: δ : vertical shrinkage or heave I_{pt} : instability index Δu : soil suction change Δz_n : thickness of the ith soil layer n : total number of soil layers considered	Aitchison (1973)
$S_f = \sum_{i=1}^{n} f_i \ (\frac{\Delta V}{V})_i \, \Delta z_i$ $(\frac{\Delta V}{V})_i = -\gamma_h \log(\frac{h_f}{h_i}) - \gamma_\sigma \log(\frac{\sigma_f}{\sigma_i})$ where: S_f : surface displacement f_i : lateral confinement factor $(\Delta V/V)_i$: average volumetric strain Δz_i : thickness of the ith soil layer n : total number of soil layers considered h_i, h_f : initial and final water potentials σ_f : applied octahedral normal stress σ_i : octahedral normal stress above which overburden pressure restricts volumetric expansion γ_h : matric suction compression index γ_σ : mean principal stress compression index	Lytton (1977)

Equation	Reference
$\Delta H = H \frac{C_\tau}{1+e_0} \log \frac{h_0}{h_f + \alpha \sigma_f}$ $C_\tau = (\alpha G_s)/(100 B)$ $\log h_0 = A - (B w_0)$ where: ΔH : soil heave H : soil layer thickness C_τ : suction index e_0 : initial void ratio h_0 : matric suction without surcharge pressure h_f : final matric suction σ_f : final applied pressure, (overburden plus external load) α : compressibility index	Johnson and Snethen (1978)
$\Delta H = \sum_{i=1}^{n} \frac{H_i}{1+e_i} [C_t \Delta \log(\sigma - u_a) + C_m \Delta \log(u_a - u_w)]_i$ where: ΔH : soil heave H_i : thickness of the i^{th} soil layer n : total number of soil layers considered e_i : void ratio for the i^{th} layer C_t : compressive index with respect to total stress C_m : compressive index with respect to matric suction $(\sigma - u_a)$: total stress $(u_a - u_w)$: matric suction	Fredlund (1979)
$\Delta H = \sum_{i=1}^{n} (I_{pt} \Delta u H_i)$ $I_{pt} = \frac{\Delta \varepsilon_y}{\Delta w} \frac{\Delta w}{\Delta u}$ where: ΔH : vertical surface movement I_{pt} : instability index Δu : soil suction change H_i : soil layer thickness over which I_{pt} can be taken constant n : number of layers to depth of the active zone Δw : moisture content change $\Delta \varepsilon_v$: change in vertical strain	Mitchell and Avalle (1984)

$$\Delta H = \sum_{i=1}^{n} \frac{H_i}{1+e_0} \times [C_h \, \Delta \log h]_i$$ where: ΔH : soil heave H_i : thickness of the i[th] layer e_0 : initial void ratio C_h : suction index with respect to void ratio h : soil suction n : number of layers to depth of the active zone	Hamberg (1985)
$$\Delta H = H \, \gamma_h \, (\Delta pF - \Delta pP)$$ where: ΔH : shrink or swell over vertical increment H : vertical increment over which shrink or swell is occurring γ_h : suction compressibility index ΔpF : change in soil suction over vertical increment ΔpP : change in soil overburden over vertical increment	Wray (1984)
$$\Delta H = H \frac{C_\psi}{1+e_0} \log \frac{\psi_i}{\psi_f}$$ $$C_\psi = (\alpha \, G_s) / (100 \, B)$$ where: ΔH : soil heave H : soil layer thickness C_ψ : suction index e_0 : initial void ratio ψ_i , ψ_f : initial and final suction α : volume compressibility factor G_s : specific gravity of solid particles B : slope of suction versus water content relationship	Dhowian (1990)
$$\Delta H = C_h \, \Delta u \, \Delta t \, f \, s$$ $$C_h = (-0.02673)(\Delta u / \Delta w) - 0.38704$$ $$f = (1 + 2K_0)/3$$ $$s = 1 - 0.01(\%SP)$$ where: ΔH : surface heave C_h : suction compression index Δu : suction change Δt : soil layer thickness	McKeen (1992)

f : lateral restraint factor

s : reduction factor to account for overburden

Δw : moisture content change

K_0 : coefficient of lateral earth pressure at rest

SP : swell pressure applied to the soil due to overburden pressure

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Cover and Lytton (2001)

$$S_f = \sum_{i=1}^{n} f_i \ (\frac{\Delta V}{V})_i \ \Delta z_i$$

$$(\frac{\Delta V}{V})_i = -\gamma_h \log(\frac{h_f}{h_i}) - \gamma_\sigma \log(\frac{\sigma_f}{\sigma_i})$$

$$\gamma_h = (\frac{\gamma(swelling\ case) + \gamma(shrinkage\ case)}{2})$$

$$\gamma(swelling\ case) = [(\frac{COLE}{100} + 1)^3 - 1]$$

$$\gamma(shrinkage\ case) = [1 - \frac{1}{(\frac{COLE}{100} + 1)^3}]$$

where:

S_f : surface displacement

f_i : lateral confinement factor

$(\Delta V / V)_i$: average volumetric strain

Δz_i : thickness of the i[th] soil layer

n : total number of soil layers considered

h_i, h_f : initial and final water potentials

σ_f : applied octahedral normal stress

σ_i : octahedral normal stress above which overburden pressure restricts volumetric expansion

γ_h : matric suction compression index

γ_σ : mean principal stress compression index

$COLE$: coefficient of linear extensibility

Lytton et al. (2004)

$$\Delta H = \sum_{i=1}^{n} f \ (\frac{\Delta V}{V})_i \ \Delta z_i$$

$$(\frac{\Delta V}{V})_{i,\ swelling} = -\gamma_h \log(\frac{h_f}{h_i}) - \gamma_\sigma \log(\frac{\sigma_f}{\sigma_i})$$

$$(\frac{\Delta V}{V})_{i,\ shrinkage} = -\gamma_h \log(\frac{h_f}{h_i}) + \gamma_\sigma \log(\frac{\sigma_f}{\sigma_i})$$

$$f = 0.67 - 0.33 \Delta pF$$

$$\gamma_\sigma = C_c / (1 + e_0)$$

where:

ΔH : surface displacement

f :	crack fabric factor, $1/3 \le f \le 1.0$
$(\Delta V/V)_i$:	volume strain
ΔZ_i :	the ith depth increment
n :	number of depth increments
h_i , h_f :	initial and final values of matric suction
σ_i , σ_f :	initial and final values of mean principal stress
γ_h :	matric suction compression index
γ_σ :	mean principal stress compression index
ΔpF :	change of suction
C_c :	compression index
e_0 :	void ratio

Because of greater sensitivity of soil suction to volume change in comparison to moisture content, soil suction-based methods have provided better characterization of expansive soil behavior and more reliable estimates of anticipated heave under field conditions than oedometer methods (Johnson and Snethen 1978, Snethen 1980, Fredlund 1983). The heave calculation procedure of suction-based methods requires both the initial and final profiles of suction along with the suction parameter. The values of initial and final suction are either estimated during some correlation (e.g., equilibrium soil suction and climate index correlation) or measured using various suction devices (e.g., tensiometers, thermocouple psychrometer, thermal conductivity sensors, filter paper, etc.) (Snethen 1980). Fredlund (1979) suggested that the initial suction is measured by using one of suitable devices; however, assumptions can be used for the final suction profile. By assuming the pore air pressure is initially to be in equilibrium and equal to atmospheric pressure, the pore water pressure can be equivalent to the matric suction, and then three different scenarios can be used for determining the final pore water pressure (i.e., the final matric suction): (i) assuming the groundwater table at the soil surface, thus a hydrostatic pore water pressure will result; (ii) the pore water pressure approaches zero throughout its depth; and (iii) the pore water pressure is slightly negative. These assumptions are simple and reasonable, but they do not provide details of how soil volume changes with respect to time.

Suction-based methods are generally considered to be simple, economical, expedient and capable of simulating field conditions. However, several possible limitations summarized below are likely when they are extended in practice:

- The relationship of the soil suction on log scale versus volume change is linear only over a certain range of suction; this range is not known or well defined in most cases.
- Soil suction has to be measured or estimated for each site. However, it is a challenge to reliably measure suction in the field especially in expansive soils.
- Although the parameters of suction-based methods (e.g., suction compression index) are functions of stress states (i.e., net normal stress and matric suction), the determination of these parameters has been done by controlling both the net normal stress and the matric suction using simple laboratory index tests.
- The prediction accuracy of suction-based methods depends on the determination of wetting depth (i.e., depth of suction variations). However, the depth of wetting is difficult to define as it varies with site and depends on the environmental changes (Snethen and Huang 1992).

2.9.2 Methods for predicting soil vertical movement over time

The soil movement over time information is required for the design of foundations placed in expansive soils. This information is also helpful for the assessment of pre-wetting and controlled wetting mitigation alternatives for expansive soils. Research particularly in the past decade has been directed by various investigators to propose methods for the prediction of the soil movement over time (e.g., Briaud et al. 2003, Vu and Fredlund 2004 and 2006, Zhang 2004, Wray et al. 2005, Overton et al. 2006, Nelson et al. 2007). Briaud et al. (2003) suggested that any method developed to predict the movement of expansive soils over time must include two components: (i) the range of water content or soil suction fluctuations as a function of time within the active zone depth; and (ii) the constitutive law that links soil state variable (i.e., water content, soil suction, and mechanical stress) to the volume change movements of the soil. The current methods of the prediction of soil movement over time can be classified into: (i) consolidation theory-based methods that use the matric suction and the mechanical stress as state variables (i.e., extending two independent stress state variables concept proposed by Fredlund and Morgenstern (1977)), (ii) water content-based methods that use water content as a state variable, and (iii) suction-based methods that use the matric suction as a state variable. The following describes some of these methods.

2.9.2.1 Consolidation theory-based methods

Vu and Fredlund (2004) method

Vu and Fredlund (2004) extended the general consolidation theory of unsaturated soils described in section (2.8.3) to develop a method for the prediction of one-, two-, or three-dimensional soil heave over time. The governing equations for soil structure (i.e., equilibrium equation, Equation 2.25) and for water phase (i.e., water continuity equation, Equation 2.30) were numerically solved using uncoupled and coupled analyses. In the uncoupled analysis, equation (2.25) was solved independently from equation (2.30). A general-purpose partial differential equation solver (FlexPDE) was used to obtain the uncoupled solution. Two key steps are used in FlexPDE; (i) soil matric suction distribution with respect to time for specified boundary conditions is determined; (ii) the soil heave is then determined taking account of the applied boundary conditions and the matric suction variations. However, in the coupled analysis, the governing equations were solved simultaneously using finite element computer program COUPSO (Pereira 1996). The results include the soil heaves and the matric suctions obtained at any time during the transient process. The uncoupled solutions can be more easily achieved than the coupled solutions because of the nonlinear functions of soil properties involved in each process (i.e., water flow or stress deformation) are considered to be independent of one another.

A case history of a floor slab of a light industrial building located in Regina, Saskatchewan, Canada, was modeled by Vu and Fredlund (2004) to test the validity of their prediction method. Figure (2.24) provides the comparison of the soil heave predicted by Vu and Fredlund (2004) at various suction conditions over time with the total heave measured by Yoshida et al. (1983) at different depths beneath the centre of the slab. The agreement between the predicted and the measured heaves differs to some degree. The amounts of heave measured at depths of 0.58 m and 0.85 m correspond to the predicted heave at 100 days. Figure (2.25) shows the comparison between the predicted heave at various matric suction conditions over time and the measured total heave at the surface of the slab. The total heave predicted under the steady state condition agrees well with the measured heave. The predicted results are in a reasonable agreement with the measured values.

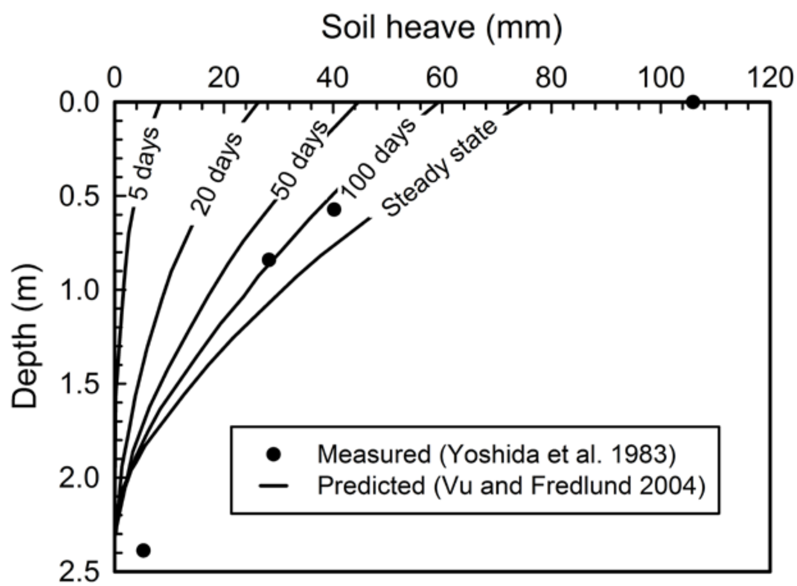


Fig. 2.24. Measured and predicted heaves with depth under the center of the slab (modified after Vu and Fredlund 2004)

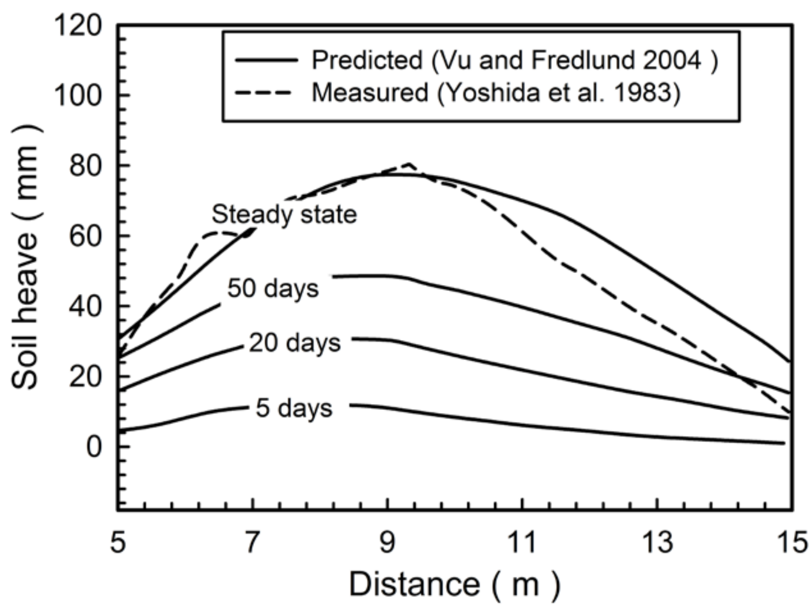


Fig. 2.25. Measured and predicted heaves at the surface of the slab (modified after Vu and Fredlund 2004)

Vu and Fredlund (2006) investigated challenges encountered by Vu and Fredlund (2004) to characterize the void ratio at low net normal stresses and (or) low matric suctions. Extremely low elastic moduli are possible for low net normal stresses or low matric suctions values which contribute to unreasonably large soil movements. These challenges have been overcome by providing a continuous, smooth void ratio constitutive surface based on the soil swelling indices obtained from conventional oedometer tests. Two typical volume change problems, water leakage from a pipe under a flexible cover and water infiltration at the ground surface, were solved by Vu and Fredlund (2006) using both the coupled and uncoupled analyses. It was suggested that the uncoupled analysis may be adequate for most heave prediction problems. However, the coupled analysis provides a more rigorous understanding of the swelling behavior of expansive soils and forms a reference for the evaluation of various uncoupled analyses.

The prediction method presented in Vu and Fredlund (2004, 2006) has been validated using Regina expansive clay. The method focuses on the prediction of soil heave, which is corresponding to a short-term condition. The soil shrinkage corresponding to a long-term condition is completely neglected. In addition, the coefficients of volume change m_1^s, m_2^s, m_1^w, and m_2^w, which are needed to perform the uncoupled/coupled analyses, require the void ratio and water content constitutive surfaces to be constructed. Those constitutive surfaces can be obtained from the consolidation tests or the triaxial shear tests with suction control. However, such tests are usually time consuming and require advanced lab equipments.

Zhang (2004) method

Unsaturated soils attain the saturated condition under different scenarios; however, researchers were unable to provide a unified theoretical framework for both saturated and unsaturated soils. Several investigators provided the coupled consolidation theory for saturated and unsaturated soils, separately. The concept of constitutive surfaces however has been provided only for unsaturated soils because of the following reasons (Zhang et al. 2005)*:* (i) the volume change theory for saturated soils is well established through the research work of Terzaghi (1936) and Biot (1941). Both Terzaghi (1936) and Biot (1941) suggested using the consolidation curve and did not use the constitutive surfaces for saturated soils; (ii) many researchers have used a single stress state variable (i.e., the effective stress principle) for interpreting the behavior of saturated soils.

Zhang (2004) provided the coupled consolidation for saturated and unsaturated soils in a unified manner. The thermodynamic analogue was used to explain the coupled consolidation process for saturated and unsaturated soils following Terzaghi (1943)'s one dimensional consolidation theory for saturated soils. Terzaghi (1943) stated that if the unit weight of water γ_w is assumed to be unity, the differential equation of Terzaghi's consolidation theory is identical to the differential equation for non-stationary, one-dimensional flow of heat through isotropic bodies. The loss of water (consolidation) corresponds to the loss of heat (cooling) and the absorption of water (swelling) to the increase of heat content of a solid body (Zhang 2004). In other words, the pore water pressure corresponds to the temperature while the water content to the heat energy per unit mass. The coupled consolidation theory for saturated and unsaturated soils includes the differential equations for soil structure and water phase. The differential equation for soil structure is given in equation (2.25). However, to derive the differential equation for water phase, it has been assumed that the continuity equation for water phase is similar to that for heat transfer (i.e., using the thermodynamics principles). As a consequence, the differential equation for water phase can be written in terms of specific water capacity of a soil (i.e., the volume of water required decreasing unit mass of soil by 1 kPa of matric suction).

$$\rho_d\, C_w \frac{d(\sigma_{mean}-u_a)}{\partial t} + m_2^w \frac{d(u_a-u_w)}{\partial t} = \frac{\partial}{\partial x}\left[k_w^x \frac{\partial}{\partial x}\left(\frac{u_w}{\rho_w g}+Y\right)\right] + \frac{\partial}{\partial y}\left[k_w^y \frac{\partial}{\partial y}\left(\frac{u_w}{\rho_w g}+Y\right)\right] + \frac{\partial}{\partial z}\left[k_w^z \frac{\partial}{\partial z}\left(\frac{u_w}{\rho_w g}+Y\right)\right] \quad (2.36)$$

where ρ_d is dry density of a soil, and C_w is specific water capacity of a soil.

Equations (2.25) and (2.36) are the differential equations for the coupled hydro-mechanical stress (consolidation) problem for unsaturated soils. However, by using the constitutive surfaces proposed by Zhang (2004) for saturated and unsaturated soils, equations (2.25) and (2.36) can also be used for saturated soils as a special case. Two stress state variables (i.e., total stress and pore water pressure) were used for saturated soils in order to develop the constitutive surfaces for both saturated and unsaturated soils mechanics in a unified system with smooth transition.

Close examination shows equations (2.30) and (2.36) are the same; the left sides of both equations are the volumetric water content variation and the right sides represent the net water flow into the soil element. If equation (2.36) is used, the water generation could be easily simulated by the heat generation based on the thermodynamic analogue. Consequently, some already well-established commercial software packages (e.g., ABAQUS, SUPER, and ANSYS) for solving those coupled thermal stress problems could be modified for solving the coupled consolidation problem to simulate several complicated problems related to the geotechnical engineering.

Zhang (2004) modelled a site in Arlington, Texas, USA, extending the coupled consolidation theory of saturated-unsaturated soils for the estimation of the soil movement over time. Four full-scale spread footings, called RF1, RF2, W1, and W2, constructed on expansive soils of the Arlington site were modelled over a period of 2 years. Factors that influence the movements of expansive soils such as the daily weather data, including the daily temperature, solar radiation, relative humidity, wind speed and rainfall, and the vegetation were considered for this field construction site. Abaqus/Standard program was used for the simulation of the soil movements at the Arlington site based on several models, including the coupled consolidation theory for saturated-unsaturated soils, potential and actual evapotranspiration estimation by using daily weather data, theories for the simulation of the soil-structure interaction at the soil-slab interface. The same site was also modeled by Briaud et al. (2003) to investigate the damage caused by expansive soils with both concrete and asphalt pavements. More details of the Arlington site are available in Briaud et al. (2003) and Zhang (2004).

For the simulation of soil volume change behavior, a coupled hydro-mechanical stress analysis was used and thermodynamic part was corresponded to the water phase continuity of the soil. By applying initial and boundary conditions and using the finite element method to solve the differential equations of the coupled consolidation theory (Equations 2.25 and 2.36), the values of mechanical stress and the matric suction can be calculated. The estimated values of mechanical stress and matric suction were used as an initial condition for the next step. This simulation technique can be continuously performed to predict the soil movement over time.

The validation of the prediction method was assessed more closely by comparing its estimations of soil movement with the long term field observations (over 2 years) at the Arlington site. Figure (2.26) shows the average values of the predicted soil movements at the four corners of the modeled footing, the measured movements of the four footings, and the average values of the measured movements of the footings over the two year's period. The comparison of the predicted movements with the measured movements of each footing did not lead to as good as the comparison based on the average values of the measured movements of the four footings (see Figure 2.26). This could be attributed to the fact that the regular measurements of soil movement can vary significantly while the average measurements are more representative of the actual variation in the movement of the soil site (Zhang 2004). The results suggested that Zhang (2004) method based on the coupled consolidation theory of saturated-unsaturated soils is a valuable tool for predicting the soil movements over time.

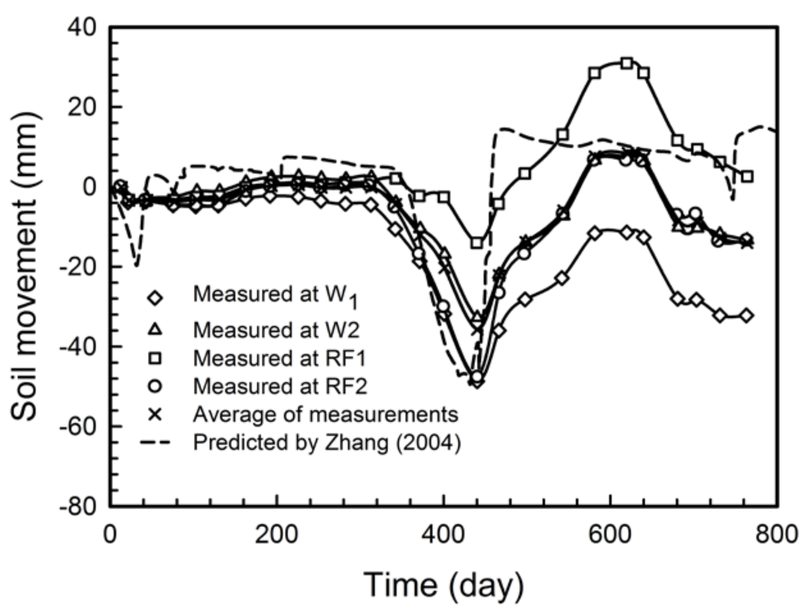


Fig. 2.26. Soil movement predicted by Zhang (2004) method and the soil movements measured at the Arlington site over two years (modified after Zhang 2004)

Zhang (2004) method is a comprehensive approach for modeling the water flow and the soil movement over time. Complex numerical solutions in finite element computer programs are required in this approach to address the analogy between the thermal and hydraulic problems. The use of the constitutive surfaces in this approach contributed to using a unified system for the first time to simulate the volume change behavior of expansive soils under both saturated and unsaturated conditions. However, there are limitations to apply the three-dimensional constitutive surface model in practice. The proposed constitutive surfaces have been developed based on testing soils under conditions not typically experienced in the field such as a shrinkage or a matric suction test at no normal stress, or a consolidation test at fully saturated conditions. Also, the conventional laboratories are not equipped to conduct the shrinkage tests or the matric suction tests which are needed for constructing the constitutive surfaces of unsaturated soils. These tests are time consuming that require sophisticated laboratory equipment and trained personnel, and hence such an approach is expensive.

2.9.2.2 Water content-based methods

Briaud et al. (2003) method

Briaud et al. (2003) proposed a method for estimating the vertical movement (shrink/swell) of the ground surface due to the variations in soil water content over time. The soil water content was used as a governing parameter; the range and the depth of water content variations can be estimated from a combination of experience, databases, observations, and calculations. The shrink test was suggested to obtain the relationship between the change in water content and the volumetric strain induced. Figure (2.27) shows the typical relationship of the water content versus the volumetric strain obtained from the shrink test. This relationship can be approximated by a straight line with the slope being the shrink–swell modulus E_w. The procedure of the method can be described as follows:

- Determine the depth of water content fluctuation and break it into an appropriate number of n layers, H_i being the thickness of the layer i.
- Collect soil specimens at the site within the depth of water content fluctuation.

- Perform the shrink test on each of the specimens; determine the shrink-swell modulus E_w and the shrinkage ratio f (i.e., the ratio of the vertical strain to the volumetric strain.)
- Determine the change in water content Δw as a function of depth and time.
- For the i^{th} layer, calculate the vertical movement of that soil layer ΔH_i as

$$\Delta H_i = H_i \ f_i \ \Delta w_i \ / \ E_{wi} \tag{2.37}$$

- Add the vertical movements of all soil layers for each date to calculate the ground surface movement for a given time as

$$\Delta H = \sum_{i=1}^{n} \Delta H_i = \sum_{1=1}^{n} (H_i \ f_i \ \Delta w_i \ / \ E_{wi}) \tag{2.38}$$

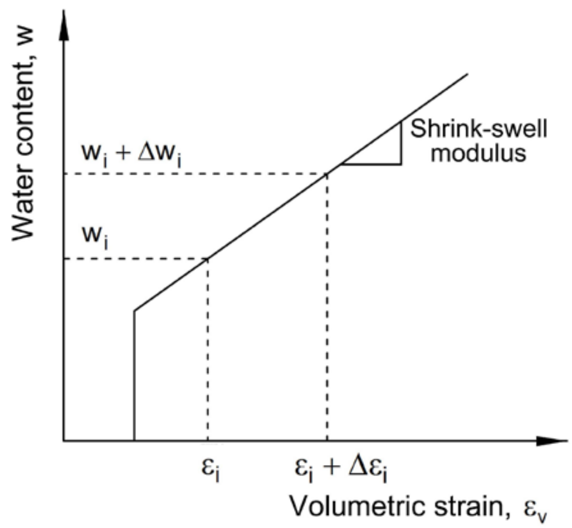


Fig. 2.27. Soil water content versus volumetric strain obtained from the shrink test (modified after Briaud et al. 2003)

This shrink test-water content method was evaluated by comparing the predictions with the measurements of the soil movements at the four full-scale spread footings (i.e., RF1, RF2, W1, and W2) constructed in the Arlington site; the same site that was modeled by Zhang (2004) (see section 2.9.2.1). Specimens were taken at the site during the 2-year period and data including water content and shrink-swell modulus were measured. Figure (2.28) shows the comparison between the soil movements predicted by Briaud et al.

(2003) method and the measured movements of footings over two years. A better simulation of the field soil movements was achieved using Zhang (2004) method in comparison to Briaud et al (2003) method.

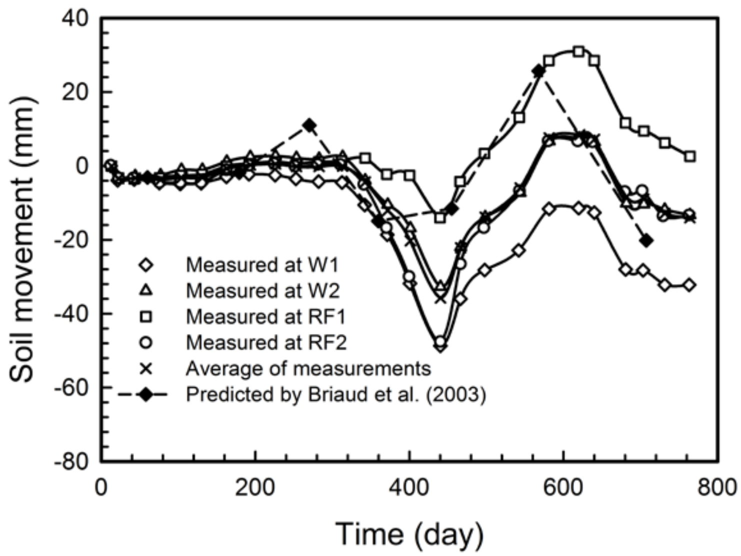


Fig. 2.28. Soil movements predicted by Briaud et al. (2003) method and the measured soil movements at the Arlington site over two years (modified after Briaud et al. 2003)

The advantage of Briaud et al. (2003) method however is its capability to predict the vertical soil swelling and soil shrinkage simultaneously using the shrink test. This method is based on the information of water content which is more reliable and simpler to measure in comparison to the soil suction. The constitutive law is obtained from the shrink test conducted on site-specific specimens instead of correlations to the index properties. However, the method is an uncoupled analysis of unsaturated soils where only the influence of moisture variation on the volume change of expansive soils is considered. In addition, when the soil is highly fractured, the shrink test is difficult to perform. Another drawback is that any theoretical consideration must make the use of the soil-water characteristic curve to transform the governing equations from suction based equations to water content based equations (Briaud et al. 2003).

Overton et al. (2006) method

The amount of soil heave at any time depends on two factors. These are the depth at which the water content in the soil has increased over time, and the expansion potential of the various soil strata. As water migrates through a soil profile, different strata become wet, some of which may have more swell potential than others. Consequently, the amount of soil heave varies with time. Overton et al. (2006) presented an approach for predicting the free field heave of expansive soils over time based on the migration of the wetting front. Analyses of the migration of the wetting front were conducted for soil profiles using the commercial software VADOSE/W (Geo-Slope 2005). VADOSE/W is a finite element program that can be used to model both saturated and unsaturated flow in response to changes in the atmospheric conditions while considering infiltration, precipitation, surface water runoff and ponding, plant transpiration and actual evaporation, and heat flow. The free field heave, which will occur at the ground surface if no stress is applied, is the fundamental parameter required in this approach. The free-field heave was predicted using the oedometer method of Nelson and Miller (1992) presented in section (2.9.1.1).

By assuming various values of swelling pressure and percent swell, the maximum free field heave and the depth of heave potential can be calculated using equation (2.34). The amount of the heave at any point in the soil profile is a function of the amount by which the water content has increased. The values of the volumetric water content are obtained from VADOSE/W at each time step. The relationship between the heave potential and the volumetric water content for a soil can be determined from oedometer tests conducted in the laboratory. For soils that are not fully wetted, the percent swell and the swelling pressure will be less than those measured after saturation in the oedometer tests. Therefore, in calculating the soil heave, those values must be corrected for the actual volumetric water content. The free field heave with respect to time is computed by multiplying the total heave potential (i.e., maximum free heave from equation (2.34)) at each soil layer by a heave factor obtained from the heave potential and volumetric water content relationship.

Overton et al. (2006) extended this approach on soil profiles in the Denver area of Colorado with good and poor drainage. Chao et al. (2006) also used this approach to investigate the effect of irrigation practices, poor drainage conditions, deep wetting from underground sources, and dipping bedrock on the heave variations over time. The results showed significant variations exist in the predicted values of heave potential versus time due to the effect of those factors.

Realistic estimates of the time rate of the migration of the wetting front and the resulting soil heave can be obtained by Overton et al. (2006) method for only ideal conditions. Such ideal conditions are only possible where sites have homogenous soil profiles with minimal macroscale fracturing or cracking, and/or where the principal direction of heave is perpendicular to the ground surface. However, if the site specific analyses have not accurately determined the rate of migration of the wetting front and the resulting time rate of heave, the entire depth of heave potential should be assumed wet during the life of a structure (i.e., maximum heave potential should be considered) (Overton et al. 2006). In addition, experimental determination of the free-field heave using oedometer tests is both time consuming and difficult to conduct. Some downsides to oedometer tests are related to the extremely long time period required for achieving the equilibrium condition and the difficulty to simulate the in situ conditions (e.g., drainage conditions and lateral pressures). Another drawback of this method is that it doesn't give any indication of possible shrinkage.

2.9.2.3 Suction-based methods

Wray et al. (2005) method

Wray et al. (2005) developed a computer program SUCH (it is named from SUCtion Heave) to predict the soil moisture changes and the resulting soil surface movements (heave/shrink), particularly under covered surfaces. The SUCH program involves two models: (i) a moisture flow model for estimating the movement of water through unsaturated expansive soils based on the diffusion equation developed by Mitchell (1993), and (ii) a volume change model developed by Wray (1997) for estimating the vertical soil movement (heave/shrink) associated with the change in soil suction over time.

The Mitchell's transient suction diffusion equation in its three dimensional takes the form

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+\frac{f(x,y,z,t)}{p}=\frac{1}{\alpha}\frac{\partial u}{\partial t} \qquad (2.39)$$

where u is total soil suction expressed in pF units (kPa = 0.1 $\times$ 10^{pF}), α is diffusion coefficient (mm^2/s) which can be measured in the laboratory (Mitchell 1993) or calculated from empirical equations (McKeen and Johnson 1990, Bratton 1991, Lytton 1994), p is unsaturated permeability (mm/s), t is time (s), x, y, and z are space coordinates, and $f(x, y, z, t)$ is internal source of moisture.

SUCH program is written in FORTRAN language, utilizing the finite difference technique to solve the transient suction diffusion equation (Equation 2.39). Two sets of information must be given: (i) the initial condition, i.e. the initial value of suction at each node in the soil mass; and (ii) the boundary conditions, i.e. the values of suction on the boundaries of the soil mass at each time step. Then, the moisture flow model can be used to determine the distribution of soil suction in the soil mass over time.

After the determination of the suction distribution through the unsaturated expansive soil mass, the resulting vertical soil movement at each nodal point associated with the change of soil suction over time can be estimated. The suction-based model (Equation 2.40) developed by Wray (1997) was used for the estimation of the resulting soil movements.

$$\Delta H_{i,j,k} = \Delta z \, \gamma_{h_{i,j,k}} [\Delta pF_{i,j,k} - \Delta pP_{i,j,k}] \qquad (2.40)$$

where $\Delta H_{i,j,k}$ is incremental volume change (heave/shrink) at grid point (i, j, k) over the increment thickness Δz, Δz is increment thickness in the z-direction over which heave or shrink occurs, $\gamma_{h_{i,j,k}}$ is suction compression index at grid point (i, j, k), McKeen (1980), Lytton (1994), and Wray (1997) presented different methods to estimate the value of $\gamma_{h_{i,j,k}}$, $\Delta pF_{i,j,k}$ is change of total soil suction expressed in pF units at grid point (i, j, k), and $\Delta pP_{i,j,k}$ is change of soil overburden over the increment thickness Δz at grid point (i, j, k). The vertical movement of each nodal point at the top surface of the soil mass was calculated as the summation of the vertical movements of the nodal points on the vertical line passing through that surface point, extending from the top to the bottom of the active zone of the soil mass (Wray et al. 2005).

The method was validated using well-documented field studies, chosen to cover widely varying climatic and soil conditions, that are located in the United States and Saudi Arabia. Two sites; namely, Amarillo test site and College Station test site, located in Texas, USA, were selected to represent a three dimensional problem (Wray 1989). The College Station site and the Amarillo site properties are similar. The only exception is that the College Station site represents a wet climate while the Amarillo site was selected to represent a dry climate. SUCH model was used for the two sites to estimate the soil suction changes and the vertical soil movements every month over a period of 5 years (from August 1985 through July 1990). Al-Ghatt, Saudi Arabia, test site investigated by Dhowian et al. (1985) was also selected to represent a two-dimensional problem. The test site was modeled over a period of 36 weeks. Comparisons were made between the estimated and the measured soil surface movements at several locations for the field studies under consideration. Figure (2.29) shows the predicted and measured monthly surface movements at 1.8 m outside slab edge along longitudinal axis at Amarillo site.

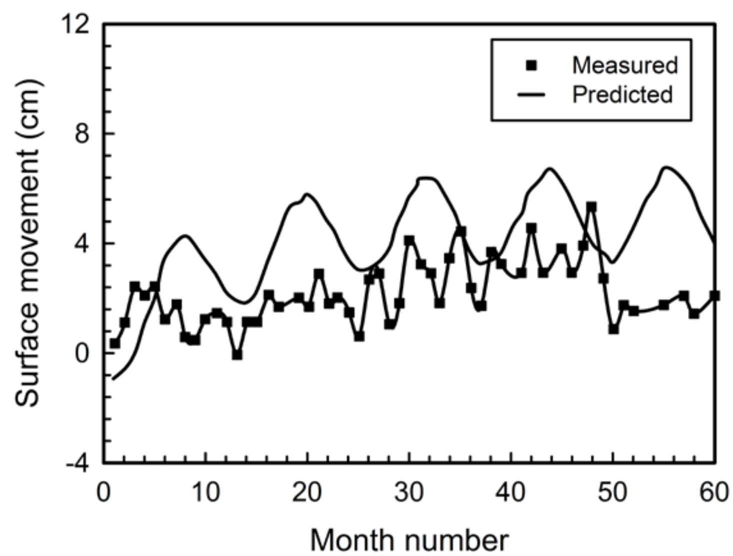


Fig. 2.29. Predicted and measured monthly surface movements at 1.8 m outside slab edge along the longitudinal axis at Amarillo site (modified after Wray et al. 2005)

The results of the SUCH model have shown a moderate to a good correlation with the reported field measurements of soil suction and the associated soil movements for the three sites (Wray et al. 2005). However, the application of the SUCH model to practical problems depends on the quantitative expression of the model parameters (i.e., diffusion coefficient, equilibrium soil suction, active zone depth, and suction compression index) and the initial and boundary conditions. Consequently, if the model parameters and the initial and boundary conditions can be accurately determined, the soil suction distribution and the resulting soil surface movements can be reasonably reproduced by the computer model SUCH. In addition, the results of the validation process revealed that Mitchell's diffusion equation for soil suction (Equation 2.39) needs to be modified to model the moisture movements in unsaturated fissured soils (soil cracks mechanism upon wetting). In SUCH model, the initial soil suction value information is required for each site. However, it is a challenge to reliably measure the field suctions especially in expansive soils.

2.10 Summary

Significant advances have been made towards the prediction of the heave and the shrink related volume change behavior of expansive soils since 1960's. The focus of most prediction methods has been towards estimating the maximum heave potential (soils under the saturation condition). These methods are classified into: empirical methods, oedometer methods, and suction-based methods. These methods are valuable; however, they do not provide information about the field soil movements as a function of time. Research studies show that the predicted soil heave by using the assumption of the saturation condition as a limiting condition is much higher than the in situ expansive soil heave. The soil moisture changes due to the environmental variations or other factors have a significant influence on the soil movement over time. Due to this reason, information related to the soil movement with respect to time is of practical interest for both the reliable design of foundations for structures on expansive soils and the assessment of mitigation alternatives for expansive soils.

In an attempt to develop both reliable and economical procedures of soil volume change analysis, few studies in recent years proposed prediction methods for estimating the expansive soil movements over time. Table (2.6) summarizes those recent methods which are classified into three main categories: consolidation theory-based methods, water content-based methods, and suction-based methods. However, there are limitations to apply the available methods in practice. The current volume change constitutive relationships are developed based on testing the soils under conditions not experienced in the field. Also, many conventional laboratories are not equipped to run the suction-controlled tests that are required for determining the parameters of the constitutive relationships. These tests generally require costly, time consuming, and difficult laboratory testing. Most importantly, few prediction methods have been validated with field measurements using one or limited number of case studies.

The prediction of volume change movements is a challenge even to date to geotechnical engineers. The available literature reviewed in this chapter highlights the need for a simple and efficient method that can be easily used in the conventional engineering practice to reliably predict the expansive soil movements with respect to environmental changes over time.

Table 2.6. Summary of the current methods for predicting the volume change movement of expansive soils over time

Description	Consolidation theory-based methods		Water content-based methods		Suction-based methods
	Vu and Fredlund (2004)	Zhang (2004)	Briaud et al. (2003)	Overton et al. (2006)	Wray et al. (2005)
Governing equations	Water continuity eq. (Eq. 2.30) Stress equilibrium eq. (Eq. 2.25)	Water continuity eq. in terms of C_w (Eq.2.36) (using thermodynamic analogue) Stress equilibrium eq. (Eq.2.25)	The constitutive equation of the soil movement formulated by extending the parallel between the shrink test–water content method and settlement methods (Eq. 2.38)	Free field heave eq. (Eq. 2.34)	Mitchell's transient suction diffusion eq. (Eq. 2.39) Suction-based model (Eq. 2.40)
State variables	Matric suction, $(u_a - u_w)$ and net normal stress, $(\sigma_{mean} - u_a)$	For saturated soils: total stress, σ and pore water pressure, u_w For unsaturated soils: $(u_a - u_w)$, $(\sigma_{mean} - u_a)$	Gravimetric water content, w	Volumetric water content, θ_w	Total soil suction
Required tests	Conventional oedometer test, oedometer or triaixal shear tests with suction control	Consolidation-swell test, free shrink test, suction test, and specific gravity test	Shrink test	Filter paper tests, consolidation-swell test, constant-volume test	Tests to measure the diffusion coefficient and the suction compression index
Soil properties	μ = Poisson's ratio Elasticity moduli: $E = fn[(u_a - u_w), (\sigma_{mean} - u_a)]$ $H = fn[(u_a - u_w), (\sigma_{mean} - u_a)]$ $E_w = fn[(u_a - u_w), (\sigma_{mean} - u_a)]$ $H_w = fn[(u_a - u_w), (\sigma_{mean} - u_a)]$ Permeability function: $k_w = fn[(u_a - u_w), (\sigma_{mean} - u_a)]$	μ E = saturated modulus of elasticity ρ_d = soil dry density α = coefficient of expansion C_w = specific water capacity m_2^w = coefficient of water volume change with respect to $(u_a - u_w)$ Permeability function $k_w = fn[(u_a - u_w), (\sigma_{mean} - u_a)]$	E_w = shrink-swell modulus f = shrinkage ratio	C_H = heave index SWCC = soil water characteristic curve k_{sat} = saturated hydraulic Conductivity, or hydraulic conductivity function : $k_w = fn[(u_a - u_w)]$	α = diffusion coefficient p = unsaturated permeability γ = suction compression index
Computer program	FlexPDE for uncoupled analysis COUPSO for coupled analysis	ABAQUS with three models: (i) coupled consolidation theory for saturated-unsaturated soils, (ii) potential and actual evapotranspiration estimation, and (iii) theories for the simulation of the soil-structure interaction	None	VADOSE/W for simulating water migration in response to atmospheric conditions	SUCH with two models: (i) moisture flow model, and (ii) a volume change model

Initial condition	Initial matric suction, $(u_a - u_w)_i$ and initial net normal stress, $(\sigma_{mean} - u_a)_i$	$(u_a - u_w)_i$, $(\sigma_{mean} - u_a)_i$	None	Initial water content profile	Initial total suction
Boundary conditions	Matric suction, water flux, applied load, and soil displacements	Matric suction, water flux, applied load, soil displacements, climate data, and vegetation data	None	Water flux, pore water pressure, climate data	Total suction
Results	1-D, 2-D, and 3-D heave over time	Vertical movement of the ground surface (heave/ shrink) over time	Vertical movement of the ground surface (heave/shrink) over time	free field heave over time	soil surface movements (heave/shrink) under covered surfaces
Application	A light industrial building in Regina, Saskatchewan, Canada	To simulate footings' movements for a site in Arlington, Texas, USA	To simulate footings' movements for a site in Arlington, Texas, USA	Soil profiles in the Denver area of Colorado, USA	Amarillo test site and College Station test site located in Texas, USA, and Al-Ghatt, Saudi Arabia, test site

CHAPTER 3

MODULUS OF ELASTICITY OF UNSATURATED EXPANSIVE SOILS

3.1 Introduction

The modulus of elasticity is conventionally used in geotechnical engineering practice in the calculation of the soil stress and deformation behavior (Davis and Poulos 1968, Schmertmann 1970, Schmertmann et al. 1978, Bowles 1987, Lade and Nelson 1987, Lade 1988, Berardi and Lancellotta 1991, Lancellotta 1995, Terzaghi et al. 1996, Mayne and Poulos 1999, Lee et al. 2008, Akbas and Kulhawy 2009, Oh et al. 2009, Vanapalli and Mohamed 2013). Prior to all these studies, Terzaghi's (1925, 1926, and 1931) pioneering modeling and experimental studies showed that the swelling and shrinkage of clay soils are essentially elastic deformations. It was shown that the swelling capacity of any soil is dependent on the elastic properties of the solid phase of the soil. Similar conclusions were derived by several investigators using different approaches (Biot 1941, Coleman 1962, Matyas and Radhakrisha 1968, Barden et al. 1969, Aitchison and Woodburn 1969, Brackley 1971, Aitchison and Martin 1973, Fredlund and Morgenstern 1976 and 1977, Vu and Fredlund 2004 and 2006, Zhang 2004, Zhang and Briaud 2010).

The total or effective stress, on the other hand, can modify the pore and soil skeleton structures and have a significant influence on the soil stiffness (the soil modulus of elasticity). However, under unsaturated conditions, the effective stress or the stress in soil skeleton is affected not only by the total stress, but also by the inter-particle stresses. The inter-particle stresses in an unsaturated soil are also referred to as suction stresses that arise due to the variation in soil water content or matric suction. The suction stresses consist of inter-particle physicochemical forces and pore water attraction due to matric suction (Lu and Likos 2006). Therefore, in fine-grained soils (e.g., silty and clayey soils), the modulus of elasticity is significantly influenced by soil water content or matric suction (Fredlund and Rahardjo 1993, Costa et al. 2003, Inci et al. 2003, Lu and Likos 2006,Yang et al. 2008).

Vanapalli and Oh (2010) proposed a semi-empirical model for predicting the modulus of elasticity of unsaturated soils with respect to matric suction using the soil-water characteristic curve (SWCC) as a tool. The validity of the model was tested for coarse- and fine-grained soils with plasticity index I_p values lower than 16%. For expansive soils with higher plasticity index, the adaptability of the Vanapalli and Oh (2010) has not been reported in the literature yet.

In this chapter, the validity of Vanapalli and Oh (2010) model to predict the modulus of elasticity of unsaturated expansive soils has been assessed using triaxial shear test results from the literature.

3.2 Background

Oh et al. (2009) proposed a semi-empirical model for predicting the variation of modulus of elasticity of unsaturated coarse-grained soils using the SWCC and the modulus of elasticity under saturated condition, extending similar concepts that were followed for the prediction of shear strength (Vanapalli et al. 1996) and bearing capacity (Vanapalli and Mohamed 2007) of unsaturated soils. The model was developed by Oh et al. (2009) using the stress versus displacement relationships from model footing tests performed on different sands under unsaturated conditions. In this model, as shown in equation (3.1), two fitting parameters α and β were used.

$$E_{unsat} = E_{sat}\left[1+\alpha\frac{(u_a-u_w)}{P_a/101.3}S^{\beta}\right] \tag{3.1}$$

where E_{unsat} is soil modulus of elasticity under unsaturated condition, E_{sat} is soil modulus of elasticity under the saturated condition, $(u_a - u_w)$ is matric suction, P_a is atmospheric pressure (P_a = 101.3 kPa) used for maintaining consistency with respect to the dimensions and units on both sides of the equation, and S is degree of saturation. Based on the results of Oh et al. (2009), the fitting parameter β = 1 was required for coarse-grained soils (i.e., plasticity index I_p = 0%). The fitting parameter α was found to be a function of footing size; values between 1.5 and 2 were recommended for large size footings to reasonably estimate the modulus of elasticity of unsaturated sandy soils E_{unsat}.

The differential form of equation (3.1), shown in equation (3.2), was used for explaining the nonlinear variations of modulus of elasticity with respect to matric suction.

$$\frac{dE_{unsat}}{d\left(u_a - u_w\right)} = \frac{E_{sat}\,\alpha}{(P_a/101.3)}\left[(S^{\beta}) + \left(u_a - u_w\right)\frac{d\left(S^{\beta}\right)}{d\left(u_a - u_w\right)}\right] \qquad (3.2)$$

Equation (3.2) shows, in coarse-grained soils, the net contribution of matric suction towards the increase in the modulus of elasticity starts decreasing as the matric suction approaches the residual suction value. Such a behavior can be attributed to both the low value of the degree of saturation S and the negative value of $d\left(S^{\beta}\right)/d\left(u_a - u_w\right)$ at the residual condition (Figure 3.1) (Oh et al. 2009).

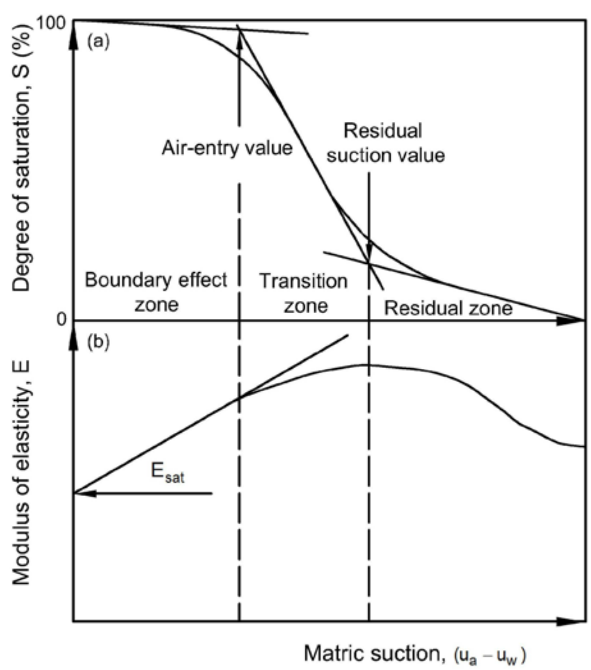


Fig. 3.1. The relationship between (a) soil-water characteristic curve (SWCC), (b) the variation of modulus of elasticity with respect to matric suction (modified after Oh et al. 2009)

Review of Figure (3.1) shows that the modulus of elasticity linearly increases up to the air-entry value of the soil; beyond this value and up to the residual suction value there is a nonlinear increase in the value of the modulus of elasticity. However, the contribution of matric suction towards the modulus of elasticity starts to decrease beyond the residual suction value for coarse-grained soils (Oh et al. 2009). This can be attributed to the variation of area of water with desaturation over the different stages of the soil-water characteristic curve (SWCC) (Vanapalli 1994, Vanapalli et al. 1996). Figure (3.1a) shows the three identifiable stages of desaturation, namely: the boundary effect stage, the transition stage, and the residual stage. In the boundary effect stage, all the soil pores are filled with water. The soil is essentially saturated, and there is no reduction in the area of water. In this stage, the single stress state $(\sigma - u_w)$ describes the behavior of the soil. Hence, there is a linear increase in the modulus of elasticity up to the air -entry value at which air starts to enter the largest pores of the soil. Then, in the transition stage, the soil starts to desaturate and the water content in the soil reduces significantly with increasing suction. Thus, there is a nonlinear increase in the modulus of elasticity associated with increasing soil suction. The amount of water at the soil particle reduces as desaturation continues. The water menisci area in contact with the soil particles is not continuous and starts reducing. Eventually, large increases in suction lead to a relatively small change in water content (or degree of saturation). This stage is referred to as the residual stage of unsaturation. The suction in the soil at the commencement of this stage is generally referred to as the residual suction. Beyond the residual suction conditions and during further desaturation, the soil modulus of elasticity may increase, decrease, or remain relatively constant. In soils that desaturate relatively fast (e.g., sands and silts), it can be expected that there is little water left in soil pores when the soil reaches the residual state and the modulus of elasticity will decrease. The water content in sands and silts at residual suction conditions can be quite low and may not transmit suction effectively to the soil particle contact points; therefore, even large increases in suction will not result in a significant increase in the modulus of elasticity. In contrast, clays may not have a well-defined residual state. Even at high values of suction there could still be considerable water available (i.e., in the form of adsorbed water) to transmit suction along the soil particles, which contributes towards an increase in the modulus of elasticity.

Vanapalli and Oh (2010) used the semi-empirical model (Equation 3.1) for estimating the modulus of elasticity for fine-grained soils with I_p values lower than 16%. The model was developed based on the results of model footing and in-situ plate load tests available in the literature. The fitting parameter $\beta = 2$ can be used for fine-grained soils, regardless of the plasticity index I_p value. The fitting parameter α was estimated based on a relationship proposed between the inverse of α (i.e., $1/\alpha$) and the soil plasticity index I_p as shown in Figure (3.2). The relationship shows that ($1/\alpha$) non-linearly increases with increasing I_p. The upper boundary relationship (Equation 3.3) was proposed for soils with I_p less than 12% and with low suction values. However, the lower boundary relationship (Equation 3.4) was proposed for soils with I_p less than 16% and with high suction values.

$$(1/\alpha) = 0.5 + 0.312(I_p) + 0.109(I_p)^2 \qquad (0 \le I_p \le 12) \tag{3.3}$$

$$(1/\alpha) = 0.5 + 0.063(I_p) + 0.036(I_p)^2 \qquad (0 \le I_p \le 16) \tag{3.4}$$

For soils with higher plasticity index (i.e., I_p > 16%), such as the expansive soils, the validity of Vanapalli and Oh (2010) model (hereafter referred as VO model) as well as its fitting parameters (i.e. α and β) values are not available.

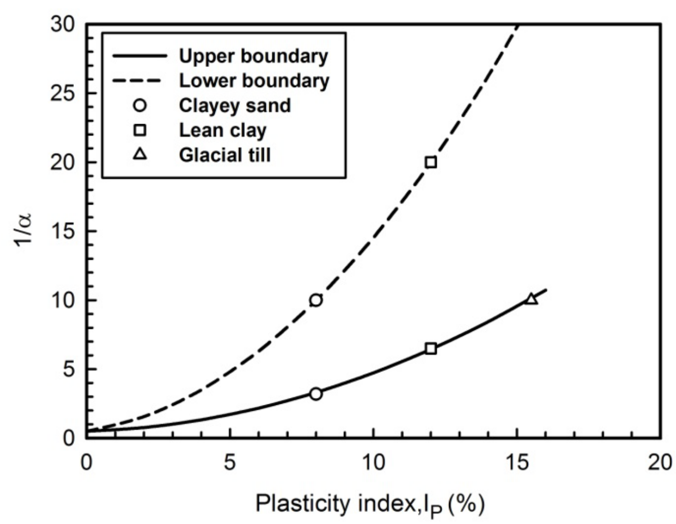


Fig. 3.2. Relationship between $1/\alpha$ and plasticity index I_p (modified after Vanapalli and Oh 2010)

In this chapter, the results of triaxial tests from the literature for three compacted expansive soils (i.e., Zao-Yang, Nanyang, and Guangxi) from China were used for examining the validity of the VO model (Equation 3.1) for unsaturated expansive soils. The elasticity moduli of soils were determined from the stress-strain curves of triaxial tests during shearing of saturated/unsaturated compacted specimens under different confining stresses and matric suctions.

Typically, stress-strain data of triaxial shear test can be represented mathematically by a hyperbola having the form below (Duncan and Chang 1970) (Figure 3.3).

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon}{\frac{1}{E} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_u}} \tag{3.5}$$

where ε is axial strain, E is initial tangent modulus of elasticity, σ_1 and σ_3 are major and minor principal stresses, respectively, and $(\sigma_1 - \sigma_3)_u$ is ultimate deviator stress at a large strain. The hyperbola is considered valid up to the actual soil failure (i.e., the actual failure deviator stress $(\sigma_1 - \sigma_3)_f$) (point A as shown in Figure (3.3)). The value of the parameters of the hyperbolic model (Equation 3.5) E and $(\sigma_1 - \sigma_3)_u$ can be determined by plotting the stress versus strain relationships on the transformed axes of (axial strain/deviator stress) $\varepsilon/(\sigma_1 - \sigma_3)$ and axial strain ε as shown in Figure (3.4), and represented by a straight line having the form below (Duncan and Chang 1970).

$$\frac{\varepsilon}{(\sigma_1 - \sigma_3)} = \frac{1}{E} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_u} \tag{3.6}$$

The intercept and the slope of the resulting straight line are the inverse of the initial tangent modulus of elasticity $1/E$ and the inverse of ultimate deviator stress $1/(\sigma_1 - \sigma_3)_u$, respectively (Duncan and Chang 1970).

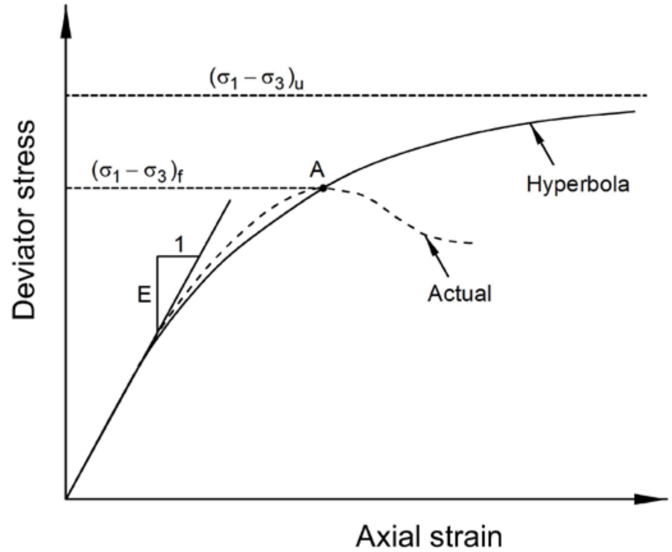


Fig. 3.3. Comparison of typical stress-strain curve with hyperbolic stress-strain curve (modified after Al-Shayea et al. 2001)

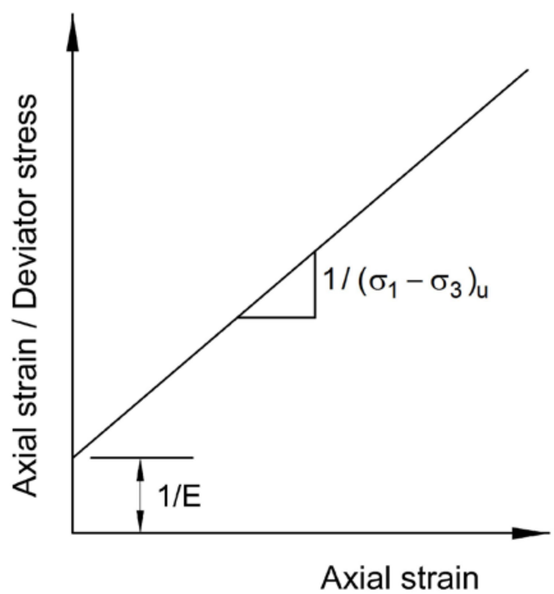


Fig. 3.4. Transformed hyperbolic stress-strain curve (modified after Duncan and Chang 1970)

The experimental values of modulus of elasticity in this study were determined as the reciprocal of intercept of the resulting straight lines. However, the predicted values of elasticity moduli were estimated using the VO model (Equation 3.1). The information required for using the VO model includes the SWCC and the modulus of elasticity of soil under the saturated condition E_{sat} along with the two fitting parameters α and β.

Comparisons were provided between the values of modulus of elasticity derived from the triaxial tests results and the VO model for the three expansive soils to check its validity.

3.3 Triaxial Shear Test Results and Soils Properties

The experimental results of triaxial shear tests carried out by Zhan (2003), Miao et al. (2002), and Miao et al. (2007) on compacted expansive soils from Zao-Yang, Nanyang, and Guangxi, respectively, were used for examining the validity of the VO model (Equation 3.1) for estimating the modulus of elasticity for unsaturated expansive soils. Figure (3.5) shows SWCCs for the three expansive soils used in this study. The key physical properties of these soils are summarized in Table (3.1).

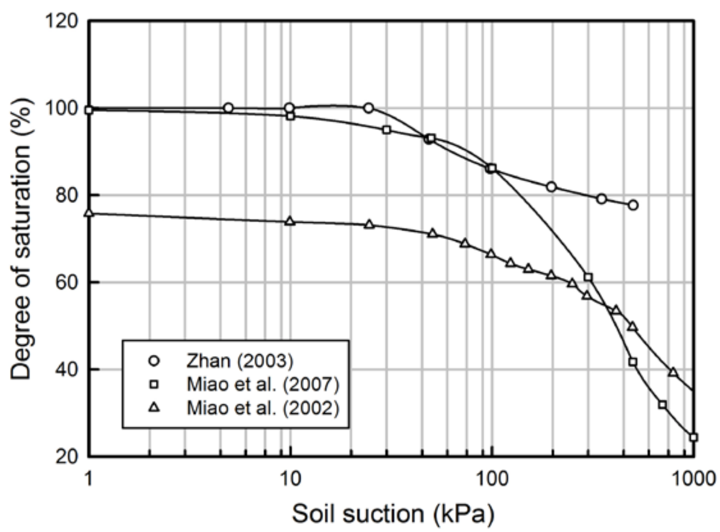


Fig. 3.5. Soil-water characteristic curves (SWCCs) for the three investigated expansive soils (Zao-Yang, Nanyang, and Guangxi expansive soils)

Table 3.1. Soil properties of Zao-Yang, Nanyang, and Guangxi expansive soils

Soil type	Plasticity index I_p %	Liquid limit w_L %	Plastic limit w_p %	Specific gravity G_s	Dry unit weight kN/m^3	Free swelling %
Zao-Yang Zhan (2003)	31.0	50.5	19.5	2.67	15.30	-
Nanyang Miao et al. (2002)	31.8	58.3	26.5	2.7	14.72	74
Guangxi Miao et al. (2007)	31.1	61.4	30.3	2.7	14.52	45

Zhan (2003) carried out conventional and suction-controlled triaxial tests on saturated and unsaturated compacted soil specimens to investigate the shear strength behavior of expansive soils. The soil samples were collected from Zao-Yang at about 400 km from Wuhan in China. The soil has 3% sand, 58% silt, and 39% clay, which can be classified as silty clay with intermediate plasticity (Zhan 2003). The SWCC for specimens of compacted Zao-Yang soils shown in Figure (3.5) was measured using a pressure plate extractor equipped with a 5 bar (1 bar =100 kPa) ceramic stone. The air-entry value of the soil is about 25 kPa. To represent the in situ state and obtain the average value of the in situ dry unit weight of 15.3 kN/m^3, the compacted specimens were prepared using a static compaction pressure of 800 kPa on the dry side of optimum at initial water content of 18%. Figure (3.6) summarizes the relationships between the deviator stress $(\sigma_1 - \sigma_3)$ and the axial strain ε during shearing of saturated compacted specimens at confining stresses σ_3 of 50, 100, 200, and 400 kPa. Figure (3.7) shows the results of the series of triaxial shear tests for unsaturated compacted specimens under two net confining stresses $(\sigma_3 - u_a)$ of 50 kPa and 200 kPa and various matric suction $(u_a - u_w)$ values of 25, 50, 100, and 200 kPa. Review of Figure (3.7a) shows an overlap in the stress versus strain relationships for the specimens tested with $(u_a - u_w)$ of 25 kPa and 50 kPa. This may be attributed to some minor differences in the initial conditions of tested specimens. Also, the low values of matric suction (25 kPa and 50 kPa) typically have limited influence on the stress-strain relationship compared to the other higher values of matric suction (100 kPa and 200 kPa).

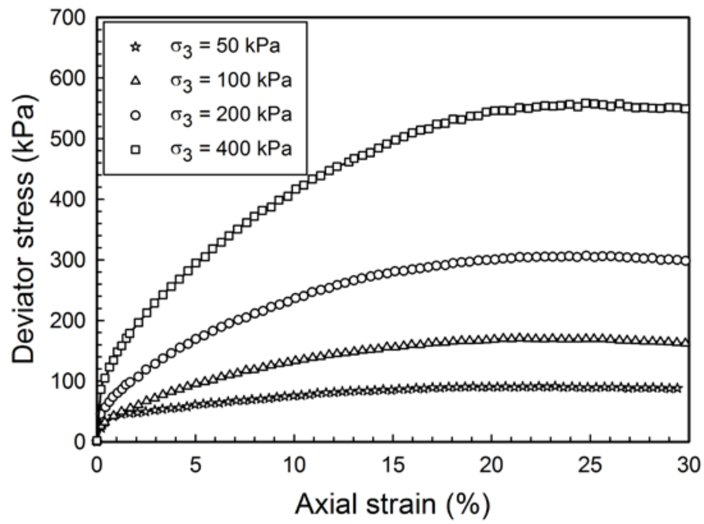
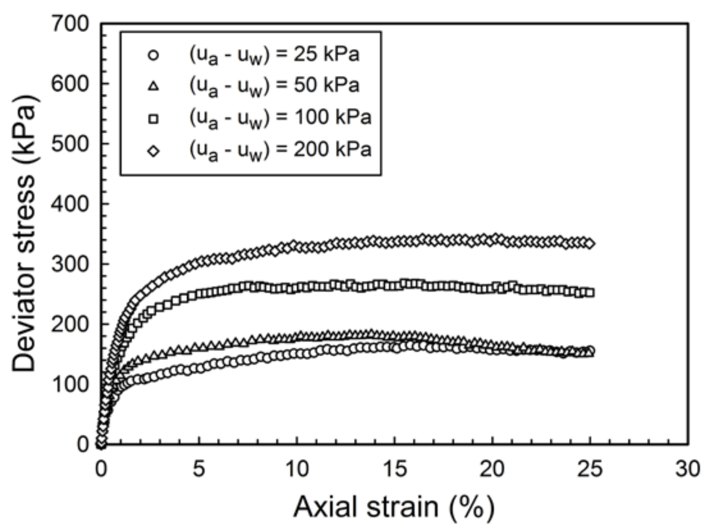
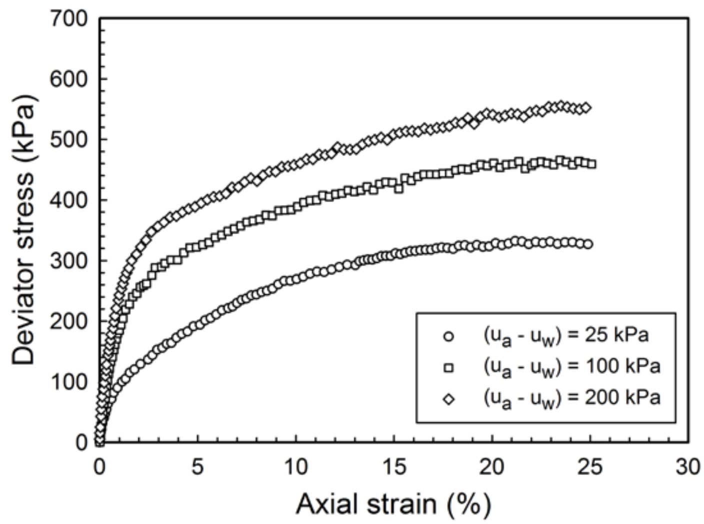


Fig. 3.6. Stress-strain curves for specimens of saturated compacted Zao-Yang soils at various confining stresses (modified after Zhan 2003)



(a) Net confining stress = 50 kPa



(b) Net confining stress = 200 kPa

Fig. 3.7. Stress-strain curves for specimens of unsaturated compacted Zao-Yang soils: (a) at net confining stress of 50 kPa, (b) at net confining stress of 200 kPa (modified after Zhan 2003)

Miao et al. (2002) studied the shear strength behavior of Nanyang expansive soils under saturated and unsaturated conditions from triaxial shear tests. The specimens were remoulded and prepared at a predetermined water content of 17% and unit weight of 14.72 kN/m^3 by the static compaction. The SWCC of the Nanyang expansive soils shown in Figure (3.5) was measured using a pressure plate with a 15 bar ceramic stone. The air-entry value and the residual suction value of the Nanyang expansive soil were 25 and 1500 kPa, respectively.

Figure (3.8) shows the conventional triaxial tests results of Nanyang soil specimens at the saturated condition under different confining stresses of 50, 100, and 150 kPa. The triaxial tests of unsaturated Nanyang soils were performed by controlling matric suction extending axis translation technique under different net confining stresses. Figure (3.9) shows the stress-strain curves for the unsaturated soil specimens with initial matric suctions of 50, 80, 120, and 200 kPa.

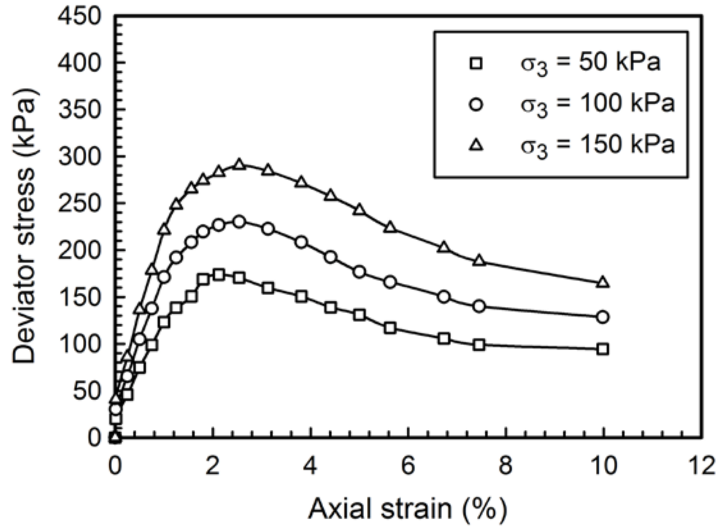
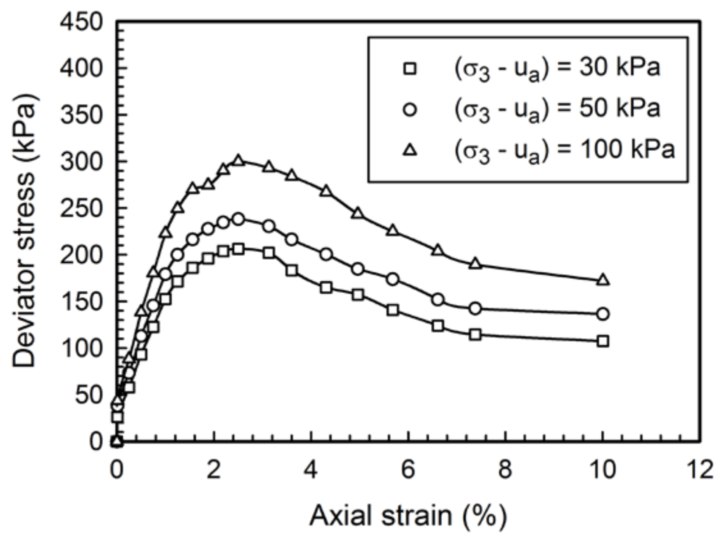
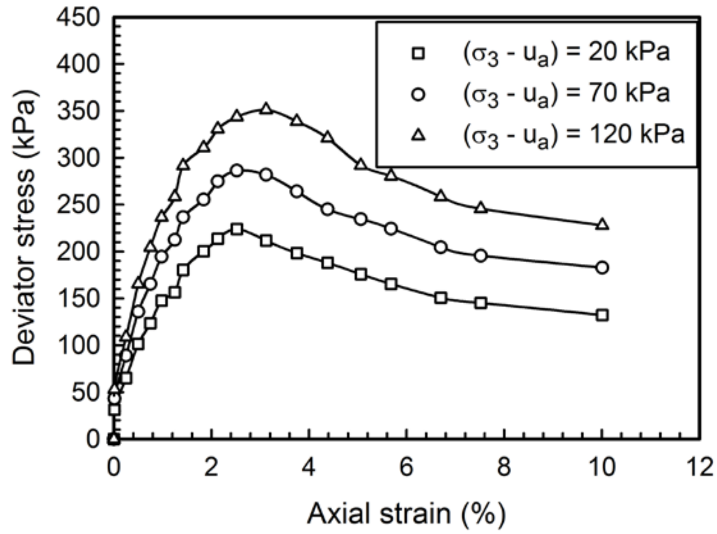


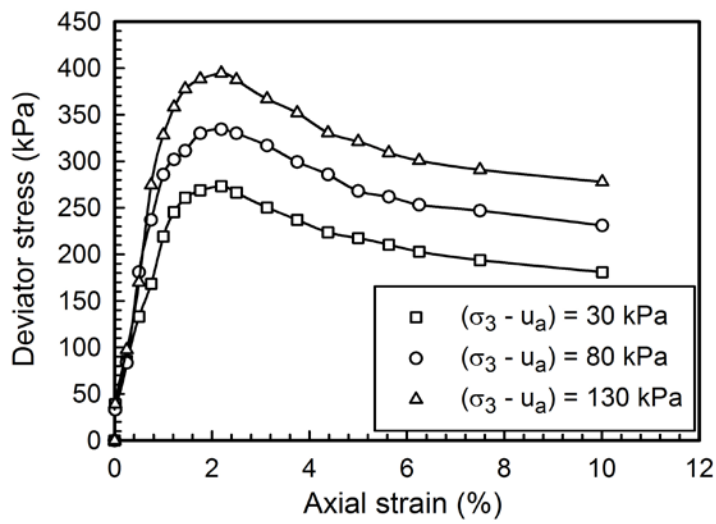
Fig. 3.8. Stress-strain curves for specimens of saturated compacted Nanyang soils (modified after Miao et al. 2002)



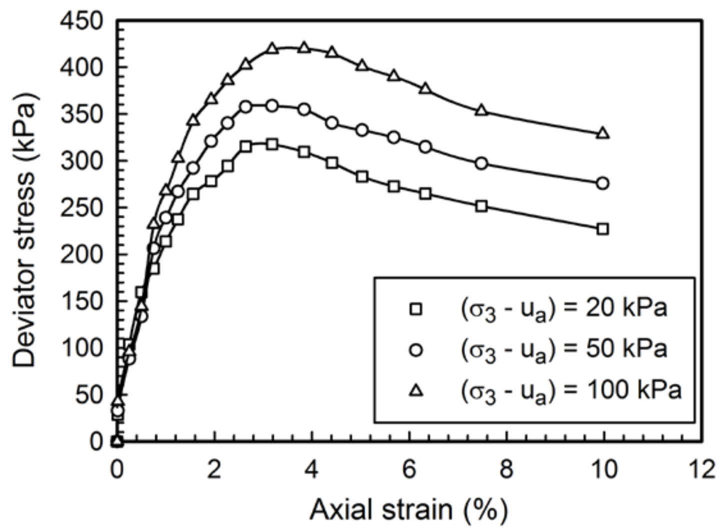
(a) Matric suction, $(u_a - u_w)$ = 50 kPa



(b) Matric suction, $(u_a - u_w)$ = 80 kPa



(c) Matric suction, $(u_a - u_w)$ = 120 kPa



(d) Matric suction, $(u_a - u_w)$ = 200 kPa

Fig. 3.9. Stress-strain curves for specimens of unsaturated compacted Nanyang soils: (a) at matric suction of 50 kPa, (b) at matric suction of 80 kPa, (c) at matric suction of 120 kPa, and (d) at matric suction of 200 kPa (modified after Miao et al. 2002)

Miao et al. (2007) also carried out triaxial shear tests to study the mechanical behavior of Guangxi soils for varying degrees of saturation (i.e., different suctions). The focus of their study was directed to understand the stress-strain-volume change behavior at different values of the degree of saturation. The specimens were statically compacted in a brass cylinder into five layers to achieve a dry unit weight of 14.52 kN/m^3. The compacted soil specimens were prepared at different initial degrees of saturation (i.e., 76.3%, 83.5%, 92.1%, and 100%). The degree of saturation was controlled by the initial water content of the soil specimens. The specimens might have some differences in the soil structure as a result of the compaction at different water contents to a certain density (Miao et al. 2007). The SWCC of the compacted Guangxi soils is shown in Figure (3.5). The air-entry value of the compacted Guangxi soils is about 30 kPa.

The conventional triaxial tests were used to study the stress-strain behavior of the saturated specimens of Guangxi soils under confining stresses of 50, 100, and 200 kPa (Figure 3.10). The triaxial tests under drained conditions were performed on specimens under unsaturated conditions. Figure (3.11) shows the stress-strain curves for Guangxi soils specimens tested under unsaturated conditions with varying initial degrees of saturation (i.e., 76.3, 83.5, and 92.1%) and net confining stresses (i.e., 50, 100, and 200 kPa).

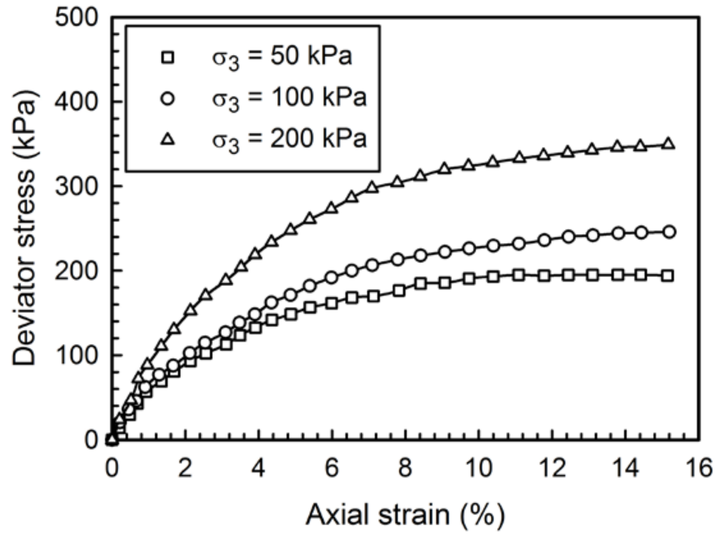
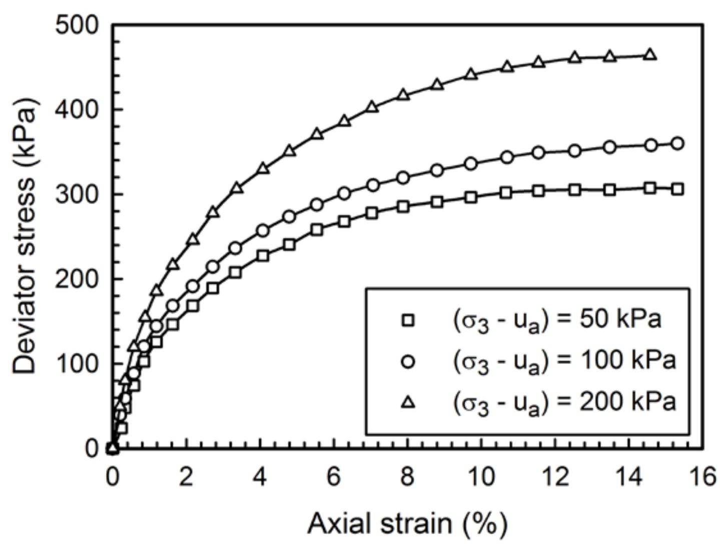
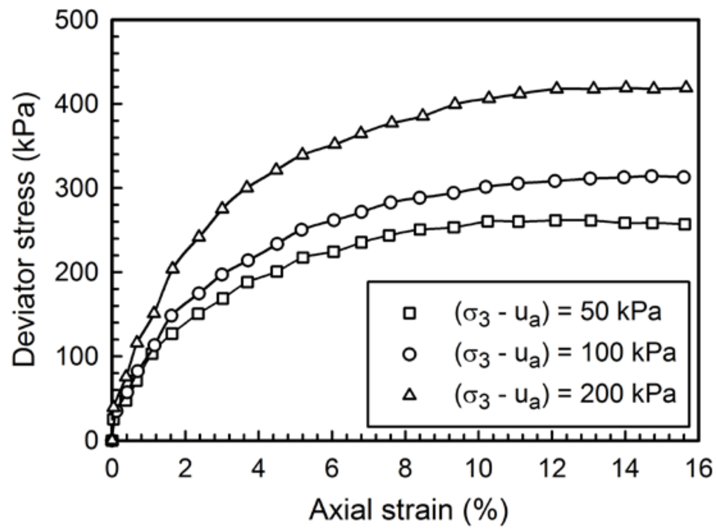


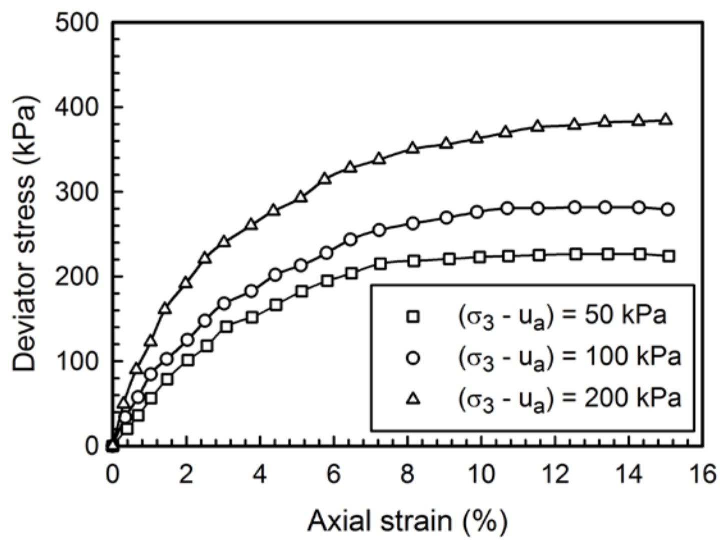
Fig. 3.10. Stress-strain curves for specimens of saturated compacted Guangxi soils (modified after Miao et al. 2007)



(a) Degree of saturation, S = 76.3%



(b) Degree of saturation, *S* = 83.5%



(c) Degree of saturation, *S* = 92.1%

Fig. 3.11. Stress-strain curves for specimens of unsaturated compacted Guangxi soils: (a) at degree of saturation of 76.3%, (b) at degree of saturation of 83.5%, and (c) at degree of saturation of 92.1% (modified after Miao et al. 2007)

3.4 Analysis of the Triaxial Tests Results

The stress-strain curves of the triaxial tests for the three expansive soils (i.e., Zao-Yang, Nanyang, and Guangxi soils) were analyzed and plotted on the transformed axes $\varepsilon/(\sigma_1-\sigma_3)$ and ε as suggested by Duncan and Chang (1970). The straight line equation

(Equation 3.6) was used to fit the data. The experimental values of modulus of elasticity were determined as the reciprocal of the intercepts of the straight lines. Figures (3.12) and (3.13) show the transformed stress-strain curves for saturated and unsaturated Zao-Yang soils specimens, respectively. The values of experimental elasticity moduli determined as the reciprocal of the intercepts of the straight lines are also shown in the Figures (3.12) and (3.13).

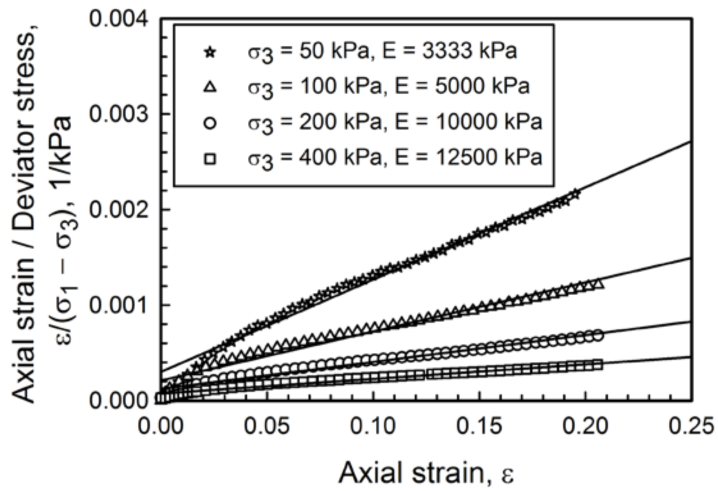
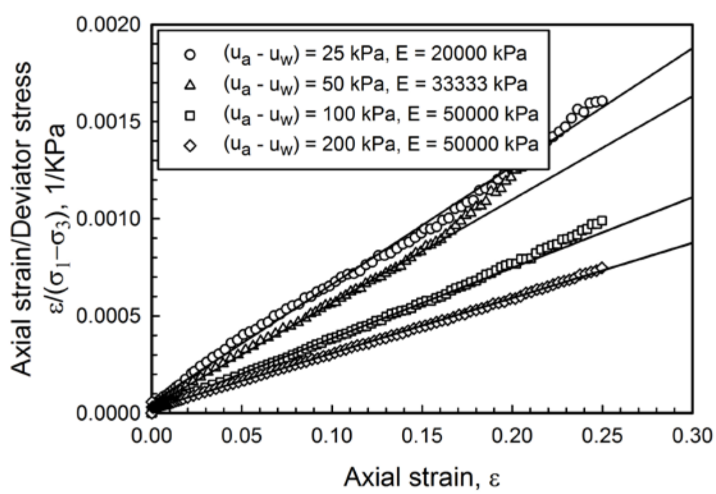
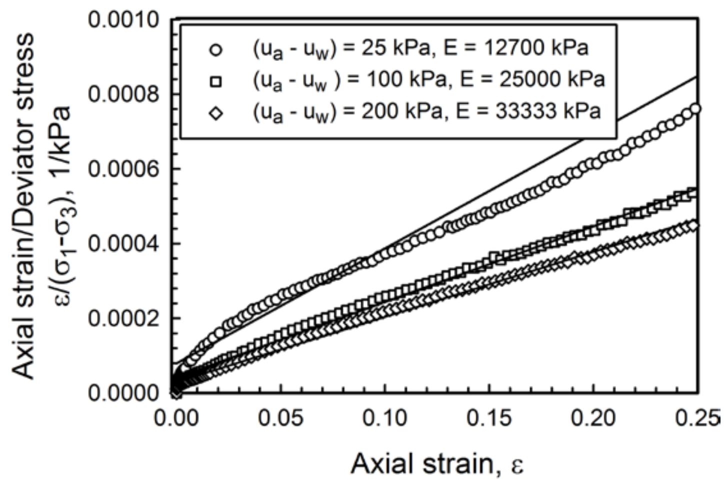


Fig. 3.12. Transformed stress-strain curve for specimens of saturated, compacted Zao-Yang soils



(a) Net confining stress = 50 kPa



(b) Net confining stress = 200 kPa

Fig. 3.13. Transformed stress-strain curves for specimens of unsaturated, compacted Zao-Yang soils: (a) at net confining stress of 50 kPa, (b) at net confining stress of 200 kPa

Review of Figure (3.12) shows that the experimental value of the saturated modulus of elasticity increases with an increase in the applied confining stress. This is consistent with Janbu's relationship (1963) (Equation 3.7), in which the soil modulus of elasticity under the saturated condition is related to the confining stress, and is nonlinearly increased with an increase in the confining stress.

$$E = K\,P_a\,(\frac{\sigma_3}{P_a})^n \tag{3.7}$$

where P_a is atmospheric pressure expressed in the same stress units as E and σ_3, K and n are fitting parameters. The Janbu's relationship (1963) (Equation 3.7) was used to fit the data of specimens of saturated, compacted Zao-Yang soils (Figure 3.14). The coefficient of determination was relatively high (R^2 = 0.97). Hence, the modulus of elasticity of saturated, compacted Zao-Yang soils at any confining stress can be estimated using equation (3.8).

$$\frac{E}{P_a} = 53.29(\frac{\sigma_3}{P_a})^{0.67} \tag{3.8}$$

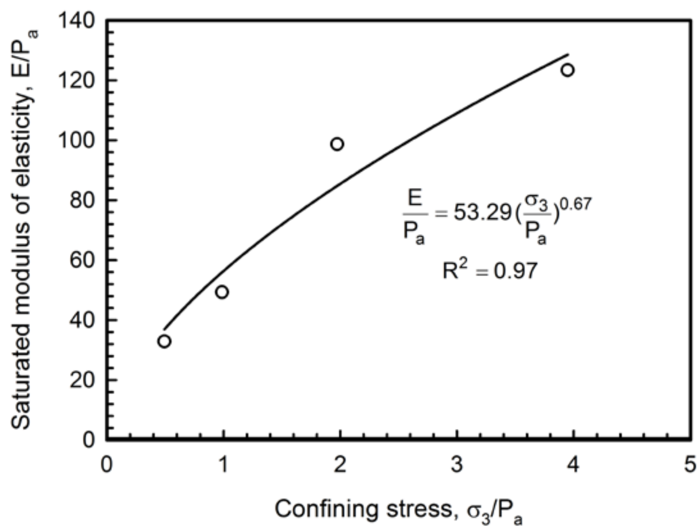


Fig. 3.14. The relationship of the saturated modulus of elasticity with the confining stress for specimens of compacted Zao-Yang soils

On the other hand, Figure (3.13) shows that the soil modulus of elasticity under an unsaturated condition increases with an increase in the matric suction $(u_a - u_w)$. This is consistent with the observations of other investigators in the literature (Fredlund and Rahardjo 1993, Costa et al. 2003, Inci et al. 2003, Lu and Likos 2006, Yang et al. 2005, Yang et al. 2008, Oh et al. 2009, Vanapalli and Oh 2010). Review of Figure (3.13) shows that some sets of data (e.g., the transformed stress-strain relationship for $(\sigma_3 - u_a)$ = 200 kPa and $(u_a - u_w)$ = 25 kPa) are not linear; such a behavior can be attributed to the stress-strain curve not following well defined hyperbolic trend with respect to its shape. Nonetheless, a straight line could be still fitted to the data. This assumption is reasonably valid as the coefficient of determination was relatively high (R^2 > 0.96). Table (3.2) summarizes the experimental values of the modulus of elasticity for Zao-Yang soils under the saturated and unsaturated conditions.

The above procedure was also applied for the other two soils (Nanyang and Guangxi soils) to determine their experimental elasticity moduli. Tables (3.3) and (3.4) summarize the experimental values of elasticity modulus for Nanyang and Guangxi soils, respectively.

Table 3.2. Experimental elasticity moduli obtained from the triaxial tests for compacted Zao-Yang soils under the saturated and unsaturated conditions (data from Zhan 2003)

Net confining stress (kPa)	Saturated modulus of elasticity (kPa)	Matric suction (kPa)			
		25	50	100	200
		Unsaturated modulus of elasticity (kPa)			
50	3333	20000	33333	50000	50000
100	5000	-	-	-	-
200	10000	12700	-	25000	33333
400	12500	-	-	-	-

Table 3.3. Experimental elasticity moduli obtained from the triaxial tests for compacted Nanyang soils under the saturated and unsaturated conditions (data from Miao et al. 2002)

Net confining stress (kPa)	Average confining stress (kPa)	Saturated modulus of elasticity (kPa)	Matric suction (kPa)			
			50	80	120	200
			Unsaturated modulus of elasticity (kPa)			
20	25	9253*	-	25000	-	50000
30		12921*	25000	-	33333	-
50	62.5	20000	33333	-	-	50000
70		25964*	-	33333	-	-
80		28982*	-	-	50000	-
100	112.5	33333	50000	-	-	50000
120		40473*	-	50000	-	-
130		43231*	-	-	50000	-
150	-	50000	-	-	-	-

* The value obtained from the best fit of the relationship between the saturated modulus of elasticity and the confining stress using Janbu's equation (1963) (Equation 3.7, the fitting parameters K = 347.47, and n = 0.82)

Table 3.4. Experimental elasticity moduli obtained from the triaxial tests for compacted Guangxi soils under the saturated and unsaturated conditions (data from Miao et al. 2007)

Net confining stress (kPa)	Saturated modulus of elasticity (kPa)	Degree of saturation (%)		
		76.3	83.5	92.1
		Unsaturated modulus of elasticity (kPa)		
50	10000	16667	20000	10000
100	10000	20000	20000	12500
200	12500	25000	25000	20000

Figure (3.9) and Table (3.3) summarize Miao et al. (2002) results of 12 triaxial tests for specimens of Nanyang soils under unsaturated conditions using different values of confining stresses and matric suctions. However, to validate Vanapalli and Oh (2010) model for estimating the modulus of elasticity of unsaturated expansive soils, at least three specimens have to be tested under the same confining stress with varying initial matric suctions. Therefore, tests conducted by Miao et al. (2002) for specimens under unsaturated conditions were divided into three groups. Every group was consisted of four tests having a close net confining stress. The net confining stress for each group was determined as an average value of the net confining stresses of the four tests in the group. Table (3.3) shows the confining stresses for the three groups used in this analysis, which are 25, 62.5, and 112.5 kPa.

3.5 Comparison between the Experimental and Predicted Values of the Modulus of Elasticity

The predicted moduli of elasticity of Zao-Yang, Nanyang, and Guangxi expansive soils were calculated using the VO model (Equation 3.1). This VO model requires the SWCC and the modulus of elasticity under the saturated condition along with the fitting parameters α and β. The saturated modulus of elasticity was determined from the stress-strain curves of the soil specimens tested under the saturated condition. The fitting parameter $\beta = 2$ was used for expansive soils regardless of the value of the plasticity index following the VO model recommendation for fine-grained soils. It was also found that $\beta = 2$ provides a reasonable estimation of the variations of soil movements with respect to time for several case studies with lightly loaded structures as will be presented later in Chapter Five. The value of the fitting parameter α was defined using a programming code for equation (3.1) that increments the value of α and calculates the predicted modulus of elasticity of unsaturated soils. The value of α was defined on the basis of the best agreement between the experimental and the predicted values of the modulus of elasticity with respect to matric suction (the coefficients of determination R^2 = 0.77–0.97). Tables (3.5)–(3.7) summarize the predicted elasticity moduli and the corresponding values of the fitting parameters α and β for Zao-Yang, Nanyang, and Guangxi expansive soils, respectively.

Table 3.5. The fitting parameters and the predicted elasticity moduli estimated using the VO model for unsaturated, compacted Zao-Yang soils

Net confining stress (kPa)	Fitting parameter values			Matric suction (kPa)			
	$1/\alpha$	α	β	25	50	100	200
				Predicted modulus of elasticity (kPa)			
50	7.4	0.135	2	14383	22526	36385	62991
200	50	0.02	2	14911	-	24690	36515

Table 3.6. The fitting parameters and the predicted elasticity moduli estimated using the VO model for unsaturated, compacted Nanyang soils

Average net confining stress (kPa)	Fitting parameter values			Matric suction (kPa)			
	$1/\alpha$	α	β	50	80	120	200
				Predicted modulus of elasticity (kPa)			
25	25	0.04	2	22215	27454	33102	44218
62.5	58.82	0.017	2	33680	38415	43521	53568
112.5	166.67	0.006	2	44122	46834	49758	55512

Table 3.7. The fitting parameters and the predicted elasticity moduli estimated using the VO model for unsaturated, compacted Guangxi soils

Net confining stress (kPa)	Fitting parameter values			Degree of saturation (%)		
	$1/\alpha$	α	β	76.3	83.5	92.1
				Unsaturated modulus of elasticity (kPa)		
50	14.29	0.07	2	17214	15867	13362
100	100	0.01	2	20306	18382	14802
200	100	0.01	2	25382	22978	18503

Tables (3.5)–(3.7) show that the predicted value of the soil modulus of elasticity increases with an increase in matric suction $(u_a - u_w)$ and net confining stress $(\sigma_3 - u_a)$ with the exception of the Zao-Yang soils specimens tested by Zhan (2003) under the net confining stress of 50 kPa having a higher elastic modulus than the specimens tested under the confining stress of 200 kPa. This exception appears to be inconsistent with the remainder of the results, and may be attributed to the effect of dilatancy of the soil at a low confining stress (i.e. $(\sigma_3 - u_a)$ = 50 kPa). During dilation, a negative pore water pressure can develop and the soil stiffness (modulus of elasticity) increases. However, the effect of dilation is negligible at high confining stresses (e.g., $(\sigma_3 - u_a)$ = 200 kPa). These results are consistent with the observations of other investigators in the literature (Zhan 2003, Vanapalli and Mohamed 2013).

Figures (3.15)–(3.17) provide the comparisons between the experimental and the predicted moduli of elasticity for Zao-Yang, Nanyang, and Guangxi soils. The results show that the soil modulus of elasticity increases with an increase in the matric suction $(u_a - u_w)$ and the net confining stress $(\sigma_3 - u_a)$ with the exception of the Zao-Yang soil data as it has been discussed above. In addition, a reasonable agreement has been observed between the experimental and the predicted values of the elasticity moduli for all the three expansive soils studied (R^2 = 0.91) (Figure 3.18). This close agreement suggests the use of the VO model (Equation 3.1) with confidence for estimating the modulus of elasticity for unsaturated expansive soils.

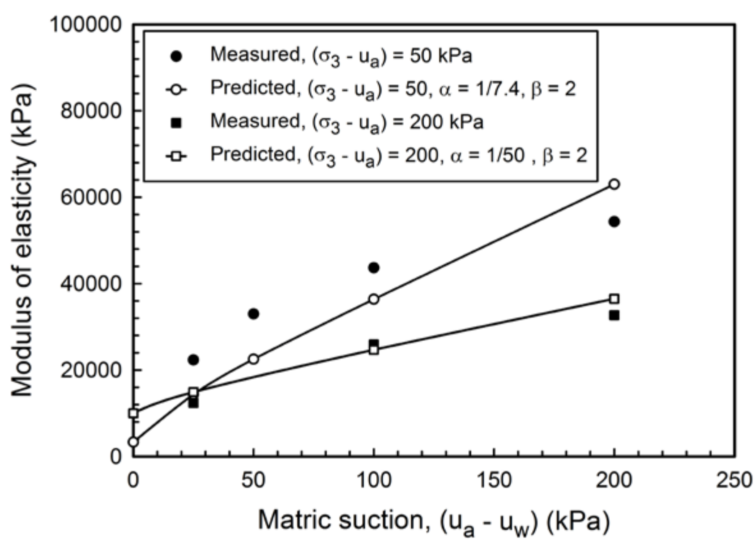


Fig. 3.15. Comparison between the experimental and predicted modulus of elasticity for Zao-Yang expansive soils

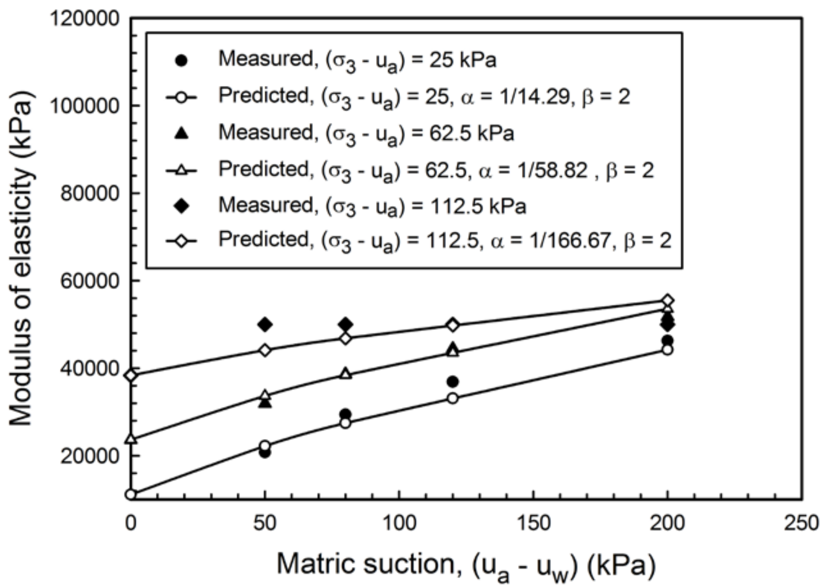


Fig. 3.16. Comparison between the experimental and predicted modulus of elasticity for Nanyang expansive soils

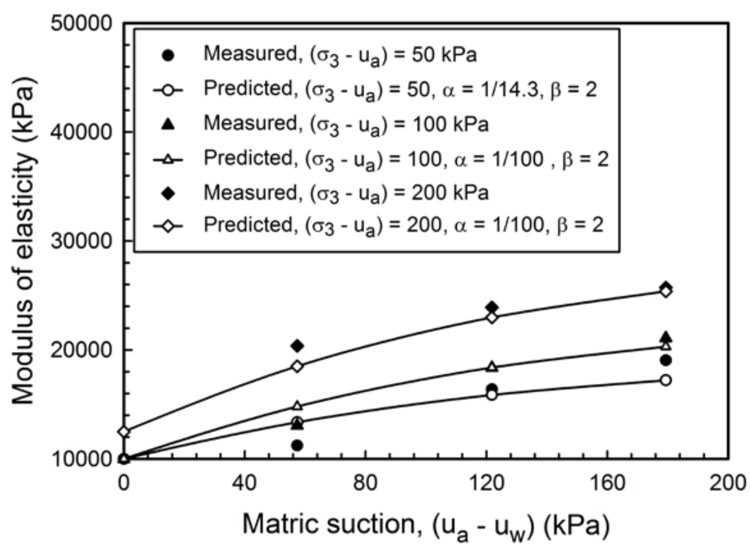


Fig. 3.17. Comparison between the experimental and predicted modulus of elasticity for Guangxi expansive soils

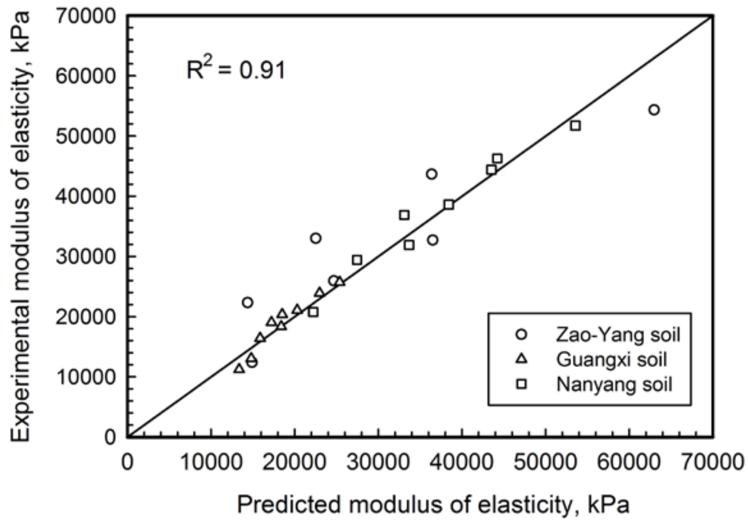


Fig. 3.18. Predicted moduli versus experimental moduli for the three investigated expansive soils (Zao-Yang, Nanyang, and Guangxi expansive soils)

3.6 The Relationship between the Plasticity Index I_P, the Net Confining Stress $(\sigma_3 - u_a)$, and the Fitting Parameter α

The values of the fitting parameter α for the three expansive soils studied, summarized in Tables (3.5)–(3.7), were plotted along with the upper and lower boundary relationships of $(1/\alpha)$ versus I_p proposed by Vanapalli and Oh (2010) for soils with plasticity index I_p lower than 16% (Figure 3.19). Review of Figure (3.19) shows that the inverse of the fitting parameter α for soils under low confining stresses are below the lower boundary relationship, and that for high confining stresses are in the range of the boundary relationships. However, there is an exception for the specimens tested by Miao et al. (2002) that have a high value of $(1/\alpha)$ when tested under the average net confining stress of 112.5 kPa.

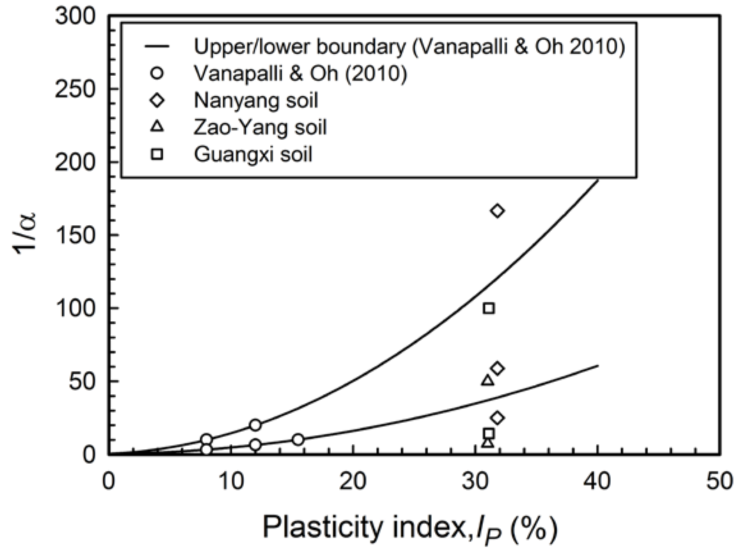


Fig. 3.19. The plot of ($1/\alpha$) versus the plasticity index I_p for the three investigated expansive soils along with the upper and lower boundary relationships of ($1/\alpha$) versus I_p proposed by Vanapalli and Oh (2010)

Vanapalli and Oh (2010) suggested that the fitting parameter α is a function of footing size and soil plasticity index I_p. Figure (3.19) shows that the value of α is also dependent on the applied net confining stress. For a soil with a certain value of I_p, the fitting parameter α may differ with the applied confining stress. Figure (3.20) shows the values of $1/\alpha$ with the associated net confining stresses for the three expansive soils under consideration. These results suggest that $1/\alpha$ may nonlinearly increase with an increase in the applied net confining stress. It is also found that the value of α between 0.05 and 0.15 with an average 0.1 provides a reasonable estimation of the modulus of elasticity for unsaturated expansive soils under a net confining stress of less than 50 kPa. This is typically the maximum overburden pressure (or the confining stress) of the soil deposit within the active zone depth (approximately 3 m) where the volume change problems of expansive soils are predominant. The values of the modulus of elasticity of unsaturated expansive soils estimated using the fitting parameters $\beta = 2$ and α = 0.05-0.15 provide reasonable predictions of the soil vertical movements with respect to time for the case studies investigated in this study (these details are presented in Chapter Five). In other words, the results of the analyses summarized in this chapter with respect to the values of the two fitting parameters (i.e., α and β) for reasonably estimating the modulus of elasticity of unsaturated expansive soils are consistent with the assumptions used in

Chapter Five for α and β values for predicting the vertical movements associated with the volume change behavior of expansive soils. This approach provides conservative estimations as the boundary restraints due to loading on the swelling behavior are not considered. Such an approach is simple for using in practice applications.

Some investigators suggest that the active zone depth for some sites may extend up to 20 ft. (~ 6 m) below the ground surface (Kalantari 2012). Nelson et al. (2001) showed, due to both soil suction and gravity, wetting extend to depths much greater than 6.1 m at sites in Denver. Diewald (2003) evaluated post-construction data from 133 investigations and determined that the depth of wetting for 7 to 10 year-old residences is approximately 12 m. Some practicing engineers in the Front Range of Colorado have used assumptions of depths of wetting of 10.4–14.0 m for their predictions of soil heave (Chao et al. 2006). For any site with deep active zone (i.e., > 3 m), the relationships between $1/\alpha$ and the net confining stress values should be established and used for a reliable estimation of soil movements using the modulus of elasticity as a tool.

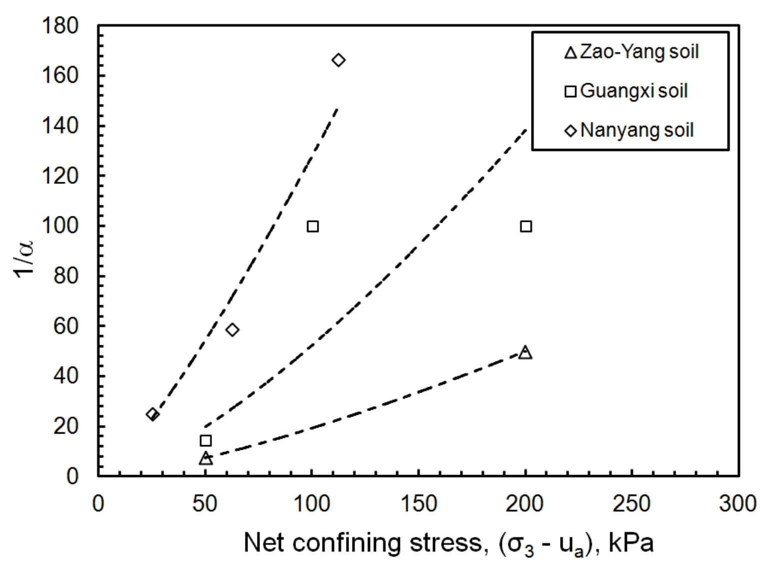


Fig. 3.20. The plot of $(1/\alpha)$ versus net confining stress $(\sigma_3 - u_a)$ for the three investigated expansive soils (Zao-Yang, Nanyang, and Guangxi expansive soils)

3.7 Summary

Vanapalli and Oh (2010) proposed a semi-empirical model (i.e., VO model) for predicting the variation of the modulus of elasticity with respect to matric suction for soils with plasticity index I_p lower than 16%. The model has been extended in this chapter to predict the modulus of elasticity of unsaturated expansive soils which, typically, have higher plasticity index. The information required for using the VO model includes the SWCC and the modulus of elasticity of the soil under the saturated condition along with two fitting parameters α and β. The experimental data of triaxial tests available in the literature for three different expansive soils were used to examine the validity of the adopted model.

The results of the study suggest that the VO model can be used for predicting the variation of the modulus of elasticity with respect to matric suction for unsaturated, compacted expansive soils (e.g., Zao-Yang, Nanyang, and Guangxi expansive soils) (the coefficient of determination R^2 = 0.91). The fitting parameter β = 2 was found to be suitable for the three expansive soils studied in this chapter. The fitting parameter α is related to the net confining stress $(\sigma_3 - u_a)$. However, to provide a generalized relationship for the fitting parameter α, more triaxial test results for different expansive soils with different plasticity index I_p tested under a large range of confining stresses and matric suctions are required. The results presented in this chapter are encouraging for use of the VO model in the modeling studies to reasonably predict the variation of soil movements with respect to time (see Chapter Five).

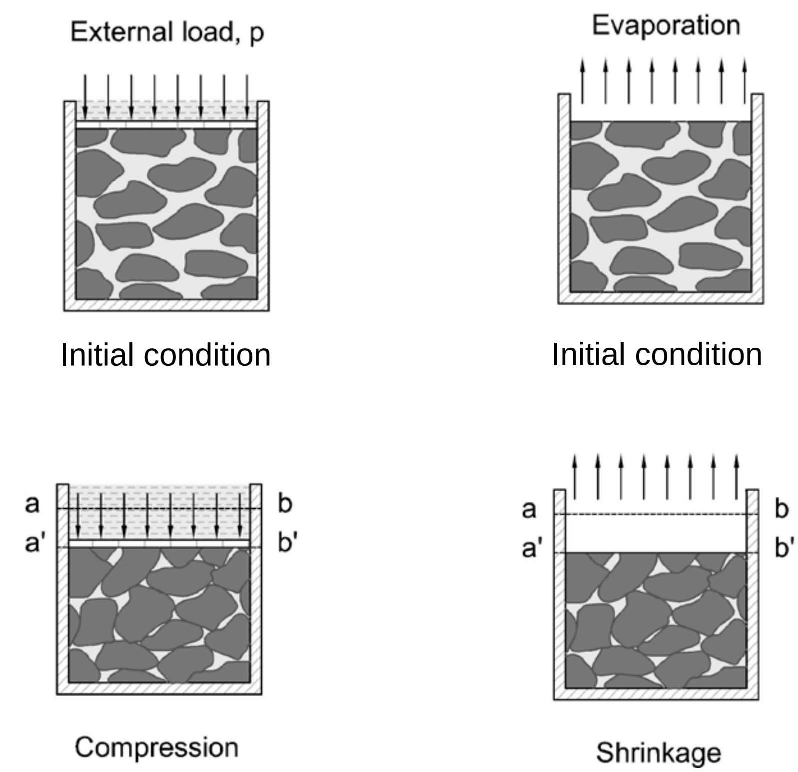
CHAPTER 4

PROPOSED APPROACH FOR PREDICTING VERTICAL MOVEMENTS OF EXPANSIVE SOILS

4.1 Introduction

Significant research has been undertaken since the last century to better understand the heave/shrink behavior of expansive soils. Terzaghi's (1925, 1926, and 1931) pioneering modeling and experimental studies showed that clay swelling and shrinkage are essentially elastic deformations caused by the clay's affinity for water. Terzaghi (1926) investigated the mechanics of the swelling of a gelatin gel, as a model for clay, using the thermodynamic principles. Empirical relationships were proposed considering many parameters that influence the swelling behavior of gel such as the concentration of gel, the size of gel micropores, and the temperature. The swelling pressure of the gel was found to be merely due to the elastic expansion of the solid phase, previously held under compression by the surface tension of the water. The swelling pressure represents the "free energy" of the system and can be entirely converted into mechanical work. The heat developed in connection with a change in the free energy with unrestricted expansion is exclusively due to liquid friction as the gel expands. In addition, Terzaghi (1931) explained the fundamental swelling behavior of a two-phase system of liquid and solid (i.e., water and soil) using two different scenarios as examples. In the first scenario, the initial water content w_0 of the submerged two-phase system was reduced to a value w_1 by applying external pressure p per unit area. The volume was reduced by $aa' - bb'$ (Figure 4.1a). In the second scenario, the water content was reduced to w_1 by drying instead of applying the external pressure p (Figure 4.1b); assuming that no air has invaded into the system (i.e., two-phase system). The volume reduction in first scenario was due to compression by load (i.e., consolidation), and it was associated with shrinkage by evaporation in the second scenario. When the applied external pressure p was removed in the first scenario, water flowed into the system accompanied by swelling. To initiate swelling in the second scenario, the surface $a' - b'$ of the system represented in Figure (4.1b) was flooded with water. In both scenarios, swelling started with identical water content w_1 and particles arrangement. In the first scenario, a load of p per unit area of surface was necessary to prevent the material from expanding. In the second scenario, there must be a force of equal intensity (as in the first scenario) acting on the surface $a' - b'$ in the system. This force is exerted by the surface tension of the capillary water (i.e., suction). The difference between the pressure in the external water (free water) and in the interstitial water causes water to flow through the surface into the two-phase system until the influence of hydraulic gradient ceases. In other words, the system expands until the suction becomes zero. The water flow is independent of the physiochemical reaction inside the two-phase system. This leads to a conclusion that the physiochemical effect could not have had any influence on the swelling behavior of the system.

Terzaghi (1931) suggested that the most important factor that contributes to swelling is the negative pressure (suction) associated with the capillary water in the interconnected pores of the clay macrostructure. Terzaghi (1925, 1931) also pointed out that the soil swelling produced by eliminating the surface tension of the capillary water (suction) was identical with the expansion produced by the removal of the external load. It was explained that any soil that is capable of swelling contains a solid phase under a pressure equal to the tension in the liquid phase. Hence, the swelling capacity of soil is dependent on the elastic properties of the solid phase of the soil. This is a fundamental concept and can be extended to all expansive soils. However, the physiochemical reactions between the solid and the liquid phases (formation of adsorption compounds within the system) can at best only play a minor role.



(a) Reducing the water content by applying external load

(b) Reducing the water content by drying

Fig. 4.1. Capillary pressure and swelling process (modified after Terzaghi 1931)

As per Terzaghi's (1925, 1926, and 1931) explanation of the swelling process of soil, a simple approach for predicting the time-dependent soil movements (heave/shrink) of natural expansive soils beneath lightly loaded structures is proposed in this research study. The proposed approach depends on the variations of the soil modulus of elasticity with respect to matric suction; it is therefore referred to as the modulus of elasticity based method (MEBM). The matric suction variations within the active zone of soil profile are simulated using the soil-atmosphere interaction model VADOSE/W. The soil vertical movements (heave/shrink) over time are estimated based on the volume change constitutive relationship for soil structure considering the variation of modulus of elasticity with respect to matric suction. This chapter details the assumptions and the fundamental concepts of the proposed MEBM along with the step-by-step procedural details for predicting the time-dependent movement of natural expansive soils.

4.2 Constitutive Relationship for Estimating Expansive Soil Movements over Time

Fredlund and Morgenstern (1976) proposed two volume change constitutive relationships as shown in equations (2.14) and (2.15) for soil structure and water phase, respectively. A total of four volume change coefficients (m_1^s, m_2^s, m_1^w, and m_2^w) were defined to completely describe the volume-mass soil properties under any set of stress conditions (Vu and Fredlund 2004).

$$d\varepsilon_v = \frac{dV_v}{V_0} = m_1^s d(\sigma_{mean} - u_a) + m_2^s d(u_a - u_w) \qquad (2.14)$$

$$d\theta_w = \frac{dV_w}{V_0} = m_1^w d(\sigma_{mean} - u_a) + m_2^w d(u_a - u_w) \qquad (2.15)$$

where ε_v is volumetric strain, θ_w is volumetric water content, (dV_v/V_0) is total volume change, (dV_w/V_0) is water volume change, $(\sigma_{mean} - u_a)$ is net normal stress, $(u_a - u_w)$ matric suction, m_1^s is coefficient of volume change with respect to net normal stress, m_2^s is coefficient of volume change with respect to matric suction, m_1^w is coefficient of water volume change with respect to net normal stress, and m_2^w is coefficient of water volume change with respect to matric suction. However, when predicting soil deformation, a change in soil volumetric strain is of main interest. Therefore, only the constitutive relationship of volume change for soil structure (Equation 2.14) is required (Fredlund et al. 1980).

Following Terzaghi (1925, 1926, 1931), and other researchers (e.g., Biot 1941, Coleman 1962, Matyas and Radhakrishna 1968, Barden et al. 1969, Aitchison and Woodburn 1969, Brackley 1971, Aitchison and Martin 1973, Fredlund and Morgenstern 1976 and 1977, Vu and Fredlund 2004 and 2006, Zhang 2004, Zhang and Briaud 2010), a simple method for predicting the vertical movements of expansive soils over time is proposed in this study based on an assumption that expansive soils are elastic in nature for a large range of loading conditions. This assumption is considered valid since it is reasonable to assume that any unsaturated expansive soil has experienced the maximum wetness and dryness in the past. In other words, if an expansive soil has some plasticity, its contribution could have been eliminated by a long history of wetting-drying cycles. This may be one of the key reasons why most expansive soils are usually heavily overconsolidated. Since the volume change behavior of expansive soil is influenced by the mechanical stress, one may argue that the soil will yield under a combination of mechanical stress and matric suction variations. However, for pavements and lightly loaded structures, where the proposed prediction method (MEBM) can be used, the mechanical stress due to the repeated traffic or the superstructure loading is not significant and will not cause soil yielding (Fredlund et al. 1980).

Assuming soils behave as incrementally isotropic, linear elastic materials, the volume change coefficients m_1^s, and m_2^s can be related to the elastic moduli E and H associated with a change in the net normal stress and a change in the matric suction, respectively. For an unsaturated soil under a general, three-dimensional loading condition, $m_1^s = 3(1-2\mu/E)$, and $m_2^s = 3/H$, where μ is Poisson's ratio; thus, the soil structure constitutive relationship can be rewritten as equation (2.12) (Fredlund and Rahardjo 1993)

$$d\varepsilon_v = 3(\frac{(1-2\mu)}{E})d(\sigma_{mean} - u_a) + \frac{3}{H}d(u_a - u_w) \quad (2.12)$$

If the soil is subjected to an increase in the matric suction, the soil volume will be the same as long as the soil remains saturated. Once the soil commences desaturation, the changes in the matric suction and the mechanical stress will affect the volume change behavior. However, as mentioned above, the influence of the mechanical stress in lightly loaded structures is insignificant in several scenarios and can be neglected. Such an assumption is conservative and can be extended in practice. In other words, the matric suction can be regarded as the only key stress state variable contributing to the soil volume change. The constitutive relationship for the soil structure (Equation 2.12) reduces to equation (4.1)

$$d\varepsilon_v = \frac{3}{H} d(u_a - u_w) \tag{4.1}$$

Equation (4.1) suggests that the matric suction changes have a direct bearing on the volume change of unsaturated expansive soils as per the earlier discussions on the swelling process provided by Terzaghi (1925, 1926, 1931). The soil swelling merely represents elastic expansion produced by lowering of the capillary pressure (suction) (Terzaghi 1925, 1926, 1931).

The movements of expansive soils associated with the changes in environmental factors often occur near the ground surface within the active zone depth. Hence, the soil movements may be assumed to be predominant in the vertical direction, and the loading condition can be assumed to be the K_0-loading. The volumetric strain $d\varepsilon_v$ is equal to the vertical strain $d\varepsilon_y$ while the soil is not permitted to deform laterally (i.e., $d\varepsilon_x$ = $d\varepsilon_z$ = 0). The volume change coefficient with respect to matric suction is $m_2^s = (1+\mu)/(H(1-\mu))$. Equation (4.1) can therefore be written in terms of the vertical strain as follows

$$d\varepsilon_y = \frac{(1+\mu)}{H(1-\mu)} d(u_a - u_w) \tag{4.2}$$

The elastic moduli H and E of unsaturated soils vary significantly with the stress state variables (i.e., the mechanical stress and the matric suction), and the elastic modulus H can be related to E and μ (Wong et al. 1998, Zhang et al. 2012) as

$$H = E/\left(1-2\mu\right) \tag{4.3}$$

Equation (4.3) has two unknowns (H and E) since Poisson's ratio μ is usually estimated or measured in the laboratory through the use of triaxial tests with the measurement of lateral strain. The relationship between H and E may be more complex for soils in a state of unsaturated condition; however, equation (4.3) that is valid for saturated soils has been applied to unsaturated soils in the present study extending the assumptions suggested by Wong et al. (1998), Zhang et al. (2012), and Geo-Slope (2007).

Substituting for H (Equation 4.3) into equation (4.2) provides equation (4.4) that relates the vertical strain in terms of the matric suction change, the associated modulus of elasticity for the soil structure E, and Poisson's ratio μ.

$$d\varepsilon_y = \frac{(1+\mu)(1-2\mu)}{E(1-\mu)} d(u_a - u_w) \tag{4.4}$$

The summation of the vertical strain changes for each an increment provides the final vertical strain of the soil.

$$\varepsilon_y = \sum d\varepsilon_y \tag{4.5}$$

To calculate the vertical movement of expansive soils with respect to time, the soil profile within the active zone are subdivided into several layers. The vertical movement for each arbitrary layer (i.e., i^{th} layer) Δh_i associated with the time increment is computed by multiplying the vertical strain ε_y at the mid-layer for the time increment with the thickness of the layer h_i.

$$\Delta h_i = h_i \left[\frac{(1+\mu)(1-2\mu)}{E(1-\mu)} \Delta(u_a - u_w) \right]_i \tag{4.6}$$

where $\Delta(u_a - u_w)$ is change in soil suction for each time increment.

The vertical movement of each soil layer at a given time is the cumulative value of the vertical movement increments prior to that given time. The total vertical movement Δh at any point in the soil profile is the summation of the vertical movements of n layers beneath.

$$\Delta h = \sum_{i=1}^{n} \Delta h_i = \sum_{i=1}^{n} \left(h_i \left[\frac{(1+\mu)(1-2\mu)}{E(1-\mu)} \Delta(u_a - u_w) \right]_i \right) \tag{4.7}$$

Since it is necessary to define a value for Poisson's ratio, equation (4.7) infers that the matric suction variations in the active zone and the associated modulus of elasticity are the key parameters to calculate the vertical soil movements (heave/shrink) over time. The matric suction variations within the active zone are simulated using the soil-atmosphere interaction model VADOSE/W, while the corresponding values of unsaturated modulus of elasticity are calculated using the VO model (Equation 3.1).

4.3 Key Parameters for Predicting the Expansive Soil Movements

4.3.1 Matric suction variations

The matric suction profile within the active zone is a representation of a state of balance of the environmental factors and the soil-water storage processes. The potential change in matric suction is generally attributed to many reasons such as the environmental conditions, human imposed irrigation, influence of vegetation, and accidental wetting due to broken pipelines. Since even small changes in soil suction may cause a significant amount of volume change, the soil suction has been considered as a more sensitive indicator and a predominant stress variable for the determination of soil movements.

To estimate the soil vertical movements over time using the proposed MEBM (Equation 4.7), the in-situ matric suction changes and the expected depth of these changes (i.e., the active zone depth) due to the variations of environmental factors are required. Several commercial programs such as SoilCover (Unsaturated Soils Group 1996), HYDRUS-2D (Simunek et al. 1999), UNSAT-H (Fayer 2000), VADOSE/W (Geo-Slope 2007), and SVFlux (SoilVision System Ltd. 2007) are available for estimating the matric suction profiles considering both the water flow in unsaturated soils and the soil-atmospheric interactions (i.e., infiltration, precipitation, surface water runoff and ponding, plant transpiration, actual evaporation, and heat flow). Such techniques are valuable, simple, and economical in comparison with the direct measurements of in-situ matric suction which are mostly unreliable.

The finite element program VADOSE/W (Geo-Slope 2007), a product of Geo-studio, is used in this research study as a tool to simulate the soil-atmospheric interactions and the water flow through unsaturated soils, and then to estimate the corresponding changes in matric suction over time. The program couples the heat transport and mass (i.e., water and vapor) transfer in unsaturated soils, together with the water and energy balance, to provide a direct and complete evaluation of soil water storage and matric suction. Critical to the formulation of VADOSE/W is its ability to predict the actual evaporation as a function of climate data applied as an upper boundary condition using the rigorous Penman-Wilson method (Wilson 1990). It has been established that VADOSE/W is capable of simulating the saturated and unsaturated flow behavior where the complex soil-atmosphere interaction is of particular interest.

VADOSE/W is primarily a *2-D* package but can be used for a *1-D* problem through the use of appropriate geometries and boundary conditions. Unsaturated flow problems with atmospheric interactions are often conducted in *2-D* when the soil surface is sloping or the layering in the profile promotes multi-dimensional flow. The VADOSE/W program requires the following input information: (*i*) material properties, namely the soil-water characteristic curve (SWCC), the coefficient of permeability function (*k* function), the soil thermal conductivity, the mass, and the specific heat capacity, (*ii*) climate data that includes the daily precipitation, the maximum and minimum daily temperature, the maximum and minimum daily relative humidity, the average daily wind speed, and the net radiation, (*iii*) vegetation data which involves the leaf area index (LAI), the plant moisture limiting point, the root depth and length in the growing season, and (*iv*) geometrical boundary conditions including the location of the ground water table.

For a proper simulation of unsaturated flow, a correct description of the boundary and initial conditions is important. Different types of boundary conditions are included in VADOSE/W to simulate various problems. The boundary conditions which are generally applied at the bottom of the soil profile include unit gradient condition (e.g., gravity driven flux = the actual hydraulic conductivity at the bottom of the domain), seepage face (e.g., flux = the saturated hydraulic conductivity when the boundary is saturated; otherwise flux = 0), prescribed head, or prescribed flux. An atmospheric flux boundary condition is another type of boundary conditions which is usually applied at the surface to simulate the atmospheric interactions. Infiltration occurs during precipitation at a rate governed by the hydraulic properties of the soil profile, and precipitation exceeding the infiltration capacity is assumed to be run off. Evaporation is assumed to occur from the soil surface and is bounded by the potential evaporation (*PE*) rate. For *2-D* simulations, the prescribed head and prescribed flux boundaries are often applied along the sides of the domain. A detailed description of different boundary conditions used in VADOSE/W is presented in Geo-Slope (2007).

The initial condition is also required for the simulation of the transient water flow through unsaturated soils, which usually includes the initial values of total head, soil temperature, and gas concentration at each nodal point within the soil profile. When this data is not available, a value of zero for the gas concentration and the soil temperature can be assumed as an initial value. However, the hydraulic initial condition state cannot be left out, and should be specified by reading the data from a file created in a separate analysis of the initial condition, by drawing the initial water table position, or by specifying the initial value as a material property.

The output of VADOSE/W includes the soil temperature, degree of saturation, water content, and, most importantly, matric suction fluctuations over time. Details of VADOSE/W simulations for the case studies used in this research are discussed in the sequent chapter.

4.3.2 Soil modulus of elasticity associated with matric suction

Matric suction may be sufficient as the sole independent variable in describing the modulus of elasticity for unsaturated soils (Fredlund and Rahardjo 1993, Costa et al. 2003, Inci et al. 2003, Lu and Likos 2006, Yang et al. 2008). The soil modulus of elasticity is significantly influenced due to the contribution of matric suction. Typically, as matric suction becomes bigger, the modulus of elasticity becomes higher.

The semi-empirical model presented in Chapter Three (i.e., VO model) (Equation 3.1) is used in the proposed MEBM for estimating the modulus of elasticity of unsaturated expansive soils associated with the change in matric suction. The information required for using the VO model include the SWCC and the saturated modulus of elasticity along with the two fitting parameters β and α. As described in Chapter Three, the VO model was validated using the experimental data of triaxial tests for three different expansive soils. Comparisons were provided between the values of the modulus of elasticity obtained from the triaxial tests and the VO model. The adopted VO model well estimates the modulus of elasticity of unsaturated expansive soils. The values of the fitting parameters of β = 2 and α = (0.05-0.15) with an average 0.1 were recommended for expansive soils. In the MEBM approach, the vertical soil movements (heave/shrink) are predicted using equation (4.7) considering the variation of modulus of elasticity with respect to matric suction estimated from the VO model (Equation 3.1). The strength of the VO model lies in its use of conventional soil properties.

4.4 Step-by-Step Procedure of the Proposed MEBM Approach

Figure (4.2) shows the step-by-step procedure of the MEBM for predicting the vertical movement of unsaturated expansive soils with respect to time. The proposed approach requires the soil matric suction variations within the active zone depth and the corresponding modulus of elasticity. The soil matric suction variations are simulated using the soil-atmosphere interaction model VADOSE/W. The corresponding values of the soil modulus of elasticity are estimated using the VO model (Equation 3.1). The soil movements (heave/shrink) over time are then predicted using equation (4.7).

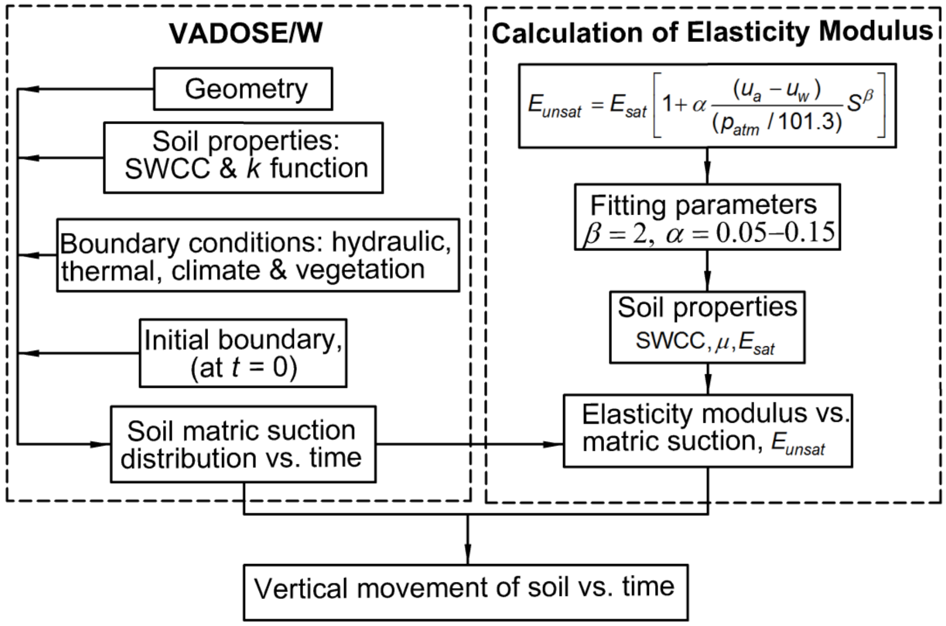


Fig. 4.2. Flowchart for the step-by-step procedure of the MEBM

To simulate the matric suction changes for each time increment in response to atmospheric conditions, the investigated soil profile for a given site may be modelled using the transient analysis with the *2-D* VADOSE/W program. Besides the soil properties, the initial and the boundary conditions of the site model are applied over the period of simulation. The matric suction variations within the active zone depth can be predicted as a response to a changing surface boundary over time. Once the matric suction variations are determined, the vertical soil movements at any depth can be predicted. The investigated soil profile is divided into a number of sub-layers based on the soil properties and the considered locations. The total soil movement at a specific location for a given time is computed by adding the movements of all layers up to the specific location for that time (Equation 4.7).

Five different case studies for studying the volume change behavior of expansive soils, from different countries that include Canada, China, and the United States, are used in this research study for testing the validity of the proposed MEBM (see the sequent Chapter). Those case studies include:

- **Case Study A** is a slab-on-ground placed on Regina expansive clay subjected to a constant infiltration rate over 175 days, which was originally modeled by Vu and Fredlund (2006).

- **Case Study B** is a typical light industrial building in north-central Regina, Saskatchewan, Canada, constructed by the Division of Building Research, National Research Council in 1961. The history of the site and the details of testing and monitoring programs are presented in Yoshida et al. (1983). The prediction of soil heave to investigate problems associated with the construction of the building was carried out by Vu and Fredlund (2004) over 150 days.

- **Case Study C** is a test site in Regina, Saskatchewan, Canada, modeled by Ito and Hu (2011) over a 1-year period to investigate the performance of water pipe lines in Regina expansive clays. Various factors influencing the soil movements such as climate condition, vegetation, watering of lawn and structural impact are considered in the modeling.

- **Case Study D** is a comprehensive field study of a cut slope in an expansive soil in Zao-Yang, Hubie, China, which was instrumented by Ng et al. (2003) to investigate the performance of the expansive soil slope over a period of one month. The effects of soil cracks, daily climate data, along with two artificial rainfall events are investigated.

- **Case Study E** is a field experiment in Arlington, Texas, USA, conducted by Briaud et al. (2003) for measuring the shrink and swell movements of four full-scale spread footings over a period of 2 years.

4.5 Summary

The modulus of elasticity based method (MEBM) is proposed for predicting the vertical movements of unsaturated expansive soils with respect to time using the modulus of elasticity as a tool. In summary, the proposed MEBM is a three step-procedure as shown in Figure (4.3). The first step is to determine the matric suction changes for each time increment. The second step is to determine the modulus of elasticity that corresponds to the matric suction value. The last step is to estimate the variation of the vertical soil movements with respect to time.

To illustrate the step-by-step procedure of the MEBM and its validation for the prediction of the vertical movements of expansive soils over time, the details of simulation of the case studies under consideration are presented in the next chapter.

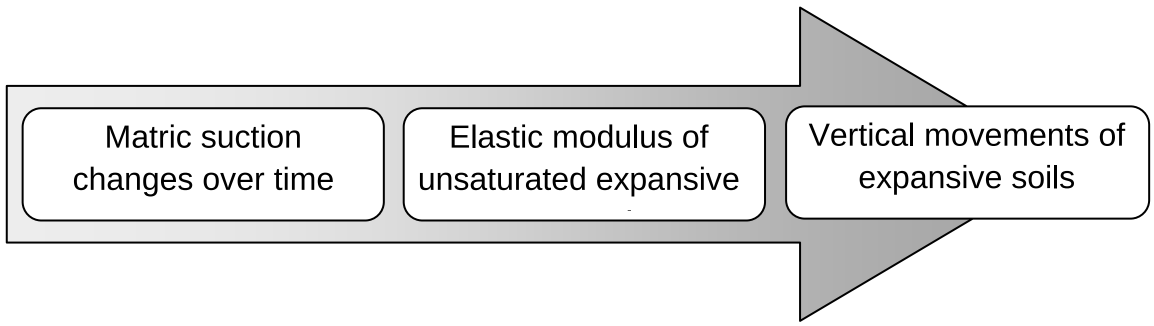


Fig. 4.3. Three step-procedure of the MEBM

CHAPTER 5

VALIDATION OF THE PROPOSED MODULUS OF ELASTICITY BASED METHOD

5.1 Introduction

The Modulus of Elasticity Based Method (MEBM) is proposed in this research study for the prediction of the volume change movement of expansive soils over time. The MEBM is tested in this chapter for its validity in several case studies collected and gathered from the literature. The vertical soil movements associated with the volume change of expansive soils for each case study are predicted using the MEBM as presented and detailed in the preceding chapter. The pore air pressure is assumed to be equal to the atmospheric pressure for all situations. Therefore, the net normal stress is equivalent to the net total stress and the matric suction is equivalent to the absolute value of the negative pore water pressure.

The following activities are carried out for each of the investigated case studies:

- Simulation of the matric suction variations over time using VADOSE/W program. The program models the water flow through unsaturated soils and the soil-atmospheric interactions over time, and then estimates the corresponding changes in soil matric suction within the active zone depth.

- Estimation of the modulus of elasticity of unsaturated expansive soils using Vanapalli and Oh (2010) model (i.e., VO model). The VO model was described and validated in Chapter Three.

- Prediction of the vertical soil movements over time using the simplified volume change constitutive relationship for soil structure (Equation 4.7). This is the first time that a simplified constitutive relationship for the volume change of soil structure has been used to estimate the soil movements in terms of the matric suction changes and the corresponding values of the modulus of elasticity.

The details of soil movement calculations using the MEBM approach, and the analysis and the discussion of the results of the MEBM for the case studies under consideration are presented in this chapter.

5.2 Case Studies

The five case studies investigated in this chapter are summarized in Table (5.1), and labelled as Case Study A, Case Study B, Case Study C, Case Study D, and Case Study E for simplicity. These case studies are used to check the validity of the MEBM for the estimation of the vertical soil movements (i.e., volume change behavior) of natural expansive soils with respect time. The case studies are chosen to be representative of a variety of site conditions from different regions of the world that include Canada, China, and the United States. Several scenarios with different boundary conditions are used to simulate each of these case studies. The predicted vertical movements of the five case studies using the proposed MEBM are compared to the measured/estimated results that are published in the literature.

Table 5.1. Case studies simulated using the proposed MEBM

Case study/ Reference	Description	Period of simulation
Case Study A (Vu and Fredlund 2006)	A problem example of a slab-on-ground placed on Regina expansive clay subjected to a constant infiltration rate.	175 days
Case Study B (Yoshida et al. 1983, Vu and Fredlund 2004)	A light industrial building in north-central Regina, Saskatchewan, Canada, subjected to a leakage from a water line below a floor slab.	150 days
Case Study C (Ito and Hu 2011)	A test site in Regina, Saskatchewan, Canada. Various factors influencing the soil	One year

	movements (such as climate changes, vegetation, watering of lawn, and structural impact) have been considered in the simulation.	
Case Study D (Ng et al. 2003)	A cut-slope in an expansive soil in Zao-Yang, Hubie, China, subjected to daily climate data with two artificial rainfall events. The effects of soil cracks and environmental conditions on the soil movements have been investigated.	One month
Case Study E (Briaud et al. 2003)	A field site in Arlington, Texas, USA, with four full-scale spread footings. Factors that influence the shrink and swell movements of expansive soils such as the daily weather, and the vegetation have been considered for this field construction site.	Two years

5.3 Case Study A: a Slab-on-Ground Placed on Regina Expansive Clay Subjected to a Constant Infiltration Rate (Vu and Fredlund 2006)

Case Study A is an example problem of volume change of unsaturated expansive soils used to validate the MEBM. Case Study A was originally modeled by Vu and Fredlund (2006), considering 5 m thick deposit of Regina expansive clay that was partially covered with a slab (i.e., lightly loaded structure). An infiltration of 2×10^{-8} m/s was imposed at the ground surface around the structure over a period of 175 days (Figure 5.1). The soil-water characteristic curve (SWCC) for Regina clay was given by Vu (2002) that fits the experimental data measured by Shuai (1996). The soil permeability function (*k* function) is estimated in this study using the software VADOSE/W, based on the input information of the saturated coefficient of permeability (0.00523 m/day = 6.053×10^{-8} m/s) and the SWCC given by Vu (2002). Figure (5.2) shows the SWCC and the permeability function used for Case Study A. The Regina expansive clay properties are presented in Table (5.2).

The heave estimation of Case Study A was performed in three steps as shown in Figure (4.3). First, the changes in soil matric suction arising from the infiltration alongside the slab were simulated using VADOSE/W. Second, the values of the unsaturated modulus of elasticity associated with the changes in matric suction were estimated using the VO model. Finally, the response of the unsaturated expansive soil during the infiltration (i.e., soil heave) was calculated from the simplified volume change constitutive equation for soil structure (Equation 4.7).

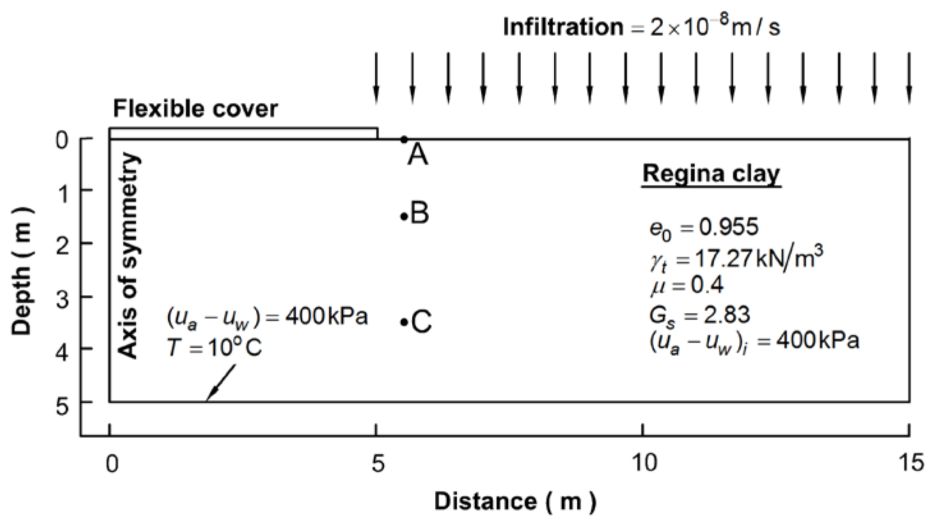


Fig. 5.1. Geometry and boundary conditions of Case Study A (modified after Vu and Fredlund 2006)

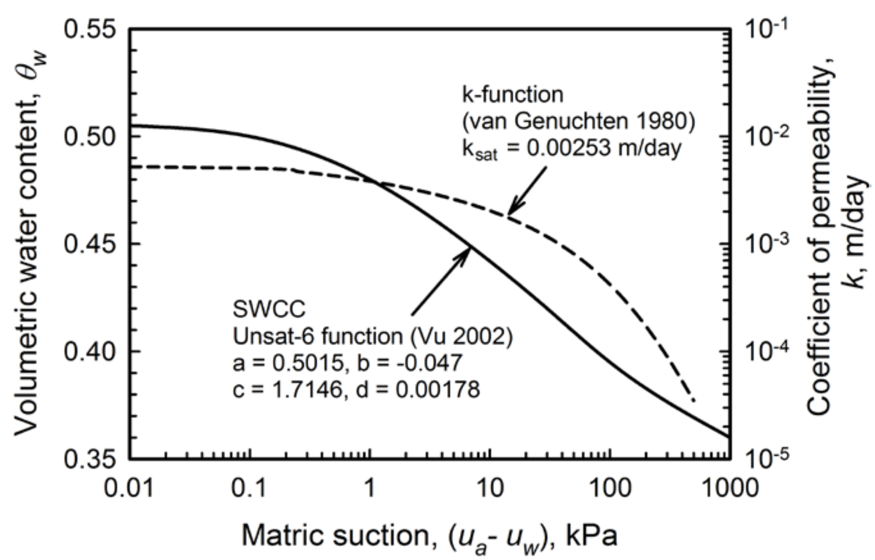


Fig. 5.2. Hydraulic characteristics of Regina expansive clay used for Case Study A (SWCC data obtained from Vu 2002)

Table 5.2. Mechanical properties of Regina expansive clay for Case Study A (modified after Shuai 1996)

Soil properties	Values
Atterberg limits	w_l = 69.9%, w_p = 31.9%, I_p = 38%
Unified Soil Classification System	CH, Inorganic clay of high plasticity
Specific gravity	G_s = 2.83
Maximum dry unit weight	γ_{dmax} = 14.01 kN/m^3
Optimum water content	w_{optm} = 28.5%
Swelling index	C_s = 0.088
Corrected swelling pressure	P_s = 300 kPa

5.3.1 Simulation of matric suction changes over time

The simulation of matric suction variations in Regina clay underlying the slab-on-ground was implemented using VADOSE/W over a period of 175 days. Case Study A was modeled as a *2-D* problem considering the transient isothermal analysis (i.e., the temperature in the soil was assumed to be constant, T = 10 ^{o}C). The infiltration of 2×10^{-8} m/s was imposed at the ground surface around the structure over 175 days. Figure (5.1) shows the initial and the boundary conditions of the simulation. For comparison purposes, the initial and the boundary conditions assumed by Vu and Fredlund (2006) for modeling Case Study A were used in the present simulation. A matric suction value of 400 kPa was applied along the bottom boundary during the infiltration. This was achieved by specifying a pressure head of -40.787 m (i.e., -400 kPa / 9.807 kN/m^3) at the bottom boundary. The initial matric suction in the soil mass was assumed to be equal to 400 kPa.

Vu and Fredlund (2006) modeled the soil heaves at three points *A*, *B*, and *C* at depths of 0 m, 1.5 m, and 3.5 m, respectively, located to the right of the outer edge of the slab as shown in Figure (5.1). The same points were also modeled in this study using the MEBM. Figure (5.3) shows the changes of matric suction with respect to time at the locations *A*, *B*, and *C*. Figure (5.4) shows the variations of matric suction profiles at the right of the outer edge of the slab in response to the infiltration over the period of simulation.

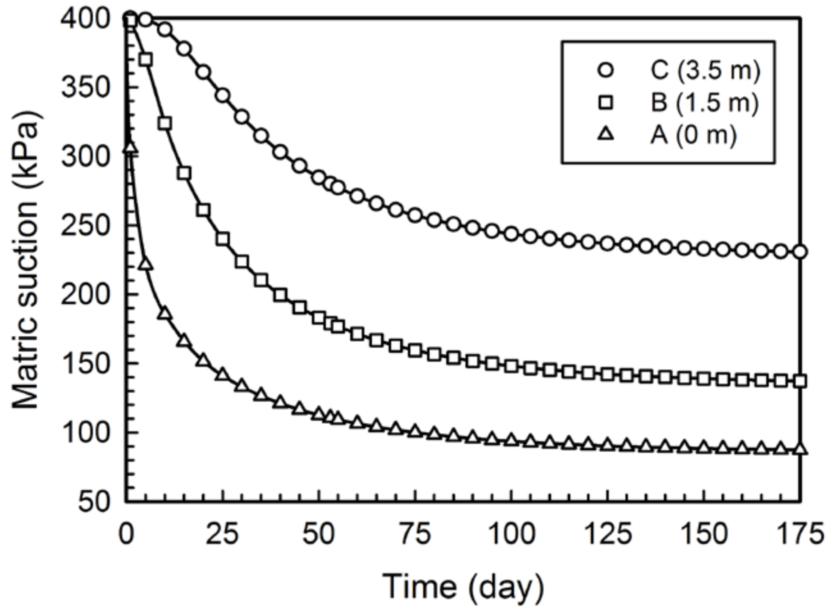


Fig. 5.3. Matric suction changes with time for the three locations *A*, *B*, and *C*

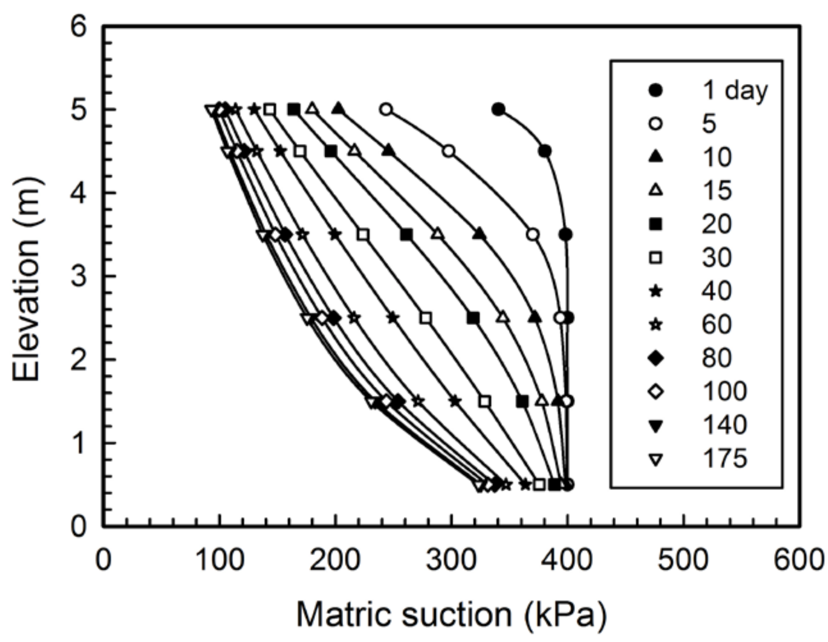


Fig. 5.4. Matric suction profiles for various elapsed times at the right of the outer edge of the slab

5.3.2 Estimation of soil modulus of elasticity associated with matric suction

The soil modulus of elasticity E and Poisson's ratio μ are the two soil property parameters required for the prediction of soil heave (see Equation 4.7). The Poisson's ratio μ does not significantly change with matric suction for expansive soils as they desaturate at a very slow rate. Vu and Fredlund (2006) calculated the value of Poisson's ratio μ in terms of the coefficient of earth pressure at rest K_0 from the following relation

$$\mu = \frac{K_0}{1+K_0} \tag{5.1}$$

Lytton (1994) presented typical values for the coefficient of earth pressure at rest K_0 which were back calculated based on field observations of soil heave and shrinkage. The coefficient of earth pressure at rest of 0.67, suggested for wetting conditions when cracks in the soil are essentially closed, was used for this case study (Case Study A). Thus, the Poisson's ratio μ = 0.4 was calculated using equation (5.1).

The modulus of elasticity of unsaturated soils is not constant but is a function of both the mechanical stress and the matric suction. The VO model (Equation 3.1) was extended in Chapter Three to estimate the soil modulus of elasticity for unsaturated expansive soils. The information required for using the model includes the SWCC and the two fitting parameters (i.e., α and β) along with the saturated modulus of elasticity E_{sat}. The value of the fitting parameters β = 2 recommended in Chapter Three for expansive soils was used for this case study. The fitting parameter α = 1/9 was assumed to provide reasonable comparisons between the predicted heave using the MEBM and Vu and Fredlund (2006) method.

The saturated modulus of elasticity E_{sat} of Regina expansive clay for the present case study can be calculated based on the constitutive surface for soil structure in terms of the soil void ratio e as shown in equation (5.2) (Zhang 2004, Vu and Fredlund 2006).

$$E_{sat} = \frac{3(1-2\mu)(1+e_0)}{(\partial e / \partial(\sigma - u_a))} \tag{5.2}$$

where $(\sigma - u_a)$ is net normal stress, and e_0 is initial void ratio.

Vu and Fredlund (2006) proposed equation (5.3) to fit the void ratio constitutive surface for unsaturated expansive soils with six fitting parameters *a*, *b*, *c*, *d*, *f*, and *g* as follows

$$e = a + b\log\left[\frac{1 + c(\sigma - u_a) + d(u_a - u_w)}{1 + f(\sigma - u_a) + g(u_a - u_w)}\right] \tag{5.3}$$

where $(u_a - u_w)$ is matric suction, and *a*, *b*, *c*, *d*, *f*, and *g* are fitting parameters of the void ratio constitutive surface for Regina expansive clay (Equation 5.3) which are shown in Table (5.3).

Table 5.3. Fitting parameters of the void ratio constitutive surface for Regina expansive clay (Vu and Fredlund 2006)

Fitting parameters of equation (5.3)					
a	*b*	*c*	*d*	*f*	*g*
1.2492	-0.0979	4.8240	3.3330	0.0009	0.0012

When the soil is saturated, equation (5.3) can be written as a function of only the net normal stress $(\sigma - u_a)$ as follows:

$$e = a + b\log\left[\frac{1 + c(\sigma - u_a)}{1 + f(\sigma - u_a)}\right] \tag{5.4}$$

Zhang (2004) also proposed equation (5.5) to fit the relationship of void ratio versus net normal stress for saturated soils.

$$e = y_1 + \frac{a_1}{1 + \exp\left(-\frac{\log(\sigma - u_a) - x_1}{b_1}\right)} \tag{5.5}$$

where a_1, b_1, x_1, and y_1 are fitting parameters. In this analysis, equation (5.5) has been used to fit the void ratio constitutive relationship for Regina expansive clay proposed by Vu and Fredlund (2006) (Equation 5.4). The fitting parameters of equation (5.5) a_1, b_1,

x_1, and y_1 were obtained to be equal to 0.5085, -1.3154, 1.1773, and 0.8128, respectively. Figure (5.5) shows the oedometer test results conducted by Shuai (1996) for Regina expansive clay along with the two fitting equations (Equations 5.4 and 5.5). The two best-fit equations are coincident as shown in Figure (5.5). Either equation (5.4) or equation (5.5) can be used to calculate the saturated modulus of elasticity of Regina expansive clay.

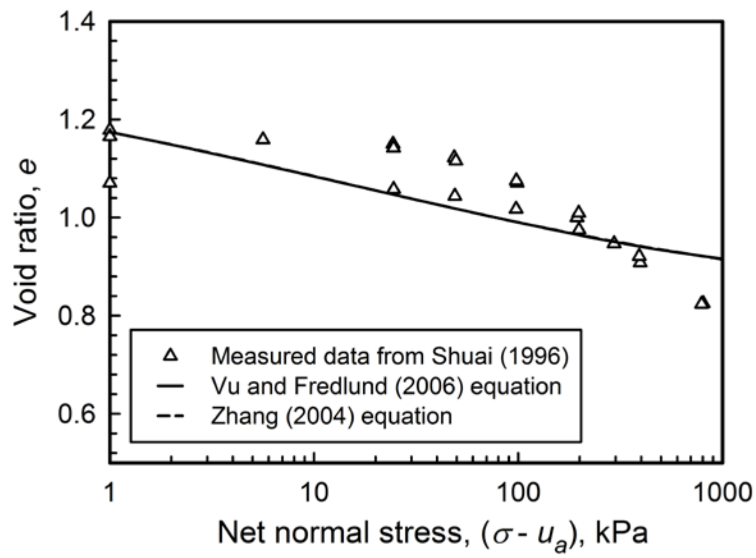


Fig. 5.5. Oedometer test results for Regina expansive clay along with the best fit equations

The calculation of the saturated modulus of elasticity E_{sat} using equation (5.2) requires the derivative of the relationship of void ratio versus net normal stress (Equation 5.4 or Equation 5.5). The derivative of equation (5.5) is as follows (Zhang 2004)

$$\frac{de}{d((\sigma-u_a)} = -\frac{a_1\left[1+\exp\left(-\frac{\log(\sigma-u_a)-x_1}{b_1}\right)\right]^{-2}\exp\left(-\frac{\log(\sigma-u_a)-x_1}{b_1}\right)}{(\sigma-u_a)b_1\ln 10} \tag{5.6}$$

Then, the saturated modulus of elasticity can be calculated by combining equations (5.2) and (5.6). The average value of the saturated modulus of elasticity of Regina expansive clay for Case Study A was calculated to be about 1100 kPa for the initial void ratio e_0 = 0.955, and Poisson's ratio μ = 0.4.

Vu and Fredlund (2004) suggested that the saturated modulus of elasticity of soil can also be calculated directly from the volume change index with respect to net normal stress (swelling index) C_s. The saturated modulus of elasticity E_{sat} can be expressed as a function of the swelling index C_s, the initial void ratio e_0, and Poisson's ratio μ, which can be written for the K_0 loading condition as follows

$$E_{sat} = \frac{2.303(1+\mu)(1-2\mu)(1+e_0)}{C_s(1-\mu)}(\sigma - u_a) \quad (5.7)$$

For Regina expansive clay having e_0 = 0.955, C_s = 0.088, and μ = 0.4, equation (5.7) can be written as

$$E_{sat} = 23.876(\sigma - u_a) \quad (5.8)$$

The average value of the saturated modulus of elasticity of Regina expansive clay for Case Study A was also obtained from equation (5.8) to be 1100 kPa. Table (5.4) summarizes the soil properties used for Case Study A.

Table 5.4. Soil properties for Case Study A (modified after Vu and Fredlund 2006)

Soil properties	Values
Total unit weight	γ_t = 17.27 kN/m^3
Initial void ratio	e_0 = 0.955
Swelling index	C_s = 0.088
Poisson's ratio	μ = 0.4
Saturated modulus of elasticity	E_{sat} = 1100 kPa
Saturated coefficient of permeability	k_{sat} = 0.00523 m/day
Saturated volumetric water content	θ_s = 0.5015
Initial matric suction	$(u_a - u_w)_i$ = 400 kPa

Equation (3.1) was then solved for each time increment over the simulation period of Case Study A, for the matric suction $(u_a - u_w)$, the degree of saturation *S*, the saturated modulus of elasticity E_{sat}, and the fitting parameters β = 2 and α = 1/9, to calculate the unsaturated modulus of elasticity E_{unsat} over time.

$$E_{unsat} = E_{sat}\left[1+\alpha\frac{(u_a-u_w)}{P_a/101.3}S^{\beta}\right] \tag{3.1}$$

5.3.3 Prediction of soil heave over time

The water infiltration into an unsaturated expansive soil leads to a decrease in matric suction, which contributes to soil volume change predominantly in the vertical direction (*1-D* heave). For Case Study A, to evaluate the *1-D* heave at any depth over 175 days, the 5 m depth of soil profile was divided into five equal layers of 1 m thickness. Based on the estimated matric suction changes within the soil profile (see Section 5.3.1), the total heave at any depth for a certain time was computed. The day to day changes of the matric suction values estimated using VADOSE/W were substituted into the volume change constitutive relationship for soil structure (Equation 4.6) to calculate the *1-D* heave of each layer Δh_i associated with the time increment (i.e., a day).

$$\Delta h_i = h_i\left[\frac{(1+\mu)(1-2\mu)}{E(1-\mu)}\Delta(u_a-u_w)\right]_i \tag{4.6}$$

where $\Delta(u_a - u_w)$ is change in matric suction for each time increment, and h_i the thickness of the i^{th} layer.

The heave of each soil layer at a given time was then calculated as the cumulated value of the heaves of the soil layer for all days prior to that given time. The total heave at any depth Δh was obtained from the summation of the heave of *n* layers beneath (Equation 4.7).

$$\Delta h = \sum_{i=1}^{n}\Delta h_i = \sum_{i=1}^{n}\left(h_i\left[\frac{(1+\mu)(1-2\mu)}{E(1-\mu)}\Delta(u_a-u_w)\right]_i\right) \tag{4.7}$$

Figure (5.6) shows the comparison of soil heaves predicted using the proposed MEBM with the numerical modeling results published in Vu and Fredlund (2006) at the three locations *A*, *B*, and *C* shown in Figure (5.1).

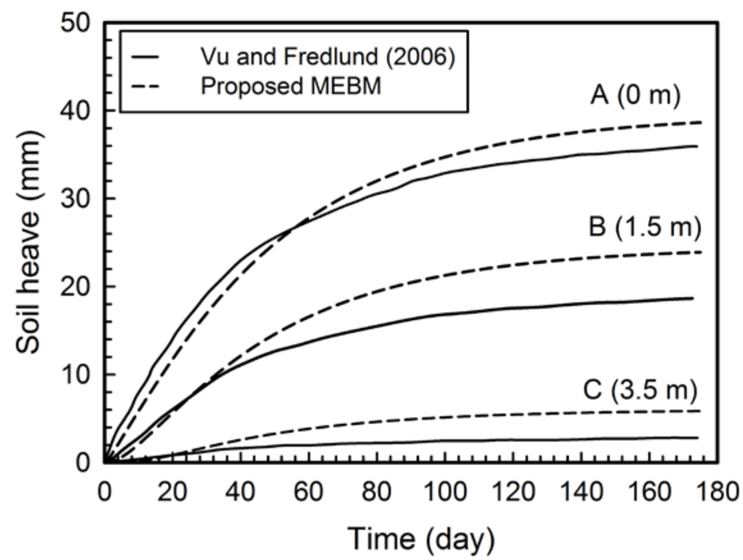


Fig. 5.6. Comparison between the predicted heaves using the proposed MEBM and Vu and Fredlund (2006) method at three locations *A*, *B*, and *C*

5.3.4 Analysis and discussion

The matric suction variations were evaluated at the three locations (*A*, *B*, and *C*) for the period of simulation (175 days) using VADSOE/W. Review of Figure (5.3) shows that the initial matric suction of 400 kPa decreases with time. The matric suction at the ground surface (at *A*) has a lower value compared to the other two locations (*B* and *C*). This reflects the effect of the water infiltration which is accompanied by a reduction in matric suctions. The infiltration influences are primarily in the upper soil layers near the ground surface, which contribute to significant changes in matric suction. Figure (5.4) shows the variations of matric suction profiles at the right of the outer edge of the slab in response to the infiltration during the period of simulation. This figure highlights the effect of the infiltration on the soil matric suction as discussed earlier.

Based on the estimated matric suction changes over time, the soil heave was calculated with respect to time at the three locations *A*, *B*, and *C*. Figure (5.6) shows the comparison between the values of the *1-D* heaves predicted using the proposed MEBM and the numerical modeling results of Vu and Fredlund (2006) for Case Study A. The total heave increased with a decrease in matric suction with most of the heave occurring in the first 60 days. The coefficient of determination between the results of the MEBM and Vu and Fredlund (2006) method was relatively high (R^2 = 0.97). However, the predicted heaves using the MEBM are slightly higher than the numerical modeling results of Vu and Fredlund (2006). The reason for this difference may be attributed to the proposed MEBM procedure which considers the *1-D* heave, whereas the numerical modeling results of Vu and Fredlund (2006) represents the soil heave in *2-D*.

5.4 Case Study B: a Light Industrial Building in North-Central Regina, Saskatchewan, Canada (Yoshida et al. 1983, Vu and Fredlund 2004)

Case Study B, a light industrial building constructed on an expansive soil deposit in Regina, Saskatchewan, Canada, has been also used in this study for testing the validity of the MEBM for the prediction of soil heave over time. The building was constructed by the Division of Building Research, National Research Council, in 1961 as a part of a comprehensive field study program to investigate the problems associated with construction in/on expansive soils in the Regina area of Saskatchewan. One year after construction, heaving and cracking in a floor slab were noticed by the building owner. The owner also noticed an unexpected loss of 35 m^3 of water. This amount of water loss was traced to a leak in a hot-water line beneath the floor slab (Yoshida et al. 1983). The maximum heave observed on the slab was found to be 106 mm. Figure (5.7) shows the geometry and the boundary conditions for Case Study B. A 2.3 m thick deposit of Regina expansive clay was considered for the estimation of *1-D* heave. This depth was considered to be equivalent to the active zone depth beyond which there will be no tendency for swelling. It was assumed that water leaked from the pipe line along a 2 m length (Yoshida et al. 1983) (see Figure 5.7). The matric suction was relatively high close to the surface and decreased with depth. The matric suction through the soil profile dissipated with time and reached steady state condition, in which the matric suction under the center of the slab was equal to 20 kPa, in about 150 days (Yoshida et al. 1983).

The initial and the boundary conditions assumed by Vu and Fredlund (2004) for modeling Case Study B are also used in the present simulation. Figure (5.8) presents the SWCC and the coefficient of permeability function (*k* function) given by Vu and Fredlund (2004). The SWCC was estimated using Fredlund and Xing (1994) equation while the permeability function was obtained from Leong and Rahardjo (1997) equation.

The soil properties used for the case history analysis are summarized in Table (5.5). The three steps of the MEBM, as stated previously, including the simulation of matric suction changes over time, the estimation of unsaturated elasticity moduli associated with the changes in matric suction, and the prediction of soil heave with respect to time, were applied on Case Study B for various elapsed times (i.e., 5, 20, 50, 100 days, and at the steady state condition).

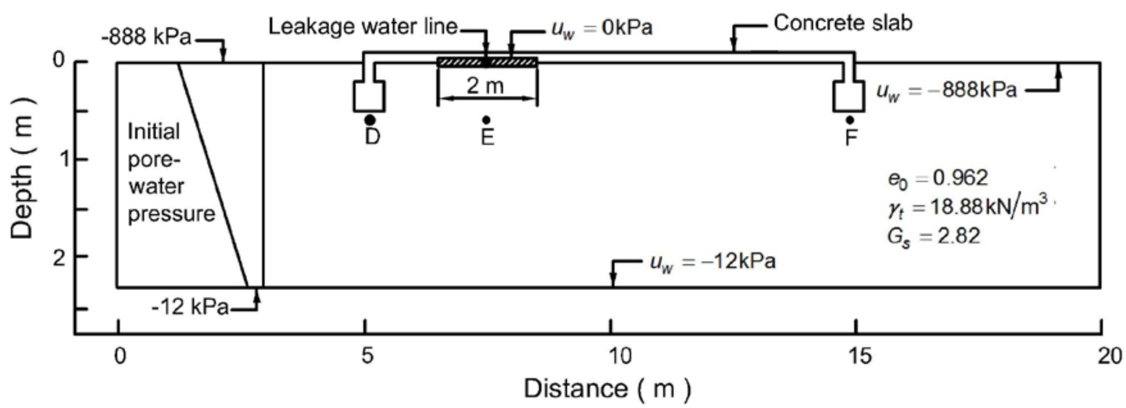


Fig. 5.7. Geometry and boundary conditions of Case Study B (modified after Vu and Fredlund 2004)

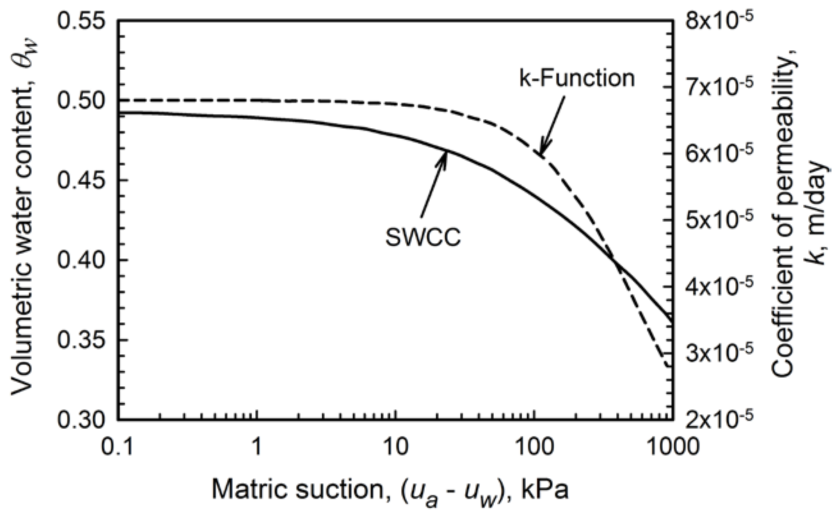


Fig.5.8. Hydraulic characteristics of unsaturated Regina expansive clay for Case Study B (modified after Vu and Fredlund 2004)

Table 5.5. Soil properties for Case Study B (Vu and Fredlund 2004)

Soil properties	Values
Atterberg limits	w_l = 77%, w_p = 33%, I_p = 44%
Specific gravity	G_s = 2.82
Total unit weight	γ_t = 18.88 kN/m^3
Initial void ratio	e_0 = 0.962
Swelling index	C_s = 0.09
Saturated coefficient of permeability	k_{sat} = 6.8×10^{-5} m/day
Saturated volumetric water content	θ_s = 0.493

5.4.1 Simulation of matric suction changes over time

The simulation of matric suction changes over time for Case Study B was carried out using the VADOSE/W program. The case study was simulated as a *2-D* problem considering the transient isothermal analysis to solve the system of equations for the unsaturated flow. The boundary conditions for the matric suction simulation are presented in Figure (5.7). The leak along the 2 m length of the hot-water line beneath the floor slab was represented by specifying zero pressure head at the desired line nodes. A "no flow" natural boundary condition on the floor slab was applied by default in VADOSE/W to represent the slab as an impervious layer. A matric suction of 12 kPa was applied along the bottom boundary during the period of simulation (150 days), which was achieved by specifying a pressure head of -1.223 m (i.e., -12/9.807) at the bottom boundary. A matric suction of 888 kPa (i.e., -90.52 m pressure head) was applied along the top boundary around the slab. The soil temperature was assumed to be constant (T = 10 ^{o}C); in other words, the temperature effects were omitted from the simulation (i.e., transient isothermal analysis). The initial matric suction given by Vu and Fredlund (2004) as a function of depth was used to represent the initial condition of the analysis (see Figure 5.7).

Vu and Fredlund (2004) predicted the changes of matric suction with time at 1.3 m depth at three locations *D*, *E*, and *F* shown in Figure (5.7). The changes of matric suction for the same locations were also obtained from VADSOE/W. Figure (5.9) shows the comparison of the predicted matric suction values over time at *D*, *E*, and *F* using VADSOE/W with those published in Vu and Fredlund (2004). Figure (5.10) presents the comparison of the predicted matric suction profiles under the center of the slab for various elapsed times (i.e., 5, 20, 50, 100 days, and at the steady state condition).

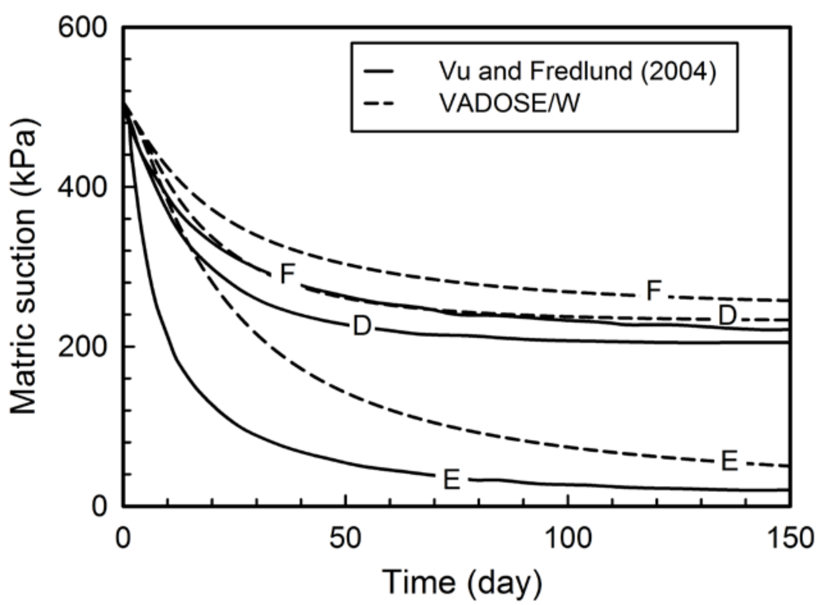


Fig. 5.9. Matric suction changes with time for the three locations *D*, *E*, and *F*

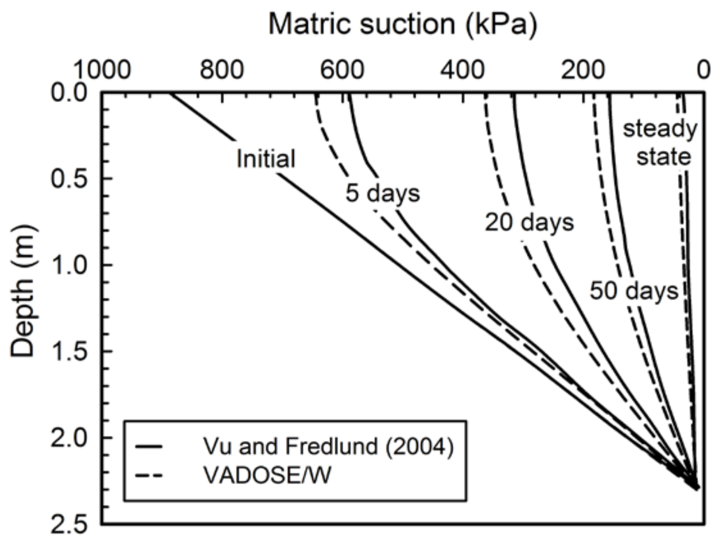


Fig. 5.10. Matric suction profiles for various elapsed times under the center of the slab

5.4.2 Estimation of soil modulus of elasticity associated with matric suction

The soil modulus of elasticity associated with the day to day changes of matric suction was estimated using the VO model (Equation 3.1). The Poisson's ratio μ of 0.4, suggested by Vu and Fredlund (2006) for the coefficient of earth pressure at rest K_0 of 0.667, was used in this analysis. The recommended value of the fitting parameter β = 2 was used for this case study. The fitting parameter α was assumed to be 1/13 in order to provide a reasonable comparison between the predicted and the measured soil heaves. The saturated modulus of elasticity E_{sat} was calculated based on the analysis of the oedometer tests results for Regina expansive clay as discussed in section (5.3.2). Equation (5.2) was combined with the best-fit equation of the void ratio constitutive relationship for the saturated Regina expansive clay (Equation 5.4 or Equation 5.5) to calculate the saturated modulus of elasticity E_{sat}. In addition, equation (5.7) was also used to calculate the saturated modulus of elasticity in terms of the swelling index C_s. The average value of the saturated modulus of elasticity of Regina expansive clay for Case Study B was calculated to be equal to 550 kPa. The variations of the unsaturated modulus of elasticity E_{unsat} with respect to matric suction over the period of simulation was estimated using equation (3.1).

5.4.3 Prediction of soil heave over time

To evaluate the soil heave resulting from the leakage in the water line below the floor slab, the 2.3 m depth of the soil (depth of matric suction variations) was subdivided into six equal layers of 0.3 m thickness and a bottom layer of 0.5 m thickness. The day to day changes of the matric suction value estimated using the VADOSE/W program were substituted into the volume change constitutive relationship (Equation 4.6) to calculate the soil layer heave for each day. The total heave at a certain depth for a given time was computed by adding the heave associated with that time for all layers below the considered depth (Equation 4.7). Figures (5.11) and (5.12) show the predicted soil heaves using the MEBM under the center of the slab and along the surface of the slab, respectively, compared with both the measurements and the numerical modeling results published in Vu and Fredlund (2004).

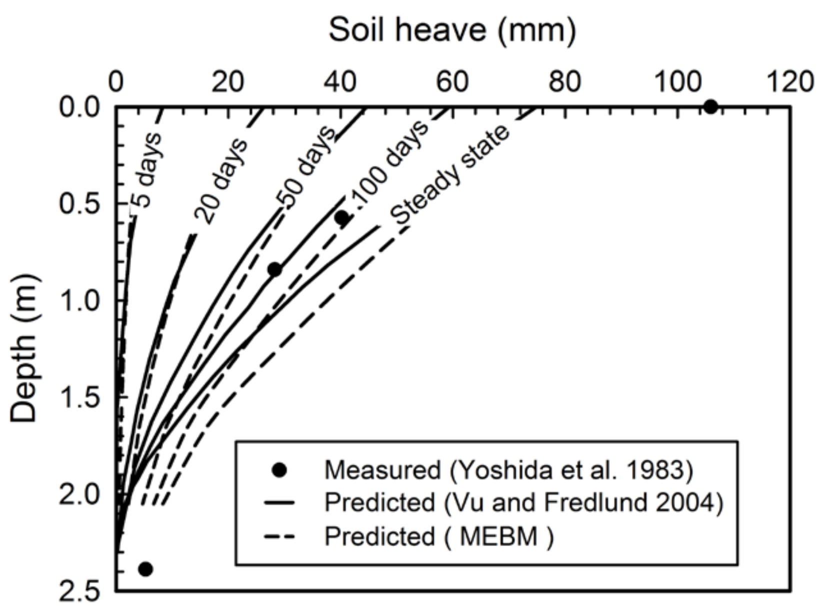


Fig. 5.11. Predicted and measured soil heave profiles under the center of the slab

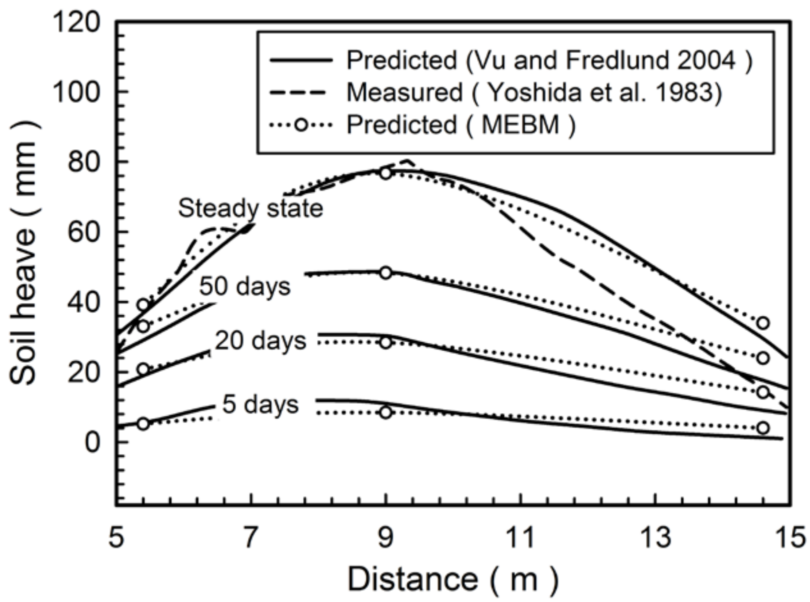


Fig. 5.12. Predicted and measured soil heave values along the surface of the slab

5.4.4 Analysis and discussion

Figure (5.9) shows the matric suction variations with respect to time at the three locations *D*, *E*, and *F*. It can be seen from Figure (5.9) that the initial matric suction significantly decreases with time and approaches a steady state condition in about 150 days. A reasonable agreement was observed between the matric suction values obtained from VADOSE/W and estimated by Vu and Fredlund (2004). The coefficient of determination was relatively high (R^2 > 0.89). However, the predicted matric suction at *E* (below the leakage position) obtained from the VADSOE/W program is slightly higher compared to the matric suction values predicted by Vu and Fredlund (2004) at the same location. The reason for this difference may be attributed to the use of different software to simulate the matric suction variations. VADOSE/W was used in the present study while FlexPDE2 (the general-purpose partial differential equation solver) was used by Vu and Fredlund (2004). Also, the differences may be due to the effect of the boundary condition that has been used to represent the leakage (i.e., u_w = 0). Review of Figure (5.9) also shows that the matric suction at *E* has a lower value compared to the other two locations *D* and *F*. This reflects the effect of wetting which is accompanied by a reduction in matric suction. Figure (5.10) shows the variations of matric suction profiles at the center of the slab in response to the wetting over the period of simulation. The results of VADOSE/W have a good match with the estimations of the matric suction variations from Vu and Fredlund (2004) method (R^2 > 0.97). The wetting primarily influences in the upper soil layers near the ground surface, which contributes to significant changes in matric suction.

Figure (5.11) shows a close agreement between the soil heave under the centre of the slab estimated using the proposed MEBM and Vu and Fredlund (2004) method with high coefficient of determination (R^2 > 0.98). The total heave increases with a decrease in matric suction, with most of the heave occurring in the upper soil layers near the ground surface where the changes of soil suction are relatively high. The heaves measured by Yoshida et al. (1983) have been also plotted on the same graph (Figure 5.11). Some differences between the predicted and the measured heave can be observed. The heave values measured at depths of 0.58 and 0.85 m correspond to the predicted heave at 100 days. Figure (5.12) compares the predicted soil heaves from the MEBM, the predicted soil heaves from Vu and Fredlund (2004) method, and the measured total heave at the surface of the slab. The soil heave predicted using the MEBM agrees well with the published data (measurements/estimates) (R^2 > 0.94).

5.5 Case Study C: a Test Site in Regina, Saskatchewan, Canada (Ito and Hu 2011)

The validity of the MEBM for predicting the vertical soil movement (i.e., heave/shrink) with respect to time is tested in an additional test site (Case Study C), taking account of the soil-environment interactions. The test site of Case Study C was previously modeled by Ito and Hu (2011) as a part of a program of studying the performance of asbestos cement (AC) water mains in Regina, Saskatchewan, Canada. Ito and Hu (2011) predicted the vertical movements of the site soil based on the matric suction data predicted from a soil-atmosphere coupled model by applying one year’s climate data (1 May, 2009–30 April, 2010). Various factors that influence the soil movements such as climate, vegetation, watering of lawn, and soil cover type were considered in modeling the case study. However, Ito and Hu (2011) approach requires oedometer tests which are costly and tedious for determining the elasticity parameters functions required for the soil–displacement analysis.

Case Study C was modelled in this section using the MEBM approach which, as previously illustrated, integrates the simplified constitutive relationship for soil structure along with the soil-atmospheric interactions model to estimate the vertical movements with respect to time. The estimated values of matric suction, volumetric water content, and vertical movement of expansive soils at different depths obtained from the MEBM approach were compared with the published results of Ito and Hu (2011). The description of the investigated test site and the details of its modeling analysis using the proposed MEBM approach are presented below.

5.5.1 Site description

The test site is located in a residential area with a high water main breakage rate in the city of Regina, Saskatchewan, Canada. It includes a park area with thick grass of 100 mm and a wide paved road with 150 mm thick asphalt pavement (Figure 5.13). The stratigraphy of the site consists of 6.4 m of highly plastic clay (w_l varies from 70 to 94% with I_p of 40 to 65%), 1.8 m of silt, and 6.8 m of till as shown in Figure (5.14). The choice of thickness and soil properties for each layer was guided by field observations from Vu et al. (2007). Figures (5.15) and (5.16) show the soil-water characteristic curve (SWCC) and the permeability function (*k* function) for each soil (Ito and Hu 2011). The properties for each soil used in modeling the soil-environment interactions for this case study are summarized in Table (5.6).

Table 5.6. Soil properties used for Case Study C (Ito and Hu 2011)

Soil properties	Clay	Silt	Till
Dry unit weight, γ_d (kN/m^{3})	12.0	13.8	15.1
Initial void ratio, e_0	1.2	0.9	0.7
Saturated coefficient of permeability, k_{sat} (m/s)	9×10^{-9}	7.56×10^{-6}	10×10^{-10}
Saturated volumetric water content, θ_s	0.56	0.48	0.42
Swelling index, C_s	0.09	0.09	0.09
Poisson's ratio, μ	0.33	0.3	0.3

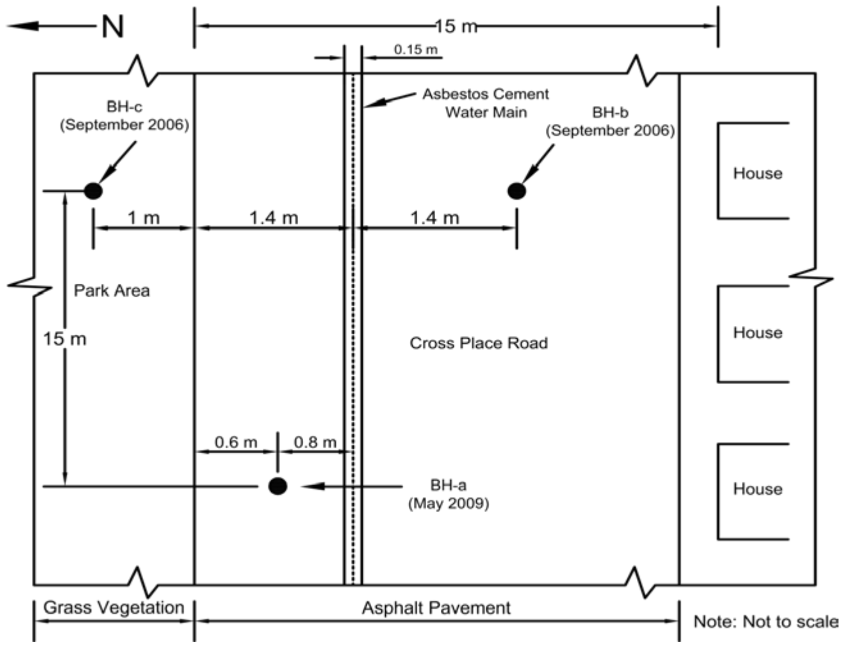


Fig. 5.13. Schematic of the test site of Case Study C (modified after Ito and Hu 2011)

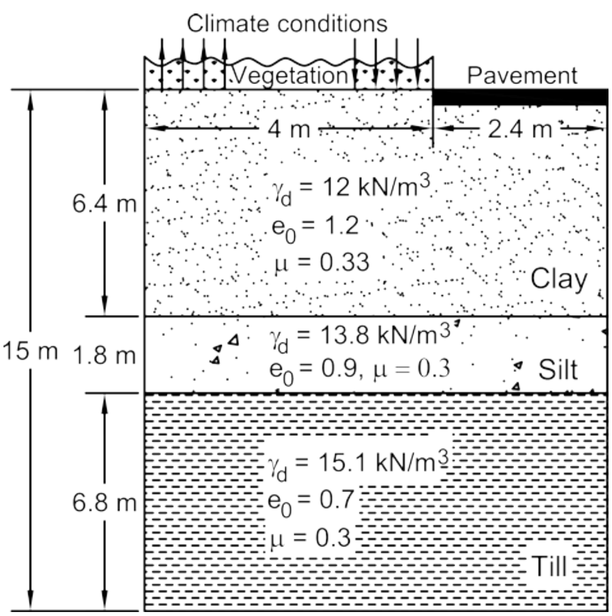


Fig. 5.14. Soil profile and soil properties of Case Study C (modified after Ito and Hu 2011)

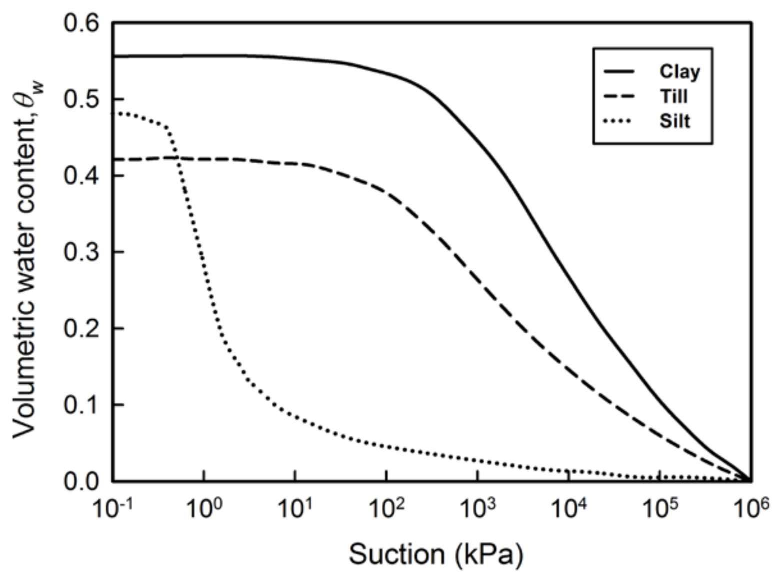


Fig. 5.15. Soil-water characteristic curves of the site soils for Case Study C (modified after Ito and Hu 2011)

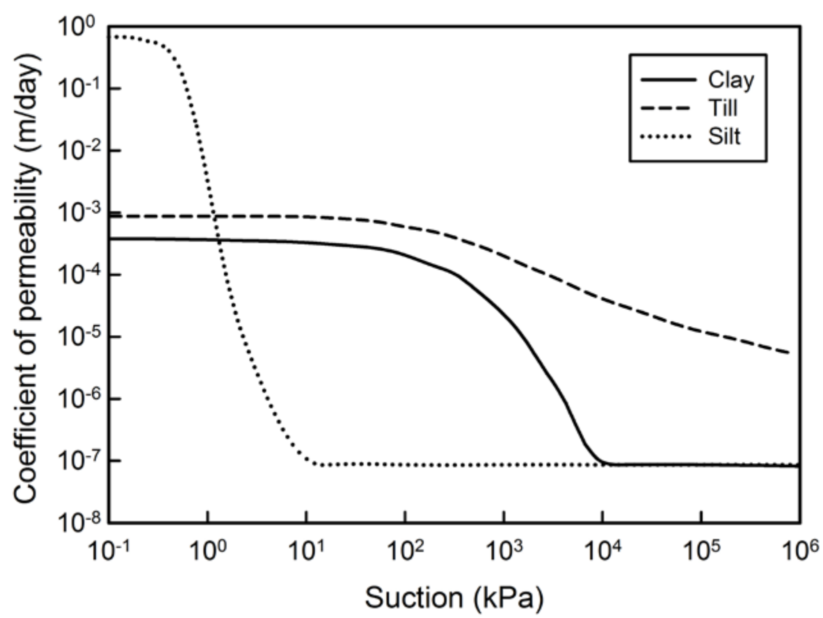


Fig. 5.16. Permeability functions of the site soils for Case Study C (modified after Ito and Hu 2011)

Figure (5.17) shows the climate data over a period of one year from 1 May, 2009 to 30 April, 2010 obtained from a weather station at Regina international airport, located at 5 km from the site (Ito and Hu 2011). The measured daily precipitation for the investigated area over the study period shows that the majority of storm events occurred during the summer, while in winter season (1 November, 2009 to 31 March, 2010) the precipitation was received as snow. The recorded average daily temperature varied from -10 °C (from November to March) to 10 °C (from April to October). The maximum difference between the high temperature (26 °C) and the low temperature (-28 °C) was 54 °C. The wind speed had no clear trend and varied between 2 and 14 m/s with an average value of 5 m/s. The relative humidity data illustrated a high average of 78% during winter and a low average of 64% during summer. The net radiation data was estimated by Ito and Hu (2011) from the corrected solar radiation; the average net radiation was 6.4 $MJ/m^2/day$ in winter and 13.8 $MJ/m^2/day$ in summer. This climate data was applied at the vegetative cover as climate boundary over the period of simulation (from 1 May, 2009 to 30 April, 2010).

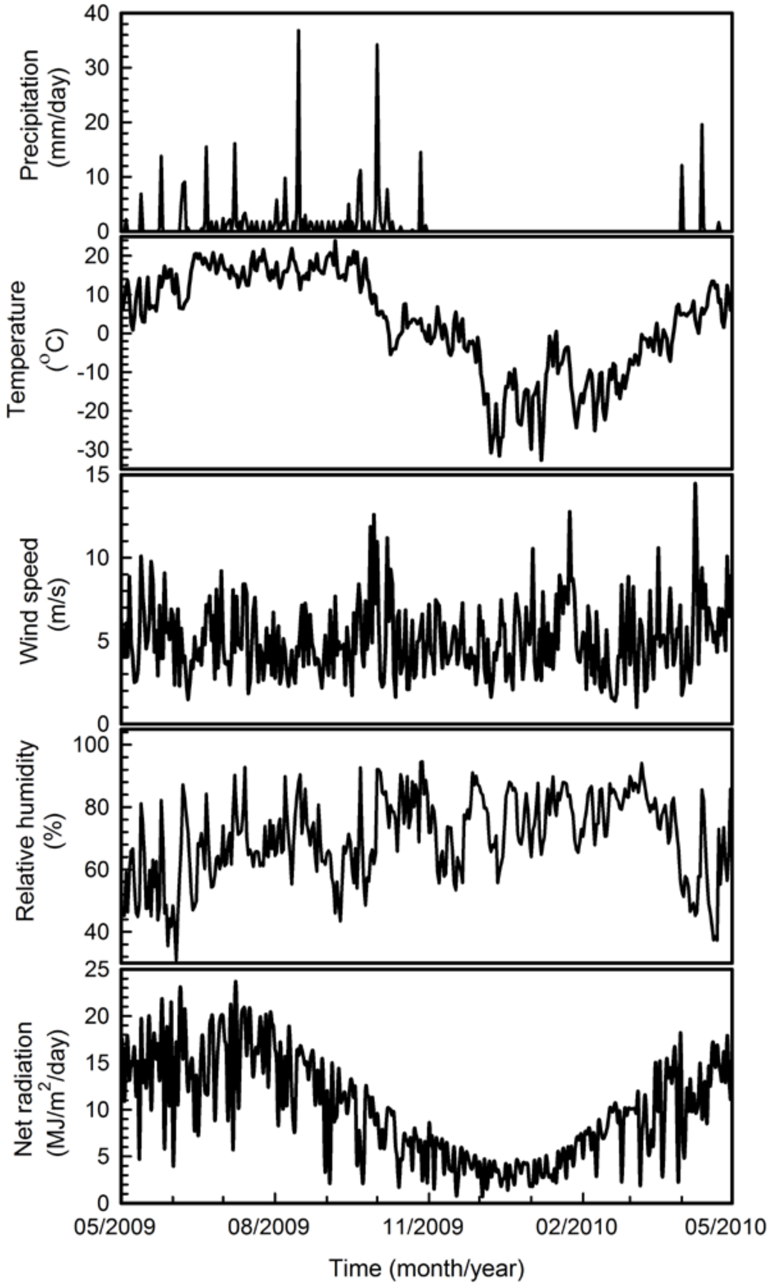


Fig. 5.17. Climate data for the Regina test site of Case Study C (modified after Ito and Hu 2011)

5.5.2 Simulation of matric suction changes over time

To simulate the soil-atmospheric interactions and estimate the corresponding matric suction changes, the soil profile shown in Figure (5.14) was modelled using the fully coupled transient analysis with the VADOSE/W software. Beside the soil properties, the initial and boundary conditions were required as input data. The initial conditions for all nodes of the model domain, including the soil matric suction and the soil temperature, were derived from implementing a steady-state analysis using the same model. Based on the field data measured by Vu et al. (2007), the initial matric suction during the steady-state analysis was set up to be 1600 kPa for the top 3 m of the clay layer, 1000 kPa for the rest of the clay, 600 kPa for the silt, and 2000 kPa for the till. These values of matric suction were achieved by specifying a pressure head of -163.15, -101.97, -61.18, and -203.94 m, respectively. The soil temperatures of nodes at the lower boundary were set up to be 10 °C.

For the fully coupled transient analysis, the climate and the vegetation data of the site were applied on the vegetated area. A "no flow" natural boundary condition was applied by default in VADOSE/W to represent the pavement as an impervious layer, while in and out moisture flows are occurring through the vegetated area. The climate data shown in Figure (5.17) was used for the simulation. However, the climate data in winter was set up to be basically constant; the temperature was assumed to be -5 °C, the relative humidity as 100%, and the other components of the climate data were set up to be zero. The cumulated winter snow precipitation was applied on a single day (1 April, 2010), when the temperature rose and remained above 0 °C. In other words, the model was not intended to simulate the soil movement activities during winter. In addition, since the site is located in a residential area with a park that has mature trees, Vu et al. (2007) and Ito and Hu (2011) specified the site vegetation as good grass and reduced the daily wind speed, precipitation, and net radiation recorded at the weather station by the scale factors of 0.3, 0.7, and 0.3, respectively. Furthermore, a park watering rate of 1.8064×10^{-3} m/day was applied every Monday and Friday for the period from 23 June to 12 October as reported in Vu et al. (2007). However, the water uptake by mature trees was not included in the simulation. The vegetation data for the site, as given by Ito and Hu (2011), includes the growing season starting in April and ending in October, the leaf area index function

(LAI) for good vegetation with a maximum LAI value of 2 as given in SoilCover (Unsaturated Soils Group 1996), the root depth that was suggested to be 150 mm with a triangular root distribution, and the plant moisture limiting point and the wilting point that were assumed to be 500 kPa and 2500 kPa, respectively.

The mass balance checking was performed on the VADOSE/W runs, and the model was solved with a total mass balance error of less than 1.5%. The responses of both the soil matric suction and the volumetric water content within the active zone to a changing surface boundary over the entire year were predicted. Figures (5.18) and (5.19) show the responses of the matric suction and the volumetric water content, respectively, under the centre of the vegetation cover. The values predicted using VADOSE/W were compared with the results of Ito and Hu (2011) as shown in Figures (5.18) and (5.19). Figure (5.20) presents the corresponding matric suction profiles for different times under the centre of the vegetation cover.

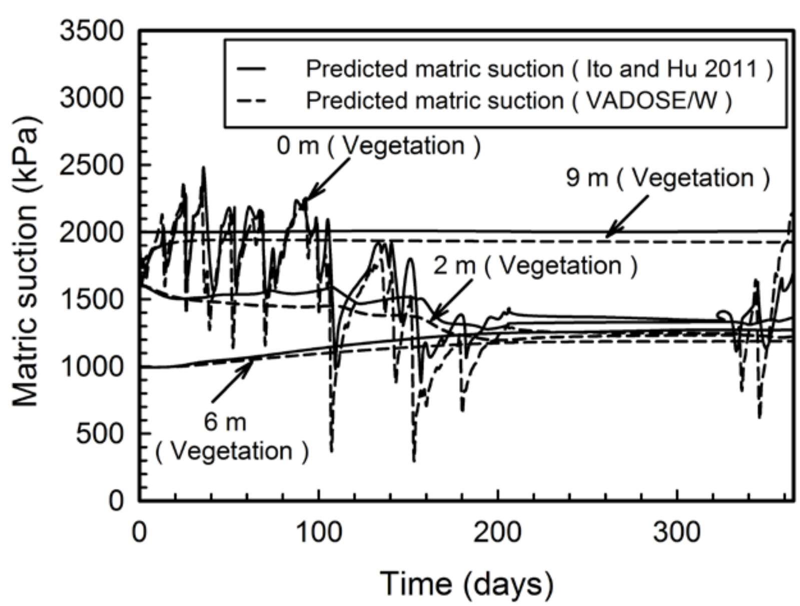


Fig. 5.18. Soil matric suction changes with respect to time at different depths under the centre point of the vegetation cover for Case Study C

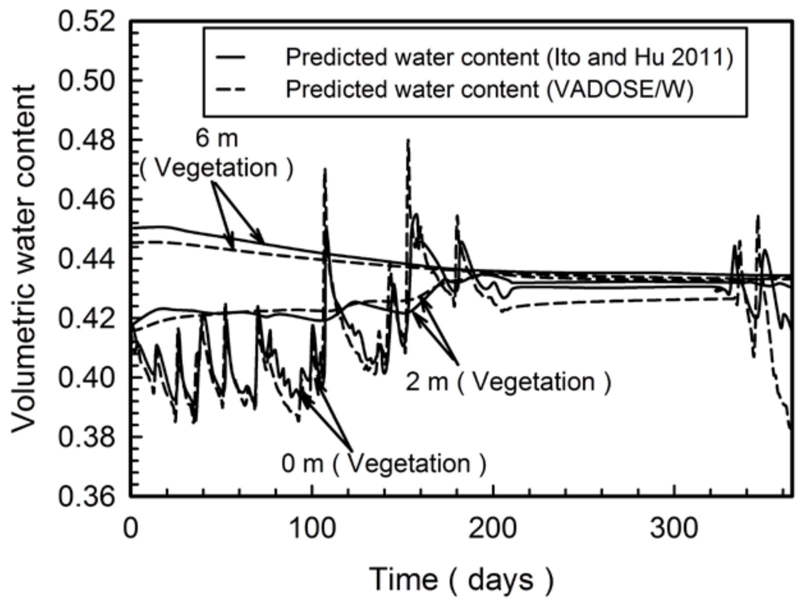


Fig. 5.19. Volumetric water content changes with respect to time at different depths under the centre point of the vegetation cover for Case Study C

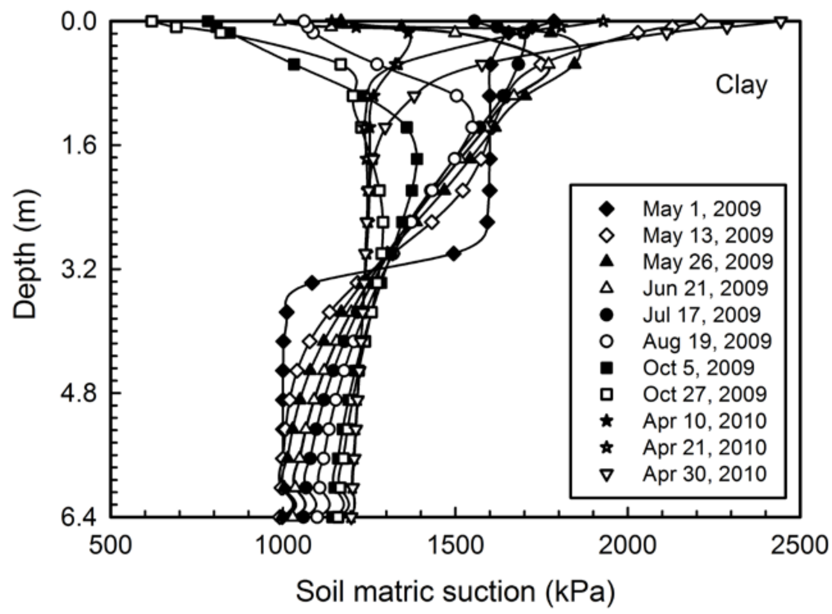


Fig. 5.20. Predicted matric suction profiles using VADOSE/W at different times under the centre point of the vegetation cover for Case Study C

5.5.3 Estimation of soil modulus of elasticity associated with mtric suction

The modulus of elasticity associated with the matric suction changes, required for predicting the soil movements, was estimated using the VO model (Equation 3.1). Poisson's ratio $\mu = 0.33$ was used for the clay layer of the site as suggested by Ito and Hu (2011). The fitting parameters β = 2 and α = 1/14.5 were chosen in order to provide a reasonable comparison between the predicted and the published results of the vertical movements of the clay layer as shown in the next section. The saturated modulus of elasticity E_{sat} for the clay layer was calculated directly from the volume change index with respect to the net normal stress (i.e., swelling index C_s) using equation (5.7). For e_0 = 1.2, C_s = 0.09, and μ = 0.33, equation (5.7) can be written as

$$E_{sat} = 38(\sigma - u_a) \tag{5.9}$$

The average value of the saturated modulus of elasticity for the top 4.3 m of the clay layer was calculated to be 1000 kPa, and that for the remainder of the clay layer was 1600 kPa. Equation (3.1) was then solved based on the matric suction changes, the SWCC in terms of the degree of saturation, and the average values of the saturated modulus of elasticity to calculate the unsaturated modulus of elasticity E_{unsat} associated with the matric suction value. The estimated values of the unsaturated modulus of elasticity provide a reasonable comparison between the predicted and the published results of the vertical soil movements for the investigated site as presented in the following section.

5.5.4 Prediction of vertical soil movement over time

Once the soil suction changes and the associated modulus of elasticity with respect to time are estimated, the vertical soil movements can be calculated at any depth and time. The Regina expansive clay layer of Case Study C was divided into 14 sub-layers (the top 2 sub-layers with 0.05 m thickness, and the other 12 sub-layers with 0.5 m thickness). The thickness of the sub-layers was chosen such that the investigated locations by Ito and Hu (2011) (i.e., 0, 0.5, 1, 2, 3, and 6 m depth) were located at the middle of the suggested soil sub-layers.

The vertical soil movement at a certain depth for each day was computed by adding the daily vertical soil movement for all layers below the considered depth. Figure (5.21) shows the vertical soil movement for each day at different depths below the centre of the vegetated cover. The vertical movement of each layer at a given time was calculated as the cumulated value of the soil layer movements for all days prior to that given time. The total vertical movement at any depth was obtained from the summation of the vertical movement of *n* layers beneath (Equation 4.7). Figure (5.22) presents the variations of the total vertical soil movement over time at different depths below the centre of the vegetation cover.

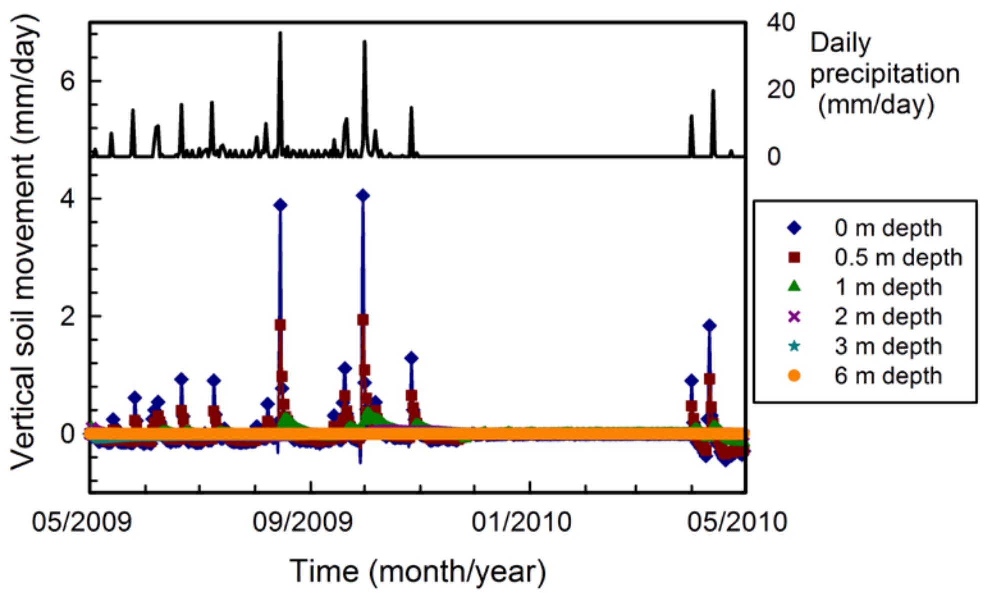


Fig. 5.21. Predicted vertical movements of clay layer for each day at different depths below the centre of the vegetation cover using the MEBM

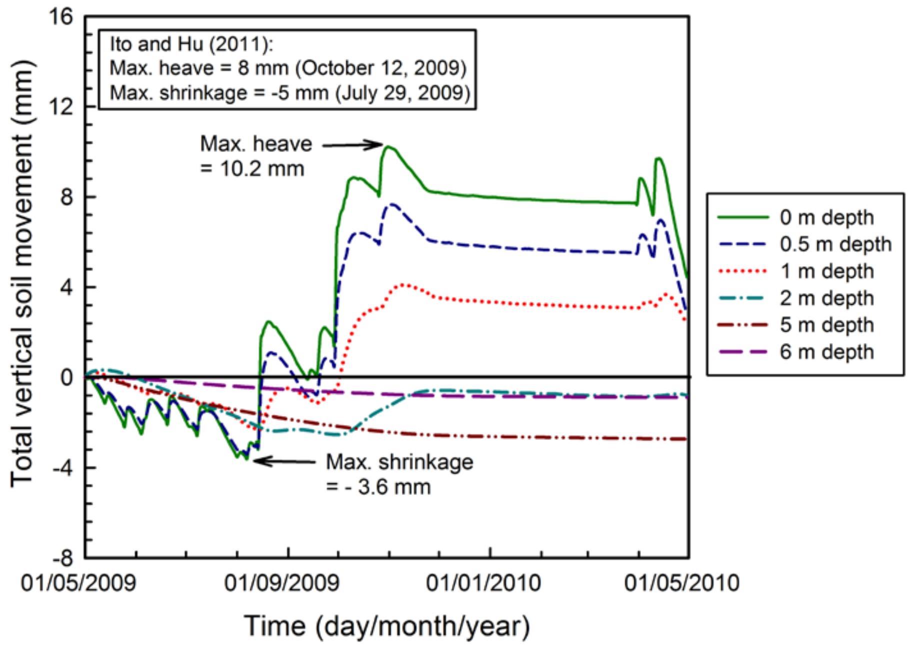


Fig. 5.22. Predicted total accumulated vertical soil movements at different depths under the centre of the vegetation cover using the MEBM

5.5.5 Analysis and discussion

Figures (5.18) and (5.19) summarize the predicted matric suction and the volumetric water content, respectively, under the centre of the vegetation cover in response to the changes in the surface boundary. These predicted values were found to vary with depth and time, and correlated well with the environmental condition on the surface boundary. The matric suction and the corresponding volumetric water content at the ground surface fluctuated widely and these fluctuations reduced with depth. There was a reasonable agreement between the predicted values of matric suction and volumetric water content of this study and Ito and Hu (2011) study (the determination coefficient $R^2 > 0.75$). The correspondence between the results was accomplished using the same meteorological data (i.e., precipitation, temperature, etc.), soil properties, and initial and boundary conditions.

Figure (5.20) shows the matric suction profile at various times under the centre of the vegetation cover of the investigated site, which represents the key information of the

MEBM approach. Extreme changes in the matric suction (vary between 600 and 2500 kPa) occurred at the ground surface. During evaporation (in summer season), the matric suction has relatively greater values at the surface, and the remainder of the profile adjusts accordingly. During infiltration (in the spring), the matric suction decreases at the surface, and it continues to decrease as water infiltrates to greater depths. The soil suction fluctuations were predominant at the surface and diminished at about 3.4 m. According to Azam and Ito (2012), such a behavior may be attributed to the interaction of environmental factors on the clay layer. The surface layer at an initially unsaturated state readily imbibes any water made available by the infiltration. Likewise the surface layer can rapidly lose water under the evaporation. With increasing depth, the overlaying soil provides a cover and the geotechnical properties of the underlying materials become progressively more significant. The high water retention capability and the low coefficient of permeability of the clay, especially under unsaturated conditions, impede variations in the suction at higher depths. The modeling results of the soil-environmental interaction in this study corroborated well with the results of Ito and Hu (2011) model thereby validating the VADOSE/W simulation. Overall, the top 3.4 m depth of this soil profile was found to be the active zone in which the environmental factors (infiltration and evaporation) are predominant.

The soil movement was predicted as a function of time and depth. Both upward and downward soil movements (i.e., heave and shrinkage) were observed over the period of the study. The soil movement has a strong correlation with the predicted values of matric suction. The soil swells with a decrease in matric suction and shrinks with an increase in matric suction. The soil movements (heave/shrink) are predominant in the upper soil layers near the ground surface in which the matric suction changes are relatively high. In addition, the vertical soil movement clearly responds to the climatic trend (i.e., infiltration and evaporation events). Figure (5.21) shows the vertical soil movement for each day at different depths under the centre point of the vegetated cover along with the daily precipitation over the period of simulation. The fluctuation in the daily vertical soil movement is active because of the higher rate of storm events. However, in winter (1 November, 2009 to 31 March, 2010), the soil response exhibited negligible variations as the precipitation was received as snow (that piled up on the ground and did not infiltrate). Figure (5.23) compares the vertical soil movements for each day at the ground surface and the 0.5 m depth estimated using the MEBM approach with those estimated using Ito and Hu (2011) model. The agreement between the results of two methods was reasonable (R^2 > 0.88). Some observed differences may be attributed to using different governing equations to estimate the soil suction profiles and the corresponding soil movements.

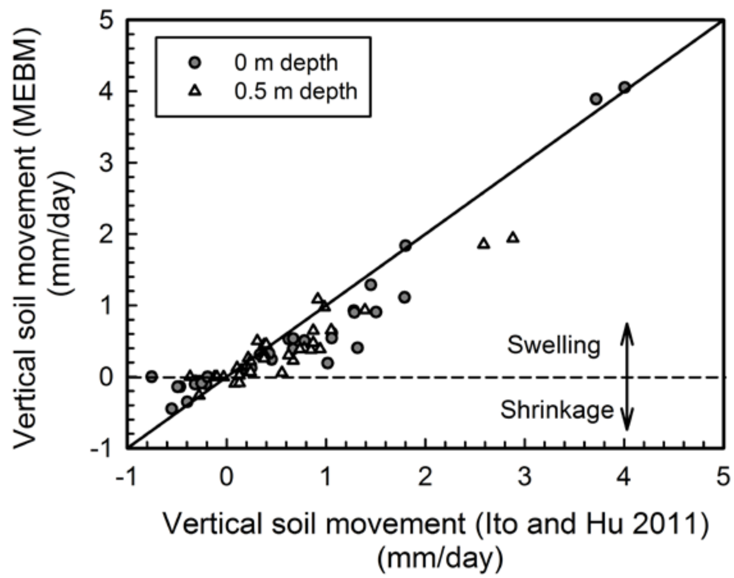


Fig. 5.23. Comparison of the vertical soil movement for each day at the ground surface and the 0.5 m depth predicted using the MEBM and Ito and Hu (2011) model

Figure (5.22) summarizes the total vertical soil movements at different depths below the centre point of the vegetation cover. The fluctuations in the total vertical soil movements were relatively large near the ground surface; however, it ceased completely at a depth of 6 m from the ground surface. The maximum heave (maximum upward soil movement) and the maximum shrinkage (maximum downward soil movement) at the ground surface beneath the center of vegetative cover were predicted to be 10.2 mm and –3.6 mm, respectively. The total vertical soil movement was estimated as 13.8 mm, which is the difference between the maximum values of soil heave and soil shrinkage. According to Ito and Hu (2011) results, the maximum values of heave and shrinkage were 8 mm and −5 mm, respectively, and the total vertical soil movement was 13 mm. It can be seen that the total soil movements predicted using the proposed MEBM was close to that predicted by Ito and Hu (2011) with a 6% difference.

5.6 Case Study D: a Cut-Slope in an Expansive Soil in Zao-Yang, Hubie, China (Ng et al. 2003)

A comprehensive field study investigated by Ng et al. (2003) has been chosen to demonstrate the validity of the MEBM for predicting the field measurements of soil movements with respect to time. The field study is a classic case in which the effect of climatic conditions, soil properties, and soil cracks are considered on the volume change behavior of expansive soils over one month (13 August to 12 September, 2001). This field study was originally performed by Ng et al. (2003) to investigate the complex soil-water interaction associated with the rainfall infiltration into a cut-slope in an expansive soil in Zao-Yang, Hubie, China (i.e., the interaction among the changes of pore water pressure (i.e., matric suction), water content, and soil movement as a result of rainfall infiltration). This field study was meant to assist in the engineering design of the 180 km portion of a canal to be excavated in unsaturated expansive soils along the middle route of the South-to-North Water Transfer Project (SNWTP).

5.6.1 Description of the field study

Figure (5.24) shows the 11 m high cut slope in expansive clay in Zao-Yang, Hubie, China (semi-arid area). The deposit is medium plastic, unsaturated, expansive clay that exhibits significant volume changes as water content changes. The initial void ratio of the soil site e_0 varies from 0.63 to 0.69, and the soil plasticity index I_p is 31%. The basic physical properties of soil specimens taken from the research slope at a depth of 1.0 m are summarized in Table (5.7). The upper soil layer with a thickness varying from 1.0 m to 1.5 m is rich in cracks and fissures likely related to the swelling and shrinkage phenomenon associated with expansive soils. Figure (5.25) shows the distributions of cracks and fissures observed within the exposed surface near the monitoring area. The maximum depth and the maximum width of the open cracks were estimated to be approximately 1.2 m and 10 mm, respectively. Figure (5.26) shows the in-situ relationship between the soil water content and the matric suction, along with the soil-water characteristic curve (SWCC) for the soil of the slope. The hysteresis between the desorption and the adsorption curves appears to be relatively insignificant since the soil specimens had experienced many wetting/drying cycles in the field (Zhan et al. 2006). The coefficient of permeability for the soil site is relatively low (less than 10^{-7} m/s) (Zhan 2003).

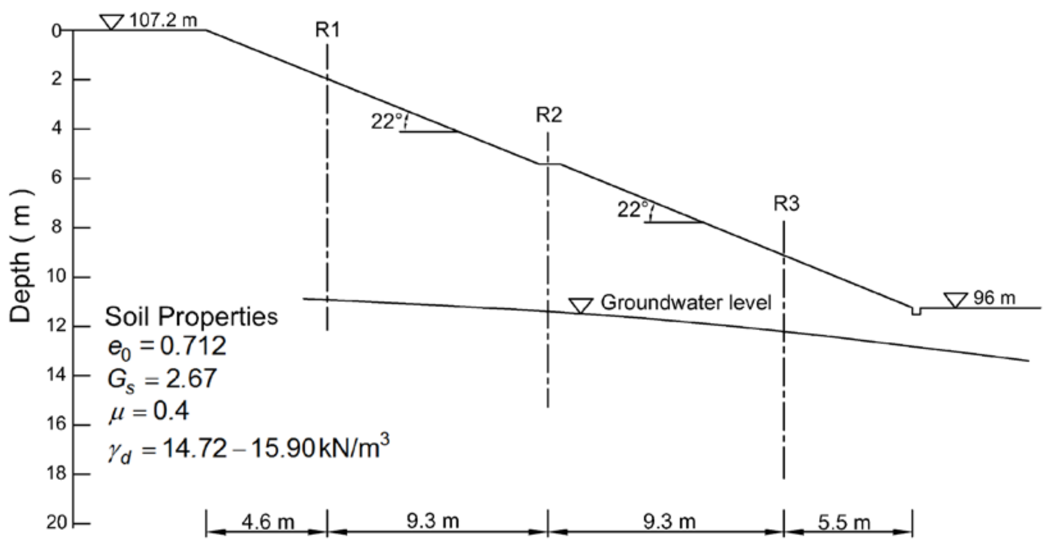


Fig. 5.24. Cross-section of the instrumented slope of Case Study D (modified after Ng et al. 2003)

Table 5.7. Physical properties of soil specimens taken from the research slope at a depth of 1.0 m (data from Zhan et al. 2007)

Clay grain content, (%)	Specific gravity, G_s	Dry unit weight, γ_d (kN/m^3)	Initial void ratio, e_0	Liquid limit, w_L(%)
39	2.67	14.72-15.90	0.712	50.5
Plasticity Index, I_p (%)	Saturated permeability, k_{sat} (m/s)	Compressibility index, C_c	Swelling index, C_s	Poisson's ratio, μ
31	$10^{-10} - 10^{-7}$	0.13	0.025	0.4

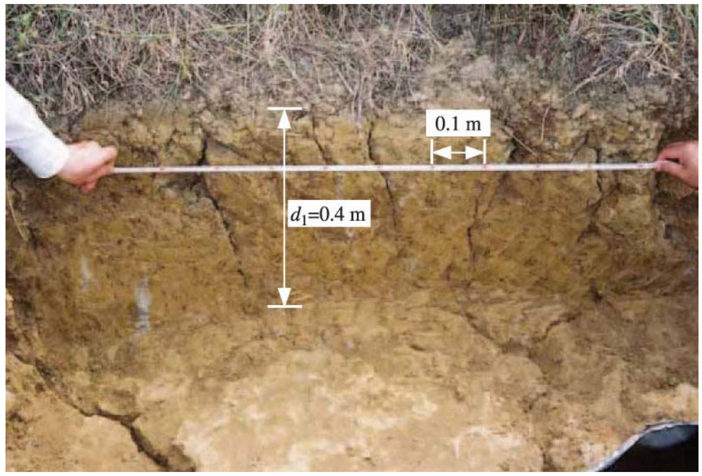


Fig. 5.25. Cracks and fissures in an excavation pit near the monitoring area of Case Study D (Zhan et al. 2007)

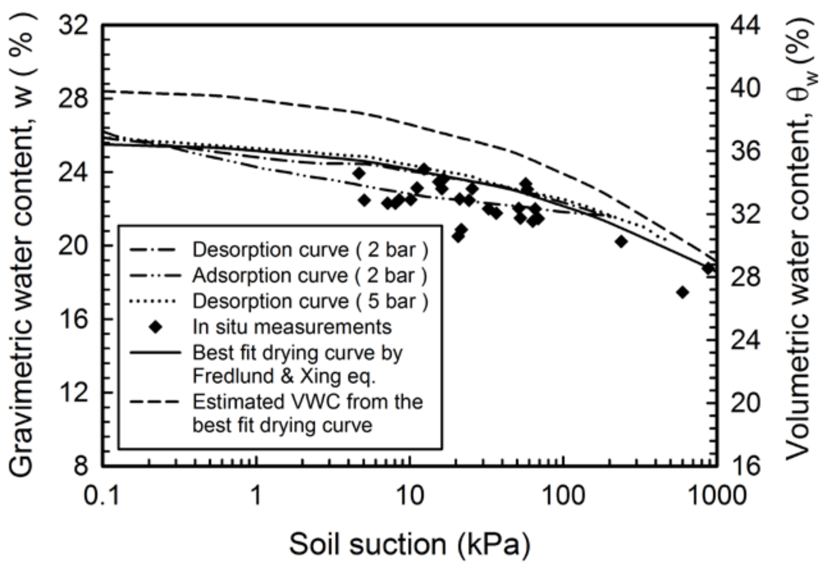


Fig. 5.26. Soil-water characteristic curves for the slope soil (modified after Zhan et al. 2006)

Two artificial rainfall events were created by Ng et al. (2003) during the one month period of field investigation including monitoring the induced rainfall infiltration through the cut-slope. Figure (5.27) shows the two simulated rainfall events during the period of monitoring (13 August to 12 September, 2001) with an average daily rainfall of 62 mm. The first rainfall lasted for 8 days from 18 to 25 August, 2001. The second simulated rainfall was applied from 8 to 10 September, 2001. To understand the soil-water interaction in an unsaturated expansive soil slope subjected to rainfall infiltration, a comprehensive instrumentation and monitoring program was carried out by Ng et al. (2003). The instrumentation included jet-filled tensiometers, thermal conductivity suction sensors, moisture probes, earth pressure cells, inclinometers, vertical movement points, an artificial rainfall simulator, a tipping bucket rain gage, a V-notch flow meter, and an evaporimeter. The outcome of the field monitoring included a collection of considerable amount of valuable data involving pore water pressure (or matric suction), soil water content, and soil movements (heave/shrink) as a result of rainfall infiltration. More details with respect to the instrumentation are available in Ng et al. (2003) and Zhan (2003).

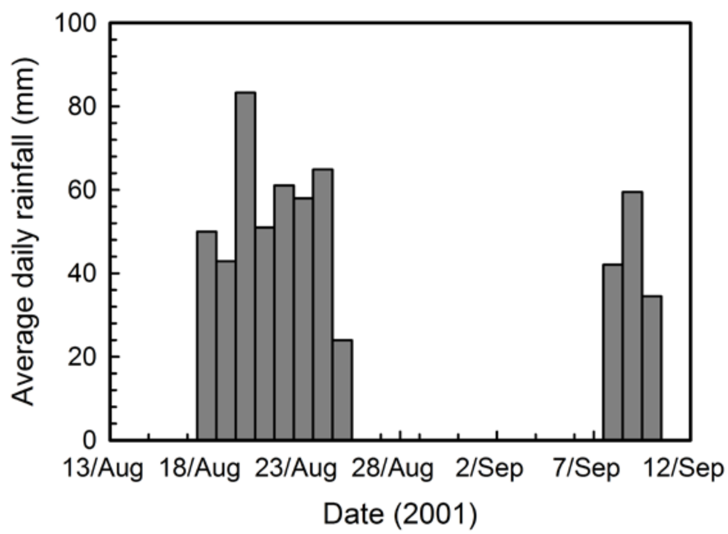


Fig. 5.27. Intensity of rainfall events during the monitoring period of Case Study D (modified after Ng et al. 2003)

The step-by-step procedure of the proposed MEBM (Figure 4.2) is applied in the following sections on the research slope of Case Study D to predict the vertical soil movement with respect to time.

5.6.2 Simulation of matric suction changes over time

The matric suction changes associated with the rainfall infiltration through the research slope were simulated using the VADOSE/W program. The program primarily requires the input of appropriate in-situ soil properties and boundary conditions to model the unsaturated and saturated flow through the slope. The research slope was modeled as a *2-D* problem using the fully coupled transient analysis. The initial condition of the model was established by conducting the steady-state analysis. The ground water table in the field was located at about 6 m depth from the ground surface. To simulate the initial condition of the ground water table at an average depth of 6 m, the pressure head in the top 1.5 m soil layer near the ground surface (i.e., the tension cracks zone) was set up to vary from -7 to -1 m while the pressure head value of the bottom boundary was set up at 1 m. As the daily fluctuation of atmosphere temperature was insignificant at the considered site, the soil slope was modeled using the simplified isothermal model. In other words, the thermal properties of the slope soil were assumed to be constant (i.e., the soil temperature = 27 ^{o}C, the thermal conductivity = 400 kJ/days/m/°C, and the volumetric heat capacity = 1875 kJ/m³/°C). The effect of vegetation on the volume change behavior of expansive soils was omitted in the analysis since the top soil to a depth of about 10 mm of the slope was removed. The two artificial rainfall events with the daily climate data obtained from a weather station in Wuhan city were applied as a climate boundary at the surface layer of the slope. Figure (5.28) shows the *2-D* model for the research slope.

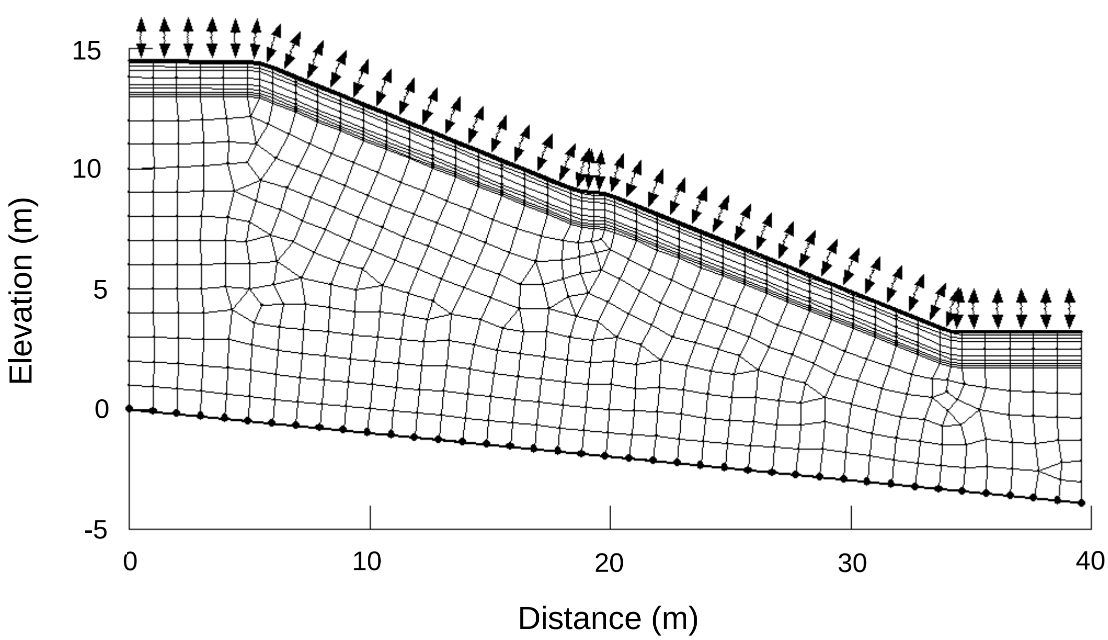
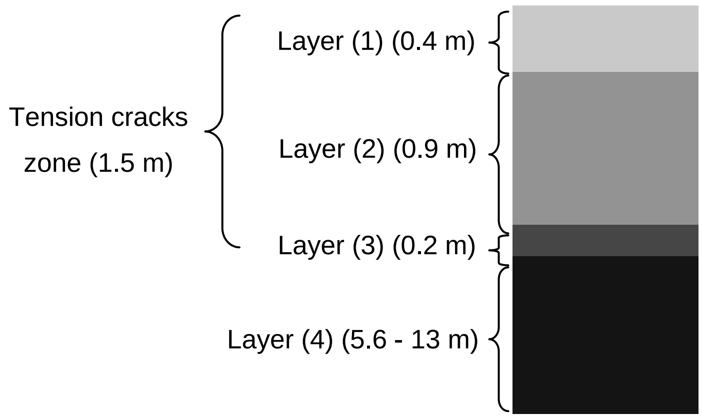


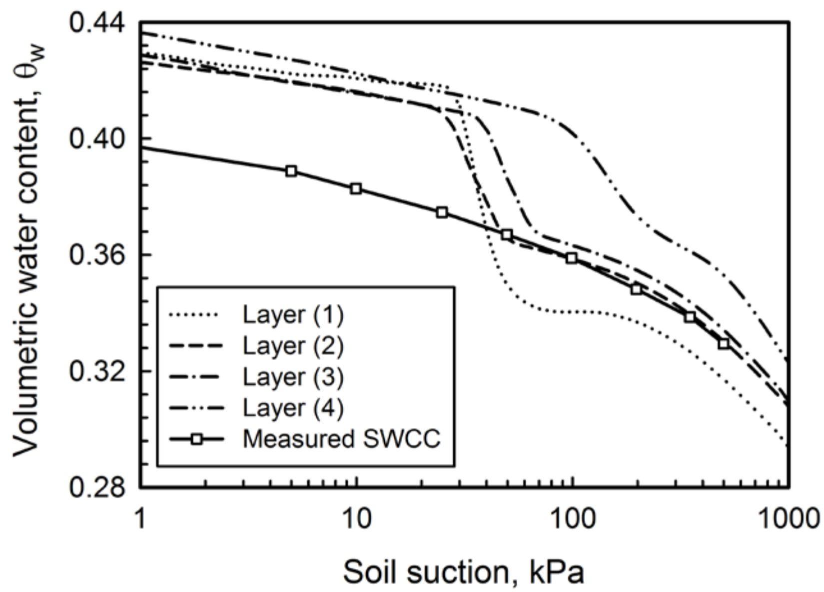
Fig. 5.28. *2-D* model for the research slope of Case Study D

5.6.2.1 Model calibration

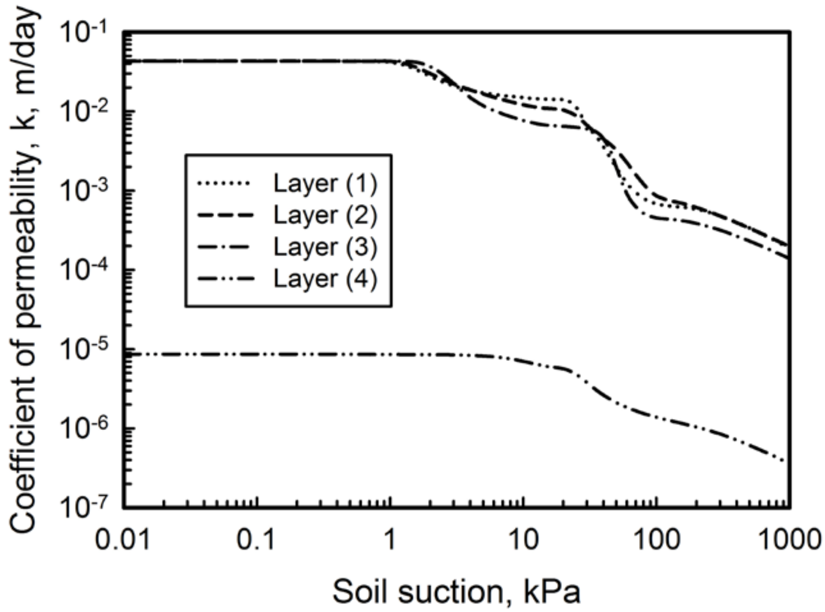
For Case Study D, VADOSE/W was calibrated to simulate the saturated and unsaturated flow through the research slope. Valid assumptions with respect to the soil properties and the boundary conditions were made such that reasonable comparisons were achieved between the predicted and measured values of the water flow properties. Similar approaches were used by Overton et al. (2006) and Diewald (2003) for the model calibration. The upper portion of the slope with a thickness varying from 1.0 to 1.5 m was observed to have cracks and fissures. Figure (5.26) shows the SWCC of the slope soil in terms of the volumetric water content that was calculated based on the SWCC in terms of the gravimetric water content estimated using the best-fit Fredlund and Xing (1994) equation. Since the abundance of cracks and fissures influences the SWCC behavior of expansive soils, a unique SWCC is not reasonable to represent the expansive soil having tension cracks. For this modeling task, the site soil was modeled by subdividing the tension cracks zone of the slope into 3 layers as shown in Figure (5.29a). VADOSE/W was calibrated by varying the SWCCs and the associated coefficient of permeability functions for each layer until the estimated values of pore water pressure (PWP) matched the measurements. Different scenarios using different assumptions related to the number of soil layers and the soil properties were implemented until the behavior of the research slope was successfully simulated. Figures (5.29b and c) show the SWCCs and the coefficient of permeability functions of the soil layers obtained from the model calibration including the cracking effects (bi-model) (i.e., two air-entry values). Figure (5.30) shows the predicted and the measured pore water pressure versus time at the mid-slope (R2) in response to the simulated rainfalls. The close agreement between the predicted and measured values of the pore water pressure at the mid-slope (R2) provides credence to the simulation based on the assumptions used in this analysis (the coefficient of determination R^2 = 0.74). The model can hence be considered to be calibrated.



(a) 4-soil layers considered



(b) Soil-water characteristic curves of the soil layers of the slope



(c) Coefficient of permeability functions of the soil layers of the slope

Fig. 5.29. Key soil properties of the 4-layers considered for the research slope: (a) 4-soil layers considered, (b) soil-water characteristic curves, and (c) coefficient of permeability functions of the slope soil

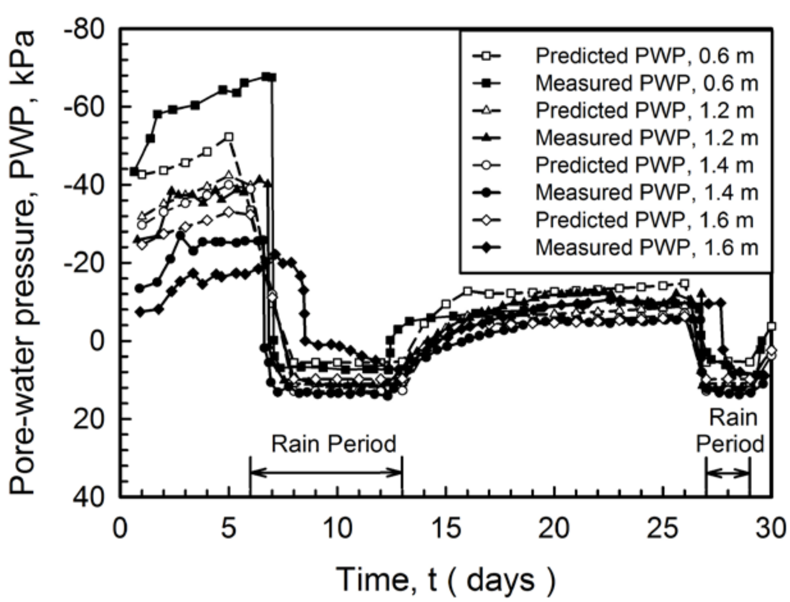


Fig. 5.30. Comparison of the predicted and the measured pore water pressure (PWP) during the rainfall events at different depths at the mid-slope

Review of Figure (5.30) shows that both the predicted and the measured pore water pressure within the crack tension zone were negative prior to the first artificial rainfall. As expected, the negative pore water pressures near the ground surface were higher than those at greater depth. After the application of the first rainfall event, the predicted pore water pressures were generally consistent with the measurements; both the predicted and the measured values of the pore water pressure changed from negative to positive. The positive pore water pressure appeared in the slope soil. After the end of the first rainfall, a recovery of negative pore water pressure was observed in the predicted values as the measured values. The recovered negative pore water pressures were much lower than the corresponding values prior to the application of the rainfall. At the application of the second rainfall, the pore water pressure values were similar to those at the first rainfall event. The predicted pore water pressure reached the equilibrium condition faster than the measured values. This can be attributed to the delay in the measured responses of the pore water pressure using the instrumentation which required at least one and a half days' time after the induced rainfall infiltration. This delay was attributed by Ng et al. (2003) to the effect of the abundant cracks and fissures in the soil on the soil-water interactions. Cracks provided an easy pathway for rainwater to infiltrate the soil. The rainwater first bypassed the regions between cracks as it entered directly through cracks. While rainwater initially flowed through cracks and fissures, the tensiometers did not register any significant changes of matric suction, and this continued until the infiltrated rainwater filled the cracks (Ng et al. 2003). In contrast, the predicted pore water pressure response was instantaneous after the commencement of the rainfall. In spite of the discussed limitations that are difficult to be introduced into the modeling, reasonable comparisons were observed between the predicted and the measured values of the pore water pressure (the determination coefficient R^2 = 0.74).

5.6.2.2 Model validation

The data from the field investigation performed by Ng et al. (2003) can be used for the validation of the model. The model validation involves the comparisons of the predicted and the measured properties of unsaturated and saturated flow through the soil, including temperature, evaporation, matric suction, and volumetric water content. A good comparison between the measured and predicted properties demonstrates that the calibrated model is reliable and can be used for predicting the water flow and soil volume change behavior considering other scenarios. The validation in the present analysis was performed by comparing the predicted and the measured values of volumetric water content (hereafter referred to as VWC) for the slope soil at section R2. Figure (5.31) shows the variations of the predicted and the measured VWC with time at R2 in response to the simulated rainfalls. The response of the VWC values was generally consistent with the corresponding pore water pressures. Prior to the first rainfall, the predicted and the measured VWCs increased with depth due to the influence of evaporation. After the rainfall, the predicted VWCs simultaneously reached the equilibrium condition; however, the measured values have the same delayed response as previously shown for the measured pore water pressures. After the cessation of the first rainfall, the predicted VWCs began to decrease progressively towards new equilibrium values, while the measured VWCs remained constant before progressively reaching the equilibrium condition. At the application of the second rainfall, the VWCs were similar to those at the first rainfall event. In general, there was a reasonable agreement between the predicted and measured values of VWC (R^2 = 0.78). However, slight differences may be attributed to the difficulty of the modeling of the natural variations in soil nature, composition, and cracks. The inconsistency in part may also be related to the accuracy of the VWC measurements in the field, which can be significantly affected by the soil compaction, soil density and soil cracks as discussed in Ng et al. (2003) and Zhan (2003).

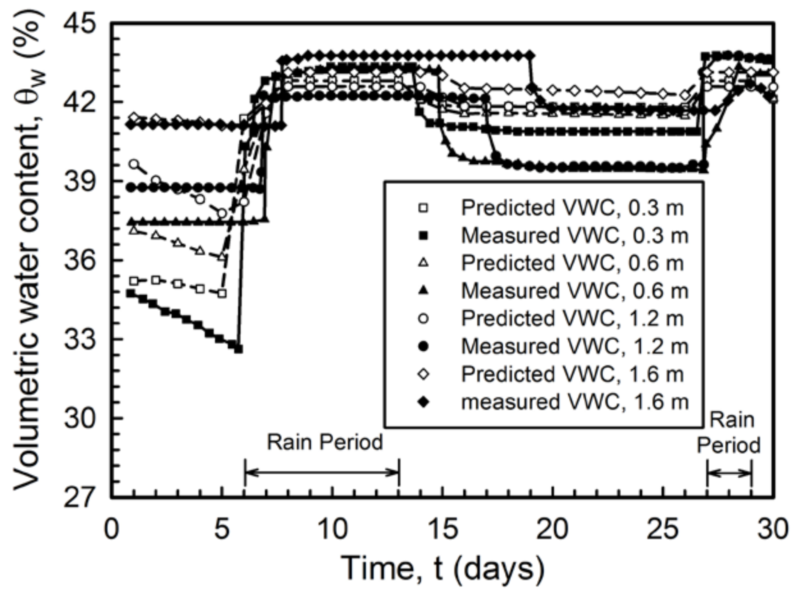


Fig. 5.31. Comparison of the predicted and the measured VWC during the rainfall events at different depths at the mid-slope

5.6.2.3 Simulation of matric suction fluctuation

After both the calibration and the validation of the model using VADOSE/W were successfully achieved, the matric suction profiles were estimated for the period of simulation (i.e., 30 days) at different sections of the slope. Figure (5.32) shows the fluctuation of matric suction profiles at the middle section of the slope (R2) in response to the rainfalls through the period of 30 days. The climate condition primarily influenced the top 2.5 m near the ground surface and resulted in the fluctuation of the soil matric suction within the active zone depth (i.e., the depth of the active zone for this site is about 2.5 m). Review of Figure (5.32) shows that the value of predicted matric suction decreased significantly after the commencement of the rainfall due to an increase in the positive pore water pressure. The continued rainfall resulted in further decrease in the matric suction till it disappeared. The largest positive pore water pressure was observed at approximately 1.5 m depth. This indicates the presence of a perched ground water table at that depth as observed in the field measurements conducted by Ng et al. (2003).

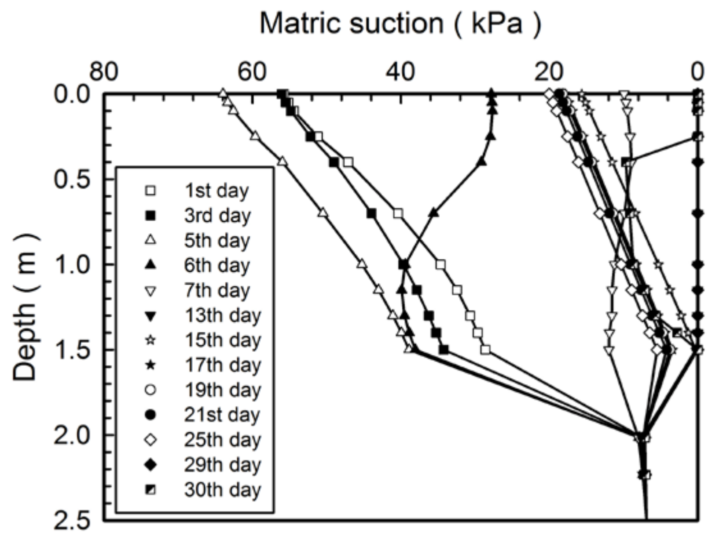


Fig. 5.32. Matric suction profiles at the middle of the slope for Case Study D

5.6.3 Estimation of soil modulus of elasticity associated with matric suction

The VO model (Equation 3.1) was used for estimating the modulus of elasticity associated with the changes in matric suction. Poisson's ratio μ for the slope soil was assumed to be a constant value of 0.4. The fitting parameters β = 2 and α = 1/20 were assumed to predict the vertical soil movements of the slope. The saturated modulus of elasticity E_{sat} for the slope soil was calculated using equation (3.8). Equation (3.8), as described in Chapter Three, was developed based on the results of the triaxial tests conducted by Zhan (2003) on saturated compacted specimens of the soil slope for various confining pressures σ_3. To estimate the saturated modulus of elasticity for each soil layer of the slope, the confining pressure σ_3 was determined as the overburden pressure at the center of the investigated soil layer but not less than 9 kPa. Equation (3.1) was then solved to calculate the unsaturated modulus of elasticity E_{unsat} associated with the changes in matric suction. Once the matric suction changes and the associated modulus of elasticity are estimated over time, the vertical soil movement can be predicted at any time.

5.6.4 Prediction of vertical soil movement over time

To evaluate the vertical soil movement with time, the soil profile within the active zone (i.e., 2.5 m) was divided into several layers with a 0.2 m thickness. The total vertical soil movement at any point in the soil profile was calculated from equation (4.7). Figure (5.33) shows the estimated vertical soil movements at different depths at the mid-slope section compared with the field measurements.

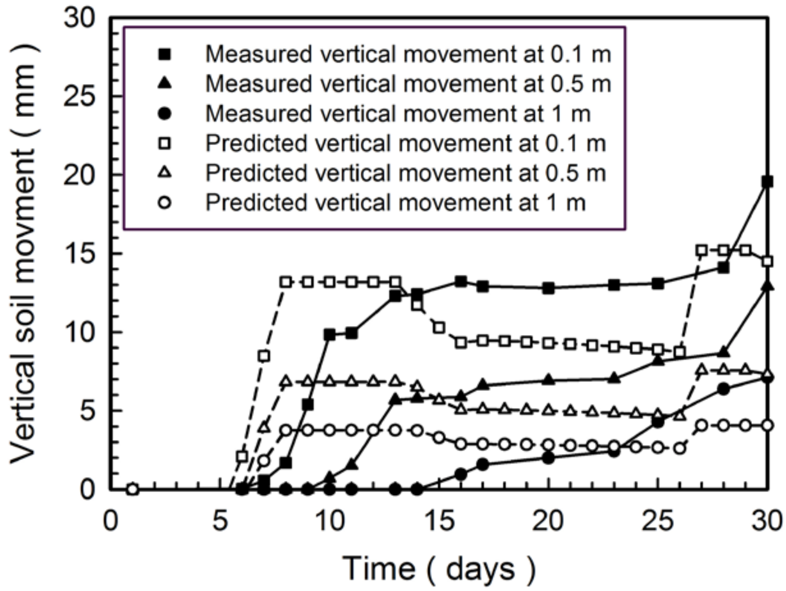


Fig.5.33. Comparison of the estimated vertical soil movements with the field measurements at the mid-slope

Figure (5.33) shows, prior to the first artificial rainfall, there was no volume change taken place in the slope since the initial matric suction was relatively high. After the commencement of the first rainfall event, the high initial matric suction values progressively reduced and the predicted soil heaves increased. After the first rainfall stopped, the predicted soil heave terminated and a significant amount of shrinkage was observed. Then, the predicted soil movements remained almost constant over the remainder of the simulation period till they started increasing again as a response to the second rainfall event.

The patterns of the predicted and the measured vertical soil movements in response to the rainfall events were quite different. The average value of the percentage difference between the predicted and the measured soil movements was 28%. It can be seen that, after the commencement of the first rainfall event, the predicted heaves had a relatively fast response to the rainfall infiltration compared to the measured values. This may be attributed to the difficulty to simulate the performance of the soil in the field considering the high intensity of the initial soil cracks. In other words, the existence of cracks changes the soil properties which, in turn, influence the soil-water interaction over time. However, the results of modeling simulation were mainly based on uniform soil properties. In addition, the field observations indicated that the soil cracks were large at the upper part of the slope and decreased towards the lower part of the slope; nevertheless, the soil layers and their properties were assumed in the simulation to be uniform over the entire slope. Figure (5.33) also shows that the predicted soil heave terminated after the first rainfall event and a significant amount of shrinkage appeared. However, the measurements show soil heaves at a low rate over the period of the slope monitoring. This can also be attributed to the uncertainty of the soil properties as the expansive soil has intensive cracks in the field, and the difficulty to model its nature (soil cracks) and response to the environmental changes.

In spite of the complexities associated with modeling the response of the natural expansive soil deposit with cracks to the rainfall infiltration, the prediction of soil movements improved when only the soil heave was considered (i.e., the soil shrinkage was omitted) in the modeling (see Figure 5.34). Figure (5.34) shows that the heave patterns from the predictions agreed with the field measurements. The average percentage difference between the predicted and the measured soil movements after the first rainfall event decreased to 21%. The results demonstrated that the MEBM could be used with a reasonable degree of confidence for estimating the soil heave with respect to time.

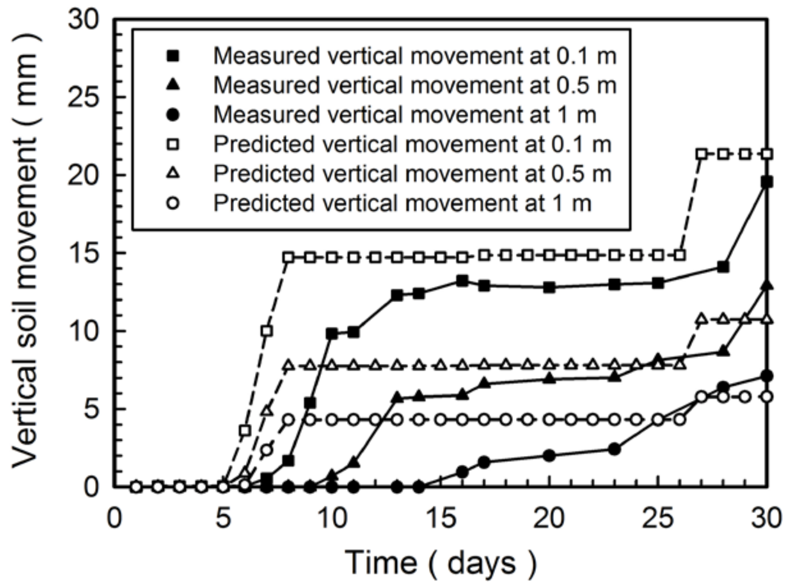


Fig.5.34. Comparison of the estimated soil heaves using the MEBM with the field measurements at the mid-slope

5.7 Case Study E: a Field Site in Arlington, Texas, USA (Briaud et al. 2003)

Case Study E is a site in Arlington, Texas, USA, selected for the validation of the MEBM for predicting the field measurements of the movement of four full-scale spread footings over a period of 2 years (1 August, 1999 to 31 October, 2001). Factors that influence the volume change behavior of expansive soils, such as daily weather condition and vegetation, were considered for this field construction site. The site was originally monitored by Briaud et al. (2003) to investigate the damage caused by expansive soils in both concrete and asphalt pavements, resulting in substantial discomfort, safety hazard, and vehicle damage. The proposed MEBM is assessed more closely by comparing its estimations of the soil movement at the considered site with the long term field observations over 2 years. In addition, to provide more evidence to use the MEBM in engineering practices, the results of the MEBM are compared with the estimated values obtained from Briaud et al. (2003) and Zhang (2004) prediction methods presented in Chapter Two.

5.7.1 Site description

Figure (5.35) shows the soil stratigraphy, and the basic soil properties for the site. The predominant soil type at the site is classified according to the Unified Soil Classification System as borderline between CL and CH (Briaud et al. 2003). The stratigraphy of the site consists of 0-1.8 m of dark gray silty clay and 1.8-4.0 m of brown silty clay. The soil surface is covered by the Johnsongrass, which is warm season, perennial grass. The grass root zone is suggested to be 0.22 m depth. The water level in the 7m deep standpipes varies between 4 m and 4.8 m below the ground surface over 2 years.

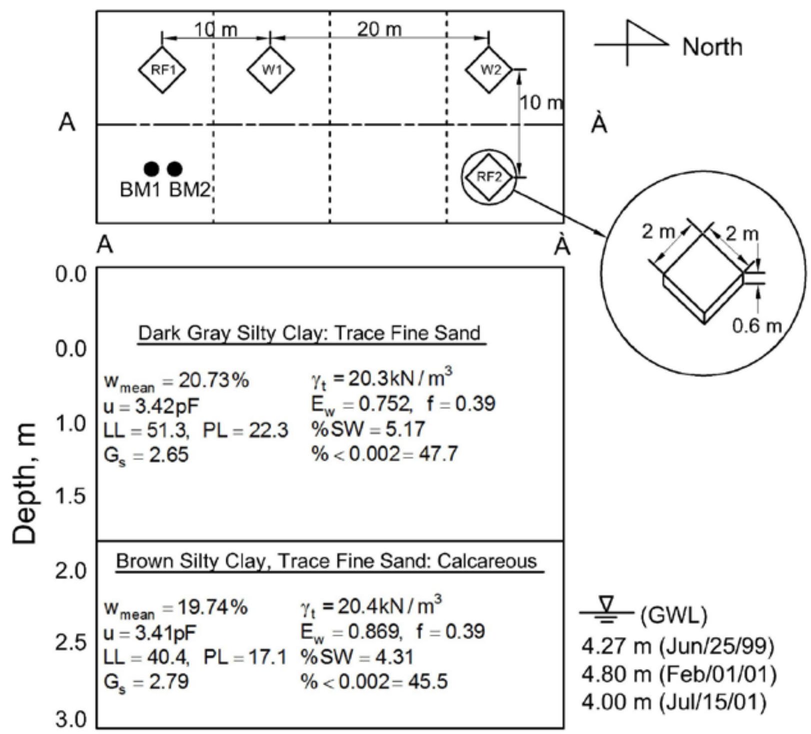
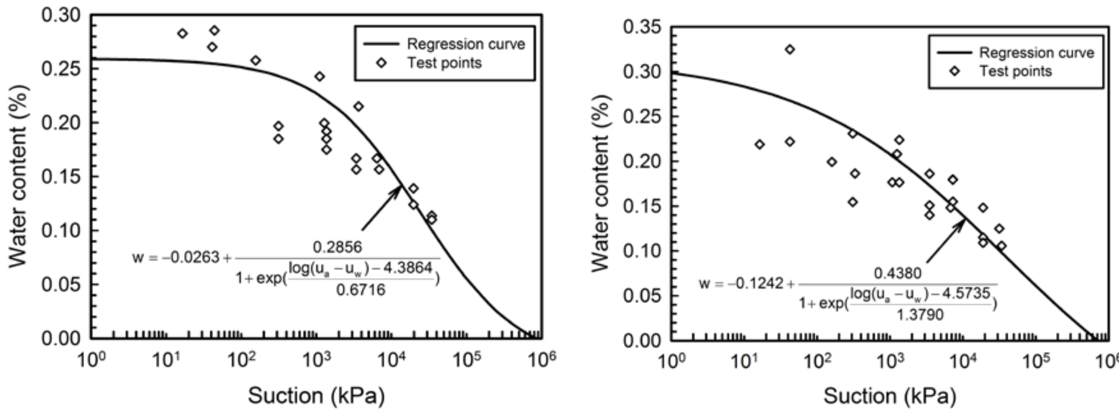


Fig. 5.35. Soil stratigraphy and basic soil properties for the site of Case Study E (modified after Briaud et al. 2003)

The SWCC was measured by Briaud et al. (2003) using the pressure plate test and the salt concentration test. Sigmaplot was used to obtain the regression curve of the all tests results. Figure (5.36) shows the SWCCs and the mathematical expressions of the best fitted SWCCs for the soil layers in the site. Permeability functions of soils under unsaturated conditions are also required for the analysis. The soil permeability functions were estimated by VADOSE/W using van Genuchten (1980) equation (see Figure 5.37).



(a) (b)

Fig. 5.36. Soil-water characteristic curves: (a) for dark gray silty clay, (b) for brown silty clay (modified after Briaud et al. 2003)

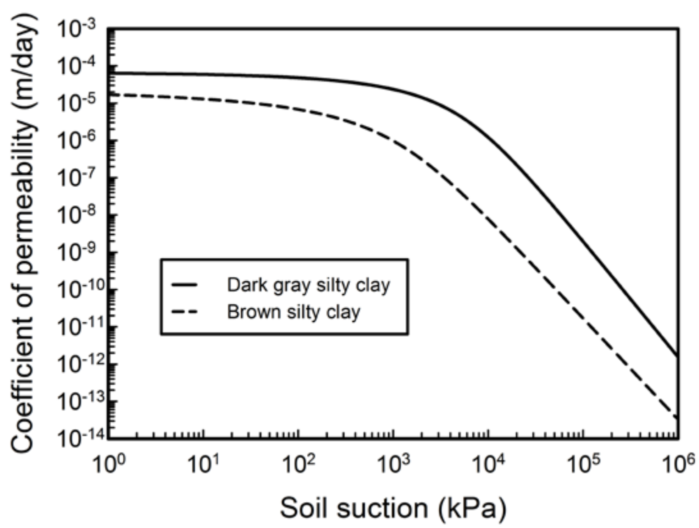


Fig. 5.37. Permeability functions estimated by VADOSE/W for the soil layers of Case Study E

Figure (5.38) shows the daily temperature, relative humidity, wind speed, rainfall, and net radiation over the period of study from 1 August, 1999 to 31 October, 2001. The daily weather data was gathered from a weather station at the Arlington Municipal Airport (Briaud et al. 2003, Zhang 2004).

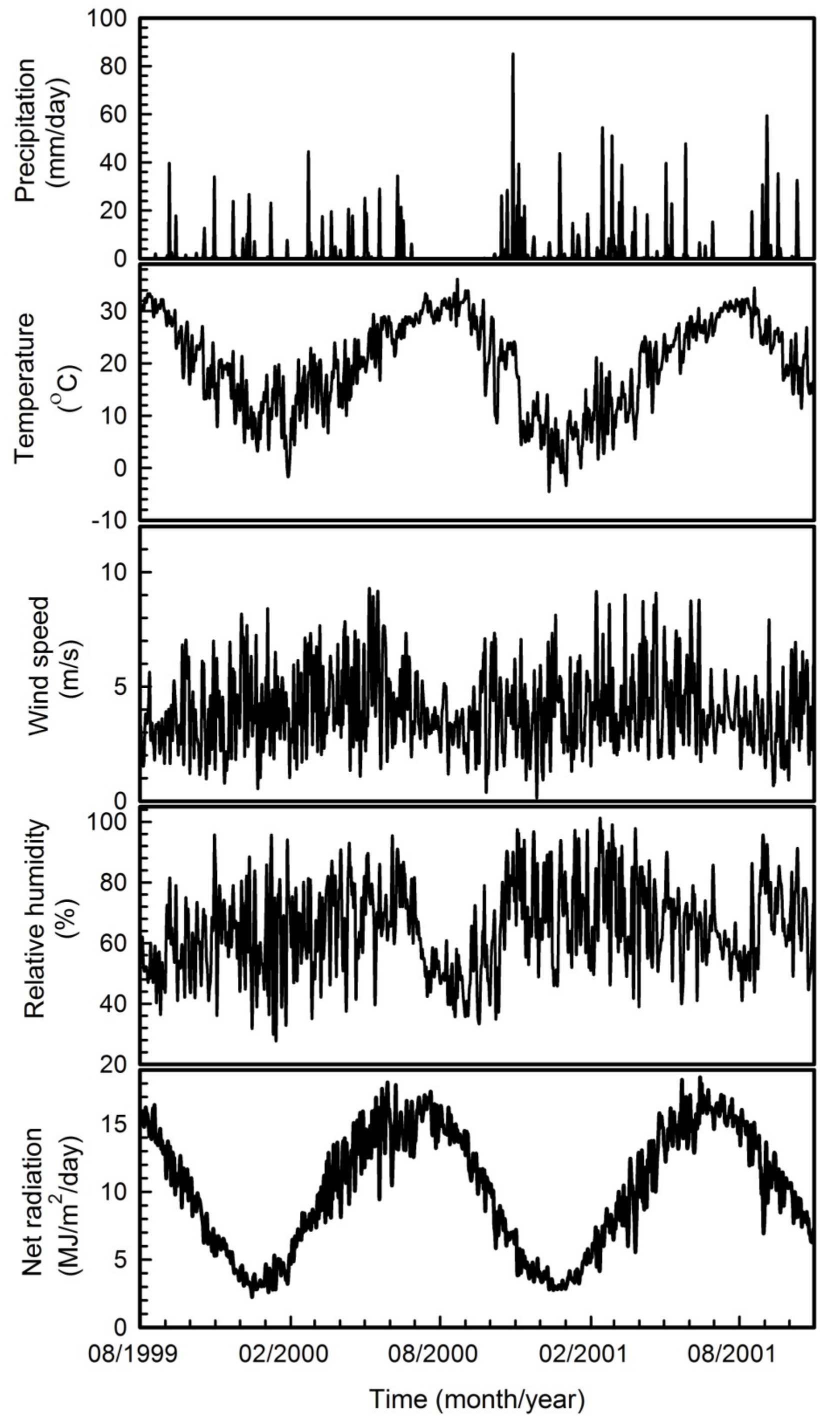


Fig. 5.38. Daily climate data over two years for the Arlington site of Case Study E (modified after Zhang 2004)

Four 10 m × 10 m areas were outlined at the site, and denoted as RF1, RF2, W1, and W2 as shown in Figure (5.35). Areas W1 and W2 were injected with 13.6 m^3 of water/day for

3 days (6–8 July, 1999) at a depth of 3 m (Briaud et al. 2003). A 2 m square footing was constructed at the center of each of the 10 m × 10 m areas between 15 and 30 July, 1999. The vertical movements at the corners of each footing were recorded every month starting from 11 August, 1999 until 1 November, 2001. The average values of the measured movements for the four footings are shown in Figure (5.39).

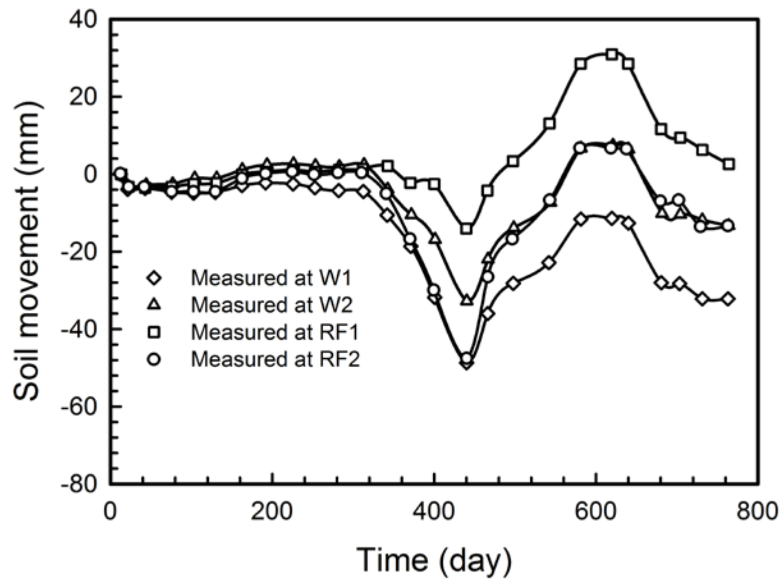


Fig. 5.39. Measured movements of the footings at the Arlington site over two years (modified after Briaud et al. 2003)

5.7.2 Simulation of matric suction changes over time

The soil domain used for the simulation of matric suction changes over time using VADOSE/W is 10 m × 10 m × 4 m (length × width × height) (Figure 5.40). A 10 m length is considered to be far away so that there is no influence of the footing (Zhang 2004). The fully coupled transient analysis was conducted on the soil profile shown in Figure (5.40) to estimate the matric suction changes over a period of 2 years. VADOSE/W requires three categories of input parameters, namely: soil properties, initial conditions, and boundary conditions. The soil properties included the SWCCs (Figure 5.36) and permeability functions (Figure 5.37). The variation of initial matric suction versus depth shown in Figure (5.40) was provided by Zhang (2004) to represent the initial condition of the simulation. The site soils were modeled using the simplified isothermal model since the fluctuation of the daily atmosphere temperature was insignificant. Thus, the soil temperature, the thermal conductivity, and the volumetric heat capacity were assumed to be constant and equal to 10 ^{o}C, 400 kJ/days/m/°C, and 1875 kJ/m³/°C, respectively. For nodes at the bottom boundary of the model domain, the soil matric suction through the simulation was assumed to be constant and equal to -10 kPa (-1 m) considering the ground water level is about 4 m deep, and the temperature was set up to be 10 ^{o}C (see Figure 5.40).

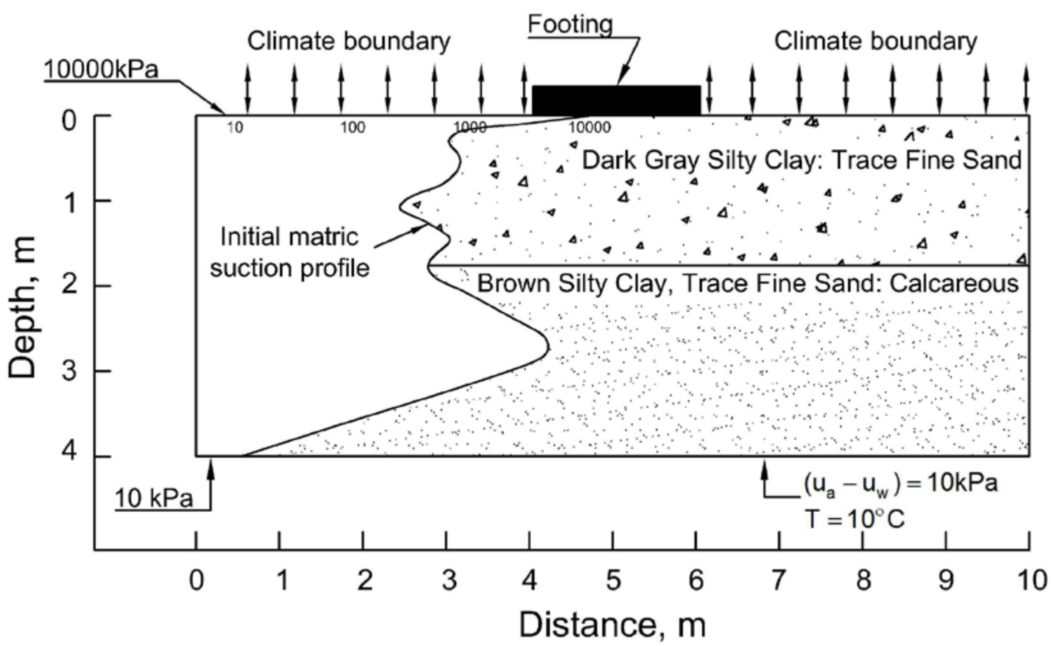


Fig. 5.40. The soil domain for Case Study E along with the initial and the boundary conditions used for the simulation of matric suction changes over time (modified after Zhang 2004)

The two year climate data (1 August, 1999 to 31 October, 2001) (Figure 5.38) was applied on the surface layer around the footing as a climate boundary condition. A “no flow” natural boundary condition was applied by default on the footing to represent the footing as an impervious layer. At the ground surface outside from the footing, the soil is covered by Johnsongrass (the most widely distributed naturalized warm-season, perennial grass in North America); therefore, the boundary conditions are controlled by the vegetation evepotranspiration. To mimic the in-situ condition, the vegetation was considered to be an excellent grass with triangular distribution roots of 0.22 m deep.

Figure (5.41) shows the leaf area index function (LAI) of the excellent vegetation with a maximum LAI value of 3 given in VADOSE/W (GeoSlope 2007), which was used for this simulation. The plant moisture limiting point and the wilting point were assumed to be 6000 kPa and 32000 kPa, respectively, as given by Zhang (2004).

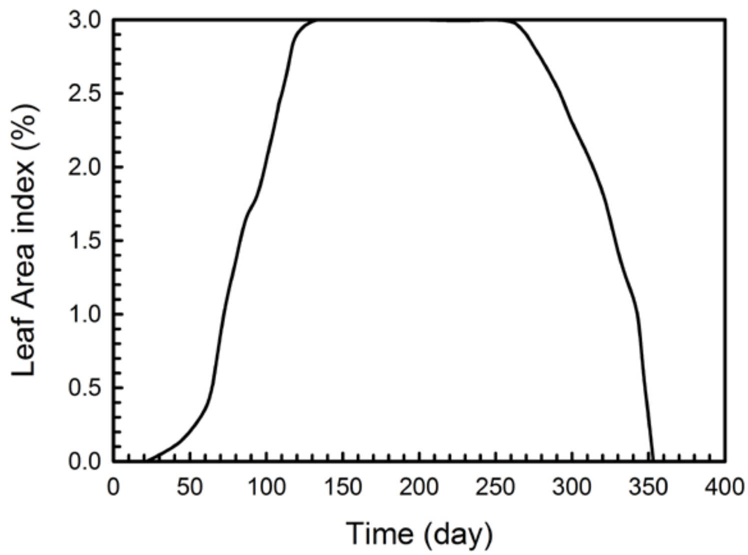


Fig. 5.41. Typical leaf area index function for excellent grass coverage (modified after GeoSlope 2007)

To check the input data used in the simulation, the initial water content profile obtained from VADOSE/W was compared with the initial water content presented in Zhang (2004) (Figure 5.42). The good comparison between the initial values of water content validates the simulation based on the used soil properties and boundary conditions. The model was also validated against the field measurements. Figure (5.43) shows a close agreement between the variations of the average values of the predicted and the measured water contents for the four footings over the 3 m depth with respect to time (the average percentage difference was 11%). This demonstrates that the soil model can be successfully used for predicting the variation of matric suction over time, which is the key information required for predicting the soil movement. Figure (5.44) shows the matric suction profiles at the corner of the modeled footing predicted over the two-year period. The matric suction profiles show significant variations near the ground surface that decrease down to a depth where the variation becomes small.

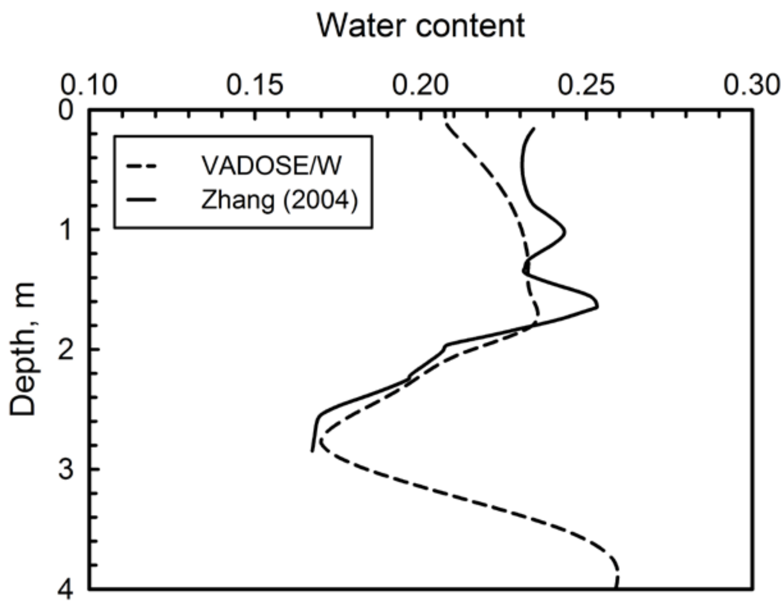


Fig. 5.42. Comparison between the initial water content profiles obtained from VADOSE/W and Zhang (2004)

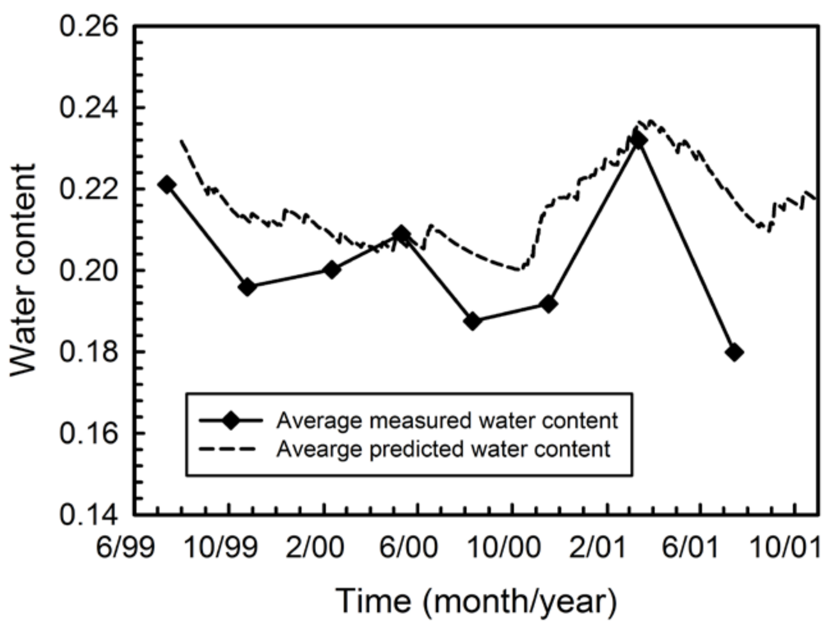


Fig. 5.43. The variations of the average values of the predicted and the measured water contents for the four footings over the 3 m depth with respect to time

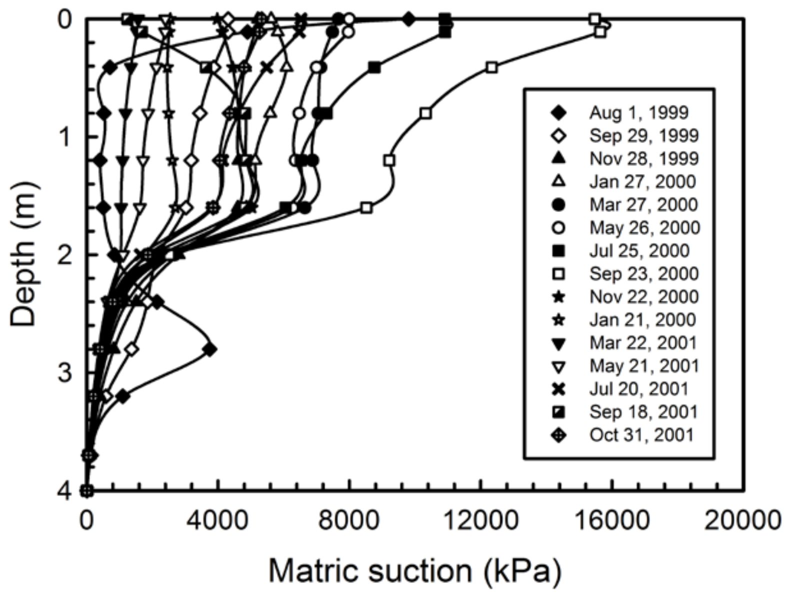


Fig. 5.44. Matric suction profiles at the corner of the footing simulated using VADOSE/W

5.7.3 Estimation of soil modulus of elasticity associated with matric suction

To estimate the modulus of elasticity associated with the changes in matric suction using the VO model (Equation 3.1), μ is assumed to be 0.4 as suggested by Zhang (2004), β is assumed to be 2 which is the recommended value for expansive soils, and α is assumed to be 1/10 in order to provide a reasonable comparison between the predicted and measured soil movements, and E_{sat} is calculated based on the analysis of the oedometer tests results given by Zhang (2004). Figure (5.45) shows the oedometer test results along with their best-fitted curves (Equation 5.5) for the investigated soils, i.e., the dark gray silty clay and the brown silty clay. Table (5.8) shows the fitting parameters of the void ratio constitutive relationships shown in Figure (5.45). Once the constitutive relationships of void ratio for the site soils are determined, the saturated modulus of elasticity E_{sat} can be calculated using equation (5.2). Figure (5.46) presents the variation of the saturated modulus of elasticity with depth for the investigated soils at the Arlington site (Case Study E).

Table 5.8. Fitting parameters of the relationship of void ratio versus the mean stress for the investigated soils at the Arlington site (Zhang 2004)

Soil material	Fitting parameters			
	a_1	b_1	x_1	y_1
Dark gray silty clay	0.49095	-0.42106	2.86157	0.19561
Brown silty clay	0.65549	-0.67522	3.48993	0.18089

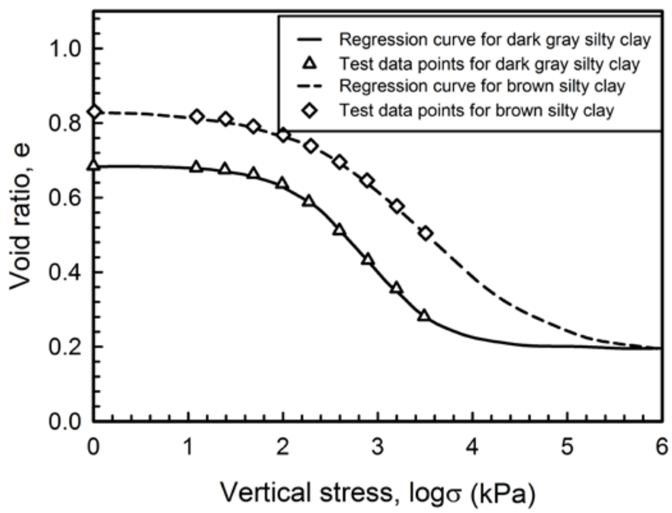


Fig. 5.45. Oedometer test results and their fitting curves for the specimens of dark gray silty clay and brown silty clay at the Arlington site (modified after Zhang 2004)

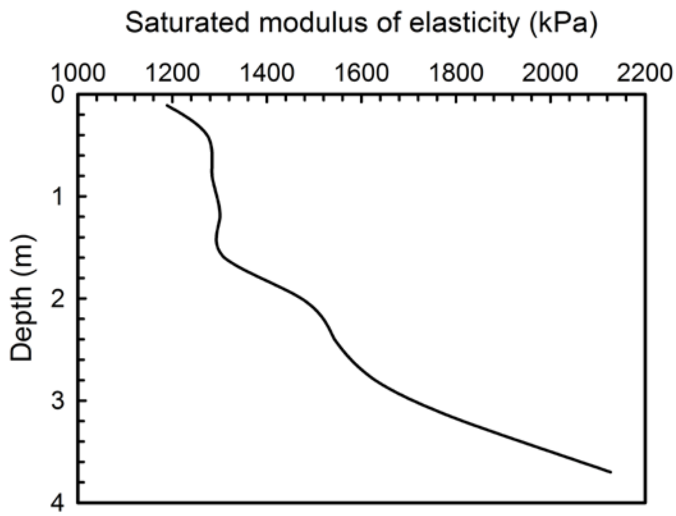


Fig. 5.46. Variation of the saturated modulus of elasticity with depth for the investigated soils at the Arlington site

As a result, the unsaturated modulus of elasticity E_{unsat} associated with the matric suction changes can be obtained from equation (3.1) in terms of the matric suction changes (Figure 5.44), the SWCCs (Figure 5.36), and the saturated modulus of elasticity values (Figure 5.46). The matric suction changes and the corresponding estimated unsaturated modulus of elasticity are then used in conjunction with the volume change constitutive relationship of soil structure to estimate the vertical soil movement over time.

5.7.4 Prediction of vertical soil movement over time

The 4 m depth of soil profile consisted of the two soils (i.e., Black gray silty clay and brown silty clay) (Figure 5.40) was subdivided into ten layers. The thickness of each layer was chosen such that the border between the two soils was located at a layer boundary. The top two layers were suggested to have a thickness of 0.22 m (equivalent to the thickness of the root depth) and 0.38 m, respectively, the bottom layer was assumed to be with 0.6 m thickness, and the thickness of the rest layers were assumed to be 0.4 m.

The amount of soil movement for each soil layer associated with the change in matric suction for each day was estimated using equation (4.6). The vertical soil movement of each layer at a given time was calculated as a cumulated value of the soil movements for all days prior to that given time. The total vertical soil movement at the ground surface was obtained from the summation of the soil movements of all layers (Equation 4.7).

Figure (5.47) shows the predicted soil movements at the corner of the modeled footing and the field measurements of the soil movement for the four footings. Compared with the field measurements, the predicted soil movements reasonably matched the measured movements in both tendency and magnitude over the first year. However, during the second year, the predicted and the measured movements did not lead as good a match as the values during the first year. It can be seen that the relatively steady evenly distributed rainfall during the first year caused very small in situ movements, whereas the worst drought in the history of the investigated area occurred during the second year caused much large soil shrinkage in the field. The findings show that the proposed MEBM has some limitations regarding its use to model the shrinkage behavior of expansive soils. This can be attributed to the complexities associated with the mechanism of soil shrinkage, and the difficulty to quantify the influence of shrinkage cracks on the soil movement.

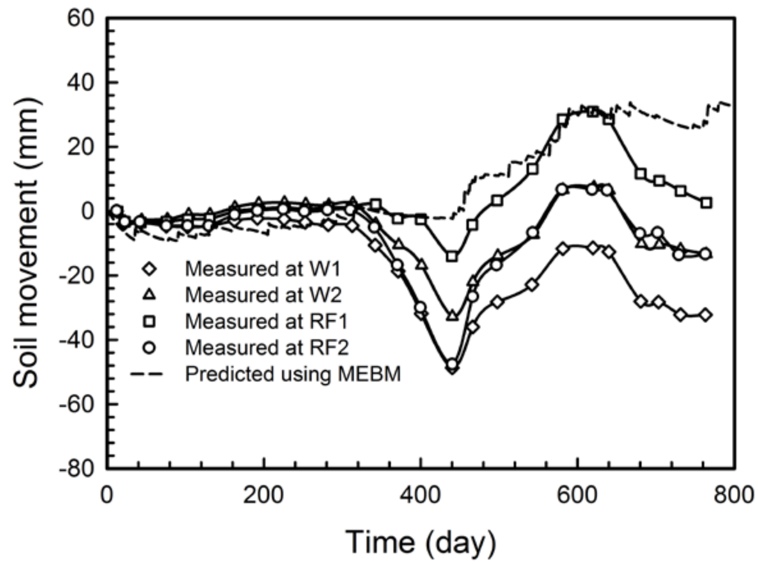


Fig. 5.47. Comparison between the predicted soil movements at the corner of the modeled footing using the MEBM and the field measurements of the soil movement for the four footings

Review of Figure (5.47) shows that the predicted soil movements, on one hand, and the measured soil movement at the footing RF1 (which doesn't experience a large amount of shrinkage), on the other hand, leads to a good comparison over the two years. The maximum values of soil heave from both the prediction and the measurement agree well and are approximately equal to 32 mm. The results demonstrate that the MEBM can be used for estimating the soil heave over time with a reasonable degree of confidence.

Briaud et al. (2003) and Zhang (2004) used two different methods to predict the field measurements of the footing movements over time at the Arlington site. The details of both methods are presented in section 2.9.2. Figure (5.48) shows the comparison of the predictions of soil movement using the MEBM, Briaud et al. (2003) method, and Zhang (2004) method with the average field measurements of soil movement for each footing.

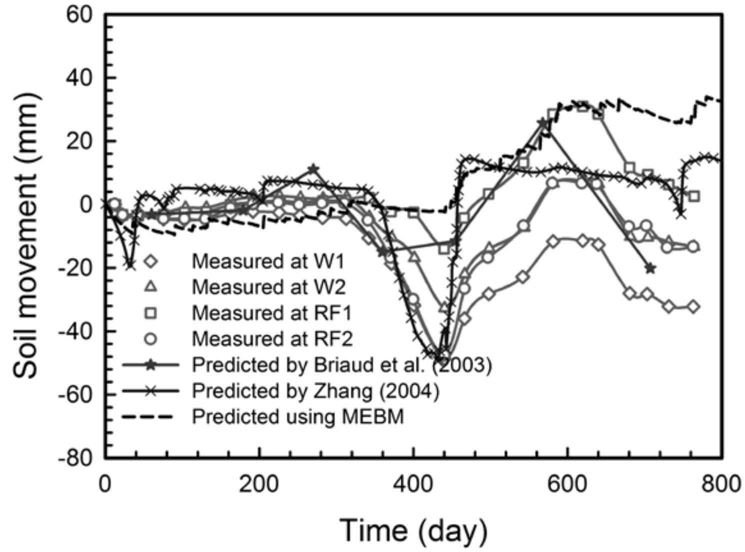


Fig. 5.48. Comparison of the soil movement predicted using different methods with the average field measurements of soil movement for the four footings

Figure (5.48) shows that, similar to the MEBM, Briaud et al. (2003) method reasonably predicts the soil heave but not the soil shrinkage. Briaud et al. (2003) method is based on the information of soil water content which is simpler and more reliable to measure in comparison to the soil matric suction. However, it is uncoupled method where only the influence of moisture variation on the soil volume change is considered. In addition, Briaud et al. (2003) method requires a shrink test which is difficult to perform when the soil is highly fractured. Another drawback is that any theoretical consideration must make use of the SWCC to transform the equations from being suction-based method to being water content-based method (Briaud et al. 2003).

Review of Figure (5.48) also shows that the predicted results of Zhang (2004) modeling study don't match the measurements of the soil movement for the four footings. The modeled footing moved upward faster than the field measurements. This reflects that the proposed approaches for modeling of grass root zone and for the construction of the constitutive surfaces of the soil properties (void ratio, water content, degree of saturation, permeability function) are not good enough for practical applications. For example, the constitutive surfaces were constructed based on testing soils under conditions not experienced in field, such as a shrinkage test at no normal stress, or consolidation test at fully saturated conditions.

5.8 Summary

The proposed Modulus of Elasticity Based Method (MEBM) is evaluated in this chapter to estimate the vertical movement of natural expansive soils associated with the changes in the environmental condition over time. The MEBM is based on the theoretical concepts of unsaturated soils. It involves integrating the numerical modeling results of the soil-atmospheric VADOSE/W program and the volume change constitutive equation for unsaturated soils. The semi-empirical model proposed by Vanapalli and Oh (2010) (VO model) was extended for unsaturated expansive soils and used as a tool to estimate the variations of the modulus of elasticity with respect to matric suction. The fitting parameters $\beta = 2$ and $\alpha = 0.05\text{-}0.15$, as suggested in Chapter Three, were also used in this chapter to provide reasonable estimations of the soil movement for the case studies under consideration. The MEBM is a simple approach that requires only the matric suction changes within the active zone depth and the associated modulus of elasticity to predict the heave-shrinkage behavior of unsaturated expansive soils.

The performance of the MEBM was tested in five case studies originally investigated in the literature by other researchers. The investigated case studies are representative candidates of a variety of site conditions. Different scenarios with different initial and boundary conditions were used to simulate each of the case studies. The results of the MEBM reasonably agree with the published results (measurements/estimates) of the case studies. The findings of this study demonstrate that the MEBM can be used with a greater degree of confidence in engineering practice to predict the in situ expansive soil movements with respect to time.

CHAPTER 6

ELASTICITY MODULI OF UNSATURATED EXPANSIVE SOILS FROM DIMENSIONAL ANALYSIS

6.1 Introduction

The modulus of elasticity of unsaturated soils depends on numerous parameters such as (i) the initial level of compaction (dry density, or void ratio), (ii) the initial state hydration (water content, degree of saturation, or matric suction), and (iii) the confinement (deviator stress, or lateral stress). The other factors that affect the modulus of elasticity include variables such as boundary conditions, Poisson's ratio, specimen dimensions, soil structure (the size of soil particles), stress path, and stress history, to list a few. The influence of all these parameters should be considered for a reliable estimation of the soil modulus of elasticity. However, accounting the influence of all these parameters requires extensive experimental programs and multi-variable regression analyses. Such an approach is cumbersome to implement in the conventional geotechnical engineering practice. Due to this reason, the modulus of elasticity of unsaturated soils has been expressed in the literature as a function of only one or two parameters. Different investigators such as Zhang et al. (2012) and Lu and Kaya (2014) proposed a power function to quantitatively describe the relationship between the soil modulus of elasticity and the soil water content. In Chapter Three, the semi-empirical model proposed by Vanapalli and Oh (2010) (i.e., VO model) was used for estimating the modulus of elasticity of unsaturated expansive soils as a function of matric suction changes, neglecting the influence of mechanical stress changes. Such an assumption is conservative and can be extended in practice for pavements and lightly loaded residential structures, as per the earlier discussions presented in Chapter Four. Some other investigators (e.g. Rahardjo et al. 2011) have linked the modulus of elasticity to the change in both the net normal stress (i.e. the mechanical stress) and the matric suction using multiple regression methods; this approach is rigorous but it is time consuming from the view point of conducting various experimental studies.

To alleviate some of the challenges associated with conducting cumbersome experimental investigations to estimate the modulus of elasticity of unsaturated expansive soils, dimensional analysis (hereafter referred to as DA) is used in this chapter as a tool to propose an alternative approach. Some researchers (Butterfield 1999, Palmer 2008, Buzzi 2010, Buzzi et al. 2011) have used the DA in a few geotechnical engineering applications; however, this approach is not widely used in practice. One of the main advantages of the DA is that it allows intelligent experiments; i.e., a reduction of the number of tests to be performed to characterize a physical phenomenon taking account of the influence of all the parameters of an activity or a phenomenon in the engineering applications. This is possible through the use of dimensionless parameters, the number and form of which can be derived from the Buckingham Pi theorem (Buckingham 1914).

For soils that are in a state of unsaturated condition, Matyas and Radhakrishna (1968), Barden et al. (1969), Fredlund and Morgenstern (1977), Alonso et al. (1990), and Gallipoli et al. (2003) proposed constitutive equations in terms of the soil void ratio with respect to the changes associated with the net normal stress (i.e. the mechanical stress) and the matric suction. Along similar lines, a dimensionless model extending the DA is proposed in this chapter to estimate the modulus of elasticity for unsaturated expansive soils, considering the influence of the state of hydration of soil expressed in terms of the matric suction and the degree of saturation, the level of compaction and the confinement described by the initial void ratio and the confining stress, respectively. Experimental results of conventional and suction-controlled triaxial tests for three expansive soils from Zao-Yang, Nanyang, and Guangxi in China, published in Zhan (2003), Miao et al. (2002), and Miao et al. (2007), respectively, that were used in Chapter Three for extending the VO model for unsaturated expansive soils, are analyzed to form the dimensionless parameters towards reliably estimating the soil modulus of elasticity. The validation of the proposed dimensionless model is conducted by comparing its estimations of the elasticity moduli with the experimental values of the elasticity moduli obtained from the triaxial shear tests. The proposed dimensionless model is also verified using the estimated values of the elasticity moduli from the semi-empirical model (i.e., VO model) presented in Chapter Three.

In addition, the dimensionless model is also used as a tool in the modulus of elasticity based method (MEBM) to estimate the modulus of elasticity of unsaturated expansive soils for the most complicated case study (Case Study D) by Ng et al. (2003), considering the influence of climatic conditions, soil properties, and soil cracks. Comparisons are provided between the resulting soil movements and the field measurements of this case study.

6.2 Dimensional Analysis Background

The dimensional analysis (DA) is a mathematical tool that shapes the general form of relations that describe natural phenomena. The application of DA to any particular physical phenomenon is based on the premise that the phenomenon can be described by a list (V) of l variables (V_1, V_2,….., V_l), encompassing a total of m independent primary dimensions (D) = (D_1, D_2, ….., D_m) (e.g. mass, length, time, temperature) which is the minimum number of reference dimensions required to describe the physical variables. The term ‘variables’ includes both the independent parameters of a specific system (e.g. size, density, mass) and the dependent quantities such as displacements, stresses, and bending moments (Butterfield 1999).

The objective of the DA is to minimize the dimension space in which the behavior of specific system might be studied by combining assumed governing variables V into N dimensionless parameters, N being less than V. In particular, Buckingham’s (1914) theorem states that an initial equation involves l variables and m dimensions can always be reduced to a dimensionless relationship involving only N dimensionless parameters, where

$$N = l - m \qquad (6.1)$$

The resulting N dimensionless parameters are conventionally labelled as $(\pi_1, \pi_2, ..., \pi_N)$ (i.e. Pi groups). As each π is dimensionless, the final function must be dimensionless, and therefore dimensionally

$$f\ (\pi_1, \pi_2, ..., \pi_N) = M^0 L^0 T^0 \qquad (6.2)$$

The form of function f is not provided by the DA but it is usually approximated by an empirical, dimensionless equation fitted to either model or prototype data. In addition, the Buckingham theorem does not provide any specific guidance related to the choice of the variables, which appear in each N parameter (i.e. Pi group) used for the reduction of the problem. In order to enable systematic computation of dimensionless numbers, the input and output variables of a concept are considered as performance variables. The choice of repeating variables should be done within the concept's internal variables and according to the unique number of the system's governing dimensions for best results (Christophe et al. 2008).

6.3 Dimensional Analysis and Combination of Parameters

The dimensional analysis (DA) is used as a tool in this chapter to propose a dimensionless model for estimating the modulus of elasticity of unsaturated expansive soils. The application of DA to account for all of the factors influencing the value of the modulus of elasticity of unsaturated expansive soils is challenging. This is due to numerous properties and parameters that influence the modulus of elasticity of unsaturated soils. In the present study, initial void ratio, matric suction, degree of saturation, confining pressure, and deviator stress changes are assumed to be primary factors. Experimental data resulting from the triaxial shear tests on unsaturated expansive soils, performed under different confining stresses with varying matric suctions, are used to apply the DA. Hence, the primary factors influencing the soil modulus of elasticity can be expressed in terms of the initial height of soil specimen h_0, the change in specimen height upon compression Δh, the volume of voids V_{vo}, the volume of water V_w, the volume of solid particles V_s, the matric suction $(u_a - u_w)$, the confining stress σ_3, and the change in deviator stress $\Delta(\sigma_1 - \sigma_3)$. The modulus of elasticity of unsaturated expansive soils can be described using the list of these parameters as shown in equation (6.3).

$$f(\Delta h, h_0, V_{vo}, V_s, V_w, (u_a - u_w), \Delta(\sigma_1 - \sigma_3), \sigma_3) = 0 \tag{6.3}$$

Equation (6.3) involves 8 independent variables and 3 dimensions (mass, length, and time). According to the Buckingham Pi theorem (Buckingham 1914), equation (6.3) can be reduced to a simpler equation involving 5 dimensionless parameters (i.e., 8 independent variables – 3 dimensions) which depend on the combination of variables present in equation (6.3). For a group of variables appearing in each dimensionless parameter, the variables have to be combined in such a way that the powers of each of the 'dimensions' appearing in the group are separately zero. Several combinations are possible to form dimensionless parameters. However, the correct sets of the dimensionless parameters are needed to be selected and have to be verified through experimental evidence. Equation (6.4) presents the used dimensionless parameters that are chosen in this study based on obtaining most satisfactory agreement between the experimental and the estimated values of the modulus of elasticity for the studied soils.

$$f\left(\frac{E_{unsat} - E_{sat}}{P_a}, \frac{(\sigma_3 - u_a)(u_a - u_w)}{P_a^2}, \frac{(u_a - u_w)}{P_a}, S, e_0\right) = 0 \tag{6.4}$$

where E_{unsat} and E_{sat} are the elasticity moduli under unsaturated and saturated conditions, respectively, S is the degree of saturation, e_0 is the initial void ratio, and P_a is the atmospheric pressure (i.e., 101.3 kPa) used for maintaining the parameters of the equation (6.4) dimensionless. The DA is applied on experimental results of conventional and controlled-suction triaxial tests available in the literature. The state of hydration of soil is expressed in terms of the matric suction and the degree of saturation, the level of compaction is described by the initial void ratio, and the confinement is described by the confining stress. For simplicity and to significantly reduce the number of tests required, the four dimensionless parameters that are related to the degree of saturation, matric suction, initial void ratio, and net confining stress for each of the tests are incorporated into one unique dimensionless parameter X. This is more convenient as the equation to estimate the modulus of elasticity of unsaturated soils can be reduced to a relationship between only two entities. Using X allows accounting for the four influencing parameters via only one dimensionless parameter, and equation (6.4) is modified to

$$f\left(\frac{E_{unsat} - E_{sat}}{P_a}, X\right) = 0 \tag{6.5}$$

The parameter X can be defined in terms of several combinations of the dimensionless parameters by a calibration procedure. The calibration in this study is achieved using a program code for the suggested formula of X that incorporates the dimensionless parameters with several exponents. The program code changes the exponents for the formula incrementally and calculates the soil modulus of elasticity. The calibrated formula of X is determined on the basis of the best agreement between the experimental and the estimated values of the soil modulus of elasticity. Based on the results of triaxial tests for the three unsaturated expansive soils from Zao-Yang, Nanyang, and Guangxi in China, presented in Chapter Three, the following relationship is suggested to be the calibrated formula of X.

$$X = \left(\frac{1}{e_0}\right)\left[\left(\frac{(\sigma_3 - u_a)(u_a - u_w)}{P_a^2}\right) + \left(S^{0.1}\frac{(u_a - u_w)}{P_a}\right)\right]^{0.7} \tag{6.6}$$

The relation of the exponents of equation (6.6) with other factors influencing the soil modulus of elasticity under unsaturated condition is not clear. The exponents of the equation could depend on soil structure, mechanical history, and other soil properties. No unique or well defined relationship could be derived despite attempts to correlate the values of the exponents to the soil properties reported from the three discussed studies (Zhan 2003, Miao et al. 2002, Miao et al. 2007).

According to the DA, equation (6.5) can be written in terms of the dimensionless parameter X as

$$\frac{E_{unsat} - E_{sat}}{P_a} = f(X) \tag{6.7}$$

The only manner to assess the proposed dimensionless model (Equation 6.7) is to use experimental data and plot the results in terms of X versus $(E_{unsat} - E_{sat})/P_a$. The results of triaxial tests presented in Chapter Three for three different expansive soils are used for examining the validity of equation (6.7). The values of elasticity moduli of soils in the first entity of equation (6.7) are experimentally determined from the stress-strain curves of triaxial tests during shearing of saturated/unsaturated compacted specimens under different confining stresses and matric suctions. As previously discussed in Chapter Three, the experimental values of modulus of elasticity are determined as the reciprocal of intercept of the straight lines resulting from plotting the stress-strain relationships on the transformed axes ε and $\varepsilon/(\sigma_1 - \sigma_3)$ (Figure 3.4). If a good correlation with a reasonable coefficient of determination R^2 could be found between the two entities X and $(E_{unsat} - E_{sat})/P_a$, equation (6.7) would be used to back-calculate the soil modulus of elasticity at any unsaturated condition.

The above analysis has been extended, and comparisons are provided between the values of the modulus of elasticity derived from the triaxial tests results and the proposed dimensionless model (Equation 6.7) to check its capability for estimating the modulus of elasticity of expansive soils.

6.4 Triaxial Tests Results Used in the Dimensional Analysis

The three data sets from Zhan (2003), Miao et al. (2002), and Miao et al. (2007) for compacted expansive soils from Zao-Yang, Nanyang, and Guangxi in China, respectively, that were analyzed in Chapter Three, are also used in the dimensional analysis. The key soil properties and the results of conventional and suction-controlled triaxial tests on saturated and unsaturated compacted specimens of these three soils under different confining stresses and matric suctions are presented in Chapter Three. The stress versus strain relationships of the triaxial tests were analyzed and plotted on the transformed axes ε and $\varepsilon/(\sigma_1 - \sigma_3)$. The straight line equation (3.6) was used to fit the data. The experimental values of soil modulus of elasticity were determined as the reciprocal of the intercepts of the resulting straight lines. The experimental values of elasticity moduli for Zao-Yang, Nanyang, and Guangxi expansive soils under saturated and unsaturated conditions are summarized in Table (3.2), (3.3), and (3.4), respectively.

6.5 The Application of Dimensional Analysis for Estimating the Soil Modulus of Elasticity

The first step to apply the DA approach for estimating the modulus of elasticity of unsaturated expansive soils involves calculating the dimensionless parameter X using equation (6.6). The data required to calculate X are available for the three soils (i.e., Zao-Yang, Nanyang, and Guangxi expansive soils), which include the initial void ratio e_0, the net confining stress $(\sigma_3 - u_a)$, the matric suction $(u_a - u_w)$, and the degree of saturation S. The second step involves plotting X versus $(E_{unsat} - E_{sat})/P_a$ obtained from the results of triaxial tests. A good correlation between the two entities X and $(E_{unsat} - E_{sat})/P_a$ validates the proposed DA approach for estimating the modulus of elasticity of unsaturated expansive soils. By using the regression analysis, the best fit equation can be found for the relationship between X and $(E_{unsat} - E_{sat})/P_a$ (Equation 6.7) (i.e., the dimensionless model).

Seven controlled-suction triaxial shear tests conducted by Zhan (2003) on unsaturated compacted specimens of Zao-Yang soils, the results of which are available in Zhan (2003), were used in the DA. First, the dimensionless parameter X was calculated using the tests data shown in Table (6.1). Then, plotting the parameter X versus $(E_{unsat} - E_{sat})/P_a$ for unsaturated specimens tested under each net confining stress led to satisfactory correlations; the coefficients of determination R^2 = 0.98 and 0.97 were obtained for the soil specimens tested under $(\sigma_3 - u_a)$ = 50 kPa and 200 kPa, respectively (Figure 6.1). In the present analysis, hyperbolic models were chosen to fit the data; however, other trends could be used if they lead to a higher coefficient of determination.

The results suggest that the proposed DA can be successfully applied on the triaxial tests results of Zao-Yang soils.

Table 6.1. Experimental data and the corresponding dimensionless parameter X for the compacted specimens of Zao-Yang soils under unsaturated conditions (data from Zhan (2003))

$(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	e_0	S	X
50	25	0.779	0.783	0.63
50	50	0.764	0.745	1.04
50	100	0.746	0.701	1.73
50	200	0.730	0.685	2.87
200	25	0.698	0.866	1.15
200	100	0.676	0.769	3.12
200	200	0.699	0.707	4.90

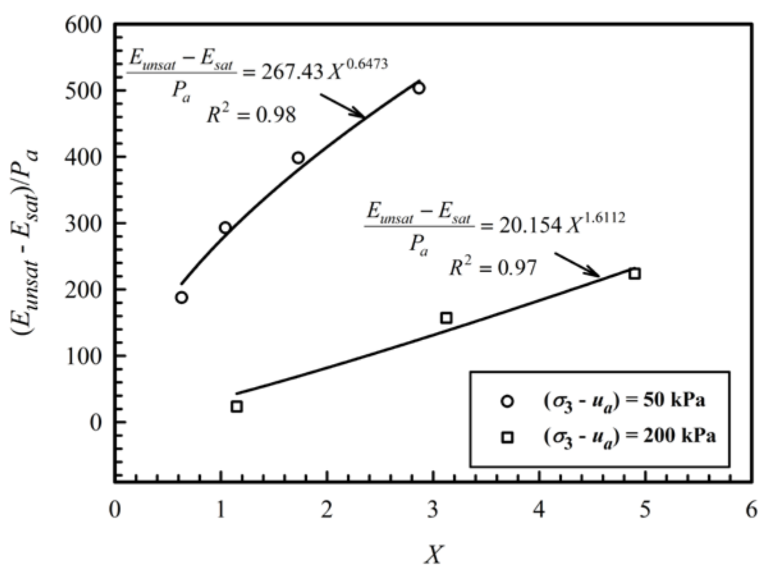


Fig. 6.1. The relationship of the dimensionless parameter X versus $(E_{unsat} - E_{sat})/P_a$ for compacted specimens of Zao-Yang soils tested under different confining stresses

Equations (6.8) and (6.9) represent the hyperbolic models that have been used to fit the relationships of X versus $(E_{unsat} - E_{sat})/P_a$ for the specimens tested under confining stress $(\sigma_3 - u_a)$ of 50 and 200 kPa, respectively.

$$\frac{E_{unsat} - E_{sat}}{P_a} = 267.43(X)^{0.65} \tag{6.8}$$

$$\frac{E_{unsat} - E_{sat}}{P_a} = 20.15(X)^{1.61} \tag{6.9}$$

Equations (6.8) and (6.9) have been used to back-calculate the modulus of elasticity of Zao-Yang soil under unsaturated condition E_{unsat} for $(\sigma_3 - u_a)$ = 50 kPa and 200 kPa, respectively. Figure (6.2) shows the comparisons between the experimental and the estimated values of the unsaturated modulus of elasticity E_{unsat}. A good agreement was observed between the elasticity moduli obtained from the experimental results and the proposed dimensionless model (R^2 = 0.98 for $(\sigma_3 - u_a)$= 50 kPa, and R^2 = 0.92 for $(\sigma_3 - u_a)$= 200 kPa).

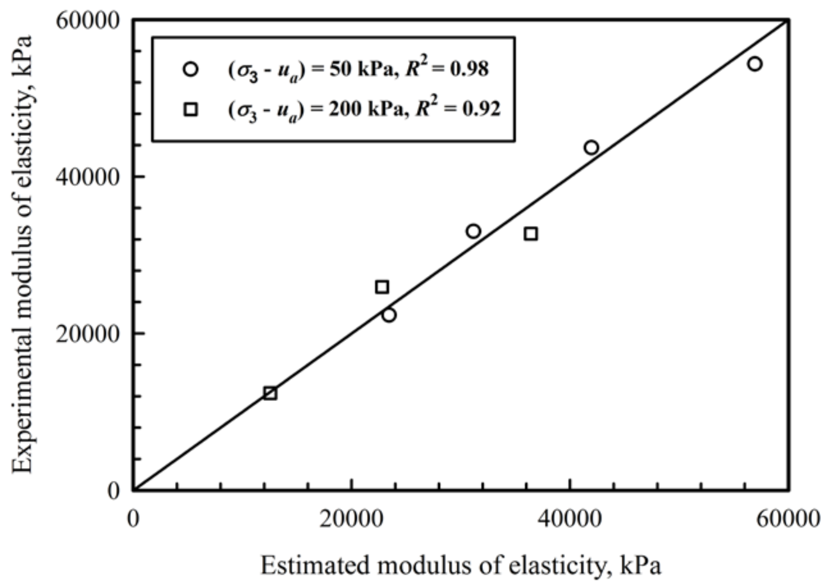


Fig. 6.2. Comparison between the experimental and the estimated values of the unsaturated modulus of elasticity for Zao-Yang soil

The proposed DA approach was also extended for the triaxial tests carried out by Miao et al. (2002) on the compacted specimens of Nanyang soil. The initial void ratio, the degree of saturation, the matric suction, and the net confining stress for each test are provided in

Table (6.2). The dimensionless parameter X was calculated for each test using equation (6.6). The experimental values of elasticity moduli of Nanyang soil summarized in Table (3.3) were used to calculate $(E_{unsat} - E_{sat})/P_a$. The relationship of X versus $(E_{unsat} - E_{sat})/P_a$ was plotted for each confining stress (i.e., each group) as shown in Figure (6.3). Good correlations were found between X and $(E_{unsat} - E_{sat})/P_a$ for the series of the tests with the exception of the tests group of 112.5 kPa net confining stress. This set of data appears to be inconsistent with the remainder of the results (the tests groups of 25 kPa and 62.5 kPa net confining stress). This may be attributed to considering the average value of the net confining stresses to represent a group of tests, or to a measuring error during some tests which leads to no change in the modulus of elasticity as a response of a change in matric suction (see Table (3.3)).

Table 6.2. Experimental data and the corresponding dimensionless parameter X for the compacted specimens of Nanyang soil under unsaturated conditions (data from Miao et al. (2002))

$(\sigma_3 - u_a)$ (kPa)	Average $(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	e_0	S	X
30	25	50	0.8	0.71	0.87
20		80	0.8	0.68	1.21
30		120	0.8	0.65	1.60
20		200	0.8	0.61	2.29
50	62.5	50	0.8	0.71	1.05
70		80	0.8	0.68	1.46
80		120	0.8	0.65	1.93
50		200	0.8	0.61	2.76
100	112.5	50	0.8	0.71	1.27
120		80	0.8	0.68	1.77
130		120	0.8	0.65	2.34
100		200	0.8	0.61	3.34

The relationships between X and $(E_{unsat} - E_{sat})/P_a$ for the specimens of Nanyang soil tested under average values of 25 kPa and 62.5 kPa confining stress (Figure 6.3) can be expressed using the best fitting equation (6.10) and equation (6.11), respectively.

$$\frac{E_{unsat} - E_{sat}}{P_a} = 114.09(X)^{1.53} \qquad (6.10)$$

$$\frac{E_{unsat} - E_{sat}}{P_a} = 96.08(X)^{1.00} \qquad (6.11)$$

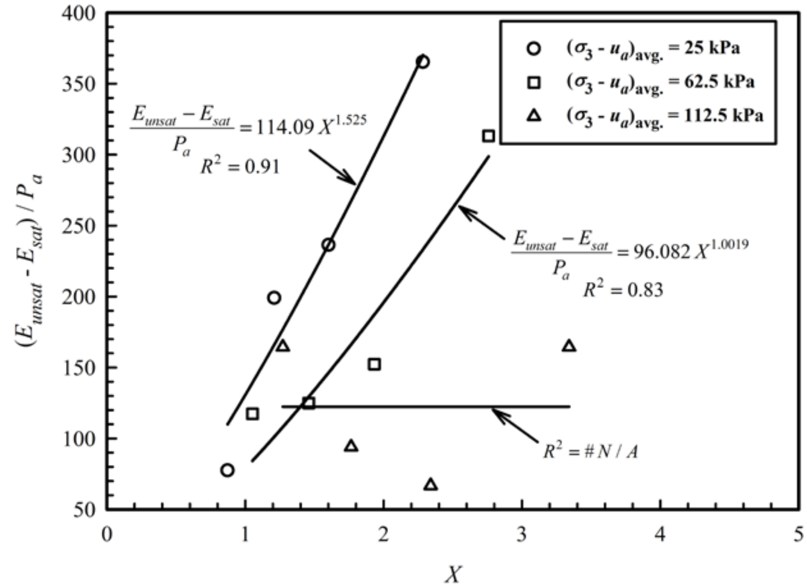


Fig. 6.3. The relationship of X versus $(E_{unsat} - E_{sat})/P_a$ for compacted specimens of Nanyang soil tested under different confining stresses

Equations (6.10) and (6.11) were used to back-calculate the elasticity moduli of Nanyang soil under unsaturated conditions E_{unsat}. Figure (6.4) presents the comparison between the experimental and estimated values of unsaturated modulus of elasticity E_{unsat}. The coefficients of determination were reasonable (R^2 = 0.92 for $(\sigma_3 - u_a)$ = 25, and R^2 = 0.8 for $(\sigma_3 - u_a)$ = 62.5 kPa). It is concluded that the unsaturated modulus of elasticity at a given confining stress for the compacted specimens of Nanyang soil can be reasonably estimated using the DA. However, the applicability of the DA to the tests group of 112.5 kPa net confining stress requires further investigation.

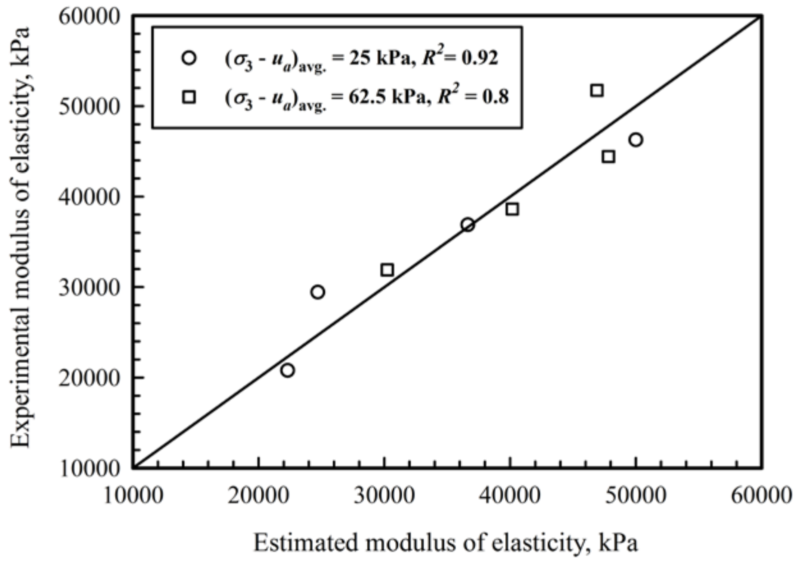


Fig. 6.4. Comparison between the experimental and the estimated values of the unsaturated modulus of elasticity for Nanyang soil

Other data sets used to validate the proposed dimensionless model for reproducing E_{unsat} were given by Miao et al. (2007) (i.e., Guangxi soil). The data shown in Table (6.3) were used to calculate the dimensionless parameter X (Equation 6.6). Figure (6.5) shows $(E_{unsat} - E_{sat})/P_a$, calculated in terms of the experimental values of saturated/unsaturated modulus of elasticity of Guangxi soil, versus X for each set of triaxial tests conducted under a certain net confining stress. Equations (6.12–6.14) represent the relationships between X and $(E_{unsat} - E_{sat})/P_a$ for the data sets under 50, 100, and 200 kPa of net confining stress, respectively.

$$\frac{E_{unsat} - E_{sat}}{P_a} = 10.74(X)^{2.62} \tag{6.12}$$

$$\frac{E_{unsat} - E_{sat}}{P_a} = 19.78(X)^{1.67} \tag{6.13}$$

$$\frac{E_{unsat} - E_{sat}}{P_a} = 54.11(X)^{0.66} \tag{6.14}$$

Table 6.3. Experimental data and the corresponding dimensionless parameter X for the compacted specimens of Guangxi soil under unsaturated conditions (data from Miao et al., 2007)

$(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	e_0	S	X
50	179.37	0.824	0.76	2.37
50	121.81	0.824	0.85	1.81
50	57.37	0.824	0.92	1.08
100	179.37	0.824	0.76	2.90
100	121.81	0.824	0.85	2.22
100	57.37	0.824	0.92	1.31
200	179.37	0.824	0.76	3.86
200	121.81	0.824	0.84	2.95
200	57.37	0.824	0.92	1.74

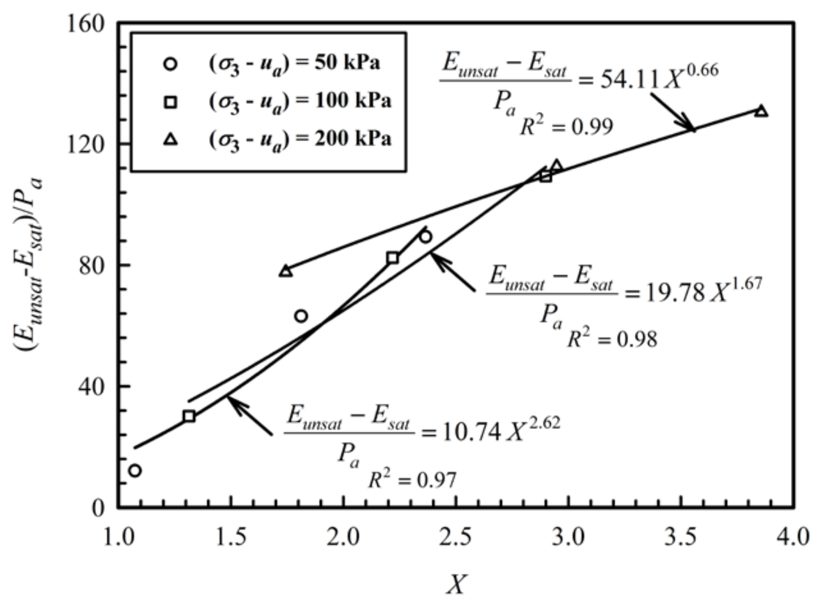


Fig. 6.5. The relationship of X versus $(E_{unsat} - E_{sat})/P_a$ for compacted specimens of Guangxi soil tested under different confining stresses

In Figure (6.5), the coefficients of determination of 0.97, 0.98, and 0.99 were obtained for specimens tested under the net confining stress of 50, 100, and 200 kPa, respectively. The excellent correlation provides a greater degree of confidence for the applicability of the *DA* approach to the experimental data of Guangxi soil.

Figure (6.6) presents the comparison between the experimental values of E_{unsat} derived from the triaxial tests and those back-calculated using equations (6.12–6.14) for $(\sigma_3 - u_a)$ = 50, 100, and 200 kPa, respectively. A close agreement was obtained between the values of the elasticity moduli. The correlation was relatively high (R^2 = 0.92, 0.97, and 0.99). Consequently, the E_{unsat} of the Guangxi soil compacted at a given net confining stress can be reliably estimated using the proposed dimensionless model.

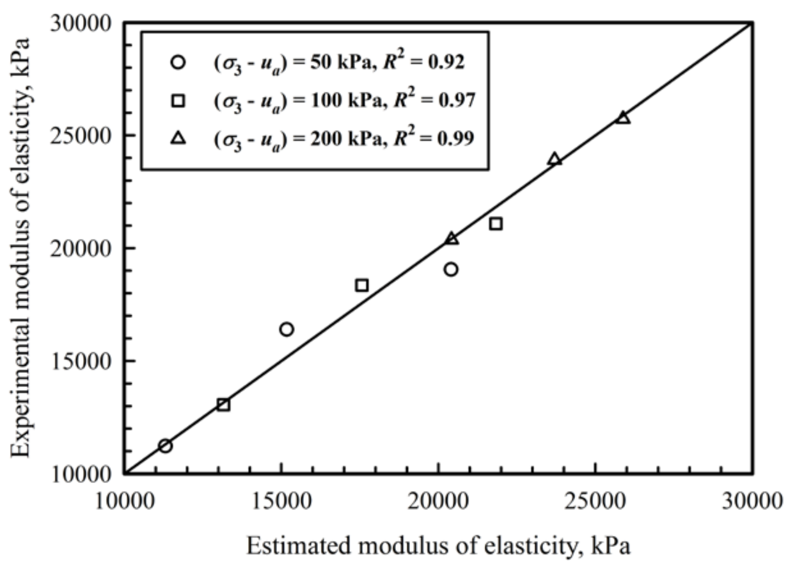


Fig. 6.6. Comparison between the experimental and estimated values of the unsaturated modulus of elasticity for Guangxi soil

6.6 Verification of the Proposed Dimensionless Model

The verification of the proposed dimensionless model was assessed by comparing its estimations of the soil modulus of elasticity with those obtained from the VO model presented in Chapter Three. The VO model (Equation 3.1) was proposed by Vanapalli and Oh (2010) for the estimation of the unsaturated modulus of elasticity of both coarse and fine-grained soils with plasticity index I_p < 16%. In Chapter Three, the VO model was extended for unsaturated expansive soils (i.e., I_p > 16%).

$$E_{unsat} = E_{sat}\left[1 + \alpha \frac{(u_a - u_w)}{P_a / 101.3} S^{\beta}\right] \quad (3.1)$$

where E_{unsat} and E_{sat} is soil modulus of elasticity under unsaturated and saturated condition, respectively, $(u_a - u_w)$ is matric suction, P_a is atmospheric pressure (P_a = 101.3 kPa), and S is degree of saturation.

The values of the unsaturated modulus of elasticity E_{unsat} for the three expansive soils (Zao-Yang, Nanyang, and Guangxi expansive soils) were reasonably estimated using the VO model (Equation 3.1) as presented in Chapter Three. The fitting parameters β = 2 and α = 0.05–0.15 were found to provide the elasticity moduli for the three expansive soils that reasonably agree with the values of the elasticity moduli obtained from the triaxial shear tests (R^2 = 0.91) (see Figure 3.18).

Figure (6.7) presents the comparison of the values of E_{unsat} for the three expansive soils obtained from the proposed dimensionless model and the VO model. It can be seen that the dimensionless model provides a good agreement with the estimations of E_{unsat} from the VO model. The coefficient of determination was relatively high (R^2 = 0.9). As a result, the proposed dimensionless model can be used for estimating the modulus of elasticity of unsaturated expansive soils under any net confining stress (i.e. any loading condition) with varying matric suctions.

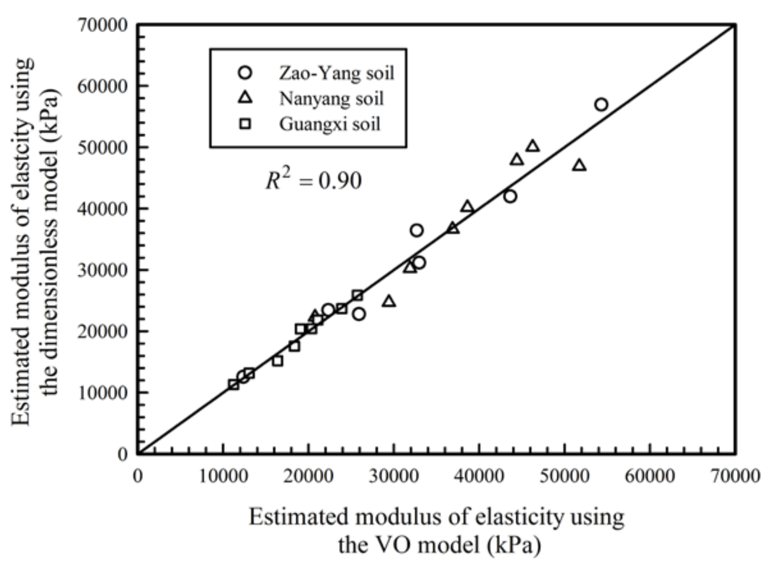


Fig. 6.7. Comparison between the values of elasticity moduli estimated from the proposed dimensionless model and the VO model for the three investigated expansive soils

The VO model (Equation 3.1) neglects the influence of the mechanical stress on the modulus of elasticity of unsaturated expansive soils. Such an assumption is conservative and can be extended in practice for pavements and lightly loaded residential structures, where the influence of net normal stress is insignificant and only matric suction changes have a predominant influence on the soil volume change. However, compared with the results of the VO model, the closer agreement between the E_{unsat} values from the dimensionless model and the triaxial tests results with higher coefficient of determination (R^2 = 0.97) (Figure 6.8) suggests that the dimensionless model (Equation 6.7) can be used with greater confidence than the VO model (Equation 3.1). In other words, the proposed dimensionless model is more rigorous and reliable, and it can be used for all scenarios of loading conditions (both lightly and heavily loaded structures) for the estimation of modulus of elasticity for unsaturated expansive soils.

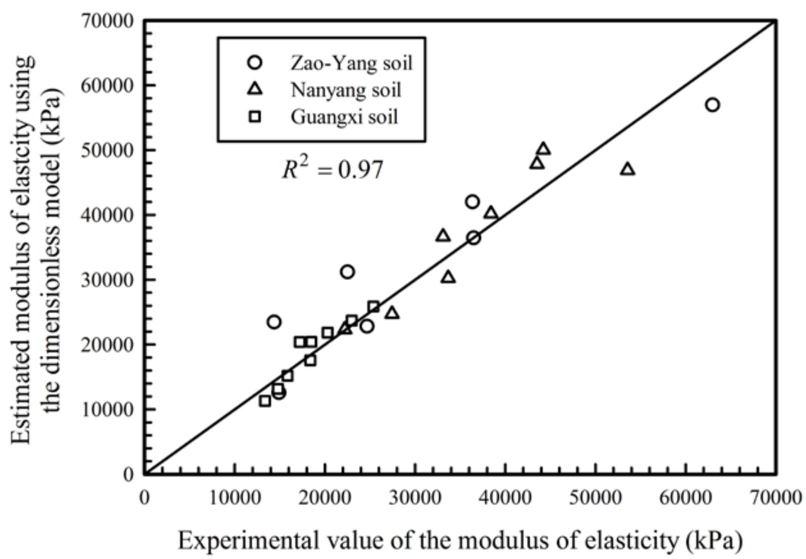


Fig. 6.8. Comparison between the values of elasticity moduli obtained from the dimensionless model and the triaxial tests for the three investigated expansive soils

6.7 Prediction of Vertical Movement of Unsaturated Expansive Soils Based on the Dimensionless Model

The modulus of elasticity based method (MEBM) has been used in this research study for predicting the long-term vertical movements of unsaturated expansive soils considering the environmental factors. The predictions can be made using only the data of initial matric suction, and the SWCC and the saturated modulus of elasticity measured from fairly routine geotechnical laboratory tests. In the MEBM, the VO model (Equation 3.1) has been used to obtain the unsaturated modulus of elasticity as a function of matric suction. The VO model can be reliably used in practice for pavements and lightly loaded residential structures. However, to conduct a reliable estimation of the soil movements, it is often desirable to effectively describe the soil modulus of elasticity as a function of all its influencing parameters. In this chapter, a new dimensionless model has been successfully proposed and used for estimating the modulus of elasticity of unsaturated expansive soils, taking into account the effect of the matric suction, the net confining stress, the initial void ratio, and the degree of saturation. The dimensionless model can be used for lightly and heavily loaded structures with a greater degree of confidence.

Case Study D by Ng et al. (2003) previously simulated in Chapter Five is revisited in this section to evaluate the MEBM approach extending the proposed dimensionless model for estimating the modulus of elasticity. Case Study D is chosen here because its triaxial tests results were analyzed dimensionally earlier in this chapter for estimating the unsaturated modulus of elasticity. Equations (6.8) and (6.9) are the dimensionless models proposed to estimate the unsaturated modulus of elasticity under confining stresses of 50 kPa and 200 kPa, respectively.

The exact value of the confining stress applied in the field is required to accurately estimate the values of the soil modulus of elasticity at any depth. However, the triaxial tests conducted by Ng et al. (2003) on compacted soil specimens for Case Study D were limited to certain values of the confining stress, and do not represent the in-situ condition. To evaluate the soil movements for the case study based on using the proposed dimensionless model for estimating the soil modulus of elasticity, it has been assumed that: i) the dimensionless model developed for specimens tested under the confining stress of 50 kPa (Equation 6.8) can be used to estimate the modulus of elasticity at the middle of the soil layers considered for Case Study D; ii) the confining stress at any depth can be calculated as the overburden pressure at that depth.

The same soil profile of Case Study D with the 2.5 m active zone depth, simulated in Chapter Five, are used here to predict the vertical soil movements over 30 days based on using the proposed dimensionless model (Equation 6.8). The daily changes in matric suction simulated by VADOSE/W and the corresponding unsaturated modulus of elasticity estimated by the dimensionless model are substituted into the soil movement constitutive relationship (Equation 4.7). Figure (6.9) shows the predicted and the measured soil movements (heave/shrink) at the mid-slope section R2 at different depths.

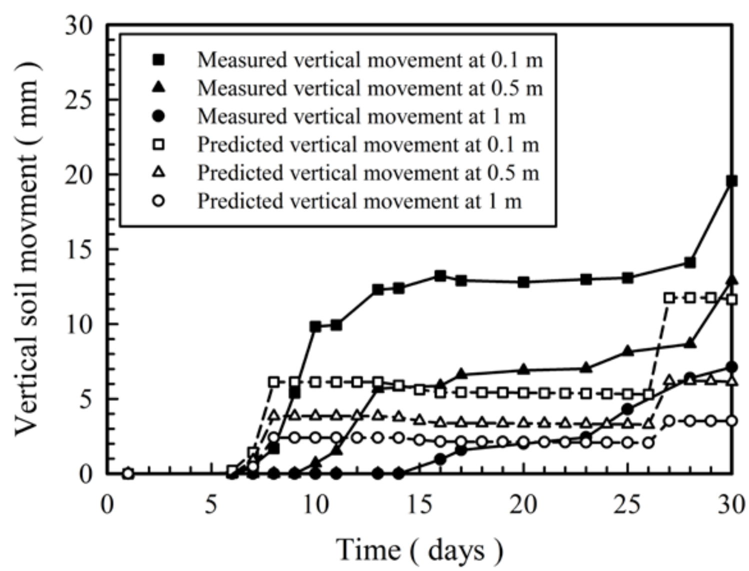


Fig. 6.9. Comparison of the predicted soil movements using the MEBM based on the dimensionless model with the field measurements at the mid-slope

The results show that the MEBM based on the proposed dimensionless model predicts the patterns of the in situ soil movements. However, the values of the predicted movements near the ground surface (0.1 m and 0.5 m) are very small compared with the field measurements. This can be attributed to the assumptions suggested for estimating the unsaturated modulus of elasticity as discussed before. In addition, the elasticity moduli estimated by the dimensionless model, that is developed based on the results of the triaxial tests for compacted specimens, are expected to have higher values compared with the elasticity moduli obtained for undisturbed specimens of soil that have cracks and fissures in the field.

6.8 Summary

The dimensional analysis (DA) was successfully used as a tool to propose a dimensionless model for estimating the modulus of elasticity of unsaturated expansive soils. The proposed model takes into account the effect of matric suction and net confining stress along with the initial void ratio and the degree of saturation towards comprehensive characterization of the unsaturated modulus of elasticity. The validation of the proposed dimensionless model was conducted using the experimental results of conventional and suction-controlled triaxial tests for the three expansive soils, namely Zao-Yang, Nanyang, and Guangxi expansive soils. A good correlation with a high determination coefficient (R^2 = 0.97) was obtained between the values of the unsaturated modulus of elasticity obtained from the dimensionless model and the triaxial tests results. However, more triaxial tests on expansive soils under different confining stresses and matric suctions are required to calibrate the fitting parameters of the dimensionless model.

The values of unsaturated modulus of elasticity E_{unsat} for the three investigated expansive soils estimated by the VO model were used to verify the proposed dimensionless model. The dimensionless model provided a greater degree of confidence for estimating the modulus of elasticity of unsaturated expansive soils compared with the results of the VO model. The VO model can be reliably used in practice for pavements and lightly loaded residential structures, while the proposed dimensionless model can be used for all scenarios of loading conditions. The proposed dimensionless model requires soil properties that can be determined from a limited number of routine laboratory tests.

Case Study D investigated by Ng et al. (2003) was revisited for the evaluation of the MEBM approach based on using the dimensionless model for estimating the modulus of elasticity. It is expected that using this innovative dimensionless model in the MEBM will provide more reliable predictions of the heave and shrink behavior of expansive soils if a limited number of triaxial shear tests can be conducted on undisturbed specimens collected from the active zone depth.

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH SUGGESTIONS

7.1 Introduction

The overall objective of this thesis is to develop a general and simple method for predicting the vertical movement of unsaturated expansive soils considering soil-atmospheric interactions within the active zone. This method is referred to as the modulus of elasticity based method (MEBM). The changes in matric suction and the associated modulus of elasticity are the key parameters required in the MEBM for estimating the vertical soil movements. The MEBM was validated for five different case studies from three countries: Canada, China, and the United States. These case studies are referred as Case Study A (Vu and Fredlund 2006), Case Study B (Yoshida et al. 1983, Vu and Fredlund 2004), Case Study C (Ito and Hu 2011), Case Study D (Ng et al. 2003), and Case Study E (Briaud et al. 2003). Several scenarios along with different boundary conditions were used to simulate each of these case studies. The step-by-step procedure of the MEBM includes: (i) simulation of the matric suction variations over time; (ii) estimation of the corresponding modulus of elasticity of the matric suction value; (iii) prediction of the vertical soil movements with respect to time.

The results of the research program are valuable to provide guidelines for rational design of lightly loaded structures placed in/on expansive soils using the mechanics of unsaturated soils. The MEBM approach is simple in comparison to other presently available methods and can be used in conventional geotechnical engineering practice.

In the following sections, conclusions derived from this research study are summarized. In addition, suggestions for future research with respect to the estimation of the vertical movement related volume change along with other applied research for expansive soils are highlighted.

7.2 Conclusions

7.2.1 Overall performance of the MEBM approach

- The MEBM is a straightforward and simple approach that has been successfully used to simulate the heave and/or shrink related volume change of expansive soils with respect to time within the active zone. The MEBM has been accurately validated for different case studies published in the literature:

  - For Case Study A, a slab-on-ground placed on Regina expansive clay subjected to a constant infiltration rate over 175 days, the *1-D* heave of the unsaturated expansive soil deposit during the infiltration were successfully predicted at different depths using the MEBM. The coefficient of determination between the results of the MEBM and the numerical modeling results of Vu and Fredlund (2006) for Case Study A was relatively high ($R^2 = 0.97$).

  - For Case Study B, a light industrial building constructed on Regina expansive clay in Saskatchewan, Canada, the soil heaves due to a water leak below the floor slab which were predicted using the MEBM agreed well with the data (measurements/estimates) published by Vu and Fredlund (2004) over 150 days ($R^2 > 0.94$).

  - For Case Study C, the other test site in Regina, Saskatchewan, Canada, the factors influencing the soil movements such as soil cracks, cover type (pavement/vegetation), lawn irrigation, climate conditions, and vegetation were successfully considered over one year (1 May, 2009–30 April, 2010). The total soil movement (i.e., the difference between the maximum values of soil heave and shrinkage) predicted using the proposed MEBM was close to that predicted by Ito and Hu (2011) with a 6% difference.

- For Case Study D, a cut-slope in an expansive soil in Zao-yang, Hubie, China, the MEBM was validated against the field measurements of the soil movements over one month (13 August–12 September, 2001), considering the effect of climatic conditions, two artificial rainfall events, and soil cracks. The average percentage difference between the predicted and the measured soil movements was 28%. The prediction of soil movements was improved when only the soil heave was considered. The average percentage difference decreased to 21%, and the heave patterns from the prediction were similar to the field measurements.

- For Case Study E, a field site in Arlington, Texas, USA, the predicted movements of the four full-scale spread footings in the site matched the measured movements reasonably well in both tendency and magnitude for the first year. However, the predicted and the measured movements during the second year did not lead as good a match as the values during the first year. This is due to the complexities associated with the mechanism of soil shrinkage, and the difficulty to quantify the influence of shrinkage cracks on the soil movement.

- The volume change constitutive equation of the MEBM approach was developed based on the assumption that the mechanical stress remains constant during the heave/shrinkage processes. The assumption is not strictly valid as the soil density changes due to a couple of factors. However, for lightly loaded structures, where the MEBM can be applied, the influence of the mechanical stress is insignificant in several scenarios and can be neglected. Such an assumption is also conservative and can be extended in practice. Furthermore, the volume change of expansive soils associated with the variations of environmental conditions occurs near the ground surface and decreases with depth.

- The matric suction variations and the corresponding values of the modulus of elasticity along with the soil-water characteristic curve (SWCC) of soils are the most important parameters that contribute to the swelling and shrinkage behavior of expansive soils. The swelling capacity of soil is essentially dependent on the elastic properties of the solid phase and caused by the expansive soil's affinity for water. The strength of the MEBM model lies in its use of the soil properties that can be determined by using conventional geotechnical testing methods.

7.2.2 Soil-atmospheric interaction

- Estimation of the soil matric suction as a function of time using the soil-atmospheric interaction model (VADOSE/W) allows for the variations of soil profile characteristics, the water infiltration/migration, and the variations of climatic conditions to be taken into account.

- The results of the VADOSE/W analyses demonstrated that rigorous soil-atmospheric interaction modeling can be performed to estimate the time evolution of matric suction profile and the depth of the active zone. For example, a reasonable agreement was observed between the matric suction values obtained from VADOSE/W and those estimated by Vu and Fredlund (2004). The coefficient of determination was relatively high ($R^2 > 0.89$).

- The results of VADOSE/W simulations for the five case studies show that the environmental conditions will primarily influence the surface layer of the soil profile, which constitutes the active zone depth that is typically 2 m to 3.5 m. The matric suction profiles were found to vary with depth and time, and correlated well with the environmental conditions on the surface boundary.

7.2.3 Unsaturated modulus of elasticity

- The VO model (i.e., Vanapalli and Oh (2010) model) with two fitting parameters α and β was extended in this study for expansive soils to estimate the variations of the modulus of elasticity with respect to matric suction. The two fitting parameters β = 2 and α = (0.05–0.15) were recommended for modeling the volume change behaviour of expansive soils. The fitting parameter β = 2 was found to be suitable for the three investigated expansive soils (i.e., Zao-Yang, Nanyang, and Guangxi expansive soils). The value of α was defined on the basis of the best agreement between the experimental and the predicted values of the modulus of elasticity with respect to matric suction (R^2 = 0.77–0.97).

- In spite of the susceptibility of the measured modulus of elasticity to measurement errors or problems associated with the experimental techniques and the difficulties of ensuring that the stress path was entirely elastic, the adopted VO model reasonably predicted the modulus of elasticity obtained from the experimental data of triaxial tests for the three unsaturated expansive soils studied (R^2 = 0.91).

- Reasonable predictions of the vertical soil movements over time for five case studies (i.e., Case Studies A, B, C, D, and E) were achieved by using the proposed MEBM extending the VO model for estimating the unsaturated modulus of elasticity. For all the case studies β was equal to 2 and the α value was between 0.05–0.15. The VO model can be reliably used in practice for pavements and lightly loaded residential structures.

- A dimensionless model for estimating the modulus of elasticity of unsaturated expansive soils was successfully developed using the dimensional analysis (DA). This dimensionless model accounts for the effect of the matric suction and the net confining stress, along with the initial void ratio and the degree of saturation. A good correlation with a high determination coefficient (R^2 =

0.97) was obtained between the values of the unsaturated modulus of elasticity obtained from the proposed dimensionless model and from the results of triaxial shear tests for the three expansive soils studied. The dimensionless model can be used in practice for all scenarios of loading conditions (both lightly and heavily loaded structures).

7.3 Recommendations and Suggestions for Future Research Studies

- The MEBM proposed in this thesis research is validated and tested in five case studies. More comprehensive field studies from different regions of the world are necessary in order to provide more evidence for the use of the MEBM in engineering practices.

- In addition to matric suction changes, it is also important to consider the influence of other parameters, such as overburden pressure, soil cracks, and soil temperature, in the numerical modeling solutions of the soil-atmospheric interactions which are subsequently used for predicting the soil movement over time.

- Changes in the soil volume as soil suction is increased/decreased can significantly affect the interpretation of soil-water characteristic curve information and result in erroneous calculations of the unsaturated soil property functions (e.g., soil permeability function and degree of saturation) (Fredlund and Houston 2013). However, the SWCC has been treated in this study as a single approximate relationship between the amount of water in a soil and soil suction. It will be very interesting to properly account for the effects of volume change on the SWCC of unsaturated expansive soils.

- Conducting a sensitivity analysis or a parametric study of the parameters that affect the volume change behavior of expansive soils would be valuable. Some of the key parameters include the saturated modulus of elasticity, the saturated coefficient of permeability, Poisson's ratio, the swelling index, and the number/thickness of soil layers.

- The fitting parameters of the VO model used in this research study are found to provide reasonable values for the modulus of elasticity which are successfully used to reproduce the in situ soil movements. However, more tests for expansive soils of various plasticity index values are needed to provide a generalized relationship of the fitting parameters in terms of the conventional soil properties such as the plasticity index, I_p.

- The dimensionless model is developed based on a limited number of the conventional and suction-controlled triaxial tests for three different expansive soils. However, more triaxial test results for the investigated soils under the same values of the confining stresses used in this study but with different matric suctions are necessary to calibrate the fitting parameters of the proposed dimensionless model for each soil.

- Case Study D has been revisited for the evaluation of the MEBM approach based on using the innovative dimensionless model as a tool for estimating the soil modulus of elasticity. It is expected that using the dimensionless model in the MEBM would provide more reliable predictions of heave and shrinkage behavior of expansive soils if the triaxial shear tests were conducted on undisturbed specimens collected from the field.

- Using an easy and simple way to obtain high quality data for the estimation of the modulus of elasticity of unsaturated expansive soils can further improve the prediction results of the MEBM.

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